

#### **Important note to students**

Who other items may be used in this section of the examination. It is **your** responsibility to ensure

<b>o be provided by the student</b>	pens, pencils, pencil sharpener, eraser, correction highlights	standard items:	glue/tape, ruler, special items:
<b>o to be used by the teacher</b>	drawing instruments, templates, notes on two unfolded sheets of highlighters	special items:	glue/tape, ruler, special items:
<b>o curriculum</b>	and up to three calculators satisfying the conditions set by the Council for this examination	special items:	glue/tape, ruler, special items:
<b>o paper,</b>			

**Materials required/recommended for this section**  
**To be provided by the supervisor**  
This Question/Answer Booklet  
formula Sheet (Detailed from Section One)

Time allowed for this section	Reading time before commenting work	Working time for this section:
10 minutes	100 minutes	100 minutes

Name of Student:

## **Calculator Two:**

MATHEMATICS 3C

(This paper is not to be released to take home before 25/6/2012)

Question/Answer Booklet

Semester I Examination 2012

REAP

Revisiōn Examination Assessment Papers

An illustration of a blue graduation cap resting on a white diploma scroll with gold lettering.

that you do not have any unauthorised notes or other items in the examination room.  
 If you have any unauthorised material with you, hand it to the supervisor **before**  
 reading any further.

### Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	12	12	100	100	
Total		150		100	

### Instructions to students

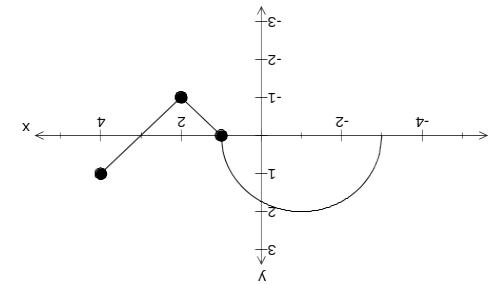
- 1 Write your answers in the spaces provided in this Question/Answer Booklet.  
 Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- 2 **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you

**Question 18 (continued)**

$$(iii) \int_2^3 f(x) dx$$

(1)

- (b) The graph of a function  $f(x)$  consists of a semi-circle and two line segments as shown.
- Find the exact value of  $\int_4^{-3} f(x) dx$



(2)

$$\int_4^{-3} f(x) dx$$

repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.  
It is recommended that you **do not use pencil**, except in diagrams.

**Section Two: Calculator-assumed  
(100 marks)**

This section has **twelve (12)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

**Question 7  
marks)**

**(10**

- (a) Emily is a very strong soccer player who has a probability of  $\frac{3}{5}$  of scoring a goal with each attempt. She has 15 attempts. Find the probability that the number of goals she scores is less than 7.  
(2)

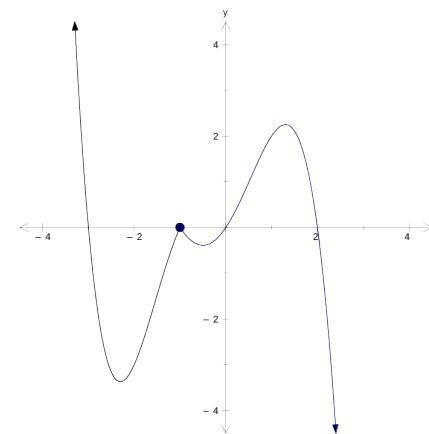
- (b) Suppose that  $Y$  is distributed normally with unknown mean  $\mu$  and standard deviation  $\sigma$ .

Given that  $P(\mu - 2.5 \leq Y \leq \mu + 2.5) = 0.9$ , find the value of  $\sigma$ . (2)

**Question 18**

**(6 marks)**

- (a) For the function  $y = f(x)$  below



It is known that

$$\int_{-3}^{-1} f(x) dx = 75$$

$$\int_{-1}^2 f(x) dx = 20$$

The area under the curve from  $x = -1$  to  $x = 2$  is 80 square units.

Use the information above and mathematical reasoning to determine the value of each of the following.

