Semester One Examination, 2013

Question/Answer Booklet

MATHEMATICS SPECIALIST 3A

Section Two: Calculator-assumed

Student's name solutions________________

Teacher : Mr White

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	6	6	50	50	33
Section Two: Calculator-assumed	12	12	100	100	67
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

Section Two: Calculator-assumed **(100 Marks)**

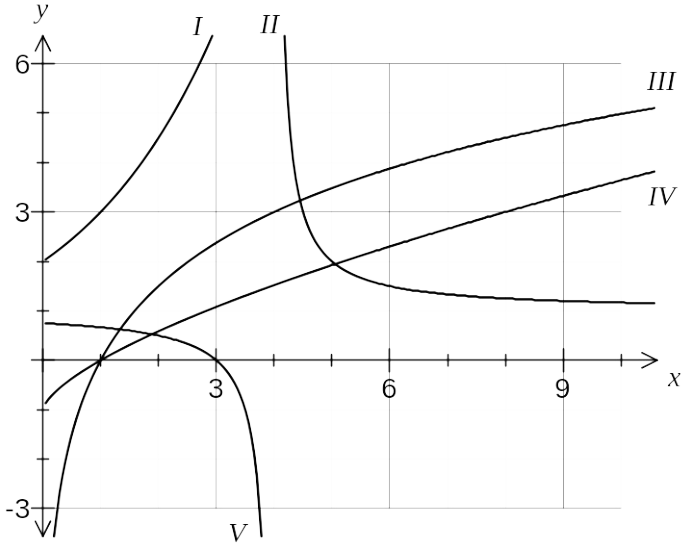
This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 7 **(8 marks)**

The graphs of the following four functions, where $a,\ b,\ c,\ d$ and e are positive constants, are shown on the axes below.

$$f(x) = c(1.5)^x \qquad g(x) = a\log_4 x \qquad h(x) = \frac{1}{x-d} + e \qquad k(x) = x^b - 1$$



(a) Match appropriate sections of the graphs (*I, II, III, IV, V*) with their functions in the table below. (3 marks)

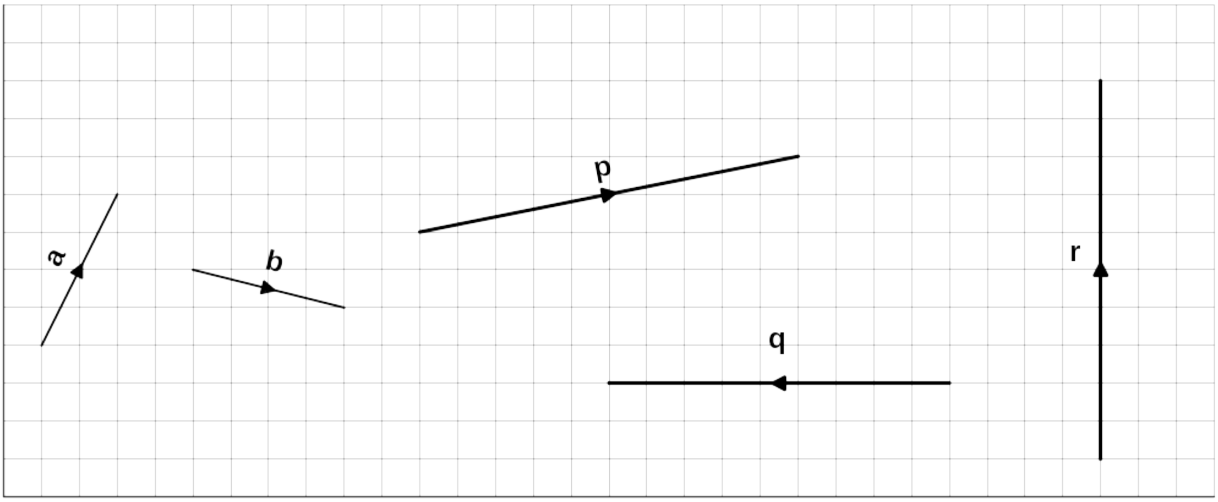
Function	$f(x)$	$g(x)$	$h(x)$	$k(x)$
Section(s)	***I***	***III***	***II, V***	***IV***

(b) Determine the values of the positive constants $a,\ b,\ c,\ d$ and e. (5 marks)

Constant	a	b	c	d	e
Value	**3**	$\frac{2}{3}$	**2**	**4**	**1**

Question 8 **(12 marks)**

Vectors **a** and **b** are as shown on the grid below.

