SHENTON
COLLEGE

ATMAM Mathematics Methods

Test 1 2019 Calculator Free

Name: SOLUTION

Teacher (Please circle name) Ai Friday Smith

Time Allowed : 30 minutes

Marks	/32

Materials allowed: Formula Sheet.

Attempt all questions. Questions 1,2, 3 ,4 and 5 are contained in this section.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given as exact values.
Marks may not be awarded for untidy or poorly arranged work.

1. [2,2,2,2]

Differentiate each of the following with respect to x, clearly showing the appropriate use of rules.
Do not simplify answers.

(a) $y = 4x^3 - \frac{1}{x^2} + \frac{1}{2}x$

$\frac{dy}{dx} = 12x^2 + \frac{2}{x^3} + \frac{1}{2}$

✓ polynomials $\frac{dy}{dx}$

✓ rational $\frac{dy}{dx}$

(b) $y = (3x+2)^3(x^4 - 3)$

$\frac{dy}{dx} = 3(3x+2)^2(3)(x^4-3) + (3x+2)^3(4x^3)$

✓ show use of product rule

✓ $\frac{dy}{dx}$ correct.

(c) $y = \frac{\cos(3x+2)}{\sin x}$

$\frac{dy}{dx} = \frac{\sin x(-\sin(3x+2))(3) - \cos(3x+2)\cos x}{\sin^2 x}$

✓ shows correct use of quotient rule

✓ $\frac{d}{dx}\cos(3x+2)$ correct

(d) $y = \sqrt{(5x-4)}$

$y = (5x-4)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(5x-4)^{-\frac{1}{2}}(5)$

✓ shows use of chain rule

✓ $\frac{d}{dx}(5x-4)^{\frac{1}{2}}$

2. [4, 6]

Consider the function $f(x) = x^3(4-x)$
$= 4x^3 - x^4$

(a) Use calculus to determine the location of all stationary points.

$f'(x) = 12x^2 - 4x^3$

$f'(x) = 0 \quad 0 = 4x^2(3-x)$

$x = 0$ or $x = 3$

Stationary Points at $(0,0)$ and $(3,27)$

✓ $f'(x)$

✓ demonstrates correct use of $f'(x)$ to determine stationary points

✓ $x = 0$ $x = 3$

✓ Give stationary Points

(b) Use the second derivative to determine the nature of the stationary points and the coordinates of any points of inflection.

$f''(x) = 24x - 12x^2$
$= 12x(2-x)$

$f''(0) = 0$
$f''(3) < 0$
$(3,27)$ local maximum

$f''(x) = 0 \quad 0 = 12x(2-x)$
$x = 0$ or $x = 2$

check concavity at $x = 0$
$f''(-1) < 0$
$f''(1) > 0$ $\therefore$ concavity changes.

$\therefore$ Hor Point of Inflection at $(0,0)$

At $x = 2$, check concavity
$f''(1) > 0$
$f''(3) < 0$ $\therefore$ Concavity changes
$\therefore$ Point of Inflection at $(2,16)$

✓ $f''(x)$

✓ correctly uses $f''(x)$ to determine nature of stationary points

✓ $(3,27)$ local Max.

✓ $f''(x) = 0$

✓ Checks concavity changes at $x = 0$ and ascertains $(0,0)$ Hor p. of i.

✓ Point of inflection at $(2,16)$

(b) If $h = 6\ cm$, then $V = 6\pi r^2 + \frac{2}{3}\pi r^3$.

For $r = 4\ cm$,

show that a small increase of k cm in the radius results in an approximate increase of $80\pi k\ cm^3$ in the volume.

$\delta r = k$ cm

$\delta V \approx \left.\frac{dV}{dr}\right|_{r=4} . \delta r$

$\approx 80\pi . k$

$= 80\pi k\ cm^3$

$V = 6\pi r^2 + \frac{2}{3}\pi r^3$

$\frac{dV}{dr} = 12\pi r + 2\pi r^2$

$\left.\frac{dV}{dr}\right|_{r=4} = 48\pi + 32\pi$
$= 80\pi$

✓ Shows use of Incremental formula with correct variables

✓ $\frac{dV}{dr}$

✓ evaluates derivative and shows $= 80\pi k\ cm^3$

8. [4 marks]

If $y = 5t^3$ use differentiation to determine the approximate percentage change in y when t changes by 4%.

