Rossmoyne Senior High School

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

Circle your Teacher's Name:	Ms Bestall	Mr Buckland	Ms Fraser-Jones
	Mr Gibbon	Ms Goh	Ms Leonard
	Mr Luzuk	Mr Ng	Ms Murray

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	54	35
Section Two: Calculator-assumed	13	13	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Supplementary page

Question number: _____

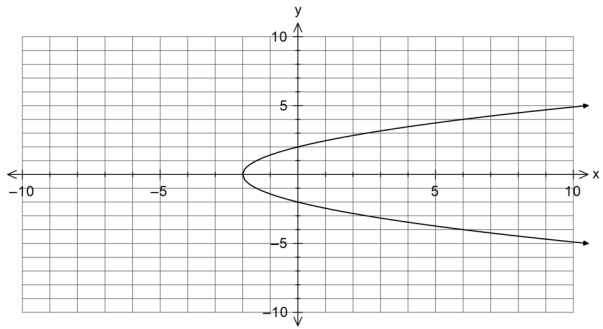
Section Two: Calculator-assumed **65% (97 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

The graph below is given in the form $y^2 = a(x-b)$.



(a) Find the values of a and b. (2 marks)

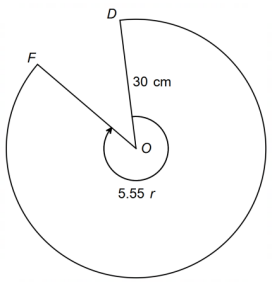
(b) State the equation of the axis of symmetry. (1 mark)

(c) Show the vertical line test on the graph above and explain how it is used to show whether this graph is a function or not. (2 marks)

See next page

Question 10 **(5 marks)**

(a) The diagram below shows a sector of a circle with centre O. The radius of the circle is 30cm and $\angle DOF = 5.55$ radians. Calculate the length of the major arc DF. (2 marks)



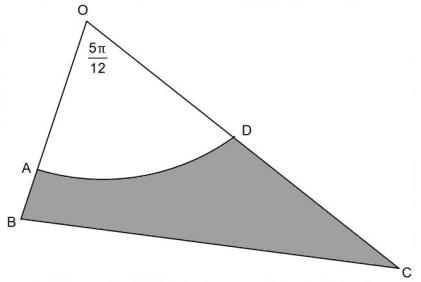
(b) In the diagram below AOD is a sector of the circle with centre O. BOC is a triangle.

In sector AOD, the radius is 30 cm and angle AOD is $\frac{5\pi}{12}$ radians.

In triangle OBC, OB = 38 cm and OC = 55 cm.

Calculate the shaded area of the shape with the vertices of ABCD rounded to 3 decimal places.

(3 marks)



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Supplementary page

Question number: _____

Supplementary page

Question number: _____

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Question 11 **(8 marks)**

A study of the achievements of 360 students enrolled in a university course yielded the following information:

- 50% of all students achieved a distinction
- 60% of those who did not achieve a distinction studied part-time
- 45% of those who studied full-time did not achieve a distinction

(a) Use the above information to complete the following table. (4 marks)

	Full-time	Part-time	Totals
Distinction			
No distinction			
Totals			360

(b) Determine the probability that a randomly chosen student from the study

(i) achieved a distinction and studied full-time. (1 mark)

(ii) achieved a distinction or studied full-time. (1 mark)

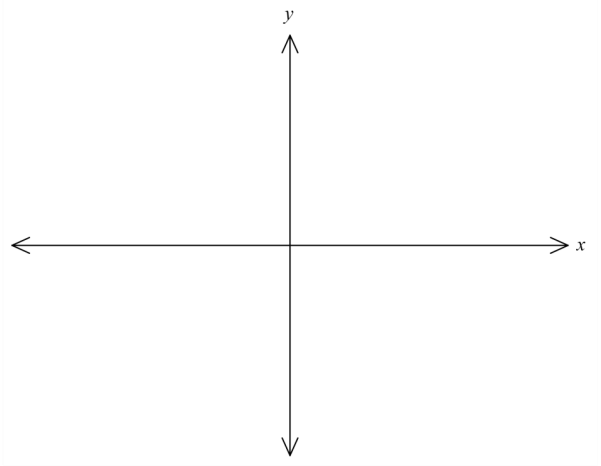
c) Sets F and D are subsets of the students in the study. A student belongs to F if they studied full-time and to D if they achieved a distinction. Use set notation to describe the subset of part-time students who achieved a distinction. (2 marks)

See next page

Question 12 **(8 marks)**

Let $f(x) = 3\sqrt{9-x} - 6$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (4 marks)



(b) Describe the transformation(s) required to obtain the graphs of the following functions from the graph of $y = 3\sqrt{9-x} - 6$:

(i) $y = \sqrt{9-x} - 2$. (2 marks)

(ii) $y = 3\sqrt{1-x} - 6$. (2 marks)

