SOLUTIONS

ROSSMOYNE SHS SEMESTER 1, 2009

UNIT 3A MATHEMATICS EXAMINATION

SECTION B APPROVED CALCULATORS PERMITTED

STUDENT'S NAME _____

TEACHER:-	Belonogoff	Goh	Hampson
(Circle one name)	Knoblauch	Lee	Robinson

TIME ALLOWED FOR THIS PAPER

Reading time before commencing Section A	Five minutes
Working time for Section A (non calculator)	Forty minutes
Changeover time between Sections A and B	Five minutes
Reading time before commencing Section B	**Five minutes**
Working time for Section B	**Eighty minutes**

Available marks: 120 marks

MATERIAL REQUIRED / RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SCHOOL:

This Question/Answer booklet

TO BE PROVIDED BY THE CANDIDATE

Standard Items Pens, pencils, eraser, ruler

Special items Curriculum Council Mathematical Formulae and Statistics Tables Book, drawing instruments, templates, notes on two sheets (4 sides) of A4 paper and calculators (Section B only) satisfying the conditions set by the Curriculum Council.

NOTE: Personal copies of the Tables Book should not contain any handwritten notes, symbols, signs, formulae or any other marks (including underlining and highlighting), except the name and address of the candidate, and may be inspected during the examination.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room.
It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor BEFORE reading any further.

STRUCTURE OF THIS PAPER

This paper consists of two sections:

Section A is a non-calculator section – **no** calculator may be used in this section.
Time allowed for this section is 40 minutes. At the end of 40 minutes, this section will be collected by the supervisor. There will be a 5 minute changeover before commencing Section B.

Section B is a section where approved calculators are permitted. Time allowed for this section is 80 minutes. At the end of 80 minutes, this section will also be collected by the supervisor.

INSTRUCTIONS TO CANDIDATES
ALL questions should be attempted. You may answer the questions in any order you wish.
Write answers in the spaces provided. Extra pages are supplied at the back of this booklet. If the extra pages are used, label the questions clearly. Indicate on the original question that your working continues at the end of this booklet.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.

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Question	Total	Mark
1	8	
2	8	
3	4	
4	6	
5	2	
6	6	
7	5	
8	6	
9	6	
10	6	
11	5	
12	5	
13	4	
14	5	
15	4	
Total	**80**	

SECTION B

APPROVED CALCULATORS PERMITTED FOR THIS SECTION

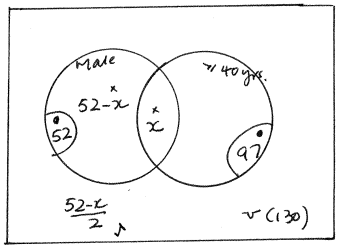
TIME: 80 minutes

MARKS ALLOCATED 80 marks

Question 1. (3, 2, 3 marks)

Of the 130 people at a conference, 52 were male and 97 were 40 years or older. The number of female delegates younger than 40 years was half that of males less than 40 years old.

(a) Show the above information in a Venn diagram.



• ✓
x ✓
✓ ✓

(b) How many delegates were male aged 40 years or more?

$97 + 52 - x + \left(\frac{52-x}{2}\right) = 130$

$149 - x + 26 - 0.5x = 130$

$175 - 1.5x = 130$

$x = 30$ ✓

30 delegates were male aged 40 yrs. ✓

F/T

(c) What is the probability that a person chosen at random will be

(i) a male aged less than 40 years? $\frac{22}{130}$ ✓

(ii) a female aged under 40 years? $\frac{11}{130}$ ✓

(iii) a female given she is at least 40 years old? $\frac{67}{97}$ ✓

(8)

Question 15 (2 , 2 marks)

Skye has inherited $540 000 from her grandmother and will invest $500 000 in an account paying 3.5% per annum compounded half yearly. She plans to make only one withdrawal each year.

(a) For how many years will she be able to withdraw $40 000 a year from this account?

CAS calc. Financial mode (P/V = 1) (C/V = 2)

N = 16.8 i.e. 16 years ✓✓

if 16.8 yrs, ✓x

(b) If she would like the money to last for 20 years, how much will she be able to withdraw from the account each year?

