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SCHOOL

Year 12 Trial WACE Examination, 2013

Question/Answer Booklet

MATHEMATICS 3C/3D
Section One: Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________

Your name ______________________

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

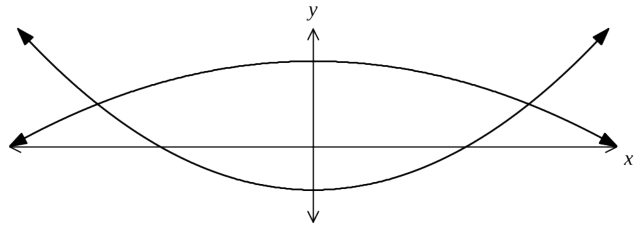
4. It is recommended that you **do not use pencil**, except in diagrams.

Additional working space

Question number: __________

Question 8 **(8 marks)**

Two functions are defined by $f(x)=x^2-2$ and $g(x)=4-\frac{1}{2}x^2$. Their graphs are shown below.



(a) State the coordinates of the minimum of $f(x)$ and the maximum of $g(x)$. (1 mark)

$\boxed{(0,-2) \text{ and } (0,4)}$

(b) Find the coordinates of the points of intersection of $f(x)$ and $g(x)$. (2 marks)

$$x^2-2=4-\tfrac{1}{2}x^2$$
$$1.5x^2=6$$
$$x^2=4$$
$$x=\pm 2$$
$$(-2,\ 2) \text{ and } (2,\ 2)$$

(c) The region enclosed by $f(x)$ and $g(x)$ is rotated about the y-axis. Find the volume of the solid formed. (5 marks)

$$\text{Volume}=\pi\int_{-2}^{2} y+2\,dy+\pi\int_{2}^{4} 8-2y\,dy$$
$$=\pi\left(\left[\frac{y^2}{2}+2y\right]_{-2}^{2}+\left[8y-y^2\right]_{2}^{4}\right)$$
$$=\pi\big((2+4)-(2-4)\quad+\quad(32-16)-(16-4)\big)$$
$$=\pi(8+4)$$
$$=12\pi$$

End of questions

Section One: Calculator-free **(50 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

The price, $\$P$, of a commodity varied continuously with time, t hours, according to the function $P(t)=t^3-9t^2+15t+30$ for $0\le t\le 3$.

Determine the lowest and highest price of the commodity.

$$P'(t)=3t^2-18t+15$$

$$3t^2-18t+15=0$$
$$t^2-6t+5=0$$
$$(t-1)(t-5)=0$$
$$t=1 \quad (\text{ignore } t=5)$$

$$P(0)=30$$
$$P(1)=37$$
$$P(3)=21$$

Lowest price is \$21 and highest is \$37

Question 2 **(6 marks)**

Differentiate each of the following with respect to x, without simplifying.

(a) $y = x^2 e^{1-2x}$ (2 marks)

$$\frac{dy}{dx} = 2x \cdot e^{1-2x} + x^2 \cdot (-2)e^{1-2x}$$

(b) $y = \dfrac{(5x-1)^3}{2x+1}$ (2 marks)

$$\frac{dy}{dx} = \frac{3(5)(5x-1)^2 \cdot (2x+1) - (5x-1)^3 \cdot 2}{(2x+1)^2}$$

(c) $y = \displaystyle\int_0^x \frac{2t}{(1-t^2)^2}\, dt$ (2 marks)

$$\frac{dy}{dx} = \frac{2x}{(1-x^2)^2}$$

Question 7 **(6 marks)**

(a) Write down the values of $m^2 + 7$ for $m = 1, 3, 5, 7$ and 9. (1 mark)

8, 16, 32, 56, 88

(b) State the largest integer, p, that $m^2 + 7$ is always divisible by, when m is a positive odd integer. (1 mark)

The largest integer is $p = 8$

(c) Prove that $m^2 + 7$ is always divisible by p when m is a positive odd integer. (4 marks)

Let the positive odd integer $m = 2n + 1$, where n is an integer greater than or equal to 0. Then:

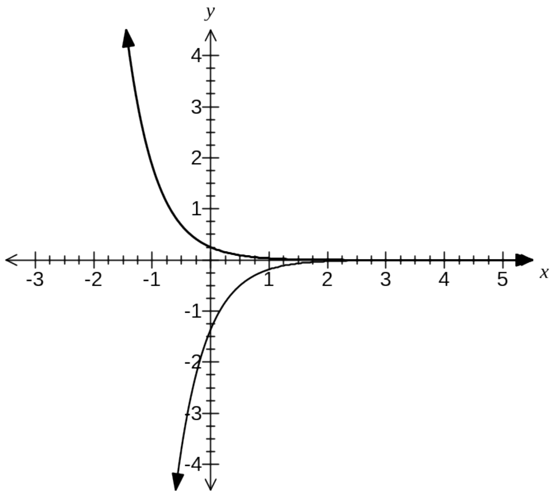
$$\begin{aligned}(2n+1)^2 + 7 &= 4n^2 + 4n + 8\\ &= 4n(n+1) + 8\\ &= 4(2k) + 8 \quad \text{(see note below)}\\ &= 8(k+1)\end{aligned}$$

Hence the expression will always be divisible by eight.

