SHENTON
COLLEGE

ATMAM Mathematics Methods

Test 3 — Calculator Assumed

Name: ....

Teacher: Friday Smith

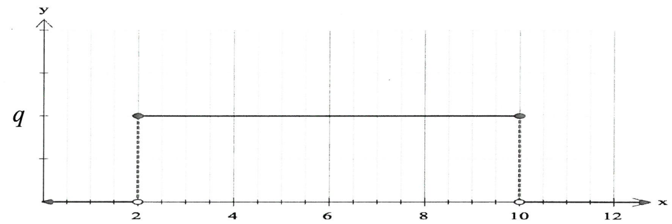
Time Allowed : 50 minutes

Marks	/56

Materials allowed: Classpad, Formula Sheet.

All necessary working and reasoning must be shown for full marks.
Where appropriate, values should be given to <u>two</u> decimal places, except for probabilities which should be given to <u>four</u> decimal places.
Marks may not be awarded for untidy or poorly arranged work.

1 The graph below shows a probability density function for a uniform random variable X, defined over $2 \le x \le 10$.



a) Determine the value of q. (1)

$\frac{1}{8}$ or 0.125

b) Determine $P(3 < x < 9)$ (1)

$\frac{3}{4}$ or 0.75

c) Find t such that $P(x < t) = 0.4$ (2)

$0.4 \times 8 = 3.2$

$t = 5.2$

✓ proportion of domain

✓ 2+

d) Determine $P(x > 8 \mid x > 5)$ (2)

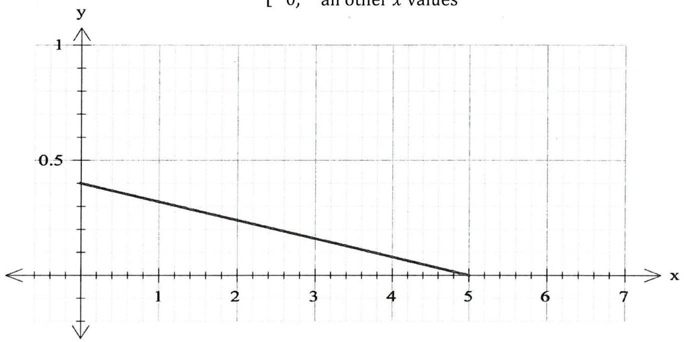
$\dfrac{\frac{1}{4}}{\frac{5}{8}} = \frac{2}{5}$ or 0.4

✓ $x > 8$ intersection

✓ condition as denominator.

2 The probability density function shown below is defined piecewise by:

$$y = \begin{cases} f(x), & 0 \le x \le 5 \\ 0, & \text{all other } x \text{ values} \end{cases}$$



a) Determine $f(x)$. (2)

$f(x) = -0.08x + 0.4$

$\frac{-2}{25}x + \frac{2}{5}$

✓ gradient

✓ intercept.

b) Determine $P(x < 2)$ (1)

$\int_0^2 f(x)\,dx = 0.64$

c) Determine $P(2 < x < 3)$ (1)

$\int_2^3 f(x)\,dx = 0.2$

d) Find k such that $P(3 < x < k) = 0.1$ (2)

$k = 3.7753$

✓ $\int_3^k f(x)\,dx$

✓ k.

12 In a game of chance, two coins are tossed at the same time and the player wins if there is at least one Head. After the game has been played 40 times, 21 wins have been recorded.

a) Determine the proportion of wins $\hat{p}$ in the sample of 40 games. (1)

$\frac{21}{40} = 0.525$

b) Determine the value of p and use this to find the standard deviation of the sample proportions. (2)

HH TH
HT TT

$p = 0.75$

$\sigma = 0.0685$

c) Calculate the number of standard deviations between p and $\hat{p}$. Use this information to comment on the results in the sample of 40 games. (3)

$\frac{0.75 - 0.525}{0.0685} = 3.286$

$\hat{p}$ is more than 3 standard deviations from p, which suggests this sample is very unlikely to be a typical random result.

13 Six different surveys were carried out to determine the number of people who have cracked screens on their mobile phones. The results of these surveys are recorded in the table to the right.

Sample	Number of people surveyed	Proportion with cracked screens
1	20	0.55
2	14	0.5
3	16	0.625
4	10	0.6
5	25	0.56
6	36	0.75

a) Use the complete set of results to determine your best estimate for p, the population proportion. (1)

weighted average from CAS, $p = 0.6198$

b) Use your value of p to determine the standard deviation of samples involving 36 people. (1)

$\sigma = 0.0809$

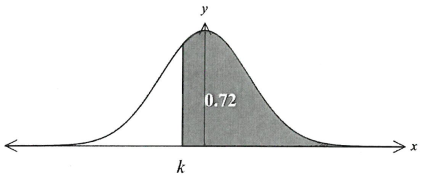
c) Calculate the number of standard deviations between p and $\hat{p}$ for the sample number 6. Use this information to comment on the results of sample 6. (2)

$\frac{0.6198 - 0.75}{0.0809} = -1.61$

While the sample proportion is higher than all the others, at $z = -1.61$ it is still within an acceptable margin from p to be considered a valid sample.

