Adaptive formation tracking control for UAV swarm systems with multiple leaders and switching topologies

1st Yuxin Xie
Sino-French Engineer School
Beihang University
Beijing, China
xie_yuxin@buaa.edu.cn

2nd Liang Han*
Sino-French Engineer School
Beihang University
Beijing, China
liang_han@buaa.edu.cn

3rd Lei Zhang
Traffic Control Technology Co.
Beijing, China
leii.zhang@bj-tct.com

4th Xiwang Dong
School of Automation Science and Electronic Engineering
Beihang University
Beijing, China

5th Qingdong Li
School of Automation Science and Electronic Engineering
Beihang University
Beijing, China

6th Zhang Ren
School of Automation Science and Electronic Engineering
Beihang University
Beijing, China

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***Abstract*—Adaptive formation tracking problems for UAV swarm systems are studied. An adaptive formation tracking protocol using neighboring relative state information is proposed, which can adjust the time-varying coupling weights among neighboring UAVs. It is proved that the adaptive formation tracking can be achieved under the proposed protocol. Finally, numerical simulation examples are given to demonstrate the effectiveness of the theoretical results.**

***Index Terms*—formation tracking control, UAV swarm systems, adaptive control, switching topology, multiple leaders**

I. INTRODUCTION

In the past decades, the cooperative control of swarm systems has attracted considerabel attention for the wide applications in unmanned aerial vehicles (UAVs) [1]-[4], mobile robots [5]-[7], unmanned ground vehicles (UGVs) [8], unmanned underwater vehicles (UUVs) [9], [10], etc. Distinct from a single UAV, a UAV swarm system can perform tasks through cooperation. With the cooperative control of UAV swarm systems, the conflicts among the UAVs are avoided and task efficency is improved [11], [12].

The formation tracking control is one of the main branches of the cooperative control, which is investigated in [13] - [20]. In [13], the formation tracking control protocol that uses only the relative state information of neighboring agents is proposed for swarm systems with switching topologies. The formation tracking is proved to be achieved under specific conditions of parameters. In [14], time-varying coupling weights are introduced to adjust the communication stength in the formation tracking control protocol for swarm systems with one leader. In [15], a time-varying adaptive formation tracking controller is designed for swarm systems with multiple leaders. It is proved that the followers can track the leaders and form the desired formation simultaneously. In [16]-[20], more formation tracking methods are considered, such as event-triggered control, fault-tolerant control, and sliding-mode control.

In the references above, the adaptive control method is not involved in [13]; only one leader is considered in [14]; the communication topology is fixed in [15]. However, UAV swarm systems with multiple leaders and switching topologies are common in engineering application. In this case, the communication topology is complex and variable, while the adaptive control method can be used to adjust the variances of the topology. In this paper, an adaptive formation tracking control protocol is designed for UAV swarm systems with multiple leaders, where the leaders have the unknown bounded control input, the followers track the weighted average position of the multiple leaders, and the communication topology is switching and weight-variable.

The paper is organized as follows: in section II, the preliminaries and problem statement are given; in section III, an adaptive formation tracking protocol is designed and the stability of the UAV swarm system using the proposed protocol is analyzed; in section IV, numerical simulation examples are given to show the effectiveness of the proposed theoretical results; in section V, a brief conclusion is made.

*Corresponding author: Liang Han(liang_han@buaa.edu.cn)

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II. PRELIMINARIES AND PROBLEM STATEMENT

In this section, the preliminaries and problem statement are given. Throughout the whole paper, I_n stands for an identify matrix of size $n \times n$, $\mathbf{1}_n$ denotes a column vector of n entries with only 1 as elements, $\mathrm{sgn}(\cdot)$ represents the sign function, and $\otimes$ denotes the Kronecker product.

A. Graph theory

The communication topology of a UAV system can be described by a weighted graph $\mathcal{G} = \{\mathcal{N}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{N} = \{n_1, n_2, \cdots, n_N\}$ is the set of the nodes corresponding to the N UAVs, $\mathcal{E} \subseteq \{(n_i, n_j) : n_i, n_j \in \mathcal{N}\}$ is the set of the edges corresponding to the interaction among the UAVs, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is a weighted adjacency matrix, gathering the nonnegative weights a_{ij}. The in-degree matrix is defined as $\mathcal{D} = \mathrm{diag}\{D_1, D_2, \cdots, D_N\}$, and the Laplacian matrix $L = [w_{ij}]_{i,j \in \{1,2,\cdots,N\}}$ is caculated by $L = \mathcal{D} - \mathcal{A}$, where $D_i = \sum_{j=1}^{N} a_{ij}$. There exists a directed path from node n_i to n_j if all the node pairs $(n_i, n_l), (n_l, \cdots n_m) \cdots (n_n, n_j)$, arranging in order end to end, belong to the node set. n_i is called as the neighbor of n_j when $(n_i, n_j) \in \mathcal{E}$. A graph is called connected if there exists a path between every two nodes. Based on the definition of neighbor, the UAVs are identified as followers or leaders, where a leader has no neighbor and a follower has at least one neighbor.

