PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST
UNITs 3 & 4
Section One:
Calculator-free

Your Name

Your Teacher's Name

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Max	Question	Mark	Max
1		6	5		6
2		5	6		11
3		9	7		6

4		7			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	11	11	100	89	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free **(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 **(6 marks)**

(a) $\int \cos^2(3x)\,dx$ (3 marks)

Solution
$\int \cos^2(3x)\,dx = \int \frac{\cos 6x + 1}{2}\,dx = \frac{1}{12}\sin 6x + \frac{1}{2}x + c$
Specific behaviours
✓ uses double angle identity ✓ anti-diffs ✓ adds a constant

b) $\int_0^{\frac{\pi}{4}} \sin^7(5x)\cos(5x)\,dx$ let $u = \sin 5x$ (3 marks)

Solution
$\int_0^{\frac{\pi}{4}} \sin^7(5x)\cos(5x)\frac{dx}{du}\,du = \int_0^{\frac{-1}{\sqrt{2}}} u^7 \cos 5x \frac{1}{5\cos 5x}\,du$ $\int_0^{\frac{-1}{\sqrt{2}}} u^7 \frac{1}{5}\,du = \left[\frac{u^8}{40}\right]_0^{\frac{-1}{\sqrt{2}}} = \frac{1}{40(16)}\ or\ \frac{1}{640}$
Specific behaviours
✓ uses change of variable ✓ obtains new integral with changed limits ✓ determines exact value

Question 2 (5 marks)

Consider the function $P(z)=z^4+10z^2+9$ where z is a complex number.

a) Show that $(z+3i)$ is a factor of $P(z)$. (2 marks)

Solution
$P(-3i)=(-3i)^4+10(-3i)^2+9=81-90+9=0$
Specific behaviours
✓ uses value of -3i ✓ evaluates each term and shows sum is zero

b) Solve for all values for $P(z)=0$ in the form $a+bi$. (3 marks)

Solution
$P(z)=(z-3i)(z+3i)(z^2+az+b)$ $=(z^2+9)(z^2+az+b)$ $z=0 \quad 9b=9 \quad b=1$ $z^3:a=0$ $P(z)=(z^2+9)(z^2+1)$ $roots: \pm 3i, \pm i$
Specific behaviours
✓ uses conjugate of -3i ✓ fully factorises ✓ states four complex roots

Question 3 **(3,4 & 2 = 9 marks)**

Given that $\dfrac{10x^2-6x+2}{(x+1)(x-2)^2}=\dfrac{a}{x+1}+\dfrac{b}{x-2}+\dfrac{c}{(x-2)^2}$ where $a,b\,\&\,c$ are constants.

a) Determine the values of $a,b\,\&\,c$. (3 marks)

Solution

$$10x^2-6x+2=a(x-2)^2+b(x+1)(x-2)+c(x+1)$$
$$x=2$$
$$30=3c \quad c=10$$
$$x=-1$$
$$18=9a \quad a=2$$
$$x=1$$
$$6=2-2b+20 \quad b=8$$

Specific behaviours

✓ solves for c with working
✓ solves for a with working
✓ solves for b with working

b) Hence determine the exact value of $\displaystyle\int_0^1 \frac{10x^2-6x+2}{(x+1)(x-2)^2}\,dx$. (simplify) (4 marks)

Solution

$$\int_0^1 \frac{2}{x+1}+\frac{8}{x-2}+\frac{10}{(x-2)^2}\,dx$$
$$=\left[2\ln|x+1|+8\ln|x-2|-\frac{10}{x-2}\right]_0^1$$
$$=(2\ln 2+10)-(8\ln 2+5)$$
$$=\ln\frac{4}{2^8}+5$$
$$=-6\ln 2+5$$

Specific behaviours

✓ integrates using logs
✓ integrates all terms
✓ subs limits and obtains an exact value
✓ simplifies result

c) Explain why $\int_{2}^{3} \frac{10x^2 - 6x + 2}{(x+1)(x-2)^2}\,dx$ does not exist. (2 marks)

