PERTH MODERN SCHOOL
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Test 1

Differentiaton , applications and Optimisation.
Basic antidifferentiation

Semester One 2017
Year 12 Mathematics Methods
Calculator Free

Name: SOLUTIONS

Teacher:

______Mr Staffe
______Mrs. Carter
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_____Ms Cheng
_____Mr McClelland
_____ Ms Skoda
_____Mr Strain

Date Monday 20th February 7.45am

You may have a formula sheet for this section of the test.

Question 1 **(4 marks)**

Find y in terms of x given that $\frac{dy}{dx} = 15x(5x^2 - 1)^2$
and $y = 40$ when $x = 1$

$y=\frac{(5x^2-1)^3}{2}+c$ √√

$40=\frac{4^3}{2}+c$

$c=8$ √

$y=\frac{(5x^2-1)^3}{2}+8$ √

Question 2 **(6 marks)**

Clearly showing your use of the product, quotient or chain rule differentiate the following. (YOU MAY LEAVE YOUR ANSWERS IN AN UNSIMPLIFIED FORM) .

a) $y=(\sqrt{x}+1)(x^2-1)$ (2)

$$\frac{dy}{dx}=\frac{(x^2-1)}{2\sqrt{x}}+2x(\sqrt{x}+1)$$

√ √

b) $y=\frac{1-t}{1-2t^2}$ (2)

$$\frac{dy}{dt}=\frac{-(1-2t^2)+4t(1-t)}{(1-2t^2)^2}$$

√ √

c) $y=(3x^2+5)^3$ (2)

$$\frac{dy}{dx}=18x(3x^2+5)^2$$

√ √

Question 3 **(4 marks)**

Given that $y = x^{\frac{1}{3}}$, use $x = 1000$ and the increments formula $\delta y \approx \frac{dy}{dx}\delta x$ to determine an approximate value for $\sqrt[3]{1006}$.

Solution
$\frac{dy}{dx} = \frac{1}{3}x^{\frac{-2}{3}}$ $\delta y \approx \frac{1}{3}x^{\frac{-2}{3}} \times 6$ When $x = 1000$, $\delta y \approx 2 \times \frac{1}{\left(\sqrt[3]{1000}\right)^2}$ $\approx \frac{2}{100}$ $\therefore \sqrt[3]{1006} \approx 10.02$
Specific behaviours
✓ substitutes for x correctly ✓ determines $\frac{dy}{dx}$ ✓ uses $\frac{\delta y}{\delta x}$ correctly ✓ determines approximate value

Question 4 **(6 marks)**

For the function $y = x^4 - 4x^3 + 1$ determine

a) The coordinates of the y- intercept $y = (0,1)$ √

b) The behaviour of the function as $x \to \pm\infty$

y *increases as* $x \to \pm\infty$ √

c) The location and nature of any turning points

$\frac{dy}{dx} = 4x^3(x-3)$

Gradient of 0 *at* $x = 0 \wedge x = 3$

$\frac{dy^2}{d^2x} = 12x^2$

$\frac{dy^2}{d^2x}(0) = 0 \therefore$ *horizontal point of inflection*

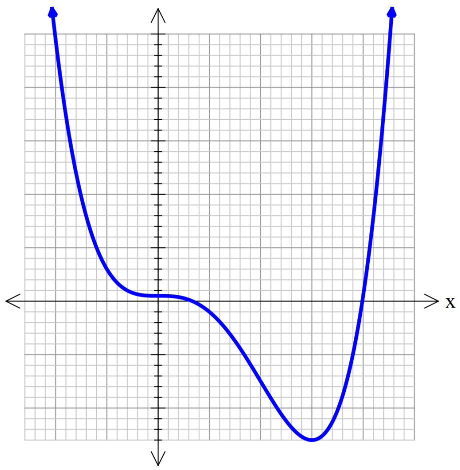
$\frac{dy^2}{d^2x}(3) > 0 \therefore$ *minimum turning point*

Minimum turning point at $(3, -26)$ √

d) Any points of inflection and what type of inflection they are.

