PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course Methods Test 3 Year 12

Student name: _______________ Teacher name: _______________

Task type: **Response/Investigation**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **____6_______**

Materials required: No classpads

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper

Marks available: **__38____ marks**

Task weighting: **_14___%**

Formula sheet provided: No but some formulae given on page 2

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Logarithms

$x = \log_a b \Leftrightarrow a^x = b$	$a^{\log_a b} = b$ and $\log_a(a^b) = b$
$\log_a mn = \log_a m + \log_a n$	$\log_a \frac{m}{n} = \log_a m - \log_a n$
$\log_a(m^k) = k \log_a m$	$\log_e x = \ln x$

$\frac{d}{dx} \ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln x + c, \quad x > 0$
$\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + c, \quad f(x) > 0$

Product rule	If $y = uv$ then $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$	or	If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$
Quotient rule	If $y = \frac{u}{v}$ then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	or	If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$
Chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or	If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$
Fundamental theorem	$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$	and	$\int_a^b f'(x)\,dx = f(b) - f(a)$

Q1 (2 & 2 = 4 marks)
Express each of the following as a **single logarithm**.

a) $\log_a b + 3\log_a(ab) - 4\log_a b$.

b) $5 + 3\log_5 c - \log_5\left(c^3\right) + \log_5 b$.

Q2 (2 & 2 = 4 marks)
Solve each of the following, giving your answer in **exact** form.

a) $2^{2x} - 12(2^x) + 32 = 0$

b) $7^x + 3\left(7^{x+2}\right) = 31$

Q3 (1, 3 & 3 = 7 marks)

The Richter scale, R, of an earthquake of intensity I is given by $R=\log_{10}\left(\frac{I}{I_o}\right)$ where I_o is a minimum intensity level used for comparison.

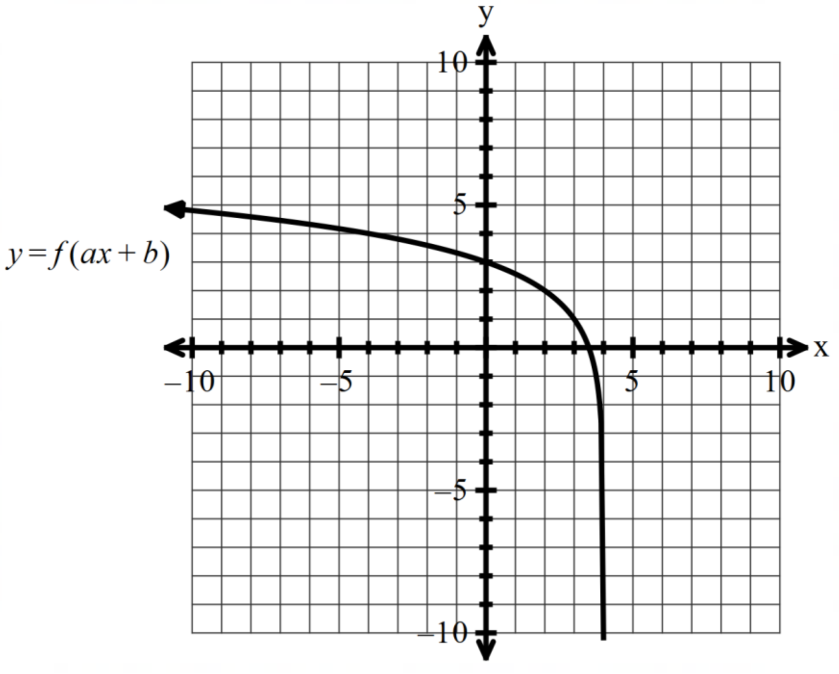
a) Determine R for an earthquake with intensity $10000I_o$.

b) An earthquake measuring 5 on the Richter scale is how many times as intense as that of one measuring 4 on the Richter scale?

c) If an earthquake registers x on the Richter scale and a second earthquake registers $x+4$ on the Richter scale, how many more times as intense is the second earthquake?

Q4 (3 marks)

Consider the function $f(x)=\log_2 x$ which undergoes a transformation $f(ax+b)$ where $a\,\&\,b$ are constants. The graph $y=f(ax+b)$ is plotted below, determine the values of $a\,\&\,b$ showing reasoning.



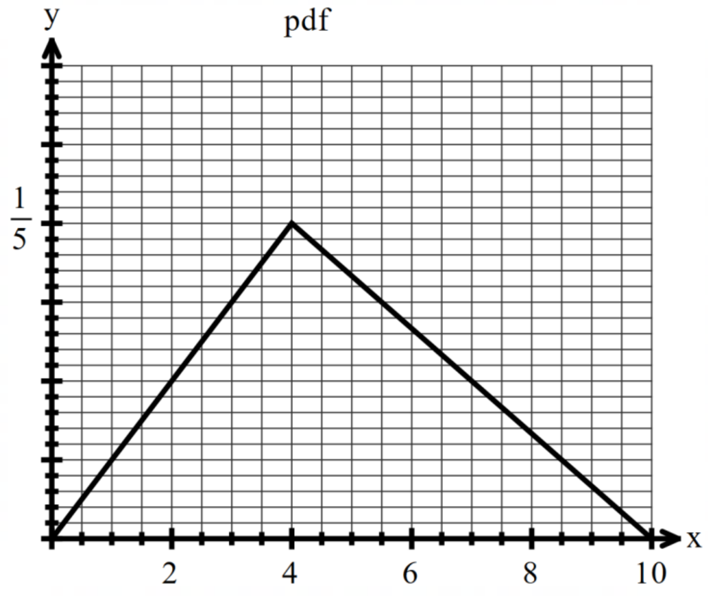
Q5 (3 & 5 = 8 marks)

Consider the function $g(x) = (x^2+3)\ln(x^3+3x)$.

a) Determine $g'(x)$.(Simplify)

b) Use the result from part a to determine $\int 2x\ln(x^3+x)\,dx$.

Q6 (3, 3, 3 & 3 = 12 marks)
Consider the continuous random variable X and its probability density function which is graphed below.



a) Determine the following **exactly**.

i) $P(2 < X < 7)$.(Simplify)

ii) $P(X > 3 | X < 5)$.(No need to simplify)

Q6 continued on next page

Q6 continued

iii) E(X) i.e the mean. (No need to simplify)

b) Derive the cumulative probability function $P\left(X \leq x\right)$ for $0 \leq x \leq 10$.

End of test
Working out space