SCOTCH
COLLEGE

**Scotch College**
**Semester One Practice Examination 2, 2016**

**Question/Answer Booklet**

# Year 12 MATHEMATICS METHODS

## Section Two:
## Calculator-assumed

**Teacher:** ____J. Fletcher
____P. Newman
____S. Reyhani

**Name:**

### Time allowed for this section
Reading time before commencing work: 10 minutes
Working time for this section: 100 minutes

### Material required/recommended for this section
***To be provided by the supervisor***
This Question/Answer Booklet
Formula Sheet (retained from Section One)

***To be provided by the candidate***
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

### Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available |
|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 52 |
| Section Two Calculator-assumed | 14 | 14 | 100 | 91 |
| | | | | 143 |

## Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

EXTRA WORKING SPACE.



Question 9. [1,3 = 4 marks]

The decibel scale can be used to describe how loud a sound is.

If $P_1$ is the intensity of the sound, then its loudness $L$, in decibels (dB), is given by the formula:

$$L = 10 \log_{10} \frac{P_1}{P_0}$$

where $P_0$ represents the intensity of the quietest sound that is audible to the human ear.

(a) If a sound has 100 times the intensity of $P_0$, so that $P_1 = 100P_0$, show that it would register as 20dB on this scale. (1)

(b) The sound of a lawnmower is measured as 85dB, and a conversation as 60dB.

How many times more intense is the lawnmower than the conversation? (3)

Question 10. [1,2,2,2 = 7 marks]

A common model of population growth is given by the formula

$$P(t) = \frac{KP_0e^{rt}}{K + P_0\left(e^{rt} - 1\right)}$$

In this equation, $P(t)$ is the population at time $t$, $P_0$ is the initial population, $K$ is the maximum population which the environment can sustain (the population ceiling), and $r$ is the rate at which the population would grow in the absence of a population ceiling.

Suppose that an ant hill can sustain a population of 50 000 ants. Initially the population is 20 000, with the growth rate $r = 0.05$, and time measured in weeks.

(a) What is the population after 10 weeks? (1)

(b) By what percentage does the population increase in the first week? (2)

(c) After how many weeks will the population reach 30 000? (2)

(d) How does the size of the population change in the long term? (2)



----------End Of Examination----------

EXTRA WORKING SPACE.

The manufacturers of a new conical ice cream, the Piccoletto, are looking to save costs on packaging. The volume of the cone is to be 188cm$^3$, and the manufacturers want to minimise the surface area.

The formulae for the volume and surface area of a cone of radius *r* and perpendicular height *h* are

$V=\frac{\pi r^2 h}{3}$ and $SA=\pi r^2+\pi r\sqrt{r^2+h^2}$ respectively.

a) Use this information to write a formula for the surface area of the cone in terms of its radius only.

(2 marks)

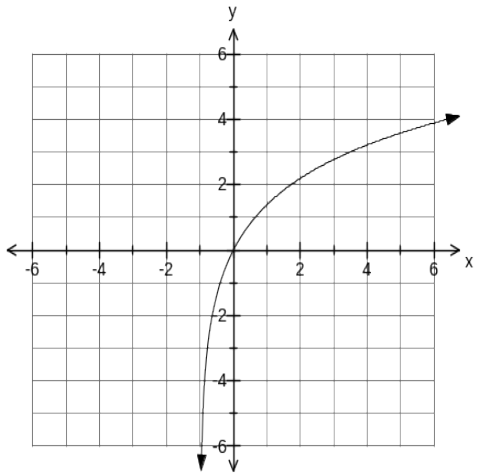
b) Determine the dimensions of the cone that will minimise the surface area. State this surface area.

(3 marks)



Question 11. [2,2,4 = 8 marks]

The following diagram shows the graph of $f(x)=2\ln(x+a)$.




(a) Determine the value of $a$. (2)

(b) Sketch the graph of $-f(x)$ on the diagram. (2)

(c) For what exact value of $x$ is $f(x)=2$? (4)

Question 12. [3,5 = 8 marks]

A dome shaped tent is being erected which is in the shape of a cylinder with a hemisphere sitting on the top. The radius of the cylinder $(r)$ and the radius of the hemisphere are equal. The height of the cylinder is twice as long as the radius.

a) Show that the volume $(v)$ of air inside the dome is $v=\frac{8\pi r^3}{3}$

b) Using differentiation determine the change in volume of the tent if the radius was increased by 10cm when the volume was $576\pi m^3$.



e) One of Barry's customers had let their coffee get cold, and asked him to re-heat it.

The re-heating process is such that the rate of change of the temperature ($℃/min$ ¿ is proportional to the temperature, with the constant of proportionality being 0.686.

(i) If the coffee was at a temperature of $25℃$ when Barry began to re-heat it, write an expression for the temperature $T$ (in $℃$ ¿ $t$ minutes after the reheating process commences.

(2 marks)

(ii) Determine how long the reheating process would take to make the coffee reach ideal serving temperature of $70℃$ once more.

