

Western Australian Certificate of Education
ATAR course examination, 2016
Question/Answer booklet

MATHEMATICS
METHODS

Section One:
Calculator-free

Student number: In figures

In words

Time allowed for this section

Reading time before commencing work: fifty minutes

Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters
Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised material. If you have any unauthorised material with you, hand
it to the supervisor **before** reading any further.



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	49	35
Section Two: Calculator-assumed	13	13	100	101	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Additional working space

Question number: _____

DO NOT WRITE IN THIS AREAS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free 35% (49 Marks)

This section has eight (8) questions. Answer all questions. Write your answers in the spaces provided.

Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

Working time: 50 minutes.

Question 1

- (a) Given that $\log_8 x = 2$ and $\log_2 y = 5$, evaluate $x - y$. (2 marks)

- (b) Express y in terms of x given that $\log_2 (x + y) + 2 = \log_2 (x - 2y)$. (3 marks)

See next page

Question 2

(5 marks)

(a)

Determine $\frac{d}{dx}(2xe^{2x})$.

(2 marks)

(b)

Use your answer in part (a) to determine $\int 4xe^{2x}dx$.

(3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

(b)

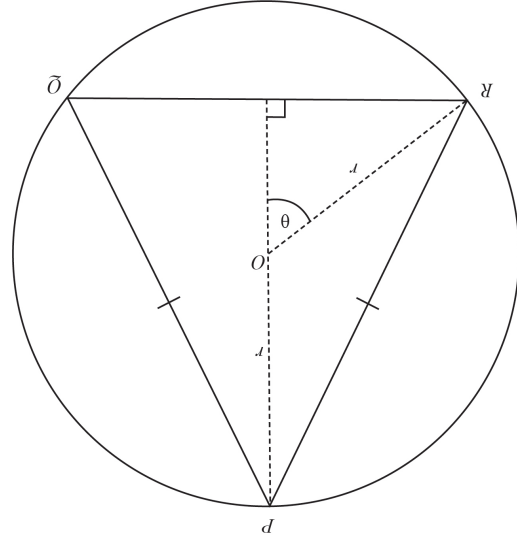
Using calculus, determine the value of θ that maximises the area A of the inscribed triangle. State this area in terms of r exactly. Justify your answer.
(Hint: you may need the identity $\sin^2x = 1 - \cos^2x$ in your working.)

(5 marks)

Question 8

(7 marks)

An isosceles triangle $\Delta P\tilde{Q}R$ is inscribed inside a circle of fixed radius r and centre O . Let θ be defined as in the diagram below.



- (a) Show that the area A of the triangle $\Delta P\tilde{Q}R$ is given by $A = r^2 \sin \theta (1 + \cos \theta)$. (2 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 3

(7 marks)

Consider the function $f(x) = \frac{e^x}{(x-1)^2}$.

- (a) Show that the first derivative is $f'(x) = \frac{-x^2 + 4x - 3}{e^x}$. (2 marks)

- (b) Use your result from part (a) to explain why there are stationary points at $x = 1$ and $x = 3$. (2 marks)

See next page

Question 3 (continued)

It can be shown that the second derivative is $f''(x) = \frac{x^2 - 6x + 7}{e^x}$.

- (c) Use the second derivative to describe the type of stationary points at $x = 1$ and $x = 3$.
(3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

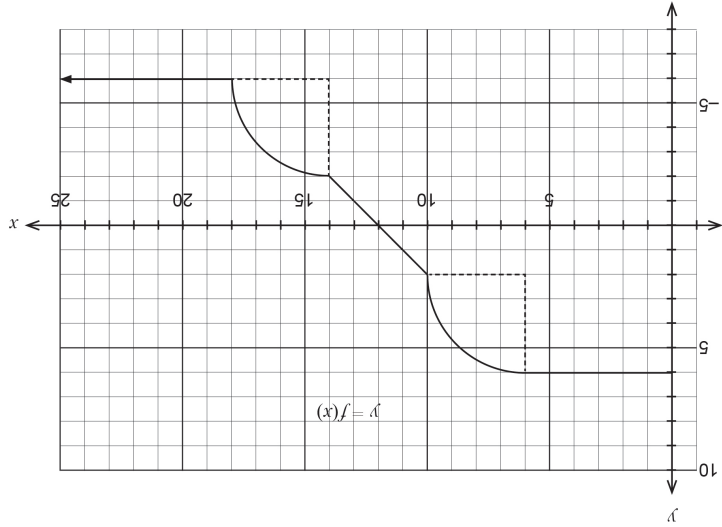
- (b) $\int_0^{18} f(x) \, dx$ (2 marks)

- (c) Determine the value of the constant α such that $\int_0^{\alpha} f(x) \, dx = 0$. There is no need to simplify your answer. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 7

Consider the graph $y = f(x)$. Both arcs have a radius of four units.



