education equals

Calculator Free
Anti-Differentiation Techniques

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 3, 3, 3, 3, 3 = 21 marks] **CF**

Anti-differentiate each of the following, showing all working. Leave all answers with positive indices.

(a) $\int \frac{4}{t^2}\, dt$

(b) $\int -\sin 2u \, du$

(c) $\int (4x-5)^3 \, dx$

(d) $\int \left(e^{-5x} + 2\pi x - \sqrt{x}\right) dx$

(e) $\displaystyle\int \frac{4t^6 - 6t^2}{8t^2}\ dt$

(f) $\displaystyle\int (x^2 - 2)^3\ dx$

(g) $\displaystyle\int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} \right) dx$

(h) $\displaystyle\int (e^{-2x} + 1)(e^{3x} - 2)\,dx$

Question Two: [3, 3, 3 = 9 marks] CF

Calculate the following integrals, showing all working.

(a) $\int_1^2 \left(x^2 - 1\right) dx$

(b) $-2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 3x \, dx$

(c) $\int_1^3 \left(-e^{4x} + 2\right) dx$

Question Three: [3 marks] CF

The derivative of $f(x)$ is given by $f'(x) = 2e^{2x} + 3x^2$. Given that $f(1) = 4 + e^2$, find an expression for $f(x)$.

Question Four: [6 marks] CF

The gradient function of $f(x)$ is given by $f'(x) = ax^2 + b$. Determine the values of a and b if $f'(-2) = 28,\ f(0) = 1$ and $f(1) = 7$.

Question Five: [1, 2, 3 = 6 marks] CF

Given that $\int_{-1}^{2} f(x)\,dx = 4$ and $\int_{-1}^{7} f(x)\,dx = 10$, determine:

(a) $2\int_{-1}^{7} f(x)\,dx$

(b) $\int_{7}^{2} f(x)\,dx$

(c)
(i) $\int_{-1}^{2} (f(x) + x)\,dx$

SOLUTIONS
Calculator Free
Anti-Differentiation Techniques

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 3, 3, 3, 3, 3 = 21 marks] **CF**

Anti-differentiate each of the following, showing all working. Leave all answers with positive indices.

(a) $\int \frac{4}{t^2}\, dt$

$\int 4t^{-2} dt$

$= \frac{4t^{-1}}{-1} + c$ ✓

$= \frac{-4}{t} + c$ ✓

(b) $\int -\sin 2u \; du$

✓

$= \frac{\cos 2u}{2} + c$ ✓

(c) $\int (4x-5)^3 \; dx$

✓

$= \frac{(4x-5)^4}{4 \times 4} + c$

$= \frac{(4x-5)^4}{16} + c$ ✓

$$\int (e^{-5x} + 2\pi x - \sqrt{x})\, dx$$

(d)

$$= \frac{e^{-5x}}{-5} + \frac{2\pi x^2}{2} - \frac{2x^{\frac{3}{2}}}{3} + c$$

$$= \frac{-1}{5e^{5x}} + \frac{2\pi x^2}{2} - \frac{2x^{\frac{3}{2}}}{3} + c$$

(e) $\int \frac{4t^6 - 6t^2}{8t^2}\ dt$

$$= \int \frac{4t^6}{8t^2} - \frac{6t^2}{8t^2} dt$$

$$= \int \frac{t^4}{2} - \frac{3}{4}\ dt$$

$$= \frac{t^5}{10} - \frac{3t}{4} + c$$

(f) $\int (x^2 - 2)^3\ dx$

$$= \int \left(x^6 + 3(x^2)^2(-2) + 3(x^2)(-2)^2 + (-2)^3\right)dx$$

$$= \int \left(x^6 - 6x^4 + 12x^2 - 8\right)dx$$

$$= \frac{x^7}{7} - \frac{6x^5}{5} + \frac{12x^3}{3} - 8x + c$$

(g) $\int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} \right) dx$

$$= 3\sin\left(\frac{x}{3}\right) + \frac{3(6x)^{\frac{4}{3}}}{8} + c$$

(h) $\int (e^{-2x} + 1)(e^{3x} - 2)dx$

$$= \int \left(e^{x} - 2e^{-2x} + e^{3x} - 2\right)dx$$

$$= e^{x} + \frac{1}{e^{2x}} + \frac{e^{3x}}{3} - 2x + c$$

Question Two: [3, 3, 3 = 9 marks] CF

Calculate the following integrals, showing all working.

