Question/Answer Booklet

MATHEMATICS
SPECIALIST 3CD
Section One
(Calculator Free)

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/Answer booklet for Section One, with a formula sheet which may also be used for Section Two.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One Calculator—free	**7**	**7**	**50 minutes**	**40**
Section Two Calculator—assumed	9	9	100 minutes	80
			Total marks	120

Instructions to candidates

1. Answer the questions in the spaces provided.

2. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

Section One: Calculator–Free   **40 marks**

This section has **seven (7)** questions. Attempt **ALL** questions.
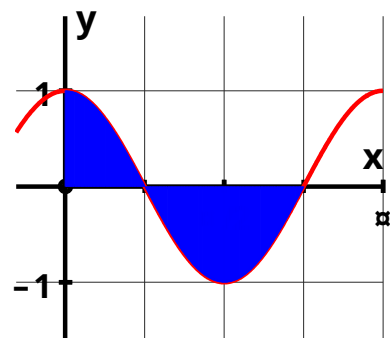

Question 1   **[9 marks]**

Give exact values for the following:

(a)
$$e^{-\frac{i\pi}{2}}$$

[1]

(b)
$$\left(\cos\left(\frac{\pi}{5}\right) - i\sin\left(\frac{\pi}{5}\right)\right)^5$$

[2]

c)
(
$$\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{3} + 2h\right) - \cos\left(\frac{\pi}{3}\right)}{h}$$

[3]

d)   The shaded area under the curve $y = \cos 2x$.
(

[3]

Question 2   [9 marks]

Given that $z = e^{ix}$ and $w = e^{-ix}$ (where x is a real number):

(a)   express   cis(3x) w   in terms of z,

[2]

(b)   determine the values of a and b if z - w is expressed in the form   a + bi,

[2]

(c)   simplify $z^3 + w^3$,

[2]

(d)   solve for x given that $z^4 + 1 = 0$

[3]

Question 3   [6 marks]

Points A, B and C have respective position vectors given by:

$$\textbf{a} \;=\; \textbf{i} + \textbf{j} - \textbf{k}$$
$$\textbf{b} \;=\; \textbf{i} + \textbf{j} + \textbf{k}$$
$$\textbf{c} \;=\; 2\textbf{i} + \textbf{j}$$

Determine :

(a)   the value of cosine of the angle between vectors **a** and **b**,

[2]

(b)   the vector equation of the line containing points A and B,

[2]

(c)   the vector equation of the plane containing vectors **a** and **b** and also containing the point C.

[2]

Question 4   **[3 marks]**

Evaluate the definite integral $\int_0^\pi \sin^2 x \, dx$ exactly.

Question 5   **[4 marks]**

(a)   Determine matrix T = $\begin{bmatrix} 2 & 0 \\ & \end{bmatrix}$

[1]

(b)   Hence if matrix T represents a transformation matrix, describe the actions of matrix T.

[3]

Question 6   **[5 marks]**

The natural logarithm function can be defined as $\ln(x) = \int_1^x \dfrac{dt}{t}$ where x > 0.

(a)   Given that a, b > 0, using the substitution $u = \dfrac{t}{a}$ find an expression for the definite
       integral $\displaystyle\int_a^{ab} \dfrac{dt}{t}$.

[3]

(b)   By considering $\displaystyle\int_1^{ab} \dfrac{dt}{t} = \int_1^a \dfrac{dt}{t} + \int_a^{ab} \dfrac{dt}{t}$ and using the result from part (a) make
       a deduction about the natural logarithm function.

[2]

Question 7   **[4 marks]**

Prove, by any method, that the cube of any number that is 2 more than a multiple of 3 is always 1 less than a multiple of 9.

END OF SECTION ONE

Additional working space

Question number(s):   ………………….

Additional working space

Question number(s):   ………………….

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Question number(s):   ………………….

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Question number(s):   ………………….