Calculator Free
Integration, Fundamental Theorem of Calculus, Area

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 =8 marks] **CF**

(a) Calculate $\int \cos\left(\frac{t}{3}\right) dt$

(b) Use your answer to part (a) to evaluate $\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt$, in terms of x

(c) Use your answer to part (b) to evaluate $\frac{d}{dx}\left(\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt\right)$

(d) Hence evaluate $\frac{d}{dx}\left(\int_{\pi}^{f(x)} \cos\left(\frac{t}{3}\right) dt\right)$

Question Two: [2, 2, 2 = 6 marks] CF

Determine each of the following:

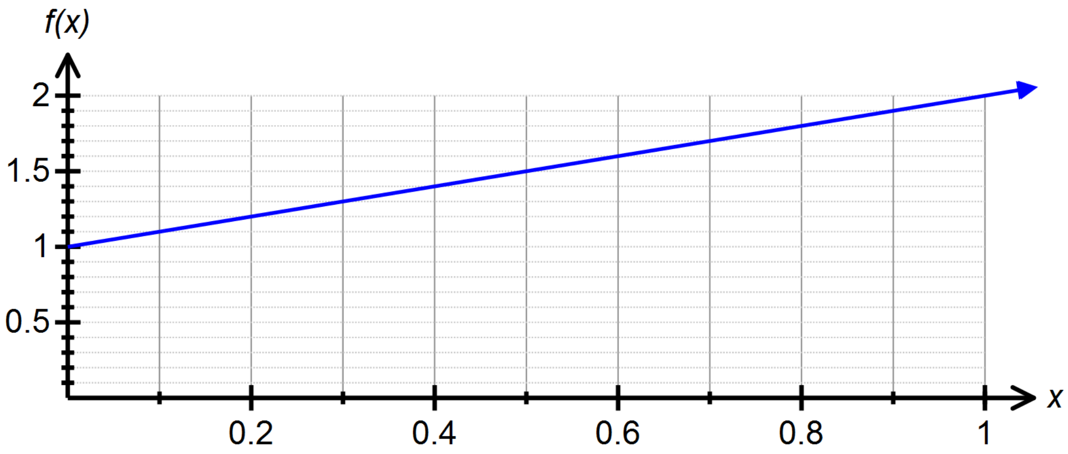
(a) $\int_{-1}^{1} 2x^3 \, dx$

(b) $\int_{-1}^{0} e^x \, dx - \int_{1}^{0} e^x \, dx$

(c) $\frac{d}{dx}\left(\int_{-3}^{x^2} \frac{\sqrt{2t-3}}{t+1} \, dt \right)$

Question Three: [2, 3, 2 = 7 marks] CF

Consider the function $f(x)$ drawn below over the domain $0 \le x \le 1$



(a) Draw rectangles on your graph that can be used to underestimate the area under $f(x)$ over the domain $0 \le x \le 1$, where $\delta x = 0.2$.

(b) Show that $\sum_5 f(x_5)\delta x_5 = \frac{7}{5}\ units^2$

(c) Use the graph of $f(x)$ above to calculate $\int_0^1 f(x)\ dx$

Question Four: [4, 5 = 9 marks] CF

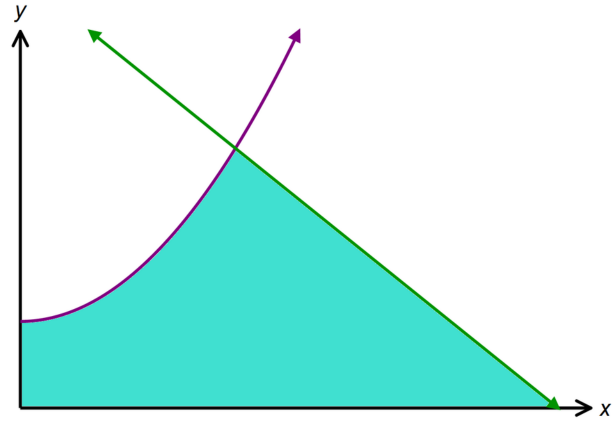
Consider the function $f(x) = x^3 + 2x^2 - x - 2$

(a) Determine the roots of the function.

