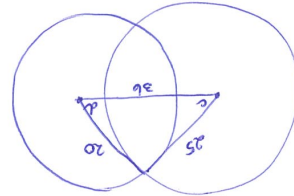


Question 9 (5 marks)

Two circles with radii 25cm and 20cm have their centres 30 cm apart. Determine the size of the common area to both circles correct to nearest square centimetre.



Big Circle

$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab} = \frac{20^2 - 25^2 - 30^2}{-2(25)(30)}$$

$$\cos C = 0.75$$

$$C = 0.7227 \text{ RAD}$$

$$aC = 1.4455 \text{ RAD}$$

$$A_1 = \frac{1}{2}(25) \times (1.4455 - \sin 1.4455)$$

$$= 141.67 \text{ cm}^2$$

$$\cos d = \frac{25^2 - 20^2 - 30^2}{-2(20)(30)}$$

$$= 0.5625$$

$$d = 0.9734 \text{ RAD}$$

$$2d = 1.9468 \text{ RAD}$$

$$A_2 = \frac{1}{2}(20)^2 \times (1.9468 - \sin 1.9468)$$

$$= 203.33 \text{ cm}^2$$

$$\therefore \text{Total Area} = 345 \text{ cm}^2$$

Small Circle

End of test

5



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Semester Two 2018 UNIT 2 METHODS

Test Four

Calculator Assumed 40 minutes / 41 marks

Scientific Calculator, ClassPad, Formula Sheet and One page one side of A4 notes is permitted

Name: Solutions.

Place a tick in the box next to your Mathematics teachers name:

- ☐ Mr Strain
- ☐ Ms Sindel
- ☐ Ms Rimando
- ☐ Mr Gannon
- ☐ Mr Young
- ☐ Mrs Flynn
- ☐ Ms Enslly

Question 1

(1, 1 = 2 marks)

A committee of two is to be chosen from a class of 20 students, 12 boys and 8 girls

- i) How many different committees could be chosen?

$${}^{20}C_2 = 190 \quad \checkmark$$

- ii) How many committees if both students are girls?

$$\binom{8}{2} = 28 \quad \checkmark$$

Question 2

(1, 1, 1 = 3 marks)

Six students, three boys and three girls, are to be seated in a row of six seats.

Find the number of ways in which they can be seated:

- i) if there are no restrictions

$$6! = 720 \quad \checkmark$$

- ii) if the boys must sit together and the girls must sit together

$$(3 \times 2 \times 1 \times 3 \times 2 \times 1) \times 2 = 72 \quad \checkmark$$

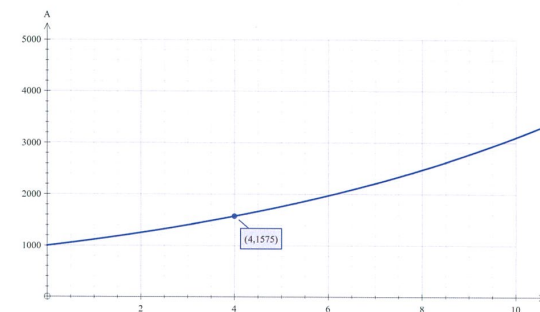
- iii) if the boys must sit together.

$$(3 \times 2 \times 1 \times 3 \times 2 \times 1) \times 4 = 144 \quad \checkmark$$

Question 8

(1, 1, 2 = 4 marks)

The graph below shows the number of assaults in a particular suburb since 1980.



- a) Find an exponential model for the number of assaults each year where t is the time since 1980.

$$\begin{aligned} A &= 1000r^t \\ 1575 &= 1000r^4 \\ \therefore r &= 1.12 \approx 1.12 \\ \therefore A &= 1000(1.12)^t \quad \checkmark \end{aligned}$$

- b) Assuming there was no intervention set up for this suburb, how many assaults would be predicted for 2020?

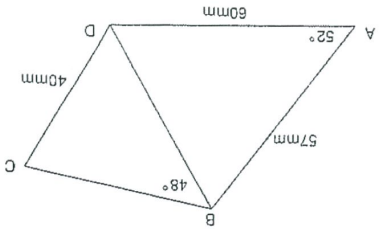
$$\begin{aligned} A &= 1000(1.12)^{40} \\ &\approx 93\,051 \text{ assaults} \quad \checkmark \end{aligned}$$

- c) When will the assault level have increased 100-fold?

$$\begin{aligned} &\text{From } 1000 \text{ to } 100\,000 \\ 100\,000 &= 1000(1.12)^t \\ t &= 40.63 \\ &\approx 41 \text{ years} \\ \therefore \text{In } 2021. &\quad \checkmark \end{aligned}$$

Question 7 (2, 2, 1, 3 = 8 marks)

Consider quadrilateral ABCD, with diagonal BD, dividing the quadrilateral into two acute-angled triangles.



a) Calculate the length of the diagonal BD.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= 57^2 + 60^2 - 2(57)(60) \cos 52^\circ$$

$$c = 51.36 \text{ mm}$$

b) Give the possible sizes of angle C.

$$\frac{\sin C}{51.36} = \frac{\sin 48^\circ}{40}$$

$$\sin C = 0.9542$$

$$C = 72.59^\circ \text{ or } 107.41^\circ$$

c) Why must one of the angles be discarded?

