

Semester 1 (Unit 3) Examination, 2019

Question/Answer Booklet

MATHEMATICS METHODS

Section One: Calculator-free

Teacher Name:

Student Name/Number:

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters
Special items: nil
Before reading any further.

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure
that you do not have any unauthorised notes or other material with you, hand it to the supervisor
in the examination room, if you have any unauthorised material with you, hand it to the supervisor
before reading any further.

- The items and the solutions/marking keys are to be kept confidentially and not copied or made available to anyone who is not a teacher at the school. Teachers may give feedback to students in the form of showing them how the work is marked but students are not to retain a copy of the items and the solutions/marking keys.
- The items that are contained in this examination are to be used solely in the school for which they are purchased.
- They are not to be shared in any manner with a school which has not purchased their own licence.
- This examination is Copyright but may be freely used within the school that purchases this licence.
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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	13	13	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the School/College assessment policy. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Additional working space

Question number: _____

(3 marks)

- Use calculus to determine when the particle first stops.
given by $x(t) = 3\sin 2t$ where x is measured in metres and t is measured in seconds.
- (d) A particle moves in a straight line such that its displacement, $x(t)$ from the origin is

(3 marks)

- Demonstrating clear use of the chain rule, determine $\frac{dy}{dx}$
- (c) Let $y(u) = \sqrt{u}$ and $u = 2 - 3x^2$.

(3 marks)

- (b) Determine $\frac{dx}{d(\cos x)}$. Simplify your answer.

(2 marks)

- (a) Differentiate $x^{e^{3x}}$ with respect to x .

(11 marks)

Question 1

Suggested working time: 50 minutes

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
This section has **(seven)** 7 questions. Answer all questions. Write your answers in the spaces provided.

Section One: Calculator-free**MATHEMATICS METHODS**
SEMESTER 1 (UNIT 3) EXAMINATION

3

CALCULATOR-FREE
SEMESTER 1 (UNIT 3) EXAMINATION**35% (50 Marks)**

(3 marks)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x \cos x + 2) dx$$

(c) Hence or otherwise, evaluate

(3 marks)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cos x dx = \frac{1}{2} \sin^2 x + C.$$

(b) Hence show clearly that

(1 mark)

$$\frac{dy}{dx} \text{ where } y = \sin^2 x.$$

(a) Determine $\frac{dy}{dx}$

(7 marks)

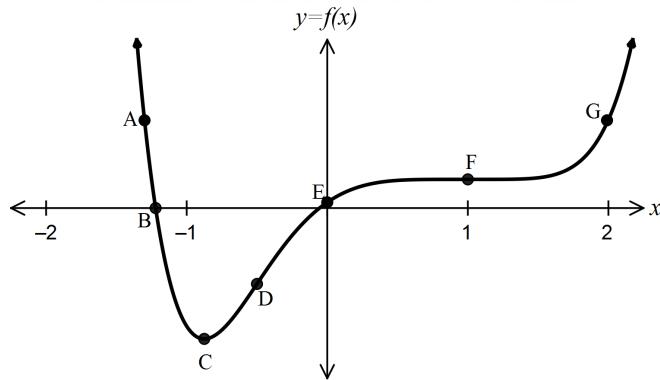
MATHEMATICS METHODS
CALCULATOR-FREE
SEMESTER 1 (UNIT 3) EXAMINATION

10

Question 2

(5 marks)

Consider the function $y = f(x)$ shown below. The points, A, B, C, D, E, F and G each lie on the graph.



(a) Which point/s labelled on the graph above satisfy the following,

(i) a point of inflection occurs?

(1 mark)

(ii) $f'(x) = 0$ and $f''(x) \neq 0$?

(1 mark)

(iii) f is increasing and $f''(x) < 0$?

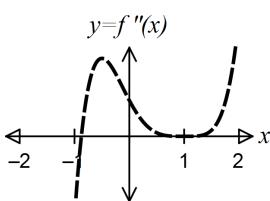
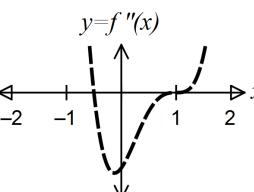
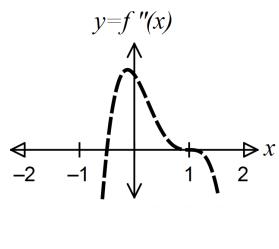
(1 mark)

(iv) $f(x) > 0$ and the function is concave up?

(1 mark)

(b) Circle the graph below that represents $f''(x)$.

