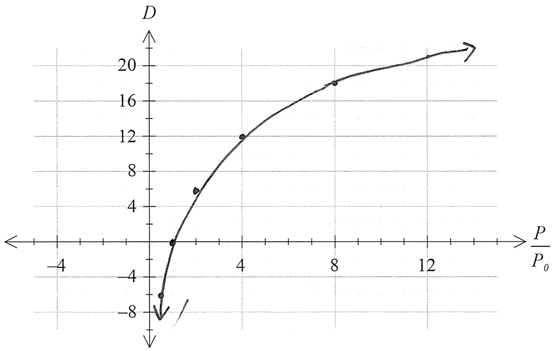
7. (8 marks)

The decibel scale for sound, measured in decibels (dB), is defined as $D = 20\log_{10}\left(\frac{P}{P_0}\right)$, where P is the pressure of the sound being measured and P_0 is a fixed reference pressure.

(a) Complete the table below, giving values rounded to one decimal place. [2]

P	$0.5P_0$	P_0	$2P_0$	$4P_0$	$8P_0$
D	-6.0	0	6.02	12.04	18.06

(b) Sketch the graph of $D = 20\log_{10}\left(\frac{P}{P_0}\right)$ on the axes below labelling all key features [3]



(c) When measured at similar distances, the sound produced by a dishwashing machine measures 47 dB, while that produced by lawn mower measures 96 dB. How many times greater is the sound pressure of the mower to that of the dishwasher? [3]

$47 = 20\log\left(\frac{P}{P_0}\right) \Rightarrow \frac{P}{P_0} = 223.87$

$96 = 20\log\left(\frac{P}{P_0}\right) \Rightarrow \frac{P}{P_0} = 63095.73$

$\therefore \frac{63095.73}{223.87} = 281.83$

≈ 282 times greater

TRINITY COLLEGE

Mathematics Methods Units 3/4
Test 4 2017

Section 1 Calculator Free

Calculus involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME Solutions

DATE: Thursday 20 July **TIME:** 25 minutes **MARKS:** 26

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (4 marks)

Determine the equation of the tangent to the curve $y = x^2 \ln x^2$ at the point $x = 1$

$y = x^2 \ln x^2$

$\frac{dy}{dx} = x^2 \cdot \frac{2x}{x^2} + 2x \ln x^2$

$= 2x + 2x \ln x^2$

Slope at pt $x = 1$

$\left.\frac{dy}{dx}\right|_{x=1} = 2(1) + 2(1)\ln(1)^2$

$= 2$

We need y-coordinate when $x = 1$

$y|_{x=1} = (1)^2 \ln(1)^2$

$= 0$

$\therefore$ pt is $(1, 0)$

Eqn of tangent

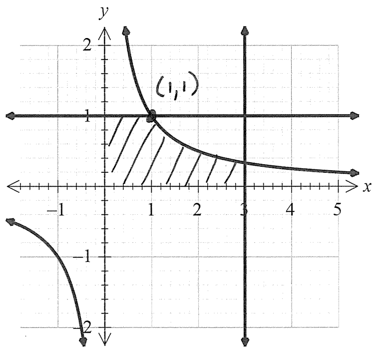
$y - y_p = m(x - x_p)$

$\Rightarrow y - 0 = 2(x - 1)$

OR $y = 2x - 2$

2. (7 marks)

(a) (i) Determine the coordinates of the point of intersection between the curve $y=\frac{1}{x}$ and the line $y=1$ [1]



$1 = \frac{1}{x}$

$\Rightarrow x = 1$

$\therefore$ pt $(1,1)$

(ii) Hence or otherwise, determine the exact area of the region trapped between the curve $y=\frac{1}{x}$, the line $x=3$, the *x-axis*, the *y-axis* and the line $y=1$. [4]

Area $=$ rectangle $+ \int_1^3 \frac{1}{x}\,dx$

$= 1 + \left[\ln|x|\right]_1^3$

$= 1 + \ln 3 - \ln 1$

$= 1 + \ln 3$ units^2

(b) $\int \frac{5x}{x^2-1}\,dx$ [2]

$f(x) = x^2 - 1$

$f'(x) = 2x$

$= \frac{5}{2}\int \frac{2x}{x^2-1}\,dx$

$= \frac{5}{2}\ln|x^2-1| + c$

(f) Determine the mean and variance of $5-2X$. [2]

$\mu = -2 \times 11.25 + 5$

$= -17.5$

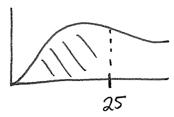
$\sigma^2 = |-2|^2 \times 13.4375$

$= 53.75$

The time (in minutes) that it takes a student to complete a second more challenging puzzle is a random variable Y with a cumulative probability distribution function given by

$F(y) = 1 - \frac{10}{y}$

(g) Determine the probability that it takes a student longer than 25 minutes to complete the second (more challenging) puzzle. [2]



$\therefore P(Y > 25) = 1 - 0.6$

$= 0.4$

$P(Y < 20) = 1 - \frac{10}{25}$

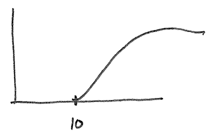
$= 0.6$

(h) Determine the quickest possible time for solving this second (more challenging) puzzle. [2]

Prob is 0 when

$0 = 1 - \frac{10}{y}$

$\Rightarrow y = 10$



$\therefore$ quickest time is when t is just greater than 10.

