SCHOOL Yr11/12

Trial WACE Examination, 2012

Question/Answer Booklet

MATHEMATICS 3A/3B
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures

In words MARKING KEY

Your name

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	12	12	100	100	67
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

Additional working space

Question number: _________

Additional working space

Question number: __________

Section Two: Calculator-assumed **(100 Marks)**

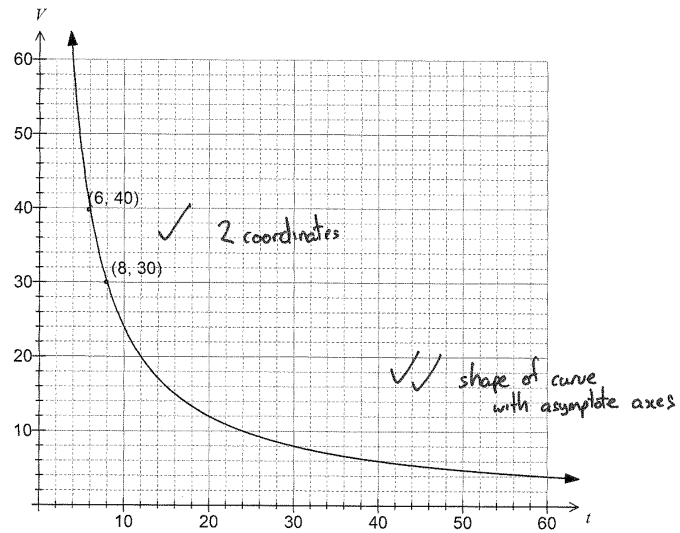
This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(3 marks)**

A quantity V is related to t by the formula $V = \frac{240}{t}$.

Sketch the graph of $V = \frac{240}{t}$ on the axes below, labelling the exact coordinates of two points on the curve.



Question 9 **(9 marks)**

The price of unleaded fuel sold at 344 outlets was recorded daily for 30 days and this data is summarised in the following table.

Price of ULP (c/L)	Frequency
$144 < x \le 146$	166
$146 < x \le 148$	805
$148 < x \le 150$	1348
$150 < x \le 152$	1947
$152 < x \le 154$	2339
$154 < x \le 156$	1485
$156 < x \le 158$	1371
$158 < x \le 160$	766
$160 < x \le 162$	93

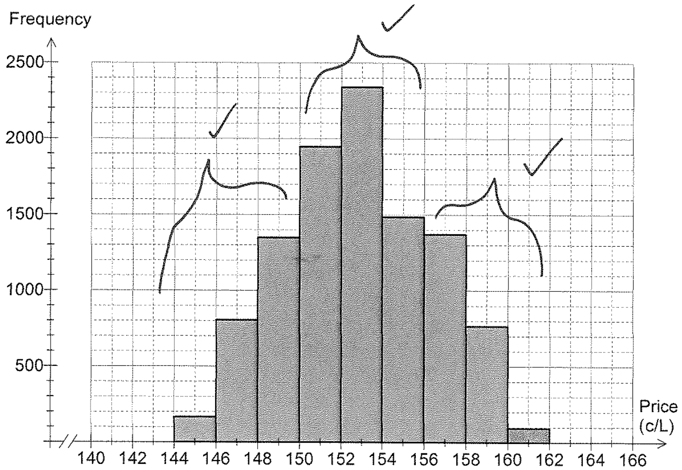
(a) Calculate the mean and standard deviation of these prices. (2 marks)

Mean = 152.84012 ✓

SD = 3.55058 ✓

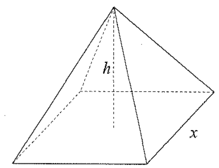
Any number of decimal places is acceptable.

(b) On the axes below, draw a histogram to show the distribution of these prices. (3 marks)



Question 19 **(7 marks)**

A pyramid has a square base of side x and perpendicular height h and is such that the sum of the side length, x, and height, h, is 15 cm.



(a) Show that the volume, V, of the pyramid is given by $V = 5x^2 - \frac{x^3}{3}$ cm³. (3 marks)

$V = \frac{1}{3}Ah$

$= \frac{1}{3}x^2 h$ but $x + h = 15 \Rightarrow h = 15 - x$ ✓

$= \frac{1}{3}x^2(15 - x)$ ✓

$= 5x^2 - \frac{x^3}{3}$ ✓

(b) Using calculus techniques, find the value of x that will maximise the volume of the pyramid, and state this maximum volume. (4 marks)

$\frac{dV}{dx} = 10x - x^2$ ✓

$= 0$ when $x = 0$, $x = 10$ ✓

$V(0) = 0$ $\therefore$ Minimum

$V(10) = \frac{500}{3}$ $\therefore$ Maximum ✓ show graph or state sign test (y' or y'')

Maximum volume of $166\frac{2}{3}$ cm³ when x is 10 cm ✓

Question 18 **(10 marks)**

A gardener makes their own blend of lawn fertiliser by mixing together x bags of Go-Green and y bags of Lush-Leaves. Each bag of Go-Green contains 40 grams of nitrogen and 20 grams of phosphorus. Each bag of Lush-Leaves contains 160 grams of nitrogen and 30 grams of phosphorus.

