Kennedy
Baptist College

**Semester One Examination, 2021**

**Question/Answer booklet**

# MATHEMATICS METHODS UNIT 1

**SOLUTIONS**

## Section One: Calculator-free

WA student number: In figures [ ][ ] [ ][ ][ ] [ ][ ][ ]

In words ______________________

______________________

Your name ______________________

### Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): [ ]

### Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet

***To be provided by the candidate***
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

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2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.



SN245-172-3

**Section One: Calculator-free** **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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**Question 1** **(6 marks)**

Solve the following equations for $x$.

(a) $(2x+5)(x-4)=0.$ (2 marks)

| Solution |
|---|
| $2x+5=0 \Rightarrow x=-\frac{5}{2}=-2.5$<br>$x-4=0 \Rightarrow x=4$<br><br>$x=-2.5, \quad x=4$ |
| **Specific behaviours** |
| ✓ first correct solution<br>✓ second correct solution |

(b) $x^2-10x-11=0.$ (2 marks)

| Solution |
|---|
| $(x-11)(x+1)=0$<br><br>$x=-1, \quad x=11$ |
| **Specific behaviours** |
| ✓ indicates correct method<br>✓ both correct solutions |

(c) $(x-8)^2-100=0.$ (2 marks)

| Solution |
|---|
| $(x-8)^2=10^2$<br>$x-8=\pm 10$<br><br>$x=18, \quad x=-2$ |
| **Specific behaviours** |
| ✓ indicates correct method<br>✓ both correct solutions |

**Question 2** **(7 marks)**

Consider the function $f(x) = \dfrac{p}{x+q}$, where $p$ and $q$ are constants. The graph of $y = f(x)$ has an asymptote with equation $x = 2$ and passes through the point $(6, -1)$.

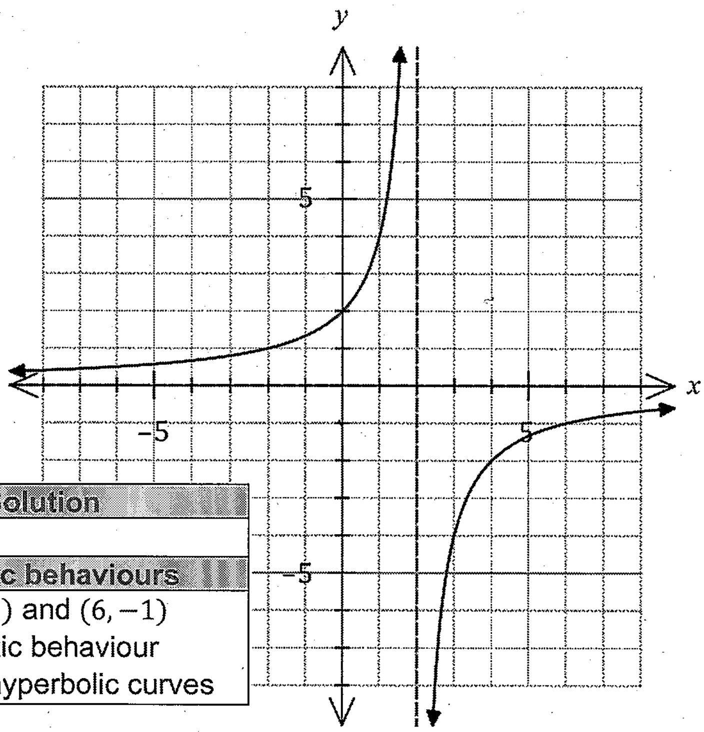
(a) Determine the value of $p$ and the value of $q$. (3 marks)

| Solution |
|---|
| Using asymptote, $2 + q = 0 \Rightarrow q = -2$.<br>Using point:<br>$$-1 = \frac{p}{6-2}$$<br>$$p = -4$$ |
| **Specific behaviours** |
| ✓ value of $q$<br>✓ forms equation using point<br>✓ calculates value of $p$ |

(b) State the equation of the other asymptote of the graph of $y = f(x)$. (1 mark)

| Solution |
|---|
| $y = 0$ |
| **Specific behaviours** |
| ✓ correct equation |

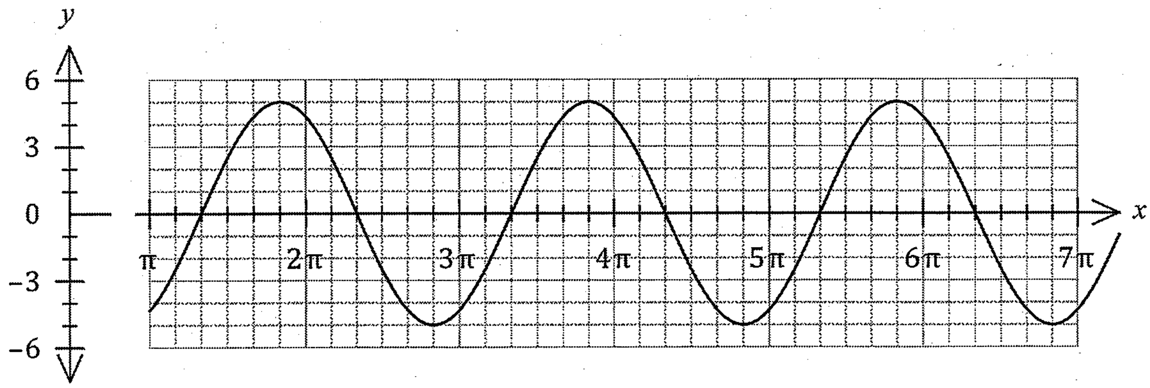
(c) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)




| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ thru' $(0, 2)$ and $(6, -1)$<br>✓ asymptotic behaviour<br>✓ smooth hyperbolic curves |

**Question 3** **(6 marks)**

(a) The graph of $y = a\sin(x + b)$ is shown below, where $a$ and $b$ are positive constants.




Determine the value of $a$ and the least value of $b$. (2 marks)

| Solution |
|---|
| $a = 5, \qquad b = \dfrac{2\pi}{3}$ |
| **Specific behaviours** |
| ✓ amplitude $a$<br>✓ least value of phase shift $b$ |

(b) Let $f(x) = 4\tan\left(x - \frac{\pi}{6}\right)$.

Determine the zeros of the graph of $y = f(x)$ for $0 \le x \le 2\pi$. (2 marks)

| Solution |
|---|
| $x - \dfrac{\pi}{6} = 0, \pi \Rightarrow x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$ |
| **Specific behaviours** |
| ✓ locates one zero<br>✓ locates second zero |

(c) Let $g(x) = 3 + \cos\left(\frac{x}{2}\right)$.

