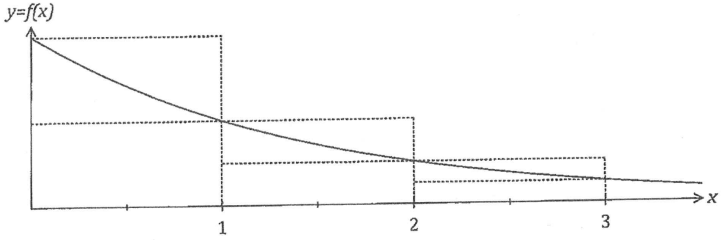
Question 10 **(3 marks)**

The function $f(x) = \frac{1}{2^x}$ is shown below.



(a) Using the sum of the inscribed rectangles shown in the diagram explain why $\int_0^3 f(x)dx > \frac{7}{8}$. (2 marks)

Underestimate : $(1 \times \frac{1}{2}) + (1 \times \frac{1}{2^2}) + (1 \times \frac{1}{2^3})$ [calculations]

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ ✓

$= \frac{7}{8}$

Since its an underestimate $\int_0^3 f(x)dx > \frac{7}{8}$ ✓ [comment]

(b) If we use the average of the sum of the areas of the inscribed rectangles and the sum of the areas of the circumscribed rectangles from the diagram above we can determine an estimate for $\int_0^3 f(x)dx$. Suggest a modification to this method to achieve a better estimate for $\int_0^3 f(x)dx$. (1 mark)

Increase number of rectangles

Smaller width rectangles ✓ [either]

etc..

3

End of questions

Christ Church Grammar School

2021
TEST 2

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

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See next page

Question 9 **(6 marks)**

The air in a hot air balloon is being inflated such that the rate of change of its volume at any time t, minutes, is given as:

$$\frac{dv}{dt} = 3t^2 - 2t$$

If initially the balloon has $3\ m^3$ of air in it, determine:

(a) the rate of change in volume when $t = 1$. Explain the meaning of this. (2 marks)

$\left.\frac{dV}{dt}\right|_{t=1} = 1\ m^3/min$ ✓ [ANSW]

Volume instantaneously increasing at $1m^3$ every minute

✓ [MUST use term 'instantaneously']

(b) for what values of t the volume is increasing. (2 marks)

$V(t) = t^3 - t^2 + 3$ ✓ [V(t)] (+3 as initially has $3m^3$)

Looking at graph on CAlc

Increasing for $t > 2/3$ ✓ [ANSW]

(c) the volume of the balloon after 5 seconds. (2 marks)

$V = \left(\frac{5}{60}\right)^3 - \left(\frac{5}{60}\right)^2 + 3$

$V = 2.99m^3$ ✓✓ [ANSW + UNITS]

6

Question 7 **(3 marks)**

(a) Evaluate the integral $\int_0^2 (\frac{1}{1+9x^2} - \frac{1}{10})\,dx$ to 4 decimal places. (1 mark)

$= 0.2685$ ✓ [value]

Just warning if not to 4 dp.

(b) Hence, or otherwise, find the area under the curve of the function $f(x) = \frac{1}{1+9x^2} - \frac{1}{10}$, from $x = 0$ *to* $x = 2$. (2 marks)

$\int_0^2 |f(x)|\,dx$ ✓ $= 0.3641$ units^2 ✓

Question 8 **(2 marks)**

A function $f(x)$ passes through the point $\left(\frac{\pi}{6}, -2\right)$. If $f'(x) = \sin(2x)$ find $f(x)$.

$f(x) = \frac{-\cos(2x)}{2} + C$

Sub in $(\pi/6, -2)$

$-2 = \frac{-\cos \pi/3}{2} + C$

$-2 = \frac{-1/2}{2} + C$

$\therefore C = -\frac{7}{4} \Rightarrow f(x) = \frac{-\cos(2x)}{2} - \frac{7}{4}$

✓ [c-value] ✓ [Equ f(x)]

5

See next page

Question 1 **(4 marks)**

Evaluate each of the following integrals (Leave answers with positive indices).

(a) $\int x^4 + \frac{1}{x^2} - \sqrt{x}\,dx$ (2 marks)

(b) $\int \frac{1}{2}\cos\left(\frac{\pi x}{4}\right)dx$ (2 marks)

See next page

Question 2 **(3 marks)**

Given that $f(x)$ is continuous everywhere and that $\int_{-4}^{6} f(x)dx = 12$, find the value of $\int_{-4}^{6} 2x - 2f(x)dx$.

