Papers written by
Australian Maths
Software

SEMESTER ONE

MATHEMATICS SPECIALIST

REVISION 2

UNIT 3

2016

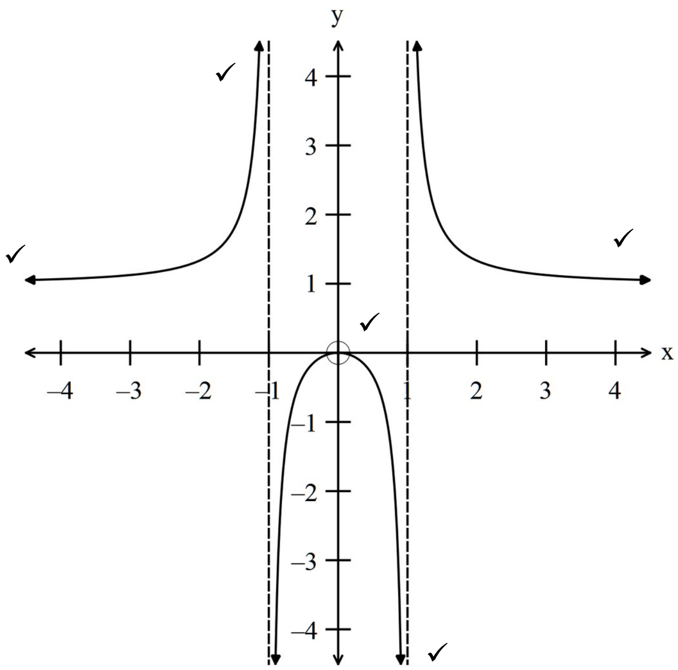
SOLUTIONS

Section One

1. (6 marks)

(a) $1+\dfrac{1}{x^2-1}=\dfrac{x^2-1+1}{x^2-1}=\dfrac{x^2}{x^2-1}$ ✓ (1)

(b) (5)



2. (10 marks)

(a)

$$\begin{bmatrix} 1 & 2 & 3 & 15 \\ 1 & -1 & -1 & -3 \\ 2 & 1 & 1 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 15 \\ 0 & 3 & 4 & 18 \\ 0 & 3 & 5 & 21 \end{bmatrix} \quad \begin{matrix} \\ R_1 - R_2 \\ 2R_1 - R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 15 \\ 0 & 3 & 4 & 18 \\ 0 & 0 & -1 & -3 \end{bmatrix} \quad \begin{matrix} \\ \\ R_2 - R_3 \end{matrix}$$

$-z=-3 \to z=3$

$3(y)+4(3)=18 \to y=2$

$x+2(2)+3(3)=15 \to x=2$

The point of intersection is $(2,2,3)$

(4)

(b) $\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 2p-7 & 2q-12 \end{bmatrix}$

(i) Exactly one solution if $2p-7 \neq 0 \Rightarrow p \neq 3.5$ ✓✓ (2)

(ii) There is no solution if $p = 3.5$ and $2q-12 \neq 0$ i.e. $q \neq 6$ ✓✓ (2)

(iii) There are inifinitely many solutions if $p = 3.5$ and $q = 6$ ✓✓ (2)

3. (13 marks)

(a) $\left(z-(1+2i)\right)\left(z-(1-2i)\right)\left(z-(3+i)\right)\left(z-(3-i)\right)$

$=\left[\left(z-(1+2i)\right)\left(z-(1-2i)\right)\right]\left[\left(z-(3+i)\right)\left(z-(3-i)\right)\right]$

$=\left[z^2-z\left(1+\cancel{2i}+1-\cancel{2i}\right)+(1+2i)(1-2i)\right]\left[z^2-z\left(3+\cancel{i}+3-\cancel{i}\right)+(3+i)(3-i)\right]$

$=\left(z^2-2z+1-4i^2\right)\left(z^2-6z+9-i^2\right)$

$=\left(z^2-2z+5\right)\left(z^2-6z+10\right)$

$=z^4-8z^3+27z^2-50z+50$

Therefore equation is $z^4-8z^3+27z^2-50z+50=0$

(3)

(b) $Let\ P(z)=z^3-z^2+3z+5$

$P(-1)=-1-1-3+5=0$

$\therefore\ z=-1$

Using synthetic division with $z=-1$ You can use long division but slower

z^3-z^2+3z+5

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 3 & 5 \\ & \downarrow & -1 & 2 & -5 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

