Working out space

End of test

PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 2 Year 12

Student name: ________________ Teacher name: ________________

Task type: **Response/Investigation**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: 7

Materials required: Upto 3 classpads/calculators

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **42 marks**

Task weighting: **13%**

Formula sheet provided: no but formulae stated on page 2

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = \lvert z\rvert = \sqrt{a^2 + b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan\theta = \dfrac{b}{a}, \quad -\pi < \theta \le \pi$
$\lvert z_1 z_2\rvert = \lvert z_1\rvert \lvert z_2\rvert$	$\left\lvert \dfrac{z_1}{z_2} \right\rvert = \dfrac{\lvert z_1\rvert}{\lvert z_2\rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\dfrac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = \lvert z\rvert^2$	$z^{-1} = \dfrac{1}{z} = \dfrac{\bar{z}}{\lvert z\rvert^2}$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$	$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
Polar form	
$z = a + bi = r(\cos\theta + i\sin\theta) = r\,cis\,\theta$	$\bar{z} = r\,cis\,(-\theta)$
$z_1 z_2 = r_1 r_2\, cis\,(\theta_1 + \theta_2)$	$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\, cis\,(\theta_1 - \theta_2)$
$cis(\theta_1 + \theta_2) = cis\,\theta_1\, cis\,\theta_2$	$cis(-\theta) = \dfrac{1}{cis\,\theta}$
De Moivre's theorem	
$z^n = \lvert z\rvert^n cis\,(n\theta)$	$(cis\,\theta)^n = \cos n\theta + i\sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\dfrac{\theta + 2\pi k}{q} + i\sin\dfrac{\theta + 2\pi k}{q}\right)$, for k an integer	

$$(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$$

Working out space

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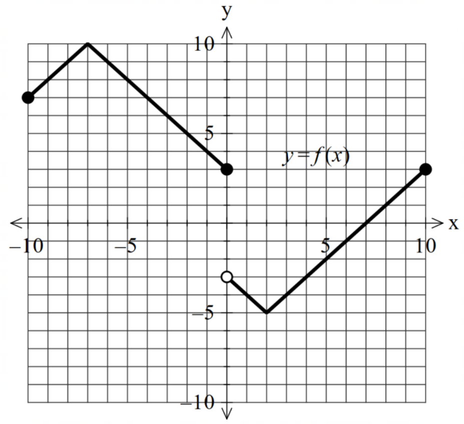
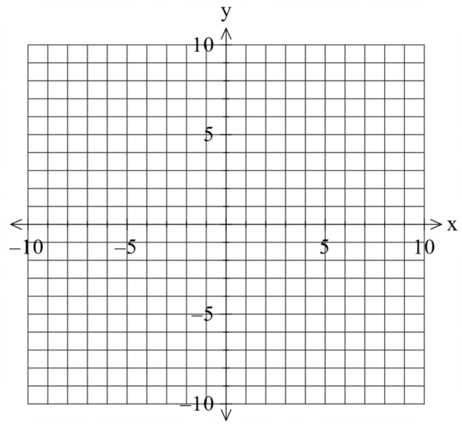
Q1 (2 & 3 = 5 marks)

Consider the function $f(x)$ plotted below.



a) Solve for $|f(x)|=5$.

b) Sketch $y=\left|f\left(|x|\right)\right|$ on the axes below.



Q2 (2, 3 & 3 = 8 marks)

Consider the functions $f(x)=\frac{1}{\sqrt{2x-9}}$ and $g(x)=\frac{1}{3x-1}$.

a) Determine the natural domain and range of $g(x)$.

b) Does $f \circ g(x)$ exist over the natural domain of $g(x)$? Explain.

c) Determine the largest possible domain for $f \circ g(x)$.

Q3 (3, 3, 1 & 2 = 9 marks)

Consider the function $f(x)=3x^2-12x+19,\ x \le 2$.

a) Determine $f^{-1}(x)$ and state its domain.

Q7 continued on next page

b) Determine the exact point in space, if any, where the smoke trails overlap at some time in the first 6 hours. (3 marks)

Q7 (3 & 3 = 6 marks)
Consider two rockets $A \& B$ that are ignited at the same time from different positions and move with constant velocities as shown below.

$$r_A = \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} km \quad ,v_A = \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} km/h$$

$$r_B = \begin{pmatrix} 5 \\ -3 \\ 15 \end{pmatrix} km \quad ,v_B = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} km/h$$

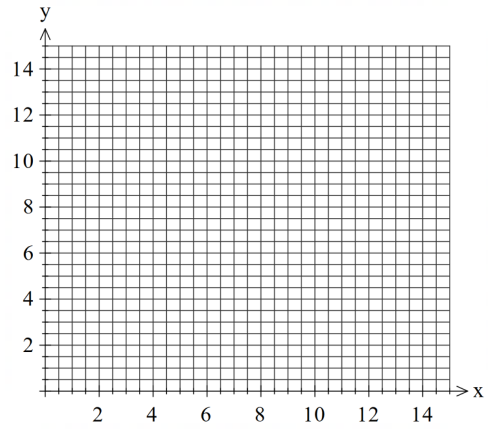
Both rockets leave a smoke trail that stays in the air for at least 6 hours.

a) Determine the distance of the closest approach between the rockets using **scalar dot** product

(3 marks)

Q3 continued

b) Sketch $f(x) \& f^{-1}(x)$ on the same set of axes below.

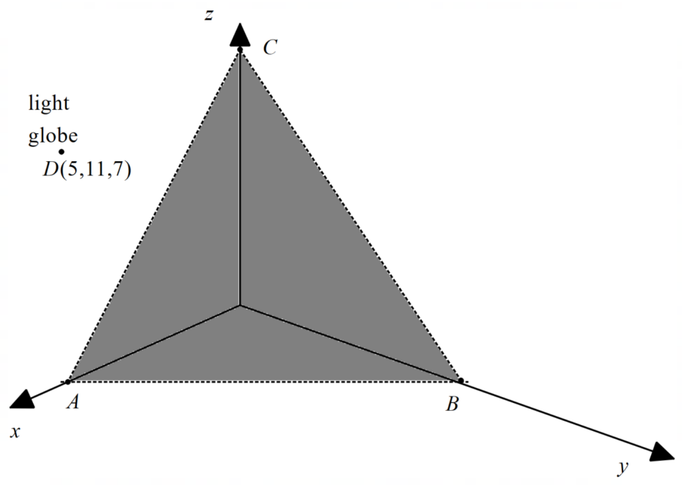


d) Determine value(s) of x, if any, such that $f \circ f(x) = x$. Explain.

Q4 (3 marks)

If $z = 27cis\frac{7\pi}{8}$ is a solution to the equation $z^n = ir$ where r is a positive real number and n is a positive integer, determine the smallest possible value for r in the form 3^p. **Justify** your answer.

Q5 (3 & 3 = 6 marks)
Consider a triangular plane with vertices $A(3,0,0), B(0,4,0) \& C(0,0,5)$ shaded as shown below. There is a light globe situated at point $D(5,11,7)$.



a) Determine the cartesian equation of the shaded plane ABC above.

Q5 continued

b) Determine the distance of the globe to the shaded plane ABC.

Q6 (5 marks)

Consider the line A $r = \begin{pmatrix} -3 \\ 0 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix}$ and the sphere B $\left| r - \begin{pmatrix} -3 \\ \alpha \\ 1 \end{pmatrix} \right| = 10$ where α is a real constant.

Determine all possible values of α ,to one decimal place such that:

i) the line misses the sphere.
ii) the line just touches the sphere.
iii) the line pierces the sphere at two points.