



PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 2 Year 12

Student name: _____ Teacher name: _____

Task type: Response

Time allowed for this task: ____40____ mins

Number of questions: ____7____

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: ____41____ marks

Task weighting: ____10____%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2, 2 & 3 = 7 marks) (3.2.1-3.2.3)

Consider the functions $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$.

- a) State the natural domain and range of $f(x)$.

Solution
$d_f : x \neq 2$ $r_f : y \neq 0$
Specific behaviours
✓ states domain ✓ states range

- b) Does $g \circ f(x)$ exist over the natural domain of $f(x)$? Explain.

Solution
$r_f \subseteq d_g$ To exist $y \neq 0 \not\subseteq y \geq 0$ therefore does not exist over natural domain
Specific behaviours
✓ states does not exist with any reason ✓ reason shows relevant domain and range

- c) State the rule and natural domain and range of $f \circ g(x)$.

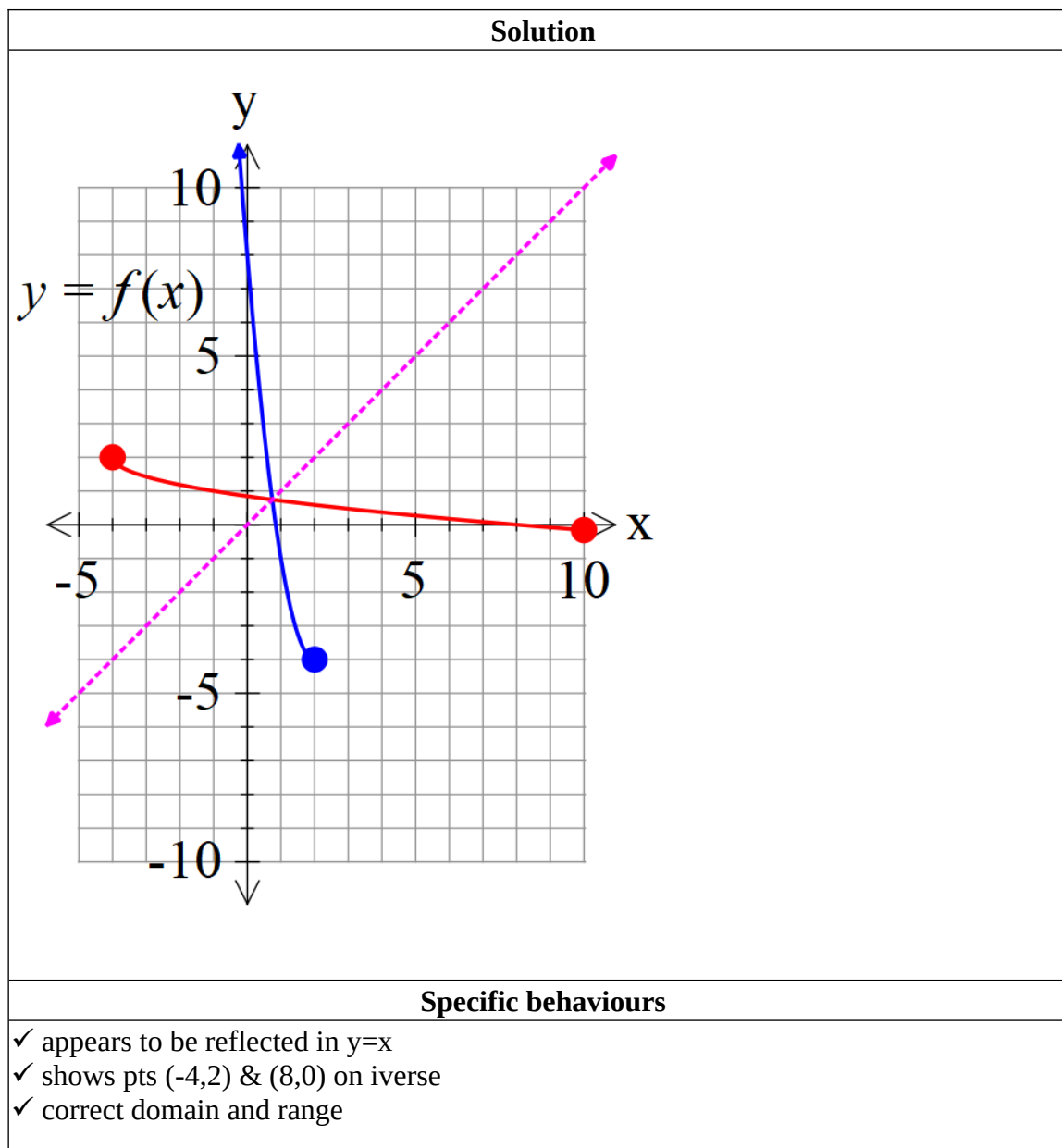
Solution
$f \circ g(x) = \frac{1}{\sqrt{x}-2}$ $d : (0 \leq x < 4) \cup (x > 4)$ $r : \mathbb{R} \setminus \left\{ -\frac{1}{2} < y \leq 0 \right\}$
Specific behaviours
✓ states rule

