

Note: All part questions worth more than 2 marks require working to obtain full marks.

Formula sheet provided: no but formulae stated on page 2

Task weighting: 13%

Marks available: 40 marks

WACE examinations

A4 paper SINGLE SIDED, and up to three calculators approved for use in the drawing instruments, templates, notes on one unfolded sheet of

Special items:

Drawing fluid/tape, eraser, ruler, highlighters, correction fluid, pens (blue/black preferred), pencils (including coloured), sharpener,

Standard items:

Up to 3 classpassads/calculators

Materials required:

Number of questions: 7

Working time allowed for this task: 40 mins

Reading time for this test: 5 mins

Task type: Response/investigation

Student name: _____ Teacher name: _____

Course Specialist Test 2 Year 12

PERTH MODERN SCHOOL
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Useful formulae**Complex numbers**

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = z = \sqrt{a^2 + b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan \theta = \frac{b}{a}, \quad -\pi < \theta \leq \pi$
$ z_1 z_2 = z_1 z_2 $	$\left \frac{z_1}{z_2} \right = \frac{ z_1 }{ z_2 }$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z \bar{z} = z ^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{ z ^2}$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$	$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
Polar form	
$z = a + bi = r(\cos \theta + i \sin \theta) = r \text{cis } \theta$	$\bar{z} = r \text{cis } (-\theta)$
$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$
$\text{cis}(\theta_1 + \theta_2) = \text{cis } \theta_1 \text{cis } \theta_2$	$\text{cis}(-\theta) = \frac{1}{\text{cis } \theta}$
De Moivre's theorem	
$z^n = z ^n \text{cis } (n\theta)$	$(\text{cis } \theta)^n = \cos n\theta + i \sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}} \left(\cos \frac{\theta + 2\pi k}{q} + i \sin \frac{\theta + 2\pi k}{q} \right), \quad \text{for } k \text{ an integer}$	

$$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta$$

c

Q2 (4 marks)

Specific behaviours	
<ul style="list-style-type: none"> ✓ rearranges inequality by taking out factor of 3 or sketches and shows reasoning ✓ determines a ✓ determines b ✓ determines c 	

c

Consider the inequality $|3x - 7| \leq a$ which is only true for $b \leq x \leq 9$ where $a \neq b$ are constants.

Determine the values of a & b .

Q1 (3 marks)

$$r_A = \begin{pmatrix} 2 \\ -18 \end{pmatrix} + t \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$r_B = \begin{pmatrix} -6 \\ 44 \end{pmatrix} + (t - \frac{1}{2}) \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$$r_A = r_B$$

$$2 + 2t = -6 + (t - \frac{1}{2})4$$

$$2 + 2t = -6 + 4t - 2$$

$$10 = 2t$$

$$t = 5$$

$$-18 + 7t = 44 + (t - \frac{1}{2})(-6)$$

$$-18 + 7t = 44 - 6t + 3$$

$$13t = 65$$

$$t = 5$$

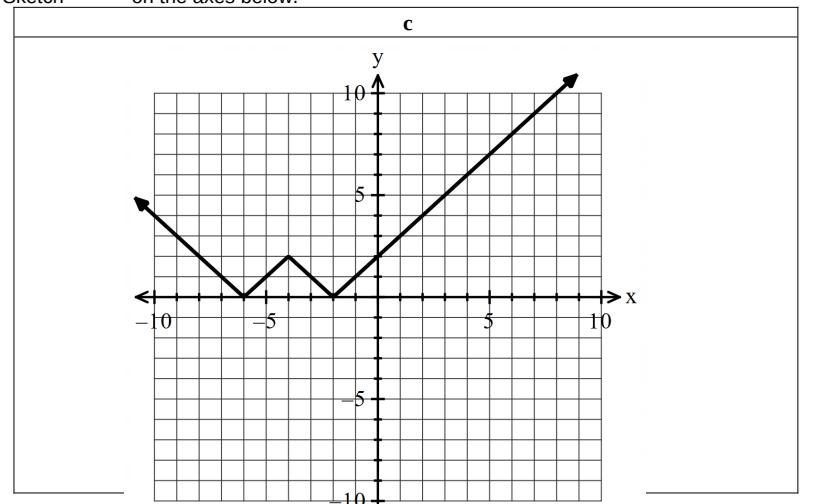
collide(12,17) at 4 pm

Specific behaviours

- ✓ takes into account different starting times
- ✓ sets up two equations for time
- ✓ solves for position of collision
- ✓ solves for time of collision in clock time

Q3 (2, 3 & 3 = 8 marks)

a) Sketch $|f(x)|$ on the axes below.



