



PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 1 Year 12

Student name: _____ Teacher name: _____

Task type: **Response/Investigation**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **7**

Materials required: **No calcs allowed!!**

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, NO notes allowed!

Marks available: **41 marks**

Task weighting: **13%**

Formula sheet provided: no, but formulae stated on page 2

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = z = \sqrt{a^2 + b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan \theta = \frac{b}{a}, \quad -\pi < \theta \leq \pi$
$ z_1 z_2 = z_1 z_2 $	$\left \frac{z_1}{z_2} \right = \frac{ z_1 }{ z_2 }$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = z ^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{ z ^2}$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$	$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
Polar form	
$z = a + bi = r(\cos \theta + i \sin \theta) = r \text{cis } \theta$	$\bar{z} = r \text{cis } (-\theta)$
$z_1 z_2 = r_1 r_2 \text{cis } (\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis } (\theta_1 - \theta_2)$
$\text{cis}(\theta_1 + \theta_2) = \text{cis } \theta_1 \text{cis } \theta_2$	$\text{cis}(-\theta) = \frac{1}{\text{cis } \theta}$
De Moivre's theorem	
$z^n = z ^n \text{cis } (n\theta)$	$(\text{cis } \theta)^n = \cos n\theta + i \sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}} \left(\cos \frac{\theta + 2\pi k}{q} + i \sin \frac{\theta + 2\pi k}{q} \right), \quad \text{for } k \text{ an integer}$	

$$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

No calcs allowed!!

Q1 (2, 2, 2 & 2 = 8 marks)

If $z = 5 - 4i$ and $w = 2 + 3i$ determine the following:

a) zw

b) $\frac{1}{w}$

c) $\frac{\bar{z}}{w}$

d) $z^2 \bar{w}$

Q2 (2 & 3 = 5 marks)

a) Determine the complex roots of $3z^2 + z + 2 = 0$.

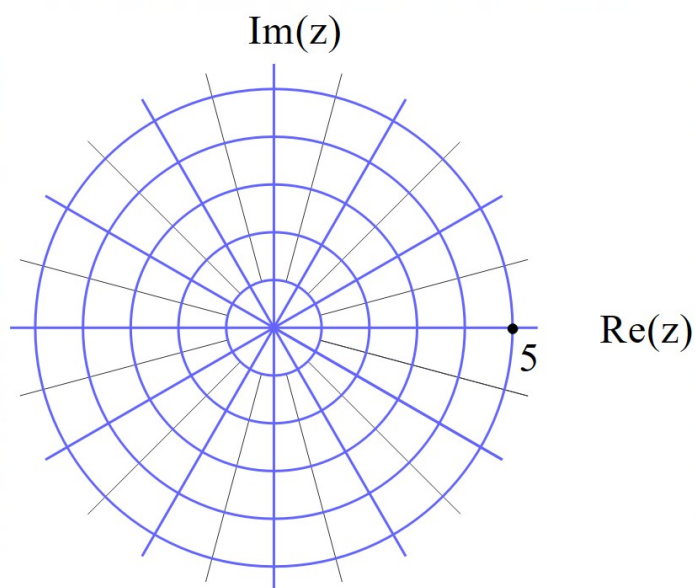
b) Use the quadratic equation to prove that if a quadratic equation with real coefficients has any non-real roots then it must have two complex roots and they must be conjugates of each other.

Q3 (4 marks)

Determine all possible real number pairs a & b such that $\frac{37+9i}{5+ai} = b-i$.

Q4 (2, 2, 2 & 2 = 8 marks)

Consider the complex number $z = \sqrt{3} + i$.



Plot the following on the axes above.

a) z

b) iz

c) $(1+i)z$

d) $\frac{z}{(1+i)}$

Q5 (5 marks)

Consider the polynomial $f(z) = az^4 + bz^3 + cz^2 + dz + e$ where a, b, c, d & e are real numbers.

Given that $f(2+i) = 0 = f(5-2i)$

and $f(0) = -290$

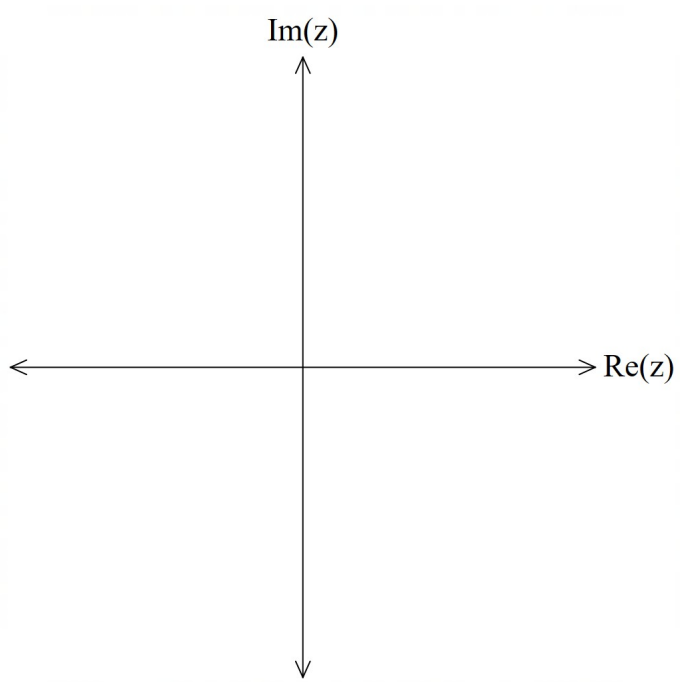
Determine the values of a, b, c, d & e .

(Note: answers without working will receive zero marks)

Q6 (2, 1, 2 & 2 = 7 marks)

Consider the locus of complex numbers z that satisfy $|z - \sqrt{3} + i| = 2$.

a) Sketch the locus on the axes below.



b) State the maximum value of $|z|$

c) State the minimum value of $\text{Arg}(z)$ such that $\text{Arg}(z) > \text{Minimum}$.

d) State the maximum value of $\text{Arg}(z)$ such that $\text{Arg}(z) < \text{Maximum}$.

Q7 (4 marks)

Consider the roots of the equation $z^n = a$ with z being a complex variable with a as a complex constant and n being an integer $n > 3$. A root is defined to be in the first quadrant if the Argument lies

in $0 < \text{Arg}(z) < \frac{\pi}{2}$.

Determine **all** the allowable values of n such that there will be **exactly** 3 roots in the first quadrant and

the smallest argument of these 3 roots will be $\frac{\pi}{10}$.

(Note: answers without working will receive zero marks)

Working out space