

No other items may be taken into the examination room. It is **your responsibility** to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Formula sheet (retained from Section One)

This Question/Answer booklet

To be provided by the supervisor

MATERIALS REQUIRED/RECOMMENDED FOR THIS SECTION

Working time: one hundred minutes
Reading time before commencing work: ten minutes

Time allowed for this section

Your name _____

In words _____

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Student Number: in figures _____

Calculator-assumed

SECTION TWO:

UNIT 3

METHODS

MATHEMATICS

If required by your examination administrator, please place your student identification label in this box

Question/Answer booklet

Semester One Examination, 2017

Mercedes College



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	11	11	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Question 9

(1 mark)

- (a) State the initial voltage between the plates.
- It was observed that after three minutes the voltage between the plates had decreased to 0.6 volts.

The voltage between the plates of a discharging capacitor can be modelled by the function $V(t) = 14e^{-kt}$, where V is the voltage in volts, t is the time in seconds and k is a constant.

Working time: 100 minutes.

This section has **eleven (11)** questions. Answer all questions. Write your answers in the spaces provided.

Section Two: Calculator-assumed

65% (98 Marks)

METHODS UNIT 3

CALCULATOR-ASSUMED

3

Question 10**(11 marks)**

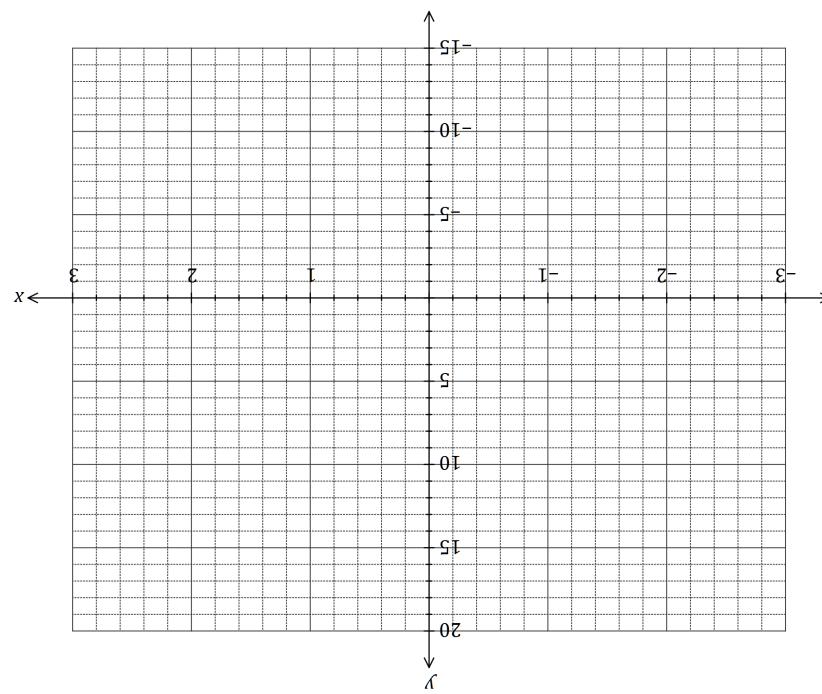
The gradient function of f is given by $f'(x) = 12x^3 - 24x^2$.

- (a) Show that the graph of $y=f(x)$ has two stationary points. (2 marks)

- (b) Determine the interval(s) for which the graph of the function is concave upward. (3 marks)

- (c) Given that the graph of $y=f(x)$ passes through $(1, 0)$, determine $f(x)$. (2 marks)

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METHODS UNIT 3

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CALCULATOR-ASSUMED

CALCULATOR-ASSUMED

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METHODS UNIT 3

Additional working space

- (d) Sketch the graph of $y = f(x)$, indicating all key features. (4 marks)

Question number: _____

Question 11**(7 marks)**

- (a) Four random variables W , X , Y and Z are defined below. State, with reasons, whether the distribution of the random variable is Bernoulli, binomial, uniform or none of these.
- (4 marks)

The dice referred to is a cube with faces numbered with the integers 1, 2, 3, 4, 5 and 6.

