PERTH MODERN SCHOOL
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Independent Public School

Course Methods Test 1 Year 12

Student name: _______________ Teacher name: _______________

Task type: **Response**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: _____**6**______

Materials required: Upto three calculators/classpads

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper,

Marks available: **41 marks**

Task weighting: **13%**

Formula sheet provided: **no but formulae listed on next page.**

Note: All part questions worth more than 2 marks require working to obtain full marks.

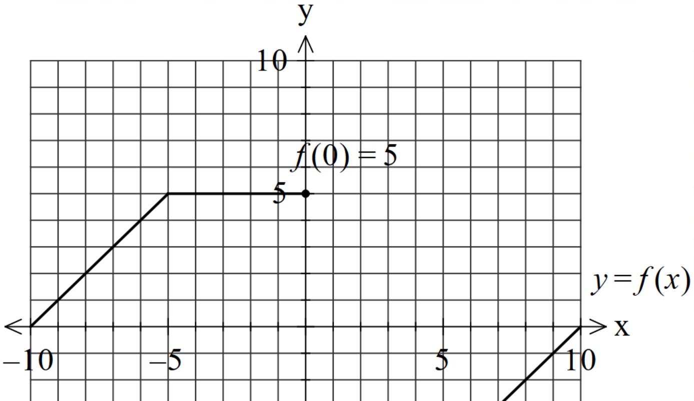
Useful formulae

$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
$\frac{d}{dx}e^{ax-b} = ae^{ax-b}$	$\int e^{ax}dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}dx = \ln x + c, \quad x > 0$
$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)}dx = \ln f(x) + c, \quad f(x) > 0$
$\frac{d}{dx}\sin(ax-b) = a\cos(ax-b)$	$\int \sin(ax-b)\,dx = -\frac{1}{a}\cos(ax-b) + c$
$\frac{d}{dx}\cos(ax-b) = -a\sin(ax-b)$	$\int \cos(ax-b)\,dx = \frac{1}{a}\sin(ax-b) + c$

Product rule	If $y = uv$ then $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$	or	If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$
Quotient rule	If $y = \frac{u}{v}$ then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	or	If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$
Chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or	If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$
Fundamental theorem	$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$	and	$\int_a^b f'(x)\,dx = f(b) - f(a)$
Increments formula	$\delta y \approx \frac{dy}{dx} \times \delta x$		
Exponential growth and decay	$\frac{dP}{dt} = kP \Leftrightarrow P = P_0e^{kt}$		

Q1 (2, 3, 2 ,2 & 3 =12 marks)

Consider the function $y = f(x)$ which is graphed below



a) $\int_{10}^{10} f(x)dx$.

Solution
Zero as right side is negative of left side
Specific behaviours
P must state zero P recognises that left = - right

b) $\int_{3}^{3} f(x) - 4\,dx$.

Solution
$\int_{3}^{3} f(x) - 4\,dx = \int_{3}^{3} f(x)dx - \int_{3}^{3} 4dx$ $= 0 - [4x]_{-3}^{3} = -(12 - -12) = -24$
Specific behaviours
P shows integral under f from x=-3 to x=3 equates to zero P states antiderivative of 4 P subs x values

c) $\frac{d}{dt}\int^{6} f(x)dx$ when $t = 8$.

Solution
$\frac{d}{dt}\int^{6} f(x)dx = \frac{d}{dt} - \int_{6}^{t} f(x)dx = - f(t)$ $= - f(8) = 2$

Specific behaviours
P uses FTC P states result

d) $\int_{-9}^{-6} f'(x)dx$.

Solution
$\int_{-9}^{-6} f'(x)dx = \left[f(x)\right]_{-9}^{-6} = f(-6) - f(-9) = 4 - 1 = 3$
Specific behaviours
P uses FTC P states result

e) $\frac{d}{dt}\int^{t^2} f(x)dx$ in terms of t for $0 < t < 2$.

Solution
$\frac{d}{dt}\int^{t^2} f(x)dx = f(t^2)2t = -10t$
Specific behaviours
P uses FTC P uses chain rule P states explicit result in terms of t only

Q2 (4 marks)
Sketch a continuous function **showing the** x **coordinates and labelling** of all special features on the axes below that meet the following requirements.

