

4					8
3					7
2					6
1					5

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Important note to Candidates

Special items: nil

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate

Formula sheet

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Time allowed for this section

Your Teacher's Name

Your Name

Calculator-free

Section One:

UNIT 3

MATHEMATICS SPECIALIST

Question/Answer booklet

Semester One Examination, 2022

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	34
Section Two: Calculator-assumed	14	14	100	97	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

See next page

	Solution
	Specific behaviours
	<p>P uses rotation of 90 degrees</p> <p>P states argument</p>
	$\operatorname{Arg}(iz) = \frac{\pi}{3}$
	$\operatorname{Arg}(iz) = \frac{4\pi}{3}$

- a) Determine the exact value of $\operatorname{Arg}(iz)$ and plot on the axes above. (2 marks)

Consider the complex number z as plotted on the complex plane below.

(4 marks)

Question 1

Working time: 50 minutes.

- number of the question that you are continuing to answer at the top of the page.
- original answer space where the answer is continued, i.e. give the page number. Fill in the original answer space for planning, indicate this clearly at the top of the page.
- Continuing an answer: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
 - Planning: if you use the spare pages for planning, indicate this clearly at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Space pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

This section has **eight (8)** questions. Answer all questions. Write your answers in the spaces provided.

Section One: Calculator-free

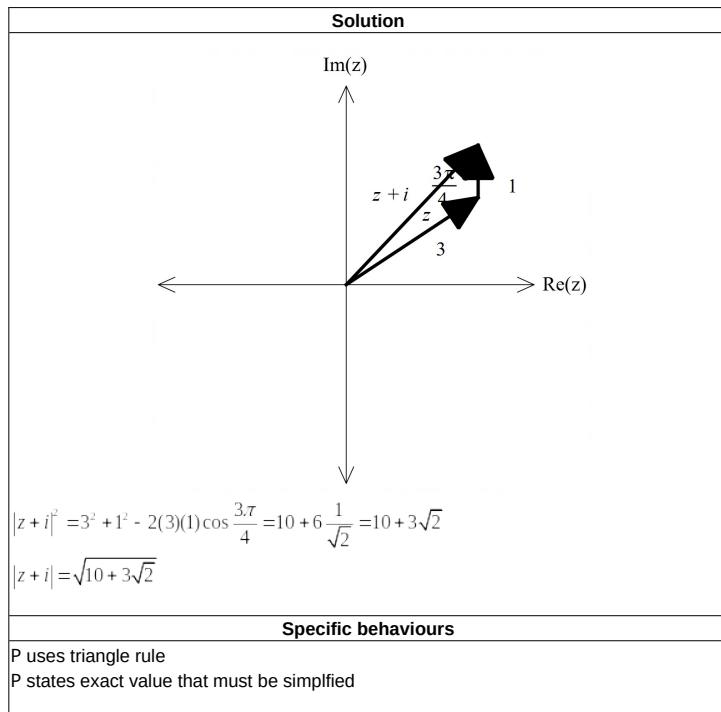
(52 Marks)

Question number:

Additional working space

b) Determine the exact value of $|z + i|$.

(2 marks)



