



**Kenedy**  
Baptist College

## Question/Answer booklet

Semester Two Examination, 2019

Please place your student identification label in this box  
if required by your examination administrator, please

# MATHEMATICS METHODS UNITS 3 AND 4 SECTION TWO: CALCULATOR-ASSUMED

**Time allowed for this section** Working time: One hundred minutes  
Reading time before commencing work: ten minutes

**Materials required/recommended for this section**

*to be provided by the supervisor*

**o be provided by the candidate**  
standard items: pens (blue/black preferred), pencils (including co-  
correction fluid/tape, eraser, ruler, highlighters)

special items: drawing instruments, templates, notes on two units and info that can't be found elsewhere.

**Important note to Candidates**  
You do not have any unauthorized material. If you have any unauthorized items may be taken into the examination room. It is your responsibility to the supervisor before reading any further.

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**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					<b>100</b>

**Instructions to candidates**

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
9	6	
10	6	
11	6	
12	6	
13	9	
14	8	
15	8	
16	8	
17	7	
18	7	
19	9	
20	9	
21	9	
S2 Total	98	
S2 Wt ( $\times 0.6633$ )	65%	

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Supplementary page

Question number: \_\_\_\_\_

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- (c) The graph of  $y=f(x)$  is congruent with the graph of  $y=\log_a g(x)$ . State a suitable function  $g(x)$ .

- (b) Determine the exact coordinates of the point on the graph where  $f(x)=\frac{1}{4}$ .

- (a) Determine the value of the constant  $a$  and hence state the equation of the asymptote of the graph.

The graph of  $y=f(x)$ , where  $f(x)=4 \log_a(x-a)$ , has a root at  $x=4$ .

Question 9

**Question 10**

(6 marks)

A machine is set to fill bottles with more than the stated capacity. The random variable  $X$  mL is the amount it overfills bottles and has probability density function  $f(x)$  shown below.

$$f(x) = \begin{cases} \frac{3\sqrt{2x-2}}{8} & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Determine
- $E(X)$
- .

(2 marks)

- (b) Determine
- $\text{Var}(X)$
- .

(2 marks)

- (c) The amount another machine overfills bottles is given by
- $Y = 2 + 0.5X$
- . Determine

- (i)
- $E(Y)$
- .

(1 mark)

- (ii)
- $\text{Var}(Y)$
- .

(1 mark)

(c) Interpret the value of  $a$  and the value of  $b$  in the context of this model.

(2 marks)

- (d) Use the model to determine

- (i) the population when
- $t = 45$
- .

(1 mark)

- (ii) the number of years for the population to reach 75 000.

(1 mark)

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## Question 11

(a) Determine the value of  $k$ .  
(2 marks)A water tank sprung a leak. The amount of water  $W$  remaining in the tank  $t$  minutes after the leak began can be modelled by the equation  $W = 30e^{-kt}$  kilolitres, where  $k$  is a constant.

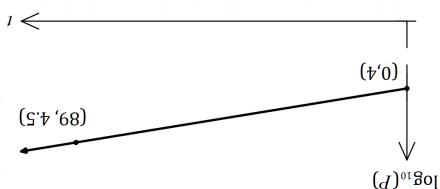
3.5 kL of water was lost from the tank in the first 10 minutes.

(b) How many kilolitres of water leaked from the tank during the first 2 hours?  
(2 marks)

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## Question 21

The population of a species  $P$  can be modelled by the equation  $P = ab^t$ , where  $a$  and  $b$  are constants and  $t$  is the number of years since the population was first recorded. The graph below shows the linear relationship between  $t$  and  $\log_{10}P$  for the population over the past 90 years and passes through the points  $(0, 4)$  and  $(89, 4.5)$ .(c) At what time, to the nearest minute, was the instantaneous rate of water loss 186 litres per minute?  
(2 marks)(a) Write an equation relating  $\log_{10}P$  and  $t$ .  
(2 marks)(b) Determine the value of  $a$  and the value of  $b$ .  
(3 marks)

(6 marks)

**Question 12**

An opinion poll found that 160 out of 386 people supported a policy to increase the minimum wage, from which a 95% approximate confidence interval for the population proportion was calculated to be

$$(0.366, 0.464)$$

- (a) Show how this interval was calculated. (4 marks)

- (b) If 15 similar opinion polls were taken and each time a 95% confidence interval calculated, determine the probability that all 15 intervals contain the true population proportion. (2 marks)

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(9 marks)

**Question 20**

Researchers in a large city wish to determine a 90% confidence interval for  $p$ , the proportion of citizens who had used the city library at least once during the previous year. The margin of error of the interval is to be no more than 5%.

- (a) If the researchers had no reliable estimate for  $p$ , determine the sample size they should take, noting all assumptions made. (5 marks)

- (b) The researchers were given access to data from a random sample of 159 citizens collected a few years earlier. Of these, 59 had used the city library at least once during the previous year.

