John Wollaston Anglican Community School

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section One: Calculator-free

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Supplementary page

Question number: _____

Question 8 **(7 marks)**

(a) Evaluate $\sin\left(\frac{39\pi}{4}\right)$. (2 marks)

Solution
$\sin\frac{39\pi}{4} = \sin\frac{(39-32)\pi}{4} = \sin\frac{7\pi}{4}$ $\sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$
Specific behaviours
✓ reduces angle ✓ exact value

(b) A is an acute angle and B is an obtuse angle such that $\cos A = \frac{1}{3}$ and $\sin B = \frac{2}{3}$.

(i) Show that $\sin A = \frac{2\sqrt{2}}{3}$ and determine the value of $\cos B$. (3 marks)

Solution
$\sin^2 A = 1 - \left(\frac{1}{3}\right)^2 = \frac{8}{9} \Rightarrow \sin A = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$ $\cos^2 B = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$ $\cos B = -\frac{\sqrt{5}}{3}$
Specific behaviours
✓ indicates how to obtain $\sin^2 A$ ✓ obtains $\cos^2 B$ ✓ correct value of $\cos B$

(ii) Determine the value of $\sin(A+B)$ as a single fraction. (2 marks)

Solution
$\sin(A+B) = \frac{2\sqrt{2}}{3} \times \left(-\frac{\sqrt{5}}{3}\right) + \frac{1}{3} \times \frac{2}{3}$ $= \frac{2-2\sqrt{10}}{9}$
Specific behaviours
✓ substitutes correctly ✓ correct value as single fraction

End of questions

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

Solve the following equations for x.

(a) $(2x-9)(x+7) = 0$. (1 mark)

Solution
$x = 4.5, \quad x = -7$
Specific behaviours
✓ both correct solutions

(b) $\frac{x}{3} = \frac{2x-1}{2}$. (2 marks)

Solution
$2x = 6x - 3$ $4x = 3 \Rightarrow x = \frac{3}{4}$
Specific behaviours
✓ cross multiplies ✓ correct solution

(c) $4x^2 = 4x$. (2 marks)

Solution
$4x(x-1) = 0$ $x = 0, \quad x = 1$
Specific behaviours
✓ one correct solution ✓ both correct solutions

See next page

Question 2 **(6 marks)**

(a) A circle of radius 4 has its centre at the point $(-2, 3)$. Determine the equation of the circle in the form $x^2 + y^2 = ax + by + c$. (3 marks)

Solution
$(x+2)^2 + (y-3)^2 = 4^2$ $x^2 + 4x + 4 + y^2 - 6y + 9 = 16$ $x^2 + y^2 = -4x + 6y + 3$
Specific behaviours
✓ writes equation of circle ✓ correctly expands ✓ writes in required form

(b) The graph of $x = y^2$ passes through the point $(4, q)$. Determine the value(s) of q and hence explain why y is a relation but not a function of x. (3 marks)

Solution
$4 = q^2 \Rightarrow q = \pm 2$ A relation exists as we are told that $x = y^2$. The relation is not a function because it is not one-to-one (for most values of x there is more than one value of y).
Specific behaviours
✓ both possible values ✓ explains relation between variables ✓ explains why relation not a function

Question 7 **(7 marks)**

(a) Complete the row of Pascal's triangle that starts $1, 6, 15, \ldots$ and express the sum of the numbers in this row as a power of 2. (2 marks)

Solution
$1, 6, 15, \mathbf{20}, \mathbf{15}, \mathbf{6}, \mathbf{1}$ $\text{Sum} = 2^6$
Specific behaviours
✓ correct bolded terms ✓ correct power of 2

(b) Determine the coefficient of

(i) the x^2 term in the expansion of $(5x-1)^2$. (1 mark)

Solution
$\binom{2}{2}(5x)^2(-1)^0 = 25x^2$ $\text{Coefficient is } 25$
Specific behaviours
✓ explicitly states coefficient

(ii) the x^4 term in the expansion of $(x+1)^6$. (1 mark)

Solution
$\binom{6}{4}(x)^4(1)^2 = 15x^4$ $\text{Coefficient is } 15$
Specific behaviours
✓ explicitly states coefficient

(iii) the x^3 term in the expansion of $(3-2x)^6$. (3 marks)

Solution
$\binom{6}{3}(3)^3(-2x)^3 = (20)(27)(-8)x^3$ $= -4320x^3$ $\text{Coefficient is: } -4\,320$
Specific behaviours
✓ indicates correct three factors of term ✓ expands each factor ✓ states coefficient

Question 6 **(7 marks)**

(a) Briefly describe the behaviour of the y values for each of the following graphs, given the behaviour of the x values:

(i) $y = x^4$, as $x \to \infty$. (1 mark)

Solution
$y \to \infty$
Specific behaviours
✓ describes correct behaviour

(ii) $y = (2 - x)^3$, as $x \to \infty$. (1 mark)

Solution
$y \to -\infty$
Specific behaviours
✓ describes correct behaviour

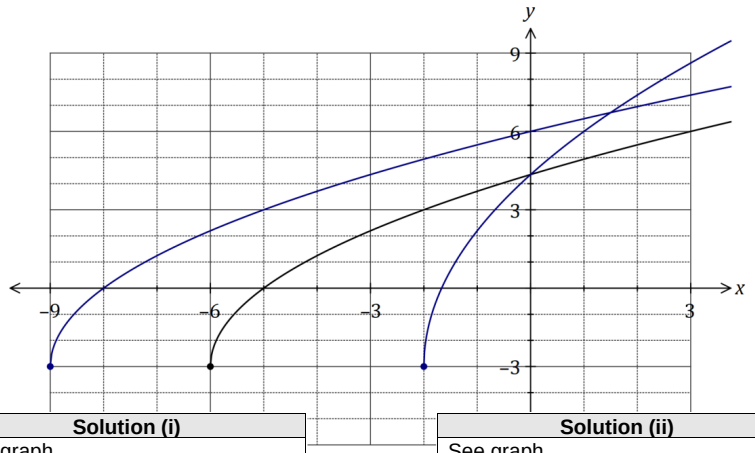
(iii) $y = \frac{1}{x}$, as $x \to -\infty$. (1 mark)

