

Marking Key

Calculator-Assumed

MAWA Semester 1 (Unit 1) Examination 2015

MATHEMATICS METHODS

Section Two: Calculator-assumed

(90 Marks)

Question 8(a)

Solution

$$m = \frac{-1 - (-3)}{4 - 3} = 2 \Rightarrow y = 2x + c$$

using $(3, -3)$; $-3 = 2(3) + c \Rightarrow c = -9$

$$\therefore y = 2x - 9$$

Marking key/mathematical behaviours	Marks
• calculates gradient	1
• uses a point to calculate c and states equation	1

Question 8(b)

Solution

$$3x + 2y + 7 = 0 \text{ has gradient } -\frac{3}{2}$$

$$\therefore y = -\frac{3}{2}x + c$$

using $(2, 3)$; $3 = -3 + c \Rightarrow c = 6$

$$\therefore y = -\frac{3}{2}x + 6$$

Marking key/mathematical behaviours	Marks
• calculates gradient	1
• uses the given point to calculate c and states equation	1

Question 8(c)

Solution

$$5x - 3y = 1 \text{ has } m = \frac{5}{3} \therefore m \perp = -\frac{3}{5}$$

$$\therefore y = -\frac{3}{5}x + c$$

using $(5, -4)$; $-4 = -\frac{3}{5}(5) + c \Rightarrow c = -1$

$$\therefore y = -\frac{3}{5}x - 1$$

Marking key/mathematical behaviours	Marks
• calculates gradient	1
• determines perpendicular gradient	1
• uses the given point to calculate c and states equation	1

1		• states the correct response
1		• uses the idea of a reduced sample space
Marks		Marking key/mathematical behaviours

Hence the probability that a selected person takes 5 attempts = $\frac{0.59}{0.59} = 0.33 = \frac{33}{59}$

Since we know (given) that the selected person takes at least 5 attempts ($P(B) = 0$), this reduces the probability sample space to 0.59

Solution
Question 9(c)

1		• provides a valid reason
1		• states events are M.E.
Marks		Marking key/mathematical behaviours

The events are mutually exclusive, since $P(A \cup B) = 0$

Solution
Question 9(b)

1		• correctly states $P(A \cup B)$
1		• states correct result for $P(A \cup B)$
1		• provides the correct result for $P(B)$
1		• reads correctly from the graph $p(A)$
Marks		Marking key/mathematical behaviours

(iii) $P(A \cup B) = 0.74$ (by the addition principle)

(ii) $= 0.41$

$P(B) = 0.01 + 0.06 + 0.11 + 0.23$ read directly from the graph

(i) $p(A) = 0.33$

Solution
Question 9(a)

1		• determines gradient of reflected line and states equation
1		• determines gradient of original line
Marks		Marking key/mathematical behaviours

\therefore gradient of reflected line is $-\frac{2}{3} \Rightarrow$ line is $y = -\frac{2}{3}x + 2$

$2y - 3x - 4 = 0 \Leftrightarrow y = \frac{2}{3}x + 2$

Solution
Question 8(d)

1		MATHEMATICS APPLICATIONS
1		SEMESTER 1 (UNIT 1) EXAMINATION
Marks		CALCULATOR-ASSUMED MARKING KEY

Question 10(a)

Solution
 $\angle ACB = 6^\circ$

Using the sine rule:

$$\frac{BC}{\sin 21^\circ} = \frac{19}{\sin 6^\circ} \Rightarrow BC = 65.14 \text{ cm}$$

Marking key/mathematical behaviours	Marks
• calculates angle ACB	1
• uses the sine rule to calculate the length of BC	1

Question 10(b)

Solution
 $\angle ABC = 153^\circ$

Using the area rule:

$$\text{Area} = \frac{1}{2}(19)(65.14)\sin 153^\circ \Rightarrow \text{Area} = 280.94 \text{ cm}^2$$