- ✓ states domain which excludes $x=4$
- ✓ states range with correct endpoints inequalities of excluded interval

Q2 (3, 3, 1 & 2 = 9 marks) (3.2.4)

Consider the function $f(x) = 3x^2 - 12x + 8$ with domain $x \leq 2$.

a) Sketch the inverse function on the axes below.



b) Determine the inverse function $f^{-1}(x)$ stating its domain. (Show all working)

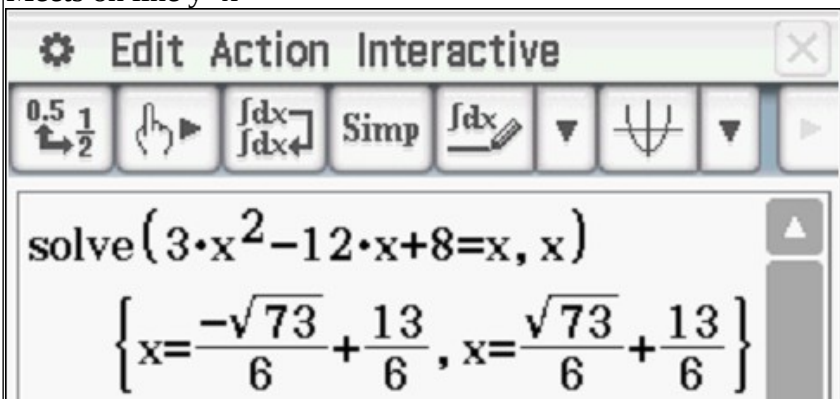
Solution

$x = 3y^2 - 12y + 8$ $3y^2 - 12y + 8 - x = 0$ $y = \frac{12 \pm \sqrt{144 - 4(3)(8 - x)}}{6} = \frac{12 \pm \sqrt{48 + 12x}}{6} = 2 \pm \frac{\sqrt{3(x+4)}}{3}$ $y \leq 2 \therefore f^{-1}(x) = 2 - \frac{\sqrt{3(x+4)}}{3}$ $d : x \geq -4$
Specific behaviours
<ul style="list-style-type: none"> ✓ shows the interchange of y & x or shows how x is made the subject of rule ✓ states inverse rule with correct sign ✓ states domain

c) Determine $f \circ f^{-1}(x)$

Solution
$f \circ f^{-1}(x) = x$
Specific behaviours
✓ states x

d) Determine when $f(x) = f^{-1}(x)$ exactly.

Solution
<p>Meets on line $y=x$</p>  <p>Discard second answer as outside domain of f</p>
Specific behaviours
✓ sets up an equation to solve for x

✓ solves for one value of x exactly

Q3 (3 marks) (3.2.6)

