SHENTON COLLEGE

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

Section One: Calculator-free

Your name:_____

Teacher name (circle one): Ai Friday Smith

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	8	8	50	51
Section Two: Calculator-assumed	13	13	100	96

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	6	
2	8	
3	8	
4	5	
5	4	
6	8	
7	5	
8	7	
S1 Total	51	

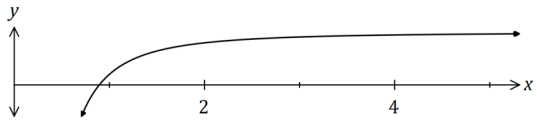
Section One: Calculator-free **(51 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

The curve shown below passes through the point $(1, 2)$ and is such that $\frac{dy}{dx} = \frac{16}{x^3}$.



(a) Determine the equation of the curve. (3 marks)

(b) Determine the area of the region enclosed by the curve, the x-axis, the line $x = 1$ and the line $x = 4$. (3 marks)

Question 2 **(8 marks)**

A calculator program will generate a single random integer n, where $3 \leq n \leq 12$. The program is run once, and the discrete random variable X is the number of fours or fives obtained.

(a) Explain why X is a Bernoulli random variable. (2 marks)

(b) Determine $P(X = 1)$. (1 mark)

(c) Determine the mean and standard deviation of X. (2 marks)

The random variable Y is the number of fours or fives obtained in three consecutive runs of the program.

(d) Determine $P(Y \leq 1)$. (3 marks)

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Question 3 **(8 marks)**

(a) Determine

(i) $\frac{d}{dx}\frac{\cos(4x)}{x^3}$ (2 marks)

(ii) $\frac{d}{dx}\int_x^5 (2\theta+5)^4\ d\theta.$ (2 marks)

(b) Find the function A(t) given that A(t) = $\int_{\frac{1}{2}}^{t} \frac{d}{dx}\ x^2\sqrt{1-x^2}\ dx$. (2 marks)

(c) Determine $\int (8x+11)^3\ dx.$ (2 marks)

Question 4 **(5 marks)**

Let $f(x) = 5x + \frac{k}{2x}$, $x < 0$ and k is a constant. The graph of $y = f(x)$ has a stationary point when $x = -3$.

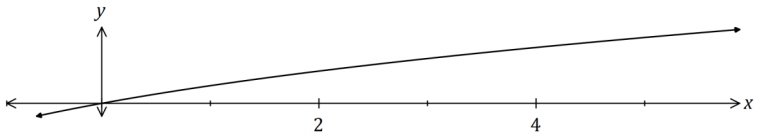
(a) Determine the value of k. (2 marks)

(b) Use the second derivative test to determine the nature of the stationary point. (3 marks)

Question 8 **(7 marks)**

(a) Determine $\frac{d}{dx}\left(2x\sqrt{4+\mathrm{x}}\right)$. (2 marks)

(b) Part of the graph of $y = \frac{x}{\sqrt{4+x}}$ is shown below.



Using your answer from part (a) or otherwise, determine $\int_0^5 \frac{x}{\sqrt{4+x}}\,dx$. (5 marks)

End of Section 1 Questions

Question 5 **(4 marks)**

A random variable X has a binomial probability distribution with a mean of 12 and variance of 3.

(a) Determine the value of n and the value of p for this distribution. (2 marks)

(b) Determine the mean and variance of the distribution Y, where $Y = 5X + 3.3$. (2 marks)

Question 6 **(8 marks)**

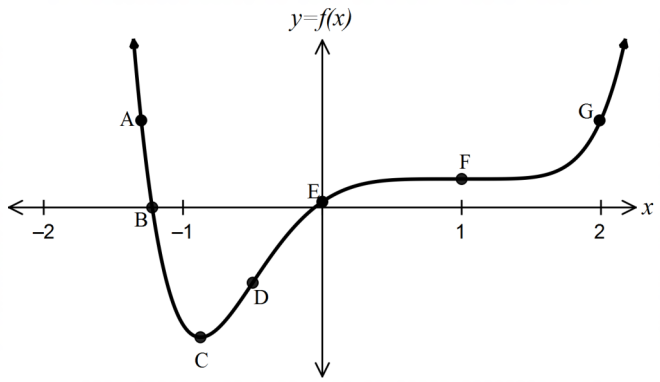
A vehicle travelling in a straight line has a velocity of 10 ms^{-1} as it leaves point Q. The acceleration of the vehicle is given by $3 - 2t$ ms^{-2}, where t is the time in seconds since the vehicle left Q.

(a) Determine the velocity of the vehicle when $t = 4$. (3 marks)

(b) Determine how far from Q the vehicle first comes to rest for $t > 0$. (5 marks)

Question 7 **(5 marks)**

Consider the function $y = f(x)$ shown below. The points, A, B, C, D, E, F and G each lie on the graph.



(a) Which point/s labelled on the graph above satisfy the following,

(i) a point of inflection occurs? (1 mark)

(ii) $f'(x) = 0$ and $f''(x) \neq 0$? (1 mark)

(iii) f is increasing and $f''(x) < 0$? (1 mark)

(iv) $f(x) > 0$ and the function is concave up? (1 mark)

(b) Circle the graph below that represents $f''(x)$. (1 mark)