Christ Church Grammar School

2019
TEST 6

MATHEMATICS METHODS Year 11

Section One:
Calculator-free

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Reading time for this section:	3 minutes
Working time for this section:	25 minutes
Marks available:	25 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items:	pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 8 **(8 marks)**

A particle is moving along in a straight line. At time t seconds its displacement s metres from a fixed point F is given by $s = t^3 - 12t^2 + 45t + 10$.

(a) Find the velocity, v, in terms of t. (1 mark)

$v(t) = 3t^2 - 24t + 45$

✓ differentiates $s(t)$ correctly

(b) Determine when the particle is stationary. (2 marks)

$v(t) = 3t^2 - 24t + 45 = 0$

$\therefore$ $t = 3$ sec or $t = 5$ sec

✓ states that $v(t) = 0$

✓ writes 2 correct times

(c) Determine the average velocity during the first 6 seconds. (2 marks)

$\bar{v} = \frac{s(6) - s(0)}{6}$

$= \frac{64 - 10}{6}$

$= 9$ m/s

✓ calculates change in displacement

✓ states correct answer with correct units

(d) Determine the total distance travelled by the particle during the first 6 seconds. (3 marks)

$s(0) = 10$ → 54

$s(3) = 64$ → 4

$s(5) = 60$ → 4

$s(6) = 64$

$\therefore$ distance travelled $= 54 + 4 + 4$

$= 62$ m

✓ calculates $s(3)$ and $s(5)$

✓ adds separate distances

✓ states correct answer

Question 7 **(3 marks)**

The sum of the first 4 terms of a geometric sequence is 20 and the sum of the first 6 terms of the sequence is 182. Determine the tenth term of the sequence.

$\frac{a(1-r^4)}{1-r} = 20$ and $\frac{a(1-r^6)}{1-r} = 182$

CAS solve: $a=-1, r=-3$ or $a=0.5, r=3$

$\therefore T_{10} = -1 \times (-3)^9$ $\quad T_{10} = 0.5 \times 3^9$

$= \underline{\underline{19683}}$ or $= \underline{\underline{9841.5}}$

✓ sets up 2 equations using sum formula

✓ solves for a and r

✓ calculates both possible values of T_{10}

* Note: deduct 1 mark if only one set of values of a and r used.

Question 1 **(4 marks)**

State whether the following sequences are arithmetic (A), geometric (G) or neither (N).

(a) $-1,\ 1,\ -1,\ 1,\ -1,\ 1, \ldots$	G	✓
(b) $\pi,\ 2\pi,\ 3\pi,\ 4\pi, \ldots$	A	✓
(c) $T_{n+1} = 3T_n - 1$	N	✓
(d) $T_n = 3n + 6$	A	✓

correct answers

Question 2 **(3 marks)**

Use the infinite sum formula to express the recurring decimal $0.2\dot{7}$ as a fraction.

$0.2\dot{7} = \frac{2}{10} + \underbrace{\frac{7}{100} + \frac{7}{1000} + \frac{7}{10000} + \ldots}$

GP: $a = \frac{7}{100}$, $r = \frac{1}{10}$ ✓ determines a and r for the GP

$S_\infty = \frac{7/100}{9/10}$ ✓ uses infinite sum formula correctly

$= \frac{7}{90}$

$\therefore 0.2\dot{7} = \frac{2}{10} + \frac{7}{90}$

$= \frac{25}{90}$ ✓ correct fraction

(OR $\frac{5}{18}$)

Question 3 **(8 marks)**

Antidifferentiate with respect to x:

(a) $\frac{dy}{dx} = \frac{1}{2}x^4$ (2 marks)

$y = \frac{x^5}{10} + c$

✓anti-differentiates
* ✓adds constant of integration

(b) $f'(x) = (3x+2)(x-5)$ (2 marks)

$= 3x^2 - 13x - 10$

$\therefore f(x) = x^3 - \frac{13x^2}{2} - 10x + c$

✓expands expression
✓antidifferentiates

(c) $g'(x) = \sqrt{x}$ (2 marks)

$= x^{\frac{1}{2}}$

$\therefore g(x) = \frac{2}{3}x^{\frac{3}{2}} + c$

✓writes $\sqrt{x}$ in index form
✓antidifferentiates

(d) $\frac{dy}{dx} = \frac{3}{x^2}$ (2 marks)

$= 3x^{-2}$

$\therefore y = \frac{-3}{x} + c$

✓writes $\frac{dy}{dx}$ using negative index
✓antidifferentiates

* Note: deduct 1 mark if constant of integration is omitted in 3(a) only.

See next page

Question 6 **(4 marks)**

A colony of rabbits on an offshore island is subjected to a controlled release of a deadly virus. The population of rabbits R on the island n months after the introduction of the virus is modelled by the pattern established in the following table.

n (months)	0	1	2	3
R (rabbits)	10 000	9 000	8 100	7290

(a) Write a recursive rule to represent the rabbit population R after n months. (2 marks)

$T_{n+1} = 0.9T_n$

Either: $T_0 = 10\,000$ or $T_1 = 9000$

✓writes recursive rule correctly
✓writes initial term correctly

(b) Find the rabbit population after 1 year. (1 mark)

$T_{12} = 2824$ rabbits

✓calculates T_{12} correctly

(c) Determine when the number of rabbits first drops below 100. (1 mark)

After 43rd month or during 44th month

✓writes correct month (either accepted)

See next page

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

Question 4 **(5 marks)**

Given the following sequence of numbers:

16, 12, 8, 4, 0, -4, ...

(a) Give the explicit formula for this sequence. (1 mark)

$T_n = 16 - 4(n-1)$ ✓correct rule

(OR) $T_n = 20 - 4n$

(b) Find the 20th term. (1 mark)

$T_{20} = 20 - 4(20)$

$= -60$ ✓correct answer

(c) Which term of the sequence is the first to be less than -99?. (3 marks)

$20 - 4(n) = -99$ ✓sets up equation

$-4n = -119$

$n = \frac{119}{4} = 29\frac{3}{4}$ ✓solves equation

∴ The 30^{th} term ✓correct term number provided

Question 5 **(5 marks)**

Determine the equation of the curve with gradient function $f'(x) = ax + b$ where a and b are constants, given $f(1) = 2$ and the curve has a turning point at $(-1, 0)$.

$f(x) = \frac{ax^2}{2} + bx + c$ ✓antidifferentiates $f'(x)$

since $(-1,0)$ is a turning point, $f'(-1) = 0$

i.e. $-a + b = 0$

$\therefore a = b$ ✓determines relationship between a and b

sub $(1,2)$: $\frac{a}{2} + b + c = 2$

i.e. $3a + 2c = 4$ ——— ①

sub $(-1,0)$: $\frac{a}{2} - b + c = 0$

i.e. $-a + 2c = 0$ ——— ② ✓sets up 2 equations between a and c

① − ②: $4a = 4$

$a = 1$

$\therefore b = 1$ ✓solves for a and b

$\therefore c = \frac{1}{2}$ ✓ solves for c

So, $f(x) = \frac{1}{2}x^2 + x + \frac{1}{2}$

End of Questions

Christ Church Grammar School

2019
TEST 6

MATHEMATICS METHODS Year 11

Section Two:
Calculator-assumed

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Reading time for this section:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items:	pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	drawing instruments, templates and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.