PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Year 12 Test Two 2022

Student name: _______________ Teacher name: _______________

Task type: **Response**

Time allowed for this task: _____40______ mins

Number of questions: _____6______

Materials required: Upto 3 Calculators with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: __41____ marks

Task weighting: _10___%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2, 3 & 3= 8 marks)

Consider the functions $f(x)=\sqrt{x-2}$ and $g(x)=\frac{1}{x}$

a) Determine the natural domains of $f \,\&\, g$.

Solution
$d_f : x \geq 2$ $d_g : x \neq 0$
Specific behaviours
✓ domain of f ✓ domain of g

b) Does $f \circ g(x)$ exist over the natural domain of g ? Explain.

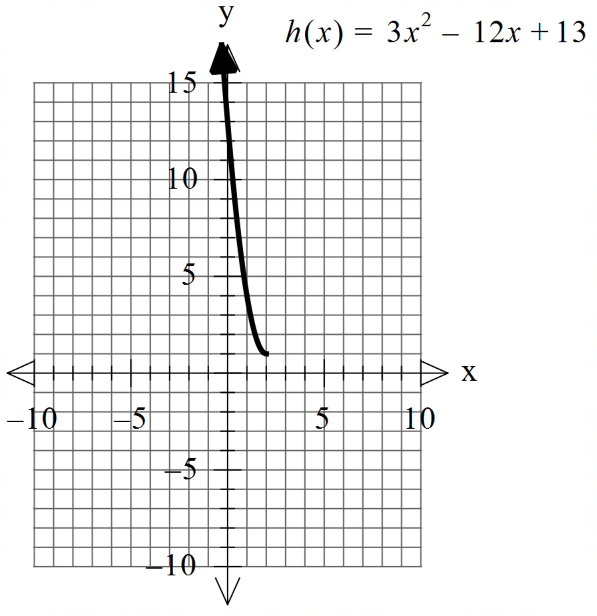
Solution
$r_g : y \neq 0$ $d_f : x \geq 2$ $not\ exist\ r_g \not\subset d_f$
Specific behaviours
✓ states range of g ✓ states condition necessary to exist ✓ shows that does not exist with actual subsets Note: zero marks for not exist with no reasoning

c) State the rule and largest possible domain for $g \circ f(x)$ and its corresponding range.

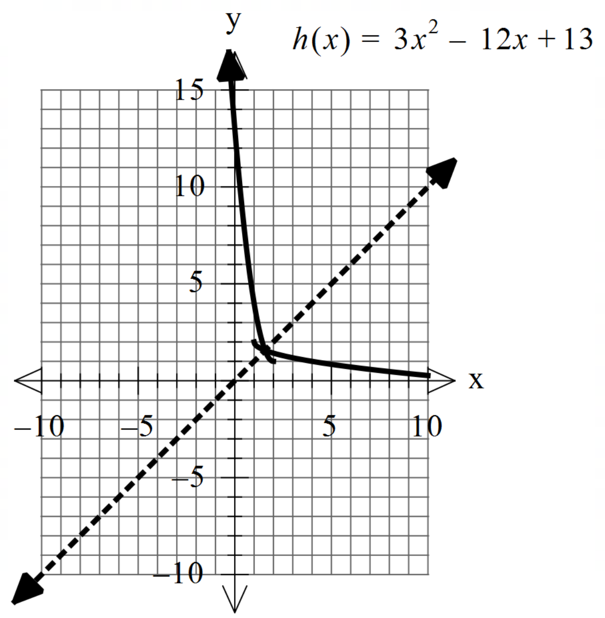
Solution
$g \circ f(x)=\frac{1}{\sqrt{x-2}}$ $d : x > 2$ $r : y > 0$
Specific behaviours
✓ states rule ✓ states largest domain ✓ states range

Q2 (2, 4, 1 & 3 = 10 marks)

The function $h(x)$ is defined below for $x \le 2$.



a) Sketch the inverse function $h^{-1}(x)$ on the axes above.

Solution

Specific behaviours
✓ endpoint (1,2) ✓ appears to be reflected in y=x, no need for dotted line

Q2 continued

b) Determine the rule for $h^{-1}(x)$ and its domain showing **full working**.

