

Important note to candidates
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the supervisor
Formula Sheet (retained from Section One)

Materials required/recommended for this section
This Question/Answer Booklet

Time allowed for this section
Working time for section: one hundred minutes
Reading time before commencing work: ten minutes

Your name
In words

Student Number: In figures

Calculator-assumed

Section Two:

UNITS 1 AND 2

METHODS

MATHEMATICS

SOLUTIONS

Question/Answer Booklet

Semester Two Examination, 2016



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
		Total		150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

65% (98 Marks)

Section Two: Calculator-assumed

provided.

Working time for this section is 100 minutes.

(a) Determine the equation of the straight line that passes through the point (8, 11) and is perpendicular to the line with equation $2x + 5y - 1 = 0$. (6 marks)

Question 9

This section has **twelve** (12) questions. Answer all questions. Write your answers in the spaces provided.

Specific behaviours	determines gradient of given line	writes equation of line
Required gradient is $-1 \div -\frac{2}{5} = \frac{5}{2}$	$y - 11 = \frac{5}{2}(x - 8)$ or $y = \frac{5}{2}x - 9$	
Solution		

- ✓ writes equation of line
- ✓ determines gradient of given line
- ✓ determines gradient of straight line

$9x^2 - 24x + 16 = 0$ <p>Calculate and use the discriminant to determine the number of real solutions.</p>
Solution

solution	$b^2 - 4ac = (-24)^2 - 4 \times 9 \times 16 \hat{=} 0$
Hence one solution	
Specifc behaviours	
substitutes into discriminant	evaluates one solution

Question 10

A walking club is planning a charity walk from Perth to Esperance. They plan to walk 20 km on the first day and 30 km every day after that. Food and camping supplies are to be set up at each overnight campsite in advance, using a vehicle based in Perth that is just large enough to carry enough for one campsite.

To leave the supplies at the first campsite, the vehicle must travel 40 km. For the second and third campsites, the vehicle must travel 100 km and 160 km respectively.

- (a) Determine the distances the vehicle will travel to set up campsites four and five. (1 mark)

Solution
220 km, 280 km
Specific behaviours
✓ states both distances

- (b) Determine, in simplified form, a rule for the distance, d km, that the vehicle will have to travel to set up campsite n . (2 marks)

Solution
$d = 40 + (n-1)(60)$
Specific behaviours
✓ uses AP general rule ✓ simplifies

- (c) The vehicle can travel a maximum of 700 km on one tank of fuel. Determine the number of the furthest campsite the vehicle can leave supplies at, using no more than one tank of fuel. (2 marks)

Solution
$60n - 20 = 700$
The 12 th campsite
Specific behaviours
✓ writes equation ✓ states number of campsite

- (d) If fuel costs 128 cents per litre and the fuel consumption of the vehicle is 9.5 litres per 100 km, determine the total fuel cost to set up the first 20 campsites. (3 marks)

Solution
$S_{20} = \frac{20}{2} [2(40) + (19)(60)] = 12200$
$12200 \div 100 \times 9.5 = 1159$ litres
$1159 \times 1.28 = \$1483.52$
Specific behaviours
✓ calculates total distance ✓ calculates number of litres ✓ calculates cost, showing units

