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UNIT 3CD MAS – 2015

TEST 1

POLAR COORDINATES, COMPLEX NUMBERS & VECTORS

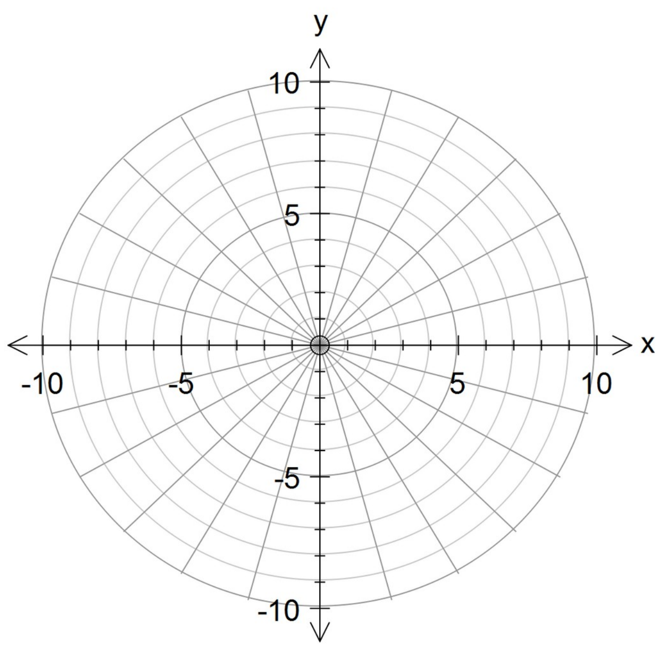
NAME: ____________________________ DATE: Tues 17th Feb.

Total: 45 marks Time: 50 min.

Question 1 **(5 marks)**

(i) Point P has polar coordinates $(4, 17^0)$ and it lies on the line $y = -x + 5$
Point Q also lies on the line and is 12cm away from P.

Find the polar coordinates of Q $[r, \theta]$ where $90^0 < \theta < 180^0$. (3 marks)



(ii) Sketch the graph of $r = \frac{4}{\pi}\theta$ on the axes above and hence state where it intersects $r = 4$ (2 marks)

Question 2 **(9 marks)**

On a 3D computer game, Chris, a keen cyclist leaves from *position* $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ metres is travelling at $(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$ m/s while his mate Dave leaves from position $(a\mathbf{i} + \mathbf{j} + b\mathbf{k})$ metres running at $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m/s.

(i) Although they do not collide, their paths do intersect at the point with coordinates $(a, 1, b)$.

Determine the values of $\boldsymbol{a}$ and $\boldsymbol{b}$. (4 marks)

(ii) Find the acute angle between these two paths. (2 marks)

(iii) Hence, determine the perpendicular distance from the point (1,-1,2) to (3 marks)

$$r = \begin{pmatrix} \frac{7}{3} \\ 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Question 3 **(5 marks)**

(a) The vectors $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} + 2\mathbf{j} + \mathbf{ak}$ are perpendicular.

Determine the value of **a**.

(1marks)

(b) Determine whether the two lines

$\mathbf{r} = 8\mathbf{i} - \mathbf{j} - 8\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{k})$ and $\mathbf{r} = \mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ intersect. (4 marks)

If they do intersect, state the position vector of their point of intersection.

If they do not intersect, justify your answer.

Question 4 **(7 marks)**

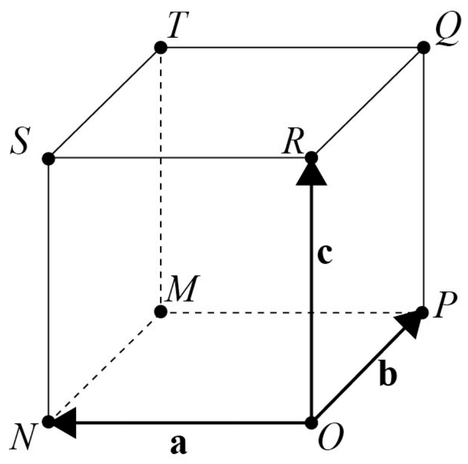
(a) If $z = 3 - 4i$, determine the reciprocal, $\frac{1}{z}$ (2 marks)

(b) Let the non-zero complex number $z = a + bi$. Show that $\frac{1}{a + bi} = \frac{\overline{Z}}{|Z|^2}$ (3 marks)

(c) Describe the geometrical relationship between any non-zero complex number and its reciprocal. (2marks)

Question 5 **(5 marks)**

Let *MNOPQRST* be a rectangular prism with sides $\overrightarrow{ON}$, $\overrightarrow{OP}$ and $\overrightarrow{OR}$ denoted by the vectors, **a**, **b**, and **c** respectively, as shown in the diagram below.



Suppose that **A** is the midpoint of $\overrightarrow{MN}$, **B** is the midpoint of $\overrightarrow{MT}$, **C** is the midpoint of $\overrightarrow{QR}$ and **D** is the midpoint of $\overrightarrow{OR}$.

(a) Express $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ *and* $\overrightarrow{OD}$ in terms of **a**, **b**, and **c.** (2marks)

(b) Prove that the quadrilateral ***ABCD*** is a parallelogram. (3 marks)

Question 6 **(5 marks)**

(a) Change the complex equation $|Z - i| = |Z - 1|$ into its Cartesian equivalent. (3 marks)

(b) Hence identify, the locus of all points Z satisfying the equation in (a). (2 marks)

Question 7 **(5 marks)**

An equilateral triangle has vertices *A, B* and *C,* where *A* is the point $\sqrt{3} - \underset{\sim}{i}$ in the Argand plane.

The circumcircle is drawn that passes through vertices *A, B,* and *C* and has a centre inside the triangle called the circumcentre.

The circumcentre is located at the origin.

Find the complex numbers z_1 and z_2 corresponding to the vertices *B* and *C*, expressing your answers in exact Cartesian form.

Question 8 **(4 marks)**

Find two numbers which have a product of 2 and a sum of 2.