Calculator-assumed Solutions

11. (a) The number of phones she is given to repair for the week. ✓

(b) She fixes 23 per day, for 4 days 23 x 4 = 92 phones ✓

(c) $\frac{108}{23} = 4.69565$ days ✓

$0.69565 \times 8 = 5.5652$ hours

5 hours and 34 minutes ✓

(d) $T_1 = 155 \quad T_2 = 128 \quad T_3 = 99 \quad T_4 = 68$

$T_{n+1} = T_n - (25 + 2n) \quad T_0 = 180$ ✓✓ [6]

12. (a) $x^2 + y^2 = 4$ ✓

(b) $\tan\theta = \sqrt{3} \quad \therefore \theta = \frac{\pi}{3}$ radians ✓

radius = 2 units ✓

(c) $OB = \dfrac{2}{\cos\frac{\pi}{3}} = 4$

Therefore area of ΔAOB = $\frac{1}{2}(4)\sqrt{3} = 2\sqrt{3}$ units^2 ✓

Area of sector = $\frac{1}{2}r^2\theta = \frac{1}{2}(2)^2\left(\frac{\pi}{3}\right)$

$= \frac{2\pi}{3}$ units^2 ✓

Area of shaded part = triangle – sector

$= 2\sqrt{3} - \frac{2\pi}{3} = 1.37$ units^2 (3 sig fig) ✓ [6]

13. (a) 5.1 seconds ✓

$v(t) = -9.8t + 25$

(b) $v(0) = 25$ ✓✓

(c) Maximum height is 31.89 m when $t = 2.55$ sec

$$\frac{31.89 \times 2}{2.55 \times 2} = 12.5 \text{ m/s}$$ ✓✓ [5]

14. (a) $f(x) = (2 + x)^2 = 4 + 4x + x^2$ ✓

$$\lim_{h \to 0} \frac{(2 + x + h)^2 - (2 + x)^2}{h} = 2x + 4$$ ✓

(b) (i) -7.5 ✓

(ii) 18 ✓

(c) $p'(x) = 3x^2 - 3a$ ✓

$0 = 3(\sqrt{2})^2 - 3a$

$a = 2$ ✓

$-\sqrt{2} = (\sqrt{2})^3 - 3(2)(\sqrt{2}) + b$

$b = 3\sqrt{2}$ ✓ [7]

15. (a) $3x^2 - \dfrac{4}{x^2} - 11 = 0$

$x = -2$ or $x = 2$ ✓✓

(b)

		-2		2	
y'	+	0	–	0	+
y	↑	-	↓	-	↑

✓✓

$x = -2$ Maximum

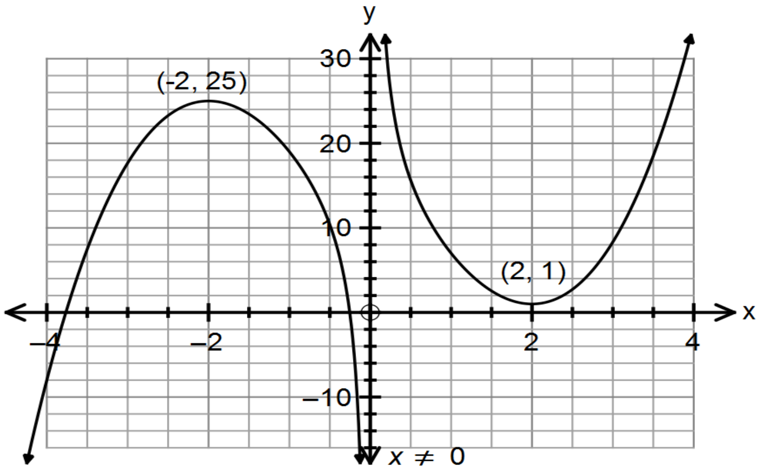
$x = 2$ Minimum ✓

(c) $y = x^3 - 11x + \dfrac{4}{x} + c$ ✓

$c = 13$

$y = x^3 - 11x + \dfrac{4}{x} + 13$ ✓

(d)



✓✓✓ [10]

16. (a) (i) $T_{100} = 23 + (99)(9)$ ✓
$= 914$ ✓

(ii) $357980 = \frac{n}{2}(23 + 2534)$ ✓
$n = 280$ ✓

(b) (i) $T_{n+1} = \frac{1}{4}T_n \quad T_1 = 12$ ✓✓

(ii) $S_\infty = \dfrac{12}{1 - \frac{1}{4}} = 16$ ✓✓ [8]

