Papers written by Australian Maths Software

SEMESTER TWO

YEAR 11

MATHEMATICS SPECIALIST
Units 1 & 2
2016
REVISION THREE

Section One
(Calculator–free)

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 5 minutes
Working time for section: 50 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items*:* pens, pencils, pencil sharpener, highlighter, eraser, ruler.

IMPORTANT NOTE TO CANDIDATES
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor
Question/answer booklet for Section One.
A formula sheet which may also be used for Section Two.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	**7**	**7**	**50**	**52**	**35**
Section Two Calculator—assumed	11	11	100	98	65
Total marks				150	

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answer in the Question/Answer booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

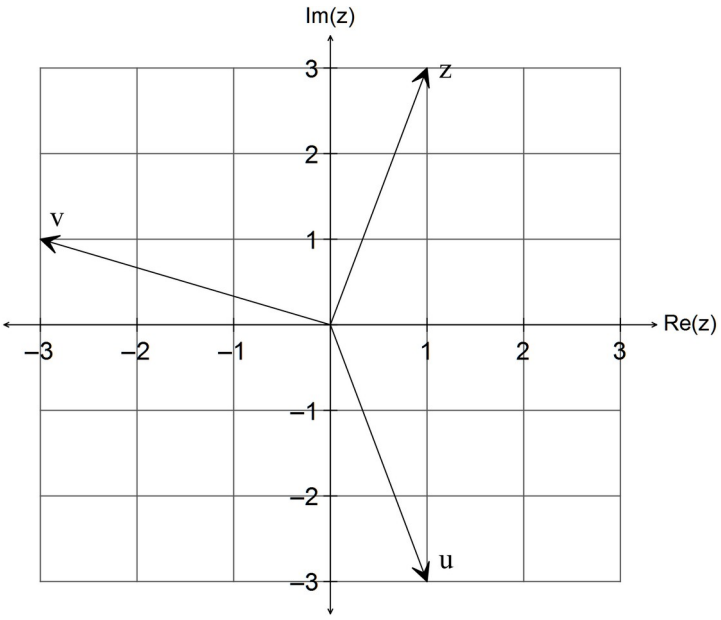
7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator–free 52 marks

This section has **seven (7)** questions. Attempt **all** questions.
Working time: 50 minutes

Question 1 (8 marks)

(a) Consider the complex numbers graphed on the set of axes below.
Express u and v in terms of z (4)



Consider the set of complex numbers $z_1 = 2 + i$, $z_2 = 1 + 3i$ and $z_3 = \overline{z_2}$.

(b) Find

(i) $(z_2)^2$ (2)

(ii) $\dfrac{z_1 - z_2}{z_3}$ (2)

Question 2 **(9 marks)**

(a) Consider the following vectors $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $\boldsymbol{i} + \boldsymbol{j}$, $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$, $2\boldsymbol{j}$, $-4\boldsymbol{i} - 3\boldsymbol{j}$.

(i) List two vectors that are parallel. (1)

(ii) List two vectors that are perpendicular. (1)

(b) Find the projection of the vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ on $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. (3)

(c) Find y given $\boldsymbol{a} = 5\boldsymbol{i} + y\boldsymbol{j}$ and $\boldsymbol{b} = 3\boldsymbol{i}$ if it is known that the angle between the two vectors is 60^0. (4)

Question 3 **(7 marks)**

Given $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$, $E = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ determine each of the following or else explain why it cannot be determined.

(a) $A + B$ (1)

(b) $B \times E$ (2)

(c) $C + D$ (2)

(d) E^{-1} (2)

Question 4 **(8 marks)**

(a) Simplify the following $\left(\begin{bmatrix} 0 & 3 \\ 7 & 1 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -5 & 2 \end{bmatrix}\right) \times \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (2)

(b) Solve for x and y $\begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$ (2)

(c) Calculate M if $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^4$. (1)

(d) Determine whether or not $\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$ (3)

Question 5 **(8 marks)**

(a) (i) Use the unit circle to explain why $\sin^2(x)+\cos^2(x)=1.$ (1)

(ii) Use the identity $\sin^2(x)+\cos^2(x)=1$ to establish and identity for $\tan^2(x)$ In terms of $\sec^2(x).$ (2)

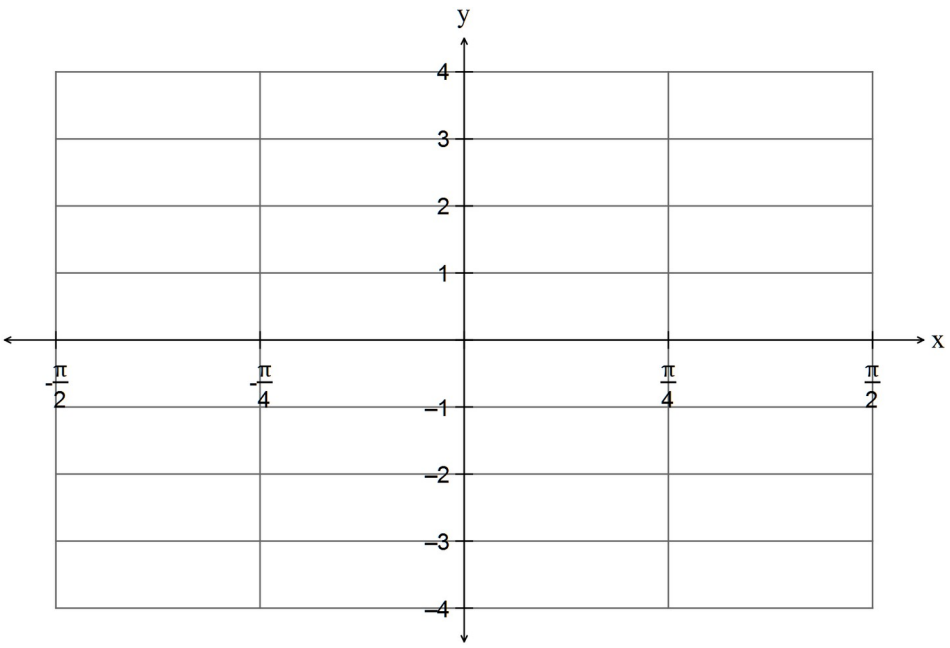
(b) (i) Show that $\tan\left(\frac{\pi}{4}+\theta\right)=\frac{1+\tan(\theta)}{1-\tan(\theta)}.$ (2)

