Calculator-free Solutions

1. (a) (i) ${}^{8}P_{5}$ or ${}^{8}C_{5}\times 5!$ ✓

(ii) ${}^{4}C_{3}\times {}^{4}C_{2}\times 5!$ ✓✓

(b) (i) $6^{4}\times 5^{2}\times 6!$ ✓

(ii) ‘9’ from first set only: ${}^{5}C_{3}\times {}^{4}C_{2}\times 6!$ ✓

‘9’ from second set only: ${}^{5}C_{4}\times {}^{4}C_{1}\times 6!$ ✓

no ‘9’ chosen: ${}^{5}C_{4}\times {}^{4}C_{2}\times 6!$ ✓

total ¿ $\left({}^{5}C_{3}\times {}^{4}C_{2}+{}^{5}C_{4}\times {}^{4}C_{1}+{}^{5}C_{4}\times {}^{4}C_{2}\right)\times 6!$ ✓ [8]

2. (a) (i) $i^{n+2}=i^{n}\times i^{2}=-i\times -1=i$ ✓

(ii) $i^{2n+1}=\left(i^{n}\right)^{2}\times i=(-i)^{2}\times i=-i$ ✓✓

(b) $\dfrac{1-i}{i+\dfrac{2}{i}}\times\dfrac{i}{i}=\dfrac{i-i^{2}}{i^{2}+2}=\dfrac{1+i}{-1+2}=1+i$ ✓✓

(c) $(1+i)^{4}-(1-i)^{4}$

¿ $\left[(1+i)^{2}+(1-i)^{2}\right]\times\left[(1+i)^{2}-(1-i)^{2}\right]$

¿ $[1+2i-1+1-2i-1]\times[1+2i-1-1+2i+1]$

¿ $0\times 4i=0$ ✓✓ [7]

3. (a) $5^{3}=5+1\times 10\times 6+1\times 4\times 15=5+60+60=125$ ✓

(b) ${}^{5}C_{2},\ {}^{4}C_{1}$ ✓✓

(c) $n^{3}=n+{}^{n-1}C_{0}\times {}^{n+1}C_{1}\times {}^{n}C_{2}+{}^{n}C_{0}\times {}^{n-1}C_{1}\times {}^{n+1}C_{2}$ ✓✓ [5]

4. (a) (i) $\begin{bmatrix}0 & -1 & 1\\ 1 & 1 & -1\\ 1 & 1 & 0\end{bmatrix}\times\begin{bmatrix}1 & 1 & 0\\ -1 & -1 & 1\\ 0 & -1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ ✓✓

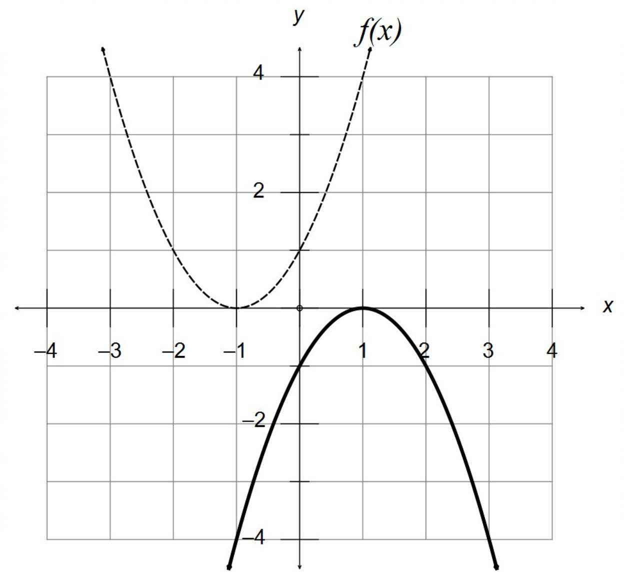
A and B are inverses of each other. ✓

(ii) $\begin{bmatrix}0 & -1 & 1\\ 1 & 1 & -1\\ 1 & 1 & 0\end{bmatrix}\times\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-6\\ 8\\ 3\end{bmatrix}$ ✓

