

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Important note to candidates

Special items: nil

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate

Formula sheet

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Time allowed for this section

Your name

In words

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WA student number: In figures

Calculator-free

Section One:

UNITS 3&4

MATHEMATICS

METHODS

SOLUTIONS

Question/Answer booklet

Semester Two Examination, 2023

INDEPENDENT PUBLIC SCHOOL

Exceptional schools. Exceptional students.

PERTH MODERN SCHOOL



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	34.5
Section Two: Calculator-assumed	11	11	100	97	65.5
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

[See next page](#)

Working time: 50 minutes.

provided.

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces

34.5% (51 Marks)

Section One: Calculator-free

Question 1**(10 marks)**(a) Solve $\ln(x-2) = \ln x + 2$.

(4 marks)

Solution

$$\begin{aligned}\ln(x-2) &= \ln x + 2\ln e \\ \ln(x-2) &= \ln x + \ln e^2 \\ \ln(x-2) &= \ln(xe^2) \\ \therefore x-2 &= xe^2 \\ -2 &= x(e^2-1) \\ x &= \frac{-2}{(e^2-1)}\end{aligned}$$

Specific behaviours

- ✓ Expresses 2 in terms of logarithms.
- ✓ Shows use of logarithm rules
- ✓ Eliminates logarithms.
- ✓ Rearranges to find correct answer

(b) Find the exact solution of $2^{x-3} = 7$ and express your answer in terms of logarithms.

(3 marks)

Solution

$$\begin{aligned}(x-3)\log 2 &= \log 7 \\ (x-3) &= \frac{\log 7}{\log 2} \\ x &= \frac{\log 7}{\log 2} + 3\end{aligned}$$

Specific behaviours

- ✓ Shows use of logarithm rules
- ✓ Rearranges equation.
- ✓ Rearranges to find correct answer

(c) Show $\log_3 5 = \frac{\ln 5}{\ln 3}$

(3 marks)

Solution

$$\begin{aligned}\log_3 5 &= y \\ 3^y &= 5 \\ \ln(3^y) &= \ln 5 \\ y \ln 3 &= \ln 5 \\ y &= \frac{\ln 5}{\ln 3}\end{aligned}$$

Specific behaviours

- ✓ Changes equation into exponential form
- ✓ Uses logarithms of both sides.
- ✓ Shows use of logarithm rules.

[See next page](#)

(2 marks)

Specific behaviours <ul style="list-style-type: none"> ✓ indicates appropriate use of previous result ✓ indicates correct antiderivative
Solution
$\int (\ln(3x) + 5) dx = \int (\ln(3x) + 1) dx + \int 4 dx$ $\ln(3x) + 5 = \ln(3x) + 1 + 4$

Specific behaviours <ul style="list-style-type: none"> ✓ indicates correct derivative ✓ indicates use of product rule
Solution
$\frac{dy}{dx} = \ln(3x) + x \cdot \frac{3}{3x} = \ln(3x) + 1$

(2 marks)

(b) Determine $\frac{dy}{dx}(x \ln(3x))$.

Specific behaviours <ul style="list-style-type: none"> ✓ indicates correct derivative
Solution
$\frac{dy}{dx} = \ln(x^2 - 3x)$

(1 mark)

(ii) $y = \int_x^z \ln(t^2 - 3t) dt$.

Specific behaviours <ul style="list-style-type: none"> ✓ indicates correct derivative
Solution
$\frac{dy}{dx} = 3x^2 - 2\sin(2x - 3)$

(1 mark)

(i) $y = x^3 + \cos(2x - 3)$

(a) Determine $\frac{dy}{dx}$ when

(6 marks)

Question number:

Supplementary page

Question 3

(7 marks)

The time in minutes that Jake takes to serve a customer at the local supermarket follows a uniform distribution defined over the interval [2,8].

(a) Determine

(i) Jake's expected checkout time.

(1 mark)

Solution

$$E(X) = \frac{2+8}{2} = 5$$

Specific behaviours

- ✓ Correctly calculates $E(X)$.

(ii) the variance of the time taken to serve a customer.

(2 marks)

Solution

$$\text{VAR}(X) = \dots$$

Specific behaviours

- ✓ Correctly calculates variance.
- ✓ States correct variance (simplified)

(iii) the probability that he will take more than 6 minutes to serve a customer.

(1 mark)

Solution

$$P(X > 6) = \frac{2}{6} = \frac{1}{3}$$

Specific behaviours

- ✓ Correctly calculates answer

(b) Given Jake has already spent 3 minutes serving a customer, find the probability he will take less than another 4 minutes to finish. (3 marks)

Solution

$$P((X < 7 | (X > 3))) = P(\dots)$$

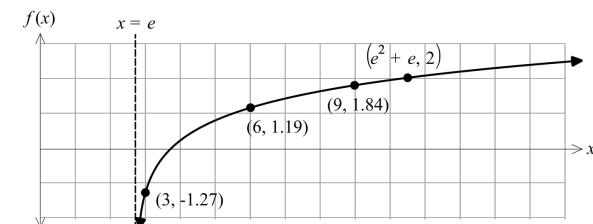
Specific behaviours

- ✓ Shows use of conditional probability
- ✓ Substitutes correctly.
- ✓ Correctly calculates answer

Question 7

(6 marks)

Part of the graph of $f(x) = \log_a(x-b)$, where $a > 1$, is shown below.

(a) State the value of b .

(1 mark)

Solution
$b = e$
Specific behaviours

- ✓ States value of b .

(b) Determine the value of a , given that the curve passes through $(e^2 + e, 2)$.

(2 marks)

Solution
$2 = \log_a(e^2)$
$a^2 = e^2$
$a = e$
Specific behaviours

- ✓ Recognises that $2 = \log_a(e^2)$ gives $a^2 = e^2$.
- ✓ Determines a .

