Applecross Senior High School

MATHEMATICS METHODS UNIT 1
Section Two:
Calculator-assumed

SOLUTIONS

Student number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
                          correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
                        and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised material. If you have any unauthorised material with you, hand
it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed   **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9   **(6 marks)**

(a) The points A and B have coordinates $(4,-6)$ and $(5,8)$ respectively. If B is the midpoint of A and C, determine the coordinates of C.   (3 marks)

Solution
If $C(x,y)$ then
$\dfrac{4+x}{2}=5 \Rightarrow x=6$ and $\dfrac{-6+y}{2}=8 \Rightarrow y=22$
$C(6,22)$
Specific behaviours
✓ writes equations for midpoints
✔ x-coordinate of C
✔ y-coordinate of C

(b) The points D and E have coordinates $(5p,-q)$ and $(2q,3p)$ respectively, where p and q are constants. Determine the value of p and the value of q if the midpoint of D and E is at $(21,17)$.   (3 marks)

Solution
$\dfrac{5p+2q}{2}=21$ and $\dfrac{-q+3p}{2}=17$
Solve simultaneously CAS to get $p=10, q=-4$
Specific behaviours
✓ equations for both coordinates of midpoint
✔ value of p
✔ value of q

Question 10 **(8 marks)**

(a) The variables C and x are directly proportional and when $x=5, C=60$.

(i) Determine an equation for the relationship between C and x. (2 marks)

Solution
$C=mx, m=\dfrac{60}{5}=12 \Rightarrow C=12x$
Specific behaviours
✓ indicates use of line through origin
◼ correct relationship

(ii) State the value of C when $x=15$. (1 mark)

Solution
$C=12(15)=180$
Specific behaviours
✓ correct value

(b) The time, t minutes, that a car takes to travel one kilometre at a constant speed of s kmh^{-1} is given by the formula $¿\dfrac{k}{s}$.

(i) Determine the value of the constant k, given that when $s=40, t=90$. (1 mark)

Solution
$90=\dfrac{k}{40} \Rightarrow k=3600$
Specific behaviours
✓ correct value

(ii) Determine the value of t when $s=30$. (1 mark)

Solution
$t=3600 \div 30=120$
Specific behaviours
✓ correct value

(iii) On the axes below, draw a graph to show how s varies with t. (3 marks)

Solution
See graph. Key points:
$(20,180),(30,120),(40,90),$ $(60,60),(90,40),(120,30)$
Specific behaviours
✓ plots at least 3 key points
◼ plots all key points
◼ smooth curve

See next page
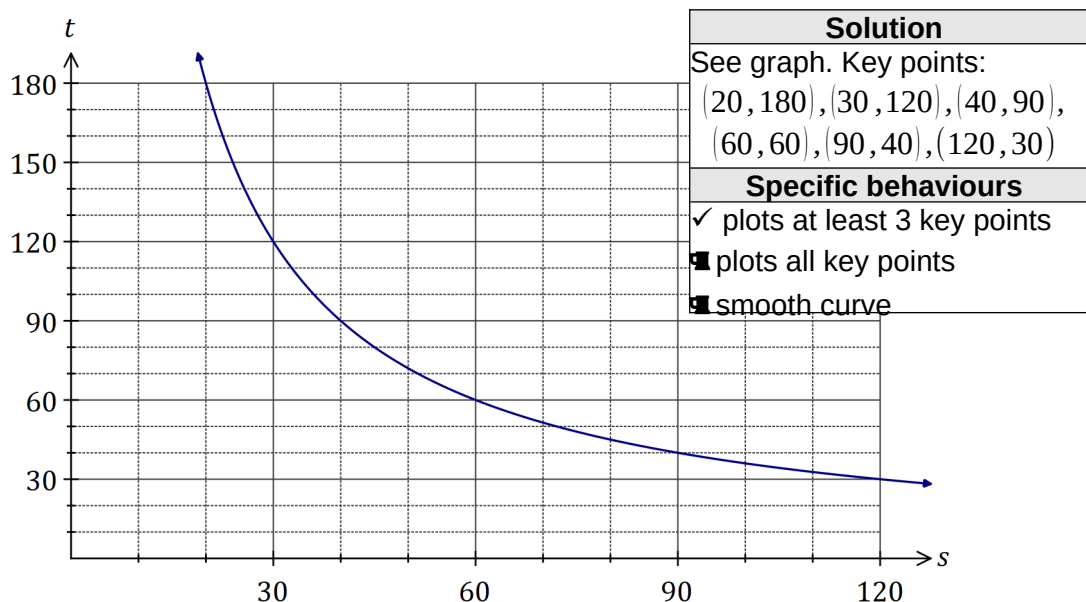
Question 11 (8 marks)

