KINGSWAY CHRISTIAN COLLEGE

MATHS DEPARTMENT

Course: Mathematics Methods Year 12

Assessment Task: Test 4 – Logarithms

Student Name: Sol key:

Date: 26th June 2017

Assessment Score: _____ / 40

Year Score: _____

Comments: _____

Teacher signature: _____

Parent/ Guardian signature: _____

Comments: _____

METHODS YEAR 12 **Test 4 2017**
Logarithms

Resource Free **Time: 35 mins** **Marks: / 40**

No notes or calculators allowed for this section.

Question 1 **(5 marks)**

Evaluate the following, giving your answer as a single log term:

$\frac{(\log 5-\log 3)^2}{\log \frac{3}{5}}$

$= \frac{(\log 5-\log 3)(\log 5-\log 3)}{-\log\left(\frac{5}{3}\right)}$ ✓

$= \frac{(\log 5-\log 3)(\log 5-\log 3)}{-(\log 5-\log 3)}$ ✓

$= \log 3-\log 5$ ✓

$= \log \frac{3}{5}$ ✓ (OR) $-\log \frac{5}{3}$

Question 2 **(9 marks)**

Solve each of the following equations. Leave answers in logarithmic form where necessary.

(a) $2^{x-3} = 5^{2x+1}$ (4 marks)

$\therefore \log 2^{x-3} = \log 5^{2x+1}$

$\Rightarrow (x-3)\log 2 = (2x+1)\log 5$ ✓

$x\log 2 - 3\log 2 = 2x\log 5 + \log 5$ ✓

$x\log 2 - 2x\log 5 = \log 5 + 3\log 2$ (OR) $\frac{1+\log 4}{\log\left(\frac{2}{25}\right)}$

$x(\log 2 - 2\log 5) = \log 5 + 3\log 2$ ✓

$x = \frac{\log 5 + 3\log 2}{\log 2 - 2\log 5}$ ✓ (OR) $\frac{\log 40}{\log\left(\frac{2}{25}\right)}$

Question 8 **(3 marks)**

The tangent to the curve $y = \ln(kx-1)$ has a gradient of 1 when $x = 2$.
Determine the value of k.

$\frac{dy}{dx} = \frac{k}{kx-1}$ ✓

$\frac{dy}{dx}\Big|_{x=2} = \frac{k}{2k-1} = 1$ ✓

$k = 2k-1$

$1 = k$ ✓

Question 9 **(2 marks)**

Determine the following anti-derivative, simplifying your answer using logarithmic laws if necessary:

$\int \frac{5e^{-2x}}{1+e^{-2x}}\,dx$

$= 5\int \frac{e^{-2x}}{1+e^{-2x}}\,dx$

$= -\frac{5}{2}\int \frac{-2e^{-2x}}{1+e^{-2x}}\,dx$ ✓

$= -\frac{5}{2}\left[\ln\left|1+e^{-2x}\right|\right] + C$ ✓

Question 7 **(9 marks)**

Differentiate each of the following with respect to x.

(a) $y = \sqrt{x}\ln\left(\frac{x}{3}\right)$ (3 marks)

$y = x^{\frac{1}{2}} \times \ln\left(\frac{x}{3}\right)$ ✓

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \cdot \ln\left(\frac{x}{3}\right) + x^{\frac{1}{2}} \times \frac{1}{\frac{x}{3}}$ ✓

$= \frac{\ln\frac{x}{3}}{2\sqrt{x}}$ ✓ $+ \frac{3}{\sqrt{x}}$ ✓.

(b) $y = \ln\left[\frac{(x+4)^2}{(3x-1)}\right]$ (3 marks)

$y = 2\ln(x+4) - \ln(3x-1)$ ✓

$\frac{dy}{dx} = \frac{2}{x+4}$ ✓ $- \frac{3}{3x-1}$ ✓.

OR $\frac{3x-14}{(x+4)(3x+1)}$.

(c) $y = \frac{\cos^2 x}{\ln x}$ *(do not simplify)* → Quotient Rule. (3 marks)

$\frac{dy}{dx} = \frac{-2\sin x\cos x(\ln x) - \frac{1}{x}\cos^2 x}{(\ln x)^2}$ ✓ ✓ ✓.

