

MATHEMATICS DEPARTMENT

MATHEMATICAL METHODS YEAR 12 – TEST 1

DATE: 2<sup>nd</sup> December 2015

Name: \_\_\_\_\_

Reading Time: 3 minutes

SECTION ONE: CALCULATOR FREE

WORKING TIME: Maximum 25 minutes

TOTAL:

24 marks

EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet (provided)

SECTION TWO: CALCULATOR ASSUMED

WORKING TIME: Minimum 25 minutes

TOTAL:

26 marks

EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing instruments, up to 3 calculators, formula sheet (provided) one A4 page of notes (one side only)

Question	Marks available	Marks awarded	Question	Marks available	Marks awarded
1	6		5	4	
2	4		6	7	
3	7		7	8	
4	7		8		
Sect 1 Total	24		Sect 2 Total	26	
			TOTAL	50	

**Question 1****(6 marks)**

Find the antiderivative of each of the following, giving all answers with positive indices.

a)  $\frac{4}{(2x - 5)^3}$

**(3 marks)**

(b)  $(10x + 5)(x^2 + x - 3)^4$

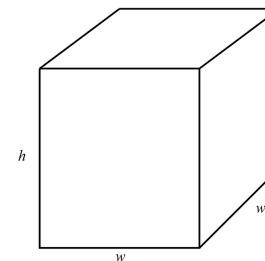
**(3 marks)**

**End of Test  
or**

**Question 8****(7 marks)**

A closed box is constructed with a square base. Exactly 10 Square metres of material is to be used in the construction of the box, without wastage.

Let  $h$  = height of box,  $w$  = width of box = length of box



(a) Show that  $5 = w^2 + 2wh$

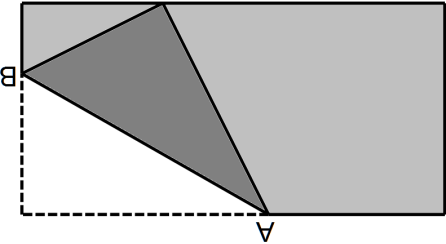
**(2 marks)**

(b) By using calculus, determine the maximum volume of the box and state the dimensions required to achieve this maximum.

**(5 marks)**

Question 8

A rectangular piece of paper measures 12 cm by 6 cm. One corner of the sheet of paper is folded up to just reach the opposite side as shown below.



What is the minimum length of the fold?

(7 marks)

Question 2

Let  $f(x) = (x + 3)(1 - x^2)$ .

The derivative of  $f(x)$  can be written in the form  $f'(x) = (1 - x^2)^4(ax^2 + bx + c)$ .

Determine the value of  $a$ ,  $b$  and  $c$ .

(4 marks)

Question 3 (7 marks)

Let A, B, C, D, E, F and G be points on the graph of a continuous function  $f(x)$ . The table below shows the information about the sign of  $f'(x)$ ,  $f''(x)$  and  $f'''(x)$  at these points.

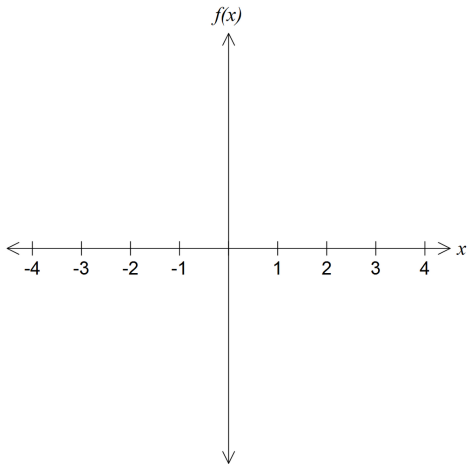
Point	A	B	C	D	E	F	G
$x$	-4	-3	-1	0	1	2	4
$f'(x)$	+	0	-	0	+	+	+
$f''(x)$	-	-	0	+	+	0	+
$f'''(x)$	+	+	+	0	-	0	+

There are no other points at which  $f'(x)$ ,  $f''(x)$  or  $f'''(x)$  are equal to zero.

