MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 2 - 2016
Discrete Random Variables and Applications of Differentiation

Resource Rich

Name: _____ **Teacher:** _____

Marks: **17**

Time Allowed: **45 minutes**

Instructions: You are allowed to Calculators and have 1 page of notes (2 sides).

You will be supplied with a formula sheet.

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1 A pair of dice is rolled and X = the maximum number. Which of the following ordered pairs would *not* be included in the probability function for X?

A $\left(3, \frac{5}{36}\right)$

B $\left(5, \frac{8}{36}\right)$

C $\left(1, \frac{1}{36}\right)$

D $\left(2, \frac{3}{36}\right)$

E $\left(6, \frac{11}{36}\right)$

[2 mark]

2 A discrete random variable X can take the values 5, 6, 7 and 8 with probabilities 0.4, 0.3, 0.2 and 0.1 respectively. What is the variance of X?

A 2

B 1.25

C 1

D 1.5

E 1.4

[2 mark]

3 Consider the probability function $p(x) = \frac{2x - 1}{36}$ for $x = 1, 2, 3, \ldots, 6$.

What is $p(2 < x \leq 5)$?

A $\frac{20}{36}$

B $\frac{7}{12}$

C $\frac{2}{3}$

D $\frac{11}{36}$

E $\frac{15}{36}$

[2 mark]

4 A discrete random variable X can take the values 0, 2, 4, 6 and 8 with probabilities 0.1, 0.15, 0.15, 0.25 and 0.35 respectively. What is the expected value of X?

A 5.2

B 5.35

C 4.7

D 10.75

E 6.1

[2 mark]

5 A driver takes the same route to a shopping centre each day, passing through two sets of traffic lights. The driver knows that that there is a 60% chance the first light will be red. If the first light is red, there is only a 20% chance the second light will be red. If the first light is green, there is 70% chance the second light will also be green. How many red lights can the driver expect on the way to the shopping centre?

A 1.00

B 1.08

C 0.76

D 0.84

E 2.00

[2 mark]

6 Given that the equation of a curve is $y = 3x^3 + 4x^2 + 5$, find the approximate increase in y as x increases from 2 to 2.03.

A 45

B 46.58

C 52

D 1.56

E 1.58

[2 mark]

7 The second derivative of $y = 4x\cos(x)$ is:

A $4\cos(x) - 4x\sin(x)$

B $-4x\cos(x) + 4\sin(x)$

C $-4x\cos(x) - 8\sin(x)$

D $-4x\cos(x) - 4\sin(x)$

E $-4x\cos(x) + 8\sin(x)$

[2 mark]

8 For what values of x is the curve $y = 2x^3 + 12x^2 - 18x - 5$ concave upwards?

A $-3 < x < -1$

B $x > -2$

C $x > 2$

D $1 < x < 3$

E $x < -2$

[2 mark]

9 The graph of the function $y = x^3 - 6x^2 - 36x + 9$ has the following stationary points:

A Minimum at (2, –79) and maximum at (–6, 207)

B Minimum at (2, –49) and maximum at (–6, 207)

C Minimum at (–6, –207) and maximum at (2, 49)

D Minimum at (6, –207) and maximum at (–2, 79)

E Minimum at (6, –207) and maximum at (–2, 49)

[2 mark]

10 The product of two positive numbers is 72. Find the numbers such that the sum of twice one number and 4 times the other is a minimum.

[3 marks]

A 6 and 12

B 4 and 18

C 8 and 9

D 2 and 36

E 3 and 24

[2 mark]

11 A spinner is in the shape of a regular pentagon. It has five equally sized segments numbered 2, 2, 5, 7 and 7. Let X be the number on which the pointer rests after a spin.

a Construct a probability distribution for X.

b Calculate $E(X)$.

c If the value of each number on the spinner is increased by 3, what is the new expected value of X?

d If the value of each of the original numbers on the spinner is increased by a factor of 5, what is the new expected value of X?

*****END OF TEST*****

[6 marks]

18 The displacement of an object from a particular point is given by $s = 2t^3 + 9t - 8$, where s is in metres and t is in seconds.

a Find the velocity at 2 s.

b Find when the velocity is 15 m/s.

c Find the acceleration at 2 s.

d Find the acceleration when the velocity is 15 m/s.

[9 marks]

19 Sketch a curve where $f''(x) > 0$ for $x < 0$, $f''(x) < 0$ for $x > 0$ and $f'(x) < 0$ for all x.

[6 marks]

12 The probability function for a discrete random variable, X, is

$p(x) = \frac{kx}{x+1}$ for $x = 1, 2, 3, 4$ and 5.

a Construct a probability distribution for X.

b Use the probability distribution to calculate the value of k.

[4 marks]

13 A random variable, X, has the probability distribution shown in the table below:

x	***0***	***1***	***2***	***3***
P(X = x)	***0.15***	***0.25***	***a***	***b***

If the expected value of X is 1.93 find the values of ***a*** and ***b***.

[4 marks]

14 The average age, *A*, of the population of a shire was studied over time, and at the start the average age was 38.

a During the study, $\frac{dA}{dt} < 0.$ What does this say about the age of the population during the study?

b If at the same time, $\frac{d^2A}{dt^2} < 0,$ what can you say about the population rate?

c Sketch the graph of the population *A* against *t*.

[5 marks]

15 A baseball player strikes the ball so that its equation of motion is given by $h = 2x - 16 - 0.05x^2$, where *h* is the height (in metres) reached by the ball and *x* is the horizontal distance (in metres) travelled by the ball. What is the greatest height reached by the ball?

[4 marks]

16 A soft drink manufacturer produces cylindrical aluminum cans, each with a volume of 500 mL (i.e. 500 cm^3). What is the radius of the can if the least amount of material is to be used in its construction? Give your answer correct to 2 significant figures.

[5 marks]

17 A slow-moving river is one kilometre wide. Tom wants to return to his camp on the opposite side of the river. He can swim at 2 km/h and walk at 3 km/h. Tom must first swim across the river to any point on the opposite bank then, from there, walk to his camp, which is one km from the point directly across the river from where he started his swim. What route will take the least amount of time? How long would this route take?