SOLUTIONS.

Mercedes College

Semester One Examination, 2016

Question/Answer Booklet

YEAR 12 MATHEMATICS METHODS
UNIT 3

Section One:
Calculator-free

Your name ________________________________

Teacher name's ________________________________

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

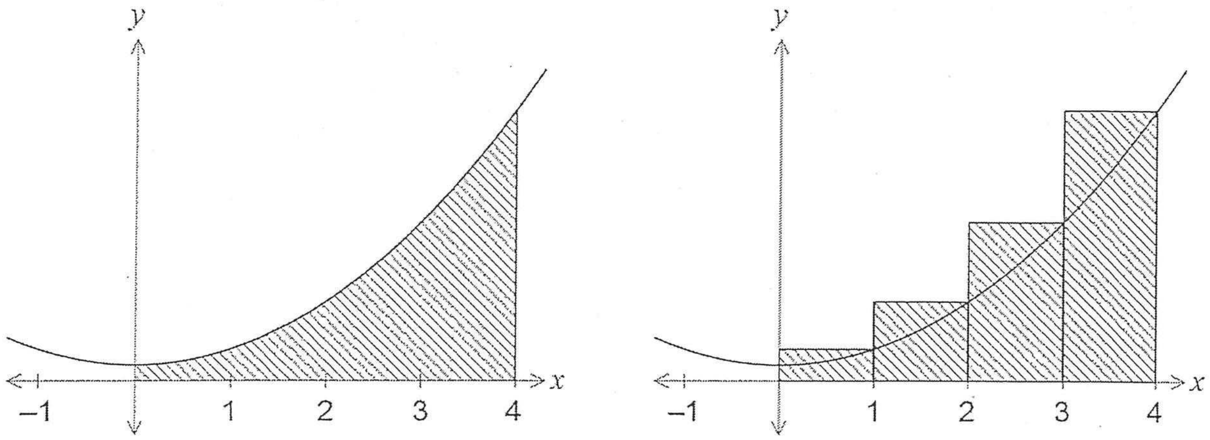
Section One: Calculator-free 35% (50 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 (5 marks)

Part of the graph of $y = x^2 + 1$ is shown in the diagrams below.



An approximation for the area beneath the curve between $x = 0$ and $x = 4$ is made using rectangles as shown in the right-hand diagram. Determine the exact amount by which the approximate area exceeds the exact area.

$$A_1 = \int_0^4 (x^2+1)\,dx$$
$$= \left[\frac{x^3}{3} + x\right]_0^4 \checkmark$$
$$= \frac{64}{3} + 4$$
$$= \frac{76}{3} \checkmark$$
$$\left(25\tfrac{1}{3}\right)$$

$$A_2 = 1\times(1^2+1) + 1\times(2^2+1) + 1\times(3^2+1) + 1\times(4^2+1)$$
$$= 2 + 5 + 10 + 17 \checkmark$$
$$= 34 \checkmark$$

$$A_2 - A_1 = 34 - \frac{76}{3}$$
$$= \frac{102-76}{3}$$
$$= \frac{26}{3} \text{ units}^2 \checkmark$$

Question 2 **(11 marks)**

(a) Differentiate the following with respect to x, simplifying your answers.

(i) $y = \int_x^1 (t - t^3).dt$. (2 marks)

$$\frac{d}{dx}\int_x^1 (t-t^3)\,dt = -\frac{d}{dx}\int_1^x (t-t^2)\,dt$$

$$= x^3 - x$$

(ii) $y = \sin^3(2x+1)$. $= [\sin(2x+1)]^3$ (3 marks)

$$\frac{dy}{dx} = 3[\sin(2x+1)]^2[\cos(2x+1)]\,2$$

$$= 6\cos(2x+1)\sin^2(2x+1)$$

(b) Evaluate $\int \sin 2x \cos^4 2x \, dx$ (2 marks)

let $y = [\cos(2x)]^5$

$$\frac{dy}{dx} = 5[\cos(2x)]^4[-\sin(2x)]\,2$$

$$= -10\sin 2x \cos^4 2x.$$

$$= \frac{-1}{10}\int -10\sin 2x\cos^4 2x\,dx$$

$$= -\frac{\cos^5(2x)}{10} + c$$

(c) Determine the values of the constants a, b and c, given that $f''(x) = e^{3x}(ax^2 + bx + c)$ when $f(x) = x^2 e^{3x}$. (4 marks)

$$f'(x) = [2xe^{3x}] + 3x^2e^{3x}$$

$$f''(x) = [2e^{3x} + 6xe^{3x}] + 3f'(x)$$

$$= 2e^{3x} + 6xe^{3x} + 3(2xe^{3x} + 3x^2e^{3x})$$

$$= 2e^{3x} + 12xe^{3x} + 9x^2e^{3x}$$

$$= e^{3x}(2 + 12x + 9x^2)$$

$a = 9 \quad b = 12 \quad c = 2$

Question 3 (6 marks)

