



**Hale School
2012**

Question/Answer Booklet

**MATHEMATICS 3CD
SEMESTER 2**

**Section One
(Calculator Free)**

Booklet 1 of 3

Circle your teacher's initials

BAH

GJ

JIB

VMU

Your name _____

Time allowed for this section

Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/answer booklet for Section One.
Formula sheet.

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

See next page

Structure of this examination

		Number of questions	Working time (minutes)	Marks available
Booklet 1 This Booklet (Section 1)	Calculator Free	9	50	50
Booklet 2 (Section 2)	Calculator Assumed	6	100	50
Booklet 3 (Section 2)		7		50
Total marks				150

Instructions to candidates

- The rules for the conduct of Western Australian external examinations are detailed in the booklet *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in the spaces provided in this Question/Answer Booklet.
- Spare answer pages are provided at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning : If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued i.e. give the page number. Fill in the number(s) of the question(s) that you are continuing at the top of the page.
- Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.

See next page

Question 1 (6 marks)

Differentiate the following with respect to x , without simplifying.

(a) $f(x) = \frac{1+e^x}{1-e^{-2x}}$ [2 marks]

(b) $g(x) = (3x+1)(1+x^2)^3$ [2 marks]

(c) $h(x) = \int_1^{x^2} (1-4t)^3 dt$ [2 marks]

Question 2 (5 marks)

(a) Evaluate $\int_{0.5}^1 (\sqrt{2x-1})^3 dx$ [3 marks]

(b) Determine $\int x e^{1+x^2} dx$ [2 marks]

Question 3 (5 marks)

Calculate the maximum and minimum values of $\frac{x^2 + 4}{x}$ in the interval $1 \leq x \leq 5$.

Question 4 (5 marks)

Solve the inequality $\frac{1-4x}{x+12} \leq 3$

Question 5 (7 marks)

Let $f(x) = e^{-x}$ and $g(x) = \frac{1}{1-x}$.

(a) Determine expressions for $f(g(x))$ and $g(f(x))$. [2 marks]

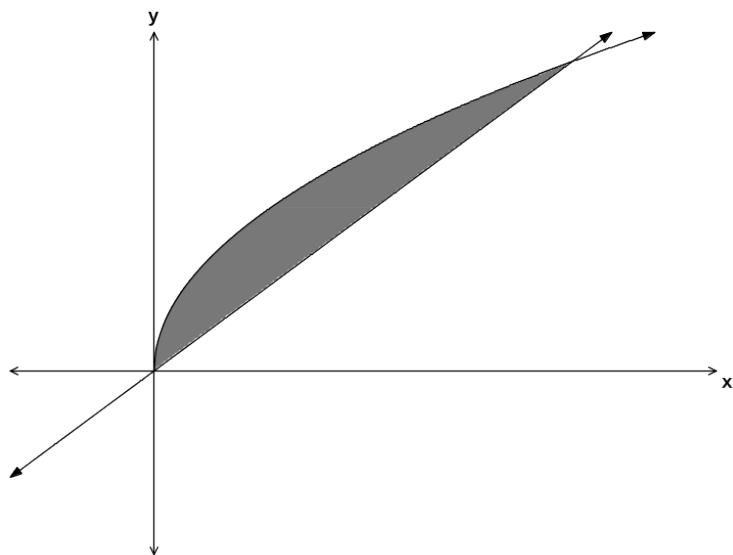
(b) Evaluate $f(g(0))$ and $g(f(0))$. [2 marks]

(c) Determine the domain of $f(g(x))$. [1 mark]

(d) Determine the range of $g(f(x))$. [2 marks]

Question 6 (6 marks)

The diagram below shows graphs of $y = \sqrt{x}$ and $y = 0.5x$. Find the shaded area.



Question 7 **(6 marks)**

Let A denote the set $\{1, 2, 3, \dots, 999, 1000\}$, the set of positive integers up to 1000.

(a) How many numbers in set A are not multiples of either 4 or 5 or both?
[3 marks]

(b) How many numbers in set A that have at least 2 digits start and finish
with the same digit? [3 marks]

Question 8 **(6 marks)**

The function $r(x) = ax + bx^2 + \frac{c}{x}$ has the following properties:

- $r(2) = 20$
- $r(x)$ has a stationary point when $x = 1$
- $r(x)$ has a point of inflection at $x = -2$

(a) Show that the constants a , b and c satisfy the simultaneous equations:

$$4a + 8b + c = 40, \quad a + 2b - c = 0, \quad 8b - c = 0. \quad [3 \text{ marks}]$$

Question 8 (cont)

- (b) Evaluate the constants a , b and c by solving the equations in part (a).
[3 marks]

Question 9 (4 marks)

Consider the conjecture:

“every prime number greater than 3 is one more or less than a multiple of 6”.

- (a) Show that the conjecture is true for three values. [1 mark]

- (b) Prove the conjecture. [3 marks]

Spare Page for Extra Working.