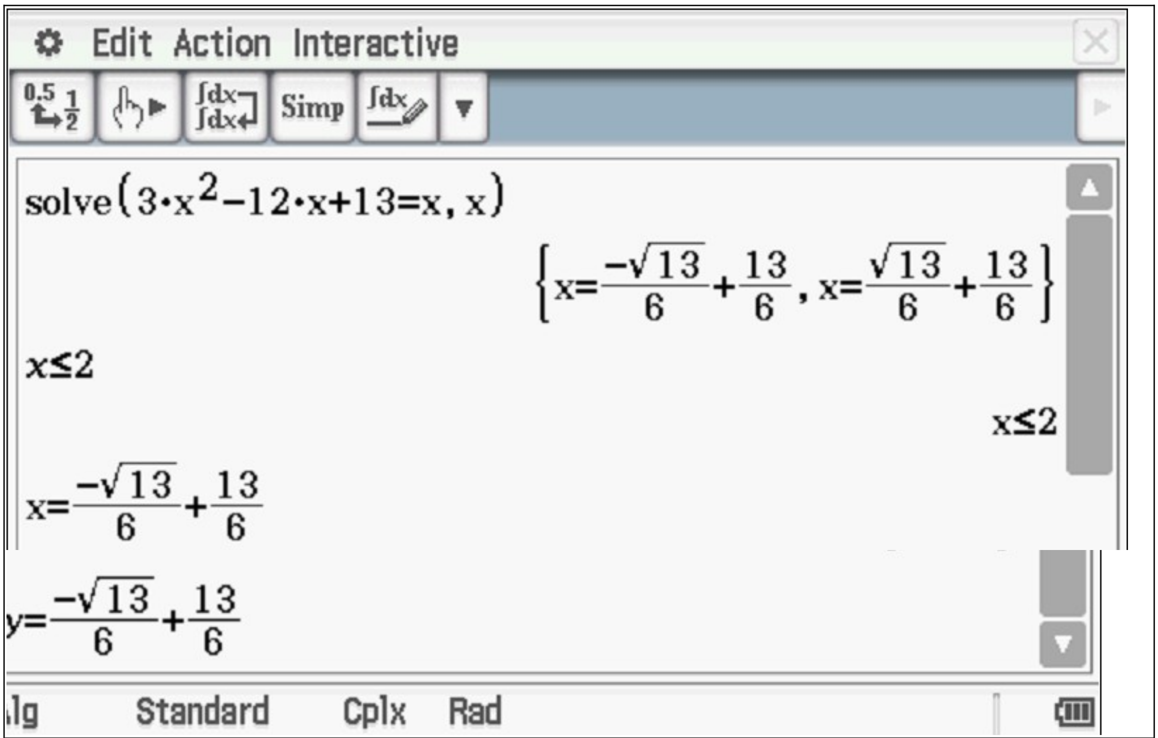
Solution
$d_h : x \le 2$ $r_h : y \ge 1$ $d_{h^{-1}} : x \ge 1$ $r_{h^{-1}} : y \le 2$ $x = 3y^2 - 12y + 13$ $0 = 3y^2 - 12y + 13 - x$ $y = \frac{12 \pm \sqrt{144 - 4(3)(13 - x)}}{6} = \frac{12 \pm \sqrt{12(x-1)}}{6} = 2 \pm \frac{\sqrt{3(x-1)}}{3}$ $h^{-1}(x) = 2 - \frac{\sqrt{3(x-1)}}{3}$
Specific behaviours
✓ states domain of inverse ✓ shows x & y interchanged or solving for x in original function ✓ shows two possibilities for rule ✓ discards the positive root Note : max 2 out of 4 if no working

c) Determine $h \circ h^{-1}(x)$.

