

2. [4 marks] Show, using the quotient rule, that $\frac{d}{dx} \tan(x) = 1 + \tan^2 x$.

[2]

$$c) y = \sin^2(4x)$$

[2]

$$\frac{1 - x_5}{x_{HS}} = \lambda \quad (q)$$

[2]

$$y(x) = -\cos\left(\frac{x}{2}\right)$$

Differentiate each of the following functions with respect to x . **Do not simplify** your answers.

Q. [6 marks]

- Complete all q
 - Show all necce
 - Total Marks =
 - 27 minutes

Name: _____

Teachers: Mr. Staffe
Mrs. Carter
Mr. Bertram
Mr. Roothi
Ms Chenege



Test Three 2016

Year 12 Mathematics Methods

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3. [9 marks]

Simplify or Evaluate the following integrals as appropriate

(a) $\int \sin 2u \, du$

[3]

(b) $\int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} \right) dx$

[3]

(c) Evaluate $-2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 3x \, dx$

[3]

4. [8 marks]

(a) Simplify $\int \cos\left(\frac{t}{3}\right) dt$

[2]

(b) Use your answer to part (a) to evaluate $\int_x^{2x+1} \cos\left(\frac{t}{3}\right) dt$, in terms of x

[2]

$$P(X \geq 1) = 0.2143$$
✓

- (d) Less than 4 students requiring the removal of at least one tooth given that at least one student required tooth removal.

$$P(X < 4 | X \geq 1) = \frac{P(1 \leq X \leq 3)}{P(X \geq 1)} = \frac{0.2142}{0.2143} = 0.9995$$
✓

Of the thirteen year-olds in Britain requiring tooth removal, the probability of them requiring just one tooth out of their 32 permanent teeth removed is 5%.

- (e) Calculate the probability of a permanent tooth in a thirteen year - old needing removal.

$$Y \sim Bin(32, 0.05)$$

$$P(Y = 1) = 0.3263$$
✓



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Name:

- Complete all questions
- Show all necessary working
- Total Marks = 23
- 23 minutes

1. [6 marks]

Determine, with reasoning, whether each of the following represent a discrete random variable.

(a)	<table border="1"> <thead> <tr> <th>x</th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th></tr> </thead> <tbody> <tr> <td>$P(X=x)$</td><td>0.3</td><td>0.1</td><td>0.4</td><td>0.05</td><td>0.15</td></tr> </tbody> </table>	x	0	1	2	3	4	$P(X=x)$	0.3	0.1	0.4	0.05	0.15
x	0	1	2	3	4								
$P(X=x)$	0.3	0.1	0.4	0.05	0.15								

(b)	<table border="1"> <thead> <tr> <th>x</th><th>-2</th><th>-1</th><th>1</th><th>3</th><th>5</th></tr> </thead> <tbody> <tr> <td>$P(X=x)$</td><td>0.2</td><td>0</td><td>-0.3</td><td>0.6</td><td>0.5</td></tr> </tbody> </table>	x	-2	-1	1	3	5	$P(X=x)$	0.2	0	-0.3	0.6	0.5
x	-2	-1	1	3	5								
$P(X=x)$	0.2	0	-0.3	0.6	0.5								

Teacher:
_____ Mr Staffe
_____ Mrs. Carter
_____ Mr Bertram
_____ Mr Roohi
_____ Ms Cheng

4. **[13 marks]**
Phoebe and Katelyn are facing a multiple choice assessment for their least favourite subject.

Marks for this test will be awarded in the following way: 4 marks will be awarded for a correct answer, 0 marks will be awarded for not attempting a question and 2 marks will be deducted for an incorrect answer.

This assessment contains 20 questions, each with four alternative answers.

Katelyn starts reading the test and is certain she knows 6 of the answers.

