Random variables, distributions, pobability and proportions

Probability: For any event A and its complement $\bar{A}$, and event B

$P(A) + P(\bar{A}) = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)\,P(B|A) = P(B)\,P(A|B)$

In a Bernoulli trial: $\bar{x}$ is the sample proportion $\hat{p}$,

Mean $\mu = p$ and standard deviation $\sigma = \sqrt{p(1-p)}$

In a binomial distribution:

$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

Mean $\mu = np$ and standard deviation $\sigma = \sqrt{np(1-p)}$

Expected value: If X is a discrete random variable,

$E(x) = \sum p_i x_i$, where x_i are the possible values of X and $p_i = P(X = x_i)$

If X is a continuous random variable,

$E(x) = \int_{-\infty}^{\infty} xp(x)dx$, where $p(x)$ is the probability density function of X.

Variance: If X is a discrete random variable,

$Var(x) = \sum p_i (x_i - \mu)^2$, where $\mu = E(X)$ is the expected value

If X is a continuous random variable,

$Var(x) = \int_{-\infty}^{\infty} (x - \mu)^2 p(x)dx$.

A confidence interval for the proportion, p, of a population is:

$$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad , \quad \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$$

where $\hat{p}$ is the sample mean,
n is the sample size and
z is the cut-off value on the standard normal distribution corresponding to the confidence level.

Margin of error: $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ is the half-width of the confidence interval

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.

Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS METHODS
ATAR COURSE

FORMULA SHEET

2016

Measurement

Circle: $C = 2\pi r = \pi D$, where C is the circumference, r is the radius and D is the diameter
$A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height

Parallelogram: $A = bh$

Trapezium: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides

Prism: $V = Ah$, where V is the volume and A is the area of the base

Pyramid: $V = \frac{1}{3}Ah$

Cylinder: $S = 2\pi rh + 2\pi r^2$, where S is the total surface area
$V = \pi r^2 h$

Cone: $S = \pi rs + \pi r^2$, where s is the slant height
$V = \frac{1}{3}\pi r^2 h$

Sphere: $S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Exponentials

Index laws: For $a, b > 0$ and m, n real,

$a^m b^m = (a\,b)^m$ $\quad$ $a^m a^n = a^{m+n}$ $\quad$ $(a^m)^n = a^{mn}$

$a^{-m} = \frac{1}{a^m}$ $\quad$ $\frac{a^m}{a^n} = a^{m-n}$ $\quad$ $a^0 = 1$

For $a > 0$ and m an integer and n a positive integer, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

See next page

Logarithims

For a, b, y, m and n positive real and k real:

$1 = a^0 \Leftrightarrow \log_a 1 = 0$ $\quad$ $y = a^x \Leftrightarrow \log_a y = x$

$\log_a mn = \log_a m + \log_a n$ $\quad$ $a = a^1 \Leftrightarrow \log_a a = 1$

$\log_e x = \ln x$ $\quad$ $\log_a(m^k) = k\log_a m$

Calculus

Differentiation:

If $f(x) = y$ then $f'(x) = \frac{dy}{dx}$ $\quad$ If $f(x) = \ln x$ then $f'(x) = \frac{1}{x}$

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ $\quad$ If $f(x) = \sin x$ then $f'(x) = \cos x$

If $f(x) = e^x$ then $f'(x) = e^x$ $\quad$ If $f(x) = \cos x$ then $f'(x) = -\sin x$

Product rule: If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$ $\quad$ or If $y = uv$ then $\frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$

Quotient rule: If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$ $\quad$ or If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$

Chain rule: If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$ $\quad$ or If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Powers: $\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$

Exponentials: $\int e^x dx = e^x + c$

Natural logarithm: $\int \frac{1}{x}\,dx = \ln|x| + c$ $\quad$ and $\int \frac{f'(x)}{fx}\,dx = \ln(f(x)) + c$

Trigonometry: $\int \sin x\,dx = -\cos x + c$ $\quad$ and $\int \cos x\,dx = \sin x + c$

Fundamental Theorem of Calculus: $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$ $\quad$ and $\int_a^b f'(x)\,dx = f(b) - f(a)$

Incremental formula: $\delta y \simeq \frac{dy}{dx}\delta x$

Exponential growth and decay: If $\frac{dy}{dt} = ky$, then $y = Ae^{kt}$

See next page