✓ method

$\therefore\ z=-1$ OR $z^2-2z+5=0$

$z=\dfrac{2\pm\sqrt{4-20}}{2}$

$z=\dfrac{2\pm\sqrt{-16}}{2}=\dfrac{2\pm\sqrt{16i^2}}{2}$

$z=\dfrac{2\pm 4i}{2}$

$z=1\pm 2i \quad or \quad z=-1$

$\therefore z=-1$ so $z+1$ is a factor

$$\begin{array}{r} z^2-2z+5 \\ z+1\overline{)z^3-z^2+3z+5} \\ -\left(z^3+z^2\right) \\ \hline -2z^2+3z \\ -\left(-2z^2-2z\right) \\ \hline 5z+5 \\ -\left(5z+5\right) \\ \hline 0 \end{array}$$

$z=-1$ or $z^2-2z+5=0$

(c) (i) $z^4=-16$

$z^4 = 16cis(\pi + n \times 2\pi) \quad n \in R$

$z = 2\left(cis(\pi + 2n\pi)\right)^{\frac{1}{4}}$

$z = 2cis\left(\frac{\pi}{4} + \frac{n\pi}{2}\right)$

$n = 0, \quad z = 2cis\left(\frac{\pi}{4}\right) = \sqrt{2} + \sqrt{2}i$

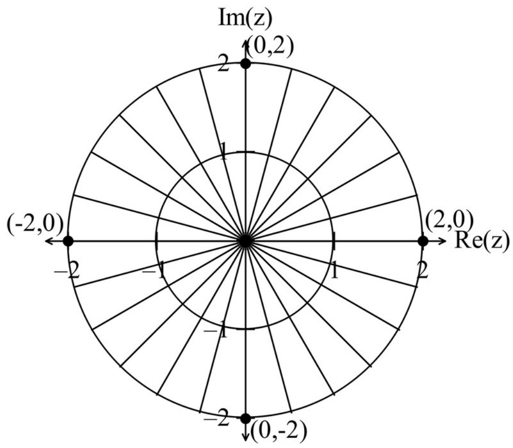
$n = 1, \quad z = 2cis\left(\frac{3\pi}{4}\right) = -\sqrt{2} + \sqrt{2}i$ ✓✓ (3)

$n = 2, \quad \cancel{z = 2cis\left(\frac{5\pi}{4}\right)}$

$n = -1, \quad z = 2cis\left(-\frac{\pi}{4}\right) = \sqrt{2} - \sqrt{2}i$

$n = -2, \quad z = 2cis\left(-\frac{3\pi}{4}\right) = -\sqrt{2} - \sqrt{2}i$

(ii)



✓✓ (2)

(iii) $z^4 = -16$ is equivalent to $z = 2cis\left(\frac{\pi}{4} + \frac{n\pi}{2}\right)$

whereas $z^4 = 16$ is equivalent to $z = 2cis\left(0 + \frac{n\pi}{2}\right)$ which means the

starting positions of the roots are different $\frac{\pi}{4}$ apart..

The roots themselves are $\frac{\pi}{2}$ apart and the roots of the two equations

start $\frac{\pi}{4}$ apart. ✓ (1)

4. (13 marks)

(a) $$\left(cis\left(\frac{\pi}{4}\right)\right)^5+(1-i)^5=\left(cis\left(\frac{\pi}{4}\right)\right)^5+\left(\sqrt{2}\,cis\left(-\frac{\pi}{4}\right)\right)^5 \quad ✓$$

$$=cis\left(\frac{5\pi}{4}\right)+\left(\sqrt{2}\right)^5 cis\left(-\frac{5\pi}{4}\right)$$

