



Rossmoyne Senior High School

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS
METHODS
UNITS 1&2
Section One:
Calculator-free

SOLUTIONS

VA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Number of additional answer booklets used (if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

Question number: _____

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

****Allow answer only for 2 mark questions unless otherwise stated**

Question 1

(a) Solve $(x - 6)^2 - 25 = 0$. (2 marks)
(6 marks)

Solution
$(x - 6)^2 = 25$ $x - 6 = \pm 5$ $x = 11, \quad x = 1$
Specific behaviours
✓ $x = 11$ ✓ $x = 1$

Let $g(x) = x^3 - 7x^2 + 7x + 15$.

(b) Evaluate $g(-1)$.

(1 mark)

Solution
$g(-1) = -1 - 7 - 7 + 15 = 0$
Specific behaviours
✓ obtains zero

(c) Factorise $g(x)$.

(3 marks)

Solution
$g(x) = (x + 1)(x^2 + bx + c)$ By inspection: $g(x) = (x + 1)(x^2 - 8x + 15)$ $= (x + 1)(x - 3)(x - 5)$
Specific behaviours
✓ uses result from (b) to obtain one factor ✓ obtains quadratic factor (or appropriate working, allow trial substitution ie use of factor theorem) ✓ correct factorisation

See next page

SN085-182-3

Question 8 Solve the following simultaneous equations for x and y .

$4x + y = \frac{4}{8}x$ and $0.1x - y = 10^{2y+4}$

(3 marks)

Solution
$2^{2x+2y} = 2^3 - x$ and $10^{y-x} = 10^{2y+4}$ $\therefore 2x + 2y = 3 - x$ and $y - x = 2y + 4$ $\therefore 3x + 2y = 3$ and $x + y = -4$ $\therefore x = 11$ and $y = -15$
Specific behaviours
✓ rewrites with same base correctly both equations ✓ states 2 linear equations to solve ✓ solve correctly

(b) Determine the coordinates of the point(s) where the line $3y - x = 10$ intersects the circle with centre $(-2, 1)$ and radius 5. (7 marks)

Solution
Equation of circle: $(x + 2)^2 + (y - 1)^2 = 25$ Use line to substitute $x = 3y - 10$: $(3y - 10 + 2)^2 + (y - 1)^2 = 25$ $(3y - 8)^2 + (y - 1)^2 = 25$ Expand: $9y^2 - 48y + 64 + y^2 - 2y + 1 - 25 = 0$ Simplify: $10y^2 - 50y + 40 = 0$ $y^2 - 5y + 4 = 0$ Solve quadratic: $(y - 4)(y - 1) = 0$ $y = 4 \Rightarrow x = 3(4) - 10 = 2$ Or $y = 1 \Rightarrow x = 3(1) - 10 = -7$ Intersect at the points $(2, 4)$ and $(-7, 1)$.
Specific behaviours
✓ writes equation of circle ✓ substitutes line to eliminate x or y ✓ expands ✓ simplifies ✓ solves quadratic ✓ one correct point ✓ second correct point

End of questions

SN085-182-3

Question 2

(6 marks)

- (a) Evaluate $f'(3)$ when $f(x) = 10x^2 - 5x^4$.

(2 marks)

Solution
$f'(x) = 20x - 20x^3$ $= 20x(1 - x^2)$ $f'(3) = 60(1 - 9)$ $= -480$
Specific behaviours
<ul style="list-style-type: none"> ✓ obtains $f'(x)$ ✓ correct value

- (b) Determine $\frac{d}{dx}((5x - 6)(5x + 6))$.

(2 marks)

Solution
$(5x - 6)(5x + 6) = 25x^2 - 36$ $\frac{d}{dx}(25x^2 - 36) = 50x$
Specific behaviours
<ul style="list-style-type: none"> ✓ expands into polynomial ✓ obtains derivative

- (c) The volume of water in a tank at time t seconds is given by $V(t) = t^3 - 3t + 1 \text{ cm}^3$. Determine the instantaneous rate of change of volume when $t = 5$.

(2 marks)

Solution
$V'(t) = 3t^2 - 3$ $V'(5) = 3(25) - 3$ $= 72 \text{ cm}^3/\text{s}$
Specific behaviours
<ul style="list-style-type: none"> ✓ obtains $V'(t)$ ✓ correct rate of change

Question 7

(7 marks)

- (a) Solve the equation $\tan(3x - 15^\circ) = 1$ when $0 \leq x \leq 90^\circ$.

(3 marks)

Solution
$3x - 15^\circ = 45, 225^\circ$ $3x = 60^\circ, 240^\circ$ $x = 20^\circ, 80^\circ$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates $\tan^{-1} 1 = 45^\circ$ ✓ one correct solution ✓ second correct solution

- (b) In triangle ABC , the length of side AB is 12 cm, $\sin A = 0.6$ and $\sin C = 0.9$. Determine the length of side BC .

(2 marks)

Solution
Using sin rule: $\frac{BC}{0.6} = \frac{12}{0.9}$ $BC = \frac{0.6 \times 12}{0.9}$ $= \frac{2}{3} \times 12 = 8 \text{ cm}$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates correct use of sin rule ✓ correct length

- (c) Triangle PQR has sides of length 3, 4 and 6 cm. Given that PR is the longest side in the triangle, determine the value of $\cos Q$.

