PERTH MODERN SCHOOL
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Independent Public School

# Course Methods Year 12

Student name: _____ Teacher name: _____

**Task type:** **Response**

**Time allowed for this task:** **____45_____ mins**

**Number of questions:** **____8_____**

**Materials required:** Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

**Marks available:** **__49___ marks**

**Task weighting:** **__10__%**

**Formula sheet provided: Yes**

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

**Q1 (3.1.7)** **(9 marks)**

Use the product rule and/or quotient rule to differentiate the following.(Simplify)

i) $y=(x-11)(x^3+2)$ (3 marks)

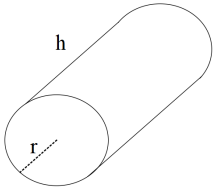
| **Solution** |
|---|
| $\frac{dy}{dx}=(x-11)3x^2+(x^3+2)(1)$<br>$=3x^3-33x^2+x^3+2$<br>$=4x^3-33x^2+2$ |
| **Specific behaviours** |
| ✓ demonstrates use of product rule<br>✓ differentiates correctly<br>✓ simplifies<br>NOTE: Zero for answer only as done by classpad |

ii) $y=\frac{2x+1}{(3-x)}$ (3 marks)

| **Solution** |
|---|
| $\frac{dy}{dx}=\frac{(3-x)2-(2x+1)(-1)}{(3-x)^2}$<br>$=\frac{6-2x+2x+1}{(3-x)^2}$<br>$=\frac{7}{(3-x)^2}$<br>(May leave denominator in expanded form) |
| **Specific behaviours** |
| ✓ demonstrates use of quotient rule<br>✓ differentiates correctly<br>✓ simplifies<br>NOTE: Zero for answer only as done by classpad |

**Q8 (3.1.16)** **(4 marks)**

Consider a closed hollow cylinder with end radius $r$ and length $h$.




If the outside of the cylinder has a surface area of $300\,m^2$ determine the dimensions of the radius and length, nearest cm, to maximise the capacity of the cylinder **using calculus techniques**.

| **Solution** |
|---|
| $2\pi r^2 + 2\pi rh = 300$<br>$h = \frac{150 - \pi r^2}{\pi r}$<br>$V = \pi r^2 h = \pi r^2 \frac{150 - \pi r^2}{\pi r} = 150r - \pi r^3$<br>$\frac{dV}{dr} = 150 - 3\pi r^2 = 0$<br>$r = \sqrt{\frac{50}{\pi}} \approx 3.98m \quad h \approx 7.99m$<br>$\frac{d^2V}{dr^2} = -6\pi r = -6\pi\sqrt{\frac{50}{\pi}} < 0 \quad \therefore local\ \max$ |
| **Specific behaviours** |
| ✓ states constraint equation in terms of r and h<br>✓ differentiates V and equates to zero<br>✓ solves for r and h, **must be in decimal** form but do not penalise if not rounded to nearest cm<br>✓uses second derivative test to show local max |





iii) $y = (5 - 2x)(x^2 + 1)^3$ (3 marks)

| **Solution** |
|---|
| $(5 - 2x)3(x^2 + 1)^2\, 2x + (x^2 + 1)^3 (-2)$<br>$2(x^2 + 1)^2 \left[3x(5 - 2x) - (x^2 + 1)\right]$<br>$2(x^2 + 1)^2 \left[15x - 6x^2 - x^2 - 1\right]$<br>$2(x^2 + 1)^2 \left[15x - 7x^2 - 1\right]$ |
| **Specific behaviours** |
| ✓ demonstrates use of product **and** chain rules correctly<br>✓ differentiates correctly for entire function<br>✓Simplifies correctly<br>NOTE: Zero for answer only as done by classpad |

**Q2** **(3 marks)**

Determine the equation of the tangent to $y = (3x + 1)^3$ at the point $(1, 64)$.

| **Solution** |
|---|
| $\frac{dy}{dx} = 3(3x + 1)^2\, 3$<br>$\left.\frac{dy}{dx}\right\|_{x=1} = 144$<br>$y = 144x + c$<br>$64 = 144 + c$<br>$c = -80$<br>$y = 144x - 80$ |
| **Specific behaviours** |
| ✓ uses chain rule to differentiate<br>✓ solves for constant<br>✓states equation |

**Q7 (3.1.11) (6 marks)**

A colony of bacteria is represented as a circle on a petri dish and is increasing in such a way that the number of bacteria present is given by $N$ where $N=\sqrt{3x+2}$, $x$ being the radius of the circle of bacteria.

a) Determine $N'(2)$ and explain its meaning. (3 marks)

| **Solution** |
|---|
| $N'=\frac{3}{2}(3x+2)^{\frac{-1}{2}}$<br>$N'(2)=\frac{3}{2\sqrt{8}}=\frac{3}{4\sqrt{2}}\approx 0.53$<br>Rate of change of N at x=2 (SCSA preferred answer) |
| **Specific behaviours** |
| ✓ states derivative in terms of x<br>✓ states value at x=2(accept approx.)<br>✓describes as rate of change **at x=2** (accept gradient of tangent at x=2) |

b) Determine $N''(2)$ and explain its meaning. (3 marks)

| **Solution** |
|---|
| $N''=\frac{-3}{4}(3x+2)^{\frac{-3}{2}}(3)$<br>$=\frac{-9}{4(8)^{\frac{3}{2}}}\approx -0.09943$<br>Rate of change of $N'(x)$ at x=2 (SCSA preferred answer) |
| **Specific behaviours** |
| ✓ states second derivative in terms of x<br>✓ states value at x=2(accept approx.)<br>✓describes as rate of change of $N'(x)$ **at x=2** (accept gradient of dy/dx at x=2) |

