

**the end of week 1 of term 4, 2017**

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## Marking Key

## Calculator-Assumed

**MAWA Semester 2 (Units 3 and 4)**  
**Examination 2017**

## MATHEMATICS METHODS

Section Two: Calculator-assumed

(99 Marks)

**Question 10(a)**

Solution

Isotope A decays faster.

Reason: Its half-life is less than the half-life of isotope B, i.e. it loses half of its mass faster than isotope B loses half of its mass.

Marking key/mathematical behaviours	Marks
• answers correctly	1
• uses the concept of half-life correctly	1

**Question 10(b)**

Solution

May assume that  $A(t) = e^{-at}$  and  $B(t) = e^{-bt}$  where  $A(t)$  and  $B(t)$  are the amounts of isotopes A and B respectively,  $t$  years from now.

$$\text{Using the half-lives: } e^{-157a} = \frac{1}{2} \text{ and } e^{-359b} = \frac{1}{2}.$$

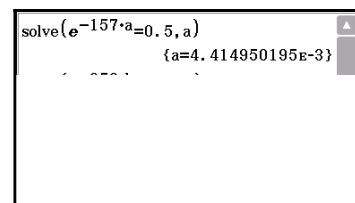
$$\text{So } a = \frac{\ln 2}{157} \approx 4.4150 \times 10^{-3} \quad \text{and}$$

$$b = \frac{\ln 2}{359} \approx 1.9308 \times 10^{-3}$$

$$\text{When } \frac{B(t)}{A(t)} = 100, \quad \frac{e^{-0.0019308t}}{e^{-0.0044150t}} = 100 \quad (\#)$$

$$\text{i.e. } e^{0.0024842t} = 100, \text{ i.e. } t \approx 1853.8$$

So it takes 1854 years before the ratio of the concentrations become 100 to 1.



Marking key/mathematical behaviours	Marks
• uses exponential models for the amounts of isotopes at time $t$	1
• uses half-lives to solve for the constants $a$ and $b$ correctly	1
• uses equation (#)	1
• solves for the time, correct to the nearest year.	1

**Question 11(a)**

Solution

Population would be all the people eligible to vote in the election

Sample is the 100 voters asked

Marking key/mathematical behaviours	Marks
• Identifies population correctly	1
• Identifies sample correctly	1

**Question 11(c)**

Use a method to randomly choose 100 people from the electoral role	Solution
For 100 estimate of proportion is 0.35	For 200 $E(p) = 0.35$
Lower 0.3	Upper 0.4
Marks	1

**Question 11(d)**

Marking key/mathematical behaviours	Solution
• states a suitable method	• states a suitable method
Marks	1

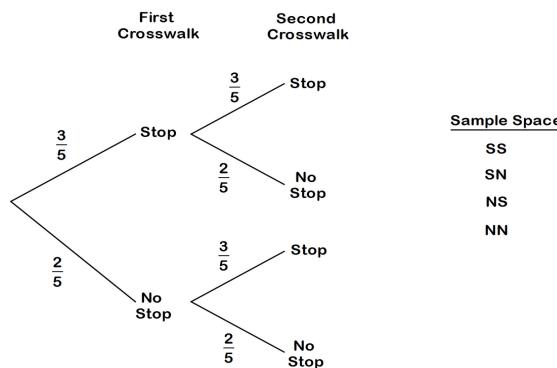
**Question 11(b)**

Marking key/mathematical behaviours	Solution
• evaluates standard deviation accurately	$\text{Std Dev}(p) = \sqrt{\frac{0.35(1-0.35)}{200}} = 0.0373$
Marks	1

**Question 11(c)**

Marking key/mathematical behaviours	Solution
• evaluates distribution of $p$ correctly	$P(0.3 < p < 0.4) = 0.8618$
Marks	1

Solution



Marking Key/mathematical behaviours

- correctly drawn and labelled tree diagram
- states the sample space

Marks

1  
1

Question 12(b)

Solution

c	0	1	2
Pr(C = c)	0.16	0.48	0.36

Marking key/mathematical behaviours

- calculates correct probabilities (if only two correct, allow 1 mark)

Marks

2

Question 12(c)

Solution

$$\begin{aligned} n &= 5 \quad p = 0.84, \quad \mu = np \\ &= 5(0.84) \\ &= 4.2 \end{aligned}$$

∴ The Bernesse family may expect to stop at least once, five times over the five days.

