Collecting a Flock With Multiple Sub-Groups by Using Multi-Robot System

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***Abstract*—In this letter, we present a distributed approach to automatically collect a flock with a group of robots. This collecting problem is challenging due to the fact that the flock size is usually much greater than that of robots, meanwhile the local sensing range of robots may not ensure a globally successful collecting. Existing literature for collecting flocks assume that the flocks are coherent, however, this limits the practicality of these approaches especially in the case where there are multiple sub-groups. To address these issues, we relax the assumption and propose a density-based strategy, which can drive the robots to move towards the edge of the flock in an edge-following behavior and ultimately encircle the flock. Once encircled, a shrink mechanism is used for squeezing the flock into a tight cluster. Collecting a single cluster and collecting a flock with multiple sub-groups can be achieved by using such edge-following behavior in combination with shrink mechanism. The lower bound on the minimum number of robots required for successfully collecting a given flock size is also theoretically investigated. We validate the performance of our approach via numerical simulations and the number of robots required for a successful collecting operation is validated by using statistical simulation results for a range of flock sizes.**

***Index Terms*—Distributed robot systems, multi-robot systems, swarm robotics, shepherding behaviours, collecting flocks.**

I. Introduction

SIMULATING the collective behaviour of a flock has attracted the attention of researchers in a wide range of fields, such as biological collectives, robotic swarms, and complex systems. Aggregation is a collective behaviour key to flocking, and is widely believed to be the evolutionary result for reducing predation risk. To reduce predation risk, some flocks exhibit aggregation due to selfish herd theory [1], i.e., the individuals are observed to move towards the center of the group. Such selfish herd theory implies that a herd (flock) can be influenced by a small group of external agents by exerting the prey-predation interaction mechanism.

Inspired by herding animals such as sheepdogs and sheep, most previous works focus on the herding problem, where one or more external agents (also called shepherds) attempt(s) to steer the movement of another group of agents (called a flock) by exerting a repulsive force so that the group can be guided from a start region to a goal region [2]–[9]. Many empirical works have been extensively studied, for example a flock of sheep flee from an approaching dog herder [10], resulting in a reduction in predation risk from shark attacks [11]. Another interesting empirical work on herding is seen in horse harem groups, in which males chase females from behind [12]. Similar but not identical, collecting is another practical problem where a group of external agents are used for gathering and restricting the flock from an initially loose configuration into a tight cluster. This behaviour can be found in nature, for example, the spotted hyenas always hunt by pushing their prey into a small area [13], similar cooperative hunting is also seen in wolf pack predation [14]. The field of robotics also requires a solution to this problem, specifically, when researchers deploy large herds of robots. Although we will use flock-robot analogy throughout this letter, the "flock" can be biological or artificial agents that satisfy the selfish herd theory, such as sheep and ground mobile robots. The applications of the collecting problem include but are not limited to collecting livestock [15], area/border coverage [16], perimeter defense tasks [17], convoy/escorting missions [18], collection of waste in ocean and sea [19], and many other potential applications. However, the research works on how to use multiple external agents to automatically collect a loose flock in a distributed way is rarely reported, especially when the flock has multiple sub-groups.

We focus on designing a distributed approach for a group of robots that can automatically collect a loose flock or a flock with multiple sub-groups that satisfy the selfish herd theory. The distributed approach is achieved by using a density-based strategy, which is able to drive the robots to move along the edge of the flock inch by inch, and ultimately encircle the flock. Once encircled, the robots shrink the encirclement and squeeze the flock into a small area. In addition, for the first time, the lower bound on the minimum number of robots required for successfully collecting a given flock size is analyzed theoretically and validated by statistical simulation results.

II. Related Work

The collecting task aims at gathering loose flock members into a designated region, it was first presented as one of shepherding behaviours in the pioneering work of [2]. Their main idea is to guide the robots into a formation, such as line and arc. Once the formation is generated, multi-task assignment methods are

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used to determine which robot approaches which steering point. The approach requires that all robots have the knowledge of the position information of all other robots and all flock members, which means the method is centralized.

With the development of shepherding algorithms, researchers began to realize that collecting those dispersed individuals is a basic requirement for further actions of shepherding behaviours. In [8] an adaptive switching method was proposed to address the herding problem while considering the collecting requirement simultaneously. The key idea is based on the global centre of mass (GCM). The robots are herding the flock to the target region if all flock members are within a predefined distance to the GCM, otherwise the robots are collecting those further away members into the neighbouring area of the GCM. By using the information of GCM, many rules-based and formation-based herding algorithms have been proposed to study the shepherding problem [3], [4], [7], [20]–[22]. Since the position information of the flock center is required, such algorithms are centralized.