Horizontal point of inflection at $(0,1)$ √

Hence sketch the curve on the axes provided. (Ensure you label all parts)



√√

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Year 12 Mathematics Methods

Calculator Assumed

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- **a formula sheet**

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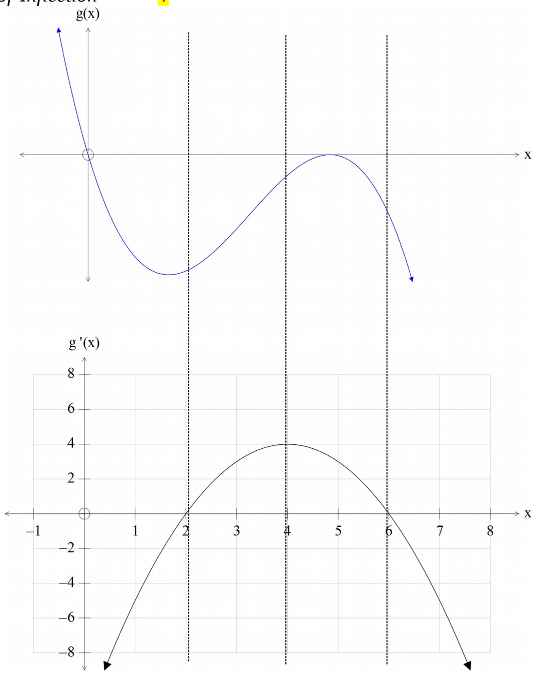
$$\therefore g(x) = \frac{-x^3}{3} + 4x^2 - 12x + \frac{1}{3}$$ √

(b) (i) Given the graph of the function y = $g'(x)$ sketch a possible graph of the function y = g(x). (3)

Shape √
Turning Points √
Horizontal Point of Inflection √



(ii) Find the equation of y = g(x) given that g(1) = - 8. (4)

$$g'(x) = -(x-2)(x-6)$$ √

$$g'(x) = -x^2 + 8x - 12$$

$$g(x) = \frac{-x^3}{3} + 4x^2 - 12x + c$$ √

$$-8 = \frac{-1}{3} + 4 - 12 + c$$

$$c = \frac{1}{3}$$ √

- **one page of A4 notes, one side**
- **a scientific calculator**
- **a classpad**

Question 1 **(7 marks)**

A small object is moving in a straight line with acceleration $a = 6t + k$ ms^{-2}, where t is the time in seconds and k is a constant. When $t = 1$ the object was stationary and had a displacement of 4 metres relative to a fixed point O on the line. When $t = 2$ the object had a velocity of 1 ms^{-1}.

(a) Determine the value of k and hence an equation for the velocity of the object at time t. (4 marks)

Solution
$v = 3t^2 + kt + c$
$t = 1,\ 3 + k + c = 0$
$t = 2,\ 12 + 2k + c = 1$
$k = -8$
$c = 5$
$v = 3t^2 - 8t + 5$
Specific behaviours
✓ antidifferentiates acceleration, adding constant ✓ derives simultaneous equations from information ✓ solves equations ✓ writes velocity equation

(b) D ...rks)

Solution
$s = t^3 - 4t^2 + 5t + c$
$t = 1,\ 4 = 1 - 4 + 5 + c$
$c = 2$
$s = t^3 - 4t^2 + 5t + 2$
$s(2) = 8 - 16 + 10 + 2$
$= 4$ m
Specific behaviours
✓ antidifferentiates velocity ✓ determines constant ✓ evaluates displacement

Question 2 [7 marks]

An open cuboid container for holding fishing equipment, is made with a base length twice as long as its width is to be made from a sheet of metal with an area of 36 m^2.

(a) Show that its height is given by the expression $h = \frac{6}{x} - \frac{x}{3}$ where x is the width of the base. [2]

$$2x^2 + 2xh + 4xh = 36$$
$$2x^2 + 6xh = 36$$
$$6xh = 36 - 2x^2$$
$$h = \frac{36}{6x} - \frac{2x^2}{6x}$$
$$= \frac{6}{x} - \frac{x}{3}$$

(b) Express the volume V, in terms of x [2]

$$V = lwh$$
$$= 2x.x.\left(\frac{6}{x} - \frac{x}{3}\right)$$
$$= 12x - \frac{2x^3}{3}$$

(c) Find the maximum Volume using Calculus techniques. [3]

$$\frac{dV}{dx} = 12 - 2x^2$$
$$\text{Put } \frac{dV}{dx} = 0$$
$$12 - 2x^2 = 0$$
$$x^2 = 6$$
$$x = \pm\sqrt{6} \quad \text{Discard negative value}$$
$$\text{Maximum volume is } 8\sqrt{6} \quad 19.60 \text{ to } 2d.p$$

Question 3 **(10 marks)**

(a) Given the sketch of the function y = f(x) on the set of axes below, use it to sketch the functions y = $f'(x)$ and y = $f''(x)$. (3)