

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

### **Important note to candidates**

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, Council for this course.  
and up to three calculators satisfying the conditions set by the Curriculum

Standard items: pens, pencils, pencil sharpener, eraser, correction/tape fluid, ruler,  
highlighters

Formula sheet (extracted from Section One)

This Question/Answer booklet  
**To be provided by the supervisor**  
**To be provided by the candidate**  
Material required/recommended for this section

Working time for paper: one hundred minutes  
Reading time before commencing work: ten minutes

Your teacher: S Edber T Hosking S Rowden

Your name: **SOLUTIONS**

Calculator-assumed  
Section Two:

**MATHEMATICS**  
**3C/3D (Year 12)**

Question/Answer Booklet

Semester 2 Examination, 2012



**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	7	7	50	50
Section Two: Calculator-assumed	13	13	100	100
				150

**Instructions to candidates**

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.

**Additional working space**

Question number(s): \_\_\_\_\_

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(5 marks)

Question 8

Suggested working time for this section is 100 minutes.

(100 Marks)

#### **Section Two: Calculator-assumed**

MATHEMATICS 3C/3D CALCULATOR-ASSUMED

### Additional working space

MATHEMATICS 3C/3D CALCULATOR-ASSUMED  
SEMESTER TWO 2012 SECTION TWO  
SEMESTER TWO 2012 SECTION TWO  
MATHEMATICS 3C/3D CALCULATOR-ASSUMED

Find the net change in the battery voltage after the toy has been used for 40 min.

increases instantaneously, at a rate modelled by  $\frac{dy}{dt} = -0.2e^{-0.02t}$  where  $t$  is time in minutes.

In an electronic toy, the voltage  $V$  volts, remains at 9 volts for 30 minutes and then the voltage rechargesable battery has a voltage of 9 volts when fully charged. When the battery is used to

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[2] Find how long the battery can be used to run this toy if a minimum voltage of 8 volts is required.

<p><b>Specified behaviours</b></p> <ul style="list-style-type: none"> <li>✓ identified = 10</li> <li>✓ recognised it was a total change question</li> <li>✓ correct answer with negative sign</li> </ul>
--

$$\int_{10}^0 -0.2e^{-0.02t} = -1.81$$

correct calculation either as above or from determining  $V(i)$  and using this to solve the answer correctly and add on 30 minutes.

$$t = 5.27 \text{ minutes}$$

See next page  
1234567-8

1234567-8

See next page

1234567-8

**Question 9**

(6 marks)

A body is moving in a straight line with velocity,  $v$  m/s, given by  $v = 2t^2 - 19t + 30$ , where  $t$  is the time, in seconds, since the body first passed through a fixed point P.

- (a) At what other time(s), if any, does the body again pass through the fixed point P?

[3]

$$\begin{aligned}x(t) &= \int 2t^2 - 19t + 30 dt \\&= \frac{2t^3}{3} - \frac{19t^2}{2} + 30t \quad (\text{NB } x(0)=0) \\&\frac{2t^3}{3} - \frac{19t^2}{2} + 30t = 0 \\&\text{when} \\&t=0, t=4.724, t=9.526 \\&\text{Hence after 4.724 and 9.526 seconds.}\end{aligned}$$

**Specific behaviours**

- ✓ determines  $x(t)$  including acknowledging that  $c = 0$
- ✓ sets  $x(t) = 0$
- ✓ solves  $x(t) = 0$ , including acknowledging  $t = 0$  is an answer to the equation, and states 2 other times

- (b) Show that the body is stationary twice and find the distance travelled by the body between these two instants.