Solution
Discontinuous at x=-1,2 which is within interval
Specific behaviours
✓ states discontinuous ✓ states x values

Question 4 (3,3 & 1 = 7 marks)

Consider the following functions:

$f(x) = e^{x+1}$

$g(x) = \frac{1}{\sqrt{x-2}}$

$h(x) = (x+3)^2$

a) Determine $f^{-1}(x)$ and its domain. (3 marks)

Solution
$x = e^{y+1}$ $lnx = y + 1$ $f^{-1}(x) = \ln x - 1$ $domain : x > 0$
Specific behaviours
✓ shows method for determining inverse ✓ states inverse rule ✓ states domain

b) Determine $g \circ h(x)$ and its domain. (3 marks)

Solution

$g \circ h(x) = \dfrac{1}{\sqrt{(x+3)^2 - 2}}$ $(x+3)^2 - 2 > 0$ $x + 3 = \pm\sqrt{2}$ $x > \sqrt{2} - 3,\ x < -\sqrt{2} - 3$
Specific behaviours
✓ states rule ✓ solves for BOTH limit values for x ✓ states both parts of domain

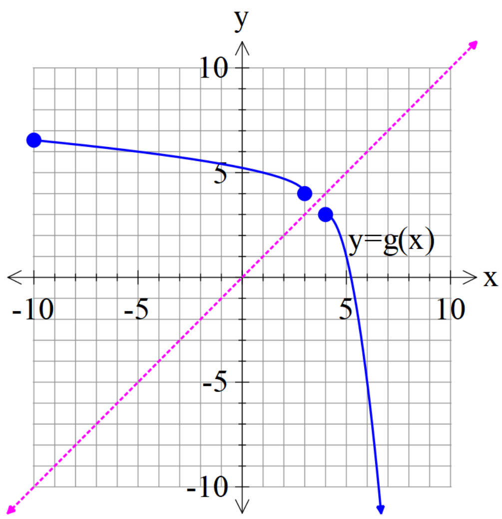
c) Determine the solution(s), if any for $f \circ h(x) = -1$, explain. (1 mark)

Solution
$f \circ h(x) = e^{(x+3)^2+1} > 0$ $\therefore y \neq -1$
Specific behaviours
✓ states that composite is always greater than zero

Question 5 **(6 marks)**

Consider the function $g(x)$ which is plotted below.

a) Plot $y = g^{-1}(x)$ on the axes above showing all major features. (3 marks)

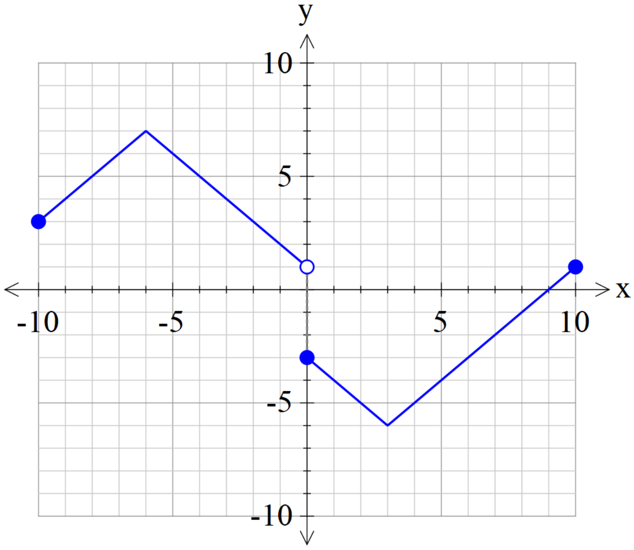
Solution

Specific behaviours
✓ reflected in line y=x ✓ shows pt (3,4) ✓ y less than 7

b) Given that $g(x) = -2(x-4)^2 + 3,\ x \geq 4$, determine the defining rule for $g^{-1}(x)$ and its domain. (3 marks)

