STRIVE FOR THE HIGHEST

Semester 2 Examination, 2012

Question/Answer Booklet

MATHEMATICS
3C/3D (Year 12)
Section One:
Calculator-free

Your name: ______________________________

Your teacher: S Ebert   T Hosking   S Rowden

Time allowed for this section

Reading time before commencing work: five minutes
Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	7	7	50	50
Section Two: Calculator-assumed	13	13	100	100
				150

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

Section One: Calculator-free **(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the space provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

For the two independent events A and B, $P(A) = 0.3$ and $P(B) = 0.1$.

Calculate

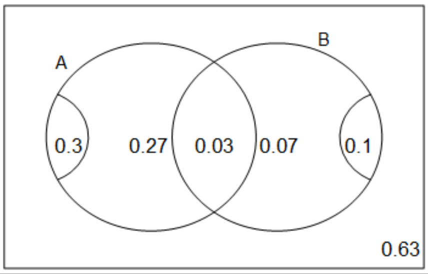
(a) $P(\overline{B})$ $\boxed{0.9}$ [1]

✓ - correct value

(b) $P(A \cap B)$ $\boxed{0.1 \times 0.3 = 0.03}$ [1]

✓ - correct value

(c) $P(A \cup \overline{B})$ [2]



$\boxed{0.3 + 0.63 = 0.93}$

✓ - working ✓ - correct value

(d) $P(\overline{A} \mid B)$ $\boxed{0.7}$ [1]

✓ - correct value (or fraction)

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Question 2 **(5 marks)**

Solve the system of equations

$$3x + 2y + 6z = 3$$
$$x + 3y + 4z = 9$$
$$2x + 8 = 2z + y$$

$$3x + 9y + 12z = 27$$
$$3x + 2y + 6z = 3$$
$$7y + 6z = 24$$

$$2x + 6y + 8z = 18$$
$$2x - y - 2z = -8$$
$$7y + 10z = 26$$

$$4z = 2$$
$$z = 0.5$$

$$7y = 24 - 6(0.5)$$
$$y = 3$$

$$x = 9 - 3(3) - 4(0.5)$$
$$x = -2$$

✓ - elimination of one variable
✓ - elimination of second variable
✓ - correct value for *x*
✓ - correct value for *y*
✓ - correct value for *z*

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Question 3 **(7 marks)**

(a) Differentiate the following with respect to x. **There is no need to simplify your answer**.

(i) $y = \frac{1}{2e^{-x^2}}$ [2]

$x.e^{x^2}$

✓ - derivative of the power part
✓ - function part

(ii) $y = 2x^3\sqrt{3 - x^2}$ [2]

$$\frac{dy}{dx} = 2x^3.\frac{1}{2}(3 - x^2). - 2x + (3 - x^2)^{\frac{1}{2}}.6x^2$$

✓ - derivative of first function
✓ - derivative of second function

(b) Simplify $\frac{d}{dx}\int_2^{x^2}\left(\frac{t^2}{3}\right)dt$ [3]

$$\frac{dy}{dx} = \frac{(x^2)^2}{3} \times 2x$$
$$= \frac{2x^5}{3}$$

✓ - substitute x^2 for t
✓ - multiply by derivative of x^2
✓ - simplify

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Question 4 **(9 marks)**

A function is defined by $f(x) = 6x^2 - 2x^3$.

(a) Find the coordinates of the turning points of $f(x)$ and state their nature.

[4]

$f'(x) = 12x - 6x^2$
$= 0$ when $x = 0$, $x = 2$
$(0, 0)$ is a minimum and $(2, 8)$ is a maximum.

✓ - derivative
✓ - x values
✓ - coordinates
✓ - identification of maximum and minimum

(b) Find the coordinates of the point of inflection of $f(x)$.

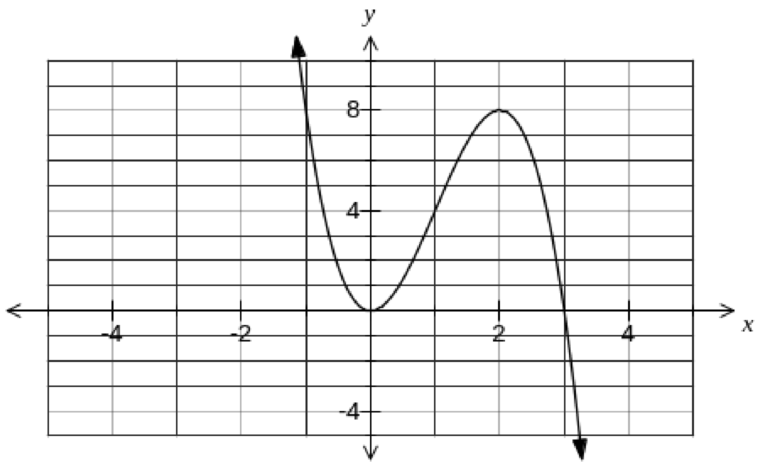
[1]

$f''(x) = 12 - 12x$
$= 0$ when $x = 1$
At $(1, 4)$

✓ - coordinates

(c) Sketch the graph of $y = f(x)$. Clearly show the key features

[3]



✓ - shape ✓ - x intercept ✓ - stationary points

(d) What is the maximum value of $f(x)$ in the interval $-2 \le x \le 4$?

