Eddie C. Fox

November 29, 2015

Username: foxed

Exercise Set 10.1: 9, 27.b, 44

9.

i. Find all edges that are incident on v1.

**E1, E2, E7.**

ii. Find all vertices that are adjacent to v3.

**V1, V2**

iii. Find all edges that are adjacent to e1.

**E2, E7**

iv. Find all loops.

**E1, E3**

V. Find all parallel edges.

**E4, E5**.

VI. Find all isolated vertices.

**V4**.

Vii. Find the degree of V3.

**2.**

VIII. Find the total degree of the graph.

7 vertices, so the total should be 14.

4 + 6 + 2 + 2 = 14.

**14 confirmed total degree of graph.**

27b.

In a group of 4 people, is it possible for each person to have exactly 3 friends? Why?

It would be possible because the total degree of each vertex would be 3. This leads to a total degree of 12, which is even, satisfying the even requirement. Furthermore, if we draw a graph of

4 vertices, each connected to the three others, we see there are 6 edges, and the total degree of a graph is twice that of the number of edges. 2 * 6 edges = 12 degrees, as was already confirmed.

Because it is the summation of 4 odd degrees, it also fulfills the requirement for there to be an even amount of odd degrees, because 4 is even. Furthermore, in a group of x vertices, it should be possible for every member to connect with every member but themselves. This would be X-1 connections, which are friends in this case. With 4 people, X is 4, and thus X-1, the maximum number of theoretical connections / friends would be 4 – 1 = 3, and the stipulation of 3 friends at the start of the problem satisfies this requirement.

44.

a. In a simple graph, must every vertex have degree that is less than the number of vertices in the graph? Why?

**Yes**. By definition, a simple graph has no parallel edges or loops, so it would be impossible for a vertex to have a degree of more than X-1, where X is the total number of vertices in the graph. This is similar to my reasoning in 27b. There is simply no way for a vertex to gain degree besides connecting with other vertices, and it cannot connect with itself because there are no loops. No parallel edges means that it cannot gain more than one degree for each vertex besides itself.

b. Can there be a simple graph that has four vertices each of different degrees?

**No.**

It is impossible for a graph of four vertices to have a vertex with 4 degrees. At most, it would have 3 degrees, which is X-1. But if it had 3 degrees, then it would be connected to all 3 other points, meaning that it would be impossible for there to be an isolated vertex, and hence it would be impossible for there to be a vertex of 0 degree. This means that even if we assign one vertex a degree of 3, there are 3 vertices left to assign and only two possible degrees, 1 and 2. By the pigeonhole principle, there must be at least 2 vertices with the same degree, because the set of possible degrees is smaller than the set of vertices that must be assigned degrees.

If one vertex isn't X-1 degrees, then this problem still arises, because we have either 0, 1, or 2 degrees, and with 4 vertices and only 3 degrees, the pigeonhole principle still applies.

c.

**No**.

My reasoning for this problem is the same as for part B. The pigeonhole principle will apply. If a simple graph has N vertices, then a vertex may be at most N-1 degrees. This prevents one from being assigned a degree of 0, because at least one vertex is connected to all others. This will leave N – 1 vertices only N-2 angles to be assigned, meaning at least 2 vertices must share the same degree by the pigeonhole principle.

If there is not a vertex of N-1 degrees, there is still N vertices that have N-1 degree choices, because we must subtract one possibility as the N-1th degree is barred as a possibility. Either way, the pigeonhole principle applies.