

Question/Answer Booklet  
Semester One Examination, 2020



**MATHEMATICS**  
**METHODS**  
**ATAR Year 12**  
**Section Two:**  
**Calculator-assumed**

Please circle your teacher's name  
Teacher: Miss Long    Miss Rowden    Miss Stone

Student Name: \_\_\_\_\_

Time allowed for this paper: 100 minutes  
Reading time before commencing work: 10 minutes  
Working time for paper: 100 minutes

**Materials required/recommended for this paper**  
**To be provided by the supervisor**  
**To be provided by the candidate**  
**Number of additional answer books used (if applicable):**

Special items:  
Standard items:  
Pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters  
Drawing instruments, emplifiers, notes on two unlined sheets of A4  
paper, and up to three calculators approved for use in this  
examination

(if applicable):

Formula Sheet (retained from Section One)

This Question/Answer Booklet

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper										
CALCULATOR-ASSUMED			MATHEMATICS METHODS							
Section										
<b>Section One:</b> Calculations available										
Number of questions available	13	13	13	100	98	65				
Number of questions to be answered	8	8	50	52	35	35				
Suggested working time (minutes)										
Marks										
Percentage of examination										
Total	100									

DO NOT WRITE THESE QUESTIONS ON THE ANSWER BOOKLET. The candidates will be given time to answer these questions in the examination. Instructions to candidates

### Instructions to candidates

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Supplementary pages for the use of planning/containing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily. Your working should be in sufficient detail to allow your answer to receive full marks. If you repeat any question, ensure that you cancel the question or part question worth more than two marks, valid working for justification is required to receive full marks.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

This section has thirteen (13) questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

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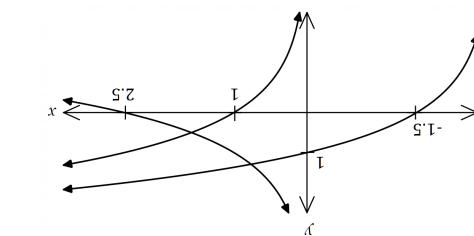
**Question 9** (7 marks)

In a sample of 1325 university students, 64 % said that they never look at their phone while driving.

- (a) Show how to use the figures from this sample to construct the 95 % confidence interval for the proportion of university students who never look at their phone while driving. (3 marks)
- (b) According to a newspaper article, "70 % of university students never look at their phone while driving". Explain whether the interval from (a) supports this claim. (2 marks)
- (c) Another source claims that "more than half of university students never look at their phone while driving". Explain whether the interval from (a) supports this claim. (2 marks)

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(1 mark)

- (a) Function  $f$  is defined by  $f(x) = 7 \log_2(x+16) - 3$  over its natural domain. Determine the value of  $y$  when  $x = 1$ .  
 (b) Function  $g$  is defined by  $g(x) = \log_n|x|$  over its natural domain, where  $n$  is a constant greater than 1. The graphs shown below have equations  $y = g(x)$ ,  $y = -g(x)$  and  $y = g(x+b)$ , where  $a$  and  $b$  are constants. Determine the value of  $n$ ,  $a$  and  $b$ .

(4 marks)

(1 mark)

- (i) the equation of the asymptote of the graph of  $y = f(x)$ .  
 (ii) the equation of the asymptote of the graph of  $y = g(x)$ .

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(1 mark)

- (i) the value of the  $y$ -intercept of the graph of  $y = f(x)$ .

(6 marks)

- (a) Function  $f$  is defined by  $f(x) = 7 \log_2(x+16) - 3$  over its natural domain. Determine

Question 10

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End of questions

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**Question 11**

(7 marks)

The percentage distribution of the number of cans of soft drink per order placed with a takeaway food company over a long period of time are shown in the following table.

Number of cans per order	0	1	2	3	4 or more
Percentage of orders	10	27	39	13	11

In the following questions, you may assume that all orders are placed with the company at random and independently.

- (a) Determine the probability that the next 4 orders all include no more than 3 cans of soft drink. (2 marks)

- (b) During a weekday, a total of 180 orders were placed. Determine the probability that

- (i) 130 of these orders included fewer than 3 cans of soft drink. (3 marks)

- (ii) less than 80 of these orders included 2 cans of soft drink. (2 marks)

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**Question 21**

(8 marks)

When a customer plays an online game of chance, a computer randomly picks one letter from those in the word LUCKY, another from those in the word BOIST, and a third from those in the word GAMER. For example, the computer might pick KSR, YBG, and so on. The customer can see the words but does not know the computer's picks and has to guess the letter it has chosen from each word. The random variable  $X$  is the number of letters correctly guessed by a customer in one play of the game.

- (a) State the distribution of  $X$  including its parameters. (1 marks)

- (b) Complete the table below to show the probability distribution of  $X$ . (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF	$x$	0	1	2	3
$P(X=x)$					

Each game costs a player 25 cents. A player wins a prize of \$14 if they guess all three letters correctly, \$1.40 if they guess two out of three letters correctly but otherwise wins nothing.

