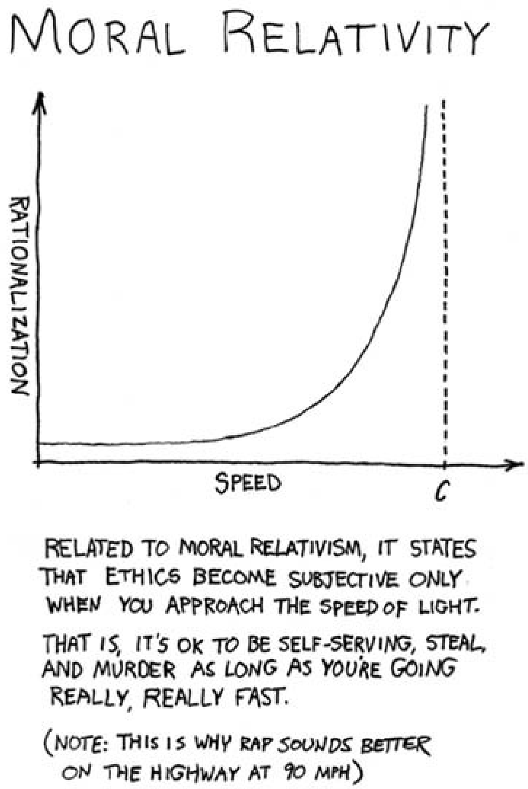
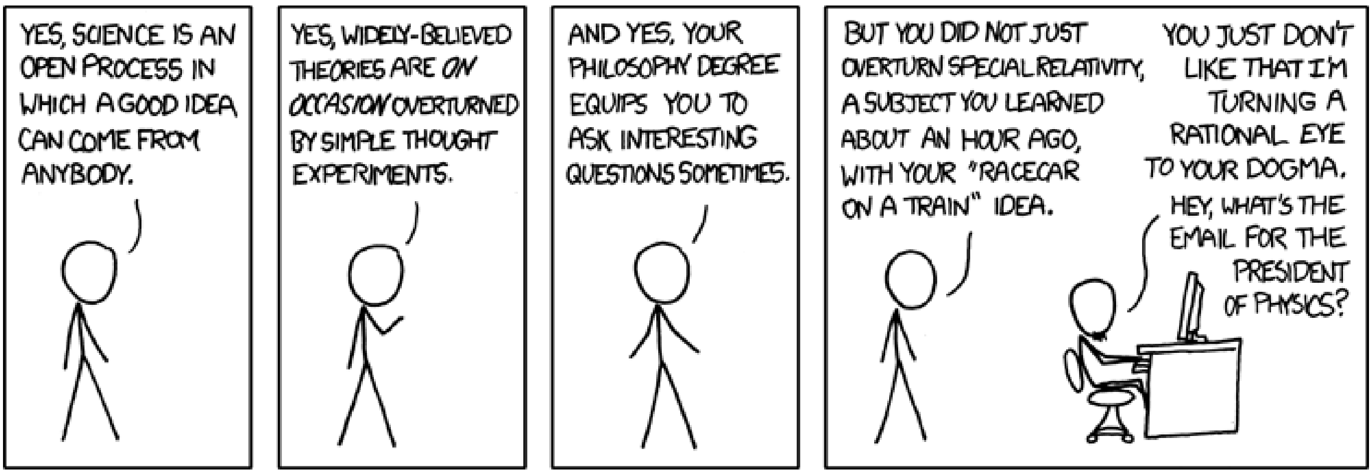
Year 12 Physics
Unit 4
Special Relativity





(Munroe, n.d.)

