

Applecross Senior High School

Question/Answer booklet

Semester One Examination, 2020

# MATHEMATICS METHODS UNIT 1 Selection Two: Calculator-assumed

SOLUTIONS

number of additional  
answer booklets used  
(if applicable):

our name

words

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WA student number: In figures

Reading time before comments:  
Working time:  
Minutes

Materials required/recommended for this section

This Question/Answer booklet  
Formula sheet (retained from Section One)

To be provided by the Canadian

To be provided by the candidate

Special items: drawing instruments, templates, notes on two unfolded sheets  
and up to three calculators approved for use in this examination.

Special items: drawing instruments, tape up to three calculators

You do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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**Structure of this paper**

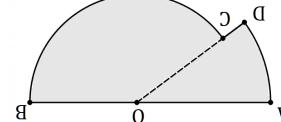
Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					<b>100</b>

Supplementary page

Question number: \_\_\_\_\_

**Instructions to candidates**

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.



(a) Determine the length OA.

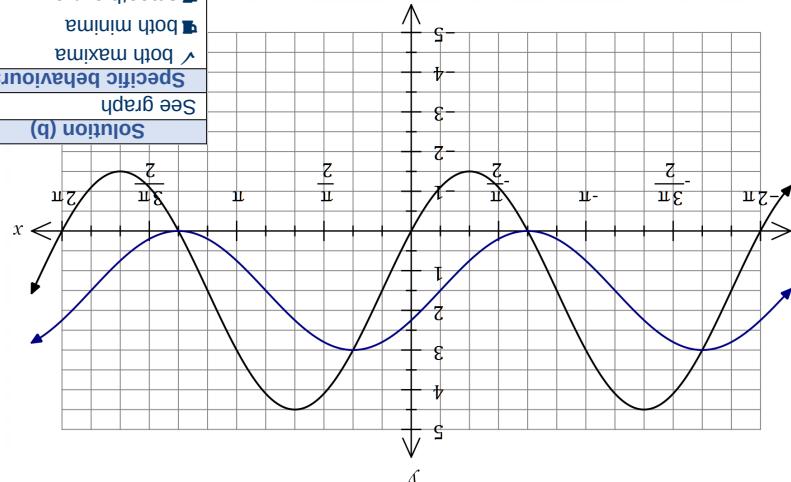
(3 marks)

<b>S</b> pecific behaviours	$OA=R$ so that	$0.32R=12R=37.5\text{ cm}$	$\checkmark$ correct use of arc length	$\checkmark$ correct length
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(b) Determine the area of the sl

, where  $c < \pi$ . (3 marks)

Solution (b)	See graph	Specific behaviours	both maxima	both minima	smooth curve
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The graph of  $y = a + b \sin(x - c)$  is drawn below, where  $a$ ,  $b$  and  $c$  are positive constants.

(8 marks)

paces

provided.

(3 marks)

**Solution**

$A_{BOA} = \frac{1}{2} \times 37.5^2 \times 0.32 = 225$

$A_{BOC} = \frac{1}{2} \times 27.5^2 [n - 0.32] = 1067$

$\text{Area of sector } DOA = 225 + 1067 = 1292 \text{ cm}^2$

**Specific behaviours**

area of sector  $DOA$

radius and angle of sector  $BOC$

area of shape

(3 marks)

<b>Solution</b>	<p>Using intersection of graphs:</p>
<b>Specific behaviors</b>	<p><math>x = -\frac{3}{2\pi}, x = \frac{3}{\pi}</math></p>

(b) On the s  
(c) Solve b

<p><b>Solution</b></p> <p>Using intersection of graphs:</p> $x = \frac{-3}{2\pi}, x = \frac{3}{\pi}$ <p><b>Specific behaviours</b></p> <ul style="list-style-type: none"> <li>✓ area of sector DOA</li> <li>✓ radius and angle of sector BOG</li> <li>✓ area of shape</li> </ul>
<p><b>Area =</b></p> $A_{BOG} = \frac{1}{2} \times 27.5^2 (\pi - 0.32) = 1067$ <p><b>Let <math>OB = r</math></b></p> $r = 65 - 37.5 = 27.5$ <p><b>Area =</b></p> $A_{DOA} = \frac{1}{2} \times 37.5^2 \times 0.32 = 225$ <p><b>Area =</b></p> $A_{BOG} + A_{DOA} = 1067 + 225 = 1292 \text{ cm}^2$ <p><b>Area =</b></p> $A = 225 + 1067 = 1292 \text{ cm}^2$ <p><b>Area =</b></p> $A = 225 + 1067 = 1292 \text{ cm}^2$