Marking key/mathematical behaviours	Marks
• calculates angle ABC	1
• uses the area formula to calculate the required area	1

Question 10(c)

Solution
 $\angle ABC = 153^\circ$

Let the mid-point of BC be D. BD = 32.57 cm

Using the cosine rule:

$$AD^2 = (32.57)^2 + (19)^2 - 2(19)(32.57)\cos 153^\circ \Rightarrow AD = 50.25 \text{ cm}$$

Marking key/mathematical behaviours	Marks
• calculates the length of BD	1
• uses the cosine rule to calculate the length of AD	1

Acknowledgements

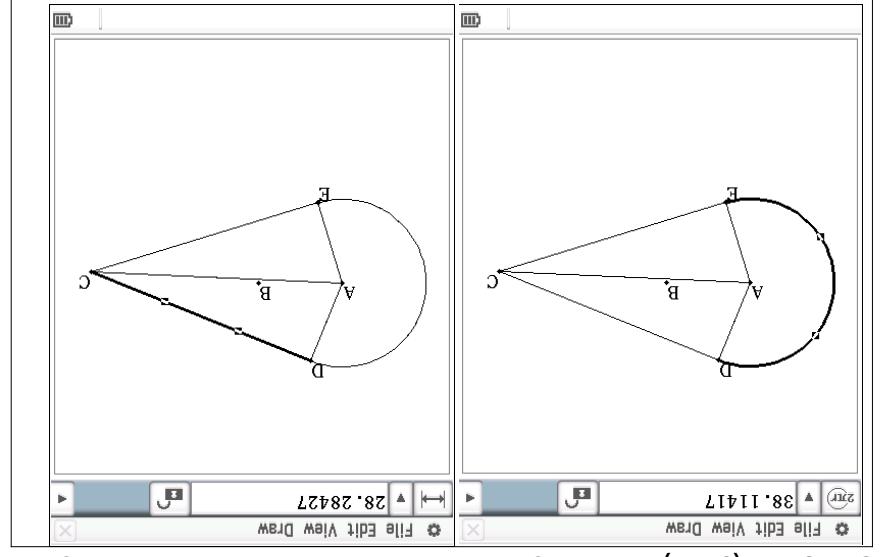
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MATHEMATICS APPLICATIONS		CALCULATOR-ASSUMED		CALCULATOR-ASSUMED	
SEMESTER 1 (UNIT 1) EXAMINATION		MATHEMATICS APPLICATIONS		MATHEMATICS APPLICATIONS	
Question 11 (a)	$\frac{4}{2}$ one mark for denominator, one mark for numerator	Question 11 (b)	$\frac{8}{2}$ one mark (no need to simplify)	Question 11 c	$\frac{8}{4}$ one mark for denominator, one mark for numerator

Marks	Marking key/mathematical behaviours	Length of rope = $38.11417 + 2(28.28427) = 94.68$ cm (which is within 1 mm of the above answer, due to rounding)
1	Indicates length of AB and shows appropriate central angle on the diagram	
1	Calculates length of tangents	
1	Calculates size of central angle	
1	Calculates appropriate arc length	
1	Determines correct length of rope (including units)	Or, calculates length of belt using CAS
2	Indicates length of major arc (MN)	Provides the correct length of the rope (including units)



Question 12(a)

Solution

$$x = 5 \cos\left(\pi t - \frac{\pi}{2}\right) \quad x = 5 \cos\left(-\frac{\pi}{2}\right) = 0$$

By substitution of $t = 0$ into we get . That is, the weight is at the rest (or 0 position).

Marking key/mathematical behaviours	Marks
• Determines that the weight is at the rest position	1

Question 12(b)

Solution

The period of the weights oscillation is 2 seconds. Hence it goes through the rest position twice every 2 seconds. That is once every second. Because it starts at the rest position and finishes at the rest position, we need to add one. Hence the answer is 6 times.

$$5 \cos\left(\pi t - \frac{\pi}{2}\right) = 0 \quad \text{for every whole number value of } t .$$

Alternatively, we note that for every whole number value of t .

i.e. when $t = 0, 1, 2, 3, 4$ and 5 . So 6 times.

