

2020
TEST 4

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

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7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

(5 marks)

Question 1

Rewrite the following as single a logarithm:

(2 marks)

(a) $1 + 2 \log(x) - \log(1 + y)$

(3 marks)

(b) $2 \ln(3) + \ln(16) - 2 \ln\left(\frac{5}{6}\right)$

See next page

Question 2

(4 marks)

Let $y = x \log_e(3x)$

(a) Find $\frac{dy}{dx}$.

(2 marks)

(b) Hence, calculate $\int_1^2 (\log_e(3x) + 1) dx$. Express your answer in the form $\log_e(a)$, where a is a positive integer.

(2 marks)

See next page

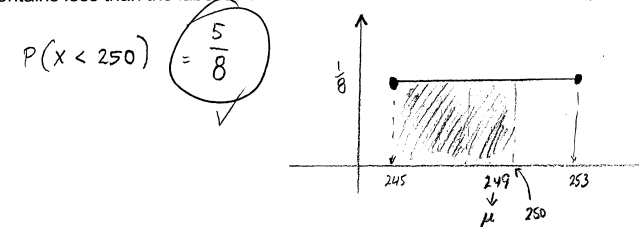
Question 11

(4 marks)

During the COVID-19 pandemic Jeff's Gin Distillery that once produced ethanol for gin, now uses it to produce sanitizer. It fills bottles of sanitizer uniformly to between 245 ml and 253 ml.

The label on the bottle states that it holds 250ml.

- (a) Determine the probability that a bottle selected randomly from the production line contains less than the labelled amount. (1 marks)



- (b) Calculate the mean and standard deviation of the amount of sanitizer in the bottles. (3 marks)

$$\bar{x} = 249 \text{ ml} \quad (\text{Uniform Dist}) \quad \checkmark \quad (\text{MEAN})$$

$$\sigma^2 = \int_{245}^{253} (x - 249)^2 \times \frac{1}{8} dx = 5\frac{1}{3} \quad \left(\frac{16}{3}\right) \quad \checkmark \quad (\text{VAR})$$

$$\therefore \sigma = \sqrt{\text{Var}}$$

$$\sigma = \sqrt{5\frac{1}{3}}$$

$$\sigma = \frac{4\sqrt{3}}{3} \quad (2.31) \quad \checkmark \quad (\text{SD})$$

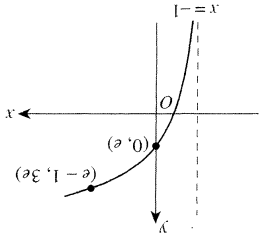
End of questions

4

Question 10

(3 marks)

The graph of $y = a \log_e(x - b) + c$ is shown below.



Calculate the exact values of b , c and a .

① Vertical asymptote $x - b = 0$

$$\therefore -1 - b = 0$$

$$b = -1$$

② Sub in $(0, e) \Rightarrow a \log_e(-b) + c$

$$\text{As } b = -1$$

$$a \log_e 1 + c = e$$

$$\text{As } \log_e 1 = 0$$

$$c = e$$

③ Sub in $(e - 1, 3e)$

$$\text{As } b = -1, c = e$$

$$3e = a \log_e(e - (-1)) + e$$

$$2e = a \log_e e$$

$$5.436$$

$$a = 2e$$

See next page

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Question 3

(3 marks)

Find the maximum value of $y = x^2 e^{-3x}$, $x > 0$. No test for the nature of the stationary point is required.

The area enclosed between the curve $y = \frac{3x-1}{2}$, the lines $x = 1$ and $x = 3$ and the $x - \text{axis}$ can be written as $\log_e B$ square units. Calculate the value of B .

(3 marks)

Question 4

End of questions

Additional working space

Question number: _____

Question 8

(3 marks)

The probability density function for a continuous random variable X has the rule

$$f(x) = \begin{cases} a \sin(\pi x) & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of a .

(1 mark)

Using ClassPad $\int_0^1 a \sin(\pi x) dx = 1$

$$\therefore a = \frac{\pi}{2} \quad \checkmark \quad (1.57)$$

(b) Find $Pr(X \leq \frac{1}{4} | X \leq \frac{1}{2})$.

