Question 6 **(2 marks)**

The gradient function of a curve is given by:

$$\lim_{h \to 0} \frac{\cos\frac{1}{2}(x+h) - \cos\frac{1}{2}x}{h}$$

Write down the equation of the curve.

$y = +\cos\frac{1}{2}x$ ✓✓

Question 7 **(5 marks)**

(a) If $y = \sin(x) - \cos(x)$, find an expression for $\frac{d^2y}{dx^2}$ in terms of y. (2 marks)

$\frac{d^2y}{dx^2} = \cos x - \sin x$ ✓

$= -y$ ✓

(b) Find the equation of the tangent to the curve $y = \cos(2x)$ at the point where $x = \frac{\pi}{4}$. (2 marks)

$2x + y - \frac{\pi}{2} = 0$ ✓✓ From CP

OR $y = \frac{\pi}{2} - 2x$

(b) Evaluate $\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3}+h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$ (1 mark)

$\frac{d}{dx} \sin x \Big|_{x=\frac{\pi}{3}} = \cos x \Big|_{x=\frac{\pi}{3}}$

$= \frac{1}{2}$ ✓

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Christ Church Grammar School

2017

Investigation 1

Year 12 MATHEMATICS METHODS
Section One:
Calculator-free

Student name - SOLUTIONS -

Teacher name ____________________

Time and marks available for this section
Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

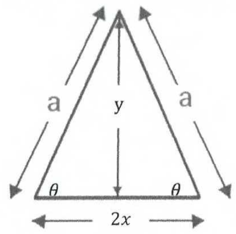
3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

Question 8 **(8 marks)**

Suppose an isosceles triangle has 2 equal sides of length a and equal base angles θ.

(a) Show that the perimeter of the triangle is P, where $P = 2a(1 + cos\theta)$. (3 marks)



$x = a \cos\theta$

$y = a \sin\theta$ ✓

$Per = 2a + 2x$

$P = 2a + 2 \times a\cos\theta$ ✓

$P = 2a(1+\cos\theta)$ ✓

As Required

(b) Deduce that, for all isosceles triangles with fixed perimeter P, the triangle of largest area is equilateral. You may use your ClassPad to determine derivatives and required values, but the essential steps and working must be shown in the space below.

(5 marks)

$A = \frac{1}{2}(2a.\cos\theta) \times \underset{}{\overset{y}{a\sin\theta}}$ ✓

$A = a^2 \cos\theta . \sin\theta$ ✓ → but $P = 2a(1+\cos\theta)$

Sub in

$a = \frac{P}{2+2\cos\theta}$ ✓ with P constant

$A = \left(\frac{P}{2+2\cos\theta}\right)^2 \times \cos\theta . \sin\theta$ ✓

$\frac{dA}{d\theta} = 0$ (P constant) on CPad

$\theta = \frac{\pi}{3}$ ✓ $(60°)$; hence Δ is equilateral that gives largest area $(0 < \theta < \frac{\pi}{2})$

NB!

$\frac{dA}{d\theta} = a^2\cos^2\theta - a^2\sin^2\theta$

$0 = a^2(1-2\sin^2\theta)$

$\theta = \pi/4$

But 'a' NOT fixed!

End of questions

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Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(8 marks)**

Determine $\frac{dy}{dx}$ in each of the following, simplify your answers.

(a) $y = \sin 4x^2$ (2 marks)

$$\frac{dy}{dx} = \cos 4x^2 \times 8x$$

$$= 8x \cdot \cos 4x^2 \quad \checkmark\checkmark$$

(b) $y = \cos^3(x)$ (2 marks)

$$\frac{dy}{dx} = 3\cos^2(x) \times (-\sin x)$$

$$= -3\sin x \cdot \cos^2 x \quad \checkmark\checkmark$$

(c) $y = cos\sqrt{2x^2 - 1}$ (4 marks)

$$y = \cos(2x^2-1)^{1/2}$$

$$\frac{dy}{dx} = -\sin(2x^2-1)^{1/2} \times \frac{1}{2}(2x^2-1)^{-1/2}(4x) \quad \checkmark \checkmark \checkmark$$

$$\frac{dy}{dx} = -\frac{2x \sin\sqrt{2x^2-1}}{\sqrt{2x^2-1}} \quad \checkmark$$

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See next page

Question 2 **(3,2 marks)**

(a) Find the gradient of the tangent to the curve $y = 3\sin(x)$ at the point where $x = \frac{\pi}{6}$.

