



Rossmyrne Senior High School

Semester One Examination, 2021

Question/Answer booklet

**MATHEMATICS**  
**METHODS**  
**UNIT 1**  
**Section Two:**  
**Calculator-assumed**

WA student number:    In figures

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In words

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Your name

\_\_\_\_\_

Reading time before commencing work:    ten minutes  
Working time:    one hundred minutes

Number of additional  
answer booklets used  
(if applicable):

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**Materials required/recommended for this section**

*To be provided by the supervisor*

This Question/Answer booklet  
Formula sheet (retained from Section One)

*To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,  
and up to three calculators, which can include scientific, graphic and

Computer Algebra System (CAS) calculators, are permitted in this ATAR  
course examination

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that  
you do not have any unauthorised material. If you have any unauthorised material with you, hand  
it to the supervisor **before** reading any further.

See next page

**SOLUTIONS**

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	54	35
Section Two: Calculator-assumed	13	13	100	97	65
<b>Total</b>					100

**Instructions to candidates**

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

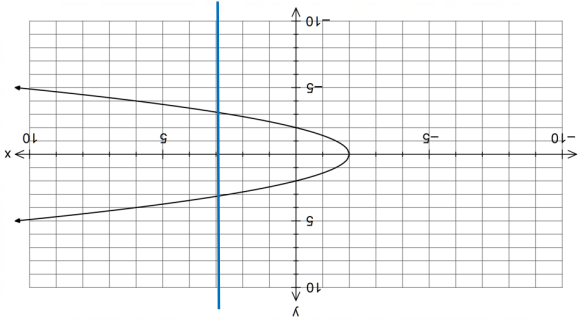
65% (97 Marks)

Working time: 100 minutes.

Question 9

(5 marks)

The graph below is given in the form  $y^2 = a(x - b)$ .



(a) Find the values of  $a$  and  $b$ .

(2 marks)

Solution
$a = 2$ $b = -2$
Specific behaviours
✓ correct value of $a$ ✓ correct value of $b$

(b) State the equation of the axis of symmetry.

(1 mark)

Solution
$y = 0$
Specific behaviours
✓ correct equation

(c) Show the vertical line test on the graph above and explain how it is used to show whether this graph is a function or not.

(2 marks)

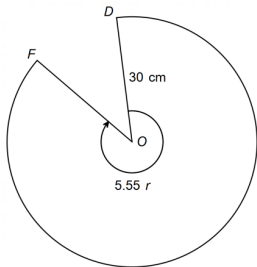
Solution
See graph. The vertical line intersects the graph twice, indicating that it is a one-to-many relation and therefore is not a function. Specific behaviours ✓ shows vertical line intersecting graph ✓ states that the vertical line intersects the graph twice

See next page

Question 10

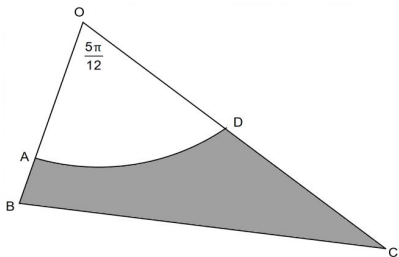
(5 marks)

- (a) The diagram below shows a sector of a circle with centre O. The radius of the circle is 30cm and  $\angle DOF = 5.55$  radians. Calculate the length of the major arc DF. (2 marks)



Solution
$L = r\theta$ $= 30 \times 5.55$ $= 166.5\text{cm}$
Specific behaviours
✓ uses arc length formula with radians ✓ correct arc length NB: Answer only ok

- (b) In the diagram below AOD is a sector of the circle with centre O. BOC is a triangle.
- In sector AOD, the radius is 30 cm and angle AOD is  $\frac{5\pi}{12}$  radians.
- In triangle OBC, OB = 38 cm and OC = 55 cm.
- Calculate the shaded area of the shape with the vertices of ABCD rounded to 3 decimal places.
- (3 marks)



