

Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS SPECIALIST UNITs 3 & 4

Section Two:

Calculator-assumed

Your Name

Your Teacher's Name

Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,

correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,

and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks awarded	Question	Marks awarded
9		15	
10		16	
11		17	
12		18	
13		19	

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14	20	
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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	12	12	100	100	65
	•			Total	100

Instructions to candidates

- 1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
- 2. Write your answers in this Question/Answer booklet.
- 3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
- 4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
- 5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- 6. It is recommended that you do not use pencil, except in diagrams.
- 7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

MATHEMATICS

4CALCULATOR-ASSUMED

Section Two: Calculator-assumed

(100 Marks)

This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the
 original answer space where the answer is continued, i.e. give the page number. Fill in the
 number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 (6 marks)

For a brand of Marvel breakfast cereal the weight of cereal in packets that claim to contain 750grams is actually normally distributed with a mean of 758 grams and standard deviation 5 grams.

(a) What percentage of cereal packets will be under the stated weight?

(2 marks)

(b) A random sample of 15 packets is selected and the sample mean is taken. What is the probability that the sample mean of these 15 packets is 760 grams when rounded to the nearest gram?

(2 marks)

(c) The confidence interval $755.99 \le \mu \le 760.01$ was obtained for a sample of 15 packets. Determine the level of confidence to 0.1%. (2 marks)

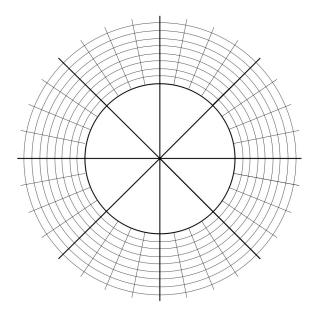
Question 10 (9 marks)

a) Determine all the roots of the equation $z^4=2$ - 2i, expressing them all in polar form with $r \ge 0$ and $-\pi < Argz \le \pi$

(3 marks)

(b) Plot all of the above roots on the diagram below.

(3 marks)



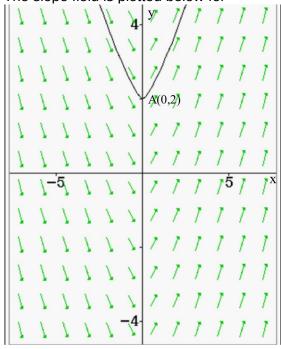
(c) Consider the roots in the first and second quadrants. Determine in polar form, the midpoint of the line joining these two roots..

(3 marks)

Question 11 (8 marks)

$$\frac{dy}{dx} = x^{\frac{1}{3}}$$

The slope field is plotted below for dx



(a) Determine the value of the slope field at point A(0,2) (2 marks)

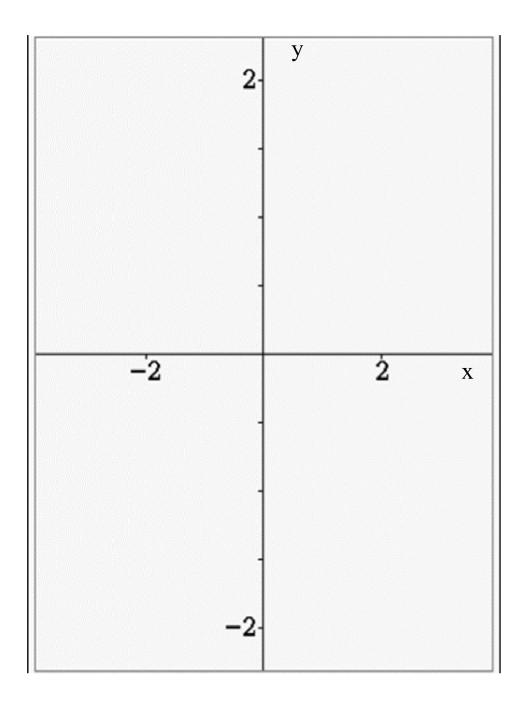
(b) Determine the equation for the line of force that passes through point A(0,2) (3 marks)

Q11 cont-

$$\frac{dy}{dx} = x(x-2)$$

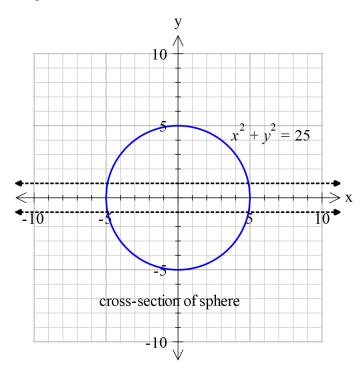
Sketch the slope field for $\,d{\it x}$ (c)

on the diagram below. (3 marks)



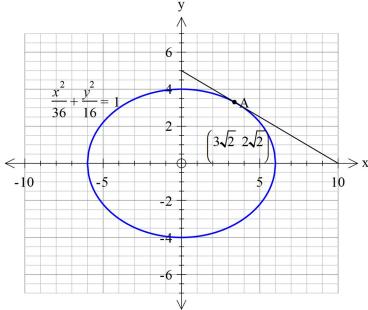
Question 12 (6 marks)

A solid sphere of radius 5 cm has a hole drilled through the centre with a diameter of 2 cm. See diagram below. Determine the exact volume of the remaining solid



Question 13 (7 marks)

Consider the ellipse
$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$
 which contains the point $A(3\sqrt{2}, 2\sqrt{2})$



(Note-tangent line is not drawn to scale)

a) Determine the equation of the tangent to the ellipse at point A. (4 marks)

b) Determine the area that is bound between the y axis, ellipse and the tangent line at A above to two decimal places. (3 marks)

Question 14 (11 marks)

