TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 3 2018

Section 1 Calculator Free

Discrete Random Variables

STUDENT'S NAME Solutions

DATE: Thursday 17 May **TIME**: 16 minutes **MARKS**: 16

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (8 marks)

The probability function of the discrete random variable M is given by:

m	0	1	2	3	4
$P(M=m)$	0.3	0.2	0.15	0.1	a

0.25

(a) Determine the value of a. [1]

$a = 0.25$

(b) Determine:

(i) $P(M=2)$ $= 0.15$ [1]

(ii) $P(M<4)$ $= 0.75$ [1]

(iii) $P(M<4 \mid M \geq 1)$ $= \dfrac{0.45}{0.7} = \dfrac{45}{70} = \dfrac{9}{14}$ [2]

(c) Determine the mean of M. [2]

$E(M) = 0\times0.3 + 1\times0.2 + 2\times0.15 + 3\times0.1 + 4\times0.25$

$= 0.2 + 0.3 + 0.3 + 1 = 1.8$

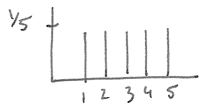
(d) Determine the most likely value of M. [1]

The most likely value is $m=0$

2. (5 marks)

The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{5}, \quad x = 1, 2, 3, 4, 5$$



(a) Determine the value of $E(X)$ [1]

$E(x) = \frac{1+5}{2}$

$= \underline{\underline{3}}$

The variance of X is 2.

(b) Determine:

(i) $E(3X-2)$ $\Rightarrow$ $3E(x) - 2$ [2]

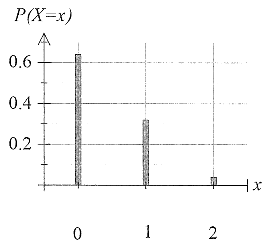
$= \underline{\underline{7}}$

(ii) $Var(4-3X)$ $= (-3)^2 \times 2$ [2]

$= \underline{\underline{18}}$

3. (3 marks)

Determine the probability of success for the binomial variable given for this distribution where $P(X = 0) = 0.64$:



$\Rightarrow$ $P(x=0) = {}^2C_0\, p^0 (1-p)^2$

$\Rightarrow$ $0.64 = (1-p)^2$

$\Rightarrow$ $0.8 = 1-p$ (ignore -ve)

$\Rightarrow$ $p = \underline{\underline{0.2}}$

7. (11 marks)

A student designed a game where two spinners each with five equally likely outcomes; 1, 2, 2, 3 and 3 are spun. Let S denote the sum of the results when these two spinners are spun.

	1	2	2	3	3
1	2	3	3	4	4
2	3	4	4	5	5
2	3	4	4	5	5
3	4	5	5	6	6
3	4	5	5	6	6

(a) Determine the largest possible value of S. [1]

6

(b) Complete the table below to give the probability associated with each value s of S [3]

s	2	3	4	5	6
$P(S=s)$	$\frac{1}{25}$	$\frac{4}{25}$	$\frac{8}{25}$	$\frac{8}{25}$	$\frac{4}{25}$
Gain	\$10	\$2	\$2	−10	−10

The player paid \$10 for each game, winning a prize of \$20 if the sum was two and \$12 if the sum was a 3 or 4.

(c) Calculate the expected gain or loss of a person who played the game once. [5]

G : Gain per \$10 game

g	10	2	−10
$P(G=g)$	$\frac{1}{25}$	$\frac{12}{25}$	$\frac{12}{25}$

$E(G) = -\$3.44$ i.e. a \$3.44 loss

(d) If the student halved the cost of the game but left the other rules unchanged, determine the new expected gain or less per game for a player. [2]

G_2 : Gain per \$5 game

g	15	7	−5
$P(G_2=g)$	$\frac{1}{25}$	$\frac{12}{25}$	$\frac{12}{25}$

$E(G_2) = \$1.56$

i.e. a gain of \$1.56

OR

$E(G) + 5$

$= -3.44 + 5$

$= 1.56$

TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 3 2018

Section 2 Calculator Assumed
Discrete Random Variables

STUDENT'S NAME Solutions

DATE: Thursday 17 May **TIME:** 33 minutes **MARKS:** 33

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

4. (5 marks)

A discrete random variable T is defined as the number of flips of a coin required before a head appears.

