SUPPLEMENTARY PAGE

Question: _____

Question: _____

PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course: Methods Year: 11

Student Name: **Marking Key** Teacher Name: ______________

Date: 09/09/22

Task Type: **Response**

Time Allowed: **40 minutes**

Number of Questions: **8**

Materials Required: CAS calculator (ClassPad) and one double-sided A4 pages of notes (to be provided by the student)

Standard Items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler and highlighters

Special Items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper (both sides) and up to three calculators approved for use in the WACE examinations

Marks Available: **40 marks**

Task Weighting: **10 %**

Formula Sheet Provided: Yes

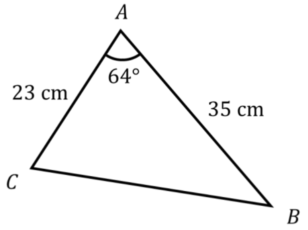
Do not penalise for missing/incorrect units.
Do not penalise for rounding to the incorrect number of decimal places.

Note: All questions worth more than 2 marks require working to obtain full marks.

TEST 4: TRIGONOMETRY AND EXPONENTIALS

Question 1 **[6 marks – 2, 2, 2]** (1.2.4)

a) Determine BC, to 1 decimal place.



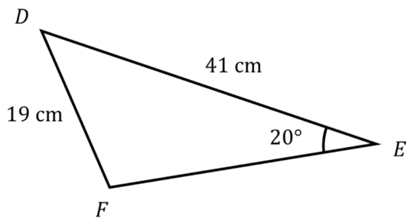
Solution

$$BC^2 = 23^2 + 35^2 - 2(23)(35)\cos 64°$$
$$BC = 32.4 \text{ cm}$$

Specific behaviours

- ✓ Substitutes into cosine rule
- ✓ Calculates length

b) Determine $\angle DFE$, to the nearest degree.



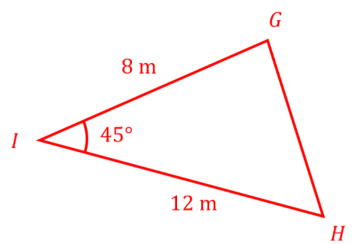
Solution

$$\frac{\sin\angle DFE}{41} = \frac{\sin 20°}{19}$$
$$\angle DFE = 48°, 132°$$

Specific behaviours

- ✓ Substitutes into sine rule
- ✓ Calculates both possible angles

c) Find the exact area of ΔGHI, given that $GI = 8$ m, $HI = 12$ m and $\angle GIH = 45°$.



Solution

$$\text{Area} = \frac{1}{2}(8)(12)\sin 45°$$
$$= 24\sqrt{2} \text{ m}^2$$

Specific behaviours

- ✓ Substitutes into area formula
- ✓ Calculates exact area

a) The temperature T°C after t hours can be modelled using the equation $T = ab^t + k$. Using the information shown, determine the equation.

Solution

From asymptote $y = k$:
$$k = 20$$
$$T = ab^t + 20$$

From y-intercept $(0, 80)$:
$$80 = ab^0 + 20$$
$$a = 60$$
$$T = 60b^t + 20$$

From $(1, 35)$:
$$35 = 60b^1 + 20$$
$$b = 0.25$$

$$T = 60(0.25)^t + 20$$

Specific behaviours

- ✓ Determines value of k
- ✓ Determines value of a
- ✓ Substitutes $(1, 35)$
- ✓ Determines value of b

Accept if equation is not stated

b) The safe drinking temperature is estimated to be about 57°C. How long does the tea need to cool for to be safe to drink, to the nearest minute?

Solution

Substituting in $T = 57$:
$$57 = 60(0.25)^t + 20$$
$$t = 0.349 \text{ hours}$$
$$t = 21 \text{ minutes}$$

Specific behaviours

- ✓ Substitutes into equation
- ✓ Determines time to the nearest minute

If estimated from graph, award 1 mark for 20 minutes or 2 marks for 21 minutes

End of Test

Question 7 **[6 marks – 3, 3]** (2.1.1-2.1.2, 2.1.7)

a) Simplify $(64a^6b^{15})^{\frac{1}{3}} \div (a^5bc^2)$, expressing your answer with positive indices.

Solution

$$(64a^6b^{15})^{\frac{1}{3}} \div (a^5bc^2) = 4a^2b^5 \div (a^5bc^2)$$
$$= 4a^{-3}b^4c^{-2}$$
$$= \frac{4b^4}{a^3c^2}$$

Specific behaviours

✓ Expands brackets
✓ Divides to combine variables
✓ Expresses with positive indices

b) Solve $16^x = 128$ for the exact value of x, showing all working.

