Mathematics Methods 3 and 4

Test 2 Calculator Free

Name: ....

SHENTON COLLEGE Teacher: Mrs Martin Dr Moore Mr Smith

Time Allowed : 30 minutes

Marks	/30

Materials allowed: Formulae Sheet provided.
Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

Question 1 **[4, 2, 2 = 8 marks]**

Determine the following:

(a) $\int\left(e^{2x}+\sqrt{x}+\pi\right)dx$

$= \frac{e^{2x}}{2} + \frac{2x^{\frac{3}{2}}}{3} + \pi x + c$ ✓ ✓ ✓ ✓

(b) $\frac{d}{dx}\int_x^3 \frac{t^2}{4}\,dt$

$-\frac{x^2}{4}$ ✓ ✓

(c) $\frac{d}{dx}\int_0^{x^2}\sqrt{1+t^2}\,dt$

$= \sqrt{1+x^4}\,(2x)$ ✓ ✓

Question 2 **[3, 4 = 7 marks]**

Evaluate

(a) $\int_0^2 \frac{3}{(2x+1)^4}dx = \int_0^2 3(2x+1)^{-4}dx$

$= \left[\frac{3(2x+1)^{-3}}{-6}\right]_0^2$ ✓ Antideriv Correct

$= \left[\frac{-1}{2(2x+1)^3}\right]_0^2$

$= \frac{-1}{250} - \frac{-1}{2}$ ✓ Sub boundaries

$= -\frac{1}{250} + \frac{125}{250}$

$= \frac{124}{250}$ ✓ $= \frac{62}{125}$

(b) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\sin 2x\,dx$

✓ $= \left[-\frac{\cancel{2}\cos 2x}{\cancel{2}}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

✓ $= -\cos 3\pi - (-\cos\pi)$

$= -(-1) + (-1)$ ✓ Correct exact Values

$= 0$ ✓

-1 -5

$\overset{-3}{\vdash}$ ——— $\overset{0}{\vdash}$ ——— $\overset{2}{\vdash}$

Question 3 **[1, 3 = 4 marks]**

Given $\int_0^{-3} f(x)dx = 1$ and $\int_0^{2} f(x)dx = -5$, find

(a) $\int_{-3}^{2} f(x)dx$

$= -6$ ✓

(b) $\int_0^{2} [3f(x) - 4]dx$

$= 3\int_0^2 f(x)dx - \int_0^2 4\,dx$ ✓

$= -15 - 8$ ✓

$= -23$ ✓

Question 4 **[5 marks]**

Given $\frac{dy}{dx} = ae^x + 1$ and when $x = 1$, $\frac{dy}{dx} = 3$ and $y = 2$

Find the value of y when $x = 0$.

$\frac{dy}{dx} = ae^x + 1$

$3 = ae + 1$

✓ $\frac{2}{e} = a$

$\frac{dy}{dx} = \frac{2}{e}e^x + 1$

$= 2e^{x-1} + 1$

$y = 2e^{x-1} + x + c$ ✓

$2 = 2e^0 + 1 + c$ ✓

$-1 = c$

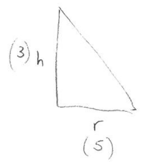
$y = 2e^{x-1} + x - 1$ ✓

at $x = 0$ $\quad y = \frac{2}{e} - 1$ ✓

Question 3 **[2, 3 = 5 marks]**

The ratio of the radius (r) to the height (h) is 5:3 for a specific cone.

(a) Show that the volume of the cone is given by $V = \dfrac{25\pi h^3}{27}$



$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{5h}{3}\right)^2 h$ ✓

$= \dfrac{25\pi h^3}{27}$ ✓

$r = \dfrac{5h}{3}$

(b) Use the method of small change to find the approximate increase in the volume of the cone if the height changes from 5 cm to 5.02 cm.

$\dfrac{dV}{dh} = \dfrac{25\pi h^2}{9}$ ✓ $\delta h = 0.02$

$\delta V \approx \dfrac{dV}{dh} \times \delta h \approx \dfrac{25\pi (5)^2}{9} \times 0.02$ ✓

$\delta V \approx 4.4 \text{ cm}^3$ ✓

Question 4 **[2, 3 = 5 marks]**

The cost, $C(x)$ (\$1000s) of manufacturing a product is given by $C(x) = 45 + 65x$. The revenue, $R(x)$, is given by the function $R(x) = 100x - 2.5x^2$. The manufacturer can only make between 2 and 10 products per week.