(2 marks)

$$= \frac{P(X \leq \frac{1}{4} \cap X \leq \frac{1}{2})}{P(X \leq \frac{1}{2})} \quad \checkmark \Rightarrow \frac{P(X \leq \frac{1}{4})}{P(X \leq \frac{1}{2})}$$

$$\frac{-\frac{\sqrt{2}}{4} + \frac{1}{2}}{\frac{1}{2}} = \frac{2 - \sqrt{2}}{4} \div 2 \Rightarrow \frac{2 - \sqrt{2}}{8}$$

(0.2929)

(3 marks)

Question 9

Jo, a physiotherapist, is undertaking a study of her patients' exercising habits. She decides to ask every eighth patient about their exercise habits.

(a) What type of sample is this?

(1 mark)

Systematic Sampling \checkmark

(b) Is this method of sampling valid? Justify your answer.

(2 marks)

(ANSW) YES, assuming that order of sampling is random. \checkmark (REASON)

See next page

Question 7

(10 marks)

Additional working space

Question number: _____

CALCULATOR-FREE

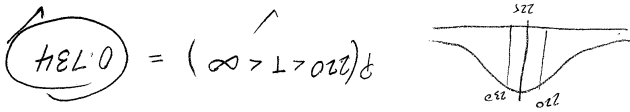
The travelling time for shuttle buses from train station A to train station B (Route AB) is found to be normally distributed with a mean arrival time of 3 hours and 45 minutes and standard deviation of 8 minutes.

- (a) Find the probability that a bus on Route AB will complete its journey in under 3 hours and 50 minutes. Give your answer correct to three decimal places.

$$\mu = 225 \text{ min}, \sigma = 8 \text{ min} \\ T \sim N(225, 8^2), P(T < 230) = 0.734 \quad \text{(parameters)}$$

- (b) Hence, find the probability that the same bus will be no earlier than 5 minutes from its mean arrival time. Give your answer correct to three decimal places.

(2 marks)



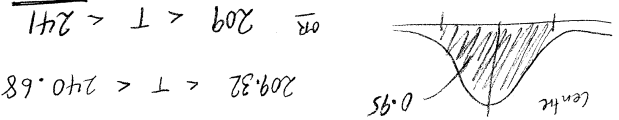
- (c) 90% of the shuttles will complete the Route AB within t minutes. Find t correct to the nearest minute.

(2 marks)



- (d) Within what range of times could we be 95% confident of completing Route AB?

(2 marks)



$$\text{OR } 209 < T < 241 \quad \checkmark \quad \left(\text{left} + \text{right} \right)$$

A random sample of the travel time to complete Route AB is taken from the time sheets of 10 shuttles.

- (e) What is the probability that of the 10 times selected, 6 of them will have completed the journey in under 3 hours and 50 minutes? Give your answer correct to four decimal places.

(2 marks)

$$\text{Binomial} \quad \binom{10}{6} (0.734)^6 (0.266)^4 = 0.16444 \quad \checkmark$$

See next page

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Question 6

(2 marks)

In a large population of fish, the proportion of angel fish is 0.25. Let \hat{p} be the random variable that represents the sample proportion of angel fish for samples of size n drawn from the population.

Find the smallest integer value of n such that the standard deviation of \hat{p} is less than or equal to 0.01.

$$sd(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$$

$$0.01 = \sqrt{\frac{\frac{1}{4} \times \frac{3}{4}}{n}} \quad \checkmark \quad (\text{Equates sd to } 0.01)$$

$$\therefore n = \underline{1875} \quad \checkmark \quad (n \text{ value})$$

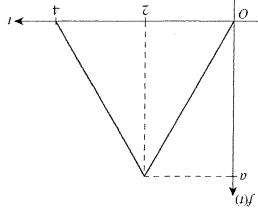
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See next page

Question 5

(5 marks)

The length of time, T minutes, that a Mathematics Methods student takes to answer an examination question is a random variable defined by the probability function whose graph is shown below.



