

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name

Teacher's name

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

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See next page

Question 10

(2 marks)

If $y = e^{\cos(x)}$ and $\frac{dy}{dx} = \frac{-1}{\cos(x)}$, then find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{3}$.

$\frac{dy}{dx} = \frac{dy}{dx} \times \frac{dx}{dt}$ but $\frac{dy}{dx} = -\sin(x) e^{\cos(x)}$

$\frac{dy}{dt} = -\sin(x) e^{\cos(x)} \times \left(\frac{-1}{\cos(x)}\right)$

$\frac{dy}{dt} = \frac{\sin(x)}{\cos(x)} e^{\cos(x)}$

or $\frac{dy}{dt} = \tan(x) e^{\cos(x)}$ ✓ (check)

$\left. \frac{dy}{dt} \right|_{x=\pi/3} = \sqrt{3}e \approx 2.855$ ✓ (answ)

2

End of questions

Question 1

(3 marks)

Consider the following discrete probability distribution for the random variable X .

x	1	2	3	4	5
$P(X = x)$	p	2p	3p	4p	5p

(a) Find the value of p.

(1 mark)

(b) Hence, find $E(X)$.

(2 marks)

See next page

Question 2

(6 marks)

- (a) If $\frac{dy}{dx} = e^{3x}$, then find an expression for y , if $y = 1$ when $x = 0$. (2 marks)

- (b) Determine $\frac{d}{dx}(5x^2e^{x^3})$. There is no need to simplify your answer.

(2 marks)

See next page

Question 8

(3 marks)

At the corner cafe, 70% of customers order a cappuccino and 30% order a latte. Of the customers who order a cappuccino, 60% order scrolls, and 25% of customers who order a latte order the scrolls.

Determine the probability that, if a scroll is ordered, the customer also ordered a cappuccino. Write your answer as a simplified fraction.

$$\begin{array}{c}
 \begin{array}{c}
 0.7 \swarrow \quad \searrow \\
 C \quad \quad \quad \bar{C} \\
 \begin{array}{c}
 0.6 \rightarrow S \\
 0.4 \rightarrow \bar{S} \\
 0.25 \rightarrow S \\
 0.75 \rightarrow \bar{S}
 \end{array}
 \end{array}
 \end{array}$$

$$P(C|S) = \frac{P(C \cap S)}{P(S)}$$

$$\begin{aligned}
 \checkmark_{(num)} &= \frac{0.7 \times 0.6}{0.7 \times 0.6 + 0.3 \times 0.25} \\
 \checkmark_{(denom)} &= \frac{0.42}{0.495} \\
 &= \frac{28}{33} \quad \checkmark \text{ (simplified fraction)} \\
 &= 0.84
 \end{aligned}$$

Question 9

(2 marks)

A discrete random variable X has the probability function $P(X = x) = k(1 - k)^x$ where $x > 0$. Write an expression for $P(X > 1)$ in terms of k , leaving your answer in simplified form.

$$\begin{aligned}
 P(X > 1) &= 1 - [P(X = 1)] \quad \text{Given } x > 0 \\
 &= 1 - (k(1 - k)^1) \quad \checkmark \text{ (complement in k's)} \\
 &= 1 - (k - k^2) \\
 &= k^2 - k + 1 \quad \checkmark \text{ (simplified)} \\
 \text{OR} \quad &= k(k - 1) + 1 \quad \text{either} \\
 &\quad \text{See next page}
 \end{aligned}$$

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Question 7

(5 marks)

A new casino gambling game is being developed. It costs \$5 to play a game. When two dice are rolled, if the uppermost numbers total 10 or more, or 4 or less, you are paid \$10. (Your \$5 cost to play and a further \$5) If the total score of 7 occurs, you receive your \$5 back. All other outcomes result in the loss of your \$5.

If X represents the amount won,

(a) complete the probability distribution for X .

(3 marks)

x	$p(X = x)$
\$5	$\frac{12}{36}$ $\left(\frac{1}{3}\right)$
\$0	$\frac{6}{36}$ $\left(\frac{1}{6}\right)$
\$-5	$\frac{18}{36}$ $\left(\frac{1}{2}\right)$

$$p(10, 11, 12) + p(4, 3, 2) = \frac{6}{36} + \frac{6}{36}$$

(b) calculate the expected amount you will win or lose if you play 6 games.

