



PERTH MODERN SCHOOL
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Independent Public School

Course Methods test 3 Year 12

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: 40 mins

Number of questions: 8

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured),
sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet
of
A4 paper, and up to three calculators approved for use in the
WACE examinations

Marks available: 46 marks

Task weighting: 10 %

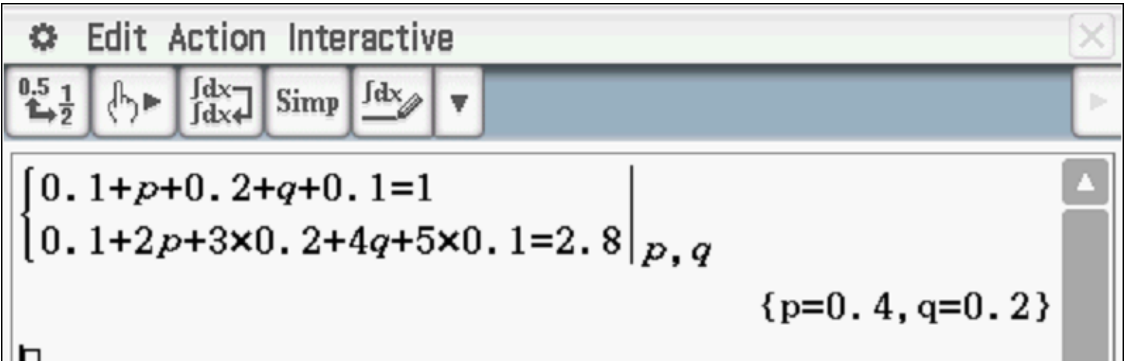
Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3 marks)

The expected value of the discrete probability distribution given below is 2.8. Determine the values of p & q and hence determine $\text{Var}(X)$, the variance of X .

x	1	2	3	4	5
$P(X=x)$	0.1	p	0.2	q	0.1

Solution

Specific behaviours
<ul style="list-style-type: none"> ✓ sets up one equation with p&q ✓ sets up two equations with p&q ✓ solves for both p&q (Note: max 2 marks if no working shown)

Q2 (9 marks)

A student wishes to play a gambling game on mufti day involving throwing two regular fair dice, each numbered 1 to 6. To play the game the student must pay \$2 for each throw of two dice. If they score a double i.e two 1s, two 2s etc they win \$6. If they throw a total of 7 they win \$11 and anything else they receive nothing.

Let X equal the profit a player receives on a single play.

a) Describe the random variable X .

(1 mark)

Solution
Discrete random variable
Specific behaviours
✓ states discrete

b) Complete the following table for X .

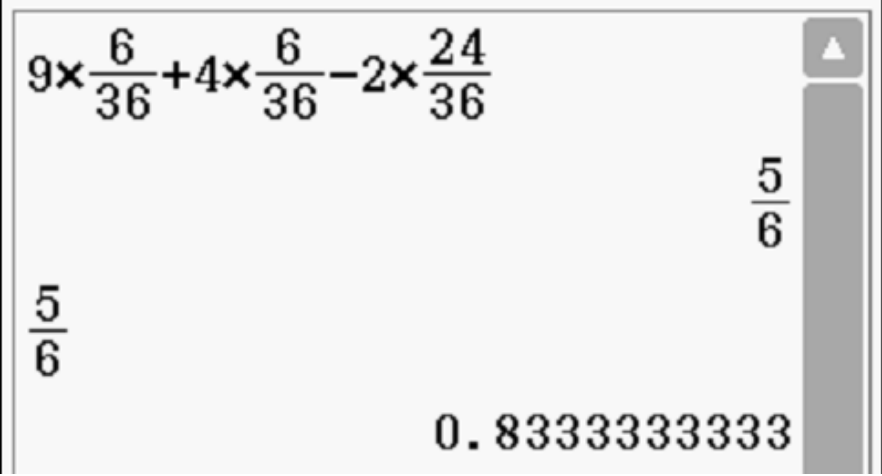
(3 marks)

