

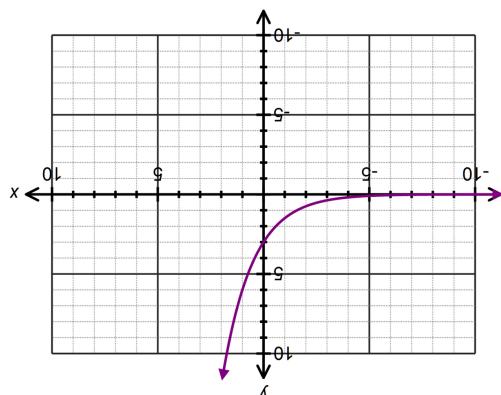
Calculator Free  
Logarithmic Graphs and Differentiation

Time: 45 minutes  
Total Marks: 45  
Your Score: / 45



Mathematics Methods Unit 4

Question One: [2, 3, 2 = 7 marks]      CR  
Consider the exponential function drawn below.



(a) State the equation of the exponential function in the form  $y = a \times b^x$ .

(b) Use the exponential graph drawn, and an appropriate mirror line, to draw the logarithmic function which is the inverse of the given exponential function.

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- (c) Hence or otherwise determine the equation of the logarithmic function,  
 $y = \log_a(bx)$  which is the inverse of the given exponential function with the same base.

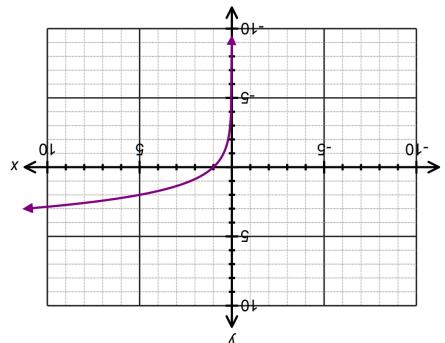
$$\begin{aligned}
 & \therefore y = 3e^x - 2e^{-x} \\
 & c = -2e^{-x} \\
 & e^x = 3e(e) + c \\
 & y = 3e^x + c \\
 & \frac{dy}{dx} = 3e^x \\
 & \left. \frac{dy}{dx} \right|_{x=0} = 2e \ln(e) + e \\
 & \frac{dy}{dx} = 2x \ln(x) + x \\
 & \frac{dy}{dx} = 2x \ln(x) + \frac{x}{x} \\
 & y = e^x \ln(e) = e^x
 \end{aligned}$$

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- (b) Determine the equation of the tangent to the curve  $y = x \ln(x)$  at the point where  $x = e$ . Leave your answers as exact simplified values.

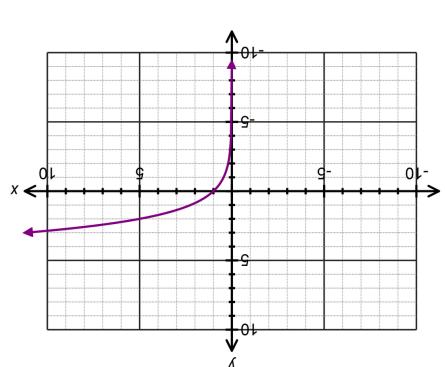
$$\begin{aligned}
 & \text{gradient of } 2. \\
 & \text{Determine the coordinates of the point(s) where the curve } y = \ln(2x - 5) + 1 \text{ has a} \\
 & \text{gradient of } 2. \\
 & \frac{dy}{dx} = \frac{2}{2x - 5} \\
 & 2 = \frac{2}{2x - 5} \\
 & 2(2x - 5) = 2 \\
 & 4x - 10 = 2 \\
 & 4x = 12 \\
 & x = 3 \\
 & y = \ln(1) + 1 = 1
 \end{aligned}$$

- (a) Determine the equation of each of the following graphs drawn below:



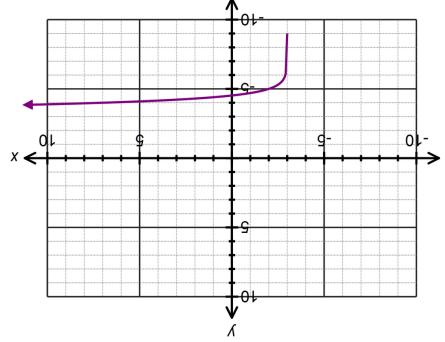
(a)

Determine the equation of each of the following graphs drawn below:



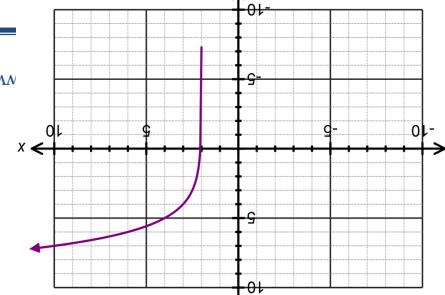
(b)

Determine the equation of each of the following graphs drawn below:



(c)

Determine the equation of each of the following graphs drawn below:



(d)

Determine the equation of each of the following graphs drawn below:

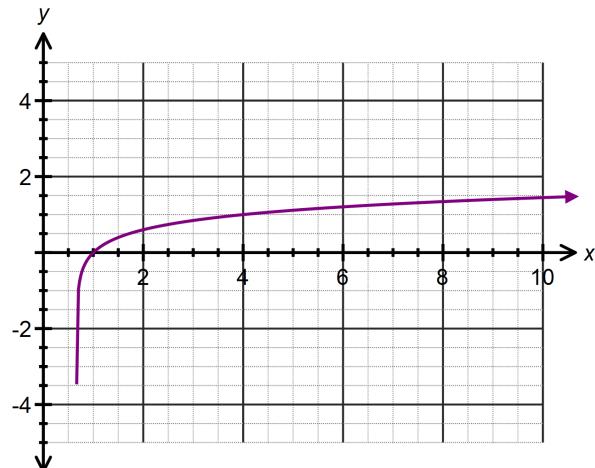
Question Two: [2, 3, 3 = 8 marks] CR

Mathematics Methods Unit 4

Mathematics Methods Unit 4

**Question Three:** [2, 1, 2 = 5 marks] CF

$f(x) = \log(ax - 2)$   
The function is drawn below.



