PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 2 Year 12

Student name: ________________ Teacher name: ________________

Task type: **Response/Investigation**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: 7

Materials required: Upto 3 classpads/calculators

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper SINGLE SIDED, and up to three calculators approved for use in the WACE examinations

Marks available: **40 marks**

Task weighting: **13%**

Formula sheet provided: no but formulae stated on page 2

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = \lvert z \rvert = \sqrt{a^2 + b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan\theta = \frac{b}{a}, \quad -\pi < \theta \le \pi$
$\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \lvert z_2 \rvert$	$\left\lvert \frac{z_1}{z_2} \right\rvert = \frac{\lvert z_1 \rvert}{\lvert z_2 \rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = \lvert z \rvert^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{\lvert z \rvert^2}$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$	$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
Polar form	
$z = a + bi = r(\cos\theta + i\sin\theta) = r\,cis\,\theta$	$\bar{z} = r\,cis(-\theta)$
$z_1 z_2 = r_1 r_2\,cis(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,cis(\theta_1 - \theta_2)$
$cis(\theta_1 + \theta_2) = cis\,\theta_1\,cis\,\theta_2$	$cis(-\theta) = \frac{1}{cis\,\theta}$
De Moivre's theorem	
$z^n = \lvert z \rvert^n cis(n\theta)$	$(cis\,\theta)^n = \cos n\theta + i\sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\frac{\theta + 2\pi k}{q} + i\sin\frac{\theta + 2\pi k}{q}\right)$, for k an integer	

$$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta$$

Q1 (3 marks)

Consider the inequality $|3x - 7| \le a$ which is only true for $b \le x \le 9$ where $a \& b$ are constants. Determine the values of $a \& b$.

c
$3\left\|x - \frac{7}{3}\right\| \le a$ $\left\|x - \frac{7}{3}\right\| \le \frac{a}{3}$ $9 - \frac{7}{3} = \frac{20}{3} = \frac{a}{3}$ $a = 20$ $9 - \frac{40}{3} = -\frac{13}{3} = b$
Specific behaviours
✓ rearranges inequality by taking out factor of 3 or sketches and shows reasoning ✓ determines a ✓ determines b

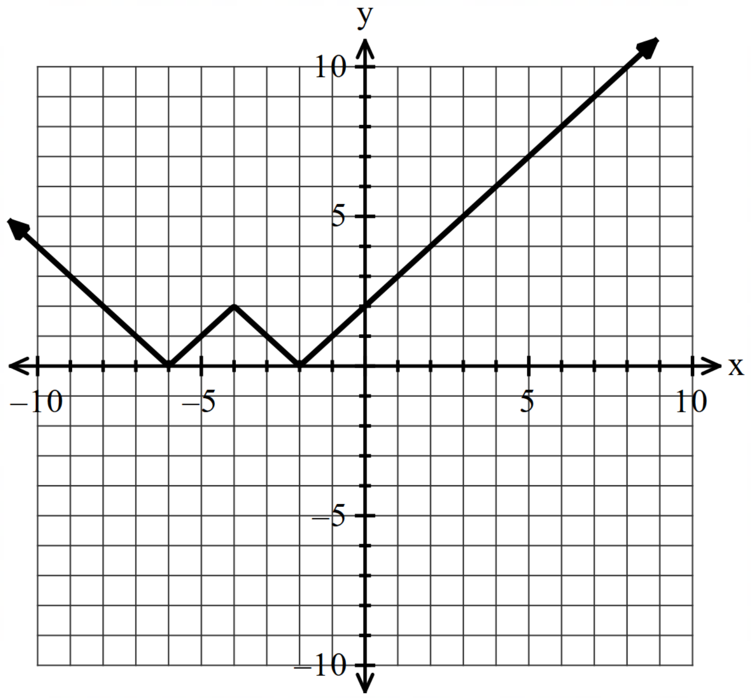
Q2 (4 marks)

c

$r_A = \begin{pmatrix} 2 \\ -18 \end{pmatrix} + t\begin{pmatrix} 2 \\ 7 \end{pmatrix}$ $r_B = \begin{pmatrix} -6 \\ 44 \end{pmatrix} + (t - \frac{1}{2})\begin{pmatrix} 4 \\ -6 \end{pmatrix}$ $r_A = r_B$ $2 + 2t = -6 + (t - \frac{1}{2})4$ $2 + 2t = -6 + 4t - 2$ $10 = 2t$ $t = 5$ $-18 + 7t = 44 + (t - \frac{1}{2})(-6)$ $-18 + 7t = 44 - 6t + 3$ $13t = 65$ $t = 5$ $collide (12,17)\ at\ 4\ pm$
Specific behaviours
✓ takes into account different starting times ✓ sets up two equations for time ✓ solves for position of collision ✓ solves for time of collision in clock time

