



PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2019

Question/Answer booklet

Yr 12 SPECIALIST UNIT 3

Section Two: Calculator-assumed

Your Name _____

Your Teacher's Name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
8		6	15		8
9		6	16		4
10		8	17		4
11		5	18		9
12		5	19		13
13		6	20		9
14		10			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	49	34.5
Section Two: Calculator-assumed	13	13	100	93	65.5
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed**(93 Marks)**

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

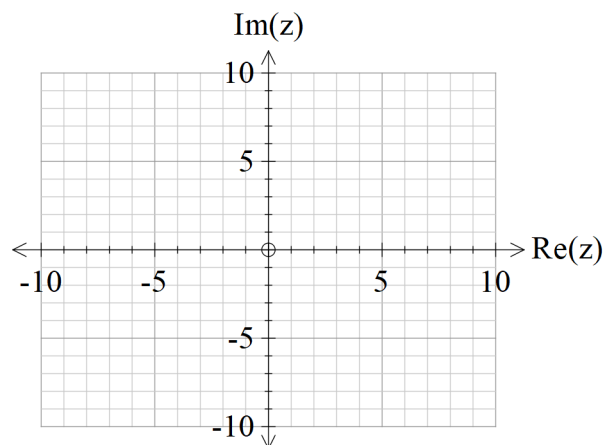
Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8**(6 marks)**

- a) Sketch the following region in the complex plane, $|z - 2 + i| \geq |z - 7 - 4i|$ (3 marks)



- b) Determine the cartesian equation of $|z - 2 + i| = |z - 7 - 4i|$ (3 marks)

Question 9**(6 marks)**

- a) Given that $|x - 4| - 5 \leq |x + a|$, where a is a constant, is only true for $1 \leq x \leq 6$, determine the value of a .

(3 marks)

- b) Given that $|2x + 6| = a|x + b| + c$, where a, b & c are constants, is only true for $-3 \leq x \leq 2$, determine the values of a, b & c .

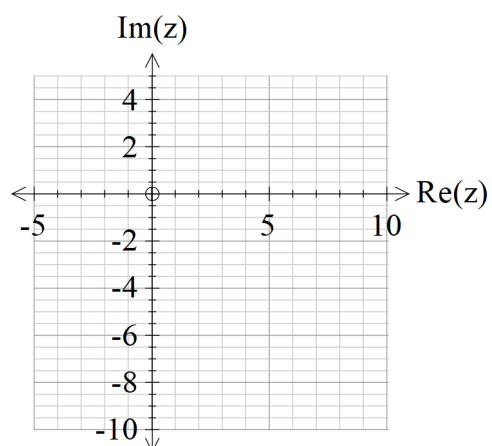
(3 marks)

Question 10**(8 marks)**

Consider the locus of points on $|z - 5 + 3i| = 2$ in the complex plane.

a) Sketch this locus below.

(2 marks)



b) Determine the minimum principal $\text{Arg}(z)$ on this locus to one decimal place.

(3 marks)

c) Determine the maximum value of $|z + 6i|$ on this locus.

(3 marks)

Question 11**(5 marks)**

Show that the line $x = 2 + 2t, y = -1 + 3t, z = -\frac{5}{2}t$ is parallel to the plane $10x - 5y + 2z = 0$ and determine its distance from the plane.

Question 12**(5 marks)**

Let $z = rcis\theta$, where $0 < \theta < \frac{\pi}{2}$, consider the sum $z + 5cis\frac{\pi}{6}$.

(a) Sketch a diagram of this sum in the complex plane.

(2 marks)

(b) Obtain an expression for the $\left| z + 5cis\frac{\pi}{6} \right|$ in terms of r & θ .