$\frac{dy}{dx} = 3\cos x$ ✓

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = 3\cos\frac{\pi}{6}$ ✓

$m = \frac{3\sqrt{3}}{2}$ ✓

(b) Evaluate $\lim_{x\to 0}\frac{2\sin(x)}{3x}$

$= \frac{2}{3}\lim_{x\to 0}\frac{\sin(x)}{x}$ ✓

$= \frac{2}{3}(1)$

$= 2/3$ ✓

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Christ Church Grammar School

2017
Investigation 1

Year 12 MATHEMATICS METHODS

Section Two:
Calculator-assumed

Student name ______________________

Teacher name ______________________

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: None

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question 3 **(6 marks)**

Find the equation of the tangent to the curve $y = 3sin^2(x)$ at the point $(\frac{\pi}{6}, \frac{3}{4})$.

$\frac{dy}{dx} = 6 \cos x . \sin x$ ✓✓

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = 6 \cos\frac{\pi}{6} . \sin\frac{\pi}{6}$ ✓

$= \cancel{6}^{3} \times \frac{\sqrt{3}}{\cancel{2}} \times \frac{1}{2}$

$m = \frac{3\sqrt{3}}{2}$ ✓

$\therefore y = mx + c \Rightarrow$ Sub in $\left(\frac{\pi}{6}, \frac{3}{4}\right)$

$\frac{3}{4} = \frac{3\sqrt{3}}{2}\left(\frac{\pi}{6}\right) + c$

$\therefore c = \frac{3}{4} - \frac{\sqrt{3}\pi}{4}$ ✓

ie Eq^n $y = \frac{3\sqrt{3}}{2}x + \frac{3 - \sqrt{3}\pi}{4}$ ✓

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Question 4 **(5 marks)**

With the help of some of the following limits,

$$\lim_{h\to 0}\frac{\sin h}{h}=1\,,\quad \lim_{h\to 0}\frac{\sin kh}{kh}=1\,,\quad \lim_{h\to 0}\frac{1-\cos h}{h}=0 \quad\text{and}\quad \lim_{h\to 0}\frac{1-\cos kh}{kh}=0$$

where k is any real number,

determine **using first principles** the derivative of $f(x)=\sin 5x$ with respect to x.

The first step has been completed for you.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin 5x \cdot \cos 5h + \cos 5x \cdot \sin 5h - \sin 5x}{h} \quad ✓$$

$$= \lim_{h\to 0}\left(\cos 5x \times \frac{\sin 5h}{h} - \sin 5x \times \frac{1-\cos 5h}{h}\right) \quad ✓$$

$$= 5\cos 5x\left(\lim_{h\to 0}\frac{\sin 5h}{5h}\right) - 5\sin 5x\left(\lim_{h\to 0}\frac{1-\cos 5h}{5h}\right) \quad ✓$$

$$= 5\cos 5x\,(1) - 5\sin 5x\,(0) \quad ✓$$

$$= 5\cos 5x \quad ✓$$

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Question 5 **(6 marks)**

Use calculus to determine the maximum and minimum points on the curve $y=\sin(x)+\cos(x)$ for $0\le x\le 2\pi$. You must use a second derivative check.

$y = \sin x + \cos x$

$y' = \cos x - \sin x$ ✓

$y'' = -\sin x - \cos x$

$\frac{dy}{dx} = 0$

$\therefore \cos x - \sin x = 0$

$\cos x = \sin x \quad (\tan x = 1)$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ ✓

When $x = \frac{\pi}{4}$ ✓ $y'' < 0$ and $y = \sqrt{2}$

$\therefore \left(\frac{\pi}{4}, \sqrt{2}\right)$ is a max ✓

When $x = \frac{5\pi}{4}$ ✓ $y'' > 0$ and $y = -\sqrt{2}$

$\therefore \left(\frac{5\pi}{4}, -\sqrt{2}\right)$ is a min ✓

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