



**MATHEMATICS SPECIALIST 3,4**  
**TEST 2 SECTION ONE 2016**  
**NON Calculator Section**  
Chapters 3 and 4

Name \_\_\_\_\_

Time: 35 minutes

Total: 35 marks

**Question 1**

**(7 marks)**

Two functions are defined as  $f(x) = \sqrt{x-1}$  and  $g(x) = \frac{1}{x-1}$

(a) Evaluate  $gf\left(\frac{13}{9}\right)$

(2 marks)

(b) Find in simplified form  $gg(x)$ .

(2 marks)

(c) Determine the domain of  $f(g(x))$

(3 marks)

**Question 2****(6 marks)**

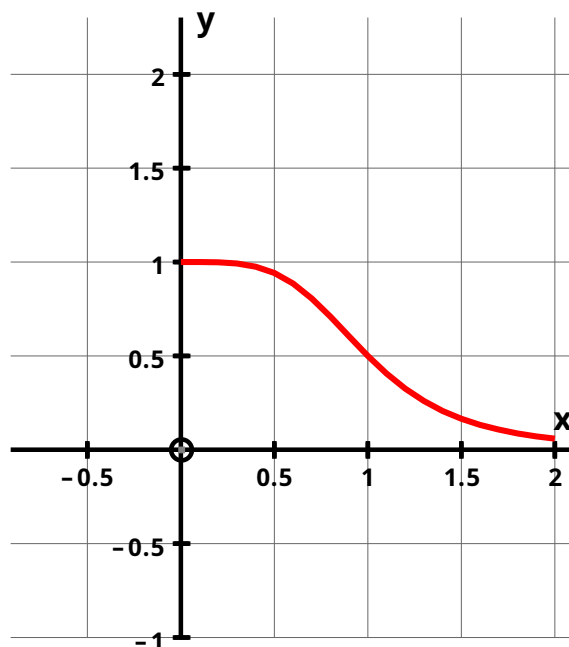
- (a) Determine the domain and range of  $f(g(x))$  given that  $f(x) = \frac{12}{x+1}$  and  $g(x) = \sqrt{x+1}$  (3)

- (b) Given that  $f(x) = 2x+3$  and  $g(f(x)) = 4x^2+12x+11$ , find  $g(x)$ . (3)

### Question 3

(6 marks)

The graph of function  $f(x) = \frac{1}{x^4 + 1}$  for the domain  $0 < x < 2$  is shown below.



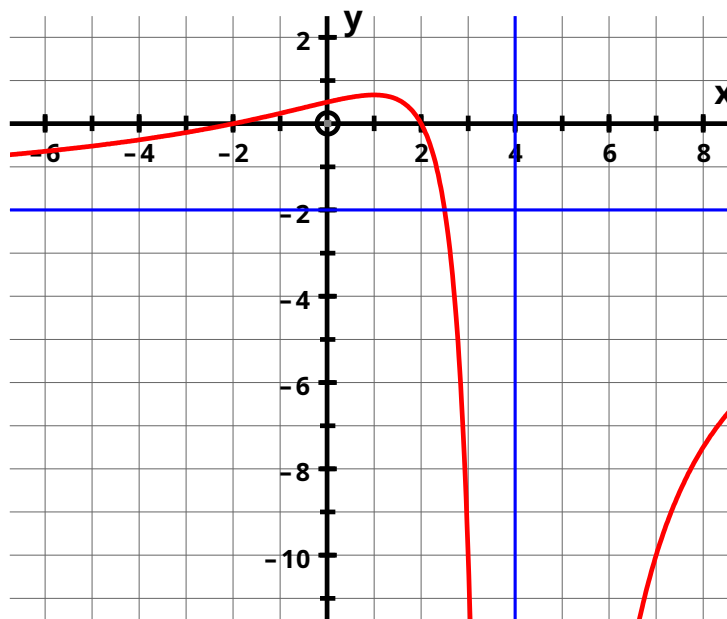
- (a) Determine the exact value for  $\lim_{x \rightarrow 2^+} f(x)$  (2)
- (b) On the axes given above, sketch the graph of the inverse function,  $y = f^{-1}(x)$  (2)
- (c) Obtain the rule for  $f^{-1}(x)$ . (2)

#### Question 4

(5 marks)

A rational function  $R(x)$  is sketched below. Function  $R(x)$  has the following properties:

- Only one pole or a discontinuity at  $x=4$
- Two horizontal intercepts at  $x=2$  and  $x=-2$ .
- A horizontal asymptote at  $y=-2$



(a) If  $R(x) = \frac{k(x^2 - a)}{(x - b)(x - c)}$  explain why  $k = -2$ ,  $a = 4$ ,  $b = 4$  and  $c = 4$

(4)

(b) Determine  $\lim_{x \rightarrow 4} R(x)$ .

(1)

### Question 5

(7 marks)

Solve the following.

