Christ Church Grammar School

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

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7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1 (6 marks)

(a) The function $g(x)$ has derivative $g'(x) = \sin(2\pi x)$.
Given that $g(1) = \frac{1}{\pi}$, find $g(x)$. (3 marks)

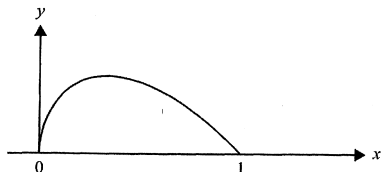(b) If $\int_1^4 f(x)\,dx = 6$, then find the value of $\int_1^4 (5 - 2f(x))\,dx$. (3 marks)

Question 2   **(4 marks)**

The graph of $f(x) = \sqrt{x}(1-x)$ for $0 \le x \le 1$ is shown below.

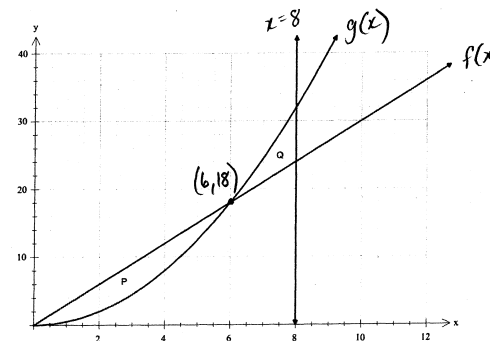Calculate the area between the graph of $f(x)$ and the $x - axis$.

Question 7   **(5 marks)**

The graph below shows the functions $f(x) = 3x$ and $g(x) = \dfrac{x^2}{2}$ and the line $x = 8$.

Region P is the area enclosed between f and g.

Region Q is the area enclosed by f, g and $x = 8$.

(a)   Determine the areas of P and Q.   (2 marks)

$$Area \quad P = 18u^2 \quad \checkmark$$

$$Area \quad Q = 7\tfrac{1}{3}u^2 \quad \checkmark$$

(b) $f(x)$ is re-defined such that $f(x) = ax$ and the area of Region P is half the area of Region Q. Calculate the value of a that makes this statement true.   (3 marks)

$$ax = \frac{x^2}{2} \quad (Intersection \ g(x) \ \& f(x)) \ \Rightarrow \ x = 2a \quad \checkmark (x\text{-value})$$

Need 'a' such that

$$2\int_0^{2a} ax - \frac{x^2}{2} \ dx = \int_{2a}^8 \frac{x^2}{2} - ax \ dx \quad \checkmark \ (Equates \ areas)$$

$$ClassPad \qquad a = 2.384 \quad \checkmark \quad (a\text{-value})$$
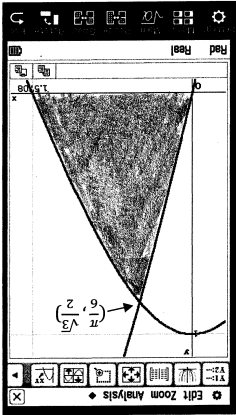
End of questions

5

Question 3 (5 marks)

The graphs of $f(x) = \cos(x)$ and $g(x) = \sqrt{3}\sin(x)$, in the first quadrant, are displayed to the right.

Find the shaded area bounded by the $x-axis$, $f(x)$ and $g(x)$.

Express your answer in its simplest form.

$\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$

End of questions

Question 6 (8 marks)

A particle moves along the $x-axis$ so that its acceleration $a(t)$ at any time t is given by $a(t) = 6t - 12m/s^2$. At time $t = 0$ the particle is instantaneously at rest at the point $x = 1$.

(a) Write formulae for the velocity $v(t)$ and the displacement $x(t)$ of the particle, t seconds from the origin. (2 marks)

$v(t) = 3t^2 - 12t$ As $t=0$ $\checkmark v(t)$

$x(t) = t^3 - 6t^2 + 1$ As $t=1$ $\checkmark a(t)$

(b) Determine when and where the particle is again instantaneously at rest. (2 marks)

when $v(t) = 0$

$3t(t-4) = 0$

$t = 4$ at rest $\checkmark$ (time)

$x(4) = -31m$ $\checkmark$ (place)

(c) Determine the speed of the particle at $t = 2$. (2 marks)

$|v(t)| = 3(2)^2 - 12(2)$

$= 12 - 24$

$= |-12|$ $\checkmark$ velocity

$= 12\, m/s$ $\checkmark$ speed

(d) Determine the distance travelled by the particle between the times $t = 0$ and $t = 8$. (2 marks)

$\int_0^8 |3t^2 - 12t|\, dt = 192m$ $\checkmark$ integral $\checkmark$ answer

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Additional working space

Question number: _____

Question 5 (5 marks)

Consider the functions $f(x) = \frac{1}{2}(x^2 - 5x)\sqrt{x}$ and $g(x) = -3\sqrt{x}$. Points O $(0,0)$, A and B are the points of intersection of the graphs of the functions f and g.

