PERTH MODERN SCHOOL
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Independent Public School

Course ___Methods_Test 3_ Year __12________

Student name: ______________ Teacher name: ______________

Date:

Task type: **Response**

Time allowed for this task: _____45______ mins

Number of questions: _____9______

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: __46____ marks

Task weighting: __10__%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3.1.6) (3 & 3 = 6 marks)

Determine the exact gradient of each of the following at the given point. Show all working.

a) $y = \cos 3x$ at the point $\left(\frac{\pi}{3}, -1\right)$

Solution
$y' = -3\sin 3x$ $=0$
Specific behaviours
✓ diff ✓ subs x value ✓ obtains derivative

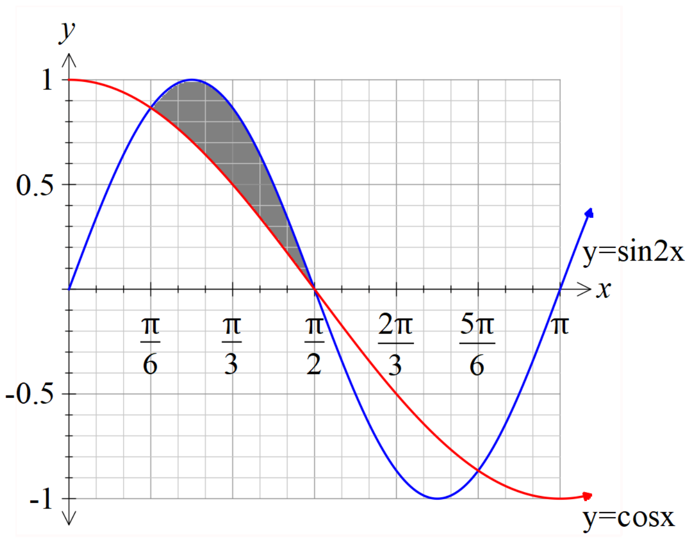
b) $y = 5\cos^2 x$ at the point $\left(\frac{\pi}{6}, \frac{15}{4}\right)$

Solution
$y' = 10\cos x(-\sin x)$ $=10\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = -\frac{5\sqrt{3}}{2}$
Specific behaviours
✓ diff ✓ subs x value ✓ obtains derivative

Q2 (3.1.6) (4 marks)

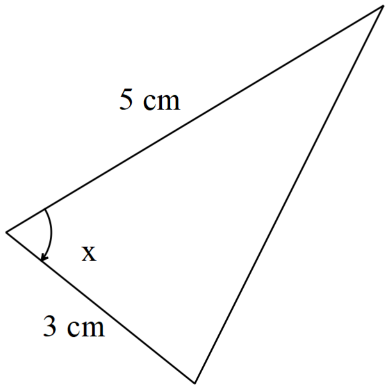
Determine the exact area shaded in the diagram below without the use of a classpad.

Solution
$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x - \cos x \, dx$ $= \left[\frac{-1}{2} \cos 2x - \sin x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \left(\frac{1}{2} - 1 \right) - \left(\frac{-1}{4} - \frac{1}{2} \right)$ $= \frac{1}{4}$
Specific behaviours
✓ sets up integral ✓ uses correct limits ✓ shows antiderivatives ✓ determines area



Q3 (3.1.6/3.1.10) (3 & 3 = 6 marks)

Consider the triangle drawn below with angle x radians and fixed length sides 5 & 3 cm. Let A represent the area of the triangle in cm^2.



a) Determine $\frac{dA}{dx}$ when $x = \frac{\pi}{4}$.

Solution
$A = \frac{1}{2}(15)\sin x$ $\frac{dA}{dx} = \frac{15}{2}\cos x$ $= \frac{15}{2}(\frac{1}{\sqrt{2}}) or \frac{15\sqrt{2}}{4} cm^2$
Specific behaviours
✓ uses area formula ✓ states derivative ✓ subs to find exact value or approx

b) Using the increments formula, determine the approximate change in the area when the angle changes from $\frac{\pi}{4}$ to $\frac{\pi}{4} + 0.01$ radians.

