PERTH MODERN SCHOOL
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Independent Public School

Course 12 Methods(Test 2 alternative) Year 12

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: **_____45_____ mins**

Number of questions: **_____9_____**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **___46___ marks**

Task weighting: **__12__%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3.2.1-3.2.3) (3 & 3 =6 marks)

Determine y in terms of x for the following.

(a) $\frac{dy}{dx} = 5x^3 - 4x^2 + 7x + 1$ given that $y = 10, x = 1$.

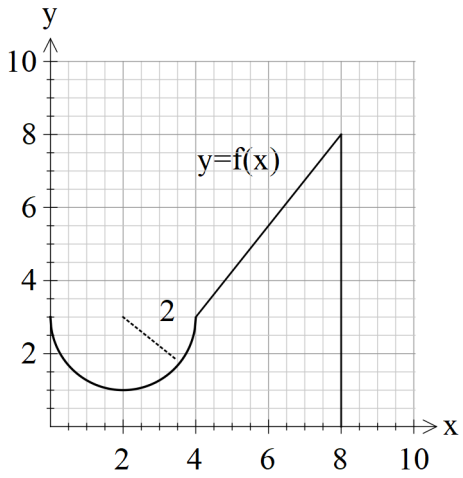
(b) $\frac{dy}{dx} = 5x^2\sqrt{6 + 2x^3}$ given that $y = 1, x = -1$.

Q2 (3.2.21-3.2.22) (4 marks)

An object is moving in a straight line such that its velocity m/s as a function time, t seconds, is given by $v = 5t^2 + pt + 1$ where p is a constant. The acceleration at time $t = 3$ seconds is $10m/s^2$ and is initially at the origin. Determine the displacement when $t = 6$ seconds.

Q3 (3.2.10-3.2.11) (3 & 4 = 7 marks)

Consider the function $f(x)$ which is graphed for $0 \leq x \leq 8$. The arc has a radius of 2 units.



(a) Determine the exact value of $\int_0^8 f(x)\,dx$.

(b) Determine α to two decimal places such that $\int_0^{\alpha} f(x)\,dx = \frac{1}{2}\int_0^8 f(x)\,dx$

Q4 (3.2.18-3.2.17) (3 & 2 = 5 marks)

A water tank has a leak and the volume of water contained, V, can be described by the following differential equation at time, t minutes, $\frac{dV}{dt} = -\frac{500t^2}{(2+t^3)^4}$. The tank is initially full but is emptied in 15 minutes.

(a) Determine the initial volume of water in the tank.

(b) Determine the change in volume in the third minute.

Q5 (3.2.11-3.2.14) (2, 2 & 2 = 6 marks)

Consider a function $f(x)$ that is defined for $0 \le x \le 13$ with the following conditions.

$f(3)=9, \quad f(10)=3$

$f(0)=0=f(5)=f(8)=f(13)$

With $f(x) \ge 0$ for $0 \le x \le 5 \quad \& \quad 8 \le x \le 13$ and $f(x) \le 0$ for $5 \le x \le 8$.

$\int_0^{13} f(x)dx = 7, \quad \int_5^8 f(x)dx = 12$.

(a) Determine $\int_3^{10} f'(x)dx$.

(b) Determine $\int_0^8 f(x)dx$ given that $\int_8^{15} f(x)dx = 6$.

(c) Determine $\frac{d}{dx}\int_0^x f(t)dt$ when $x = 10$.

Q6 (3.2.20) (4 marks)

Determine to two decimal places the area between the curves $y = x^2 + 6x + 2$ and $y = -x^2 - 7x + 5$.
(Hint- Sketch the curves first on your classpad)

Q7 (3.2.16) (1 & 3 = 4 marks)

Consider $y = \int_b^x f(t)\,dt$

a) In terms of f, express $\frac{d^2y}{dx^2}$.

b) If $f''(x) = 3x + 1$ and $f'(0) = 0 = f(0)$, determine y in terms of x only.

Q8 (3.1.4) (4 marks)

A radioactive substance ZZZ initially has a mass of 230 grams and decays according to $\frac{dN}{dt}=kN$ where N equals the mass at time t minutes and k is a constant. After 6 minutes the mass is 176 grams. Determine the time taken for half the mass to decay(half-life) and the value of k to three decimal places.

Q9 (3.2.6) (2 & 4 =6 marks)

(a) Determine $\frac{d}{dx}\left(x\sqrt{5-2x}\right)$.

(b) Using your result from part (a) and **without using your classpad** determine $\int\frac{x}{\sqrt{5-2x}}dx$.

Working out space

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