In an experiment, the sound intensity, S, can be modelled by $S(x)=3.6-2.4x+1.5x^2-0.2x^3$, where x is the distance from the sound source in metres and $0\le x\le 6$.
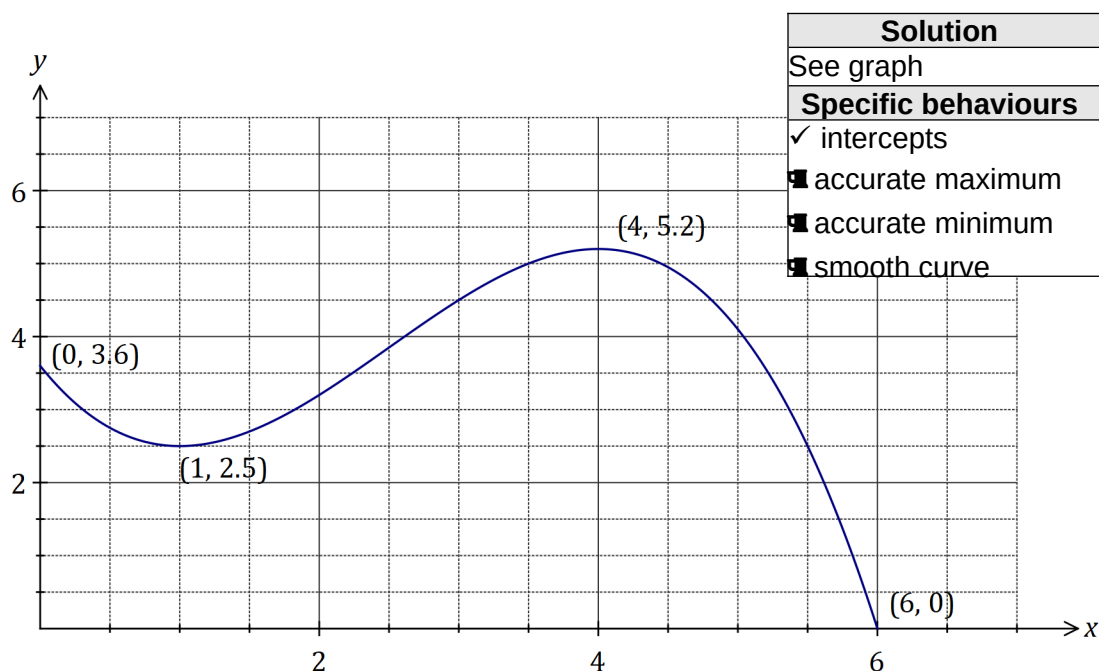
(a) Determine S when $x=3$. (1 mark)

Solution
$S(3)=4.5$

Specific behaviours
✓ correct value

(b) Draw the graph of $y=S(x)$ on the axes below. (4 marks)

Solution
See graph
Specific behaviours
✓ intercepts
❏ accurate maximum
❏ accurate minimum
❏ smooth curve

(c) Determine the equation of the straight line L that passes through the x-intercept and the y-intercept of the graph of $y=S(x)$. (2 marks)

Solution
$(0,3.6)\wedge(6,0)\Rightarrow m=-3.6\div 6=-0.6$
$y=-0.6x+3.6$

Specific behaviours
✓ gradient
❏ y-intercept and equation

(d) Determine the coordinates of the point of intersection of L with the graph of $y=S(x)$ where $x>0$ and $y>0$. (1 mark)

Solution
$(1.5,2.7)$

Specific behaviours
✓ correct coordinates

See next page

Question 12   **(8 marks)**

(a)   A function is given by $f(x) = \dfrac{3}{x+1} + 2$.