(3 marks)

Question 13**(6 marks)**

$$p = \begin{pmatrix} -2 \\ 11 \\ 7 \end{pmatrix} \quad \text{and} \quad q = \begin{pmatrix} -6 \\ 15 \\ 9 \end{pmatrix}$$

- a) Determine a vector that is perpendicular to both p and q and has a magnitude of $3(|p| - |q|)$. (Do not simplify) (3 marks)

$$a = \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 3 \\ m \\ 3 \end{pmatrix}$$

- b) Let a and b where m is a real constant. In terms of m , determine an expression for the angle between a & b . (3 marks)
- c)

Question 14**(10 marks)**

$$r = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -9 \\ 13 \end{pmatrix} + \lambda \begin{pmatrix} 10 \\ 3 \\ -7 \end{pmatrix}.$$

Consider a plane defined by

a) Determine a normal vector to this plane.

(1 mark)

b) Determine the cartesian equation of this plane.

(2 marks)

c) Show how to determine the distance of point $P(-1, 3, 4)$ from the plane above **using** scalar dot product and the normal vector. (4 marks)

d) Consider a general plane $Ax + By + Cz + D = 0$, where A, B, C & D are constants.

Show that the distance of point $Q(x_1, y_1, z_1)$ from this plane is given by the expression

$$\frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

(3 marks)

e)

Question 15**(8 marks)**

In deep space an astronaut is space walking outside a stationary space station. At time $t = 0$ seconds the astronaut is positioned at $(22, 10, -7)$ metres relative to the space station and is

moving with a velocity of $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$ metres per second. A rogue satellite is observed to be at

position $(33, -44, 9)$ at time $t = 0$ with a velocity of $\begin{pmatrix} 8 \\ -5 \\ 3 \end{pmatrix}$ metres per second relative to the space station.

The satellite emits radiation and if the astronaut comes within 70 metres of the satellite the dosage will be harmful.

a) Determine the distance between the astronaut and satellite at $t = 3$ seconds. (3 marks)

b) Determine if the astronaut is in danger and if so for how long in seconds to two decimal places. (Justify your answer). (5 marks)

Question 16**(4 marks)**

Consider the two spheres $\left| r - \begin{pmatrix} 9 \\ -1 \\ 3 \end{pmatrix} \right| = \sqrt{21}$ and $x^2 + y^2 + z^2 + 8x - 2y + 2z = 66$.
Determine whether there are any common points on both spheres. Justify your answer.

Question 17**(4 marks)**

Show **using vector cross product**, how to determine the distance of point $A(11, -33, 7)$ from

the line $r = \begin{pmatrix} 17 \\ -11 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -8 \\ 2 \end{pmatrix}$.

Question 18**(9 marks)**

$$r = \begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} \quad \text{and the sphere} \quad \left| r - \begin{pmatrix} 2 \\ -9 \\ 7 \end{pmatrix} \right| = \alpha$$

Consider the line and the sphere where α is a constant.

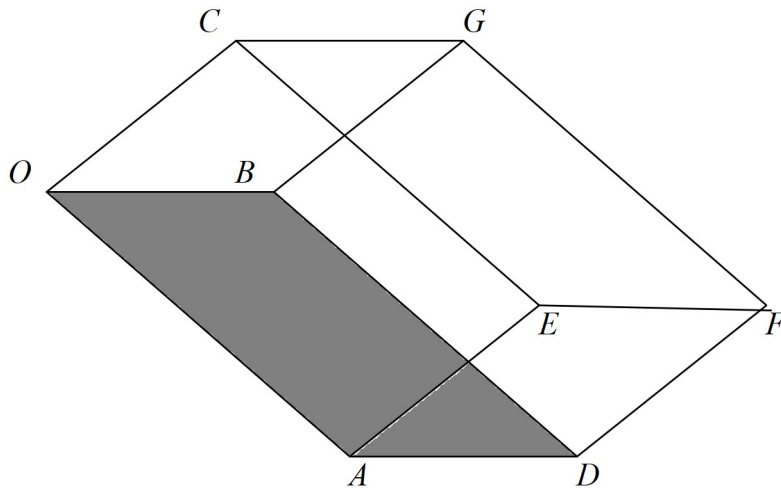
Determine the values of α , to two decimal places, for each of the following scenarios:

(Justify your answers)

- i) the line does not meet the sphere at all.
- ii) the line meets the sphere at two points.
- iii) the line is a tangent to the sphere.

