Question 6 [2,2=4 marks]

A discrete random variable X has E(X) =100 and Var(X) =100. Suppose that Y is a random variable such that Y = 2.5 X + 10.

$\sigma = 10$

Determine

(a) E(Y)

$2.5 \times 100 + 10$

$= 260$

(b) σ(Y)

2.5×10

$= 25$

END OF TEST

YEAR 12 MATHEMATICS
METHODS UNIT 3

APPLECROSS
SENIOR HIGH SCHOOL

TEST 3
TERM 2, 2017
Test date: Friday 5th of May

STUDENT NAME: ______________________

All working must be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks.

	Total	Result	
Section 1	27		%
Section 2	40		
Total	67		

Section 1: Resource – Free

Working time: 20 minutes

Question 1 [2, 2, 2, 1= 7 marks]

A particle moves in a straight line according to the function $x(t) = e^{sint}$, t ≥ 0, where t is in seconds and x is in metres.

(a) Determine the velocity function for this particle.

$V = \frac{dx}{dt} = (\cos t\, e^{\sin t})$ m/s

(b) Determine the instantaneous rate of change of the velocity. a

$a = \frac{dv}{dt} = -\sin t\, e^{\sin t} + \cos t\, e^{\sin t} \cos t$

90

(c) Evaluate exactly $\int_0^{\frac{\pi}{2}} x'(t)\ dt$.

$\left[e^{\sin t}\right]_0^{\frac{\pi}{2}}$

$= e^{\sin\frac{\pi}{2}} - e^{\sin 0}$

$= e - 1$

(d) What does the answer to part (c) represent in terms of the context of the particle moving according to the function $x(t) = e^{sint}$, t ≥ 0 seconds.

the displacement from $t = 0$ to $\frac{\pi}{2}$ seconds

Question 2 [1, 2, 2, 3, = 8 marks]

Differentiate each of the following functions with respect to x. Do not simplify your answers.

(a) $y = e^{-3x^2}$

$\frac{dy}{dx} = -6x e^{-3x^2}$

(b) $g(x) = -\cos\left(\frac{x}{2}\right)$

$g'(x) = + \sin\left(\frac{x}{2}\right) \times \frac{1}{2}$

(c) $f(x) = x^2 e^{2x-1}$

$f'(x) = 2x\, e^{2x-1} + x^2 (2) e^{2x-1}$

(d) $y = \sin^2(4x)$

let $\sin 4x$ be u $\quad \frac{du}{dx} = 4\cos 4x$

$\therefore y = u^2$

$\frac{dy}{du} = 2u$ $\quad \therefore \frac{dy}{dx} = 2\sin 4x \times 4\cos 4x$

Question 3 [3 marks]

Given $f'(g(x)) = e^{0.5x}\cos(2e^{0.5x})$ and $g(x) = e^{0.5x}$, determine $f(x)$.

$f(g(x)) = \int e^{0.5x} \cos(2e^{0.5x})$

$= \frac{e^{0.5x} \sin(2e^{0.5x})}{0.5\,2e^{0.5x}}$

$= \sin(2e^{0.5x})$

$g(x) = e^{0.5x}$

$f(x)\ \sin 2x$

$\sin^2 x + \cos^2 x = 1$

Question 4 [4 marks]

Show, by using the quotient rule, that $\frac{d}{dx}\tan(x) = 1 + \tan^2 x$.

$\tan x = \frac{\sin x}{\cos x}$

$\frac{d}{dx} = \frac{\cos x(\cos x) - \sin(x)(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$

$= 1 + \tan^2 x$

Question 5 [3,2,1=6 marks]

Fermium-257 is a radioactive substance that decays continuously such that $\frac{dQ}{dt} = kQ$,

where Q is the mass in grams and t is measured in days and Q_0 = the original amount and k is the rate of decay. The time taken to decay to half of the original amount is known as a substance's half-life. The half-life of Fermium-257 is 100.5 days.

(a) Determine the value of k to three significant figures.

$\frac{1}{2} = e^{100.5k}$

$k = -0.00690$

(b) How many days will it take for 100 grams of the substance to first decay below five grams?

$5 = 100e^{-0.00690t}$

$= 434$ days

(c) Determine the rate of change of the amount of Fermium on the day found in part (b).

$\frac{dQ}{dt} = -0.00690 \times 5$

$= -.00345$ g/day

Question 3 [3, 1, 1, 2, 2 = 9 marks]

At the local school fete, Daniel plays a game where he gambles on the roll of two dice.

Each time the two dice are rolled, he places a bet. The sum of the uppermost faces are noted and the prizes are awarded as follows:

- \$0 if the sum is 7 $\frac{6}{36}$
- \$1 if the sum is even $\frac{1}{2}$
- \$4 if the sum is 3 or 5
- \$6 if the sum is 9 or 11

Let Y represent the value of the prizes offered.

