

Papers written by
Australian Maths
Software

SEMESTER ONE

YEAR 12

**MATHEMATICS SPECIALIST
REVISION 2
UNIT 3**

2016

**Section One
(Calculator-free)**

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work:

5 minutes

Working time for section:

50 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: nil.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section One.

A formula sheet which may also be used for Section Two.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	5	5	50	50	35
Section Two Calculator— assumed	11	11	100	100	65
Total marks				150	100

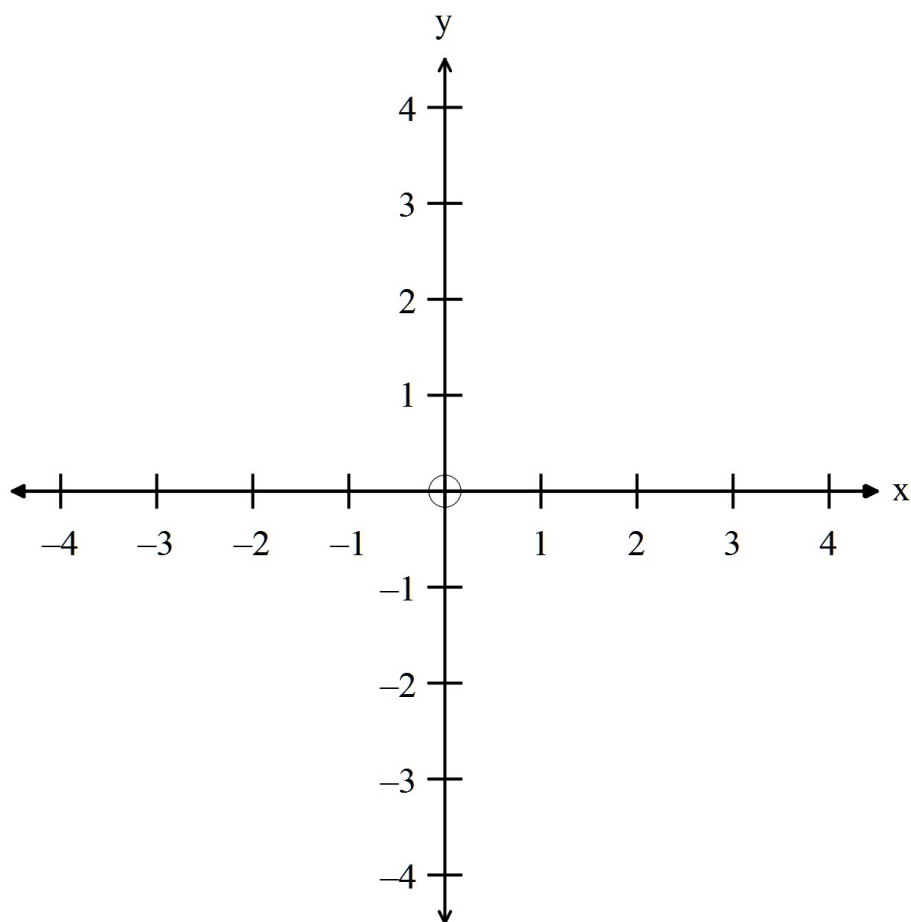
Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answer in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

1. (6 marks)

(a) Show that $1 + \frac{1}{x^2 - 1} = \frac{x^2}{x^2 - 1}$ (1)

(b) Sketch $y = \frac{x^2}{x^2 - 1}$ on the set of axes below. (5)



2. (10 marks)

(a) Use Gaussian elimination to solve the system of equations

$$x + 2y + 3z = 15$$

$$x - y - z = -3$$

$$2x + y + z = 9$$

(4)

- (b) Given the system of equations,

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 2p - 7 & 2q - 12 \end{bmatrix}$$

find possible values of p and q if

- (i) there is exactly one solution. (2)

- (ii) there is no solution. (2)

- (i) there are infinitely many solutions. (2)

