

(c) Assuming that the model continues to be valid, during which year will the cost of computer memory fall below 20 cents per gigabyte? (2 marks)

$$0.2 = 18.5 \times 0.37^x$$

$$x = 14.6 \text{ years}$$

By the year 2019 (round 14)
or the year 2020 rounded

End of section 2

Mark Available:

Time Allowed: 25 minutes

Student's Name: *Chen Minh Phung*

Question 1

(a) Simplify $\sqrt{4-5}$.

$$(4-5)^{\frac{1}{2}}$$

$$4-5 = -1$$

(2 marks)

(7 marks)

10/17

31/40

(b) Write the value of xy in scientific notation when $x = 2.5 \times 10^3$ and $y = 5 \times 10^7$. (2 marks)

$$2.5 \times 5 \times 10^3 \times 10^7$$

$$12.5 \times 10^{10}$$

$$1.25 \times 10^{11}$$

1/1

(3 marks)

(c) Determine the value of n given that $9n+1 = \sqrt{27}$.

$$(3^2)^{n+1} = 27^{\frac{1}{2}}$$

$$3^{2n+2} = (3^3)^{\frac{1}{2}}$$

$$2n+2 = \frac{3}{2}$$

$$2n = -\frac{1}{2}$$

$$n = -\frac{1}{4}$$

1/1

Question 2

a. Simplify the following, leaving all indices positive.

$$(1) \left(\frac{8a^{-1}b^2}{27a^2b^{-1}} \right)^{-\frac{1}{3}} \left(\frac{2^3 a^1 b^2}{3^3 a^2 b^{-1}} \right)^{\frac{1}{3}} = \frac{3a}{2b} \quad (10 \text{ marks})$$

$$= \frac{2a^{\frac{1}{3}}b^{\frac{2}{3}}}{3a^{\frac{2}{3}}b^{-\frac{1}{3}}} = \frac{2b^{\frac{2}{3}} \times b^{\frac{1}{3}}}{3a^{\frac{2}{3}} \times a^{\frac{1}{3}}} = \frac{2b}{3a} \quad (2 \text{ marks})$$

$$(2) \frac{3^n + 3^{n+2}}{3^{n-1}}$$

$$\frac{3^n + 3^{n+2}}{3^{n-1}} = \frac{3^n(1+3^2)}{3^{n-1}} = \frac{3^n \cdot 10}{3^{n-1}} = \frac{3+9+3}{3^{-1}} = 15 \quad (2 \text{ marks})$$

b. Solve for x.

$$4^x = \sqrt{8}$$

$$(2^2)^x = 2^{\frac{1}{2}} \\ (2^2)^{\frac{1}{2}} = (2^3)^{\frac{1}{2}} \\ 2^{\frac{2}{2}} = \frac{2}{2}$$

$$\frac{2}{x} = \frac{3}{2} \quad (1) \\ x = \frac{4}{3} \quad (2 \text{ marks})$$

(c) Solve algebraically for x.

$$16^{\frac{x-5}{2}} = \sqrt[3]{64} \quad (2 \text{ marks}) \\ (4^2)^{\frac{x-5}{2}} = (4^3)^{\frac{1}{3}} \\ 4^{\frac{2x-5}{2}} = 4^1 \\ \frac{2x-5}{2} = 1 \\ 2x-5 = 2 \\ 2x = 7 \\ x = \frac{7}{2} \quad (1) \text{ c.f.}$$

(d) Given

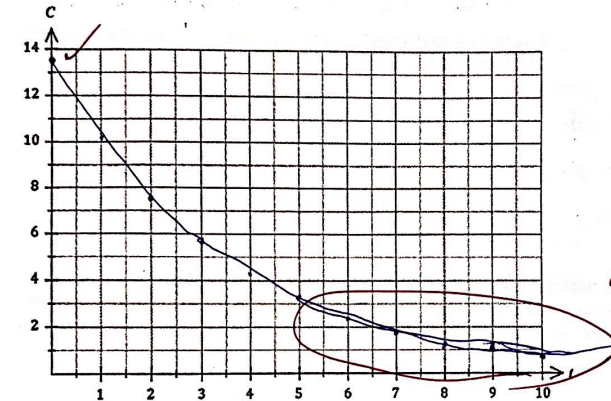
Question 6

The cost, C dollars, for a gigabyte of computer memory between the end of year 2005 ($t = 0$) and the end of year 2015 ($t = 10$) can be modelled by the equation $C = 13.5(0.75)^t$. (6 marks)

