PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Specialist TEST 1 Friday 8 February 2019 TIME: 45 minutes working **No Classpads nor calculators allowed!** 37 marks 8 Questions

Name: Marking Key

Teacher: ____________________

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (1 & 2 = 3 marks)
Express each of the following in the form $a+bi$ where $a\&b$ are real numbers.

a) $(3-4i)(5i)$ $20+15i$ ✓

b) $\frac{2-3i}{5+i}$ $\frac{5-i}{5-i}$ ✓ $= \frac{7-17i}{26}$ ✓ f/t

Q2 (3 marks)
Determine the remainder when $3x^2-5x+7$ is divided by $(x+3-2i)$

$= 3(-3+2i)^2 - 5(-3+2i) + 7$ ✓

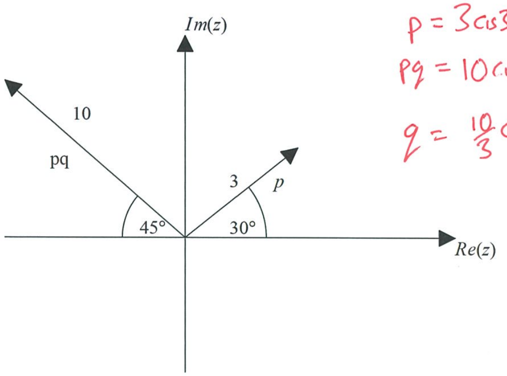
$= 3(9-4-12i) + 15 - 10i + 7$ ✓

$= 3(5-12i) + 15 - 10i + 7$

$= 15 - 36i + 15 - 10i + 7$

$= 37 - 46i$ ✓ f/t

Q3 (3 marks)
Determine the complex number q in polar form.



$p = 3\operatorname{cis}30°$ ✓

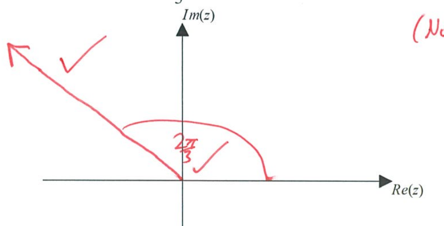
$pq = 10\operatorname{cis}(135°)$ ✓

$q = \frac{10}{3}\operatorname{cis}(105°)$ ✓

Must show reasoning for full marks otherwise 2 marks

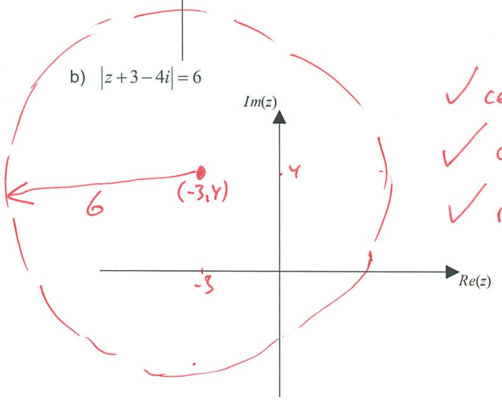
Q4 (2 & 3 = 5 marks)
Sketch the following in the complex plane showing all major features.

a) $\arg(z) = \dfrac{2\pi}{3}$



(Note: origin maybe open or close)

b) $|z+3-4i| = 6$



✓ centre (-3,4)

✓ circle

✓ radius of 6 units

Q5 (2, 3 & 3 = 8 marks)

If $z = a + ib$ and $w = p + iq$ where $a, b, p \& q$ are real numbers, show the following:

a) $\overline{z+w} = \overline{z} + \overline{w}$

$LHS = \overline{a+ib + p+iq}$

$= \overline{(a+p) + i(b+q)}$ ✓

$= (a+p) - i(b+q)$ ✓

$= a - ib + p - iq$ ✓

$= \overline{z} + \overline{w} = RHS$

b) $\overline{zw} = \overline{z}\,\overline{w}$

$LHS = \overline{(a+ib)(p+iq)}$

$= \overline{(ap-bq) + i(bp+aq)}$ ✓

$= (ap-bq) - i(bp+aq)$.

