Christ Church Grammar School

2020
TEST 4

MATHEMATICS METHODS Year 11

Section One:
Calculator-free

Your name SOLUTIONS.

Teacher's name _____

Time and marks available for this section

Working time for this section:	25 minutes
Marks available:	20 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

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3. Answer all questions.

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6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

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See next page

Question 8 continued

(c) Determine the probability that only one feature works. (2 marks)

identifies correct probabilities and adds them

$P(V \cap \overline{IR}) + P(\overline{V} \cap IR)$ ✓

$= 0.08 + 0.17$

$= 0.25$ ✓

Obtains correct answer

identifies 1 - given probability

or $1 - [P(\overline{V} \cap \overline{IR}) + P(V \cap IR)]$ ✓

$1 - (0.72 + 0.03)$

$= 0.25$ ✓

Obtains correct answer

(d) Given that only one feature works, justify whether it is more likely that the Voice Recognition Command or the Infra-red Obstacle Avoidance is the component that works. (3 marks)

$P(\overline{V} \mid IR) = \dfrac{P(\overline{V} \cap IR)}{P(IR)}$

$= \dfrac{0.17}{0.17 + 0.72}$

$= \dfrac{17}{89}$

$= 0.191$ ✓

evaluates probability that one works $P(\overline{V} \mid IR)$

Note: If they calculate $P(V \mid \overline{IR})$ and $P(IR \mid \overline{V})$ only $\frac{1}{3}$ if correct comparison.

$P(\overline{IR} \mid V) = \dfrac{P(\overline{IR} \cap V)}{P(V)}$

$= \dfrac{0.08}{0.8}$

$= 0.1$ ✓

evaluates probability that the other works $P(\overline{IR} \mid V)$

∴ It is more likely the infra-red obstacle avoidance component works as $P(\overline{V} \mid IR) > P(\overline{IR} \mid V)$

$(0.191 > 0.1)$

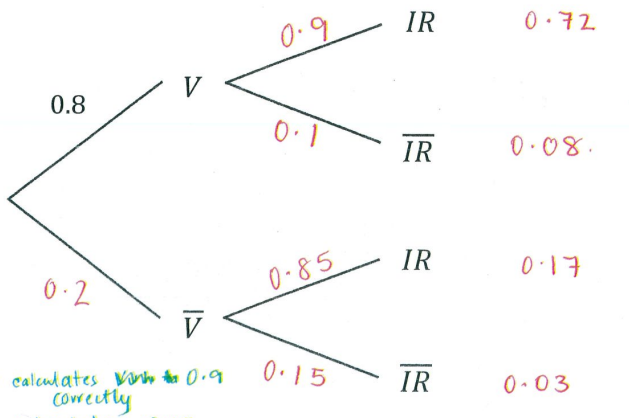
Final statement with numerical probability comparison.

End of questions

Question 8 **(10 marks)**

A DIY robot assembly kit has two main electronic features that can either work or be defective because of operator error when assembled. The features are Voice Recognition Command (V) and Infra-red Obstacle Avoidance (IR). The Voice Recognition Command feature works 80% of the time. The probability that both will work is 0.72 and the probability that both will be defective is 0.03.

(a) Complete the tree diagram with the correct probabilities on each branch. (3 marks)



✓ calculates 0.9 correctly
✓ calculates 0.15
✓ completes remaining values on tree diagram

(b) Determine the probability that the Infra-red Obstacle Avoidance works. (2 marks)

$P(V \cap IR) + P(\overline{V} \cap IR)$ ✓ identifies correct probabilities to add

$= 0.72 + 0.17$

$= 0.89$ ✓ obtains solution Probability is 0.89

Question 1 **(5 marks)**

(a) Express the following in scientific notation. (2 marks)

$(6.3 \times 10^5) \div (3.15 \times 10^{-2})$

$= \frac{6.3 \times 10^5}{3.15 \times 10^{-2}}$

$= 2 \times 10^7$

✓ determines the correct constant ✓ determines the correct power of 10.

(b) Simplify the following and express with positive indices where required. (3 marks)

$\frac{(16y^3z)^{\frac{1}{2}}}{8\sqrt{yz^{-3}}}$

$= \frac{16^{1/2} y^{3/2} z^{1/2}}{8 y^{1/2} z^{-3/2}}$ ✓ distributes the power of $\frac{1}{2}$ correctly to the indices in the numerator

$= \frac{4 y^{2/2} z^{4/2}}{8}$

$= \frac{yz^2}{2}$ ✓ obtains the correct positive indices for y, z. ✓ obtains the correct factor of 2 in the denominator.

