

Question	Marks	Max	Question	Marks	Max
4		8			
3		10			
2		10	6		
1		5			
8		8			

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material if you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Important note to candidates**

Special items:      **nil**

Standard items:      pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

**To be provided by the candidate**

Formula sheet

This Question/Answer booklet

**To be provided by the supervisor**

Materials required/recommended for this section

Reading time before commencing work:      five minutes

Working time:      fifty minutes

Materials required/recommended for this section

Time allowed for this section

Your Teacher's Name:

Your Name:

Calculator-free

Section One:

UNIT 3 & 4

MATHEMATICS METHODS

Question/Answer booklet

Semester Two Examination, 2022



Perth Modern School

Mathematics Department

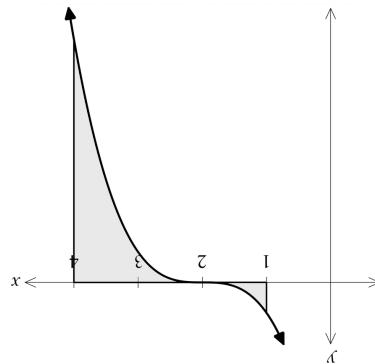
**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	49	33
Section Two: Calculator-assumed	11	11	93	100	67
<b>Total</b>					<b>100</b>

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

If the area of the shaded region is  $17 \text{ units}^2$ , determine the value of  $k$ .



The graph with equation  $y = k(2x - 4)^3$  is shown below.  
Shaded region is between the curve and the  $x$  axis from  $x = 1$  to  $x = 4$ .

(5 marks)

**Question 1**

Working time: 50 minutes.

- number of the question that you are continuing to answer at the top of the page.  
original answer space where the answer is continued, i.e. give the page number. Fill in the  
continuing an answer: if you need to use the space to continue an answer, indicate in the  
planning: if you use the spare pages for planning, indicate this clearly at the top of the page.  
responses and/or additional space if required to continue an answer.

Space pages are included at the end of this booklet. They can be used for planning your

responses and/or additional space if required to continue an answer.

(49 marks)

**Section One: Calculator-free**

Mathematics Department

Perth Modern School

This section has six questions. Answer all questions. Write your answers in the spaces provided.

Perth Modern School

Mathematics Department

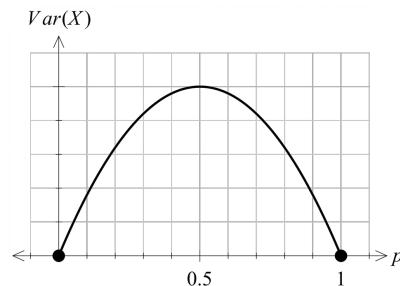
**Question 2****(10 marks)**

A random experiment can either result in a success with a probability of  $p$  or a failure.

In an event, this random experiment is conducted twice. Each experiment is independent of the other. Let the number of successes be represented by  $X$ .

- (a) State the distribution, and its parameters, that can be used to model the event described above. (2 marks)

The graph below shows of the variance for the distribution in part (a) for various values of  $p$ .



DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

- (b) Explain why the graph is continuous, despite the distribution being discrete. (1 mark)

- (c) (i) State the value of  $p$  for which the standard deviation of  $X$  is maximised. (1 mark)

- (ii) Hence determine the exact value of the standard deviation, for the value of  $p$  in part (c)(i). (1 mark)

(1 mark)

(f) Explain why  $\text{Var}(Y) = \text{Var}(X)$ .Given that  $\text{Var}(X) = 0.42$ .A second random variable  $Y$  is defined as  $Y = B(2, 0.7)$ .

(1 mark)

(e) Determine  $E(X)$  for this distribution.

(1 mark)

(ii) Hence show that the value of  $p$  is 0.3.

(2 marks)

(d) (i) Show that  $(1-p)^2 = 0.49$ 

For the distribution in part (a), the probability that at least one of these experiments results in a success is 0.51.

