

Western Australian Certificate of Education

Examination, 2015

Question/Answer Booklet

MATHEMATICS

3C/3D

Section Two:

Calculator-assumed

Place one of your candidate identification labels in this box.

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Ensure the label is straight and within the lines of this box.

Student Number:

In figures

In words

Time allowed for this section

Reading time before commencing work:

Working time for section:

ten minutes

one hundred minutes

Number of additional answer booklets used (if applicable):

**Materials required/recommended for this section**  
*To be provided by the supervisor*  
This Question/Answer Booklet  
Formula Sheet (retained from Section One)

**To be provided by the candidate**  
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters  
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

**Important note to candidates**  
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Ref: 15-082  
MAT3CD-S2



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33⅓
Section Two: Calculator-assumed	13	13	100	100	66⅔
Total					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

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Additional working space

Question number: \_\_\_\_\_

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MATHEMATICS 3C/3D	18	CALCULATOR-ASSUMED	<div> <div>DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF</div> <div> </div> </div>
MATHEMATICS 3C/3D 66⅔% (100 Marks) This section has 13 questions. Answer all questions. Write your answers in the spaces provided. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. • Planning: If you use the spare pages for planning, indicate this clearly at the top of the page. • Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page. Working time: 100 minutes.	3	CALCULATOR-ASSUMED	<div> <div>DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF</div> <div> <p>Question 9 (6 marks)</p> <p>For any two numbers <math>a &gt; b &gt; 0</math>, it is conjectured that <math>\sqrt{a} - \sqrt{b} &lt; \sqrt{a - b}</math>.</p> <p>(a) Provide two pairs of numbers to demonstrate that the conjecture appears to be true. (2 marks)</p> <p>(b) If <math>a - b = c</math> where <math>c &gt; 0</math>, show that the conjecture is equivalent to <math>\sqrt{b + c} &lt; \sqrt{b} + \sqrt{c}</math>. (1 mark)</p> <p>(c) Prove algebraically that the conjecture in part (b) is true for all positive numbers <math>b</math> and <math>c</math>. (3 marks)</p> <p>See next page</p> </div> </div>

**Question 10** (10 marks)

The events  $A$  and  $B$  have probabilities  $P(A) = 0.3$ ,  $P(\bar{B} | \bar{A}) = 0.2$  and  $P(B | A) = 0.4$ .

(a) Show that  $P(A \cap B) = 0.12$ . (1 mark)

(b) Show that  $P(A \cup B) = 0.86$ . (3 marks)

(c) Determine  $P(B)$ . (2 marks)

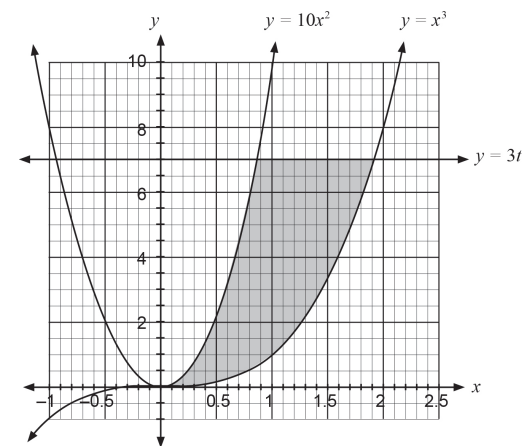
(d) Determine  $P(A | B)$ . (2 marks)

(e) Are events  $A$  and  $B$  independent? Justify your answer. (2 marks)

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The line  $y = 5$  is replaced with the line  $y = 3t$  where  $0 \leq t \leq 3$ , as can be seen in the diagram below for a particular value of  $t$ . The area enclosed is revolved around the  $y$  axis, forming a solid revolution.



