

Section	Number of questions available	Time (minutes)	Marks	Percentage of examination
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Structure of this paper

METHODS UNITS 1 AND 2 CALCULATOR-ASSUMED

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate

Formula sheet (retained from Section One)

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time: one hundred minutes

Reading time before commencing work: ten minutes

Time allowed for this section

Your name _____

In words _____

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Student number: In figures _____

SOLUTIONS

MATHEMATICS
METHODS
UNITS 1 AND 2 Section Two:
Calculator-assumed

Question/Answer booklet

WAEF Semester Two Examination, 2018



	s available	answered			
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question

have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

(ii) the sum of the first 120 terms of this sequence. (2 marks)

Solution	
	Specific behaviours
$\begin{aligned} \diamond\diamond_{120} &= 2 \times (975 + 760.8) \\ &= 104148 \end{aligned}$	<input type="checkbox"/> uses sum formula <input type="checkbox"/> correct sum
	Specific behaviours

Question 9 (6 marks) (a) A sequence is defined by $\diamond\diamond_{n+1} = \diamond\diamond_n - 1.8$, $\diamond\diamond_1 = 975$. Determine the value of $\diamond\diamond_{120}$. (1 mark)

Solution	
	Specific behaviours
$\diamond\diamond_{120} = 760.8$	<input type="checkbox"/> correct value
	Specific behaviours

This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

Section Two: Calculator-assumed 65% (98 Marks)

Working time: 100 minutes.

(b) Another sequence is defined by $\ddot{d}_n = 975(0.2)^{n-1}$. Determine

CALCULATOR-ASSUMED 19 METHODS UNITS 1 AND 2

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Supplementary page

Question number: _____

(i) the value of \ddot{d}_5 . (1 mark)

Solution
$\ddot{d}_5 = 1.56$
Specific behaviours
✓ correct value

(ii) the value \ddot{d}_∞ approaches as $\ddot{d} \rightarrow \infty$. (2 marks)

Solution
$\ddot{d}_\infty = 975$ $1 - 0.2$ $= 1218.75$
Specific behaviours
✓ uses sum to infinity formula ✓ correct value

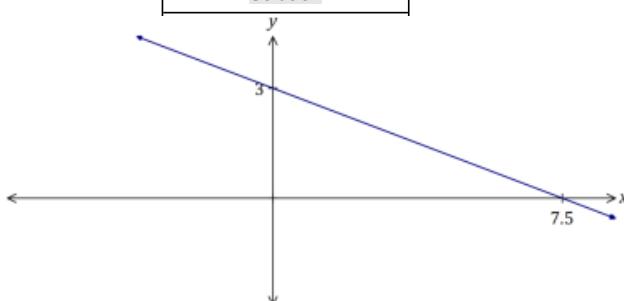
SN078-122-4 See next page

METHODS UNITS 1 AND 2 4 CALCULATOR-ASSUMED

Question 10 (6 marks) The variables \ddot{d}_1 and \ddot{d}_2 are related by the equation $2\ddot{d}_1 + 5\ddot{d}_2 = 15$.

(a) Sketch the graph of this relationship. (2 marks)

Solution



(c) The domain of $\frac{1}{x}$ is restricted to $-5 \leq x < 10$. State the range of $\frac{1}{x}$. (2 marks)

Solution	$x = -5, x = 5, x = 10$, $x = -1 - 1 < x \leq 5$
Specific behaviours	
Correct values	\checkmark correct values \checkmark correct inequalities

(b) Express $\frac{1}{x}$ in terms of y and briefly explain why $\frac{1}{x}$ is a function of y . (2 marks)

Solution	$y = 3 - 5x$ For each y value there is just one x value, so one-to-one relationship.
Specific behaviours	
Correct rule	\checkmark correct rule \checkmark indicates use of vertical line test or one-to-one, relationship

Question number: _____

Supplementary page

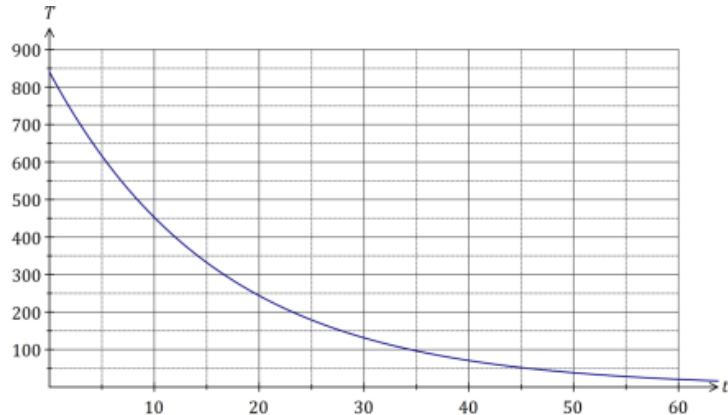
Question 11 (9 marks)

