

Mathematics Methods Unit 4

Calculator Free
The Natural Logarithm and Anti-Differentiation

Total Marks: 45
Time: 45 minutes
Your Score: / 45



Question One: [2, 3 = 8 marks] C.R.

$$(a) \int \frac{2x - 1}{x} dx$$

$$(b) \int \frac{\cos x}{\sin x} dx$$

$$(c) \int x p_{x-2} dx$$

Determine each of the following anti-derivatives, simplifying your answer where possible:

Question Two:[4, 4 = 8 marks]**CF**

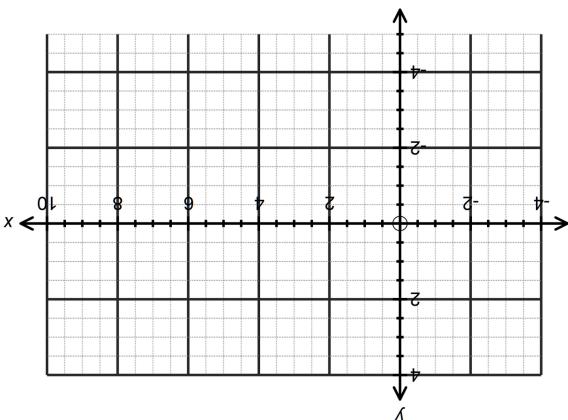
Calculate each of the following definite integrals, simplifying your answers using logarithmic laws.

(a) $\int \frac{6}{3x-1} dx$

(b) $\int_1^6 \frac{x-1}{x^2 - 2x} dx$

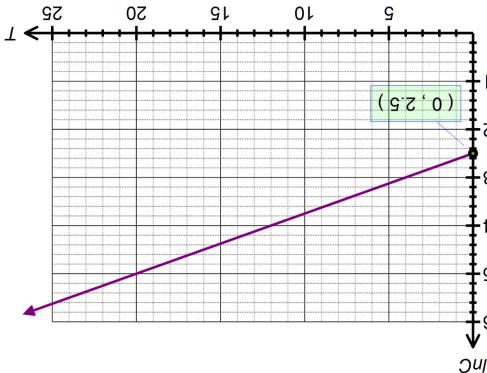
Consider the function $f(x) = -\frac{1}{4}x + 2$

(a) Sketch the function on the axes below.



- (b) Calculate the area bounded by the function, the x -axis and the lines $x = 0$ and $x = 2$. Simplify your answer.

Synergy, the provider of electricity in Perth, monitor the maximum consumption of electricity over summer measured against the maximum temperatures. Graphing the data provides us with the following graph, where C is maximum consumption in megawatts and T is the maximum temperature in degrees Celsius.



(a) Determine the equation of $\ln C$ in terms of T .

$$\ln C = \frac{m}{T} + b$$

$$C = e^{\frac{m}{T} + b}$$

(b) Use your answer to (a) to determine the exponential function which models the energy consumption based on the maximum temperature recorded.

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- (c) Calculate the area bounded by the function, the y -axis and the lines $y = 1$ and $y = 4$

Question Four: [4 marks] CF

Show that the exponential rule used to calculate compound interest, $A = P(1+r)^t$ can be

$$t = \frac{\ln A - \ln P}{\ln(1+r)}$$

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- (d) Hence or otherwise determine the values of k and c .

$$\ln 175 = c \quad \checkmark$$

$$\ln 95 = 5k + \ln 175$$

$$5k = \ln 95 - \ln 175$$

$$k = \frac{\ln 95 - \ln 175}{5} \quad \checkmark$$

- (e) Hence determine when the pizza has reached room temperature.

$$\ln 0 = \frac{\ln 95 - \ln 175}{5} t + \ln 175 \quad \checkmark$$

$$1 - \ln 175 = \frac{\ln 95 - \ln 175}{5} t$$

$$t = \frac{5(1 - \ln 175)}{\ln 95 - \ln 175} \quad \checkmark$$

- (c) After 5 minutes, what is the value of θ ?

- (b) Initially, what is the value of θ ?

A pizza is removed from a 200°C oven and put on the bench in a 25°C room. After 5 minutes, the temperature of the pizza is 120°C .

- (a) Integrate and equate each side to show that $\ln \theta = kt + c$

$$\int_1^\theta \frac{d\theta}{\theta} = \int k dt$$

We can then integrate both sides, as follows:

In order to find a rule modelling θ in terms of t , we can first separate the variables as follows:

This can be defined as: $\frac{d\theta}{dt} = k\theta$ where θ is the difference between the temperature of the body and the surrounding room temperature and t is the time in minutes since the body was introduced to the room.

Newton's Law of Cooling allows us to monitor the rate at which the difference between the temperature of a body and its surroundings will cool over time.

Question Five: [3, 1, 2, 2 = 9 marks] GR

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$$\theta = 120 - 25 = 95 \text{ C} \quad \checkmark$$

- (c) After 5 minutes, what is the value of θ ?

$$\theta = 200 - 25 = 175 \text{ C} \quad \checkmark$$

- (b) Initially, what is the value of θ ?

A pizza is removed from a 200°C oven and put on the bench in a 25°C room. After 5 minutes, the temperature of the pizza is 120°C .

$$\int k dt = kt + c$$

$$\int_1^\theta d\theta = \ln \theta$$

- (a) Integrate and equate each side to show that $\ln \theta = kt + c$

$$\int_1^\theta d\theta = \int k dt$$

We can then integrate both sides, as follows:

$$\frac{\theta}{d\theta} = k dt$$

In order to find a rule modelling θ in terms of t , we can first separate the variables as follows:

This can be defined as: $\frac{d\theta}{dt} = k\theta$ where θ is the difference between the temperature of the body and the surrounding room temperature and t is the time in minutes since the body was introduced to the room.

