

MELVILLE
SENIOR HIGH SCHOOL

MATHEMATICS METHODS
YEAR 12, UNIT 4
Semester 2, 2021
Test 4
Weighting 7%
[Curriculum references: Logarithms 4.1.1-4.1.14]

Section One: Calculator-free

Student Name:

Solutions

Teacher's Name:

Time allowed for this section

Reading time before commencing work: one minute

Working time for paper: forty minutes

Total Marks: forty-six marks

Materials required/recommended for this section

To be provided by the supervisor

- This Question/Answer booklet
- Formula sheet
- To be provided by the candidate
- Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
- Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be allocated for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. It is recommended that students **do not use a pencil**, except in diagrams

Question 1. [2, 2 = 4 marks]

Express each of the following as a single logarithm.

a) $2\log 3 + \log 2 - \log 6$

$$\begin{aligned} &= \log 9 + \log 2 - \log 6 \\ &= \log \left(\frac{9 \times 2}{6} \right) \\ &= \log 3 \quad \checkmark \end{aligned}$$

b) $(\log x)^3 \div (\log x)^2 + \log x^2$

$$\begin{aligned} &= \log x + 2\log x \\ &= 3\log x \quad \checkmark \end{aligned}$$

Question 2. [2, 2 = 4 marks]

Evaluate each of the following, showing full working.

a) $1.5\log_8 4$

$$\begin{aligned} &= \log_8 4^{3/2} \\ &= \log_8 (2^3) \\ &= \log_8 8 \\ &= 1 \quad \checkmark \end{aligned}$$

b) $\frac{\log 135 - \log 5}{\log 3^2}$

$$\begin{aligned} &= \frac{\log \left(\frac{135}{5} \right)}{2\log 3} \\ &= \frac{\log 27}{2\log 3} \\ &= \frac{3\log 3}{2\log 3} = \frac{3}{2} \quad \checkmark \end{aligned}$$

Question 3. [1, 3 = 4 marks]

If $\log x = y$, where x is positive, express each of the following in terms of y .

a) $\log x^2$

$$= 2y \quad \checkmark$$

b) $\log x m^3 - 3\log m$

$$\begin{aligned} &= \log x + 3\log m - 3\log m \\ &= \log x \\ &= y \quad \checkmark \end{aligned}$$

Question 4. [3, 3 = 6 marks]

Give an exact value for x in terms of \log_{10} (simplify your logs)

b) $2^{x-3} = 5^{2x+1}$

$4^{2x} - 4^x - 6 = 0$

Let $y = 4^x$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3$ or $y = -2$

$4^x = 3$ or $4^x = -2$

reject

$x = \frac{\log 3}{\log 4}$

$2^x \cdot 2^{-3} = 5^{2x+1}$

$\frac{2^x}{5^{2x}} = \frac{5}{2^{-3}}$

$\left(\frac{2}{25}\right)^x = 40$

$\log_{\frac{2}{25}} 40 = x$

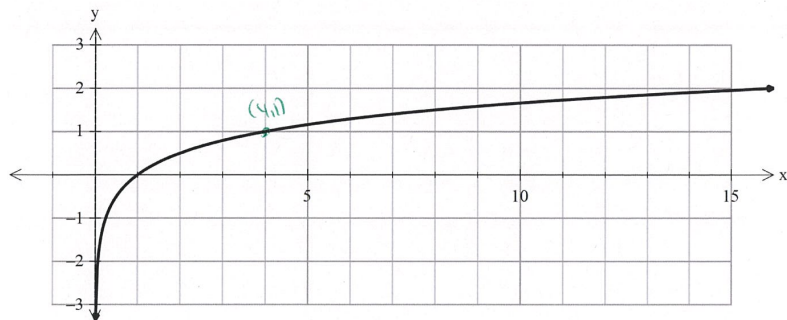
$\therefore x = \frac{\log 40}{\log \frac{2}{25}}$

$= \frac{\log 2 - 2 \log 5}{\log 40}$

or $\frac{\log 2 - 2 \log 5}{\log 5 + 3 \log 2}$

Question. 5. [1, 1, 1, 2 = 5 marks]

The function $y = \log_a x$ is graphed below. Use the graph to answer the questions that follow.



- a) Find the value of a .

$$\log_a 4 = 1$$

$$a = 4$$

- b) Use the graph to approximate to 1 d.p. the solution to $\log_a x = 1.6$

- c) Approximate the solution for x if $x = a^{1.2}$

$$x \approx 9$$

(within ± 0.5)

- d) Use the graph to find an approximation to $\log_3 7$

$$= \frac{\log 7}{\log 3} \checkmark \approx \frac{1.4}{0.8} = \frac{14}{8} \checkmark$$

Additional working space.

