

Insert School Logo

Semester Two Examination 2018 Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 & 4

Section One: Calculator-free

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula Sheet

To be provided by the candidate

Standard items: pens(blue/black preferred), pencils(including coloured), sharpener,
correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	9	9	50	50	35
Section Two Calculator—assumed	11	11	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2018*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section One: Calculator-free**50 marks**

This section has **nine (9)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 50 minutes

Question 1 (7 marks)

Let $p = \log_3 x$.

(a) State the value $\log_x 9x^2$ in terms of p . (3 marks)

(b) Evaluate 3^{2p} if $x = 3$. (2 marks)

(c) Evaluate $\frac{d}{dx} \int_1^{x^2} 3^{2p} dp$ (2 marks)

Question 2 (8 marks)

Petra inflates her beach ball. Over the first six seconds, the volume of the ball is

given by $V = \frac{4\pi}{3}(12t - t^2)^3$.

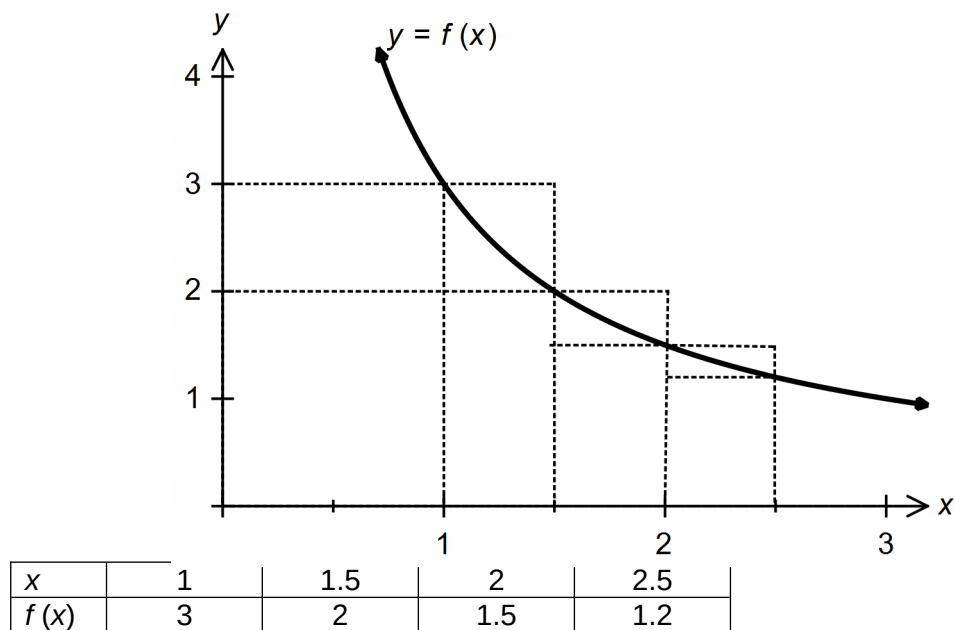
- (a) Use Calculus to determine when the maximum volume is achieved. (4 marks)

- (b) (i) Determine the relationship between the radius of the ball (r) and t . (2 marks)

- (ii) Show that this verifies your answer from (a). (2 marks)

Question 3 (6 marks)

Consider the graph of $f(x)$, and the table of values of $f(x)$ for certain x values.



- (a) By considering the rectangles drawn on the graph, calculate the value of p and q , where:

$$p < \int_1^{2.5} f(x) \, dx < q$$

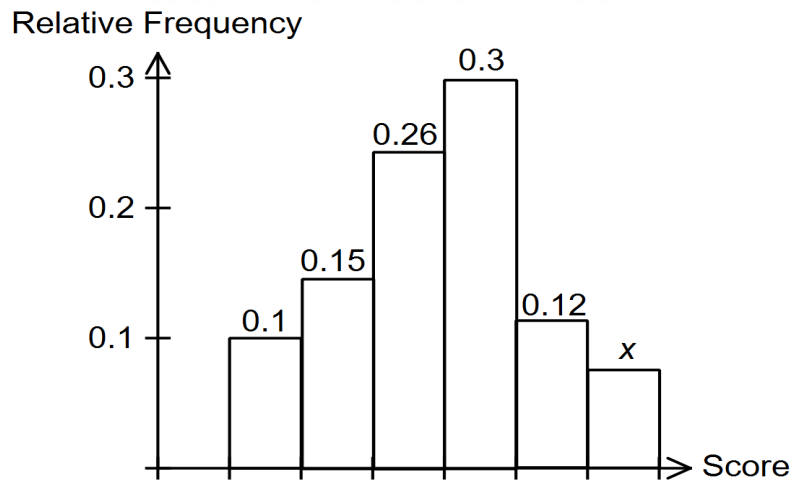
(4 marks)

(b) Given that $f(x) = \frac{3}{x}$ determine exactly, the true values of $\int_1^{2.5} f(x) \, dx$.

(2 marks)

Question 4 (5 marks)

Albert discovered an old die, which wobbled when he rolled it. He rolled it many times and recorded the results. The relative frequencies are shown in the histogram as shown.



- (a) What is the value of x ? (1 mark)
- (b) What is the probability that the next roll would result in a 1 or a 2? (1 mark)
- (c) What is the probability that the next two rolls would both result in a 1 or both in a result of a 2? (2 marks)
- (d) How does this histogram compare with a histogram for an unbiased die? (1 mark)

Question 5 (6 marks)

(a) Differentiate $x \cos 5x$ with respect to x .

(2 marks)

Hence,

(b) show that $\int x \sin 5x \, dx = \frac{\sin 5x}{25} - \frac{x \cos 5x}{5} + c$

(4 marks)

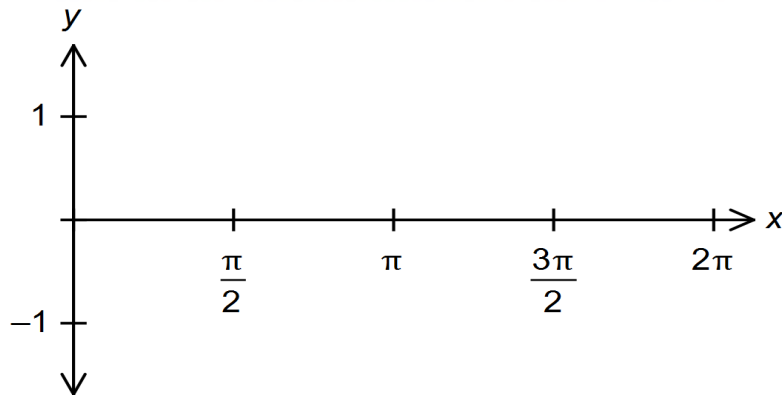
Question 6 (5 marks)

Solve $4e^{2x} - 9 = 9e^x$ given that $\ln 3 = 1.0986$.

(5 marks)

Question 7 (5 marks)

- (a) Sketch the curves $y = \sin x$ and $y = \cos x$ on the axes below, for the domain given. (2 marks)



Hence, or otherwise,

- (b) determine the area enclosed between the curves between $x = \frac{\pi}{2}$ and π . (3 marks)

Question 8 (3 marks)

Two independent samples of different sizes were taken from a population.
The first sample had sample size n_1 , and the second sample had sample size n_2 ,
where $n_2 = 9n_1$.

The sample proportions of adults in the samples were the same.
Confidence intervals were calculated for both samples at the 90% level.

Determine the ratio of the two margins of error. That is $\frac{E_2}{E_1}$. (3 marks)

Question 9 (5 marks)

- (a) Determine $\int \frac{\cos x}{\sin x} dx$. (2 marks)

Hence, or otherwise,

- (b) evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} dx$ exactly. (3 marks)

Additional working space

Question number(s):