Solution		
$A_a = 0.5ab \sin \theta$ $= (0.5)(38)(55) \sin \left( \frac{5\pi}{12} \right)$ $= 1009.3925 \text{ cm}^2$	$A_{\text{sector}} = 0.5r^2\theta$ $= (0.5)(30)^2 \left( \frac{5\pi}{12} \right)$ $= 589.0486 \text{ cm}^2$	$A_{\text{shaded}} = 1009.3925 - 589.0486$ $= 420.344 \text{ cm}^2$
Specific behaviours		
<ul style="list-style-type: none"><li>✓ area of triangle</li><li>✓ area of sector</li><li>✓ shaded area, correct to 3dp</li></ul> <p>NB: No marks awarded if calculator is in degree mode as it will give a negative answer for the final area (area of triangle will be 23.87cm<sup>2</sup>)</p>		

Supplementary page

Question number: \_\_\_\_\_

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**Question 11** (8 marks)  
A study of the achievements of 360 students enrolled in a university course yielded the following information:

- 50% of all students achieved a distinction
- 60% of those who did not achieve a distinction studied part-time
- 45% of those who studied full-time did not achieve a distinction

(a) Use the above information to complete the following table. (4 marks)

	Full-time	Part-time	Totals
Distinction	88	92	180
No distinction	72	108	180
Totals	160	200	360

Solution		
$360 \times 0.5 = 180,$	$180 \times 0.6 = 108,$	$180 - 108 = 72$
$\frac{72}{x} = 0.45 \Rightarrow x = 160,$	$160 - 72 = 88,$	$180 - 88 = 92$
Specific behaviours		
✓ splits total to D/ND; ✓ splits no distinction to FT/PT ✓ calculates PT total; ✓ completes table		

(b) Determine the probability that a randomly chosen student from the study

(i) achieved a distinction and studied full-time. (1 mark)

Solution
$P = \frac{88}{360} = \frac{11}{45} = 0.2\bar{4}$
Specific behaviours
✓ correct probability

(iii) achieved a distinction or studied full-time. (1 mark)

Solution
$P = \frac{360 - 108}{360} = \frac{252}{360} = \frac{7}{10}$
Specific behaviours
✓ correct probability

(c) Sets  $F$  and  $D$  are subsets of the students in the study. A student belongs to  $F$  if they studied full-time and to  $D$  if they achieved a distinction. Use set notation to describe the subset of part-time students who achieved a distinction. (2 marks)

Solution
$F \cap D$
Specific behaviours
✓ uses complement notation ✓ correct set notation

See next page

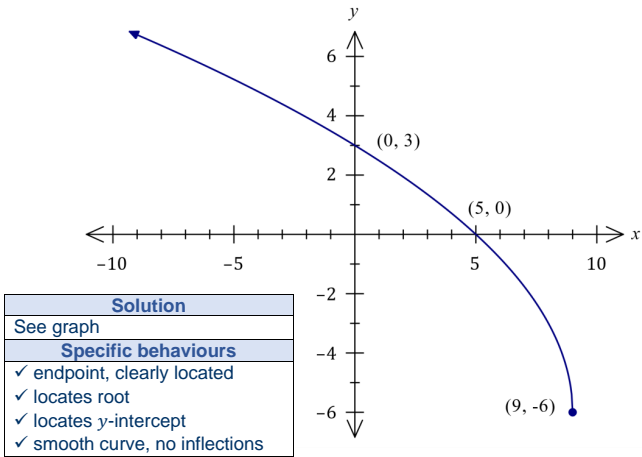
Question 12

(8 marks)

Let  $f(x) = 3\sqrt{9-x} - 6$ .

(a) Sketch the graph of  $y = f(x)$  on the axes below.

(4 marks)



(b) Describe the transformation(s) required to obtain the graphs of the following functions from the graph of  $y = 3\sqrt{9-x} - 6$ :

(i)  $y = \sqrt{9-x} - 2$ .

(2 marks)

Solution
$y = \frac{1}{3}f(x)$ . Vertical dilation OR dilation parallel to the y-axis OR dilation from the x-axis of scale factor $\frac{1}{3}$ .
Specific behaviours
✓ both "dilation" and correct direction in description
✓ correct scale factor

(ii)  $y = 3\sqrt{1-x} - 6$ .