(ii) Show that $\tan\left(\frac{5\pi}{12}\right)=-2-\sqrt{3}.$

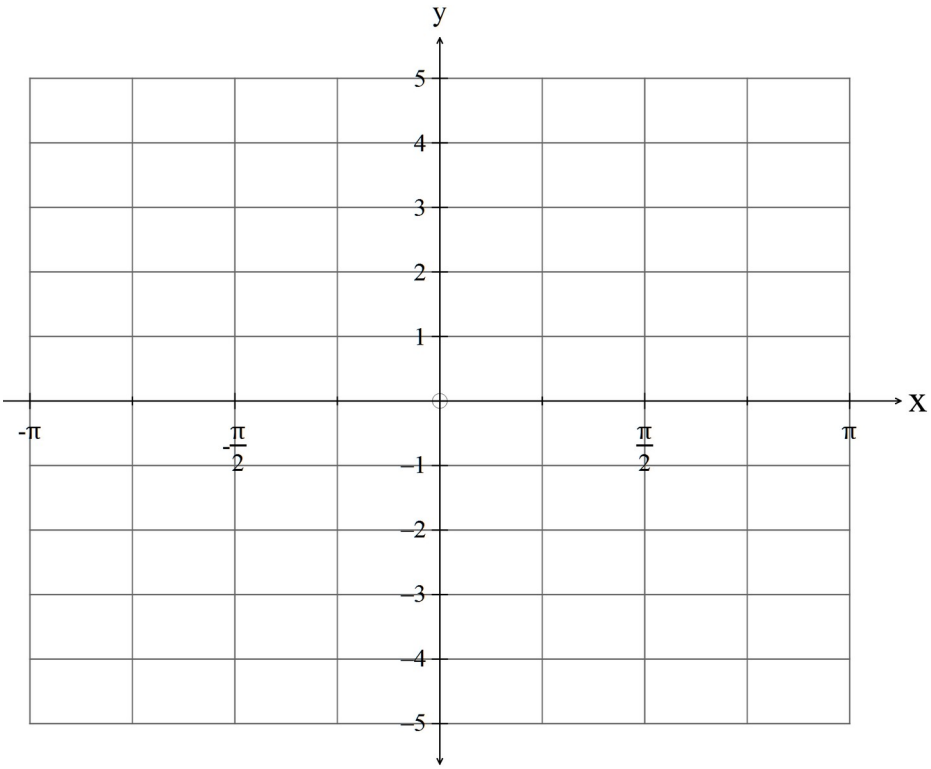
HINT: $\frac{5\pi}{12}=\frac{\pi}{4}+\frac{\pi}{6}$ (3)

Question 6 **(4 marks)**

(a) Sketch the function $y = \tan\left(2x + \frac{\pi}{2}\right)$ on the domain $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. (2)



(b) Sketch the function $y = 1 - \text{cosec}(x)$ on the domain $-\pi \le x \le \pi$ (2)



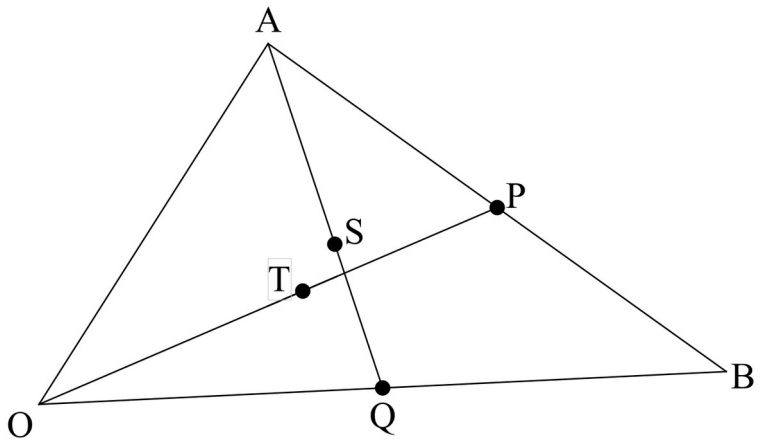
Question 7 **(8 marks)**

(a) Provide a counter example to show that the following statement is false. (1)

"*If a quadrilateral has four right angles and equal diagonals, then it is a square."*

(b) Prove that "*The opposite angles of a cyclic quadrilateral are supplementary*." (3)

(c) Prove the following using vectors
"*The three medians of a triangle are concurrent at the point of trisection*." (4)



Let ABC be the triangle. P and Q are the midpoints of sides AB and AB respectively.
Let T and S be the points of trisection of **OP** and A**Q** respectively.

END OF SECTION ONE