$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.24 + 0.44 - 0.63$

$= 0.68 - 0.63$

$= 0.05$

Year 12 Methods — Maraitai Tutoring

Lesson 9: Probability (Review)

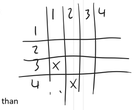
Practice Test

1. List the sample space for
   a. The toss of a coin — $\xi = \{H, T\}$
   b. The roll of 2 dice — $\xi = \{(1,1), (1,2), (2,1) \cdots\}$
   c. The number of 10s in a hand of 5 playing cards (from a standard deck) — $\{0, 1, 2, 3, 4\}$

2. A random experiment results in 1, 2, 3, 4, or 5. If 1, 2 and 3 are equally likely to occur, 4 is twice as likely to occur as 3, and the probability of 1 is 15%.
   a. Find the probability of the result "5". — 25%
   b. How was the complementary events rule used? — $100\% - 75\%$

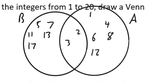
3. A dark is throw at random onto a board of two concentric circles of radius 5, and 10 respectively. Given the dart lands in the larger circle, what is the probability it lands in the smaller circle?

4. A bag contains four balls. A ball is chosen at random, the number is noted, and the ball is replaced. A second ball is then chosen at random and its number noted.
   a. Draw up the ordered pairs to show the sample space for the experiment
   b. Find the probability that
      i. The numbers sum to 5 — $\frac{4}{16} = \frac{1}{4}$
      ii. The numbers are different — $\frac{3}{4}$
      iii. The second number is two more than the first — $\frac{1}{8}$



5. Suppose $\Pr(A) = 0.24$, $\Pr(B) = 0.44$, $\Pr(A \cup B) = 0.63$. Find $\Pr(A \cap B)$ — $= 0.05$

6. Suppose that A is the set of factors of 24, and B is the set of prime numbers less than 18. If a number is chosen at random from the integers from 1 to 20, draw a Venn diagram and use it to find:
   a. Pr(A) — $\frac{8}{20}$
   b. Pr(B) — $\frac{7}{20}$
   c. $\Pr(A \cup B)$ — $\frac{12}{20}$



7. If A and B are events such that $\Pr(A) = 0.34$, $\Pr(A \cap B) = 0.3$, $Pr(A' \cap B) = 0.1$, find
   a. $\Pr(A' \cap B)$ — $= 0.04$
   b. $\Pr(A \cup B)$ — $= 0.44$
   c. $Pr(A' \cap B') = 0.56$

	B	B′	
A	0.3	0.04	0.34
A′	0.1	0.56	
	0.4	0.6	

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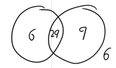
8. At a concert attended by 50 people. 35 people knew the first song, and 38 people knew the second song. 6 people knew neither. Find the probability that a given person know:
   a. Only one of the two songs — $\frac{3}{10}$
   b. Only the first song, but not the second song — $\frac{3}{21}$



9. Wen throws 30 shots every basketball game, and scores 24 of those shots on average. Charley shoots 20 shots each game, 60% of which go in the hoop. You watch a game that Wen and Charley are playing. Let W represent the event that Wen threw the shot, and let G be the event that that the shot missed.
   a. Find for any given shot.
      i. $\Pr(W)$ — $\frac{24}{30} = \frac{4}{5}$
      ii. $\Pr(G|W)$ — $\frac{1}{5}$
      iii. $\Pr(G|W')$ — $0.4 = \frac{2}{5}$
      iv. $\Pr(G)$ — 0.28
      v. $\Pr(W|G)$ — $P_r(W|G) = \frac{P(W \cap G)}{P(G)} = \frac{P(W)}{P(G)} = \frac{2}{7}$

10. What is the probability that a first coin flip was heads given that the second one was heads? — $\frac{1}{2}$

11. What is the probability that a first dice roll was 5 or 6 given the second one was 5 or 6? — $\frac{1}{3}$

12. Questions 10 and 11 featured two independent events. What conclusions can we make about $\Pr(A|B)$ for independent events?
    $P_r(A|B) = P_r(A)$

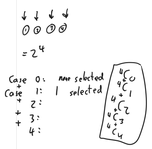
13. Suppose the probability of it raining Pr(R) is 0.35, and the probability of bob wearing a yellow shirt Pr(Y) is 0.2. These two events are independent. Find:
    a. $\Pr(A \cup B)$ — $0.55 = 0.07 = 0.48$
    b. $\Pr(A' \cap B')$ — $0.65 \cdot 0.8 = 0.52$

We know that $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ if $\Pr(B) \neq 0$

Two events are independent if the occurrence of one event has no effect on the other. I.e.

Events are independent if and only is $\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$

1.



10a) ${}^6P_4 = \frac{6!}{(6-4)!} = \frac{720}{2} = 360$

b) ${}^6C_3 = \frac{6!}{(6-3)! \cdot 3!} = \frac{720}{2 \cdot 3!} = \frac{720}{36} = 20$

c) ${}^{300}C_1 = 300$

d) ${}^{400}C_0 = 1$

e) ${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 =$

2. $n: {}^nC_2 = 55$

$\frac{n!}{(n-2)! \cdot 2!} = \frac{(n)(n-1)(n-2)!}{(n-2)! \cdot 2!}$

$\rightarrow (n)(n-1) = 110$

$n = 11$

$3^3 = 27$

$7! = 6$

$(A \cup B) = A + B - (A \cap B)$

$(A \cup B \cup C) = A + B + C - (A \cap B) - (A \cap C) - (B \cap C) + (A \cap B \cap C)$

$(A \cup B \cup C \cup D) = A + B + C + D - (A \cap B) - (A \cap C) - (A \cap D) - (B \cap C) - (B \cap D) - (C \cap D) + (A \cap B \cap C) + (B \cap C \cap D) + (C \cap D \cap A) + (B \cap A \cap D)$

$P \in \{A \cap B \cap C \cap D\}$

↓ ↓ ↓ ↓

A B C D

A set of all arrangement where A is in the same spot.

$n(A) = 6$

B: $n(B) = 6$

C: $n(C) = 6$

D: $n(D) = 6$

$n(A) + n(B) + n(C) + n(D) - {}^6C_2 + {}^4C_1 - 1$

$4 \cdot 6 - 6 \cdot 2 + 4 - 1$

$= 24 - 12 + 4 - 1$

$= 12 + 4 - 1$

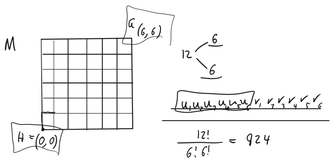
$= 16 - 1$

$= 15$

$24 - 15 = 9$

Ⓟ

Ⓒ



$\frac{12!}{6! \cdot 6!} = 924$

1 2 3 4 5 6 7 8 9 10 11 12

$\boxed{{}^{12}C_6}$ $\{1,7\} = \{2,13\}$

$= 924$