(i)  $\int_{-1}^0 f(x) dx$  (2)

- (ii) the area between the curve and the x-axis from  $x = -3$  to  $x = 0$  (1)

- Question 17 (continued)**
- (a) A group of anthropologists found that human tooth size is continuing to decrease, such that  $\frac{dS}{dt} = kS$  per 1000 years. In Northern Europeans, for example, tooth size reduction now has a rate of 1% per 1000 years, and  $S$  represents time in years and  $k$  represents tooth size, find the value of  $k$ .
- (i) If  $t$  represents time in years and  $S$  represents tooth size, find the value in how many years will human tooth size be 90% of their present size?
- (ii) What will be our descendants tooth size 20 000 years from now? (1)
- (iii) What will be our descendants tooth size as a percentage of our present tooth size?
- (iv) In how many years will human tooth size be 90% of their present size? (2)
- (c) Alice, Brownlyn and Cathy independently each think of an integer in the set  $\{1,2,3,4,5,6,7\}$ . Find the probability that, of the three integers selected, all three are greater than 4 (1)
- (b) A group of anthropologists found that human tooth size is continuing to decrease, such that  $\frac{dS}{dt} = kS$  per 1000 years. In Northern Europeans, for example, tooth size reduction now has a rate of 1% per 1000 years, and  $S$  represents time in years and  $k$  represents tooth size, find the value in how many years will human tooth size be 90% of their present size?

**Question 7 (continued)**

(c) (ii) all three are greater than 5

(1)

(iii) the least integer is 5.

(1)

(iv) the three integers are different given that the least integer selected is 5.

(2)

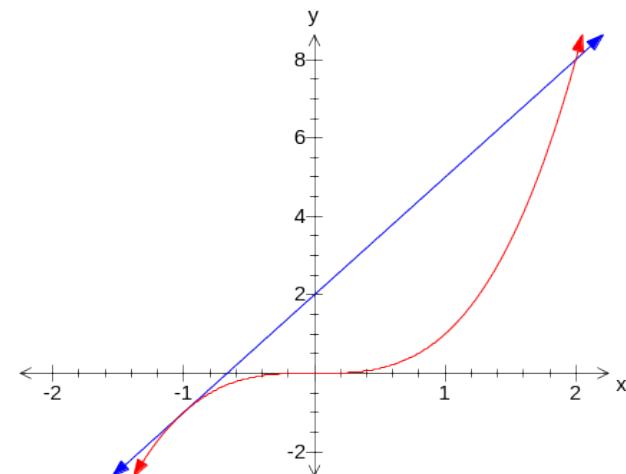
(v) the sum of the three integers is more than 15.

(1)

**Question 17****(9 marks)**(a) Shade the region, R, bounded by the curves,  $y = x^3$ ,  $y = 3x + 2$ , and  $x = 0$  in the diagram.

Find the area of the region R, showing all working steps.

(4)



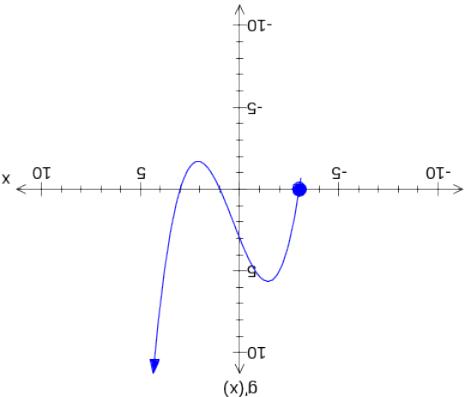
(2)

- (i) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The fluid in the tank "t" hours after the valve is opened is given by

$$y = 6 \left( 1 - \frac{t}{12} \right)^2 \text{ metres.}$$

(2)

- (ii) Find the rate  $\frac{dy}{dt}$  m/hour at which the tank is draining at time,  $t$ .

**Question 16**

CALCULATOR-ASSUMED

**Question 8**

The graph of  $g(x)$  is given below.

**Question 8**

MATHEMATICS 3C

- (a) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The fluid in the tank "t" hours after the valve is opened is given by

- (iii) When is the fluid in the tank falling fastest and slowest?

