

Question	Marks	Max	Question	Marks	Max
14	8		13	9	20
12	10	19	11	4	18
10	6	17	9	8	8
8	8	15	8	8	8
7	6	16	6	16	10
5	8	17	4	18	8
4	6	19	3	10	8
3	9	20	2	12	8
2	8	13	1	10	8

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination.

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate
Formula sheet (retained from Section One)

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time: one hundred minutes
Reading time before commencing work: ten minutes

Your Teacher's Name:

Your Name:

Calculator-assumed

Section Two:

UNIT 3

MATHEMATICS METHODS

Question/Answer booklet

Semester One Examination, 2019



PERTH MODERN SCHOOL

Structure of this paper

Section	Percentage of marking available	Marks available	Number of questions to be answered	Working time available (minutes)	Number of questions to be answered	Working time available (minutes)	Section One:	Section Two:	Total
CALCULATOR-ASSUMED	3	34	52	50	7	7	Calculator-free	Calculator-assumed	66
MATHEMATICS METHODS	100	103	100	13	13	13	Section Two:	Section One:	100

CALCULATOR-ASSUMED**MATHEMATICS METHODS**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning, incorrect answers to be checked readily and for marks to be awarded for reasoning, incomplete answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-assumed

(103 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8

(6 marks)

Consider the following table with

$$P(X=0)=0$$

x	1	2	3	4	5
$P(X \leq x)$	0.1	0.4	0.7	0.9	1

- a) Complete the probabilities in the table below

(2 marks)

Solution					
x	1	2	3	4	5
$P(X=x)$	0.1	0.3	0.3	0.2	0.1
Specific behaviours					
<input checked="" type="checkbox"/> determines probs for $x=2$ & 3 <input checked="" type="checkbox"/> determines probs for all x values					

Additional working space

Question number: _____

(a) Determine $\frac{dA}{dr}$ when $r = 5$ metres. (2 marks)

A liquid is spilled onto a floor forming a circle of radius r metres. The surface area, A , square metres, of the spill liquid is given by $A = \int_0^r 15\pi x^2 dx$.

(8 marks) Question 9

Solution $(x > 2 x < 4) = \frac{P(x = 3)}{P(x = 3)} = \frac{p(x < 4)}{0.3} = \frac{0.7}{0.3} = \frac{7}{3}$	Specific behaviours uses conditional prob formula determines correct quotient of probs expresses as simple fraction of integers
---	---

(3 marks)

Find $P(x > 2 \vee x < 4)$ (simplify)

Solution	$P(X=4) + P(X=5) = 0.3$
Specific behaviours	determines prob by adding for x=4 & 5

(1 mark)

Find $P(x \leq 4)$

Solution
$\frac{d}{dr} \int_0^r 15e^{\frac{x^3}{20}} dx = 15e^{\frac{r^3}{20}}$
$= 15e^{\frac{125}{20}} = 15e^{\frac{25}{4}}$
Approx. 7770.2
Specific behaviours
✓ uses fundamental theorem ✓ states an approx value for rate

- (b) What is the meaning of your answer in (a) above? (2 marks)

Solution
rate of change of area with respect to radius.
Specific behaviours
✓ states a rate ✓ with respect to radius

The radius, r , varies with time, t seconds, by the model $r = (5t^2 + 1)^4$.

- (c) Determine $\frac{dA}{dt}$ when $t = 1$ second as an exact value. (4 marks)

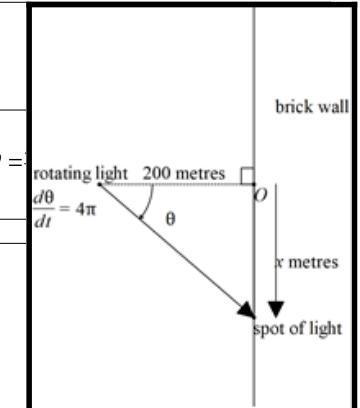
Solution

See next page

- ✓ differentiates x wrt θ
- ✓ uses chain rule with correct angle
- ✓ obtains approx value of velocity

