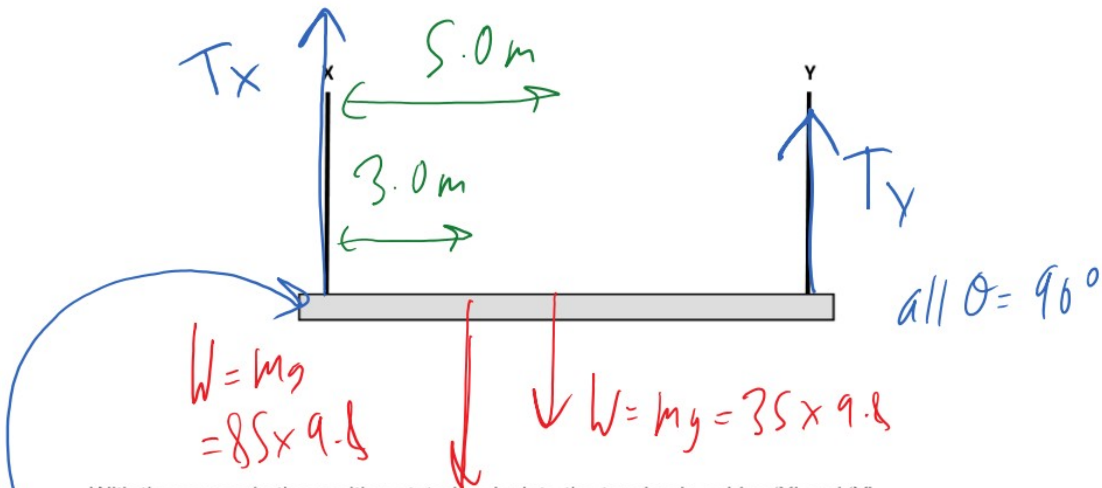
Question 12

A uniform, 35.0 kg horizontal platform is supported by two vertical steel cables 'X' and 'Y' situated 10.0 m apart as shown. A person with a mass of 85.0 kg stands 3.00 m from 'X'.



With the person in the position stated, calculate the tension in cables 'X' and 'Y'. (4 marks)

Taking moments from x $\tau = r.F.\sin\theta$

Σ acwm = Σ cwm

$10 \times T_y = 3 \times (85 \times 9.8) + 5 \times (35 \times 9.8)$

$T_y = 421.4 = 421\ N$ (up)

$\Sigma F_{vertical} = 0$ $\Sigma F_{up} = \Sigma F_{down}$

$T_x + T_y = W_1 + W_2$

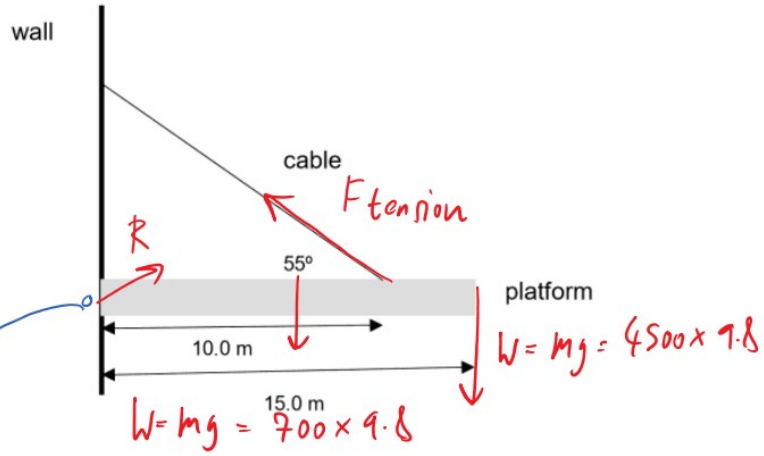
$T_x + 421.4 = (35 + 85) \times 9.8$

$T_x = 754.6\ N$

$T_x = 755\ N$ up

Question 16 **(16 marks)**

A platform has been constructed so people can walk out over a gorge and view it. The platform structure is shown in the diagram below.



