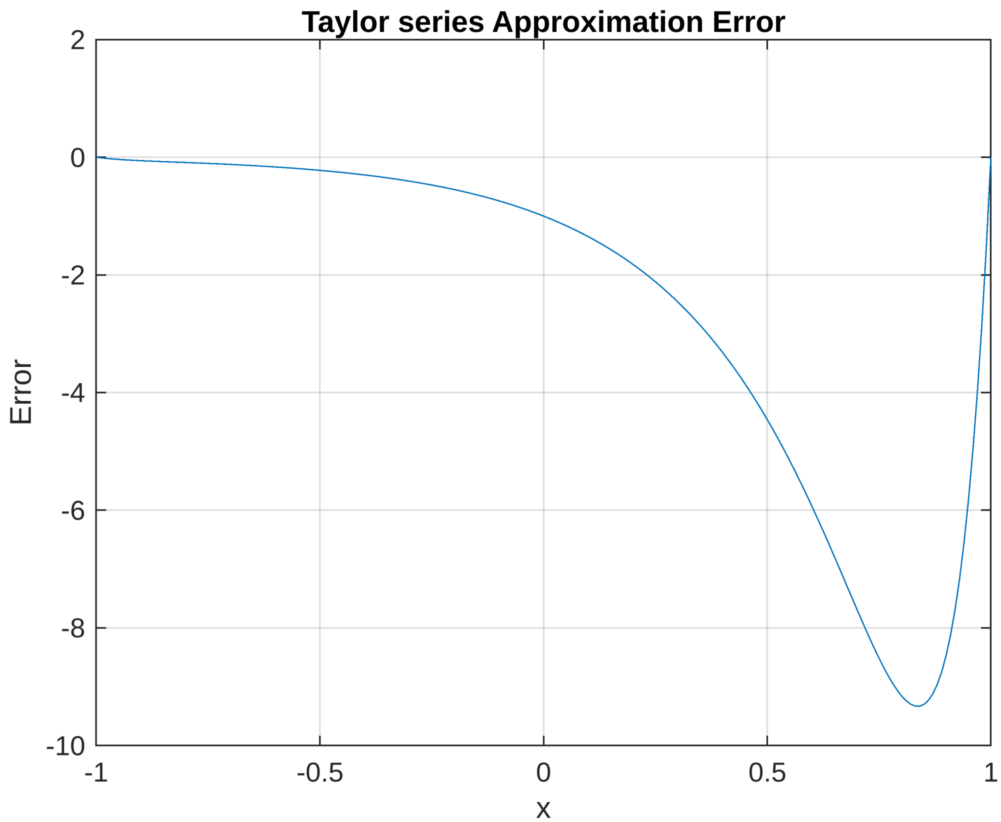
Problem 1: (Approximation theory, 25pts)

```
equispaced = zeros(1, 15);
for i = 0:14;
    j = i+1;
equispaced(j) = -1+2*i/14;
end

Chebyshev = zeros(1, 15);
for i = 0:14;
    j = i+1;
Chebyshev(j) = -cos((i*pi)/14);
end

Legendre =  [-0.987992518020485 -0.937273392400706 -0.848206583410427
-0.724417731360170 -0.570972172608539...
-0.394151347077563 0.570972172608539 -0.201194093997435 0.724417731360170 0
0.848206583410427 0.201194093997435 0.937273392400706...
0.394151347077563 0.987992518020485];
```

Part A

```
%f = exp(3*x);
val = linspace(-1, 1, 201);
pow = 3.^(1:14);
factorial = cumprod(1:14);
Coefficients = pow./factorial;
coeffs = [ones(1), Coefficients];
ex = exp(3*val);
taylor = polyval(coeffs, val);
error = taylor-ex;

figure;
plot(val, error)
xlabel('x');
ylabel('Error');
title('Taylor series Approximation Error');
grid on;
```