**Question 3****(10 marks)**

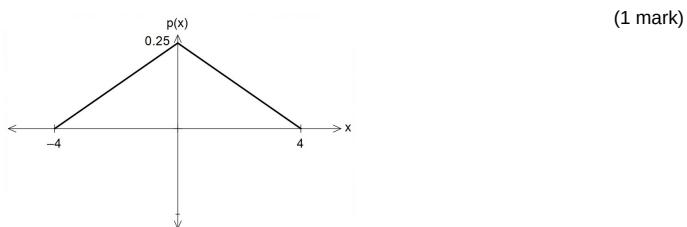
- (a) Determine whether the following represent or do not represent a probability distribution.  
Justify each answer.

$$f(x) = \frac{x}{x+2}, x = 0, 1, 2.$$

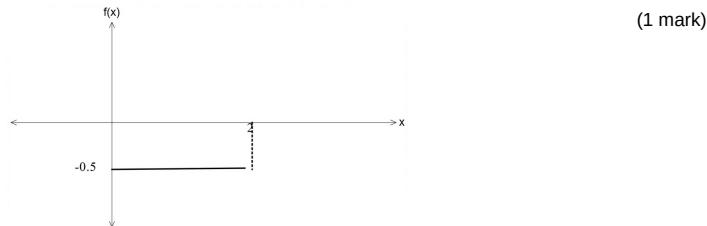
(i)

(1 mark)

(ii)



(iii)

**End of questions**

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(2 marks)

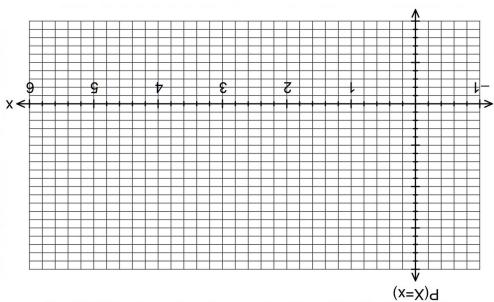
- (iv) State the cumulative probability distribution for  $X$ .

(2 marks)

- (iii) Determine  $P(X > 1 \mid X \leq 4)$

(1 mark)

- (ii) Determine the probability of obtaining a value of  $X$  less than 3.



(2 marks)

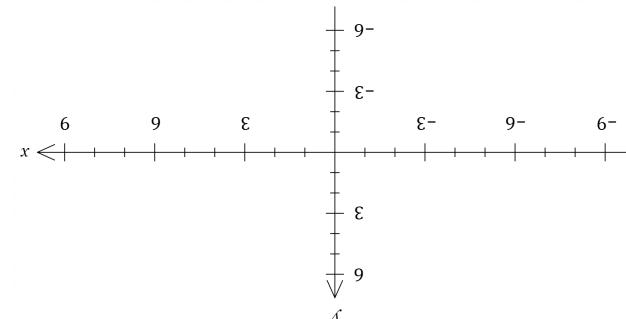
- (i) Graph the probability function on the axes below.

$$P(X = x) = \frac{15}{x} \text{ where } x = 1, 2, 3, 4, 5$$

- (b) A probability distribution of a random variable  $X$  is given by

(2 marks)

- (c) Given that  $\log_6(x+6) = \frac{\ln(6)}{\ln(x+6)}$ , determine the value of  $x$  where the slopes of  $y = f(x)$  and line  $L$  are the same.



(3 marks)

- (b) Sketch the graph of  $y = f(x)$  on the axes below.

(3 marks)

- (a) Determine the value of the constant  $C$  and the value of the constant  $k$ .

The graph of  $y = f(x)$  intersects line  $L$  with equation  $5y + 2x + 15 = 0$  when  $x = 0$  and  $x = -5$ .

(8 marks)

- Let  $f(x) = k \log_6(x+6) + C$ , where  $k$  and  $C$  are constants.

**Question 4**

Determine the following:

(a)  $\int 6 e^{2x-3} dx.$

**(8 marks)**

(1 mark)

(b)  $\int_0^{\frac{\pi}{8}} \sin(4x) dx.$

(2 marks)

(c)  $f\left(\frac{\pi}{6}\right)$  when  $f(x) = \frac{\cos(3x)}{2+\sin(x)}.$

(3 marks)

(d)  $\frac{d}{dx} \int_1^x \cos(t+1) dt.$

(1 mark)

(e)  $\int_0^3 \frac{d}{dx} (x e^{2x}) dx.$

(1 mark)

**Question 5**Components A and B form part of an electronic circuit, and properties of these components are measured  $t$  seconds after the circuit is turned on.

(a) The rate of change of temperature,  $T$  °C, of component A is given by  $\frac{dT}{dt} = \frac{18t}{3t^2+8}.$

Determine, in simplest form, the increase in temperature of this component during the first 4 seconds. (4 marks)

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(b) The current,  $I$  amps, flowing through component B reaches a peak very quickly and then declines as time goes on, as modelled by  $I(t) = \frac{2+ln(t)}{4t}$ . Determine, in simplest form, the maximum current that flows through this component. (4 marks)