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No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Important note to candidates

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in this examination

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate

Formula sheet (retained from Section One)

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time: one hundred minutes
Reading time before commencing work: ten minutes

Time allowed for this section

Student Name: _____

Solutions

MATHEMATICS METHODS

UNIT 1

Calculator Assumed
Section Two:

Q

Question/Answer Booklet

Semester 1 Examination, 2020

Year 11 Mathematics Methods
Mathematics Department



Christ Church
Grammar School

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	14	14	100	97	65
Total					100

Supplementary page

Question number: _____

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the Christ Church Grammar School reporting and assessment policy. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Working time: 100 minutes.

Original answer where the answer is continued, i.e., give the page number.

If you use these pages to continue an answer, indicate at the end of this Questionnaire/Answer booklet.

Opposite many pages for planning/containing your answers to questions are provided at the end of this Questionnaire/Answer booklet.

1 mark)

(1) two chocolates are chosen from the box.

(marks)

1 mark)

(4) (3) (2) (1)

1 mark)

Solution

(2 marks)

the dis (!!)

A straight line makes an angle of 30° with the positive x -axis and passes through the point with coordinates $(0, 1)$. Determine the exact equation of the line. (2 marks)

Algebraic Solution	$F = 1.965d$	$d = \frac{26.72}{1.965} = 13 \text{ km}$	Specified behaviour	✓ indicates use of direct proportion	✓ correct distance
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26.72	$\times 8.5 = 131.6 \text{ km}$	16.70	Specific behaviours	Indicates use of direct proportion	Correct distance
-------	---------------------------------	-------	---------------------	------------------------------------	------------------

Solution	$\binom{9}{3} = \binom{r}{7}$ then $r = 6$	Specifc behaviours correct value
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Specified behaviours	$\binom{9}{1} + \binom{9}{2} + \binom{9}{3} = 9 + 36 + 84 = 129$
Solution	Correct total value correct selects, 1, 2 and 3 chocolates

(2 marks)

The last (1)

(marks)

✓ correct value
Specific behaviours

✓ correct total value

1000000

The values in Pascal's triangle are $\binom{n}{k}$, which is the same value as $\binom{9}{9}$. Just like the other side of Pascal's triangle.

(iii) Explain how Pascal's triangle can be used to find the solution to part (i). (1 mark)

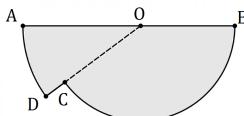
Solutions	$m = \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
Specific behaviours	$y = \sqrt[3]{x+1}$ gradient \neq equation in exact form

			valid explanation
			Specific behaviours
			other side of Pascal's triangle.
		(3)	is the same value as (3). Just starting from the

Question 10

(5 marks)

Shape $AOBDA$ below consists of sector BOC of circle centre O joined to sector DOA of a different circle, also centre O . AB is a straight line of length 65 cm, arc AD is 12 cm long and $\angle AOD = 0.32$ radians.



- (a) Determine the length
- OA
- .

(2 marks)

Solution
Let $OA = R$ so that $0.32R = 12$ $R = 37.5 \text{ cm}$
Specific behaviours
✓ correct use of arc length ✓ correct length

- (b) Determine the area of the shape.

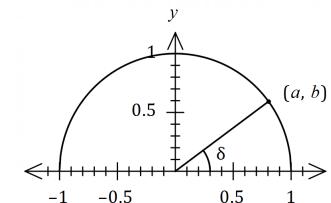
(3 marks)

Solution
$A_{DOA} = \frac{1}{2} \times 37.5^2 \times 0.32$ $= 225$
Let $OB = r$ $r = 65 - 37.5$ $= 27.5$
$A_{BOC} = \frac{1}{2} \times 27.5^2(\pi - 0.32)$ $= 1067$
Area = $225 + 1067$ $= 1292 \text{ cm}^2$
Specific behaviours
✓ area of sector DOA ✓ radius and angle of sector BOC ✓ area of shape

Question 21

(5 marks)

Consider part of the unit circle shown below.



Determine, in terms of a and / or b , an expression for each of the following:

- (a)
- $\cos \delta^\circ$
- .

Solution
$\cos \delta^\circ = a$
Specific behaviours
✓ correct expression

- (b)
- $\sin(180^\circ - \delta^\circ)$
- .

Solution
$\sin(180^\circ - \delta^\circ) = \sin \delta^\circ$ $= b$
Specific behaviours
✓ correct expression

- (c)
- $\cos(\delta^\circ - 90^\circ)$
- .

