PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Specialist TEST 1 Friday 9 February 2018 TIME: 5 mins reading 40 minutes working Classpads **allowed!** 37 marks 7 Questions

Name:________________________________

Teacher: _____________________________

Note: All part questions worth more than 2 marks require working to obtain full marks.

Some useful Formulae

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = \lvert z\rvert = \sqrt{a^2+b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan\theta = \frac{b}{a}, \quad -\pi < \theta \le \pi$
$\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$	$\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = \lvert z\rvert^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{\lvert z\rvert^2}$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$	$\overline{z_1 z_2} = \bar{z}_1\bar{z}_2$
Polar form	
$z = a + bi = r(\cos\theta + i\sin\theta) = r\,cis\,\theta$	$\bar{z} = r\,\text{cis}(-\theta)$
$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$
$\text{cis}(\theta_1 + \theta_2) = \text{cis}\,\theta_1\,\text{cis}\,\theta_2$	$\text{cis}(-\theta) = \frac{1}{\text{cis}\,\theta}$
De Moivres theorem	
$z^n = \lvert z\rvert^n\,\text{cis}(n\theta)$	$(\text{cis}\,\theta)^n = \cos n\theta + i\sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\frac{\theta+2\pi k}{q} + i\sin\frac{\theta+2\pi k}{q}\right)$, for k an integer	

$\cos^2 x + \sin^2 x = 1$	$1 + \tan^2 x = \sec^2 x$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	$\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$
$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$	$\sin 2x = 2\sin x\cos x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$	$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$
$\cos A\cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$	$\sin A\cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$
$\sin A\sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$	$\cos A\sin B = \frac{1}{2}(\sin(A+B) - \sin(A-B))$

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1) (2, 2, 2, 2 & 1 = 9 marks)
If $w = 2 - 2i$ and $z = 9 - 5i$ determine exactly:

a) wz

b) $\frac{w}{z}$

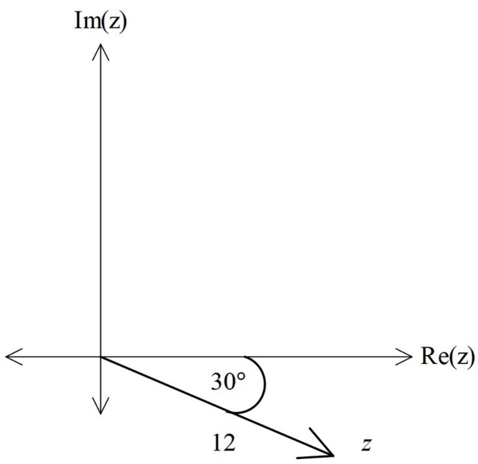
c) $z\overline{w}$

d) $w\overline{z}$

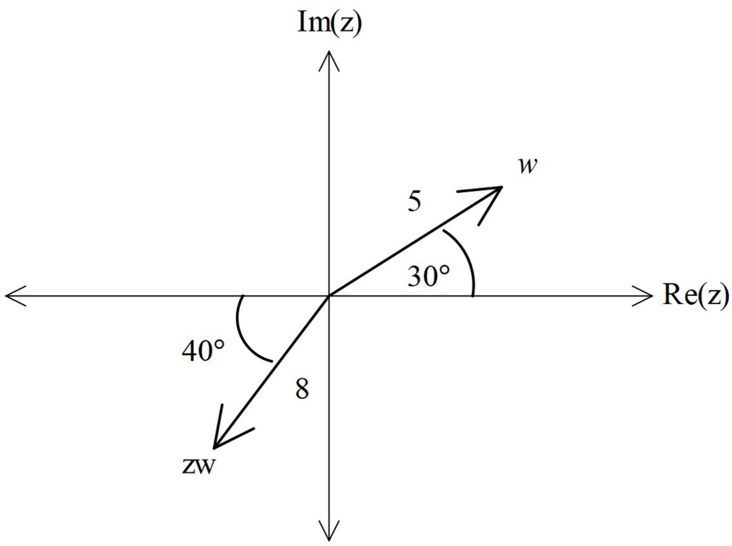
e) What do you notice about (c) and (d)?

Q2 (2 & 2 = 4 marks)
Express each of the following into Cartesian form, $a + bi$

a) $7cis\left(-\frac{2\pi}{3}\right)$

b) 

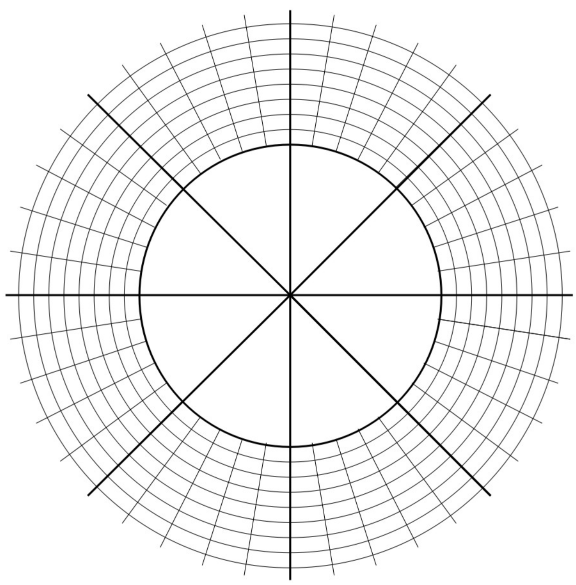
Q4 (3 marks)
Determine z in polar form given that w and zw have been drawn below.



Q5 (5, 3 & 3 =11 marks)

a) Determine all the roots of the equation $z^5 = 1 - i$, expressing them all in polar form with $r \geq 0$ and $-\pi < Arg z \leq \pi$

b) Plot the roots on the diagram below. (Note: each minor angle is $\frac{\pi}{20}$ radians.)



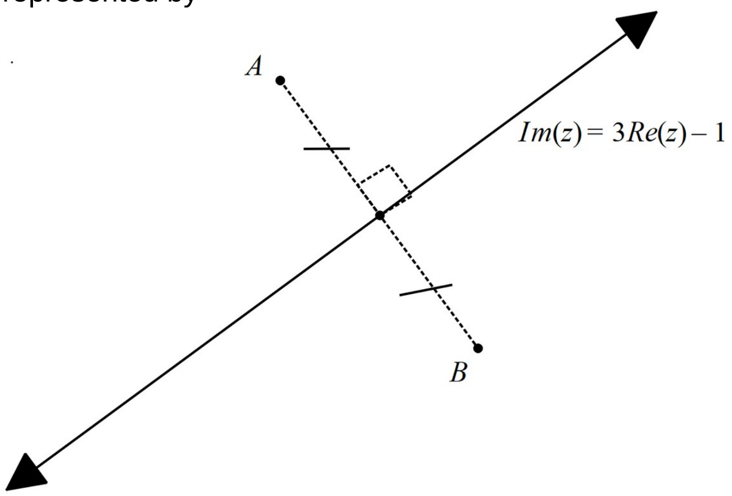
c) The roots form the vertices of a pentagon. Determine the value for the perimeter of the pentagon to two decimal places.

Q6 (5 marks)

Determine, **using de Moivre's theorem**, an expression for $\sin 3\theta$ in terms of $\sin\theta$ only.

{Hint: start with $(\cos\theta + i\sin\theta)^3$ }

Q7 (5 marks)
Consider the points A and B in the complex plane. The perpendicular bisector of the line AB is represented by $\mathrm{Im}(z) = 3\mathrm{Re}(z) - 1$



If point A is $5 + ci$ and point B is $d - 7i$ in the complex plane, determine the values of the constants c and d.