Section Two: Calculator-assumed **(80 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Suggested working time for this section is 100 minutes.

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Question 7 **(8 marks)**

A basketball training squad consists of 4 guards, 2 centres and 3 forwards. A team of 5 is to be chosen to start the game.

In how many ways can this starting team be chosen if:

(a) there are no restrictions? (1 mark)

$^9C_5 = \underline{126}$

(b) the team must consist of 2 guards, 1 centre and 2 forwards? (2 marks)

$(^4C_2)(^2C_1)(^3C_2) = 6 . 2 . 3$ ✓
$= \underline{36}$ ✓

(c) the team includes at most 2 centres? (3 marks)

$^2C_2\,^7C_3 + {}^2C_1\,^7C_4 + {}^2C_0\,^7C_5$ ✓
$= 35 + 70 + 21$ ✓
$= \underline{126}$ ✓

Julie is a centre player and is chosen in the team. If the other players are selected at random, what is the probability that:

(d) Julie is the only centre in the team? (2 marks)

$P(\text{Julie only centre}) = \dfrac{(^7C_4)(^1C_1)}{^9C_5}$
$= 35/126 = \underline{5/18}$

See Next Page

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Question 8 **(5 marks)**

(a) State the natural domain and corresponding range for gof(x) given that (2 marks)

$f(x) = x - 5$ and $g(x) = \frac{1}{x-1}$

$g(f(x)) = \frac{1}{(x-5)-1} = \frac{1}{x-6}$

$R_x \to x - 5 \to R_y \longrightarrow \frac{1}{x-1} \to y \neq 0$

$\{x \in R : x \neq 6\}$ ✓ $\{y \in R : y \neq 0\}$ ✓

(b) If $f(x) = 3x^2 - 2$ and $h(x) = \frac{3}{1 - x}$ find h(f(x)). (1 mark)

$h\left(\frac{3}{1-(3x^2-2)}\right)$ $h(f)x = \frac{3}{3 - 3x^2}$

$= \frac{1}{1-x^2}$ ✓

(c) A composite function is defined by the equation $h(f(x)) = \sqrt{x - 3} - 4$. Determine the domain and range of this function for x real. (2 marks)

$h(f(x)) = \sqrt{x-3} - 4$

$\{x \in R: x \geqslant 3\}$ ✓

$\{y \in R: y \geqslant -4\}$ ✓

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Additional working space

Question 10 **(5 marks)**

Consider the function $f(x) = x^3 + \boldsymbol{a}x^2 + 2x + \boldsymbol{b}$ where ***a*** and ***b*** are constants

(a) Find an expression for the gradient of the curve (1 mark)

$f'(x) = 3x^2 + 2ax + 2$ ✓

(b) Given that the tangents at A(0, ***b***) and B(2, 5) are parallel, find the value of ***a*** and ***b***. (4 marks)

$f'(0) = 2$

$f'(2) = 12 + 4a + 2$

$f'(2) = 14 + 4a$

$f'(0) = f'(2)$
Same gradient!

$\therefore\ 2 = 14 + 4a$ ✓

$4a = -12$

$a = -3$ ✓

$f(2) = 5$

$5 = 8 - 12 + 4 + b$

$b = 5$ ✓

$a = -3$

$b = 5$ ✓

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Question 9 **(5 marks)**

In the first five seconds of inflation, the relationship between the radius (r cm) and time (t sec) of a spherical party balloon are related by the formula

$$r = -t(t - 10)$$

(a) Show that the relationship between volume (V cm^3) and time is given by $V = \dfrac{4\pi\left(10t - t^2\right)^3}{3}$ (1 mark)

$V_{sp} = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi\left(-t^2 + 10t\right)^3$

$V = \dfrac{4\pi\left(-t^2 + 10t\right)^3}{3}$ ✓

(b) Determine the exact volume of the balloon 3 seconds after first being inflated. (1 mark)

$V(3) = \dfrac{4\pi\left(-(3^2) + 30\right)^3}{}$ ✓

$V = \dfrac{4\pi(21)^3}{3}$

$V = 12348\pi$ (Exact)

(c) Determine the approximate change in volume as t increases from 3 to 3.01 sec. (3 marks)