A function $P(x)$ is such that $\frac{dP}{dx} = ax^2 - 12x$, where a is a constant and the graph of $y = P(x)$ has a stationary point at (4, 8). Determine $P(10)$.

$$\left.\frac{dP}{dx}\right|_{x=4} = a(4)^2 - 12(4) = 0 \checkmark$$

$$16a - 48 = 0$$

$$\underline{\underline{a = 3}} \checkmark$$

$$\frac{dP}{dx} = 3x^2 - 12x$$

$$P = \int 3x^2 - 12x \, dx \checkmark$$

$$= x^3 - 6x^2 + c$$

subst (4,8)

$$8 = (4)^3 - 6(4)^2 + c$$

$$8 = 64 - 96 + c \checkmark$$

$$\underline{\underline{c = 40}}$$

$$P(x) = x^3 - 6x^2 + 40 \checkmark$$

$$P(10) = 1000 - 600 + 40 \checkmark$$

$$= 440$$

Question 4 **(7 marks)**

Consider the function defined by $f(x)=\dfrac{x}{2}-\sqrt{x},\ x\geq 0$.

(a) Determine the coordinates of the stationary point of $f(x)$. (3 marks)

$f'(x) = \frac{1}{2} - \frac{1}{2\sqrt{x}}$ ✓

$\frac{1}{2} - \frac{1}{2\sqrt{x}} = 0$

$\underline{\underline{x = 1}}$ ✓

$f(1) = \frac{1}{2} - \sqrt{1}$

$= -\frac{1}{2}$ ✓

$\therefore \quad (1, -\frac{1}{2})$

(b) Use the second derivative test to determine the nature of the stationary point found in (a). (3 marks)

$f''(x) = \frac{1}{4\sqrt{x^3}}$ ✓

$f''(1) = \frac{1}{4} > 0 \quad \therefore$ LOCAL MINIMUM.

✓ ✓

(c) State the global minimum of $f(x)$. (1 mark)

$-\frac{1}{2}$ ✓

Question 5 **(5 marks)**

The area of a segment with central angle θ (in radians) in a circle of radius r is given by $A = \frac{r^2}{2}(\theta - \sin\theta)$. Use the increments formula to approximate the increase in area of a segment in a circle of radius 10 cm as the central angle increases from $\frac{\pi}{3}$ to $\frac{11\pi}{30}$.

$$A = \frac{100}{2}(\theta - \sin\theta)$$

$$A = 50(\theta - \sin\theta)$$

$$\frac{dA}{d\theta} = 50(1 - \cos\theta) \checkmark$$

$$\left.\frac{dA}{d\theta}\right|_{\theta = \frac{\pi}{3}} = 50(1 - 0.5) = 25 \checkmark$$

$$\delta\theta = \frac{11\pi}{30} - \frac{\pi}{3} = \frac{\pi}{30} \checkmark$$

$$\delta A \approx \frac{dA}{d\theta} \times \delta\theta \checkmark$$

$$\approx 25 \times \frac{\pi}{30}$$

$$\checkmark$$

$$\approx \frac{5\pi}{6} \text{ cm}^2$$

Question 6 **(5 marks)**

Given $y = \dfrac{2x+1}{e^x}$

(a) Calculate $\dfrac{dy}{dx}$, simplifying your answer. (3 marks)

$$\frac{dy}{dx} = \frac{2e^x - (2x+1)e^x}{e^x e^x} \checkmark \qquad u = 2x+1 \quad v = e^x$$
$$u' = 2 \quad v' = e^x$$

$$= \frac{\cancel{e^x}(2 - 2x - 1)}{\cancel{e^x} e^x} \checkmark$$

$$= \frac{(1-2x)}{e^x} \checkmark$$

(b) Hence or otherwise evaluate $\int_1^2 \left(\dfrac{1-2x}{e^x}\right) dx$. (2 marks)

$$\int_1^2 \left(\frac{1-2x}{e^x}\right) dx = \left[\frac{2x+1}{e^x}\right]_1^2 \checkmark$$

$$= \frac{5}{e^2} - \frac{3}{e} \checkmark$$

$$= \frac{5-3e}{e^2}$$

Question 7 (6 marks)

