Heat Problems

specific heat of water = 4.18×10^3 $Jkg^{-1}K^{-1}$
specific heat of ice = 2.10×10^3 $Jkg^{-1}K^{-1}$
specific heat of steam = 2.00×10^3 $Jkg^{-1}K^{-1}$
specific heat of steel = 4.50×10^2 $Jkg^{-1}K^{-1}$
latent heat of vaporization water = 2.26×10^6 Jkg^{-1}
latent heat of fusion water = 3.34×10^5 Jkg^{-1}
specific heat of copper = 3.85×10^2 $Jkg^{-1}K^{-1}$
specific heat of aluminium = 8.80×10^2 $J\,kg^{-1}\,K^{-1}$

NOTE: Value for Aluminium is 9.00×10^2 $J\,kg^{-1}\,K^{-1}$ in Exploring Physics.

1. 0.1 kg of an unknown metal is found to require 3.5 kJ to change its temperature from 25^0C to 82^0C. What is the specific heat of the metal?

$$\mathbf{Q = mc\Delta T}$$
$$\mathbf{3500 = 0.1 \times c \times 57}$$
$$c = \frac{3500}{(0.1 x 57)}$$
$$\mathbf{c = 6.1 \times 10^2\ J\ kg^{-1}\ K^{-1}}$$

2. The specific heat of copper is 3.85×10^2 $J\,kg^{-1}\,K^{-1}$. A specific mass of copper has 1.74×10^4 J of energy added to it to change its temperature from 20^0C to 80^0C. What was the mass of copper?

$$\mathbf{Q = mc\Delta T}$$
$$\mathbf{1.74 \times 10^4 = m \times 385 \times 60}$$
$$m = \frac{1.74 x 10^4}{(385 x 60)}$$
$$\mathbf{m = 0.75\ kg}$$

3. If 15.7 kJ of heat energy is added to 250 mL of water at 20^0C, what will the new temperature be?

$$\mathbf{Q = mc\Delta T}$$
$$\mathbf{15.7 \times 10^3 = 0.25 \times 4180 \times (T_f - 20)}$$
$$\mathbf{T_f - 20} = \frac{15.7 x 10^3}{(0.25 x 4180)}$$
$$\mathbf{T_f - 20 = 15.024}$$
$$\mathbf{T_f = 35^0\ C}$$

4. Over a period of 6 hours, a hot water bottle cools from 95^0C to 20^0C. If the hot water bottle held 2.5 L water, what is the rate of cooling in Js^{-1}?

$$\mathbf{Q = 2.5 \times 4180 \times 75}$$
$$\mathbf{= 783750\ J}$$

$$\textbf{Energy per second} = \frac{783750}{(6 \, x \, 60 \, x \, 60)}$$
$$= \mathbf{36\ Js^{-1}}$$

5. A kettle rated at 2000 W contains 1.8 L water at 15^0C. If it runs for 3.5 minutes, will the water boil?

$$P = \frac{W}{t} = \frac{Q}{t}$$

Q = P x t

= 2000 x 3.5 x 60

= 420000 L

Q = mcΔT

420000 = 1.8 x 4180 x (T_f – 15)

$$T_f - 15 = \frac{420000}{(1.8 \, x \, 4180)}$$

T_f – 15 = 55.82

T_f = 55.82 + 15

T_f = 71^0C

So water will not boil.

6. How much heat energy is released when 423 g of steam at 100^0C condenses to water also at 100^0C?

Q = mL

= 0.423 x 2.26 x 10^6

Q = 9.6 x 10^5 J

7. 4.87 x 10^5 J of heat are added to a mass of ice at 0^0C. If the ice melts and becomes water at 21.5^0C, what was the mass of ice?

Q = melt ice + heat water

Q = mLf + mcΔT

4.87 x 10^5 = (m x 3.34 x 10^5) + (m x 4180 x 21.5)

4.87 x 10^5 = 3.34 x 10^5m + 89870m

4.87 x 10^5 = 423870 m

m = 1.15 kg ice

8. At what rate in Js^{-1} is a refrigerator absorbing heat if 2.15 kg of water at 21.5^0C is just frozen in 2.0 hours?

$Q =$ (cool water) + (freeze water)

$= (mc\Delta T) + (mL_f)$

$= (2.15 \times 4180 \times 21.5) + (2.15 \times 3.34 \times 10^5)$

$= 193220.5 + 718100$

$= 911320.5$ J per 2 hours

rate in Js^{-1} $= \dfrac{911320.5}{(2 \times 60 \times 60)}$

$= 127\ Js^{-1}$

9. 20 g of milk at 5.0^0C is added to 250 g of coffee at 90^0C. What is the final temperature of the drink? (Specific heats: milk: $3.9 \times 10^3\ Jkg^{-1}K^{-1}$, coffee $4.10 \times 10^3\ Jkg^{-1}K^{-1}$.)

Heat lost = Heat gained

$mc\Delta T_{coffee} = mc\Delta T_{milk}$

$0.25 \times 4100 \times (90 - T_f) = 0.02 \times 3900 \times (T_f - 5)$

$92250 - 1025T_f = 78T_f - 390$

$92250 + 390 = 78T_f + 1025T_f$

$92640 = 1103T_f$

$T_f = \dfrac{92640}{1103}$

$T_f = 84^0C$

10. 100 g of a metal at 95^0C is added to 500 mL of water at 2.0^0C. If the final temperature of the water is 3.6^0C, what is the specific heat of the metal?

