Question One: [2, 2, 2, 1 = 7 marks]   CA

A random variable X is normally distributed with a mean of 40 cm and a standard deviation of 3 cm.

(a)   Calculate the value of x associated with a standardised score of $\dfrac{2}{3}$.

(b)   Determine the value of the 85th percentile.

(c)   Calculate $P(X < 45 \,|\, X > 38)$.

(d)   Determine k for $P(X > k) = 0.8$.

Question Two: [3, 2, 2, 3 = 10 marks]　　　　CA

A new battery in an electric car has a charge that can last on average for 150 km of travel with a standard deviation of 21 km. Testing is underway to evaluate the performance of these batteries.

(a)　Determine the probability that a randomly selected battery:

　　(i)　Can be used for at least 140 km before it needs to be charged.

　　(ii)　Can be used for more than 165 km if it is known that it can be used for at most 180 km.

(b)　The worst performing 3% of batteries will be studied for their deficiencies. What is the maximum distance one of these batteries can be used for?

(c)　In a sample of 250 batteries, how many would you expect to be in the 0.85 quantile?

(d)　To improve the consistency of the battery performance, a team of engineers decide that at most 0.27% of batteries should last less than 100 km. Calculate the value of the new standard deviation for this normal distribution if the mean remains the same.

Question Three: [3, 3, 5 = 11 marks]   **CA**

(a)   Given that $Y \sim N(\mu,16)$, calculate μ given that $P(Y > 25) = 0.89435$.

(b)   Given that $X \sim N(100, \sigma^2)$, calculate σ given that $P(X < 80) = 0.04779$.

(c)   Given that $T \sim N(\mu, \sigma^2)$, calculate μ and σ given that

$$P(X < 15.5 \mid X > 14) = \frac{8186}{9772}$$

Question Four: [3 marks]   CA

In a state spelling competition, results were normally distributed with a mean of 58% and a standard deviation of 14%.

Participants who scored between 85% and 95% received a certificate of distinction. In the state, 103 participants were awarded a certificate of distinction.

How many students participated?

Question Five: [3 marks]   CA

The amount of milk sold in 2L containers is normally distributed with a mean of 1998 mL.

Out of 1000 bottles tested, 248 contained between 1998 mL and 2001 mL.

Calculate the standard deviation of the amount of milk contained in these 2L containers.

Mathematics Methods Unit 4

Question Six: [2, 3, 3, 3 = 11 marks] CA

To apply to be an international flight attendant for QANTAS, your height must be between 163 cm and 183 cm.

Data collected in 2011 indicates that the mean height of adult males in Australia is 175.6 cm with and the mean height of adult females in Australia is 161.8 cm with a standard deviation of 3.8 cm.

(a) Out of a random sample of 2000 Australian adult females applicants, how many would you expect to meet the QANTAS height requirements?

(b) The proportion of adult male Australian applicants who are below the maximum height requirement is 0.96096. Calculate the standard deviation for the height of the applicants.

(c) An applicant is 170 cm tall. Are they more likely to be a tall female or a short male?

(d) QANTAS are interviewing potential applicants, all of whom meet the height requirement. The first three interviewees are female. What is the probability that the first two are less than 168 cm and the third is taller than 165 cm?

Question One: [2, 2, 2, 1 = 7 marks]   CA

A random variable X is normally distributed with a mean of 40 cm and a standard deviation of 3 cm.

(a) Calculate the value of x associated with a standardised score of $\frac{2}{3}$.

$$\frac{2}{3} = \frac{x-40}{3}$$
$$2 = x - 40$$
$$x = 42$$

(b) Determine the value of the 85th percentile.

$$P(X < k) = 0.85$$
$$k = 43.11$$

(c) Calculate $P(X < 45 \,|\, X > 38)$.

$$\frac{P(38 < X < 45)}{P(X > 38)}$$
$$= \frac{0.6997}{0.7475}$$
$$= 0.9361$$

(d) Determine k for $P(X > k) = 0.8$.

$$k = 37.48$$

Question Two: [3, 2, 2, 3 = 10 marks]   CA

A new battery in an electric car has a charge that can last on average for 150 km of travel with a standard deviation of 21 km. Testing is underway to evaluate the performance of these batteries.

