PENRHOS COLLEGE

STRIVE FOR THE HIGHEST

Semester One Examination 2011
Question/answer booklet

YEAR 12 MATHEMATICS 3C/DMAT

Section One (Calculator-Free)

Student Name: **SOLUTIONS**

Circle your teacher's name

S. ROWDEN N. EDMUNDS

Time allowed for this section

Reading time before commencing work: 5 minutes
Working time for section: 50 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section One
Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One: Calculator-free	**10**	**10**	**50 minutes**	**40**
Section Two: Calculator-assumed	15	15	100 minutes	80
			Total marks	120

Instructions to candidates

1. Answer the questions in the spaces provided.

2. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

3. **Show all your working clearly.**
   Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.
   For any question or part question worth more than two marks, valid working or justification is required to receive full marks.
   If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

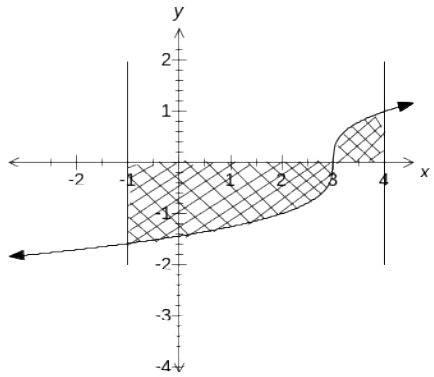
SEE NEXT PAGE

Section One: Calculator-Free 40 Marks

This section has **Ten (10)** questions. Attempt **all** questions.

Question 1 (3 marks)

Explain how you would find the area of the region bounded by the x-axis and the equations $y = (x - 2)^{\frac{1}{3}}$, $x = -1$ and $x = 4$. You are not required to find the area.



Solution
Find the definite integral over the boundary -1 to 4 of the absolute value of the function Or Find the opposite of the definite integral for the boundary -1 to 3 for function plus the definite integral for the boundary 3 to 4 for the function
Specific behaviours
✓definite integral ✓correct boundaries ✓absolute value of function Or ✓ opposite of definite integral of the function between -1 to 3 ✓ definite integral of the function between 3 to 4 ✓ add two areas

SEE NEXT PAGE

Question 2 (5 marks)

Differentiate the following: (do not simplify your answers)

(a) $y = 8x^4 + \frac{2}{x} - \frac{3}{7}$

[1]

Solution
$\frac{dy}{dx} = 32x^3 - 2x^{-2}$
Specific behaviours
✓correct derivative

(b) $y = \sqrt{x^2 - \frac{1}{x^2}}$

[2]

Solution
$y = \left(x^2 - x^{-2}\right)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}\left(x^2 - x^{-2}\right)^{-\frac{1}{2}}\left(2x + 2x^{-3}\right)$
Specific behaviours
✓$nf(x)^{n-1}$ ✓$f'(x)$

(c) $y = \frac{2x+1}{(3x+2)^2}$

[2]

Solution
$\frac{dy}{dx} = \frac{2(3x+2)^2 - (2x+1)2(3x+2)3}{(3x+2)^4}$ $= \frac{2(3x+2)^2 - 6(2x+1)(3x+2)}{(3x+2)^4}$
Specific behaviours
✓applies quotient rule correctly ✓applies chain rule correctly

SEE NEXT PAGE

Additional working space

Question number: _____

Additional working space

Question number: _____

Question 3 (4 marks)

In a probability experiment, events M and N are such that

$P(M)=\frac{1}{4}$, $P(N|M)=\frac{1}{3}$ and $P(\overline{M}\cap\overline{N})=\frac{1}{4}$

Find

(a) $P(M\cup N)$

[2]

Solution
$P(M\cup N)=1-P(\overline{M}\cap\overline{N})$ $=1-\frac{1}{4}$ $=\frac{3}{4}$
Specific behaviours
✓ identifies $P(\overline{M}\cap\overline{N})$ as the complement of $P(M\cup N)$ ✓ correct $P(M\cup N)$

(b) $P(N)$

[2]

Solution
$P(M\cup N)=P(M)+P(N)-P(M\cap N)$ $=P(M)+P(N)-P(N\|M)\times P(M)$ $\frac{3}{4}=\frac{1}{4}+P(N)-\frac{1}{3}\times\frac{1}{4}$ $P(N)=\frac{7}{12}$
Specific behaviours
✓ determines $P(M\cap N)$ uses addition rule to correctly determine $P(N)$ ✓

SEE NEXT PAGE

Question 4 (7 marks)

Determine the following integrals:

(a) $\int(3x + x^{\frac{-2}{3}})\,dx$

[1]

Solution	
$\frac{3x^2}{2} + 3x^{\frac{1}{3}} + c$	If c is omitted from this question 0
Specific behaviours	
✓correct answer	

(b) $\int\frac{x - 2x^3}{3x^5}\,dx$

[2]

