Section One: Calculator-free **35% (50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(6 marks)**

Determine the derivative for each of the following, ensuring all answers are expressed with positive indices.

(a) $f(x) = 3x^5 - \frac{2}{x^3}$ (2 marks)

$$= 15x^4 + \frac{6}{x^4}$$

(b) $g(x) = \frac{2x^2}{(3-2x)^3}$ (Do not simplify) (2 marks)

$$= \frac{(3-2x)^3(4x) - 2x^2(3)(3-2x)^2(-2)}{(3-2x)^6} = \frac{(3-2x)^3(4x) + 12x^2(3-2x)^2}{(3-2x)^6}$$

OR $\boxed{\frac{4x}{(3-2x)^3} + \frac{12x^2}{(3-2x)^4}}$

(c) $h(x) = \sqrt[4]{x^3} - \sqrt[3]{x^4}$ (2 marks)

$$h(x) = x^{\frac{3}{4}} - x^{\frac{4}{3}}$$

$$h'(x) = \frac{3}{4}x^{-\frac{1}{4}} - \frac{4}{3}x^{\frac{1}{3}}$$

$$= \frac{3}{4x^{\frac{1}{4}}} - \frac{4x^{\frac{1}{3}}}{3}$$

Question 2 **(8 marks)**

Millie got 90% on the Law Admission Test (LAT). The average LAT applicant got 80% and the standard deviation was 5. Malcolm got 80% on the Undergraduate Medicine and health Science Admission Test (UMAT). The UMAT applicants result was 70% and standard deviation was 4.

(a) Who did relatively better their respective test, Millie or Malcolm? (3 marks)

$Z_{Mi} = \frac{90-80}{5}$

$= 2$ ✓

$Z_{Ma} = \frac{80-70}{4}$

$= 2.5$ ✓

∴ Malcolm did better ✓ as 2.5 std dev. above Mean

(b) Molly walks to work five days a week day, going through two sets of traffic lights. Let X be the probability density function that gives the probability of the number of red lights Molly encounters in one day.

x	0	1	2
$P(X = x)$	0.5	0.3	0.2

(i) Determine the expected number of red lights per trip. (2 marks)

$E(X) = 0 \times 0.5 + 1 \times 0.3 + 2 \times 0.2$ ✓

$= 0 + 0.3 + 0.4$

$= 0.7$ ✓

(ii) What is the probability that ~~Molly~~ had 2 red lights two days in a row? (1 marks)

$P(R_1, R_2) = 0.2 \times 0.2$

$= 0.04$ ✓

(iii) Determine the standard deviation of the distribution of X. Leave your answer un-simplified. (2 marks)

OR

$Var(x) = E(x^2) - [E(x)]^2$

$= 0.3 \times 1^2 + 0.2 \times 2^2$

$= 0.3 + 0.8$

$= 1.1$

$Var(x) = 1.1 - 0.7^2$

$= 1.1 - 0.49$

$= 0.61$

$\sigma = \sqrt{\Sigma (x_i - \mu)^2 p(x)}$ ✓ ✓

$= \sqrt{(0-0.7)^2 \times 0.5 + (1-0.7)^2 \times 0.3 + (2-0.7)^2 \times 0.2}$

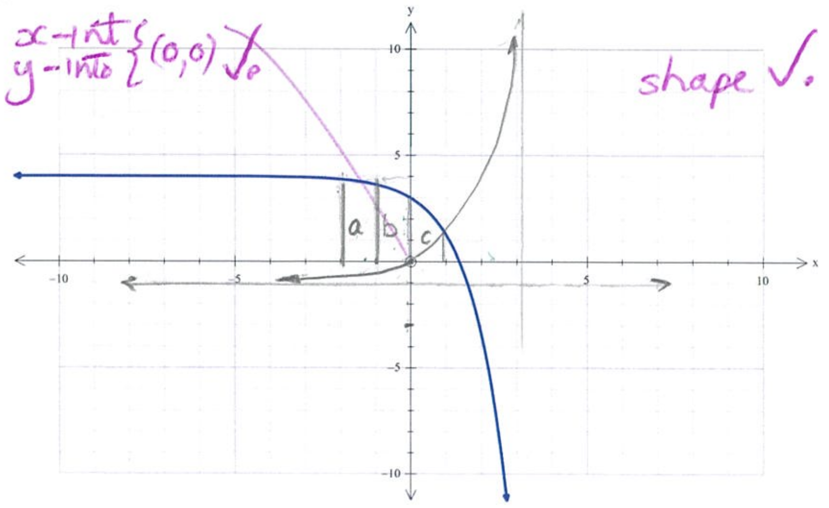
$= \sqrt{-0.7^2 \times 0.5 + 0.3^2 \times 0.3 + 1.3^2 \times 0.2}$

$= \sqrt{0.245 + 0.027 + 0.338}$

∴ std dev $= \sqrt{0.61}$

Question 3 **(8 marks)**

The function $f(x) = -e^x + 4$ is graphed on the axes below.



✓ asymptote: $y = -1$

(a) If $\int_{-2}^{-1} f(x)\,dx = a$, $\int_{-1}^{0} f(x)\,dx = b$ and $\int_{0}^{1} f(x)\,dx = c$, evaluate each of the following definite integrals in terms of the constants a, b and c.