Marking key/mathematical behaviours	Marks
• Provides a reasonable explanation as to how arrived at the number of times the weight is at the rest position	1
• Determines the correct number of times (i.e. 6 times)	1

Question 12(c)

Solution

Negative values of x represent the distance that the weight is below the rest position. The negative represents 'below the rest position' the magnitude of the number represents the distance.

Marking key/mathematical behaviours	Marks
• Indicates that the negative represents the distance 'below'	1

Question 18(a)

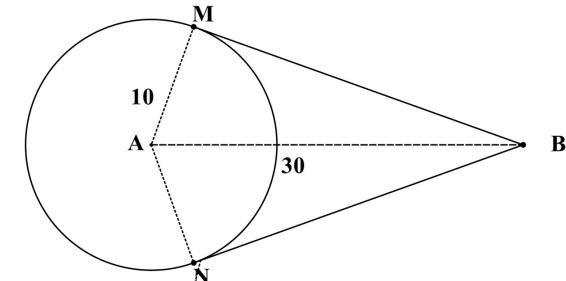
Solution

$$\text{If } C = -B \text{ then } \sin(A+C) = \sin(A+(-B)) \\ \therefore \sin(A-B) = \sin A \cos(-B) + \cos A \sin(-B) \\ = \sin A \cos B - \cos A \sin B \\ (\because \cos(-B) = \cos B \text{ and } \sin(-B) = -\sin B)$$

Marking key/mathematical behaviours	Marks
• Substituting $-B$ for C into given identity	1
• Indicating $\cos(-B) = \cos B$ and $\sin(-B) = -\sin B$ and simplifying	1

Question 18(b)

Solution



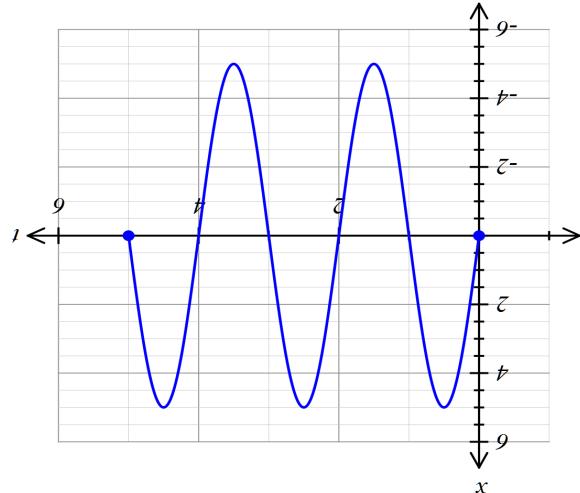
$$\Delta AMB \text{ has a rt angle at } M \Rightarrow BM = \sqrt{30^2 - 10^2} = 20\sqrt{2} \approx 28.284$$

$$\angle MAN = 2 \left(\operatorname{inv} \left(\cos \left(\frac{1}{3} \right) \right) \right) = 2.462$$

$$\therefore \text{length of major arc } MN = (2\pi - 2.462) \times 10 = 38.213$$

$$\text{Hence the length of the rope } \approx 2 \times 28.284 + 38.213 \approx 94.78 \text{ cm}$$

