Mathematics: Specialist
Formula sheet Units 3C and 3D

Vectors

$|(a,b,c)| = \sqrt{a^2+b^2+c^2}$ $\qquad |\mathbf{a}+\mathbf{b}| \le |\mathbf{a}|+|\mathbf{b}|$

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta = a_1b_1 + a_2b_2 + a_3b_3$

Vector equation of a line in space:

one point and the slope: $\mathbf{r} = \mathbf{r}_1 + \lambda\,\mathbf{l}$

two points: $\mathbf{r} = \mathbf{r}_1 + \lambda\,(\mathbf{r}_2 - \mathbf{r}_1)$

Cartesian equations of a line in space $\dfrac{x-a}{p} = \dfrac{y-b}{q} = \dfrac{z-c}{r}$

Parametric form of vector equation of a line in space

$x = a + \lambda p$...............(1)

$y = b + \lambda q$...............(2)

$z = c + \lambda r$...............(3)

Equation of a plane $\mathbf{r} \cdot \mathbf{n} = c$

$(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$

Trigonometry

In any triangle ABC

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad$ Area $= \dfrac{1}{2}ab\sin C$

$a^2 = b^2 + c^2 - 2bc\cos A$ $\qquad \cos A = \dfrac{b^2+c^2-a^2}{2bc}$

In a circle of radius r, for an arc subtending angle θ (radians) at the centre:

Length of arc $= r\theta$ $\qquad$ Area of sector $= \dfrac{1}{2}r^2\theta$ $\qquad$ Area of segment $= \dfrac{1}{2}r^2(\theta - \sin\theta)$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$ $\qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan(\theta \pm \phi) = \dfrac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$ $\qquad \lim\limits_{x\to 0} \dfrac{1-\cos x}{x} = 0$,

Simple Harmonic Motion: If $\dfrac{d^2x}{dt^2} = -k^2x$ then $x = a\cos(kt+\alpha) = a\sin(kt+\beta)$

Exponentials and logarithms

If $\dfrac{dP}{dt} = kP$, then $P = ae^{kt}$

Functions

Differentiation

If $f(x) = \sin x$, then $f'(x) = \cos x$

If $f(x) = \cos x$, then $f'(x) = -\sin x$

If $f(x) = \tan x$, then $f'(x) = \sec^2 x = \dfrac{1}{\cos^2 x}$

	Function notation		Leibniz Notation	
	y	y'	y	y'
Product rule	$f(x)\, g(x)$	$f'(x)\, g(x) + f(x)\, g'(x)$	$u v$	$\dfrac{du}{dx} v + u \dfrac{dv}{dx}$
Quotient rule	$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$	$\dfrac{u}{v}$	$\dfrac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2}$
Chain rule	$f(g(x))$	$f'(g(x))\, g'(x)$	$y = f(u)$ and $u = g(x)$	$\dfrac{dy}{du} \times \dfrac{du}{dx}$

Integration

$\int \sin x \, dx = -\cos x + c$ $\qquad$ $\int \cos x \, dx = \sin x + c$ $\qquad$ $\int \sec^2 x \, dx = \tan x + c$

Fundamental Theorem of Calculus: $\dfrac{d}{dx} \int_a^x f(t)\, dt = f(x)$ and $\int_a^b f'(x)\, dx = f(b) - f(a)$

Complex numbers

For $z = a + ib$, where $i^2 = -1$

Argument: $\arg z = \theta$, where $\tan \theta = \dfrac{b}{a}$ and $-\pi < \theta < \pi$

Modulus: $\operatorname{mod} z = |z| = |a + ib| = \sqrt{a^2 + b^2} = r$

$\arg z_1 z_2 = \arg z_1 + \arg z_2$

Product: $|z_1 z_2| = |z_1||z_2|$

$\arg \dfrac{z_1}{z_2} = \arg z_1 - \arg z_2$

Polar form: $z = r \operatorname{cis}\theta$, where $r = |z|$ and $\theta = \arg z$

$\operatorname{cis}\theta = \cos\theta + i \sin\theta$ $\qquad$ $\operatorname{cis}\theta \operatorname{cis}\phi = \operatorname{cis}(\theta + \phi)$

$\operatorname{cis}(-\theta) = (\operatorname{cis}\theta)^{-1}$ $\qquad$ $\operatorname{cis} 0 = 1$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta + \phi)$ $\qquad$ $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \operatorname{cis}(\theta - \phi)$

For complex conjugates $z = a + ib$ *and* $\bar{z} = a - ib$,

$\bar{z} = r \operatorname{cis}(-\theta)$ $\qquad$ $z \bar{z} = |z|^2$ $\qquad$ $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ $\qquad$ $z^{-1} = \dfrac{1}{z} = \dfrac{\bar{z}}{|z|^2}$

Matrices

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $|A| = ad - bc$ and $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Transformations

Dilation $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

Shear $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$

Rotation $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Reflection $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

Mathematical reasoning

De Moivre's theorem: $(\text{cis}\,\theta)^n = (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$z^n = |z|^n \text{ cis } (n\theta)$

$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\frac{\theta + 2\pi k}{q} + i\sin\frac{\theta + 2\pi k}{q}\right)$ for $k = 0, 1, 2, \ldots$

Measurement

Trapezium: Area = $\frac{1}{2}(a + b) \times$height, where a and b are the lengths of the parallel sides

Prism: Volume = Area of base $\times$ height

Cylinder: Total surface area = $2\pi r h + 2\pi r^2$

Volume = $\pi r^2 \times h$

Pyramid: Volume = $\frac{1}{3} \times$ area of base $\times$ height

Cone: Total surface area = $\pi r s + \pi r^2$, s is the slant height

Volume = $\frac{1}{3} \times \pi r^2 \times h$

Sphere: Total surface area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.