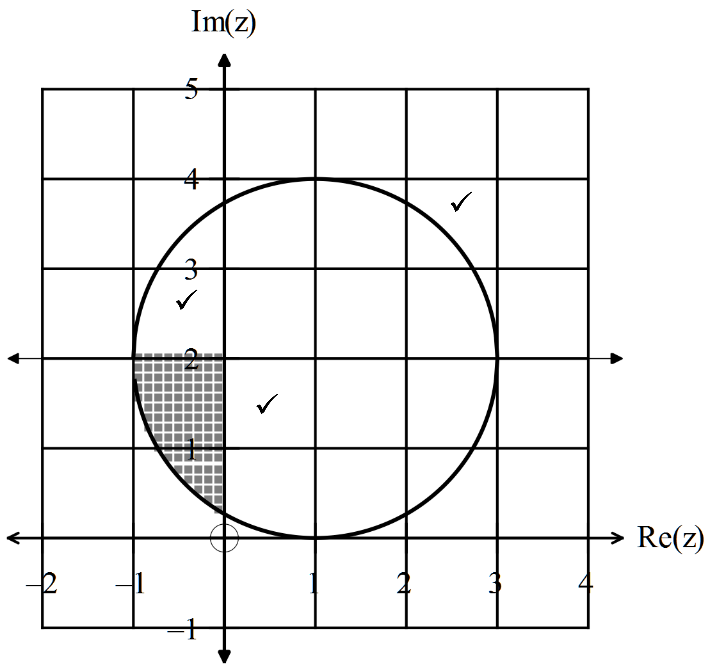
$$=-\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}+4\sqrt{2}\left(-\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right) \quad ✓$$

$$=-\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}+4(-1+i) \quad ✓$$

$$\left(cis\left(\frac{\pi}{4}\right)\right)^5+(1-i)^5=\left(-4-\frac{1}{\sqrt{2}}\right)+i\left(4-\frac{1}{\sqrt{2}}\right)$$

(3)

(b)



(c) $|z+1|=|z-i|$ ✓✓ (2)

(d) $$z=\frac{\left(cos\left(\frac{\pi}{3}\right)+i\,sin\left(\frac{\pi}{3}\right)\right)}{\left(cos\left(\frac{4\pi}{3}\right)+i\,sin\left(\frac{4\pi}{3}\right)\right)}=cis\left(\frac{\pi}{3}-\frac{4\pi}{3}\right)=cis(-\pi) \quad ✓$$

mod$(z)=1$ arg$(z)=\pi$ ✓

(e) $z=\dfrac{(3-2i)}{(4+3i)}\times\dfrac{(4-3i)}{(4-3i)}=\dfrac{6-17i}{25}$ ✓ ✓ $\quad Re(z)=\dfrac{6}{25}$ ✓ (2)

5. (8 marks)

(a) $(g(x))^2=(1-x)^2 \qquad f^{-1}(x)=x-1$ ✓

$$(g(x))^2=f^{-1}(x)$$

$$(1-x)^2=x-1$$

$$i.e.\ (x-1)^2=x-1$$ ✓

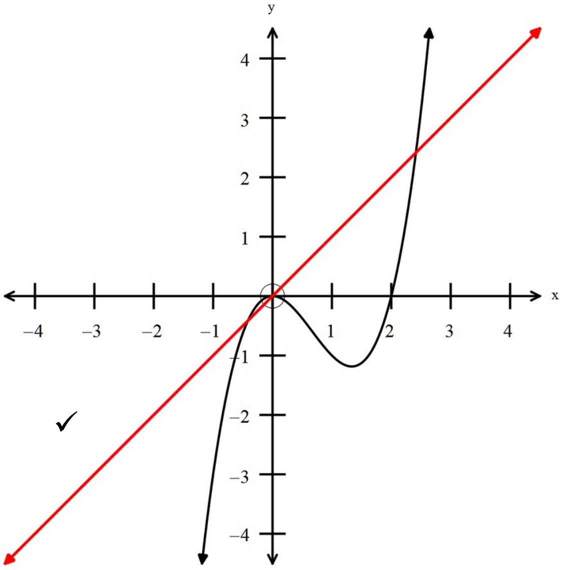
$$(x-1)^2-(x-1)=0$$

$$(x-1)[x-1-1]=0$$

$$x=1 \ or \ x=2$$ ✓

(b) (i) $x\le 0$ ✓✓ *Answers will vary* (2)

(ii) $y=x$ ✓



✓

(2)

(iii) $f^{-1}(32)=4$ ✓ (1)

END OF SECTION ONE

Section Two

6. (6 marks)

(a) $\int_1^3 (2-t)\boldsymbol{i}+(3t^2+1)\boldsymbol{j}\,dt$

$=\left[\left(2t-\frac{t^2}{2}\right)\boldsymbol{i}+(t^3+t)\boldsymbol{j}\right]_1^3$ ✓

$=\left(\left(6-\frac{9}{2}\right)\boldsymbol{i}+(27+3)\boldsymbol{j}\right)-\left(\left(2-\frac{1}{2}\right)\boldsymbol{i}+(1+1)\boldsymbol{j}\right)$ ✓

