Marking

PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester One
Examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST
UNIT 3
Section One:
Calculator-free

Your Name

Your Teacher's Name

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	49	34.5
Section Two: Calculator-assumed	13	13	100	93	65.5
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free **(49 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 **(8 marks)**

Let $f(x)=\sqrt{x-4}$ and $g(x)=\frac{1}{x-2}$.

a) Determine $g\circ g(x)$ (2 marks)

Solution
$g\circ g=\dfrac{1}{\frac{1}{x-2}-2}=\dfrac{1}{\frac{1-2(x-2)}{x-2}}=\dfrac{x-2}{5-2x}$
Specific behaviours
✓ subs g into g ✓ simplifies expression

b) Does $g\circ f(x)$ exist over the natural domain of f ? Explain. (2 marks)

Solution
$r_f : y\geq 0$ $d_g : x\neq 2$ $r_f \not\subset d_g \quad \therefore does\ not\ exist$
Specific behaviours
✓ states range and domain ✓ states not exist

c) Determine $f\circ g(x)$ (simplify) and its natural domain. (4 marks)

Solution

$f \circ g = \sqrt{\frac{1}{x-2} - 4} = \sqrt{\frac{1}{x-2} - 4\frac{(x-2)}{(x-2)}} = \sqrt{\frac{9-4x}{x-2}}$ $\frac{1}{x-2} \geq 4$ $2 < x \leq \frac{9}{4}$
Specific behaviours
✓ subs g into f ✓ simplify expression ✓ lower limit of domain (excluded) ✓upper limit of domain (included)

Acknowledgements

Additional working space

Question number: ________

Question 2 **(4 marks)**

Let $\frac{11-16i}{a-2i}=3+bi$ where $a\ \&\ b$ are real constants. Determine all possible value(s) of $a\ \&\ b$.

Solution
$\frac{11-16i}{a-2i}=3+bi$ $11-16i=(3+bi)(a-2i)=3a+2b+i(ab-6)$ $11=3a+2b$ $16=6-ab\rightarrow ab=-10\rightarrow b=\frac{-10}{a}$ $11=3a-2\frac{10}{a}$ $11a=3a^2-20$ $3a^2-11a-20=0$ $(3a+4)(a-5)=0$ $a=5,\frac{-4}{3}$ $b=-2,\frac{15}{2}$
Specific behaviours
✓ multiplies both sides by denominator ✓ obtains two simultaneous eqns ✓ obtains quadratic eqn for one variable ✓obtains two pairs of values

Question 3 **(6 marks)**

Solve for $x, y \& z$ in the following system of linear equations.

$$2x + z = 4$$
$$2x + 3y + 3z = 3$$
$$5x + y + 3z = 10$$

Solution
$\begin{bmatrix} 2 & 0 & 1 & 4 \\ 2 & 3 & 3 & 3 \\ 5 & 1 & 3 & 10 \end{bmatrix}$ $\begin{bmatrix} 2 & 0 & 1 & 4 \\ 0 & -3 & -2 & 1 \\ 0 & -2 & -1 & 0 \end{bmatrix}$ $\begin{bmatrix} 2 & 0 & 1 & 4 \\ 0 & -3 & -2 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ $1y = 1$ $y = 1$ $-2(1) - z = 0$ $z = -2$ $2x + -2 = 4$ $x = 3$ $solution\ (3, 1, -2)$
Specific behaviours
✓ eliminates one variable from one equation ✓ eliminates one variable from two equations ✓ eliminates two variables from one equation ✓solves for one variable ✓ solves for two variables ✓solves all three variables NOTE: No need to use gaussian elimination method

