MATHEMATICS METHODS

MAWA Semester 2 (Unit 3&4) Examination 2019 Calculator-free

Marking Key

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The release date for this exam and marking scheme is • the end of week 1 of term 4, Fri October 18th 2019

CALCULATOR-FREE SEMESTER 1 (UNIT 3&4) EXAMINATION

135

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MATHEMATICS METHODS

(z marks	iestion 9(a)
	noitulo2
	$\Delta rea of triangle = 1$
	$\frac{1}{3} = \frac{1}{3}$
	$6 \div \frac{1-}{\varepsilon} = 1$ = 3 sadient
	· · · · · · · · · · · · · · · · · · ·
	$6 \ge t \ge 0$ for $\frac{1}{8} + t \frac{1}{8t} = (t)$
	СОТ
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Marks	Mathematical behaviours
т	determines k value
τ	states the probability density function

τ	 states the cumulative distribution function
τ	• integrates f(t)
Marks	Mathematical behaviours
	$\frac{1}{5} = (1) + \frac{1}{3} \frac{1}{6} + \frac{1}{3} \frac{1}{6} + \frac{1}{3} \frac{1}{6} + \frac{1}{3} \frac{1}{6} + \frac{1}{6} = \frac{1}{6}$
	noitulo2
(z marks)	nesuou a(p)

τ	 calculates the conditional probability
τ	 determines the intersection of probabilities for t > 1 and t < 3
Marks	Mathematical behaviours
	$P(t>1 \cap t<3) = \frac{16}{36}$ $P(t>3 \lor t>1) = \frac{16}{25}$
	$\frac{3e}{5z} = (z < z)d$
	$\frac{11}{86} = (1 > 1)q$ $\frac{27}{86} = (1 > 1)q$
	11 -(1>4)q
	noitulo2
(S marks)	(c) (c)

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SEMESTER 1 (UNIT 3&4) EXAMINATION

Section One: Calculator-free (50 Marks)

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(3 marks) Question 1 (a)

Solution		
$f^{' x } = -2x \cdot e^{-x^2} \sqrt{2x-5} + \frac{1}{2} (2x-5)^{\frac{-1}{2}} \cdot 2 \cdot e^{-x^2} \left(2x \sqrt{2x-5} - \frac{1}{\sqrt{2x-5}} \right)$		
Mathematical behaviours	Marks	
uses product rule correctly	1	
• differentiates e^{-x^2} correctly	1	
• differentiates $\sqrt{2x-5}$ correctly	1	

Question 1 (b)

Question 1 (b)	(3 marks)
Solution	
Let $u = x^2 + 16.(*)$	
Then $\frac{du}{dx} = 2x$, and so	
$g(x) = \int \frac{xdx}{x^2 + 16} = \int \frac{du}{2u} = \frac{1}{2} \ln u + c = \frac{1}{2} \ln(x^2 + 16) + c(**)$	
Since $g(0) = \ln 5 \ln 5 = \frac{1}{2} \ln 16 + c \ln 5 = \ln 4 + cc = \ln 5 - \ln 4c = \ln \frac{5}{4} g(x) = \frac{1}{2} \ln (x^2 + c) =$	$+16$) $+ \ln \frac{5}{4}$
Mathematical behaviours	Marks
makes substitution (*)	1
integrates correctly (**)	1
(use of rule $\int \left[\frac{f'(x)}{f(x)} \right] dx = \ln f(x) + c$ to integrate correctly – award both marks) • evaluates integration constant correctly	
- evaluates integration constant correctly	1

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Question 7 (3 marks)

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Solution	
$\int_{0}^{\ln 2} e^{-2x} dx \dot{\zeta} \left[\frac{e^{-2x}}{-2} \right]_{0}^{\ln 2} \dot{\zeta} \frac{e^{-2\ln 2}}{-2} - \frac{e^{0}}{-2} \dot{\zeta} \frac{e^{\ln\left(\frac{1}{4}\right)}}{-2} - \frac{1}{-2} \dot{\zeta} - \frac{1}{8} + \frac{1}{2} \dot{\zeta} \frac{3}{8}$	
Mathematical behaviours	Marks
anti-differentiates exponential function	1
substitutes correctly	1
simplifies correctly	1

Question 8	(4 marks)
Solution	
$V = \frac{4}{3}\pi r^3$, and so $\frac{dV}{dr} = 4\pi r^2$	
By the increments formula, $\delta V \simeq \frac{dV}{dr} \delta r = 4 \pi r^2 \delta r$ (*)	
So $\frac{\delta V}{V} \approx \frac{4\pi r^2}{\frac{4}{3}\pi r^3} \delta r = 3\frac{\delta r}{r} $ (**)	
Now $\frac{\delta V}{V} = \frac{-12}{800} = -0.015$	
So $\frac{\delta r}{r} \approx \frac{-0.015}{3} = -0.005$	
So the percentage change in the radius r is a decreases of 0.5 %.	
Mathematical behaviours	Marks
differentiates correctly	1
• finds approximation (*)	1
• evaluates $\frac{\delta V}{V}$	1
evaluates V	1
obtains correct answer	_

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Question 2 (3 marks)

