



Semester Two Examination, 2020

Question/Answer booklet

SHENTON
COLLEGE

MATHEMATICS
METHODS
UNITS 3 & 4

Section One:
Calculator-free

Student Name: _____

Teacher: Ai Friday White

Time allowed for this section

Reading time before commencing work: five (5) minutes
Working time: fifty (50) minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your responsibility** to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisors before reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	36%
Section Two: Calculator-assumed	12	12	100	93	64%
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your response to the specific questions asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. Do **not** place the Formula Sheet inside your Question/Answer booklet. It will be collected separately.

Markers Use Only		
Question	Max.	Mark
1	7	
2	6	
3	5	
4	7	
5	7	
6	7	
7	7	
8	6	
Section One Total	52	
Section One %		
Section Two Total	92	
Section Two %		
Overall Deductions	Units	
	Rounding	
	Notation	
Total	144	
Overall %		

Section One: Calculator-free**(52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1**(7 marks)**

The function f is defined by $f(x) = \frac{x^2 - 5}{3 - x}$, $x \neq 3$.

The second derivative of f is $f''(x) = 8(3 - x)^{-3}$.

Determine the coordinates and nature of all stationary points of the graph of $y = f(x)$.

Question 2**(6 marks)**

The continuous random variable X takes values in the interval 3 to 8 and has cumulative distribution function $F(x)$ where

$$F(x) = P(X \leq x) = \begin{cases} 0 & x < 3 \\ \frac{x-3}{5} & 3 \leq x \leq 8 \\ 1 & x > 8. \end{cases}$$

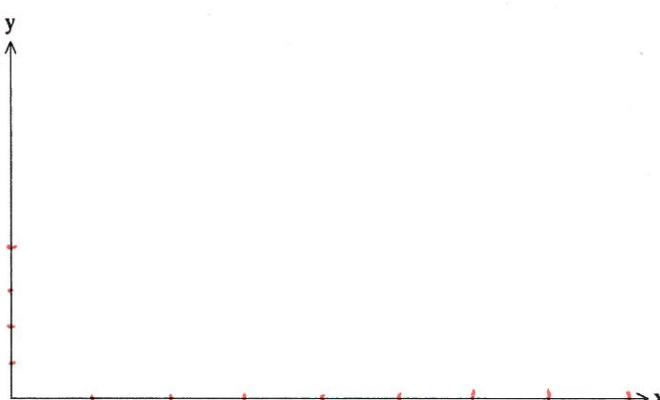
(a) Determine

(i) $P(X \leq 4.5)$. (1 mark)

(ii) the value of k , if $P(X > k) = 0.75$. (2 marks)

(b) Determine $f(x)$, the probability density function of X , and use the axes below to sketch the graph of $y = f(x)$.

(3 marks)



Question 3**(5 marks)**

The rate of change of pressure in an air tank is given by $P'(t) = -2e^{-\frac{t}{25}}$, where t is the time in minutes since it began emptying from an initial pressure of 65 psi. (*psi is a unit of pressure expressed in pounds of force per square inch of area*).

(a) Determine an expression for the pressure P in the tank at any time t , $t \geq 0$. (2 marks)

(b) Determine

(i) the exact time taken for the pressure in the tank to fall to 25 psi. (2 marks)

(ii) the long term behaviour of the pressure in the tank as $t \rightarrow \infty$ for $t \geq 0$. (1 mark)

Question 4

(7 marks)

- (a) Determine an expression for $f'(x)$ for each of the following functions.
DO NOT SIMPLIFY YOUR ANSWERS.

(i) $f(x) = \ln(1 - \cos 3x)$. (2 marks)

(ii) $f(x) = e^{5x}(5 - 2x)^3$. (3 marks)

- (b) For the positive number x , let $A(x) = \int_0^x (8 - 2t^2) dt$.

Determine the value(s) of x for which $\frac{dA}{dx} = 0$. (2 marks)

Question 5

(7 marks)

- (a) Simplify $3 \log 5 + \log 4 - \log \frac{1}{2}$. (2 marks)
- (b) Given that $\log_a x = 0.8$, determine the value of $\log_a(x^2\sqrt{x})$. (2 marks)
- (c) Determine the solution to the equation $4^{2x} = 3^{2-x}$ in the form $x = \frac{\log a}{\log b}$. (3 marks)

Question 6

(7 marks)

The discrete random variable X is defined by

$$P(X = x) = \begin{cases} \frac{2x + k}{3} & x = 0, 1 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Determine the value of the constant k .

(2 marks)

- (b) Determine

(i) $P(X = 0)$.

(1 mark)

(ii) $E(3X - 1)$.