**Question 10**

The height  $h$  metres of a particle above level ground is defined as a function of time  $t$  seconds as follows:

$$h(t) = 68.75 + 15t - 5t^2, 0 \leq t \leq 5.5.$$

- (a) Determine the height of the particle when

(i)  $t=0$ .

Solution	
$h(0) = 68.75 \text{ m}$	(1 mark)
$h(4.5) = 35 \text{ m}$	
Specific behaviours	
✓ (i) correct ✗ (ii) correct	

(ii)  $t=4.5$ .

(1 mark)

- (b) Determine the maximum height reached by the particle and the time it reached this height.

Solution	
From graph of $h(t)$ :	(2 marks)
Specific behaviours	
✓ correct height ✗ correct time	

Maximum height:  $h=80 \text{ m}$  when  $t=1.5 \text{ s}$ .

- (c) Determine the time(s) that the particle was at a height of 75 m.

(2 marks)

Solution	
From graph of $h(t)$ :	
Specific behaviours	
✓ one time ✗ both times	

$h=75$  when  $t=0.5 \text{ s}, 2.5 \text{ s}$

- (d) State the range of the function  $h(t)$  for the given domain.

(2 marks)

Solution	
Range of $h$ :	
Specific behaviours	
✓ upper limit ✗ lower limit, correct inequality	

$0 \leq h \leq 80$

**CALCULATOR-ASSUMED**

- (c) In triangle  $ABC$ ,  $AC=65 \text{ cm}$ ,  $AB=44 \text{ cm}$  and  $\angle ACB=37^\circ$ . Determine the smallest possible area of the triangle. (3 marks)

Solution	
$\frac{65}{\sin B} = \frac{44}{\sin 37^\circ}$	
$B=62.75^\circ \text{ or } 117.25^\circ$	
For smallest area need $\angle A$ to be small as possible:	

$$A=180^\circ-37^\circ-117.25^\circ=25.75^\circ$$

$$\text{Area}=\frac{1}{2}(65)(44)\sin 25.75^\circ \approx 621 \text{ cm}^2$$

Specific behaviours	
✓ size of angle $B$	
✗ smallest size of angle $A$	
✗ correct area	

**Question 20** (6 marks)

(a) Two events are such that  $P(X)=0.2$ ,  $P(Y)=0.5$  and  $P(Y|X)=0.1$ . Determine the probability that both events occur. (2 marks)

**Solution**

$P(Y X)=\frac{P(X \cap Y)}{P(X \cup Y)}$	
$\Rightarrow P(X \cap Y)=P(X) \cdot P(Y X)=0.2 \times 0.1=0.02$	
Indicates use of conditional formula	
Correct probability	
<b>Specific behaviours</b>	
Indicates use of rule of product	

(b) at least one event occurs. (2 marks)

**Solution**

$P(X \cup Y)=P(X)+P(Y)-P(X \cap Y)$	
$?=0.2+0.5-0.02=0.68$	
Indicates use of rule of sum	
Correct probability	
<b>Specific behaviours</b>	
Indicates use of rule of sum	

(c) neither event occurs. (1 mark)

**Solution**

$P(X \cap Y)=1-P(X \cup Y)$	
$?=1-0.68=0.32$	
Indicates use of rule of complement	
Correct probability	
<b>Specific behaviours</b>	
Indicates use of rule of complement	

(d)  $X$  occurs given that  $Y$  has occurred. (1 mark)

**Solution**

$P(X Y)=\frac{P(X \cap Y)}{P(X \cup Y)}$	
$=\frac{0.02}{0.5}=0.04$	
Indicates use of rule of conditional probability	
Correct probability	
<b>Specific behaviours</b>	
Indicates use of rule of conditional probability	

