## Calculator Assumed Sample Proportions and the Central Limit Theorem

Time: 45 minutes Total Marks: 45 Your Score: / 45



## Question One: [3, 3 = 6 marks] CA

(a) Use the fact that a linear transformation of a random variable X of the form Y = aX + b yields E(Y) = aE(X) + b and  $Var(Y) = a^2Var(X)$  to show that

E(p) = p , where  $\, p \,$  and  $\, p \,$  are the sample proportion and expected probability

respectively.

(d) Using the same facts as above, also establish that  $Var(p) = \frac{p(1-p)}{n}$ 

Mathematics Methods Unit 4

Question 6: [1, 2, 2, 2, 3 = 10 marks] CA

interval 2 to 25 minutes.

The waiting times of 15 samples of 30 patients each are recorded.

a) Calculate the mean waiting time for all of the patients at this dentist.

w nim 
$$\partial . \xi I = \Delta + \partial . II = \frac{\Delta - \partial \Delta}{\Delta}$$

Calculate the standard deviation of the waiting times for all of the patients at this dentist

The time patients wait for their appointments at a dentist is uniformly distributed over the

$$\operatorname{nim} + 6.6 = \frac{{}^{2}(2 - 82)}{21}$$

$$\text{constraint} \text{ timid feating VA}$$

$$2212.1 = \frac{46.0}{08} = 0..$$

 Define the probability distribution that best models the distribution of the means of the samples.

$$V \sim N(13.5, 1.2122) N \sim X$$

Find the probability that a random sample has a mean waiting time of less than 15 minutes.

$$0.0268.0 = (21 > X)q$$

(e) Calculate the probability that at most two samples had mean waiting times of less than 15 minutes.

than 15 minutes. Y ~ 
$$Bin(15,0.8920)$$
 Y  $V(Y = 2) = 2.27 \times 10^{-11}$  V

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#### Question Two: [1, 3, 1, 2, 2, 2 = 11 marks] CA

On a Saturday night in Perth, a Random Breath Test (RBT) location is set up on a busy Perth street and 3000 RBTs are conducted by police. 45 drivers were found to have a blood alcohol limit in excess of 0.05.

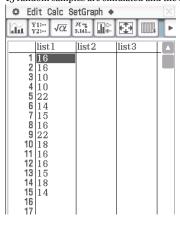
- (a) State the sample proportion of drivers with a blood alcohol concentration in excess of the limit of 0.05.
- (b) Police conduct further RBTs in similar locations over the next 15 Saturday nights in Perth. Calculate the mean and standard deviation of the proportion of samples if each sample is 3000 drivers.

Police decide to conduct RBTs in this same location on a Sunday morning. They first decide to model the scenario using a binomial random variable X.

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(c) If each sample is to be of size 1000 on a Sunday morning, define X.

15 random samples are simulated and the results are given below.



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## Mathematics Methods Unit 4

## **Question 4:** [2, 2, 3 = 7 marks] **CA**

Samples of size 70 are taken of a Bernoulli variable with p = 0.6.

State the approximate mean and standard deviation of the samples.

$$\mu = 0.6$$
  $\checkmark$ 

$$\sigma = \sqrt{\frac{0.6 \times 0.4}{70}} = 0.0586$$
  $\checkmark$ 

(b) Use an appropriate distribution to calculate P(0.55 < P < 0.7)

$$X \sim N(0.6, 0.0586^2)$$
  
 $P(0.55 < X < 0.7) = 0.7593$ 

(c) Show the use of a standard normal distribution to obtain the same answer as in (b).

$$z_1 = \frac{0.55 - 0.6}{0.0586} = -0.8532$$

$$z_2 = \frac{0.7 - 0.6}{0.0586} = 1.7065$$

$$P(-0.8532 < Z < 1.7065) = 0.7593$$

### Question 5: [2 marks] CA

One hundred samples of 10 people are tested for a gene which occurs in 5.5% of the population.

Another one hundred sample of 100 people are tested for the same gene.

If both sets of samples were graphed, which would most look like a normal distribution? Explain your answer.

The second set of samples – the sample size of 100 is much larger than 10 and the sample proportions would better approximate a normal distribution.

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(d) Calculate the sample proportions from the simulations given above.

(e) Calculate the mean and standard deviation of the sample proportions.

(f) Comment on your answers to part (b) and (f).

Mathematics Methods Unit 4

Question Three: [1, 1, 4, 1, 2 = 9 marks] CA

A random sample of three items is selected from a batch of 15 items which contains five defective.

(a) What is p, the proportion of defectives in the batch?

$$\frac{\varepsilon}{I} = \frac{\varsigma I}{\varsigma} = d$$

(d) What are the possible values of the sample proportion p of defective in the sample?

$$1, \frac{2}{\varepsilon}, \frac{1}{\varepsilon}, 0$$

 Construct a probability distribution table which summarises the sampling distribution of the sample proportion of defectives in the sample.

