### MATHEMATICS METHODS

# MAWA Semester 1 (Unit 3) Examination 2020 Calculator-free

# Marking Key

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The release date for this exam and marking scheme is

June 12<sup>th</sup> the end of week 7 of term 2, 2020

CALCULATOR-FREE SEMESTER 1 (UNIT 3) EXAMINATION

MATHEMATICS METHODS

the coordinates of B are (-1,-2)

Question 6(b) Solution (2 marks)

Solution

Solution

Solution

Solution

Solution

Solution

Solution

8

So

Mathematical behaviours

equates derivative to 0 and solves

states co-ordinates of B

states co-ordinates of B

Question 7(a) (2 marks)

Solution It is the area between the two curves from x=0 to  $x=\pi$ . Markematical behaviours

• states it is the area between the two given curves

• states the area is from x=0 to  $x=\pi$ • states the area is from x=0 to  $x=\pi$ 

Question 7(b) (3 marks)

T T T	• anti-differentiates $X = X = X = X = X = X = X = X = X = X $
Marks	Mathematical behaviours
	$\int x p_{xx} \cdot 3x \cdot \sqrt{\frac{1}{2}} - \int -x \cos x = x p_{xx} \cdot 3x - x \text{ mis}$

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(50 Marks)

1

Oi	estion 1(a)	(2 marks)	
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Solution		
$f(x) = (3 + x^3)^{\frac{1}{2}}$		
$f'(x) = \frac{1}{2}(3+x^{3})^{\frac{1}{2}}(3x^{2})$		
$=\frac{3x^2}{2\sqrt{3}+x^3}$		
Mathematical behaviours	Mark	
applies chain rule	1	
• obtains correct result 1		

Question 1(b) (2 marks)

Solution	
$z = t^2 \cos(2t - 1)$	
$\frac{dz}{dt} = \cos(2t-1) \times 2t + t^2 \times (-2)\sin(2t-1)$	
$=2t\cos(2t-1)-2t^{2}\sin(2t-1)$	
Mathematical behaviours	Marks
differentiates cos term correctly	1
applies product rule and states result	1

Question 1(c) (3 marks)

₹#00#0# =(0)	(0)
Solution	
$y = 5\sin(4x + 3)$	
$\frac{dy}{dx} = 5\cos(4x + 3^2 + 16 \times (5\sin(4x + 3))^2$	
$=400\cos^2(4x+3)+400\sin^2(4x+3)$	
$=400(\cos^2(4x+3)+\sin^2(4x+3)) \qquad(*)$	
=400 $\because \cos^2(4x+3) + \sin^2(4x+3) = 1$	
Mathematical behaviours	Marks
differentiates correctly	1
substitutes and simplifies to (*)	1

evaluates correctly, stating Pythagorean identity

Question 5(c) (5 marks)

7

Solution	
$y = \frac{1}{e^{2x} + 1} = (e^{2x} + 1)^{-1}$ $\frac{dy}{dx} = \frac{-2e^{2x}}{(e^{2x} + 1)^2} = -2\left(\frac{e^x}{(e^{2x} + 1)}\right)^2$	
$\int \frac{dy}{dx} dx = \int 2\left[\frac{e^x}{(e^{2x}+1)}\right]^2 dx$	
$ie   y+C=-2\int \left(\frac{e^x}{\left(e^{2x}+1\right)}\right)^2 dx$	
$ie \frac{1}{e^{2x} + 1} + C = -2 \int \left( \frac{e^x}{(e^{2x} + 1)} \right)^2 dx$	
$ie\left(\frac{-1}{2}\right)\frac{1}{e^{2x}+1}+C=\int \left(\frac{e^{x}}{(e^{2x}+1)}\right)^{2}dx \Rightarrow A=\frac{-1}{2}$	
Mathematical behaviours	Marks
applies the chain rule to the derivative	1
• differentiates $e^{2x}$ correctly	1
- uniciditates confectly	1 1

Mathematical behaviours	Marks
applies the chain rule to the derivative	1
<ul> <li>differentiates e<sup>2x</sup> correctly</li> <li>recognises application of the Fundamental Theorem</li> </ul>	1 1
• factors out - 2 and re-writes fraction involving $e^{2x}$ in numerator and denominator as one fraction squared	1
• multiplies both sides of expression by $-\frac{1}{2}$ to obtain desired result	1

Question 6(a) (3 marks)

Solution	
$y = \frac{8x}{(x-1)^2} \Rightarrow \frac{dy}{dx} = \frac{(x-1)^2 \cdot 8 - 8x \times (2)(x-1)}{(x-1)^4}$	
$= \frac{8(x-1)-16x}{(x-1)^3} \Rightarrow c = 1, d = -1$	
$= \frac{-8x - 8}{(x - 1)^3} \Rightarrow a = b = -8.$	

	Mathematical behaviours	Marks
<ul> <li>applies</li> </ul>	quotient rule	1
<ul> <li>different</li> </ul>	iates both parts correctly and states the value of $c$ and $d$	1
<ul> <li>simplifie</li> </ul>	s result and states value of $a$ and $b$	1

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Question 2(a) (2 marks)

