Temporal autocorrelation

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2024-07-18

```
library(dplyr)
library(tidyverse)
library(lme4)
library(nlme)
library(xts)
library(ModelMetrics)
library(zoo)
library(lubridate)
library(ggpubr)
library(astsa) #for lag scatterplot
```

```
get.traintest <- function(master.df, year.list, fold) {

  # create output
  combined_training_list <- vector("list",fold)
  combined_testing_list <- vector("list",fold)

  ## Loop 10 folds
  for (f in 1:fold) {

    # dataframe to store the sampled years for each location
    dfind = data.frame(location=character(),
                       year=integer(), stringsAsFactors = FALSE)
    dfindt = data.frame(location=character(),
                       year=integer(), stringsAsFactors = FALSE)

    for (i in 1:length(year.list)){ # i loops through each location
      smp = year.list[[i]] #train
      if (length(smp)>1){
        indx.train=(sample(smp, 1))
        ytrain=indx.train
      } else if (length(smp)==1){
        indx.train=smp
        ytrain=indx.train
      }

      smpt = year.list[[i]][year.list[[i]] != indx.train] #test
      if (length(smpt)>1){
        indx.test=(sample(smpt, 1))
        ytest=indx.test
      } else if (length(smpt)==1){
        indx.test=smpt
        ytest=indx.test
```

```
      }

      dfind[i,] = c(names(year.list[i]), ytrain)
      dfindt[i,] = c(names(year.list[i]), ytest)
    }

    # subset the dataframe
    awf_train <- merge(master.df, dfind, by=c("location", "year"))
    awf_test <- merge(master.df, dfindt, by=c("location", "year"))

    combined_training_list[[f]] <- awf_train
    combined_testing_list[[f]] <- awf_test
  }
  return(list(combined_training_list, combined_testing_list))
}
good_year <- function(master.df, dayln) {
  # get table of sample years by location
  yr_out=table(master.df$location, master.df$year)

  # select sample years with more than X days (x=250)
  full_year=list()
  sind=vector()
  cnt=0
  ind=0

  for (i in seq_along(loc_seq)){
   rm(ind)
   if (ncol(yr_out)>=1) {
     ind=which(yr_out[row.names(yr_out)==loc_seq[i],]>dayln)

     if (length(ind)>0) {
       sind=c(sind,i)
       cnt=cnt+1
       full_year[[cnt]]=colnames(yr_out)[ind]
     }
   }
  }
  full_year=setNames(full_year,loc_seq[sind])
  print(full_year)
}
```

Data importing and cleaning

```
## Data import
air <- read.csv("tributary air temperatures clean.csv", stringsAsFactors=F)
water <- read.csv("tributary water temperature/water_temperature_d.csv",
                  stringsAsFactors=F)
flow <- read.csv("tributary discharge.csv", stringsAsFactors=F)

# Convert to date format
air$date <- as.Date(air$date, "%m/%d/%Y")
water$date <- as.Date(water$date, "%m/%d/%Y")
flow$date <- as.Date(flow$date, "%m/%d/%Y")
```

```
## Calculate the MEAN airT for US locations
for (i in which(is.na(air$mean_temp))) {
  air$mean_temp <- (air$max_temp + air$min_temp) / 2
}
```

Data combining

Now we want to combine air, water, and flow into a master dataframe

```
## Combine water temperature and discharge
aw <- merge(air, water, by = c("station_name","date"))
awf <- merge(aw, flow, by = c("location","date"))
awf <- awf %>% arrange(location, date)

## Change into factor
awf$location <- as.factor(awf$location)
awf$station_name <- as.factor(awf$station_name)

# Get location sequence
loc_seq=levels(awf$location)


## Check if there are any duplicates
duplicates <- awf %>%
  group_by(location, date) %>%
  filter(n() > 1) # Duplicates should be NA...


## Print the result
table(awf$location)
```

```
##
##   bigcreek   bigotter        fox    genesee     humber mississagi    nipigon
##       4554        919       1374       1095       4446       1434        848
##    portage    saginaw      still    stlouis  vermilion
##        730       1095       1246       1349       1095
```

Check for imputed values in water temp

Use a 7-days rolling mean to check for possibly imputed temperatures. If the variance of a certain day's temperature is very close to zero, then it is likely that this particular data is imputed.

```
# make sure that no initial NA values in awf water temperature
which(is.na(awf$temp))
```

```
## integer(0)
```

```
# Need to calculate the rolling mean for each location separately...
for(loc in loc_seq) {
  sub <- awf[awf$location == loc,]
  sub$rolling_mean <- rollmean(sub$temp, k = 7, fill = NA, align = "right")
  awf[awf$location == loc, "rolling_mean"] <- sub$rolling_mean
}

# Calculate variance
```

```
awf$variance <- (awf$temp - awf$rolling_mean)^2

# Assign NA when variance is very small
awf$temp[which(awf$variance < 1e-10)] <- NA

# Check results
awf %>%
  group_by(location) %>%
  summarise(na_count = sum(is.na(temp)))
```

```
## # A tibble: 12 x 2
##    location   na_count
##    <fct>         <int>
##  1 bigcreek        101
##  2 bigotter          0
##  3 fox               6
##  4 genesee           6
##  5 humber           12
##  6 mississagi        6
##  7 nipigon           0
##  8 portage           2
##  9 saginaw          17
## 10 still           226
## 11 stlouis         144
## 12 vermilion       137
```

Calculate lagged days

```
## Get the time lag day variables
awf <- awf %>%
  group_by(location) %>%
  mutate(dmean_1 = lag(mean_temp, 1),
         dmean_2 = lag(mean_temp, 2),
         dmean_3 = lag(mean_temp, 3),
         dmean_4 = lag(mean_temp, 4),
         dmean_5 = lag(mean_temp, 5))
```

Get final master temp dataframe

```
master.temp <- awf[complete.cases(cbind(awf$mean_temp,awf$temp)),] %>%
  select(location, date, station_name, country,
         year, month = month.x, day = day.x,
         water = temp, air = mean_temp, dmean_1, dmean_2, dmean_3,
         dmean_4, dmean_5, flow)

master.temp <- master.temp[complete.cases(master.temp[,8:15]),]
```

Get training and testing (for season and annual)

Subsetting seasonal scales

Now we want to subset four master dataframes that contains seasonal-scale data. We categorize the data into four different seasonal categories:

1. spring: 3,4,5
2. summer: 6,7,8
3. fall: 9,10,11
4. winter: 12,1,2

```
master.sum <- master.temp %>%
  filter(month == 6 | month == 7 | month == 8)

master.win <- master.temp %>%
  filter(month == 12 | month == 1 | month == 2)

master.spring <- master.temp %>%
  filter(month == 3 | month == 4 | month == 5)

master.fall <- master.temp %>%
  filter(month == 9 | month == 10 | month == 11)

master.annual <- master.temp %>%
  filter(month != 12 & month != 1 & month != 2)
```

Identify good years

We want the seasonal data to be greater than 60 days, annual data (winter removed) more than 180 days.

```
## Annual (250 days)
fulyear <- good_year(master.annual, 180)
```

```
## $bigcreek
##  [1] "2000" "2001" "2002" "2003" "2004" "2005" "2006" "2007" "2008" "2009"
## [11] "2012" "2013" "2014"
##
## $bigotter
## [1] "2012" "2013" "2014"
##
## $fox
## [1] "2011" "2012" "2013" "2014"
##
## $genesee
## [1] "2011" "2012" "2013"
##
## $humber
##  [1] "1998" "1999" "2000" "2001" "2002" "2003" "2005" "2006" "2007" "2008"
## [11] "2009" "2011"
##
## $mississagi
## [1] "2011" "2013"
##
## $nipigon
## [1] "2008" "2009"
##
## $portage
## [1] "2011" "2012"
##
## $saginaw
## [1] "2013" "2014"
```

```
##
## $still
## [1] "2002" "2004" "2005" "2008"
##
## $stlouis
## [1] "2011" "2012" "2013" "2014"
##
## $vermilion
## [1] "2012" "2013" "2014"
```

```
## Seasonal (60 days)
fulspring <- good_year(master.spring, 60)
```

```
## $bigcreek
##  [1] "2001" "2002" "2003" "2004" "2005" "2006" "2007" "2008" "2009" "2012"
## [11] "2013" "2014"
##
## $bigotter
## [1] "2012" "2013" "2014"
##
## $fox
## [1] "2011" "2012" "2013" "2014"
##
## $genesee
## [1] "2011" "2012" "2013"
##
## $humber
##  [1] "1999" "2000" "2001" "2002" "2003" "2005" "2006" "2007" "2008" "2009"
## [11] "2011" "2013"
##
## $mississagi
## [1] "2011" "2013" "2014"
##
## $nipigon
## [1] "2008" "2009"
##
## $portage
## [1] "2011" "2012"
##
## $saginaw
## [1] "2013" "2014"
##
## $still
## [1] "2005" "2008"
##
## $stlouis
## [1] "2012" "2013"
##
## $vermilion
## [1] "2012" "2013" "2014"
```

```
fulsum <- good_year(master.sum, 60)
```

```
## $bigcreek
##  [1] "2000" "2001" "2002" "2003" "2004" "2005" "2006" "2007" "2008" "2009"
```

```
## [11] "2012" "2013" "2014"
##
## $bigotter
## [1] "2012" "2013" "2014"
##
## $fox
## [1] "2011" "2012" "2013" "2014"
##
## $genesee
## [1] "2011" "2012" "2013"
##
## $humber
##  [1] "1998" "1999" "2000" "2001" "2002" "2003" "2005" "2006" "2007" "2008"
## [11] "2009" "2011"
##
## $mississagi
## [1] "2011" "2012" "2013"
##
## $nipigon
## [1] "2008" "2009"
##
## $portage
## [1] "2011" "2012"
##
## $saginaw
## [1] "2014"
##
## $still
## [1] "2002" "2004" "2005" "2008"
##
## $stlouis
## [1] "2011" "2012" "2013" "2014"
##
## $vermilion
## [1] "2013" "2014"
```

```
fulfall <- good_year(master.fall, 60)
```

```
## $bigcreek
##  [1] "2000" "2001" "2002" "2003" "2004" "2005" "2006" "2007" "2008" "2009"
## [11] "2012" "2013"
##
## $bigotter
## [1] "2012" "2013"
##
## $fox
## [1] "2011" "2012" "2013" "2014"
##
## $genesee
## [1] "2011" "2012" "2013"
##
## $humber
##  [1] "1998" "1999" "2000" "2001" "2002" "2003" "2005" "2006" "2007" "2008"
## [11] "2009" "2011"
##
```

```
## $mississagi
## [1] "2010" "2011" "2012" "2013"
##
## $nipigon
## [1] "2008" "2009"
##
## $portage
## [1] "2011" "2012"
##
## $saginaw
## [1] "2012" "2014"
##
## $still
## [1] "2002" "2004"
##
## $stlouis
## [1] "2011" "2012" "2013" "2014"
##
## $vermilion
## [1] "2012" "2013" "2014"
```

```
fulwin <- good_year(master.win, 60)
```

```
## $bigcreek
##  [1] "2001" "2002" "2004" "2005" "2006" "2007" "2008" "2009" "2012" "2013"
##
## $bigotter
## [1] "2013"
##
## $fox
## [1] "2012" "2013" "2014"
##
## $genesee
## [1] "2011" "2012" "2013"
##
## $humber
##  [1] "1999" "2000" "2001" "2002" "2003" "2005" "2006" "2007" "2008" "2009"
## [11] "2011"
##
## $mississagi
## [1] "2011" "2012" "2013"
##
## $nipigon
## [1] "2008" "2009"
##
## $portage
## [1] "2011" "2012"
##
## $saginaw
## [1] "2012" "2014"
##
## $stlouis
## [1] "2012"
##
## $vermilion
```

```
## [1] "2012" "2013"
```

Get training and testing

Get training and testing for the specific season, and for annual. We store them in one master dataframe called grand_training and grand_testing, each is a list of five sub-temporal data.

```
fold = 10

## Get training and testing for annual
annual <- get.traintest(master.temp, fulyear, 10)

combined_training_list <- annual[[1]]
combined_testing_list <- annual[[2]]


## Get training and testing for each season
sum <- get.traintest(master.sum, fulsum, 10)
win <- get.traintest(master.win, fulwin, 10)
spr <- get.traintest(master.spring, fulspring, 10)
fall <- get.traintest(master.fall, fulfall, 10)


combined_training_list_sp <- spr[[1]]
combined_testing_list_sp <- spr[[2]]

combined_training_list_su <- sum[[1]]
combined_testing_list_su <- sum[[2]]

combined_training_list_fa <- fall[[1]]
combined_testing_list_fa <- fall[[2]]

combined_training_list_w <- win[[1]]
combined_testing_list_w <- win[[2]]


## Create a list to store all the combined training and testing lists
grand_training <- list(combined_training_list_sp, combined_training_list_su,
                       combined_training_list_fa, combined_training_list_w,
                       combined_training_list)

grand_testing <- list(combined_testing_list_sp, combined_testing_list_su,
                       combined_testing_list_fa, combined_testing_list_w,
                       combined_testing_list)
```

NOTE: All following models are done for one iteration only (using only i=1)!

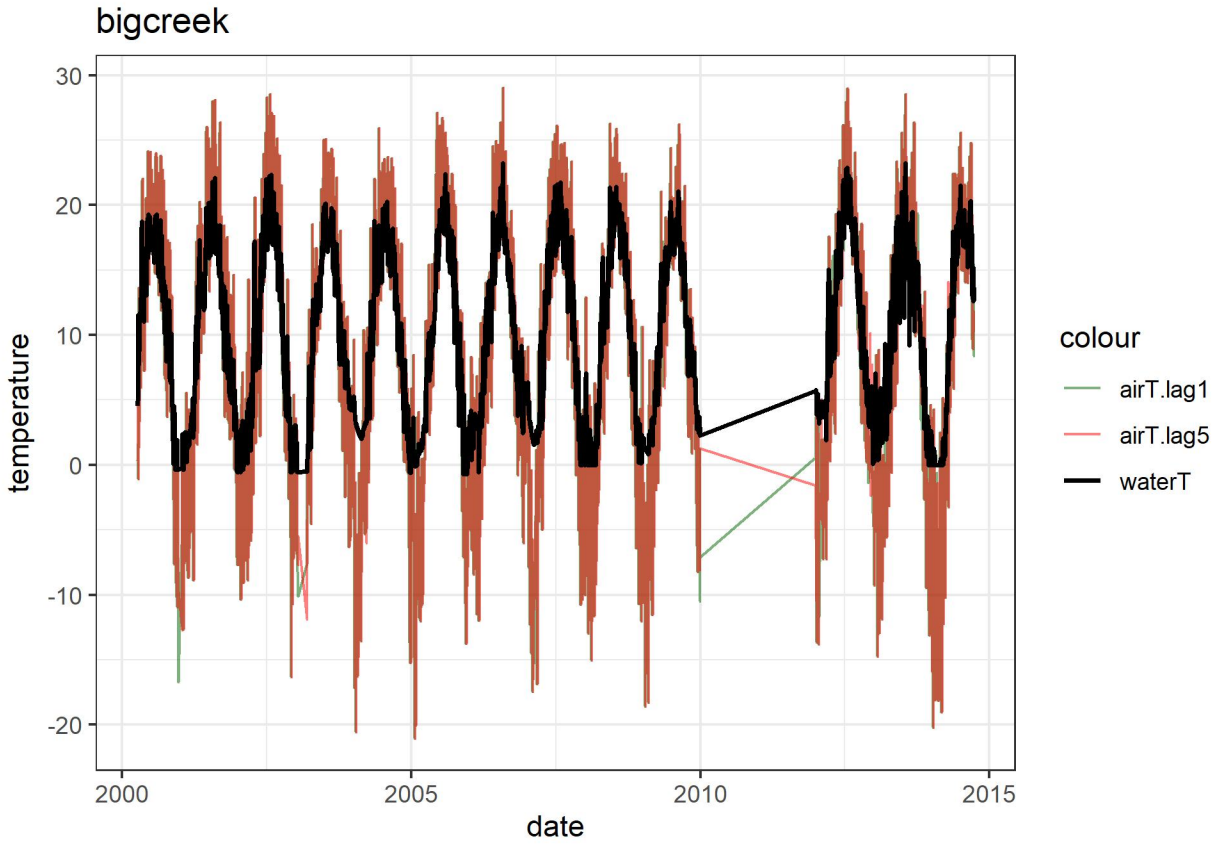
Data plots

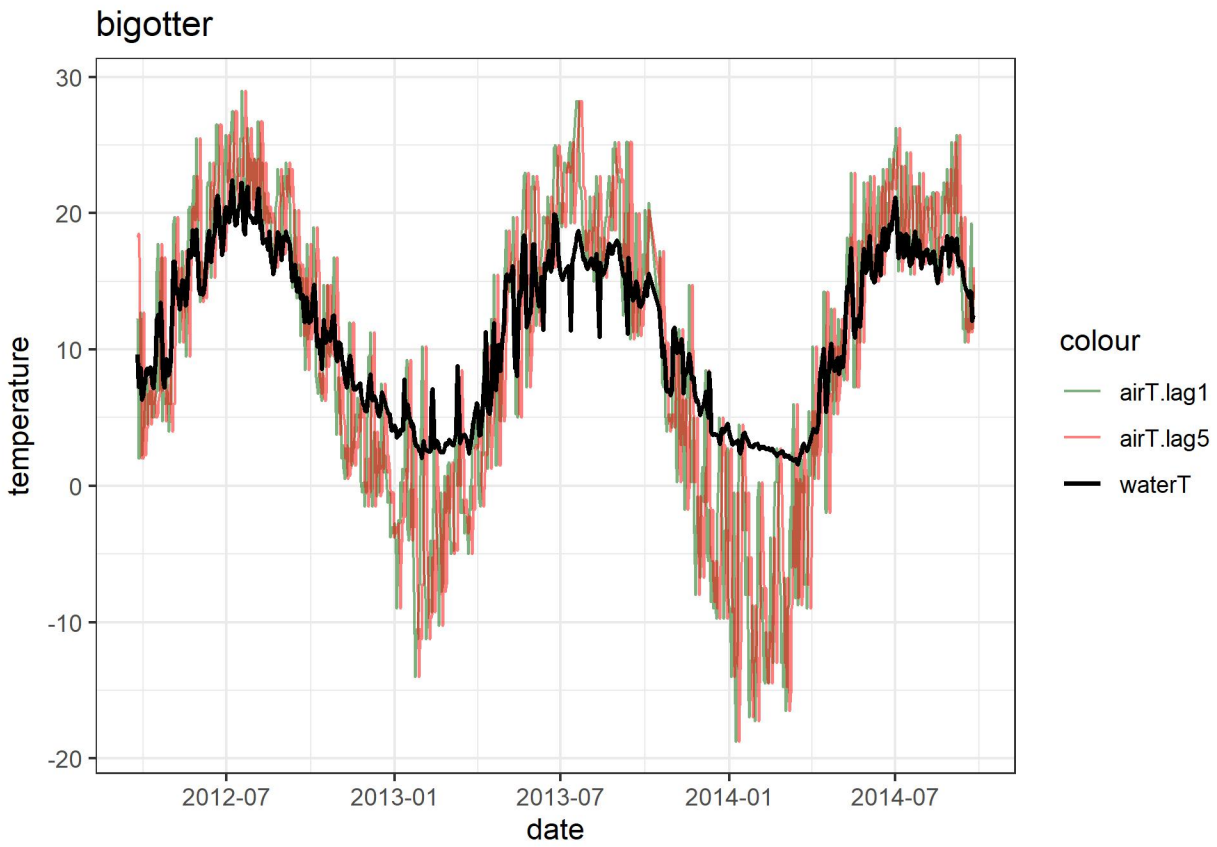
Temperature plot

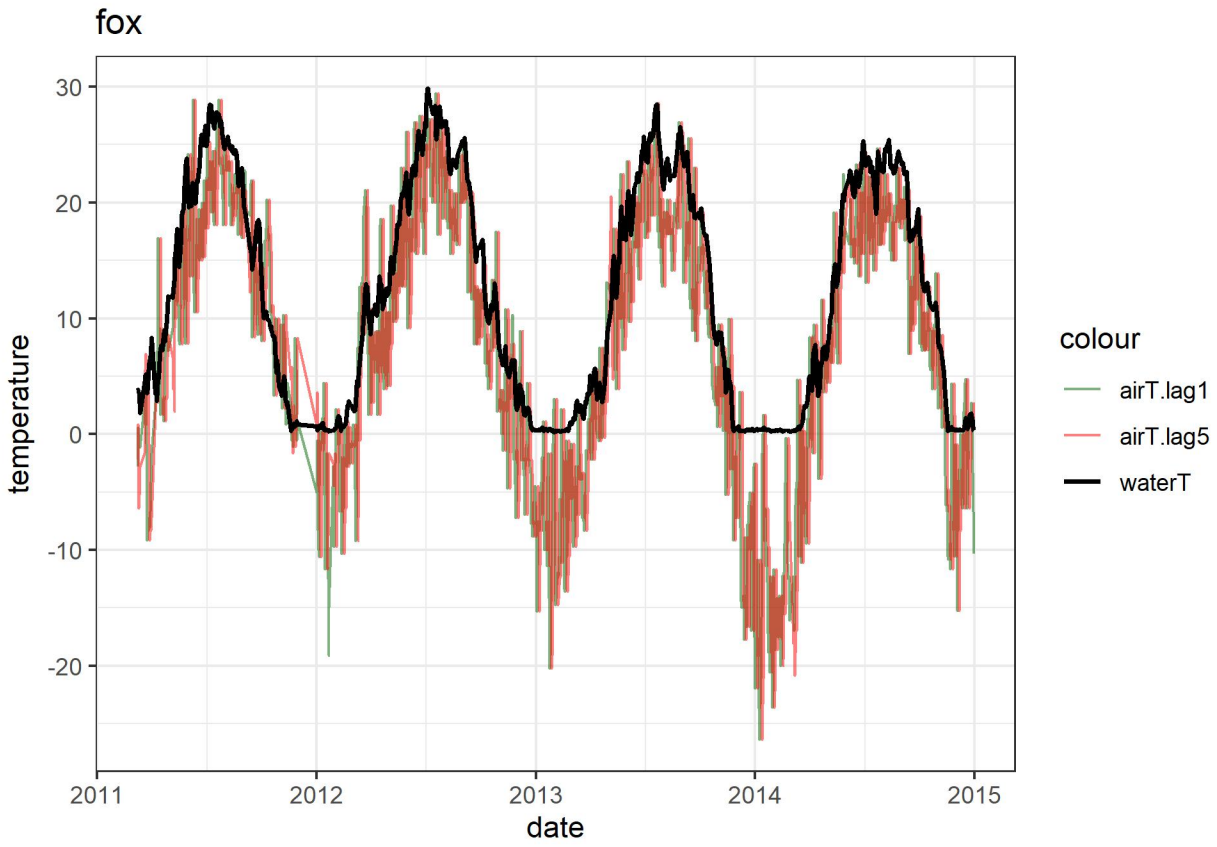
Before we run any analysis, we first want to check whether the air temperature and lags are corrected calculated. Here, we plot the lag1 air temperature, lag5 air temperature, and water temperature verus the times series for each location.

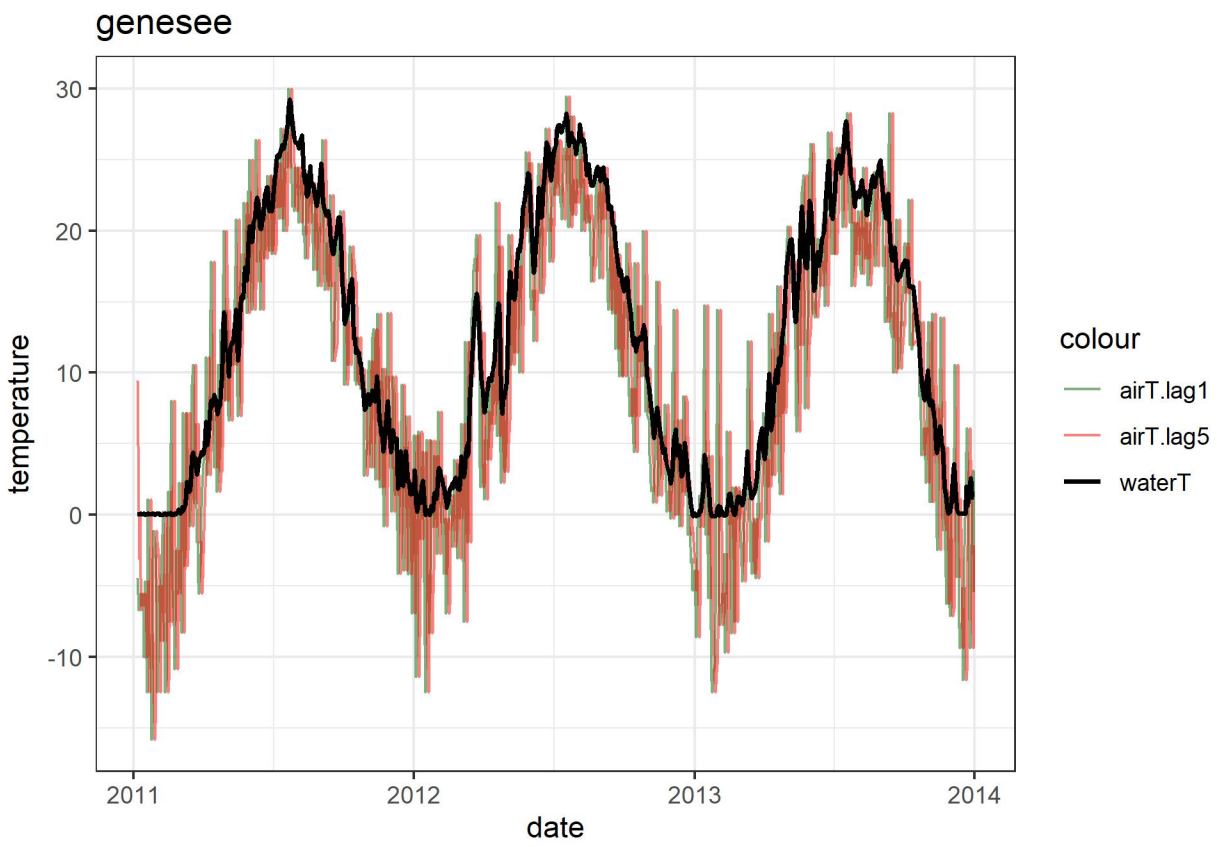
Notice here NA values are indicated by gaps in the plots.

