Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3 Year 12

If required by your examination administrator, please place your student identification label in this box

Section One: Calculator-free

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	5	
2	6	
3	7	
4	6	
5	6	
6	7	
7	6	
8	7	
S1 Total	50	
S1 Wt (×0.7)	35%	
S2 Wt	65%	
Total	100%	

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Section One: Calculator-free **35% (50 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

The displacement vector of a particle at time t seconds is given by $r(t)=\begin{pmatrix}\sin(2t)\\3t\\\cos(2t)\end{pmatrix}$ cm.

Show that the particle is moving at a constant speed and determine this speed.

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Question 2 **(6 marks)**

Let $p(z)=z^4-2z^3+14z^2-18z+45$.

(a) Show that $z-3i$ is a factor of $p(z)$. (2 marks)

(b) Solve the equation $p(z)=0$. (4 marks)

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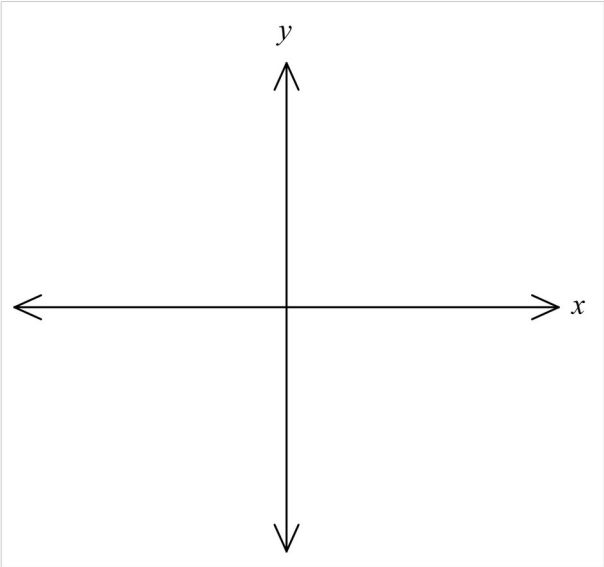
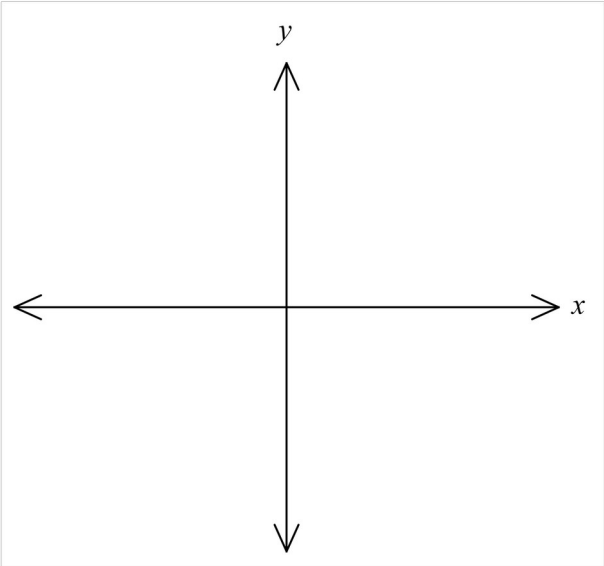
Question 3 **(7 marks)**

The functions f and g are defined as $f(x)=\dfrac{x-2}{x+2}$ and $g(x)=|x|$.

(a) Sketch the graph of $y=f(x)$ on the axes below. (3 marks)



(b) Sketch the graph of $y=f(g(x))$ on the axes below. (3 marks)



(c) Determine the range of function h, where $h(x)=\left|f(g(x))\right|$. (1 mark)

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Question 4 **(6 marks)**

Let $u=\sqrt{3}-i$ and $v=1-i$.

(a) Express u and v in polar form and hence show that $u \div v=\sqrt{2}\,cis\left(\frac{\pi}{12}\right)$. (3 marks)

(b) Hence show that $\sin\left(\frac{\pi}{12}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$. (3 marks)

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Question 5 **(6 marks)**

The equations of planes Π_1, Π_2 and Π_3 are $x - z = 1$, $y + 2z = 3$ and $x + y + z = 4$ respectively.