Solution
$\int\frac{x}{3x^5} - \frac{2x^3}{3x^5}\,dx$ $= \int 3x^{-4} - \frac{2x^{-2}}{3}\,dx$ $= -\frac{1}{9x^3} + \frac{2}{3x} + c$
Specific behaviours
✓separates into expression with 2 terms ✓ correct answer

SEE NEXT PAGE

Additional working space

Question number: _____

Question 10 (5 marks)

Simplify the following:

$$\frac{x^3 - 5x^2 - 14x}{x^2 - 2x - 3} \div \frac{x^2 - 4}{2x - 6}$$

Solution	
$\frac{x(x-7)(x+2)}{(x-3)(x+1)} \times \frac{2(x-3)}{(x-2)(x+2)}$	
$= \frac{x(x-7)\cancel{(x+2)}}{\cancel{(x-3)}(x+1)} \times \frac{2\cancel{(x-3)}}{(x-2)\cancel{(x+2)}}$	$x - 3 \neq 0$ $x + 2 \neq 0$
$= \frac{2x(x-7)}{(x+1)(x-2)}$	

Specific behaviours
✓ correct factorisation for first fraction
✓ correct factorisation for second fraction
✓ invert second fraction and multiply
✓ simplifies division correctly
✓ states restrictions on division

END OF QUESTIONS

SEE NEXT PAGE

Question 4 Continued

(c) $\int \frac{1}{(3x-2)^4}\,dx$

[2]

Solution
$\int (3x-2)^{-4}\,dx$ $=\frac{(3x-2)^{-3}}{-3\times 3}+c$ $=-\frac{1}{9(3x-2)^3}+c$
Specific behaviours
✓raise the power of the function by one ✓divides by both raised power and derivative of $3x-2$

(d) $\int \frac{2}{\sqrt{x}}(3-\sqrt{x})^2\,dx$

[2]

Solution
$\int 2x^{-\frac{1}{2}}\left(3-x^{\frac{1}{2}}\right)^2 dx$ $=\frac{-4\left(3-\sqrt{x}\right)^3}{3}+c$
Specific behaviours
✓✓applies chain rule

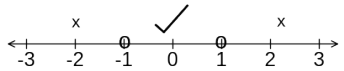
Question 9 (2 marks)

Find $\frac{d}{dx}\left(\int_3^{2x^3} t(6-t)\,dt\right)$

Solution
$\frac{d}{dx}\left(\int_3^{2x^3} t(6-t)dt\right)$ $=2x^3(6-2x^3)6x^2$ $=24x^5(3-x^3)$
Specific behaviours
✓correctly uses the Fundamental theorem of Calculus ✓uses the chain rule

Question 8 (3 marks)

Solve for y the inequality $\frac{1}{y-1} < \frac{1}{y+1}$

Solution
$\frac{1}{y-1} - \frac{1}{y+1} < 0$ $\frac{2}{(y-1)(y+1)} < 0$ $y \neq 1$ *or* $y \neq -1$  $-1 < y < 1$
Specific behaviours
✓simplifies inequality ✓identifies critical values ✓tests critical regions and states correct answer

Question 5 (3 marks)

Find the equation of the tangent to the curve $y = \frac{1}{3x+2}$ at (-1, -1).

Solution
$\frac{dy}{dx} = -3(3x+2)^{-2}$ $x = -1 \quad \frac{dy}{dx} = -3$ $-1 = -3(-1) + c$ $-4 = c$ $y = -3x + 4$
Specific behaviours
✓correct derivative ✓correct gradient at given point ✓determines correctly the y intercept and hence states equation

Question 6 (5 marks)

Solve the system of equations

$$\begin{aligned} x + y + z &= 9 \\ 3x + y - 2z &= 1 \\ x + z &= 2y \end{aligned}$$

Solution
$x + y + z = 9 \quad (1)$ $3x + y - 2z = 1 \quad (2)$ $x - 2y + z = 0 \quad (3)$ $3(1) - (2) \quad 2y + 5z = 26 \quad (4)$ $2 - 3(3) \quad 7y - 5z = 1 \quad (5)$ $(4) + (5) \quad 9y = 27$ $y = 3$ $2 \times 3 + 5z = 26$ $z = 4$ $x + 3 + 4 = 9$ $x = 2$ $x = 2, y = 3, z = 4$
Specific behaviours
✓✓reduces the system of equations to 2 equations with 2 variables ✓reduces to 1 equation with 1 variable and solves equation ✓✓back substitutes to find other 2 variables

Question 7 (3 marks)

Determine the value of b for the following, given b is a positive integer.

$$10 = \int_4^b \frac{1}{\sqrt{x}}\, dx$$

Solution
$10 = \int_4^b x^{-\frac{1}{2}}\, dx$ $10 = \left[2\sqrt{x} \right]_4^b$ $10 = 2\sqrt{b} - 2\sqrt{4}$ $10 = 2\sqrt{b} - 4$ $14 = 2\sqrt{b}$ $7 = \sqrt{b}$ $49 = b$
Specific behaviours
✓integrates function ✓substitutes upper and lower boundaries ✓solves correctly for b