See next page

Supplementary page

Question number: _____

Question 13 **(8 marks)**

(a) Triangle ABC is such that $b = 15$ cm, $c = 18$ cm and $\angle A = 125°$. Determine, with justification, the length of side a. (2 marks)

(b) Triangle PQR is such that $p = 48.1$ cm, $q = 41.5$ cm and $\angle Q = 45°$. Determine all possible areas of this triangle. (6 marks)

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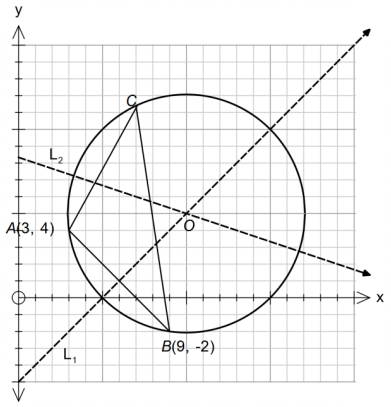
Question 14 **(5 marks)**

Triangle *ABC* is shown below where $A(3,4)$ and $B(9,-2)$.
The line L_1 is the perpendicular bisector of side *AB*.
The line L_2 intersects side *AC* and has the equation $3y+x=25$.
O is the centre of the circle, $(x-10)^2+(y-5)^2=50$ which passes through the vertices of $\triangle ABC$.



Show algebraically that *O* is the intersection of L_1 and L_2.

Question 21 (cont.)

(ii) Determine $n(C \cup M \cup P)$ (1 mark)

(iii) If one student is selected at random from the group, determine the probability of the following scenarios, leaving your answers as unsimplified fractions:

(a) They elected to study Maths but not Physics. (2 marks)

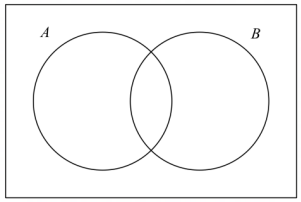
(b) They elected to study Maths and Physics, given that they did not study chemistry. (2 marks)

(c) They elected to study two of the subjects, given that they did not elect to study all three subjects. (2 marks)

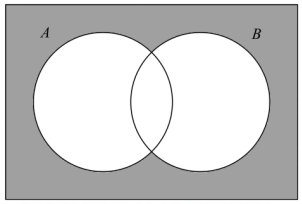
Question 21 **(10 marks)**

(a) Consider the two Venn diagrams below:

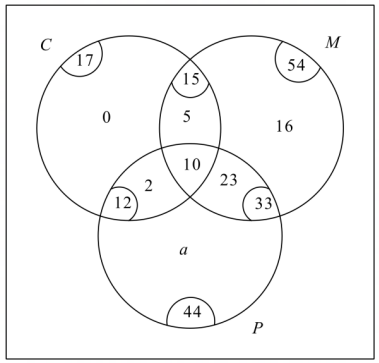
(i) Shade the region corresponding to $A \cap \overline{B}$. (1 mark)



(ii) Use symbolic set notation to describe the shaded regions below. (1 mark)



(b) The following Venn diagram shows the number of students electing to study at least one of Chemistry (C), Maths (M) or Physics (P).



(i) Determine the value of a. (1 mark)

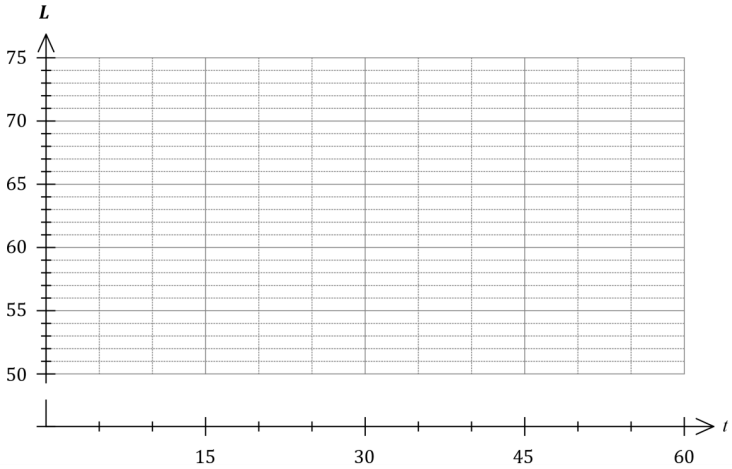
Question 15 **(9 marks)**

The loudness L of sound, in decibels, emitted by a machine t minutes after it is switched on can be modelled by

$$L = 62 - 11\cos\left(\frac{\pi t}{10}\right)$$

(a) Determine the initial loudness emitted by the machine. (1 mark)

(b) Draw the graph of L against t on the axes below for the first 60 minutes. (3 marks)



(c) State the maximum loudness emitted by the machine and the time this maximum was first reached. (2 marks)

(d) A health and safety inspector can deem a machine unserviceable if the loudness it emits exceeds 70 dB for more than 15 minutes in any hour that it is running. Determine, with justification, whether this machine could be deemed unserviceable. (3 marks)

Question 16 **(8 marks)**

A souvenir shop sells T-shirts in two colours and three sizes. Sales records for the past year are shown below.