PMT $35 278.10 ✓✓

if P/V = C/V = 1, then $35 180.54 ✓x

if P/V = C/V = 2, then $29 845.61 ✓x (4)

End of Part B

Question 14. (5 marks)

Tyson, a young man about town, is in the market for a new car and has narrowed his choice to four cars. The different aspects of the cars and the respective weightings placed by him are as follows.

	Macho rating	Acceleration	Price	All terrain capability	Fuel economy
Weighting	**6**	**5**	**4**	**3**	**2**
Car A	7	6	6	4	7
Car B	8	5	7	5	2
Car C	8	7	6	5	5
Car D	6	8	6	6	5

Calculate the weighted scores of the 4 cars.

A $\dfrac{(7\times6)+(6\times5)+(6\times4)+(4\times3)+(7\times2)}{20} = 6.1$ ✓

B $\dfrac{(8\times6)+(5\times5)+(7\times4)+(5\times3)\times(2\times2)}{20} = 6$ ✓

C $\dfrac{(8\times6)+(7\times5)+(6\times4)+(5\times3)\times(5\times2)}{20} = 6.6$ ✓

D $\dfrac{(6\times6)+(8\times5)+(6\times4)+(6\times3)+(5\times2)}{20} = 6.4$ ✓

Tyson will buy the car with the highest weighted score. Which one will he buy?

Tyson will buy car C ✓

(5)

Question 2. (2, 1, 1, 2, 2 marks)

A doctor notes that in the last two years, 24 of his patients were diagnosed with some form of cancer. 18 of these patients had been exposed to harmful chemicals in their workplace. 5 had not been exposed to chemicals but were smokers. A total of 20 patients were smokers. Represent this information in a two way table and use it to answer the following.

Chemicals \ Smokers	Yes	No	Total
Yes	15	3	18 •
No	5 •	1	6
Total	20 •	4	24 •

• given
others ✓✓
−1 each error.

What is the probability that one of his cancer patients

(a) is neither a smoker nor has been exposed to chemicals? $\frac{1}{24}$ ✓ PIT from table.

(b) is a smoker but has not been exposed to chemicals? $\frac{5}{24}$ ✓

(c) is a smoker given that he/she has not been exposed to chemicals? $\frac{5}{6}$ ✓

(d) is not a smoker given he/she has been exposed to chemicals? $\frac{3}{18}$ ✓

(8)

Question 3. (1, 1, 2 marks)

Aziz is driving across the Nullabor to Melbourne and taken a collection of these CDs – 2 Country and Western, 3 Jazz, 2 modern music and 4 classical music.

In how many ways can he listen to all of them if

(a) he does not mind the order? $11! = 39\,916\,800$ ✓

(b) the Country and Western CDs are the first and the last CDs? $2\times9!\times1 = 725\,760$ ✓

(c) he listens to the Country and Western ones first, then the Jazz, then the modern music ones and finally the classical CDs? $2!\times3!\times2!\times4! = 576$ ✓✓

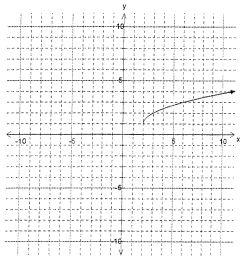
(4)

12

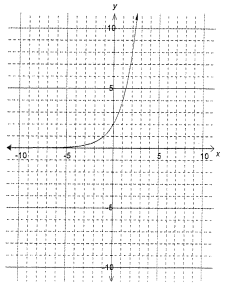
Question 4. (1, 1, 1, 1, 1, 1 marks)

Match the correct equations with the graphs below. Use the **letter** (eg A, B etc) and not the equation.