(2 marks)

Solution
$y = f(x-8)$ . Horizontal translation of 8 units to the left.
Specific behaviours
✓ states a translation
✓ correct distance and direction

Supplementary page

Question number: \_\_\_\_\_

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Question 13 (8 marks)

(a) Triangle  $ABC$  is such that  $b = 15$  cm,  $c = 18$  cm and  $\angle A = 125^\circ$ . Determine, with justification, the length of side  $a$ . (2 marks)

Solution
$a^2 = 15^2 + 18^2 - 2(15)(18) \cos 125^\circ$ $a = 29.3 \text{ cm}$
Specific behaviours
✓ clearly shows use of cosine rule ✓ correct length

(b)

Triangle  $PQR$  is such that  $p = 48.1$  cm,  $q = 41.5$  cm and  $\angle Q = 45^\circ$ . Determine all possible areas of this triangle. (6 marks)

Solution
$\frac{\sin P}{48.1} = \frac{\sin 45^\circ}{41.5}$ First solution: $\angle P_1 = 55^\circ$ $\angle R_1 = 180^\circ - 45^\circ - 55^\circ = 80^\circ$ $A_1 = \frac{1}{2}(48.1)(41.5) \sin 80^\circ$ $A_1 = 983 \text{ cm}^2$ Second solution: $\angle P_2 = 180^\circ - 55^\circ = 125^\circ$ $\angle R_2 = 180^\circ - 45^\circ - 125^\circ = 10^\circ$ $A_2 = \frac{1}{2}(48.1)(41.5) \sin 10^\circ$ $A_2 = 174 \text{ cm}^2$ Areas are $174 \text{ cm}^2$ and $983 \text{ cm}^2$ .
Specific behaviours
✓ shows use of sine rule ✓ calculates one value for $\angle P$ ✓ derives $\angle R$ and shows use of area formula ✓ calculates one correct area ✓ calculates second set of values for $\angle P$ and $\angle R$ ✓ calculates second area

See next page

Question 14

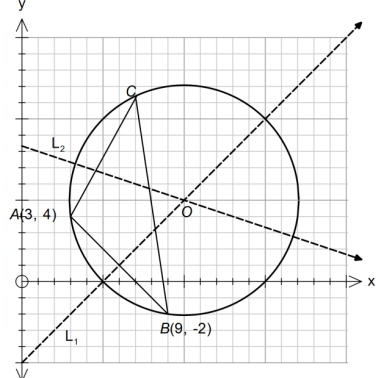
(5 marks)

Triangle  $ABC$  is shown below where  $A(3,4)$  and  $B(9,-2)$ .

The line  $L_1$  is the perpendicular bisector of side  $AB$ .

The line  $L_2$  intersects side  $AC$  and has the equation  $3y + x = 25$ .

$O$  is the centre of the circle  $(x-10)^2 + (y-5)^2 = 50$  which passes through the vertices of  $\triangle ABC$ .



Show algebraically that  $O$  is the intersection of  $L_1$  and  $L_2$ .

Solution	
$m_{AB} = \frac{-2-4}{9-3} = \frac{-6}{6} = -1$	$L_1 = L_2$ $y+5 = 25-3y$ $4y = 20$ $y = 5$ $x = 10$ $\therefore \text{Int @ } (10,5)$
$m_{L_1} = 1$	$L_1 : y = x - 5$
From the circle equation, the centre is also at $(10,5)$ .	
$\therefore$ Lines intersect at $O$ .	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ gradient of <math>L_1</math></li> <li>✓ equation of <math>L_1</math></li> <li>✓ equates <math>L_1</math> and <math>L_2</math></li> <li>✓ solves to find the coordinates of the intersection</li> <li>✓ states that the intersection is the same point as the centre of the circle, as can be determined from the circle equation</li> </ul>	