Question 11 CALCULATOR-ASSUMED METHODS UNITS 1 AND 2													
(a) Records show that of the 1756 washing machines sold by a retailer during 2015, 464 were deluxe models and the rest were standard. Of all the machines sold, 42 were returned and 31 of those returned machines were standard models.	(a) Determine how many of the standard models were not returned. (2 marks)												
$N_s = 1756 - 464 = 1292$ $1292 - 31 = 1261$	<table border="1"> <tr> <td>models</td> </tr> <tr> <td>✓ calculates number of standard models</td> </tr> <tr> <td>Specified behaviours</td> </tr> <tr> <td>Solution</td> </tr> </table>	models	✓ calculates number of standard models	Specified behaviours	Solution								
models													
✓ calculates number of standard models													
Specified behaviours													
Solution													
(b) Calculate, to three decimal places, the probability that a randomly chosen machine from those sold	(b) Calculate, to three decimal places, the probability that a randomly chosen machine from those sold (2 marks)												
$\frac{1292}{1756} = 0.736$ $\frac{42}{1756} = 0.024$	<table border="1"> <tr> <td>(i) was a standard model.</td> </tr> <tr> <td>✓ calculates probability to 3 decimal places</td> </tr> <tr> <td>Specified behaviours</td> </tr> <tr> <td>Solution</td> </tr> </table> <table border="1"> <tr> <td>(ii) was returned.</td> </tr> <tr> <td>✓ calculates probability to 3 decimal places</td> </tr> <tr> <td>Specified behaviours</td> </tr> <tr> <td>Solution</td> </tr> </table> <table border="1"> <tr> <td>(iii) was returned given that it was a deluxe model.</td> </tr> <tr> <td>✓ calculates probability to 3 decimal places</td> </tr> <tr> <td>Specified behaviours</td> </tr> <tr> <td>Solution</td> </tr> </table>	(i) was a standard model.	✓ calculates probability to 3 decimal places	Specified behaviours	Solution	(ii) was returned.	✓ calculates probability to 3 decimal places	Specified behaviours	Solution	(iii) was returned given that it was a deluxe model.	✓ calculates probability to 3 decimal places	Specified behaviours	Solution
(i) was a standard model.													
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Solution													
(iii) was returned given that it was a deluxe model.													
✓ calculates probability to 3 decimal places													
Specified behaviours													
Solution													
(c) Is there any indication that the likelihood of a machine being returned is independent of the model type? Explain your answer. (2 marks)	(c) Is there any indication that the likelihood of a machine being returned is independent of the model type? Probability of being returned is the same for just deluxe models (0.024) and for all models (0.024) and so events are likely to be independent. Yes. <table border="1"> <tr> <td>✓ states YES</td> </tr> <tr> <td>Specified behaviours</td> </tr> <tr> <td>Solution</td> </tr> </table>	✓ states YES	Specified behaviours	Solution									
✓ states YES													
Specified behaviours													
Solution													

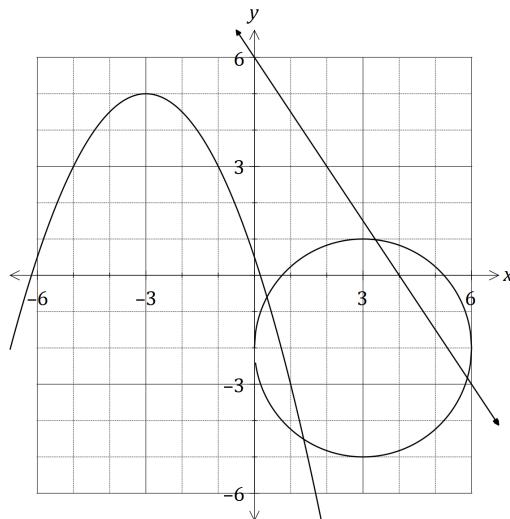
Question number: _____

Additional working space

(10 marks)

Question 12

The graph of two functions and a circle of radius 3 units are shown.



- (a) One function is $f(x) = ax + b$. Determine the values of the constants a and b . (2 marks)

Solution
$a = \frac{-6}{4} = \frac{-3}{2}, b = 6$
Specific behaviours
✓ states value of a
✓ states value of b

- (b) The relation can be written in the form $x^2 + px + y^2 + qy + r = 0$.

Determine the values of the constants p , q and r .

(3 marks)

Solution
Centre at $(3, -2)$
$(x-3)^2 + (y+2)^2 = 3^2$
$x^2 - 6x + y^2 + 4y + 4 = 0$
$x^2 - 6x + y^2 + 4y + 4 = 0$
$p = -6, q = 4, r = 4$
Specific behaviours
✓ identifies centre of circle
✓ writes in standard circle form

Additional working space

Question number: _____

(1 mark)

(iii) State the range of the function $y = -g(x)$.

Solution	Specific behaviours
Reflects in x -axis: $y \in \mathbb{R}, y \geq -5$	states range
	states coordinates of Tp

(1 mark)

(ii) State coordinates of the turning point of the graph of $y = g(x-7)$.

Solution	Specific behaviours
Translates 7 units right: $(4, 5)$	states coordinates of Tp
	states range

(3 marks)

(i) Determine the values of the constants c , d and e , given that $g(x)$ has a maximum at $(-3, 5)$.

Solution	Specific behaviours
$g(x) = c(x+3)^2 + 5$	uses turning point form
$g(-1) = 3 \Leftrightarrow c(4) + 5 = 3 \Leftrightarrow c = -\frac{1}{2}$	determines c
$g(x) = -\frac{1}{2}(x+3)^2 + 5 = -\frac{1}{2}x^2 - 3x + \frac{1}{2}$	
$c = -\frac{1}{2}, d = -3, e = \frac{1}{2}$	

(c) The other function is $g(x) = c x^2 + dx + e$.(i) Determine the values of the constants c , d and e , given that $g(x)$ has a maximum at $(-3, 5)$.(c) The other function is $g(x) = c x^2 + dx + e$.