Newton's Law of Cooling allows us to monitor the rate at which the difference between the temperature of a body and its surroundings will cool over time.

Question Five: [3, 1, 2, 2 = 9 marks] GR

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(d) Hence or otherwise determine the values of k and c .

(e) Hence determine when the pizza has reached room temperature.

(c) Calculate the area bounded by the function, the y -axis and the lines $y=1$ and $y=4$.

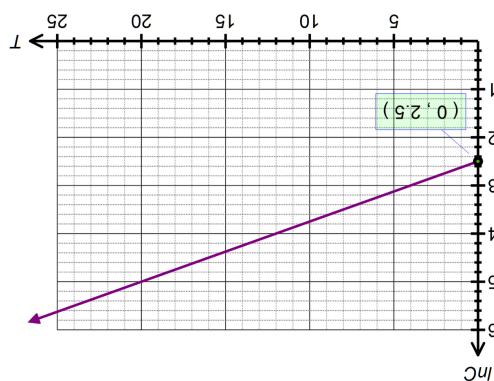
$$\begin{aligned}x &= \frac{-1}{y+2} + 4 \checkmark \\ \int^4_{1} \frac{-1}{y+2} + 4 \, dy &\checkmark \\ &= \left[-\ln|y+2| + 4x \right]_1^4 \checkmark \\ &= (-\ln 6 + 16) - (-\ln 3 + 4) \\ &= \ln 3 - \ln 6 + 16 - 4 \checkmark \\ &= \ln \frac{1}{2} + 12 \text{ units}^2 \checkmark\end{aligned}$$

Question Four: [4 marks] CF

Show that the exponential rule used to calculate compound interest, $A = P(1+r)^t$ can be written as $t = \frac{\ln A - \ln P}{\ln(1+r)}$.

$$\begin{aligned}\ln A &= \ln(P(1+r)^t) \checkmark \\ \ln A &= \ln P + t \ln(1+r) \checkmark \\ t \ln(1+r) &= \ln A - \ln P \checkmark \\ t &= \frac{\ln A - \ln P}{\ln(1+r)} \checkmark\end{aligned}$$

(a) Graphing the data provides us with the following graph, where C is maximum consumption in megawatts and T is the maximum temperature in degrees Celsius.



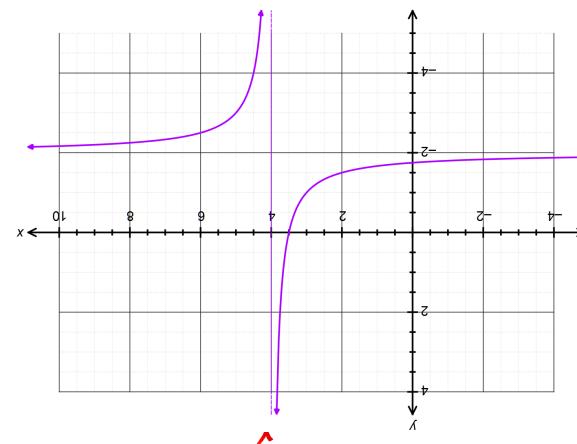
Graphing the data provides us with the following graph, where C is maximum consumption in megawatts and T is the maximum temperature in degrees Celsius.
Electricity over summer measured against the maximum consumption of energy, the provider of electricity in Perth, monitor the maximum consumption of

Question Six: [3, 2 = 5 marks] C/P

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$$\begin{aligned}
 \text{Area} &= \ln 2 - 4 \text{ units}^2 \\
 &= \ln 2 - 4 \\
 &= (\ln 2 - 4) - (-\ln 4) \\
 &= [\ln(x-4) - 2x^2]_0^2 \\
 &= \int_{-1}^2 x - 4 - 2x^2 \, dx
 \end{aligned}$$

(b) Calculate the area bounded by the function, the x -axis and the lines $x = 0$ and $x = 2$. Simplify your answer.



(a) Sketch the function on the axes below.

$$f(x) = \frac{x-4}{-1} - 2$$

Question Three: [3, 4 = 11 marks] C/P

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Question One: [2, 3, 3 = 8 marks] CF

Determine each of the following anti-derivatives, simplifying your answer where possible:

$$(a) \int \frac{2}{2x-1} dx$$

$$= \ln|2x-1| + c$$
✓ ✓

$$(b) \int \frac{\sin x}{\cos x} dx$$

$$= -\ln|\cos x| + c$$
✓ ✓

$$(c) \int \frac{e^x}{e^x - 2} dx$$

$$= \ln|e^x - 2| + c$$
✓ ✓

Question Two: [4, 4 = 8 marks]

CF

Calculate each of the following definite integrals, simplifying your answers using logarithmic laws.

$$(a) \int \frac{6}{3x-1} dx$$

$$\begin{aligned} &= \left[2 \ln|3x-1| \right]_2^4 \\ &= 2 \ln|11| - 2 \ln|5| \\ &= 2(\ln 11 - \ln 5) \\ &= 2 \ln\left(\frac{11}{5}\right) \end{aligned}$$
✓ ✓

$$(b) \int_1^5 \frac{x-1}{x^2-2x} dx$$

$$\begin{aligned} &= \left[0.5 \ln|x^2 - 2x| \right]_1^5 \\ &= 0.5(\ln 15 - \ln 3) \\ &= 0.5 \ln 5 \end{aligned}$$
✓ ✓