PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course _ Mathematics Methods____ Year _11_

Student name: Mark Inguide Teacher name: ______________

Date: 21 September 2020_____

Task type: Response

Time allowed for this task: ___45____ mins

Number of questions: ___7________

Materials required: ***This assessment is calculator-free***

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper (double sided)

Marks available: __44____ marks

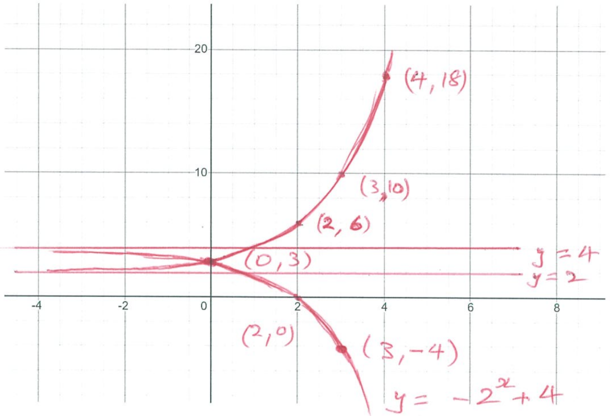
Task weighting: _16___%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 (2.1.1- 2.1.7) [5+1+4 = 10 marks]

(a) Sketch the graphs of $y = 2^x + 2$ and $y = -2^x + 4$ on the axes below, showing important features of each graph.



Asymptotes
✓ $y=2$
✓ $y=4$

Two points on each
✓ ✓

Shape ✓

(b) Using your graph (or otherwise), find the intersection point of these two functions.

From the graph, intersection is $(0,3)$ ✓

or $2^x + 2 = -2^x + 4$ OR

$\Leftrightarrow 2^x + 2^x = 2$ ✓

$\Leftrightarrow 2^{x+1} = 2^1 \Rightarrow x + 1 = 1 \Rightarrow x = 0$

(c) Solve for x: $9^{2x-1} = 243$

$9^{2x-1} = 243$ ✓ $9 = 3^2$ $243 = 3^5$ ✓

Thus $3^{2(2x-1)} = 3^5$

Equate indices

$4x - 2 = 5$ ✓

$\therefore 4x = 7$

$x = 7/4$ ✓

Question 2 (2.3.1, 2.3.4, 2.3.5) [4+2 = 6 marks]

(a) For the function $f(x) = 3x^2$, use *first principles* to find $\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ and hence show that $f'(x) = 6x$

$f(x) = 3x^2 \qquad f(x+h) = 3(x+h)^2$

$\therefore \lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to0}\frac{3(x^2+2xh+h^2)-3x^2}{h}$ ✓

$= \lim_{h\to0}\frac{3x^2+6xh+3h^2-3x^2}{h}$ ✓

$= \lim_{h\to0}\frac{\cancel{h}(6x+3h)}{\cancel{h}}$ ✓ $= 6x$ as $h\to0$. ✓

(b) Briefly describe what $\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ represents on a graph of $f(x)$.

Instantaneous rate of change of $f(x)$ at x. or

OR Gradient of tangent to $f(x)$ at point x.

Question 3 (2.3.7, 2.3.13 – 2.3.17) [4+4 = 8 marks]

The curve with the equation $y = (x+1)(x-2)(x-5)$ cuts the $x - axis$ at the points $A\,(-1,0)$, $B\,(2,0)$ and $C\,(5,0)$. The expanded equation is $y = x^3 - 6x^2 + 3x + 10$

(a) Find $\frac{dy}{dx}$ and hence show that the *tangents* to the curve at points A and C are parallel.

