

MATHEMATICS
3C/3D

Section Two:

Calculator-assumed

--	--	--	--	--	--	--	--	--	--

Student Number: In figures

In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

This examination paper – apart from any third party copyright material contained in it – may be freely copied, or communicated on an intranet, for non-commercial purposes in educational institutions, provided that it is not changed and that the Curriculum Council is acknowledged as the copyright owner. Teachers in schools offering the Western Australian Certificate of Education (WACE) may change the examination paper, provided that the Curriculum Council's moral rights are not infringed. Copying or communication for any other purpose can be done only within the terms of the Copyright Act or with prior written permission of the Curriculum Council. Copying or communication of any third party copyright material can be done only within the terms of the Copyright Act or with permission of the copyright owners.

Published by the Curriculum Council of Western Australia
27 Walters Drive
OSBORNE PARK WA 6017

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	40	
Section Two: Calculator-assumed	12	12	100	80	
Total				120	100

Instructions to candidates

- The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2011*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.

Additional working space

Question number: _____

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Section Two: Calculator-assumed

(80 Marks)

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9

(5 marks)

For events A and B ,

$$P(A) = 0.5, \quad P(B | A) = 0.3 \quad \text{and} \quad P(A \cup B) = 0.8.$$

(a) Calculate $P(A \cap B)$ (1 mark)

(b) Calculate $P(B)$ (1 mark)

(c) Calculate $P(\bar{A} \cap B)$ (1 mark)

(d) Are events A and B independent? Justify your answer. (2 marks)

See next page

Question 10**(8 marks)**

Suppose that 9% of people in a certain community live alone.

Sixty percent of people who live alone own pets, whereas only 3% of people who do not live alone own pets.

- (a) What is the probability that a person chosen at random from this community owns a pet? (2 marks)
- (b) What is the probability that a randomly-selected person from this community neither lives alone nor owns a pet? (1 mark)
- (c) What percentage of people from this community who own a pet live alone? (2 marks)
- (d) In a group of 30 people in this community, four live alone. If six people are selected from this group, what is the probability that no more than two of them live alone? (3 marks)

Additional working space

Question number: _____

Question 11

(5 marks)

When an amount \$ A is invested at an interest rate of $r\%$ per annum, compounded n times per year, the value \$ V of the investment after one year is given by $V = A \left(1 + \frac{100r}{n} \right)^n$.

Kelvin invests \$6000 at 8% per annum interest for one year.

- (a) What is the value of the investment at the end of the year if interest is compounded twice per year? (1 mark)

- (b) If interest is compounded monthly, by what percentage does the investment increase over the course of the year? (2 marks)

- (b) Use your answer from part (a) to show that the volume V of a cone is given by $V = \frac{1}{3}Ah$ where A is the area of the base and h is the height. (2 marks)

(4 marks)

- (a) Find the volume of revolution obtained when the line $y = mx$, between the limits $y = 0$ and $y = h$, is rotated about the y -axis. (2 marks)

- (c) If the investment were to be compounded more frequently, could the value of Kelvin's investment rise above \$6500 at the end of the year? Justify your answer. (2 marks)

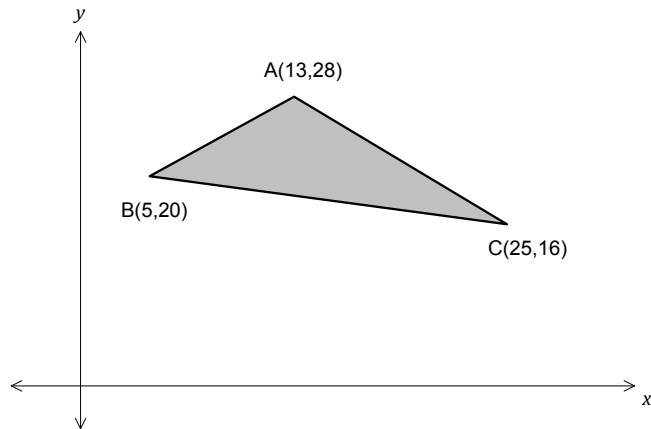
Question 12

(9 marks)

Each day, a company produces x thousand units of commodity X and y thousand units of commodity Y.