(a)   Determine the probability that a randomly selected battery:

   (i)   Can be used for at least 140 km before it needs to be charged.

   $$P(X > 140) = 0.6830 \quad \checkmark$$

   (ii)   Can be used for more than 165 km if it is known that it can be used for at most 180 km.

   $$P(X > 165 \,|\, X < 180) = \frac{P(165 < X < 180)}{P(X < 180)} \quad \checkmark$$

   $$= \frac{0.1610}{0.9234}$$
   $$= 0.1744 \quad \checkmark$$

(b)   The worst performing 3% of batteries will be studied for their deficiencies. What is the maximum distance one of these batteries can be used for?

   $$P(X < k) = 0.03 \quad \checkmark$$
   $$k = 110.5 \; km \quad \checkmark$$

(c)   In a sample of 250 batteries, how many would you expect to be in the 0.85 quantile?

   $$0.85 \times 250 = 212.5 \quad \checkmark$$

   212 or 213 batteries $\quad \checkmark$

(d)   To improve the consistency of the battery performance, a team of engineers decide that at most 0.27% of batteries should last less than 100 km. Calculate the value of the new standard deviation for this normal distribution if the mean remains the same.

   $$P(Z < k) = 0.0027$$
   $$k = -2.7822 \quad \checkmark$$
   $$-2.7822 = \frac{100 - 150}{\sigma} \quad \checkmark$$
   $$\sigma = 15.19 \; km \quad \checkmark$$

Question Three: [3, 3, 5 = 11 marks] CA

(a) Given that $Y \sim N(\mu, 16)$, calculate μ given that $P(Y > 25) = 0.89435$.

$P(Z > k) = 0.89435$

$k = -1.25$ ✓

$-1.25 = \dfrac{25 - \mu}{4}$ ✓

$\mu = 30$ ✓

(b) Given that $X \sim N(100, \sigma^2)$, calculate σ given that $P(X < 80) = 0.04779$.

$P(Z < k) = 0.04779$

$k = -1.6667$ ✓

$-1.6667 = \dfrac{80 - 100}{\sigma}$ ✓

$\sigma = 12$ ✓

(c) Given that $T \sim N(\mu, \sigma^2)$, calculate μ and σ given that

$P(X < 15.5 \mid X > 14) = \dfrac{8186}{9772}$

$P(Z > k) = 0.9772$

$k = -1.999$ ✓

$-1.999 = \dfrac{14 - \mu}{\sigma}$

$P(Z > c) = 0.1534$ ✓

$c = 1.022$ ✓

$1.022 = \dfrac{15.5 - \mu}{\sigma}$

$\mu = 15$ ✓

$\sigma = 5$ ✓

Question Four: [3 marks] CA

In a state spelling competition, results were normally distributed with a mean of 58% and a standard deviation of 14%.

Participants who scored between 85% and 95% received a certificate of distinction. In the state, 103 participants were awarded a certificate of distinction.

How many students participated?

$P(85 < X < 95) = 0.0228$ ✓

$0.0228x = 103$ ✓

$x = 4518$ ✓

Question Five: [3 marks] CA

The amount of milk sold in 2L containers is normally distributed with a mean of 1998 mL.

Out of 1000 bottles tested, 248 contained between 1998 mL and 2001 mL.

Calculate the standard deviation of the amount of milk contained in these 2L containers.

$P(Z < k) = 0.5 + 0.248$

$k = 0.6682$ ✓

$0.6682 = \dfrac{2001 - 1998}{\sigma}$ ✓

$\sigma = 4.49\ ml$ ✓

Mathematics Methods Unit 4

Question Six: [2, 3, 3, 3 = 11 marks]　　　CA

To apply to be an international flight attendant for QANTAS, your height must be between 163 cm and 183 cm.

Data collected in 2011 indicates that the mean height of adult males in Australia is 175.6 cm with and the mean height of adult females in Australia is 161.8 cm with a standard deviation of 3.8 cm.

(a)　Out of a random sample of 2000 Australian adult females applicants, how many would you expect to meet the QANTAS height requirements?

$P(163 < F < 183) = 0.3761$ ✔

$0.3761 \times 2000 = 752$ ✔

(b)　The proportion of adult male Australian applicants who are below the maximum height requirement is 0.96096. Calculate the standard deviation for the height of the applicants.

$P(Z < k) = 0.96096$

$k = 1.7619$ ✔

$1.7619 = \dfrac{183 - 175.6}{\sigma}$ ✔

$\sigma = 4.2 \ cm$ ✔

(c)　An applicant is 170 cm tall. Are they more likely to be a tall female or a short male?

$P(F > 170) = 0.0155$ ✔

$P(M < 170) = 0.0912$ ✔

✔

More likely to be a short male.

(d)　QANTAS are interviewing potential applicants, all of whom meet the height requirement. The first three interviewees are female. What is the probability that the first two are less than 168 cm and the third is taller than 165 cm?

$P(F < 168) = 0.9845$ ✔

$P(F > 165) = 0.1999$ ✔

$= 0.9845^2 \times 0.1999$

$= 0.1938$ ✔