Determine the coordinates of the minimum of the graph of $y = g(x)$ for $0 \le x \le 4\pi$. (2 marks)

| Solution |
|---|
| Minimum of $y = \cos x$ when $x = \pi$, but period doubled and so now when $x = 2\pi$.<br><br>Hence minimum at $(2\pi, 3 - 1) = (2\pi, 2)$. |
| **Specific behaviours** |
| ✓ correct $x$-coordinate<br>✓ correct $y$-coordinate |

**Question 4** **(7 marks)**

The straight line $L$ has equation $3x - 2y = 1$.

(a) Write the equation of $L$ in the form $y = mx + c$ to show that its gradient is 1.5. (1 mark)

| Solution |
|---|
| $2y = 3x - 1 \Rightarrow y = \frac{3}{2}x - \frac{1}{2} \Rightarrow m = \frac{3}{2} = 1.5$ |
| **Specific behaviours** |
| ✓ correct values of $m$ and $c$ |

Line $L_1$ is parallel to $L$ and passes through the point $(2, -3)$.

Line $L_2$ is perpendicular to $L$ and passes through the point $(9, 1)$.

(b) Determine the point of intersection of $L_1$ and $L_2$. (6 marks)

| Solution |
|---|
| $L_1: (y - -3) = \frac{3}{2}(x - 2) \Rightarrow y = \frac{3}{2}x - 6$ <br> $L_2: (y - 1) = -\frac{2}{3}(x - 9) \Rightarrow y = -\frac{2}{3}x + 7$ <br> $\frac{3}{2}x - 6 = -\frac{2}{3}x + 7$ <br> $\left(\frac{3}{2} + \frac{2}{3}\right)x = 13$ <br> $\frac{13}{6}x = 13$ <br> $x = 6$ <br> $y = \frac{3}{2}(6) - 6 = 3$ <br> Lines intersect at $(6, 3)$. |
| **Specific behaviours** |
| ✓ equation of $L_1$ <br> ✓ gradient of $L_2$ <br> ✓ equation of $L_2$ <br> ✓ equates lines and groups like terms <br> ✓ solves for $x$ <br> ✓ solves for $y$ and states point of intersection |

**Question 5** **(7 marks)**

**(a)** In a right triangle, one angle measures $x°$, where $\sin x = \frac{4}{5}$.

State the value of $\cos(90 - x°)$. (1 mark)

| Solution |
|---|
| $\frac{4}{5}$ |
| **Specific behaviours** |
| ✓ exact answer stated |

**(b)** Solve the equation $(\tan\theta + 1)(\sin^2\theta - \sin\theta) = 0$ for $\theta$, given that $-\pi \le \theta \le 2\pi$. (3 marks)

| Solution |
|---|
| $\tan\theta = -1$ or $\sin\theta = 0$ or $\sin\theta = 1$ <br> $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, -\frac{\pi}{4}, 0, \pi, 2\pi, -\pi, \frac{\pi}{2}$ |
| **Specific behaviours** |
| ✓ three equations stated <br> ✓ all solutions for solving tan or sin equation stated <br> ✓ all solutions stated |

**(c)** By using the appropriate addition formula find the exact value of $\cos\left(\frac{5\pi}{12}\right)$. (3 marks)

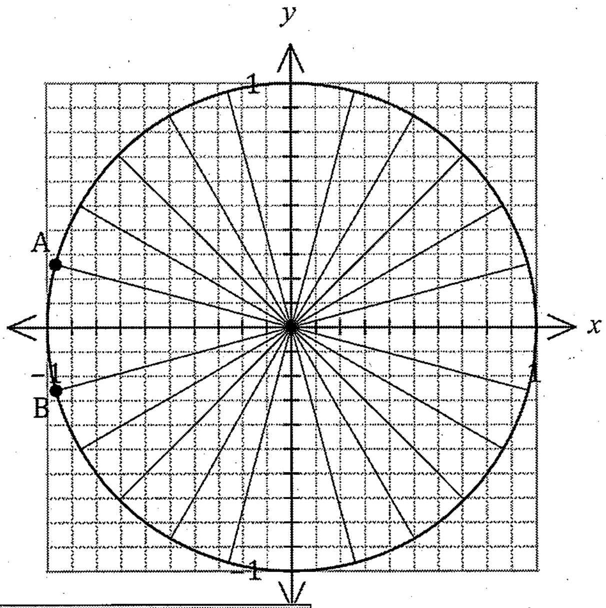
| Solution |
|---|
| $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$ <br> $= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$ <br> $= \frac{\sqrt{6} - \sqrt{2}}{4}$ |
| **Specific behaviours** |
| ✓ correct identity with two appropriate angles stated <br> ✓ exact values for sin and cos correctly substituted <br> ✓ final solution stated |

**Question 6** **(6 marks)**

(a) A unit circle is shown.

Mark on the circumference of the circle the points A and B so that rays drawn from the origin to each point make anti-clockwise angles of $165°$ and $\frac{13\pi}{12}$ from the positive $x$-axis respectively.

Hence estimate the value of $\cos 165°$ and the value of $\sin\left(\frac{13\pi}{12}\right)$.




(3 marks)

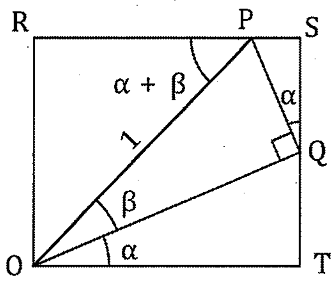
| Solution |
|---|
| See graph for points. |
| $\cos 165° = x$, where $-0.98 \le x \le -0.95$ |
| $\sin\left(\frac{13\pi}{12}\right) = y, -0.28 \le y \le -0.24$ |
| **Specific behaviours** |
| ✓ both points located correctly |
| ✓ value of cosine within range |
| ✓ value of sine within range |

(b) Solve the equation $3\tan(2x - 10°) = \sqrt{3}$ for $0° \le x \le 180°$. (3 marks)

| Solution |
|---|
| $\tan(2x - 10°) = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$<br>$2x - 10° = 30°, 210°$<br>$2x = 40°, 220°$<br>$x = 20°, 110°$ |
| **Specific behaviours** |
| ✓ eliminates tan from equation |
| ✓ one correct solution |
| ✓ second correct solution |

**Question 7** **(7 marks)**

Consider rectangle $ORST$ that contains the right triangle $OPQ$ as shown.




Let the length of $OP = 1$,
$\angle QOT = \angle SQP = \alpha$,
$\angle POQ = \beta$ and
$\angle OPR = \alpha + \beta$.

