



Name: \_\_\_\_\_

Marks: /35

Time: 45 minutes

Calculator Free (25 marks)

1. [7 marks]

Differentiate the following functions and simplify:

a)  $y = (1 + 3x^3)^5$

[2]

b)  $y = \sqrt{x}e^{x^2+1}$

[2]

c)  $y = (1 - x^2)e^{4x}$

[3]

2. [5 marks]

a) Consider  $f(x) = \frac{(x-2)^2}{e^{x-2}}$ , clearly show that  $f'(x) = \frac{-x^2 + 6x - 8}{e^{x-2}}$  [3]

b) Determine the x-ordinates of the point(s) where the gradient of the curve is zero. [2]

3. [3 marks]

Determine the equation of the tangent to the curve  $y = 3x^2 + e^{2x} + 3$  at the point  $(1, 6 + e^2)$ .

8. [4 marks]  
 The rate of decay of a radio-active material is proportional to the amount present  
 i.e.  $\frac{dM}{dt} = -kM$  where  $M$  is the amount of radio-active material in grams and  $t$  is in  
 years.

Given that it takes 100 years for ten grams of the materials to decay to eight grams,  
 determine:

a) the mass present after 50 years, if ten grams were originally present

b) the material's half-life.

a)

$$\frac{dz}{dx}$$

5. [4 marks]  
 If  $z = 6 - x^2$  and  $y = \sqrt{z}$  determine:

b)

$$\frac{dz}{dy}$$

c)

$$\frac{dp}{dx}$$

6.

3.

4. [3 marks]

The curve  $y = a\sqrt{x} + 3x$  has a gradient of 4 when  $x = 1$ .  
 Calculate the value of  $a$ .

6. [3 marks]

Given  $y = x + \sqrt{x^2 - 4}$  show that  $\frac{d^2y}{dx^2} = \frac{-4}{(\sqrt{x^2 - 4})^3}$

**Calculator Section (10 marks)**

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7. [6 marks]

The temperature,  $T$  °C, of a bronze casting  $t$  seconds after being removed from a kiln was modelled by  $T = T_0 e^{-0.0034t}$  for  $0 \leq t \leq 800$ .

a) How long, to the nearest second, did it take for the initial temperature of the casting to halve? [2]

b) Determine the initial temperature of the casting, given that it had cooled to 787 °C after one minute. [2]

c) Can the above rate of change model be used to calculate how long it takes the temperature of the casting to fall below 40 °C? Explain your answer. [2]