Year 12 Examination, 2017

Question/Answer Booklet

MATHEMATICS SPECIALIST

Section One: Calculator-free

Student Name/Number: _____

Teacher Name: _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

To be provided by the candidate:
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	54	35
Section Two: Calculator-assumed	11	11	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the _____*School/College assessment policy*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **35% (54 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Suggested working time: 50 minutes.

Question 1 **(4 marks)**

Solve the differential equation

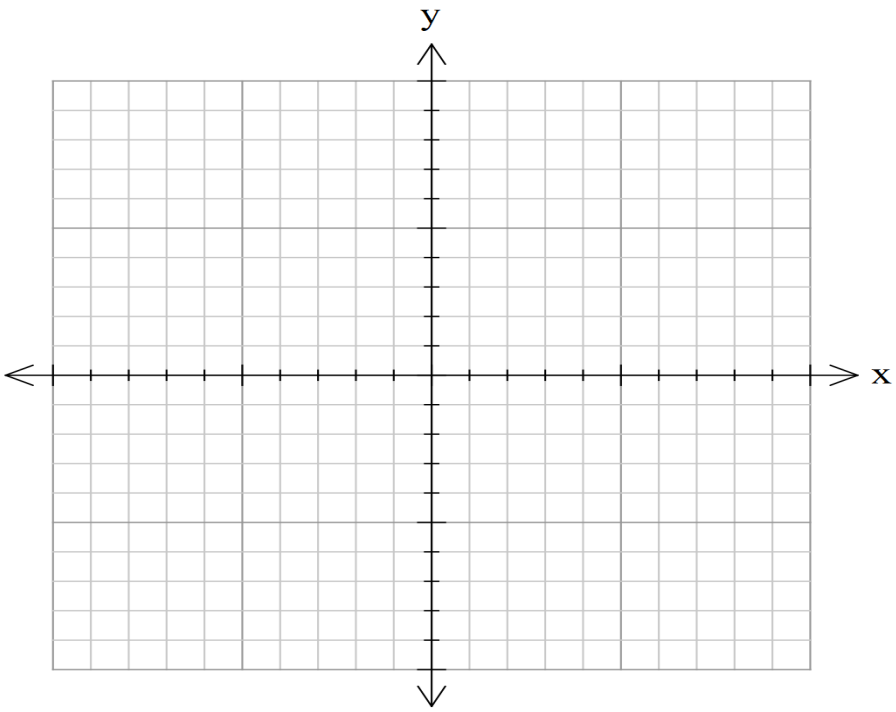
$$\frac{dy}{dx} = -2xy^2$$

given that $y(0) = 2.$

See next page

Question 2 **(6 marks)**

Sketch the function $f(x) = \frac{x}{x^2 + 4}$ paying particular attention to the location of its turning point(s) and the behaviour as $x \to \pm\infty$.



See next page

Question 3 **(8 marks)**

(a) Determine the two points P and Q where the curve

$$x^2 + xy + y^3 = 9$$

cuts the x-axis. (2 marks)

