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Rossmoyne Senior High School

Year 12 Trial WACE Examination, 2014

Question/Answer Booklet

MATHEMATICS 3A/3B
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures

In words

Your name MARKING KEY

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	6	6	50	50	33⅓
Section Two: Calculator-assumed	13	13	100	100	66⅔
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

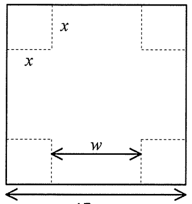
4. It is recommended that you **do not use pencil**, except in diagrams.

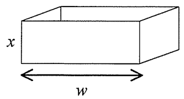
Additional working space

Question number: ________

Question 19 **(7 marks)**

A square sheet of metal has sides of length 45 cm. An open box, with a square base of side w cm, is made by cutting squares with sides of x cm out of the corners of the metal sheet and folding up the sides.





(a) Explain why $w = 45 - 2x$. (1 mark)

Width of box is width of sheet (45 cm) less two corners ($2x$).

(b) Show that the volume of the open box is given by $V = 4x^3 - 180x^2 + 2025x$ cm³. (2 marks)

$V = LWH$
$= w \cdot w \cdot x$
$= (45-2x)(45-2x)x$
$= 4x^3 - 180x^2 + 2025x$

(c) Using calculus techniques, determine the dimensions of the open box that has the maximum possible volume and state what this volume is. (4 marks)

$\frac{dV}{dx} = 12x^2 - 360x + 2025$ Must state derivative for this mark

$0 = 3x^2 - 180x + 2025 \Rightarrow x = 7.5,\ \cancel{x = 22.5}$

$w = 45 - 2(7.5) = 30$

$V_{max} = 6750$ cm³ when box is 30 by 30 by 7.5 cm

Note: sign test not needed for 3ABMAT, only needed for 3CDMAT

Section Two: Calculator-assumed **(100 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

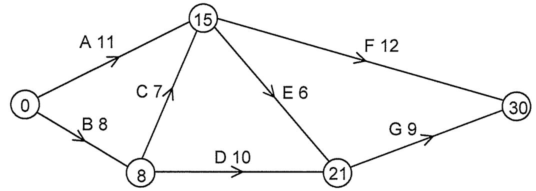
Working time for this section is 100 minutes.

Question 7 **(7 marks)**

The tasks involved in a project, their immediate predecessors and duration, are shown below.

Task	A	B	C	D	E	F	G
Immediate predecessor	-	-	B	B	A, C	A, C	D, E
Duration (days)	11	8	7	10	6	12	9

(a) Use the above information to complete the project network below. (3 marks)



-1 each error/omission

(b) State the critical path and the minimum completion time for the project. (2 marks)

Path: B - C - E - G

MCT: 30 days

(c) What is the minimum time that two people would take to complete all tasks if only one person can be allocated to each task at any one time? Justify your answer. (2 marks)

33 days.

Person A: Critical path (30 days)

Person B: A - D - F (33 days)

Question 8 **(6 marks)**

The net contents of packets of breakfast cereal filled by a machine are normally distributed with a mean of 574.7 g and a standard deviation of 6.3 g.

(a) If a packet is chosen at random from the production line, determine the probability that the contents of the packet

(i) are less than 565 g. (1 mark)

$\boxed{P(x<565)=0.062}$ ✓

(ii) lies between 570 g and 580 g. (1 mark)

$\boxed{P(570<x<580)=0.572}$ ✓

(b) During one shift, the machine filled 2 400 packets with cereal. Estimate how many of these packets have contents of less than 555 g. (2 marks)

$P(X<555)=0.000883$ ✓
$0.000883\times 2400=2\,packets$ ✓ tion here.

(c) Determine, to the nearest gram, the contents exceeded by 5% of all packets of cereal produced. (2 marks)

$P(x>k)=0.05$ ✓
$k=585$ g ✓

Question 18 **(6 marks)**

Three nurses were comparing their earnings, during the last tax year, to the national average for their occupation. Ali earned 20% more than the average, Ben earned 15% more than Ali and Chad earned $550 more than Ben.

