

Date: 13 Sept 2019  
 Time: 45 minutes  
 Total: 18 + 27 = 45 marks  
 Weight: 8%

Name:

TOPICS: Continuous Random Variables, Normal Distribution, Sampling, Sample Proportions

## SECTION A – NON CALCULATOR

### INSTRUCTIONS:

- Show all necessary working out
- Approved Formula sheet allowed
- Calculators are not allowed
- No Notes allowed

You may assume the following z scores for normal distributions and confidence intervals

- For 68% of scores  $-1 \leq z \leq 1$
- For 95% of scores  $-2 \leq z \leq 2$
- For 99.7% of scores  $-3 \leq z \leq 3$

### Student Reflection

Q1	Q2.5	Q3.7	Q4	Q6	Total
Uniform Distribution	Normal Distribution	Confidence Intervals	Sample proportions	Continuous Random Variables	
4					
12					
16					
5					
8					
45					

What went well:
   
 I did well at...

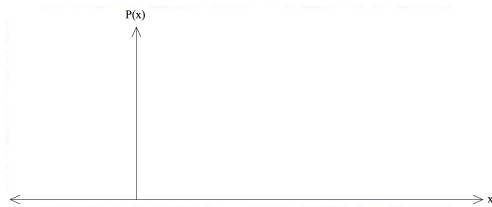
Areas for development:
   
 I need to improve...

1. [4 marks]

Anna arrives at 8.38 am, two minutes early for her maths methods class and knows that it is equally likely for her maths teacher to arrive at class anywhere from 1 minute to 6 minutes later.

Let the continuous random variable  $X$  be the number of minutes taken for Anna's teacher to arrive after 8.38 am.

- a) Draw a graph on the axes below that shows the probability density function of the random variable  $X$ . [2]



- b) What is the probability that Anna's maths teacher arrives after 8.40 am? [1]

- c) What is the probability that Anna's maths teacher arrives before 8.42 am given that he arrives after 8.40 am? [1]

**2. [7 marks]**

The maximum temperatures of Perth days in the month of April can be modelled using a normal distribution with a mean of  $26^{\circ}\text{C}$  and a standard deviation of 3.

Using this model, answer the following.

- a) If the first day of April had a standardized score of  $-1.25$ , what was the maximum temperature on this day? [1]

- b) What is the probability that the maximum temperature of an April day will be between  $20^{\circ}\text{C}$  and  $29^{\circ}\text{C}$ ? [1]

- c) Below what temperature do the lowest 16% of the daily maximums in April lie? [1]

- d) How many April days in the next decade would you expect to have maximum temperatures above  $32^{\circ}\text{C}$ ?  
(Note: there are 30 days in the month of April) [2]

- e) The lowest recorded maximum for an April day is  $16.3^{\circ}\text{C}$ . Is this consistent with the use of the described model? Explain your answer. [2]

**3. [7 marks]**

- a) A random sample of size  $n_1$  was taken and the proportion of people who had cycled in the last week was  $m$ .

Determine a 68% confidence interval for the proportion of the population who had cycled in the last week in terms of  $n_1$  and  $m$ . [2]

- b) A new sample of size  $n_2$  was taken and the proportion of people who had cycled in the last week was again  $m$ . When a 95% confidence interval was determined it was found to be the same as the interval determined in part (a).

(i) Is  $n_2$  larger or smaller than  $n_1$ ? Explain [2]

(ii) What is the relationship between  $n_1$  and  $n_2$ ? [3]

**7. [9 marks]**

In a random sample of 200 Year 12 ATAR students, it was found that 28 of the students received extra tutoring outside of school.

- a) Calculate the sample proportion of these students who received extra tutoring outside of school. [1]

- b) Calculate the 90% confidence interval for the population proportion and interpret your answer. [3]

- c) A second survey of Year 12 ATAR students is planned; however, it is decided that the 90% confidence interval should involve a maximum margin of error of 3%. Determine the sample size required for such a survey. [3]

- (d) If ten surveys were taken and for each a 90% confidence interval for the population proportion was calculated, determine the probability that at most seven of the intervals included the true value of the population proportion. [2]

6. [8 marks]

The time  $X$  minutes for a meal to be delivered by an uber eats driver is modelled using a continuous random variable with probability density function given by

$$f(x) = \begin{cases} k(x - 30)^2 & : 0 < x < 30, \\ 0 & : \text{elsewhere} \end{cases}$$

a) Find the value of  $k$

[2]

b) What is the probability of the driver delivering a meal within 15 minutes?

[2]

c) Calculate the mean delivery time for the driver

[2]

d) Calculate the standard deviation of the delivery time for the driver.

[2]

SECTION B – CALCULATOR ALLOWED

INSTRUCTIONS:

- Show all necessary working out
- Approved Formula sheet allowed
- Scientific and CAS Calculators are allowed
- One A4 page of notes (both sides) is allowed

TOPICS: Continuous Random Variables, Normal Distribution, Sampling, Sample Proportions

Date: 12 Sept 2019  
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Weight: 8%

Name:



Corpus Christi College  
Year 12 Mathematics Methods

2019 Test 5

**4. [5 marks]**

It is known that 12% of the population are left handed.

- a) Describe the distribution of the proportions of left handers in samples of size 500.

[2]

- b) 500 major league baseballers were surveyed and it was found that 95 of them were left handed. Comment on this result.

[3]

**5. [5 marks]**

The horn length of adult black rhinos is normally distributed with 38% of adult black rhinos having a horn length above 75 cm and 12% of adult black rhinos having a horn length below 61 cm.

Above what length are the longest 10% of adult black rhino horns?