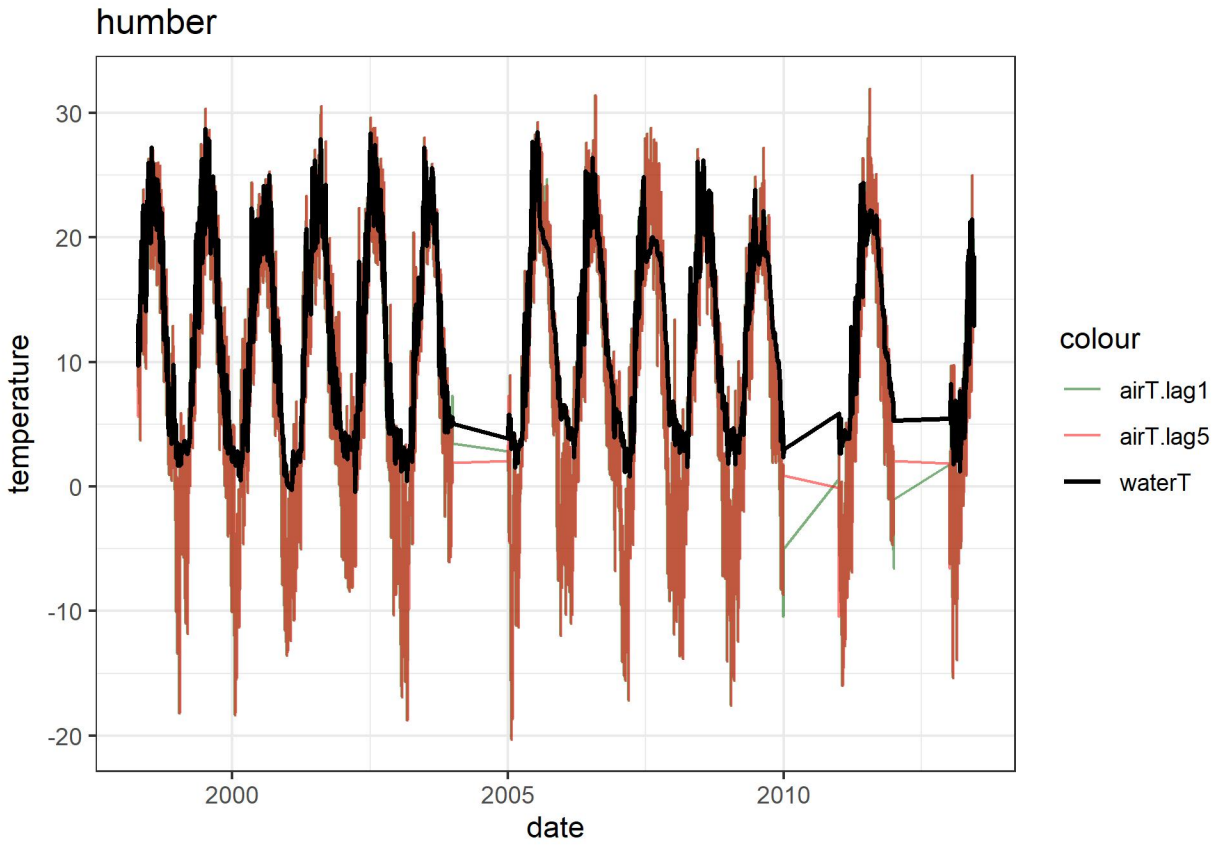
```r
colors <- c("airT.lag1" = "darkgreen", "airT.lag5" = "red", "waterT" = "black")

for (loc in levels(master.temp$location)) {

  plot.df <- master.temp %>% filter(location == loc)

  pl <- ggplot(plot.df, aes(x = date))+
          geom_line(aes(y = dmean_1, color = "airT.lag1"), alpha = 0.5)+
          geom_line(aes(y = dmean_5, color = "airT.lag5"), alpha = 0.5)+
          geom_line(aes(y = water, color = "waterT"), linewidth = 0.8)+
          labs(x="date", y="temperature")+
          ggtitle(paste(loc))+
          scale_color_manual(values = colors)+
          theme_bw()

  print(pl)
}
```

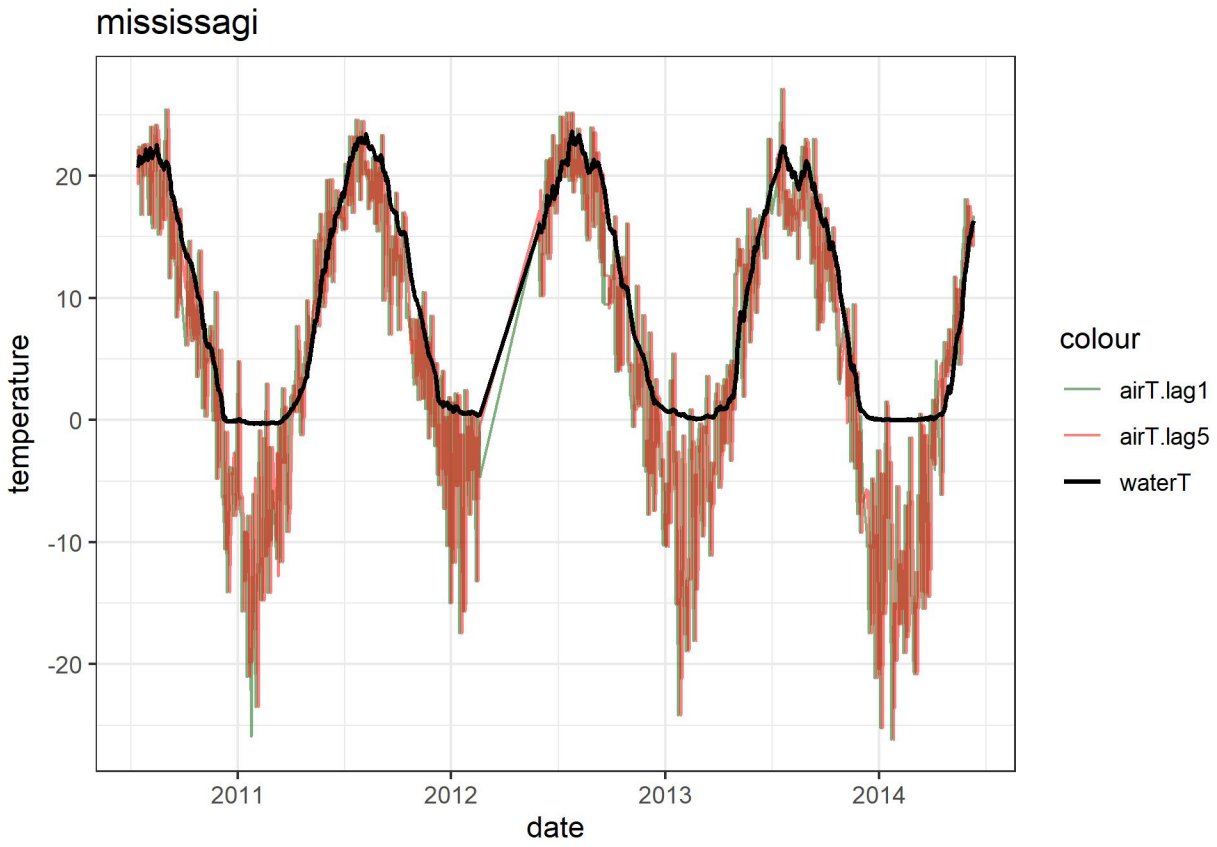


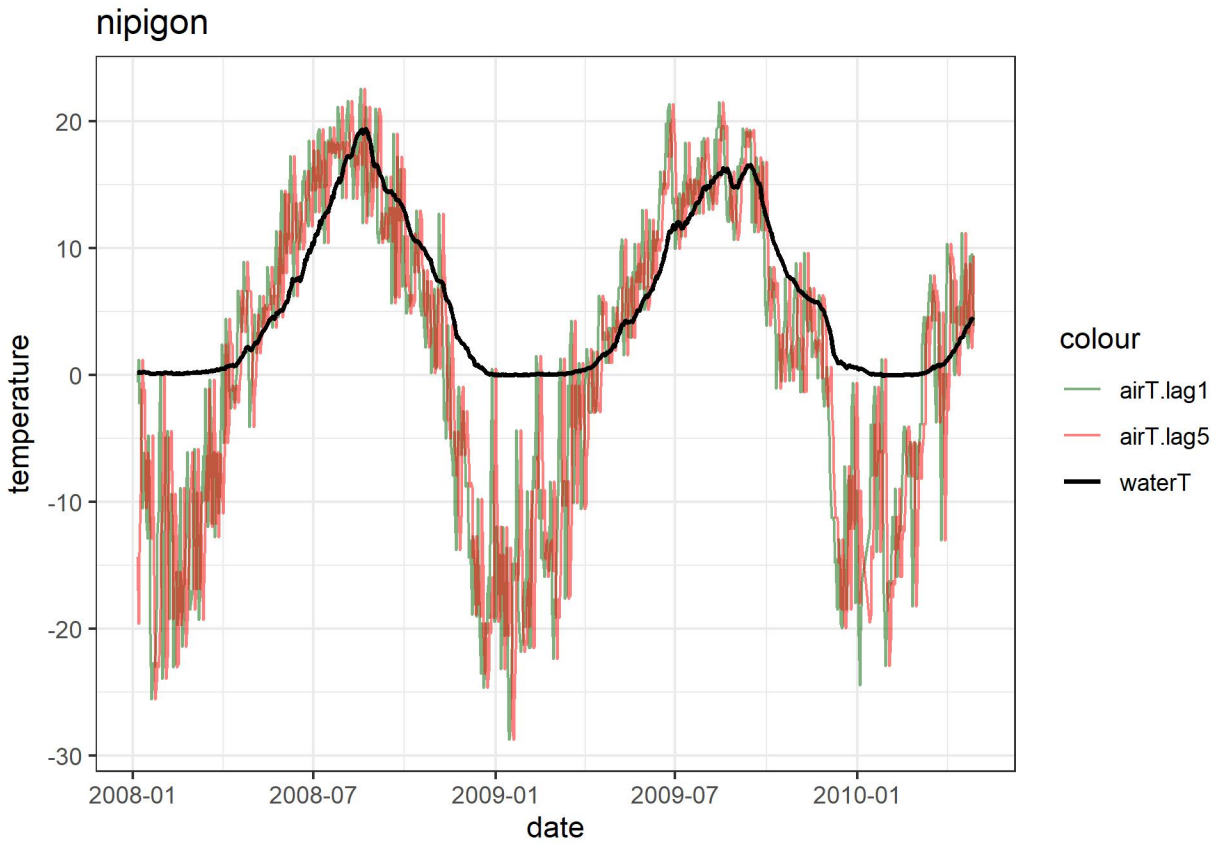

bigotter
30
20
10
0
-10
-20
temperature
2012-07
2013-01
2013-07
2014-01
2014-07
date
colour
airT.lag1
airT.lag5
waterT

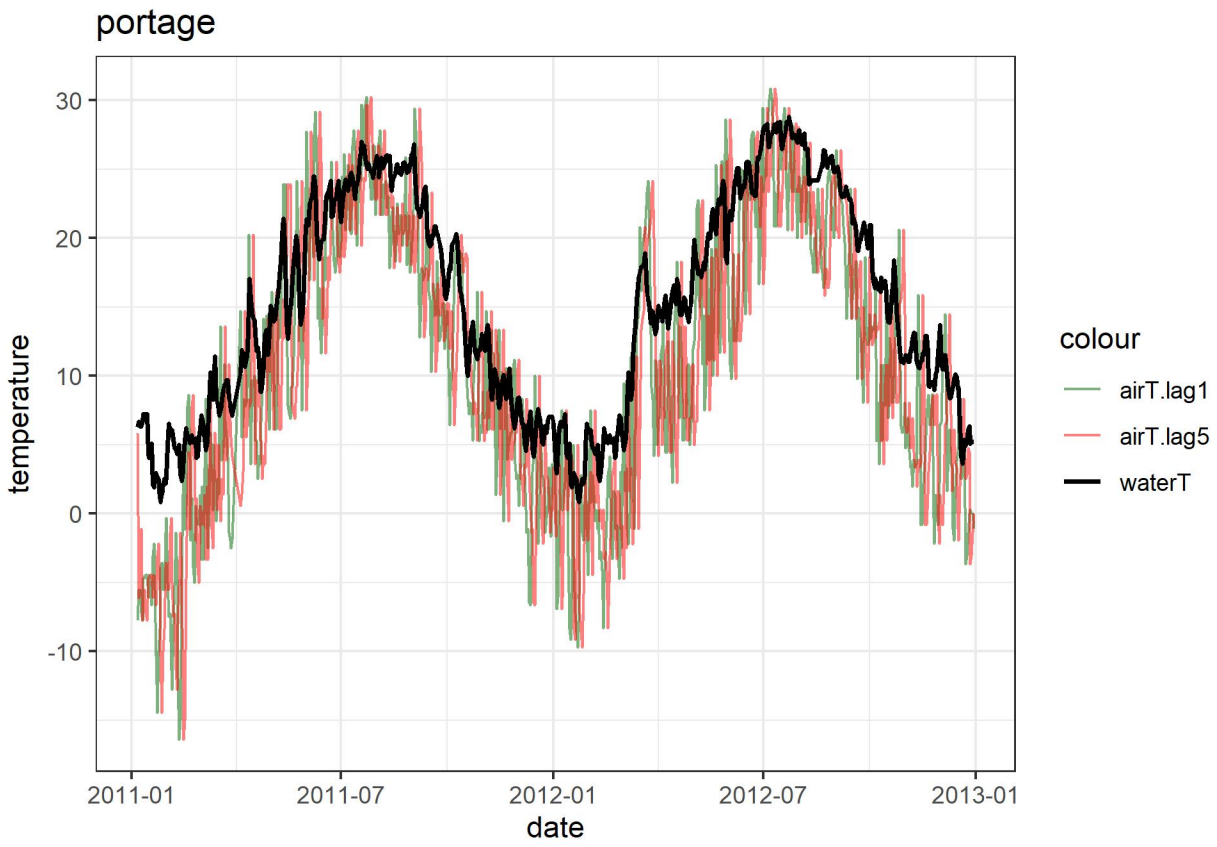

fox
30
20
10
0
-10
-20
temperature
2011
2012
2013
2014
2015
date
colour
airT.lag1
airT.lag5
waterT

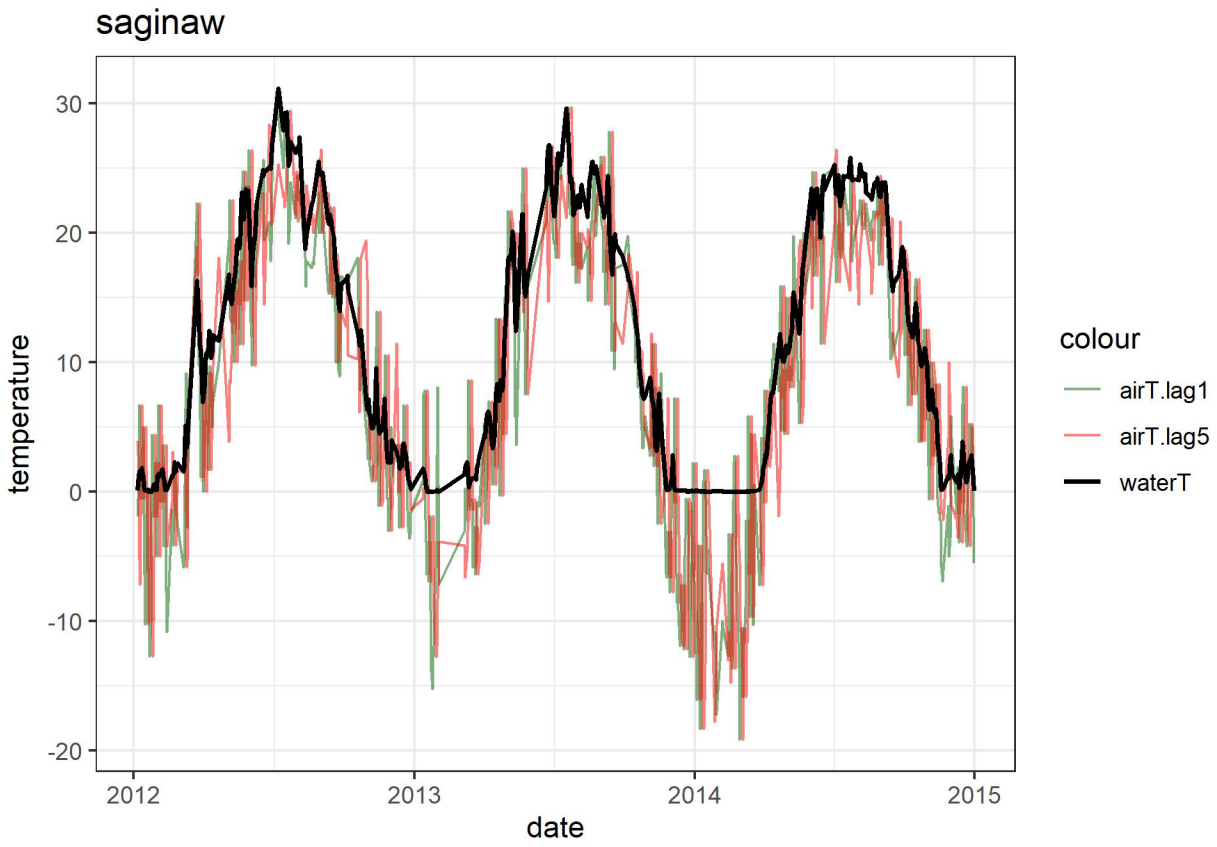

genesee
30
20
10
0
-10
temperature
2011
2012
2013
2014
date
colour
airT.lag1
airT.lag5
waterT

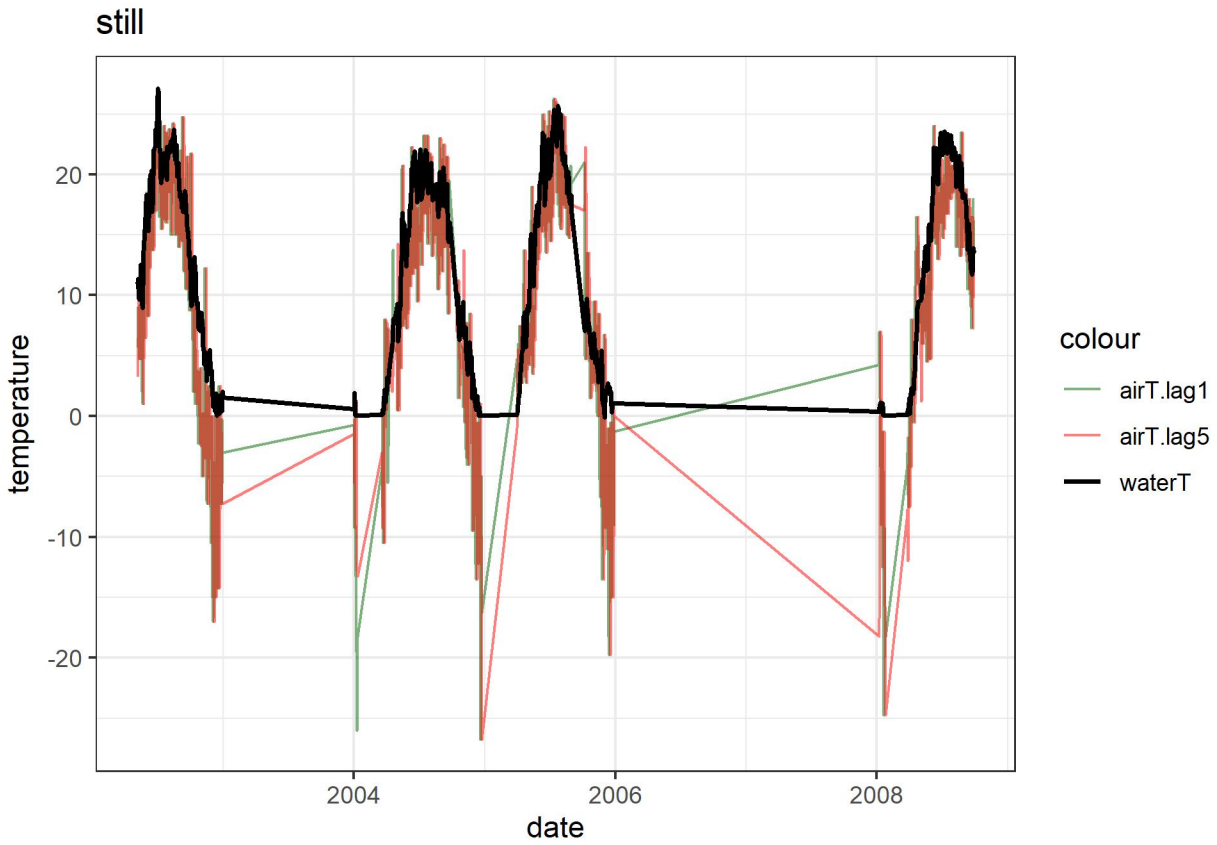

humber
temperature
30
20
10
0
-10
-20
2000
2005
2010
date
colour
airT.lag1
airT.lag5
waterT

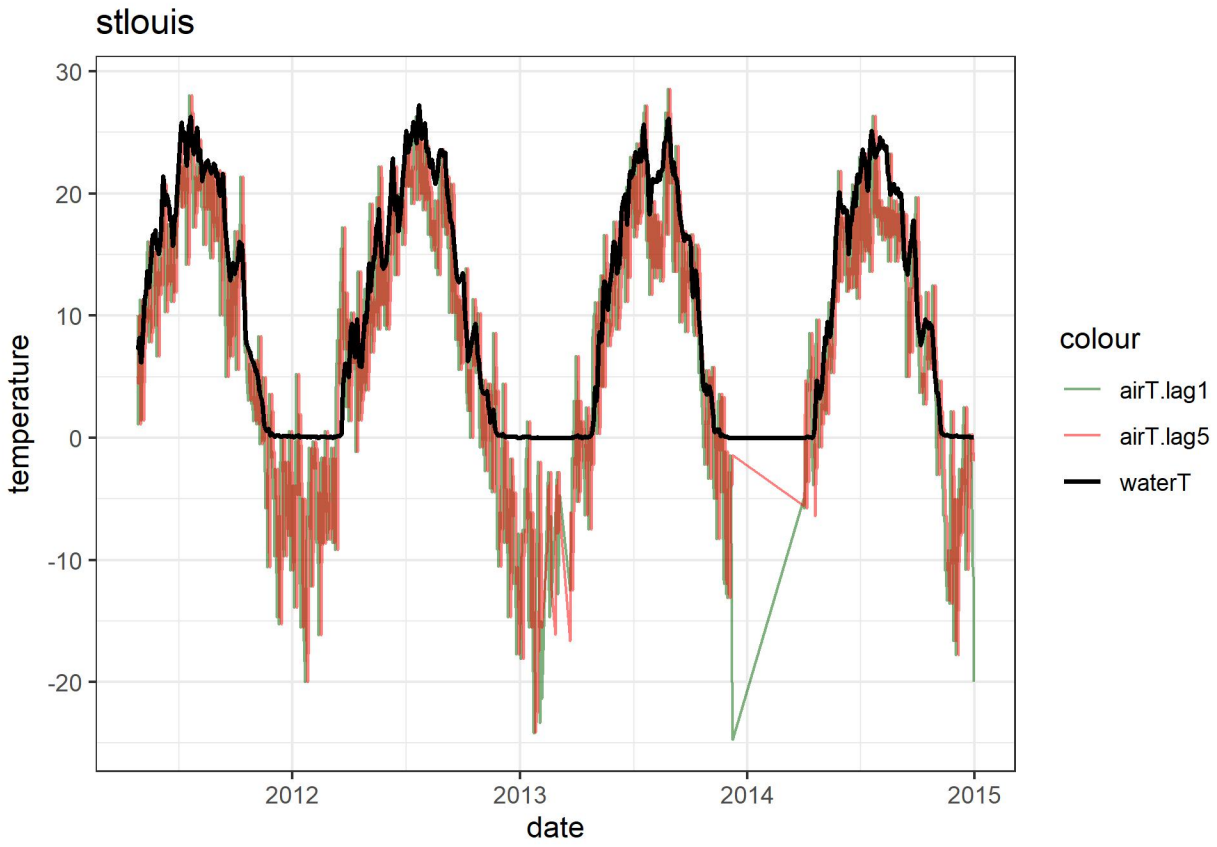

mississagi
temperature
20
10
0
-10
-20
2011
2012
2013
2014
date
colour
airT.lag1
airT.lag5
waterT

