WESLEY COLLEGE
By daring & by doing

**YEAR 12 MATHEMATICS METHODS**
CRV's & Normal Probability
**Test 5**

Name: Solutions Marks: /48

## Calculator Free (22 marks)

**1. [2, 2. 1, 2 = 7 marks]**

A continuous random variable $X$ has the probability function $f$:

$$f(x)=\begin{cases}0 & ,x<0\\ h(10-x) & ,0\le x\le 10\\ 0 & ,x>0\end{cases}$$

a) Determine the constant $h$.

$\frac{1}{2}(10)(10h) = 1$ or $\left[10hx - \frac{hx^2}{2}\right]_0^{10} = 1$

$h = \frac{1}{50}$ $\qquad$ $\therefore h = \frac{1}{50}$

b) State the cumulative function $F(x)$ for the PDF $f(x)$.

$$F = \begin{cases} 0, & x<0 \\ \frac{1}{50}\left(10x - \frac{x^2}{2}\right), & 0 \le x \le 10 \\ 1, & x > 10\end{cases}$$

Hence or otherwise, determine:

c) $P(X = 3)$ $\quad 0$

d) $P(X \ge 1)$ $\quad 1 - F(1)$ or $\frac{1}{2}\times 9 \times \frac{1}{50}\times 9$

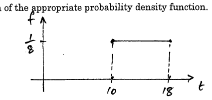
$= 1 - 0.19$ $\qquad = \frac{81}{100}$

$= 0.81$ $\qquad = 0.81$

**2. [2, 2, 1, 1, 1, 2, 1 = 10 marks]**

Mandy catches the bus to and from school each day. It can be assumed that the length of time, $T$, to wait for the bus between home and school follows a uniform distribution and takes between 10 and 18 minutes.

a) Make a quick sketch of the appropriate probability density function.




b) State the probability density function for $T$.

$$f(t) = \begin{cases}\frac{1}{8}, & 10 \le t \le 18\\ 0, & \text{elsewhere}\end{cases}$$

c) For a randomly chosen waiting time between home and school, calculate the probability that the time taken by Mandy waits:

(i) more than 16 minutes $\quad \frac{2}{8} = \frac{1}{4}$

(ii) less than 13 minutes $\quad \frac{3}{8}$

(iii) less than 9.5 minutes $\quad 0$

(iv) more than 15 minutes given that she waits more than 12 minutes.

$P(X>15 \mid X>12) = \frac{P(X>15)}{P(X>12)}$

$= \frac{3}{8} \Big/ \frac{6}{8} = \frac{1}{2}$

d) What was the average time that Mandy waits for the bus?

14 minutes

**3. [2, 3 = 5 marks]**

In a certain PDF the distribution is defined by:

$$f(x)=\begin{cases}A\sin(x) & ,0\le x\le\pi\\ 0 & ,elsewhere\end{cases}$$

a) Show that the value of $A = \frac{1}{2}$.

$\int_0^{\pi} A\sin(x)\,dx = \left[-A\cos(x)\right]_0^{\pi} = 1$

$\Rightarrow -A(\cos(\pi) - \cos(0)) = 1$

$\Rightarrow 2A = 1$

$A = \frac{1}{2}$

b) Determine exactly $P(X < \frac{\pi}{4})$

$\int_0^{\pi/4} \frac{1}{2}\sin(x)\,dx = -\frac{1}{2}\left(\cos(\frac{\pi}{4}) - \cos(0)\right)$

$= -\frac{1}{2}\left(\frac{\sqrt{2}}{2} - 1\right)$

$= \frac{2-\sqrt{2}}{4}$

**End of Part A**

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## Calculator Section (26 marks)

**4. [2, 2 = 4 marks]**

Pierre spends $X$ hours gaming during the day.

The probability distribution of $X$ is given by:

$$f(x)=\begin{cases}2(1-x) & ,0\le x\le 1\\ 0 & ,elsewhere\end{cases}$$

a) Evaluate $E(X)$, the expected value of $X$, to the nearest minute.

$\int_0^1 2x(1-x)\,dx$

$= \frac{1}{3}$

b) Determine the variance of $X$.

$\int_0^1 2x^2(1-x)\,dx - \left(\frac{1}{3}\right)^2$

$= \frac{1}{6} - \frac{1}{9}$

$= \frac{1}{18}$

**5. [1, 1, 1, 2, 2, 2, 2 = 11 marks]**

Main Roads Western Australia recently installed new radar devices in the Northbridge Tunnel on the Graham Farmer Freeway. During the first week of monitoring the average speed was determined to be 82 km/h with a standard deviation of 5.1 km/h.

$X \sim N(82, 5.1^2)$

a) If vehicle speeds can be considered normally distributed, determine the probability that a randomly chosen vehicle was travelling:

(i) less than 80 km/h

$P(X < 80) = 0.3475$

(ii) at 90 km/h

$P(X = 90) = 0$

(iii) between 85 km/h and 90 km/h

$P(85 < X < 90) = 0.2198$

(iv) faster than 90 km/h given the vehicle was travelling in excess of 85 km/h.

$$P(X > 90 / X > 85) = \frac{P(X > 90)}{P(X > 85)}$$

$$= \frac{0.0584}{0.2782} = 0.2099$$

b) The fastest 4% of vehicles were issued with speeding fines.
Above what speed would you calculate a driver to be fined?

$P(Z < k) = 0.96$

$\therefore$ speed $= 90.92 \sim 91$ km/h

c) Determine the probability that in a randomly chosen group of 12 cars

(i) Exactly three drivers were fined

$Y \sim b(12, 0.04)$

$P(Y=3) = 0.0098 \quad (9.75 \times 10^{-3})$

(ii) At least one driver was fined

$P(Y \geq 1) = 0.3873$

$(1 - 0.96^{12})$

**6. [2, 2, 1, 1 = 6 marks]**

a) A continuous random variable X has a mean of 12 and a standard deviation of 4.

Determine the expected value and ~~variance~~ std dev. in each part below if $X$ is transformed to the random variable $Y$ by each of the following:

(i) $Y = 5X$

$E(Y) = 60$

$SD(Y) = 20$

(ii) $Y = 5 - 2X$

$E(Y) = -2 \times 12 + 5$

$= -19$

$SD(Y) = 8$

b) If $X \sim N(28, 7^2)$ determine the:

(i) 31th percentile

$P(X < k) = 0.31$

$\therefore k \sim 24.53$

(ii) 0.73 quantile

$P(X < k) = 0.73$

$\therefore k \approx 32.29$

**7. [5 marks]**

The speed limit along the Kwinana Freeway is 100 km/h. Speeds are normally distributed with mean $\mu$ km/h and standard deviation $\sigma$ km/h. If it is known that 12.5% of drivers drive in excess of 100km/h and that 5% drive at less than 88 km/h, calculate $\mu$ and $\sigma$ to an accuracy of two decimal places.

$P(X > 100) = 0.125 \qquad P(X < 88) = 0.05$

ie $\frac{100 - \mu}{\sigma} = 1.15$ (1) $\qquad \frac{88 - \mu}{\sigma} = -1.645$ (2)

Solve simultaneously:

$\mu = 95.06$

$\sigma = 4.29$

**End of Part B**