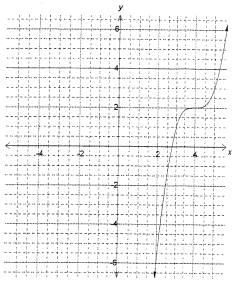
A	$y = \frac{2}{x}$	**B**	$y = \frac{1}{x+2}$	**C**	$y = 2\sqrt{x}$
D	$y = 2^{(x+1)}$	**E**	$y = \sqrt[3]{x-3} + 4$	**F**	$y = (x+4)^2 - 9$
G	$y = (x-4)^3 + 2$	**H**	$y = \sqrt{x-2} + 1$	**I**	$y = \frac{1}{(x-4)^2} + 3$



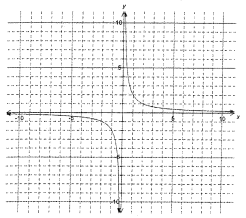
H ✓



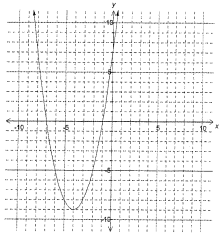
D ✓



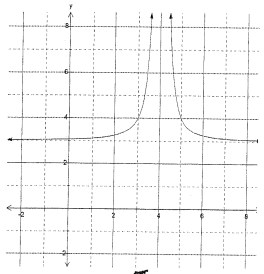
G ✓



A ✓



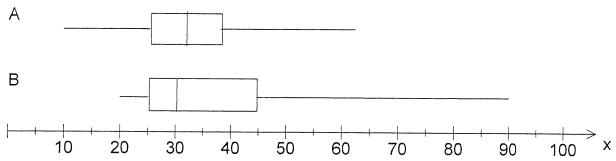
F ✓



I ✓

(6)

Question 13. (4 marks)



Write four statements comparing the distributions shown in the box plots above.

Spread . B has a larger spread and IQR than A

Skewness . B is positively skewed (to the right). A is slightly skewed to the right

medians . The medians of A & B are similar

. B has a higher maximum mark, Q3 and lowest mark than A

symmetry . A is more symmetrical than B

any 4

(4)

Question 11. (2, 3 marks)

The frequency table below shows the weekly wages earned by some students who have part-time jobs.

Weekly earnings ($)	Number of students
0 - 9	10
30 - 39	3
40 - 49	5
50 - 59	4
60 - 69	11
70 - 79	7
80 - 89	x

(a) If x = 2, find the mean and standard deviation for the students' weekly earnings.

$\bar{x}$ = $47.36 ✓ s = $26.84 ✓

(b) Find the value of x if the mean is 54.5. Show clearly how you obtained your answer.

$$54.5 = \frac{10(4.5) + 3(34.5) + 5(44.5) + 4(54.5) + 11(64.5) + 7(74.5) + 84.5x}{40 + x} \quad ✓$$

$54.5(40+x) = 1820 + 84.5x$

$2180 + 54.5x = 1820 + 84.5x$ ✓

$360 = 30x$

$x = 12$ ✓

(5)

Question 12. (2, 1, 2 marks)

Kumar borrows $25 000 at interest rate of 5.5% p.a.. He agrees to pay $5 000 at the end of each year.

(a) Write the recursive rule used to calculate the amount Kumar will owe at the end of each year.

$T_{n+1} = 1.055\,T_n - 5000$ ✓ $T_0 = 25000$ ✓

(b) In which year will he owe less than $10 000?

after the 4th year, i.e. in the 5th year. ✓

(c) After how many years will he finally pay off his loan? What will Kumar's final payment be?

$T_6 = \$30.82$ ✓ T_7

last payment $= 1.055(30.82)$ ✓

$= \$32.52$ in 7th year ✓

Question 5. (2 marks)

The Health Department needs an estimate of patients infected with Porcine Flu. The names on List A are patients who exhibit symptoms of the disease, List B has some names of people from the Procine Flu Association. List A had 263 names. List B had 127 names of whom 81 were also on List A.

(a) How many sufferers of Porcine Flu do these figures suggest exist in the population as a whole?

$\frac{263}{x} = \frac{81}{127}$

$x \approx 412$ ✓ There are approx. 412 sufferers

(b) How many of these do not feature on either list?

$412 - (263 + 127 - 81) = 103$ ✓

(2)

Question 6. (2, 1, 2, 1 marks)

In 1990 it was noticed that the population of rock wallabies in Wayugal Reserve was increasing according to the equation $y_w = 40 + 0.1(x^2 + 20x)$ whilst the population of feral goats in the same area was increasing according to the equation $y_g = 20\,(1.09)^x$ where x is the number of years after 1990. Use your CAS calculator to answer the following questions.