- (i) W is the number of throws of a dice until a six is scored.
- (ii) X is the score when a dice is thrown.
- (iii) Y is the number of odd numbers showing when a dice is thrown.
- (iv) Z is the total of the scores when two dice are thrown.
- (b) Pegs produced by a manufacturer are known to be defective with probability p , independently of each other. The pegs are sold in bags of n for \$4.95. The random variable X is the number of faulty pegs in a bag.

If $E(X)=1.8$ and $\text{Var}(X)=1.728$, determine n and p .

(3 marks)

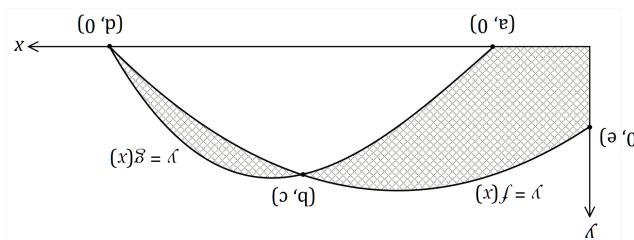
Additional working space

Question number: _____

(4 marks)

- (b) Evaluate the area when $f(x) = 15 + 12x - 3x^2$ and $g(x) = -x^3 + 3x^2 + 13x - 15$.

- (a) Using definite integrals, write an expression for the area of the shaded region. (3 marks)



The graphs of the functions f and g are shown below, intersecting at the points (b, c) and $(d, 0)$.
 Question 12
 (7 marks)

(9 marks)

Question 13

75% of the avocados produced by a farm are known to be first grade, the rest being second grade. Trays of 24 avocados are filled at random in a packing shed and sent to market.

Let the random variable X be the number of first grade avocados in a single tray.

- (a) Explain why X is a discrete random variable, and identify its probability distribution. (2 marks)

- (b) Calculate the mean and standard deviation of X . (2 marks)

- (c) Determine the probability that a randomly chosen tray contains

- (i) 18 first grade avocados. (1 mark)

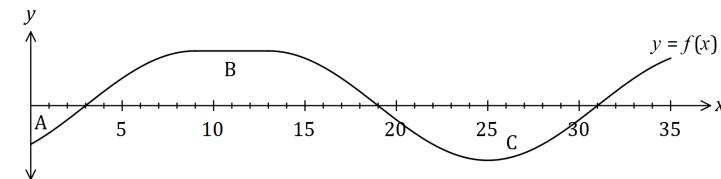
- (ii) more than 15 but less than 20 first grade avocados. (2 marks)

- (d) In a random sample of 1000 trays, how many trays are likely to have fewer first grade than second grade avocados. (2 marks)

(9 marks)

Question 19

The graph of $y=f(x)$ is shown below. The areas between the curve and the x -axis for regions A, B and C are 3, 20 and 12 square units respectively.



- (a) Evaluate

$$(i) \int_0^{31} f(x) dx. \quad (1 \text{ mark})$$

$$(ii) \int_{19}^0 f(x) dx. \quad (2 \text{ marks})$$

$$(iii) \int_3^{31} 2 - 3f(x) dx. \quad (3 \text{ marks})$$

It is also known that $A(31)=0$, where $A(x)=\int_{10}^x f(t) dt$.

- (b) Evaluate

$$(i) A(19). \quad (1 \text{ mark})$$

$$(ii) A(0). \quad (2 \text{ marks})$$

(1 mark)

- (c) Suggest one change to the above procedure to improve the accuracy of the estimate.

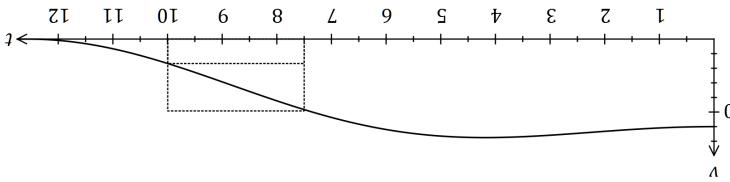
Interval	0 - 2.5	2.5 - 5	5 - 7.5	7.5 - 10	Inscribed area	Circumscribed area			24.15
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- (b) Complete the following table and hence estimate the distance travelled by the car during the first ten seconds by calculating the mean of the sums of the inscribed areas and the circumscribed areas, using four rectangles of width 2.5 seconds.
- The rectangles for the 7.5 to 10 second interval are shown on the graph. (5 marks)

t	0	2.5	5	7.5	10	$v(t)$			3.34
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- (a) Complete the table below, rounding to two decimal places. (2 marks)

The area under the curve for any time interval represents the distance travelled by the car.



