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6.1 - 3, 7, 13, 18, 33, 34

3. a. Is  $R \subseteq T$ ? Explain.

$R$  is not  $\subseteq T$  because there are elements in  $R$  that are not in  $T$ . For example, 4 is divisible by 2 but not 6.

b. Is  $T \subseteq R$ ? Explain.

Yes. Every number that is divisible by 6 is also divisible by 2.

c. Is  $T \subseteq S$ ? Explain.

Yes. Every number that is divisible by 6 is also divisible by 3.

7. Let  $A = \{x \in \mathbb{Z} \mid x = 6a + 4 \text{ for some integer } a\}$ ,

$B = \{y \in \mathbb{Z} \mid y = 18b - 2 \text{ for some integer } b\}$ , and  $C = \{z \in \mathbb{Z} \mid z = 18c + 16 \text{ for some integer } c\}$

a.  $A \subseteq B$

This is false. We can easily find a counter example with the number of 1.  $6a+4 = 10$ . 10 is not an element in  $B$ . If  $b = 1$ , then  $Y = 16$ . If  $b = 0$ , then  $y = -2$ .

b.  $B \subseteq A$

True.

Suppose  $x$  is a particular but arbitrary element of  $B$ .

By definition, there is an integer  $b$  such that  $x = 18b - 2$ .

If  $y$  is an element of  $A$ , there is also an integer  $a$  such that  $y = 6a + 4$

Let  $A$  equal  $3b - 1$ .  $6(3b - 1) + 4 = 18b - 6 + 4 = 18b - 2$ , which is identical to the equation for set  $B$ . This proves that all elements of  $B$  can be contained within set  $A$ , and that set  $B$  is a subset of set  $A$ .

C.  $B = C$

$Y = 6*(3b) - 2$  and  $Z = 6*(3c) + 6*(3) - 2$ .  $Z$  can be simplified to  $6(3c+3) - 2$ .

If we define  $c$  as  $b-1$ , then we get  $Z = 6(3b-3) + 18 - 2$ . This simplifies to  $18b - 18 + 18 - 2$ , further simplifying to  $18b - 2$ , which is identical to  $Y$  from set  $B$ , meaning the sets are equivalent.

13. Indicate whether the following relationships are true or false.

a.  $Z^+ \subseteq Q$ . True. All positive integers are rational numbers.

b.  $R^- \subseteq Q$ . False. Not all negative real numbers are rational.

c.  $Q \subseteq Z$ . False. Not every rational number is an integer.

d.  $Z^- \cup Z^+ = Z$  False. All negative and positive real numbers does not include the number 0.

e.  $Z^- \cap Z^+ = \text{Null set}$ . True. The set of all positive and negative numbers have nothing in common.

f.  $Q \cap R = Q$ . True, because  $Q$  is a subset of  $R$ .

g.  $Q \cup Z = Q$  True, because integers are a proper subset of rational numbers, the union would just be all rational numbers.

h.  $Z^+ \cap R = Z^+$  True, because all positive integers are a subset of all real numbers, the intersection would just be all positive integers.

i.  $Z \cup Q = Z$  False. The union would be  $Q$  because all rational numbers includes all integers.

18. a. Is the number 0 in the empty set? Why

0 is not in the empty set because the null set contains no elements and 0 is an element.

b. Is empty set = { null set} ? Why?

No, because the empty set has no elements, and the other is a set with the empty set as an element.

c. Is empty set an element in { empty set}? Why?

Yes, because the empty set itself can be an element in a set, even if itself has no elements.

D. Is the empty set an element in empty set? Why?

No, because the empty set contains no elements.

33. a. The power set of  $\emptyset$  is  $\{ \emptyset \}$

b. The power set of the power set of  $\emptyset$  is  $\{ \emptyset, \{\emptyset\} \}$

c. The power set of the power set of the power set of  $\emptyset$  is  $\{ \emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\} \}$

34. Let  $A_1 = \{1, 2, 3\}$ ,  $A_2 = \{u, v\}$ , and  $A_3 = \{m, n\}$ . Find each of the following sets:

a.  $A_1 \times (A_2 \times A_3)$

$\{ (1, (u, m)), (1, (v, m)), (1, (u, n)), (1, (v, n)), (2, (u, m)), (2, (v, m)), (2, (u, n)), (2, (v, n)), (3, (u, m)), (3, (v, m)), (3, (u, n)), (3, (v, n)) \}$

b.  $(A_1 \times A_2) \times A_3 \{ ((1, u), m), ((1, v), m), ((1, u), n), ((1, v), n), ((2, u), m), ((2, v), m), ((2, u), n), ((2, v), n), ((3, u), m), ((3, v), m), ((3, u), n), ((3, v), n) \}$

c.  $A_1 \times A_2 \times A_3$

$\{ (1, u, m), (1, v, m), (1, u, n), (1, v, n), (2, u, m), (2, v, m), (2, u, n), (2, v, n), (3, u, m), (3, v, m), (3, u, n), (3, v, n) \}$