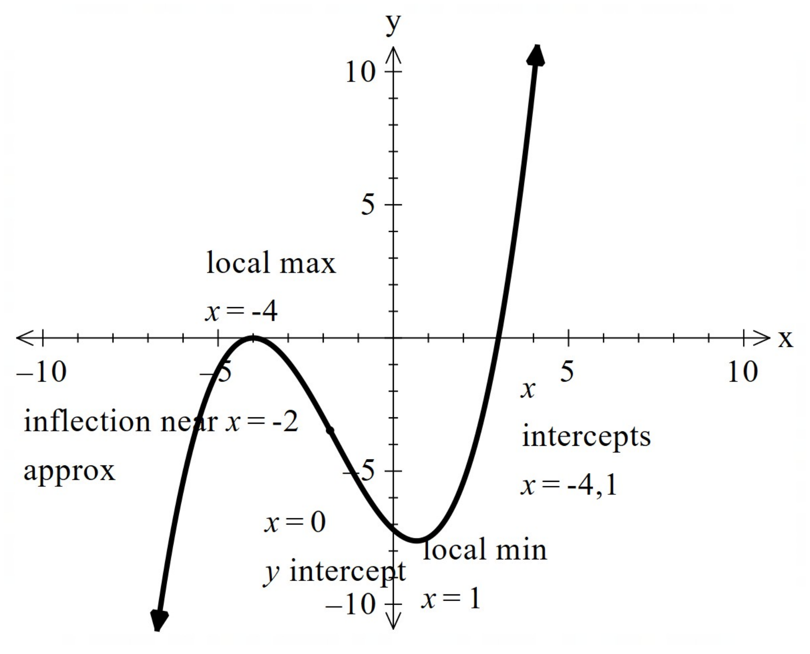
$f(-4)=0=f(3)$

$f(0)=-7$

$f'(-4)=0=f'(1)$

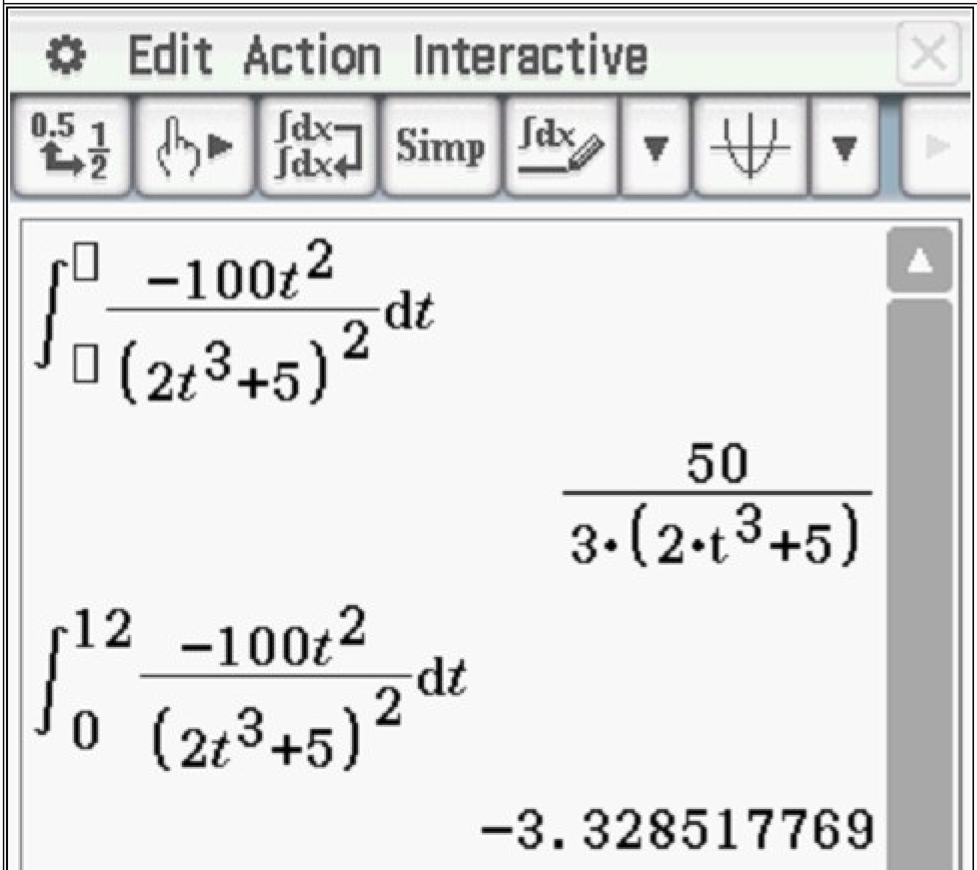
$f''(1)>0,\ f''(-4)<0$

Has **exactly** two stationary points.

