

**ALL SAINTS' COLLEGE**



Question/Answer Booklet

Semester I Examination, 2016

MATHEMATICS

UNIT 3

## Section Two:

### **Student Number**

our name  
in words

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**Time allowed for this section**: Reading time before commencing work: Working time for section:

Matters referred/recommended for this section

## This Question/Answer Booklet

*to be provided by the student* Standard items: pens (blue/black preferred) fluid/tape, eraser, ruler, highlighter.

fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper and up to three calculators approved for use in the WACE examinations

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Whole paper - MARKING KEY

**Student Name:**

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	48	35
Section Two: Calculator-assumed	13	13	100	101	65
		<b>Total</b>		149	100

**Additional working space**

Question number: \_\_\_\_\_

**Instructions to candidates**

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

- (b) Assuming the current rate of growth continues, how long will it take for the population to increase from 24 million to 25 million people? (2 marks)

- (a) Assuming that the rate of growth of the population can be modelled by the equation
- $$\frac{dp}{dt} = kp$$
- , where  $P$  is the population of Australia at time  $t$  months, determine the value of the constant  $k$ . (3 marks)
- A recent news report said that it took 34 months for the population of Australia to increase from 23 to 24 million people.
- Working time for this section is 100 minutes.
- Question 9 (5 marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Section Two: Calculator-assumed  
METHODS UNIT 3  
CALCULATOR-ASSUMED  
Additional working space  
65% (101 Marks)

METHODS UNIT 3  
CALCULATOR-ASSUMED  
3

CALCULATOR-ASSUMED

18

METHODS UNIT 3  
CALCULATOR-ASSUMED  
Additional working space  
Question number: \_\_\_\_\_

(7 marks)

**Question 10**

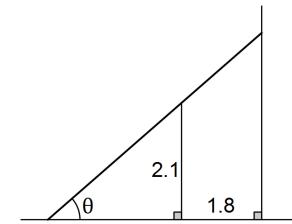
A small object is moving in a straight line with acceleration  $a = 6t + k \text{ ms}^{-2}$ , where  $t$  is the time in seconds and  $k$  is a constant. When  $t = 1$  the object was stationary and had a displacement of 4 metres relative to a fixed point  $O$  on the line. When  $t = 2$  the object had a velocity of  $1 \text{ ms}^{-1}$ .

- (a) Determine the value of  $k$  and hence an equation for the velocity of the object at time  $t$ .  
(4 marks)

**Question 21**

(7 marks)

A vertical wall, 2.1 metres tall, stands on level ground and 1.8 metres away from the wall of a house. A ladder, of negligible width, leans at an angle of  $\theta$  to the ground and just touches the ground, wall and house, as shown in the diagram.



$$L = \frac{2.1}{\sin \theta} + \frac{1.8}{\cos \theta}$$

(3 marks)

- (b) Determine the displacement of the object when  $t = 2$ .  
(3 marks)

- (a) Show that the length of the ladder,  $L$ , is given by  
$$L = \frac{2.1}{\sin \theta} + \frac{1.8}{\cos \theta}$$
.  
(3 marks)

- (b) Use a calculus method to determine the length of the shortest ladder that can touch the ground, wall and house at the same time.  
(4 marks)

(7 Mars)

METHODS UNIT 3

9

CALCULATOR-ASSUMED

## Question 1

A random sample of  $n$  students is selected from all Year 12s in this country, and the random variable  $X$  is the number of those in the sample who study advanced mathematics.

