PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Methods TEST 1 Friday 22 February 2019 TIME: 45 minutes working One page Notes allowed **Calculator Assumed** 39 marks 7 Questions

Name: Marking Key Teacher: ______________

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 **(4 marks)**

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-1	-2	-1
2	2	-1	1	0
3	1	-1	2	1

(a) Define $h(x) = \frac{f(x)}{g(x)}$, use the table to find the value for $h'(2)$. (2 marks)

$$h' = \frac{gf' - fg'}{g^2} = \frac{1(-1) - 2(0)}{1^2}$$

$$= -1$$

✓ uses quotient rule

✓ subs correct values

(b) Define $I(x) = f(g(x))$, use the table to find the value for $I'(3)$. (2 marks)

$$I' = f'g'$$

$$= (-1)(1)$$

$$= -1$$

✓ uses chain rule

✓ subs correct values

Question 2 **(3 marks)**

Find the equation of the line tangent to the function $y = (3x^2 - 2)^3$ at the point $(2, 2)$. Give your answer in the gradient-intercept form.

$y' = 3(3x^2-2)^2(6x)$

$x = 2 \qquad y' = 3600$

$y = 3600x + c$
$2 = 7200 + c$
$c = -7198$

$y = 3600x - 7198$

✓ obtains $\frac{dy}{dx}$
✓ solves for constant
✓ states eqn of tangent.

Question 3 **(3 marks)**

The time period T for a simple pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{g}}$ where g is a constant.

If the length changes by 3%, use the incremental formula to estimate the percentage change in the period.

$$\Delta T \approx \frac{dT}{dl}\Delta l$$
$$= \frac{\pi}{\sqrt{g}} l^{-\frac{1}{2}} \Delta l$$

$$\frac{\Delta T}{T} = \frac{\frac{\pi}{\sqrt{gl}}\Delta l}{2\pi\sqrt{\frac{l}{g}}} = \frac{\Delta l}{2l}$$
$$= \frac{1}{2}(3\%)$$
$$= 1.5\%$$
$$\left(\frac{3}{2}\right)\%$$

✓ uses incremental formula
✓ obtains expression for $\frac{\Delta T}{T}$
✓ determines % change.

Question 4 **(7 marks)**

A company is purchasing a type of thin sheet metal required to make a closed cylindrical container with a capacity of 4000π cm^3.

(a) Let the radius of the cylindrical base be r. Find the expression for the height h in terms of r. (1 mark)

$$4000\pi = \pi r^2 h \qquad h = \frac{4000}{r^2} \checkmark$$

(b) Hence, find the expression for the surface area of the cylinder in terms of r. (2 marks)

$$S = 2\pi r^2 + 2\pi r h$$
$$= 2\pi r^2 + 2\pi r \frac{4000}{r^2}$$
$$= 2\pi r^2 + \frac{8000\pi}{r}$$

✓ determines surface area in terms of r & h

✓ expresses in terms of r only

(c) Therefore, find the least area of metal required to make a closed cylindrical container from thin sheet metal in order that it will have a capacity of 4000π cm^3. (4 marks)

$$S = 2\pi r^2 + 8000\pi r^{-1}$$

$$\frac{dS}{dr} = 4\pi r - \frac{8000\pi}{r^2}$$

$$4\pi r - \frac{8000\pi}{r^2} = 0 \quad , r \neq 0$$

$$r = \frac{2000}{r^2}$$

$$r^3 = 2000$$

$$r = \sqrt[3]{2000} \approx 12.60 \text{ cm}$$

$$\frac{d^2S}{dr^2} = 4\pi + \frac{16000\pi}{r^3} > 0 \quad \therefore \text{local min}$$

$$S \approx 2992.2 \text{ cm}^2$$

✓ obtains $\frac{dS}{dr}$

✓ equates $\frac{dS}{dr}$ to zero AND solves for r

✓ uses first or second derivative test to determine nature

✓ determines least surface area

Question 5 **(8 marks)**

The position of a train on a straight mono rail, x metres at time t seconds, is modelled by the following formula for the velocity, v in metres/second, $v = pt^2 - 12t + q$ where $p \& q$ are constants. The deceleration of the train is $8ms^{-2}$ when $t = 1$, has a position $x = \frac{4}{3}$ when $t = 2$ and is initially at the origin ($x = 0$).

a) Determine the values of the constants $p \& q$. (4 marks)