Let x be the national average wage for nurses.

(a) Write a simplified expression, in terms of x, for the earnings of each of the nurses. (3 marks)

Ali: $1.2x$ ✓
Ben: $1.15(1.20x)=1.38x$ ✓
Chad: $1.38x+550$ ✓

Chad actually earned $10 360 more than Ali.

(b) Use this information to write an equation, solve it for x and hence determine the earnings of each of the nurses. (3 marks)

$1.38x+550=1.2x+10360$ ✓

$x=54500$ ✓ f/t

Ali earned $65400
Ben earned $75210
Chad earned $75760 ✓

Question 17 **(6 marks)**

To gain entry to a new apartment building, tenants are given an access code consisting of a non-zero digit, followed by a letter, followed by another non-zero digit (e.g. 3F2, 7Q7, etc).

(a) Show that 2106 access codes are possible. (1 mark)

$\boxed{9\times 26\times 9=2106}$ ✓

(b) How many different access codes are possible that contain a vowel and start and end with an even digit? (1 mark)

$\boxed{4\times 5\times 4=80}$ ✓

(c) What is the probability that a randomly selected access code

(i) contains a vowel? (1 mark)

$\boxed{\frac{5}{26}}$ ✓

(ii) starts and ends with an even digit? (1 mark)

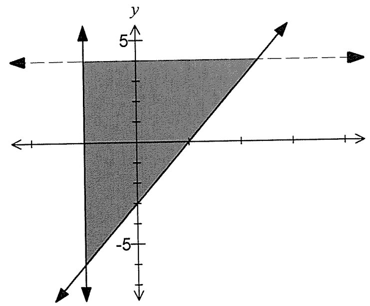
$\boxed{\frac{4\times 26\times 4}{2106}=\frac{16}{81}}$ ✓

(iii) contains a vowel or starts and ends with an even digit? (2 marks)

$\boxed{\frac{5}{26}+\frac{16}{81}-\frac{80}{2106}=\frac{19}{54}}$ ✓ ✓ f/t

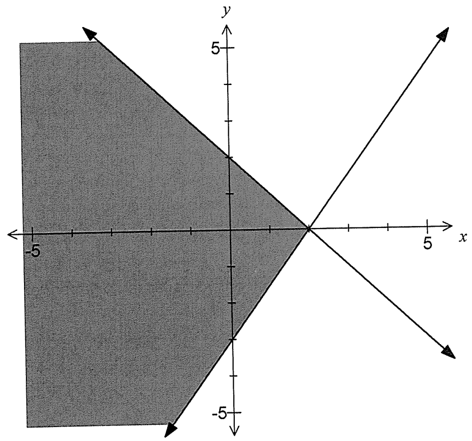
Question 9 **(6 marks)**

(a) Determine the inequalities that define the shaded region below. (3 marks)



$x\ge -1$ ✓
$y<4$ ✓
$y\ge 3x-3$ ✓

(b) Shade the region satisfied by the inequalities $x+y\le 2$ and $3x-2y\le 6$ on the axes below. (3 marks)



✓ line $x+y=2$

✓ line $3x-2y=6$

✓ shaded region

Question 10 **(8 marks)**

The ages of the 45 students attending a Statistics evening class are listed below in ascending order. The mean and standard deviation of these ages are 26.6 and 10.8 respectively.

15, 16, 16, 17, 17, 17, 18, 18, 18, 19, 19, 19, 20, 20, 21,
21, 21, 21, 22, 22, 24, 24, 24, 24, 25, 25, 25, 25, 25, 26,
27, 28, 29, 30, 32, 33, 33, 34, 34, 37, 37, 39, 56, 58, 65.

(a) Determine the mode and the median of these ages. (2 marks)

Mode: 25 ✓

Median: 24 ✓

The ages of students attending a Mathematics evening class are summarised in the frequency table below.