Solution			
x	\$9	\$4	-\$2
$P(X=x)$	$\frac{6}{36}$	$\frac{6}{36}$	$\frac{24}{36}$

Specific behaviours			
✓ correct values for x ✓ at least 1 probs correct ✓ all probs correct			

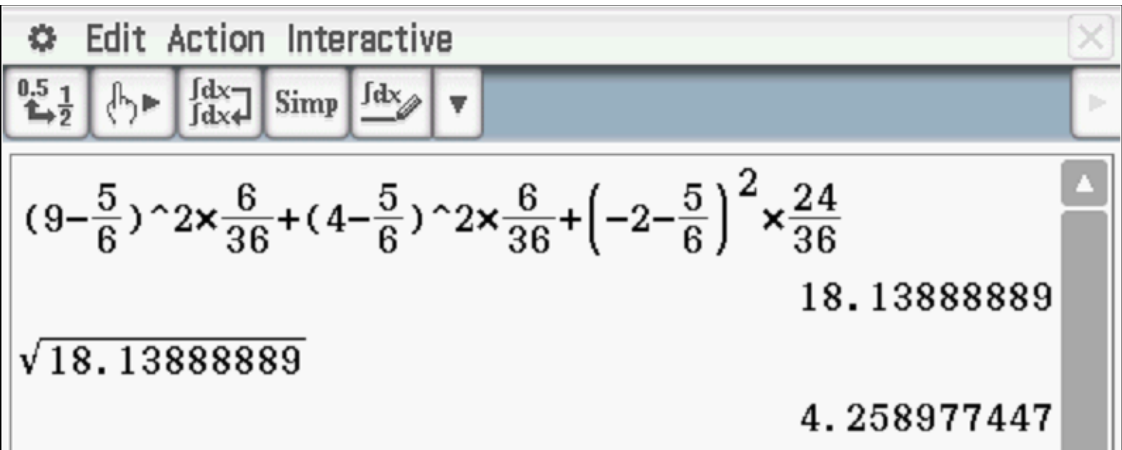
c) Determine the expected profit by a player on a single game.

(3 marks)

Solution	
	
Specific behaviours	
✓ multiplies x by prob ✓ shows sum of products ✓ states expected profit, approx. or exact (no need for units) (Note two marks for answer only)	

d) Determine the standard deviation of X .

(3 marks)

Solution

Specific behaviours
<ul style="list-style-type: none"> ✓ shows calculation ✓ determines variance ✓ states standard deviation <p>Note: Answer only two marks- third mark refers to shown working</p>

Q3 (7 marks)

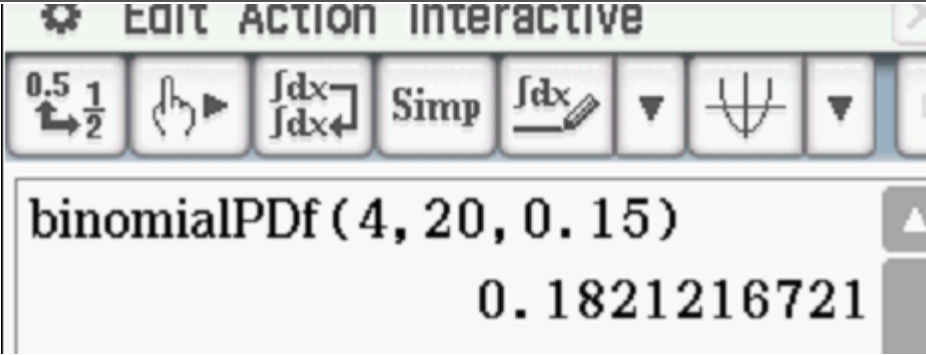
A factory produces toy cars. The probability that any toy car being defective is 0.15. If 20 toy cars are selected at random, let X equal the number of defective cars out of 20.