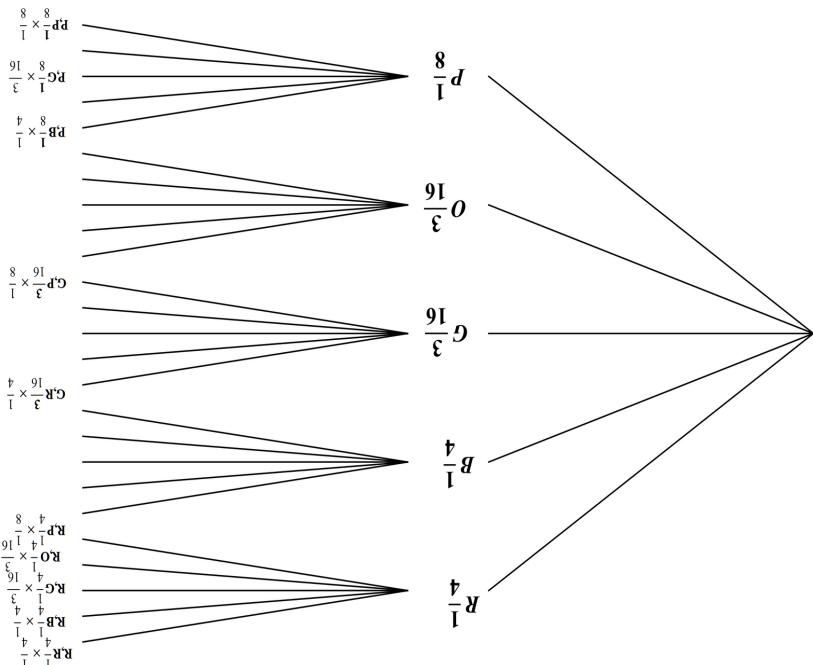
Alternatively, using the geometry app on a CAS



Solution
Question 12(d)

Here, it is best to draw a tree diagram to represent what happens

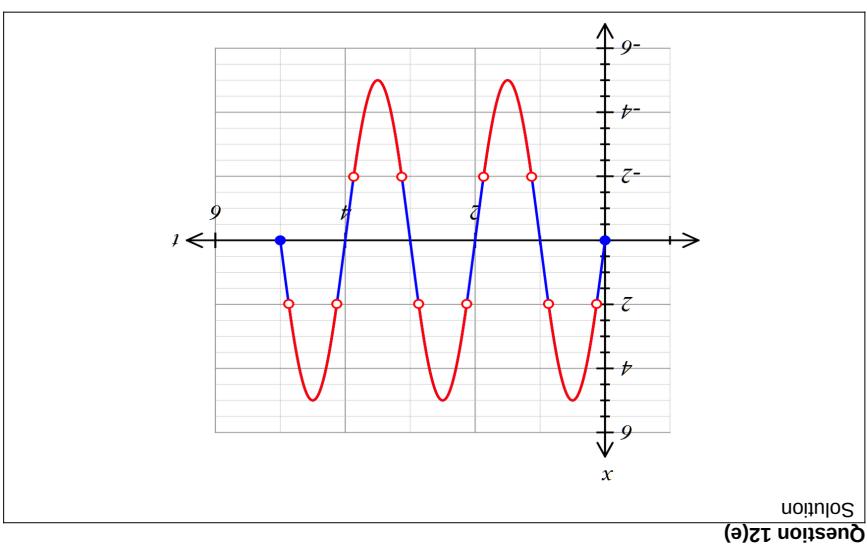
What we want is the probability of Blue and Red or Red and Green or Green and Orange and Orange and Pink and Pink



Marking key/mathematical behaviours

Marks

- applies the multiplication principle for simultaneously occurring independent events
- adds the mutually exclusive events
- defines all the possibilities (by listing or other sample space representation)
- calculates correctly to give the correct result



Solution
Question 12(e)

MATHEMATICS APPLICATIONS
SEMESTER 1 (UNIT 1) EXAMINATION

CALCULATOR-ASSUMED
MARKING KEY

Marking key/mathematical behaviours	Marks
• Indicates (by use of colour or otherwise), the points of the curve where the magnitude of x is greater than 2	1
• Excludes when $x = 2$ or -2	1

Question 12(f)

Solution

From the graph, the values of t for which $x > 2$ is approximately $0.13 < t < 0.87$ for the first second of motion. This is approx. 75% of the time. This is repeated during every second of

the motion. Hence the fraction requested is approximately $\frac{3}{4}$.

