WESLEY COLLEGE

By daring & by doing

2018 YEAR 12 MATHEMATICS METHODS

Logarithms & Differentiation Applications

Test 2

Name: ______________________________ Marks: /50

Calculator Free (22 marks)

1. [2, 2 = 4 marks]

The displacement for an object is given by $x = e^{2t}\sin(t)$, where x is in metres and t is in seconds.

a) Determine the velocity equation.

b) Show that when the object is at rest, $\tan(t) = -\frac{1}{2}$

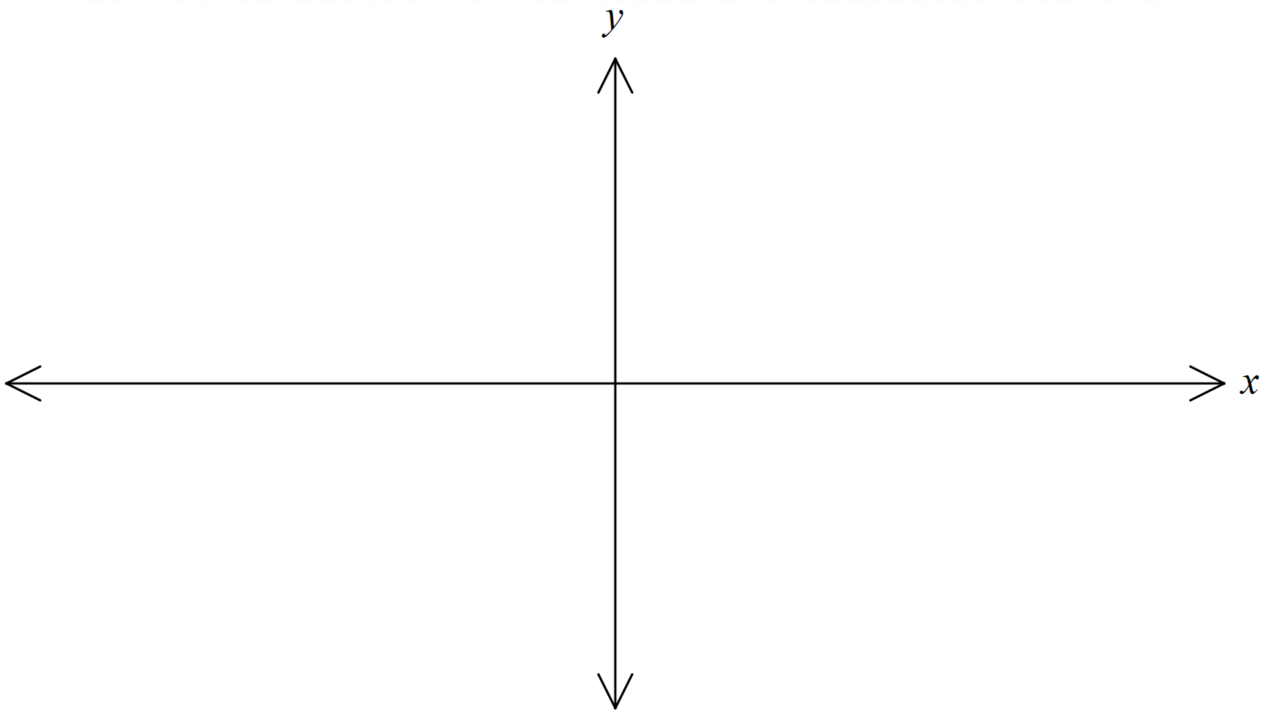
2. [2, 3, 2 = 7 marks]

Given the function: $y = x^4 + 4x^3 - 16x + 3$.

a) The gradient function, $y' = 4x^3 + 12x^2 - 16$ can be factorised as $y' = 4(x+2)^2(x-1)$
By using relevant tests to justify your answers, find and state the nature of all stationary points.

b) Using relevant tests to justify your answers, find and state all points of inflection.

c) Sketch the graph of $y = f(x)$, showing <u>all important features</u>.



3. [3 marks]

Given that $y = \sqrt{x}$, use the incremental formula $\delta y \approx \frac{dy}{dx} \times \delta x$ to determine an approximate value for $\sqrt{50}$.

4. [2, 2, 4 = 8 marks]

a) Evaluate $\log_x x^2 - 6\log_x y + 3\log_x (xy)^2$

b) Given $2\log_n x - 1 = \log_n 25$, write x in terms of n.

c) If $5^x = 3$ and $5^y = 4$, express in terms of x and/or y:

(i) $\log_5 0.75$

(ii) $\log_5 100$

End of Part A

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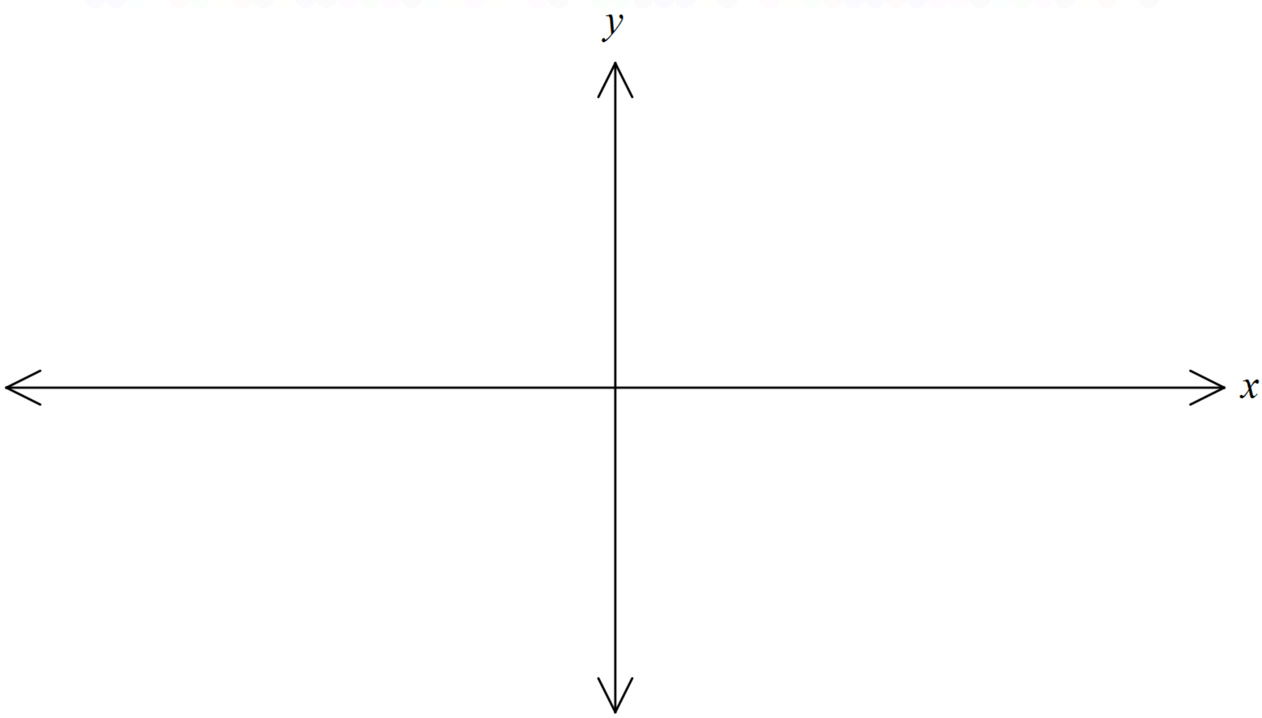
Calculator Section **(28 marks)**

5. [4 marks]

Draw one possible function with all the following features:

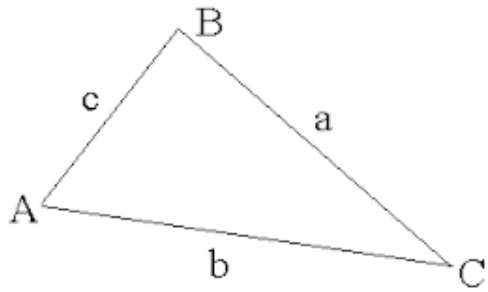
$$\frac{dy}{dx} = 0 \quad for \quad x = -3, \quad x = 2, \quad x = 5$$

$$\frac{d^2y}{dx^2} < 0 \;\; for \;\; x = -3 \qquad \frac{d^2y}{dx^2} > 0 \;\; for \;\; x = 2 \;\; and \;\; \frac{d^2y}{dx^2} = 0 \;\; for \;\; x = 5$$



6. [3 marks]

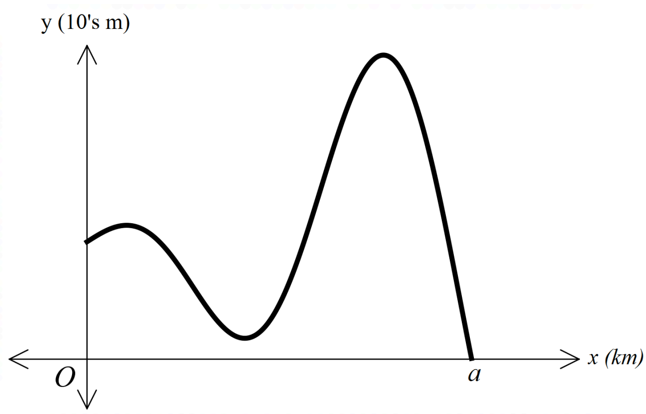
The area of a triangle can be calculated by the formula $Area = \dfrac{ab\sin C}{2}$.



Using the incremental formula, determine the approximate change in area of an equilateral triangle, with each side 20 cm, when each side increases by 0.1 cm.

7. [1, 1, 3 = 5 marks]

The cross-section of the Darling Range, east of Perth is shown below.



The cross-sectional curve is given by $y = x\cos(x) + 4,\ 0 \le x \le a$.

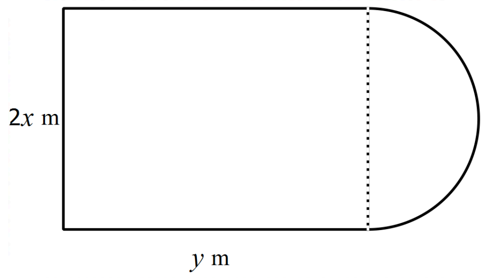
a) Determine the value of a to two decimal places.

b) Determine the height of the highest point on the range.

c) Moving away from O, down the far side of the range from the highest point, where is the steepest part of the hill side?

8. [2, 1, 4 = 7 marks]

A 200 m fence is placed around a lawn which has the shape of a rectangle with a semi-circle on one of its sides.



a) Using the dimensions on the diagram, clearly show that $y = 100 - x - \dfrac{\pi}{2}x$

b) Hence, determine the area of the lawn $A(x)$, in terms of x only.

c) Using calculus techniques determine the dimensions of the lawn if it has a maximum area and state this area.

9. [2, 2, 2, 3 = 9 marks]

A Cobalt projectile is travelling along a magnetic rail. It moves in a straight line such that its displacement from O, after t seconds is x metres where $x = t^3 - 10t^2 + 29t - 20$.

Determine, writing all relevant equations and giving answers correct to 2 decimal places:

a) the initial speed and acceleration of projection,

b) when the particle is at rest,

c) the particle's velocity when the acceleration is 0,

d) the total distance travelled in the first 6 seconds.

End of Part B