(a) State the value of a so that f is a probability density function. (1 mark)

$$1 = \frac{1}{2}(4) \times a$$

$$\therefore a = \frac{1}{2}$$

(b) Hence state the rule for the probability function f . (2 marks)

Line from (0,0) to (2,1) $\Rightarrow f(t) = t/2$
 " " (2,1) to (4,0) $\Rightarrow f(t) = 1 - t/4$

$$f(t) = \begin{cases} t/2 & \text{for } 0 \leq t \leq 2 \\ 1 - t/4 & \text{for } 2 < t \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

-1 for each missing or incorrect domain

(c) Find $\Pr(T \leq 1)$. (2 marks)

$$\Pr(T \leq 1) = \int_0^1 \frac{t}{2} dt$$

See next page

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MATHEMATICS METHODS Year 12

Section Two:

Calculator-assumed

Christ Church
Grammar School



2020
TEST 4

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 Working time for this section: 30 minutes
 Marks available: 30 marks

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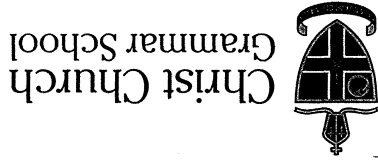
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Calculator-assumed

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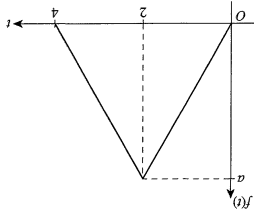
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MATHEMATICS METHODS Year 12

Question 5

(5 marks)

The length of time, T minutes, that a Mathematics Methods student takes to answer an examination question is a random variable defined by the probability function whose graph is shown below.



(a) State the value of a so that f is a probability density function.

(1 mark)

(b) Hence, state the rule for the probability function, f .

(2 marks)

(c) Find $P(T \leq 1)$.

(2 marks)

Question 6

(2 marks)

In a large population of fish, the proportion of angel fish is 0.25. Let \hat{p} be the random variable that represents the sample proportion of angel fish for samples of size n drawn from the population.

Find the smallest integer value of n such that the standard deviation of \hat{p} is less than or equal to 0.01.

See next page

Additional working space

Question number: _____

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Question 7 (10 marks)

The travelling time for shuttle buses from train station A to train station B (Route AB) is found to be normally distributed with a mean arrival time of 3 hours and 45 minutes and standard deviation of 8 minutes.

(a) Find the probability that a bus on Route AB will complete its journey in under 3 hours and 50 minutes. Give your answer correct to three decimal places. (2 marks)

(b) Hence, find the probability that the same bus will be no earlier than 5 minutes from its mean arrival time. Give your answer correct to three decimal places. (2 marks)

(c) 90% of the shuttles will complete the Route AB within t minutes. Find t correct to the nearest minute. (2 marks)

(d) Within what range of times could we be 95% confident of completing Route AB? (2 marks)

A random sample of the travel time to complete Route AB is taken from the time sheets of 10 shuttles.

(e) What is the probability that of the 10 times selected, 6 of them will have completed the journey in under 3 hours and 50 minutes? Give your answer correct to four decimal places. (2 marks)

See next page

Question 8

(3 marks)

The probability density function for a continuous random variable X has the rule

$$f(x) = \begin{cases} \sin(\pi x) & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of a .

(1 mark)

(b) Find $P\left(X \leq \frac{1}{4} \mid X \leq \frac{1}{2}\right)$.

(2 marks)

Question 9

(3 marks)

Jo, a physiotherapist, is undertaking a study of her patients' exercising habits. She decides to ask every eighth patient about their exercise habits.

(a) What type of sample is this?

(1 mark)

(b) Is this method of sampling valid? Justify your answer.

(2 marks)

See next page

Question 3

(3 marks)

Find the maximum value of $y = x^2 e^{-3x}$, $x > 0$. No test for the nature of the stationary point is required.

