Saigon International College

Saigon International College
Department of Mathematics and Science
Semester 1, 2022
Year 11 Mathematics Methods ATAR
Test 2
Trigonometric Functions

Section One (Calculator Free)

$\frac{44}{98}$

Time Allowed: (3+25) minutes **Total mark:** $\frac{24}{41}$

Name: . . Chu . . Minh . . Đăng

$\cos\frac{2}{3} = -$

Question 1 **(8 marks)**

(a) If α and β are acute angles such that $\cos\alpha = \frac{2}{3}$ and $\sin\beta = \frac{3}{5}$, determine the value of $\cos(\alpha - \beta)$ as a single fraction.

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\cos A - B = \cancel{\cos}\frac{2}{5} \cancel{\cos}\frac{3}{5} + \cancel{\sin}\frac{2}{3} \cancel{\sin}\frac{3}{5}$

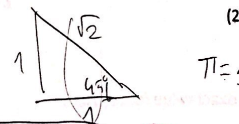
(b) Solve the following equations.

(i) $\sqrt{2}\sin x = -1$ *where* $0 \le x \le 2\pi$. **(2 marks)**

$\sin x = \frac{-1}{\sqrt{2}}$

$\sin x = 45°$

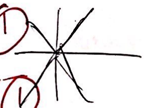
$\sin x = \frac{\pi}{4}$



$\pi = \frac{180}{2} = \frac{90}{2} = 45$

$\boxed{x = \frac{\pi}{4} \quad x = \frac{5\pi}{4} \quad x = \frac{3}{4}\pi \quad x = \frac{7\pi}{4}}$

1



1

(ii) tan (2x) = 0.4 where $0 \le x \le 180°$ and given that $\tan 22° = 0.4$. **(2 marks)**

$\tan 2x = 0.4$

~~tan 2~~

$2x = 22°$

$\boxed{x = 11°}$

1

Question 3
(a) P

Question 2. **(8 marks)**

(a) Solve the equation $\sqrt{3}\tan(x) - 3 = 0$ *for* $0 \le x \le 2\pi$. **(3 marks)**

$\sqrt{3}\tan x - 3 = 0$

$\sqrt{3}\tan x = 3$

$\tan x = \frac{3}{\sqrt{3}}$ ✓ 1

$\tan x = \frac{3\sqrt{3}}{3} \times \frac{180}{\pi}$ ✗

(b) A function has a period of k and is defined by f(x) = 4 cos (2x).

(i) State the value of k. (1 mark)

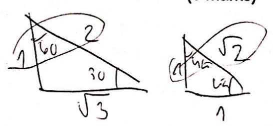
2 ✗

(ii) State the amplitude of f(x). (1 mark)

4 ✓ 1

(c) Determine an exact value for $\cos 105°$. (3 marks)

soh cah toa



$\cos 105 = \cos(60 + 45)$

~~cos 105~~

$\cos(60+45) = \cos 60 \cos 45 - \sin 60 \sin 45$

$\cos(60+45) = \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$\cos(60+45) = \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$

$\cos(60+45) = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$\cos(60+45) = \frac{\sqrt{2} - \sqrt{6}}{4}$ ✓ 3

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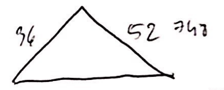
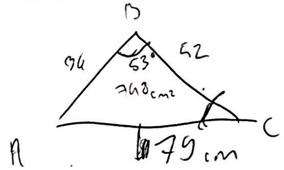


Question 13 **(7 marks)**

An obtuse angled triangle ABC has a = 36 cm, c = 52 cm and area of 748 cm².

(a) Sketch a triangle to show this information. (1 mark)



1

(b) Determine the size of $\angle B$. (2 marks)

$748 = \frac{1}{2} \times 36 \times 52 \times \sin\theta$

$748 = 936 \sin\theta$

$\frac{748}{936} = \sin\theta$

$\theta = 53°$

$\angle B = 180 - 53 = 127$

(c) Show that b ≈ 79 cm. (2 marks)

$\frac{36}{\sin 32} = \frac{79}{\sin x}$

$\cancel{c^2 = 36^2 + 52^2 - 2 \times 52 \times 36 \times \cos 53}$

$\cancel{c = 41.8 \text{ cm}}$

(d) Show that ∠C ≈ 32°. (2 marks)

