

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

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Instructions to candidates

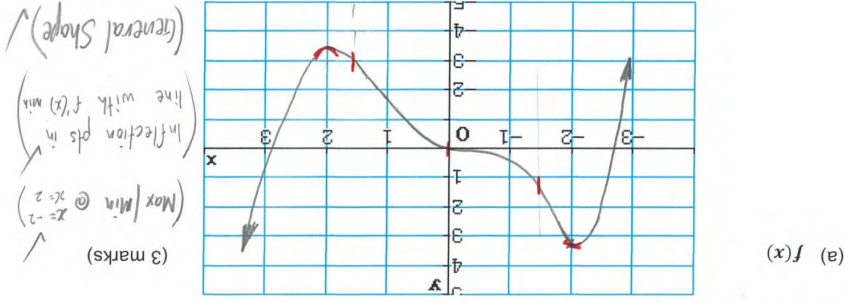
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See next page

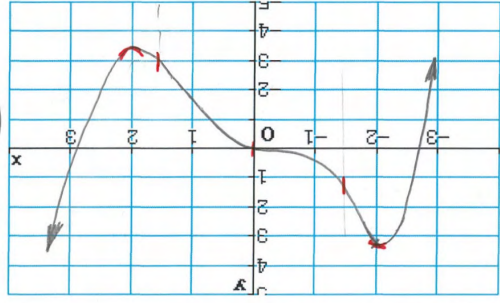
Question 9

Sketch the possible graphs of $f(x)$ and $f''(x)$ on the axes provided below given the graph of the derivative function $f'(x)$.

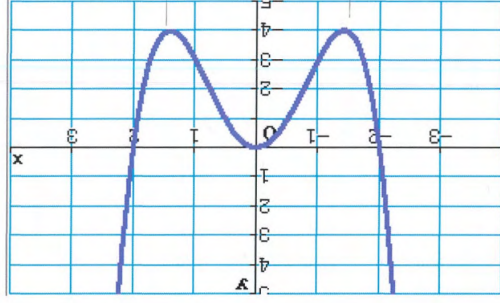
(5 marks)



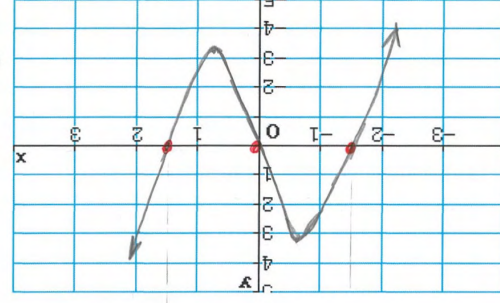
(a) $f(x)$



$f'(x)$



(b) $f''(x)$



End of questions

5

(2 marks)
Shape ✓
Roots ✓

Question 1

(2 marks)

The curve $y = kx^2 - 7x + 6$ has a gradient of 11 when $x = 3$. Find the value of k .

(3 marks)

Question 2

Find the derivative of $\frac{\sin(2\theta)}{\cos(2\theta)}$ with respect to θ . You must show full working with use of the Quotient Rule. Simplify your answer.

Question 3

(6 marks)

Consider the function $f(x) = \frac{1}{\sqrt{x}}$.

(2 marks)

(a) Calculate the derivative of $f(x)$.

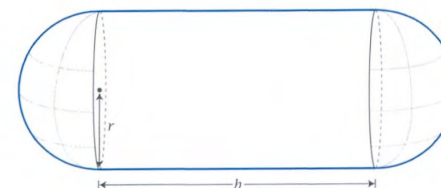
(b) Using your answer to part (a) and the function $f(x)$, calculate the approximate value of $\frac{1}{\sqrt{100.5}}$, leaving your answer as a fraction. (4 marks)

See next page

Question 6

(7 marks)

A tank is to be built with a volume of $200\pi \text{ m}^3$. It is to be built with metal at a cost of $\$p$ per m^2 of cylindrical surface and $\$2p$ per m^2 of hemispherical surface. The tank is to be built according to the diagram shown below:



(a) Show that the cost function $C = \frac{400\pi p}{r} + \frac{16\pi r^2 p}{3}$. (3 marks)

$$V = 200\pi$$

$$\therefore 200\pi = \frac{4}{3}\pi r^3 + \pi r^2 h \rightarrow h = \frac{200\pi - \frac{4}{3}\pi r^3}{\pi r^2}$$

✓ (isolates h from volume)

$$\text{Cost } SA = 2\pi r h (p) + 4\pi r^2 (2p)$$

sub in h .