The discrete random variable X has the probability distribution shown in the table below.

x	0	1	2	3
$P(X=x)$	$\frac{2a^2}{3}$	$\frac{1-3a}{3}$	$\frac{1+2a}{3}$	$\frac{4a^2}{3}$

Determine the value of the constant a.

$$\frac{2a^2 + 1-3a + 1+2a + 4a^2}{3} = 1 \checkmark$$

$$6a^2 - a - 1 = 0 \checkmark$$

$$(3a+1)(2a-1) = 0$$

$$a = -\frac{1}{3} \quad a = \frac{1}{2} \checkmark$$

$\checkmark$

check $a = -\frac{1}{3}$ $\checkmark$

$$\left[\frac{2}{27}, \frac{2}{3}, \frac{1}{9}, \frac{4}{27}\right]$$

check $a = \frac{1}{2}$

$$\left[\frac{1}{6}, \cancel{-\frac{1}{6}}, \frac{2}{3}, \frac{1}{3}\right]$$

< 0

$$\therefore a = -\frac{1}{3} \checkmark$$

Question 8 (5 marks)

The area bounded by the curve $y = e^{2-x}$ and the lines $y = 0$, $x = 1$ and $x = k$ is exactly $e - 1$ square units. Determine the value of the constant k, given that $k > 1$.

$$\int_1^k e^{2-x}\,dx = \left[-e^{2-x}\right]_1^k \checkmark$$

$$= -e^{2-k} - (-e^1)$$

$$= -e^{2-k} + e$$

$$\therefore -e^{2-k} + e = e - 1 \checkmark$$

$$\therefore e^{2-k} = 1$$

$$e^{2-k} = e^0 \checkmark$$

$$2 - k = 0$$

$$\underline{k = 2} \checkmark$$

Mercedes College

Semester One Examination, 2016

Question/Answer Booklet

YEAR 12 MATHEMATICS METHODS
UNIT 3

Section Two:
Calculator-assumed

Your name ____________________________________

Teacher name's ____________________________________

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time for section:	one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items:	pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

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Section Two: Calculator-assumed **65% (100 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(5 marks)**

A recent news report said that it took 34 months for the population of Australia to increase from 23 to 24 million people.

(a) Assuming that the rate of growth of the population can be modelled by the equation $\frac{dP}{dt} = kP$, where P is the population of Australia at time t months, determine the value of the constant k accurate to 5 significant figures. (3 marks)

$$24 = 23e^{k(34)}$$

$$e^{34k} = \frac{24}{3}$$

$$k = 0.0012518$$

(b) Assuming the current rate of growth continues, how long will it take (to the nearest month) for the population to increase from 24 million to 30 million people? (2 marks)

$$30 = 24e^{0.0012518t}$$

$$t = 178.26$$

≈ 178 months.

OR 179 months.

Question 10 **(7 marks)**

A small object is moving in a straight line with acceleration $a = 6t + k$ ms^{-2}, where t is the time in seconds and k is a constant. When $t = 1$ the object was stationary and had a displacement of 4 metres relative to a fixed point O on the line. When $t = 2$ the object had a velocity of 1 ms^{-1}.

(a) Determine the value of k and hence an equation for the velocity of the object at time t. (4 marks)

$a = 6t + k$ $\left(\begin{matrix} t=1 \\ v=0 \\ x=4 \end{matrix}\right)$ $\left(\begin{matrix} t=2 \\ v=1 \end{matrix}\right)$

$v = 3t^2 + kt + c$

$\left(\begin{matrix} t=1 \\ v=0 \end{matrix}\right)$ $0 = 3 + k + c$ ✓

$\left(\begin{matrix} t=2 \\ v=1 \end{matrix}\right)$ $1 = 12 + 2k + c$ ✓

$\boxed{k = -8}$ ✓, $c = 5$

$\boxed{v = 3t^2 - 8t + 5}$ ✓

(b) Determine the displacement of the object when $t = 2$. (3 marks)

$x = t^3 - 4t^2 + 5t + q$ ✓

$\left(\begin{matrix} t=1 \\ x=4 \end{matrix}\right)$ $4 = 1 - 4 + 5 + q$

$q = 2$ ✓ $\Longleftrightarrow$ $x = t^3 - 4t^2 + 5t + 2$

$x\Big|_{t=2} = 2^3 - 4(2)^2 + 5(2) + 2$

$= 4\text{m}$ ✓

Question 11 **(7 marks)**

It is known that 15% of Year 12 students in a large country study advanced mathematics.

