Christ Church Grammar School

2016
UNIT TEST 2

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Student name Solution

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

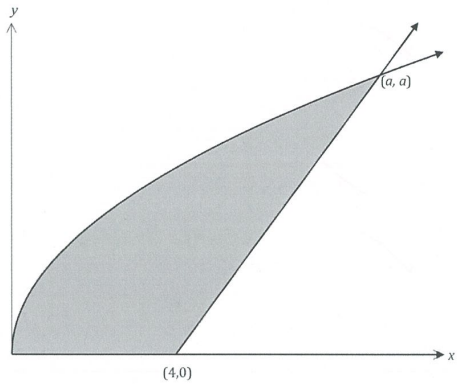
3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 8 **(6 marks)**

The diagram below shows an area bounded by the x-axis, the function $y = \sqrt{ax}$ and the function $ax - (a-4)y = 4a$.



(a) Write an expression involving integrals to calculate this shaded area. (3 marks)

$\text{Area} = \int_0^a \sqrt{ax}\,dx - \int_4^a \frac{ax-4a}{a-4}\,dx$ ✓ ✓ ✓

(b) Given that the shaded area is 31.5 square units, determine the value of a. (3 marks)

$\int_0^a \sqrt{ax}\,dx - \int_4^a \frac{ax-4a}{a-4}\,dx = 31.5$ ✓

$a = 9$ ✓ ✓

End of questions

Question 1 **(4 marks)**

Evaluate each of the following integrals:

(a) $\int (1+5x)^3\,dx$ (1 mark)

$= \frac{1}{20}(1+5x)^4 + c$ ✓

(b) $\int xe^{x^2-1}\,dx$ (1 mark)

$= \frac{1}{2}e^{x^2-1} + c$ ✓

(c) $\int \cos 2x \sin^3 2x\,dx$ (2 marks)

$= \frac{1}{8}\sin^4 2x + c$ ✓ ✓

See next page

Question 2 **(3 marks)**

It is given that $f(x) = x\cos x$ and $f'(x) = \cos x - x\sin x$.

Use the above facts to find $\int x\sin x\,dx$.

$f'(x) = \cos x - x\sin x$

$x\sin x = \cos x - f'(x)$

$\int x\sin x\,dx = \int \cos x\,dx - f(x)$ ✓

$= \sin x - x\cos x + c$

✓ ✓

Question 7 continued

(c) $\int_0^2 2f(-x)dx$ (2 marks)

$= 2(-5 - 1)$ ✓

$= -12$ ✓

(d) $\int_{-1}^{1} f(x-1)dx$ (2 marks)

$= -5 - 1$ ✓

$= -6$ ✓

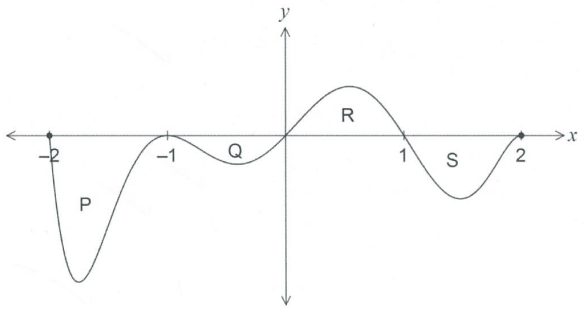
(e) $\int_0^1 \sin \pi x - f(x)dx$ (2 marks)

$= \int_0^1 \sin \pi x\,dx - \int_0^1 f(x)\,dx$ ✓

$= \frac{2}{\pi} - 2$ ✓

Question 7 **(10 marks)**

The graph of the function $y = f(x)$ is shown below over the domain $-2 \leq x \leq 2$.



The area of the regions P, Q, R and S enclosed by the curve and the x-axis are 5, 1, 2, and 3 squared units respectively.

Determine

(a) the area enclosed by the curve and the x-axis for $-1 \leq x \leq 1$. (2 marks)

Area = 1 + 2 ✓
= 3 units² ✓

(b) $\int_{-2}^{2} f(x)dx$ (2 marks)

= −5 − 1 + 2 − 3 ✓
= −7 ✓

Question 3 **(4 marks)**

The area of the region bounded by the curve with equation $y = kx^{\frac{1}{2}}$, where k is a positive constant, the x-axis and the line with equation $x = 9$ is 27 $units^2$. Find the value of the constant k.

