

MATHEMATICS 3C/3D SAMPLE EXAMINATION

RESOURCE-RICH

Further information about the WACE Examination policy can be accessed from the Curriculum Council website at http://newwace.curriculum.wa.edu.au/pages/about_wace_manual.asp.

Section 7 of the New WACE Manual: General Information 2006–2009 outlines the policy on WACE examinations.



ACKNOWLEDGEMENTS

SAMPLE EXAM

The secretary of the Seaviews Golf Club visits the manufacturer and is assured that the mean distance travelled by their golf balls is 263 metres. He makes a random selection of 64 golf balls.

- (d) If the sample mean is 262.8 metres, determine a 99% confidence interval for the mean distance travelled by the golf balls.

[3 marks]

- (e) Determine the probability that the sample mean will lie between 262.5 metres and 263.5 metres.

[3 marks]

- (f) How large a sample should the manufacturer use to be 95 percent sure that the sample mean is within 1 metre of 263 metres?

[2 marks]

DRAFT

Consultant Draft

Structure of this paper

Working time	Number of questions available	Number of questions to be attempted	Marks
100 minutes	11	11	80
	[Total marks]		80

This paper has **ELEVEN (11)** questions. Attempt **ALL** questions.

Question	Marks
1	5
2	6
3	7
4	6
5	6
6	7
7	5
8	5
9	7
10	12
11	14
Total marks	80

Instructions to candidates

- The rules for the conduct of Curriculum Council examinations are detailed in the *Student Information Handbook*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages may be found at the end of the booklet. If you need to use them, indicate in the original answer space where the answer is continued (i.e. give the page number).
- A blue or black ballpoint or ink pen should be used.
- It is recommended that you **do not use pencil** except in diagrams.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Correct answers given without supporting reasoning may not be allocated full marks. Incorrect answers given without supporting reasoning cannot be allocated any marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked
- On the front cover you are asked to state the kinds of calculator that you brought into the examination. This information is required to ensure the examination is fair for all students. Please complete the box. Note that the same marking procedure will apply to all scripts, whatever calculator you use.

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- (c) The tangent to $y = f(x)$ at $x = 5$ intersects the x -axis at E and the right-angled triangle CDE may be drawn, as shown in the diagram.

Find the exact length of ED.

[2 marks]

An infinite number of tangents to $y = f(x)$ can be drawn. Each tangent forms a right-angled triangle CDE, where E is the x -axis intercept.

Jason investigates the tangents at $x = 1$, $x = 3$ and $x = 12$. His findings are summarised in the table below:

x	Length of ED
1	$\frac{1}{3}$
3	1
12	4

- (d) (i) Using Jason's findings, make a conjecture about the value of the length of ED for the tangent to $y = f(x)$ at $x = k$ in terms of k .

[1 mark]

- (ii) By considering the tangent at $x = k$, prove the conjecture you made in part (d)(i).

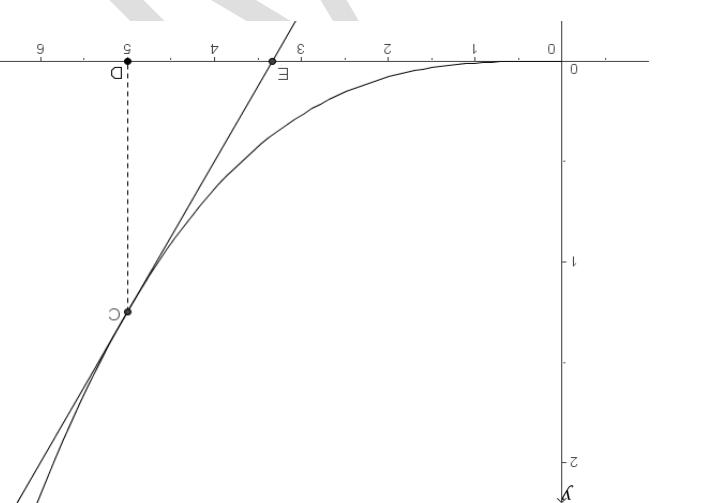
[5 marks]

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The graph of $y = f(x)$, where $f(x) = \frac{100}{x^3}$, $x > 0$, is shown below.

The tangent to $y = f(x)$ at $x = 5$ is also shown.



[3 marks]

(a) Find $f'(x)$.

[1 mark]

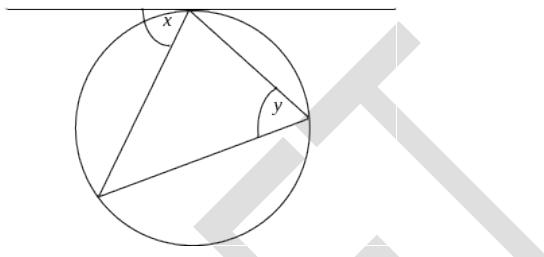
(b) Find the equation of the tangent to $y = f(x)$ at $x = 5$.

RESOURCE-RICH

This paper has **ELEVEN (11)** questions. Attempt **ALL** questions.