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Proposed timeline

Wk	#	Topic	Practical activities	STAWA Questions	Pearson Physics
5	5	Developing classical relativity			
6	1	Staff PL			
6	2	Length contraction and time dilation			
6	3	Lorentz transformation of velocity			
6	4	Relativistic momentum			
6	5	Mass-energy equivalence			
7	1	Standard model			
7	2	Standard model			
7	3	Standard model			
7	4	**Relativity and Universe Topic Test**			
7	5	Standard Model			

SCSA ATAR Syllabus

https://senior-secondary.scsa.wa.edu.au/syllabus-and-support-materials/science/physics

Science Understanding

- observations of objects travelling at very high speeds cannot be explained by Newtonian physics. These include the dilated half-life of high-speed muons created in the upper atmosphere, and the momentum of high-speed particles in particle accelerators
- Einstein's special theory of relativity predicts significantly different results to those of Newtonian physics for velocities approaching the speed of light
- the special theory of relativity is based on two postulates: that the speed of light in a vacuum is an absolute constant, and that all inertial reference frames are equivalent
- motion can only be measured relative to an observer; length and time are relative quantities that depend on the observer's frame of reference

This includes applying the relationships

$$l=l_0\sqrt{1-\frac{v^2}{c^2}} \quad t=\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad u=\frac{v+u'}{1+\frac{vu'}{c^2}} \quad u'=\frac{u-v}{1-\frac{uv}{c^2}}$$

- relativistic momentum increases at high relative speed and prevents an object from reaching the speed of light

This includes applying the relationship

$$p_v=\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

- the concept of mass-energy equivalence emerged from the special theory of relativity and explains the source of the energy produced in nuclear reactions

This includes applying the relationship

$$E=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Science as a Human Endeavour

Research studies of cosmic rays show that interactions between cosmic rays and the upper atmosphere produce muons. These particles have a lifetime of about two microseconds and should have ceased to exist before reaching the surface of the Earth. However, because they are travelling near the speed of light, the time dilation effect allows them to complete their journey. Continuing research in the field of high-energy physics is important for improving our understanding of our world and its origins.

Classical relativity

- The velocity of an object must be measured relative to a reference point or reference grid – frame of reference
- The velocity varies based on which frame of reference is selected
- Consider a flight attendant walking along the cabin of a cruising plane
- Relative to the plane they are moving at 0.5 m s^{-1}
- Relative to the surface of the Earth they are moving at 250 m s^{-1}
- Relative to the sun they are moving at 30000 m s^{-1}

Examples

1. Ike is running North at 10 m s^{-1} and he throws a ball to Timmy. The ball leaves Ike's hand at 20 m s^{-1} (relative to Ike).
    a. If Timmy is standing still when he catches the ball how fast is it travelling relative to Timmy?

    b. If Timmy is running towards Ike at 5 m s^{-1} when he catches the ball, how fast is it travelling relative to Timmy?

    c. If Timmy is running away from Ike at 5 m s^{-1} when he catches the ball, how fast is it travelling relative to Timmy?

2. Angela and Bill are throwing a 200 g ball back and forth in a train moving at a steady speed of 30 ms^{-1} to the right. Angela in the front carriage throws the ball at 10 m s^{-1}. Bill catches it and throws it back at the same speed.
    a. Determine the velocity of the ball in Clare's frame of reference who is watching from outside on the grass.

    b. Bill catches and throws the back in one steady movement that takes 2s. Determine the constant force Bill applies from his point of view.

    c. Determine the constant force from Clare's point of view.

Formalising classical relativity

$$u = v + u'$$

$$u = velocity\ of\ object \in frame\ S$$
$$v = relative\ velocity\ between\ frame\ S \wedge frame\ S'$$
$$u' = velocity\ of\ object \in frame\ S'$$

Example

1. Matt is watching a sailboat from a nearby jetty as it moves directly away from him at 12.0 m s^{-1} due east. His friend John is walking at 3.0 m s^{-1} to the front of the boat. Determine John’s velocity relative to Matt.

