



**PERTH MODERN SCHOOL**  
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**Independent Public School**

**Course** \_\_\_\_\_ **Specialist** \_\_\_\_\_ **Year** 12

Student name: \_\_\_\_\_ Teacher name: \_\_\_\_\_

Date: 24 Feb

**Task type:** \_\_\_\_\_ **Response**

**Time allowed for this task:** 45 mins

**Number of questions:** 8

**Materials required:** Calculator with CAS capability (to be provided by the student)

**Standard items:** Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

**Special items:** Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

**Marks available:** 50 marks

**Task weighting:** 10%

**Formula sheet provided:** Yes

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

Q1 (3.1.1, 3.1.2, 3.1.3)

(2, 2, 3 &amp; 3 = 10 marks)

If  $z = 2 + 3i$  and  $w = -1 + 2i$  determine exactly the following. (Simplify)

a)  $\overline{zw}$

b)  $\overline{ww}$

c)  $w \div \overline{w}$

d)  $\frac{1}{z} + \frac{1}{w}$

Q2 (3.1.3)

(3 marks)

Determine all possible real values of  $a$  &  $b$  such that  $\frac{43 - i}{a + 4i} = 5 + bi$

Q3 (3.1.14, 3.1.15)

(3 &amp; 3 = 6 marks)

Consider the quadratic equation  $x^2 + bx + c = 0$  where  $b$  &  $c$  are real.

- a) If one root of the above equation is  $x = 4 - 2i$ , determine  $b$  &  $c$ .

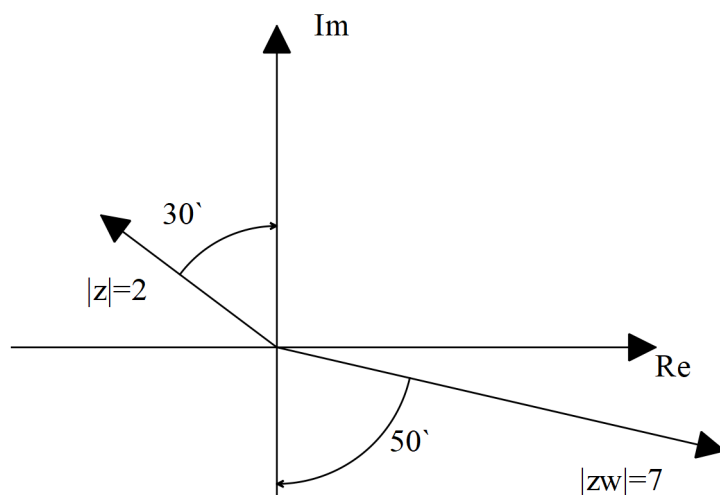
Consider the equation  $x^3 + px^2 + qx + w = 0$  where  $p, q$  &  $w$  are real.

- b) If the cubic equation above has roots  $x = 2$  &  $x = \sqrt{3}i$ , determine  $p, q$  &  $w$ .

Q4 (3.1.3, 3.1.3, 3.1.3)

(2 marks)

Determine  $z$  &  $w$  in the form  $rcis\theta$  with  $-\pi < \theta \leq \pi$ . (Note: diagram not drawn to scale)

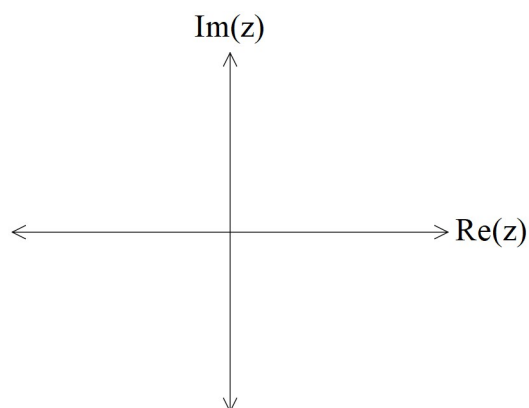


Q5 (3.1.10)