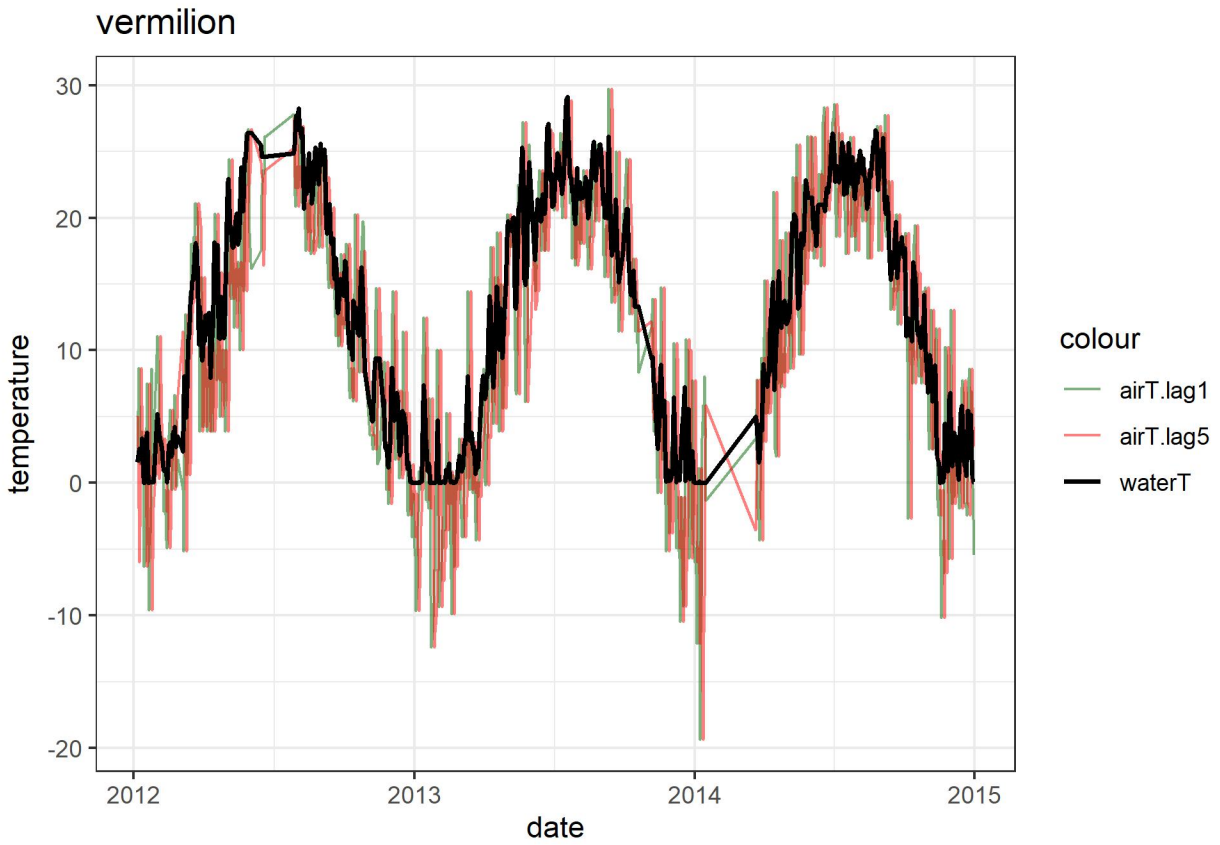

nipigon
temperature
20
10
0
-10
-20
-30
2008-01
2008-07
2009-01
2009-07
2010-01
date
colour
airT.lag1
airT.lag5
waterT


portage
temperature
30
20
10
0
-10
2011-01
2011-07
2012-01
2012-07
2013-01
date
colour
airT.lag1
airT.lag5
waterT


saginaw
30
20
10
0
-10
-20
temperature
2012
2013
2014
2015
date
colour
airT.lag1
airT.lag5
waterT


still
temperature
20
10
0
-10
-20
2004
2006
2008
date
colour
airT.lag1
airT.lag5
waterT


stlouis
30
20
10
0
-10
-20
temperature
2012
2013
2014
2015
date
colour
airT.lag1
airT.lag5
waterT

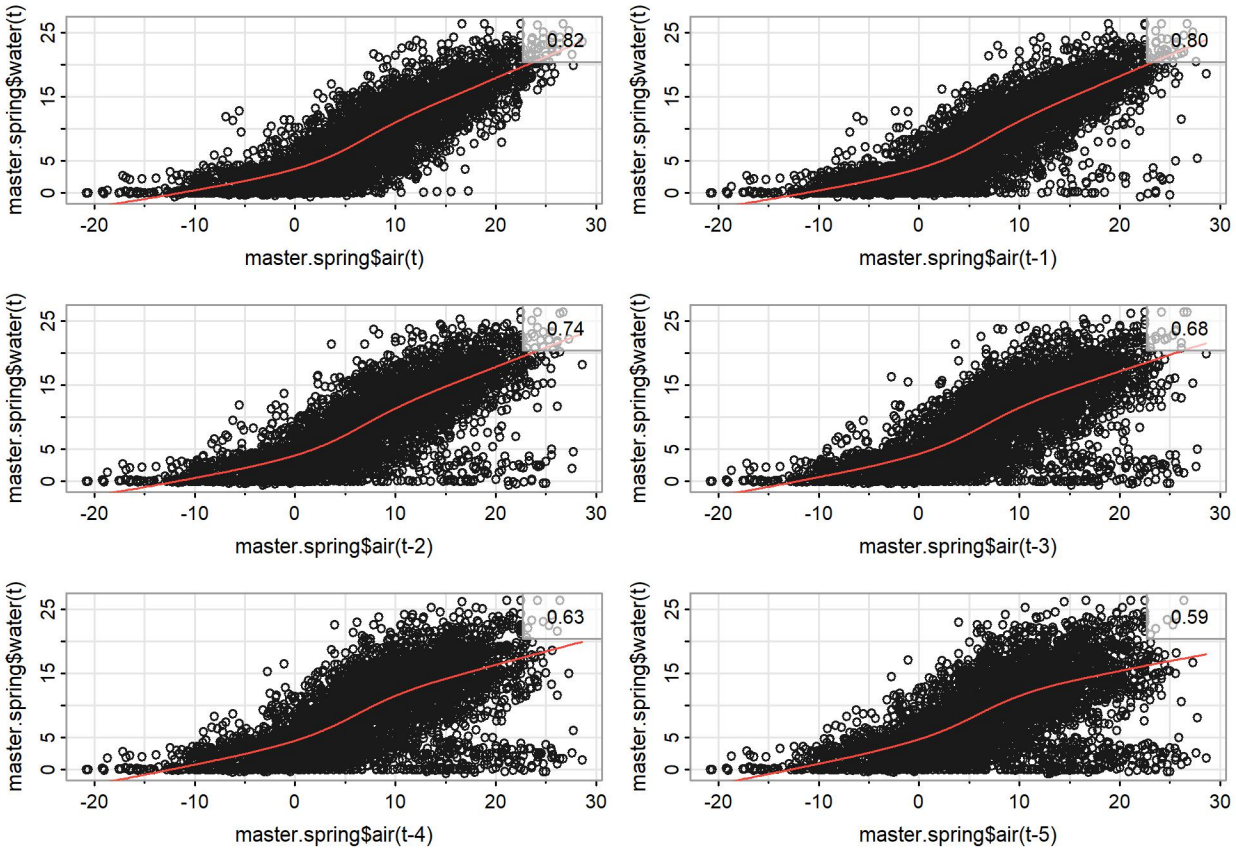


It seems that the lagged day temperatures are working fine.

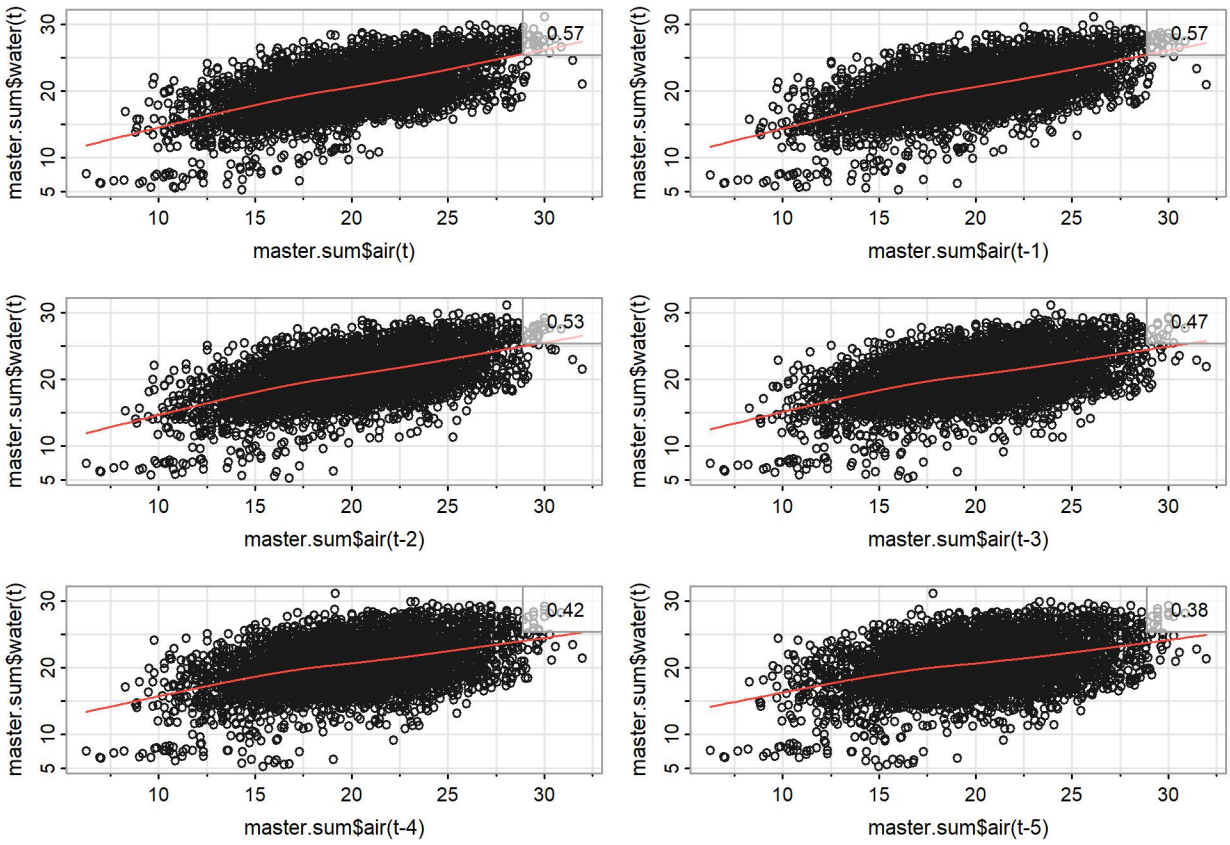
Lagged scatterplot

Then, we want to use the relationship between our response variables (water temperature), and our predictors (air temperature at various lags) for each season. All locations are plotted on one graph for easier demonstration.

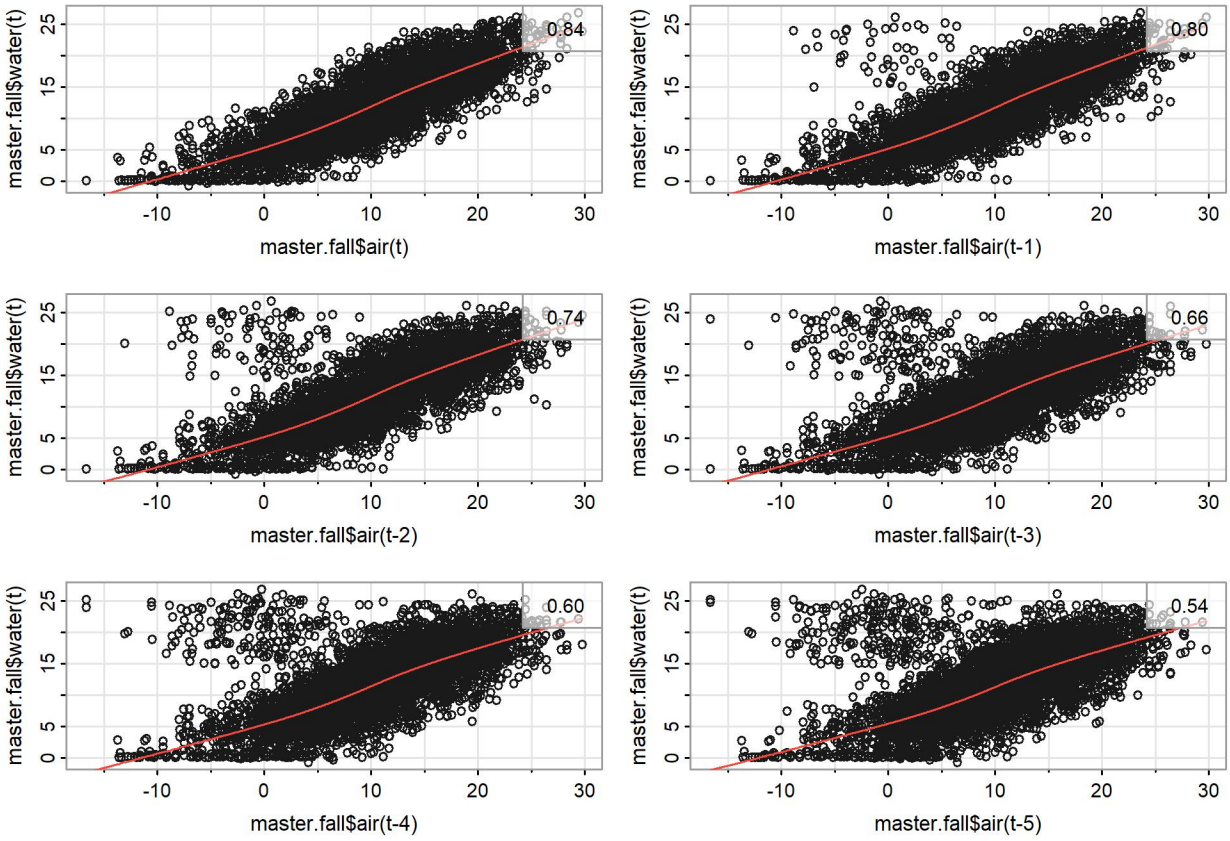
```
lag2.plot(master.spring$air, master.spring$water, 5)
```



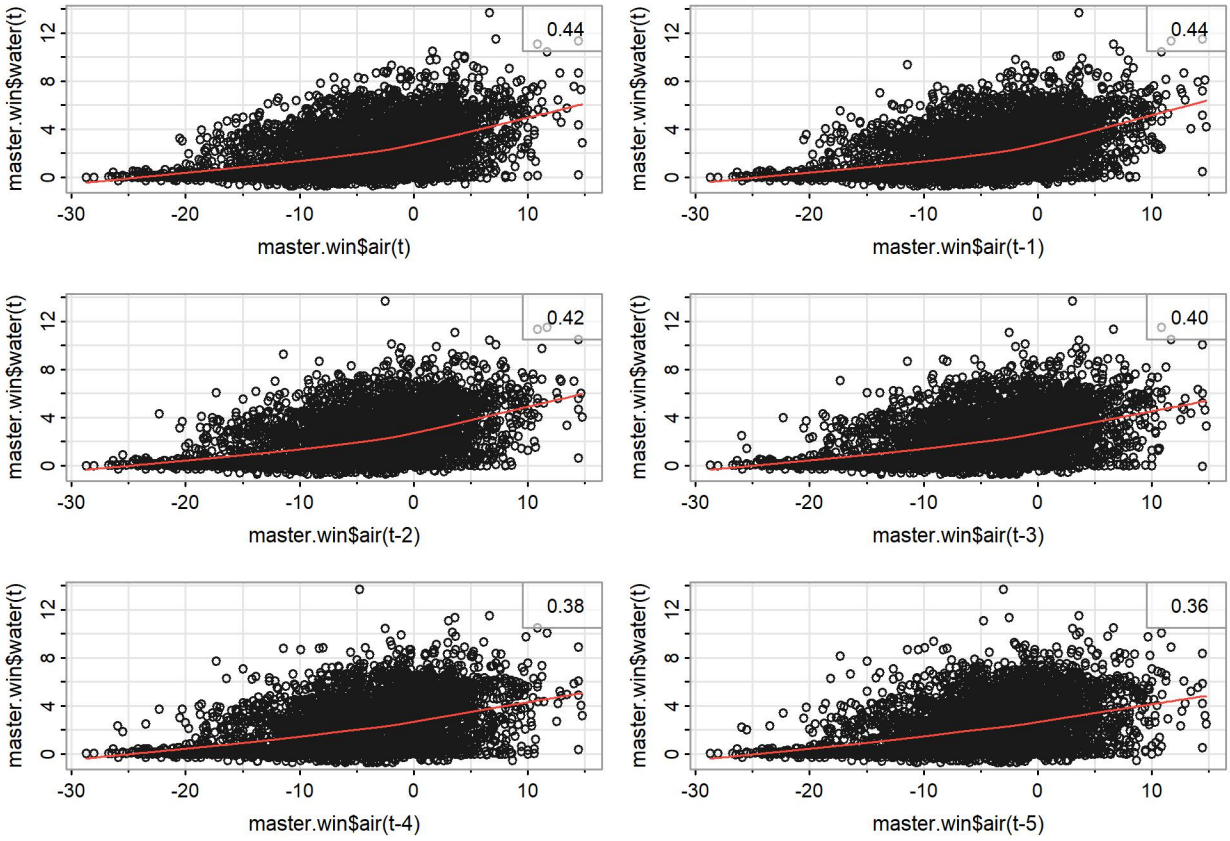

```
lag2.plot(master.sum$air, master.sum$water, 5)
```




```
lag2.plot(master.fall$air, master.fall$water, 5)
```




```
lag2.plot(master.win$air, master.win$water, 5)
```



Specify the season

Here, we use this code to identify the specific season used to run model and check temporal autocorrelation.

Note: season 1-5: spring, summer, fall, winter, annual

```
i = 1
season = 1
ctrl = lmeControl(opt='optim')

# select current dataset, and all unique location levels
current.training <- grand_training[[season]][[i]] %>% arrange(location, date)
current.testing <- grand_testing[[season]][[i]] %>% arrange(location, date)
```

Multiple linear regression models with lag

Linear regression lag 3

```
## Forms
form.locrandom <- water ~ air + dmean_1 + dmean_2
compare <- NA

# model training and predicting
model <- lme(form.locrandom,
             random = ~1 | location,
             na.action = na.omit, data = current.training)
```

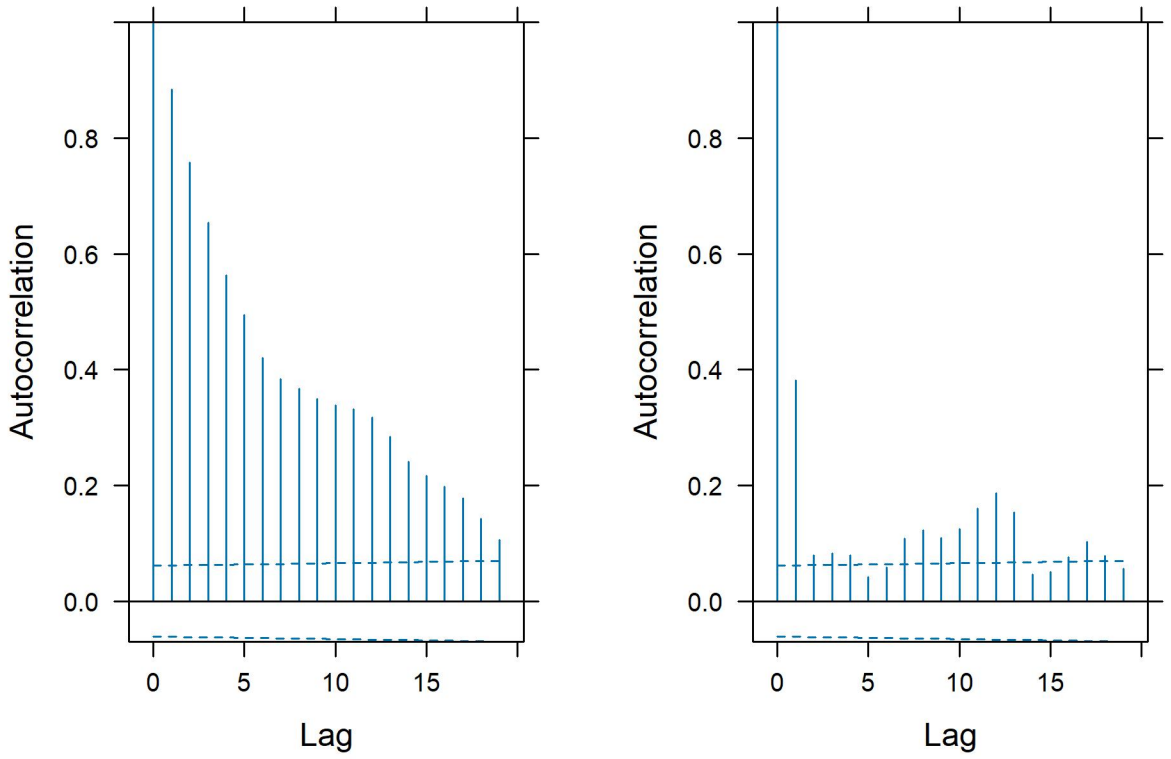
```r
current.testing$preds <- as.vector(predict(model, newdata = current.testing))
current.training$preds <- as.vector(predict(model))

# Include AR term
model.ar <- lme(form.locrandom,
                random = ~1 | location,
                na.action = na.omit, data = current.training,
                correlation=corAR1(form=~1|location))

current.testing$preds.ar <- as.vector(predict(model.ar, newdata = current.testing, re.form = ~1|locatio
current.training$preds.ar <- as.vector(predict(model.ar, re.form = ~1|location))


## compare dataframe output
comp.fit <- current.training %>%
  select(location, year, date, obs = water, preds, preds.ar)

comp.pred <- current.testing %>%
  select(location, year, date, obs = water, preds, preds.ar)


## Plot ACF
ggarrange(plot(ACF(model,resType="normalized"),alpha=0.05),
        plot(ACF(model.ar,resType="normalized"),alpha=0.05),
        nrow = 1)


## Plots
for (loc in loc_seq) {
  plot.df <- comp.fit[comp.fit$location == loc,]
  diff <- round(sqrt(mean((plot.df$preds-plot.df$obs)^2)),2)
  diff.ar <- round(sqrt(mean((plot.df$preds.ar-plot.df$obs)^2)),2)

  pl <- ggplot(data=plot.df, aes(x=date))+
          geom_line(aes(x=date, y=obs), color = "black")+
          geom_line(aes(x=date, y=preds), color = "red")+
          geom_line(aes(x=date, y=preds.ar), color = "blue")+
          ggtitle(paste(
            loc, ": ", plot.df$year, " (RMSE:", diff, "&", diff.ar, ")"))
  print(pl)
}


## Metrics calculation
#ModelMetrics::rmse(compare$preds, compare$obs)
#ModelMetrics::rmse(compare$preds.ar, compare$obs)

anova(model, model.ar)
```

Linear lag 3 model definitely does not perform better compared to the linear lag 5 model.

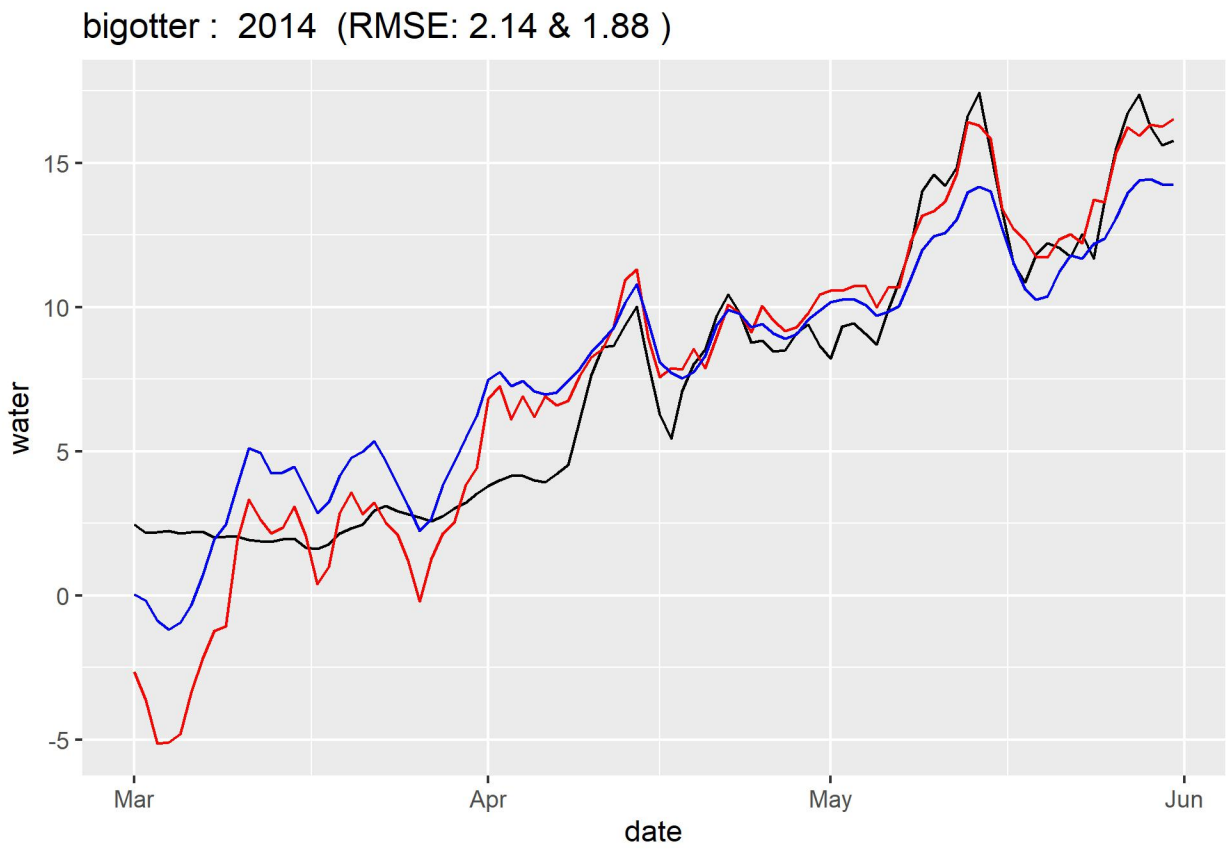
Linear regression lag 5

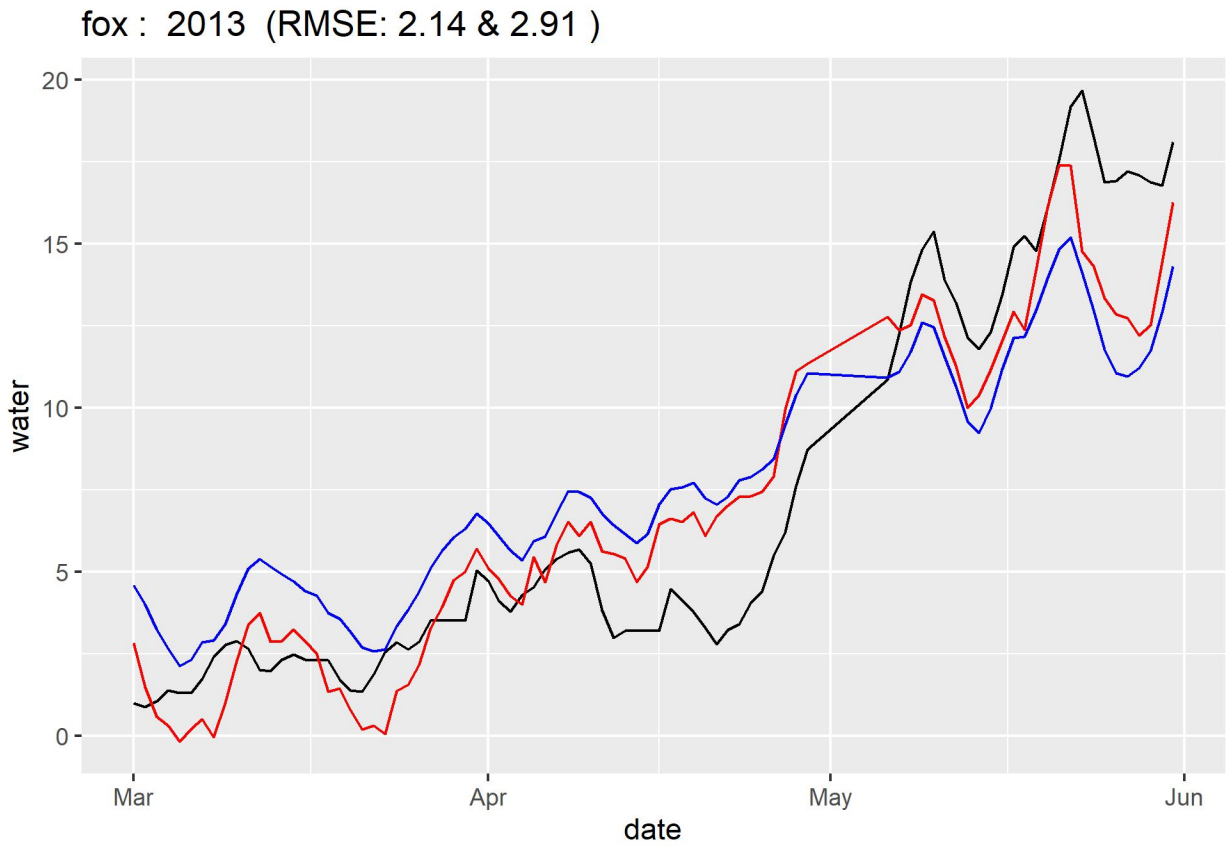
Now, let's use a multiple linear (lag 5) mixed model with location as random effect.