Question 6 **(8 marks)**

A small body is moving in a straight line with velocity $v = 2t^2 - 19t + 30$ m/s, where t is the time, in seconds, since the body first passed through the origin, O.

(a) Determine an expression for $x(t)$, the displacement of the body at time t. (2 marks)

$x(t) = \int v(t)\,dt$

$= \int 2t^2 - 19t + 30\,dt$ ✓ [integral]

$x(t) = \frac{2}{3}t^3 - \frac{19t^2}{2} + 30t$ ✓ [Answ] [No +c as $t=0$ $x=0$]

(b) Show that the body is stationary twice and find the change in displacement of the body between these two instants. (3 marks)

Stationary: $V=0 \Rightarrow 2t^2 - 19t + 30 = 0$ OR $\Delta = 121$, $\Delta > 0 \therefore$ 2 sol^{ns}

$t=2$ and $t=15/2$ ✓ [both t values]

$\int_2^{7/2} x(t)\,dt = \frac{-1331}{24}$ ✓ [integral]

$\frac{82}{3}$ (27.3) → $t=2$

$-\frac{225}{8}$ (28.125) → $t=7.$

≈ -55.46 m ✓ [ANSW]

(c) Determine the position of the body when its velocity is a minimum. (3 marks)

$V'(t) = 4t - 19$ ✓ [V'(t)] Min $V'(t) = 0$

$t = \frac{19}{4}$ (4.75) ✓ [t-value]

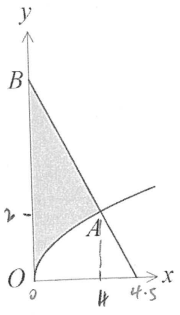
$x(4.75) = -\frac{19}{48}$

≈ -0.396m ✓ [ANSW]

8

Question 5 **(8 marks)**

The diagram below shows the graph of the function $y = \sqrt{x}$ and the straight line AB that is perpendicular to the curve at A, where $x = 4$.



(a) Determine the equation of AB. (3 marks)

$y = x^{1/2}$

$\frac{dy}{dx}\Big|_{x=4} = \frac{1}{4}$ ✓ (m_{AB})

$\therefore m_{AB} = -4$

$y = -4x + c$ (sub in 4, 2)

$2 = -4(4) + c$

$c = 18$ ✓ (c-value)

Equ $y = -4x + 18$ ✓ [Equ]

(b) Determine the shaded area in the diagram, enclosed by the curve $y = \sqrt{x}$, the straight line AB and the y-axis. (2 marks)

$\int_0^4 (-4x+18) - \sqrt{x}\, dx = \frac{104}{3}$ ✓ [integral]

$= 34.6\dot{6}$ units^2 ✓ [Answ]

(c) Determine the area enclosed by the curve $y = \sqrt{x}$, the straight line AB and the x-axis. (3 marks)

AB cuts x-axis at 4.5 $(0 = -4x + 18)$ ✓ [x-int]

$\therefore A = \frac{1}{2}(4.5) \times 18 - \frac{104}{3}$ ← from (b) ✓ [Subtract areas]

$= 40.5 - \frac{104}{3}$

$\Rightarrow 5.8\dot{3}$ units^2 $(35/6)$ ✓ [ANSW]

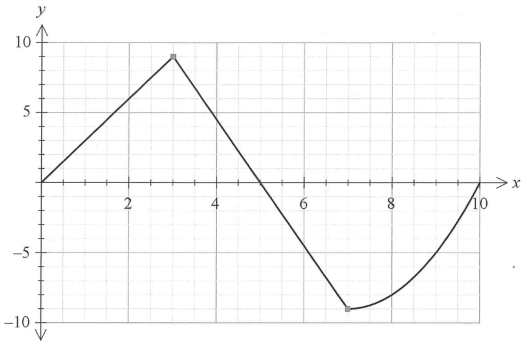
R
$\int_0^4 \sqrt{x}\, dx + \frac{1}{2} \times 1\frac{1}{2} \times 2$
$= 5\frac{1}{3} + \frac{1}{2}$
$= 5.8\dot{3}$ u^2

8

See next page

Question 3 **(4 marks)**

The graph of $y = f(x)$ is shown below. It consists of two straight lines followed by a curve. The area between the function and the x-axis is equal to 50 square units.