Alternative Solution

$$\begin{aligned} 2 &= 2\log_a(e) \\ 1 &= \log_a e \\ \Rightarrow a &= e \end{aligned}$$

Specific behaviours
✓ Writes that $2 = \log_a(e^2)$ gives $1 = \log_a e$.
✓ Determines a .

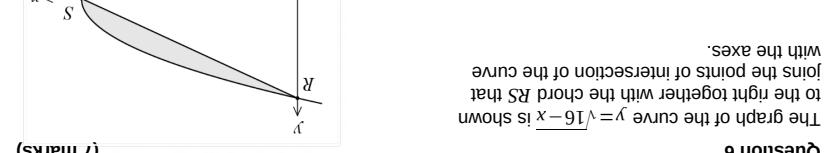
(c) Using the graph, determine an approximation to the following definite integral: (3 marks)

$$\int_3^{9+e} \frac{1}{x-e} dx$$

Solution
$\int_{3+e}^{9+e} \frac{1}{x-e} dx$
$\ln(x-e) _3^{9+e} = \ln(9+e) - \ln(3+e)$
$1.84 - (-1.27) = 3.11$
Specific behaviours

- ✓ Correctly integrates.
- ✓ Substitutes in boundaries
- ✓ Uses values from graph and determines answer.

Question 6
(8 marks)



(1 mark)

(a) Determine the slope of the curve at R.

(a) $V'(4)$.

Determine

$$V(t) = \frac{t^2+4}{10t}, 0 \leq t \leq 20.$$

A tank initially contains 24 L of water. Let $V(t)$ be the volume, in litres, of water in the tank t seconds after it is ruptured, so that

t seconds after it is ruptured, so that

Solution	
SPECIFIC BEHAVIOURS	
$V(4) = \frac{4^2+4}{10 \cdot 4} = -2 \text{ L/s}$	
(a) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$V(t) = \frac{-10(t^2+4)-10t(2t)}{-40-10t^2} = \frac{(t^2+4)}{-40-10t^2}$	
(b) $V''(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$V''(4) = \frac{-40-10(4)^2}{-40-160} = \frac{400}{120} = \frac{10}{3} \text{ L/s}^2$	
(c) $V''(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$V(4) = 24 + \int_4^{24} dt = 24 - 5 \int_4^{24} \frac{2t}{t^2+4} dt$	
(d) $V(4)$.	

(4 marks)

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \int [t^2+4]^{24}_4 dt = ?24 - 5 \ln [t^2+4]^{24}_4$	
(e) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [t^2+4]^{24}_4 = ?24 - 5 \ln [5^2+4] - 5 \ln [20^2+4]$	
(f) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [5^2+4] - 5 \ln [4^2+4] = ?24 - 5 \ln [25+4] - 5 \ln [16+4]$	
(g) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(h) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(i) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(j) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(k) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(l) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(m) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(n) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(o) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(p) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(q) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(r) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(s) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(t) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(u) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(v) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(w) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(x) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(y) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(z) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(aa) $V(4)$.	

Solution	
SPECIFIC BEHAVIOURS	
$?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4] = ?24 - 5 \ln [25+4] - 5 \ln [4]$	
(ab) $V(4)$.	

Solution

Question 5

(7 marks)

The random variable X takes the values 0, 1, 2, 3 only and its probability distribution is shown below.

x	0	1	2	3
$P(X=x)$	a	b	0.05	0.15

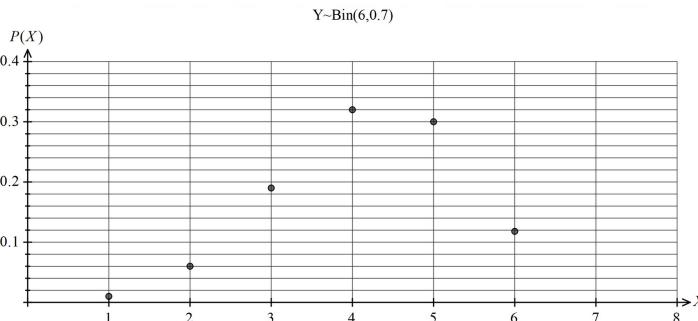
- (a) (i) Given that $E(X)=1.05$, find the values of a and b . (3 marks)

Solution
$b+0.1+0.45=1.05$
$b=0.5 \therefore a=0.3$
Specific behaviours
✓ Recognises correct equation for expected value. ✓ Correctly solves for b . ✓ States correct value for a .

- (ii) Does X have a binomial distribution? Justify your answer. (2 marks)

Solution
No
Possible answer: $n=3$, $np=1.05$ gives $p=0.35$
$P(X=3)=0.15 \neq 0.35^3$
Specific behaviours
✓ Gives correct answer. ✓ Justifies answer mathematically.

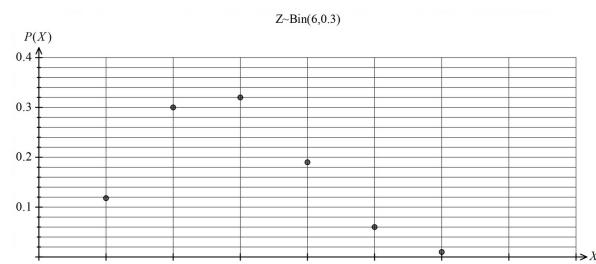
- (b) A binomial distribution for $Y \sim Bi(6, 0.7)$ is shown below.



Draw a graph of $Z \sim Bi(6, 0.3)$ on the axes below. (2 marks)

See next page

Solution



Specific behaviours

- ✓ Recognises graph reflects the previous graph.
- ✓ Plots graph correctly.

See next page