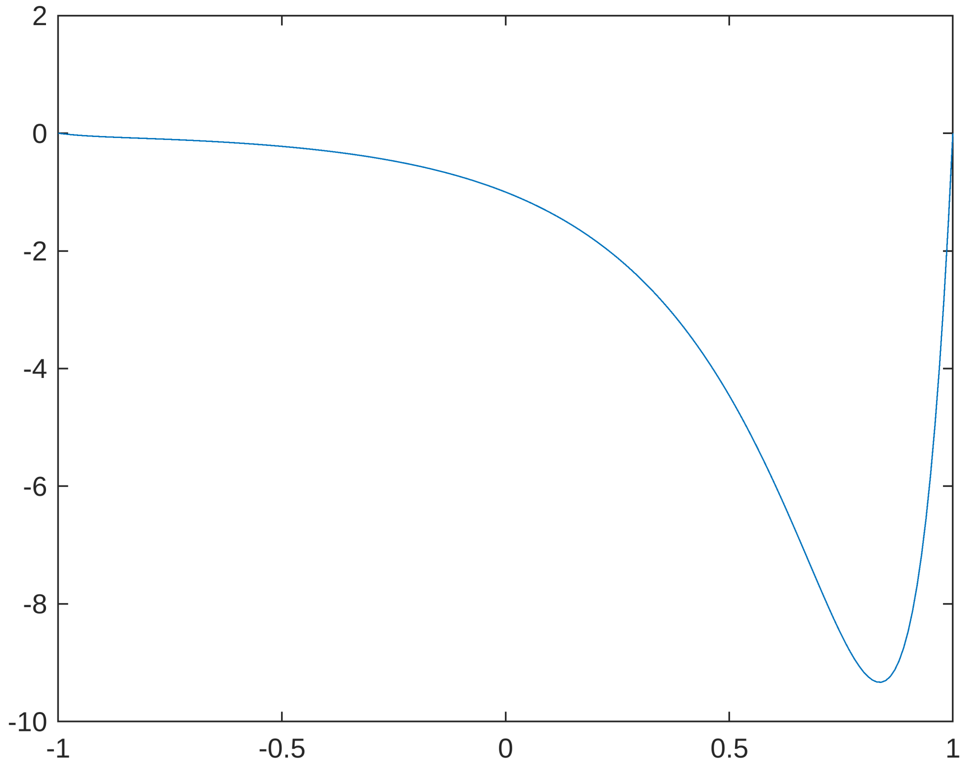

```
% calculate norm
norm_1 = norm(error, 1);
norm_2 = norm(error, 2);
norm_inf = norm(error, Inf);
table(norm_1,norm_2,norm_inf)
```

ans = 1x3 table

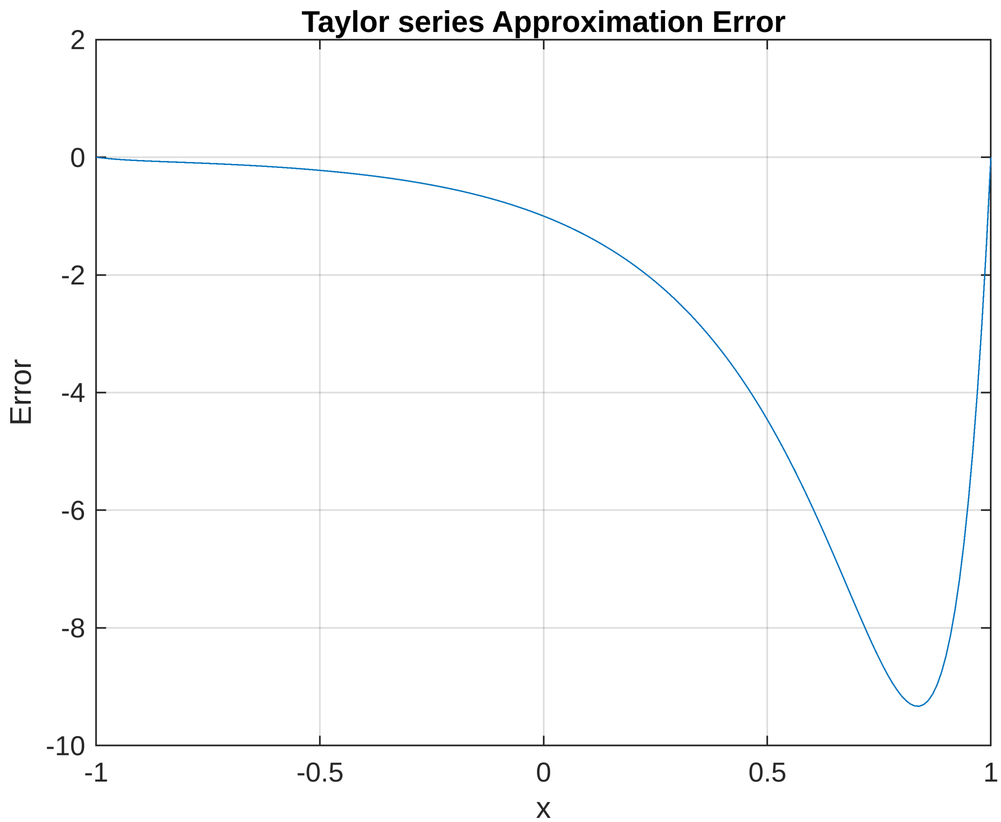
	norm_1	norm_2	norm_inf
1	495.9108	54.4286	9.3315