$\delta V = \dfrac{dV}{dt} \cdot \delta t$ ✓ ✓

$= 4\pi\left(10t - t^2\right)^2 . (10 - 2t) . 0 \cdot 01$ (at 3)

$= 70 \cdot 56\pi \text{ cm}^3$ ✓

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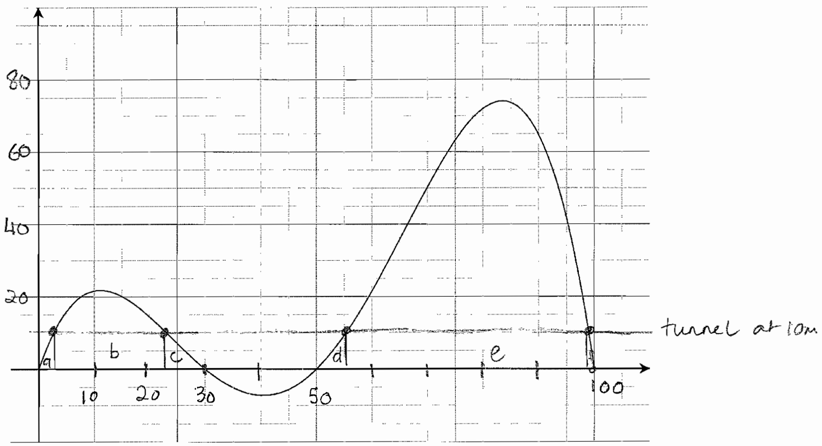
Question 18 **(5 marks)**

The cross section of land can be modeled by the equation

$H = 0.00003d(d - 30)(d - 50)(100 - d)$ where H and d are, respectively, the height (in metres) above a fixed horizontal level and the distance (in metres) from a fixed point.

The cross section has been shown in the diagram below.



A tunnel, 10m high, will be constructed through the two hills. Show how the cross sectional area of soil removed can be determined using integrals and mensuration (measurement) formula. There is no need to evaluate your answer. (5 marks)

Solve $0 \cdot 00003d(d-30)(d-50)(100-d) = 10$ ✓

Poi $2 \cdot 64,\ 23 \cdot 03,\ 55 \cdot 33,\ 99$ ✓✓

$\text{Area} = \underbrace{\int_0^{2.64} H}_{a} + \underbrace{(23 \cdot 03 - 2 \cdot 64) \times 10}_{b} + \underbrace{\int_{23.03}^{30} H}_{c} + \underbrace{\int_{50}^{55.33} H}_{d} + \underbrace{(99 - 5.33) \times 10}_{e} + \int_{99}^{100} H$

✓ ✓

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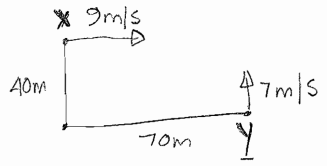
Question 17 **(6 marks)**

Two competing cyclist are riding with constant speed. At 12 midday cyclist X is 40 metres north of a judge and is riding east at 9m/s, while cyclist Y is 70 metres east of the judge and is riding north at 7m/s.

(a) Show diagrammatically this situation (a scale diagram is not required) (1 mark)



(b) If the distance between the cyclist **t** seconds later is **D** metres, show that

$\mathbf{D}^2 = 6500 - 1820t + 130t^2$ (3 marks)

$D^2 = (70-9t)^2 + (40-7t)^2$ ✓

$= 4900 - 1260t + 81t^2 + 1600 - 560t + 49t^2$

$= 6500 - 1820t + 130t^2$ ✓

(c) Determine the time the cyclists are closest together and determine the minimum distance between them. (2 marks)

Find minimum turning point

$x = \frac{-(-1820)}{2(130)}$

$x = 7$ secs

Minimum time 7 seconds ✓

minimum distance

$\underline{130\text{m}}$ ✓

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Question 11 **(5 marks)**

A mathematics teacher, in conversation with a colleague explained that her Year 10 class of students could be classified as

Well behaved (Group A consisting of 15 students)

Moderately behaved (Group B consisting of 10 students)

Poorly behaved (Group C consisting of 5 students)

She also mentioned that when there is a full moon on any particular lunar cycle the probability that a student will misbehave one or more times is 0.05, 0.15 and 0.3 for a randomly selected student from Group A, B and C respectively.

