

**NAME:** \_\_\_\_\_

**MATHEMATICS METHODS Unit 1**

**2022 Investigation 1**

**Take Home Section**

**Kennedy**  Baptist College

**Validation Test: Monday 9<sup>th</sup> May 2022**

**Weighting: 8%**

**INSTRUCTIONS:**

- No calculators will be allowed in the validation test.
- You are encouraged to use your ClassPad to help you to complete the Take Home part.
- The Take Home part may be used in the validation test.
- The Take Home part is not worth any marks.
- It is recommended you work in small groups after school hours and come to tutoring.

**Introduction:**

A function is an equation for which any  $x$  that can be plugged into the equation will yield exactly one  $y$  out of the equation. We can draw a graph of a function on the Cartesian plane.

**TRANSFORMATIONS** include:

There are certain actions we can perform on a function that change the graph in specific ways. These are

1. Transformations: (horizontal or vertical)
2. Dilations: (horizontal or vertical)
3. Reflections: (horizontal or vertical)

called **TRANSFORMATIONS**.

Function notation is the way a function is written. It is meant to be a precise way of giving information about the function without a rather lengthy written explanation.

The most popular function notation is  $f(x)$

$f(x) = 3x + 1$

This is NOT the multiplication of  $f$  times  $x$ .  
which is read " $f$  of  $x$ ".

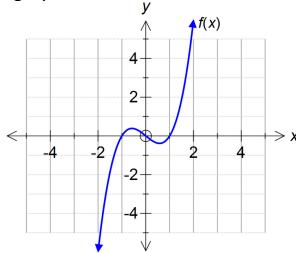
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Traditionally, functions are referred to by single letter names, such as  $f$ ,  $g$ ,  $h$  and so on.

Any letter(s), however, may be used to name a function. Examples:

$f(x) = x^2 - 7$        $g(x) = 3x^2 - 2x + 1$        $S(t) = \frac{1}{t^2} - 3t + 1$

1. The function  $f(x) = x^3 - x$  is graphed below.

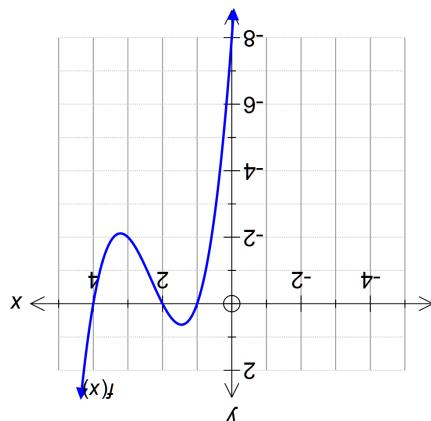


Use your Classpad to investigate how each of the following changes to  $f(x) = x^3 - x$  affects the graph. Make sure you consider both **positive** and **negative** numbers. It is recommended that you draw your results on graph paper.

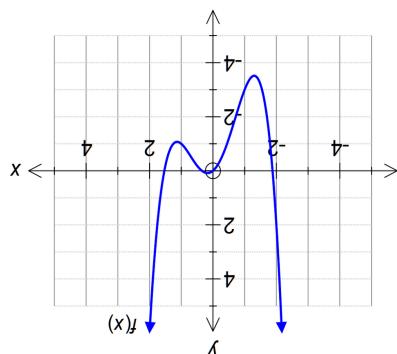
- a) Adding a number (constant) to the function. Eg. Adding 2 to the function  $f(x) = x^3 - x$   
results in  $f(x) + 2 = x^3 - x + 2$ .
- b) Adding a number to the variable. Eg. Adding the number 2 to the  $x$  value in the function  
 $f(x) = x^3 - x$   
so that every  $x$  in the function has been replaced by  $(x+2)$ . This results in the transformed function:  
 $f(x+2) = (x+2)^3 - (x+2)$ .
- c) Multiplying the function by a constant Eg. If the function is multiplied by 2,  $f(x)$  transforms to  $2f(x)$ . Thus using  $f(x) = x^3 - x$  our transformed function becomes  $2f(x) = 2x^3 - 2x$ .
- d) Multiplying the variable by a number. Eg. Multiplying the variable by 2 using  $f(x) = x^3 - x$   
our transformed function becomes  $f(2x) = \textcolor{red}{\cancel{x}}$  which simplifies to  $8x^3 - 2x$ . Consider also multiplying the variable by a fraction  $< 1$  Eg. Multiplying the variable by  $\frac{1}{2}$  using  $f(x) = x^3 - x$   
our transformed function becomes  $f\left(\frac{x}{2}\right) = \textcolor{red}{\cancel{x}}$  which simplifies to  $\frac{x^3}{8} - \frac{x}{2}$ .