Solution
$\Delta A \simeq \frac{dA}{dx}\Delta x$ $= \frac{15}{2}\left(\frac{1}{\sqrt{2}}\right)0.01 \approx 0.053$

Specific behaviours
✓ uses increments formula ✓ subs correct values ✓ determines approx. change

Q4 (3.3.1) (4 marks)

The expected value of the discrete probability distribution, X given below, is $3\frac{2}{3}$. Determine the values of the constants $p \,\&\, q$ and the variance of X to 3 decimal places.

x	1	2	3	4	5
$P(X=x)$	0.1	P	0.1	q	0.3

Solution
$p+q=0.5$ $\frac{11}{3}=0.1+2p+0.3+4q+1.5$ $\left.\begin{cases} p+q=0.5 \\ \frac{11}{3}=2p+4q+1.9 \end{cases}\right\vert_{p,q}$ $\left\{p=\frac{7}{60}, q=\frac{23}{60}\right\}$ Variance =1.655
Specific behaviours
✓states one equation with p & q ✓ states second equation with p&q ✓ solves for p&q ✓ states variance to 3 dp

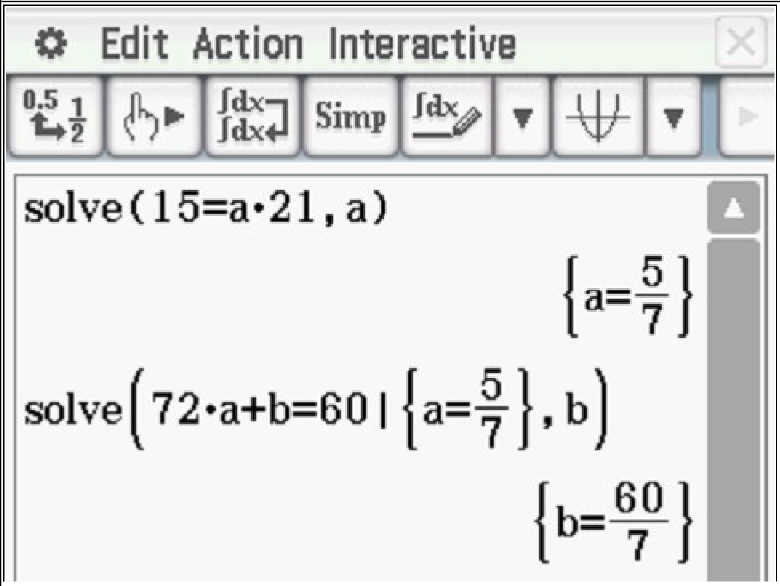
Q5 (3.3.13) (3 marks)

A binomial distribution has a mean of 6 and a standard deviation pf 1.9. Determine the values of $n \& p$, the number of trials and the probability of a success.

Solution
$\begin{cases} 6=np \\ 1.9=\sqrt{np(1-p)} \end{cases} \Big\vert_{n,\,p}$ $\{n=15.01504007, p=0.3995993333\}$ n = 15 & p=0.4
Specific behaviours
✓ states two equations for n and p   ✓ solves approx. values   ✓ rounds n to an integer

Q6 (3.3.7) (4 marks)

A teacher needs to scale the results of her class by first multiplying be a constant and then adding a second constant. The original mean was 72 with a standard deviation of 21, the teacher needs the scaled results to have a mean of 60 and a standard deviation of 15. Determine the values of $a \& b$.

Solution

Specific behaviours
✓ states one equation with constant   ✓ states two equations with constants   ✓ solves for one constant

✓ solves for second constant

Q7 (4.1.11) (3 & 3 = 6 marks)

The displacement of a car moving in straight line is given by $s(t)$ km at t hours, where $s(t) = 55 + t\ln(31t^2)$.

The following questions require full working and answers only given by the classpad will not receive full marks.

a) Determine the velocity at $t = 3.5$ hours.

