PERTH COLLEGE

Year 12

Semester One Examination 2012

Question/Answer booklet

MATHEMATICS 3CMAS/3DMAS

Section Two (Calculator - assumed)

Student Name: ___________________________________

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section Two
Formula sheet

To be provided by the candidate
Standard items: *pens, pencils, pencil sharpener, highlighter, eraser, ruler*

Special items: *drawing instruments, templates, notes (both sides of two unfolded sheets of A4 paper) and up to three calculators (CAS, graphic or scientific) which satisfy the conditions set by the Curriculum Council for this course.*

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.
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Structure of this paper

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One Calculator-free	5	5	50 minutes	50
Section Two Calculator-assumed	**12**	**12**	**100 minutes**	**100**
			Total marks	150

Instructions to candidates

1. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer
    a. Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    b. Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.

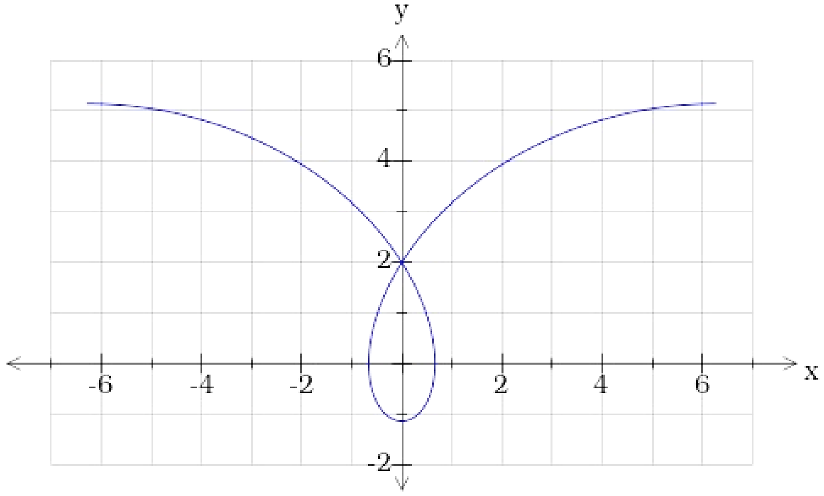
3. It is recommended that you **do not use pencil,** except in diagrams.

Question 6 **[7 marks]**

The *prolate cycloid* represented in the diagram below is defined by the parametric equations

$x=2t-\pi\sin t$ and $y=2-\pi\cos t$ for $-\pi\leq t\leq\pi$.

It crosses itself at the point $(0,2)$. At this point there are two tangent lines.
Find (in exact terms) the equations of each of these lines.



Question 7 **[6 marks]**

Determine, exactly, the equation of the tangent to the curve defined by $e^{y+3} = ln\,(x-3)$ at the point where $x = e + 3$.

Question 8 [8 marks]

a) Differentiate with respect to x: $y = x^{\sin(2x)}$

[4]

b) Determine each of the following indefinite integrals:

(i) $\int \left(x^e + e^x \right) dx$

[2]

(ii) $\int \left(4x+6\right)\left(e^{x^2+3x}\right) dx$

[2]

Question 9 [11 marks]

Evaluate each of the following definite integrals, using the suggested substitutions. You must show all of your working.

a) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin^3 x}{\cos^4 x}\,dx$ using the substitution $u = \cos x$.

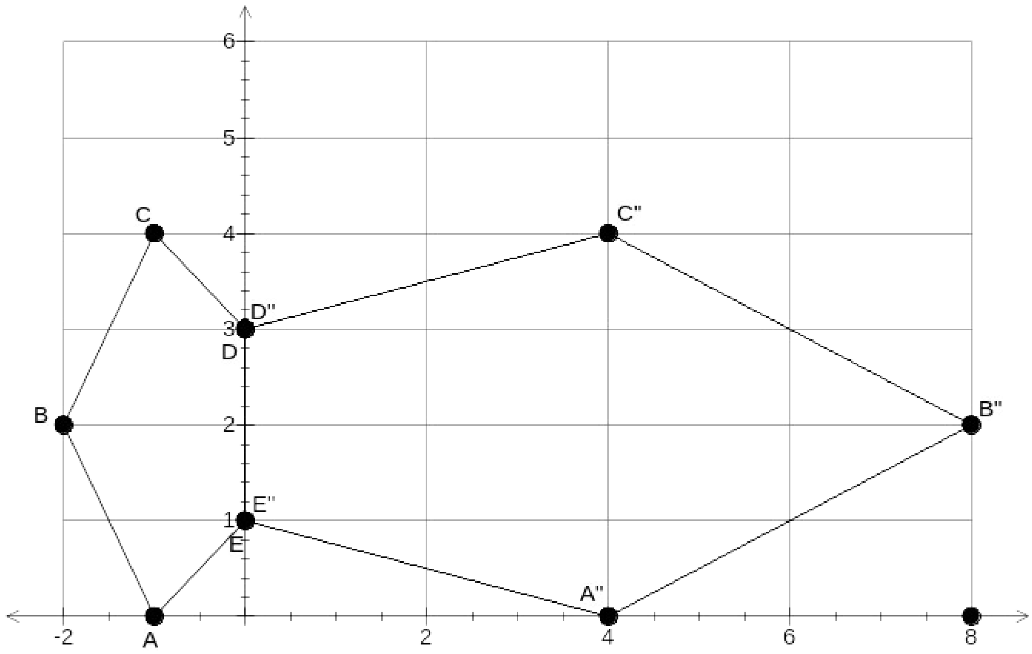
[6]

