Rossmoyne Senior High School

Year 11 Examination, 2015

Question/Answer Booklet

MATHEMATICS SPECIALIST UNITS 1 AND 2

Section One:
Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free **(52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

(a) If $\tan A = \frac{1}{2}$, determine the exact value of $\tan 2A$. (2 marks)

$$\tan 2A = \frac{2\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{\frac{3}{4}}$$

$$= \frac{4}{3}$$

(b) Solve $\cos\left(2\left(x - 10°\right)\right) = 0.5$, $0° \le x \le 180°$. (3 marks)

$$2(x - 10) = \{60, 300\}$$

$$x - 10 = \{30, 150\}$$

$$x = \{40°, 160°\}$$

Question 2 **(6 marks)**

If $A=\begin{bmatrix}2\\1\\3\end{bmatrix}$, $B=\begin{bmatrix}4 & -1 & 2\end{bmatrix}$, $C=\begin{bmatrix}3 & 3 & -2\\5 & 4 & -3\end{bmatrix}$ and $D=\begin{bmatrix}1 & 2\\4 & 0\\7 & 3\end{bmatrix}$, state whether the following are true or false. If false, clearly explain your reasoning.

(a) $A+D=\begin{bmatrix}2 & 1 & 2\\1 & 4 & 0\\3 & 7 & 3\end{bmatrix}$. (1 mark)

False – cannot add different size matrices.

(b) $AB=\begin{bmatrix}8 & -2 & 4\\4 & -1 & 2\\12 & -3 & 6\end{bmatrix}$. (1 mark)

True.

(c) $a_{12}+b_{21}+c_{11}+d_{22}=3$. (1 mark)

False, as a_{12} and b_{21} do not exist.

(d) $BA=13$. (1 mark)

False, result is the matrix [13].

(e) $CD=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$. (1 mark)

True.

(f) $C^{-1}=D$. (1 mark)

False, matrix C must be square to have an inverse.

Question 3 **(7 marks)**

Two vectors are given by $\mathbf{a} = 9\mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j}$. Determine

(a) a vector parallel to $a - b$ of magnitude 25. (3 marks)

$$\begin{bmatrix} 9 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\left\| \begin{bmatrix} 6 \\ 8 \end{bmatrix} \right\| = 10$$

$$\frac{25}{10} \times \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \end{bmatrix}$$

(b) $\mathbf{a}$ in terms of $\mathbf{d}$ and $\mathbf{e}$, where $\mathbf{d} = 3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{e} = 5\mathbf{i} - 2\mathbf{j}$. (4 marks)

$$\begin{bmatrix} 9 \\ 4 \end{bmatrix} = x \begin{bmatrix} 3 \\ -5 \end{bmatrix} + y \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$$3x + 5y = 9$$
$$-5x - 2y = 4$$

$$15x + 25y = 45$$
$$-15x - 6y = 12$$

$$19y = 57 \Rightarrow y = 3,\ x = -2$$

$$\mathbf{a} = -2\mathbf{d} + 3\mathbf{e}$$

Question 4 **(8 marks)**

(a) Evaluate $\frac{3!7!}{9!}$. (1 mark)

$$x = \frac{3 \times 2 \times 7!}{9 \times 8 \times 7!} = \frac{1}{12}$$

(b) Determine the number of different permutations of the letters in the word NEEDLED. (2 marks)

$$n = \frac{7!}{3!2!}$$

$$= \frac{7 \times 6 \times 5 \times 4}{2} = 420$$

(c) A password is formed using all seven of the characters $, %, @, Y, Z, 8 and 9 just once. Determine the number of different passwords that are possible in which all the symbols are adjacent, all the letters are adjacent and all the digits are adjacent. (3 marks)

CLD: $3! \times 2! \times 2! = 24$

CLD arranged $3!$ ways

$24 \times 3! = 144$

(d) Determine the least number of randomly chosen integers between 10 and 99 required to be certain that the difference of the digits in at least two of the integers is the same. (*For example, the difference of the digits in the integer* 49 *is* $9 - 4 = 5$). (2 marks)

There are 10 possible differences (from 0 to 9), which give us 10 pigeonholes to fill.

If more than 10 numbers are chosen, then at least one pigeonhole must contain two or more numbers. So at least 11 numbers must be chosen.

Question 5 **(5 marks)**

A proposition states that for any integer n, if $n^2 - 4n - 3$ is even, then n is odd.

(a) Write the contrapositive of this proposition. (1 mark)

If n is not odd, then $n^2 - 4n - 3$ is not even.

