Year 12 – Physics
Motion and Forces in a Gravitational Field
Part 1 – Projectile Motion
Notes - 2013

Reviewed 27/10/2009

Outcomes / Objectives from Motion and Forces in a Gravitational Field Covered in this booklet

- *describe and apply the principle of conservation of energy*
- *resolve, add and subtract vectors in one plane*
- draw free body diagrams, showing the forces acting on objects, from descriptions of real life situations involving forces acting in one plane
- explain and apply the concept of centre of mass
- describe and apply the concepts of distance and displacement, speed and velocity, acceleration, energy and momentum in the context of motion in a plane, including the trajectories of projectiles in the absence of air resistance—this will include *applying the relationships*:

$$v_{av} = \frac{s}{t}, \quad v_{av} = \frac{v+u}{2}, \quad a = \frac{v-u}{t},$$

$$s = ut + \tfrac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

$$p = mv, \quad \sum p_{before} = \sum p_{after}, \quad F\Delta t = mv - mu$$

$$E_k = \tfrac{1}{2}mv^2, \quad E_p = mg\Delta h, \quad W = Fs, \quad W = \Delta E$$

- describe qualitatively the effects of air resistance on projectile motion

Context

Student unit learning contexts for **motion and forces in a gravitational field** may include**:**
- playground equipment
- physics in sport
- space travel
- planetary motion
- fairground physics
- bridges and buildings.

Texts
- Heinemann
- Stawa

Name	____

What are projectiles?

A projectile is any object that is in un-powered flight and has a negligible air resistance. Some examples of situations involving projectile motion are…

- Balls in Flight

E.g. netball, basketball, golf, cricket, shot put etc.

- Sports People in Flight

E.g. pole vault, diving, ballet, trampolining, ski jumping, sky diving

Some non-examples of projectiles are rockets, aircraft and parachutes because they use engines and wings etc, to alter their direction, range and height after they have taken off from the ground.

Activities

1. Draw the forces acting on a canon shell in mid flight

2. Draw the forces acting on a glider in mid flight

3. What is the only significant force acting on the canon shell?

4. What are the forces acting on the glider?

Rules

5. The only (significant) force acting on a projectile undergoing projectile motion is ____________________

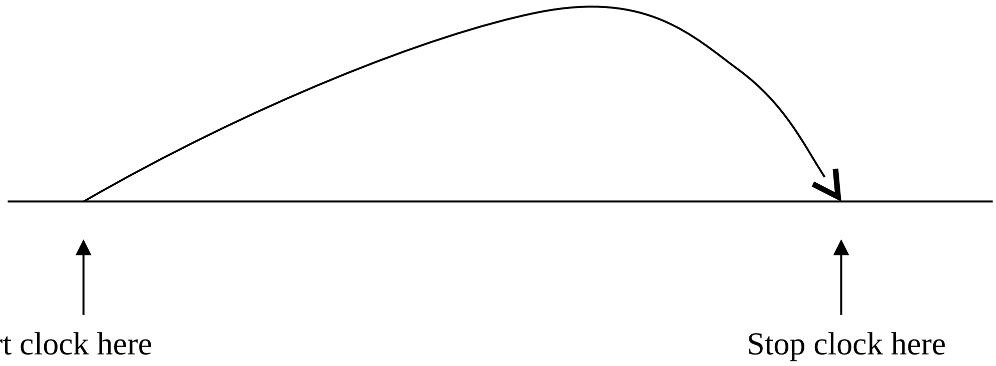
6. The forces (other than weight) that may act on non-projectiles are …
____

7. What makes a projectile different from any other flying object?
____
____

What do the terms time of flight, range and maximum height mean?

<u>Time of Flight</u>

The time of flight is the timing of how long the projectile (object) is in the air. The timing starts when the projectile leaves the ground, and finishes when the object reaches its destination.



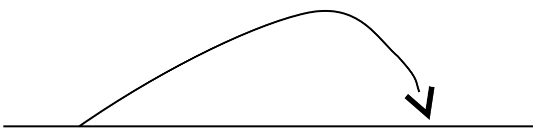
<u>Range</u>

The range is how far away from the launch position (start) that the projectile lands. The distance is measured along a horizontal line from the launch position. To be technical, it is the x – component of the projectiles final displacement.