The blend must contain at least 800 grams of nitrogen, at least 300 grams of phosphorus, and must use at least twice as many bags of Go-Green as Lush-Leaves.

(a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy the above constraints. (3 marks)

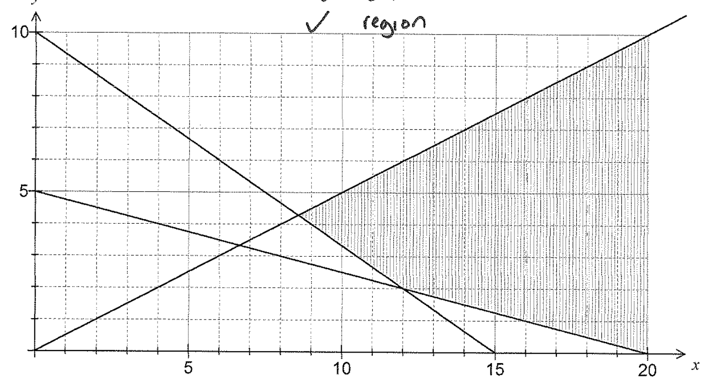
$40x + 160y \geq 800$ ✓

$20x + 30y \geq 300$ ✓

$x \geq 2y$ ✓

(b) Draw the three inequalities from (a) on the graph and shade the region that satisfies these inequalities. (4 marks)

✓✓✓ 3 lines
✓ region



(c) If one bag of Go-Green costs \$3 and one bag of Lush-Leaves costs \$8, how many of each should the gardener use to make their blend for the cheapest possible cost, and what is this cost? (3 marks)

$C(x, y) = 3x + 8y$
$C(9,5) = 67$ accept $(8.57, 4.29)$ or $(9,4)$ ✓
$C(12,2) = 52$ ✓
$C(20,0) = 60$ (optional pt)
Use 12 bags of Go-Green and 2 bags of Lush-Leaves for a minimum cost of \$52. ✓

(c) What features of the histogram suggest that this data may be modelled by a normal distribution? (2 marks)

The histogram has a roughly symmetrical, bell-shaped outline, with most prices clustered around the mean and then tailing off on either side. ✓ ✓

(d) Over the following 30 day period, prices were again recorded daily from all 344 outlets. The mean and standard deviation of the prices changed to 155.3 and 3.29 cents per litre respectively. Briefly compare the distribution of petrol prices in the first and second sets of observations. (2 marks)

Petrol prices increased by 2.5c/L from the first to second period, whilst the spread of prices decreased slightly, reflected in the smaller standard deviation. ✓ ✓

Question 10 **(8 marks)**

A $5000 loan is to be repaid by fourteen monthly payments of $360 and a final, fifteenth, payment to bring the loan balance to zero. Interest, of 9% pa of the balance, is added at the end of each month before the payment is made.

The table below shows the amount in the loan account at the start and end of each month, together with the monthly interest payable.

Month	Amount at start of month	Interest	Repayment	Amount at end of month
n	A_n	I_n	R_n	A_{n+1}
1	5000.00	37.50	360.00	4677.50
2	4677.50	35.08	360.00	4352.58
3	4352.58	32.64	360.00	4025.23
4	4025.23	30.19	360.00	3695.41
5	3695.41	27.72	360.00	3363.13
6	3363.13	25.22	360.00	3028.35
7	3028.35	22.71	360.00	2691.07
8	2691.07	20.18	360.00	2351.25
9	2351.25	17.63	360.00	2008.88
10	2008.88	15.07	360.00	1663.95
11	1663.95	12.48	360.00	1316.43
12	1316.43	9.87	360.00	966.30
13	966.30	7.25	360.00	613.55
14	613.55	4.60	360.00	258.15
15	258.15	1.94	260.09	0.00

(a) Write a recursive rule to determine the amount of the loan at the end of each month. (2 marks)

$A_{n+1} = A_n \times 1.0075 - 360$ ✓

$A_1 = 5000$ ✓

(b) Complete the table for months 14 and 15, clearly showing the amount of the final repayment. (3 marks)

