Semester 2 (Unit 3&4) Examination, 2019

Question/Answer Booklet

MATHEMATICS METHODS

Section One: Calculator-free

Student Name/Number: _____

Teacher Name: _____

Time allowed for this section
Reading time before commencing work: five minutes

Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	9	9	50	50	35
Section Two: Calculator-assumed	13	13	100	103	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the School/College assessment policy. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (50 Marks)**

This section has **(nine) 9** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Suggested working time: **50 minutes**.

Question 1 **(6 marks)**

(a) Determine $f'(x)$, given that $f(x)=e^{-x^2}\sqrt{2x-5}$. (3 marks)

(b) Determine $g(x)$,given that $g'(x)=\frac{x}{x^2+16}$ and $g(0)=\ln 5$. (3 marks)

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Acknowledgements

Published by The Mathematical Association of WA
12 Cobbler Place, MIRRABOOKA 6061

Additional working space

Question number: _____

Question 2 **(3 marks)**

Prior to an election, two samples of voters were selected independently from the Australian public. The first sample was three times as big as the second. The sample proportions of undecided voters were the same in each sample. When 90% confidence intervals were calculated for each sample, the margin of error for the first sample was equal to k multiplied by the margin for the second. What was the exact value of k?

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Question 3 **(7 marks)**

A discrete random variable X has probability distribution $P(x)$ given by

$P(x) = k \log_e e^x$ where $x = 1, 2, a.$

(a) Complete the table for $P(x).$ (2 marks)

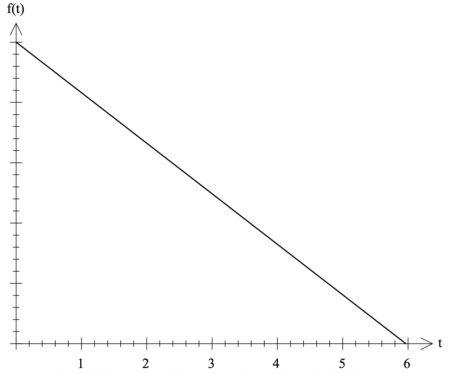
x	1	2	a
$P(x)$	k		

(b) Show that $a = \frac{1 - 3k}{k}.$ (2 marks)

(c) Determine the expected value of X when $k = \frac{1}{3}.$ (3 marks)

Question 9 **(6 marks)**

The new programmable calculator, CALC100X, can complete a calculation in anytime from 0 to 6 seconds, once the execute button has been pressed. A probability density function that the calculator will complete an operation t seconds after the execute button has been pressed, starts at $f(t) = k$ and falls at a constant rate until $t = 6$. Once six seconds has passed the calculator will respond with a message that the calculation is too complex to solve.



(a) Find the equation for the probability density function, given $0 \le t \le 6$. (2 marks)

(b) Determine the rule for the cumulative distribution function for T. (2 marks)

(c) Determine the probability that the calculator will complete a calculation within 3 seconds,
given it has not displayed the solution in the first second. (2 marks)

Question 7 **(3 marks)**

Determine the following definite integral,

$$\int_0^{\ln 2} e^{-2x}\,dx.$$

Question 8 **(4 marks)**

A spherical balloon is leaking gas. Use the increments formula to estimate the percentage change in the radius if its volume decreases from $800\ cm^3$ to $788\ cm^3$.

Question 4 **(11 marks)**

(a) By taking logs of both sides and isolating x, solve exactly $2^x = 3^{x-1}$. (3 marks)

(b) Solve for x, $\log_{10}(x+2) + \log_{10}(2x-3) = 2\log_{10} x$. (4 marks)

(c) Show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos x}{\sin x}\,dx = a\ln b$ and evaluate a and b, where b is an integer. (4 marks)

Question 5 **(4 marks)**

Given the equation below, for $x \geq 0$

$$\int_0^x f(t)dt = \left[f(x) \right]^2.$$

Differentiate to find $f'(x)$ and hence $f(x)$, if it is known that $f(x) \geq 0$.

Question 6 **(6 marks)**

Joggers, who are part of a fitness program, have been jogging at a constant speed of 4 m/s. They are then told to accelerate. During a five second period their acceleration increases at a constant rate from 0 m/s^2 to 2 m/s^2.

(a) At what speed are the joggers moving at the end of the five second period? (3 marks)

(b) How far do the joggers travel over the five second period? (3 marks)