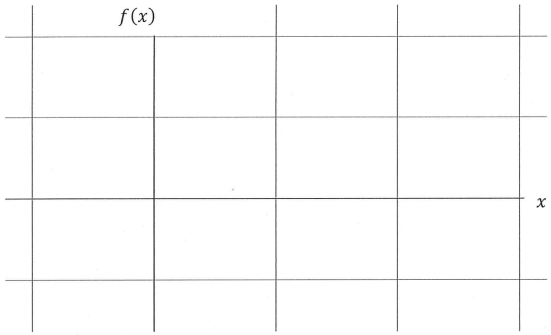
(a) $\int_0^5 f(x)dx$ (2 marks)

(b) $\int_7^{10} f(x)dx$ (2 marks)

See next page

Question 4 **(4 marks)**

(a) Sketch the curves $f(x) = \frac{x}{2}$ and $g(x) = x^2 - 2x$ on the axes below, shade the area between the curves and indicate the point(s) of intersection. (2 marks)



(b) Determine a definite integral that represents the area between the curves.
(There is no need to evaluate the integral) (2 marks)

End of Questions

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See next page

Christ Church Grammar School

2021
TEST 2

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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Christ Church Grammar School

2021
TEST 2

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section
Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

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See next page

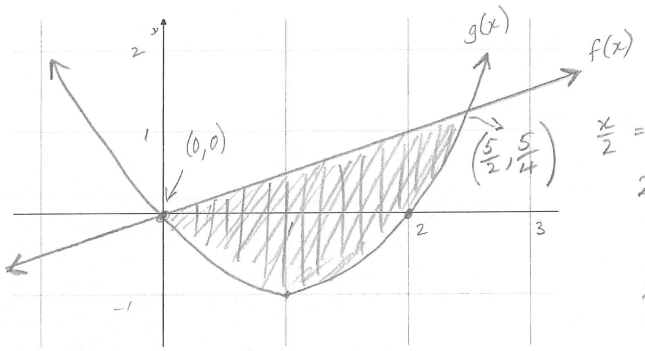
Additional working space

Question number: _____

Question 4 **(4 marks)**

(a) Sketch the curves $f(x) = \frac{x}{2}$ and $g(x) = x^2 - 2x$ on the axes below, shade the area between the curves and indicate the point(s) of intersection. (2 marks)

$g(x) = x(x-2)$



$\frac{x}{2} = x^2 - 2x$

$2x^2 - 4x - x = 0$

$2x^2 - 5x = 0$

$x(2x-5) = 0$

$x = \frac{5}{2}$

✓ [shaded region]

✓ [Points of intersection]

(b) Determine a definite integral that represents the area between the curves. (There is no need to evaluate the integral) (2 marks)

$\int_0^{5/2} \frac{x}{2} - (x^2 - 2x)\,dx$ or $\int_0^{5/2} f(x) - g(x)\,dx$ 4

✓ [x-values]

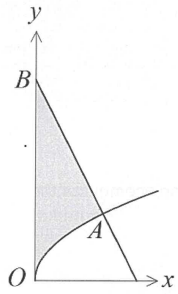
✓ [f(x) - g(x)]

End of Questions

Question 5 **(8 marks)**

The diagram below shows the graph of the function $y = \sqrt{x}$ and the straight line AB that is perpendicular to the curve at A, where $x = 4$.

(a) Determine the equation of the line AB. (3 marks)



(b) Calculate the shaded area in the diagram, enclosed by the curve $y = \sqrt{x}$, the straight line AB and the y-axis. (2 marks)

(c) Determine the area enclosed by the curve $y = \sqrt{x}$, the straight line AB and the x-axis. (3 marks)

See next page

Question 6 **(8 marks)**

A small body is moving in a straight line with velocity $v = 2t^2 - 19t + 30$ m/s, where t is the time, in seconds, since the body first passed through the origin, O.

(a) Determine an expression for $x(t)$, the displacement of the body at time t. (2 marks)

(b) Show that the body is stationary twice and find the change in displacement of the body between these two instants. (3 marks)

(c) Determine the position of the body when its velocity is a minimum. (3 marks)

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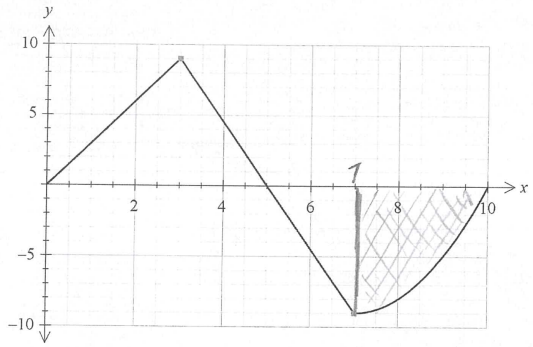
Question 3 **(4 marks)**

The graph of $y = f(x)$ is shown below. It consists of two straight lines followed by a curve. The area between the function and the x-axis is equal to 50 square units.



