

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

### **Important note to candidates**

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters  
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,  
and up to three calculators satisfying the conditions set by the Curriculum Council for this course.  
*To be provided by the candidate*

This Question/Answer booklet  
Formula sheet (retained from Section One)  
*To be provided by the supervisor*

### **Material required/recommended for this section**

Reading time before commencing work: 10 minutes  
Working time for paper: 100 minutes

Your name \_\_\_\_\_  
In words \_\_\_\_\_  

--	--	--	--	--	--	--	--

  
Student Number: \_\_\_\_\_ In figures \_\_\_\_\_

**MATHEMATICS**  
**3C/3D**  
**Section Two:**  
**Calculator-assumed**

# **SOLUTIONS**

Question/Answer Booklet

Trial WACE Examination, 2010

Question number(s): \_\_\_\_\_

Additional working space

**CALCULATOR-ASSUMED**

**MATHEMATICS 3C/3D**

**SECTION TWO - SOLUTIONS**

**TRIAL EXAMINATION 2010**

**SCHOOL**

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	8	8	50	40
Section Two: Calculator-assumed	12	12	100	80
				120

**Instructions to candidates**

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.

**Question 20**

(5 marks)

A random variable  $X$  is normally distributed such that the mean is twice the variance and the probability that  $X$  is greater than 21.5 is 0.231. Find the mean and standard deviation of  $X$ .

Let  $x = \text{standard deviation}$  and  $y = \text{mean}$ .

$$y = 2x^2 \quad (1)$$

$$P(Z > z) = 0.231 \Rightarrow z = 0.73556$$

$$\frac{21.5 - y}{x} = 0.73556 \quad (2)$$

Solve 1 & 2 simultaneously to get

$$x = 3.099982$$

$$y = 19.2197$$

(Ignore other solution as standard deviation must be positive)  
Hence mean is 19.22 and standard deviation is 3.10 (2dp).

(a) If 7.2% of the trees in the plantation were affected today, what percentage is expected to be affected by the disease in one and a half years time?

Better, as the rate of change of diseased trees with time is negative ( $-0.017$ ) and hence decreasing.

(a) Was the health of the plantation getting better or worse? Briefly justify your answer by referring to the above relationship.

The percentage of trees,  $P$ , in a plantation affected by a disease was changing with time,  $t$ , in months, according to the relationship  $\frac{dp}{dt} = -0.017p$ .

### Question 9

Working time for this section is 100 minutes.

This section has twelve (12) questions. Answer all questions. Write your answers in the space provided.

Section Two: Calculator-assumed (80 Marks)

TRIAL EXAMINATION 2010 SECTION TWO - SOLUTIONS SECTION TWO: CALCULATOR-ASSUMED MATHEMATICS 3C/3D

(3 marks)

Region A is the area trapped by  $f$  and  $g$ .  
Region B is the area trapped by  $f$ ,  $g$  and the line  $x=2$ .

$f$  and  $g$  intersect when  $x=1$ .

$$\text{Region A} = \int_1^6 (f - g) dx = \frac{1}{2}$$

$$\text{Region B} = \int_1^2 (f - g) dx = \frac{1}{12}$$

$$\int_0^{10} (f - g) dx = \int_0^{10} (g - f) dx$$

$$\frac{15}{2} = 50k^3 - 2k + \frac{15}{4}$$

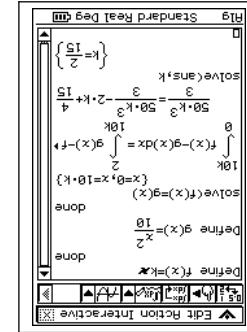
$$10k = 50k^3 - 2k + 15$$

$$50k^3 = 50k^3 - 2k + 15$$

$$0 = -2k + 15$$

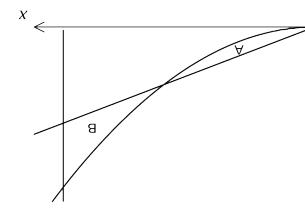
$$2k = 15$$

$$k = \frac{15}{2}$$



(b)  $f(x)$  is modified to become the line  $f(x) = kx$ , so that the area of region A is exactly the same as the area of region B. Determine the value of  $k$ . (4 marks)

(3 marks)



The graph below, not to scale, shows the functions  $f(x) = \frac{10}{x^2}$  and the line  $x=2$ .

