

before reading any further.
examination room. If you have any unauthorised notes or other items of a non-personal nature in the examination room, it is **your responsibility** to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room.

Important note to candidates

Special items: and up to three calculators approved for use in the WACE examinations
drawing instruments, templates, notes on two unfolded sheets of A4 paper,

Standard items: ruler, highlighters
ruler, pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser,

To be provided by the candidate
Formula Sheet (removed from Section One)
This Question/Answer Booklet
To be provided by the supervisor

Materials required/recommended for this section

Working time for Section: one hundred minutes
Reading time before commencing work: ten minutes

Time allowed for this section

Your Teacher's name

Student's Name

Calculator-assumed
Section Two:

MATHEMATICS
METHODS

Question/Answer Booklet

Semester Two Examination, 2018

PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.



Perth Modern School

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|------------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|--------------------|
| Section One: Calculator-free | 7 | 7 | 50 | 50 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 100 | 65 |
| Total | | | 149 | 100 | |

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

(6 marks)

- (a) Find the first and second derivatives of the profit function and explain exactly how these derivatives could help you graph the function.

(11 marks)

Question 8

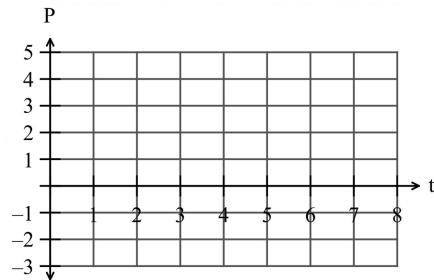
Working time for this section is 100 minutes.

This section has **thirteen (13)** questions. Answer all questions. Write your answers in the spaces provided.

Section Two: Calculator-assumed **65% (100 marks)**

- (b) Sketch the profit equation on the set of axes.

(3 marks)



After the first two months when the profit had been increasing, the owner employed more staff and it took a little while for sales to start to increase again.

- (c) Determine when the profit started to increase again.

(1 mark)

- (d) Determine when the break-even point was reached i.e. when profit again became positive.

(1 mark)

(2 marks)

- (a) Eight red marbles and 12 yellow marbles are placed in a bag and mixed thoroughly. One marble is selected, and its colour noted then it is replaced in the bag and mixed thoroughly with the other marbles again. This process is repeated several times.

A marble can be drawn up to four times. The draw stops once a yellow marble is selected.

- (b) What is the probability that a yellow marble is not drawn?

(2 marks)

(2 marks)

- (a) What is the probability that the first yellow marble drawn is the fourth?

Question number: _____

Additional working space

(4 marks)

Question 10

In 1880, the population in the United States was 50 189 209.
In 1930, the population had increased to 123 202 624.

- (a) Taking $t=0$ in 1880, set up an equation in the form $P=P_0e^{kt}$ that can be used to estimate the population in the United States during the 50-year period. (2 marks)

- (b) Write down the average annual population growth over that period. (1 mark)

Over the next 60 years to 1990, the population grew from 123 202 624 to 248 709 873.

- (c) Determine if the rate of growth during the 60 years from 1930 to 1990 is the same as the rate of growth from 1880 to 1930. (1 mark)

- (d) Use the data from 1930 to 1990 to predict the population in 2016. (1 mark)

NB. The actual population in 2016 was 324 118 787.

(2 marks)

(iii) In what year will the population reach 100?

(2 marks)

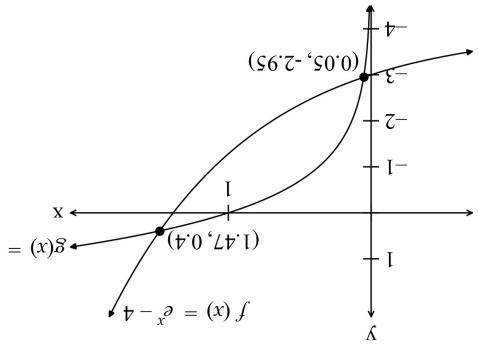
(i) What was the population in 2002?

Population can be modelled by the equation $P(t) = 22(\ln(t+3))$ where t is in years starting in 2002.

(b) A small colony of quolls live in hummock grasslands on the sand plains not far from Port Hedland. The population of this colony was studied in 2002. The

population can be modelled by the equation $P(t) = 22(\ln(t+3))$ where t is in

(3 marks)



The points of intersection are shown.