(a) Explain why $QT = \sin\alpha\cos\beta$. (2 marks)

| Solution |
|---|
| In triangle $OPQ$, $OQ = \cos\beta$.<br><br>Hence, in triangle $OQT$, $QT = OQ\sin\alpha = \cos\beta\sin\alpha$. |
| **Specific behaviours** |
| ✓ uses $\Delta OPQ$ for length of $OQ$<br>✓ uses $\Delta OQT$ to obtain result |

(b) Determine expressions for the lengths of $QS$ and $OR$ and hence prove the angle sum identity $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$. (3 marks)

| Solution |
|---|
| $QS = PQ\cos\alpha$<br>$= \sin\beta\cos\alpha$ ✓<br><br>$OR = \sin(\alpha+\beta)$ ✓<br><br>Because $ORST$ is a rectangle then<br><br>$OR = SQ + QT$<br>$\sin(\alpha+\beta) = \sin\beta\cos\alpha + \cos\beta\sin\alpha$ ✓ |
| **Specific behaviours** |
| ✓ length of $QS$<br>✓ length of $OR$<br>✓ uses congruent sides of rectangle to complete proof |

(c) Use the identity from part (b) to show that $\sin\left(x + \frac{3\pi}{2}\right) = -\cos x$. (2 marks)

| Solution |
|---|
| $\sin\left(x+\frac{3\pi}{2}\right) = \sin x\cos\frac{3\pi}{2} + \cos x\sin\frac{3\pi}{2}$<br>$= \sin x \times 0 + \cos x \times -1$<br>$= -\cos x$ |
| **Specific behaviours** |
| ✓ expands using identity<br>✓ clearly shows both known values and simplifies |

**Question 8** **(6 marks)**

Two polynomial functions are defined by $f(x) = (3x - 2)(x + 4)$ and $g(x) = x^3 - x^2 + 3x + 2$.

Determine the coordinates of the point(s) of intersection of $f(x)$ and $g(x)$.

| Solution |
|---|
| Expand $f(x)$ ✓<br>$$f(x) = (3x-2)(x+4) = 3x^2 + 10x - 8$$<br>Equate functions:<br>$$x^3 - x^2 + 3x + 2 = 3x^2 + 10x - 8$$<br>Equate to zero: ✓<br>$$x^3 - 4x^2 - 7x + 10 = 0$$<br>Find root: ✓<br>$$x = 1 \Rightarrow 1 - 4 - 7 + 10 = 0$$<br>Start factorising: ✓<br>$$x^3 - 4x^2 - 7x + 10 = (x-1)(x^2 - 3x - 10)$$<br>Complete factorising: ✓<br>$$x^3 - 4x^2 - 7x + 10 = (x-1)(x-5)(x+2)$$<br>Coordinates: ✓<br>$$f(1) = (1)(5) = 5$$<br>$$f(5) = (13)(9) = 117$$<br>$$f(-2) = (-8)(2) = -16$$<br>Intersect at $(1, 5)$, $(5, 117)$ and $(-2, -16)$. |
| **Specific behaviours** |
| ✓ expands quadratic<br>✓ equate functions and then to zero<br>✓ finds first root<br>✓ factors into linear and quadratic<br>✓ completes factorisation<br>✓ determines $y$-coordinates and states coordinates of all points |

**End of questions**

Kennedy
Baptist College

**Semester One Examination, 2021**

**Question/Answer booklet**

# MATHEMATICS METHODS UNIT 1

## Section Two: Calculator-assumed

**SOLUTIONS**

WA student number: In figures

In words

Your name

Number of additional answer booklets used (if applicable):

### Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

### Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet
Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

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| | | | | **Total** | 100 |

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7. The Formula sheet is not to be handed in with your Question/Answer booklet.

## Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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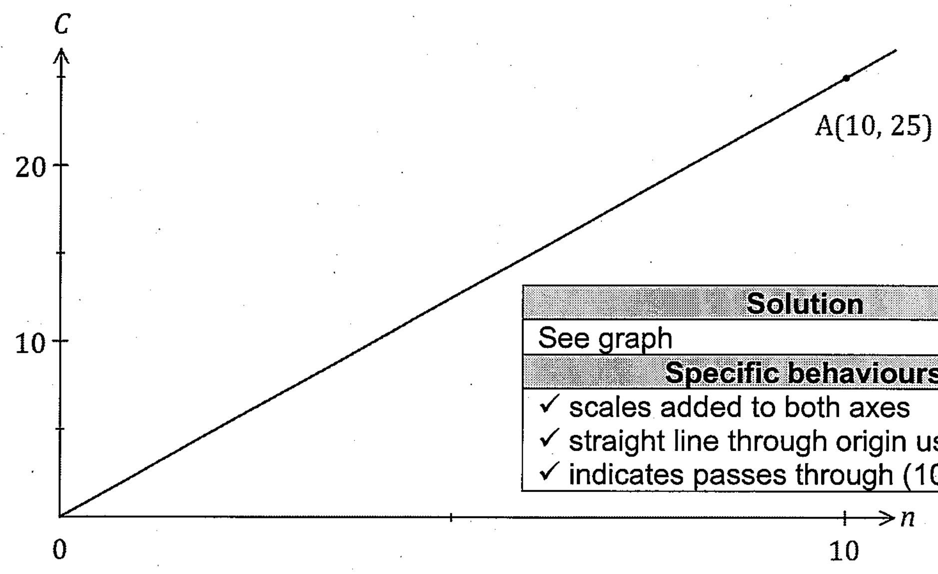
### Question 9 **(6 marks)**

(a) The variables $C$ and $n$ are directly proportional to each other, so that when $n = 10$, it is known that $C = 25$. Sketch a graph of the relationship between $C$ and $n$ on the axes below. (3 marks)




| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ scales added to both axes<br>✓ straight line through origin using ruler<br>✓ indicates passes through (10, 25) |

(b) The variables $A$ and $n$ are inversely proportional to each other, so that when $n = 10$, it is known that $A = 60$.

(i) Write an equation that relates $A$ and $n$. (2 marks)

| Solution |
|---|
| $A \propto \dfrac{1}{n} \Rightarrow A = \dfrac{k}{n}$<br>$k = 10 \times 60 = 600 \Rightarrow A = \dfrac{600}{n}$ |
| **Specific behaviours** |
| ✓ correct form of equation<br>✓ determines constant |

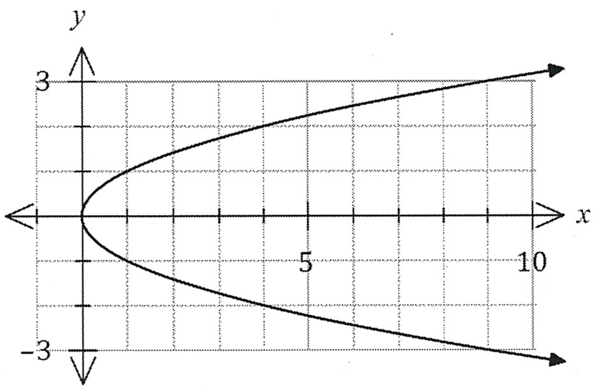
(ii) Determine the value of $n$ when $A = 15$. (1 mark)

| Solution |
|---|
| $n = \dfrac{600}{15} = 40$ |
| **Specific behaviours** |
| ✓ states value |

ft from (i)

**Question 10** **(6 marks)**

(a) The parabolic graph of a relation is shown below.