Solution
$h \circ h^{-1}(x) = x$
Specific behaviours
✓ states x

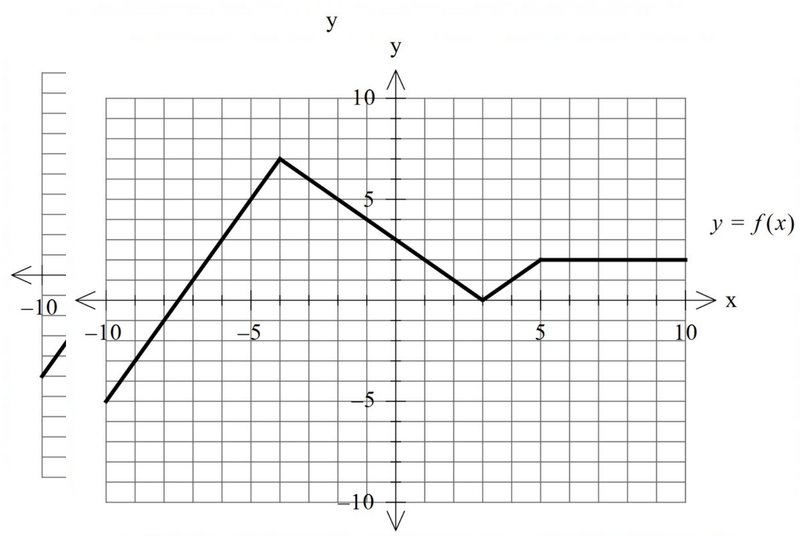
d) Determine the exact coordinates (if any) for where $h(x) = h^{-1}(x)$.

Solution

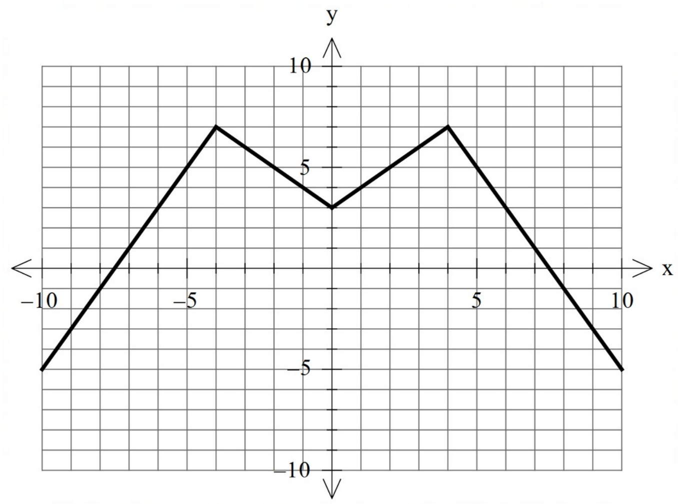
Specific behaviours

✓ equates to x ✓ solves for two x values ✓ discards larger and give y coordinate

Q3 (2 & 3 = 5 marks)

Consider the function $y = f(x)$ which is plotted below.

