

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material if you have any unauthorised material with you, hand it to the supervisor before reading any further.

### Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate  
Formula sheet (retained from Section One)  
This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time: one hundred minutes  
Reading time before commencing work: ten minutes

Your name \_\_\_\_\_

In words \_\_\_\_\_

Student number: In figures \_\_\_\_\_

Calculator-assumed

Section Two:

UNIT 1

METHODS

MATHEMATICS

# SOLUTIONS

Question/Answer booklet

Semester One Examination, 2018

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					<b>100</b>

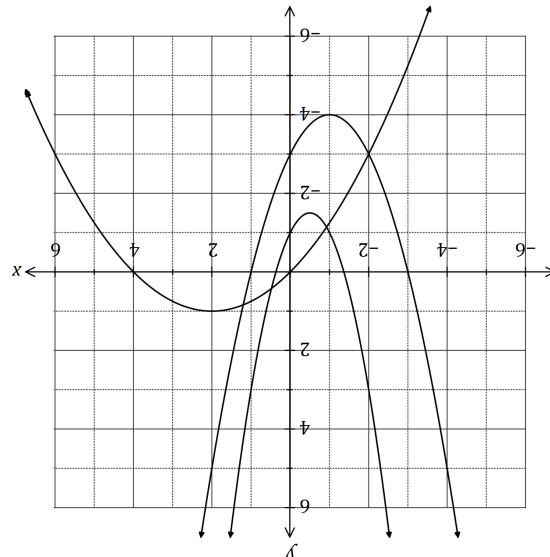
**Instructions to candidates**

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

<b>Specific behaviours</b> <ul style="list-style-type: none"> <li>➤ uses point on inverted parabola</li> <li>➤ value of <math>a</math></li> <li>➤ value of <math>b</math></li> <li>➤ value of <math>c</math></li> </ul>
$c = -1$ ( $y$ -intercept)
$b = -1$ (Other root at $-3$ )
$a = \frac{4}{-1}$ $x = 4 \Rightarrow 0 = a(4 - 2)^2 + 1$

**Solution**

Determine the values of the constants  $a$ ,  $b$  and  $c$ .



The graphs of  $y = 2x^2 + 2x + c$ ,  $y = a(x - 2)^2 + 1$  and  $y = (x + b)(x + 3)$  are shown below.

(4 marks)

**Question 9**

Working time: 100 minutes.

provided.

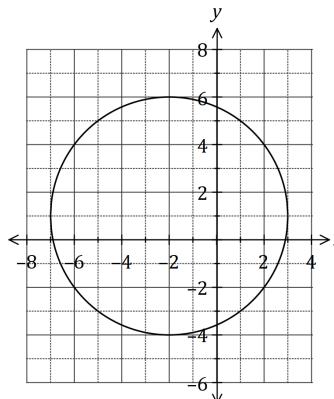
This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

**Section Two: Calculator-assumed**

(7 marks)

**Question 10**

- (a) The graph of a relationship is circular, as shown below.



Determine the equation of this circle in the form  $x^2 + y^2 = a + bx + cy$ , where  $a$ ,  $b$  and  $c$  are constants.

(4 marks)

**Solution**

Centre at  $(-2, 1)$  and  $r=5$

$$(x+2)^2 + (y-1)^2 = 5^2$$

$$x^2 + y^2 = 20 - 4x + 2y$$

**Specific behaviours**

- ✓ indicates centre
- ✓ indicates radius
- ✓ factored form
- ✓ re-arranges as required

- (b) The line  $x+y+1=0$  intersects the circle at the points  $A$  and  $B$ . Show that the line passes through the centre of the circle, and hence determine the distance  $AB$ .

(3 marks)

**Solution**

$$\text{Sub centre } (-2, 1) : -2 + 1 + 1 = 0$$

Hence line passes through centre

$AB$  is a diameter

$$AB = 2 \times 5 = 10$$

**Specific behaviours**

- ✓ correct substitution
- ✓ indicates  $AB$  is diameter
- ✓ correct distance

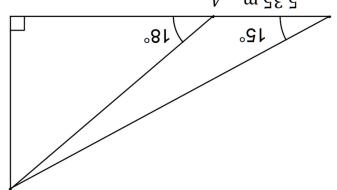
Supplementary page

Question number: \_\_\_\_\_

<b>Solution</b> $\sin 15^\circ = \frac{5.35}{AT} \Leftrightarrow AT = 26.46$ $h = 26.46 \times \sin 18^\circ = 8.18$	<b>Specific behaviours</b> □ angle $BTA$ □ equation using sine rule □ solution for $AT$ □ use of trig in right triangle □ determines $h$
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(5 marks)

(b) Showing use of trigonometry, determine the height of the post.

