



**Semester Two Examination, 2021**

**Question/Answer Booklet**

# **MATHEMATICS METHODS**

**ATAR Year 12**

**Section One:**

**Calculator-free**

Student Name: \_\_\_\_\_

Please circle your teacher's name

**Teacher: Miss Hosking**

**Miss Rowden**

## **Time allowed for this paper**

Reading time before commencing work:

5 minutes

Working time for paper:

50 minutes

## **Materials required/recommended for this paper**

### ***To be provided by the supervisor***

This Question/Answer Booklet

Formula Sheet

Number of additional  
answer booklets used  
(if applicable):

### ***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## **Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Suggested working time (minutes)	Marks available	Percentage of examination
Section One: Calculator free	8	8	50	51	35
Section Two: Calculator-assumed	13	13	100	97	65
<b>Total</b>					100

## Instructions to candidates

1. The rules for the conduct of the ATAR course examinations are detailed in the *Year 12 Information Handbook 2021*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Supplementary pages for the use planning/continuing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free****35% (51 Marks)**

This section has eight (8) questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes

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**Question 1****(5 marks)**

(a) Determine  $\int \frac{4x+1}{2x^2+x-5} dx, x > 1$ .

**(2 marks)**

(b) The line  $y = 12 - 2x$  intersects the curve  $y = \frac{10}{x}$  at  $(1, 10)$  and  $(5, 2)$ . Determine the area trapped between line and the curve.

**(3 marks)**

**Question 2**

**(5 marks)**

A summary of the lengths of a large sample of nails from a production line are shown below.

Length, $L$ mm	Relative frequency
$147 < L \leq 148$	0.17
$148 < L \leq 149$	0.13
$149 < L \leq 150$	0.21
$150 < L \leq 151$	0.19
$151 < L \leq 152$	0.16
$152 < L \leq 153$	0.14

- (a) What proportion of nails are longer than 149 mm? (1 mark)
- (b) Determine the probability that a randomly selected nail from the production line is longer than 150 mm given that it is no longer than 152 mm. (2 marks)
- (c) State, with reasons, whether the data suggests that the nail lengths are normally distributed. (2 marks)

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**Question 3****(7 marks)**

The curve  $y = 8x - \frac{4}{x^2}$  has one stationary point.

- (a) Obtain expressions for  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . (2 marks)

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- (b) Determine the coordinates of the stationary point and determine its nature. (4 marks)

- (c) Explain why the curve has no point of inflection. (1 mark)

## Question 4

(7 marks)

(a) Let  $F(x) = \int_0^x \sin 2\theta \, d\theta$ .

Express  $F(x)$  as a function of  $x$  and hence evaluate  $F\left(\frac{\pi}{6}\right)$ .

(3 marks)

**Question 4 continued**

(b) Let  $g(x) = \frac{e^{2x-1}}{2x+1}$ .

(i) Show that  $g'(x) = \frac{4xe^{2x-1}}{(2x+1)^2}$ . (2 marks)

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(ii) Hence, or otherwise, evaluate  $\int_0^1 \frac{xe^{2x-1}}{(2x+1)^2} dx$ . (2 marks)

**Question 5****(7 marks)**

- (a) By first using log laws, or otherwise, determine  $\frac{d}{dx} \left( \ln \left( e^{3x} \sqrt{x^2 + 3} \right) \right)$  in simplest form.

**(3 marks)**

- (b) The function  $f(x) = x^2 \ln(2x)$  for  $x > 0$  has one stationary point, a global minimum.

Determine the minimum value of the function.

**(4 marks)**

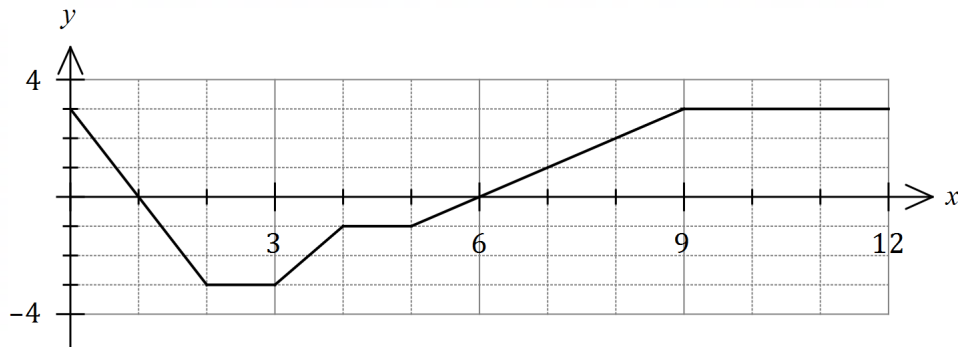


**Question 6**

**(6 marks)**

The graph of  $y=f(x)$  consists of line segments, as shown below.

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Evaluate each of the following:

(a)  $\int_7^{10} f(x) dx.$

(1 mark)

(b)  $\int_3^8 f(x) dx.$

(2 marks)

(c)  $\int_0^9 (f(x)+2) dx.$

(3 marks)

**Question 7****(8 marks)**

The random variable  $X$  is defined by  $P(X=x) = \begin{cases} k \log_3(x+2) & x=1, 25, 79 \\ \text{elsewhere} \end{cases}$

(a) Determine the value of the constant  $k$ . (2 marks)

(b) Calculate the expected value of  $X$ . (2 marks)

The Bernoulli random variable  $Y$  is solely dependent on  $X$ , so that  $Y=1$  when  $X=1$ , and  $Y=0$  for all other values of  $X$ .

(c) Determine

(i)  $P(Y=0)$ . (1 mark)

(ii)  $E(Y)$ . (1 mark)

See next page

(iii)  $\text{Var}(3Y+1)$ .

(2 marks)

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**Question 8****(6 marks)**

In triangle  $ABC$ , the length of the side opposite angle  $A$  is given by  $a = \sqrt{13 - 6 \cos A}$  cm.

Use the increments formula to calculate the approximate change in length of  $a$  as the size of angle  $A$  decreases from  $\frac{20\pi}{30}$  to  $\frac{19\pi}{30}$ .

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**End of questions**

Supplementary page

Question number: \_\_\_\_\_

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