Question/Answer Booklet

MATHEMATICS 3CD SEMESTER 2
Section Two
(Calculator Assumed)

Circle your teacher's initials

GJ   JIB   BAH

Booklet 1 of 2

Your name _____

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section
To be provided by the supervisor
Two Question/Answer Booklets
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

	Number of questions	Working time (minutes)	Marks available
Section 1 Calculator Free	8	50	40
This Section (Section 2) **Calculator Assumed**	**11**	**100**	**80**
		Total marks	120

Instructions to candidates

1. The rules for the conduct of WACE external examinations are detailed in the booklet *WACE Examinations Handbook*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions in the spaces provided.

3. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

4. Show all working clearly. Any question, or part question, worth more than 2 marks requires valid working or justification to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

Question 9   **(5 marks)**

The area enclosed between the curve $y = 4 - x^2$, the line $y = 3x$ and the y – axis, is rotated through 360˚ about the y – axis.
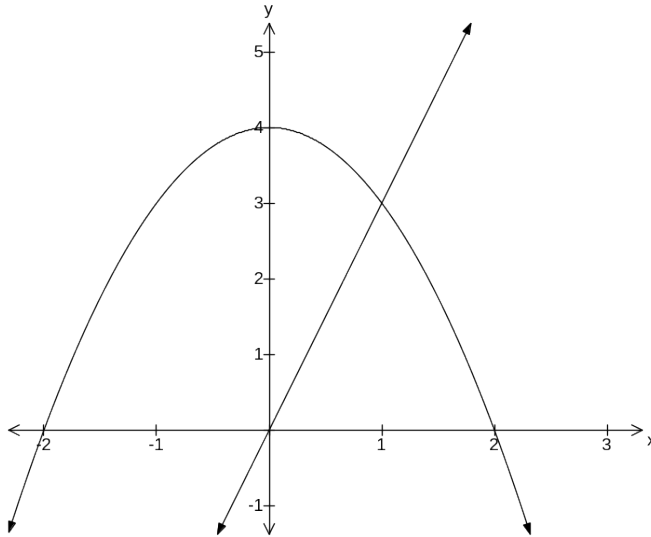
a)   shade the appropriate region on the graph above

[1]

b)   write down an integral expression that will evaluate the volume of the shape produced.

[3]

c)   find the exact value of the integral calculation in part b)

[1]

Question 10   **(7 marks)**

The Longlife Tyre Company produces a radial tyre which has a mean lifetime of 50 000 km and a standard deviation of 5 000 km. The lifetime of a tyre is normally distributed.

a)   Find the probability that any one tyre will last longer than 40 000 km.
[1]

b)   Find the probability that all four tyres bought by a particular customer will last more than 40 000 km.   [2]

c)   Find the probability that a tyre lasts more than 60 000 km given that it has already lasted 40 000 km.   [2]

d)   Approximately fifty percent of the tyres last between 45,000 km and x km. Find the value of x accurate to the nearest one hundred.   [2]

Question 11   (8 marks)

An army detachment contains 2 sergeants, 4 corporals and 12 privates.
A guard consists of 6 people selected from the group.

a)   In how many ways can the guard be formed

    i)   without restriction?   [1]

    ii)   If 1 sergeant, 1 corporal and 4 privates need to be put together
       to form the guard?   [2]

    iii)   If 1 sergeant, 1 corporal and 4 privates need to be put together
       but Sergeant Bilco refuses to serve with Private Ryan?   [3]

b)   If 1 sergeant, 1 corporal and 4 privates are selected at random from
their respective categories what is the probability that Sergeant Bilco and
Private Ryan are selected together?   [2]

Question 12   (6 marks)

Six dice with six sides numbered from 1 to 6 are rolled 500 times and the number of sixes recorded. The table below shows the results.

Number of sixes	0	1	2	3	4	5	6
Observed Frequency	89	178	148	66	17	2	0
Expected Frequency							

a) Explain why a Binomial distribution is suitable to model this situation.

