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Section 2.1: 5, 10, (25-31), 52, 54

5. Indicate which of the following sentences are statements.

a. 1024 is the smallest four-digit number that is a perfect square.

This is a statement because it is a sentence that is either true or false, but not both. Either 1024 is the smallest number or it is not.

b. She is a mathematics major.

This is not a statement because the reference of the pronoun "she" is unclear. For some values, the sentence be true, and for other values, the sentence may be false.

c. $128 = 2^6$.

This is a statement because even though it is false, it is a sentence that is either true or false, but not both at the same time. 2^6 has a definite value which can be compared to 128.

d. $x = 2^6$

This is not a statement because changing the value of X changes whether or not the sentence is true, similar to question 5B. It could be both true and false at the same time, rather than remaining statically one.

10. Let p be the statement "DATAENDFLAG is off," q the statement "ERROR equals 0," and r the statement "SUM is less than 1,000." Express the following sentences in symbolic notation.

a. DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.

$p \wedge q \wedge r$. Because all statements are true, we can symbolically notate it as the conjunction between all three of them.

b. DATAENDFLAG is off but ERROR is not equal to 0.

$p \wedge \sim q$. p is true, and but is equivalent to and. The ERROR not being equal to 0 indicates $\sim q$. Therefore, we can combine the two to make $p \wedge \sim q$

c. DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.

$p \wedge (\sim q \vee \sim r)$. P is true. Error is not 0 indicates $\sim q$. Sum is greater than or equal to 1,000 indicates $\sim r$. The or in between error or sum tells me that I should group them together in parentheses, separate from P, and use the or operator.

d. DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.

$\sim p \wedge q \wedge \sim r$. Data flag is on, so P isn't true. And links it to error, so I include that symbol. Q is true because error equals 0. R is false because the sum is greater than or equal to 1,000. The phrase but tells me to link it to p and q with the \wedge operator.

e. Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

$\sim p \vee (q \wedge r)$. And between error and sum tells me to group q and r together with parentheses. They both evaluate to true. P is false because data flag is on, and I separate it from Q and R with the \vee symbol because of the either or wording.

25. Hal is a math major and Hal's sister is a computer science major.

$M \wedge C$ (where M is hal being a math major and C is Hal's sister being a computer science major)

De Morgan's laws say $\sim(M \wedge C) = \sim M \vee \sim C$

Translated to English: **Hal is not a math major or Hal's sister is not a computer science major.**

26. Sam is an orange belt and Kate is a red belt.

$O \wedge R$ (where O is Sam being an orange belt and R is Kate being a red belt)

De Morgan's laws say $\sim(O \wedge R) = \sim O \vee \sim R$

Translated to English: **Sam is not an orange belt or Kate is not a red belt.**

27. The connector is loose or the machine is unplugged.

L = Connector is loose. M = Machine is unplugged

$L \vee M$

De Morgan's Laws say $\sim(L \vee M) = \sim L \wedge \sim M$

Translated into English: **The connector is not loose and the machine is not unplugged.**

28. The units digit of 4^{67} is 4 or it is 6.

4 = Units digit is 4. 6 = Units digit is 6

$4 \vee 6$

De Morgan's Laws say $\sim(4 \vee 6) = \sim 4 \wedge \sim 6$

Translated into English: **The units digit of 4⁶⁷ is not 4 and not 6.**

29. This computer program has a logical error in the first ten lines or it is being run with an incomplete data set.

LE = Logical Error ID = Incomplete Data

$LE \vee ID$

De Morgan's laws say $\sim(LE \vee ID) = \sim LE \wedge \sim ID$

Translated into English: **This computer program does not have a logical error in the first ten lines and it is not being run with an incomplete data set.**

30. The dollar is at an all-time high and the stock market is at a record low.

DH = Dollar High SL = Stock Low

$DH \wedge SL$

De Morgan's laws say $\sim(DH \wedge SL) = \sim DH \vee \sim SL$

Translated into English: **The dollar is not at an all-time high or the stock market is not at a record low.**

31. The train is late or my watch is fast.

TL = Train late WF = Watch fast

$TL \vee WF$

De Morgan's laws say $\sim(TL \vee WF) = \sim TL \wedge \sim WF$

Translated into English: **The train is not late and my watch is not fast.**

Use theorem 2.1.1 to verify the logical equivalencies in 50-54. Supply a reason for each step.

52. $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) = \sim p$

$(\sim p \wedge \sim(\sim q)) \vee (\sim p \wedge \sim q)$ De Morgan's law

$(\sim p \wedge q) \vee (\sim p \wedge \sim q)$ Double Negative Law

$(\sim p \wedge (q \vee \sim q))$ Distributive Law

$(\sim p \wedge t)$ Negation Law. Q or not Q is a tautology.

$= \sim p$. Identity law says $p \wedge t = p$, so tautology is just canceled out.

$$54. (P \wedge (\sim(\sim p \vee q))) \vee (P \wedge Q) = P$$

$$(P \wedge (\sim(\sim p) \wedge \sim Q))) \vee (P \wedge Q) = P \text{ De Morgan's Laws}$$

$$(P \wedge (P \wedge \sim Q)) \vee (P \wedge Q) = P \text{ Double negative law}$$

$$(\sim Q \wedge (P \wedge P)) \vee (P \wedge Q) = P \text{ Associative law}$$

$$(\sim Q \wedge P) \vee (P \wedge Q) = P \text{ Idempotent law}$$

$$(P \wedge \sim Q) \vee (P \wedge Q) = P \text{ Commutative Law}$$

$$P \wedge (Q \vee \sim Q) = P \text{ Distributive Law}$$

$$P \wedge T \text{ Negation Law. Q or not Q is a tautology}$$

$$P = P \text{ Identity law. } P \wedge T = P$$