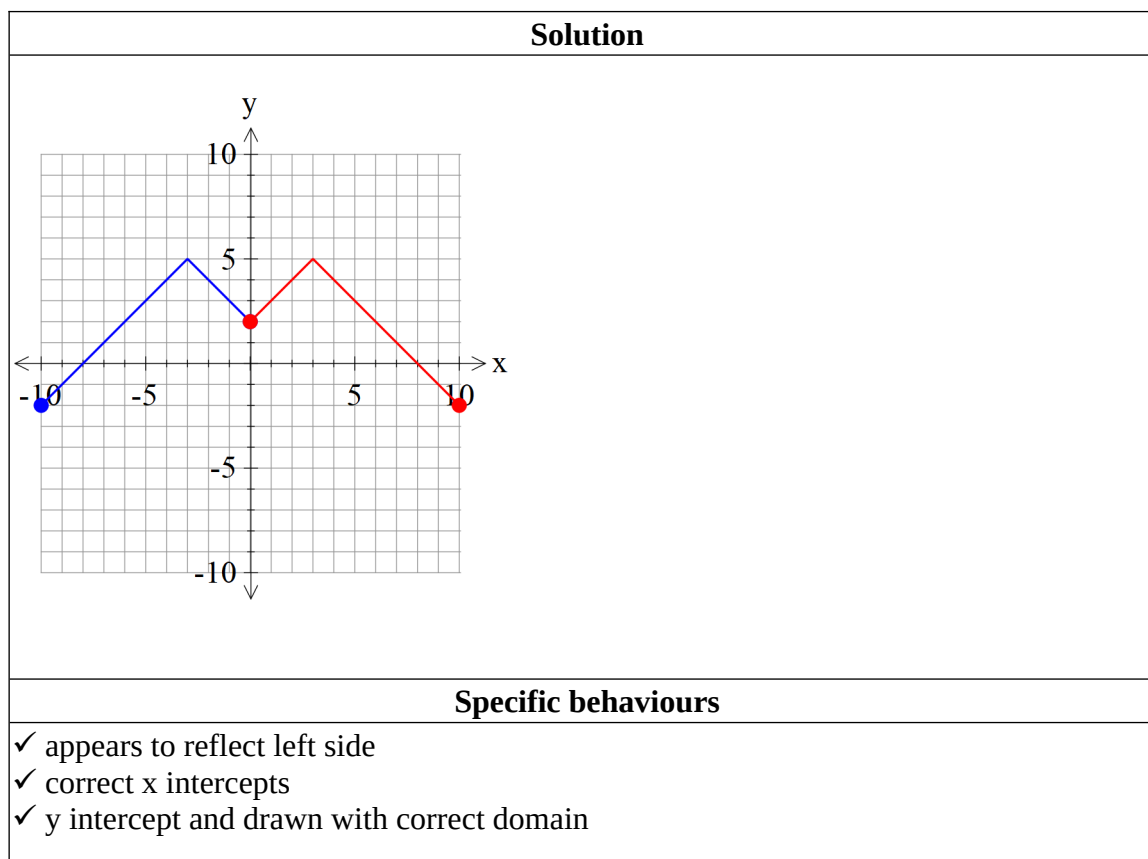
Consider the inequality $\left| \frac{3}{2}x + b \right| \leq 4.5$ is **only true** for $4 \leq x \leq 10$ with b a constant.
Determine the value of b .

Solution
$\left \frac{3}{2}x + b \right \leq 4.5$ $\frac{3}{2} \left x + \frac{2b}{3} \right \leq \frac{9}{2}$ $\left x + \frac{2b}{3} \right \leq 3$ $\frac{2b}{3} = -7 \quad \text{as } 4 \leq x \leq 10$ $b = -\frac{21}{2}$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses midpoint of solution interval ✓ rearranges inequality to identify centre in terms of x ✓ states correct value of constant

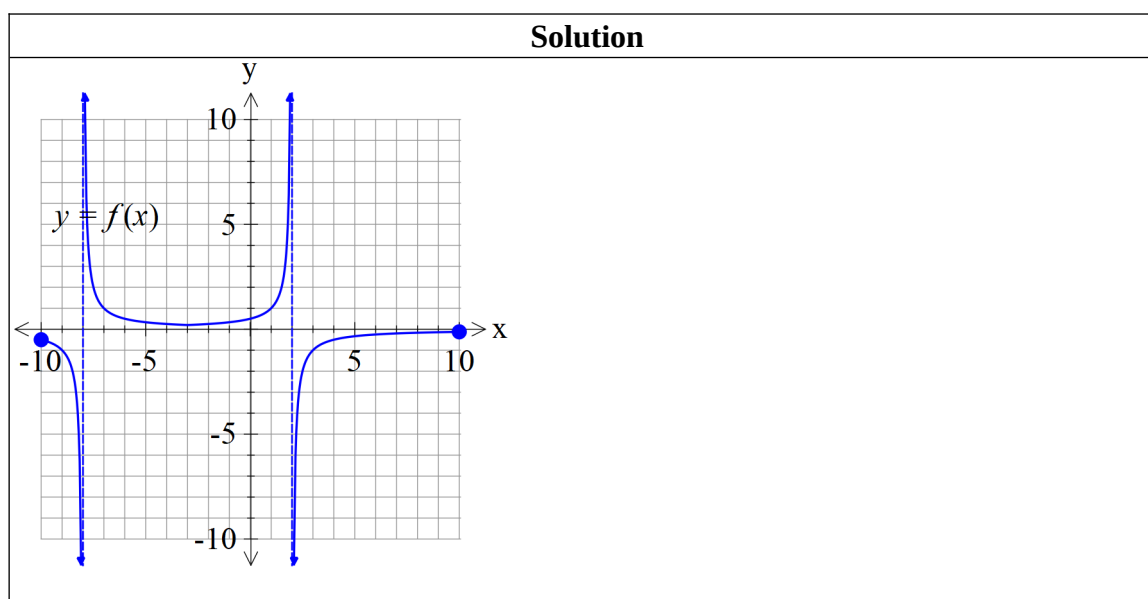
Q4 (3 & 3 = 6 marks) (3.2.7)