(a) Calculate the probabilities required to complete the probability distribution table below: [2]

t	1	2	3	4	5	6
$P(T=t)$	0.5	0.25	0.125	0.0625	0.0313	0.0156

(b) Explain why the sum of these probabilities is not one. [1]

The random variable T continues to ∞

(c) Determine:

(i) $P(T \le 4) = 0.9375$ [1]

(ii) $P(T \ge 6) = 1 - P(T \le 5)$ [1]

$= 1 - 0.96875$

$= 0.03125$

≈ 0.0313

5. (8 marks)

Rabbit offspring born on a particular farm have a probability of 20% of being albino. For each breeding cycle, in preparation for the Christmas season, the young rabbits are shipped off to veterinaries in groups of 16.

(a) Determine the probability that in a group there are

(i) an equal number of albino rabbits and non-albino rabbits. [2]

A: # rabbits that are albion

$A \sim B(16, 0.2)$

$P(A = 8) = 5.5276 \times 10^{-3} \doteq \underline{\underline{0.0055}}$

(ii) fewer than 5 albino rabbits. [1]

$P(A \leq 4) = \underline{\underline{0.7982}}$

(iii) exactly 2 albino rabbits. [1]

$P(A = 2) = \underline{\underline{0.2111}}$

(b) A random sample of 4 groups of 16 is taken from the farm. Determine the probability that exactly 2 groups contain fewer than 5 albino rabbits. [2]

G: # groups with fewer than 5 albino

$G \sim B(4, 0.7982)$

$P(G = 2) = \underline{\underline{0.1556}}$ (0.1557 if used rounded p)

(c) The farm decides to ship n rabbits per shipment to veterinaries, so that the chance of there being at least one albino rabbit in a shipment is no more than 99%. Determine the largest possible value of n. [2]

$P(A \geq 1) < 0.99$

$\Rightarrow P(A = 0) > 0.01$

$\Rightarrow {}^{n}C_{0} (0.2)^{0} (0.8)^{n} > 0.01$

$\Rightarrow n < 20.63$

$\therefore n = \underline{\underline{20}}$

6. (9 marks)

A new treatment for back pain is being tested.

There is a 25% chance that a patient will report an improvement after one month if no treatment is given.

Let X denote a patient who will report an improvement after one month, assuming that no treatment is given.

(a) Explain why the random variable X is discrete. [1]

You either have improvement or don't have improvement. This is two outcoms, ∴ discrete

(b) State the probability distribution for X. [2]

X is Bernoulli with $p = 0.25$ or $P(X = n) = \begin{cases} 0.25 & n = 1 \\ 0.75 & n = 0 \end{cases}$

(c) Calculate the

(i) Mean of X $E(X) = \underline{\underline{0.25}}$ [1]

(ii) Standard Deviation of X $\sigma(X) = \sqrt{0.25 \times 0.75}$ [1]

$= \underline{\underline{0.4330}}$

A trial group consists of 100 randomly chosen patients with back pain.

(d) What is the probability that 35 or more of the patients in the trial group will report an improvement after one month, assuming no treatment is given? [2]

BP: # people with back pain improvement

$BP \sim B(100, 0.25)$

$P(BP \geq 35) = \underline{\underline{0.0164}}$

(e) Now suppose that each patient in the trial group is given the new treatment and that 35 of them report an improvement after one month. Is this strong evidence that the treatment is effective? Justify your answer. [2]

Yes. 35% improved with this treatment, while only 25% improved without treatment. There is only a 1.64% chance that this improvement for 35 people would have occured with no treatment.