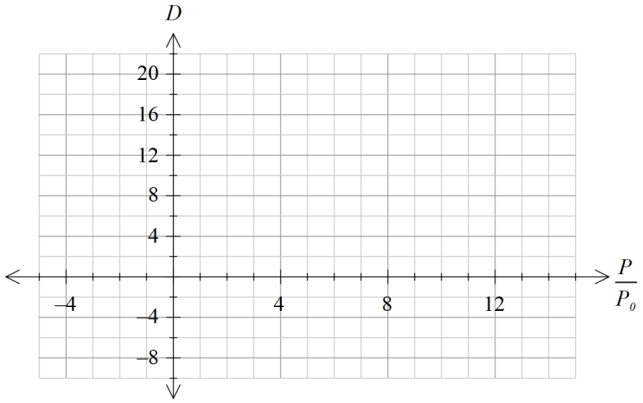
7. (8 marks)

The decibel scale for sound, measured in decibels (dB), is defined as $D = 20\log_{10}\left(\frac{P}{P_0}\right)$, where P is the pressure of the sound being measured and P_0 is a fixed reference pressure.

(a) Complete the table below, giving values rounded to one decimal place. [2]

P	$0.5P_0$	P_0	$2P_0$	$4P_0$	$8P_0$
D	-6.0				

(b) Sketch the graph of $D = 20\log_{10}\left(\frac{P}{P_0}\right)$ on the axes below labelling all key features [3]



(c) When measured at similar distances, the sound produced by a dishwashing machine measures 47 dB, while that produced by lawn mower measures 96 dB. How many times greater is the sound pressure of the mower to that of the dishwasher? [3]

TRINITY COLLEGE

Mathematics Methods Units 3/4
Test 4 2017

Section 1 Calculator Free

Calculus involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME _____

DATE: Thursday 20 July **TIME:** 25 minutes **MARKS**: 26

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

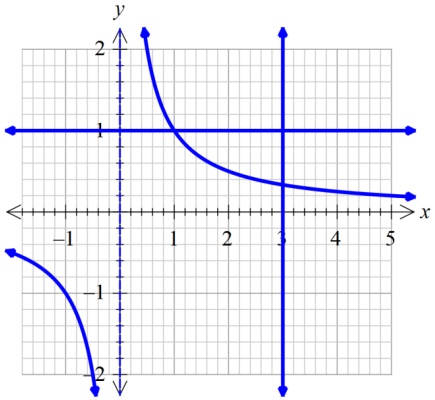
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (4 marks)

Determine the equation of the tangent to the curve $y = x^2 \ln x^2$ at the point $x = 1$

2. (7 marks)

(a) (i) Determine the coordinates of the point of intersection between the curve $y = \frac{1}{x}$ and the line $y = 1$ [1]



(ii) Hence or otherwise, determine the exact area of the region trapped between the curve $y = \frac{1}{x}$, the line $x = 3$, the *x-axis*, the *y-axis* and the line $y = 1$. [4]

(b) $\int \frac{5x}{x^2 - 1} dx$ [2]

(f) Determine the mean and variance of $5 - 2X$. [2]

The time (in minutes) that it takes a student to complete a second more challenging puzzle is a random variable Y with a cumulative probability distribution function given by

$$F(y) = 1 - \frac{10}{y}$$

(g) Determine the probability that it takes a student longer than 25 minutes to complete the second (more challenging) puzzle. [2]

(h) Determine the quickest possible time for solving this second (more challenging) puzzle. [2]

6. (15 marks)

The time (in minutes) that it takes a student to complete a puzzle is a random variable X with a probability density function given by:

$$f(x)=\begin{cases}\dfrac{20x-x^2}{1125} & 5\le x\le 20\\ 0 & \textit{elsewhere}\end{cases}$$

(a) Determine the probability that it takes exactly 6 minutes to complete the puzzle. [1]

(b) Determine the probability that it takes less than 10 minutes to complete the puzzle. [2]

(c) Determine the probability that it takes between 8 and 10 minutes to complete the puzzle given that it takes less than 10 minutes. [2]

(d) Determine the expected time it takes to complete the puzzle. [2]

(e) Determine the standard deviation of the random variable X. [2]

3. (11 marks)

(a) Differentiate each of the following with respect to x.

(i) $y=\dfrac{\ln x}{x^3}$ [3]

(ii) $y=(x+\ln\sin x)^4$ [3]

(iii) $y=\ln\sqrt{\dfrac{e^{5x}}{x^2-1}}$ [3]

(b) If $f(x)=\int_1^x \ln\sqrt{t}\,dt$, determine $f'\left(e^2\right)$ [2]

4. (4 marks)

A continuous random variable, X , has a probability density function given by

$$f(x)=\begin{cases}\frac{1}{5}e^{-\frac{x}{5}} & x\geq 0\\ 0 & x<0\end{cases}$$

The median of X is m. Determine the exact value of m.

TRINITY COLLEGE

Mathematics Methods Units 3/4
Test 4 2017

Section 2 Calculator Assumed

Calculus involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME _____

DATE: Thursday 20 July **TIME:** 25 minutes **MARKS**: 29

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (6 marks)

Let $x=\log_n 5$ and $y=\log_n 4$.

(a) Write $x-\frac{y}{2}$ as a single logarithmic term. [2]

(b) Express the following in terms of x and/or y.

(i) $\log_n 100$ [2]

(ii) $\log_5 4$ [2]