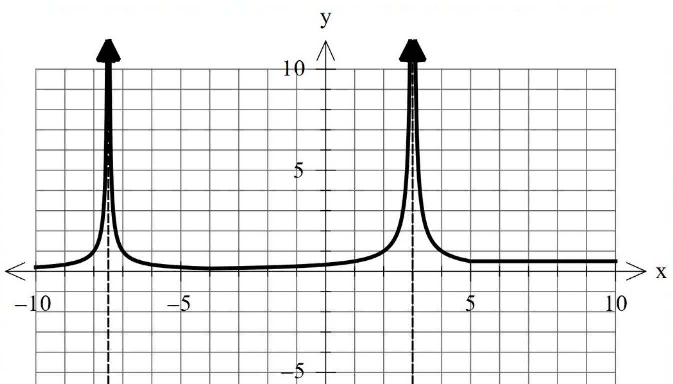


a) Sketch $y = f(-|x|)$



Solution
Specific behaviours
✓ reflects left side ✓ x & y intercepts accurate

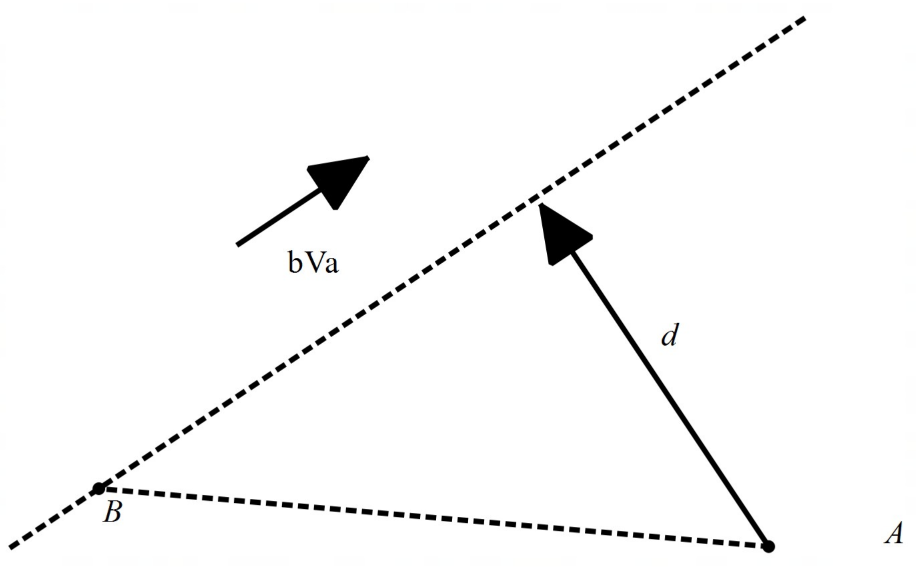
b) Sketch $y = \dfrac{1}{|f(x)|}$

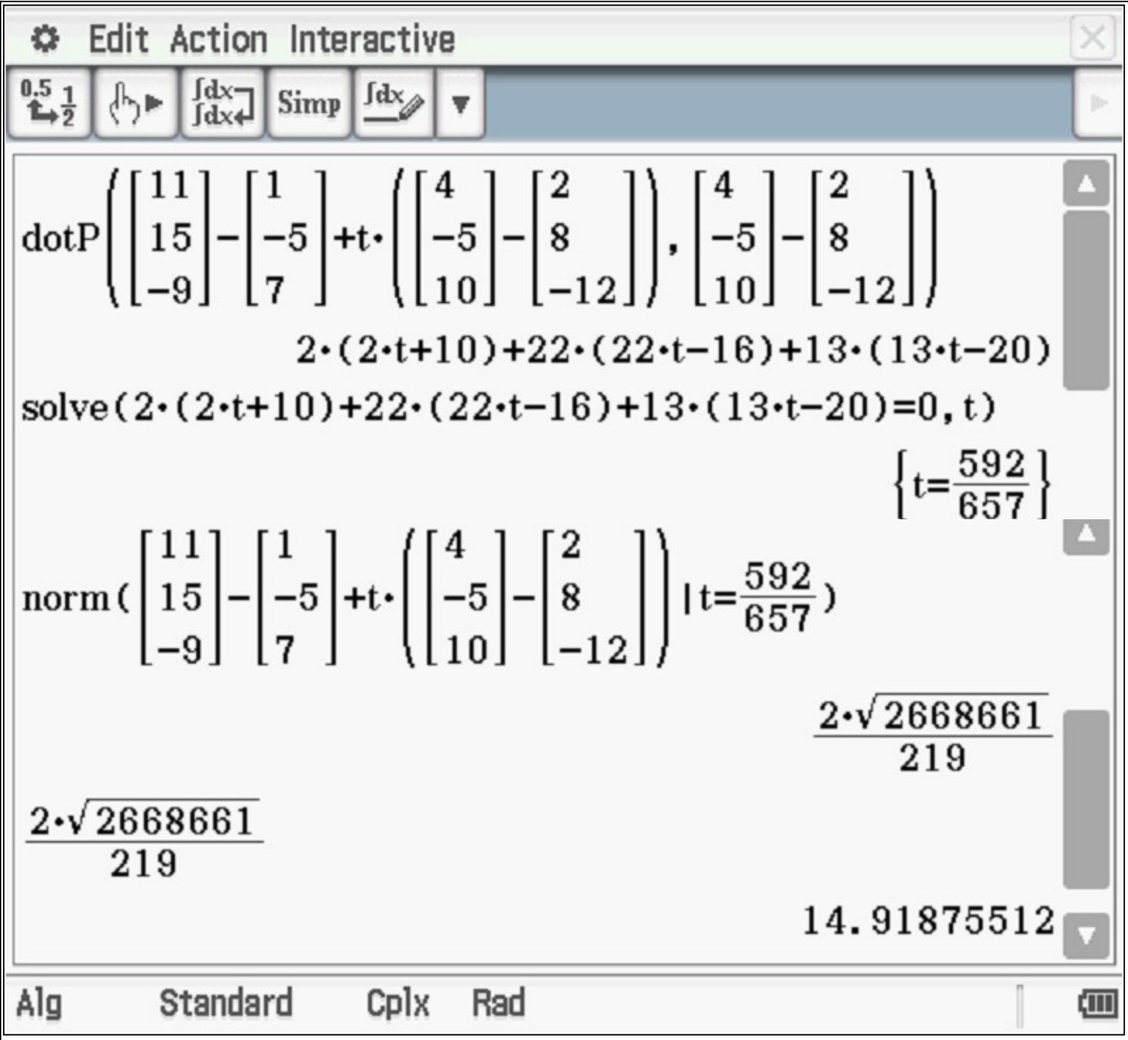


Solution
Specific behaviours
✓ two vertical asymptotes at correct positions ✓ shape correct between asymptotes ✓ y=0.5 for x>5

