This page is available for extra working if needed. Remember to number questions carefully if you use this page.

CHRIST CHURCH GRAMMAR SCHOOL

12_3CD MAT Examination Mid-Year 2012

Insert your examination number here:

This paper is in two parts: Section A (40 marks) in which calculators may not be used, and Section B (80 marks) in which calculators may be used.

SECTION A
CALCULATORS MAY NOT BE USED IN THIS SECTION

MATERIAL REQUIRED / RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR Curriculum Council Formula Sheet.

TO BE PROVIDED BY THE CANDIDATE

Standard Items: Pens, pencils, eraser or correction fluid, ruler

Special Items: Drawing instruments, templates and up to 2 calculators satisfying the conditions set by the Curriculum Council, plus one sheet of A4 paper, i.e. 2 pages of A4 notes.

IMPORTANT NOTE TO CANDIDATES

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INSTRUCTIONS TO CANDIDATES

Show all working clearly, in sufficient detail to allow your answers to be checked readily, and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

Question	Maximum Mark	Mark Achieved
A1	6	
A2	10	
A3	6	
A4	6	
A5	8	
A6	4	
TOTAL	40	

Section A Question 1 [3x2=6 marks]

Simplify (i) $\dfrac{x^2-2x-3}{9-x^2}$

(ii) $\dfrac{\binom{6}{2}}{\binom{5}{2}}$

(iii) $\dfrac{(3x+1)^2-(1-3x)^2}{-3x}$

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Section B Question 16 [1+2=3 marks]

An economics model once trialled by the Department of Treasury and Finance in Canberra calculated the annual inflation rate *I(x)* based on the GST rate *x*%.

The *marginal* inflation rate was defined as $I'(x) = \dfrac{9}{2\sqrt{x}}$

(a) Use the incremental formula $\delta I \approx \dfrac{dI}{dx}.\ \delta x$ to estimate the change in the annual inflation rate if the GST was increased from 9% to 9.5%

(b) Apply an integration method to calculate the predicted inflation rate for a GST of 16%, given that a 9% GST is associated with an inflation rate of 3.5%

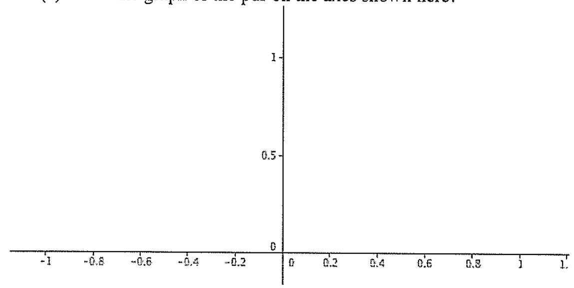
END OF CALCULATOR ALLOWED SECTION

Section A Question 2 [1+1+2=4 marks]

A random variable X has a probability density function defined by

$$f(x) = \begin{cases} 1+x, & -1 \le x \le 0 \\ 1-x, & 0 < x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Draw the graph of the pdf on the axes shown here.



(b) On the graph above show the region that represents p(0.2<x<1)

(c) Find p(0.2<x<1)

Section A Question 3 [3+3+2+2=10 marks]

(i) Showing all algebraic steps use the quotient rule to differentiate $\frac{3x^2+2}{2x-1}$. Simplify your answer.

(ii) Find $\frac{dy}{dx}$ if $y= e^x(2e^{-x}-3)^3$ (You do not need to simplify your answer)

(iii) Showing all algebraic steps find $\frac{dy}{dx}$ if $y= \frac{3e^3}{2x^2}$

(iv) Find $\int(3x^2-2)(4x-1)dx$ Simplify your answer.

Section B Question 15 [2+2=4 marks]

After complaints from customers, "Greenmoores" decides to check its glucosamine hydrochloride tablets and finds that 30% of the bottles of tablets have incorrect amounts.

If KINGS CHEMIST has 15 bottles on the self,

a) Find the probability that exactly 5 bottles have incorrect amounts.

b) Find the probability that more than 10 bottles have correct amounts.

Section B Question 14 [2+2+2+2=8 marks]

A squad of 18 soccer players consists of 3 goalkeepers, called Andy, Bandy and Candy, 9 backs and 6 forwards. From these, a team of 11 must be chosen.

(a) How many of these possible teams have

(i) 1 goalkeeper, 6 backs and 4 forwards.

(ii) at least 1 goalkeeper.

(b) Find the probability that in a team, Andy and Bandy are chosen, but not Candy.