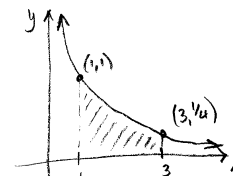
$$\begin{aligned} \text{Max when } \frac{dy}{dx} &= 0 & u'v + uv' \\ 2x e^{-3x} + x^2(-3)e^{-3x} &= 0 & \checkmark \left(\frac{dy}{dx} = 0\right) \\ x e^{-3x} (2 - 3x) &= 0 \\ x = 0 &\text{ or } x = \frac{2}{3} & \checkmark \text{ (solves for } x) \\ \therefore y &= \left(\frac{2}{3}\right)^2 e^{-3\left(\frac{2}{3}\right)} \\ \therefore \text{Max value } &\left(\frac{4}{9e^2}\right) & \checkmark \text{ (MAX)} \end{aligned}$$

Question 4

(3 marks)

The area enclosed between the curve $y = \frac{2}{3x-1}$, the lines $x = 1$ and $x = 3$ and the x -axis can be written as $\log_e B$ square units. Calculate the value of B .

$$\begin{aligned} \int_1^3 \frac{2}{3x-1} dx &= \frac{2}{3} \int_1^3 \frac{3}{3x-1} dx \\ &= \frac{2}{3} \left[\ln(3x-1) \right]_1^3 & \checkmark \text{ (Anti-diff)} \\ &= \frac{2}{3} [\ln 8 - \ln 2] & \checkmark \text{ (sub in)} \\ &= \frac{2}{3} \ln 4 \\ &= \ln 4^{\frac{2}{3}} \\ \therefore B &= 4^{\frac{2}{3}} & \checkmark \text{ (B-value)} \end{aligned}$$

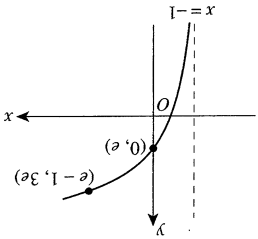


End of questions

Question 10

(3 marks)

The graph of $y = a \log_e(x - b) + c$ is shown below.



Calculate the exact values of b , c and a .

See next page

Question 2

Let $y = x \log_e(3x)$

(a) Find $\frac{dy}{dx}$.

(4 marks)

(2 marks)

$$\frac{dy}{dx} = \ln(3x) + x \cdot \frac{1}{x} \quad \checkmark \quad \text{(uses product Rule)}$$

$$= \ln(3x) + 1 \quad \checkmark \quad \text{(simplifies)}$$

(b) Hence, calculate $\int_1^2 (\log_e(3x) + 1) dx$. Express your answer in the form $\log_e(a)$, where a is a positive integer.

(2 marks)

$$\therefore \int_1^2 \ln(3x) + 1 \, dx = \left[x \ln(3x) \right]_1^2 \quad \checkmark \quad \text{(integrates)}$$

$$= 2 \ln 6 - \ln 3$$

$$= \ln 36 - \ln 3$$

$$= \ln 12 \quad \checkmark \quad \text{(ANSWER is } \ln(a) \text{ for } a = 12)$$

See next page

4

Question 11

(4 marks)

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The label on the bottle states that it holds 250ml.

- (a) Determine the probability that a bottle selected randomly from the production line contains less than the labelled amount. (1 marks)

- (b) Calculate the mean and standard deviation of the amount of sanitizer in the bottles. (3 marks)

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Question 1

(5 marks)

Rewrite the following as single a logarithm:

- (a) $1 + 2 \log(x) - \log(1+y)$ (2 marks)

$$= \log 10 + \log x^2 - \log(1+y) \quad \checkmark \left(\begin{array}{l} \text{convert 1 to } \log 10 \\ \frac{1}{2} \text{ uses 1 log law.} \end{array} \right)$$

$$= \log 10x^2 - \log(1+y)$$

$$= \log \left(\frac{10x^2}{1+y} \right) \quad \checkmark \text{ (ANSW)}$$

- (b) $2 \ln(3) + \ln(16) - 2 \ln\left(\frac{6}{5}\right)$ (3 marks)

$$= \ln 3^2 + \ln 16 - \ln \left(\frac{6}{5}\right)^2 \quad \checkmark \text{ (Brings 2 to index)}$$

$$= \ln 9 + \ln 16 - \ln \frac{36}{25}$$

$$= \ln \left(\frac{9 \times 16}{\frac{36}{25}} \right) \quad \checkmark \text{ (2x log laws)}$$

$$= \ln \frac{9 \times 16^4 \times 25}{36^4}$$

$$= \ln 100 \quad \checkmark \text{ (ANSW) as single logarithm}$$

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CALCULATOR-ASSUMED

10 MATHEMATICS METHODS Year 12

Additional working space

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Christ Church
Grammar School

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MATHEMATICS METHODS Year 12

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