- (a) If Katelyn attempts all questions, what is the chance she'll answer 15 out of 20 correctly?

$$X \sim Bin(14, 0.25)$$

$$P(X = 9) = 0.00181$$
✓

- (b) If Katelyn attempts all questions, what is the most likely number of questions she'll answer correctly?

$$P(X = 3) = 0.2402$$
✓

[3]

x	0	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$

(c)

- (c) At least one student requiring the removal of at least one tooth.

$$P(X=0) = 0.7857 \quad \text{Red mark}$$

- (b) No students requiring the removal of any teeth.

$$P(X=2) = 0.02213 \quad \text{Red mark}$$

$$X \sim \text{Bin}(24, \frac{1}{100})$$

- (a) 2 students needing at least one tooth removed.

Determine the probability of:

[3]

$$(b) P(x) = \frac{2k-1}{x}; \quad x = 1, 2, 3, 4, 5, 6$$

A typical primary school class of 24 eight year-olds are investigated for the need to remove at least one tooth.

Studies in Britain have recorded that 1 in 100 eight year-old children need at least one tooth removed caused by sugary drinks and severe tooth decay.

Studies in Britain have recorded that 1 in 100 eight year-old children need at least one tooth removed caused by sugary drinks and severe tooth decay.

[2]

x	0	1	2	3	4	0.15
$P(X=x)$	0	0.2	k	0.3	0.1	0.15

(a)

Each of the following represent discrete probability functions. Determine the value of k for each.

2. [8 marks]

$$k = \frac{15}{1} \quad \text{Red mark}$$

$$2k + k + 5k + 6k = 1 \quad \text{Red mark}$$

$$(c) P(X=x) = \left(\frac{2}{1}\right)^x; \quad x = 1, 2, 3, 4, \dots$$

x	$P(X=x)$	1	$2k$	k	3	k	$5k$	$6k$

(c)

Yes this table does represent a DRV, all probabilities add to 1 and there are no negative values. ✓✓

(b)	x	-2	-1	1	3	5
	$P(X=x)$	0.2	0	-0.3	0.6	0.5

No, this table does not represent a DRV. Despite all the probability values adding to 1, as one is negative, this cannot represent a DRV. ✓✓

(c) $P(X=x) = \left(\frac{1}{2}\right)^x ; x=1,2,3,4\dots$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \dots$
The sequence of probabilities is:
 $S_x = \frac{0.5}{1 - 0.5} = 1$

Therefore all probabilities will add to 1, and none are negative. ✓

3. [9 marks]

Studies in Britain have recorded that 1 in 100 eight year-old children need at least one tooth removed caused by sugary drinks and severe tooth decay.

A typical primary school class of 24 eight year-olds are investigated for the need to remove at least one tooth.

Determine the probability of:

(a) 2 students needing at least one tooth removed.

[1]

(b) No students requiring the removal of any teeth.

[1]

(c) At least one student requiring the removal of at least one tooth.

[2]

(d) Less than 4 students requiring the removal of at least one tooth given that at least one student required tooth removal.

[2]

2. [8 marks]

(a)

x	0	1	2	3	4
$P(X=x)$	0.2	k	0.3	0.1	0.15

$k = 1 - 0.2 - 0.3 - 0.1 - 0.15$

$k = 0.25$ ✓✓

(b) $P(x) = \frac{x}{2k-1}; x=1,2,3,4,5,6$

$1 = \frac{1+2+3+4+5+6}{2k-1}$ ✓

$1 = \frac{21}{2k-1}$ ✓

$k = 11$ ✓

2. [4 marks]

(a) $y = \cos\left(\frac{x}{2}\right)$

(b) $y = \sin\left(\frac{5x}{2}\right)$

(c) $y = \sin\left(\frac{\pi}{4}x\right)$

$\frac{dy}{dx} = -\frac{1}{2}\sin\left(\frac{x}{2}\right)$

$\frac{dy}{dx} = 2\sin(x)(4\cos(4x))$

$\frac{dy}{dx} = (5x - 1)(\cos x) - (5\sin x)$

Differentiate each of the following functions with respect to x . Do not simplify your answers.