(a) Which point is a local minimum? (1 mark)

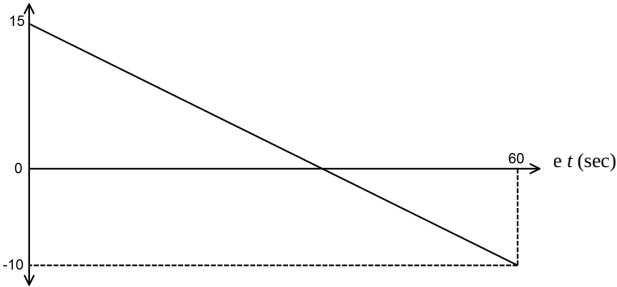
(b) Describe the nature of the graph at point F. (2 marks)

(c) Sketch the function on the axes below. (4 marks)



Question 7 (8 marks)

A jet plane travels horizontally along a straight path for one minute, starting at time  $t = 0$ , where  $t$  is measured in seconds. The acceleration,  $a$ , measured in  $\text{ms}^{-2}$ , of the jet plane is given by the straight line graph below.



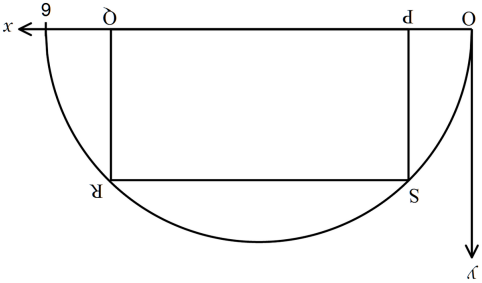
(a) Find an expression for the acceleration of the jet plane during this time, in terms of  $t$ . (2 marks)

(b) Given that when  $t = 0$  the jet plane is travelling at  $125 \text{ ms}^{-1}$ , find its maximum velocity in  $\text{ms}^{-1}$  during the minute that follows. (3 marks)

(c) Given that the jet plane breaks the sound barrier at  $295 \text{ ms}^{-1}$ , find out for how long the jet plane is travelling greater than this speed. (3 marks)

Question 6

Consider the graph of  $y = \sqrt{6x - x^2}$  shown in the following figure. A rectangle PQRS is drawn with its base on the x-axis, as shown in the following figure.



Let  $OP = x$ .

(a) Explain why the area of the rectangle PQRS is given by

$$Area = (6 - 2x)\sqrt{6x - x^2}$$

(3 marks)

(7 marks)

(b) (i)

Find the rate of change of area when  $x = 2$ .

(2 marks)

(iii)

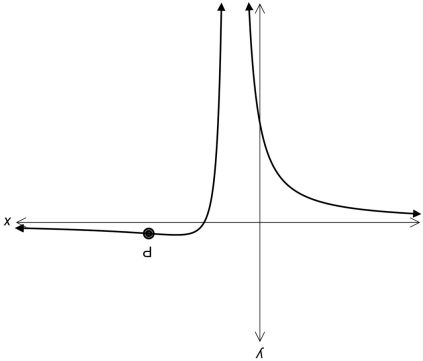
The area is decreasing for  $a < x < b$ . Find the value of  $a$  and of  $b$ .

(2 marks)

Question 4

$$h(x) = \frac{x-2}{x-1}, x \neq 1$$

Consider the function  $h(x) = \frac{x-2}{x-1}$ . A sketch of part of the graph of  $h$  is given below. The point P is a point of inflection on the graph.



(a) Find  $h'(x)$ , writing your answer in the form  $\frac{a-x}{(x-1)^n}$  where  $a$  and  $n$  are constants to be determined.

(4 marks)

(7 marks)

Question 4 continued on next page...

Question 4 continued...

- (b) Given that  $h''(x) = \frac{2x-8}{(x-1)^4}$ , calculate the coordinates of P. (3 marks)

End of Section 1

**Section 2 Calculator Assumed.** Name: \_\_\_\_\_

**Question 5** (4 marks)

Given that  $y = x^{\frac{1}{3}}$ , use  $x = 1000$  and the increments formula  $\delta y = \frac{dy}{dx} \delta x$  to determine an appropriate value for  $\sqrt[3]{1006}$ .