PERTH MODERN SCHOOL
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Independent Public School

Year 12 Specialist
TEST 2
2018
TIME: 5 mins reading 40 minutes working
Classpads **allowed!**
36 marks 8 Questions

Name:_____

Teacher: _____

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2 & 2 = 4 marks)

Consider $f(x) = x^3 - x^2 + 4x - 4$.

i) Show that $(x - 2i)$ is a factor of $f(x)$

ii) Determine three linear factors of $f(x)$

Q2 (5 marks)

Consider $f(x) = x^3 + bx^2 + cx + 8$ where b & c are constants. Given that $(x + 2)$ is a factor of $f(x)$ and when $f(x)$ is divided by $(x - 3)$ has a remainder of -10. Determine b & c.

Q3 (3 marks)

Given that $f(x) = \sqrt{x + 2}$ and $g(x) = 5x - 3$. Does $f \circ g(x)$ exist over the natural domain of g? Explain your answer.

Q4 (2 & 2 = 4 marks)

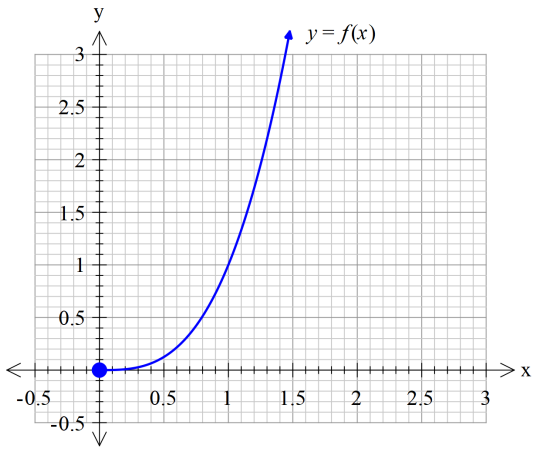
Given that $f(x)=\sqrt{x}$ and $h(x)=\frac{1}{x^2+5}$:

i) Determine the rule of $h \circ f(x)$

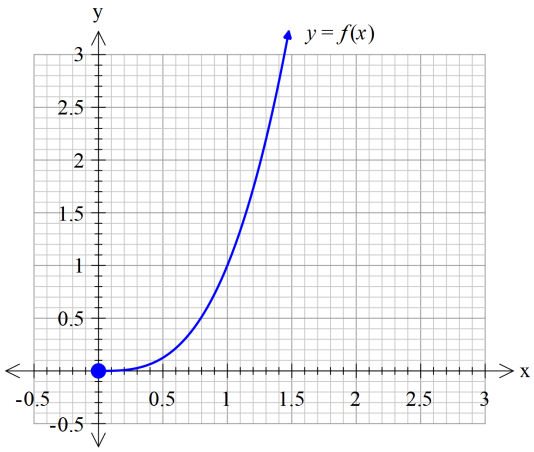
ii) State the natural domain and range of $h \circ f(x)$

Q5 (3 & 3 = 6 marks)

i) On the diagram, sketch the inverse function $f^{-1}(x)$



ii) On the diagram below, sketch $y = \frac{1}{f(x)}$



Q6) (1, 1, 2 & 2= 6 marks)

Consider the function $f(x) = \frac{cx + d}{ax + b}$ where $a, b, c \& d$ are non-zero constants.

i) Determine the natural domain of f

ii) Determine the limit that f approaches as $x \to \pm\infty$

iii) Determine the inverse function $f^{-1}(x)$ in terms of $a, b, c \& d$.

iv) Determine the possible values of $a, b, c \& d$ if $f = f^{-1}$.

Q7 (4 marks)

Consider the equation $|2x+5| = p|x+q| + r$ which is true and only true for $\frac{-5}{2} \le x \le 3$.

Determine the possible values of the constants $p, q \& r$.

Q8 (4 marks)

Let $z = \cos(2\theta) + i\sin(2\theta)$, prove that $\frac{1+z}{1-z} = \frac{i}{\tan\theta}$