✓ (SA expression)

$$h = \frac{200}{r^2} - \frac{4}{3}r$$

$$SA = 2\pi r p \left(\frac{200}{r^2} - \frac{4}{3}r \right) + 8\pi r^2 p$$

$$= \frac{400\pi p}{r} - \frac{8\pi r^2 p}{3} + \frac{24}{3}\pi r^2 p \quad \checkmark \quad \text{(Subs in } h \text{ and simplifies)}$$

$$= \frac{400\pi p}{r} + \frac{16\pi r^2 p}{3}$$

As required.

See next page

Question 3

Consider the function $f(x) = \frac{1}{\sqrt{x}}$. (6 marks)

(a) Calculate the derivative of $f(x)$.

$$f(x) = x^{-1/2}$$

$$f'(x) = -\frac{1}{2}x^{-3/2}$$

(correct derivative)

$$\text{or } f'(x) = -\frac{1}{2\sqrt{x^3}}$$

(b) Using your answer to (a) and the function $f(x)$, calculate the approximate value of $\frac{1}{\sqrt{100.5}}$, leaving your answer as a fraction. (4 marks)

$$\text{When } x=100 \Rightarrow y = \frac{1}{10} \neq \frac{dy}{dx} = -\frac{1}{2}(100)^{-3/2}$$

$$\frac{dy}{dx} = -\frac{1}{2000}$$

$$\left. \frac{dy}{dx} \right|_{x=1000}$$

$$\text{Using } \frac{dy}{dx} \approx \frac{\delta y}{\delta x} \Rightarrow \delta y \approx \frac{dy}{dx} \times \delta x$$

$$\delta y \approx -\frac{1}{2000} \times 0.5$$

[incremental formula]

for $x=100$
 $\delta x = 0.5$

$$\therefore \delta y \approx -\frac{1}{4000}$$

$$\text{Hence } \frac{1}{\sqrt{100.5}} = \frac{1}{10} - \frac{1}{4000}$$

$$= \frac{399}{4000}$$

(fraction)

6

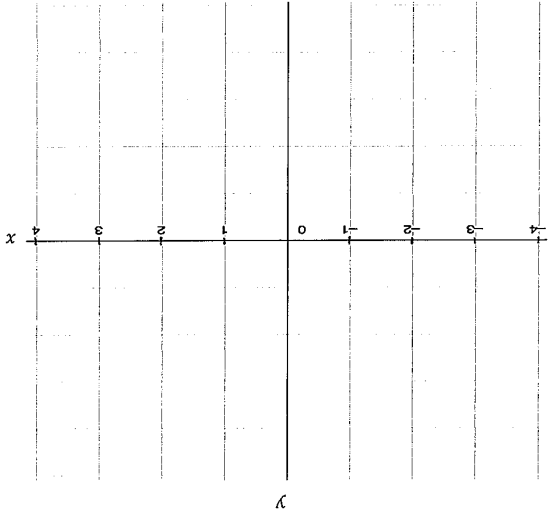
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Question 4

(4 marks)

Sketch a function $y = f(x)$ with all of the following features.

- $f(1) = f(-1) = 0$
- $f'(0) = f'(2) = 0$
- $f''(2) = 0$
- $f'(x) < 0$ for $x < 0$
- $f'(x) > 0$ for $x > 2$, $0 < x < 2$



End of Questions

Additional working space

Question number: _____

Question 8

(5 marks)

Two variables x and y are such that $x^4y = 8$. A third variable z is defined by $z = x + y$.(a) Find the values of x and y that give z a stationary value.