2. If John then runs towards the back of the boat at 5.0 m s^{-1}. Determine Matt’s velocity relative to John.

Developing relativity further

- Recall the flight attendant travelling 0.5 m s^{-1} relative to the plane, 250 m s^{-1} relative to the Earth and 300000 m s^{-1} relative to the Sun
- Is one of those frames of reference better than the others?
- Is there an absolute frame of reference? One that all velocities can/should be measured relative to?
- 1687 Newton (drawing on Galileo) describes space and time as absolute and unchanging, the backdrop for motion
- 1704 Newton suggest an aethereal medium (a medium for light – later called luminiferous aether) to explain some of light's behaviour

Away from classical relativity

- 1861 Maxwell's equations describing the behaviour of light show it moves at a constant $3x10^8$ m s^{-1} in a vacuum – classical relativity suggests that a frame of reference moving relative to the aether should show a different velocity for light
- 1887 Michelson-Morley experiment completely fails to detect the aether
- 1892 Lorentz devises Lorentz Ether Theory that includes an undetectable aether and length contraction of moving objects to reconcile old ideas of an aether with Maxwell's equations
- 1905 Einstein's theory of Special Relativity explains all testable predictions of Lorentz Ether Theory extending beyond light, without the need for an undetectable ether
- 1907 Minkowski demonstrates that Special Relativity has a very natural interpretation in terms of a 4-D spacetime

Special relativity

- Lorentz Ether Theory required length contraction and time dilation to justify an undetectable aether
- In Einstein's theory of Special Relativity length contraction and time dilation arose very naturally from two key ideas:

1. The laws of physics are the same in all non-accelerating (inertial) frames of reference
2. The speed of light in a vacuum is the same for all observers regardless of the motion of the observer or light source

- The name 'Special Relativity' refers to the fact that this applies in the special case of flat spacetime (no gravity) – need General Relativity to account for the effect of gravity (curvature of spacetime)

Implications of Special Relativity

- Anyone can assume they are stationary, as long as they aren't accelerating – all inertial (non-accelerating) frames of reference are equally valid
- The speed of light in a vacuum is absolute – it will never be observed to be anything other than $3x10^8$ m s^{-1}
- Absolute speed limit – no physical object, field lines or information can travel faster than the speed of light in a vacuum (e.g. the effect of gravity is not instantaneous – travels at the speed of light)
- Space and time are relative – measurements of length and time may differ between observers in different frames of reference
- Simultaneity is relative - two events that occur simultaneously for one observer may not for another in another frame of reference – this requires the absolute speed limit to preserve causality
- Mass and energy are equivalent – small quantities of mass represent large quantities of energy and vice versa
- Momentum, and energy are relative – increasing disproportionately to rest mass as relative velocity increases – effectively enforcing absolute speed limit – increasingly difficult to accelerate

Length Contraction

- A moving object's length is measured to be less than its proper length by an observer at rest
- This contraction only occurs parallel to the direction of movement – a fast rocket is measured to be shorter but not narrower

$$l=l_0\sqrt{1-\frac{v^2}{c^2}}$$

l_0 = proper length of an object – object's length measured in its rest frame (frame of reference in which it is stationary)

l = length of the moving object measured by a stationary observer

v = velocity of object relative to the observer

- Note that for small values of v, $l \approx l_0$, it is only as v approaches c that length contraction becomes significant these velocities are known as relativistic velocities

Example

1. A 120 m long, 25 m wide rocket passes the Earth at 0.4c, what will the dimensions of the rocket be viewed from Earth?

2. What would the dimensions of Earth be viewed from the rocket?

Time dilation

- An observer will measure a moving clock as ticking more slowly than a clock at rest in the observer's own frame of reference
- Often stated as; "moving clocks run slowly"