- a) Describe the distribution X . (2 marks)

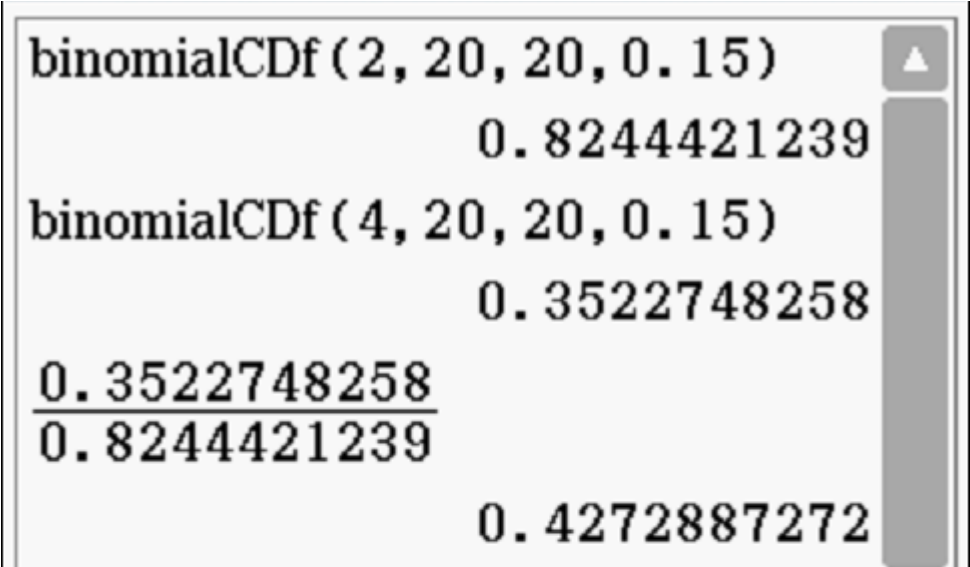
Solution
$X \sim \text{Bin}(20, 0.15)$
Specific behaviours
<ul style="list-style-type: none"> ✓ states Binomial ✓ states n & p

- b) Determine that probability that exactly 4 cars will be defective. (2 marks)

Solution

	
Specific behaviours	
<ul style="list-style-type: none"> ✓ uses correct parameters ✓ states prob 	

- c) Determine the probability that at least 4 cars will be defective given that we know at least 2 cars are defective. (3 marks)

Solution	
$P(X \geq 4 X \geq 2) = \frac{P(X \geq 4)}{P(X \geq 2)}$ 	
Specific behaviours	
<ul style="list-style-type: none"> ✓ uses conditional prob reasoning ✓ shows numerator and denominator values ✓ states prob 	

Q4 (4 marks)

Sound loudness, L dB, is measured by comparing the intensity of the sound, I , with the intensity of a sound that is just detectable by the human ear, I_o .

$$L = 10 \log_{10} \left(\frac{I}{I_o} \right)$$

- a) If the noise level in a room was 65 dB, express the intensity of sound in this room in terms of I_o . (1 mark)

Solution
$65 = 10 \log_{10} \left(\frac{I}{I_o} \right)$ $\left(\frac{I}{I_o} \right) = 10^{6.5}$ $I = I_o 10^{6.5}$
Specific behaviours
✓ states expression

- b) How many times is the intensity of a 105 dB noise level that of the intensity of a 35 dB noise level? (3 marks)