(1 mark)



(b) Show that by using three trapeziums an estimate for the area under $f(x) = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$ is given by $\frac{\pi}{6} \left[\frac{2 + \sqrt{3}}{2} \right]$.

(2 marks)

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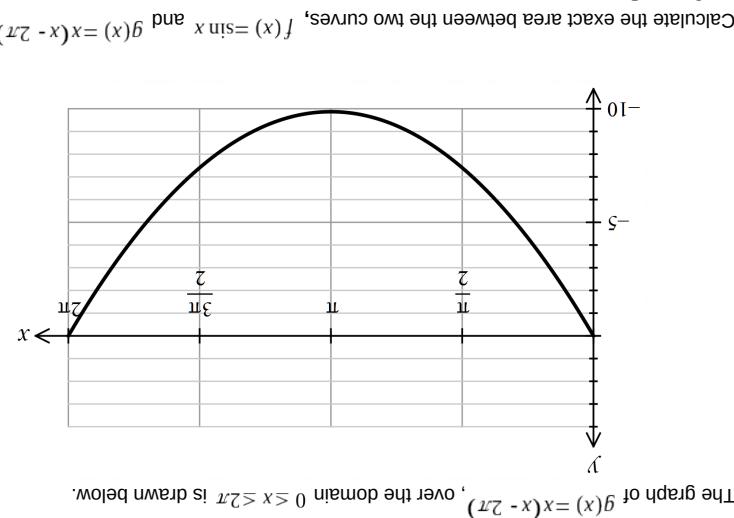
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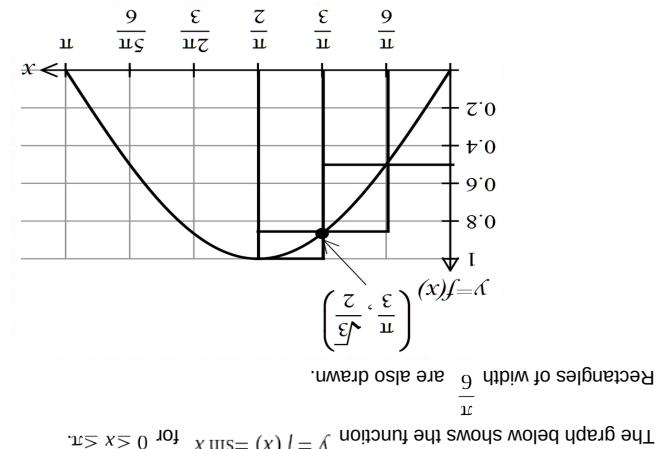
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Question 3

Question 3

(4 marks)

The graph below shows the function $y = f(x) = \sin x$ for $0 \leq x \leq \pi$.

- (i) an underestimate for the area under $f(x) = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. (1 mark)

- (ii) an overestimate for the area under $f(x) = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. (1 mark)

(a) Use these rectangles to determine

- (i) an underestimate for the area under $f(x) = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. (1 mark)

- (ii) an overestimate for the area under $f(x) = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. (1 mark)

Question 4**(9 marks)**

(a) Determine $\int \left(2e^{2x} - \frac{3}{\sqrt{x}} \right) dx.$

(2 marks)

(b) Evaluate $\int_0^1 (3 - 2x)^2 dx.$

(2 marks)

(c) Determine $F'(x)$ if $F(x) = \int_x^1 \frac{dt}{1 + \sqrt{1-t}}.$

(2 marks)

(d) Solve for m given $\int_m^{-m} (m^3 - x^3) dx = 1250$.

(3 marks)

Question 5**(10 marks)**

Melinda and Matt are two students of Mathematics Methods. They are playing a board game using two unbiased dice. The person who throws the first double six gets to start the game.

Melinda and Matt decide to apply their knowledge of probability distributions to their game. Before Matt throws the two dice for the first time he defines a random variable S . The random variable S is assigned the value 1 if two sixes are thrown and 0 otherwise.

- (a) Describe the distribution of S by stating its name, mean and variance.
(3 marks)

Melinda and Matt only have a short time to play the game so decide they will stop after each player has had 15 turns of rolling the dice.

Define the random variable W as the total number of double sixes that Melinda throws in her 15 rolls of the dice.

- (b) Describe the distribution of W including its parameters.
(3 marks)

- (c) State an expression to determine the probability that Melinda throws just one pair of double sixes in her 15 throws.
(1 mark)

- (d) State an expression to determine the probability that the total number of double sixes thrown by Melinda and Matt in their 15 rolls each of the dice is exactly 2 given that Melinda threw at least one double six.
(3 marks)