

SOLUTIONS

MATHEMATICS
METHODS
UNITS 1 AND 2
Section One:
Calculator-free

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Student Number: In figures

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total				150	100

Instructions to candidates

- The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Additional working space

Question number: _____

Section One: Calculator-free
(52 Marks)

Working time for this section is 50 minutes.

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Question 7

(a)

Determine the coefficient of the x^3 term in the expansion of $(3 - 2x)^5$.

(2 marks)

(6 marks)

$$\dots + \left(5 \right) (3)^2 (-2x)^3 + \dots$$
$$\dots + 10 \times 9 \times (-8)x^3 + \dots$$
$$\dots - 720x^3 + \dots$$

Coefficient is -720

(b)

Solve $\sin 2x = \frac{1}{2}$ for $0 \leq x \leq 90^\circ$.

$$2x = 30, 150$$
$$x = 15^\circ, 75^\circ$$

(c)

Simplify $\cos\left(\frac{7}{x}\right)\cos\left(\frac{7}{x}\right) + \sin\left(\frac{7}{x}\right)\sin\left(\frac{7}{x}\right)$.

$$\cos\left(\frac{7}{x}\right)\cos\left(\frac{7}{x}\right) + \sin\left(\frac{7}{x}\right)\sin\left(\frac{7}{x}\right) = \cos\left(\frac{7}{x} - \frac{7}{x}\right) = \cos\left(-\frac{2x}{35}\right) = \cos\left(\frac{2x}{35}\right)$$

(2 marks)

Question 1

(8 marks)

A quadratic function is given by $f(x) = (x - 2)^2 - 9$.

(a)

The function can also be written in the form $f(x) = x^2 + bx + c$. Determine the values of b and c .

(2 marks)

$$f(x) = (x - 2)(x - 2) - 9$$
$$= x^2 - 4x - 5$$
$$b = -4, c = -5$$

(b)

Solve the equation $f(x) = 0$.

$$x - 2 = \pm 3$$
$$x = -1 \text{ or } x = 5$$

(2 marks)

(c)

For the graph of $y = f(x)$, state:

$$(2, -9)$$

(1 mark)

(iii)

the equation of the line of symmetry.

$$x = 2$$

(1 mark)

(iiii)

the coordinates of all axes intercepts.

$$(0, -5), (-1, 0) \text{ and } (5, 0)$$

(2 marks)

Question 2

(7 marks)

- (a) Determine the coordinates of the midpoint of A(-12, 3) and B(8, -9). (1 mark)

(-2, -3)

- (b) Are the straight lines given by $3x + 4y = 12$ and $y = 0.75x + 1.25$ parallel, perpendicular or neither? Justify your answer. (2 marks)

Neither

$$3x + 4y = 12$$

$$4y = -3x + 12$$

$$y = -0.75x + 3$$

Gradients are not the same (-0.75 and 0.75) so not parallel.

Gradients do not have a product of -1 ($-0.75 \times 0.75 = 0.5625$) so not perpendicular.

- (c) Determine the equation of the straight line perpendicular to the line $y = 8 - \frac{1}{3}x$ and passing through the point (2, 1). (2 marks)

Required gradient $-\frac{1}{3} \times m = -1 \Rightarrow m = 3$.

$$1 = 3(2) + c$$

$$c = -5$$

$$y = 3x - 5$$

- (d) Solve $2(3x - 2) = \frac{2x + 11}{2}$. (2 marks)

$$4(3x - 2) = 2x + 11$$

$$12x - 8 = 2x + 11$$

$$10x = 19$$

$$x = 1.9$$

- (ii) $f(x)$.

(2 marks)

$$f(x) = \frac{x^4}{8} - \frac{x}{3} + c$$

$$4 = \frac{(-2)^4}{8} - \frac{-2}{3} + c$$

$$c = 4 - 2 - \frac{2}{3}$$

$$= \frac{4}{3}$$

$$f(x) = \frac{x^4}{8} - \frac{x}{3} + \frac{4}{3}$$

Question 6

(a) Differentiate the following with respect to t :

(i) $x = 1 + t - t^2$.

$$\frac{dx}{dt} = 1 - 2t$$

(ii) $v = \frac{6}{t^2} + \frac{4t^3}{9}$.

$$\frac{dv}{dt} = \frac{6}{t^3} + \frac{4t^2}{3}$$

(1 mark)

(1 mark)

(9 marks)

(b)

State whether the graph of $y = x^3 - 2x^2 - 3x - 2$ is increasing, decreasing or stationary at the point $(-1, 1)$. Justify your answer.

$$\frac{dy}{dx} = 3x^2 - 4x - 3 \Big|_{x=-1} = 3 + 4 - 3 = 4$$

Graph is increasing as has a +ve gradient.

The tangent to the curve $y = f(x)$ at the point A is $13x + 3y + 14 = 0$.