(c) The players want to set up a committee of 4 which must include at least one goalkeeper, 1 back, and 1 forward. How many such committees are possible?

Section A Question 4 [6 marks]

The points $P(-4,3)$, $Q(6,3)$ and $R(-2,-1)$ all lie on the graph of $f(x) = ax^2 + bx + c$

Find a, b and c.

Section A Question 5 [6 marks]

Two events A and B have probabilities given by $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{5}$, and $P(\overline{A \cup B}) = \frac{2}{15}$

(a) Find (i) $P(A \mid B)$ (ii) $P(B \mid A)$

Answers: 5(a) (i) $P(A|B) =$ (ii) $P(B|A) =$

(b) Find, showing evidence, whether events A and B are independent.

Section B Question 13 [1+2+2+2+1=8 marks]

A manufacturer produces a type of device that is a component of an electronic product. The selling price, Q, in dollars, of each device is given by the function with rule $Q(z) = 400 - 2z$ where z is the number of devices produced. The cost, C, in dollars, of producing z devices is given by the function with rule $C(z) = 0.2z^2 + 4z + 400$.

a. i. Find the rule for the function R that gives the revenue in dollars from producing z devices.

ii. Show that the profit, $\$P$, from producing and selling z devices, is given by the function with rule $P(z) = -2.2z^2 + 396z - 400$.

b. i. Find the value of z for which the profit is maximised.

ii. Find the selling price per device if this maximum profit is obtained.

iii. Find the maximum profit.

Section B Question 12 [1+2+2+2+1+2 =10 marks]

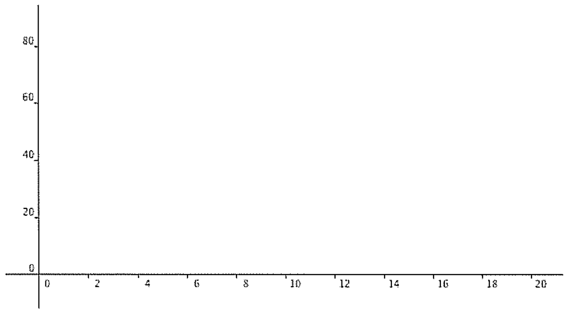
As a substance cools its temperature, T^0 C, is related to the time (t minutes) for which it has been cooling. The relationship is given by the equation

$T = 20 + 60\,e^{-0.1t}$, $t \geq 0$.

. (a) What was the value of T when the substance started to cool?.

. (b) Explain why the temperature of the substance is always above 20° C

. (c) Sketch the graph of T against t.



. (d) Find the value, to 2 significant figures, of t at the instant when T = 40.

. (e) Find $\frac{dT}{dt}$

(f) Hence find the value of t at which the temperature is decreasing at a rate of 1.8° C per minute.

Section A Question 6 [8 marks]

A furniture shop has an offer that when a customer buys either a small or large table they can choose either a small or a big rug at a discount price.

Records show that two out of every three customers buy a big table and also show that of these customers who buy a big table, one quarter of them choose a small rug. The records also show that after choosing a table, seven out of every ten customers choose a big rug.

(a) Draw a probability tree showing the possible table and rug choices and their associated probabilities.

(b) Determine the probability that a randomly selected customer:

. (i) will choose both a big table and a big rug

. (ii) will choose a small rug given that they have chosen a small table

. (iii) will choose a big table given that they have chosen a small rug

END OF THE NON-CALCULATOR SECTION

This page is available for extra working if needed. Remember to number questions carefully if you use this page.

Section B Question 11 [5+2=7 marks]

Use calculus techniques to solve the following problem:

A delivery vehicle uses fuel at the rate of $\frac{x^2+3000}{300x}$ litres per kilometer where ix is the speed in kilometres per hour. Diesel fuel costs \$1.30 per litre. At what speed should the vehicle travel for a 180km trip if the driver wants to minimize fuel costs?

What is the minimum fuel cost for the 180km trip?

Section B Question 10 [4+2=6 marks]

27% of people who have a current driving licence require visual aids whilst driving their vehicle.

(a) Determine the probability that in a group of 10 people who have a current driving licence

(i) at least two require visual aids.

(ii) at least two require visual aids if it is known that at least five do not.

(b) Many people renew their driving licence at their local Post Office. In a line of 10 people waiting to renew their driving licence, what is the probability that the second and the eighth person require visual aids and all of the others do not?