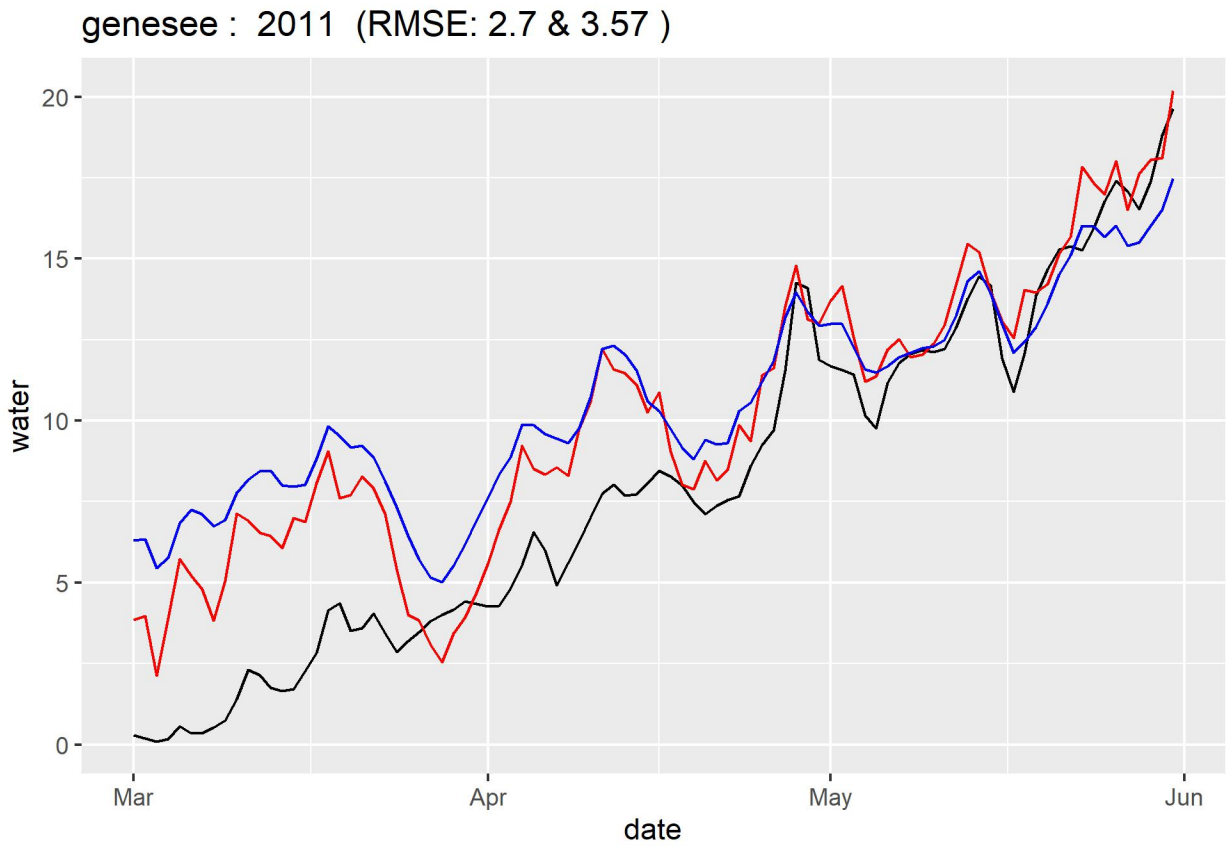
```r
## Forms
form.locrandom <- water ~ air + dmean_1 + dmean_2 + dmean_3 + dmean_4 + dmean_5
compare <- current.testing


# model training and predicting
model <- lme(form.locrandom,
             random = ~1 | location,
             na.action = na.omit, data = current.training)

compare$preds <- as.vector(predict(model, newdata = current.testing))

# Include AR term
model.ar <- lme(form.locrandom,
                random = ~1 | location, control = ctrl,
                na.action = na.omit, data = current.training,
                correlation=corAR1(form=~1|location,0.85,fixed=T))

compare$preds.ar <- as.vector(predict(model.ar, newdata = current.testing, re.form = ~1|location))


## Plot ACF
ggarrange(plot(ACF(model,resType="normalized"),alpha=0.05),
        plot(ACF(model.ar,resType="normalized"),alpha=0.05),
        nrow = 1)
```

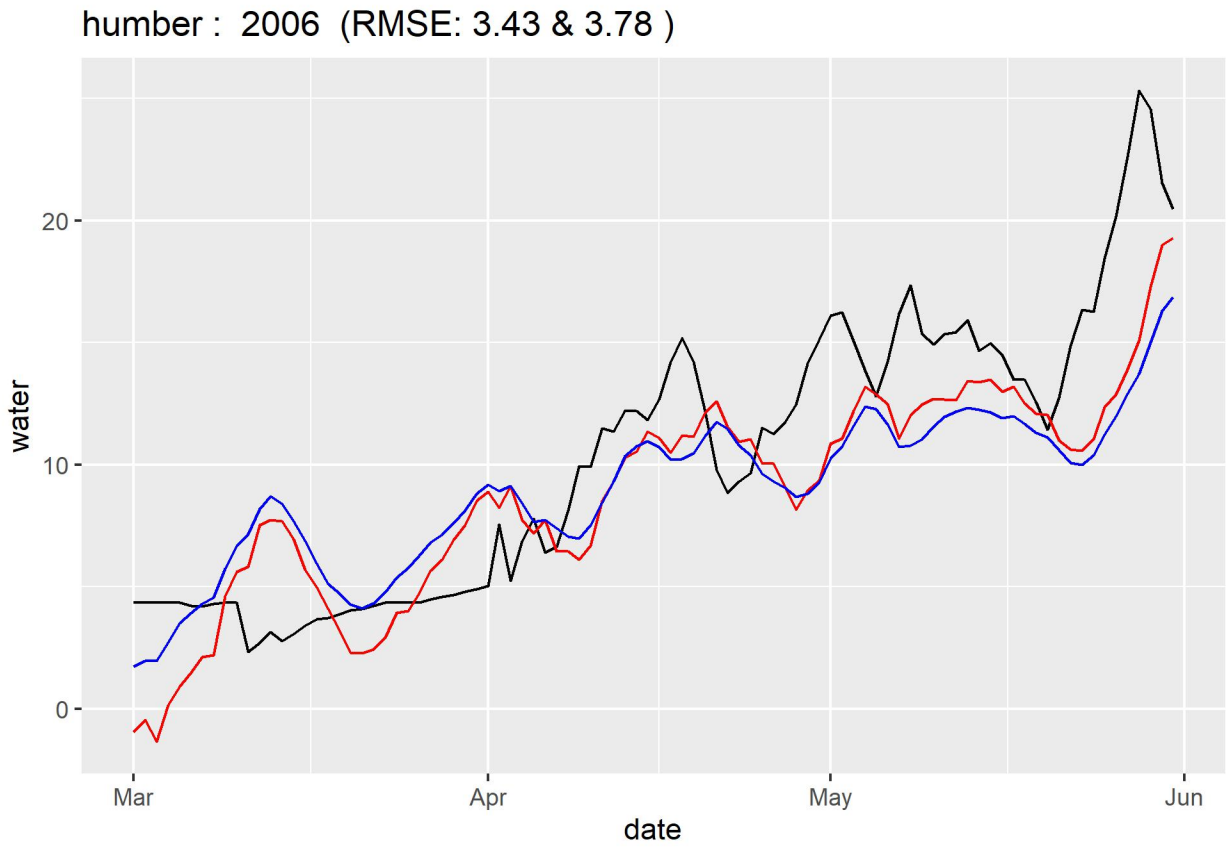


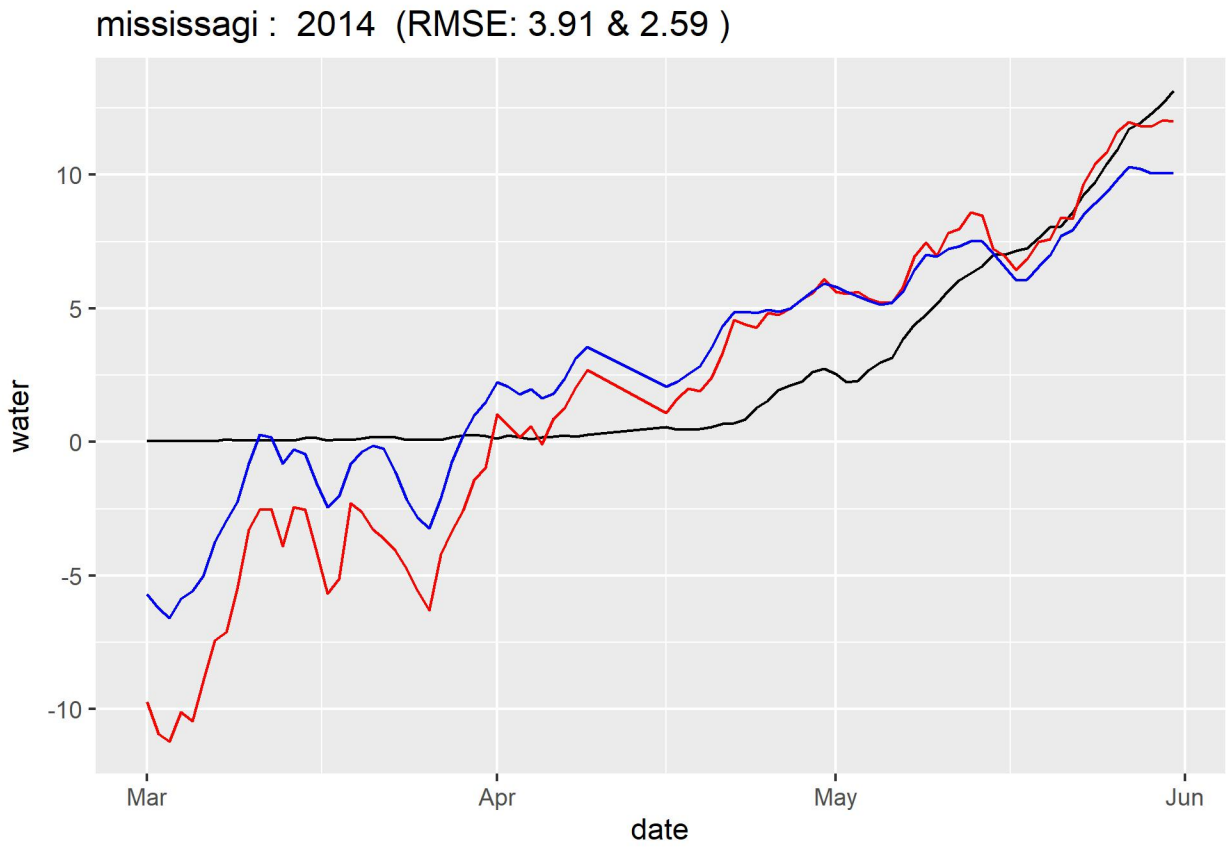
```r
## Plots
for (loc in loc_seq) {
  plot.df <- compare[compare$location == loc,]
  diff <- round(sqrt(mean((plot.df$preds-plot.df$water)^2)),2)
  diff.ar <- round(sqrt(mean((plot.df$preds.ar-plot.df$water)^2)),2)

  pl <- ggplot(data=plot.df, aes(x=date))+
          geom_line(aes(x=date, y=water), color = "black")+
          geom_line(aes(x=date, y=preds), color = "red")+
          geom_line(aes(x=date, y=preds.ar), color = "blue")+
          ggtitle(paste(
            loc, ": ", plot.df$year, " (RMSE:", diff, "&", diff.ar, ")"))
  print(pl)
}
```