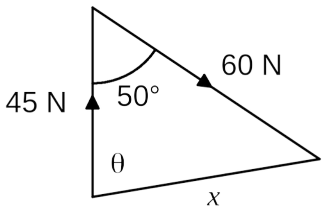


(a) On the grid above, sketch and label the vectors **p**, **q** and **r** where

$$\mathbf{p} = \mathbf{a} + 2\mathbf{b}$$
$$\mathbf{q} = 2\mathbf{a} - \mathbf{b}$$
$$\mathbf{r} = -2\mathbf{b} - 0.5\mathbf{a}$$

(7 marks)

(b) Two vectors have magnitudes of 45 N and 60 N and the angle between their directions is 130°. Sketch a diagram to show their sum and use trigonometry to calculate the magnitude of the resultant and the angle it makes with the smaller vector. (5 marks)



$$x^2 = 45^2 + 60^2 - 2 \times 45 \times 60 \cos 50$$
$$x = 46.41 \text{ N}$$

$$\frac{\sin\theta}{60} = \frac{\sin 50}{46.41}$$
$$\theta = 82.0^\circ$$

Question 9 **(11 marks)**

The latitude and longitude of three cities are given in this table, to the nearest degree.

City	*Latitude*	*Longitude*
Chicago (USA)	42°N	88°W
Luanda (Angola)	9°S	13°E
Rome (Italy)	42°N	13°E

Assume the radius of the earth is 6350 km. Calculate all distances to three significant figures and times to the nearest minute.

(a) Calculate the distance between Chicago and Rome along their common line of latitude. (4 marks)

$$d = \frac{88+13}{360} \times 2 \times \pi \times 6350 \times \cos(42)$$
$$= 8318.5$$
$$\approx 8320 \text{ km}$$

(b) A small plane leaves Rome to fly directly to Luanda at an average speed of 485 kilometres per hour. Ignoring the effects of altitude and any wind, determine

(i) how far the plane is from Luanda after seven hours. (3 marks)

$$d_{RL} = \frac{42+9}{360} \times 2 \times \pi \times 6350 = 5652.2 \qquad d_{flown} = 485 \times 7 = 3395$$

$$d = 5652.2 - 3395$$
$$= 2257.2 \approx 2260 \text{ km}$$

(ii) how long after leaving Rome the plane crosses the equator (line of latitude 0°N). (2 marks)

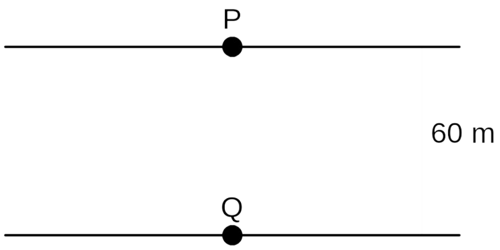
$$d = \frac{42}{360} \times 2 \times \pi \times 6350$$
$$= 4654.8$$
$$t = 4654.8 \div 485$$
$$= 9.598$$
$$\approx 9 \text{ hours } 36 \text{ minutes}$$

(iii) what time the plane reaches Luanda, if it left Rome at 7:30 am. (2 marks)

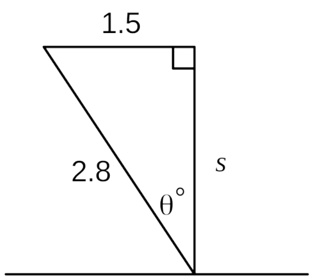
$$5652.2 \div 485 = 11.654$$
$$= 11 \text{ hours } 39 \text{ minutes after 7:30 am}$$
$$= 7{:}09 \text{ pm}$$

Question 10 **(7 marks)**

P and Q are two points directly across from each other on opposite banks of a river. P and Q are 60 m apart and a steady current flows along the river at 1.5 ms^{-1}.



A boat with a speed of 2.8 ms^{-1} must travel directly from P to Q. Find the angle the path of the boat must make with the river bank, to the nearest degree, and determine the time the boat will take, to the nearest tenth of a second.



$$\theta = \sin^{-1}\frac{1.5}{2.8}$$
$$= 32.39$$
$$\approx 32^\circ$$

$$s = \sqrt{2.8^2 - 1.5^2}$$
$$= 2.36431$$

Steer at $90 - 32 = 58^\circ$ to the bank

$$t = 60 \div 2.36431$$
$$= 25.377$$
$$\approx 25.4 \text{ seconds}$$

Question 11 **(10 marks)**

Two vectors are $\mathbf{c} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{d} = -12\mathbf{i} + 5\mathbf{j}$.