(i) $\int_{0}^{1} f(-x)\,dx$. (1 mark)

⇒ Reflection in y-axis

$\therefore \int_0^1 f(-x) = \int_{-1}^{0} f(x) = b$ ✓

(ii) $\int_{-2}^{0} -f(x)\,dx$. (2 marks)

⇒ Reflection in x-axis

$\therefore \int_{-2}^{0} -f(x) = -(a+b)$ ✓✓ or $-a-b$

(iii) $\int_{-1}^{0} 2f(x-1)\,dx$. (2 marks)

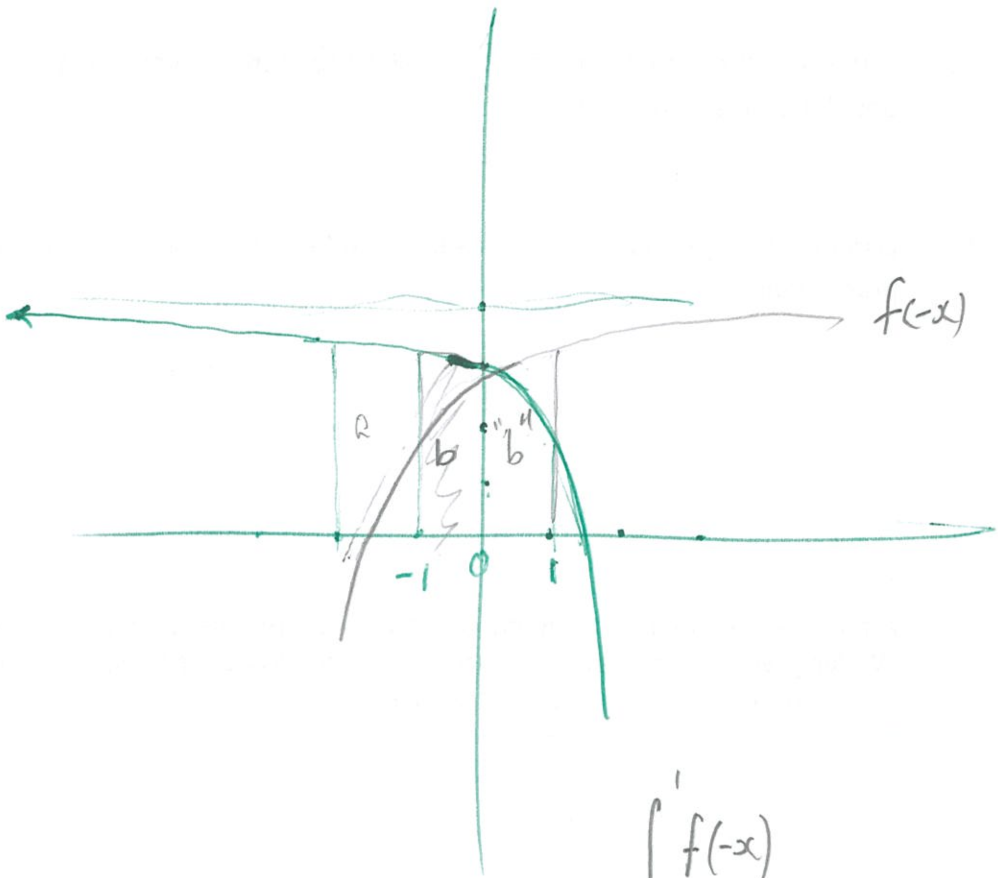
⇒ (i) Translate 1 → right
(ii) Dilate × 2 // y-axis

$= 2\int_{-2}^{-1} f(x) = \boxed{2a}$ ✓✓ ✱: Note "2c" etc ✱

Gross-error −2

(b) On the axes above, sketch the graph of $y = 3 - f(x)$, showing all relevant features. (3 marks)

$y = -f(x) + 3$



$\int_0^1 f(-x)$

Q3.

(a) (i) b

(This is correct)

$$\int_{-1}^{0} -e^{x}+4\,dx = e^{-1}+3 = \boxed{b}$$

$$\int_{0}^{1} -e^{-x}+4\,dx = e^{-1}+3 = \boxed{b}$$

Question 4 **(5 marks)**

(a) Determine $\int \sqrt{x} - \frac{1}{x^2}\,dx$ (2 marks)

$= \int x^{\frac{1}{2}} - x^{-2}\,dx$

$= \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{x} + c$ ✓ ✓

−1 if NO "c"

$6x^5 - 84x^3 + 294x$

$f(x) = x^6 - 21x^4 + 147x^2 + c$ ✓

(b) If $f'(x) = 6x(x^2 - 7)^2$ and $f(2) = -20$, determine $f(3)$. (3 marks)

$\int 6x(x^2-7)^2\,dx = 3\int 2x(x^2-7)^2\,dx$

ie $f(x) = (x^2-7)^3 + c$ ✓

$f(2) = -20 \Rightarrow -20 = (2^2-7)^3 + c$

ie $-20 = -27 + c$

ie $c = 7$ ✓

$\therefore f(x) = (x^2-7)^3 + 7$

$f(3) = (3^2-7)^3 + 7$

$= 15$ ✓

Question 5 **(7 marks)**

Two hundred and fifty randomly selected students were surveyed to determine if an overseas trip for students should be planned for students in Year 10, or in Year 11 or in Year 12.