Marking key/mathematical behaviours

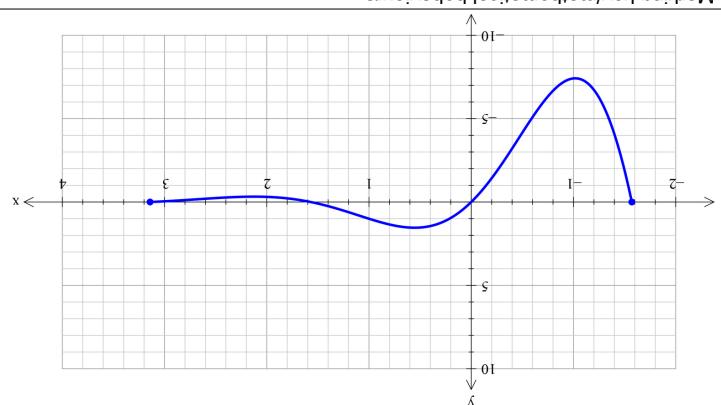
- recognises the binomial distribution and correctly calculates the expected value

Marks

1+1

## Question 22(a)

MATHEMATICS METHODS	CALCULATOR-ASSUMED	MARKING KEY
SEMESTER 1 (UNITS 3 AND 4) EXAMINATION	MATHEMATICS METHODS	SEMESTER 1 (UNITS 3 AND 4) EXAMINATION



## Question 22(a)

Marks	Marks
Marks	Marks
1+1	1

## Solution 13(a)

$$\Pr(\text{train is late 4 times out of 15}) = {}^{15}C_4(0.7)^4(0.3)^{11}$$

Solution  
Pr (train is late 4 times out of 15)

$$= 0.219$$

Value  
• recognises the binomial distribution and correctly calculates the expected

## Question 22(b)

Marks	Marks
Marks	Marks
1	1

## Solution 13(b)

$$\Pr(\text{train is late once over the 8 days}) = 8C_1(0.219)^1(0.781)^7$$

Solution  
Pr that train is not late over the 8 days

$$= 0.138$$

Pr that train is late over the 8 days

$$= 8C_0(0.219)^0(0.781)^8$$

Pr (train is late 4 times for at least 2 of the next 8 days):

$$\text{late 4 times per day} = 0.219 \dots \text{from part (a)}$$

Pr (train is late 4 times for at least 2 of the next 8 days):

$$= 1 - 0.138 - 0.311$$

∴ Pr train is late 4 times over the 8 days

$$= 0.551$$

## Question 22(c)

Marks	Marks
Marks	Marks
1	1

## Solution 13(c)

$$\Pr(\text{train late once}) = 0.7(0.3)(0.7) = 0.103$$

Solution  
Pr train late once

$$= 0.103$$

Marketing key/mathematical behaviours

• recognises ordered probability and uses appropriate calculation

## Question 22(d)

Marks	Marks
Marks	Marks
2	2

## Using the CAS calculator to solve for a:

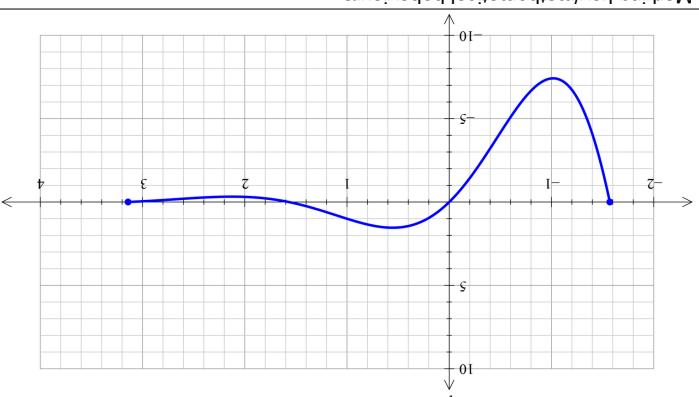
```
a=-14.69074126, a=-13.113133929, a=-0.6044564071
solve(∫a2 3xe-xsin(2x)dx=0, a)
solve(∫a2 3xe-xsin(2x)dx=0, a)
```

