WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2017
TEST 2: Functions

Name: _____

Monday 3rd April

Time: 50 minutes **Mark** **/40 =** **%**

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet in **both** sections of the test.
- You are permitted one A4 page (one side) of notes in the calculator assumed section.

Calculator free section **Suggested time: 30 minutes** **/26**

1. [9 marks]

Two functions *f* and *g* are defined by $f(x) = \sqrt{x+4}$ and $g(x) = e^x - 1$

a) Express $g \circ f(x)$ in terms of *x*

[1]

b) What is the natural domain of $y = g \circ f(x)$

[2]

c) What is the range (co-domain) of $y = g \circ f(x)$

[2]

A third function $y = h(x)$ is such that $f(h(x)) = \sqrt{x^2 - 4}$.

d) Express $h(x)$ in terms of *x*.

[1]

e) Clearly define $y = f^{-1}(x)$ and specify both its domain and range.

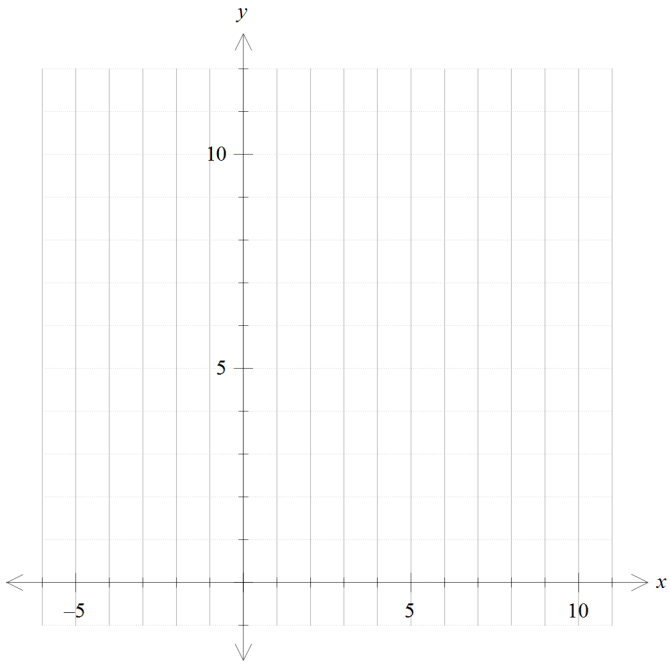
[3]

2. [7 marks]

a) Solve the inequality $|2 - x| \geq 5$

[3]

b) Calculate where the line $y = |2x - 6|$ intersects $y = |x + 2| + 1$ and illustrate your solution on the axes provided.

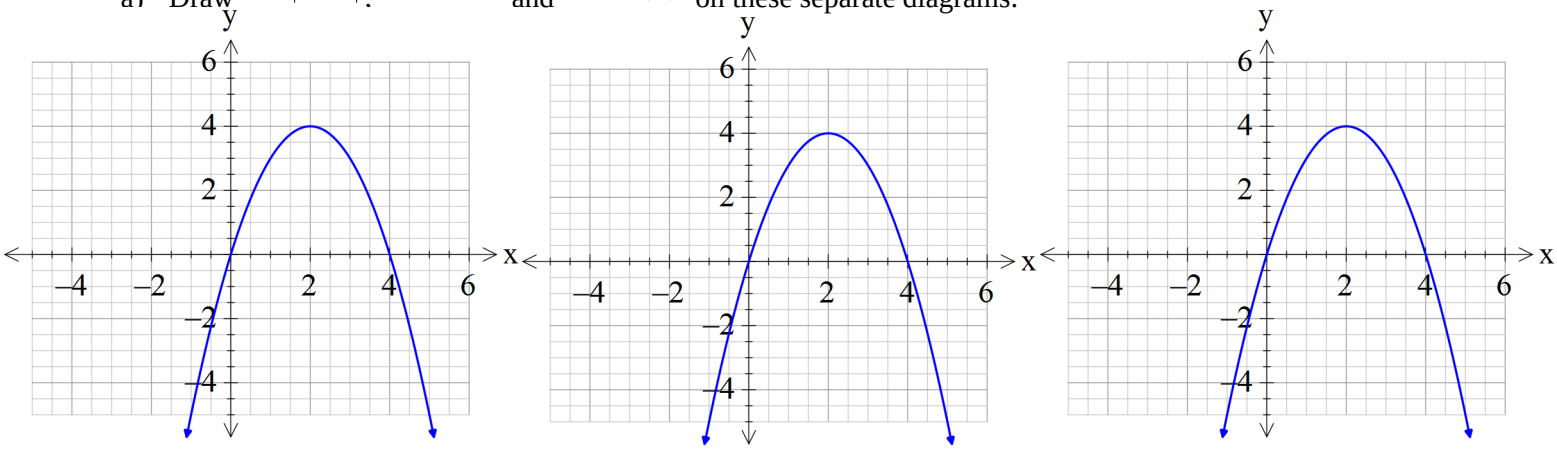


[4]

3. [10 marks]

The function $f(x) = 4x - x^2$ is represented by the graph of $y = f(x)$ shown on each set of axes provided.

a) Draw $y = |f(x)|$, $y = \frac{1}{f(x)}$ and $y = f(|x|)$ on these separate diagrams:



[4]

The domain of $f(x) = 4x - x^2$ is restricted to $\{x : x \in \mathbb{R}, x \leq k\}$ so that $y = f^{-1}(x)$ can be defined as a function.

b) Determine the largest possible value of *k*

[1]

c) Define $y = f^{-1}(x)$ and specify its domain and range.

