

Papers written by
Australian Maths
Software

SEMESTER TWO

YEAR 12

MATHEMATICS METHODS

REVISION 1

Units 3-4

2016

**Section One
(Calculator-free)**

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work:

5 minutes

Working time for section:

50 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section One.

A formula sheet which may also be used for Section Two.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	5	5	50	52	35
Section Two Calculator— assumed	12	12	100	98	65
Total marks				150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

1. (8 marks)

Evaluate the following

(a) $\int (2x + 4)^6 dx$ (2)

(b) $\int_{\pi/4}^{\pi/2} (2 \sin(x) - \cos(x)) dx$ (4)

(c) $\int \left(x^4 + e^{2x} + \frac{2}{x} \right) dx$ (2)

2. (16 marks)

(a) Find the derivative of

(i) $f(x) = \ln\left(\frac{x^2 - 3}{1 + x}\right)$ (2)

(ii) $g(x) = \frac{e^{\sin(x)}}{\cos(x)}$ (3)

(iii) $h(x) = e^x \times \ln(x^2)$ (2)

(b) (i) Given $g(x) = \sqrt{\sin(x)}$
show that $g'(x) = \frac{\cos(x)}{2\sqrt{\sin(x)}}$. (2)

(ii) Hence determine $\int \frac{3\cos(x)}{\sqrt{\sin(x)}} dx$ (2)

(c) Find $2 \int_0^4 (1 - f(x)) dx$ given $\int_0^{10} f(x) dx = -6.4$ and $\int_4^{10} f(x) dx = 2.3$. (2)

(d) Given $r = \sqrt{t}$, $t = 4x$, $x = \cos(\theta)$
find an expression for $\frac{dr}{d\theta}$ as a function of θ . (3)

3. (6 marks)

(a) Simplify

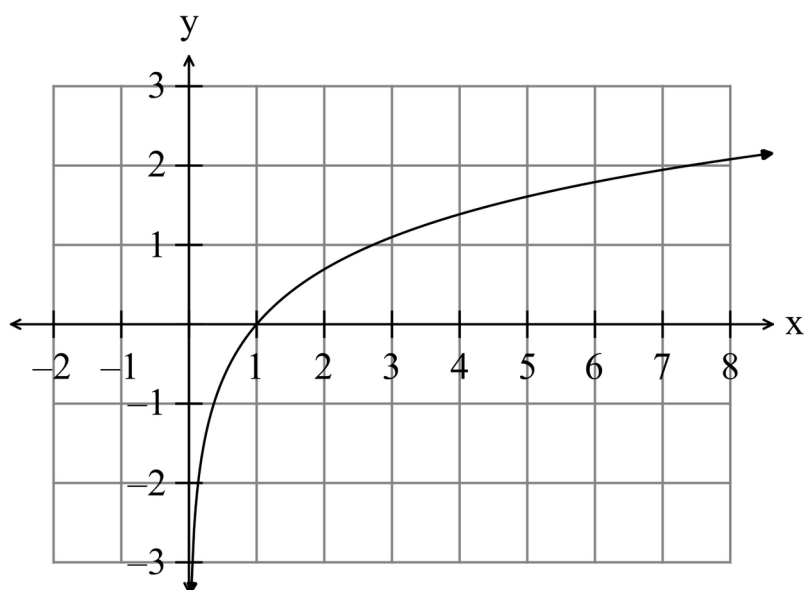
$$\frac{\log_{10}(4 \times 3^2) - \log_{10}(3 \times 6) - 3\log_{10} 2}{-2\log_{10} 2} \quad (3)$$

(b) Find x given

$$(\log_3(x) - 1)(\ln(x) - 1) = 0 \quad (3)$$

4. (6 marks)

(a) The graph of $y = \ln(x)$ is shown on the set of axes below,



(i) Sketch on the same set of axes

$$y = 2\ln(x) \quad \text{and} \quad y = 1 - \ln(x) \quad (2+2)$$

(ii) Find the inverse of f given $f(x) = \ln(x)$.

State the domain of the inverse. (2)

5. (16 marks)

- (a) Which of the following represent probability density functions?
Give your reasons.

(i)

x	3	4	5	6
$P(X = x)$	0.3	0.4	0.5	-0.2

(2)

(ii)

x	0	1	2	3
$P(X = x)$	0.3	0.1	0.2	0.4

(1)

- (b) Thirty samples of size 10 are drawn from a population where each outcome is equally likely. The mean of the population is 15.
Sketch the shape of the distribution of the sample means.

(2)

- (c) (i) Show that the function defined by $p(x) = \frac{1}{x}$ for $1 \leq x \leq e$ is a continuous probability density function. (3)

- (ii) Find $P(1 \leq x \leq 2)$. (2)

- (d) Find the cumulative probability density distribution for the probability density function $f(x) = \frac{x}{2}$ defined on $[0, 2]$. (2)

- (e) Calculate the mean and write down the expression for the variance for the uniform continuous probability function defined as $p(x) = \begin{cases} 0.2 & \text{for } 5 \leq x \leq 10 \\ 0 & \text{otherwise} \end{cases}$. (4)

END OF SECTION ONE