2. Investigate the transformation from Q1 above with other graphs such as:

- a)  $f(x) = \sqrt{x}$
- b)  $f(x) = \frac{1}{x}$  or  $f(x) = x^{-1}$



- a)
- Drawn to the right is the graph of  $f(x) = x^3 - 7x^2 + 14x - 8$
  - Write down the new equation for  $f(x+1)$
  - Add the sketch of  $f(x+1)$  to the same set of axes.
  - Fully describe the transformation.



- i.
- Using the terminology for transformations from page 1, describe the transformation on  $f(x)$  if the equation becomes  $f(x) + 3 = x^4 - 3x^2 + x + 3$ .
  - Drawn to the right is the graph of  $f(x) = x^4 - 3x^2 + x$
  - Add the sketch of  $f(x) + 3$  to the same set of axes.

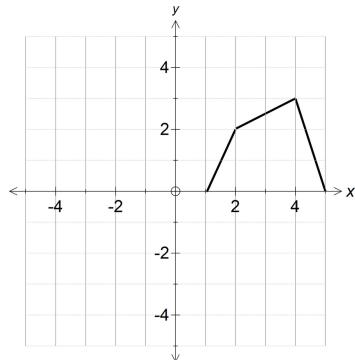
### PRACTICE PROBLEMS 1

c)  $f(x) = x^2$

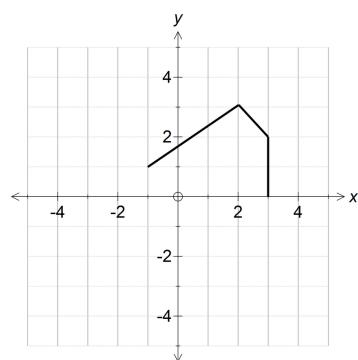
**PRACTICE PROBLEMS 2**

Draw each transformation on the original axes shown.

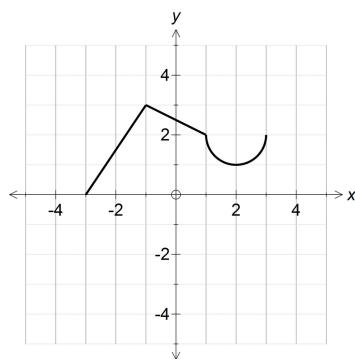
$$f(x) \rightarrow f(x+4)$$



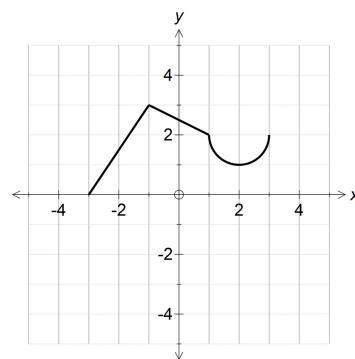
$$f(x) \rightarrow f(x)+1$$



$$f(x) \rightarrow f(x-2)$$

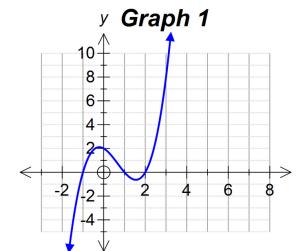
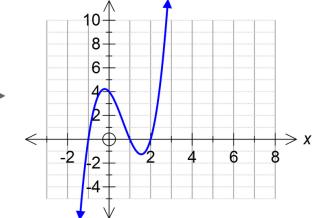


$$f(x) \rightarrow f(x-1)-4$$

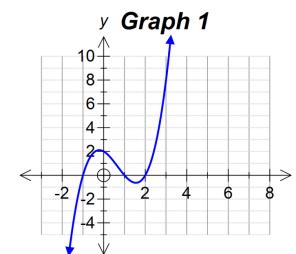
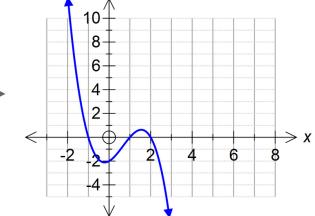
**PRACTICE PROBLEMS 7**

