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Exercise Set 6.1: 12, 16 and Exercise Set 6.2 : 4, 10, 14

6.1 #12:

Let the universal set be the set R of all real numbers and let $A = \{x \in R \mid -3 \leq x \leq 0\}$, $B = \{x \in R \mid -1 < x < 2\}$, and $C = \{x \in R \mid 6 < x \leq 8\}$. Find each of the following.

a. $A \cup B$. **[-3, 2)**

b. $A \cap B$ (-1,0]

c. A^c (- infinity, -3) ∪ (0, infinity)

d. $A \cup C$ = [-3, 0] ∪ (6, 8]

e. $A \cap C$ = null set

f. B^c = (- infinity to -1] ∪ [2, infinity)

g. $A^C \cap B^C$ = (- infinity to -1] U [2, infinity)

h. $A^c \cup B^c$ = (- infinity, -1] ∪ (0, infinity)

i. $(A \cap B)^c$ = (- infinity to -1] ∪ (0, infinity)

j. $(A \cup B)^c$ = (-infinity, -3) ∪ [2, infinity)

6.1 #16:

a. Find $A \cup (B \cap C)$, $(A \cup B) \cap C$ and $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?

$A \cup (B \cap C) = \{a,b,c\}$, $(A \cup B) \cap C = \{b,c\}$ and $(A \cup B) \cap (A \cup C)$. $\{a, b, c, d\} \cap \{a, b, c, e\} = \{a,b\ c\}$

$A \cup (B \cap C)$, and $(A \cup B) \cap (A \cup C)$ are equal.

b. $A \cap (B \cup C) = \{b, c\}$. $(A \cap B) \cup C = \{b,c,e\}$. $(A \cap B) \cup (A \cap C) = \{b,c\}$

$A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$ are equal.

c. $(A-B) - C = A$. $A - (B - C) = A$. The sets are equal.

6.2 #4:

Bold is blank.

Proof: Suppose A and B are any sets and $A \subseteq B$. [*We must show* **that $A \cup B \subseteq B$.**]. Let **$x \in A \cup B$**. [We must show that **$x \in B$**]. By definition of union $x \in$ **A, or** $x \in$ **B**. In case $x \in$ **A**, then since $A \subseteq B$, $x \in$ **A**. In case $x \in B$, then clearly $x \in B$. So in either case, $x \in$ **B** [as was to be shown.]

Note **A** and **or** are separate blanks, meant to fill answers D and E.

#10: For all sets A, B and C,

$(A- B) \cap (C - B) = (A \cap C) - B$.

For these two expressions to be equal, they must be subsets of each other.

First, we will show that is $(A- B) \cap (C - B \subseteq$). $(A \cap C) - B$.

Let $x \in (A- B) \cap (C- B)$. We must show that $x \in (A \cap C) - B$

By definition of intersection, for $x \in (A \cap C) - B$, x must $\in (A-B)$ and $\in (C-B)$

By definition of intersection $x \in (A-B)$ and $x \in (C-B)$, as was to be shown.

Next, we need to show that is $(A \cap C) - B$. $\subseteq (A- B) \cap (C - B)$.

Let $x \in (A \cap C) - B$. We must show that $x \in (A -B) \cap (C - B)$

By definition of intersection, for $x \in (A- B) \cap (C- B)$, x must show that $x \in (A- B)$ and $x \in (C- B)$.

By definition of intersection, $x \in (A- B)$ and $x \in (C-B)$, as was to be shown.

Because both expressions are subsets of each other, they are equivalent.

#14: For all sets A, B, and C, if $A \subseteq B$ then $A \cup C \subseteq B \cup C$.

$A \cup C \subseteq B \cup C$, we must show that if $x \in A \cup C$, then $x \in B \cup C$.

Let $x \in A \cup C$.

Because $x \in A \cup C$, $x \in A$ or $x \in C$ by definition of union.

If $x \in C$, then clearly x is in $B \cup C$ as an element of C.

If $x \in A$, then x also $\in B$ because $A \subseteq B$, meaning there is no element in A that is not in B. Because $x \in B$, clearly $x \in B \cup C$ as an element of B.

In either case, if $x \in A \cup C$, then $x \in B \cup C$.

Thus, $A \cup C \subseteq B \cup C$, as was to be shown.