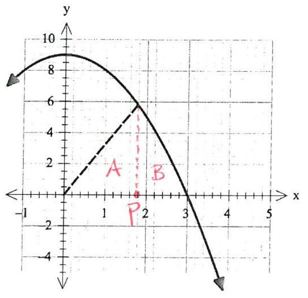
9 The curve $y = 9 - x^2$ is shown on the diagram below. A line is drawn from the origin to a point on the curve such that the area trapped between the line, the curve and the y-axis is the same as the area trapped between the curve, the line and the positive x-axis. Determine the equation of the line needed to achieve the equal areas. (5)

[HINT: Divide the half on the right into a triangle and a curved section]



✓ total area under parabola

✓ expression for triangle

✓ $A+B = 9$

✓ solve for p

✓ equation of line

Total area under curve $= \int_0^3 9 - x^2\,dx$

$= 18$

Triangle $A = \frac{1}{2}p(9-p^2)$

Region $B = \int_p^3 9 - x^2\,dx$

$\frac{1}{2}p(9-p^2) + \int_p^3 9 - x^2\,dx = 9$

$p \approx 1.788215$

At point (p, y) on $9 - x^2$, $y \approx 5.802287$

Gradient $= 3.24$ to 2dp

$\therefore$ Equation of line $y = 3.24x$

SHENTON COLLEGE

ATMAM Mathematics Methods

Test 2 (2019) Calculator Free

Name: ....

Teacher: Friday Smith Ai

Time Allowed : 18 minutes

Marks	/23

+c −1 whole paper

Materials allowed: Formula Sheet.

Attempt all questions. Questions 1 to 4 are in this section.
All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

1 Determine the following indefinite integrals.

a) $6\int 3x^2 - 4x\,dx$ (2)

$= 6x^3 - 12x^2 + c$

✓ integrate terms
✓ factor of 6

b) $\int \cos^2(x)\sin(x) - \sin(x)\,dx$ (3)

$= -\frac{\cos^3 x}{3} + \cos x + c$

✓ integrate trig
✓ correct signs
✓ ÷3 from chain rule

c) $\int 2x(2x+1)^2 dx$ (2)

$= \int 2x(4x^2 + 4x + 1)\,dx$

$= \int 8x^3 + 8x^2 + 2x\,dx$

$= 2x^4 + \frac{8x^3}{3} + x^2 + c$

✓ expand
✓ integrate

d) $\int (3x+5)^4 dx$ (1)

$= \dfrac{(3x+5)^5}{15} + c$

e) $\int \dfrac{4x-1}{x^3} dx$ (2)

$= \int 4x^{-2} - x^{-3} dx$

✓ separate fractions

✓ integrate terms

$= -4x^{-1} + \frac{1}{2}x^{-2} + c$

or $\dfrac{-4}{x} + \dfrac{1}{2x^2} + c$

2 Evaluate the following definite integrals.

a) $\int_1^5 \sqrt{3x+1}\, dx$ (4)

$= \left[\dfrac{2}{3} \times \dfrac{(3x+1)^{\frac{3}{2}}}{3}\right]_1^5$

✓ integrate bracket

✓ ÷3 from chain rule

✓ substitute

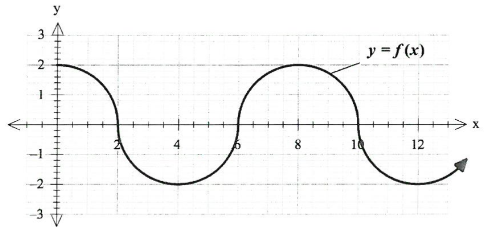
✓ evaluate

$= \frac{2}{9}(16)^{\frac{3}{2}} - \frac{2}{9}(4)^{\frac{3}{2}}$

$= \dfrac{128}{9} - \dfrac{16}{9}$

$= \dfrac{112}{9}$

8 The graph below is made from sections of a circle with radius 2 units.



a) Determine $\int_0^4 f(x)dx$ (1)

0

b) The function A(p) is defined as $A(p) = \int_0^p f(x)dx$.
For the questions below, we will only consider the values $0 \le p \le 12$.

(i) Determine the value(s) for p such that A(p) < 0. (1)

$4 < p < 8$

(ii) Determine the value(s) for p such that A(p) is at its maximum. (1)

$p = 2,\ p = 10$

(iii) Determine the value(s) of p, $p > 0$, where the value of A(p) is increasing at its fastest rate. (1)

$p = 8$

c) Evaluate $\int_2^{10} |f(x)| dx$ (1)

$\pi(2)^2$

$= 4\pi$ or 12.57 to 2dp

7 Using appropriate algebra and calculus techniques, show how you would calculate the area trapped between the curves given by $f(x) = x(x-5)^2$ and $g(x) = 8x - 12$. (4)

Intersection $x(x-5)^2 = 8x - 12$

$x = 3, \quad \frac{7+\sqrt{65}}{2}, \quad \frac{7-\sqrt{65}}{2}$

✓ intersections

$$\int_{\frac{7-\sqrt{65}}{2}}^{3} f(x) - g(x)\,dx + \int_{3}^{\frac{7+\sqrt{65}}{2}} g(x) - f(x)\,dx$$