The temperature θ of a cast taken out of an oven cools according to the model $\theta = 840(0.94)^t$, where t is the time in minutes since the cast was removed from the oven. θ is measured in $^{\circ}\text{C}$.

- (a) Determine the fall in temperature of the cast during the first 5 minutes. (2 marks)

Solution
$\theta = 840(0.94)^5 \approx 616^{\circ}\text{C}$ $\Delta\theta = 840 - 616 = 224^{\circ}\text{C}$
Specific behaviours
✓ value of θ when $t = 5$ ✓ correct drop

- (b) Graph the temperature of the cast against time on the axes below. (4 marks)



(a) Calculate the area of the minor segment that subtends an arc of 72° in a circle of diameter 170 cm. (2 marks)

Question 12 (8 marks)

METHODS UNITS 1 AND 2 & CALCULATOR-ASSUMED

SN07G-T22-A

170 cm. (2 marks)

Solution	
	See next page, states not valid, with reason
	Specific behaviours
	For large values of θ , the model shows that $\theta \rightarrow 0$ but the temperature of the cast only rises to 15°C and so model not valid for large θ .

(ii) Comment on the usefulness of the model for large values of θ . (1 mark)

Solution	
	Specific behaviours
	$840(0.94)^\theta = 15 \Rightarrow \theta = 65.1 \text{ m}$
	✓ correct time

(i) Determine the time taken for the cast to reach room temperature. (1 mark)

(d) The temperature of the cast falls to room temperature of 15°C .

Solution	
	Exponential.
	Specific behaviours
	✓ correct name

(c) State the name of this type of function. (1 mark)

Question number: _____

Supplementary page

METHODS UNITS 1 AND 2 & CALCULATOR-ASSUMED

$72^\circ = \frac{2\pi}{5}$, $\theta = \frac{170}{2} = 85$
$\text{Area} = \frac{1}{2}(85)^2 \left(\frac{2\pi}{5} - \sin \frac{2\pi}{5} \right) \approx 1104 \text{ cm}^2$
Specific behaviours
✓ converts angle, uses correct radius ✓ calculates area

(b) A chord of length 26 cm subtends an angle of θ at the centre of a circle. Calculate the radius of the circle. (2 marks)

Solution
$26 = 2\theta \sin \left(\frac{1}{2} \times \theta \right)$ $\theta \approx 141 \text{ cm}$
Specific behaviours
✓ substitutes into formula ✓ calculates radius

(c) Use calculus to determine the maximum volume of the pyramid and state the dimensions required to achieve this volume. (4 marks)

Solution
$\text{Volume} = 360\theta - 12\theta^2$ $360\theta - 12\theta^2 = 0 \Rightarrow \theta = 0, 30$ $\text{Volume} = 180(30)^2 - 4(30)^3 = 54000 \text{ cm}^3$ $\theta = 30 \text{ cm}, \text{Height} = 90 \text{ cm}, h = 60 \text{ cm}$
Specific behaviours
✓ correct derivative using given variables ✓ solves derivative equal to zero ✓ correct maximum volume ✓ correct dimensions

(c) Parallelogram ABCDEF has side AB = 28 cm, side BC = 19 cm and an area of 400 cm².

Determine the lengths of the diagonals of ABCDEF. (4 marks)

Solution
$2(28)(19) \sin \theta = 400$
$\theta = 48.75^\circ, 131.25^\circ$

Bought outright	108	◆◆	232
Aged under 30	Aged 30 or over	Total	

A mobile phone retailer classified recent sales of 625 phones by the age of customer and if the phone was bought outright or on a plan. A summary of the data is shown in the table below.

