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Mathematics Methods Unit 3

Calculator Assumed Applications of Anti-Differentiation 1

Time: 45 minutes Total Marks: 45 Your Score: \ 45



Question One: [3 marks] CA

The area under the curve $f(x) = 4e^{\kappa x}$ over the domain $0 \le x \le 0$ is $\frac{40}{3} = \frac{4}{3} = \frac{1}{3} = \frac{4}{3} =$

Determine the value of k.

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Mathematics Methods Unit 3

Question Seven: [8 marks] CA

The area bounded by the curve $f(x) = ax^2 + b$ and the x axis over the domain $-1 \le x \le 1$ is 10.5 units².

. $\mathfrak{I} + x = \emptyset$ si I = x is $(x) \mathfrak{f}$ of the ganst and in ordinary and I

Determine the values of a, b and c.

$$xn2 = (x)'$$

$$xn2 = (x)'$$

$$I = (I)'$$

$$I = n2$$

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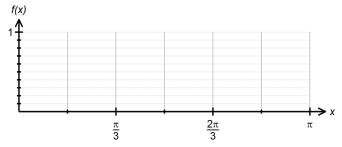
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Question Two: [2, 2, 3, 3 = 10 marks]

CA

Consider the function $f(x) = \sin\left(\frac{x}{2}\right)$

Sketch f(x) over the domain $0 \le x \le \pi$



Draw rectangles on your graph that can be used to overestimate the area under f(x) over the domain $0 \le x \le \pi$, where $\delta x = \frac{\pi}{6}$.

Hence approximate the area under the curve over the domain $0 \le x \le \pi$.

Calculate the margin of error between your answer in part (c) and the exact value of the area under the curve over the domain $0 \le x \le \pi$.

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Question Five: [4 marks]

CA

Calculate the area enclosed between the two curves $y = \cos x$ and $y = 3\sin(2x)$ over the domain $0 \le x \le \pi$.

Draw a sketch to support your solution.

Area =
$$2\int_{0.1674}^{\frac{\pi}{2}} 3\sin(2x) - \cos x \, dx = 4.17 \, units^2$$

Question Six: [4 marks] CA

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The area of the shaded region of $y = a \sin bx$ below is 6 units².

Determine the values of a and b.

$$\int_{0}^{\frac{2\pi}{b}} a \sin bx \, dx = 6$$

$$\int_{0}^{\frac{\pi}{b}} a \sin bx \, dx = 3$$

$$\int_{0}^{\pi} \sin x \, dx = 2$$

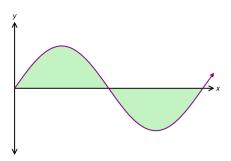
$$\int_{0}^{\pi} 1.5 \sin x \, dx = 3$$

$$a = 1.5$$

$$b = 1$$

$$or where \frac{2a}{b} = 3$$

$$a = 1.5b$$



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Mathematics Methods Unit 3

of the particle is -4 m/s.

Question Three: [1, 2, 2, 2, 2 = 9 marks] CA

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The acceleration of a particle moving in rectilinear motion is given by $a(t) = -4\cos(2t) + 12t$, where t is time in seconds and a(t) is \max^2 . The initial velocity

(a) Determine the initial acceleration of the particle.

(b) Determine an expression for the velocity of the particle.

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(c) Calculate when the speed of the particle is 4 m/s.

d) Calculate the change in displacement in the first second.

(e) Calculate the distance travelled in the third second.

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Question Four: [2, 2, 3 = 7 marks] CA

The marginal cost of producing x units of a certain product is $120 - 0.5x + 0.01x^2$ dollars per unit.

(a) Determine the extra cost associated with producing the 31^{st} item.

$$C(30) = 120 - 0.5(30) + 0.01(30)^2$$

Find the increase in cost if the production level is increased from 200 units to 500 units.

$$008 \text{ EYE} = xb^{-2}x10.0 + x2.0 - 021 \int_{002}^{002}$$

(c) The marginal revenue from producing and selling x units of a certain product is $x+2x^2$. Determine the profit function if the profit from producing 10 items

$$(^{z}x_{1}0.0 + x_{2}.0 - 0.21) - ^{z}x_{2} + x = (x)^{\prime}q$$

$$0.021 - x_{2}.1 + ^{z}x_{2} + 0.01$$

$$0.021 - \frac{x}{4}.000 + \frac{x}{4} + \frac{x}{6}.000$$

$$0.021 - \frac{x}{4}.000 + \frac{x}{4}.000$$

$$0.021 - \frac{x}{4}.000 + \frac{x}{4}.00$$

$$0.021 - \frac{x}{4}.000 + \frac{x}{4}.00$$

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$$0.021 - \frac{x}{4}.$$

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Question Four: [2, 2, 3 = 7 marks]

CA

The marginal cost of producing x units of a certain product is $120-0.5x+0.01x^2$ dollars per unit.