**Question 11** (11 marks)

(a) Determine the area of triangle  $PQR$  when  $\angle PQR=26^\circ$ ,  $\angle PRQ=122^\circ$  and  $PQ=57$  cm. (4 marks)

**Solution**

$\frac{q}{\sin 26^\circ}=\frac{57}{\sin 122^\circ} \Rightarrow q=29.46$	
$\text{Area}=\frac{1}{2}(57)(29.46)\sin(32^\circ)=445 \text{ cm}^2$	
Sketch of triangle	
Length of second side	
Correct area	
<b>Specific behaviours</b>	
Sketch of triangle	

**Diagram**

(b) The area of triangle  $ABC$  is  $96 \text{ cm}^2$ ,  $\angle ACB=30^\circ$  and  $2BC=3AC$  as shown in the diagram. Determine the length of  $AB$ . (4 marks)

**Solution**

$AB^2=16^2+24^2-2(16)(24)\cos(30^\circ)$	
$?=12.92 \text{ cm}$	
Length of $AB$	
Cosine rule	
Value of $x$	
Area equation	
Length of $AB$	
<b>Specific behaviours</b>	
Length of $AB$	

**Diagram**

**Question 12**

(8 marks)

The height above ground level,  $h$  m, of a seat on a steadily rotating Ferris wheel  $t$  minutes after the wheel begins to move is given by  $h=21.5-18.5\cos\left(\frac{\pi t}{6}+\frac{\pi}{3}\right)$ .

- (a) Determine the initial height of the seat.

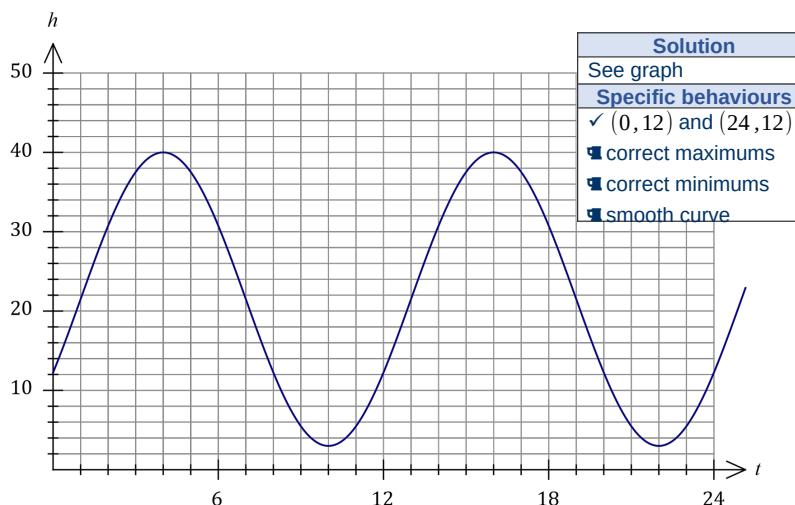
(1 mark)

<b>Solution</b>
$h(0)=12.25$ m
<b>Specific behaviours</b>

✓ correct height

- (b) Graph the height of the seat against time on the axes below.

(4 marks)



- (c) Determine

- (i) the maximum height above ground reached by the seat.

(1 mark)

<b>Solution</b>
$h_{MAX}=40$ m
<b>Specific behaviours</b>

✓ correct height

- (ii) the time taken, to the nearest second, for the seat to first reach a height of 4 m above ground level.

(2 marks)