3

T.	$\overline{SIV} = X$ satisfy •
τ	$0=x$ sequetes function to 0 and obtains $\bullet$
Marks	Mathematical behaviours
	$\overline{\text{CI}} \neq 0 = x \Leftarrow$
	$0 = (I - {}^{z}X)X \Leftarrow$
	$0 = x \le 1 - {\varepsilon} x \Leftarrow 0 = (x) $
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Question 2(b) (4 marks)

τ	locates point of inflection	
	stnioq	
τ	<ul> <li>uses second derivative test (or sign test) to determine nature of stationary</li> </ul>	
τ	<ul> <li>obtains correct y values of the stationary points</li> </ul>	
τ	<ul> <li>differentiates, equates to 0 and solves</li> </ul>	
Marks	Mathematical behaviours	
	$\int_{\mathbb{R}^n} f(x) = 0$ $f(0) = 0$ point of inflection	
	$xem \leftarrow 2I -= (2 -)^n $ $f = (2 -)^1 $ $f = (3 -)^2 $	
	$aim = 0 < \Sigma I = (\Sigma)^n $ $aightarrow $ $aig$	
	$X \subset I - {}_{\varepsilon}X = (X) $	
	UOIINIOS	

Question 2(c) (1 mark)

τ	$ullet$ determines $^{\{oldsymbol{1}\}}$ and concludes maximum
Mathematical behaviours Marks	
	31 osls $si(S-1)$ since $f(S-1)$
	91=84 -43=(4)
	91 -= 84 + 49 -= (4 -)
	xzz - x = (x)
noitulo2	

T T	• plots zeros at 0 and such that $-4 < x < -3$ and $3 < x < 4$
Marks	Mathematical behaviours
	$x \in \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$
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Question 5(b) (3 marks)

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τ	<ul> <li>integrates correctly and calculates the result</li> </ul>		
τ	<ul> <li>applies linearity of integrals correctly</li> </ul>		
	(ii)		
τ	the correct result		
	applies linearity of integrals, swaps bounds of integration and determines		
	(1)		
Marks	Mathematical behaviours		
	I -=		
	(\tau - I) + \textit{Z} =		
$= \sum_{i} [x] + (i) =$			
$xp\int_{x}^{\infty} + xp(x)\int_{x}^{\infty} z = xp(z+x)\int_{x}^{\infty} z = xp(z+x)\int_{x}^{\infty}$			
t t			
£=			
	†+I-=		
	$xp(x) \int_{0}^{p} + xp(x) \int_{0}^{p} - = xp(x) \int_{0}^{1}$		
	9 1 9		
	(i)		
noitulo2			

1

1

•	plots stationary points and point of inflection accurately	1
•	obtains correct shape for the graph, scale and end points	

Question 2(d)	(3 marks)
Question 3(a)	(1 mark)

Solution	
x=2,	
$\frac{dc}{dx} = 2(8+1)^{\frac{1}{2}} = 6$	
Mathematical behaviours	Mark
states correct answer	1

Question 3(b) (4 marks)

Solution	
$\int_{0}^{2} x(2x^{2}+1)^{\frac{1}{2}} dx$	
$= \frac{1}{4} \int_{0}^{2} 4x (2x^{2} + 1)^{\frac{1}{2}} dx$	
$=\frac{1}{4}\left[\left(2x^2+1\right)^{\frac{3}{2}}\cdot\frac{2}{3}\right]_0^2$	
$=\frac{1}{6}(27-1)$	
$=\frac{13}{3}$	
Mathematical behaviours	Marks
$\int_{0}^{2} \int x (2x^{2} + 1)^{\frac{1}{2}} dx$ • states the change as	1
anti-differentiates correctly	1

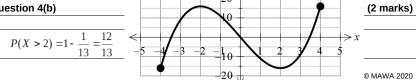
Question 4(a)	(2 marks)

substitutes correct limits of integration

determines correct answer

4	Question 4(a)	(Z IIIdiks)
	Solution	
	$k + 3k + 5k + 4k = 1 \Rightarrow k = \frac{1}{13}$	
	Mathematical behaviours	Marks
	states the sum of probabilities is 1	1
	deduces k value	1
	v	

## Question 4(b)



	Mathematical behaviours	Marks
•	states an expression to calculate required probability	1
•	determines probability	1

5

Question 4(c)	(2 marks)
Solution	
8	
$P(X \le 5 \mid X > 2) = \frac{\overline{13}}{12} = \frac{8}{12} = \frac{2}{3}$	
13	
Mathematical behaviours	Marks
writes fraction with the correct denominator	1
obtains simplified result	1

Question 5(a) (4 marks)

C (- )	
Solution	
(i)	
$2\pi$	
$\int_{0} 2\sin(4x)  dx$	
$= \left[\frac{-2\cos(4x)}{4}\right]_0^{2\pi}$	
$=-\frac{1}{2}[\cos 8\pi - \cos 0]$	
=0	
(ii)	
$\int_{-\infty}^{\infty} \frac{1}{x} dx$	
$= \int 1 + x^{\frac{-1}{2}} dx$ $= x + 2\sqrt{x} + c$	
$=x+2\sqrt{x}+c$	

	Mathematical behaviours	Marks	
(i)			
•	states anti-derivative	1	
•	evaluates result	1	
(ii)			
•	rewrites fraction as sum of two functions	1	
•	anit-differentiates including $^{\mathcal{C}}$	1	