A random sample of n students is selected from all Year 12's in this country, and the random variable X is the number of those in the sample who study advanced mathematics.

(a) Describe the distribution of X. (2 marks)

$X \sim B(n, 0.15)$ ✓✓

(b) If $n = 22$, determine the probability that

(i) three of the students in the sample study advanced mathematics. (1 mark)

$X \sim B(22, 0.15)$

$P(X=3) = 0.23700$ ✓

(ii) more than three of the students in the sample study advanced mathematics. (1 mark)

$P(X \geq 4) = 0.42482$ ✓

(iii) none of the students in the sample study advanced mathematics. (1 mark)

$P(X=0) = 0.02800$ ✓

(c) If ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study advanced mathematics. (2 marks)

$Y \sim B(10, 0.0280)$ ✓

$P(Y \geq 1) = 0.24723$ ✓

Question 12 **(8 marks)**

The height of grain in a silo, initially 0.4 m, is increasing at a rate given by $h'(t) = 0.55t - 0.05t^2$ for $0 \le t \le 11$, where h is the height of grain in metres and t is in hours.

(a) At what time is the height of grain rising the fastest? (2 marks)

$h''(t) = 0.55 - 0.1t$ ✓

$0.55 - 0.1t = 0$

$t = 5.5$ hours ✓

(b) Determine the height of grain in the silo after 11 hours. (3 marks)

$\int_0^{11} (0.55t - 0.05t^2)\,dt = 11.0917$ ✓

$\therefore$ HEIGHT $= 11.0917 + 0.4$ ✓

$= 11.4917$ metres ✓

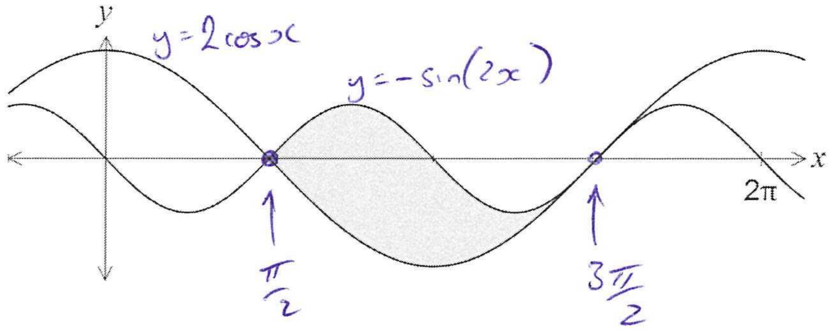
(c) Calculate the time taken for the grain to reach a height of 4.45 m. (3 marks)

$\int_0^{x} (0.55t - 0.05t^2)\,dt + 0.4 = 4.45$ ✓ ✓

$x = 4.5$ hrs ✓

Question 13 **(5 marks)**

The shaded region on the graph below is enclosed by the curves $y = -\sin(2x)$ and $y = 2\cos x$.



Calculate the area of the shaded region.

Intersection, solve $-\sin(2x) = 2\cos x$ ✓

$\therefore \quad x = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$

$x = \frac{\pi}{2}$ ✓

$$\text{Area} = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\sin(2x) - 2\cos x \, dx \quad ✓✓$$

$= 4 \text{ units}^2$ ✓

5

Question 14 **(14 marks)**

(a) Determine the mean of a Bernoulli distribution with variance of 0.24. (3 marks)

$0.24 = p(1-p)$ ✓

$\therefore p = 0.4$ or 0.6 ✓

$\Rightarrow$ mean $= 0.4$ or 0.6 ✓

(b) A Bernoulli trial, with probability of success p, is repeated n times. The resulting distribution of the number of successes has an expected value of 5.76 and a standard deviation of 1.92. Determine n and p. (4 marks)

$E(x) = np = 5.76$ ✓

$\sigma_x = \sqrt{np(1-p)} = 1.92$ ✓

$\therefore n = 16$ ✓

$p = 0.36$ ✓

7

(c) The probability that a student misses their bus to school is 0.2, and the probability that they miss the bus on any day is independent of whether they missed it on the previous day.

