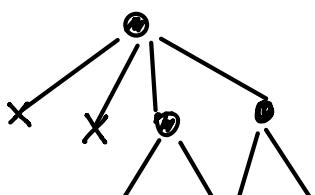
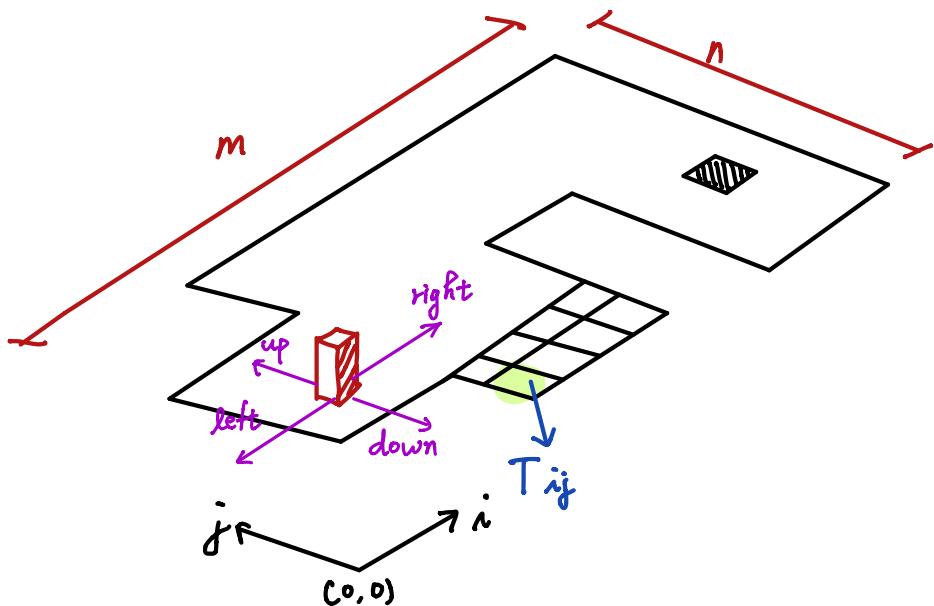


排除 Illegal Move

固定 size B , an upper bound of move $B \Rightarrow$
4 choices for each decision



Total 步數 從 1 步 開始, SAT solver can learn invalid pattern gradually



1. Set path length range :

$$\text{Distance} \leq \text{Path length } P_n \leq \text{Size of map}$$

2. For each P_n , determine variables

$$A = \{Lx, Ly, S\}$$

$$\text{Each tile } T_{ij} \Rightarrow v_{ija \in A} = \begin{cases} 1, & \text{state } A \text{ on tile } T_{ij} \text{ is valid} \\ 0, & \text{not valid} \end{cases}$$

路

$$\text{Block's State (变数)} b_{ija \in A} = \begin{cases} 1, & \text{block at the } T_{ij} \text{ with state } a \\ 0, & \text{otherwise} \end{cases}$$

Move , for each node (变数)

$$\# : 3 \times |B|$$

$$M = \{ \text{left, right, down, up} \}$$
$$move_{ija, m \in M} = \begin{cases} 1, & \text{Given block's state } b_{ija}, \text{ the player next take action } m \in M \\ 0, & \text{otherwise} \end{cases}$$

Rules

- Matching in v_{ija} and $bija$

$$\bigwedge_{i \in [m], j \in [n]} \bigwedge_{a \in A} (bija \rightarrow v_{ija}) \quad \text{略}$$

Bij on the map

- b_{ijLx} , b_{ijLy} , b_{ijS} choose one among them

$$\bigwedge_{i \in [m], j \in [n]} (b_{ijLx} \rightarrow \bigwedge_{\substack{a \in A \\ a \neq Lx}} (\overline{bija}) \wedge_{a \in A} (\overline{b_{ij+1}a}))$$

$$\bigwedge_{i \in [m], j \in [n]} (b_{ijLy} \rightarrow \bigwedge_{\substack{a \in A \\ a \neq Ly}} (\overline{bija}) \wedge_{a \in A} (\overline{b_{ij+1}a}))$$

$$\bigwedge_{i \in [m], j \in [n]} (b_{ijS} \rightarrow \bigwedge_{\substack{a \in A \\ a \neq S}} (\overline{bija}))$$

$$b_{i \text{start} j \text{start} S} = 1, b_{i \text{end} j \text{end} S} = 1$$

- Chosen $bija$ can form a single path

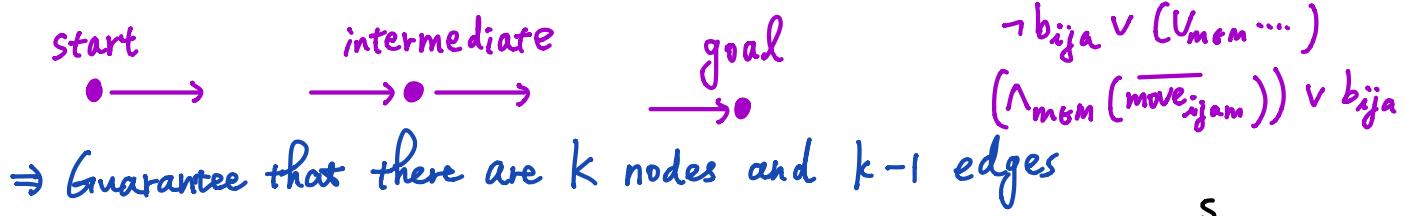
Given a node v_{ija} and its outgoing edge ε_k , incoming edge d_k