For a more accurate answer, use a CAS calculator as follows:

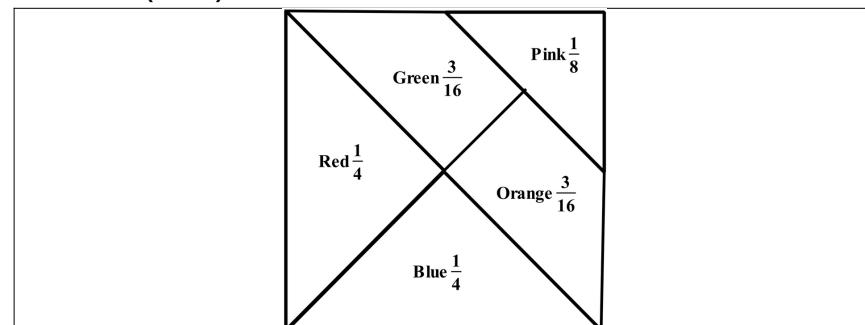
```
Solve(5*cos(pi*t-pi/2)=2, t, 0, 0, 1)
{t=0.1309898804, t=0.8690101196}
(0.8690101196-0.1309898804)
0.7380202392
```

This indicates that the weight is further than 2 cm from the rest position for approximately 73.8% of the time.

Marking key/mathematical behaviours	Marks
• Attempts to estimate the correct fraction of any of the cycles from the graph	1
• Provides an reasonably accurate estimate (70-80%)	1
• Use a calculator to refine the result to 73.8%	1

MATHEMATICS APPLICATIONS
SEMESTER 1 (UNIT 1) EXAMINATION

CALCULATOR-ASSUMED
MARKING KEY



Firstly we determine the proportional areas for each colour (gives the probability sample

space). Hence the Probability that the coin lands on Blue is $\frac{1}{4}$

Marking key/mathematical behaviours	Marks
• represents sample space	1
• determines the correct probability	1

Question 17(b)

Solution

$$\frac{1}{4} + \frac{3}{16} = \frac{7}{16}$$

Marking key/mathematical behaviours	Marks
• identifies that needs to add the proportional areas	1
• adds the appropriate proportional areas correctly	1

Question 17(c)

Solution

$$\frac{1}{2} \times \frac{3}{16} = \frac{3}{32}$$

Marking key/mathematical behaviours	Marks
• identifies that two events need to occur simultaneously	1
• multiplies the appropriate probabilities to get the correct result	1

Question 17(d)

Solution

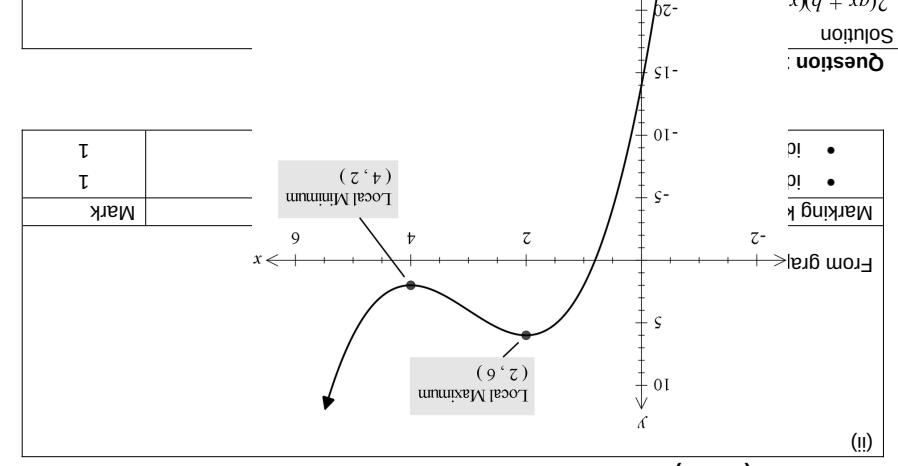
The events are independent i.e. probability of a tail is not affected by the colour it lands on

