



PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester two Examination,  
2019

Question/Answer booklet

## MATHEMATICS SPECIALIST

### UNITs 3 & 4

Section One:

Calculator-free

Your Name \_\_\_\_\_

Your Teacher's Name \_\_\_\_\_

#### Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

#### Materials required/recommended for this section

*To be provided by the supervisor*

This Question/Answer booklet

Formula sheet

*To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

#### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Max	Question	Mark	Max
1		6	5		9
2		6	6		7
3		12	7		5
4		6			

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	34
Section Two: Calculator-assumed	13	13	100	100	66
Total					100

## Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

**Section One: Calculator-free****(51 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

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**Question 1****(6 marks)**

Let  $f(x) = \sqrt{3-2x}$  and  $g(x) = \frac{1}{x-2}$ .

Determine:

a) the natural domain and range of  $f$ . (2 marks)

b)  $g \circ f(x)$  (1 mark)

c) the natural domain and range of  $g \circ f(x)$  (3 marks)

**Question 2****(6 marks)**

Consider the following system of linear equations.

$$2x + 5y - 4z = 7$$

$$x + 2y - 3z = 1$$

$$5x - 3y + 2z = -17$$

a) Solve for  $x, y$  &  $z$ .

(3marks)

$$2x + 5y + pz = 7$$

$$x + 2y - 3z = 1$$

b) For the following system  $5x - 3y + 2z = q$  where  $p$  &  $q$  are constants, determine

the possible values of  $p$  &  $q$  for i) no solutions

ii) infinite solutions.

(3 marks)

**Question 3****(12 marks)**

Determine the following integrals.

a)  $\int (x+2)\sqrt{x-3} \, dx$  using  $u = x - 3$

**(3 marks)**

b)  $\int 3\sin^2(2x) \, dx$

**(3 marks)**

c)  $\int \cos^3(5x) \, dx$

**(3 marks)**

d)  $\int \frac{1}{\cos^2(3x)} \, dx$

**(3 marks)**

**Question 4****(6 marks)**

$$\frac{-x^2 + 10x + 23}{(x-1)(x+3)^2}$$

- a) Express the following expression into partial fractions (3 marks)

$$\int_3^4 \frac{-x^2 + 10x + 23}{(x-1)(x+3)^2} dx$$

- b) Hence evaluate (Do not simplify) (3 marks)

**Question 5****(9 marks)**

Consider a plane that contains the following points,  $A(-1, 2, 4)$ ,  $B(0, 1, -3)$  &  $C(5, -2, 3)$ .

a) Determine a vector equation for all points in the plane above. (3 marks)

b) Determine the distance of point  $D(-7, 1, 2)$  from the plane in part a. (3 marks)  
(Do **not** simplify)

c) Determine the cosine of the angle that the line  $r = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$  makes with the plane in  $2x - y + z = 3$ . (3 marks)

**Question 6****(7 marks)**

Consider the logistical model defined by  $\frac{dN}{dt} = aN - bN^2$  where  $a$  &  $b$  are positive constants.

- a) Determine the value of  $N$ ,  $N > 0$ , where  $\frac{dN}{dt} = 0$ . Explain the significance of this value. (2 marks)

- b) Derive the logistical formula  $N = \frac{a}{b + Ce^{-at}}$  showing steps in your working. (5 marks)



**Question 7****(5 marks)**

Evaluate the following integral  $\int_0^{\frac{\pi}{2}} \cos^4(2x) dx$  showing all working. (Simplify)

**End of section one**

## Acknowledgements