Find

(a) a simplified expression for the Profit if x units are made and sold.

$P(x) = 100x - 2.5x^2 - (45 + 65x)$ ✓

$= 35x - 2.5x^2 - 45$ ✓

(b) the minimum and maximum profit possible each week.

Min Profit \$15 000 ✓

Max Profit \$ 77 500 ✓

✓ (in 1000s)

Question 5 **[1, 1, 2, 2 = 6 marks]**

Given $h(x) = \int_0^x \cos(2t)\,dt$, determine

a) $h'(x)$

$h'(x) = \cos 2x$ ✓

b) $h'\left(\frac{\pi}{2}\right)$

$h'\left(\frac{\pi}{2}\right) = \cos\left(\frac{2\pi}{2}\right) = -1$ ✓

c) $h\left(\frac{\pi}{2}\right) = \int_0^{\frac{\pi}{2}} \cos(2t)\,dt$

$= \left[\frac{\sin 2t}{2}\right]_0^{\frac{\pi}{2}}$ ✓

$= \frac{\sin\pi}{2} - \frac{\sin 0}{2}$

$= 0 - 0 = 0$ ✓

$\left(\frac{\pi}{2}, 0\right)$

d) the equation of the tangent to the curve $h(x)$ at $x = \dfrac{\pi}{2}$

$m = -1$ $\left(\frac{\pi}{2}, 0\right)$

$y = -x + c$ ✓

$0 = -\frac{\pi}{2} + c$

$\frac{\pi}{2} = c$

$y = -x + \frac{\pi}{2}$ ✓

Mathematics Methods 3 and 4

Test 2 Calculator Assumed

SHENTON COLLEGE

Name: ....

Teacher: Mrs Martin Dr Moore Mr Smith

Time Allowed: 20 minutes

Marks	/27

Materials allowed: Formulae Sheet provided. Classpad, calculators, 1 A4 page of notes, one side.
Attempt all questions.
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Question 1 [2, 2, 2, 2 = 8 marks]

The acceleration (m/s^2) of a particle moving in a straight line is given by $a = 2t - 4$. The particle's initial velocity is 3 m/s. Its initial displacement from the origin is -15 m.

(a) Find the expression for the particle's velocity at any time.

$v(t) = t^2 - 4t + 3$ ✓ ✓

(b) Find the time(s), if any, when the particle comes to rest.

$t = 1$ ✓ and $t = 3$ s ✓

(c) Find its displacement when $t = 3$

$x(t) = \frac{t^3}{3} - 2t^2 + 3t - 15$ ✓

$x(3) = -15$ m ✓

(d) Find the distance travelled in the first 3 seconds.

$$\text{Dist} = \int_0^3 \left| t^2 - 4t + 3 \right| dt$$

$$= \frac{8}{3} \text{ m}$$ ✓✓

Question 2 [3, 6 = 9 marks]

Consider the functions: $f(x) = x(5-x)$ and $g(x) = x(x-3)$

(a) Write down an integral which when evaluated will determine the area trapped between the two functions and calculate the area.

$$A = \int_0^4 x(5-x) - x(x-3)\,dx = 21\tfrac{1}{3} \text{ square units}$$ ✓ ✓ ✓

Limits ✓

(b) Within the area trapped between the two functions a vertical line is drawn, intersecting $f(x)$ at Point P and intersecting $g(x)$ at Point Q.

(i) Show use of calculus to find the value of x for which the length of line segment PQ is a maximum.

Let L be the length $L = x(5-x) - x(x-3)$ ✓

$\frac{dL}{dx} = -4x + 8 = 0$ ✓

$x = 2$ ✓

(ii) Use the second derivative test to show that this value of x does indeed produce a maximum value.

$\frac{d^2L}{dx^2} = -4$ ✓

For all values of x $\frac{d^2L}{dx^2} < 0$ ∴ Concave down

∴ Maximum ✓

(iii) State the maximum length possible.

$L = 8$ units ✓