(a) $$\int_1^2 (x^2 - 1)\,dx$$

$$= \left[\frac{x^3}{3} - x\right]_{-1}^{2} \checkmark$$

$$= \left(\frac{8}{3} - 2\right) - \left(\frac{-1}{3} + 1\right) \checkmark$$

$$= \frac{9}{3} - 3$$

$$= 3 - 3$$

$$= 0 \checkmark$$

(b) $$-2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 3x\,dx$$

$$= -2\left[\frac{-\cos 3x}{3}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \checkmark$$

$$= -2\left[\frac{-\cos \pi}{3} - \frac{-\cos\frac{\pi}{2}}{3}\right] \checkmark$$

$$= -2\left(\frac{1}{3} + 0\right)$$

$$= \frac{-2}{3} \checkmark$$

(c) $$\int_1^3 \left(-e^{4x} + 2\right)dx$$

$$=\left[\frac{-e^{4x}}{4}+2x\right]_{-1}^{3}$$

$$=\left(\frac{-e^{12}}{4}+6\right)-\left(\frac{-e^{-4}}{4}-2\right)$$

$$=\frac{-e^{12}+e^{4}}{4}+8$$

Question Three: [3 marks] CF

The derivative of $f(x)$ is given by $f'(x) = 2e^{2x} + 3x^2$. Given that $f(1) = 4 + e^2$, find an expression for $f(x)$.

$f(x) = \int 2e^{2x} + 3x^2 \; dx$

$f(x) = e^{2x} + x^3 + c$ ✓

$4 + e^2 = e^2 + 1 + c$ ✓

$c = 3$

$f(x) = e^{2x} + x^3 + 3$ ✓

Question Four: [6 marks] CF

The gradient function of $f(x)$ is given by $f'(x) = ax^2 + b$. Determine the values of a and b if $f'(-2) = 28,\ f(0) = 1$ ✓ and $f(1) = 7$.

$28 = 4a + b$

$f(x) = \frac{ax^3}{3} + bx + c$ ✓

$1 = c$ ✓

$7 = \frac{a}{3} + b + 1$ ✓

$6 = \frac{a}{3} + b$

$28 = 4a + b$

$22 = \frac{11}{3}a$

$\frac{66}{11} = a$

$6 = a$ ✓

$28 = 24 + b$

$b = 4$ ✓

Question Five: [1, 2, 3 = 6 marks] CF

Given that $\int_{-1}^{2} f(x)\,dx = 4$ and $\int_{-1}^{7} f(x)\,dx = 10$, determine:

(a) $2\int_{-1}^{7} f(x)\,dx$

$=2\times 10$

$=20$ ✓

(b) $\int_{7}^{2} f(x)\,dx$

$=\int_{-1}^{7} f(x)dx - \int_{-1}^{2} f(x)dx$ ✓

$=10-4$

$=6$

$\therefore -6$ ✓

(c) $\int_{-1}^{2}\left(f(x)+x\right)dx$

$$= \int_{-1}^{2} f(x)dx + \int_{-1}^{2} x\,dx$$

$$=4+\left[\frac{x^2}{2}\right]_{-1}^{2}$$

$$=4+\left(\frac{4}{2}-\frac{1}{2}\right)$$

$$=4+\frac{3}{2}$$

$$=5\frac{1}{2}$$