The results are in the table below.

Year	10	11	12
Preference	90	90	70

(a) (i) Convert the data to form a probability density function. (2 marks)

x	10	11	12
$P(X=x)$	0·36	0·36	0·28
	$\frac{90}{250}$	$\frac{90}{250}$	$\frac{70}{250}$

(ii) If two students from the school were selected at random, what is the probability that they both thought Year 12 students should not go on an overseas trip.
(Do not simplify) (2 marks)

$0{\cdot}28 \times 2$ 1 mark

$P(Yr12, Yr12) = 0{\cdot}28 \times 0{\cdot}28$

OR $\underbrace{{}^2C_2\left(\frac{140}{500}\right)^2\left(\frac{360}{500}\right)^0 = 0{\cdot}28^2 = \left(\frac{7}{25}\right)^2 = \frac{49}{625}}_{\text{any of these}}$

Which of the following represents a probability density function? Give your reasons. (3 marks)

(b) (i)

x	0	1	2	3
$P(X = x)$	0.1	0.2	0.3	0.4

$\sum P(X=x) = 1$

$\therefore$ yes

If an No reasons for (ii) & (iii) 1 mark only

(ii)

x	10	11	12	13
$P(X = x)$	0.2	0.3	0.1	0.5

$\sum P(X=x) > 1$

$\therefore$ No

(iii)

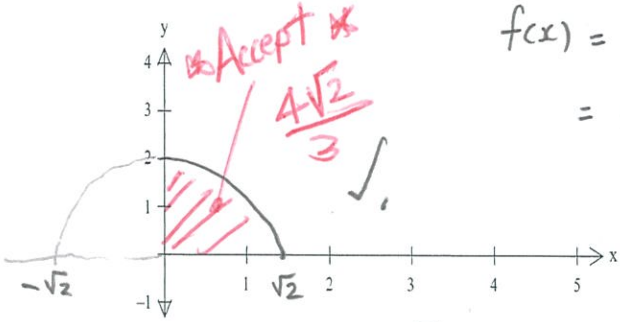
x	6	7	8	9
$P(X = x)$	0.4	0.5	-0.3	0.4

$P(X=8) < 0$

$\therefore$ No

Question 6 **(6marks)**

(a) Find the area between the function $f(x) = 2 - x^2$ and the x axis. (4 marks)

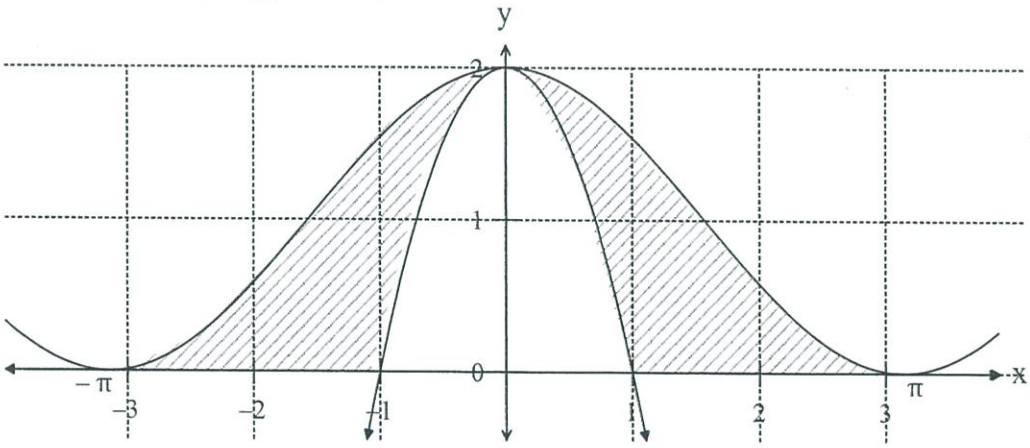


$$f(x) = 2 - x^2$$
$$= (\sqrt{2} - x)(\sqrt{2} + x)$$

roots at $\pm\sqrt{2} \Rightarrow (-\sqrt{2}, 0)$ and $(\sqrt{2}, 0)$

$$A = \int_{-\sqrt{2}}^{\sqrt{2}} f(x)$$
$$= \int_{-\sqrt{2}}^{\sqrt{2}} (2 - x^2)\,dx \checkmark$$
$$= \left[2x - \frac{x^3}{3}\right]_{-\sqrt{2}}^{\sqrt{2}} \checkmark$$
$$= \left(2\sqrt{2} - \frac{\sqrt{2}^3}{3}\right) - \left(2\sqrt{2} - \frac{(-\sqrt{2})^3}{3}\right)$$
$$= \left(2\sqrt{2} - \frac{2\sqrt{2}}{3} + 2\sqrt{2} - \frac{2\sqrt{2}}{3}\right)$$
$$= \boxed{4\sqrt{2} - \frac{4\sqrt{2}}{3}}$$
$$= \frac{12\sqrt{2} - 4\sqrt{2}}{3} = \boxed{\frac{8\sqrt{2}}{3}} \checkmark$$

(b) Write down the expression for the area between the function $y = 2 - 2x^2$ and $y = 1 + cos(x)$ between $-\pi \le x \le \pi$ that is illustrated in the diagram below. (2 marks)



$$\int_{-\pi}^{\pi} 1 + \cos(x)\,dx \checkmark - \int_{-1}^{1} 2 - 2x^2\,dx \checkmark$$

-1 mark $= \int_{-\pi}^{\pi} 1 + \cos x - 2 + 2x^2\,dx$ ✓ only.

