Tree

a. $n \rightarrow n^2$

$\frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4} \rightarrow 3 \cdot \frac{n^2}{16} = \frac{3}{16} n^2 = \frac{3^i}{4^{2i}} n^2$

$$= n^2 \sum_{i=0}^{\log_4(n)} \frac{3^i}{4^{2i}} = \frac{n^2}{16} \sum_{i=0}^{\log_4(n)} \left(\frac{3}{4}\right)^i \quad \frac{3}{4} < 1 \Rightarrow \frac{1}{1-\frac{3}{4}} = 4$$

$$\sum_{i=0}^{\log_4(n)} \left(\frac{3}{4}\right)^i < 4 \Rightarrow \frac{n^2}{4} \Rightarrow \boxed{O(n^2)}$$

b. $n \rightarrow n \log n$

$\frac{n}{3} \quad \frac{2n}{3} \rightarrow \frac{n}{3} \log\left(\frac{n}{3}\right) + \frac{2n}{3} \log\left(\frac{2n}{3}\right)$

$< \frac{n}{3} \log\left(\frac{2n}{3}\right) + \frac{2n}{3} \log\left(\frac{2n}{3}\right)$

$= n \log\left(\frac{2n}{3}\right) \Rightarrow n\left[\sum_{i=0}^{\log(n)} \log\left(\left(\frac{2}{3}\right)^i\right) + \log(n)\right]$

$\Rightarrow \frac{1}{\frac{1}{3}} = 3 \qquad < 3$

$= n(3\log n + \log n)$

$= O(n \log(n))$ or $n \log\left(\frac{n}{3}\right) \Rightarrow < \frac{3}{2}$

Still $O(\log n \cdot n)$

$= \boxed{O(n \log n)}$

C. $2W(n/2) + n/\log(n)$

$n \longrightarrow n/\log(n)$

$\frac{n}{2} \quad \frac{n}{2} \longrightarrow \frac{n/2}{\log(n/2)} \cdot 2 = n/\log(\frac{n}{2})$

$\frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4} \Rightarrow n/\log(\frac{n}{2^i})$

$$= n\sum_{i=0}^{\log_2 n} \frac{1}{\log(\frac{n}{2^i})} = n\sum_{i=0}^{\log_2(n)} \frac{1}{\log(n) - i}$$

$\Rightarrow$ Harmonic series

$$\sum_{i=1}^{\lg n} \frac{1}{i} < \int_1^{\lg(n)} \frac{1}{i} = \ln i \Big|_1^{\lg(n)}$$

$$\ln(\lg n)$$

$$= \boxed{O(n\log(\log(n)))}$$

Brick Method

a. $1.01n \rightarrow$ cost of root

$1.01(.49n) + 1.01(.49n) = .9898n$

$1.01n > .9898n$

root dominated

Only care about root

$1.01n \rightarrow \boxed{O(n)}$

b. $.999n \rightarrow$ Cost of root

$\frac{.999}{2}n + \frac{.999}{4}n = .74925n$

root dominated

$\Rightarrow \boxed{O(n)}$

c. Cost of root $= \sqrt{n} = 4$

Suppose $n = 16$

Leaf dominated.

16 $\rightarrow$ 16

4 4 4 4 $\rightarrow 4 \cdot 2 = 8$

2 2 2 2 2 2 2 2 $\quad \sqrt{2} \cdot 8 = 8\sqrt{2}$

16 ones $\rightarrow$ 1 1 1 1 1 1 1 1 1 $\rightarrow 16 \cdot 1 = 16$

Cost of each leaf: 1

Depth of tree $\rightarrow \sqrt{n}$

Number of leaves $\rightarrow n$

$\boxed{O(n\sqrt{n})}$