(3 marks)

$$x^4y = 8 \Rightarrow y = \frac{8}{x^4}$$

$$\therefore \underline{z = x + 8x^{-4}} \quad \checkmark \quad (z \text{ in terms of 1 variable})$$

$$\frac{dz}{dx} = 1 - \frac{32}{x^5} \quad \text{for stat pt} \quad \frac{dz}{dx} = 0$$

$$\therefore \frac{32}{x^5} = 1$$

$$\underline{x = 2}, \text{ hence } \underline{y = \frac{1}{2}}$$

\checkmark \checkmark
 (x-value) (y-value)

(b) Use the second derivative test to show that this value of z is a minimum.

(2 marks)

$$\underline{\frac{d^2z}{dx^2} = \frac{160}{x^6}} \quad \checkmark \quad (\text{shows } \frac{d^2z}{dx^2})$$

$$\left. \frac{d^2z}{dx^2} \right|_{x=2} = \frac{160}{2^6} > 0 \quad \therefore \text{Min}$$

\checkmark
($\frac{d^2z}{dx^2} > 0$)

See next page

Question 7

(a) Find $f'(\pi)$, if $f(x) = \frac{3x^2+1}{\cos(x)}$, leaving your answer as an exact value. (2 marks)

$$f'(x) \Big|_{x=\pi} \text{ on Grad} = -6\pi //$$

(b) A curve with equation $y = ax^3 + bx^2 + cx + d$ has zero gradient at the point $(\frac{1}{4}, \frac{3}{4})$ and also touches, but does not cross, the x-axis at the point $(1,0)$. Find the values of x for which the curve has a negative gradient.

$$y' = 3ax^2 + 2bx + c$$

$$\left. \begin{array}{l} \text{Sub } y' \\ (1,0) \Rightarrow 0 = a + b + c + d \\ \text{Sub } y \\ (1,0) \Rightarrow 0 = a + b + c + d \\ \text{Sub } y' \\ (\frac{1}{4}, \frac{3}{4}) \Rightarrow \frac{3}{4} = \frac{3}{4}a + \frac{1}{4}b + \frac{1}{4}c + d \\ \text{Sub } y \\ (\frac{1}{4}, \frac{3}{4}) \Rightarrow \frac{3}{4} = \frac{3}{4}a + \frac{1}{4}b + \frac{1}{4}c + d \end{array} \right\} \begin{array}{l} \text{Solve on} \\ \text{Grad} \\ \text{Sets up} \\ \text{equations} \end{array}$$

(3 marks)

$$a=1, b=-2, c=1, d=0 \quad \checkmark \quad \text{[Solves for variables]} \quad y = x^3 - 2x^2 + x \quad \checkmark \quad \text{graph on Grad}$$

$$\therefore \text{Neg. Gradient for } \{x: \frac{1}{3} < x < 1\}$$

See next page

5

See next page



Christ Church
Grammar School

2019
TEST 1

MATHEMATICS METHODS Year 12

Section Two:

Calculator-assumed

Your name _____

Teacher's name _____

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Working time for this section: 30 minutes
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MATHEMATICS METHODS Year 12

5

CALCULATOR-ASSUMED

Question 6 continued

(b) Find the radius for the minimum cost of the tank, verifying that it is a minimum. (3 marks)

$$\frac{dc}{dr} = \frac{32pr^3\pi - 1200\pi p}{3r^2} \quad \checkmark \quad \left[\text{Shows } \frac{dc}{dr} \right]$$

$$\frac{dc}{dr} = 0 \Rightarrow r = \sqrt[3]{\frac{75}{2}}$$

$$r = \underline{3.347\text{m}} \quad \checkmark \quad (\text{radius})$$

$$\text{Check } \left. \frac{d^2c}{dr^2} \right|_{r=3.347} = \underline{100.54p} > 0 \quad \checkmark \quad \left[\begin{array}{l} 2^{\text{nd}} \text{ deriv.} \\ \text{check} \end{array} \right]$$

∴ Min

(c) Find the cost when $p = 10$.

