



Perth Modern School

PERTH MODERN SCHOOL
Exceptional schooling Exceptional students.

Semester One Examination, 2016

Question/Answer Booklet

**MATHEMATICS
METHODS
UNIT 3
Section Two:
Calculator-assumed**

If required by your examination administrator, please
place your student identification label in this box

Student Number: In figures

<input type="text"/>					
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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes

Working time for section:

one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	48	35
Section Two: Calculator-assumed	13	13	100	101	65
Total				149	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Please indicate if you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula Sheet is not to be handed in with your Question/Answer Booklet.

Additional working space

Question number: _____

Question 9 (5 marks)

A recent news report said that it took 34 months for the population of Australia to increase from 23 to 24 million people.

- (a) Assuming that the rate of growth of the population can be modelled by the equation $\frac{dP}{dt} = kP$, where P is the population of Australia at time t months, determine the value of the constant k . (3 marks)

(c)

Show that the acceleration of the particle is always positive.

(2 marks)

- (b) Assuming the current rate of growth continues, how long will it take for the population to increase from 24 million to 25 million people? (2 marks)

(d)

Determine the displacement of the particle at the instant it is stationary.

(2 marks)

(ii) Calculate k . (2 marks)

(e)

Determine the velocity function, $v(t)$, for the particle.

(2 marks)

(i) Explain why $x = \int v(t) dt$. (1 mark)

(f)

A particle moves in a straight line according to the function $x(t) = t^2 - 4t + 3$, where t is in seconds and x is the displacement of the particle from a fixed point O , in metres.

(1 mark)

(g) After five seconds, the particle has moved a distance of m metres. (9 marks)

See next page

(h) Determine the velocity function, $v(t)$, for the particle.

(2 marks)

(i) Calculate s . (3 marks)

(2 marks)

(j) Calculate s . (3 marks)

(2 marks)

(k) Calculate s . (3 marks)

(2 marks)

(l) Calculate s . (3 marks)

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(2 marks)

Question 10

A small object is moving in a straight line with acceleration $a = ct + k \text{ ms}^{-2}$, where t is the time in seconds and k is a constant. When $t = 1$ the object was stationary and had a displacement of 4 metres relative to a fixed point O on the line. When $t = 2$ the object had a velocity of 1 ms^{-1} .

- (a) Determine the value of k and hence an equation for the velocity of the object at time t . (4 marks)

Question 20

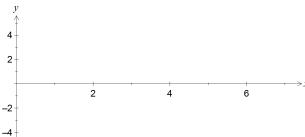
Consider the function $f(t) = \frac{t+4}{2}$ and the function $A(x) = \int f(t) dt$.

- (a) Complete the table below. (2 marks)

x	0	1	2	3	4	5	6
$A(x)$	0	-1.75					

- (b) For what value(s) of x is the function $A(x)$ increasing? (1 mark)

- (c) On the axes below, sketch the graph of $y = A(x)$ for $0 \leq x \leq 6$. (2 marks)



- (d) Determine

(i) when $A'(x) = 0$. (1 mark)

- (c) If ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study advanced mathematics. (2 marks)

- (ii) the function $A(x)$ in terms of x . (1 mark)

See next page

Question 11

It is known that 15% of Year 12 students in a large country study advanced mathematics.

A random sample of n students is selected from all Year 12's in this country, and the random variable X is the number of those in the sample who study advanced mathematics.

- (a) Describe the distribution of X . (2 marks)

(2 marks)

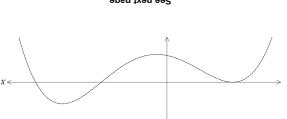
- (b) If $n = 22$, determine the probability that

- (i) three of the students in the sample study advanced mathematics. (1 mark)

- (c) On the axes below, sketch the graph of $y = f(x)$ for $0 \leq x \leq 6$. (2 marks)

- (ii) more than three of the students in the sample study advanced mathematics. (1 mark)

- (iii) none of the students in the sample study advanced mathematics. (1 mark)



- (d) On the axes below, sketch a possible graph of $y = f(x)$. The graph of $y = f(x)$ is

(5 marks)

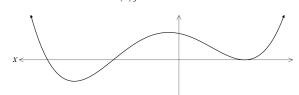
- (e) Does the graph of $y = f(x)$ have a horizontal point of inflection? Justify your answer. (2 marks)

(2 marks)

- (f) Does the graph of $y = f(x)$ have a local maximum? Justify your answer. (2 marks)

(2 marks)

- (g) For what values of a does the graph of $y = f(x)$ have a point of inflection? (1 mark)



(1 mark)

- (h) The graph of $y = f(x)$, the derivative of a polynomial function f , is shown below. The graph of $y = f(x)$ has stationary points when $x = a$, $x = b$, and $x = c$, where $a < b < c < d < e$, and $x = d$, and roots when $x = u$, $x = v$, and $x = w$, where $u < v < w < d$. Determine the mean of a Bernoulli distribution with probability of success p is repeated n times. The resulting

- (a) distribution of the number of successes has an expected value of 5.76 and a standard deviation of 1.24. Determine n and p . (4 marks)

(4 marks)

- (a) Determining the mean of a Bernoulli distribution with variance of 0.24. (3 marks)

(3 marks)

- Question 17 (1 mark)

METHODS UNIT 3

CALCULATOR-ASSUMED

CALCULATOR-ASSUMED

METHODS UNIT 3

See next page

See next page

Question 12

(8 marks)

The height of grain in a silo, initially 0.4 m, is increasing at a rate given by $h'(t) = 0.55 + 0.05t^2$ for $0 \leq t \leq 11$, where h is the height of grain in metres and t is in hours.

- (a) At what time is the height of grain rising the fastest? (2 marks)

(2 marks)

- (b) Determine the height of grain in the silo after 11 hours. (3 marks)

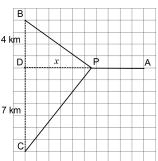
(3 marks)

Question 19

(7 marks)

Three telecommunication towers, A, B and C, each need underground power cable connections directly to a new power station, P, that is to be built x km from depot D on a 10 km road running east-west between D and A.

Tower B lies 4 km due north of depot D and tower C lies 7 km south of the depot, as shown in the diagram.



- (a) Determine an expression for the total length of underground cable required to connect A, B and C directly to P. (2 marks)

(2 marks)

- (b) Show that the minimum length of cable occurs when $\frac{x}{\sqrt{16+x^2}} + \frac{x}{\sqrt{49+x^2}} = 1$. (3 marks)

(3 marks)

- (c) Calculate the time taken for the grain to reach a height of 4.45 m. (3 marks)

(3 marks)

- (c) Determine the minimum length of cable required, rounding your answer to the nearest hundred metres. (2 marks)

(2 marks)

See next page

See next page

(2 marks)

See next page

(2 marks)