

Insert School Logo

**Semester Two
Examination 2018
Question/Answer booklet**

**MATHEMATICS
SPECIALIST UNITS 1 & 2**

**Section One:
Calculator-free**

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	7	7	50	50	35
Section Two Calculator—assumed	11	11	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.



Section One: Calculator–free**35% (50 marks)**

This section has **seven (7)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes

Question 1 (8 marks)

In each case below write expressions using the notation nC_r and/or nP_r as needed. Do NOT evaluate.

- (a) Five character passwords are created using the letters of the word QUESTION without repetition of characters. How many different passwords can be created if:
- (i) no other restrictions apply? (1 mark)
 - (ii) it contains 3 vowels and 2 consonants? (2 marks)
- (b) Six-digit PIN numbers are created using four digits from the set $\{1, 2, 4, 6, 8, 9\}$ and two digits from the set $\{0, 3, 5, 7, 9\}$. How many different PIN numbers can be created if:
- (i) repetition of digits is allowed? (1 mark)
 - (ii) repetition of digits is not allowed? (4 marks)

Question 2 (7 marks)

(a) Given that $i^n = -i$ determine each of the following:

(i) i^{n+2} (1 mark)

(ii) i^{2n+1} (2 marks)

(b) Rationalise and simplify. (2 marks)

$$\frac{1-i}{i+\frac{2}{i}}$$

(c) Simplify. (2 marks)

$$(1+i)^4 - (1-i)^4$$

Question 4 (12 marks)

(a) Consider the 3x3 matrices $A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$.

- (i) Obtain the matrix AB and conclude a relationship between A and B . (3 marks)

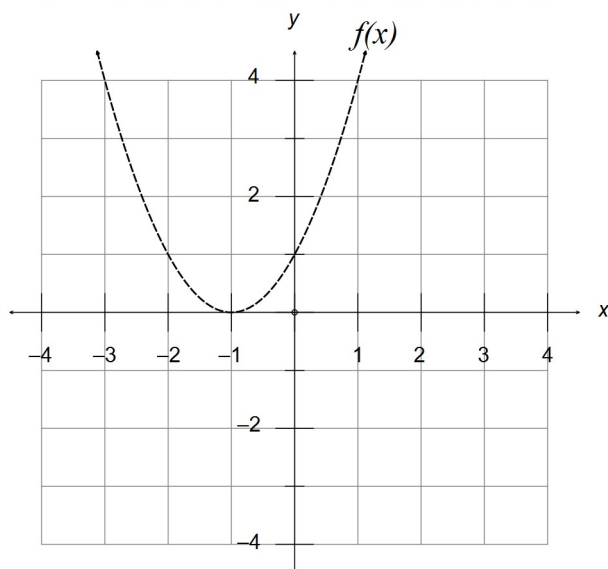
- (ii) Using your answers in (i), solve the system of linear equations below. (3 marks)

$$\begin{array}{rcl} -y+z & \text{?} & -6 \\ x+y-z & \text{?} & 8 \\ x+y & \text{?} & 3 \end{array}$$

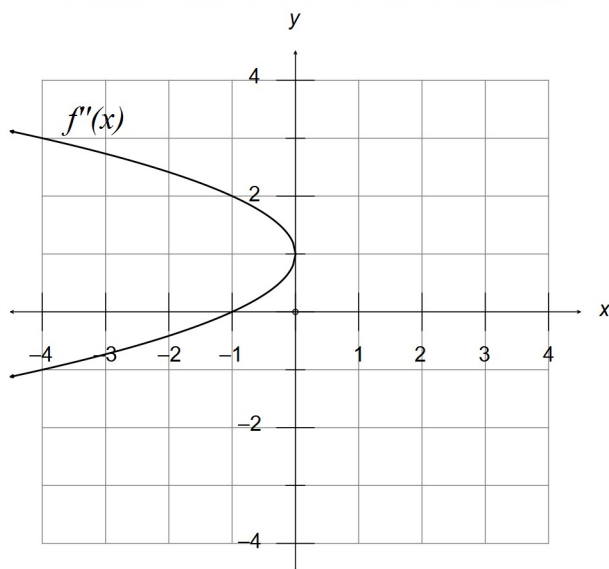
Question 4 (Continued)

(b) The function $f(x) = (x+1)^2$ is transformed by $T_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ to obtain $f'(x)$, the image of $f(x)$.

- (i) Sketch the graph of $f'(x)$ on the axes below and state its equation.
 $f(x)$ is shown as a reference. (2 marks)



- (ii) Transformation matrix T_2 is then applied to the function $f'(x)$ from (i) and gives the curve shown below. State the transformation matrix T_2 and describe its effect. (2 marks)