Age	15-19	20-24	25-29	30-34	35-39	40-44
Frequency	5	12	15	9	3	1

(b) Estimate the mean and standard deviation for the ages of the students attending the Mathematics evening class, rounding both figures to three significant figures. (3 marks)

Mean: 26.6 ✓

SD: 5.85 ✓

✓ 3 SFs for both

(c) Use the mean and standard deviation to compare the above age datasets for the Statistics and Mathematics classes, explaining any significant differences. (3 marks)

By comparing the means, the average age of students attending both classes can be seen to be the same. ✓

By comparing the standard deviations, it can be seen that the spread of ages of students in the Statistics class was much larger than that of the students in the Math class. ✓

This larger spread in the Statistics class is caused by the three students aged over 50, who are outliers for this dataset. ✓

1 mark for each reasonable correct statement

Be lenient!

(b) Prove algebraically that the value of $a^2 + b^2 - 2$ will always be a multiple of eight if a is an odd integer and b is four more than a.

(4 marks)

$a = 2n+1$
$b = 2n+5$ ✓

$a^2 + b^2 - 2 = (2n+1)^2 + (2n+5)^2 - 2$ ✓
$= 8n^2 + 24n + 24$
$= 8(n^2 + 3n + 3)$ ✓

Hence always a multiple of 8. ✓

Question 16 **(10 marks)**

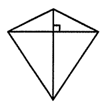
(a) Consider the validity of each of the following geometric arguments and state whether each one is **always true** or **sometimes false**.

If an argument is always true, sketch one example that demonstrates that it is true. If an argument is sometimes false, sketch one example that shows it is false.

Clearly label key features of your sketch that support your answer.

(i) 'If a quadrilateral has perpendicular diagonals, then it is a rhombus.' (2 marks)

Sometimes false:

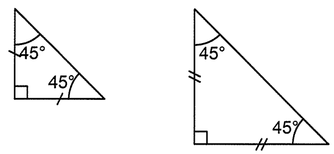


Shape is a kite, not a rhombus.

✓✓ -1 for lack of clarity

(ii) 'Two right isosceles triangles will always be similar.' (2 marks)

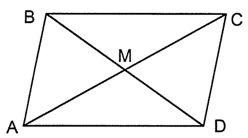
Always true:



✓✓

(iii) 'If M is the point of intersection of the diagonals AC and BD of parallelogram ABCD, then triangle ABM must be an obtuse triangle.' (2 marks)

Sometimes false:



Triangle ABM clearly acute angled.

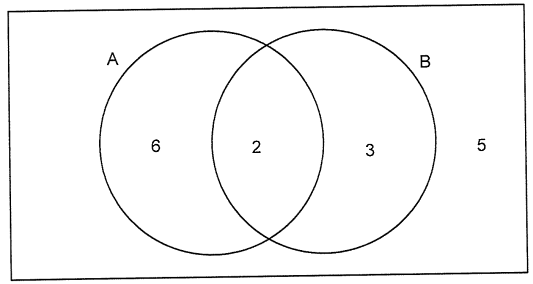
✓✓

See next page

Question 11 **(7 marks)**

A bag contains 16 balls, numbered from 1 to 16. When one ball is selected at random from the bag, event A is that the number on the ball is even and event B is that the number on the ball is a multiple of three.

(a) Complete the Venn diagram below to show $n(A \cap B)$, $n(A \cap \overline{B})$, $n(\overline{A} \cap B)$ and $n(\overline{A} \cap \overline{B})$. (3 marks)



✓✓✓ -1 each error omission

A={2, 4, 6, 8, 10, 12, 14, 16}

B={3, 6, 9, 12, 15}

f/t

(b) Determine

(i) $P(B)$. (1 mark)

$\boxed{\frac{5}{16}}$ ✓

(ii) $P(A \cup B)$. (1 mark)

$\boxed{\frac{11}{16}}$ ✓

(iii) $P(B|\overline{A})$. (2 marks)

$\boxed{\frac{3}{8}}$ ✓ numerator ✓ denominator

See next page

Question 12 **(7 marks)**

Angela buys a motorbike for $14 500. She borrows the full amount from a credit company, making monthly repayments of $1 500. Initially, the interest rate was 13.2% per annum.