Describe the transformation(s) that take place to create the second graph from the first:

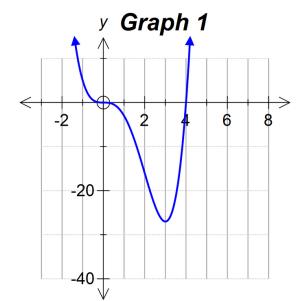
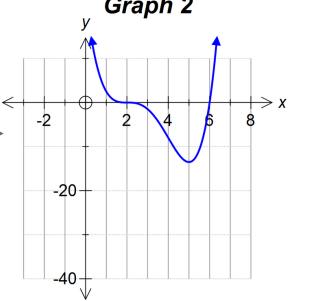
1.

**Graph 2**

2.

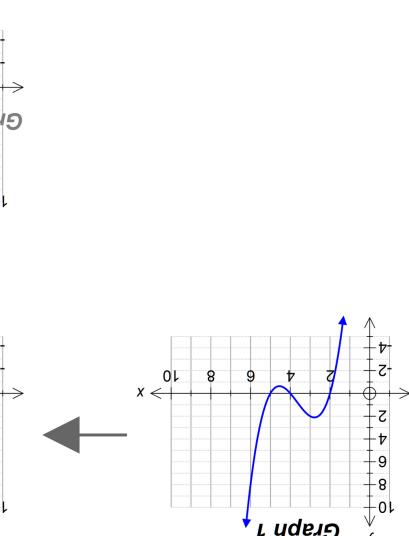
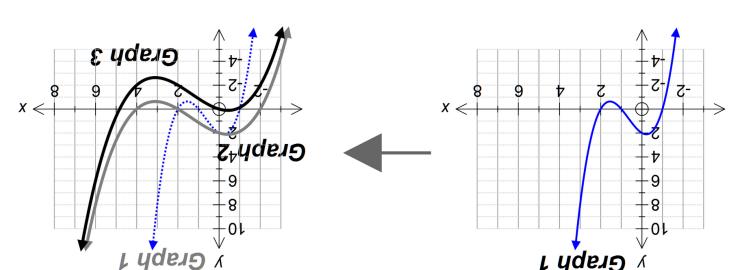
**Graph 2**

3.

**Graph 2**

**END OF INVESTIGATION**

2.



1.

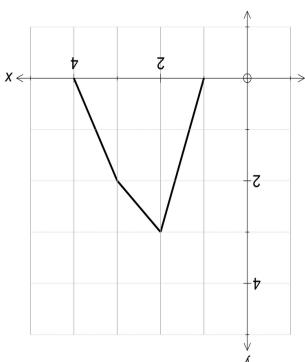
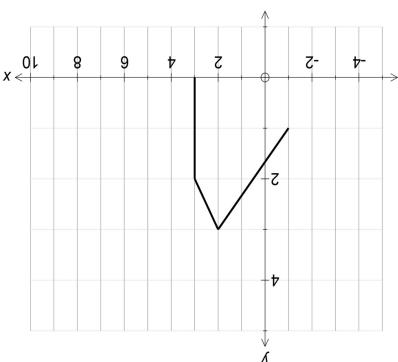
Describe in order the transformations that take place from graph 1 to graph 3 in each case:

### PRACTICE PROBLEMS 3

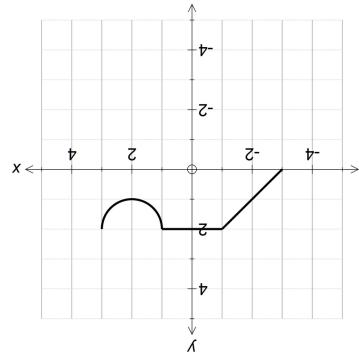
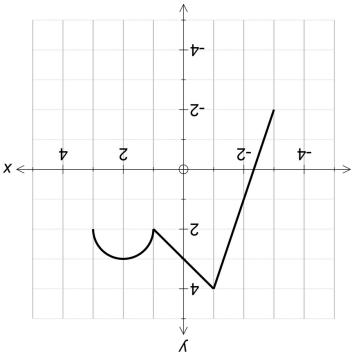
Draw each transformation on the original axes shown.

$$\left( f(x) \rightarrow f\left(\frac{1}{3}x\right) \right)$$

$$f(x) \rightarrow f(2x)$$



$$f(x) \rightarrow 2f(x)$$



$$f(x) \rightarrow \frac{1}{2}f(x)$$

### PRACTICE PROBLEMS 6

**PRACTICE PROBLEMS 4**

For each of the following:

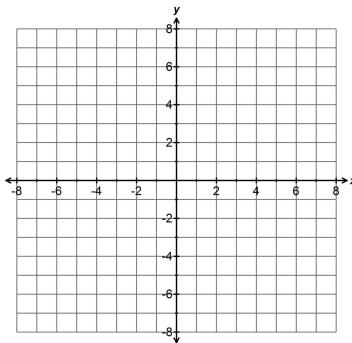
- a Use your classpad or otherwise to draw each of these original functions.  
 b Write down the new equation according to the transformation shown.  
 c Sketch the graph of both the original function and the transformation on the axes provided.

