

Question	Marks	Max	Question	Marks	Max
17			12		
16			4		
15			23		
8			22		
14			21		
13			20		
12			19		
11			18		
10			9		

You do not have any unauthorised material if you have any unauthorised material with you, hand it to the supervisor **before** reading any further.
No other items may be taken into the examination room. It is your responsibility to ensure that

Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

Materials required/recommended for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes
Time allowed for this section

Teacher's Name: _____

Name: _____ SOLUTIONS

Calculator-assumed

Section Two:

UNIT 1 AND 2

MATHEMATICS METHODS

Question/Answer booklet

Semester Two Examination, 2020



INDEPENDENT PUBLIC SCHOOL

Executive school, Exceptional students.

Perth Modern School

Perth, Western Australia

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	44	34
Section Two: Calculator-assumed	14	14	100	85	66
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Additional working space

MATHEMATICS

26

CALCULATOR ASSUMED

MATHEMATICS

3

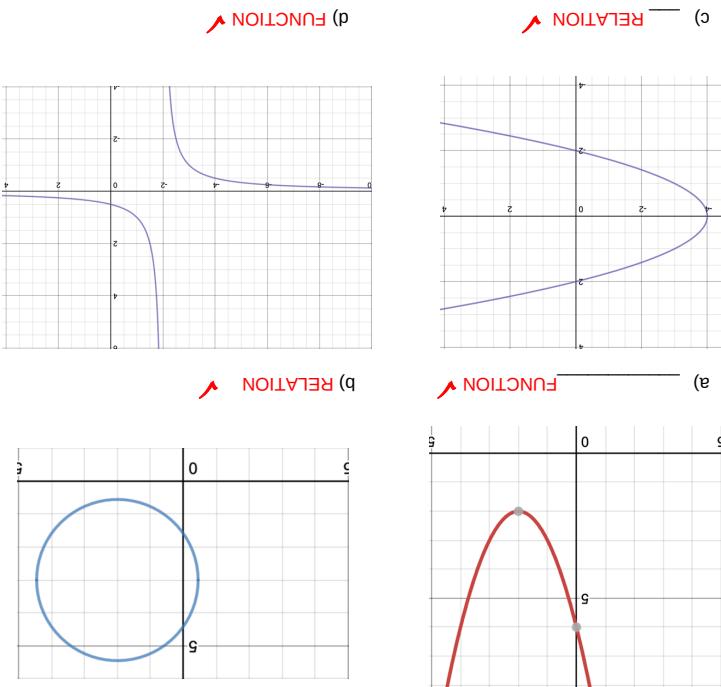
(85 Marks)

This section has 14 questions. Answer all questions. Write your answers in the spaces

responses and/or as additional space if required to continue an answer.

Planning: if you use the spare pages for planning, indicate this clearly at the top. Continuing an answer: if you need to use the space to continue an answer, indicate this clearly at the top. Answering questions: if the question requires a numbered answer, give the page number of the question at the top of the page.

Working time: 100 minutes.



Apply the vertical line test to each of the following graphs and conclude whether the graph represents a function or a function.

Responses and/or as additional space if required to continue an answer.

Planning: If you use the spare pages for planning, indicate this clearly at the top of the page

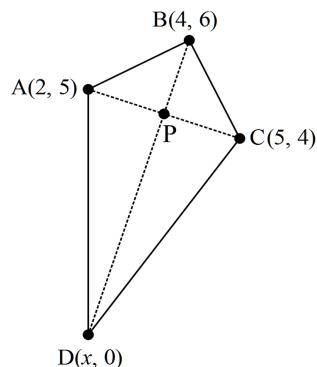
Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e., give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Question 11 (1.1.1 – 1.1.6)**(2, 2, 1, 3, 2 = 10 marks)**

The kite ABCD is graphed below.

The coordinates of the vertices are A(2, 5), B(4, 6), C(5, 4) and D(x, 0) as shown below.

NB Kites have diagonals that meet at right angles.



