



ALL SAINTS'
COLLEGE

Mathematics
Specialist
Test 3 2016

Vectors in 3D

NAME: _____

TEACHER: MLA

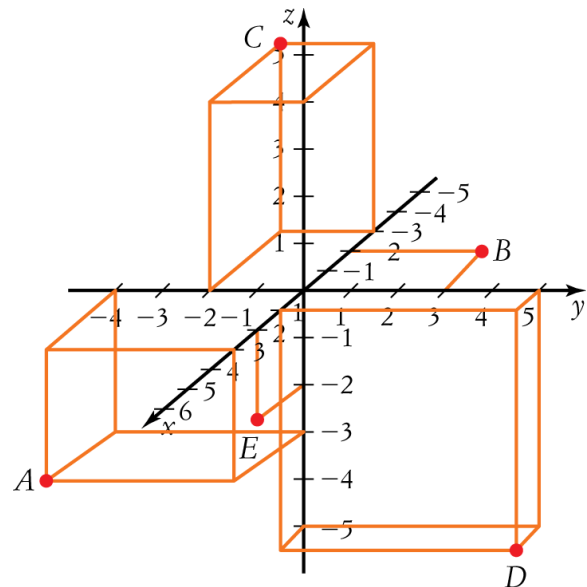
45 marks

45 minutes

One unfolded A4 page of notes, SCSA formulae booklet and ClassPad calculator permitted

Question 1 [2 marks]

Use the diagram below to determine the (exact) distance from C to D:



Question 2 [1, 1 & 1 = 3 marks]

Vector \mathbf{u} has norm 8, and both azimuthal (in the x-y plane) and altitude angles of 60° .

Vector $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

- (a) State the (exact) rectangular form for \mathbf{u}

- (b) Find the acute angle between \mathbf{u} and \mathbf{v} , correct to two decimal places.

- (c) Determine the (exact) vector orthogonal to both \mathbf{u} and \mathbf{v}

Question 3 [4 marks]

Find the projection of $\mathbf{a} = (-4, 2, -3)$ on $\mathbf{b} = (2, -5, 1)$

Question 4 [4 marks]

C is the midpoint of the line segment AB. D is a point **not** on the line AB such that $DC=CA$.

Use vector methods to prove that DA is perpendicular to DB.

Question 5 [4 marks]

Find the angle between the line $r = \begin{bmatrix} 1-2\lambda \\ \lambda \\ 2+3\lambda \end{bmatrix}$ and the plane $r \cdot \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = -4$

Question 6 [5 marks]

Find the equation of the plane containing the points E $(4, -3, 1)$, F $(5, 0, 2)$ and G $(3, 2, 5)$

Question 7 [4 & 1 = 5 marks]

- (a) Determine the vector equation of the plane consisting of all points that are equidistant from the points P $(-1, 2, -3)$ and Q $(4, -2, 2)$.
- (b) Hence, or otherwise, state the Cartesian form of the plane in (a)

Question 8 [5 & 2 = 7 marks]

Use elementary row operations to reduce the following system of equations to echelon form:

$$\begin{aligned}x + y + (k - 3)z &= 1 \\ 2x + 4y + 4z &= 3 \\ -x + y + (7 - 2k)z &= m\end{aligned}$$

(a) State the values of k and m if the system is to have:

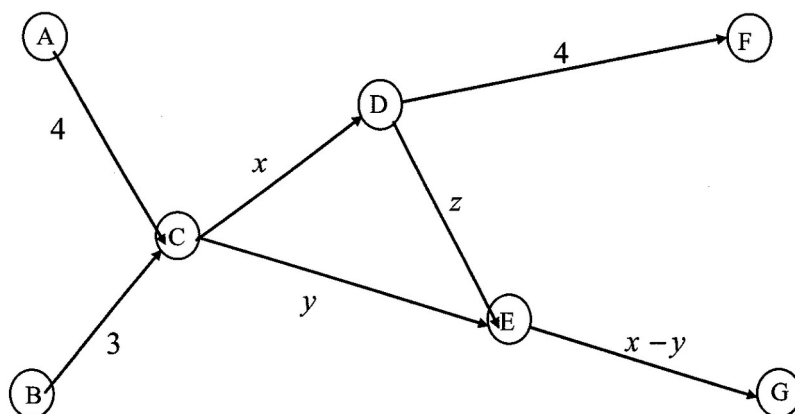
- (i) A unique solution
- (ii) Infinite solutions
- (iii) No solutions

(b) In the case of infinite solutions, find the particular solution for which $x=1$.

Question 9 [6 marks]

The schematic diagram below shows the volume of passengers (in tens of thousands) through airports A, B, C, D, E, F and G in the month of January.

Use Gaussian elimination to find the values of x , y and z if there are 10 000 less arrivals than departures at C, an equal number of arrivals and departures at D, and 20 000 more departures than arrivals at E.



Question 10 [5 marks]

At $t=0$, the position (\mathbf{r}) and velocity (\mathbf{v}) vectors of a British warship (B) and German submarine (G) are as follows:

$$r_B = \begin{bmatrix} 1150 \\ 827 \\ 0 \end{bmatrix} m; v_B = \begin{bmatrix} 10 \\ -2 \\ 0 \end{bmatrix} m/sec \qquad r_G = \begin{bmatrix} 1345 \\ 970 \\ 0 \end{bmatrix} m; v_G = \begin{bmatrix} -5 \\ -13 \\ -4 \end{bmatrix} m/sec$$

If both vessels maintain their respective velocities, determine the time at which the submarine is directly beneath the warship, and the depth of the submarine at this instant.

Note. The x-y plane represents the surface of the ocean

End of Test 3