Suppose that the communication graph is switching. Let t_k denote the time of the k^{th} $(k \in \mathbb{N})$ switch, where $0 < \tau_0 \leq t_{k+1} - t_k$ for $k \in \mathbb{N}$, and τ_0 is the dwell time. The communication graph is supposed to be fixed during the time interval τ_0. Let $\mathcal{G}^{\sigma(t)}$ denote the graph at time t, where $\sigma : [0, +\infty) \to \{1, 2, \cdots, s\}$ is the switching signal function and $s \in \mathbb{N}^*$ is the number of the possible switching graphs. $\sigma(t)$ is equal to the superscript of the graph at time t. The related Laplacian matrix of $\mathcal{G}^{\sigma(t)}$ is denoted as $L^{\sigma(t)} = [w_{ij}^{\sigma(t)}] \in \mathbb{R}^{N \times N}$, which is written as

$$L^{\sigma(t)} = \begin{bmatrix} L_1^{\sigma(t)} & L_2^{\sigma(t)} \\ 0_{N_E \times N_F} & 0_{N_E \times N_E} \end{bmatrix}, \tag{1}$$

where $L_1^{\sigma(t)} \in \mathbb{R}^{N_F \times N_F}$ reflects the interaction among the followers and $L_2^{\sigma(t)} \in \mathbb{R}^{N_F \times N_E}$ reflects the interaction between each follower and each leader. Suppose that the edges connecting the followers are undirected, then $L_1^{\sigma(t)} \in \mathbb{R}^{N_F \times N_F}$ is symmetric.

B. Problem statement

Consider a UAV swarm system, including of N UAVs with N_E leaders and N_F followers satisfying $N = N_F + N_E$, where $N, N_F, N_E \in \mathbb{N}^*$. Based on the graph theory, the UAV swarm system is described by a connected and switching graph. The nodes $n_1, n_2, \cdots, n_{N_F}$ of the topology graph represent the followers, and the nodes $n_{N_F+1}, n_{N_F+2}, \cdots, n_N$ represent the leaders. The weight of the edge between the node n_i and n_j is defined as

$$a_{ij} = \begin{cases} b_i b_j \neq 0 \text{ if } j \in \{N_F+1, N_F+2, \cdots, N\} \text{ and } \\ \qquad (j,i) \in \mathcal{E}; \\ b_i = 0 \text{ if } j \in \{N_F+1, N_F+2, \cdots, N\} \text{ and } \\ \qquad (j,i) \notin \mathcal{E}; \\ a_{ji} \neq 0 \text{ if } i,j \in \{1,2,\cdots,N_F\} \text{ and } (j,i) \in \mathcal{E}; \\ a_{ji} = 0 \text{ if } i,j \in \{1,2,\cdots,N_F\} \text{ and } (j,i) \notin \mathcal{E}. \end{cases} \tag{2}$$

where $a_{ij}, b_i \in \mathbb{R}$ are nonnegative constants, and $b_j \in \mathbb{R}$ satisfies that $b_j > 0$ and $\sum_{j=N_F+1}^{N} b_j = 1$. The switching topology of the UAV swarm system is represented by the graph $\mathcal{G}^{\sigma(t)}$, where $\mathcal{G}^{\sigma}(t) \in \{\mathcal{G}^1, \mathcal{G}^2, \cdots, \mathcal{G}^s\}$, and the corresponding Laplacian matrix is wriiten in Eq. 1.

