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Test 5

Continuous Random Variables

The Normal Disribution

Sample Proportions

Semester Two 2018
Year 12 Mathematics Methods
Calculator Assumed

Name:

Teacher:

_____ **Mr McClelland**

_____ **Miss Berry**

_____ **Mr Gannon**

_____ **Ms Cheng**

_____ **Mr Staffe**

_____ **Mr Strain**

Date: Fri 17th Aug. **7:45am**

You may have a formula sheet for this section of the test.

Classpad Calculators

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Total_____/47 **50 minutes**

Question 1 **(5 marks)**

The life of an electronic component is given by the probability density function:

$$f(x) = \begin{cases} \frac{100}{x^2} & x > 100 \\ 0 & \text{otherwise} \end{cases}$$

Find:

(a) the probability that a component lasts for more than 250 hours. (2 marks)

(b) the median life of a component. (2 marks)

(c) the lifetime for 95% of components. (1 mark)

Question 2 **(4 marks)**

(a) Pr($Z < -0.376$), where Z is a standard normal random variable is: (1 mark)

(b) If Z is a standard normal random variable, and Pr($Z > c$) = 0.75, then the value of c is? (1 mark)

(c) If X is a normally distributed random variable with mean $\mu = 4$ and standard deviation, $\sigma = \sqrt{2}$, then the transformation that maps the curve of the density function of X, $f(x)$, to the curve of the standard normal distribution is: (2 marks)

Question 3 **(2 marks)**

The weight of a population of teenage females is normally distributed with a mean of 55 kg and a standard deviation of 8 kg. If the lowest 5% of teenage females is classified as underweight, what is the cut-off weight for this group?

Question 4 **(6 marks)**

A probability density function is given by

$$f(x) = Ax(6 - x)^2 \qquad 0 < x < 6$$

Find the value of A and hence the mean and the standard deviation of this distribution.

Question 5 **(10 marks)**

A taxi company determined that on an annual basis the distance travelled per taxi is normally distributed with a mean of 92 000 kilometres and a standard deviation of 23 500 kilometres.

(a) What is the probability, correct to four decimal places, that a taxi travels less than 75 000 kilometres per year?

(b) What is the probability, correct to four decimal places, that a taxi travels more than 80 000 kilometres per year?

(c) What is the probability, correct to four decimal places, that a taxi travels between 60 000 and 100 000 kilometres in the year?

(d) Find the minimum mileage that could be expected by 95% of taxis, to the nearest km.

(e) Fred runs a fleet of 10 taxis. What is the probability that at least four of the taxis travel more than 80 000 kilometres in a year?

Question 6 **(1 marks)**

A bag contains 4 black balls and three blue balls. If a random sample of four balls is taken from the bag, without replacement, the possible values of the sample proportion of blue balls in the sample are:

Question 7 **(9 marks)**

A random sample of 100 people indicated that 19% had taken a plane flight in the last year.

(a) Determine a 90% confidence interval for the proportion of the population that had taken a plane flight in the last year. (3 marks)

Assume the 19% sample proportion applies to the whole population.

(b) A new sample of 200 people was taken and X= the number of people who had taken a plane flight in the last year was recorded. Give a range, using the 90% confidence internal, within which you would expect X to lie. (1 mark)

(c) Determine the probability that in a random sample of 120 people, the number who had taken a plane flight in the last year was greater than 26. (3 marks)

(d) If seven surveys were taken and for each a 95% confidence interval for p was calculated, determine the probability that at least four of the intervals included the true value of p. (2 marks)

Question 8 **(10 marks)**

A random survey was conducted to estimate then proportion of mobile phone users who favoured standard smart phones over the new *phablet* style smart phones. It was found that 283 out of 412 people surveyed preferred the new *phablet* style smart phones.

(a) Determine the sample proportion $\hat{p}$ of those in the survey who preferred a phablet style smart phone. (1 mark)

(b) Use the survey results to estimate the standard deviation of $\hat{p}$, for the sample proportions. (2 marks)

(c) A follow – up survey is to be conducted to confirm the results of the initial survey. Working with a confidence interval of 95%, estimate the sample size necessary to ensure margin of error of at most 4%. (3 marks)

The 90% confidence interval of the sample proportion $\hat{p}$, from the initial survey is $0.649 \leq \hat{p} \leq 0.725$.

(d) Use the 90% confidence interval of the initial sample to compare the following samples:

(i) A random sample of 365 people at a shopping centre found that 258 had a preference for the phablet style smart phone. (2 marks)

(ii) A random sample of 78 people at a retirement village for Maths teachers 52 had a preference for the phablet style smart phone. (2 marks)