$$f(x) = (x-1)(x-2)(x-4)$$

with transformation

$$f(x) \rightarrow 2f(x).$$

$$2f(x) = \underline{\hspace{2cm}}$$

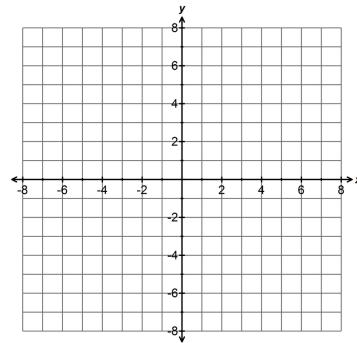


$$f(x) = x^3 - 3x$$

with transformation

$$f(x) \rightarrow -f(x).$$

$$-f(x) = \underline{\hspace{2cm}}$$



$$f(x) = \frac{1}{x-3}$$

with transformation

$$f(x) \rightarrow f(x)+2$$

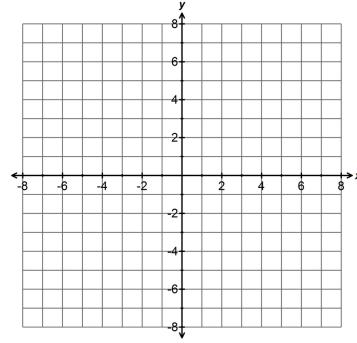
$$f(x)+2 = \underline{\hspace{2cm}}$$

$$f(x) = \sqrt{x-4}$$

with transformation

$$f(x) \rightarrow f(x+2)$$

$$f(x+2) = \underline{\hspace{2cm}}$$

**PRACTICE PROBLEMS 5**Describe *in order* the transformations shown by the following functions:

1.  $f(x) \rightarrow -f(x)+3$

2.  $f(x) \rightarrow 2f(x)-1$

3.  $f(x) \rightarrow f(x-1)+4$

4.  $f(x) \rightarrow 2f(x-3)$

5.  $x^2(x-5) \rightarrow (x-2)^2(x-2-5)-1$

6.  $\frac{1}{x+3} \rightarrow \frac{1}{2x+6}+5$

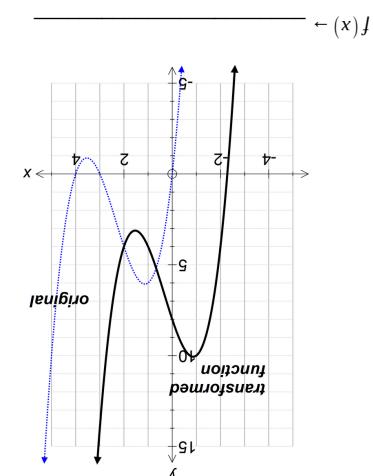
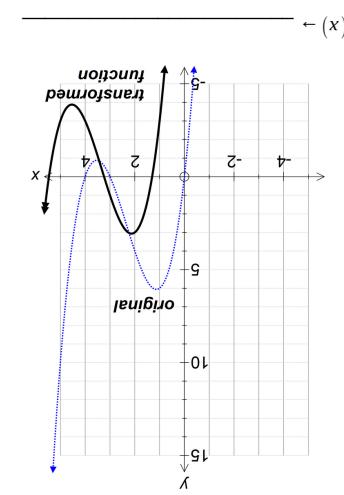
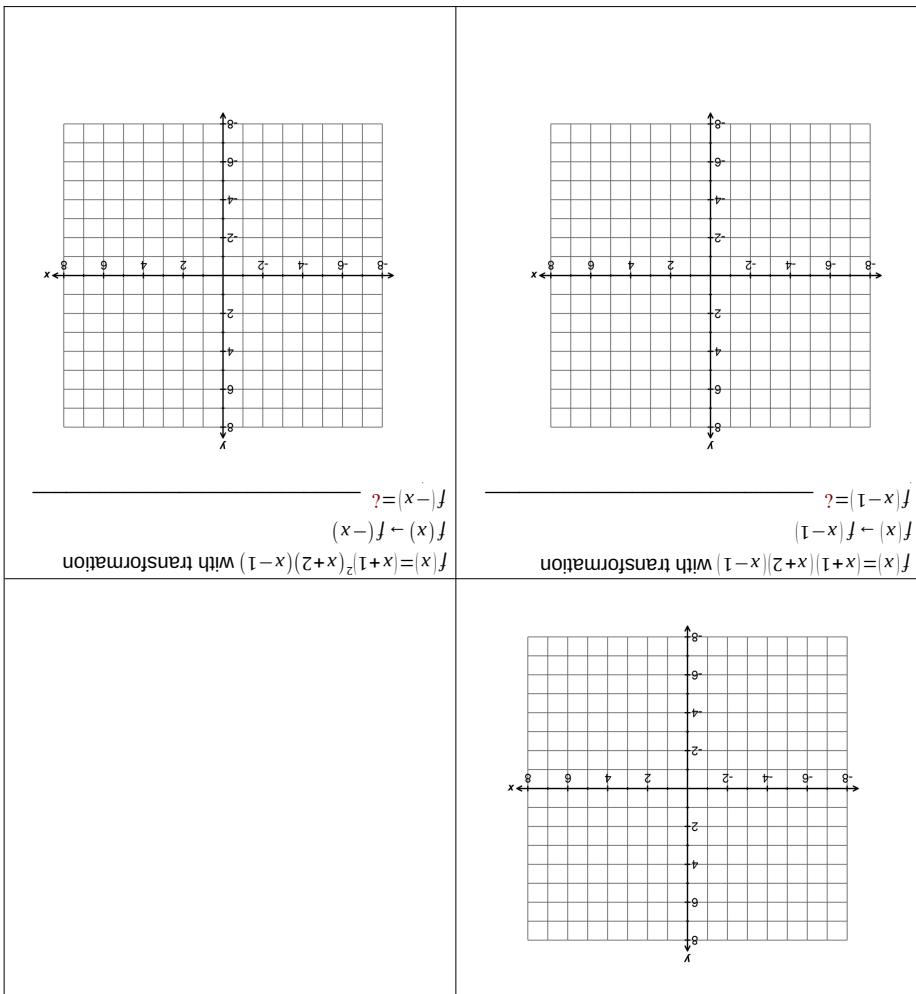
7.  $\sqrt{x} \rightarrow 2\sqrt{x-4}$