(2 marks)

Solution
Using cosine rule: $\cos Q = \frac{3^2 + 4^2 - 6^2}{2(3)(4)}$ $= -\frac{11}{24}$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates correct use of cosine rule ✓ correct value

Question 3 (5 marks)

The quadratic function $f(x) = ax^2 + bx - 6$ has roots at $x = 1$ and $x = -3$.

(a) Determine the value of the constant a and the value of the constant b . (3 marks)

Solution
$a(-3)^2 + b(-3) - 6 = 0$ $9a - 3b - 6 = 0 \dots\dots\dots 1$ $a(1)^2 + b(1) - 6 = 0$ $3a + 3b - 18 = 0 \dots\dots\dots 2$ $12a - 24 = 0 \dots\dots\dots 1 + 2$ $a = 2$ $b = 4$
Specific behaviours
✓ subst in roots equates to 0 ✓ value of a ✓ value of b

Solution
Roots \rightarrow factors: $f(x) = a(x - 1)(x + 3)$ $= a(x^2 + 2x - 3)$ Using last term: $-6 = -3a \Rightarrow a = 2$ $b = 2a = 4$
Specific behaviours
✓ uses factors to expand ✓ value of a ✓ value of b

(b) State the range of the function f . (2 marks)

Solution
Minimum turning point midway between roots: $x = \frac{1 - 3}{2} = -1$ $f(-1) = 2(-2)(2) = -8$ Hence range is $y = \{y \in \mathbb{R}, y \geq -8\}$
Specific behaviours
✓ locates turning point ✓ obtains range, allow just $y \geq -8$

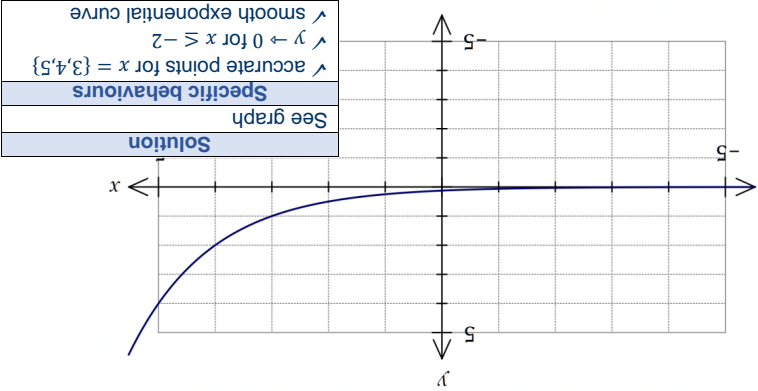
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SN085-182-3

Question 6 (5 marks)

Let $f(x) = 2^{x-3}$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



(b) Solve $f(x) = \sqrt[3]{2}$ for x . (2 marks)

Solution
$2^{x-3} = 2^{\frac{1}{3}}$ $x - 3 = \frac{1}{3}$ $x = 3\frac{1}{3} = \frac{10}{3}$
Specific behaviours
✓ forms equation with fractional index ✓ correct solution

SN085-182-3

See next page

Question 4

(7 marks)

- (a) The first term of an arithmetic sequence is 4 and the 11th term is three times the 4th term. Determine the sum of the first 10 terms of this sequence. (4 marks)

Solution
$T_{11} = 3T_4$ $4 + (11 - 1)d = 3(4 + (4 - 1)d)$ $4 + 10d = 12 + 9d$ $d = 8$ $S_{15} = \frac{10}{2}(2(4) + (10 - 1)(8))$ $= 5(8 + 72)$ $= 5 \times 80$ $= 400$
Specific behaviours
<ul style="list-style-type: none"> ✓ formulates equation ✓ solves for d ✓ correct use of sum formula ✓ calculates sum

- (b) Determine S_{∞} for the following geometric sequence:

$$\frac{7}{4}, \frac{7}{16}, \frac{7}{64}, \frac{7}{256}, \dots$$

(3 marks)

Solution
$a = \frac{7}{4}, \quad r = \frac{1}{4}$ $S_{\infty} = \frac{7}{4} \div \left(1 - \frac{1}{4}\right)$ $= \frac{7}{4} \div \frac{3}{4}$ $= \frac{7}{3}$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates a and r ✓ correct use of formula ✓ correct sum to infinity

Question 5

(7 marks)

- (a) Determine the function f given that $f(3) = 2$ and $f'(x) = 11 - 8x$. (3 marks)

Solution
$f(x) = 11x - 4x^2 + c$ $f(3) = 11(3) - 4(3)^2 + c = 2$ $c = 5$ $\therefore f(x) = 11x - 4x^2 + 5$
Specific behaviours
<ul style="list-style-type: none"> ✓ obtains antiderivative ✓ evaluates constant ✓ clearly states function

- (b) Determine the equation of the tangent to the curve $y = x^4 - 4x^2 + 19x + 42$ at the point where $x = -2$. (4 marks)

Solution
<p>Gradient function:</p> $\frac{dy}{dx} = 4x^3 - 8x + 19$ <p>Gradient of tangent:</p> $m = 4(-2)^3 - 8(-2) + 19$ $= 4(-8) + 16 + 19$ $= 3$ <p>y-coordinate of point of tangency:</p> $y = (-2)^4 - 4(-2)^2 + 19(-2) + 42$ $= 16 - 16 - 38 + 42$ $= 4$ <p>Hence tangent:</p> $y - 4 = 3(x - (-2))$ $y = 3x + 10$
Specific behaviours
<ul style="list-style-type: none"> ✓ obtains gradient function ✓ calculates gradient of tangent ✓ obtains y-coordinate ✓ obtains equation of tangent