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4.6 #12, #16, and #28 (prove by contradiction)

12. The product of any two irrational numbers is irrational.

Proof by counter example.

Theorem 4.7.1 says that $\sqrt{2}$ is irrational.

The product of $\sqrt{2}$ and $\sqrt{2}$ is a rational number, 2. 2 is rational because it can be expressed as a ratio of 2 integers (Itself divided by 1).

Because there is an instance of the proposition being false, it cannot hold true universally.

16. a. Use proof by contradiction to show that for any integer n, it is impossible for n to equal both 3q1 + r1 and 3q2 +r2, where q1, q2, r1, and r2 are integers, $0 \le r1 < 3$, $0 \le r2 <3$, and $r1 \ne r2$.

a. Suppose that n is equal to both 3q1 +r1 and 3q2 +r2.

This means that 3q1+r1 and 3q2+r2 must be equal.

Because r1 and r2 cannot equal each other, they must have different values.

Because they must have different values, q1 and q2 cannot be the same.

> Adding different values of r1 and r2 to a values where q1 and q2 are equal would produce a different result. This would mean that 3q1 + r1 and 3q2 + r2 would not be equal.

The least q1 and q2 can be different is 1, because they are integers.

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The least the values of 3q1 and 3q2 can be different is 3, because 3 multiplies the least difference of q1 and q2.

The most r1 and r2 can be different is 2, if one of them were 0 and the other were 2. They must be integers and neither of them can be 3.

Because 3q1 and 3q2 have to be at least 3 in value apart from each other, and r1 and r2 can only cover at most 2 of that difference, 3q1 +r1 and 3q2+r2 cannot be equal.

So 3q1 +r1 and 3q2+r2 are not equal, and yet the must be equal in order for n to equal both of them. This is a contradiction, thus proving it is possible for n to equal both 3q1 + r1 and 3q2 + r2.

b. Use proof by contradiction, the quotient-remainder theorem, division into cases, and the result of part (a) to prove that for all integers n, if $n^2$ is divisible by 3, then n is divisible by 3.

Suppose that if $n^2$ is divisible by 3, then it is not necessarily true that n is divisible by 3.

Either n is divisible by 3 or it is not divisible by 3.

Let X be the result of dividing N by X.

If n is not divisible by 3, then it can be written as either 3x -1 or 3x-2, where X is any integer. 3x would be the case where N is divisible by 3, but that would not prove the supposition false.

3x-1 or 3x-2 must be integers because they are equal to N, which is an integer.

N^2 must also be an integer because it is equivalent to N * N, and integers are closed with multiplication.

In case 1, where N = 3X – 1, $N^2$ = 9X^2 – 6X + 1. $N^2$ cannot be divisible by 3 because it would equal 3X^2 -2X + (1/3). Because a fraction is added, the result is not an integer.

In case 2, where N = 3x -2, $N^2$ = 9X^2 – 12x +4, which is not divisible by 3 either because 4/3 is a fraction that would make the result not an integer.

Both cases are contradictions because we suppose that there are cases where N is not divisible by 3, but both cases of that violate the known knowledge that $N^2$ is divisible by 3. $N^2$ cannot be both divisible and 3 and not divisible by 3 at the same time.

Because my supposition is not true due to the contradictions shown, it must be necessarily true that if $n^2$ is divisible by 3, then so is n.

C. Suppose that √3 is rational. Then √3 can be expressed as the ratio of 2 integers A and B, such that (A / B) = √3

Then 3 = $A^2$ / $B^2$

$3B^{2 =} A^2$

If B is odd, then $B^{2,}$ $A^2$, and A must also be odd. Similarly if B is even, $B^2$, $A^2$ and A must also be even.

A and B cannot be even because they would be further reducible by a common factor of 2, therefore they must be odd.

Because A and B are odd, they can be defined as A = 2X +1 and B = 2Y + 1, where X and Y are some integers.

If we substitute this into the original expression $3B^2 = A^2$, we get $12X^2 + 12X+3 = 4Y^2 + 4Y + 1$.

We can subtract 1 from both sides and then divide by 2 to get $6X^2 + 6X + 1 = 2Y^2 + 2Y$

This can be simplified as $2(3X^2 + 3X) + 1 = 2 (Y^2 +1 )$.

The left must be even and the right must be odd, but this is a contradiction because for them to equal each other, they must be the same value, and no value is both even and odd.

This contradiction shows that my supposition is false, and thus the proposition is that √3 is irrational is true.

28. n! = n(n-1) … 3 * 2 * 1.

An alternative proof of the infinitude of the prime numbers begins as follows.

Proof: Suppose there are only finitely many prime numbers. Then one is the largest. Call it p. Let M = p! +1. We will show there is a prime number q such that q> p. Complete this proof.

If P is the largest prime number, then there is no prime number q such that q > p.

M = p! +1

If M is a prime, then M would be a prime number larger than P, which would contradict the statement that there is no larger prime number than p.

If M is not a prime, then it must be divisible by a prime number, because all non-prime numbers are divisible by a prime number. Let this prime number be Q.

This prime number is not divisible by any of the prime numbers leading up to and including m, because those are factors of p, not p+1. Q must be outside the finite set of prime numbers leading up to, and including p.

Because !p includes every prime number leading up to and including p, and Q is outside of the set, Q must be greater than p. Q would not be lesser than p because any less primes would be included in the finite set.

Q > P contradicts the necessary condition of p being the largest that there is no prime Q such that Q > P.

Therefore, my supposition is false and the proposition that there is an infinitude of prime numbers is true.