PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 1 Year 12

Student name: ________________ Teacher name: ________________

Task type: **Response/Investigation**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **7**

Materials required: No cals allowed!!

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **42 marks**

Task weighting: **13%**

Formula sheet provided: **no but formulae stated on page 2**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = \lvert z\rvert = \sqrt{a^2+b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan\theta = \frac{b}{a}, \quad -\pi < \theta \le \pi$
$\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$	$\left\lvert \frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = \lvert z\rvert^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{\lvert z\rvert^2}$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$	$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
Polar form	
$z = a + bi = r(\cos\theta + i\sin\theta) = r\,cis\,\theta$	$\bar{z} = r\,cis\,(-\theta)$
$z_1 z_2 = r_1 r_2\,cis\,(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,cis\,(\theta_1 - \theta_2)$
$cis(\theta_1 + \theta_2) = cis\,\theta_1\,cis\,\theta_2$	$cis(-\theta) = \frac{1}{cis\,\theta}$
De Moivre's theorem	
$z^n = \lvert z\rvert^n\,cis\,(n\theta)$	$(cis\,\theta)^n = \cos n\theta + i\sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\frac{\theta+2\pi k}{q} + i\sin\frac{\theta+2\pi k}{q}\right)$, for k an integer	

$$(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$$

No cals allowed!!

Q1 (2, 2, 2 & 2 = 8 marks)

If $z = 3 + 4i$ and $w = 1 - i$ determine the following exactly.

a) zw

b) $z^2 w$

c) $\dfrac{1}{\bar{z}}$

d) $\dfrac{z}{w}$

Q2 (4 marks)

Determine all possible real number pairs $a \& b$ such that $\dfrac{22-3i}{a+i} = 5 + bi$.

Q3 (2, 3 & 3 = 8 marks)
Consider the function $f(z) = z^3 + 2z^2 + 9z + 18$.

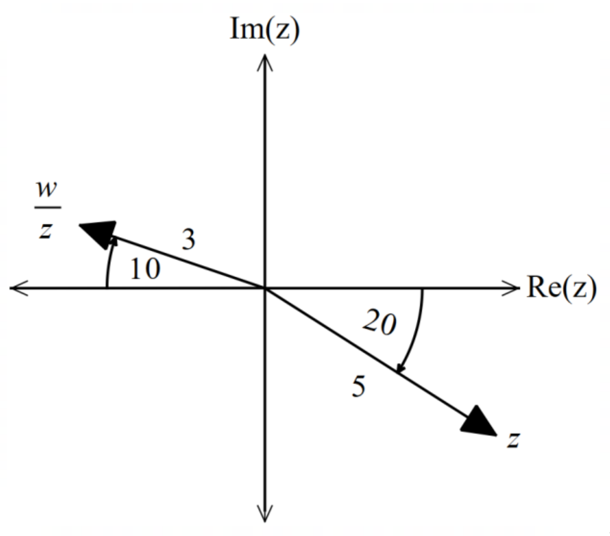
a) Determine $f(3i)$.

b) Hence solve $z^3 + 2z^2 + 9z + 18 = 0$

c) Consider $g(z) = (z^2 + bz + c)(z^2 + dz + e)$ where $b, c, d \,\&\, e$ are real constants and $g(3+i) = 0 = g(2-3i)$. Determine the values of $b, c, d \,\&\, e$.

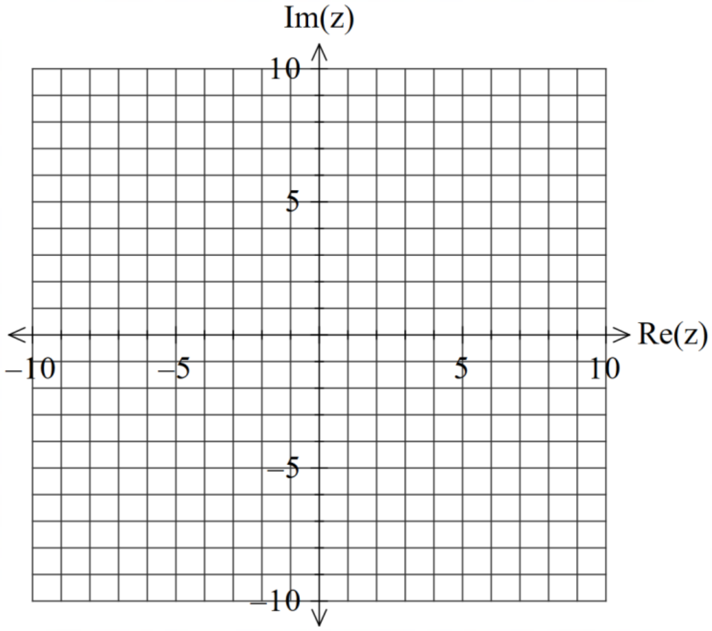
Q4 (3 marks)
Use the diagram below to determine the complex number w in polar form with a principal argument.