Alternate Solution	
$m_{AB} = \frac{-2-4}{9-3} = \frac{-6}{6} = -1$	$L_2 : y = -\frac{x}{3} + \frac{25}{3}$ Centre $\rightarrow (10,5)$ $5 = -\frac{10}{3} + \frac{25}{3}$ $5 = \frac{15}{3}$ $5 = 5$
$m_{L_1} = 1$	$L_1 : y = x - 5$
From the circle equation, the centre is also at $(10,5)$ .	
$\therefore$ Lines intersect at $O$ .	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ gradient of <math>L_1</math></li> <li>✓ equation of <math>L_1</math></li> <li>✓ checks centre lies on <math>L_1</math></li> <li>✓ checks centre lies on <math>L_2</math></li> <li>✓ concludes that the lines intersect at the centre of the circle</li> </ul>	

Question 21 (cont.)

- (ii) Determine  $n(C \cup M \cup P)$  (1 mark)

Solution
$n(C \cup M \cup P) = 54 + 2 + 9 = 65$
Specific behaviours
✓ correct value

- (iii) If one student is selected at random from the group, determine the probability of the following scenarios, leaving your answers as unsimplified fractions:

- (a) They elected to study Maths but not Physics. (2 marks)

Solution
$P(M \cap \bar{P}) = \frac{16+5}{65}$ $= \frac{21}{65}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correct numerator</li> <li>✓ correct denominator</li> </ul>

- (b) They elected to study Maths and Physics, given that they did not study chemistry. (2 marks)

Solution
$P(M \cap P   \bar{C}) = \frac{23}{16+23+9}$ $= \frac{23}{48}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correct numerator</li> <li>✓ correct denominator</li> </ul>

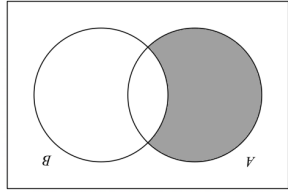
- (c) They elected to study two of the subjects, given that they did not elect to study all three subjects. (2 marks)

Solution
$P(\text{two}   \bar{C} \cap M \cap P) = \frac{2+5+23}{55}$ $= \frac{30}{55}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correct numerator</li> <li>✓ correct denominator</li> </ul>



Question 21

(a) Consider the two Venn diagrams below:

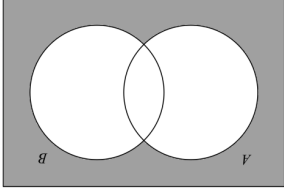


(i) Shade the region corresponding to  $A \cap \overline{B}$ .

(1 mark)

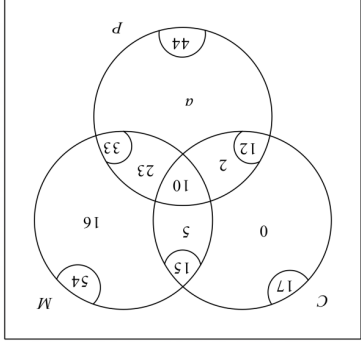
(ii) Use symbolic set notation to describe the shaded regions below.

(1 mark)



<b>Solution</b>
$\overline{A \cup B}$ or $\overline{A} \cap \overline{B}$
<b>Specific behaviours</b>
✓ correct notation

(b) The following Venn diagram shows the number of students electing to study at least one of Chemistry (C), Maths (M) or Physics (P).



(i) Determine the value of  $a$ .

(1 mark)

<b>Solution</b>
$a = 44 - 2 - 10 - 23 = 9$
<b>Specific behaviours</b>
✓ correct value of $a$

See next page

Question 15

(9 marks)

The loudness  $L$  of sound, in decibels, emitted by a machine  $t$  minutes after it is switched on can be modelled by

$$L = 62 - 11 \cos\left(\frac{\pi t}{10}\right)$$

(a)

Determine the initial loudness emitted by the machine.

