2019 Year 11 Mathematics Methods

PERTH MODERN SCHOOL
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INVESTIGATION 1: Transformations

Calculator Assumed

TAKE HOME SECTION

NAME: ____ TEACHER: ____ DUE DATE: Friday 15 March 2019

INSTRUCTIONS:

Complete this take home section BEFORE the in-class validation on the morning of Friday 15 March.

You may bring your ClassPad and this take home section with you for the validation. You will have access to this take home and your work in the validation.

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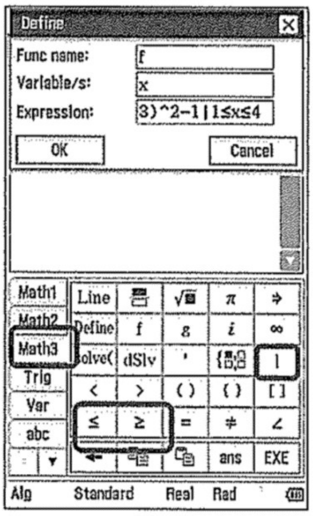
Consider the function $f(x) = (x-3)^2 - 1$ with restricted domain $1 \le x \le 4$.

Define the function

- Open Main $\sqrt{\alpha}$
- Select [Interactive | Define]
- Type the function into the Expression box
  - Enter (x-3)^2–1
  - Press [Keyboard] to open the keyboard
  - Tap [Math3]
  - Tap | (this means "given")
  - Complete the expression using the the inequality keys to enter the restricted domain.
- Tap OK.

By default, the calculator will call the function *f* and use variable *x*



1. Complete the table.

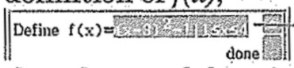
Expression	ClassPad output	By hand explanation
$f(1)$	3	$f(1) = (3-1)^2 - 1 = 2^2 - 1 = 4 - 1 = 3$
$f(2)$		
$f(3)$		
$f(4)$		
$f(5)$		
$f(3)+1$		
$2f(3-2)$		

Graph the function

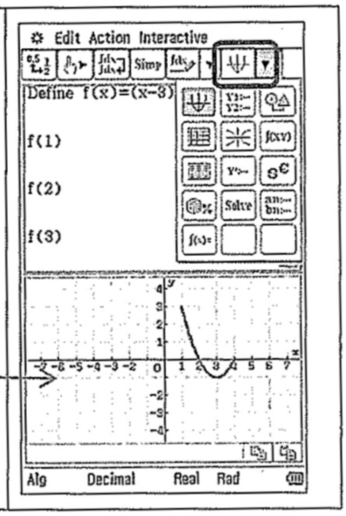
Bring up a graphing window

- Select from the pull down menu (if required)
- Tap

- Select [Zoom | Initialize]
- Tap back in the Main window, select the definition of *f(x)*,

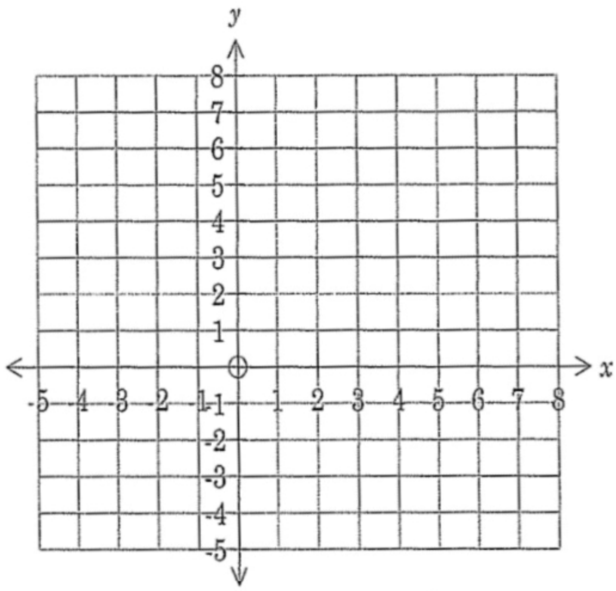


- then drag and drop it in the graph window



2.

a) Draw the resulting graph on the axes below, labelling the key features (i.e. turning point and axis intercepts).



b) State the range of the function over its restricted domain.

c) Use the graph to find the approximate solution to the equation $f(x) = 2$.

d) Give an example of an equation involving $f(x)$ that would have:

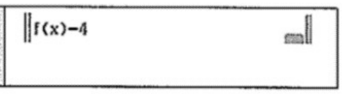
i) exactly two solutions;

ii) no solution.

Translations

We can easily apply transformations to the function in the Main screen using the function notation.