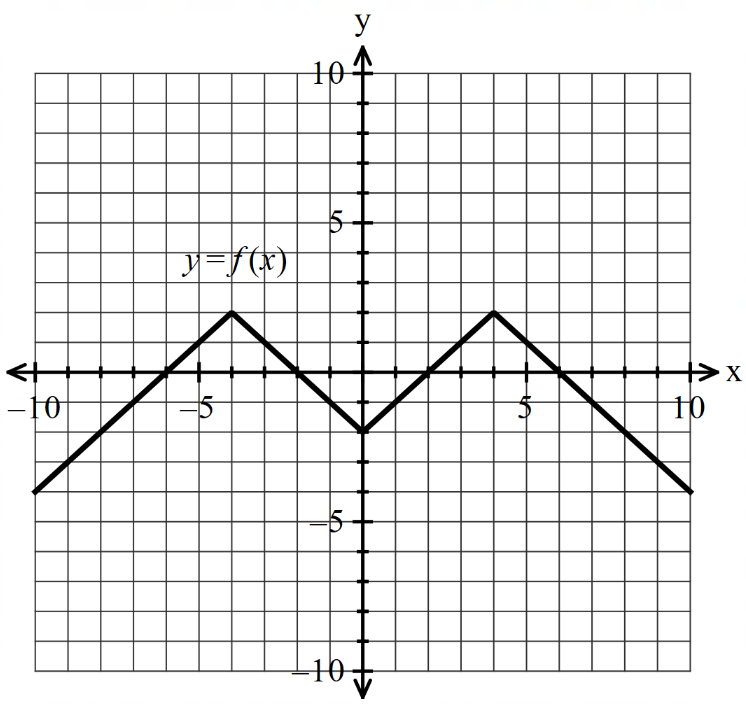
Q3 (2, 3 & 3 = 8 marks)

a) Sketch $|f(x)|$ on the axes below.

c


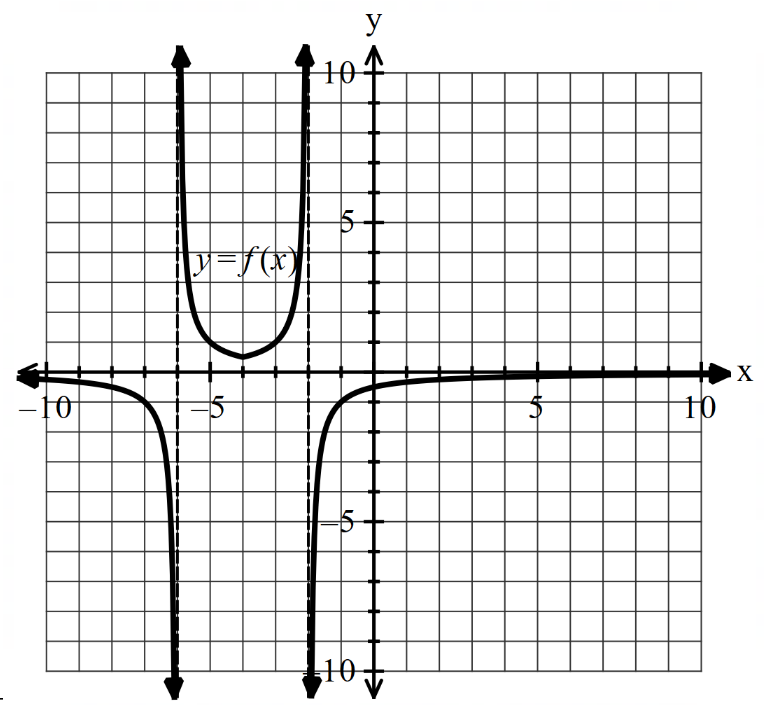
Specific behaviours
✓ shape ✓ correct x & y intercepts

b) Sketch $f(-|x|)$ on the axes below.