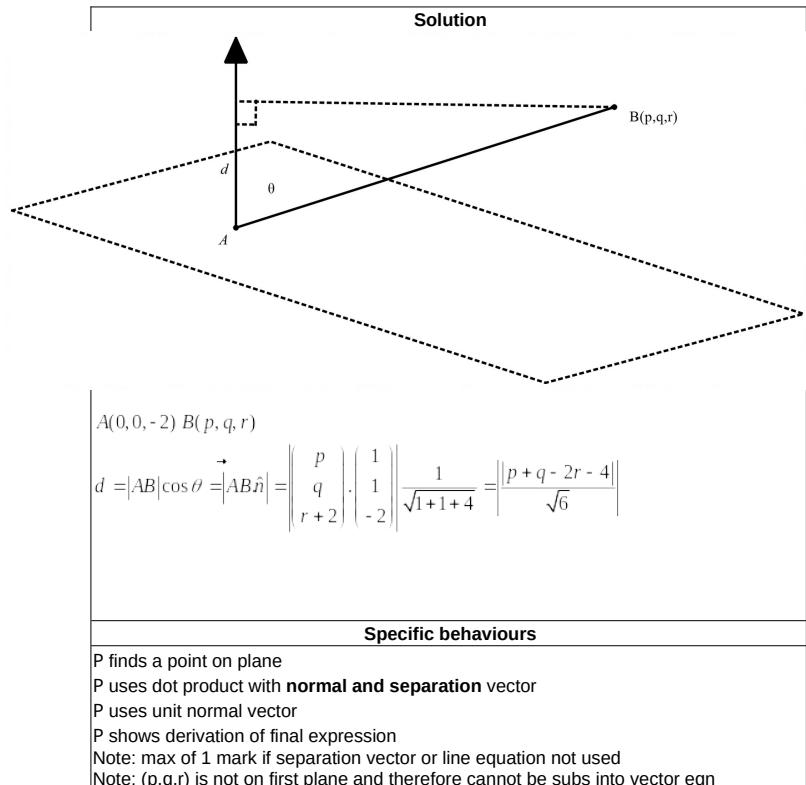
Question 8

(4 marks)

$$r \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = 4$$

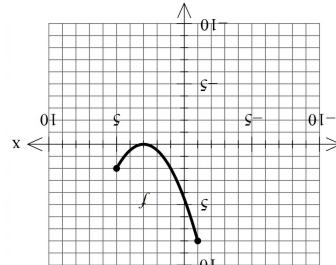
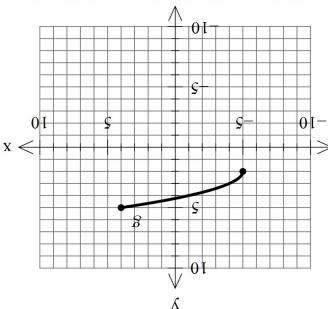
Consider two parallel planes Λ & Ψ . Plane Λ is given by the point (p, q, r) .

$$\text{Show that the distance between the two planes is given by } \left| \frac{p+q-2r-4}{\sqrt{6}} \right|$$



(8 marks)

Consider the functions $f_{\alpha, g}$ with domains shown in the graphs below.



Question 2

a) Sketch the inverse functions above.

(a) Sketch the inverse functions above.
(b) If they exist. (2 marks)

1 mark)

c) Determine $f \circ f^{-1}(x)$

d) Determine the exact coordinates, if any, where $f(x) = f_{-1}(x)$

(3 marks)

(3 marks)

P states x

speciale edizioni

$$x = (x)_{\downarrow \downarrow} \circ \downarrow$$

Solution

(3 marks)

(x)₁, $f = (x) f$

d) Determine the exact

Solution

Specific behaviours

P states that f does not exist
P states that g with correct endpoints is inverse of f

Solution	$f(x) = 3x^2 - 12x + 11 = 0$	$x = \frac{13 \pm \sqrt{169 - 4(3)(11)}}{6} = \frac{13 \pm \sqrt{37}}{6}$ as $x \leq 2$ $x = \frac{6}{13 - \sqrt{37}}$ $x = \frac{6}{13 - \sqrt{37}}, \frac{6}{13 + \sqrt{37}}$	States simplified exact coordinates for point discards the positive value equates to x and solves for two values specific behaviours
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d) Determini