Part B

```
a = equispaced;
for k = 1:15
    f(k) = exp(3*equispaced(k));
end

Aproximation = baryinterp(a,f,val);
error = taylor - Aproximation;
plot(val, error);
```




```
% calculate norm
norm_1 = norm(error, 1);
norm_2 = norm(error, 2);
norm_inf = norm(error, Inf);
table(norm_1,norm_2,norm_inf);

b = Chebyshev;
for k = 1:15
    f(k) = exp(3*Chebyshev(k));
end

Aproximation = baryinterp(b,f,val);
error = taylor - Aproximation;

figure;
plot(val, error)
xlabel('x');
ylabel('Error');
title('Taylor series Approximation Error');
grid on;
```



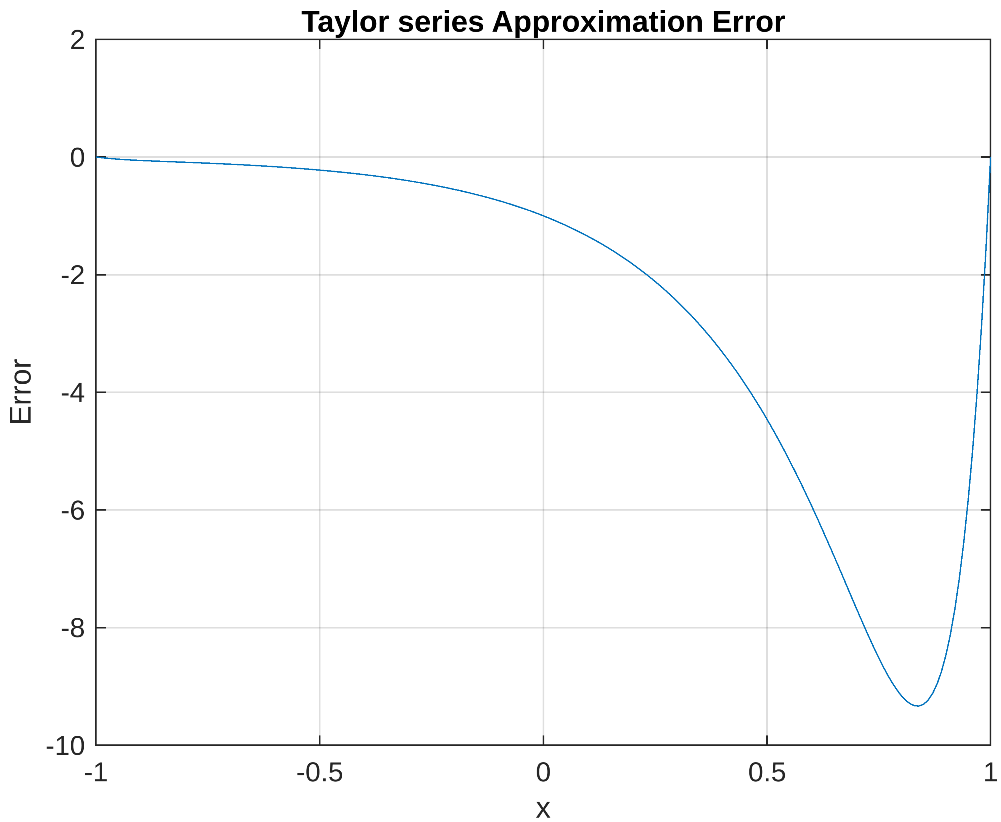

```
% calculate norm
norm_1 = norm(error, 1);
norm_2 = norm(error, 2);
norm_inf = norm(error, Inf);
table(norm_1,norm_2,norm_inf)
```

ans = 1x3 table

	norm_1	norm_2	norm_inf
1	495.9108	54.4286	9.3315

```
c = Legendre;
for l = 1:15
    f(l) = exp(3*Legendre(l));
end

Aproximation = baryinterp(c,f,val);
error = taylor - Aproximation;

figure;
plot(val, error)
xlabel('x');
ylabel('Error');
title('Taylor series Approximation Error');
```

```
grid on;
```




```
% calculate norm
norm_1 = norm(error, 1);
norm_2 = norm(error, 2);
norm_inf = norm(error, Inf);
table(norm_1,norm_2,norm_inf)
```