   (i)   State the natural domain and corresponding range of f.   (2 marks)

Solution
Domain: $\{x : x \neq -1\}$
Range: $\{y : \neq 2\}$
Specific behaviours
✓ Domain
� Range

   (ii)   The   graph of $f(x)$ is
           dilated vertically by a scale factor of 2. Determine the coordinates of the y-axis
           intercept.   (2 marks)

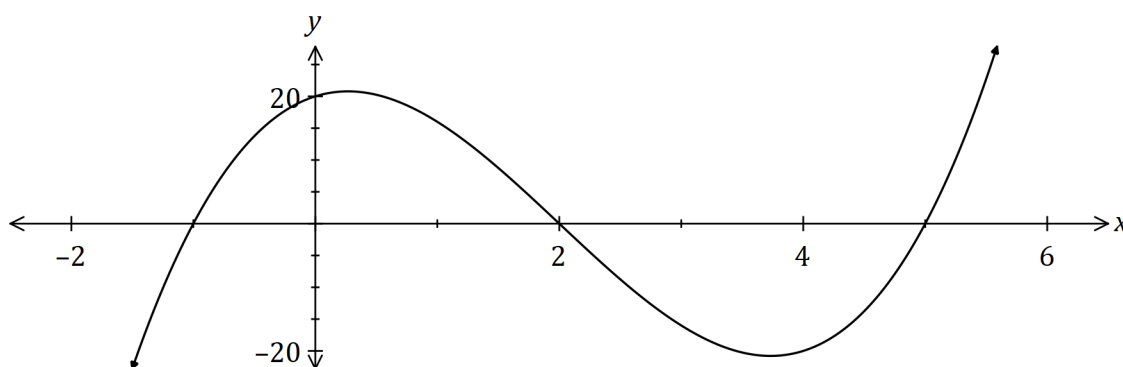| Solution |
|---|
| Y intercept is $(0, 10)$ |
| **Specific behaviours** |
| ✓ ✓ correct coordinates |

   (iii)   The graph of $f(x)$ is translated 3 units to the right. Determine the equation of the
            translated function.   (1 mark)

Solution
$(x-3)\, replaces\, x \in f(x)$
$f(x-3) = \dfrac{3}{x-2} + 2$
Specific behaviours
✓ writes correct equation

(b)   The graph of the   cubic function $y = g(x)$ is
       shown below. Determine $g(10)$.   (3 marks)
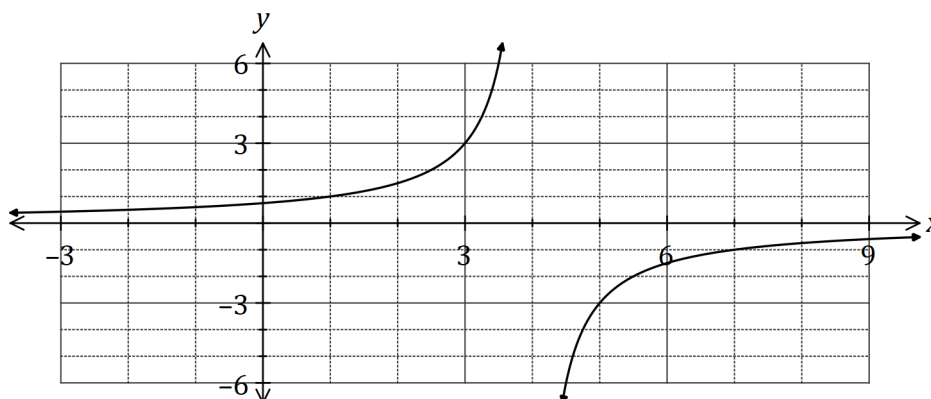

