Matrices

Questions are taken from VCE Secondary Papers

2009

Question 2

Tickets for the function are sold at the school office, the function hall and online.

Different prices are charged for students, teachers and parents.

Table 1 shows the number of tickets sold at each place and the total value of sales.

Table 1

	School office	Function hall	Online
Student tickets	283	35	84
Teacher tickets	28	4	3
Parent tickets	5	2	7
Total sales	\$8712	\$1143	\$2609

For this function

- student tickets cost $\$x$
- teacher tickets cost $\$y$
- parent tickets cost $\$z$.

a. Use the information in Table 1 to complete the following matrix equation by inserting the missing values in the shaded boxes.

$$\begin{bmatrix} 283 & 28 & 5 \\ \boxed{} & 4 & \boxed{} \\ 84 & 3 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8712 \\ 1143 \\ 2609 \end{bmatrix}$$

b. Use the matrix equation to find the cost of a teacher ticket to the school function.

Question 3

In 2009, the school entered a Rock Eisteddfod competition.

When rehearsals commenced in February, all students were asked whether they thought the school would make the state finals. The students' responses, 'yes', 'no' or 'undecided' are shown in the initial state matrix S_0.

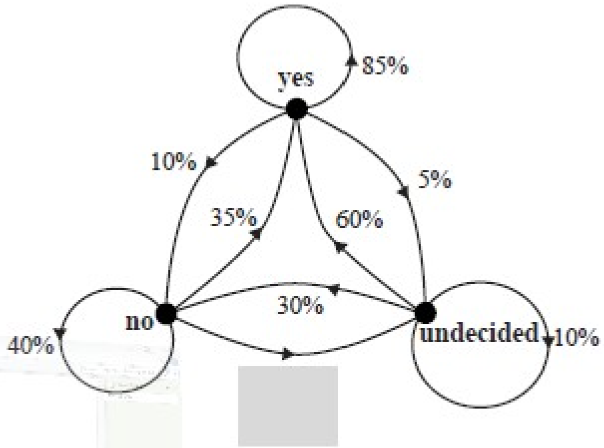
$$S_0 = \begin{array}{c} \begin{bmatrix} 160 \\ 120 \\ 220 \end{bmatrix} \end{array} \begin{array}{l} yes \\ no \\ undecided \end{array}$$

a. How many students attend this school?

Each week some students are expected to change their responses. The changes in their responses from one week to the next are modelled by the transition matrix T shown below.

$$\begin{array}{cc} & \textit{response this week} \\ & \begin{array}{ccc} \textit{yes} & \textit{no} & \textit{undecided} \end{array} \\ T = & \begin{bmatrix} 0.85 & 0.35 & 0.60 \\ 0.10 & 0.40 & 0.30 \\ 0.05 & 0.25 & 0.10 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \textit{yes} \\ \textit{no} \\ \textit{undecided} \end{array} \quad \begin{array}{l} \textit{response} \\ \textit{next week} \end{array}$$

The following diagram can also be used to display the information represented in the transition matrix T.



b. **i.** Complete the diagram above by writing the missing percentage in the shaded box.

ii. Of the students who respond 'yes' one week, what percentage are expected to respond 'undecided' the next week when asked whether they think the school will make the state finals?

iii. In total, how many students are not expected to have changed their response at the end of the first week?

c. Evaluate the product $S_1 = T\,S_0$, where S_1 is the state matrix at the end of the first week.

d. How many students are expected to respond 'yes' at the end of the third week when asked whether they think the school will make the state finals?

Question 4

A series of extra rehearsals commenced in April. Each week participants could choose extra dancing rehearsals or extra singing rehearsals.

A matrix equation used to determine the number of students expected to attend these extra rehearsals is given by

$$L_{n+1} = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \times L_n - \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

where L_n is the column matrix that lists the number of students attending in week n.

The attendance matrix for the first week of extra rehearsals is given by

$$L_1 = \begin{bmatrix} 95 \\ 97 \end{bmatrix} \begin{array}{l} \textit{dancing} \\ \textit{singing} \end{array}$$

a. Calculate the number of students who are expected to attend the extra singing rehearsals in week 3.

b. Of the students who attended extra rehearsals in week 3, how many are not expected to return for any extra rehearsals in week 4?

