Christ Church
Grammar School

Mathematics Department
Year 11 Mathematics Methods

Semester Two Examination, 2018

Question/Answer booklet

**MATHEMATICS
METHODS
UNITS 1 AND 2**
Section One:
Calculator-free

SOLUTIONS

Student Name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	14	14	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet, preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free 35% (52 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (7 marks)

Solve each equation below for x.

(a) $\dfrac{3x}{x-5} = \dfrac{2}{3}$. (2 marks)

Solution
$9x = 2x - 10$
$7x = -10$
$x = -\dfrac{10}{7}$
Specific behaviours
✓ cross multiplies
✓ correct solution

(b) $(x+3)(x-3) = 8x$. (3 marks)

Solution
$x^2 - 9 = 8x$
$x^2 - 8x - 9 = 0$
$(x+1)(x-9) = 0$
$x = -1, \qquad x = 9$
Specific behaviours
✓ expands and equates to zero
✓ factorises
✓ correct solutions

(c) $\sqrt{2}\sin x + 1 = 0$, $0° \le x \le 360°$. (2 marks)

Solution
$\sin x = -\dfrac{1}{\sqrt{2}}$
$x = 225°, \qquad x = 315°$
Specific behaviours
✓ one correct solution
✓ both correct solutions

See next page

Question 2   **(3 marks)**

Expand $(2x^2 - \dfrac{1}{x})^4$, giving your answer in simplified form.

Solution
$(1)(2x^2)^4 + (4)(2x^2)^3\left(\dfrac{-1}{x}\right) + (6)(2x^2)^2\left(\dfrac{-1}{x}\right)^2 + (4)(2x^2)\left(\dfrac{-1}{x}\right)^3 + (1)\left(\dfrac{-1}{x}\right)^4$
$= 16x^8 - 32x^5 + 24x^2 - \dfrac{8}{x} + \dfrac{1}{x^4}$

Specific behaviours
✓ uses correct row of Pascal's Triangle
✓ uses correct powers of x in each term of initial expansion
✓ gives correct final simplified expansion

Additional working space

Question number: _____

Question 3 (6 marks)

(a) Evaluate $\frac{m^{0.5}}{n^2}$ when $m = 4 \times 10^6$ and $n = 5 \times 10^2$, writing your answer without the use of scientific notation. (3 marks)

Solution
$\dfrac{m^{0.5}}{n^2} = \dfrac{\sqrt{4} \times 10^3}{25 \times 10^4}$
$= \dfrac{2}{25} \times \dfrac{1}{10}$
$= \dfrac{1}{125}$

Specific behaviours
✓ simplifies $m^{0.5}$
✓ simplifies n^2
✓ correct value

(b) Determine the value of x when $4^x = 32\sqrt{2}$. (3 marks)

Solution
$2^{2x} = 2^5 \times 2^{\frac{1}{2}}$
$= 2^{\frac{11}{2}}$
$2x = \dfrac{11}{2} \Rightarrow x = \dfrac{11}{4}$

Specific behaviours
✓ LHS as power of 2
✓ RHS as power of 2
✓ equates indices and solves

Additional working space

Question number: _____

See next page

Question 4   (7 marks)

(a)

    (i)   Calculate $\dfrac{d}{dx}(3x^4 - 2x + 12)$.   (1 mark)

Solution
$12x^3 - 2$
Specific behaviours
✓ correct derivative

    (ii)   Simplify $\displaystyle\lim_{h \to 0} \dfrac{(x+h)^3 - x^3}{h}$.   (1 mark)

Solution
$3x^2$
Specific behaviours
✓ correct derivative

(b)   Determine the equation of the tangent to the curve $y = x^3 + 2x + 5$ when $x = -1$.

                                                              (3 marks)

Solution
$\dfrac{dy}{dx} = 3x^2 + 2$
$x = -1, \quad y = -1 - 2 + 5 = 2, \quad \dfrac{dy}{dx} = 3 + 2 = 5$
$y - 2 = 5(x + 1) \Rightarrow y = 5x + 7$
Specific behaviours
✓ correct derivative
✓ calculates y-coordinate and gradient
✓ correct equation of tangent, in any form

(c)   Determine $f(x)$ given $f'(x) = 8x + 3$ and $f(-2) = 5$.   (2 marks)

Solution
$f(x) = 4x^2 + 3x + c$
$4(-2)^2 + 3(-2) + c = 5 \Rightarrow c = -5$
$f(x) = 4x^2 + 3x - 5$
Specific behaviours
✓ correct antiderivative with constant
✓ correct $f(x)$

Question 8 continued

(b)

    (i)   Use the formula $f'(x) = \displaystyle\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to determine $\dfrac{dy}{dx}$ for the curve.