The dynamics of each UAV i $(i \in \{1, 2, \cdots, N\})$ in the space rectangular coordinate system O-XYZ is described by the following equations [?]

$$\begin{aligned} \dot{x}_i(t) &= v_{ix}(t) \\ \dot{v}_{ix}(t) &= (-\phi_i(t)\sin(\varphi_i(t)) - \theta_i(t)\cos(\varphi_i(t)))g \\ \dot{y}_i(t) &= v_{iy}(y) \\ \dot{v}_{iy}(t) &= (-\vartheta_i(t)\sin(\varphi_i(t)) + \phi_i\cos(\varphi_i(t)))g \\ \dot{z}_i(t) &= v_{iz}(t) \\ \dot{v}_{iz}(t) &= \frac{\Delta F_i(t)}{m_i} \end{aligned} \tag{3}$$

where $x_i(t), y_i(t)$, and $z_i(t)$ are the positions in X, Y, and Z directions respectively. $v_{ix}(t), v_{iy}(t)$, and $v_{iz}(t)$ are the velocities in X, Y, and Z directions respectively. $\phi_i(t), \vartheta_i(t)$, and $\varphi_i(t)$ are the roll, pitch, and yaw angle respectively. g is the gravity acceleration. $\Delta F_i(t)$ is the vertical resultant force. m_i is the mass of UAV i.

Define the state information of each UAV as $s_i(t) = [s_{ix}^T(t), s_{iy}^T(t), s_{iz}^T(t)]^T$, where $s_{ix}(t) = [x_i(t), v_{ix}(t)]^T$,$s_{iy}(t) = [y_i(t), v_{iy}(t)]^T$, and $s_{iz}(t) = [z_i(t), v_{iz}(t)]^T$. Let $u_i(t) = [u_{ix}(t), u_{iy}(t), u_{iz}(t)]^T$ be the control input of each UAV i. As the attitude dynamics is negligible compared to the trajectory dynamics, only the positions and velocities are considered in this paper. Then the dynamic model is simplified into the following double integrator:

$$\dot{s}_i(t) = A s_i(t) + B u_i(t) \tag{4}$$

where $i \in \{1, 2, \cdots, N\}$, $A = I_3 \otimes \bar{A} = I_3 \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = I_3 \otimes \bar{B} = I_3 \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. For each leader l $(l \in \{N_F+1, N_F+2, \cdots, N\})$, the control input $u_l(t)$ is a bounded unknown vector.

Let time-varying vector $h_i(t) = [h_{ix}^T(t), h_{iy}^T(t), h_{iz}^T(t)]^T$ denote the formation vector of the follower i $(i \in \{1, 2, \cdots, N_F\})$, where $h_{ix}(t) = [h_{ipx}(t), h_{ivx}(t)]^T$, $h_{iy}(t) = [h_{ipy}(t), h_{ivy}(t)]^T$, and $h_{iz}(t) = [h_{ipz}(t), h_{ivz}(t)]^T$, and the time-varying formation vectors of all the followers are represented by $h_F(t) = [h_1^T(t), h_2^T(t), \cdots, h_{N_F}^T(t)]^T$.

Definition 1. *The time-varying formation tracking for the UAV swarm system (4) with multiple leaders is achieved if for any given bounded initial states and any $i \in \{1, 2, \cdots, N_F\}$*

$$\lim_{t\to\infty}(s_i(t) - h_i(t) - \sum_{l=N_F+1}^{N} \delta_l s_l(t)) = 0, \tag{5}$$

where δ_l ($l \in \{N_F+1, N_F+2, \cdots, N\}$) are positive constants satisfying $\sum_{l=N_F+1}^{N} \delta_l = 1$.

This paper aims at studying the formation tracking problems for UAV swarm systems with switching topologies and multiple leaders by: (i) designing a control protocol using neighboring state information, and coupling weights of the communication graph; (ii) analyzing the stability of the UAV swarm systems under the formation tracking protocol; (iii) performing simulation experiments to verify the formation tracking control method.

III. ADAPTIVE FORMATION TRACKING CONTROL

Consider the following adaptive formation tracking protocol of the follower i ($i \in \{1, 2, \cdots, N_F\}$)

$$\begin{cases} u_i(t) = K\sum_{j=1}^{N_F} c_i(t)a_{ij}((s_i(t) - h_i(t)) - (s_j(t) - h_j(t))) \\ \qquad + K\sum_{l=N_F+1}^{N} c_i(t)a_{il}(s_i(t) - h_i(t) - s_l(t)) \\ \qquad + f(t)\mathrm{sgn}(K\xi_i(t)) \\ \dot{c}_i(t) = \xi_i^T(t)T\xi_i(t) \end{cases}, \tag{6}$$

where $K = I_3 \otimes \bar{K}$ is the constant gain matrix with $\bar{K} \in \mathbb{R}^{1\times 2}$, $c_i(t)$ is the coupling weight of the UAV i satisfying $c_i(0) > 0$, $T = I_3 \otimes \bar{T}$ is the constant controller matrix with $\bar{T} \in \mathbb{R}^{2\times 2}$, $f(t)$ is a positive function to be determined, and $\xi_i(t)$ is the formation tracking error which is defined as:

$$\begin{aligned} \xi_i(t) =& \sum_{j=1}^{N_F} a_{ij}((s_i(t) - h_i(t)) - (s_j(t) - h_j(t))) \\ &+ \sum_{l=N_F+1}^{N} a_{il}(s_i(t) - h_i(t) - s_l(t)). \end{aligned} \tag{7}$$