[1]

$f(-2) = 6(4) - 2(-8) = 40$

Max value is 40.

✓ - identifies 40

Question 5 **(8 marks)**

Let $f(x)=\dfrac{1}{1-x}$ and $g(x)=e^{2x}$.

(a) Determine the domain of $f(g(x))$.

[2]

$$1-e^{2x}\neq 0$$
$$x\neq 0$$

✓ - $1-e^{2x}\neq 0$
✓ - $x\neq 0$

(b) Determine the range of $g(f(x))$.

[2]

$g(f(x))=e^{\frac{2}{1-x}} \Rightarrow y>0$

But $\dfrac{2}{1-x}\neq 0 \Rightarrow y\neq 1$

Hence range: $y>0, y\neq 1$

✓ - $y>0$
✓ - $y\neq 1$

(c) Solve $f(x)\geq 3-2x$.

[4]

$$\frac{1}{1-x}-3-2x\geq 0$$
$$\frac{1-(3-5x+2x^2)}{1-x}\geq 0$$
$$\frac{2x^2-5x+2}{x-1}\geq 0$$
$$\frac{(2x-1)(x-2)}{x-1}\geq 0$$

Critical points when $x=\frac{1}{2}, 1$ and 2

$\frac{1}{2}\leq x<1$ or $x\geq 2$

✓ - rearranges and factorises
✓ - critical points
✓ - first interval
✓ - second interval

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Question 6 **(9 marks)**

(a) Determine $\int x(3x^2+6x)^4+(3x^2+6x)^4\,dx$

[3]

$$\begin{aligned}
&=\int (x+1)(3x^2+6x)^4 dx\\
&=\frac{1}{6}\int (6x+6)(3x^2+6x)^4 dx\\
&=\frac{(3x^2+6x)^5}{6\times 5}+c\\
&=\frac{(3x^2+6x)^5}{30}+c
\end{aligned}$$

✓ - factorise
✓✓ - answer

(b) Calculate the area bounded by the functions $f(x)=(x-2)^2-3$ and $g(x)=2x-4$.

[6]

Solve $f(x)=g(x)$

$$\begin{aligned}
x^2-4x+4-3-2x+4&=0\\
x^2-6x+5&=0\\
(x-1)(x-5)&=0\\
x=1,\ x&=5
\end{aligned}$$

Integrate to find area

$$\int_1^5 g(x)-f(x)dx$$

$$=-\int_1^5 x^2-6x+5dx$$

$$=-\left[\frac{x^3}{3}-3x^2+5x\right]_1^5$$

$$=-\left(\frac{125}{3}-75+25\right)+\left(\frac{1}{3}-3+5\right)$$

$=10\frac{2}{3}$ square units

✓ - write equation to solve for intersection points
✓ - find correct intersection points
✓ - write integral as g(x) – f(x)
✓ - integrate correctly
✓ - substitute limits
✓ - correct value

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Question 7 **(7 marks)**

A closed cylindrical can of radius r cm has a volume of 250π cm^3.

(a) Show that the total surface area, A cm^2, of this can is given by $A = \dfrac{500\pi}{r} + 2\pi r^2$. [2]

$$V = \pi r^2 h$$
$$250\pi = \pi r^2 h \Rightarrow h = \frac{250}{r^2}$$
$$A = 2\pi r^2 + 2\pi r h$$
$$= 2\pi r^2 + 2\pi r \frac{250}{r^2}$$
$$= \frac{500\pi}{r} + 2\pi r^2$$

✓ - rearrange volume formula to make h the subject
✓ - substitute h into A equation (MUST show working for mark)

(b) Determine the minimum possible surface area of the can and the radius and height required to achieve this optimum area. [5]

$$\frac{dA}{dr} = -\frac{500\pi}{r^2} + 4\pi r$$
$$-\frac{500\pi}{r^2} + 4\pi r = 0$$
$$r^3 = 125 \Rightarrow r = 5 \text{ cm}$$
$$h = \frac{250}{5^2} = 10 \text{ cm}$$
$$A = \frac{500\pi}{5} + 2\pi \times 5^2$$
$$A = 150\pi$$

✓ - differentiate A
✓ - make dA/dr = 0
✓ - find value of r
✓- find value of h
✓ - find value of A

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End of Section One

Additional working space

Question number(s):_______________

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Additional working space

Question number(s):_______________

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