Nowadays, the rule of boundary flock members attract more and more attention. In [9] a n-wavefront algorithm was presented which is a boundary control method. More recently, in [15] the extended hull was derived from a given convex hull of the herding animals by means of geometric methods. By navigating the barking drones to fly to (on) the extended hull, an edge-searching behavior is generated. However, there is no research details about how to collect a flock with multiple sub-groups by using the convex hull. Instead of focusing on the convex hull, a distributed method was proposed in [23] where the steering robots only need to focus on guiding the nearest flocking agent to the desired location. Without centralized coordination, it was shown that multiple steering agents produced an arc formation to control the flock effectively.

The major difference between all the above mentioned works and ours is that (1) we designed a distributed collecting algorithm without requiring the information of GCM of flocks. (2) We relaxed the assumption that flocks are coherent and focused on the collecting problem with multiple flocks that satisfy selfish herd theory. (3) Instead of using all flock members or simply using the convex hull of flocks, we proposed a novel density-based strategy to better reflect the distribution character of the boundary of flocks. (4) The edge-following behavior and the shrink mechanism were used to encircle and squeeze the flock into a tight cluster.

III. Problem Formulation

Consider a 2-D obstacle-free space where there are M robots and a flock of N agents. A *robot* is an external agent that influences the movement of the flock. A *flock* consists of agents which are known to have selfish herd theory strategies, such as sheep, zebra, cattle, etc. These herding animals usually have a large scale and are not easily controlled directly. Each agent (robot or flock member) has a limited sensing range to detect its local environment and to share the information with its neighbours. The robots are presumed to be controllable in a direct way. Assume that the maximum speed and the sensing range of robots are not less than that of flock members. Each agent has no knowledge of the group size of its own group or the other group caused by the limited sensing range. The collecting problem can be specifically described as

Problem (Collecting): Find control laws for M robots to gather and restrict the flock from an initially loose configuration into a tight cluster.

The challenge of the collecting problem lies on the fact that the flock size is much greater than that of robots, while the local (limited) sensing range of robots may not ensure globally successful collecting. Meanwhile, the dispersion preference of the flock may cause the collected members disperse again when the robots disappear from their views.

IV. General Model of Flock

Without loss of generality, we assume that the flock members prefer to move away from others for more substantial living areas in the absence of threat of robots. Under threat of robots, the selfish herd hypothesis suggests that gregarious behaviour has evolved [1], which can be characterised by a Hamiltonian movement rule: each flock member moves towards the gap between its nearest neighbour and its nearest neighbour's nearest neighbour. A flight distance is also assumed at which the flock will regard the threat as great enough to move away. Specifically, we model the flock by three behavioral rules:

1) Dispersion preference: each flock member moves away from others for more substantial living areas in the absence of robots.
2) Predator avoidance: each flock member moves away from the robots when the perceived threat is great enough.
3) Aggregation preference: the flock exhibit Hamiltonian movement rule in the presence of robots.

The movement of the flock then can be described as the weighted sum of the above three forces by the first-order integrator dynamics in $\mathbb{R}^2$:

$$\frac{dS_i}{dt} = k_d\hat{D}_i + \alpha_i \cdot (k_p\hat{P}_i + k_a\hat{A}_i) + \mathfrak{R}(\eta).$$

Here, S_i represents the position of the flock member i. $\hat{D}_i$, $\hat{P}_i$ and $\hat{A}_i$ are unit vectors, indicating the directions of dispersion preference, predator avoidance and aggregation preference, respectively. $k_d > 0$, $k_p > 0$ and $k_a > 0$ are the weights. $k_d = 1$ is chosen as a reference value in the following of letter. α is an indicator function, where $\alpha = 1$ indicates the situation that there exist robots within the sensing range of the flock member i, otherwise $\alpha = 0$. The effect of external disturbance is simulated as a velocity disturbance $\mathfrak{R}(\eta)$ that alters the desired velocity by using a random noise with a uniform distribution in the interval $[-\eta, \eta]$, where η is the noise strength.

In the absence of robots (i.e., $\alpha = 0$), flock members move away from others for either more substantial living areas (if they are herding animals) or monitoring areas (if they are robots doing environmental monitoring). Without loss of generality, we use repulsive force which is common in robotics and in models of collective behaviour to simulate the dispersion preference of the flock:

$$D_i = \frac{1}{\mathcal{N}_{i,s}} \sum_{j=1}^{\mathcal{N}_{i,s}} \frac{1}{\|S_i - S_j\|} \frac{S_i - S_j}{\|S_i - S_j\|},$$

where $\|S_i - S_j\|$ represents the distance between S_i and S_j. $\mathcal{N}_{i,s}$ and $\mathcal{N}_{i,r}$ are the sets of nearby flock members and nearby robots that are within the sensing range of the flock member i. Let R_s denote the sensing range of flock members. For a uniform circular sensor model, the neighbourhood of flock member i is defined as

$$\begin{aligned} \mathcal{N}_{i,s} &= \{ S_j \mid \|S_i - S_j\| < R_s, \, j = 1, 2, \cdots N, \, j \neq i\}, \\ \mathcal{N}_{i,r} &= \{ R_j \mid \|S_i - R_j\| < R_s, \, j = 1, 2, \cdots M\}, \\ \mathcal{N}_i &= \mathcal{N}_{i,s} + \mathcal{N}_{i,r}. \end{aligned}$$

The neighbourhood of a robot can be defined in a similar way.