- d) Determine the acceleration of the spot of light when

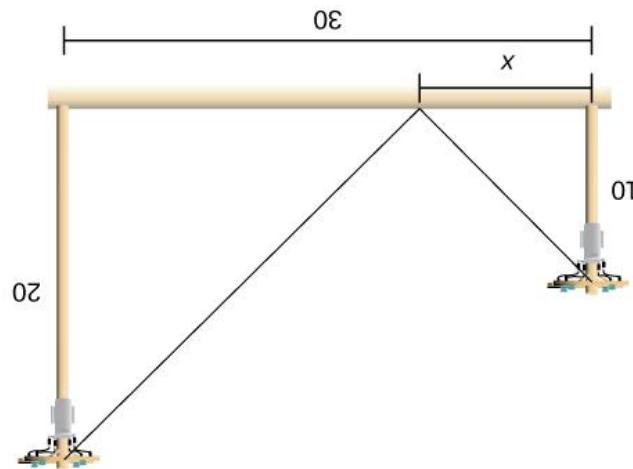
Solution
$v = 200 \frac{1}{\cos^2 \theta} 4\pi = 800\pi \cos^{-2} \theta$ $a = \frac{d^2x}{dt^2} = \frac{dv}{d\theta} \frac{d\theta}{dt} = \frac{-1600\pi}{\cos^3 \theta} (-\sin \theta) 4\pi$ $= -6400\pi^2 \left(\frac{-1}{2}\right) = \frac{25600\pi^2}{3\sqrt{3}} \approx 48624.8 \text{ m/s}^2$



Specific behaviours
✓ differentiates velocity wrt time ✓ uses chain rule ✓ subs required angle and rate ✓ obtains an approx value for acceleration (no need of units) or exact un-simplified

See next page

(2 marks)



Question 10

Two power poles need to be joined using a wire that is also connected to the ground, as shown below. The two poles are 10 and 20 metres high, and are separated by 30 metres.

- ✓ states derivative of radius wrt time and radius at $t=1$
- ✓ uses chain rule with dA
- ✓ uses fundamental theorem
- ✓ gives an exact expression (no need to simplify)

Specific behaviours

$$\begin{aligned} dA/dt &= \frac{r^3}{1296} \cdot 8640 = 15e^{20} \cdot 8640 \\ r &= 6 = 1296 \end{aligned}$$

Specific behaviors

c) Determine the velocity of the spot of light, $\frac{dx}{dt}$ in metres/second, when $\theta = \frac{\pi}{6}$ radians.

$$\frac{dx}{dt} = \frac{dx}{d\theta} \frac{d\theta}{dt} \text{ with } \frac{d\theta}{dt} = 4\pi$$
 Hint-use $\frac{dx}{dt} = \frac{dx}{d\theta} \frac{d\theta}{dt}$ with $\frac{d\theta}{dt} = 4\pi$ (3 marks)

(4) A lampshade is placed 200 metres in front of a long brick wall as shown in the diagram below. As the light is shone against the wall, the spot of light from the lampshade will move across the wall. Let x = the displacement of the spot of light from the point closest to the lamp to the point furthest from the lamp. The light that has two complete revolutions per second.

Question 20

Solution
$\sqrt{10^2 + x^2} + \sqrt{20^2 + (30 - x)^2}$
Specific behaviours
✓ uses Pythagoras with x ✓ states total length in terms of x

- (b) Using **calculus**, show how to determine the value of x to minimize the length of wire required. Determine this length to the nearest centimetre.
(Use of a classpad is required)

(4 marks)

Solution
$l = \sqrt{10^2 + x^2} + \sqrt{20^2 + (30 - x)^2}$
$\frac{dl}{dx} = \frac{2x}{2\sqrt{100+x^2}} + \frac{-2(30-x)}{2\sqrt{400+(30-x)^2}}$
$\frac{d}{dx} \left(\sqrt{100+x^2} + \sqrt{400+(30-x)^2} \right) = \frac{x\sqrt{x^2-60x+1300} + x\sqrt{x^2+100} - 30\sqrt{x^2+100}}{\sqrt{x^2-60x+1300}\sqrt{x^2+100}}$
$\text{solve } \left\{ \frac{x\sqrt{x^2-60x+1300} + x\sqrt{x^2+100} - 30\sqrt{x^2+100}}{\sqrt{x^2-60x+1300}\sqrt{x^2+100}} = 0 \right\} \Rightarrow \{x=10\}$