(7 marks)

SECTION TWO - SOLUTIONS SECTION TWO: CALCULATOR-ASSUMED MATHEMATICS 3C/3D

(b) If 18 months  $t = 18$  months  
 $P = 7.2e^{-0.017t} = 7.2e^{-0.017 \times 18} = 5.3\%$

(b) If 7.2% of the trees in the plantation were affected today, what percentage is expected to be affected by the disease in one and a half years time?

$$P = 7.2e^{-0.017t}$$

$$t = 18 \text{ months}$$

$$= 5.3\%$$

(3 marks)

(5 marks)

**Question 10**

- (a) A and B are two independent events such that  $P(A) = 0.2$  and  $P(B) = 0.15$ .

Evaluate

(i)  $P(A|B)$

$$\boxed{0.2}$$

(1 mark)

(ii)  $P(A \cap B)$

$$\boxed{0.2 \times 0.15 = 0.03}$$

(1 mark)

(iii)  $P(A \cup B)$

$$\boxed{0.2 + 0.15 - 0.03 = 0.32}$$

(1 mark)

- (b) The probability that a door to door salesman convinces a customer to buy is 0.4.

Assuming that sales are independent, find the probability that the salesman makes at least one sale before reaching the fourth house. (2 marks)

$$\boxed{1 - (0.6)^3 \\ = 0.784}$$

**Question 18**

(7 marks)

Climbing rope produced by a manufacturer is known to be such that over a long production run, ten-metre lengths have breaking strengths that are normally distributed with a mean of 180.2kg and standard deviation of 9.5kg.

- (a) Find the probability that a randomly chosen ten-metre length will have a breaking strength of less than 160kg. (1 mark)

$$\boxed{0.0167}$$

- (b) At the start of a production run, a quality control officer at the factory randomly samples 20 ten-metre lengths and after testing, determines that the mean breaking strength of the sample is 176.9kg. Construct a 90% confidence interval for the population mean based on this sample. (3 marks)

$$\begin{aligned} \bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}} &= 176.9 \pm 1.645 \frac{9.5}{\sqrt{20}} \\ &= 176.9 \pm 3.49 \\ 173.41 \leq \mu &\leq 180.39 \end{aligned}$$

- (c) If the quality control officer repeated the same sampling process in (b) every day for 30 consecutive days, how many of the intervals constructed would be expected to include the known mean breaking strength of 180.2kg? You may assume there were no problems with the manufacturing process. (1 mark)

$$\boxed{0.9 \times 30 = 27 \text{ days.}}$$

- (d) How large a sample should the quality control officer take, if the probability that the estimated mean breaking strength is in error by more than 2.5kg is to be at most 5%? (2 marks)

$$\begin{aligned} n &= \left( \frac{z_{crit} \times \sigma}{e} \right)^2 = \left( \frac{1.96 \times 9.5}{2.5} \right)^2 \\ &= 55.47 \end{aligned}$$

Hence sample 56 lengths of rope.



**Question 12** (10 marks)

Every weekday the chef at a restaurant sends out an apprentice to the local market to spend as little as possible and at the same time come back with at least 16kg of onions, at least 17kg of carrots and at least 21kg of potatoes.

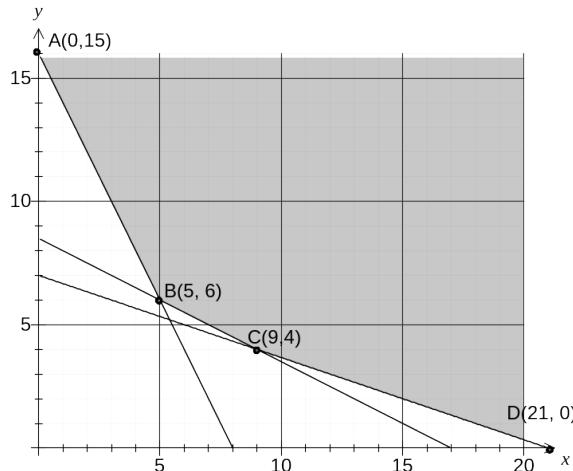
One stall at the market sells 'Best Buy' packs consisting of 2kg of onions, 1kg of carrots and 1kg of potatoes for \$3.50 each. Another stall sells 'Chefs Choice' packs consisting of 1kg of onions, 2kg of carrots and 3kg of potatoes for \$6.50 each.

The apprentice buys  $x$  'Best Buy' packs and  $y$  'Chefs Choice' packs.