$=0\,\boldsymbol{i}+28\,\boldsymbol{j}$ ✓

(3)

(b) $\int_0^{\pi/2} (sin(3t))\boldsymbol{i}+(-cos(3t))\boldsymbol{j}\,dt$

$=-\left[\frac{cos(3t)}{3}\mathbf{i}+\frac{sin(3t)}{3}\mathbf{j}\right]_0^{\pi/2}$ ✓

$=-\frac{1}{3}\left(\left(cos\left(\frac{3\pi}{2}\right)\mathbf{i}+sin\left(\frac{3\pi}{2}\right)\mathbf{j}\right)-\left(cos(0)\mathbf{i}+sin(0)\mathbf{j}\right)\right)$ ✓

$=-\frac{1}{3}(-\mathbf{j}-\mathbf{i})$

$=\frac{1}{3}\mathbf{i}+\frac{1}{3}\mathbf{j}$ ✓

(3)

7. (27 marks)

(a) (i) $\boldsymbol{r}(t)=(10\,cos(t))\boldsymbol{i}+(10\,sin(t))\boldsymbol{j}$

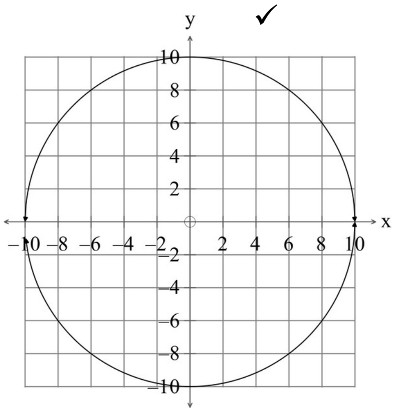
$x=10cos(t) \quad y=10sin(t)$ ✓

$sin^2(t)+cos^2(t)=1$

$\therefore\left(\frac{x}{10}\right)^2+\left(\frac{y}{10}\right)^2=1$

$x^2+y^2=100$ ✓

✓



(3)

(ii) $\boldsymbol{r}(t) = (10\cos(t))\boldsymbol{i} + (10\sin(t))\boldsymbol{j}$

$\boldsymbol{v}(t) = (-10\sin(t))\boldsymbol{i} + (10\cos(t))\boldsymbol{j}$ ✓

$\boldsymbol{r}(t) \bullet \boldsymbol{v}(t) = \begin{pmatrix} 10\cos(t) \\ 10\sin(t) \end{pmatrix} \bullet \begin{pmatrix} -10\sin(t) \\ 10\cos(t) \end{pmatrix}$

$\boldsymbol{r}(t) \bullet \boldsymbol{v}(t) = -100\cos(t)\sin(t) + 100\sin(t)\cos(t) = 0$ ✓

$|\boldsymbol{r}(t)| \neq 0,\quad |\boldsymbol{v}(t)| \neq 0 \quad \therefore \cos(t) = 0 \Rightarrow t = \frac{\pi}{2}$ ✓

Therefore the position vector is always at right angles to the velocity vector.

(3)

(iii) $\boldsymbol{a}(t) = (-10\cos(t))\boldsymbol{i} + (-10\sin(t))\boldsymbol{j}$ ✓

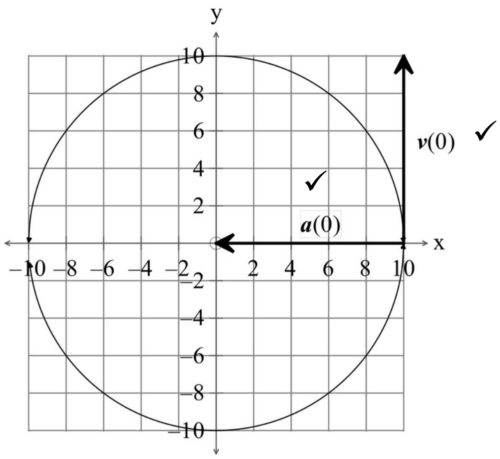
$\boldsymbol{a}(t) = -\left((10\cos(t))\boldsymbol{i} + (10\sin(t))\boldsymbol{j}\right)$ ✓

$\boldsymbol{a}(t) = -\boldsymbol{r}(t)$

$\boldsymbol{r}(t)$ is a position vector, i.e. it goes out from the origin.