(2 marks)

$$E(X) = 5\left(\frac{1}{3}\right) + 0\left(\frac{1}{6}\right) - 5\left(\frac{1}{2}\right)$$

$$= -\frac{5}{6} \quad \checkmark E(X)$$

ie losing $\frac{5}{6}$ per game

\therefore $\overline{\$5 \text{ loss}}$ for 6 games \checkmark (answ)

5

See next page

See next page

Question 2 continued

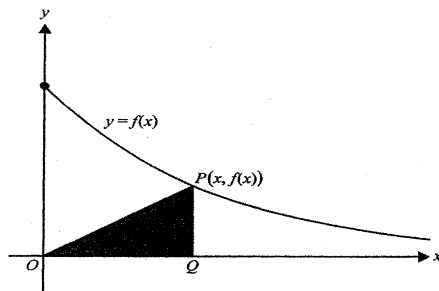
(c) Determine $\int_0^{\pi} 2\cos(3x)dx$

(2 marks)

Question 3

(6 marks)

Let $f(x) = 2e^{-\frac{x}{5}}$. A right-angled triangle OQP has point O at the origin, point Q on the x -axis and point P on the graph of f , as shown. The coordinates of P are $(x, f(x))$.



- (a) Find an expression for the area, A , of the triangle OQP in terms of x . (2 marks)

- (b) Find the maximum area of triangle OQP and the value of x for which the maximum occurs. (4 marks)

End of questions

Question 6

(7 marks)

Scientists are studying a complex biological process. They have measured the **rate of change** of concentration of two enzymes, A and B , over a period of time and have found that these rates are approximately modelled by the two functions

Enzyme A: $R_A'(t) = 4e^{-0.5t}, t \geq 0$

Enzyme B: $R_B'(t) = \frac{8}{t^2+2}, t \geq 0$

Where $R_A'(t)$ is the rate of change of concentration of enzyme A and $R_B'(t)$ is the rate of change of concentration of enzyme B after t minutes. Both rates of change of concentrations are measured in grams per litre per minute.

- (a) Find the initial rate of change of concentration of each of the enzymes. (2 marks)

$$R_A'(0) = 4 \text{ g/L/min} \quad \checkmark$$

$$R_B'(0) = 4 \text{ g/L/min} \quad \checkmark$$

- (b) State the derivative of $R_A'(t)$. (1 mark)

$$R_A''(t) = -2e^{-t/2} \quad \checkmark$$

- (c) Determine if the functions $R_A'(t)$ and $R_B'(t)$ are increasing, decreasing or neither for $t > 0$. (2 marks)

$$R_A'(t) \Rightarrow \text{Decreasing} \quad \checkmark$$

$$R_B'(t) \Rightarrow \text{Decreasing} \quad \checkmark$$

- (d) State the time(s), to 4 decimal places, where $R_A'(t) = R_B'(t)$ for $0 < t < 10$. (2 marks)

$$\begin{aligned} t &= 1.4817 \text{ min} \quad \checkmark \\ t &= 5.6853 \text{ min} \quad \checkmark \end{aligned}$$

(-1 if not to 4 dp)

See next page

Question 5

(6 marks)

The table shows the probability distribution of a discrete random variable X .

x	1	2	3	4
$P(X = x)$	$\frac{2}{p}$	$2p$	$p + 0.1$	p

Calculate:

(a) the value of p . (1 mark)

$$\frac{2}{p} + 2p + p + 0.1 + p = 1 \Rightarrow p = 0.2$$

(b) $E(X^2)$ (1 mark)

$$E(X^2) = 1^2 \cdot \frac{2}{p} + 2^2 \cdot 2p + 3^2 \cdot (p + 0.1) + 4^2 \cdot p = 7.6$$

(c) $E(2X + 1)$ (2 marks)

$$E(X) = 2.6$$

$$\text{Grad. } \checkmark (E(X))$$

(d) $\text{Var}(2X + 1)$ (2 marks)

$$\therefore E(2X + 1) = 2(2.6) + 1 = 6.2$$

$$\checkmark (E(2X + 1))$$

$$\left\{ \begin{array}{l} \sigma^2 = 0.84 \\ \text{Grad. } \checkmark (\text{var}) \end{array} \right.$$

$$\therefore \text{Var}(2X + 1) = 2^2(0.84)$$

$$= 3.36 \checkmark (\text{answ})$$

See next page

6

Additional working space

Question number: _____

See next page

Additional working space

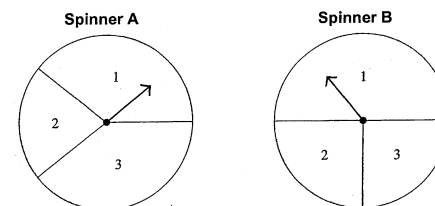
Question number: _____

Question 4

(5 marks)

Two different spinners are constructed by dividing a circular card into 3 sectors scoring 1, 2 and 3. Each spinner has a rotating pointer pivoted at the centre, as shown below.

(Diagrams are not drawn to scale)



Spinner A has angles 135° , 90° and 135° for the sectors scoring 1, 2 and 3 respectively.

