Supplementary page

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Semester Two Examination, 2020

Question/Answer Booklet

MATHEMATICS METHODS

ATAR Year 12
Section One: Calculator-free

Student Name: _____

Please circle your teacher's name

Teacher: Miss Long Miss Rowden Ms Stone

Time allowed for this paper

Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Materials required/recommended for this paper

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

Number of additional answer booklets used (if applicable): ☐

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Suggested working time (minutes)	Marks available	Percentage of examination
Section One: Calculator free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the ATAR course examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Supplementary pages for the use planning/continuing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

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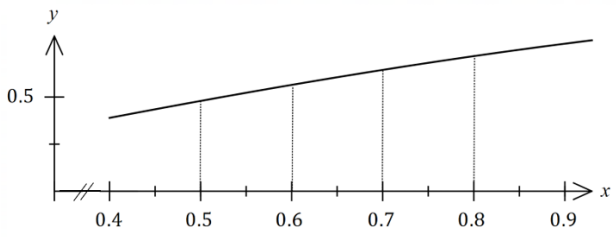
Question 8 **(7 marks)**

The graph and a table of values for $y=f(x)$ is shown below, where $f(x)=\sin x$.



x	y
0.4	0.39
0.5	0.48
0.6	0.56
0.7	0.64
0.8	0.72
0.9	0.78

Let $I=\int_{0.5}^{0.8} \sin x\,dx$.

(a) By using the information shown and considering sums of the form $\sum_i f(x_i)\delta x_i$ explain why $I<0.192$.

(3 marks)

(b) In a similar manner to (a), determine the lower estimate, L, for the value of I. That is, the value of L for which $I>L$.

(2 marks)

(c) Use your previous answers to determine a numerical estimate for I and explain whether your estimate is smaller or larger than the exact value of I. (2 marks)

End of questions

Section One: Calculator-free **35% (52 Marks)**

This section has eight (8) questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 **(7 marks)**

(a) Determine an expression for $f'(x)$ when

(i) $f(x)=\ln(1-\cos 3x)$. (2 marks)

(ii) $f(x)=e^{5x}(5-2x)^3$. (3 marks)

(b) For the positive number x, let $A(x)=\int_0^x (8-2^{t^2})\,dt$.

Determine the value(s) of x for which $\frac{dA}{dx}=0$. (2 marks)

See next page

Question 2 **(7 marks)**

The discrete random variable X is defined by

$$P(X=x)=\begin{cases}\frac{x+b}{5x+2} & x=0,1\\ \text{elsewhere} & \end{cases}$$

(a) Determine the value of the constant b. (2 marks)

(b) Determine

(i) $P(X=0)$. (1 mark)

(ii) $E(7X-3)$. (2 marks)

(iii) $Var(7X-3)$. (2 marks)

See next page

Question 7 **(6 marks)**

The acceleration at time t seconds of a small body travelling in a straight line is given by

$$a(t)=\frac{-27}{\sqrt{3t+1}}\text{cm/s}^2, t\geq 0.$$

When $t=1$ the body was at the origin and 7 seconds later its displacement was 22 cm.

Determine the velocity of the body when $t=5$.

See next page

Question 6 **(7 marks)**

(a) Determine the value of $3\log 20 - \log 4 + \log 5$. (2 marks)

(b) Given that $\log_a x = 0.82$, determine the value of $\log_a\left(\frac{\sqrt{x}}{x}\right)$. (2 marks)

(c) Determine the solution to the equation $2^{3x} = 5^{2-x}$ in the form $x = \frac{\log a}{\log b}$. (3 marks)

See next page

Question 3 **(5 marks)**

The rate of change of pressure in an air tank is given by $P'(t) = -3e^{-0.05t}$, where t is the time in minutes since it began emptying from an initial pressure of 70 psi.

(a) Determine an expression for the pressure P in the tank at any time t, $t \geq 0$. (2 marks)

(b) Determine

(i) the time taken for the pressure in the tank to fall to 40 psi. (2 marks)

(ii) the minimum pressure in the tank for $t \geq 0$. (1 mark)

See next page

Question 4 **(6 marks)**

The continuous random variable X takes values in the interval 3 to 8 and has **cumulative** distribution function $F(x)$ where

$$F(x)=P(X\leq x)=\begin{cases}0 & x<3\\ \frac{x-3}{5} & 3\leq x\leq 8\\ 1 & x>8.\end{cases}$$

(a) Determine

(i) $P(X\leq 4.5)$. (1 mark)

(ii) the value of k, if $P(X>k)=0.75$. (2 marks)

(b) Determine $f(x)$, the probability density function of X, and sketch the graph of $y=f(x)$. (3 marks)

Question 5 **(7 marks)**

The function f is defined by $f(x)=\frac{x^2-7}{4-x}$, $x\neq 4$.

The second derivative of f is $f''(x)=18(4-x)^{-3}$.

Determine the coordinates and nature of all stationary points of the graph of $y=f(x)$.