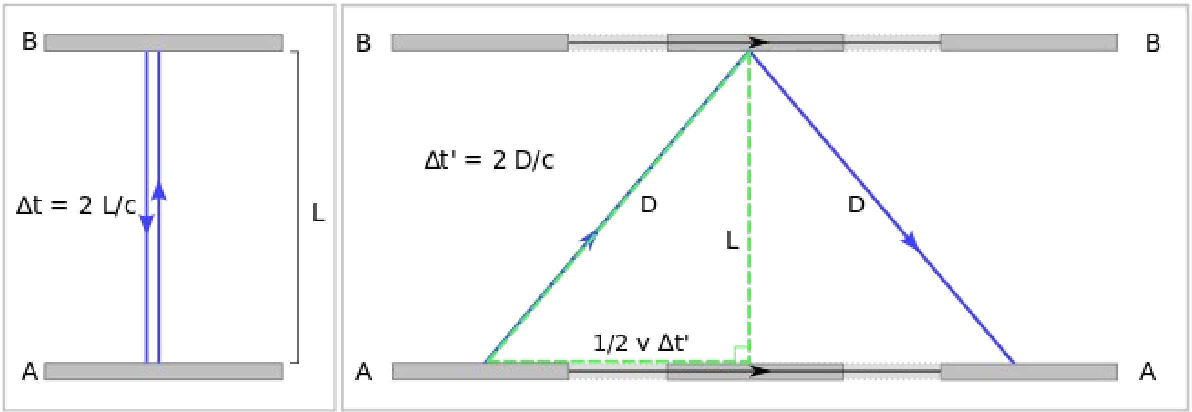
$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

t_0 = proper time – time interval measured in an object's rest frame (frame of reference in which it is stationary)
t = time interval measured in a stationary observer's frame of reference
v = velocity of object relative to the observer

- Note that for small values of v, $t \approx t_0$, it is only as v approaches c that time dilation becomes significant these velocities are known as relativistic velocities

Thought experiment for time dilation

- Imagine a clock that functions by bouncing a photon between mirrors – 'ticking' every time the photon strikes one of the mirrors
- When the clock is at rest the photon travels twice the distance between the mirrors each 'tick'
- If the clock is moving parallel to the mirrors the photon travels a greater distance, since photons must always travel at the same speed the 'ticking' of the clock will slow



Examples

1. Timmy gets stuck on a planetoid travelling away from Earth at 0.7c relative to Earth. An observer on Earth sees Timmy as having aged 1 year, how much time has passed on Earth?

2. Timmy gets stuck on a planetoid travelling away from Earth at 0.7c relative to Earth. Once Timmy has aged 1 year, how much time does he see as having passed on Earth?

3. A colony ship travels to Alpha Centauri (4.26 ly away) at 0.85c relative to Earth.
   a. How much time passes on the colony ship?

   b. What length of space does the colony ship see itself cross?

4. A spaceship passing earth sends a flash of light that lasts for 2 s. The velocity of the spaceship is 0.60 c. For the observer on earth determine the time duration of the signal.

5. Muons are produced naturally by cosmic ray bombardment in the upper atmosphere. Their mean lifetime is 2.2×10^{-6} s as measured in the lab. A muon is created 12.4 km above the surface of the Earth, moving towards the surface at 0.9997c.
    a. How long will it take the muon to reach the ground in Earth's frame of reference.

    b. Should the muon reach the surface?

    c. How long will the muon last on average from the Earth's frame of reference?

    d. Should the muon reach the surface?

Lorentz transformation of velocities

- If two spaceships were travelling towards each other, each at 0.6c, what would one measure the other's speed as?
- Classical relativity would tell us 1.2c, but that is faster than the speed of light, so not possible
- Special relativity shows us how to determine the relative velocity at these relativistic speeds
- In fact, each spaceship would observe the other as moving towards it at 0.882c

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}} \quad u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$u = velocity\ of\ object\ 1 \in frame\ of\ reference\ S$$
$$v = velocity\ of\ object\ 2 \in frame\ of\ reference\ S\left(velocity\ of\ S'\ relative¿\ S\right)$$
$$u' = velocity\ of\ object\ 1 \in frame\ of\ reference\ S'$$

Examples

1. A spaceship approaching Earth at 0.5c fires a projectile at 0.3c (relative to the ship) towards Earth. How fast does Earth measure the projectile as travelling?

2. Spaceship A is travelling away from Earth at 0.7c relative to Earth. Spaceship B is further out moving away from Earth in the same direction at 0.6c relative to spaceship A. Determine the velocity of spaceship B relative to Earth.

