

Question/Answer Booklet

Semester 1 (Unit 3) Examination, 2020

MATHEMATICS METHODS

Section One: Calculator-free

Student Name/Number: _____

Teacher Name: _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	100	65
					100

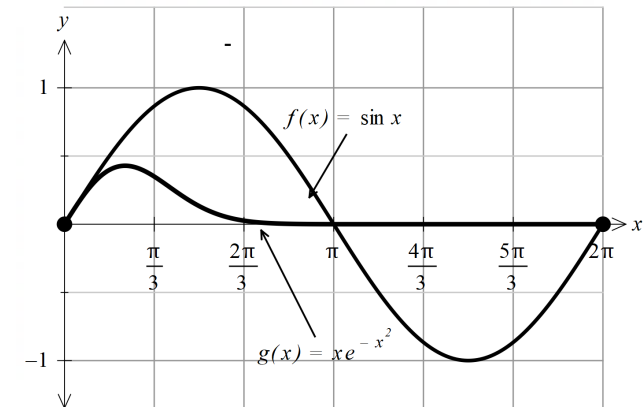
Instructions to candidates

- The rules for the conduct of School exams are detailed in the School/College assessment policy. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Question 7

(5 marks)

The graphs of $f(x) = \sin x$ and $g(x) = x e^{-x^2}$, over the domain $0 \leq x \leq 2\pi$ are drawn below.



- (a) Explain what is meant by the expression given below:

(2 marks)

$$\int_0^{\pi} (\sin x - x e^{-x^2}) dx$$

- (b) Determine $\int (\sin x - x e^{-x^2}) dx$

(3 marks)

Question 6

(5 marks)

(a) The derivative function $\frac{dy}{dx}$ for the function $y = \frac{8x}{(x - 1)^2}$ is of the form, $\frac{dy}{dx} = \frac{cx + d}{ax + b}$

Determine the value of a, b, c and d .

(3 marks)

Question 1

(7 marks)

(a) Determine $f'(x)$ given that $f(x) = \sqrt[3]{3 + x^3}$.

(2 marks)

(b) Determine $\frac{dz}{dt}$ given that $z = t^2 \cos(2t - 1)$.

(2 marks)

(b) The tangent to the curve $y = \frac{8x}{(x - 1)^2}$ at the point B is parallel to the x axis. State the coordinates of B.

(2 marks)

(c) Evaluate $\left(\frac{dy}{dx}\right)_z + 16y^2_z$ given that $y = 5\sin(4x + 3)$.

(3 marks)

Question 2

(10 marks)

Suppose that $f(x) = x^3 - 12x$.

(a) Determine the exact values of all zeros of f .

(2 marks)

(b) Determine the location and nature of all stationary points and points of inflection of f .

(4 marks)

(c) By considering the derivative of $y = \left(\frac{1}{e^{2x} + 1} \right)$ show that the value $A = -\frac{1}{2}$ makes

$$\int \left(\frac{e^x}{e^{2x} + 1} \right)^2 dx = \frac{A}{(e^{2x} + 1)} + C$$

true, where C is an arbitrary constant.

(5 marks)

Question 5

(12 marks)

(a) (i) Evaluate $\int_{2\pi}^0 2\sin(4x) \, dx$

(2 marks)

(ii) Determine $\int \frac{x}{x + \sqrt{x}} \, dx$

(2 marks)

(b) Given that $f(x)$ is continuous everywhere and $\int_6^7 f(x) \, dx = 4$ and $\int_1^4 f(x) \, dx = 1$, determine,

(i) $\int_6^1 f(x) \, dx$

(1 mark)

(ii) $\int_1^4 (2f(x) + 1) \, dx$

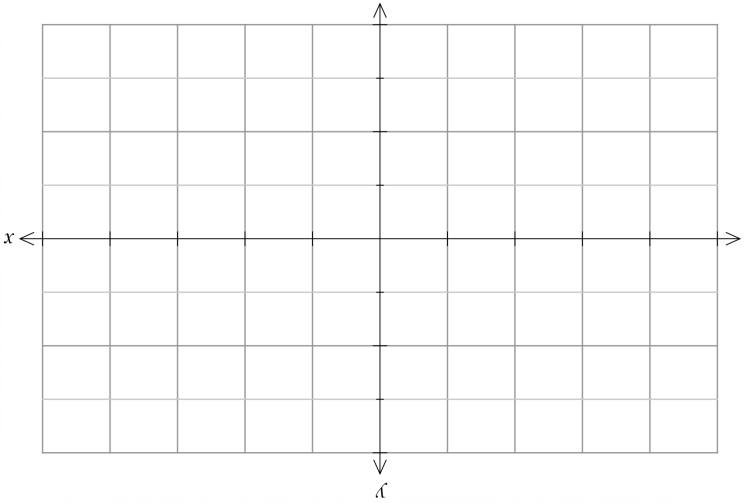
(2 marks)

(c) Determine the maximum value of f for $-4 \leq x \leq 4$.

(1 mark)

(d) Sketch on the axes below the graph of $y = f(x)$ where $-4 \leq x \leq 4$.

(3 marks)



Question 3

(5 marks)

Given that $\frac{dc}{dx} = x(2x^2 + 1)^{\frac{1}{2}}$, determine

(a) the instantaneous rate of change of c with respect to x when $x = 2$. (1 mark)

(b) the change in c when x changes from $x = 0$ to $x = 2$. (4 marks)

Question 4

(6 marks)

A discrete random variable X has the probability function, $P(X = x)$ given by

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5 \\ k(x - 3) & x = 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{1}{13}$. (2 marks)

(b) Determine $P(X > 2)$ (2 marks)

(c) Determine $P(X \leq 5 | X > 2)$ (2 marks)