Note: Since n is an integer, then one of n and $n+1$ will be even and the other odd. Hence the product $n(n+1)$ will have a factor of 2, and so $n(n+1) = 2k$, where k is an integer greater than or equal to 0.

Question 6 **(7 marks)**

Let $f(x) = ae^{bx+c}$, where a, b and c are constants. The graph of $y = f(x)$ is shown below.



(a) Write down the range of

(i) $f(x)$ (1 mark)

$\boxed{y < 0}$

(ii) $1 - f(x)$ (1 mark)

$\boxed{y > 1}$

(b) The graph passes through the point $(0.5, -0.5)$. Explain why $b + 2c = 0$ and hence state the value of a and whether c is positive or negative. (3 marks)

> e^k is irrational for all values of k except 0. When $x = 0.5$, $y = -0.5$ which is rational, so $b(0.5) + c = 0$ and so $b + 2c = 0$.
>
> Hence $a = -0.5$.
>
> The graph of $-f(x)$ would show decay, thus b must be negative and so c must be positive.

(c) Sketch the graph of $y = af(x - \frac{c}{b})$ on the axes above. (2 marks)

$$\begin{aligned} y &= a \times ae^{b(x-\frac{c}{b})+c} \\ &= a^2 e^{bx} \\ &= \tfrac{1}{4} e^{bx} \end{aligned}$$

Question 3 **(5 marks)**

As part of a product recall, a shop removed all sizes of a variety of soup from its shelves. The soup was sold in 300 mL, 500 mL and 800 mL sizes for \$4.50, \$6.00 and \$7.50 respectively. The total volume of soup in all 42 cans removed was 25 L and the value of these cans was to \$267.

If x 300 mL cans, y 500 mL cans and z 800 mL cans were removed, then some of the above information can be expressed by the equations $3x + 4y + 5z = 178$ and $3x + 5y + 8z = 250$.

Write down a third equation from the information and use it to find how many of each size of can were removed.

$x + y + z = 42 \quad (1)$

$3x + 5y + 8z = 250$
$3x + 4y + 5z = 178$
$y + 3z = 72$

$3x + 4y + 5z = 178$
$3x + 3y + 3z = 126 \quad 3 \times (1)$
$y + 2z = 52$

$y + 3z = 72$
$y + 2z = 52$
$z = 20$
$y = 52 - 2(2) = 12$
$x = 42 - 20 - 12 = 10$

10 @ 300 mL, **12** @ 500 mL and **20** @ 800 mL.

Question 4 **(5 marks)**

(a) Determine $\int \frac{1}{2\sqrt{x}} - \frac{1}{x^2}\, dx$ (2 marks)

$$\boxed{\sqrt{x} + \frac{1}{x}}$$

(b) If $f'(x) = 6x(x^2 - 7)^2$ and $f(2) = -20$, determine $f(3)$. (3 marks)

$f(x) = (x^2 - 7)^3 + c$

$-20 = (2^2 - 7)^3 + c \Rightarrow c = 7$

$f(3) = (3^2 - 7)^3 + 7$
$= 15$

Question 5 **(8 marks)**

The events A and B have the properties $P(A) = \frac{3}{8}$ and $P(A \cup B) = \frac{1}{2}$.

(a) Determine $P(B)$ in each of these cases:

(i) If A and B are mutually exclusive. (1 mark)

$P(B) = \frac{1}{2} - \frac{3}{8}$
$= \frac{1}{8}$

(ii) If $P(A \cap B) = \frac{3}{40}$. (2 marks)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\frac{1}{2} = \frac{3}{8} + P(B) - \frac{3}{40}$
$P(B) = \frac{20-15+3}{40}$
$= \frac{8}{40}$
$= \frac{1}{5}$

(iii) If $P(B \mid A) = \frac{1}{6}$. (3 marks)

$P(B \cap \overline{A}) = \frac{1}{8}$
$x = P(B)$

$P(A \cap B) = x - \frac{1}{8}$
$P(B \mid A) = \left(x - \frac{1}{8}\right) \div \frac{3}{8}$
$\frac{1}{6} \times \frac{3}{8} = x - \frac{1}{8}$
$x = P(B) = \frac{3}{16}$

(b) For the case where $P(A \cap B) = \frac{3}{40}$, are A and B independent? Justify your answer.
(2 marks)

Yes, as $P(A) \times P(B) = P(A \cap B)$
$\frac{3}{8} \times \frac{1}{5} = \frac{3}{40}$