<b>Solution</b> 	<b>Specific behaviours</b> □ sketch with right-angle, two given angles and distance $AB$
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(1 mark)

(a) Draw a sketch to represent this information.

A thin pole stands vertically in the middle of a level playing ground. From point  $A$  on the ground, the angle of elevation to the top of the pole is  $18^\circ$ . From point  $B$ , also on the ground but  $5.35$  metres further from the foot of the pole than  $A$ , the angle of elevation to the top of the pole is  $15^\circ$ .

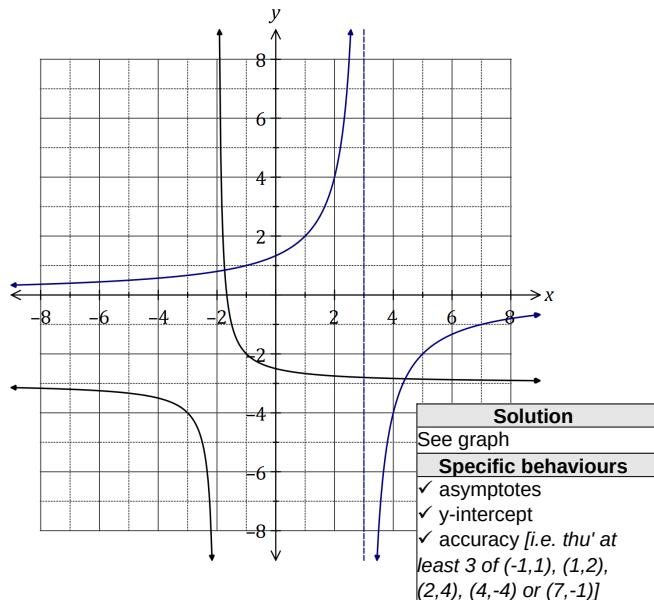
(6 marks)

From point  $T$ , which is  $5.35$  metres from point  $A$ , the angle of elevation to the top of the pole,  $T$ , is  $18^\circ$ . A thin pole stands vertically in the middle of a level playing ground. From point  $A$  on the ground, the angle of elevation to the top of the pole,  $T$ , is  $18^\circ$ .

(7 marks)

Let  $f(x) = \frac{4}{3-x}$  and  $g(x) = \frac{1}{x+p} + q$ , where  $p$  and  $q$  are constants.

The graph of  $y=g(x)$  is shown below.



- (a) Sketch the graph of  $y=f(x)$  on the axes above. (3 marks)

- (b) Determine the values of  $p$  and  $q$ . (2 marks)

<b>Solution</b>
$p=2, q=-3$
<b>Specific behaviours</b>
✓ value of $p$ ✓ value of $q$

- (c) Solve the equation  $f(x)=g(x)$ , giving your solution(s) to one decimal place. (2 marks)

<b>Solution</b>
$x=-1.7, x=4.4$
<b>Specific behaviours</b>
✓ one solution ✓ second solution <i>(Rounding for guidance only but penalise answers given as coordinates)</i>

(6 marks)

(2 marks)

(2 marks)

(4 marks)

(2 marks)

(2 marks)

(2 marks)

(2 marks)

**Question 21**

(8 marks)

(2 marks)

- ✓ states a valid reason
- ✓ indicates that  $Q$  is a function and that  $Q$  is not

**Specific behaviours**

$Q$  is a function,  $Q$  is not as it does not satisfy the vertical line test when graphed (or it has multiple  $y$ -values for some  $x$ -values i.e.  $(2,3)$  and  $(2,4)$  etc.)

Justify your comment.

Comment on whether either of  $Q$  or  $Q'$  would qualify to be called a function?

(c) Justify your comment.

(ii) State the Range of  $Q$ .