(a) What is the probability that a randomly selected student will misbehave at least once within a lunar cycle? (2 marks)

30 Students

A WB $\frac{1}{20}$ B $\frac{3}{20}$ C $\frac{3}{10}$ | WB 15 MB 10 PB 5

$\therefore$ P (Anyone misbehaves) ✓

$= \frac{1}{20} \cdot \frac{15}{30} + \frac{3}{20} \cdot \frac{10}{30} + \frac{3}{10} \cdot \frac{5}{30}$

$= \underline{\tfrac{1}{8}}$ ✓

(b) If a randomly selected student had misbehaved at least once during a lunar cycle, what is the probability the student was from Group C? (3 marks)

$P(C \mid M) = \frac{P(C \cap M)}{P(M)}$ ✓

$= \frac{3/10 \times \frac{1}{6}}{1/8}$ ✓

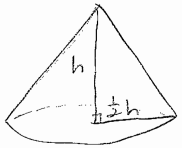
$= 2/5$

$= 0.4$ ✓

5/28

Question 12 **(8 marks)**

a) Sand is falling onto the top of a pile at the rate of 2 cubic centimetres per hour. The pile maintains a conical shape in which the radius of the base is always one half of the height. How fast is the height of the pile growing when it is 5 metres high? (4 marks)



$$\frac{dV}{dt} = 2\,cm^3/hr$$

$$\frac{dh}{dt} = ?$$

$$V = \frac{1}{3}\left(\pi\left(\frac{h}{2}\right)^2\right)h$$

$$V = \frac{\pi h^3}{12} \checkmark$$

$$\frac{dV}{dh} = \frac{\pi h^2}{4} \checkmark$$

$$\frac{dh}{dt} = \frac{dV}{dt} \cdot \frac{dh}{dV}$$

$$= 2 \cdot \frac{4}{\pi h^2}$$

$$= 8/\pi h^2 \checkmark$$

When $h = 5m$ $\frac{dh}{dt} = \frac{8}{\pi(500)^2}$

Change in height with respect to time $\frac{1}{31250\pi}$ cm

$= 1.0186 \times 10^{-5}$ cm $\checkmark$

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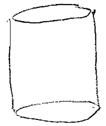
Question 16 **(8 marks)**

A closed cylindrical can is to be made to hold 1 litre of oil.

Find the dimensions that will minimise the cost of the metal to make the can. Assume the metal joins perfectly and no overlaps are required.



$V = 1000cm^3$

$1000 = \pi r^2 h$ $\checkmark$

$h = \frac{1000}{\pi r^2}$ $\checkmark$

Sub Vol form $\checkmark$

finding h in terms of r $\checkmark$

$TSA = 2\pi r^2 + 2\pi r h$ $\checkmark$

$= 2\pi r^2 + 2\pi r \frac{1000}{\pi r^2}$

$= 2\pi r^2 + \frac{2000}{r}$ $\checkmark$

Sub h in TSA form $\checkmark\checkmark$

Diff TSA $\checkmark$

$A' = 4\pi r - \frac{2000}{r^2}$ $\checkmark$

Equate to 0 $\checkmark$

$A' = 0$

$\Rightarrow 4\pi r^3 - 2000 = 0$ $\checkmark$

$r = 5.42cm$ $\checkmark$

Value of r $\checkmark$

$h = \frac{1000}{\pi \cdot (5.42)^2}$ $\checkmark$

Value of h $\checkmark$

$h = 10.84$

$SA = 553.58\ cm^2$

$A'' = 4\pi + \frac{2000}{r^3}$ When $r = 5.42$ $A'' > 0$

$\therefore$ Minimum

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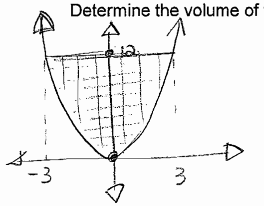
Question 15 **(8 marks)**

a) A "paraboloid" is formed by revolving a parabola, $y = kx^2$, about its axis of symmetry. The paraboloid is bounded by a plane cutting the axis of symmetry perpendicularly at the point (0,12). The intersection of this plane and the paraboloid is a circle of radius 3 units.
Determine the volume of the paraboloid. (5 marks)



