SEMESTER TWO

MATHEMATICS
METHODS
UNITS 1 & 2

2018

SOLUTIONS

Calculator-free Solutions

1. (a) $\frac{2^{3x}}{2^y} = 2^4 = 16$ ✓✓

(b) $u^{\frac{7}{2}-\left(\frac{5}{2}+\frac{1}{2}\right)}$ ✓

$= u^{\frac{1}{2}} = \sqrt{u}$ ✓ [4]

2. $ab = 15$ and $a + b = 8$
$a = 3$, $b = 5$ or $a = 5$, $b = 3$ ✓
$abx^2 + (2b + 7a)x + 14 = 15x^2 + cx + 14$
$2b + 7a = 2(5) + 7(3)$ or $2b + 7a = 2(3) + 7(5)$ ✓
$c = 31$ or 41 ✓✓ [4]

3. (a) $m = \frac{4}{3}$ ✓

$-5 = \frac{4}{3}(3) + c$

$\therefore c = -9$ $\therefore y = \frac{4}{3}x - 9$ ✓

(b) $-4x + 3y = -27$ ✓
$-8x + 6y = -14$
$9x - 6y = 12$
$x = -2$ $y = -5$ ✓
$D(-2, -5)$ ✓

(c) $4(k - 2) - 3(2k - 3) = 7$ ✓
$-2k + 1 = 7$
$k = -3$ ✓ [7]

4. $\frac{1.2 \times 10^{-4}}{3 \times 10^{-7}} = 4 \times 10^2 = 400$ ✓✓ [2]

5. (a) $2(3xy) + 2(3x^2) + 2xy = 32$ ✓
$3xy + 3x^2 + xy = 16$ ✓
$3x^2 + 4xy = 16$

(b) $V = 3x^2y$ and $y = \frac{16 - 3x^2}{4x}$ ✓

$V = 3x^2\left(\frac{16}{4x} - \frac{3x^2}{4x}\right)$ ✓

$V = 12x - \frac{9x^3}{4}$

(c) $V' = 12 - \frac{27x^2}{4}$

$12 - \frac{27x^2}{4} = 0$ for stationary point ✓

$x^2 = \frac{16}{9}$ ✓

$x = \frac{4}{3}$ (discard $-\frac{4}{3}$) ✓

(d)

$V(x)$	↑	$\frac{4}{3}$	↓
$V'(x)$	+	0	–

V has a maximum value ✓✓ [9]

6. (a) $T_{n+1} = T_n + 10 \quad T_1 = -3$ ✓✓

(b) $T_2 = -8$ ✓

$T_3 = -32$ ✓ [4]

7. (a) $y = 10x^2 - 2x^3 - 16x + c$ ✓

$3 = 10(2)^2 - 2(2)^3 - 16(2) + c$ ✓

$c = 11$

$y = 10x^2 - 2x^3 - 16x + 11$ ✓

(b) Gradient of x − axis = 0

$\frac{dy}{dx} = 20(2) - 6(2)^2 - 16 = 0$ ✓

$\therefore m = 0$ Tangent is horizontal and parallel to the x−¿axis. ✓ [5]

8. (a) Vertical translation 12 units down ✓

Horizontal dilation by factor $\frac{1}{2}$. ✓

(b) $7^x = 7^{2x} - 12$

Let 7^x be k. $k^2 - k - 12 = 0$ ✓

$(k - 4)(k + 3) = 0$ ✓

$7^x = 4 \qquad 7^x \neq -3$ ✓

One solution, therefore intersects at only one point. [5]