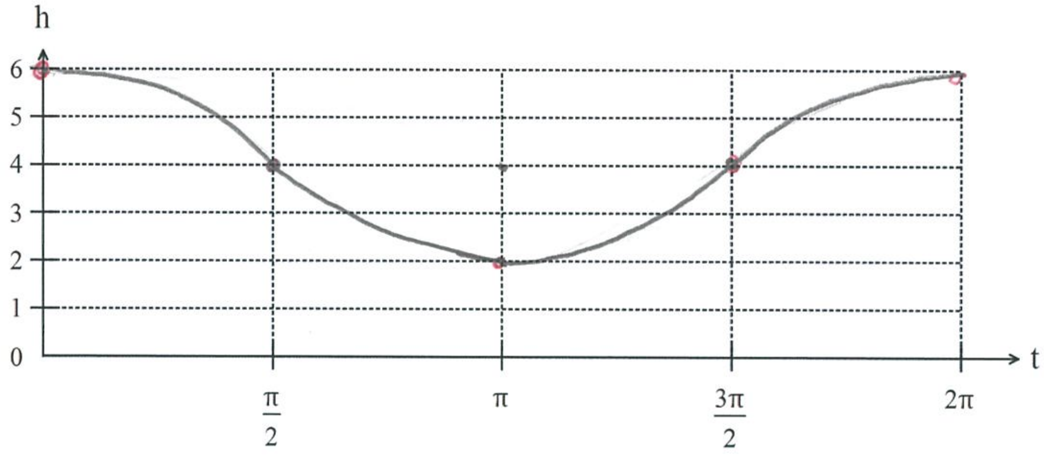
Question 7 **(10 marks)**

The height of the acrobat on a circus swing is given by $h(t) = 2\cos(t) + 4$ where h is measured in metres and t in seconds.

(a) Sketch the height of the acrobat on the set of axes below for $0 \le t \le 2\pi$. (2 marks)

passes thru critical pts ✓.

shape ✓.



(b) At what rate is the acrobat rising at $t = \frac{3\pi}{2}$? (2 marks)

$h(t) = 2\cos(t) + 4$

$h'(t) = -2\sin(t)$ ✓

when $t = \frac{3\pi}{2}$ $\quad h'(t) = -2\sin\left(\frac{3\pi}{2}\right) = 2\,m/s$ ✓.

(c) Explain using the graph in (a) why your answer to (b) gives the maximum rate of rise of the acrobat. (3 marks)

$\therefore$ P.O.I $\begin{cases} \text{Concave Up} \Rightarrow \pi \le t < \frac{3\pi}{2} \Rightarrow \text{Velocity increasing} ✓. \\ \text{Concave Down} \;\; \frac{3\pi}{2} \le t < 2\pi \Rightarrow \text{Velocity decrease} \end{cases}$ ✓.

$\therefore$ when $t = \frac{3\pi}{2}$ Velocity is Max. ✓.

OR $h''(t) = -2\cos(t)$ ✓ and $h''\left(\frac{3\pi}{2}\right) = 0$ ✓ which is a Max ✓ (P.O.I)

(d) At what rate is the vertical velocity of the acrobat changing at $t = \frac{3\pi}{2}$? (3 marks)

$h''(t) = -2\cos(t)$ ✓

$\therefore h''\left(\frac{3\pi}{2}\right) = 0\,m/s^2$ ✓

If $0\,m/s^2$ only (No statement) -1

$\therefore$ Velocity is Not changing ✓

Section Two: Calculator-assumed **65% (100 marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(11 marks)**

The cash balance C (in thousands of dollars) for the first few months of a start-up company vary according to the function $C = -e^{0.1t}sin(t)$, where t represents months. **Hint: Use radians.**

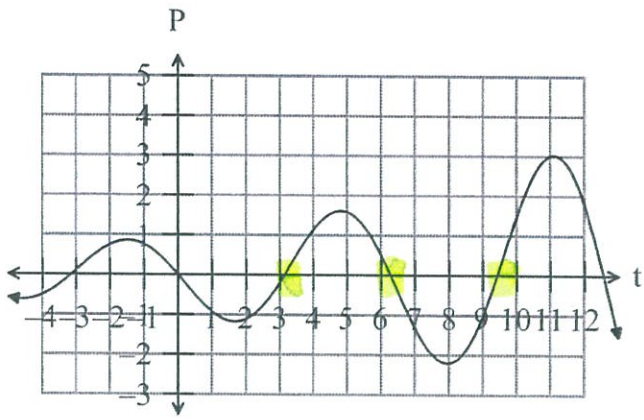
(a) State the first and second derivatives of the cash balance function. (3 marks)

Solution
$C = -e^{\frac{t}{10}}Sin(t)$ $\frac{dC}{dt} = -e^{\frac{t}{10}}\left(\frac{Sin(t)}{10} + Cos(t)\right)$ $\frac{d^2C}{dt^2} = -\frac{1}{100}e^{\frac{t}{10}}\left(20\,Cos(t) - 99\,Sin(t)\right)$
Specific behaviours
✓ states first derivative   ✓✓ states second derivative

(b) State when in the first year when either the cash balance function equals zero or its first derivative equal zero. (3 marks)

Solution
Cash Balance is zero at 3.14, 6.28, 9.42   Stationary Points at 1.67, 4.81, 7.95, 11.09
Specific behaviours
✓ identifies when cash balance is zero   ✓identifies stationary point   ✓ Need all points.