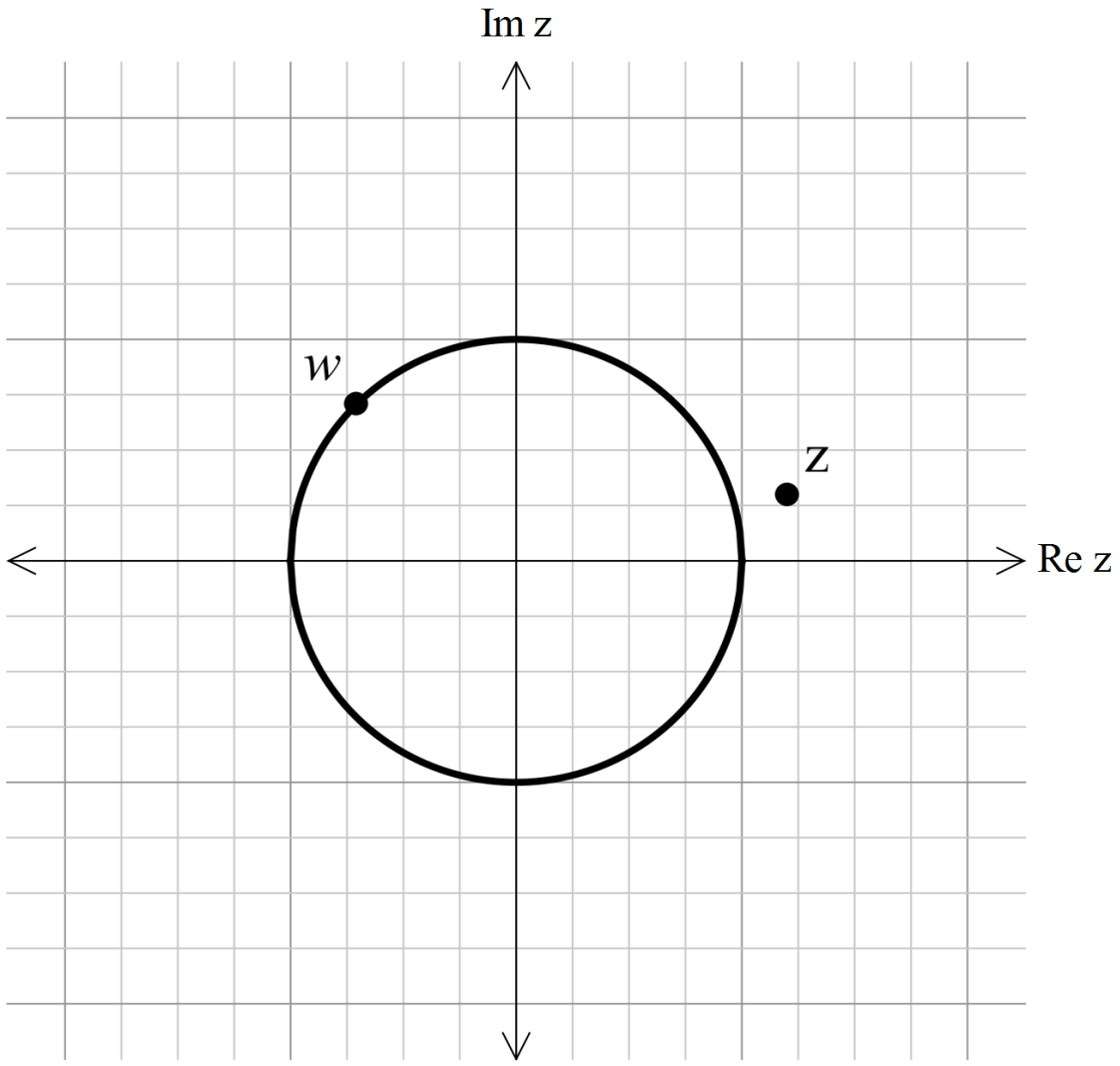
(b) Show that the tangents to the curve at P and Q are parallel. (4 marks)

(c) The point R on the curve has x co-ordinate 2.99. Use increments to deduce an approximation for the y co-ordinate of R. (2 marks)

See next page

Question 4 **(7 marks)**

The positions in the complex plane of the numbers w and z together with the unit circle are shown below.



See next page

(a) Indicate, as accurately as possible, the positions in the complex plane of the numbers:

(i) $\overline{z}$ (1 mark)

(ii) wz (1 mark)

(iii) z^{-1} (1 mark)

(iv) $w+z$ (1 mark)

(v) w^4 (1 mark)

(b) Sketch on the diagram the set of complex numbers Ω which satisfy both the inequalities

$|\Omega| \leq 1$ and $\frac{\pi}{3} \leq \arg(\Omega) \leq \frac{2\pi}{3}$. (2 marks)

See next page

Question 5 **(12 marks)**

Determine the following integrals:

(a) $\int \sin^2 x \cos^2 x \, dx$ (3 marks)

(b) $\int_0^{\pi/4} \tan^2 x \, dx$. (3 marks)

See next page

(c) Use the substitution $u = x^5 + 4$ to evaluate (3 marks)

$$\int_0^2 \frac{x^4}{\sqrt{x^5+4}}\,dx$$

.

(d) Use the substitution $v = \ln x$ to determine the value of Q if

$$\int_e^Q \frac{dx}{x \ln x} = 1$$

. (3 marks)

Question 6 **(8 marks)**

(a) The formula for a confidence interval for the mean μ of a population is often expressed in the form

$$\bar{X} - E \leq \mu \leq \bar{X} + E$$

where $\bar{X}$ is the sample mean and E is called the 'margin of error'.

Write down a formula for E assuming that the sample mean $\bar{X}$ is normally distributed and the standard deviation of the population is known. (1 mark)

(b) Describe the effect on the margin of error if

(i) the sample size doubles. (1 mark)

(ii) the level of confidence increases. (1 mark)

(iii) the standard deviation of the underlying population doubles. (1 mark)

 See next page

(c) State whether the following statements about confidence intervals for μ are true or false. Justify your answers.

(i) If there are 10 independent samples from the same population, then exactly 9 of the corresponding 90% confidence intervals will contain μ. (2 marks)

(ii) Each 95% confidence interval for μ contains 95% of the underlying population. (2 marks)

See next page

Question 7 **(9 marks)**

Two vectors $\mathbf{a}$ and $\mathbf{b}$ are defined to be $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
For some scalar value λ, the vector $\mathbf{a} + \lambda\mathbf{b}$ is orthogonal to $\mathbf{c} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(a) Determine the value of λ. (3 marks)

(b) Determine the equation of the line parallel to **c** that passes through the origin. Express the equation of the line in both vector and parametric forms. (2 marks)

(c) Determine the vector product of $\mathbf{a}$ and $\mathbf{b}$. (2 marks)

See next page

(d) Write down the equation of the plane that is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ and that contains the origin. (2 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Acknowledgements

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