In the presence of robots (i.e., $\alpha = 1$), in addition to the dispersion preference (repulsive force) mentioned above, the predator avoidance and aggregation preference have a combined influence on the movement of the flock. For predator avoidance, we model the fleeing behaviour using an artificial potential field which is widely used in prey-predator interaction models [5], [24]:

$$P_i = -\frac{1}{\mathcal{N}_{i,r}} \sum_{j=1}^{\mathcal{N}_{i,r}} \frac{1}{\|S_i - S_j\|^3} \frac{S_i - S_j}{\|S_i - S_j\|}.$$

For aggregation preference, a closely approximate Hamiltonian rule is used where each flock member moves towards the midpoint of two nearest neighbours. Though there are various models for aggregation rules, we believe that the approximate Hamiltonian rule captures the essential features of most previous algorithms. Let S_1 be the midpoint of two nearest neighbours of the flock member i, the Hamiltonian rule can be described as

$$A_i = -\|S_i - S_1\| \frac{S_i - S_1}{\|S_i - S_1\|}.$$

Note that if there is only one neighbour within the sensing rang of the focal member, then it moves towards this single neighbour.

V. Distributed Collecting Approach

In the collecting problem, we study how multiple robots gather an initially loose flock into a tight cluster. Different from the herding task, in the collecting task there is no predefined goal region which restricts the movement of the flock members inside of a limited area.

A. Definition of Robot Density

The density of an agent is defined as the weighted sum of distance to its nearby agents within the sensing range (in practice the density can be obtained by distance-relevant quantities instead of using relative distance directly, such as light intensity and sound intensity, etc.), which, as a comprehensive property of the local group, represents the neighbourhood distribution characteristics. Many core functions can be adopted to estimate such local distribution density. Without loss of generality, we use the local crowded horizon function as the weight function [25]:

$$W(S_i, S_j) = \frac{1}{1 + k\|S_i - S_j\|} \frac{S_i - S_j}{\|S_i - S_j\|},$$

where S_i is the position of agent i and S_j is a neighbouring agent. $k = 0.375$, it is chosen such that an agent 20 body length away has 11.76% as much influence on the direction of the focal agent i as one right next to the agent i. By using the local crowded horizon function, an agent that is close to the focal agent has more contribution to the integrated density. Specifically, the density of a robot is defined as the weighted sum of distance to its nearby robots as well as to the flock members. We use

$$\rho_r = \sum_{j}^{\mathcal{M}_{i,r}} W(R_i, R_j) \text{ and } \rho_s = \sum_{j}^{\mathcal{M}_{i,s}} W(R_i, S_j)$$

to represent the density attributions from other robots and the flock members respectively, where $\mathcal{M}_{i,r}$ and $\mathcal{M}_{i,s}$ are the set of neighbouring robots and neighbouring flock members within the sensing range of robot i, respectively. They have similar definitions to (1). The definition of density ρ for a robot is given by the sum of two parts

$$\rho = C_r \rho_r + C_s \rho_s,$$

where C_r and C_s are the weight parameters of two attributions. We have $0 < C_r < C_s$ such that the robots pay more attention to the distribution of the flock than that of themselves.

B. Density-Based Edge-Following Behavior

The movement of robots is guided by the overlap avoidance force and the density-based force as

$$\frac{dR_i}{dt} = k_o \hat{O}_i + k_f \hat{F}_i,$$

where $k_o > 0$ and $k_f > 0$ are the weights of two forces, representing the payoff of exploration and exploitation. R_i is the position of the robot i. O_i is the overlap avoidance force

$$O_i = \frac{1}{\mathcal{M}_{i,r}} \sum_{j=1}^{M_{i,r}} \frac{1}{\|R_i - R_j\|} \frac{R_i - R_j}{\|R_i - R_j\|}.$$

F_i is the density-based force, steering the robots to move towards the edge of the flock. It is given by

$$\begin{aligned} F_i = \sum_{j=1}^{\mathcal{M}_i} \frac{1}{\rho_i} \left[\left(\frac{\rho_i}{\rho_0} \right)^7 - 1 \right] \cdot \Big[& C_r \nabla_i W(R_i, R_j) \\ & + C_s \nabla_i W\left(R_i, S_j\right) \Big], \end{aligned}$$

where $\mathcal{M}_i$ is the sum of neighbouring robots and flock individuals, ρ_i is the current density of robot i, and ρ_0 is a reference density that controls the expected distance between the robot and the flock. The physical meaning of the above density-based force is that the robots converge to the outside edge of the flock corresponding to the predefined density ρ_0 from the gradient descent direction of the density field. With the overlap avoidance force and the density-based force, an edge-following behavior can be generated.