Second derivative at $x=10$ is positive as shown below:

See next page

- c) The area trapped between the graphs of $y=f(x)$ and $y=h(x)$ is 36 square units.
Determine the value of m .
(4 marks)

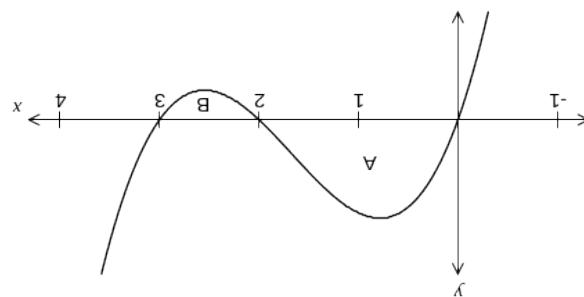
Solution
$\int_0^{3+m} mx + 4 - x^2 + 3x - 4 dx = 36$
$\int_0^{3+m} -x^2 + (3+m)x dx = 36$
$\left[-\frac{x^3}{3} + (3+m)\frac{x^2}{2} \right]_0^{3+m} = 36$
$\frac{-(3+m)^3}{3} + (3+m)\frac{(3+m)^2}{2} = \frac{(3+m)^3}{6} = 36$
$3+m = 6$
$m = 3$
Specific behaviours
✓ shows integral in terms of m ✓ sets up equation with integral for m ✓ shows integration equation for m ✓ solves for m

See next page

(1 mark)

$$dx \int_3^2 f(x) \quad \text{(a)}$$

Evaluate:



Part of the graph of $y = f(x)$ is shown below. The areas of regions A and B, bounded by the curve and the x -axis, are 16 and 2 square units respectively.

(4 marks)

Question 11

Specific behaviours	
$\text{length} = 42.43 \text{ metres}(4243\text{cm})$	42.42640687
$30\sqrt{2}$	$30\sqrt{2}$
$\sqrt{100+x^2 + 400+(30-x)^2} _{x=10}$	
$x\sqrt{x^2-60x+1300+x\sqrt{x^2+100-30\sqrt{x^2+100}}}$	$ _{x=10}$
$3\sqrt{2}$	80
All Standard Cplx Deg	

$x^2 - 3x + 4 = mx + 4$ $x^2 - (3+m)x = 0$ $x(x - (3+m)) = 0$ $x = 0 \text{ or } x = (3+m)$	Solution $x^2 - 3x + 4 = mx + 4$ $x(x - (3+m)) = 0$ $x = 0 \text{ or } x = (3+m)$	<u>Specific behaviours</u> <u>equates functions and factorises</u> <u>states both x values</u>
---	---	--

Show that the graphs of $y = l(x)$ and $y = n(x)$ intersect when $x = 0$ and $x = m + s$. (2 marks)

The figure shows a TI-Nspire CX CAS calculator displaying a step-by-step solution for a definite integral problem.

Step 1: Shows the integral $\int_6^0 [3x+4 - (x^2 - 3x + 4)] dx$.

Step 2: Shows the integral $\int_6^0 [3x + 4 - x^2 + 3x - 4] dx$.

Step 3: Shows the integral $\int_6^0 [6x - x^2] dx$.

Step 4: Shows the integral $\left[6x^2 - \frac{x^3}{3} \right]_6^0$.

Step 5: Shows the result $6(6^2) - \frac{6^3}{3} = -120$.

Step 6: Shows the final answer -120 .

Step 7: Shows the integral $\int_6^0 [3x+4 - (x^2 - 3x + 4)] dx$ again, with the condition $\{x=0, x=6\}$ indicated.

Step 8: Shows the integral $\int_0^6 [3x+4 - (x^2 - 3x + 4)] dx$.

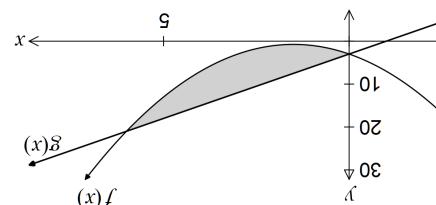
Step 9: Shows the integral $\int_0^6 [6x - x^2] dx$.

Step 10: Shows the integral $\left[6x^2 - \frac{x^3}{3} \right]_0^6$.