$LHS = RHS$

$RHS = \overline{(a+ib)}\ \overline{(p+iq)}$

$= (a-ib)(p-iq)$ ✓

$= ap - bq - i(bp+aq)$ ✓

c) Hence or otherwise show that if there is a complex root to the quadratic equation $ax^2 + bx + c = 0$ with real coefficients, then the conjugate is also a root.
(Hint: Take the conjugate of both sides of the quadratic equation)

$\overline{ax^2 + bx + c} = \overline{0}$

$= \overline{axx + bx + c}$

$= \overline{axx} + \overline{bx} + \overline{c} = 0$ ✓

$= a\overline{x}\,\overline{x} + b\overline{x} + c = 0$ ✓ as a, b, c are real

$= a(\overline{x})^2 + b(\overline{x}) + c = 0$ ✓

hence $\overline{x}$ is a root of quadratic.

OR

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, if complex $b^2 - 4ac = -n^2$ ✓ $n \in \mathbb{R}$

$x = \dfrac{-b \pm \sqrt{-n^2}}{2a} = \dfrac{-b \pm in}{2a}$ ✓ $x_1 = \dfrac{-b+in}{2a}$ ✓ $x_2 = \dfrac{-b-in}{2a}$

conjugates

Q6 (4 marks)
Consider the set of complex numbers $z = x + iy$ that satisfy the following equation:
$|z+1-i| = |z-3-7i|$.
Determine the cartesian equation, in terms of x & y, of these numbers.

$(3,7)$

$(-1,1)$

midpoint $\left(\frac{3+-1}{2}, \frac{1+7}{2}\right) \Rightarrow (1,4)$ ✓

gradient $= \frac{7-1}{3--1} = \frac{3}{2}$ ✓

$\perp$ gradient $= -\frac{2}{3}$ ✓

$y = -\frac{2}{3}x + C$

$4 = -\frac{2}{3} + C$

$C = \frac{14}{3}$

$y = -\frac{2}{3}x + \frac{14}{3}$ ✓

ft

Q7 (2 & 4 = 6 marks)
Consider the function $f(z) = az^3 + bz^2 + cz + d$ where a, b, c & d are real constants.
It is known that $(z-1)$ is a factor and when $f(z)$ is divided by $(z-1)$ there is a remainder of -32. Also $f(0) = -18$ & $f(3i) = 0$.

a) Determine all three factors of $f(z)$.

$(z-1)(z-3i)(z+3i)$ ✓ ✓

b) Determine the values of a, b, c & d.

$f(z) = a(z-1)(z^2+9)$ ✓

$-18 = -9a$

$\therefore a = 2$ ✓

$f(z) = 2(z-1)(z^2+9)$

$= 2(z^3 + 9z - z^2 - 9)$

$= 2z^3 - 2z^2 + 18z - 18$ ✓

$\left.\begin{aligned} a &= 2 \\ b &= -2 \\ c &= 18 \\ d &= -18 \end{aligned}\right\}$ ✓

Q8 (4 & 1 = 5 marks)
Consider the set of complex numbers, z, that satisfy the following:
$|z - 2\sqrt{2} - 2\sqrt{2}i| \le c$, $c \ge 0$ and real, and $0 < Arg(z) < \frac{\pi}{2}$.

Determine:

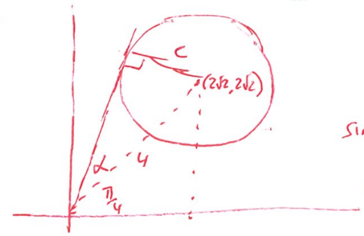
a) The value of c given that the Maximum value of $Arg(z) = \frac{5\pi}{12}$.

✓ diagram

$\alpha = \frac{5\pi}{12} - \frac{\pi}{4}$

$= \frac{5\pi}{12} - \frac{3\pi}{12}$

$= \frac{\pi}{6}$ ✓



$\sin\frac{\pi}{6} = \frac{c}{4}$ ✓

$c = 2$ ✓

b) Maximum value of $|z|$.

$|z| = 4 + 2$

$= 6$ ✓