Semester Two Examination, 2016

Question/Answer Booklet

MATHEMATICS METHODS UNITS 1 AND 2

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(6 marks)**

(a) Determine the equation of the straight line that passes through the point $(8, 11)$ and is perpendicular to the line with equation $2x+5y=1$. (3 marks)

(b) Calculate and use the discriminant to determine the number of solutions to the equation $9x^2-24x+16=0$. (3 marks)

Question 10 **(8 marks)**

A walking club is planning a charity walk from Perth to Esperance. They plan to walk 20 km on the first day and 30 km every day after that. Food and camping supplies are to be set up at each overnight campsite in advance, using a vehicle based in Perth that is just large enough to carry enough for one campsite.

To leave the supplies at the first campsite, the vehicle must travel 40 km. For the second and third campsites, the vehicle must travel 100 km and 160 km respectively.

(a) Determine the distances the vehicle will travel to set up campsites four and five. (1 mark)

(b) Determine, in simplified form, a rule for the distance, d km, that the vehicle will have to travel to set up campsite n. (2 marks)

(c) The vehicle can travel a maximum of 700 km on one tank of fuel. Determine the number of the furthest campsite the vehicle can leave supplies at, using no more than one tank of fuel. (2 marks)

(d) If fuel costs 128 cents per litre and the fuel consumption of the vehicle is 9.5 litres per 100 km, determine the total fuel cost to set up the first 20 campsites. (3 marks)

Question 11 **(8 marks)**

Records show that of the 1756 washing machines sold by a retailer during 2015, 464 were deluxe models and the rest were standard. Of all the machines sold, 42 were returned and 31 of these returned machines were standard models.

(a) Determine how many of the standard models were not returned. (2 marks)

(b) Calculate, to three decimal places, the probability that a randomly chosen machine from those sold

(i) was a standard model. (1 mark)

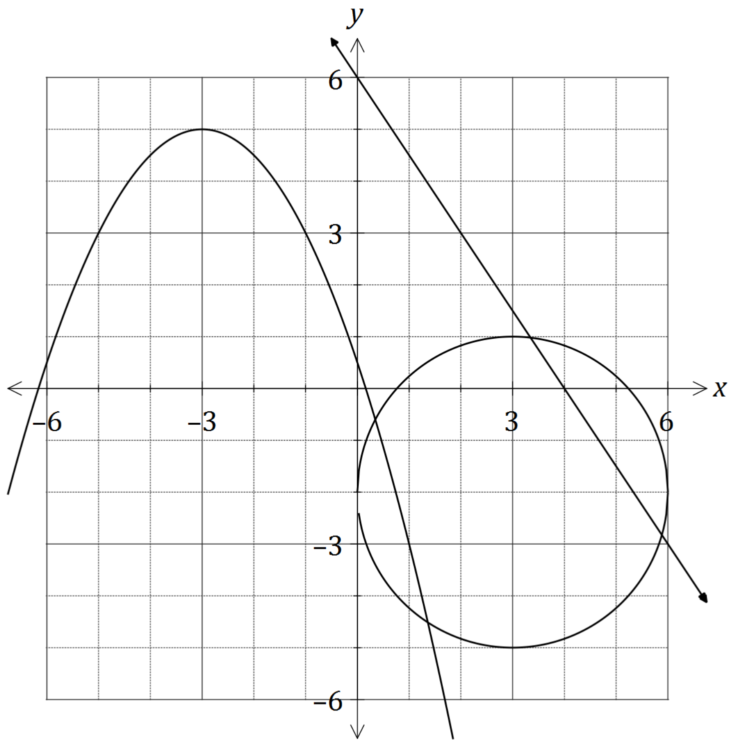
(ii) was returned. (1 mark)

(iii) was returned given that it was a deluxe model. (2 marks)

(c) Is there any indication that the likelihood of a machine being returned is independent of the model type? Explain your answer. (2 marks)

Question 12 **(10 marks)**

The graph of two functions and a circle of radius 3 units are shown.