Using the graph of $y = f(x)$, $x \geq 0$, evaluate exactly the following integrals.

(a) $\int_{12}^0 f(x) \, dx$

(3 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 4

The displacement x micrometres at time t seconds of a magnetic particle on a long straight superconductor is given by the rule $x = 5 \sin 3t$.

(a) Determine the velocity of the particle when $t = \frac{\pi}{2}$.

(3 marks)

(b) Determine the rate of change of the velocity when $t = \frac{\pi}{2}$.

(3 marks)

(c) Determine $\int_{\frac{3\pi}{2}}^0 \frac{dv}{dt} dt$.

(2 marks)

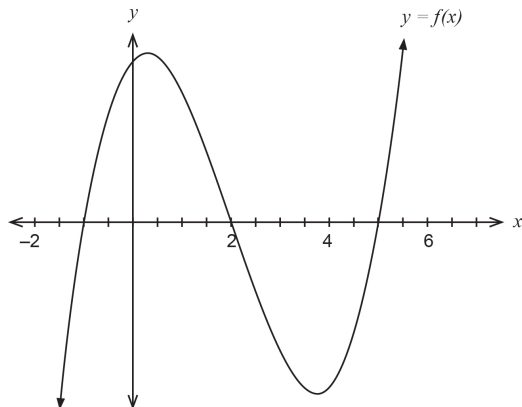
Let v = velocity of the particle at t seconds.

See next page

Question 5

(6 marks)

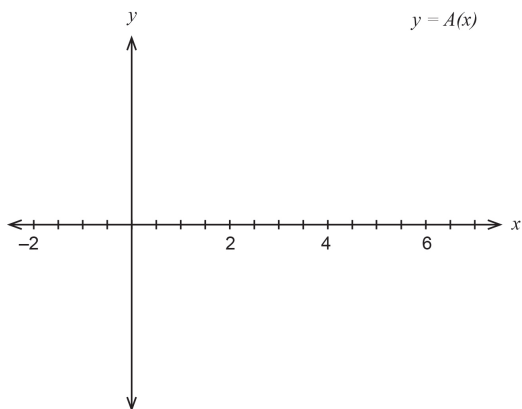
Consider the graph of $y = f(x)$ which is drawn below.



Let $A(x)$ be defined by the integral $A(x) = \int_{-1}^x f(t) dt$ for $-1 \leq x \leq 6$.

It is known that $A(2) = 15$, $A(5) = 0$ and $A(6) = 8$.

Sketch on the axes below the function $A(x)$ for $-1 \leq x \leq 6$ labelling clearly key features such as x intercepts, turning points and inflection points if any.



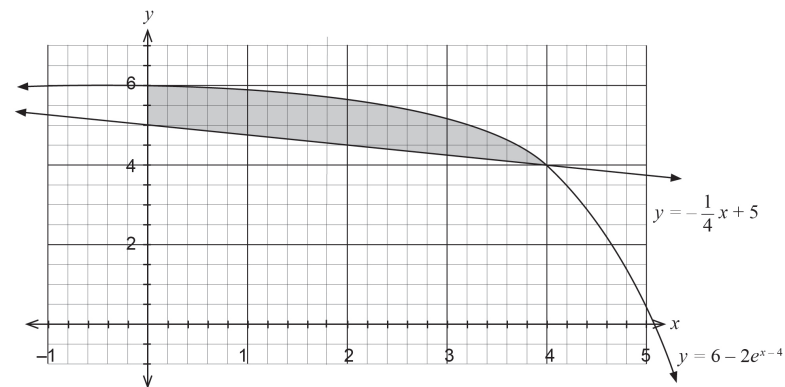
See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 6

(4 marks)

The graphs $y = 6 - 2e^{x-4}$ and $y = -\frac{1}{4}x + 5$ intersect at $x = 4$ for $x \geq 0$.



Determine the exact area between $y = 6 - 2e^{x-4}$, $y = -\frac{1}{4}x + 5$ and the y axis for $x \geq 0$.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page