c

Specific behaviours
✓ uses reflection of left side or original

✓ shape ✓ correct x & y intercepts

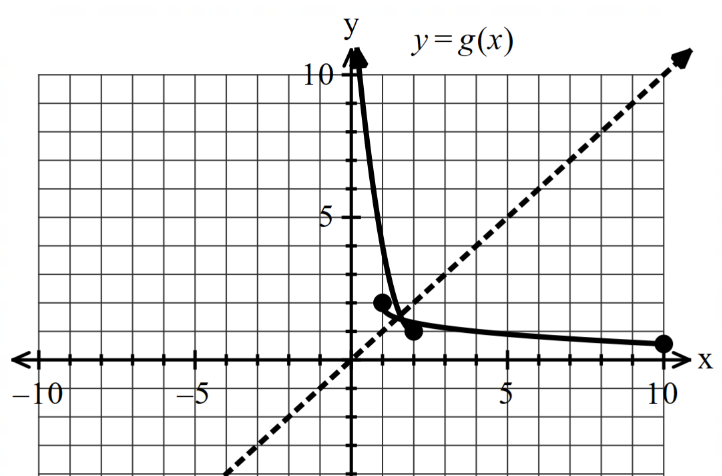
c) Sketch $\dfrac{1}{f(x)}$ on the axes below.

c

Specific behaviours
✓ vertical and horizontal asymptotes ✓ approx. y intercept ✓ shape in all 3 sections

Q4 (2, 3, 1 & 3 =9 marks)

Consider $g(x) = 3x^2 - 12x + 13$ for $x \le 2$ which is plotted below.

a) Sketch $g^{-1}(x)$ on the axes above.

c


Specific behaviours
✓ contains endpoint (1,2) ✓ appears reflected in line y=x

b) Determine the rule for $g^{-1}(x)$ showing full working and the domain.(Simplify)

c
$x = 3y^2 - 12y + 13$ $3y^2 - 12y + 13 - x = 0$ $y = \dfrac{12 \pm \sqrt{144 - 4(3)(13 - x)}}{6}$ $f^{-1}(x) = 2 - \sqrt{\dfrac{x-1}{3}}, x \geq 1$
Specific behaviours
✓ swaps x and y ✓ solves for y with reasoning ✓ uses minus in rule and states domain

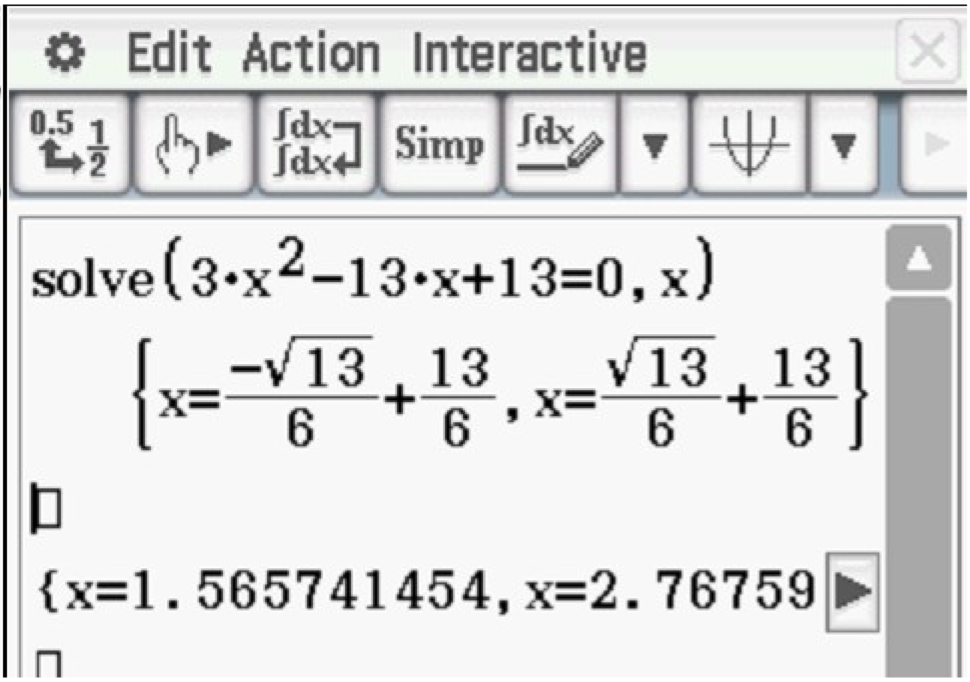
Q4 cont-

c) Determine $g^{-1} \circ g(x)$.