9. (a) $y = \frac{x^3}{2} + 2x - 7$ ✓

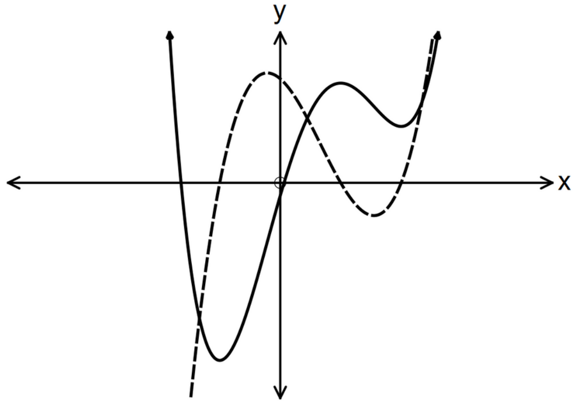
$\frac{dy}{dx} = \frac{3}{2}x^2 + 2$ ✓

$m = 8$ ✓

$1 = 8(2) + c$

$y = 8x - 15$ ✓

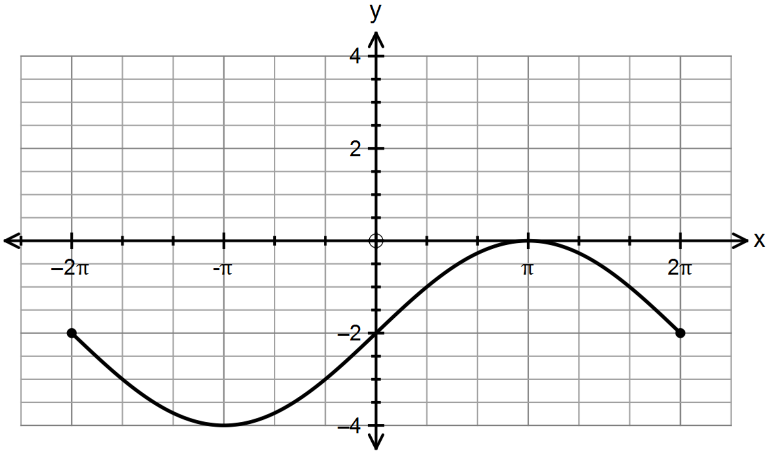
(b)



✓✓ [6]

10. (a) Period = 4π ✓
Amplitude = 2 ✓

(b)



✓✓✓ [5]

Calculator-assumed Solutions

11. (a) The number of phones she is given to repair for the week. ✓

(b) She fixes 23 per day, for 4 days 23 x 4 = 92 phones ✓

(c) $\frac{108}{23} = 4.69565$ days ✓

$0.69565 \times 8 = 5.5652$ hours

5 hours and 34 minutes ✓

(d) $T_1 = 155$ $T_2 = 128$ $T_3 = 99$ $T_4 = 68$

$T_{n+1} = T_n - (25 + 2n)$ $T_0 = 180$ ✓✓ [6]

12. (a) $x^2 + y^2 = 4$ ✓

(b) $\tan\theta = \sqrt{3}$ $\therefore \theta = \frac{\pi}{3}$ radians ✓

radius = 2 units ✓

(c) $OB = \frac{2}{\cos\frac{\pi}{3}} = 4$

Therefore area of ΔAOB = $\frac{1}{2}(4)\sqrt{3} = 2\sqrt{3}$ units^2 ✓

Area of sector = $\frac{1}{2}r^2\theta = \frac{1}{2}(2)^2\left(\frac{\pi}{3}\right)$

= $\frac{2\pi}{3}$ units^2 ✓

Area of shaded part = triangle – sector

= $2\sqrt{3} - \frac{2\pi}{3} = 1.37$ units^2 (3 sig fig) ✓ [6]

13. (a) 5.1 seconds ✓

$v(t) = -9.8t + 25$

(b) $v(0) = 25$ ✓✓

(c) Maximum height is 31.89 m when $t = 2.55$ sec

$$\frac{31.89 \times 2}{2.55 \times 2} = 12.5 \text{ m/s}$$ ✓✓ [5]

14. (a) $f(x) = (2 + x)^2 = 4 + 4x + x^2$ ✓

$$\lim_{h \to 0} \frac{(2 + x + h)^2 - (2 + x)^2}{h} = 2x + 4$$ ✓

(b) (i) -7.5 ✓

(ii) 18 ✓

(c) $p'(x) = 3x^2 - 3a$ ✓

$0 = 3(\sqrt{2})^2 - 3a$

$a = 2$ ✓

$-\sqrt{2} = (\sqrt{2})^3 - 3(2)(\sqrt{2}) + b$

$b = 3\sqrt{2}$ ✓ [7]

15. (a) $3x^2 - \dfrac{4}{x^2} - 11 = 0$

$x = -2$ or $x = 2$ ✓✓

(b)

		–2		2	
y'	+	0	–	0	+
y	↑	-	↓	-	↑

✓✓

$x = -2$ Maximum

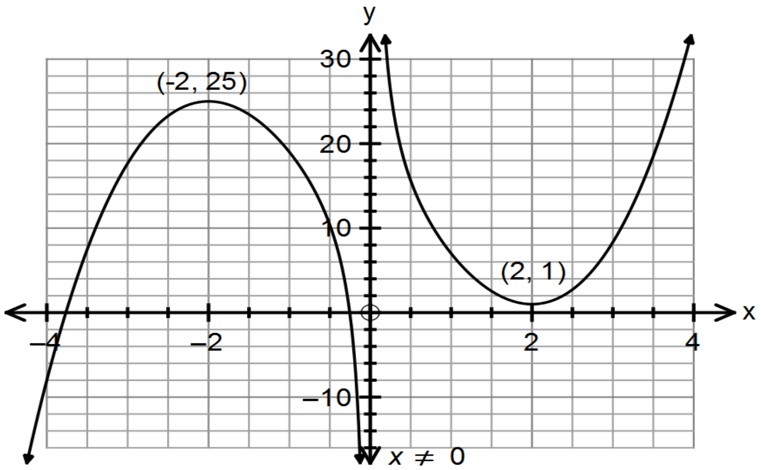
$x = 2$ Minimum ✓

(c) $y = x^3 - 11x + \dfrac{4}{x} + c$ ✓

$c = 13$

$y = x^3 - 11x + \dfrac{4}{x} + 13$ ✓

(d)



