PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Year 12 Test Three 2022

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: **_____40_____ mins**

Number of questions: **_____6_____**

Materials required: NO classpads nor calculators

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **__40____ marks**

Task weighting: **_10___%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

NO classpads nor calculators!

Q1 (3 & 3= 6 marks) (3.3.9-3.3.10)

a) Solve the following set of linear equations.

$$3x - 2y + z = -8$$
$$x + 2y - 3z = -14$$
$$2x + y - z = -9$$

b) Consider the system below,

$$3x - 2y + z = p$$
$$x + 2y - 3z = -14$$
$$2x + y + qz = -9$$

Determine the values of $p \& q$ such that there are:

i) Unique solution
ii) Infinite solutions
iii) No solutions.

Q2 (2, 2, 2 & 3 = 9 marks) (3.3.11, 3.3.13)

A particle moves such that at time t seconds the velocity is $v = \begin{pmatrix} t \\ -t^2 \\ -3 \end{pmatrix} m/s$. The particle is initially at the origin.

Determine:

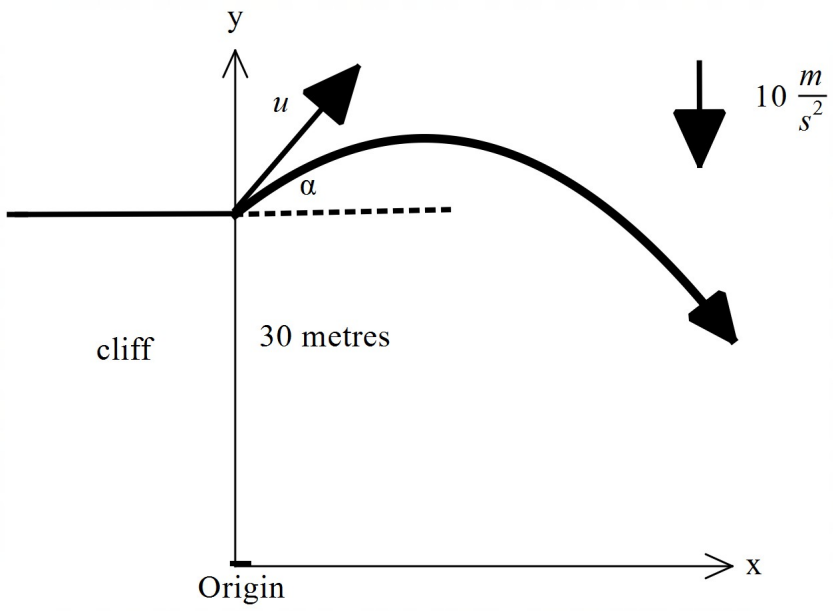
a) The position vector at time $t = 1$ second.

b) The acceleration of the particle at $t = 1$ second.

c) The speed of the particle at $t = 2$ seconds.

d) The time(s) when the velocity is perpendicular to the acceleration.

Q3 (4, 3 & 2 = 9 marks) (3.3.12, 3.3.13, 3.3.15)
Consider a particle that is projected from the top of a cliff of height 30 metres with a speed of u metres per second at an angle of α to the horizontal. Assume that the acceleration is constant at $10\,m/s^2$ towards the centre of the Earth. Let the origin of cartesian axes be at the base of the cliff as shown below with the appropriate unit vectors $i \& j$.



Let $\ddot{r} = \begin{pmatrix} 0 \\ -10 \end{pmatrix} m/s^2$.

a) Using vector integration, show how to derive the position vector r at time t seconds in terms of $u \& \alpha$. Show all steps.

b) Show how to derive the cartesian equation for the path of the particle in terms of $u \& \alpha$.

c) Set up an equation in terms of $u \& \tan \alpha$ ONLY, but do not solve, that would allow the range (x) to be determined where the particle hits the floor from the base of the cliff.

Q4 (4 marks) (4.2.1)

If $y^2 - \sin x = 1 - 5y$, determine $\frac{dy}{dx} \& \frac{d^2y}{dx^2}$ in terms of $x \& y$ only.

Q5 (3 & 4 = 7 marks) (4.2.1)
Determine the following integrals:

a) $\int \frac{5x}{\sqrt{x+1}} dx \quad u = x+1$

b) $\int \frac{8x^2 - 6x + 5}{(x - 2)(x^2 + 1)} dx$

Q6 (5 marks) (4.1.1, 4.2.2)
Consider a lighthouse that is 50 metres away from the shore. On the shore is a long brick wall. The light on the lighthouse is rotating at 4 revolutions per second. Determine the exact speed of the dot of light on the wall at a point 3 metres from the point directly opposite the lighthouse as shown below.