ans = 1x3 table

	norm_1	norm_2	norm_inf
1	495.9108	54.4286	9.3315

Part C

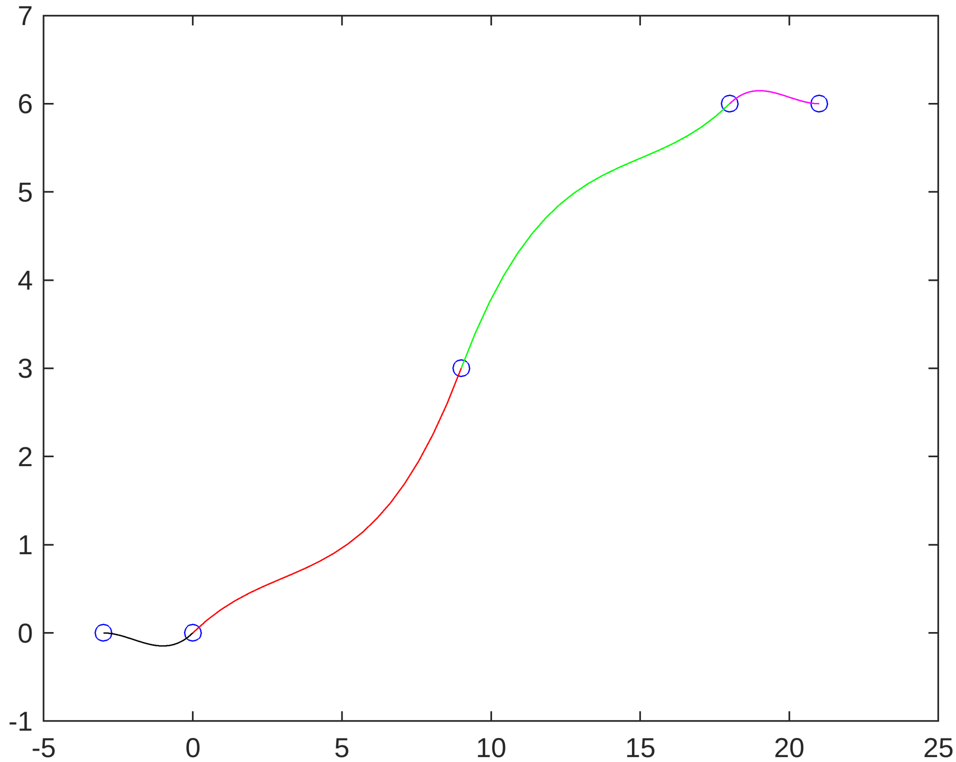
```
disp('I want to say that the when you have more nodes then Chebyshev is the best approach')
```

I want to say that the when you have more nodes then Chebyshev is the best approach

Problem 2: (Piecewise cubic Hermite interpolation, 15pts)

```
%% Define the data
x = [-3 0 9 18 21];
f = [0 0 3 6 6];
df = [0 1/3 19/21 1/3 0];
```

```
%% Find piecewise polynomial on the first interval:
h1 = x(2)-x(1);
delta1 = (f(2)-f(1))/h1;
e1 = f(1);
d1 = df(1);
c1 = (3*delta1 - 2*d1 - df(2))/h1;
b1 = (d1 - 2*delta1 + df(2))/h1^2;
H1 = @(t) e1 + (t-x(1)).*(d1 + (t-x(1)).*(c1 + (t-x(1))*b1));
xx1 = linspace(x(1),x(2),20);
%plot(x(1:2),f(1:2),'o',xx1,H1(xx1),'b-');

%% Find piecwise polynomial on the second interval:
h2 = x(3)-x(2);
delta2 = (f(3)-f(2))/h2;
e2 = f(2);
d2 = df(2);
c2 = (3*delta2 - 2*d2 - df(3))/h2;
b2 = (d2 - 2*delta2 + df(3))/h2^2;
H2 = @(t) e2 + (t-x(2)).*(d2 + (t-x(2)).*(c2 + (t-x(2))*b2));
xx2 = linspace(x(2),x(3),20);
%plot(x(2:3),f(2:3),'o',xx2,H2(xx2),'r-');
%% Find piecwise polynomial on the third interval:
h3 = x(4)-x(3);
delta3 = (f(4)-f(3))/h3;
e3 = f(3);
d3 = df(3);
c3 = (3*delta3 - 2*d3 - df(4))/h3;
b3 = (d3 - 2*delta3 + df(4))/h3^2;
H3 = @(t) e3 + (t-x(3)).*(d3 + (t-x(3)).*(c3 + (t-x(3))*b3));
xx3 = linspace(x(3),x(4),20);
%plot(x(3:4),f(3:4),'o',xx3,H3(xx3),'r-');
%% Find piecwise polynomial on the fourth interval:
h4 = x(5)-x(4);
delta4 = (f(5)-f(4))/h4;
e4 = f(4);
d4 = df(4);
c4 = (3*delta4 - 2*d4 - df(5))/h4;
b4 = (d4 - 2*delta4 + df(5))/h4^2;
H4 = @(t) e4 + (t-x(4)).*(d4 + (t-x(4)).*(c4 + (t-x(4))*b4));
xx4 = linspace(x(4),x(5),20);
%plot(x(4:5),f(4:5),'o',xx4,H4(xx4),'r-');
%% Plot the polynomials together
plot(x,f,'bo',xx1,H1(xx1),'k-',xx2,H2(xx2),'r-',xx3,H3(xx3),'g-',xx4,H4(xx4),
'm-')
```