Question 13 (8 marks)

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Δ = $3\Delta_1 + h = 180 - 4\Delta_1 - 3\Delta_2 = 180 - 4\Delta_1$ $\Delta_1 = 3(\Delta_2 \times 3\Delta_1) / (180 - 4\Delta_1)$ $\Delta_1 = 3\Delta_2^2 / (180 - 4\Delta_1)$	Solution
$\Delta_1 = 3\Delta_2^2 / (180 - 4\Delta_1)$ $\Delta_1 = 3\Delta_2^2 / (180 - 4(3\Delta_2^2 / (180 - 4\Delta_1)))$ $\Delta_1 = 3\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 4\Delta_1))$ $\Delta_1 = 3\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 4(3\Delta_2^2 / (180 - 4\Delta_1))))$ $\Delta_1 = 3\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 4\Delta_1)))$ $\Delta_1 = 3\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 4(3\Delta_2^2 / (180 - 4\Delta_1)))))$ $\Delta_1 = 3\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 4(3\Delta_2^2 / (180 - 4\Delta_1)))))$	Specific behaviours $\Delta_1 = 3\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 12\Delta_2^2 / (180 - 4(3\Delta_2^2 / (180 - 4\Delta_1)))))$

(b) Show that the volume of the pyramid is given by $V = 180x^2 - 4x^3$. (2 marks)

Specific behaviours

- ✓ equation for half area
- ✓ both angles of parallelogram
- ✓ correct length of one diagonal
- ✓ second correct length

specific behaviours

Question 21 (8 marks)

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A pyramid with a rectangular base of length ℓ and width w has perpendicular height h . The length of the base is three times its width and the sum of the width, length and height is 180 cm.

- ✓ indicates correct method
- ✓ correct probability

Bought on a plan	$\frac{1}{2}$	152	$\frac{1}{2}$
Total			625

(a) Determine the values of $\frac{1}{2}$, $\frac{1}{2}$ and $\frac{1}{2}$ shown in the table. (3 marks)

Solution
$\frac{1}{2} = 232 - 108 = 124$
$\frac{1}{2} = 625 - 232 = 393$
$\frac{1}{2} = 393 - 152 = 241$
Specific behaviours
✓ each correct value

Solution
$\frac{1}{2}(\frac{1}{2} \cap \frac{1}{2}) = 0.005 \times 0.97 = 0.00485$
$\frac{1}{2}(\frac{1}{2} \cap \frac{1}{2}) = 0.995 \times 0.03 = 0.02985$
$\frac{1}{2}(\frac{1}{2}) = 0.00485 + 0.02985 = 0.0347$
$\frac{1}{2}(\frac{1}{2} \frac{1}{2}) = 0.00485$
$0.0347 = 97$
$694 \approx 0.140$
Specific behaviours
✓ calculates $\frac{1}{2}(\frac{1}{2} \cap \frac{1}{2})$
✓ calculates $\frac{1}{2}(\frac{1}{2} \cap \frac{1}{2})$
✓ indicates $\frac{1}{2}(\frac{1}{2})$
✓ correct probability

(b) A recent sale is selected at random from those recorded above. Event $\frac{1}{2}$ occurs if the customer was aged under 30 and event $\frac{1}{2}$ occurs if the phone was bought outright.

Determine the following probabilities:

(i) $\frac{1}{2}(\frac{1}{2})$. (1 mark)

Solution
$\frac{1}{2}(\frac{1}{2}) = 393$
$625 (= 0.6288)$
Specific behaviours
✓ correct probability

(b) Two sheep are randomly selected for the test from those on the station. Determine the probability that just one of the sheep is diagnosed correctly. (2 marks)

Solution
$= 0.97 \times 0.03 = 0.0291$
$\frac{1}{2}(\frac{1}{2}_1 \cap \frac{1}{2}_2)$
$\frac{1}{2}(\frac{1}{2}_1 \cap \frac{1}{2}_2) = 0.03 \times 0.97 = 0.0291$
$\frac{1}{2} = 0.0291 + 0.0291$
$= 0.0582$
Specific behaviours

(iv) $\frac{d}{dx}(x^3 + 1)^{-1} = 3x^2(-1)$. (2 marks)

Solution
$\frac{d}{dx}(x^3 + 1)^{-1}$
$= -3x^2(x^3 + 1)^{-2}$
$= -3x^2(\frac{1}{x^3 + 1})$

(a) Use the derivative $\frac{d}{dx}(f(x))$ to determine the coordinates of all stationary points of the function. (3 marks)

Question 14 (7 marks) A function is defined by $f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^3$.

