St Mary's Anglican Girls' School

Semester II, 2011

Question/Answer Booklet

MATHEMATICS

SPECIALIST 3CD

Section Two
(Calculator Assumed)

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/Answer booklet for Section Two, formula sheet retained from Section One.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: drawing instruments, templates, notes on 2 unfolded sheets of A4 paper, and up to 3 calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One Calculator—Free	7	7	50 minutes	40
Section Two Calculator—Assumed	**9**	**9**	**100 minutes**	**80**
			Total marks	120

Instructions to candidates

1. Answer the questions in the spaces provided.

2. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

Section Two: Calculator–Assumed 80 marks

This section has **NINE (9)** questions. Attempt **ALL** questions.

Question 8 [7 marks]

The current $I(t)$ amperes, in an electrical circuit, t milliseconds after a switch is turned on, obeys the differential equation :

$$\frac{dI}{dt} = 5 - 2I \quad \text{amperes/millisecond}$$

Initially there is no current flowing when the switch is turned on.

(a) Give the initial rate of change of the current when the switch is turned on.

[1]

(b) Using the separation of variables technique, determine the defining rule for $I(t)$ in terms of t.

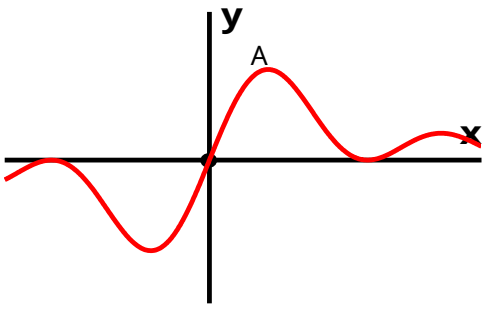
[4]

(c) Explain what is happening to the current after 1 second.

[2]

Question 9 [8 marks]

The curve given by $\sin^2(2x) = xy$ is shown.



(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y using implicit differentiation.

[4]

(b) Give the x co-ordinate for point A, the global maximum of the curve, correct to 0.01.

[2]

(c) Determine the slope of the curve as $x \to 0$.

[2]

Question 10 [10 marks]

Using Calculus techniques, find the following indefinite integrals :

(a) $\int \sqrt{2x + 3}\, dx$

[2]

(b) $\int \frac{8}{4x + 1}\, dx$

[2]

(c) $\int 4x\, e^{-x^2}\, dx$

[2]

(d) $\int x\sqrt{x + 2}\, dx$ Put u = x + 2

[4]

Question 11 [6 marks]

Using the method of proof by induction, prove that for all counting numbers n, the expression $3^{4n} - 1$ is divisible by 80.

Question 12 [8 marks]

A particle experiencing simple harmonic motion has a displacement of 10 cm when its velocity is $\sqrt{1200}$ cm s^{-1} and when the particle is at its equilibrium position it has a velocity of 40 cm s^{-1}.

(a) Determine the period and amplitude of its motion.

[5]

(b) Determine the distance travelled by the particle, correct to the nearest 0.1 cm, over any period of time equal to $\frac{9\pi}{2}$ seconds.

[3]

Question 13 [9 marks]

(a) Solve the equation $z^5 = 1$ over the set of complex numbers, giving solutions in exact exponential form.

[3]

(b) Prove that the sum of the complex roots to part (a) is zero.

[3]

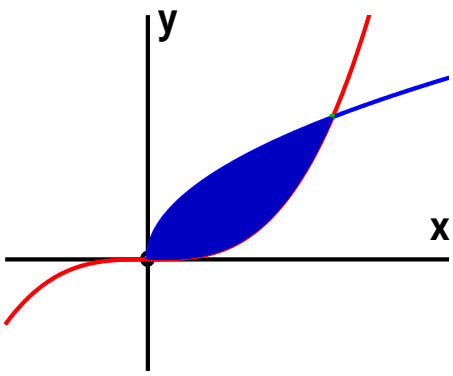
(c) Hence, or otherwise, solve exactly the equation $z^7 + z^5 - z^2 - 1 = 0$.

[3]

Question 14 [10 marks]

Consider the functions f(x) = x^n and g(x) = $\sqrt{x}$ and the area trapped between these curves.

The graph on the right shows the case where n = 3 i.e. f(x) = x^3



(a) Using Calculus, determine the exact area between the curves for the case when n = 3.

[4]

(b) If the exact area between the curves is $\frac{28}{51}$ square units, determine the value of n.