Answers

2. a. 35 and 2

b. $32

$$\begin{bmatrix} 283 & 28 & 5 \\ 35 & 4 & 2 \\ 84 & 3 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 8712 \\ 1143 \\ 2609 \end{bmatrix} = \begin{bmatrix} 27 \\ 32 \\ 35 \end{bmatrix}$$

3. a. 500

b. i. 25%

ii. 5%

iii. 206

c. $$S_1 = \begin{bmatrix} 310 \\ 130 \\ 60 \end{bmatrix}$$

d. 361

$$T^3 S_0 = \begin{bmatrix} 361 \\ 91.1 \\ 47.9 \end{bmatrix}$$

4. a. 68

$$L_2 = T \times L_1 - \begin{bmatrix} 5 \\ 7 \end{bmatrix} \Rightarrow L_2 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 95 \\ 97 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 80 \end{bmatrix}$$

$$\therefore L_3 = T \times L_2 - \begin{bmatrix} 5 \\ 7 \end{bmatrix} \Rightarrow L_3 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 100 \\ 80 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 68 \end{bmatrix}$$

b. 12

The subtracted column matrix $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$ indicates that, each week, another 5 + 7 = 12 students will no longer turn up to any rehearsal.

2008

Question 1

Two subjects, Biology and Chemistry, are offered in the first year of a university science course.
The matrix N lists the number of students enrolled in each subject.

$$N = \begin{array}{c} \begin{bmatrix} 460 \\ 360 \end{bmatrix} \end{array} \begin{array}{l} \text{Biology} \\ \text{Chemistry} \end{array}$$

The matrix P lists the proportion of these students expected to be awarded an A, B, C, D or E grade in each subject.

$$P = \begin{array}{ccccc} A & B & C & D & E \\ \left[0.05\right. & 0.125 & 0.175 & 0.45 & \left.0.20\right] \end{array}$$

a. Write down the order of matrix P.

b. Let the matrix $R = NP$.

i. Evaluate the matrix R.

ii. Explain what the matrix element R_{24} represents.

c. Students enrolled in Biology have to pay a laboratory fee of $110, while students enrolled in Chemistry pay a laboratory fee of $95.

i. Write down a clearly labelled row matrix, called F, that lists these fees.

ii. Show a matrix calculation that will give the total laboratory fees, L, paid in dollars by the students enrolled in Biology and Chemistry. Find this amount.

Question 2

The following transition matrix, T, is used to help predict class attendance of History students at the university on a lecture-by-lecture basis.

$$\begin{array}{cccc} & \text{this lecture} & & \\ & \begin{array}{cc} \textit{attend} & \textit{not attend} \end{array} & & \\ T = & \begin{bmatrix} 0.90 & 0.20 \\ 0.10 & 0.80 \end{bmatrix} & \begin{array}{l} \textit{attend} \\ \textit{not attend} \end{array} & \text{next lecture} \end{array}$$

S_1 is the attendance matrix for the first History lecture.

$$S_1 = \begin{bmatrix} 540 \\ 36 \end{bmatrix} \begin{array}{l} \textit{attend} \\ \textit{not attend} \end{array}$$

S_1 indicates that 540 History students attended the first lecture and 36 History students did not attend the first lecture.

a. Use T and S_1 to

i. determine S_2 the attendance matrix for the second lecture

ii. predict the number of History students attending the fifth lecture.

b. Write down a matrix equation for S_n in terms of T, n and S_1.

The History lecture can be transferred to a smaller lecture theatre when the number of students predicted to attend falls below 400.

c. For which lecture can this first be done?

d. In the long term, how many History students are predicted to attend lectures?

Question 3

The bookshop manager at the university has developed a matrix formula for determining the number of Mathematics and Physics textbooks he should order each year.

For 2009, the starting point for the formula is the column matrix S_{2008}. This lists the number of Mathematics and Physics textbooks sold in 2008.

$$S_{2008} = \begin{bmatrix} 456 \\ 350 \end{bmatrix} \begin{array}{l} \textit{Mathematics} \\ \textit{Physics} \end{array}$$

O_{2009} is a column matrix listing the number of Mathematics and Physics textbooks to be ordered for 2009.

O_{2009} is given by the matrix formula

$$O_{2009} = A\,S_{2008} + B \qquad \text{where } A = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.68 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 18 \\ 12 \end{bmatrix}$$

a. Determine O_{2009}

The matrix formula above only allows the manager to predict the number of books he should order one year ahead. A new matrix formula enables him to determine the number of books to be ordered two or more years ahead.