Under protocol (6), Eq. (4) becomes into

$$\begin{cases} \dot{s}_i(t) = As_i(t) + Bf(t)\mathrm{sgn}(K\xi_i(t)) \\ \qquad + BK\sum_{j=1}^{N_F} c_i(t)a_{ij}((s_i(t) - h_i(t)) - (s_j(t) - h_j(t))) \\ \qquad + BK\sum_{l=N_F+1}^{N} c_i(t)a_{il}(s_i(t) - h_i(t) - s_l(t)) \\ \dot{c}_i(t) = \xi_i^T(t)T\xi_i(t). \end{cases} \tag{8}$$

Denote $U_E(t) = [u_{N_F+1}(t)^T, u_{N_F+2}^T(t), \cdots, u_N^T(t)]^T$. Then the system (4) turns to

$$\begin{cases} \dot{X}_F(t) = (I_{N_F} \otimes A + L_1^{\sigma(t)}C \otimes BK)X_F(t) \\ \qquad - (CL_1^{\sigma(t)} \otimes BK)h_F(t) \\ \qquad + (I_{N_F} \otimes B)f(t)\mathrm{sgn}((I_{N_F} \otimes K)\xi(t)) \\ \qquad + (CL_2^{\sigma(t)} \otimes BK)X_E(t) \\ \dot{X}_E(t) = (I_{N_E} \otimes A)X_E(t) + (I_{N_E} \otimes B)U_E(t). \end{cases} \tag{9}$$

Assumption 1. *At least one leader can provide a directed path to the followers. Each uninformed follower can provide at least one directed path to a well-informed one.*

Assumption 2. *Each row sum of the Laplacian matrix $L^{\sigma(t)}$ is zero.*

Lemma 1. *If Assumption 1 and Assumption 2 hold simultaneously, $-L_1^{\sigma(t)^{-1}} L_2^{\sigma(t)}$ can be represented by*

$$-L_1^{\sigma(t)^{-1}} L_2^{\sigma(t)} = \mathbf{1}_{N_F}[b_{N_F+1}^{\sigma(t)}, b_{N_F+2}^{\sigma(t)}, \cdots, b_N^{\sigma(t)}], \tag{10}$$

where b_l are positive constants satisfying $\sum_{l=N_F+1}^{N} b_j = 1$.

Define

$$\begin{aligned} &\varphi_i(t) = s_i(t) - h_i(t) \quad \text{for } i \in \{1, 2, \cdots, N_F\}, \\ &\varphi_F(t) = [\varphi_1^T(t), \varphi_2^T(t), \cdots, \varphi_{N_F}^T(t)]^T, \\ &C(t) = \mathrm{diag}\{c_1(t), c_2(t), \cdots, c_{N_F}(t)\}. \end{aligned} \tag{11}$$

The UAV swarm system (9) is transformed into

$$\begin{cases} \dot{\varphi}(t) = (I_{N_F} \otimes A + CL_1^{\sigma(t)} \otimes BK)\varphi(t) \\ \qquad + (I_{N_F} \otimes A)h_F(t) - \dot{h}_F(t) \\ \qquad + (I_{N_F} \otimes B)f(t)\mathrm{sgn}((I_{N_F} \otimes K)\xi(t)) \\ \qquad + (CL_2^{\sigma(t)} \otimes BK)X_E(t) \\ \dot{X}_E(t) = (I_{N_E} \otimes A)X_E(t) + (I_{N_E} \otimes B)U_E(t). \end{cases} \tag{12}$$

Let $\xi_F(t) = [\xi_1^T(t), \xi_2^T(t), \cdots, \xi_{N_F}^T(t)]^T$, the formation tracking error can be defined by

$$\xi_F(t) = (L_1^{\sigma(t)} \otimes I_n)\varphi(t) + (L_2^{\sigma(t)} \otimes I_n)X_E(t) \tag{13}$$

and we have

$$\varphi(t) = (L_1^{\sigma(t)^{-1}} \otimes I_n)\xi_F(t) - (L_1^{\sigma(t)^{-1}} L_2^{\sigma(t)} \otimes I_n)X_E(t). \tag{14}$$