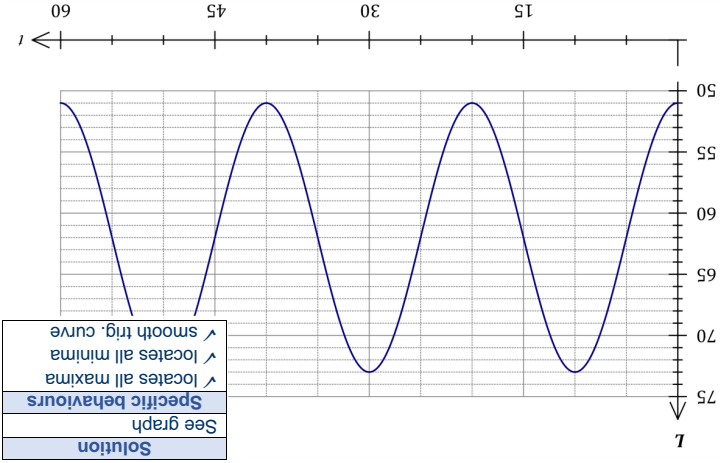
(1 mark)

<b>Solution</b>
$L(0) = 51$ dB
<b>Specific behaviours</b>
✓ correct value with units

(b)

Draw the graph of  $L$  against  $t$  on the axes below for the first 60 minutes.

(3 marks)



<b>Solution</b>
See graph
<b>Specific behaviours</b>
✓ locates all maxima
✓ locates all minima
✓ smooth trig. curve

(c) State the maximum loudness emitted by the machine and the time this maximum was first reached.

(2 marks)

<b>Solution</b>
$L_{max} = 73$ dB when $t = 10$ mins.
<b>Specific behaviours</b>
✓ correct maximum with units (penalise once in this question)
✓ correct time

(d) A health and safety inspector can deem a machine unserviceable if the loudness it emits exceeds 70 dB for more than 15 minutes in any hour that it is running. Determine, with justification, whether this machine could be deemed unserviceable.

(3 marks)

<b>Solution</b>
Exceeds value for $7.59 < t < 12.41$ during first cycle.
$3(12.41 - 7.59) = 14.5$ minutes per hour - and so machine could not be deemed unserviceable.
<b>Specific behaviours</b>
✓ identifies interval endpoints
✓ calculates minutes per hour (allow 14.4 mins)
✓ uses calculations to draw conclusion

See next page

Question 16

(8 marks)

A souvenir shop sells T-shirts in two colours and three sizes. Sales records for the past year are shown below.

	Small	Medium	Large
Blue	210	420	310
White	230	450	180

Assume that the shop holds a large stock and that sales continue in similar proportions. Where relevant, round your answers in this question to three decimal places.

(a) A customer randomly selects a T-shirt for purchase. Determine:

- (i) the size and colour of the least likely T-shirt and the probability that this T-shirt is selected. (2 marks)

Solution
Least likely: White, large.
$P = \frac{180}{1800} = 0.1$
Specific behaviours
✓ type of T-shirt ✓ calculates probability

- (ii) the probability that the T-shirt selected is not small. (2 marks)

Solution
$P = \frac{870 + 490}{1600} = \frac{1360}{1800} = \frac{34}{45} \approx 0.756$
Specific behaviours
✓ counts required sizes (1360) ✓ calculates probability NB: Answer only ok, don't penalise rounding here

(b) A customer randomly selects two T-shirts for purchase. Determine the probability that the T-shirts are:

- (i) both medium. (2 marks)

Solution
$P(MM) = \left(\frac{870}{1800}\right)^2 = \frac{841}{3600} \approx 0.234$
Specific behaviours
✓ probability of one medium ✓ calculates probability

- (ii) of different colours. (2 marks)

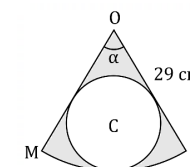
Solution
$P(BW) = \frac{940}{1800} \times \frac{860}{1800} = \frac{2021}{8100} (\approx 0.2495)$
$\therefore P(WB \cup BW) = 2 \times \frac{2021}{8100} = \frac{2021}{4050} \approx 0.499$
Specific behaviours
✓ probability of one then the other ✓ calculates probability

Question 20

(7 marks)

The diagram shows sector  $OMN$  of a circle centre  $O$  of radius 29 cm and  $\alpha = 68^\circ$ .

