ALL SAINTS' COLLEGE

Mathematics Specialist

Test 1 2016

Complex Numbers

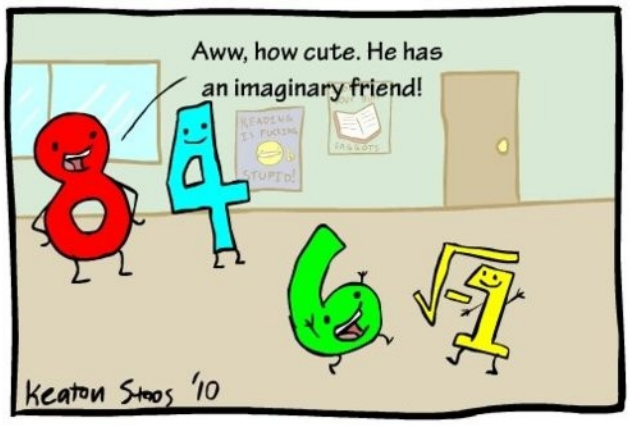
Resource FREE

NAME: _____

TEACHER: MLA

28 marks

28 minutes

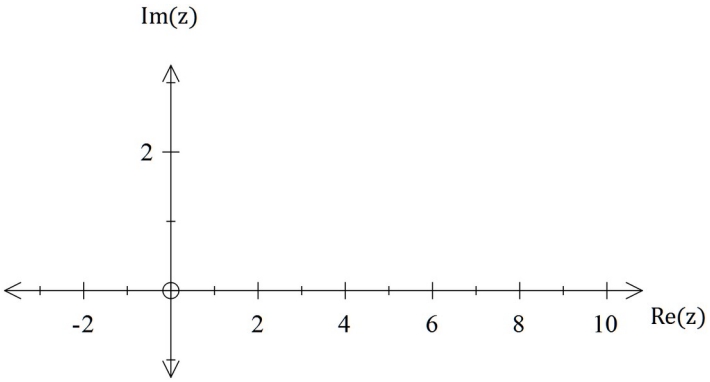


SCSA formulae sheets may be used in this section

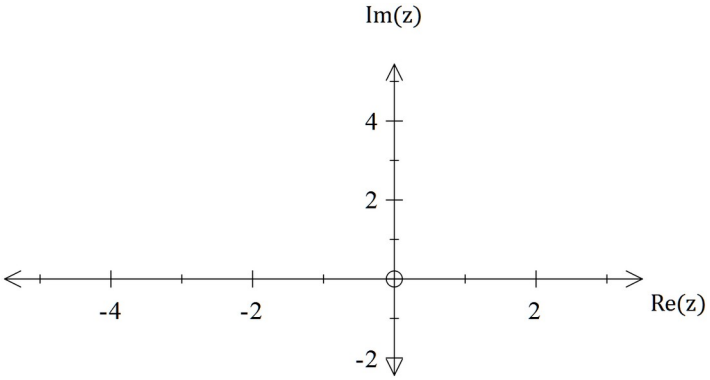
Question 1 [3, 2, 2 and 2 = 9 marks]

Represent the following regions on separate Argand diagrams:

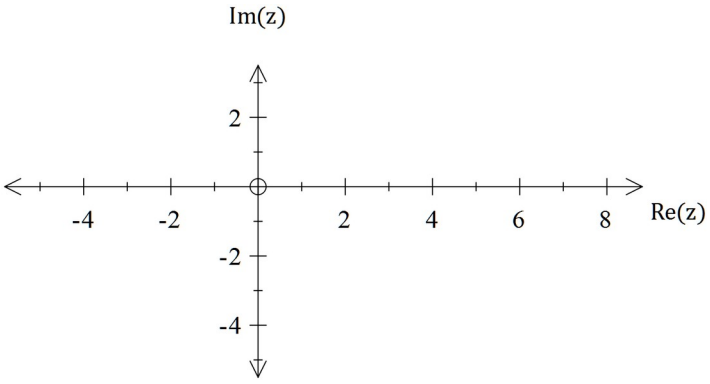
(a) $\Im(z) \leq 2\Re(z)+1 \cap 1 \leq \Im(z) < 2$



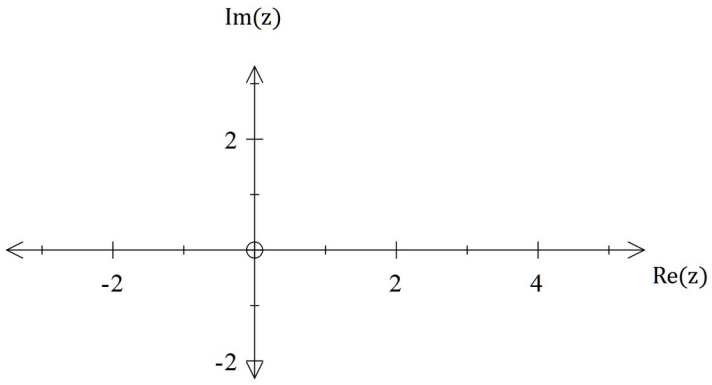
(b) $arg(z-3i) = arg\ ¿) + \pi$



(c) $|z| = arg\,(z)$



(d) ¿



Question 2 [3, 2 & 3 = 8 marks]

(a) If $z = r\,cis(\alpha)$, *prove that* $z^{-1} = \dfrac{\overline{z}}{r^2}$

(b) Show that $\cos(\theta) - i\sin(\theta) = cis(-\theta)$

(c) Express $z + \overline{z} = (z)(\overline{z})$ in Cartesian form. Describe the locus of z.