Lemma 2. *The UAV swarm system (4) achieves the time-varying formation tracking if*

$$\lim_{t\to\infty} \xi_F(t) = 0. \tag{15}$$

Proof. According to the definition of the formation tracking error (13), by pre-multiplying $(L_1^{\sigma(t)^{-1}} \otimes I_n)$, one can get

$$\begin{aligned} &(L_1^{\sigma(t)^{-1}} \otimes I_n)\xi_F(t) \\ =&X_F(t) - h_F(t) + (L_1^{\sigma(t)^{-1}} L_2^{\sigma(t)} \otimes I_n)X_E(t). \end{aligned} \tag{16}$$

If the condition $\lim_{t\to\infty} \xi_F(t) = 0$ is satisfied, one can get that

$$\lim_{t\to\infty}(X_F(t) - h_F(t) - (L_1^{\sigma(t)^{-1}} L_2^{\sigma(t)} \otimes I_n)X_E(t)) = 0. \tag{17}$$

By applying Lemma 1, one can obtain that

$$\lim_{t\to\infty}(\varphi(t) - ([b_{N_F+1}^{\sigma(t)}, b_{N_F+2}^{\sigma(t)}, \cdots, b_N^{\sigma(t)}] \otimes I_n) X_E(t)) = 0, \tag{18}$$

which leads to

$$\lim_{t\to\infty}(s_i(t) - h_i(t) - \sum_{l=N_F+1}^{N} \alpha_l^{\sigma(t)} s_l(t)) = 0. \tag{19}$$

Therefore, the proof of Lemma 2 is completed.

To analyze the time limit of the formation tracking error, the derivate of $\xi_F(t)$ can be calculated by

$$\begin{aligned}\dot{\xi}_F(t) =& (I_{N_F} \otimes A + L_1^{\sigma(t)} C \otimes BK)\xi_f(t) \\ &+ (L_2^{\sigma(t)} \otimes B)U_E(t) \\ &+ f(t)(L_1^{\sigma(t)} \otimes B)\mathrm{sgn}((I_{N_F} \otimes K)\xi_F(t)) \\ &+ (L_1^{\sigma(t)} \otimes I_n)[(I_{N_F} \otimes A)h_F(t) - \dot{h}_F(t)].\end{aligned} \tag{20}$$

Theorem 1. *The parameters K and T in the protocol (6) are given by $\bar{K} = -\beta R^{-1} B^T P$, and $\bar{T} = \beta R^{-1} P B B^T P$, where β is a positive constant and P is the solution of the following algebraic Riccati equation (ARE):*

$$A^T P + PA - R^{-1} P B B^T P + Q = 0 \tag{21}$$

with $Q, R \in \mathbb{R}^2$ satisfying $Q = Q^T > 0$ and $R > 0$. Then the UAV swarm system (4) under the protocol (6) achieves the time-varying formation tracking control if the following conditions hold:

1) *the formation feasibility condition:*

$$\lim_{t\to\infty}(Ah_i(t) - \dot{h}_i(t)) = 0, \tag{22}$$

2) *the dwell-time condition*

$$\tau_0 > \ln r_2 / r_1, \tag{23}$$

where $r_1 \in \mathbb{R}$ satisfying $r_1 > 0$ and $r_2 = \lambda_{\max}(L_1^{\sigma(t)^{-1}} \otimes P)/\lambda_{\min}(L_1^{\sigma(t)^{-1}} \otimes P)$.

Proof. If the formation feasibility condition is satisfied, it yields that

$$\lim_{t\to\infty}(L_1^{\sigma(t)} \otimes I_n)[(I_{N_F} \otimes A)h_F(t) - \dot{h}_F(t)] = 0. \tag{24}$$

Next analyze the stability of the following system

$$\begin{aligned}\dot{\xi}_F(t) =& (I_{N_F} \otimes A + L_1^{\sigma(t)} C \otimes BK)\xi_F(t) \\ &+ (L_2^{\sigma(t)} \otimes B)U_E(t) \\ &+ f(t)(L_1^{\sigma(t)} \otimes B))\mathrm{sgn}((I_{N_F} \otimes K)\xi_F(t)).\end{aligned} \tag{25}$$

Consider the following Lyapunov function

$$V(t) = \xi_F^T(t)(L_1^{\sigma(t)^{-1}} \otimes P)\xi_F(t) + (C - \gamma I_{N_F})^T (C - \gamma I_{N_F}). \tag{26}$$

