

Question/Answer Booklet

(This paper is not to be released to take home before 25/6/2012)

MATHEMATICS 3C

Section Two: Calculator-assumed

Name of Student: _____

Time allowed for this section

Reading time before commencing work: 10 minutes

Working time for this section: 100 minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the student

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

Important note to students

No other items may be used in this section of the examination. It is **your** responsibility to ensure

that you do not have any unauthorised notes or other items in the examination room.
If you have any unauthorised material with you, hand it to the supervisor **before**
reading any further.

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|-----------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|--------------------|
| Section One Calculator-free | 6 | 6 | 50 | 50 | |
| Section Two Calculator-assumed | 12 | 12 | 100 | 100 | |
| | | Total | 150 | 100 | |

Instructions to students

- 1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- 2 **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you

Question 18 (continued)

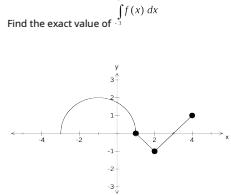
(iii) $\int_{-1}^t f(x) dx$

(1)

repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

- (b) The graph of a function $f(x)$ consists of a semi-circle and two line segments as shown.



(2)

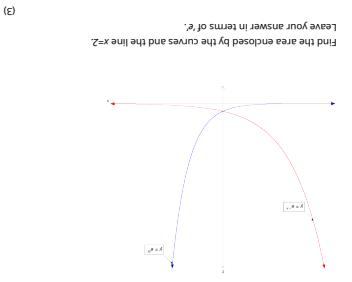
Find the exact value of $\int_{-1}^t f(x) dx$

Leave your answer in terms of π .

(i) Find a given that $C_1(0.2)^a(0.8)^{16} = 0.167772$

(1)

(ii) Explain the meaning of your answer to part (b) with respect to the fruit balls.



Leave your answer in terms of e .

Find the area enclosed by the curves and the line $x=2$.

(i) Calculate $C_1(0.2)^a(0.8)^b$

(1)

(ii) A random sample of ten fruit balls is taken from the box.

(i) Explain the meaning of $C_1(0.2)^a(0.8)^b$ with respect to this sample.

(1)

(ii) Calculate $C_1(0.2)^a(0.8)^b$

(i) Calculate $C_1(0.2)^a(0.8)^b$

(1)

(ii) Calculate $C_1(0.2)^a(0.8)^b$

**Section Two: Calculator-assumed
(100 marks)**

This section has **twelve (12)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

**Question 7
marks)**

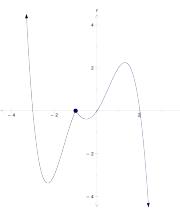
(10)

- (a) Emily is a very strong soccer player who has a probability of $\frac{3}{5}$ of scoring a goal with each attempt. She has 15 attempts. Find the probability that the number of goals she scores is less than 7.
(2)

Question 18

(6 marks)

- (a) For the function $y = f(x)$ below

**Question 18**

(6 marks)

- (a) For the function $y = f(x)$ below

Find the expected number of these garden gnomes that will be Superior.
(2)

$P(2 \text{ Superior}) = 0.65$
 $P(1 \text{ Superior}) = 0.28$
 $P(0 \text{ Superior}) = 0.09$

It is also found that of these three gnomes,
(c) A group of 3 consecutive garden gnomes is inspected and the first is a Regular.

It is also found that of these three gnomes,
(b) If the first garden gnome inspected is Superior, find the probability that the next three gnomes are Superior.

- (b) Suppose that Y is distributed normally with unknown mean μ and standard deviation σ .

Given that $P(\mu - 2.5 \leq Y \leq \mu + 2.5) = 0.9$, find the value of σ .
(2)

Use the information above and mathematical reasoning to determine the value of each of the following.

$$(i) \int_{-1}^0 f(x) dx$$

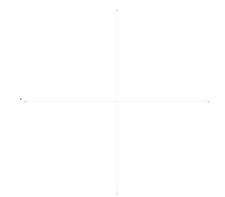
(2)

- (ii) the area between the curve and the x -axis from $x = -3$ to $x = 0$
(1)

Question 11 (continued)

$$(iii) \text{ Scale the coordinates of } A \text{ if the curve is transformed to } y = -\sqrt{2}x + 2.$$

(2)



If the first garden gnome inspected is Superior, find the probability that the next three garden gnomes are Superior.
(2)

If the first garden gnome is Superior, then the probability that the next three garden gnomes are Superior is 0.7.

If the garden gnome is Superior, then the probability that the next three garden gnomes are Superior is 0.3. If the garden gnome is Regular, then the probability that the next three garden gnomes are Superior is 0.5. If the garden gnome is Superior, then the probability that the next three garden gnomes are Superior is 0.3. If the garden gnome is Regular, then the probability that the next three garden gnomes are Superior is 0.5.

If the garden gnome is Superior, depending on the quality of their work, Adam awards quality control in the order of completion, who classifies them as either Superior, or Regular.

Adam awards quality control in the order of completion, who classifies them as either Superior, or Regular, depending on the quality of their work.