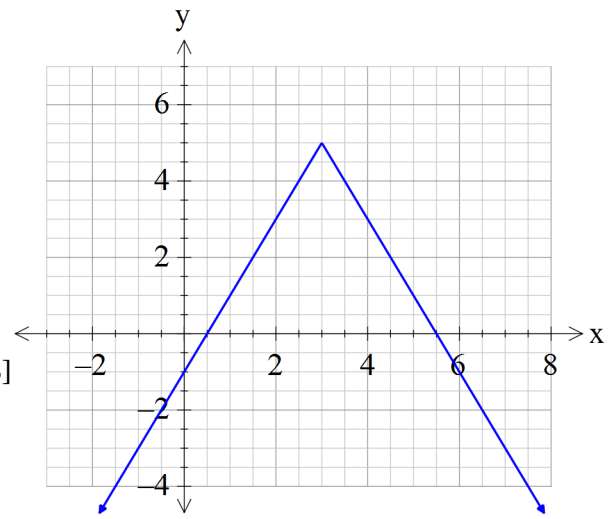
[5]

Calculator assumed section **Suggested time: 20 minutes** **/14**

Name: _____

4. [7 marks]

The graph of $y = f(x)$ for $f(x) = a|x + b| + c$ has a y-intercept of $(0, -1)$ and a maximum point at $(3, 5)$, as shown.



a) Evaluate a, b and c.

[3]

b) For which value(s) of d does $|f(x)| = d$ have exactly four solutions?

[2]

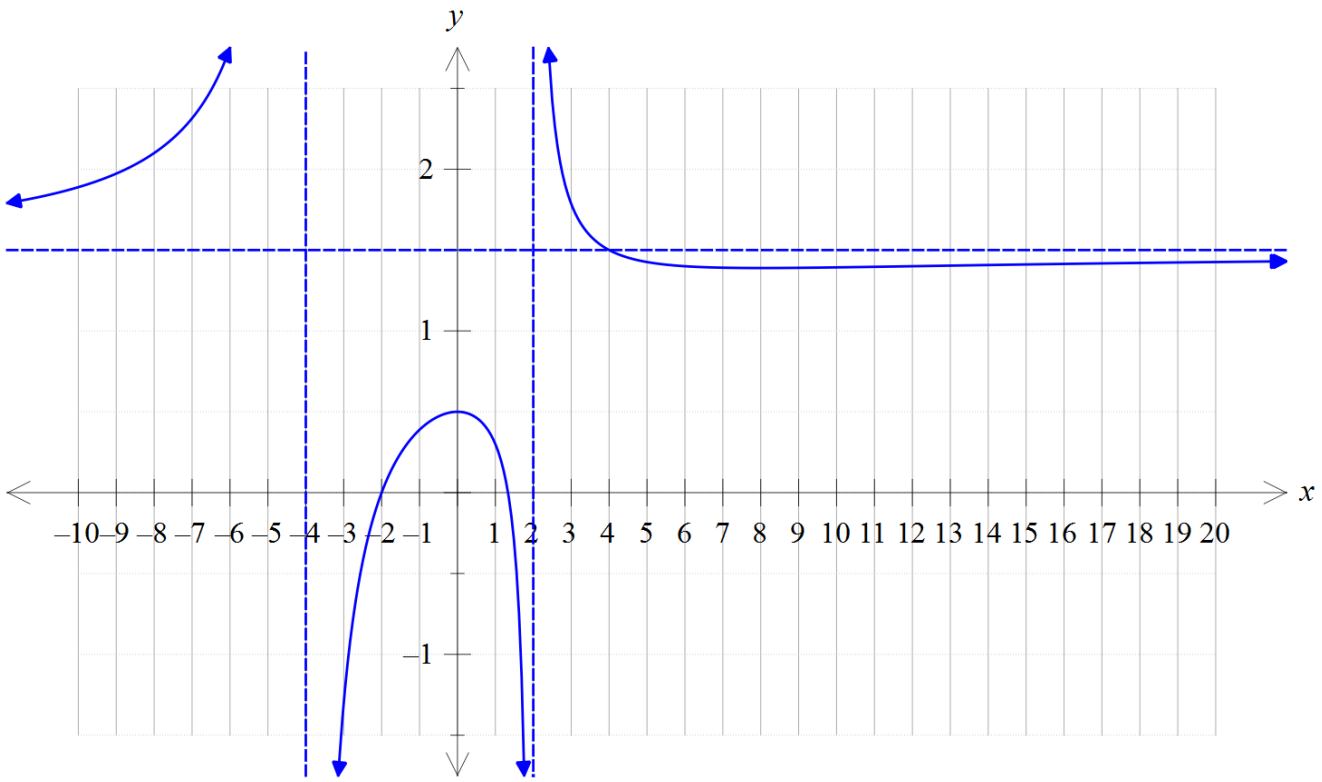
c) Add a graph of $y = g(x)$ to the axes above so that $\{x : x \in \mathbb{R} \text{ and } f(x) = g(x)\} = \{x : x \in \mathbb{R} \text{ and } -1 \leq x \leq 3\}$

[2]

5. [7 marks]

$$y = f(x) = \frac{ax^2}{(x+b)(x-c)} + d$$

This graph represents a function of the form



$\left(\frac{4}{3}, 0\right)$

The asymptotes are as shown and the unmarked x intercept is .

(a) Determine the values of the constants a, b, c and d.

[4]

$y = f(x)$

(b) What is the exact range of ?

[3]