Qu 9b Alternative Sdn

$$\frac{d}{dx} \left(\frac{\ln x}{x} \right) = \frac{1}{x^2} - \frac{\ln x}{x^2}$$

$$\int_1^2 \frac{d}{dx} \left(\frac{\ln x}{x} \right) dx = \int_1^2 \frac{1}{x^2} dx - \int_1^2 \frac{\ln x}{x^2} dx \checkmark$$

$$\frac{\ln 2}{2} - \frac{\ln 1}{1} = \left[\frac{x^{-1}}{-1} \right]_1^2 - \int_1^2 \frac{\ln x}{x^2} dx$$

$$\int_1^2 \frac{\ln x}{x^2} dx = \left[-\frac{1}{x} \right]_1^2 - \left(\frac{\ln 2}{2} - 0 \right) \checkmark$$

$$\int_1^2 \frac{\ln x}{x^2} dx = -\frac{1}{2} - \left(-\frac{1}{1} \right) - \frac{\ln 2}{2} \checkmark$$

$$= -\frac{1}{2} + 1 - \frac{\ln 2}{2}$$

$$= \frac{1}{2} - \frac{\ln 2}{2}$$

$$= \frac{1 - \ln 2}{2} \checkmark$$

Question 9. [2, 4 = 6 marks]

Given $y = \frac{x}{\ln x}$

a) Find $\frac{dy}{dx}$

$$= \frac{x^2}{x^2 - \ln x \cdot (1)} = \frac{x^2}{1 - \ln x}$$

b)

Hence, or otherwise, use $\frac{dy}{dx}$ to show that $\int_{1-\ln 2}^{1-\ln x} \frac{1}{x^2} dx = \frac{1-\ln 2}{2}$

$$\frac{d}{dx} \left(\frac{x^2}{1 - \ln x} \right) = \frac{2x}{1 - \ln x} - \frac{x^2}{(1 - \ln x)^2}$$

$$\frac{dy}{dx} = \frac{1}{x^2} - \frac{1}{1 - \ln x}$$

$$\frac{1}{1 - \ln x} = \frac{1}{x^2} - \frac{dy}{dx}$$

$$\text{If } x=1, y=0 \quad \text{If } x=a, y=\frac{1}{2} \ln 2$$

$$= \int_{\frac{1}{2} \ln 2}^0 \frac{1}{x^2} dx - \int_{\frac{1}{2} \ln 2}^0 dy$$

$$= \left[-\frac{1}{x} \right]_{\frac{1}{2} \ln 2}^0 - \left[y \right]_{\frac{1}{2} \ln 2}^0$$

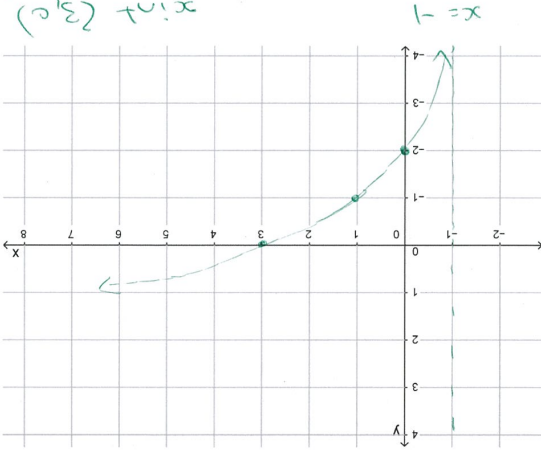
$$= -\frac{1}{\frac{1}{2} \ln 2} + 1 - \frac{1}{2}$$

End of Calculator Free Section

Additional working space.

Question 6 [3 marks]

Sketch the graph of $y = \log_2(x+1) - 2$. Clearly label the key features.



Point (3, 0)
Point (0, -2)

$x = -1$ asymptote

General shape

Question 7. [3, 3, 3 = 9 marks]

Differentiate each of the following with respect to x , showing full working.

(DO NOT SIMPLIFY YOUR ANSWER)

a) If $y = x^2 \ln(\sin x)$ find $\frac{dy}{dx}$

$$\frac{dy}{dx} = 2x \ln(x) + \frac{x^2 \cos x}{\sin x}$$

b) $g(x) = \ln[(x^2+1)(x-1)^4]$

$$g(x) = \ln(x^2+1) + \ln(x-1)^4$$

$$= \ln(x^2+1) + 4\ln(x-1)$$

$$g'(x) = \frac{2x}{x^2+1} + \frac{4}{(x-1)}$$

c) $y = \log_2(x^3 - 2x)$

$$y = \frac{\ln(x^3 - 2x)}{\ln 2}$$

$$\frac{dy}{dx} = \frac{1}{\ln 2} \cdot \frac{(3x^2 - 2)}{(x^3 - 2x)}$$

Question 8. [2, 3 = 5 marks]

Integrate each of the following, showing full working.

a) $\int \frac{x}{x^2-4} dx$

$$= \frac{1}{2} \int \frac{2x}{x^2-4} dx$$

$$= \frac{1}{2} \ln|x^2-4| + C$$

b) $\int \frac{4x^4 - 3x^2 + x}{x^3} dx$

$$= \int 4x - \frac{3}{x} + \frac{1}{x^2} dx$$

$$= 2x^2 + 3\ln|x| - \frac{1}{x} + C$$

-1 no +C