The platform is designed to support a load of 4.5 tonnes and is 15 m long. A single steel cable supports the platform and is attached 10.0 m from the end at 55° as shown in the figure. The platform has a mass of 0.7 tonnes that is uniformly distributed. The platform is pivoted to the wall at the left hand edge.

The 4.5 tonne design load is acting at the right-hand edge of the platform.

The steel cable shown has a maximum tensile strength of 1.50×10^5 N.

a) Draw a free-body diagram showing all the forces acting on the platform. Label the forces appropriately on the diagram above.

all correct forces ✓ correct location ✓ labelled ✓

(3 marks)

b) Show that with the design load acting at the right of the platform, the cable will be able to support the platform. Support your answer with calculations to determine the tension in the cable.

(4 marks)

pivot $\tau = r.F.\sin\theta$ $\Sigma M = 0$ ✓

$\Sigma\, acwm = \Sigma\, cwm$ ✓

$10 \times F_t \times \sin 55° = 7.5 \times 700 \times 9.8 \times \sin 90° + 15 \times 4500 \times 9.8 \times \sin 90°$ ✓

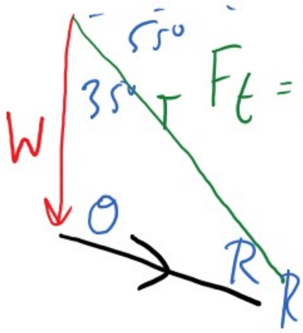
$F_t = 8.70 \times 10^4$ N ✓

this is less than 1.50×10^5 N maximum

c) Calculate the magnitude of the force that the wall exerts on the platform. [If you could not calculate an answer for the previous question, use a value of 8.70 x 10⁴ N for the tension in the cable] (4 marks)

$\Sigma F = 0$ $\quad W = (4500 + 700) \times 9.8$

$W = 50960\ N$



$F_t = 87000\ N$

By Cosine rule

$R^2 = W^2 + F_t^2 - 2W.F_t \cos 35°$

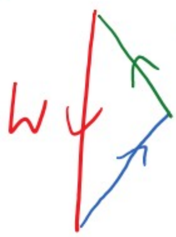
$R^2 = 50960^2 + 87000^2 - 2 \times 50960 \times 87000 \times \cos 35°$

$R = 53874.55 = 5.39 \times 10^4\ N$

d) If the design load of 4.5 tonnes is moved towards the middle of the platform, describe what happens the magnitude and direction of the reaction force you calculated in part c). (2 marks)

Magnitude: decreases

Direction: more upright



F_t decreases

e) If the design load is exceeded, calculate how much mass the platform can support at the right-hand edge before the cable snaps (3 marks)

let $F_t = 1.50 \times 10^5$

$10 \times 1.50 \times 10^5 \times \sin 55° = 7.5 \times 700 \times 9.8 + 15 \times m \times 9.8$