Teacher:	Mr Staffe
	Mrs Carter
	Mr Bertram
	Mr Roothi
	Ms Chene

Name: **Mark King-Gyde**

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[3]

(e) Calculate the probability of a permanent tooth in a thirteen year - old needing removal.

Of the thirteen year-olds in Britain requiring tooth removal, the probability of them requiring

any one tooth out of their 32 permanent teeth removed, independently of the others is 5%.

$P(X=x)$	0	0.1	0.4	0.05	0.15
x	0	1	2	3	4

Determine, with reasoning, whether each of the following represent a discrete random variable.

1. [6 marks]

- Complete all questions
- Show all necessary working
- Total Marks = 36
- 36 minutes

Name: **Mark King-Gyde**

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Teacher:	Mr Staffe
	Mrs Carter
	Mr Bertram
	Mr Roothi
	Ms Chene

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- Complete all questions
- Show all necessary working
- Total Marks = 36
- 36 minutes

Show, using the quotient rule, that $\frac{d}{dx} \tan(x) = 1 + \tan^2 x$.

$$\begin{aligned}y &= \tan x = \frac{\sin x}{\cos x} \quad \checkmark \\ \frac{dy}{dx} &= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2 x} \quad \checkmark \\ \frac{dy}{dx} &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \quad \checkmark \\ \frac{dy}{dx} &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \quad \checkmark \\ \frac{dy}{dx} &= 1 + \tan^2 x\end{aligned}$$

3. [9 marks] Simplify or Evaluate the following integrals as appropriate

(a) $\int \sin 2u \, du = \frac{-\cos 2u}{2} + c \quad \checkmark$

(b) $\int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} \right) dx = 3\sin\left(\frac{x}{3}\right) + \frac{3(6x)^{\frac{4}{3}}}{8} + c \quad \checkmark$

(c)
$$\begin{aligned}-2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 3x \, dx \\ = -2 \left[\frac{-\cos 3x}{3} \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \quad \checkmark \\ = -2 \left[\frac{-\cos \pi}{3} - \frac{-\cos \frac{\pi}{2}}{3} \right] \quad \checkmark \\ = -2 \left(\frac{1}{3} + 0 \right) \\ = -\frac{2}{3} \quad \checkmark\end{aligned}$$

4. [8 marks]

(a) Calculate $\int \cos\left(\frac{t}{3}\right) dt$

$$= 3\sin\frac{t}{3} + c \quad \checkmark$$

(b) Use your answer to part (a) to evaluate $\int_{\pi}^{2\pi+1} \cos\left(\frac{t}{3}\right) dt$, in terms of x

$$\begin{aligned}&= \left[3\sin\frac{t}{3} + c \right]_{\pi}^{2\pi+1} \\ &= \left(3\sin\frac{2x+1}{3} + c \right) - \left(3\sin\frac{\pi}{3} + c \right) \\ &= 3\sin\frac{2x+1}{3} - 3\sin\frac{\pi}{3} \quad \checkmark\end{aligned}$$

(c) Use your answer to part (b) to evaluate $\frac{d}{dx} \left(\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt \right)$

$$\begin{aligned}&\frac{d}{dx} \left(3\sin\frac{2x+1}{3} - \frac{3\sqrt{3}}{2} \right) \\ &= 3\cos\frac{2x+1}{3} \times 2 \quad \checkmark \\ &= 6\cos\frac{2x+1}{3} \quad \checkmark\end{aligned}$$

(d) Hence evaluate $\frac{d}{dx} \left(\int_{\pi}^{f(x)} \cos\left(\frac{t}{3}\right) dt \right)$

$$\begin{aligned}&= \cos\left(\frac{f(x)}{3}\right) \times f'(x) \quad \checkmark\end{aligned}$$