(a) Explain whether any of these planes are parallel. (2 marks)

(b) Solve the system of linear equations for the three planes. (3 marks)

(c) Describe the geometric interpretation of the solution of the system of equations. (1 mark)

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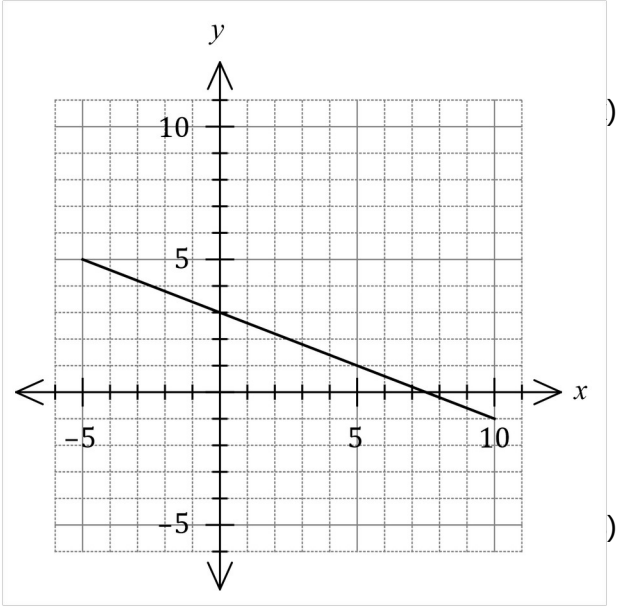
Question 6 **(7 marks)**

Linear function f has domain $-5 \le x \le 10$ and is shown on the graph at right.



(a) Determine $f \circ f(7.5)$.)

(b) Draw the graph of $y = f^{-1}(x)$ on the same axes.)

Function g is defined by $g(x) = \frac{4+3x}{5-x}$, $x \neq 5$.

(c) Determine $g^{-1}(x)$. (2 marks)

(d) Solve the equation $g \circ f(x) = 0.8$. (2 marks)

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Question 7 **(6 marks)**

The point A lies on the surface of a sphere with diameter OB. The position vectors of A and B relative to O are a and b respectively.

(a) Prove that $a \cdot b = |a|^2$. (2 marks)

The point P lies on the diameter of the sphere such that OB is perpendicular to AP and $\overrightarrow{OP} = \lambda b$.

(b) When $a = i + 2j - k$ and $b = 2i + j - 2k$, determine the value of the constant λ and the position vector of P relative to O. (4 marks)

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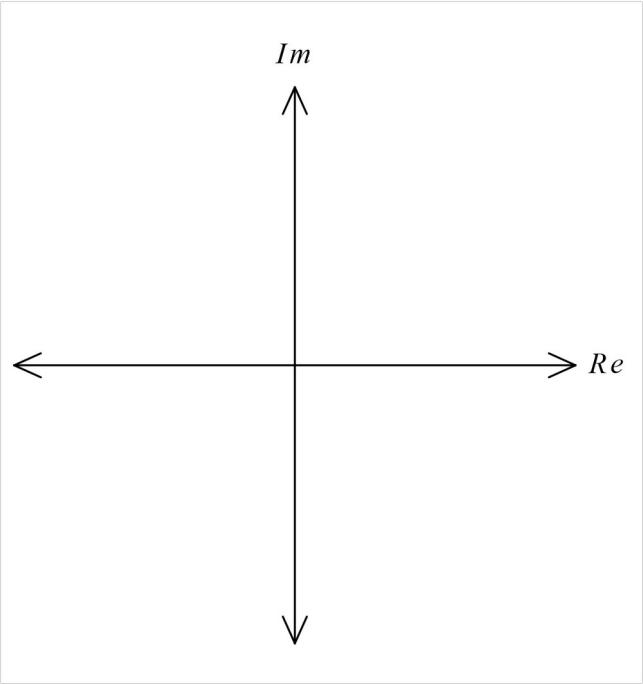
Question 8 **(7 marks)**

Let u and v be the two square roots of the complex number $3-4i$. On the diagram below, indicate the locus of a complex number z which satisfies $|z|\leq\sqrt{5}$ and $|z-u|=¿\, z-v \vee ¿$.



End of questions

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Supplementary page

Question number: _____

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