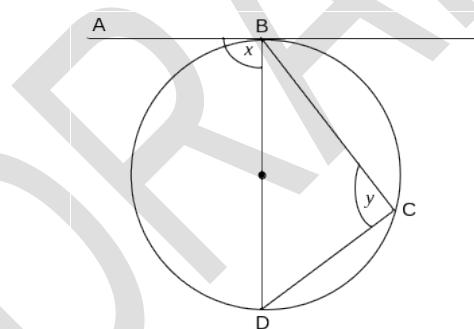
Question 1 [5 marks]

There is a theorem called the Alternate Segment Theorem which says that the angle between the tangent and the chord at the point of contact is equal to the angle in the alternate segment. In the diagram below, $x = y$.



A student who was asked to prove the Alternate Segment Theorem gave this 'proof':

Since the Alternate Segment Theorem has to be true for all triangles, it has to be true when one side of the triangle is the diameter of the circle.



$\angle ABD = 90^\circ$ (angle between a radius and a tangent)
 $\therefore x = 90^\circ$
 $\angle BCD = 90^\circ$ (angle in a semicircle)
 $\therefore y = 90^\circ$
 and $x = y$

- (i) Write a formula for the sum of n consecutive numbers starting at x .

[2 marks]

- (ii) Using algebra and the formula that you wrote in (i) above, prove Zaki's conjecture.

[4 marks]

- Zak also recognises from the table that he could write a formula for the sum of n consecutive numbers starting at x . He used the facts that:
 - 1, 3, 6, 10, 15, 21... are triangular numbers, $T_1 = 1$, $T_2 = 3$, $T_3 = 6$
 - the n th triangle number can be calculated using $T_n = \frac{n(n+1)}{2}$

- The sum of n consecutive numbers:
 - is divisible by n for odd values of n and
 - is not divisible by n for even values of n .

On the basis of the table Zaki conjectured that:

$3x + 3$	3	n
$4x + 6$	4	
$5x + 10$	5	
$6x + 15$	6	
$7x + 21$	7	

(b) Zaki continued to add consecutive numbers and summarised his results in a table:

[1 mark]

(a) Use algebra to show that the sum of five consecutive numbers is divisible by 5.

$= 3(x + 1)$ which is divisible by 3.

$$3 + x =$$

$$(2+x) + (1+x) + x$$

Zakl likes to add consecutive numbers together, for example $7 + 8 + 9 = 24$. When Zakl adds three consecutive numbers together, the sum always appears to be divisible by 3. This does not surprise Zakl, because he realises that if the first of the three numbers is, then

Question 9 [7 marks]

[2 marks]

(a) Discuss the validity of the student's proof.

Question 2 [6 marks]

Events A and B are such that $P(A) = 0.45$, $P(B) = 0.52$ and $P(A \cup B) = 0.736$

(a) Evaluate

(i) $P(\bar{A})$

[1 mark]

(ii) $P(A \cap B)$

[2 marks]

(iii) $P(B|\bar{A})$

[1 mark]

(b) Determine whether or not events A and B are independent.

[2 marks]

Question 8 [5 marks]

A new drug is claimed to improve the hand-eye coordination of children. In order to test the drug, ten children who suffer hand-eye coordination difficulties participated in a six-week study. An appropriate study design was used which included measuring hand-eye coordination with a standard test.

The manager of the study recognises that improvement on a test could be due to the drug or due to chance alone and that the probability of any one child improving their score by chance alone is 0.5. Hence, the manager decides to accept that the drug improves hand-eye coordination only if the probability that the results could have been produced by chance alone is less than 10%.

(a) Use your knowledge of the binomial distribution to find the probability that eight or more children would show improvement by chance alone.

[3 marks]

(b) If eight or more children show improvement, would the manager conclude that the drug improves hand-eye coordination, using his 10% criterion? Explain.

[2 marks]

[3 marks]

Compare and contrast the distribution of mean values that you would expect for Ms Smith's and Mr Brown's classes.

- (d) Ms Smith's class of 15 students explores the same experiment—each student rolls a die mean number of successes. Ms Smith records the number of successes for each student and calculates the mean number of successes. Ms Smith repeats this exercise 10 times.

[2 marks]

Sketch a graph showing the distribution that you would expect for the 10 mean values.

- (e) As part of exploring the outcomes of the above experiment, Mr Brown asks each student in this class of 30 to roll a die 48 times. He records the number of successes for each student, and calculates the mean number of successes. The class repeats this exercise 10 times.

[1 mark]

- (b) the expected standard deviation for X .

[1 mark]

- (a) the expected mean value for X .

An experiment involves rolling a fair die and when 6 is on the uppermost face the outcome is called a success; if the random variable X represents the number of successes when the die is rolled 48 times, calculate:

Question 3 [7 marks]

about whether or not the variable is normally distributed.