- (i) Determine the margin of error for a 90% confidence interval for  $p$  based on this sample. (2 marks)

- (ii) The researchers used this data to decrease the sample size calculated in part (a). By how much did the sample size decrease? (2 marks)

**Question 13**

- The time taken to answer a customer call at a large business can be modelled by the continuous random variable  $T$  that is uniformly distributed between  $7$  and  $32$  seconds.
- (a) Sketch a diagram of the associated probability density function for  $T$ . (3 marks)

**Question 14**

- A particle moves along the  $x$ -axis with initial position  $x(0) = 3.2$  m and velocity  $v(0) = 1.2$  m/s. The acceleration of the particle after  $t$  seconds is given by  $a(t) = m - 0.2t$  m/s<sup>2</sup>.
- Between  $t = 1$  and  $t = 4$  the particle undergoes a change in displacement of 51 m.
- (a) Determine the value of the constant  $m$ . (4 marks)
- (b) Determine  $P[7 < T < 23 | T < 28]$ . (2 marks)
- (c) A simulation involves taking a random sample from the uniform distribution, recording the time and repeating a total of 500 times. The times are then grouped into 5 equal width classes, from which a frequency histogram is constructed.
- (i) Sketch a likely histogram on the axes below. (3 marks)
- 
- (ii) Briefly explain how your sketch would change if the simulation was repeated a second time. (1 mark)

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**Question 15**

- (a) Determine the maximum velocity of the particle. (2 marks)
- (b) Determine the distance of the particle from the origin after 6 seconds. (3 marks)
- (c) A simulation involves taking a random sample from the uniform distribution, recording the time and repeating a total of 500 times. The times are then grouped into 5 equal width classes, from which a frequency histogram is constructed.
- (i) the maximum velocity of the particle. (2 marks)
- (ii) the distance of the particle from the origin after 6 seconds. (3 marks)

(8 marks)

**Question 14**

The table below shows the probability distribution for a random variable  $X$ .

$x$	0	1	2	3
$P(X=x)$	$2k^2+2k$	$k^2$	$2k^2+k$	$k$

- (a) Determine the value of the constant  $k$ . (2 marks)

- (b) Determine  $E(X)$  and  $\text{Var}(X)$ . (3 marks)

- (c) Given that  $E(aX+b)=5$  and  $\text{Var}(aX+b)=38$ , determine all possible values of the constants  $a$  and  $b$ . (3 marks)

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(7 marks)

**Question 18**

A citrus farm grows Eureka lemons. Their weights are normally distributed with a mean of 172 g and a standard deviation of 8.6 g.

- (a) Determine the probability that  
 (i) a randomly chosen lemon has a weight that exceeds 175 g. (1 mark)

- (ii) in a random sample of 12 lemons, exactly 4 have a weight that exceeds 175 g. (2 marks)

The farm classifies their lemons by size, so that the ratio of the number of small to medium to large lemons is 1:2:4.

- (b) Determine the upper and lower bounds for the weight of a medium sized lemon. (2 marks)

- (c) Determine the probability that when lemons are picked at random, the first small lemon is chosen on the 5<sup>th</sup> pick. (2 marks)

## Question 15

(8 marks)

- (a) It is known that 80 % of a large population of animals carry microfilariae in their blood (are carriers). A student must stimulate 50 animals that either are or are not carriers.

- (b) Describe a method that the student could use.

(2 marks)

- (c) The random variable  $X$  is the number of animals in a random sample of size 200 that are carriers. Describe the distribution of  $X$  and determine  $E(X)$ .

(2 marks)

- (d) 225 students carry out the simulation so that they each have a sample of size 200. Then each student calculates  $p$ , the proportion of animals in their sample that are carriers. The distribution of these 225 values of  $p$  will be approximately normal.
- (e) Determine the parameters of the normal distribution the 225 values of  $p$  will approximate.

(1 mark)

- (f) (i) the sample size was larger.  
(ii) the percentage of animals that are carriers was higher.

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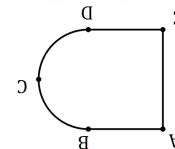
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NS245-1A5-3

## Question 17

(7 marks)

When seen from above, an evaporation tank of area  $320 \text{ m}^2$  has the shape of rectangle  $ABDE$  and semicircle  $BCD$  of radius  $r$ .



$$P = \frac{320}{r} + 2r + \pi r$$

- (a) If length  $AB = x$ , express  $x$  in terms of  $r$  and hence show that the perimeter,  $P$ , of the tank is given by

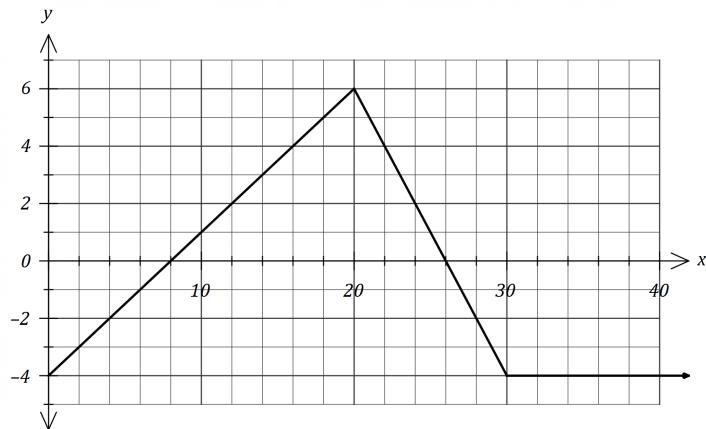
(4 marks)

- (b) Use a calculus method to determine the minimum perimeter of the tank.

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**Question 16**

The graph of  $y=f(x)$  is shown below.



- (a) Determine  $\int_4^{14} f(x) dx$ . (2 marks)

Let  $A(x) = \int_0^x f(t) dt$ .

- (b) Determine

(i)  $A(8)$ . (1 mark)

(ii)  $A'(8)$ . (1 mark)

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- (c) Determine the coordinates of the maximum of the graph of  $y=A(x)$ . (2 marks)

- (d) Determine the root of the graph of  $y=A(x)$  for  $x > 16$ . (2 marks)