PERTH MODERN SCHOOL
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Independent Public School

# Program: MATHEMATICS SPECIALIST 3 & 4
## 2022
# Note: This outline and assessment dates are subject to change!

The program you have been issued references all objectives in the Curriculum Council syllabus. The syllabus may be easily downloaded from the Curriculum Council website www.scsa.wa.edu.au

**Unit 3: Description**

Unit 3 of the Mathematics Specialist ATAR course contains three topics: Complex numbers, Functions and sketching graphs and Vectors in three dimensions. The study of vectors was introduced in Unit 1 with a focus on vectors in two-dimensional space. In this unit, three-dimensional vectors are studied and vector equations and vector calculus are introduced, with the latter extending students' knowledge of calculus from the Mathematics Methods ATAR course. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space. The Cartesian form of complex numbers was introduced in Unit 2, and the study of complex numbers is now extended to the polar form.

The study of functions and techniques of graph sketching, begun in the Mathematics Methods ATAR course, is extended and applied in sketching graphs and solving problems involving integration.

**Unit 4: Description**

Unit 4 of the Mathematics Specialist ATAR course contains three topics: Integration and applications of integration, Rates of change and differential equations and Statistical inference.

In Unit 4, the study of differentiation and integration of functions continues, and the calculus techniques developed in this and previous topics are applied to simple differential equations, in particular in biology and kinematics. These topics demonstrate the real-world applications of the mathematics learned throughout the Mathematics Specialist ATAR course.

In this unit, all of the students' previous experience working with probability and statistics is drawn together in the study of statistical inference for the distribution of sample means and confidence intervals for sample means.

Access to technology to support the computational aspects of these topics is assumed.

**SYLLABUS CONTENT**

| Topics | Syllabus Time Allotment |
|---|---|
| 3.1: Complex numbers | 18 hours |
| 3.2: Functions and sketching graphs | 16 hours |
| 3.3: Vectors in three dimensions | 21 hours |
| 4.1: Integration and applications of integration | 20 hours |
| 4.2: Rates of change and differential equations | 20 hours |
| 4.3: Statistical inference | 15 hours |

# Note: Year 12 course begins late in term 4 2021

# Course Outline

**TEXT** Mathematics Specialist Units 3 & 4 by A.J. Sadler

**WEBSITES** www.scsa.wa.edu.au/

| Week | Content | | Text |
|---|---|---|---|
| **TERM 4 - 2021** | **Unit 3 Preliminary work.** | | **Preliminary Work** |
| | Number. | 7 | |
| | The absolute value. | 8 | p 7 – 22 |
| | Function. | 8 | |
| | Algebra. | 9 | |
| | Vectors. | 10 | |
| | Complex numbers. | 15 | |
| | Circles. | 17 | |
| | Differentiation. | 18 | |
| | Sketching graphs. | 19 | |
| | Antidifferentiation. | 20 | |
| | Trigonometrical identities. | 21 | |
| | Matrices. | 21 | |
| | Use of technology. | 22 | |

| Week | Content | Text |
| --- | --- | --- |
| Wk 5-8 | **Topic 3.1: Complex numbers (18 hours)**<br>**Cartesian forms**<br>3.1.1 review real and imaginary parts Re(z) and Im(z) of a complex number z<br>3.1.2 review Cartesian form<br>3.1.3 review complex arithmetic using Cartesian forms<br><br>**Factorisation of polynomials**<br>3.1.13 prove and apply the factor theorem and the remainder theorem for polynomials<br>3.1.14 consider conjugate roots for polynomials with real coefficients<br>3.1.15 solve simple polynomial equations<br><br>**Complex arithmetic using polar form**<br>3.1.4 use the modulus \|z\| of a complex number *z* and the argument Arg (z) of a non-zero complex number z and prove basic identities involving modulus and argument<br>3.1.5 convert between Cartesian and polar form<br>3.1.6 define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these<br>**The complex plane (The Argand plane)**<br>3.1.8 examine and use addition of complex numbers as vector addition in the complex plane<br>3.1.9 examine and use multiplication as a linear transformation in the complex plane<br>3.1.10 identify subsets of the complex plane determined by relations such as<br>$\lvert z-3i\rvert \le 4,\ \frac{\pi}{4} \le Arg(z) \le \frac{3\pi}{4}$ and $\lvert z-1\rvert = 2\lvert z-i\rvert$ | **Ch 1**<br><br><br><br><br><br><br><br><br><br>**Ch 2**<br>**Ex 2A to 2D only** |
| **TERM 1 - 2022**<br><br>**Wk 1-2**<br><br><br><br><br>**Wk 3-5** | **Roots of complex numbers**<br>3.1.11 determine and examine the $n^{th}$ roots of unity and their location on the unit circle<br>3.1.12 determine and examine the $n^{th}$ roots of complex numbers and their location in the complex plane<br>3.1.7 prove and use De Moivre's theorem for integral powers<br><br>**Topic 3.2: Functions and sketching graphs (16 hours)**<br>**Functions**<br>3.2.1 determine when the composition of two functions is defined<br>3.2.2 determine the composition of two functions<br>3.2.3 determine if a function is one-to-one<br>3.2.4 find the inverse function of a one-to-one function<br>3.2.5 examine the reflection property of the graphs of a function and its inverse | **Ch 2**<br>**Ex 2E,F**<br><br><br><br>**Ch 3**<br><br>**TEST 1 (Wk3)** |

