PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section One: Calculator-free

SOLUTIONS

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	47	33
Section Two: Calculator-assumed	12	12	100	94	67
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Supplementary page

Question number: _____

End of questions

Section One: Calculator-free **33% (47 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

Solve each of the following equations.

(a) $\frac{x}{2}+\frac{2x}{3}=\frac{x+1}{2}$. (2 marks)

Solution
$\frac{x}{2}+\frac{2x}{3}=\frac{x+1}{2}$ $\frac{3x+4x}{6}=\frac{x+1}{2}$ $7x=3x+3$ $4x=3$ $x=\frac{3}{4}$
Specific behaviours
✓ simplifies equation ✓ obtains correct solution

(b) $3x^3=12x^2$. (2 marks)

Solution
$3x^3=12x^2$ $3x^3-12x^2=0$ $3x^2(x-4)=0$ $x=0, x=4$
Specific behaviours
✓ factorises ✓ both correct solutions

(c) $(x+5)^2-49=0$. (2 marks)

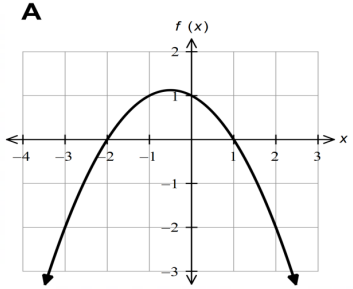
Solution
$(x+5)^2-49=0$ $(x+5)^2=49$ $x+5=\pm 7$ $x=-5\pm 7$ $x=-12, x=2$
Specific behaviours
✓ arranges equation into form $a^2=b^2$ ✓ both correct solutions

See next page

Question 2 **(7 marks)**

Consider the graphs drawn below.

A



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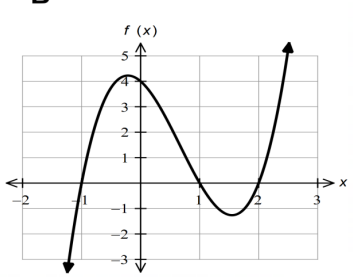
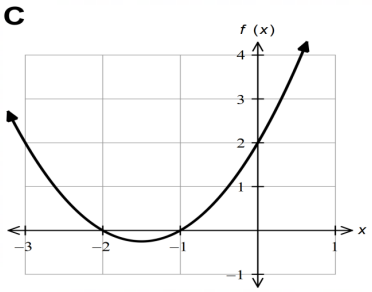


C



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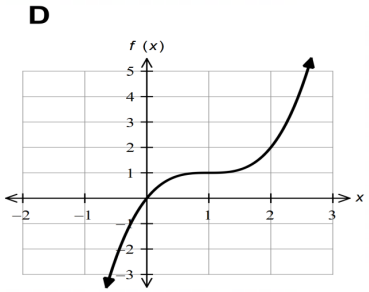

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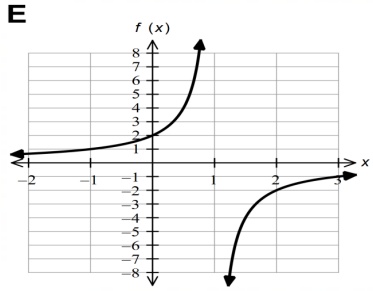
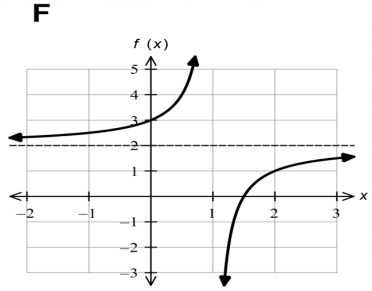


F



(c) Hence, using your answers from parts a) and b), determine the values of a and b under the following conditions:

(i) $P(\overline{A} \cap \overline{B}) = 0.97$ (3 marks)

Solution

If $P(\overline{A} \cap \overline{B}) = 0.97$ then $P(A \cap B) = 1 - 0.97 = 0.03$

From part b), $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$0.2 = \frac{0.03}{b}$$
$$\therefore b = 0.15$$

From part a), $P(A \cap B) = a + b - 0.68$

$$0.03 = a + 0.15 - 0.68$$
$$\therefore a = 0.56$$

Specific Behaviours

✓ Determines the intersection correctly
✓ Determines the value of b
✓ Determines the value of a

(ii) The events A and B are independent (3 marks)

Solution

If A and B are independent then $P(A|B) = P(A)$
Thus from part b), $a = 0.2$

$$P(A \cap B) = P(A) \times P(B)$$
$$P(A \cap B) = 0.2b$$

Substituting into our answer from part a),
$$0.2b = 0.2 + b - 0.68$$
$$0.8b = 0.48$$
$$\therefore b = 0.6$$

Specific Behaviours

✓ Determines the value of a
✓ Determines an expression for the intersection, given independent events
✓ Determines the value of b

Question 7 **(10 marks)**

The events A and B have the probabilities: $P(A) = a$, $P(B) = b$ and $P(\bar{A} \cap \bar{B}) = 0.32$

(a) Determine an expression for $P(A \cap B)$ in terms of a and b. (2 marks)

