WESLEY COLLEGE

By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2017
QUESTIONS OF REVIEW 3:
Vectors in 3 dimensions

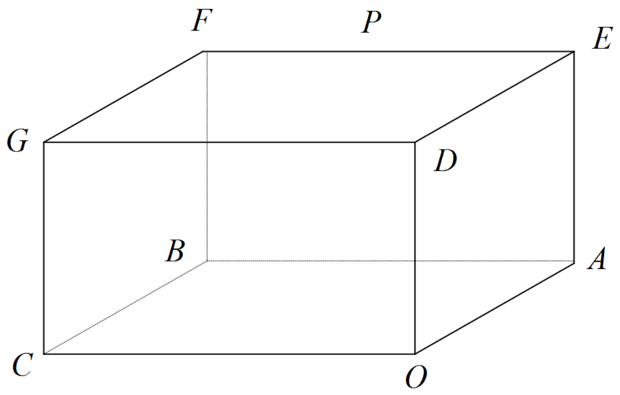
Name: ________________________

Thursday 11th May   Time: 40 minutes   Mark   /30

Calculator allowed.

1. [6 marks – 1, 1, 2 and 2]

The right rectangular prism shown has vertices $O(0,0,0)$, $A(3,0,0)$, $C(0,4,0)$ and $D(0,0,2)$.



Use appropriate vector methods to represent:

a) the position of point *G*

b) the position of point *P*, the mid-point of *EF*

c) an equation for the line through points *C* and *P*

d) the angle the line *CP* makes with the base *OABC*

2. [5 marks – 2 and 3]

Use $OA \otimes OB$ and the vectors $OA = 3i - 4j + k$ and $OB = 5i + 4j - 2k$ to calculate

a) the area of ΔOAB

b) $\angle AOB$

3. [8 marks – 1, 3, 2, 1 and 1]

a) Express the vector equation $\left| \vec{r} - \begin{bmatrix} 1 \\ 3 \\ 1.5 \end{bmatrix} \right| = \frac{9}{2}$ in Cartesian form

b) Calculate the point(s) of intersection of $\vec{r} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix}$ and $\left| \vec{r} - \begin{bmatrix} 1 \\ 3 \\ 1.5 \end{bmatrix} \right| = \frac{9}{2}$

c) Describe, by a suitable vector or algebraic equation, the locus of points that are equidistant from the points of intersection found in (b)

d) Show that $\vec{r} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix}$ is a part of a diameter of $\left| \vec{r} - \begin{bmatrix} 1 \\ 3 \\ 1.5 \end{bmatrix} \right| = \frac{9}{2}$

e) Calculate the distance between any pair of parallel planes tangential to opposite sides of $\left| \vec{r} - \begin{bmatrix} 1 \\ 3 \\ 1.5 \end{bmatrix} \right| = \frac{9}{2}$

4. [7 marks – 2, 2 and 3]

The vector $\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$ is perpendicular to the plane Γ_1, which is itself parallel to both $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$.

a) Use scalar (dot) product calculations to set up equations sufficient to evaluate *a* and *b*

b) Use a vector (cross) product calculation to set up an equation to evaluate *a* and *b*

c) Solve for *a* and *b* and hence develop a Cartesian equation for Γ_1, which passes through $(1,2,3)$

5. [4 marks]

The position vectors of three non-collinear points *A*, *B* and *C*, with respect to an origin *O*, are $\underset{\sim}{\mathbf{a}}$, $\underset{\sim}{\mathbf{b}}$ and $\underset{\sim}{\mathbf{c}}$ respectively.

O does not lie in the plane *ABC*.
The point *Q* with position vector $\underset{\sim}{\mathbf{q}} = \alpha\underset{\sim}{\mathbf{a}} + \beta\underset{\sim}{\mathbf{b}} + \gamma\underset{\sim}{\mathbf{c}}$ does lie in the plane *ABC*.

Show that $\alpha + \beta + \gamma = 1$

Formulae:

Vectors

Magnitude:	$\lvert(a_1, a_2, a_3)\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$	
Dot product:	$\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$	
Triangle inequality:	$\lvert\mathbf{a} + \mathbf{b}\rvert \leq \lvert\mathbf{a}\rvert + \lvert\mathbf{b}\rvert$	
Vector equation of a line in space:	one point and the slope: two points A and B:	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$
Cartesian equations of a line in space:	$\dfrac{x - a_1}{b_1} = \dfrac{y - a_2}{b_2} = \dfrac{z - a_3}{b_3}$	
Parametric form of vector equation of a line in space:	$x = a_1 + \lambda b_1 ......(1)$ $y = a_2 + \lambda b_2 ......(2)$ $z = a_3 + \lambda b_3 ......(3)$	
Vector equation of a plane in space:	$\mathbf{r}\bullet\mathbf{n} = c$ or	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$
Cartesian equation of a plane:	$ax + by + cz = d$	

Vector cross product $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$ and $\lvert a \times b\rvert = \lvert a\rvert\lvert b\rvert\sin\theta$

The sphere defined by $\lvert r - a\rvert = k$ has a centre with position vector a and radius k