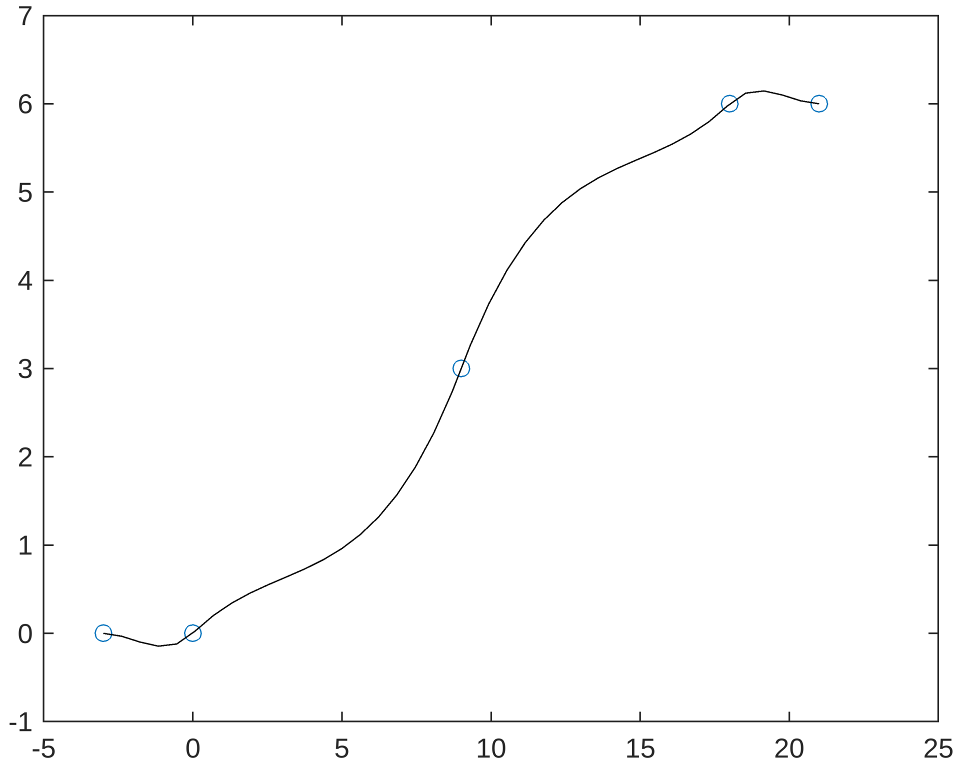


Part B

```
%% Matlab's piecewise polynomial features:
coeffs = [b1 c1 d1 e1; b2 c2 d2 e2; b3 c3 d3 e3; b4 c4 d4 e4];
pp = mkpp(x,coeffs);
xx = linspace(x(1),x(end),40);
```

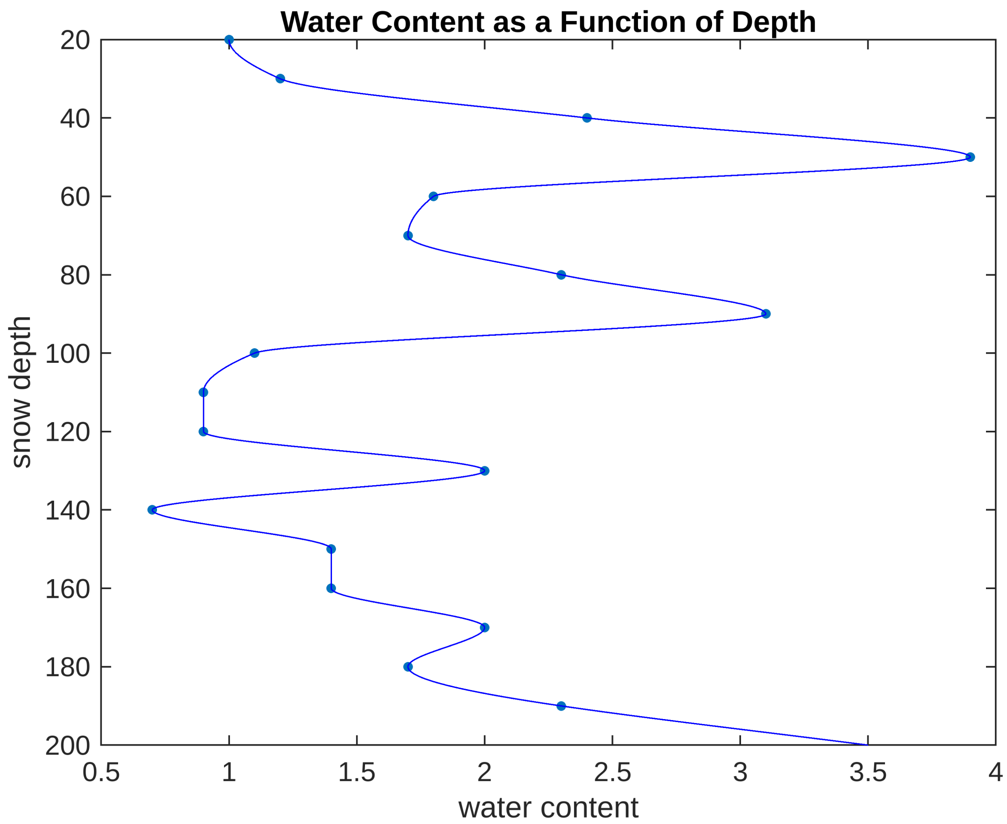
Part C

```
q = ppval(pp,xx);
plot(x,f,'o',xx,q,'k-')
```