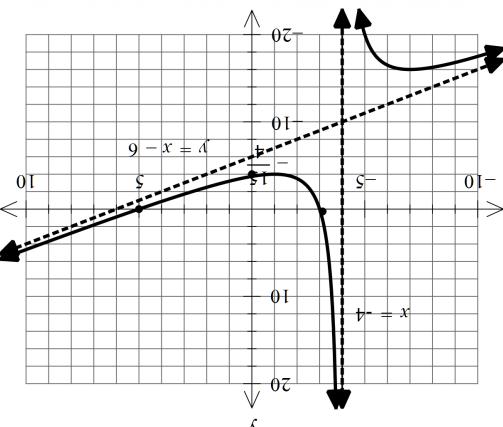
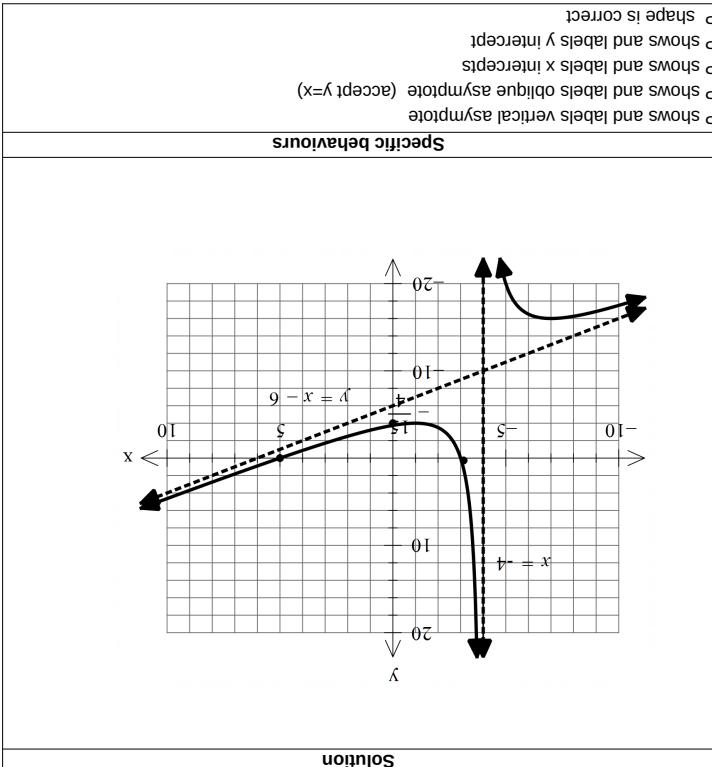
- b) Which of the following exist over natural domains, $f \circ g$, $g \circ f$? Explain.
 State the natural domain and corresponding ranges for those that exist. (3 marks)

P states inverse rule with two possibilities P discards positive to give only one rule

Solution
$d_f : -1 \leq x \leq 5 \quad r_f : 0 \leq y \leq 8$ $d_g : -5 \leq x \leq 4 \quad r_g : 2 \leq y \leq 5$ $f \circ g \quad r_g \subseteq d_f \therefore \text{exists}$ $d : -5 \leq x \leq 4 \quad r : 0 \leq y \leq 2$ $g \circ f \quad r_f \not\subseteq d_g \therefore \text{not exist}$
Specific behaviours
P states domain and ranges of both functions in argument P shows that fog exists with condition P shows that gof does not exist with condition

- c) The rule for g is $2 + \sqrt{x+5}$. State the inverse rule g^{-1} and its domain. (3 marks)

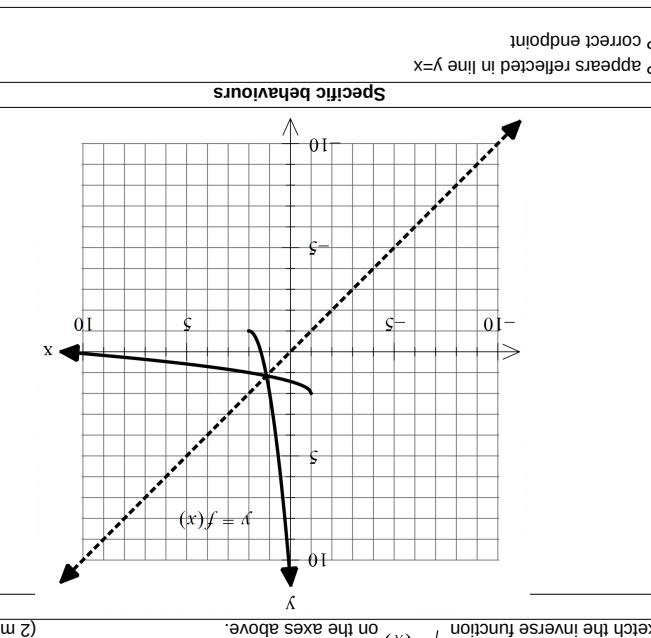
Solution
$d_{g^{-1}} = r_g = 2 \leq x \leq 5$ $x = 2 + \sqrt{y+5}$ $(x-2)^2 = y+5$ $g^{-1}(x) = (x-2)^2 - 5$
Specific behaviours
P swaps x & y P solves for inverse P states domain