(1 mark)

**Question 22** **(5 marks)**

b) What is the initial temperature of the coffee when it is made? (1 mark)

c) If left to cool, eventually the temperature of the coffee will be the same as the temperature of the cafe. What is this temperature?
(1 mark)

d) The ideal serving temperature for a cup of black coffee is $70\,℃$. For how many minutes after the coffee is made should Barry wait before serving it?
(1 mark)



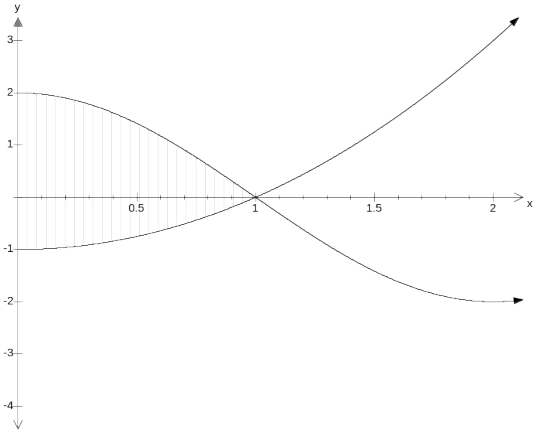
Question 13. [4,2 = 6 marks]

(a) If $y = \sin(x^2)$, show that $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = 0$ **[4]**

(b) Find the derivatives of $f(x) = \int_{\pi}^{x^2} \frac{1}{\sin^3 t}\,dt$ **[2]**

Question 14. [4 marks]

The shaded region in the diagram below is bounded by the y-axis, and the curves with equations $y = x^2 - 1$ and $y = 2\cos\left(\frac{\pi x}{2}\right)$. Find the EXACT value of the area of the shaded region.






c) How far does the particle travel in total in the first 10 seconds? (2 marks)

**Question 21 (8 marks)**

At his part-time job working in a cafe, mathematician Barry Easter noticed that, as cups of black coffee cooled, the temperature ($T$ ℃) $t$ minutes after they had been made follows the exponential function

$$T = 75e^{-0.1t} + 20$$

a) Describe the transformations of the function $T = e^t$ required to produce this function.

(2 marks)

b) Use the marginal rate to estimate the cost of printing one more book at the stage in the printing when 1000 copies have been produced. Compare this cost with the average cost of producing the second 500 copies of the book.
(2 marks)

Question 20 **(6 marks)**

A particle, originally at rest at the origin, moves in such a way that its acceleration $a$ m/s$^2$, $t$ seconds later is given by the formula $a = 12t - 18$.

a) State expressions for the velocity and displacement of the particle in terms of $t$.
(2 marks)

b) State the time and position when the particle is next at rest. (2 marks)



Question 15. [2,2 = 4 marks]

Find the following:

**(a)** $\frac{d}{dt}\int_1^{t^3}\left[\frac{\sqrt{x^2+1}}{x}\right]dx$ **[2]**

**(b)** $\int_1^{t^3} \frac{d}{dx}\left(\frac{\sqrt{x^2+1}}{x}\right)dx$ **[2]**

Question 16. [3,4 = 7 marks]

The cross section of the blade of a paddle is shown below:



Question 19. [2,2 =4 marks]

The marginal costs involved in printing *x* copies of a particular book follow the rule $C'(x) = \frac{2.5}{\sqrt[3]{x}} + 3$.

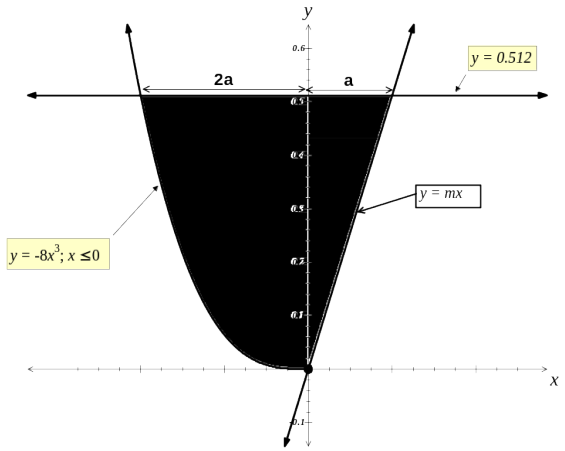
a) Write an expression involving integration for the extra cost incurred by producing 1000 copies rather than 500. Use this expression to determine the average cost per book of producing the second 500 books.

(2 marks)

The manufacturers decided each flower was too small and did not weigh enough and were therefore blown around by the wind. It was decided to double all the measurements of the flowers and to make them 1.5cm thick.

c) Determine the volume of plastic used to produce one of the new flowers.






The blade is determined by the curved edge $y=-8x^3$ and the straight edges, $y=mx$ and $y=0\cdot512$.

a) Determine the value of $m$ and $a$.

b) Using calculus, determine the area of the shaded region.

Question 17. [4,5 = 9 marks]

An internet marketing company has found that important news spreads through a university population according to the formula:

$\frac{dN}{dt} = k(P - N)$ where $N$ is the number of people who know the important news $t$ hours after the important news is announced. $P$ and $k$ are constants where $P$ is the total population of the university and $k$ is the growth factor.

**(a)** Let $A_0$ be the initial number of people not knowing the news. Use integration to show that $N = P - A_0 e^{-kt}$ is the solution of the differential equation.

**[4]**

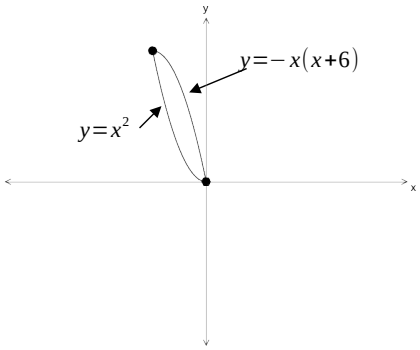
**(b)** Challenger University has a student population of 18 000 students. News that boy band, No Direction, would be visiting the campus was announced at noon. Three hours later, 60% of the student population knew about the news. At what time, to the nearest hour, will 95% of the population know the news?

**[5]**



Question 18. [2,5,4 = 11 marks]

A manufacturing company has been hired to make flowers from coloured plastic sheeting. The flowers have **four** identical petals each. One such petal is shown below.




a) Show that the two equations that make up the petal intersect at $(0,0)$ and $(-3,9)$.

Note: Each unit of the graph represent **5cm** in real life.

b) Determine the amount of plastic used to create 50 entire flowers. Give your answer to the nearest $mm^2$.