(a) One function is $f(x)=ax+b$. Determine the values of the constants a and b. (2 marks)

(b) The relation can be written in the form $x^2+px+y^2+qy+r=0$.

Determine the values of the constants p, q and r. (3 marks)

(c) The other function is $g(x)=cx^2+dx+e$.

(i) Determine the values of the constants c, d and e, given that $g(x)$ has a maximum at $(-3, 5)$. (3 marks)

(ii) State coordinates of the turning point of the graph of $y=g(x-7)$. (1 mark)

(iii) State the range of the function $y=-g(x)$. (1 mark)

Question 13 **(7 marks)**

In triangle PQR, $PR = 50$ cm, $QR = 30$ cm and $\angle QPR = 25°$.

(a) Show that $\angle PQR$ can be $44.8°$ or $135.2°$. (3 marks)

(b) Showing use of trigonometry, determine

(i) the largest possible length of PQ. (2 marks)

(ii) the smallest possible area of triangle PQR. (2 marks)

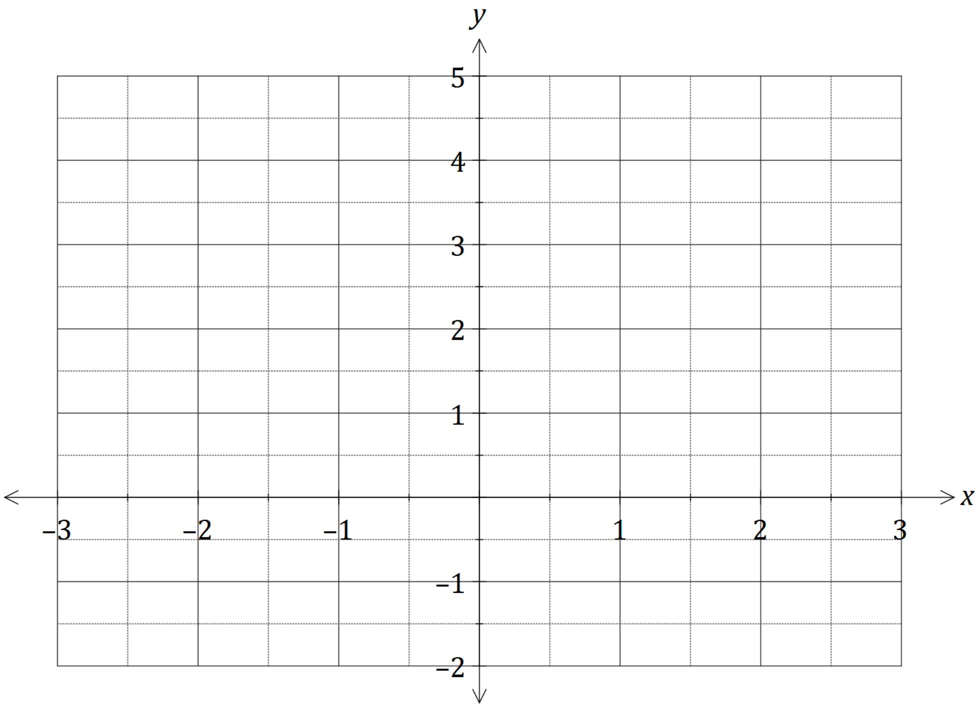
Question 14 **(9 marks)**

The function f is given by $f(x)=x^3-3x+2$.

