

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Important note to candidates

Special items: nil

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate

Formula sheet

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time: fifty minutes
Reading time before commencing work: five minutes

Time allowed for this section

Your name _____

In words _____



Student Number: in figures _____

Calculator-free

Section One:

UNIT 3

METHODS

MATHEMATICS

SOLUTIONS

Question/Answer booklet

Semester One Examination, 2017

Mercedes College



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	11	11	100	98	65
Total					100

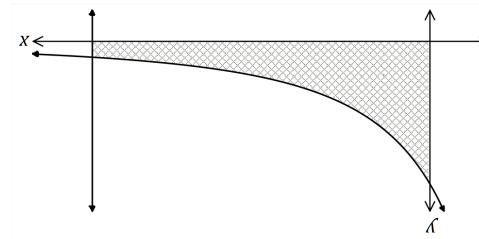
Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

<ul style="list-style-type: none"> ✓ simplifies ✓ substitutes bounds ✓ multiplies ✓ antiderivatives - correct ✓ antiderivatives - correct power ✓ writes integral
Specific behaviours

$\text{?} - 90 \times \frac{15}{2} \text{?} 12 \text{ sq units}$ $\left[\frac{1}{2} \times (2x+5) \right]_0^{15} - 90 \left[\frac{1}{180} \times (2x+5)^2 \right]_0^5$ $A = \int_0^5 180(2x+5)^2 dx$

Solution



Determine the area of the shaded region, enclosed by the $x-\text{?}$ axis, the $y-\text{?}$ axis, the line $x=5$ and the curve.

The graph below shows the curve $y = \frac{180}{2} (2x+5)^2$ and the line $x=5$.

(5 marks)

Question 1

Working time: 50 minutes.

provided.

This section has **eight (8)** questions. Answer all questions. Write your answers in the spaces

35% (52 Marks)

Section One: Calculator-free

Question 2**(8 marks)**

A small body, initially at the origin, moves in a straight line with acceleration $a(t)=6t-10 \text{ ms}^{-2}$, where t is the time in seconds, $t \geq 0$. When $t=5$, it was observed to have a velocity of 31 ms^{-1} .

- (a) Determine an expression for $v(t)$, the velocity of the body. (2 marks)

Solution
$v(t)=3t^2-10t+c$
$31=75-50+c \Rightarrow c=6$
$v(t)=3t^2-10t+6$
Specific behaviours
✓ antidifferentiates
✓ evaluates constant and states

- (b) Determine the acceleration of the body when $v=19$. (3 marks)

Solution
$3t^2-10t+6=19$
$3t^2-10t-13=0$
$(3t-13)(t+1)=0 \Rightarrow t=-1, t=\frac{13}{3}$
$a=6 \times \frac{13}{3}-10=16 \text{ m/s}^2$
Specific behaviours
✓ uses $v=19$ to obtain quadratic equal to zero
✓ solves quadratic for t (+ve only)
✓ determines

- (c) Determine the velocity of the body as it passes through the origin for the last time. (3 marks)

Solution
$x(t)=t^3-5t^2+6t$
$0=t(t-2)(t-3)$
$t=3$
$v(3)=27-30+6=3 \text{ m/s}$
Specific behaviours
✓ antidifferentiates to obtain displacement equation
✓ solves for last

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$f''(1) = 1 + 4 - 1 = 4 \Rightarrow$ Local minimum when $x = 1$	$f''(-3) = 9 - 12 - 1 = -4 \Rightarrow$ Local maximum when $x = -3$
Solution	
Specific behaviours	
\checkmark clearly shows $f''(-3)$ is $-ve$ \checkmark clearly shows $f''(1)$ is $+ve$ \checkmark interprets signs of second derivative as required	

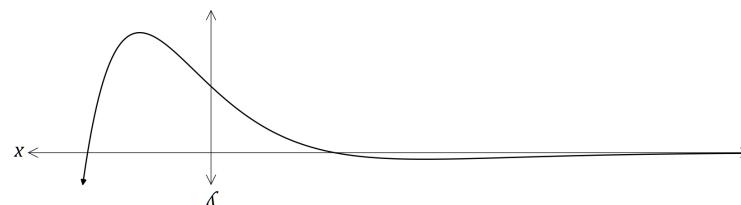
- (c) Given that $f(x) = e^x(x^2 + 4x - 1)$, use the second derivative to justify that one of the stationary points is a local minimum and that the other is a local maximum. (3 marks)

$e^x(x^2 + 2x - 3) = 0$ $(x+3)(x-1) = 0 \Rightarrow x = -3, 1$	\checkmark states $x = ?$ values \checkmark factorises
Solution	
Specific behaviours	
\checkmark determines $x = ?$ coordinates of the stationary points of $f(x)$.	