Step 11: Shows the result $6(6^2) - \frac{6^3}{3} = 120$.

Step 12: Shows the final answer 120 .

State the integral required to calculate the shaded area. (2 marks)



The graphs of $y = f(x)$ and $y = g(x)$ are shown on the axes below.

$t+xw = (x)l$

✓ uses second derivative sign test(or first) to verify a maximum

Solution	
-2	
Specific behaviours	

✓ states integral value

b) $\int_0^3 f(x)dx$ (1 mark)

Solution	
16-2=14	
Specific behaviours	

✓ states total value

c) $\int_0^2 f(x)-3dx$ (2 marks)

Solution	
$\int_0^2 f(x)-3dx = \int_0^2 f(x)dx - \int_0^2 3dx = 14 - [3x]_0^2 = 14 - [3x^2]_0^2 = 14 - [3(2)^2 - 3(0)^2] = 14 - [12 - 0] = 14 - 12 = 2$	
Specific behaviours	

✓ uses linearity and breaks into two integrals
✓ states value

Question 12 (10 marks)

A train on a monorail moves with velocity, v metres per second, at time t seconds, in a straight line, with acceleration, a metres per second squared, given by:

$$a=5 \sin \sin\left(3t + \frac{\pi}{2}\right)$$

The train begins at the origin and at rest.

Determine:

(a) The greatest speed of the train. (2 marks)

Solution	

See next page

- b) Determine the selling price per unit to establish the maximum profit in (a) above. (2 marks)

Solution	
$x(3x^2+19x+4)$	
$x \cdot (3x^2+19x+4)$	
$(3x^2+19x+4) x=50$	
8454	
Selling price per unit=\$8454	
Specific behaviours	
✓ factorises revenue ✓ determines selling price for $x=50$	

- c) What is the average cost of producing 100 items? (2 marks)

Solution	
3110400	
3110400/100	
31104	
Average cost is \$31104	
Specific behaviours	
✓ subs $x=100$ into cost function ✓ divides total cost by 100 items	

Question 19 (8 marks)

Consider the following 3 equations, where m is a positive real constant:

$$\begin{aligned}f(x) &= x^2 - 3x + 4 \\g(x) &= 3x + 4\end{aligned}$$

See next page

Solution

- (c) An expression for the displacement of the train from the origin. (3 marks)

Solution

- (b) The first time that the train begins to decelerate. (3 marks)

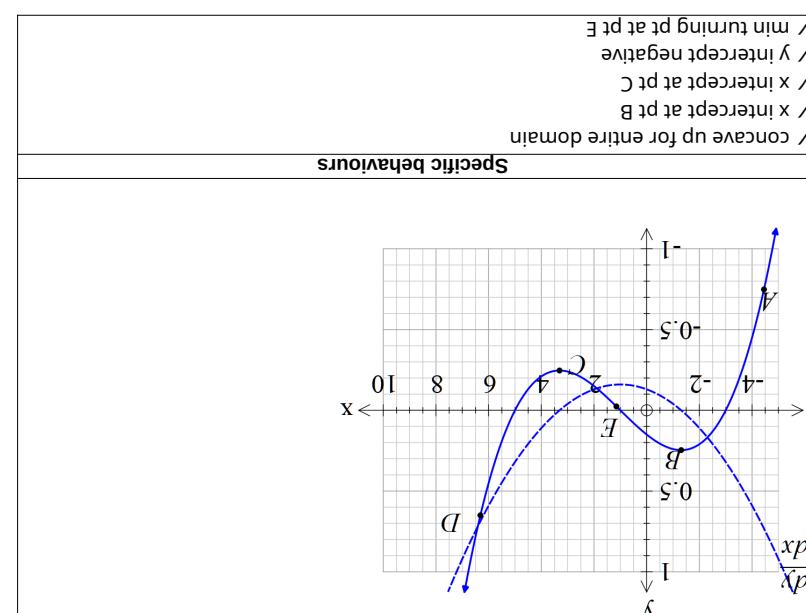
Solution

<ul style="list-style-type: none"> ✓ solves for x ✓ determines derivative and equates to zero ✓ states profit function
$\frac{dp}{dx} = -2x + 100$ $-2x + 100 = 0$ $x = 50$ $p(x) = R - C = -x^2 + 100x + 80000$