(a) Determine the coordinates of the points A and B, correct to 2 decimal places.

(2 marks)

Class Pad $A\left(2, -4.24\right)$ ✓

$B\left(3, -5.20\right)$ ✓

(b) Use a definite integral to write an expression for the area enclosed by the graphs of the functions f and g and evaluate this integral, correct to 2 decimal places.
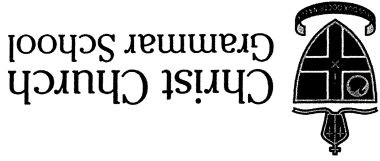
(3 marks)

$$\int_0^2 f(x) - g(x)\, dx + \int_2^3 g(x) - f(x)\, dx$$

$\left(1^{st}\ part\right)$ ✓ $\left(2^{nd}\ part\right)$ ✓

OR $\int_0^3 \left|g(x) - f(x)\right| dx$

$= 1.616244 + 0.131629$

$= 1.75\ u^2$ (to 2dp) ✓

$\left[\begin{array}{l}-1\ \text{overall if} \\ \text{not to 2dp}\end{array}\right]$

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Christ Church Grammar School

2020   TEST 2

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section
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Working time for this section: 30 minutes
Marks available: 25 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
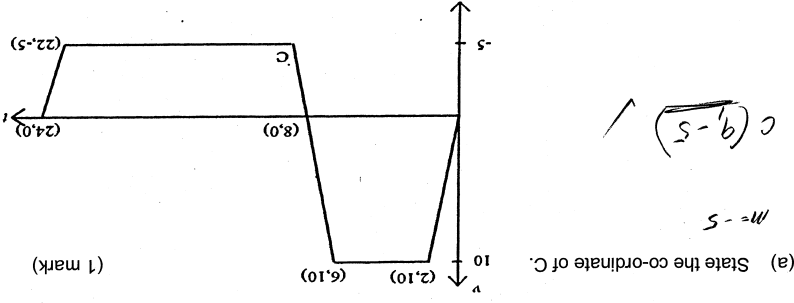Formula Sheet (retained from Section One)

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Question 4   (7 marks)

The graph below shows the velocity v (m/s) of a particle graphed against t (seconds) initially at the origin.

(a)   State the co-ordinate of C.   (1 mark)

$m = -5$

$C(9, -5)$ ✓

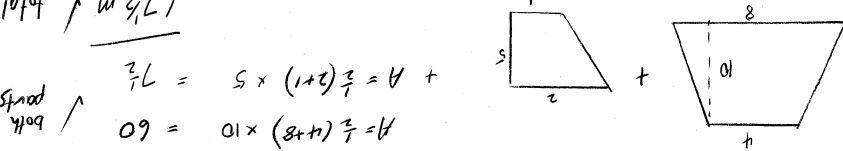(b)   State the acceleration at $t = 3$.   (1 mark)

$a = 0\ m/s^2$ ✓

(c)   Determine the acceleration during $0 \le t \le 2$.   (1 mark)

$a(t) = 5\ m/s^2$ ✓

(d)   Determine the distance travelled in the first 10 seconds.   (2 marks)

$A = \frac{1}{2}(4+8) \times 10 = 60$ } both parts ✓

$+$ $A = \frac{1}{2}(2+1) \times 5 = 7\frac{1}{2}$

$67\frac{1}{2}\ m$ ✓ $= |a|$

(e)   Determine the time when the particle returns to its starting point?   (2 marks)

Area Above = Area below. ✓   (equate areas)
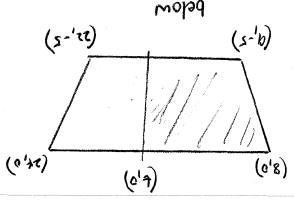
$60 = \frac{1}{2}[(t-8)+(t-9)] \times 5$