(a) Find

(i) $5\mathbf{c} + \mathbf{d}$ (2 marks)

$$\boxed{5\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} -12 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ -15 \end{bmatrix}}$$

(ii) $|\mathbf{d}|$ (2 marks)

$$\boxed{13}$$

(iii) $-|\mathbf{c}|\mathbf{d}$ (2 marks)

$$\boxed{-5\begin{bmatrix} -12 \\ 5 \end{bmatrix} = \begin{bmatrix} 60 \\ -25 \end{bmatrix}}$$

(b) Find $\mathbf{e}$ and $\mathbf{f}$ if $2\mathbf{e} + \mathbf{f} = 2\mathbf{c}$ and $\mathbf{e} - \mathbf{f} = \mathbf{d}$. (4 marks)

$$2\mathbf{e} + \mathbf{f} + \mathbf{e} - \mathbf{f} = 2\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} -12 \\ 5 \end{bmatrix}$$

$$3\mathbf{e} = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$$

$$\mathbf{e} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \mathbf{f} = \begin{bmatrix} -12 \\ 5 \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} 10 \\ -6 \end{bmatrix}$$

Question 12 **(9 marks)**

Let $a = \log_m 4$ and $b = \log_m 5$.

(a) Write the following as a single logarithmic term.

(i) $b - a$ (1 mark)

$$\boxed{\log_m 5 - \log_m 4 = \log_m \tfrac{5}{4}}$$

(ii) $2a + 1$ (2 marks)

$$\boxed{\log_m 4^2 + \log_m m = \log_m 16m}$$

(b) Express the following in terms of a and b.

(i) $\log_m 64$ (1 mark)

$$\boxed{\begin{aligned}\log_m 64^3 &= 3\log_m 4\\ &= 3a\end{aligned}}$$

(ii) $\log_m \dfrac{m^2}{25}$ (2 marks)

$$\boxed{\begin{aligned}\log_m m^2 - \log_m 5^2 &= 2\log_m m - 2\log_m 5\\ &= 2 - 2b\end{aligned}}$$

(iii) $\log_m 20 \times \log_m 0.8$ (3 marks)

$$\boxed{\begin{aligned}(\log_m 4 + \log_m 5)\times(\log_m 4 - \log_m 5) &= (a+b)(a-b)\\ &= a^2 - b^2\end{aligned}}$$

Question 13 **(6 marks)**

The Richter scale is used to measure the intensity of an earthquake.

R, the magnitude of an earthquake on the Richter scale, is given by $R = \log I$, where I is the intensity of the earthquake measured relative to the smallest seismic activity that can be measured. R is usually rounded to one decimal place.

The location, year and magnitude of three large earthquakes are given in this table.

Earthquake	Meeberrie, 1979	Sumatra, 2005	Valdivia, 1960
R, magnitude	7.2	8.6	9.5

(a) Calculate the magnitude of the 1968 earthquake in Meckering, 130 km from Perth, which had an intensity of 7 940 000. (1 mark)

$$R = \log(7940000)$$
$$= 6.9$$

(b) What was the intensity of the 2005 earthquake in Sumatra, rounded to 3 significant figures? (2 marks)

$$8.6 = \log(I)$$
$$I = 10^{8.6}$$
$$= 398\ 000\ 000$$

(c) The strongest earthquake ever recorded in the world was at Valdivia, Chile, in 1960. The strongest onshore earthquake ever recorded in Australia was at Meeberrie in 1979.

Express the ratio of the intensity of these two earthquakes in the form $1:n$, where $n > 1$. (3 marks)

$$I_m = 10^{7.2} \qquad I_v = 10^{9.5}$$

$$\text{Ratio is } 1:\frac{10^{9.5}}{10^{7.2}} = 1:10^{2.3}$$
$$= 1:199.53$$
$$= 1:200$$

Question 14 **(6 marks)**

The points A, B and C have coordinates (4, 6), (10, -2) and (7, 10) respectively.