- (a) Show that $AB = BC$.

$$AB = \sqrt{(4-2)^2 + (6-5)^2} = \sqrt{5} \quad \checkmark$$

$$BC = \sqrt{(5-4)^2 + (4-6)^2} = \sqrt{5} \quad \checkmark$$

$$\therefore AB = BC$$

- (b) Find the gradient of AC and hence the equation of the diagonal BD.

$$m_{AC} = \frac{4-5}{5-2} = -\frac{1}{3}$$

$$\therefore m_{BD} = 3 \quad \checkmark$$

$$y = 3x + c$$

$$(4, 6) \quad 6 = 12 + c \quad c = -6$$

$$y = 3x - 6 \quad \checkmark$$

Additional working space

Question number: _____

- (d) Find the midpoint of AC and show it belongs to diagonal BD.

$$\text{if } y = 0 \quad 0 = 3x - 6 \quad x = 2$$

- (c) Determine the value of d, the x co-ordinate of point D.

Question number:

Additional working space

- (e) Show how the midpoint of AC could have been used to determine the value of x

value of
x

$$\begin{aligned} \text{if } y = 0 \quad 0 &= 3x - 6 \quad x = 2 \\ y &= 3x - 6 \\ (4,6) \quad 6 &= 12 + c \quad c = -6 \\ y &= 3x + c \\ m &= \frac{4 - 3.5}{6 - 4.5} = \frac{0.5}{1.5} = 3 \\ p(3.5,4.5) \quad B(4,6) \end{aligned}$$

$$\begin{aligned} \therefore (3.5,4.5) &\in BD \\ \text{If } x = 3.5, \quad y &= 3 \times 3.5 - 6 = 4.5 \\ p(3.5,4.5) \end{aligned}$$

Question 12 {1.1.26}

(4 marks)

- a) Give the equation of the image $y=x^2$ after the following transformation:
Translate 3 units horizontally left followed by reflection in the x -axis.

Solution
$y=-(x+3)^2$
Specific behaviours
<input checked="" type="checkbox"/> + 3 <input checked="" type="checkbox"/> Applies negative to the whole function

- b) Give the equation of the image $y=\frac{1}{x}$ after the following transformation:
Translate vertically up 2 units then reflect in the y -axis.

Solution
$y=\frac{-1}{x}+2$
Specific behaviours
<input checked="" type="checkbox"/> + 2 <input checked="" type="checkbox"/> Applies negative to the x only

Question 13 {2.3.4}

(3 marks)

Determine the gradient of the secant passing through the graph $y=x^3-2x^2-4$ at the points where $x=6$ and $x=9$.

$$\begin{aligned} \text{AT } x = 6: \\ y &= 6^3 - 2(6)^2 - 4 \\ &= 140. \quad \checkmark \end{aligned} \qquad \begin{aligned} \therefore m &= \frac{563 - 140}{3} \\ &= \underline{\underline{141}}. \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{AT } x = 9: \\ y &= 9^3 - 2(9)^2 - 4 \\ &= 563. \quad \checkmark \end{aligned}$$

Question 14 {2.3.1}

(3,2 = 5 marks)

Additional working space

Question number: _____

By first calculating the relevant positions of the car, determine its average velocity in:

$$x(t) = 30t + \frac{t+1}{30} - 30, \text{ for } 0 \leq t \leq 4$$

(a) the first 2 seconds.

B By first calculating the relevant positions of the car, determine its average velocity in:

$$\begin{aligned} x(0) &= 30(0) + \frac{0+1}{30} - 30 \\ &= 0 \text{ m.} \\ x(2) &= 30(2) + \frac{2+1}{30} - 30 \\ &= 40 \text{ m.} \\ x(4) &= 30(4) + \frac{4+1}{30} - 30 \\ &= 96 \text{ m.} \end{aligned}$$

(b) the last 2 seconds.

No penalty for missing units.

$$\begin{aligned} x(2) &= 30(2) + \frac{2+1}{30} - 30 \\ &= 40 \text{ m.} \\ x(4) &= 30(4) + \frac{4+1}{30} - 30 \\ &= 96 \text{ m.} \\ \Delta x &= 96 - 40 = 56 \text{ m.} \\ \Delta t &= 4 - 2 = 2 \text{ s.} \\ \text{AV} &= \frac{\Delta x}{\Delta t} = 28 \text{ m/s.} \end{aligned}$$

Question 15 {2.3.5, 2.3.8)

(3, 1 = 4 marks)

