$Arg(z_1) = \frac{\pi}{10}$ $Arg(z_2) = \frac{\pi}{10} + \frac{2\pi}{n}$ $Arg(z_3) = \frac{\pi}{10} + \frac{4\pi}{n}$ $Arg(z_4) = \frac{\pi}{10} + \frac{6\pi}{n}$ $\frac{\pi}{10} + \frac{4\pi}{n} < \frac{\pi}{2}, \frac{4}{n} < \frac{1}{2} - \frac{1}{10} = \frac{2}{5}$ $\frac{n}{4} > \frac{5}{2}, n > 10$ $\frac{\pi}{10} + \frac{6\pi}{n} > \frac{\pi}{2}, \frac{6}{n} > \frac{1}{2} - \frac{1}{10} = \frac{2}{5}$ $\frac{n}{6} < \frac{5}{2}, n < 15$ $10 < n < 15$ Note; accept n=15 though point out that SCSA would not!
Specific behaviours
✓ uses correct difference in arguments ✓ sets up inequality for lower n value using 3rd root ✓ sets up inequality for upper n value using 4th root ✓ solves for interval of n values NOTE: any statement that is not supported receives zero marks)

PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

Independent Public School

Course Specialist Test 1 Year 12

Student name: ______________ Teacher name: ______________

Task type: **Response/Investigation**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: 7

Materials required: No cals allowed!!

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, NO notes allowed!

Marks available: 41 marks

Task weighting: 13%

Formula sheet provided: no, but formulae stated on page 2

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = \lvert z\rvert = \sqrt{a^2+b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan\theta = \frac{b}{a}, \quad -\pi < \theta \le \pi$
$\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$	$\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = \lvert z\rvert^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{\lvert z\rvert^2}$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$	$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
Polar form	
$z = a + bi = r(\cos\theta + i\sin\theta) = r\,cis\,\theta$	$\bar{z} = r\,cis(-\theta)$
$z_1 z_2 = r_1 r_2\,cis(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,cis(\theta_1 - \theta_2)$
$cis(\theta_1 + \theta_2) = cis\,\theta_1\,cis\,\theta_2$	$cis(-\theta) = \frac{1}{cis\,\theta}$
De Moivre's theorem	
$z^n = \lvert z\rvert^n\,cis(n\theta)$	$(cis\,\theta)^n = \cos n\theta + i\sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\frac{\theta+2\pi k}{q} + i\sin\frac{\theta+2\pi k}{q}\right)$, for k an integer	

$$(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$$

$$ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$m\frac{-1}{\sqrt{3}} = -1$ $m = \frac{\sqrt{3}}{1} = \tan\theta$ $\theta = \frac{\pi}{3}, \frac{-2\pi}{3}$
Specific behaviours
✓ determines gradient of tangent ✓ determines min argument

d) State the maximum value of $Arg(z)$

Solution
Max = $\frac{\pi}{3}$ See above
Specific behaviours
✓ determines gradient of tangent ✓ determines max argument

Q7 (4 marks)

Consider the roots of the equation $z^n = a$ with z being a complex variable with a as a complex constant and n being an integer $n > 3$. A root is defined to be in the first quadrant if the Argument lies in $0 < Arg(z) < \frac{\pi}{2}$.

Determine **all** the allowable values of n such that there will be **exactly** 3 roots in the first quadrant and the smallest argument of these 3 roots will be $\frac{\pi}{10}$.

Solution

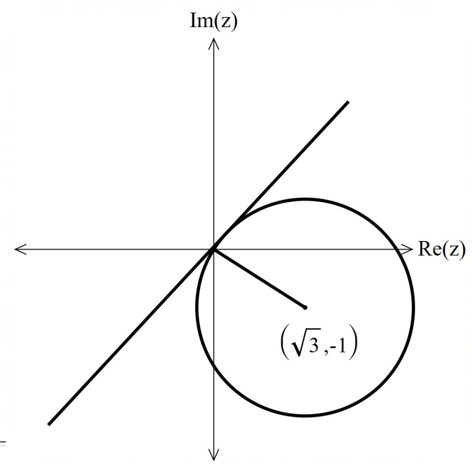
Specific behaviours
✓ circle with centre coordinates stated ✓ goes through origin

b) State the maximum value of $|z|$

Solution
$|z| = 4$
Specific behaviours
✓ states maximum

c) State the minimum value of $Arg(z)$

Solution



No cals allowed!!