Question 13

In triangle PQR , $PR=50$ cm, $QR=30$ cm and $\angle QPR=25^\circ$.

- (a) Show that $\angle PQR$ can be 44.8° or 135.2° .

Solution
$\frac{50}{\sin Q} = \frac{30}{\sin 25}$
$\angle Q = 44.8 \text{ or } 180 - 44.8 = 135.2$
$\angle Q = 44.8 \Rightarrow \angle R = 110.2 \text{ and } \angle Q = 135.2 \Rightarrow \angle R = 19.8$
Specific behaviours
✓ substitutes into sine rule
✓ shows values are supplementary solutions
✓ shows that both triangles exist by checking

- (b) Showing use of trigonometry, determine

- (i) the largest possible length of PQ .

(2 marks)

Solution
$\frac{PQ}{\sin 110.2} = \frac{30}{\sin 25} PQ = 66.6 \text{ cm}$
Specific behaviours
✓ shows use of largest value of $\angle R$ in sine or cosine rules
✓ calculates length

- (ii) the smallest possible area of triangle PQR .

(2 marks)

Solution
$A = \frac{1}{2} \times 50 \times 30 \times \sin 19.8 A = 254 \text{ cm}^2$
Specific behaviours
✓ shows use of smallest value of $\angle R$ in area formula
✓ calculates area

(7 marks)

(3 marks)

Question 20

- (a) Show that the equation of the tangent to the curve $y = \frac{x+x^3}{2}$ at the point where $x=2$ is $13x-2y=16$.

(4 marks)

Solution
$\frac{dy}{dx} = \frac{1+3x^2}{2} x=2 \Rightarrow y=5 \text{ and } \frac{dy}{dx} = \frac{13}{2}$
$y-5 = \frac{13}{2}(x-2) \quad 2y-10 = 13x-26$
$13x-2y=16$
Specific behaviours
✓ differentiates curve
✓ calculates y and $\frac{dy}{dx}$
✓ substitutes into equation of line
✓ simplifies into required form

Question 19

(2 marks)

1 mark)

(a) A sequence is defined by $T_{n+1} = T_n - 7$, $T_1 = 111$.

Solution	Specific behaviours	Roots
$x^2 - 3x + 2 = (x+2)(x-1)^2$ $2(x-1)^2 = 0 \Leftrightarrow x = -2, 1$ roots at $(-2, 0)$ and $(1, 0)$	specific behaviours	Roots coordinates of roots arises

Solution	$111 + [19](-7) = -22$	\checkmark Calculates term
Specific behaviours		\checkmark 40 terms, S_{40} .

(b) Use calculus techniques to determine the coordinates of all stationary points of the graph
of $y = f(x)$. (4 marks)

$$\frac{40}{2}(2[111] + [39](-7)) = -1020$$

✓ Calculates sum
✓ Specific behaviours

(c) Sketch the graph of $y = f(x)$ on the axes below for $-2 \leq x \leq 2$. (3 marks)

(d) A geometric sequence with $T_2=87.5$ has a sum to infinity of 800. Determine all possible values of T_1 for this sequence. (3 marks)

<u>Solution</u> $f(x) = 3x^2 - 3$ $3(x-1)(x+1) = 0 \Leftrightarrow x = -1, 1$ $A(-1, 4) \text{ and } (1, 0)$	<u>Specific behaviours</u> ∇ differentiable ∇ shows that $f'(x) = 0$ ∇ solves $f'(x) = 0$
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Solution	<p>By inspection of table of values, $S_{15} = 930$, $S_{16} = 936$ and $S_{17} = 935$. Hence maximum value is 936 when $n = 16$.</p>
Specific behaviours	<ul style="list-style-type: none"> ✓ identifies maximum states value of n

