

SOLUTIONS

MATHEMATICS
METHODS
UNITS 1 AND 2
Section One:
Calculator-free

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Student Number: In figures

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total				150	100

Instructions to candidates

- The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Additional working space

Question number: _____

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

CALCULATOR-FREE

Question 1

(a) The table shows the temperature of a liquid over a period of time.

Time (minutes)	0	5	10	15	20	25
Temperature (°C)	58	44	32	25	21	19

Determine the average rate of change of temperature of the liquid

(i) _____ over the first ten minutes. (1 mark)

$$\frac{32-58}{10-0}=-\frac{26}{10}=-2.6\text{ }^{\circ}\text{C/min}$$

(iii) _____ between 15 and 20 minutes. (1 mark)

$$\frac{21-25}{20-15}=-\frac{4}{5}=-0.8\text{ }^{\circ}\text{C/min}$$

(b) Determine the equation of the tangent to the curve $y = x^3 - 2x^2 + x + 2$ when $x = 2$. (4 marks)

$$\begin{aligned}x^3 - 2x^2 + x + 2 \Big|_{x=2} &= 4 \\ \frac{dy}{dx} &= 3x^2 - 4x + 1 \Big|_{x=2} = 5 \\ y - 4 &= 5(x - 2) \\ y &= 5x - 6\end{aligned}$$

See next page

CALCULATOR-FREE

Question 8

(a) Calculate the gradient of $y = x^2 - 3x - 10$ at the points where $y = 8$. (3 marks)

(7 marks)

$$\begin{aligned}x^2 - 3x - 10 &= 8 \\ x^2 - 3x - 18 &= 0 \\ (x + 3)(x - 6) &= 0 \\ \frac{dy}{dx} &= -3 \Rightarrow x = -9 \\ \frac{dy}{dx} &= 6 \Rightarrow x = 9\end{aligned}$$

(b) The function $f(x) = \frac{x^2}{2}(x - 6)$ has a local minimum at (p, q) , where $p > 0$. Determine the values of p and q . (4 marks)

$$\begin{aligned}f(x) &= \frac{x^3}{2} - 3x^2 \\ f'(x) &= \frac{3x^2}{2} - 6x \\ 0 &= \left(\frac{3x}{2} - 6\right)x \Rightarrow x \leq 0, x = 4 \\ d &= 4 \\ b &= \frac{2}{4^2}(4 - 6) = -16\end{aligned}$$

End of questions

Question 2

(7 marks)

(a) The vertices of three points are A(1, 1), B(-1, 2) and C(-2, -1).

(i) Use gradients to explain whether the lines AB and BC are perpendicular. (2 marks)

$$m_{AB} = -\frac{1}{2} \quad m_{BC} = 3 \quad -\frac{1}{2} \times 3 = -\frac{3}{2}$$

No, since perpendicular gradients have a product of -1.

(ii) Determine the equation of the line through A that is parallel to the line BC. (1 mark)

$$\begin{aligned} y &= 3x + c \\ 1 &= 3(1) + c \\ c &= -2 \end{aligned} \quad \therefore y = 3x - 2$$

(iii) If B is the mid-point of A and D, determine the coordinates of D. (2 marks)

$$D(-1-2, 2+1)$$

$$D(-3, 3)$$

(b) Solve $\frac{x-3}{3} - 3x = 4$.

(2 marks)

$$\begin{aligned} x - 3 - 9x &= 12 \\ -8x &= 15 \\ x &= -\frac{15}{8} \end{aligned}$$

See next page

Question 7

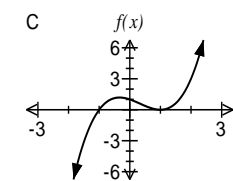
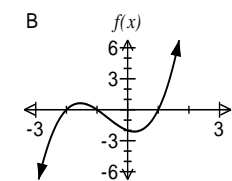
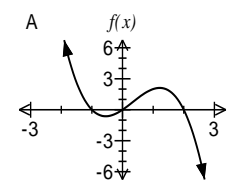
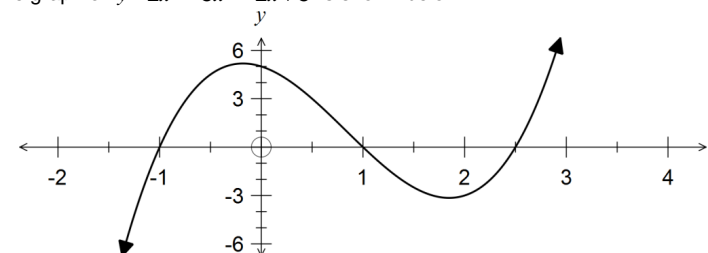
(8 marks)

(a) If $(x-2)(x+2)(x+3) = ax^3 + bx^2 + cx + d$, determine the value of c . (2 marks)

$$(x^2 - 4)(x + 3) = \dots - 4x - \dots \Rightarrow c = -4$$

(b) Match each function in the table below with its graph. (2 marks)

Function	Graph (A, B or C)
$f(x) = (x+1)(x-1)^2$	C
$f(x) = x(1+x)(2-x)$	A
$f(x) = (x-1)(x+1)(x+2)$	B

(c) The graph of $y = 2x^3 - 5x^2 - 2x + 5$ is shown below.(i) Solve $2x^3 - 2x = 5x^2 - 5$. (2 marks)

$$\begin{aligned} 2x^3 - 5x^2 - 2x + 5 &= 0 \\ x &= -1, x = 1, x = 2.5 \end{aligned}$$

(ii) Factorise $2x^3 - 5x^2 - 2x + 5 = 0$. (2 marks)

$$2(x+1)(x-1)(x-2.5) = (x+1)(x-1)(2x-5)$$

See next page

(5 marks)

(1 mark)

Question 3

Solve the following equations.

