

Note: All part questions worth more than 2 marks require working to obtain full marks.

Formula sheet provided: Yes

Task weighting: 10%

Marks available: 41 marks

Examinations

A4 paper, and up to three calculators approved for use in the WACE

Drawing instruments, templates, notes on one unfolded sheet of

Special items:

Correction fluid/tape, eraser, ruler, highlighters

Pens (blue/black preferred), pencils (including coloured), sharpener,

Calculator with CAS capability (to be provided by the student)

Number of questions: 8

Time allowed for this task: 40 mins

Task type: Response

Student name: _____ Teacher name: _____

Course Methods test 2 Year 12

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Q1 (3 & 3 = 6 marks) (3.2.9)

Determine y in terms of x for the following. Show all working.

a) $\frac{dy}{dx} = 15x^2 + 14x$ and $y = 13$ when $x = 1$.

Solution
$\frac{dy}{dx} = 15x^2 + 14x$ $y = 5x^3 + 7x^2 + C$ $13 = 5 + 7 + C$ $C = 1$ $y = 5x^3 + 7x^2 + 1$
Specific behaviours
<ul style="list-style-type: none"> ✓ anti-diffs terms ✓ introduces an unknown constant and subs to solve ✓ states value of constant

b) $\frac{dy}{dx} = 10(2x+1)^4$ and $y = 10$ when $x = -1$.

Solution
$\frac{dy}{dx} = 10(2x+1)^4$ $y = \frac{10(2x+1)^5}{2(5)} + C = (2x+1)^5 + C$ $10 = (-1)^5 + C$ $C = 11$ $y = (2x+1)^5 + 11$
Specific behaviours
<ul style="list-style-type: none"> ✓ anti-diffs terms ✓ introduces an unknown constant and subs to solve ✓ states value of constant

$$\begin{aligned}
 0 &= -128a + 32b - 8c + 48 \\
 \frac{-368}{3} &= 64a - \frac{64}{3}b + 8c - 96 \\
 \text{add} \\
 \frac{-368}{3} &= -64a + \frac{32}{3}b - 48 \\
 a &= -b \\
 \frac{-368}{3} &= 64b + \frac{32}{3}b - 48 \\
 \left(\frac{-368}{3} + 48\right) &= \frac{224}{3}b \\
 \frac{-224}{3} &= \frac{224}{3}b \\
 b &= -1 \\
 a &= 1 \\
 a = 1, b = -1, c = -14 &\& d = 24
 \end{aligned}$$

Specific behaviours
<ul style="list-style-type: none"> ✓ solves for d ✓ derives $a = -b$ using inflection point ✓ sets up linear equation using x intercept ✓ uses definite integral and then integrates and sets up a linear equation for unknowns ✓ states values for all 4 unknowns <p>NOTE: follow through does not apply if mistake makes easier!</p>

Solution
<p>What is the velocity of the car at $t = \frac{3}{4}$ seconds?</p> <p>✓ states the correct rule for x ✓ integrates to find v and shows solving for constant with subs ✓ states exact value for x at required time, no need for units</p>
Specific behaviours
$x = -\frac{9}{4} \cos(\frac{3t}{4}) + C$ $k = \frac{9}{4}$ $0 = -\frac{9}{4} \cos(0) + C$ $C = \frac{9}{4}$ $v = \frac{3}{4} \sin(\frac{3t}{4}) + C$ $a = \frac{9}{4} \cos(\frac{3t}{4})$
<p>Solution</p>

Solution

Given quadratic equation: $a = 1, b = -1, c = -14 \Rightarrow d = 24$

OR solving without classpad

$f(x) = ax^2 + bx + c$

$$f(x) = x^2 - x - 14$$

$$x^2 - x - 14 = 0$$

$$(x^2 - 4x) + (3x - 12) = 0$$

$$x(x - 4) + 3(x - 4) = 0$$

$$(x + 3)(x - 4) = 0$$

$$x_1 = -3, x_2 = 4$$

Graphing calculator screen showing the steps:

- Equation input: $0 = -64a + 16b - 4c + 24$
- Equation input: $64a - \frac{64}{3}b + 8c - 96 = -\frac{368}{3}$
- Equation input: $a = -b$
- Equation input: $\{a=1, b=-1, c=-14\}$
- Equation input: $a = -3, b = 4$
- Equation input: $0 = 64a + 2b$
- Equation input: $0 = 64(-3) + 2(4)$
- Equation input: $0 = -192 + 8$
- Equation input: $0 = -184$
- Equation input: $0 = 0$

$$v = \frac{4}{3} \sin(3t)$$

$$t = \frac{\pi}{3}$$

$$v = 0$$

Specific behaviours

- ✓ subs into v
- ✓ states velocity, no need for units

Q3 (2 marks) (3.2.19)

Determine the exact area between $y = x^3 + x^2 - 37x + 35$ and the x axis from $x = -10$ to $x = 10$.**Solution**

Edit Action Interactive

0.5 1 $\frac{1}{2}$ $\int_{-10}^{10} |x^3 + x^2 - 37x + 35| dx$ 5689 2

Specific behaviours

- ✓ writes a correct integral for the area
- ✓ states exact value

Q4 (2, 2 & 3 = 7 marks) (3.2.18)

A factory produces electric vehicles. The total number, E , that the company has produced t months after production commenced is such that:

$$\frac{dP}{dt} = 450 - \frac{800}{(t+5)^3}$$

Determine the number produced in
a) The first 6 months**Solution**

Consider the function $G(x) = \int f(t) dt$ such that $G''(x) = \frac{3}{4x^{\frac{5}{2}}}$ and $G(4) = \frac{79}{2}$. Determine the rule for the function $f(x)$.

Solution

$$G(x) = \int f(t) dt$$

$$G'(x) = f(x)$$

$$G''(x) = f'(x)$$

$$f'(x) = \frac{3}{4}x^{-\frac{5}{2}}$$

$$f(x) = -\frac{2}{3}\left(\frac{3}{4}x^{-\frac{3}{2}} + c\right) = -\frac{1}{2}x^{-\frac{3}{2}} + c$$

$$\int f(t) dt = \frac{79}{2} = \left[x^{-\frac{1}{2}} + cx \right]_1^4 = \left(\frac{1}{2} + 4c \right) - (1+c) = 3c - \frac{1}{2}$$

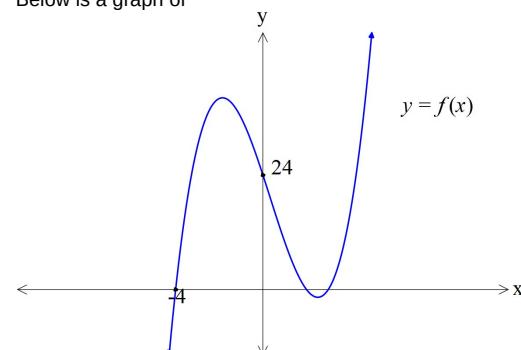
$$c = \frac{40}{3}$$

$$f(x) = -\frac{1}{2}x^{-\frac{3}{2}} + \frac{40}{3}$$

Specific behaviours

- ✓ uses fundamental theorem to express $G'' = f'$
- ✓ integrates to express f in terms of x and a constant
- ✓ uses definite integral to set up equation for constant
- ✓ solves for constant and express f in terms of x in full.

Q8 (5 marks) (3.1.15)

Consider the function $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c & d are constants.Below is a graph of $f(x)$ 

Q7 (4 marks) (3.2.16)

- Determine the minimum number of months required to produce:
- c) 10000 vehicles.

Specific behaviours

- ✓ states change, accept decimal
- ✓ writes a correct integral with limits or anti-diff with a constant
- ✓ sets up an equation
- ✓ sets number of years to one decimal place

Solution

The calculator screen shows the integral $\int_0^6 450 - \frac{800}{(x+5)^3} dx$ and the result 2687.305785. The interface includes a toolbar with various mathematical functions like ln, log, sin, cos, etc., and a menu bar at the top.

b) The third month

Specific behaviours

- ✓ states number of years to one decimal place
- ✓ sets up an equation
- ✓ sets number of years in one decimal place

Solution

The calculator screen shows the equation $\text{solve}(e^{0.08x}=3, x)$ and the result {x=13.73265361}. The interface includes a toolbar with various mathematical functions and a menu bar at the top.

b) Determine the time taken (years in one decimal place) to triple the number with the disease.