107.41° must be discarded as both triangles are acute.

d) Calculate the area of quadrilateral ABCD.

$$\text{Area} = \frac{1}{2} ab \sin C + \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} (57)(60) \sin 52^\circ + \frac{1}{2} (51.36)(40) \sin 59.41^\circ$$

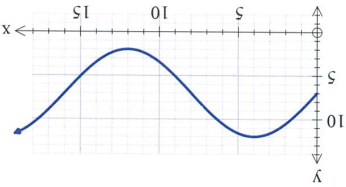
$$= 1347.5 + 884.25$$

$$= 2231.7 \text{ mm}^2$$

8

Question 3 (1, 1, 3 = 5 marks)

Consider the graph below



i) State the amplitude of the graph.

5

ii) State the period of the graph

16

iii) Write an equation that will result in the graph.

$$y = 5 \sin \left(\frac{11x}{8} \right) + 7 \text{ or } 5 \cos \left(\frac{11x}{8} - \frac{\pi}{2} \right) + 7$$

Question 4 (2, 2 = 4 marks)

a) Simplify into index form.

$$\frac{3^{3x+10} + 154x-6}{3^{3x+10}} = \frac{3^{4x-6} \cdot 3^{4x-6}}{3^{4x-6}} = \frac{3^{4x-6}}{3^{4x-6}} = 1$$

b) Solve

$$\sqrt{2} \cos 2x - 1 = 0 \text{ for } -2\pi \leq x \leq 0$$

$$\text{Solve } (\sqrt{2} \cos(ax) - 1 = 0 \mid -2\pi \leq x \leq 0)$$

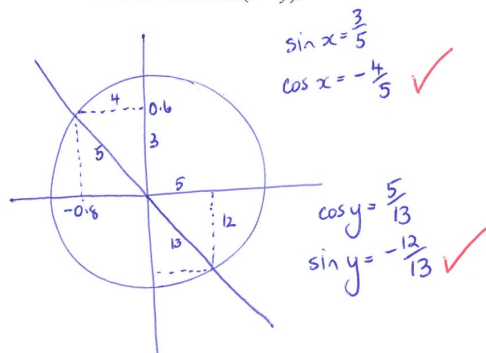
$$-15\frac{\pi}{8}, -9\frac{\pi}{8}, -7\frac{\pi}{8}, -\frac{\pi}{8}$$

9

Question 5

(4 marks)

If $\sin(x) = \frac{3}{5}$ and $\cos(y) = \frac{5}{13}$, where x is in the second quadrant and y in the fourth, find the exact value of $\sin(x - y)$.



$$\begin{aligned}
 \sin(x - y) &= \sin x \cos y - \sin y \cos x \\
 &= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) \checkmark \\
 &= \frac{15}{65} - \frac{48}{65} \\
 &= -\frac{33}{65} \checkmark
 \end{aligned}$$

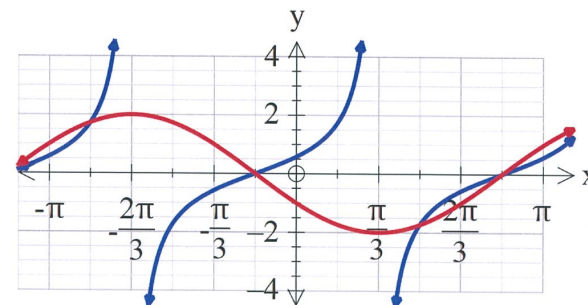
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Question 6

(4, 2 = 6 marks)

Shown below are the graphs of

$f(x) = \tan(ax + b)$ and $h(x) = e \cos(x + f)$ where x is in radians.



- i) Determine the values of the constants a , b , e and f .

$$\begin{aligned}
 a &= 1 \checkmark \\
 b &= \frac{\pi}{6} \checkmark \\
 e &= -2 \checkmark \\
 f &= -\frac{\pi}{3} \checkmark
 \end{aligned}$$

- ii) Use the graph to solve $f(x) = h(x)$, $-\pi \leq x \leq \pi$.

$$-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \checkmark \checkmark$$

6