b) $\int_0^1 \frac{1}{\sqrt{4-x^2}}\ dx$ using the substitution $x = 2\cos\theta$.

[5]

Question 10 [11 marks]

The diagram below shows pentagon ABCDE, and the resulting pentagon A"B"C"D"E", after having undergone two matrix transformations, T_1 followed by T_2.



a) Pentagon ABCDE is reflected in the y-axis (T_1). Draw the resulting image A'B'C'D'E' on the diagram above.

[1]

b) The pentagon A'B'C'D'E' is then transformed by matrix T_2 to A"B"C"D"E", as shown in the diagram above. Find and describe the transformation represented by matrix T_2.

[3]

c) Show how T_1 and T_2 combine to map ABCDE directly to A"B"C"D"E" and give the resulting matrix.

[2]

d) Determine the ratio Area ABCDE : Area A''B''C''D''E''.

[1]

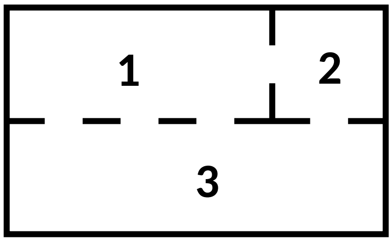
e) What matrix would transform A''B''C''D''E'' back to ABCDE?

[2]

f) Consider $T_3 = \begin{bmatrix} 0 & k \\ k & 0 \end{bmatrix}$. If T_3 is applied to ABCDE, the area of the resulting pentagon will be double the area of ABCDE. Determine the value(s) of k.

[2]

Question 11 **[8 marks]**



A mouse is placed in the maze shown on the right.

During a fixed time interval, the mouse randomly chooses one of the doors (openings) available to it, and moves into the next room. It ***does not*** remain in the room it occupies.

If the mouse started in ***Room 1***, the probabilities that the mouse will be in each room after 1 transition are:

Room 1:	0	(since it must move to another room)
Room 2:	0.25	(since 1 out of the 4 doors available lead to Room 2)
Room 3:	0.75	

a) The first column of the transition matrix is show below. Complete the remainder of the transition matrix.

$$\begin{array}{ccccc} & & & \text{From room} & \\ & & 1 & 2 & 3 \\ & 1 & & & \text{¿} \\ \text{To room} & 2 & \text{¿.¿.} & & \\ & 3 & & & \end{array}$$

¿.

[2]

b) If the mouse starts in Room 3, what is the probability that it is in Room 1 after 4 transitions?

[2]

c) Using appropriate rounding, determine the "**Stable State Matrix**".

[2]

d) As the number of transitions becomes large, what is the probability that the mouse will be in Room 2?

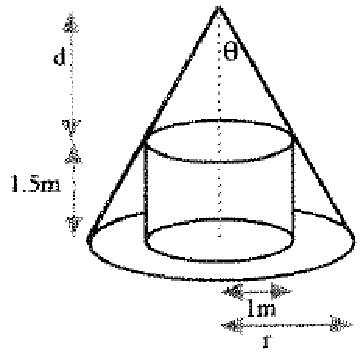
[1]

e) In the long run, what percentage of the time will the mouse spend in Rooms 1 or 2?

[1]

Question 12 [8 marks]

A cylindrical platform of height 1.5 metres and base radius 1 metre sits on a stage with a vertical axis. To experiment with lighting effects, a point source of light casting a shadow of the cylinder on the stage, is moved upwards so that the angle θ shown in the diagram decreases at a rate of 0.1 radians per second.



a) Show that $d = \dfrac{1.5}{r-1}$. [2]

b) Express r in terms of θ. [2]

c) Find the rate at which the radius of the shadow is decreasing when $\theta = \dfrac{\pi}{6}$. [4]

Question 13 [9 marks]

a) Consider the following information about matrices ***A***, ***B*** and ***C*** :

- They are all 2 x 2 matrices
- Matrix ***B*** is a non-singular matrix
- $\boldsymbol{A} = \boldsymbol{BCB}^{-1}$

Find a simplified expression for $\boldsymbol{A}^2$ and $\boldsymbol{A}^3$. Use your results to deduce $\boldsymbol{A}^n$.