$\frac{36}{\sin 32} = \frac{79}{\cdot}$

$\cancel{\frac{79}{\sin 53} = \frac{36}{\sin x}}$

$\cancel{98.918 = \frac{36}{\sin x}}$

$\sin x = 0.363$

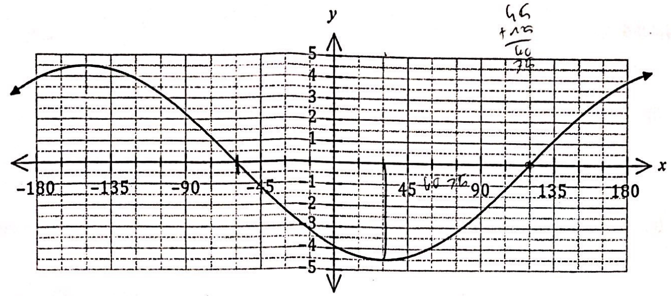
$x =$

End of section two

Question 3 **(7 marks)**

(a) Part of the graph of $y = c\sin(x - \theta)$ is shown below.

46
+ 75
60
76



State the value of the constant c and the value of the constant θ, $0° \le \theta \le 180°$. (2 marks)

$c = 4.5$ 1

$\theta = 120$ 1

(b) Show that $\sin(x - y) + \sin(x + y) = b\sin x\cos y$ and state the value of the constant b. (2 marks)

sin

b =

(c) Determine an exact value for $\sin 15° + \sin 105°$. (3 marks)

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S | A
T | C

Question 4

(a) State the exact value of

(i) $\cos(-\frac{\pi}{3})$ (1 mark)

$-\frac{\pi}{3} \times \frac{180}{\pi} = -\frac{180}{3} = -60$

$\cos(-60°) = \cos 60 = \frac{1}{2}$ 1

(ii) $\cos 15°$ (3 marks)

$\cos 15 = \cos(45 - \cos 30)$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \frac{1}{2} = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$ 3

(b) Solve for θ,

(i) $\sin(\theta + 90°) = 0$ $\quad 0° \le \theta \le 360°$ (2 marks)

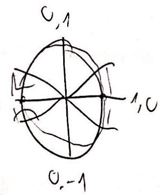
$\sin(\theta + 90°) = 0$

(ii) $3\tan^2\theta - 1 = 0$ $\quad -\pi \le \theta \le \pi$ (3 marks)

$3\tan^2\theta = 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \frac{1}{\sqrt{3}}$



$\tan\theta = 30°$ $\quad \tan 30$

$\cancel{\tan}\ \theta = \frac{\pi}{6}$ 1

$30 \times \frac{\pi}{180} = \frac{30\pi}{180} = \frac{\pi}{6}$

$\theta = \boxed{\frac{\pi}{6}, \frac{5}{6}\pi}, \frac{7\pi}{6}, \frac{11\pi}{6}, \boxed{-\frac{\pi}{6}, -\frac{5}{6}\pi}$ 1

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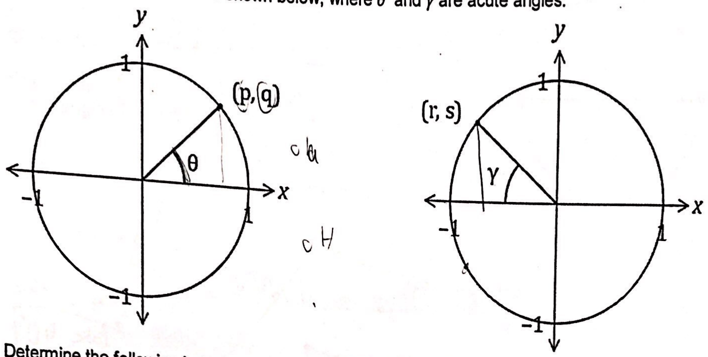
Question 12

(7 marks)

Consider the points with coordinates (p, q) and (r, s) that lie in the first and second quadrants respectively of the unit circles shown below, where θ and γ are acute angles.



Determine the following in terms of p, q, r and s, simplifying your answers where possible.