<u>Activities</u>

a) Onto the below diagrams draw the range.

i)



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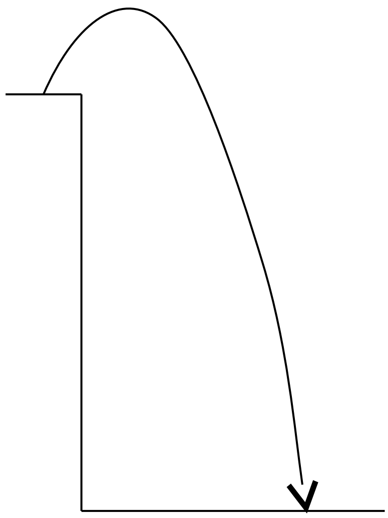

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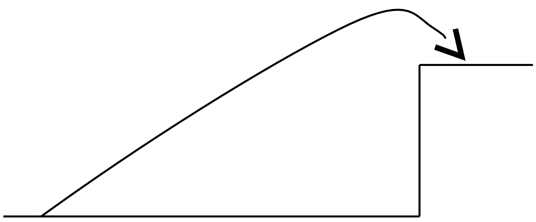


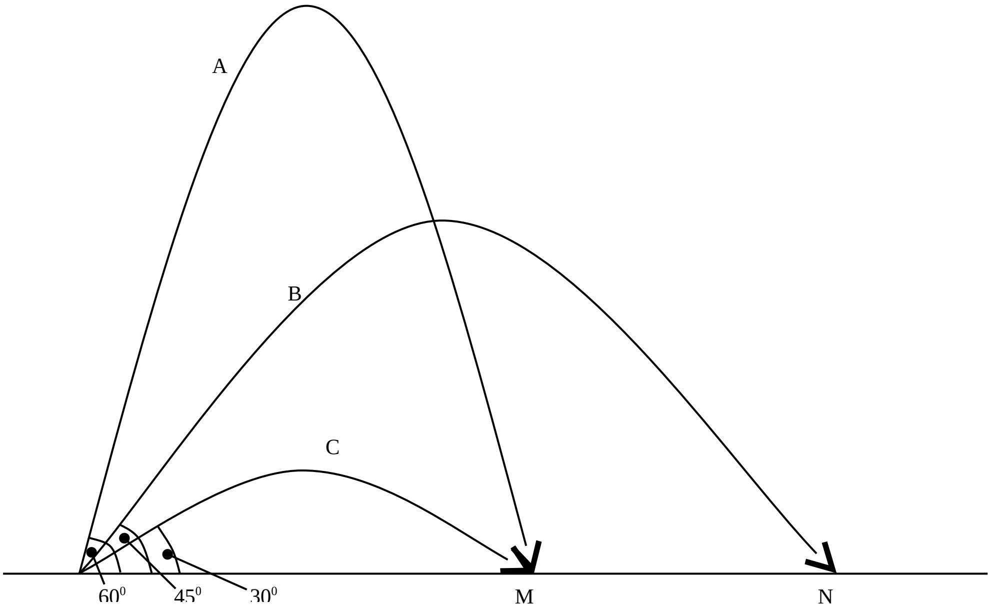
b) Using a different colour label what the range is <u>not</u>.

c) Draw a key (colour code) on the side of the diagrams indicating which is the correct definition of range and which is the incorrect definition of range.

Maximum Height

The maximum height is the greatest height above the launch position that the projectile (object) will reach on its path through the air. The distance is measured along a vertical line from the launch position. To be technical, it is the y – component of the projectiles highest displacement.

Activities



The following pathways are drawn ignoring air resistance and wind. Which pathway has the …

(i) largest maximum height. ________________________________

(ii) smallest maximum height. ________________________________

(iii) shortest range. ________________________________

(iv) longest range. ________________________________

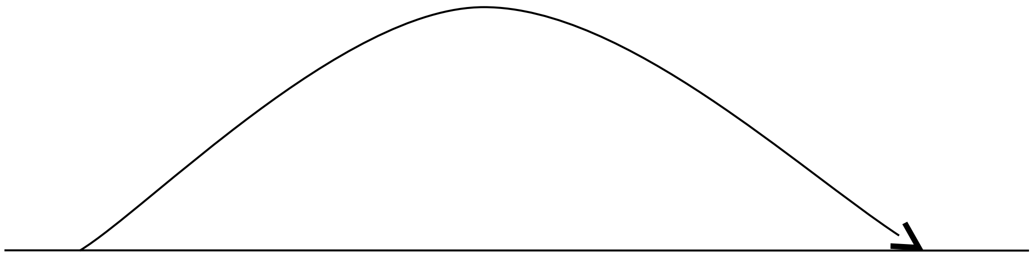
(v) For each of the questions (i - iv) above, write in the angle of projection next to your answer.