(5 marks)

Question 3

Solution	p appears reflected in line $y=x$ p correct endpoint
Specific behaviours	$x = d \Rightarrow x \geq -1$ $r = d \Rightarrow r \leq 2$ $x = 3y^2 - 12y + 11$ $0 = 3y^2 - 12y + 11 - x$ $y = \frac{12 \pm \sqrt{44 - 4(3)(11-x)}}{6} = \frac{12 \pm \sqrt{12 + 12x}}{6} = 2 \pm \sqrt{3(x+1)}$ $f_{-1}(x) = 2 - \frac{\sqrt{3(x+1)}}{3}$
Specific behaviours	p swaps x & y of solves for x
States domain	p shows and labels x intercepts p shows and labels y intercepts p shape is correct



Question 7

(10 marks)

Consider the function $f(x) = 3x^2 - 12x + 11$ for $x \leq 2$ which is plotted below.

$$f(x) = \frac{(x+4)}{(x-5)}$$

Question 4**(8 marks)**

Consider the polynomial $P(z) = 3z^4 - 18z^3 - 3z^2 + 258z + 510$ for the complex variable z .

- a) Given that $P(5+3i)=0$, determine a quadratic factor of $P(z)$. (3 marks)

Solution
$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta$
$\alpha = 5 + 3i$
$\beta = 5 - 3i$
$(x^2 - 10x + 34)$ or $3(x^2 - 10x + 34)$
Specific behaviours
P uses conjugate
P correct coefficient of x
P correct quadratic factor

- b) If $P(5+3i)=0=P(-2-i)$, determine all solutions to $3z^4 - 18z^3 - 3z^2 + 258z + 510 = 0$. (2 marks)

Solution
$5 \pm 3i, -2 \pm i$
Specific behaviours
P states 2 solns
P states all 4 roots

- c) $P(z)$ can be expressed as $a(z^2 + bz + c)(z^2 + dz + e)$ where $a, b, c, d & e$ are real integers. Determine the values of $a, b, c, d & e$. (3 marks)

Solution
$a(z^2 + bz + c)(z^2 + dz + e) = 3(x^2 - 10x + 34)(x^2 + 4x + 5)$
Specific behaviours
P uses conjugate of $-2-i$

Specific behaviours
P obtains an equation with two variable eliminated and in terms of p&q
P states requirement for uniqueness
P states requirements for infinite and no solns

none: $p = \frac{19}{7}$ and $q = \frac{10}{22}$
inf init: $p = \frac{19}{7}$ and $q = \frac{10}{22}$
unique: $p \neq -\frac{19}{7}$
$0 \quad 0 \quad 19p + 7 \quad 10q - 22$
$0 \quad 13 \quad 2p - 2 \quad 10 - q$
$1 \quad 4 \quad 3 \quad 5$
$0 \quad 10 \quad 3p - 1 \quad 6$
$0 \quad 13 \quad 2p - 2 \quad 10 - q$
$1 \quad 4 \quad p \quad 5$
$3 \quad 2 \quad 1 \quad 9$
$2 \quad -5 \quad 2 \quad q$
$1 \quad 4 \quad p \quad 5$

Solution

- (i) Unique solution
(ii) Infinitesimal solutions
(iii) No solutions.
- b) Solve for all possible values of $p \neq q$ for the following scenarios:

b) Consider the following system $\begin{cases} 2x - 5y + 2z = p \\ x + 4y + pz = q \end{cases}$ where $p \neq q$ are constants. (3 marks)

