



Time Allowed : 30 minutes

Marks /32
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*Materials allowed: Formula Sheet.*

*Attempt all questions. Questions 1,2, 3 ,4 and 5 are contained in this section.  
All necessary working and reasoning must be shown for full marks.  
Where appropriate, answers should be given as exact values.  
Marks may not be awarded for untidy or poorly arranged work.*

1.      [2,2,2,2]

Differentiate each of the following with respect to  $x$ , clearly showing the appropriate use of rules.  
Do not simplify answers.

(a)     $y = 4x^3 - \frac{1}{x^2} + \frac{7}{2}x$       (b)     $y = (3x + 2)^3(x^4 - 3)$

(c)     $y = \frac{\sin x}{\cos(3x+2)}$

(d)     $y = \sqrt[3]{(5x - 4)}$

2. [4,6]

Consider the function  $f(x) = x^3(4 - x)$

(a) Use calculus to determine the location of all stationary points.

(b) Use the second derivative to determine the nature of the stationary points and the coordinates of any points of inflection.

(b) If  $h = 6 \text{ cm}$ , then  $V = 6\pi r^2 + \frac{2}{3}\pi r^3$ .

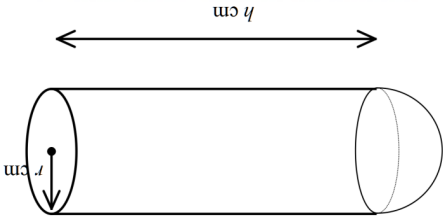
For  $r = 4 \text{ cm}$ ,

show that a small increase of  $k \text{ cm}$  in the radius results in an approximate increase of  $80\pi k \text{ cm}^3$  in the volume.

8. [4 marks]

If  $y = 5t^3$  use differentiation to determine the approximate percentage change in  $y$  when  $t$  changes by 4%.

7. [2,1,4,3]



- A solid wooden peg consists of a cylinder of length  $h$  cm and a hemispherical cap of radius  $r$  cm. The volume,  $V\text{ cm}^3$ , of the peg is given by  $V = \pi r^2 h + \frac{3}{2}\pi r^3$ .
- (a) If the surface area of the peg is  $100\pi\text{ cm}^2$ .

(i) Show that  $h = \frac{100-3r^2}{2r}$

(ii) Determine  $V$  as a function of  $r$ .

(iii) Show the use of calculus to determine the dimensions required to obtain the maximum volume and state the maximum volume.

4. [4 marks]

Determine  $\frac{dy}{dx}$  if  $y = \sqrt{u}$ ,  $u = v^2 + 1$  and  $v = x + x^{-1}$ . Do not simplify your answer.

3. [3 marks]

If  $y = 3 \sin 2x + 2 \cos 2x$  show that  $4y + \frac{d^2y}{dx^2} = 0$

5. [1,1,1,4]

The table below contains information about the sign of  $f(x)$ ,  $f'(x)$  and  $f''(x)$  at seven points on the graph of the continuous function  $f(x)$ . Apart from those in the table, there are no other points where  $f(x)$ ,  $f'(x)$  or  $f''(x)$  are equal to zero.

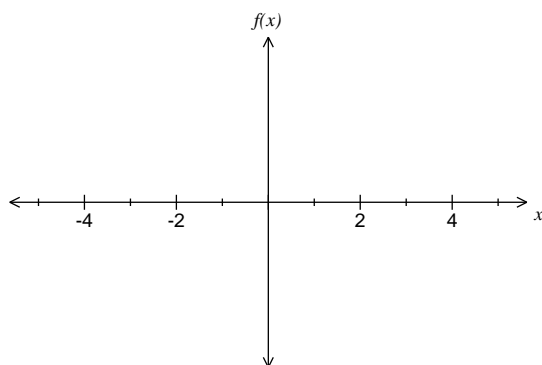
$x$	-3	-1	0	1	2	3	4
$f(x)$	—	0	+	+	+	0	—
$f'(x)$	+	0	+	+	0	—	—
$f''(x)$	—	0	+	0	—	—	—

(a) Describe the nature of the graph when  $x=2$

(b) At what value(s) of  $x$  is  $f(x)$  concave up?

(c) Describe the nature of the graph when  $x=-1$ .

(d) Sketch the function on the axes below.



SHENTON  
COLLEGE

# ATMAM Mathematics Methods

Test 1 2019

Calculator Assumed

Name: .....

Teacher (Please circle name)    Ai    Friday    Smith

Time Allowed : 20 minutes

Marks /19

**Materials allowed:** Classpad calculator, Formula Sheet.

**Attempt all questions. Questions 6, 7 and 8 are contained in this section.**

**All necessary working and reasoning must be shown for full marks.**

*Where appropriate, answers should be given as exact values.*

*Marks may not be awarded for untidy or poorly arranged work.*

6. [1,1,1,2]

A particle is moving in a straight line so that at time  $t$ , in seconds, its position from the origin  $O$  is given by  $x(t) = 7.2 - 3 \cos(0.65t)$  metres,  $t \geq 0$

(a) State the initial position of the particle.

(b) Determine the velocity function for this particle.

(c) At what time does the particle first come to rest after  $t = 0$ ?

(d) At what time does the particle first reach its maximum velocity? Justify your choice.