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(ii) the sheep actually has the disease if the test indicates that it does. (4 marks)

Solution
$P(\text{test positive} \text{sheep has disease}) = 0.97$
$P(\text{test negative} \text{sheep has disease}) = 0.03$
$P(\text{test positive} \text{sheep does not have disease}) = 0.05$

(i) the sheep has the disease, but the test indicates that it does not. (2 marks)

(a) A sheep is randomly selected for the test from those on the station. Determine the probability that

A diagnostic test for a disease has a 97% chance of giving the correct outcome and it is known that 0.5% of all sheep on a station have the disease. It can be assumed that the correct outcome of the test is independent of whether a sheep has the disease.

Question 20 (8 marks)

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(ii) Use the tangent to estimate $\frac{dy}{dx}(-0.5)$. (2 marks)

Solution
$y = x^2$
$\frac{dy}{dx} = 2x$

(i) Draw the tangent to the curve when $x = -0.5$. (1 mark)

Solution
$\frac{dy}{dx} = 2x$
$625 = 473$
$625 = 625 - 152$

Solution
$\frac{dy}{dx} = 0.005 \times 0.03 = 0.00015$
$20000 = 0.00015$
$\frac{dy}{dx} = 0.005 \times 0.03 = 0.00015$

(i) the sheep has the disease, but the test indicates that it does not. (2 marks)

Solution
$625 (= 0.1984)$
$\frac{dy}{dx} (x=0) = 0.1984$
$\frac{dy}{dx} (x=0) = 124$

Solution
$393 (\approx 0.613)$
$\frac{dy}{dx} (x=0) = 241$
$\frac{dy}{dx} (x=0) = 3$

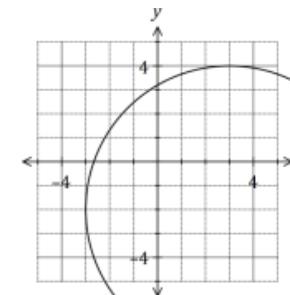
(i) the sheep has the disease, but the test indicates that it does not. (2 marks)

Solution
$625 = 473$
$625 = 625 - 152$
$\frac{dy}{dx} (x=0) = 625 - 152$

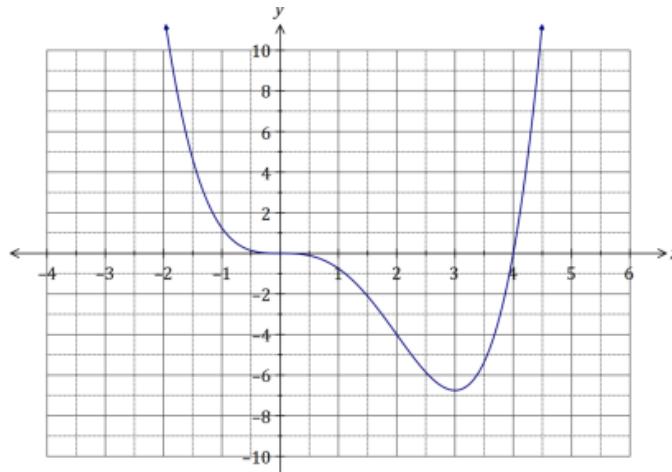
Solution
$\frac{dy}{dx}(x) = x^3 - 3x^2$
$x^3 - 3x^2 = 0 \Rightarrow x = 0, x = 3$
$y(0) = 0, y(3) = -6.75$
Stationary points at $(0, 0)$ and $(3, -6.75)$
Specific behaviours
✓ correct derivative ✓ correct zeros of derivative ✓ correct coordinates

Question 19 (7 marks)
 (a) Part of the circle $(x - 3)^2 + (y + 2)^2 = 6^2$ is shown below. Determine the values of the constants a , b and c . (4 marks)

Solution
$(x - 3)^2 + (y + 2)^2 = 6^2$
$x^2 - 6x + 9 + y^2 + 4y + 4 = 36$
$x^2 + y^2 - 6x + 4y - 23 = 0$
$x^2 + y^2 - 6x - 4y + 23 = 0$
$x^2 - 6x + 9 + y^2 - 4y + 4 = 0$
$(x - 3)^2 + (y - 2)^2 = 9$
Specific behaviours
✓ circle in factored form ✓ correct radius and centre ✓ expands into required form ✓ correct values of a , b and c

