Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2020

Question/Answer booklet

MATHEMATICS
METHODS

**Section Two:
Calculator-assumed**

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

WA student number:	In figures							
	In words							

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	10	10	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2020: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

Question number: _____

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Section Two: Calculator-assumed 65% (97 Marks)

This section has **10** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 **(7 marks)**

The weight, X, of chicken eggs from a farm is normally distributed with mean 60 g and standard deviation 5 g. Eggs with a weight of more than 67 g are classed as 'jumbo'.

(a) What proportion of eggs from the farm are 'jumbo'? (2 marks)

(b) What proportion of 'jumbo' eggs are less than 75 g in weight? (3 marks)

(c) The heaviest 0.05% of eggs fetch a higher price. What is the minimum weight of these eggs? (2 marks)

Supplementary page

Question number: ——————

Question 9 (8 marks)

A cake shop makes birthday cakes. The probability distribution of the number of birthday cakes sold in a day, X, is given below.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.25	0.35	0.1

(a)   Calculate the mean number or birthday cakes sold in a day.   (1 mark)

(b)   On Monday, the cake shop makes four birthday cakes. If each birthday cake costs $20 to make and sells for $50, what is the expected profit or loss on that day?   (3 marks)

On Tuesday, the shop makes three birthday cakes. Let the random variable Y denote the number of birthday cakes **not** sold on that day.

(c)   Explain why $P(Y = 0) = 0.45$.   (2 marks)

(d)   Obtain the probability distribution of Y.   (2 marks)

(b)   Determine, to the nearest metre, the value of q that will allow the sheep to graze over the maximum area and state this maximum area.   (4 marks)

The length of the fence from E to H is given by the equation:

$$L_{EH} = \int_0^q \sqrt{1 + (R'(x))^2} \, dx$$, where $R'(x)$ is the first derivative of $R(x)$.

(c)   (i)   Determine $R'(x)$.   (1 mark)

(ii)   Hence determine the total length of fencing required by David and Katrina to enclose their sheep with maximum area for grazing.   (3 marks)

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Question 10 (7 marks)

Water flows into a bowl at a constant rate. The water level, h, measured in centimetres, increases at a rate given by

$$h'(t) = \frac{4t+1}{2t^2+t+1}$$

where the time t is measured in seconds.

(a) Determine the rate that the water level is rising when $t = 2$ seconds. (1 mark)

(b) Explain why $h(t) = \ln(2t^2+t+1) + c$. (2 marks)

(c) Determine the total change in the water level over the first 2 seconds. (1 mark)

The bowl is filled when the water level reaches $\ln(56)$ cm.

(d) If the bowl is initially empty, determine how long it takes for the bowl to be filled. (3 marks)

Question 17 (12 marks)

David and Katrina have a small farm and wish to fence off an area of their land so they can raise sheep. The area they have chosen has one border along a road as shown in the diagram below.
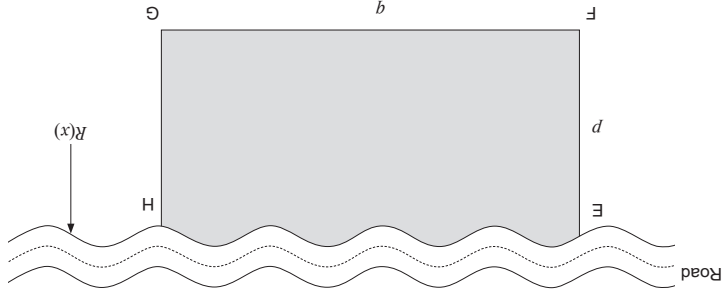
The enclosure is shown as the shaded area above and has right angles at points F and G.
David and Katrina want the combined lengths of the fencing from E to F and F to G to equal 500 metres. Let the length of fence EF be equal to q metres and the length of fence FG be equal to p metres. If we locate the origin at point F and the x-axis along the line FG, the equation defining the fence along the road is given by:

$$R(x) = 10\sin\left(\frac{x}{15}\right) + p$$

(a) Show that the equation defining the area of the enclosure, $A(q)$, can be given in terms of q as follows:

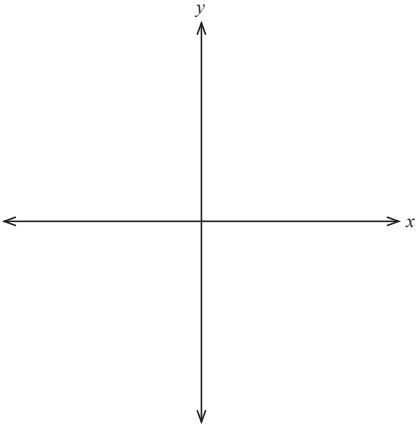$$A(q) = 500q - q^2 - 150\cos\left(\frac{q}{15}\right) + 150$$

(4 marks)

Question 11 (9 marks)

The line $y = x + c$ is tangent to the graph of $f(x) = e^x$.

(a) Obtain the coordinates of the point of intersection of the tangent with the graph of $f(x)$.
(2 marks)

(b) What is the value of c? (1 mark)

(c) Sketch the graph of $f(x)$ and the tangent on the axes below. (1 mark)

An alternative probability density function proposed to model the refrigerator temperature, in degrees Celcius, is given by:

$$p(t) = \frac{3}{4} t^3 - 3t^2 + 3t, \ \ 0 \le t \le 2$$

(d) Determine the probability that the refrigerator temperature is above $1\ °C$ using the new model. (2 marks)

Question 16 (7 marks)

A large refrigerator in a scientific laboratory is always required to maintain a temperature between 0 °C and 1 °C to preserve the integrity of biological samples stored inside. A scientist working in the laboratory suspects that the refrigerator is not maintaining the required temperature and decides to record the temperature every hour for seven days. Based on these measurements, the scientist concludes that the temperature, T, in the refrigerator is normally distributed with a mean of 0.8 °C and a standard deviation of 0.4 °C.

(a) Temperature in degrees Fahrenheit, T_f, is given by $T_f = \frac{9}{5}T + 32$. Determine the mean and standard deviation of the refrigerator temperature in degrees Fahrenheit. (2 marks)

(b) Determine the probability that the refrigerator temperature is above 1 °C. Give your answer rounded to four decimal places. (1 mark)

The histogram of data gathered by the scientist is shown below. N denotes the number of observations in each temperature interval.

N axis: 0, 10, 20, 30
T axis: −0.2 0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2
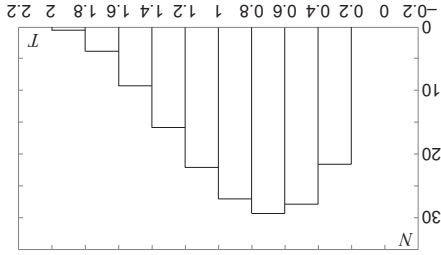
(c) Do you agree that the normal distribution was an appropriate model to use? Provide a reason to justify your response. (2 marks)

(d) Evaluate the exact area between the graph of $f(x)$, the tangent line, and the line $x = \ln 2$. (3 marks)

(e) Given that $g(x)$ is the inverse function of $f(x)$, write a definite integral that could be used to determine the area between the graph of $g(x)$, the x-axis, and the line $x = \ln 2$. (2 marks)

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Question 12   **(21 marks)**

It is estimated that 20% of small businesses fail in the first year. A business advisory group takes a random sample of 500 new businesses that started in January 2018. An analyst employed by the group suggests the use of the binomial distribution is appropriate in this case.