Solution
$g(x) = a(x+1)(x-2)(x-5)$
$20 = a(1)(-2)(-5) \Rightarrow a = 2$
$g(10) = 2(11)(8)(5) = 880$
Specific behaviours
✓ cubic in factored form
� correct value of a
� required value

Question 13   **(8 marks)**

The graph of $y=f(x)$ is shown below where $f(x)=\dfrac{c}{a-x}$.

(a) State the value of the constant a and the value of the constant c.   (3 marks)

Solution
$a=4$
Using $(3,3)\Rightarrow 3=c\div 1\Rightarrow c=3$
Specific behaviours
✓ value of a
✓ uses point on curve
✓ value of c

(b) The hyperbola shown above has two asymptotes. State their equations.   (2 marks)

Solution
$x=4, y=0$
Specific behaviours
✓ vertical asymptote
✓ horizontal asymptote

(c) Describe the transformation required to transform the graph of $y=f(x)$ to obtain the graph of $y=f(x-3)$ and state the domain and range of the transformed function.   (3 marks)

Solution
Translate the graph 3 unit to the right.
Domain: $x\neq 7$
Range: $y\neq 0$
Specific behaviours
✓ transformation
✓ domain
✓ range

See next page

Question 14 **(8 marks)**

(a) Convert, giving an exact answer

(i) 16° to radians. (1 mark)

Solution
$\dfrac{4\pi}{45}$
Specific behaviours
◼ exact value

(ii) 0.4 radians to degrees. (1 mark)

Solution
$\dfrac{72}{\pi}$
Specific behaviours
◼ exact value

(b) Calculate, to the nearest degree, the acute angle between the line $y=1.5x-4$ and the line $y=-0.5x+4$. (3 marks)

Solution
$\theta_1=\tan^{-1}1.5=56.3$
$\theta_2=\tan^{-1}(-0.5)=-26.6$
$\theta_2-\theta_1=83° \, Not \, 97°$
Specific behaviours
✓ indicates use of $\tan^{-1}m$
◼ one correct angle with x-axis
◼ correct angle between lines

(c) The sides adjacent to the right-angle in a right triangle have lengths 65 cm and 72 cm.

If the smallest angle in the triangle is α, determine an exact value for

(i) $\tan\alpha$. (1 mark)

Solution
$\tan\alpha=\dfrac{65}{72}$
Specific behaviours
✓ correct ratio

(ii) $\sin(90°-\alpha)$. (2 marks)

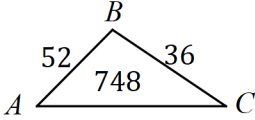| Solution |
|---|
| $\sqrt{65^2+72^2}=97$ |
| $\sin(90°-\alpha)=\dfrac{72}{97}$ |
| **Specific behaviours** |
| ✓ calculates length of hypotenuse |
| ◼ correct ratio for small |

See next page

Question 15   **(7 marks)**

An **obtuse** angled triangle ABC has $a=36$ cm, $c=52$ cm and an area of 748 cm², given the length of b is the longest length of a side of triangle ABC.

(a)   Sketch a triangle to show this information.   (1 mark)

Solution
Specific behaviours
✓ triangle with all information

(b)   Determine the size of $\angle B$.   (2 marks)

Solution
$748 = \dfrac{1}{2}(52)(36)\sin B$ $B = 126.95 \approx 127\,°$
Specific behaviours
✓ substitutes into area equation
✗ correct (obtuse) angle

(c)   Show that $b \approx 79$ cm.   (2 marks)

Solution
$b^2 = 52^2 + 36^2 - 2(52)(36)\cos 126.95$ $b = 79.06 \approx 79\,\text{cm}$
Specific behaviours
✓ uses appropriate equation that includes b
✗ substitutes correctly and solves to at least 1 dp