(a) $\int_0^5 f(x)dx$ (2 marks)

Area of ΔABC

$= \frac{1}{2}(5)(9)$ ✓ [working]

$= 22.5$ ✓ [Answ]

(b) $\int_7^{10} f(x)dx$ (2 marks)

$= -(\text{Shaded Area})$

$= 50 - (22\frac{1}{2} + \frac{1}{2}(2)(9))$

$= 50 - 31\frac{1}{2}$ ✓ [working]

$= 18\frac{1}{2}$

$\therefore \int_7^{10} f(x)\,dx = -18\frac{1}{2}$ ✓ [Answ] 4

See next page

Question 2 **(3 marks)**

Given that $f(x)$ is continuous everywhere and that $\int_{-4}^{6} f(x)dx = 12$, find the value of $\int_{-4}^{6} 2x - 2f(x)dx$.

$= \int_{-4}^{6} 2x\,dx - 2\int f(x)\,dx$

$= [x^2]_{-4}^{6} - 2\times 12$ ✓ $[\int 2x\,dx]$ ✓ $[-2\times\int f(x)\,dx]$

$= 36 - 16 - 24$

$= -4$ ✓ [Answ]

3

Question 7 **(3 marks)**

(a) Evaluate the integral $\int_0^2 (\frac{1}{1+9x^2} - \frac{1}{10})\,dx$ to 4 decimal places. (1 marks)

(b) Hence, or otherwise, find the area under the curve of the function $f(x) = \frac{1}{1+9x^2} - \frac{1}{10}$, from $x = 0$ to $x = 2$. (2 marks)

Question 8 **(2 marks)**

A function $f(x)$ passes through the point $\left(\frac{\pi}{6}, -2\right)$. If $f'(x) = \sin(2x)$ then determine $f(x)$.

Question 9 **(6 marks)**

The air in a hot air balloon is being inflated such that the rate of change of its volume at any time t, minutes, is given as:

$$\frac{dv}{dt} = 3t^2 - 2t$$

If initially the balloon has $3\ m^3$ of air in it, then determine

(a) the rate of change in volume when $t = 1$ and interpret this value. (2 marks)

(b) the values of t when the volume is increasing. (2 marks)

(c) the volume of the balloon after 5 seconds. (2 marks)

Question 1 **(4 marks)**

Evaluate each of the following integrals (Leave answers with positive indices)

(a) $\int x^4 + \frac{1}{x^2} - \sqrt{x}\ dx$ (2 marks)

$\int x^4 + x^{-2} - x^{\frac{1}{2}}\ dx$

$= \frac{x^5}{5} - \frac{x^{-1}}{-1} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$ ✓

$= \frac{x^5}{5} - \frac{1}{x} - \frac{2x^{\frac{3}{2}}}{3} + c$ ✓ [pos ind. +c]

(b) $\int \frac{1}{2}\cos\left(\frac{\pi x}{4}\right) dx$ (2 marks)

$= \frac{1}{2}\int \cos\frac{\pi x}{4}\ dx$

$= \frac{\frac{1}{2}\sin\frac{\pi x}{4}}{\frac{\pi}{4}} + c$ ✓ [Integrates]

$= \frac{2}{\pi}\sin\frac{\pi x}{4} + c$ ✓ [Tidy up]

4

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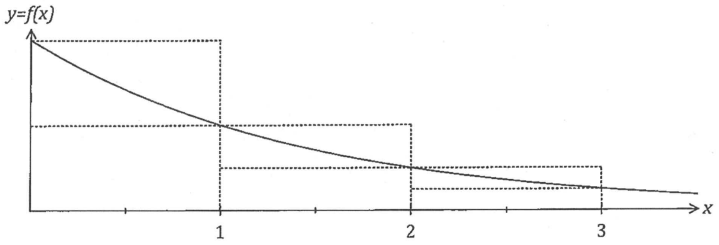

(a) Using the sum of the inscribed rectangles shown in the diagram explain why $\int_0^3 f(x)dx > \frac{7}{8}$. (2 marks)

(b) If we use the average of the sum of the areas of the inscribed rectangles and the sum of the areas of the circumscribed rectangles from the diagram above we can determine an estimate for $\int_0^3 f(x)dx$. Suggest a modification to this method to achieve a better estimate for $\int_0^3 f(x)dx$. (1 mark)

Additional working space

Question number: _____

See next page

Christ Church Grammar School

2021
TEST 2

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name - SOLUTIONS -

Teacher's name _____

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