Heat lost = Heat gained

$(mc\Delta T)_{metal}$ $(mc\Delta T)_{water}$

$0.1 \times c \times (95 - 3.6) = 0.5 \times 4180 \times (3.6 - 2)$

$9.14c = 3344$

$c = \dfrac{3344}{9.14}$

11. How much heat energy is needed to change 1.0 kg of ice at –3.0^{0}C to steam at 107^{0}C?

Q = heat ice + melt ice + heat water + boil water + heat steam

$= (mc\Delta T)_{ice} + (mL_f) + (mc\Delta T)_w + (mL_v) + (mc\Delta T)_{steam}$

$= (1 \times 2100 \times 3) + (1 \times 3.34 \times 10^5) + (1 \times 4180 \times 100) + (1 \times 2.26 \times 10^6) + (1 \times 2000 \times 7)$

$= 6300 + 3.34 \times 10^5 + 418000 + 2.26 \times 10^6 + 14000$

$= 3032300$

$Q = 3.03 \times 10^6 \text{ J}$

12. How much ice at 0^{0}C must be added to 250 mL of coffee (specific heat: 4.10 x 10^3 Jkg^{-1}K^{-1}) in an insulated cup (assume no loss of heat to the container and surroundings) to cool the coffee from 95^{0}C to 65^{0}C?

Q lost = Q gained

coffee at 95°C = ice at 0°C

$(mc\Delta T)_{coffee} = (mL_f)_{ice} + (mc\Delta T)_{ice\ water}$

$0.25 \times 4100 \times (95-65) = (m \times 3.34 \times 10^5) + (m \times 4180 \times 65)$

$= 3.34 \times 10^5 m + 271700m$

$30750 = 601700m$

$m = \dfrac{30750}{601700}$

$m = 0.051$ kg ice

13. Copper calorimeters are used to determine the specific heat of unknown substances. A calorimeter of mass 41 g, has 100 mL of water at 15^{0}C placed in it. 50 g of iron is heated to 160^{0}C then carefully lowered into the water. What would be the final temperature of the water? (specific heat of copper is 385 Jkg^{-1}K^{-1} , iron is 477 Jkg^{-1}K^{-1})

Heat lost = Heat gained

iron = water + calorimeter

$(mc\Delta T)_{iron} = (mc\Delta T)_w + (mc\Delta T)_{cal}$

$0.05 \times 477 \times (160 - T_f) = (0.1 \times 4180 \times [T_f - 15]) + (0.041 \times 385 \times [T_f - 15])$

$3816 - 23.85T_f = 418T_f - 6270 + 15.785T_f - 236.775$

$3816 + 6270 + 236.775 = 23.85T_f + 15.785T_f + 418T_f$

$10322.775 = 457.635T_f$

$T_f = 22.6^oC$

14. 5.0 g of ice at –2.0^{0}C is placed into a 78 g copper calorimeter containing 120 mL of water at 90^{0}C. The water is stirred until all the ice has dissolved. What is the final temperature of the water?

Heat lost = Heat gained

$(mc\Delta T)_{water} + (mc\Delta T)_{cal} = (mc\Delta T)_{ice} + (mL_f) + (mc\Delta T)_w$

$(0.12 \times 4180 \times (90-T_f)) + (0.078 \times 385 \times (90 - T_f))$
$= (0.005 \times 2100 \times 2) + (0.005 \times 3.34 \times 10^5) + (0.005 \times 4180 \times T_f)$

$45144 - 501.6T_f + 2702.7 - 30.03T_f = 21 + 1670 + 20.9T_f$

$-501.6T_f - 30.03T_f - 20.9T_f = -45144 - 2702.7 + 21 + 1670$

$-552.53T_f = -45603.17$

$T_f = \dfrac{46156.7}{552.53}$

$T_f = 83.5^oC$

15. A 5.45 kg steel container contains 12.0 kg of water at 22.0^{0}C. When 2.65 kg of molten alloy (latent heat of fusion 2.50 x 10^4 J kg^{-1} K^{-1}) at its melting point of 327^{0}C is poured into the water the final temperature reached is 27.8^{0}C. Find the specific heat of the alloy.

Heat lost = Heat gained

(Solidify alloy) + (cool alloy) = heat container + heat water

$(mL_f) + (mc\Delta T)_{alloy} = (mc\Delta T)_{steel} + (mc\Delta T)_{water}$

$(2.65 \times 2.50 \times 10^4) + (2.65 \times c \times 299.2) = (5.45 \times 450 \times 5.8) + (12 \times 4180 \times$

$66250 + 792.88c = 14224.5 + 290928$

$792.88c = 14224.5 + 290928 - 66250$

$= 238902.5$

$c = \dfrac{238902.5}{792.88}$

$c = 301\ J\,kg^{-1}K^{-1}$

16. How much ice at -4.00^{0}C must be added to an aluminium calorimeter of mass 47.0 g containing 150 g of water at 95.0 ^{0}C so that the final temperature once the ice has fully melted is 70 ^{0}C?

Heat lost = heat gained

$(0.047 \times 880 \times 25) + (0.15 \times 4180 \times 25) = (m \times 2100 \times 4)$
$+ (m \times 4180 \times 70) + (m \times 3.34 \times 10^5)$

$1034 + 15675 = 8400m + 292600m + 3.34 \times 10^5 m$

$16709 = 635000m$

$m = 0.0263\ kg$