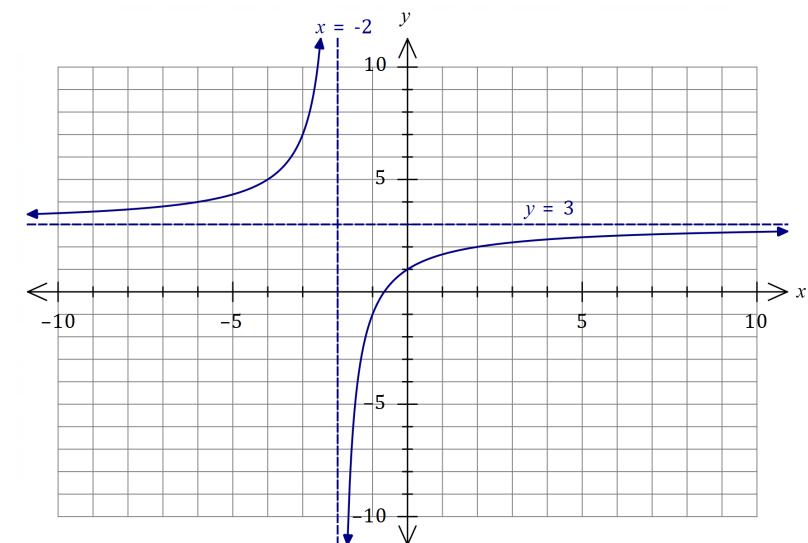
<b>Solution</b>
$h=4 \Rightarrow t=9.37$
$0.37 \times 60=22$
$t=9$ m 22 s (562 s)
<b>Specific behaviours</b>

✓ time as decimal

■ time to nearest second

- (c) On the axes below, draw the graph of  $y=2-f(x)$  and label all asymptotes with their equations.

(4 marks)



<b>Solution</b>
See graph
<b>Specific behaviours</b>
✓ vertical asymptote with equation
✓ horizontal asymptote with equation
■ y-intercept and graph for $x>-2$
■ remainder of graph

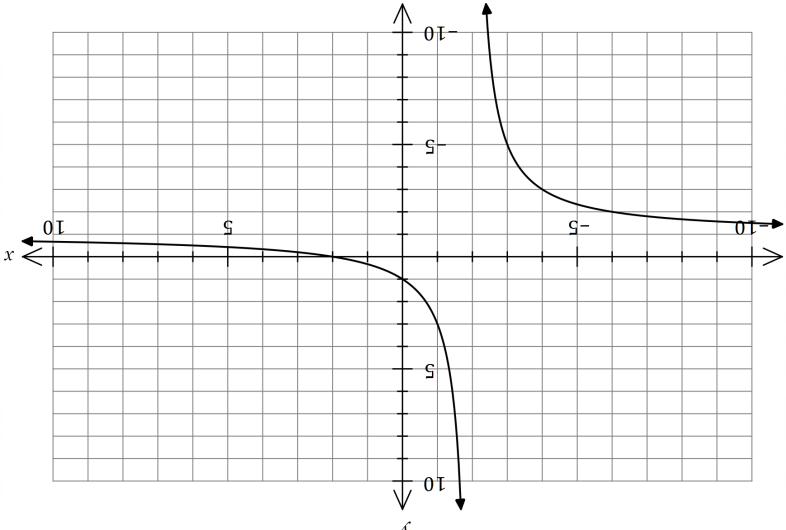
<input checked="" type="checkbox"/> correct range
<input checked="" type="checkbox"/> correct domain
<b>Specific behaviours</b>
Range: $y \neq -1$

(2 marks)

(b) State the domain and range of  $f(x)$ .

<input checked="" type="checkbox"/> value of $c$
<input checked="" type="checkbox"/> value of $b$
<input checked="" type="checkbox"/> value of $a$
<b>Specific behaviours</b>
$a = 4$
$(0, 1) \Leftrightarrow \frac{a}{2} = 1$
$b = 2, c = -1$

(3 marks)

(a) Determine the value of  $a$ , the value of  $b$  and the value of  $c$ .

The graph of  $y = f(x)$  is shown, where  $f(x) = \frac{x+b}{a} + c$  and  $a, b$  and  $c$  are constants.

Question 19  
(9 marks)**CALCULATOR-ASSUMED**

14

(1 mark)

(a) State the equation of the line of symmetry for the graph of  $y = f(x)$ .The graph  $y = f(x)$ , where  $f(x) = x^2 + bx + c$  has a turning point at  $(2, -7)$ .