Solution

Specific behaviours
P labels and shows x coordinates of x & y intercepts P labels and show approx. x value of inflection pt P labels and shows x values of both turning pts P correct shape of curve

Q3 (3 marks)

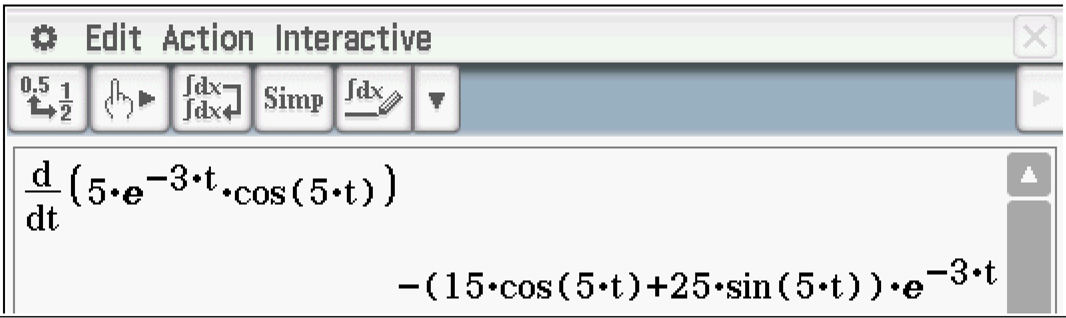
Consider a balloon whose volume V, litres, varies with time, t seconds, such that $\dfrac{dV}{dt} = \dfrac{-100t^2}{(2t^3+5)^2}$.
If the balloon fully deflates after 12 seconds, determine the initial volume. Full reasoning must be shown for full marks.

Solution
 Initial volume = 3.33 litres
Specific behaviours
P uses a **definite** integral (Must be shown) P states antiderivative P states initial volume with UNITS and to at least 2 dp

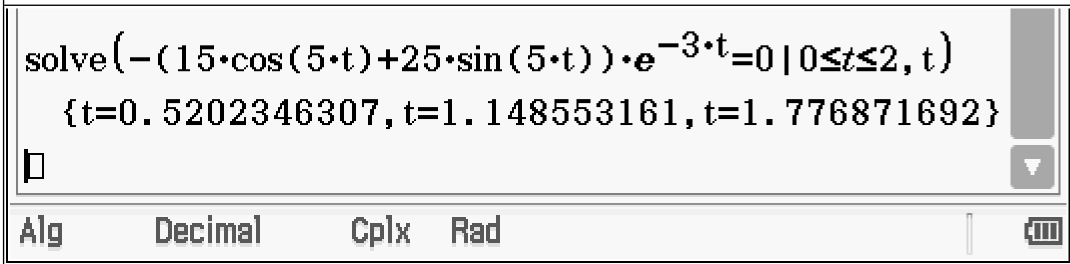
Q4 (2, 2 & 3 = 7 marks)

An object's displacement, x metres at t seconds, from the origin is $x = 5e^{-3t}\cos(5t)$ metres.