$y = 5t^3$ $\qquad \frac{\delta t}{t} = \frac{4}{100}$

$\frac{dy}{dt} = 15t^2$

$\frac{\delta y}{y} \approx \frac{\frac{dy}{dt} . \delta t}{y}$

$\approx \frac{\cancel{15}^{3}\cancel{t^2} . \delta t}{\cancel{5}\cancel{t^3}_{t}}$

$= 3 . \frac{\delta t}{t}$

$= 3 \frac{4}{100}$

$= 12\%$.

When t changes by 4% approximate change in y is 12%

✓ Demonstrates Incremental process

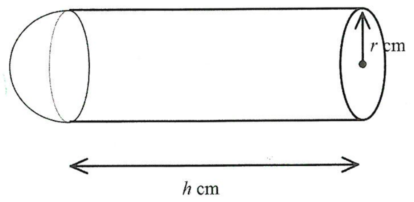
✓ $\frac{\delta t}{t} = \frac{4}{100}$

✓ $\frac{\delta y}{y}$ substitution and correct algebra

✓ $3\frac{\delta t}{t} = 12\%$

End of Questions

7. [2,1,4,3]



A solid wooden peg consists of a cylinder of length h cm and a hemispherical cap of radius r cm.
The volume, V cm^3, of the peg is given by $V = \pi r^2 h + \frac{2}{3}\pi r^3$.

(a) If the surface area of the peg is 100π cm^2.

(i) Show that $h = \frac{100-3r^2}{2r}$

$100\pi = 2\pi r h + \pi r^2 + 2\pi r^2$ ✓ correct S.A $100\pi =$ rule

$100\pi = 2\pi r h + 3\pi r^2$

$100\pi - 3\pi r^2 = 2\pi r h$ ✓ $h =$ Show transformation clearly

$\frac{100\pi - 3\pi r^2}{2\pi r} = h$

$\therefore h = \frac{100 - 3r^2}{2r}$

(ii) Determine V as a function of r.

$V = \pi r^2 \left(\frac{100-3r^2}{2r}\right) + \frac{2}{3}\pi r^3$ ✓ substitution No need to SIMPLIFY

(iii) Show the use of calculus to determine the dimensions required to obtain the maximum volume and state the maximum volume.

For Max $\frac{dV}{dr} = 0$ ✓ use of $\frac{dV}{dr}$

ie. $\cdots \left(\frac{5r^2\pi - 100\pi}{2}\right) = 0$

$r = \pm 2\sqrt{5}$ cm $\pm 4{\cdot}472$ cm. ✓ r

When $r = 4{\cdot}472$ cm $f''(4{\cdot}472) < 0$ ✓ Confirm Max.

$\therefore$ Max when $r = 4{\cdot}472$ cm or $2\sqrt{5}$ cm

$V = 468{\cdot}32$ cm^3 ✓ Vol.

$V = \frac{200\sqrt{5}\cdot\pi}{3}$ cm^3.

3. [3 marks]

If $y = 3\sin 2x + 2\cos 2x$ show that $4y + \frac{d^2y}{dx^2} = 0$

$\frac{dy}{dx} = 3\cos 2x\,(2) - 2\sin 2x\,(2)$ ✓ $\frac{dy}{dx}$

$= 6\cos 2x - 4\sin 2x$ ✓ $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = -6\sin 2x\,(2) - 4\cos 2x\,(2)$

$= -12\sin 2x - 8\cos 2x$

$4(3\sin 2x + 2\cos 2x) + (-12\sin 2x - 8\cos 2x)$ ✓ Show clearly $= 0$

$= 12\sin 2x + 8\cos 2x - 12\sin 2x - 8\cos 2x$

$= 0$

$\therefore 4y + \frac{d^2y}{dx^2} = 0$

4. [4 marks]

Determine $\frac{dy}{dx}$ if $y = \sqrt{u}$, $u = v^2 + 1$ and $v = x + x^{-1}$. Do not simplify your answer.