Question 2 **(5 marks)**

Solve the following equations.

(a) $\frac{1}{x^2} = 0.25$ (2 marks)

$\frac{1}{x^2} = \frac{1}{4}$

$4 = x^2$ ✓

$\therefore x = \pm 2$ ✓

re-writes the equation to obtain $x^2 = 4$

obtains both solutions $x = \pm 2$.

Note: Answer only is 2 marks
$x = 2$ 0 marks without $x^2 = 4$.

(b) $(3^x)^2 + 6(3^x) - 27 = 0$ (3 marks)

let $y = 3^x$

$y^2 + 6y - 27 = 0$

$(y + 9)(y - 3) = 0$ ✓ factorises expressions and has $= 0$.

$y = -9$ or 3

If $y = -9$ then $3^x = -9$ no solution ✓ states $3^x = -9$ has no solution

If $y = 3$ then $3^x = 3$ $\therefore x = 1$ ✓ obtains solution $x = 1$

or

✓ $(3^x + 9)(3^x - 3) = 0$.

$3^x + 9 = 0$ or $3^x - 3 = 0$

$3^x = -9$ $3^x = 3$

no solution ✓ states $3^x = -9$ has no solution

$\therefore x = 1$ ✓ obtains solution $x = 1$.

See next page

Question 7 continued

(c) Justify if the events A and B are mutually exclusive. (1 mark)

A and B are not mutually exclusive as $P(A \cap B) \neq 0$. ✓ correct statement and must have $P(A \cap B) \neq 0$.

(d) Justify if the events A and B are independent. (2 marks)

If independent then

$P(A) \times P(B) = P(A \cap B)$

$\therefore$ LHS : $P(A) \times P(B)$

$= \frac{3}{10} \times \frac{1}{2}$

$= \frac{3}{20}$ ✓ evaluates $P(A) \times P(B)$ correctly

RHS : $P(A \cap B)$

$= \frac{1}{5}$

$\therefore P(A) \times P(B) \neq P(A \cap B)$ shows $P(A) \times P(B) \neq P(A \cap B)$

as $\frac{3}{20} \neq \frac{1}{5}$ ✓ and makes final statement.

$\therefore$ Events A and B are not independent.

Note: Cannot be awarded 2 marks if just state not independent, must justify
Can use other independent rules to show.

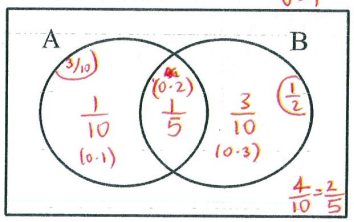
See next page

Question 7 **(8 marks)**

The following information regarding events A and B is known.

$P(A) = \frac{3}{10}$ $P(B) = \frac{1}{2}$ $P(A|B) = \frac{2}{5}$

(a) Complete the Venn diagram below. (2 marks)



calculates A∩B correctly ✓

completes all other regions. ✓

$U = 1$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\frac{1}{2} \times \frac{2}{5} = P(A \cap B)$

$P(A \cap B) = \frac{1}{5}$

(b) Determine

(i) $P(\overline{A})$ (1 mark)

$= \frac{7}{10}$ (0.7) ✓ obtains correct value

(ii) $P(A \cap B)$ (1 mark)

$= \frac{1}{5}$ (0.2) ✓ obtains correct value

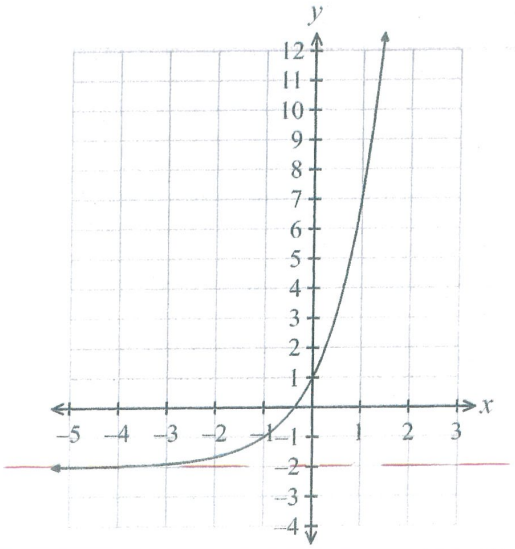
(iii) $P(A \cap \overline{B})$ (1 mark)

$= \frac{1}{10}$ (0.1) ✓ obtains correct value

See next page

Question 3 **(3 marks)**

The graph of $y = a^{x-c} + b$ is shown below.