The new matrix formula is

$$O_{n+1} = C\,O_n - D$$

where O_n is a column matrix listing the number of Mathematics and Physics textbooks to be ordered for year n.

Here, $C = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$ and $D = \begin{bmatrix} 40 \\ 38 \end{bmatrix}$

The number of books ordered in 2008 was given by

$$O_{2008} = \begin{bmatrix} 500 \\ 360 \end{bmatrix} \begin{array}{l} \textit{Mathematics} \\ \textit{Physics} \end{array}$$

b. Use the new matrix formula to determine the number of Mathematics textbooks the bookshop manager should order in 2010.

Question 4

By the end of each academic year, students at the university will have either passed, failed or deferred the year.

Experience has shown that

- 88% of students who pass this year will also pass next year
- 10% of students who pass this year will fail next year
- 2% of students who pass this year will defer next year

- 52% of students who fail this year will pass next year
- 44% of students who fail this year will fail next year
- 4% of students who fail this year will defer next year

- 65% of students who defer this year will pass next year
- 10 % of students who defer this year will fail next year
- 25% of students who defer this year will defer next year.

Twelve hundred and thirty students began a business degree in 2007.

By the end of the 2007 academic year, 880 students had passed, 230 had failed, while 120 had deferred the year.

No students have dropped out of the business degree permanently.

Use this information to predict the number of business students who will **defer** the 2009 academic year.

Answers

1. a. 1 x 5

b. i. $\begin{bmatrix} 23 & 57.5 & 80.5 & 207 & 92 \\ 18 & 45 & 63 & 162 & 72 \end{bmatrix}$

ii. The number of students who are expected to get a D in Chemistry

c. i. $\begin{array}{cc} B & C \end{array}$
$\begin{bmatrix} 110 & 95 \end{bmatrix}$

ii. $\begin{bmatrix} 110 & 95 \end{bmatrix}\begin{bmatrix} 460 \\ 360 \end{bmatrix} = \begin{bmatrix} 84800 \end{bmatrix}$

2. a. i. $\begin{bmatrix} 493.2 \\ 82.8 \end{bmatrix}$

$$S_2 = T\ S_1 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}\begin{bmatrix} 540 \\ 36 \end{bmatrix} = \begin{bmatrix} 493.2 \\ 82.8 \end{bmatrix}$$

ii. 421

$$S_5 = T^4 S_1 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}^4 \begin{bmatrix} 540 \\ 36 \end{bmatrix} = \begin{bmatrix} 421.46 \\ 154.54 \end{bmatrix}$$

b. $S_n = T^{n-1} \times S_1$

c. Lecture 8

d. 384

When two different calculations involving $S_n = T^{n-1} \times S_1$ produce the same result, the long-term state matrix has been produced. High powers for n, such as $n = 50$ and 51, could be used.

$$T^{50}S_1 = \begin{bmatrix} 384.000 \\ 191.999 \end{bmatrix} \text{ and } T^{51}S_1 = \begin{bmatrix} 384.000 \\ 191.999 \end{bmatrix} \text{ both give the same result}$$

3. a. $\begin{bmatrix} 360 \\ 250 \end{bmatrix}$

$$O_{2009} = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.68 \end{bmatrix}\begin{bmatrix} 456 \\ 350 \end{bmatrix} + \begin{bmatrix} 18 \\ 12 \end{bmatrix}$$

b. 248

$$O_{2009} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}\begin{bmatrix} 500 \\ 360 \end{bmatrix} - \begin{bmatrix} 40 \\ 38 \end{bmatrix} = \begin{bmatrix} 360 \\ 250 \end{bmatrix}$$

$$O_{2010} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}\begin{bmatrix} 360 \\ 250 \end{bmatrix} - \begin{bmatrix} 40 \\ 38 \end{bmatrix} = \begin{bmatrix} 248 \\ 162 \end{bmatrix}$$

The added column matrix at each year is applied after each transition matrix is applied. Consequently, simply squaring the transition matrix is not appropriate in this question.

4. 42

$$S_{2009} = \begin{bmatrix} 0.88 & 0.52 & 0.65 \\ 0.10 & 0.44 & 0.10 \\ 0.02 & 0.04 & 0.25 \end{bmatrix}^2 \begin{bmatrix} 880 \\ 230 \\ 120 \end{bmatrix} = \begin{bmatrix} 996.9 \\ 191.4 \\ 41.7 \end{bmatrix}$$

2007

Question 1

The table below displays the energy content and amounts of fat, carbohydrate and protein contained in a serve of four foods: bread, margarine, peanut butter and honey.