C. Shrink Mechanism and Shrink Condition

Given a specific value of ρ_0, with (1) - (1), the robots can be guided to the edge of the flock. This kind of edge-following behaviour ensures a group of robots to move around the edge of the flock inch by inch and ultimately encircle the flock with a uniform ring. In order to collect the flock into a tight cluster and restrict the movement of the flock within a small area, we shrink the ring generated by the robots slowly so that the flock will aggregate due to the behavioral rules of predator avoidance and aggregation preference.

An event trigger is designed to implement such a shrink mechanism. Let Q_0 represent the trigger event, after which happens the robot system starts its shrink mechanism. The trigger event Q_0 is defined as

Definition 1 (Shrink condition): Let d_i be the distance between robot i and its nearest neighbour robot, $\bar{d}$ be the mean of all d_i (by means of information sharing), the distribution evenness is defined as

$$s^2 = \sum_{i=1}^{M} \frac{(d_i - \bar{d})^2}{M}$$

The event Q_0 is triggered if $s^2 < \sigma$. Here $\sigma \geq 0$ is a relaxation factor, where $\sigma \to 0$ means the shrink condition is more exacting.

Assume that Q_0 happens at $t = t_0$, then the shrink mechanism is implemented by regulating the expected reference density $\rho_{i,0}(t)$

$$\rho_{i,0}(t) = \begin{cases} \rho_0 \cdot \frac{1}{\mathcal{M}_{i,s}^2}, & \text{if} \quad t < t_0 \\ (\rho_0 + h(t - t_0)) \cdot \frac{1}{\mathcal{M}_{i,s}^2}, & \text{otherwise.} \end{cases}$$

where $\rho_0 > 0$ is a constant that controls the initial distance between the robots and the flocks. $h > 0$ is a parameter that controls the shrink rate.

The expected reference density is influenced by the group size of neighbouring flock members $\mathcal{M}_{i,s}$, while it is independent of the group size of neighbouring robots $M_{i,r}$. This allows the robots to follow the edge of the flock and to be less affected by other robots. An inverse square law is used such that the robot is forced to slow the movement speed when the perceived density of the flock is higher.

D. Minimum Number of Robots Required

The proposed density-based edge-following strategy is promising in shepherding behaviors which provides topological cooperation among robots by producing enclosed formations [4]–[6]. Despite its robustness and efficiency, the large increase in controlling robots over other partial-circle formation-based methods (e.g., V-formation [3] and arc-formation [21]) should be taken into account. The swarm robotic system could be leveraged to this issue. In this section, we theoretically investigate what is the minimum number of robots required for successfully collecting a given flock size.

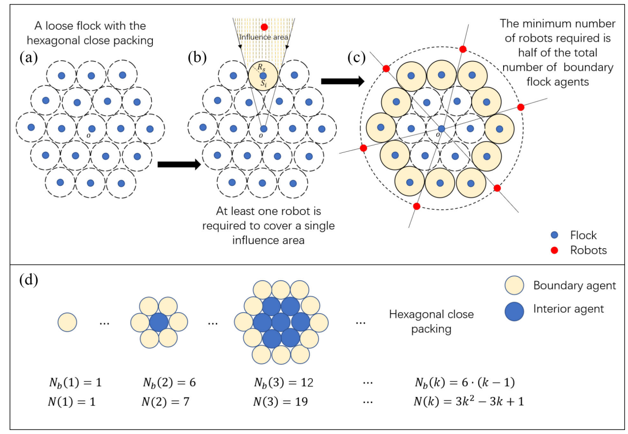

Fig. 1. Illustration of how to get the minimum number of robots required. (a) A loose flock with the hexagonal close packing is generated by the dispersion model. (b) In order to influence and collect the flock, at least one robot is required to move inside the influence area of each boundary flock agent. (c) The minimum number of robots required for collecting the whole flock is merely determined by the number of boundary flock agents. (d) The number of boundary flock agents for a given flock size can be obtained by an arithmetic sequence.

Theorem 1: Let $N \in \mathbb{N}_+$ be the flock size and $N_b \in \mathbb{N}_+$ be the number of boundary individuals in the flock, then the number of robots required $M_{min} \in \mathbb{N}_+$ for a successful collecting is largely determined by the number of boundary individuals. The lower bound on the minimum number of robots required is $M_{min} = \frac{N_b}{2}$.

Proof: Under ideal conditions, a flock will spread into a loose pattern via a hexagonal close packing, as shown in Fig. 1(a).

Each robot moves along the gradient descent direction of the density field, which points from itself to the flock center O. To influence the flock individual S_i and push it towards the center O, it is necessary that one or more robots need(s) to move inside the influence area of the individual S_i, as shown in the shaded area of Fig. 1(b). The boundary condition is obtained when every two common borders of influence area is deployed one robot, such that at least one robot always covers two influence areas, as shown in Fig. 1(c). Since the influence areas of different boundary agents do not overlap, at least $\frac{N_b}{2}$ robots are required to collect all boundary flock agents. Due to the self-herding behaviour of the flock, the interior flock members are going to move towards the center and show aggregation during the collecting process. Therefore, the lower bound on the minimum number of robots required for a successful collecting operation is determined by the number of boundary flock agents with $M_{min} = \frac{N_b}{2}$.