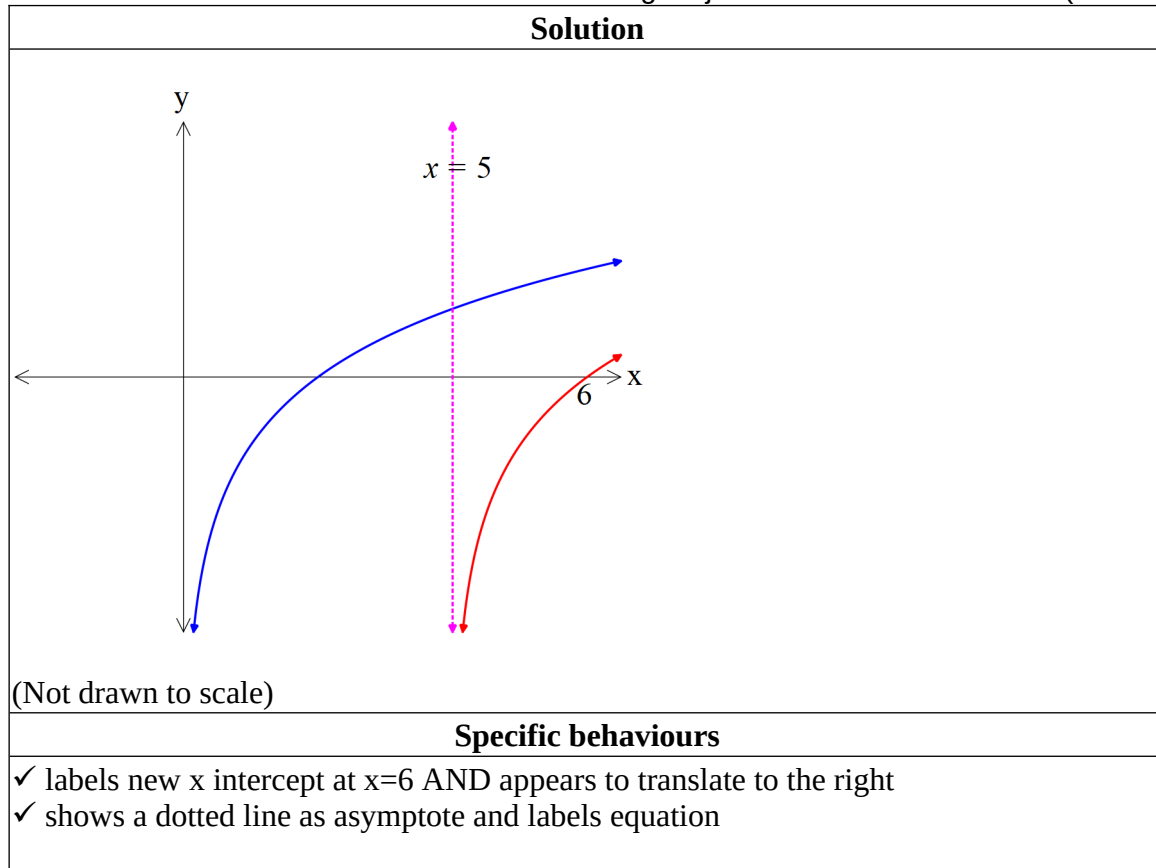
Solution
$L = 10 \log_{10} \left(\frac{I}{I_o} \right)$ $\frac{I}{I_o} = 10^{\frac{L}{10}}$ $\frac{10^{10.5}}{10^{3.5}} = 10^7$
Specific behaviours
✓ uses index form ✓ shows the powers of 10 for both levels ✓ states simplified ratio