(a) How many

(i) rock wallabies were there in 1990? 40 ✓

(ii) feral goats were there in 1990? 20 ✓

(b) In which year and month will the two populations equal each other if these rates of increase are maintained?

graphing curves $x = 21.85$ ✓ must have month and year

i.e. November 2011 ✓

(c) In 2005, the shire council removed forty goats from the population in an eradication programme. If, after the eradication, the two populations will still continue to increase at the same rates, write the equations of the growths of the rock wallabies and feral goats from 2005 onwards.

In 2005 there were 72.8 i.e. 73 goats, so, 33 goats left.

$y_w = 40 + 0.1[(x+15)^2 + 20(x+15)]$ ✓

$y_g = 33(1.09)^x$ ✓

(d) In which year will the two populations equal each other? _____

$x = 24.7$ i.e. 2029 ✓F/T

if $y_w = 93 + 0.1(x^2 + 20x)$, then 2023 (F/T) ✓

(6)

Question 7. (2, 3 marks)

L varies directly proportional to x and inversely proportional to $\sqrt{y}$. If L = 24 when x = 2 and y = 9

(a) Find L in terms of x and y.

$L = \frac{kx}{\sqrt{y}}$ ✓ ✓

(b) Find the **exact** value of x when L = 8 and y = 72.

$24 = \frac{2k}{3}$

$k = 36$ ✓

i.e. $L = \frac{36x}{\sqrt{y}}$

$8 = \frac{36x}{\sqrt{72}}$ ✓

$x = \frac{8.6\sqrt{2}}{36} = \frac{4\sqrt{2}}{3}$ ✓

(5)

Question 8. (3, 3 marks)

In the first month of operation a small gold mine produced 200 troy ounces of gold. (The mass of gold is defined in troy ounces where 1 troy ounce = 31.103 g.) In the second month 280 troy ounces were produced. In the third month it produced 392 troy ounces of gold. Successive monthly production figures formed a geometric sequence until full production is reached in the fifth month. Thereafter, this amount is maintained.

(a) What is the maximum monthly production of the gold mine?

200, 280, 392 . . . .

$r = 1.4$ ✓

$T_5 = 392(1.4)^2$ ✓

$= 768.32$ troy ounces ✓

(b) What total amount of gold in troy ounces was produced in the first twelve years of its operation?

$S_5 = 2189.12$ ✓

$139 \times T_5 = 139(768.32)$

$= 106\,796.48$ ✓

Total in 12 yrs $= 108\,985.6$ troy ounces ✓

(6)

Question 9. (3, 3 marks)

Solve the following. All working must be shown.

(a) $7^{3n+2} \times 7^{1-n} = \frac{1}{16807}$

$7^{3n+2+1-n} = 7^{-5}$ ✓

$2n + 3 = -5$ ✓

$n = -4$ ✓

(a) $3^{2n} - 12(3^n) = -27$

let $y = 3^n$, $y^2 - 12y = -27$

$y^2 - 12y + 27 = 0$ ✓ use of calculator OK

$(y - 9)(y - 3) = 0$

i.e. $3^n = 9$ or $3^n = 3$ ✓

$n = 2$ or $n = 1$ ✓ both answers for 1 mark

(6)

Question 10. (2, 2, 2 marks)

All the letters of the word FORMULA are to be arranged at random in a row.

(a) How many of these arrangements have the three vowels next to each other?

$3! \times 4! \times 5 = 720$ ✓✓

vowels consonants

(b) How many of these arrangements have the vowels and the consonants occupying alternate positions?

$\frac{4}{C} \cdot \frac{3}{V} \cdot \frac{3}{C} \cdot \frac{2}{V} \cdot \frac{2}{C} \cdot \frac{1}{V} \cdot \frac{1}{C} = 4! \times 3!$

$= 144$ ✓✓

(c) What is the probability that the 'word' commences with an M and the vowels are together?

$\frac{1 \times 3! \times 4!}{7!}$ ✓ $= \frac{6 \times 24}{5040}$ unsimplified OK

$= \frac{1}{35}$ ✓

(6)