Test 6
Continuous Random Variables

This assessment contributes 7% towards the final year mark.
Time Allowed : 40 minutes
This is a Calculator-Assumed Task.
No notes of ANY nature are permitted for this assessment.

Full marks may not be awarded to correct answers unless sufficient justification is given.

Name : ______________________ **Score :** (out of 35) ____

Do not turn over this page until you are instructed to do so.

1. (5 marks)

The lifetimes (t hours) of a consignment of electric light bulbs are displayed below.

t	$t \le 50$	$t \le 100$	$t \le 150$	$t \le 200$	$t \le 250$	$t \le 300$	$t \le 350$	$t \le 400$
Number of bulbs	8	20	60	180	250	300	380	410

Define the random variable T: Lifetime of light bulbs.

Estimate

a) The mean and variance of T. (2 marks)

b) The lifetime exceeded by 10% of the light bulbs. (3 marks)

6. (7 marks)

The Longlife Tyre Company produces a radial tyre which has a mean lifetime of 50 000 km and a standard deviation of 5 000 km. The lifetime of a tyre is normally distributed.

a) Find the probability that any one tyre will last longer than 40 000 km. (1 mark)

b) Find the probability that all four tyres bought by a particular customer will last more than 40 000 km. (2 marks)

c) Find the probability that a tyre lasts more than 60 000 km given that it has already lasted 40 000 km. (2 marks)

d) Approximately fifty percent of the tyres last between 45,000 km and x km. Find the value of x accurate to the nearest one hundred. (2 marks)

5. (4 marks)

A continuous random variable X has a mean of 55 and a standard deviation of 5. With a change of scale and origin the random variable Y= aX + b. Find a and b if the mean and standard deviation of Y are 173 and 15 respectively.

2. (7 marks)

A continuous random variable, X, has a probability density function given

by $$f(x)=\begin{cases} kx & 0\le x\le 2 \\ 2k & 2<x\le 6 \\ 0 & otherwise \end{cases}$$

a) Determine the value of k. (3 marks)

b) Find

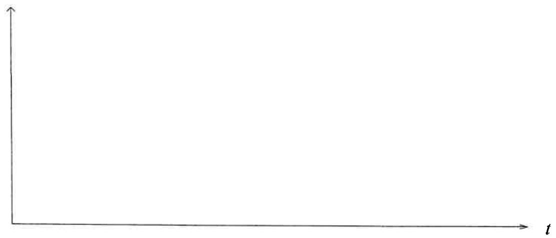
i) $P(X\le 4)$ (1 mark)

ii) M, the median of the distribution. (3 marks)

3. (8 marks)

The serving time, T seconds, for a customer at an automatic banking machine is a uniformly distributed random variable, with lower and upper limits 50 and 150. The mean serving time is 100 seconds and the standard deviation is 28.9 seconds. The serving times for different customers are independent.

a) Sketch the graph of the distribution function for T. (2 marks)



b) Evaluate $P(T = 120)$. (1 mark)

c) Evaluate $P(T \geq 120)$. (1 mark)

d) What is the probability that exactly 3 of the next 5 customers will require at least 2 minutes to be served? (2 marks)

e) Find the cumulative distribution function for T. (2 marks)

4. (4 marks)

The queuing time, X minutes, of a traveller at the ticket office of a large railway station has a probability density function f defined by

$$f(x)=\begin{cases} 0.0004x(100-x^2) & 0 \leq x \leq 10 \\ 0 & otherwise \end{cases}$$

a) Find the mean of the distribution. (2 marks)

b) Find the standard deviation of the distribution, correct to 2 decimal places. (2 marks)