17. (a) 92°C (initial temp of tea) ✓
22°C (room temp) ✓

(b) After 4.12 mins and before 7.45 mins ✓✓
$4.12 \leq t \leq 7.45$

(c) Horizontal asymptote $y = 22$ ✓
The tea will cool at a decreasing rate as it approaches room temperature which is 22°. ✓ [6]

18. (a) $y = \dfrac{2}{x-3} + 2$ ✓✓

(b) $y = -3\sqrt{x+4}$ ✓✓

(c) $y = \left(\dfrac{1}{2}\right)^x - 4$ ✓✓ [6]

19. (a) 0.2 ✓
(b) 0.5 ✓
(c) 0.3 ✓
(d) Pr(X ∩ Y) = Pr(X) x Pr(Y) if independent
Pr(X ∪ Y) = Pr(X) + P(Y) - Pr(X ∩ Y) ✓
Pr(X) + Pr(Y) - Pr(X ∪ Y) = Pr(X) x Pr(Y) ✓
Let Pr(Y) = k
0.5+ k - 0.8 = 0.5k
k = 0.6 = Pr (Y) ✓ [6]

20. (a) 1, 3, 7, 15,…
$T_{n+1} = T_n + 2^n \quad T_1 = 1$ ✓✓
(b) 20 sets ✓ [3]

21. (a) (i) $y = 16 - 6 - w$
$y = 10 - w$ ✓
(ii) $y^2 = 36 + w^2 - 12w \cos Y$ ✓
(iii) $(10 - w)^2 = 36 + w^2 - 12w \cos Y$ ✓
$64 - 20w = -12w \cos Y$ ✓
$\therefore \cos Y = \dfrac{5w - 16}{3w}$

(b) (i) $A = \dfrac{6w \sin Y}{2} = 3w \sin Y$ ✓
$A^2 = 9w^2 \sin^2 Y$ ✓
(ii) $9w^2 \sin^2 Y = 9w^2(1 - \cos^2 Y)$ ✓
$A^2 = 9w^2\left(1 - \left(\dfrac{5w - 16}{3w}\right)^2\right)$ ✓
$A^2 = -16w^2 + 160w - 256$ ✓

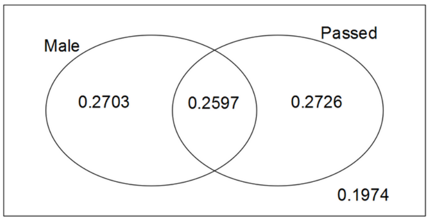
(c) (i) $A = \sqrt{-16w^2 + 160w - 256}$
$A' = 0$ when $w = 5$ ✓
Maximum area = 12 units2 ✓
(ii) $y = 10 - w = 5$
The triangle is isosceles. ✓ [12]

22. $y' = 3x^2 - 12x + k$ ✓
$b^2 - 4ac = 0$ for one solution
$144 - 4(3)k = 0$ ✓
$k = 12$ ✓ [3]

23. (a) $W = W_0\,(1.085)^t$
$R = R_0\,(0.95)^t$
$10W_0 = R_0$
$W_0\,(1.085)^t = 10 \times W_0\,(0.95)^t$ ✓
$(1.085)^t = 10(0.95)^t$
$t = 17.329$ years ✓
After 18 years there will be more wallabies. ✓
(b) $W_{n+1} = 1.085\,W_n \quad W_0 = 655$ ✓
$W_5 = 985$ ✓ [5]

24. (a) $r = -\dfrac{3}{2}$ ✓
$$\frac{-2\left(1 - \left(-\frac{3}{2}\right)^n\right)}{1 - \left(-\frac{3}{2}\right)} = 30$$ ✓
$n = 9.003$
$\therefore$ 10 terms ✓
(b) $T_5 = a + 4d$ and $T_7 = a + 6d$
$\therefore 2a + 10d = 38$ (eq 1) ✓
$S_{15} = 375 = \dfrac{15}{2}(2a + 14d)$ (eq 2) ✓
$a = 4 \quad d = 3$ ✓
$S_{30} = 15(2(4) + 29(3)) = 1425$
Sum of next 15 terms = $S_{30} - S_{15} = 1050$ ✓ [7]

25. (a)



✓✓✓

(b) (i) 0.5323 ✓

(ii) 0.2726 ✓

(iii) 0.51 ✓

(iv) 0.8026 ✓ [7]

26. The particle's initial displacement is 5 m to the right of the origin. ✓

$v = 3t^2 - 12t$ $\therefore$ Initial velocity = 0 ✓ [2]