Christ Church Grammar School

2016
UNIT TEST 1

# MATHEMATICS METHODS Year 12

**Section One:**
**Calculator-free**

Student name _____

Teacher name _____

## Time and marks available for this section

| | |
|---|---|
| Reading time before commencing work: | 2 minutes |
| Working time for this section: | 15 minutes |
| Marks available: | 15 marks |

## Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer Booklet
Formula Sheet

## *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

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**Question 1** **(4 marks)**

Differentiate with respect to $x$. Do not simplify your answers.

(a) $x^2e^{-2x}$ (1 mark)

(b) $e^{\tan 2x}$ (1 mark)

(c) $\frac{\sqrt{x}}{\cos(2x-1)}$ (2 marks)

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**Question 2** **(3 marks)**

Find $\frac{dy}{dx}$ given that $x = e^{\sin\theta}$ and $y = e^{\cos\theta}$.

**See next page**



**Additional working space**

Question number: _____

**Additional working space**

Question number: _____



**Question 3** **(4 marks)**

Find the minimum and maximum values of $f(x) = \frac{x^3}{3} - x^2 + 4$ over the interval $-3 \leq x \leq 3$ .

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**Question 4** **(4 marks)**

For the function $f(x) = (x - 200)^6 + 300$,

(a) find the value of $a$ for which $f''(a) = 0$ (1 mark)

(b) determine the concavity of $y = f(x)$ when $x < a$ and when $x > a$ (2 marks)

(c) hence determine if $x = a$ is a point of inflection or not, giving a reason for your answer.
(1 mark)

**End of questions**



**Question 8** **(6 marks)**

Consider two circles, the first having a radius $r_1$ and the other radius $r_2$ , with the sum of the two radii being constant, that is, $r_1 + r_2 = c$.

(a) Find an expression for the sum of the areas of the two circles in terms of $r_1$ and $c$. (2 marks)

(b) Use calculus to prove that if the sum of the radii of two circles is constant, then the sum of the areas of the two circles is at a minimum when the circles have equal radii. (4 marks)

**End of questions**

**Question 7 continued**

(e) Find how many items were manufactured and sold if the profit associated with the sale of the next item is approximately $10, given that more than 100 items were manufactured and sold. (2 marks)

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**2016**
UNIT TEST 1

# MATHEMATICS METHODS Year 12

**Section Two:**
**Calculator-assumed**

Student name _____

Teacher name _____

## Time and marks available for this section

| | |
|---|---|
| Reading time before commencing work: | 3 minutes |
| Working time for this section: | 30 minutes |
| Marks available: | 30 marks |

## Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer Booklet
Formula Sheet (retained from Section One)

### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

## Important note to candidates

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## Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

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**Question 7** **(7 marks)**

KSL Productions sells a product at a unit price of \$30. The cost of producing $x$ items is given by $C(x) = \frac{80x}{x+1} + 0.04x^2 + 500$.

(a) Find an expression for the profit $P(x)$ corresponding to the manufacture and sale of $x$ items. (1 mark)

(b) Find an expression $P'(x)$. (1 mark)

(c) Find $P'(100)$. Interpret this value. (2 marks)

(d) Find the average profit per item associated with the manufacture and sale of 100 items. (1 mark)





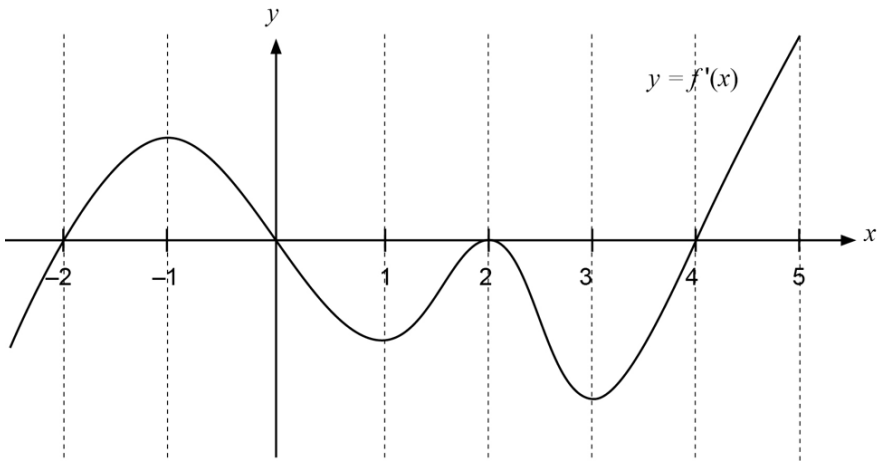
**Question 5** **(8 marks)**

(a) Use the method of small changes to find the approximate change in the radius of a spherical balloon corresponding to a change in its volume from $500\ cm^3$ to $485\ cm^3$. (4 marks)

(b) The displacement of a body at time $t$ seconds is given by $x = 4t + \frac{1}{1+t}$ metres. Find an expression for the velocity of the body at time $t$ seconds and then show that the body is never stationary. (4 marks)

**Question 6** **(9 marks)**

The diagram below shows the graph of $y = f'(x)$ of a function $y = f(x)$.




(a) For what values of $x$ does $y = f(x)$ have a local maximum or minimum? (2 marks)

(b) For what values of $x$ does $y = f(x)$ have inflection points? (2 marks)

(c) Does $y = f(x)$ have a horizontal point of inflection? Explain (2 marks)





**Question 6 continued**

(d) On the axis below, sketch the graph of $y = f''(x)$. (3 marks)