PERTH MODERN SCHOOL

UNIT 3C/3D MAS – 2012

PERTH MODERN SCHOOL
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TEST 1 – POLAR COORDINATES & COMPLEX NUMBERS

NAME: SOLUTIONS **DATE: 14/15 February 2012**

[To achieve full marks, working and reasoning should be shown.]
[A maximum of 2 marks will be deducted for incorrect rounding, units, notation, etc.]

This is *Resource Free – 40 minutes for 36 marks*:

1. [2, 2, 2 = 6 marks]

   Determine $\frac{dy}{dx}$ for each of the following

   a) $y = (e^{2x} + 1)^3$

   $$\frac{dy}{dx} = 3 \times 2e^{2x}(e^{2x} + 1)^2 \quad \checkmark$$

   $$= 6e^{2x}(e^{2x} + 1)^2 \quad \checkmark$$

   b) $y = \frac{3x - 1}{x^2 + 1}$

   $$\frac{dy}{dx} = \frac{3(x^2 + 1) - 2x(3x - 1)}{(x^2 + 1)^2} \quad \checkmark$$

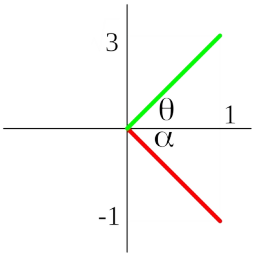
   $$= \frac{3 + 2x - 3x^2}{(x^2 + 1)^2} \quad \checkmark$$

   c) $y = ln[x^2(x + 1)]$

   $$y = 2 \ln x + \ln(x + 1) \quad \checkmark$$

   $$\frac{dy}{dx} = \frac{2}{x} + \frac{1}{x + 1} \quad \checkmark$$

2. [4 marks]
Express (1, -1) and (1, $\sqrt{3}$) into ***exact*** polar form for $-\pi < \theta \leq \pi$.



$r_1 = \sqrt{1^2 + 1^2} = \sqrt{2}$ ✓

$\alpha = \tan^{-1} 1 = \frac{\pi}{4} \Rightarrow \theta_1 = -\frac{\pi}{4}$ ✓

$\therefore (1, -1) = [\sqrt{2}, -\frac{\pi}{4}]$ -1 overall if omit this line or not [].

$r_2 = \sqrt{(\sqrt{3})^2 + 1^2} = 2$ ✓

$\theta = \tan^{-1} \sqrt{3} = \frac{\pi}{3}$ ✓

$\therefore (1, \sqrt{3}) = [2, \frac{\pi}{3}]$

3. [2 marks]
Find the **exact** distance between the points A [6, 25^0] and B [10, 145^0].

$AB = \sqrt{6^2 + 10^2 - 2(6)(10)\cos(-120^\circ)}$ ✓

$= \sqrt{136 - 120 \times (-0.5)}$

$= \sqrt{196}$

$= 14$ units ✓

4. [3 marks]
Find the polar equation and the Cartesian equation of a circle of centre (0,0) and radius 3.

Polar equation: $r = 3$ ✓

Cartesian equation: $\sqrt{x^2 + y^2} = 3$ ✓

$\Rightarrow x^2 + y^2 = 9$ ✓

5. [1, 2, 2 = 5 marks]
Given $z = 3 - 3i$, calculate:

a) $\bar{z} = 3 + 3i$ ✓

b) $z^2 = (3 - 3i)(3 - 3i)$
$= 9 - 9i - 9i - 9$
$= -18i$ ✓

c) $z \times \bar{z} = (3 - 3i)(3 + 3i)$ ✓
$= 9 + 9i - 9i + 9$
$= 18$ ✓

6. [1, 1, 1 = 3 marks]
For each of the following, express p in terms of q.

a) $q^4 = \frac{p^3}{8}$
$p^3 = 8q^4$
$p = 8q^{\frac{4}{3}}$ ✓

b) $\log_e p = 2 \log_e q$
$\log_e p = \log_e q^2$
$p = q^2$ ✓

c) $\frac{e^{2p}}{3} = q$
$e^{2p} = 3q$
$2p = \ln 3q$
$p = \frac{\ln 3q}{2}$ ✓

7. [3 marks]
The Cartesian equation of a circle is $x^2 + y^2 = 10$. Find the polar equation of this circle.

$x^2 + y^2 = 10 \Rightarrow$ circle centre (0,0) and radius $\sqrt{10}$ ✓✓
$\therefore$ Polar equation is $r = \sqrt{10}$. ✓

8. [1, 4, 2 = 7 marks]

If $z = \text{cis}\,\frac{\pi}{4}$ and $w = \text{cis}\,\frac{\pi}{6}$,

a) express $\frac{z}{w}$ in polar form,

$\frac{z}{w} = \text{cis}\left(\frac{\pi}{12}\right)$ ✓ -1 overall if given as co-ordinates in [].

b) express z, w and $\frac{z}{w}$ in Cartesian form, and

$z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ ✓ or $z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$\mathbf{w} = \frac{\sqrt{3}}{2} + \frac{1}{2}\mathbf{i}$ ✓

$\frac{z}{w} = \frac{\sqrt{2} + \sqrt{2}\,i}{\sqrt{3} + i}$ ✓✓

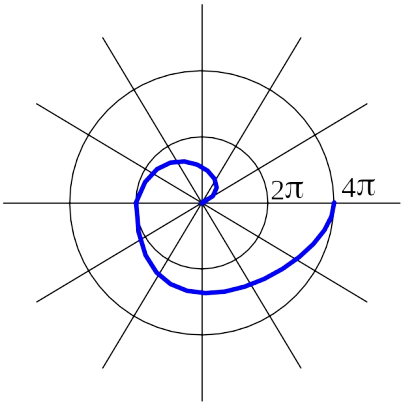
c) give $\frac{z}{w}$ with a rationalised denominator.

$\frac{z}{w} = \frac{\sqrt{2} + \sqrt{2}\,i}{\sqrt{3} + i} \times \frac{\sqrt{3} - i}{\sqrt{3} - i}$ ✓

$= \frac{(\sqrt{6} + \sqrt{2}) + (\sqrt{6} - \sqrt{2})\mathbf{i}}{4}$ ✓

9. [3 marks]

Find the polar equation of this curve.



Equation is in the form $r = k\theta$.

First x-intercept is $[2\pi,\pi]$. ✓

$\Rightarrow k = 2$ ✓

$\therefore$ Polar equation is $r = 2\theta$. ✓