^	^	^_	^	
$\frac{SS^{\dagger}}{0I} = \frac{\begin{pmatrix} \varepsilon \\ \varsigma I \end{pmatrix}}{\begin{pmatrix} 0 \\ 0 I \end{pmatrix} \begin{pmatrix} \varepsilon \\ \varsigma \end{pmatrix}}$	$\frac{SSt}{001} = \frac{\begin{pmatrix} \varepsilon \\ \varsigma_1 \end{pmatrix}}{\begin{pmatrix} 1 \\ 01 \end{pmatrix} \begin{pmatrix} \zeta \\ \varsigma \end{pmatrix}}$	$\frac{\zeta_{S}}{\zeta_{S}} = \frac{\zeta_{S}}{\zeta_{S}} = \frac{\zeta_{S}}{\zeta_{S}}$	$\frac{SS^{\dagger}}{071} = \frac{\begin{pmatrix} \varepsilon \\ \varsigma I \end{pmatrix}}{\begin{pmatrix} \varepsilon \\ \varsigma I \end{pmatrix} \begin{pmatrix} 0 \\ \varsigma \end{pmatrix}}$	(d = d)d
I	$\frac{\varepsilon}{2}$	$\frac{\varepsilon}{\varepsilon}$	0	d
				defective items in sample
3	2	Ţ	0	Number of

Use you sampling distribution to determine the probability that the proportion of defectives is less than 0.4.

$$\frac{422}{150 + 252} = \frac{455}{345}$$

 $(0 < q \mid 1.0 > 4)$  briff (9)

$$\frac{722}{577} \div \frac{722}{332} = \frac{332}{577} \div \frac{722}{577}$$

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## Question Three: [1, 1, 4, 1, 2 = 8 marks] CA

A random sample of three items is selected from a batch of 15 items which contains five defective.

- (a) What is *p*, the proportion of defectives in the batch?
- (b) What are the possible values of the sample proportion p of defective in the sample?
- (c) Construct a probability distribution table which summarises the sampling distribution of the sample proportion of defectives in the sample.

- (d) Use you sampling distribution to determine the probability that the proportion of defectives is less than 0.4.
- (e) Find P(P < 0.4 | P > 0)

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# Mathematics Methods Unit 4

(d) Calculate the sample proportions from the simulations given above.

0.016, 0.016, 0.01, 0.01, 0.022, 0.014, 0.015, 0.016, 0.022, 0.018, 0.016, 0.016, 0.015, 0.018, 0.014



(e) Calculate the mean and standard deviation of the sample proportions.

$$\mu = 0.0159$$
  $\checkmark$ 
 $\sigma = 0.003284$   $\checkmark$ 

(f) Comment on your answers to part (b) and (e).

The mean and standard deviation of the proportion of samples in (b) is very close to the mean and standard deviation of the sample proportions.



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[2, 2, 3 =7 marks]  $\mathbf{C}\mathbf{V}$ Question 4:

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One hundred samples of 10 people are tested for a gene which occurs in 5.5% of the

If both sets of samples were graphed, which would most look like a normal distribution?

State the approximate mean and standard deviation of the samples.

Show the use of a standard normal distribution to obtain the same answer as in (b).

[2 marks] Question 5: **KA** 

population.

Another one hundred sample of 100 people are tested for the same gene.

Explain your answer.

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Mathematics Methods Unit 4

Question Two: [1, 3, 1, 2, 2, 2 = 11 marks]

limit in excess of 0.05. street and 3000 RBTs are conducted by police. 45 drivers were found to have a blood alcohol On a Saturday night in Perth, a Random Breath Test (RBT) location is set up on a busy Perth

the limit of 0.05. State the sample proportion of drivers with a blood alcohol concentration in excess of

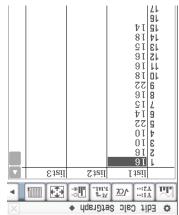
$$210.0 = \frac{24}{0000} =$$

sample is 3000 drivers. Perth. Calculate the mean and standard deviation of the proportion of samples if each Police conduct further RBTs in similar locations over the next 15 Saturday nights in

to model the scenario using a binomial random variable X. Police decide to conduct RBTs in this same location on a Sunday morning. They first decide

If each sample is to be of size 1000 on a Sunday morning, define X.

15 random samples are simulated and the results are given below.



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CA

The time patients wait for their appointments at a dentist is uniformly distributed over the interval 2 to 25 minutes.

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(a) Calculate the mean waiting time for all of the patients at this dentist.

(b) Calculate the standard deviation of the waiting times for all of the patients at this dentist.

(c) Define the probability distribution that best models the distribution of the means of the samples.

(d) Find the probability that a random sample has a mean waiting time of less than 15 minutes.

(e) Calculate the probability that at most two samples had mean waiting times of less than 15 minutes. Mathematics Methods Unit 4



### SOLUTIONS Calculator Assumed Mixed Applications of Discrete and Continuous Random Variables

Time: 45 minutes Total Marks: 45 Your Score: / 45

### Question One: [3, 3 = 6 marks]

CA

(a) Use the fact that a linear transformation of a random variable X of the form Y = aX + b yields E(Y) = aE(X) + b and  $Var(Y) = a^2Var(X)$  to show that E(p) = p, where p and p are the sample proportion and expected probability respectively.

$$P = \frac{X}{n}$$

$$E(X) = np \checkmark$$

$$E(P) = E\left(\frac{X}{n}\right) = \frac{1}{n}E(X) = \frac{1}{n} \times np = p \checkmark$$

(b) Using the same facts as above, also establish that  $Var(p) = \frac{p(1-p)}{n}$ 

$$Var(X) = np(1-p)$$

$$Var(P) = Var\left(\frac{X}{n}\right)$$

$$= \left(\frac{1}{n}\right)^{2} Var(X)$$

$$= \frac{1}{n^{2}} np(1-p)$$

$$= \frac{p(1-p)}{n}$$