$60 = \frac{1}{2}(2t-17) \times 5$

$t = 20.5\ sec.$ (time) ✓

See next page

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Christ Church Grammar School

2020
TEST 2

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name   • *Solutions* •

Teacher's name

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 25 marks

Materials required/recommended for this section

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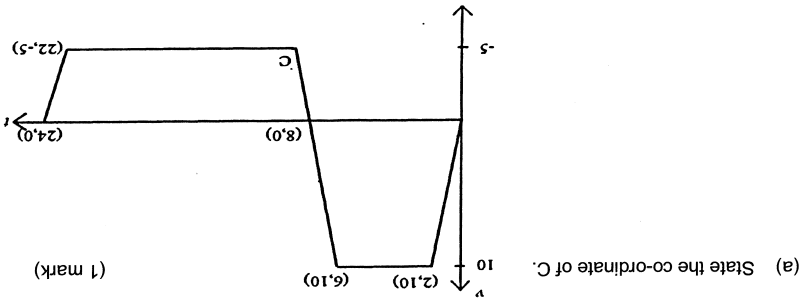Formula Sheet (retained from Section One)

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(a)   State the co-ordinate of C.   (1 mark)

(b)   State the acceleration at $t = 3$.   (1 mark)

(c)   Determine the acceleration during $0 \le t \le 2$.   (1 mark)

(d)   Determine the distance travelled in the first 10 seconds.   (2 marks)

(e)   Determine the time when the particle returns to its starting point?   (2 marks)

Question 5   **(5 marks)**

Consider the functions $f(x) = \frac{1}{2}(x^2 - 5x)\sqrt{x}$ and $g(x) = -3\sqrt{x}$. Points O $(0,0)$, A and B are the points of intersection of the graphs of the functions f and g.

(a)   Determine the coordinates of the points A and B, correct to 2 decimal places.

                                                              (2 marks)

(b)   Use a definite integral to write an expression for the area enclosed by the graphs of the functions f and g and evaluate this integral, correct to 2 decimal places.

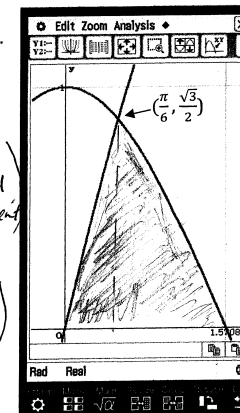   **(3 marks)**

Question 3   **(5 marks)**

The graphs of $f(x) = \cos(x)$ and $g(x) = \sqrt{3}\sin(x)$, in the first quadrant, are displayed to the right.

Find the shaded area bounded by the $x - axis$, $f(x)$ and $g(x)$.

Express your answer in its simplest form.

$\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$

$$\sqrt{3}\int_0^{\frac{\pi}{6}} \sin(x)\, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(x)\, dx \qquad \checkmark \left(\begin{array}{l}\text{correct} \\ \text{integral} \\ \text{statement}\end{array}\right)$$

$$= \sqrt{3}\left[-\cos(x)\right]_0^{\frac{\pi}{6}} + \left[\sin(x)\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \qquad \checkmark \left(\begin{array}{l}\text{Integrates} \\ \text{correctly}\end{array}\right)$$

$$= \sqrt{3}\left[-\cos\frac{\pi}{6} - \left(-\cos 0\right)\right] + \left[\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right]$$
$$\phantom{= \sqrt{3}\left[-\cos\frac{\pi}{6}\right]}{}_{1 - \frac{1}{2}}$$

$$= \sqrt{3}\left(-\frac{\sqrt{3}}{2} + 1\right) + \left(\frac{1}{2}\right) \qquad \checkmark\checkmark \left(\begin{array}{l}\text{uses exact values} \\ \text{in each bracket}\end{array}\right)$$

$$= -\frac{3}{2} + \sqrt{3} + \frac{1}{2}$$

$$= \underline{\sqrt{3} - 1 \quad \text{units}^2} \qquad \checkmark \left(\text{simplest form}\right)$$

• End of questions •

5

Question 6   (8 marks)

A particle moves along the $x-axis$ so that its acceleration $a(t)$ at any time t is given by $a(t) = 6t - 12 m/s^2$. At time $t = 0$ the particle is instantaneously at rest at the point $x = 1$.