(a)   What is the probability that at most 120 of the businesses fail in the first year?   (2 marks)

(b)   What is the approximate distribution of the sample proportion of small businesses that fail by the end of the year in this sample? Justify your answer.   (3 marks)

(c)   What is the probability that the sample proportion of businesses that fail by the end of the year is less than 0.18?   (2 marks)

(d)   By January 2019, 90 of the 500 new businesses had failed. Calculate a 95% confidence interval for the proportion of new businesses that fail in the first year.   (2 marks)

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Assume that T_0 is still 200 °C.

(d)   Determine the rate of increase in temperature of the water five minutes after being placed in the oven. Give your answer rounded to two decimal places.   (2 marks)

(e)   Explain what happens to the rate of change in the temperature of the water as time increases and how this relates to the temperature of the water.   (3 marks)

Question 15 (9 marks)

A chef needs to use an oven to boil 100 mL of water in five minutes for a new experimental recipe. The temperature of the water must reach 100 °C in order to boil. The temperature, T, of 100 mL of water after t minutes after being placed in an oven set to T_0 °C can be modelled by the equation below.

$$T(t) = T_0 - 175e^{-0.07t}$$

In a preliminary experiment, the chef placed a 100 mL bowl of water into an oven that had been heated to $T_0 = 200$ °C.

(a) What is the temperature of the water at the moment it is placed into the oven? (1 mark)

(b) What is the temperature of the water five minutes after being placed in the oven? (1 mark)

(c) What change could be made to the temperature at which the oven is set in order to achieve the five-minute boiling requirement? (2 marks)

The business advisory group believes that the proportion of new businesses that fail within a year can be reduced by providing financial advice. They took another random sample of 500 businesses that started in January 2019 and provided them with regular financial advice. In this random sample, at the end of the year 80 businesses had failed.

(e) Calculate the sample proportion and its margin of error at the 95% confidence level. (2 marks)

(f) Calculate a 95% confidence interval for the proportion of businesses that failed. What do you conclude regarding the value of the financial advice provided to the new businesses? (4 marks)

(g) If the sample size was reduced, what would be the effect on the confidence interval? Justify your answer. (2 marks)

(h) State **two** assumptions that the analyst made in recommending the use of the binomial model in this case and discuss whether they are valid. (4 marks)
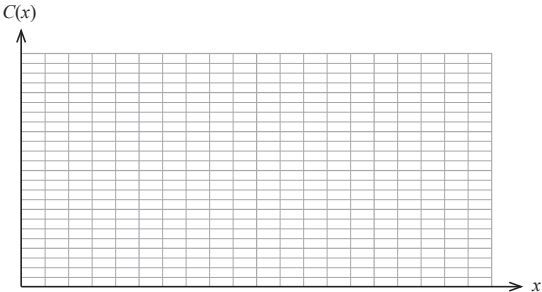
Question 13 (7 marks)

A company manufactures small machine components. They can manufacture up to 200 of a particular component in one day. The total cost, in hundreds of dollars, incurred in manufacturing

the components is given by: $C(x) = \dfrac{x \ln(2x + 1)}{3} - 2x + 120$, where x is the number of

components that will be produced on that day.

(a) Determine the total cost of manufacturing 20 components in one day. (1 mark)

(b) On the axes below, sketch the graph of $C(x)$. (3 marks)

(c) With reference to your graph in part (b), explain how many components the company should manufacture per day if the total cost is to be as low as possible. (3 marks)

Question 14 (10 marks)

A suburban council hires a consultant to estimate the proportion of residents of the suburb who use its library.

(a) The consultant decides to estimate a 95% confidence interval for the proportion to within an error of 0.01. What minimum sample size should be selected? (3 marks)

(b) If resource limitations dictate that the maximum sample size that can be managed is 500, what is the maximum margin of error in estimating a 99% confidence interval? (3 marks)

The consultant decides to select the sample by standing on the roadside outside the library at lunchtime and asking a random sample of the passers-by whether they use the library.

(c) Identify and explain **two** possible sources of bias with this sampling scheme. (4 marks)