Insert School Logo

Semester One Examination 2019
Question/Answer Booklet

MATHEMATICS
METHODS UNIT 3

Section One:
Calculator–free

Student Name: _____

Teacher's Name: _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula Sheet

To be provided by the candidate
Standard items: pens(blue/black preferred), pencils(including coloured), sharpener,
                              correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	**9**	**9**	**50**	**50**	**35**
Section Two Calculator—assumed	15	15	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2019.* Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

    Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

    Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

    It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section One: Calculator–free

50 marks

This section has **nine (9)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 50 minutes

Question 1 **(3 marks)**

Given the function $y = -\dfrac{1}{\sqrt{4x + 3}}$ then $\dfrac{dy}{dx} = \dfrac{m}{(4x + 3)^n}$

(a) State the values of m and n respectively. (2)

(b) Determine the instantaneous rate of change of y when $x = 1.5$ (1)

SEE NEXT PAGE

Question 2 (11 marks)

Consider the function $f(x) = x^3(x - 4)$.

(a) State the coordinates of the stationary points. (3)

(b) Apply the second derivative test to determine the nature of those stationary points. (4)

(c) (i) Show that an oblique point of inflection exists at $x = 2$. (2)

(ii) Find the equation of the tangent at $x = 2$. (2)

SEE NEXT PAGE

Question 3   **(4 marks)**

Differentiate the following. (Solutions should have positive indices where appropriate.)

(a) $y = \left(\dfrac{1}{e^{3x}}\right)^4$ (2)

(b) $y = 2e^{\cos x}$ (2)

Question 4   **(5 marks)**

The distribution function of a certain random variable X is given by:

$f(x) = \dfrac{6 - x}{15}$ where $x = 1, 2, 3, 4, 5$.

(a)   Show that a probability distribution is formed by the values of $f(x)$.   (2)

(b)   What is the probability of obtaining a value less than 3?   (1)

(c)   Determine $P(X > 2 \mid X \leq 4)$.   (2)

Question 5   (3 marks)

Mani is working with a function, g, which passes through the point ($\frac{\pi}{2}$, 1).

She differentiates the function and finds that $g'(x) = \sin x + \cos 2x$. Determine the primitive function.

(3)

Question 6   (4 marks)

Use Calculus rules to find the gradient functions of the following. (Do not simplify.)

(a) $f(x) = \dfrac{\cos x}{1 - e^{2x}}$

(2)

(b) $y = 3x^2 \sin^4 (2x - 3)$

(2)

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Question 7 **(8 marks)**

(a) Given that $\int_0^4 h(x)\ dx = 2$ and $\int_{-4}^{4} h(x)\ dx = -1$, calculate:

(i) $\int_{-4}^{0} h(x)\ dx$ (1)

(ii) $3\int_0^4 h(x)\ dx + 2\int_4^{-4} h(x)\ dx$ (2)

(iii) the area bounded by the curve $h(x)$ and the x – axis, between $x = $ -4 and $x = 4$. (2)

(b) Find x in terms of t, given that $\dfrac{dx}{dt} = 4e^{2t} + \sqrt{1 + t}$ and $x = 1$ when $t = 0$. (3)

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Question 8   **(7 marks)**

(a) The discrete random variable X takes the values of 0, 1 and 2 only. The probability distribution of X is shown in the table, where p is a constant and $0 < p < \dfrac{1}{3}$.

x	0	1	2
$P(X = x)$	$1 - 3p$	$2p$	p

Given that Var$(X) = \dfrac{1}{2}$ find the two possible values of E(X).   (4)

(b) A discrete random variable Y has an expected value of 12 and a variance of 9.
Find:

     (i)   E$(2Y - 3)$ (1)

     (ii)   Standard deviation$(2Y - 3)$ (2)

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Question 9   **(5 marks)**

(a)   Evaluate $\displaystyle\int_{-1}^{3} \frac{\pi}{\sqrt{2x + 3}}\, dx$ (2)

(b)   A digital beetle has been programmed such that its position is given by the equation

$$\frac{dy}{dx} = 1 + \sin\left(\frac{\pi x}{2}\right).$$

The beetle changes its direction as the value of $\frac{dy}{dx}$ changes.
If it is at the point (1, 4), find the equation of the path it travels.   (3)

End of Section One

Additional working space

Question number(s): ………………….

Additional working space

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