MATHEMATICS METHODS

MAWA Semester 1 (Unit 3) Examination 2018
Calculator-free

Marking Key

Section One: Calculator-free (50 Marks)

Question 1 (a) (2 marks)

Solution	
$\frac{d}{dx}(x\cos x) = x(-\sin x)+\cos x$ $=\cos x - x\sin x$	
Mathematical behaviours	Marks
• applies product rule • differentiates $\cos x$ term	1 1

Question 1 (b) (2 marks)

Solution	
$\frac{d}{dx}(x^3+4\sin x)^5 = 5(x^3+4\sin x)^4 \cdot \frac{d}{dx}(x^3+4\sin x)$ $=5(x^3+4\sin x)^4(3x^2+4\cos x)$	
Mathematical behaviours	Marks
• applies the chain rule • differentiates $\sin x$ term	1 1

Question 1 (c) (3 marks)

Solution	
$\frac{d}{dx}\left(\frac{e^{-2x}}{4x+2}\right)$ $f(x)=e^{-2x}$ $f'(x)=-2e^{-2x}$ $g(x)=4x+2$ $g'(x)=4$ $=\frac{(4x+2)\cdot(-2e^{-2x})-e^{-2x}\cdot 4}{(4x+2)^2}$ $=\frac{-2(4x+2)(e^{-2x})-4e^{-2x}}{(4x+2)^2}$	
Mathematical behaviours	Marks
• applies chain rule to obtain $f'(x)$ • applies quotient rule • correct answer	1 1 1

Question 2 **(4 marks)**

Solution

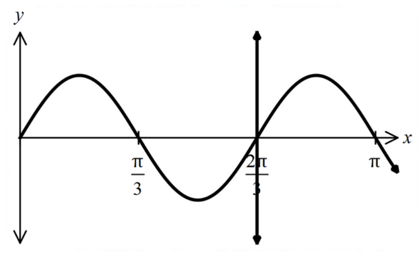
$$A = 2\int_0^{\frac{\pi}{3}} \sin 3x\,dx$$

$$= 2\left[\frac{-\cos 3x}{3}\right]_0^{\frac{\pi}{3}}$$

$$= \frac{2}{3}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{2}{3}\left[1+1\right]$$

$$= \frac{4}{3}$$



Mathematical behaviours	Marks
• states a correct expression using integrals to determine the area	1
• anti-differentiates integral correctly	1
• subs in limits of integration correctly	1
• determines correct result	1

Question 3 **(3 marks)**

Solution	
$f'(x) = x + \sqrt{3+6x}$ $\therefore f(x) = \frac{x^2}{2} + \frac{(3+6x)^{\frac{3}{2}}}{6} . \frac{2}{3} + c$ $f(1) = 10 \Rightarrow 10 = \frac{1}{2} + \frac{(3+6(1))^{\frac{3}{2}}}{6} . \frac{2}{3} + c$ ie $10 = \frac{1}{2} + \frac{9^{\frac{3}{2}}}{9} + c$ ie $c = 6\frac{1}{2}$ $\therefore f(x) = \frac{x^2}{2} + \frac{(3+6x)^{\frac{3}{2}}}{9} + 6.5$	
Mathematical behaviours	**Marks**
• anti-differentiates square root term • uses anti-derivative and $f(1) = 10$ to determine c • states $f(x)$	1 1 1

Question 4 (a) **(1 mark)**

Solution	
X has a discrete uniform distribution	
Mathematical behaviours	**Marks**
• states that the distribution is uniform	1

Question 4 (b) **(1 mark)**

Solution	
There are $550 - 250 + 1 = 301$ whole numbers in the interval $250 \le X \le 550$. So $P(250 \le X \le 550) = 0.301$	
Mathematical behaviours	**Marks**
• correct answer	1

Question 4 (c) **(2 marks)**

Solution	
There are $\frac{1000}{7} = 142\frac{6}{7}$, and so there are 142 whole numbers in the interval $1 \le X \le 1000$ that are divisible by 7. So $P¿$.	
Mathematical behaviours	**Marks**
• obtains 142 whole numbers divisible by 7 • divides by 1000	1 1

Question 4 (d) **(4 marks)**

Solution
In the interval $1 \le X \le 1000$there are: 100 whole numbers that are divisible by 10, 40 whole numbers that are divisible by 25, and 20 whole numbers that are divisible by both 10 and 25,(i.e. divisible by $50¿$ So there are $100+40-20=120$ whole numbers that are divisible by 10 or 25 . and so $P¿$.