Theorem 2: If the distribution of flock approximately obeys the hexagonal close packing, let k be the lap number of the flock, N and $N_b(k)$ be the flock size and the number of boundary agents, then $N_b(k)$ can be selected from a singleton or an interval:

$$\begin{cases} \left\{ \frac{2(N-1)}{k} \right\}, & \text{if } N = N(k) \\ \left(\frac{2(N(k)-1)}{k}, \frac{2(N(k+1)-1)}{k+1} \right), & \text{if } N \in (N(k), N(k+1)) \end{cases}$$

where $N(k) = 3k^2 - 3k + 1, \quad k \in \mathbb{N}_+$.

Proof: As shown in Fig. 1(d), for a flock approximately obeying hexagonal close packing, the number of boundary individuals is an arithmetic sequence with the lap number $k \geq 2$. The general term formula can be expressed as

$$N_b(k) = 6(k-1), \quad k \geq 2.$$

Considering when the lap number $k = 1$, the flock size $N(1) = 1$, the total number of flock members within the first k laps can be expressed by utilising the sum of the first k terms of this arithmetic sequence as

$$N(k) = 3k^2 - 3k + 1$$

Substituting (1) into (1) yields

$$N_b(k) = \frac{2(N-1)}{k}$$

where $N = N(k)$. If $N(k) < N < N(k+1)$, then those flock individuals outside the k-th lap will become additional boundary agents, such that

$$N_b(k) \in \left(\frac{2(N(k)-1)}{k}, \frac{2(N(k+1)-1)}{k+1} \right).$$

Theorem 3: Let $N \in \mathbb{N}_+$ be the flock size and $N_b \in \mathbb{N}_+$ be the number of boundary individuals in the flock, then the number of boundary individuals can be approximately obtained by

$$N_b \approx \pi\sqrt{N}.$$

Proof: Due to the fact of noise in the movement of flock individuals, the flock usually does not obey the hexagonal close packing. For a general distribution of flock, we use the area equivalence principle to approximately estimate the number of boundary individuals.

Let D be the stable distribution range of flock under dispersion preference, the circle area (referred to as the big circle) covered by the whole flock can be characterised by πD^2. Such area can also be characterised by the sum of each individual's living area, which is a small circle area within a radius of sensing range R_s, i.e., $N\pi R_s^2$. According to the area equivalence principle, we have

$$\pi D^2 \approx N\pi R_s^2.$$

Since the boundary individuals cover the circumference of the big circle, we have the following equation

$$2R_s N_b \approx 2\pi D.$$

Substituting (1) to (1) yields

$$N_b \approx \pi\sqrt{N}.$$

E. Collecting a Flock With Multiple Sub-Groups

Existing literature for collecting flocks assumes that flocks are coherent, which limits the practicality of these approaches especially in the case where there are multiple sub-groups. To address this issue, we propose a novel distributed collecting algorithm for the flock with multiple sub-groups.

Algorithm 1: Collecting Algorithm.

```
Input: Initial configurations of flock and robots.
Output: Actions: edge-following, shrink and
        searching.
1  Initial robot action: edge-following.
2  while Successful collecting condition is not satisfied do
3      for each robot do
4          Implement edge-following behavior to its
           nearest sub-group i of the flock.
5          if Shrink condition then
6              Switch edge-following behavior to shrink
               behavior.
7          if Searching condition then
8              Switch shrink behavior to searching
               behavior. The robots move towards
               another sub-group j of the flock while
               keeping the current flock members inside
               their encirclement.
9          if Edge-following condition then
10             Switch searching behavior to
               edge-following behavior.
11         Repeat the actions 3-11 until all flock members
           are collected into a small area.
```

The main idea is to use edge-following behavior and shrink mechanism as building blocks, encircle each sub-group, merge two sub-groups into a single one sequentially and ultimately collecting all flock members into one single flock. The overall algorithm is shown in Algorithm 1. Note that, to collect the multiple sub-groups into a single flock, a searching behavior is necessary for a distributed approach. In this letter, we assume that an unmanned aerial vehicle is used to undertake this work.

The algorithm can be described as follows: At the beginning, the robots implement edge-following behavior to its nearest sub-group. They switch to shrink behavior when shrink condition (as defined in Definition 1) is met. The flock cannot split or scatter as a result of executing the shrink mechanism. If $\rho_{max} = f(N_i) = N_i$, where ρ_{max} is the maximum value of density perceived by the robot group, N_i is the total number of sub-group i, then the robots switch to searching phase. During the searching phase, the robots move towards another sub-group while keeping the current flock members inside of their encirclement. When the robots meet another sub-group of the flock, i.e., $N' > N_i$ where N' is the maximum number the robot group can perceive (by means of information sharing), then the robots switch to edge-following phase again. The phases of edge-following, shrink, searching and edge-following will be implemented repeatedly until all flock members are gathered into a single tight cluster.