(6 marks)

<input checked="" type="checkbox"/> correct equation
<b>Specific behaviours</b>
$x = 2$
<b>Solution</b>

**METHODS UNIT 1**

7

**CALCULATOR-ASSUMED**

<input checked="" type="checkbox"/> correct equation (either form)
<input checked="" type="checkbox"/> identifies new turning point
<b>Specific behaviours</b>
Equation is $y = (x+1)^2 - 5 = x^2 + 2x - 4$
New turning point at $(-3, -7+2) = (-1, -5)$ .
<b>Solution</b>

(c) The graph of  $y = f(x)$  is translated 3 units to the left and 2 units upwards. Determine the equation of the resulting curve.  
(2 marks)

<input checked="" type="checkbox"/> value of $c$
<input checked="" type="checkbox"/> value of $b$
<input checked="" type="checkbox"/> writes $f(x)$ in squared form
<b>Specific behaviours</b>
$c = -3$
$b = -4$
$f(x) = (x-2)^2 - 7$
<b>Solution</b>

(b) Determine the value of the constant  $b$  and the value of the constant  $c$ .  
(3 marks)

<input checked="" type="checkbox"/> correct range
<input checked="" type="checkbox"/> correct domain
<b>Specific behaviours</b>
Range: $y \neq -1$
Domain: $x \neq -2$
<b>Solution</b>

(2 marks)

(b) State the domain and range of  $f(x)$ .**METHODS UNIT 1**

7

**CALCULATOR-ASSUMED****CALCULATOR-ASSUMED**

14

Question 13  
(9 marks)

(6 marks)

**Question 14**

When a random sample of 173 people from a university were classified according to whether they had a driver's licence (event  $D$ ) and whether they wore spectacles (event  $S$ ), it was observed that  $n(D)=140$ ,  $n(S)=53$  and  $n(S \cap D)=10$ .

(a) Determine

(i)  $n(\bar{S})$ .

(1 mark)

Solution
$n(\bar{S})=173-53=120$
Specific behaviours
✓ correct number

(ii)  $n(D \cap S)$ .

(1 mark)

Solution
$n(D \cap S)=53-10=43$
Specific behaviours
✓ correct number

(b) Determine the probability that a randomly chosen person from the sample

(i) does not have a driver's licence.

(2 marks)

Solution
$n(\bar{D})=173-140=33$
$P(\bar{D})=\frac{33}{173} \approx 0.191$
Specific behaviours
✓ numerator ☒ denominator

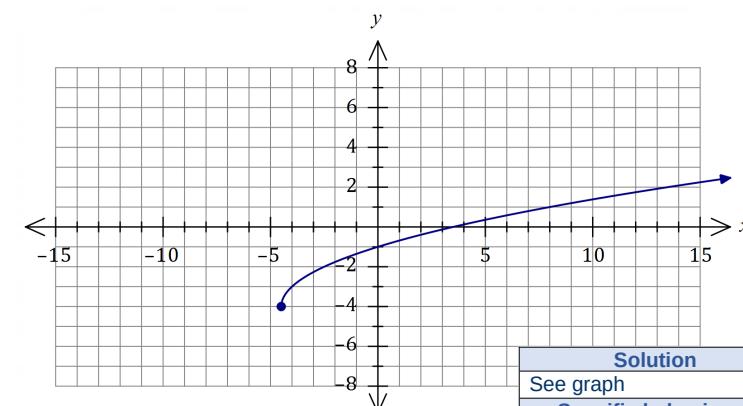
(ii) wears spectacles given that they have a driver's licence.

(2 marks)

Solution
$P(S D)=\frac{n(D \cap S)}{n(D)}=\frac{43}{140} \approx 0.307$
Specific behaviours
✓ numerator ☒ denominator

(c) Draw the graph of  $y=f(2x)$  on the axes below.

(3 marks)



Solution
See graph
Specific behaviours
✓ endpoint at $(-4.5, -4)$ ☒ thru' $(0, -1)$ and $(3.5, 0)$ ☒ smooth curve

Question 18 (8 marks)

Consider part of the unit circle shown below, where  $\theta = 51^\circ$ .