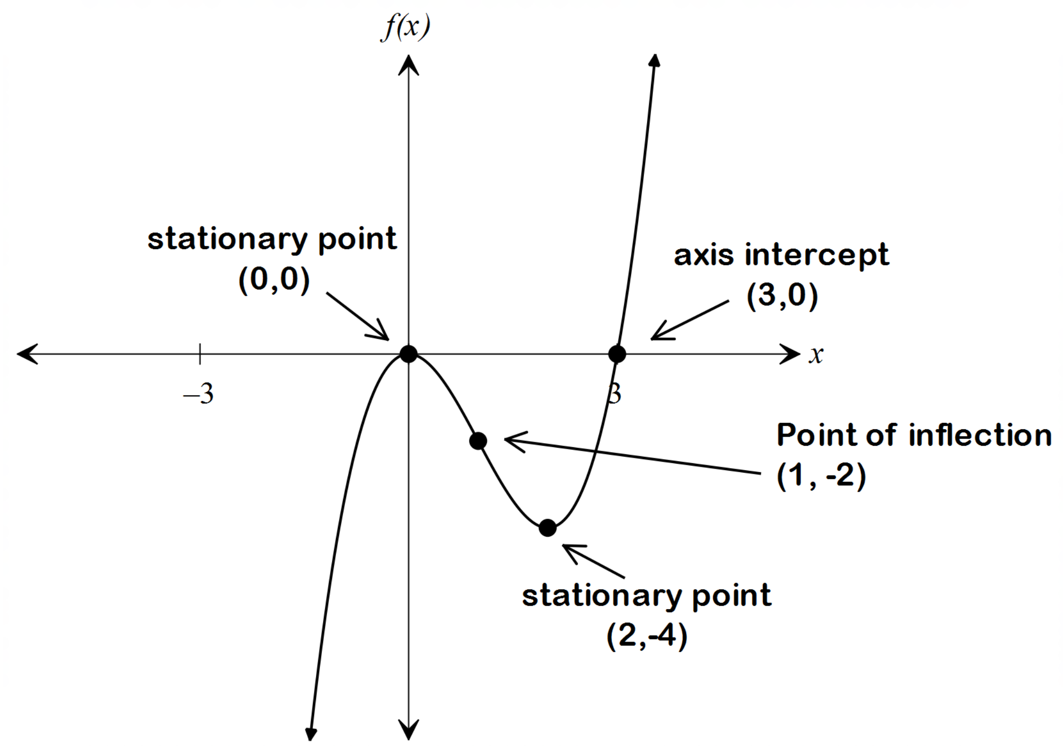
Mathematical behaviours	Marks
• correct numbers for divisibility by 10 and by 25	1+1
• uses $¿(A \cup B)=¿(A)+¿(B)-¿(A \cap B)$	1
• divides by 1000	1

Question 4 (e) **(2 marks)**

Solution
The following numbers have exactly two 3's in their decimal expansion: $33, 133, 233, 433, \ldots, 933$, $303, 313, 323, 343, \ldots, 393$, and $330, 331, 332, 334, \ldots 339$ So $P¿$.

Mathematical behaviours	Marks
• obtains 27 whole numbers with the desired property	1
• divides by 1000	1

Question 5 **(3 marks)**

Solution


Mathematical behaviours	Marks
• correctly identifies stationary points	1
• correctly identifies point of inflection	1
• accurate sketch of the curve including x axis intercepts	1

Question 6 (a) **(2 marks)**

Solution
$\int \frac{1-2x}{x^3}\,dx = \int x^{-3} - 2x^{-2}\,dx = \frac{2}{x} - \frac{1}{2x^2} + c$

Mathematical behaviours	Marks
• splits the fraction into two parts and anti-differentiates x^{-3}	1
• states anti-derivative including $+c$	1

Question 6 (b) **(2 marks)**

Solution
$\int \sin\left(x - \frac{\pi}{4}\right) - \cos \pi x \quad dx = -\cos\left(x - \frac{\pi}{4}\right) - \frac{\sin \pi x}{\pi} + c$