Over five consecutive weekdays, what is the probability that the student

(i) only misses the bus on Tuesday? (2 marks)

$= 0.8 \times 0.2 \times 0.8^3$

$= \underline{0.08192}$

(ii) misses the bus at least twice? (2 marks)

let X = no. of times missing bus

$X \sim B(5, 0.2)$

$P(X \geq 2) = \underline{0.26272}$

(iii) misses the bus on Tuesday and on two other days? (3 marks)

ie From 4 days, missing bus twice

$Y \sim B(4, 0.2)$

$P(Y = 2) = 0.1536$

$\therefore$ P(Tuesday + two other days)

$= 0.2 \times 0.1536$

$= \underline{0.03072}$

7

Question 15 **(9 marks)**

A particle moves in a straight line according to the function $x(t) = \dfrac{t^2+3}{t+1}$, $t \geq 0$, where t is in seconds and x is the displacement of the particle from a fixed point O, in metres.

(a) Determine the velocity function, $v(t)$, for the particle. (2 marks)

$$v(t) = \frac{dx}{dt} = \frac{(2t)(t+1) - (1)(t^2+3)}{(t+1)^2}$$

$$= \frac{t^2+2t-3}{(t+1)^2}$$

(b) Determine the displacement of the particle at the instant it is stationary. (2 marks)

Stationary @ $v(t) = 0$

$\therefore t = -3$ or 1
(ignore)

$x(1) = \underline{\underline{2\text{ m}}}$

(c) Show that the acceleration of the particle is always positive. (2 marks)

$$a(t) = \frac{dv}{dt} = \frac{8}{(t+1)^3}$$

Given $t \geq 0 \Rightarrow$ denominator is always > 0

$\therefore a(t)$ is always positive.

6

(d) After five seconds, the particle has moved a distance of k metres.

(i) Explain why $k \neq \int_0^5 v(t)\,dt$. (1 mark)

Because the particle has stopped @ $t = 1$ second to change direction (worked out in part(b))

(ii) Calculate k. (2 marks)

$$k = \int_0^5 |v(t)|\,dt$$

$$= \int_0^5 \left| \frac{t^2+2t-3}{(t+1)^2} \right| dt$$

$$= \underline{3.\dot{6}\ m}$$

3

Question 16 **(8 marks)**

The discrete random variable Y has the probability distribution shown in the table below.

y	-2	-1	0	1	2
$P(Y = y)$	0.4	0.2	0.1	0.1	0.2

(a) Determine $P(Y \geq 0 \mid Y \leq 1)$. (2 marks)

$$= \frac{P(Y \geq 0 \cap Y \leq 1)}{P(Y \leq 1)}$$

$$= \frac{0.2}{0.8}$$

$$= \frac{1}{4}$$

(b) Calculate

(i) $E(Y)$. (2 marks)

$E(Y) = \text{mean} = \underline{-0.5}$ (stats - CAS)

(ii) $E(1-2Y)$. (1 mark)

$$E(-2 \times Y + 1) = -2 \times (-0.5) + 1$$
$$= \underline{2}$$

(c) Calculate

(i) $Var(Y)$. (2 marks)

$$\sigma_Y = 1.565$$
$$Var(Y) = (\sigma_Y)^2$$
$$= \underline{2.45}$$

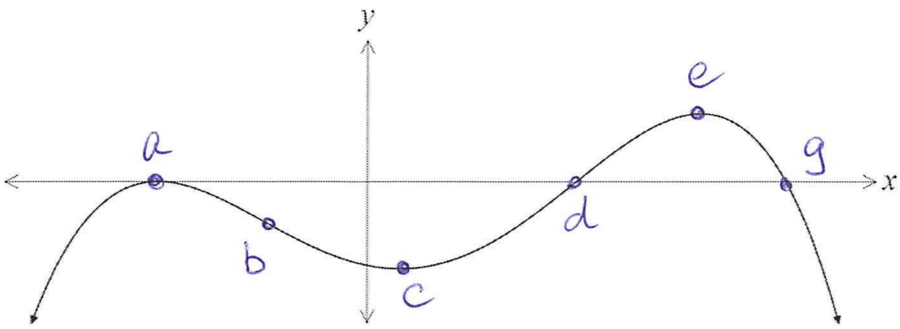
(ii) $Var(1-2Y)$. (1 mark)