Solution
$\cos(\delta^\circ - 90^\circ) = \sin \gamma$ $= b$
Specific behaviours
✓ correct expression

- (d)
- $\sin(2\delta^\circ)$
- .

Solution
$\sin(2\delta^\circ) = \sin(\delta^\circ + \delta^\circ)$ $= \sin \delta^\circ \cos \delta^\circ + \cos \delta^\circ \sin \delta^\circ$ $= (b)(a) + (a)(b)$ $= 2ab$
Specific behaviours
✓ halves angle and uses sum identity ✓ correct expression

(1 mark)

Solution
(i) $t = 0$. $h(0) = 68.75 \text{ m}$ $h(4.5) = 35 \text{ m}$

(a) Determine the height of the particle when

$$h(t) = 68.75 + 15t - 5t^2, \quad 0 \leq t \leq 5.5.$$

follows:

The height h metres of a particle above level ground is defined as a function of time t seconds as follows:The graph of $y = a + b \cos(x + c)$ is drawn below, where a , b and c are positive constants.

Question 11

(8 marks)

(2 marks)

Solution (b)
(i) Determine the maximum height reached by the particle and the time it reached this height. $\text{From graph of } h(t):$ $\text{Maximum height: } h = 80 \text{ m when } t = 1.5 \text{ s.}$

(b) Determine the maximum height reached by the particle and the time it reached this height.

Solution
(i) $t = 4.5$. $h(4.5) = 35 \text{ m}$

(i) $t = 0$.(a) Determine the value of a , the value of b and the value of c , where $c < \pi$. (3 marks)

Solution
$a = 1.5$, $b = 3$, $c = \frac{\pi}{6}$

(b) On the same axes, draw the graph of $y = a + \frac{b}{2} \cos(x - c)$.

Specific behaviours
\checkmark value of a \checkmark value of b \checkmark value of c

(3 marks)

(2 marks)

(2 marks)

(d) State the range of the function $h(t)$ for the given domain.

Solution
$0 \leq h \leq 80$ $\text{Range of } h:$

(2 marks)

Solution
$0 \leq h \leq 80$ $\text{OR Accept } \{h \in \mathbb{R} : 0 \leq h \leq 80\}$

(2 marks)

Specific behaviours
\checkmark upper limit, correct inequality \checkmark lower limit, correct inequality \checkmark no penalty for just writing inequality in this context

(2 marks)

(2 marks)

Specific behaviours
\checkmark two solutions as given (as exact values or decimals; decimals to at least 2 decimal places)

(2 marks)

Specific behaviours
\checkmark a correct solution, anywhere

(2 marks)

Solution
$x = -2.6180$, $x = 0.5236$

(2 marks)

Solution
$x = -\frac{5\pi}{6}$, $x = \frac{\pi}{6}$ both times

(2 marks)

Using intersection of graphs:
\checkmark both times

(2 marks)

Specific behaviours
\checkmark one time

(2 marks)

Specific behaviours
\checkmark lower limit

(2 marks)

Specific behaviours
\checkmark upper limit

(2 marks)

Specific behaviours
\checkmark correct inequality

(2 marks)

Specific behaviours
\checkmark correct time

(2 marks)

Specific behaviours
\checkmark correct height

(2 marks)

Specific behaviours
\checkmark both times

(2 marks)

Smooth curve
\checkmark both minima

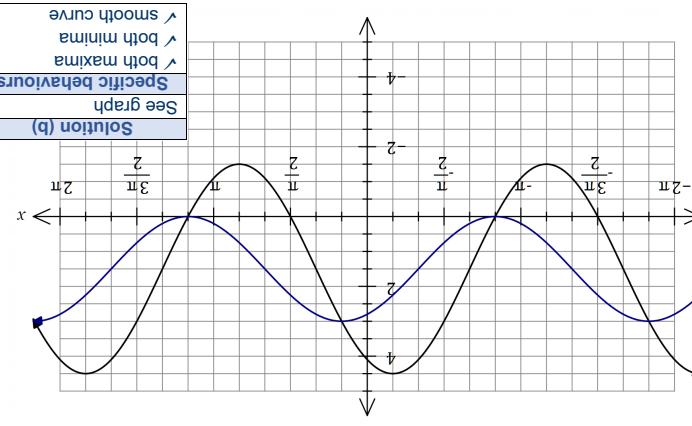
(2 marks)

Smooth curve
\checkmark both maxima

(2 marks)

Smooth curve
\checkmark smooth curve

(2 marks)

The graph of $y = a + b \cos(x + c)$ is drawn below, where a , b and c are positive constants.