Denote $V_1(t) = \xi_F^T(t)(L_1^{\sigma(t)^{-1}} \otimes P)\xi_F(t)$ and $V_2(t) = (C - \gamma I_{N_F})^T (C - \gamma I_{N_F})$. By taking the derivative of $V_1(t)$ and $V_2(t)$, one gets

$$\begin{aligned}\dot{V}_1(t) =& \xi_F^T(t)(L_1^{\sigma(t)^{-1}} \otimes (A^T P + PA)) + 2C \otimes PBK)\xi_F(t) \\ &+ 2\xi_F^T(t)(L_1^{\sigma(t)^{-1}} L_2^{\sigma(t)} \otimes PB)U_E(t) \\ &+ 2f(t)\xi_F^T(t)(I_{N_F} \otimes PB)\mathrm{sgn}((I_{N_F} \otimes K)\xi_F(t))\end{aligned} \tag{27}$$

and

$$\dot{V}_2(t) = \xi_F^T(t)(2C \otimes T - 2\gamma I_{N_F} \otimes T)\xi_F(t). \tag{28}$$

Note that $T = -PBK$, we have

$$\begin{aligned}\dot{V}(t) =& \xi_F^T(t)(L_1^{\sigma(t)^{-1}} \otimes (A^T P + PA)) - 2\gamma I_{N_F} \otimes T)\xi_F(t) \\ &+ 2\xi_F^T(t)(L_1^{\sigma(t)^{-1}} L_2^{\sigma(t)} \otimes PB)U_E(t) \\ &+ 2f(t)\xi_F^T(t)(I_{N_F} \otimes PB)\mathrm{sgn}((I_{N_F} \otimes K)\xi_F(t)).\end{aligned} \tag{29}$$

As K is chosen as negative, by applying Lemma 1, one can obtain

$$\begin{aligned}\dot{V}(t) \leq& \xi_F^T(t)(L_1^{\sigma(t)^{-1}} \otimes (A^T P + PA) - 2\gamma I_{N_F} \otimes T))\xi_F(t) \\ &- 2(f(t) - \|u_l(t)\|_\infty) \sum_{i=1}^{N_F} \|\xi_i^T(t) PB\|_1,\end{aligned} \tag{30}$$

as u_l is bounded. Choose $f(t) > \|u_l(t)\|_\infty$, the inequality (30) is simplified into

$$\dot{V}(t) \leq \xi_F^T(t)[L_1^{\sigma(t)^{-1}} \otimes (A^T P + PA) - 2\gamma I_{N_F} \otimes T]\xi_F(t). \tag{31}$$

Denote $Q_F^{\sigma(t)}$ as a nonsingular matrix satisfying

$$Q_F^{\sigma(t)^{-1}} L_1^{\sigma(t)^{-1}} Q_F^{\sigma(t)} = J_F^{\sigma(t)^{-1}}, \tag{32}$$

where $J_F^{\sigma(t)^{-1}}$ is the Jordan matrix related to $L_1^{\sigma(t)^{-1}}$. Let $\{\lambda_1^{\sigma(t)}, \lambda_2^{\sigma(t)}, \cdots, \lambda_{N_F}^{\sigma(t)}\}$ denote the eigenvalues of $L_1^{\sigma(t)}$, then $J_F^{\sigma(t)^{-1}} = \mathrm{diag}\{\lambda_1^{\sigma(t)^{-1}}, \lambda_2^{\sigma(t)^{-1}}, \cdots, \lambda_{N_F}^{\sigma(t)^{-1}}\}$. Let $\eta_F(t) = (Q_F^{\sigma(t)^{-1}} \otimes I_n)\xi_F(t)$, one can get that

$$\begin{aligned}&\xi_F^T(t)(L_1^{\sigma(t)^{-1}} \otimes (A^T P + PA) - 2\gamma I_{N_F} \otimes T)\xi_F(t) \\ =& \eta_F^T(t)(J_F^{\sigma(t)^{-1}} \otimes (A^T P + PA) - 2\gamma I_{N_F} \otimes T)\eta_F(t).\end{aligned} \tag{33}$$

The inequality (31) is transformed into

$$\dot{V}(t) \leq \eta_F^T(t) S^{\sigma(t)} \eta_F(t), \tag{34}$$

where

$$\begin{aligned}S^{\sigma(t)} =& J_F^{\sigma(t)^{-1}} \otimes (A^T P + PA) - 2\beta\gamma R^{-1} I_{N_F} \otimes (PBB^T P) \\ =& \mathrm{diag}_i\{\lambda_i^{\sigma(t)^{-1}}(A^T P + PA - 2\beta\gamma R^{-1} \lambda_i^{\sigma(t)} PBB^T P)\} \\ \leq& \mathrm{diag}_i\{\lambda_i^{\sigma(t)^{-1}}(-Q)\} < 0.\end{aligned} \tag{35}$$