(i) State the equation of its axis of symmetry. (1 mark)

| Solution |
|---|
| $y = 0$ |
| **Specific behaviours** |
| ✓ correct equation |

(ii) State the equation of the relationship between $x$ and $y$. (1 mark)

| Solution |
|---|
| $x = y^2$ |
| **Specific behaviours** |
| ✓ correct equation |

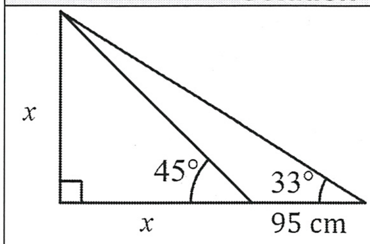
(b) Points $A$ and $B$ have coordinates $(7, 8)$ and $(-3, 2)$ respectively. Determine the equation of the circle that has diameter $AB$. (4 marks)

| Solution |
|---|
| Midpoint: $$\left(\frac{7-3}{2}, \frac{8+2}{2}\right) = (2, 5)$$ Radius: $$r^2 = (7-2)^2 + (8-5)^2$$ $$= 34$$ Equation: $$(x-2)^2 + (y-5)^2 = 34$$ Must be 34 – not decimal value squared |
| **Specific behaviours** |
| ✓ identifies midpoint as centre<br>✓ calculates coordinates of centre<br>✓ calculates square of radius<br>✓ writes equation of circle |

**Question 11** **(8 marks)**

(a) At 3 pm, the length of the shadow of a thin vertical pole standing on level ground is the same as the height of the pole. A while later, the angle of elevation of the sun has decreased by 12° and the length of the shadow has increased by 95 cm. Determine the height of the pole. (4 marks)

**Solution**

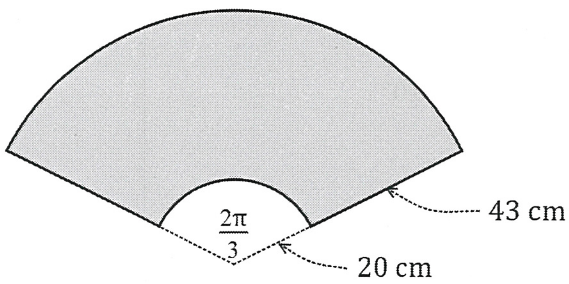



$$\tan(33) = \frac{x}{x+95} \Rightarrow x = 176 \text{ cm}$$

**Specific behaviours**

✓ initial info sketch
✓ extra info added to sketch
✓ writes equation with $x$ as a variable
✓ solves equation with CAS (AWRT 176)

(b) A windscreen wiper on a car is 43 cm long and rotates through one-third of a circle, as shown below. The inner and outer radii of the arcs are 20 cm and 63 cm. Determine the shaded area, rounding your answer to a reasonable degree of accuracy. (4 marks)




**Solution**

$$\tfrac{1}{2} \times 20^2 \times \frac{2\pi}{3} = 418.9$$
$$\tfrac{1}{2} \times 63^2 \times \frac{2\pi}{3} = 4156.3$$
$$\text{Diff} = 3737.4$$
$$\text{Area} = 3740 \text{ cm}^2$$

**Specific behaviours**

✓ inner sector
✓ outer sector
✓ works throughout to at least 4 sf
✓ rounds answer to 2 or 3 sf

**Question 12** **(8 marks)**

(a) Triangle $ABC$ is such that $b = 15$ cm, $c = 18$ cm and $\angle A = 125°$. Determine, with justification, the length of side $a$. (2 marks)

| Solution |
|---|
| $a^2 = 15^2 + 18^2 - 2(15)(18)\cos 125°$<br>$a = 29.3 \text{ cm}$ |
| **Specific behaviours** |
| ✓ clearly shows use of cosine rule<br>✓ correct length |

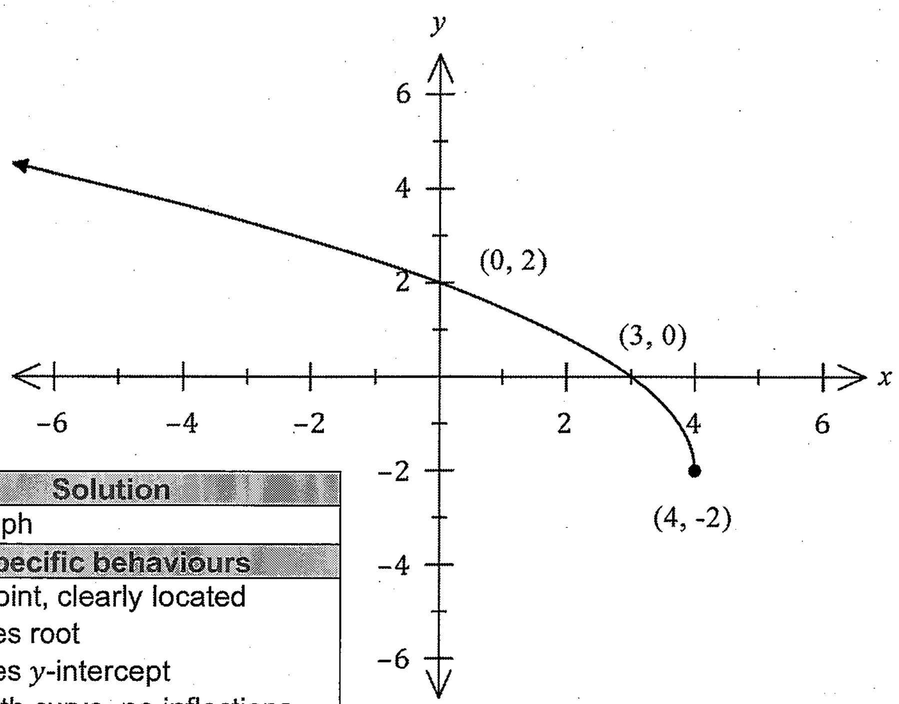
(b) Triangle $PQR$ is such that $p = 48.1$ cm, $q = 41.5$ cm and $\angle Q = 45°$. Determine all possible areas of this triangle. (6 marks)

| Solution |
|---|
| $\frac{\sin P}{48.1} = \frac{\sin 45°}{41.5}$<br>First solution:<br>$\angle P_1 = 55°$<br>$\angle R_1 = 180° - 45° - 55° = 80°$<br>$A_1 = \frac{1}{2}(48.1)(41.5)\sin 80°$<br>$A_1 = 983 \text{ cm}^2$<br><br>Second solution:<br>$\angle P_2 = 180° - 55° = 125°$<br>$\angle R_2 = 180° - 45° - 125° = 10°$<br>$A_2 = \frac{1}{2}(48.1)(41.5)\sin 10°$<br>$A_2 = 174 \text{ cm}^2$<br><br>Areas are $174 \text{ cm}^2$ and $983 \text{ cm}^2$. |
| **Specific behaviours** |
| ✓ shows use of sine rule<br>✓ calculates one value for $\angle P$<br>✓ derives $\angle R$ and shows use of area formula<br>✓ calculates one correct area<br>✓ calculates second set of values for $\angle P$ and $\angle R$<br>✓ calculates second area |

