Year 12 Physics 2011

Motion and Forces Unit Test

Name: SOLUTION

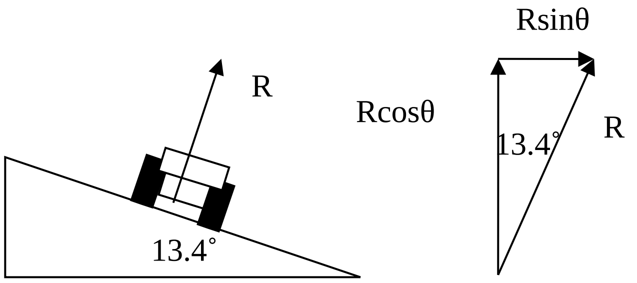
Mark: / 50

= %

Notes to Students:

- You must include **all** working to be awarded full marks for a question.
- Marks will be deducted for incorrect or absent units.
- Answers should be given to 3 significant figures.

1. By banking the curves of racetracks at an angle to the horizontal, it possible for vehicles to turn in a horizontal circle without relying on friction.



(a) For a car of mass 1 700 kg, if the angle of banking is set at 13.4° above the horizontal for a curve of radius 171 m, calculate the optimum speed that a car can go around the curve without relying on friction. [4]

$\tan\theta = v^2 / g.r$ 1 $\tan\theta = R\sin\theta / R\cos\theta \therefore v^2 = g.r.\tan\theta$ 1

$v = (g.r.\tan\theta)^{1/2} = (9.8 \times 171 \times \tan 13.4)^{1/2}$ 1

$= 19.98 \text{ m s}^{-1}$ 1

(b) Calculate the normal reaction force acting on the car from the track. [3]

$R\cos\theta = m.g \therefore R = m.g / \cos\theta$ 1

$= 1\,700 \times 9.80 / \cos 13.4$ 1

$= 1.71 \times 10^4$ N perpendicular to the surface out of track 1

(no direction = -½)

(c) For a mass sliding down a frictionless inclined plane, the normal reaction force from the plane acting on the mass is less than the weight of the mass. In the above example of circular motion on a banked track the reaction force is now greater than the weight.

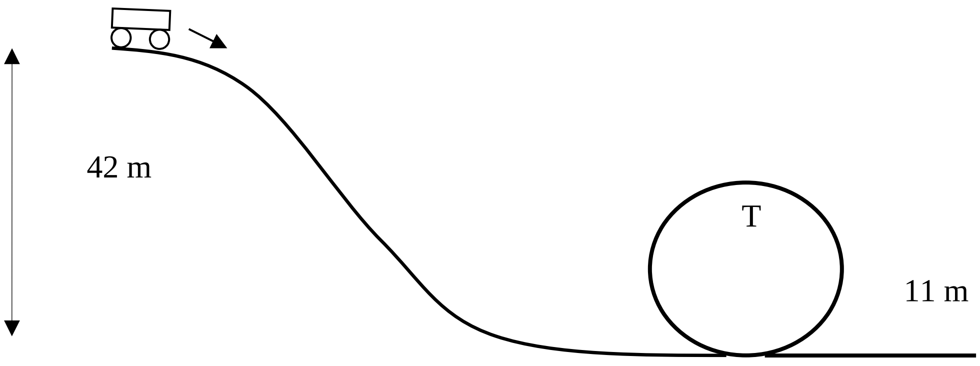
Explain how the bank can apply a normal reaction force greater than the weight. [3]

• The car is essentially driving into the curve and applying an action force, the reaction force from the curve pushes the car round the curve. This reaction force increases as speed around the circumference increases.

• Once the optimum speed has been reached the vertical component of the reaction force balances the weight force down and keeps the car at a constant height on the bank.

• These two components add together to exceed the weight force of the car.

2. A roller coaster car has a mass of 470 kg and starts from a height of 42.0 m above the ground. The car relies on mechanical energy only to go around the loop. The bottom of the circular loop is at ground level and the loop has a radius of 11.0 m as shown in the diagram below. The car is initially moving at a speed of 6.10 m s^{-1}.