(c) Sketch the cash balance equation on the set of axes. (3 marks)



x-int ✓

T.P. ✓

shape/accuracy ✓

After the first five months, the owner employed more staff and it took a little while for sales to start to increase again.

(d) Determine when the cash balance started to increase again. (1 mark)

Solution
Cash starts to increase after 7.95 months
Specific behaviours
✓ identifies when cash balance begins to increase

(e) Determine when the cash balance become positive again. (1 mark)

Solution
Cash Balance become positive after 9.42 months
Specific behaviours
✓ identifies when cash balance returns to positive

Question 9 **(4 marks)**

Michelle collected comic book trading cards from two brands, Marvel and DC. She has eight (8) Marvel cards and twelve (12) DC cards. She shuffles the cards in a deck.
One card is selected, and its brand colour noted then it is replaced in the deck and mixed thoroughly with the other cards again. This process is repeated several times.

(a) What is the probability that the first DC card drawn is the third card? (2 marks)

$\left(\frac{2}{5}\right)^2 \times \left(\frac{3}{5}\right) = \frac{12}{125}$ or 0.096 ✓✓

Up to three cards can be drawn. The draw stops once a DC card is selected.

(b) What is the probability that a DC card is not drawn? (2 marks)

0.4	0.6

$1 - [P(DC\ 1^{st}) + P(DC\ 2^{nd}) + P(DC\ 3^{rd})]$

$= 1 - 0.6 - 0.4 \times 0.6 - 0.4^2 \times 0.6$

$= 0.064$ ✓✓

Question 10 **(7 marks)**

Australian population in 1880 was 2,231,489.
The population in 1930 had grown to 6,501,012.

(a) Taking $t = 0$ in 1880, set up an equation in the form $P = P_0 e^{kt}$ that can be used to estimate the population growth during the 50-year period. (2 marks)

$$6\,501\,012 = 2\,231\,489\, e^{50k}$$

Solve: $\frac{6\,501\,012}{2\,231\,489} = e^{50k}$

$k \approx 0.021386$ ✓.

$\therefore P = 2\,231\,489\, e^{0.021386t}$ ✓.

(b) Write down the average annual population growth over that period. (1 mark)

$$\text{Ave Growth} = \frac{6\,501\,012 - 2\,231\,489}{50}$$

$\approx 85\,390.46$

Over the next 60 years to 1990, the population grew to 17,169,181.

(c) Determine if the rate of growth during the 60 years from 1930 to 1990 is the same as the rate of growth from 1880 to 1930. (2 mark)

Solve $\frac{17\,169\,181}{6\,501\,012} = e^{60k}$

$k \approx 0.016186$ (Not the Same)

(d) Use the data from 1930 to 1990 to predict the ~~taxable income~~ population in 2016. (2 mark)

$P = 17\,169\,181\, e^{0.016186 \times 26}$

2016
$t = 26$

$\approx 26\,152\,666.6$

$\approx 26\,152\,700$

NB. Australia's population in 2016 was 24,129,300.

Question 11 **(10 marks)**

(a) (i) Find the expected value and variance of the probability density function in the table below.
~~(5 marks)~~ (2 marks)

x	1	2	3	4
$P(X = x)$	0.3	0.2	0.2	0.3

$E(X) = 1\times 0.3 + 2\times 0.2 + 3\times 0.2 \times 4\times 0.3$

$= 0.3 + 0.4 + 0.6 + 1.2$

$= \underline{\underline{2.5}}$ ✓

$Var(X) = 0.3(1-2.5)^2 + 0.2(2-2.5)^2 + 0.2(3-2.5)^2 + 0.3(4-2.5)^2$

$= 0.3(-1.5)^2 + 0.2(0.5)^2 + 0.2(0.5)^2 + 0.3(1.5)^2$

$= \underline{1.45}$

$\boxed{\text{Classpad}}$ $\sigma x = 1.204$ $\therefore \underline{Var(X) = 1.450}$ ✓

(ii) The values of set X are transformed so that $Y = 2X + 1$.
Write down the expected value and variance of set Y. (2 marks)

$E(2X+1) = 2\times 2.5 + 1$

$= 6$ ✓

$Var(2X+1) = 2^2 \times 1.45$

$= \underline{\underline{5.8}}$ ✓

(b) Fiona bet on the outcome of an unfair 4-sided tetrahedral die with probabilities as in the table below.

	-1	1	1	-1
x	1	2	3	4
$P(X = x)$	0.3	0.2	0.2	0.3

It costs Fiona \$1 per roll and the payout is \$2 for a 2 or a 3 and nothing otherwise.

What is Fiona's average payout? (3 marks)

$-0.3 + 0.2 + 0.2 - 0.3$

$= \$ -0.2$ $(\$0.8)$

11.4 ~~9.77~~ 0.8

Question 12 **(6 marks)** ✓

The speeds of 250 vehicles, on a section of freeway undergoing roadworks with a speed limit of 12 ~~60~~ kmh^{-1}, had a mean and standard deviation of ~~56.9 kmh^{-1}~~ and 3.6 kmh^{-1} respectively. A summary of the data is shown in the table below.