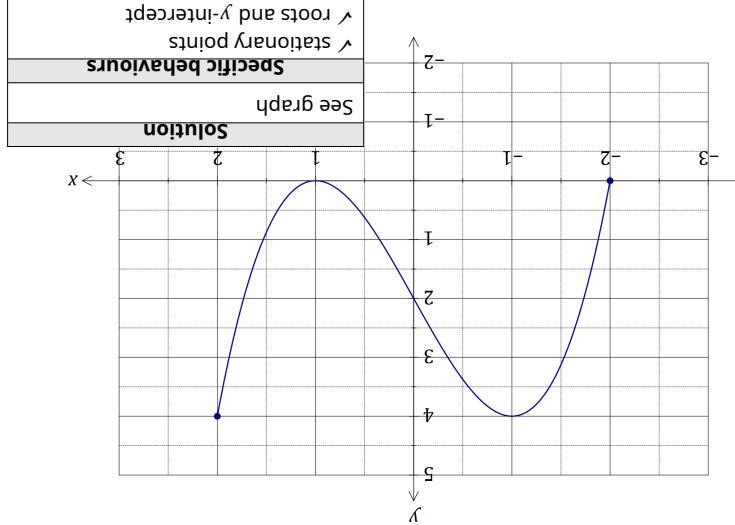
(iii) The value of n that maximises S_n .

1 mark)

Solution	$111 + [19](-7) = -22$
Specific behaviours	✓ calculate terms

[see next page](#) smooth curve within given

[See next page](#)



(3 marks)

values of T_1 for this sequence. (3 marks)

Question 18

(8 marks)

- (a) Two students are to be chosen from a class of 18.

- (i) Determine how many different pairs of students may be chosen. (1 mark)

Solution
$\binom{18}{2} = 153$
Specific behaviours

✓ calculates combinations

- (ii) One of the students in the class is the oldest in the school. What is the probability that this student is included in the pair chosen? (2 marks)

Solution
$\binom{1}{1} \times \binom{17}{1} = 17$
$\frac{17}{153} = \frac{1}{9}$
Specific behaviours

✓ calculates ways to choose oldest

- (b) A box contains 13 cans of soup, four of which have tomato as an ingredient and the remainder that do not. Four cans are to be selected at random from the box.

- (i) Calculate how many different selections of four cans can be made from the box. (1 mark)

Solution
$\binom{13}{4} = 715$
Specific behaviours

✓ calculates combinations

- (ii) Determine the probability that none of the four cans will have tomato as an ingredient. (2 marks)

Solution
$\binom{9}{4} = 126; \frac{126}{715} = 0.176$
Specific behaviours

✓ chooses four from the 9 non-tomato

✓ calculates probability

- (iii) Determine the probability that in the selection of four cans, there will be an equal number of cans with and without tomato as an ingredient. (2 marks)

Solution
$\binom{4}{2} \binom{9}{2} = 6 \times 36 = 216; \frac{216}{715} = 0.302$
Specific behaviours

✓ chooses 2 tomato and 2 non-tomato

✓ calculates probability

(2 marks)

(iii) Use the graph to solve $|x| = g(x)$, $0^\circ \leq x \leq 180^\circ$.

Solution
$x = 54^\circ, 84^\circ, 133^\circ$
Specific behaviours
✓ three solutions, two within $+2^\circ$.

Solution
$\frac{dA}{dr} = 30 - 2r$
$30 - 2r = 0 \Leftrightarrow r = 15$
$A = 30(15) - 15^2 = 225 \text{ cm}^2$
$\theta = \frac{60}{60} = 2 \text{ radians}$
✓ differentiates
✓ solves derivative to zero, starting r
✓ calculates area

(c) Use calculus to determine the maximum area of the sector and state the values of r and θ that achieve this maximum.

(4 marks)

(4 marks)

(i) Determine the values of the constants a, b, c and d .

Solution
$a = -3, b = -\frac{9}{360} = -4, c = 2, d = -30$
✓ ✓ ✓ each value

(2 marks)

(b) Show that the area of the sector is given by $30r - r^2$.

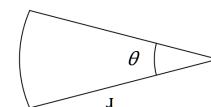
Solution
$A = \frac{1}{2}r^2\theta$
$\frac{1}{2}r^2\left(\frac{60}{r}\right) = \frac{30r - r^2}{r}$
✓ substitutes θ from (a) into area

(2 marks)

(a) Show that $\theta = \frac{60}{r} - 2$.

