Working out space

PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two: Calculator-assumed

Your Name _____

Your Teacher's Name_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
9			16		
10			17		
11			18		
12			19		
13			20		
14			21		
15			22		

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	49	34
Section Two: Calculator-assumed	14	14	100	96	66
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

See next page

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Section Two: Calculator-assumed **(96 Marks)**

This section has **14** questions. Answer **all** questions. Write your answers in the spaces provided.

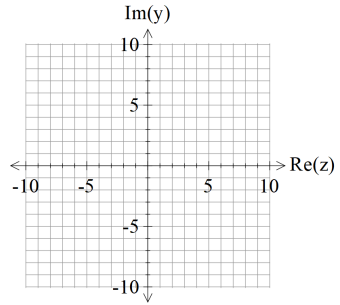
Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(6 marks)**

a) Sketch the locus of the equation $|z - 5 + 4i| = |z + 6 - 2i|$ on the axes below. (3 marks)



b) Determine the cartesian equation of this locus in terms of $x \& y$. (3 marks)

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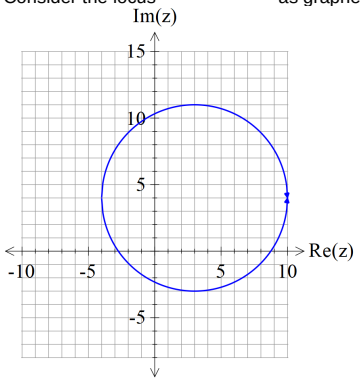
Question 10 **(9 marks)**

Consider the locus $|z - 3 - 4i| = 7$ as graphed below.



Determine the following.

a) Maximum value of $|z|$. (2 marks)

b) Minimum value of $|z + 8 - 12i|$ (3 marks)

c) Sketch the region defined by $|z - 3 - 4i| \leq 7$ and $\text{Im}(z) + \text{Re}(z) \geq 6$ on the axes above stating the coordinates of all boundary points. (4 marks)

Question 22 **(6 marks)**

a) Using De Moivre's theorem, derive an expression for $\cos(3\theta)$ in terms of $\cos\theta$ only.
(3 marks)

b) **Using** the result from (a) above, show how to obtain **all** solutions to $8z^3 - 6z - 1 = 0$ in the form $\cos\phi$. Express possible values of ϕ in **exact** form.
(3 marks)

Question 11 **(6 marks)**

Consider the line $r = \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -4 \\ 3 \end{pmatrix}$ and the point A $(11, -3, 4)$.

a) Using **scalar dot** product show how to find the closest distance of point A to the line above.
(3 marks)

b) Using vector **cross** product show how to find the closest distance of point A to the line above.
(3 marks)

Question 12 **(9 marks)**

Consider the sphere $\left| r - \begin{pmatrix} 7 \\ -5 \\ 1 \end{pmatrix} \right| = \alpha$ with α being a positive constant and the line $r = \begin{pmatrix} 9 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -4 \\ 1 \end{pmatrix}$.

Determine all possible values of α,(2 decimal places) for the following.

i) The line does not meet the sphere at all.

ii) The line just touches the sphere at one point only.

iii) The line meets the sphere at two points.

Question 21 **(4 marks)**

Consider the polynomial $P(z) = z^5 - z^4 + az^3 + bz^2 + cz + d$ where $a, b, c \& d$ are real constants.

Given that $P(2i) = 0 = P(-3i)$ and $a + b + c + d = 0$ determine the values of $a, b, c \& d$.

Question 20 **(6 marks)**

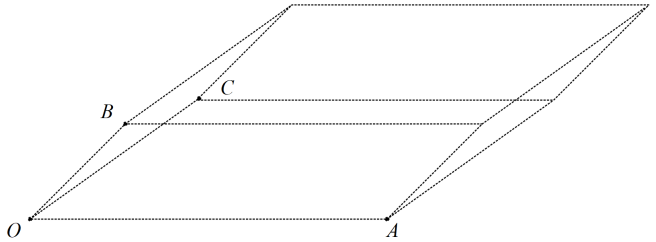
Let $z = r\,cis\theta$ be a complex number such that $r > 0$ and $0 < \theta < \frac{\pi}{2}$.

a) Express in terms of r & θ the complex number $\frac{\left(\sqrt{3}+i\right)z^3}{\overline{z}(1-i)}$. (Simplify) (3 marks)

b) Express $\beta = Arg(z-r)$ in terms of θ. (3 marks)

Question 13 **(4 marks)**

Consider a prism where each side is a parallelogram with opposites sides congruent. The units given are in metres.