(8 marks)

Question 20

(8 marks)

Question 12

(6 marks)

The graph $y = f(x)$, where $f(x) = x^2 + bx + c$ has a turning point at $(-2, -1)$.

- (a) State the equation of the line of symmetry for the graph of $y = f(x)$.

(1 mark)

Solution
$x = -2$
Specific behaviours
✓ correct equation

- (b) Determine the value of the constant b and the value of the constant c .

(3 marks)

Solution
$f(x) = (x + 2)^2 - 1$
$= x^2 + 4x + 4 - 1$
$b = 4$
$c = 3$
Specific behaviours
✓ writes $f(x)$ in squared form
✓ value of b
✓ value of c

- (c) The graph of $y = f(x)$ is translated 3 units to the right and 5 units upwards. Determine the equation of the resulting curve.

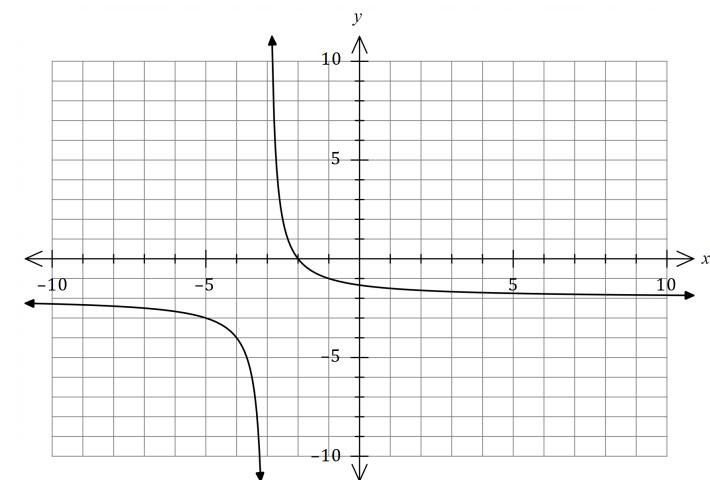
(2 marks)

Solution
New turning point at $(-2 + 3, -1 + 5) = (1, 4)$.
Equation is $y = (x - 1)^2 + 4 = x^2 - 2x + 5$
Specific behaviours
✓ identifies new turning point
✓ correct equation (either form)

Question 19

(5 marks)

The graph of $y = f(x)$ is shown, where $f(x) = \frac{a}{x+b} + c$ and a, b and c are constants.



- (a) Determine the value of a , the value of b and the value of c .

(3 marks)

Solution
$b = 3, c = -2$
$(-2, 0) \Rightarrow 0 = \frac{a}{-2+3} - 2$
$a = 2$
Specific behaviours
✓ value of a
✓ value of b
✓ value of c

- (b) State the domain and range of $f(x)$.

(2 marks)

Solution
Domain: $\{x \in \mathbb{R}: x \neq -3\}$
Range: $\{y \in \mathbb{R}: y \neq -2\}$
Specific behaviours
✓ correct domain
✓ correct range
Note: must have curly brackets and should have $x \in \mathbb{R}$ and $y \in \mathbb{R}$ and only penalise once.

Question 18 (8 marks) Determine the area of triangle PQR when $\angle PQR = 32^\circ$, $\angle PRQ = 114^\circ$ and $PR = 37$ cm.

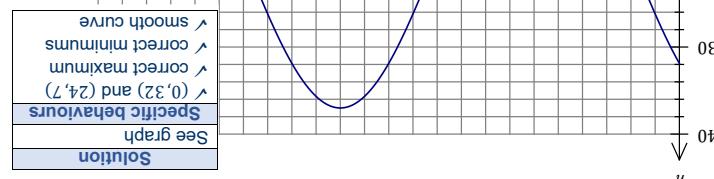
Solution	
<i>Note to scale</i>	
$\text{Area} = \frac{1}{2}(37)(21.46) \sin(34^\circ)$ $= 222 \text{ cm}^2$	
<i>Used in rule</i> <i>Sketch of triangle or evidence of correct values</i> <i>Used in rule</i> <i>Correct use of sine rule</i> <i>Length of second side</i> <i>Correct area</i>	

(a) Determine the area of triangle PQR when $\angle PQR = 32^\circ$, $\angle PRQ = 114^\circ$ and $PR = 37$ cm. (4 marks)

