PERTH MODERN SCHOOL
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Independent Public School

Course 12 Methods(Test 2 alternative) Year 12

Student name: ______________ Teacher name: ______________

Task type: **Response**

Time allowed for this task: **______45_____ mins**

Number of questions: **______9_____**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **___46___ marks**

Task weighting: **__10__%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3.2.1-3.2.3) (3 & 3 =6 marks)

Determine y in terms of x for the following.

(a) $\frac{dy}{dx}=5x^3-4x^2+7x+1$ given that $y=10,\ x=1$.

Solution
$\frac{dy}{dx}=5x^3-4x^2+7x+1$ $y=\frac{5x^4}{4}-\frac{4}{3}x^3+\frac{7}{2}x^2+x+c$ $10=\frac{5}{4}-\frac{4}{3}+\frac{7}{2}+1+c$ $c=\frac{67}{12}$ $y=\frac{5x^4}{4}-\frac{4}{3}x^3+\frac{7}{2}x^2+x+\frac{67}{12}$
Specific behaviours
✓ ant differentiates correctly ✓ uses a constant ✓ solves for constant correctly

(b) $\frac{dy}{dx}=5x^2\sqrt{6+2x^3}$ given that $y=1,\ x=-1$.

Solution

Working out space

Working out space

$\frac{dy}{dx} = 5x^2\sqrt{6+2x^3}$ $y = A(6+2x^3)^{\frac{3}{2}} + c$ $y' = \frac{A3}{2}(6+2x^3)^{\frac{1}{2}}(6x^2) \Rightarrow 5 = 9A \Rightarrow A = \frac{5}{9}$ $y = \frac{5}{9}(6+2x^3)^{\frac{3}{2}} + c$ $1 = \frac{5}{9}(8) + c$ $c = \frac{-31}{9}$ $y = \frac{5}{9}(6+2x^3)^{\frac{3}{2}} + \frac{-31}{9}$
Specific behaviours
✓ ant differentiates correctly ✓ solves for multiped constant correctly ✓ solves for added constant correctly

Q2 (3.2.21-3.2.22) (4 marks)

An object is moving in a straight line such that its velocity m/s as a function time, t seconds, is given by $v = 5t^2 + pt + 1$ where p is a constant. The acceleration at time $t = 3$ seconds is $10m/s^2$ and is initially at the origin. Determine the displacement when $t = 6$ seconds.

Solution
$v = 5t^2 + pt + 1$ $a = 10t + p$ $10 = 30 + p$ $p = -20$ $v = 5t^2 - 20t + 1$ $x = \frac{5t^3}{3} - 10t^2 + t + c$ $0 = c$ $x = \frac{5(6)^3}{3} - 10(6)^2 + 6 = 6$ x= 6 metres
Specific behaviours

✓ differentiates to determine acceleration ✓ solves for p correctly ✓ integrates to determine displacement **and states** a constant c ✓ determines displacement

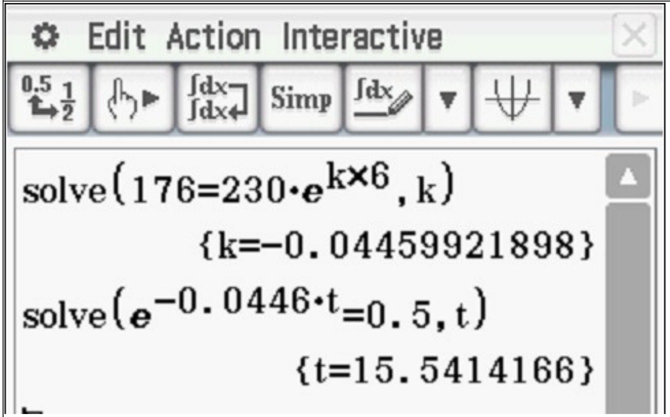
Specific behaviours
✓ uses product rule ✓ determines derivative

(b) Using your result from part (a) and **without using your classpad** determine $\int \frac{x}{\sqrt{5-2x}}dx$.

Solution
$\frac{d}{dx}\left(x\sqrt{5-2x}\right)=x\frac{1}{2}(5-2x)^{\frac{-1}{2}}(-2)+\sqrt{5-2x}$ $=\frac{-x}{\sqrt{5-2x}}+\sqrt{5-2x}$ $\frac{x}{\sqrt{5-2x}}=\sqrt{5-2x}-\frac{d}{dx}\left(x\sqrt{5-2x}\right)$ $\int\frac{x}{\sqrt{5-2x}}dx=\frac{-1}{3}(5-2x)^{\frac{3}{2}}-x\sqrt{5-2x}+c$
Specific behaviours
✓ attempts to integrate both sides of result in a (lineraity) ✓ uses fundamental theorem ✓ integrates all terms correctly ✓ determines required integral

Q8 (3.1.4) (4 marks)

A radioactive substance ZZZ initially has a mass of 230 grams and decays according to $\frac{dN}{dt}=kN$ where N equals the mass at time t minutes and k is a constant. After 6 minutes the mass is 176 grams. Determine the time taken for half the mass to decay(half-life) and the value of k to three decimal places.