$y = kx^2$
$12 = 9k$
$k = 4/3$ ✓

$y = \frac{4}{3}x^2$
$x = \sqrt{\frac{3}{4}y}$ ✓

$V = \int_0^{12} \pi \left(\sqrt{3/4\, y}\right)^2 . dy$ ✓

$V = \int_0^{12} \pi \frac{3}{4} y \cdot dy$

$= \pi \frac{3}{8} y^2 \Big|_0^{12}$ ✓

$= 54\pi\ u^3$ ✓

(b) The population of a particular country (P million people) was changing such that t years after records were first kept

$$\frac{dP}{dt} \approx 5.1 + 0.04t$$

If it is now 20 years since records were first kept use the above rule to determine an approximate value, to the nearest half million, for the increase in population in the next eight years. (3 marks)

$\int_{20}^{28} 5.1 + 0.04t = 5.1t + 0.08t^2 \Big|_{20}^{28}$

$= \left[5.1(28) + (0\ 08 . 28^2)\right] - \left[5.1(20) +\right.$

$= 48.5$ million

8/60

b) Given that $dV/_{dt} = (t-2)^2 + 1$, find

I. The instantaneous rate of change of V with respect to t when t=4 (1 mark)

$\frac{dV}{dt} = (4-2)^2 + 1$

Instantaneous rate of change when $t = 4$ is 5 units

II. The net change in V when t changes from t=1 to t=4 (2 marks)

$V = \frac{(t-2)^3}{3} + t$

$V(1) = 2/3$

$V(4) = 6\,2/3$ ✓✓

Net change is 6 units

$\int_1^4 (t-2)^2 + 1 . dt = \underline{6}$

III. The average rate of change of V in the interval $1 \le t \le 4$ seconds (1 mark)

$\frac{6}{3} = 2$ units per second ✓

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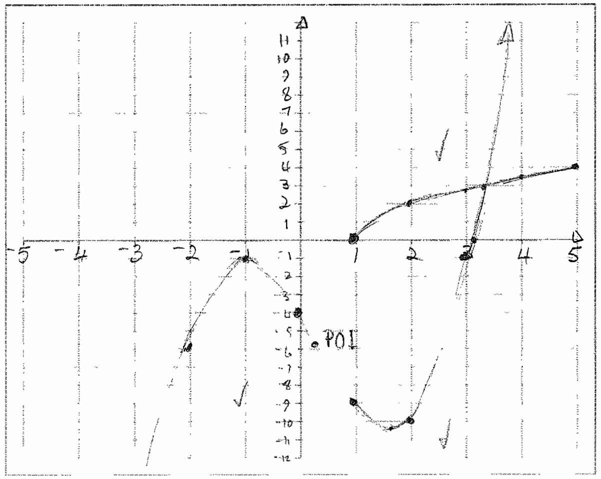
Question 13 **(8 marks)**

On the axes below draw the curves $y = 2\sqrt{x-1}$ and $y = x^3 - x^2 - 5x - 4$.



(3 marks)

(a) Determine any points of inflection. (1 mark)

POI = $(0.\dot{3}, -5.74)$ ✓

(b) Explain why there will only be one turning point. (1 mark)

As x tends to infinity y is increasing in both graphs ✓ (accept reasonable comment

(c) Use calculus techniques to determine where the exact turning points occur. (3 marks)

$y = x^3 - x^2 - 5x - 4$

$\frac{dy}{dx} = 3x^2 - 2x - 5$ ✓ When $3x^2 - 2x - 5 = 0$

$(3x - 5)(x + 1) = 0$ ✓

$\therefore$ Tpts at $x = 5/3$ and $x = -1$ ✓

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Question 14 **(8 marks)**

A function is defined as $y = pxe^{qx}$ where p and q are constants.

(a) Determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (3 marks)

$\frac{dy}{dx} = pqxe^{qx} + p.e^{qx}$ ✓

$pe^{qx}(qx + 1)$

$\frac{d^2y}{dx^2} = pq^2xe^{qx} + 2pqe^{qx}$ ✓✓

$pqe^{qx}(qx + 2)$

(b) Using the results found in (a), determine the values for p and q so that y has a maximum of 1 when $x = \frac{1}{2}$ (5 marks)

$1 = 0.5pe^{0.5q}$ ✓

and $pe^{0.5q} + 0.5pqe^{0.5q} = 0$ ✓✓

Solve simultaneously

$p \approx 5.4366$ $q = -2$ ✓✓

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