- (c) Determine  $E(Y)$  and  $\text{Var}(Y)$ , where the random variable  $Y$  is the gain, in cents, made by the customer in one play of the game. (4 marks)

- (d) If an average of 250 people from around the world play the game once every 20 seconds, calculate the gross profit expected by the game owners in any 24-hour period. (1 mark)

MATHEMATICS METHODS	18	CALCULATOR-ASSUMED	Question 12	MATHEMATICS METHODS
			<p>(a) The voltage generated by a circuit at time <math>t</math> seconds is given by <math>V(t) = e^{0.2t} \cos(3t)</math> for <math>0 \leq t \leq 4</math>. Show that the voltage is initially increasing. (2 marks)</p> <p>(b) Using a graphical method, or otherwise, determine the voltage at the instant the rate of change of voltage first starts to increase. (3 marks)</p>	

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(a) Determine the probability that a randomly selected girl from the study has a height greater than 95 cm.	(i) $e^{0.2t} \cos(3t) > 95$	(ii) $\int e^{0.2t} \cos(3t) dt$	$\left[ \frac{1}{3} e^{0.2t} \sin(3t) + \frac{1}{6} e^{0.2t} \cos(3t) \right]_0^4$	$\left[ \frac{1}{3} e^{0.8} \sin(12) + \frac{1}{6} e^{0.8} \cos(12) \right] - \left[ \frac{1}{3} e^0 \sin(0) + \frac{1}{6} e^0 \cos(0) \right]$
(b) The heights of girls $H$ in a large study of 3-year-old children are normally distributed with a mean of 94.5 cm and a standard deviation of 3.15 cm.	$P(H > 95) = P\left(\frac{H - 94.5}{3.15} > \frac{95 - 94.5}{3.15}\right) = P(Z > 0.16)$	$Z = \frac{H - 94.5}{3.15}$	$Z = \frac{95 - 94.5}{3.15} = 0.16$	$P(Z > 0.16) = 1 - P(Z \leq 0.16) = 1 - 0.5636 = 0.4364$
(c) The shortest 1.5% of girls were classified as unusually short. Determine the greatest height of a girl to be classified as unusually short. (1 mark)	$P(H < h) = 0.015$	$P\left(\frac{H - 94.5}{3.15} < \frac{h - 94.5}{3.15}\right) = 0.015$	$P(Z < \frac{h - 94.5}{3.15}) = 0.015$	$\frac{h - 94.5}{3.15} = -2.17$
(d) The shortest 3.5% of boys in the study are normally distributed with mean of 96.4 cm and the shortest. Determine the standard deviation of the boys' heights. (3 marks)	$P(H < h) = 0.035$	$P\left(\frac{H - 96.4}{s} < \frac{h - 96.4}{s}\right) = 0.035$	$P(Z < \frac{h - 96.4}{s}) = 0.035$	$\frac{h - 96.4}{s} = -2.05$

- (a) The shortest 1.5% of girls were classified as unusually short. Determine the greatest height of a girl to be classified as unusually short. (1 mark)
- (b) The heights of boys in the study are normally distributed with mean of 96.4 cm and the shortest. Determine the standard deviation of the boys' heights. (3 marks)
- (c) The heights of boys in the study are normally distributed with mean of 96.4 cm and the shortest. Determine the standard deviation of the boys' heights. (3 marks)
- (d) The shortest 3.5% of boys in the study are normally distributed with mean of 90.2 cm, were classified as unusually short. Determine the greatest height of a girl to be classified as unusually short. (1 mark)

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- (e) Use the increments formula to estimate the change in voltage in the one hundredth of a second after  $t = 2$ . (3 marks)

## MATHEMATICS METHODS

## Question 13

The diagram shows a flag design, with dimensions in centimetres.

The shaded region is bounded by the  $y$ -axis,  $y=f(x)$ ,  $y=g(x)$  and  $y=h(x)$  where

$$f(x) = \frac{x}{3},$$

$$g(x) = 8 + 4\sqrt{2} \sin\left(\frac{\pi x}{24}\right)$$

$$h(x) = 12 + 4 \cos\left(\frac{\pi x}{12}\right).$$

- (a) Let  $A$  be the area of another region on the graph, where  $A = \int_{24}^{30} [f(x) - g(x)] dx$ .

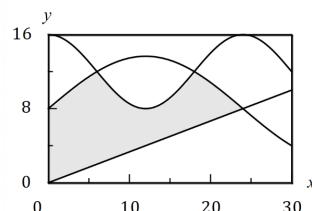
- (i) Clearly mark the region on the diagram with the letter  $A$ . (1 mark)
- (ii) Determine the value of  $A$ , rounded to one decimal place. (1 mark)

- (b) Using calculus determine the exact area of the shaded region. (5 marks)

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## CALCULATOR-ASSUMED

(7 marks)



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## CALCULATOR-ASSUMED

17

## MATHEMATICS METHODS

(8 marks)

## Question 19

The cross section of a triangular prism with a volume of  $54 \text{ cm}^3$  is an equilateral triangle of side length  $x \text{ cm}$ .