Circle  $C$  is inside the sector and just touches  $OM$ ,  $ON$  and arc  $MN$ .



- (a) Determine the area of sector  $OMN$ . (2 marks)

Solution
$A = \frac{68^\circ}{360^\circ} \times \pi(29)^2 = \frac{14297\pi}{90} \approx 499 \text{ cm}^2$
Specific behaviours
✓ indicates suitable method ✓ calculates area

- (b) Show that the radius of circle  $C$  is 10.4 cm, correct to one decimal place. (3 marks)

Solution
Extend sector sides to form isosceles triangle:
$PN' = 29 \tan\left(\frac{68^\circ}{2}\right) = 19.56 \text{ cm}$
$r = 19.56 \tan\left(\frac{56^\circ}{2}\right) = 10.4 \text{ cm}$
Specific behaviours
✓ forms isosceles triangle, shows angles ✓ calculates $PN'$ ✓ calculates radius

Alternate Solution
$2r + y = 29$ $(r + y)^2 = x^2 + r^2$ $\tan 34^\circ = \frac{r}{x}$
CAS solve: $r = 10.4 \text{ cm}$
Specific behaviours
✓ forms triangle OCQ ✓ sets up three equations to solve simultaneously ✓ calculates radius

- (c) Determine the area of the shaded region, inside sector  $OMN$  but outside circle  $C$ . (2 marks)

Solution
$A_C = \pi(10.4)^2 \approx 340$
Shaded area = $499 - 340 = 159 \text{ cm}^2$
Specific behaviours
✓ calculates area of circle ✓ calculates shaded area, with units NB: Answer only ok

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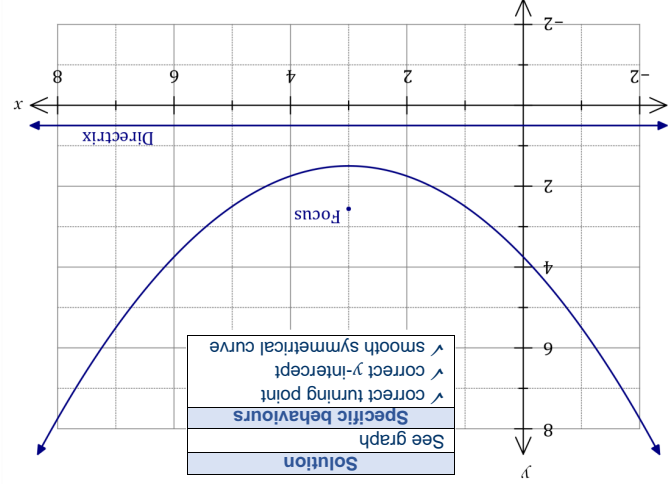
Question 19

(7 marks)

The equation of a parabola is  $y = \frac{1}{4}(x^2 - 6x + 15)$ .

(a) Sketch the parabola on the axes below.

(3 marks)



All parabolas have a focal point and a directrix. For a parabola with equation  $y = a(x - p)^2 + q$ , the focal point is at  $(p, q + \frac{1}{4a})$  and the equation of the directrix is  $y = q - \frac{1}{4a}$  where  $a, p$  and  $q$  are constants.

(b) Determine the focal point and directrix for this parabola and add them, with labels, to your sketch above.

(4 marks)

<b>Solution</b>
From graph, turning point at (3, 1.5). Hence $a = \frac{1}{4}, p = 3, q = 1.5$
<b>Specific behaviours</b>
✓ indicates turning point ✓ indicates values of all constants ✓ plots focus ✓ draws directrix

See next page

Question 17

(8 marks)

(a) Let  $f(x) = x^2 + bx + c$ , where  $b$  and  $c$  are constants. The graph of  $y = f(x)$  has an axis of symmetry with equation  $x = -3$  and an axis intercept at (0, 5).

(i) State the value of the constant  $c$ .