| Week | Content | Text |
| --- | --- | --- |
| Wk 6-9<br><br><br><br><br>Wk 10 | **Sketching graphs**<br>3.2.6 use and apply $\lvert x \rvert$ for the absolute value of the real number $x$ and the graph of $y = \lvert x \rvert$<br>3.2.7 examine the relationship between the graph of $y = f(x)$ and the graphs of $y = \frac{1}{f(x)}$, $y = \lvert f(x) \rvert$ and $y = f(\lvert x \rvert)$<br>3.2.8 sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree<br><br>Topic 3.3: Vectors in three dimensions (21 hours)<br>**The algebra of vectors in three dimensions**<br>3.3.1 review the concepts of vectors from Unit 1 and extend to three dimensions, including introducing the unit vectors i, j and k<br>3.3.2 prove geometric results in the plane and construct simple proofs in 3 dimensions<br>**Vector and Cartesian equations**<br>3.3.3 introduce Cartesian coordinates for three dimensional space, including plotting points and equations of spheres<br>3.3.4 use vector equations of curves in two or three dimensions involving a parameter and determine a 'corresponding' Cartesian equation in the two-dimensional case<br>3.3.5 determine a vector equation of a straight line and straight line segment, given the position of two points or equivalent information, in both two and three dimensions<br>3.3.6 examine the position of two particles, each described as a vector function of time, and determine if their paths cross or if the particles meet<br>3.3.7 use the cross product to determine a vector normal to a given plane<br>3.3.8 determine vector and Cartesian equations of a plane | **INVEST 1 Wk6**<br><br>**Ch 4 Vectors in 2D**<br><br><br><br>**Ch 5 Vectors in 3D** |

| Week | Content | Text |
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| **Term 2**<br><br>**1-2**<br><br>**3** | **Systems of linear equations**<br>3.3.9 recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations<br>3.3.10 examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables<br>**Vector calculus**<br>3.3.11 consider position vectors as a function of time<br>3.3.12 derive the Cartesian equation of a path given as a vector equation in two dimensions, including ellipses and hyperbolas<br>3.3.13 differentiate and integrate a vector function with respect to time<br>3.3.14 determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration<br>3.3.15 apply vector calculus to motion in a plane, including projectile and circular motion | **Ch6**<br><br>**Test 2 week 3 Monday**<br><br>**Ch7** |
| Weeks 4 & 5<br><br>**Week 6-7** | **Exam weeks**<br><br>**Unit Four: Preliminary work.**<br>***Differentiation.*** ***198***<br>***Antidifferentiation and integration.*** ***(To be completed at Home)*** ***198***<br>***Statistics.*** ***198***<br>***Trigonometric identities.*** ***199***<br>***Limit of a sum.*** ***200***<br>***Use of technology***.<br><br>Exam Review<br><br>**Topic 4.2: Rates of change and differential equations (20 hours)**<br>**Applications of differentiation**<br>4.2.1 use implicit differentiation to determine the gradient of curves whose equations are given in implicit form<br>4.2.2 examine related rates as instances of the chain rule: $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$<br>4.2.3 apply the incremental formula $\partial y \approx \frac{dy}{dx}\partial x$ to differential equations | **Preliminary Work**<br><br>**Ch 7** |

| Week | Content | Text |
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| **Week 8** | **Topic 4.1: Integration and applications of integration (20 hours)** | **Ch 8** |
| | **Integration techniques** | |
| | 4.1.1 integrate using the trigonometric identities $\sin^2 x = \frac{1}{2}(1-\cos 2x)$, $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ and $1+\tan^2 x = \sec^2 x$ | |
| | 4.1.2 use substitution $u$ = g(x) to integrate expressions of the form $f(g(x))g'(x)$ | |
| | 4.1.3 establish and use the formula $\int \frac{1}{x}dx = \ln\|x\| + c$ for $x \neq 0$ | |
| **Weeks 9 & 10** | 4.1.4 use partial fractions where necessary for integration in simple cases | **Ch9** |
| | **Applications of integral calculus** | |
| | 4.1.5 calculate areas between curves determined by functions | |
| | 4.1.6 determine volumes of solids of revolution about either axis | |
| | 4.1.7 use technology with numerical integration | |