(b) Hence determine the area bounded by the curve and the $x-$ axis.

Question Five: **[1, 2, 4 = 7 marks]** **CF**

The functions $f(x) = x^2 + 2$ and $h(x) = -2x + 10$ are drawn below.



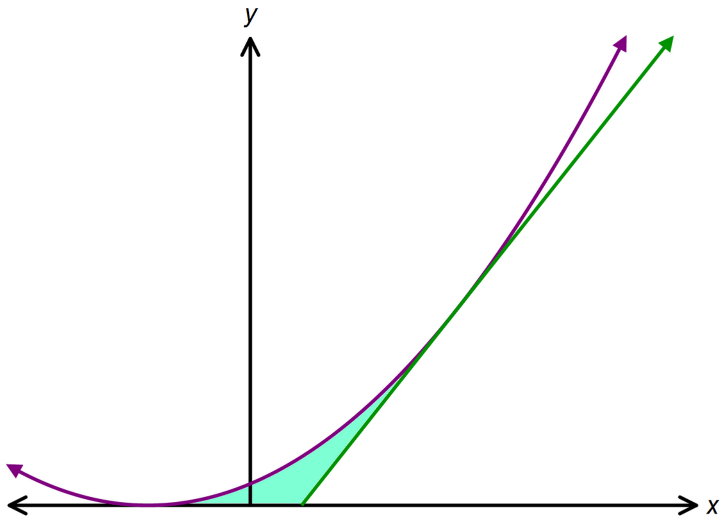
(a) Solve $h(x) = 0$

(b) Solve $f(x) = h(x)$

(c) Hence find the area shaded on the graph above.

Question Six: [3, 5 = 8 marks] **CF**

The curve $y = (x+1)^2$ and the tangent line at $x = 2$ are graphed below.



(a) Determine the equation of the tangent to $y = (x+1)^2$ drawn above.

(b) Hence find the area shaded on the graph above.

Mathematics Methods Unit 3

SOLUTIONS
Calculator Free
Integration, Fundamental Theorem of Calculus, Area

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 =8 marks] **CF**

(a) Calculate $\int \cos\left(\frac{t}{3}\right) dt$

$$=3\sin\frac{t}{3}+c$$

(b) Use your answer to part (a) to evaluate $\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt$, in terms of x

$$=\left[3\sin\frac{t}{3}+c\right]_{\pi}^{2x+1}$$

$$=\left(3\sin\frac{2x+1}{3}+c\right)-\left(3\sin\frac{\pi}{3}+c\right)$$

$$=3\sin\frac{2x+1}{3}-\frac{3\sqrt{3}}{2}$$

(c) Use your answer to part (b) to evaluate $\frac{d}{dx}\left(\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt\right)$

$$\frac{d}{dx}\left(3\sin\frac{2x+1}{3}-\frac{3\sqrt{3}}{2}\right)$$

$$=3\cos\frac{2x+1}{3}\times 2$$

$$=6\cos\frac{2x+1}{3}$$

(d) Hence evaluate $\dfrac{d}{dx}\left(\int_{\pi}^{f(x)} \cos\left(\dfrac{t}{3}\right) dt\right)$

$$=\cos\left(\frac{f(x)}{3}\right)\times f'(x)$$

Question Two: [2, 2, 2 = 6 marks] CF

Determine each of the following:

(a) $\int_{-1}^{1} 2x^3\, dx$

$$= \left[\frac{2x^4}{4}\right]_{-1}^{1}$$

$$= \frac{1}{2} - \frac{1}{2}$$

$$= 0$$

(b) $\int_{-1}^{0} e^x\, dx - \int_{1}^{0} e^x\, dx$

$$= \int_{-1}^{0} e^x\, dx + \int_{0}^{1} e^x\, dx$$

$$= \int_{-1}^{1} e^x\, dx$$

$$= \left[e^x\right]_{-1}^{1}$$

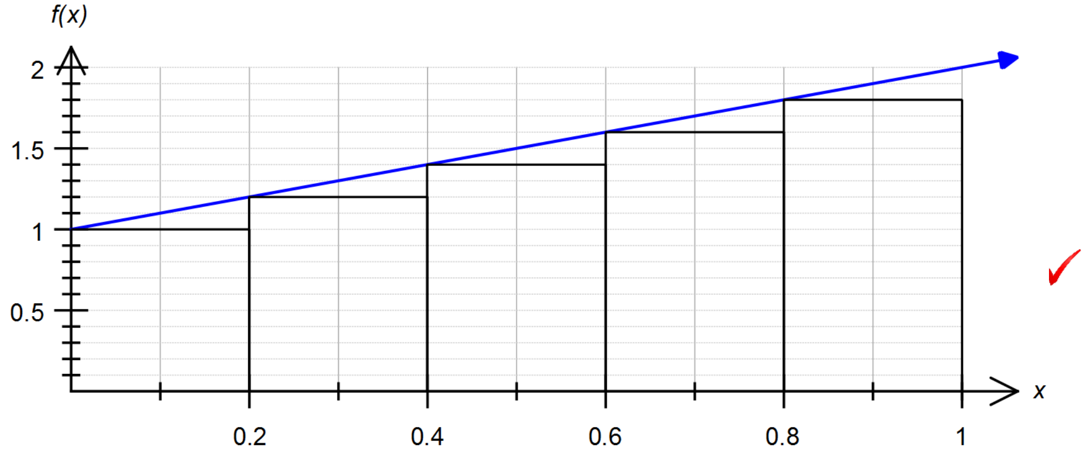
$$= e^1 - e^{-1}$$

(c) $\frac{d}{dx}\left(\int_{-3}^{x^2} \frac{\sqrt{2t-3}}{t+1}\, dt\right)$

$$= \frac{\sqrt{2x^2-3}}{x^2+1} \times 2x$$

Question Three: **[2, 3, 2 = 7 marks]** **CF**

Consider the function $f(x)$ drawn below over the domain $0 \le x \le 1$



(a) Draw rectangles on your graph that can be used to underestimate the area under $f(x)$ over the domain $0 \le x \le 1$, where $\delta x = 0.2$.

(b) Show that $\sum_5 f(x_5)\delta x_5 = \frac{7}{5}\ units^2$

$\sum_5 f(x_5)\partial x_5 = 0.2\times 1 + 0.2\times 1.2 + 0.2\times 1.4 + 0.2\times 1.6 + 0.2\times 1.6$

$= 0.2(1 + 1.2 + 1.4 + 1.6 + 1.8)$

$= \frac{1}{5}\times 7$

$= \frac{7}{5} units^2$

(c) Use the graph of $f(x)$ above to calculate $\int_0^1 f(x)\, dx$

$= \frac{1(1+2)}{2} = \frac{3}{2}$

Question Four: [4, 5 = 9 marks] CF

Consider the function $f(x) = x^3 + 2x^2 - x - 2$

(a) Determine the roots of the function.

$x = 1 \text{ is a factor}$

$$\begin{array}{r}
x^2 + 3x + 2 \\
x - 1 \overline{)\, x^3 + 2x^2 - x - 2} \\
\underline{x^3 - x^2 \quad\quad\quad\;\;} \\
3x^2 - x \quad\;\; \\
\underline{3x^2 - 3x \quad\;\;} \\
2x - 2 \\
\underline{2x - 2} \\
0
\end{array}$$

$f(x) = (x - 1)(x^2 + 3x + 2)$

$f(x) = (x - 1)(x + 2)(x + 1)$

$roots = (1, 0)\ (-2, 0)\ (-1, 0)$

(b) Hence determine the area bounded by the curve and the $x-$ axis.

$$= \int_{-2}^{-1} f(x)\,dx + \left|\int_{-1}^{1} f(x)\,dx\right|$$

$$= \left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} - 2x\right]_{-2}^{-1} + \left|\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{1}\right|$$

$$= \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right) - \left(4 - \frac{16}{3} - 2 + 4\right) + \left|\left(\frac{1}{4} + \frac{2}{3} - \frac{1}{2} - 2\right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right)\right|$$