[4]

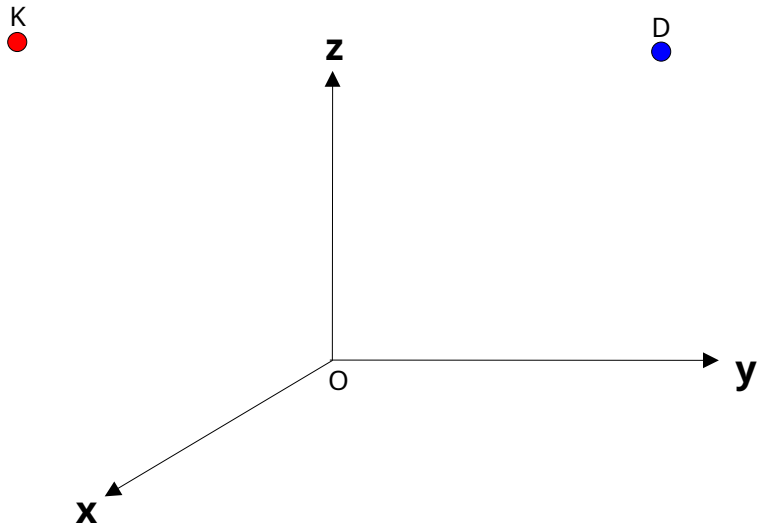
(c) If we consider an extremely high power for n, explain what happens to the area between the two curves.

[2]

Question 15 [10 marks]

A co-ordinate system is defined showing the positive co-ordinate axes with O being the origin. Two part time rock-climbers Des Duller and Kev Krudder are each attached to two straight wires that allow them to slide down within a wide canyon.

At exactly 0930 hours, Des is at a position of -250**i** + 350**j** + 700**k** metres and is sliding down his wire with velocity 2.5**i** – **j** – **k** metres per second. Meanwhile Kev is stationary at a position 500**i** – 200**j** + 800**k** metres admiring the view. At exactly 0935 hours, Kev begins to slide down his wire at a velocity of -0.5**i** + **j** – 1.5**k** metres per second.



(a) What is Kev's speed (correct to the nearest 0.01 m/sec) as he slides down his wire ?

[1]

(b) At what angle to the horizontal plane does Kev slide down, correct to the nearest degree ?

[2]

Question 15 [contd.]

(c) It is known that Des and Kev do not collide. Determine the distance of their closest approach (correct to the nearest metre) and when this occurs (correct to the nearest second).

[3]

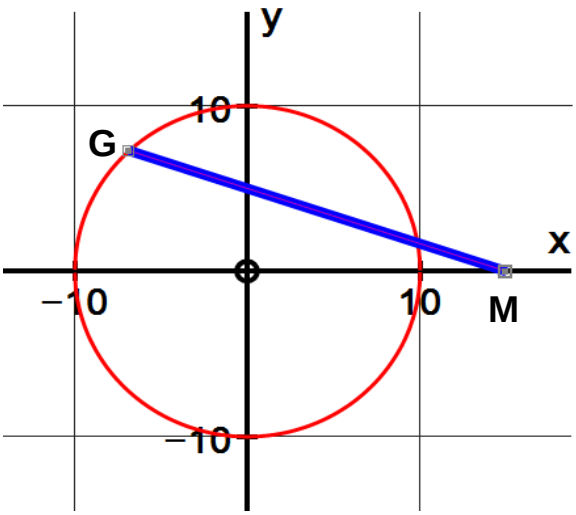
(d) If Kev was able to select both the speed and the time at which he commenced sliding down his wire, determine the distance, correct to the nearest metre, he would be able to get closest to Des ?

Explain showing your method.

[4]

Question 16 [12 marks]

A small girl G is riding on a merry-go-round and her mother is watching from a fixed position at point M that is 5 metres from the merry go-round. The merry-go-round rotates at a speed of 1 revolution every 10 seconds in an anti-clockwise direction.



Let G have co-ordinates (x, y) at any time t seconds after the start. Assume that the girl begins the merry-go-round ride at (10, 0), the closest position to the mother.

(a) Given that we can use parametric equations for the position of the point G in the form of :

$x = a\cos bt$ (metres)

$y = a\sin bt$

Explain why $a = 10$ and $b = \frac{\pi}{5}$.

[2]

(b) If the mother is directly east of the merry-go-round, at what rate, correct to the nearest cm/second, is the girl travelling in a westerly direction 3 seconds after the start of the ride ?

[3]

Question 16 (contd.)

The distance that the girl is from the mother will fluctuate during the merry-go-round ride.

We can define S(t) = the distance MG (distance between the girl from the mother).

(c) Express S(t) as a function of t.

[1]

(d) Determine the rate, correct to the nearest cm/second, at which the distance between the small girl and the mother is changing after 3 seconds.

[2]

(e) Determine when the small girl is moving away from the mother at the fastest rate, correct to the nearest 0.01 seconds.

[2]

(f) How far is the small girl from the mother at the moment she is moving away from the mother at the fastest rate ?

[1]

(g) Explain the geometric significance of the position of the small girl when she is moving away from the mother at the fastest rate.

[1]

END OF SECTION TWO

Additional working space

Question number(s): …….

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Question number(s): …….

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