Each unit of commodity X earns a profit of \$21, and each unit of commodity Y earns a profit of \$15.

The feasible region for the company's daily production schedule is the triangle shown below.



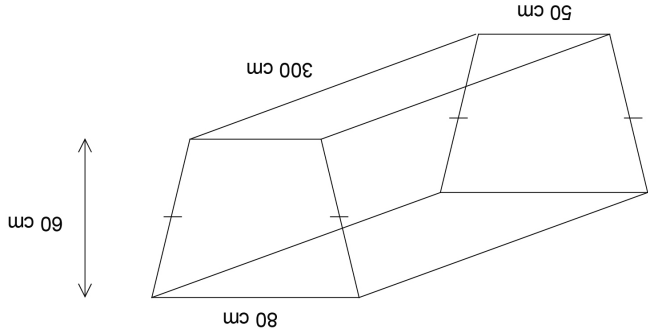
- (a) Determine the inequality satisfied by x and y that corresponds to the edge AB of the feasible region. (2 marks)

- (b) Determine the maximum possible daily profit. (2 marks)

- (c) Water is being pumped into the trough at a rate of 40 litres per minute. At what rate is the depth of the water increasing at the instant when the trough is half full by volume? Give your answer correct to the nearest centimetre per minute. (4 marks)

Question 19

A water trough has the shape of a trapezoidal prism. It is 300 centimetres long, 60 centimetres high, 80 centimetres wide at the top and 50 centimetres wide at the bottom. A sketch of the trough is shown below (not to scale).



(8 marks)

(a)

The top surface of the water in the trough has the shape of a rectangle whose length is 300 cm. Show that if the water in the trough is h cm deep, then the width of this rectangle is $50 + 0.5h$ cm.

(2 marks)

(b)

Show that if water in the trough is h cm deep, then the volume of water, V litres, is given by $V = h(15 + 0.075h)$.

(2 marks)

See next page

See next page

(c)

The company decides that the amount of commodity Y produced cannot be more than three times the amount of commodity X.

How does this additional constraint affect the maximum possible profit? Justify your answer.

(2 marks)

(d)

Changing market conditions will cause changes to the unit profits for commodities X and Y.

On each of the next seven days, the unit profit for commodity X will fall by \$1, and the unit profit for commodity Y will rise by \$1.

(3 marks)

Describe how these changes will affect the maximum possible daily profit over the course of the next week.

Question 13

(7 marks)

The lifetimes of Glowbrite light bulbs are normally distributed with mean 3500 hours and standard deviation 200 hours.

- (a) What is the probability that the lifetime of a randomly-selected bulb is at least 3400 hours? (1 mark)

- (b) Calculate t , given that 5% of Glowbrite bulbs last longer than t hours. (1 mark)

- (c) What is the probability that a Glowbrite bulb will last no more than 3500 hours, if it has already lasted 3200 hours? (3 marks)

- (b) Use calculus to show that the lowest point on the cable occurs where it crosses the y -axis, that is, where $x = 0$. (3 marks)

- (c) The length s of the curve $y = f(x)$, between the limits $x = a$ and $x = b$, is given by the formula

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Use this formula to determine the length of the cable if the lowest point of the cable is 10 units below the level of the supports A and B . (3 marks)

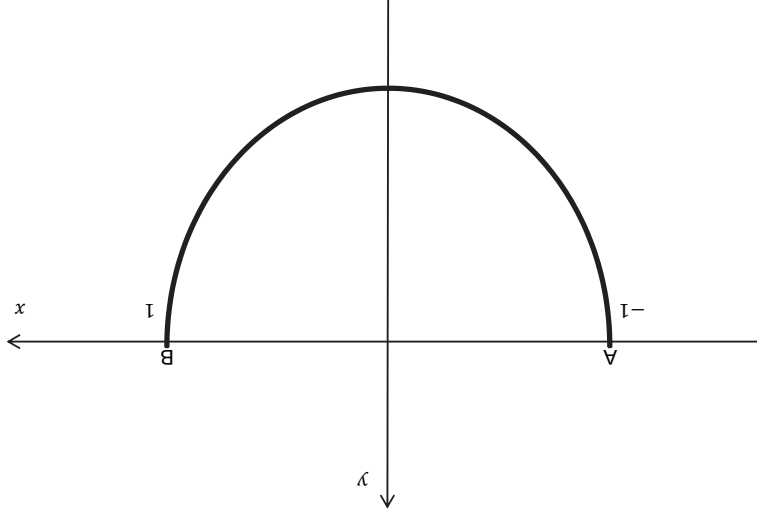
Question 18

(7 marks)