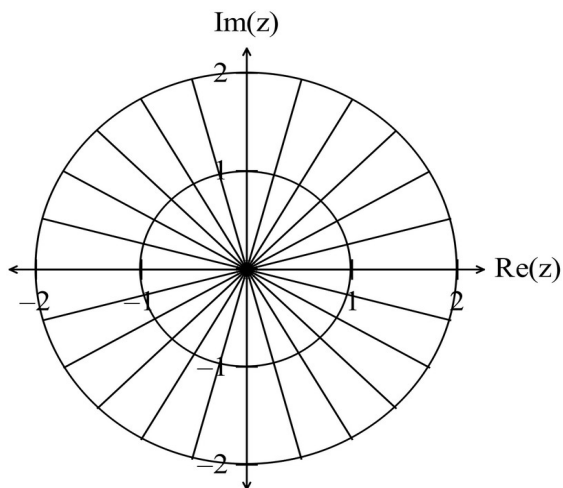
3. (13 marks)

(a) Find the equation that has as its roots $z = 1 \pm 2i$ and $z = 3 \pm i$. (3)

(b) Factorize the equation $z^3 - z^2 + 3z + 5 = 0$ then solve it. (4)

- (c) (i) Solve the equation $z^4 = -16$ for $-\pi < \theta \leq \pi$. Show all working. (3)

- (ii) Sketch the roots of the (different) equation $z^4 = 16$ on the diagram below. (2)

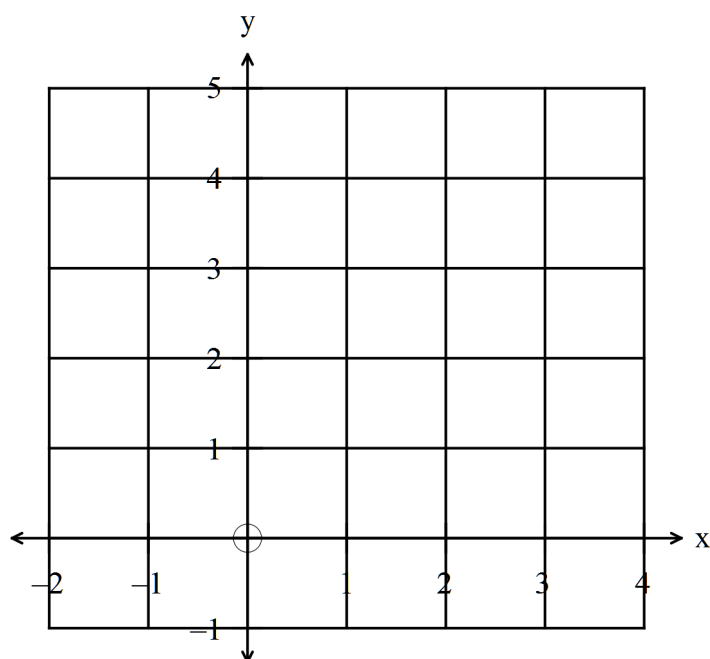


- (iii) Explain why the two sets of roots differ and how they are related. (1)

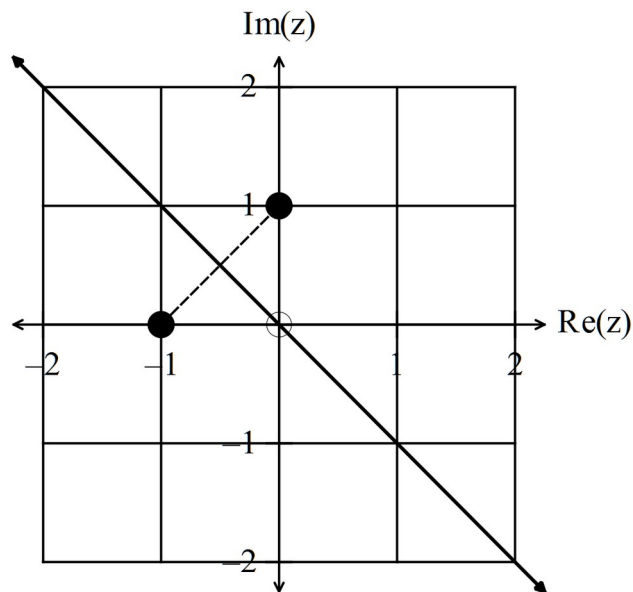
4. (13 marks)

