

# Test 4

## (Matrices, Exponentials and Logarithms, Functions)



This assessment contributes 5% towards the final year mark.

Name :

Score :  
(out of 45)

**Part A**      **The use of a CAS calculator is assumed.**  
(12 minutes permitted)

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1. A native reptile of MathMagic Isle called the hypottentot has a generation change every 2 years. The table below shows the survival rates, breeding rates and the initial population profile for 5 of the age groups :

Age (years)	0-2	2-4	4-6	6-8	8-10
Survival Rate	0.6	0.8	0.7	0.4	0
Breeding Rate	0.1	0.9	1.4	0.5	0.4
Initial Population	10	12	15	20	10

- a. State the Leslie matrix  $L$  for this colony of hypottentots.

[2]

- b. Use matrix  $L$  to determine the population profile after 10 years.

[3]

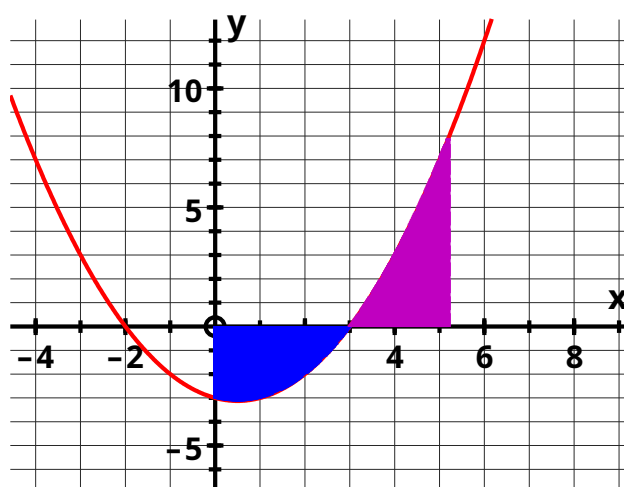
- c. Determine the long term inter-generational growth rate, as a percentage.

[3]

2. The graph of function  $f(x) = 0.5(x + 2)(x - 3)$  is shown below.

Region A is bounded by the curve, the x axis, and the lines  $x = 0$  and  $x = 3$ .

Region B is bounded by the curve, the x axis, and the lines  $x = 3$  and  $x = a$ .



If the areas of regions A and B have the same area, determine the value of the constant  $a$  correct to 0.001.

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It may be used for spare working.

# Test 4

## (Matrices, Exponentials and Logarithms, Functions)



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**Part B**      **No calculator to be used.**  
(33 minutes permitted)

Name :

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3. Consider the following transformation matrices in the co-ordinate plane :

- R rotates  $180^\circ$  about the origin
- D dilates vertically about  $y = 0$  with factor 1.5
- S downward shear parallel to the vertical axis with factor 1

a. Give matrices R, D and S.

[3]

The 3 diagrams below show a parallelogram.

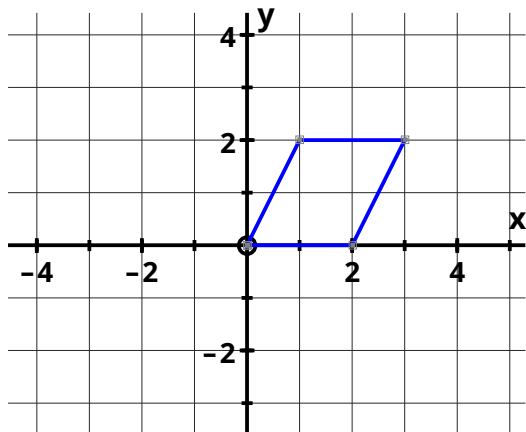


Diagram 1

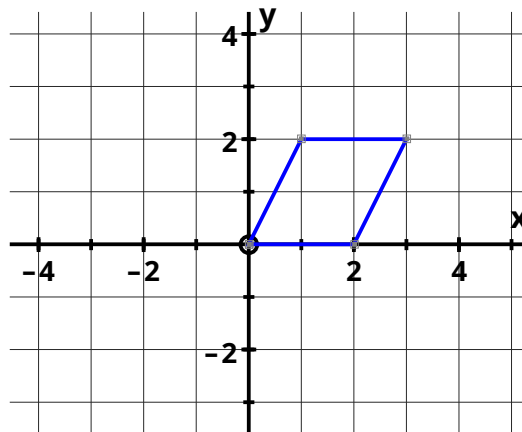


Diagram 2

Draw the image of this parallelogram under the action of transformation :

b. D (on Diagram 1)

[1]

c. S (on Diagram 2)

[2]

d. R then S (on Diagram 3)

[2]

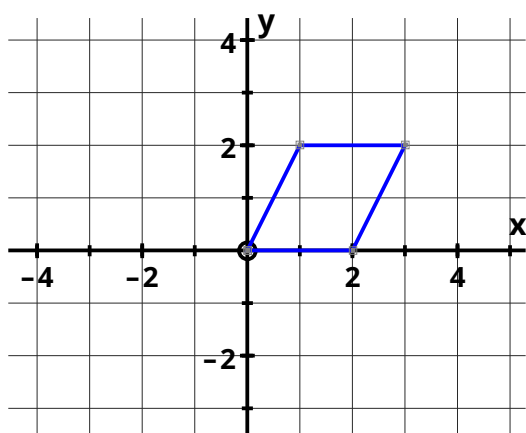


Diagram 3

3. e. Does transformation matrix  $S$  change the area of any object it transforms ?  
Explain.

[2]

- f. If the parallelogram is transformed by matrix D then S, what matrix will return the resultant image back to the original parallelogram ?

[3]

See next page

4. Find the following indefinite integrals, using an appropriate Calculus technique :  
[ONE mark will be given for a correct answer only]

a.  $\int \frac{4x + 8}{(x^2 + 4x)^5} dx$



[3]

b.  $\int \frac{e^{2x}}{e^{2x} + 1} dx$

[3]

c.  $\int 30x\sqrt{x+2} dx$  Put  $u = x+2$

[4]

5. Evaluate the following definite integrals, using the given substitution and an anti-derivative technique : [ONE mark will be given for a correct answer only]

a.  $\int_{\frac{1}{6}}^{\frac{1}{3}} \frac{dx}{\sqrt{1-9x^2}}$  Put  $3x = \cos \theta$

5.      b.      Given that  $\frac{d}{du} [\tan^{-1}(u)] = \frac{1}{1+u^2}$  evaluate  $\int_0^{\frac{\pi}{4}} \tan^2 \theta \, d\theta$   
by using  $u = \tan \theta$ .

End of Assessment Task

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