**Question 13** **(8 marks)**

Let $f(x) = 2\sqrt{4-x} - 2$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (4 marks)




| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ endpoint, clearly located |
| ✓ locates root |
| ✓ locates $y$-intercept |
| ✓ smooth curve, no inflections |

(b) Describe the transformation(s) required to obtain the graphs of the following functions from the graph of $y = f(x)$:

(i) $y = 2\sqrt{1-x} - 2$. (2 marks)

| Solution |
|---|
| $y = f(x-3)$. Horizontal translation of 3 units to the right. |
| **Specific behaviours** |
| ✓ states a translation |
| ✓ correct distance and direction |

(ii) $y = \sqrt{4-x} - 1$. (2 marks)

| Solution |
|---|
| $y = \frac{1}{2}f(x)$. **Vertical dilation** of scale factor $\frac{1}{2}$. |
| **Specific behaviours** |
| ✓ both bolded words in description |
| ✓ correct scale factor |

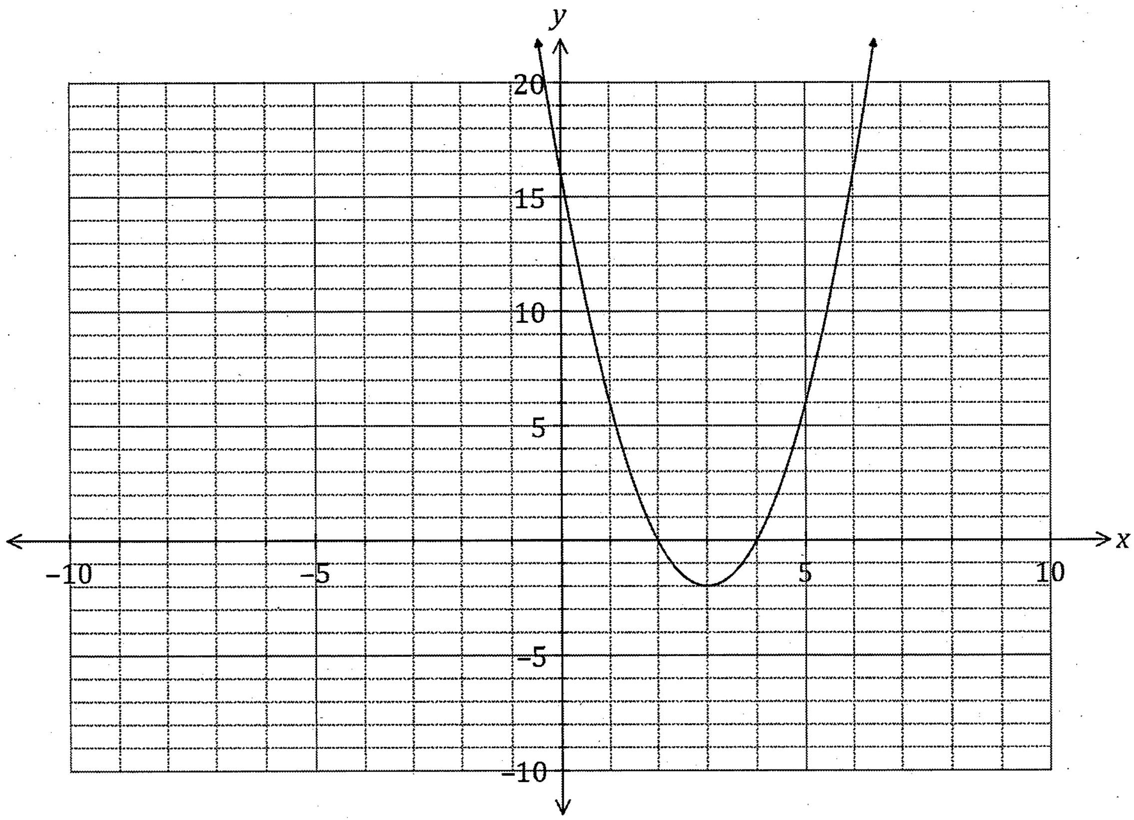
**Question 14** **(8 marks)**

(a) The graph of $y = 2x^2 + bx + 16$ has a line of symmetry with equation $x = 3$.

(i) Determine the value of $b$. (2 marks)

| Solution |
|---|
| $x = -\dfrac{b}{2a} \Rightarrow b = -3 \times 2 \times 2 = -12$ |
| **Specific behaviours** |
| ✓ uses line of symmetry<br>✓ value of $b$ |

(ii) Draw the graph of the parabola on the axes below. (3 marks)




| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ turning point<br>✓ three axes intercepts<br>✓ smooth curve |

(b) One of the solutions to the equation $2x^3 + 21x^2 + cx - 495 = 0$ is $x = 5$. Determine the value of $c$ and all other solutions. (3 marks)

| Solution |
|---|
| Using CAS, when $x = 5, 5c + 280 = 0 \Rightarrow c = -56$<br>Use CAS to solve $2x^3 + 21x^2 - 56x - 495 = 0$<br>$x = -11, x = -4.5$ and $x = 5$ |
| **Specific behaviours** |
| ✓ substitutes $x = 5$<br>✓ determines $c$<br>✓ states other two solutions |

**Question 15** **(9 marks)**

The loudness $L$ of sound, in decibels, emitted by a machine $t$ minutes after it is switched on can be modelled by