Consider the following function $f(x)$.

- a) Sketch $y = f(-|x|)$ on the axes below.



- b) Sketch $y = \frac{1}{f(x)}$ on the axes below.



Specific behaviours
<ul style="list-style-type: none"> ✓ both asymptotes shown ✓ y intercept correct with y endpoints ✓ shape in all 3 section

Q5 (3 & 3 = 6 marks) (3.3.3-3.3.6)

Consider two rockets A&B, moving with constant velocities such that at time $t=0$ hours their positions and velocities are as follows:

$$r_A = \begin{pmatrix} -2 \\ 3 \\ 7 \end{pmatrix} \text{ km} \quad r_B = \begin{pmatrix} 6 \\ -2 \\ -1 \end{pmatrix} \text{ km}$$

$$v_A = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} \text{ km/h} \quad v_B = \begin{pmatrix} -2 \\ 5 \\ 2 \end{pmatrix} \text{ km/h}$$

- a) Determine the time and distance of their closest approach.

Solution
<div style="border: 1px solid #ccc; padding: 5px; margin-bottom: 5px;"> ⚙ Edit Action Interactive </div> <div style="border: 1px solid #ccc; padding: 5px; margin-bottom: 5px;"> 0.5 1/2 ↩ ↪ ∫dx ∫dx↩ Simp ∫dx ▼ </div> <div style="border: 1px solid #ccc; padding: 10px;"> $\text{dotP}\left(\begin{bmatrix} 6 \\ -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix} + t \cdot \left(\begin{bmatrix} -2 \\ 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}\right), \begin{bmatrix} -2 \\ 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}\right)$ <div style="text-align: right; margin-top: -20px;">$6 \cdot (3 \cdot t - 8)$</div> $\text{solve}(6 \cdot (3 \cdot t - 8) = 0, t)$ <div style="text-align: right; margin-top: 20px;">$\left\{t = \frac{8}{3}\right\}$</div> </div>

$$\text{norm}\left(\begin{bmatrix} 6 \\ -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix} + t \cdot \left(\begin{bmatrix} -2 \\ 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}\right) \mid t = \frac{8}{3}\right)$$

Distance=5 mins

Specific behaviours

- ✓ uses displacement vector with dot product OR calculus with separation vector
- ✓ states time of closest approach (no need for units)
- ✓ states distance approx. or exact (no need for units)

- b) Given that the rockets leave smoke trails that stays in the air for a long period of time, determine if the smoke trails cross at all and if they do, its position in space. Justify.

Solution

$$r_A = \begin{pmatrix} -2 \\ 3 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$$

$$r_B = \begin{pmatrix} 6 \\ -2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 5 \\ 2 \end{pmatrix}$$

<div> </div>	
<div> $\begin{cases} -2 + \lambda = 6 - 2\mu \\ 3 + 5\lambda = -2 + 5\mu \\ 7 - \lambda = -1 + 2\mu \end{cases} \quad \lambda, \mu$ $\{ \lambda = 2, \mu = 3 \}$ $\begin{bmatrix} 0 \\ 13 \\ 5 \end{bmatrix}$ </div>	
Specific behaviours	
<ul style="list-style-type: none"> ✓ uses lines with different parameters ✓ shows solution to stated simultaneous equations(all 3) with values of parameters ✓ states point of intersection of smoke trails 	

Q6 (6 marks) (3.3.4, 3.3.6)

$$r = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ -2 \end{pmatrix} \quad \text{and the sphere} \quad \left| r - \begin{pmatrix} 6 \\ \beta \\ -7 \end{pmatrix} \right| = 5$$

Consider the line and the sphere with β a constant.

