PERTH COLLEGE
YR 12 3CD SPECIALIST MATHEMATICS
SEMESTER ONE 2010
TEST 1
VECTORS (60%) & DIFFERENTIATION (40%)

Name:_____

SECTION TWO: CALCULATOR ALLOWED

TIME: 35 minutes **TOTAL MARKS: 30**

- Answer all questions neatly in the spaces provided.
- **Show all working** where appropriate.

Question 5 (3 marks)

Find the distance between the parallel planes $r \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 10 \wedge r \cdot \begin{pmatrix} 9 \\ 6 \\ 3 \end{pmatrix} = 22$.

(Show clear justification to gain full marks. You do not need to rationalise your solution).

Question 6 (1, 1, 2, 2 = 6 marks)

Line L has a vector equation $r = 4i - j + 2k + \lambda(2i + 3j - 4k)$

Plane P_1 has a vector equation $r \cdot (3i - 2j + k) = 5$

Plane P_2 has a vector equation $r \cdot (-2i + a\,j + 2k) = -8$

a) Clearly show whether the point A, with position vector $(8i + 5j - 6k)$,lies on the Line L.

b) Clearly show whether the point B, with position vector $(3i + 3j + 2k)$,lies on the plane P_1.

c) Clearly show that Line L does not lie in Plane P_1.

d) Determine the value of "*a*", such that Line L lies in the plane P_2.

Question 7 (1, 1, 2 = 4 marks)

If $a = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} \wedge b = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}$ find;

a) The angle between ***a*** and ***b*** to the nearest degree.

b) The vector equation of the line passing through point A, position vector ***a***, and point B, position vector ***b***.

c) The vector equation of the plane containing point A and perpendicular to line AB. *(To gain full marks you must fully justify your solution).*

Question 8 (3, 1, 1 = 5 marks)

a) **Show** that the equation of the tangent to $y = tan\ (x)$ at the point where $x=\frac{\pi}{4}$ is $y=2x+(\frac{2-\pi}{2})$

b) If f(x) = 4 $\sin^5$ (x) find: *(to 3 DP)*

(i) $f'\left(\frac{\pi}{3}\right)$

(ii) $f''(\frac{\pi}{3})$

Question 9 (3, 2 = 5 marks)

a) Find the points on the curve $xy - y - x = 1$ where the tangent is parallel to the line $x + 4y = 1$.
(To gain full marks you must fully justify your solution).

b) Indentify the function being differentiated and hence find its exact value:

$$\lim_{h \to 0} ¿¿$$

Question 10 (7 marks)

Luke and Han are flying their B wing aircraft. The initial position vectors relative to the home base (the origin) are:

$$r_{Luke} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} kilometres \wedge ¿ \qquad r_{Han} = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} kilometres .$$

They start flying at the same time with velocities:

$$v_{Luke} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix} kilometres\ per\ hour \wedge v_{Han} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} kilometres\ per\ hour .$$

If they collide, state the time of the collision after the aircraft left their initial position.
If they do not collide, state the minimum distance they are apart and the time when this occurs (after the aircraft leave their initial position).
(Show clear justification to gain full marks).