

it to the supervisor **before** reading any further.
 You do not have any unauthorised material. If you have any unauthorised material with you, hand
 over other items may be taken into the examination room. It is **your** responsibility to ensure that

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Important note to Candidates

Special items:
 drawing instruments, templates, notes on two unruled sheets of A4 paper,

Standard items:
 correction fluid/tape, eraser, ruler, highlighters,
 pens (blue/black preferred), pencils (including coloured), sharpener,

To be provided by the candidate

Formula sheet (referred from Section One)

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time:
 Reading time before commencing work: ten minutes

Working time:
 one hundred minutes

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Your name
 in words

Student number:
 in figures

Calculator-assumed

Section Two:

MATHEMATICS **UNIT 3** **METHODS**

If required by your examination administrator, please
 place your student identification label in this box

Question/Answer booklet

Semester One Examination, 2019

COLLEGE
ALL SAINTS'



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|------------------|----------|--------------|--------------------|
| Markers use only | Question | Maximum mark | Score |
| 10 | 7 | 5 | 9 |
| 11 | 8 | 5 | 12 |
| 12 | 8 | 5 | 13 |
| 13 | 5 | 7 | 14 |
| 14 | 7 | 7 | 15 |
| 15 | 7 | 12 | 16 |
| 16 | 9 | 9 | 17 |
| 17 | 9 | 7 | 18 |
| 18 | 9 | 7 | 19 |
| 19 | 7 | 7 | 20 |
| 20 | 7 | 7 | 21 |
| 21 | 7 | 7 | 2 Total |
| 2 Total | 98 | 65% | S2 Wt (x0.6633) |

Student Name: _____

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|------------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|---------------------------|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
| Total | | | | | 100 |

Supplementary page

Question number: _____

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

METHODS UNIT 3

3

CALCULATOR-ASSUMED

65% (98 Marks)

SECTION 1 W8: CALCULATOR-ASSUMED

CALCULATOR-ASSUMED

87

Section 1 (MCQs): Calculations-assumed
63% (98 marks)

Quarantine TZ
(7 marks)

This section has **thirteen** (13) questions. Answer all questions. Write your answers in the spaces provided.

(a) Given that $f(t) = \sin\left(4t + \frac{\pi}{4}\right)$ and $F(x) = \int_x^0 f(t) dt$, determine the exact value of

Useful flows into a storage tank that is initially empty at a rate of $\sqrt{9+4t}$ litres per minute, where t is time in minutes and $0 \leq t \leq 110$.
Question 9 (5 marks)

Determine how much fuel is in the tank after 18 minutes. (2 marks)

• $\left(\frac{8}{\pi}\right) F$ (ii)

Question 10

X is a uniform discrete random variable where $x=1,2,3,4,6,8,11$.

(a) Determine

(i) $P(X \geq 3)$.

(1 mark)

Question 20

An aquarium, with a volume of $80\,000 \text{ cm}^3$, takes the shape of a rectangular prism with square ends of side $x \text{ cm}$ and no top. The glass for the base costs 0.05 cents per square cm and for the four vertical sides costs 0.08 cents per square cm. The cost of glue to join the edges of two adjacent pieces of glass is 0.6 cents per cm. Assume the glass has negligible thickness and ignore any other costs.

(a) Show that $C = \frac{x^2}{625} + \frac{9x}{250} + \frac{168}{x^2} + \frac{960}{x^2}$, where C is the cost, in dollars, to make the aquarium.
(4 marks)

(ii) $P|X > 2|X \leq 8$.

(2 marks)

(b) Calculate the exact value of

(i) $E(X)$.

(2 marks)

(b) Show use of a calculus method to determine the minimum cost of making the aquarium.
(3 marks)

(ii) $\text{Var}(X)$.