(d)   Show that $\angle C \approx 32°$.   (2 marks)

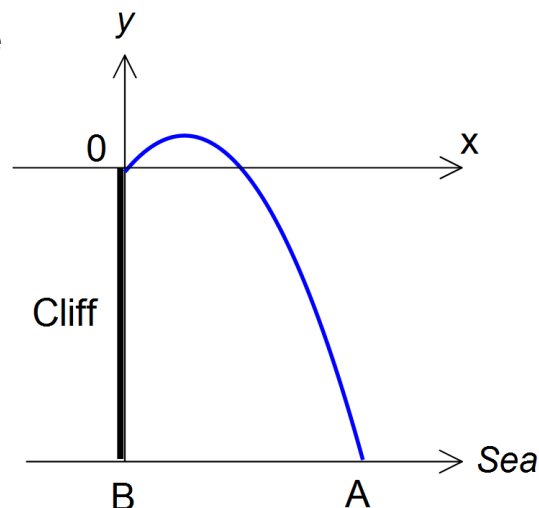| Solution |
|---|
| $\dfrac{52}{\sin C} = \dfrac{79.06}{\sin 126.95}$

 $\angle C = 31.71 \approx 32\,°$ |
| **Specific behaviours** |
| ✓ uses appropriate equation that includes C |
| ✗ substitutes correctly and solves to at least 1 dp |

Question 16 **(8 marks)**

A ball is thrown off the top of a cliff, 100m above sea level. Taking the point of projection O as the origin of the coordinate axes, the path taken by the ball is given as

$$y = 0.1x(30-x).$$

The ball hits the surface of the sea at A.

(a) Find the height above sea level for the highest point reached by the ball. (2 marks)

Solution
Maximum turning point (15, 22.5)
Highest point above sea level is 100 + 22.5 = 122.5 m
Specific behaviours
✓ finds the turning point
🎯 correct distance

(b) Find the distance from A to B, the base of the cliff. (3 marks)

Solution
A: y = -100   x = 50
B: y = -100   x = 0
Distance AB = 50 m
Specific behaviours
✓ ✓ finds correct x coordinate for points A and B
🎯 correct distance AB

(c) Find the horizontal distance from O, correct to one decimal place, when the ball is 110m above sea level. (3 marks)
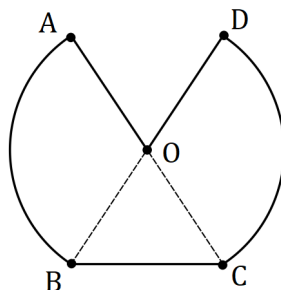
Solution
Y = 10 ⇒ x = 3.82, 26.18
Horizontal distance = 26.18 – 3.82
= 22.36
= 22.4 m (correct to 1 d.p.)
Specific behaviours
✓ understands 110 m above sea level is y = 10
🎯 correct x values
🎯 correct distance

See next page

Question 17 **(7 marks)**

In shape $OABCD$ below, $\angle AOB = 126°$ and AC, BD are diameters of the circle with centre O and radius 35 cm.

(a) Calculate the perimeter of $OABCD$. (4 marks)

Solution
$126° = \dfrac{7\pi}{10}, \pi - \dfrac{7\pi}{10} = \dfrac{3\pi}{10} = 54°$
$AB, DC: 35 \times \dfrac{7\pi}{10} = \dfrac{49\pi}{2} \approx 76.97$
$BC^2 = 35^2 + 35^2 - 2(35)(35)\cos 54 \quad BC \approx 31.78$
$P_{TOTAL} = 2(35) + 2(76.97) + 31.78 \approx 256 \, cm$
Specific behaviours
✸ indicates length of arc AB
✸ indicates use of cosine rule for BC
✸ correct length BC
✸ correct total perimeter

(b) Calculate the area of $OABCD$. (3 marks)

Solution
$AOB + DOC: 2 \times \dfrac{1}{2} \times 35^2 \times \dfrac{7\pi}{10} = \dfrac{1715\pi}{2} \approx 2694$
$BOC: \dfrac{1}{2}(35)(35)\sin 54 \approx 496$
$A_{TOTAL} = 2694 + 496 = 3190 \, cm^2$
Specific behaviours
✸ sector area AOB
✸ triangle area BOC
✸ correct total area

See next page

Question 18   **(6 marks)**

Let $a = \sin 50°$ and $b = \cos 100°$.

