PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course ___Methods_Test 2_ Year __12________

Student name: ______________ Teacher name: ______________

Date: 30 March

Task type: **Response**

Time allowed for this task: ____**45**_____ **mins**

Number of questions: ____**9**_____

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: __**46**___ **marks**

Task weighting: __**10**__**%**

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q9 (3.2.6) (6 marks)

(a) Determine $\frac{d}{dx}\left(x(x+1)^{\frac{1}{3}}\right)$.

Solution
$\frac{d}{dx}\left(x(x+1)^{\frac{1}{3}}\right) = x\frac{1}{3}(x+1)^{-\frac{2}{3}} + (x+1)^{\frac{1}{3}}$
Specific behaviours
✓ uses product rule correctly ✓ determines derivative

(b) Using your result from part (a) and without using your classpad determine $\int \frac{x}{3(x+1)^{\frac{2}{3}}}dx$

Solution
$\int \frac{d}{dx}\left(x(x+1)^{\frac{1}{3}}\right)dx = \int x\frac{1}{3}(x+1)^{-\frac{2}{3}}dx + \int (x+1)^{\frac{1}{3}}dx$ $\int \frac{x}{3(x+1)^{\frac{2}{3}}}dx = x(x+1)^{\frac{1}{3}} - \frac{3}{4}(x+1)^{\frac{4}{3}} + c$
Specific behaviours
✓ Uses linearity of antidifferentiation ✓ uses Fundamental Theorem of Calculus ✓integrates (x+1)1/3 term correctly ✓Determines integral with a constant

Q1 (3.2.1-3.2.3) (3 & 3 = 6 marks)

Determine y in terms of x for the following.

a) $\frac{dy}{dx} = 5x^3 - \frac{2}{x^2}$ given that $y = 10$ when $x = 2$.

Solution
$\frac{dy}{dx} = 5x^3 - \frac{2}{x^2} = 5x^3 - 2x^{-2}$ $y = \frac{5x^4}{4} + 2x^{-1} + c$ $10 = \frac{5(16)}{4} + 1 + c$ $c = -11$ $y = \frac{5x^4}{4} + 2x^{-1} - 11$
Specific behaviours
✓ uses negative indices ✓ anti-differentiates ✓solves for constant

b) $\frac{dy}{dx} = \frac{50x^2}{(5 - x^3)^5}$ given that $y = 100$ when $x = 2$.

Solution

Solution
$\dfrac{dy}{dx}=\dfrac{50x^2}{\left(5-x^3\right)^5}$ $y=A\left(5-x^3\right)^{-4}+c$ $y'=-4A\left(5-x^3\right)^{-5}\left(-3x^2\right)$ $50=12A$ $A=\dfrac{25}{6}$ $100=\dfrac{25}{6}(-3)^{-4}+c$ $c=\dfrac{48575}{486}\approx 99.948...$ $y=\dfrac{25}{6}\left(5-x^3\right)^{-4}+\dfrac{48575}{486}$
Specific behaviours
✓ recognises that numerator is proportional to derivative of brackets ✓ solves for multiplier constant ✓solves for added constant, accept approx

Q2 (3.2.21-3.2.22) (4 marks)
A particle travels along a straight line such that its acceleration at time t seconds is equal to $(5t-1)m/s^2$. When $t=1$ the displacement is 22 metres and when $t=3$ The displacement is -10 metres. Determine the displacement when $t=6$.

Solution
$a=(5t-1)$ $v=\dfrac{5t^2}{2}-t+c$ $x=\dfrac{5t^3}{6}-\dfrac{t^2}{2}+ct+k$

b) $\dfrac{d^2y}{dx^2}$

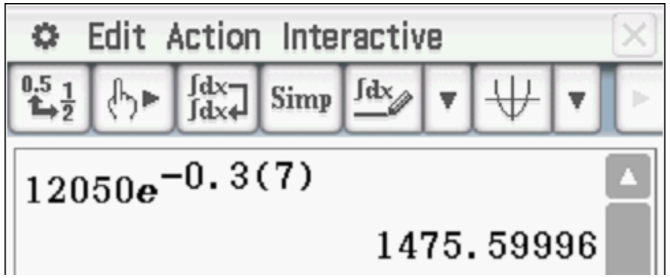
Solution
$3x^2+15(1+4e^{2x})^4 8e^{2x}$
Specific behaviours
✓ uses chain rule correctly ✓ determines derivative