$a = 2pt - 12$

$-8 = 2p(1) - 12$

$p = 2$

$v = 2t^2 - 12t + q$

$x = \frac{2t^3}{3} - 6t^2 + qt + c$

$c = 0$

$\frac{4}{3} = \frac{2}{3}(2)^3 - 6(2)^2 + 2q$

$q = 10$

✓ Solves for p using acceleration

✓ integrates to find x

✓ states constant = 0 for x

✓ determines q

b) Determine the time(s) that the velocity is zero. (2 marks)

$v = 2t^2 - 12t + 10$

$= (2t - 2)(t - 5)$

$t = 1$ or 5

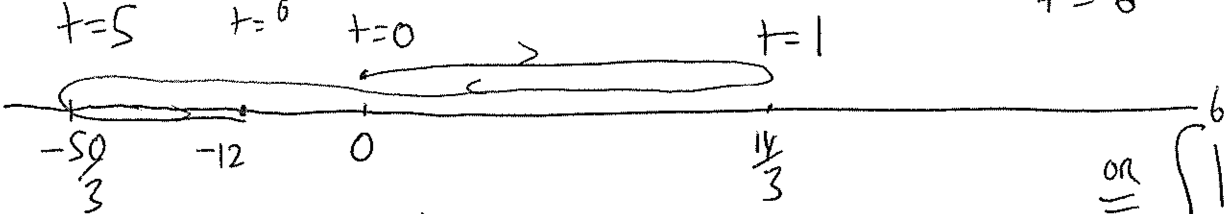
✓ obtains expression for velocity

✓ states both times

c) The distance travelled when the acceleration is $12ms^{-2}$. (2 marks)

$12 = 4t - 12$

$t = 6$



✓ diagram (labelled)

✓ distance

distance $= \frac{14}{3} + \frac{14}{3} + \frac{50}{3} + \left(\frac{50}{3} - 12\right)$

$= \frac{92}{3}$ $(30.\hat{7})$

✓ $\int |v| \, dt$

✓ distance

OR $\int_0^6 |2t^2 - 12t + 10| \, dt$

$= \frac{92}{3}$ (30.7)

Question 6 **(8 marks)**

The volume, V in cubic metres and radius R metres, of a spherical balloon are changing with time, t seconds. $V = \frac{4\pi R^3}{3}$. The radius of the balloon at any time is given by $R = 2t(t+3)^3$.

Determine the following:

a) The value of $\frac{dR}{dt}$ when $t = 1$. (3 marks)

$$\frac{dR}{dt} = 2t3(t+3)^2 + 2(t+3)^3$$
$$= 6(4)^2 + 2(4)^3$$
$$= 224$$

✓ uses product rule
✓ determines exp for $\frac{dR}{dt}$
✓ obtains rate at $t=1$

b) The value of $\frac{dV}{dt}$ when $t = 1$. (3 marks)

$$R = 2(4)^3$$
$$= 128$$

$$\frac{dV}{dt} = \frac{dV}{dR}\frac{dR}{dt}$$
$$= 4\pi R^2 (224)$$
$$= 4\pi (128)^2 (224)$$
$$= 46118781 \cdot 22$$

✓ uses chain rule
✓ determines R at $t=1$
✓ obtains $\frac{dV}{dt}$ at $t=1$

Consider the volume of the balloon at $t = 1$.

c) Use the incremental formula to estimate the change in volume 0.1 seconds later (i.e $t = 1.1$) (2 marks)

$$\Delta V \approx \frac{dV}{dt}\Delta t$$
$$= 46118781 \cdot 22 (0 \cdot 1)$$
$$= 4611878 \cdot 122$$
$$(4611878 \pm 0 \cdot 2)$$

✓ uses incremental formula
✓ obtains approx change in volume.

Question 7 **(6 marks)**

A share portfolio, initially worth \$26000, has a value of f dollars after t months, and begins with a negative rate of growth. The rate of growth remains negative until after 20 months $(t = 20)$ when the value of the portfolio is momentarily stationary and then continues with negative growth for the life of the investment. The value of the portfolio, $f(t)$ after t months can be modelled by the following model, $f(t) = -2t^3 + bt^2 + ct + d$, $0 \le t \le 37$ months where $b, c \,\&\, d$ are constants.

Determine the values of the constants.

$f(0) = 26000$

$d = 26000$

$f(t) = -2t^3 + bt^2 + ct + d$

$f'(t) = -6t^2 + 2bt + c$

$f''(t) = -12t + 2b$

$0 = f'(20) = f''(20)$ Inflection pt (horiz)

$0 = -12(20) + 2b$

$b = 120$

$0 = -6(20)^2 + 240(20) + c$

$c = -2400$

✓ determines d

✓ identifies horiz inflection at $t = 20$

✓ determines exp for $f'(t)$

✓ determines exp for $f''(t)$

✓ solves for b

✓ solves for c.