[2]

b) Complete the table above to show the expected frequencies assuming that the dice are fair.

[2]

c) Comment on whether the observed data is consistent with the expected distribution.

[2]

See next page

Question 13   (7 marks)

A packaging company makes cans of salmon. Each can is supposed to contain 378g of salmon but due to variation in the canning process the actual net weight of the salmon has a normal distribution with mean 380g and standard deviation of 4g.

a)   Find the probability that an individual can of salmon has less than
     the stated amount of 378g.   [1]

b)   What would be the distribution of the sample mean if a sample of 10
     cans of salmon was taken?   [2]

c)   Find the probability that a sample of 10 cans has a mean weight of
     below 378g.   [1]

The packaging company has found a way to reduce the standard deviation of the weight of salmon in each can without affecting the mean.

d)   To what value should the standard deviation be set so that there
     is only a 1% chance that a can of salmon is underweight?   [3]

Question 14 (7 marks)

A particular manufacturer makes two types of outdoor settings, known as Standard and Deluxe. The number of Standard settings is called x and the number of Deluxe settings is called y.

The following inequalities describe the constraints of manufacturing.

$$x + 2y \leq 40 \qquad x \geq 6$$
$$x + 4y \leq 60 \qquad y \geq 4$$

Each Standard setting makes a profit of $85 and each Deluxe setting makes a profit of $63.

The graph below shows the lines equating to the inequalities above.
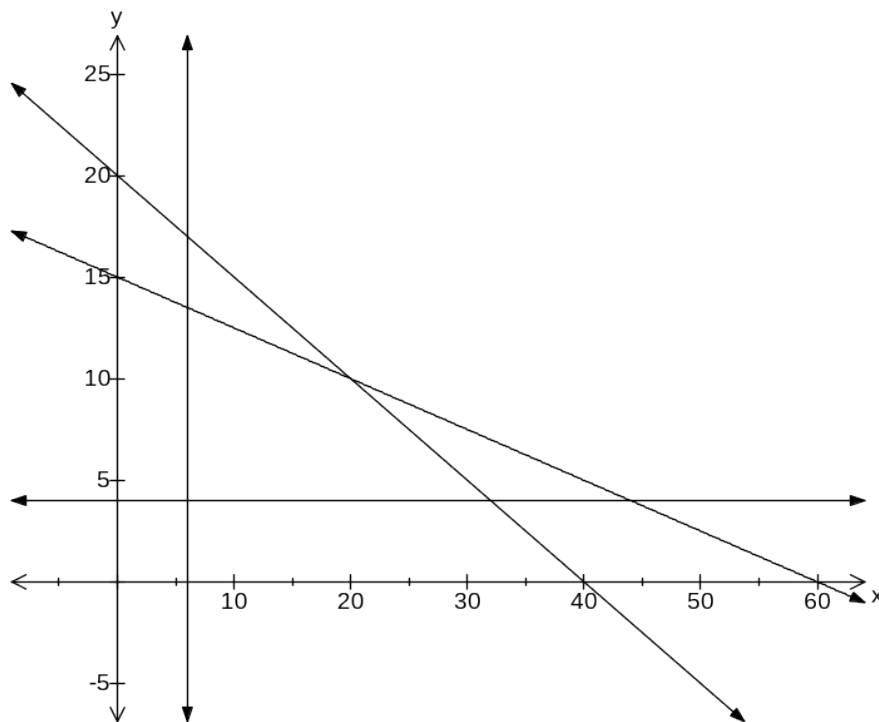
a)   Shade the feasible region on the graph.   [1]

b)   Write down the objective function for the profit.   [1]

See next page

c)	Determine how many of each type of setting should be made to maximise the profit. Show your working clearly.

[3]

d)	If the profit on a Standard setting remains the same, to what value does the profit on a Deluxe setting need to rise so that the optimal solution is changed?