(a) $\tan\theta = \frac{q}{p}$ 1 (1 mark)

(b) $\sin(180 - \theta) = -\frac{q}{1}$ (1 mark)

(c) $\cos\gamma$. r (1 mark)

(d) $\sin(\pi + \gamma)$ $-r$ (1 mark)

(e) $\cos(\gamma - \theta)$ (3 marks)

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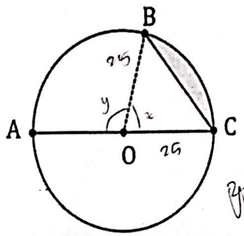
Question 11 **(8 marks)**

(a) The circle shown has centre O and diameter AC of length 50 cm. Determine the shaded area given that $2 \times \angle AOB = 3 \times \angle BOC$. **(4 marks)**



Diameter : 50 cm

$2\angle AOB = 3\angle BOC$

$2 \times 40 = 3 \times 40$

$x = 80°$

$y = 120$

$2y = 3x$

$y = \frac{3x}{2}$

$\frac{3x}{2} + 3x = 180$

$\frac{3}{2}x + 3x = 180°$

$\frac{9}{2}x = 180°$

$\boxed{x = 40°}$

(b) A sector of a circle has a perimeter of 112 cm and an area of 735 cm². Determine the radius of the circle. **(4 marks)**

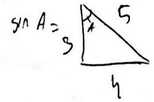
Question 5 **(9 marks)**

sch

Given that sin A = $\frac{4}{5}$ and $0 < A < \frac{\pi}{2}$, find the exact value of:

(a) cos A **(2 marks)**

$\sin A = \frac{4}{5}$



$\cos A = \frac{3}{5}$ 2

(b) tan A **(2 marks)**

$\tan A = \frac{4}{3}$ 1

(c) sin $(\frac{\pi}{2} + A)$ **(2 marks)**

$\sin\left(\frac{\pi}{2} + A\right) = \sin\frac{\pi}{2}\cos A + \cos\frac{\pi}{2} \times \sin A$ 1

$\sin\left(\frac{\pi}{2} + A\right) = 1 \times \frac{3}{5} + 0 \times \frac{4}{5}$

$\sin\left(\frac{\pi}{2} + A\right) = \frac{3}{5}$ 1

0.71

45°

$\cos \frac{\pi}{4}$

cos 30

(d) cos $(\frac{\pi}{4} - A)$ **(3 marks)**

$\cos\left(\frac{\pi}{4} - A\right) = \cos\frac{\pi}{4}\cos A + \sin\frac{\pi}{4}\sin A$

$= \frac{1}{\sqrt{2}} \cdot \frac{3}{5} + \frac{1}{\sqrt{2}} \cdot \frac{4}{5}$ 1

$= \frac{3}{5\sqrt{2}} + \frac{4}{5\sqrt{2}} = \frac{7}{5\sqrt{2}}$ 1 1

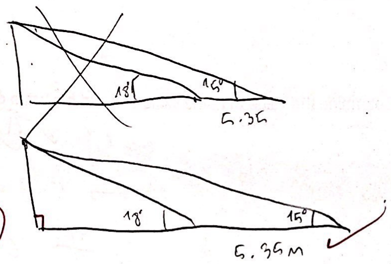
End of section one

Question 10 **(6 marks)**

A thin pole stands vertically in the middle of a level playing ground. From point A on the ground, the angle of elevation to the top of the pole, T, is 18°.

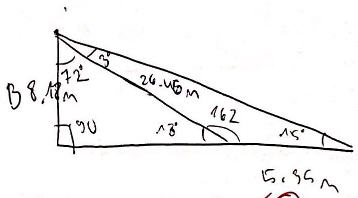
From point B, also on the ground but 5.35 metres further from the foot of the pole than A, the angle of elevation to the top of the pole is 15°.

(a) Draw a diagram to represent this information. **(1 marks)**



1

(b) Showing use of trigonometry, determine the height of the post. **(5 marks)**



$$\frac{5.35}{\sin 3} = \frac{x}{\sin 15} = 26.46\text{ m}$$ 1 1

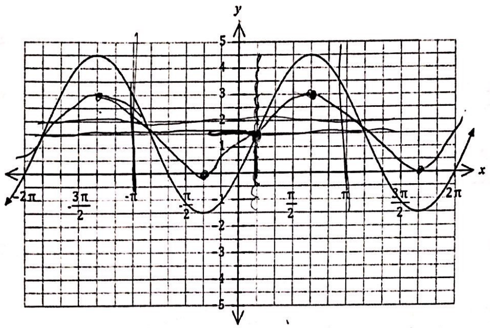
$$\frac{26.46}{90} = \frac{B}{\sin 18} = \boxed{8.1758}$$ 1 1

$\downarrow$

$\boxed{8.18\text{ m}}$ 1

Question 9 **(8 marks)**

The graph of $y = a + b\sin(x - c)$ is drawn below, where a, b and c are positive constants.



