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5.2: 9, 27, 35, Set 5.3: 10, 18, 23.b

5.2 #9: Prove using mathematical induction. Do not derive them from Theorem 5.2.2 or Theorem 5.2.3.

For all integers $n \geq 3$,

$4^3 + 4^4 + 4^5 + 4^n = (4(4^n – 16)) / 3$

Let property P(n) be: $4^3 + 4^4 + 4^5 + 4^n = (4(4^n – 16)) / 3$

To establish P(3), we must show that P(3) = $4(4^{n – 16)} / 3 = 64$

$4^3 = 64$, which is equal to the 64 we get from $(4(4^n – 16)) / 3$, therefore P(3) is true.

Suppose that P(k) is true for a particular but arbitrarily chosen integer $k \geq 3$, such that

$4^3 + 4^4 + 4^5 \ldots 4^k = (4(4^K – 16) / 3$.

We must show that P(k+1) is true.

That is, that

$4^3 + 4^4 + 4^5 \ldots 4^{(k+1)} = (4(4^{(k+1)} – 16)) / 3$

We will show that the left hand and right hand equations are equal to each other.

$4^3 + 4^4 + 4^5 \ldots 4^k = (4(4^k – 16) / 3$, and we are only adding on 4^{K+1} so we have as a starting point:

$4^3 + 4^4 + 4^5 \ldots 4^K = (4(4^K – 16) / 3 + 4^{(K+1)}$

We must show that $(4(4^K – 16) / 3) + 4^{(K+1)} = (4(4^{(k+1)} – 16)) / 3$

First, we can multiply $4^{(k+1)}$ by 3 and put it over 3 to give the two terms on the left hand a common denominator.

$(4(4^k – 16) / 3) + (3 *(4^{(k+1)}) / 3$. We can group the terms.

$(4(4^k – 16) + (3 *(4^{(k+1)}) / 3$

$4(4^k) = 4^{k+1}$, so we can rewrite the equation as $(4^{k+1} – 4(16) + (3 * (4^{k+1})) /3$

We can group the 4^{k+1}'s together to give us $((4 * 4^{k+1}) – 4(16)) / 3$. We can take the 4 out to give us

$4(4^{k+1} – 16) / 3$, which is identical to P(K+1), as was to be shown.

Therefore we have proved the statement with mathematical induction.

5.2 #27

Use the formula for the sum of the first *n* integers and/or the formula for the sum of a geometric sequence to evaluate the sums in 20-29 or to write them in closed form.

$5^3 + 5^4 + 5^5 + \ldots + 5^{K,}$ where k is any integer with $k \geq 3$

The base formula gives us

$= (5^{k+1} – 1) / (5-1) =$

$(5^{k+1} – 1) / 4$

We can factor out a 5^3 from the sequence to get

$1 + 5^1 + 5^2 \ldots 5^{k-2}$, which means we need to subtract 2 from the exponents in the base formula.

$5^3 = 125$, which is what we factored out. Also 5^{k+1} will turn to 5^{k-1}. z

$(125 (5^{k-1} – 1)) / 4$

5.2 #35

Find the mistakes in the proof fragment.

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1$$

"Proof" (by mathematical induction): Let the property

P(n) be

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1$$

Show that P(1) is true: When n = 1

$$\sum_{i=1}^{1} i(i!) = (1+1)! - 1$$

So $1(1!) = 2! - 1$

And $1 = 1$. Thus P(1) is true."

This method starts from a statement and deduces a true conclusion, but that doesn't prove the statement to be true, because true conclusions can also be deduced from false statements.

The method should not prove that $1 = 1$, but show that the statements are equivalent to each other. P(1) would be true not because $1= 1$, but because *i (i!)* $= (2!) - 1$. It would be a proof to say that $i(i!) = 1$ and $(2!) - 1$ is also $= 1$, not proving the true conclusion that $1 = 1$.

5.3 #10

Prove that $n^3 - 7n + 3$ is divisible by 3, for each integer ≥ 0, by mathematical induction.

Let the property p(n) be the sentence "$n^3 - 7n + 3$ is divisible by 3 for each integer ≥ 0.

To establish P(0), we must show that $0^3 - 7(0) + 3$ is divisible by 3.

$0 - 0 + 3 = 3$, which is divisible by 3.

Suppose that P(k) is true for a particular but arbitrarily chosen integer $k \geq 0$ and suppose that

$K^3 - 7k + 3$ is divisible by 3.

By the definition of divisibility, this means that $K^3 - 7k + 3 = 3r$ for some integer r.

We must show that $(K+1)^3 - 7(k+1) + 3$ is divisible by 3.

