Christ Church Grammar School

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
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Special items: nil

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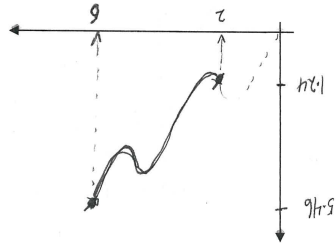to the supervisor **before** reading any further.

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7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1   (7 marks)

Differentiate with respect to x. Simplify your answers, leaving them in fully factorised form where appropriate.

(a) $\dfrac{2x+1}{8-3x}$ (2 marks)

(b) $(2x^4+2)(4+x)^8$ (3 marks)

(c) $\sin(3x) - 4\cos 2x$ (2 marks)

Question 9   (8 marks)

The population of a certain bacteria in a culture (in hundreds) is modelled by

$P = t + \sin(2t)$ for $2 \leq t \leq 6$, where t is time in weeks.

(a)   Sketch the population on the axes below   (2 marks)

$\checkmark$ [General Shape]

$\checkmark$ [Has end pts [2,6]]

(handwritten sketch with values 5.46, 1.74 on vertical axis; 2 and 6 on horizontal axis)

(b)   Find the exact value of t when the bacteria achieve a temporary peak in its population. State the population at this time.   (4 marks)

$\checkmark$ $\left[\dfrac{dP}{dt} = 0\right]$

$\dfrac{dP}{dt} = 0 \implies 1 + 2\cos 2t = 0$ for $4 \leq 2t \leq 12$

$\cos 2t = -\tfrac{1}{2}$

$2t = \dfrac{8\pi}{3},\ \dfrac{10\pi}{3}$

$\therefore t = \dfrac{4\pi}{3}$ or $\dfrac{5\pi}{3}$ $\checkmark$ [t-value]

       4.188   5.23

MAX $\dfrac{4\pi}{3}$ $\dfrac{5\pi}{3}$ represents min

Population at this time $P = \dfrac{4\pi}{3} + \sin\left(\dfrac{8\pi}{3}\right)$ $\checkmark$ [Sub in]

$p = 5.0548$ (x100)

$P = 505$ at this time $\sqrt{}$ [population]

(b)   Find the maximum population of the bacteria in the interval $2 \leq t \leq 6$.   (2 marks)

$P\big|_{t=6} = \boxed{546}$ $\checkmark\checkmark$ [only mark if 5.46]

End of questions

8

Question 2   **(2 marks)**

Find the value of x, in terms of a and b, where the equation $y = 2ax^2 + b^2x$ has a derivative equal to zero given that a and b are positive constants.

Question 7   **(4 marks)**

A particle moves such that its displacement from the origin O, at time t seconds, is x metres, where:
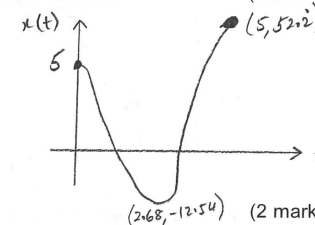
$$x(t) = \frac{2t^4 - t^3 - 28t^2}{t+4} + 5 \qquad \text{for } t \geq 0.$$

Determine

(a)   the distance travelled by the particle in the first 5 seconds.   **(2 marks)**

$Dist = (2 \times 12.54) + (52.2) + 5$ ✓

$= \boxed{82.3 \, m}$ ✓

[graph of $x(t)$ showing point $(5, 52.2)$, value 5 on axis, minimum at $(2.68, -12.54)$]

(b)   the acceleration of the particle when $t = 5$.   **(2 marks)**

$\ddot{x}(5) = \underline{42.35 \, ms^{-2}}$

on CPad ✓   ✓[correct units]

Question 8   **(3 marks)**

KTL Productions sells a product at a unit price of $30. The cost of producing x items is given by $C(x) = \frac{80x}{x+1} + 0.04x^2 + 500$.

