# Explaining Default Intuitions using Maximum Entropy

## Explaining default intuitions using maximum entropy

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## Outline

- Defaults & default reasoning
- Maximum entropy & variable strength defaults
- The ME-solution and ranked model(s)
- How ME handles default intuitions
- Conclusion
  - ME as normative default reasoning?
  - Eliciting default information



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## What is a default?

- A general rule for jumping to conclusions, e.g.,

woman → ¬good_driver

racing_driver → good_driver

default connective "→" read as "usually" or "normally"

- Defaults are defeasible
  - more information may lead us to change our minds

How about Penelope_Pitstop ??

woman ^ racing_driver → good_driver ??



## How to reason using defaults?

- Need a means to systematically manipulate a set of defaults to arrive at defeasible conclusions
  - e.g. from the defaults

bird → fly

parrot → bird

  - if we know an object is a parrot, we might conclude that, by virtue of it being a bird, it can fly
  - i.e. we infer another default parrot → fly



## Extending a set of defaults

- A default reasoning system can be viewed as a way of extending a set of defaults into a larger set
  - just those defaults sanctioned by the system

| KB | Conclusions |
|---|---|
| | bird → fly |
| bird → fly | parrot → bird |
| parrot → bird | parrot → fly |
| | ...etc... |

## Systems of default reasoning

- Originally, many different approaches were taken
  - » **McCarthy's circumscription**
  - » **Reiter's default logic**
  - » **Nonmonotonic logic, inheritance networks, conditional logics...**
- Later, attempts were made to standardise
  - » **Gabbay & Makinson's inference rules for nonmonotonic systems**
  - » **Shoham's preferential logics**
  - » **Kraus, Lehman and Magidor's preferential reasoning**
  - » **also, preferential and rational consequence relations, rank-based systems, epsilon semantics, etc.**

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## Preferential reasoning

- Using this framework and applying a set of fairly uncontentious inference rules, a consensus was reached about minimal/basic default reasoning
  - » **inference rules of System P**
- Many different semantics for System P all leading to the same conclusions
  - » **preferential consequence relations, system P, probabilistic or e-semantics, possibilistic semantics**
- BUT these conclusions are extremely conservative
  - » **can't even infer** parrot $\rightarrow$ fly

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## Probabilistic or e-semantics

- Interprets a default as a statement of extremely high conditional probability, so bird $\rightarrow$ fly means

$$P(\text{fly} \mid \text{bird}) \geq 1 - e \qquad (\text{or} \quad P(\neg\text{fly} \mid \text{bird}) \leq e)$$

$$e > 0$$

- Sanctions conclusions that also have extremely high probability as $e \rightarrow 0$
  - » **exactly the same conclusions as System P**

**Need to be more adventurous!**

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## How to capture stronger conclusions justifiably?

- Use principle of maximum entropy (ME).
- Intuitively, the principle is simple
  - » **model all that is known and assume nothing about that which is unknown**
  - » **given a set of defaults, choose a distribution that is consistent with all the defaults, but otherwise as uniform as possible**
- ME-solution is both
  - » **adventurous (so leads to more conclusions) and**
  - » **justifiable (since assumes no more than is known)**

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## Applying maximum entropy to default reasoning

- ε-semantics based on probability so can apply ME
- First done by Goldszmidt, Morris & Pearl (1993)
  - » **derived unique ME solution + algorithm to find it in small class of problems**
- Bourne & Parsons (1999) adjusted underlying semantics
  - » **found more general solution that allows greater expressiveness of defaults + algorithm to find it**
  - » **multiple solutions in cases of redundancy**

Both cases, result in a ranked model (or models)

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## New semantics: Adding strengths to defaults

| Old semantics | New semantics | Differences |
|---|---|---|
| $a_i \rightarrow b_i$ | $a_i \rightarrow_{si} b_i$ | $s_i$ is a (specified) relative strength |
| $P(b_i \mid a_i) \geq 1 - \varepsilon$ | $P(b_i \mid a_i) = 1 - O_i(\varepsilon^{si})$ | Constraints are equalities |
| $P(\neg b_i \mid a_i) \leq \varepsilon$ | $P(\neg b_i \mid a_i) = O_i(\varepsilon^{si})$ | $O_i$ is an (unspecified) convergence function |

