

Note: All part questions worth more than 2 marks require working to obtain full marks.

Formula sheet provided: Yes

Task weighting: 10%

Marks available: 43 marks

Special items: Drawing instruments, templates, one page of A4 notes double-sided

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Materials required: Up to 3 calculators/classpads allowed

Number of questions: 6

Time allowed for this task: 40 mins

Task type: Response

Student name: _____ Teacher name: _____

Course Methods Year 12 test three 2022

PERTH MODERN SCHOOL



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Q1 (3, 3 & 2 = 8 marks)

Consider the discrete random variable X and the table of probabilities below.

X	0	1	2	3	4
$P(X = x)$	0.2	a	0.3	b	0.25

- a) Given that the expected value of X is 2.15, determine the values of a & b .

Solution

Specific behaviours

- ✓ sets up one equation for a & b
- ✓ sets up two equations for a & b
- ✓ solves for a & b

- b) Determine the standard deviation of X to 3 dp, showing all reasoning.

Solution

$$V(x) = (0 - 2.15)^2 \cdot 0.2 + (1 - 2.15)^2 \cdot 0.1 + (2 - 2.15)^2 \cdot 0.3 + (3 - 2.15)^2 \cdot 0.2 + (4 - 2.15)^2 \cdot 0.25 \\ = 2.028$$

$$std = 1.424$$

Specific behaviours

- ✓ shows a sequence for variance
- ✓ determines variance
- ✓ determines std to 3 dp

- c) Determine the $E(3X + 4)$ and $\text{Variance}(3X + 4)$.

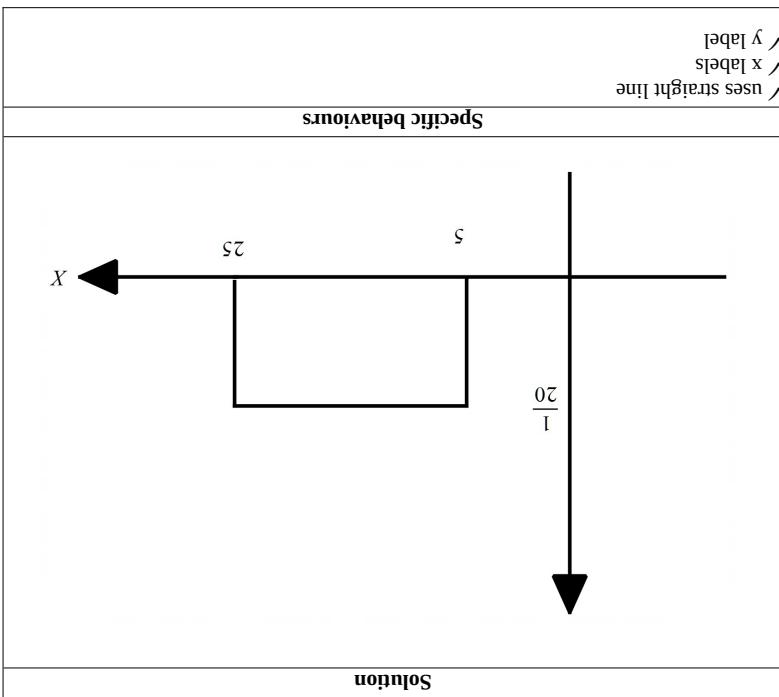
Solution

$$E(3X + 4) = 10.45$$

$$\text{Variance}(3X + 4) = 18.2975$$

Solution

- Q2 continued
 b) Determine the probability that the train will be less than 12 mins late given that it is at least 7 mins late.



- The number of X minutes late a train arrives at a particular station is a uniform probability distribution from 5 mins to 25 mins.
 a) Sketch the probability density function for X showing all relevant features and labels.