Given that $\overrightarrow{OA} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} m, \overrightarrow{OB} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix} m, \overrightarrow{OC} = \begin{pmatrix} -11 \\ 1 \\ 8 \end{pmatrix} m$ and **using vector** methods, determine the volume of the prism. (Hint: Volume = area of face multiplied by perpendicular width)

Question 14 **(9 marks)**

Consider the plane Π $5x - 2y + 6z = 9$.

a) Determine the distance of point A $(11, -3, 4)$ from the plane Π. (4 marks)

b) Determine an expression in terms of $x, y \& z$ for the distance of point P (x, y, z) from the plane Π. (3 marks)

c) If point A $(11, -3, 4)$ is on a plane parallel to Π, determine a vector equation for this parallel plane. (2 marks)

Question 19 **(7 marks)**

a) Consider the cartesian equation $x^2 + y^2 + z^2 - 6x + 8y - 3z + 20 = 0$. Describe what this locus of points represents and state major features and give the **vector** equation. (4 marks)

b) Consider the equation $x^2 + y^2 + z^2 + 4x - 2y + 6z = \alpha$ where α is a constant. Determine the values of α for which the equation would be a sphere giving the centre and radius in terms of α. (3 marks)

Question 18 **(7 marks)**

$$a=\begin{pmatrix}4\\-3\\2\end{pmatrix}, b=\begin{pmatrix}-2\\\frac{3}{2}\\q\end{pmatrix}, c=\begin{pmatrix}7\\r\\5\end{pmatrix} \& d=\begin{pmatrix}s\\-11\\7\end{pmatrix}$$

Consider the vectors

a) Determine $q, r \& s$ given that $a \& b$ are parallel, c is perpendicular to a and d is perpendicular to b. (4 marks)

b) Given that $e=\begin{pmatrix}6\\-4\\5\end{pmatrix}$, determine a vector parallel to a but equal in magnitude to e. (3 marks)

Question 15 **(7 marks)**

Consider two submarines A & B moving in deep ocean with constant velocities

$$v_A=\begin{pmatrix}9\\-2\\5\end{pmatrix} km/h \quad v_b=\begin{pmatrix}12\\5\\\frac{2}{3}\end{pmatrix} km/h$$

.

$$r_A=\begin{pmatrix}11\\8\\-5\end{pmatrix} km$$

At 12:30am submarine A is at position and at 1am the same day

$$r_B=\begin{pmatrix}2\\-5.5\\1\end{pmatrix} km$$

submarine B is at position .

a) Determine the time of day, to nearest minute, that the submarines are closest to each other stating this distance to the nearest metre, (4 marks)

b) If both submarines leave a lasting water trail of bubbles, determine if the trails cross and if they do at which position under water. (3 marks)

Question 16 **(5 marks)**

Consider the complex numbers $s, p, w \& z$ such that:

$w = 1 + \sqrt{3}i$

$p = \sqrt{5}\left| -\frac{\sqrt{3}}{2} + \frac{1}{2}i \right|$

$Arg(p\bar{z}) = \frac{7\pi}{12}$

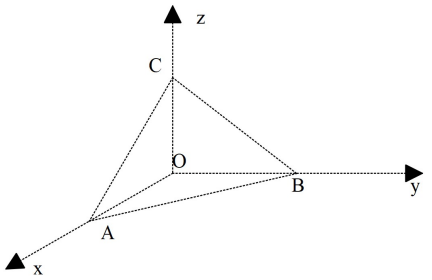
$s = \frac{pw}{z}$

$|s| = \sqrt{10}$

Determine z in the form $z = x + iy$ where $x \& y$ are real numbers.

Question 17 **(11 marks)**

Consider the 3D object $OABC$ as drawn below with O the origin and $A(5,0,0), B(0,4,0) \& C(0,0,3)$



a) Determine the vectors $AB \& AC$. (2 marks)

b) Determine to the nearest degree the angle $\angle CAB$ (2 marks)

c) Determine the exact area of triangle ΔABC using vectors. (3 marks)

d) Determine the cartesian equation of the plane containing triangle ΔABC. (4 marks)