- (a) Using calculus methods, determine the number of units, x , to maximise the profit. (4 marks)

A hovercraft company has established that for selling x units, their revenue function, in dollars, can be given by $R(x) = 3x^3 + 20x^2 - 96x - 80000$. (Profit=Revenue-Cost)

(b) (8 marks)



Solution

<ul style="list-style-type: none"> ✓ min turning pt at pt E ✓ intercept negative ✓ intercept at pt C ✓ intercept at pt B ✓ concave up for entire domain
$\frac{dp}{dx} = -2x + 100$ $-2x + 100 = 0$ $x = 50$ $p(x) = R - C = -x^2 + 100x + 80000$

- (c) An expression for the displacement of the train from the origin. (3 marks)

A train starts from rest and moves with a constant acceleration of 3 m/s^2 for 2 seconds. It then moves with a constant velocity for 4 seconds before decelerating with a constant negative acceleration of -5 m/s^2 until it comes to rest.

(d) (8 marks)

MATHEMATICS METHODS

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$$\frac{dv}{dt} = 5 \sin\left(3t + \frac{\pi}{2}\right)$$

$$v = -\frac{5}{3} \cos\left(3t + \frac{\pi}{2}\right) + c$$

$$c = 0$$

$$v = -\frac{5}{3} \cos\left(3t + \frac{\pi}{2}\right)$$

$$x = -\frac{5}{9} \sin\left(3t + \frac{\pi}{2}\right) + c$$

$$t = 0, x = 0$$

$$0 = -\frac{5}{9} + c$$

$$c = \frac{5}{9}$$

$$x = -\frac{5}{9} \sin\left(3t + \frac{\pi}{2}\right) + \frac{5}{9} \quad OR \quad -\frac{5}{9} \cos(3t) + \frac{5}{9}$$

Specific behaviours

- ✓ integrates velocity
- ✓ solves for constant
- ✓ states displacement function

(d) The maximum distance that the train is from the origin.

(2 marks)

Solution

$$x = \frac{5}{9} + \frac{5}{9} = \frac{10}{9} \text{ m}$$

Specific behaviours

- ✓ recognizes that sin function equals -1
- ✓ states max distance (no need for units)

Question 13

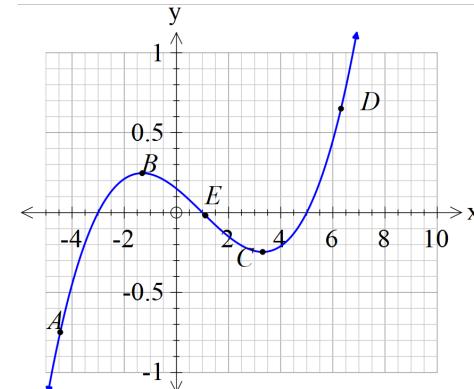
(9 marks)

Consider the function $f(x)$ shown graphed below. The table gives the value of the function at the given x values.

See next page

CALCULATOR-ASSUMED

21MATHEMATICS METHODS



c) What is the average cost of producing 100 items? (2 marks)

Solution

Edit Action Interactive
0.5 1 $\frac{dy}{dx}$ $\int_{a}^{b} \frac{f(x)}{dx}$ Simp $\int_{a}^{b} f(x) dx$

$3x^3+20x^2-96x-80000 | x=100$
3110400
 $3110400/100$
31104

Average cost is \$31104

Specific behaviours

- ✓ subs $x=100$ into cost function
- ✓ divides total cost by 100 items

Specific behaviours

- ✓ shows one approx inflection pt

c) Sketch the graph of f' on the same axes as the graph of f above. (5 marks)

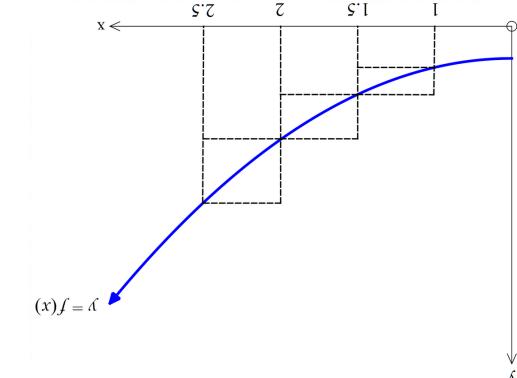
Solution

See next page

Consider the table of further values of $f(x)$ given below.

Solution		Specific behaviours			
Point	f'	f''	D	C	B
A	-	-ve			
B	+	-ve			
C	+	+ve			
D	+	+ve			

x	$f(x)$
1	1
1.5	1.8
2	2.6
2.5	3.8
3	5.4



- a) By considering the areas of the rectangles shown, demonstrate and explain why

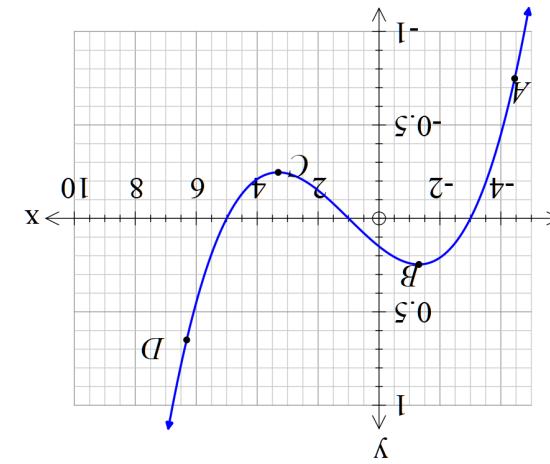
$$4.1 < \int_1^2 |f(x)| dx < 5.9.$$

- (3 marks)

- A, B, C, D are points on the graph of f . Determine whether the first and second derivatives are positive, negative or equal to zero at these points. Record your findings in the table below.

(4 marks)

in the table below.



The graph of a function f is given below.

x	0	1	1.5	2	2.5	3	3.5
$f(x)$	1	1.8	2.6	3.8	5.4	7.3	9.6

- b) Use the table values to determine the best estimate possible for $\int_1^3 2f(x)dx$ (4 marks)

Solution

$$0.5 \times 1.8 + 0.5 \times 2.6 + 0.5 \times 3.8 + 0.5 \times 5.4$$

$$< \int_1^3 f(x)dx < 0.5 \times 2.6 + 0.5 \times 3.8 + 0.5 \times 5.4 + 0.5 \times 7.3$$

$$6.8 < \int_1^3 f(x)dx < 9.55$$

$$\int_1^3 f(x)dx \approx \frac{6.8 + 9.55}{2} = 8.175$$

$$\int_1^3 2f(x)dx \approx 16.35$$

Specific behaviours

- ✓ shows sum of lower rectangles
- ✓ shows sum of upper rectangles
- ✓ uses average
- ✓ states estimate for $2f(x)$

- c) State two ways in which you could determine a more accurate value for $\int_1^3 2f(x)dx$ (2 marks)

Solution

- *use smaller widths of rectangles
- *model a rule for function and integrate

Specific behaviours

- ✓ states one reasonable method
- ✓ states two reasonable methods

- b) Hence determine the profit from selling 80 items. (2 marks)

Solution

4922

Profit of \$4922

Specific behaviours

- ✓ subs $x=80$
- ✓ states profit

- c) Find the net change in profit if the number of items sold changes from 80 to 160 items. (3 marks)

Solution

16768

□

Specific behaviours

- ✓ uses integration(or antiderivative)
- ✓ uses correct limits
- ✓ states net change

Question 14

A realtor's sales history over any month can be represented by the following probability distribution:

Number of houses sold in a month	0	1	2	3	4
Probability	0.15	0.4	0.3	0.1	0.05

The realtor is paid \$1000 every month with a bonus of \$1500 for every house sold up to three houses and a special bonus of \$1800 if four or more houses are sold in a month.

Let $X = ?$ the monthly earning of the realtor.

Determine:

(a) The expected monthly earning of the realtor, $E(X)$. (4 marks)

Solution

$E(X) = 1000 \times 0.15 + 2500 \times 0.4 + 4000 \times 0.3 + 5500 \times 0.1 + 7300 \times 0.05$
 $= 3265$

Specific behaviours

- states two values of earnings, X
- states all earnings values for X
- shows calculation for expected value of X
- states expected value

Question 16

The marginal profit from the sale of x_{th} item is given by $P(x) = 0.0015x^2 + 1.6x - 4.8$, where $P(x)$ is the profit from selling x items.

