PERTH MODERN SCHOOL
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Independent Public School

Course Methods Test 1 Year 12

Student name: ________________ Teacher name: ________________

Task type: **Response**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **_____8______**

Materials required: No Cals allowed at all!

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper,

Marks available: **40 marks**

Task weighting: **13%**

Formula sheet provided: no but formulae listed on next page.

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$	
$\frac{d}{dx}e^{ax-b} = ae^{ax-b}$		$\int e^{ax}dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\ln x = \frac{1}{x}$		$\int \frac{1}{x}dx = \ln x + c, \quad x > 0$	
$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$		$\int \frac{f'(x)}{f(x)}dx = \ln f(x) + c, \quad f(x) > 0$	
$\frac{d}{dx}\sin(ax-b) = a\cos(ax-b)$		$\int \sin(ax-b)\,dx = -\frac{1}{a}\cos(ax-b) + c$	
$\frac{d}{dx}\cos(ax-b) = -a\sin(ax-b)$		$\int \cos(ax-b)\,dx = \frac{1}{a}\sin(ax-b) + c$	
Product rule	If $y = uv$ then $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$	or	If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$
Quotient rule	If $y = \frac{u}{v}$ then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	or	If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$
Chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or	If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$
Fundamental theorem	$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$	and	$\int_a^b f'(x)\,dx = f(b) - f(a)$
Increments formula	$\delta y \approx \frac{dy}{dx} \times \delta x$		
Exponential growth and decay	$\frac{dP}{dt} = kP \Leftrightarrow P = P_0e^{kt}$		

No calculators allowed!!!

Q1 (2 & 3 = 5 marks)

Determine the equation of the tangent to the following curves at the stated point:

a) $y = 2x^3 - 3x + 1$ at the point $(1, 0)$

b) $y = -5x^3 + \frac{1}{x^2}$ at the point $(-1, 6)$

Q2 (3 & 3 = 6 marks)

Determine the derivatives of the following using the quotient rule and simplify your answer.

a) $f(x) = \frac{x+3}{2x^3+2}$

b) $f(x) = \frac{3x^2+1}{(5x-1)^3}$

Q3 (5 marks)
Determine the coordinates of the stationary points of $f\left(x\right)=x^{3}-3x+2$ using calculus and justify their nature.

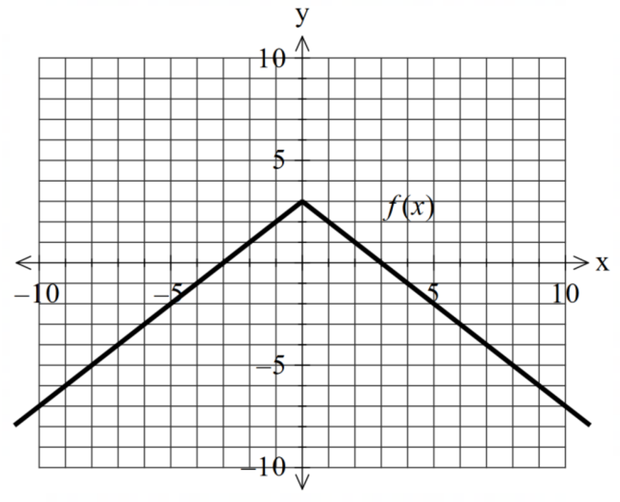
Q4 (1, 2 & 3 = 6 marks)
Consider an object initially at the origin that moves only in a straight line with displacement from origin, x, given by $x=\frac{t^{3}}{3}-\frac{3t^{2}}{2}+2t$ at time, t seconds.
Determine:

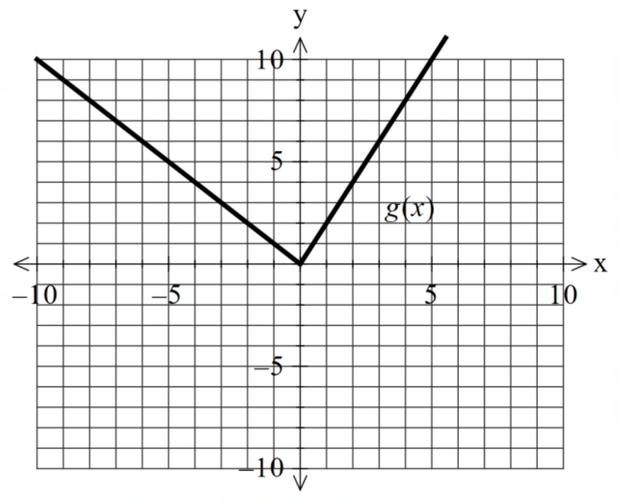
a) Acceleration at $t=1$ second.

b) The times the object is at rest.

c) The distance travelled in the first 3 seconds.