VI. Simulation Results

The flock is initially deployed in a random configuration inside a 20 m square. The robots are initially deployed in a random configuration inside a 10 m square near the flock. The simulation results are not sensitive to the initial configurations

TABLE I
SUMMARY OF VALUES OF CONTROL PARAMETERS

Parameter	Description	Values
Flock		
R_s	Sensing range of flock	3 m
V_s	Maximum velocity of flock	1 m ts^{-1}
k_a	Strength of aggregation	2
k_d	Strength of dispersion	1
k_p	Strength of predator avoidance	3
Robot		
R_r	Sensing range of robot	15 m
V_r	Maximum velocity of robot	1.5 m ts^{-1}
k_o	Strength of overlap avoidance	1
k_h	Strength of density force	2
ρ_0	Initial reference density	20
h	Shrink rate	100
σ	Relaxation factor	0.2
C_s	Flock's density attribution	1
C_r	Robot's density attribution	3
Environment		
Δt	Time interval	0.1 s
T	Maximum simulation timestep	1×10^4
η	Amplitude of random noise	1

of robots even they are deployed inside of the flock. Unless otherwise stated, the values of parameters are summarized in Table I.

A. Edge-Following Behavior and Shrink Mechanism

We first verify the edge-following behaviour and the shrink mechanism in the case of single cluster. Fig. 2(a) presents the snapshots of a collecting process by using edge-following behaviour and shrink mechanism. At the early stages ($TS \leq 1147$), the robots spread out from their initial configuration and stretch in an arc on one side of the flock. The density-based force drives the robots towards the edge of the flock, while the overlap avoidance force pushes the robots to move around the edge and explore new areas. Such edge-following behaviour ultimately encircle the flock with a ring pattern even though the final configuration of robots does not exactly match with the expected density due to the fact of noise.

The trigger event happens at $TS = 1147$ where the distribution evenness $s^2 = 0.2$. After that, the shrink mechanism is triggered, the expected reference density increases according to (1), the robots move close to the flock and squeeze the flock into a tight cluster. As confirmed in Fig. 2(a), the proposed method successfully collects all flock members into a tight cluster by sequentially implementing edge-following behaviour and shrink mechanism.

To quantitatively evaluate the collecting process, the distribution range is defined as the distance between the group center (GC) and the outmost group member, presenting the tightness of the group. The distribution evenness is defined as the variance of nearest neighbour distance from others to each robot, supporting the definition of the shrink condition. The mean distance to the group center presents the average level of the whole flock distribution. The isolated rate is defined as the proportion of isolated members in the flock, demonstrating the effectiveness of the collecting method.

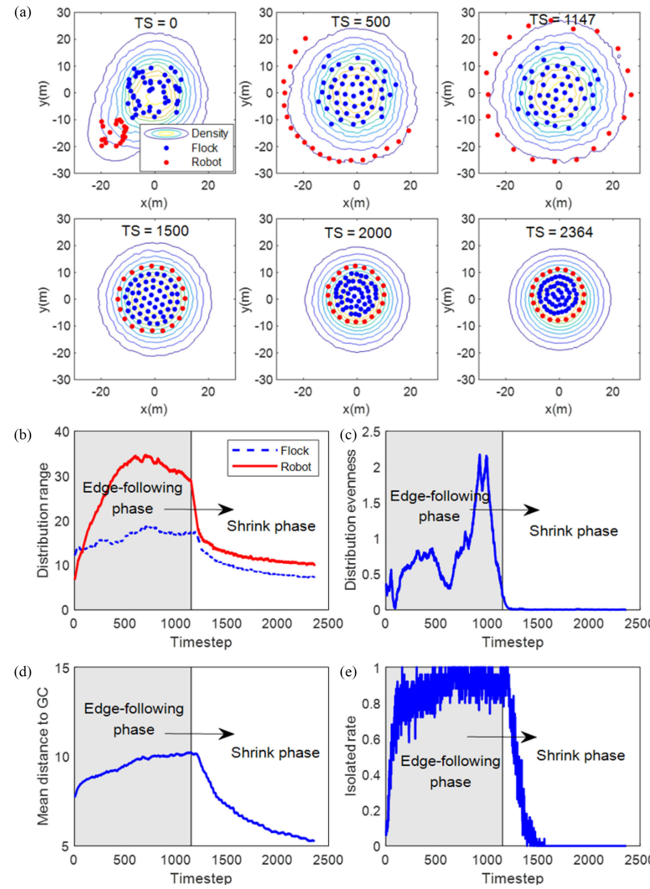

Fig. 2. Collecting a flock with 50 flock members and 20 robots in the case of single cluster. (a) The proposed distributed collecting approach successfully collects all flock members into a tight cluster by sequentially implementing edge-following behaviour and shrink mechanism. (b)-(e) The history of order parameters of robots and flock in the case of single cluster.

