

Tree

a. $n \rightarrow n^2$

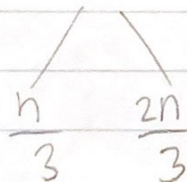


$$n/4 \quad n/4 \quad n/4 \rightarrow 3 \cdot \frac{n^2}{16} = \frac{3}{16} n^2 = \frac{3^1}{4^2} n^2$$

$$= n^2 \sum_{i=0}^{\log_4(n)} \frac{3^i}{4^{2i}} = \frac{n^2}{16} \sum_{i=0}^{\log_4(n)} \left(\frac{3}{4}\right)^i \quad \frac{3}{4} < 1 \Rightarrow \frac{1}{1 - \frac{3}{4}} = 4$$

$$\sum_{i=0}^{\log_4(n)} \left(\frac{3}{4}\right)^i < 4 \Rightarrow \frac{n^2}{4} \Rightarrow \boxed{O(n^2)}$$

b. $n \rightarrow n \log n$



$$\frac{n}{3} \quad \frac{2n}{3} \rightarrow \frac{n}{3} \log\left(\frac{n}{3}\right) + \frac{2n}{3} \log\left(\frac{2n}{3}\right)$$

$$< \frac{n}{3} \log\left(\frac{2n}{3}\right) + \frac{2n}{3} \log\left(\frac{2n}{3}\right)$$

$$= n \log\left(\frac{2n}{3}\right) \Rightarrow n \left[\sum_{i=0}^{\log(n)} \log\left(\left(\frac{2}{3}\right)^i\right) + \log(n) \right]$$

$$\Rightarrow \frac{1}{\frac{1}{3}} = 3 \quad \hookrightarrow < 3 = n(3 \log n + \log n)$$

$$= O(n \log(n)) \quad \text{or} \quad n \log\left(\frac{n}{3}\right) \Rightarrow < \frac{3}{2}$$

Still $O(\log n \cdot n)$

$$= \boxed{O(n \log n)}$$

$$C. 2W(n/2) + n/\log(n)$$

$$\begin{array}{c}
 n \rightarrow n/\log(n) \\
 \swarrow \searrow \\
 \frac{n}{2} \quad \frac{n}{2} \rightarrow \frac{n/2}{\log(n/2)} \cdot 2 = n/\log(n/2) \\
 \swarrow \searrow \quad \swarrow \searrow \\
 \frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4} \Rightarrow \frac{n}{\log(n/4)}
 \end{array}$$

$$= n \sum_{i=0}^{\log_2 n} \frac{1}{\log(n/2^i)} = n \sum_{i=0}^{\log_2(n)} \frac{1}{\log(n) - i}$$

\Rightarrow Harmonic series

$$\sum_{i=1}^{\lg n} \frac{1}{i} < \int_1^{\lg(n)} \frac{1}{i} = \ln i \Big|_1^{\lg(n)}$$

$$= \boxed{O(n \log(\log(n)))}$$

Brick Method

a. $1.01n \rightarrow$ cost of root

$$1.01(.49n) + 1.01(.49n) = .9898n$$

$$1.01n > .9898n$$

root dominated

Only care about root

$$1.01n \rightarrow \boxed{O(n)}$$

b. $.999n \rightarrow$ Cost of root

$$\frac{.999}{2}n + \frac{.999}{4}n = .74925n$$

root dominated

$$\Rightarrow \boxed{O(n)}$$

c. Cost of root = ~~4~~ $\sqrt{n} = 4$

Suppose $n = 16$

Leaf dominated.

$$16 \rightarrow 16$$

$$\begin{array}{c} \diagup \diagdown \diagup \diagdown \\ 4 \ 4 \ 4 \ 4 \end{array} \rightarrow 4 \cdot 2 = 8$$

Cost of each leaf: 1

Depth of tree $\rightarrow \sqrt{n}$

Number of leaves $\rightarrow \sqrt{n}$

$$\begin{array}{c} \diagup \diagdown \diagup \diagdown \diagup \diagdown \diagup \diagdown \\ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \end{array} \quad \sqrt{2} \cdot 8 = 8\sqrt{2}$$

$$\begin{array}{c} \diagup \diagdown \diagup \diagdown \diagup \diagdown \diagup \diagdown \diagup \diagdown \diagup \diagdown \diagup \diagdown \\ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \end{array} \rightarrow 16 \cdot 1 = 16$$

$$\boxed{O(n\sqrt{n})}$$