$$Var(-2 \times Y + 1) = (-2)^2 \times 2.45$$
$$= \underline{9.8}$$

8

Question 17 **(8 marks)**

The graph of $y = f'(x)$, the derivative of a polynomial function f, is shown below. The graph of $y = f'(x)$ has stationary points when $x = a$, $x = c$ and $x = e$, points of inflection when $x = b$ and $x = d$, and roots when $x = a$, $x = d$ and $x = g$, where $a < b < c < d < e < g$.



(a) For what value(s) of x does the graph of $y = f(x)$ have a point of inflection? (1 mark)

Inflection @ $f''(x) = 0$
ie @ $x = a, c$ and e ✓

(b) Does the graph of $y = f(x)$ have a local maximum? Justify your answer. (2 marks)

Max

YES. At point g. ✓

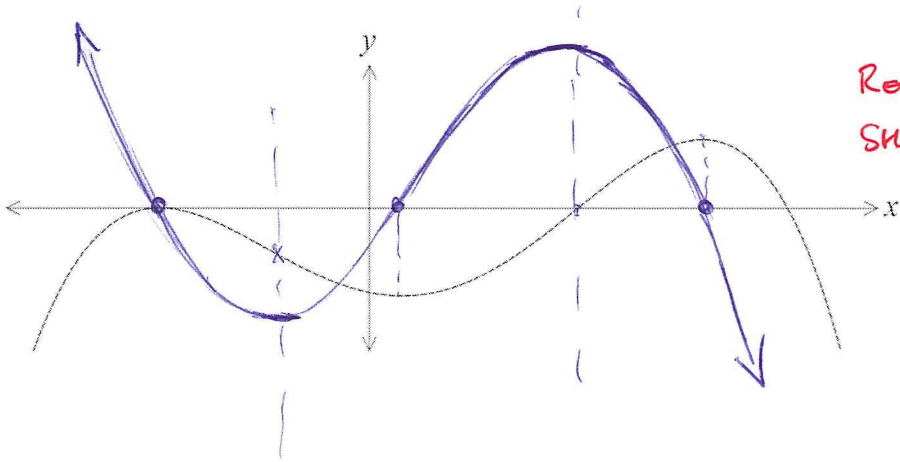
Slope to left of g is +ve
Slope @ g = 0
Slope to right of g is −ve
} Max @ g ✓

(c) Does the graph of $y = f(x)$ have a horizontal point of inflection? Justify your answer. (2 marks)

Point "a" ✓
YES

Slope to left of "a" = −ve
Slope @ "a" = 0
Slope to right of "a" = −ve
} Inflection at "a" ✓

(d) On the axis below, sketch a possible graph of $y = f''(x)$. The graph of $y = f'(x)$ is shown with a broken line for your reference. (3 marks)

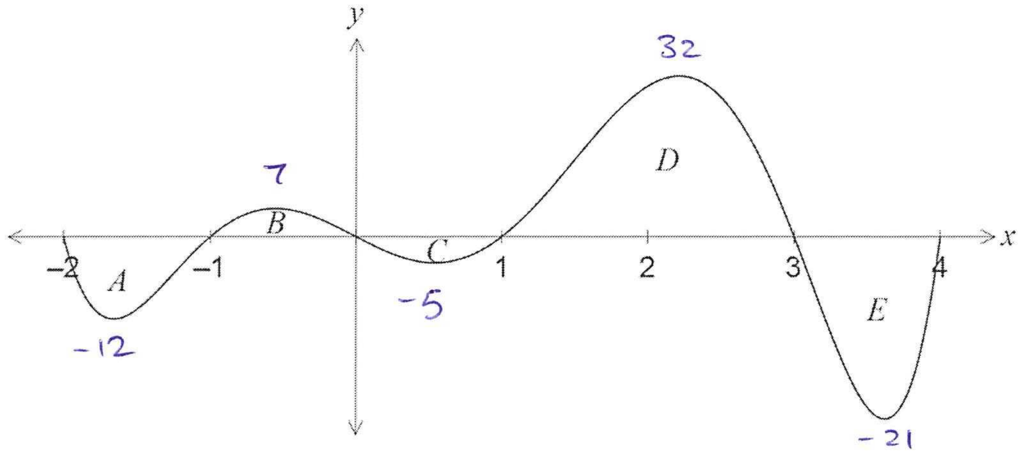


ROOTS ✓✓
SHAPE ✓

8

Question 18 **(8 marks)**

The graph of the function $y = f(x)$ is shown below for $-2 \le x \le 4$.