Solution
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $P(A \cap B) = a + b - (1 - 0.32)$ $P(A \cap B) = a + b - 0.68$
Specific Behaviours
✓ Uses the correct addition rule to establish an expression for the intersection
✓ Obtains the correct expression for the intersection in terms of a and b

If $P(A|B) = 0.2$

(b) Determine an expression for a in terms of b. (2 marks)

Solution
$P(A\|B) = \frac{P(A \cap B)}{P(B)}$ $0.2 = \frac{a + b - 0.68}{b}$ $0.2b = a + b - 0.68$ $a = 0.2b - b + 0.68$ $\therefore a = 0.68 - 0.8b$
Specific Behaviours
✓ Uses the conditional probability formula with substituted values
✓ Obtains the correct expression for a in terms of b

The equations of the graphs drawn are given below.

$f(x) = a(x-1)(x+2)$; $f(x) = \frac{b}{x-1}$; $f(x) = (x-1)^3 + c$

$f(x) = x^2 + dx + 2$; $f(x) = e^{(x^2-1)}(x+f)$; $f(x) = \frac{1}{x-1} + g$

Determine the values of *a*, *b*, *c*, *d*, *e*, *f* and *g*. (7 marks)

$a = -\frac{1}{2}$ ✓

b = -2 ✓
c = 1 ✓
d = 3 ✓
e = 2 ✓
f = –2 ✓
g = 2 ✓

Note: Accept $g = 0$ *or* 4

Question 3 **(4 marks)**

A cubic polynomial exists such that *f* (1) = 0, *f* (3) = 0, *f* (4) = 6 and the coefficient of x^3 is 1. State the polynomial in factorised form.

$(x - 1)(x - 3)(x - a) = 0$ ✓

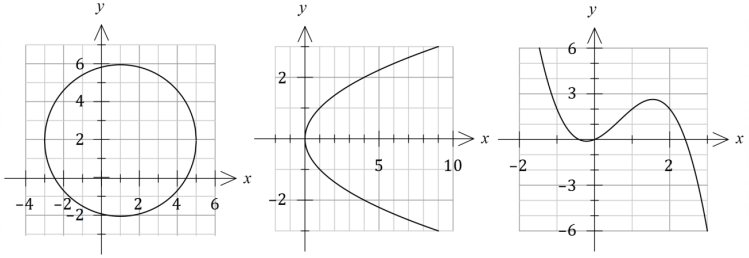
$f(4) = 6 \rightarrow (3)(1)(4 - a) = 6$ ✓

$\therefore \quad a = 2$ ✓

$\therefore \quad f(x) = (x - 1)(x - 3)(x - 2)$ ✓

Question 4 **(8 marks)**

The graphs of the function $y = f(x)$ and two relations are shown below.



(a) Explain how the vertical line test can be used to distinguish a function from a relation. (1 marks)

Solution
The test concludes that a relation is a function if and only if no vertical line intersects the relation more than once. Otherwise, graph is simply a relation.
Specific behaviours
✓ includes reference to all possible vertical lines (if and only if...) or includes reference to no more than one intersection for function

(b) State the equation of the parabolic relationship. (1 mark)

Solution

$y^2 = x$

Specific behaviours

✓ correct equation

(c) Determine $f(3)$. (1 mark)

Solution
$f(3) = -6$
Specific behaviours
✓ correct value

(d) Solve $f(x) = 2$. (1 mark)

Solution
$x = -1, \quad x = 1, \quad x = 2$
Specific behaviours
✓ all correct solutions

(e) The equation of the circle is $x^2 + y^2 + ax + by = c$, where a, b and c are constants. Determine the value of each constant. (3 marks)

Solution
$(x-1)^2 + (y-2)^2 = 4^2$
$x^2 - 2x + 1 + y^2 - 4y + 4 = 16$
$x^2 + y^2 - 2x - 4y = 11$
See next page
$a = -2, \quad b = -4, \quad c = 11$

Question 5 **(8 marks)**

(a) $b = 5$ ✓

and $h(2) = -\frac{2}{3}(2 + a) + 5 = 9$ ✓

$\therefore \quad 2 + a = -6 \rightarrow a = -8$ ✓

(b) Ball is back at height of 5 m when $-\frac{t}{3}(t - 8) = 0$

ie $t = 8$ ✓

$\therefore$ Max height occurs when $t = 4$ seconds ✓

(c) At the same height when $2t = -\frac{t}{3}(t - 8) + 5$ ✓

$\therefore \quad 6t = -t^2 + 8t + 15 \rightarrow t^2 - 2t - 15 = 0$ ✓

$\therefore \quad (t - 5)(t + 3) = 0 \rightarrow t = 5$ ✓ [8]

6. (a) (i) $m = \frac{6}{6} = 1$

$\therefore \quad y = x + c \rightarrow 2 = 4 + c \rightarrow c = -2$ ✓

$\therefore \quad y = x - 2$ ✓

(ii) Mid–point AB is $(0, -\frac{1}{2})$ ✓

$m = \frac{7}{8} \rightarrow y = -\frac{8}{7}x + c$ ✓

$\therefore \quad y = -\frac{8}{7}x - \frac{1}{2}$ ✓

[5]

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