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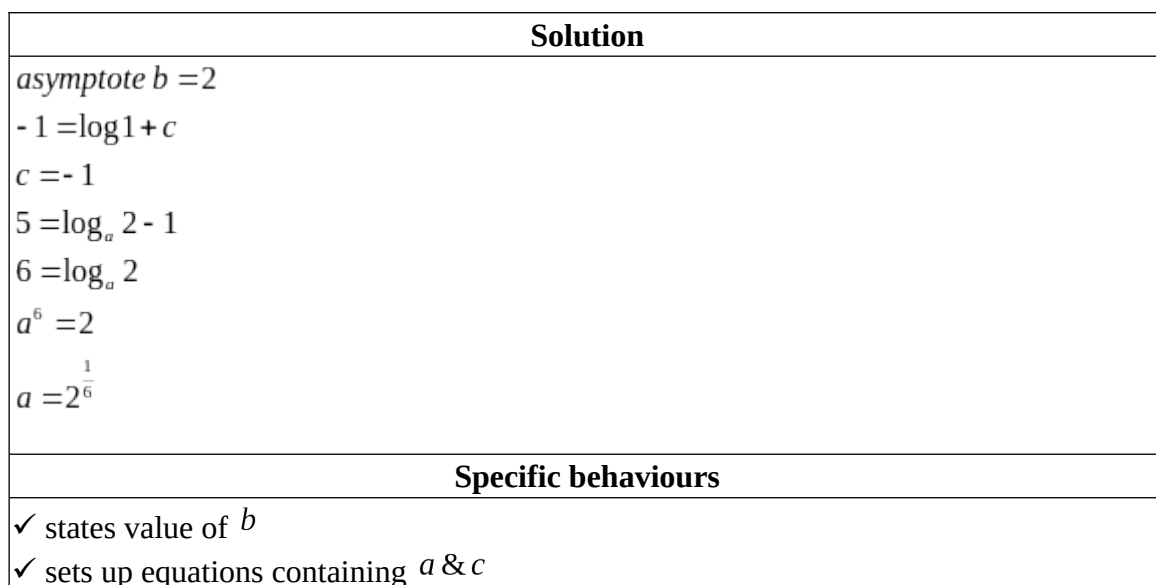
Q5 (5 marks)

Below is a graph of $y = \log_a x$ where a is a positive constant.

- a) Sketch on the axes above $y = \log_a (x - 5)$ labelling major features. (2 marks)



- b) Determine the values of a, b & c given that $y = \log_a(x + b) + c$ contains points $(-1, -1)$ & $(0, 5)$ and has a vertical asymptote at $x = -2$. (3 marks)



✓ states exact values of a & c

Q6 (6 marks)

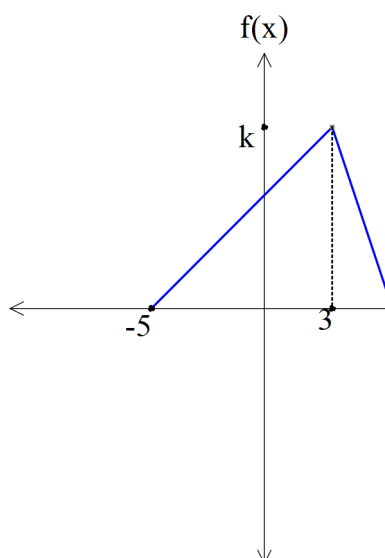
- a) Determine $\frac{d}{dx}(x^3 \ln x)$. (simplify) (3 marks)

Solution
$\frac{d}{dx}(x^3 \ln x) = x^3 \frac{1}{x} + 3x^2 \ln x$ $= x^2 + 3x^2 \ln x$
Specific behaviours
<ul style="list-style-type: none"> ✓ shows use of product rule ✓ at least one product correct ✓ states simplified derivative

- b) Using your result in a) above and **NOT using your classpad** determine $\int 10x^2 \ln x \, dx$. Show all working. (3 marks)

Solution
$\frac{d}{dx}(x^3 \ln x) = x^2 + 3x^2 \ln x$ $x^3 \ln x = \int x^2 \, dx + 3 \int x^2 \ln x \, dx$ $\int x^2 \ln x \, dx = \frac{1}{3} \left(x^3 \ln x - \frac{x^3}{3} \right) + c$ $\int 10x^2 \ln x \, dx = \frac{10}{3} \left(x^3 \ln x - \frac{x^3}{3} \right) + c$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses linearity (integrates exp in (a) above) ✓ integrates squared term and adds a constant ✓ obtains exp for required integral (no need to factorise) (zero marks for answer only- from classpad)

Q7 (6 marks)

Consider the continuous random variable X and its probability density function shown below.a) Determine the exact value of k . (2 marks)

Solution
Specific behaviours
<ul style="list-style-type: none"> ✓ uses total area of one ✓ solves for exact value of k

b) Determine Prob ^{$(1.5 < X < 4.5)$}
(4 marks)