**Specific behaviours**
 $\{0,1,2,\dots,10\}$ 

(i) List the elements of  $Q$ .

**Specific behaviours**
 $\{0,0,(0,1),(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,8),(8,9)\}$ 

For  $K = \{0,1,2,\dots,10\}$ ,  $Q = \{(b,a) : a \in K, b = \text{number of primes that do not exceed } a\}$

If the set is redefined as follows:

(b) List the elements of  $Q$ .

**Specific behaviours**
 $\{0,1,2,3\}$ 

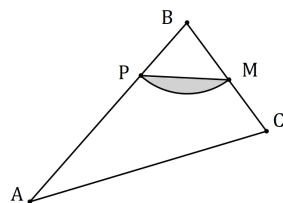
(i) Solve simultaneously using CAS

$a = -2, b = 6$

## Question 14

(10 marks)

A logo with triangular outline  $ABC$  contains a shaded segment bounded by the straight line  $PM$  and the circular arc  $PM$  with centre  $B$  and radius  $BM = 18$  cm, as shown below.



Given that  $\angle ABC = \frac{5\pi}{12}$ ,  $\angle BCA = 2\angle BAC$  and  $M$  is the midpoint of  $BC$ , determine

- (a) the size of  $\angle ABC$  in degrees. (1 mark)

Solution	
$\frac{5\pi}{12} \times \frac{180}{\pi} = 75^\circ$	
Specific behaviours	
✓ converts angle	

- (b) the area of the shaded segment. (2 marks)

Solution	
$A = \frac{1}{2}(18)^2 \left( \frac{5\pi}{12} - \sin\left(\frac{5\pi}{12}\right) \right) \approx 55.6 \text{ cm}^2$	
Specific behaviours	
✓ indicates substitution into segment area formula ✓ evaluates area	

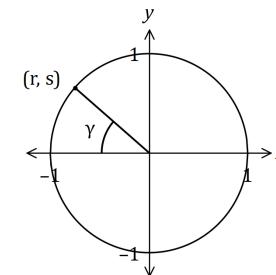
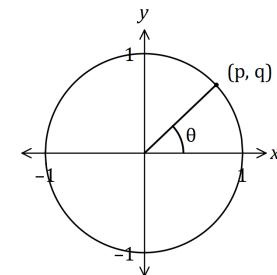
- (c) the perimeter of the shaded segment. (3 marks)

Solution	
$P M_{arc} = 18 \times \frac{5\pi}{12} = \frac{15\pi}{2} \approx 23.56$	
$b = \sqrt{18^2 + 18^2 - 2[18][18]\cos 75} \approx 21.92$	
Perimeter	$= 23.56 + 21.92 \approx 45.5 \text{ cm}$
Specific behaviours	
✓ calculates arc length ✓ indicates use of cosine rule to find $PM$ ✓ evaluates $PM$ and states perimeter	

## Question 20

(7 marks)

Consider the points with coordinates  $(p, q)$  and  $(r, s)$  that lie in the first and second quadrants respectively of the unit circles shown below, where  $\theta$  and  $\gamma$  are acute angles.



Determine the following in terms of  $p, q, r$  and  $s$ , simplifying your answers where possible.

- (a)  $\tan \theta$ . (1 mark)

Solutions	
(i)	$\frac{q}{p}$
(ii)	$q$
(iii)	$-r$
(iv)	$-s$
Specific behaviours	
✓ each correct response	

- (b)  $\sin(180^\circ - \theta)$ . (1 mark)

- (c)  $\cos \gamma$ . (2 mark)

- (d)  $\sin(\pi + \gamma)$ . (1 mark)

- (e)  $\tan(-\gamma)$ . (2 marks)