	A(x)	0	-1.75				
x	0	1	2	3	4	5	6

2 miles)

Consider the function  $f(t) = \frac{t-4}{2}$  and the function  $A(x) = \int f$

(b) If  $n = 22$ , determine the probability that three of the students in the sample study advanced mathematics. (1 mark)

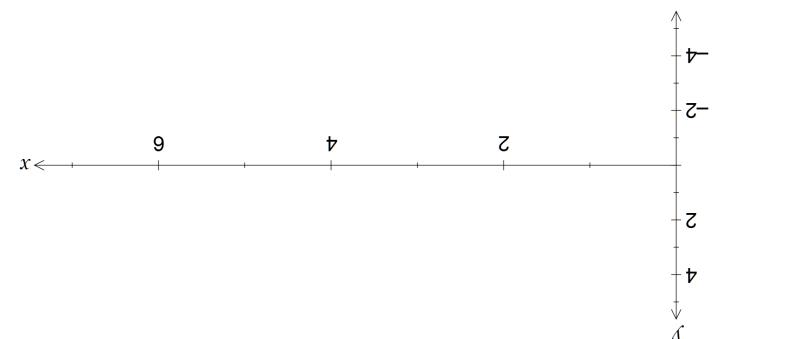
(1 mark)

(c) On the axes below, sketch the graph of  $y = A(x)$  for  $0 \leq x \leq 6$ .

(ii) more than three of the students in the sample study advanced mathematics. (1 mark)

2 marks)

(!!!) none of the students in the sample study advanced mathematics. (1 mark)



(c) If ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study advanced mathematics. (2 marks)

(1 mark)

When  $\alpha = (\gamma)_V$  (1)

(d) Determine

(ii) the function  $A(x)$  in terms of  $x$ .

(1 mark)

(8 marks)

The height of grain in a silo, initially 0.4 m, is increasing at a rate given by  $h'(t) = 0.55t - 0.05t^2$  for  $0 \leq t \leq 11$ , where  $h$  is the height of grain in metres and  $t$  is in hours.

- (a) At what time is the height of grain rising the fastest? (2 marks)

- (b) Determine the height of grain in the silo after 11 hours. (3 marks)

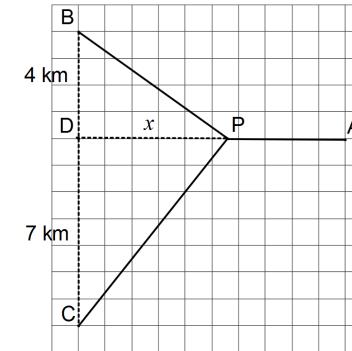
- (c) Calculate the time taken for the grain to reach a height of 4.45 m. (3 marks)

Question 19

(7 marks)

Three telecommunication towers, A, B and C, each need underground power cable connections directly to a new power station, P, that is to be built  $x$  km from depot D on a 10 km road running east-west between D and A.

Tower B lies 4 km due north of depot D and tower C lies 7 km south of the depot, as shown in the diagram.



- (a) Determine an expression for the total length of underground cable required to connect A, B and C directly to P. (2 marks)

- (b) Show that the minimum length of cable occurs when  $\frac{x}{\sqrt{16+x^2}} + \frac{x}{\sqrt{49+x^2}} = 1$ . (3 marks)

- (c) Determine the minimum length of cable required, rounding your answer to the nearest hundred metres. (2 marks)

**Question 18** METHODS UNIT 3

**Calculator-Assumed**

**7**

**METHODS UNIT 3**

**CALCULATOR-ASSUMED**

**14**

**Question 13** (8 marks)

The graph of the function  $y = f(x)$  is shown below for  $-2 \leq x \leq 4$ .  
The shaded region on the graph below is enclosed by the curves  $y = -\sin(2x)$  and  $y = 2\cos x$ .  
Show that the area of the region is 4 square units.

The area of regions enclosed by the  $x$ -axis and the curve, A, B, C, D and E, are 12, 7, 5, 32 and 21 square units respectively.

**(a)** Determine the value of  $\int_{\frac{\pi}{4}}^{\pi} f(x) dx$  (2 marks)

**(b)** Determine the area of the region enclosed between the graph of  $y = f(x)$  and the  $x$ -axis from  $x = 0$  to  $x = 4$ . (2 marks)

**(c)** Determine the values of  $\int_{\frac{1}{2}}^{\frac{3}{2}} f(x) + 3 dx$  (2 marks)

**(d)**  $\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{2}{f(x)} dx$  (2 marks)

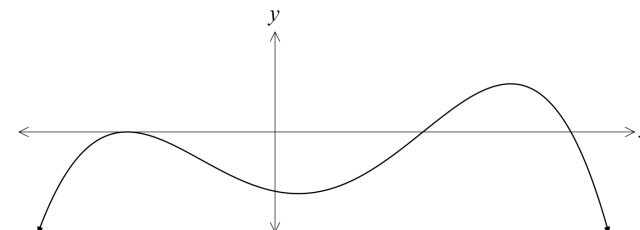
**Question 14**

- (a) Determine the mean of a Bernoulli distribution with variance of 0.24.