Solution
$\frac{ds}{dt} = t\frac{62t}{31t^2} + \ln(31t^2)$ $= 2 + \ln\left(\frac{1519}{4}\right) \simeq 7.9$
Specific behaviours
✓ uses product rule ✓ diff log term ✓ obtains speed

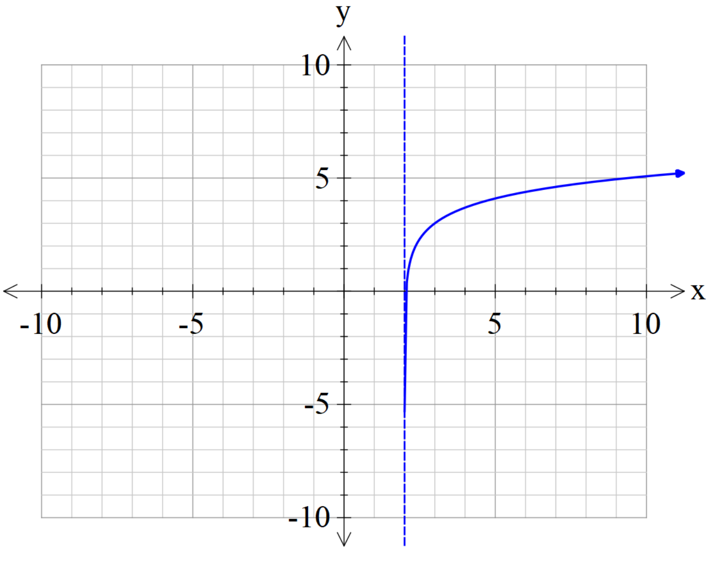
b) Determine the time that the acceleration will be 0.2 km/h^2.

Solution
$v = 2 + \ln(31t^2) = 2 + \ln 31 + 2\ln t$ $a = \frac{2}{t} = 0.2$ $t = 10$
Specific behaviours
✓ shows how to diff velocity ✓ sets up equation ✓ solves for t

Q8 (4.1.6) (3 & 3 = 6 marks)

Consider the function $f(x)=\ln(x-2)+3$.

a) Sketch the function on the axes below showing all major features.

Solution

Specific behaviours
✓ asymptote ✓ shape ✓ y less than 6 at x=10

b) In terms of the constants p & q, determine the x intercept of the function $f(x+2p)-q$.

Solution
$f(x)=\ln(x-2)+3$ $f(x+2p)-q=\ln(x+2p-2)+3-q$ $0=\ln(x+2p-2)+3-q$ $q-3=\ln(x+2p-2)$ $x+2p-2=e^{q-3}$ $x=e^{q-3}+2-2p$
Specific behaviours
✓ replaces x with x+2p ✓ rearranges to an exponential equation ✓ obtains expression for x

Q9 (4.1.11/3.2.16) (3 & 4 = 7 marks)
This question must be answered without the use of a classpad to receive full marks.

a) $\frac{d}{dx}\left[(x+1)\ln(1+x)\right]$ (Simplify)

Solution
$\frac{d}{dx}\left[(x+1)\ln(1+x)\right] = (x+1)\frac{1}{1+x} + \ln(1+x) = 1 + \ln(1+x)$
Specific behaviours
✓ uses product rule ✓ diff log term ✓ obtains simplified expression

b) Use the result from (a) above to determine $\int_2^3 \ln(1+x)\,dx$ in exact simplified form.

Solution
$\int \frac{d}{dx}\left[(x+1)\ln(1+x)\right] dx = \int 1 + \ln(1+x)$ $(x+1)\ln(1+x) = x + \int \ln(1+x)\,dx$ $\int \ln(1+x)\,dx = (x+1)\ln(1+x) - x$ $\int_2^3 \ln(1+x)\,dx = \left[(x+1)\ln(1+x) - x\right]_2^3 = (4\ln 4 - 3) - (3\ln 3 - 2)$ $= \ln 4^4 - \ln 3^3 - 1$ $= \ln\left(\frac{4^4}{3^3}\right) - 1 = \ln\left(\frac{4^4}{3^3}\right) - 1ne$ $= \ln\left(\frac{4^4}{3^3 e}\right)$
Specific behaviours
✓ uses linearity principle (first line) ✓ uses fundamental theorem ✓ obtains antiderivative and subs correct limits

✓ gives simplified exact log expression