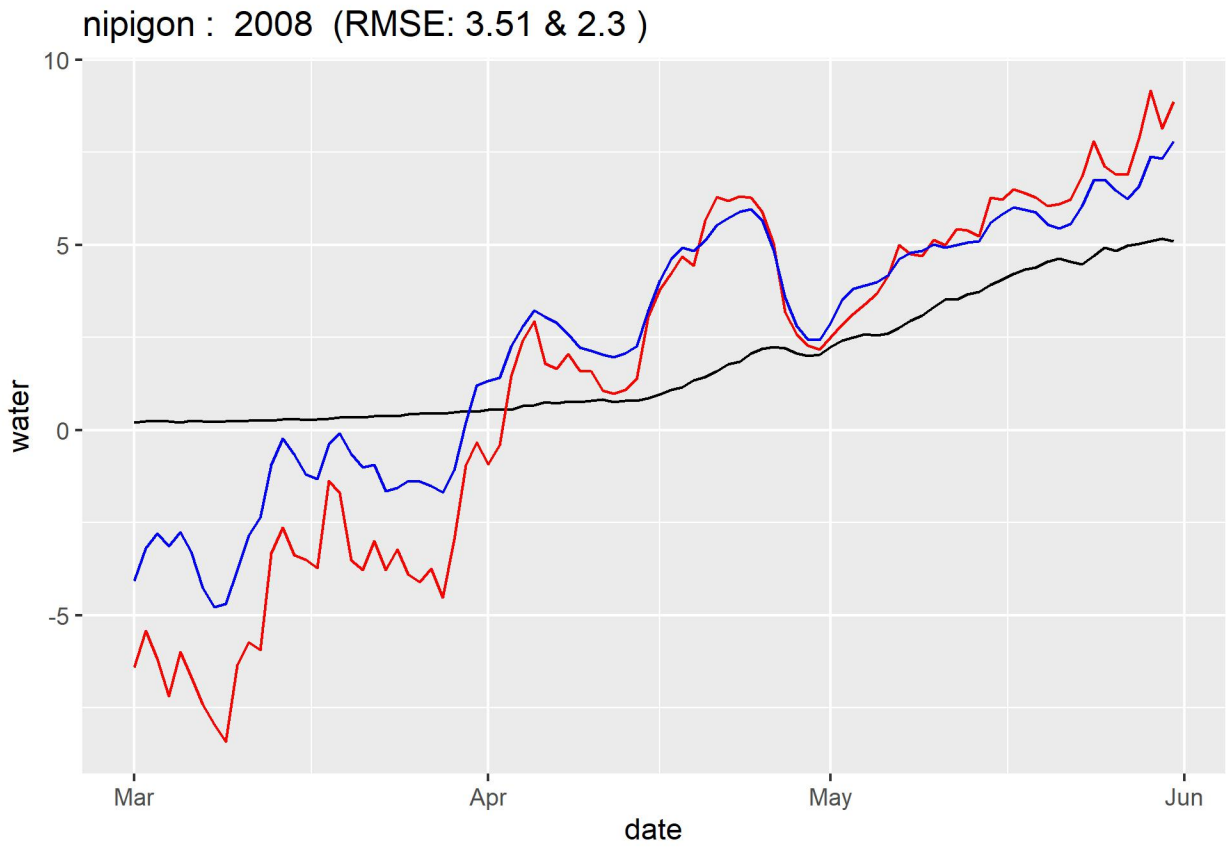

bigotter : 2014 (RMSE: 2.14 & 1.88)
water
15
10
5
0
-5
Mar
Apr
May
Jun
date

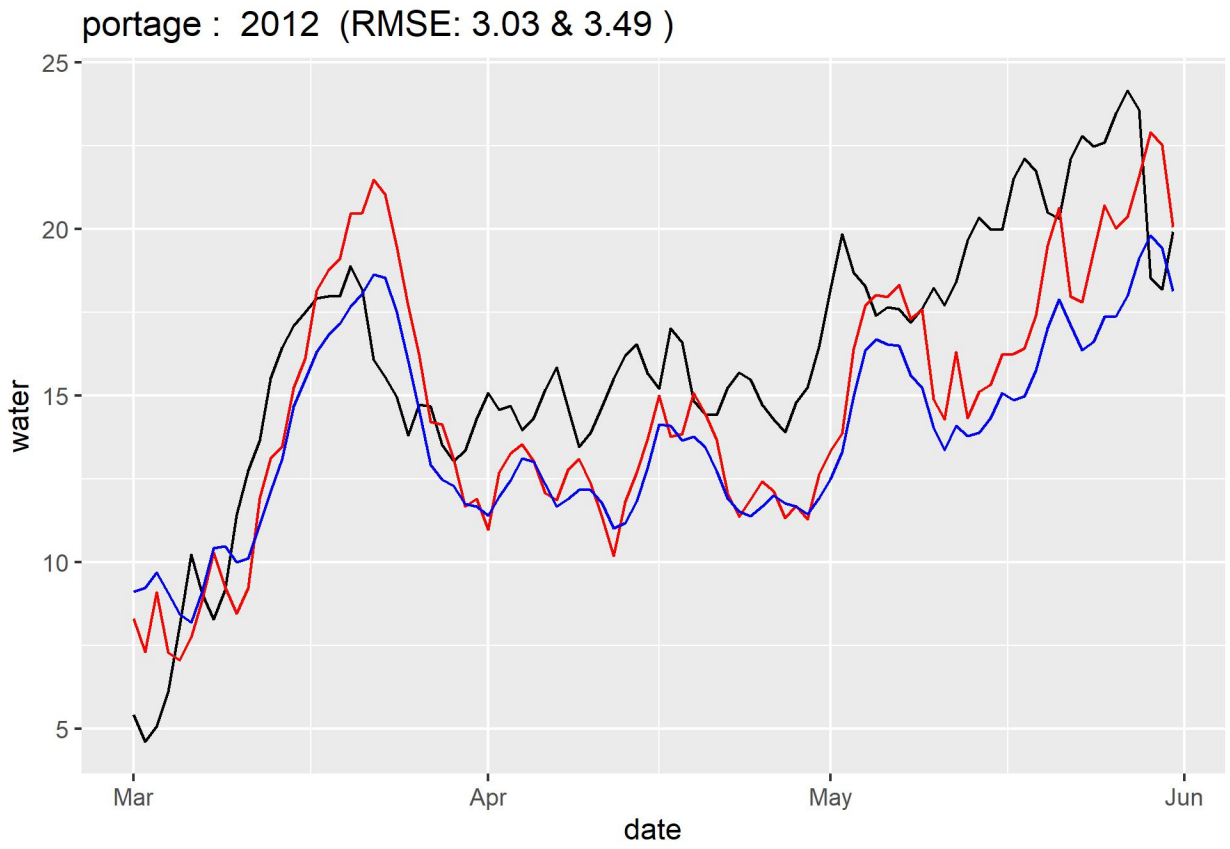

fox : 2013 (RMSE: 2.14 & 2.91)
20
15
10
5
0
water
Mar
Apr
May
Jun
date

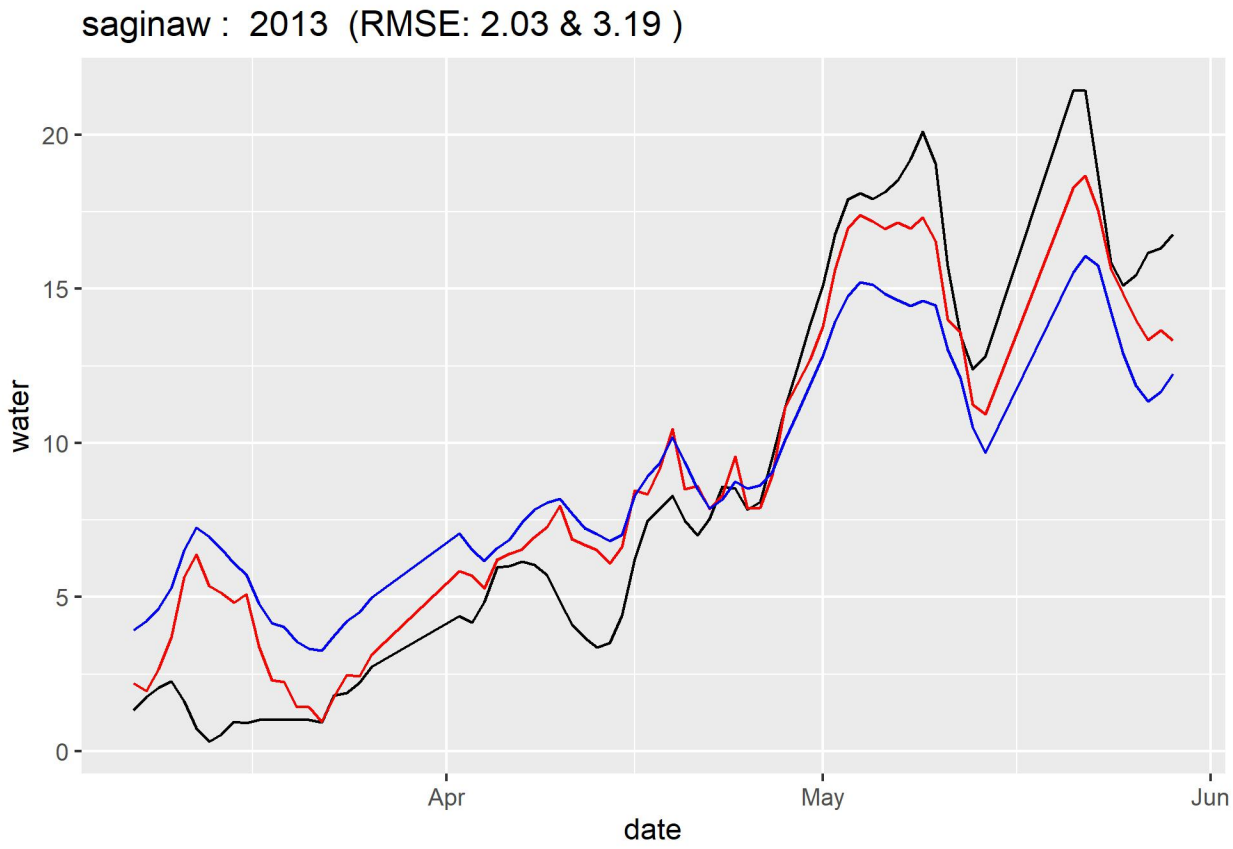

genesee : 2011 (RMSE: 2.7 & 3.57)
water
20
15
10
5
0
Mar
Apr
May
Jun
date

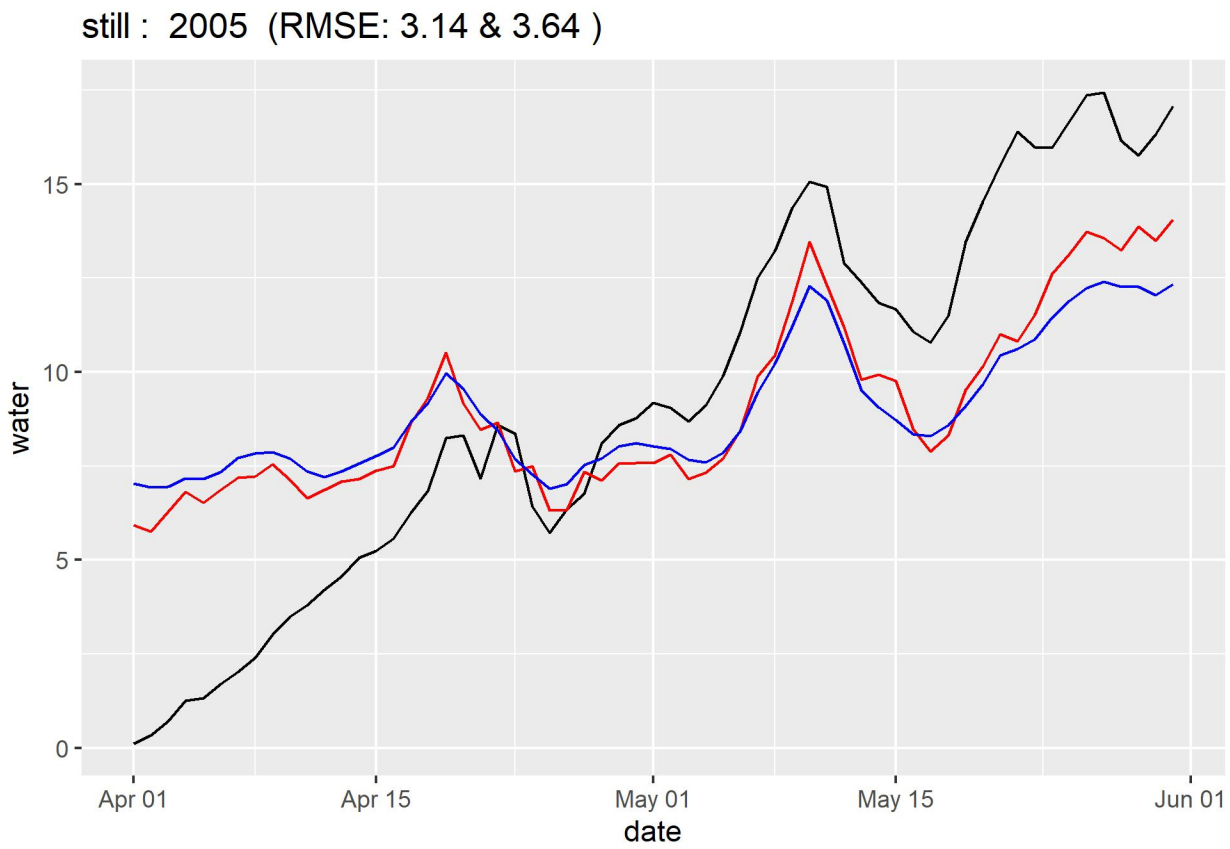

humber : 2006 (RMSE: 3.43 & 3.78)
water
20
10
0
Mar
Apr
May
Jun
date

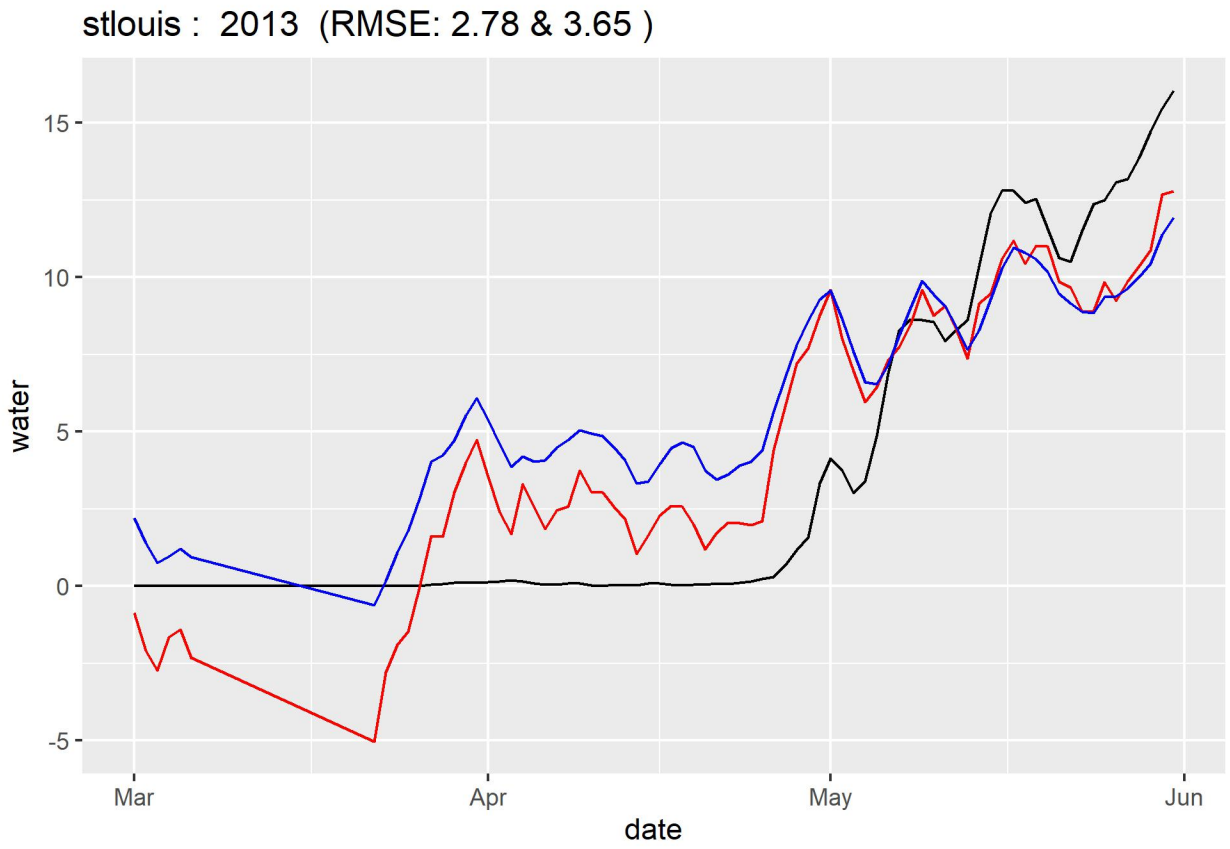

mississagi : 2014 (RMSE: 3.91 & 2.59)
water
10
5
0
-5
-10
Mar
Apr
May
Jun
date

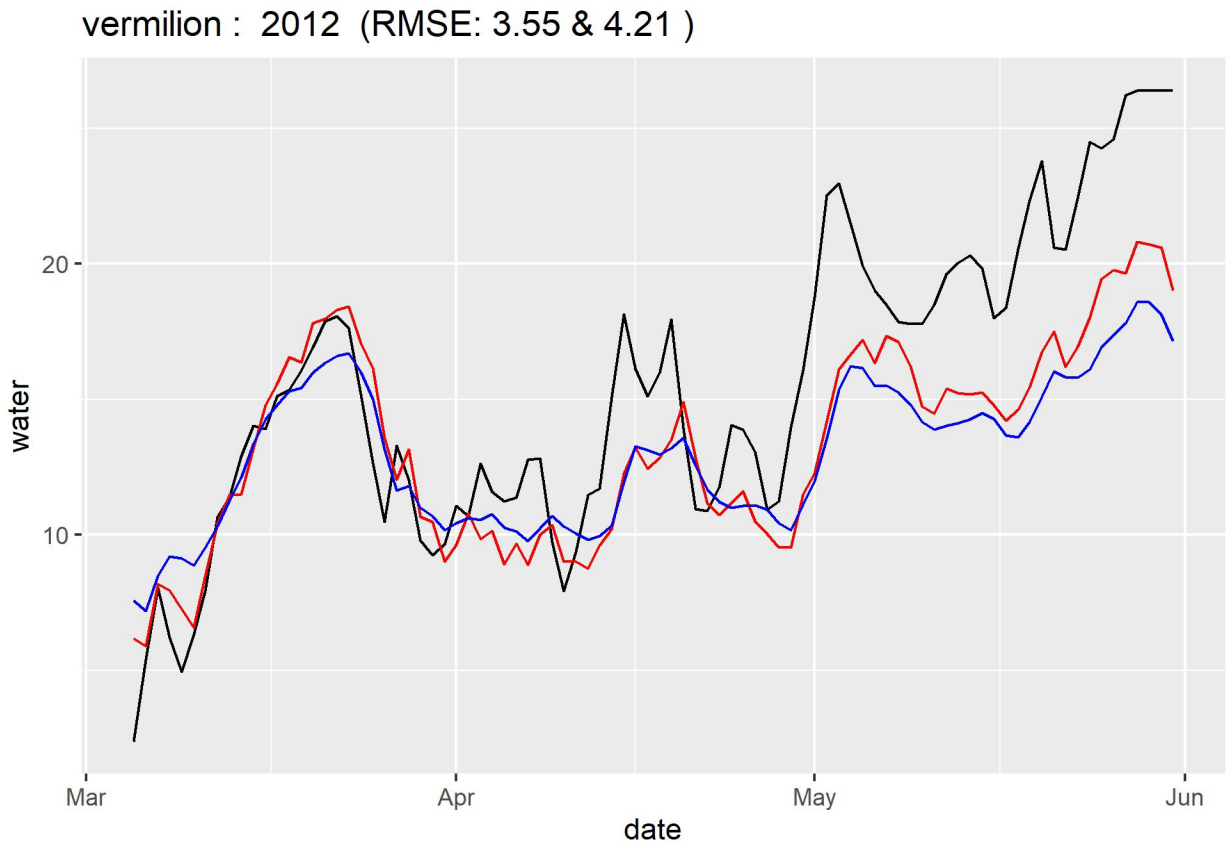

nipigon : 2008 (RMSE: 3.51 & 2.3)
10
5
0
-5
water
Mar
Apr
May
Jun
date


portage : 2012 (RMSE: 3.03 & 3.49)
25
20
15
10
5
water
Mar
Apr
May
Jun
date


saginaw : 2013 (RMSE: 2.03 & 3.19)
water
20
15
10
5
0
Apr
May
Jun
date


still : 2005 (RMSE: 3.14 & 3.64)
water
15
10
5
0
Apr 01
Apr 15
May 01
May 15
Jun 01
date


stlouis : 2013 (RMSE: 2.78 & 3.65)
15
10
5
0
-5
water
Mar
Apr
May
Jun
date




```
## Metrics calculation
#ModelMetrics::rmse(compare$preds, compare$obs)
#ModelMetrics::rmse(compare$preds.ar, compare$obs)

anova(model, model.ar)
```

```
##          Model df      AIC      BIC    logLik
## model        1  9 4854.969 4899.220 -2418.485
## model.ar     2  9 2947.169 2991.419 -1464.584
```

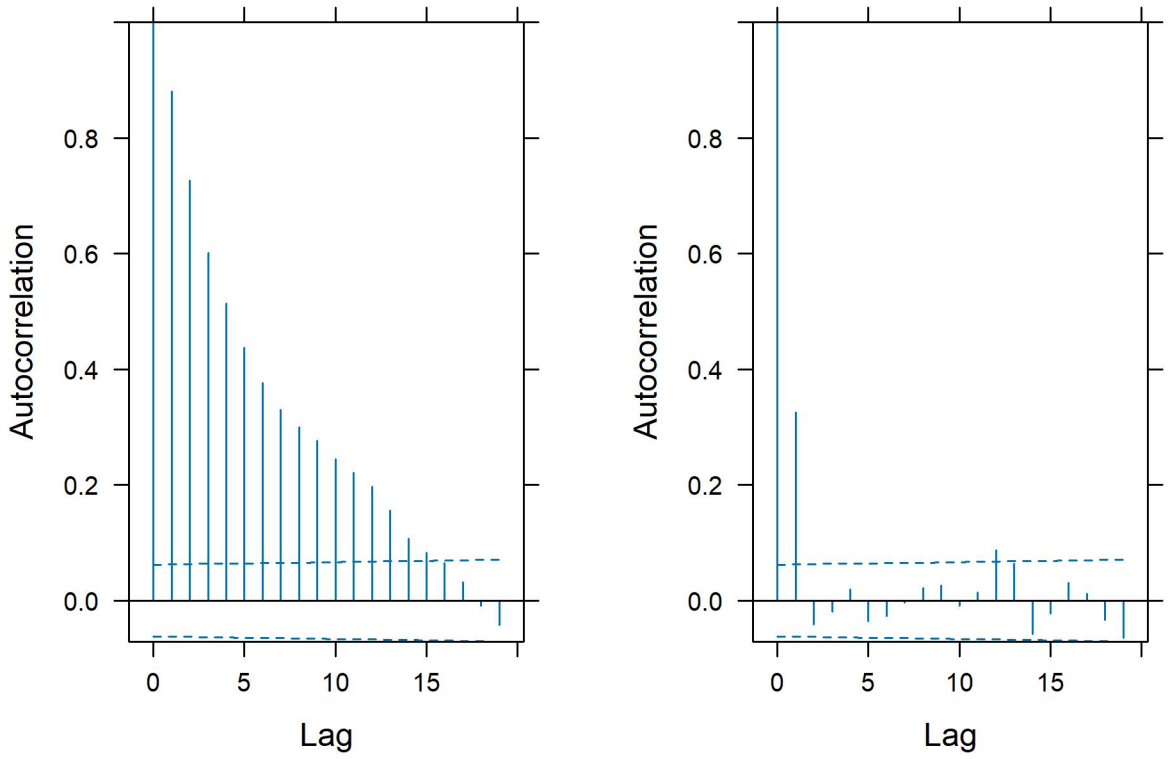
1. Temporal autocorrelation: The autocorrelation in summer and winter is reduced to a decent amount. In spring and fall, it has some degree of autocorrelation at day 6-10, but low at day 5.
2. The winter prediction is relatively good, but worse than simply using the mean. Also, it does not capture large daily variation. Meanwhile, residuals at two ends seems not normally distributed.

Reasonal residual

```
compare <- NA
current.testing <- grand_testing[[season]][[i]] %>% arrange(location, date)


## TRAINING
# get the model annual component
annual.comp <- nls(air ~ a+b*sin(2*pi/365*(yday(date)+t0)),
                   start = list(a=0.05, b=5, t0=-26),
                   data=current.training)
```

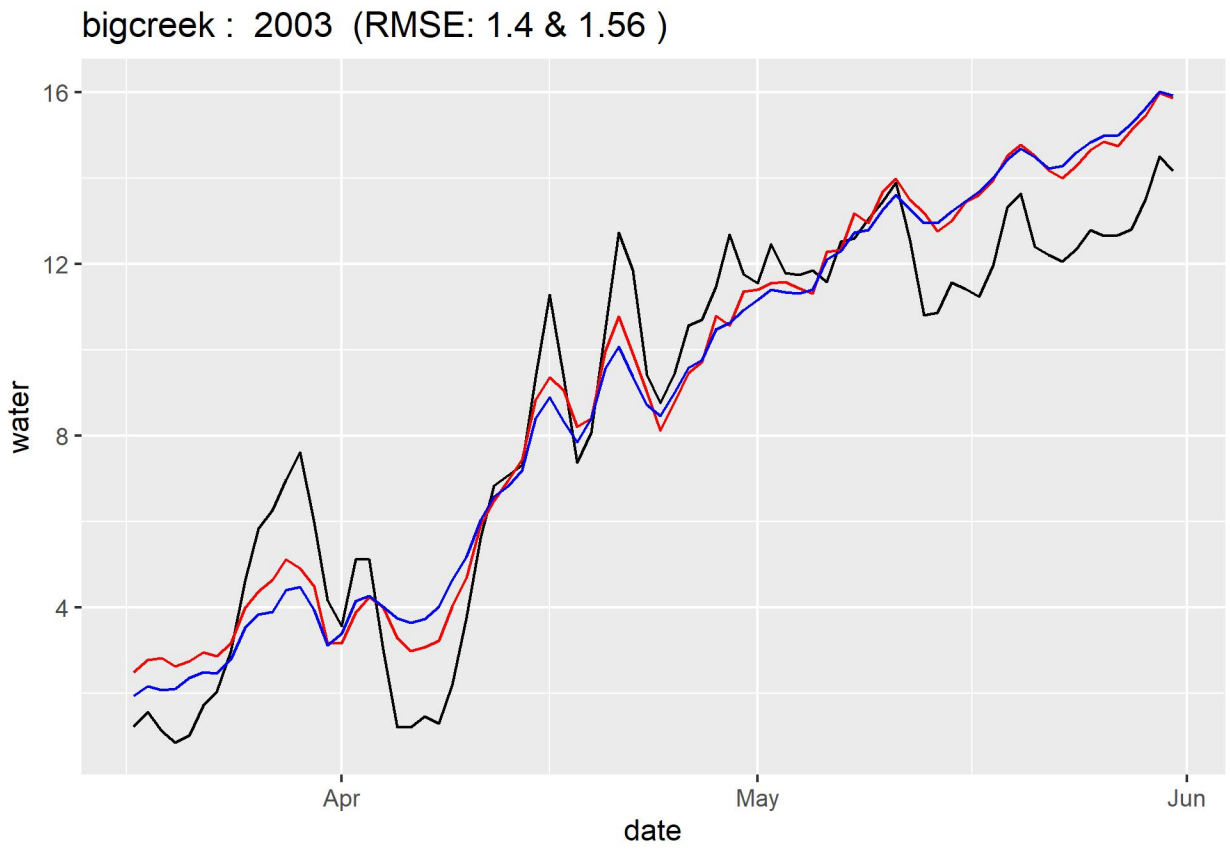
```r
# get the air temperature residuals
res <- as.data.frame(matrix(NA, ncol = 2,
                            nrow = length(na.omit(current.training$air))))
# dataframe to store the residuals
colnames(res) <- c("res.t", "location")
res[,"location"] <- na.omit(current.training$location)
res[,"res.t"] <- as.vector(residuals(annual.comp))
res <- res %>% group_by(location) %>%
  mutate(res.t1 = lag(res.t, 1),
         res.t2 = lag(res.t, 2))
res[,"res.w"] <- residuals(nls(water ~ a+b*sin(2*pi/365*(yday(date)+t0)),
                               start = list(a=0.05, b=5, t0=-26),
                               data = current.training))

# get the water temperature residual component
residual.comp <- lme(fixed = res.w ~ res.t + res.t1 + res.t2,
                     random = ~ 1|location,
                     data = res, na.action = na.omit,
                     control = ctrl)

residual.comp.ar <- lme(fixed = res.w ~ res.t + res.t1 + res.t2,
                        random = ~ 1|location,
                        correlation = corAR1(form=~1|location,0.85,fixed=T),
                        data = res, na.action = na.omit,
                        control = ctrl)


## TESTING
# Annual
preds.annual <- as.data.frame(predict(annual.comp, newdata=current.testing))
preds.annual <- cbind(preds.annual,
                      current.testing$location, current.testing$date)
colnames(preds.annual) <- c("preds.annual", "location", "date")

# Residuals data
res <- as.data.frame(matrix(NA, ncol = 3,
                            nrow = length(current.testing$air))) #residuals
colnames(res) <- c("res.t", "location", "date")
res[,"location"] <- current.testing$location
res[,"date"] <- current.testing$date
res[,"res.t"] <- current.testing$air - preds.annual$preds.annual
res <- res %>% group_by(location) %>%
  mutate(res.t1 = lag(res.t, 1),
         res.t2 = lag(res.t, 2))

# Residuals predictions
pres <- predict(residual.comp, newdata=res, na.action=na.omit,
                re.form=~(1|location))
pres.ar <- predict(residual.comp.ar, newdata=res, na.action=na.omit,
                   re.form=~(1|location))
preds.residuals <- cbind(na.omit(res)[,"location"],
                         na.omit(res)[,"date"],
                         as.data.frame(pres), as.data.frame(pres.ar))
```

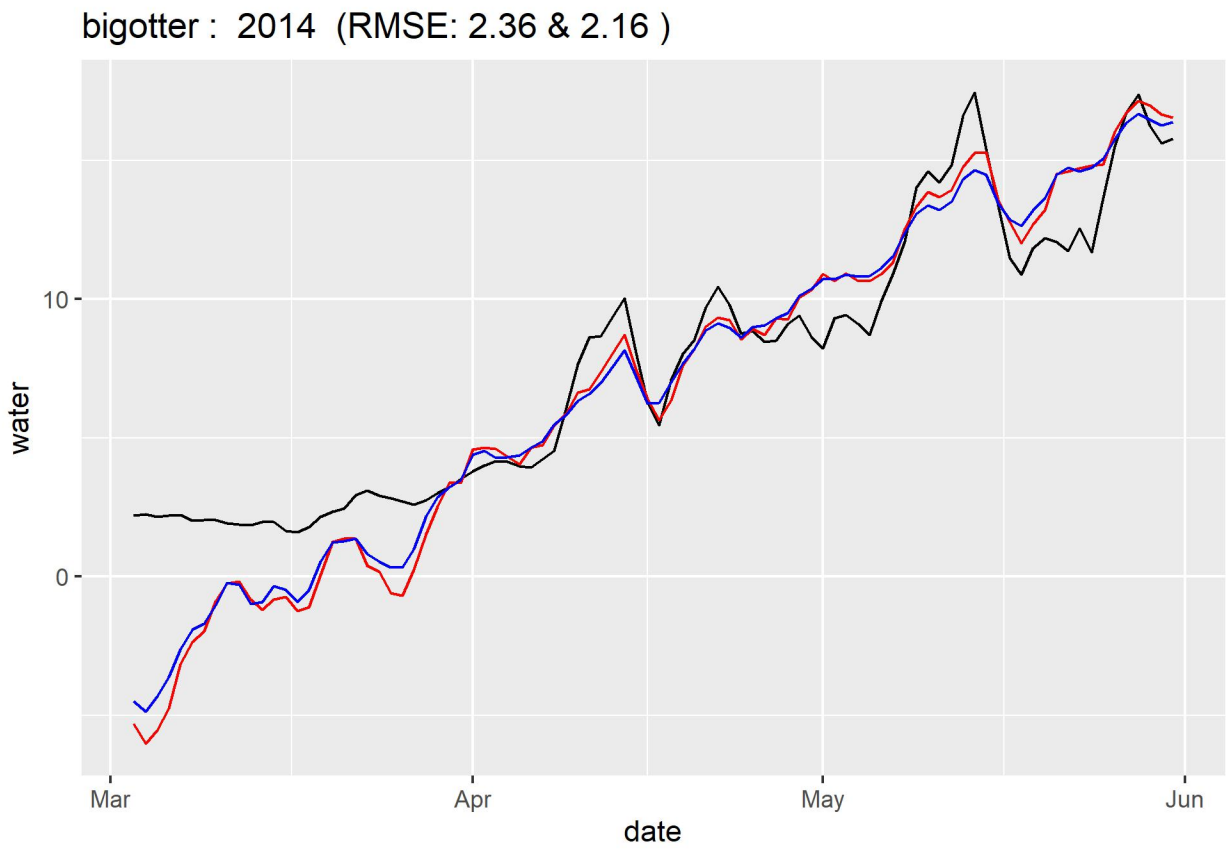
```r
# Combine and add up both components
p <- merge(preds.annual, preds.residuals, by=c("location","date"))
p[,"preds"] <- p$preds.annual + p$pres
p[,"preds.ar"] <- p$preds.annual + p$pres.ar

compare <- merge(current.testing, p, by=c("location","date")) %>%
  select(location, year, date, water, preds, preds.ar, preds.annual)


## Plot ACF
ggarrange(plot(ACF(residual.comp,resType="normalized"),alpha=0.05),
          plot(ACF(residual.comp.ar,resType="normalized"),alpha=0.05),
          nrow = 1)
```

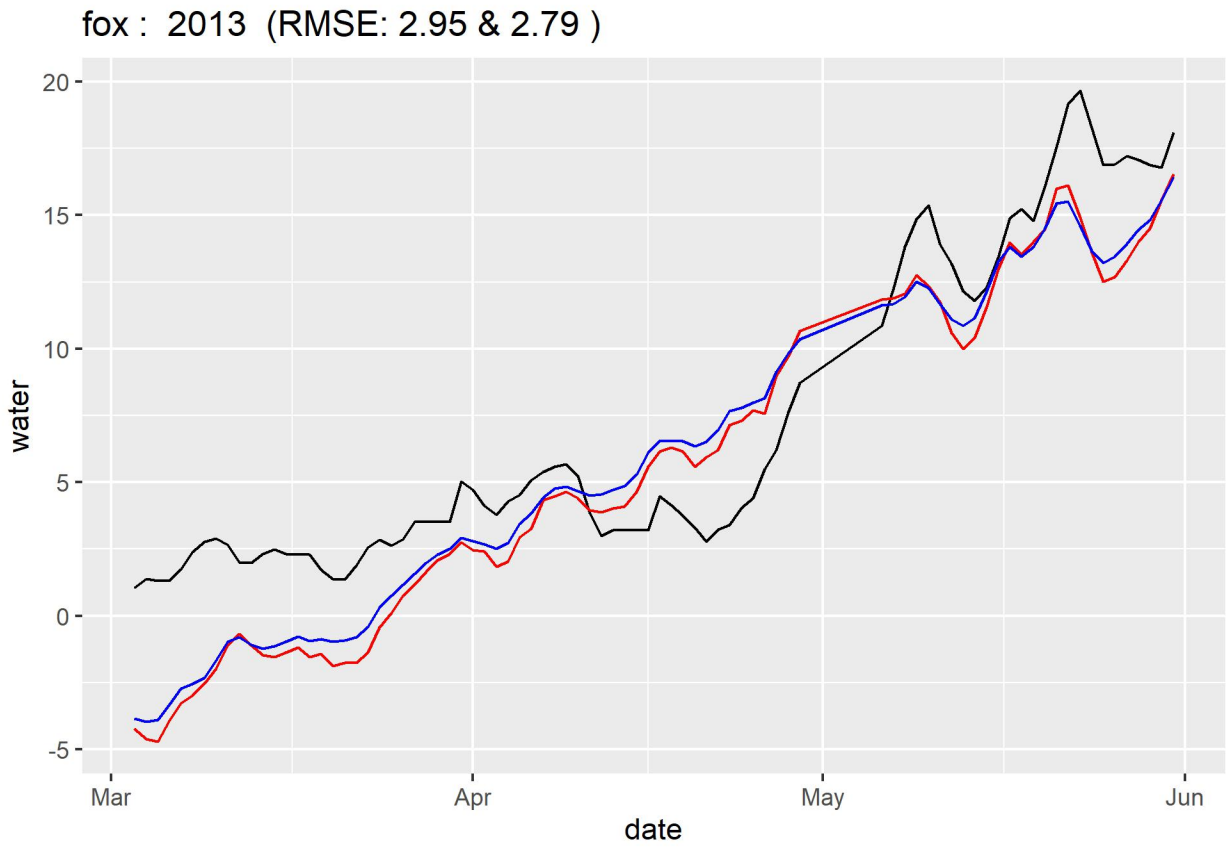


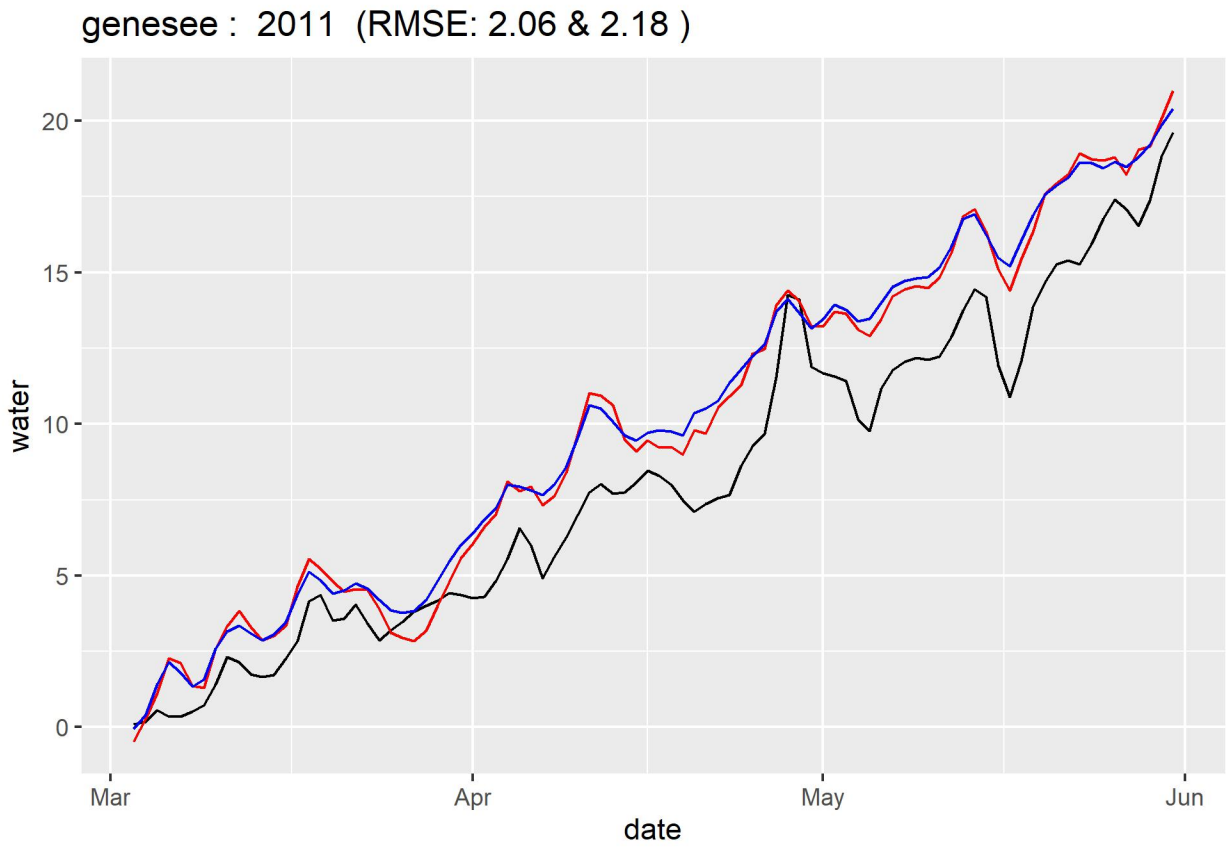

```r
## Plots
for (loc in loc_seq) {
  plot.df <- compare[compare$location == loc,]
  diff <- round(sqrt(mean((plot.df$preds-plot.df$water)^2)),2)
  diff.ar <- round(sqrt(mean((plot.df$preds.ar-plot.df$water)^2)),2)

  pl <- ggplot(data=plot.df, aes(x=date))+
          geom_line(aes(x=date, y=water), color = "black")+
          geom_line(aes(x=date, y=preds), color = "red")+
          geom_line(aes(x=date, y=preds.ar), color = "blue")+
          ggtitle(paste(
            loc, ": ", plot.df$year, " (RMSE:", diff, "&", diff.ar, ")"))
```