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## Applying maximum entropy to default reasoning

- Find that distribution with maximum entropy subject to the constraints imposed by the defaults

$$H[P] = -\Sigma_m P(m)\log P(m)$$

- As e → 0, find the order of magnitude abstraction of this distribution
  - » **the ME-solution or ME-ranked model**

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## Implication of assumptions

- By being vague about what the input defaults represent, we can nevertheless obtain a concrete and unique result in many cases
  - » **i.e. can obtain firm results without considering the convergence functions themselves, just their order of magnitude**
  - » **BUT sometimes this vagueness leads to multiple solutions**
  - » **AND since we assume the constraints are satisfied as equalities, it's possible they could be inconsistent and lead to no solution (not all strength assignments are valid)**

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## Problems with redundancy

- Troublesome cases are caused by defaults being redundant (i.e. ME-entailed by other defaults)
  - » **in such a case, constraints may not be satisfied and model breaks down**
- By identifying and ignoring redundant defaults the ME-solution can again be computed
  - » **[though this can be subjective since under the revised semantics there may be several candidates for redundancy]**

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## Unique ME-solutions

- When all goes well, obtain a unique ME-solution/ranked model, based on the relative strengths assigned to defaults.
  - » **each default has an ME-rank (different from its strength)**
  - » **ranked model assigns a rank to each possible world being the sum of ranks of those defaults it falsifies (i.e. contradicts)**
- ME-ranking determines defaults belonging to the extension
  - » **a default is ME-entailed if its minimal verifying model is lower than minimal falsifying model**
  - » **also can determine degree of entailment (or strength)**

» **some defaults ME-entailed under *any* strength assignment (we term this uncontroversial entailment)**

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## Answering queries using ME-ranking

**knowledge base:**

r1: bird => fly

r2: parrot => bird

**query:**

parrot => fly

is ME-entailed to degree 1

| | $r_1$ | $r_2$ | ME | q? |
|---|---|---|---|---|
| ¬bird^¬fly ^¬parrot | . | . | 0 | . |
| ¬bird^¬fly ^ parrot | . | f | 1 | f |
| ¬bird^ fly ^¬parrot | . | . | 0 | . |
| ¬bird^ fly ^ parrot | . | f | 1 | v |
| bird^¬fly ^¬parrot | f | . | 1 | . |
| bird^¬fly ^ parrot | f | v | 1 | f |
| bird^ fly ^¬parrot | v | . | 0 | . |
| bird^ fly ^ parrot | v | v | 0 | v |



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## Benchmark patterns of default reasoning

- Without a general theory of default reasoning, what are 'correct' patterns?
- Soundness of systems can only be tested against (subjective) intuitions.
- No one system has handled all the benchmark problems correctly.
- Despite this, let's look at the common default intuitions and see whether the ME can help us understand them.



## How ME handles benchmark default examples

- Property inheritance, transitivity and irrelevance
  - » **do parrots fly?**
- Specificity and conflicting inheritance
  - » **do penguins fly?**
  - » **was Nixon a pacifist?**
- Exceptional inheritance
  - » **do emus have wings?**
- Multiple inheritance
  - » **do marine chaplains drink beer?**



## Property inheritance & transitivity

- Main use of default reasoning is to encode general rules concisely (but they must be defeasible).

| KB | Conclusion? |
|---|---|
| bird $\rightarrow$ fly<br>parrot $\rightarrow$ bird | parrot $\rightarrow$ fly |

- This is harder than it looks!
  - » **e.g., not a conclusion of preferential reasoning.**

(though parrot ^ bird $\rightarrow$ fly is)



## Other systemic approaches

- But since defaults can be overridden, when exactly should property inheritance occur?

Default logic: unless it leads to inconsistency

Circumscription: unless there is an abnormality

- Unfortunately, this leads to multiple extensions and/or need for extra defaults defining what is meant by abnormal (can be an endless task!).

# Explaining Default Intuitions using Maximum Entropy

## Transitivity usually happens under ME

| KB | ME-consequence |
| --- | --- |
| bird $\rightarrow_{s1}$ fly<br>parrot $\rightarrow_{s2}$ bird | parrot $\rightarrow_{min(s1,s2)}$ fly |

- Inheritance normally occurs uncontroversially (i.e., regardless of strengths assigned).
- Strength of inference determined by 'weakest link'.
- No need to impose it, so no unwanted side effects.

