Government of **Western Australia**
Curriculum Council

WACE
Western Australian Certificate of Education

Mathematics: Specialist
Formula sheet Units 3A and 3B

Vectors

$|(a,b)| = \sqrt{a^2 + b^2}$ $\qquad |\mathbf{a}+\mathbf{b}| \leq |\mathbf{a}|+|\mathbf{b}|$

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta = a_1b_1 + a_2b_2$

Vector equation of a line in the plane:

one point and the slope: $\mathbf{r} = \mathbf{r}_1 + \lambda\,\mathbf{l}$

two points: $\mathbf{r} = \mathbf{r}_1 + \lambda\,(\mathbf{r}_2 - \mathbf{r}_1)$

Vector form of the equation of a circle in the plane: $|\mathbf{r} - \mathbf{d}| = \rho$

Trigonometry

In any triangle ABC

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad$ Area $= \dfrac{1}{2}ab\sin C$

$a^2 = b^2 + c^2 - 2bc\cos A$ $\qquad \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

In a circle of radius r, for an arc subtending angle θ (radians) at the centre:

Length of arc $= r\theta$ $\qquad$ Area of sector $= \dfrac{1}{2}r^2\theta$ $\qquad$ Area of segment $= \dfrac{1}{2}r^2(\theta - \sin\theta)$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$ $\qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan(\theta \pm \phi) = \dfrac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Exponentials and logarithms

For $a, b > 0$ and m, n real,

$a^m\,a^n = a^{m+n}$ $\qquad a^m\,b^m = (ab)^m$ $\qquad (a^m)^n = a^{mn}$

$a^{-m} = \dfrac{1}{a^m}$ $\qquad \dfrac{a^m}{a^n} = a^{m-n}$ $\qquad a^0 = 1$

For m an integer and n a positive integer: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

For $a, y > 0$, $\qquad x = \log_a y \Leftrightarrow y = a^x$

$\log_a 1 = 0$ $\qquad \log_a a = 1$

$\log_a cd = \log_a c + \log_a d$ $\qquad \log_a(c^b) = b\log_a c$

Functions

Differentiation

If $f(x) = y$, then $f'(x) = \frac{dy}{dx}$

If $f(x) = x^n$, then $f'(x) = n x^{n-1}$

If $f(x) = e^x$, then $f'(x) = e^x$

If $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$

	Function notation		Leibniz Notation	
	y	y'	y	y'
Product rule	$f(x)\, g(x)$	$f'(x)\, g(x) + f(x)\, g'(x)$	uv	$\frac{du}{dx} v + u \frac{dv}{dx}$
Quotient rule	$\frac{f(x)}{g(x)}$	$\frac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$	$\frac{u}{v}$	$\frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2}$
Chain rule	$f(g(x))$	$f'(g(x))\, g'(x)$	$y = f(u)$ and $u = g(x)$	$\frac{dy}{du} \times \frac{du}{dx}$

Integration

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$$

$$\int e^x\, dx = e^x + c$$

$$\int \frac{1}{x}\, dx = \ln x + c$$

Fundamental Theorem of Calculus: $\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$ and $\int_a^b f'(x)\, dx = f(b) - f(a)$

Piece-wise defined functions

Absolute value function: $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

Sign function: $\operatorname{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$

Greatest integer function: int (x) = greatest integer $\leq x$ for all x

Measurement

Trapezium: Area = $\frac{1}{2}(a + b) \times$height, where a and b are the lengths of the parallel sides

Prism: Volume = Area of base $\times$ height

Cylinder: Total surface area = $2\pi r h + 2\pi r^2$ Volume = $\pi r^2 \times h$

Pyramid: Volume = $\frac{1}{3} \times$ area of base $\times$ height

Cone: Total surface area = $\pi r s + \pi r^2$, s is the slant height Volume = $\frac{1}{3} \times \pi r^2 \times h$

Sphere: Total surface area = $4\pi r^2$ Volume = $\frac{4}{3}\pi r^3$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.