A balloon's volume at time t seconds is represented by $V(t) \text{ cm}^3$, where:

$$V(t) = -2t^2 + 800, \text{ for } 0 \leq t \leq 20$$

- a) Using the difference quotient, determine the average rate of change over $[5, 5+h]$.

$$\begin{aligned} \frac{f(5+h) - f(5)}{h} &= \frac{-2(5+h)^2 + 800 - (-2(5)^2 + 800)}{h} \quad \checkmark \\ &= \frac{-2(25+10h+h^2) + 800 - (-50+800)}{h} \quad \checkmark \\ &= \frac{-50-20h-2h^2+800+50-800}{h} \\ &= \frac{-20h-2h^2}{h} \\ &= \frac{-20-2h}{1} \text{ cm}^3/\text{s.} \quad \checkmark \end{aligned}$$

- b) Hence, determine the rate of change as $h \rightarrow 0$.

$$\lim_{h \rightarrow 0} (-20-2h) = -20 \text{ cm}^3/\text{s.} \quad \checkmark$$

END of SECTION

(4 marks)

Question 23 (1.3.7, 1.3.11, 1.3.14, 1.3.16)

(1, 1, 1, 1 = 6 marks)

Differentiate $f(x) = 5x^3$ by first principles.

$$\begin{aligned} f(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) \\ &= 3x^2 \end{aligned}$$

$$\begin{aligned} f(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) \\ &= 3x^2 \end{aligned}$$

$$\begin{aligned} f(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) \\ &= 3x^2 \end{aligned}$$

a) Events A and B are such that $P(A) = 0.4$, $P(B) = 0.7$ and $P(\underline{A} \cup B) = 0.4$. Determine:

Solution

$P(A \cap B) = 0.1$

$P(A \cup B) = 0.2$

Solution

$P(C \cap D) = \frac{5}{2}$

Solution

$P(C \cup D) = \frac{1}{15}$

Solution

$P(C) = \frac{15}{7}$

Solution

$P(D|C) = \frac{1}{7}$

Solution

✓ correct answer
Specific Behaviours

✓ correct answer
Solution

iv. $P(D|C)$

iii. $P(C)$

ii. $P(C \cup D)$

ii. $P(A \cup B)$

i. $P(A \cup B)$

✓ correct answer
Specific Behaviours

Solution

$P(C \cap D) = \frac{5}{2}$

Solution

$P(C \cup D) = \frac{1}{15}$

Solution

$P(C) = \frac{15}{7}$

Solution

$P(D|C) = \frac{1}{7}$

Solution

✓ correct answer
Specific Behaviours

✓ correct answer
Solution

iv. $P(D|C)$

iii. $P(C \cup D)$

ii. $P(C \cap D)$

i. $P(A \cup B)$

b) Given $P(D) = \frac{2}{3}$, $P(C|D) = \frac{3}{5}$ and $P(C|\underline{D}) = \frac{1}{5}$. Determine:

Solution

$P(A \cap B) = 0.1$

Solution

$P(A \cup B) = 0.2$

Solution

$P(C \cap D) = \frac{5}{2}$

Solution

$P(C \cup D) = \frac{1}{15}$

Solution

$P(C) = \frac{15}{7}$

Solution

$P(D|C) = \frac{1}{7}$

Solution

✓ correct answer
Specific Behaviours

✓ correct answer
Solution

MATHEMATICS

10

CALCULATOR ASSUMED

MATHEMATICS

19

CALCULATOR ASSUMED

e)