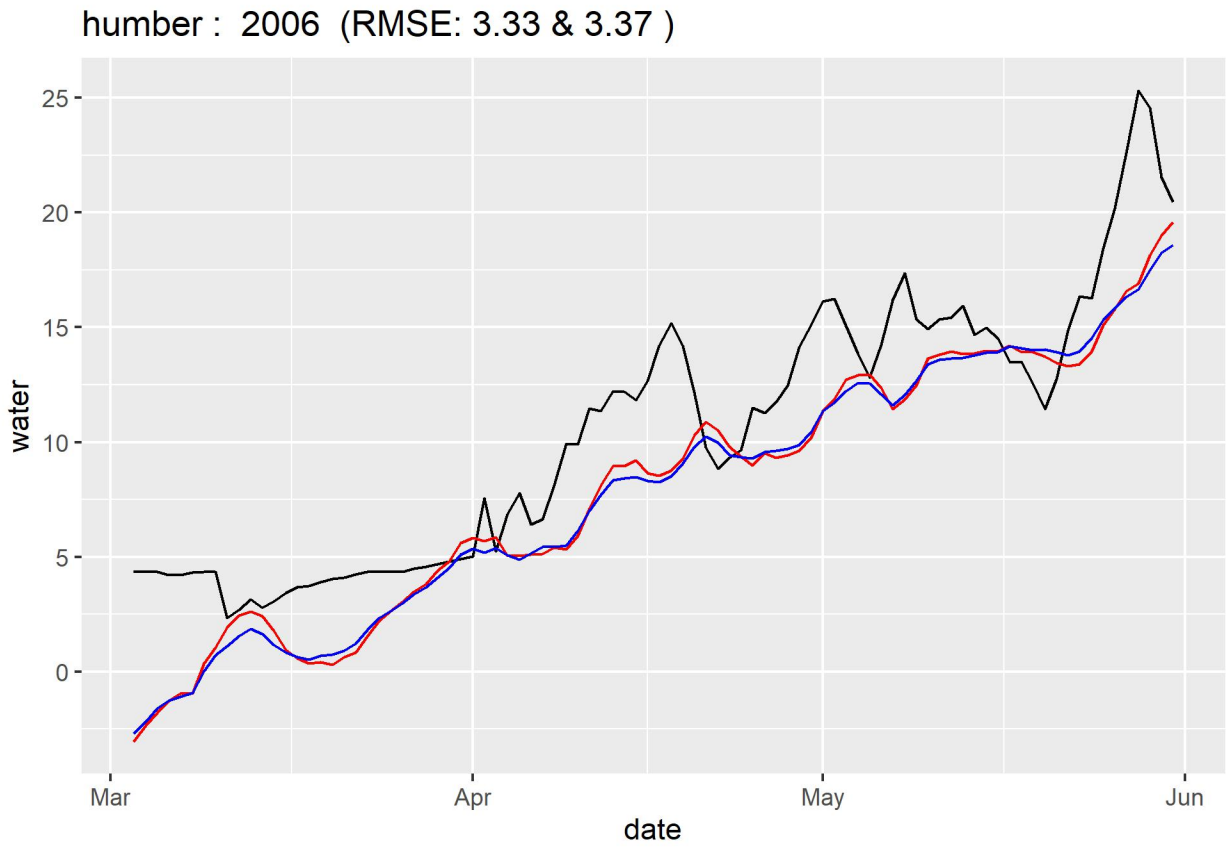
```
  print(p1)
}
```

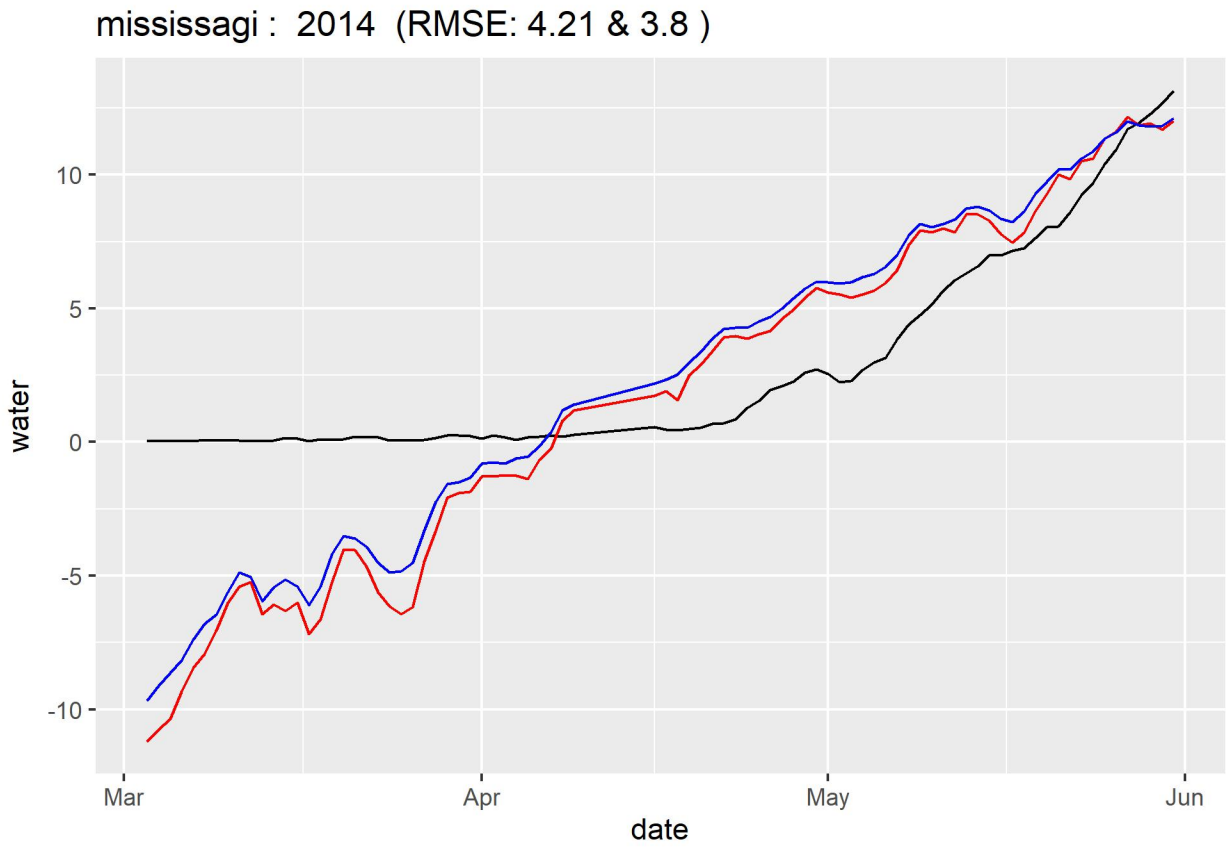


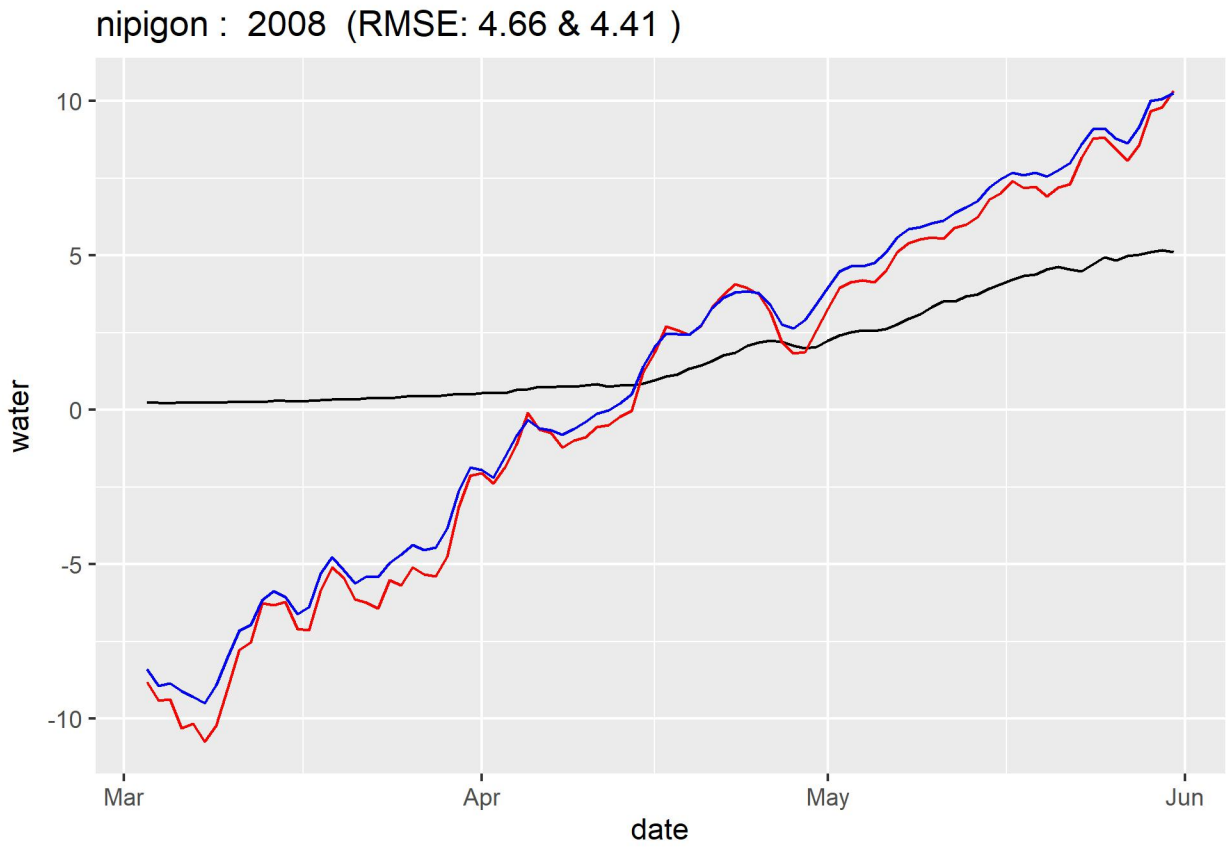

bigotter : 2014 (RMSE: 2.36 & 2.16)
water
10
0
Mar
Apr
May
Jun
date

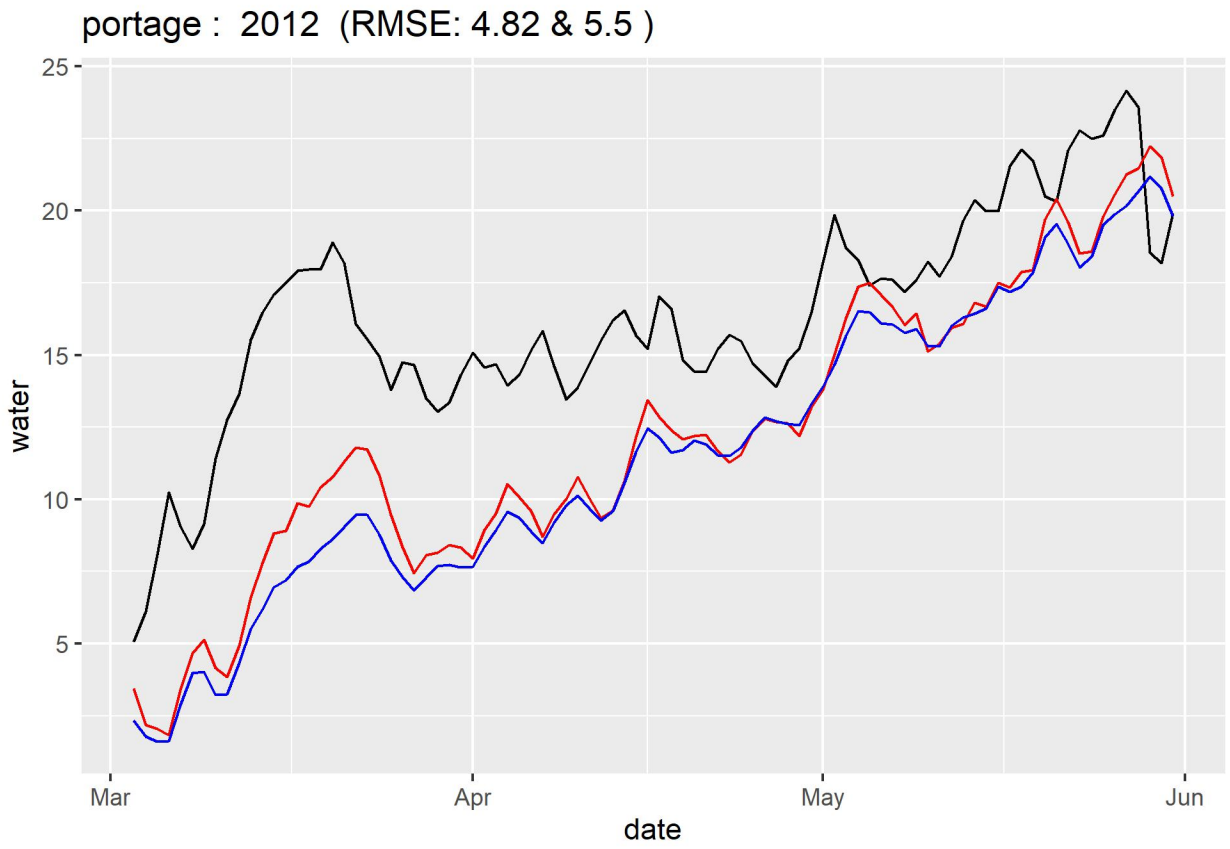

fox : 2013 (RMSE: 2.95 & 2.79)
20
15
10
5
0
-5
water
Mar
Apr
May
Jun
date

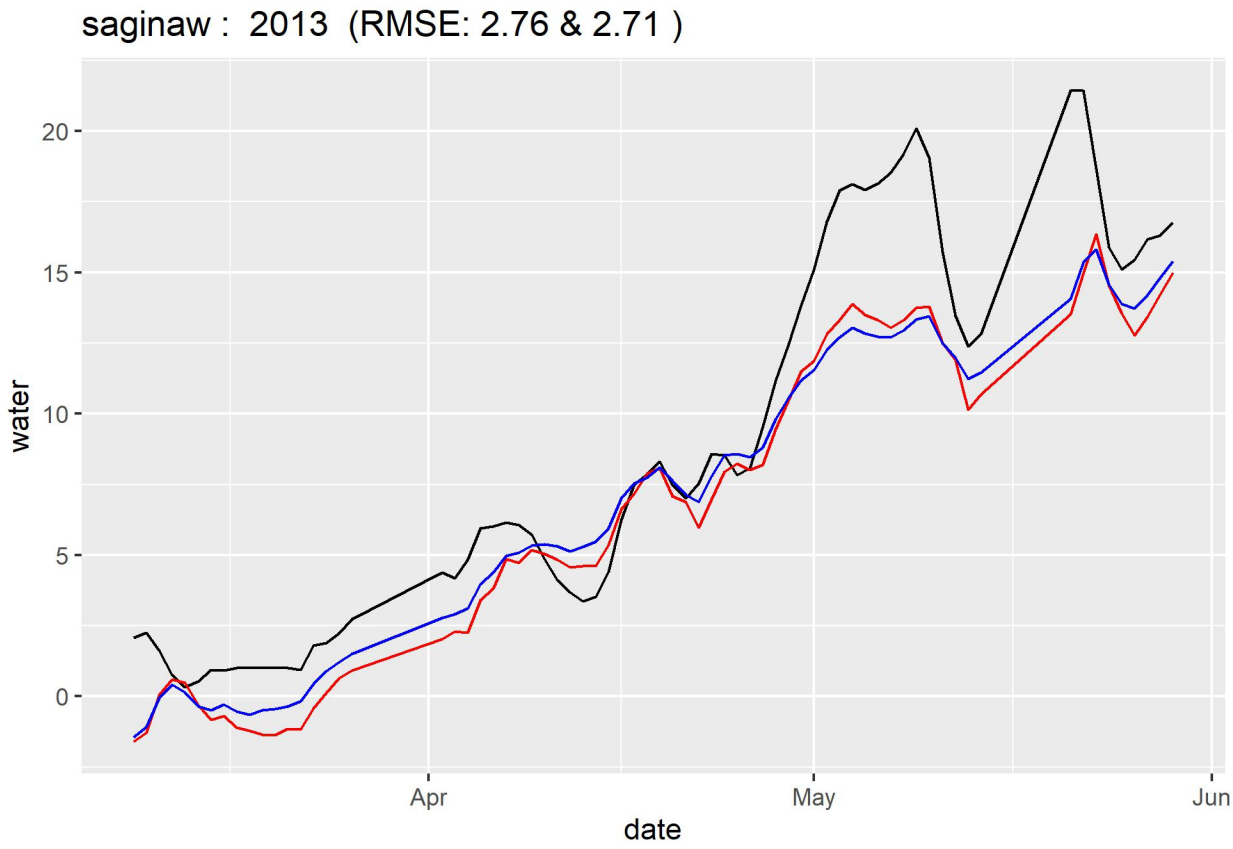

genesee : 2011 (RMSE: 2.06 & 2.18)
water
20
15
10
5
0
Mar
Apr
May
Jun
date

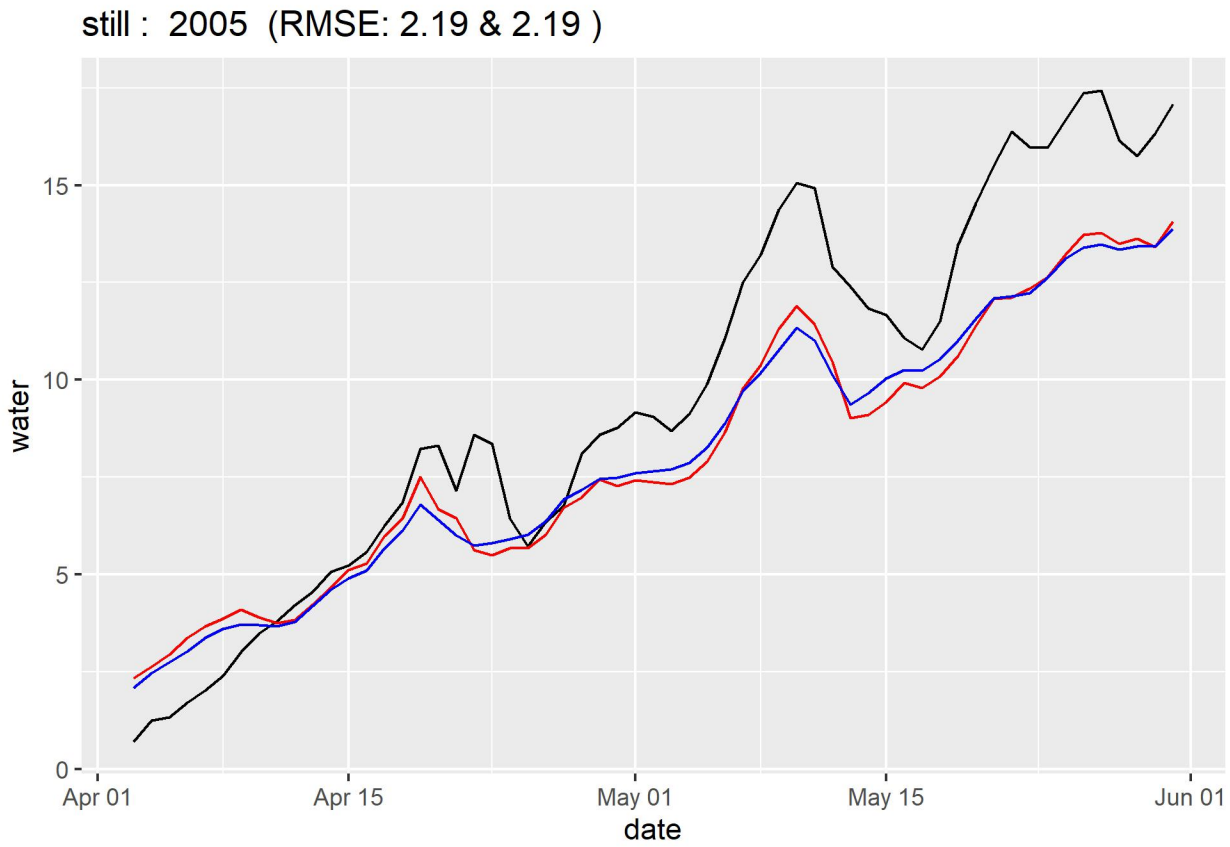

humber : 2006 (RMSE: 3.33 & 3.37)
water
25
20
15
10
5
0
Mar
Apr
May
Jun
date

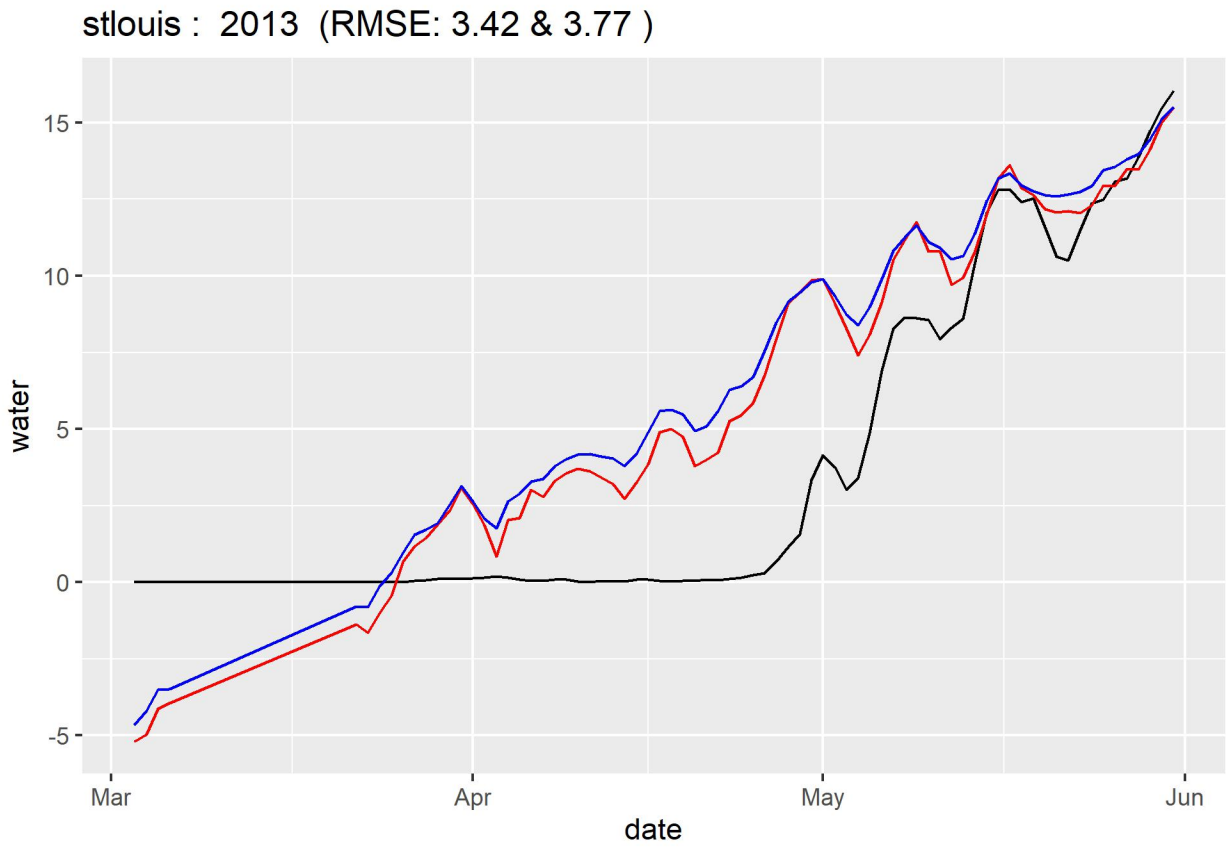

mississagi : 2014 (RMSE: 4.21 & 3.8)
water
10
5
0
-5
-10
Mar
Apr
May
Jun
date

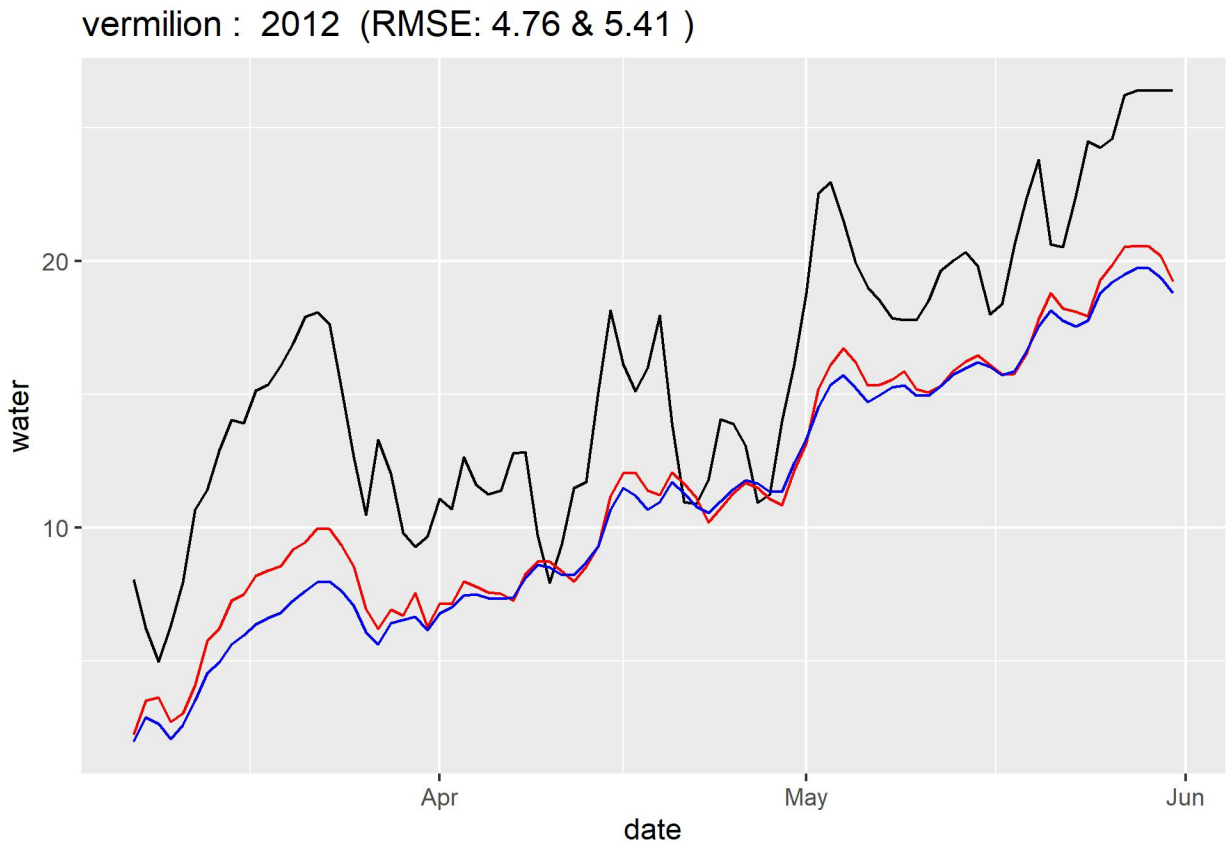

nipigon : 2008 (RMSE: 4.66 & 4.41)
10
5
0
-5
-10
water
Mar
Apr
May
Jun
date


portage : 2012 (RMSE: 4.82 & 5.5)
25
20
15
10
5
water
Mar
Apr
May
Jun
date


saginaw : 2013 (RMSE: 2.76 & 2.71)
water
20
15
10
5
0
Apr
May
Jun
date


still : 2005 (RMSE: 2.19 & 2.19)
water
15
10
5
0
Apr 01
Apr 15
May 01
May 15
Jun 01
date


stlouis : 2013 (RMSE: 3.42 & 3.77)
15
10
5
0
-5
water
Mar
Apr
May
Jun
date




```
## Ploting only the annual component
# for (loc in loc_seq) {
#   plot.df <- compare[compare$location == loc,]
#
#   pl <- ggplot(data=plot.df, aes(x=date))+
#           geom_line(aes(x=date, y=water), color = "black")+
#           geom_line(aes(x=date, y=preds.annual), color = "blue")+
#           ggtitle(paste(loc, ": ", plot.df$year))
#   print(pl)
# }


## Metrics calculation
#ModelMetrics::rmse(compare$preds, compare$obs)
#ModelMetrics::rmse(compare$preds.ar, compare$obs)

anova(residual.comp, residual.comp.ar)
```

```
##                  Model df      AIC      BIC    logLik
## residual.comp        1  6 4294.371 4323.745 -2141.186
## residual.comp.ar     2  6 2684.097 2713.471 -1336.049
```

1. Temporal autocorrelation: Set phi=0.85 largely reduced temporal autocorrelation to an acceptable level for all seasons.
2. Gives good estimation for spring, summer, and fall. In late fall and winter, the model tends to underestimate the temperature.