Solution

Specific behaviours
✓ uses an exponential model ✓ solves for k ✓ sets up an equation to solve for half life ✓ solves for half life (no need to round nor units)

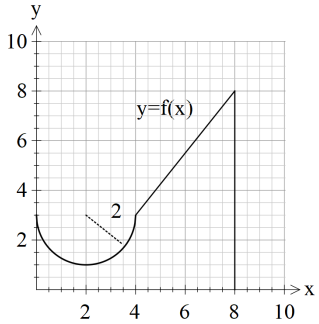
Q9 (3.2.6) (2 & 4 =6 marks)

(a) Determine $\frac{d}{dx}\left(x\sqrt{5-2x}\right)$.

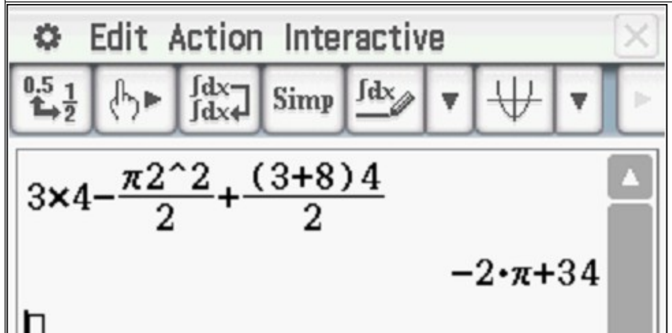
Solution
$\frac{d}{dx}\left(x\sqrt{5-2x}\right)=x\frac{1}{2}(5-2x)^{-\frac{1}{2}}(-2)+\sqrt{5-2x}$ $=\frac{-x}{\sqrt{5-2x}}+\sqrt{5-2x}$

Q3 (3.2.10-3.2.11) (3 & 4 = 7 marks)

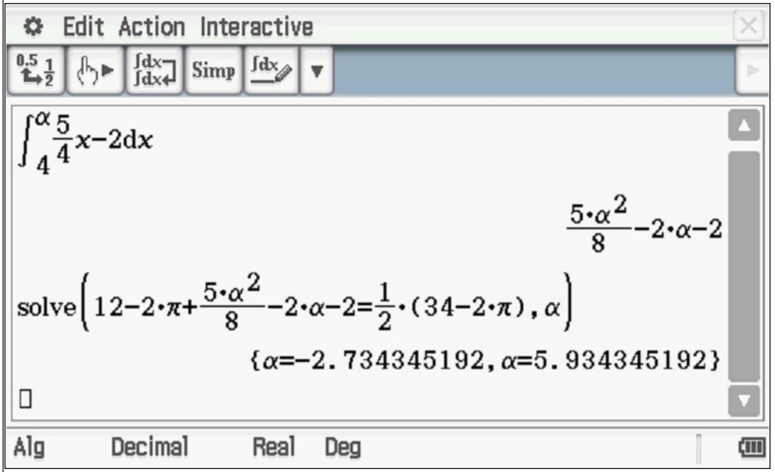
Consider the function $f(x)$ which is graphed for $0 \le x \le 8$. The arc has a radius of 2 units.



(a) Determine the exact value of $\int_0^8 f(x)\,dx$.

Solution

Specific behaviours
✓ determines area under arc ✓ determines area of trapezium ✓ express the exact value in terms of pi

(b) Determine α to two decimal places such that $\int_0^\alpha f(x)\,dx = \frac{1}{2}\int_0^8 f(x)\,dx$

Solution

Specific behaviours
✓ determines equation of line ✓ determines an expression in terms of alpha for area between 4 and alpha ✓ determines an equation for alpha ✓ solves for alpha to two decimal places

Q7 (3.2.16) (1 & 3 = 4 marks)

Consider $y = \int_0^x f(t)\,dt$

a) In terms of f, express $\frac{d^2y}{dx^2}$.

Solution
$y = \int_0^x f(t)\,dt$ $y' = f(x)$ $y'' = f'(x)$
Specific behaviours
✓ determines expression

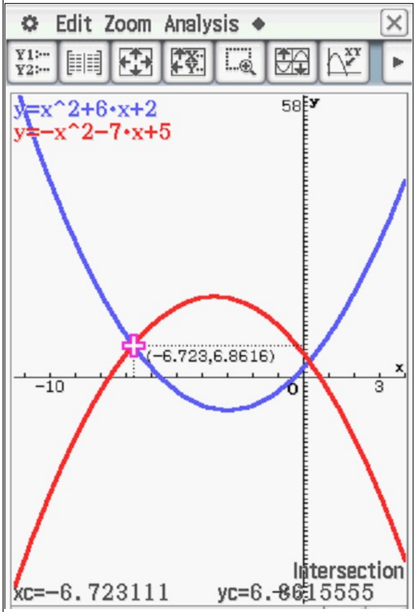
b) If $f''(x) = 3x+1$ and $f'(0) = 0 = f(0)$, determine y in terms of x only.