Q2 (3, 2, 1 & 3 = 9 marks)

Specific behaviours

The calculator screen shows the following steps:

$$\frac{(12-7)\frac{1}{20}}{(25-7)\frac{1}{20}}$$

$$\frac{5}{18}$$

$$\frac{5}{18}$$

$$0.277777778$$

At the top, the menu bar includes "Edit", "Action", "Interactive", and various function keys like $\frac{1}{2}$, $\int \! dx$, $\int \! dx \leftarrow$, Simp , $\frac{d}{dx}$, and $\frac{d}{dx} \leftarrow$.

Specific behaviours

- correct denominator
- correct numerator

c) Determine the mean number of minutes late.

Solution
Mean = 15 mins
Specific behaviours
<input checked="" type="checkbox"/> states midpoint

d) Determine the standard deviation of X showing all reasoning.

Solution

- Q3 (3, 3 & 2 = 8 marks)
- Consider a game where two ordinary dice are thrown into the air and then land and the sum of the two top numbers is added. If the sum is a prime number (2, 3, 5, 7,...) etc then this is considered a win.
- a) Determine the probability of a win. Show reasoning.

Specific behaviours

- ✓ sets up integral
- ✓ determines variance
- ✓ determines standard deviation
- ✓ determines stdev

5. 773502692

$$\int_{-5}^5 (x-15)^2 \cdot 20 dx$$

$$\frac{3}{10 \cdot \sqrt{3}}$$

$$\frac{3}{100}$$

$$\frac{\sqrt{3}}{100}$$

✓ evaluates $x=0$ and $x=1$ showing both values for two terms after integration
✓ changes factor to give required definite integral (no need to simplify)

Solution																																																							
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Prime numbers 2,3,5,7,11																																																							
Pr(prime)=15/36																																																							
Specific behaviours																																																							
<ul style="list-style-type: none"> ✓ shows sample space ✓ number of favourable outcomes shown ✓ states probability 																																																							

- b) If this game was played 10 times, determine the probability that a win occurs at least 7 times.
Show all reasoning.

Solution						
$X \sim B(10, \frac{15}{36})$						
$P(X \geq 7)$						
Specific behaviours						
<ul style="list-style-type: none"> ✓ states Binomial with parameters ✓ uses cumulative expression ✓ states prob 						

- a) Let X = sum of the top numbers of both dice. Determine the mean and standard deviation for X .

a) Show without the use of a classpad how to $\frac{d}{dx} [5x \ln(3x+1)]$

Solution	
$\frac{d}{dx} [5x \ln(3x+1)] = 5x \frac{3}{3x+1} + 5 \ln(3x+1)$	
Specific behaviours	
<ul style="list-style-type: none"> ✓ uses product rule ✓ obtains correct expression 	

- b) Using (a) above and without the use of a classpad, show how to evaluate $\int 3 \ln(3x+1) dx$

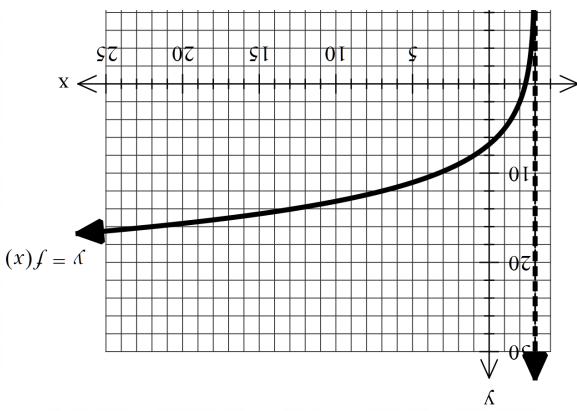
Hint-use $\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$