3. Spaceship A is travelling away from Earth at 0.7c relative to Earth. Spaceship B is further out moving towards Earth at 0.5c relative to Earth. Determine the velocity of spaceship B relative to Spaceship A.

4. In the future it is possible that humans may travel to distant places like Alpha Centauri 4.13 x10^{13} km from Earth. Imagine you are on a spacecraft travelling past Earth towards Alpha Centauri at 0.720c relative to Earth. On your journey you pass another spacecraft travelling parallel and in the opposite direction to you. You measure the relative velocity of the other spacecraft as 0.695c.
    a. Calculate the velocity of the other spacecraft relative to Earth. (3 marks)

    b. Calculate the number of years for your spacecraft to journey from Earth Alpha Centauri as measured by an observer on Earth. (3 marks)

    c. Calculate the number of years the journey will take as measured by those on the spacecraft travelling at 0.720c relative to Earth. (3 marks)

    d. For those on the spacecraft travelling to Alpha Centauri at 0.720c relative to Earth calculate the time they would have observed to have elapsed on Earth during the journey from Earth to Alpha Centauri. (3 marks)

Relativistic momentum

- At relativistic speeds momentum increases
- Has the effect of making further acceleration increasingly difficult, effectively enforcing the speed limit
- Since momentum is no longer just velocity multiplied by a constant (rest mass), it implies an apparent increase in mass

$$p_v = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Often observed in particle accelerators – relativistic particles have more momentum than they ought to according to classical mechanics

Example

- Determine the momentum of a 2.7×10^{19} kg asteroid travelling at 0.3c.

Mass-energy equivalence

- Special relativity suggests that anything with mass has an equivalent amount of energy and vice-versa
- The conversion between energy and mass at non-relativistic speed is just by a constant factor of c^2 – making mass and energy equivalent aside from units
- At relativistic speeds the energy of an object increases – implies an apparent increase in mass lining up perfectly with the relativistic momentum

$$E=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Rest energy

- For an object at rest, the mass-energy equivalence equation on the left simplifies to the more familiar equation on the right

$$E=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\quad E=mc^2$$

- This gives the rest energy of the object – the energy associated with its mass
- At relativistic speeds, the extra energy can be thought of as kinetic energy

$$E_{total}=E_{rest}+KE$$
$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}=mc^2+KE$$

Example

1. A proton is accelerated to 0.95c.
   a. What is its energy?

   b. How much kinetic energy does the proton have?

2. In a particle accelerator, an alpha particle with a mass of 6.64424×10^{-27} kg is accelerated from rest to high speed. The total work done on the alpha particle is equal to 7.714×10^{-10} J. Determine its final speed.

Empirical evidence of length contraction and time dilation – muons

- Interaction between cosmic rays and the upper atmosphere creates muons (particles with a lifespan of about 2 μs), they should cease to exist before reaching the surface of the Earth but they don't
- From their frame of reference, they are stationary, the atmosphere moves past them at close to the speed of light so it is length contracted shortening the trip to the surface
- From the Earth's frame of reference, time is observed to pass more slowly for the muons, meaning that they reach the surface before their lifespan has elapsed

Other empirical evidence for special relativity

- Modern technologies including GPS satellites, electron microscopes and particle accelerators do not function unless we account for the effects of special relativity
- 1971 Hafele and Keating flew atomic clocks around the Earth, East and West to compare with a clock remaining at the US Naval Observatory - the lost time of the Eastward bound clock and the gained time of the Westward bound clock were in agreement with theoretical predictions from the theories of special and general relativity
- Many other experiments going back as far as 1810 have led to the formation of special relativity and have then continued to support its predictions
- Experiments involving relativity continue as we search for a quantum theory of gravity

Alleged paradoxes

- Special relativity can make people uncomfortable, many have proposed "paradoxes" that allegedly disprove special relativity
- These paradoxes rely on an incomplete understanding of special relativity to create impossible interpretations of special relativity
- All of these "paradoxes' can be addressed with a good understanding of special relativity; none stand up as serious arguments against the theory