Solution	
$\begin{cases} \frac{6}{31} = m \times 3 + c \\ 0 = m \times (-5) + c \end{cases} \bigg _{m, c}$	$\left\{ m = \frac{3}{124}, c = \frac{15}{124} \right\}$
$\begin{cases} \frac{6}{31} = m \times 3 + c \\ 0 = m \times \left(\frac{16}{3}\right) + c \end{cases} \bigg _{m, c}$	$\left\{ m = -\frac{18}{217}, c = \frac{96}{217} \right\}$
$\int_{1.5}^3 \frac{3}{124}x + \frac{15}{124} dx$	$\frac{261}{992}$
$\int_3^{4.5} -\frac{18}{217}x + \frac{96}{217} dx$	$\frac{171}{868}$
$\frac{261}{992} + \frac{171}{868}$	$\frac{3195}{6944}$
$\frac{3195}{6944}$	0.460109447
Prob = 0.4601	
Specific behaviours	
<ul style="list-style-type: none"> ✓ determines equation of one side ✓ determines equations of both sides ✓ states integrals with correct limits for total area ✓ states approx. area to 4 decimal places (accept exact) 	

Q8 (5 marks)

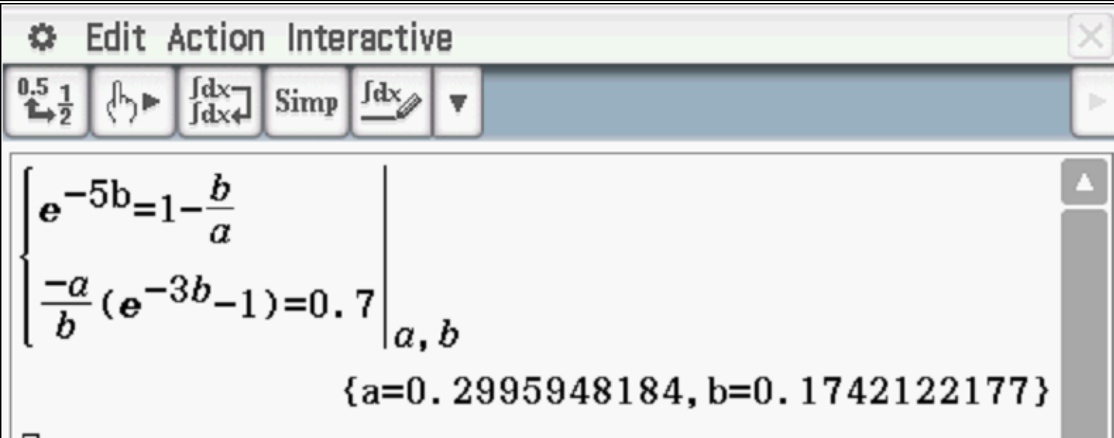
Consider a continuous random variable, X , that has the following probability density function.

$$f(x) = \begin{cases} ae^{-bx} & , 0 \leq x \leq 5 \\ 0 & , \text{elsewhere} \end{cases} \text{ with } a \text{ \& } b \text{ being constants.}$$

- a) Determine the cumulative distribution function, $P(X \leq x)$, in terms of a & b .
(2 marks)

Solution
$P(X \leq x) = \int_0^x ae^{-bu} du$ $= \left[\frac{-a}{b} e^{-bu} \right]_0^x = \frac{-a}{b} (e^{-bx} - 1)$
Specific behaviours
<ul style="list-style-type: none"> ✓ integrates with correct limits (no need to change variables) ✓ states cumulative function

- b) Given that $P(X \leq 3) = 0.7$ solve for approximate values of a & b to two decimal places.
(3 marks)

Solution
 <p>a=0.30 & b=0.17</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ sets up equation using total prob of one with x=5 ✓ sets up second equation at x=3

✓ solves for a & b to two decimal places (**must round**)