$1177278.066 = 15 \times m \times 9.8$

$m = 8008.69 = 8.01 \times 10^3\ kg$

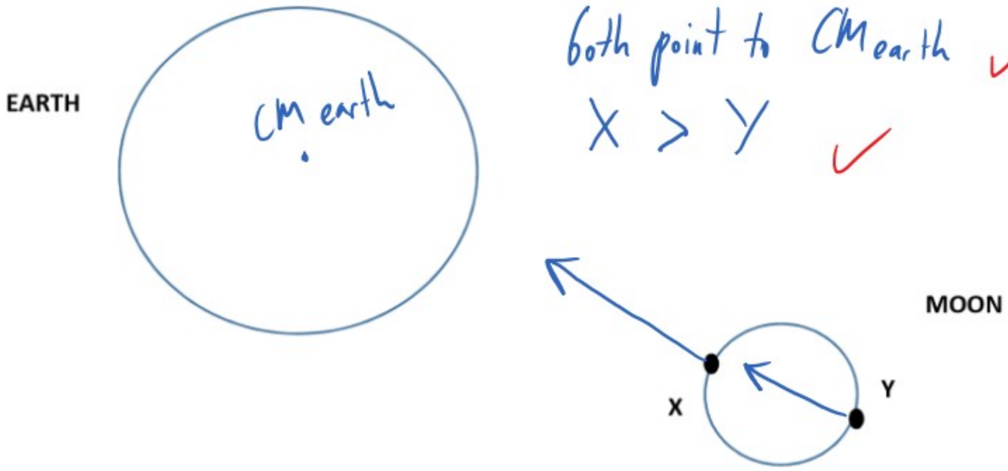
Question 17 **(14 marks)**

In astrophysics, the 'Roche Limit' or 'Roche Radius' is the distance within which a natural satellite (eg – the Moon) will disintegrate due to its host celestial body (eg – the Earth) exerting a 'tidal force' on it. Disintegration occurs because the tidal forces from the host exceed the gravitational force holding the natural satellite together.

Inside the Roche Limit or Roche Radius, satellites break up into particles and dust and typically form rings (eg – like those around Saturn); outside of it, satellites tend to form almost perfect spherical shapes.

a) The diagram below shows the Moon in orbit around the Earth (Not to scale). Consider the two points shown: 'X' and 'Y'. On the diagram, draw vectors with arrows to show the relative magnitude and direction of the Earth's gravitational field acting at these points.
(2 marks)

both point to CM earth ✓
X > Y ✓



b) The 'Roche Radius' for the Moon orbiting around the Earth is 9492 km. With reference to the Formulae and Data Sheet and information in this question, clearly explain why the Moon has not disintegrated. No calculations are required but you must make 2 separate points to justify your response.
(2 marks)

9492 km is less than 3.84×10^8 m
∴ Moon is outside Roche limit ✓
So tidal forces do not exceed gravitational forces holding moon together. ✓

c) The Roche Limit is different if the natural satellite is composed of gas or liquid. Explain whether the value would be higher or lower than for a natural satellite composed of rock.
(2 marks)

The value would be higher as fluids not held together as strongly ✓ as solid rock, so disintegration can occur at a greater separation. ✓

d) Use the data supplied in your Formulae and Data Sheet to calculate the Earth's gravitational field strength at position 'X' (g_x) on the Moon's surface.

(3 marks)

$$r_x = r_{moon-earth} - r_{moon}$$

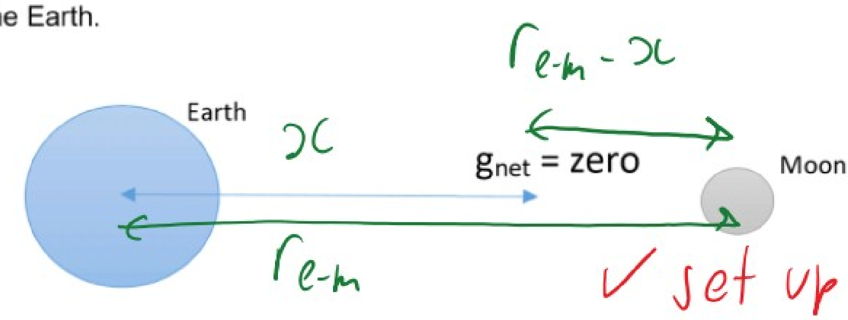
$$r_x = 3.84 \times 10^8 - 1.74 \times 10^6 = 382260000 \text{ m} ✓$$

$$g_x = G\frac{M_{earth}}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(382260000)^2} ✓$$

$$g_x = 2.7251 \times 10^{-3} = 2.73 \times 10^{-3} \text{ N kg}^{-1} ✓$$

e) There is point between the Earth and the Moon where the net gravitational field strength due to the Earth and the Moon is zero. Calculate how far this point is from the centre of the Earth.