- (a) Determine the extra cost associated with producing the 31st item.
- (b) Find the increase in cost if the production level is increased from 200 units to 500 units.

(c) The marginal revenue from producing and selling x units of a certain product is $x+2x^2$. Determine the profit function if the profit from producing 10 items is \$38.33.

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Question Three: [1, 2, 2, 2, 2 = 9 marks]

CA

The acceleration of a particle moving in rectilinear motion is given by $a(t) = -4\cos(2t) + 12t$, where t is time in seconds and a(t) is ms^{-2} . The initial velocity of the particle is -4 m/s.

(a) Determine the initial acceleration of the particle.

$$a(0) = -4ms^{-2}$$

(b) Determine an expression for the velocity of the particle.

$$v(t) = \int -4\cos(2t) + 12t \ dt$$

$$v(t) = -2\sin(2t) + 6t^{2} + c$$

$$-4 = -2\sin(0) + 6(0)^{2} + c$$

$$c = -4$$

$$v(t) = -2\sin(2t) + 6t^{2} - 4$$

(c) Calculate when the speed of the particle is 4 m/s.

$$|v(t)| = 4$$
 \checkmark
 $t = 0s, 0.543s, 1.24s$ \checkmark

(d) Calculate the change in displacement in the first second.

$$\int_{0}^{1} v(t) dt = -3.42m$$

(e) Calculate the distance travelled in the third second.

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$$\int_{2}^{3} |v(t)| dt = 35.62m$$

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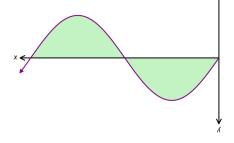
Question Five: [4 marks] $\mathbf{C}\mathbf{V}$

. $\pi \ge x \ge 0$ nismob sht Calculate the area enclosed between the two curves $y = \cos x$ and $y = 3\sin(2x)$ over

Six: Question Six: $\mathbf{C}\mathbf{V}$ [4 marks]

The area of the shaded region of $y = a \sin bx$ below is 6 units².

Determine the values of a and b.



Draw a sketch to support your solution.

CA

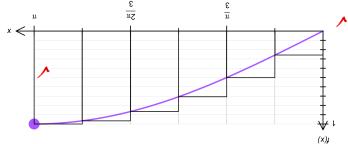
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Mathematics Methods Unit 3

Consider the function $f(x) = (x) \int \sin x dx$

Sketch f(x) over the domain $0 \le x \le \pi$

Question Two: [2, 2, 3, 3 = 10 marks]



Draw rectangles on your graph that can be used to overestimate the area

 $\frac{\pi}{6} = x\delta$ brack $\pi \ge x \ge 0$ nismob and reverse $x \ge 0$ nismob and reverse $x \ge 0$

. $\pi \ge x \ge 0$ mis mode the curve over the domain $0 \le x \ge 0$

 $\sum_{n=0}^{\infty} stinu \, \delta L. \Delta = n \, \text{soft}$ $\left(\left(\frac{\pi}{2} \right) \text{nis} + \left(\frac{\pi \xi}{21} \right) \text{nis} + \left(\frac{\pi}{\xi} \right) \text{nis} + \left(\frac{\pi}{4} \right) \text{nis} + \left(\frac{\pi}{4} \right) \text{nis} + \left(\frac{\pi}{21} \right) \text{nis} \right) \frac{\pi}{6} = n 9 M$

value of the area under the curve over the domain $0 \le x \le \pi$. Calculate the margin of error between your answer in part (c) and the exact

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Question Seven: [8 marks] CA

The area bounded by the curve $f(x) = ax^2 + b$ and the *x* axis over the domain $-1 \le x \le 2$ is 10.5 units².

The equation of the tangent to f(x) at x = 1 is y = x + c.

Determine the values of a, b and c.

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SOLUTIONS Calculator Assumed Applications of Anti-Differentiation 1

Time: 45 minutes Total Marks: 45 Your Score: / 45

Question One: [3 marks] CA

The area under the curve $f(x) = 4e^{kx}$ over the domain $0 \le x \le 10$ is $\frac{40}{3} \left(-e^{-3} + 1 \right)$.

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Determine the value of k.

$$\int_{0}^{10} 4e^{kx} dx = \frac{40}{3} \left(-e^{-3} + 1 \right)$$

$$\checkmark \left[\frac{4e^{kx}}{k}\right]^{10} = \frac{40}{3}(-e^{-3}+1)$$

$$\frac{4e^{10k}}{k} - \frac{4}{k} = \frac{40}{3} \left(-e^{-3} + 1 \right) \checkmark$$

$$k = -0.3$$