SCHOOL

Trial WACE Examination, 2010

Question/Answer Booklet

MATHEMATICS 3C/3D

Section One: Calculator-free

SOLUTIONS

Student Number: In figures

In words ____________________

Your name ____________________

Time allowed for this section

Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	8	8	50	40
Section Two: Calculator-assumed	12	12	100	80
				120

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

Section One: Calculator-free **(40 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the space provided.

Working time for this section is 50 minutes.

Question 1 **(4 marks)**

Determine the equation of the tangent to the curve $y = 1 - \dfrac{9}{2x - 1}$ at the point (2, -2).

$$y = 1 - 9(2x - 1)^{-1}$$

$$\frac{dy}{dx} = 9(2)(2x - 1)^{-2} \Big|_{x=2}$$

$$= \frac{18}{3^2} = 2$$

Hence equation of tangent given by

$$y - (-2) = 2(x - 2)$$

$$y = 2x - 6$$

Question 2 **(4 marks)**

Differentiate the following, without simplifying:

(a) $y = \dfrac{3}{\sqrt{1+e^{5x}}}$ (2 marks)

$$y = 3(1+e^{5x})^{-0.5}$$
$$y' = 3(-0.5)(5e^{5x})(1+e^{5x})^{-1.5}$$

(b) $y = \dfrac{x^3 - 4}{x - 2}$ (2 marks)

$u = x^3 - 4 \qquad v = x - 2$

$u' = 3x^2 \qquad v' = 1$

$$y' = \frac{3x^2(x-2) - 1(x^3 - 4)}{(x-2)^2}$$

Question 3 **(4 marks)**

Determine the domain and range of $f \circ g(x)$, where $f(x) = 2^{x+2}$ and $g(x) = \sqrt{x+1}$.

$f \circ g(x) = f(\sqrt{x+1})$

$= 2^{\sqrt{x+1}+2}$

Domain: $x + 1 \ge 0 \Rightarrow x \ge -1$.

Range: $y \ge 2^2 \Rightarrow y \ge 4$.

Question 4 **(6 marks)**

In a foreign country, a student had a number of \$5, \$2 and \$1 notes with a total value of \$40. The number of \$1 notes was one more than the total number of \$2 and \$5 notes, with a total of 19 notes altogether.

Let x, y and z be the number of \$5, \$2 and \$1 notes respectively.

(a) Write down three equations using the above information. (2 marks)

$$
\boxed{\begin{array}{ll}
5x+2y+z=40 & (1)\\
z=x+y+1 & (2)\\
x+y+z=19 & (3)
\end{array}}
$$

(b) Solve the system of equations in part (a). (4 marks)

$$
\boxed{\begin{array}{lll}
5x+2y+z=40 & (1) & \\
x+y-z=-1 & (2) & \\
x+y+z=19 & (3) & \\
 & & \\
6x+3y=39 & (1)+(2) & \\
2x+y=13 & (4) & \\
 & & \\
2x+2y=18 & (2)+(3) & (5)\\
 & & \\
y=5 & (5)-(4) & \\
x=4 & & \\
z=10 & &
\end{array}}
$$

Question 5 **(5 marks)**

Determine the following integrals:

(a) $\int (6x+9)(3x+x^2)^2\,dx$ (2 marks)

$$= 3\int (3+2x)(3x+x^2)^2\,dx$$
$$= 3\frac{(3x+x^2)^3}{3} + c$$
$$= (3x+x^2)^3 + c$$

(b) $\int_1^4 3\sqrt{x}\,dx$ (3 marks)

$$= \left[\frac{3x^{3/2}}{3/2}\right]_1^4$$
$$= \left[2(x)^{3/2}\right]_1^4$$
$$= 2(4)^{3/2} - 2(1)^{3/2}$$
$$= 16 - 2$$
$$= 14$$

Question 6 **(4 marks)**

The volume, V in cm^3, of an object is changing with time, t in seconds, so that the volume at any time is given by $V = 5t + \frac{12}{t}$. Use the incremental formula to find the approximate change in volume of the object between $t = 2$ and $t = 2.01$ seconds.

For small change, $\partial V = \frac{dV}{dt}\partial t$.

$$\frac{dV}{dt} = 5 - \frac{12}{t^2}$$

$$\begin{aligned}\partial V &= \left(5 - \frac{12}{t^2}\right)\partial t \\ &= \left(5 - \frac{12}{2^2}\right)\times(2.01 - 2) \\ &= 2\times 0.01 \\ &= 0.02 \text{ cm}^3\end{aligned}$$

Question 7 **(6 marks)**

Solve for x the inequality $\frac{1}{2x-1} \geq \frac{2}{x+2}$.

$$\frac{1}{2x-1} \geq \frac{2}{x+2}$$

$$\frac{1}{2x-1} - \frac{2}{x+2} \geq 0$$

$$\frac{x+2-4x+2}{(2x-1)(x+2)} \geq 0$$

$$\frac{4-3x}{(2x-1)(x+2)} \geq 0$$

Critical points when $x = -2$, $\frac{1}{2}$ and $\frac{4}{3}$.

$x < -2$	$-2 < x < \frac{1}{2}$	$\frac{1}{2} < x \leq \frac{4}{3}$	$x \geq \frac{4}{3}$
+ve	-ve	+ve	-ve

Solution:

$x < -2$ or $\frac{1}{2} < x \leq \frac{4}{3}$

Question 8 **(7 marks)**

Determine the coordinates of all roots, stationary points and points of inflection of the function $y = x^3(4 + x)$. Justify the nature of the stationary points found using a standard test.

Roots:
$x^3 = 0$ or $4 + x = 0$.
Hence roots at (0, 0) and (-4, 0).

Stationary points:
$y = 4x^3 + x^4$
$\frac{dy}{dx} = 12x^2 - 4x^3$
$= 4x^2(3 - x)$
$\therefore x^2 = 0$ or $3 - x = 0$
Hence stationary points at (0, 0) and (-3, -27).

Nature:
$\frac{d^2y}{dx^2} = 24x + 12x^2$
$= 12x(2 + x)$
$y''(0) = 0 \Rightarrow$ Horizontal pt of inflection (as $y'(0) = 0$)
$y''(-3) = +\text{ve} \Rightarrow$ Minimum
Hence horizontal point of inflection at (0, 0) and minimum at (-3, -27).

Points of inflection:
$12x(2 + x) = 0$
when $x = 0$ or $x = -2$.
(0, 0) already stated above, another point of inflection at (-2, -16).

End of questions

Additional working space

Question number(s):_________

Additional working space

Question number(s):_________