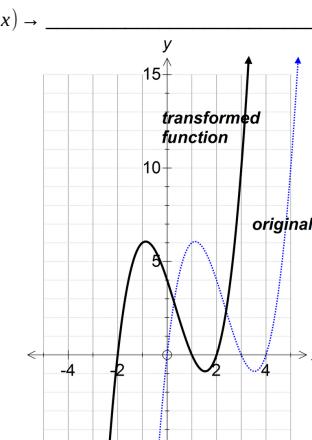
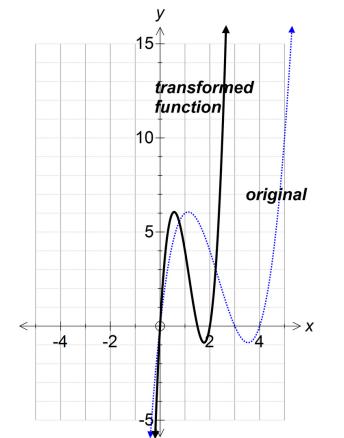
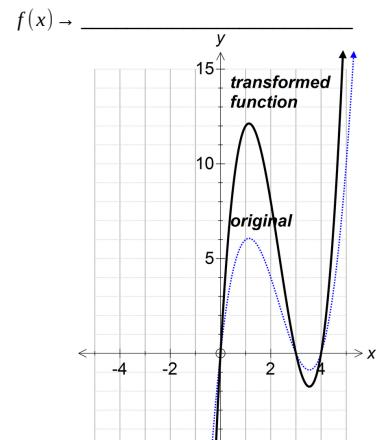
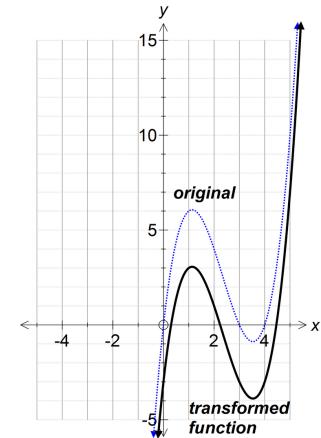
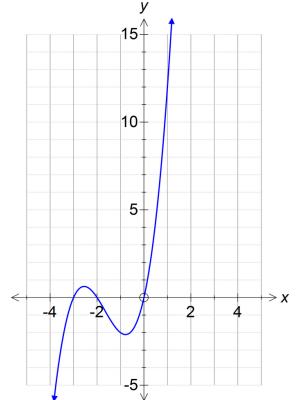
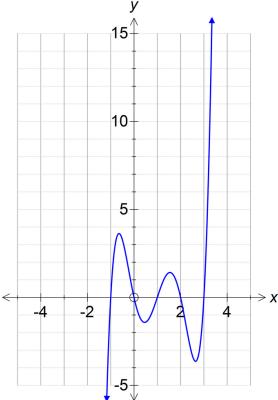
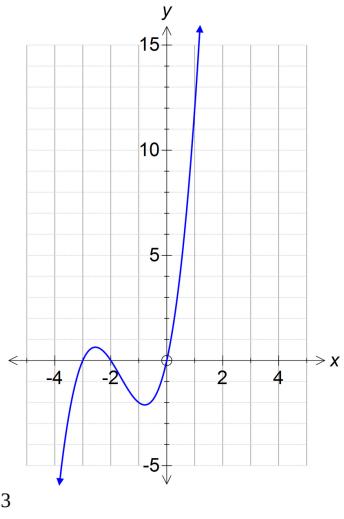
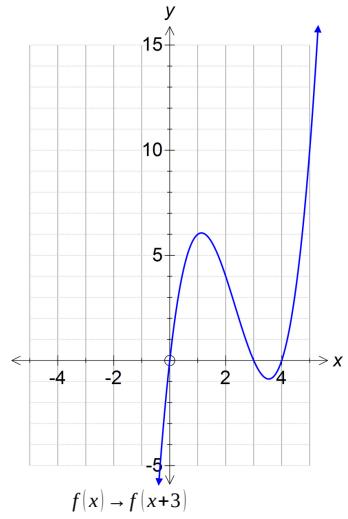
8.  $x^3(x-1) \rightarrow 4(x+1)^3x$

$$(x-f) \leftarrow (x-f)$$

$$f(x) \leftarrow 2f(x)$$

Sketch the graphs of the following transformations:





For each of the following, describe the transformation in words AND using function notation  
(e.g.  $f(x) \rightarrow 2f(x)$  or  $f(x) \rightarrow f(x-3)$ )