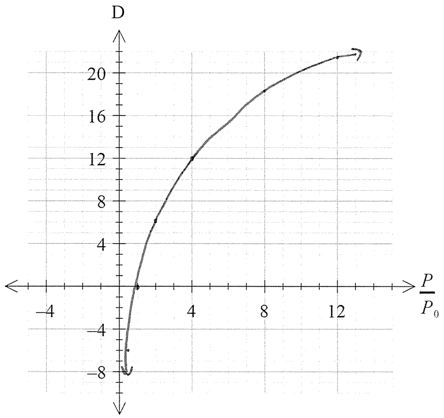
9. (9 marks)

The decibel scale for sound, measured in decibels (dB), is defined as $D = 20\log_{10}\left(\frac{P}{P_0}\right)$, where P is the pressure of the sound being measured and P_0 is a fixed reference pressure.

(a) Complete the table below, giving values rounded to one decimal place. [3]

P	$0.5P_0$	P_0	$2P_0$	$4P_0$	$8P_0$
D	-6.0	0	6.0	12.0	18.1

(b) Sketch the graph of $D = 20\log_{10}\left(\frac{P}{P_0}\right)$ on the axes below. [3]



(c) When measured at similar distances, the sound produced by a dishwashing machine measures 47 dB, while that produced by a mowing machine measures 96 dB. How many times greater is the sound pressure of the mower to that of the dishwasher? [3]

$47 = 20\log_{10}\left(\frac{P}{P_0}\right)$ $\quad$ $96 = 20\log_{10}\left(\frac{P}{P_0}\right)$

$\Rightarrow \frac{P}{P_0} = 223.87$ $\quad$ $\Rightarrow \frac{P}{P_0} = 63095.73$

$\therefore$ factor $= \frac{63095.73}{223.87}$

$= 281.8$

≈ 280 times greater

TRINITY COLLEGE

Mathematics Methods Year 12
Test 3 2016

Section 1 Calculator Free

Area Under Curves, Discrete Random Variables and Logarithms

STUDENT'S NAME Solns

DATE: Friday 20 May **TIME:** 20 minutes **MARKS**: 22

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, Formula page

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (2 marks)

The table below describes the probability distribution for a discrete random variable X .

x	4	5	6	8	9
$P(X=x)$	0.2	a	b	0.5	0.15

Determine the values of a and b if $P(X \le 5) = 0.3$.

$P(X \le 5) = 0.3$

$\Rightarrow 0.2 + a = 0.3$

$\therefore a = 0.1$

$\sum P(X=x) = 1$

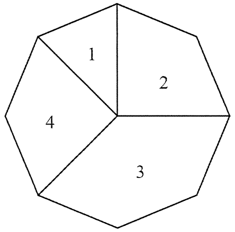
$\Rightarrow b = 0.05$

2. (4 marks)

The following regular octagon is used as a spinner.

(a) Determine, in the form of a table, the probability distribution table [2]

x	1	2	3	4
$P(X=x)$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{2}{8}$



(b) Determine the mean [2]

$$\mu(X) = 1\times\frac{1}{8} + 2\times\frac{2}{8} + 3\times\frac{3}{8} + 4\times\frac{2}{8}$$
$$= \frac{1}{8} + \frac{4}{8} + \frac{9}{8} + \frac{8}{8}$$
$$= \frac{22}{8}$$
$$= \frac{11}{4}$$

3. (3 marks)

Determine p and n for $X \sim B(n,p)$ where $\sigma(X)=\sqrt{8}$ and $\mu(X)=12$

$\sigma(x) = \sqrt{8}$

$\Rightarrow \sqrt{8} = \sqrt{np(1-p)}$

$\Rightarrow 8 = 12(1-p)$

$\Rightarrow \frac{8}{12} = 1-p$

$\Rightarrow p = \frac{4}{12}$

$\therefore p = \frac{1}{3}$

$\mu(x) = 12$

$\Rightarrow np = 12$

$\Rightarrow n = 36$

8. (11 marks)

Rainfall records for Perth indicate that, on average, the probability of rain falling on any one day in June is 0.2. Assuming that the days on which rain falls are randomly distributed, determine:

(a) the probability that rain will fall on the first three days of a given week in June, but not the other four days. [2]

P(rain first 3 and none)

$= (0.2)^3(0.8)^4 = 0.0032$

(b) the probability that rain will fall on exactly three days of a given week in June. [2]

X: # days rain in 7 days

$X \sim B(7, 0.2)$

$P(X=3) = 0.1147$

(c) the probability that rain will fall on at least three days in a given week in June. [2]

$P(X \geq 3) = P(3 \leq X \leq 7)$

$= 0.1480$

(d) the first day in June where the probability of it having rained in this month exceeds 70%. [3]

