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Set 9.6 - 4, 12, 18

9.6 #4:

A camera shop stocks eight different types of batteries, one of which is type A7b. Assume there are at least 30 batteries of each type.

a. How many ways can a total inventory of 30 batteries be distributed among the eight different types?

Repetition allowed. Order does not matter.

$N = 8$  different types

$R = 30$  different batteries

$C(N+R-1, R) = C(30+8-1, 30) = C(37, 30) = 37! / (30! * 7!) = \mathbf{10,295,472}$  ways.

b. How many ways can a total inventory of 30 batteries be distributed among the eight different types if the inventory must include at least four A76 batteries?

4 batteries set, so the other 26 can be distributed.

$N = 8$  different types

$R = 26$  remaining batteries

$C(26+8-1, 26) = C(33, 26) = 33! / (26! * 7!) = \mathbf{4,272,048}$  ways.

c. How many ways can a total inventory of 30 batteries be distributed among the eight different types if the inventory includes at most three A7b batteries?

Ways with no a7b batteries + ways with 1 a7b battery + ways with 2 a7b batteries + ways with 3 a7b batteries.

In each of these ways,  $N = 7$  battery types

No a7b batteries

$R = 30$  batteries

$30 + 7 - 1 = 36$   $C(36, 30) = 36! / (30! * 6!) = 1,947,792$  ways

1 a7b battery

$R = 29$  batteries

$$29+7-1 = 35 \quad C(35,29) = 35! / (29! * 6!) = 1,623,160 \text{ ways}$$

2 a7b battery. R = 28 batteries

$$28 + 7 - 1 = 34. \quad C(34,28) = 34! / (28! * 6!) = 1,344,904 \text{ ways}$$

3 a7b battery. R = 27 batteries.

$$27+7-1 = 33 \quad C(33,27) = 33! / (27! * 6!) = 1,107,568 \text{ ways}$$

$$1,947,792 + 1,623,160 + 1,344,904 + 1,107,568 = \mathbf{6,023,424 \text{ ways.}}$$

9.6 #12:

Find how many solutions there are to the given equation that satisfy the given condition.

$$Y_1 + Y_2 + Y_3 + Y_4 = 30, \text{ each } Y_i \text{ is a nonnegative integer.}$$

$$\text{Each } Y_i \geq 0$$

$$30 + 4 - 1 = 33$$

$$C(33,30) = 33! / (30! * 3!) = \mathbf{5,456 \text{ different collections.}}$$

9.6 #18:

A large pile of coins consists of pennies, nickels, dimes, and quarters.

a. How many different collections of 30 coins can be chosen if there are at least 30 of each kind of coin?

N = 4 types of coins. Pennies, nickels, dimes, quarters.

R = 30 coins being chosen.

$$30+4-1 = C(33,30) = 33! / (30! * 3!) = 5,456 \text{ different collections.}$$

b. If the pile contains only 15 quarters but at least 30 of each other kind of coin, how many collections of 30 coins can be chosen?

Total set of collections of 30 coins – collections where there are at least 16 quarters.

If there are at least 16 quarters, then there are 14 other coins

$$C(14+4-1,14) = C(17,14) = 17! / (14! * 3!) = 680 \text{ collections.}$$

$$5456 - 680 = \mathbf{4,776 \text{ different collections}}$$

c. If the pile contains only 20 dimes but at least 30 of each other kind of coin, how many collections of 30 coins can be chosen?

Only 20 dimes = Total collection – collections with at least 21 dimes.

At least 21 dimes = 9 other coins being chosen.

$(4+9-1,9) = (12,9) = 12! / (9! * 3!) = 220$  different collections.

$5456 - 220 = \mathbf{5236}$  different collections

d. If the pile contains only 15 quarters and only 20 dimes but at least 30 of each other kind of coin, how many collections of 30 coins can be chosen?

Total collection – (At least 16 quarters + At least 21 dimes) =  $5456 - (680 + 220) = 5456 - 900 = \mathbf{4556}$  different collections