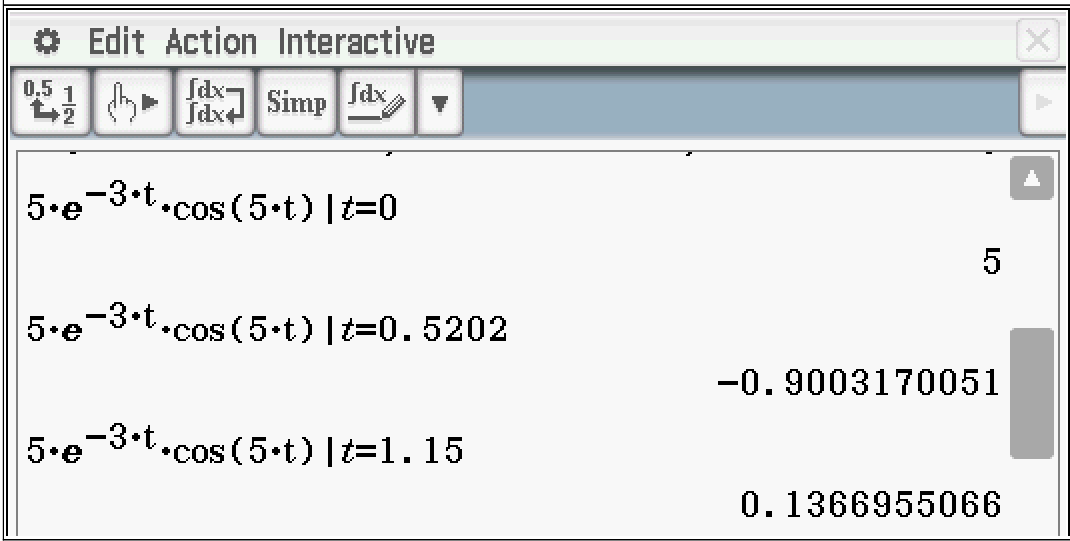
a) Determine the velocity function.

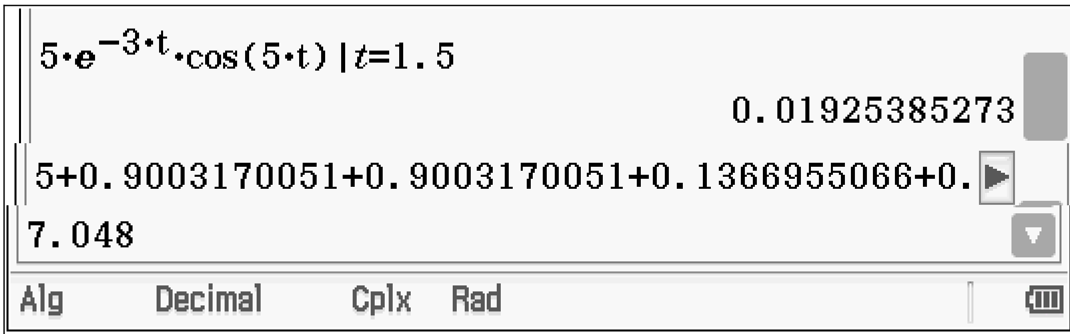
Solution

Specific behaviours
P differentiates P determines velocity function , no need for units nor simplifying

b) Determine the first two times that the object changes direction.

Solution
 Time= 0.52 & 1.15 seconds
Specific behaviours
P equates velocity to zero P states at least the first two times

c) Determine the distance travelled in the first 1.5 seconds.

Solution




Accept 7.02 to 7.25 due to rounding errors

Specific behaviours
P determines initial position P determines positions when velocity = 0 P states distance, no need for units & between 7.02 and 7.25 Accept integration of absolute velocity for full marks if stated in full with correct limits (Answer only – 2 marks)

Q5 (2 & 4 = 6 marks)

a) Determine $\frac{d}{dx}\left(3x\cos\frac{\pi x}{6}\right)$ **without the use of a classpad**. Full reasoning must be given.

Solution
$\frac{d(uv)}{dx} = u\frac{dv}{dx} + \frac{du}{dx}v$ $\frac{d}{dx}\left((3x)(\cos\frac{\pi x}{6})\right) = -(3x)\frac{\pi}{6}\sin\frac{\pi x}{6} + 3(\cos\frac{\pi x}{6})$ $= -x\frac{\pi}{2}\sin\frac{\pi x}{6} + 3\cos\frac{\pi x}{6}$
Specific behaviours
P uses product rule, clearly shown via brackets or defining u & v functions P at least one term correct (Note- zero marks if answer given only)