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## Irrelevance

- Related problem of how to ignore irrelevant information.

| KB | Conclusion? |
| --- | --- |
| bird $\rightarrow$ fly<br>parrot $\rightarrow$ bird | parrot ^ red $\rightarrow$ fly |

- This *is* handled correctly by some systems but is a problem for preferential reasoning.
- But is also an uncontroversial ME-consequence.

| ME-consequence |
| --- |
| parrot ^ red $\rightarrow$ fly |

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## Specificity and conflicting inheritance

- Defaults may lead to contradictory conclusions
  - » **directly (ambiguity)**
  - » **indirectly (specificity)**
- When should inheritance be blocked?

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## Ambiguity: preservation or resolution?

| KB | Conclusion? |
| --- | --- |
| quaker $\rightarrow_{s1}$ pacifist<br>republican $\rightarrow_{s2}$ ¬pacifist | quaker ^ republican $\rightarrow$ ??? |

- Nixon was both a Republican and a Quaker.
- Should either default take priority or should no conclusion be drawn?
- Under ME, result depends on the strengths of the defaults, the stronger one dominating.

| ME-consequence |
| --- |
| quaker ^ republican $\rightarrow_{s1-s2}$ pacifist (s1>s2)<br>quaker ^ republican $\rightarrow_{s2-s1}$ ¬pacifist (s2>s1) |

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## Specificity

| KB | Conclusion? |
| --- | --- |
| bird $\rightarrow_{s1}$ fly<br>penguin $\rightarrow_{s2}$ bird<br>penguin $\rightarrow_{s3}$ ¬fly | penguin ^ bird $\rightarrow$ ??? |

- Main argument in favour of preferential reasoning systems is that they correctly preserve specificity (and so does ME since it extends such systems).
- Structural prioritisation of defaults carries through and conclusions hold regardless of strength assignments.

| ME-consequence |
| --- |
| penguin ^ bird $\rightarrow_{s3}$ ¬fly |

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## Exceptional inheritance

| KB | Conclusion? |
| --- | --- |
| bird $\rightarrow$ fly<br>emu $\rightarrow$ bird<br>emu $\rightarrow$ ¬fly<br>bird $\rightarrow$ wings | emu $\rightarrow$ wings |

- When should inheritance to exceptional subclasses occur?
- Very difficult behaviour to obtain
  - » **blocking abnormal properties also blocks normal ones**

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## Exceptional inheritance

- Also, not easy to define behaviour required
  - » **e.g. how exceptional can a subclass be before we doubt its classification rather than its inherited properties?**
- Intuition here is that objects belonging to the same class should be similar in all features that define the class
  - » **as many typical features as possible should be inherited**

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## Exceptional inheritance under ME

| KB | ME-consequence |
|---|---|
| bird $\rightarrow_{s1}$ fly<br>emu $\rightarrow_{s2}$ bird<br>emu $\rightarrow_{s3}$ ¬fly<br>bird $\rightarrow_{s4}$ wings | emu $\rightarrow_{min(s2,s4)}$ wings |

- A unique ME-solution exists, and result is uncontroversial
- BUT strength of ME-entailment depends on which of emu $\rightarrow_{s2}$ bird and bird $\rightarrow_{s4}$ wings is weaker.

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## Exceptional inheritance under ME

- While ME gets it 'right' for this simple example, it throws up an interesting issue…

| |
|---|
| bird $\rightarrow_{s1}$ fly<br>emu $\rightarrow_{s2}$ bird<br>bird $\rightarrow_{s4}$ wings<br>emu $\rightarrow_{min(s2,s4)}$ wings |

- » **minimal falsifying model falsifies**
  - – **either** emu $\rightarrow$ bird
  - – **or** bird $\rightarrow$ fly **and** bird $\rightarrow$ wings
- » **which of these is relevant depends on strengths**
- » **emus without wings, while abnormal may be more normal than emus that are not birds**

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## Exceptional inheritance under ME

- So are wingless emus birds?
  - » **Is** emu ^ ¬wings $\rightarrow$ bird **ME-entailed?**

```
ME(emu ^ ¬wings ^  bird) = s1 + s4
ME(emu ^ ¬wings ^ ¬bird) = s1 + s2
```

  - » **depends on how strongly emus are birds relative to birds having wings**
  - » **ME balances these considerations**
  - » **if they are equal, no conclusion can be reached**
  - » **otherwise either default or converse can be entailed – it's a controversial consequence**

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## Multiple inheritance

- As seen in the 'Nixon Diamond', some cases involve inheriting from different sources leading to conflict and confusion.
- As problems get bigger, it is practically impossible to determine what the 'correct' inferences are.
- In the past this has lead to much debate and controversy…[cf. OO programming]
- The following example has been much discussed...