Therefore $\boldsymbol{a}(t)$ is directed towards the origin. ✓ (3)

(iv) $\boldsymbol{r}(0) = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$ ✓, $\boldsymbol{v}(0) = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$, $\boldsymbol{a}(0) = \begin{pmatrix} -10 \\ 0 \end{pmatrix}$ ✓



(4)

(v) Speed = $|\boldsymbol{v}(t)|$ ✓

$|\boldsymbol{v}(t)| = \sqrt{(-10\sin(t))^2 + (10\cos(t))^2}$ ✓

$= \sqrt{100(\sin^2(t) + \cos^2(t))}$

$= 10\sqrt{1}$

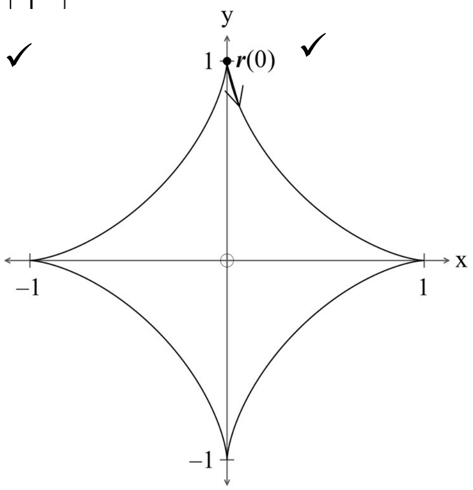
$= 10$ ✓

The speed is constant. (3)

(b) (i) $\boldsymbol{r}(t) = (sin^3(t))\boldsymbol{i} + (cos^3(t))\boldsymbol{j}$.

$\boldsymbol{r}(0) = (sin^3(0))\boldsymbol{i} + (cos^3(0))\boldsymbol{j}$

$\boldsymbol{r}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ✓ $\quad \boldsymbol{r}(0^+) = \begin{pmatrix} 0^+ \\ 1^- \end{pmatrix}$ ✓



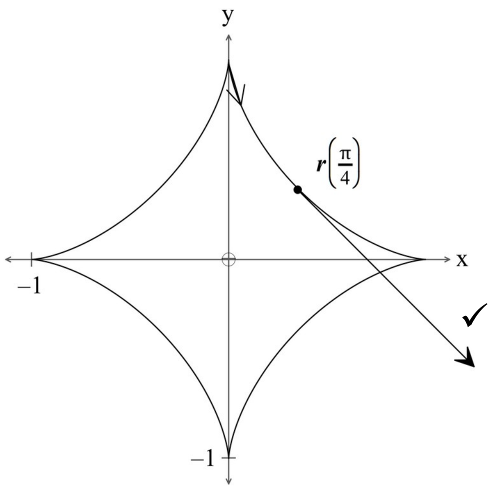
(3)

(ii) $\boldsymbol{v}(t) = (3sin^2(t)cos(t))\boldsymbol{i} - (3cos^2(t)sin(t))\boldsymbol{j}$ ✓✓✓ (2)

-1/error

(iii) $\boldsymbol{r}\left(\frac{\pi}{4}\right) = \begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix} \approx \begin{pmatrix} 0.35 \\ 0.35 \end{pmatrix}$ ✓

$\boldsymbol{v}\left(\frac{\pi}{4}\right) \approx \begin{pmatrix} 1.06 \\ -1.06 \end{pmatrix}$ ✓



(3)

(iv) $\boldsymbol{v}(t)=\begin{pmatrix}0\\0\end{pmatrix}$ t = ?

$\boldsymbol{v}(t)=\left(3sin^2(t)cos(t)\right)\boldsymbol{i}\ -\ \left(3cos^2(t)sin(t)\right)\boldsymbol{j}$

$x=\left(3sin^2(t)cos(t)\right)=1.5sin(2t)sin(t)$

$If\ \ x=0\quad sin(2t)=0\ \ or\quad sin(t)=0$

$2t=0,\pi,2\pi\qquad t=0,\pi,2\pi\ldots$

$If\ \ x=0\qquad t=0,\frac{\pi}{2},\pi$ ✓

$y=\left(3cos^2(t)sin(t)\right)=1.5sin(2t)cos(t)$ (3)

