

YEAR 12 MATHEMATICS SPECIALIST SEMESTER TWO 2016

TEST 4: Motion and Differential Equations

		Na	me:	_		
Monday 12 th September		Time: 50 minutes	Mark	/45		
Sect	tion 1 – Calculator free 20	marks				
1.	[5 marks – 4 and 1]					
	The noise level, in decibels, of the Year 9 class next door is increasing at a rate					
		$\frac{dN}{dt}$ o	$\propto \sqrt{N}$			
	proportional to the squa and rose to 100dB in 20	•	. The noise level started a	t 64dB		
	(a) Write and solve an a	annronriate differential ec	mation to model this situation			

(b) How long from the start will it take to exceed the pain threshold by reaching 144 dB?

2. [6 marks – 1 each]

A particle is moving in simple harmonic motion with its acceleration at time *t* given by

$$\frac{d^2x}{dt^2} = 4\cos(kt)$$
, for k a constant.

- (a) Express each of these quantities in terms of *k*:
 - (i) the period of motion
 - (ii) the frequency of motion
 - (iii) the amplitude of the motion
 - (iv) the displacement x(t)
- (b) If the maximum speed of the particle is 6 units, evaluate:
 - (*v*) *k*
 - (vi) the amplitude

3. [3 marks]

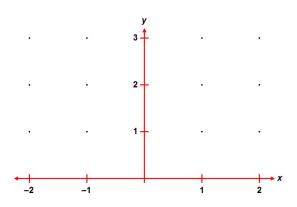
An object, with displacement x and velocity v, moves so that v = 3x - 5 m/s. What is the acceleration of this object it is 2 metres from the origin?

- [6 marks 1, 2 and 3]4.
 - Enter the values of $\frac{dy}{dx} = xy$ in this table. (a)

<u>, </u>	III tilis tab	ic.
x	У	$\frac{dy}{dx}$
1	2	
- 2	3	
3	0	

Use these values and others from the 20 integer (b) points marked to draw the slope field for the

differential equation $\frac{dy}{dx} = xy$



- Solve the differential equation $\frac{dy}{dx} = xy$, y > 0 if y = 2 when x = 0. (c)

5. [3 marks - 2 and 1]

$$\frac{dy}{dx} = xy$$

Continuing with $\frac{dy}{dx} = xy$, complete the table to find the coordinates of the next (a) two points, starting from (2, 1), when the incremental formula (Euler's method) is applied.

X	у	δx	δy
2	1	0.1	
		-	-

- What can be said about the initial (boundary) condition if \overline{dx} (b) graph produced using Euler's method is a horizontal line?
- 6. [3 marks - 2 and 1]

The velocity of particle P_1 is given by $v_1(t) = 2t$, $t \ge 0$ while that of P_2 is given by $v_2(t) = t^2 + 2t - 1, t \ge 0$

Both particles are moving along the same straight line and are initially at the origin O.

When is the velocity of the two particles the same? (a)

(b) What distance is covered by P_2 up to and when the velocities are equal?

7. [12 marks – 2, 6, 2, 1 and 1]

An advertising executive commissioned a mathematical analysis of the effectiveness of a particular television campaign.

The rate of increase in the percentage of the market (*P*) aware of the product was

The rate of increase in the percentage of the following
$$\frac{dP}{dt} = 2P - 0.025P^2$$
 , at t weeks

(a) This equation has the rate of increase proportional to two basic quantities. What are they?

(b) Use appropriate calculus techniques to derive
$$P(t) = \frac{2}{0.025 + Ce^{-2t}}.$$

If 20% of the market was initially aware of the product, determine:
(c) the proportion aware after 2 weeks of advertising
(d) how long before 75% of the market is aware
(e) the maximum or limiting value of market awareness
(e) the maximum of minting value of market awareness
PTO for question 8

8. [7 marks – 2, 1, 1, 1 and 2]

$$x = 2\cos 2t - \sqrt{5}\sin 2t$$

A steam-driven piston has a displacement, x, given by

(a) Show that the piston is in simple harmonic motion.

Determine each of:

(b) the amplitude.

(c) the first two values of t, $t \ge 0$, when the piston is in its central (mean) position

(d) the initial direction of travel

(e) the velocity when $\chi = \sqrt{6}$