1,5

(a) Determine the value of a, the value of b and the value of c, where $c < \pi$. (3 marks)

$a = 1,5$ ✓ 1

$3\sin\left(x - \frac{\pi}{6}\right) + 1.5$ ~~b = 6~~ $b = 3$ ✓ 1

90°

$c = \frac{\pi}{6}$ ✓ 1

(b) On the same axes, draw the graph of $y = a + \frac{b}{2}\sin(x + c)$. (3 marks)

$y = 1.5\sin\left(x + \frac{\pi}{6}\right) + 1.5$ ✗

(c) Solve $b\sin(x - c) = \frac{b}{2}\sin(x + c)$ for $-\pi \le x \le \pi$. (2 marks)

✓

$3\sin\left(x - \frac{\pi}{6}\right) = 1.5\sin\left(x + \frac{\pi}{6}\right) + 1.5$

intersection: $(1.04719, 3)$

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Test 2

Trigonometric functions

Section Two (Calculator assumed)

20

Time Allowed: (5+50) minutes **Total mark available: 57**

Name: Chu Minh Phú

Question 6 **(8 marks)**

(a) Use the formula for sin (A + B) to show that sin 2A = 2sin A cos A. (2 marks)

$\sin A + B = \sin A \cos B + \cos A \sin B$

$\sin A + B = 1$ ✗

(b) Use the formula in (a) to solve the x in the trigonometric equation:

cos x. + sin 2x = 0 for $0 \le x \le 360°$. (3 marks)

$2A = 2\sin A \cos A$ ✗

(c) Use the formula in (a) to solve for x in the trigonometric equation:

sin 2x – sin x = 0 for $0 \le x \le 2\pi$. (3 marks)

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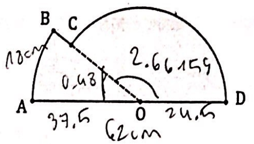
Question 7 **(5 marks)**

Shape $ABCDOA$ below consists of sector AOB of circle centre O joined to sector COD of a different circle, also centre O. AD is a straight line of length 62 cm, arc AB is 18 cm long and $\angle AOB = 0.48$ radians.



(a) Determine the length OA. (2 marks)

$l = r\theta$

$18 = r\,0.48$ ✓ (1)

$\frac{18}{0.48} = r$

$r = 37.5$ ✓ (1)

(b) Determine the area of the shape. (3 marks)

$180 \times \frac{\pi}{180} = \pi$

Area of sector small

$A = \frac{1}{2} \times 37.5^2 \times 0.48$

~~A = cm²~~

$A = 337.5\ cm^2$ ✓ (1)

Area big

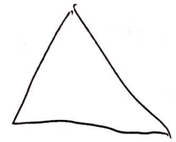
$A = \frac{1}{2}\,24.5^2 \times (\pi - 0.48)$

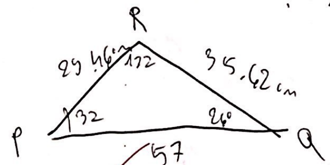
$A = 798.81\ cm^2$ ✓ (1)

$\boxed{A\ total} = 1136.3\ cm^2$ ✓ (1)

Question 8 **(8 marks)**

(a) Determine the area of triangle PQR when $\angle PQR = 26°$, $\angle PRQ = 122°$ and $PQ = 57$ cm. (4 marks)





$\frac{a+b+c}{2} = \frac{122.1}{2} = 61.0$

$s = 61.04$ (1)

~~$A = \sqrt{61.04}$~~

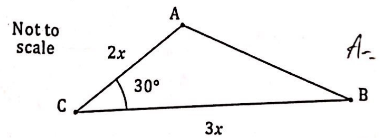
✓ (1)

$\frac{57}{\sin(122)} = \frac{x}{\sin 26}$ $x = 29.46$ cm ✓ (1)

$A = \sqrt{61.04(61.04 - 29.46)\ldots}$

$\boxed{A = 445\ cm^2}$ ✓ (1)

(b) The area of triangle ABC is 96 cm², $\angle ACB = 30°$ and $2BC = 3AC$ as shown in the diagram. Determine the length of AB. (4 marks)



$A = 96$

$2BC = 3AC$

$2(3x) = 3(2x)$ ✗

$6x = 6x$

$c^2 =$