(1 mark)

<b>Solution</b>
$c$ is the y-intercept: $c = 5$ .
<b>Specific behaviours</b>
✓ correct value

(ii) Determine the value of the constant  $b$ .

(2 marks)

<b>Solution</b>
Axis of symmetry has equation $x = -\frac{b}{2a}$ : $-3 = -\frac{b}{2} \Rightarrow b = 6$
<b>Specific behaviours</b>
✓ indicates appropriate method ✓ calculates value

(b) Let  $g(x) = 2(x - 2)^2 - 7$ . Determine

(i) the coordinates of the turning point of the graph of  $y = g(x)$ .

(1 mark)

<b>Solution</b>
Turning point is at (2, -7).
<b>Specific behaviours</b>
✓ correct coordinates

(ii) the domain and range of  $g(x)$ .

(2 marks)

<b>Solution</b>
Domain: $x \in \mathbb{R}$ , and range: $y \geq -7$ .
<b>Specific behaviours</b>
✓ states domain ✓ states range

(iii) the coordinates of the turning point of the graph of  $y = g(x - 3) + 2$ .

(2 marks)

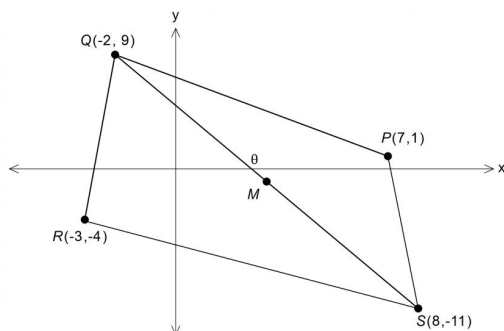
<b>Solution</b>
Graph has been translated 3 units right and 2 units upwards and so new turning point at (5, -5).
<b>Specific behaviours</b>
✓ indicates correct use of one translation ✓ correct coordinates

See next page

Question 18

(9 marks)

In the diagram  $PQRS$  is a quadrilateral having vertices  $P(7, 1)$ ,  $Q(-2, 9)$ ,  $R(-3, -4)$  and  $S(8, -11)$ .  $M$  is the midpoint of  $QS$ .



- (a) If a line is drawn from  $P$  to  $R$ , determine the equation of the line  $PR$ .

(2 marks)

Solution	
$m_{PR} = \frac{1 - (-4)}{7 - (-3)}$	$y = mx + c$
$= \frac{5}{10}$	$= \frac{1}{2}x + c$
$= \frac{1}{2}$	$1 = \frac{7}{2} + c$
	$c = -\frac{5}{2}$
	$\therefore y = \frac{x-5}{2}$
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ correct gradient</li> <li>✓ states equation of line</li> </ul>	

- (b) Determine whether  $M$  lies on the line  $PR$ . (3 marks)

Solution	
$M @ \left( \frac{-2+8}{2}, \frac{9-11}{2} \right)$	$y = \frac{x-5}{2}$
$M @ (3, -1)$	$y_{ x=3} = \frac{3-5}{2}$
	$= -1$
	$\therefore M \text{ lies on the line } PR.$
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ correct coordinates for M</li> <li>✓ substitutes M into equation and makes correct conclusion</li> </ul>	

Question 18 (cont.)

- (c) Show that  $QS$  is perpendicular to  $PR$ .

(2 marks)

Solution	
$m_{PR} = \frac{1}{2}$	$m_{PR} \times m_{QS} = 0.5 \times -2$
$m_{QS} = \frac{-11-9}{8-(-2)}$	$= -1$
$= -2$	$\therefore QS \perp PR$
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ gradient of QS</li> <li>✓ multiplies by gradient of PR to get -1 and concludes that the lines are perpendicular</li> </ul>	

- d) Calculate  $\theta$ , the angle of inclination of  $QS$ .

(2 marks)

Solution	
$\tan \theta = -2$	
$\theta = 116.57^\circ$	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ equation for angle of inclination</li> <li>✓ correct angle (accept any rounding)</li> </ul>	
NB: answer only ok	