Determine the value(s) of β , to one decimal place, such that:

- The line is a tangent to sphere.
- The line meets the sphere in two places.
- The line misses the sphere completely.

Solution

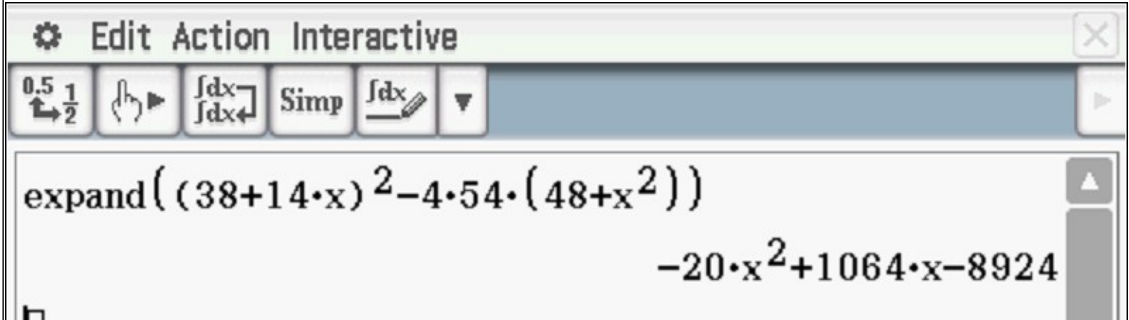
$$\left| \begin{pmatrix} 3+\lambda \\ 7\lambda \\ 1-2\lambda \end{pmatrix} - \begin{pmatrix} 6 \\ \beta \\ -7 \end{pmatrix} \right| = 5$$

$$\sqrt{(\lambda - 3)^2 + (7\lambda - \beta)^2 + (8 - 2\lambda)^2} = 5$$

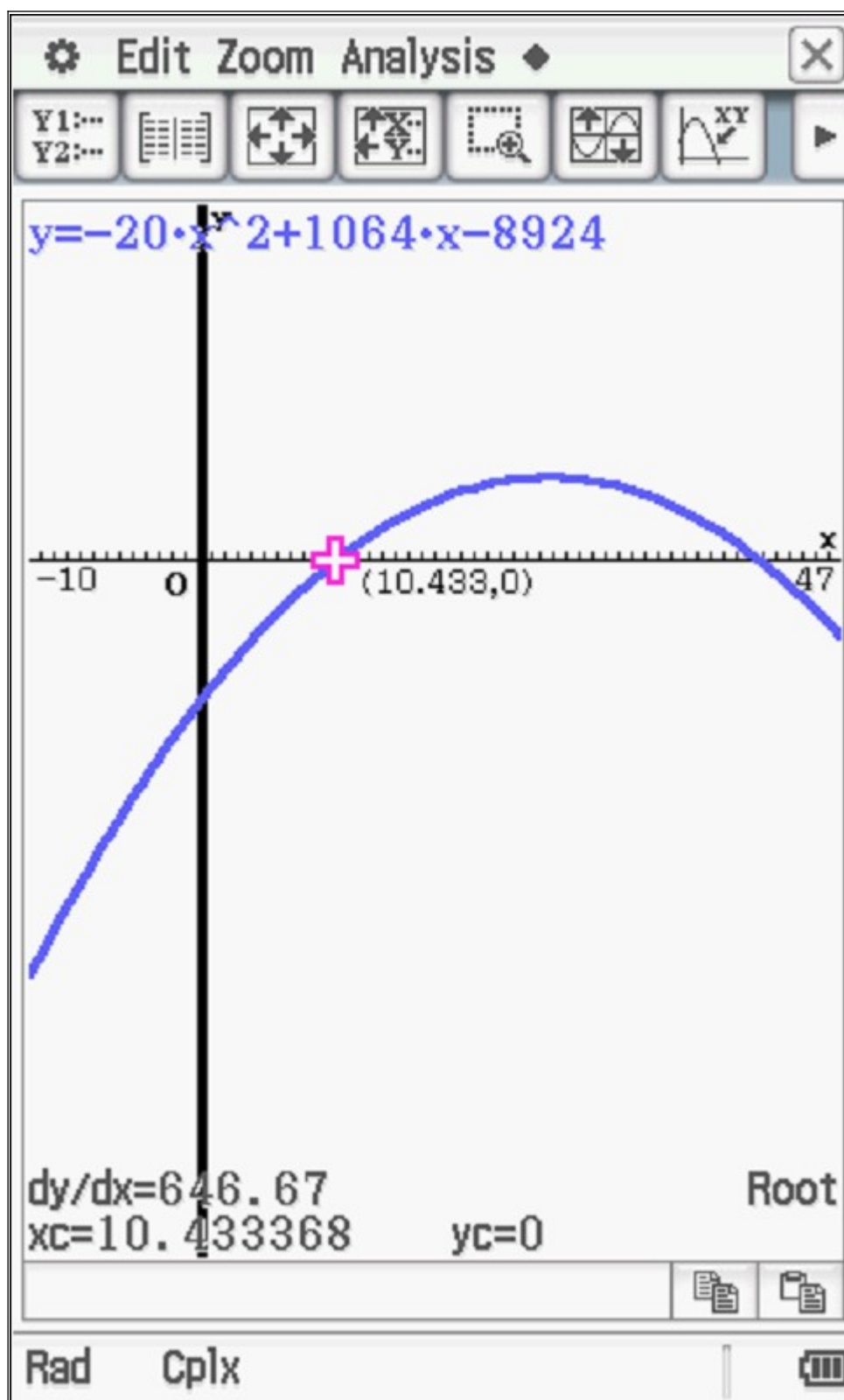
$$\lambda^2 - 6\lambda + 9 + 49\lambda^2 - 14\beta\lambda + \beta^2 + 64 - 32\lambda + 4\lambda^2 = 25$$