Q8 (3.1.4) (4 marks)
The instantaneous rate of decline in the number of kangaroos on a particular park is 30% of the population per year. If there were 12 050 kangaroos on the park 3 years ago, how many will be on the park in four years from now

Solution

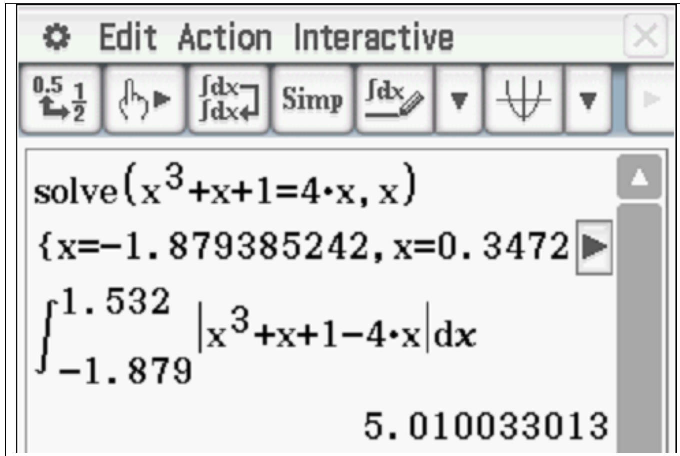
$\dfrac{dN}{dt}=-0.3N$

$N=12050e^{-0.3t}$



Specific behaviours

✓ recognizes exponential decay
✓ uses correct model of rule
✓ uses correct parameters (both)
✓determines final population (no need to round)



Area = 5.01 sq units

Specific behaviours
✓ determines points of intersection ✓ uses integral with difference between functions OR sets up integral from ✓ uses integrals with absolute values ✓determines area no need to round to 2 dp

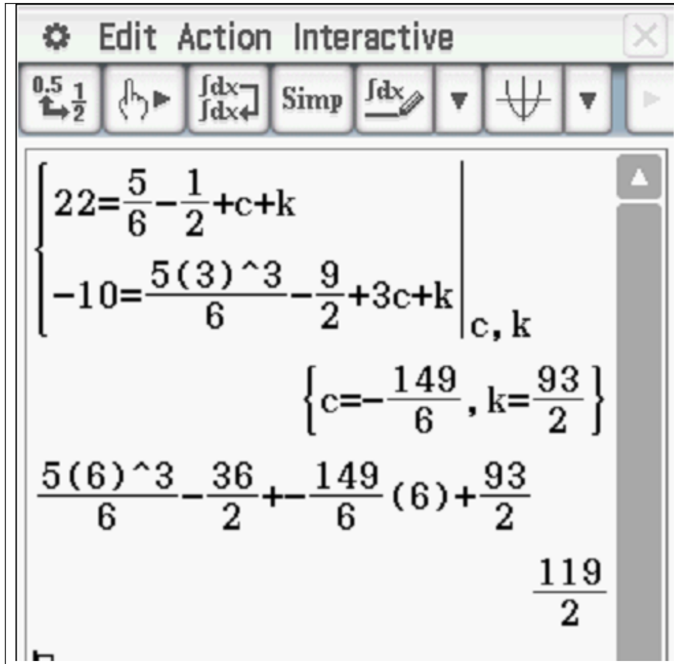
Q7 (3.2.16) (2 & 2 = 4 marks)

Consider $y = \int_0^x t^3 + 3\left(1+4e^{2t}\right)^5 dt$

Determine.

a) $\dfrac{dy}{dx}$

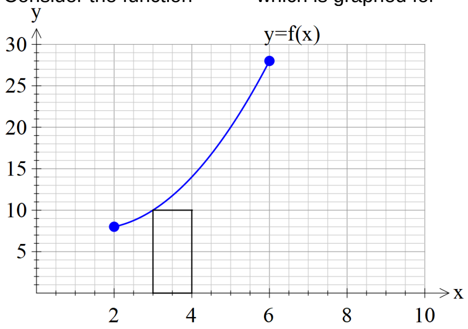
Solution
$\dfrac{d}{dx}\int_0^x t^3 + 3\left(1+4e^{2t}\right)^5 dt = x^3 + 3(1+4e^{2x})^5$
Specific behaviours
✓ uses fundamental theorem ✓ determines derivative in terms of x



Specific behaviours
✓ determines velocity function ✓ determines displacement function with two constants ✓ solves for both constants ✓determines displacement at t=6

Q3 (3.2.10-3.2.11) (2. 2, 1 & 2 = 7 marks)

Consider the function $f(x)$ which is graphed for $2 \le x \le 6$.



a) By using rectangles of width one unit, as shown above, determine a lower estimate for the area under $f(x)$ for $2 \le x \le 6$.