From the graph in part (a) it is obvious that  $\frac{2}{x} < a < 0$  so, need to select

## Question 22(e)

Marks	Marks
Marks	Marks
1	1

## Solution 22(e)



## Solution 13(e)

$$\Pr(\text{train is late once}) = 1 - 0.138 - 0.311$$

Solution  
Pr train is late once

$$= 0.551$$

Marketing key/mathematical behaviours

• recognises the binomial distribution and correctly calculates the expected

minimum and maximum points are reasonably accurate

• clearly shows the correct intercepts

• graph is appropriately smooth

## Question 22(f)

## Solution 22(f)

## Question 22(g)

## Solution 22(g)

Marks	Marks
Marks	Marks
1	1

## Solution 13(g)

$$\Pr(\text{train is late once}) = 1 - 0.138 - 0.311$$

Solution  
Pr train is late once

$$= 0.551$$

Marketing key/mathematical behaviours

• recognises the binomial distribution and correctly calculates the expected

minimum and maximum points are reasonably accurate

• clearly shows the correct intercepts

• graph is appropriately smooth

## Question 22(h)

## Solution 22(h)

## Question 22(i)

## Solution 22(i)

## Question 22(j)

## Solution 22(j)

## Question 22(k)

## Solution 22(k)

## Question 22(l)

## Solution 22(l)

## Question 22(m)

## Solution 22(m)

## Question 22(n)

## Solution 22(n)

## Question 22(o)

## Solution 22(o)

## Question 22(p)

## Solution 22(p)

## Question 22(q)

## Solution 22(q)

## Question 22(r)

## Solution 22(r)

## Question 22(s)

## Solution 22(s)

## Question 22(t)

## Solution 22(t)

## Question 22(u)

## Solution 22(u)

## Question 22(v)

## Solution 22(v)

## Question 22(w)

## Solution 22(w)

## Question 22(x)

## Solution 22(x)

## Question 22(y)

## Solution 22(y)

## Question 22(z)

## Solution 22(z)

## Question 22{aa}

## Solution 22{aa}

## Question 22{ab}

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## Solution 22{cp}

## Question 22{cq}

## Solution 22{cq}

## Question 22{cr}

## Solution 22{cr}

MATHEMATICS METHODS  
SEMESTER 1 (UNITS 3 AND 4) EXAMINATION  
Question 14(a)

Solution

$$N \propto \log_{10} \left( \frac{P}{P_0} \right)$$

Since  $N$  is the noise level in decibels and  $P$  is the power and  $P_0$  is a reference power level, and since  $N$  increases by 10 if the power increases by a factor of 10,  $N = 10(\log_{10} P - \log_{10} P_0)$ , (#)

So if  $P$  increases by a factor of 40,  $N$  increases by  $10\log_{10} 40 \approx 16.02 \text{ dB}$

Marking key/mathematical behaviours	Marks
• obtains equation (#) or equivalent	1
• obtains correct answer	1

Question 14(b)(i)

Solution

$$\text{Since } 2 \times 7^2 = 98 \approx 100 = 10^2$$

it follows that  $\log_{10} 2 + 2 \log_{10} 7 \approx 2$  (#)

$$\text{i.e. } \log_{10} 7 \approx 1 - \frac{\log_{10} 2}{2} \approx 1 - \frac{0.30}{2} = 0.85$$

Marking key/mathematical behaviours	Marks
• obtains approximation (#)	1
• obtains correct answer	1

Question 14(b)(ii)