The spreadsheet below shows the balance and monthly interest for the life of the loan.

Month (n)	Balance at start of month (T_n)	Interest
1	14 500.00	159.50
2	13 159.50	144.75
3	11 804.25	129.85
4	10 434.10	114.78
5	9 048.88	99.54
6	7 648.42	91.78
7	6 240.20	74.88
8	4 815.08	57.78
9	3 372.86	40.48
10	1 913.34	22.96
11	436.30	5.24

(a) Calculate the eleventh (final) payment made by Angela. (1 mark)

$$\boxed{436.30 + 5.24 = \$441.54}$$

(b) Write a recursive rule to determine the balance at the start of each month, when the interest rate was 13.2% per annum. (2 marks)

$1 + 13.2 \div 12 \div 100 = 1.011$

$T_{n+1} = T_n \times 1.011 - 1500,\ T_1 = 14500$

(c) The interest rate increased at the start of one month and then did not change for the remainder of the loan period.

(i) In which month did the rate increase? (1 mark)

Month 6

(ii) What was the increase in the annual interest rate? (2 marks)

$\frac{91.78}{7648.42} \times 12 \times 100 = 14.4\%$

Increase is 1.2%

f/t

(d) What was the total amount of interest paid by Angela? (1 mark)

$$\boxed{10 \times 1500 + 441.54 - 14500 = \$941.54}$$

(c) Calculate the value of the missing entries **A** and **B** in the table. (3 marks)

$A = 647.9 - 639.0 \Rightarrow A = 8.9$

$$\frac{\frac{664.1}{2} + 649.7 + 647.5 + 642.6 + \frac{B}{2}}{4} = 652.7 \Rightarrow B = 677.9$$

(d) Use your calculator to determine the equation of the linear regression line that could be used to predict the four-point centred moving average (n) from time (t) and state the correlation coefficient for this association. (3 marks)

$n = 2.225t + 618.503$

$r = 0.878$

(e) Use appropriate data from the residuals in the table to calculate the seasonal component for February. (1 mark)

3.567

(f) Predict the number of people employed in the accommodation and food services industry in Australia in February 2006 and comment on the reliability of your prediction. (3 marks)

$t = 21$

$n = 2.225(21) + 618.503 + 3.567$

$= 668.342$

Or

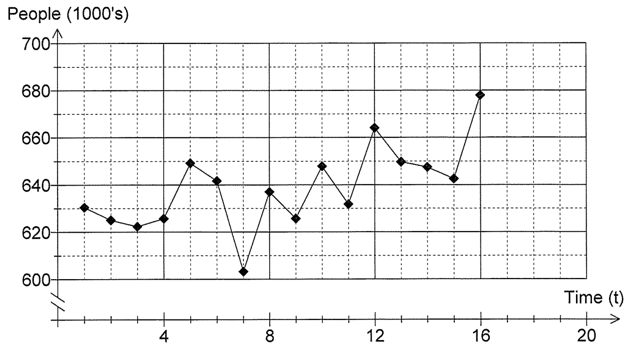
668342 *people(approx)*

Despite strong correlation for this relationship, the prediction involves considerable extrapolation and so should be treated with caution.

Question 15 **(12 marks)**

The table and graph below show the number of people employed in the accommodation and food services industry in Australia from February 2001 to November 2004.

Year	Quarter	Time (t)	People (1000's)	Four-point centred moving average (n)	Residual
2001	Feb	1	630.4		
2001	May	2	625.0		
2001	Aug	3	622.3	628.2	-5.9
2001	Nov	4	625.7	632.6	-6.9
2002	Feb	5	649.2	632.3	16.9
2002	May	6	641.7	631.4	10.3
2002	Aug	7	603.2	629.8	-26.6
2002	Nov	8	637.0	627.7	9.3
2003	Feb	9	625.7	632.0	-6.3
2003	May	10	647.9	639.0	**A**
2003	Aug	11	631.8	645.4	-13.6
2003	Nov	12	664.1	648.3	15.8
2004	Feb	13	649.7	649.6	0.1
2004	May	14	647.5	652.7	-5.2
2004	Aug	15	642.6		
2004	Nov	16	**B**		



(a) How do the data points for August support the use of a four-point centred moving average to smooth the entire data set? (1 mark)

The data points for August tend to be the lows for each year, suggesting that there is a cycle of four quarters to the data.