- P1 A node on the path puts its “one” outgoing edge on the path
(expect goal)
- Covered by Successor function

$$\bigwedge_{i \in [m], j \in [n], a \in A} \{ bija \leftrightarrow V_{m \in M} (move_{ijam}) \}$$

$$\bigwedge_{m \in M} (\overline{move_{i \text{end} j \text{end} S^m}})$$

$$\bigwedge_{i \in [m], j \in [n], a \in A} (\bigwedge_{m_1, m_2 \in M, m_1 \neq m_2} (\overline{move_{ijam_1}} \vee \overline{move_{ijam_2}}))$$

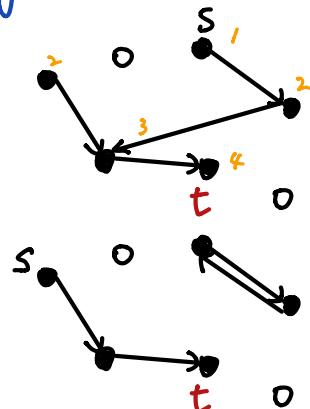


Start → $v_2 \rightarrow v_3 \rightarrow \dots \rightarrow v_k \rightarrow \text{end}$

$$p(v_2) = S(\text{start})$$

$$p(v_i) = S(p(v_{i-1})) \text{ for } i \in 3 \dots k$$

$$p(v_k) = S^{k-1}(\text{start})$$



Successor Function (Unary Encoding)

$U_{ijap} = 1$ iff v_{ija} 's position is p. $p \in 1 \dots \text{mapsize}$

△ 1-1 mapping between bija and U_{ijap} ($\# \text{ of bija} = \# \text{ of } U_{ijap}$)

$$\wedge_{i \in [m], j \in [n], ij \neq \text{End}, a \in A} (\text{bija} \leftrightarrow \vee_{p=1}^{\text{mapsize}} (U_{ijap}))$$

$$\begin{array}{c} \Rightarrow \\ \text{unique} \\ \Leftarrow \end{array} \begin{array}{l} \text{bija} (\overline{\text{bija}} \vee (\vee_{p=1}^{\text{mapsize}} (U_{ijap}))) \\ \text{bija} (\wedge_{p_1 \neq p_2} (\overline{U_{ijap_1}} \vee \overline{U_{ijap_2}})) \\ \text{bija} \wedge_{p=1}^{\text{mapsize}} (\overline{U_{ijap}} \vee b_{ij,a}) \end{array}$$

P's Continuity

- Each P maps to at most one node

$$\wedge_{p=1}^{\text{mapsize}} \left(\wedge_{i_1 \neq i_2, j_1 \neq j_2, a_1 \neq a_2} (\overline{U_{ij_1 a_1 p}} \vee \overline{U_{ij_2 a_2 p}}) \right)$$

- Recursion (State + action → Next State)

$$U_{i \text{ start} j \text{ start} S^0} = 1$$

$$U_{i \infty j, a, p} \wedge \text{move}_{i, j, a, m} \xrightarrow[m \in M]{} U_{i+1, a_2, p+1}$$

$(i, j) \neq \text{End}$

$(i, j) \neq \text{End}$

$$(U_{i j S^p}) \wedge \begin{cases} \text{stand} \\ \text{move}_{i j S} \text{ up} \longrightarrow U_{i j+1 L y} p+1 \\ \text{move}_{i j S} \text{ down} \longrightarrow U_{i j-1 L y} p+1 \\ \text{move}_{i j S} \text{ right} \longrightarrow U_{i+1 j L x} p+1 \\ \text{move}_{i j S} \text{ left} \longrightarrow U_{i-1 j L x} p+1 \end{cases}$$

$$(U_{i j L x} p) \wedge \begin{cases} \text{move}_{i j L x} \text{ up} \longrightarrow U_{i j+1 L x} p+1 \\ \text{move}_{i j L x} \text{ down} \longrightarrow U_{i j-1 L x} p+1 \\ \text{move}_{i j L x} \text{ right} \longrightarrow U_{i+1 j S} p+1 \\ \text{move}_{i j L x} \text{ left} \longrightarrow U_{i-1 j S} p+1 \end{cases}$$

$(U_{ij} L_y p) \wedge$
 move _{i,j} L_y up $\rightarrow U_{i,j+2} L_y p+1$
 move _{i,j} L_y down $\rightarrow U_{i,j-1} L_y p+1$
 move _{i,j} L_y right $\rightarrow U_{i+1,j} L_y p+1$
 move _{i,j} L_y left $\rightarrow U_{i-1,j} L_y p+1$

Some moves are INVALID!!
 depend on i, j, a