(b) Use the contrapositive statement to prove the proposition is true. (4 marks)

If n is not odd:

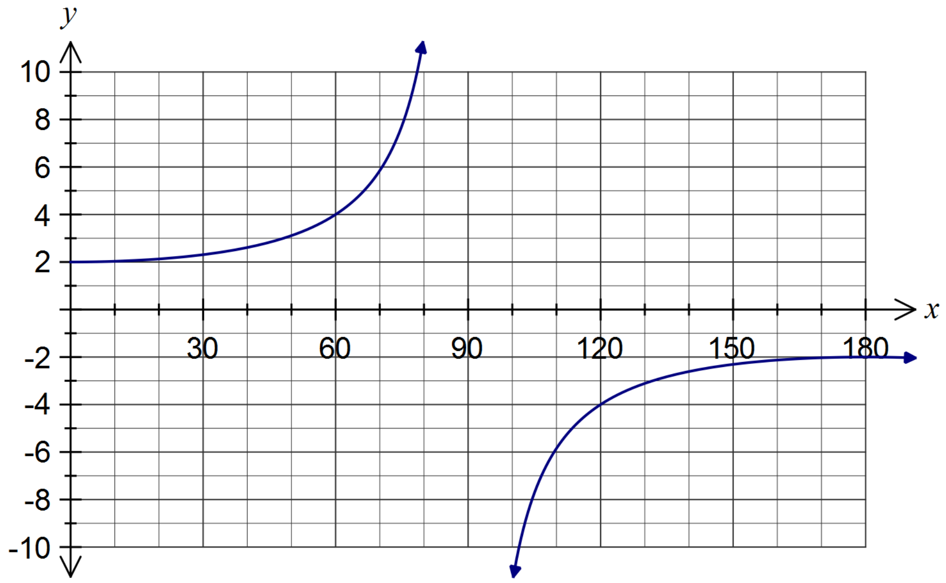
$$n = 2k \quad (k \in \mathbb{Z})$$

$$\begin{aligned} n^2 - 4n - 3 &= (2k)^2 - 4(2k) - 3 \\ &= 4k^2 - 8k - 3 \\ &= 2(2k^2 - 4k - 2) + 1 \Rightarrow \text{ odd} \end{aligned}$$

Hence, as the contrapositive has been shown to be true, the original proposition must also be true.

Question 6 **(7 marks)**

(a) Sketch the graph of $y = 2cosec(x+90)$ for $0^\circ \leq x \leq 180^\circ$. (3 marks)



(b) Prove the identity $\cot A + \tan A = sec\,A\,cosec\,A$. (4 marks)

$$LHS = \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$$

$$= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$$

$$= \frac{1}{\cos A \sin A}$$

$$= \sec A \operatorname{cosec} A$$

$$= RHS$$

Question 7 **(7 marks)**

(a) Matrix A represents a rotation of 180º about the origin. Determine

(i) matrix A. (1 mark)

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

(ii) the exact coordinates of the point (-2, 3) after transformation by matrix A. (1 mark)

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \Rightarrow A'(2,-3)$$

(iii) the determinant of matrix A. (1 mark)

1

(b) Matrix $B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Describe the transformation represented by B and calculate its determinant. (2 marks)

B is a reflection in the y-axis.

$det(B) = -1$

(c) Use an example to show that two non-singular square matrices C and D exist such that the determinant of their sum is equal to the sum of their determinants. (2 marks)

If $C = A$ and $D = B$ from above, then $\det(C) + \det(D) = -1 + 1 = 0$.

Also, $C + D = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ and $\det(C + D) = 0$.

Hence $\det(C + D) = \det(C) + \det(D)$.

Question 8 **(7 marks)**

The complex number $z = \dfrac{1+i}{1-i} - \dfrac{4+3i}{a-i}$, where a is a real constant.

(a) Show that $\mathrm{Re}(z) = \dfrac{3-4a}{a^2+1}$ and that $\mathrm{Im}(z) = \dfrac{a^2-3a-3}{a^2+1}$. (4 marks)

$$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i} - \frac{4+3i}{a-i} \times \frac{a+i}{a+i}$$
$$= \frac{2i}{2} - \frac{4a-3+(3a+4)i}{a^2+1}$$

$$\mathrm{Re}(z) = \frac{3-4a}{a^2+1}$$

$$\mathrm{Im}(z) = 1 - \frac{3a+4}{a^2+1}$$
$$= \frac{a^2+1-3a-4}{a^2+1}$$
$$= \frac{a^2-3a-3}{a^2+1}$$

(b) Determine the value(s) of a when $\mathrm{Im}(z) + \mathrm{Re}(z) = 0$. (3 marks)

$$\mathrm{Im}(z) + \mathrm{Re}(z) = 0$$
$$a^2 - 3a - 3 + 3 - 4a = 0$$

$$a^2 - 7a = 0$$
$$a(a-7) = 0$$

$$a = 0,\ a = 7$$

End of questions

Additional working space

Question number: _________

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