c
$g^{-1} \circ g(x) = x$
Specific behaviours

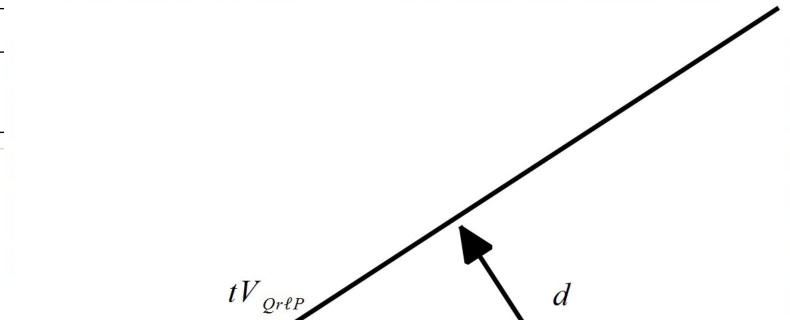
✓ states x

d) Determine all value(s) of x such that $g(x) = g^{-1}(x)$

c
$g(x) = 3x^2 - 12x + 13 = x$ $3x^2 - 13x + 13 = 0$  Discard x=2.77 as not within domain of f(x)
Specific behaviours
✓ equates g(x) to x ✓ sets up an equation to solve for x ✓ solves for one approx. value and states that the second value MUST be discarded Note: 2 marks max if using classpad to solve function=inverse without reasoning

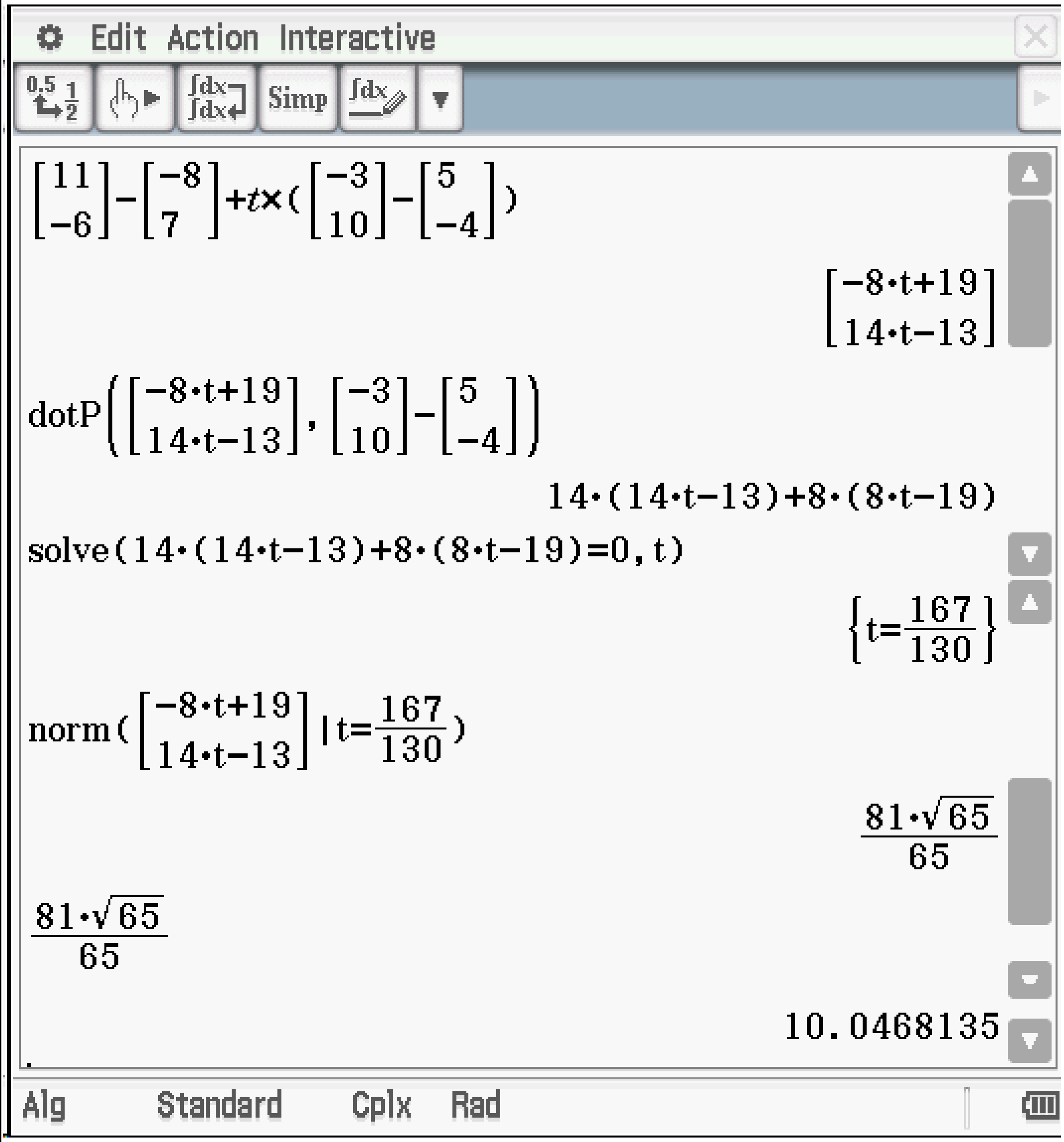
Q5 (5 marks)

