



Date: 12 Sept 2019  
Time: 45 minutes  
Total: 18 + 27 = 45 marks  
Weight: 8%

Name: MARKING KEY

TOPICS: Continuous Random Variables, Normal Distribution, Sampling,  
Sample Proportions

## SECTION A – NON CALCULATOR

## INSTRUCTIONS:

- Show all necessary working out
- Approved Formula sheet allowed
- Calculators are not allowed
- No Notes allowed

You may assume the following z scores for normal distributions and confidence intervals

- For 68% of scores  $-1 \leq z \leq 1$
- For 95% of scores  $-2 \leq z \leq 2$
- For 99.7% of scores  $-3 \leq z \leq 3$

## Student Reflection

|                      |                     |                      |                    |                             |       |
|----------------------|---------------------|----------------------|--------------------|-----------------------------|-------|
| Q1                   | Q2.5                | Q3.7                 | Q4                 | Q6                          | Total |
| Uniform Distribution | Normal Distribution | Confidence Intervals | Sample proportions | Continuous Random Variables |       |
| 4                    |                     |                      |                    |                             |       |
| 12                   |                     |                      |                    |                             |       |
| 16                   |                     |                      |                    |                             |       |
| 5                    |                     |                      |                    |                             |       |
| 8                    |                     |                      |                    |                             |       |
| 45                   |                     |                      |                    |                             |       |

What went well:  
I did well at...

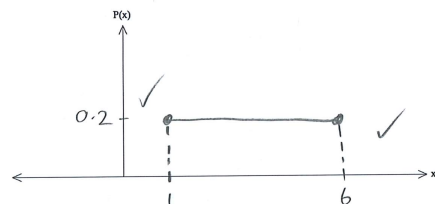
Areas for development:  
I need to improve...

1. [4 marks]

Anna arrives at 8.38 am, two minutes early for her maths methods class and knows that it is equally likely for her maths teacher to arrive at class anywhere from 1 minute to 6 minutes later.

Let the continuous random variable  $X$  be the number of minutes taken for Anna's teacher to arrive after 8.38 am.

- a) Draw a graph on the axes below that shows the probability density function of the random variable  $X$ . [2]



- b) What is the probability that Anna's maths teacher arrives after 8.40 am? [1]

$$\frac{4}{5} \quad \checkmark$$

- c) What is the probability that Anna's maths teacher arrives before 8.42 am given that he arrives after 8.40 am? [1]

$$\frac{P(2 < X < 4)}{P(X > 2)} = \frac{2}{4} = \frac{1}{2} \quad \checkmark$$

4

2. [7 marks]

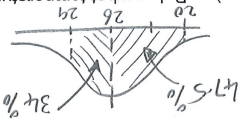
The maximum temperatures of Perth days in the month of April can be modelled using a normal distribution with a mean of  $26^{\circ}\text{C}$  and a standard deviation of 3.

Using this model, answer the following.

- a) [1] If the first day of April had a standardized score of  $-1.25$ , what was the maximum temperature on this day?

$$26 - 3 \times 1.25 = 22.25^{\circ}\text{C}$$

- b) [1] What is the probability that the maximum temperature of an April day will be between  $20^{\circ}\text{C}$  and  $29^{\circ}\text{C}$ ?



$$47.5 + 34 = 81.5\%$$

- c) [1] Below what temperature do the lowest 16% of the daily maximums in April lie?

$23^{\circ}\text{C}$

- d) How many April days in the next decade would you expect to have maximum temperatures above  $32^{\circ}\text{C}$ ?  
(Note: there are 30 days in the month of April)

$$300 \times 0.025 = 300 \div 10 \div 2 \div 2 = 7.5$$

7 or 8 days

- e) [2] The lowest recorded maximum for an April day is  $16.3^{\circ}\text{C}$ . Is this consistent with the use of the described model? Explain your answer.

✓ We would expect 0.15% of the days to be below  $17^{\circ}\text{C}$ .  
✓ Hence a lowest temperature of  $16.3^{\circ}\text{C}$  is consistent with the model.

3. [7 marks]

- a) A random sample of size  $n_1$  was taken and the proportion of people who had cycled in the last week was  $m$ .

Determine a 68% confidence interval for the proportion of the population who had cycled in the last week in terms of  $n_1$  and  $m$ . [2]

$$m - \sqrt{\frac{m(1-m)}{n_1}} \leq p \leq m + \sqrt{\frac{m(1-m)}{n_1}} \quad \checkmark \checkmark$$

- b) A new sample of size  $n_2$  was taken and the proportion of people who had cycled in the last week was again  $m$ . When a 95% confidence interval was determined it was found to be the same as the interval determined in part (a).

- (i) Is  $n_2$  larger or smaller than  $n_1$ ? Explain [2]

$n_2$  is larger, to compensate for the larger % value.   
 ✓

- (ii) What is the relationship between  $n_1$  and  $n_2$ ? [3]

$$\sqrt{\frac{m(1-m)}{n_1}} = 2 \sqrt{\frac{m(1-m)}{n_2}} \quad \checkmark$$

$$\frac{m(1-m)}{n_1} = 4 \frac{m(1-m)}{n_2} \quad \checkmark$$

$$\frac{n_2}{n_1} = 4 \quad \checkmark$$

$n_2$  is 4 times larger than  $n_1$    
 7

7. [9 marks]

In a random sample of 200 Year 12 ATAR students, it was found that 28 of the students received extra tutoring outside of school.