(5 marks)



$$g_{earth} = g_{moon}$$

$$\frac{G \times M_{earth}}{x^2} = \frac{G \times M_{moon}}{(r_{em} - x)^2} ✓$$

$$\frac{M_{earth}}{M_{moon}} = \frac{x^2}{(r_{em} - x)^2} ✓$$

$$\sqrt{\frac{M_{earth}}{M_{moon}}} = \frac{x}{r_{em} - x} \rightarrow \sqrt{\frac{5.97 \times 10^{24}}{7.35 \times 10^{22}}} = \frac{x}{r_{em} - x}$$

$$9.012463 = \frac{x}{r_{em} - x} \qquad 9.012463 \times r_{em} - 9.012463x = x ✓$$

$$9.012463 \times r_{em} = 10.012463\, x ✓$$

$$x = \frac{9.012463 \times 3.84 \times 10^8}{10.012463} = 3.46 \times 10^8 \text{ m}$$

Question 20 **(15 marks)**

Our Sun is a medium sized star that is part of a spiral galaxy called the Milky Way. Like all spiral galaxies, the stars in the Milky Way rotate around a galactic centre.

Our Sun's orbit is circular with a radius of 2.50 x 10^{20} m (about 26000 light years); its average orbital speed is 2.20 x 10^{5} m s^{-1}.

a) Demonstrate by calculation that the orbital period of the Sun around the galactic centre of the Milky Way is 2.26 x 10^{8} years.

(3 marks)

$$V = \frac{2\pi r}{T} \qquad T = \frac{2\pi r}{V} = \frac{2\pi \, 2.50 \times 10^{20}}{2.20 \times 10^{5}}$$

$$T = 7.14 \times 10^{15} \, s$$

$$T = \frac{7.14 \times 10^{15}}{365 \times 24 \times 60 \times 60} = 2.26 \times 10^{8} \, yrs$$

b) The circular orbit of the Sun around the galactic centre of the Milky Way is due to the gravitational force of attraction between the Sun's mass and centre of mass of the Milky Way. Calculate the mass of the Milky Way based on this information.

(4 marks)

$$T^2 = \frac{4\pi^2}{GM} r^3 \qquad T = 7.14 \times 10^{15} \, s \qquad r = 2.50 \times 10^{20} \, m$$

$$M = \frac{4\pi^2 r^3}{G . T^2}$$

$$M = \frac{4 \times \pi^2 \times (2.50 \times 10^{20})^3}{6.67 \times 10^{-11} \times (7.14 \times 10^{15})^2}$$

$$M = 1.81 \times 10^{41} \, Kg$$

The planet Mercury is a natural satellite of the Sun. Mercury has a mass of 3.29×10^{23} kg and a radius of 2 440 km. Mercury is travelling around the Sun at an orbital radius of 57 909 050 km

c) Calculation that the centripetal acceleration of Mercury around the Sun.

(3 marks)

$$a = \frac{V^2}{r} = g = \frac{GM}{r^2} \quad (\text{for satellite})$$

$$a = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{(57\,909\,050\,000)^2}$$

$$a = 0.0396 \text{ ms}^{-2}$$

d) Calculate the orbital speed of Mercury around the Sun.

(3 marks)

$$\frac{V^2}{r} = \frac{GM}{r^2} \qquad V = \sqrt{\frac{GM}{r}}$$

$$V = \sqrt{\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{57\,909\,050\,000}}$$

$$V = 4.79 \times 10^4 \text{ ms}^{-1}$$

e) Over billions of years the mass of the Sun will decrease. What effect this will have on the orbit of Mercury? You must refer to the orbital radius and the orbital period in your response.