Additional working space

Question number: ________

Additional working space

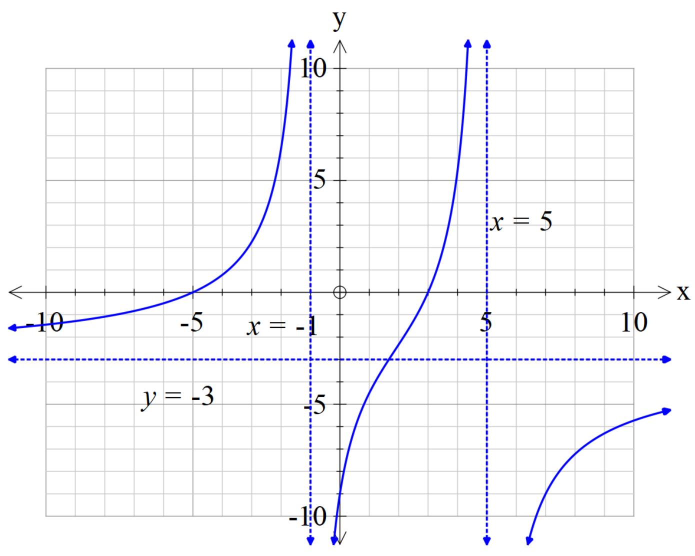
Question number: ________

Question 4 **(6 marks)**

Sketch the following function on the axes below showing all major features.

$$f(x) = \frac{3(3-x)(x+5)}{(x+1)(x-5)}$$

Solution



Specific behaviours

✓ shows dotted lines for both vertical asymptotes
✓ shows dotted line for horizontal asymptote for far left and right
✓ labels equation of all 3 asymptotes
✓shows y intercept
✓ correct shape in 2 sections
✓correct shape in all 3 sections

Question 5 **(9 marks)**

a) Determine all roots of $z^6 = -2 + 2\sqrt{3}i$ in exact polar form with principal arguments. (4 marks)

Solution
$z^6 = 4cis(\frac{2\pi}{3} + 2n\pi) \quad , n = 0, \pm 1, \pm 2...$ $z = 4^{\frac{1}{6}} cis(\frac{2\pi}{18} + \frac{6n\pi}{18})$ $z_1 = 2^{\frac{1}{3}} cis(\frac{\pi}{9})$ $z_2 = 2^{\frac{1}{3}} cis(\frac{4\pi}{9})$ $z_3 = 2^{\frac{1}{3}} cis(\frac{-2\pi}{9})$ $z_4 = 2^{\frac{1}{3}} cis(\frac{7\pi}{9})$ $z_5 = 2^{\frac{1}{3}} cis(\frac{-5\pi}{9})$ $z_6 = 2^{\frac{1}{3}} cis(\frac{-8\pi}{9})$
Specific behaviours
✓ determines modulus(no need to simplify) ✓ uses De Moivre's Theorem to find one argument ✓ determines three principal arguments(no need to simplify) ✓determines all six principal arguments(no need to simplify)

b) Plot all of these roots on the graph below showing all major features. (2 marks)

Solution

Specific behaviours
✓ uses distributive law ✓ negative used when changing order of cross ✓shows parallel

Question 7 **(9 marks)**

A, B & C are 3 distinct points with non-zero position vectors $\underset{\sim}{a}, \underset{\sim}{b} \,\&\, \underset{\sim}{c}$ respectively.

a) If $\underset{\sim}{b} \times \underset{\sim}{c} = \underset{\sim}{a} \times \underset{\sim}{c}$ what can be deduced about OC and AB? Explain. (3 marks)

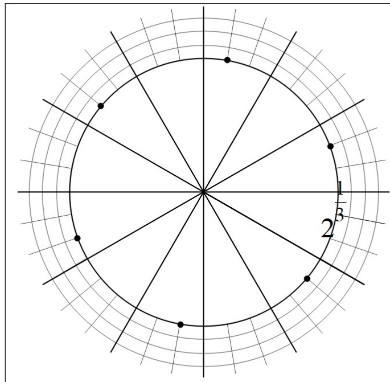
Solution
$\underset{\sim}{b} \times \underset{\sim}{c} = \underset{\sim}{a} \times \underset{\sim}{c}$ $\underset{\sim}{b} \times \underset{\sim}{c} - \underset{\sim}{a} \times \underset{\sim}{c} = \underset{\sim}{0}$ $(b - a) \times c = 0$ $AB \quad parallel \quad c$
Specific behaviours
✓ uses distributive law ✓ shows AB cross c is zero ✓states parallel

b) If $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = 0$ what relationship exists between $\underset{\sim}{a} \times \underset{\sim}{b} \;\&\, \underset{\sim}{b} \times \underset{\sim}{c}$? (3 marks)