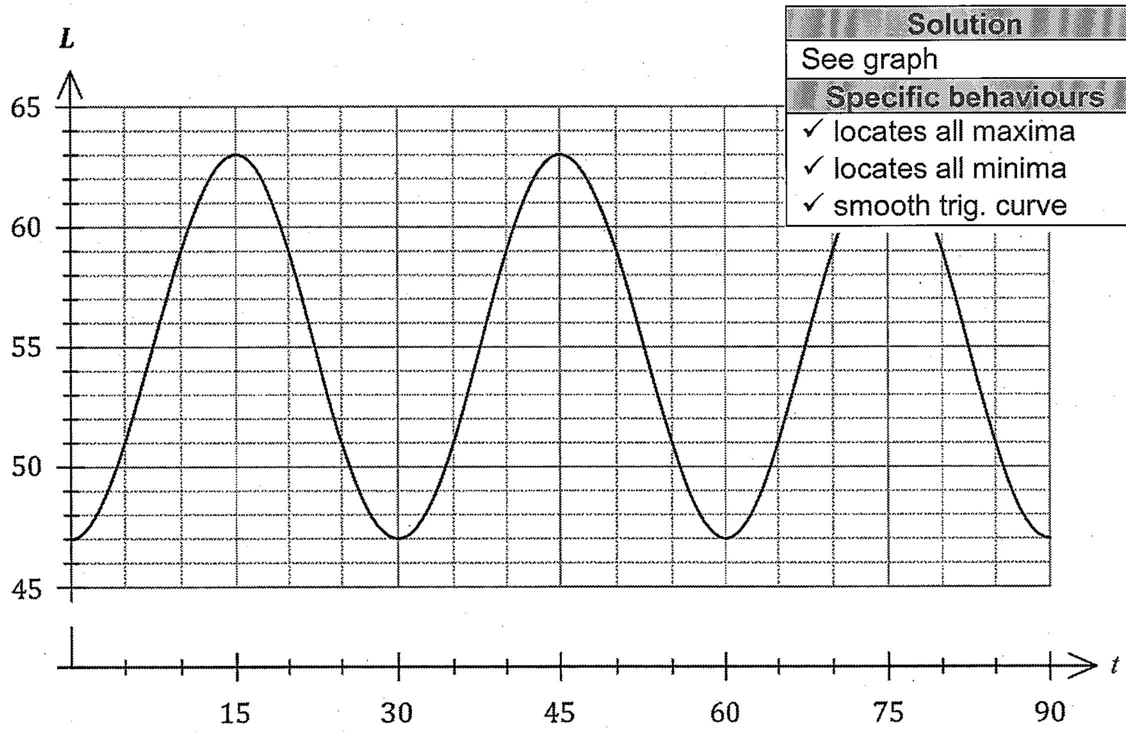
$$L = 55 - 8\cos\left(\frac{\pi t}{15}\right)$$

(a) Determine the initial loudness emitted by the machine. (1 mark)

| Solution |
|---|
| $L(0) = 47$ dB |
| **Specific behaviours** |
| ✓ correct value |

(b) Draw the graph of $L$ against $t$ on the axes below for the first 90 minutes. (3 marks)

| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ locates all maxima |
| ✓ locates all minima |
| ✓ smooth trig. curve |




(c) State the maximum loudness emitted by the machine and the time this maximum was first reached. (2 marks)

| Solution |
|---|
| $L_{MAX} = 63$ dB when $t = 15$ min |
| **Specific behaviours** |
| ✓ correct maximum |
| ✓ correct time |

(d) A health and safety inspector can deem a machine unserviceable if the loudness it emits exceeds 60 dB for more than 15 minutes in any hour that it is running. Determine, with justification, whether this machine could be deemed unserviceable. (3 marks)

| Solution |
|---|
| Exceeds value for $10.73 < t < 19.28$ during first cycle.<br><br>$2(19.28 - 10.73) = 17.1$ minutes per hour - and so machine could be deemed unserviceable. |
| **Specific behaviours** |
| ✓ identifies interval endpoints |
| ✓ calculates minutes per hour |
| ✓ uses calculations to draw conclusion |

**Question 16** **(8 marks)**

(a) Let $f(x) = x^2 + bx + c$, where $b$ and $c$ are constants. The graph of $y = f(x)$ has an axis of symmetry with equation $x = -3$ and an axis intercept at $(0, 5)$.

Determine the value of *f(1)* (3 marks)

| Solution |
|---|
| $c$ is the $y$-intercept: $c = 5$.<br>Axis of symmetry has equation $x = -\frac{b}{2a}$:<br>$$-3 = -\frac{b}{2} \Rightarrow b = 6$$<br>f(1) = 1 + b + c = 12 |
| **Specific behaviours** |
| ✓ correct value for c<br>✓ calculates value of b<br>✓ calculates *f(1)* correctly |

(b) Let $g(x) = 2(x-2)^2 - 7$. Determine

(i) the coordinates of the turning point of the graph of $y = g(x)$. (1 mark)

| Solution |
|---|
| Turning point is at $(2, -7)$. |
| **Specific behaviours** |
| ✓ correct coordinates |

(ii) the domain and range of $g(x)$. (2 marks)

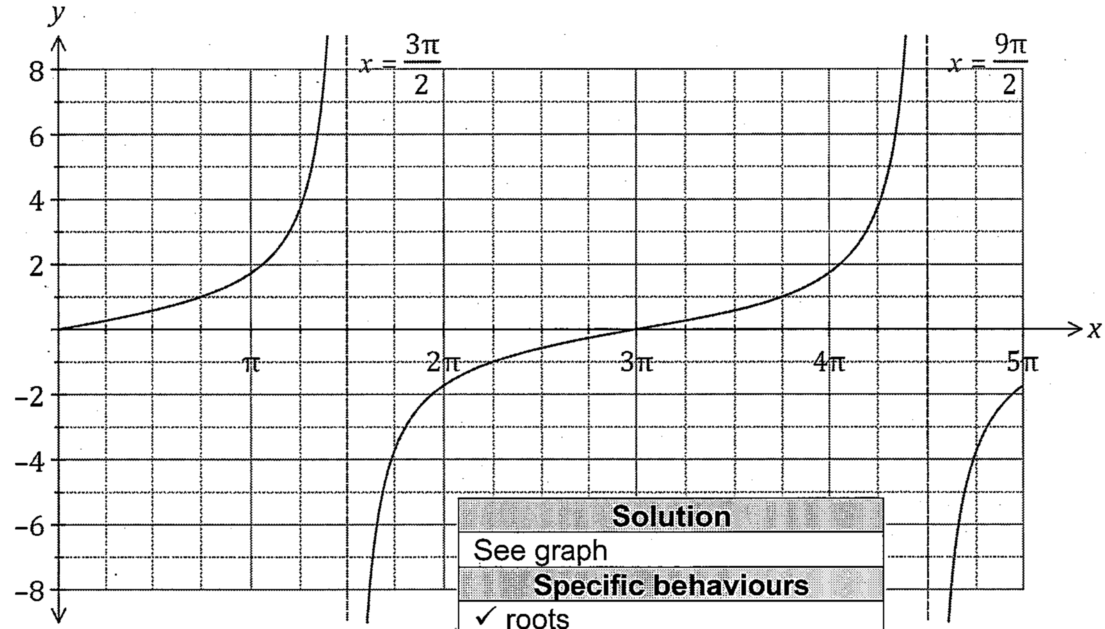
| Solution |
|---|
| Domain: $x \in \mathbb{R}$, and range: $y \geq -7$. |
| **Specific behaviours** |
| ✓ states domain<br>✓ states range |

(iii) the coordinates of the turning point of the graph of $y = g(x-3) + 2$. (2 marks)

| Solution |
|---|
| Graph has been translated 3 units right and 2 units upwards and so new turning point at $(5, -5)$. |
| **Specific behaviours** |
| ✓ indicates correct use of one translation<br>✓ correct coordinates |

**Question 17** **(8 marks)**

(a) On the axes below, draw the graph of $y = \tan\left(\frac{x}{3}\right)$ over the interval $0 \le x \le 5\pi$, clearly indicating the equations of any asymptotes. (3 marks)




| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ roots<br>✓ general shape<br>✓ equations of asymptotes |

(b) Solve the following equations over the interval $0 \le x \le 5\pi$.