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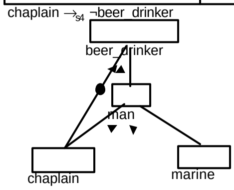
## Do marine chaplains drink beer?

| KB | Conclusion? |
|---|---|
| chaplain $\rightarrow_{s1}$ man<br>marine $\rightarrow_{s2}$ man<br>man $\rightarrow_{s3}$ beer_drinker<br>chaplain $\rightarrow_{s4}$ ¬beer_drinker | marine ^ chaplain $\rightarrow$ ??? |




- » **unique ME-solution**
- » **no they don't (usually).**
- » **really it's just a simple case of specificity**

| ME-consequence |
|---|
| marine ^ chaplain $\rightarrow_{s4}$ ¬beer_drinker |

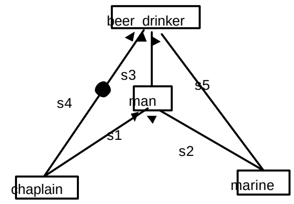
## Do marine chaplains drink beer?

- But aren't marines usually heavy drinkers?
- Need to model this by adding a new default…

| KB | Conclusion? |
|---|---|
| chaplain $\rightarrow_{s1}$ man | |
| marine $\rightarrow_{s2}$ man | |
| man $\rightarrow_{s3}$ beer_drinker | marine ^ chaplain $\rightarrow$ ??? |
| chaplain $\rightarrow_{s4}$ ¬beer_drinker | |
| marine $\rightarrow_{s5}$ beer_drinker | |



## Do marine chaplains drink beer?






## Do marine chaplains drink beer?

- Now the problem potentially contains redundancy
  - » **if strength of new default is lower than it was previously ME-entailed, it can't be a constraint and therefore doesn't affect solution**
  - » **if same strength, multiple solutions possible**
  - » **if stronger, unique solution, but the query is now a controversial ME-consequence**

**ME(chaplain ^ marine ^ ¬beer_drinker) = s3 + s5 – min(s2,s3)**

**ME(chaplain ^ marine ^ beer_drinker) = s4**



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## ME as normative default reasoning?

- Main problem with default reasoning is that people can't always agree on what correct inferences are.
- ME matches intuitions in simple cases and demonstrates an intuitively acceptable way of examining complex ones.
- Can be used as a means of disambiguating and resolving arguments, since it only uses the information that has been supplied and not hidden semantics of propositions.



## Eliciting default information

- Remember that ME assumes just what is known and uniformly distributes rest of the uncertainty.
- So can be used to help clarify our own biases
  - » **e.g. in multiple inheritance case, adding the extra default helped us to better understand the problem**
- If we can't accept a particular inference sanctioned by ME, it may be that we are loading a proposition with extra semantic information
  - » **can use ME to help design default knowledge bases**

## Are high-school dropouts usually employed?

- From a set of defaults
  adult → employed
  high_school_dropout → adult
- Reiter & Criscuolo (1983) were reluctant to accept the default conclusion
  high_school_dropout → employed
  - » **implies they must have had an additional constraint**
  - » **(even though their constraint was in fact to remain agnostic about the employment status of high-school dropouts)**

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## ME as a benchmark system

- ME-reasoning is intractable (& likely to remain so)
- But if ME is normative, it can be used in comparison with other default reasoning systems to examine what biases they contain
  - » **understanding biases of other systems may lead to their acceptance under certain conditions**
  - » **e.g. can show that lexicographic entailment is a form of ME-reasoning, whereas Systems Z/Z+ are not**

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**Thank you!**

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http://www2.elec.qmul.ac.uk/~rach/drs.html

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