- (b) Determine the $x = ?$ coordinates of the stationary points of $f(x)$. (2 marks)

$f(x) = e^x(x^2 + 2x - 3)$	\checkmark indicates use of product rule
Solution	
Specific behaviours	

- (a) Show that $f(x) = e^x(x^2 + 2x - 3)$. (1 mark)



The graph of $y = f(x)$ is shown below, where $f(x) = e^x(x^2 - 3)$.

(6 marks)

Question number: _____

Question 4

(8 marks)

(a) Determine $\frac{d}{dx} \left(\frac{1+e^{2x}}{1+\sqrt{x}} \right)$.

(3 marks)

Solution $\frac{d}{dx} \left(\frac{1+e^{2x}}{1+\sqrt{x}} \right) = \frac{2e^{2x}(1+\sqrt{x}) - (1+e^{2x})(\frac{1}{2\sqrt{x}})}{(1+\sqrt{x})^2}$
Specific behaviours <ul style="list-style-type: none"> ✓ obtains $u'v$ ✓ obtains uv' ✓ uses correct form of quotient rule <p>(simplification not required)</p>

(b) Determine $\frac{d}{dx} [2x \sin(3x)]$.

(2 marks)

Solution $\frac{d}{dx} [2x \sin(3x)] = 2 \sin(3x) + 2x \cdot 3 \cdot \cos(3x)$ <p style="color: red;">$\cancel{2} \sin(3x) + 6x \cos(3x)$</p>
Specific behaviours <ul style="list-style-type: none"> ✓ applies product rule ✓ differentiates correctly <p>(simplification not required)</p>

(c) Use your answer from (b) to determine $\int 6x \cos(3x) dx$.

(3 marks)

Solution $\int 6x \cos(3x) dx = \int 2 \sin(3x) + 6x \cos(3x) - 2 \sin(3x) dx$ <p style="color: red;">$\cancel{\int 2 \sin(3x) + 6x \cos(3x) dx} - \int 2 \sin(3x) dx$</p> <p style="color: red;">$\cancel{2} x \sin(3x) + \frac{2}{3} \cos(3x) + c$</p>
Specific behaviours <ul style="list-style-type: none"> ✓ uses linearity of anti-differentiation ✓ integrates using reverse differentiation ✓ obtains expression, including constant

(4 marks)

x	$P(X=x)$	a	$a+b$	b	$2a$
0					
1					
2					
3					

(a) Determine the values of the constants a and b .

$$0(a+1(a+b)+2(b+3(2a))=1.7$$

$$\begin{aligned} 6a+3b &= 1.5 \\ 7a+3b &= 1.7 \end{aligned}$$

$$a=0.2, b=0.1$$

- ✓ equation using sum of probabilities
- ✓ equation using expected value
- ✓ determines a and b

Solution

$$a=0.2, b=0.1$$

$$\begin{aligned} 6a+3b &= 1.5 \\ 7a+3b &= 1.7 \end{aligned}$$

$$0(a+1(a+b)+2(b+3(2a))=1.7$$

(4 marks)

(2 marks)

$$\int_0^2 f(t) dt = \text{Area} = \frac{2}{4+3} \times 2 = 7$$

Solution	Correct estimate
$\int_0^2 f(t) dt = \text{Area} = \frac{2}{4+3} \times 2 = 7$	<ul style="list-style-type: none"> ✓ indicates area calculation ✓ specific behaviour