Speed (x kmh^{-1})	9 10 $\cancel{45} \le x < \cancel{50}$	10 11 $\cancel{50} \le x < \cancel{55}$	11 12 $\cancel{55} \le x < \cancel{60}$	12 13 $\cancel{60} \le x < \cancel{65}$	13 14 $\cancel{65} \le x \le \cancel{70}$
Relative frequency	0.024	0.272	0.504	0.188	0.012

(a) Use the table of relative frequencies to estimate the probability that the next vehicle to pass the roadworks

(i) was not exceeding the speed limit. (1 mark)

$0.024 + 0.272 + 0.504 = 0.8$ ✓

(ii) had a speed of less than 13 ~~65~~ kmh^{-1}, given they were exceeding the speed limit. (1 mark)

$$\frac{0.188}{0.188+0.012} = \frac{0.188}{0.2} = 0.94$$ ✓

(b) Subsequent tests on the measuring equipment discovered that it had been wrongly calibrated. The correct speed of each vehicle, v, could be calculated from the measured speed, x, by increasing x by 6% and then adding 1.7.

(i) Calculate the adjusted mean and standard deviation of the vehicle speeds. (2 marks)

$Y = 1.06x + 1.7$

$\mu(Y) = 1.06 \times 11.4 + 1.7 = 13.78$ ✓

$\sigma(Y) = 1.06 \times 0.8 = 0.848$ ✓

(ii) Determine the correct proportion of vehicles that were speeding. (2 marks)

Let the incorrect speeding be z.

$1.06z + 1.7 = 12$

$z = 9.72$ km/h ✓

$0.272 + 0.504 + 0.188 + 0.012 =$

$= \underline{\underline{0.976}}$ ✓

Question 13 **(3 marks)**

A lottery sells 1000 tickets and claims that there will be 10 winners. How many tickets should you buy so that you have a 20% chance of winning at least 1 prize.

$X \sim Bin(1000, 0.01)$

$P(X \geq 1) = 0.20$

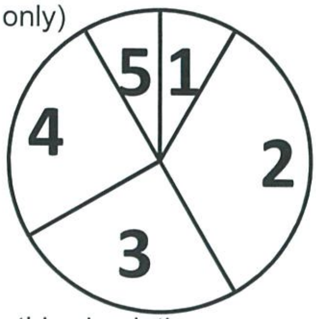
BinCDF(1, 23, 23, 0.01) = 0.2064

∴ need to purchase 23 tickets

Question 14 **(9 marks)**

The simulation of a loaded (unfair) spinner is spun 60 times and the results recorded with the following results. (Below diagram is for illustrative purposes only)

Result	Count
1	5
2	20
3	15
4	15
5	5



(a) Calculate the proportion of even numbers recorded in this simulation. (2 mark)

$\frac{35}{60} = \frac{7}{12} \approx 0.583$

(b) Determine the mean and standard deviation for the sample proportion of even numbers in 60 tosses, using the results above. (2 marks)

MEAN: $\hat{p} = \frac{7}{12} \approx 0.583$ ✓

STD. DEV.: $\sigma_{\hat{p}} \approx \sqrt{\frac{0.583(1-0.583)}{60}} \approx 0.0636$ or $\frac{\sqrt{21}}{72}$ ✓

(c) It has been decided to create a confidence interval for the proportion of even numbers using the simulation results. The level of confidence will be chosen from 90% or 95%. Explain which level of confidence will give the smallest margin error. State the margin of error. (3 marks)

90% confidence level has the smallest margin of error ✓ as it has the smallest range ✓

90% Margin of Error $= 1.645 \times \sigma \approx 0.1047$ ✓ | OR 95% Margin of Error $= 1.96 \times \sigma \approx 0.1247$ ✓

This simulation of 60 spins of the spinner is performed another 200 times, with the proportion of even numbers recorded each time and graphed.

(d) Comment briefly on the key features of this graph. (2 marks)

Binomial distribution approaches Normal when n is large ✓

Distribution centred on $\hat{p} \approx 0.583$ ✓

etc.

Question 15 **(8 marks)**

From a random sample of n people, it was found that ~~540~~ 54 of them watch the AFL grand final. A symmetric confidence interval for the true population proportion who watched the grand final is $0.1842 < p < 0.2958$.

(a) Determine the value of n, by first finding the mid-point of the interval. (4 marks)

midpoint $\rightarrow \dfrac{0.1842 + 0.2958}{2} = \underline{\underline{0.24}}$

$$\frac{54}{n} = 0.24$$

$\therefore n = 225$ people

(b) Determine the confidence level of the interval. (4 marks)

Margin of error (MOE)

$$= \frac{0.2958 - 0.1842}{2}$$

$$= 0.0558$$

$$\text{MOE} = z\sqrt{\frac{(0.24)(1-0.24)}{225}} = 0.0558$$

$z = 1.9598$ (CAS)

~ 1.960

$\therefore$ 95% confidence level

Question 16 **(8 marks)**

The moment magnitude scale M_w is used by seismologists to measure the size of earthquakes in terms of the energy released. It was developed to succeed the 1930's-era Richter magnitude scale.