(a)   Find an expression for the profit $P(x)$ corresponding to the manufacture and sale of x items.   **(1 mark)**

$P(x) = 30x - \left(\frac{80x}{x+1} + 0.04x^2 + 500 \right)$ ✓

(b)   Find how many items were manufactured and sold if the profit associated with the sale of the next item is approximately $10, given that more than 100 items were manufactured and sold.   **(2 marks)**
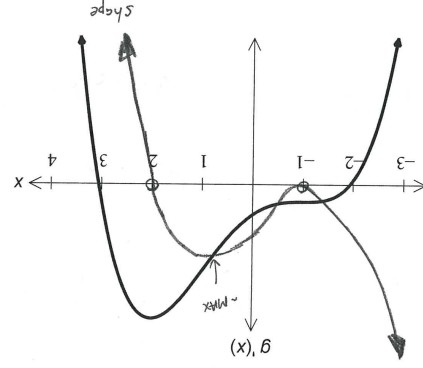
✓ $\boxed{P'(x) = 10} \quad \Rightarrow \quad x = 1 \text{ or } 250$

$\underline{250}$ items manufactured & sold   (Accept 249)

✓

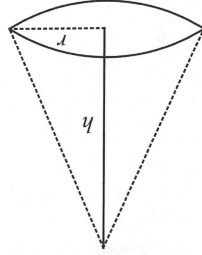## Question 3   (6 marks)

A tent in the shape of a cone is to be pitched. A bamboo frame is needed for the circumference of the base and the height of the cone. 8 metres of bamboo is to be used for the framework, represented by the solid lines in the diagram below.

(a)   Show that the volume V, of the tent in terms of its radius r, is given by:

$$V = \frac{8}{3}\pi r^2 - \frac{2}{3}\pi^2 r^3$$

                                                  (2 marks)

(b)   Determine the radius of the tent that will maximise the volume, leaving your answer in terms of π. You **are not** required to prove it is a maximum.   (4 marks)

End of questions

Question 6   (4 marks)

The graph of $y = g'(x)$ is sketched below.
On the same axes, sketch $y = g''(x)$.

Additional working space

Question number: _____

Question 5   (4 marks)

Consider the functions $f(x) = ax^3 + \frac{b}{x}$ with $f'(1) = 9$ and $f''(1) = 6$. Determine the values of a and b.

$$f'(x) = 3ax^2 - bx^{-2} \quad \checkmark \quad \left[\begin{array}{l}\text{Shows} \\ \text{1st derivative}\end{array}\right]$$

$$9 = 3a - b \quad -\text{①} \quad \checkmark \quad [\text{forms equ}]$$

$$f''(x) = 6ax + 2bx^{-3}$$

$$6 = 6a + 2b \quad -\text{②} \quad \checkmark \quad [\text{Forms 2}^{nd}\text{ Equ}]$$

Solve ① & ② Simultaneously

Clad

$$\begin{array}{l} a = 2 \\ \hline b = -3 \end{array} \quad \checkmark \quad [\text{both correct}]$$

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Christ Church Grammar School

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section

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Question 4   (7 marks)

An explosion produces a sound wave which expands through the air as a sphere. The radius increases at a rate of 300 m/s.

(a) At what rate is the sphere increasing the volume of the sphere increasing two seconds after the explosion?   (3 marks)

We have $\dfrac{dr}{dt} = 300 \, \text{m/s}$

$\dfrac{dv}{dt} = \dfrac{dv}{dr} \times \dfrac{dr}{dt}$ [Chain Rule]

And $V = \dfrac{4\pi r^3}{3}$ $\therefore \dfrac{dv}{dr} = 4\pi r^2$ $\left[\dfrac{dv}{dr}\right]$

$= 4\pi r^2 \times 300 = 1200\pi r^2$ $\boxed{1200\pi r^2}$

At $t=2$, $r = 600$

$\therefore \dfrac{dv}{dt} = 1200\pi (600)^2$

$= 432\,000\,000\,\pi \, m^3/sec$

$= 1{,}357\,168\,026 \, m^3/sec$ [Answ]