Spinner B has angles 180° , 90° and 90° for the sectors scoring 1, 2 and 3 respectively.

After being set in motion, the pointers come to rest independently in random positions.

The random variable X is the **larger** of the two scores if they are different, and their common value if they are the same.

- (a) Find $P(X = 2)$.

	Spinner A	Spinner B
$P(1)$	$\frac{3}{8}$	$\frac{1}{2}$
$P(2)$	$\frac{1}{4}$	$\frac{1}{4}$
$P(3)$	$\frac{3}{8}$	$\frac{1}{4}$

(3 marks)

$$\begin{aligned}
 P(X=2) &= P(1,2) + P(2,1) + P(2,2) \quad \checkmark \text{ (All combinations)} \\
 &= \left(\frac{3}{8} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{2}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right) \quad \checkmark \text{ (working)} \\
 &= \frac{9}{32} \quad (0.28125) \quad \checkmark \text{ (Answer)}
 \end{aligned}$$

- (b) Complete the probability distribution for X .

(2 marks)

x	1	2	3
$P(X = x)$	$\frac{3}{8} \times \frac{1}{2} = \frac{6}{32}$	$\frac{9}{32}$	$\frac{17}{32}$

OR $\frac{3}{16} \checkmark$ See next page \checkmark
 0.1875 0.28125 0.53125

5

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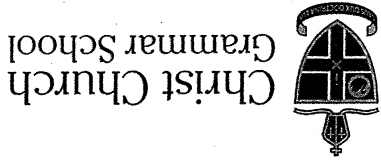
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See next page



2020
TEST 3

MATHEMATICS METHODS Year 12

Section Two:

Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section
Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

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Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

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See next page

**Christ Church
Grammar School****2020
TEST 3****MATHEMATICS METHODS Year 12****Section Two:****Calculator-assumed**Your name • SOLUTIONS •

Teacher's name _____

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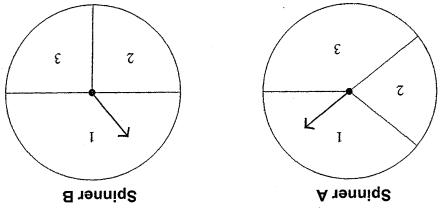
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(5 marks)

Two different spinners are constructed by dividing a circular card into 3 sectors scoring 1, 2 and 3. Each spinner has a rotating pointer pivoted at the centre, as shown below.

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Spinner A has angles 135° , 90° and 135° for the sectors scoring 1, 2 and 3 respectively. **Spinner B** has angles 180° , 90° and 90° for the sectors scoring 1, 2 and 3 respectively. After being set in motion, the pointers come to rest independently in random positions. The random variable X is the **larger** of the two scores if they are different, and their common value if they are the same.

(a) Find $P(X = 2)$. (3 marks)

x	$P(X = x)$
1	
2	
3	

(b) Complete the probability distribution for X . (2 marks)

See next page

Question number: _____

Additional working space

Question 5

(6 marks)

The table shows the probability distribution of a discrete random variable X .

x	1	2	3	4
$P(X = x)$	$\frac{p}{2}$	$2p$	$p + 0.1$	p

Calculate:

(a) the value of p .

(1 mark)

(b) $E(X^2)$

(1 mark)

(c) $E(2X + 1)$

(2 marks)

(d) $Var(2X + 1)$

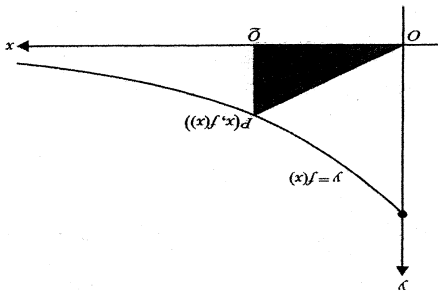
(2 marks)

Additional working space

Question number: _____

Question 3

Let $f(x) = 2e^{-\frac{x}{5}}$. A right-angled triangle OQP has point O at the origin, point Q on the x -axis and point P on the graph of f , as shown. The coordinates of P are $(x, f(x))$.