(c) How much interest will be paid over the life of this loan? (2 marks)

$14 \times 360 + 260.09 - 5000 = \$300.09$ ✓ ✓

full marks for answer only

(d) Comment on the time taken to repay the loan if the repayment was changed to $36 per month. (1 mark)

The loan will never be repaid, as the monthly interest is greater than the repayment. ✓

(e) Describe how the graph of $y = f(x+1)$ may be obtained from the graph of $y = f(x)$ and hence state the coordinates of the root of the graph $y = f(x+1)$. (2 marks)

Translate $y = f(x)$ 1 unit to the left. ✓

Root at (-1, 0) ✓

(f) State the equation of the line of symmetry of the graph of $g(x)$. (1 mark)

$x = 1$ ✓

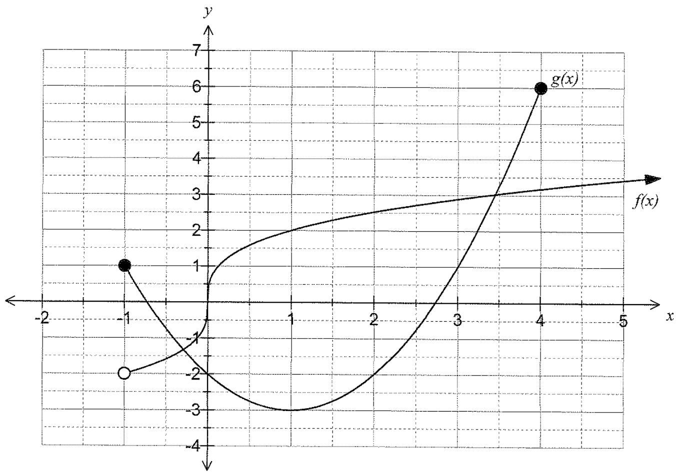
(g) State the solution(s) to the equation $f(x) = g(x)$. (2 marks)

$x = -0.3$, $x = 3.5$ (to 1 decimal place) ✓ ✓

Accept any number of decimal places

Question 17 **(10 marks)**

The graphs of the functions $f(x)$ and $g(x)$ are shown below.



(a) State the domain of $f(x)$. (1 mark)

$\boxed{\{x : x > -1\}}$ ✓

No need to use set notation

(b) State the range of $g(x)$. (1 mark)

$\boxed{\{y : -3 \le y \le 6\}}$ ✓

(c) Which function(s), if any, have a point of inflection? (1 mark)

$\boxed{f(x)}$ ✓

(d) Describe how the graph of $y = -g(x)$ may be obtained from the graph of $y = g(x)$ and hence state the coordinates of the turning point of the graph $y = -g(x)$. (2 marks)

Reflect $y = g(x)$ in the x-axis. ✓
TP at (1, 3) ✓

See next page

Question 11 **(8 marks)**

(a) A student sat three tests, scoring 29 out of 45 in Test A, 28 out of 38 in test B and 45 out of 60 in Test C.

The following summary statistics were available for these three tests:

Test	Maximum	Mean	Standard Deviation
A	45	24.8	8.2
B	38	22.5	8.9
C	60	36.2	12.8

Use standard scores (number of standard deviations from the mean) to compare the performance of the student in these three tests, explaining in which test the student performed the best. (4 marks)

Test A: $\frac{29-24.8}{8.2} = 0.51$ ✓

Test B: $\frac{28-22.5}{8.9} = 0.62$ ✓

Test C: $\frac{45-36.2}{12.8} = 0.69$ ✓

The student performed best in Test C as their standard score in that test was higher than in the other two tests. ✓

(b) 50 tagged trout were released into a lake. Over weekly intervals, small samples of trout were caught from various locations around the lake, with the following results:

	Number of trout in sample	Percentage of sample tagged
Week 1	40	5%
Week 2	20	10%
Week 3	50	4%

(i) Use the capture/recapture technique to estimate the number of trout in the lake. (3 marks)

$$\frac{50}{x} = \frac{40\times.05 + 20\times.10 + 50\times.04}{40+20+50}$$ ✓

$$\frac{50}{x} = \frac{6}{110}$$ ✓

$x = 916.\dot{6} \approx 920$ trout ✓

2 marks for working
1 mark for answer

(ii) Give a possible reason why the percentage of the sample tagged in Week 2 was much higher than in the other two weeks. (1 mark)

Sample size was much smaller in week 2 than other weeks, and so more prone to error. ✓

See next page

Question 12 **(6 marks)**

A survey of car owners in a city determined that the annual distance they drove in 2010 was normally distributed with a mean of 12 600 km and a standard deviation of 3 750 km.