Time dilation "paradoxes"

- One member of a pair of twins sets out from Earth on a spaceship travelling at 0.9c relative to the Earth before returning at 0.9c. The twin on Earth sees the spaceship twin take 20 years to return but since the spaceship twin is travelling at a relativistic speed, the Earth twin will see the spaceship twin as having experienced time more slowly so the spaceship twin should now be younger.
- However, the spaceship twin could say that the Earth moved away from him at 0.9c before returning, he should expect that the Earth twin will now be younger
- Some say resolved by acceleration of one of the twins while the other remains at constant velocity – more accurately resolved by the fact that one twin occupies two different inertial frames of reference, not one
- More generally 2 people each view each other's clocks as running slow, must consider location as well as time, complicates issue.
- Detailed explanation: https://www.youtube.com/watch?v=kN_d7eknfYk

Length contraction "paradoxes"

- If a 300 m long spaceship travelling at 0.9c flew through a 150 m long hangar that is open at both ends, then an observer standing outside to the side of the hangar would see the spaceship is sufficiently length contracted that for a split second, the whole spaceship would be inside then hangar.
- The spaceship however would see the hangar as length contracted so it should not all fit in at the same time.
- Resolved by the relativity of simultaneity
- To the outside observer the nose of the spaceship is inside the hangar at the same time as the tail is inside the hangar.
- For someone on the spaceship these two events are not simultaneous, the nose of the spaceship is no longer inside the hangar when the tail of the spaceship enters the hangar
- Detailed explanation: https://www.youtube.com/watch?v=0TU1tKTOIj4

Other resources

- http://www.youtube.com/watch?v=j72bPmXsyvk (Time: Travel : Einstein's big idea)
- http://www.youtube.com/watch?v=V7vpw4AH8QQ (Einstein's special relativity E = mc2)
- http://www.youtube.com/watch?v=KHjpBjgIMVk (Time dilation – Alb Einstein and theory of rel)
- http://www.youtube.com/watch?v=HHRK6ojWdtU&NR=1 (2nd half)
- http://www.youtube.com/watch?v=tpbGuuGosAY (The Elegant Universe: Einstein's relativity)
- http://www.onestick.com/relativity/
- Special Relativity: Train/Lightning Paradox and Simultaneity by Maxwell Fazio Mar 17, 2018
- https://www.youtube.com/watch?v=bRxfxhJBm4g
- Einstein's Relativistic Train in a Tunnel Paradox: Special Relativity by Eugene Khutoryansky
- Oct 7, 2015
- https://www.youtube.com/watch?v=Xrqj88zQZJg

Relativity revision

1. What is meant by the term ‘no absolute zero velocity’?
2. What is meant by classical relativity?
3. What are Einstein’s two postulates for his special theory of relativity?
4. A car moving at 55 km/h and a person in the car shoots an arrow forwards at 15 km/h.
    a) What is the speed of the arrow relative to the car?
    b) What is the speed of the arrow relative to the person standing on the side of the road?
    c) Repeat a and b if the arrow was shot backwards.
5. Two spaceships approach each other moving at 0.5 c. One sends a radio signal to the other. At what speed does the signal reach the other? Why?
6. A 150 km long, 20km wide spaceship flies past earth, at 0.6 c, how will its dimensions appear to a man on earth?
7. A light is switched on the middle of the train and its beam travels to both ends of the long carriage. Will the beam of light reach the ends of the carriage at the same time for:
    a) A person in the train? Explain what they observe.
    b) A person on the ground outside the train? Explain what they observe.
8. Time seems to slow down for a spaceship travelling away from earth at 0.5 c. Explain how this is possible.
9. A star, Proxima Centauri, is 4.35 light years from Earth.
    a) When you see this star in the night sky, when did that light originate from the star. Explain.
    b) How far is Proxima Centauri form Earth in km?
    c) If you could travel at 1/10 the speed of light, how long would it take to reach it?