Determine the minimum distance between them **using vectors** and the time that this occurs.



$d = PQ + t\,{}_{Q}V_{P}$

$d \cdot {}_{Q}V_{P} = 0$



Distance = 10.05 metres at time 1.28 seconds

Specific behaviours
✓ uses relative velocity ✓ determines a separation vector d

✓ uses dot product and equates to zero ✓ solves for time stating in seconds ✓ solves for distance stating in metres Note: max -1 if units not stated Max 3 out of 5 if vector method not used

Q6 (5 marks)

i) The line will be a tangent to the circle.
ii) The line crosses the circle at two points.
iii) The line will never meet the circle.

c

$$\left|\begin{pmatrix}-2+4\lambda\\ \lambda\end{pmatrix}-\begin{pmatrix}3\\ \alpha\end{pmatrix}\right|=9$$

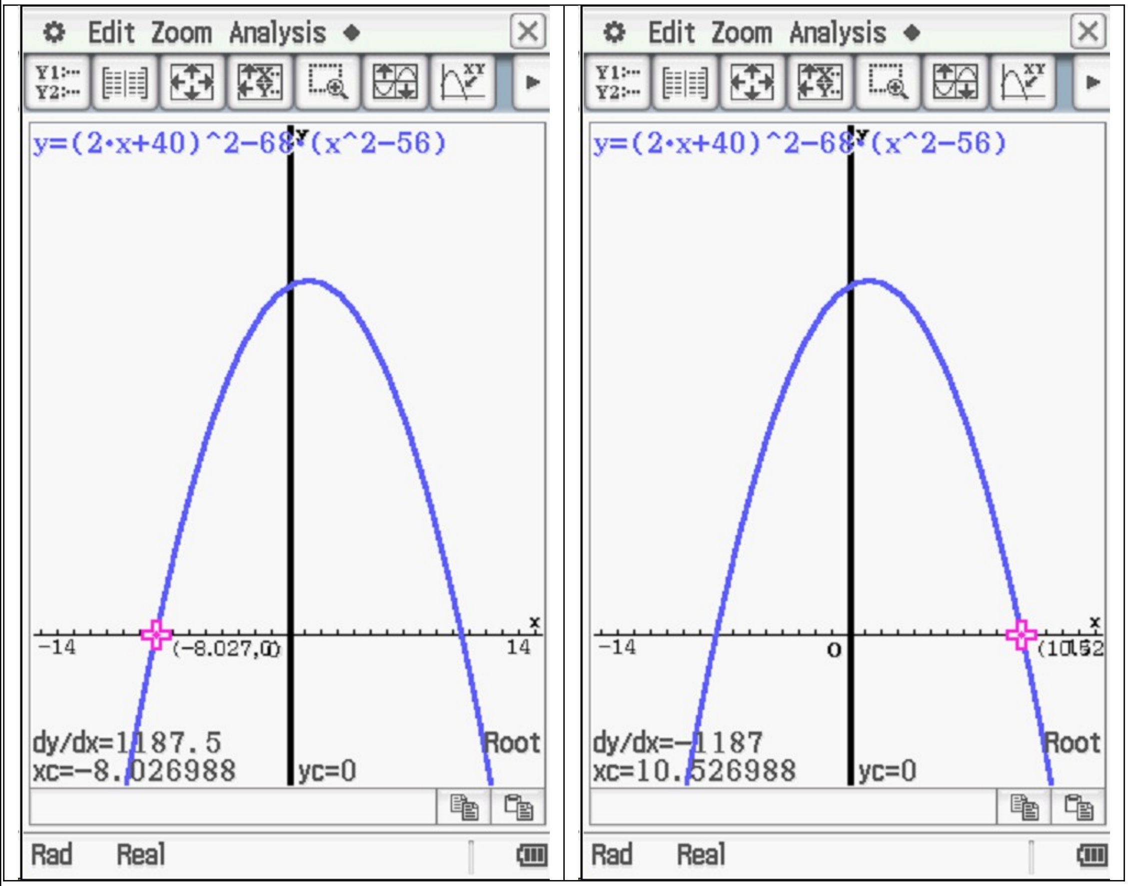
$$\left|\begin{pmatrix}-5+4\lambda\\ \lambda-\alpha\end{pmatrix}\right|=9$$