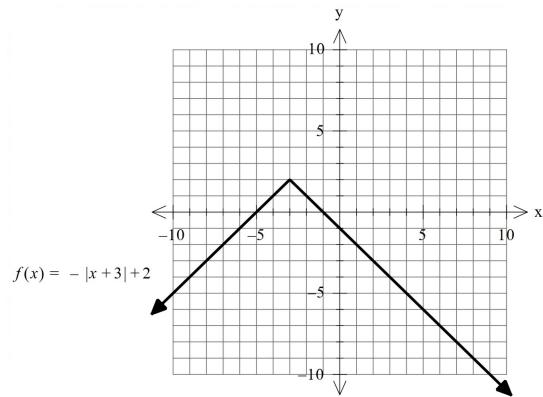
$$3x + 2y + z = 9$$

Specific behaviours			
$x - 4 + 6 = 5 \Rightarrow x = 3$			
$-13 + 4z = -5 \Rightarrow z = 2$			
$16y = -16 \Rightarrow y = -1$			
$0 \quad 16 \quad 0 \quad -16$			
$0 \quad 13 \quad 4 \quad -5$			
$1 \quad 4 \quad 3 \quad 5$			
$0 \quad 10 \quad 8 \quad 6$			
$0 \quad 13 \quad 4 \quad -5$			
$1 \quad 4 \quad 3 \quad 5$			
$3 \quad 2 \quad 1 \quad 9$			
$2 \quad -5 \quad 2 \quad 15$			
$1 \quad 4 \quad 3 \quad 5$			

P uses a=3	P states both quadratic factors
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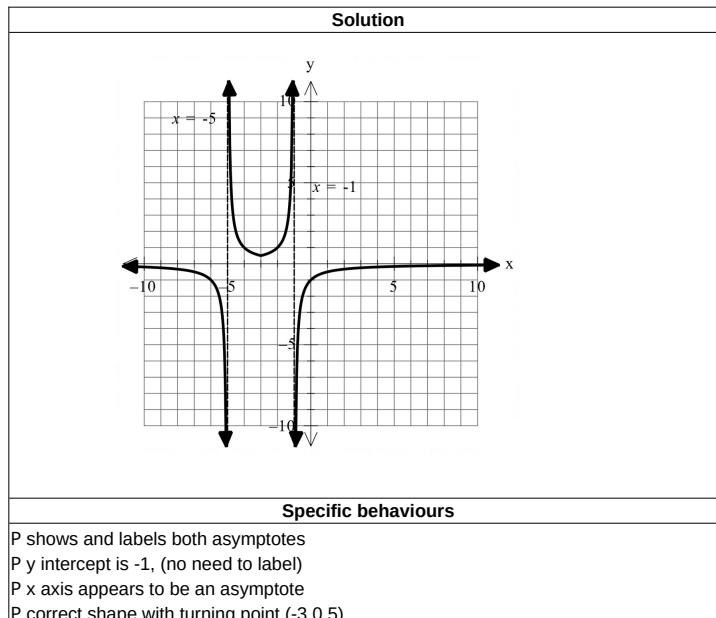
Question 5

(7 marks)

Consider the function $f(x) = -|x+3| + 2$.

- a) Sketch $y = \frac{1}{f(x)}$ on the axes below, labeling important features.

(4 marks)



b) Hence determine the natural domain and range of $g(x) = \frac{1}{3|x+3|-6}$ (3 marks)

Solution
$g(x) = \frac{1}{-3f(x)}$
$d_g : x \neq -5, -1$
$r_g : R \setminus \left(\frac{-1}{6} < y < 0 \right)$
$\frac{-1}{6} < y \leq 0$
i.e Range of g is all Real numbers excluding $y \leq -\frac{1}{6}, y > 0$
i.e $y \leq -\frac{1}{6}, y > 0$
Specific behaviours
P shows use of factor -1/3 and graph in (a) or other stated reasoning
P states domain
P states range

Question 6

(6 marks)

$$3x + 2y + z = 9$$

$$x + 4y + 3z = 5$$

$$2x - 5y + 2z = 15$$

Consider the linear equations

- a) Solve for $x, y \& z$.

(3 marks)

Solution