$If\ \ y=0\quad sin(2t)=0\ \ or\quad cos(t)=0$

$2t=0,\pi,2\pi\qquad t=\frac{\pi}{2},\frac{3\pi}{2}\ldots$

$If\ \ y=0\qquad t=0,\frac{\pi}{2},\pi$ ✓

So for $\boldsymbol{v}(t)=\begin{pmatrix}0\\0\end{pmatrix}$ for $t>0$, first time is $t=\frac{\pi}{2}$ ✓

8. (3 marks)

(a) $\mathbf{AG}=\mathbf{AO}+\mathbf{OG}=-\mathbf{OA}+\mathbf{OG}$ ✓

$=-\mathbf{a}+\mathbf{g}$ ✓ (2)

(b) $\mathbf{OM}=\mathbf{OA}+\frac{1}{2}\mathbf{AG}=\mathbf{a}+\frac{1}{2}(-\mathbf{a}+\mathbf{g})=\frac{1}{2}(\mathbf{a}+\mathbf{g})$ ✓ (1)

9. (13 marks)

(a) (i) $C(-1,4,0)$ ✓ $\quad r^2=(1-(-1))^2+(2-4)^2+(4-0)^2=4+4+16=24$ ✓

$(x+1)^2+(y-4)^2+z^2=24$ ✓ (3)

(ii) $\mathbf{PQ}=\begin{pmatrix}-4\\4\\-8\end{pmatrix},\ \mathbf{PR}=\begin{pmatrix}0\\-1\\-3\end{pmatrix}$ ✓

$\mathbf{r}=\begin{pmatrix}1\\1\\1\end{pmatrix}+t\begin{pmatrix}-4\\4\\-8\end{pmatrix}+s\begin{pmatrix}0\\-1\\-3\end{pmatrix}$ ✓✓

Other solutions are possible (3)

(iii) $\mathbf{PQ} = \begin{pmatrix} -4 \\ 4 \\ -8 \end{pmatrix}$, $\mathbf{PR} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$

$\mathbf{PQ} \times \mathbf{PR} = \begin{pmatrix} -20 \\ -12 \\ 4 \end{pmatrix}$ ✓ (1)

(b) (i) $\mathbf{r}_{bird}(t) = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} + t\begin{pmatrix} 2.5 \\ -1 \\ -3 \end{pmatrix}$

$x = 4 + 2.5t, \quad y = 5 - t, \quad z = 6 - 3t$

$If \quad z = 0, t = 2$ check $x = 4 + 5 = 9$

$z = 6 - 6 = 0$

So at (9,0,3) $t = 2$ ✓ (1)

(ii) $(9,3,0)$ to $(9,4,0)$ Mouse takes 1 second to get to its hole. ✓ (1)

(iii) $\left|\begin{pmatrix} 2.5 \\ -1 \\ -3 \end{pmatrix}\right| = 4.03$ m/s ✓ $\left|\begin{pmatrix} 2.5 \\ 0 \\ -3 \end{pmatrix}\right| = 3.91$ m/s ✓

Change in speed is 0.12 m/s ✓ (2)

(iv) At $t = 1$ the bird is at $P(6.5,\ 4,\ 3)$

$\mathbf{r}_{bird}(t) = \begin{pmatrix} 6.5 \\ 4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2.5 \\ 0 \\ -3 \end{pmatrix}$

After one second (when the mouse gets to its hole) ✓

$\mathbf{r}_{bird}(1) = \begin{pmatrix} 6.5 \\ 4 \\ 3 \end{pmatrix} + 1\begin{pmatrix} 2.5 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \\ 0 \end{pmatrix}$ the bird arrives at the nest,

so they both arrive at the hole together. ✓

Let's hope the mouse does not have a long tail!!! (2)

10. (12 marks)

(a)

$$Re\left(\frac{(1+i)^6\, cis\left(\frac{\pi}{2}\right)}{(1-i)^2}\right) = Re\left(\frac{\left(\sqrt{2}\, cis\left(\frac{\pi}{4}\right)\right)^6 cis\left(\frac{\pi}{2}\right)}{\left(\sqrt{2}\, cis\left(-\frac{\pi}{4}\right)\right)^2}\right)$$

$$= \frac{8}{2} Re\left(cis\left(\frac{6\pi}{4}+\frac{\pi}{2}+\frac{2\pi}{4}\right)\right) \quad ✓$$