- (a) Show that the surface area  $S \text{ cm}$  of the prism is given by  $S = \frac{\sqrt{3}}{2}x^2 + \frac{216\sqrt{3}}{x}$ . (4 marks)

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- (b) Use calculus to determine the minimum surface area of the triangular prism. (4 marks)

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- (d) After five seconds, the particle has moved a distance of  $k$  metres.  
A student was set the task of determining the proportion of people in their suburb who use public transport at least once a week.  
(a) Briefly discuss the main source of bias in each of the following sampling methods.  
(i) The student invites people via social media to respond to their survey. (1 mark)
- Question 14  
(6 marks)

(i) Explain why  $k \neq \int_0^5 V(t) dt$ .

(d) After five seconds, the particle has moved a distance of  $k$  metres.

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- (ii) Calculate  $k$ .  
(2 marks)
- Determine the level of confidence of this interval.  
least once a week and went on to construct the confidence interval  $[0.49, 0.81]$ .  
(b) The student noted that 39 out of all those sampled said they used public transport at least once a week and went on to construct the confidence interval  $[0.49, 0.81]$ .  
(4 marks)

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**MATHEMATICS METHODS****10****CALCULATOR-ASSUMED****Question 15****(8 marks)**

A cooling system maintains the temperature  $T$  of an integrated circuit between  $0.5^\circ C$  and  $1^\circ C$ . At any instant,  $T$  is a continuous random variable defined by the probability density function

$$f(t) = \begin{cases} \frac{a}{t} & 0.5 \leq t \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Determine the exact value of the constant  $a$ .

(2 marks)

- (b) Determine a decimal approximation for the probability that a temperature taken at random exceeds  $0.85^\circ C$ .

(2 marks)

- (c) Determine decimal approximations for the mean and standard deviation of the temperature of the integrated circuit.

(4 marks)

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**CALCULATOR-ASSUMED****15****MATHEMATICS METHODS****Question 18****(8 marks)**

A particle moves in a straight line according to the function  $x(t) = \frac{t^2 + 3}{t + 1}$ ,  $t \geq 0$ , where  $t$  is in seconds and  $x$  is the displacement of the particle from a fixed point  $O$ , in metres.

- (a) Determine the velocity function,  $v(t)$ , for the particle.

(1 mark)

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- (b) Determine the displacement of the particle at the instant it is stationary.

(2 marks)

- (c) Show that the acceleration of the particle is always positive.

(2 marks)

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A charged capacitor discharges through a resistor. Some readings of the deflection  $D$  cm of the pointer are given below.

97 11015303

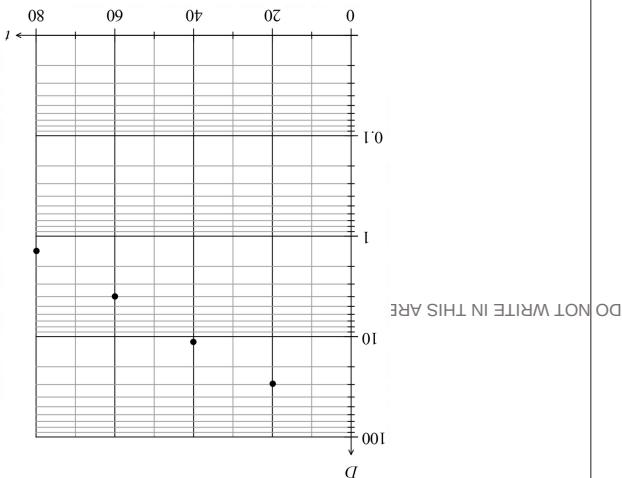
(c) Discuss how the factors such as addressability and applicability of the approach make it suitable for distributed decision making. (2 marks)

A galvanometer scale in the circuit  $t$  seconds after the discharge began are shown on the same graph below.

(b) Discuss how this evidence uses the approaches of the approachable distribution in (a) to the true distribution of proportions.

(a) Use the above relationships to obtain an expression for  $L$  in terms of  $a$ ,  $b$ , and  $t$ .  
 and  $y$ . Explain why plotting the data using a logarithmic scale on the vertical axis aligns the points in a straight line.

The relationship between the variables is of the form  $D = ae^{\frac{x}{k}}$ , where  $a$  and  $k$  are constants.



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- (b) Show how to use the relationship and the galvanometer readings at  $t=20$  and  $t=60$  to determine estimates for  $a$  and  $k$ . (4 marks)

- (c) Determine

(i) the deflection after 90 seconds. (1 mark)

(ii) the time for the deflection to reach 1 mm. (1 mark)

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**Question 17**

Random samples of 165 people are taken from a large population. It is known that 8% of the population have blue eyes.

- (a) Use a discrete probability distribution to determine the probability that the proportion of people in one sample who have blue eyes is less than 7%. (3 marks)

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- (b) Ten consecutive random samples are taken. Determine the probability that the proportion of those with blue eyes is less than 7% in exactly half of these samples. (2 marks)

A large number of random samples of 165 people are taken, the proportion of blue eyed people calculated for each sample and the distribution of these sample proportions analysed.

- (c) Describe the continuous probability distribution that these sample proportions approximate, including any parameters. (3 marks)

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