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Assignment 2 Part 2: Set 4.1: 32, 61, Set 4.2: 20, 25, Set 4.6: 28 (Prove by Contraposition)

4.1 #32:

If a is any odd integer and b is any even integer, then 2a+3b is even.

We know that B is even. Any even number twice of some integer.

Let Q be an integer such that 2Q = B

2a+3b = 2a+6q

Using distributive law, 2a+6q = 2 (a+3q).

A+3q is an integer because all integers are closed under addition, subtraction, and multiplication.

Because 2a+3b = 2(a+3q), 2a+3b is twice that of an integer.

All numbers twice that of an integer are even.

Therefore 2a+3b is even.

4.1 #61:

Suppose that integer's m and n are perfect squares. Then m + n + 2 √mn is also a perfect square. Why?

If M and N are perfect squares, then they are integers such that P^2 = M and Q^2 = N. For example, if P is 3 and Q is 4, then M is 9 and N is 16.

M+ n+ 2 √mn = P^2 + Q^2 + 2√P^2Q^2

This can be simplified to P^2 + Q^2 + 2PQ.

This can be re-arranged to P^2 + 2PQ + Q^2

In general, (P+Q)^2 = P^2 + 2PQ + Q^2. We can see this is exactly the form of our re-arrangement.

Therefore, m+n+2 √mn is equal to (P+Q)^2, which makes it a perfect square.

4.2 #20:

Determine which of the statements in 15-20 are true and which are false. Prove each true statement directly from the definitions, and give a counterexample for each false statement. In case the statement is false, determine whether a small change would make it true. If so, make the change and prove the new statement. Follow the directions for writing proofs on page 154.

20. Given any two rational numbers r and s with r < s, there is another rational number between r and s.

I assume we are allowed to use the results of #18 and #19 as part of our proof.

As a result of #19, we know that for all real numbers a and b, if a < b, then a < (a+b) / 2 < b.

Let us convert a and b to r and s for the purposes of this problem. In any case, both sets AB and RS represent any arbitrary values, where one number (B or S) is greater than the other (A or R).

r < (r+s) / 2 < s

Let T = (r+s) / 2, the number in between r and s.

As a result of #18, we know that T is rational when R and S are any two rational numbers where R < S.

We have thus shown that T is both rational and in between R and S. This counts as a universal proof because r and s are specific but arbitrary values.

4.2 #25:

Derive the statements in 24-26 as corollaries of theorems 4.2.1, 4.2.2, and the results of exercises 12, 13, 14, 15 and 17.

If r is any rational number, then 3r^2 – 2r + 4 is rational.

Theorem 4.2.1 tells us that all integer is rational, so if we show that 3r^2 -2r + 4 is an integer, we have proven that 3r^2 -2r +4 is rational.

Theorem 4.2.2 tells us that the sum of all rational numbers are rational.

During the proof to theorem 4.2.2, we can see that the product of rational numbers are rational. I will prove this now.

> **Let R * S = (A / B) * (C / D).**
>
> **A, B, C, and D are integers because in 4.2 we defined a rational number as a fraction with integers in the numerator and denominator.**
>
> **We know from the assumptions of 4.1, as a law of basic algebra, that integers are closed among addition, subtraction, and multiplication, meaning that it would take division to possibly turn an integer into a non-integer. (I assume I can call upon this**

law as a corollary because it is listed as an assumption before we do ANY proofs. This would seem to indicate it is essential to do proofs.)

(A/B) * (C/D) = AC / BD because integers are closed during multiplication, meaning their products stay integers.

AC / BD is rational because it is the quotient of two integers, AC and BD.

Thus we can see for any arbitrary values, the product of two rational numbers are rational, thus it is universally proven.

3r^2 – 2r +4 is rational because we know r is rational, and we can completely break the expression down into the sums and products of rational numbers.

R ^ 2 = R * R, and R is a rational number, so R ^ 2 is rational.

3R^ 2 = a rational number because both 3 and R ^ 2 are rational.

3r^2 – 2R is rational because it would be equivalent to adding a -2R to 3r^2, and we have proven that the sum of 2 rational numbers is rational.

-2R is rational because both -2 and R are rational.

Finally, 3r^2 – 2R + 4 is rational because 4 is a rational number, and 3r^2 – 2R is also a rational number. Thus, their sums, (3r^2 – 2R + 4) is also rational.

4.6 #28:

28. Prove statement 28 by contraposition.

For all integers m and n, if mn is even, then m is even or n is even.

Suppose neither m nor n is even.

By De Morgan’s Law’s the negation of (even OR even) is NOT (Even ^ Even)

NOT (Even and Even) equates to ~Even and ~Even.

As mentioned on page 202, the quotient remainder law shows us by using a denominator of 2 that any integer is either even or odd.

Therefore, any integers that are not even or odd.

Therefore, ~Even and ~ Even = Odd and Odd.

Thus, if M and N are not even, they are odd.

Property #3 on page 167 tells us that the product of two odd numbers is also an odd number.

Thus, if neither m nor n are even, then the product, mn, must be odd.

This can be expressed in the form: If M and N are odd, the product is odd.

We have proven that if M and N are odd, then MN is odd. Therefore, we have also proven the logically equivalent contrapositive: If MN is even, M or N is even.

It is M or N is even, not M and N are even because according to De Morgan's Law's, the negation of $(X \wedge Y) = \sim (X \vee Y)$, where X and Y are two statements.