PENRHOS COLLEGE

STRIVE FOR THE HIGHEST

Semester 1 Examination, 2012

Question/Answer Booklet

MATHEMATICS
3C/3D (Year 12)
Section One:
Calculator-free

Your name: **SOLUTIONS**

Please circle your teacher's name: S Ebert T Hosking S Rowden

Time allowed for this section
Reading time before commencing work: five minutes
Working time for paper: fifty minutes

Material required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	8	8	50	50
Section Two: Calculator-assumed	13	13	100	100
				150

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

See next page

Section One: Calculator-free **(50 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time for this section is 50 minutes.

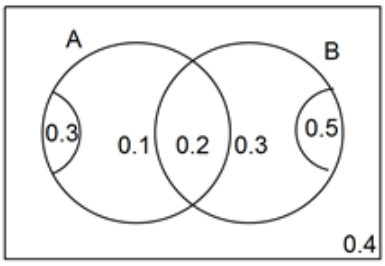
Question 1 **(4 marks)**

Given A and B are events with P(A) = 0.3 and P(B) = 0.5

(a) State the values of m and n for $m \leq P(A \cup B) \leq n$. [1]

Solution
$m = 0.5$ $n = 0.8$
Specific behaviours
✓ correctly determines the values of m and n

(b) If $P(A \cup B') = 0.7$ determine $P(B/A')$. [3]

Solution
$P(B/A') = \frac{P(B \cap A')}{P(A')}$ $= \frac{0.3}{0.7}$ $= \frac{3}{7}$ 
Specific behaviours
✓ determines the probability of the given condition ✓ determines the intersection ✓ determines the correct probability

Question 2 **(7 marks)**

Find the derivative of each of the following: [DO NOT SIMPLIFY YOUR ANSWER]

(a) $7x^3 - \frac{3}{x} + 4\pi$

[1]

Solution
$21x^2 + \frac{3}{x^2}$
Specific behaviours
✓ correctly derivative

(b) $(\sqrt{2x})(1+x)^2$

[2]

Solution
$\sqrt{2}.\frac{1}{2}x^{-\frac{1}{2}}(1+x)^2 + \sqrt{2x}.2(1+x)$
Specific behaviours
✓ applies product rule correctly ✓ differentiates each term correctly

(c) $\frac{x}{\sqrt{2x-3}}$

[2]

Solution
$\frac{1\sqrt{2x-3} - x.\frac{1}{2}(2x-3)^{-\frac{1}{2}}.2}{2x-3}$ *or* $1(2x-3)^{-\frac{1}{2}} + -x.\frac{1}{2}(2x-3)^{-\frac{3}{2}}.2$
Specific behaviours
✓ applies quotient (product) rule correctly ✓ differentiates each term correctly

(d) $\int_1^{2x}(t^3 + t^2)\,dt$

[2]

Solution
$[(2x)^3 + (2x)^2].2$
Specific behaviours
✓ recognises the fundamental theorem, substituting t with 2x ✓ multiplies by the derivative of 2x

See next page

Question 3 **(5 marks)**

Make use of the chain rule (with Leibniz notation), to determine:

$$\frac{dz}{dy}, \text{ if } z = \frac{1}{3x} \text{ and } y = \frac{4}{x - 3}$$

Solution
$\frac{dz}{dy} = \frac{dz}{dx} \cdot \frac{dx}{dy}$ $= \frac{-1}{3x^2} \cdot \frac{(x-3)^2}{-4}$ $= \frac{(x-3)^2}{12x^2}$ $= \frac{\left(\frac{4}{y} + 3 - 3\right)^2}{12\left(\frac{4}{y} + 3\right)^2}$ $= \frac{\left(\frac{4}{y}\right)^2}{12\left(\frac{4}{y} + 3\right)^2}$ $z = \frac{x^{-1}}{3}$ $\quad y = 4(x-3)^{-1}$ $\frac{dz}{dx} = -\frac{1}{3x^2}$ $\quad \frac{dy}{dx} = -4(x-3)^{-2}$ $= -\frac{4}{(x-3)^2}$
Specific behaviours
✓ applies chain rule correctly
✓ determines $\frac{dz}{dx}$ correctly
✓ determines $\frac{dy}{dx}$ correctly
✓ determines $\frac{dz}{dy}$ correctly in terms of x
✓ determines $\frac{dz}{dy}$ correctly in terms of y

Question 4 **(7 marks)**

Given $f(x) = x + 3$ and $g(x) = \frac{2}{x - 3} + x$:

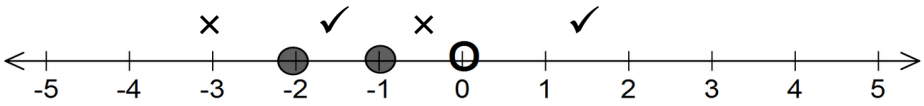
(a) show that $g \circ f(x) = \frac{x^2 + 3x + 2}{x}$

[3]

Solution
$\frac{2}{(x+3) - 3} + x + 3$ $= \frac{2}{x} + x + 3$ $= \frac{2 + x^2 + 3x}{x}$ $= \frac{x^2 + 3x + 2}{x}$
Specific behaviours
✓ substitutes f(x) into g(x) ✓ simplifies and determines LCD ✓ states final simplification is $\frac{x^2 + 3x + 2}{x}$

(b) determine when $g \circ f(x) \geq 0$

[4]

Solution
$\frac{x^2 + 3x + 2}{x} \geq 0$ $\frac{(x+2)(x+1)}{x} \geq 0$ Critical values x = 0, -2 and -1  $-2 \leq x \leq -1$ or $x > 0$
Specific behaviours
✓ factorises numerator ✓ identifies critical values ✓ tests values ✓ correct boundaries (including 'or')

See next page

Question 5 **(5 marks)**

Find the maximum and minimum values of the function $f(x) = 104 + 8x + \frac{288}{x}$, over the interval $1 \leq x \leq 7$.