Mathematical behaviours	Marks
• anti-differentiates sin or cos part of expression correctly	1
• states correct solution	1

Question 6 (c) **(2 marks)**

Solution
$\int \left(e^x - \frac{1}{e^x} \right)^2 \ dx = \int \ e^{2x} - 2 + e^{-2x} \ dx = \frac{e^{2x}}{2} - 2x - \frac{e^{-2x}}{2} + c$

Mathematical behaviours	Marks
• expands brackets correctly	1
• anti-differentiates each part correctly	1

Question 7 (a) **(3 marks)**

Solution
$y = \sec(\frac{\pi}{3} - x)$ $u(x) = \frac{\pi}{3} - x \qquad u'(x) = -1 \qquad \frac{d}{dx}\sec x = \frac{\sin x}{\cos^2 x}$ $\frac{dy}{dx} = \frac{\sin(u(x))}{\cos^2(u(x))} \cdot u'(x)$ $\frac{dy}{dx} = \frac{\sin\left(\frac{\pi}{3} - x\right)}{\cos^2\left(\frac{\pi}{3} - x\right)} \cdot (-1) = -\frac{\sin\left(\frac{\pi}{3} - x\right)}{\cos^2\left(\frac{\pi}{3} - x\right)}$

Mathematical behaviours	Marks
• correctly differentiates $sec\, x$	1
• applies chain rule	1
• correct answer	1

Question 7 (b) **(4 marks)**

Solution
$\frac{dy}{dx} = -\frac{\sin\left(\frac{\pi}{3} - x\right)}{\cos^2\left(\frac{\pi}{3} - x\right)}$, when $x = \frac{2\pi}{3}$ $= -\frac{\sin\left(\frac{\pi}{3} - \frac{2\pi}{3}\right)}{\cos^2\left(\frac{\pi}{3} - \frac{2\pi}{3}\right)} = -\frac{\sin\left(-\frac{\pi}{3}\right)}{\cos^2\left(-\frac{\pi}{3}\right)}$ $= -\frac{\frac{-\sqrt{3}}{2}}{\left(-\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} \div \frac{1}{4} = 2\sqrt{3}$

Mathematical behaviours	Marks
• correct substitution and subtraction of fractions	1+1
• both exact values correct	1
• correct simplified answer	1

Question 8 (a) (i) **(1 mark)**

Solution
$\int_0^4 f(x)\ dx = A - B$

Mathematical behaviours	Marks
• determines expression	1

Question 8 (a) (ii) **(3 marks)**

Solution
$\int_0^4 2f(x)\ dx + \int_8^4 f(x)\ dx$ $= 2\int_0^4 f(x)\ dx - \int_4^8 f(x)\ dx$ $= 2(A - B) - 2A = -2B$

Mathematical behaviours	Marks
• uses linearity to deduce $\int_0^4 2f(x)\ dx = 2(A - B)$	1
• uses relationship $\int_8^4 f(x)\ dx = -\int_4^8 f(x)\ dx$	1
• sums expressions and simplifies	1

Question 8 (b) **(2 marks)**

Solution	
$\int_6^8 f'(x)\ dx = f(8) - f(6) = 0 - 3 = -3$	
Mathematical behaviours	Marks
• applies the Fundamental Theorem • evaluates result	1 1

Question 8 (c) (i) **(2 marks)**

Solution	
Area $\Delta = 8$ $\therefore \int_2^3 f(x)\ dx = -4 \Rightarrow \int_0^3 f(x)\ dx = 0$ $\therefore$ one value of m is $m = 3$. Also, $\int_0^0 f(x)\ dx = 0$ for any function hence, $m = 0$ is another solution. From the symmetry of the graph, $m = 6, 9, 12$ Hence $m = 0, 3, 6, 9, 12$.	
Mathematical behaviours	Marks
• states $m = 0$ or $m = 3$ • states all correct values for m	1 1

Question 8 (c) (ii) **(2 marks)**

Solution	
$\int_0^4 g(x)\ dx = \int_0^4 [f(x) + 2]\ dx$ $= \int_0^4 f(x)\ dx + \int_0^4 2\ dx$ $= (-4) + 2(4 - 0)$ $= 4$	
Mathematical behaviours	Marks
• uses linearity to split $g(x)$ • evaluates sum of integrals	1 1