(a) Find the vector $\overrightarrow{BC}$. (1 mark)

$$\begin{aligned} \overrightarrow{BC} &= \overrightarrow{OC} - \overrightarrow{OB} \\ &= \begin{bmatrix} 7 \\ 10 \end{bmatrix} - \begin{bmatrix} 10 \\ -2 \end{bmatrix} \\ &= \begin{bmatrix} -3 \\ 12 \end{bmatrix} \end{aligned}$$

(b) Find $|\overrightarrow{AB}|$ (2 marks)

$$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\ &= \begin{bmatrix} 10 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} 6 \\ -8 \end{bmatrix} \\ |\overrightarrow{AB}| &= 10 \end{aligned}$$

(c) The point D divides the line segment CB internally in the ratio 2:3.

Find the position vector of the point D. (3 marks)

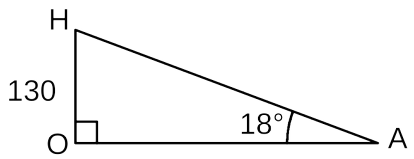
$$\overrightarrow{CB} = \begin{bmatrix} 3 \\ -12 \end{bmatrix}$$

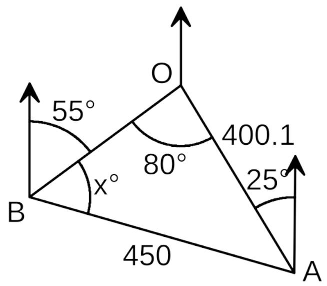
$$\begin{aligned} \overrightarrow{OD} &= \overrightarrow{OC} + \frac{2}{5}\overrightarrow{CB} \\ &= \begin{bmatrix} 7 \\ 10 \end{bmatrix} + \frac{2}{5}\begin{bmatrix} 3 \\ -12 \end{bmatrix} \\ &= \begin{bmatrix} 8.2 \\ 5.2 \end{bmatrix} \end{aligned}$$

Question 15 **(9 marks)**

A helicopter hovering at a height of 130 m above level ground is sighted at the same instant from two points A and B. From A, the helicopter has a bearing of 335° and an angle of elevation of 18°. From B, 450 metres away from A, the bearing of the helicopter is 055°.

(a) Determine the bearing of A from B. (6 marks)





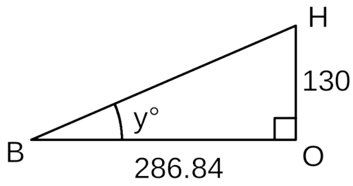
$OA = 130 \div \tan(18) = 400.1$

$$\frac{\sin x}{400.1} = \frac{\sin 80}{450}$$

$x = 61.1$

Bearing is $55 + 61.1 \approx 116^\circ$

(b) Determine the angle of elevation of the helicopter from B. (3 marks)



From diagram 2:

$$\frac{OB}{\sin(38.9)} = \frac{450}{\sin 80}$$

$OB = 286.84$

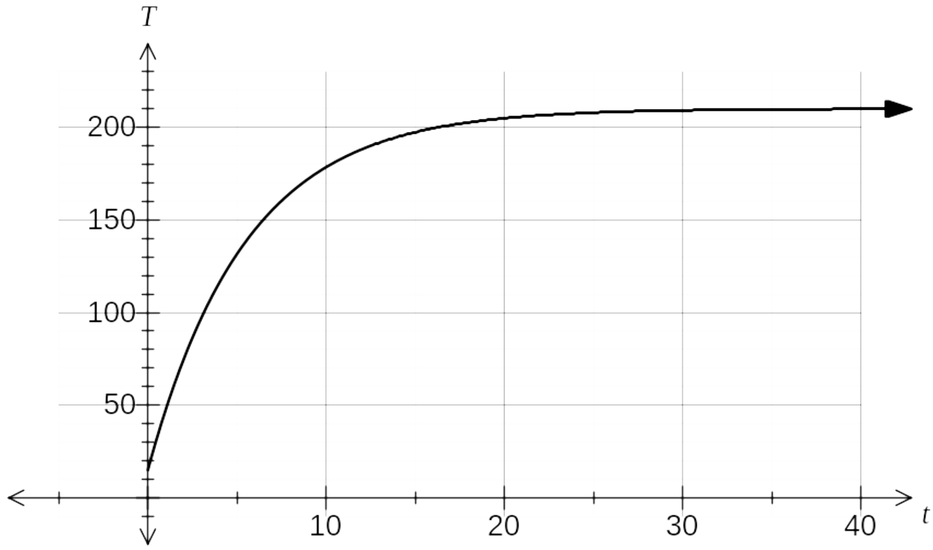
$$\tan y = \frac{130}{286.84}$$

$y = 24.4^\circ$

Question 16 **(8 marks)**

The internal temperature, T ˚C, of a 200 gram potato placed into a hot oven is observed to change with time according to the rule $T = 210 - 195(1.2)^{-t}$, where t is the time, in minutes, after being placed in the oven.