[3]

$$\begin{aligned}2t^2 - 19t + 30 &= 0 \text{ when } t = 2, t = 7.5 \text{ seconds.} \\D &= \left| \int_2^{7.5} 2t^2 - 19t + 30 dt \right| \\&= \left| -\frac{1331}{24} \right| \approx 55.46 \text{ metres.}\end{aligned}$$

**Specific behaviours**

- ✓ determines when body is stationary
- ✓ Appropriate formula/calculation to determine distance
- ✓ determines distance travelled between these two instances correctly

See next page

**Additional working space**

Question number(s): \_\_\_\_\_

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1234567-8

[2]

- (d) Using the sample mean from (a) as the best estimate for the population mean, what is the probability that the sample mean of a larger sample of 225 bags is less than 516 g?

✓ Uses correct parameters  
✓ Specific behaviours

$$X \sim N(517.5, 13.5^2)$$

$$P(X < 516) = 0.0478$$

[2]

- (c) Calculate the 99% confidence interval for the mean weight of flour in a bag.

✓ Appropriate formula/calculation to determine the standard deviation  
✓ Correct interval  
✓ Specific behaviours

$$517.5 \pm 2.576 \frac{13.5}{\sqrt{81}}$$

$$= (513.636 \text{ g}, 521.364 \text{ g})$$

[2]

- (b) Find the value of  $\sigma$ , the standard deviation of the normal population from which the sample is drawn.

✓ Correct answer  
✓ Specific behaviours

$$\bar{x} = \frac{514.56 + 520.44}{2} = 517.5 \text{ g}$$

[1]

- (a) Show the value of  $\bar{x}$ , the mean weight of the sample is 517.5 g.

On the basis of the results obtained from a random sample of 81 bags produced by a mill, the 95% confidence interval for the mean weight of flour in a bag is found to be (514.56g, 520.44g).

Additional working space  
Question 10  
(7 marks)

SECTION TWO  
SEMESTER TWO 2012  
MATHEMATICS 3C/3D  
CALCULATOR-ASSUMED

SECTION TWO  
SEMESTER TWO 2012  
MATHEMATICS 3C/3D  
CALCULATOR-ASSUMED

See next page

- [2]
- (d) Using the sample mean from (a) as the best estimate for the population mean, what is the probability that the sample mean of a larger sample of 225 bags is less than 516 g?

✓ Uses correct parameters  
✓ Specific behaviours

$$X \sim N(517.5, 13.5^2)$$

$$P(X < 516) = 0.0478$$

✓ Appropriate formula/calculation to determine the interval  
✓ Correct interval  
✓ Specific behaviours

$$517.5 \pm 2.576 \frac{13.5}{\sqrt{81}}$$

$$= (513.636 \text{ g}, 521.364 \text{ g})$$

- (c) Calculate the 99% confidence interval for the mean weight of flour in a bag.

✓ Appropriate formula/calculation to determine the standard deviation  
✓ Correct interval  
✓ Specific behaviours

$$520.44 - 517.5 = 1.96 \frac{\sigma}{\sqrt{81}}$$

$$\sigma = 13.5 \text{ g}$$

- (b) Find the value of  $\sigma$ , the standard deviation of the normal population from which the sample is drawn.

✓ Correct answer  
✓ Specific behaviours

$$\bar{x} = \frac{514.56 + 520.44}{2} = 517.5 \text{ g}$$

- (a) Show the value of  $\bar{x}$ , the mean weight of the sample is 517.5 g.

On the basis of the results obtained from a random sample of 81 bags produced by a mill, the 95% confidence interval for the mean weight of flour in a bag is found to be (514.56g, 520.44g).

Additional working space  
Question 10  
(7 marks)

SECTION TWO  
SEMESTER TWO 2012  
MATHEMATICS 3C/3D  
CALCULATOR-ASSUMED

20

Question number(s): \_\_\_\_\_

SECTION TWO  
SEMESTER TWO 2012  
MATHEMATICS 3C/3D  
CALCULATOR-ASSUMED

Additional working space

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Atmospheric pressure,  $P$  (kPa), decreases approximately exponentially with increasing height  $h$  (m), above sea level according to the relationship  $\frac{dP}{dh} = kP$ , where  $k$  is a constant. Atmospheric pressure at sea level is 101.3 kPa, and halves with every 5 800 m increase in height.