The working length of an electric cell is found to be Normally distributed. A sample of 20 such electric cells were found to have a sample mean of 1138 working hours and a sample deviation of 250 hours. The customer needs to purchase electric cells that will have a mean working time in excess of 1250 hours.

a)	Determine a 95% confidence interval for the population mean working time decimal places.	to two
	·	(4 marks)
b)	Should the customer buy this type of electric cell? Justify your answer.	(2 marks)

- c) If a single cell was selected, would you expect the working time to lie in the confidence interval in part a above? Explain. (2 marks)
- d) A second sample of n cells gave a sample deviation of 250 hours. A 86% confidence interval was calculated to be $1069.5 \le \mu \le 1234.5$. Determine the value of n. (3 marks)

Question 15 (12 marks)

Consider a particle that is projected into the air such that its position vector at any time,

 $r = \begin{pmatrix} 50t \\ 30t - 5t^2 \end{pmatrix}_{\text{metres.}}$

(a) Determine the exact initial velocity.

(2 marks)

(b) Determine the time taken to hit the ground and the horizontal distance covered.

(3 marks)

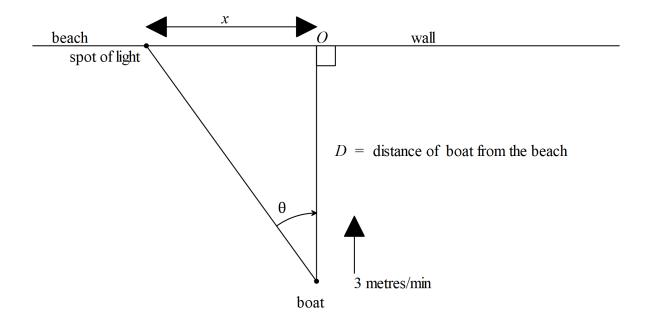
(c) Determine $\frac{dy}{dx} & \frac{d^2y}{dx^2}$ when t = 2 seconds. (4 marks)

(d) Determine the total distance travelled through the air to the nearest cm. (3 marks)

Question 16 (6 marks)

A boat is moving towards the beach line at 3 metres/minute. On the boat is a rotating light, revolving at 4 revolutions per minute clockwise, as observed from the beach. There is a long straight wall on the beach line, as the boat approaches the beach, the light moves along the wall. Let X = the displacement of the light from point O on the wall, which faces the boat directly. See diagram below.

Determine the velocity, in metres/minute, of the light on the wall when $^\chi=5$ metres and the distance of the boat from the beach, D, is 12 metres.



Question 17 (8 marks)

The number of bacterial cells, $\,^{N}$ grams, present in a petri dish in a science lab is given by the

$$\frac{dN}{dN} = 200N - 4N^2$$

 $\frac{dN}{dt} = \!\! 200\,N - 4\,N^2$ logistical growth model $\frac{dN}{dt}$, at time t hours. The initial number of bacteria cells is 5 grams.

- Determine the positive value of $\frac{dN}{dt}$ =0 . What does the value represent? (a) (2 marks)
- $N = \frac{50}{1 + Ce^{-200t}}$ By using separation of variables and partial fractions show that (b) Determine the value of ${\cal C}$. (4 marks)

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(c) Determine the time taken for the number of cells to triple.

(2 marks)

Question 18 (10 marks)

An object oscillates around a fixed point, O such that its displacement from O, x metres, is given by \ddot{x} =- 16x at time t seconds. The maximum speed is 20 metres/second and is also the initial velocity.

(a) Determine the distance from O where the speed of the particle will be half of the maximum. (4 marks)

(b) Determine the distance travelled by the particle in the first 2 seconds to the nearest cm. (3 marks

(c) Determine the percentage of the time that the particle is more than one quarter of the amplitude away from O . (3 marks)

Question 19 (11 marks)

Consider a Cartesian equation of the plane x + z = 0 and a vector equation of the line

$$r = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ -5 \end{pmatrix}$$

Determine

(a) the position vector of the point of intersection of the line and the plane above.

(3 marks)

(b) the length of the perpendicular from the origin to the line above, to two decimal places. (3 marks)

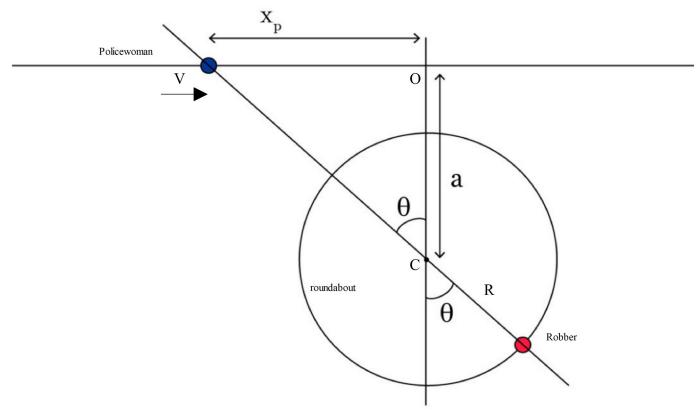
(c) a cartesian equation of a new plane which contains the line above and the origin. (3 marks)

(d) the acute angle between the planes from (a) & (c) above.

(2 marks)

Question 20 (6 marks)

A policewoman moves along a straight road on a motorcycle with constant speed V. Let X_P = distance of policewoman from point O on the road. A robber is on a second motorcycle moving on a roundabout of radius R. The robber wishes to stay on a line of sight with the policewoman such that this line will always go through the centre of the roundabout C as the policewoman approaches point O on the road. The centre C of the roundabout is C units away from the road.



Determine the speed on the roundabout that the robber must move at to maintain the line of sight with the policewoman through the centre of the roundabout in terms of X_P,V,a & R only.

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Additional working space

Question number:

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Additional working space

Question number:

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