(a) Show that the graph of $y=f(x)$ has two roots and state their coordinates. (2 marks)

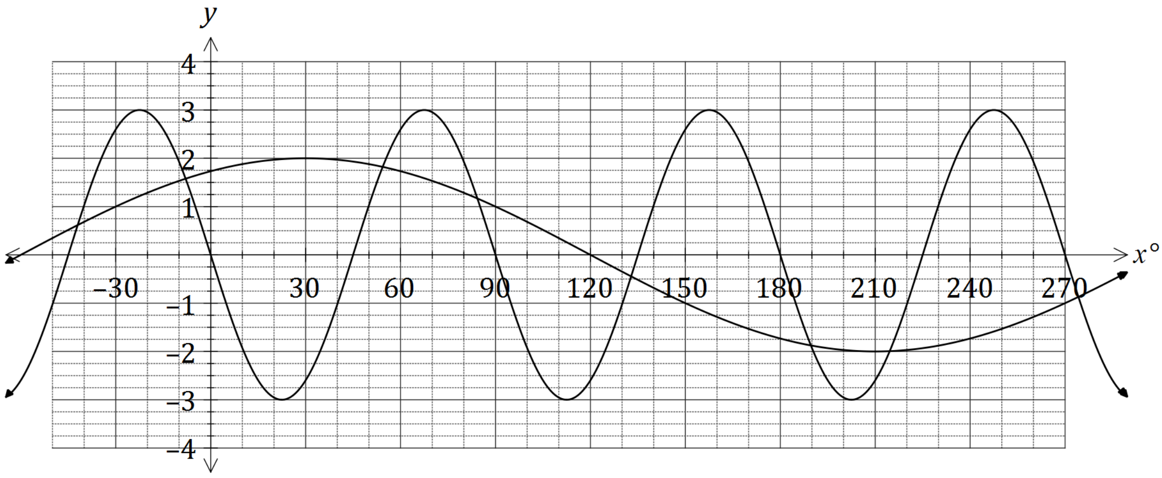
(b) Use calculus techniques to determine the coordinates of all stationary points of the graph of $y=f(x)$. (4 marks)

(c) Sketch the graph of $y=f(x)$ on the axes below for $-2\le x\le 2$. (3 marks)



Question 15 **(10 marks)**

(a) The graphs of $f(x)=a\sin(bx)$ and $g(x)=c\cos(x+d)$, where x is in degrees, are shown below.



(i) Determine the values of the constants a, b, c and d. (4 marks)

(ii) Use the graph to solve $f(x)=g(x)$, $0° \leq x \leq 180°$. (2 marks)

(b) P and Q are acute angles with $\sin P = \frac{12}{13}$ and $\cos Q = \frac{15}{17}$. Determine the **exact** value of $\cos(P - Q)$. (4 marks)