- (a) Write down three inequalities to represent the above constraints, apart from  $x \geq 0$  and  $y \geq 0$ . (2 marks)

$$\begin{aligned}2x + y &\geq 16 \\x + 2y &\geq 17 \\x + 3y &\geq 21\end{aligned}$$

- (b) Complete the constraints on the graph below and indicate the feasible region. (3 marks)



See next page

**Question 16** (7 marks)

A factory uses three machines to produce one type of plastic bottle. Of the total production, machine A produces 35%, machine B produces 25% and machine C the rest. Due to the age of the machines, they all produce some defective bottles. Of their production, machines A and B produce 3% and 6% defective respectively.

- (a) Find the probability that a randomly selected bottle is produced by machine A and is defective. (1 mark)

$$\begin{aligned}P(A \cap D) &= P(A) \times P(D|A) \\&= 0.35 \times 0.03 \\&= 0.0105\end{aligned}$$

- (b) If the probability of a randomly selected bottle being defective is 0.0455, what percentage of the production of machine C is defective? (4 marks)

Machine C has  $100\% - 35\% - 25\% = 40\%$  of production.

$$\begin{aligned}P(A \cap D) &= 0.0105 \\P(B \cap D) &= 0.25 \times 0.06 = 0.015 \\P(D) &= P(A \cap D) + P(B \cap D) + P(C \cap D) \\0.0455 &= 0.0105 + 0.015 + P(C \cap D) \\ \therefore P(C \cap D) &= 0.02 \\P(C \cap D) &= P(C) \times P(D|C) \\0.02 &= 0.4 \times P(D|C) \\ \therefore P(D|C) &= 0.05\end{aligned}$$

5% of output from C is defective.

(Tree diagram or 2-way table are alternative methods here)

- (c) Given that a randomly selected bottle is not defective, find the probability that it was produced by either machine A or machine B. (2 marks)

$$\begin{aligned}\frac{P(A \cap \bar{D}) + P(B \cap \bar{D})}{1 - 0.0455} \\= \frac{0.35 \times 0.97 + 0.25 \times 0.94}{0.9545} \\= 0.6019\end{aligned}$$

See next page

(2 marks)

Point	Cost = $3.5x + 6.5y$
A(0, 16)	104
B(5, 6)	56.5
C(9, 4)	57.5
D(21, 0)	73.5

(3 marks)

(c) How many of each pack should the apprentice buy to minimise the purchase cost and what is the minimum cost?

CALCULATORS ASSUMED  
MATHEMATICS 3C/3DSECTION TWO - SOLUTIONS  
TRIAL EXAMINATION 2010

Let objective function be $C = kx + 6.5y$
Then buy fewer Best Buys, so
$16 \times 6.5 = 5k + 6 \times 6.5$
$k = 13$
$13 - 3.5 = 9.5$
Price can increase by \$9.50 before optimum solution will change.

(d) By how much can the price of a 'Best Buy' pack rise without changing the optimum number of packs found in your answer to (c)?

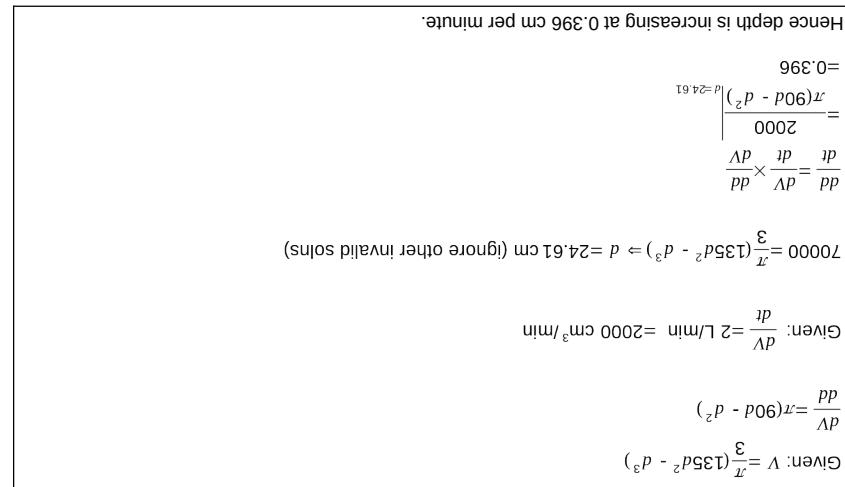
70000 =  $\frac{3}{\pi} (35d^2 - d^3) \Rightarrow d = 24.61 \text{ cm (ignore other invalid solutions)}$