(b) By using the incremental formula, determine the percentage increase in the surface area of the sphere when the radius increases by two percent.   (4 marks)

$\delta A \approx \dfrac{dA}{dr} \times \delta r$ $A_{\text{sphere}} = 4\pi r^2$

$= 8\pi r \times 0.02 r$ $\boxed{\dfrac{dA}{dr} = 8\pi r}$ $\left[\dfrac{dA}{dr}\right]$

$= 0.16\pi r^2$ [obtain δA]

But $\dfrac{\delta A}{A}$ $= \dfrac{0.16\pi r^2}{4\pi r^2}$ $\left[\dfrac{\delta A}{A}\right]$

$= \dfrac{0.16}{4}$

$= \dfrac{0.16}{4}$ ← 4% increase   [%]

See next page
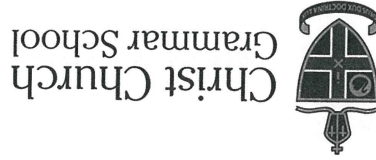
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(b) By using the incremental formula, determine the percentage increase in the surface area of the sphere when the radius increases by two percent. (4 marks)

Question 5   **(4 marks)**

Consider the functions $f(x) = ax^3 + \frac{b}{x}$ with $f'(1) = 9$ and $f''(1) = 6$. Determine the values of a and b.

$2\pi r + h = 8 \quad \Rightarrow \quad \underline{h = 8 - 2\pi r}$

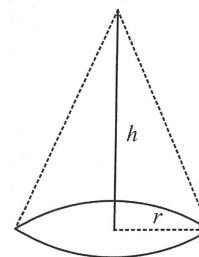**Question 3**   **(6 marks)**

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(a)   Show that the volume V, of the tent in terms of its radius r, is given by:

$$V = \frac{8}{3}\pi r^2 - \frac{2}{3}\pi^2 r^3$$

                                                          (2 marks)

$2\pi r + h = 8 \quad \Rightarrow \quad \underline{h = 8 - 2\pi r}$

$V = \frac{1}{3}\pi r^2 h$ [h subject]

$V = \frac{1}{3}\pi r^2 (8 - 2\pi r)$ ✓   [sub in]

✓ $V = \frac{8}{3}\pi r^2 - \frac{2}{3}\pi^2 r^3$

(b)   Determine the radius of the tent that will maximise the volume, leaving your answer in terms of π. You **are not** required to prove it is a maximum.   (4 marks)

$\frac{dV}{dr} = \frac{16}{3}\pi r - 2\pi^2 r^2 \quad \Rightarrow \quad \frac{dV}{dr} = 0$ ✓ $[\frac{dV}{dr}]$

$\therefore \frac{16}{3}\pi r = 2\pi^2 r^2$ ✓ $[\frac{dV}{dr} = 0, \text{manipulates equation}]$

$\frac{16}{3} = 2\pi r$

$r = \frac{16}{6\pi}$ ✓ [isolates r]

$\boxed{r = \frac{8}{3\pi} \; m}$ ✓ [Answer with units]

End of Questions

Question 2   (2 marks)

Find the value of x, in terms of a and b, where the equation $y = 2ax^2 + b^2x$ has a derivative equal to zero given that a and b are positive constants.

If $y = 2ax^2 + b^2x$

$\dfrac{dy}{dx} = 4ax + b^2$

$0 = 4ax + b^2$ ✓ $[dy/dx = 0]$

$4ax = -b^2$

$\boxed{x = -\dfrac{b^2}{4a}}$ ✓ $[x=]$
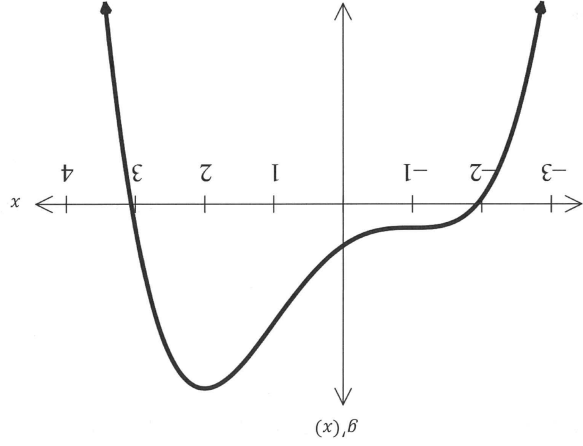
2