$$k^2 = 10^2 + 10^2 - 200 \cos(2\theta - .7)$$

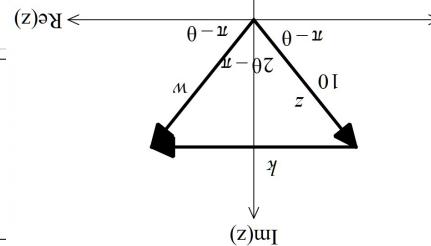
$$k^2 = 200 + 200 \cos 2\theta = 200(1 + \cos 2\theta)$$

$$k^2 = 200(2\cos^2 \theta)$$

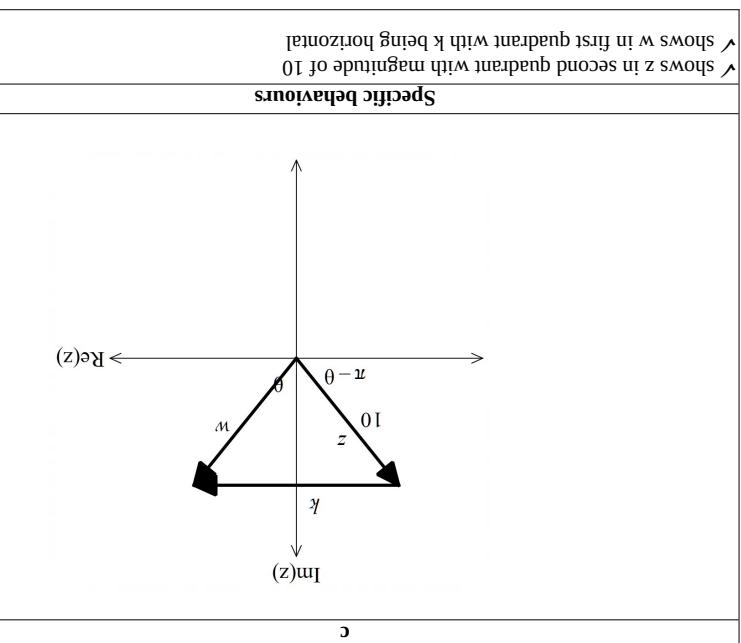
$$k = -20 \cos \theta$$

Specific behaviours

- ✓ identifies symmetry of triangles
- ✓ uses cosine rule or other trig identity
- ✓ obtains expression for k squared
- ✓ states simplified expression for k in terms of theta with a negative

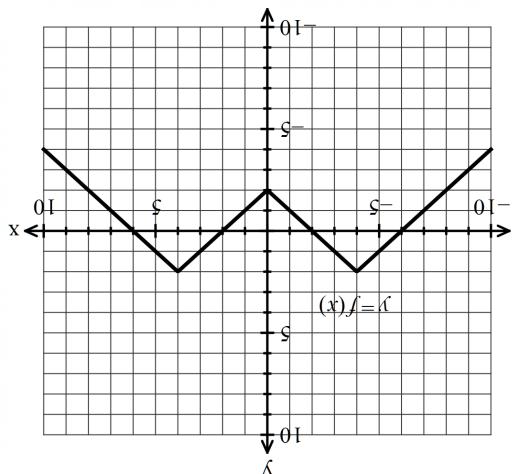
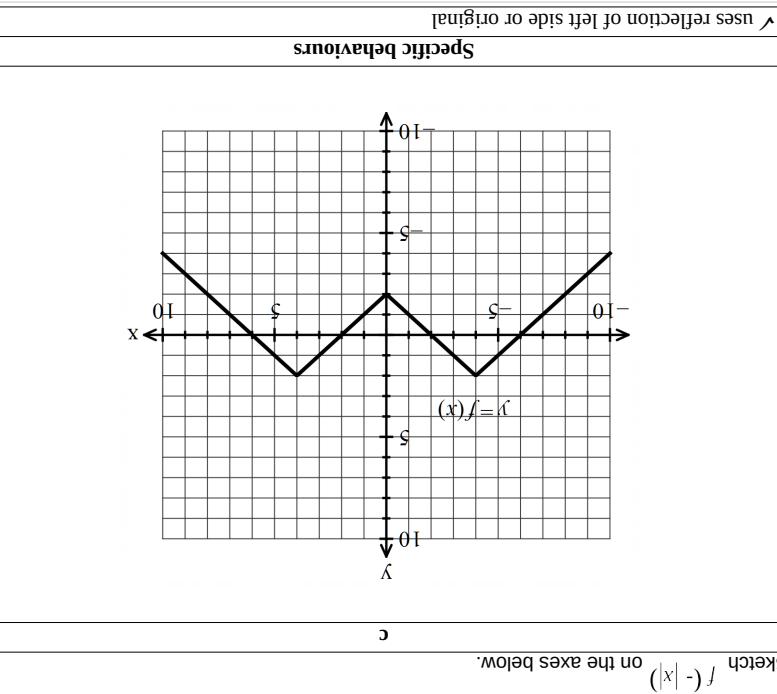


- b) Determine a simplified expression for k in terms of θ . Justify your answer.