(1 mark)

$$c|_{p=10} \text{ on CPad} = \underline{\$5631.50} \quad \checkmark$$

See next page

4

Question 5 (8 marks)

A particle moves in such a way that its displacement x metres from the origin is given by $x = t^3 - 6t^2 + 9t - 1$, where t is the time in seconds.

Determine:

(a) where the particle is initially. (1 mark)

$$x|_{t=0} = -1 \text{ m} \quad \checkmark \quad \text{or } 1 \text{ m to left.}$$

(b) an expression for the velocity of the particle in terms of t . (1 mark)

$$x'(t) = 3t^2 - 12t + 9 \text{ m/s} \quad \checkmark$$

(c) when the particle is at rest. (2 marks)

$$\text{When } x'(t) = 0$$

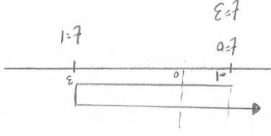
$$0 = 3t^2 - 12t + 9$$

$$t = 1, 3 \text{ sec} \quad \checkmark$$

(d) an expression for the acceleration of the particle and the acceleration when $t = 2$ seconds. (2 marks)

$$a = 6t - 12 \text{ ms}^{-2} \quad \checkmark$$

$$a|_{t=2 \text{ sec}} = 6(2) - 12 = 0 \text{ ms}^{-2} \quad \checkmark$$



(e) the distance travelled in the first 3 seconds.

$$\text{total dist} = 8 \text{ m} \quad \checkmark$$

$$\left[-1 \text{ overall for missing units more than once} \right]$$

See next page

8

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(b) an expression for the velocity of the particle in terms of t . (1 mark)

(c) when the particle is at rest. (2 marks)

(d) an expression for the acceleration of the particle and the acceleration when $t = 2$ seconds. (2 marks)

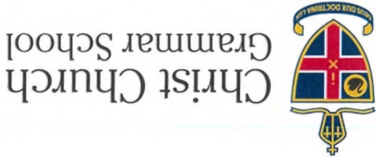
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MATHEMATICS METHODS Year 12
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Calculator-assumed

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Teacher's name _____

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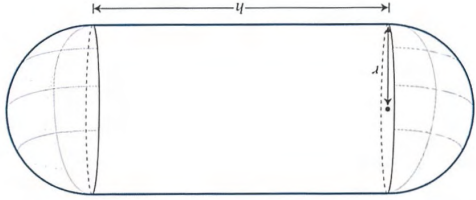
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See next page

MATHEMATICS METHODS Year 12 4
CALCULATOR-ASSUMED
Question 6 (7 marks)

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(a) Show that the cost function $C = \frac{400\pi p}{r} + \frac{16\pi r^2 p}{3}$. (3 marks)

Question 6 continued

- (b) Find the radius for the minimum cost of the tank, verifying that it is a minimum.
(3 marks)

- (c) Find the cost when $p = 10$.
(1 mark)

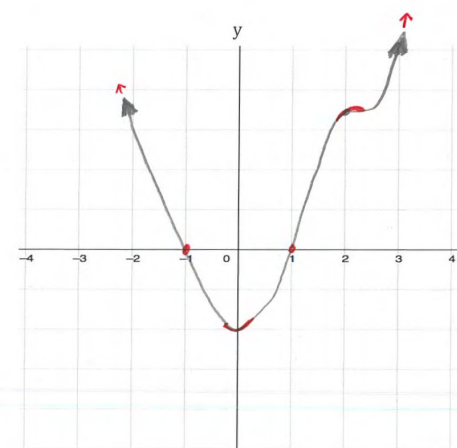
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Question 4

(4 marks)

Sketch a function $y = f(x)$ with all of the following features.

