CHURCHLANDS SENIOR HIGH SCHOOL
MATHEMATICS SPECIALIST 3,4 TEST 1 SECTION TWO 2016 Year 12
Calculator Section

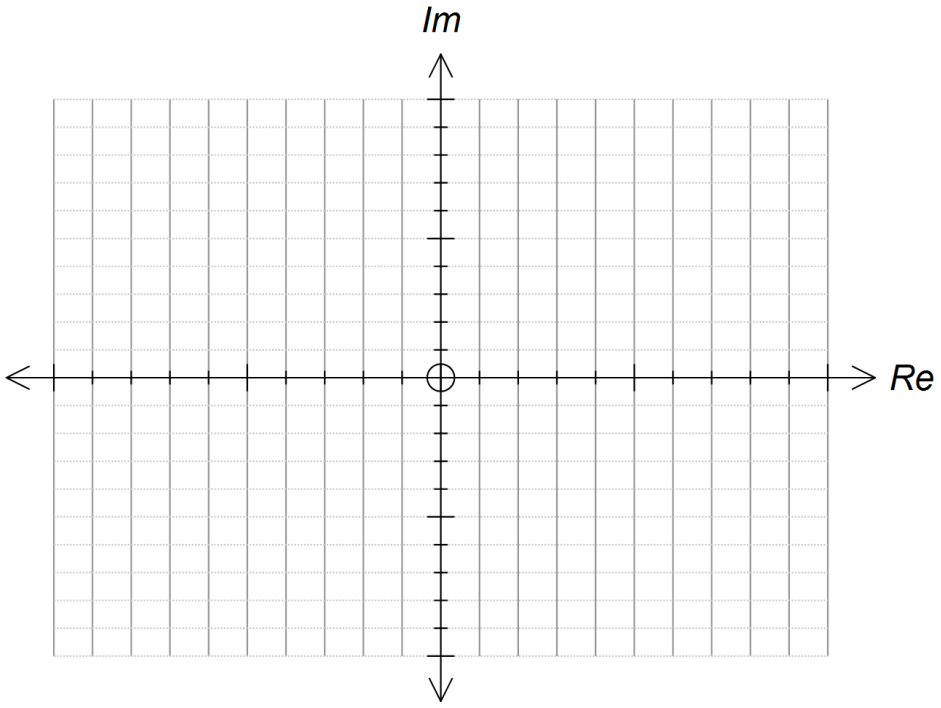
Name_____ **Time: 35 minutes**
Total: 35 marks

1. [1, 5, 3 marks]

(a) State the exact value of $(2+2\sqrt{3}i)^4$ in **Cartesian form**. (1 mark)

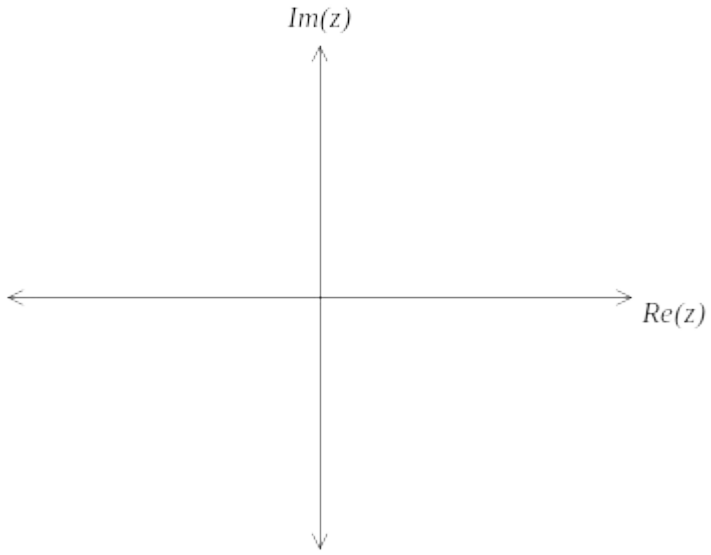
(b) Hence determine exact values for all the roots of $z^4=-8-8\sqrt{3}i$. (5 marks)

(c) Sketch all the roots from (b) on the Argand diagram below. Identify all the important features. (3 marks)

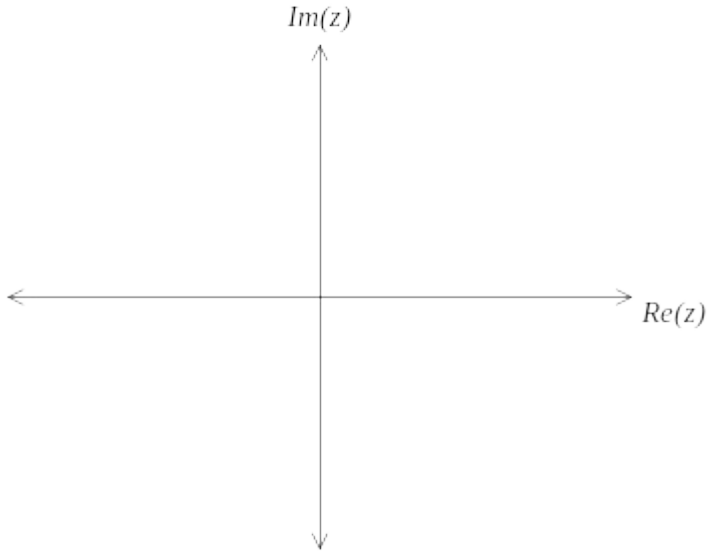


2. [2, 3, 2 marks]
Sketch the following regions in the complex plane.

(a) $-\frac{2\pi}{3} \le \arg(z) \le \frac{\pi}{3}$. (2 marks)



(b) $|z+1-2i| \ge |z+2-4i|$. (3 marks)



(c) For the region in (b) above, state the minimum value of $|z|$. (2 marks)

3. [6, 2 marks]

(a) Use de Moivre's Theorem to solve $z^5 = -iz$
Give your answers in the form $rcis\theta$ where $r \geq 0$ and $-\pi < \theta \leq \pi$
[To obtain full marks for this question, you must show clearly the use of de Moivre's Theorem.]

(b) An exact solution to $z^5 = -iz$ is $z = \left(\frac{-\sqrt{2-\sqrt{2}}}{2}\right) + \left(\frac{-\sqrt{2+\sqrt{2}}}{2}\right)i$

Given that $cos\theta = \left(\frac{-\sqrt{2-\sqrt{2}}}{2}\right)$, use your answer in (a) and the above solution to z to show that $\theta = \frac{-5\pi}{8}$. Explain clearly how you arrived at your answer.

4 [4, 3, 3 marks]

(a) Express each of the following in polar form such that $r \geq 1$ and $0 \leq \theta \leq 2\pi$. (4)

(i) $(1-i)^5$

(ii) $(-\sqrt{3}-i)^4$

(iii) $(-1+i\sqrt{3})$

(iv) $(-2+2i)^3$

(b) **Hence,** simplify $\dfrac{(1-i)^5(-\sqrt{3}-i)^4}{(-1+i\sqrt{3})(-2+2i)^3}$ giving your answer in Cartesian form.
Your working steps must show clearly how you multiply and divide complex numbers expressed in polar form. (3)

(c) The complex number z is given such that $\overline{z}=\frac{-1-i}{1-\sqrt{3}i}$.

Find z ¿, $\frac{|z|^2}{\overline{z}}$ and hence state a relationship between them. (3)