- a) Calculate the sample proportion of these students who received extra tutoring outside of school. [1]

$$\frac{28}{200} = 0.14 \quad \checkmark$$

- b) Calculate the 90% confidence interval for the population proportion and interpret your answer. [3]

$$0.14 - 1.645 \sqrt{\frac{0.14 \times 0.86}{200}} \leq p \leq 0.14 + \sqrt{\frac{0.14 \times 0.86}{200}}$$

$$0.0996 \leq p \leq 0.1804 \quad \checkmark \checkmark$$

90% certain that this confidence interval contains the population proportion!

- c) A second survey of Year 12 ATAR students is planned; however, it is decided that the 90% confidence interval should involve a maximum margin of error of 3%. Determine the sample size required for such a survey. [3]

$$1.645 \sqrt{\frac{0.5 \times 0.5}{n}} \leq 0.03 \quad \checkmark$$

$$\text{solve on CAS} \Rightarrow n \geq 751.54 \quad \checkmark$$

$$n = 752 \quad \checkmark$$

$$\text{Accept } 1.645 \sqrt{\frac{0.14 \times 0.86}{n}} \leq 0.03$$

$$n \geq 361.94 \Rightarrow n = 362$$

$$\text{Also Accept } n \geq 362.006 \Rightarrow n = 363$$

- (d) If ten surveys were taken and for each a 90% confidence interval for the population proportion was calculated, determine the probability that at most seven of the intervals included the true value of the population proportion. [2]

$$X \sim \text{Bin}(10, 0.9) \quad \checkmark$$

$$P(X \leq 7) = 0.0702 \quad \checkmark$$

6. [8 marks]

The time  $X$  minutes for a meal to be delivered by an uber eats driver is modelled using a continuous random variable with probability density function given by

$$f(x) = \begin{cases} k(x-30)^2 & : 0 < x < 30, \\ 0 & : \text{elsewhere} \end{cases}$$

[2]

a) Find the value of  $k$

$$\int_{30}^0 k(x-30)^2 dx = 1$$

$$9000k = 1$$

$$k = \frac{1}{9000}$$

[2]

b) What is the probability of the driver delivering a meal within 15 minutes?

$$\int_0^{15} \frac{1}{9000} (x-30)^2 dx = \frac{8}{7}$$

[2]

c) Calculate the mean delivery time for the driver

$$\text{Mean} = \int_{30}^0 x \cdot \frac{1}{9000} (x-30)^2 dx = 7.5$$

[2]

d) Calculate the standard deviation of the delivery time for the driver.

$$\sigma^2 = \int_{30}^0 \frac{1}{9000} (x-30)^2 (x-7.5)^2 dx$$

$$\sigma^2 = 33.75 \Rightarrow \sigma = 5.809 \quad \left( \text{or } \sigma = 3\sqrt{15} \right)$$

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## SECTION B – CALCULATOR ALLOWED

### INSTRUCTIONS:

- Show all necessary working out
- Approved Formula sheet allowed
- Scientific and CAS Calculators are allowed
- One A4 page of notes (both sides) is allowed

CORPUS CHRISTI COLLEGE  
REGENT GARDENS



Corpus Christi College

Year 12 Mathematics Methods

2019 Test 5

4. [5 marks]

It is known that 12% of the population are left handed.

- a) Describe the distribution of the proportions of left handers in samples of size 500.

[2]

Approximately normal with ✓

mean of sample proportions = 0.12

$$\sigma \text{ of sample proportions} = \sqrt{\frac{0.12(0.88)}{500}} = 0.0145 \quad \checkmark$$

- b) 500 major league baseballers were surveyed and it was found that 95 of them were left handed. Comment on this result.

[3]

$$\hat{p} = \frac{95}{500} = 0.19 \quad \checkmark$$

$$z = \frac{0.19 - 0.12}{0.0145} = 4.817 \quad \checkmark$$

Sample proportion is almost  $5 \times \sigma$  above population proportion.

It appears that there is a greater proportion of left handers in MLB compared to the proportion of left handers in the population. ✓  
(or similar)

5. [5 marks]

The horn length of adult black rhinos is normally distributed with 38% of adult black rhinos having a horn length above 75 cm and 12% of adult black rhinos having a horn length below 61 cm.

Above what length are the longest 10% of adult black rhino horns?

$$\text{invnorm}(R, 0.38, 1, 0) \Rightarrow z = 0.3055$$

$$\text{invnorm}(L, 0.12, 1, 0) \Rightarrow z = -1.17499 \quad \checkmark \checkmark$$

$$\left. \begin{aligned} 0.3055 &= \frac{75 - \mu}{\sigma} \\ -1.175 &= \frac{61 - \mu}{\sigma} \end{aligned} \right\} \Rightarrow \begin{aligned} \mu &= 72.11 \quad \checkmark \\ \sigma &= 9.46 \quad \checkmark \end{aligned}$$

$$\text{invnorm}(R, 0.1, 9.46, 72.11) \Rightarrow K = 84.2$$

$$84.2 \text{ cm} \quad \checkmark$$