PERTH MODERN SCHOOL
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Independent Public School

Course Methods Year 12 test four 2022

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: **_____40_____ mins**

Number of questions: **____6_____**

Materials required: **Upto 3 calculators/classpads allowed**

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, **one page of A4 notes doublesided**

Marks available: **___44___ marks**

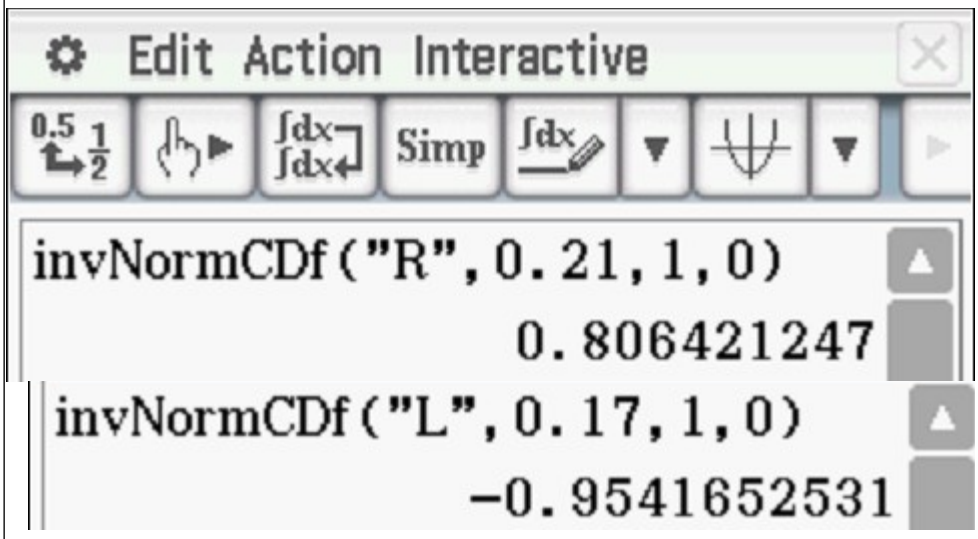
Task weighting: **_10___%**

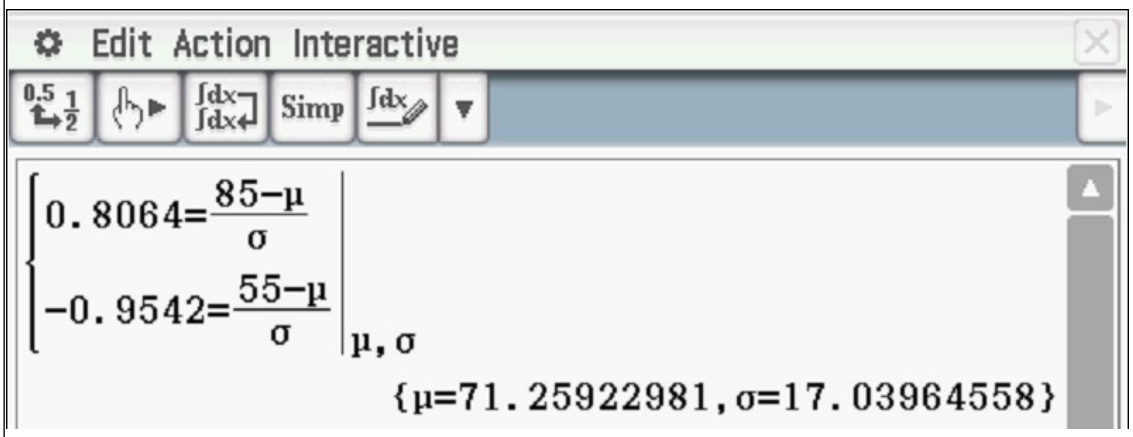
Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (4 marks) 4.2.5
The exam results, out of a 100, for a Methods exam at a particular school was found to be Normally Distributed. It was found that 21% of the students scored a result greater than 85 and 17% scored a mark less than 55. Determine the mean and standard deviation.