Q4 (4 marks)

Consider two moving objects A & B such that at $t=0$ seconds $r_A = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix}, r_B = \begin{pmatrix} 11 \\ 15 \\ -9 \end{pmatrix}$ metres and $v_A = \begin{pmatrix} 2 \\ 8 \\ -12 \end{pmatrix}, v_B = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix}$ metres per second. Determine the closet approach using **vector** methods.

Solution
 $d = AB + t\,V_{BrelA}$ $d.V_{BrelA} = 0$



Specific behaviours

✓ uses dot product
✓ solves for time of closest approach
✓ determines separation vector d
✓ determines closest approach (approx.)

Q5 (6 marks)

Consider a sphere $\left| r - \begin{pmatrix} 1 \\ -5 \\ \alpha \end{pmatrix} \right| = 7$ with α a constant and the line $r = \begin{pmatrix} 4 \\ -9 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}$.

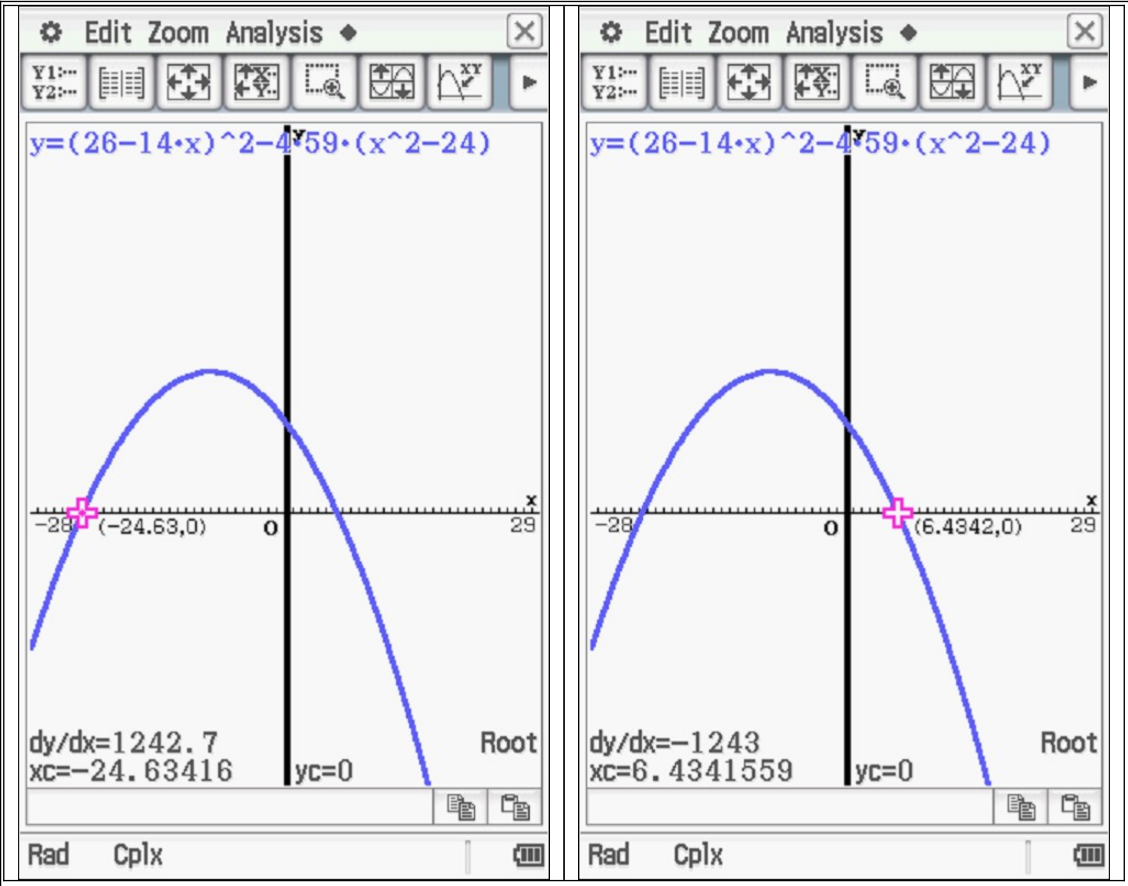
Determine all possible real values of α such that:

(i) the line meets the sphere at two pints.