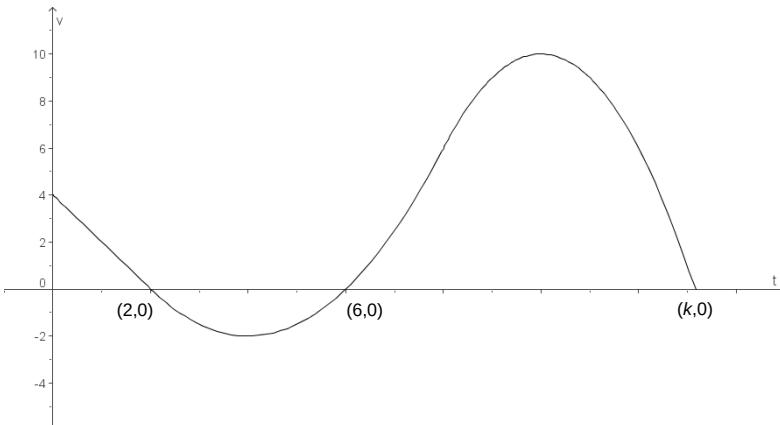
A workshop received the volume of oil removed from the most recent 120 cars serviced. The mean volume was 4.44 L and standard deviation 0.80 L. The lower quartile was 3.72 L, median 4.40 L, and upper quartile 5.12 L. Examine these statistics and make and justify a conclusion

[5 marks]

Question 7 [5 marks]

Question 4 [6 marks]

An object moves along a straight line for a period of k seconds. The graph of its velocity, v m/s, as a function of time, t seconds, is shown in the diagram.



Given, $v(t) = \begin{cases} 4 - 2t & 0 \leq t \leq 2 \\ \frac{1}{2}(t-4)^2 - 2 & 2 < t < 8 \\ -t^2 + 20t - 90 & 8 \leq t \leq k \end{cases}$

Determine:

(a) k

[1 mark]

(b) the distance travelled by the object in the first 8 seconds.

[3 marks]

Let t be the number of months since January.

The median monthly on-peak energy consumption rate in the 1990s, $f(t)$, can be modelled by

$$f(t) = 4.5t^3 - 45.4t^2 - 101.1t + 1774.5 \text{ for } 0 \leq t \leq 11.$$

The median monthly off-peak energy consumption rate in the 1990s, $g(t)$, can be modelled by

$$g(t) = 2.0t^3 - 25.5t^2 + 38.0t + 521.3 \text{ for } 0 \leq t \leq 11.$$

(c) (i) Write an expression which would give the area between the graphs of $y = f(t)$ and $y = g(t)$ for $0 \leq t \leq 11$.

[2 marks]

(ii) Find the area between the graphs of $y = f(t)$ and $y = g(t)$ for $0 \leq t \leq 11$. Give your answer correct to five significant figures.

[1 mark]

(iii) Interpret the area between the graphs of $y = f(t)$ and $y = g(t)$ for $0 \leq t \leq 11$.

[2 marks]

[2 marks]

- (c) the time(s) at which the acceleration of the object is 0 m/s^2 .

The graph below shows the median monthly on-peak and off-peak energy consumption rates in kilowatts per month at an industrial site in the 1990s.

Question 6 [7 marks] The graph below shows the median monthly on-peak and off-peak energy consumption rates in kilowatts per month at an industrial site in the 1990s.

SAMPLE EXAM

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SAMPLE EXAM

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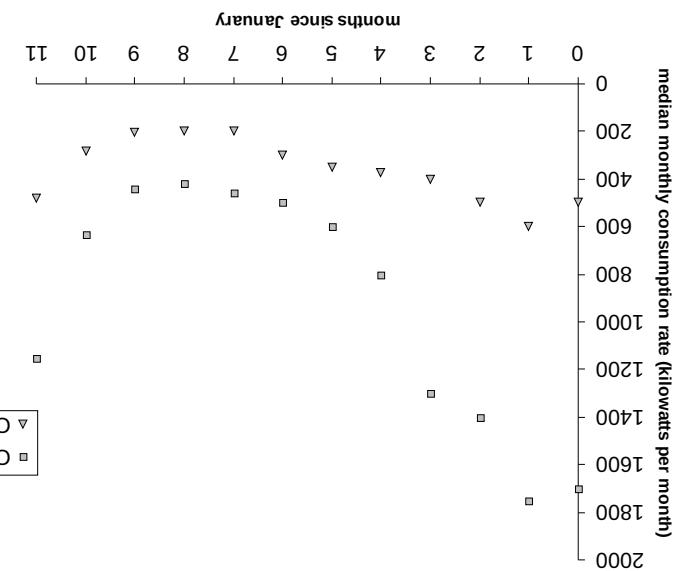
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[1 mark]

- (b) Which two consecutive months produced the largest decrease in median monthly on-peak energy consumption rate in the 1990s?

[1 mark]

- (a) Which month had the greatest median monthly on-peak energy consumption rate in the 1990s?



SAMPLE EXAM

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SAMPLE EXAM

Question 5 [6 marks]

A spherical metal ball of radius 4 cm is coated with a layer of ice of uniform thickness. The ice is melting such that when the ice is 6 cm thick, the thickness of the ice is decreasing at the rate of 0.008 cm/h.

When the ice is 6 cm thick, calculate the rate at which the volume of the ice is decreasing.

[6 marks]

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