CHRIST CHURCH GRAMMAR SCHOOL

12_3CD MAT Examination Mid-Year 2010

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SECTION B

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B7	8	
B8	14	
B9	12	
B10	6	
B11	7	
B12	10	
B13	8	
B14	8	
B15	4	
B16	3	
TOTAL	80	

d) Between what values can the profit on iPod shuttles range so that the optimal solution in c) does not need to change?

e) To what value would the profit on iPod maxis need to drop before it becomes unprofitable to produce any iPod maxis at all?

Section B Question 9 [1+2+2+4+3=12 marks]

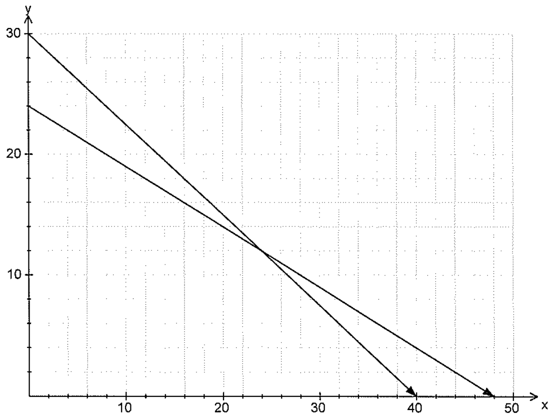
The Pear Computer Company wants to produce two new types of iPods, the shuttle and the maxi. Construction of the iPod shuttle requires 3 man hours and construction of the iPod maxi requires 4 man hours. There is a maximum of 120 man hours available each day. A maximum of 24 hours has been put aside for testing. The iPod shuttle requires 0.5 hrs for testing whereas the iPod maxi requires one hour. Market research suggests that there will be demand for at least as many iPod shuttles as iPod maxis.

Let x be the number of iPod shuttles that should be produced each day.
Let y be the number of iPod maxis that should be produced each day.

Some of the constraints given in the information above are provided below:
$x \geq 0 \qquad y \geq 0 \qquad 0.5x + y \leq 24 \qquad 3x + 4y \leq 120$

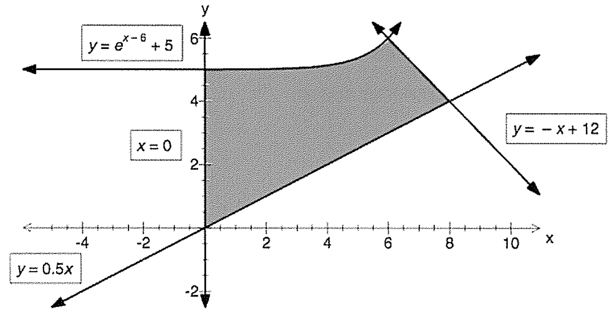
a) Write down the remaining constraint.

b) Draw the constraint from a) on the graph below and shade the feasible region.



c) The profit for each iPod shuttle is $99 and each iPod maxi is $149. How many of each type of iPod should be produced each day in order to maximise profits?

Section B Question 7 [4 +4=8 marks]



(a) The graphs of $y = -x + 12$ and $y = e^{x-6} + 5$ intersect at the point (6,6), and the graphs of $y = 0.5x$ and $y = -x + 12$ intersect at the point (8,4). Write down the integral used to find the **exact** area of the shaded region from x=0 to x=6 in the diagram above and solve it using your calculator correct to two decimal places.

(b) The shaded region from x=6 to x=8 in the diagram above is rotated 360° about the x-axis. Write down the integral used to find the **exact** volume of the solid formed and solve it using your calculator correct to two decimal places.

Section B Question 8 [3+3+2+3+3=14 marks]

At a building site the probability, P(A), that all materials arrive on time is 0.85. The probability, P(B), that the building will be completed on time is 0.60. The probability that the materials arrive on time and that the building is completed on time is 0.55.

(i) Illustrate this in a Venn diagram

(ii) Show that events A and B are not independent.

(ii) All the materials arrive on time. Find the probability that the building will not be completed on time.

Question 8 continues on the next page

(b) There was a team of ten people working on the building, including three electricians and two plumbers. The architect called a meeting with five of the team, and randomly selected people to attend. Calculate the probability that exactly two electricians and one plumber were called to the meeting.

(c) The number of hours worked per week by the 10 people in the team is normally distributed with a mean of 42 hours. 10% of the team work 48 hours or more a week. Find the probability that both plumbers work more than 40 hours in a given week.