Solution
$\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = 0$ $c = -a - b$ $b \times c = b \times (-a - b) = a \times b - b \times b = a \times b$
Specific behaviours
✓ express c in terms of a & b ✓ expands b cross c in terms of a & b ✓shows pairs are equal

c) If $\underset{\sim}{c} \neq 0$ and $\underset{\sim}{b} \times \underset{\sim}{c} = \underset{\sim}{c} \times \underset{\sim}{a}$, prove that $\underset{\sim}{a} + \underset{\sim}{b} = k\underset{\sim}{c}$ for some scalar k. (3 marks)

Solution
$\underset{\sim}{b} \times \underset{\sim}{c} = \underset{\sim}{c} \times \underset{\sim}{a}$ $b \times c - c \times a = 0$ $b \times c + a \times c = 0$ $(b + a) \times c = 0$ $b + a \;\; parallel \; c \therefore \underset{\sim}{a} + \underset{\sim}{b} = k\underset{\sim}{c} \quad k \quad scalar$



Specific behaviours
✓ shows scale ✓ six points equally spaced and positioned correctly

c) If all the consecutive roots are joined by a straight line, determine the exact area of the polygon formed. (3 marks)

Solution
6 equilateral triangles $6\frac{1}{2}(2^{\frac{1}{3}})^2 \sin\left(\frac{2\pi}{6}\right)$ $= \frac{3\sqrt{3}\left(4^{\frac{1}{3}}\right)}{2}$
Specific behaviours
✓ uses equilateral triangles ✓ determines area of one triangle ✓determines simplified exact area in total

Question 6 **(7 marks)**

Determine all solutions of the following equations.

a) $z^4 - 2z^2 + 4 = 0$ and express in exact polar form with principal arguments. (4 marks)

Solution
$z^4 - 2z^2 + 4 = 0$ $z^2 = \frac{2 \pm \sqrt{4 - 4(4)}}{2} = \frac{2 \pm \sqrt{12i^2}}{2} = 1 \pm \sqrt{3}i$ $z^2 = 2cis\left(\frac{\pi}{3}\right) \quad or \quad 2cis\left(-\frac{\pi}{3}\right)$ $z^2 = 2cis\left(\frac{\pi}{3} + 2n\pi\right)$ $z = \sqrt{2}cis\left(\frac{\pi}{6} + n\pi\right)$ $z_1 = \sqrt{2}cis\left(\frac{\pi}{6}\right) \quad z_2 = \sqrt{2}cis\left(-\frac{5\pi}{6}\right)$ $z^2 = 2cis\left(\frac{-\pi}{3} + 2n\pi\right)$ $z = \sqrt{2}cis\left(\frac{-\pi}{6} + n\pi\right)$ $z_3 = \sqrt{2}cis\left(\frac{-\pi}{6}\right) \quad z_4 = \sqrt{2}cis\left(\frac{5\pi}{6}\right)$
Specific behaviours
✓ uses quadratic formula to express z^2 in two polar forms ✓ uses De Moivre's to square root ✓ obtains two roots in polar principle arguments ✓obtains the conjugates of these roots

b) $z^2 + 2z - \sqrt{3}i = 0$ and express in exact cartesian form. (3 marks)

Solution
$z^2 + 2z - \sqrt{3}\,i = 0$ $z = \frac{-2 \pm \sqrt{4 + 4\sqrt{3}i}}{2} = \frac{-2 \pm 2\sqrt{1 + \sqrt{3}i}}{2} = -1 \pm \sqrt{2cis\left(\frac{\pi}{3}\right)} = -1 \pm \sqrt{2}cis\left(\frac{\pi}{6}\right)$ $z = -1 \pm \sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$ $z_1 = -1 + \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i \quad z_2 = -1 - \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$
Specific behaviours
✓ uses quadratic formula ✓ expresses discriminant in polar form ✓obtains two roots in exact cartesian form