PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Specialist TEST 1 Friday 8 February 2019 TIME: 45 minutes working **No Classpads nor calculators allowed!** 37 marks 8 Questions

Name:________________________________

Teacher: _____________________________

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (1 & 2 = 3 marks)

Express each of the following in the form $a+bi$ where $a \& b$ are real numbers.

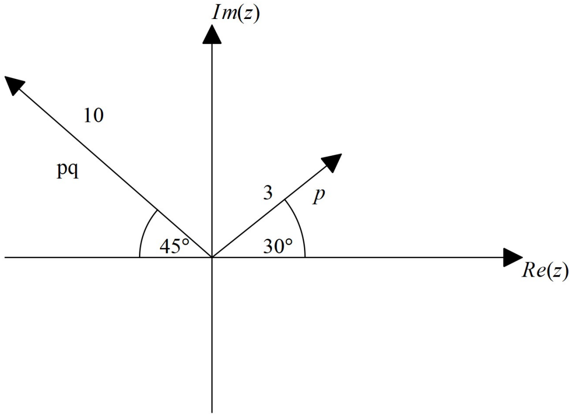
a) $(3-4i)(5i)$

b) $\dfrac{2-3i}{5+i}$

Q2 (3 marks)

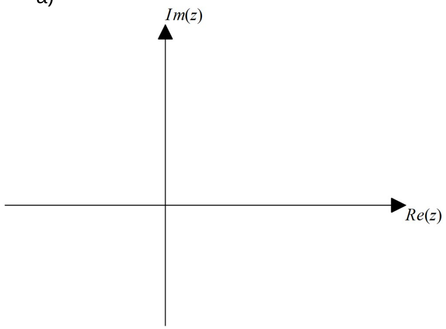
Determine the remainder when $3x^2-5x+7$ is divided by $(x+3-2i)$

Q3 (3 marks)
Determine the complex number q in polar form.

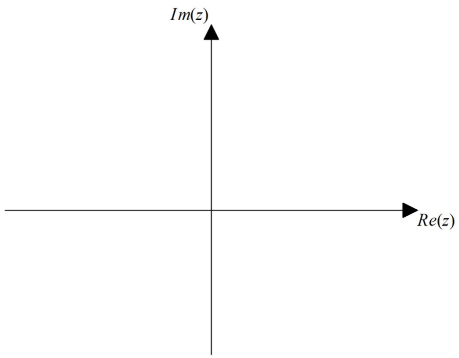


Q4 (2 & 3 = 5 marks)
Sketch the following in the complex plane showing all major features.

a) $\arg(z) = \dfrac{2\pi}{3}$



b) $|z + 3 - 4i| = 6$



Q5 (2, 3 & 3 = 8 marks)

If $z = a + ib$ and $w = p + iq$ where $a, b, p \,\&\, q$ are real numbers, show the following:

a) $\overline{z + w} = \bar{z} + \bar{w}$

b) $\overline{zw} = \bar{z}\,\bar{w}$

c) Hence or otherwise show that if there is a complex root to the quadratic equation $ax^2 + bx + c = 0$ with real coefficients, then the conjugate is also a root.
(Hint: Take the conjugate of both sides of the quadratic equation)

Q6 (4 marks)

Consider the set of complex numbers $z = x + iy$ that satisfy the following equation:

$|z+1-i| = |z-3-7i|$.

Determine the cartesian equation, in terms of $x \,\&\, y$, of these numbers.

Q7 (2 & 4 = 6 marks)

Consider the function $f(z) = az^3 + bz^2 + cz + d$ where $a, b, c \,\&\, d$ are real constants.

It is known that $(z-1)$ is a factor, and $f(0) = -18$ & $f(3i) = 0$.

a) Determine all three factors of $f(z)$.

b) Determine the values of $a, b, c \,\&\, d$.

Q8 (4 & 1 = 5 marks)

Consider the set of complex numbers, z, that satisfy the following:

$|z - 2\sqrt{2} - 2\sqrt{2}i| \le c$, $c \ge 0$ a real constant, and $0 < Arg(z) < \frac{\pi}{2}$.

Determine:

a) The value of c given that the Maximum value of $Arg(z) = \frac{5\pi}{12}$.

b) Maximum value of $|z|$.