Question Seven: [8 marks] **CA**

The area bounded by the curve $f(x)=ax^2+b$ and the x axis over the domain $-1\le x\le 2$ is 10.5 units².

The equation of the tangent to $f(x)$ at $x=1$ is $y=x+c$.

Determine the values of a, b and c.

$f'(x)=2ax$ ✓

$f'(1)=1$

$2a=1$ ✓

$a=\frac{1}{2}$ ✓

$\int_{-1}^{2}\frac{1}{2}x^2+b\ dx=10.5$

$\left[\frac{x^3}{6}+bx\right]_{-1}^{2}=10.5$ ✓

$\frac{8}{6}+2b+\frac{1}{6}-b=10.5$

$\frac{9}{6}+b=10.5$ ✓

$b=9$ ✓

$f(1)=\frac{1}{2}+9=9.5$ ✓

$9.5=1+c$

$c=8.5$ ✓

education equals

Calculator Assumed
Applications of Anti-Differentiation 1

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [3 marks] **CA**

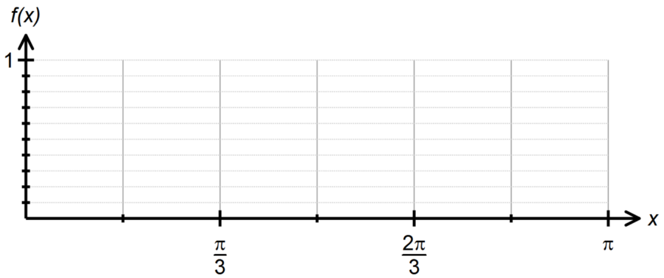
The area under the curve $f(x)=4e^{kx}$ over the domain $0\le x\le 10$ is $\frac{40}{3}\left(-e^{-3}+1\right)$.

Determine the value of k.

Question Two: [2, 2, 3, 3 = 10 marks] CA

Consider the function $f(x)=\sin\left(\frac{x}{2}\right)$

(a) Sketch $f(x)$ over the domain $0 \le x \le \pi$



(b) Draw rectangles on your graph that can be used to overestimate the area under $f(x)$ over the domain $0 \le x \le \pi$, where $\delta x = \frac{\pi}{6}$.

(c) Hence approximate the area under the curve over the domain $0 \le x \le \pi$.

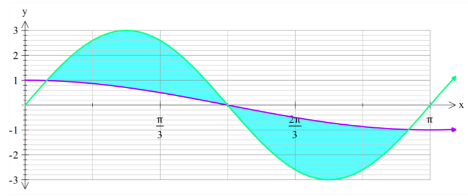
(d) Calculate the margin of error between your answer in part (c) and the exact value of the area under the curve over the domain $0 \le x \le \pi$.

Question Five: [4 marks] CA

Calculate the area enclosed between the two curves $y=\cos x$ and $y=3\sin(2x)$ over the domain $0 \le x \le \pi$.

Draw a sketch to support your solution.

$$Area = 2\int_{0.1674}^{\frac{\pi}{2}} 3\sin(2x)-\cos x \; dx = 4.17\,units^2$$



Question Six: [4 marks] CA

The area of the shaded region of $y=a\sin bx$ below is 6 units².

Determine the values of a and b.

$$\int_0^{\frac{2\pi}{b}} a\sin bx \; dx = 6$$

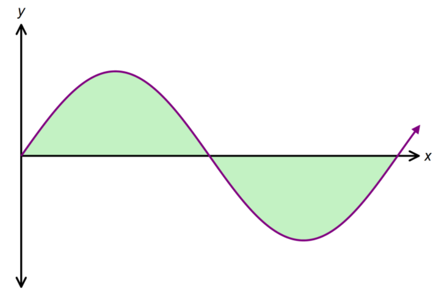
$$\int_0^{\frac{\pi}{b}} a\sin bx \; dx = 3$$

$$\int_0^{\pi} \sin x \; dx = 2$$

$$\int_0^{\pi} 1.5\sin x \; dx = 3$$

$a=1.5$
$b=1$

$or\ where\ \frac{2a}{b}=3 \quad a=1.5b$



Question Four: [2, 2, 3 = 7 marks] CA

The marginal cost of producing x units of a certain product is $120-0.5x+0.01x^2$ dollars per unit.

(a) Determine the extra cost associated with producing the 31st item.

$C'(30)=120-0.5(30)+0.01(30)^2$
$C'(30)=\$114$

(b) Find the increase in cost if the production level is increased from 200 units to 500 units.

$\int_{200}^{500} 120-0.5x+0.01x^2 \; dx=\$373\,500$

(c) The marginal revenue from producing and selling x units of a certain product is $x+2x^2$. Determine the profit function if the profit from producing 10 items is \$38.33.

$P'(x)=x+2x^2-(120-0.5x+0.01x^2)$
$=1.99x^2+1.5x-120$
$P(x)=\dfrac{1.99x^3}{3}+\dfrac{3x^2}{4}-120x+c$
$38.33=\dfrac{1.99(10)^3}{3}+\dfrac{3(10)^2}{4}-120(10)+c$
$c=500$
$P(x)=\dfrac{1.99x^3}{3}+\dfrac{3x^2}{4}-120x+500$

Question Three: [1, 2, 2, 2, 2 = 9 marks] CA

The acceleration of a particle moving in rectilinear motion is given by $a(t)=-4\cos(2t)+12t$, where t is time in seconds and $a(t)$ is ms^{-2}. The initial velocity of the particle is -4 m/s.

