TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 6 2018

Section 1 Calculator Free
Sampling

STUDENT'S NAME Solutions

DATE: Thursday 6 September **TIME:** 10 minutes **MARKS**: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (5 marks)

A researcher wishes to evaluate how well the local library is catering to the needs of a town's residents. To do this she hands out a questionnaire to each person entering the library over the course of a week.

(a) Discuss whether this method will result in a random sample. [2]

The sample will not be random for <u>all</u> residents. It will only capture residents that currently use the library. It will not capture non-user residents.

(b) Discuss any improvements that could be made to increase the validity of the results. [2]

Survey more residents. Survey could be included in the council rates.

(c) Suggest a possible question that the researcher can ask to gauge whether the library is catering to the needs of the town's residents. [1]

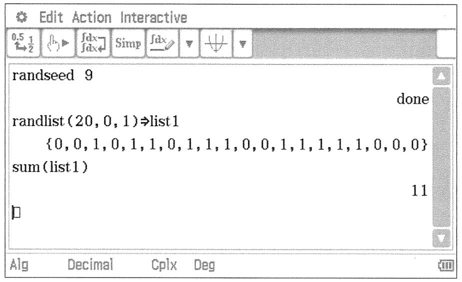
Are you happy with the current facilities at the library?

2. (5 marks)

The following question was asked:

Are you happy that Scott Morrison is the 31st Prime Minister of Australia?

The following simulation for 20 people was run where a 1 indicates 'yes' and a 0 indicates 'no'.



(a) Determine the sample proportion of people who answered 'yes'. [1]

$$\hat{p} = \frac{11}{20}$$

(b) Determine the standard deviation in the form $\sqrt{\frac{a}{b}}$ of the sampling distribution for the people who answered 'yes'. [4]

$$\sigma = \sqrt{\frac{\frac{11}{20} \times \frac{9}{20}}{20}}$$

$$= \left(\frac{11}{20} \times \frac{9}{20}\right) \times \frac{1}{20}$$

$$= \sqrt{\frac{99}{8000}}$$

7. (5 marks)

A sample of 30 fair coins is taken. They are shaken in a container and then poured onto the floor.

(a) Determine the probability that at least 15 coins land head side up. [2]

X : # coins head side up from 30

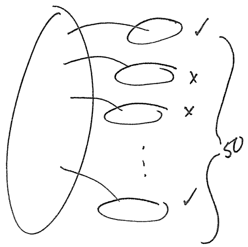
$X \sim B(30, 0.5)$

$P(X \geq 15) = 0.5722$

(b) The coins are returned to the container, shaken, and poured onto the floor a total of 50 times.

(i) Determine the distribution of sample proportions that have at least 15 coins which have landed head side up. [2]

$\hat{p}$: proportion of pours that have at least 15 heads



$$\hat{p} \sim N\left(0.5722, \sqrt{\frac{0.5722(1-0.5722)}{50}}^{\,2}\right)$$

$$\hat{p} \sim N\left(0.5722, 0.0700^2\right)$$

(ii) Determine the probability that the sample proportion has a value between 0.4 and 0.6. [1]

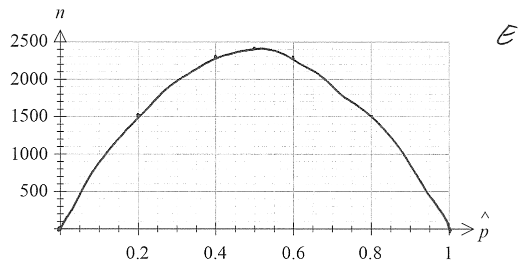
$P(0.4 \leq \hat{p} \leq 0.6) = 0.6474$

6. (9 marks)

A survey is being planned to estimate the proportion of people in Australia who think that university fees should be abolished. The organisers of the survey want the error in the approximate 95% confidence interval for this proportion to be no more that $\pm 2\%$.

They have no prior information about the value of the proportion.

(a) Plot the sample size, $n = \left(\frac{1.96}{E}\right)^2 \hat{p}(1-\hat{p})$, against $\hat{p}$ for $0 \le \hat{p} \le 1$ [3]



$E = 0.02$

(b) Determine the value of $\hat{p}$ where the sample size is a maximum [1]

$\hat{p} = 0.5$

(c) Determine the value of n that you would recommend be used for the survey [1]

$n = 2401$

(d) Show that the maximum sample size required for the error in an approximate 95% confidence interval to be no more than E is approximately $n = \frac{1}{E^2}$ [2]