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- $f'(0) = f'(2) = 0$
- $f''(2) = 0$
- $f'(x) < 0$ for $x < 0$
- $f'(x) > 0$ for $x > 2$, $0 < x < 2$



roots ✓
Horiz infl $x=2$ ✓
Min TP ✓
Shape as $x \rightarrow \pm\infty$ ✓

4

End of Questions

Question 1

(2 marks)

The curve $y = kx^2 - 7x + 6$ has a gradient of 11 when $x = 3$. Find the value of k .

$$\begin{aligned} \frac{dy}{dx} &= 2kx - 7 \\ \Rightarrow \frac{dy}{dx} \bigg|_{x=3} &= 11 \\ \therefore 11 &= 2k(3) - 7 \\ k &= 3 \end{aligned}$$

Question 2

(3 marks)

Find the derivative of $\frac{\sin(2\theta)}{\cos(2\theta)}$ with respect to θ . You must show full working with use of the Quotient Rule. Simplify your answer.

$$\begin{aligned} & \frac{u'v - uv'}{v^2} = \frac{2\cos(2\theta) \cdot \cos(2\theta) - \sin(2\theta) \cdot (-2\sin(2\theta))}{\cos^2(2\theta)} \\ &= \frac{2\cos^2(2\theta) + 2\sin^2(2\theta)}{\cos^2(2\theta)} \\ &= \frac{2[\cos^2(2\theta) + \sin^2(2\theta)]}{\cos^2(2\theta)} \\ &= \frac{2}{\cos^2(2\theta)} \end{aligned}$$

Simplifies

$$\cos^2(2\theta) + \sin^2(2\theta) = 1$$

Pythagorean Identity

Quotient Rule

See next page

5

Question 7

(5 marks)

(a) Find $f'(\pi)$, if $f(x) = \frac{3x^2+1}{\cos(x)}$, leaving your answer as an exact value. (2 marks)

(b) A curve with equation $y = ax^3 + bx^2 + cx + d$ has zero gradient at the point $(\frac{1}{3}, \frac{7}{4})$ and also touches, but does not cross, the x -axis at the point $(1,0)$. Find the values of x for which the curve has a negative gradient. (3 marks)

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Question 8

(5 marks)

Two variables x and y are such that $x^4y = 8$. A third variable z is defined by $z = x + y$.

- (a) Calculate the values of x and y at the point where z is a stationary point.

(3 marks)

- (b) Use the second derivative test to show that this value of z is a minimum.

(2 marks)

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Christ Church
Grammar School

2019
UNIT TEST 1

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

SOLUTIONS

FINAL

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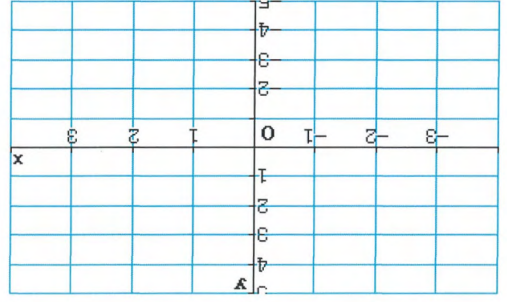
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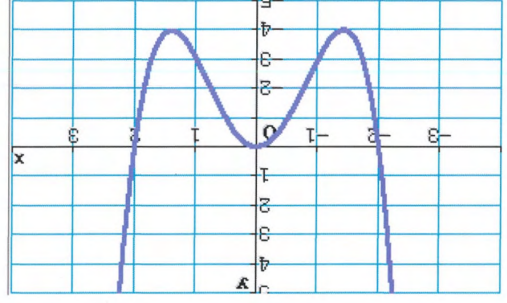
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Question 9 (3, 2 marks)

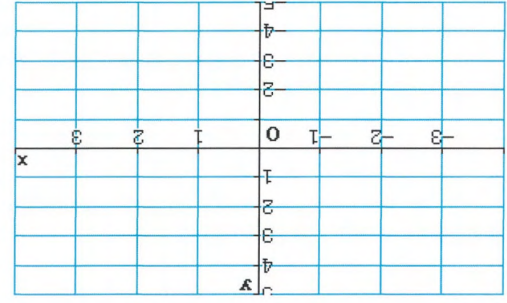
Sketch the possible graphs of $f(x)$ and $f''(x)$ on the axes provided below given the graph of the derivative function $f'(x)$.



$f(x)$



$f'(x)$



$f''(x)$

End of questions

Additional working space

Question number: _____

See next page

Additional working space

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See next page