(2 marks)

less tightly bound by gravity

$\therefore$ radius will increase

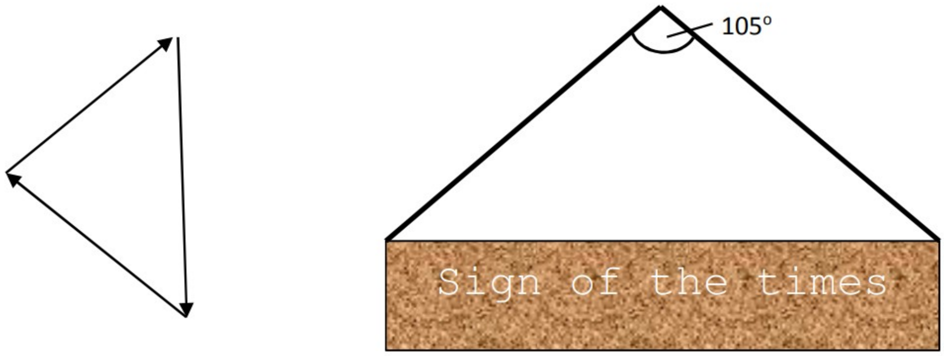
orbital period will increase

(If gravity = zero → Straight line motion

T → infinity r → infinity)

Question 5 **(3 marks)**

A 25.0 kg sign is suspended by two cables as shown in the diagram. The cables make an angle of 105° with each other and have the same length. Find the tension in each cable.



Each cable will have the same tension "T" (symmetry)

$\Sigma F_y = 0$

$2T\sin 37.5$ ✓ $= 25 \times 9.8$ ✓

$T = 201$ N ✓

Question 11 **(3 marks)**

Kepler's third law is defined as: "The square of the orbital period of a planet is directly proportional to the cube of the radius of its orbit", or as is on the data sheet:

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Beginning with Newton's Universal Law of gravitation **and** the formula for centripetal force (both on the data sheet); derive Kepler's third law. *Full working must be shown.*

$F_c = F_g$

$mv^2/r = GMm/r^2$ ✓

$v = 2\pi r/T$

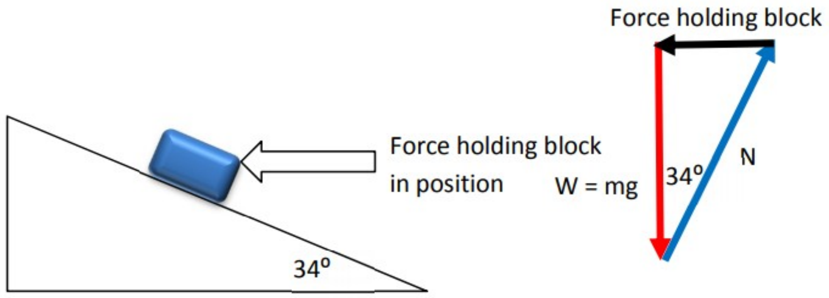
$(2\pi r/T)^2 = GM/r$ ✓

$4\pi^2 r^2/T^2 = GM/r$

$T^2/r^2 = 4\pi^2/GM$ ✓

Question 15 **(4 marks)**

A metal block of mass 12.0 kg is at rest on a frictionless inclined plane. It is held in position by a horizontal force acting as shown on the diagram. The angle of incline of the plane is 34.0°



a) Calculate the magnitude of the holding force

(2 marks)

Reference to vector diagram

$F = mg \tan 34 = 12 \times 9.8 \times \tan 34$ ✓

$F = 79.3$ N ✓

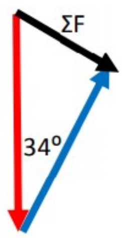
There is a normal reaction force acting on the block both when the holding force is present and when the holding force is withdrawn allowing the block to accelerate along the inclined plane.

b) What happens to the magnitude of the normal reaction force when the holding force is withdrawn? The normal reaction force: (circle a response)

(2 marks)