$\int_0^9 k x^{\frac{1}{2}} dx = 27$ ✓

$\left[k \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^9 = 27$ ✓

$\frac{2}{3} k (9)^{\frac{3}{2}} = 27$

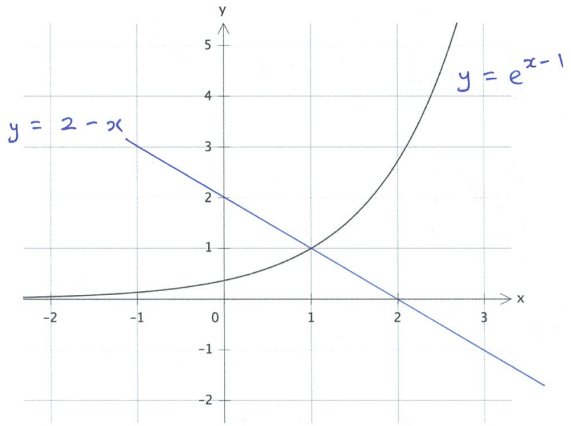
$\frac{2}{3} k (27)$ ✓ $= 27$

$k = \frac{3}{2}$ ✓

Question 4 **(4 marks)**

The graph of $y = e^{x-1}$ is shown in the diagram below.

Calculate the exact area between the graphs of $y = e^{x-1}$, $y = 2 - x$ and the two axes.



Area $= \int_0^1 e^{x-1}\,dx + \int_1^2 2 - x\,dx$ ✓

$= \left[e^{x-1}\right]_0^1$ ✓ $+ \left[2x - \frac{x^2}{2}\right]_1^2$ ✓

$= 1 - \frac{1}{e} + 2 - \frac{3}{2}$

$= \frac{3}{2} - \frac{1}{e}$ ✓

End of questions

Question 6 continued

(c) What was the initial speed of the particle? (2 marks)

$v(0) = -2$ ✓

initial speed $= 2$ m/s ✓

(d) Find the value(s) of t when the particle comes to rest, and the distance(s) from the origin at that times(s). (3 marks)

$6t^2 - 10t - 2 = 0$ ✓

$t = 1.85$s or -0.18 (reject) ✓

$x = -7.15$ m

$\therefore$ distance $= 7.15$ m ✓

(e) Calculate the acceleration of the particle when the velocity is $6\ m/s$. (3 marks)

$6t^2 - 10t - 2 = 6$ ✓

$t = 2.26$ s ✓

$\frac{dv}{dt} = 17.12$ m/s^2 ✓

See next page

Question 6 **(11 marks)**

A particle moves in rectilinear motion, such that its displacement (x) from the origin O, at any time t (seconds), is given as $x = t(2t^2 - 5t + b) + 1$ metres.

It is known that at $t = 3$, the particle is $4\ m$ to the right of the origin.

(a) Determine the value of b. (1 mark)

$4 = 3(2(3)^2 - 5(3) + b) + 1$

$b = -2$ ✓

(b) Determine the velocity of the particle when $t = 3$. (2 marks)

$x = t(2t^2 - 5t - 2) + 1$

$\frac{dx}{dt} = 6t^2 - 10t - 2$ ✓

$\left.\frac{dx}{dt}\right|_{t=3} = 22\ m/s$ ✓

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2016
UNIT TEST 2

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Student name Solution

Teacher name _____

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Working time for this section:	30 minutes
Marks available:	30 marks

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Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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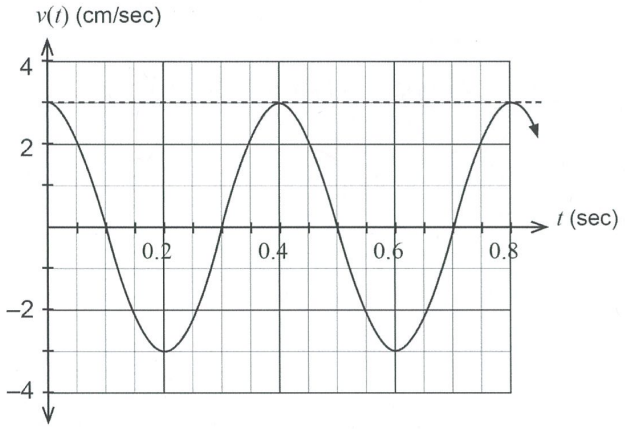
Question 5 **(3 marks)**

The velocity function of a particle is given by $v(t) = 3\cos 5\pi t$ cm/sec.



Find the distance travelled by the particle from time $t = 0$ to 0.8 sec.

exact

distance $= \int_0^{0.8} |3\cos 5\pi t|\, dt$ ✓ ✓ (method)

$= \frac{24}{5\pi}$ units ✓

See next page