If you were sending a projectile to the letter M, which angle of projection would you use if…

(vi) M has many tall trees growing near it. ______________________

(vii) M is inside a long room with a ceiling. ______________________

What are the forces acting on a projectile …?



At the start?	In flight?	On landing?
mg	**mg**	**mg**
and	and	and
A getting started force to give the projectile an initial velocity E.g. a kick or explosion	Air resistance. This will be ignored when you are doing calculations.	A landing force where the ground exerts a force on the projectile to bring it to a stop

For which part(s) of the above journey do the equations of motion apply?

From the moment the take off force stops till just before landing. Highlight this on the above diagram.
i.e. the pathway but not the end points.
i.e. (…….) not […..]

What is the difference between a qualitative answer and a quantitative answer?

A <u>qualitative</u> answer is an answer that includes sentences, formulae, and diagrams but no calculations. When providing a qualitative answer to a projectiles motion question **you must include a discussion about air resistance.**

A <u>quantitative</u> answer is an answer that includes sentences, formulae, and diagrams and calculations. When providing a quantitative answer to a projectiles motion question **you can not include air resistance in you calculation (it is too complicated – requires calculus)**

Summary

Type of Q	**Calculation**	**Air resistance**
Qualitative		
Quantitative		

Activity

1. When I do projectile motion calculations do I have to include air resistance?

2. When I answer projectile motion descriptions do I have to include air resistance?

3. What does the word qualitative mean?

4. What does the word quantitative mean?

Do projectiles obey Newton's Laws of motion? (Yes)

1st and 2nd Law

1. What is Newton's First Law?

2. What is Newton's Second Law?

3. Why do projectiles start moving in the first place?

4. Why do projectiles follow a curved path and not a straight line?

5. Why do projectiles stop moving when they land?

3rd Law

What is Newton's Third law?

What are the action / reaction pair(s) involving projectiles near the surface of the earth?

Explain the action and reaction force at takeoff.

Explain the action and reaction force(s) in flight.

Explain the action and reaction force on landing.

What are the mathematical (calculate-able) patterns in projectile motion (ignore air resistance)?

1. Velocity in the horizontal is constant.
This means initial velocity in the horizontal direction = final velocity in the horizontal direction.

E.g. $v_x = u_x = \frac{s_x}{t}$

or

$v_h = v_h = \frac{s_h}{t}$

2. Velocity in the vertical is accelerated at 9.8 m/s^2 downwards.
This means from just after takeoff to just before landing the projectile is constantly accelerating downwards to the unbalanced weight force (mg) acting on the projectile.

e.g.	v_y	=	$u_y + a_y t$	or	v_v	=	$u_v + a_v t$
	v_y^2	=	$u_y^2 + 2a_y s_y$	or	v_v^2	=	$u_v^2 + 2a_v s_v$
	s_y	=	$u_y t + ½a_y t^2$	or	s_v	=	$u_v t + ½a_v t^2$

3. A projectile shot and landing at the same height will travel the furthest (greatest range) when shot at 45.0 ^{o} to the horizontal.

Hyperlink to Projectile Motion Spreadsheet Activity

Spread Sheet Questions – to be answered while observing changes in the spread sheet.

1. Confirm that the spacing of the points on the x axis is even, indicating that the velocity in the horizontal is constant.

2. Confirm that increasing the acceleration due to gravity decreases the max height.

3. Confirm that for a projectile taking off and landing at the same height, confirm that a 45.0 degree angle of projection produces the greatest range.

4. Confirm that for a projectile landing below its take off point, an angle below 45.0 ^{0} degrees gives the optimum range.

5. Confirm that for a projectile landing above its take off point, an angle above 45.0 ^{0} degrees gives the optimum range.

Why does air resistance effect light projectiles more than heavy projectiles?