(a) Use your calculator to find the area enclosed between the two functions $f(x) = e^x - 4$ and $g(x) = \ln(x)$ as shown in the diagram below.

(5 marks)

| x | 1 | 2 | 3 | 4 |
|----------|-----|-----|-----|-----|
| $P(X=x)$ | 0.3 | 0.2 | 0.2 | 0.3 |

(10 marks)

Question 20 Question 11

(a) (i) Find the expected value and variance of the probability density function in the table below.

(5 marks)

(a) (ii) Use your calculator to find the area enclosed between the two functions

$f(x) = e^x - 4$ and $g(x) = \ln(x)$ as shown in the diagram below.

(5 marks)

- (b) Sam bet on the outcome of a spinner with probabilities as in the table below.

| | | | | |
|------------|-----|-----|-----|-----|
| x | 1 | 2 | 3 | 4 |
| $P(X = x)$ | 0.3 | 0.2 | 0.2 | 0.3 |

It costs Sam \$1 per spin and the payout is \$2 for a 2 or a 3 and nothing otherwise.

What is Sam's average payout?

(3 marks)

The set of samples obtained from the second potential site has a mean sample grade of 4.7 g/t. The data was given to Kheng as a box-plot with the median of 4.72 g/t, the lower quartile of 4.2 g/t and the upper quartile of 5.2 g/t.

- (d) Examine the above statistics to determine if the data for the second potential site could be represented by a normal distribution.
Justify your conclusion.

(2 marks)

- (e) Which of the mine sites should be sold? Use your knowledge of statistics and probability to support your choice.

(2 marks)

(2 marks)

(ii) Determine the correct proportion of vehicles that were speeding.

(2 marks)

(i) Calculate the adjusted mean and standard deviation of the vehicle speeds.

(ii) Subsequent tests on the measuring equipment discovered that it had been wrongly calibrated. The correct speed of each vehicle, v , could be calculated from the measured speed, x , by increasing x by 6% and then adding 1.7.

(iii) The probability that another sample contained less than the particular grade was 0.25 or 25%. Determine the maximum grade for the sample. (1 mark)

(1 mark)

(ii) had a speed of less than 65 km h^{-1} , given they were exceeding the speed limit.

(1 mark)

(i) Use the table of relative frequencies to estimate the probability that the next vehicle to pass the roadworks

| Speed (km h^{-1}) | $45 \leq x < 50$ | $50 \leq x < 55$ | $55 \leq x < 60$ | $60 \leq x < 65$ | $65 \leq x < 70$ |
|------------------------------|------------------|------------------|------------------|------------------|------------------|
| Relative frequency | 0.024 | 0.022 | 0.504 | 0.188 | 0.012 |

The speeds of 250 vehicles, on a section of freeway undergoing roadworks with a speed limit of 60 km h^{-1} , had a mean and standard deviation of 56.9 km h^{-1} and 3.6 km h^{-1} respectively. A company is examining samples from a recent drilling program for both sites to help with his decision. A sample taken from the first site has a mean sample grade of 4.6 g/t with a standard deviation of 0.56 g/t . Kheung found that the data for the samples are normally distributed.

Kheung is employed by BLP Limited as a geologist and is in charge of choosing between two of its sites for the construction of an open-pit mine.

One of the sites will be sold to pay for the construction of the open-pit mine.

It is known that the cost of an open-pit mine is proportional to the volume of the site.

The volume of the site is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base of the site and h is the height of the site.

The radius of the base of the site is 100 m and the height of the site is 50 m .

Determine the volume of the site.

(6 marks)

Question 19

MATHEMATICS METHODS

CALCULATOR-ASSUMED

6

CALCULATOR-ASSUMED

6

MATHEMATICS METHODS

6

CALCULATOR-ASSUMED</p

Question 13

A lottery sells 1000 tickets and claims that there will be 10 winners. How many tickets should you buy so that you have a 20% chance of winning at least 1 prize.

Question 18

Cans of soft drinks are advertised as containing 375 mL. However there have been complaints for one particular brand that several cans contained less than 375 mL causing the manufacturer to investigate the settings on the machines that fill the cans. It turns out that the amount the machines are filling the cans with follows a normal distribution with a mean of 377 mL and a standard deviation of 3 mL.