$$= 4\, Re\left(cis\left(\frac{5\pi}{2}\right)\right)$$

$$= 4\, Re\left(cis\left(\frac{\pi}{2}\right)\right) \quad ✓$$

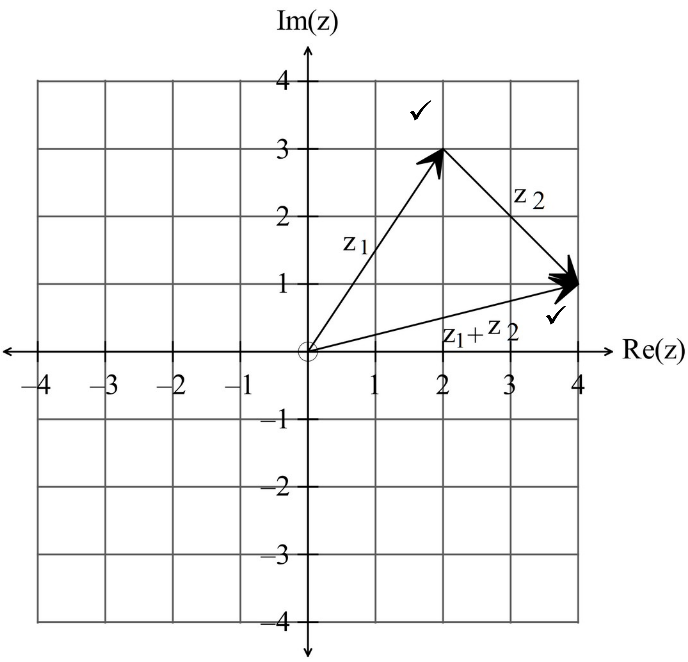
$$= 4\, Re\left(cos\left(\frac{\pi}{2}\right) + i\, sin\left(\frac{\pi}{2}\right)\right)$$

$$= 4\, Re(0+i)$$

$$= 0 \quad ✓$$

(3)

(b)



(2)

(c) (i) $x+yi=\dfrac{2+3i}{1+i}-\dfrac{1+5i}{3-i}.$

$$\frac{2+3i}{1+i}-\frac{1+5i}{3-i}=\frac{2+3i}{1+i}\times\frac{1-i}{1-i}-\frac{1+5i}{3-i}\times\frac{3+i}{3+i} \quad ✓ \quad ✓$$

$$=\frac{2+3i-2i-3i^2}{1-i^2}-\frac{3+15i+i+5i^2}{9-i^2} \quad ✓ \quad ✓$$

$$=\frac{5+i}{2}-\left(\frac{-2+16i}{10}\right)$$

$$=\frac{5}{2}+\frac{1}{5}+i\left(\frac{1}{2}-\frac{8}{5}\right)$$

$$=\frac{27}{10}-\frac{11i}{10}$$

$$x=2.7 \ \ and \ \ y=-1.1 \quad ✓✓$$

(6)

(ii) $x+yi=\sqrt{4+3i}$ $\quad x=2.12,\ y=0.71$ ✓ (1)

11. (6 marks)

(a) $$\cos(\theta)=\frac{\begin{pmatrix}2\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}3\\-2\\4\end{pmatrix}}{\left|\begin{pmatrix}2\\-3\\1\end{pmatrix}\right|\left|\begin{pmatrix}3\\-2\\4\end{pmatrix}\right|} \quad ✓$$

$$\cos(\theta)=\frac{6+6+4}{\sqrt{4+9+1}\sqrt{9+4+16}}$$

$$=\frac{16}{\sqrt{14}\sqrt{29}}$$

$$\theta=37.43^{\circ} \quad ✓$$

(2)

(b) The projection of $\boldsymbol{a}$ on $\boldsymbol{b}=\dfrac{\boldsymbol{a}\bullet\boldsymbol{b}}{|\boldsymbol{b}|}=\dfrac{16}{\sqrt{29}}$ (2)

✓ ✓

(c) $\begin{pmatrix}3\\-2\\4\end{pmatrix}\bullet\begin{pmatrix}4\\0\\-3\end{pmatrix}=0$ $\qquad \mathbf{p}=\begin{pmatrix}4\\0\\-3\end{pmatrix}$ ✓✓ *Answers will vary* (2)

12. (17 marks)

(a) $p(q(x))=(x+1)(x+3)$ and $p(x)=x^2-1$ find $y=q(x)$.