Solution	
$\frac{d}{dx} [5x \ln(3x+1)] = 5x \frac{3}{3x+1} + 5 \ln(3x+1)$	
$\int \frac{d}{dx} [5x \ln(3x+1)] dx = 5 \int \frac{3x}{3x+1} dx + 5 \int \ln(3x+1) dx$	
$[5x \ln(3x+1)]_0^1 = 5 \left[1 - \frac{1}{3x+1} \right]_0^1 + 5 \int \ln(3x+1) dx$	
$5 \ln 4 = 5 \left[x - \frac{1}{3} \ln(3x+1) \right]_0^1 + 5 \int \ln(3x+1) dx$	
$5 \ln 4 = 5(1 - \frac{1}{3} \ln 4) + 5 \int \ln(3x+1) dx$	
$3 \ln 4 = 3(1 - \frac{1}{3} \ln 4) + 3 \int \ln(3x+1) dx$	
$3 \int \ln(3x+1) dx = 3 \ln 4 - 3 + \ln 4 = 4 \ln 4 - 3$	
Note- zero marks if answer given without any working!	
Specific behaviours	
<ul style="list-style-type: none"> ✓ integrates expression from part a ✓ uses FTC ✓ changes $\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$ and shows integration of these two terms (or uses $u=3x+1$) 	

The figure shows a TI-Nspire CX handheld calculator displaying a statistical analysis for a single variable. The data list, labeled 'List1', contains 12 entries ranging from 0.0278 to 0.1667. The calculator has calculated several statistics:

- Mean (\bar{x}): 0.0833
- Standard Deviation (s_x): 0.0556
- Sum of Squares (Σx^2): 54.83333
- Sum (Σx): 2.4152295

The top right corner of the screen shows the text "states mean of 7" and "states stdv of 2.415". The bottom of the screen features a menu bar with options like Edit, Calc, SetGraph, and Solution.



Q44 (5 marks) Consider $f(x) = r \log_5(x + p) + q$ where $r, p \neq q$ are constants.

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Q6 (2 & 5 = 7 marks)

c) Determine the median.

The figure shows a mobile application interface for solving mathematical problems. The main screen displays a problem involving definite integration:

$$\int_{-3}^2 x^3 (x-5) dx$$

Below the problem, there is a toolbar with various mathematical operations and functions:

- Left arrow (\leftarrow)
- Right arrow (\rightarrow)
- Up arrow (\uparrow)
- Down arrow (\downarrow)
- Integral symbol (\int)
- Derivative symbol ($\frac{dy}{dx}$)
- Simplify (Simp)
- Definite integral (\int_a^b)
- Sum (\sum)
- Product (\prod)
- Quotient ($\frac{\cdot}{\cdot}$)
- Root ($\sqrt{\cdot}$)
- Exponent ($\cdot^{\frac{1}{2}}$)
- Logarithm ($\ln \cdot$)

At the bottom of the screen, there is a navigation bar with the following labels:

- Edit
- Action
- Interactive

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Using the graph above and given that the following points
 $(22, 16)$ & $(2, 9)$ lie on the curve $y = f(x)$, determine the values of r, p & q .

Solution
$f(x) = r \log_5(x + p) + q$
Asymptote $x = -3$, $p = 3$
$(2, 9)$
$9 = r \log_5(5) + q = r + q$
$(22, 16)$
$16 = r \log_5(25) + q = 2r + q$
$16 - 9 = r$
$r = 7$
$16 = 14 + q$
$q = 2$
Specific behaviours
<ul style="list-style-type: none"> ✓ identifies asymptote at $x = -3$ ✓ determines p ✓ sets up one equation with two unknowns ✓ sets up two equations with two unknowns ✓ states r & q

Q5 (2, 2 & 2 = 6 marks)

Consider the probability density function $f(x) = ax^3(x - 5)$, $1 \leq x \leq 5$ and zero for all other values of x .

a) Show that $a = \frac{-5}{776}$.

Solution

Specific behaviours
<ul style="list-style-type: none"> ✓ sets up integral for total area = 1 ✓ shows equation to solve for a

b) Determine the probability $\Pr(X \leq 3 | 2 \leq X \leq 4)$ for the above function.

Solution
$\Pr(X \leq 3 2 \leq X \leq 4) = \frac{\Pr(2 \leq X \leq 3)}{\Pr(2 \leq X \leq 4)}$