- (a) What percentage of cans contain less than the advertised quantity? (2 marks)

- (b) If 10 cans are chosen at random, what is the probability that at least half of them have less than the advertised quantity? (2 marks)

- (c) A random sample of 10 cans is selected and the quantity in each can is measured. What is the probability that the mean of the measurements is: (3 marks)

i) More than the advertised amount

ii) Equal to the advertised quantity when rounded to the nearest mL

The manufacturer knows sufficient statistics to realise that if the machine is set to fill the same average amount but the exact amount in each can is more consistent (i.e. the standard deviation is smaller) there will be less chance of cans being under filled.

- (d) What is the standard deviation (correct to 2 dp) would result in at most 10 in 1000 cans being underfilled. Show all working and give your answer accurate to 2 dp. (4 marks)

(2 marks)

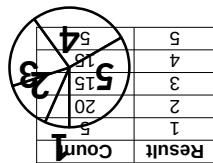
- (d) Comment briefly on the key features of this graph.

This simulation of 60 spins of the spinner is performed another 200 times, with the proportion of composite numbers recorded each time and graphed.

95%. Explain which level of confidence will give the smallest margin error. State the numbers using the simulation results. The level of confidence will be chosen from 90% or 95%. It has been decided to create a confidence interval for the proportion of composite numbers in 60 tosses, using the results above.

Determine the mean and standard deviation for the sample proportion of composite numbers in 60 tosses, using the results above.

- (a) Calculate the proportion of composite numbers recorded in this simulation.



The simulation of a loaded (unfair) spinner is spun 60 times and the results recorded with the following results. (Below diagram is for illustrative purposes only)

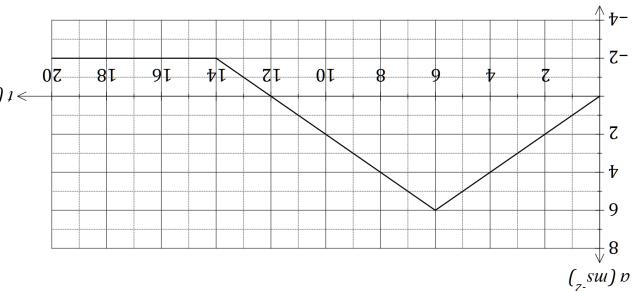
Question 14
CALCULATOR-ASSUMED
14
MATHEMATICS METHODS
(9 marks)

- (c) Determine the distance of the particle from the origin after 3 seconds. (3 marks)

- (b) At what time is the velocity of the body a maximum, and what is the maximum velocity? (2 marks)

- (ii) $t=20$. (2 marks)

- (i) $t=6$. (1 mark)
- (a) Determine the velocity of the object when



A particle, initially stationary and at the origin, moves subject to an acceleration, $a \text{ ms}^{-2}$, as shown in the graph below for $0 \leq t \leq 20$ seconds.

(8 marks)

Question 15

From a random sample of n people, it was found that 54 of them subscribe to a streaming music service. A symmetric confidence interval for the true population proportion who subscribe is $0.1842 < p < 0.2958$.

- (a) Determine the value of n , by first finding the mid-point of the interval. (3 marks)

- (b) Determine the confidence level of the interval. (4 marks)

(8 marks)

Question 16

The moment magnitude scale M_w is used by seismologists to measure the size of earthquakes in terms of the energy released. It was developed to succeed the 1930's-era Richter magnitude scale.

The moment magnitude has no units and is defined as $M_w = \frac{2}{3} \log_{10}(M_0) - 10.7$, where M_0 is the total amount of energy that is transformed during an earthquake, measured in dyn-cm.

- (a) On 28 June 2016, an estimated 2.82×10^{21} dyn-cm of energy was transformed during an earthquake near Norseman, WA. Calculate the moment magnitude for this earthquake. (1 mark)

- (b) A few days later, on 8 July 2016, there was another earthquake with moment magnitude 5.2 just north of Norseman. Calculate how much energy was transformed during this earthquake. (2 marks)

- (c) Show that an increase of 2 on the moment magnitude scale corresponds to the transformation of 1000 times more energy during an earthquake. (4 marks)