(a) Simplify $\left(\operatorname{cis}\left(\frac{\pi}{4}\right) \right)^5 + (1-i)^5$ (3)

(b) Sketch $\left\{ z: |z - (1+2i)| \leq 2 \cap \frac{\pi}{2} \leq \arg(z) \leq \pi \cap \operatorname{Im}(z) \leq 2 \right\}$ on the set of axes below. (3)



- (c) Define the line that bisects the distance between the two given points on the set of axes below. (2)



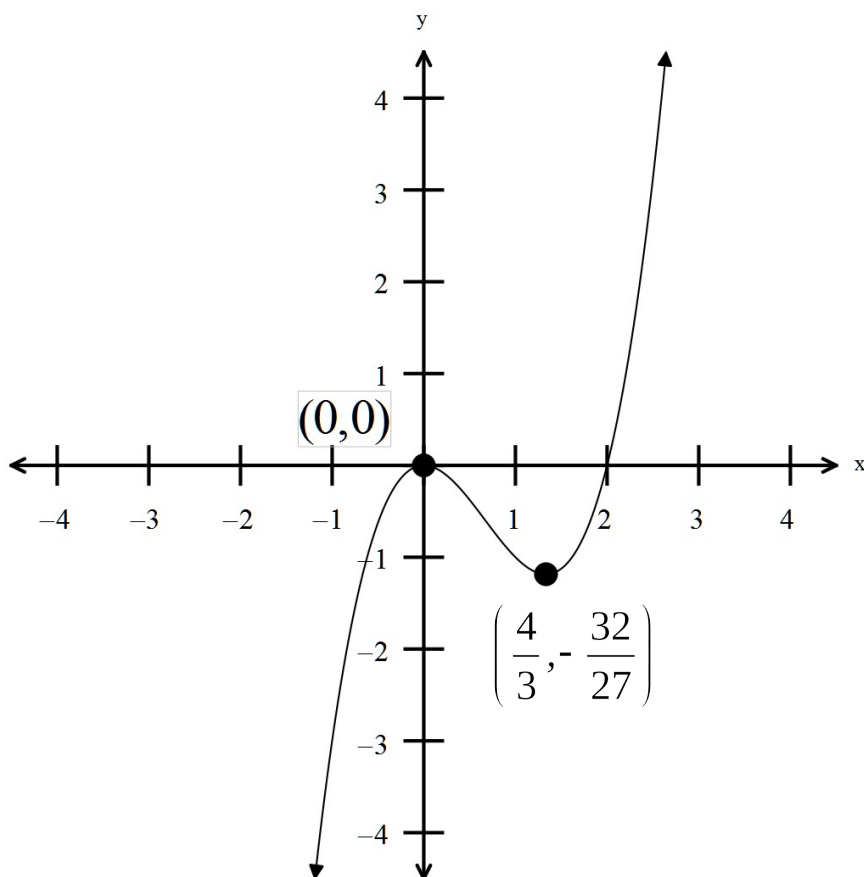
- (d) Find the modulus and argument of z given $z = \frac{\left(\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right)}{\left(\cos\left(\frac{4\pi}{3}\right) + i \sin\left(\frac{4\pi}{3}\right) \right)}$ (2)

- (e) If $z = \frac{(3 - 2i)}{(4 + 3i)}$ then determine $\text{Re}(z)$. (3)

5. (8 marks)

(a) Given $f(x) = 1 + x$ and $g(x) = 1 - x$ solve the equation $(g(x))^2 = f^{-1}(x)$. (3)

(b) The function $f(x) = x^2(x - 2)$ is graphed below. The function has a local minimum at $\left(\frac{4}{3}, -\frac{32}{27}\right)$ and a local maximum at $(0, 0)$.



(i) Find a domain for f such that f^{-1} exists. (2)

(ii) Write down the equation of the line of reflection between f and f^{-1} and sketch it on the set of axes. (2)

(iii) Given $f(4)=32$ then find $f^{-1}(32)$. (1)

END OF SECTION ONE