- (a) Find the value of  $k$ , rounded to four significant figures.

$$0.5 = e^{5800k}$$

$$k = -0.0001195$$

[2]

**Specific behaviours**

- ✓ determines appropriate equation to solve
- ✓ correct value of  $k$

- (b) Calculate the atmospheric pressure at the top of a mountain of height 3 785 m.

$$P = 101.3e^{-0.0001195(3785)}$$

$$= 64.44 \text{ kPa}$$

[2]

**Specific behaviours**

- ✓ Substitutes all information into equation
- ✓ correct answer

- (c) Use the increments formula to find the approximate change in pressure as a climber descends 250 m from the top of a mountain of height 3 785 m.

[3]

$$\frac{\partial P}{\partial h} \approx \frac{dP}{dh}$$

$$\approx kP\partial h$$

$$\approx -0.0001195 \times 64.44 \times -250$$

$$\approx 1.93 \text{ kPa}$$

(An increase in pressure)

**Specific behaviours**

- ✓ identifies negative change of -250
- ✓ recognises  $\frac{dP}{dh} = -0.0001195 \times 64.44$
- ✓ correct answer

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A function is such that  $f'(x) = x^2 - 2x - 3$ .

- (a) State the  $x$ -coordinate of the minimum of  $f(x)$ .

[2]

$$f'(x) = 0 \text{ when } x = -1, x = 3$$

$$f''(x) = 2x - 2$$

$$f''(3) > 0 \Rightarrow \text{minimum when } x = 3$$

**Specific behaviours**

- ✓ determines values of  $x$  for  $f'(x) = 0$
- ✓ determines value of  $x$  for which  $f(x)$  is a minimum

- (b) Justify that  $f(x)$  has a point of inflection when  $x = 1$ .

$$f''(x) = 0 \text{ when } x = 1$$

$$f'(1) = -4 \Rightarrow f(x) \text{ has a PI when } x = 1, \text{ as } f''(1) = 2$$

**Specific behaviours**

- ✓ shows  $f''(1) = 0$  and  $f'(1) = 4$
- ✓ concluding statement

- (c) Find  $f(-1) - f(2)$ .

$$f(-1) - f(2) = \int_{-1}^2 f'(x) dx$$

$$= 9$$

[2]

**Specific behaviours**

- ✓ identifies as the definite integral from 2 to -1
- ✓ correct answer





**Question 13** (12 marks)

- (a) A pottery produces souvenir coffee mugs, of which it is known that 5% are defective.

- (i) In a box of 24 mugs, what is the probability that there are at least 4 defectives? [2]

$$\begin{aligned} X &\sim B(24, 0.05) \\ P(X \geq 4) &= 0.0298 \end{aligned}$$

**Specific behaviours**

- ✓ correctly identifies the distribution and parameters
- ✓ correct probability

- (ii) In a box of 12 mugs, what is the probability that there are no defectives? [1]

$$\begin{aligned} Y &\sim B(12, 0.05) \\ P(Y = 0) &= 0.5404 \end{aligned}$$

**Specific behaviours**

- ✓ correct probability

- (iii) What is the probability that in 10 boxes, each containing 12 mugs, that either two or three of the boxes contain no defectives? [2]

$$\begin{aligned} W &\sim B(10, 0.54036) \\ P(2 \leq W \leq 3) &= 0.1082 \end{aligned}$$

**Specific behaviours**

- ✓ correctly identifies the parameters
- ✓ correct probability

- (iv) The pottery decides to pack  $n$  mugs per box for wholesale clients, so that the chance of there being at least one defective mug in a box is no more than 50%. Find the largest value of  $n$ . [2]

$$\begin{aligned} 0.95^n &\leq 0.5 \\ n &\leq 13.51 \\ \text{Hence, } 13 \text{ mugs per box.} \end{aligned}$$

**Specific behaviours**

- ✓ recognises  $0.95^n \leq 0.5$
- ✓ solves for  $n$  and states correct largest value

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**Question 18 (continued)**

- (d) Prove the conjecture in part (c). [4]

If  $5xy - x^2 - 3$  is even, then since 3 is odd,  $5xy - x^2$  must also be odd.