Increases Stays the same Decreases

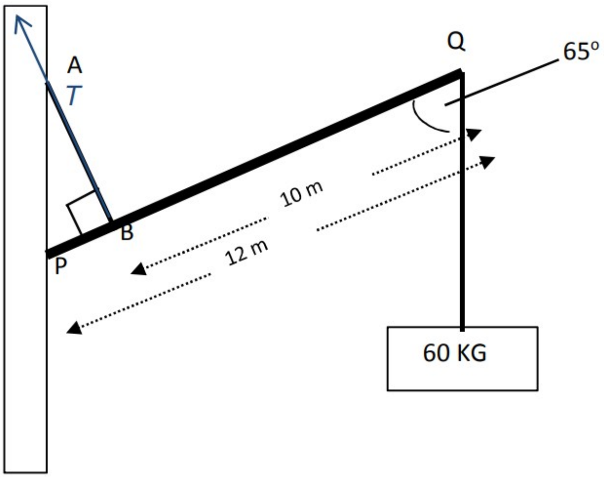
Explain briefly:



Reference to vector diagram
Now only 2 forces act such that sum of forces parallel to plane, previously a component of the normal reaction was acting against the holding force ✓
Any acceptable explanation

Question 18 **(14 marks)**

A 25.0 kg uniform beam PQ is supporting a 60 kg load as shown in the diagram. A cable AB is attached 2.0 m from a frictionless hinge at P, at right angles.



(a) Find the tension in the cable AB for the position shown. (3 marks)

Take torques about P:

$\Sigma\tau_{cw} = \Sigma\tau_{acw}$

$T(2.0) = (245)(6)(\sin 65°) + (588)(12)(\sin 65°)$ ✓✓

$T = 3863.59 = 3{,}860\ \text{N}$ ✓

(b) Find the (reaction) force exerted on the beam by the hinge at P. Be sure to find the magnitude and direction of this force. (5 marks)

Note: T is at 25º to the vertical (90-65)

ΣF_h	$= 0$	ΣF_v	$= 0$
R_h	$= 3863 \times \sin 25º$	R_v	$= 588 + 245 - (3{,}860 \times \cos 25º)$
	$= 1632.82$ N ✓		$= -2{,}668.6$ N (i.e. down) ✓

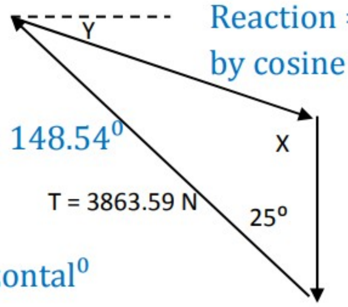
$R^2 \quad = 1632.82^2 + (-2{,}668.6)^2$

$R \quad = 3128.5 = 3{,}130$ N ✓

$\Theta \quad = \tan\text{-}1(2{,}668/1{,}632.82)$ ✓

$\quad\; = 58.5º$ ✓

i.e. Reaction force is 3,128 N, 58.5º below horizontal and to the right.



Reaction = 3128.5 = 3130 N
by cosine rule ✓✓✓

By sine rule angle X = 148.54°
=from vert. ✓✓
Or Y = 148.54 – 90
Y = 58.5° below horizontal°

(c) The beam is then lowered by lengthening cable AB. Explain the effect that this change will have (if any) on the following. Clearly explain your reasoning. (6 marks)

	Change	Explanation
Magnitude of tension in AB	*Increase* ✓	*CW torque increases as both weights lever arm angle is now 90°. T increases to provide opposing ACW torque.* ✓
Horizontal component of reaction force on beam at P.	*Increase* ✓	*As T increases, so does its horizontal component. So R_h must increase to maintain horizontal equilibrium.* ✓
Vertical component of reaction force on beam at P.	*Increase* ✓	*As T increases, so does vertical component of T (which is up). So R_v (which is down) must increase to maintain vertical equilibrium.* ✓