✓✓✓ [10]

16. (a) (i) $T_{100} = 23 + (99)(9)$ ✓
$= 914$ ✓

(ii) $357980 = \frac{n}{2}(23 + 2534)$ ✓
$n = 280$ ✓

(b) (i) $T_{n+1} = \frac{1}{4}T_n \quad T_1 = 12$ ✓✓

(ii) $S_\infty = \dfrac{12}{1 - \frac{1}{4}} = 16$ ✓✓ [8]

17. (a) 92°C (initial temp of tea) ✓
22°C (room temp) ✓

(b) After 4.12 mins and before 7.45 mins ✓✓
$4.12 \leq t \leq 7.45$

(c) Horizontal asymptote $y = 22$ ✓
The tea will cool at a decreasing rate as it approaches room temperature which is 22°. ✓ [6]

18. (a) $y = \dfrac{2}{x - 3} + 2$ ✓✓

(b) $y = -3\sqrt{x + 4}$ ✓✓

(c) $y = \left(\frac{1}{2}\right)^x - 4$ ✓✓ [6]

19. (a) 0.2 ✓
(b) 0.5 ✓
(c) 0.3 ✓
(d) Pr(X ∩ Y) = Pr(X) x Pr(Y) if independent
Pr(X ∪ Y) = Pr(X) + P(Y) - Pr(X ∩ Y) ✓
Pr(X) + Pr(Y) - Pr(X ∪ Y) = Pr(X) x Pr(Y) ✓
Let Pr(Y) = k
0.5+ k - 0.8 = 0.5k
k = 0.6 = Pr (Y) ✓ [6]

20. (a) 1, 3, 7, 15,…
$T_{n+1} = T_n + 2^n \quad T_1 = 1$ ✓✓
(b) 20 sets ✓ [3]

21. (a) (i) $y = 16 - 6 - w$
$y = 10 - w$ ✓

(ii) $y^2 = 36 + w^2 - 12w\cos Y$ ✓

(iii) $(10 - w)^2 = 36 + w^2 - 12w\cos Y$ ✓
$64 - 20w = -12w\cos Y$ ✓
$\therefore \cos Y = \dfrac{5w - 16}{3w}$

(b) (i) $A = \dfrac{6w\sin Y}{2} = 3w\sin Y$ ✓
$A^2 = 9w^2\sin^2 Y$ ✓

(ii) $9w^2\sin^2 Y = 9w^2(1 - \cos^2 Y)$ ✓
$A^2 = 9w^2\left(1 - \left(\dfrac{5w - 16}{3w}\right)^2\right)$ ✓
$A^2 = -16w^2 + 160w - 256$ ✓

(c) (i) $A = \sqrt{-16w^2 + 160w - 256}$
$A' = 0$ when $w = 5$ ✓
Maximum area = 12 units2 ✓

(ii) $y = 10 - w = 5$
The triangle is isosceles. ✓ [12]

22. $y' = 3x^2 - 12x + k$ ✓
$b^2 - 4ac = 0$ for one solution
$144 - 4(3)k = 0$ ✓
$k = 12$ ✓ [3]

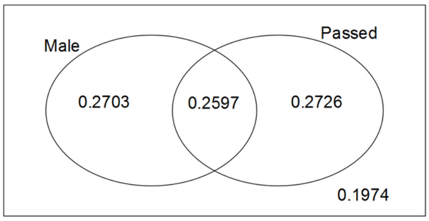
23. (a) $W = W_0(1.085)^t$
$R = R_0(0.95)^t$
$10W_0 = R_0$
$W_0(1.085)^t = 10 \times W_0(0.95)^t$ ✓
$(1.085)^t = 10(0.95)^t$
$t = 17.329$ years ✓
After 18 years there will be more wallabies. ✓

(b) $W_{n+1} = 1.085\,W_n \quad W_0 = 655$ ✓
$W_5 = 985$ ✓ [5]

24. (a) $r = -\dfrac{3}{2}$ ✓
$$\frac{-2\left(1 - \left(-\frac{3}{2}\right)^n\right)}{1 - \left(-\frac{3}{2}\right)} = 30$$ ✓
$n = 9.003$
$\therefore$ 10 terms ✓

(b) $T_5 = a + 4d$ and $T_7 = a + 6d$
$\therefore 2a + 10d = 38$ (eq 1) ✓
$S_{15} = 375 = \dfrac{15}{2}(2a + 14d)$ (eq 2) ✓
$a = 4 \quad d = 3$ ✓
$S_{30} = 15(2(4) + 29(3)) = 1425$
Sum of next 15 terms = $S_{30} - S_{15} = 1050$ ✓ [7]

25. (a)



✓✓✓

(b) (i) 0.5323 ✓

(ii) 0.2726 ✓

(iii) 0.51 ✓

(iv) 0.8026 ✓ [7]

26. The particle's initial displacement is 5 m to the right of the origin. ✓

$v = 3t^2 - 12t$ $\therefore$ Initial velocity = 0 ✓ [2]