$p(q(x))=(q(x))^2-1$ ✓

$p(q(x))=(x+1)(x+3)$

$=x^2+4x+3$

$=x^2+4x+4-1$ ✓

$p(q(x))=(x+2)^2-1$ ✓

$\therefore\ q(x)=x+2$ ✓

(4)

(b) (i) $f(x)=|x(x-1)(x+1)|$ ✓ (1)

(ii) $f(x)=|x|(|x|-1)(|x|+1)$ ✓✓ (2)

(iii) $f(x)=\dfrac{1}{x(x-1)(x+1)}$ ✓✓✓ (3)

(c) (i) $2f(1)=2\times(-3)=-6$ ✓ (1)

(ii) $f(|-1|)=f(1)=-3$ ✓ (1)

(iii) $|f^{-1}(3)|=|-1|=1$ ✓ (1)

(iv) True ✓

(as $f(-1)=3$ and $f(1)=-3$

and f^{-1} is monotonically decreasing) (1)

(v) True ✓ (1)

(d) (i) $f(x)=e^{2x} \quad \Rightarrow \quad y=e^{2x}$

To get inverse $x=e^{2y}$

$2y=ln(x)$

$y=f^{-1}(x)=\dfrac{ln(x)}{2}$

✓ (1)

(ii) $f\left(f^{-1}\left(f^{-1}(1)\right)\right)=f^{-1}(1)=\dfrac{ln(1)}{2}=0$ ✓ (1)

13. (4 marks)

(a) $f\left(g(x)\right)=f\left(x^2\right)=\sqrt{1-x^2}$ $\quad -1\le x\le 1 \quad 0\le f\left(g(x)\right)\le 1$

✓ ✓ (2)

(b) (i) $h(x)=1+e^x$

To get inverse:

$x=1+e^y$

$x-1=e^y$

$ln(x-1)=y$

$y=h^{-1}(x)=ln(x-1)$

✓ (1)

(ii) $h^{-1}(2)=ln(2-1)=0$

✓ (1)

14. (4 marks)

(a) $y=-2|x|+2=\begin{cases}\underline{-2x+2} & \text{for } x\ge 0\\ \underline{2x+2} & \text{for } x<0\end{cases}$ (1)

(b) $y=|1-x|=\begin{cases}1-x & \text{for } \underline{x\le 1}\\ x-1 & \text{for } \underline{x>1}\end{cases}$ ✓ (1)

(c)

$For\ \ x>1 \qquad -2x+2=x-1$

$3=3x$

$1=x$

$For\ \ 0<x<1 \qquad -2x+2=1-x$

$1=x$

$For\ \ x<0 \qquad 2x+2=1-x$

$3x=-1$

$x=-\frac{1}{3}$

(2)

15. (3 marks)

$a=-1,\ b=-2,\ c=0$

✓ ✓ ✓ (2)

16. (5 marks)

Prove that $\cos(5\theta) = 16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta)$

$$\cos(5\theta) = Re\left(cis(5\theta)\right)$$

$$= Re\left(\cos(\theta) + i\sin(\theta)\right)^5$$

$$= Re\left(\cos^5(\theta) + 5\cos^4(\theta)\left(i\sin(\theta)\right) + 10\cos^3(\theta)\left(i\sin(\theta)\right)^2\right.$$

$$\left. + 10\cos^2(\theta)\left(i\sin(\theta)\right)^3 + 5\cos(\theta)\left(i\sin(\theta)\right)^4 + \left(i\sin(\theta)\right)^5\right)$$

$$= \cos^5(\theta) - 10\cos^3(\theta)\sin^2(\theta) + 5\cos(\theta)\sin^4(\theta)$$

$BUT \quad \sin^2(\theta) = 1 - \cos^2(\theta)$

$$\cos(5\theta) = \cos^5(\theta) - 10\cos^3(\theta)\left[1 - \cos^2(\theta)\right] + 5\cos(\theta)\left[1 - \cos^2(\theta)\right]^2$$

$$= \cos^5(\theta) - 10\cos^3(\theta) + 10\cos^5(\theta) + 5\cos(\theta)\left[1 - 2\cos^2(\theta) + \cos^4(\theta)\right]$$

$$= 11\cos^5(\theta) - 10\cos^3(\theta) + 5\cos(\theta) - 10\cos^3(\theta) + 5\cos^5(\theta)$$

$$\cos(5\theta) = 16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta)$$

(5)

END OF SECTION TWO