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Assignment 7 Part 2 - Set 9.4 - 6, 7, 16, 27

9.4 #6:

a. Given any set of seven integers, must there be two that have the same remainder when divided by 6? Why?

Yes. By the quotient remainder theorem, there are only six possible remainders when a number is divided by 6. No remainder, 1, 2, 3, 4 or 5. Because there are seven numbers, and only six possible remainders, by the pigeonhole principle, at least two numbers must be represented by the same remainder when divided by 6.

b. Given any set of seven integers, must there be two that have the same remainder when divided by 8? Why?

No. When a number is divided by 8, there are 8 possible remainders. No remainder, 1, 2, 3, 4, 5, 6, and 7. It is possible with 7 numbers that they all have entirely different remainders and one remainder possibility is not yet represented. There would need to be at least 8 integers for us to apply the Pidgeon hole principle here.

9.4 #7:

Let S = {3,4,5,6,7,8,9,10,11,12}. Suppose six integers are chosen from S. Must there be two integers whose sum is 15? Why?

Yes. We can divide S into 5 mutually disjoint subsets. {3,12}, {4,11}, {5,10}, {6,9}, and {7,8}. Because six integers are being chosen, then at least two of them must fall into the same subset by the pigeonhole principle. There are 5 subset slots, but 6 numbers chosen. Because there are at least two numbers in the same subset, then there are at least two integers whose sum is 15 because every subset has a sum of 15.

9.4 #16:

How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

First, let us find out how many integers are divisible by 5. These are integers that end in either 5 or 0. There are 100 integers from 1 to 100, but integers ending in 1, 2, 3, 4, 6, 7, 8, or 9 are barred. This means that only 2 out of every 10 numbers are allowed, and thus 20 out of 100 numbers are divisible by 5.

You would need to pick **81** numbers to be sure of picking at least one divisible by 5.

100 – 20 = 80 integers from 1 to 100 that are not divisible by 5. Worse case scenario, you pick all of those, and would need to pick one more on top of that to pick the first number guaranteed to be divisible by 5.

9.4 #27:

In a group of 2,000 people, must at least 5 have the same birthday? Why?

Yes. By the generalized Pidgeonhole principle, we can consider set X to be the finite set of 2,000 people and set Y to be the finite set of 366 possible birthdays (each day of the year is a different possible birthday).

N = 2000 elements of X

M = 366 elements of Y

For this problem, K = 4, where K asserts there must be at least 4 elements of X that share the same Y.

We must now find N / M and compare the value to K. N / M = 2000 / 366 = 5.46, which is greater than K.

Because K < N / M (4 < 5.46), then there is some Y that is the image of at least K + 1 (4+1) elements of X. In other words , there is some day of the year where at least 5 people share the same birthday.

Actually, because the value of N / M is 5.46, we could actually set K to 5 and suppose that in a group of 2,000 people, at least 6 must share the same birthday. Since this is true, a lesser truth, that there are at least 5 people sharing the same birthday, would necessarily arise as well.