Solution

$$\text{Since } 2^{12} \times 3^5 = 995328$$

$$\text{and } 995328 \approx 1000000 = 10^6$$

it follows that  $12 \log_{10} 2 + 5 \log_{10} 3 \approx 6$  (#)

$$\log_{10} 3 \approx \frac{6 - 12 \log_{10} 2}{5} \approx \frac{6 - 12 \times 0.30}{5} = 0.48$$

and so

Marking key/mathematical behaviours	Marks
• evaluates $2^{12} \times 3^5$ correctly	1
• obtains approximation (#)	1
• obtains correct answer	1

CALCULATOR-ASSUMED  
MARKING KEY

MATHEMATICS METHODS  
SEMESTER 1 (UNITS 3 AND 4) EXAMINATION  
Question 21(c)(ii)

Solution:

```
define f(x)=2+π/(x+5)+sin(π/90*x)
done
∫₀¹²⁰ f(x)dx
293.0842313
□
```

The heat loss is ~293 kilojoules.

Marking key/mathematical behaviours	Marks
• indicates that the heat loss in the integral from 0 to 120 of $\frac{dH}{dt}$	1
• states the correct result	1
• states the correct units	1

Question 21(c)(iii)

Solution

```
define f(x)=2+π/(x+5)+a*sin(π/90*x)
done
solve(∫₀¹²⁰ f(x)dx=300, a)
{a=1.160937246}
□
```

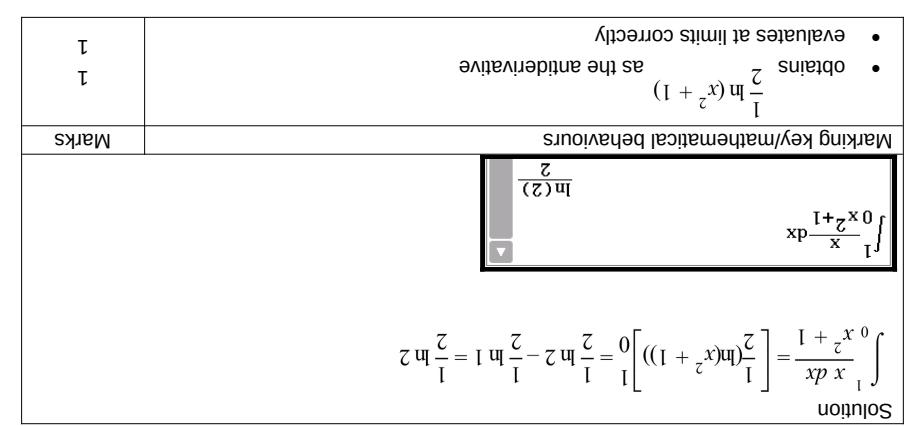
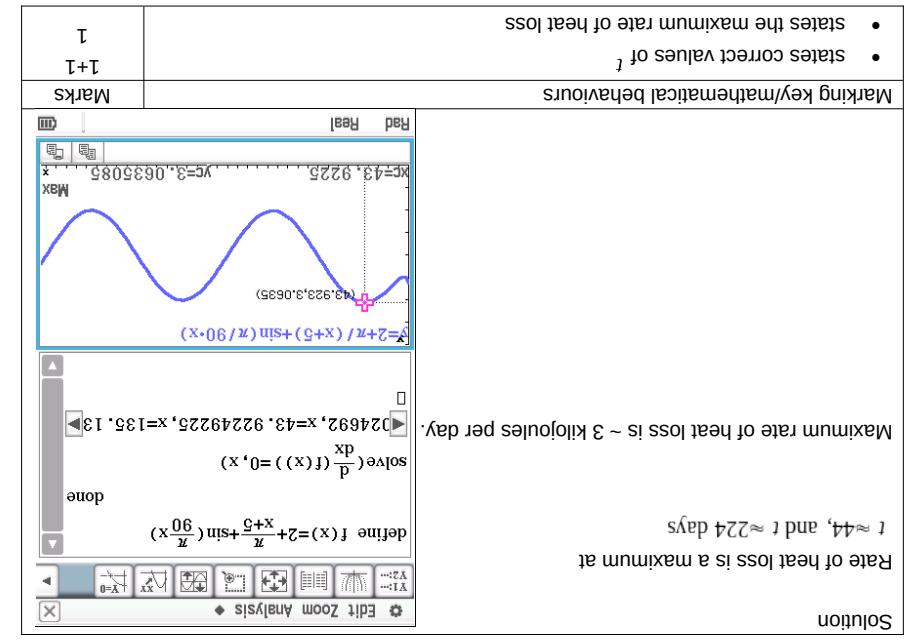
$$a \approx 1.16$$