Solution	
$\frac{q}{\sin 32^\circ} = \frac{37}{\sin 114^\circ} \Rightarrow q = 21.46$	
$\text{Area} = \frac{1}{2}(37)(21.46) \sin(34^\circ)$ $= 222 \text{ cm}^2$	
<i>Used in rule</i> <i>Sketch of triangle or evidence of correct values</i> <i>Used in rule</i> <i>Correct use of sine rule</i> <i>Length of second side</i> <i>Correct area</i>	

(a) Determine the initial height of the seat. (1 mark)

(b) Graph the height of the seat against time on the axes below. (4 marks)



(c) Determine the maximum height above ground reached by the seat. (1 mark)

(ii) the maximum height above ground reached by the seat. (1 mark)

(iii) the time taken, to the nearest second, for the seat to first reach a height of 5 m above ground level. (2 marks)

Solution	
$h = 5 \Rightarrow t = 4.49$ $0.49 \times 60 = 29$ $t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Specific behaviours</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$h_{\text{max}} = 37 \text{ m}$	
<i>Correct height</i> <i>Specific behaviours</i>	
Specific behaviours	
Solution	

Solution	
$AB^2 = 15^2 + 20^2 - 2(15)(20) \cos(30^\circ)$ $AB = 10.27 \text{ cm}$	
<i>Length of AB</i> <i>Cosine rule</i> <i>Value of x</i> <i>Area equation</i>	
Specific behaviours	
Solution	

Solution	
$x = 5$	
<i>Length of AB</i> <i>Cosine rule</i> <i>Value of x</i> <i>Area equation</i>	
Specific behaviours	
Solution	

Solution	
$\frac{1}{2}(4x)(3x) \sin(30^\circ) = 75$	
<i>Length of AB</i> <i>Cosine rule</i> <i>Value of x</i> <i>Area equation</i>	
Specific behaviours	
Solution	

Solution	
$h(0) = 31.87 \text{ m}$	
<i>Correct height</i> <i>Accept 1 or 2 decimal places, but do not penalise if more decimal places</i> <i>Graph the height of the seat against time on the axes below.</i>	
Specific behaviours	
Solution	

Solution	
$h_{\text{max}} = 37 \text{ m}$	
<i>Correct height</i> <i>Specific behaviours</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$h = 5 \Rightarrow t = 4.49$	
<i>Correct height</i> <i>Specific behaviours</i>	
Specific behaviours	
Solution	

Solution	
$0.49 \times 60 = 29$	
<i>Correct height</i> <i>Specific behaviours</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$h = 5 \Rightarrow t = 4.49$	
<i>Correct height</i> <i>Specific behaviours</i>	
Specific behaviours	
Solution	

Solution	
$0.49 \times 60 = 29$	
<i>Correct height</i> <i>Specific behaviours</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	
Solution	

Solution	
$t = 4 \text{ m } 29 \text{ s } (269 \text{ s})$	
<i>Time to nearest second</i> <i>Time as decimal</i> <i>Time to nearest second</i>	
Specific behaviours	

Question 14

(a) Express

(i) 35° in radians.

Solution
$35^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{36}$
Or $0.6109 r$ or (radians)
✓ correct measure

(1 mark)

(ii) $\frac{11\pi}{15}$ in degrees.

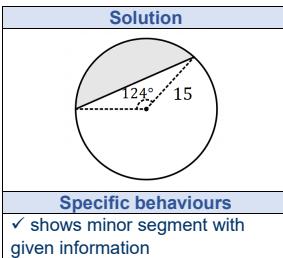
Solution
$\frac{11\pi}{15} \times \frac{180^\circ}{\pi} = 132^\circ$
✓ correct measure

(1 mark)

(b) A minor segment subtends an angle of 124° in a circle of radius 15 cm.

(i) Sketch a diagram to show the circle and minor segment.

(1 mark)



Specific behaviours
✓ shows minor segment with given information

(ii) Determine the area of the minor segment.