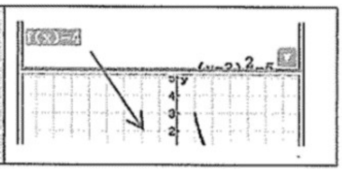
- Type $f(x) - 4$ into the Main screen and press EXE



3.

a) Write down the calculator output for $f(x) - 4$.

- Select the $f(x) - 4$ and drag it into the Graph window.
  Observe the graph and compare it to the graph of the original function.



b) Describe the transformation using appropriate mathematical language. (see Learning Notes)

c) The domain is unchanged. Write down the range of $y = f(x) - 4$.

4. Type $f(x + 4)$ into the Main screen and tap EXE.

a) Write down the ClassPad output for $f(x + 4)$.

Select the $f(x + 4)$ and drag it into the Graph window. Observe the graph and compare it to the graph of the original function.

b) Describe the transformation using appropriate mathematical language.

c) Explain why the restricted domain is now $-3 \le x \le 0$.

Dilations

5. Type $2f(x)$ into the Main screen and tap $\boxed{\text{EXE}}$.

   a) Write down the calculator output for $2f(x)$.

   b) Generate the graph then describe the transformation using appropriate mathematical language.

   c) Write down the range of $y = 2f(x)$.

6. Type $f(2x)$ into the Main screen and tap $\boxed{\text{EXE}}$.

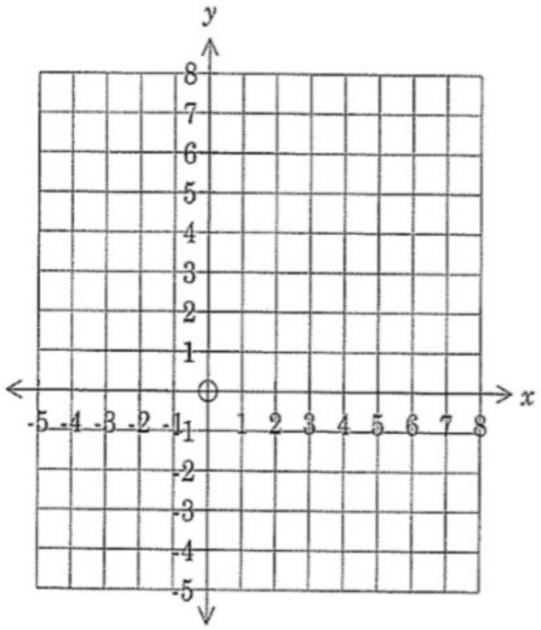
   a) Write down the ClassPad output for $f(2x)$.

   b) Generate the graph then describe the transformation using appropriate mathematical language.

   c) Write down the domain and explain why this should be the case.

7. Graph $y = f(x)$ on the axes below.

a) Draw, in different colours, the graphs of $y = 3 + f(x)$ and $y = f(x-2)$.
Pay particular attention to the location of the key points that you labelled in Q1.



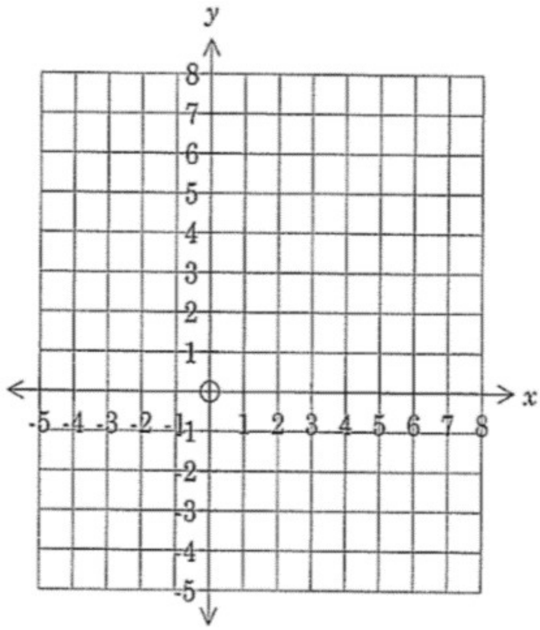
b) Describe the transformations of

i) $f(x) \to 3 + f(x)$

ii) $f(x) \to f(x-2)$

8. Graph $y = f(x)$ on the axes below.

a) Draw, in different colours, the graphs of $y = 2f(x)$ and $y = -f(x)$.



b) Describe the transformations of:

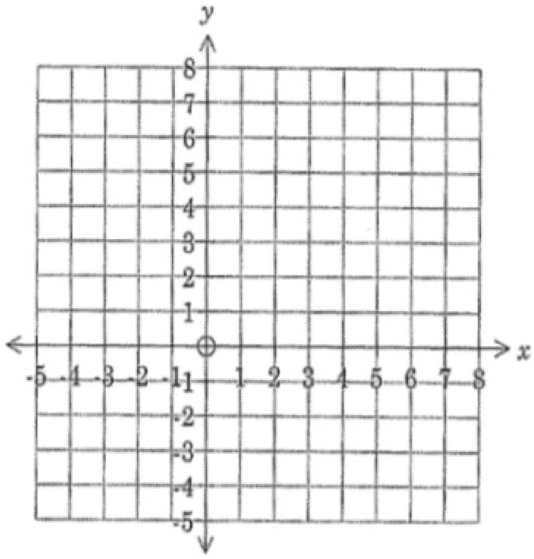
i) $y = f(x) \to y = 2f(x)$

ii) $y = f(x) \to y = -f(x)$

Combine transformations

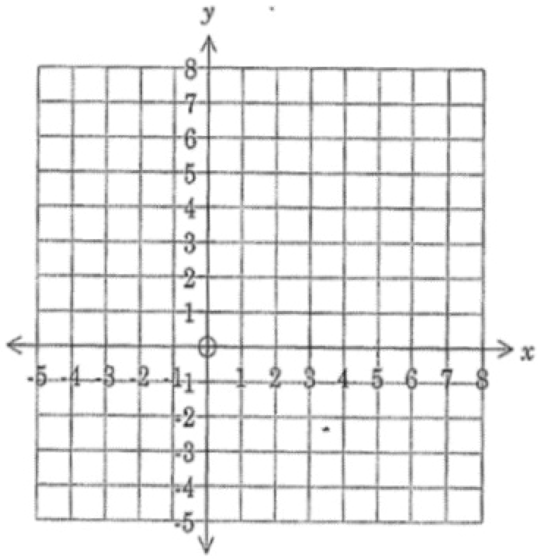
9. Draw each pair of functions and describe the transformations required to move $f(x)$ to the second function.

a) $f(x)$ and $2f(x)-3$



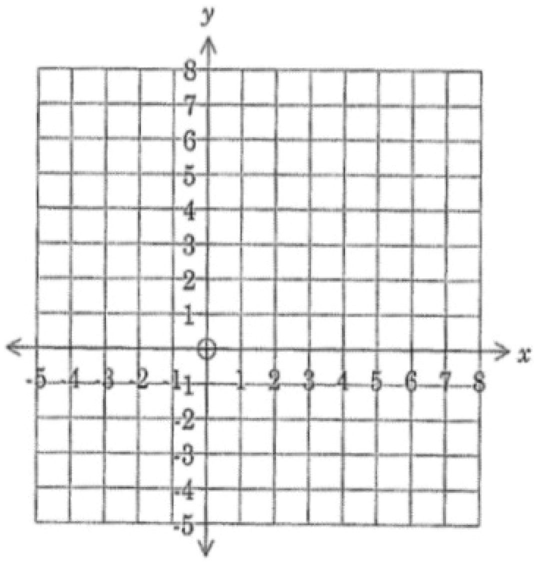
Transformations:

b) $f(x)$ and $f(2(x+3))$



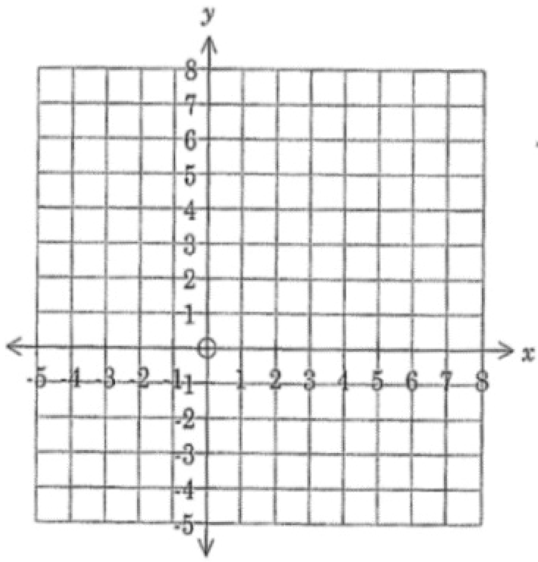
Transformations:

c) $f(x)$ and $f(x+2)-4$



Transformations:

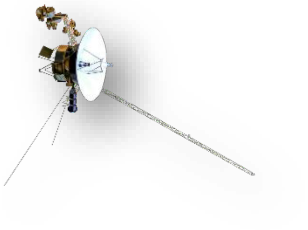
d) $f(x)$ and $-2f(x)+5$



Transformations:

Modelling with Transformations

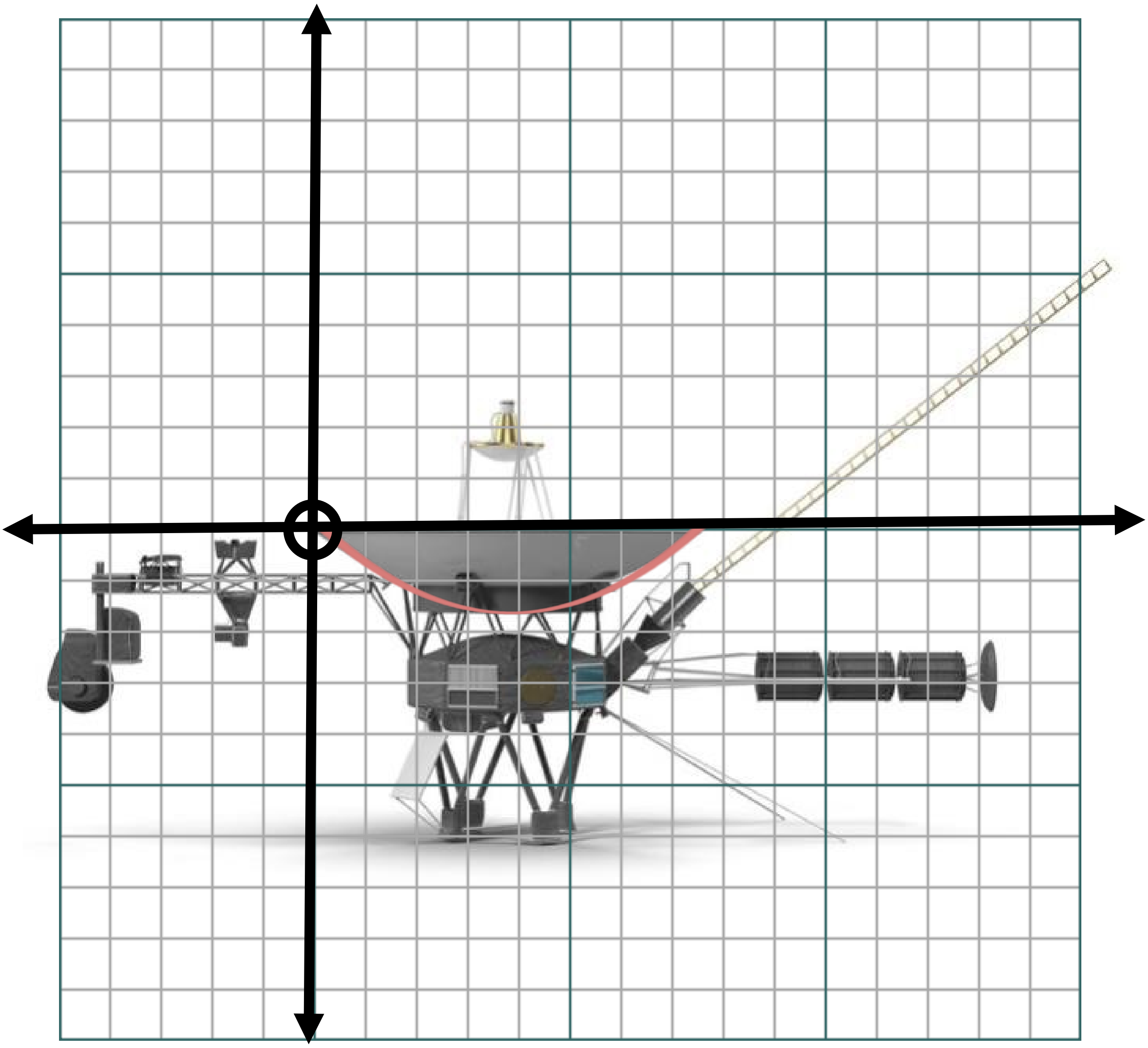
10. Using the transformations of functions we are able to model real world objects. This is useful in modern engineering to simulate and test designs before starting expensive building processes.



In this exercise we will use the Voyage 1 spacecraft as an example. This space craft was launched by NASA in 1977 and in 2012 become the first human built object to pass the Heliopause and move into interstellar space. It is expected that Voyage 1 will continue to provide scientific data until 2025.

One of the defining features of Voyager 1 is the large parabolic reflector dish mounted on it. This dish is part of the communications system and supports the extreme long range communication.

Given the parabolic dish has a diameter of 3.66 meters and depth of 0.8 meters (use these values for this exercise).



For the function that represents the Reflector Dish, as shown on the graph on the previous page.

a) State the coordinate of all intercepts.

b) State the coordinate of the vertex for the Reflector Dish Function

Use the turning point form : $y = a(x - h)^2 + k$

c) Calculate **a** (to 4 decimal places)

d) If $f(x) = x^2$, State the transformations required to make f(x) transform into the Reflector Dish Function.

e) State the Reflector Dish function in terms of f(x)