Give your answers to the following in terms of a and/or b.

(a)   Write down an expression for

    (i) $\sin 130°$.   (1 mark)

Solution
$\sin 130° = \sin 50° = a$
Specific behaviours
✓ correct expression

    (ii) $\cos 80°$.   (1 mark)

Solution
$\cos 80° = -\cos 100° = -b$
Specific behaviours
✓ correct expression

(b)   Determine an expression for $\cos 130°$.   (3 marks)

Solution
$\cos^2 130° + \sin^2 130° = 1$
$\cos^2 130° = 1 - a^2$
$\cos 130° = \pm\sqrt{1 - a^2}$
but $\cos 130° < 0 \Rightarrow \cos 130° = -\sqrt{1 - a^2}$
Specific behaviours
✓ indicates use of $\sin^2 \theta + \cos^2 \theta = 1$
✓ expression for $\cos^2 130°$
✓ correct expression

(c)   Determine an expression for $\tan 130°$.   (1 mark)

Solution
$\tan 130° = \dfrac{\sin 130°}{\cos 130°} = \dfrac{-a}{\sqrt{1 - a^2}}$
Specific behaviours
✓ correct expression

Question 19   **(8 marks)**

(a)   The equation of the axis of symmetry for the graph of $y=3x^2+6x+7$ is $x=k$. Determine the value of k, using a method that does not refer to the graph of the parabola.

(2 marks)

Solution	
$x=\dfrac{-6}{2\times 3}=-1$ $k=-1$	$y=3(x+1)^2+c$ $\therefore k=-1$
Specific behaviours	
✓ uses $x=-b\div(2a)$ or partially completes the square ◼ value of k	

(b)   A parabola with equation $y=ax^2+bx+c$ has a turning point at $(6,-5)$ and passes through the point $(-2,-37)$. Determine the value of a, the value of b and the value of c.

(3 marks)

Solution
$y=a(x-6)^2-5$ $-37=a(-2-6)^2-5 \Rightarrow a=-0.5$ $y=-0.5(x-6)^2-5$ $¿-0.5x^2+6x-23$ $a=-0.5, b=6, c=-23$
Specific behaviours
✓ correctly writes in turning point form   ◼ solves for a using point   ◼ expands and states all values

(c)   Determine the value of the discriminant for the quadratic equation $16x^2-24x+9=0$ and use it to explain how many solutions the equation $(x+1)(16x^2-24x+9)=0$ will have.

(3 marks)

Solution
$d=(-24)^2-4(16)(9)=0$ When $d=0$, quadratic will have one solution.   Hence equation will have two solutions - one from linear factor and one from quadratic factor.
Specific behaviours
✓ value of discriminant   ◼ uses discriminant to say quadratic will have one solution   ◼ explains why equation has two solutions