Solution

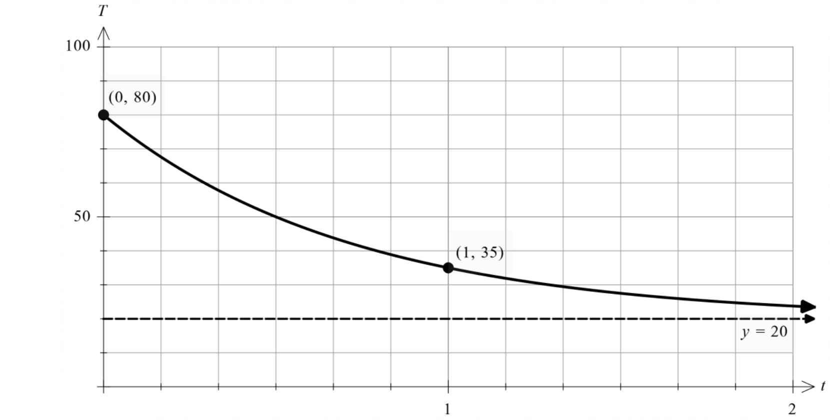
$$(2^4)^x = 2^7$$
$$2^{4x} = 2^7$$
$$4x = 7$$
$$x = \frac{7}{4}$$

Specific behaviours

✓ Expresses using a base of 2
✓ Equates indices
✓ Solves for exact value of x

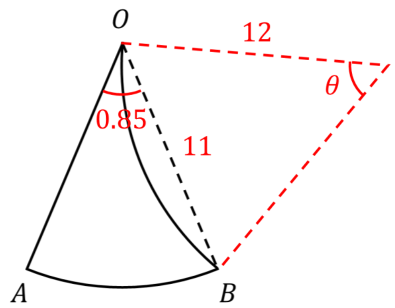
Question 8 **[6 marks – 4, 2]** (2.1.1-2.1.2, 2.1.7)

A cup of green tea is poured at 80°C and cools down towards room temperature at an exponential rate, as shown below.



Question 2 **[4 marks]** (1.2.5-1.2.6)

For the shape below, arc AB has radius 11 cm, arc OB has radius 12 cm and $\angle AOB = 0.85$. Find the area of the shape to 1 decimal place.



Solution

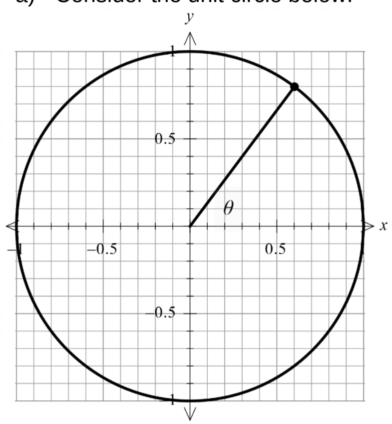
$$11 = 2(12)\sin\frac{\theta}{2}$$
$$\theta = 0.95$$
$$A = \frac{1}{2}(11)^2(0.85)$$
$$-\frac{1}{2}(12)^2(0.95 - \sin 0.95)$$
$$= 41.5 \text{ cm}^2$$

Specific behaviours

✓ Substitutes into chord formula
✓ Calculates angle subtended by arc OB
✓ Substitutes into sector and segment area formulas
✓ Calculates area (difference)
Award 1 mark for finding sector area AOB

Question 3 **[4 marks – 2, 2]** (1.2.7-1.2.8)

a) Consider the unit circle below.



i) Find $\cos(180° + \theta)$ to 1 decimal place.

Solution

$$\cos(180° + \theta) = -\cos\theta$$
$$= -0.6$$

Specific behaviours

✓ States value

ii) Find $\sin(-\theta)$ to 1 decimal place.

Solution

$$\sin(-\theta) = -\sin\theta$$
$$= -0.8$$

Specific behaviours

✓ States value

b) Determine the exact values of the following:

i) $\sin 135°$

ii) $\tan 300°$

Solution

$$\sin 135° = \frac{\sqrt{2}}{2} \quad \text{or} \quad \frac{1}{\sqrt{2}}$$

Specific behaviours

✓ States value

Solution

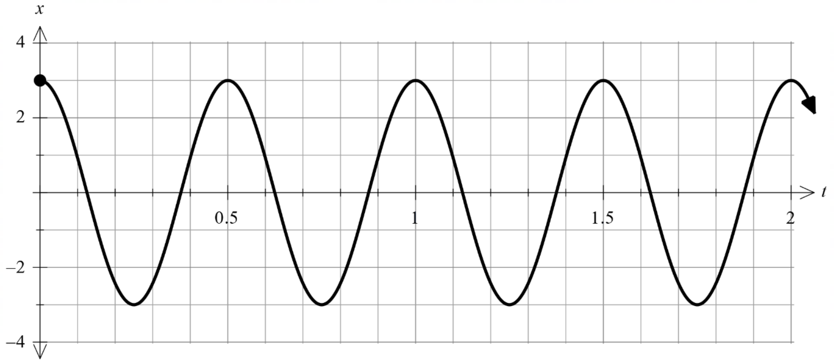
$$\tan 300° = -\sqrt{3}$$

Specific behaviours

✓ States value

Question 4 [3 marks – 1, 2] (1.2.9-1.2.12, 1.2.15)

A pendulum oscillates such that its horizontal position x cm with respect to time t seconds is as shown in the graph below.



a) State the amplitude and period of the pendulum.

Solution
$\text{Amplitude} = 3\text{ cm} \quad \text{Period} = 0.5\text{ seconds}$
Specific behaviours
✓ States amplitude and period

b) Given that $x(t) = a\cos(bt)$, state the equation of the pendulum's motion.

Solution
$x(t) = 3\cos(4\pi t) \quad \text{or} \quad 3\cos(720t)$
Specific behaviours
✓ Determines value of a
✓ Determines value of b
Accept if equation is not stated

Question 5 [7 marks – 3, 4] (1.2.16, 1.2.14)

a) Given that $\sin a = b$, where a is a positive acute angle, determine the exact solutions of $\sin 2\theta = -b$ where $0 \le \theta \le 2\pi$.

Solution
$2\theta = \pi + a,\ 2\pi - a,\ 3\pi + a,\ 4\pi - a$ $\theta = \frac{\pi + a}{2},\ \frac{2\pi - a}{2},\ \frac{3\pi + a}{2},\ \frac{4\pi - a}{2}$
Specific behaviours
✓ States first two solutions of 2θ
✓ States second two solutions of 2θ
✓ Divides by 2 to determine solutions of θ

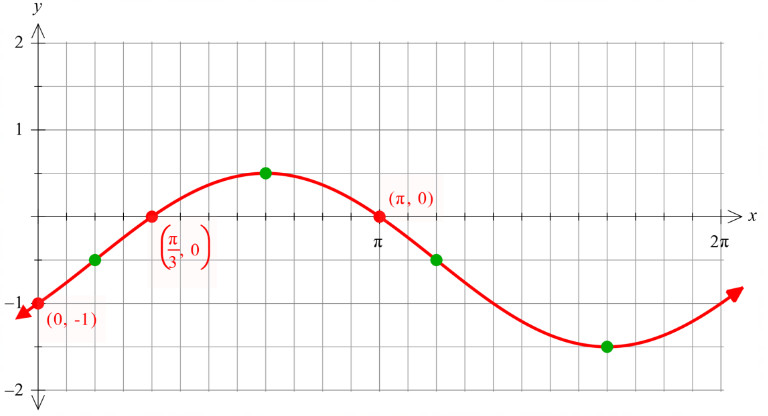
Question 5 (continued)

b) If $\cos A = -\frac{12}{13}$ where $180° < A < 270°$ and $\sin B = \frac{15}{17}$ where B is obtuse, determine the exact value of $\cos(A - B)$.

Solution
$\sin A = -\frac{5}{13}$ $\cos B = -\frac{8}{17}$ $\cos(A - B) = \cos A\cos B + \sin A \sin B$ $= \left(-\frac{12}{13}\right)\left(-\frac{8}{17}\right) + \left(-\frac{5}{13}\right)\left(\frac{15}{17}\right)$ $= \frac{21}{221}$
Specific behaviours
✓ Determines exact value of $\sin A$ or the correct magnitudes of $\sin A$ and $\cos B$
✓ Determines exact value of $\cos B$ or the correct signs of $\sin A$ and $\cos B$
✓ Substitutes into identity
✓ Calculates exact value of $\cos(A - B)$

Question 6 [4 marks] (1.2.9-1.2.12)

Graph $y = \sin\left(x - \frac{\pi}{6}\right) - \frac{1}{2}$ on the axes below, labelling the exact coordinates of all intercepts.



Specific behaviours
✓ Passes through points marked in green
✓ Correct y-intercept *(accept if not labelled)*
✓ Correct x-intercepts *(accept if not labelled)*
✓ Correct general shape
Follow through up to 2 out of 4 marks from any incorrect transformations.