Determine, in terms of  $p$  and/or  $q$ , an expression for each of the following:

(a)  $\sin 51^\circ$ . (1 mark)

**Solution**

$\sin 51^\circ = q$	✓ correct expression
<b>Specific behaviours</b>	
$\cos 129^\circ = -\cos 51^\circ = -p$	✓ uses reflection
<b>Specific behaviours</b>	
$\cos 129^\circ = -\cos 51^\circ = -p$	✓ correct expression

(b)  $\cos 129^\circ$ . (2 marks)

**Solution**

$\cos 129^\circ = -\cos 51^\circ = -p$	✓ correct expression
<b>Specific behaviours</b>	
$\sin 141^\circ = \sin (51^\circ + 90^\circ) = \cos 51^\circ$	✓ uses fundamental relationship
<b>Specific behaviours</b>	
$\sin 141^\circ = \cos 51^\circ$	✓ correct expression

(c)  $\sin 141^\circ$ . (2 marks)

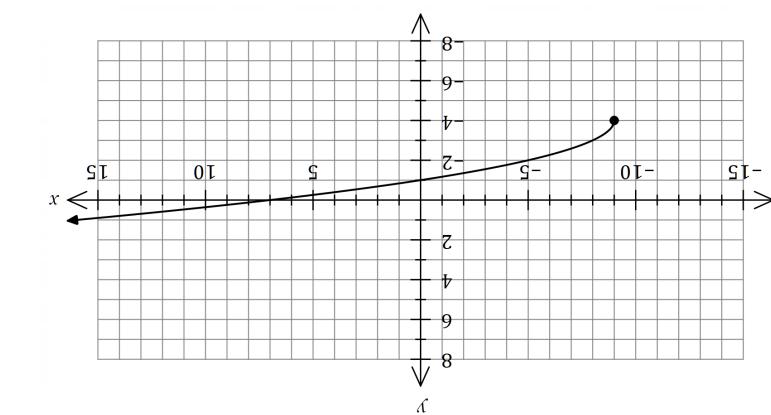
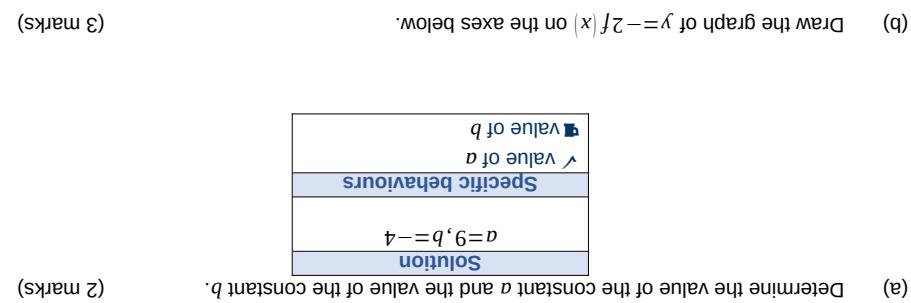
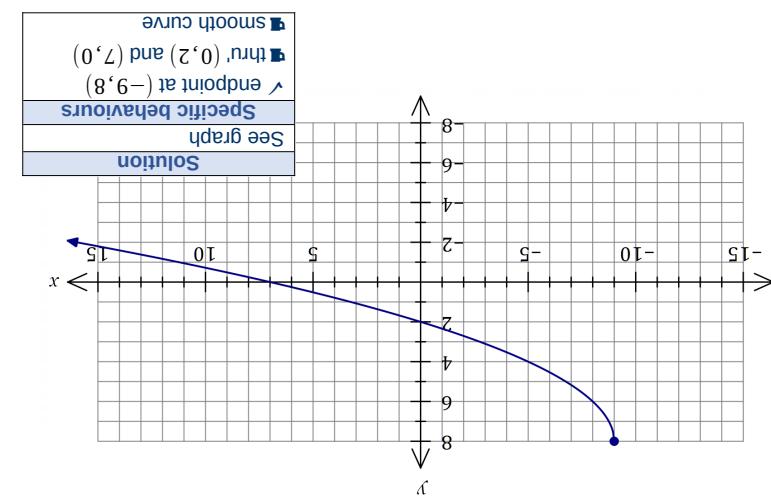
**Solution**

$\sin 141^\circ = \cos 51^\circ$	✓ correct expression
<b>Specific behaviours</b>	
$\cos 258^\circ = \cos (129^\circ + 129^\circ) = \cos 258^\circ$	✓ uses sum identity
<b>Specific behaviours</b>	
$\cos 258^\circ = \cos (129^\circ + 129^\circ) = \cos 258^\circ$	✓ correct expression