Solution
$f(x) = 104 + 8x + 288x^{-1}$ $f'(x) = 8 - 288x^{-2}$ $0 = 8 - \frac{288}{x^2}$ $x^2 = 36$ $x = \pm 6$ x = -6 f(x) = 8 (outside the domain) x = 1 f(x) = 400 x = 6 f(x) = 200 $201\frac{1}{7}$ x = 7 f(x) = Max value = 400 Min value = 200
Specific behaviours
✓ derives f(x) ✓ solve f'(x) = 0 correctly ✓ determines f(x) at x = ± 6 (note : only need x=6 as other outside domain) ✓ determines f(x) at boundaries ✓ states maximum and minumum value of f(x)

DO NOT WRITE IN THIS AREA

Question 6 **(8 marks)**

(a) Integrate each of the following with respect to x. DO NOT SIMPLIFY

(i) $3 - 2x$

[1]

Solution
$3x - x^2 + c$
Specific behaviours
✓ correctly integrates

(ii) $\dfrac{6x - 4}{\sqrt{x}}$

[2]

Solution
$\int 6x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} dx$ $= 4x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + c$
Specific behaviours
✓ simplifies ✓ correctly integrates

(iii) $-3(5 - x)^{\frac{3}{4}} + \dfrac{1}{2x^2}$

[3]

Solution
$\int -3(5 - x)^{\frac{3}{4}} + \dfrac{1}{2x^2}\, dx$ $= \dfrac{12(5 - x)^{\frac{7}{4}}}{7} - \dfrac{1}{2x} + c$
Specific behaviours
✓ correct derivative of the 1st term ✓ correct derivative of the 2nd term ✓ includes constant (only penalise in this question)

See next page

Question 6 continued

(b) $\int \frac{2ax+b}{(ax^2+bx+c)^2}\,dx$ where *a* and *b* are constants. Simplify the answer.

[2]

Solution
$\int \frac{2ax+b}{(ax^2+bx+c)^2}\,dx$ $= \int (2ax+b)(ax^2+bx+c)^{-2}\,dx$ $= \frac{-1}{ax^2+bx+c} + k$
Specific behaviours
✓ recognises numerator is the derivative of denominator ✓ hence integrates function correctly

DO NOT WRITE IN THIS AREA

Question 7 **(7 marks)**

(a) Solve $\frac{x}{3x+9} - \frac{6}{x^2-9} = 0$

[4]

Solution
$\frac{x}{3x+9} - \frac{6}{x^2-9} = 0$ $3(x+3)(x-3) \times \frac{x}{3(x+3)} - \frac{6}{(x-3)(x+3)} \times 3(x+3)(x-3) = 0 \times 3(x+3)(x-3)$ $\quad x \neq \pm 3$ $x(x-3) - 18 = 0$ $x^2 - 3x - 18 = 0$ $(x-6)(x+3) = 0$ $x = 6$ or ~~$x = -3$~~ $\therefore x = 6$
Specific behaviours
✓ factorises denominator and multiplies throughout by LCD ✓ simplifies expression ✓ factorises quadratic and solves x ✓ identifies restrictions and recognises only one solution

See next page

Question 7 continued

(b) Simplify $\dfrac{x-2}{x+3} - \dfrac{4}{5x^2+14x-3} + 1$

[3]

Solution
$\dfrac{x-2}{x+3} - \dfrac{4}{5x^2+14x-3} + 1$ $= \dfrac{x-2}{x+3} - \dfrac{4}{(x+3)(5x-1)} + 1$ $= \dfrac{(x-2)(5x-1) - 4 + (x+3)(5x-1)}{(x+3)(5x-1)}$ $= \dfrac{5x^2 - 11x + 2 - 4 + 5x^2 + 14x - 3}{(x+3)(5x-1)}$ $= \dfrac{10x^2 + 3x - 5}{(x+3)(5x-1)}$
Specific behaviours
✓ factorises quadratic ✓ identifies LCD and uses to simplify expression ✓ correct solution

DO NOT WRITE IN THIS AREA

Question 8 **(7 marks)**

Lumen (*lm*) is the unit of total light output from a light source. Three stage lights, Aa, Bb and Cc, of different light outputs are positioned above the stage in the Rixon Theatre. If all of the three stage lights are put on at 50% capacity a total output of 1200 *lm* lights up the stage. If Aa is turned on 50% and Bb is turned on 100% capacity then 1000 *lm* lights the stage and if 50% of Bb and 100% of Cc are turned on, 1300 *lm* lights the stage.

(a) Write three equations to represent this information.

[2]

Solution
$0.5A + 0.5B + 0.5C = 1200$ $0.5A + B = 1000$ $0.5B + C = 1300$
Specific behaviours
✓ ✓determines all three equations ✓ determines 2 of the 3 equations

(b) Solve the system of equations

$$x + y + z = 24$$
$$x + 2y = 20$$
$$y + 2z = 26$$

[3]

Solution
$x + y + z = 24$ [1] $x + 2y = 20$ [2] $y + 2z = 26$ [3] [1] - [2] $-y + z = 4$ [4] [4] + [3] $3z = 30$ $z = 10$ $y = 6$ $x = 8$
Specific behaviours
✓ determines z ✓ determines y ✓ determines x

Question 8 continued

(c) Hence determine the Lumen capacity of each of the stage lights Aa, Bb and Cc.

[2]

Solution
Aa = 800 Bb = 600 Cc = 1000
Specific behaviours
✓ recognises the relationhips between the equations in part (a) and (b) ✓ correct Lumen capacity of each of the stage lights