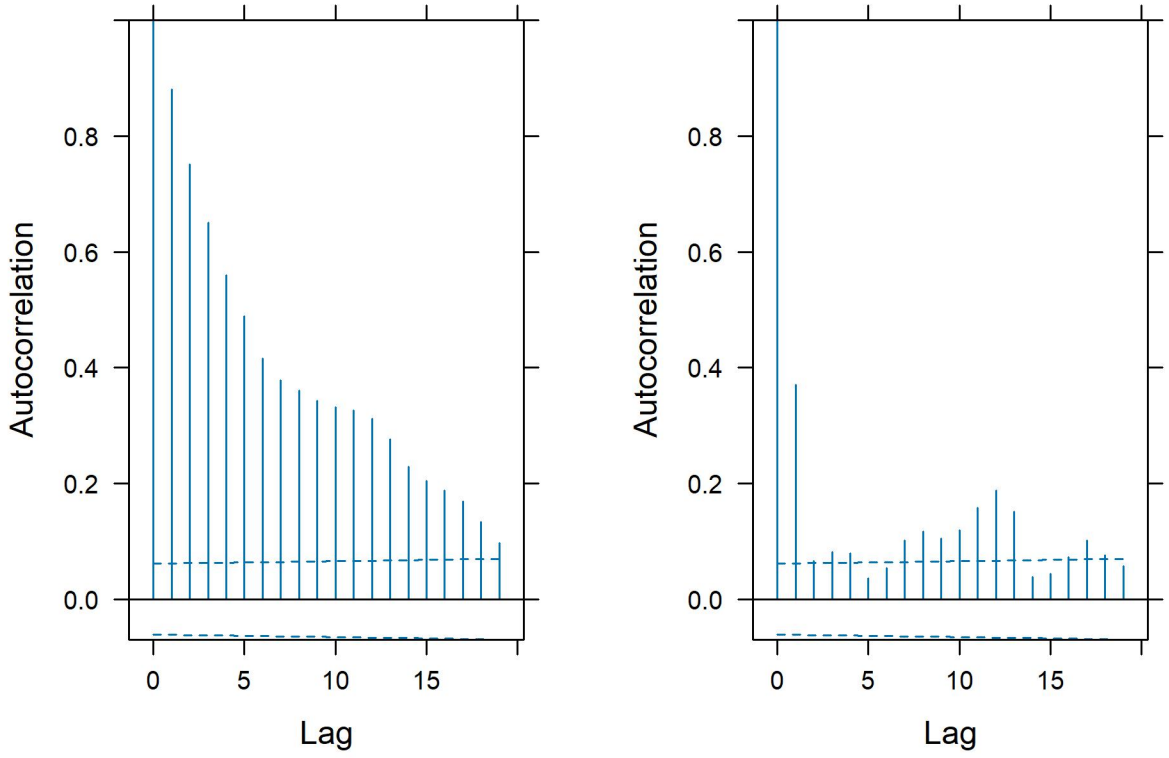
3. In general, this model cannot capture the detailed daily variation when set phi at a reasonably high level.

Lag model with flow variable

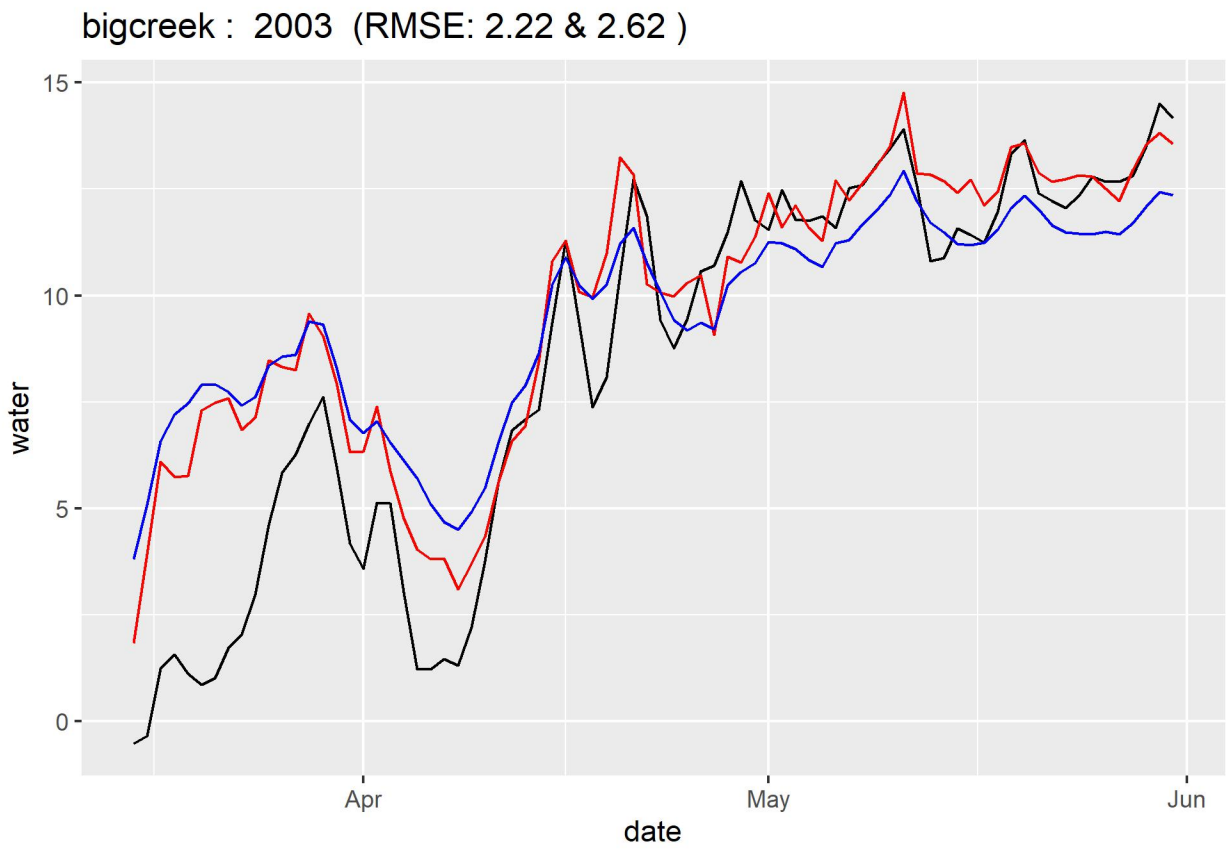
We determined in the "TEMP simple regression with flow.R" that including inverse flow as input is the best model with AIC selection.

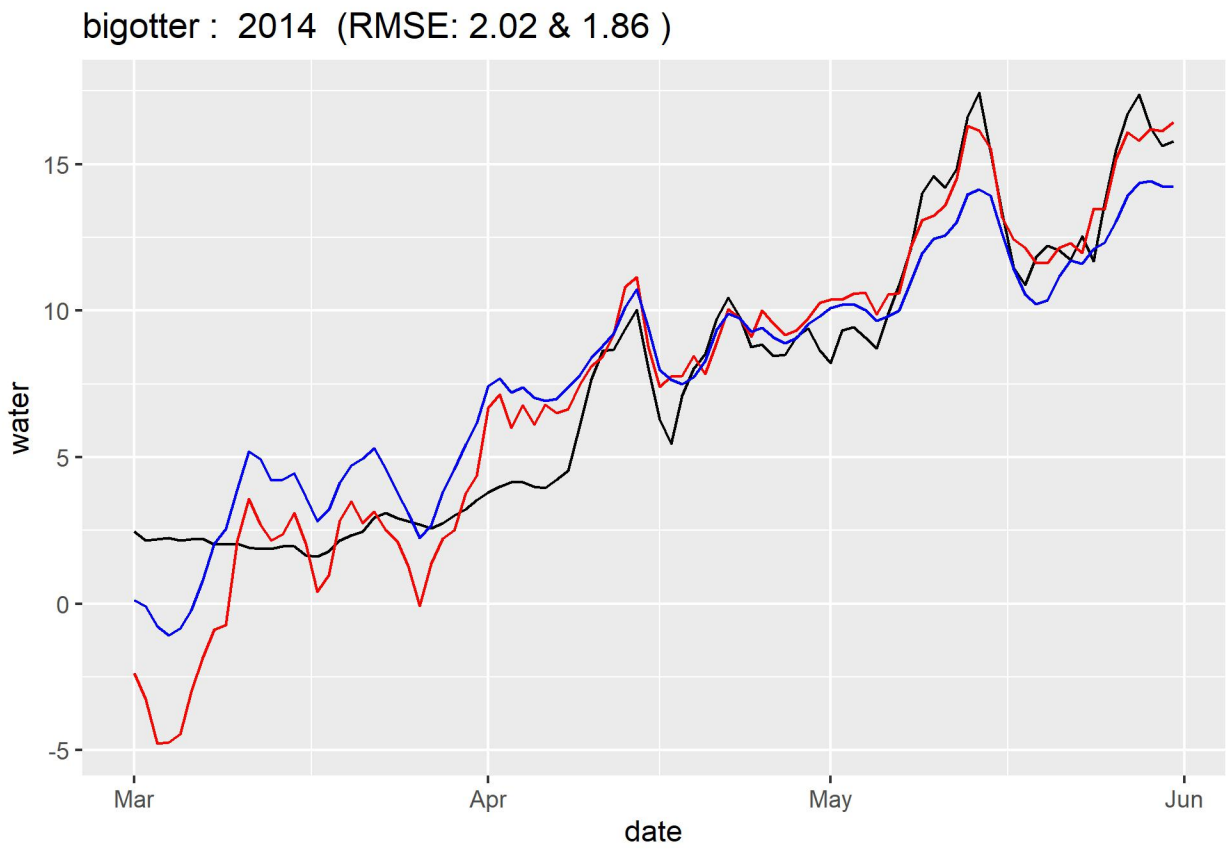
So here, let's try the multiple linear regression lag 5 model with daily inverse flow as a separate input.

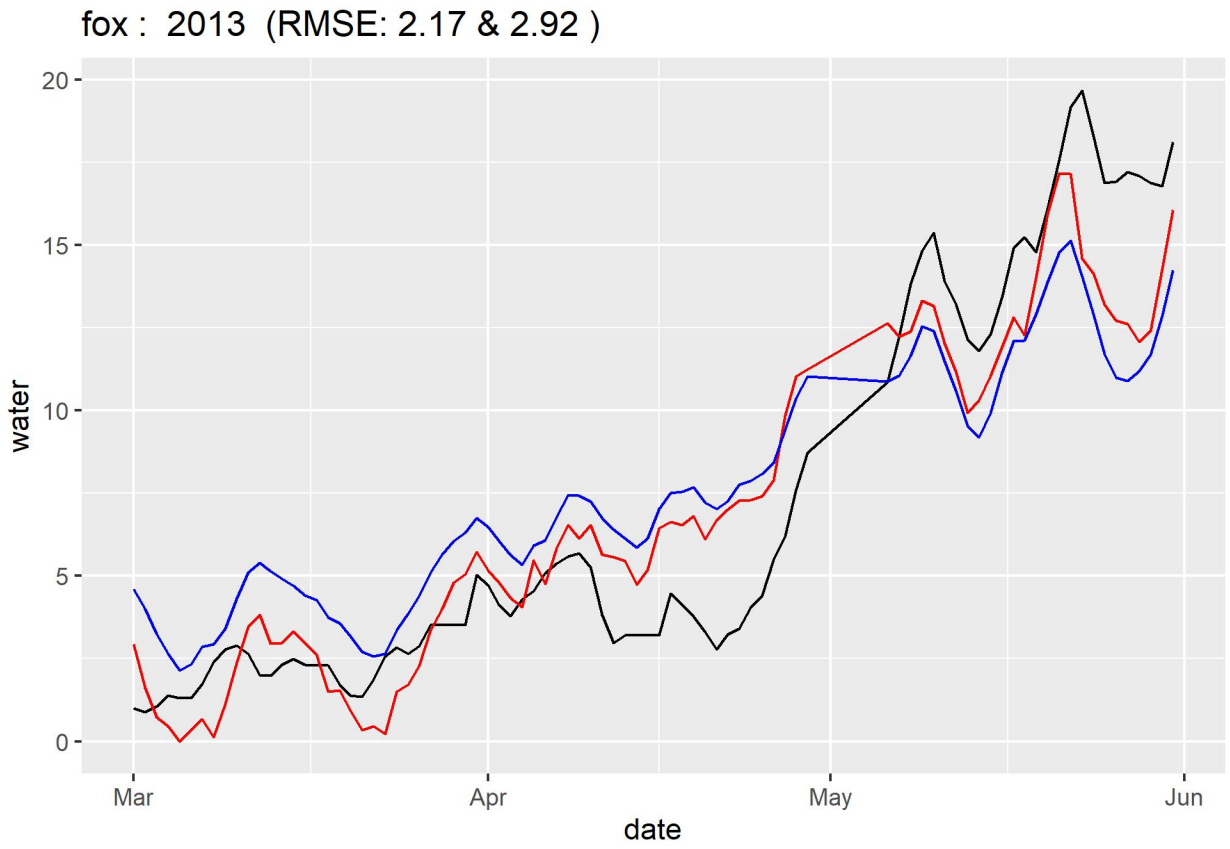
```
## Forms
form.locrandom <- water ~ air + dmean_1 + dmean_2 + dmean_3 + dmean_4 + dmean_5 + I(1/flow)
compare <- current.testing


# model training and predicting
model <- lme(form.locrandom,
             random = ~1 | location,
             na.action = na.omit, data = current.training)

compare$preds <- as.vector(predict(model, newdata = current.testing))

# Include AR term
model.ar <- lme(form.locrandom,
                random = ~1 | location, control = ctrl,
                na.action = na.omit, data = current.training,
                correlation=corAR1(form=~1|location,0.85,fixed=T))

compare$preds.ar <- as.vector(predict(model.ar, newdata = current.testing, re.form = ~1|location))


## Plot ACF
ggarrange(plot(ACF(model,resType="normalized"),alpha=0.05),
        plot(ACF(model.ar,resType="normalized"),alpha=0.05),
        nrow = 1)
```

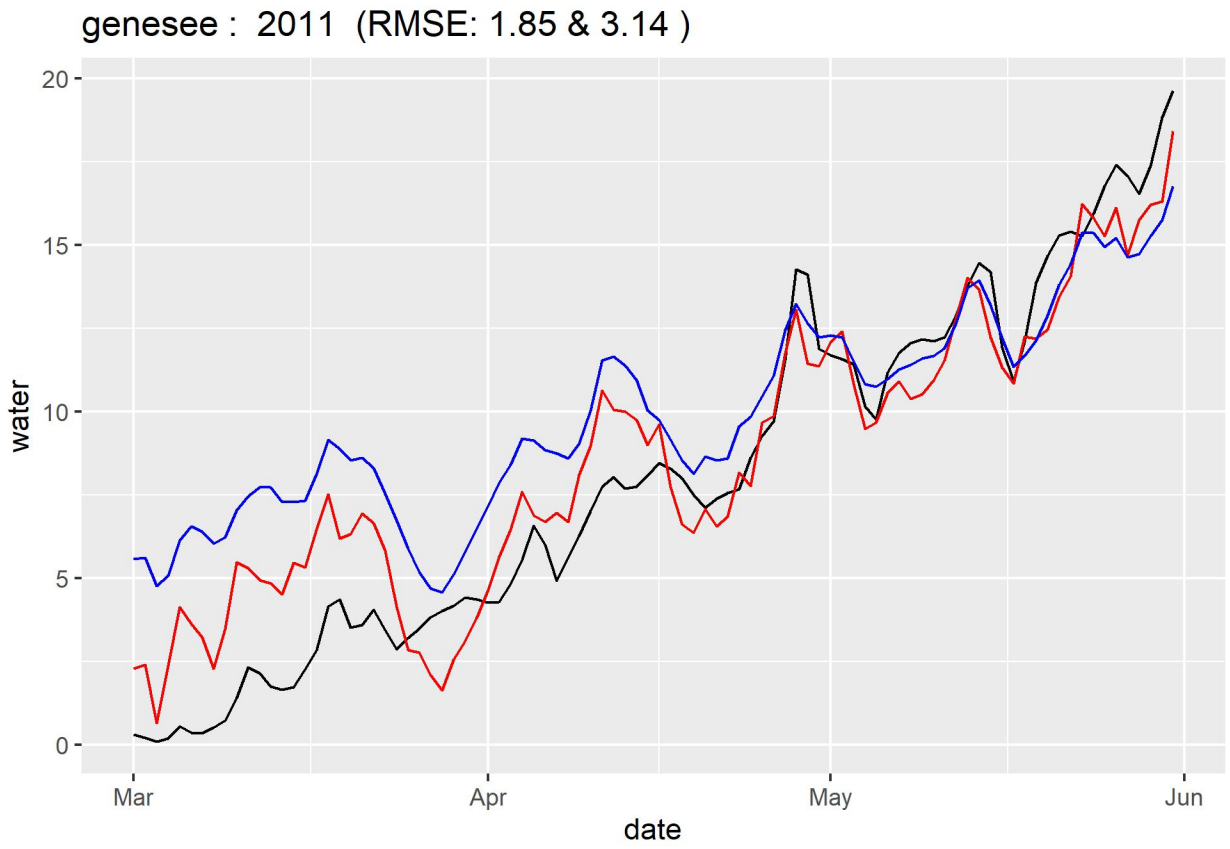


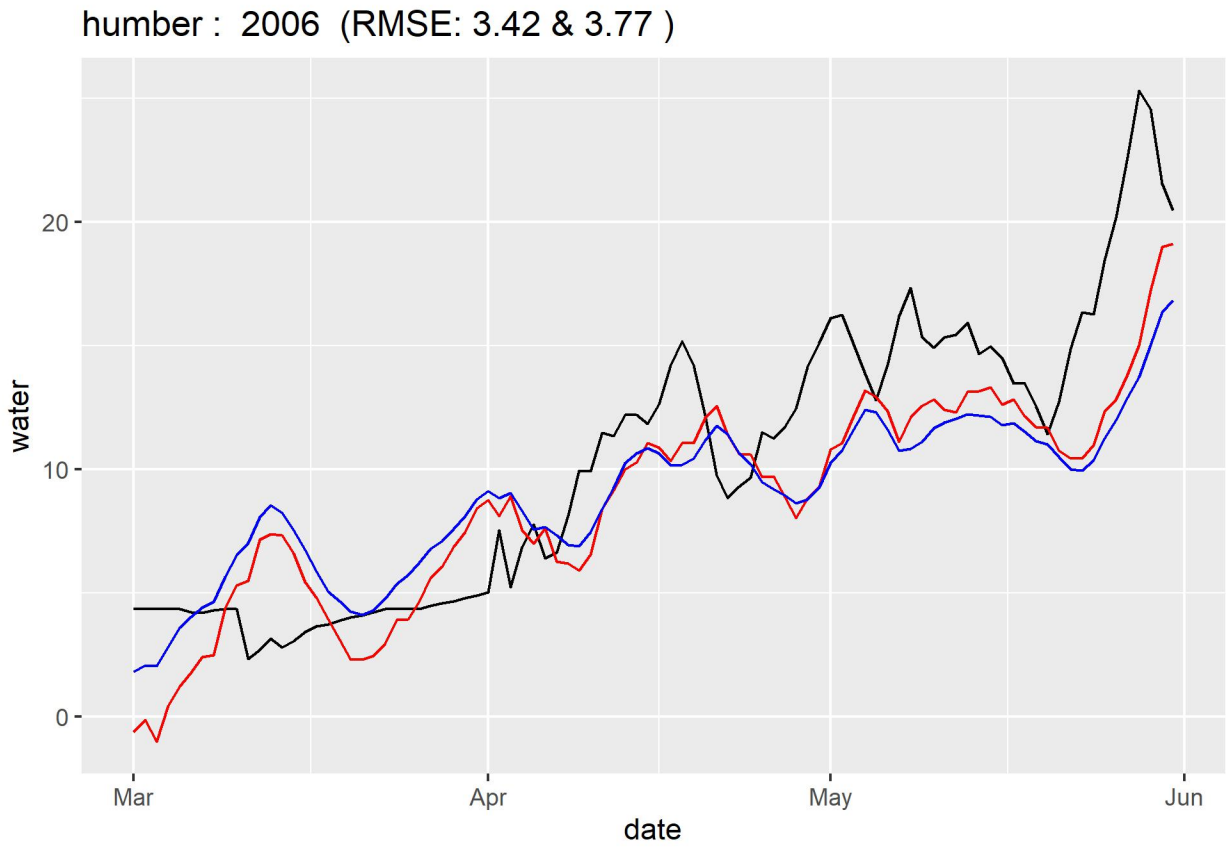

```
## Plots
for (loc in loc_seq) {
  plot.df <- compare[compare$location == loc,]
  diff <- round(sqrt(mean((plot.df$preds-plot.df$water)^2)),2)
  diff.ar <- round(sqrt(mean((plot.df$preds.ar-plot.df$water)^2)),2)

  pl <- ggplot(data=plot.df, aes(x=date))+
          geom_line(aes(x=date, y=water), color = "black")+
          geom_line(aes(x=date, y=preds), color = "red")+
          geom_line(aes(x=date, y=preds.ar), color = "blue")+
          ggtitle(paste(
            loc, ": ", plot.df$year, " (RMSE:", diff, "&", diff.ar, ")"))
  print(pl)
}
```