- The speed, in metres per second, of a car approaching a stop sign is shown in the graph below and can be modelled by the equation $v(t) = 6(1 + \cos(0.25t + \sin^2(0.25t)))$, where t represents the time in seconds. (8 marks)

Question 14

CALCULATOR-ASSUMED

METHODS UNIT 3

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(a) Show that the cost of materials for the container is $12\pi r^2 + \frac{648\pi}{r}$ cents, where r is the radius of the cylinder. (4 marks)

A storage container of volume $36\pi \text{ cm}^3$ is to be made in the form of a right circular cylinder with one end open. The material for the circular end costs 12π cents per square centimetre and for the curved side costs 9c per square centimetre.

(b) Use calculus techniques to determine the dimensions of the container that minimises its material costs and state this minimum cost. (4 marks)

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CALCULATOR-ASSUMED

METHODS UNIT 3

METHODS UNIT 3

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Question 14

CALCULATOR-ASSUMED

METHODS UNIT 3

Question 15**(10 marks)**

A slot machine is programmed to operate at random, making various payouts after patrons pay \$2 and press a start button. The random variable X is the amount of the payout, in dollars, in one play of the machine. Each payout can be assumed to be independent of other payouts.

The probability, P , that the machine makes a certain payout, x , is shown in the table below.

Payout (\$) x	0	1	2	5	10	20	50	100
Probability $P(X=x)$	0.25	0.45	0.2125	0.0625	0.0125	0.005	0.005	0.0025

(a) Determine the probability that

(i) in one play of the machine, a payout of more than \$1 is made. (1 mark)

(ii) in ten plays of the machine, it makes a payout of \$5 no more than once. (2 marks)

(iii) in five plays of the machine, the second payout of \$1 occurs on the fifth play. (3 marks)

(e) Let Y be a Bernoulli random variable with parameter $p=P(A)$. Determine the mean and standard deviation of Y . (2 marks)

(f) Determine the probability that A occurs no more than twice in ten random selections of four letters from those in the word LOGARITHM. (2 marks)

(b) Calculate the mean and standard deviation of X . (2 marks)

Let the random variable X be the number of vowels in a random selection of four letters from those in the word LOGARITHM, with no letter to be chosen more than once.

(a) Complete the probability distribution of X below. (1 mark)

x	0	1	2	3	$\frac{1}{21}$	$\frac{10}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$P(X=x)$
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(c) In the long run, what percentage of the patron's money is returned to them? (2 marks)

(b) Show how the probability for $P(X=1)$ was calculated. (2 marks)

(c) Determine $P(X \geq 1 \vee X \leq 2)$. (2 marks)

(d) State $P(\underline{A})$. (1 mark)

Let event A occur when no vowels are chosen in random selection of four letters from those in the word LOGARITHM.

Question 16

Particle P leaves point A at time $t=0$ seconds and moves in a straight line with acceleration given by

$$a = \frac{16}{(2t+1)^3} \text{ ms}^{-2}.$$

Particle P has an initial velocity of -3 ms^{-1} and point A has a displacement of 4 metres from the origin.

- (a) Calculate the initial acceleration of P .

(1 mark)

- (b) Is P ever stationary? If your answer is yes, determine the time(s) when this happens. If your answer is no, explain why.

(3 marks)

- (c) Calculate the displacement of P when $t=12$ seconds.

(2 marks)

- (d) Calculate the change of displacement of P during the third second.

(2 marks)

- (e) Determine the maximum speed of P during the first three seconds and the time when this occurs.

(2 marks)

- (f) Calculate the total distance travelled by P during the first three seconds.

(2 marks)