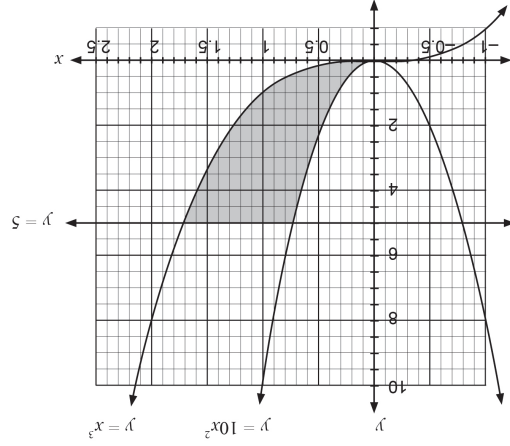
(b) Derive an expression for the volume,  $V$ , of the solid of revolution as a function of  $t$  (may be left as an integral). (2 marks)

(c) Determine  $\frac{dV}{dt}$  when  $t = 2$ . (2 marks)

End of questions

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- (a) Determine the volume of the solid generated when the shaded area enclosed by the curves and line  $y = 10x^2$ ,  $y = x^3$ , and  $y = 5$  (see below) is revolved around the  $y$ -axis. (4 marks)



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The points  $P(-2, 1)$  and  $Q(6, 9)$  lie on the parabola  $y = \frac{x^2}{4}$ .

- (a) Find the equations of the tangents to the parabola at  $P$  and  $Q$ . (2 marks)

- (b) The tangents to the parabola at  $P$  and  $Q$  meet at point  $R$ . Find the coordinates of  $R$ . (2 marks)

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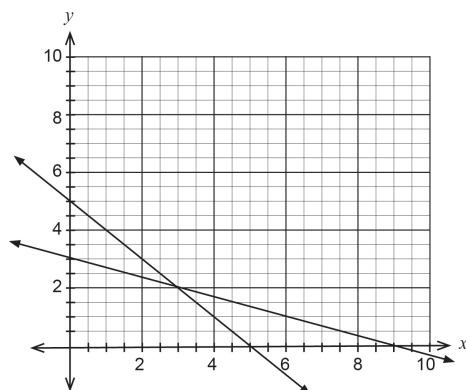
**Question 12** (12 marks)

A farmer needs to buy two types of fertiliser, 'A' and 'B', to meet the needs of the next season of crops. Under a marketing deal with his supplier, the farmer must buy a total of at least five kilograms of fertiliser each season. Both of the fertilisers contain small amounts of poison to deter a particular type of bug. The farmer knows that at least 36 micrograms in total of this poison is needed to deter this bug from the farm.

Each kilogram of Type A fertiliser contains 12 micrograms of this poison, while each kilogram of Type B contains six micrograms. The farmer needs to plant crops over at least 90 hectares to ensure a viable crop. It is known that each kilogram of Type A can cover 10 hectares, while each kilogram of Type B can cover 30 hectares.

Let  $x$  = the number of kilograms of Type A fertiliser  
and  $y$  = the number of kilograms of Type B fertiliser.

- (a) Two of the three above-mentioned constraints have been drawn on the axes below. Write down the missing constraint in terms of  $x$  and  $y$ . (Note :  $x \geq 0, y \geq 0$ .) (2 marks)



- (b) Draw in the missing constraint on the axes above, and then shade the feasible region that satisfies the constraints. (3 marks)

See next page

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- (ii) What would you advise the engineer regarding the suitability of the cables for the crane? Justify your answer. (2 marks)

- (iii) The engineer wants to obtain a 99% confidence interval no wider than 20 tonnes for the population mean strength of the cables. What sample size should she take? (3 marks)

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Question 20 (15 marks)

The strength of steel cables produced by a manufacturer is normally distributed, with a specified mean of 1000 tonnes and a standard deviation of 100 tonnes.

- (a) Of 50 steel cables, how many would be expected to have a strength of less than 990 tonnes? (3 marks)

- (b) What is the probability that out of 10 cables selected at random, at least nine have a strength of less than 990 tonnes? (3 marks)

- (c) A civil engineer wants to use these cables for a crane that requires the mean strength of the cables to be at least 1014 tonnes. She tests the strength of a random sample of 200 cables, and obtains a sample mean strength of 995 tonnes.

- (i) Obtain a 99% confidence interval for the population mean strength of the cables, correct to two decimal places. (4 marks)

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(c) Given that each kilogram of Type A fertiliser costs \$12 and each kilogram of Type B fertiliser costs \$15, determine the number of kilograms that the farmer must buy so as to minimise the cost and still satisfy all constraints. State this minimum cost. (4 marks)

- (d) By how much can the price of Type B fertiliser change so as to increase the amount of the fertiliser while still maintaining the minimum cost found in part (c)? (3 marks)

See next page

Question 13

(4 marks)

The area bound by the parabola  $y = 6x^2 - 6x$ , the  $x$  – axis and the lines  $x = 1$  and  $x = c$ , ( $c > 1$ ), is equal to 1 unit<sup>2</sup>. Find the value of the constant.