(2 marks)

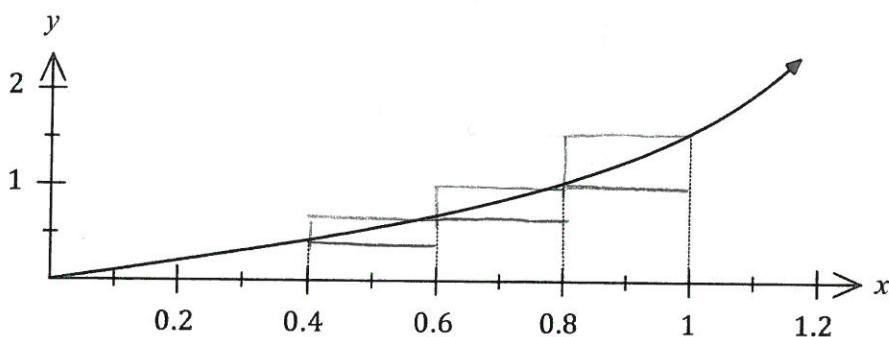
(iii) $\text{Var}(3X - 1)$.

(2 marks)

Question 7

(7 marks)

The graph and a table of values for $y = f(x)$ is shown below, where $f(x) = \tan x$.



x	y
0.2	0.2
0.4	0.42
0.6	0.68
0.8	1.03
1	1.56
1.2	2.57

$$\text{Let } I = \int_{0.4}^1 \tan x \, dx.$$

- (a) By using the information shown and considering sums of the form $\sum_i f(x_i) \delta x_i$, explain why $I > 0.426$. (3 marks)
- (b) In a similar manner to (a), determine the best estimate for the value of the constant U , where $I < U$. (2 marks)
- (c) Use your previous answers to determine a numerical estimate for I and explain whether your estimate is smaller or larger than the exact value of I . (2 marks)

Question 8**(6 marks)**

The acceleration at time t seconds of a small body travelling in a straight line is given by

$$a(t) = \frac{-12}{\sqrt{4t+5}} \text{ cm/s}^2, \quad t \geq 0.$$

When $t = 1$ the body was at the origin and 4 seconds later its displacement was 2 cm.

Determine the velocity of the body when $t = 11$.

Supplementary page

Supplementary page

Question number: _____



SHENTON
COLLEGE

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS
METHODS
UNITS 3 & 4

Section Two:
Calculator-assumed

Student Name: _____

Teacher: Ai Friday White

Time allowed for this section

Reading time before commencing work: ten (10) minutes

Working time: one hundred (100) minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templated, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

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Markers Use Only		
Question	Max.	Mark
9	6	
10	7	
11	7	
12	8	
13	8	
14	6	
15	10	
16	10	
17	8	
18	8	
19	8	
20	6	
Section Two Total	92	
Section Two %		

Section Two: Calculator-assumed

(93 Marks)

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(6 marks)

- (a) Function f is defined by $f(x) = 3 \log_6(x + 6) - 2$ over its natural domain. Determine

- (i) the value of the y -intercept of the graph of $y = f(x)$. (1 mark)

- (ii) the equation of the asymptote of the graph of $y = f(x)$. (1 mark)

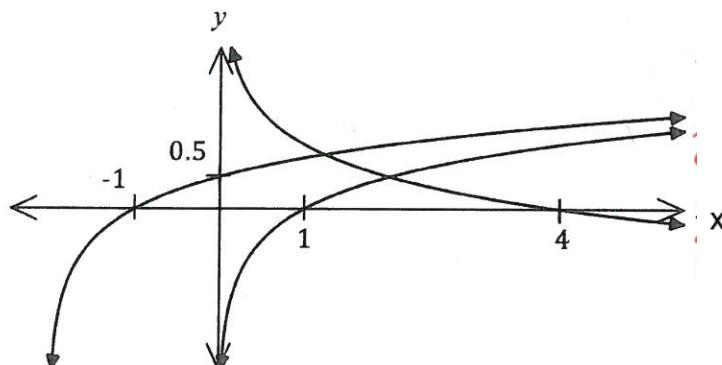
- (b) Function g is defined by $g(x) = \log_n x$ over its natural domain, where n is a constant greater than 1.

The graphs shown below have equations $y = g(x)$, $y = a - g(x)$ and $y = g(x + b)$, where a and b are constants.

- (i) Label each graph with the appropriate equation from those listed above.

- (ii) Determine the value of b and hence determine the value of n and a .

(4 marks)



Question 10**(7 marks)**

The percentage distribution of the number of cans of soft drink per order placed with a takeaway food company over a long period of time are shown in the following table.

Number of cans per order	0	1	2	3	4 or more
Percentage of orders	14	24	45	5	12

In the following questions, you may assume that all orders are placed with the company at random and independently.