$\frac{dy}{dx} = 3x^2 - 12x + 3$ ✓

At A, $x = -1 \rightarrow \frac{dy}{dx} = 3 + 12 + 3 = 18$ ✓ (both)

At C, $x = 5 \rightarrow \frac{dy}{dx} = 3(25) - 12(5) + 3$

$= 75 - 60 + 3 = 18$

Tangents have the same gradient ✓

$\therefore$ tangents are parallel. ✓

(b) Find the equation of the *tangent* to the curve at the point C and find the point (x, y) where the *tangent* crosses the $y - axis$.

$y = 18x + c$ at $(5,0)$ Substitute (x,y)

$0 = 18(5) + c$

$\Rightarrow c = -90$

$\boxed{y = 18x - 90}$

y intercept $= (0, -90)$

Question 4 (2.3.8 - 2.3.11) [3+3 = 6 marks]

A jet pilot follows a flight path defined by $f(x) = x^3 - 9x^2 + 15x - 8$.

(a) Is the gradient of the flight path positive (going up) or negative (down) at the point $(2, -6)$? Explain your answer.

$f(x) = x^3 - 9x^2 + 15x - 8$

$f'(x) = 3x^2 - 18x + 15$ Substitute $x = 2$

$= 12 - 36 + 15$

$= -9$

Negative gradient shows that the flight path is downwards at $(2,-6)$ (or $x = 2$)

(b) At what $x - \boldsymbol{values}$ on the curve $f(x)$ is the tangent *parallel* to the line $y = 3$?

$y = 3 \Rightarrow y' = 0$

$\therefore$ Solve $f'(x) = 3x^2 - 18x + 15 = 0$

$= 3(x^2 - 6x + 5)$

$= 3(x-5)(x-1) \Rightarrow x = 5$ or 1 both

Question 5 (2.3.3 - 2.3.7, 2.3.22) [4 marks]

Find y in terms of x if $\frac{dy}{dx} = 3x^2 - 2x - 6$ and the function $f(x)$ passes through the point $(2, 4)$.

$f'(x) = 3x^2 - 2x - 6$

Write Anti-derivative
Add c
Substitute
State answer

$f(x) = \int (3x^2 - 2x - 6)\,dx$

$= \frac{3x^3}{3} - \frac{2x^2}{2} - 6x + c$

At $(2,4)$, $\Rightarrow 4 = 2^3 - 2^2 - 6(2) + c \Rightarrow c = 8$

$\therefore y = x^3 - x^2 - 6x + 8$

Question 6 (2.3.10) [4 marks]

A section of roller coaster has been constructed using the function:

$$f(x) = x^3 + 3x^2 - 4$$

An amusement park photographer is taking "action shots" near the roller coaster where the gradient is equal to -3 ("*negative* 3"). In terms of $x - values$, where is the photographer working? Explain your answer with suitable working.

$f(x) = x^3 + 3x^2 - 4$

$f'(x) = 3x^2 + 6x$

Set $f'(x) = -3$

$\therefore 3x^2 + 6x = -3$

or $3x^2 + 6x + 3 = 0$

$\therefore 3(x^2 + 2x + 1) = 0$

$\therefore 3(x+1)^2 = 0 \Rightarrow x = -1$ — Ans

(check)

$f''(x) = 6x + 6$

at $x = -1$, $f''(x) = 0$

Hence point of inflection

Question 7 (2.3.19, 2.3.22) [3+3 =6 marks]

A function $V(t)$ for which $V'(t) = 4t + k$, (where k is a constant), has a turning point at $(1, -2)$. Find:

(a) The value of k

$V'(t) = 4t + k = 0$ at $(1,-2) \Rightarrow \boxed{k = -4}$

$\therefore V(t) = \frac{4t^2}{2} + kt + c = 2t^2 + kt + c$

(b) The value of $V(t)$ when $t = 4$

$V(t) = 2t^2 - 4t + c$ Subs $(1,-2)$

$-2 = 2 - 4 + c \Rightarrow c = 0$

$V(t) = 2t^2 - 4t$

When $t = 4$, $V(t) = 2(16) - 4(4) = 16$

END OF ASSESSMENT