(b) $3^{2x+1} - 5(3^x) - 2 = 0$ (5 marks)

$3.3^{2x} - 5.3^x - 2 = 0$

let $3^x = c$ ✓

$\therefore 3c^2 - 5c - 2 = 0$

$(3c+1)(c-2) = 0$.

$3c = -1$ or $c = 2$

$c = -\frac{1}{3}$ ✓ or $c = 2$ ✓.

$\therefore 3^x \neq -\frac{1}{3}$ ✓ or $3^x = 2$ ✓

↓ N.A.

$x = \log_3 2$ ✓.

(−1) if you didn't reject the $-\frac{1}{3}$.

Question 3 **(5 marks)**

If $\log_{10} 2 = x$ and $\log_{10} 3 = y$. Express the following in terms of x and y

(a) $\log_{10} 0.6$ (2 marks)

$\log_{10}\frac{6}{10}$

$= \log_{10}\frac{3\times 2}{10}$ ✓

$= \log_{10} 3 + \log_{10} 2 - \log_{10} 10$

$= y + x - 1$. ✓

(b) $\log_{10} 45$ (3 marks)

$= \log_{10}(3^2 \times 5)$ ✓

$= \log_{10} 3^2 + \log_{10} 5$

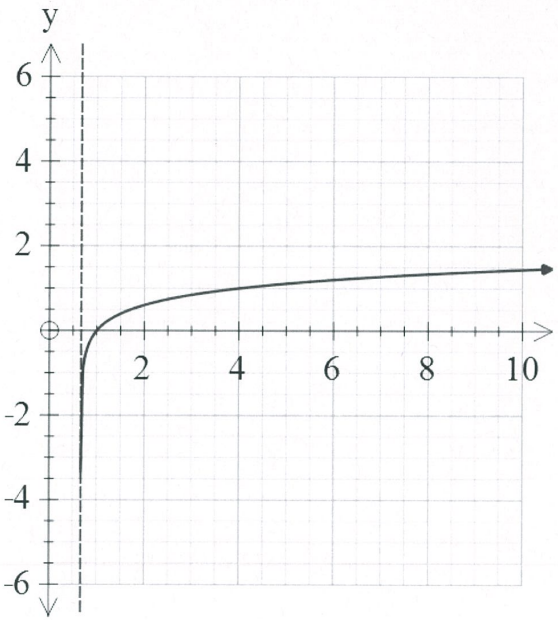
$= 2\log_{10} 3 + \log_{10}\frac{10}{2}$ ✓

$= 2\log_{10} 3 + \log_{10} 10 - \log_{10} 2$

$= 2y + 1 - x$. ✓

Question 4 **(3 marks)**

The function $f(x) = \log(bx - 2)$ is drawn below.



(a) Determine the value of b. (2 marks)

$y = \log(bx - 2)$

through $(4,1)$ $1 = \log(4b - 2)$ ✓

$10^1 = 4b - 2$

$\therefore 4b = 12$ ✓

$b = 3$

(b) Use the graph to approximate the solution to $\log(bx - 2) = 1$ (1 marks)

$\log(3x - 2) = 1$

$10^1 = 3x - 2$

$12 = 3x$ ✓

$x = 4$

Question 5 **(3 marks)**

If $x = \frac{1}{\sqrt{3}}$, show that $\log(1 - x^4) - \log(1 - x) - \log(1 + x) = 2\log 2 - \log 3$.

$LHS = \log\left(\frac{1-x^4}{(1-x)(1+x)}\right)$

$= \log\left(\frac{(1+x^2)(1+x)(1-x)}{(1-x)(1+x)}\right)$ ✓

$= \log(1+x^2)$

$= \log\left(1 + \left(\frac{1}{\sqrt{3}}\right)^2\right)$

$= \log\left(1 + \frac{1}{3}\right)$

$= \log\left(\frac{4}{3}\right)$ ✓

$= \log 4 - \log 3$

$= 2\log 2 - \log 3$

$= RHS$ ✓

Question 6 **(4 marks)**

State the following as y in terms of x

$2\log_2(xy) = 5\log_2 x$

$\log_2(xy)^2 = \log_2 x^5$

$\therefore (xy)^2 = x^5$ ✓

$x^2y^2 = x^5$ ✓

$y^2 = \frac{x^5}{x^3}$ ✓

$y^2 = x^3$

$y = \sqrt{x^3}$ or $x^{\frac{3}{2}}$ ✓

if $\pm$ then (-1) mark → as $x > 0$ then y also > 0