Solution





Specific behaviours

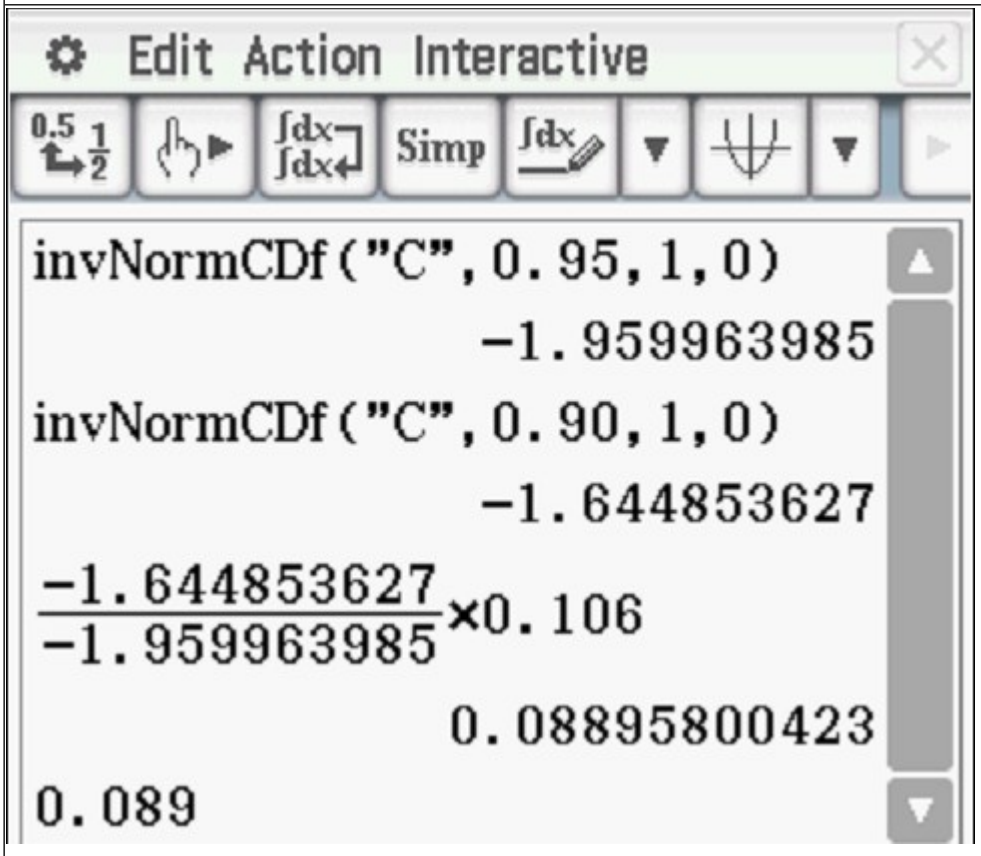
✓ determines both z scores
✓ sets up one equation for mean and stdev
✓ sets up two equations for mean and stdev
✓ solves for mean and stdev
Note- answer only max of 2 marks

Q2 (2, 2, 3 & 2 = 9 marks) 4.3.8
A 95% confidence interval was determined for the proportion of faulty factory parts made at a company. The interval length is 0.106 and the sample size of 200.
Determine the **expected length** of the interval for each change in isolation to 3 decimal places.

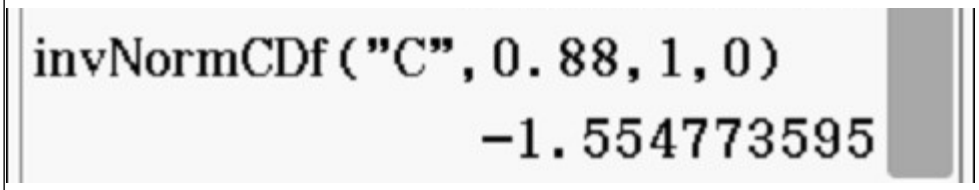
a) A sample size of 300 was used.