Determine the values of the constants a, b, and c.

$y \to -2$ as $x \to -\infty$ $\therefore$ $b = -2$

$y = a^{x-c} - 2$

when $x = 0, y = 1$

$1 = a^{-c} - 2$

$3 = a^{-c}$

$3 = a^{-(-1)}$

$3 = a^{1}$

when $x = -1, y = -1$

$-1 = a^{-1-c} - 2$

$1 = a^{-1-c}$

$\therefore$ power must be 0

$-1 - c = 0$

$c = -1$

$\therefore$ $a = 3$, $b = -2$, $c = -1$

Note: Can get FT marks for a an c if working clear.

See next page

✓ ✓ ✓ Obtains values for $a = 3$, $b = -2$, $c = -1$. Must state values explicitly.

Question 4 **(4 marks)**

The rising water level in a storm drain after a thunderstorm is measured and given by the equation $H = \frac{1}{8}(2^{t+1}) + 3$, where H is the depth of the water in the storm drain in millimetres, t minutes after the thunderstorm began.

(a) Determine the depth of the water 2 minutes after the thunderstorm began. (1 mark)

$t=2 \quad H = \frac{1}{8}(2^3)+3$

$= \frac{1}{8} \times 8 + 3$

$= 4$ mm ✓ states the correct answer.

(b) Determine the amount of time that the height of the water in the storm drain is less than 11 mm. (3 marks)

$H < 11$.

Solve for $H = 11$

$11 = \left(\frac{1}{8}\right)2^{t+1} + 3$ substitutes $H=11$ and obtains $64 = 2^{t+1}$

$8 = \left(\frac{1}{8}\right)2^{t+1}$

$64 = 2^{t+1}$ ✓

$64 = 2^t \cdot 2$

$32 = 2^t$

$2^5 = 2^t$

$\therefore t = 5$ ✓ solves equation correctly for t.

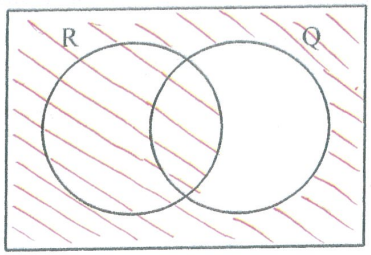
∴ The height of the water in the storm drain is less than 11 mm when $0 < t < 5$ mins (or when $t < 5$ mins). ✓ states final solution with correct statement about time.

See next page

Question 6 **(3 marks)**

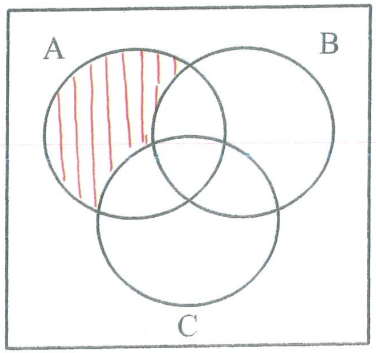
Shade the region indicated by

(a) $R \cup \overline{Q}$ (1 mark)



✓ shades correct region

(b) $A \cap (\overline{B \cup C})$ (2 marks)



✓ identifies region $(\overline{B \cup C})$

✓ shades correct final region $A \cap$ with $\overline{B \cup C}$.

See next page

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Question 5 **(3 marks)**

The expansion of $(2x-1)^5$ is given by $ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, where a, b, c, d, e, and f are constants.

Determine the value of d.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1 *
x^5 x^4 x^3 x^2

required value.

$10.(2x)^2(-1)^3$ ✓ obtains correct term

$= 10.4.x^2.-1$

$= -40x^2$ ✓ simplifies expansion correctly

$\therefore d = -40$ ✓ states $d=-40$.

or

$\binom{5}{2}(2x)^2(-1)^3$ ✓ obtains correct term

$= 10.4x^2.-1$ ✓ evaluates $\binom{5}{2}$ and 2^2 correctly

$= -40x^2$ ✓ states $d=-40$

$\therefore d = -40$

$-20 = (2)$

Additional working space

Question number: _____

Christ Church Grammar School

2020
TEST 4

MATHEMATICS METHODS Year 11

Section Two:
Calculator-assumed

Your name SOLUTIONS.

Teacher's name _____

Time and marks available for this section

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Marks available:	21 marks

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Special items: drawing instruments, templates and up to three calculators approved for use in this assessment

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