Christ Church Grammar School

2015
EPW 2

Year 12 MATHEMATICS 3CD
Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section
Reading time before commencing work: 5 minutes
Working time for this section: 45 minutes
Marks available: 45 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

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Question 1 **(6 marks)**

The set of counting numbers $C = \{1, 2, 3, 4, 5, \dots\}$.

(a) If a and b are consecutive odd counting numbers with $a > b$, prove that $a^2 - b^2$ is a multiple of 8. (3 marks)

(b) Prove that all two-digit counting numbers whose sum of digits is a multiple of three are divisible by three. (3 marks)

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Question 2 **(7 marks)**

Let $B = n^2 + an + a$, where a and n are both positive integers.

(a) For each conjecture below state whether it is true or false. If a conjecture is true, give **an example** that shows it is true. If a conjecture is false, give **an example** that shows it is false. (4 marks)

(i) B is always odd when n is even.

(ii) B is always odd when a is odd.

(b) By letting $a = 2k + 1$, where k is an integer, $k \geq 0$, prove the conjecture from part (a) that is true. (3 marks)

Question 7 **(5 marks)**

Let $f(n) = n^2 + n + 2$, where n is a positive integer. It is conjectured that $f(n)$ can **never** be a multiple of 3 for all positive integers n.

Let k be an integer ≥ 0.

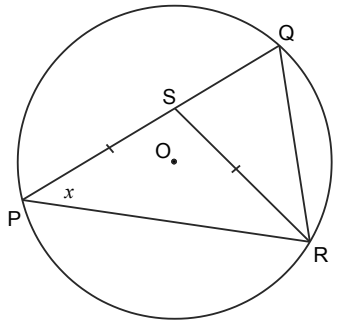
(a) Show that $f(3k) = 3(3k^2 + k) + 2$. (1 mark)

(b) Show that $f(3k + 1) = 3(3k^2 + 3k + 1) + 1$. (1 mark)

(c) Hence prove that $f(n)$ can **never** be a multiple of 3 for all positive integers n. (3 marks)

Question 6 **(7 marks)**

In the diagram below, the three vertices of ΔPQR lie on a circle with centre O. Point S lies on PQ such that the lengths PS and RS are equal and $\angle QPR = x$.



(a) Explain why $\angle QOR = 2x$. (1 mark)

(b) Find, in terms of x, $\angle PRS$ and $\angle QSR$. (2 marks)

(c) Prove that $QROS$ is a cyclic quadrilateral. (4 marks)

Question 3 **(6 marks)**

For any two unequal positive numbers a and b, the arithmetic mean is defined by $\frac{a+b}{2}$ while the geometric mean is defined by $\sqrt{ab}$.

It is conjectured that the arithmetic mean of two unequal positive numbers is always greater than their geometric mean.

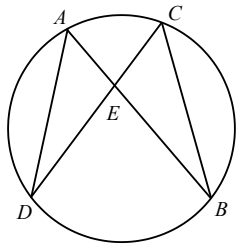
(a) Provide one pair of numbers to demonstrate that the conjecture is true. (2 marks)

(b) Prove algebraically that the conjecture is true for all unequal positive numbers a and b.
Hint: Start by considering that the square of the difference of $\sqrt{x}$ and $\sqrt{y}$ is always greater than or equal to zero.
(4 marks)

Question 4 **(6 marks)**

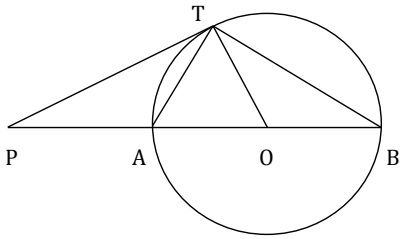
(a) In triangles ABC and DEF, $AC = DF$ and $\angle BAC = \angle EDF$.
Is the additional fact that $BC = EF$ enough to prove that triangle ABC is congruent with triangle DEF? Justify your answer. (2 marks)

(b) In the circle shown below, not to scale, AB and CD are chords that intersect at E.
Find a pair of similar triangles, giving reasons for their similarity. Hence if $AE = 4$ cm, $BE = 8$ cm and $CE = 6$ cm, determine the length of DE (4 marks)



Question 5 **(8 marks)**

In the diagram below PT is a tangent to a circle whose centre is O. The points P, A, O and B are collinear. The lengths of the line segments PA and PT are 4 cm and 6 cm, respectively.



(a) Name another angle whose size is equal to ∠PTA?
Give a reason for your answer. (2 marks)

(b) Prove that triangles PAT and PTB are similar. (3 marks)

(c) Deduce that $(PT)^2 = PA \times PB$. (1 mark)

(d) Determine the radius of the circle. (2 marks)