Marking key/mathematical behaviours	Marks
• indicates solving the integral of from 0 to 120 of $\frac{dH}{dt} = 300$	1
• states the correct result	1

<p><b>Solution 15(c)</b></p> <p>Marking key/mathematical behaviours</p> <ul style="list-style-type: none"> <li>• differentiates correctly</li> <li>• details correct answer</li> <li>• details equation (#)</li> <li>• details mathematical behaviours</li> </ul> <p>So the number of daylight hours will be decreasing fastest on the 82<sup>nd</sup> day, i.e. on March 23<sup>rd</sup>.</p> $\text{So } \frac{dy}{dt}(t) = -\frac{365}{2\pi} \sin \frac{365}{2\pi(t+9)} = -\frac{365}{2\pi} \sin \frac{365}{2\pi(t+9)}$ $\text{i.e. when } t + 9 = \frac{\pi}{2} \text{ min when } t = 82.25$ <p><b>Marks</b></p>	<p>1 1 1 1 Marks</p>
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<p><b>Solution 15(b)</b></p> <p>Marking key/mathematical behaviours</p> <ul style="list-style-type: none"> <li>• differentiates correctly</li> <li>• details equation (#)</li> <li>• details mathematical behaviours</li> </ul> <p>So the 356<sup>th</sup> day, (December 22<sup>nd</sup>) will be the longest day.</p> $\text{When } y(t) = y_{\max} \text{ we have } \frac{365}{2\pi(t+9)} = 2\pi$ $\text{i.e. } t + 9 = 365 \text{ i.e. } t = 356$ <p><b>Marks</b></p>	<p>1 1 1 1 Marks</p>
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<p><b>Solution 15(a)</b></p> <p>Marking key/mathematical behaviours</p> <ul style="list-style-type: none"> <li>• differentiates correctly</li> <li>• solves for <math>a</math> and <math>b</math> correctly</li> <li>• details equations (#)</li> <li>• details mathematical behaviours</li> </ul> <p>Since the period is 1 year, i.e. 365 days, <math>c = 365</math></p> $\text{and so } a = 12 \text{ and } b = 2.5$ $y_{\max} = a + b = 14.5 \text{ and } y_{\min} = a - b = 9.5 \text{ (#)}$ <p><b>Marks</b></p>	<p>1 1 1 1 Marks</p>
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**Question 15(d)**

Solution

$$y'_{\min} = -\frac{5\pi}{365} \approx -0.0430$$

By the increments formula  $\delta y \approx y' \times \delta t$  and so if  $\delta t = 1$   $\delta y \approx y' \approx -0.0430$

So the largest difference in the number of daylight hours in successive days is 0.043 hours, i.e. 2.6 minutes.

Marking key/mathematical behaviours	Marks
• correctly calculates $y'_{\min}$	1
• uses increments formula correctly	1

**Question 16(a)**

Solution

$$X \sim N(3.5, 0.2^2)$$

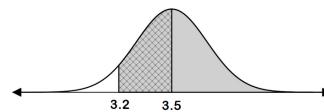
$$(i) P(X = 3.5) = 0$$

$$X \sim N(3.5, 0.2^2)$$

$$(ii) P(X > 3.2) = 0.93$$

(iii)

$$\begin{aligned} P(X < 3.5 | X > 3.2) &= \frac{P(3.2 < X < 3.5)}{P(X > 3.2)} \\ &= \frac{0.4332}{0.9332} \\ &= 0.4642 \end{aligned}$$



Marking key/mathematical behaviours	Marks
• recognises exact probabilities are equal to zero	1
• calculates correct probability	1
• applies the appropriate formula and associated probabilities leading to the correct answer and correct diagram	1+1+1