Adam awards quality control in the order of completion, who classifies them as either Superior, or Regular, depending on the quality of their work.

Question 7 (continued)

- (c) (ii) all three are greater than 5

(1)

- (iii) the least integer is 5.

(1)

- (iv) the three integers are different given that the least integer selected is 5.
-
- (2)

- (v) the sum of the three integers is more than 15.

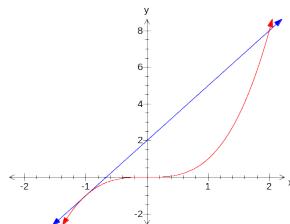
(1)

Question 17**(9 marks)**

- (a) Shade the region, R, bounded by the curves,
- $y = x^3$
- ,
- $y = 3x + 2$
- , and
- $x = 0$
- in the diagram.

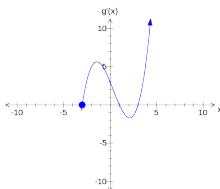
Find the area of the region R, showing all working steps.

(4)



(d) Adam's little brother, Brodie, joins in this business venture. The probability that any one of Brodie's painted garden gnomes is Regular is 0.8. He wants to ensure that the minimum number of gnomes that Brodie would need to paint to achieve this aim is 3. Calculate the probability that he paints at least two Superior is at least 0.9. Calculate the minimum number of gnomes that Brodie would need to paint to achieve this aim.

Question 13 (continued)

Question 16The graph of $g(x)$ is given below.

- (a) What can be said about the gradient of the function
- $g(x)$
- between
- $x = -3$
- to
- $x = 1$
- ?

- (1) When does the function, $g(x)$ have a negative gradient? (1)
- (c) State an equation for the tangent to the graph of $g(x)$ at $x = 3$. (2)
- (d) Find the value of x at which $g(x)$ has a relative maximum for $-3 \leq x \leq 4$ (1)

- (e) Find the x -coordinate of each point of inflection of the graph of $g(x)$ for $-3 \leq x \leq 4$ (2)

Question 8**(7 marks)**

- (a) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth y of fluid in the tank t hours after the valve is opened is given by

$$y = 6 \left| 1 - \frac{t}{12} \right|^3 \text{ metres.}$$

- (i) Find the rate $\frac{dy}{dt}$ m/hour at which the tank is draining at time, t . (2)

- (ii) Using calculus, find the length of each part of the wire when the sum of the areas is a minimum. (5)

- (iii) When is the fluid in the tank falling fastest and slowest? (1)
- What are the values of $\frac{dy}{dt}$ at these times? (2)

- (b) When does the function, $g(x)$ have a negative gradient? (1)

- (c) State an equation for the tangent to the graph of $g(x)$ at $x = 3$. (2)

- (d) Find the value of x at which $g(x)$ has a relative maximum for $-3 \leq x \leq 4$ (1)

- (e) Find the x -coordinate of each point of inflection of the graph of $g(x)$ for $-3 \leq x \leq 4$ (2)

Question 8 (continued)

- (b) If the volume of a cylinder is given by $V = \pi r^2 h$, find the appropriate percentage change in V when r changes by 2%
 $\frac{1}{3}$
 (3)

- (iii) If event A is $X < 3$ and event B is $X < 10$, are these two events independent? Justify your answer.
 (4)
- (ii) Do the packages contain bolts that are more consistent in length?
 (2)

- (i) Do the manufacturer better off justify
 (1)
- (ii) Do the manufacturer better off justify.
 (2)
- (iii) Do the manufacturer better off justify.
 (1)

- (d) A new quality controller suggests adjusting the settings on the machines so that the standard deviation becomes 0.55 mm and that only the shortest 5% and the longest 5% of the bolts are rejected.
- Find the new minimum and maximum acceptable lengths correct to the nearest 0.1 mm.
 (3)
- (e) Find the probability that a randomly selected bolt is longer than 10 mm.
 (2)
- (f) The probability distribution of X where random variable X is the sum of the uppermost numbers when two fair die are rolled is tabulated below.
- | $P(X=x)$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|----------|----|----|----|----|----|----|----|----|----|----|----|----|
| x | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 1 |
- (g) Is this a reasonable outcome for the company? Justify your answer.
 (2)
- (h) Find the expected number of acceptable bolts in a batch of 100 000.
 (1)
- (i) Find the probability of a randomly selected bolt being an acceptable length.
 (2)
- (j) Nuts and Bolts Company manufactures 120mm bolts which are normally distributed with a mean length of 118 mm and a standard deviation of 1.1 mm. Only bolts which are between 118 mm and 121 mm pass inspection and are packaged as 120mm bolts.
- (k) Give two reasons why the following cannot be a probability distribution.
 (2)
- | $P(X=x)$ | 0.0 | 0.1 | 0.4 | 0.1 | 0.5 | 0.3 |
|----------|-----|-----|-----|-----|-----|-----|
| x | 3 | 4 | 5 | 6 | 7 | 8 |
- (l) The probability distribution of X where random variable X is the sum of the marks obtained in two questions. Calculate the expected value of X .
 (10)