Problem 3: (Interpolating snow data, 10 pts)

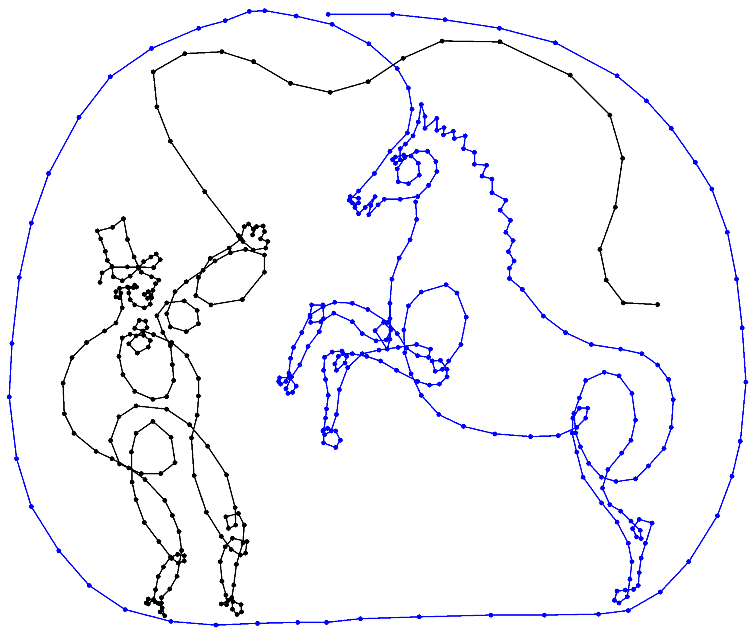
```
load snow.mat

d = snow(:,1);
p = snow(:,2);
s = linspace(20,200,1000);
ch = pchip(d,p,s);


plot(p,d,'.', 'markerSize', 12)
set(gca,'ydir','reverse')
hold on
plot(ch,s,'b')
set(gca,'ydir','reverse')
hold off
xlabel('water content')
ylabel('snow depth')
title('Water Content as a Function of Depth')
```



Problem 4: (Parametric interpolation, 15 pts)

Part A

```
cheval = load('cheval.dat');
dresseur = load('dresseur.dat');



plot(cheval(:,1),cheval(:,2),'.-b')
hold on;
plot(dresseur(:,1),dresseur(:,2),'.-k')


set(gca,'ydir','reverse');
axis off;
daspect([1 1 1])
hold off;
```



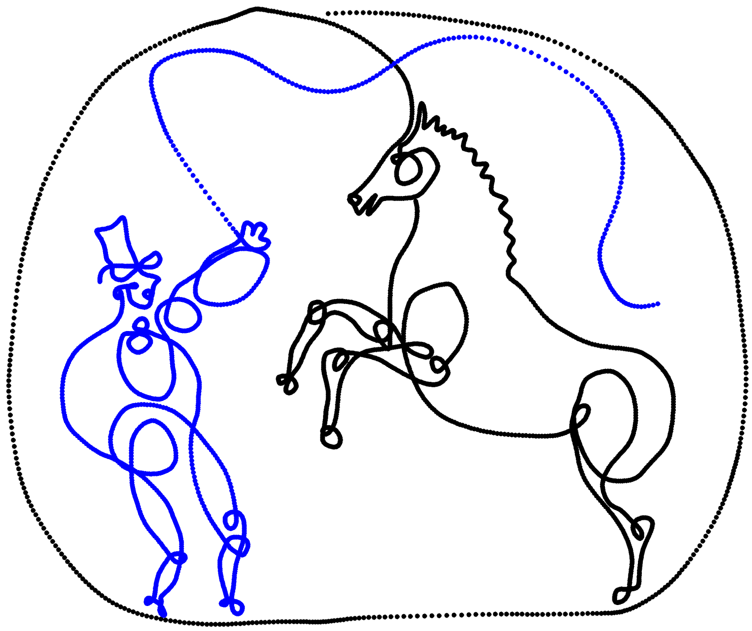
Part B

```
cd_lin = linspace(1,291,10*291);
zz = (1:size(cheval,1))';
chevalx = spline(zz,cheval(:,1));
chevaly = spline(zz,cheval(:,2));
ppx = ppval(chevalx,cd_lin);
ppy = ppval(chevaly,cd_lin);
plot(ppx,ppy,'.k')
set(gca,'ydir','reverse');
axis on;
daspect([1 1 1])
hold on;
```