A cable hanging between two points $A(-1,0)$ and $B(1,0)$ lies on the curve

$$y = e^{cx} - d + e^{-cx},$$

where c and d are positive constants.



- (a) Show that $d = e^c + e^{-c}$. (1 mark)

See next page

(d)

The company also produces Ultrabrite light bulbs, whose lifetimes are also normally distributed, with the same standard deviation of 200 hours but with a possibly different mean μ hours.

A quality control expert at the company wishes to estimate μ using the mean lifetime of a random sample of Ultrabrite bulbs.

How large should the sample be in order to be 95% confident that the estimate will be no more than 10 hours in error? (2 marks)

See next page

Question 14

(5 marks)

During a volcanic eruption a rock is ejected from the top of the volcano. The rock rises upward and then falls onto a flat plain 1500 metres below the top of the volcano. During its flight, the vertical velocity of the rock, v m/s, is given by

$$v = 160 - 9.8t$$

where t seconds is the time after the ejection of the rock.

(a) How high does the rock rise above the top of the volcano?

(3 marks)

(b) How long does it take for the rock to reach the plain below?

(2 marks)

(c) Examine the difference between A and B for various values of x and y and state a conjecture about A and B . (1 mark)

(d) Prove the conjecture in part (c).

(3 marks)

Question 17

Let A denote the average of the squares of two numbers x and y :

$$A = \frac{1}{2}(x^2 + y^2),$$

and let B denote the square of the average of x and y :

$$B = \left(\frac{x + y}{2}\right)^2.$$

(a) Evaluate A and B in the case $x = 5$ and $y = 7$.

(1 mark)

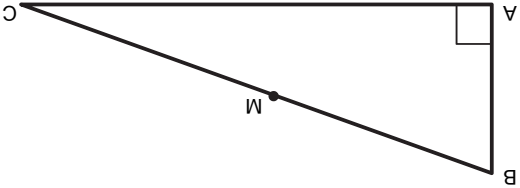
(b) What can be said about A and B if $x = y$? Justify your answer.

(2 marks)

See next page

Question 15

In the diagram below ABC is a right-angled triangle, and M is the mid-point of the hypotenuse BC .



Prove that M is equidistant from each of the vertices A , B and C .

Hint: Start by drawing the line through M that is parallel to the side AB .

See next page

Question 16

(11 marks)

The mean μ and standard deviation σ of the uniform distribution on the interval $[a, b]$ are given by

$$\mu = \frac{a+b}{2} \text{ and } \sigma = \frac{b-a}{2\sqrt{3}}.$$

A calculator can generate random numbers that are uniformly distributed between 0 and 1.

(a) For this distribution of the random numbers generated by the calculator, calculate

(i) the mean. (1 mark)

(ii) the standard deviation (to **three (3)** decimal places). (1 mark)

(b) What is the probability that a randomly-generated number lies between $\frac{1}{4}$ and $\frac{1}{3}$? (1 mark)

(c) What is the probability that a randomly-generated number contains no seven in its first **five (5)** decimal places? (1 mark)

See next page

(d) What is the probability that a randomly-generated number contains at most three odd digits in its first five decimal places? Give your answer to **four (4)** decimal places. (2 marks)

(e) What is the probability that the sum of 500 randomly-generated numbers exceeds 260? Give your answer to **four (4)** decimal places. (3 marks)

(f) Another uniform distribution on an interval $[a, b]$ has a standard deviation of $2\sqrt{3}$. How wide is the interval? (2 marks)

See next page