PERTH MODERN SCHOOL
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Independent Public School

Mathematics Specialist Year 11

Student name: _____ Teacher name: _____

Date: Monday 10 August 2020

Task type:	**Response + Investigation**
Time allowed:	**45 minutes (for the entire booklet)**
Number of questions:	**5**
Materials required:	Calculator with CAS capability (to be provided by the student)
Standard items:	Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	Drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations
Marks available:	**40 marks**
Task weighting:	**14% combined (8% for Test 2 and 6% for investigation 2)**
Formula sheet provided:	**Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 {1.3.4, 1.3.5} **(4 marks)**

(a) Let $x \epsilon R$.Prove that that $x^2 > x$is false by giving a counterexample. (1 mark)

(b) Disprove the following statement: There exists $x \epsilon R$ such that $5+x^2=1-x^2$
(3 marks)

Question 2 {2.1.1} **(5 marks)**

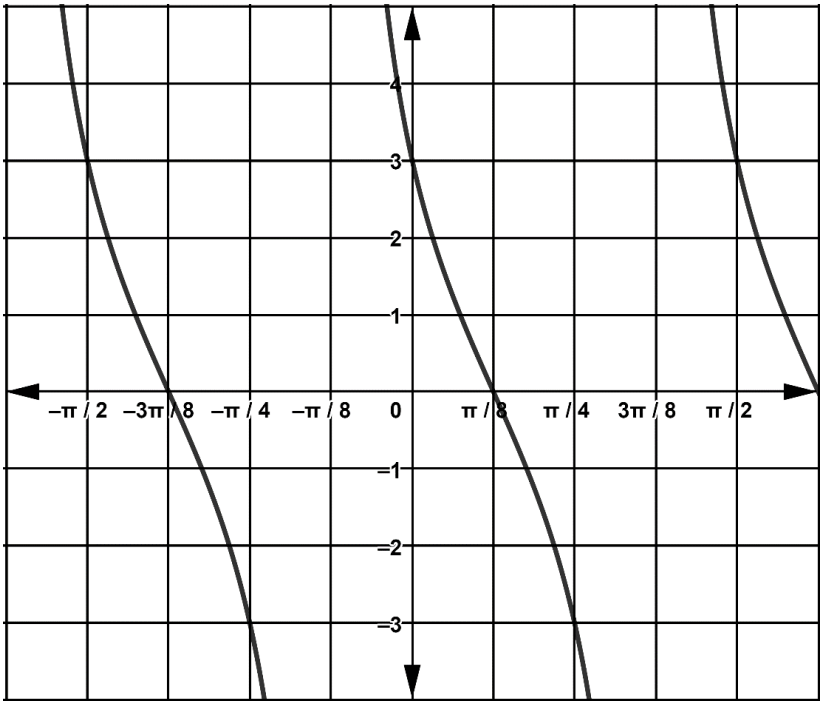
Solve $2\cos\left(2\left(x+\frac{\pi}{3}\right)\right)=-1$ given that $x \in [0,2\pi]$. Show your working.

Question 3 {2.3.4, 2.3.6} **(7 marks)**

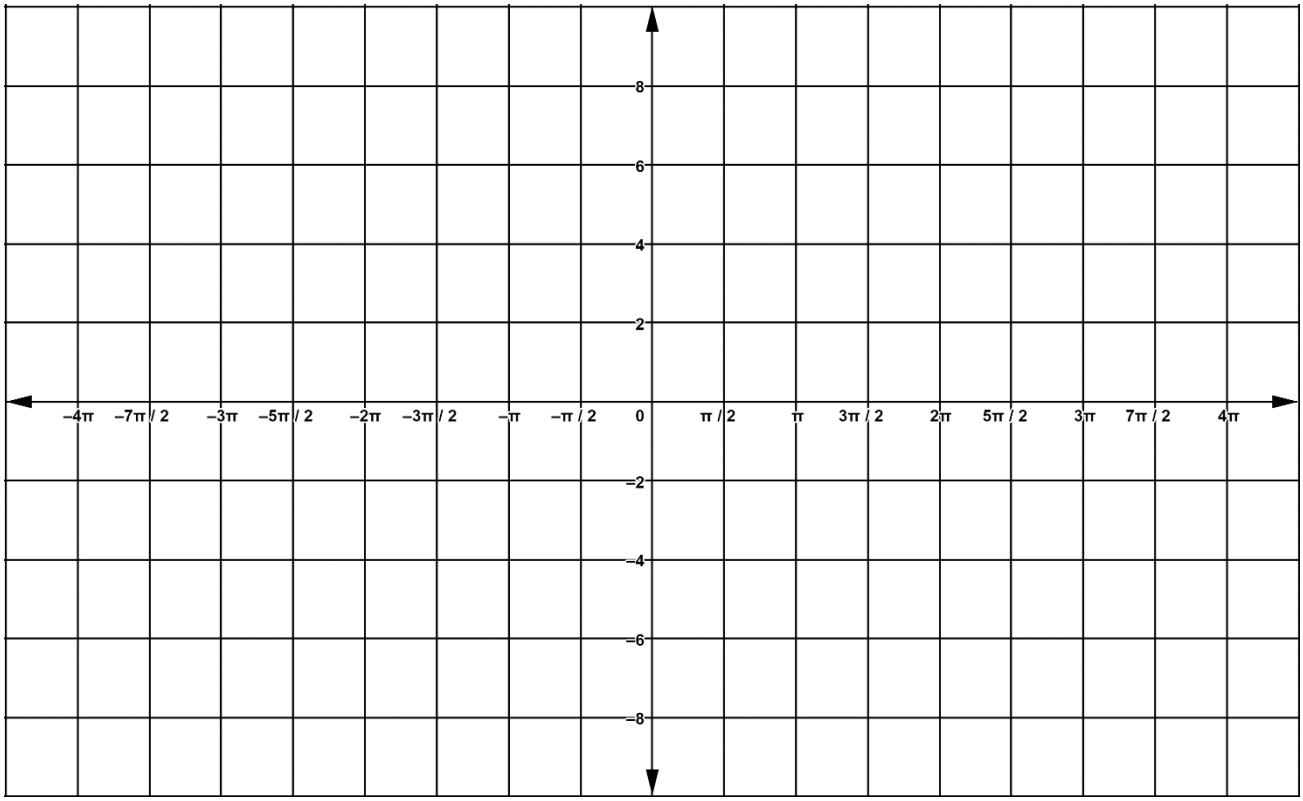
Use mathematical induction to prove that that 4^n+6n-1 is divisible by 3 for all $n \in N$

Question 4 {2.1.2} **(8 marks)**

(a) The function $f(x)=a\tan(b(x-c))$ has been graphed below. Determine the values of the constants a, b and c. (4 marks)



(b) Sketch the graph of $y = 6\cos\left(\frac{1}{2}x + \frac{\pi}{4}\right)$. (4 marks)



Investigation Validation {2.1.3, 2.3.4, 2.3.5} **(16 marks)**

a) Use the identity

$$2\,sinAcosB = \sin(A+B)+\sin(A-B)$$

(or otherwise) to show that

$$2\sin[x]\cos[(2k+1)x] = \sin[2(k+1)x] - \sin[2kx]$$

b) Given that $\sin(x)\neq 0$ prove, by mathematical induction, that for all positive integers *n*,

$$\cos(x)+\cos(3x)+\ldots+\cos[(2n-1)x]=\frac{\sin(2nx)}{2\sin(x)}$$

You may find the identity $\sin(2A)=2sinAcosA$ useful.

(additional working space)