The moment magnitude has no units and is defined as $M_w = \frac{2}{3}\log_{10}(M_0) - 10.7$, where M_0 is the total amount of energy that is transformed during an earthquake, measured in dyn·cm.

(a) On 21 December 1937, an estimated 6.31×10^{13} dyn·cm of energy was transformed during an earthquake in the Simpson Desert in NT. Calculate the moment magnitude for this earthquake. (2 mark)

$\frac{2}{3}\log_{10} 6.31\times 10^{13} - 1.07$ ✓

$M_W = \cancel{8.13}$ ✓. -1.5

(b) A few years later, on 27 July 1941, there was another earthquake with moment magnitude 6.5 in the Simpson Desert. Calculate how much energy was transformed during this earthquake. (2 marks)

$6.5 = \frac{2}{3}\log_{10}(M_o) - 10.7$ ✓.

$17.2 = \frac{2}{3}\log_{10}(M_o)$

$25.8 = \log_{10}(M_o)$

$\therefore M_o = 6.31\times 10^{25}$ dyn.cm ✓.

(c) Show that an increase of 2 on the moment magnitude scale corresponds to the transformation of 1000 times more energy during an earthquake. (4 marks)

$M_W = 6.5 \Rightarrow M_{6.5} = 6.31\times 10^{25}$ ✓

$M_W = 8.5 \Rightarrow M_{8.5} = 10^{(8.5+10.7)\times 1.5}$

$= 6.31\times 10^{28}$ ✓

$\therefore \frac{M_{8.5}}{M_{6.5}} = 1000$ times greater energy ✓✓

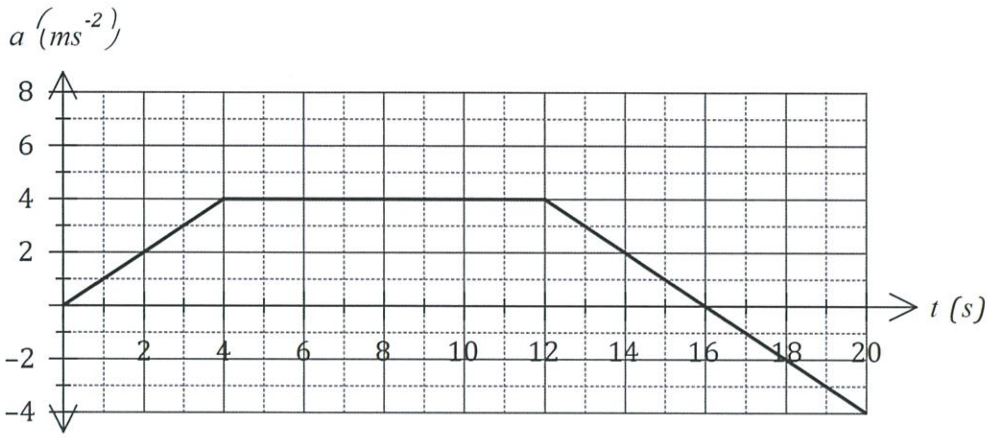
M_x $M_w = x$

$M_{x+2} =$ $M_{w+2} = x+2$

$= 2$

Question 17 **(8 marks)**

A particle, initially stationary and at the origin, moves subject to an acceleration, a ms^{-2}, as shown in the graph below for $0 \leq t \leq 20$ seconds.



(a) Determine the velocity of the object when

(i) $t = 4$ (1 mark)

Solution	v(4) = 8 ms^{-1}
Behaviour	Calculate area under graph

(ii) $t = 12$ (1 marks)

Solution	v(12) = 40 ms^{-1}
Behaviour	Calculate area under graph

(iii) $t = 20$ (1 marks)

Solution	v(20) = 40 ms^{-1}
Behaviour	Calculate area under graph

(b) At what time is the velocity of the body a maximum, and what is the maximum velocity? (2 marks)

Solution	t=16 v(16) = 48 ms^{-1}
Behaviour	Identify time when velocity is at max Velocity at that time

(c) Determine the distance of the particle from the origin after 20 seconds. (3 marks)

Solution	s(t) = $\iint_0^4 t\ dt + \iint_4^{12} 4\ dt \iint_{12}^{20} (-t + 16)\ dt$ s(t) = 1269.33m
Behaviour	Integrate to get velocity, Integrate again to identify displacement Calculate the intervals to obtain total displacement.

(d)

Question18 **(11 marks)**

In a certain library, there is one computer with internet access that is open for public use. The internet usage (in minutes) is a continuous random variable X with pdf given by

$$f(x) = \begin{cases} 0.047e^{-0.047x}, & x \geq 0, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the probability that the visitor will use library internet access for:

(i) less than 15 minutes (2 marks)

$$P(x<15) = \int_0^{15} 0.047e^{-0.047x}\,dx$$

$$= 0.5059$$

(ii) longer than 30 minutes. (2 marks)

$$P(X>30) = 1 - P(x \leq 30) = 1 - \int_0^{30} 0.047e^{-0.047x}\,dx$$

$$= 0.2441$$

(b) Calculate the rate of internet usage after 30 minutes. (2 marks)

paper said at 30min. Should this be zero?