**Question 16(b)**

Solution

Tail setting <b>Left</b>	<input type="checkbox"/> Help
prob 0.8	<input type="checkbox"/> Next >>
σ 0.2	<input type="checkbox"/> Back
μ 3.5	<input type="checkbox"/> Help

x <sub>1</sub> InvN <b>3.6683242</b>	<input type="checkbox"/> Help
prob 0.8	<input type="checkbox"/> Back
σ 0.2	<input type="checkbox"/> Help
μ 3.5	<input type="checkbox"/> Help

$P(X \leq m) = 0.8$   
 $\Rightarrow m = 3.668$

Marking key/mathematical behaviours	Marks
• states probability condition involving $m$	1
• calculates the correct value for $m$	1

**Question 20(b)**

Solution

The area of the triangle formed by  $g(x)$  and the  $x$ -axis (between  $x = 0$  and  $x = 2$ ) = 1 square unit.

Hence,

$$(i) \text{ region A} = \left| \int_0^2 f(x)dx \right| - 1 = 5.1 - 1 = 4.1$$

(ii)

$$\begin{aligned} \text{Region B} &= \int_0^4 f(x)dx - \int_0^4 g(x)dx \\ &= \int_0^4 f(x)dx - \int_0^4 f(x) - \int_0^4 g(x)dx \\ &= 2.18 - (-5.1) - 1 \\ &= 1.92 \end{aligned}$$

Marking key/mathematical behaviours	Marks
• Calculates the area of the triangle	1
• Calculates the area of region A	1
• Defines region B in terms of integrals of $f(x)$ and $g(x)$	1
• Re-arranges the integrals using the integral properties so as to be able to use the information given	2
• Shows the required result.	1

**Question 21(a)**

Solution

$$\int_0^1 \frac{dx}{x+1} = [\ln(x+1)]_0^1 = \ln 2 - \ln 1 = \ln 2$$

$$\boxed{\int_0^1 \frac{1}{x+1} dx} \quad \boxed{\ln(2)}$$

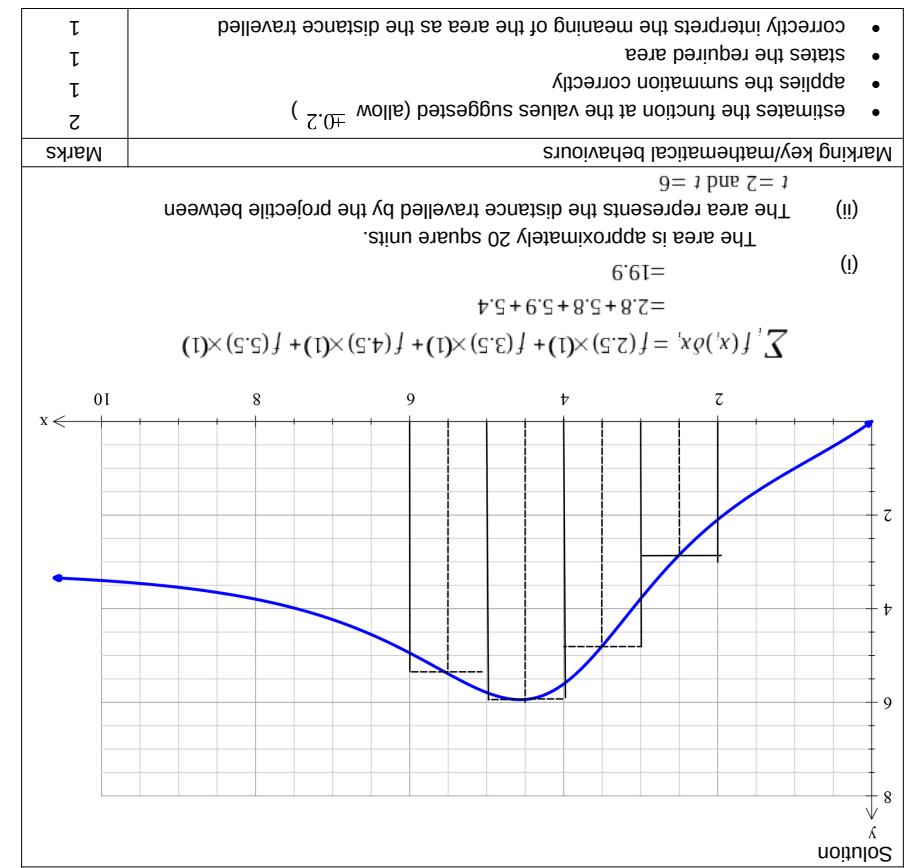
Marking key/mathematical behaviours	Marks
• obtains $\ln(x+1)$ as the antiderivative	1
• evaluates at limits correctly	1