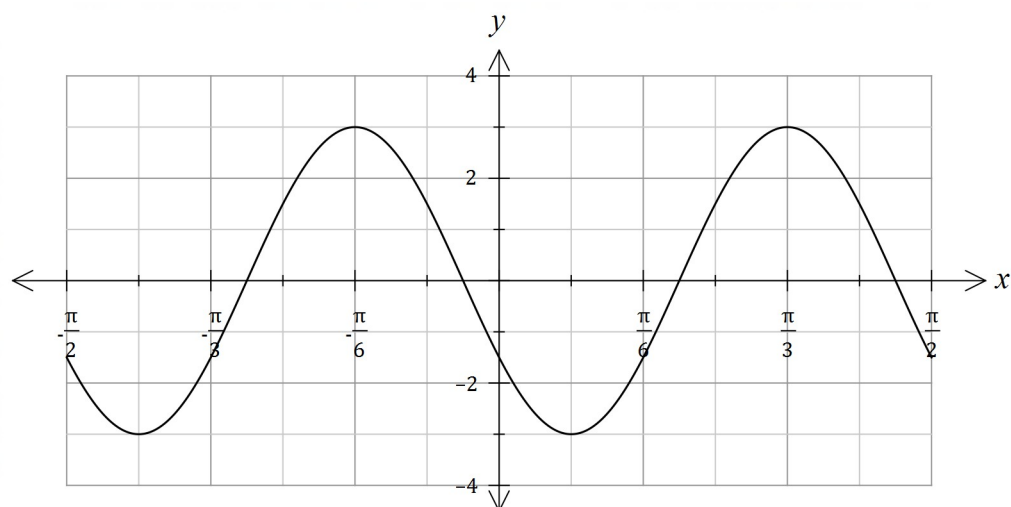


(c) The area enclosed by the function $f(x)$ and the line $y=4$ is $10\frac{2}{3}$ units².

Determine the new area of this region when $f(x)$ is transformed by $T_3 = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$. (2 marks)

Question 5 (8 marks)

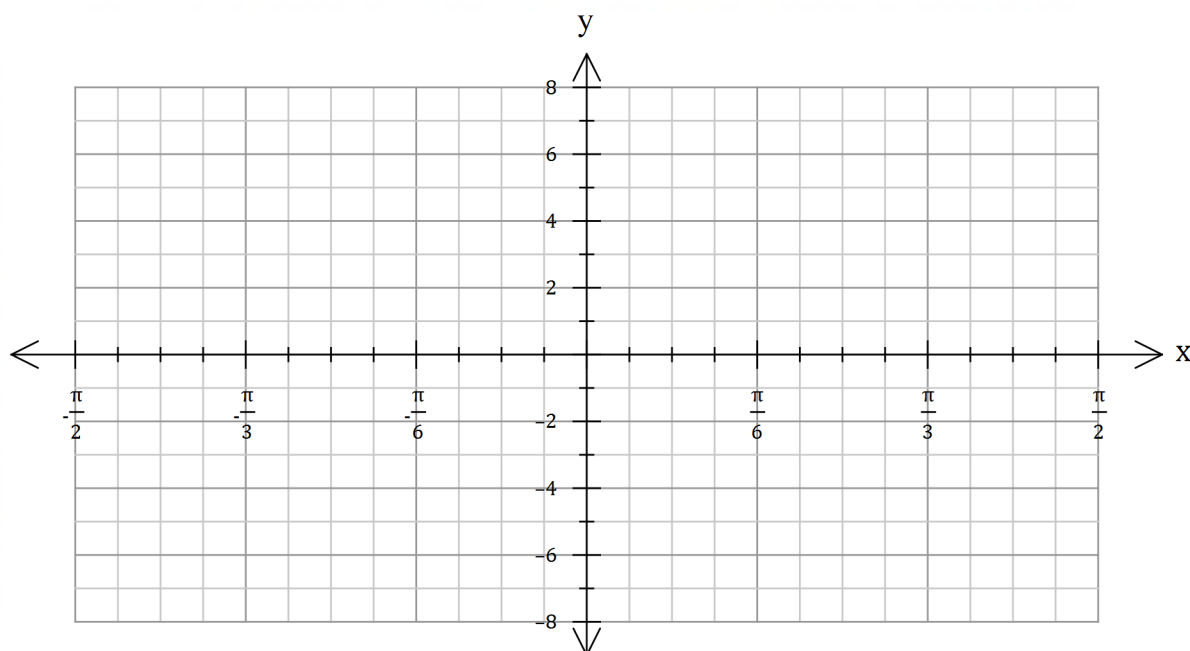
- (a) The curve below is of the format $y = A \cos(\omega x + \theta)$, where $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.



- (i) State the value of A , ω and θ . (3 marks)

- (ii) State the equation of the curve as a sine function with an acute phase shift. (2 marks)

- (b) Sketch the graph of $y = 2 \cot\left(2x - \frac{\pi}{6}\right)$ on the axes below, where $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. (3 marks)



Question 6 (6 marks)

(a) Consider the following conjecture: If a is irrational and b is irrational, then ab is irrational.

(i) Provide a counterexample to disprove it. (1 mark)

(ii) State the converse of this conjecture. (1 mark)

(b) Determine whether or not $A \Leftrightarrow B$, by considering the validity of $A \Rightarrow B$ and $B \Rightarrow A$.

A : The triangle has two equal sides.

B : The triangle has two congruent angles.

(2 marks)

(c) Rewrite the following statement using only mathematical symbology notation. (2 marks)

"For all irrational numbers x there exists an irrational number y such that their product is rational"

Question 7 (4 marks)

Use proof by contradiction to show that if n is an odd number, then n^3 is also an odd number.

(4 marks)

End of Questions

Additional working space

Question number(s):

Additional working space

Question number(s):