Solution
$L \propto \frac{1}{\sqrt{n}}$ $\frac{L_1}{L_2} = \sqrt{\frac{n_2}{n_1}}$ $= \sqrt{\frac{200}{300}}(0.106) = 0.1224$ ≈ 0.087
Specific behaviours
✓ uses correct ratio ✓ states length rounded to 3 dp

b) A 90% confidence was used.

Solution

Specific behaviours
✓ uses z quantiles ✓ states length to 3dp (Do not double penalise if not 3dp)

c) A 88% confidence AND a sample size of 150 was used.

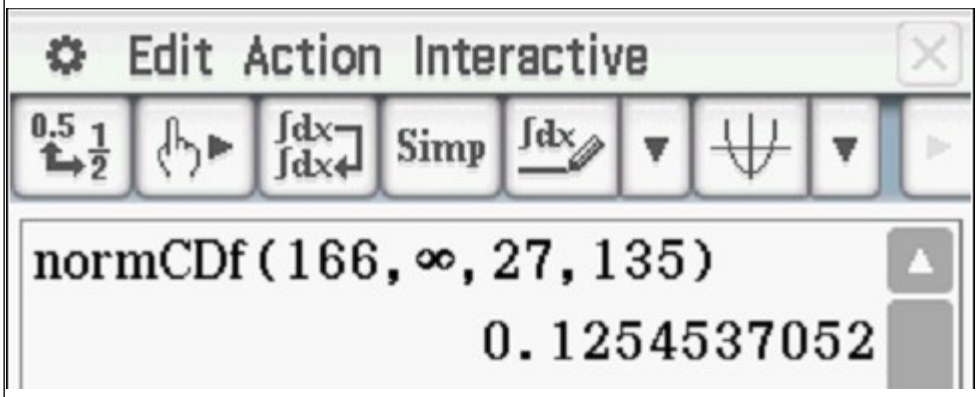
Solution
 $L \propto z\frac{1}{\sqrt{n}}$ $\frac{L_1}{L_2} = \frac{z_1}{z_2}\sqrt{\frac{n_2}{n_1}}$ $= \frac{1.5548}{1.960}\sqrt{\frac{200}{150}}(0.106) = 0.1224$ ≈ 0.097
Specific behaviours
✓ states quantile for 0.88 ✓ shows ratio involving sample sizes ✓ states length

d) The true proportion of faulty parts does not lie in the stated interval. Does this suggest a sampling error was made? Justify.

Solution
Cannot tell if there is a sampling error as not all confidence intervals contain the true value of population proportion p.
Specific behaviours
✓ states no with any reason ✓ states reason as above (Note – zero marks if no without any reason)

Q3 (2, 2 ,2, 3 ,3, 3 & 3 = 18 marks) 4.2.5, 4.2.3, 3.3.1, 3.3.6, 3.3.7
A parcel making factory makes boxes of the same width and heights nut the lengths vary and are found to be Normally Distributed with a mean of 135 mm and a standard deviation of 27 mm.

a) Determine the percentage of boxes that are longer than 166mm.

Solution

Specific behaviours
✓ uses correct parameters ✓ states prob

The boxes can be classified as the following.

b) Complete the missing probabilities in the above table.