[2]

Spare page

Hale School
2010

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Booklet 2 of 2

Your name _____

Question 15 **(5 marks)**

a)

At a fast food store, a sample of 100 customers is observed and their times waiting for service recorded; the sample has a mean value of 3 minutes with a standard deviation of 30 seconds.

Determine a 95% confidence interval for the mean time that it takes to serve customers in the store. Give your answer in seconds to the nearest second. [3]
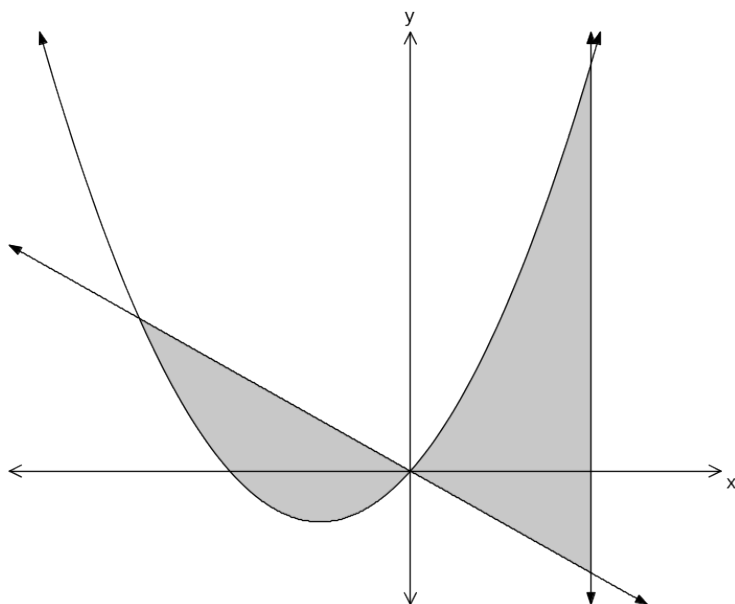
b)

At another store, the standard deviation of waiting times is also 30 seconds. How large a sample needs to be taken to be 90% confident that the sample mean will be within 8 seconds of the true mean? [2]

Question 16   (5 marks)

Write down, in the correct order, the transformations that are needed to change the graph of $y = e^{2(x-1)}$ into the graph of $y = 4e^{-x} + 1$.

Question 17   (7 marks)

The graph shows two shaded areas. A is the area bounded by the curve
y = x(x+2) and the line y = - x. B is the area between the curve y = x(x+2),
the line y = -x and the line x = a.

a)   Write down an integral that will determine the area of A.   [2]

b)   Write down an integral that will determine the area of B.   [2]

c)   Find the value of a so that area B is twice the size of area A, giving
     your answer accurate to 3 decimal places.   [3]

See next page

Question 18   **(8 marks)**

The population of Australia from 2010 onwards is expected to be modelled by the equation $P = 50 - 30e^{kt}$ where P is the population in millions and t is the time in years after the start of 2010.

a) Show that $\dfrac{dP}{dt} = k(P - 50)$ [2]

b) Given that the population is expected to reach 35 million in 2040, determine the value of k accurate to 4 decimal places.   [2]

c) Find the rate at which the population is changing when the population reaches 40 million.   [2]

d) Explain what happens to the rate of change of population, $\dfrac{dP}{dt}$ over time.   [2]

Question 19   (7 marks)

A particle has an acceleration given by the formula $a = 36 - 6t$ where a is measured in ms^{-2}. After 4 seconds the particle is instantaneously stationary.

a)   Find the velocity of the particle at time t.   [3]

b)   Find the total distance travelled in the first 15 seconds of the motion.
                                                                                        [2]

c)   When, if ever, does the particle return to its starting position?
                                                                                        [2]

Question 20   (8 marks)

Consider the functions $f(x) = 3x - 1$ and $g(x) = \sqrt{x}$

a)   Find   i) $f \circ g(4)$ [1]

       ii)   x such that $g \circ f(x) = 4$ [2]

b)   State the domain and range of $g \circ f(x)$ [2]

c)   Solve the equation $f(f(x)) = (g(x))^2$ [3]