$$\sqrt{(4\lambda-5)^2+(\lambda-\alpha)^2}=9$$

$$16\lambda^2-40\lambda+25+\lambda^2-2\alpha\lambda+\alpha^2-81=0$$

$$17\lambda^2-(40+2\alpha)\lambda-56+\alpha^2=0$$

$$\Delta=(40+2\alpha)^2-4(17)(\alpha^2-56)$$



i) Alpha=-8.03, 10.53
ii) -8.03 <alpha< 10.53
iii) Alpha< -8.03, alpha> 10.53

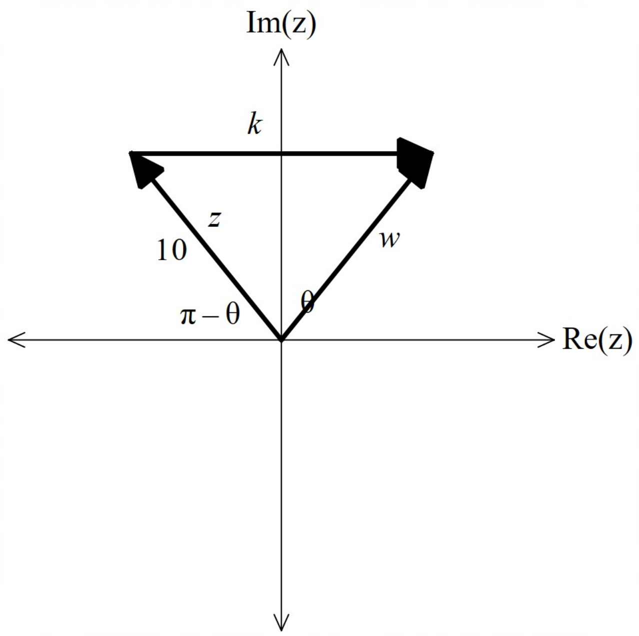
Specific behaviours

✓ subs line into circle

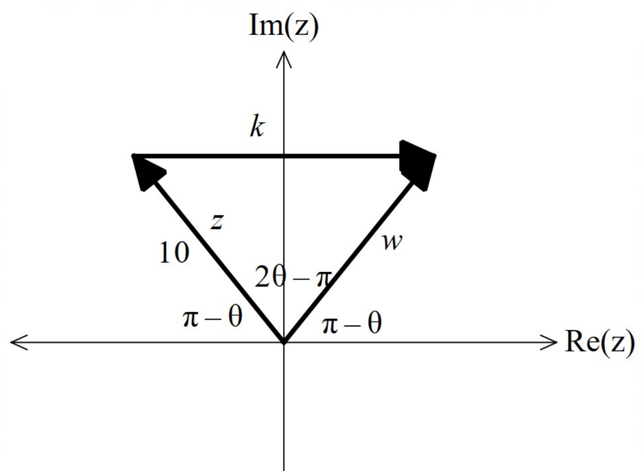
✓ sets up a quadratic equation for lambda ✓ determines an expression for discriminant in terms of alpha ✓ states values for tangent **with discriminant = zero (stated)** ✓states values for intersecting at two points AND no intersection and states conditions for discriminant. Note: max -1 if discriminant values not stated

Q7 (2 & 4 = 6 marks)

a) Represent this information on the Argand Diagram below.

c

Specific behaviours
✓ shows z in second quadrant with magnitude of 10 ✓ shows w in first quadrant with k being horizontal

b) Determine a simplified expression for k in terms of θ. Justify your answer.



$k^2 = 10^2 + 10^2 - 200\cos(2\theta - \pi)$ $k^2 = 200 + 200\cos 2\theta = 200(1 + \cos 2\theta)$ $k^2 = 200(2\cos^2\theta)$ $k = -20\cos\theta$
Specific behaviours
✓ identifies symmetry of triangles ✓ uses cosine rule or other trig identity ✓ obtains expression for k squared ✓ states simplified expression for k in terms of theta with a negative