- (a) Determine the probability that the next 10 orders all include at least one can of soft drink. (2 marks)

- (b) During a weekday, a total of 225 orders were placed. Determine the probability that
- (i) 40 of these orders included 3 or more cans of soft drink. (3 marks)
 - (ii) more than 25 of these orders included no cans of soft drink. (2 marks)

Question 11**(7 marks)**

In a sample of 1 325 university students, 64% said that they never look at their phone while driving.

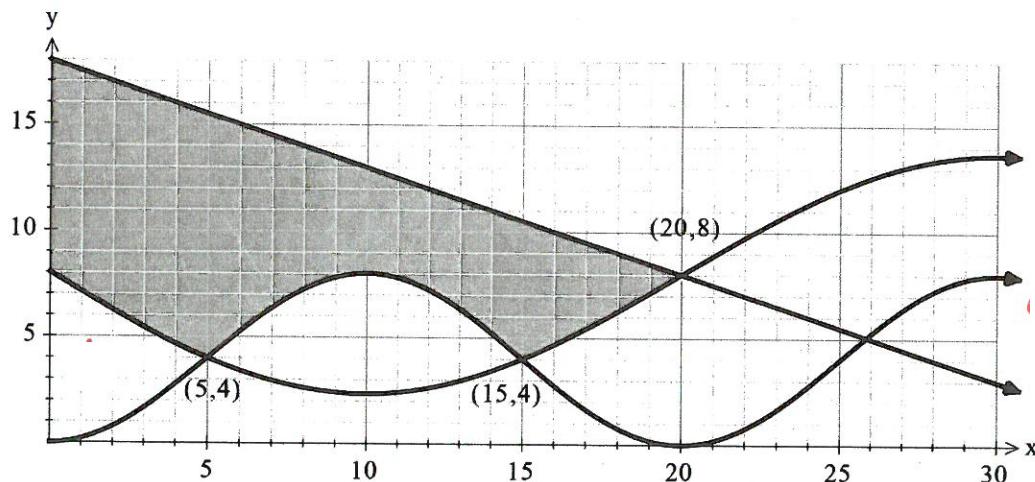
- (a) Show how to use the figures from this sample to construct the 95% confidence interval for the proportion of university students who never look at their phone while driving. (3 marks)

- (b) According to a newspaper article, "70% of university students never look at their phone while driving". Explain whether the interval from (a) supports this claim. (2 marks)
- (c) Another source claims that "the majority of university students never look at their phone while driving". Explain whether the interval from (a) supports this claim. (2 marks)

Question 12

(8 marks)

The diagram shows a flag design, with dimensions in centimetres.



The shaded region is bounded by the y -axis, $y = f(x)$, $y = g(x)$ and $y = h(x)$ where

$$f(x) = 18 - 0.5x,$$

$$g(x) = 4 - 4\cos\left(\frac{\pi x}{10}\right) \text{ and}$$

$$h(x) = 8 - 4\sqrt{2} \sin\left(\frac{\pi x}{20}\right).$$

- (a) Label each graph on the diagram above with the correct function, $f(x)$, $g(x)$, $h(x)$. (1 mark)
- (b) Let A be the area of another region on the graph, where $A = \int_0^5 [h(x) - g(x)] dx$.
- (i) Clearly mark the region on the diagram with the letter A . (1 mark)
 - (ii) Determine the value of A , rounded to one decimal place. (1 mark)
- (c) Show a calculus method to determine the area of the shaded region. (5 marks)

Question 13

(8 marks)

The heights of girls H in a large study of 3-year-old children are normally distributed with a mean of 94.5 cm and a standard deviation of 3.15 cm.

- (a) Determine the probability that a randomly selected girl from the study has a height
- (i) that rounds to 93 cm, to the nearest cm. (2 marks)
- (ii) of at least 90 cm given that they are shorter than 94.5 cm. (2 marks)
- (b) The shortest 1.5% of girls were classified as unusually short. Determine the greatest height of a girl to be classified in this manner. (1 mark)
- (c) The heights of boys in the study are normally distributed with mean of 96.4 cm and the shortest 3.5% of boys, with a height less than 90.2 cm, were classified as unusually short. Demonstrate use of the standard normal distribution to determine the standard deviation of the boys' heights. (3 marks)

Question 14**(6 marks)**

The voltage (*in volts*) generated by a circuit at time t seconds is given by

$$V(t) = e^{0.2t} \cos(3t) \text{ for } 0 \leq t \leq 4.$$

- (a) Show that the voltage is initially increasing. (2 marks)

- (b) Determine the voltage at the instant the rate of change of voltage first starts to increase. (2 marks)

- (c) Show use of the incremental formula to estimate the change in voltage in the one hundredth of a second after $t = 2$. (2 marks)

Question 15

(10 marks)

The probability density function for a continuous random variable T is given by:

$$f(t) = \begin{cases} at(t - 3) & 0 \leq t \leq 2 \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Show use of calculus to determine the value of the constant a . (4 marks)

- (b) Determine $P(1 \leq T \leq 2)$ (2 marks)

- (c) If $E(T) = \frac{6}{5}$ determine the variance of T . (2 marks)

- (d) Find the median of T . (2 marks)

Question 16

(10 marks)

Random samples of 165 people are taken from a large population. It is known that 8% of the population have blue eyes.