$$54\lambda^2 - (38 + 14\beta)\lambda + 48 + \beta^2 = 0$$

$$\Delta = (38 + 14\beta)^2 - 4(54)(48 + \beta^2)$$



The screenshot shows a TI-Nspire calculator interface. At the top, there is a menu bar with 'Edit', 'Action', and 'Interactive' options. Below the menu bar is a toolbar with various mathematical symbols and functions. The main display area shows the expression $\text{expand}((38+14 \cdot x)^2 - 4 \cdot 54 \cdot (48+x^2))$ and the result $-20 \cdot x^2 + 1064 \cdot x - 8924$.



Tangent $\beta = 10.4, 42.8$

Two places $10.4 < \beta < 42.8$

No intersection $\beta < 10.4, \beta > 42.8$

Specific behaviours

- ✓ subs line into sphere equation
 - ✓ sets up equation for λ & β
 - ✓ sets up a quadratic equation for λ in terms of β
 - ✓ determines expression for discriminant in terms of β and solves when $\Delta = 0$
 - ✓ states values for one of the three scenarios with reasoning (no need for rounding)
 - ✓ states values for all three scenarios with **reasoning for each** (no need for rounding)
- NOTE: No follow through if mistake makes problem easier.

Q7 (4 marks) (3.1.4)

The solutions to the complex equation $z^n = k$ are plotted in the complex plane. (n is an integer & k is a complex constant). Exactly **four** of the solutions are plotted in the second quadrant, $\frac{\pi}{2} < \text{Arg}(z) < \pi$, and **no more**. Of these four solutions, the smallest argument is $\frac{7\pi}{12}$. Determine all possible values of n .

Solution	
Consecutive roots arguments separated by	$\frac{2\pi}{n}$
	$\frac{7\pi}{12}$
	$\frac{7\pi}{12} + \frac{2\pi}{n}$
	$\frac{7\pi}{12} + \frac{4\pi}{n}$
	$\frac{7\pi}{12} + \frac{6\pi}{n}$
Four arguments are:	
	$\frac{7\pi}{12} + \frac{6\pi}{n} < \pi$
	$\frac{6\pi}{n} < \frac{5\pi}{12} \quad 72 < 5n \quad n > 14.4$
5th	
	$\frac{7\pi}{12} + \frac{8\pi}{n} > \pi$
	$\frac{8\pi}{n} > \frac{5\pi}{12} \quad 5n < 96 \quad n < 19.2$
values	$15 \leq n \leq 19$

Specific behaviours
$\frac{2\pi}{n}$ ✓ uses additions of $\frac{2\pi}{n}$ ✓ sets up equation for lower boundary for n ✓ sets up equation for upper boundary for n ✓ states all allowed integer values for n