(2)

- (i) When does the function,  $g(x)$  have a negative gradient?

(1)

- (b) State an equation for the tangent to the graph of  $g(x)$  at  $x = 3$ .

(2)

- (c) Find the value of  $x$  at which  $g(x)$  has a relative maximum for  $-3 \leq x \leq 4$

(1)

- (d) Find the  $x$ -coordinate of each point of inflection of the graph of  $g(x)$  for  $-3 \leq x \leq 4$

- (e) Find the  $x$ -coordinate of each point of inflection of the graph of  $g(x)$  for  $-3 \leq x \leq 4$

**Question 8 (continued)**

- (b) If the volume of a cylinder is given by  $V = 2\pi r^3$ , find the appropriate percentage

change in  $V$  when  $r$  changes by  $\frac{1}{2}\%$   
(3)



**Question 10****(7 marks)**

- (a) The function  $f(x)$  is differentiable for all  $x \in \mathbb{R}$  and satisfies the conditions

$$f'(x) < 0 \text{ where } x < 2$$

$$f'(x) = 0 \text{ where } x = 2$$

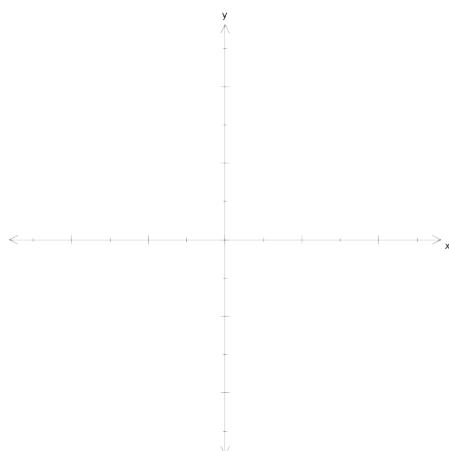
$$f'(x) = 0 \text{ where } x = 4$$

$$f'(x) > 0 \text{ where } 2 < x < 4$$

$$f'(x) > 0 \text{ where } x > 4$$

- (i) Draw a sketch of this function  $f(x)$ .

(3)



- (ii) State whether the following statement is true or false.

"The graph  $f(x)$  has a stationary point of inflection where  $x=4$ ".

(1)

- (b) If  $\int_0^a f(x) dx = a$ , find  $2 \int_0^{5a} \left[ f\left(\frac{x}{5}\right) + 3 \right] dx$

(3)

**Question 14****(9 marks)**

A piece of wire 8cm long is cut into two unequal parts. One part is used to form a rectangle that has a length three times its width. The other part of the wire is used to form a square.

- (i) If the width of the rectangle is  $x$  units, determine an equation that will give the sum of the areas of the rectangle and the square in terms of  $x$ .

(4)

- (ii) Using Calculus, find the length of each part of the wire when the sum of the areas is a minimum.

(5)

**CALCULATOR-ASSUMED**

MATHEMATICS 3C

CALCULATOR-ASSUMED

MATHEMATICS 3C

- (d) Adams' little brother, Brodie joins in this business venture. The probability that any one of Brodie's painted gnomes is Regular is 0.8. He wants to ensure that the probability that he paints at least two Superior is at least 0.9. Calculate the minimum number of gnomes that Brodie would need to paint to achieve this aim.

- (e) Brodie's garden gnomes are painted in three colors: red, blue, and green. The probability that a randomly selected gnome is red is 0.4. The probability that a randomly selected gnome is blue is 0.3. The probability that a randomly selected gnome is green is 0.2. If Brodie has 100 gnomes in his garden, calculate the expected number of each color.

**QUESTION 13 (continued)**

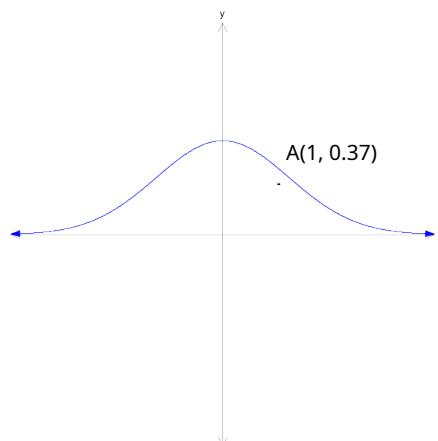
**Question 11****(7 marks)**

- (a) The function
- $y = e^{x(x-1)(x+1)}$
- is transformed to
- $y = -e^{x(1-x^2)}$
- .