- a) Represent this information on the Argand diagram below.

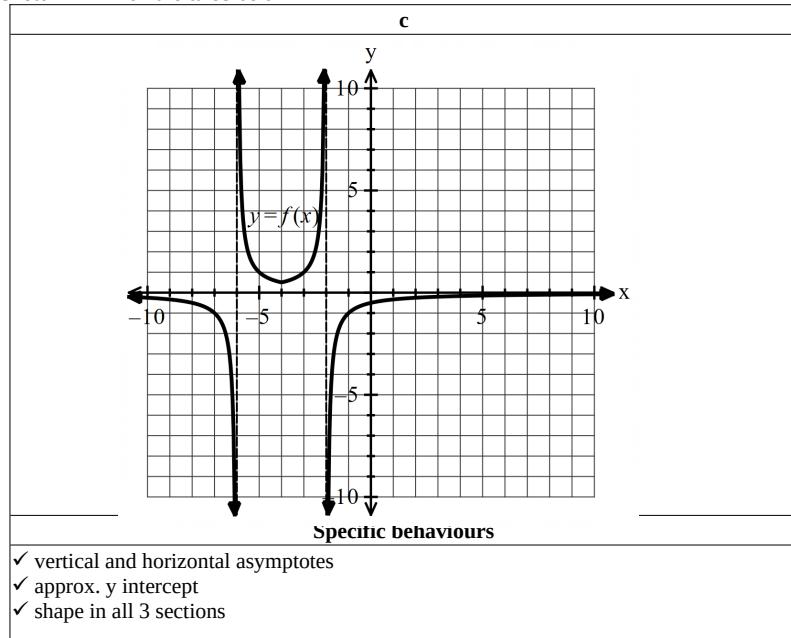
Specific behaviours	<ul style="list-style-type: none"> ✓ sets up a quadratic equation for λ ✓ determines an expression for discriminant in terms of alpha ✓ states values for tangent with discriminant = zero (stated) ✓ states values for intersecting at two points AND no intersection and states conditions for discriminant. ✓ note: max -1 if discriminant values not stated
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Q7 (2 & 4 = 6 marks)	<ul style="list-style-type: none"> ✓ sets up a quadratic equation for λ ✓ determines an expression for discriminant in terms of alpha ✓ states values for intersecting at two points AND no intersection and states conditions for discriminant. ✓ note: max -1 if discriminant values not stated
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- ✓ shape
- ✓ correct x & y intercepts

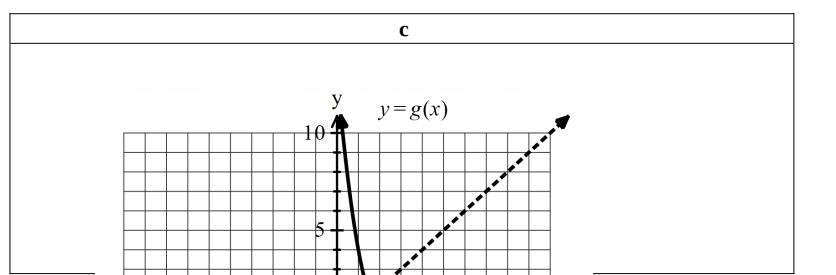
c) Sketch $\frac{1}{f(x)}$ on the axes below.



Q4 (2, 3, 1 & 3 = 9 marks)

Consider $g(x) = 3x^2 - 12x + 13$ for $x \leq 2$ which is plotted below.

a) Sketch $g^{-1}(x)$ on the axes above.



Q6 (5 marks)

- The line will be a tangent to the circle.
- The line crosses the circle at two points.
- The line will never meet the circle.

c

$$\left| \begin{pmatrix} -2+4\lambda \\ \lambda \end{pmatrix} - \begin{pmatrix} 3 \\ \alpha \end{pmatrix} \right| = 9$$

$$\left| \begin{pmatrix} -5+4\lambda \\ \lambda - \alpha \end{pmatrix} \right| = 9$$

$$\sqrt{(4\lambda - 5)^2 + (\lambda - \alpha)^2} = 9$$

$$16\lambda^2 - 40\lambda + 25 + \lambda^2 - 2\alpha\lambda + \alpha^2 - 81 = 0$$

$$17\lambda^2 - (40 + 2\alpha)\lambda - 56 + \alpha^2 = 0$$

$$\Delta = (40 + 2\alpha)^2 - 4(17)(\alpha^2 - 56)$$

Edit Zoom Analysis

$y=(2\cdot x+40)^2-68\cdot(x^2-56)$

$dy/dx=1187.5$
 $xc=-8.026988$
 $yc=0$

Root

Edit Zoom Analysis

$y=(2\cdot x+40)^2-68\cdot(x^2-56)$

$dy/dx=-1187$
 $xc=10.526988$
 $yc=0$

Root

Specific behaviours

- ✓ subs line into circle

Specific behaviours	
	$x = (x) \circ_{\perp} g$
c	

c) Determine $g_{\perp} \circ g(x)$.