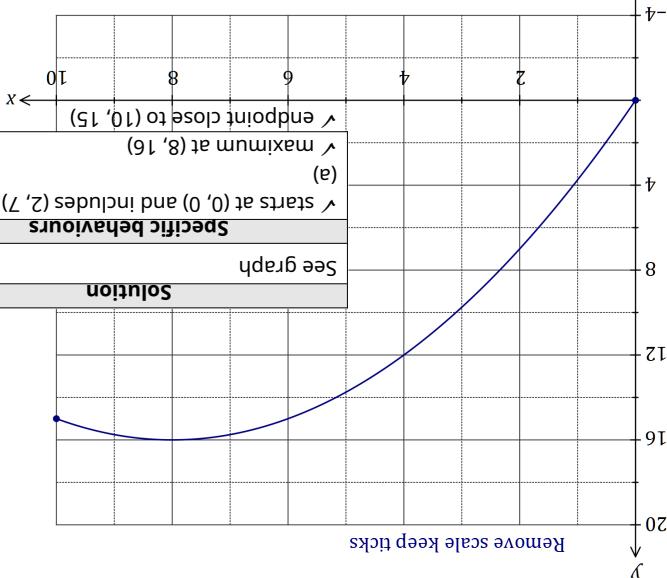
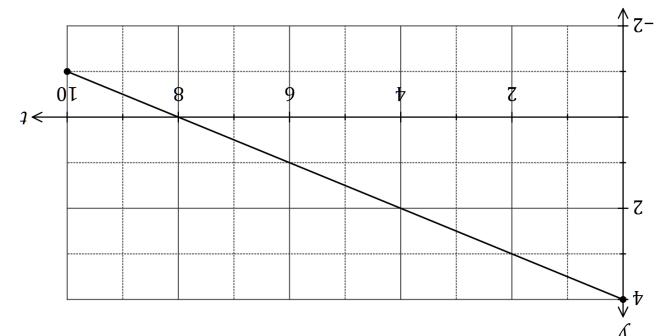
(4 marks)

$$\int_0^x f(t) dt = F(x)$$

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SN064-95-2

End of questions(b) On the axes below, sketch the graph of $y=F(x)$ for $0 \leq x \leq 10$, where $F(x)=\int_0^x f(t) dt$.The graph of $y=f(t)$ is shown below over the interval $0 \leq t \leq 10$.

(6 marks)

(6 marks)

The table below shows the probability distribution for a random variable X .It is known that $E(X)=1.7$ and $\text{Var}(X)=1.41$.

(6 marks)

Question 8

Question 5

Question 6

(7 marks)

- (a) The function f is such that $f(1) = -2$ and $f'(x) = \sqrt{3+x^2}$. Use the increments formula to determine an approximate value for $f(1.05)$. (3 marks)

Solution

$$y = f(x) \Rightarrow \delta y \approx f'(x) \delta x$$

$$x = 1, \delta x = 0.05$$

$$\delta y \approx \sqrt{3+1^2} \times 0.05 \approx 0.1$$

$$f(1.05) \approx -2 + 0.1 \approx -1.9$$

Specific behaviours

- ✓ identifies values of x and δx
- ✓ uses formula to calculate increment

- (b) The function C is such that $C(1) = 10$ and $C'(x) = 3\sqrt{x+3}$.

- (i) Explain why the increments formula would not yield an approximate value for $C(6)$. (1 mark)

Solution

The increment in x from 1 to 6 is not small.

Specific behaviours

- (ii) Determine $C(6)$. (3 marks)

Solution

$$\Delta C = \int_1^6 3\sqrt{x+3} dx \rightarrow \int_1^6 3(x+3)^{\frac{1}{2}} dx \rightarrow \left[2(x+3)^{\frac{3}{2}} \right]_1^6 \rightarrow 54 - 16 = 38$$

$$C(6) = C(1) + \Delta C = 10 + 38 = 48$$

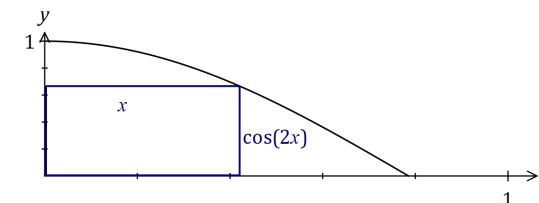
Specific behaviours

- ✓ antidifferentiates
- ✓ evaluates total change
- ✓ correct value

Question 7

(6 marks)

- A rectangle has its base on the x -axis, its lower left corner at $(0, 0)$ and its upper right corner on the curve shown below, $y = \cos 2x$, $0 \leq x \leq \frac{\pi}{4}$.



- (a) Sketch a possible rectangle on the graph above and explain why the perimeter of the rectangle is given by the function $p(x) = 2x + 2\cos 2x$. (2 marks)

Solution

See diagram.
Perimeter is twice base ($2x$) plus twice height ($2\cos 2x$).

Specific behaviours

- ✓ rectangle as required

- (b) Determine the largest perimeter of the rectangle. (4 marks)

Solution

$$p'(x) = 2 - 4\sin 2x \quad p'(x) = 0 \text{ when } \sin 2x = \frac{1}{2}$$

$$x = \frac{\pi}{12}, p\left(\frac{\pi}{12}\right) = \frac{\pi}{6} + 2\cos\frac{\pi}{6} \cdot \frac{\pi}{6} + \sqrt{3}$$

Specific behaviours

- ✓ derivative
- ✓ equates to zero and obtains trig equation
- ✓ solves for x within domain
- ✓ determines P_{MAX}