

**Insert School Logo**

**Semester One Examination 2020  
Question/Answer Booklet**

**MATHEMATICS SPECIALIST  
UNIT 3**

**Section One:  
Calculator-free**

Student Name: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: five minutes

Working time for paper: fifty minutes

**Material required/recommended for this section**

**To be provided by the supervisor**

This Question/Answer booklet

Formula Sheet

**To be provided by the candidate**

Standard items: pens(blue/black preferred), pencils(including coloured), sharpener, correction  
tape/fluid, erasers, ruler, highlighters

Special Items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available	Weighting
<b>Section One Calculator—free</b>	<b>7</b>	<b>7</b>	<b>50 minutes</b>	<b>53</b>	<b>35%</b>
Section Two Calculator—assumed	10	10	100 minutes	97	65%
				150	100%

## Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

**Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

**Section One: Calculator–free**

**53 marks**

This section has **seven (7)** questions. Attempt **all** questions.  
Write your answers in the spaces provided.

Working time: 50 minutes

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**Question 1 (7 marks)**

Functions  $f$  and  $g$  are defined such that:

$$f(x) = \sqrt{x}$$

$$g(x) = \frac{2}{x-2}$$

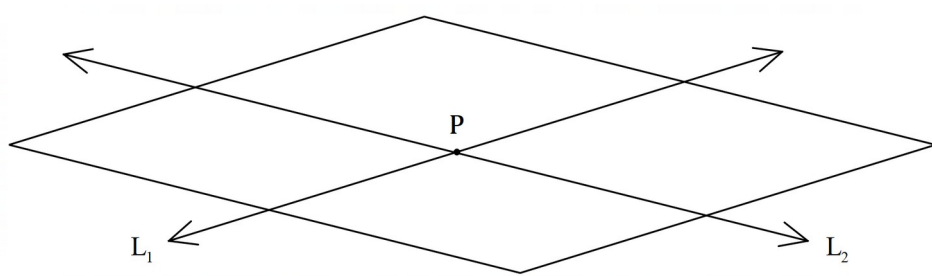
(a) Determine  $gf(x)$ . (1 mark)

(b) Determine the domain and range for  $gf(x)$ . (4 marks)

(c) Given that  $f^{-1}(x) = 4 - x^2$ , is it true that  $f^{-1}(-1) = 3$ ? Explain. (2 marks)

**Question 2 (9 marks)**

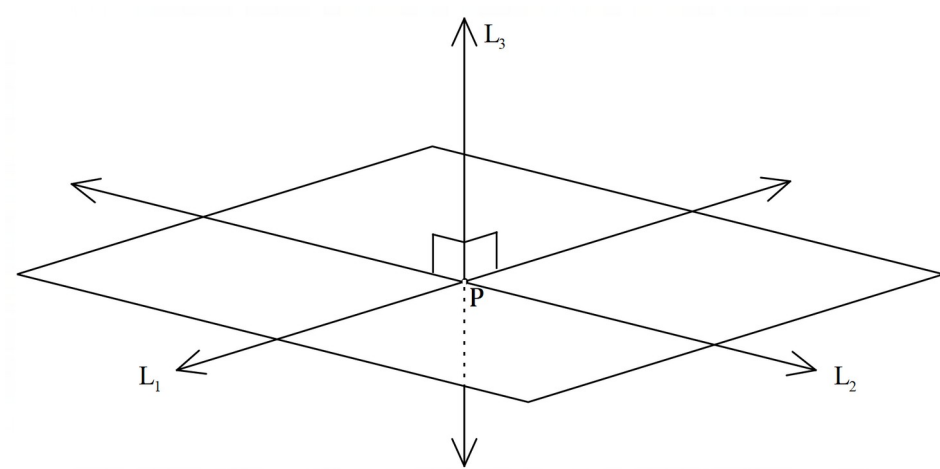
The lines  $L_1$  and  $L_2$  have equations  $r = (3 + \lambda)i + (1 + \lambda)j - 2\lambda k$  and  $r = (a\ b\ c) + \mu(4 - 2\ 1)$  respectively, with  $a, b, c \in R$ , and they lie on the same plane  $\Pi$  as shown.



- (a) Given that the lines intersect at the point  $P$  when  $\lambda = 3 = \mu + 2$ , determine the value of the constants  $a, b$  and  $c$ , and the exact distance of point  $P$  from the origin. (4 marks)

(Question 2 – Continued)

- (b) A third line  $L_3$  is perpendicular to the plane formed by  $L_1$  and  $L_2$ , and passes through  $P$ . Determine the vector equation of  $L_3$  and the Cartesian equation of the plane  $\Pi$ .