✓ split at appropriate bounds

✓ manage negative regions

Area $= 136.08$ or $\frac{1633}{12}$

✓ area

b) $\int_0^{\pi} \sin(x)\,dx$ (3)

$= \left[-\cos x\right]_0^{\pi}$

$= -\cos\pi + \cos 0$

$= 1 + 1$

$= 2$

✓ integral

✓ substitute

✓ exact values & evaluate

3 a) Find, in terms of x,

$\frac{d}{dx}\int_x^1 (u^2-4)^3\,du$ (2)

$= -(x^2-4)^3$

✓ recognise FToC & substitute for x

✓ -ve due to reversed bounds

b) $\frac{d}{dx}\left(x^2\int_0^{\pi}\sin y\,dy\right)$ (2)

$= \frac{d}{dx}\left(2x^2\right)$

$= 4x$

✓ use $\int \sin$ from 2b.

✓ differentiate $2x^2$

4 If $f(x) = \frac{1-x}{\sqrt{1+x}}$, evaluate $\int_1^3 f'(x)\,dx$ (2)

$f(3) - f(1)$

$= -1 - 0$

$= -1$

✓ recognise FToC & substitute bounds

✓ evaluate

ATMAM Mathematics Methods

SHENTON COLLEGE

Test 2 (2019) Calculator Assumed

Name: ....

Teacher: Friday Smith Ai

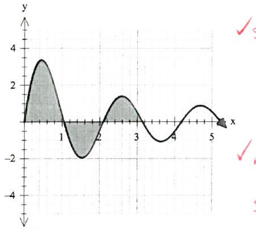
Time Allowed : 25 minutes

Marks	/22

Materials allowed: Classpad, Formula Sheet.

Attempt all questions. Questions 5 to 9 are in this section.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given to two decimal places.
Marks may not be awarded for untidy or poorly arranged work.

5 a) Below is a graph of the function $y = \frac{5\sin(3x)}{x+1}$, $x \geq 0$. Without using absolute values, write an expression to calculate the area shown below. (2)



$\frac{5\sin(3x)}{x+1} = 0 \Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

Area $= \int_0^{\frac{\pi}{3}} y\,dx + \int_{\frac{2\pi}{3}}^{\frac{\pi}{3}} y\,dx + \int_{\frac{2\pi}{3}}^{\pi} y\,dx$

reverse bounds, subtract integral or -ve function all ok.

✓ split integral at roots

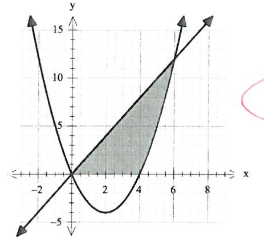
✓ manage -ve sections

b) Calculate $\int_0^{\pi} \frac{5\sin(3x)}{x+1}\,dx$ on your Classpad and explain why it gives a different result to your expression in part a). (2)

The region below the axis (from $\frac{\pi}{3}$ to $\frac{2\pi}{3}$) has a negative value, which works to negate some of the positive values above the axis, unless the regions above and below the axis are evaluated separately and their sign accounted for.

6 For each of the following diagrams, circle the integral that would give the indicated area. If neither integral would give the correct area, cross out all integrals and write "neither". (4)

a)

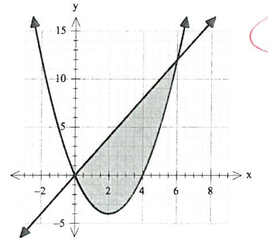


$\int_0^6 2x\,dx - \int_0^6 x(x-4)\,dx$

$\int_0^6 2x\,dx - \int_4^6 x(x-4)\,dx$

$\int_0^6 2x\,dx - \int_0^4 x(x-4)\,dx$

b)

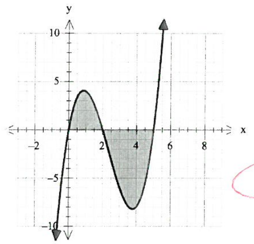


$\int_0^6 2x\,dx + \int_6^0 x(x-4)\,dx$

$\int_0^6 2x\,dx - \int_4^6 x(x-4)\,dx$

$\int_0^6 2x\,dx + \int_0^4 x(x-4)\,dx$

c)

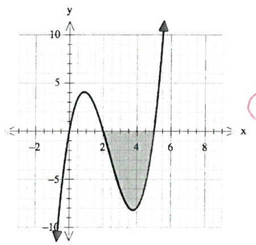


$\int_0^5 x^3 - 7x^2 + 10x\,dx$

$\int_0^2 x^3 - 7x^2 + 10x\,dx + \int_2^5 x^3 - 7x^2 + 10x\,dx$

$\int_0^2 x^3 - 7x^2 + 10x\,dx + \int_5^2 x^3 - 7x^2 + 10x\,dx$

d)



$\int_2^5 x^3 - 7x^2 + 10x\,dx$

$\int_2^5 -x^3 + 7x^2 - 10x\,dx$

$-\int_0^5 x^3 - 7x^2 + 10x\,dx$