

MATHEMATICS METHODS Year 11
Section One:
Calculator-free

Your name _____

Solutions _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 29 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet using blue/black pen. Do not use erasable or gel pen.
3. Answer all questions.
4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
6. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
7. It is recommended that **you do not use pencil**, except in diagrams.

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Question 1

(6 marks)

A trigonometric function is given as $f(x) = 5 - 3 \cos(2x)$, for $0 \leq x \leq 2\pi$.

(a) Describe the transformations which have been applied to the graph of $y = \cos x$ to obtain $f(x)$. (3 marks)

- horizontal dilation scale factor $\frac{1}{2}$
- vertical dilation scale factor (-3) \swarrow reflection about x axis
- vertical translation 5 units up

Behaviours: must have all 3 compound transformations w/ correct order ✓ (2 marks)

$$\min \cos x = -1 \quad \therefore \min 5 - 3 \cos(2x) = 2.$$

Co-ordinates of minimum points: $(0, 2)$ $(\pi, 2)$ $(2\pi, 2)$

Behaviours: All three points w/ (working not required)

(c) Describe the phase shift required to express $f(x)$ as a function of sine resulting in the same graph over the given domain. (1 mark)

Horizontal translation of $\frac{\pi}{4}$ to the right ✓

Behaviours: correct solution

Not: Equation 1: $3 \sin(2x - \frac{\pi}{2}) + 5$ (not required.)
or $y = -3 \sin(2(x + \frac{\pi}{4})) + 5$

See next page

Question 2

(3 marks)

Determine the centre and radius of the circle given by the equation:

$$x^2 + y^2 + 6x - 10y = 2.$$

$$x^2 + 6x + 9 - 9 + y^2 - 10y + 25 - 25 = 2.$$

$$(x+3)^2 + (y-5)^2 - 9 - 25 = 2.$$

$$(x+3)^2 + (y-5)^2 = 36.$$

Hence circle centre = $(-3, 5)$

radius = 6.

Behaviours : • completes the square ✓
(working)

• states solution in form ✓

$$(x-h)^2 + (y-k)^2 = r^2$$

• states correct radius ✓

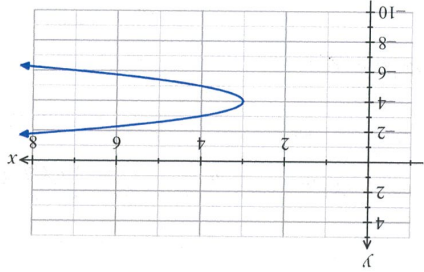
• states correct circle centre ✓

See next page

Additional working space

Question number: _____

A graph has been drawn below.



Determine:

- (a) with reason(s), if the graph is a relation or function. (2 marks)

relation ✓

fails vertical line test ✓

- (b) the equation of the axis of symmetry. (1 mark)

$$y = -4$$

- (c) the equation which represents the graph. (2 marks)

$$(y + 4)^2 = x - 3$$

$$(or) (y + 4)^2 + 3 = x$$

Behaviours : recognises { vertical translation (4 units down) ✓
horizontal translation (3 units right) ✓
: final solution statement ✓

See next page

Question 4

(8 marks)

Solve:

(a) $(2 \cos x + 1)(\sin x - 2) = 0$ for $0 \leq x \leq 3\pi$.

(4 marks)

NFL. either $2 \cos x + 1 = 0$ or $\sin x - 2 = 0$ ✓

$\therefore \cos x = -1/2$.

$\sin x = 2$

no solution ✓

$x = 2\pi/3, 4\pi/3, 8\pi/3$ ✓

Behaviour: NFL ✓

: no solution $\sin x = 2$ ✓

: all three solutions ✓

(b) $4(2 \sin^2 x + \cos^2 x) - 6 = 0$ for $0 \leq x \leq 2\pi$.

(4 marks)

either $2 \sin^2 x + \cos^2 x = 3/2$

or $2(1 - \cos^2 x) + \cos^2 x = 3/2$

$\therefore 2 \sin^2 x + 1 - \sin^2 x = 3/2$

$2 - 2 \cos^2 x + \cos^2 x = 3/2$

$\sin^2 x = 3/2 - 1$

$2 - 1.5 = \cos^2 x$

$\sin^2 x = 1/2$

$\therefore \cos x = \pm 1/\sqrt{2}$

$\sin x = \pm 1/\sqrt{2}$

Hence $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

Behaviour:

✓ 1. substitutes trig identity

✓ 2. reduces to $\sin x = \pm 1/\sqrt{2}$

✓ 3. all 4 solutions (2)

• only 2 solutions (1)

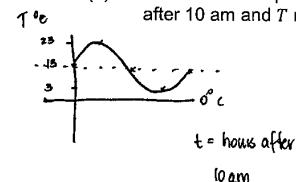
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Question 9

(6 marks)

In a particular city in South Africa it is known that the outdoor temperature ranges from a low of 3°C to a high of 23°C each day. Scientists measure the outdoor temperature throughout the day and discover that it can be modelled by a sine function. The temperature midpoint occurs at 10 am and at 10 pm with the temperature peaking in the afternoon.