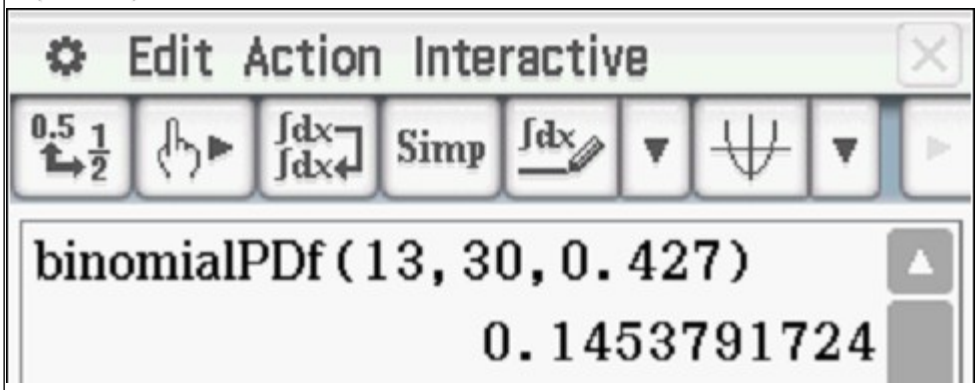
Solution				
Box	Short	Long	Very Long	Gigantic
Length	0 to 45 mm	45 to 100 mm	100 to 140 mm	Greater than 140mm
Probability	0.00043 (0.000)	0.097	0.476	0.427
Specific behaviours				
✓ Prob for short ✓ Prob for Gigantic (Must be at least 3 dp- otherwise -1 mark) Note SCSA do penalise if not sufficient dp regardless of whether asked or not.				

c) Comment on the appropriateness of the Normal Model for the lengths of the boxes.

Solution
Normal model goes from – infinity to plus infinity, both of which are impossible/impractical.
Specific behaviours

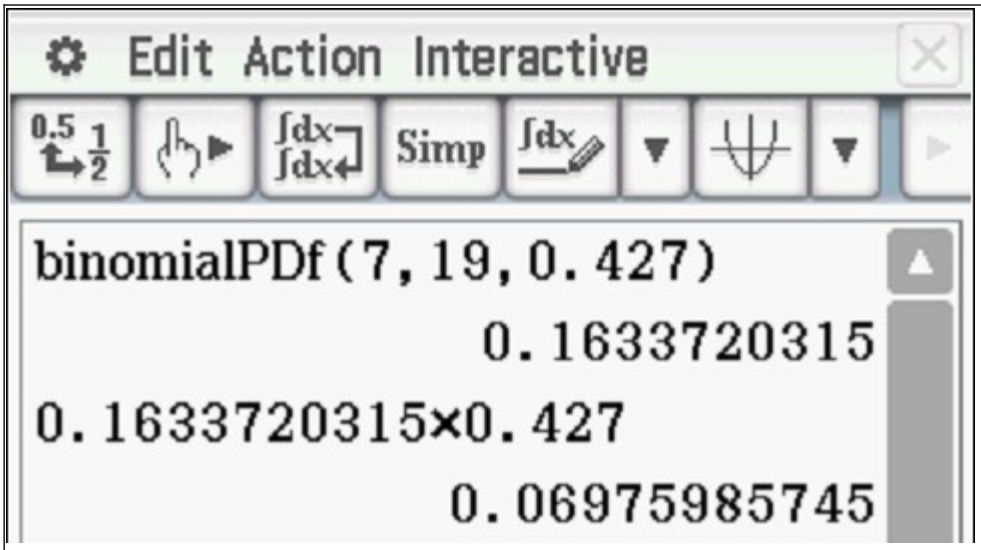
✓ discusses negative values ✓ discusses unlimited positive values

d) If 30 boxes were taken off the assembly line, determine the probability that exactly 13 were gigantic lengths.

Solution
$X \sim Bin(30, 0.427)$ $P(X = 13)$ 
Specific behaviours
✓ states Binomial with parameters ✓ uses x=13 ✓ states prob (no need to round)

e) Determine the probability that it would take 20 boxes in a row off the assembly line before 8 gigantic boxes were found.

Solution
$X \sim Bin(19, 0.427)$ $P(X = 7) \times 0.427$



Specific behaviours
✓ states Binomial with n=19 ✓ determines prob for x=7 ✓ states total prob (no need to round)

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Q3 cont
If the costs of each box were determined as follows.