Solution
$g(x) = -2(x-4)^2 + 3,\ x \geq 4$ $x = -2(y-4)^2 + 3,\ x \leq 3$ $\dfrac{x-3}{-2} = (y-4)^2$ $\sqrt{\dfrac{3-x}{2}} = \pm(y-4)$ $y = 4 + \sqrt{\dfrac{3-x}{2}} = f^{-1}(x)$ $d: x \leq 3$
Specific behaviours
✓ rearranges x and y ✓ states rule ✓ states domain

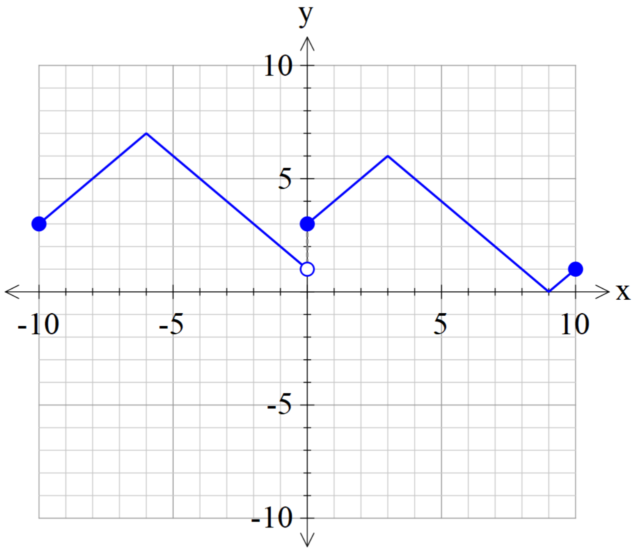
Question 6 **(11 marks)**

Consider the function below.

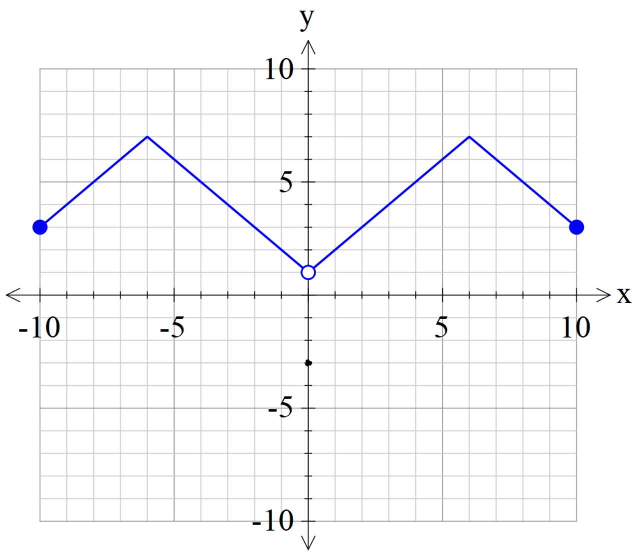


Sketch the following functions showing all major features.

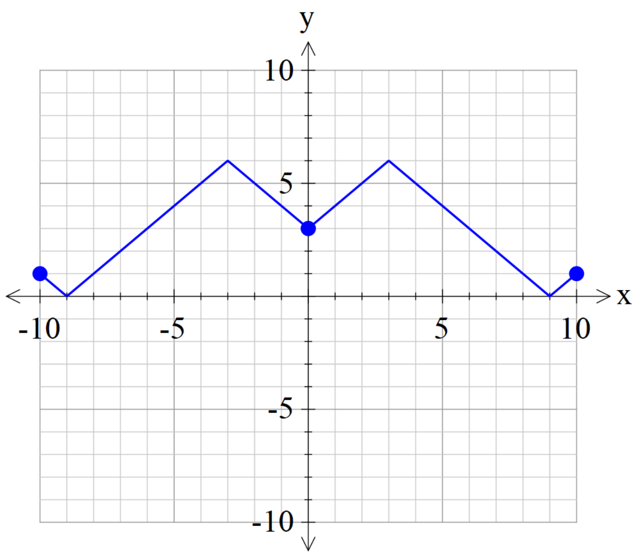
a) $y = \left| f(x) \right|$ (2 marks)