(a) Determine an equation of this function where t represents the time, in hours, after 10 am and T represents the outdoor temperature in $^\circ\text{C}$. (3 marks)



$T = 10 \sin(\pi/12 t) + 13$

Behaviour: centrelines ✓

: vertical dilat ✓

: period π ✓

: final equation

(b) When the outdoor temperature falls below 10°C heating systems are used. Determine the times, to the nearest minute, at which heating systems are in use. (3 marks)

let $T^\circ = 10^\circ\text{C}$

$t = 13.164$ } hours after 10am ✓

Hours after 10am.

$= 22.836$

$\therefore 23.16 = 11:10\text{pm}$

$32.84 = 8:50\text{am}$

Hence

$11:10\text{pm} < t < 8:50\text{am}$

↑
on

↑
off

✓ calculation of both 't' values

✓ converts 't' into time

✓ lists both boundaries to nearest minute

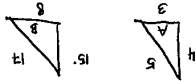
End of questions

Question 8

(4 marks)

A and B are both acute angles with $\cos(B) = \frac{17}{8}$ and $\sin(A) = \frac{4}{5}$.

Determine the exact value of $\sin(A+B)$.



$$\begin{aligned}\sin A &= \frac{4}{5} \\ \cos A &= \frac{3}{5} \\ \sin B &= \frac{15}{17} \\ \cos B &= \frac{8}{17}\end{aligned}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{4}{5} \times \frac{15}{17} + \frac{3}{5} \times \frac{8}{17}$$

$$= \frac{32}{45} + \frac{85}{45}$$

$$= \frac{77}{85}$$

$$\cos A$$

Behaviour: calculates $\sin B$

uses correct substitution

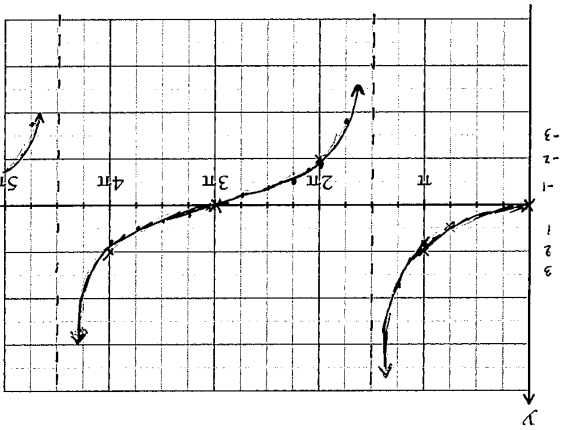
final solution

See next page

Question 5

(7 marks)

(a) On the axes below, sketch the graph of $y = \tan\left(\frac{x}{3}\right)$ over the interval $0 \leq x \leq 5\pi$, clearly indicating the equations of any asymptotes. (3 marks)



Behaviour
 ✓ correct asymptotes
 ✓ correct intercepts
 ✓ correct shape

(b) Solve the following equations over the interval $0 \leq x \leq 5\pi$, giving exact answers.

(i) $\tan\left(\frac{x}{3}\right) = -1$

$$\begin{aligned}\tan y &= -1 \\ y &= \frac{3\pi}{4}\end{aligned}$$

$$\frac{x}{3} = \frac{3\pi}{4}$$

$$\therefore x = \frac{9\pi}{4}$$

(iii) $\tan\left(\frac{x}{3}\right) - \sqrt{3} = 0$

$$\tan \frac{x}{3} = \sqrt{3}$$

$$\begin{aligned}\tan y &= \sqrt{3} \\ y &= \frac{\pi}{3}\end{aligned}$$

$$\therefore \frac{x}{3} = \frac{\pi}{3}$$

$$x = \pi, 4\pi$$

$$\therefore x = \pi$$

End of questions

Note: no penalty (i) if correct solutions (ii) (no working)

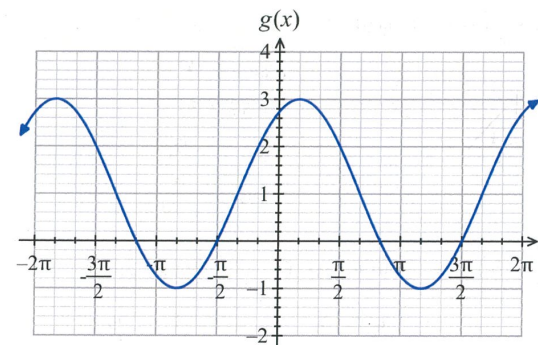
Additional working space

Question number: _____

Question 6

(3 marks)

Determine the equation of the trigonometric function, $g(x)$, shown below in terms of a sine function.



amplitude = 2

period = 2π

vertical translation = 1

horizontal translation = $\pi/3$ (left)

$$g(x) = 2 \sin\left(x + \frac{\pi}{3}\right) + 1$$

Behaviours: • amplitude

• phase shift

• centre line

(2 marks)

Question 7

The length of a string in musical instruments varies inversely to the frequency of the vibrations. This generates the sound or pitch of the note.

If an 11-inch string has a frequency of 400 cycles per second, then determine the frequency of a bass guitar which uses 30-inch strings.

let f = frequency

l = length

$$f = \frac{k}{l}$$

$$\therefore k = f \times l$$

$$= 400 \times 11$$

$$= 4400$$

$$\therefore f_2 = \frac{4400}{30}$$

$$= 146\frac{2}{3}$$

cycles per second.

Note: no penalty for rounding

See next page

Behaviours

- calculates k ✓
- calculates f_2 ✓

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Christ Church
Grammar School

2020
TEST 3

MATHEMATICS METHODS Year 11

Section Two:

Calculator-assumed

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Teacher name _____

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