(diagram not drawn to scale)

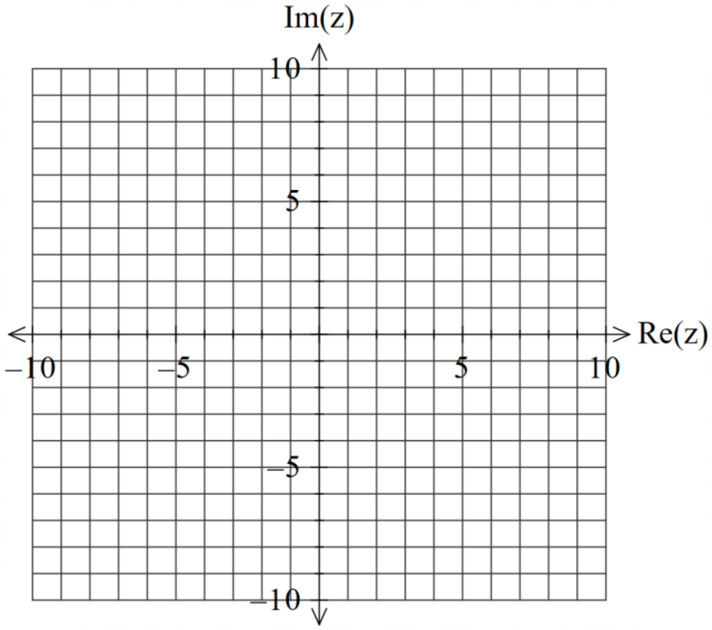


Q5 (2 & 3 = 5 marks)
Sketch the following locus of points on the axes below.

a) $|z-2-3i|+|z-5-7i|=5$



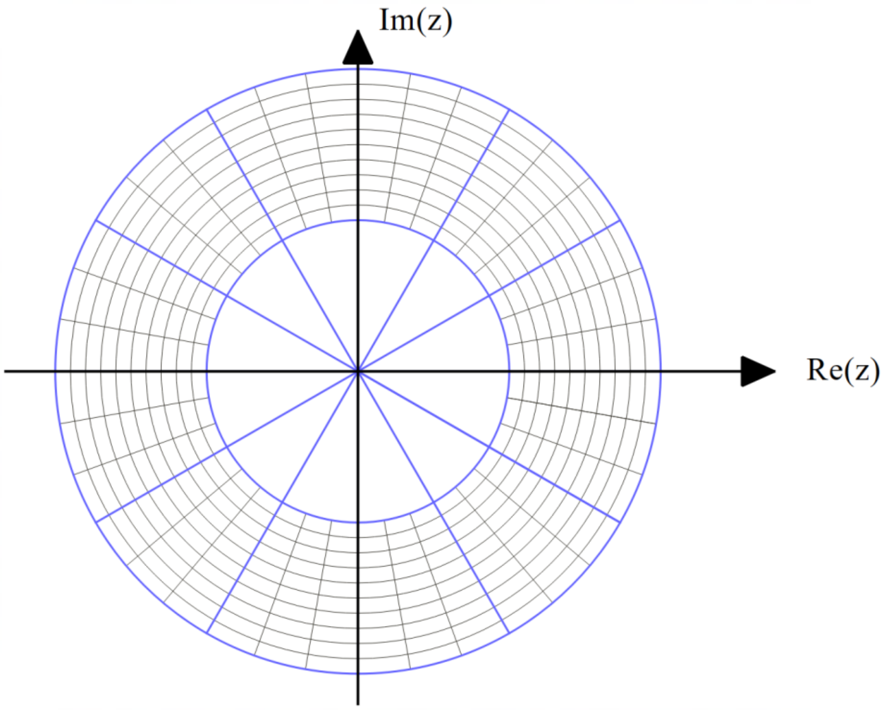
b) $|z-7|=|z-3i|+\sqrt{58}$



Q6 (5, 2 & 2 = 9 marks)

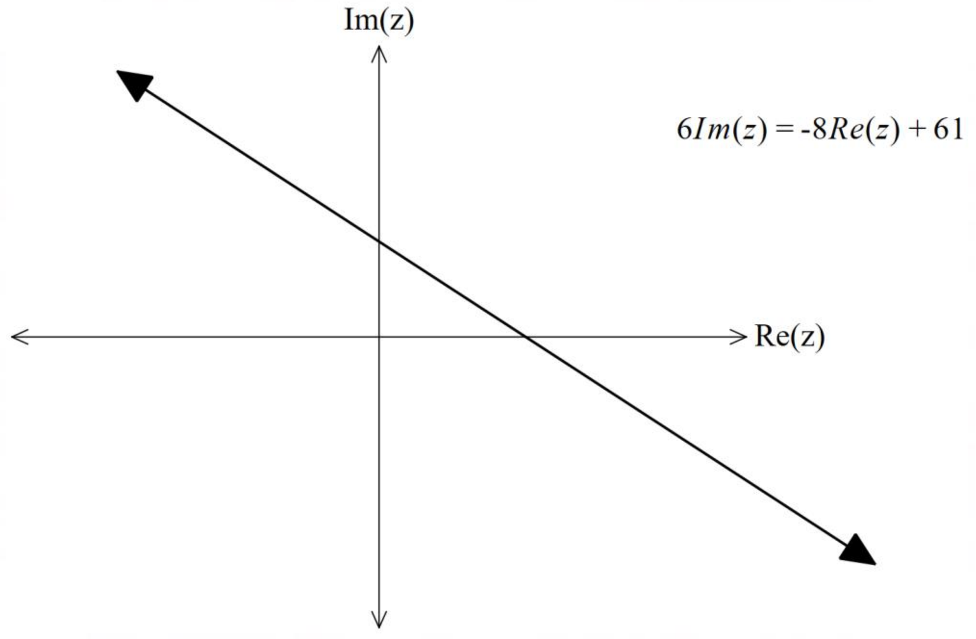
a) Solve $z^6 = 2 + 2\sqrt{3}i$ in polar form with principal arguments.

b) Plot these points on the axes below.



c) Determine the area of the polygon formed by joining the points in (b) above.

Q7 (5 marks)
The locus of $|z-a-2i|=|z-7-bi|$ where $a\,\&\,b$ are real constants is plotted below and can also be defined as $6\,\mathrm{Im}(z)=-8\,\mathrm{Re}(z)+61$. Determine the values of $a\,\&\,b$ showing full reasoning.
(Not drawn to scale)



Working out space