- (a) Use a discrete probability distribution to determine the probability that the number of people in one sample who have blue eyes is less than 7%.

(3 marks)

- (b) Ten consecutive random samples are taken. Determine the probability that the number of those with blue eyes is less than 7% in exactly half of these samples.

(2 marks)

A large number of random samples of 165 people are taken. The proportion of blue eyed people calculated for each sample and the distribution of these sample proportions analysed.

- (c) Describe the continuous probability distribution that these sample proportions approximate, including any parameters. (3 marks)

- (d) Describe how two factors affect the closeness of the approximate distribution in (c) to the true distribution of proportions. (2 marks)

Question 17

(8 marks)

The cross section of a triangular prism with a volume of 54 cm^3 is an equilateral triangle of side length $x \text{ cm}$.

- (a) Show that the surface area $S \text{ cm}$ of the prism is given by $S = \frac{\sqrt{3}x^2}{2} + \frac{216\sqrt{3}}{x}$. (4 marks)

(HINT: Use the formula Area of a Triangle = $\frac{1}{2}abs\sin C$.)

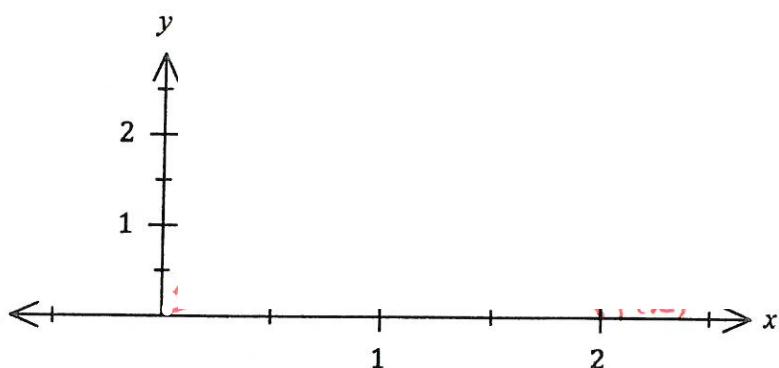
- (b) Use calculus to determine the minimum surface area of the triangular prism. (4 marks)

Question 18

(8 marks)

Let $f(x) = 2 - 2(x - 1)^2$ and $g(x) = \ln(x + 1)$.

- (a) Sketch the graphs of $y = f(x)$ and $y = g(x)$ for $x \geq 0$ on the axes below. (2 marks)



- (b) Show that $\frac{d}{dx}((x + 1)\ln(x + 1) - (x + 1)) = \ln(x + 1)$. (2 marks)

- (c) Show that the area of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$, and the straight line $x = 1$ is exactly $\frac{7}{3} - 2\ln 2$ square units. (4 marks)
(HINT: use your answer in part (b).)

Question 19

(8 marks)

A customer plays an online game of chance. In this game, the computer randomly picks one letter from the letters of the word LUCKY, one letter from the letters of the word BOIST, and one letter from the letters of the word GAMER.

For example, the computer might pick K from LUCKY, S from BOIST and R from GAMER, making KSR.

The customer can see the words but does not know the computer's 3-letter picks and has to guess the letter it has chosen from each word. The customer can guess 0 letters, 1 letter, 2 letters or 3 letters correctly.

The random variable X is the number of letters correctly guessed by a customer in one play of the game.

- (a) Complete the table below to show the probability distribution of X . (3 marks)

x	0	1	2	3
$P(X = x)$				

Each game costs a player 25 cents. A player wins a prize of \$14 if they guess all three letters correctly, \$1.40 if they guess two out of three letters correctly but otherwise wins nothing.

- (b) Determine $E(Y)$ and $\text{Var}(Y)$, where the random variable Y is the gain, in cents, made by the customer in one play of the game. (4 marks)

- (c) If 1 000 people played the game, calculate the expected gross profit for the game owners. (1 mark)

Question 20

(6 marks)

A student was set the task of determining the proportion of people in their suburb who use public transport at least once a week.

- (a) Briefly discuss the main source of bias in each of the following sampling methods.
- (i) The student invites people via social media to respond to their survey. (1 mark)
- (ii) The student asks everyone she meets until she has a large enough sample. (1 mark)
- (b) The student noted that 39 out of all those sampled said they used public transport at least once a week and went on to construct the confidence interval $(0.49, 0.81)$. Determine the level of confidence of this interval. (4 marks)

Supplementary page

Question number: _____