If $f'(x) = \frac{2}{x^3} - \frac{1}{3}$ find

(i) the coordinates of point A.

$$\begin{aligned} \text{Gradient of tangent is } -\frac{13}{3}. \\ x^3 - \frac{2}{x^3} - \frac{1}{3} &= -\frac{13}{3} \\ 3x^3 - 2 &= -26 \\ x^3 &= -8 \Rightarrow x = -2 \\ 13(-2) + 3y + 14 &= 0 \\ 3y &= 12 \Rightarrow y = 4 \\ A(-2, 4) \end{aligned}$$

See next page

(b)

Determine the value of n if $\frac{\sqrt[4]{x^3}}{1} = x^n$.

$$\frac{1}{1} = \frac{\sqrt[4]{x^3}}{x^{-\frac{3}{4}}} = x^{-\frac{3}{4}}$$

(2 marks)

(c)

Solve

(i) $8^{2x} = 4\sqrt{2}$.

$$\begin{aligned} 2^{3 \times 2x} &= 2^2 \times 2^{0.5} \\ 6x &= 2.5 \\ x &= \frac{2.5}{6} = \frac{5}{12} \end{aligned}$$

(2 marks)

(iii)

$\sqrt[3]{(4x-1)} + 2 = 0$

$$\begin{aligned} \sqrt[3]{4x-1} &= -2 \\ 4x-1 &= (-2)^3 \\ &= -8 \\ 4x &= -7 \\ x &= -\frac{7}{4} \end{aligned}$$

(2 marks)

(a)

Evaluate 0.0007^2 , writing your answer in scientific notation.

$$(7 \times 10^{-5})^2 = 49 \times 10^{-10} = 4.9 \times 10^{-9}$$

(1 mark)

(7 marks)

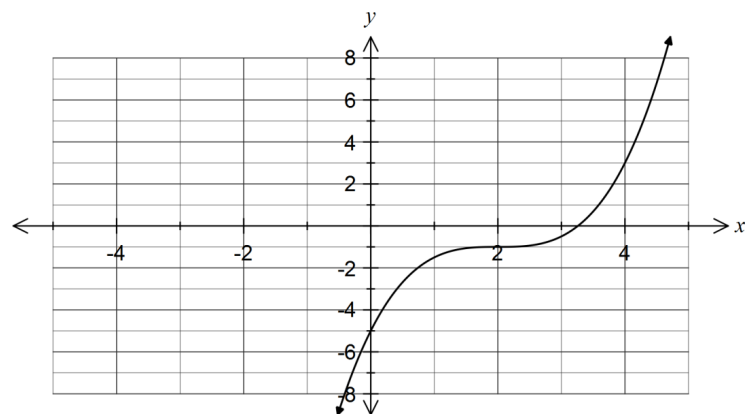
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Question 4

(8 marks)

- (a) Sketch the graph of
- $y = 0.5(x - 2)^3 - 1$
- .

(3 marks)



- (b) Expand
- $(3x - 1)(3x + 1)(x + 3)$
- .

(2 marks)

$$(9x^2 - 1)(x + 3) = 9x^3 + 27x^2 - x - 3$$

- (c) Solve
- $x^3 + 6x^2 + 5x - 12 = 0$
- .

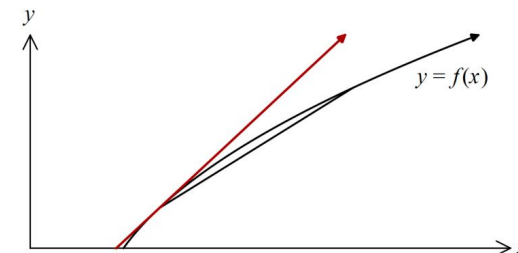
(3 marks)

$$\begin{aligned} f(1) = 0 &\Rightarrow (x - 1)(x + 7x + 12) = 0 \\ (x - 1)(x + 3)(x + 4) &= 0 \\ x = 1, x = -3, x = -4 \end{aligned}$$

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Question 5

(7 marks)

The graph of $y = f(x)$ and a chord of the graph from $(2.5, 7.5)$ to $(5.5, 19.5)$ is shown below.

- (a) Use the ratio
- $\frac{f(x+h) - f(x)}{h}$
- to determine the gradient of the chord. Clearly state the values of
- x
- and
- h
- that you use. (2 marks)

$$\begin{aligned} x &= 2.5 \\ h &= 3 \\ \frac{f(2.5 + 3) - f(2.5)}{3} &= \frac{19.5 - 7.5}{3} = 4 \end{aligned}$$

- (b) As the value of
- h
- used in (a) decreases towards zero and the value of
- x
- remains unchanged, will
- $\frac{f(x+h) - f(x)}{h}$
- increase, decrease or stay the same? Explain your answer. (2 marks)

Increase.

As h decreases, it can be seen from the graph that the gradient of the chord will increase.

- (c) Clearly describe what feature of the graph of
- $y = f(x)$
- will be found by evaluating
- $\lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$
- when
- $x = 4$
- . (2 marks)

The gradient (or derivative) of $y = f(x)$ at the point where $x = 4$.

- (d) On the axes above, draw the tangent to the graph of
- $y = f(x)$
- at the point
- $(2.5, 7.5)$
- . (1 mark)

See next page