Q1 (2, 2, 2 & 2 = 8 marks)

If $z = 5 - 4i$ and $w = 2 + 3i$ determine the following:

a) zw

Solution
$(5 - 4i)(2 + 3i) = 10 + 12 - 8i + 15i$ $= 22 + 7i$
Specific behaviours
✓ real part ✓ Imaginary part

b) $\frac{1}{w}$

Solution
$\frac{1}{2+3i}\frac{2-3i}{2-3i} = \frac{2-3i}{13}$
Specific behaviours
✓ uses conjugate ✓ express answer

c) $\frac{\overline{z}}{w}$

Solution
$\frac{5+4i}{2+3i}\frac{2-3i}{2-3i} = \frac{22-7i}{13}$
Specific behaviours
✓ numerator ✓ denominator

d) $z^2\overline{w}$

Solution

$$(5-4i)^2(2-3i)=(25-16-40i)(2-3i)$$
$$(9-40i)(2-3i)$$
$$=18-120-80i-27i$$
$$=-102-107i$$

Specific behaviours

✓ evaluates square term
✓ determines answer

Q2 (2 & 3 = 5 marks)

a) Determine the complex roots of $3z^2+z+2=0$.

Solution

$$3z^2+z+2=0$$
$$z=\frac{-1\pm\sqrt{1-24}}{6}$$
$$z=\frac{-1\pm\sqrt{23}i}{6}$$

Specific behaviours

✓ uses quadratic formula
✓ has two complex roots

b) Use the quadratic equation to prove that if a quadratic equation with real coefficients has any non-real roots then it must have two complex roots and they must be conjugates of each other.

Solution

$$ax^2+bx+c=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$b^2-4ac=-n^2=i^2n^2$$
$$x=\frac{-b\pm\sqrt{i^2n^2}}{2a}=\frac{-b\pm in}{2a}$$

$$(x-\alpha)(x-\beta)=x^2-(\alpha+\beta)x+\alpha\beta$$
$$-(\alpha+\beta)=-2\operatorname{Re}al,\ \alpha\beta=|z|^2$$
$$f(z)=a(z^2-4z+5)(z^2-10z+29)$$
$$z=0,\ f(z)=-290\ \therefore a=-2$$
$$f(z)=-2(z^4-14z^3+74z^2-166z+145)$$
$$a=-2$$
$$b=28$$
$$c=-148$$
$$d=332$$
$$e=-290$$

Specific behaviours

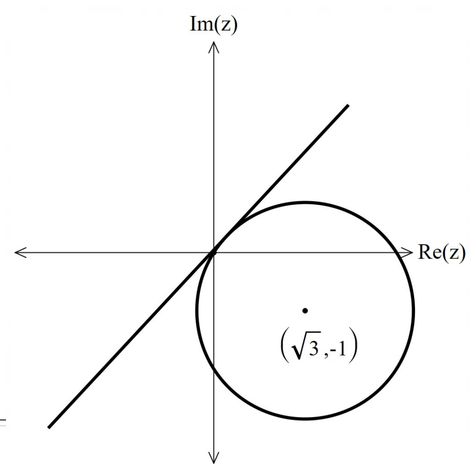
✓*shows reasoning for determining value of a*
✓*uses ONE quadratic factor*
✓*uses two quadratic factors*
✓*shows reasoning in determining quadratic factors (i.e roots in brackets)*
✓*shows reasoning on how to determine quartic polynomial.*
Note: Any statement of values without reasoning will NOT receive any marks!

Q6 (2, 1, 2 & 2 = 7 marks)

Consider the locus of complex numbers z that satisfy $|z-\sqrt{3}+i|=2$.

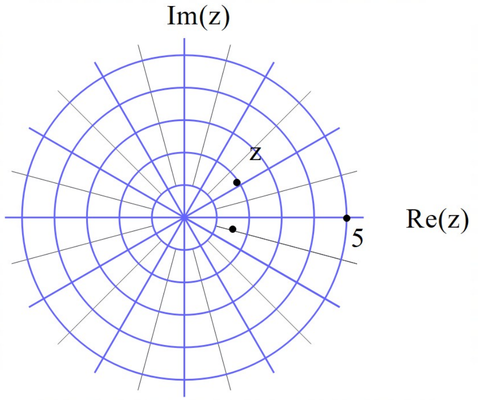
a) Sketch the locus on the axes below.