Question 3 [3 & 2 = 5 marks]

(a) Use de Moivre's theorem to solve $z^3 = -8$, leaving answers in polar form.

(b) Determine the exact area of the polygon whose vertices are the solutions found above.

Question 4 [6 marks]

Consider the identities $z^n+\frac{1}{z^n}=2\cos(n\theta)$ and $z^n-\frac{1}{z^n}=2i\sin(n\theta)$.

Use one or both of these identities to prove that $6\sin(2\theta)+3\sin(4\theta)=12\sin(2\theta)\cos^2(\theta)$.

End of Section 1

ALL SAINTS' COLLEGE

Mathematics Specialist

Test 1 2016

Complex Numbers

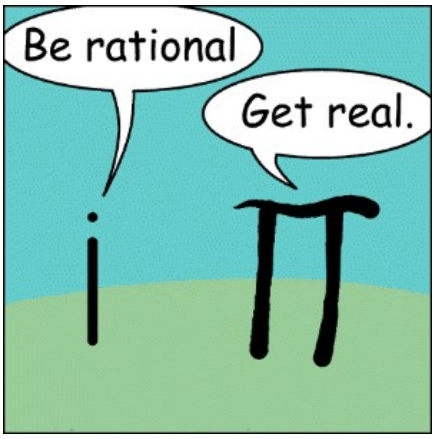
Resource RICH

NAME: _____

TEACHER: MLA

22 marks

22 minutes



SCSA formulae sheets, ClassPads and a single A4 sheet of notes may be used in this section

Clear working must be shown in order to be awarded full marks

Question 5 [3 & 3 = 6 marks]

(a) The polynomial $2x^3+bx^2+c$ has a factor $(x+1)$ and leaves a remainder of 16 when it is divided by $(x-3)$. Find the values of b and c.

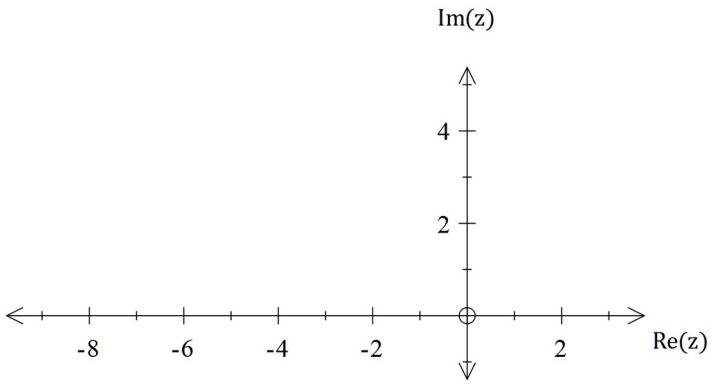
(b) If $(x-a)^2$ *is a factor of the real polynomial* $f(x)$, *then* $(x-a)$ is a factor of $f'(x)$, where $f'(x)$ is the derivative of $f(x)$ with respect to x.

Knowing this, if $(x+2)^2$ *is a factor of* $2x^4+bx^3+cx^2-4$, determine the values of b and c.

Question 6 [2, 1, 2 = 5 marks}

For $\{z:|z+5-2i|=2\}$, determine:

(a) The exact maximum possible value of $|z|$



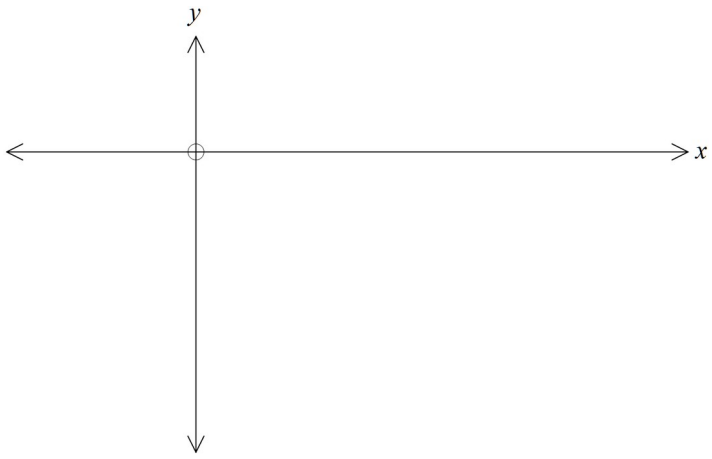
(b) The maximum possible value of $arg\ (z)$

(c) The minimum possible value of $arg\ (z)$, correct to 1 decimal place.

Question 7 [4 & 1 = 5 marks]

(a) Determine the Cartesian equation represented by $\{z : |z-(10+5i)| = 3|z-(2-3i)|\}$

(b) Sketch the locus defined in (a)



Question 8 [6 marks]

Solve $z^3+(1+i)z^2+(2+i)z+2=0$, $\forall\, z \in \mathrm{C}$, leaving answers in exact form.

End of Section 2