	1	2	3	4	5	6
1	2	3		5		
2	3		5			
3	4	5				9
4	5				9	
5	6			9		11
6	7		9		11	

(a) Represent the probability density function Y in a table below.

y	\$0	\$1	\$4	\$
$P(Y=y)$	$\frac{6}{36}$	$\frac{18}{36}$	$\frac{6}{36}$	$\frac{6}{36}$

Daniel bets \$1 for every roll of the two dice.

(b) What is the probability Daniel makes a loss?

$P(Y=0) = \frac{6}{36}$

(c) What is the probability Daniel breaks even?

$P(Y=1) = \frac{1}{2}$

(d) What is the probability that Daniel makes a profit given that he didn't make a loss?

$\frac{\frac{12}{36}}{\frac{30}{36}} = \frac{12}{36} \times \frac{36}{30} = \frac{12}{30} = \frac{6}{15} = \frac{2}{5}$

(e) What are Daniel's expected winnings?

$E(Y) = \$1.17$

Question 4 [2,2,2,3 = 9 marks]

Determine the following integrals

(a) $\int \sin(2x+1)\ dx$

$= \frac{-\cos(2x+1)}{2} + c$

(b) $\int (sin^3 x)(\cos x)\ dx$

$= \frac{\sin^4 x}{4} + c$

(c) $\int \frac{6}{e^{5x}}\ dx$

$= \int 6e^{-5x}\ dx$

$= \frac{6e^{-5x}}{-5} + c$

(d) $\int \frac{8e^{2x}+e^{-x}}{e^{-x}}\ dx$

$= \int \left(\frac{8e^{2x}}{e^{-x}} + \frac{e^{-x}}{e^{-x}} \right) dx$

$= \int 8e^{3x} dx + \int 1\ dx$

$= \frac{8e^{3x}}{3} + x + c$

END OF SECTION 1

YEAR 12 MATHEMATICS
METHODS UNIT 3

TEST 3
TERM 2, 2017
Test date: Friday 5th of May

APPLECROSS
SENIOR HIGH SCHOOL

STUDENT NAME: ______________________

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Section 2: Resource – Rich
Working time: 40 minutes

To be provided by the student:
ClassPad and/or Scientific Calculators
1 sheet of A4–sized paper of notes, double-sided

Question 1 [3, 3, 3 = 9 marks]

A carton contains 12 eggs, 5 of which are brown and 7 white. A chef selects 4 eggs at random, to use in an omelette.

(a) Determine the discrete probability distribution for X which represents the number of white eggs chosen, giving your answer in fraction form.

$P(0) = \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} \times \frac{2}{9}$

$P(1) = \frac{7}{12} \times \frac{5}{11} \times \frac{4}{10} \times \frac{3}{9} \times 4$

$P(2) = \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} \times 6$

$P(3) = \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10} \times \frac{5}{9} \times 4$

$P(4) = \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10} \times \frac{4}{9}$

x	0	1	2	3	4
P(X = x)	$\frac{1}{99}$	$\frac{14}{99}$	$\frac{14}{33}$ $\frac{42}{99}$	$\frac{35}{99}$	$\frac{7}{99}$

(b) Determine :

(i) $P(X \geq 2)$

$\frac{42+35+7}{99} = \frac{84}{99}$

(ii) $P(X \leq 3 \mid X \geq 2)$

$\frac{\frac{42}{99} + \frac{35}{99}}{\frac{84}{99}} = \frac{11}{12}$

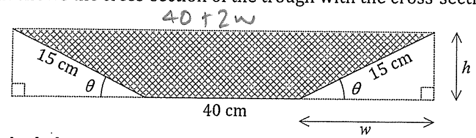
(c) Calculate the mean and standard deviation of the probability distribution.

$E(x) = 2.\dot{3}$

$\sigma_x = 0.84$

Question 2 [1,3,4 = 8 marks]

A trough for holding water is to be formed by taking a length of metal sheet 70 cm wide and folding 15 cm on either end, up through an angle of θ.
The following diagram shows the cross-section of the trough with the cross-sectional area, A, shaded.



(a) Determine the shaded area A in terms of w and h. (1 mark)

$A = \frac{1}{2}(40 + 40 + 2w) \times h$

$= (40 + w)h$

(b) Show that $A = 600 \sin\theta + 225 \sin\theta \cos\theta$. (3 marks)

$\sin\theta = \frac{h}{15}$

$h = 15 \sin\theta$

$\cos\theta = \frac{w}{15}$

$w = 15\cos\theta$

$A = [40 + (15\cos\theta)]\, 15\sin\theta$

$= 40 \times 15 \sin\theta + 15 \times 15 \sin\theta \cos\theta$

$= 600 \sin\theta + 225 \sin\theta \cos\theta$

(c) Use calculus to determine the maximum possible cross-sectional area. (4 marks)

$\frac{dA}{d\theta} = 600\cos\theta + 225\sin\theta(-\sin\theta) + 225\cos\theta\cos\theta$

$0 = 600\cos\theta - 225\sin^2\theta + 225\cos^2\theta$

$\theta = 0$ or 1.26

$A = 637\ cm^2$