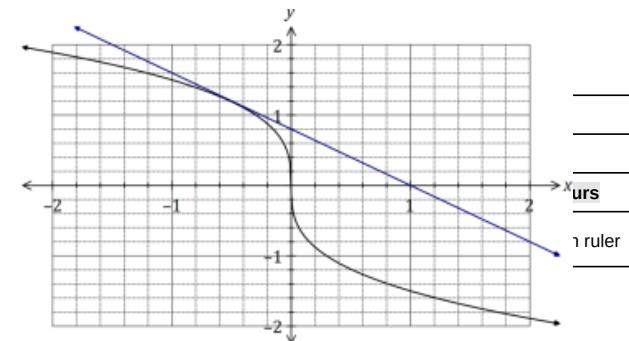


(b) Sketch the graph of $y = f(x)$ on the axes below. (4 marks)



- ✓ minimum at $(3, -7)$
- ✓ root at $(4, 0)$

(b) The graph of a power function $y = f(x)$ is shown below.



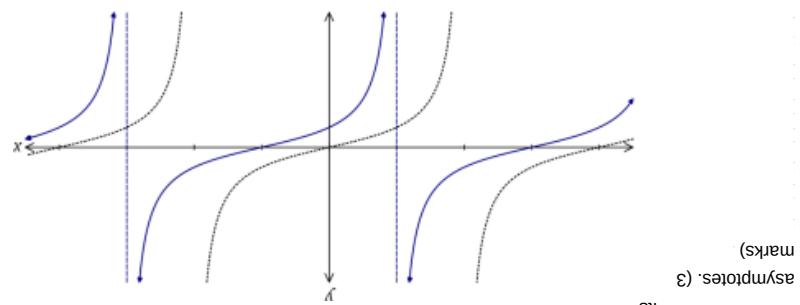
1 ruler

✓ determines both heights
✓ states difference to nearest cm
Solution

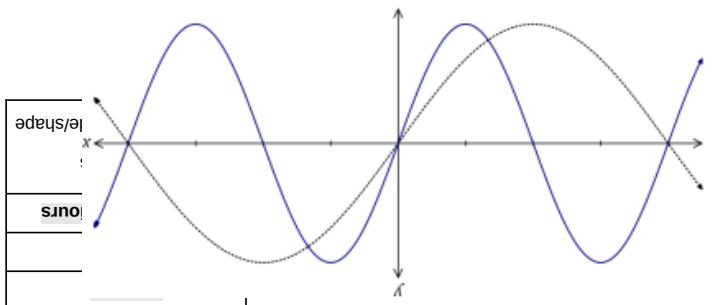
$\Delta h = 3.26 - 2.74 = 0.52 \text{ m}$
 $h(130) = 2.74, h(131) = 3.26$
(d) Determine the change in height of the seat between $\theta = 130$ and $\theta = 131$, giving your answer rounded to the nearest cm. (2 marks)

✓ time that rounds to 41 s
✓ specific behaviours
Solution

(c) At what time, when the seat was rising, did it first reach a height of 11 metres? (1 mark)



(b) The graph of $\theta = \tan \phi$ is shown below. On the same axes, sketch $\theta = \tan(\phi - \frac{\pi}{4})$, and all its asymptotes. (3 marks)



Question 15 (7 marks) (a) The graph of $\theta = \sin 2\phi$ is shown below. On the same axes, sketch $\theta = \sin 2(\phi - \frac{\pi}{4})$. (2 marks)

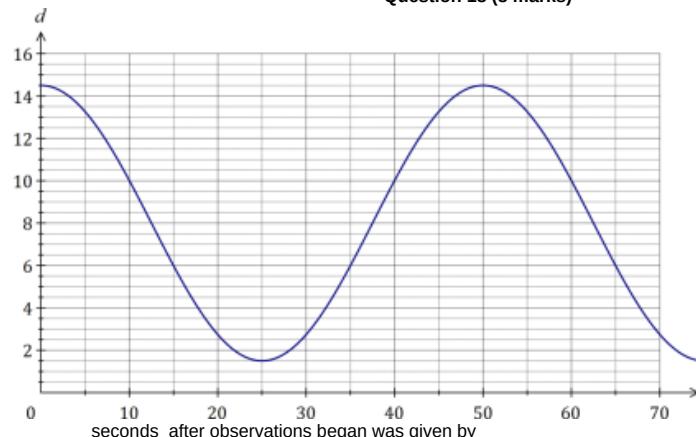
(b) How long did the Ferris wheel take to complete one revolution? (1 mark)