	Small	Medium	Large
Blue	210	420	310
White	230	450	180

Assume that the shop holds a large stock and that sales continue in similar proportions. Where relevant, round your answers in this question to three decimal places.

(a) A customer randomly selects a T-shirt for purchase. Determine:

(i) the size and colour of the least likely T-shirt and the probability that this T-shirt is selected. (2 marks)

(ii) the probability that the T-shirt selected is not small. (2 marks)

(b) A customer randomly selects two T-shirts for purchase. Determine the probability that the T-shirts are:

(i) both medium. (2 marks)

(ii) of different colours. (2 marks)

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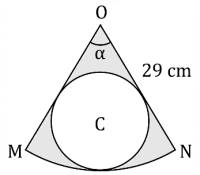
Question 20 **(7 marks)**

The diagram shows sector OMN of a circle centre O of radius 29 cm and $\alpha = 68°$.

Circle C is inside the sector and just touches OM, ON and arc MN.



(a) Determine the area of sector OMN. (2 marks)

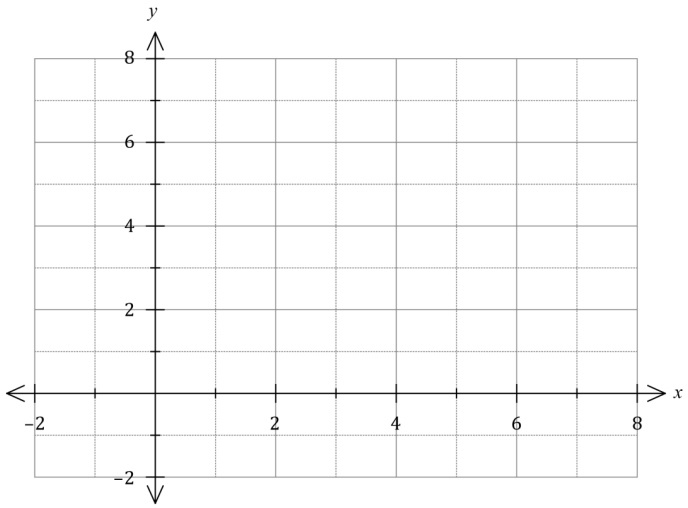
(b) Show that the radius of circle C is 10.4 cm, correct to one decimal place. (3 marks)

(c) Determine the area of the shaded region, inside sector OMN but outside circle C. (2 marks)

Question 19 **(7 marks)**

The equation of a parabola is $y = \frac{1}{4}(x^2 - 6x + 15)$.

(a) Sketch the parabola on the axes below. (3 marks)



All parabolas have a focal point and a directrix. For a parabola with equation $y = a(x - p)^2 + q$, the focal point is at $\left(p, q + \frac{1}{4a}\right)$ and the equation of the directrix is $y = q - \frac{1}{4a}$, where a, p and q are constants.

(b) Determine the focal point and directrix for this parabola and add them, with labels, to your sketch above. (4 marks)

Question 17 **(8 marks)**

(a) Let $f(x) = x^2 + bx + c$, where b and c are constants. The graph of $y = f(x)$ has an axis of symmetry with equation $x = -3$ and an axis intercept at $(0, 5)$.

(i) State the value of the constant c. (1 mark)

(ii) Determine the value of the constant b. (2 marks)

(b) Let $g(x) = 2(x - 2)^2 - 7$. Determine

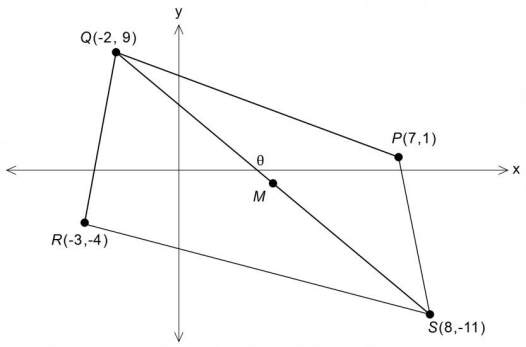
(i) the coordinates of the turning point of the graph of $y = g(x)$. (1 mark)

(ii) the domain and range of $g(x)$. (2 marks)

(iii) the coordinates of the turning point of the graph of $y = g(x - 3) + 2$. (2 marks)

Question 18 **(9 marks)**

In the diagram *PQRS* is a quadrilateral having vertices *P*(7, 1), *Q*(−2, 9), *R*(−3, −4) and *S*(8, −11). *M* is the midpoint of *QS*.



(a) If a line is drawn from *P* to *R*, determine the equation of the line *PR*. (2 marks)

(b) Determine whether *M* lies on the line PR. (3 marks)

Question 18 (cont.)

(c) Show that QS is perpendicular to PR. (2 marks)

(d) Calculate θ, the angle of inclination of QS. (2 marks)