$(K+1)^3 - 7k + 3 = (K^2 + 2k + 1)(K+1) - 7K - 7 + 3$

If we distribute the terms out, we get:

$K^3 + K^2 + 2k^2 + 2K + K + 1 - 7k - 7 + 3$. Contained within this expression is P(K), which is $K^3 - 7k + 3$. We can group the terms of P(k) separately.

$P(K+1) = P(k) + 3k^2 + 3k - 6$.

We know that by the definition of divisibility, there is some integer r such that $p(k) = 3r$.

We can create a new expression

$3r + 3k^2 + 3k – 6$, which can be written as

$3(r + k^2 + k – 2)$.

This is divisible by 3 because we could eliminate the 3 and just have $(r + k^2 + k -2)$.

Further, we know that $r + k^2 + k – 2$ is an integer because it is composed of sums, products, and differences of integers, which does not change its integer status.

Since we have proved the basis step and the inductive step, we can conclude that the proposition is true.

5.3 #18

Prove by mathematical induction

$5^n + 9 < 6_n$, for all integers $n \geq 2$

Let the property p(n) be the sentence "$5^n + 9 < 6^n$, for all integers $n \geq 2$

To establish P(2), we must show that $5^2 + 9 < 6^2$.

$5^2 + 9 = 25 + 9 = 34$. $6^2 = 36$

$34 < 36$, so P(2) is true.

Suppose that P(k) is true for a particular but arbitrarily chosen integer $k \geq 2$ and suppose that

$5^k + 9 < 6^k$

We must show that $5^{k+1} + 9 < 6^{k+1}$

$6 (5^k + 9) < 6 (6^k)$ Multiply both sides by 6

$(5+1) (5^k + 9) < 6^{k+1}$ exponential laws

$(5 * 5^k) + 5^k + 9 + 45$ expanding 6 to 5 + 1 and distributing

$(5^{k+1}) + 5^k + 9 + 45$. $5^{k+1} = 5 * 5^k$

$(5^{k+1} + 9) + (5^k + 45) < 6^{k+1}$ Regrouping terms.

$5^{k+1} + 9$ must be $< 6^{k+1}$ because $5^k + 45 > 0$. If 6^{k+1} is greater than a larger total value $(5^{k+1} + 5^k + 45 +9)$, surely it must also be greater than a lesser valued part of the total $(5^{k+1} + 9)$

Thus, we have demonstrated that $5^{k+1} + 9 < 6^{k+1}$ for integers ≥ 2, as was to be shown.

5.3 #23b

Prove by mathematical induction

$N! > N^2$ for all integers $n \geq 4$.

Let the property p(n) be the sentence "$N! > N^2$, for all integers $n \geq 4$"

To establish P(4), we must show that $4! > 4^2$

$4*3*2*1 = 24$ $4^2 = 16$

24 > 16, so P(4) is true.

Suppose that P(k) is true for a particular but arbitrarily chosen integer $k \geq 4$ and suppose that

$K! > K^{2=}$

We must show that $(K+1)! > (K+1)^2$

We start with (K+1)! And $(K+1)^2$. We can subtract the later expression from the former to get

$(K+1)! – (K+1)^2$, which is equal to $(K+1)(K!) – (K+1)^2$ by factoring the K+1 out of (K+1)!

We can further factor a (K+1) out of $(K+1)^2$ and group the terms to get $(K+1)[K! – (K+1)]$.

Finally, we distribute the negative sign to get $(K+1) [K! – K – 1]$

We know from our supposition that $K! > K^2$, so we can say that

$(K+1)[K! – (K+1)] > (K+1)[K^2 – K – 1]$

We can complete the square on the right hand side to get

$(K+1)[(K – 1/2)^2 – 1 – 1/4]$, which simplifies to $(K+1)[(K – ½)^2 – 5/4]$

We know that $K \geq 4$, so $K- ½ \geq 3 ½$.

Therefore, $(K – ½)^2$ must be greater than 12 ¼ , by squaring both sides.

When we subtract the -5/4 from $)[(K – ½)^2 – 5/4]$, we get that $)[(K – ½)^2 – 5/4] \geq 11$

$(K + 1) \geq 5$, so if we multiplied $)[(K – ½)^2 – 5/4]$ and (K+1), the result must be ≥ 55.

If $)[(K – ½)^2 – 5/4] \geq 55$, surely it must be > 0

Coming back to the left hand of this inequality, we have

$(K+1)[K! – (K+1)] > [(K – ½)^2 – 5/4] > 0$

By chaining these comparisons, and changing $(K+1)[K! – (K+1)]$ back to its equivalent form $(K+1)! – (K+1)^2$, we can conclude that $(K+1)! – (K+1)^2 > 0$.

With this expression, we can simply add $(K+1)^2$ to both sides to get

$(K+1)! > (K+1)^2$, as was to be shown.

Thus, by the principal of mathematical induction, we have proven that $K! > K^2$ for all integers $\geq$ 4.