



# MATHEMATICS DEPARTMENT

## Year 12 Methods - Test Number 1 - 2017

### Differentiation of Exponential and Trigonometric Functions

### Resource Free

Name: \_\_\_\_\_ Teacher: \_\_\_\_\_

Marks: 18

Time Allowed: 20 minutes

**Instructions:** You are NOT allowed any Calculators or notes.

You will be supplied with a formula sheet.

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1. Find  $\frac{dy}{dx}$  for

a)  $y = \frac{16e^x}{4e^{5x}}$

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.....

.....

b)  $y = 2\sin(e^{2x})$

c)  $y = 3x^2 e^{2x}$  [simplify your answer]

d)  $y = 3\pi \tan(1+e)^2$

**[3,3,3,3 = 12 Marks]**

2. Find the equation of the tangent to the curve defined by  $h = (e^{2t})(e^t + 1)^2$  at the point (0,4).



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**COLLEGE**

# **MATHEMATICS DEPARTMENT**

**Year 12 Methods - Test Number 1 - 2017**

**Differentiation of Exponential and  
Trigonometric Functions**

**[6 Marks]**

**Resource Rich**

**Name:** \_\_\_\_\_ **Teacher:** \_\_\_\_\_

**Marks:** 26

**Time Allowed:** 25 minutes

**Instructions:** You are allowed a ClassPad and 1 page of notes (both sides).

You will be supplied with a formula sheet.

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- 1) It is known that the amount of a dangerous 'recreational drug' (in mg) left unabsorbed in the bloodstream after  $t$  hours is given by

$$U = 100e^{-0.05t}$$

- a) Show that the rate of change of  $U$  with respect to time is proportional to the amount of the drug remaining.

- b) Find the time taken for 90% of the initial amount of the drug to be absorbed by the bloodstream. Give your answer to the nearest hour.

- c) Find an expression that describes the amount of the drug absorbed by the bloodstream after  $t$  hours.

**[3,2,1 = 6 Marks]**

- 2) a) The normal to a given curve at a point is defined as the perpendicular to the tangent at that point. Find the equation of the normal to the curve  $y = \frac{e^x}{2 - x}$  at the point where  $x = 1$ .

- b)  $y = x + 1$  is a tangent to the curve  $y = ax + b \sin x$  at the point  $(\frac{\pi}{2}, 1 + \frac{\pi}{2})$ . Find  $a$  and  $b$ .

**[4,4 = 8 Marks]**

- 3) Fishermen monitored the growth of the population of sardines in a particular location over a 30 year period from 1985 when the population was estimated to be 2 000 000 . They found that the population was continuously growing with the instantaneous rate of increase in the population

per year  $\frac{dP}{dt}$  , always close to  $\frac{P}{20}$  .

- a) Estimate the population of sardines at the end of the 30 year period.

- b) If this pattern of growth continues estimate the population of sardines in 2040.

**[3,3 = 6 marks]**

4) The displacement,  $x$  cm, of a particle from a fixed point O,  $t$  seconds after it is released is modelled by the equation  $x = -5 \cos \frac{\pi t}{4}$ . Use a calculus method to determine:

a) The velocity of the particle after 2 seconds,

b) When during the interval  $0 \leq t \leq 8$ , the particle travels with a speed of  $1 \text{ cm s}^{-1}$ .

**[2,4 = 6 marks]**

**\*\*End of Test\*\***