$E = 1.96\sqrt{\frac{0.5 \times 0.5}{n}}$

$\Rightarrow \left(\frac{E}{1.96}\right)^2 = \frac{0.25}{n}$

$\Rightarrow n = \frac{0.25 \times 1.96^2}{E^2}$

$= \frac{0.9604}{E^2}$

$\approx \frac{1}{E^2}$

(e) You want to halve the margin of error. Determine the factor increase in the sample size required to achieve this at a 95% confidence level. [2]

at 2% , $n = 2401$

$\Rightarrow 0.01 = 1.96\sqrt{\frac{0.5 \times 0.5}{n}}$

$\Rightarrow n = 9604$

$\therefore$ sample size is 4 times large

TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 6 2018

Section 2 Calculator Assumed
Sampling

STUDENT'S NAME ______________________

DATE: Thursday 6 September **TIME:** 40 minutes **MARKS**: 41

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

3. (5 marks)

A company has 2000 employees, 700 of whom are female. A random sample of 100 employees was selected, and 40 of them were female.

In this example of female employees,

(a) What is the size of the population? [1]

2000

(b) What is the value of the population proportion p ? [1]

$p = \frac{700}{2000} = 0.35$

(c) What is the value of the sample proportion $\hat{p}$? [1]

$\hat{p} = \frac{40}{100} = 0.4$

(d) Describe the distribution of the sample proportions. [2]

$\hat{p} \sim N\left(0.35, \sqrt{\frac{0.35 \times 0.65}{100}}^2\right)$

$\sim N(0.35, 0.0477^2)$

4. (11 marks)

A ReachTel poll published on Tuesday 28 August, 2018, surveyed 886 residents in the Federal seat of Wentworth. It reported that 443 respondents would support Labor at the next election.

(a) Calculate the sample proportion of Labor supporters. [1]

$$\hat{p} = \frac{443}{886} = 0.5$$

(b) Calculate an approximate 90% confidence interval for the proportion of respondents that would support Labor at the next election. Give your answer to four decimal places. [3]

$$\hat{p} - 1.645\sqrt{\frac{0.5\times0.5}{886}} \le p \le \hat{p} + 1.645\sqrt{\frac{0.5\times0.5}{886}}$$

$$0.4724 \le p \le 0.5276$$

(c) Determine the margin of error in the above estimate. [2]

$$\frac{0.5276 - 0.4724}{2} = 0.0276$$

or

$$E = 1.645\sqrt{\frac{0.5\times0.5}{886}} = 0.0276$$

(d) Determine the number of people that need to be surveyed to achieve a margin of error of at most 2% at a 90% confidence level. [2]

$$\Rightarrow \quad 0.02 > 1.645\sqrt{\frac{0.5\times0.5}{n}}$$

$$\Rightarrow \quad n > 1692 \qquad (n = 1691.2656)$$

(e) Assume that $\hat{p}$ applies to the whole population. Determine the probability that in a random sample of 150 people, the number of people who vote Labor is greater than 75. [3]

X : # of people who support Labor from 150

$X \sim B(150, 0.5)$

$P(X > 75) = P(76 \le X \le 150)$

$= 0.4675$

if $X \sim N(75, \sigma^2)$
$P(X > 75) = 0.5$
1 mark only

5. (11 marks)

In a study of the prevalence of red hair in a certain country, researchers collected data from a random sample of 1800 urban adults from major capital cities.

Of the 1000 females in the sample, they found that 10% had red hair.

(a) Determine a point estimate for the proportion of people with red hair. [1]

$$p \approx 0.1$$

(b) Determine with reasons if the point estimate in part (a) would be a fair estimate for the population. [3]

Not a fair estimate for total population.
- only females in sample
- only urban areas in sample (no country)

(c) Calculate an approximate 95% confidence interval for the proportion with red hair in the female population. [3]

$$0.1 - 1.96\sqrt{\frac{0.1\times0.9}{1000}} \le p \le 0.1 + 1.96\sqrt{\frac{0.1\times0.9}{1000}}$$

$$0.0814 \le p \le 0.1186$$

Of the 800 males in the sample, they found that 10% had red hair. A 95% confidence interval for the proportion with red hair in the male population is $0.0792 \le p \le 0.1208$

(d) How should the sample of 1800 adults be chosen to ensure that the widths of the two confidence intervals (for males and females) are the same when the sample proportions are the same? [1]

Sample 900 of each

(e) Assume that there are 1000 females and 800 males in the sample, and that the proportion of females in the sample with red hair is 10%. Determine the sample proportion of red-headed males that would result in the 95% confidence interval for the proportion with red hair in the female population and the 95% confidence interval for the proportion with red hair in the male population being of the same width. [3]

Same width $\Rightarrow$ same margin of error

$$\Rightarrow \quad \cancel{z}\sqrt{\frac{0.1\times0.9}{1000}} = \cancel{z}\sqrt{\frac{\hat{p}(1-\hat{p})}{800}}$$

$$\Rightarrow \quad \hat{p} = 0.0781 \text{ or } 0.9219$$