P(rain after x days) $= 1 - P($no rain in x days$)$

$= 1-(0.8)^x$

$\Rightarrow 1-(0.8)^x > 0.7$

$x > 5.396$

$\therefore$ on the 6^{th} day of June

(e) the probability that from the four weeks in June, at least two of the weeks have rain on exactly three days. [2]

Y: # weeks in June with exactly 3 rain days from 4 wks

$Y \sim B(4, 0.1147)$

$P(Y \geq 2) = P(2 \leq Y \leq 4)$

$= 0.0674$

7. (8 marks)

When a biased six-faced die is rolled, the value, X, on the uppermost face occurs according to the following distribution.

x	1	2	3	4	5	6
$P(X \leq x)$	0.1	0.3	0.5	0.6	0.7	1
$P(X = x)$	0.1	0.2	0.2	0.1	0.1	0.3

(a) Calculate the probability that in two rolls of the die a "3" is followed by a "6". [2]

$P(3 \cap 6) = 0.2 \times 0.3$

$= \underline{\underline{0.06}}$

(c) If the die is rolled fifteen times, what is the probability that

(i) three sixes occur? (remember to define your variable) [2]

X: # of 6's from 15 rolls

$X \sim B(15, 0.3)$

$P(X = 3) = \underline{\underline{0.1700}}$

(ii) at least three sixes occur? [2]

$P(X \geq 3) = P(3 \leq X \leq 15)$

$= \underline{\underline{0.8732}}$

(iii) only a six occurs on each of the first three rolls of these fifteen rolls? [2]

P(sixes in first 3 and none)

$= (0.3)^3 \times (0.7)^{12}$

$= 0.0003737$

$\approx \underline{\underline{0.0004}}$

4. (7 marks)

Let $x = \log_n 3$ and $y = \log_n 4$

(a) Write $2x - y$ as a single logarithmic term. [2]

$\Rightarrow 2\log_n 3 - \log_n 4$

$\Rightarrow \log_n\left(\frac{9}{4}\right)$

(b) Express the following in terms of x and/or y.

(i) $\log_n 0.25 = \log_n 4^{-1}$ [1]

$= -y$

(ii) $\log_n(12n) = \log_n 4 + \log_n 3 + \log_n n$ [2]

$= y + x + 1$

(c) Evaluate exactly n^{2x}. [2]

let $z = n^{2\log_n 3}$

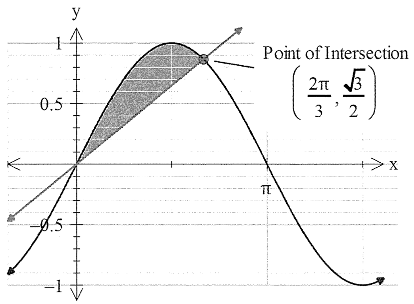
$\Rightarrow \log_n z = 2\log_n 3$

$\Rightarrow z = 9$

$\therefore n^{2x} = 9$

5. (6 marks)

Determine the exact shaded area between the two functions, $f(x) = \sin x$ and $g(x) = \frac{3\sqrt{3}}{4\pi}x$



$$\text{Area} = \int_0^{2\pi/3} \sin x - \frac{3\sqrt{3}}{4\pi}x \; dx \quad ✓$$

$$= \left[-\cos x - \frac{3\sqrt{3}}{\cancel{8}\pi}x^2\right]_0^{2\pi/3} \quad ✓ \quad ✓$$

$$= \left(-\cos\frac{2\pi}{3} - \frac{3\sqrt{3}}{8\pi}\left(\frac{2\pi}{3}\right)^2\right) - \left(-\cos 0 - 0\right)$$

$$= \frac{1}{2} - \frac{12\sqrt{3}\pi^2}{72\pi} + 1 \quad ✓ \quad ✓$$

$$= \frac{3}{2} - \frac{\sqrt{3}\pi}{6} \quad \text{units}^2 \quad ✓$$

TRINITY COLLEGE

Mathematics Methods Year 12
Test 3 2016

Section 2 Calculator Assumed

Area Under Curves, Discrete Random Variables and Logarithms

STUDENT'S NAME ________________________________

DATE: Friday 20 May **TIME:** 30 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, Formula page (retain from Section 1)

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

6. (2 marks)

A Probability Distribution Function, X, for the distribution of marks in the Mathematics Methods Unit 3/4 course has a mean of 62% and a standard deviation of 11%. The marks are scaled to form a new distribution, $Y = 1.1X + 4\%$

Determine

(a) $\mu(Y) = 1.1 \times 62 + 4$ [1]

$= 72.2\ \%$

(b) $\sigma(Y) = 11 \times 1.1$ [1]

$= 12.1\ \%$