The air resistance force experienced by a projectile depends most importantly on two key factors…

(i) Cross sectional area
(ii) Speed

The air resistance formula is …

$$F_{air} = k A v^2$$

Where …

Symbol	Definition	Units
F_{air}	Air resistance acting in the opposite direction to the motion of the object	N
k	Constant of proportionality	
A	Cross sectional area of the projectile. Relates to the amount of surface on the object facing the oncoming wind.	m^2
v	Speed of the projectile	$m\ s^{-2}$

The air resistance force causes the object to accelerate according to the formula …

$$F = ma$$

So, in summary, the retarding effects (deceleration) of air resistance on a projectile can be predicted using the formula…

$$ma = \ = k A v^2$$

or when re - arranged

$$a = k A v^2 / m$$

Activity

1. What happens to the acceleration due to air resistance on a projectile when …

Change	Effect on balls acceleration (due to air resistance)	Proof using formula
m increase		$m\ a = k A v^2$
v increase		$m\ a = k A v^2$
A increase		$m\ a = k A v^2$

2. What is the direction of the air resistance at any given instant in time?

Past exam Question

3a) Two balls of identical volume and surface area but very different mass are dropped from a height of 1000 m above the ground through the earth's atmosphere. Which ball hits the ground first? Explain why.

How does the pathway taken by a projectile with air resistance differ from a projectile without air resistance?

The best way to go about this is to draw the pathway of a projectile without air resistance, and then draw the pathway with air resistance on the same diagram (on the same axis). Then, in words, compare the 2 paths mentioning…

Shape \ Symmetry
Max Height
Range

P.S. Don't forget to label the two pathways and the forces acting at various points along the pathways.

Activity

1. Describe qualitatively the effect of air resistance on projectile motion

	With air resistance	Without air resistance
Range		
Max height		
Time of flight		
Symmetry		

Explanation

2 Compare the flight path taken by a projectile on the earth with that taken by a projectile on a planet with 1/3 the gravity of earth. Assume this planet has no atmosphere and that the earth does.

Past Exam Question

3. A large beach ball and a tennis ball have the same mass but very different volumes and surface areas. In the space provided below qualitatively show the pathway (trajectory) taken by each ball. Assume that both balls are launched at identical angles and identical speeds from ground level.

(1 mark)

b) Show the approximate size and direction of the individual forces acting on the balls on your diagrams drawn above…

i) just <u>after</u> launch.

ii) at their maximum height.

(2 marks)

c) Explain why the different balls have different ranges.

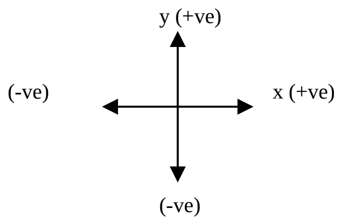
(1 mark)

What is the sign convention that we use when doing projectile motion questions?

- The start point is the origin
- Up and right are positive +ve. Down and left are negative (-ve).

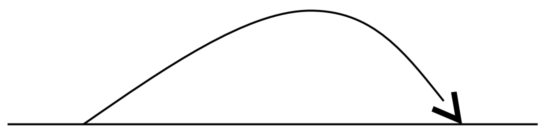
Activity For each of the above projectile diagrams…

a) Draw a co-ordinate axes at the take of site (origin) like so…

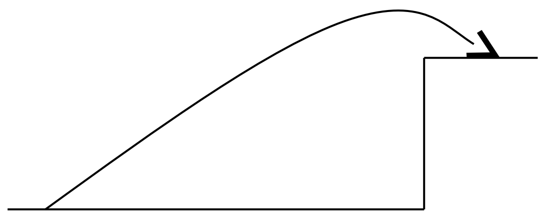


b) Predict whether each variable listed will be +ve, -ve or zero.

