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Exercise Set 10.2: 2, 14

2.

In the graph below, determine whether the following walks are trails, paths, closed walks, circuits, simple circuits, or just walks.

a. v1e2v2e4v3e4v4e5v2e2v1e1v0

Several vertices are repeated multiple times, such as v1 and v2. E2 is repeated once. It contains at least one edge, and does not start and end at the same point, as it starts at v1 and ends at v0.

This means that it is **simply a walk**.

b. v2v3v4v5v2.

Including edges, we get v2 e3 v3 e4 v4 e6 v5 e7 v2.

No repeating edges, starts and ends on the same vertex, only the first and last vertex repeat. Not a path because a vertex repeats.

This means it is **a trail, closed walk, circuit, and simple circuit.**

C. v4v2v3v4v5v2v4

Including edges, we get: v4 e5 v2 e3 v3 e4 v4 e6 v5 e7 v2 e5 v4

E5 is repeated. Starts and ends on the same vertex. First and last vertex is repeated, but also v2.

Because there is a repeated edge, (e5), it cannot be a circuit, simple circuit, trail, or path. It is a **closed walk** because it begins and ends on the same vertex.

d. v2v1v5v2v3v4v2

Including edges, we get: v2 e2 v1 e9 v5 e7 v2 e3 v3 e4 v4 e5 v2

No repeating edges. Repeated vertex. Both first and last, and in middle. Begins and ends in same vertex, so it can’t be a path. Repeats vertex in middle, so can’t be simple circuit.

This means it is a **trail**, **closed walk**, and **circuit**.

e. Including edges, we get:

v0 e8 v5 e7 v2 e3 v3 e4 v4 e5 v2 e2 v1

Does not begin or end at the same vertex, so it is not a circuit, simple circuit, or closed walk. There are repeated vertices (v2), so it is not a path. This means it is a **trail.**

f. v5v4v2v1

Including edges, we get: V5 E6 V4 E5 V2 E2 V1

Does not begin or end on the same vertex, so it is not a circuit, simple circuit, or closed walk. There are no repeated vertices or edges, so it is **both a trail and path**.

14.

Yes, it has an Euler circuit. Firstly, the entire graph is connected, one of the pre-requisites of having a Euler circuit. Secondly, the degrees of all vertices are even.

List of degrees:

A: 2

B: 4

C: 4

D: 4

E: 2

F: 2

G: 4

H: 4

I: 4

Because the degree of every vertex of the graph is a positive even number, and the graph is connected, it must have an Euler circuit by Theorem 10.2.3.

Here is an example of an Euler circuit of the graph

I A B I H B C H G C D G F D E F I.