| Week | Content | Text |
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| **Term 3**<br><br>**Week 1** | 4.2.4 solve simple first order differential equations of the form $\frac{dy}{dx}=f(x)$; differential equations of the form $\frac{dy}{dx}=g(y)$; and, in general, differential equations of the form $\frac{dy}{dx}=f(x)g(y)$, using separation of variables<br>4.2.5 examine slope (direction or gradient) fields of a first order differential equation<br>4.2.6 formulate differential equations, including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved | **Ch 10**<br><br>**TEST 3 (Wk1)** |
| **Weeks 2-3** | **Modelling motion**<br>4.2.7 consider and solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and the use of expressions, $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ for acceleration | **Ch 11** |
| | **Topic 4.3: Statistical inference (15 hours)** | **Inv 2 (wk 3) Fri** |
| Weeks 4-5 | **Sample means**<br>4.3.1 examine the concept of the sample mean $\overline{X}$ as a random variable whose value varies between samples where $X$ is a random variable with mean $\mu$ and the standard deviation $\sigma$<br>4.3.2 simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of $\overline{X}$ across samples of a fixed size *n*, including its mean $\mu$ its standard deviation $\frac{\sigma}{\sqrt{n}}$ (where $\mu$ and $\sigma$ are the mean and standard deviation of *X*), and its approximate normality if *n* is large<br>4.3.3 simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate the approximate standard normality of $\frac{\overline{X}-\mu}{s/\sqrt{n}}$ for large samples $(n \geq 30)$, where *s* is the sample standard deviation | **Ch 12** |
| Weeks 6-7 | **Confidence intervals for means**<br>4.3.4 examine the concept of an interval estimate for a parameter associated with a random variable<br>4.3.5 examine the approximate confidence interval $\left(\overline{X}-\frac{zs}{\sqrt{n}}, \overline{X}+\frac{zs}{\sqrt{n}}\right)$ as an interval estimate for the population mean $\mu$, where $z$ is the appropriate quantile for the standard normal distribution<br>4.3.6 use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain $\mu$ | **Test 4 week 7 Weds** |

| Week | Content | Text |
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| **Week 8** | 4.3.7 use $\bar{x}$ and $s$ to estimate $\mu$ and $\sigma$ to obtain approximate intervals covering desired proportions of values of a normal random variable, and compare with an approximate confidence interval for $\mu$<br><br>Revision | |
| **9-10** | **Mock Exam weeks** | **SEM 2 EXAM** |
| **TERM 4**<br>**1-2** | **Exam review** | |

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| **A** | **Identifies and organises relevant information**<br>Identifies and organises information from previous parts of a problem to carry through and solve new and different problems.<br>Defines variables and equations from text and diagrams.<br>Organises data in a concise, clear format and appropriately presents it in tabular, diagrammatic and/or graphical form.<br>Identifies the underlying assumptions related to the relevant mathematics of an investigation. |
| | **Chooses effective models and methods and carries through the methods correctly**<br>Selects an appropriate strategy and applies mathematical knowledge to solve non-routine problems.<br>Generalises and extends models from previous parts of the question.<br>Translates between representations in unpractised ways.<br>Selects appropriate calculator techniques to solve multi-step problems in unfamiliar contexts.<br>Solves unstructured problems and carries through an extended response, using deductive reasoning.<br>Produces results, carries out analysis and generalises in situations requiring investigative techniques. |
| | **Follows mathematical conventions and attends to accuracy**<br>Consistently uses mathematical conventions to link expressions in clearly defined steps that are easily followed.<br>Works fluently with exact values such as surds, radian values or natural logarithms and exponentials, and expresses answers accurately using these forms as appropriate.<br>Completes concise and accurate solutions to mathematical problems set in applied and theoretical contexts. |
| | **Links mathematical results to data and contexts to reach reasonable conclusions**<br>Identifies and explains the limitations of complex models.<br>Recognises the domain as implied by the context of the question, and excludes any results outside it.<br>Interprets the result and draws the correct conclusion about the effect of changing conditions.<br>Uses counter-examples and general cases in mathematical analysis of an investigation. |
| | **Communicates mathematical reasoning, results and conclusions**<br>Sets out the steps of the solution using deductive reasoning in a clear and logical sequence, including suitable justification and explanation of methods and processes used.<br>Adds a detailed diagram to illustrate and use in the solution of a problem.<br>Presents work with the final answer clearly identified, using the correct units and related to the context of the question.<br>Communicates investigation findings with a comprehensive interpretation of mathematical results in the context of the investigation. |

