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Assignment 6 Part 2: Set 5.9 - 6, 10, 13.b, 16

5.9 #6:

Proof by structural induction: Let the property be the following sentence: The string begins in A.

Show that each object in the BASE for S satisfies the property: The only element in the base is a, and the string a begins in a.

Show that for each rule in the RECURSION for S, if the rule is applied to an object in S that satisfies the property then the objects defined by the rule also satisfy the property: The recursion for S consists of two rules denoted II(a) and II(b). Suppose s is a string in S that begins in the letter a. In the case where rule II(a) is applied to s, the result is the string aa, which also ends in a. In the case where rule II(b) is applied to s, the result is the string ab, which also begins in the letter a. Thus, when each rule in the recursion is applied to a string in S that begins in an A, the result is also a string that ends in an A. Because there is nothing in S except for the objects defined in I and II, every object in S satisfies the property of beginning in a.

10. Use structural induction to prove that every integer in S is also divisible by 5.

Show that each object in the BASE for S satisfies the property: There are two elements in the base for S, 0 and 5. 0 is divisible by everything with no remainder, so clearly 0 is divisible by 5. 5 is also divisible by 5, having a result of 1 with no remainder.

Show that for each rule in the RECURSION for S, if the rule is applied to an object in S that satisfies the property, then the objects defined by the rule also satisfy the property: The recursion for S consists of two rules denoted II(a) and II(b). Suppose that s is one element in S and t is another element in S. We can use 5 as s and 0 as t. We have already proven in the base step they satisfy the property. If we apply rule II(a), then s + t = 5 + 0 = 5, and we have already proven that 5 is divisible by 5, and hence fulfills the property. If we apply rule II(b), we have s – t = 5 – 0 = 5. We have already proven that 5 is divisible by 5, and hence fulfills the property. When each rule in the recursion is applied to any two elements in S, the result is a number that satisfies the property of being divisible of 5. Furthermore, because the Restriction stipulates that nothing is in S other than the objects defined in I and II above, we know that all integers in S satisfy the property of being divisible by 5.

13b. (())(())

1. By I, () is in P.

2. By I and II.a, (()) is also in P as (E), where E is defined as () as per 1.

3. By 2 and II.b (())(()) is in P as the product EF, where E is defined as (()) and F is defined as (()).

16. Give a recursive definition for the set of all strings of 0's and 1's for which all the 0's precede all the 1's.

BASE:

1. the null string $\varepsilon \in S$.

RECURSION: If $s \in S$, then

IIa. $0s \in S$ and b. $s1 \in S$

Where 0s and s1 are the concatenations of s with 0 and 1 respectively.

RESTRICTION

III. Nothing is in S other than the objects defined in I and II above.