(3 marks)

**Question 17**

The graph of  $y = f'(x)$ , the derivative of a polynomial function  $f$ , is shown below. The graph of  $y = f'(x)$  has stationary points when  $x = a$ ,  $x = c$  and  $x = e$ , points of inflection when  $x = b$  and  $x = d$ , and roots when  $x = a$ ,  $x = d$  and  $x = g$ , where  $a < b < c < d < e < g$ .



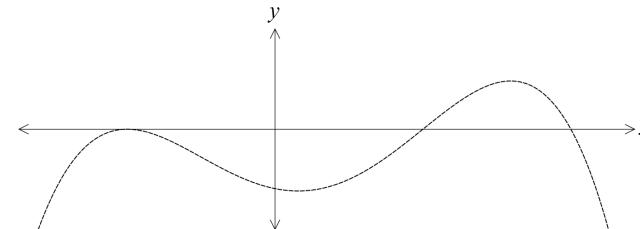
- (b) A Bernoulli trial, with probability of success  $p$ , is repeated  $n$  times. The resulting distribution of the number of successes has an expected value of 5.76 and a standard deviation of 1.92. Determine  $n$  and  $p$ . (4 marks)

- (a) For what value(s) of
- $x$
- does the graph of
- $y = f(x)$
- have a point of inflection? (1 mark)

- (b) Does the graph of
- $y = f(x)$
- have a local maximum? Justify your answer. (2 marks)

- (c) Does the graph of
- $y = f(x)$
- have a horizontal point of inflection? Justify your answer. (2 marks)

- (d) On the axis below, sketch a possible graph of
- $y = f''(x)$
- . The graph of
- $y = f'(x)$
- is shown with a broken line for your reference. (3 marks)



(3 marks)

(iii) misses the bus on Tuesday and on two other days?

(2 marks)

(ii) misses the bus at least twice?

(2 marks)

(i)  $E(Y)$ 

(b) Calculate

(c) Calculate

(iii)  $E(1 - 2Y)$ 

(1 mark)

(i)  $Var(Y)$ 

(2 marks)

(iii)  $Var(1 - 2Y)$ 

(1 mark)

**Question 16** The probability that a student misses their bus to school is 0.2, and the probability that they miss the bus on any day is independent of whether they missed it on the previous day.

Over five consecutive weekdays, what is the probability that the student only misses the bus on Tuesday? (2 marks)

$P(Y = y)$	0.4	-0.2	0.0.2	0.1	1	2	0.2
$y$	-2	-1	0	1	2	1	2

(a) Determine  $P(Y \leq 0 | Y \leq 1)$ 

(2 marks)

(b) Calculate

(i)  $E(Y)$ 

(2 marks)

(iii)  $E(1 - 2Y)$ 

(1 mark)

(i)  $Var(Y)$ 

(2 marks)

(9 marks)

**Question 15**

A particle moves in a straight line according to the function  $x(t) = \frac{t^2 + 3}{t + 1}$ ,  $t \geq 0$ , where  $t$  is in seconds and  $x$  is the displacement of the particle from a fixed point  $O$ , in metres.

- (a) Determine the velocity function,  $v(t)$ , for the particle.

(2 marks)

- (b) Determine the displacement of the particle at the instant it is stationary.

(2 marks)

- (c) Show that the acceleration of the particle is always positive.

(2 marks)

- (d) After five seconds, the particle has moved a distance of  $k$  metres.

(i) Explain why  $k \neq \int_0^5 v(t) dt$ .

(1 mark)

- (ii) Calculate  $k$ .

(2 marks)