Question 14

(8 marks)

The probability that a potential buyer viewing a property buys it is 0.1. Assume people viewing the property decide independently of each other whether to buy the property or not, and that only one person views the property at a time. If the person does not buy the property, only then will a new person be allowed to view the property.

- (a)     What is the probability that the property is sold to the second person viewing it? (2 marks)
- (b)     What is the probability that more than two people view the property before it is sold?  
(3 marks)
- (c)     Four people are scheduled to view the property. What is the probability that one of them buys it?  
(3 marks)

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Question 19

(10 marks)

A monorail services two mining towns A and B on a straight line from a depot in the city. At time  $t = 0$ , the monorail passes through the depot with velocity  $v = 6t^2 - 60t + 126$ , with velocity in km/h and time in hours. The monorail is on a test run and will travel through Town A without stopping, moving on to Town B, then returning to Town A and then Town B for a second time, without stopping.

- (a)     Determine the displacement function  $x$  from the depot, in terms of  $t$ .  
(2 marks)
- (b)     Determine the times that the monorail will stop at Towns A and B.  
(3 marks)
- (c)     What is the distance between the two towns?  
(2 marks)
- (d)     Determine the distance travelled and the time taken when the monorail enters Town B for the second time.  
(3 marks)

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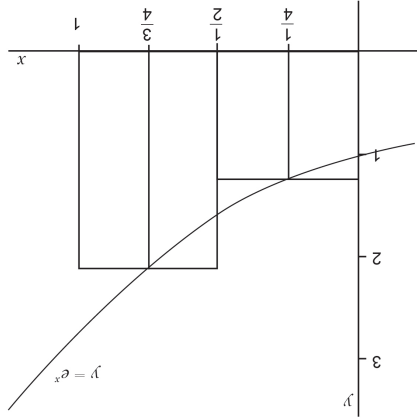


Consider two circles, the first having a radius  $R_1$  and the other radius  $R_2$ , with the sum of the two radii being constant,  $R_1 + R_2 = C$ .  
Use calculus to prove that if the sum of the radii of two circles is constant, then the sum of the areas of the two circles is at a minimum when the circles have equal radii.

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A plot of the exponential function  $y = e^x$  is shown below.



The integral  $\int_1^0 e^x dx$  may be approximated by the areas of the rectangles as shown above.  
(a) Show that the value of the integral  $\int_1^0 e^x dx$  is approximately given by  $\int_1^0 e^x dx \approx \frac{1}{2}(e^{\frac{1}{4}} + e^{\frac{3}{4}})$ . (2 marks)

(b) Determine upper and lower limits for the integral  $\int_1^0 e^x dx$  using the areas of the rectangles. (2 marks)

See next page

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**Question 16** (7 marks)

A toy manufacturer buys pre-assembled robotic arms from three different suppliers: 50% of the total order comes from Supplier A, 30% from Supplier B, and the remaining 20% from Supplier C. Past data shows that the quality control standards of the three suppliers are different. While 2% of the arms produced by Supplier A are defective, Suppliers B and C produce defective arms at rates of 3% and 4%, respectively.

- (a) Construct a probability tree diagram for the above information. (4 marks)

- (b) Given that a robotic arm is defective, determine the probability that the arm did not come from Supplier A. (3 marks)

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**Question 17** (6 marks)

A pendulum consists of a bob connected to a rope of length  $\ell$  metres, where  $\ell$  is a function of time  $t$ .

The time  $T$  seconds taken for a complete swing (back and forth) is given by the formula

$$T = 2\pi \sqrt{\frac{\ell}{10}}.$$

- (a) When  $\ell = 12$  metres, the length of rope is changing at a rate of  $\frac{d\ell}{dt} = 0.1$  metres per second. Determine  $\frac{dT}{dt}$ . (3 marks)

- (b) Use the increments formula  $\delta T \approx \frac{dT}{d\ell} \delta \ell$  to determine the approximate percentage change in  $T$  if  $\ell$  changes by 2% (that is,  $\frac{\delta \ell}{\ell} = 0.02$ ). (3 marks)

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