The area of regions enclosed by the x-axis and the curve, A, B, C, D and E, are 12, 7, 5, 32 and 21 square units respectively.

(a) Determine the value of $\int_{-2}^{4} f(x)\,dx$. (2 marks)

$= -12 + 7 - 5 + 32 - 21$

$= 1$

(b) Determine the area of the region enclosed between the graph of $y = f(x)$ and the x-axis from $x = 0$ to $x = 4$. (2 marks)

$5 + 32 + 21 = 58$ units^2

(c) Determine the values of

(i) $\int_0^3 f(x) + 3\,dx$. (2 marks)

$= \int_0^3 f(x)\,dx + \int_0^3 3\,dx$

$= -5 + 32 + 3 \times 3$

$= 36$

(ii) $\int_{-2}^{3} \frac{f(x)}{2}\,dx$. (2 marks)

$= \frac{1}{2} \times \int_{-2}^{3} f(x)\,dx$

$= \frac{1}{2} \times (-12 + 7 - 5 + 32)$

$= \frac{1}{2} \times 22$

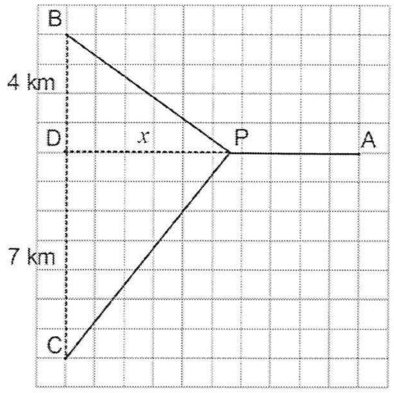
$= 11$

8

Question 19 **(7 marks)**

Three telecommunication towers, A, B and C, each need underground power cable connections directly to a new power station, P, that is to be built x km from depot D on a 10 km road running east-west between D and A.

Tower B lies 4 km due north of depot D and tower C lies 7 km south of the depot, as shown in the diagram.



(a) Determine an expression for the total length of underground cable required to connect A, B and C directly to P. (2 marks)

$BP = \sqrt{x^2+16}$

$CP = \sqrt{x^2+49}$

$AP = 10 - x$

$\therefore L = \sqrt{x^2+16} + \sqrt{x^2+49} + 10 - x$

(b) **Show** that the minimum length of cable occurs when $\dfrac{x}{\sqrt{16+x^2}} + \dfrac{x}{\sqrt{49+x^2}} = 1$. (2 marks)

$\dfrac{dL}{dx} = \frac{1}{2}(x^2+16)^{-\frac{1}{2}} \times 2x + \frac{1}{2}(x^2+49)^{-\frac{1}{2}} \times 2x - 1$

$= \dfrac{x}{\sqrt{x^2+16}} + \dfrac{x}{\sqrt{x^2+49}} - 1 = 0$ (Min @ $\frac{dL}{dx} = 0$)

Min @ $\dfrac{dL}{dx} = 0$ $\therefore$ $1 = \dfrac{x}{\sqrt{x^2+16}} + \dfrac{x}{\sqrt{x^2+49}}$

(c) Determine the minimum length of cable required, rounding your answer to the nearest hundred metres. Use the second derivative test to justify that your solution is a minimum. (3 marks)

Solving $\Rightarrow x = 3.02554$ km

Second derivative $\dfrac{d^2L}{dx^2}$ @ $x = 3.02554$ $= 0.24$

> 0

$\therefore L$ is min @ $x = 3.026$

$L = 19.616$ km

$\simeq 19.6$ km (19600m)

7

Question 20 **(7 marks)**

Consider the function $f(t) = \dfrac{t-4}{2}$ and the function $A(x) = \int_0^x f(t)\,dt$.

