



Mathematics: Specialist Formula sheet Units 3C and 3D

Vectors

$$|(a, b, c)| = \sqrt{a^2 + b^2 + c^2}$$

$$|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Vector equation of a line in space:

one point and the slope: $\mathbf{r} = \mathbf{r}_1 + \lambda \mathbf{l}$

two points: $\mathbf{r} = \mathbf{r}_1 + \lambda (\mathbf{r}_2 - \mathbf{r}_1)$

Cartesian equations of a line in space

$$\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$$

$$x = a + \lambda p \dots \dots \dots (1)$$

Parametric form of vector equation of a line in space

$$y = b + \lambda q \dots \dots \dots (2)$$

$$z = c + \lambda r \dots \dots \dots (3)$$

Equation of a plane $\mathbf{r} \cdot \mathbf{n} = c$

$$(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$$

Trigonometry

In any triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

In a circle of radius r , for an arc subtending angle θ (radians) at the centre:

Length of arc = $r\theta$ Area of sector = $\frac{1}{2} r^2 \theta$ Area of segment = $\frac{1}{2} r^2 (\theta - \sin \theta)$

$$\sin (\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos (\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan (\theta \pm \phi) = \frac{\tan \theta \pm \tan \phi}{1 \mp \tan \theta \tan \phi}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0,$$

Simple Harmonic Motion: If $\frac{d^2 x}{dt^2} = -k^2 x$ then $x = a \cos (kt + \alpha) = a \sin (kt + \beta)$

Exponentials and logarithms

If $\frac{dP}{dt} = kP$, then $P = ae^{kt}$

Functions

Differentiation

If $f(x) = \sin x$, then $f'(x) = \cos x$

If $f(x) = \cos x$, then $f'(x) = -\sin x$

If $f(x) = \tan x$, then $f'(x) = \sec^2 x = \frac{1}{\cos^2 x}$

	Function notation		Leibniz Notation	
	y	y'	y	y'
Product rule	$f(x) g(x)$	$f'(x) g(x) + f(x) g'(x)$	uv	$\frac{du}{dx} v + u \frac{dv}{dx}$
Quotient rule	$\frac{f(x)}{g(x)}$	$\frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$	$\frac{u}{v}$	$\frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2}$
Chain rule	$f(g(x))$	$f'(g(x)) g'(x)$	$y = f(u)$ and $u = g(x)$	$\frac{dy}{du} \times \frac{du}{dx}$

Integration

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

Fundamental Theorem of Calculus: $\frac{d}{dx} \int_a^x f(t) dt = f(x)$ and $\int_a^b f'(x) dx = f(b) - f(a)$

Complex numbers

For $z = a + ib$, where $i^2 = -1$

Argument: $\arg z = \theta$, where $\tan \theta = \frac{b}{a}$ and $-\pi < \theta < \pi$

Modulus: $\text{mod } z = |z| = |a + ib| = \sqrt{a^2 + b^2} = r$

$$\arg z_1 z_2 = \arg z_1 + \arg z_2$$

Product: $|z_1 z_2| = |z_1| |z_2|$

$$\arg \frac{z_1}{z_2} = \arg z_1 - \arg z_2$$

Polar form: $z = r \text{ cis } \theta$, where $r = |z|$ and $\theta = \arg z$

$$\text{cis } \theta = \cos \theta + i \sin \theta$$

$$\text{cis } \theta \text{ cis } \phi = \text{cis } (\theta + \phi)$$

$$\text{cis } (-\theta) = (\text{cis } \theta)^{-1}$$

$$\text{cis } 0 = 1$$

$$z_1 z_2 = r_1 r_2 \text{ cis } (\theta + \phi)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{ cis } (\theta - \phi)$$

For complex conjugates $z = a + ib$ and $\bar{z} = a - ib$,

$$\bar{\bar{z}} = z \text{ cis } (-\theta)$$

$$z \bar{z} = |z|^2$$

$$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$$

$$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$$

$$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{|z|^2}$$

Matrices

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $|A| = ad - bc$ and $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Transformations

Dilation $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

Shear $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$

Rotation $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

Reflection $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

Mathematical reasoning

De Moivre's theorem: $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$

$$z^n = |z|^n \operatorname{cis}(n\theta) \qquad z^{\frac{1}{q}} = r^{\frac{1}{q}} \left(\cos \frac{\theta + 2\pi k}{q} + i \sin \frac{\theta + 2\pi k}{q} \right) \text{ for } k = 0, 1, 2, \dots$$

Measurement

Trapezium: Area = $\frac{1}{2}(a + b) \times \text{height}$, where a and b are the lengths of the parallel sides

Prism: Volume = Area of base \times height

Cylinder: Total surface area = $2\pi r h + 2\pi r^2$

$$\text{Volume} = \pi r^2 \times h$$

Pyramid: Volume = $\frac{1}{3} \times \text{area of base} \times \text{height}$

Cone: Total surface area = $\pi r s + \pi r^2$, s is the slant height

$$\text{Volume} = \frac{1}{3} \times \pi r^2 \times h$$

Sphere: Total surface area = $4\pi r^2$

$$\text{Volume} = \frac{4}{3} \pi r^3$$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.