$y = u^{\frac{1}{2}}$ $\quad u = v^2 + 1$ $\quad v = x + x^{-1}$

$\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$ $\quad \frac{du}{dv} = 2v$ $\quad \frac{dv}{dx} = 1 - x^{-2}$

$= 2(x + x^{-1})$

$= \frac{1}{2}(v^2+1)^{-\frac{1}{2}}$ ✓ all derivatives correct not necessarily in terms of x.

$= \frac{1}{2}\left[(x+x^{-1})^2 + 1\right]^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$ ✓ shows correct use of chain rule.

$= \frac{1}{2}\left[(x+x^{-1})^2+1\right]^{-\frac{1}{2}} \cdot 2(x+x^{-1})(1-x^{-2})$.

✓ $\frac{dy}{du}$ in terms of x

✓ $\frac{du}{dv}$ in terms of x

5. [1,1,1,4]

The table below contains information about the sign of $f(x)$, $f'(x)$ and $f''(x)$ at seven points on the graph of the continuous function $f(x)$. Apart from those in the table, there are no other points where $f(x)$, $f'(x)$ or $f''(x)$ are equal to zero.

x	-3	-1	0	1	2	3	4
$f(x)$	–	0	+	+	+	0	–
$f'(x)$	+	0	+	+	0	–	–
$f''(x)$	–	0	+	0	–	–	–

(a) Describe the nature of the graph when $x=2$

Maximum stationary Point ✓

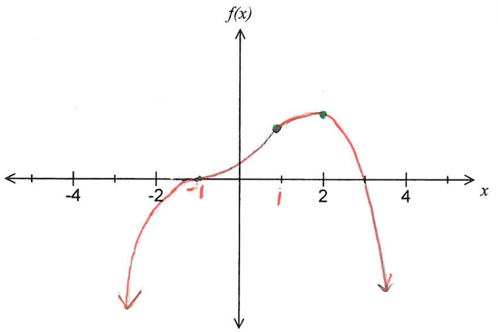
(b) At what value(s) of x is $f(x)$ concave up?

$-1 < x < 1$ ✓

(c) Describe the nature of the graph when $x=-1$.

Horizontal point of inflection ✓

(d) Sketch the function on the axes below.



✓ shape ↓ $x<-1$ ↓ $x>3$

✓ Horizontal P. of I at $x=-1$

✓ Point of Inflection change of concavity at $x=1$

✓ Max Stationary Point at $x=2$

SHENTON COLLEGE

ATMAM Mathematics Methods

Test 1 2019 Calculator Assumed

-1 overall Section 2 units

if missing > 1 unit

Name: SOLUTION

Teacher (Please circle name) Ai Friday Smith

Time Allowed : 20 minutes

Marks /19

Materials allowed: *Classpad calculator,* Formula Sheet.

Attempt all questions. Questions 6, 7 and 8 are contained in this section.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given as exact values.
Marks may not be awarded for untidy or poorly arranged work.

6. [1,1,1,2]

A particle is moving in a straight line so that at time t, in seconds, its position from the origin O is given by $x(t) = 7.2 - 3\cos(0.65t)$ metres, $t \geq 0$

(a) State the initial position of the particle.

$x(0) = 4.2\text{m}$ ✓ correct position

(b) Determine the velocity function for this particle.

$v(t) = 1.95\sin(0.65t)$ ✓ correct function

(c) At what time does the particle first come to rest after $t = 0$?

$t = 4.83\text{ s}$ ✓ correct time

(d) At what time does the particle first reach its maximum velocity? Justify your choice.

Max Velocity at 2.42 s ✓ correct time

$v'(t) = 0$ or $v(t)$ max from graph on calc ✓ Justify