Fig. 2(b) shows the histories of distribution range of robots and flock. It is seen that the trigger event happens at $TS = 1147$. After the trigger event ($TS > 1147$), the robots and the flock step into the shrink phase where the distribution range of both robots and flock decreases with timestep. Assume that the allowed minimum distance between the flock members is d_0, then the minimum distribution range D_0 satisfies

$$\pi D_0^2 \approx N \cdot \pi \left(\frac{d_0}{2}\right)^2 \quad \Rightarrow \quad D_0 \approx \frac{d_0}{2}\sqrt{N}.$$

The distribution range of robots is greater than that of flock during the whole collecting period, demonstrating that all flock members are successfully collected into a tight cluster.

Fig. 2(c) shows the histories of distribution evenness. Generally, it first increases and then decreases in the edge-following phase. The trigger condition happens when $s^2 = 0.2$, after which the system steps into the shrink phase and the distribution evenness gradually converges to zero. This result indicates that the robots go on to encircle the flock with an uniform ring during the shrink phase. From Fig. 2(d) we can see the mean distance to the flock center increases in the edge-following phase while decreases in the shrink phase. This indicates that the flock has

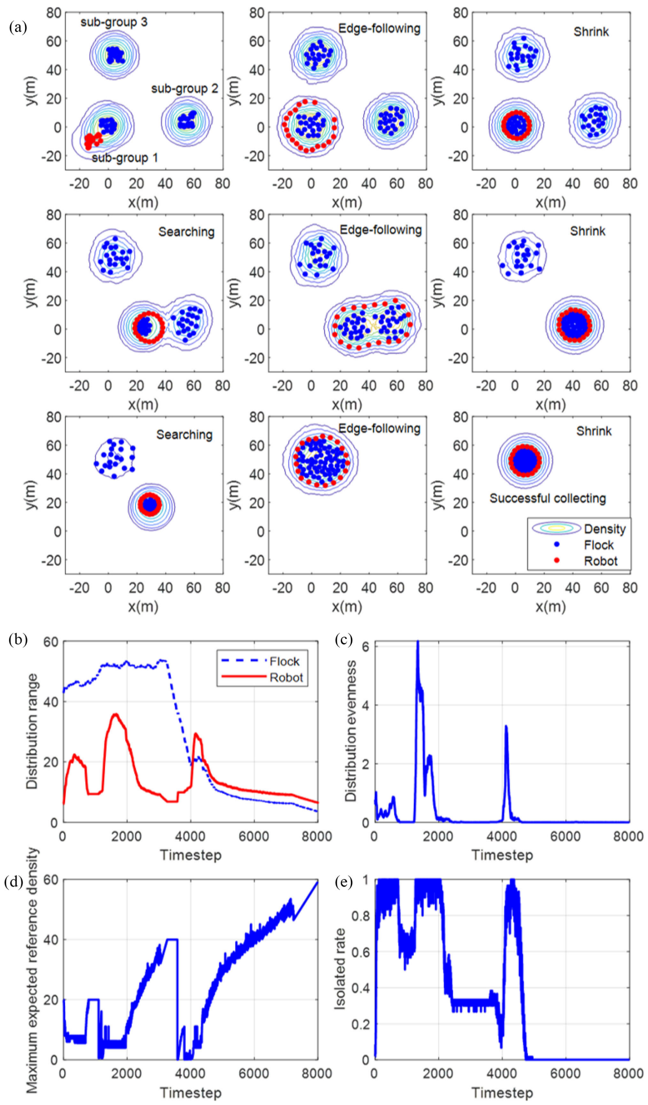

Fig. 3. Collecting a flock with 3 sub-groups. $N = 60$, $M = 20$. (a) The proposed distributed collecting approach successfully collects all flock members in three independent sub-groups into a single tight cluster by sequentially implementing edge-following behaviour, shrink behavior and searching behavior. (b)-(e) The history of order parameters of robots and flock in the case of multiple sub-groups.

a dispersion trend when the robots implement edge-following behaviour and has a monotone shrink trend due to the fact of shrink mechanism of the robots. The isolated rate has a similar trend to the curve of mean distance to the flock center, as shown in Fig. 2(e). The isolated rate finally converges to zero, indicating that all flock members are collected into a tight cluster and no one is lost.

B. Collecting a Flock With Multiple Sub-Groups

We further verify the case of collecting a flock with multiple sub-groups. Fig. 3(a) presents the snapshots of a collecting process by using the general collecting approach. At the beginning, there are 60 flock members in three independent sub-groups and 20 robots near one of the sub-groups.

The robots spread out from its initial configuration and implement edge-following behavior and shrink mechanism to its nearest sub-group 1. After the sub-group 1 is well encircled, the robots switch to the searching behavior and move towards the sub-group 2. With the edge-following behavior and shrink mechanism again, a merging phenomenon is observed and a new subgroup that combines sub-group 1 and sub-group 2 is produced. The robots begin the searching behavior while keeping this new sub-group inside their encirclement until they meet sub-group 3. The edge-following behavior and shrink mechanism again allow the robots to move close to the group and ultimately squeeze the flock into one single tight cluster.