(b) Given that the company incurs a loss of \$70 if no items are sold, find an expression for $P(x)$ in terms of x . (3 marks)

Solution

$P(x) = 0.0015x^2 + 1.6x - 4.8$
 $- 70 = c$
 $\frac{dp}{dx} = 0.003x + 1.6 = 4.8x + c$
 $p(x) = 0.0005x^3 + 0.8x^2 - 4.8x + c$
 $p(x) = 0.0005x^3 + 0.8x^2 - 4.8x - 70$

Specific behaviours

- uses a constant
- solves for constant
- integrates

Solution

The standard deviation of X . (2 marks)

Specific behaviours

- calculates variance
- standard deviation = 1574.57

Solution

$\sqrt{3265} = 57.14$
 $\sum x^2 = 3265$
 $\sum x^3 = 13139500$
 $\sum x^4 = 1574.5714$
 $\max = 1000$
 $\min = 1$
 $Sx = 57.14$
 $Q_1 = 3265$
 $Q_3 = 13139500$
 $Med = 5714$

Start Calculation

Question 16

Shows at least two attempts with differing n values

Shows value of $n=7$

States value of $n=8$

States $n=8$

8 throws

binomialCDF(3, 8, 8, 0.5)	0.7734375
binomialCDF(3, 7, 7, 0.5)	0.65625
binomialCDF(3, 6, 6, 0.5)	0.5
binomialCDF(3, 5, 5, 0.5)	0.15

0.85546875

Calculator Interface

Question 16

The marginal profit from the sale of x_{th} item is given by $P(x) = 0.0015x^2 + 1.6x - 4.8$, where $P(x)$ is the profit from selling x items.

(a) Given that the company incurs a loss of \$70 if no items are sold, find an expression for $P(x)$ in terms of x . (3 marks)

Solution

$P(x) = 0.0015x^2 + 1.6x - 4.8$
 $- 70 = c$
 $\frac{dp}{dx} = 0.003x + 1.6 = 4.8x + c$
 $p(x) = 0.0005x^3 + 0.8x^2 - 4.8x + c$
 $p(x) = 0.0005x^3 + 0.8x^2 - 4.8x - 70$

Specific behaviours

- uses a constant
- solves for constant
- integrates

✓ states standard deviation (answer only required for full marks)

- (c) Variance ($5X - 3$). (2 marks)

Solution
$Variance = 5^2 \times V_{old} = 25 \times 1574.57^2 = 61981767.12$
Specific behaviours
✓ multiplies by 25 and does NOT subtract 3 ✓ states variance(accept standard notation)

Question 15 (8 marks)

Consider a fair die with the numbers $\{1, 2, 3, 4, 5, 6\}$. The random variable X is defined as the number of trials of the thrown die showing an even number on top.

- (a) If you throw the die 11 times,

- (i) Determine the probability that you will end up with more even numbers than odd numbers. (2 marks)

The figure shows the TI-Nspire CX CAS software interface. The top menu bar has 'Solution' selected. Below it is a toolbar with various icons: a gear for settings, 'Edit', 'Action', and 'Interactive'. The main workspace contains the following input:

```
binomialCDF(6, 11, 11, 0.5)
```

Below the input, there is a numeric entry field containing '0.5'.

- (ii) Determine the probability, to 4 decimal places, that there are in total, even number of times that the die shows an even number.

(3 marks)

Solution
$X \sim Bin(11, 0.5)$
$P(X = 2) + P(X = 4) + P(X = 6) + P(X = 8) + P(X = 10)$
$= 0.49951$
$= 0.4995$
<i>OR</i>
$P(X = 0) + P(X = 2) + P(X = 4) + P(X = 6) + P(X = 8) + P(X = 10)$
$= 0.5000$
Specific behaviours
<ul style="list-style-type: none"> ✓ states binomial dist ✓ states all even values of X ✓ states prob to 4 decimal places

- (b) If you would like to get at least three times an even number. Find the minimum times that you need to throw the die for which the probability of three or more even numbers is at least 85%. (3 marks)

Solution