(2 marks)

Question 19**Question 11****(8 marks)**

The potential difference, V , volts, across the terminals of an electrical capacitor t seconds after it begins to discharge through a resistor can be modelled by the equation

$$V = V_0 e^{-kt}$$

(2 marks)**(3 marks)**(a) Determine an equation for $x(t)$, the displacement of the body after t seconds. (3 marks)

$$v(t) = 20 \sin\left(\frac{\pi t}{18}\right) \text{ cm/s}$$

After t seconds is given byA small body has displacement $x=0$ when $t=0$ and moves along the x -axis so that its velocity**(7 marks)**(b) Describe, with justification, how the speed of the body is changing when $t=30$. (4 marks)

- (i) the time taken for the potential difference to drop from 17.5 to 12.5 volts. (3 marks)
- (ii) the time taken for the potential difference to change from 12.5 to 20 volts. (3 marks)

- (iii) the rate of change of V when the potential difference is 20 volts. (1 mark)

(b) Another capacitor takes 66 seconds for its maximum potential difference to halve. It is instantaneously recharged to its maximum every 3 minutes, which is the time required for the potential difference to fall from its maximum to 1.8 volts. Determine the maximum potential difference for this capacitor. (2 marks)

Question 12**(8 marks)**

A manufacturing process begins and the rate at which it produces gas after t minutes ($t \geq 0$) is modelled by

$$r(t) = 45(1 - e^{-0.4t}) \text{ m}^3/\text{minute}$$

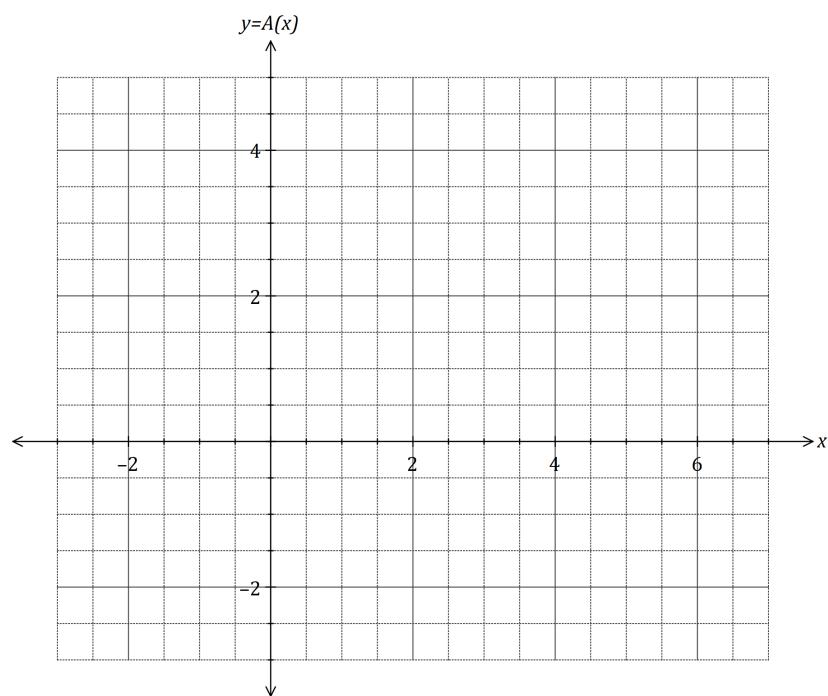
- (a) State the maximum rate that gas can be produced at. (1 mark)

- (b) Calculate the rate that gas is being produced after 2 minutes. (1 mark)

- (c) Use the increments formula to determine the approximate change in r between 30 and 33 seconds after production began. (3 marks)

- (d) Use the increments formula to determine the approximate volume of gas produced in the 5 seconds following $t=2$. (3 marks)

- (c) Sketch the graph of $y = A(x)$ on the axes below, indicating and labelling the location of all key features. (5 marks)



(1 mark)

- (c) Suggest a modification to the method used in (b) to achieve a better estimate for

$$\int_0^3 f(x) dx.$$

(2 marks)

- (b) Use the average of the sum of the areas of the inscribed rectangles and the sum of the areas of the circumscribed rectangles shown to determine an estimate for $\int_0^3 f(x) dx$.

(2 marks)

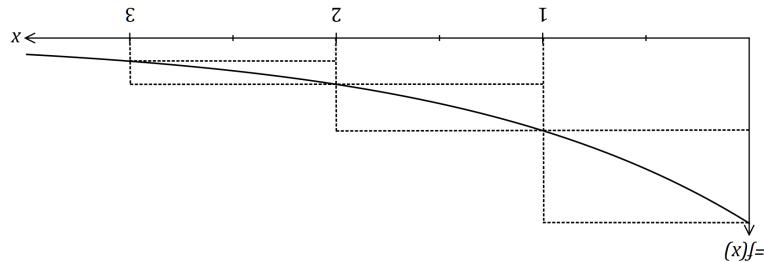
- (b) Using the graph of $y = f(x)$ or otherwise, explain why $A[5] = 3$.

It is also known that $A[2] = 0$.

(2 marks)

- (a) Use the sum of the areas of the inscribed rectangles shown in the diagram to explain why

$$\int_0^3 f(x) dx < \frac{9}{7}.$$

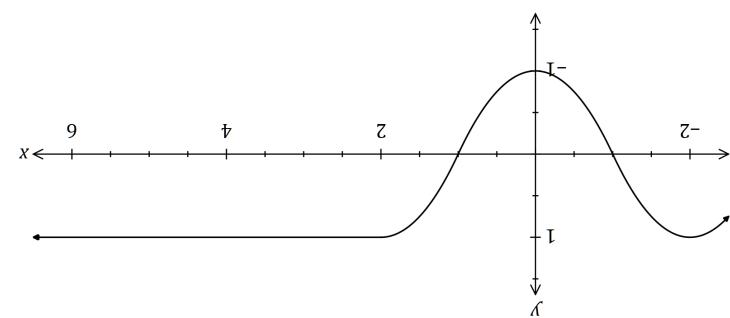


The function $f(x) = \frac{\sqrt{x}}{1+x}$ is shown below.

(5 marks)

- (a) Use the graph of $y = f(x)$ to identify all the turning points of the graph of $y = A(x)$, stating the x -coordinate and nature of each point.

Let $A(x)$ be defined by the integral $A(x) = \int_x^{-2} f(t) dt$ for $x \geq -2$.



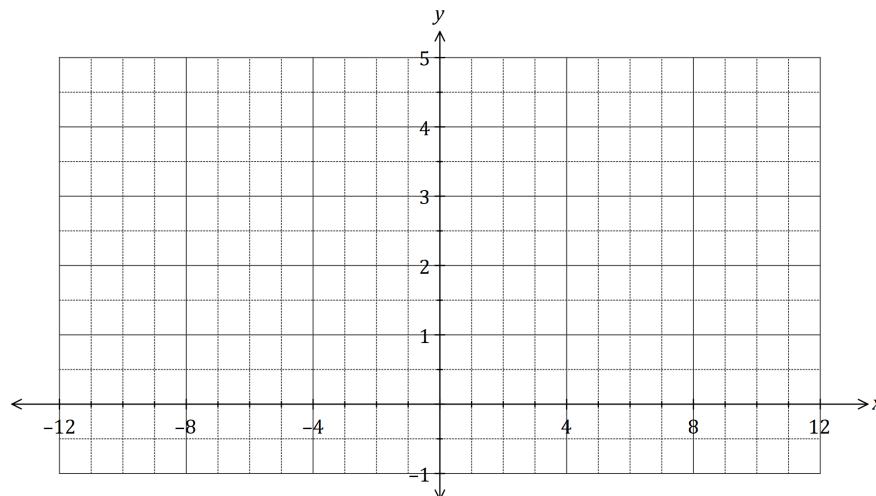
The graph of $y = f(x)$ is shown below.

(7 marks)

Question 14Let $f(x) = 3 + e^{-0.2x-2}$.

- (a) Sketch the graph of
- $y=f(x)$
- on the axes below.

(2 marks)



- (b) The line
- $y=2-0.2x$
- is tangential to the curve
- $y=f(x)$
- at
- $x=-10$
- , and it intersects the
- x
- axis at the point
- $(k, 0)$
- . Add the line to the graph above and shade the area enclosed by the line, the curve and
- $x=k$
- . (2 marks)

- (c) Determine the area enclosed by the line, the curve and
- $x=k$
- . (3 marks)

Suppose it is known that 66% of all seeds planted will germinate and that seeds are now planted in rows of 16.

