PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

WAEP Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS UNITS 1 AND 2 Section One: Calculator-free

SOLUTIONS

Student number: In figures

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination

	available				
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

CALCULATOR-FREE 3 METHODS UNITS 1 AND 2

Section One: Calculator-free 35% (52 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (4 marks) (a) Expand $(2\text{}+1)^3$. (2 marks)

Solution
$(2\text{}+1)^3=(1)(2\text{})^3(1)^0+(3)(2\text{})^2(1)^1+(3)(2\text{})^1(1)^2+(1)(2\text{})^0(1)^3=8\text{}^3+12\text{}^2+6\text{}+1$
Specific behaviours
✓ correct method ✓ correct expansion

(b) Determine the gradient of the curve $\text{}=(2\text{}+1)^3$ at the point $(1, 27)$. (2 marks)

Solution
$\frac{\text{}}{\text{}}=24\text{}^2+24\text{}+6$ $\text{}=1\Rightarrow\frac{\text{}}{\text{}}$ $\frac{\text{}}{\text{}}=24+24+6=54$
Specific behaviours
✓ differentiates expression from

(a) ✓ evaluates gradient

Question 2 (6 marks) (a) Evaluate $\text{}^{0.5}$ when $= 6 \times 10^2$ and $= 9 \times 10^4$, writing your answer without the use of scientific notation. (3 marks)

Solution
$$\frac{^{0.5}}{^2} = \frac{6^2 \times 10^4}{\sqrt{9 \times 10^2}}$$ $$= \frac{36}{3 \times 10^2}$$ $$= 1\,200$$
Specific behaviours
✓ simplifies ^{2}
✓ simplifies $^{0.5}$
✓ correct value

✓ substitutes function into quotient ✓ correctly combines difference of fractions ✓ eliminates division by h ✓ evaluates limit

(b) Determine the value of x when $9^{x} = 27\sqrt{3}$. (3 marks)

Solution
$3^{2x} = 3^{3} \times 3^{\frac{1}{2}}$ $= 3^{\frac{7}{2}}$ $2x = \frac{7}{2} \Rightarrow x = \frac{7}{4}$
Specific behaviours
✓ LHS as power of 3 ✓ RHS as power of 3 ✓ equates indices and solves

End of questions SN078-122-2

Supplementary page

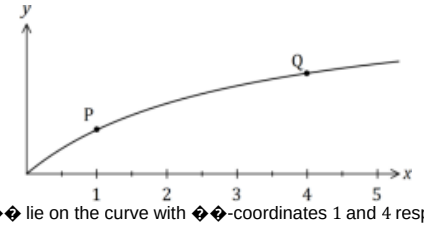
Question number: _____

CALCULATOR-FREE 5 METHODS UNITS 1 AND 2

Question 3 (7 marks) Solve each equation below for ��.

(a) $3$$$ $$$$ $$$$ $

Question 8 (8 marks) Let $f(x) = x^3 + 3$. The graph of $y = f(x)$ is shown below.



(a) Points A and B lie on the curve with x-coordinates 1 and 4 respectively.

(i) Determine $f(1)$ and $f(4)$. (1 mark)

Solution
$f(1) = \frac{1}{4}, f(4) = \frac{4}{7}$
Specific behaviours
✓ both values correct

(c) $\sqrt{2}\sin x + 1 = 0, 0° \leq x \leq 360°$. (2 marks)

Solution
$\sin x = -\frac{1}{\sqrt{2}}$ $x = 225°, x = 315°$
Specific behaviours
✓ one correct solution ✓ both correct solutions

Question 4 (7 marks) (a) Simplify

(i) $\frac{d}{dx}(10 - 3x + 4x^2)$. (1 mark)

Solution
$-3 + 8x$
Specific behaviours
✓ correct derivative

(ii) $\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h}$. (1 mark)

Solution
$2x$
Specific behaviours
✓ correct derivative

(b) Determine the equation of the tangent to the curve $\text{}=\text{}^3-9+15$ when $\text{}=2$.

(3 marks)

Solution
$$\frac{}{}=3\text{}^2-9$$ $$\text{}=2,\text{}=8-18+15=5,\frac{}{}=12-9=3$$ $$\text{}-5=3(\text{}-2)\Rightarrow\text{}=3\text{}-1$$
Specific behaviours
✓ correct derivative ✓ calculates $\text{}$-coordinate and gradient ✓ correct equation of tangent, in any form

(c) Determine $\text{}(\text{})$ given $\text{}'(\text{})=6\text{}-2$ and $\text{}(-1)=6$. (2 marks)

Solution
$$\text{}(\text{})=3\text{}^2-2\text{}+\text{}$$ $$3(-1)^2-2(-1)+\text{}=6\Rightarrow\text{}=1\ \text{}(\text{})=3\text{}^2-2\text{}+1$$
Specific behaviours
✓ correct antiderivative with
✓ equates differences ✓ solves for $\text{}$ ✓ states $\text{}$ ✓ correct $\text{}_4$

