



**ALL SAINTS'**  
**COLLEGE**

Mathematics  
Specialist

Test 4 2017

# Integration Techniques & Applications of Integral Calculus

NAME: \_\_\_\_\_  
Mrs Da Cruz

TEACHER:

Resource Free Section

33 marks  
35 minutes

Question 1

[3 & 3 = 6 marks]

(a) Express  $\frac{2}{x^2 - 1}$  as partial fractions.

(b) Hence determine  $\int_2^8 \frac{2}{x^2 - 1} dx$ . Give your answer in the form  $\ln \frac{p}{q}$ .

**Question 2****[3, 2 & 2 = 7 marks]**

Determine the following indefinite integrals:

(a)  $\int 1 - \cos^2(5x) dx$

-

(b)  $\int \frac{1}{2}(2x-4)(x^2-4x+1)^6 dx$

(c)  $\int \sin^3(2x)\cos(2x) dx$

**Question 3**

**[5 marks]**

Use the substitution  $u=4+\sqrt{x}$  to evaluate  $\int \sqrt{4+\sqrt{x}} dx$ .

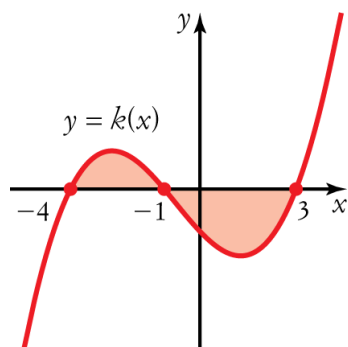
Do not factorize or simplify your answer.



Question 4

[1 mark]

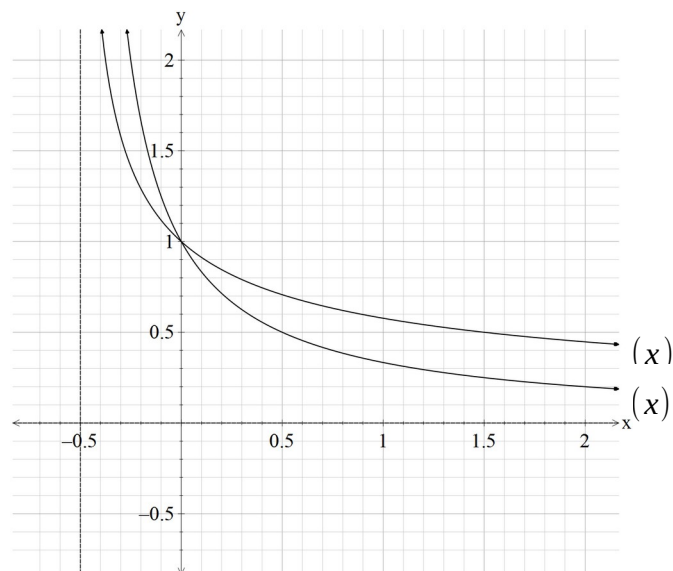
The area under the curve  $y = k(x)$  can be described by:



- A**  $\int_{-4}^3 k(x) dx$
- B**  $\int_{-4}^{-1} k(x) dx + \int_{-1}^3 k(x) dx$
- C**  $\left| \int_{-1}^3 k(x) dx \right| + \int_{-4}^{-1} k(x) dx$
- D**  $\left| \int_{-1}^3 k(x) dx \right|$
- E**  $1 - \int_{-1}^3 k(x) dx$

Question 5

[3 & 4 = 7 marks]



(a) Find the area under the curve, in square units, for the function  $f(x) = \frac{1}{2x+1}$  from  $x=0$  to  $x=1$ .

(b) Find the area enclosed by the curves, in square units, of the graphs  $f(x) = \frac{1}{2x+1}$ ,  $g(x) = \frac{1}{\sqrt{2x+1}}$  and the line  $x=1$ .

### Question 6

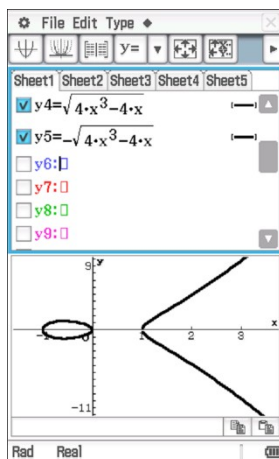
[2 marks]

Find  $\frac{d}{dx}(x^2 e^x)$  and use your answer to evaluate  $\int 4x e^x (x+2) dx$ .

### Question 7

[5 marks]

The region bounded by the lines  $x=k$  and  $x=1$  and the curve  $y^2=4x^3-4x$  is rotated about the  $x$ -axis  $180^\circ$ . The volume formed is  $9\pi$ . Determine the value of  $k$  where  $k$  is a positive integer.







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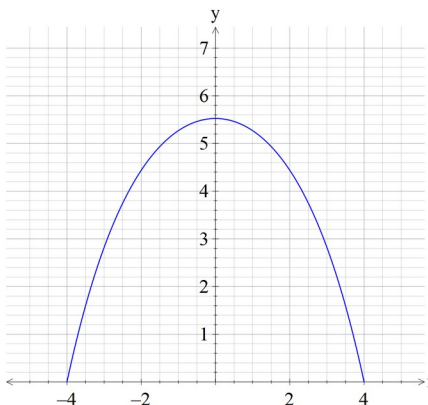
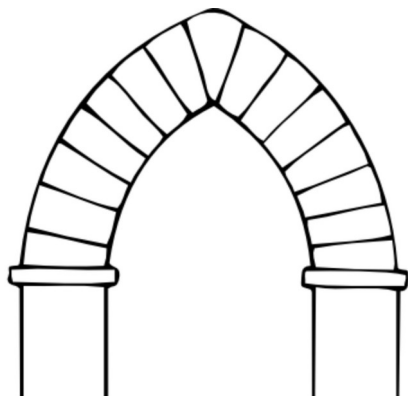
TEACHER:

Resource Rich Section

11 marks  
15 minutes

Question 8

[1, 1 & 2 = 4 marks]



The upper component of an archway is designed to bear the load of the wall above and around it. For this, the best shape is a catenary. A catenary is the name given to the curve formed by two simple exponential terms added together. The equation of the upper arch is  $f(x) = -e^{\frac{x}{2}} - e^{-\frac{x}{2}} + c$ . The x-intercepts of the catenary are  $(-4, 0)$  and  $(4, 0)$ .

**a** Use this information to determine the exact value of  $c$ .

Paint has to be applied to the area under the catenary curve.

**b** State a definite integral that will find the area of paint required.

**c** Calculate the exact area to be painted, giving your answer with positive indices.

**Question 9****[2 marks]**

Use a suitable definite integral to find the **exact** volume, in cubic units, that is formed by rotating about the  $x$ -axis the following curves between the limits shown.

$$y = x^3, \text{ from } x = 1 \text{ and } x = 3.$$

**Question 10****[3 marks]**

Use a suitable definite integral to find the exact volume, in cubic units, that is formed by rotating about the  $y$ -axis the following curves between the limits shown.

$$y = \frac{1}{5} \log_e(2x - 1), \text{ from } y = 0 \text{ and } y = 1.$$

Question 10

[2 marks]

Use a suitable definite integral to find the exact volume, in cubic units, that is formed by rotating the shaded area about the  $y$ -axis.