(a) Write formulae for the velocity $v(t)$ and the displacement $x(t)$ of the particle, t seconds from the origin.   (2 marks)

(b) Determine when and where the particle is again instantaneously at rest.   (2 marks)

(c) Determine the speed of the particle at $t = 2$.   (2 marks)

(d) Determine the distance travelled by the particle between the times $t = 0$ and $t = 8$.   (2 marks)

Question 2   (4 marks)

The graph of $f(x) = \sqrt{x}(1 - x)$ for $0 \leq x \leq 1$ is shown below.

Calculate the area between the graph of f and the $x-axis$.
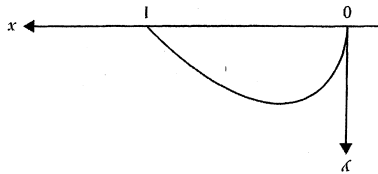
$$A = \int_0^1 \sqrt{x}\,(1-x)\,dx$$

$$A = \int_0^1 x^{1/2} - x^{3/2}\,dx \quad \checkmark \quad (\text{re-write integral})$$

$$= \left[\frac{2}{3} x^{3/2} - \frac{2}{5} x^{5/2}\right]_0^1 \quad \checkmark \quad (\textstyle\int \text{ correctly})$$

$$= \frac{2}{3} - \frac{2}{5} \quad \checkmark \quad (\text{subs in correctly})$$

$$= \frac{4}{15} \text{ units}^2 \quad \checkmark \quad (\text{Answer})$$

Question 7   **(5 marks)**

The graph below shows the functions $f(x) = 3x$ and $g(x) = \dfrac{x^2}{2}$ and the line $x = 8$.
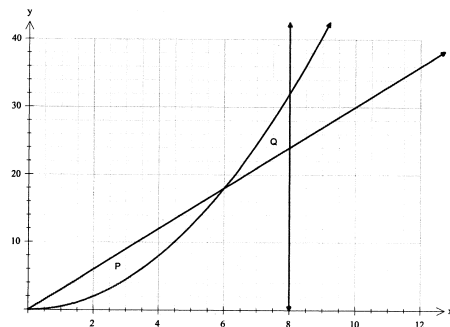
Region P is the area enclosed between f and g.

Region Q is the area enclosed by f, g and $x = 8$.

(a)   Determine the areas of P and Q.   **(2 marks)**

(b) $f(x)$ is re-defined such that $f(x) = ax$ and the area of Region P is half the area of Region Q. Calculate the value of a that makes this statement true.   **(3 marks)**

End of questions

Question 1   **(6 marks)**

(a)   The function with rule $g(x)$ has derivative $g'(x) = \sin(2\pi x)$.
Given that $g(1) = \frac{1}{\pi}$, find $g(x)$.   **(3 marks)**

$$g(x) = -\frac{1}{2\pi}\cos(2\pi x) + c \qquad \checkmark \quad (\text{Anti-diff})$$

$$g(1) = -\frac{1}{2\pi}\cos(2\pi) + c$$

$$\therefore \quad -\frac{1}{2\pi}(1) + c = \frac{1}{\pi}$$

$$c = \frac{3}{2\pi} \qquad \checkmark \quad (c\text{-value})$$

$$\therefore \quad \boxed{g(x) = -\frac{1}{2\pi}\cos(2\pi x) + \frac{3}{2\pi}} \quad \checkmark \quad \left(\begin{array}{c} g(x) \\ \text{with 'c'} \end{array}\right)$$

(b)   If $\int_1^4 f(x)\,dx = 6$, then find the value of $\int_1^4 (5 - 2f(x))\,dx$.   **(3 marks)**

$$\int_1^4 5 - 2f(x)\,dx = \int_1^4 5\,dx - 2\int_1^4 f(x)\,dx$$

$$\checkmark \;(\text{re-write} \int)$$

$$= \Big[5x\Big]_1^4 - 2 \times 6 \qquad \checkmark \quad (\text{Subs correctly})$$

$$= 20 - 5 - 12$$

$$\boxed{= 3} \qquad \checkmark \quad (\text{Answ})$$

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See next page

Additional working space

Question number: _____

Additional working space

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Christ Church Grammar School

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

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Teacher's name_____

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