



**PERTH MODERN SCHOOL**  
Exceptional schooling. Exceptional students.  
Independent Public School

Year 12 Methods  
TEST 7 June 2019

TIME: 45 minutes working

**Calculator Assumed**  
44 Marks 6 Questions

Name: \_\_\_\_\_ Teacher: \_\_\_\_\_

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

**Question 1**

**(5 marks)**

(a) Differentiate  $x \sin x$

**(2 marks)**

(b) Hence find  $\int_0^{\frac{\pi}{2}} x \cos x \, dx$  **using** the result in (a) above.

**(3 marks)**

**Question 2****(3 marks)**

Determine the  $x$ -coordinates of all points on the graph of  $f(x) = 2\cos(x) + x$  for  $-\pi \leq x \leq \pi$  where the tangent line is horizontal. (Justify your answers)

**Question 3****(7 marks)**

A survey conducted by a local bank shows that 75% of its customers use an ATM at least once a month.

- (a) Find the probability that in a random sample of 8 customers, **at least 75%** of them use an ATM machine at least once a month. (2 marks)

- (b) If the random variable  $X$  follows a binomial distribution with  $n=12$  and  $p=0.75$ , what is the mean of this distribution and what is  $P(X \geq \text{mean})$ ? (3 marks)

- (c) If the sample size became very large what would you expect  $P(X \geq \text{mean})$  to approach? Briefly explain your answer. (2 marks)

**Question 4****(10 marks)**

The discrete random variable  $X$  can only take the values 2, 3 or 4. For these values the cumulative distribution function is defined by

$$P(X \leq x) = \frac{(x+k)^2}{25}$$

for  $x=2, 3 \wedge 4$ , where  $k$  is a positive constant integer.

(a) Find the value for  $k$ .

(3 marks)

(b) Complete the following table for  $X$ .

(3 marks)

$X$	2	3	4
$P(X \leq x)$			
$P(X = x)$			

(c) Hence find  $E(X)$  and  $SD(X)$ .  
marks)

(2

(d) Calculate  $Var(3 - 2X)$  giving your answer to 2 decimal places.

(2 marks)

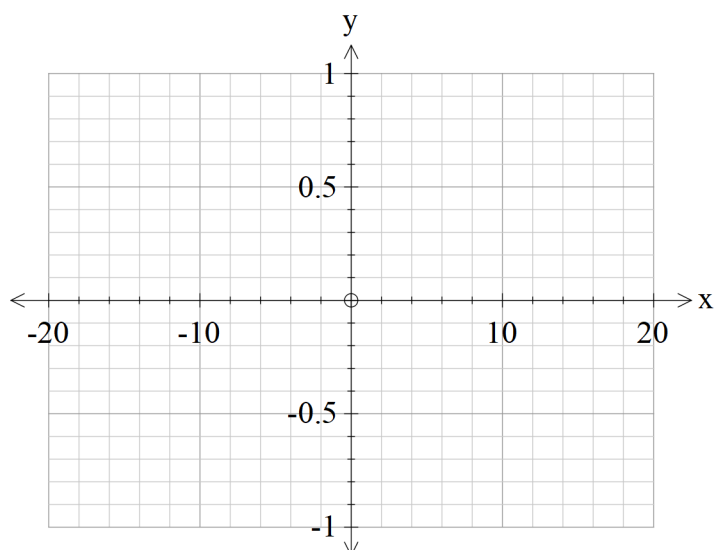
## Question 5

(8 marks)

Consider the function  $f(x) = \frac{1 - \cos x}{x}$  where  $x$  is in radians.

- a) Sketch  $f(x)$  on the axes below for  $-20 \leq x \leq 20$  on the axes below. Clearly label undefined points (if any).

(3 marks)



- b) As  $x$  approaches zero from the positive side, state the value that  $f(x)$  approaches. (1 mark)

- c) As  $x$  approaches zero from the negative side, state the value that  $f(x)$  approaches. (1 mark)

- d) Use the above to define a value for  $f(x)$  as  $x$  approaches zero, that is the following limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$$

(1 mark)

It can be shown that  $\frac{d}{dx}(\cos x) = -\cos x \lim_{h \rightarrow 0} \frac{1 - \cosh}{h} - \sin x \lim_{h \rightarrow 0} \frac{\sinh}{h}$ .

- e) Using the fact that  $\lim_{h \rightarrow 0} \frac{\sinh}{h} = 1$  and the above results, show that  $\frac{d}{dx}(\cos x) = -\sin x$ .

(2 marks)

**Question 6****(11 marks)**

A game is played by throwing two standard six-sided dice into the air once. The sum of the uppermost numbers are added together and if the sum is greater than 8 the player wins \$5.

Determine:

- a) the probability of winning \$5 in one game. (2 marks)
  
  
  
  
  
  
  
  
  
  
- b) the probability of winning exactly \$15 in 5 games. (3 marks)
  
  
  
  
  
  
  
  
  
  
- c) the probability of winning at least \$15 in at most 5 games. (3 marks)
  
  
  
  
  
  
  
  
  
  
- d) the minimum number of games to be played so that the probability of winning at least \$15 is greater than 0.47. (Justify) (3 marks)

**Working out space**

**Working out space**