(a) Calculate the speed of the car at point T, the top of loop. [5]

Total Mechanical Energy = constant = mgh + ½ m v^2 1

As mass is constant, gh + ½ v^2 = constant

$(g.h_{top} + \frac{1}{2} v^2)_{top} = (g.h_{loop} + \frac{1}{2} v^2_{loop})$ 1 (or indication that TME = constant)

$(9.8 \times 42 + \frac{1}{2} . 6.1^2) = 430.205 = (9.8 \times 22) + \frac{1}{2} v_{loop}^2$ 1

$\rightarrow v_{loop}^2 = 429.21$ 1 $\therefore v_{loop} = 20.7$ m/s 1

(b) Calculate the normal reaction force acting on the car at the top of the loop. [3]

$\Sigma F = m.v^2 /r = R + m.g$ (towards the centre of the circle) $R = m.v^2 /r - m.g$ 1

$R = 470 \times 429.21 /11 - 470 \times 9.8$ 1

$R = 1.37 \times 10^4$ N **towards centre of circle** 1

(c) Determine the minimum speed that the car can have at the top of the loop before it starts to fall away from the track. [4]

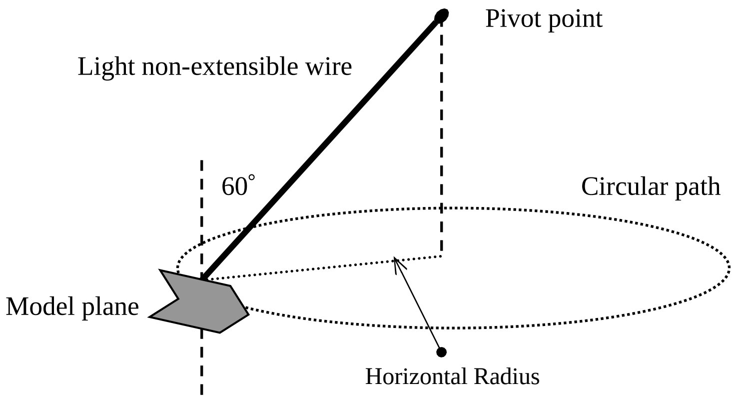
This occurs when R is reduced to zero $\Sigma F = m.v^2/r = R + m.g$ (to centre of loop)

$R = m.v^2/r - m.g$ 1

$0 = m.v^2/r - m.g \rightarrow \therefore m.v^2/r = m.g$ 1

$v = (r \times g)^{½} = (11 \times 9.8)^{½}$ 1 $= 10.4\ m\ s^{-1}$ 1

3. A model plane of mass 160 g is suspended from a light non-extensible wire. When in horizontal circular motion it is noted that it makes ten revolutions in 15 seconds and that the wire is at an angle θ of 60.0° to the vertical.



(a) Calculate the **tension** along the wire. [4]

$m = 0.160$ kg $g = 9.80$ N kg^{-1} $T_{period} = 15/10 = 1.50$ s

Forces acting on model plane are balanced in the vertical 1

∴ $W_{down} = T.\cos\theta$ so $m.g = T.\cos\theta$ 1

$T = (0.160 \times 9.80) / \cos 60°$ 1 $= 3.14$ N 1

(b) Calculate the horizontal radius of circular motion. [4]

$\tan\theta = v^2 / g.r$ and $v = 2\pi r/T_{period}$ 1 $\tan\theta = (2\pi r/T_{period})^2 \times 1/g.r$ 1

$\tan\theta = 4.\pi^2.r^2/T_{period}^2 \times 1/g.r$ so $\tan\theta = 4.\pi^2.r/(T_{period}^2.g)$ 1

$r = (\tan 60 \times 1.5^2 \times 9.80) / 4.\pi^2$ $= 0.967$ m 1

4. A 5.00 kg lump of rock dropped near the surface of Mars reaches a speed of 14.8 ms^{-1} in 4.00 seconds.

(a) Calculate the acceleration due to gravity near the surface of Mars. [2]

$v = u + at$ 1 $a = (v-u)/t$

$a = 14.8/4$ $a = 3.70$ ms^{-2} 1

(b) Given that Mars has a radius of 3400 km calculate the mass of Mars. [3]

$F_g = Gm_{Mars}m_2/R^2$ $\quad 3.70 = Gm_{Mars}/R^2$ 1

(c)

$3.70 \times R^2/G = m_{mars}$ 1 $\quad m_{mars} = (3.70 \times (3.40 \times 10^6)^2)/(6.67 \times 10^{-11})$ 1

$= 6.41 \times 10^{23}$ kg

$F = \frac{GM_S M_{Mars}}{R^2} = \frac{M_{Mars}v^2}{R}$ 1 $\quad$ where $v = \frac{2\Pi R}{T}$

$\frac{GM_{Mars}}{R^2} = (2\Pi R/T)^2$

$\frac{GM_{Mars}}{R^2} = \frac{(4\Pi^2R^2/T^2)}{R}$ $\quad M_{Mars} = \frac{4\Pi^2R^3}{GT^2}$ 1

