PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course ________Specialist_______ Year __12______

Student name: ______________ Teacher name: ______________

Date: 17 June Weds p3 (Advo)

Task type: **Response**

Time allowed for this task: **_____45______ mins**

Number of questions: **_____7______**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **__42____ marks**

Task weighting: **_12___%**

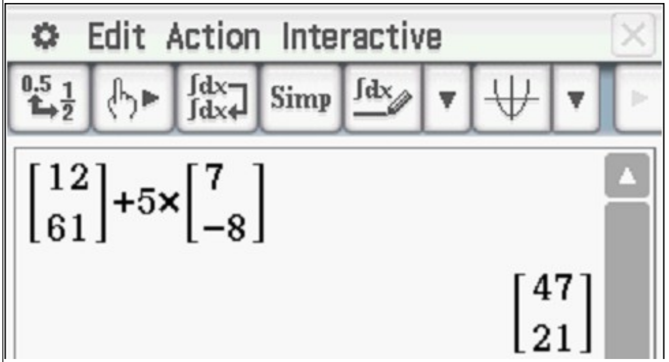
Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3.3.5- 3.3.6) **(2 & 3 = 5 marks)**

Consider a car A that has an initial position vector $\begin{pmatrix} 12 \\ 61 \end{pmatrix}$ km and moving with a constant velocity of $\begin{pmatrix} 7 \\ -8 \end{pmatrix}$ km/h.

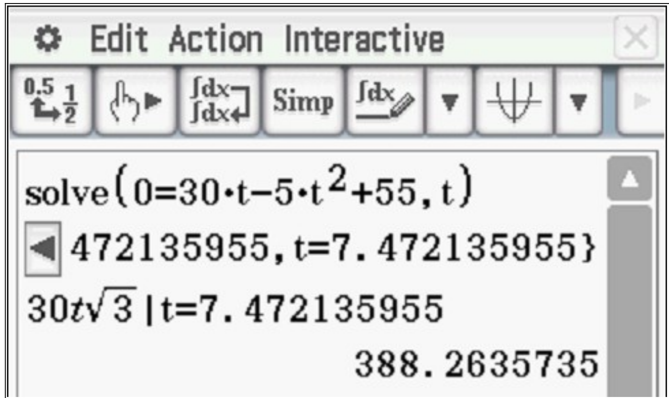
(a) Determine the position vector in 5 hours from now.

Solution

Specific behaviours
✓ multiplies velocity by time ✓ states position vector

Consider a second car B that has an initial position $\begin{pmatrix} 57 \\ -29 \end{pmatrix}$ km and a constant velocity of $\begin{pmatrix} -2 \\ 10 \end{pmatrix}$ km/h.

(b) Determine if the two cars collide and if they do the position vector of this point of collision and the time it occurs.

End of test

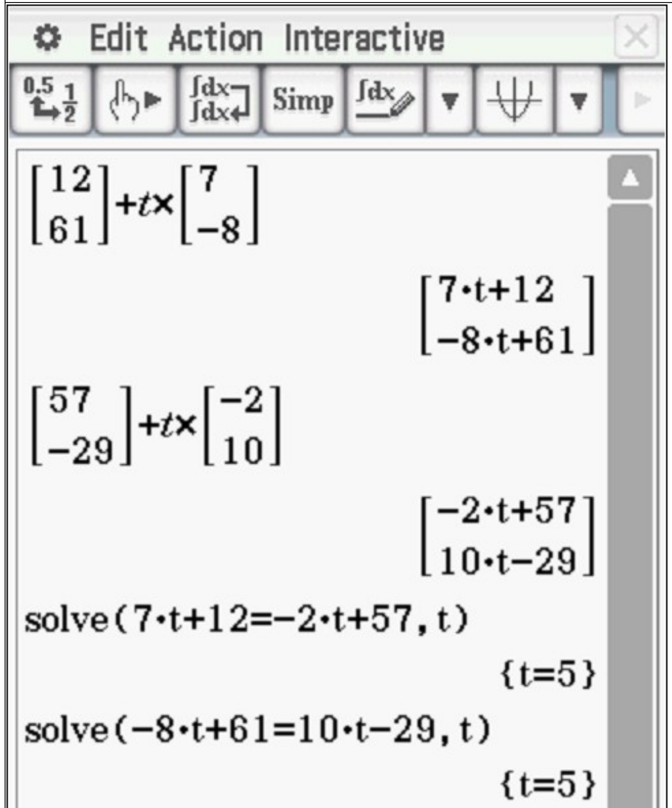


Specific behaviours

✓ equates y parametric equation to zero
✓ solves for time to one decimal place
✓ states approx. horizontal distance

Mathematics Department Perth Modern

Solution



Collide at (47,21) Km

Specific behaviours

✓ obtains expression for position vectors in terms of time
✓ solves for i components
✓ solves for j component and states pt of intersection

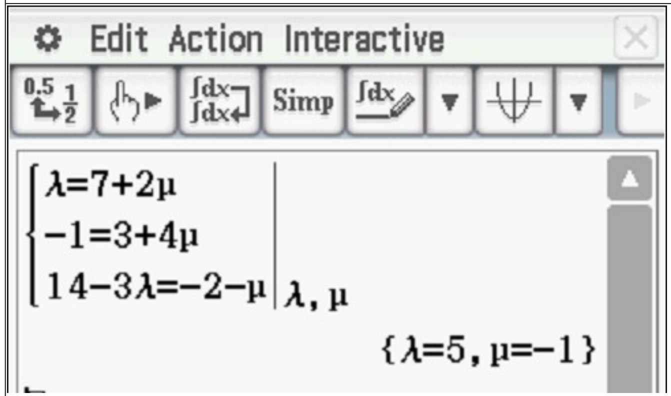
Q2 (3.3.1, 3.3.3) (3 & 2 = 5 marks)

Consider the two lines

$$L_1 : r = \begin{pmatrix} 0 \\ -1 \\ 14 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} \qquad L_2 : r = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$$

(a) Determine the point of intersection, if any.

Solution



Intersect at (5,-1,-1)

Specific behaviours

✓ uses two parameters
✓ sets up three simultaneous equations
✓ states pt of intersection

(b) Determine to the nearest degree the acute angle between the two lines.
(Consider the plane that contains both lines)

Solution

$\ddot{r} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$

$\dot{r} = \begin{pmatrix} 0 \\ -10t \end{pmatrix} + \underset{\sim}{c}$

$\begin{pmatrix} 60\cos 30 \\ 60\sin 30 \end{pmatrix} = \underset{\sim}{c} = \begin{pmatrix} 30\sqrt{3} \\ 30 \end{pmatrix}$

$\dot{r} = \begin{pmatrix} 30\sqrt{3} \\ 30 - 10t \end{pmatrix}$

$r = \begin{pmatrix} 30t\sqrt{3} \\ 30t - 5t^2 \end{pmatrix} + \underset{\sim}{w}$

$\underset{\sim}{w} = \begin{pmatrix} 0 \\ 55 \end{pmatrix}$

$r = \begin{pmatrix} 30t\sqrt{3} \\ 30t - 5t^2 + 55 \end{pmatrix}$

$x = 30t\sqrt{3} \quad t = \dfrac{x}{30\sqrt{3}}$

$y = 30t - 5t^2 + 55 = \dfrac{x}{\sqrt{3}} - \dfrac{5x^2}{2700} + 55$

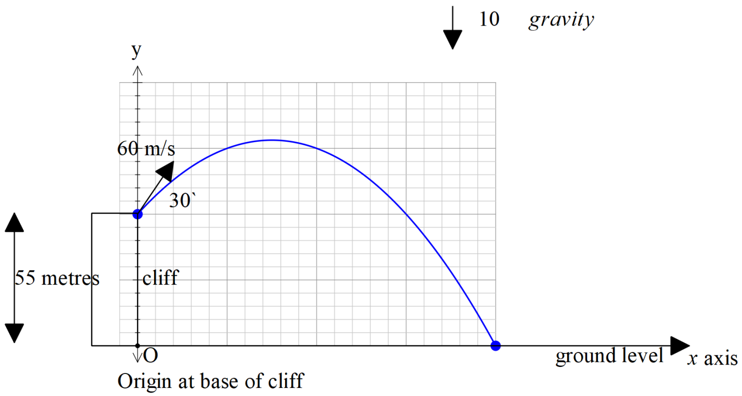
Specific behaviours

✓ integrates acceleration and solves for vector constant
✓ integrates velocity and solves for vector constant
✓ obtains expression for t in terms of x
✓ obtains exact cartesian equation

(b) Determine the time, one decimal place, taken to hit the ground and the horizontal distance of this point from the base of the cliff.

Solution

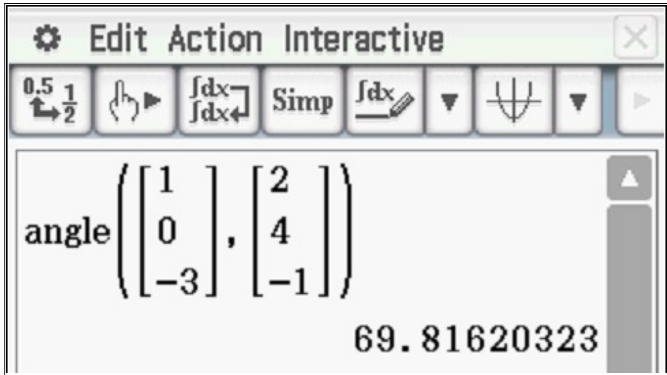
Q7 (3.3.15) **(4 & 3 = 7 marks)**



Consider a football that is kicked off the top of a cliff of height 55 metres with an initial speed of 60 m/s at an angle of 30° with the horizontal. The acceleration due to gravity is $-10m/s^2$.

(a) Show using **vector integration** how to determine the exact cartesian equation of the path using the base of the cliff as the origin.

Solution



Specific behaviours
✓ uses vectors parallel to lines ✓ states acute angle between lines

Q3 (3.3.8) **(2, 3 & 3 = 8 marks)**

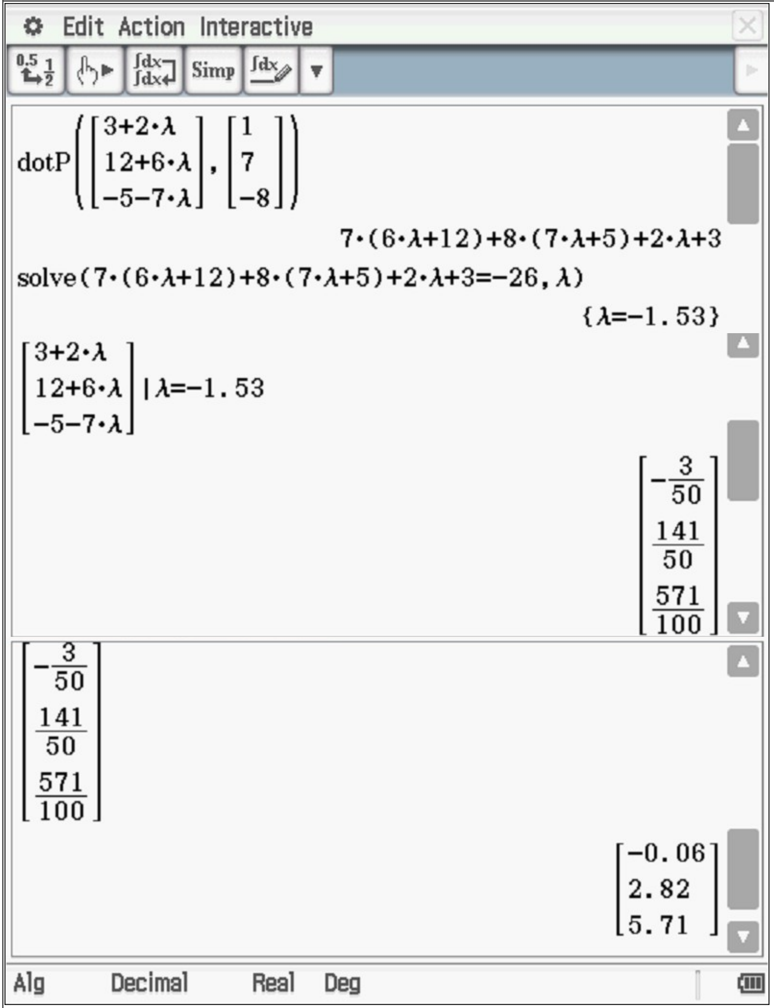
Consider a plane that contains the point $(5,-1,3)$ and has a normal vector $\begin{pmatrix}1\\7\\-8\end{pmatrix}$.

(a) Determine the vector equation of the plane.

Solution
$r.\begin{pmatrix}1\\7\\-8\end{pmatrix}=\begin{pmatrix}5\\-1\\3\end{pmatrix}.\begin{pmatrix}1\\7\\-8\end{pmatrix}=-26$ $r.\begin{pmatrix}1\\7\\-8\end{pmatrix}=-26$
Specific behaviours
✓ uses dot product with normal ✓ right hand side correct scalar

(b) Determine the point of intersection of the line $r = \begin{pmatrix} 3 \\ 12 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 6 \\ -7 \end{pmatrix}$ with the plane above.

Solution



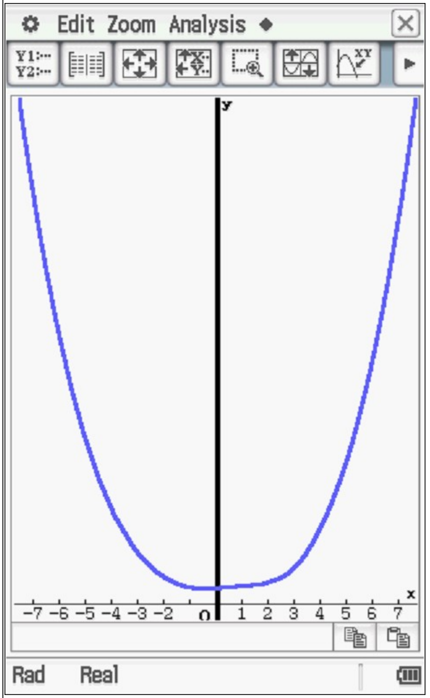
Specific behaviours

✓ subs line into plane equation

Solution

$$\text{norm}\left(\begin{bmatrix}3x^2\\3+x\\x^3-2x\end{bmatrix}-\begin{bmatrix}-5\\7\\13\end{bmatrix}\right)$$

$$\left(x^6+5\cdot x^4-26\cdot x^3+35\cdot x^2+44\cdot x+210\right)^{0.5}$$



Min distance at t=zero seconds

Specific behaviours

✓ graphs or uses fmin of expression for distance apart from part a
✓ states t=0 & 14.5 km

✓ uses dot product and solves for parameter
✓ states pt of intersection, allow approx. decimal

(c) Determine the distance of point $(11,-3,6)$ from the plane above.

Solution

Choose any point on plane (0,0,26/8)

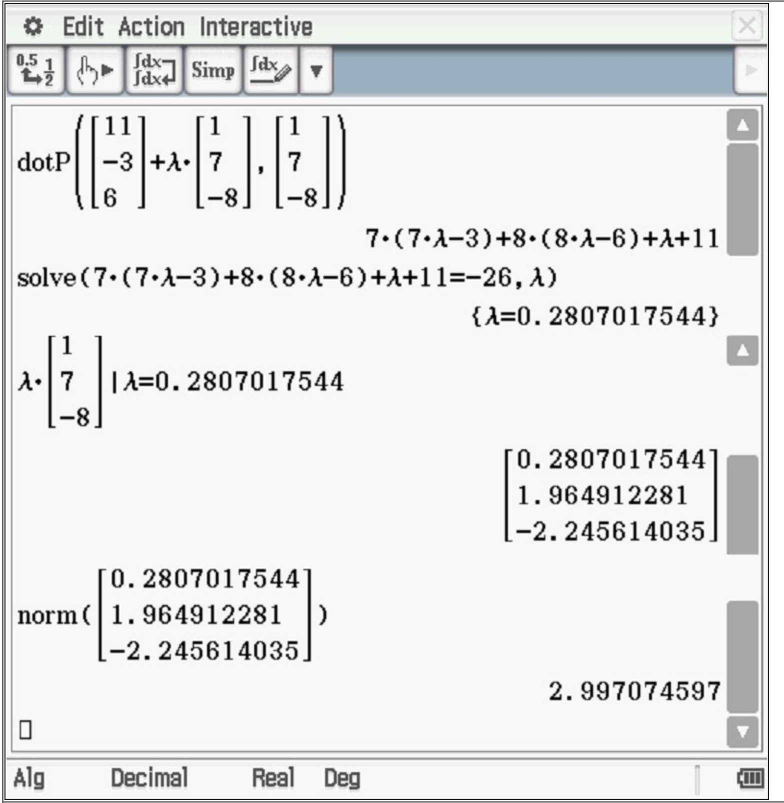
$$\text{dotP}\left(\begin{bmatrix}0\\0\\\frac{26}{8}\end{bmatrix}-\begin{bmatrix}11\\-3\\6\end{bmatrix},\begin{bmatrix}1\\7\\-8\end{bmatrix}\cdot\frac{1}{\sqrt{1^2+7^2+8^2}}\right)$$

$$2.997074597$$

$$2.997074597$$

$$\frac{17368344}{5795099}$$

OR



Specific behaviours
✓ chooses any point on plane OR line parallel to normal through point ✓ uses dot product ✓ states approx distance

Q4 (3.3.9-3.3.10) **(3 & 3 = 6 marks)**

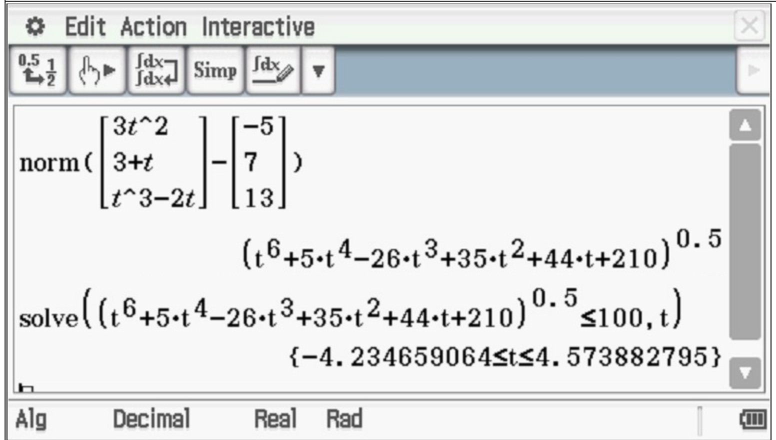
(a) Solve the following system of linear equations. Working must be shown.

$$\begin{aligned} 3x - 5y + 7z &= 43 \\ x + 2y + 3z &= 9 \\ 2x - 3y + 2z &= 20 \end{aligned}$$

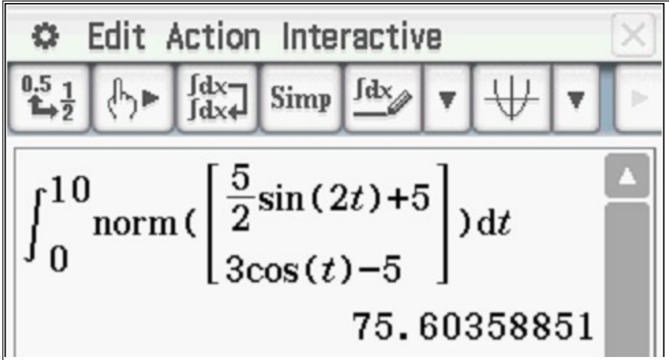
Q6 (3.3.15) **(3 & 2 = 5 marks)**

Consider an aircraft with position vector $r = \begin{pmatrix} 3t^2 \\ 3+t \\ t^3 - 2t \end{pmatrix} km$ at time t hours. At the top of a building stands an antenna with the position vector of the highest point being $r = \begin{pmatrix} -5 \\ 7 \\ 13 \end{pmatrix} km$.

(a) Determine the times the aircraft is less than 100 km from the top of the antenna.

Solution
 **Time between zero and 4.57 seconds**
Specific behaviours
✓ uses vector subtraction ✓ determines expression for distance apart ✓ solves for less than 10 km and states non negative values of time

(b) Determine the closest approach of the aircraft and the time it occurs.



Distance =75.6 metres

Specific behaviours
✓ uses magnitude of velocity(shown) ✓ states integral ✓ states distance to one decimal place

Solution
$\begin{bmatrix} 1 & 2 & 3 & 9 \\ 2 & -3 & 2 & 20 \\ 3 & -5 & 7 & 43 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 7 & 4 & -2 \\ 0 & 11 & 2 & -16 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 7 & 4 & -4 \\ 0 & -15 & 0 & 30 \end{bmatrix}$ $y=-2$ $-14+4z=-2$ $z=3$ $x-4+9=9$ $x=4$ $(4,-2,3)$

Specific behaviours
✓ eliminates one variable from two equations ✓ eliminates two variables from one equation ✓ solves for all three variables

Consider the constants $p \& q$ in the system below.

$$3x-5y+7z=p$$
$$x+2y+qz=9$$
$$2x-3y+2z=20$$

(b) Determine all the value(s) of $p \& q$ such that:

(i) There will be an unique solution

(ii) There will be infinite solutions

(iii) There will be no solutions

`

Solution
$\begin{bmatrix} 1 & 2 & q & 9 \\ 2 & -3 & 2 & 20 \\ 3 & -5 & 7 & p \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & q & 9 \\ 0 & 7 & 2q-2 & -2 \\ 0 & 11 & 3q-7 & 18-p \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & q & 9 \\ 0 & 7 & 2q-2 & -2 \\ 0 & 0 & 27+q & 7p-211 \end{bmatrix}$ $i)q \neq -27$ $ii)q=-27 \ \& \ p=\frac{211}{7}$ $iii)q=-27 \ \& \ p \neq \frac{211}{7}$
Specific behaviours
✓ obtains row with two variables eliminated ✓ determines values for infinite solns ✓ determines values for unique and no solution

Q5 (3.3.11 – 3.3.15) **(3 & 3 = 6 marks)**

Consider an object moving with acceleration $\ddot{r} = \begin{pmatrix} 5\cos(2t) \\ -3\sin t \end{pmatrix} m/s^2$ at time t seconds. The initial velocity is $\begin{pmatrix} 5 \\ -2 \end{pmatrix} m/s$ and initial displacement $\begin{pmatrix} -7 \\ 5 \end{pmatrix} m$.

(a) Determine the position vector at time t seconds.

Solution
$\ddot{r} = \begin{pmatrix} 5\cos(2t) \\ -3\sin t \end{pmatrix}$ $\dot{r} = \begin{pmatrix} \frac{5}{2}\sin(2t) \\ 3\cos t \end{pmatrix} + \underset{\sim}{c}$ $\begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \underset{\sim}{c} \quad \underset{\sim}{c} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$ $\dot{r} = \begin{pmatrix} \frac{5}{2}\sin(2t)+5 \\ 3\cos t - 5 \end{pmatrix}$ $r = \begin{pmatrix} \frac{-5}{4}\cos(2t)+5t \\ 3\sin t - 5t \end{pmatrix} + \underset{\sim}{w}$ $\begin{pmatrix} -7 \\ 5 \end{pmatrix} = \begin{pmatrix} \frac{-5}{4} \\ 0 \end{pmatrix} + \underset{\sim}{w} \quad \underset{\sim}{w} = \begin{pmatrix} \frac{-23}{4} \\ 5 \end{pmatrix}$ $r = \begin{pmatrix} \frac{-5}{4}\cos(2t)+5t-\frac{23}{4} \\ 3\sin t - 5t + 5 \end{pmatrix}$
Specific behaviours
✓ integrates to find velocity with a vector constant ✓ integrates to find position with a vector constant ✓ solves correctly for both vector constants

(b) Determine the distance travelled in the first 10 seconds.(One decimal place)

Solution