**Note must mention at x=2 otherwise max 4 out of 6 marks**

c) The distance travelled in the first 12 seconds. (2 marks)

| Solution |
|---|
| t=0 x=18<br>t=5.5 x=-12.25 turns around<br>t=12 x=30<br>Distance equals 18 +12.25 +12.25 +30=72.5 metres |
| **Specific behaviours** |
| ✓ determines distance from start to turning pt<br>✓ determines total distance, no need for units. |

d) $x''(1)$ and explain its meaning. (2 marks)

| Solution |
|---|
| Acceleration of 2 at t=1 second |
| **Specific behaviours** |
| ✓ states acceleration at time t=1 (accept rate of change of v at t=1)<br>✓ states 2 for second derivative |

**Q6 (3.1.10) (3 marks)**

If $y=3x^5$ use the small increments formula $\partial y \approx \frac{dy}{dx}\partial x$ to determine the approximate percentage change in $y$ when $x$ decreases by $2\%$.

| Solution |
|---|
| $\frac{\Delta y}{y} \approx \frac{\frac{dy}{dx}\Delta x}{y}$<br>$= \frac{15x^4\Delta x}{3x^5} = 5\frac{\Delta x}{x} = 10\%$ |
| **Specific behaviours** |
| ✓ uses increments formula<br>✓ obtains expression for approx. percentage change for y in terms of x<br>✓obtains approx. percentage change for y |





**Q3 (3.1.8) (8 marks)**

Consider the functions $P(x) \& Q(x)$ and their derivatives $P'(x) \& Q'(x)$ with values given for the following x values.

| X value | -1 | 3 | 7 |
|---|---|---|---|
| $P(x)$ | 5 | 2 | -4 |
| $P'(x)$ | 0 | 1 | -2 |
| $Q(x)$ | 2 | 5 | -3 |
| $Q'(x)$ | -1 | -2 | 6 |

Determine the following **derivatives** at the given x values.'

a) $P(x)Q(x)$ at $x=3$ (2 marks)

| Solution |
|---|
| $P(x)Q(x)$<br>$P(x)Q'(x)+Q(x)P'(x)$<br>$(2)(-2)+(5)(1)$<br>$1$ |
| **Specific behaviours** |
| ✓ uses product rule<br>✓states result |

b) $\left[Q(x)\right]^3$ at $x=-1$ (3 marks)

| Solution |
|---|
| $3\left[Q(x)\right]^2 Q'(x)$<br>$3\left[2\right]^2(-1)$<br>$-12$ |
| **Specific behaviours** |
| ✓ demonstrates chain rule<br>✓ subs values correctly<br>✓states final result |

c) $\frac{[P(x)]^2}{Q(x)}$ at $x=7$ (3 marks)

| Solution |
|---|
| $\frac{Q(x)2P(x)P'(x) - P^2(x)Q'(x)}{Q^2(x)}$<br>$\frac{(-3)2(-4)(-2) - (-4)^2(6)}{9}$<br>$-16$ |
| **Specific behaviours** |
| ✓ demonstrates quotient **and** chain rule<br>✓ subs values correctly<br>✓states final result |

**Q4 (3.1.14, 3.1.15) (7 marks)**

Use calculus techniques to determine the **exact** coordinates of any stationary points on the following curves and use the second derivative test to determine the nature of the stationary point.

a) $y=(x-4)^3-1$ (3 marks)

| Solution |
|---|
| $y'=3(x-4)^2=0$<br>$x=4$<br>$y''=6(x-4) \quad x=4 \Rightarrow y''=0$<br>(4,-1) inflection |
| **Specific behaviours** |
| ✓ determines first derivative<br>✓ equates to zero and solves for stationary pt and states y value<br>✓determines value of second derivative and states horizontal inflection |





b) $y=2x^3+9x^2-60x+12$ (4 marks)

| Solution |
|---|
| $y'=6x^2+18x-60=0$<br>$x^2+3x-10=(x+5)(x-2)=0$<br>$x=-5,2$<br>$y''=12x+18$<br>$x=-5 \; y''=-42 \quad \therefore local \max(-5,287)$<br>$x=2 \; y''=42 \quad \therefore local \min(2,-56)$ |
| **Specific behaviours** |
| ✓ determines first derivative and equates to zero<br>✓ solves for stationary pts including y value<br>✓ determines second derivative for stationary pts<br>✓identifies nature for each stationary point |

**Q5 (3.1.12) (7 marks)**

The displacement of a body from an origin O, at time $t$ seconds, is $x$ metres where $x=t^2-11t+18$, $t \geq 0$.

Determine the following.

a) The velocity function. (2 marks)

| Solution |
|---|
| $v=2t-11$ |
| **Specific behaviours** |
| ✓ differentiates<br>✓ expresses in terms of t |

b) The times and displacements when the body is at rest. (3 marks)

| Solution |
|---|
| $2t-11=0$<br>$t=5.5$<br>$x=-12.25$ |
| **Specific behaviours** |
| ✓ equate velocity to zero<br>✓ solves for time<br>✓determines displacement |