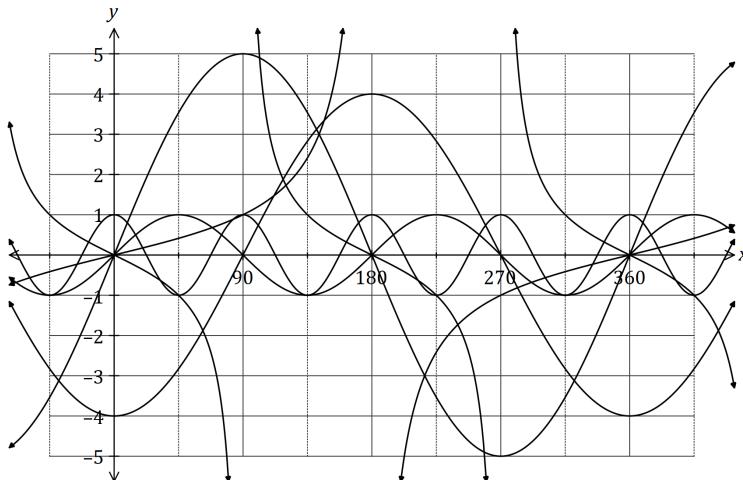
Solution	
$\tan(-\gamma) = \frac{-s}{-r}$	
Specific behaviours	
✓ ✓ correct expression	



## Question 15

- (a) The graphs of the following, where  $a, b, c, d, e$  and  $f$  are constants, are shown below.

$$y = \sin(ax), y = b \cos(x), y = \tan(cx), y = d \sin(x), y = \cos(ex), y = f \tan(x)$$



State the values of  $a, b, c, d, e$  and  $f$ .

(6 marks)

Solution	
See table	
<b>Specific behaviours</b>	

✓ each value

Constant	Value
$a$	2
$b$	-4
$c$	0.5
$d$	5
$e$	4
$f$	-1

- (b) Calculate the acute angle in degrees between the lines  $y = x + 5$  and  $y = 3x - 1$ , rounding your answer to one decimal place.

(3 marks)

Solution	
$\alpha = \tan^{-1}(1) = 45^\circ$	
$\beta = \tan^{-1}(3) = 71.565^\circ$	
$\beta - \alpha = 26.6^\circ$ (1 dp)	
<b>Specific behaviours</b>	

✓ angle of inclination of first line  
✓ angle of inclination of second line  
✓ acute angle, to one decimal place

See next page

SN002-112-4

## Specific behaviours

- ✓ states equation of new upper curve and states equation of new lower curve

- (b) Determine the equation

- (i) of the line passing through the point  $(2, 3)$  and parallel to the line with equation  $3x + 2y + 7 = 0$

## Solution

$$3x + 2y + 7 = 0 \text{ has gradient } -\frac{3}{2}$$

$$\therefore y = -\frac{3}{2}x + c$$

$$\text{using } (2, 3); 3 = -3 + c \Rightarrow c = 6$$

$$\therefore y = -\frac{3}{2}x + 6$$

## Specific behaviours

- ✓ calculates gradient
- ✓ uses the given point to calculate  $c$  and states equation

- (ii) of the line which is the reflection of the line  $2y - 3x - 4 = 0$  in the  $y$ -axis

## Solution

$$2y - 3x - 4 = 0 \Rightarrow y = \frac{3}{2}x + 2$$

$$\therefore \text{gradient of reflected line is } -\frac{3}{2} \Rightarrow \text{line is } y = -\frac{3}{2}x + 2$$

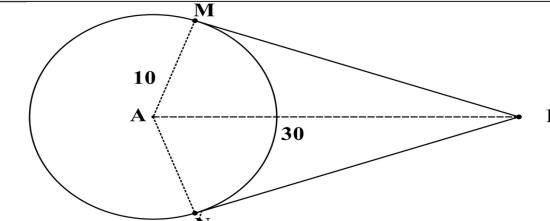
Or note the  $x$  is replaced with  $-x$ .  $\Rightarrow$  line is  $2y + 3x - 4 = 0$

## Specific behaviours

- ✓ calculates gradient
- ✓ uses the given point to calculate  $c$  and states equation

- (c) Consider a rope fixed at B and tightly wrapped around a disc A as shown in the diagram below.

