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Assignment 6 Part 1 5.6 #2 and #4. 5.7 #4, #6, and #7

5.6

Find the first four terms of each of the recursively defined sequences in 1-8.

#2: $b_k = b_{k-1} + 3k$, for all integers $k \geq 2$

$B_1 = 1$

$B_2 = 7$

$B_3 = 16$

$B_4 = 28$

#4: $d_k = k(d_{k-1})^2$ for all integers $k \geq 1$

$D_0 = 3$

$D_1 = 1 * (3)_2 = 9$

$D_2 = 2 * (9)^2 = 2 * 81 = 162$

$D_3 = 3 * (162)^2 = 3 * 26244 = 78732$

$D_{4\,=}\ 4 * (78732)^2 = 4 * 6{,}198{,}727{,}824 = 24{,}794{,}911{,}296$

5.7

In each of 3-15 a sequence is defined recursively. Use iteration to guess an explicit formula for the sequence. Use the formulas from Section 5.2 to simplify your answers whenever possible.

#4:

$B_k = (b_{k-1}) / (1 + b_{k-1})$ for all integers $k \geq 1$

$B_0 = 1$

$B_1 = 1 / (1+1) = 1 / 2$

$B_2 = ½ / (1/2 + 1) = ½ / 3/2 = ½ * 2/3 = 2 /6 = 1/3$

$B_3 = 1/3 / (1/3 +1) = 1/3 / 4/3 = 1/3 * ¾ = 3 /12 = ¼$

Explicit formula guess:

B_n=

$$\frac{1}{n+1}$$

#6:

$D_k = 2d_{k-1} + 3$, for all integers $k \geq 2$

$D_0 = 2$

$D_1 = 7$

$D_2 = 17$

$D_3 = 37$

$D_4 = 77$

7-2 = 5

17- 7 = 10

37 – 17 = 20

77-37 = 40

All differences are multiples of 5. Assuming the formula is (something) + 2.

Subtracting by 2 gives us

0, 5, 15, 35, 75.

Dividing by 5 (because they are multiples of 5), gives us

0, 1, 3, 7, 15.

For terms 0, 1, 2, 3, 4

What seems to fit this is $2^n – 1$.

$2^0 – 1= 1-1 = 0$

$2^1 – 1 = 2-1 = 1$

$2^2 -1 = 4-1 = 3$

$2^3 -1 = 8-1 = 7$

$2^4 -1 = 16-1 = 15$

Then we add back in the 5x multiplier and the +2 to get my guess at the explicit formula.

$D_n = 5 * (2^n – 1) + 2$

#7:

$E_k = 4e_{k-1} + 5$ for all integers $k \geq 1$

$E_0 = 2$

$E_1 = 4*2 + 5 = 13$

$E_2 = 4 * 13 + 5 = 57$

$E_3 = 4 * 57 + 5 = 233$

$E_4 = 4 * 233 + 5 = 937$

The difference between every term divided by 4 is equal to the difference between the two previous terms. For example (937 – 233) / 4 = 176, and 233 – 57 = 176. Similarly, (233- 57) / 4 = 44, and (57 – 13) = 44

Another observation I can make is that every term is equal to 11 * some number + 2.

$E_1 = 11 * 1 + 2 = 11 +2 = 13$

$E_2 = 11 * 5 +2 = 55 + 2 = 57$

$E_3 = 11 * 21 + 2 = 231 +2 = 233$

$E_4 = 11 * 85 + 2 = 935 + 2 = 937$

Furthermore, the number you multiply 11 by in each term is equal to that in the previous term * 4 + 1. For example E_2 is 5 and E_3 is 21. 5*4 +1 = 20 + 1= 21

Also every previous term is equal to the next term divided by 4, minus 5/4. For example 937 / 4 234.25 – 5/4 = 233. 233 / 4 = 58.25 – 5/4 = 57

I can’t figure out what to make of all these things.

After the deadline, I looked on Piazza and saw the professor had posted a solution to #6. The following is an extrapolation of the solution to #7.

$E_0 = 2$

$E_1 = 4 (E_0) + 5 = (4* 2) + 5$

$E_2 = 4 (E_1) + 5 = 4 * (4 * 2 + 5) + 5 = (4^2 * 2)+ (4 * 5) + 5 = 57$

$E_3 = 4 (E_2) + 5 = 4 * [(4_2 * 2) + (4 * 5) + 5)] + 5 = (4^3 * 2) + (4^2 * 5) + (4*5) = 128 + 80 + 20 + 5 = 128 + 125 = 233$

E_4 = 4 (E_3) + 5 = 4[(4^3 * 2) + (4_2 * 5) + (4*5) + 5)] + 5 = (4^4 * 2) + (4^3 * 5) + (4_2 *5) + (4 *5) + 5 = (2*256) + (64 * 5) + (16 * 5) + (4*5) + 5 = 512 + 320 + 80 + 20 + 5 = 937

E_5 = 4(E_4) + 5 = 4[(4^4 * 2) + (4^3 * 5) + (4^2 *5) + (4*5) + 5] + 5 = (4^5 * 2) + (4^4*5) + (4^3 *5) + (4^2 * 5) + (4 *5) + 5 = (2 * 1024) + (256 * 5) + (64 *5) + (16 *5) + (4*5) + 5 = 2048 + 1280 + 320 + 80 + 20 + 5 = 3753

Guess E_n = (4^n * 2) + (4^{n-1} * 5) + (4^{n-2} * 5) ……. (4^2 *5) + (4*5) + 5

(2 * 4^n) + 5 * $\sum_{i=0}^{n-1} 4^i$

= (2 *4^n) + 5 * ($\frac{4^n-1}{4-1}$)

= (2 * 4^n) + 5 * ($\frac{4^n-1}{3}$)

= (2 * 4^n) + $\boldsymbol{\frac{(5*4^n - 5)}{3}}$ Note, there is a *4^n there, not a +.

Test: E_4 = 937.

N = 4

(2 * 4^4) + ((5* 4^4 -5) / 3) = 512 + ((1280 – 5) / 3)

= 512 + (1275 / 3) = 512 + 425 = 937

Explicit formula confirmed