(a) $x(x+2)(2x-3)=0$.

$$x=0, x=-2, x=\frac{3}{2}$$

(2 marks)

(b) $x^2+4x+6=2x^2+5x-6$.

$$\begin{aligned} 0 &= x^2 + x - 12 \\ (x+4)(x-3) &= 0 \\ x &= -4, x = 3 \end{aligned}$$

(2 marks)

(c) $2(x-2)^2=100$.

$$\begin{aligned} (x-2)^2 &= 50 \\ x-2 &= \pm 5\sqrt{2} \\ x &= 2+5\sqrt{2}, x=2-5\sqrt{2} \end{aligned}$$

See next page

(5 marks)

(1 mark)

(a) Determine the exact value of $\sin 210^\circ$.

$$-\frac{1}{2}$$

(2 marks)

(b) Solve the equation $\cos\left(\frac{1}{2}x\right)=\frac{\sqrt{3}}{2}$ for $-\pi \leq x \leq \pi$.

$$\begin{aligned} \frac{x}{2} &= \pm \frac{6}{\pi} \\ x &= \pm \frac{3}{\pi} \end{aligned}$$

(2 marks)

(c) Expand $(n-1)^4$.

$$\begin{aligned} (n-1)^4 &= n^4 + 4n^3(-1) + 6n^2(-1)^2 + 4n(-1)^3 + (-1)^4 \\ &= n^4 - 4n^3 + 6n^2 - 4n + 1 \end{aligned}$$

See next page

Question 4

(7 marks)

- (a) Determine $\frac{dy}{dx}$ in simplified form if

(i) $y = 2x^3 - x + 3.$

(1 mark)

$$\frac{dy}{dx} = 6x^2 - 1$$

(ii) $y = \frac{5x^3}{6} - \frac{x^4}{12}.$

(1 mark)

$$\frac{dy}{dx} = \frac{5x^2}{2} - \frac{x^3}{3}$$

- (b) Determine the coordinates of the point on the curve $y = 3x^2 - 7x - 10$ where the gradient is 5.

(2 marks)

$$\begin{aligned}\frac{dy}{dx} &= 6x - 7 \\ 6x - 7 &= 5 \Rightarrow x = 2 \\ y &= 3(4) - 7(2) - 10 = -12 \\ \text{At } (2, -12)\end{aligned}$$

- (c) Determine $f(x)$ given that $f'(x) = 5 + 2x - 6x^2$ and $f(1) = 0.$

(3 marks)

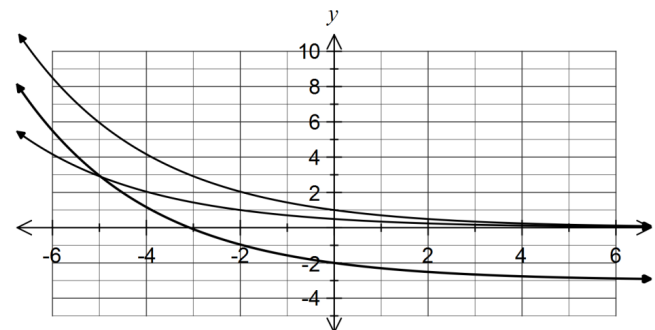
$$\begin{aligned}f(x) &= 5x + x^2 - 2x^3 + c \\ c &= 0 - 5 - 1^2 + 2 = -4 \\ f(x) &= 5x + x^2 - 2x^3 - 4\end{aligned}$$

See next page

Question 5

(7 marks)

- (a) The graph of $y = a^x$ is shown below.



On the same axes, sketch the graphs of

(i) $y = a^{x+2}.$

(1 mark)

(ii) $y = a^x - 3.$

(1 mark)

- (b) Evaluate $(3.6 \times 10^{-3}) \div (1.2 \times 10^{-4}).$

(1 mark)

$$\frac{3.6}{1.2} \times \frac{10^{-3}}{10^{-4}} = 3 \times 10 = 30$$

- (c) Solve for x :

(i) $27^{2x-1} = 81.$

(2 marks)

$$\begin{aligned}3^{3(2x-1)} &= 3^4 \\ 6x - 3 &= 4 \\ x &= \frac{7}{6}\end{aligned}$$

(ii) $x^{-2} = 6\frac{1}{4}.$

(2 marks)

$$\frac{1}{x^2} = \frac{25}{4} \Rightarrow x^2 = \frac{4}{25} \Rightarrow x = \pm \frac{2}{5}$$

See next page