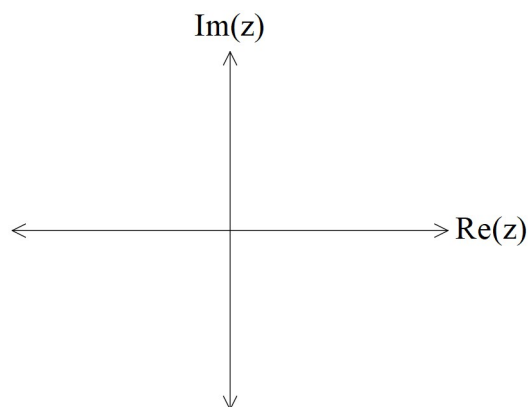
(2, 2 &amp; 3 = 7 marks)

Sketch the following regions in the complex plane showing major features.

a)  $\text{Arg}(z) = \frac{3\pi}{4}$



b)  $|z + 3 + 4i| \geq |z - 5 + i|$



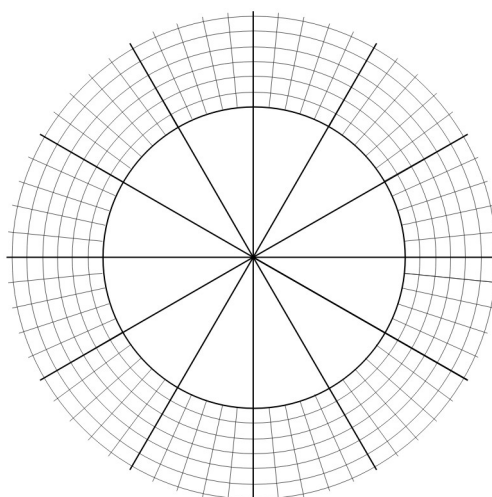
- c) Consider all the complex numbers  $z$  that satisfy  $|z - (2 + 5i)| = 3$ , determine the maximum possible value of  $\text{Arg}(z)$ , giving your answer in radians correct to two decimal places.

Q6 (3.1.7, 3.1.12)

(4 &amp; 3=7 marks)

a) Determine all the roots of  $z^5 = \sqrt{3} + i$  expressing in the form  $rcis\theta$  with  $-\pi < \theta \leq \pi$ .

b) Plot all of these roots on the diagram below.



Q7 (3.2.1, 3.2.2)

(1, 2, 2 &amp; 2 = 7 marks)

Consider the functions  $f(x) = \sqrt{x-8}$  &  $g(x) = x^3$ .

a) Give the defining rule for  $f \circ g(x)$ .

b) Does  $f \circ g(x)$  exist over the natural domain of  $g(x)$ ? Explain

c) State the natural domain and range for  $f \circ g(x)$ .

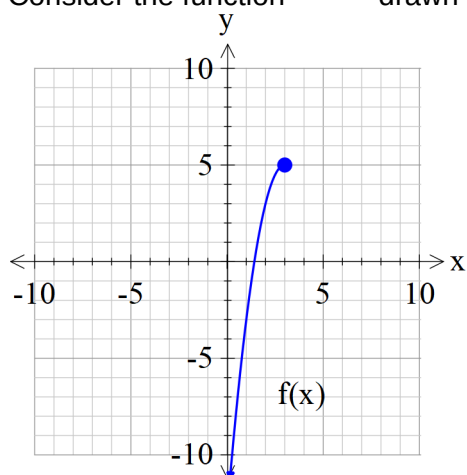
Consider the function  $h(x) = x - 8$ .

d) Does the function  $[f(x)]^2 = h(x)$ ? Justify your answer.

Q8 (3.2.3, 3.2.4)

(2, 3 &amp; 3 = 8 marks)

Consider the function  $f(x)$  drawn below.



a) Sketch  $y = f^{-1}(x)$  on the axes above.

b) Given that  $f(x) = -2x^2 + 12x - 13$ ,  $x \leq 3$ , determine the defining rule for  $y = f^{-1}(x)$ .  
Show working for full marks.

c) Consider the function  $h(x) = ax^3$  where  $a$  is a positive constant. Solve in terms of  $a$ , the solution(s) to  $h(x) = h^{-1}(x)$ .

**Working out space**