✓ correct time
✓ specific behaviours
Solution

✓ correct curvature

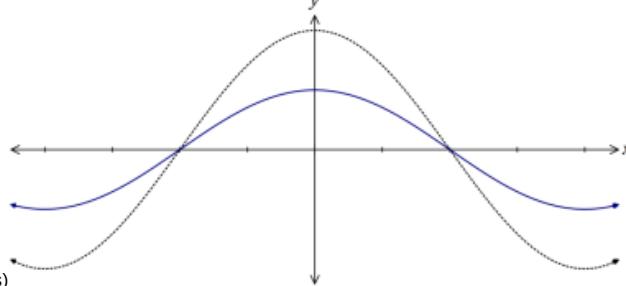
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Question 18 (8 marks)



The height, h metres, above level ground of a seat on a steadily rotating Ferris wheel

- (c) The graph of $y = \cos x$ is shown below. On the same axes, sketch $y = -\sin(x + \frac{\pi}{2})$. (2 marks)



+ (2 marks)

Solution
See graph
Specific behaviours
✓ location of roots ✓ correct amplitude/shape

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Question 16 (8 marks)

A council took a random sample of 154 and 127 properties from suburbs 1 and 2 respectively. A total of 49 of the properties in the sample were in arrears with their rates, and 27 of these properties were in suburb 1. 'In arrears' means that payment of rates is overdue.

- (a) Council officers wanted to choose 5 of the properties that were in arrears. How many different selections of properties are possible? (2 marks)

Solution
$\binom{49}{5} = 1906884$
Specific behaviours
✓ indicates use of formula ✓ correct number

Solution
See graph
Specific behaviours
✓ correct x -intercept ✓ accurate curve near maximum ✓ accurate curve near minimum ✓ smooth curve throughout

with rates is independent of the suburb the property is in. (3 marks)

(c) Justifying your answer with conditional probabilities, comment on whether being in arrears

✓ correct probability
Specific behaviours
$154 \approx 0.825$
Solution

(ii) is not in arrears given that it is in suburb . (1 mark)

(1 mark)
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✓ denominator
Specific behaviours
$281 \approx 0.078$
Solution

(i) is in suburb and is in arrears. (2 marks)

(b) Determine the probability that one randomly chosen property from the sample

✓ correct value
Specific behaviours
$5516 - 5310 = 206 \text{ mL}$
Solution

(iii) State, to the nearest mL, how much more water contains than at this time.

✓ correct value
Specific behaviours

Solution
$\text{P}(\text{Arrears} \text{Suburb}) = 27$ $154 \approx 17.5\%$
$\text{P}(\text{Arrears} \text{Suburb}) = 22$ $127 \approx 17.3\%$
Hence being in arrears is independent of suburb, as conditional probabilities are very similar.
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates $\text{P}(\text{Arrears} \text{Suburb})$ ✓ calculates $\text{P}(\text{Arrears} \text{Suburb})$ ✓ correct conclusion

Solution
$\text{P}_{25} = 2(1 - 1.1^{25})$ $1 - 1.1$ $= 197 \text{ mL}$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses sum formula ✓ correct amount

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CALCULATOR-ASSUMED 11 METHODS UNITS 1 AND 2

Question 17 (8 marks)

Two water containers, initially empty, are being filled with water. The amount of water added to container A each minute follows an arithmetic sequence, with 3 mL poured in during the first minute and 6 mL poured in during the second minute. The amount of water added to container B each minute follows a geometric sequence, with 2 mL poured in during the first minute and 2.2 mL poured in during the second minute.

(a) The amount of water poured into container B during the n^{th} minute is given by

$$\text{P}_n = (\text{P}_1)^{n-1}. \text{ State the value of the constants } \text{P}_1 \text{ and } r. \text{ (2 marks)}$$

Solution
$\text{P}_1 = 2$ $r = \frac{2.2}{2} = 1.1$
Specific behaviours
<ul style="list-style-type: none"> ✓ value of P_1 ✓ value of r

(b) Determine the total amount of water in container B at the end of the 25th minute.

(2 marks)

(c) How long does it take to fill container B with 360 mL of water? (2 marks)

Solution
$\text{P}_{25} = 2(2(3) + (25 - 1)(3))$ $= 360 \text{ mL} = 15 \text{ minutes}$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses sum formula ✓ correct time

(d) Container A first holds more water than container B at the end of minute n .

(i) Determine the value of n . (1 mark)

Solution
$\text{P}_n = 59$