Specific behaviours

- ✓ uses an exponential function
- ✓ writes a correct integral with limits or anti-diff with a constant
- ✓ states change, accept decimal
- ✓ sets number, accept decimal

Solution

The calculator screen shows the equation $2300e^{0.08x}=5$ and the result Number of kangaroos 3431. The interface includes a toolbar with various mathematical functions and a menu bar at the top.

Mathematics Department

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Specific behaviours

- ✓ states number, accept decimal
- ✓ sets number of years in one decimal place

Solution

The calculator screen shows the equation $2300e^{0.08x}=5$ and the result 3431.196805. The interface includes a toolbar with various mathematical functions and a menu bar at the top.

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Solution

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$\int_0^x 450 - \frac{800}{(t+5)^3} dt = 10000, x$

{x=-5.179990402, x=-4.81881312, x=22.2565813}

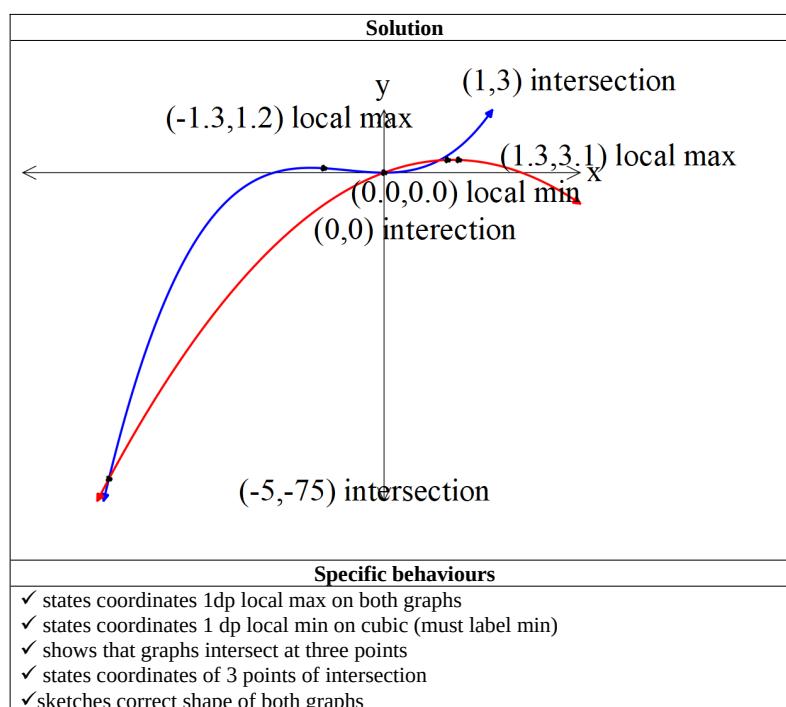
Need at least 22.27 OR 23 months

Specific behaviours

- ✓ sets up an integral with unknown upper limit
- ✓ solves for a decimal number of months
- ✓ states number of months

Q5 (5 & 3 = 8 marks) (3.2.20)

- a) On the axes below, sketch the following graphs: $y = x^3 + 2x^2$ and $y = 5x - 2x^2$. Indicate on your sketch coordinates (one decimal place) of any stationary points and label their nature and of any points where the graphs intersect each other.



NOTE: follow through does not apply if mistake makes easier!

- b) Determine the exact area between $y = x^3 + 2x^2$ and $y = 5x - 2x^2$.

Solution

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$$\int_{-5}^1 |x^3 + 2x^2 - (5x - 2x^2)| dx$$

$\frac{443}{6}$

Specific behaviours

- ✓ sets up correct integral(s)
- ✓ uses correct limits for integrals
- ✓ states exact area

Q6 (2 & 2 = 4 marks) (3.1.3, 3.1.4)

The number of kangaroos, N in a particular site that have developed disease W are increasing such that $\frac{dN}{dt} = 0.08N$ with t the time in years. There are initially 2300 kangaroos.

- a) Determine the number of kangaroos with disease W in 5 years' time.

Solution