- (a) Find an expression for the area, A , of the triangle OQP in terms of x . (2 marks)

$$A = \frac{1}{2} b \times h$$

$$A = \frac{1}{2} x \times 2e^{-x/5}$$

$$A = x \cdot e^{-x/5}$$

- (b) Find the maximum area of triangle OQP and the value of x for which the maximum occurs. (4 marks)

$$\frac{dA}{dx} = e^{-x/5} + x(-1/5 e^{-x/5})$$

$$= e^{-x/5} (1 - x/5)$$

$$\frac{dA}{dx} = 0 \Rightarrow x = 5$$

$$\text{When } x = 5 \Rightarrow A = 5e^{-1}$$

$$A = \frac{e}{5} \text{ units}^2$$

End of questions

(correct value)

6

Question 6

Scientists are studying a complex biological process. They have measured the rate of change of concentration of two enzymes, A and B, over a period of time and have found that these rates are approximately modelled by the two functions

$$\text{Enzyme A: } R_A'(t) = 4e^{-0.5t}, t \geq 0$$

$$\text{Enzyme B: } R_B'(t) = \frac{t}{t^2+2}, t \geq 0$$

- Where $R_A'(t)$ is the rate of change of concentration of enzyme A and $R_B'(t)$ is the rate of change of concentration of enzyme B after t minutes. Both rates of change of concentrations are measured in grams per litre per minute.
- (a) Find the initial rate of change of concentration of each of the enzymes. (2 marks)

- (b) State the derivative of $R_A'(t)$ (1 mark)

- (c) Determine if the functions $R_A'(t)$ and $R_B'(t)$ are increasing, decreasing or neither for $t > 0$. (2 marks)

- (d) State the time(s), to 4 decimal places, where $R_A'(t) = R_B'(t)$ for $0 < t < 10$. (2 marks)

See next page

Question 7

(5 marks)

A new casino gambling game is being developed. It costs \$5 to play a game. When two dice are rolled, if the uppermost numbers total 10 or more, or 4 or less, you are paid \$10. (Your \$5 cost to play and a further \$5) If the total score of 7 occurs, you receive your \$5 back. All other outcomes result in the loss of your \$5.

If X represents the amount won,

- (a) complete the probability distribution for X .

(3 marks)

x			
$P(X = x)$			

- (b) calculate the expected amount you will win or lose if you play 6 games.

(2 marks)

See next page

Question 2 continued

- (c) Determine $\int_0^{\pi/6} 2\cos(3x)dx$

(2 marks)

$$= \left[\frac{2\sin(3x)}{3} \right]_0^{\pi/6} \quad \checkmark \text{ (Anti-diff)}$$

$$= \frac{2\sin \pi/2}{3} - 0$$

$$= \underline{\underline{2/3}} \quad \checkmark \text{ (Ans)} \quad \underline{2}$$

See next page

Question 2

- (a) If $\frac{dy}{dx} = e^{3x}$, then find an expression for y , if $y = 1$ when $x = 0$. (2 marks)

$$\int e^{3x} dx = \frac{e^{3x}}{3} + C \quad (\text{Anti-diff})$$

$$1 = \frac{e^0}{3} + C$$

$$\therefore C = \frac{2}{3}$$

$$y = \frac{1}{3} e^{3x} + \frac{2}{3} \quad (\text{Ans})$$

- (b) Determine $\frac{d}{dx}(5x^2 e^{x^3})$. There is no need to simplify your answer.

$$y' = u'v + uv'$$

$$\frac{dy}{dx} = 10x \cdot e^{x^3} + 5x^2 \cdot (3x^2)e^{x^3}$$

$$\text{or } 5x e^{x^3} (2 + 3x^3) \quad (\text{not required})$$

See next page

4

Question 8

At the corner cafe, 70% of customers order a cappuccino and 30% order a latte. Of the customers who order a cappuccino, 60% order scrolls, and 25% of customers who order a latte order the scrolls.

Determine the probability that, if a scroll is ordered, the customer also ordered a cappuccino. Write your answer as a simplified fraction.

Question 9

A discrete random variable X has the probability function $P(X = x) = k(1 - k)^x$ where $x > 0$. Write an expression for $P(X > 1)$ in terms of k , leaving your answer in simplified form.

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If $y = e^{\cos(x)}$ and $\frac{dx}{dt} = \frac{-1}{\cos(x)}$, then find the value of $\frac{dy}{dt}$ when $x = \frac{\pi}{3}$.

End of questions

Question 1

(3 marks)

Consider the following discrete probability distribution for the random variable X .

x	1	2	3	4	5
$P(X = x)$	p	$2p$	$3p$	$4p$	$5p$

(a) Find the value of p .

(1 mark)

$$15p = 1$$

$$\therefore p = \frac{1}{15} \quad \checkmark$$

(b) Hence, find $E(X)$.

(2 marks)

$$E(X) = 1 \times p + 2 \times 2p + 3 \times 3p + 4 \times 4p + 5 \times 5p \quad \checkmark \left(\sum x \cdot p(x) \right)$$

$$= 1p + 4p + 9p + 16p + 25p$$

$$= 55p$$

$$= \frac{55}{15} \quad \checkmark \quad (\text{ANSW})$$

[OR 2 mks for just ans.]

See next page

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Additional working space

Question number: _____

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Christ Church
Grammar School

2020
TEST 3

MATHEMATICS METHODS Year 12

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Teacher's name _____

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