Part C

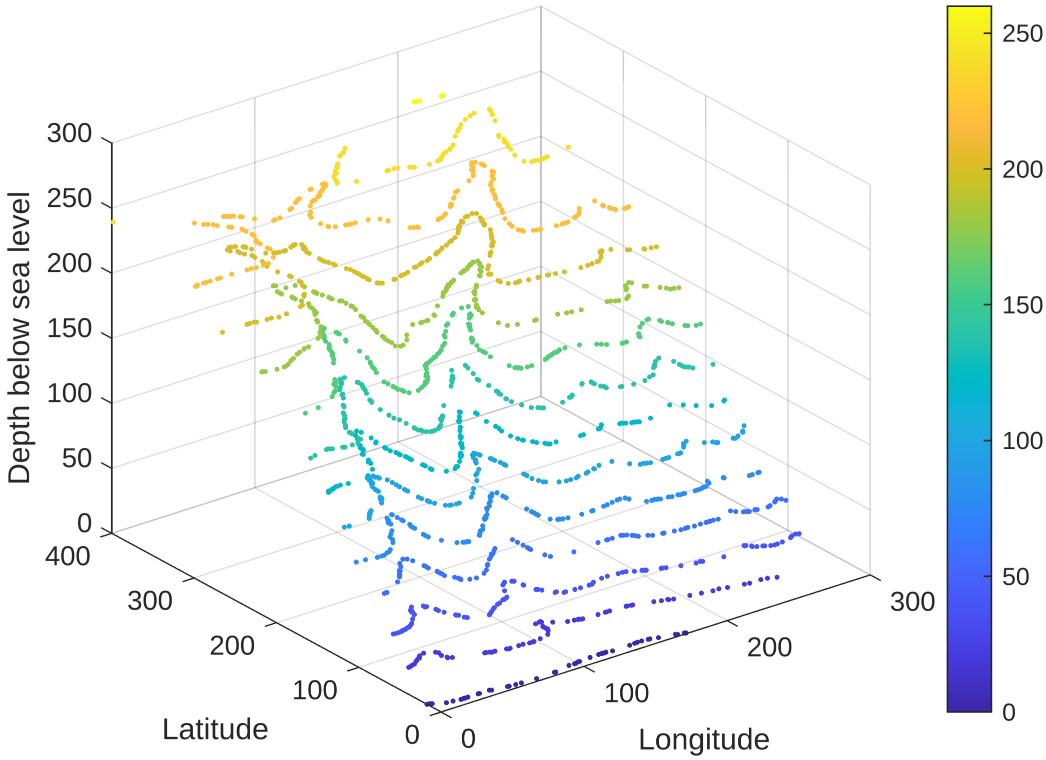
```
cd_lin = linspace(1,223,10*223);
zz = (1:size(dresseur,1))';
dresseurx = spline(zz,dresseur(:,1));
dresseury = spline(zz,dresseur(:,2));
ppx = ppval(dresseurx,cd_lin);
```

```
ppy = ppval(dresseury,cd_lin);
plot(ppx,ppy,'.b')
set(gca,'ydir','reverse');
axis off;
daspect([1 1 1])
hold off;
```