Question 19**(13 marks)**

Consider a prism where opposite sides are congruent parallelograms(parallelepiped) with coordinates $O(0,0,0)$ $A(-4,1,5)$ $B(7,2,-8)$ $C(11,-5,1)$.



- a) Determine a unit normal vector to the base $OADB$. (3 marks)
- b) Using this unit normal, determine the distance between the faces $OADB$ & $CEFG$.
(Hint-use vector OC) (3 marks)
- c) Show using cross product how to determine the area of the base $OADB$. (2 marks)
- d) Hence or otherwise, determine the volume of the prism. (3 marks)
- e) In terms of the vectors OA, OB & OC write an expression using cross and dot products to represent the volume of the prism. (2 marks)

Question 20

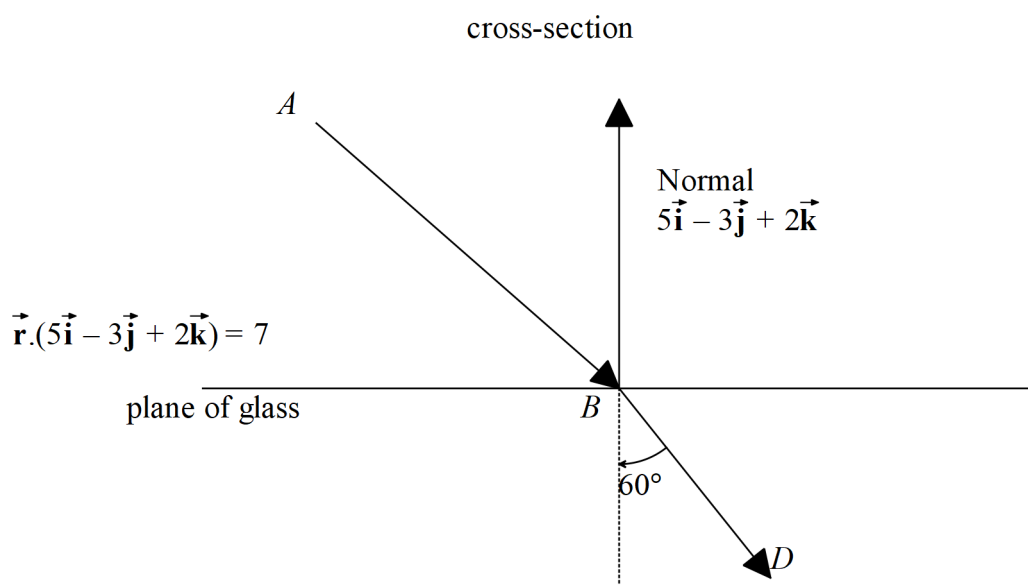
(9 marks)

Consider a single photon of light that is released from a box positioned at point A $(-2, 3, 7)$ and

moves in a direction of $\begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix}$ hitting a planar sheet of glass at point B. The planar sheet of glass

is given by $\mathbf{r} \cdot \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} = 7$. The photon is refracted, that is changes direction, through the glass such that the angle with the perpendicular is 60° and passes through point D.

It is given that the vectors $\overrightarrow{AB}, \overrightarrow{BD}$ and the normal are all in the same plane.



a) Determine the point B.

(3 marks)

Assume that \overrightarrow{BD} is a unit vector and be represented as $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ with $a^2 + b^2 + c^2 = 1^2$

b) Determine two other independent equations for a, b & c .

(4 marks)

Question continued

- c) Hence solve for vector BD using your CAS calculator (simultaneous) to 2 decimal places.

(2 marks)

Additional working space

Question number: _____

Additional working space

Question number: _____