[3]

b) Let $\mathbf{A}^2 - 6\mathbf{A} + 5\mathbf{I} = \mathbf{O}$ where $\mathbf{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

(i) Show that $\mathbf{I} = \frac{1}{5}\mathbf{A}(6\mathbf{I} - \mathbf{A})$.

[2]

(ii) Express $\mathbf{A}^4$ in the form $p\mathbf{A} + q\mathbf{I}$ (ie… find the values of p and q).

[4]

Question 14 **[8 marks]**

a) Let $\boldsymbol{A} = \begin{bmatrix} -4 & 20 & 2 \\ -3 & 15 & -3 \\ 7 & -17 & 1 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 1 & 1 \\ 3 & -4 & 0 \end{bmatrix}$.

Determine $\boldsymbol{C} = \boldsymbol{AB}$.

[1]

b) Tickets to a concert cost $2 for children, $3 for teenagers and $5 for adults. 570 people attended the concert and the total ticket receipts were $1950. The ratio of teenagers to children attending was 3 to 4.

(i) Write 3 equations to represent this information, using the variables *c*, *t* and *a* to represent the number of children, teenagers and adults respectively.

[3]

(ii) Represent your answer to b) (i) to write a matrix equation in the form ***PX = Q***, where $\boldsymbol{X} = \begin{bmatrix} c \\ t \\ a \end{bmatrix}$.

[1]

(iii) **Use your answer from a)** to determine how many children, teenagers and adults attended the concert.

[3]

Question 15 [11 marks]

A rocket ship leaves space station A, which is located at $\begin{pmatrix} -20 \\ 40 \\ 20 \end{pmatrix}$ km, at 9 am. It travels with a constant velocity of $\begin{pmatrix} 60 \\ 120 \\ 360 \end{pmatrix}$ km h^{-1}. At some time it is supposed to reach the neighbouring space station B, which is located at $\begin{pmatrix} 80 \\ 160 \\ 2020 \end{pmatrix}$ km.

a) Show that this rocket ship will not reach space station B on its current trajectory.

[3]

b) Find the closest distance between the rocket ship and space station B and the time when this occurs. Answer correct to the nearest minute.

[4]

A second rocket ship is launched from space station B at 9 am with constant velocity and is aimed to collide with the first rocket at exactly 1 pm.

c) Determine the velocity of the second rocket ship that will ensure collision takes place at the required time.

[4]

Question 16 [6 marks]

The table below shows the details of a population of kangaroos in a region of Western Australia in 2000.

Age (years)	0 – 2	2 – 4	4 – 6	6 - 8	8 - 10
Initial population	1200	1400	1600	810	425
Breeding Rate	0	0.1	3.5	2.5	0.5
Survival Rate	0.4	0.5	0.7	0.2	0

a) Write down the Leslie matrix, L, for this population.

[1]

b) What was the expected total population in 2006?

[1]

c) Find the percentage growth rate between the 3rd and 4th generation.

[2]

Due to complaints from local property owners, a decision was made to control the population by culling.

d) To reduce the population growth, 10 % of kangaroos aged between 4 and 8 years are culled at the beginning of every second year. What will the expected population be in 2016?

[2]

Question 17 [7 marks]

The lines l_1 and l_2 have equations

$\mathbf{r}_1 = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ respectively, where λ and μ are parameters.

a) Find the acute angle between l_2 and the line joining the points P$(1,-1,1)$ and Q $(2,-1,-4)$, giving your answer correct to the nearest degree.

[2]

b) Determine the position vector of the point R that lies on the line joining P$(1,-1,1)$ and Q $(2,-1,-4)$ such that PR : RQ = 1 : 2.

[3]

c) Find an equation in the form $r \cdot n = \rho$, of the plane that passes through Q$(2,-1,-4)$ and is perpendicular to l_1.

[2]

EXTRA PAGES FOR WORKING
Clearly number any questions you do here.

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Clearly number any questions you do here.

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Clearly number any questions you do here.

Your Name: ________________________________

	Question	Marks Available	Your Mark
	6	7	
	7	6	
	8	8	
	9	11	
	10	11	
	11	8	
	12	8	
	13	9	
	14	8	
	15	11	
	16	6	
	17	7	
	TOTAL SECTION 2	100	