(a) If one of the car owners from the survey was randomly chosen, what is the probability that they drove between 10 000 km and 15 000 km? (2 marks)

$$P(10000 < X < 15000) = 0.4949$$
$$\approx 0.49$$

✓✓ Accept any number of decimal places

(b) Find the minimum distance driven by at least 90% of drivers in the survey. (2 marks)

$$P(X > k) = 0.9$$ ✓
$$k = 7794$$
$$\approx 7800 \text{ km}$$ ✓

full marks for answer only + below (c) as well

(c) If 2 400 car owners took part in the survey, estimate how many of them had driven more than 20 000 km. (2 marks)

$$P(X > 20000) = 0.02423$$ ✓
$$0.02423 \times 2400 \approx 58 \text{ owners}$$ ✓

Question 16 **(7 marks)**

An employer is considering two pay schemes for new employees:

Scheme A
For the first month, a new employee is paid $750. In subsequent months, their pay is increased so that it is $40 more than the previous month.

Scheme B
For the first month, a new employee is paid $750. In subsequent months, their pay is increased so that it is 5% more than the previous month.

(a) Write a recursive rule for A_n, the payment made in month n to an employee using Scheme A and another recursive rule for B_n, the payment made in month n to an employee using Scheme B. (3 marks)

$$A_{n+1} = A_n + 40 \quad A_1 = 750$$ ✓
$$B_{n+1} = B_n \times 1.05 \quad B_1 = 750$$ ✓

✓ for both first terms

(b) In which month does the payment using Scheme B first exceed that using Scheme A, and by how much? (2 marks)

Month 5:

$A_5 = 910.00$
$B_5 = 911.63$ ✓

$911.63 - 910.00 = \$1.63$ ✓

(c) If an employee only worked for 6 months, which pay scheme should they choose in order to gain the most pay? Justify your answer. (2 marks)

Choose Scheme B. ✓

Sum of first 6 months for

A: $5100.00
B: $5101.43 ✓

Question 15 **(10 marks)**

A function has equation $y = 2x^3 - 3x^2 - 12x + 20$.

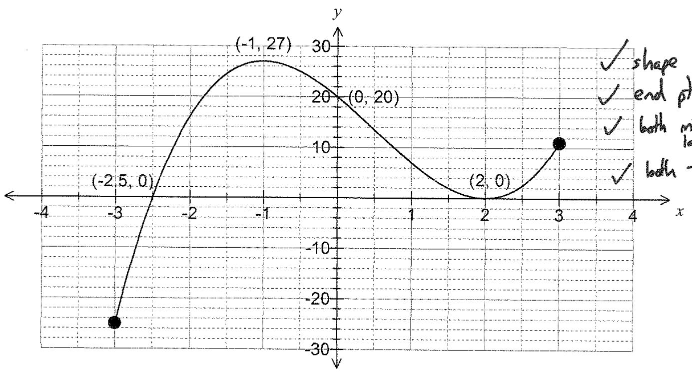
(a) Using calculus techniques, determine the coordinates of both stationary points of the function. (3 marks)

$\frac{dy}{dx} = 6x^2 - 6x - 12$ ✓

$= 0$ when $x = -1$, $x = 2$

Points are (-1, 27) and (2, 0) ✓ ✓

(b) Sketch the graph of the function over the domain $-3 \le x \le 3$, labelling all intercepts and stationary points. (4 marks)



✓ shape
✓ end pts
✓ both intercepts labelled
✓ both turning pts labelled

(c) Determine the coordinates of the point on the curve, other than the y-intercept, where the gradient is equal to that of the tangent at the y-intercept. (3 marks)

From gradient function above, when $x = 0$, $y' = -12$. ✓

$6x^2 - 6x - 12 = -12$

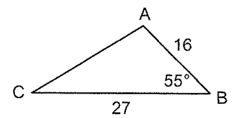
$x = 0$, $x = 1$ ✓

Hence at the point (1, 7) ✓

Question 13 **(8 marks)**

A triangle ABC has sides AB = 16 cm and BC = 27 cm and angle ABC = 55°.