- (c) Assuming that seeds germinate independently of each other, determine

- (i) the most likely number of seeds to germinate in a row. (1 mark)

- (ii) the probability that at least 9 seeds germinate in a randomly chosen row. (2 marks)

- (iii) the probability that in eight randomly chosen rows, exactly six rows have at least 9 seeds germinating in them. (2 marks)

(2 marks)

$$\cdot xp(x), \int_5^0 +xp(x) \int_0^{-4} \quad (\text{IV})$$

(2 marks)

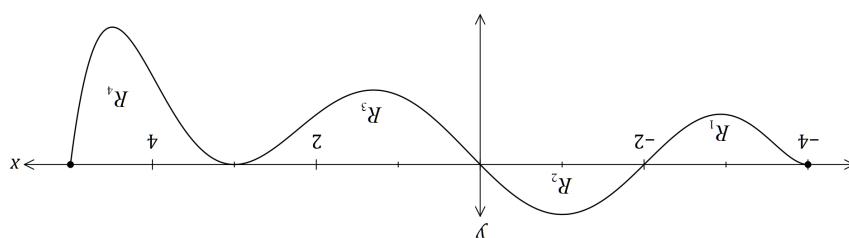
$$\cdot \exp\left((x)f(x) - \int_{\xi}^x\right) \quad (\text{iii})$$

(2 marks)

$$\cdot xp(x) f \int_{\xi}^{-} \quad (\text{iii})$$

(a) Determine the value of

and 32 square units respectively.



The graph of $y = f(x)$ is shown below for $-4 \leq x \leq 5$.

(7 marks)

Question 15

CALCULATOR-ASSUMED

6

METHODS UNIT 3

12

CALCULATOR-ASSUMED

Seeds were planted in rows of five and the number of seeds that germinated in each of the 120 rows are summarised below.

(3 marks)

| Number of germinating seeds | 0 | 1 | 1 | 3 | 16 | 46 | 53 |
|-----------------------------|---|---|---|---|----|----|----|
| Number of rows | | 1 | 1 | 3 | 16 | 46 | 53 |

(a) Use the results in the table to determine

(i) the probability that no more than 4 seeds germinate in a randomly selected row.

(iii) the mean number of seeds that germinated per row.

(1 mark)

(1 mark)

$$\cdot xp(x) f \int_0^x$$

(i) the probability that no more than 4 seeds germinate in a randomly selected row.

(a) Use the results in the table to determine
 (i) the probability that no more than 4 seeds germinate in a randomly selected row.

(i) the probability that no more than 4 seeds germinate in a randomly selected row.

Question 16

The random variable X is the number of goals scored by a team in a soccer match, where

$$P(X=x) = \frac{2.2^x e^{-2.2}}{x!} \text{ for } x=0, 1, 2, 3, \dots \text{ to infinity}$$

- (a) Determine the probability that the team scores at least one goal in a match. (2 marks)

The random variable Y is the bonus each player is paid after a match, depending on the number of goals the team scored. For four or more goals \$500 is paid, for two or three goals \$250 is paid and for one goal \$100 is paid. No bonus is paid if no goals are scored.

- (b) Complete the probability distribution table for Y . (3 marks)

| Goals scored | $x=0$ | $x=1$ | $2 \leq x \leq 3$ | $x \geq 4$ |
|--------------|-------|-------|-------------------|------------|
| $y (\$)$ | 0 | | | 500 |
| $P(Y=y)$ | | | | 0.1806 |

- (c) Calculate

- (i) the mean bonus paid per match. (2 marks)

- (ii) the standard deviation of the bonus paid per match. (2 marks)

- (d) The owner of the team plans to increase the current bonuses by \$50 next season (so that the players will get a bonus of \$50 even when no goals are scored) and then further raise them by 12% the following season. Determine the mean and standard deviation of the bonus paid per match after both changes are implemented. (3 marks)