MATHEMATICS SPECIALIST 3CD

SEMESTER 1 2010

EPW 2

INTEGRATION BY PARTS INVESTIGATION

TOTAL MARKS: 35 **TIME ALLOWED: 1 HOUR**

Can you determine $\int x\cos x\,dx$?

One possible method would be trial and error. Obviously the appropriate result would have the form

$x \sin x$

Test this by differentiating with respect to x which results in the expression

$1.\sin x + x\cos x$

which looks right but has the extra term $\sin x$.

Try $x\sin x + \cos x$

Testing this by differentiating with respect to x results in the expression

$1.\sin x + x\cos x - \sin x$ or $x\cos x$ as required.

$$\therefore \int x\cos x\,dx = x\sin x + \cos x + c$$

1. [3 marks]

Determine $\int x\sin x\,dx$

An alternative method for finding integrals of this type utilises the product rule.

$$\frac{d}{dx}(uv) = v.\frac{du}{dx} + u.\frac{dv}{dx}$$

Integrating with respect to x gives

$$\int \frac{d}{dx}(uv)dx = \int v.\frac{du}{dx}dx + \int u.\frac{dv}{dx}dx$$

ie $uv = \int v.\frac{du}{dx}dx + \int u.\frac{dv}{dx}dx$ or rearranging

$$\int u.\frac{dv}{dx}dx = uv - \int v.\frac{du}{dx}dx .$$

To use integration by parts to determine $\int x\cos x\,dx$

Let $u = x$ and $\frac{dv}{dx} = \cos x$

then $\frac{du}{dx} = 1$ and $v = \sin x$ (neglecting the constant term)

Using $\int u.\frac{dv}{dx}dx = uv - \int v.\frac{du}{dx}dx$

$$\int x\cos x\,dx = x\sin x - \int \sin x.1\,dx$$
$$= x\sin x - \int \sin x\,dx$$
$$= x\sin x + \cos x + c$$
as before.

2. [4 marks]

Show that the result would not change if we used $v = \sin x + k$ in place of $v = \sin x$.

Generally u is the function which produces a simpler result when differentiated while $\frac{dv}{dx}$ is the more complex part which can still be integrated.

Integrate by parts, each of the following.

3. [4, 4, 4, 4, 4, 8 marks]

(a) $\int x e^x\,dx$

(b) $\int 3x\sin x\,dx$

(c) $\int x\sqrt{2x-1}\,dx$

(d) $\int x^3\ln x\,dx$

(e) $\int 3x(2x+3)^5\,dx$

(f) $\int e^x\cos x\,dx$ using $u = e^x$ and $\frac{dv}{dx} = \cos x$

IMPORTANT RESULTS:

$\frac{d}{dx}e^x = e^x$ $\int e^x\,dx = e^x + c$

$\frac{d}{dx}\ln x = \frac{1}{x}$ $\int \frac{1}{x}dx = \ln|x| + c$