Question 17(a)	
Solution	$v = \int 3 \sin(2t) dt$ $\therefore v = -\frac{3}{2} \cos(2t) + C$ $t=0 \leftarrow -\frac{2}{3} + C = 4$ $C = 5.5$ $\therefore v = -\frac{2}{3} \cos(2t) + 5.5$ Marks
	• correctly integrates to find equation for $v$ involving $c$
	• writes an expression for $v$
1	• evaluates $c$
1	• finds $c = 5.5$
1	• marks key/mathematical behaviours

**Question 17(a)**

Marks	Marking key/mathematical behaviours	
1	uses the correct formula and substitutes values	
1	calculates the standard score	
1	states the correct answer	
•		

**Question 16(c)**



**Question 20(a)**

**Question 17(b)**

Solution

$$x = \int \left[ \frac{-3}{2} \cos(2t) + 5.5 \right] dt$$

$$\frac{3}{4} \sin(2t) + 5.5t + c$$

When  $t=0 \rightarrow c=2$  or  $-2$

$$\text{When } t=2 \rightarrow x = \frac{-3}{4} \sin(4) + 5.5(2) \mp 2$$

$$= 13.57 \text{ m or } 9.57 \text{ m}$$

Marking key/mathematical behaviours

	Marks
• determines correct integral of function plus $c$	1
• calculates a value for $c$	1
• calculates $x$ accurately when $t = 2$ and includes both possible values	1+1

**Question 18(a)**

Solution

$$X \sim N(0,1)$$

$$P(X \leq x) = 0.3$$

$$x = -0.5244$$

Marking key/mathematical behaviours

	Marks
• identifies the parameters of the standard normal and states the problem in terms of probability	1
• states the correct result	1

**Question 18(b)**

Solution

$$X \sim N(16, 3^2)$$

$$P(X > x) = 0.6$$

$$\therefore x = 15.24$$

$$\text{So, } 3k - 1 = 15.24$$

$$\therefore k = 5.41$$

Marking key/mathematical behaviours

	Marks
• applies the normal distribution to determine $x$	1
• states the correct result for $k$	1

**Question 18(c)**

Solution

The  $x$ -value of 6 is 2.4 standard deviations away from the mean.

Marking key/mathematical behaviours

- provided an acceptable explanation

1

**Question 18(d)**

$$F(x) = \int_0^x 3x^2 \, dx = \left[ x^3 \right]_0^x = x^3 \quad (0 < x < 1)$$

$$\therefore F(x) = \begin{cases} 0 & x \leq 0 \\ x^3 & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Solution

Marking key/mathematical behaviours

- evaluates the correct integral
- defines  $F(x)$
- states the three domains correctly for  $F(x)$

1

1

1

**Question 19**

Solution

Check sample size is large enough for normal approximation  $np > 10$  and  $n(1-p) > 10$ .

$$\text{In this case, } 1000 \times 0.48 = 480 > 10 \\ 1000 \times 0.52 = 520 > 10$$

Therefore, normal approximation can be applied.

$$CI = 0.48 \pm 1.96 \sqrt{\frac{0.48 \times 0.52}{1000}}$$

$$= 0.48 \pm 0.03097$$

$$= (0.45, 0.51)$$

(0.45, 0.51) is a 95% Confidence Interval for the true proportion of students excited by the upcoming concert.

Marking key/mathematical behaviours

- Checks the sample size for normal approximation
- Sets up CI and evaluates correctly
- correctly interprets result

1+1

1+1

1