Q5 (2, 2 & 2 = 6 marks)
The graphs of f and g are displayed below.





a) Determine the derivative of $f(x)g(x)$ at $x=3$.

b) Determine the derivative of $\dfrac{f(x)}{g(x)}$ at $x=2$.

c) Determine the derivative of $f(g(x))$ at $x=-1$

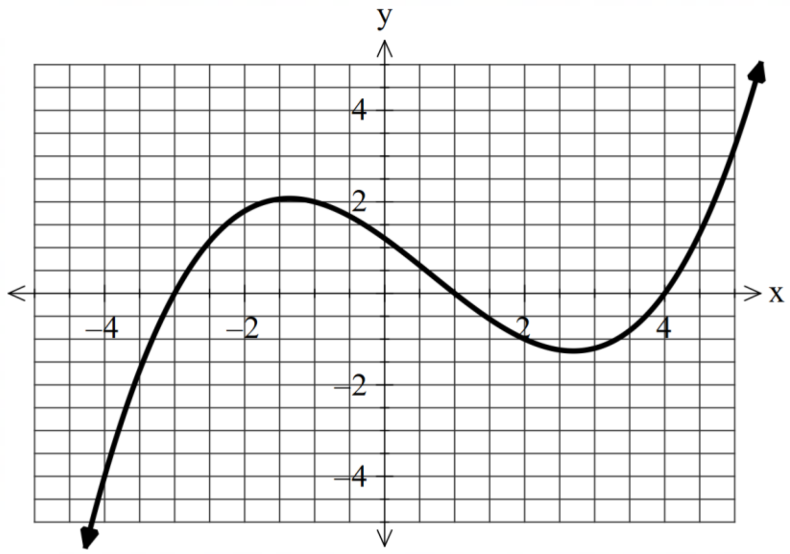
Q6 (3 marks)

If $q = \dfrac{5}{t^{\frac{3}{2}}}$ use differentiation to determine the approximate percentage change in q when t increases by 3%.

Q7 (5 marks)

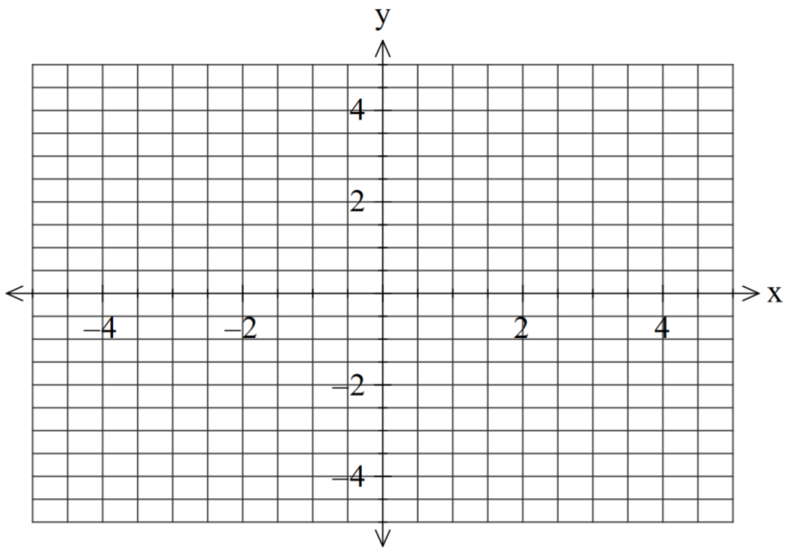
Consider the function $f(x)$ as graphed below. On the axes below sketch the function $y = f'(x)$ and **on this graph** label and show the coordinates and nature of all important features of $f(x)$.

(Do not write on original function graph)

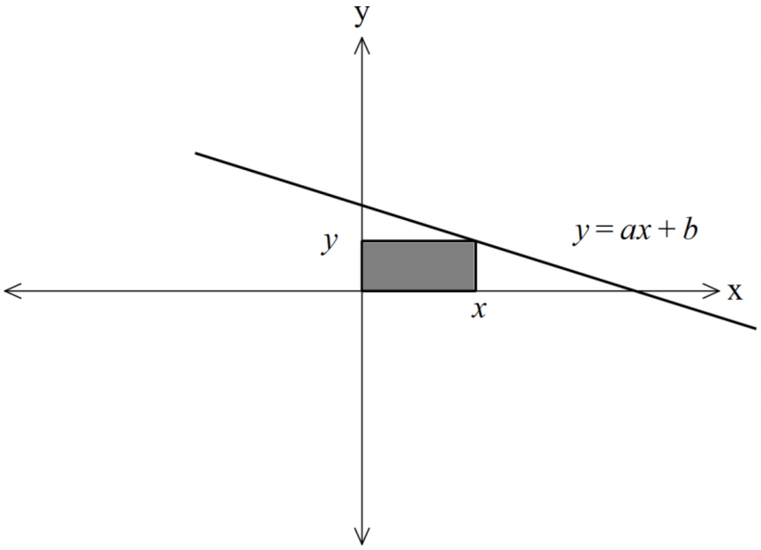


$f(x)$

$y = f'(x)$



Q8 (4 marks)
A rectangle has one vertex at the origin, another on the positive x-axis, another on the positive y-axis and a fourth on the line $y = ax + b$ where $a \& b$ are constants.



The greatest area occurs when $x = 8$ with an area of 32 sq units**. Using calculus**, determine the values of the constants $a \& b$.