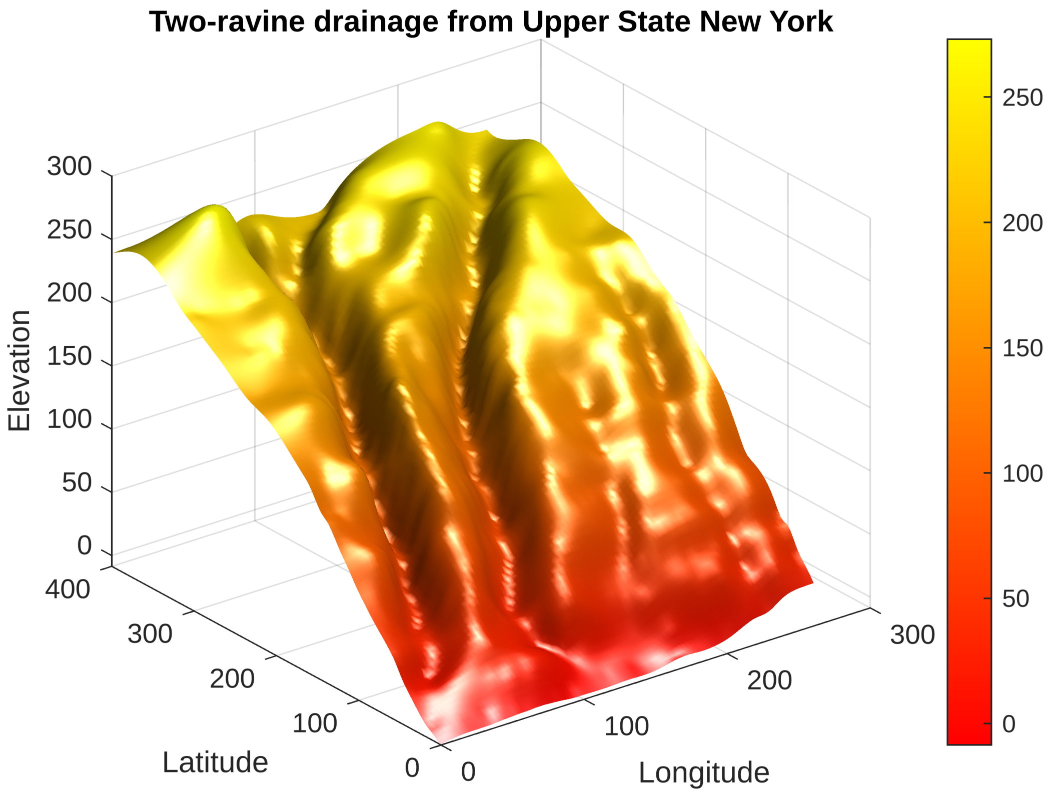
Problem 5: (Scattered data interpolation, 15pts)

```
%% Load and plot data
load topodata.mat
scatter3(x,y,z,40,z, '.');
colorbar
xlabel("Longitude")
ylabel("Latitude")
zlabel("Depth below sea level")
```

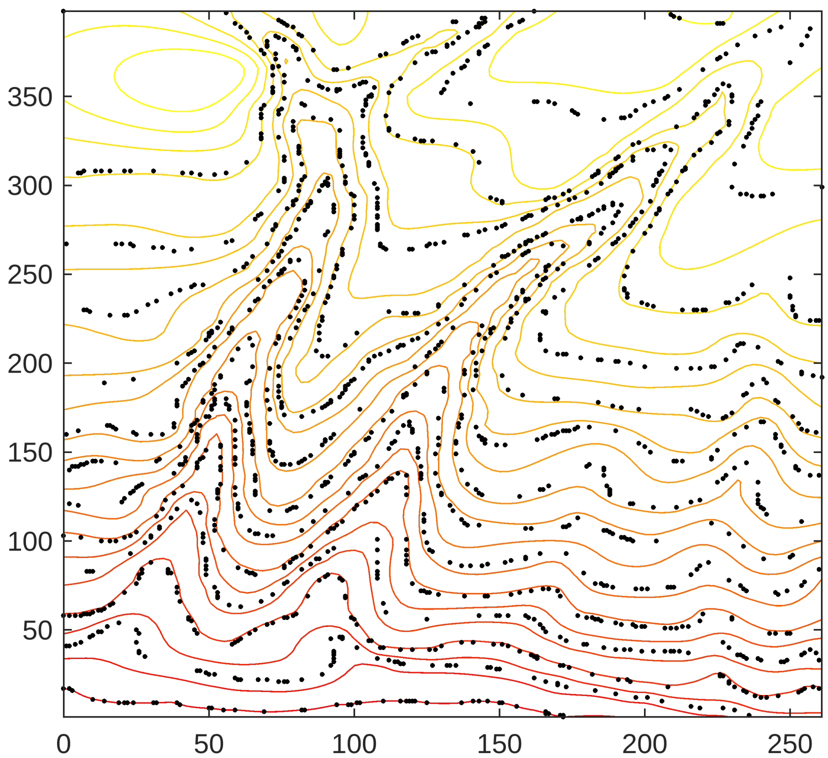



```
%% Fit the data with a radial basis function
coeffs = rbffit(x,y,z);
xmin = min(x);
xmax = max(x);
ymin = min(y);
ymax = max(y);
xgrid = linspace(xmin,xmax,100);
ygrid = linspace(ymin,ymax,100);
[xx,yy] = meshgrid(xgrid,ygrid);
ss = rbfval(coeffs,x,y,xx,yy);

%% Surface plot
surf(xx,yy,ss)
xlabel("Longitude")
ylabel("Latitude")
zlabel("Elevation")
title('Two-ravine drainage from Upper State New York')


% Beautify the plot
shading interp
camlight right
lighting phong
colorbar
```

```
colormap(autumn)
```




```
clc;

%% Contour plot
contour(xx,yy,ss,linspace(0,260,20))
hold on
plot(x,y,'k.')
hold off
```


350
300
250
200
150
100
50
0
50
100
150
200
250