If  $x(5y - x)$  is odd, then both  $x$  and  $5y - x$  must be odd.

Hence  $x$  is odd.

If  $5y - x$  is odd, but  $x$  is odd, then  $5y$  must be even.

Since 5 is odd, then  $y$  must be even.

Since  $x$  is odd and  $y$  is even then  $B = x + y$ , will always be odd.

**Specific behaviours**

- ✓ identifies  $5xy - x^2$  must be odd
- ✓ explains why  $x$  must be odd
- ✓ explains why  $y$  must be even
- ✓ concluding statement

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- (a) Evaluate  $A$  and  $B$  when  $x = 3$  and  $y = 2$ .  
 Let  $A = 5xy - x^2 - 3$  and  $B = x + y$ , where  $x$  and  $y$  are integers.  

$$A = 5 \times 3 \times 2 - 3^2 - 3 = 18$$
  

$$B = 3 + 2 = 5$$
- (b) A worker at the pottery took 150 of the defective mugs, filled them with soil and then planted four seeds in each. After 14 days, the number of seeds which germinated in each of the mugs was noted, with these results:  
 Number of germinating seeds      0      1      2      3      4      5      16      19      23  
 Number of mugs      47      67      57      16      19      1      1      0
- (i) What is the mean number of seeds germinating per mug?  

$$\bar{x} = 3.2$$

- (ii) Show the probability of one seed germinating is 0.8.  
 If  $X$  is the random variable, number of seeds germinating out of four, then assume that  $X \sim \text{Bin}(4, p)$ .  $\bar{x} = np$  and so  $p = \frac{\bar{x}}{n} = \frac{3.2}{4} = 0.8$
- (iii) Use an associated binomial distribution to calculate the theoretical frequency distribution of seeds germinating in the 150 mugs and comment on how well your distribution models the observed results above.
- (iv) Use a normal distribution to calculate the theoretical frequency distribution for the number of seeds germinating in the 150 mugs and hence state a conjecture about the parity of  $B$  when  $A$  is even.

- [3]
- Specific behaviours**
- ✓ correct justification
- Specific behaviours**
- ✓ correct party for subtraction
- Specific behaviours**
- ✓ correct party for multiplication
- Specific behaviours**
- ✓ conjecture  
✓ various values of  $x$  and  $y$  (minimum 2)
- When  $A$  is even,  $B$  is always odd.
- |   |    |    |   |
|---|----|----|---|
| 3 | 4  | 50 | 7 |
| 3 | 3  | 35 | 6 |
| 3 | 2  | 20 | 5 |
| 4 | 35 | 6  |   |
| 2 | 3  | 25 | 4 |
| 2 | 2  | 15 | 4 |
| 1 | 4  | 18 | 5 |
| 1 | 3  | 13 | 4 |
| 1 | 2  | 8  | 3 |
| x | y  | A  | B |
- Specific behaviours**
- ✓ uses binomial distribution  
✓ gives expect values to whole numbers and ensure  $+ 150$   
✓ comment on distribution
- The theoretical results are a reasonably close match to the observed results, suggesting that the binomial model is appropriate.
- |          |   |   |   |   |    |    |    |
|----------|---|---|---|---|----|----|----|
| Expected | 0 | 1 | 2 | 3 | 23 | 62 | 61 |
| Seeds    |   |   |   |   |    |    |    |
- $P(X=0) = 0.0016 \times 150 = 0.24$   
 $P(X=1) = 0.0256 \times 150 = 3.84$   
 $P(X=2) = 0.1536 \times 150 = 23.04$   
 $P(X=3) = P(X=4) = 0.4096 \times 150 = 61.44$

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- Question 13 (continued)**
- (a) Evaluate  $A$  and  $B$  when  $x = 3$  and  $y = 2$ .  
 $A = 5xy - x^2 - 3$  and  $B = x + y$ , where  $x$  and  $y$  are integers.