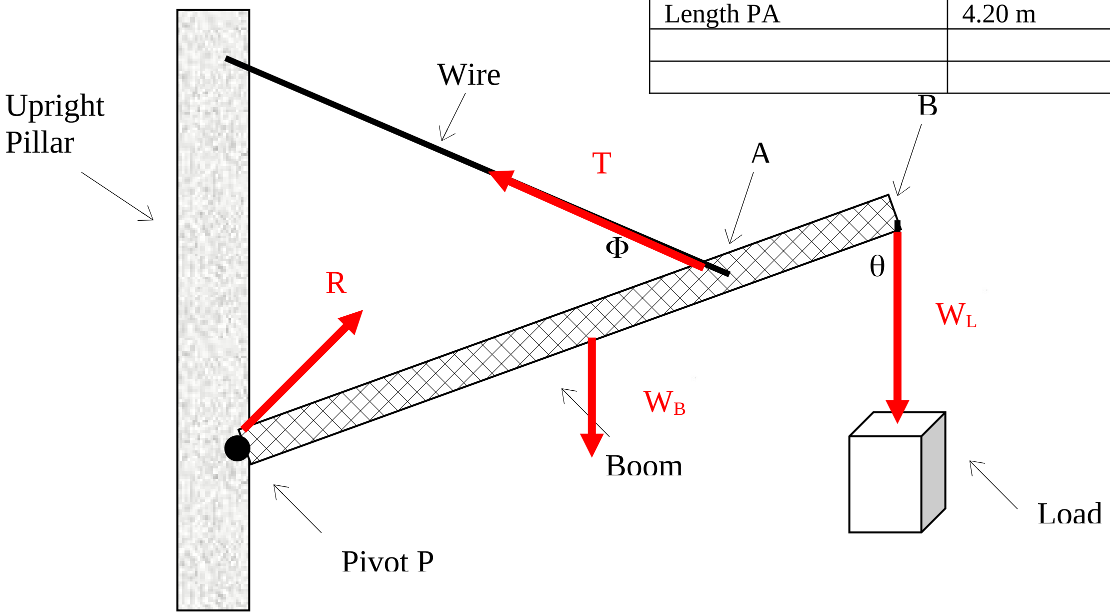
$T^2 = \frac{4\Pi^2R^3}{GM_{Mars}}$ 1

$T^2 = \frac{1.92 \times 10^{21}}{4.28 \times 10^{13}}$ 1 $\quad T = \sqrt{4.49 \times 10^7}$ $\quad T = 6.70 \times 10^3$ s 1

Calculate the orbital period of a 50 kg satellite if it was put into orbit about the equator of Mars at an altitude of 250 km. [5]

5. Consider the non-parallel forces acting on the boom of a crane. The wire that lifts the crane connects to the boom at point A. The boom pivots at point P. A load is suspended from the boom at point B. The masses, dimensions and angles for this set up are shown in the following table.

Mass of boom	150 kg
Mass of load	880 kg
Angle Φ	50°
Angle θ	70°
Boom Length PB	5.40 m
Length PA	4.20 m



(a) Calculate the tension in the wire. [5]

Let lever arm to tension T = r_T = 4.20 m Let lever arm to load W_{LOAD}= r_L = 5.40 m

Let lever arm to centre of mass of boom W_{BOOM} = r_B = 2.70 m

Take moments about P to eliminate R from calculation.

By principle of moments ΣM = 0 ∴ Σacwm = Σcwm 1

$r_T.T.\sin \Phi = r_L.W_L.\sin \theta + r_B.W_B.\sin \theta$ 1

4.20 × T × sin 50 = 5.40 × (880 × 9.8) × sin 70 + 2.70 × (150 × 9.8) × sin 70 1

4.20 × T × sin 50 = 43 761.11 + 3 729.64 4.20 × T × sin 50 = 47 490.75

T = 47 490.75 ÷ (4.20 × sin 50) 1 T = 14 760.66 N

<u>Tension = 1.48 × 10^4 N</u> 1 <u>(because a flexible component then along the wire – NB direction is not strictly required but a good idea to state this anyway)</u>

(b) Calculate reaction force R acting on the boom at the pivot point P. [5]

Consider components of R

Angle ρ b/w T and horizontal = $(\Phi+\theta) - 90^\circ$

$\rho = (50 + 70) - 90 = 30^\circ$ 1

$\Sigma F_{horizontal} = 0$ $T_{horizontal}$ to LHS = $R_{horizontal}$ to RHS

$R_{hor} = T \cos 30^\circ = 14\ 761 \times \cos 30 = 12783.4$ 1

$\Sigma F_{vertical} = 0$

W_{B+L} down = $R_v + T_v$ up

$(880 + 150) \times 9.8 = R_v + T.\sin 30$

$10\ 094 = R_v + (14\ 761 \times \sin 30)$

$R_V = 2713.5$ N 1

$R = (12783.4^2 + 2713.5^2)^{0.5}$

R = 13 068 N = 1.31×10^4 N magnitude 1

$\gamma = \arctan(R_h/R_v) = \arctan(12783.4/2713.5)$

$\gamma = 78.0^\circ$ to the vertical – direction 1