(5 marks)

**Question 3 (12 marks)**

(a) Give an exact expression for each of the following in the form  $a+bi$ .

(i)

(3 marks)

(ii)  $\left(\frac{1}{\sqrt{2}}\right)^{\square}$

(3 marks)

**(Question 3 – Continued)**

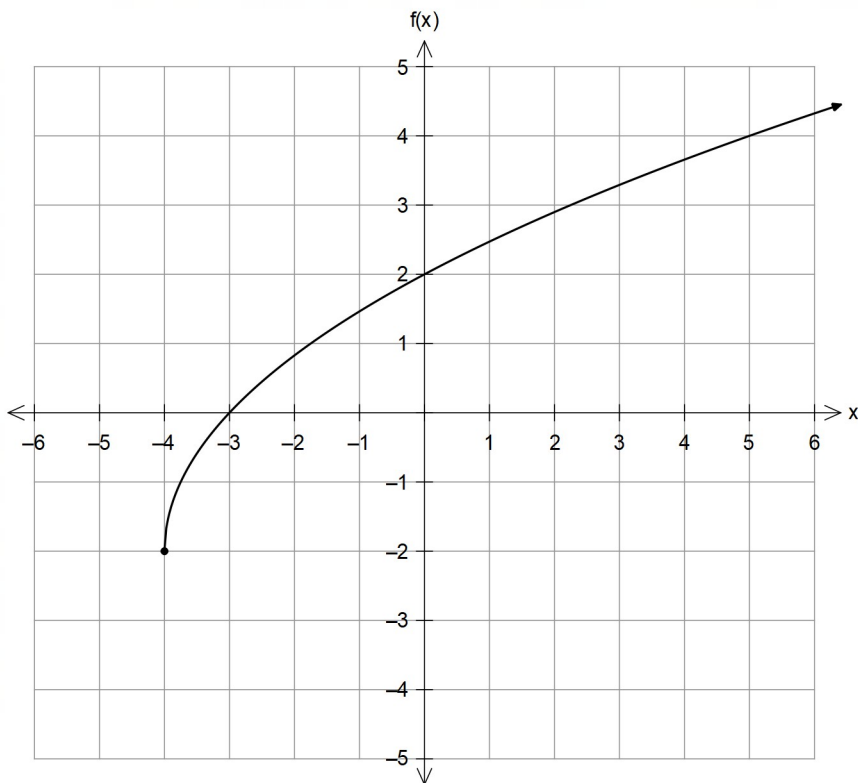
(b) Consider  $f(z) = z^3 - 5z^2 + 8z - 6$  where  $z$  is a complex number.

(i) Show that  $(z - 1 - i)$  is a factor of  $f(z)$ . (2 marks)

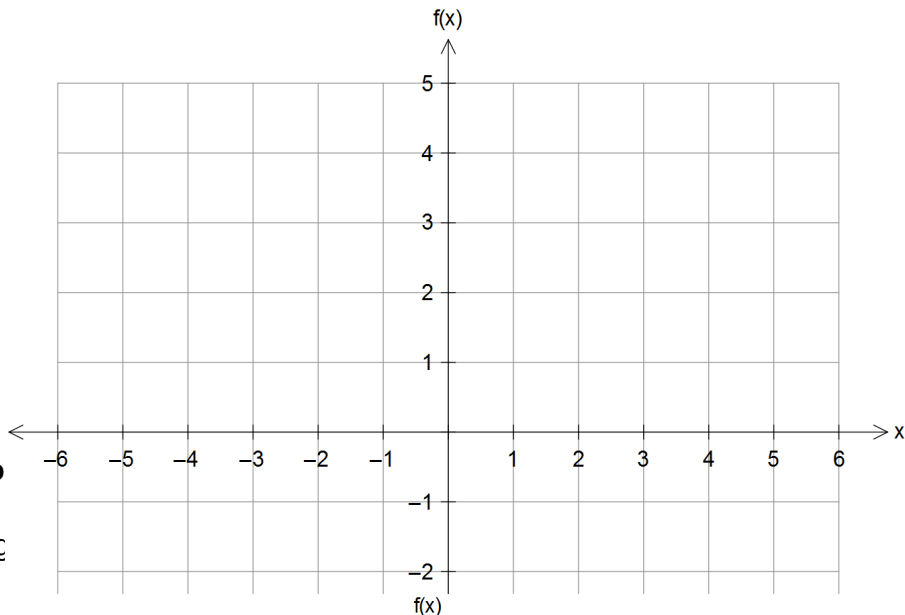
(ii) Solve the equation  $z^3 - 5z^2 + 8z - 6 = 0$ . (4 marks)

Question 4 (9 marks)

The graph of  $f(x)=2\sqrt{x+4}$  is shown below.