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τ	 re-arranges equation to determine the value of k 		
τ	simplifies equation by squaring and dividing		
τ	 uses the formula for margin of error to compare each sample 		
Marks	Mathematical behaviours		
$\frac{1}{2\sqrt{1-\frac{1}{2}}} = \frac{1}{2\sqrt{1-\frac{1}{2}}} $			
noitulo2			

Question 3(a) (2 marks)

τ				.(£)9, P(3).	 uses log laws to find probabi
τ				ائلى, ٩(٤).	 uses log laws to find probabi
Marks	Mathematical behaviours				
	дқ	7K 7	noitu 1	(x) _d	$P(x = 1) = k \log_{e} e^{x} = 2k$ $P(x = 2) = k \log_{e} e^{x} = 2k$

Question 3(b) (2 marks)

τ	$^{ m A}$ fo emitsh in Express $^{ m D}$ in terms of $^{ m K}$
τ	• sums probabilities and equates 1
Marks	Mathematical behaviours
	$a = \frac{1 - 3k}{k} + ak = 1$ $3k + ak = 1$
	noituloS

Question 6(a) (3 marks)

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τ	 states the correct velocity at 5 seconds 	
τ	 anti-differentiates to find the velocity equation 	
τ	determines the acceleration equation	
Marks	Mathematical behaviours	
$4 + \sqrt{1} \frac{1}{S} = v \therefore 4 = 0 \text{ inside } 0 = 1 \text{ inside } 0 = 1$		
noitulo2		

Question 6(b) (3 marks)

$$x = \frac{1}{5} \frac{1}{3} + 4t + cLet x = 0, when t = 0 : c = 0x = \frac{1}{15}t^3 + 4t When t = 5x = \frac{1}{15}(125) + 20$$

$$x = \frac{25}{3} + 20$$

$$x = \frac{85}{3} \sqrt{28.33} m$$
Mathematical behaviours
$$x = \frac{85}{3} \sqrt{28.33} m$$

$$x = \frac{1}{3} \sqrt{28.33} m$$

$$x = \frac{1$$

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(3 marks)

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Question 3(c)

	Solution
$k = \frac{1}{3} \Rightarrow a = 0$	
$E(X) = 1 \times k + 2 \times 2k$	
=5k	
$k = \frac{1}{3} \Rightarrow E(X) = \frac{5}{3}$	

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Mathematical behaviours	Marks
determines value of ^a	1
substitutes into the expected value formula	1
states expected value	1

Question 4(a) (3 marks)

Solution $2^{x} = 3^{x-1}$ $ie \ x \log 2 = (x - 1) \log 3$ $ie \ x \log 2 - x \log 3 = -\log 3$ $ie \ x(\log 2 - \log 3) = -\log 3$

Mathematical behaviours	Marks
• rewrites equation by taking logarithms of each side and applying log laws	1
 rearranges equation to isolate ^X 	1
• solves for ^X	1

Question 4(b) (4 marks)

Ques	ction 4(D)	(4 marks)
	Solution	
	$\log_{10}(x+2) + \log_{10}(2x-3) = 2\log_{10}x$	
	$ie \log_{10}(x+2)(2x-3) = \log_{10}x^2$	
	$ie(x+2)(2x-3)=x^2$	
	$ie 2x^2 + x - 6 = x^2$	
	$ie x^2 + x - 6 = 0$	
	ie(x+3)(x-2)=0	
	ie $x = -3, 2$	
	$2x - 3 > 0 \Rightarrow x = 2$	
	Mathematical behaviours	Marks
	uses ^{log} laws to simplify both sides of equation	1
	obtains quadratic equation	1
	obtains quadratic equation	1

İ	•	simplifies quadratic and solves	1
	•	solves for $^{\chi}$, justifying answer	

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Question 4(c) (4 marks)

Solution
$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos x}{\sin x} dx = \left[\ln \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$
$=\ln\sin(\frac{\pi}{3})-\ln\sin\frac{\pi}{6}$
$=\ln\frac{\sqrt{3}}{2}-\ln\frac{1}{2}$
$=\ln\sqrt{3}$ - $\ln 2$ - $(\ln 1$ - $\ln 2)$
$=\ln\sqrt{3}$
$=\frac{\ln 3}{2}$
∴ $a = 0.5, b = 3$

	Mathematical behaviours	Marks	
	anti-differentiates to obtain ln expression	1	
	substitutes exact values and evaluates expression	1	
	uses ^{log} laws to simplify expression	1 1	
	• states the value of a and b	-	

Ouestion 5

Question 5	(4 marks)
Solution	
$\frac{d}{dx}\left(\int_{a}^{x} f(t)dt\right) = \frac{d}{dx}\left(\left[f(x)\right]^{2}\right)$	
$f(x)=2f(x).f'(x)f'(x)=\frac{1}{2}$	
$f(x) = \frac{x}{2} + c \int_{0}^{0} f(t) dt = [f(0)]^{2} :: 0 = f(0) f(x) = \frac{x}{2}$	
Mathematical behaviours	Marks
uses Fundamental Theorem of Calculus	1
• uses chain rule to differentiate $[f(x)]^2$	1
• determines $f(x)$	1
• determines $f(x)$ and shows how to calculate the constant, c.	1