Fig. 3(b) shows the histories of distribution range of robots and flock. It is seen that the distribution range of robots is greater than that of flock after $TS > 4000$, demonstrating that all flock members are finally successfully collected into a single tight cluster. The distribution evenness increases significantly at the beginning of the phase of edge-following, otherwise, it remains close to zero, as shown in Fig. 3(c). From Fig. 3(d) we can see the maximum expected reference density increases during the phase of shrink and remains unchanged during the phase of searching. The isolated rate has a stair-like trend as shown in Fig. 3(e). The number of isolated members decreases with the merging phenomenon. It reaches zero when the flock members finally merge into a single flock, indicating that all flock members are collected into one single tight cluster and no one is lost.

The agents in the three sub-groups have limited freedom to move, as a result, it seems like the three sub-groups were already aggregated and stationary before the collection. Some more interesting scenarios could be imaged. For example, the flock is moving as a collective, the agents in the flock are moving more freely, or the agents in the flock are more spread out. However, they are beyond the scope of this letter.

C. Minimum Number of Robots Required

We further investigate the minimum number of robots required for a successful collecting. Let $D_R(t)$ and $D_S(t)$ denote the distribution range of robots and flock respectively, if $\exists$ $0 < t < T$ (T means the maximum time), such that $D_R(t) < D_S(t)$, then the collecting is failed, otherwise it is successful. We implement 300 independent simulation runs for each value of M, and record the successful rate of a given value of M for a specific flock size N. This statistical result is regarded as the ground truth for comparing the predicted results of *hexagonal close packing estimation* and *area equivalence principle estimation*. Fig. 4 shows the comparison results of ground truth and two estimates. It is seen that the minimum number of robots required increases almost linearly with the flock size. Both estimations underestimate the real values of the minimum number of robots required. This is because the boundary condition for obtaining the minimum number of robots is hard to realize and the movement uncertainty of flock individuals even makes it impossible. We believe that the two estimations capture the essential trend of minimum number of robots required for successful collecting and can be used as a reference for practical applications.

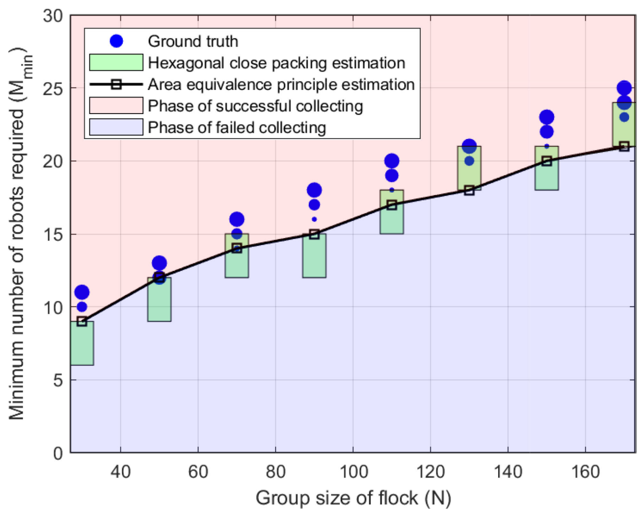

Fig. 4. The comparison results of the minimum number of robots required. The position of blue dots denotes the number of robots required for a given flock size, and the size of each blue dot indicates the successful rate of collecting in this case. The two estimations capture the essential trend of the minimum number of robots required for successfully collecting a given flock size.

VII. Conclusion

We consider a collecting scenario in which a group of robots seek to gather and restrict a flock from an initial loose configuration into a tight cluster. We proposed a distributed collecting approach that only utilizes local position information of nearby agents. The main idea of this approach is to use edge-following behavior and shrink behavior based on the local distribution density to achieve collecting assignment. Without the prerequisite that the flock is maintained well after the flock is merged, the proposed algorithm can collect flocks not only with single cluster but also with multiple sub-groups. In addition, the minimum number of robots required for successfully collecting a given size of flock is investigated from both theoretical analysis and statistical simulations. We find that there is a lower bound on the minimum number of robots required, and it is the half of the number of boundary individuals in the flock.

In our future work, we plan to validate the performance of the approach in real robotic experiments. There are two potential experimental scenarios. One is to equip a group of mobile robots (e.g., quadruped robots) with loud speakers that can mimic the barking of sheepdog, and then use these mobile robots to collect real herding animals (such as sheep). The other simpler scenario is to use a small group of robots to gather another flock of robots, such as a flock of environmental monitoring robots. At the end of the deployment, we let the flock of robots behave like herding animals and use the proposed collecting method to squeeze them into small areas. Another interesting idea is to consider adversarial flock agents in collecting problem who are willing to stand up to predators and not just escape.

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