(a) Calculate C at the end of year 2010. (1 mark)

$$C = 13.5(0.75)^5 \\ C = 49.20 \quad (1)$$

(b) Draw the graph of C against t on the axes below. (3 marks)



(1) (2)

$$105216 = 185430 \times (1 - 0.827)^3$$

$$105216 = 185430 \times 0.173$$

$$105216 = 32177.31$$

$$105216 \div 32177.31 = 3.27$$

17% decrease each year

(b) The population of a mining town in northern Australia decreases from 185430 people to 105216 over a period of 3 years. What was the constant percentage rate of decrease in the population per year over that period? (2 marks)

ii. How long would she need to sleep if she wanted to wake up with at least \$1,000,000 in the bank? (2 marks)

$$1000000 = 27(1.05375)^t$$

$$1000000 \div 27 = 37037.04$$

$$37037.04 = (1.05375)^t$$

$$\ln 37037.04 = t \ln 1.05375$$

$$t = \frac{\ln 37037.04}{\ln 1.05375} = 277.1$$

277 years

(a) Sleeping Beauty slept for 200 years. She had \$2 in the bank where she started sleeping. The interest rate during those years remained at a constant 5.75% compounded annually. i. How much would she have in her bank account when she wake up? (2 marks)

(6 marks)

Question 5

End of section 1

$$3^x = 5$$

$$\ln 3^x = \ln 5$$

$$x \ln 3 = \ln 5$$

$$x = \frac{\ln 5}{\ln 3} = 1.465$$

1.465

(d) Given that $3^x = 5$, determine the value of 9^{x+1} . (2 marks)

(2 marks)

Saigon International College
Department of Mathematics and Science
Semester 2, 2022
Year 11 ATAR Mathematics Methods
Test 4
(Indices, exponential functions)

Section Two (Calculator assumed)

21

Mark Available: 23

Time Allowed: 45 minutes

Student's Name: ... Chu Minh Day

Question 3

(5 marks)

The area of forest in Methodland is estimated to be decreasing at a rate of 12% per year. In January 2010 the area of the forest was 275 km².

- i. Write down an equation in the form $A = A_0 k^t$, where A_0 is the initial area, t is the time in years, after 2010 and k is a fixed constant. [2 marks]

$$A = 275 \times 0.88^t$$

- ii. What is the area of the forest expected to be in 2020? [1 mark]

$$275 \times 0.88^{10}$$

$$76.58 \text{ km}^2$$

- iii. In what year is the area of the forest expected to be 50 km²? [2 marks]

$$50 = 275 \times 0.88^t$$

$$t = 12.3$$

beginning of year 2024 rounded

or April of 2023 to be exact

Question 4

(6 marks)

- (a) Determine the solution(s), if any exist, when $3^{x-1} = 6$. Give your answer correct to one decimal place. (2 marks)

$$3^{x-1} = 6$$

one solution

$$x = 2.6$$

- (b) (i) Determine the coordinates of the points of intersection of the functions $y = 3^{x-1}$ and $y = x + 1$. (2 marks)

$$y = 3^{x-1} \text{ and } y = x + 1$$

$$x + 1 = 3^{x-1}$$

$$x = -0.8721, 0.12784$$

$$(-0.8721, 0.12784)$$

$$x = 2$$

$$y = 2 + 1$$

$$y = 3$$

$$(2, 3)$$

- (ii) Calculate the distance between the points of intersection in correct to 2 significant figures. (2 marks)

$$3 - 0.12784 = 2.87216$$

$$2 + 0.87213 = 2.87213$$

$$y = 3 - 0.12 = 2.88$$

$$x = 2 - 0.87 = 2.87$$

$$4.06$$

$$\text{answer: } 4.06$$

2. sig fig