Solution



d) $\frac{z}{(1+i)}$

Solution



Specific behaviours

✓ argument
✓ modulus

Q5 (5 marks)

Consider the polynomial $f(z)=az^4+bz^3+cz^2+dz+e$ where $a,b,c,d\ \&\ e$ are real numbers.

Given that $f(2+i)=0=f(5-2i)$

and $f(0)=-290$

Determine the values of $a,b,c,d\ \&\ e$.
(Note: answers without working will receive zero marks)

Solution

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Specific behaviours

✓ sets up equation with a negative discriminant
✓ uses $i^2=-1$ with discriminant
✓ derives two complex roots which are conjugates of each other

Q3 (4 marks)

Determine all possible real number pairs $a\ \&\ b$ such that $\frac{37+9i}{5+ai}=b-i$.

Solution

$\frac{37+9i}{5+ai}=b-i$

$37+9i=(5+ai)(b-i)=5b+a+i(ab-5)$

$37=5b+a$

$9=ab-5, ab=14, a=\frac{14}{b}$

$37=5b+\frac{14}{b}$

$37b=5b^2+14$

$5b^2-37b+14=0$

$(5b-2)(b-7)=0$

$b=7, a=2$

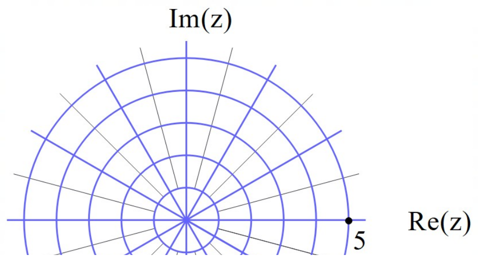
$b=\frac{2}{5}, a=35$

Specific behaviours

✓ sets up equation and equates real and imaginary
✓ obtains two simultaneous equations
✓ solves for one pair of values
✓ solves for two pairs of values

Q4 (2, 2, 2 & 2 = 8 marks)

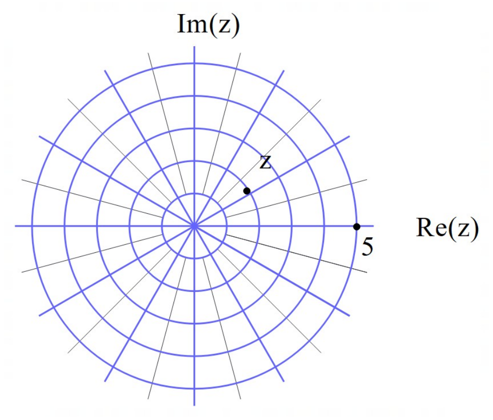
Consider the complex number $z=\sqrt{3}+i$.



Plot the following on the axes above.

a) z

Solution

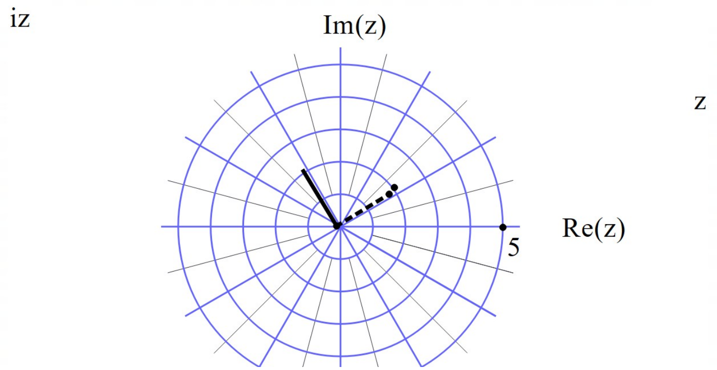


Specific behaviours

✓ argument
✓ length of 2 units

b) iz

Solution

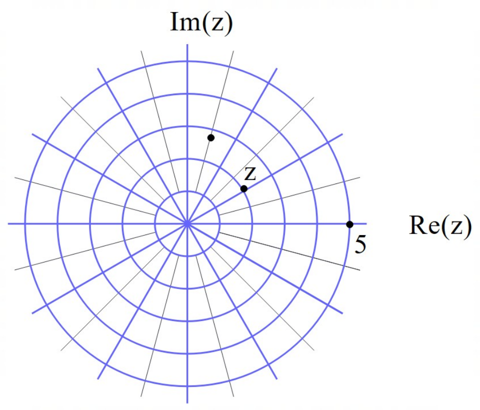


Specific behaviours

✓ uses right angle
✓ rotates anticlockwise with unchanged length

c) $(1+i)z$

Solution



Specific behaviours

✓ argument
✓ modulus