(a) Determine the initial acceleration of the particle.

(b) Determine an expression for the velocity of the particle.

(c) Calculate when the speed of the particle is 4 m/s.

(d) Calculate the change in displacement in the first second.

(e) Calculate the distance travelled in the third second.

Question Four: [2, 2, 3 = 7 marks] CA

The marginal cost of producing x units of a certain product is $120 - 0.5x + 0.01x^2$ dollars per unit.

(a) Determine the extra cost associated with producing the 31st item.

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(c) The marginal revenue from producing and selling x units of a certain product is $x + 2x^2$. Determine the profit function if the profit from producing 10 items is $38.33.

Question Three: [1, 2, 2, 2, 2 = 9 marks] CA

The acceleration of a particle moving in rectilinear motion is given by $a(t) = -4\cos(2t) + 12t$, where t is time in seconds and $a(t)$ is ms^{-2}. The initial velocity of the particle is -4 m/s.

(a) Determine the initial acceleration of the particle.

$a(0) = -4ms^{-2}$ ✓

(b) Determine an expression for the velocity of the particle.

$v(t) = \int -4\cos(2t) + 12t \; dt$

$v(t) = -2\sin(2t) + 6t^2 + c$ ✓

$-4 = -2\sin(0) + 6(0)^2 + c$

$c = -4$

$v(t) = -2\sin(2t) + 6t^2 - 4$ ✓

(c) Calculate when the speed of the particle is 4 m/s.

$|v(t)| = 4$ ✓

$t = 0s,\ 0.543s, 1.24s$ ✓

(d) Calculate the change in displacement in the first second.

$\int_0^1 v(t)\; dt = -3.42m$

✓ ✓

(e) Calculate the distance travelled in the third second.

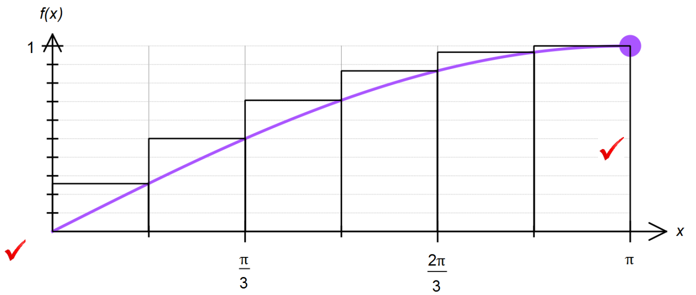
$\int_2^3 |v(t)| dt = 35.62m$

✓ ✓

Question Two: [2, 2, 3, 3 = 10 marks] CA

Consider the function $f(x)=\sin\left(\frac{x}{2}\right)$

(a) Sketch $f(x)$ over the domain $0 \le x \le \pi$



(b) Draw rectangles on your graph that can be used to overestimate the area under $f(x)$ over the domain $0 \le x \le \pi$, where $\delta x = \frac{\pi}{6}$.

(c) Hence approximate the area under the curve over the domain $0 \le x \le \pi$.

$$Area = \frac{\pi}{6}\left(\sin\left(\frac{\pi}{12}\right)+\sin\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{3}\right)+\sin\left(\frac{5\pi}{12}\right)+\sin\left(\frac{\pi}{2}\right)\right)$$

$$Area = 2.25 units^2$$

(d) Calculate the margin of error between your answer in part (c) and the exact value of the area under the curve over the domain $0 \le x \le \pi$.

$$\int_0^\pi \sin\left(\frac{x}{2}\right)dx = 2$$

$$2.25-2=0.25$$

Question Five: [4 marks] CA

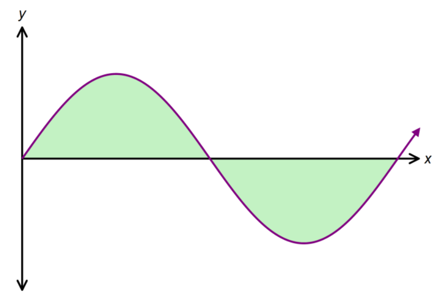
Calculate the area enclosed between the two curves $y=\cos x$ and $y=3\sin(2x)$ over the domain $0 \le x \le \pi$.

Draw a sketch to support your solution.

Question Six: [4 marks] CA

The area of the shaded region of $y=a\sin bx$ below is 6 units².

Determine the values of a and b.



Question Seven: [8 marks] **CA**

The area bounded by the curve $f(x)=ax^2+b$ and the x axis over the domain $-1\le x\le 2$ is 10.5 units².

The equation of the tangent to $f(x)$ at $x=1$ is $y=x+c$.

Determine the values of *a, b* and *c*.

SOLUTIONS
Calculator Assumed
Applications of Anti-Differentiation 1

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [3 marks] **CA**

The area under the curve $f(x)=4e^{kx}$ over the domain $0\le x\le 10$ is $\frac{40}{3}\left(-e^{-3}+1\right)$.

Determine the value of *k*.

$$\int_0^{10} 4e^{kx}\,dx=\frac{40}{3}\left(-e^{-3}+1\right)$$

$$\left[\frac{4e^{kx}}{k}\right]_0^{10}=\frac{40}{3}\left(-e^{-3}+1\right) \checkmark$$

$$\frac{4e^{10k}}{k}-\frac{4}{k}=\frac{40}{3}\left(-e^{-3}+1\right) \checkmark$$

$$k=-0.3 \checkmark$$