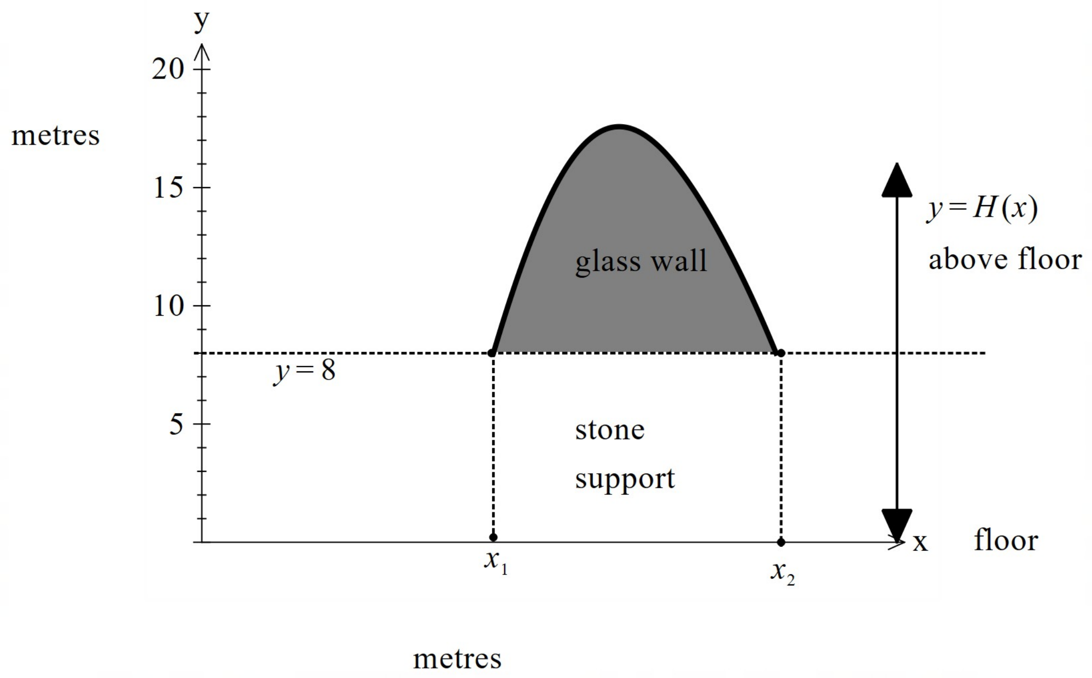
b) Hence show how to determine $\int_{6}^{\pi} \frac{\pi}{6} x\sin\frac{\pi x}{6}\,dx$ **without the use of a classpad**. Full reasoning must be given **using** the result from part a.

Solution

$\frac{d}{dx}\left(3x\cos\frac{\pi x}{6}\right) = -3x\frac{\pi}{6}\sin\frac{\pi x}{6} + 3\cos\frac{\pi x}{6}$ $\int\frac{d}{dx}\left(3x\cos\frac{\pi x}{6}\right)dx = \int -3x\frac{\pi}{6}\sin\frac{\pi x}{6}dx + \int 3\cos\frac{\pi x}{6}dx$ $\int\frac{d}{dx}\left(-x\cos\frac{\pi x}{6}\right)dx = \int x\frac{\pi}{6}\sin\frac{\pi x}{6}dx - \int\cos\frac{\pi x}{6}dx$ $-x\cos\frac{\pi x}{6} = \int x\frac{\pi}{6}\sin\frac{\pi x}{6}dx - \frac{6}{\pi}\sin\frac{\pi x}{6} + c$ $\int x\frac{\pi}{6}\sin\frac{\pi x}{6}dx = \frac{6}{\pi}\sin\frac{\pi x}{6} - x\cos\frac{\pi x}{6} + c$
Specific behaviours
P shows with **integral signs** that both sides of part a are integrated P uses FTC P integrates cosine term P rearranges to show value of required integral

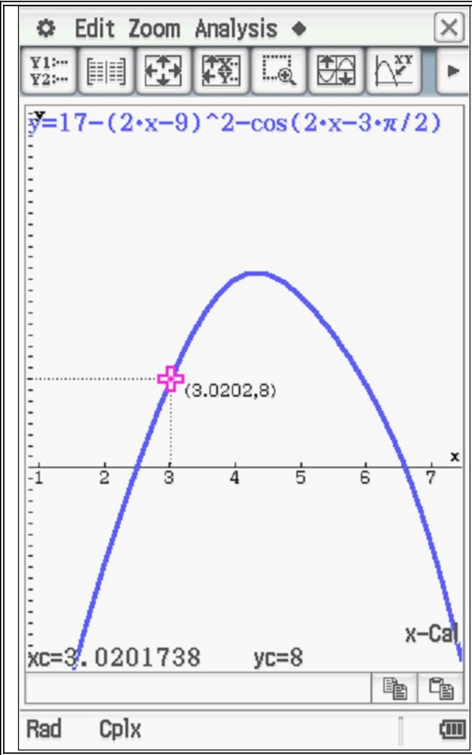
Q6 (2, 4 & 3 = 9 marks)

