WAEP Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section One: Calculator-free

SOLUTIONS

WA student number: In figures | | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

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Supplementary page

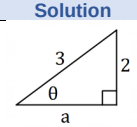
Question number: _____

Question 8 **(8 marks)**

(a) Determine an exact value for $\cos 103° \cos 58° + \sin 103° \sin 58°$. (2 marks)

Solution
$\cos 103° \cos 58° + \sin 103° \sin 58° = \cos(103° - 58°)$ $= \cos 45°$ $= \frac{\sqrt{2}}{2}$
Specific behaviours
✓ uses double angle formula ✓ states exact value

(b) Determine all possible values of $\tan\theta$ when $\sin\theta = \frac{2}{3}$. (3 marks)

Solution
 $a = \sqrt{9-4} = \sqrt{5}$ Note that θ can be in quadrant 1 or 2. In Q1: $\tan\theta = \frac{2}{\sqrt{5}}$, in Q2 $\tan\theta = -\frac{2}{\sqrt{5}}$
Specific behaviours
✓ relevant use of right triangle ✓ one correct value ✓ both correct values

(c) Determine an exact value for $\sin 75°$. (3 marks)

Solution
$\sin 75° = \sin(30° + 45°)$ $= \sin 30° \cos 45° + \cos 30° \sin 45°$ $= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$ $= \frac{\sqrt{2} + \sqrt{6}}{4}$
Specific behaviours
✓ uses double angle formula ✓ uses correct exact values ✓ simplifies to obtain final value

End of questions

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Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

The point $M(-2, 5)$ is the midpoint of point $A(-6, 3)$ and point B.

(a) Determine the coordinates of point B. (2 marks)

Solution
$\frac{-6 + x}{2} = -2 \Rightarrow x = 2$ $\frac{3 + y}{2} = 5 \Rightarrow y = 7$ $B(2, 7)$
Specific behaviours
✓ correct x-coordinate ✓ correct y-coordinate

(b) Determine the equation of the straight line that passes through point $C(4, -1)$ and is perpendicular to the line through points A and M. (3 marks)

Solution
$m_{AM} = \frac{5 - 3}{-2 - (-6)} = \frac{1}{2}$ $m_{\perp} = -1 \div \frac{1}{2} = -2$ $y - (-1) = -2(x - 4)$ $y = -2x + 7$
Specific behaviours
✓ gradient of AM ✓ perpendicular gradient ✓ correct equation

See next page

Question 2 **(4 marks)**

The expansion of $(x+1)^{11}$ is

$$x^{11}+11x^{10}+55x^9+165x^8+330x^7+462x^6+462x^5+330x^4+165x^3+55x^2+11x+1.$$

(a) Determine the number of combinations of 7 objects taken from a set of 11 distinct objects. (1 mark)

Solution
x^4 (or x^7) coefficient: $\binom{11}{7}=330$
Specific behaviours
✓ correct number

(b) Consider the simplified expansion of $(x+1)^{12}$. The first four terms in descending powers of x are

$$x^{12}+px^{11}+qx^{10}+rx^9.$$

(i) State the number of terms in the complete simplified expansion. (1 mark)

Solution
13 terms.
Specific behaviours
✓ correct number

(ii) Determine the value of each of the coefficients p, q and r. (2 marks)

Solution
Using coefficients from expansion of $(x+1)^{11}$ and properties of Pascal's triangle: $p=1+11=12$ $q=11+55=66$ $r=55+165=220$
Specific behaviours
✓ at least 2 correct ✓ all three correct

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Question 7 **(8 marks)**

Solve the following equations for x.

(a) $x^2+20x-21=0$. (2 marks)

Solution
$x^2+20x-21=(x-1)(x+21)$ Hence $x=1,\ x=-21$.
Specific behaviours
✓ factorises ✓ states both solutions

(b) $(x-1)^2-4=2x-3$. (3 marks)

Solution
$x^2-2x+1-4=2x-3$ $x^2-4x=0$ $x(x-4)=0$ $x=0,\quad x=4$
Specific behaviours
✓ expands and equates to zero ✓ factorises ✓ states both solutions

(c) $x^3-2x^2-11x+12=0$. (3 marks)

Solution
$x^3-2x^2-11x+12\big\vert_{x=1}=13-13=0$ $x^3-2x^2-11x+12=(x-1)(x^2-x-12)$ $=(x-1)(x+3)(x-4)$ $x=-3,\quad x=1,\quad x=4$
Specific behaviours
✓ indicates linear factor ✓ factorises ✓ states all solutions

Question 6 **(7 marks)**

(a) The variable V is inversely proportional to the variable t, so that when $t = 3.6$, $V = 10$.

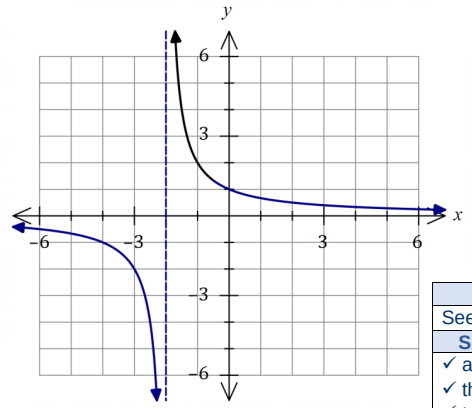
(i) Explain how V will change as t increases. (1 mark)

Solution
As t increases V will decrease.
Specific behaviours
✓ correct explanation

(ii) Determine t when $V = 3$. (2 marks)

Solution
$V \times t = k \Rightarrow k = 3.6 \times 10 = 36$ $3t = 36 \Rightarrow t = 12$
Specific behaviours
✓ indicates appropriate method
✓ correct value

(b) Part of the graph of $y = \dfrac{a}{x+2}$ is drawn below.