Solution

Specific behaviours
✓ shows left is unchanged ✓ reflects right in x axis

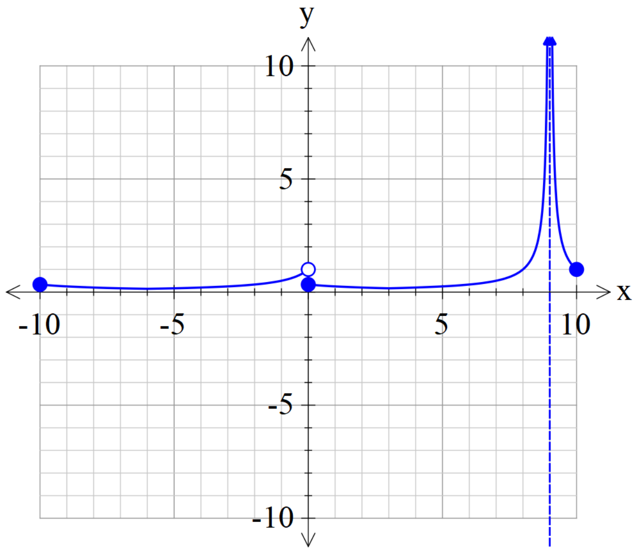
b) $y = f(-|x|)$ (3 marks)

Solution

Specific behaviours
✓ uses left side ✓ reflects to give new right side ✓ shows open circle on y axis and y intercept (0,-3)

c) $y = \left| f\left(|x|\right) \right|$ (3 marks)

Solution

Specific behaviours
✓ uses right side ✓ reflects in x axis ✓ reflects in y axis for new left

d) $y = \dfrac{1}{|f(x)|}$ (3 marks)

Solution

Specific behaviours
✓ shows asymptote with curve either side above x axis ✓ shape ✓ y intercept and open circle

Question 7 **(6 marks)**

Using the substitution $u = \sin x$, evaluate the integral $\int_0^{\frac{\pi}{6}} \frac{-2\cos x}{1-\sin^2 x}\,dx$. (simplify)

Solution
$\int_0^{\frac{\pi}{6}} \frac{-2\cos x}{1-\sin^2 x}\frac{dx}{du}\,du$ $-2\int_0^{\frac{1}{2}} \frac{\cos x}{1-u^2}\frac{1}{\cos x}\,du$ $-2\int_0^{\frac{1}{2}} \frac{1}{1-u^2}\,du$ $\frac{1}{1-u^2} = \frac{1}{(1-u)(1+u)} = \frac{A}{1-u} + \frac{B}{1+u}$ $1 = A(1+u) + B(1-u)$ $u = 1$ $1 = 2A \quad , A = \frac{1}{2}$ $u = -1$ $1 = 2B \quad , B = \frac{1}{2}$ $-2\int_0^{\frac{1}{2}} \frac{\frac{1}{2}}{1-u} + \frac{\frac{1}{2}}{1+u}\,du = -\left[-\ln(1-u) + \ln(1+u)\right]_0^{\frac{1}{2}} = -\left[\ln\frac{1+u}{1-u}\right]_0^{\frac{1}{2}} = -\ln 3$
Specific behaviours
✓ uses du/dx ✓ changes limits ✓ obtains integral in terms of u only ✓ uses partial fractions ✓ obtains anti-derivative and subs limits ✓obtains final value simplified NOTE: Follow through only if partial fractions used)

Additional working space

Question number:

Additional working space

Question number:

Additional working space

Question number: