PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Year 11

Student name: ________________________ Teacher name: _______________

Date: 18 Sep 2020

Task type: **Response**

Time allowed for this task: _____45______ **mins**

Number of questions: _____6______

Materials required: Calculator-Free

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates

Marks available: __45____ **marks**

Task weighting: __10__%

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 (2.2.1, 2.2.2, 2.2.3) (6 marks)

If $A=\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}$, O is the 2×2 zero matrix and I is the 2×2 identity matrix, find

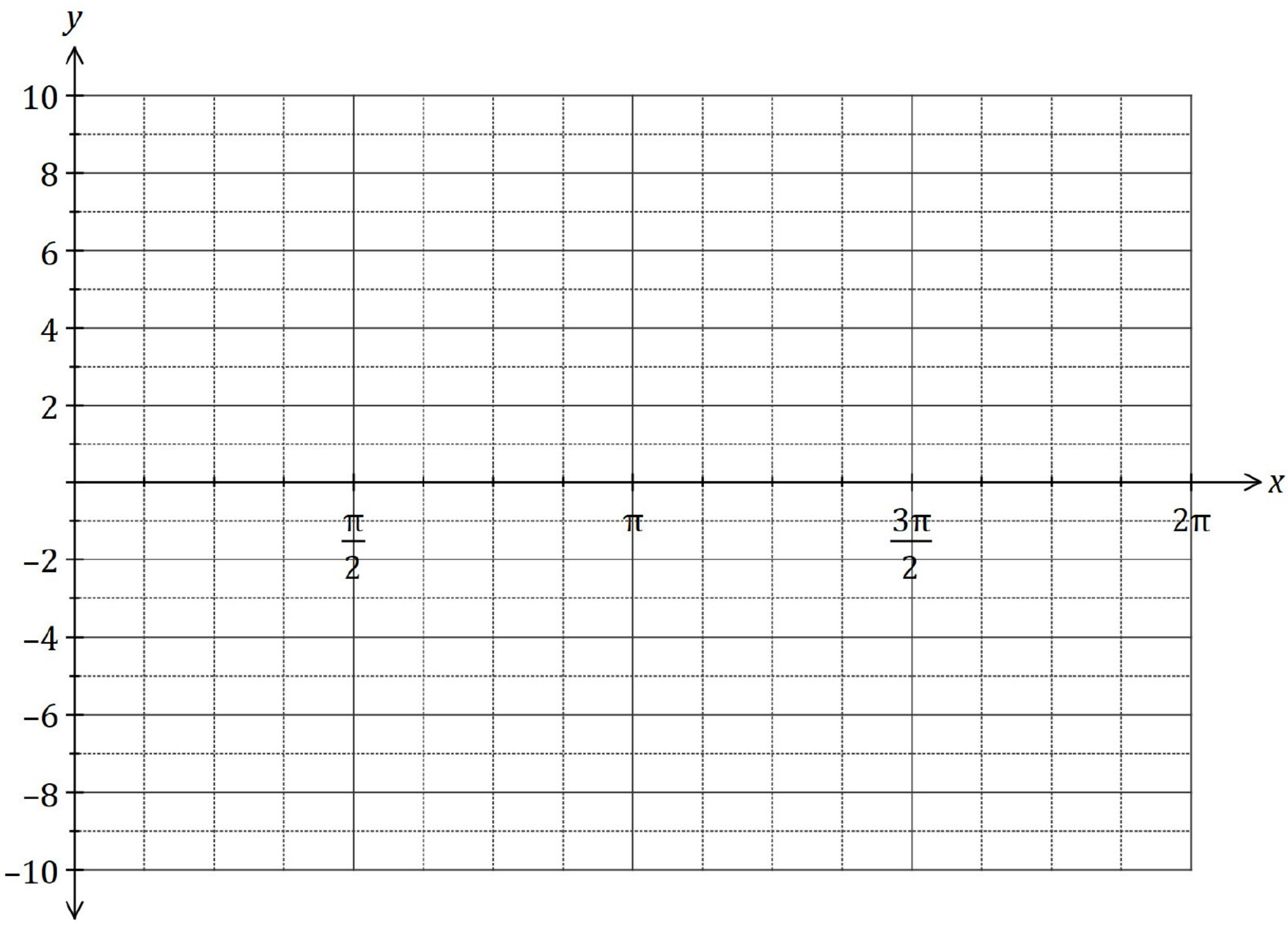
a) Matrix B given that $A-B=I$ (1 mark)

b) Matrix C given that $2A+C=O$ (1 mark)

c) Matrix D given that $D=B-AD$ (4 marks)

Question 2 (2.1.4, 2.1.7) (7 marks)

a) On the axes below, sketch the graph of $y=5\,sec(x-\pi),\,0\le x\le 2\pi$. (3 marks)



b) Find the general solution for $\sqrt{3}\cos(x)-\sin(x)=1$. (4 marks)

Question 3 (2.2.3, 2.1.3) (6 marks)

Let $A=\begin{bmatrix}\cos(\alpha) & \sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{bmatrix}$ and $B=\begin{bmatrix}\cos(\beta)\\ \sin(\beta)\end{bmatrix}$, such that $AB=\begin{bmatrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{bmatrix}$.

Find α and β for $\alpha,\beta\in\left[0,\frac{\pi}{2}\right]$.

Question 4 (2.1.3, 2.1.5) (6 marks)

Prove the following identity:

$$\tan\left(\theta - \frac{\pi}{4}\right) = \frac{\sin(2\theta) - 1}{1 - 2\sin^2\theta}$$

Question 5 (2.2.11) (9 marks)

If $A=\begin{bmatrix} 4 & 1 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $B=\begin{bmatrix} -2 & 2 & 4 \\ 2 & -2 & -2 \\ 8 & -6 & -14 \end{bmatrix}$

a) Determine AB. (2 marks)

b) Express A^{-1} in terms of B. (3 marks)

c) Solve the system $\begin{cases} 4x+y+z=8 \\ 3x-y+z=4 \\ x+y=3 \end{cases}$, clearly showing your use of A^{-1}. (4 marks)

Question 6 (2.2.4, 2.2.5, 2.2.6, 2.2.7, 2.2.8, 2.2.9, 2.2.10) (11 marks)

a) Determine the matrices that produce each of the transformations described below:

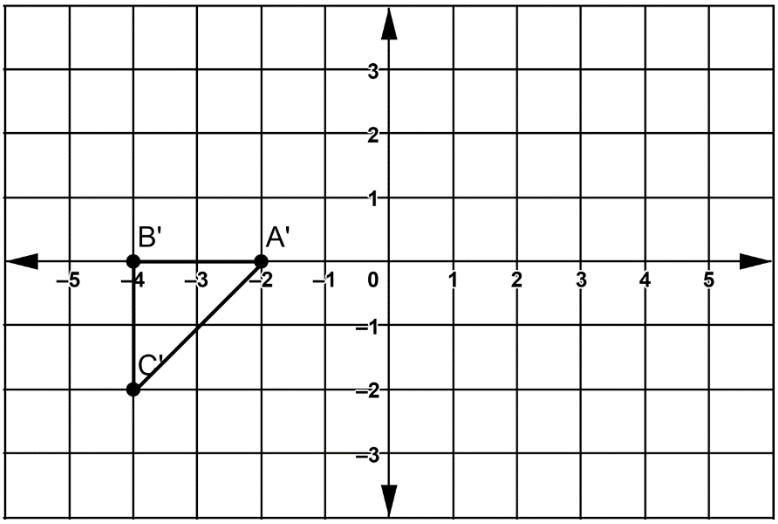
i. a rotation clockwise about the origin by $90°$ (1 mark)

ii. a dilation parallel to the y-axis by a scale factor of 2 (1 mark)

iii. a reflection in the line $y=x$ (1 mark)

b) Show how to obtain the single transformation matrix T, given that T is the result of applying the transformations given in part a) in the order listed [i.e. a rotation clockwise about the origin by 90°, followed by a dilation parallel to the y-axis by a scale factor of 2, then a reflection in the line $y=x$]. (2 marks)

c) ΔABC is translated left by 1 unit and down by 2 units, then the transformation matrix T in part b) is applied to it. The final image $\Delta A'B'C'$ is shown below:



i. Determine the coordinates of points A, B and C in exact form. (4 marks)

ii. Determine the exact area of $\Delta\, ABC$. (2 marks)