Solution (b)(ii)
See graph
Specific behaviours
✓ asymptotes
✓ thru' $(0,1), (-4,-1)$
✓ two smooth curves

(i) Determine the value of a. (1 mark)

Solution
$(-1,2) \Rightarrow 2 = \dfrac{a}{-1+2} \Rightarrow a = 2$
Specific behaviours
✓ correct value

(ii) Draw the remainder of the graph. (3 marks)

Question 3 **(5 marks)**

Functions f and g are defined by $f(x) = 4x^2 - 4x + 5$ and $g(x) = 2x^2 - 8x + 6$.

(a) Determine the discriminant of f and the discriminant of g. (2 marks)

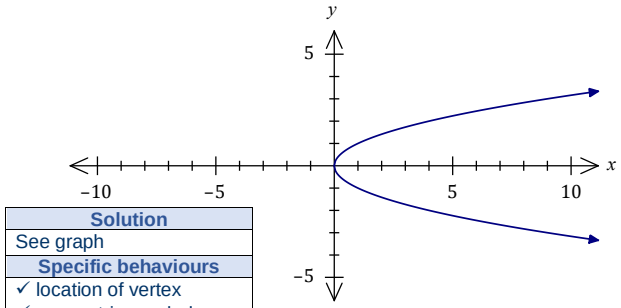
Solution
$\Delta_f = (-4)^2 - 4(4)(5) = -64$ $\Delta_g = (-8)^2 - 4(2)(6) = 16$
Specific behaviours
✓ discriminant of f
✓ discriminant of g

(b) State, with justification, which function has no zeros and determine all zeros of the other function. (3 marks)

Solution
f has no zeroes as $\Delta_f < 0$. $g(x) = 0$ when $x = \dfrac{-(-8) \pm \sqrt{16}}{2(2)}$ $x = 2 \pm 1$ g has zeros when $x = 1, x = 3$.
Specific behaviours
✓ states f has no zeroes
✓ indicates appropriate method to find zeros
✓ both zeros of g

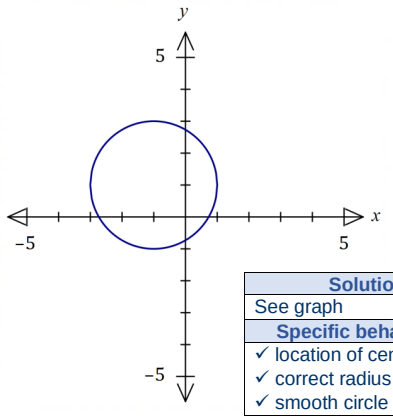
Question 4 **(7 marks)**

(a) Sketch the graph of $y^2 = x$ on the axes below. (2 marks)



Solution
See graph
Specific behaviours
✓ location of vertex
✓ symmetric parabola

(b) Sketch the graph of $(x+1)^2 + (y-1)^2 = 4$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ location of centre
✓ correct radius
✓ smooth circle

(c) Explain whether y is a function of x in the relationship graphed in (a). (2 marks)

Solution
y is NOT a function of x. This is because the graph of the relationship fails the vertical line test.
Specific behaviours
✓ states not a function
✓ explanation (VLT, one-to-many, etc)

Question 5 **(8 marks)**

(a) A periodic function is defined by $f(x) = 2 - 2\sin(3x)$.

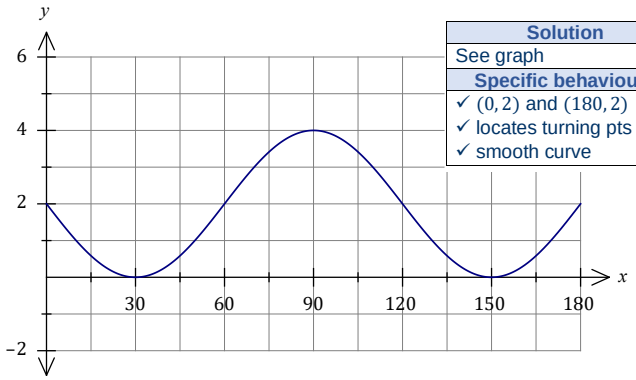
(i) State the amplitude of the function. (1 mark)

Solution
Amplitude is 2.
Specific behaviours
✓ correct amplitude

(ii) State the period of the function in degrees. (1 mark)

Solution
Period is $360 \div 3 = 120°$.
Specific behaviours
✓ correct period

(iii) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ $(0, 2)$ and $(180, 2)$
✓ locates turning pts
✓ smooth curve

(b) Solve the equation $2\cos(x - 15°) = \sqrt{3}$ where $0 \leq x \leq 360°$. (3 marks)

Solution
$\cos(x - 15°) = \frac{\sqrt{3}}{2}$
$x - 15° = 30°, 330°$
$x = 45°, 345°$
Specific behaviours
✓ determines an angle for $\frac{\sqrt{3}}{2}$
✓ determines one solution
✓ both correct solutions