For each time interval $t \in [t_k, t_{k+1})$, the communication graph is fixed, thus $V(t)$ is a continuous function. During $t \in [t_k, t_{k+1})$, $V(t)$ is bounded as $V(t) \geq 0$ and $\dot{V}(t) < 0$, then $V_2(t)$ is bounded. For $t \in [0, +\infty)$, $V_2(t)$ is a continuous function, $\dot{V}_2(t) > 0$, and $\dot{c}_i(t) > 0$ ($i \in \{1, 2, \cdots, N_F\}$), thus $V_2(t)$ is bounded for all $t \in [0, \infty)$. It follows that there exists a constant $r_1 > 0$ such that $V_2(t) \le r_1$. It can be obtained then $\lim_{t\to\infty} \dot{V}_2(t) = 0$, which leads to $\lim_{t\to\infty} \dot{V}(t) = \dot{V}_1(t)$. There exists a fixed time T_0 such that $\dot{V}_1(t) = \dot{V}(t)$, when $t \in [T_0, \infty)$. From the definition of $V_1(t)$, one can obtain that for $t \in [t_k, t_{k+1})$ and $t_k > T_0$,

$$\dot{V}_1(t) = \dot{V}(t) \le -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} V_1(t). \tag{36}$$

It follows that

$$V_1(t_{k+1}^-) \le e^{-\alpha(t_{k+1}-t_k)} V_1(t_k). \tag{37}$$

As the communication graph switches at time t_k,

$$V_1(t_k) \le r_2 V_1(t_k^-), \tag{38}$$

where $r_2 = \lambda_{\max}(L_1^{\sigma(t)^{-1}} \otimes P)/\lambda_{\min}(L_1^{\sigma(t)^{-1}} \otimes P)$. From inequalities (34)-(35), one gets that

$$V_1(t_{k+1}) \le r_2 e^{-r_1(t_{k+1}-t_k)} V_1(t). \tag{39}$$

Suppose that $t_0 > T_0$, it can be obtained by iterations that

$$\begin{aligned} V_1(t_k) \le& r_2^k e^{-r_1(t_k - t_0)} V_1(t_0) \\ \le& e^{-k(r_1\tau_0 - \ln r_2)} V_1(t_0). \end{aligned} \tag{40}$$

If the dwell time satisfies that $\tau_0 > \ln r_2/r_1$, then $r_1\tau_0 - \ln r_2 > 0$ and $\lim_{t\to\infty} V_1(t) = 0$, which leads to $\lim_{t\to\infty} \xi_{N_F}(t) = 0$. From Lemma 2 and Definition 1, the adaptive formation tracking under control protocol (6) is achieved and the Theorem 1 is proved.

IV. NUMERICAL SIMULATION

In this section, numerical simulation examples are given to show the effectiveness of the adaptive formation tracking protocol. Suppose the UAV swarm system (4) contains three leaders and four followers. There are three topologies $\mathcal{G}_1$, $\mathcal{G}_2$, and $\mathcal{G}_3$ (shown in Fig. 1) with weighted adjacency matrices, containing only 0 and 1 as elements. The switching signal function is shown in Fig. 2. The solid lines show the interaction among the followers and the dotted arrows show the interaction between the followers and leaders. The dwell-time τ_0 is chosen as $5s$. The function $f(t)$ is chosen as $f(t) = 30$

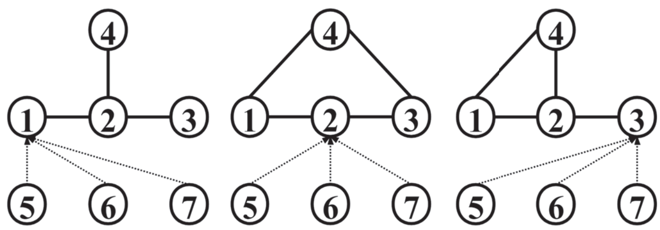

Fig. 1. Switching topologies $\mathcal{G}^1$, $\mathcal{G}^2$, and $\mathcal{G}^3$.

satisfying $f(t) > \|u_l(t)\|_\infty$ before time $t = 15$. For each

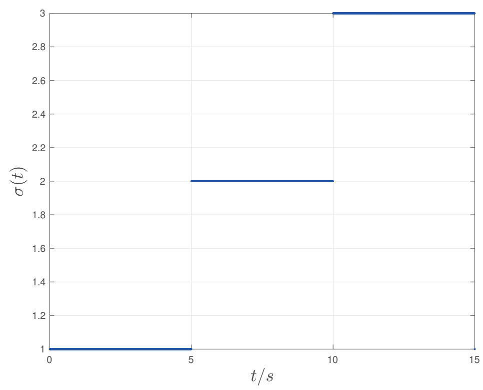

Fig. 2. The switching signals.