$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0 & -1 & 1\\ 1 & 1 & -1\\ 1 & 1 & 0\end{bmatrix}^{-1}\times\begin{bmatrix}-6\\ 8\\ 3\end{bmatrix}=\begin{bmatrix}1 & 1 & 0\\ -1 & -1 & 1\\ 0 & -1 & 1\end{bmatrix}\times\begin{bmatrix}-6\\ 8\\ 3\end{bmatrix}=\begin{bmatrix}2\\ 1\\ -5\end{bmatrix}$ ✓✓

$\therefore x=2, y=1, z=-5$

4. (b) (i) T_1 performs a rotation of 180°



$\therefore\ y=-(x-1)^2$ ✓✓

(ii) Reflection of the line $y=x$ ✓

T_2¿$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ✓

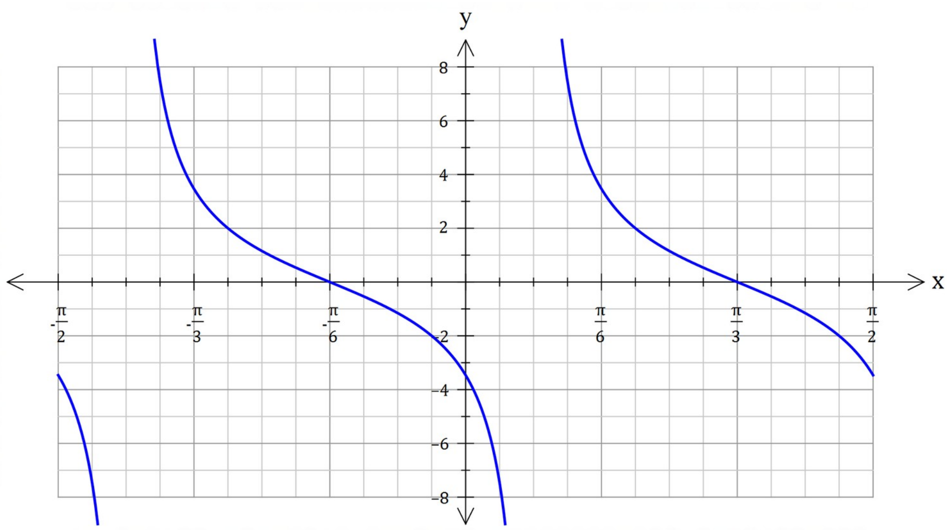
(iii) New Area ¿$|T_3|\times 10\frac{2}{3}=6\times\frac{32}{3}=64\,unit\,s^2$ ✓✓ [12]

5. (a) (i) $y=-3\cos\left[4\left(x-\frac{\pi}{12}\right)\right]=-3\cos\left[4x-\frac{\pi}{3}\right]$

$\therefore\ A=-3\ \omega=4\ \theta=\frac{-\pi}{3}$ ✓✓✓

(ii) $y=-3\sin\left(4x+\frac{\pi}{6}\right)$ ✓✓

(b)



✓ Scale factor ($y=2$ at $x=\frac{5\pi}{24}$)

✓ Period of $\frac{\pi}{2}$

✓ Vertical Asymptote

[8]

6. (a) (i) $\sqrt{2}\times\sqrt{8}=\sqrt{16}=4$ ✓

(ii) "If ab is irrational, then both a and b must be irrational" ✓

(b) A $\Rightarrow$ B: If the triangle has two equal sides, then it is isosceles, and therefore it has two congruent sides.

$\therefore$ A $\Rightarrow$ B is valid and True. ✓

B $\Rightarrow$ A: If the triangle has two congruent sides, then it is isosceles, and therefore it has two equal sides.

$\therefore$ B $\Rightarrow$ A is valid and True. ✓

$\therefore$ A $\Leftrightarrow$ B

(c) $\forall\, x\in P, \exists\, y\in P: xy\in Q$ ✓✓ [6]

7. Assume n is even and that n^3 is odd. ✓

Let $m\in N: n=2m$ is even. ✓

$n^3=(2m)^3=8m^3=2(4m^3)$ ✓

Since n^3 cannot be both even and odd simultaneously, then this is a contradiction. And therefore n must be odd. ✓ [4]