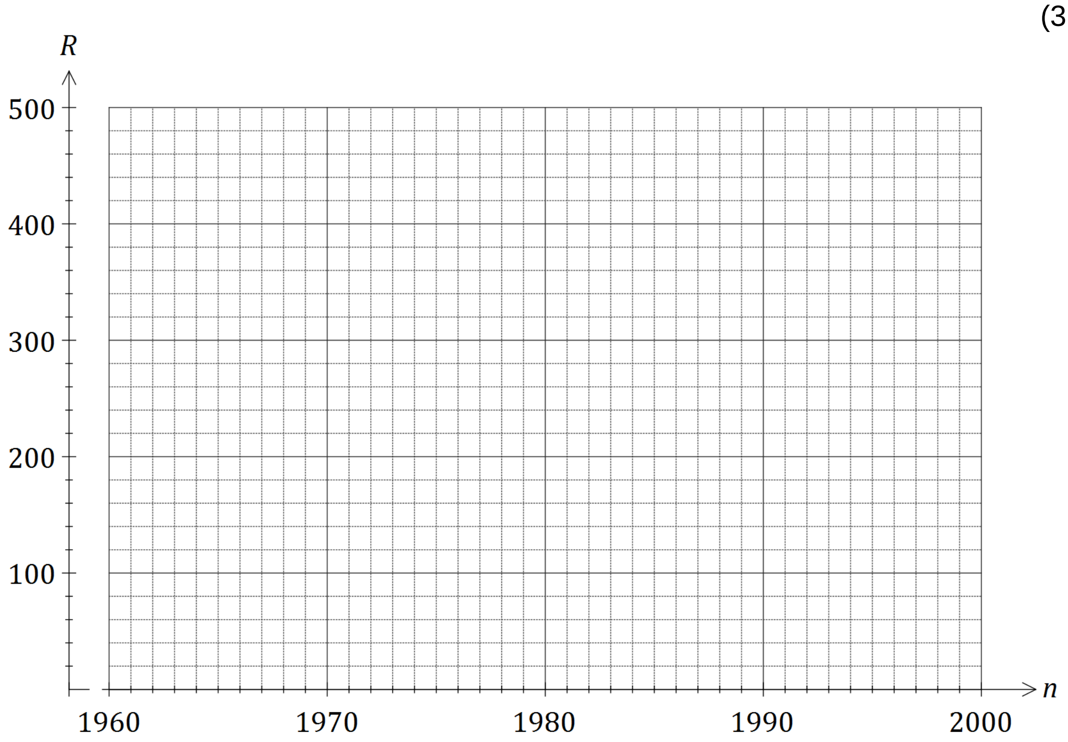
Question 16 **(8 marks)**

The imprisonment rate R, in number of prisoners per 100 000 people, in the US between the years 1960 and 2000, can be modelled by the following equation, where n is the year.

$$R = 85\left(1.038\right)^{n-1960}$$

(a) Calculate the imprisonment rate in the year 2000. (1 mark)

(b) Draw the graph of the imprisonment rate for $1960 \le n \le 2000$ on the axes below. (3 marks)

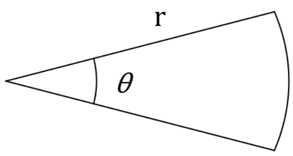


(c) The population of the US was 266 million in 1995. Determine the number of prisoners in the US at this time, to the nearest 1 000. (3 marks)

(d) When R first exceeded 500, steps were taken to address the exponential growth in the prison population and the model no longer applied. In what year did this occur? (1 mark)

Question 17 **(8 marks)**

The perimeter of a sector of a circle, of radius r cm and central angle θ radians, is 60 cm.



(a) Show that $\theta = \frac{60}{r} - 2$. (2 marks)

(b) Show that the area of the sector is given by $30r - r^2$. (2 marks)

(c) Use calculus to determine the maximum area of the sector and state the values of r and θ that achieve this maximum. (4 marks)

Question 18 **(8 marks)**

(a) Two students are to be chosen from a class of 18.

(i) Determine how many different pairs of students may be chosen. (1 mark)

(ii) One of the students in the class is the oldest in the school. What is the probability that this student is included in the pair chosen? (2 marks)

(b) A box contains 13 cans of soup, four of which have tomato as an ingredient and the remainder that do not. Four cans are to be selected at random from the box.

(i) Calculate how many different selections of four cans can be made from the box. (1 mark)

(ii) Determine the probability that none of the four cans will have tomato as an ingredient. (2 marks)

(iii) Determine the probability that in the selection of four cans, there will be an equal number of cans with and without tomato as an ingredient. (2 marks)

Question 19 **(7 marks)**

(a) A sequence is defined by $T_{n+1}=T_n-7, T_1=111$.

(i) Determine T_{20}. (1 mark)

(ii) The sum of the first 40 terms, S_{40}. (1 mark)

(iii) The value of n that maximises S_n. (2 marks)

(b) A geometric sequence with $T_2=87.5$ has a sum to infinity of 800. Determine all possible values of T_1 for this sequence. (3 marks)

Question 20 **(9 marks)**

(a) Show that the equation of the tangent to the curve $y=\frac{x+x^3}{2}$ at the point where $x=2$ is $13x-2y=16$. (4 marks)

(b) The line with equation $y=5x+c$ is a tangent to the curve $y=x^3+3x^2-4x-12$.

Determine the value(s) of c. (5 marks)

End of questions

Additional working space

Question number: __________

Additional working space

Question number: _________

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