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Section 2.2: 11, 13.b, 15, 20, 43, 45

11. ( p → (q→ r)) ←→ (( p ^ q) → r)

| P | Q | R | P ^ Q | ( P ^ Q) → R | Q→R | P→(Q→R) | ( p → (q → r)) ←→ (( p^q) → r) |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | F | T | T | T | T |
| F | T | F | F | T | F | T | T |
| F | F | T | F | T | T | T | T |
| F | F | F | F | T | T | T | T |

**The truth table for problem 11 has the final answer in the right most column, indicating that the statement is regardless of the truth values of P, Q, and R.**

13b. Use truth tables to verify the following logical equivalencies. Include a few words of explanation with your answers.

b. ~ ( p→ q) = p ^ ~q

| P | Q | ~Q | P ^ ~ Q | P → Q | ~ P -→ Q |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | T | T | F | T |
| F | T | F | F | T | F |
| F | F | T | F | T | F |

**This problem asks us to verify the logical equivalence of the negation of a conditional. This can be proven in several ways, including De Morgan's laws, but the problem asks us to use truth tables. First I start by listing the standard truth values for P and Q, and find the negation of Q by taking the opposite of Q's truth values. I then find P ^ ~Q because ^ and V are handled after negation.**

**~ (p → q) and p ^ q are logically equivalent because they have the same truth values (F, T, F, F) in order.**

15. Determine whether the following statements are logically equivalent.

P → (Q → R) and (P → Q) → R

| P | Q | R | Q → R | P → (Q → R) | P → Q | (P → Q) → R |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | F | F | T | F |
| T | F | T | T | T | F | T |
| T | F | F | T | T | F | T |
| F | T | T | T | T | T | T |
| F | T | F | F | T | T | F |
| F | F | T | T | T | T | T |
| F | F | F | T | T | T | F |

Final Truth Table of the two statements

| P → (Q → R) | (P → Q) → R |
|---|---|
| T | T |
| F | F |
| T | T |
| T | T |
| T | T |
| T | F |
| T | T |
| T | F |

**We can see that the truth values of the statements do not match in row 7 and 9, therefore the statements are not logically equivalent.**

20. Write negations for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)

a. If P is a square then P is a rectangle.

**P is a square, and not a rectangle.**

b. If today is New Year's Eve, then tomorrow is January.

**Today is New Year's Eve, and tomorrow is not January.**

c. If the decimal expansion of r is terminating, then r is rational.

**The decimal expansion of r is terminating and r is not rational.**

d. If n is prime, then n is odd or n is 2.

**N is prime, and neither odd nor 2.**

e. If x is nonnegative, then x is positive or x is 0.

**X is nonnegative, and X is not positive nor zero.**

f. If Tom is Ann's father, then Jim is her Uncle and Sue is her Aunt.

**Tom is Ann's father, and either Jim is not her uncle or Sue is not her aunt.**

**Every other negation problem follows the translation of P→ Q to P ^ ~Q, but this problem is slightly different. Assume P is "Tom is Ann's father." And Q is "Either Jim is not her uncle or Sue is not her Aunt."**

**~ (P → Q) = P ^ ~ Q is the standard negation. Q is a compound statement. (Jim is her Uncle AND ^ Sue is her Aunt). By De Morgan's Laws, the negation would be ~ (Jim is her Uncle) V OR (Sue is her Aunt), explaining why I phrased my answer the way I did rather than "Tom is Ann's father, and neither is Jim her Uncle nor Sue her Aunt." Hopefully I am right.**

43. Use the contrapositive to rewrite the statement in if-then form in two ways.

Doing homework regularly is a necessary condition for Jim to pass the course.

**First way: If Jim does not do homework regularly, he will not pass the course.**

**Second way: If Jim passes the course, then Jim did homework regularly.**

**Let "doing homework regularly" be P and "Jim passes the course" be Q. Because P is a necessary condition for Q, it means if not P, then not Q. ~ P → ~ Q**

**The contrapositive of this would be ~(~Q) → ~(~P), which can be simplified to Q→ P using the double negative laws to eliminate the negations.**

**We then translate ~P → ~Q and Q → P into English as our two different ways of using the contrapositive to rewrite the statement.**

I now check #42 to see if my reasoning is correct by breaking down the statement and coming up with expected statements by copying exactly my form from question 43, and comparing it to the answers in the appendix. It is not part of my homework, but I put it here for review purposes.

Being divisible by 3 = P

Being divisible by 9 = Q

Following same process.

Expected First way: If it is not divisible by 3, it is not divisible by 9.

Expected Second way: If it is divisible by 9, it is divisible by 3.

Reasoning confirmed.

45. Rewrite the statement in if-then form.

A necessary condition for this computer program to be correct is that it not produce error messages during translation.

**First way: If the computer program produces error messages during translation, then the program is not correct.**

**Second way: If the program is correct, then the program did not produce error messages during translation.**

**Let S be: The computer program is correct and R be: It does not produce error messages during translation. A necessary condition for S is R, meaning that R is a necessary condition for S, meaning if not R, then not S. ~R → ~S. An alternative is to take the contrapositive, which gives us S → R**