(ii) the line is a tangent to the sphere.

(iii) the line misses the sphere completely.

Solution
$\left\| \begin{pmatrix} 4+3\lambda \\ -9-\lambda \\ 7\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ -5 \\ \alpha \end{pmatrix} \right\| = 7$ $\left\| \begin{pmatrix} 3+3\lambda \\ -4-\lambda \\ 7\lambda-\alpha \end{pmatrix} \right\| = 7$ $(3+3\lambda)^2 + (-4-\lambda)^2 + (7\lambda-\alpha)^2 = 49$ $9+18\lambda+9\lambda^2+16+8\lambda+\lambda^2+49\lambda^2-14\lambda\alpha+\alpha^2 = 49$ $59\lambda^2+(26-14\alpha)\lambda+\alpha^2-24=0$ $\Delta = (26-14\alpha)^2 - 4(59)(\alpha^2-24)$



i) $-24.63 < \beta < 6.43$

ii) $\alpha = -24.63,\ 6.43$

iii) $\alpha < -24.63, \alpha > 6.43$

Specific behaviours
✓ sets up an equation with both unknows ✓ sets up a quadratic equation ✓ obtains expression for discriminant ✓ graphs discriminant or solves equalling zero ✓ solves for tangent ✓ solves for all 3 scenarios

Q6 (2, 3 & 3 = 8 marks)

Consider the plane Ω given by $2x - 3y + 5z = 11$.

a) The point $A(5,-8,3)$ is on a plane parallel to Ω. Determine the cartesian equation of this plane.

Solution

$$r.\begin{pmatrix}2\\-3\\5\end{pmatrix}=\begin{pmatrix}5\\-8\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\5\end{pmatrix}=49$$

$$2x-3y+5z=49$$

Specific behaviours

✓ identifies normal
✓ states cartesian

b) Determine the distance between these two planes. Show full reasoning.