                                                                    (4 marks)

Solution
$f'(x) = \displaystyle\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$
$= \displaystyle\lim_{h \to 0} \left(\dfrac{x+h}{x+h+2} - \dfrac{x}{x+2} \right) \div h$
$= \displaystyle\lim_{h \to 0} \left(\dfrac{(x+h)(x+2) - x(x+h+2)}{(x+h+2)(x+2)} \right) \div h$
$= \displaystyle\lim_{h \to 0} \dfrac{2}{(x+h+2)(x+2)}$
$= \dfrac{2}{(x+2)^2} = \dfrac{dy}{dx}$
Specific behaviours
✓ substitutes function into quotient
✓ combines into one fraction
✓ simplifies fraction
✓ evaluates limit

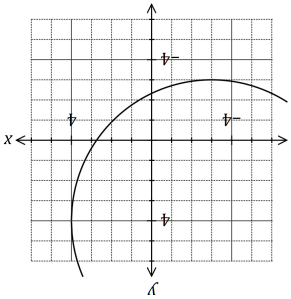   (ii)   Calculate the gradient of the curve at **P**.   (1 mark)

Solution
$f'(3) = \dfrac{2}{(3+2)^2} = \dfrac{2}{25}$
Specific behaviours
✓ correct value

Question 5 (7 marks)

(a) Part of the circle $x^2 + y^2 = ax + by + c$ is shown below. Determine the values of the constants a, b and c. (3 marks)

Solution

$(x+3)^2 + (y-4)^2 = 7^2$

$x^2 + y^2 = -6x + 8y + 24$

$a = -6$, $b = 8$, $c = 24$

Specific behaviours
- ✓ circle in factored form
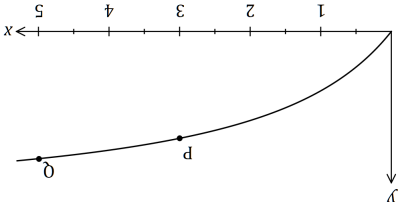- ✓ expands into required form
- ✓ correct values of a, b and c

(b) Solve the following quadratic equation by **completing the square**. Give your answer in exact form. (4 marks)

$$2x^2 + 6x - 16 = 0.$$

Solution

$x^2 + 3x - 8 = 0$

$x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 8 = 0$

$\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 8 = 0$

$\left(x + \frac{3}{2}\right)^2 - \frac{41}{4} = 0$

$x = -\frac{3}{2} \pm \sqrt{\frac{41}{4}}$ or $x = -\frac{3}{2} \pm \frac{\sqrt{41}}{2}$ (either answer is acceptable)

Specific behaviours
- ✓ Divides both sides of equation by 2 to make a monic equation
- ✓ correct value added to and subtracted from LHS
- ✓ written as difference of perfect square and fraction
- ✓ final answer (both solutions)

Question 8 (8 marks)

Let $f(x) = \dfrac{x}{x+2}$. The graph of $y = f(x)$ is shown below.

(a) Points P and Q lie on the curve with x-coordinates 3 and 5 respectively.

(i) Determine $f(3)$ and $f(5)$. (1 mark)

Solution

$f(3) = \frac{3}{5}$, $f(5) = \frac{5}{7}$

Specific behaviours
- ✓ both values correct

(ii) Determine the gradient of the straight line through P and Q. (2 marks)

Solution

$m = \left(\frac{5}{7} - \frac{3}{5}\right) \div 2 = \left(\frac{25 - 21}{35}\right) \div 2 = \frac{2}{35}$

Specific behaviours
- ✓ substitutes correctly into gradient formula
- ✓ correct value

Question 6   (6 marks)

The derivative of a cubic polynomial is given by $\dfrac{dy}{dx} = 3x^2 - 2x - 24$.

The cubic passes through the point $(-1, -14)$.

(a)   Determine the equation of the cubic.   (2 marks)

Solution
$y = x^3 - x^2 - 24x + c$
$-14 = -1 - 1 + 24 + c \Rightarrow c = -36$
$y = x^3 - x^2 - 24x - 36$
Specific behaviours
✓ antidifferentiates correctly
✓ determines constant

(b)   Show that the cubic has a root when $x = -2$.   (1 mark)

Solution
$x = -2, \quad y = -8 - 4 + 48 - 36 = 48 - 48 = 0$
Specific behaviours
✓ substitutes and obtains zero

(c)   Determine the coordinates of the other two roots of the cubic.   (3 marks)

Solution
$(x^3 - x^2 - 24x - 36) \div (x + 2) = x^2 - 3x - 18$
$y = (x + 2)(x^2 - 3x - 18)$
$y = (x + 2)(x + 3)(x - 6)$
Other roots at $(-3, 0)$ and $(6, 0)$.
Specific behaviours
✓ obtains quadratic factor by algebraic long division or inspection
✓ factorises quadratic
✓ states both roots as coordinates

Question 7   (8 marks)

The first three terms, in order, of a sequence are $4x - 1, 2x - 5$ and $x - 4$.

Determine the fourth term of the sequence if

(a)   the sequence is arithmetic.   (4 marks)

Solution
$d = (2x - 5) - (4x - 1) = (x - 4) - (2x - 5)$
$-2x - 4 = -x + 1 \Rightarrow x = -5$
$d = (-5 - 4) - (-10 - 5) = 6$
$T_4 = (-5 - 4) + 6 = -3$
Specific behaviours
✓ equates differences
✓ solves for x
✓ states d
✓ correct T_4

(b)   the sequence is geometric.   (4 marks)

Solution
$r = \dfrac{2x - 5}{4x - 1} = \dfrac{x - 4}{2x - 5}$
$(2x - 5)(2x - 5) = (4x - 1)(x - 4)$
$4x^2 - 20x + 25 = 4x^2 - 17x + 4$
$3x = 21$
$x = 7$
$r = \dfrac{7 - 4}{14 - 5} = \dfrac{3}{9} = \dfrac{1}{3}$
$T_4 = (7 - 4) \times \dfrac{1}{3} = 1$
Specific behaviours
✓ equates ratios
✓ solves for x
✓ states r
✓ correct T_4