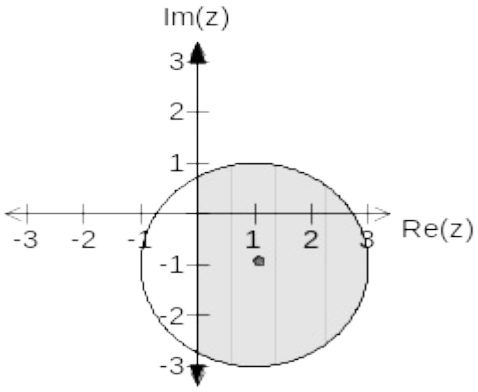
CHURCHLANDS SENIOR HIGH SCHOOL
MATHEMATICS SPECIALIST 3, 4 TEST ONE 2017
Calculator Section
Chapters 1, 2,

Name_____ **Time: 40 minutes**
Total: 36 marks

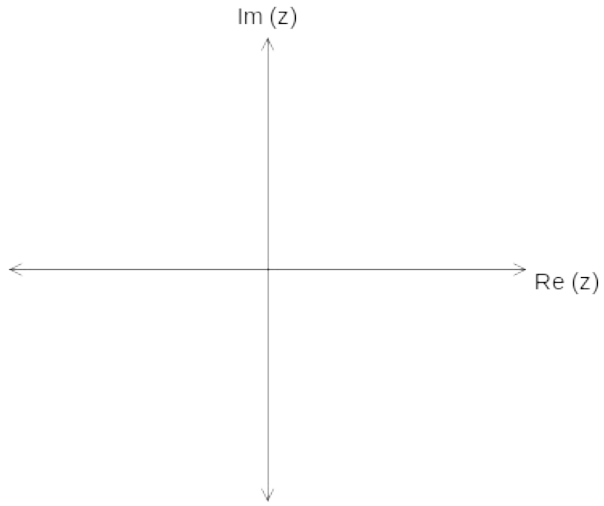
1. [5 marks:3,2]

a) State the complex relationship represented by the shaded region.



b) Sketch the following regions in the complex plane.

$$\left\{ z : -\frac{\pi}{2} < arg\,(z) \le \frac{3\pi}{4} \right\}$$



2. [3 marks]
If $z=\sqrt{2}\,cis(\frac{-4\pi}{5})$, find $w=z^9$ expressing your answer in exact polar form.

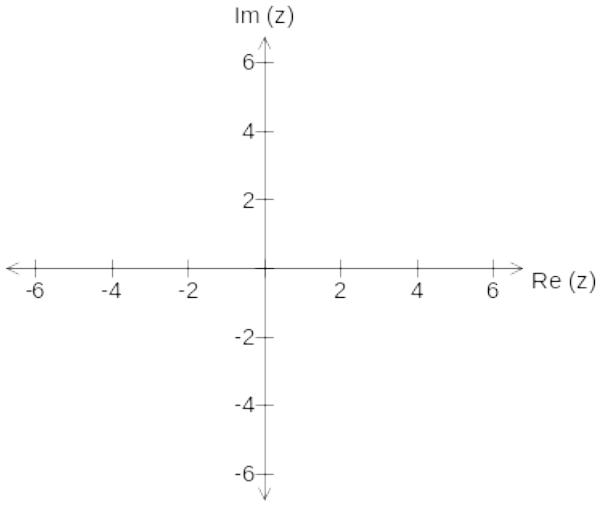
3. [6 marks]
Find the 4 fourth roots of -4 in the form $z=rcis\theta$ where $r\geq 0$ and $-\pi<\theta\leq\pi$. You need to show evidence of having used De Moivre's theorem to gain full marks.

4.[10 marks: 2,5,3]