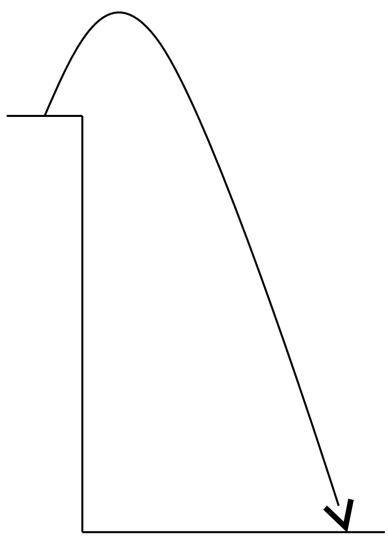
(i)	s_v	=	s_h	=
	u_v	=	$u_h = v_h$	=
	v_v	=	a_h	=
	a_v	=	t	=



(ii)	s_v	=	s_h	=
	u_v	=	$u_h = v_h$	=
	v_v	=	a_h	=
	a_v	=	t	=



(iii)	s_v	=	s_h	=
	u_v	=	$u_h = v_h$	=
	v_v	=	a_h	=
	a_v	=	t	=



What are the steps to solving a projectile motion calculation?

Step 1 Read question carefully (at least twice) and underline key words / numbers.

Step 2 Draw / Sketch a diagram of the situation showing flight path.

Step 3 Mark onto the diagram the information provided in the question

Step 4 Draw the co-ordinate axis at the takeoff point.

Step 5 Divide page below the diagram in half by drawing a vertical line down the page. Label the left column "vertical" and the right column "horizontal".

Step 6 Write down vertical information in the vertical column next to algebraic letters. Do the same for the horizontal information in the horizontal column.

Step 7 Write vertical formula with subscripts in the vertical column. Do the same for the horizontal information in the horizontal column.

Step 8 Select the appropriate formula(e) to use.

Step 9 Solve.

Step 10 Update the diagram with each answer as you calculate it because this is the best way of keeping track of your answers.

Note – DO NOT SPLIT THE PROJECTILE MOTION **PATHWAY** INTO PARTS AND CALCULATE EACH PART SEPARATELY.

Try the activities that follow with the help of your teacher.

Activity - Try these

(Start and finish same height)

1. A ball is kicked at an angle of 30.0 ^{0} to the horizontal with an initial velocity of 18.0 m/s. The ball flies through the air and lands on the ground some distance away at the same height from which it was kicked.

a) Draw a diagram of the situation below.

b) What is the initial velocity of the ball in the vertical and horizontal?

(u_v = 9.00 m/s, u_h = 15.6 m/s)

c) What is the maximum height of the ball?

(s_{max} = 4.13 m)

d) What is the velocity of the ball when it returns to the ground?

(18.0 m/s at 30.0 ^{0} below the horizontal)

e) What is the velocity of the ball at its maximum height?

(15.6 m/s horizontally)

f) What is the final range of the ball?

(28.7 m to the right)

g) At what times is the ball 3 m above the ground?

($t = 0.438$ s and 1.40 s)

h) If the ball was kicked at the same speed but at an angle of $40.0\ ^0$ to the horizontal, would its final range be greater of smaller?

(The range at $40.0\ ^0$ is greater. The new range is 32.5 m)

(Finish below where it started)

2. A rock is thrown of a 22.0 m tall cliff with an initial velocity of 4.00 m/s and an angle of projection of 40.0^0 to the vertical.

a) Draw a diagram of the situation.

b) What is the final velocity of the rock just before it strikes the ground?

(21.1 m/s down 6.98^0 to the right)

c) What is the range of the rock?

(6.30 m to the right)

d) What is the maximum height of the rock …

(i) Above the level from which it was thrown.
(ii) Above the level on which it lands.

(0.478 m above takeoff point, 22.478 above landing point)

e) At what time is the rock at a height of 5 m below its starting height?

(t = 1.37 s)

f) Will an angle above, below or equal to 45.0 ^{0} produce the maximum range for the rock of the initial speed of throwing is not changed?

(below)

(Finish above where it starts)

3. A student is watering a hanging basket using a stream of water from a garden hose. The water leaves the hose with a velocity of 5.00 m/s at an angle of 75.0^0 to the horizontal. The hose nozzle is 0.900 m from the ground and the hanging basket is 1.70 m form the ground.

a) Draw a diagram of the situation.

b) What is the maximum height of the stream of water?

(1.19 m above nozzle, 2.09 m above the ground)

c) At what time is that maximum height achieved after leaving the nozzle?

(t = 0.493 s)

d) At what time does the water enter the basket?

(t = 0.775 s)

e) What is the final velocity of the water as it enters the basket?