Box	short	long	Very long	gigantic
Cost $	$3.21	$4.12	$5.20	$6.30

i) Determine the mean cost to two decimal places. Show all working.

Solution
$3.21\times0.00043+4.12\times0.097+5.20\times0.476+6.30\times0.427=5.57$
Specific behaviours
✓ multiplies each x score by prob ✓ shows total series ✓ states mean to 2 dp

ii) Determine the standard deviation to two decimal places. Show all working.

Solution

$(3.21 - 5.57)^2 \times 0.00043 + (4.12 - 5.57)^2 \times 0.097 + (5.20 - 5.57)^2 \times 0.476 + (6.30 - 5.57)^2 \times 0.427 = s^2$ $s = 0.71$
Specific behaviours
✓ subtracts mean from each x value and squares ✓ shows total series ✓ states stdev to 2dp (Note max of -1 for total question for rounding errors)

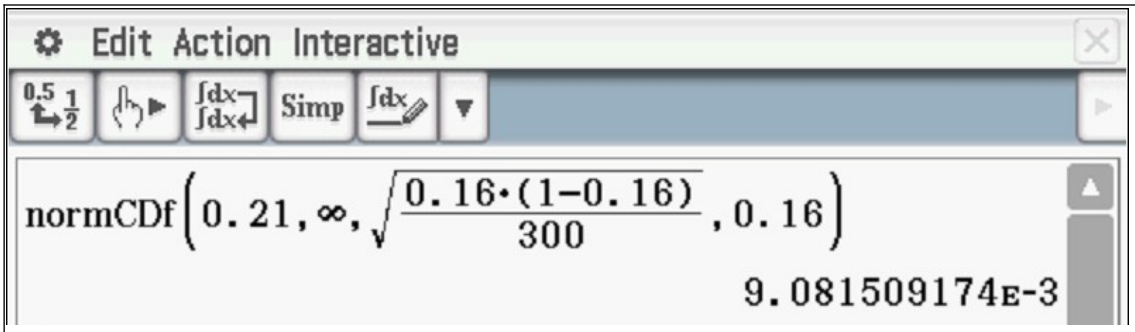
Q4 (3, 2 ,3, 3 & 2 =13 marks) 4.3.4, 4.3.5, 4.3.6, 4.3.9, 4.3.10
In Australia it has been found that 16% of people are left-handed. Samples of people are surveyed to ascertain the proportion that are left-handed. Let $\hat{p}$ denote the proportion of people in the sample who are left handed.

a) State the approximate distribution of $\hat{p}$ for sample sizes of 100.

Solution
$\hat{p} \sim N\left(0.16, \left[\sqrt{\frac{0.16(1 - 0.16)}{100}}\right]^2\right)$ $stdev = 0.037$
Specific behaviours
✓ states normal ✓ states mean ✓ states stdev

b) Determine the approximate probability that in a sample of 300 people that the proportion of left handed people is greater than 0.21.