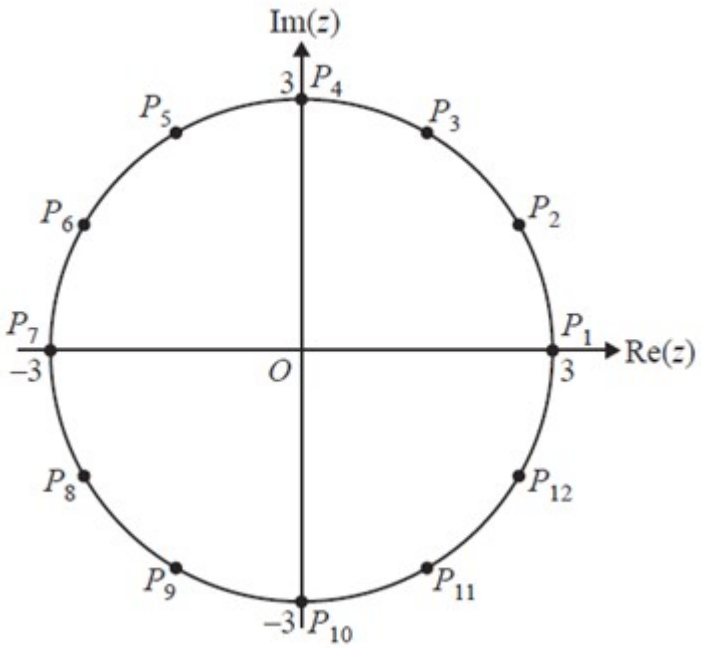
a) State the exact value of $(1-\sqrt{3}i)^4$ in Cartesian form.

b) Hence, determine exact values for all the roots of $z^4=-648+648\sqrt{3}i$

c) Sketch all the solutions from your answer above on the Argand diagram below.



5.[3 marks]
On the argand diagram below, the 12 points p_1, p_2, p_3, …p_{12} are evenly spaced around the circle of radius 3.

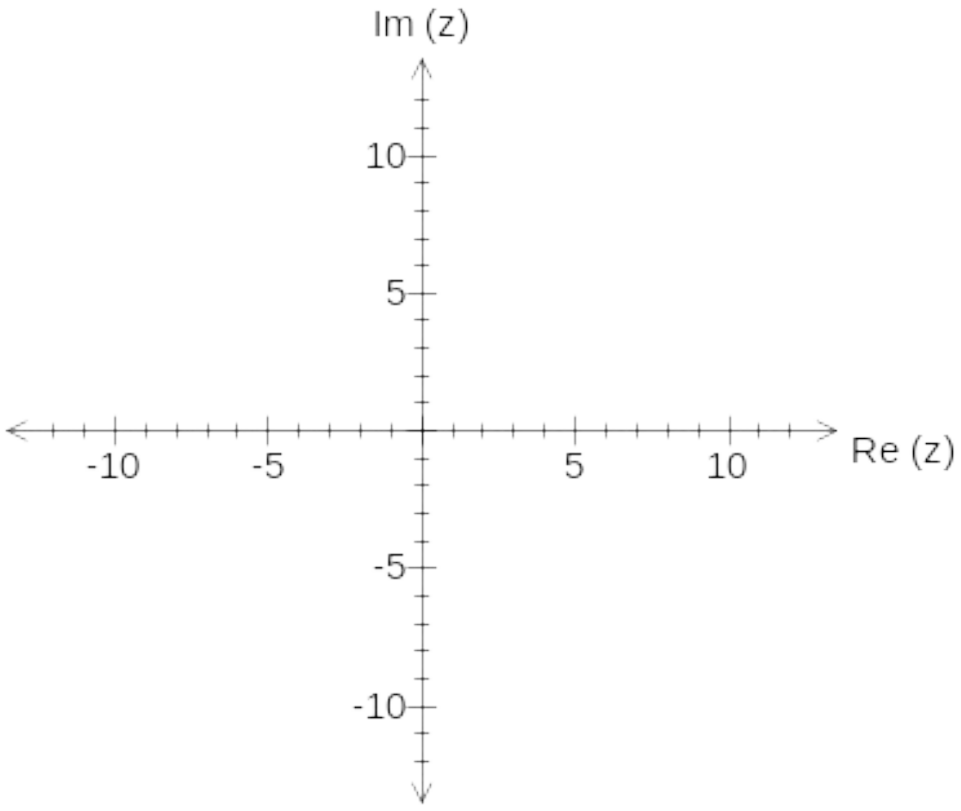


Find the points which represent complex numbers such that $z^3=-27i$

6.[3marks]

Consider $f(z)=z^3+9z^2+28z+20, z \in C(\text{complex numbers})$.
Given $f(-1)=0$, factorize $f(z)$ over C.

7. [6marks:3,3]
a) Sketch in the complex plane the region defined by $1 \leq \vee z-8-6i \vee \leq 4$.



b) Determine in polar form $rcis\theta$, $-\pi < \theta \leq \pi$, the complex number z that satisfies $|z-8-6i|=4$ and has the minimum argument.