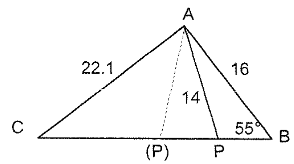
(a) Calculate the length of AC. (3 marks)



$AC^2 = 27^2 + 16^2 - 2 \times 27 \times 16 \times \cos 55°$ ✓✓

$AC = 22.1$ cm ✓

(b) P is a point on the side BC so that the distance AP = 14 cm. Find the largest possible area of triangle APC. (5 marks)



$\frac{16}{\sin \angle APB} = \frac{14}{\sin 55}$

$\angle APB = 110.58$ ✓

$\angle PAB = 180 - 110.58 - 55$

$\angle PAB = 14.42$ ✓

$PB^2 = 14^2 + 16^2 - 2 \times 14 \times 16 \times \cos 14.42$

$PB = 4.256$ ✓

$CP = 27 - 4.256$

$= 22.74$

$\angle APC = 180 - 110.58$

$= 69.42$ ✓

Area $= 0.5 \times 14 \times 22.74 \times \sin 69.42$

$= 149.0$ cm^2 ✓

Any number of decimal places

OR

$\triangle APC = \text{Area } ABC - \text{Area } ABP$

$= \frac{1}{2}(16)(27)\sin 55° - \frac{1}{2}(16)(14)\sin 14.42$ ✓ ✓

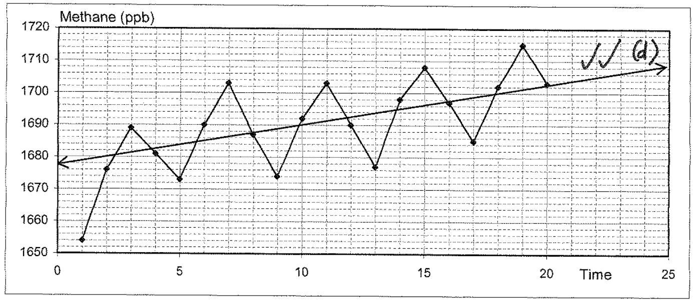
$\approx 176.94 - 27.89$

≈ 149.05 cm^2 ✓

Question 14 **(14 marks)**

The methane concentration in parts per billion (ppb) measured quarterly at a weather station over a period of five years from 1991 to 1995 is shown in the graph and table below.

Time period (t)	Year	Month	Methane concentration (ppb)	4-Point Centred Moving Average (m)	Residual
1	1991	Mar	1654	-	-
2	1991	Jun	1676	-	-
3	1991	Sep	1689	1677.4	11.6
4	1991	Dec	1681	1681.5	-0.5
5	1992	Mar	1673	**A**	**B**
6	1992	Jun	1690	1687.5	2.5
7	1992	Sep	1703	1688.4	14.6
8	1992	Dec	1687	1688.8	-1.8
9	1993	Mar	1674	1689.0	-15
10	1993	Jun	1692	1689.4	2.6
11	1993	Sep	1703	1690.1	12.9
12	1993	Dec	1690	1691.3	-1.3
13	1994	Mar	1677	1692.6	-15.6
14	1994	Jun	1698	1694.1	3.9
15	1994	Sep	1708	1696.0	12
16	1994	Dec	1697	1697.5	-0.5
17	1995	Mar	1685	1698.9	-13.9
18	1995	Jun	1702	1700.5	1.5
19	1995	Sep	**C**	-	-
20	1995	Dec	1703	-	-



(a) Explain why calculating a centred 4-point moving average is an appropriate way to smooth this data. (1 mark)

From the graph there is an obvious 4-part cycle of data reflecting the 4 quarters of the year. ✓

(b) Determine the values of A, B and C in the table above. (3 marks)

$$A=\frac{\frac{1689}{2}+1681+1673+1690+\frac{1703}{2}}{4}$$

$=1685$ ✓

$$1700.5=\frac{\frac{1697}{2}+1685+1702+C+\frac{1703}{2}}{4}$$

$C=1715$ ✓

$B=1673-1685$
$=-12$ ✓

OR $1698.9 = \dfrac{0.5(1708)+1697+1685+1702+0.5\ldots}{4}$

(c) Calculate the linear regression model for the smoothed data (m) against time (t) and write down the value of the correlation coefficient for this relationship. (3 marks)

$m=1.2547t+1677.3$ ✓ ✓

$r=0.97$ ✓

Allow any number of decimal places

(d) Add your linear model from (c) to the graph. (2 marks)

see graph

Passes through (0, 1677) and (25, 1709)

(e) Calculate the seasonal component for the September quarter and explain what this value means in the context of this question. (2 marks)

$$\frac{11.6+14.6+12.9+12}{4}=12.775$$ ✓

During this part of the cycle, the methane level is usually about 13 ppb above the moving average. ✓

(f) Predict the methane concentration for September 1996 and comment on how reliable this prediction is. (3 marks) ✓

$1.2547(23)+1677.3+12.775=1718.9$

≈ 1719 ppb ✓

As this prediction involves only a small degree of extrapolation and the correlation coefficient indicates a strong relationship, then we can be reasonably confident of this value. ✓