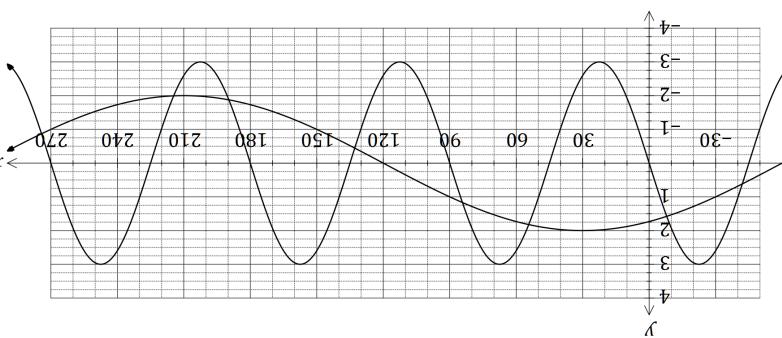
Solution
$P = 2r + r\theta$
$Hence 60 = r(2 + \theta) \Leftrightarrow \theta = \frac{60}{r} - 2$

(2 marks)

The perimeter of a sector of a circle, of radius r cm and central angle θ radians, is 60 cm.

(8 marks)

(10 marks)

Question 17
Question 15
CALCULATOR-ASSUMED
METHODS UNITS 1 AND 2
METHODS UNITS 1 AND 2Question 15
CALCULATOR-ASSUMED
METHODS UNITS 1 AND 2
METHODS UNITS 1 AND 2

- (b) P and Q are acute angles with $\sin P = \frac{12}{13}$ and $\cos Q = \frac{15}{17}$. Determine the **exact** value of $\cos(P-Q)$. (4 marks)

Solution

P is a 5-12-13 triangle and Q is a 8-15-17 triangle

$$\text{Hence } \cos P = \frac{5}{13} \text{ and } \sin Q = \frac{8}{17}$$

$$\cos(P-Q) = \cos P \cos Q + \sin P \sin Q = \frac{5}{13} \cdot \frac{15}{17} + \frac{12}{13} \cdot \frac{8}{17} = \frac{171}{221}$$

Specific behaviours

- ✓ obtains exact values for P
- ✓ obtains exact values for Q
- ✓ substitutes into double angle formula
- ✓ evaluates correctly

Solution**Question 16**

(8 marks)

The imprisonment rate R , in number of prisoners per 100 000 people, in the US between the years 1960 and 2000, can be modelled by the following equation, where n is the year.

$$R = 85(1.038)^{n-1960}$$

- (a) Calculate the imprisonment rate in the year 2000. (1 mark)

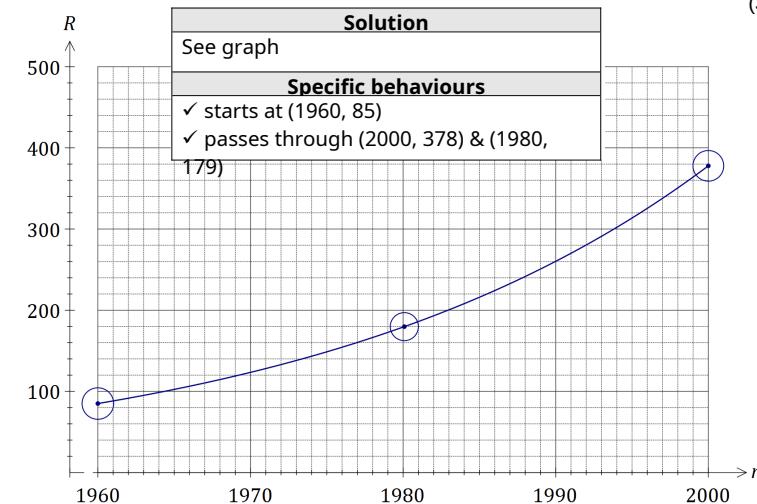
Solution

$$R = 378$$

Specific behaviours

- ✓ calculates rate

- (b) Draw the graph of the imprisonment rate for $1960 \leq n \leq 2000$ on the axes below. (3 marks)



- (c) The population of the US was 266 million in 1995. Determine the number of prisoners in the US at this time, to the nearest 1 000. (3 marks)

Solution
 $R|_{1995} = 313.564$
 $313.564 \div 100000 \times 266000000 = 834080$
834 000 prisoners

Specific behaviours

- ✓ calculates rate, maintaining accuracy
- ✓ adjusts for population
- ✓ rounds at end as required

- (d) When R first exceeded 500, steps were taken to address the exponential growth in the prison population and the model no longer applied. In what year did this occur? (1 mark)

Solution
 $500 = 85(1.038)^{n-1960} \Rightarrow n = 2007.5 \Rightarrow \text{During 2007}$

Specific behaviours

- ✓ solves for n