- (b) A worker at the pottery took 150 of the defective mugs, filled them with soil and then planted four seeds in each. After 14 days, the number of seeds which germinated in each of the mugs was noted, with these results:  
 Number of germinating seeds      0      1      2      3      4      5      16      19      23  
 Number of mugs      47      67      57      16      19      1      1      0
- (i) What is the mean number of seeds germinating per mug?  

$$\bar{x} = 3.2$$
- (ii) Use a normal distribution to calculate the theoretical frequency distribution for the number of seeds germinating in the 150 mugs and hence state a conjecture about the parity of  $B$  when  $A$  is even.
- (iii) Use an associated binomial distribution to calculate the theoretical frequency distribution of seeds germinating in the 150 mugs and comment on how well your distribution models the observed results above.
- (iv) Use a binomial distribution to calculate the theoretical frequency distribution for the number of seeds germinating in the 150 mugs and hence state a conjecture about the parity of  $B$  when  $A$  is even.
- [3]
- Specific behaviours**
- ✓ correct justification
- Specific behaviours**
- ✓ correct party for subtraction
- Specific behaviours**
- ✓ correct party for multiplication
- Specific behaviours**
- ✓ conjecture  
✓ various values of  $x$  and  $y$  (minimum 2)
- When  $A$  is even,  $B$  is always odd.
- |   |    |    |   |
|---|----|----|---|
| 3 | 4  | 50 | 7 |
| 3 | 3  | 35 | 6 |
| 3 | 2  | 20 | 5 |
| 4 | 35 | 6  |   |
| 2 | 3  | 25 | 4 |
| 2 | 2  | 15 | 4 |
| 1 | 4  | 18 | 5 |
| 1 | 3  | 13 | 4 |
| 1 | 2  | 8  | 3 |
| x | y  | A  | B |
- Specific behaviours**
- ✓ uses binomial distribution  
✓ gives expect values to whole numbers and ensure  $+ 150$   
✓ comment on distribution
- The theoretical results are a reasonably close match to the observed results, suggesting that the binomial model is appropriate.
- |          |   |   |   |   |    |    |    |
|----------|---|---|---|---|----|----|----|
| Expected | 0 | 1 | 2 | 3 | 23 | 62 | 61 |
| Seeds    |   |   |   |   |    |    |    |
- $P(X=0) = 0.0016 \times 150 = 0.24$   
 $P(X=1) = 0.0256 \times 150 = 3.84$   
 $P(X=2) = 0.1536 \times 150 = 23.04$   
 $P(X=3) = P(X=4) = 0.4096 \times 150 = 61.44$

**Question 14**

(10 marks)

- (a) A spherical snowball is melting at a rate of  $18\ 000 \text{ cm}^3$  per hour. At the instant the volume of the snowball is  $4\ 000 \text{ cm}^3$ , calculate the rate of change of radius of the snowball, in cm per minute.

[4]

$$\begin{aligned}\frac{4}{3}\pi r^3 &= 4000 \Rightarrow r = 9.84745 \text{ cm} \\ \frac{dV}{dt} &= -\frac{18000}{60} = -300 \text{ cm}^3 \text{ per minute} \\ V &= \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2 \\ \frac{dr}{dt} &= \frac{dr}{dV} \times \frac{dV}{dt} \\ &= \frac{1}{4\pi \times 9.84745^2} \times -300 \\ &= -0.246 \text{ cm per minute}\end{aligned}$$

**Specific behaviours**

- ✓ negative rate of change
- ✓ correct derivative of volume formula
- ✓ correct formula for  $\frac{dr}{dt}$  including value of radius
- ✓ correct answer in  $\text{cm}^3/\text{minute}$

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**Question 17 (continued)**

- (d) The objective function is changed to  $Q = ax + 30y$ .