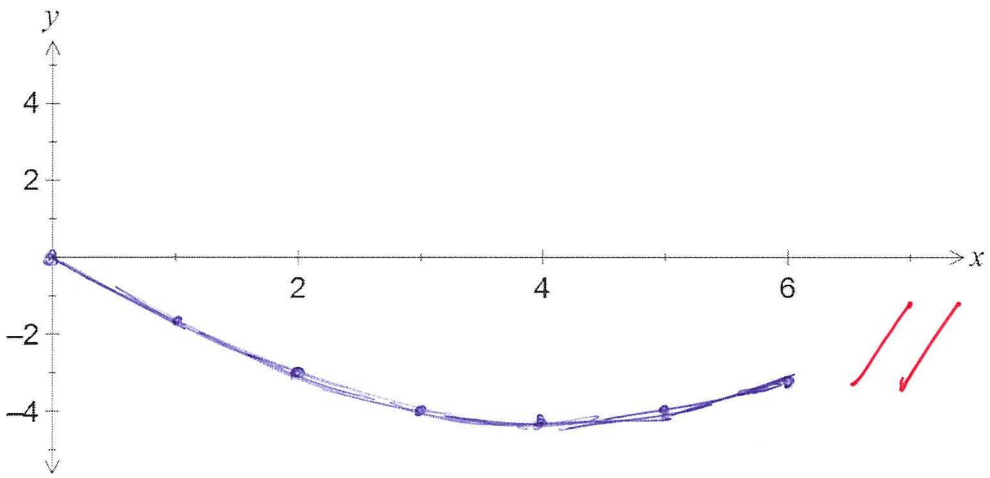
(a) Complete the table below. $A(x) = \int_0^x \frac{t-4}{2}\,dt$ (2 marks)

x	0	1	2	3	4	5	6
$A(x)$	0	-1.75	-3	-3.75	-4	-3.75	-3

(b) For what value(s) of x is the function $A(x)$ increasing? (1 mark)

$x > 4$

(c) On the axes below, sketch the graph of $y = A(x)$ for $0 \le x \le 6$. (2 marks)



(d) Determine

(i) when $A'(x) = 0$. (1 mark)

$x = 4$ (ie minimum)

(ii) the function $A(x)$ in terms of x. (1 mark)

$A(x) = \frac{x^2}{4} - 2x$

$\frac{x^2}{4} - \frac{8x}{4}$

$= \frac{x^2 - 8x}{4}$ OR $\frac{x(x-8)}{4}$

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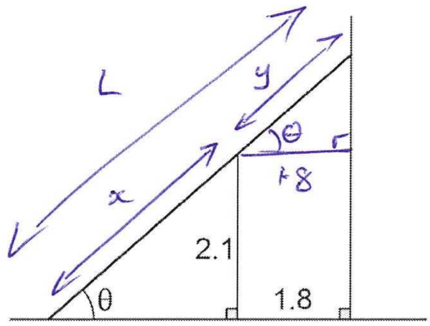
Question 21 **(7 marks)**

A vertical wall, 2.1 metres tall, stands on level ground and 1.8 metres away from the wall of a house. A ladder, of negligible width, leans at an angle of θ to the ground and just touches the ground, wall and house, as shown in the diagram.



(a) Show that the length of the ladder, L, is given by $L = \dfrac{2.1}{\sin\theta} + \dfrac{1.8}{\cos\theta}$. (3 marks)

$\sin\theta = \dfrac{2\cdot1}{x}$ $\qquad$ $\cos\theta = \dfrac{1\cdot8}{y}$

$x = \dfrac{2\cdot1}{\sin\theta}$ $\qquad$ $y = \dfrac{1\cdot8}{\cos\theta}$

$\therefore L = x + y = \dfrac{2\cdot1}{\sin\theta} + \dfrac{1\cdot8}{\cos\theta}$

(b) Use a calculus method to determine the length of the shortest ladder that can touch the ground, wall and house at the same time. (4 marks)

$$\frac{dL}{d\theta} = \frac{-0\cdot1\left(21\cos^3\theta - 18\sin^3\theta\right)}{\cos^2\theta\sin^2\theta}$$

Min @ $\dfrac{dL}{d\theta} = 0$

$\therefore 21\cos^3\theta - 18\sin^3\theta = 0$

$\Rightarrow \theta = 0\cdot81108$ radians

$\dfrac{d^2L}{d\theta^2}$ @ $\theta = 0\cdot81108 = 16\cdot53 > 0$ $\therefore$ Min @ $\theta = 0\cdot81108$

$\Rightarrow L = 5\cdot50999 \approx \underline{5\cdot51\text{ m}}$

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End of questions