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Question 7 (4 marks)

A particle moves such that its displacement from the origin O, at time t seconds, is x metres, where:

$$x(t) = \frac{2t^4 - t^3 - 28t^2}{t+4} + 5 \qquad \text{for } t \geq 0.$$

Determine

(a) the distance travelled by the particle in the first 5 seconds. (2 marks)

(b) the acceleration of the particle when $t = 5$. (2 marks)

Question 8 (3 marks)

KTL Productions sells a product at a unit price of $30. The cost of producing x items is given by $C(x) = \frac{80x}{x+1} + 0.04x^2 + 500$.

(a) Find an expression for the profit $P(x)$ corresponding to the manufacture and sale of x items. (1 mark)

(b) Find how many items were manufactured and sold if the profit associated with the sale of the next item is approximately $10, given that more than 100 items were manufactured and sold. (2 marks)

Question 1 (7 marks)

Differentiate with respect to x. Simplify your answers, leaving them in fully factorised form where appropriate.

(a) $\frac{2x+1}{8-3x}$ (2 marks)

$$\frac{u'v - uv'}{v^2} \Rightarrow \frac{2(8-3x) - (2x+1)(-3)}{(8-3x)^2} \quad \checkmark \quad \left[\text{Quotient Rule}\right]$$

$$= \frac{16 - 6x + 6x + 3}{(8-3x)^2}$$

$$= \boxed{\frac{19}{(8-3x)^2}} \quad \checkmark \quad \left[\text{Fully Simplified}\right]$$

(b) $(2x^4 + 2)(4 + x)^8$ (3 marks)

$$u'v + uv'$$

$$= 8x^3(4+x)^8 + (2x^4+2)8(4+x)^7 \times 1 \quad \checkmark \quad \left[\text{Product Rule}\right]$$

$$= 8(4+x)^7\left[x^3(4+x) + 2x^4 + 2\right] \quad \checkmark \quad \left[\text{HCF}\right]$$

$$= 8(4+x)^7\left[4x^3 + x^4 + 2x^4 + 2\right]$$

$$= \boxed{8(4+x)^7(3x^4 + 4x^3 + 2)} \quad \checkmark \quad \left[\text{Fully factorised}\right]$$

(c) $\sin(3x) - 4\cos 2x$ (2 marks)

$$= \underline{3\cos(3x)} + \underline{8\sin(2x)} \qquad \left[\text{Each part}\right]$$
$$\quad\quad \checkmark \quad\quad\quad \checkmark$$

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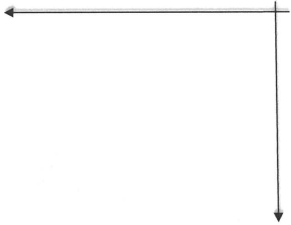7. It is recommended that **you do not use pencil**, except in diagrams.

Question 9   (8 marks)

The population of a certain bacteria in a culture (in hundreds) is modelled by

$P = t + \sin(2t)$ for $2 \leq t \leq 6$, where t is time in weeks.

(a) Sketch the population on the axes below   (2 marks)

(b) Find the exact value of t when the bacteria achieves a temporary peak in its population. State the population at this time.   (4 marks)

(c) Find the maximum population of the bacteria in the interval $2 \leq t \leq 6$.   (2 marks)

End of questions

Additional working space

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Grammar School

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Teacher's name _____ MAR _____

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