(d)  $\cos 258^\circ$ . (3 marks)

**Solution**

$\cos 258^\circ = \cos (129^\circ + 129^\circ) = \cos 258^\circ$	✓ correct expression
<b>Specific behaviours</b>	
$\frac{1}{2}(\cos 129^\circ + \cos 129^\circ) = \cos 129^\circ$	✓ halves angle
<b>Specific behaviours</b>	
$\cos 129^\circ = -\cos 51^\circ = -p$	✓ correct expression



**Question 16**

(8 marks)

A polynomial of degree 3 passes through the points with coordinates  $(0, 4)$ ,  $(-2, 0)$ ,  $(2, 0)$  and  $(0.5, 0)$ .

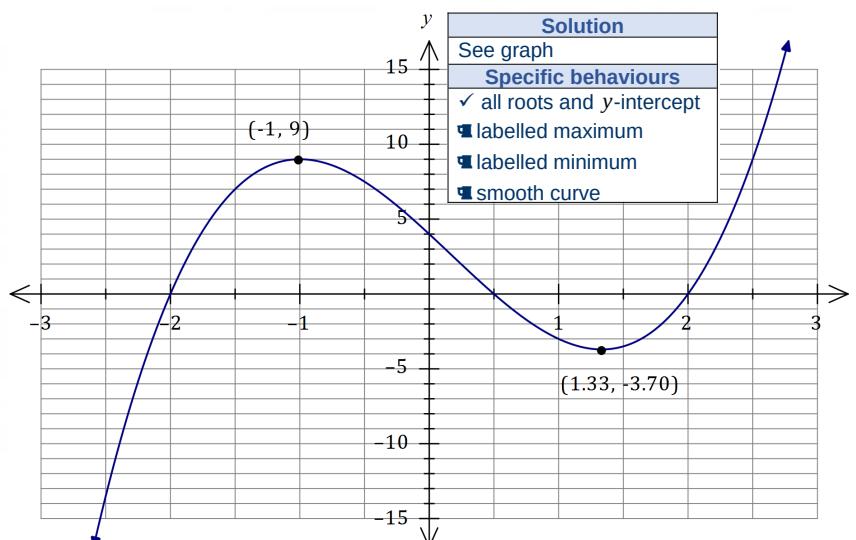
- (a) Determine the equation of the polynomial in expanded form.

(4 marks)

Solution	
Using roots:	
$y = a(x+2)(x-2)(x-0.5)$	
Use 4th point:	
$x=0 \Rightarrow 4=a(2)(-2)(-0.5) \therefore a=2$	
Expand:	
$y=2(x+2)(x-2)(x-0.5) \therefore 2x^3 - x^2 - 8x + 4$	
Specific behaviours	
✓ factored form using roots	
✗ substitutes fourth point	
✗ correct value of $a$	
✗ correct expanded form	

- (b) Draw the graph of the polynomial on the axes below, indicating the coordinates of all turning points.

(4 marks)

**Question 17**

(7 marks)

Let  $f(x)=1+\sqrt{6-2x}$  and  $g(x)=x+2$ .

- (a) Evaluate  $f(-5)-g(-5)$ .

(2 marks)

Solution	
$f(-5)-g(-5)=1+\sqrt{16}-(-5+2)\therefore 5-(-3)$	
8	

Specific behaviours	
✓ evaluates $f$ correctly	
✗ correct value	

- (b) State the domain of  $f(x)$ .

(2 marks)

Solution	
$6-2x \geq 0 \therefore 2x \geq -6 \therefore x \leq 3$	
Specific behaviours	
✓ correct inequality	
✗ correct bound	

- (c) State the range of  $g(x)$ .

(1 mark)

Solution	
$y \in R$	
Specific behaviours	
✓ correct range (symbols or words)	

- (d) Determine the coordinates of the point(s) of intersection of  $y=f(x)$  and  $y=g(x)$ .

(2 marks)

Solution	
Using graph/CAS: $(1, 3)$	
Specific behaviours	
✓ x-coordinate	
✗ y-coordinate	