(i) $\tan\left(\frac{x}{3}\right) = -1.$ (1 mark)

| Solution |
|---|
| $x = \frac{9\pi}{4}$ |
| **Specific behaviours** |
| ✓ one solution |

(ii) $\tan\left(\frac{x}{3}\right) - \sqrt{3} = 0.$ (2 marks)

| Solution |
|---|
| $x = \pi, \qquad x = 4\pi$ |
| **Specific behaviours** |
| ✓ first solution<br>✓ second solution |

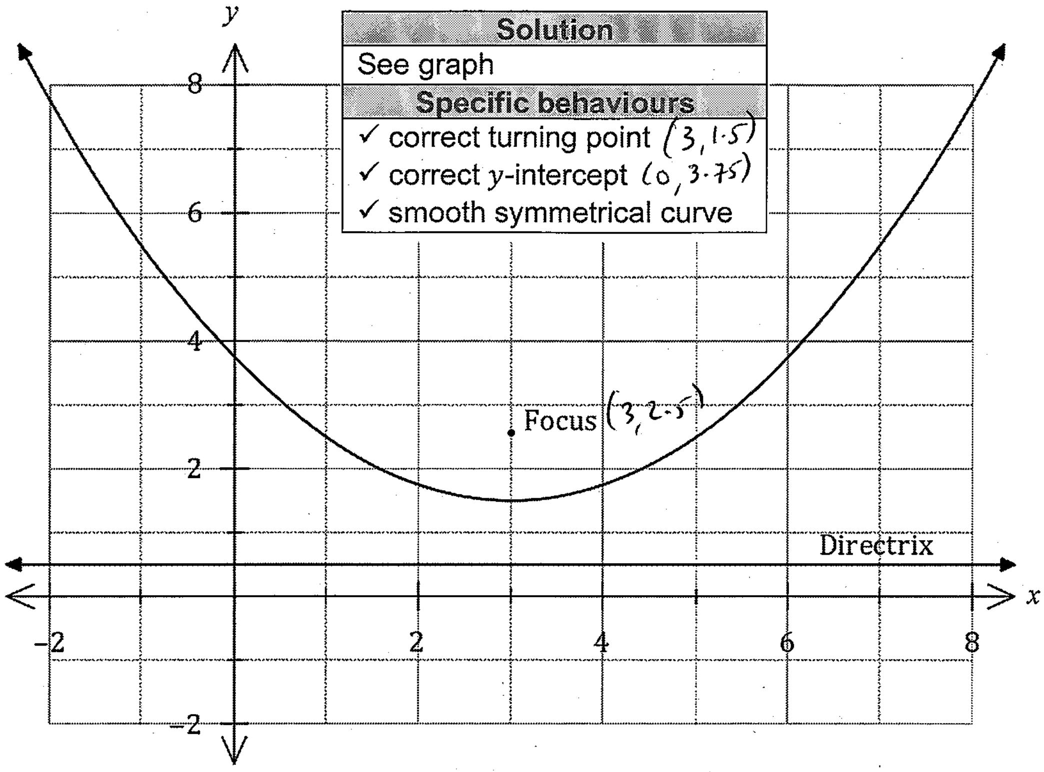
(c) Determine the smallest positive value of $\alpha$ so that $\tan\left(x - \frac{5\pi}{6}\right) = \tan(x + \alpha)$. (2 marks)

| Solution |
|---|
| Function has period of $\pi$ and so $\alpha = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$ |
| **Specific behaviours** |
| ✓ uses period<br>✓ determines $\alpha$ |

**Question 18** **(7 marks)**

The equation of a parabola is $y = \frac{1}{4}(x^2 - 6x + 15)$.

(a) Sketch the parabola on the axes below. (3 marks)




| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ correct turning point (3, 1.5) |
| ✓ correct $y$-intercept (0, 3.75) |
| ✓ smooth symmetrical curve |

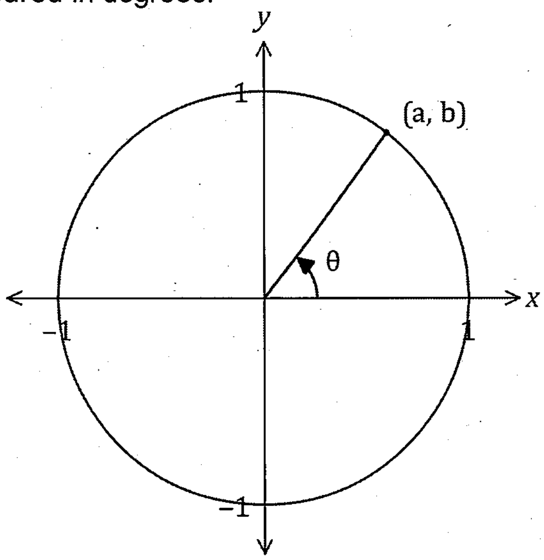
All parabolas have a focal point and a directrix. For a parabola with equation $y = a(x-p)^2 + q$, the focal point is at $\left(p, q + \frac{1}{4a}\right)$ and the equation of the directrix is $y = q - \frac{1}{4a}$, where $a, p$ and $q$ are constants.

(b) Determine the focal point and directrix for this parabola and add them, with labels, to your sketch above. (4 marks)

| Solution |
|---|
| From graph, turning point at $(3, 1.5)$. Hence<br>$a = \frac{1}{4}, \quad p = 3, \quad q = 1.5$<br>Focal point: $(3, 2.5)$ and directrix: $y = 0.5$. |
| **Specific behaviours** |
| ✓ indicates turning point<br>✓ indicates values of all constants<br>✓ plots focus<br>✓ draws directrix |

**Question 19** **(8 marks)**

(a) Using the unit circle shown, determine the following in terms of $a$ and/or $b$, given that $\theta$ is an acute angle measured in degrees.