$$y = 0.047e^{-0.047x}$$

$$\frac{dy}{dx} = -0.047^2 e^{-0.047(30)}$$

$$= -5.39 \times 10^{-4}$$

(c) Alex has been waiting for his turn to use the internet for at least 10 minutes. What is the probability that he will have to wait for at least 10 more minutes? (3 marks)

$$\frac{P(x>20)}{P(x>10)} = \frac{1 - \int_0^{20} 0.047e^{-0.047x}\,dx}{1 - \int_0^{10} 0.047e^{-0.047x}\,dx}$$ ✓ ✓

$$= \frac{0.3906}{0.6250}$$

$$= 0.62496.$$ ✓

(d) $0.8 = \int_0^a 0.047e^{-0.047x}\,dx \Rightarrow a = 34.24$ $\therefore$ return in 35min

Question 19 **(7 marks)**

Sophie is a petroleum engineer working for Vechron Limited in charge of choosing between two of its sites for the construction of an offshore drilling rig. The government will only allow drilling of one site.

Sophie is examining recently taken samples for both sites to help with his decision. A sample taken from the first site has a mean sample grade of 4.6 millilitre per cubic metre (mLm^{-3}) with a standard deviation of 0.56 mLm^{-3}. Sophie found that the data for the samples are normally distributed.

(a) Determine the probability that a randomly chosen sample contains a grade that is

(i) exactly 4.6 mLm^{-3}. (1 mark)

Solution	$p(X=4.6)=0$
Behaviour	Continuous data has zero probability at point

(ii) greater than 3.5 mLm^{-3}. (1 mark)

Solution	$p(X>3.5)=0.975$
Behaviour	Calculate probability distribution,

(b) Determine the median score. (1 mark)

Solution	$\mu=4.6mLm^{-3}$
Behaviour	Notes the mean is median in normal distribution.

(c) The probability that another sample contained less than the particular grade was 0.25 or 25%. Determine the maximum grade for the sample. (1 mark)

Solution	invNormCDF (“L”,0.25,0.56,4.6) = 4.22mLm^{-3}
Behaviour	Calculate max grade sample from probability distribution,

The set of samples obtained from the second site has a mean sample grade of 4.7 mLm^{-3}. The data was given to Sophie as a box-plot with the median of 4.72 mLm^{-3}, the lower quartile of 4.2 mLm^{-3} and the upper quartile of 5.2 mLm^{-3}.

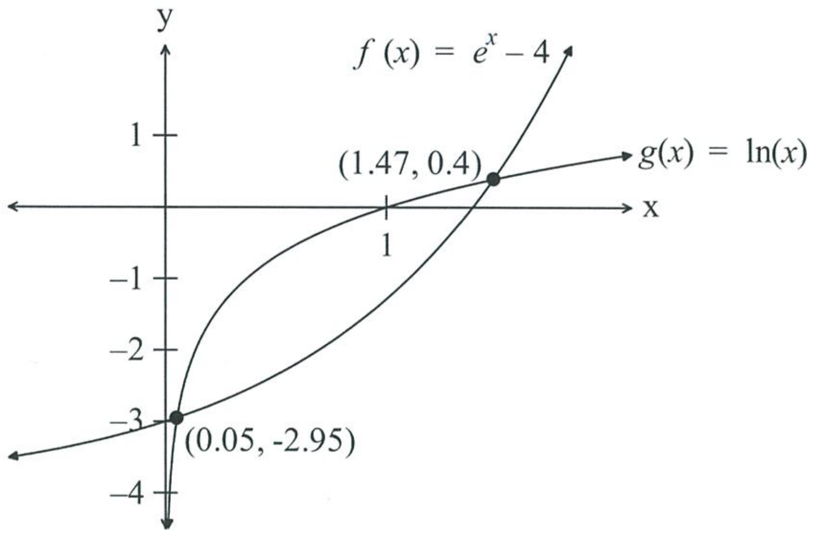
(d) Examine the above statistics to determine if the data for the second potential site could be represented by a normal distribution. **Justify your conclusion.** (3 marks)

Solution	4.72 (Median) ~ 4.7 (Mean), not equal but close (1 mark) 4.72 – 4.2 ~ 5.2 – 4.72, not symmetrical, but close (1 mark) Conclusion can be Yes/No, as long as justified. (1 mark)
Behaviour	States conclusion. Justifies position with understanding of characteristics of normal distribution.

Question 20 **(7 marks)**

(a) Use your calculator to find the area enclosed between the two functions $f(x) = e^x - 4$ and $g(x) = ln(x)$ as shown in the diagram below.

The points of intersection are shown.



(2 marks)

Solution	$\left\lvert \int_{0.05}^{1.47} f(x)dx - \int_{0.05}^{1.47} g(x)dx \right\rvert$ Area = 1.67839 =1.68 units^2
Behaviour	Calculates value.

(b) Keiko is a Biologist studying a bacteria. She has a colony of an experimental bacteria that she is testing for growth characteristics. The population of this colony was studied in over a month. The total population (in millions) can be modelled by the equation $P(t) = 22(ln(t+3))$ where t is in days starting on 1st September 2018.

(i) What was the population on the 1st of September? (2 marks)

Solution	P(0)=22(ln(0+3) = 24.17million
Behaviour	Sets up equation States correct population

(ii) On what day will the population reach 100 million? (3 marks)

Solution	P(t)=22(ln(t+3) = 100 million *t=91.2* *92nd day or 1st December*
Behaviour	Sets up equation Solves equation States the correct day