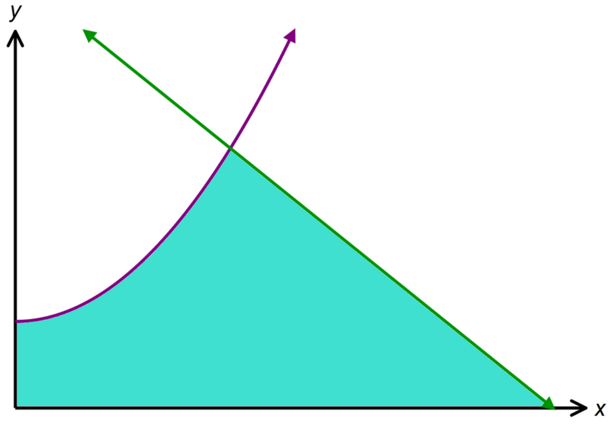
$$= \frac{-1}{4} - 4 + \frac{14}{3} + \left|\frac{4}{3} - 4\right|$$

$$= \frac{5}{12} + 2\frac{2}{3}$$

$$= 3\frac{1}{12} units^2$$

Question Five: [1, 2, 4 = 7 marks] CF

The functions $f(x) = x^2 + 2$ and $h(x) = -2x + 10$ are drawn below.



(a) Solve $h(x) = 0$

$-2x + 10 = 0$

$-2x = -10$

$x = 5$ ✓

(b) Solve $f(x) = h(x)$

$x^2 + 2 = -2x + 10$

$x^2 + 2x - 8 = 0$ ✓

$(x + 4)(x - 2) = 0$

$x = -4,\ x = 2$ ✓

(c) Hence find the area shaded on the graph above.

$$Area = \int_0^2 x^2 + 2\,dx + \int_2^5 -2x + 10\,dx$$

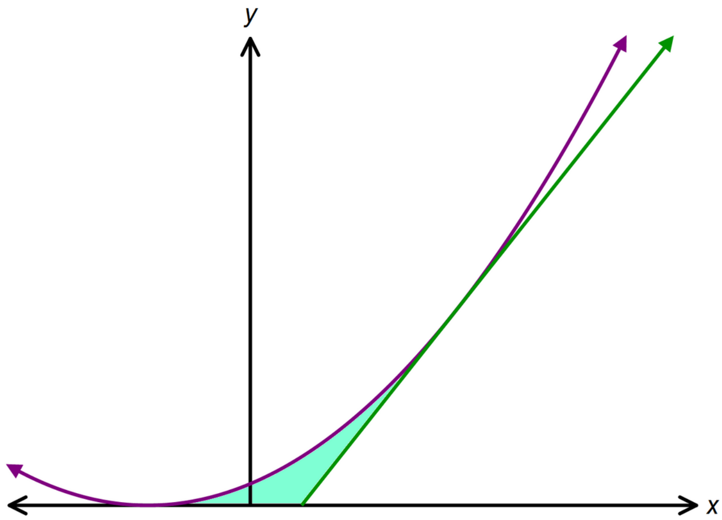
$$= \left[\frac{x^3}{3} + 2x\right]_0^2 + \left[-x^2 + 10x\right]_2^5$$

$$= \left(\frac{8}{3} + 4\right) - (0+0) + (-25+50) - (-4+20)$$

$$= 15\frac{2}{3}\ units^2$$

Question Six: [3, 5 = 8 marks] **CF**

The curve $y=(x+1)^2$ and the tangent line at $x=2$ are graphed below.



(a) Determine the equation of the tangent to $y=(x+1)^2$ drawn above.

$\frac{dy}{dx}=2(x+1)$ ✓

$x=2 \quad \frac{dy}{dx}=2(2+1)=6$ ✓

$x=2 \quad y=(2+1)^2=9$

$y=6x+c$

$9=6\times2+c$

$c=-3$

$\therefore y=6x-3$ ✓

(b) Hence find the area shaded on the graph above.

✓ ✓

$$Area = \int_{-1}^{2}(x+1)^2 \ dx - \int_{0.5}^{2} 6x - 3 \ dx$$

$$= \left[\frac{(x+1)^3}{3}\right]_{-1}^{2} - \left[3x^2 - 3x\right]_{0.5}^{2}$$

$$= (9+0) - \left(6 + \frac{3}{4}\right)$$

$$= 2\frac{1}{4} \ units^2$$