(a)  $\lim_{x \rightarrow 2} \frac{x^3 - 2x^2}{x^2 - 4}$

(1)

(b)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$

(2)

(c )
$$\left| \frac{1}{3x'+4} \right| \geq \left| \frac{1}{5x'+2} \right|$$
(2)

(d)
$$\left| \frac{1}{x'-6} \right| \leq \left| \frac{1}{4x'+3} \right|$$
(2)



**MATHEMATICS SPECIALIST 3,4**  
**TEST 2 SECTION TWO 2016**  
**Calculator Section**  
Chapters 3 and 4

Name \_\_\_\_\_

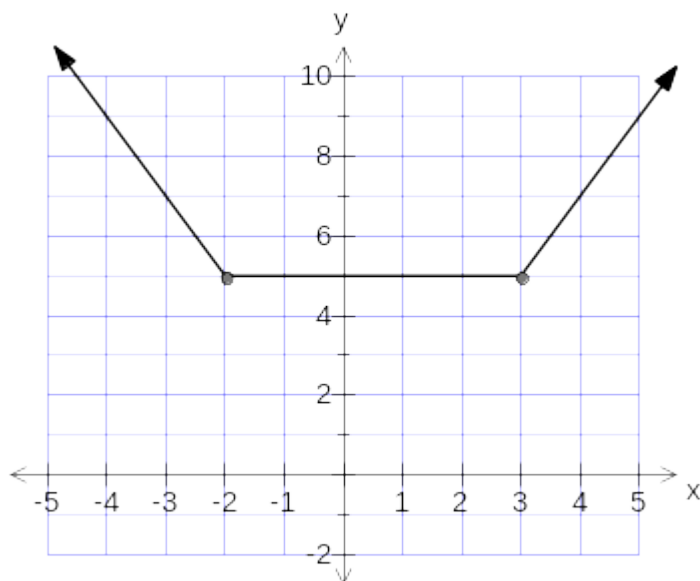
Time: 20 minutes

Total: 20 marks

**Question 1**

**(5 marks)**

The function  $f$ , defined for all real  $x$  by  $f(x) = |x - a| + |x + b|$ , where  $a$  and  $b$  are positive integers, has the following graph.



(a) Find the values of  $a$  and  $b$ .

(b) Express  $f(x)$  as a piecewise function.

**Question 2****(5 marks)**

At 10.00am, two bumper cars at the royal show, G and T, have position vectors,  $\mathbf{r}$  m, and velocity vectors,  $\mathbf{v}$  m/s, as shown below:

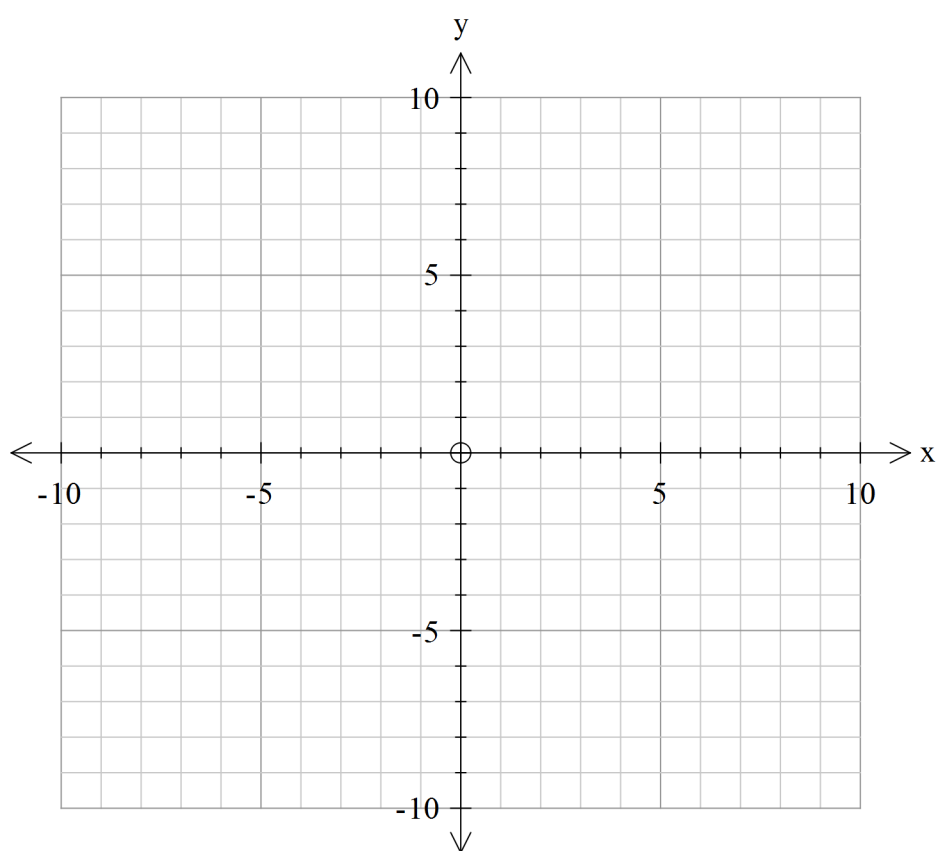
$$\begin{aligned} \mathbf{r}_G &= 3\mathbf{i} + 9\mathbf{j} & \mathbf{v}_G &= -\mathbf{i} - \mathbf{j} \\ \mathbf{r}_T &= 9\mathbf{i} & \mathbf{v}_T &= -5\mathbf{i} + 5\mathbf{j} \end{aligned}$$

Prove that the bumper cars will collide if they continue with these velocities and find the time and location of the collision.



**Question 3****(5 marks)**

Sketch the graph  $y = \frac{x^3}{(x+4)(2x-3)}$ , the asymptotes and describe the behaviour of the graph as  $x \rightarrow \pm\infty$ . Give the equations for the vertical and other asymptotes.



**Question 4**

**(5 marks)**

Find the Cartesian equation of the line perpendicular to the vector  $7\mathbf{i} + 5\mathbf{j}$  and passing through the point  $(-1, 3)$