Describe the transformation in order.

(3)

- (b) The curve C has equation
- $y = e^{-x^2}$
- and is drawn below



- (i) Sketch the graph of
- $y = f(-x+1)$
- .

(2)

(c) Find the expected number of these gnomes that will be Superior.

$$\begin{aligned}P(\text{2 Superior}) &= 0.63 \\P(\text{1 Superior}) &= 0.28 \\P(\text{no Superior}) &= 0.09\end{aligned}$$

It is also found that of these three gnomes,  
(c) A group of 3 consecutive garden gnomes is inspected and the first is a Regular.

- (b) If the first garden gnome inspected is Superior, find the probability that the next three gnomes are Superior.

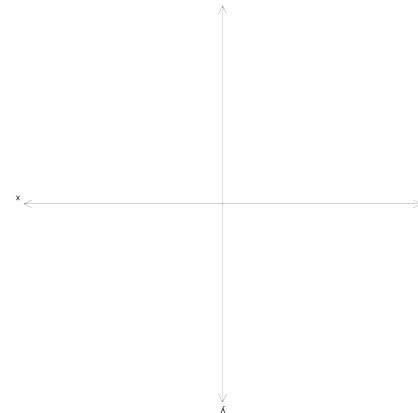
(1)

(2)

(z)

(iii) State the coordinates of A if the curve is transformed to  $y = -f\left(\frac{1}{2}x + 2\right)$

### Question 11 (continued)



### Question 13 (8 marks)

- (a) If the first garden gnome inspected is Superior, find the probability that the third garden gnome is Superior.
- If the garden gnome is Superior, then the probability that the next garden gnome is Superior is 0.9. If the garden gnome is Regular, then the probability that the next garden gnome is Superior is 0.7.
- If the garden gnome is Superior, then the probability that the next garden gnome is Superior, or 'Regular', depending on the quality of their finish.
- Adam paints garden gnomes to sell. He sends the garden gnomes to his father (a qualified quality controller) in the order of completion, who classifies them as either Superior, or 'Regular', depending on the quality of their finish.

(2)

**Question 12****(9 marks)**

- (a) A company produces fruit balls coated in either dark chocolate or milk chocolate. A large number of these fruit balls are placed in a box. Twenty per cent of the fruit balls in the box are coated with dark chocolate.

(i) Calculate  $C_4^{10}(0.2)^4(0.8)^6$  (1)

- (ii) A random sample of ten fruit balls is taken from the box.  
Explain the meaning of  $C_4^{10}(0.2)^4(0.8)^6$  with respect to this sample.

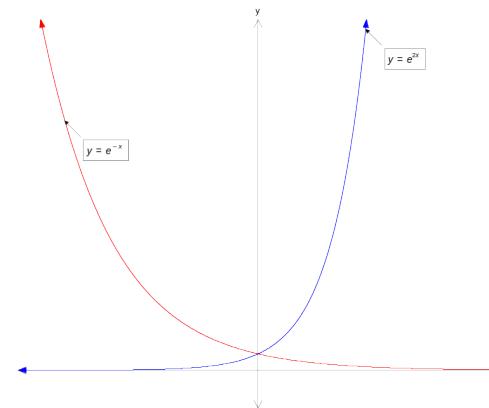
(2)

(b) (i) Find  $n$  given that  $C_0^n(0.2)^0(0.8)^n = 0.167\ 772\ 16$  (1)

- (ii) Explain the meaning of your answer to part (b) with respect to the fruit balls. (2)

**Question 12 (continued)**

- (c) The curve  $y = e^{2x}$  and  $y = e^{-x}$  intersect at the point  $(0, 1)$  as shown in the diagram.



Find the area enclosed by the curves and the line  $x=2$ .  
Leave your answer in terms of 'e'. (3)