(3 marks)

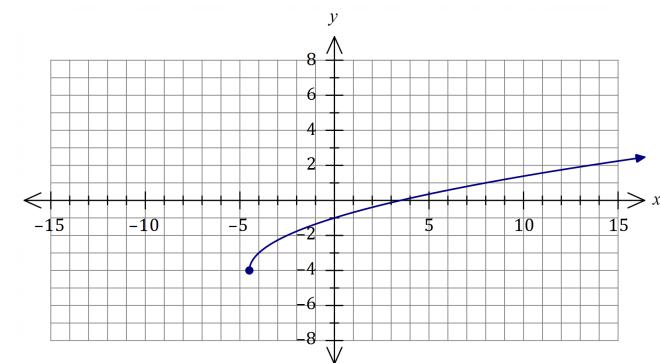
Solution
$124^\circ \times \frac{\pi}{180^\circ} = \frac{31\pi}{45} (2.164r)$
$A = \frac{1}{2}(15)^2 \left(\frac{31\pi}{45} - \sin \frac{31\pi}{45} \right)$
$A = 150 \text{ cm}^2$
Specific behaviours ✓ converts degrees to radian measure and uses radian measure (exact or decimal value) ✓ substitutes correctly (exact or decimal value) ✓ correct area, any rounding (units not required, but preferred) Note: if degree is used in radian formula, then no 2 nd mark

Alternative Solution
$\text{Area sector} = \frac{124}{360} \times \pi \times 15^2 = 243.473$
$\text{Area triangle} = \frac{1}{2} \times 15 \times 15 \times \sin 124 = 93.2667$
$\text{Area segment} = \text{Area sector} - \text{area triangle} = 150 \text{ cm}^2 (150.2 \text{ cm}^2)$
Specific behaviours ✓ calculates area of sector correctly ✓ calculates area of triangle correctly ✓ correct area, any rounding (units not required, but preferred)

See next page

(c) Draw the graph of $y = f(2x)$ on the axes below.

(3 marks)



Solution
See graph
Specific behaviours
✓ endpoint at $(-4.5, -4)$ ✓ thru' $(0, -1)$ and $(3.5, 0)$ ✓ smooth curve

See next page

(7 marks)

Question 15Let $f(x) = \sqrt{12 - 2x} - 2$ and $g(x) = 16 + x$.

- (a) Evaluate
- $f(-2) - g(-2)$
- .

Solution
$f(-2) - g(-2) = \sqrt{16} - 2 - (16 - 2)$ $= 2 - 14$ $= -12$
Specific behaviours
✓ evaluates f correctly ✓ correct value

(2 marks)

- (b) State the domain of
- $f(x)$
- .

Solution
$12 - 2x \geq 0$ $-2x \geq -12$ $x \leq 6$
Domain: $\{x \in \mathbb{R} : x \leq 6\}$
Specific behaviours

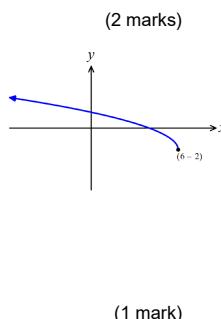
✓ correct inequality
✓ correct notation
Note: penalise for incorrect notation and make a note so not penalised in question 19 as well.

- (c) State the range of
- $g(x)$
- .

Solution
$y \in \mathbb{R}$
Specific behaviours

✓ correct range (symbols or words)

(2 marks)



(1 mark)

- (d) Determine the coordinates of the point(s) of intersection of
- $y = f(x)$
- and
- $y = g(x)$
- .

(2 marks)

Solution
Using graph/CAS: $(-2, 4)$
Specific behaviours

✓ x -coordinate
✓ y -coordinate

See next page

Question 16A polynomial of degree 3 passes through the points with coordinates $(0, -3)$, $(1, 0)$, $(-3, 0)$ and $(-0.5, 0)$.

- (a) Determine the equation of the polynomial in expanded form.

(4 marks)

Solution
Using roots:
$y = a(x - 1)(x + 3)(x + 0.5)$
Use 4th point: $x = 0 \Rightarrow -3 = a(-1)(3)(0.5)$ $a = 2$
Expand: $y = 2(x - 1)(x + 3)(x + 0.5)$ $= 2x^3 + 5x^2 - 4x - 3$
Specific behaviours
✓ factored form using roots ✓ substitutes fourth point ✓ correct value of a ✓ correct expanded form

- (b) Draw the graph of the polynomial on the axes below, indicating the coordinates of all turning points.

(4 marks)

Solution
See graph
Specific behaviours

✓ all roots and y -intercept $(-3, 0)$, $(0.5, 0)$, $(1, 0)$ and $(0, -3)$
✓ labelled maximum $(-2, 9)$
✓ labelled minimum $\left(\frac{1}{3}, -\frac{100}{27}\right) = (3.3, -3.7)$
✓ smooth curve