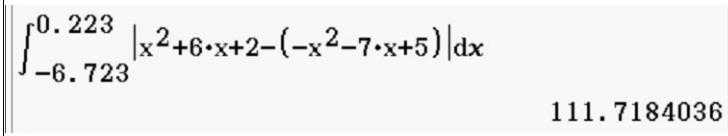
Solution
$f''(x) = 3x+1$ $f'(x) = \frac{3}{2}x^2 + x + c$ $c = 0$ $f(x) = \frac{x^3}{2} + \frac{x^2}{2} + c$ $c = 0$ $y = \int_0^x \frac{t^3}{2} + \frac{t^2}{2}\,dt = \left[\frac{t^4}{8} + \frac{t^3}{6}\right]_0^x = \frac{x^4}{8} + \frac{x^3}{6}$
Specific behaviours
✓ determines f(x) ✓ uses an integral with parameter t and rule f to define y ✓ determines expression of y in terms of x only

Q6 (3.2.20) (4 marks)

Determine to two decimal places the area between the curves $y = x^2 + 6x + 2$ and $y = -x^2 - 7x + 5$.
(Hint- Sketch the curves first on your classpad)

Solution





Area equals 111.72 units

Specific behaviours

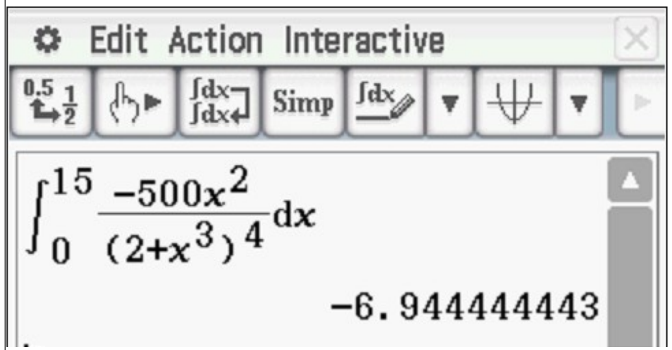
✓ determines both points of intersection
✓ identifies which curve is on top or uses absolute value
✓ writes an appropriate integral
✓ determines area rounded to 2 dp

Q4 (3.2.18-3.2.17) (3 & 2 = 5 marks)

A water tank has a leak and the volume of water contained, V, cubic metres, can be described by the following differential equation at time, t minutes, $\frac{dV}{dt} = -\frac{500t^2}{(2+t^3)^4}$. The tank is initially full but is emptied in 15 minutes.

(a) Determine the initial volume of water in the tank to the nearest cubic metre..

Solution



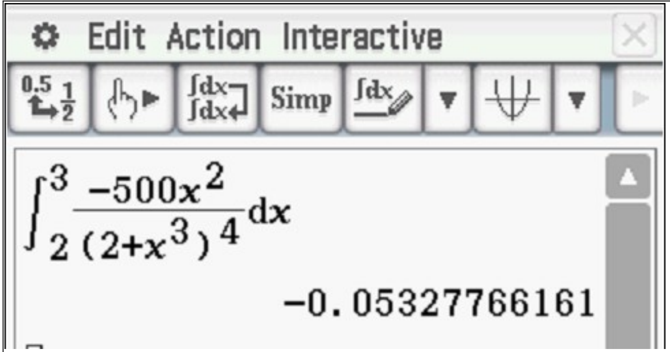
Volume = 7 cubic metres

Specific behaviours

✓ integrates or writes an integral
✓ uses domain of 0 to 15 mins
✓ determines initial volume (no need to round nor units)

(b) Determine the change in volume in the third minute.

Solution



Specific behaviours
✓ writes an integral ✓ uses correct limits from 2 to 3 minutes

Q5 (3.2.11-3.2.14) (2, 2 & 2 = 6 marks)

Consider a function $f(x)$ that is defined for $0 \le x \le 13$ with the following conditions.

$f(3)=9, \quad f(10)=3$

$f(0)=0=f(5)=f(8)=f(13)$

With $f(x)\ge 0$ for $0 \le x \le 5 \quad \& \quad 8 \le x \le 13$ and $f(x) \le 0$ for $5 \le x \le 8$.

$\int_0^{13} f(x)dx=7, \quad \int_0^5 f(x)dx=12$.

(a) Determine $\int_3^{10} f'(x)dx$.

Solution
$\int_3^{10} f'(x)dx = f(10) - f(3)$ $=3-9$ $=-6$
Specific behaviours
✓ uses fundamental theorem ✓ determines integral

(b) Determine $\int_5^8 f(x)dx$ given that $\int_8^{13} f(x)dx=6$.

(c) Solution
$\int_5^8 f(x)dx=7-12-6$ $=-11$
Specific behaviours
✓ uses additive property ✓ determines integral

(d) Determine $\frac{d}{dx}\int_0^x f(t)dt$ when $x=10$.

(d) Solution
$\frac{d}{dx}\int_0^x f(t)dt = f(x)$ $f(10)=3$
Specific behaviours
✓ uses fundamental theorem ✓ determines value