(a) Determine the value of  $a$ .

$$\log(ax - 2) = -1$$

(b) Use the graph to approximate the solution to

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$$\frac{dy}{dx} = \frac{3 \cos 3x}{\sin 3x}$$

(e)  $y = \log_2(x^3 - 2x)$

$$y = \frac{\ln(x^3 - 2x)}{\ln 2}$$

$$\frac{dy}{dx} = \frac{3x^2 - 2}{(x^3 - 2x)\ln 2}$$

(f)  $y = 5^x$

$$\ln y = x \ln 5$$

$$y = e^{x \ln 5}$$

$$\frac{dy}{dx} = \ln 5 e^{x \ln 5} = \ln 5 (5^x)$$

**Question Four:** [1, 3, 3, 2, 3, 3 = 15 marks] **CF**

(c) Solve  $\log(ax - 2) = 2$  algebraically.

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Differentiate each of the following with respect to  $x$ , showing full working:

(a)  $y = \ln(4x - 5)$

(b)  $y = \int x e^{-x} dx$

(c)  $\left( \frac{1-x}{e^x} \right)^n = (x)b$

(d)  $y = \ln(\sin(x))$

(e)  $y = \log_e(x - 2x)$

(f)  $y = \ln(\sin(x))$

$$\begin{aligned} y &= \ln(\sin(x)) \\ y &= \ln\left(\frac{1-x}{e^x}\right) \\ y &= \ln(x) - \ln(e^x) \\ y &= \ln(x) - \ln(e^{x-1}) \end{aligned}$$

$$\left( \frac{1-x}{e^x} \right)^n = (x)b$$

$$\begin{aligned} \frac{x}{e^{x-1}} + (x)e^{-x-1} &= (x) \\ (x)e^{-x+1} &= (x) \\ (x)e^{-x+1} &= (x) \end{aligned}$$

$$\frac{dy}{dx} = \frac{4x-5}{4}$$

(g)  $y = \ln(4x - 5)$

Differentiate each of the following with respect to  $x$ , showing full working:

**Question Four:** [1, 3, 3, 2, 3, 3 = 15 marks] **CF**

$$x = \frac{3}{98}$$

$$3x = 98$$

$$3x - 2 = 100$$

$$\log(3x - 2) = 2$$

$$\begin{aligned} \log(ax - 2) &= 2 \\ ax - 2 &= 100 \\ ax &= 102 \\ a &= 102/a \end{aligned}$$

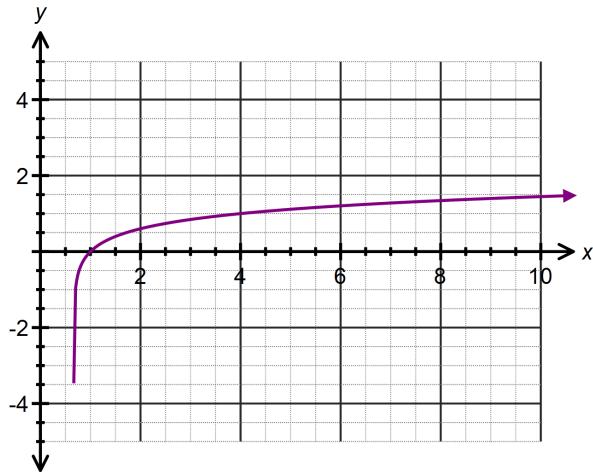
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(f)  $y = 5^x$

**Question Three:** [2, 1, 2 = 5 marks]**CF**

$f(x) = \log(ax - 2)$

The function is drawn below.

(a) Determine the value of  $a$ .

$0 = \log(a - 2)$  ✓

$1 = a - 2$

$3 = a$  ✓

$\log(ax - 2) = -1$

(b) Use the graph to approximate the solution to

$x \approx 0.8$  ✓



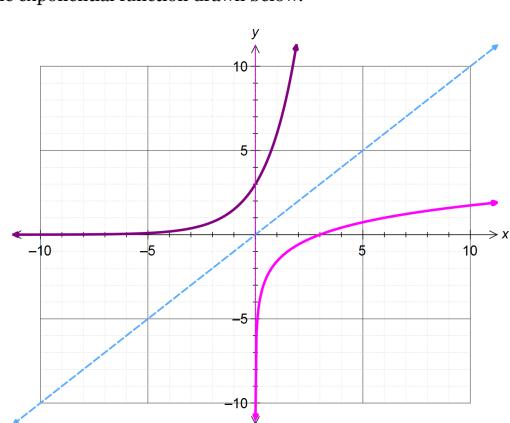


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**Question One: [2, 3, 2 = 7 marks]****CF**

Consider the exponential function drawn below.



- (a) State the equation of the exponential function in the form  $y = a \times b^x$ .

$$y = 3 \times 2^x$$

✓ ✓

- (b) Use the exponential graph drawn, and an appropriate mirror line, to draw the logarithmic function which is the inverse of the given exponential function.

- (c) Hence or otherwise determine the equation of the logarithmic function,  $y = \log_a(bx)$ , which is the inverse of the given exponential function with the same base.

$$y = \frac{\log(x)}{\log 2} = \log_2\left(\frac{x}{3}\right)$$

✓ ✓