Solution
$8\times1+10\times1+14\times1+20\times1=52$ accept(50 to 54)
Specific behaviours
✓ uses y intercepts from the left of each rectangle ✓ determines sum of areas

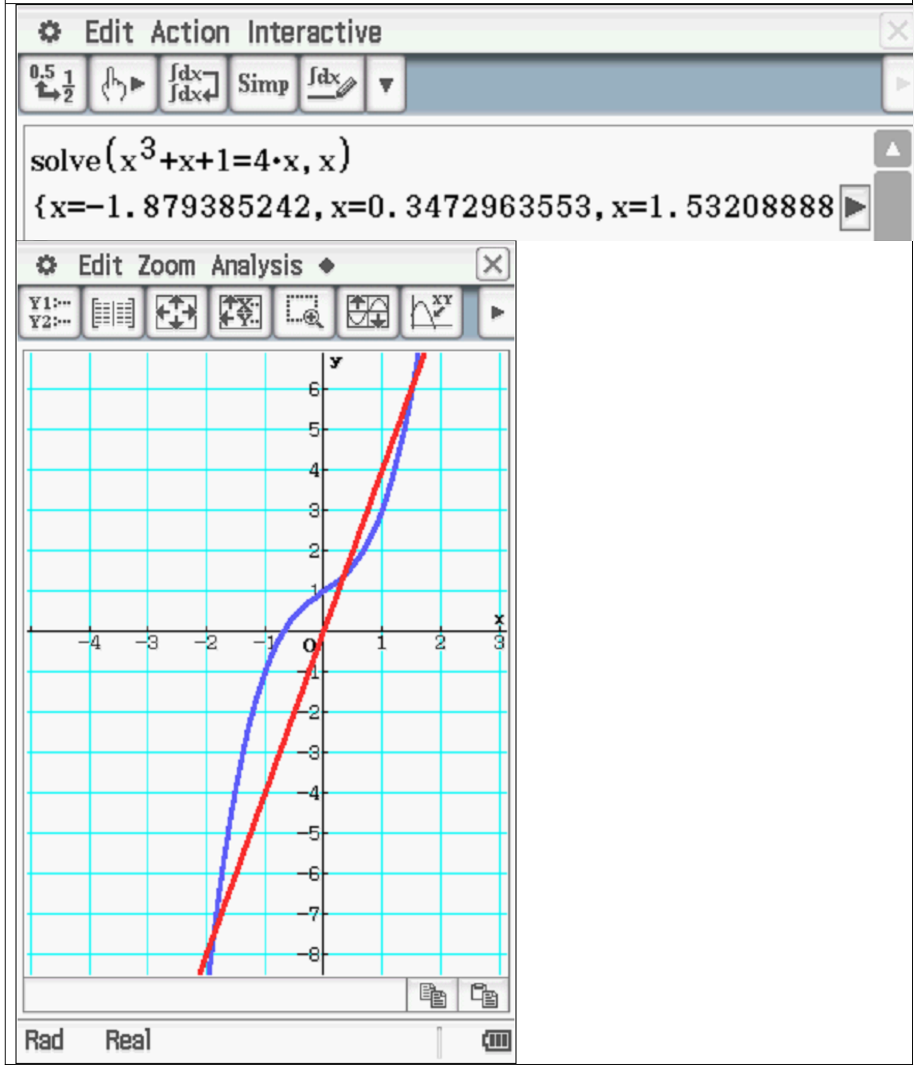
b) By using rectangles of width one unit, as shown above, determine an upper estimate for the area under $f(x)$ for $2 \le x \le 6$.

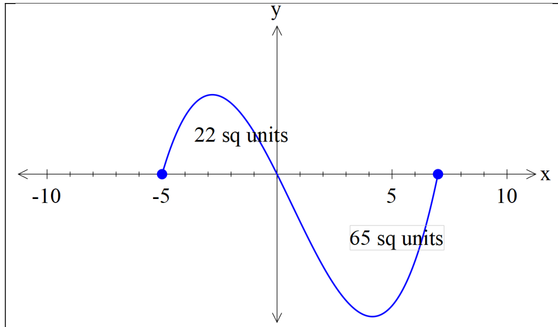
Solution
$10\times1+14\times1+20\times1+28\times1=72$ accept (70 to 75)
Specific behaviours
✓ uses y intercepts from the right of each rectangle ✓ determines sum of areas

Q6 (3.2.20) (4 marks)

Determine to two decimal places the area between the curves $y = x^3 + x + 1$ and $y = 4x$.
(Hint- Sketch the curves first on your classpad)

Solution





Area= 22 + 65=87 sq units
Specific behaviours
✓ uses absolute value of each integral ✓ determines area

c) Determine a better approximation for the area under $f(x)$ for $2 \le x \le 6$.

