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Set 9.2- 32c, 33, 36, 39b, 39d, Set 9.5 - 7b, 14, and 20

9.2 #32c

How many ways can the letters of the word ALGORITHM be arranged in a row if the letters must remain together (in order) as a unit?

Algorithm is 9 letters.

A l g o r I t h m is 7 units instead of 9, because gor is treated as one unit.

Because there is no repetition, and order does not matter, there is  $7!$  Ways.

**5,040 ways.**

9.2 #33:

Six people attend the theater together and sit in a row with exactly six seats.

a. How many ways can they be seated together in the row?

6 elements. Repetition not allowed. Order does not matter, so the number of ways is  $6!$

**720 ways.**

b. Supposed one of the six is a doctor who must sit on the aisle in case she is paged. How many ways can the people be seated together in the row with the doctor in an aisle seat?

Because the doctor has to sit in the aisle seat, there are 5 elements that need to fit in 5 seats. Repetition not allowed and order does not matter, so  $5!$  Ways.

**120 ways**

c. Suppose the six people consist of three married couples and each couple wants to sit together with the husband on the left. How many ways can the six be seated together in the row?

We can treat each couple as a unit, so there are 3 units, meaning there are  $3!$  Ways.

**6 ways.**

9.2 #36:

Write all the 3-permutations of  $\{s, t, u, v\}$

$$P(4,3) =$$

$$4! / (4-3)! = 4! / 1! = 24 \text{ permutations.}$$

{s, t, u, v}

**S as first letter**

1. s,t,u (v excluded)
2. s,t,v (u excluded)
3. s,u,t (v excluded, letters switched)
4. s,u,v (t excluded)
5. s,v,t (u excluded, letters switched)
6. s,v,u (t excluded, letters switched)

**T as first letter**

7. t,s,u (v excluded)
8. t,s,v (u excluded)
9. t,u,s (v excluded, letters switched)
10. t,u,v (s excluded)
11. t,v,s (u excluded, letters switched)
12. t,v,u (s excluded, letters switched)

**U as first letter**

13. u,s,t (V excluded)
14. u,s,v (T excluded)
15. u,t,s (v excluded, letters switched)
16. u,t,v (s excluded)
17. u,v,s (t excluded, letters switched)
18. u,v,t (s excluded, letters switched)

**V as first letter**

19. v,s,t (u excluded)
20. v,s,u (t excluded)

21. v,t,s (u excluded, letters switched)

22. v,t,u (s excluded)

23. v,u,s (t excluded, letters switched)

24. v,u,t (s excluded, letters switched)

9.2 #39b:

No repetition, 9 elements.

$$P(9,6) = 9! / (9-6)! = 9! / 3! = \mathbf{60,480 \text{ ways}}$$

9.2 #39d:

How many ways can six of the letters of the word ALGORITHM be selected and written in a row if the first two letters must be OR.

9 elements.

2 letters predetermined.

7 letters left to take 4 slots.

$$P(7,4) = 7! / 3! = \mathbf{840 \text{ ways}}$$

9.5 #7b

A programming team has 13 members.

Suppose 7 team members are women and six are men.

i. How many groups of seven can be chosen that contain 4 women and 3 men?

There are  $(6,3)$  ways to choose the men and  $(7, 4)$  ways to choose the women.

$$(6! / 3! * 3!) * (7! / 4! * 3!) = (6! * 7!) / (3! * 3! * 4! * 3!) = 20 * 35 = \mathbf{700 \text{ ways}}$$

ii. How many groups of seven can be chosen that contain at least one man?

$$\begin{aligned} \text{Total teams of 7} &= (13, 7) = 13! / 6! = 13 * 12 * 11 * 10 * 9 * 8 * 7 / 6! = 13 * 12 * 11 * 10 * 9 * 8 / 6 * 5 * 4 * 3 * 2 * 1 \\ &= 13 * 11 * 9 * 8 / 6 * 1 \quad (5 * 2 \text{ cancels } 10 \text{ and } 4 * 3 \text{ cancels } 12) = 13 * 12 * 11 * 10 * 9 * 8 / 6 \\ &= 1716 \text{ ways} \end{aligned}$$

$$\text{Number of ways with no men} = (7, 7) = 7! / 7! * 0! = 7! / 7! * 1 = 7! / 7! = 1$$

$$\text{Total ways} - \text{ways with no men} = 1716 - 1 = \mathbf{1715 \text{ ways}}$$

iii. How many groups of seven can be chosen that contain at most three women?

At most three women. There are only 6 men, so there must be at least 1 woman in each group.

$$(7,1) * (6,6) + (7,2) * (6,5) + (7,3) * (6 * 4) = (7 * 1) + (21 * 6) + (35 * 15) = 7 + 126 + 525 = \mathbf{658 \text{ ways}}$$

9.5 #14

a. How many 16-bit strings contain exactly seven 1's?

$$C(16,7) = 16! / (7! * 9!) = \mathbf{11,440}$$

b. How many 16-bit strings contain at least thirteen 1's?

$$C(16,13) = 16! / 13! * 3! = \mathbf{560}$$

c. How many 16-bit strings contain at least one 1?

At least one 1 = Total 16 bit strings – 16 bit strings with no ones.

$$\text{Total 16 bit strings} = 2^{16}$$

Only one 16 bit string with no 1's: 0000000000000000

$$\text{At least one: } 2^{16} - 1 = 65,536 - 1 = \mathbf{65,535}$$

d. How many 16-bit strings contain at most one 1?

Sum of ways that contain no 1's (1) and 16 bit strings that have 1 1.

$$C(16,1) = 16$$

$$16 + 1 = \mathbf{17 \text{ ways}}$$

9.5 #20

a. How many distinguishable ways can the letters of the word Millimicron be arranged in order?

3 letter I's

2 letter M's

2 letter L's

1 C

1 R

1 O

1 N

11 letters total.

$$C(11,3) * C(8,2) * C(6,2) * C(4,1) * C(3,1) * C(2,1) * (1,1) = (11! * 8! * 6! * 4! * 3! * 2! * 1!) / (3! * 8! * 2! * 6! * 2! * 4! * 1! * 3! * 1! * 2! * 1! * 1! * 1! * 1!) = \mathbf{1,663,200 \text{ ways}}$$

$$165 * 28 * 15 * 4 * 3 * 2 * 1 =$$

b. How many distinguishable orderings of the letters of Millimicron begin with M and end with N?

M is at the end and N is at the end.

3 letter I's

2 letter L's

1 M

1 C

1 R

1 O

2 letters set out of 11, so only 9 objects left.

$$C(9,3) * C(6,2) * C(4,1) * C(3,1) * C(2,1) * C(1,1) =$$

$$84 * 15 * 4 * 3 * 2 * 1 = \mathbf{30,240 \text{ ways}}$$

c. How many distinguishable orderings of the letters of millimicron contain the letters CR next to each other in order and also the letters ON next to each other in order?

CR and ON are set. 4 out of 11 letters. 7 letters left.

3 letter I's

2 letter M's

2 letter L's

$$C(7,3) * C(4,2) * C(2,2) = (7! * 4! * 2!) / (3! * 4! * 2! * 2! * 2!) = \mathbf{210 \text{ ways}}$$