(3.05 m/s Right 64.9 ^{0} Down)

f) The water entering the hanging basket with speed is causing the dirt in the basket to splash out and create a mess. If the gardener cannot alter the initial velocity of the water, what can the gardener to reduce the velocity at which the water enters the basket?

(Lower the nozzle closer to the ground to increase the distance between the nozzle and the basket)

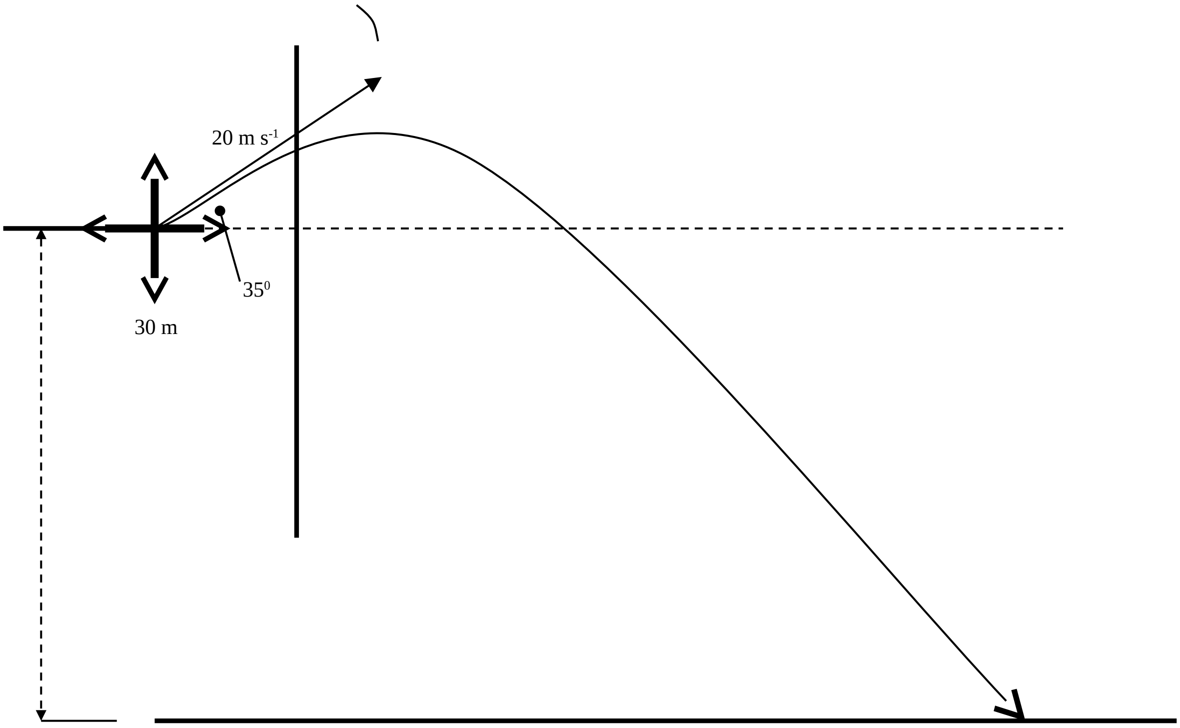
The law of conservation of energy can also be used to solve some projectile motion questions. In fact $v^2 = u^2 + 2as$ is actually the conservation of energy formula in disguise.

The law of conservation of energy as applied to these questions is of the form …

$$E_k + E_p = E_T = E_k + E_p$$

A stone is kicked off a cliff at an angle of 35^0 above the horizontal with a velocity of 20 m s^{-1}. The cliff's height is 30 m and the stone's mass is 300 g.

Diagram



a) What is the initial kinetic energy of the ball as it leaves the top of the cliff?

E_k = 60.0 J

b) What is the total energy of the stone as it leaves the top of the cliff?

E_T = 148.2 J

c) What is the final speed of the stone as it strikes the ground?

Speed = 31.4 m/s

d) What is the final velocity of the stone as it strikes the ground?

Velocity = 31.4 m/s Right 58.6 ^{0} Down

e) At what height above the ground is the kinetic energy of the stone twice that at take off?

h = 9.59 m

What Do We Do Now? (W^2D^2N)

Heinemann 3AB – Book – Chapter 1.1 – Projectile Motion p26 - 31.

Heinemann 3AB – Book - Chapter 1.1 – Questions p32.

Stawa = Set 2 – Page ____ – ___.