Solution
$y \to 0$
Specific behaviours
✓ describes correct behaviour

(b) The graph of $y = f(x)$ is shown below. On the same axes sketch the graph of

(i) $y = f(x + 3)$. (2 marks)

(ii) $y = f(3x)$. (2 marks)

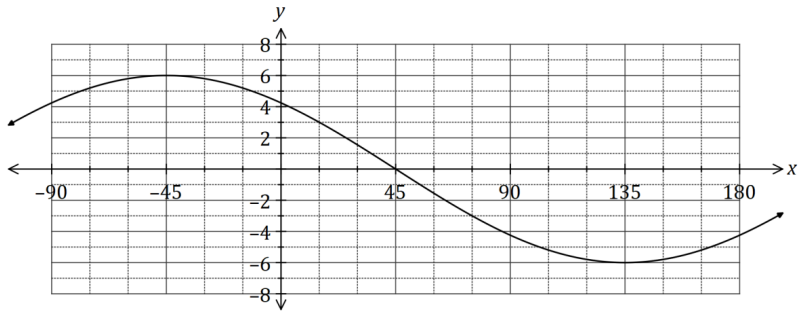


Solution (i)
See graph
Specific behaviours
✓ smooth curve starting at $(-9, -3)$
✓ intercepts at $(-8, 0)$ and $(0, 6)$

Solution (ii)
See graph
Specific behaviours
✓ smooth curve starting at $(-2, -3)$
✓ same y-intercept as $f(x)$

Question 3 **(6 marks)**

(a) The graph of $y = a\cos(x + b)$ is shown below, where a and b are constants.



Determine the value of a and the value of b, where $-90° \leq b \leq 180°$. (2 marks)

Solution
$a = 6, \quad b = 45$
Specific behaviours
✓ value of a ✓ value of b

(b) Given that $0° \leq x \leq 360°$, solve

(i) $\cos(x) = \frac{1}{2}$. (1 mark)

Solution
$x = 60°, 300°$
Specific behaviours
✓ correct solutions

(ii) $8\cos(x + 30°) + 4\sqrt{3} = 0$. (3 marks)

Solution
$\cos(x + 30°) = -\frac{\sqrt{3}}{2}$
$x + 30° = 150°, 210°$
$x = 120°, 180°$
Specific behaviours
✓ simplifies equation
✓ solves for angle sum
✓ correct solutions

Question 4 **(7 marks)**

(a) Determine the coordinates of the

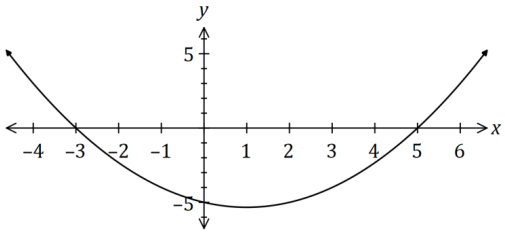
(i) y-intercept of the graph of $y = 4(x-3)^2 - 26$. (1 mark)

Solution
$x = 0, y = 4(-3)^2 - 26 = 36 - 26 = 10$ At $(0, 10)$
Specific behaviours
✓ correct coordinates

(ii) turning point of the graph of $y = (x+4)(x-2)$. (2 marks)

Solution
$x = (-4+2) \div 2 = -1$ $y = (-1+4)(-1-2) = -9$ At $(-1, -9)$
Specific behaviours
✓ correct x-coordinate ✓ correct y-coordinate

(b) The graph of $y = ax^2 + bx + c$ is shown below. Determine the value of the coefficients a, b and c. (4 marks)



Solution
$y = a(x+3)(x-5)$ $-5 = a(3)(-5) \Rightarrow a = \frac{1}{3}$ $y = \frac{1}{3}(x^2 - 2x - 15)$ $a = \frac{1}{3}, \quad b = -\frac{2}{3}, \quad c = -5$
Specific behaviours
✓ uses roots to write in factored form ✓ uses y-intercept to determine a ✓ expands quadratic ✓ states all coefficients

Question 5 **(7 marks)**

(a) Expand $x(x+4)^2$. (2 marks)

Solution
$x(x^2 + 8x + 16) = x^3 + 8x^2 + 16x$
Specific behaviours
✓ expands quadratic correctly ✓ correct expansion

(b) Let $f(x) = x^3 + 2x^2 - 5x - 6$.

(i) Determine $f(2)$. (1 mark)

Solution
$f(2) = 2^3 + 2(2)^2 - 5(2) - 6$ $= 8 + 8 - 10 - 6$ $= 0$
Specific behaviours
✓ correct value

(ii) Solve $f(x) = 0$. (4 marks)

Solution
$x^3 + 2x^2 - 5x - 6 = (x-2)(x^2 + bx + 3)$ $-5x = -2bx + 3x \Rightarrow b = 4$ $x^2 + 4x + 3 = (x+3)(x+1)$ $(x-2)(x+3)(x+1) = 0 \Rightarrow x = -3, -1, 2$
Specific behaviours
✓ uses (a) to write cubic as linear and quadratic factor ✓ determines entire quadratic factor ✓ factorises quadratic ✓ all correct solutions