(b) the sequence is geometric. (4 marks)

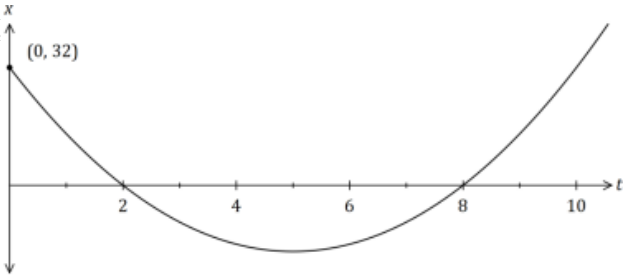
Solution
$$\text{}=2\text{}-1$$ $$4\text{}+3=\text{}-8$$ $$2\text{}-1$$ $$(2\text{}-1)(2\text{}-1)=(4\text{}+3)(\text{}-8)$$ $$4\text{}^2-4\text{}+1=4\text{}^2-29\text{}-24$$ $$25\text{}=-25$$ $$\text{}=-1$$ $$\text{}=-2-1$$ $$-4+3=3$$ $$\text{}_4=(-1-8)\times3=-27$$
Specific behaviours
✓ equates ratios ✓ solves for $\text{}$ ✓ states $\text{}$ ✓ correct $\text{}_4$

Other roots at $(-2, 0)$ and $(-4, 0)$.
Specific behaviours
✓ obtains quadratic factor by inspection ✓ factorises quadratic ✓ states both roots as coordinates

See next page SN078-122-2

Question 5 (6 marks)

A small body moves in a straight line so that its displacement s from a fixed point O after t seconds is given by $s = at^2 + bt + c$ metres.



The position-time graph of the body is shown below.

(a) Determine the values of the constants a, b and c. (3 marks)

Solution
$s = a(t - 2)(t - 8)$ $32 = a(-2)(-8) \Rightarrow a = 2$ $s = 2(t^2 - 10t + 16)$ $= 2t^2 - 20t + 32$ $a = 2, b = -20, c = 32$
Specific behaviours
✓ writes equation using roots ✓ uses s-intercept to find c ✓ expands and states three values

constant ✓ correct $s(t)$

See next page SN078-122-2

Question 7 (8 marks) The first three terms, in order, of a sequence are $4t + 3, 2t - 1$ and $t - 8$.

Determine the fourth term of the sequence if

(a) the sequence is arithmetic. (4 marks)

Solution
$d = (2t - 1) - (4t + 3) = (t - 8) - (2t - 1)$ $-2t - 4 = -t - 7 \Rightarrow t = 3$ $d = (6 - 1) - (12 + 3) = -10$ $T_4 = (3 - 8) - 10 = -15$
Specific behaviours

$\diamond\diamond = \diamond\diamond^3 + 3\diamond\diamond^2 - 10\diamond\diamond + \diamond\diamond$ $-30 = 1 + 3 - 10 + \diamond\diamond \Rightarrow \diamond\diamond = -24$ $\diamond\diamond = \diamond\diamond^3 + 3\diamond\diamond^2 - 10\diamond\diamond - 24$
Specific behaviours
✓ antidifferentiates correctly ✓ determines constant

(b) Determine the displacement of the body when its velocity is 24 ms^{-1}. (3 marks)

Solution
$\diamond\diamond = 4\diamond\diamond - 20$ $4\diamond\diamond - 20 = 24 \Rightarrow \diamond\diamond = 11$ $\diamond\diamond(11) = 2(11-2)(11-8) = 54$ m
Specific behaviours
✓ equation for velocity ✓ solves for time ✓ substitutes for displacement

(b) Show that the cubic has a root when $\diamond\diamond = 3$. (1 mark)

Solution
$\diamond\diamond = 3, \diamond\diamond = 27 + 27 - 30 - 24 = 54 - 54 = 0$
Specific behaviours
✓ substitutes and obtains zero

 See next page

Question 6 (6 marks) The derivative of a cubic polynomial is given by $\diamond\diamond\diamond\diamond$

$$\diamond\diamond\diamond\diamond = 3\diamond\diamond^2 + 6\diamond\diamond - 10.$$

The cubic passes through the point $(1, -30)$.

(a) Determine the equation of the cubic. (2 marks)

Solution

(c) Determine the coordinates of the other two roots of the cubic. (3 marks)

Solution
$\diamond\diamond^3 + 3\diamond\diamond^2 - 10\diamond\diamond - 24 = (\diamond\diamond - 3)(\diamond\diamond^2 + \diamond\diamond\diamond\diamond + 8) = (\diamond\diamond - 3)(\diamond\diamond^2 + 6\diamond\diamond + 8)$ $= (\diamond\diamond - 3)(\diamond\diamond + 4)(\diamond\diamond + 2)$