6. (15 marks)

The time (in minutes) that it takes a student to complete a puzzle is a random variable X with a probability density function given by:

$$f(x) = \begin{cases} \dfrac{20x - x^2}{1125} & 5 \le x \le 20 \\ 0 & \textit{elsewhere} \end{cases}$$

(a) Determine the probability that it takes exactly 6 minutes to complete the puzzle. [1]

$$P(X = 6) = 0$$

(b) Determine the probability that it takes less than 10 minutes to complete the puzzle. [2]

$$\int_5^{10} f(x)\,dx = 0.4074$$

(c) Determine the probability that it takes between 8 and 10 minutes to complete the puzzle given that it takes less than 10 minutes. [2]

$$\frac{\int_8^{10} f(x)\,dx}{\int_5^{10} f(x)\,dx} = \frac{0.1754}{0.4074} = 0.4305$$

(d) Determine the expected time it takes to complete the puzzle. [2]

$$E(x) = \int_5^{20} x\, f(x)\,dx$$

$$= 11.25$$

(e) Determine the standard deviation of the random variable X. [2]

$$\sigma(x)^2 = \int_5^{20} (x - 11.25)^2 f(x)\,dx$$

$$= 13.4375$$

$$\therefore \sigma(x) = 3.67$$

3. (11 marks)

(a) Differentiate each of the following with respect to x.

(i) $y = \dfrac{\ln x}{x^3}$ [3]

$$\frac{dy}{dx} = \frac{(x^3)(\frac{1}{x}) - (\ln x)(3x^2)}{(x^3)^2}$$

(ii) $y = (x + \ln \sin x)^4$ [3]

$$\frac{dy}{dx} = 4(x + \ln \sin x)^3 \times \left(1 + \frac{\cos x}{\sin x}\right)$$

(iii) $y = \ln\sqrt{\dfrac{e^{5x}}{x^2 - 1}}$ [3]

$$= \frac{1}{2}\left(\ln e^{5x} - \ln (x^2 - 1)\right)$$

$$= \frac{1}{2}\left(5x - \ln(x^2 - 1)\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(5 - \frac{2x}{x^2 - 1}\right)$$

(b) If $f(x) = \int_1^x \ln\sqrt{t}\,dt$, determine $f'(e^2)$ [2]

$$\Rightarrow f'(x) = \ln(\sqrt{x})$$

$$\Rightarrow f'(e^2) = \ln\sqrt{e^2}$$

$$= \ln e$$

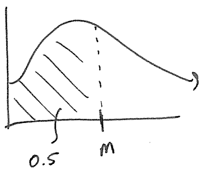
$$= 1$$

4. (4 marks)

A continuous random variable, X , has a probability density function given by

$$f(x)=\begin{cases}\frac{1}{5}e^{-\frac{x}{5}} & x\ge 0\\ 0 & x<0\end{cases}$$

The median of X is m. Determine the exact value of m.



$\Rightarrow \quad 0.5 = \int_0^m \frac{1}{5} e^{-\frac{1}{5}x} \, dx$

$\Rightarrow \quad 0.5 = \left[-e^{-\frac{1}{5}x}\right]_0^m$

$\Rightarrow \quad 0.5 = \left[\frac{-1}{e^{1/5 x}}\right]_0^m$

$\Rightarrow \quad 0.5 = \frac{-1}{e^{m/5}} - \frac{-1}{e^0}$

$\Rightarrow \quad 0.5 = \frac{-1}{e^{m/5}} + 1$

$\Rightarrow \quad \cancel{-}\frac{1}{2} = \cancel{-}\frac{1}{e^{m/5}}$

$\Rightarrow \quad e^{m/5} = 2$

$\Rightarrow \quad m = 5\ln 2$

TRINITY COLLEGE

Mathematics Methods Units 3/4
Test 4 2017

Section 2 Calculator Assumed
Calculus involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME ______________________________

DATE: Thursday 20 July **TIME:** 25 minutes **MARKS:** 29

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (6 marks)

Let $x=\log_n 5$ and $y=\log_n 4$.

(a) Write $x-\frac{y}{2}$ as a single logarithmic term. [2]

$= x - \frac{1}{2}y$

$= \log_n 5 - \frac{1}{2}\log_n 4$

$= \log_n 5 - \log_n 2$

$= \log_n \frac{5}{2}$

(b) Express the following in terms of x and/or y.

(i) $\log_n 100 = \log_n(4\times 5\times 5)$ [2]

$= y + 2x$

(ii) $\log_5 4 = \frac{\log_n 4}{\log_n 5}$ [2]

$= \frac{y}{x}$