Solution



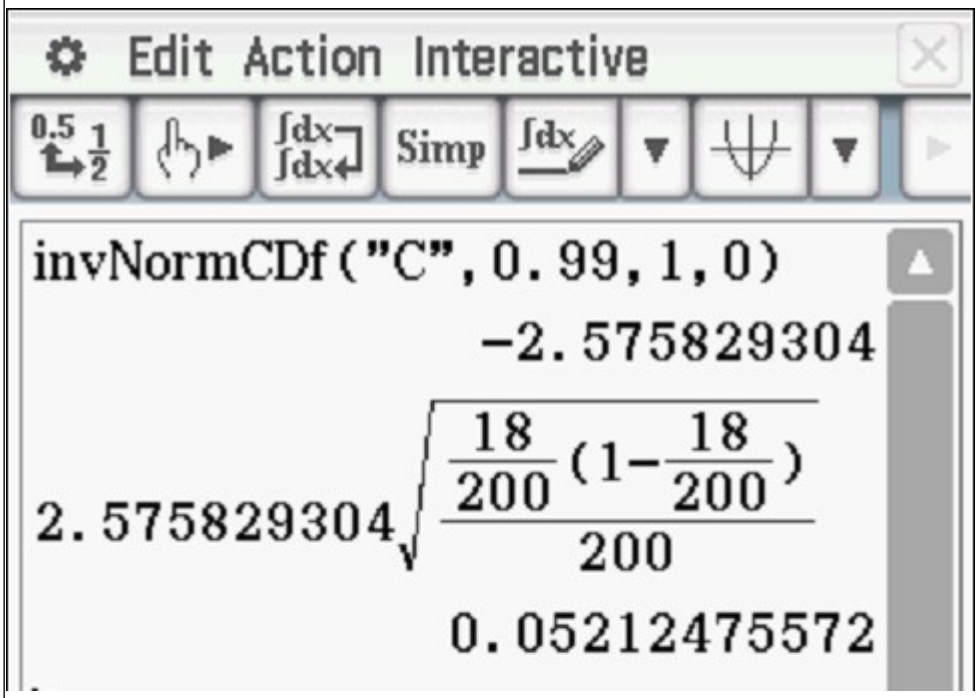
Specific behaviours
✓ uses correct paremeters ✓ states prob

Q4 cont-

In a recent survey it was found that 18 people out of a sample of 200 were left handed.

c) For a 99% confidence interval, what is the margin of error based on this recent sample of 200?

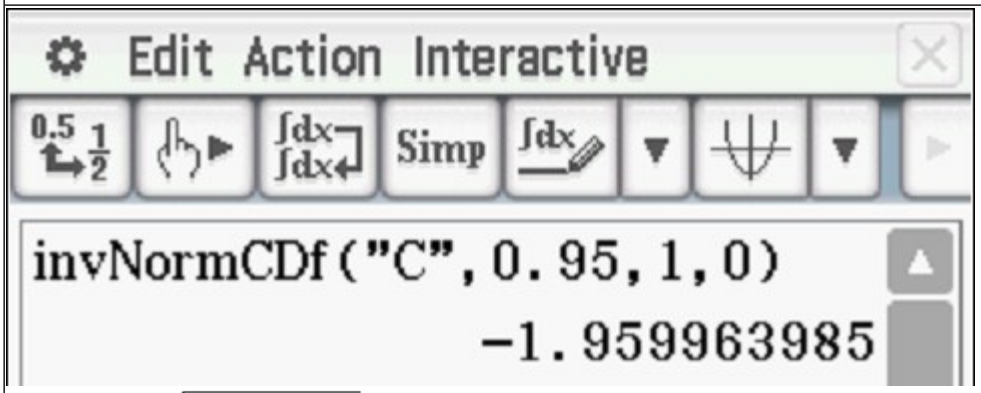
Solution



Specific behaviours
✓ determines z quantile

✓ shows working for margin of error ✓ determines margin of error

d) Determine a 95% confidence interval based on the recent sample of 200 people.

Solution
 $\frac{18}{200} \pm 1.960\sqrt{\frac{\frac{18}{200}(1-\frac{18}{200})}{200}}$ (0.0503 - 0.1297)
Specific behaviours
✓ determines z quantile ✓ shows working for confidence interval ✓ states interval Note: 2 marks for answer only

e) Does the recent sample support the assumed proportion of 16% for left handed people? Explain.

Solution
The recent sample does not support the assumed model as 0.16 does not fit in the interval. (SCSA preferred answer) OR One sample cannot be used to make an inference as not all confidence intervals contain the true population p

Specific behaviours
✓ states that support cannot be made with any reason ✓ one of the reasons stated above

Working out space