Q4 cont-

Specific behaviours	
	<ul style="list-style-type: none"> ✓ uses minus in rule and states domain ✓ solves for y with reasoning ✓ swaps x and y
c	

$$\begin{aligned} f_{\perp}(x) &= 2 - \sqrt{\frac{x}{x-1}}, x \geq 1 \\ y &= \frac{12 \pm \sqrt{44 - 4(3)(13-x)}}{6} \\ 3y^2 - 12y + 13 - x &= 0 \\ x = 3y^2 - 12y + 13 & \end{aligned}$$

b) Determine the rule for $g_{\perp}(x)$ showing full working and the domain. (Simplify)

Specific behaviours	
	<ul style="list-style-type: none"> ✓ appears reflected in line $y=x$ ✓ contains endpoint (1,2)

Max 3 out of 5 if vector method not used
 Note: max -1 if units not stated

- ✓ solves for distance starting in metres
- ✓ solves for time starting in seconds
- ✓ uses dot product and equates to zero

✓ states x

- d) Determine all value(s) of x such that $g(x) = g^{-1}(x)$

c

$$g(x) = 3x^2 - 12x + 13 = x$$

$$3x^2 - 13x + 13 = 0$$

Edit Action Interactive

$\text{solve}(3 \cdot x^2 - 13 \cdot x + 13 = 0, x)$

$$\left\{ x = \frac{-\sqrt{13}}{6} + \frac{13}{6}, x = \frac{\sqrt{13}}{6} + \frac{13}{6} \right\}$$

$\boxed{x=1.565741454, x=2.76759}$

Discard $x=2.77$ as not within domain of $f(x)$

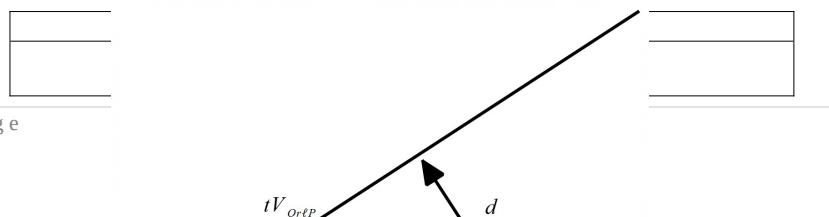
Specific behaviours

- ✓ equates $g(x)$ to x
- ✓ sets up an equation to solve for x
- ✓ solves for one approx. value and states that the second value MUST be discarded

Note: 2 marks max if using classpad to solve function-inverse without reasoning

Q5 (5 marks)

Determine the minimum distance between them **using vectors** and the time that this occurs.



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$d = PQ + t_q V_p$

$$d \cdot {}_q V_p = 0$$

Edit Action Interactive

$$\begin{bmatrix} 11 \\ -6 \end{bmatrix} - \begin{bmatrix} -8 \\ 7 \end{bmatrix} + t \times (\begin{bmatrix} -3 \\ 10 \end{bmatrix} - \begin{bmatrix} 5 \\ -4 \end{bmatrix})$$

$$\begin{bmatrix} -8 \cdot t + 19 \\ 14 \cdot t - 13 \end{bmatrix}$$

$$\text{dotP}\left(\begin{bmatrix} -8 \cdot t + 19 \\ 14 \cdot t - 13 \end{bmatrix}, \begin{bmatrix} -3 \\ 10 \end{bmatrix} - \begin{bmatrix} 5 \\ -4 \end{bmatrix}\right)$$

$$14 \cdot (14 \cdot t - 13) + 8 \cdot (8 \cdot t - 19)$$

$$\text{solve}(14 \cdot (14 \cdot t - 13) + 8 \cdot (8 \cdot t - 19) = 0, t)$$

$$\left\{ t = \frac{167}{130} \right\}$$

$$\text{norm}\left(\begin{bmatrix} -8 \cdot t + 19 \\ 14 \cdot t - 13 \end{bmatrix} \mid t = \frac{167}{130}\right)$$

$$\frac{81 \cdot \sqrt{65}}{65}$$

$$10.0468135$$

Alg Standard Cplx Rad

Distance = 10.05 metres at time 1.28 seconds

Specific behaviours

- ✓ uses relative velocity
- ✓ determines a separation vector d

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