follower i ($i \in \{1, 2, 3, 4\}$), the desired formation functions $h_i(t)$ are predefined by

$$\begin{aligned} h_{ipx}(t) =& 2\sin(10t + \frac{i-1}{2}\pi) \\ h_{ivx}(t) =& 20\cos(10t + \frac{i-1}{2}\pi) \\ h_{ipy}(t) =& 2\cos(10t + \frac{i-1}{2}\pi) \\ h_{ivy}(t) =& -20\sin(10t + \frac{i-1}{2}\pi) \\ h_{ipz}(t) =& 10 \\ h_{ivz}(t) =& 0. \end{aligned} \tag{41}$$

The gain matrix K and controller matrix T are given by solving the ARE (21), where the parameters are chosen by $Q = I_2$ and $R = 1$. The solution $P = \begin{bmatrix} 1.7321 & 1 \\ 1 & 1.7321 \end{bmatrix}$, then it is obtained that $K = I_3 \otimes [-10 - 17.3205]$ and $T = I_3 \otimes \begin{bmatrix} -10 & -17.3205 \\ -17.3205 & -30 \end{bmatrix}$ by choosing $\beta = 10$. The initial states $x_i(0)$ and $y_i(0)$ in X and Y directions are chosen by bounded vectors.

In Fig. 3, the positions of each UAV at time $t = 0$, $t = 5$, $t = 10$, and $t = 15$ are shown in O-XY plane, where the crosses represent the leaders, the asterisks represent the followers, and the pentagram represents the average position of the leaders. At time $t = 15$, the four followers are distributed as four vertices of a square and the average position of the leaders was in the center of all followers, which agrees to the predefined formation. In Fig. 4, the curve of the formation tracking error is given, and one can observe that the formation tracking error converges to zero. Based on Lemma 2, the formation tracking is achieved if and only if the limit of the formation tracking error is zero as time tends to infinity. Therefore, the adaptive formation tracking with switching topologies and multiple leaders of unknown bounded input is achieved.

V. CONCLUSION

Adaptive formation tracking problems for UAV swarm systems with multiple leaders and switching topologies were

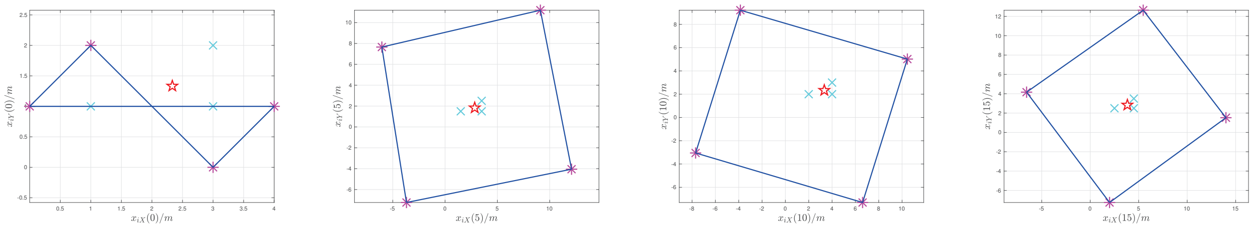

Fig. 3. Position of UAVs at $t = 0$, $t = 5$, $t = 15$, and $t = 20$.

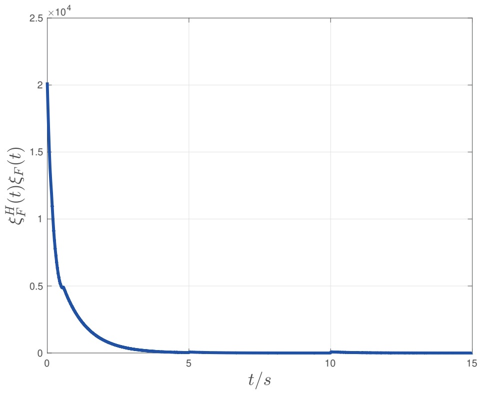

Fig. 4. The curve of formation tracking error.

investigated. In order to achieve the formation tracking, the adaptive formation tracking protocol with switching topology was designed based on neighboring relative state information. Sufficient conditions for UAV swarm systems to achieve the formation tracking were concluded. The numerical simulation examples were presented to verify the given results. Based on the study of this paper, more problems of time-varying formation tracking with communication delay and disturbances are worth further studying in the future.

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