What is the minimum possible value of the constant  $a$ , given that the minimum value of  $Q$  still occurs at the same corner point?

[3]

$$\begin{aligned}30a + 30(15) &\leq 42a + 30(13) \\ 12a &\geq 60 \\ a &\geq 5 \\ a &\text{ is } 5\end{aligned}$$

**Specific behaviours**

- ✓✓ appropriate working
- ✓ correct answer

- (e) An additional constraint  $x + y \geq 45$  is imposed. How does this additional constraint affect the minimum value of  $Q$  in the feasible region?

[2]

$$\begin{aligned}\text{No change in } Q \text{ since at } C(30, 15), \text{ where the minimum occurs,} \\ x + y &= 30 + 15 \\ &= 45 \\ \text{which satisfies the new constraint.}\end{aligned}$$

**Specific behaviours**

- ✓ comment on effect of additional constraint
- ✓ justification of comment





**Question 15**

(9 marks)

At the end of a technology course, all students sat a practical and a theory examination, with 20% achieving a distinction in the practical examination, 3% of students achieving distinctions in both examinations and 76% achieving no distinction in either examination.

- (a) What is the probability that a student chosen at random from the course achieved a distinction in the theory examination?

$$\begin{aligned} P(P) + P(\bar{P} \cap \bar{T}) &= 0.96 \\ P(T \cap \bar{P}) &= 1 - 0.96 \\ &= 0.04 \\ P(T) &= 0.03 + 0.04 = 0.07 \end{aligned}$$

[4]

**Specific behaviours**

- ✓ determines  $P(P) + P(\bar{P} \cap \bar{T})$
- ✓ determines  $P(T \cap \bar{P})$
- ✓ recognises  $P(T \cap P) + P(T \cap \bar{P}) = P(T)$
- ✓ correct probability

- (b) Are the events 'achieving a distinction in the practical examination' and 'achieving a distinction in the theory examination' independent? Explain your answer.

No. From above it can be seen that  $P(T) \neq P(T | \bar{P})$ .

[2]

**Specific behaviours**

- ✓ calculation/explanation to justify answer
- ✓ answer

- (c) In a group of 14 students who took the course, three achieved a distinction in the practical examination. If five students are selected at random from this group, what is the probability that at least two of them achieved a distinction in the practical examination?

[3]

$$\begin{aligned} P(X=2) &= \frac{\binom{11}{3} \binom{3}{2}}{\binom{14}{5}} = \frac{495}{2002} \\ P(X=3) &= \frac{\binom{11}{2} \binom{3}{3}}{\binom{14}{5}} = \frac{55}{2002} \\ P(X \geq 2) &= \frac{550}{2002} = \frac{25}{91} \approx 0.2747 \end{aligned}$$

**Specific behaviours**

- ✓ determines number of combinations for 2 getting a distinction

See next page

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- ✓ determines number of combinations for 3 getting a distinction
- ✓ correct probability

**Question 16**

(5 marks)

A continuous random variable  $X$  has the probability distribution function  $f(x) = 0.04$ ,  $14 \leq x \leq 39$ .

- (a) Calculate

$$(i) P(21 < X < 22.5).$$

[1]

$$(22.5 - 21) \times 0.04 = 0.06$$

**Specific behaviours**

- ✓ correct probability

$$(ii) P(X < 29 | X > 25).$$

[2]

$$\frac{29 - 25}{39 - 25} = \frac{2}{7}$$

**Specific behaviours**

- ✓ correct numerator
- ✓ correct denominator

- (b) If  $P(20 < X < k | X < k) = 0.75$ , find the value of  $k$ .

[2]

$$\begin{aligned} \frac{k - 20}{k - 14} &= 0.75 \\ k - 20 &= 0.75k - 10.5 \\ 0.25k &= 9.5 \\ k &= 38 \end{aligned}$$

**Specific behaviours**

- ✓ identifies appropriate equation to solve for  $k$
- ✓ correct value of  $k$



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See next page