Food	Energy content (kilojoules/serve)	Fat (grams/serve)	Carbohydrate (grams/serve)	Protein (grams/serve)
Bread	531	1.2	20.1	4.2
Margarine	41	6.7	0.4	0.6
Peanut butter	534	10.7	3.5	4.6
Honey	212	0	12.5	0.1

a. Write down a 2×3 matrix that displays the fat, carbohydrate and protein content (in columns) of bread and margarine.

b. A and B are two matrices defined as follows.

$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 531 \\ 41 \\ 534 \\ 212 \end{bmatrix}$$

i. Evaluate the matrix product AB.

ii. Determine the order of matrix product BA.

Matrix A displays the number of servings of the four foods: bread, margarine, peanut butter and honey, needed to make a peanut butter and honey sandwich.

Matrix B displays the energy content per serving of the four foods: bread, margarine, peanut butter and honey.

iii. Explain the information that the matrix product AB provides.

c. The number of serves of bread (b), margarine (m), peanut butter (p) and honey (h) that contain, in total, 53 grams of fat, 101.5 grams of carbohydrate, 28.5 grams of protein and 3568 kilojoules of energy can be determined by solving the matrix equation

$$\begin{bmatrix} 1.2 & 6.7 & 10.7 & 0 \\ 20.1 & 0.4 & 3.5 & 12.5 \\ 4.2 & 0.6 & 4.6 & 0.1 \\ 531 & 41 & 534 & 212 \end{bmatrix} \begin{bmatrix} b \\ m \\ p \\ h \end{bmatrix} = \begin{bmatrix} 53 \\ 101.5 \\ 28.5 \\ 3568 \end{bmatrix}$$

Solve the matrix equation to find the values b, m, p and h.

Question 2

To study the life-and-death cycle of an insect population, a number of insect eggs (E), juvenile insects (J) and adult insects (A) are placed in a closed environment.

The initial state of this population can be described by the column matrix

$$S_0 = \begin{array}{c} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} \end{array} \begin{array}{l} E \\ J \\ A \\ D \end{array}$$

A row has been included in the state matrix to allow for insects and eggs that die (D).

a. What is the total number of insects in the population (including eggs) at the beginning of the study?

In this population

- eggs may die, or they may live and grow into juveniles
- juveniles may die, or they may live and grow into adults
- adults will live a period of time but they will eventually die.

In this population, the adult insects have been sterilised so that no new eggs are produced. In these circumstances, the life-and-death cycle of the insects can be modelled by the transition matrix

$$\begin{array}{cc} & \text{this week} \\ & \begin{array}{cccc} E & J & A & D \end{array} \\ T = & \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix} \end{array} \begin{array}{l} E \\ J \\ A \\ D \end{array} \quad \text{next week}$$

b. What proportion of eggs turn into juveniles each week?

c. **i.** Evaluate the matrix product $S_1 = T\,S_0$

$$S_1 = T\,S_0 = \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix} \begin{array}{l} E \\ J \\ A \\ D \end{array}$$

ii. Write down the number of live **juveniles** in the population after one week.

iii. Determine the number of live **juveniles** in the population after four weeks. Write your answer correct to the nearest whole number.

iv. After a number of weeks there will be no live eggs (less than one) left in the population.

When does this first occur?

v. Write down the exact steady-state matrix for this population.

$$S_{\text{steady state}} = \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix} \begin{array}{l} E \\ J \\ A \\ D \end{array}$$

d. If the study is repeated with unsterilised adult insects, eggs will be laid and potentially grow into adults. Assuming 30% of adults lay eggs each week, the population matrix after one week, S_1, is now given by

$$S_1 = T\,S_0 + B\,S_0$$

where $B = \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $S_0 = \begin{array}{c} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} \end{array}\begin{array}{l} E \\ J \\ A \\ D \end{array}$

i. Determine S_1

$$S_1 = \begin{bmatrix} \\ \\ \\ \end{bmatrix}\begin{array}{l} E \\ J \\ A \\ D \end{array}$$

This pattern continues. The population matrix after n weeks, S_n, is given by

$$S_n = T\,S_{n-1} + B\,S_{n-1}$$

ii. Determine the number of live eggs in this insect population after two weeks.