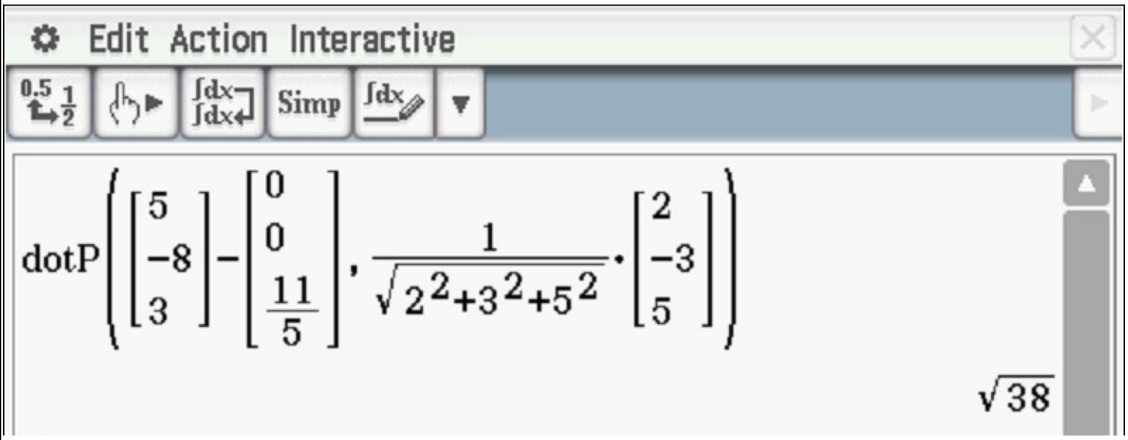
Solution

$$2x-3y+5z=11$$

$$Let\ x=0,\ y=0\Rightarrow\ z=\frac{11}{5}\ ptB$$

$$\hat{n}=\frac{1}{\sqrt{2^2+3^2+5^2}}\begin{pmatrix}2\\-3\\5\end{pmatrix}$$

$$dis\tan ce=\left|\overrightarrow{AB}.\hat{n}\right|$$



Specific behaviours

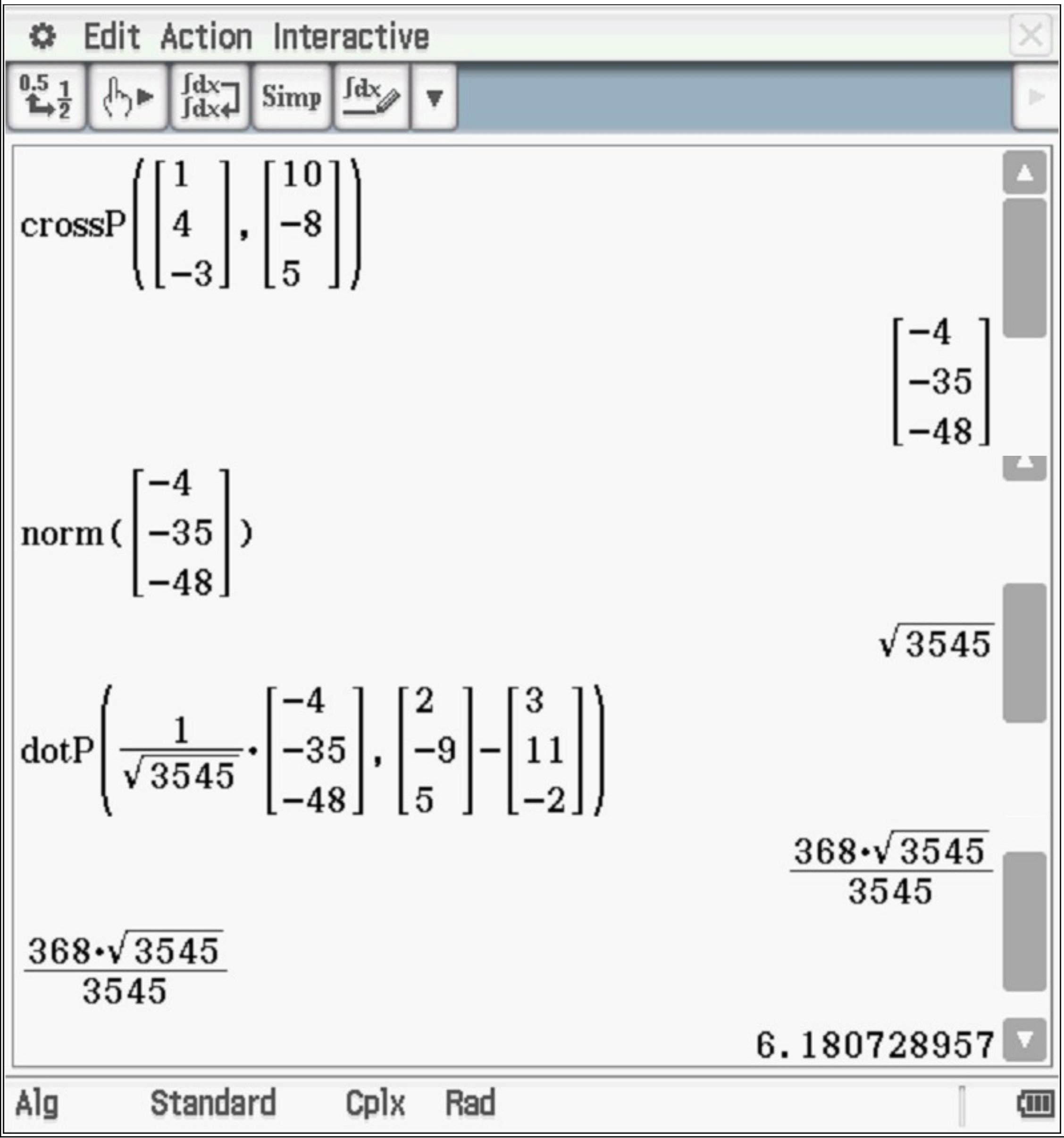
✓ uses dot product
✓ sets up expression for distance or subs line into vector plane equation

✓ solves for distance, accept approx. Note- formula used with derivation max 1 out of 3

c) Consider the lines $r_A = \begin{pmatrix} 2 \\ -9 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$ and $r_B = \begin{pmatrix} 3 \\ 11 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ -8 \\ 5 \end{pmatrix}$. Determine the distance between these lines.

Solution

$AB.\hat{n}$



Specific behaviours
✓ determines normal vector to both planes ✓ uses dot product with normal ✓ determines approx. distance Note: zero marks if closest approach method is used

Extra working space