## Solution



$$\Delta AMB \text{ has a rt angle at } M \Rightarrow BM = \sqrt{30^2 - 10^2} = 20\sqrt{2} \approx 28.284$$

$$\angle MAN = 2 \left( \operatorname{inv}(\cos(\frac{1}{3})) \right) = 2.462$$

$$\therefore \text{length of major arc MN} = (2\pi - 2.462) \times 10 = 38.213$$

$$\text{Hence the length of the rope } \approx 2 \times 28.284 + 38.213 \approx 94.78 \text{ cm}$$

## Specific behaviours

- ✓ calculates the straight lengths MB and MN
- ✓ calculates the angle MAN
- ✓ calculates the reflex angle MAN
- ✓ calculates the length of the major arc

See next page

SN002-112-4



## Question 17

(9 marks)

The wind speed at a weather station,  $v$  metres per second,  $t$  hours after recording began, can be modelled by the function

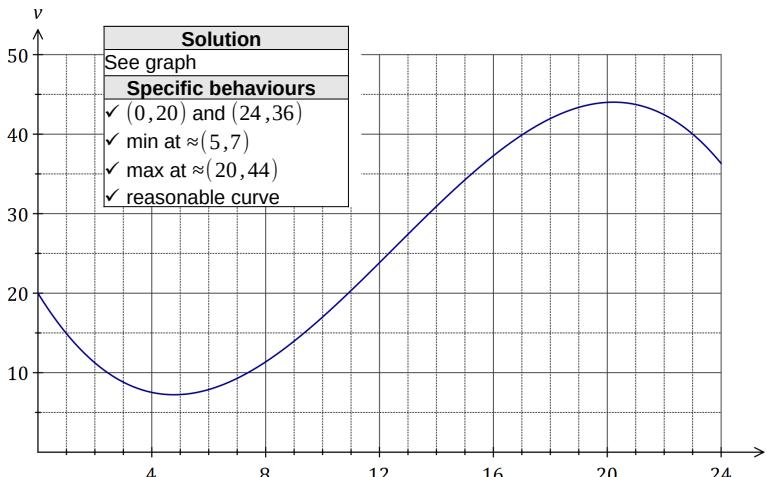
$$v=20-5.8t+0.75t^2-0.02t^3, 0 \leq t \leq 24$$

- (a) Calculate the wind speed when  $t=11$ .

Solution
$v(11)=20.33 \text{ m/s}$
<b>Specific behaviours</b>

(1 mark)

- (b) Sketch the graph of wind speed against time on the axes below.



(4 marks)

- (c) During the 24-hour period, determine

- (i) the time at which the wind speed was greatest.

(1 mark)

Solution
$t=20.2 \text{ h}$
<b>Specific behaviours</b>

Solution
$v_{\text{MIN}} \approx 7.23 \text{ m/s}$
<b>Specific behaviours</b>

- (ii) the minimum wind speed.

(1 mark)

- (iii) the length of time, in hours and minutes, that the wind speed was increasing.

(2 marks)

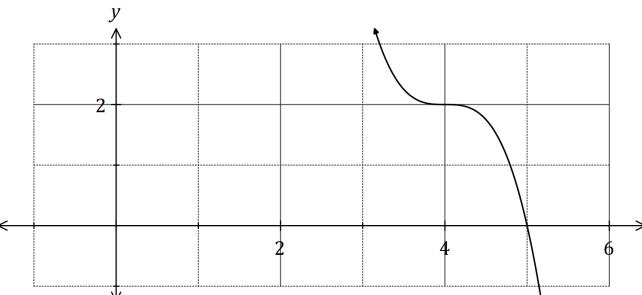
Solution
$20.219 - 4.781 = 15.438$
$\textcolor{red}{\cancel{15}} \text{ } 15 \text{ h } 26 \text{ min}$
<b>Specific behaviours</b>

See next page

## Question 18

(6 marks)

- (a) Part of the graph of  $y=f(x)$  is shown below, where  $f(x)=-2(x-b)^3+c$ , and  $b$  and  $c$  are constants.



- (i) State the degree of  $f(x)$ .

Solution
3
<b>Specific behaviours</b>

(1 mark)

- (ii) Determine the value of  $b$ .

Solution
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(1 mark)

- (iii) Determine  $f(0)$ .

Solution
$f(x)=-2(x-4)^3+2$
$f(0)=-2(-4)^3+2=130$
<b>Specific behaviours</b>

(2 marks)

- (b) Another function is given by  $g(x)=f(x+8)$ .

Describe how to obtain the graph of  $y=g(x)$  from the graph of  $y=f(x)$ .

Solution
Translate graph 8 units to the left.
<b>Specific behaviours</b>

(2 marks)