Answers

1. a. $\begin{bmatrix} 1.2 & 20.1 & 4.2 \\ 6.7 & 0.4 & 0.6 \end{bmatrix}$

   b. i. [1890]

   ii. 4 x 4

   iii. The total energy content of the servings of these four foods in one sandwich.

   c. $b = 4,\ m = 4,\ p = 2,\ h = 1$

2. a. 700

   b. 0.5

   c. i. $\begin{bmatrix} 160 \\ 280 \\ 180 \\ 80 \end{bmatrix}$

   $$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}\begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 160 \\ 280 \\ 180 \\ 80 \end{bmatrix}$$

   ii. 280

   iii. 56

   $$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}^4\begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 10.24 \\ 56.32 \\ 312.96 \\ 320.48 \end{bmatrix}$$

   iv. 7 weeks

   v. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 700 \end{bmatrix}$

   $$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}^{80}\begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \\ 0 \\ 700 \end{bmatrix}$$

   d. i. $\begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix}$

   $$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}\begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix}$$

   ii. 130

   $$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}\begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix} = \begin{bmatrix} 130 \\ 207 \\ 284 \\ 163 \end{bmatrix}$$

2006

Question 2

A new shopping centre called Shopper Heaven (S) is about to open. It will compete for customers with Eastown (E) and Noxland (N).

Market research suggests that each shopping centre will have a regular customer base but attract and lose customers on a weekly basis as follows.

80% of Shopper Heaven customers will return to Shopper Heaven next week
12% of Shopper Heaven customers will shop at Eastown next week
8% of Shopper Heaven customers will shop at Noxland next week

76% of Eastown customers will return to Eastown next week
9% of Eastown customers will shop at Shopper Heaven next week
15% of Eastown customers will shop at Noxland next week

85% of Noxland customers will return to Noxland next week
10% of Noxland customers will shop at Shopper Heaven next week
5% of Noxland customers will shop at Eastown next week

a. Enter this information into transition matrix T as indicated below (express percentages as proportions, for example write 76% as 0.76).

$$\begin{array}{rcl} & \textbf{this week} & \\ & \begin{array}{ccc} S & \quad E \quad & N \end{array} & \\ T = & \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} & \begin{array}{l} S \\ E \quad \textbf{next week} \\ N \end{array} \end{array}$$

During the week that Shopper Heaven opened, it had 300 000 customers.
In the same week, Eastown had 120 000 customers and Noxland had 180 000 customers.

b. Write this information in the form of a column matrix, K_0, as indicated below.

$$K_0 = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \begin{array}{l} S \\ E \\ N \end{array}$$

c. Use T and K_0 to write and evaluate a matrix product that determines the number of customers expected at each of the shopping centres during the following week.

d. Show by calculating at least two appropriate state matrices that, in the long term, the number of customers expected at each centre each week is given by the matrix

$$K = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$

Question 3

Market researchers claim that the ideal number of bookshops (x), sports shoe shops (y) and music stores (z) for a shopping centre can be determined by solving the equations

$$2x + y + z = 12$$
$$x - y + z = 1$$
$$2y - z = 6$$

a. Write the equations in matrix form using the following template.

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

b. Do the equations have a unique solution? Provide an explanation to justify your response.

c. Write down an inverse matrix that can be used to solve these equations.

d. Solve the equations and hence write down the estimated ideal number of bookshops, sports shoe shops and music stores for a shopping centre.

Answers

2. a. $T = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix}$ b. $K_0 = \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix}$

c. $T\,K_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix}\begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 268800 \\ 136200 \\ 195000 \end{bmatrix}$

d. Any two products $T^n\,K_0$ where $n \geq 38$. For example:

$$K_{38} = T^{38}\,K_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix}^{38}\begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$

and

$$K_{39} = T^{39}.K_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix}^{39}\begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$ and this is the same as K_{38}

3. a. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \\ 6 \end{bmatrix}$ b. There is a unique solution since $\det\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix} = 1 \neq 0$

c. $\begin{bmatrix} -1 & 3 & 2 \\ 1 & -2 & -1 \\ 2 & -4 & -3 \end{bmatrix}$

d. 3 bookshops
4 sports shoe shops
2 music stores