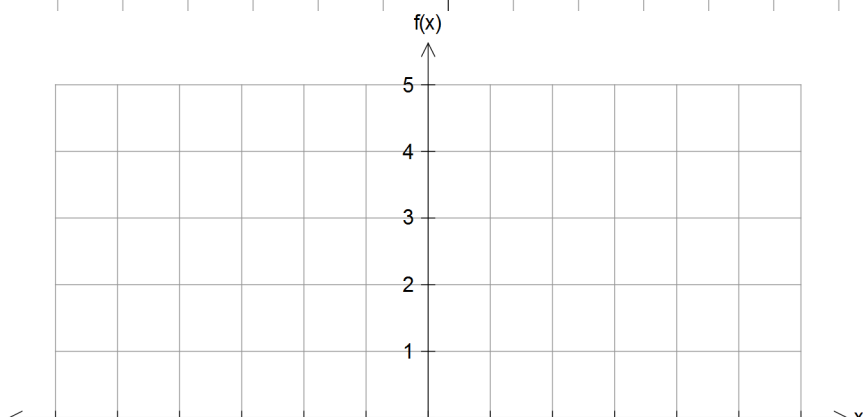


- (a) Sketch the graph of  $\frac{1}{f(x)}$  on the grid below. (3 marks)



(Question 4 – Co

- (b) Sketch the graph of  $\frac{1}{f(x)}$  on the grid below. (3 marks)





- (c) The domain of  $f(x)$  is restricted to  $x \geq k$  so that the inverse of  $g(x) = |f(x)|$  exists. Determine the value of  $k$  and state the domain and range of  $g^{-1}(x)$ . (3 marks)

**Question 5 (6 marks)**

(a) Solve the system of linear equations below.

(3 marks)

$$\begin{aligned} 2x + y - 3z + 7 &= 0 \\ x + z - 7 &= 0 \\ 3y - z + 7 &= 0 \end{aligned}$$

**(Question 5 – Continued)**

Consider another system of linear equations, where one of the coefficients is  $k \in \mathbb{R}$ .

$$\begin{aligned}x + y + z &= 0 \\ 2x + z &= 2 \\ y + kz &= -5\end{aligned}$$

It can be shown that the solutions to this system of equations are given in terms of  $k$ :

$$x = \frac{2k+3}{2k-1}$$

$$y = \frac{5-2k}{2k-1}$$

$$z = \frac{8}{1-2k}$$

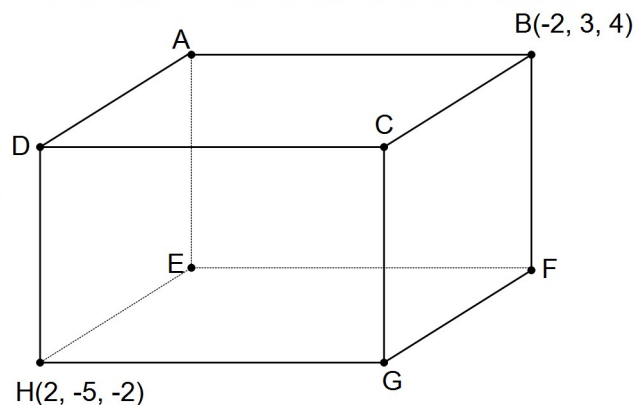
- (b) Explain whether this system of equations will have a unique solution for all real values of  $k$ .  
If not, then explain the geometric interpretation of this.

(3 marks)

**Question 6 (6 marks)**

The right rectangular prism  $ABCDEFGH$  shown is positioned in the Cartesian coordinate system such that  $AD$  is parallel to the  $x$ -axis,  $AB$  is parallel to the  $y$ -axis, and  $EA$  is parallel to the  $z$ -axis.

The vertices  $B$  and  $H$  have coordinates  $(-2, 3, 4)$  and  $(2, -5, -2)$  respectively.



(a) State the coordinates of vertex  $E$ . (1 mark)

(b) Determine the percentage of the prism that lies in the first octant. (2 marks)

**(Question 6 – Continued)**

- (c) Determine the vector equation of the sphere that has HB as its diameter. (3 marks)

**Question 7 (4 marks)**

Consider the set of complex numbers that is a solution to the equation  $z^n = -1$ , with  $n \in \mathbb{N}$ .

Determine the value(s) of  $n$  such that two of its solutions are  $\left(\frac{\pi}{3}\right)$  and  $\pi$ .

Justify/explain your answer.

(4 marks)

**Additional working space**

Question number(s): .....

**Additional working space**

Question number(s): .....