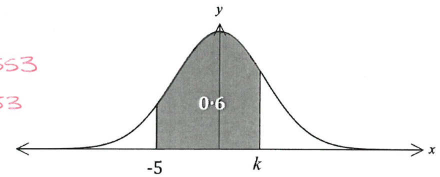
9 Determine the value of k in the normal distributions below.

a) $X \sim N(14, 3^2)$ (1)



$k = 12.25$

b) $X \sim N(-1, 4^2)$ (2)



$P(x < -5) = 0.1586553$

$\Rightarrow$ left tail $P = 0.7586553$

$k = 1.81$

10 A normally distributed random variable R has distribution $R \sim N(12, 2.5^2)$. Determine

a) $P(R < 8)$ (1)

0.0548

b) $P(10 < R < 14)$ (1)

0.5763

c) $P(R > 10 \mid R < 14)$ (1)

$\dfrac{0.5763}{0.7881} = 0.7312$

d) Find k if $P(R > k) = 0.15$ (1)

$k = 14.59$

11 H is a normally distributed random variable, with 20% of values greater than 70 and 40% of values less than 50. Determine the mean and standard deviation of H, to two decimal places. (4)

20% > 70 , $z = 0.8416212$

40% < 50 , $z = -0.253347$

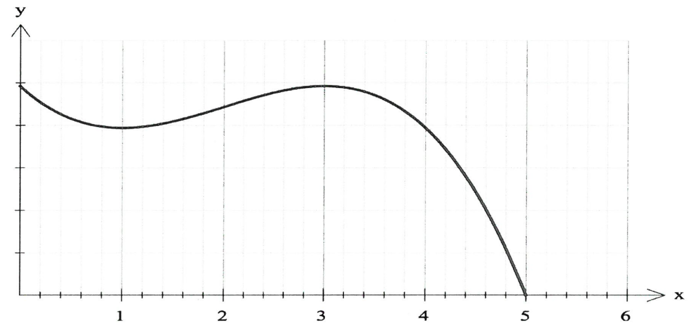
$70 = \mu + 0.8416212\,\sigma$ ✓ z scores

$50 = \mu - 0.253347\,\sigma$ ✓ simultaneous

$\mu = 54.63$ ✓ μ

$\sigma = 18.27$ ✓ σ

3 The graph below shows a probability density function for a random variable X, defined by the function $f(x) = k(x^3 - 6x^2 + 9x - 20)$.



a) Find the value of k (2)

$\int_0^5 k(x^3 - 6x^2 + 9x - 20)\,dx = 1$ ✓ $\int = 1$

$\Rightarrow k = \dfrac{-4}{325}$ ✓ k

b) Find the expected value of X. (Give your answer as an exact value.) (2)

$E(X) = \int_0^5 x f(x)\,dx = \dfrac{30}{13}$ ✓ uses $xf(x)$ ✓ $E(x)$

4 A triangular random variable is defined by the function $f(x)$. It has an expected value $E(X) = 7$ and standard deviation $\sigma_X = 3$. The mode m has a frequency $f(m) = k$.

If $f(x)$ is transformed into $f(3X - 1)$, determine the new…

a) expected value (1)

20

b) standard deviation (1)

9

c) value for the mode in terms of m, and the new frequency in terms of k. (2)

$3m - 1$ $\qquad \dfrac{k}{3}$

5 "Fun Size" Ganymede™ bars are sold in bags of 13. There is a 1% chance that any individual Fun Size bar contains a whole rat foot or a rat tooth.

a) What is the probability that a bag of Fun Size Ganymede™ bars contains at least one bar with a whole rat foot or rat tooth? (1)

0.1225

(0.122479)

b) The bags of Fun Size bars are packed together into boxes of 15 for delivery to supermarkets. What is the probability that a box chosen at random contains no more than three bags that contain at least one bar with a whole rat foot or rat tooth? (2)

$X \sim Bin(15, 0.122479)$ ✓ distribution

$P(0 \le X \le 3) = 0.8983$ ✓ P

c) Rob, a Ganymede™ bar addict, regularly eats a whole bag of Fun Size bars as a snack. Given that Rob has already eaten 8 bars that have not had any rat body parts in them, what is the probability that the next bar he eats contains a whole rat foot or rat tooth? (1)

0.01

6 A random binomial variable V has the distribution $V \sim \text{Bin}(4, p)$.

a) What is $P(V = 1)$? (1)

$4p(1-p)^3$

b) What is $P(V > 2)$? (2)

$4p^3(1-p) + p^4$

7 A random binomial variable G has the distribution $H \sim \text{Bin}(n, 0.8)$

Determine the smallest value of n such that the chance of there being more than 4 successes is at least 95%. (2)

Trial and error on CAS

for $n = 8$, $P = 0.9437$ (too small)

for $n = 9$, $P = 0.98$ (sufficient).

$\Rightarrow n = 9$.

8 Discuss the following methods of sample selection. In particular, consider the method of selection in terms of any possible introduced bias.

a) A local council is considering increasing the width of a road by removing a footpath and decreasing the size of the verge. They use a systematic sampling technique to get feedback from local residents on those streets, selecting every fourth house. (2)

YES Doesn't consider non resident road users

Every 4th means just one side.

Possibly affected by corner blocks vs non,

Home during the survey.

NO Only ¼ of people – not everyone

The fact that people may have different opinions is <u>not</u> bias

b) A fast food restaurant that offers dine-in, take-away and drive-thru options has posters in its dining area inviting customers to rate their satisfaction with their meal. They offer a range of social media formats as well as a paper form that customers can fill out. (2)

YES Excludes drive-thru.

Possible multiple reviews by determined individuals (or robots)

Self selection bias, likely to be those with strong opinions

Literacy / English

NO Excludes those w/o social media

Excludes those who never go to this restaurant

c) A small island, known to be a popular penguin breeding spot, has a perimeter of approximately 4km. To try and determine the number of penguins on the island, a group of volunteers walk along 1km of the coast, carefully counting any penguins they can see. They then multiply this result by 4 to determine the total penguin population. (2)

YES Penguins may gather at specific locations.

(Not that part of the coast, In the centre, etc)

Relies on eyesight & visibility

Time of day / habits of penguins.

Miscount / losing count / double count etc.

NO Volunteers might get tired and start slacking off.