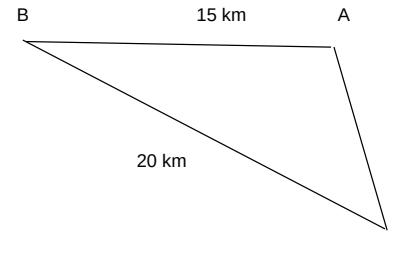
<p>Question 17 {2.3.19} (3, 2, 3, 4 = 12 marks)</p> <p>A particles position (in metres) with respect to time (in seconds) is given by the following equation:</p> $S(t) = \frac{3}{t^3} - 5t^2 + at + b$ <p>Given that the particle is initially at the origin and has a displacement of 27 metres at 3 seconds, determine the values of a & b.</p> <p>Solution</p> <p>Substitutes in $t = 0$ to find b</p> $S(0) = 0$ $\therefore b = 0$ <p>Substitutes in $t = 3$</p> $S(3) = 27$ $\therefore 27 = \frac{3}{(3)^3} - 5(3)^2$ $\therefore a = 21$ <p>Find a</p> <p>Answer</p> <p>2 d.p.</p> <p>Solution</p> <p>c) Use calculus techniques to determine the particle is at rest.</p> <p>Solution</p> <p>$S'(t) = t^2 - 10t + 21$</p> $(t-7)(t-3) = 0$ $\therefore t = 7 \text{ or } t = 3$ <p>$\therefore \text{particle at rest when } t = 3 \text{ v/t}$</p> <p>Differentiates</p> <p>Solves</p> <p>Equates to zero</p> <p>Solution</p> <p>d) Use calculus techniques to determine the maximum displacement of the particle.</p> <p>Solution</p> <p>$S'(t) = 3t^2 - 10t + 21$</p> $S''(t) = 6t - 10$ $6t - 10 = 0$ $\therefore t = \frac{10}{6} = \frac{5}{3}$ $S\left(\frac{5}{3}\right) = \frac{3}{\left(\frac{5}{3}\right)^3} - 5\left(\frac{5}{3}\right)$ $\therefore S\left(\frac{5}{3}\right) = 21.67 \text{ m}$ <p>$\therefore \text{maximum displacement of the particle is } 21.67 \text{ m}$</p> <p>Differentiates</p> <p>Solves</p> <p>Equates to zero</p> <p>Solution</p> <p>Specific behaviours</p> <p>Notes:</p> <p>Determines nature of t.p at $t=3$ (sign test, double derivative test or checking function values either side acceptable).</p> <p>Solves for the displacement when $t=3$.</p> <p>States maximum displacement and time.</p> <p>Notes:</p> <p>Determines nature of t.p at $t=3$ (sign test, double derivative test or checking function values either side acceptable).</p> <p>Solves for the displacement when $t=3$.</p> <p>States maximum displacement and time.</p>	<p>Solution</p> <p>$P(C M) = \frac{91}{125}$</p> <p>Specific Behaviours</p> <p>Correct numerator</p> <p>Correct denominator</p>	<p>Given that a student in Year 11 studies Chemistry, what is the probability that they also study Mathematics Methods?</p>
<p>Solution</p> <p>$P(M C) = \frac{91}{205}$</p> <p>Specific Behaviours</p> <p>Correct numerator</p> <p>Correct denominator</p>	<p>Given that a student in Year 11 studies Mathematics Methods, what is the probability that they also study Chemistry?</p>	
<p>Solution</p> <p>$P(M \cup C) = \frac{11}{250}$</p> <p>Specific Behaviours</p> <p>Correct numerator</p> <p>Correct denominator</p>	<p>What is the probability that a student in Year 11 studies neither Mathematics Methods nor Chemistry?</p>	
<p>Solution</p> <p>Venn has 4 fields completed.</p> <p>Specific Behaviours</p> <p>Correct numbers</p>	<p>Represent this information in a completed Venn diagram.</p>	

Minus one mark for not supplying values for tests and simply saying +ve or -ve.
Not required to check boundary points of domain but should be mentioned after the examination that a local maximum isn't necessarily a global maximum.

Question 21 {1.2.4}

(4 marks)

A ground search for a lost hiker is being organised using three camping sites in a national park as bases. It is known that the hiker is within the triangular area formed with the three campsites as vertices. Campsite A is 15km due east from campsite B. Campsite C is on a bearing of 170° from campsite A (hint : $\angle BAC = 100^\circ$). Campsite B is 20km from campsite C.
Note: The diagram is not to scale.