Hence depth is increasing at 0.396 cm per minute.

$$\begin{aligned} \frac{dd}{dt} &= \frac{dV}{dt} \times \frac{dy}{dd} \\ &= \frac{2000}{\pi(90d - d^2)} \Big|_{d=24.61} \\ &= 0.396 \end{aligned}$$

$$\text{Given: } \frac{dt}{dV} = 2 \text{ L/min} = 2000 \text{ cm}^3/\text{min}$$

$$\text{Given: } V = \frac{3}{\pi} (35d^2 - d^3)$$



If water is poured into a hemispherical container of radius 45cm at a constant rate of 2 litres per minute, how fast is the depth of water increasing at the instant that the hemisphere contains 70L of water?

When fluid rests in the bottom of a hemisphere of radius  $r$ , the volume of fluid  $V$ , can be calculated using the formula  $V = \frac{\pi r^2 (3r - d)}{3}$ , where  $d$  is the depth of the fluid.

of fluid  $V$ , can be calculated using the formula  $V = \frac{\pi r^2 (3r - d)}{3}$ ,

When fluid rests in the bottom of a hemisphere of radius  $r$ , the volume

of fluid  $V$ , can be calculated using the formula  $V = \frac{\pi r^2 (3r - d)}{3}$ ,

where  $d$  is the depth of the fluid.

of fluid  $V$ , can be calculated using the formula  $V = \frac{\pi r^2 (3r - d)}{3}$ ,

where  $d$  is the depth of the fluid.

**Question 13** (5 marks)

A sub-committee of four, consisting of a chairperson, a secretary and two ordinary members is to be chosen from a committee of 20 people.

- (a) Find the number of possible choices for

- (i) the posts of chairperson and secretary,

(1 mark)

$${}^{20}P_2 = 20 \times 19 = 380$$

- (ii) the two ordinary members,

(1 mark)

$${}^{20}C_2 = 190$$

- (iii) the chairperson, secretary and two ordinary members.

(1 mark)

$${}^{20}C_1 \times {}^{19}C_1 \times {}^{18}C_2 = 58140$$

- (b) If all possible sub-committees are equally likely to be chosen, what is the probability that the chairman of the main committee is not selected in the sub-committee? (2 marks)

$$\frac{{}^{19}C_1 \times {}^{18}C_1 \times {}^{17}C_2}{58140} = \frac{46512}{58140} = \frac{4}{5}$$

or

$$\frac{19}{20} \times \frac{18}{19} \times \frac{17}{18} \times \frac{16}{17} = \frac{4}{5}$$

**Question 14** (11 marks)

A manufacturer of chocolate produces 3 times as many soft centred chocolates as hard centred ones. The chocolates are randomly packed in boxes of 20.

- (a) Find the probability that in a box there are

- (i) an equal number of soft centred and hard centred chocolates

(3 marks)

Let the rv  $X$  be the number of hard centred chocolates per box of 20.

Then  $X \sim \text{Bin}(20, 0.25)$

$$P(X = 10) = 0.00992$$

- (ii) fewer than 5 hard centred chocolates.

(1 mark)

$$P(X < 5) = P(X \leq 4) = 0.41484$$

- (b) Determine the mean and standard deviation of the number of hard centred chocolates in a box of 20. (2 marks)

$$\text{Mean: } np = 20 \times 0.25 = 5$$

$$\text{SD: } \sqrt{np(1-p)} = \sqrt{20 \times 0.25 \times 0.75} = \sqrt{3.75} = 1.9365$$

- (c) A random sample of 5 boxes is taken from the production line. Find the probability that exactly 3 of them contain fewer than 5 hard centred chocolates. (2 marks)

Let the rv  $Y$  be the number of boxes out of 5 with fewer than 5 hard centres.

Then  $Y \sim \text{Bin}(5, 0.41484)$

$$P(Y = 3) = 0.21115$$

- (d) A random sample of 30 boxes is taken from the production line. Find the probability that the mean number of hard centred chocolates per box in the sample exceeds 5.5. (3 marks)

For samples of size 30 or more, CLT says distribution of sample means  $\bar{X}$  will be approximately normally distributed as follows:

$$\bar{X} \sim N\left(5, \frac{1.9365^2}{30}\right) \sim N(5, 0.35355^2)$$

$$\text{Hence } P(\bar{X} > 5.5) = 0.07865$$