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| **B** | **Identifies and organises relevant information**<br>Identifies and organises key information from previous parts of a problem and brings it together to solve subsequent parts of the problem.<br>Draws a diagram and labels it with appropriate variables.<br>Organises data clearly and appropriately presents it in tabular, diagrammatic and/or graphical form.<br>Identifies suitable variables and constant parameters related to various aspects of an investigation. |
| | **Chooses effective models and methods and carries the methods through correctly**<br>Selects an appropriate strategy and applies mathematical knowledge to solve simple non-routine problems.<br>Translates between representations in practised ways.<br>Selects appropriate calculator techniques to solve multi-step problems.<br>Attempts to analyse and calculate specific cases of generalisation in situations requiring investigative techniques. |
| | **Follows mathematical conventions and attends to accuracy**<br>Interprets and uses mathematical terminology, symbols and conventions in unpractised situations.<br>Defines introduced variables.<br>Uses exact values, such as surds and radian values to express answers accurately when specified.<br>Completes mostly accurate solutions to mathematical problems set in applied and theoretical contexts. |
| | **Links mathematical results to data and contexts to reach reasonable conclusions**<br>Identifies and explains the limitations of simple models.<br>Takes account of the domain as defined in the problem, and excludes results outside it.<br>Links the effect of changing conditions to the original solution.<br>Uses examples in mathematical analysis of an investigation and draws valid conclusions related to a given context. |
| | **Communicates mathematical reasoning, results and conclusions**<br>Carries through calculations and simplifications in a clear sequence, showing a logical line of reasoning.<br>Defines variables associated with a given diagram and uses these in the working of a problem.<br>Presents work with the final answer clearly identified and using the correct units.<br>Communicates investigation findings in a systematic and concise way using mathematical language and relating the solution to the original problem or statement. |

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| **C** | **Identifies and organises relevant information from information that is relatively narrow in scope**<br>Identifies and organises key information needed to solve a familiar problem.<br>Identifies variables in a given diagram or draws a simple diagram from given information.<br>Organises some data and presents it in tabular, diagrammatic and/or graphical form.<br>Identifies some mathematical content related to various aspects of an investigation in a given context. |
| | **Chooses effective models and methods and carries through the methods correctly**<br>Selects from a range of strategies and formulae and applies mathematical knowledge in practised ways to solve routine problems.<br>Recognises and uses information in different representations.<br>Uses familiar calculator applications to solve routine problems.<br>Selects an appropriate strategy to carry out analysis in situations requiring investigative techniques. |
| | **Follows mathematical conventions and attends to accuracy**<br>Recognises and applies mathematical definitions, rules and procedures in practised situations.<br>Applies basic conventions for diagrams and graphs.<br>Uses calculus, vector and complex number notation correctly.<br>Rounds to suit context and specified accuracy.<br>Generates some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts. |
| | **Links mathematical results to data and contexts to reach reasonable conclusions**<br>Identifies and describes limitations of simple models.<br>Shows some recognition of the domain as defined in the problem.<br>Recognises that changing conditions will affect the outcome. |

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| | Makes inferences from analysis and uses these to draw conclusions related to a given context for investigation. |
| | **Communicates mathematical reasoning, results and conclusions**<br>Shows adequate working and supports answers with simple or routine statements.<br>Relates the working to a labelled diagram that has been given as part of the question.<br>Presents work that includes the required answer but does not clearly identify it as such.<br>Communicates investigation findings in a systematic way using some mathematical expression and everyday language. |

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| **D** | **Identifies and organises relevant information**<br>Identifies and attempts to use given information to solve a simple routine problem.<br>Identifies variables in a simple diagram.<br>Displays data using an inappropriate presentation format.<br>Identifies limited mathematical content of an investigation. |
| | **Chooses effective models and methods and carries the methods through correctly**<br>Follows an appropriate strategy to solve practised problems.<br>Deals with information in familiar representations only.<br>Uses a calculator for straightforward problems.<br>Attempts to select an appropriate strategy to carry out analysis in situations requiring investigative techniques. |
| | **Follows mathematical conventions and attends to accuracy**<br>Applies limited mathematical conventions to practised problems.<br>Expresses answers with limited accuracy.<br>Generates partly accurate and generally incomplete solutions to mathematical problems set in applied and theoretical contexts. |
| | **Links mathematical results to data and contexts to reach reasonable conclusions**<br>Identifies limitations of simple models and, on occasions, recognises specified conditions.<br>Makes some inferences from analysis of an investigation. |
| | **Communicates mathematical reasoning, results and conclusions**<br>Shows some working in an attempt to answer simple questions.<br>Sets out calculations in a manner that is difficult to check for accuracy.<br>Presents working with no clear indication of the final answer evident.<br>Offers simple conclusions that are not supported by data or calculations. |

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| **E** | Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade. |