Question 20   **(8 marks)**

(a)   The graphs of three functions,

$f(x) = a\sin(x) + b$, $g(x) = c\tan(x + d)$ and $h(x) = \cos(px + q)$

     are shown below.   (6 marks)
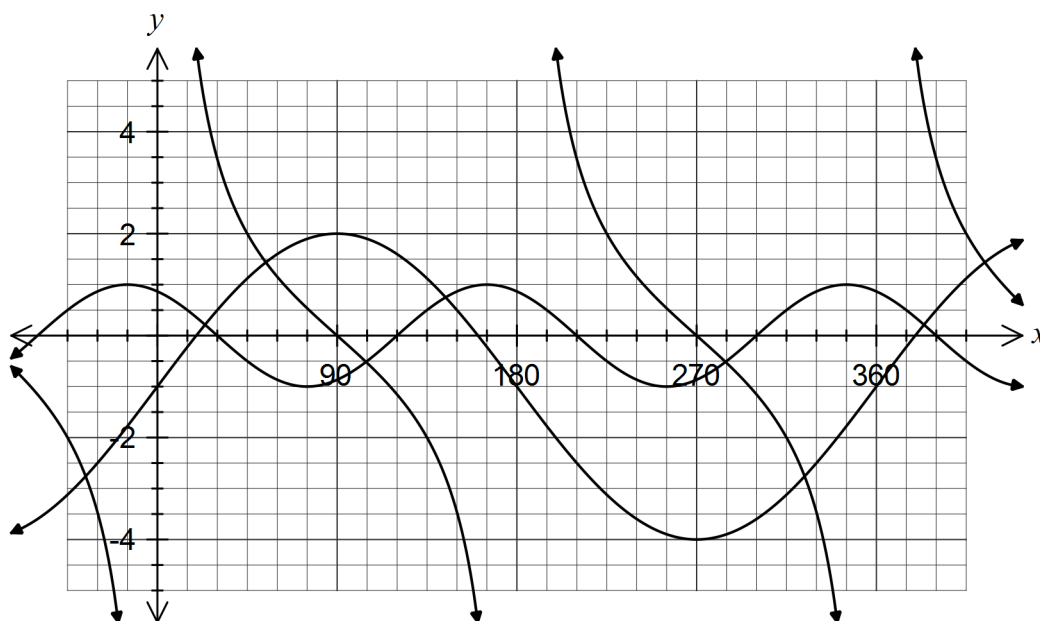
Determine the values of the real constants a, b, c, d, p and q.

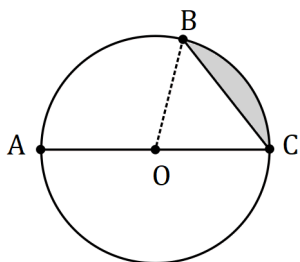| Solution |
| --- |
| $a = 3, b = -1,$ |
| $c = -2, d = 90,$ |
| $p = 2, q = 30°$ |
| **Specific behaviours** |
| ✏ each correct number |

(b)   Determine the equation of a line which is inclined at an angle of 120° to the positive
     x-axis and cuts the y-axis at (0, -2).   (2 marks)

Solution
$m = \tan 120°$
$m = -\sqrt{3}$
$y = -\sqrt{3}\,x - 2$
Specific behaviours
✓ correct gradient
✏ correct equation

Question 21 **(8 marks)**

(a) The circle shown has centre O and diameter AC of length 50 cm. Determine the shaded area given that $2 \times \angle AOB = 3 \times \angle BOC$. (4 marks)

Solution
$\angle AOB = \dfrac{3}{2} \angle BOC$
$\angle BOC + \dfrac{3}{2} \angle BOC = \pi \Rightarrow \angle BOC = \dfrac{2\pi}{5} = 72°$
$A = \dfrac{1}{2}(25)^2 \left(\dfrac{2\pi}{5} - \sin \dfrac{2\pi}{5} \right) ¿ 95.5 \, cm^2$
Specific behaviours
✓ equation using angles
✔ correct angle for segment
✔ substitutes correctly into formula

(b) A sector of a circle has a perimeter of 112 cm and an area of 735 cm². Determine the radius of the circle. (4 marks)

Solution
$2r + r\theta = 112$
$\dfrac{1}{2}r^2\theta = 735$
Solving simultaneously gives
$r = 21, \theta = \dfrac{10}{3}$ or $r = 35, \theta = \dfrac{6}{5}$
Hence $r = 21$ or $r = 35 \, cm$
Specific behaviours
✓ equation for perimeter
✔ equation for area
~~✔ solution of equations~~

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____