(i) $\sin(\theta)$. (1 mark)

(ii) $\cos(180 - \theta)$. (1 mark)

(iii) $\tan(90 + \theta)$. (1 mark)

| Solution |
|---|
| (i) $b$ <br> (ii) $-a$ <br> (iii) $-\frac{a}{b}$ |
| **Specific behaviours** |
| ✓ (i) <br> ✓ (ii) <br> ✓ (iii) |

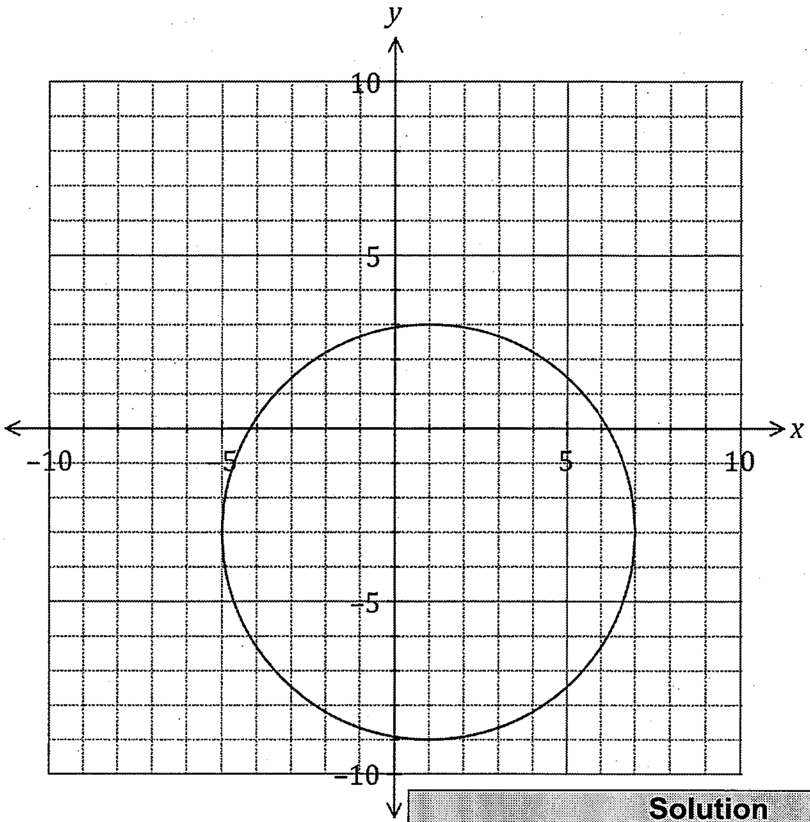
(b) Determine $x$ in each of the following cases, where $0 \le x \le \frac{\pi}{2}$.

(i) $\sin x = \sin 17\pi$. (1 mark)

(ii) $\cos x = \cos \dfrac{23\pi}{6}$. (1 mark)

| Solution |
|---|
| (i) $x = 0$ <br> (ii) $x = \frac{\pi}{6}$ |
| **Specific behaviours** |
| ✓ (i) <br> ✓ (ii) |

(c) Draw the graph of the relation $x^2 + y^2 = 2x - 6y + 26$. (3 marks)

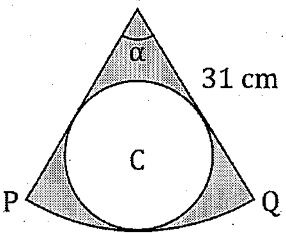



| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ centred at (1, -3) and radius of 6 |
| ✓ at least three axes-intercepts within $\pm 0.5$ |
| ✓ smooth circle |

**Question 20** **(7 marks)**

The diagram shows sector $OPQ$ of a circle centre $O$ of radius 31 cm and $\alpha = 40°$.

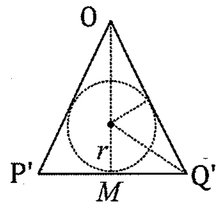
Circle $C$ is inside the sector and just touches $OP, OQ$ and arc $PQ$.




(a) Determine the area of sector $OPQ$. (2 marks)

| Solution |
|---|
| $A = \frac{40°}{360°} \times \pi(31)^2 = \frac{961\pi}{9} \approx 335 \text{ cm}^2$ |
| **Specific behaviours** |
| ✓ indicates suitable method<br>✓ calculates area |

(b) Show that the radius of circle $C$ is 7.9 cm, correct to one decimal place. (3 marks)

| Solution |
|---|
| Extend sector sides to form isosceles triangle: |
|  |
| $MQ' = 31\tan\left(\frac{40°}{2}\right) = 11.28 \text{ cm}$<br>$r = 11.28\tan\left(\frac{70°}{2}\right) = 7.9 \text{ cm}$ |
| **Specific behaviours** |
| ✓ forms isosceles triangle, uses half-angle<br>✓ calculates $MQ'$<br>✓ calculates radius |

(c) Determine the area of the shaded region, inside sector $OPQ$ but outside circle $C$. (2 marks)

| Solution |
|---|
| $A_C = \pi(7.9)^2 \approx 196$<br><br>Shaded area $= 335 - 196 = 139 \text{ cm}^2$ |
| **Specific behaviours** |
| ✓ calculates area of circle<br>✓ calculates shaded area, with units |

**Question 21** **(7 marks)**

The equation $f(x) = k$ has two solutions, where $f(x) = ax^3 + bx^2 - 12x + 8$ and $a, b$ and $k$ are constants.

The graph of $y = f(x)$ cuts the $x$-axis at $x = 2$, $x = -2$, and at one other point.

Determine the value(s) of the constant $k$, rounded to 2 decimal places. Explain your reasoning.

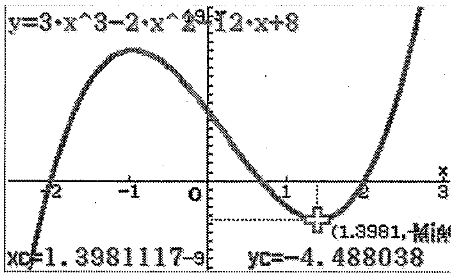
**Solution**

Use roots to solve for $a$ and $b$:

$$f(2) = 0 \Rightarrow 8a + 4b - 24 + 8 = 0$$

$$f(-2) = 0 \Rightarrow -8a + 4b + 24 + 8 = 0$$

Solving simultaneously with CAS gives $a = 3$ and $b = -2$.




For two solutions, $k$ must equal the local maximum or equal the local minimum of $f(x)$ - found using CAS.

Local maximum is $y = 15.0230$

Local minimum is $y = -4.4880$

Hence $k = -4.49$ or $k = 15.02$.

**Specific behaviours**

✓ indicates solving for $a$ and $b$
✓ equates $f(1) = 0, f(-2) = 0$
✓ identifies equations as simultaneous
✓ solves equations for $a$ and $b$
✓ describes case for one solution
✓ states value of local minimum, maximum
✓ correct solutions for $k$



**End of questions**