Hence, answer is $\frac{1}{2}$

Marking key/mathematical behaviours	Marks
• identifies that events are independent	1
• states the correct result	1

Question 17(e)

Solution



MATHEMATICS APPLICATIONS
SEMESTER 1 (UNIT 1) EXAMINATION

Question 14(b)

Solution

readable points from the graph are (0,1), (1,3) and (2,3)

\Rightarrow axis of symmetry is at $x = 1.5$

$$\therefore y = a(x - 1.5)^2 + c$$

$$\text{at } x = 0, y = 1 \Rightarrow 1 = \frac{9a}{4} + c$$

$$\text{at } x = 2, y = 3 \Rightarrow 3 = \frac{a}{4} + c$$

$$a = -1 \text{ and } c = \frac{13}{4}$$

Solving simultaneously we get that

$$y = -(x - 1.5)^2 + \frac{13}{4} = -x^2 + 3x + 1$$

Hence the equation is

Marking key/mathematical behaviours	Marks
• uses known points off graph	1
• determines axis of symmetry	1
• substitutes two points into a general, appropriate equation of a parabola	1
• solves for a and c	1
• states the equation in the required form	1

Question 14(c)

Solution

$$\Delta = b^2 - 4ac = 3^2 - 4(-1)(1)$$

$$= 13$$

> 0 , but not a perfect square \Rightarrow irrational roots

Marking key/mathematical behaviours	Marks
• determines the discriminant (of the parabola function equated to 0)	1
• interprets $\Delta > 0$	1

Question 15 (a)

Solution

substitute $x = 9$ into upper curve and get

$$y = \frac{-2(9)^2}{27} + \frac{8(9)}{3} - 9 = 9$$

therefore coordinates are (9, 9)

Marking key/mathematical behaviours	Marks
• substitutes $x = 9$ into upper curve	1
• states coordinates	1

CALCULATOR-ASSUMED
MARKING KEY

MATHEMATICS APPLICATIONS
SEMESTER 1 (UNIT 1) EXAMINATION

Question 15 (b)

Solution

using the coordinate (9, 9)

$$9 = a(9)^2 \Rightarrow a = \frac{1}{9}$$

$$\therefore \text{equation is } y = \frac{1}{9}x^2$$

Marking key/mathematical behaviours

- substitutes $x = 9$ into lower curve to obtain value of a
- states equation of lower curve

CALCULATOR-ASSUMED
MARKING KEY

Question 15 (c)

Solution

$$y = \frac{-2x^2}{27} + \frac{8x}{3} - 9 + 3 \therefore y = \frac{-2x^2}{27} + \frac{8x}{3} - 6$$

$$\text{new upper curve: } y = \frac{1}{9}x^2 + 3$$

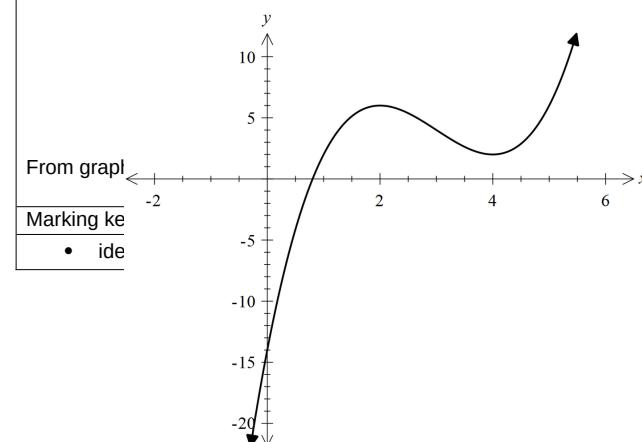
Marking key/mathematical behaviours

- states equation of new upper curve
- states equation of new lower curve

Question 16 (a)

Solution

(i)



From graph

Marking key

- ide

Mark
1