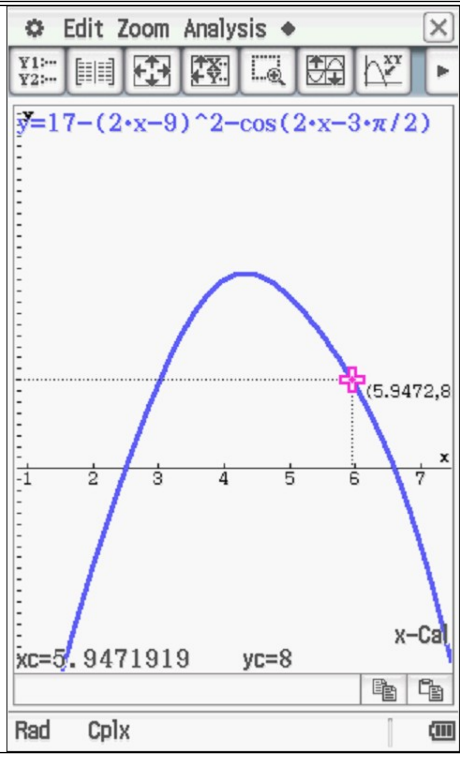
Consider a glass wall with the height $H(x)$ metres **above floor** at x metres along the floor according to
$H(x) = 17 - (2x-9)^2 - \cos(2x - \frac{3\pi}{2})$. The glass wall sits on a stone support of height 8 metres.



a) Determine the values x_1 & x_2 to the nearest cm.

Solution





X1=3.02metres X2=5.95metres

Specific behaviours
P states both x values P rounded to nearest cm, 2 dp m or whole cm, no need for units only rounding.

b) Using calculus, determine the maximum height of the wall. Justify.

Solution

$$H(x) = 17 - (2x - 9)^2 - \cos(2x - \frac{3\pi}{2})$$

$$\frac{dH}{dx} = -4(2x - 9) + 2\sin(2x - \frac{3\pi}{2})$$

$$OR - 4(2x - 9) + 2\cos(2x)$$

$$\frac{d^2H}{dx^2} = -8 + 4\cos(2x - \frac{3\pi}{2})$$

$$\frac{d}{dx}\left(17-(2\cdot x-9)^2-\cos\left(2\cdot x-\frac{3\cdot\pi}{2}\right)\right)$$

$$-8\cdot x+2\cdot\cos(2\cdot x)+36$$

solve$(-8\cdot x+2\cdot\cos(2\cdot x)+36=0 \mid 0\le x\le 7, x)$

$\{x=4.322302151\}$

$$\frac{d}{dx}(-8\cdot x+2\cdot\cos(2\cdot x)+36)$$

$$-4\cdot\sin(2\cdot x)-8$$

$$-4\cdot\sin(2\cdot x)-8\,|\,x=4.322302151$$

$$-10.81361146$$

$$17-(2\cdot x-9)^2-\cos\left(2\cdot x-\frac{3\cdot\pi}{2}\right)\,|\,x=4.322302151$$

$$17.57709676$$

Max height = 17.58 metres

Specific behaviours
P states first derivative function P equates derivative to zero and solves for x P uses derivative test **with result** to show nature P states y value of turning point, no need for units

c) If the wall is 5 cm wide determine the volume of glass with units, needed to make the wall.

Solution
Edit Action Interactive Simp $\int_{3.02}^{5.947} 17-(2\cdot x-9)^2-\cos\left(2\cdot x-\frac{3\cdot\pi}{2}\right)dx$ $\frac{-(8\cdot x^3-108\cdot x^2+384\cdot x+3\cdot\cos(2\cdot x))}{6}$ 41.49106944 41.49106944

41.491−(5.95−3.02)×8

18.051

18.051×.05

0.90255

Volume = 0.903 cubic metres

Specific behaviours

P sets up definite integral for area

P uses correct limits in definite integral

P changes 5 cm into metres and states volume **with units cubic metres or cubic cm**

Q6c continue.

End of test.