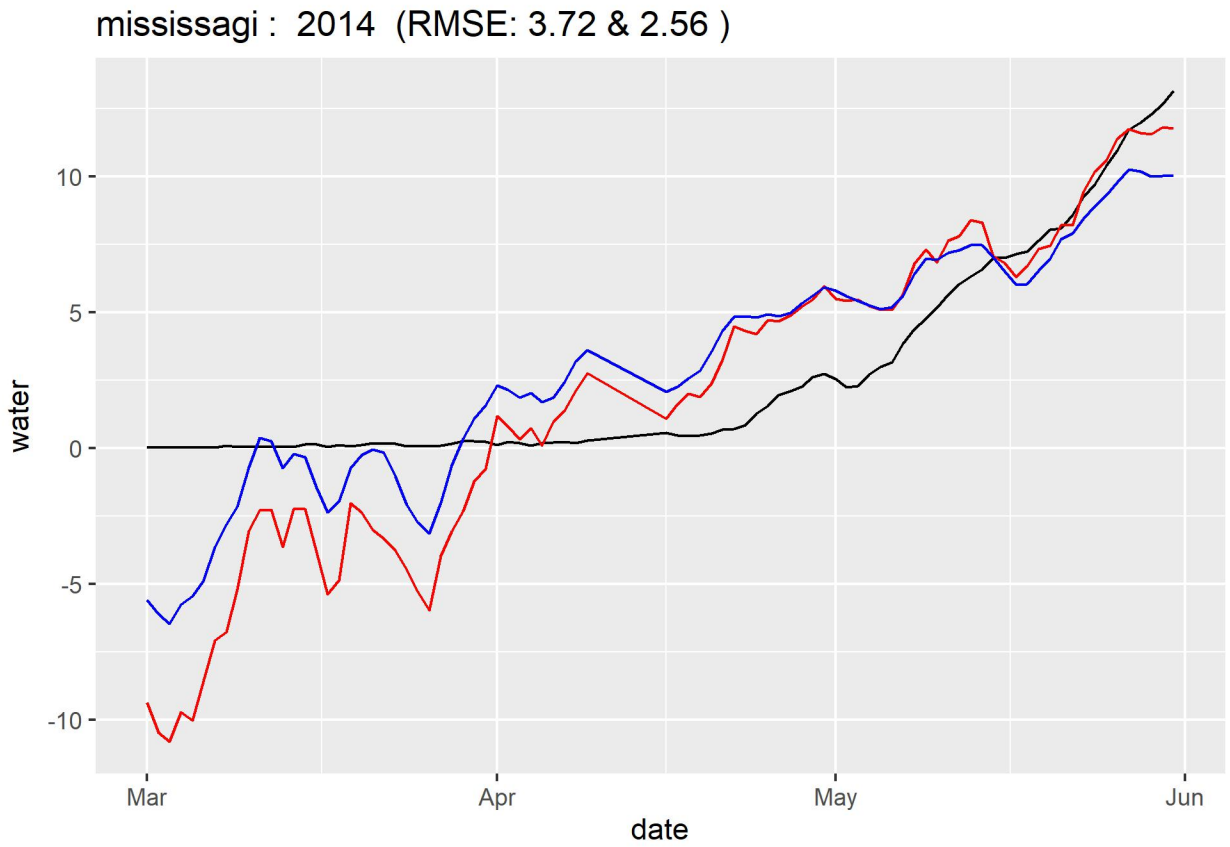

bigcreek : 2003 (RMSE: 2.22 & 2.62)
15
10
5
0
water
Apr
May
Jun
date

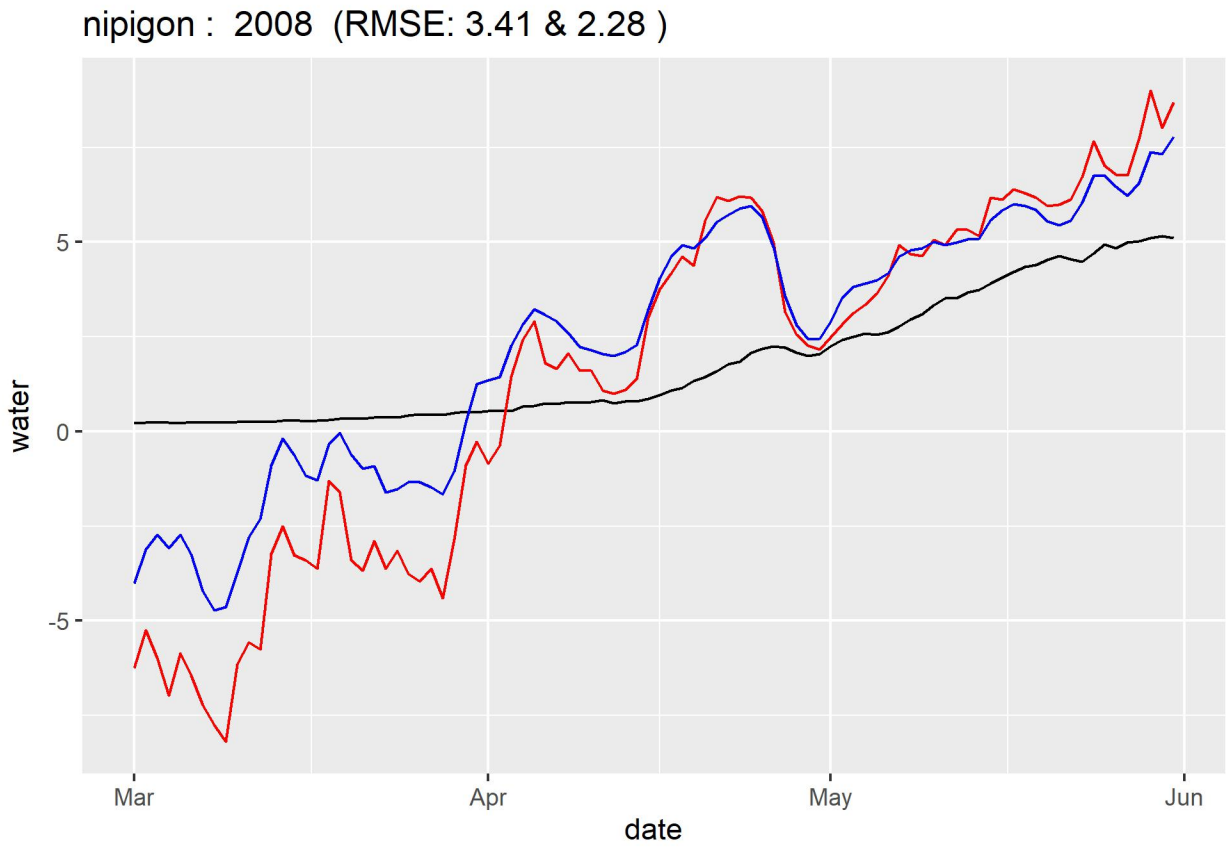

bigotter : 2014 (RMSE: 2.02 & 1.86)
water
15
10
5
0
-5
Mar
Apr
May
Jun
date

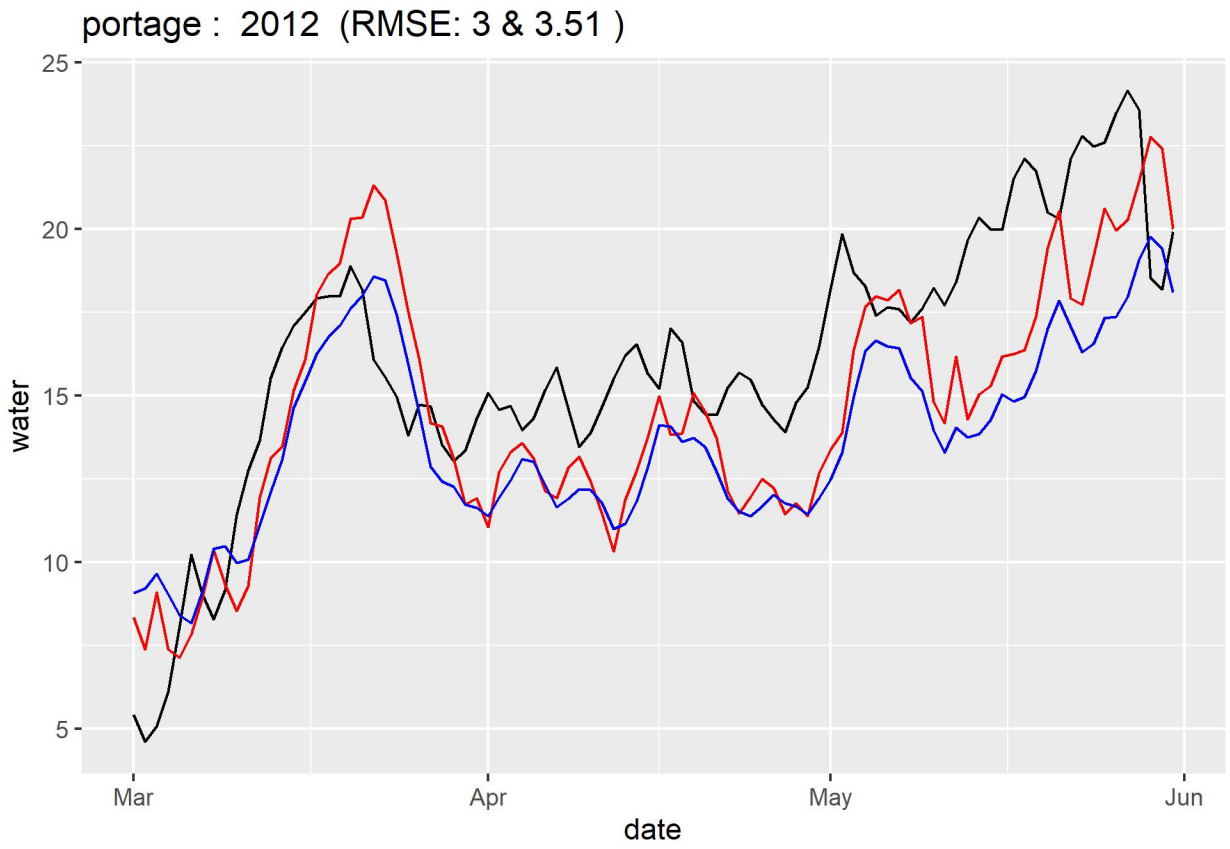

fox : 2013 (RMSE: 2.17 & 2.92)
20
15
10
5
0
water
Mar
Apr
May
Jun
date

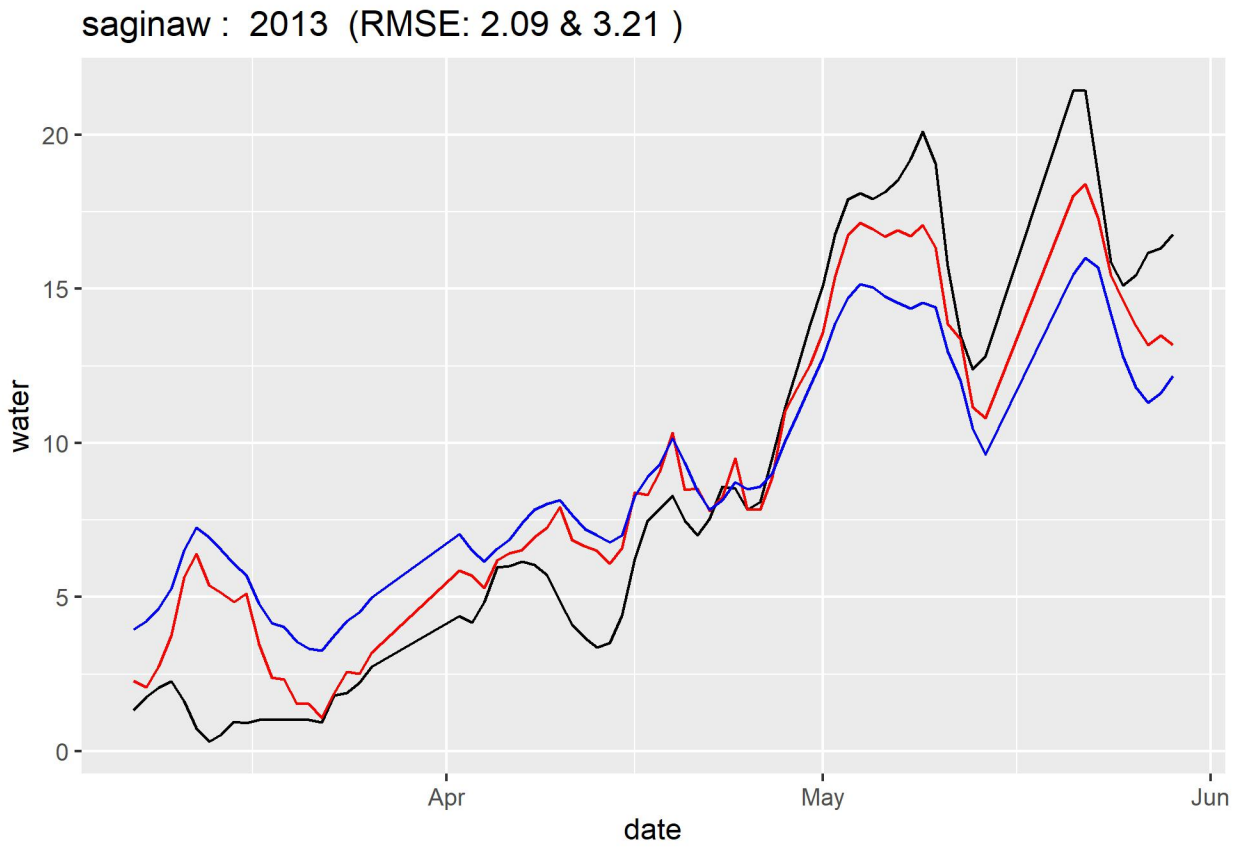

genesee : 2011 (RMSE: 1.85 & 3.14)
20
15
10
5
0
water
Mar
Apr
May
Jun
date

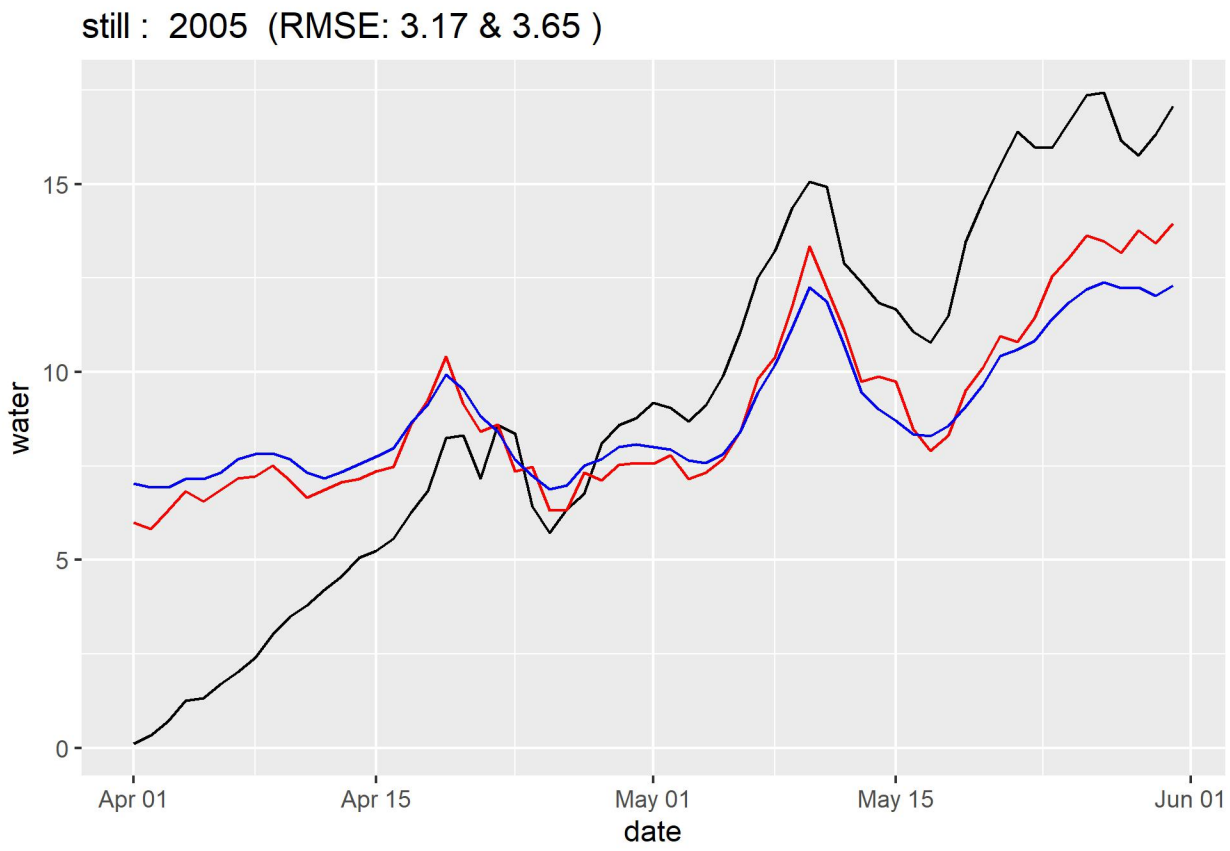

humber : 2006 (RMSE: 3.42 & 3.77)
water
20
10
0
Mar
Apr
May
Jun
date

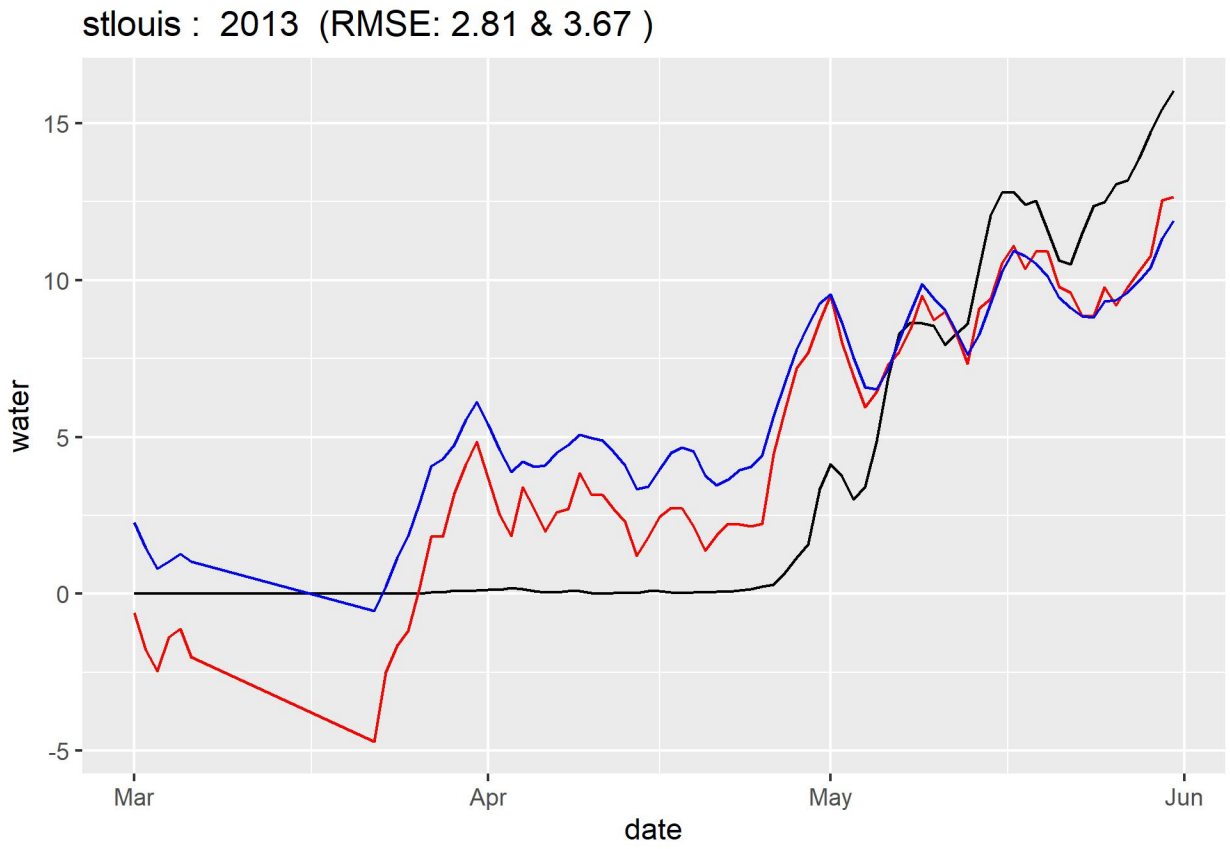

mississagi : 2014 (RMSE: 3.72 & 2.56)
water
10
5
0
-5
-10
Mar
Apr
May
Jun
date

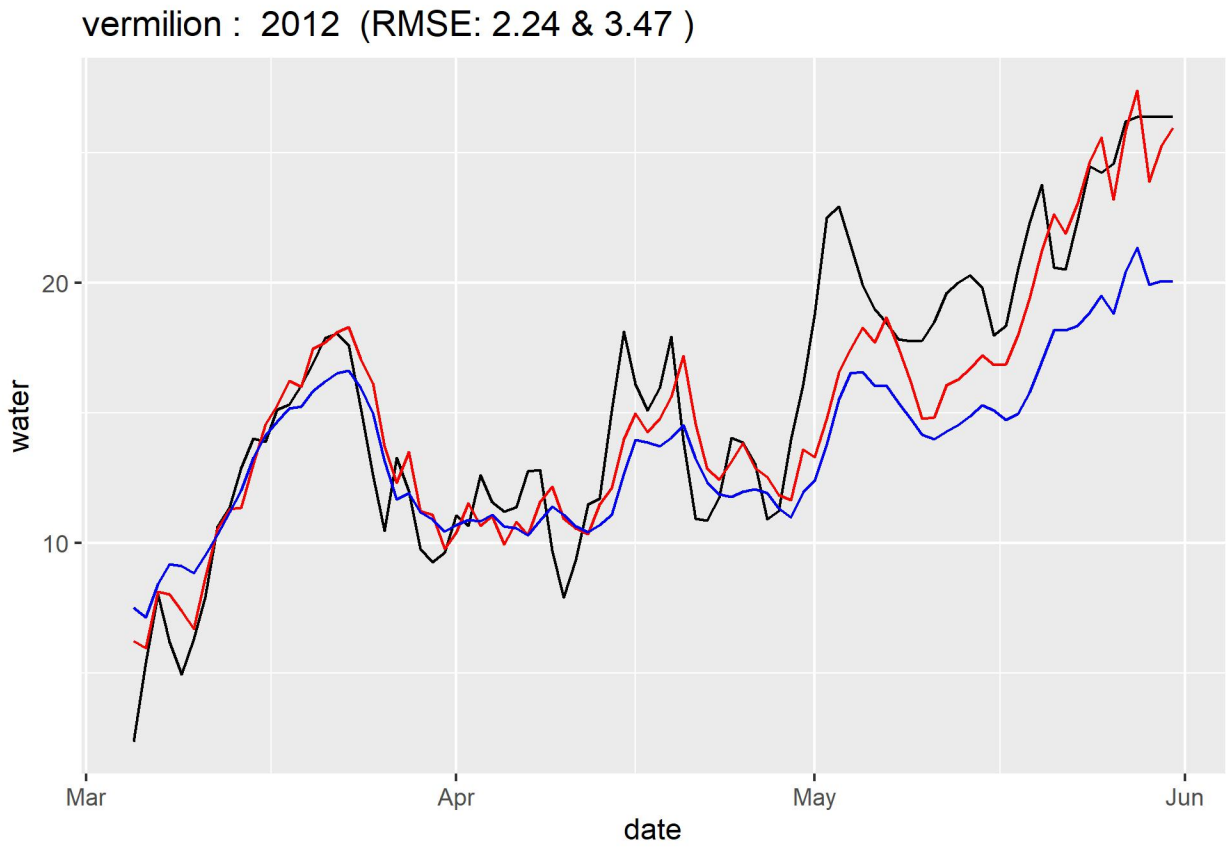

nipigon : 2008 (RMSE: 3.41 & 2.28)
water
5
0
-5
Mar
Apr
May
Jun
date


portage : 2012 (RMSE: 3 & 3.51)
25
20
15
10
5
water
Mar
Apr
May
Jun
date


saginaw : 2013 (RMSE: 2.09 & 3.21)
water
20
15
10
5
0
Apr
May
Jun
date


still : 2005 (RMSE: 3.17 & 3.65)
water
15
10
5
0
Apr 01
Apr 15
May 01
May 15
Jun 01
date


stlouis : 2013 (RMSE: 2.81 & 3.67)
15
10
5
0
-5
water
Mar
Apr
May
Jun
date




```
## Metrics calculation
#ModelMetrics::rmse(compare$preds, compare$obs)
#ModelMetrics::rmse(compare$preds.ar, compare$obs)

anova(model, model.ar)
```

```
##           Model df      AIC      BIC    logLik
## model         1 10 4805.030 4854.187 -2392.515
## model.ar      2 10 2940.742 2989.899 -1460.371
```

1. Overall, including a temporal autocorrelation AR1 correction at phi=0.85 at seasonal scale yields relatively good prediction and low ACF.
2. At annual scale (winter removed), we are unable to obtain a good predictive model while fully reducing temporal autocorrelation under significant level. Yet, it is sensible to use one (still set phi=0.85) solely for the purpose of prediction.