(b) Write down the calculation used to evaluate the four-point centred moving average for August 2001. (1 mark)

$$\frac{\frac{630.4}{2}+625.0+622.3+625.7+\frac{649.2}{2}}{4}$$

See next page

Question 13 **(7 marks)**

In triangle ABC, $a=15.4$ cm, $b=12.8$ cm and $\angle B=50°$.

(a) If ABC is an **acute-angled** triangle,

(i) write down an equation that could be solved to determine the size of $\angle A$. (1 mark)

$$\frac{15.4}{\sin A}=\frac{12.8}{\sin 50}$$

(ii) determine the size of $\angle A$. (1 mark)

$$A=67.2°$$

(b) If ABC is an **obtuse-angled** triangle,

(i) write down an equation that could be solved to determine the length c. (2 marks)

$\angle A=180-67.2=112.8° \Rightarrow \angle C=180-112.8-50=17.2°$

$\frac{c}{\sin 17.2}=\frac{12.8}{\sin 50}$ or $12.8^2=c^2+15.4^2-2c(15.4)\cos 50$

(ii) determine the length c. (1 mark)

$c=4.93$ cm

(c) Determine the largest possible area of triangle ABC. (2 marks)

Largest if triangle ABC is acute angled.

$\angle C=180-67.2-50=62.8°$

Area $=0.5(15.4)(12.8)\sin 62.8$

$=87.7$ cm^2

See next page

Question 14 **(11 marks)**

Consider the function $f(x) = 136 - 96x - 4x^2 + 24x^3 - 4x^4$.

(a) Calculate $f(0)$ and $f(5)$. (1 mark)

$f(0) = 136$

$f(5) = 56$

✓ Both

(b) Using calculus techniques, determine the coordinates of all stationary points of the graph of $y = f(x)$ in the interval $0 \le x \le 5$. (4 marks)

$\frac{dy}{dx} = -16x^3 + 72x^2 - 8x - 96$ ✓ Must show derivative for the mark

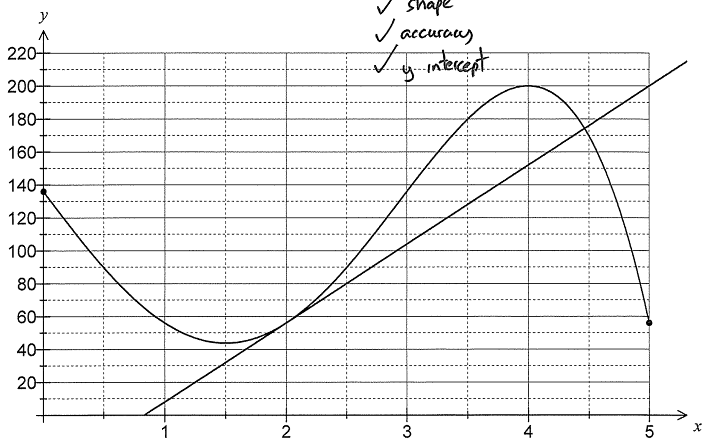
Solve $0 = -16x^3 + 72x^2 - 8x - 96$ ✓

$\cancel{x=-1}$, $x = 4$, $x = 1.5$ ✓

Stationary points at $(4, 200)$ and $(1.5, 43.75)$ ✓

(c) Sketch the graph of $y = f(x)$ over the interval $0 \le x \le 5$. **(Ignore tangent line drawn)**(3 marks)

✓ shape
✓ accuracy
✓ y intercept



(d) Calculate the gradient of the curve $y = f(x)$ at the point (2, 56). (1 mark)

$f'(2) = 48$ ✓

(e) State the equation of the tangent to the curve $y = f(x)$ at the point (2, 56). (2 marks)

$y = 48x - 40$ ✓✓