(a) Draw the graph of this relationship on the axes below. (3 marks)



(b) What is the initial temperature of the potato? (1 mark)

$$T = 210 - 195(1.2)^0$$
$$= 15^\circ\text{C}$$

(c) Explain why the temperature of the potato can never exceed 210˚C, according to this model. (1 mark)

As $t \to \infty$, $195(1.2)^{-t} \to 0$ and so $T \to 210^\circ\text{C}$

(d) How long, to the nearest minute, will it take for the temperature of the potato to increase from 100˚C to 200˚C? (3 marks)

$100 = 210 - 195(1.2)^t$ $\quad$ $200 = 210 - 195(1.2)^t$

$t = 3.14$ $\quad$ $t = 16.29$

Time to increase $= 16.29 - 3.14 \approx 13$ minutes

Question 17 **(6 marks)**

Two vectors are given by $\mathbf{p} = 6\mathbf{i} + (3x+7)\mathbf{j}$ and $\mathbf{q} = (x+2)\mathbf{i} + 4\mathbf{j}$.

(a) If $x = -1$, find a vector parallel to $\mathbf{q}$ that has the same magnitude as $\mathbf{p}$. Give your answer in exact form. (3 marks)

$$\mathbf{p} = 6\mathbf{i} + 4\mathbf{j} \qquad \mathbf{q} = \mathbf{i} + 4\mathbf{j}$$
$$|\mathbf{p}| = 2\sqrt{13} \qquad |\mathbf{q}| = \sqrt{17}$$

Required vector is $= \dfrac{\sqrt{17}}{2\sqrt{13}}(\mathbf{i} + 4\mathbf{j})$

$$= \frac{\sqrt{221}}{26}(\mathbf{i} + 4\mathbf{j})$$

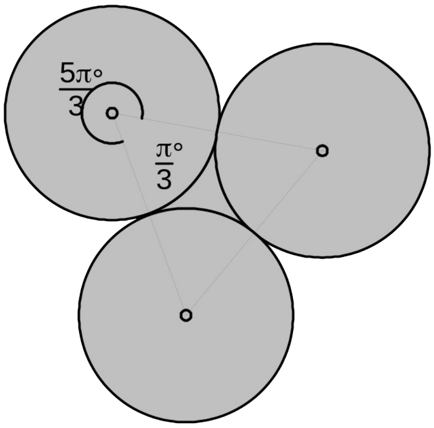
(or in opposite direction)

(b) Determine all possible values of x so that $\mathbf{p}$ and $\mathbf{q}$ are parallel. (3 marks)

$$\frac{6}{x+2} = \frac{3x+7}{4}$$
$$24 = (3x+7)(x+2)$$
$$3x^2 + 13x - 10 = 0$$
$$(3x-2)(x+5) = 0$$
$$x = -5, \ x = \tfrac{2}{3}$$

Question 18 **(8 marks)**

The cross-section of a plastic moulding is shown below. It is made from three circles, each of radius $\frac{15}{\pi}$ cm, that just touch each other.



(a) Find the perimeter of the cross-section. (3 marks)

3 arcs subtending $\frac{5\pi}{3}$

$$3 \times \frac{15}{\pi} \times \frac{5\pi}{3} = 75 \text{ cm}$$

(b) Find the exact area of the cross-section. (5 marks)

3 sectors + 1 triangle (dotted):

$$3 \times \frac{1}{2} \times \left(\frac{15}{\pi}\right)^2 \times \frac{5\pi}{3} \quad + \quad \frac{1}{2} \times \frac{30}{\pi} \times \frac{30}{\pi} \times \sin\left(\frac{\pi}{3}\right)$$

$$= 3 \times \frac{375}{2\pi} \quad + \quad \frac{225\sqrt{3}}{\pi^2} \quad (\approx 3 \times 59.683 + 39.486)$$

$$= \frac{1125}{2\pi} + \frac{225\sqrt{3}}{\pi^2}$$

$$= \frac{1125\pi + 450\sqrt{3}}{2\pi^2} \text{ cm}^2 \quad (\approx 218.54)$$

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