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Assignment 10

14.

Inf = infinity

Italicized = Already a part of T

Step	V(T)	E(T)	F	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)	L(z)
0	{a}	empty	{a}	**0**	inf	inf	inf	inf	inf	inf	inf
1	{a}	empty	{b,e}	*0*	**1**	inf	inf	4	inf	inf	inf
2	{a,b}	{{a,b}}	{c,e,f}	*0*	*1*	**2**	inf	4	8	inf	inf
3	{a,b,c}	{{a,b},{b,c}}	{d,e,f, g}	*0*	*1*	*2*	**3**	4	8	10	inf
4	{a,b,c,d}	{{a,b},{b,c},{c,d}}	{e,z}	*0*	*1*	*2*	*3*	**4**	8	10	23
5	{a,b,c,d,e}	{{a,b},{b,c},{c,d}, {a,e}}	{f,z}	*0*	*1*	*2*	*3*	*4*	**5**	10	23
6	{a,b,c,d,e,f}	{{a,b},{b,c},{c,d}, {a,e}, {e,f}}	{g,z}	*0*	*1*	*2*	*3*	*4*	*5*	**6**	23
7	{a,b,c,d,e,f, g}	{{a,b},{b,c},{c,d}, {a,e}, {e,f}, {f,g}}	{z}	*0*	*1*	*2*	*3*	*4*	*5*	*6*	**7**
Summary	{a,b,c,d,e,f, g,z}	{{a,b},{b,c},{c,d}, {a,e}, {e,f}, {f,g},{g,z}}	{}	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*

The process can now terminate because Z is an element of V(t). **The shortest path from A to Z is of length 7, and consists of the path: A-> E-> F->G->Z**

15. The graph of exercise 9 with a = a and z = f. This means the endpoint we are trying to get to is vertex f. Italicized = Already a part of T

Step	V(T)	E(T)	F	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)
0	{a}	empty	{a}	**0**	infinity	infinity	infinity	infinity	infinity	infinity
1	{a}	empty	{b,e,g}	*0*	3	infinity	infinity	**3**	infinity	4

L(b) and L(e) are equal, so I just choose one at random.

Step	V(T)	E(T)	F	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)
2	{a,e}	{{a,e}}	{b,c,d,f,g}	*0*	**3**	10	14	*3*	7	4
3	{a,b,e}	{{a,e},{a,b}}	{c,g}	*0*	*3*	10	14	*3*	7	**4**
4	{a,b,e,g}	{{a,e},{a,b},{a,g}}	{c,f}	*0*	*3*	10	14	*3*	**5**	*4*
Summary	{a,b,e,g,f}	{{a,e},{a,b},{a,g},{a,f}}	{c,d}	*0*	*3*	10	14	*3*	*5*	*4*

The shortest path from A to Z (which is equal to F) is of length 5, with the path being A->G->F

18. Given any two vertices of a tree, there exists a unique path from one to the other.

a. Write an informal justification for the above statement.

Trees are not supposed to have paths that connect from the starting point back to itself. Suppose there were two paths from Vertex A to Vertex B, You could take path 1 from A to B, and then follow path 2 in reverse back to A, since both paths start at the same place. This would mean that the graph would have a cyclic path that starts and ends on the same vertex, which means it wouldn't be a tree. This means that there must be a unique path from one vertex to another.

b. Write a formal proof of the above statement.

I'm actually using a somewhat different proof here, but the informal justification was my first thoughts on the matter.

Suppose that Graph T is a Tree. A tree is by definition connected, therefore there must exist at least one path between each pair of vertices. Let Tree T include, but not be limited to vertices A and B. Suppose that there are distinct paths between A and B. Because the paths are distinct, they must diverge at a certain vertex X. Because the paths have the same end destination, they must meet up again at or before vertex B. Let the point the two paths converge be vertex Y. Because these two paths are disjoint, they form a cycle that repeats vertices, and thus Tree T has a circuit.

But this contradicts the assumption that trees cannot have circuits. Thus, the supposition is false, and there is a unique path between any two vertices of a tree.

In numerical terms, let Path 1 be vertex A, J_1, J_2, J_3, J_{X-1}, J_X... JY … vertex B. Let Path 2 be K_1, K_2 K_3… K_{X-1}, K_X… K_Y.. vertex B. Let point X be the vertex at which the two paths diverge, the least integer such that J_X is not equal to K_X. Let Y be the smallest integer greater than I such that $J_y = K_y$. This must exist because eventually, the two paths converge upon Vertex B. Either $J_y = K_y$ = vertex B, or the two paths converge before vertex B. Let path 3 be defined as taking path 1 from Vertex A to J_Y, and taking path 2 from K_Y to Vertex A. Paths 1 and 2 were combined for J_1,J_2,J_3….J_{X-1} and $K_{1,}$$K_2$,$K_3$… K_{X-1}. Therefore path 3 would eventually repeat the vertices of path 1 and 2 that are J_{X-1} / K_{X-1} and lower. Path 3 forms a circuit, which trees cannot have. Therefore, there is only 1 unique path between two vertices A and B.