Calculate the area of the search to the nearest square kilometre.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ 20^2 &= b^2 + 15^2 - 2b(15) \cos 100^\circ \\ b &= 10.8780 \text{ km} \end{aligned}$$

$$\begin{aligned} A(\Delta) &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2}(15)(10.8780) \sin 100^\circ \\ &= 80.345 \\ &\approx 80 \text{ km}^2 \text{ (nearest km)} \end{aligned}$$

- ✓ Cosine Rule
- ✓ length
- ✓ area of triangle formula
- ✓ area to the nearest km

- Note: Alternative method
- ✓ Sine rule $c=47.61$
 - ✓ $B = 32.39$
 - ✓ Area triangle formula
 - ✓ area to the nearest km

<p>Solution</p> $3x^2 - 3 = 0$ $x^2 = 1$ $x = \pm 1$ $f(1) = 0$ $f(-1) = 4$ $f'(0) = -3$ $f'(2) = 9$ $\therefore \text{min} f(x) \in (1, 0)$ $f(-2) = 9$ $\therefore \max f(x) \in (-1, 4)$
<p>Specific behaviors</p> <p>Determines location of l.p. at $x = -1$ (sign test or double derivative test).</p> <p>Determines nature of l.p. at $x = 1$ (sign test or double derivative test).</p> <p>Solves correctly differential equation to zero.</p>
<p>Notes:</p> <p>No mark for not supplying values for tests and simply saying +ve or -ve.</p>
<p>.</p>

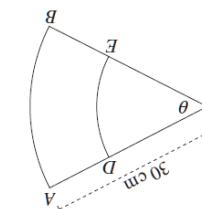
b) Using calculus techniques find the location and nature of any stationary points.

<p>Solution</p> $f(x) = x^3 - 3x + c$ $\text{since } f(2) = 4$ $2^3 - 3(2) + c = 4$ $\therefore c = 2$ $\therefore f(x) = x^3 - 3x + 2$
<p>Anti-differentiates</p> <p>Substitutes in $x=2$ to find c</p> <p>States $f(x)$</p>
<p>Specific behaviors</p>

a) Determine $f(x)$
Given that $f(x)$ runs through the point $(2, 4)$ and $f'(x) = 3x^2 - 3$:

(3, 6 = 9 marks)

The arc DE divides the sector ABC into two regions of equal area.
Find the length of the interval CD.



The region ABC is a sector of a circle with radius 30cm, centred at C. The angle in the sector is θ . The arc DE lies on a circle also centred at C, as shown in the diagram.

<p>Solution</p> $x = 8.79$ $\therefore \frac{1}{2}(30-x)^2\theta = \frac{1}{2}(30)^2\theta$ $\therefore \frac{1}{2}(30-x)^2 = \frac{1}{2}(30)^2$ $\therefore x = 21.21 \text{ cm} \quad (15 \sqrt{2} \text{ cm})$
<p>Specific behaviours</p> <p>Substitutes correctly for large sector</p> <p>Substitutes correctly for small sector</p> <p>divides large sector by 2</p>
<p>.</p>

<p>Solution</p> $x = 21.21 \text{ cm}$ $\frac{1}{2}x^2\theta = \frac{1}{2}30^2\theta$ $\therefore x = \pm 1$ OR
<p>Specific behaviours</p> <p>Substitutes correctly for small sector</p> <p>Substitutes correctly for large sector</p> <p>divides large sector by 2</p>
<p>.</p>

Question 19 {2.3.22}

(8 marks)

Anika has her own private plane that she uses to travel from Perth to Melbourne. The amount of fuel she consumes on the journey and therefore the cost of her flight, C (\$), is dependent on the speed at which she flies, x (km/h). The relationship between the overall cost of the flight is modelled by the equation below.

$$C = \frac{x^2}{200} - 8x + 20400 \quad \text{where } 550 \leq x \leq 900$$

Use calculus techniques to determine both the **optimal speed** to travel in order to minimise the cost of her journey and the **cost of that flight**.

Solution
$\frac{dC}{dx} = \frac{x}{100} - 8$
$\frac{x}{100} - 8 = 0$
$x = 800$
$\frac{dC}{dx} \vee x = 799 = -0.01$
$\frac{dC}{dx} \vee x = 801 = 0.01$
$\therefore \min @ x = 800$
$C \vee x = 800 = 17200$
$\therefore \text{the cost will be minimised at \$17200 when travelling at a speed of } 800 \text{ km/h}$

Specific behaviours
Differentiates
Equates differential equation to zero
Solves correctly
(2 marks) Performing sign test or double derivative test correctly.
Stating / confirming that it is a minimum
Determines cost at $x = 800$
Stating optimal speed and associated cost
Notes:
No mark for not supplying values for tests and simply saying +ve or -ve.