Solution
$\frac{52+72}{2}=62$
Specific behaviours
✓ determines average

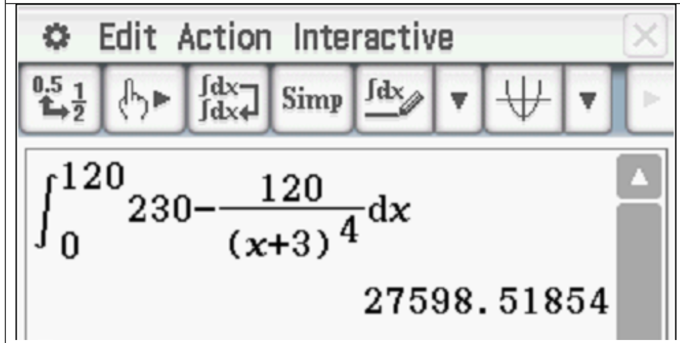
d) Describe two different methods to improve the approximation for the area under $f(x)$ for $2 \le x \le 6$.

Solution
Use rectangles of smaller widths Use calculus with an accurate rule for function Model parabolas for the top of each rectangle and then integrate (Note: Trapezium method is the same as averaging upper & lower rectangles therefore do NOT accept)
Specific behaviours
✓ at least one appropriate method ✓ at least two appropriate methods

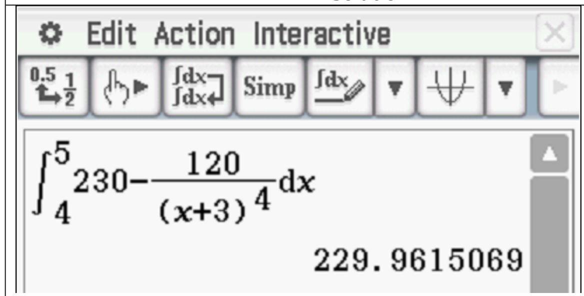
Q4 (3.2.18-3.2.17) (3 & 2 = 5 marks)

An oil tank is drained of oil such that if $V\ kL$ of oil in the tank t seconds after draining commences is described by $\frac{dV}{dt}=230-\frac{120}{(t+3)^4}$.
The initially full tank is emptied in 2 mins.

a) How much oil was in the full tank? (nearest kL)

Solution
 27599 KL
Specific behaviours
✓ uses an integral OR anti-differentiates using 0 to 120 seconds ✓ determines change ✓ rounds change to nearest KL (no need to state units)

b) How much oil was drained from the tank in the fifth second, nearest kL.

Solution
 230 KL
Specific behaviours
✓ sets up integral with correct limits OR uses antiderivative with correct limits ✓ states units with answer (no need for nearest KL)

Q5 (3.2.11-3.2.14) (2, 2 & 2 = 6 marks)

Consider a function $f(x)$ which is only defined for $-5\le x\le 7$ with

$f(-5)=0=f(0)=f(7)$

$f(-4)=8$

$f(-1)=11$

$\int_{-5}^{0} f(x)dx=22$

$\int_{-5}^{7} f(x)dx=-43$

It is known that $f(x)\ge 0$ for $-5\le x\le 0$ and $f(x)\le 0$ for $0<x\le 7$.
Determine.

a) $\int_{-4}^{-1} f'(x)dx$

Solution
$\int_{-4}^{-1} f'(x)dx=\left[f(x)\right]_{-4}^{-1}=f(-1)-f(-4)$ $=11-8=3$
Specific behaviours
✓ uses fundamental theorem ✓ evaluates integral

b) $\int_{0}^{7} f(x)dx$

Solution
$\int_{-5}^{7} f(x)dx=\int_{-5}^{0} f(x)dx+\int_{0}^{7} f(x)dx$ $-43=22+\int_{0}^{7} f(x)dx$ $\int_{0}^{7} f(x)dx=-65$
Specific behaviours
✓ uses linearity principle ✓ solves for required integral

c) The area between $y=f(x)$ and the x axes for $-5\le x\le 7$.

Solution