Test 2
(Vectors and Mathematical Reasoning)

This assessment contributes 5% towards the final year mark.
40 minutes are allocated for this task.
The use of a CAS calculator is assumed for this task.

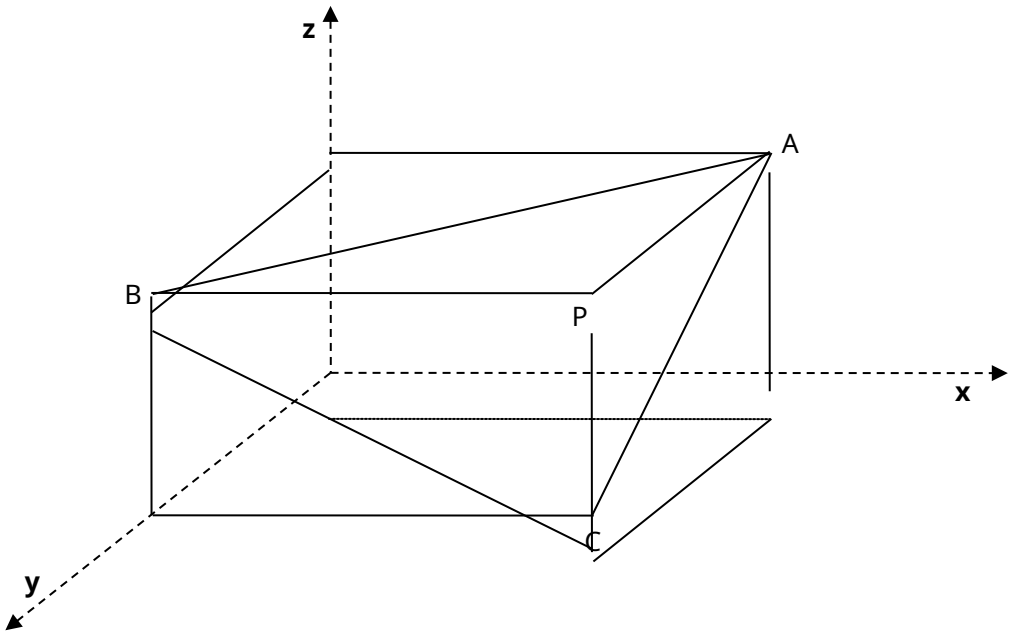
Name : _____

Score : _____
(out of 50)

Note : There are spare pages at the back of this booklet, if you require additional working space.

Do NOT turn over this page until you are instructed to do so.

1. The diagram shows a rectangular prism with dimensions 10 cm x 8 cm x 5 cm. Point P has position vector $10\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$.



a. Determine the position vectors for points A, B and C.

[2]

b. Determine the vector equation of the line containing points A and C.

[3]

Consider the plane determined by the points A, B and C. Hereafter in this question, this is the plane referred to as "the plane". Shade the plane in the diagram above.

c. Show that the normal vector to the plane can be given by $\mathbf{n} = 4\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$.

[3]

1. Given that point R is any point in the plane having position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$,

d. Show that $4x + 5y + 8z = 80$.

[3]

e. Give the expression for the vector $\overrightarrow{PR}$

[1]

f. Find the value of k such that $\overrightarrow{PR} = k\,\mathbf{n}$

i.e. $\overrightarrow{PR}$ is parallel to the normal vector **n**

[4]

g. Hence, or otherwise, determine the position vector for the point R such that R is the closest point in the plane to the point P.

[2]

h. Give the shortest distance, correct to 0.01 cm, from point P to the plane.

[1]

2. A plane is flying in a straight line with velocity vector k**i** + 50**j** – 2**k** km/hr. It is noticed that a car is travelling (on the ground) with velocity 100**j** km/hr.

At 1030 hrs the car is at position 100**i** – 80**j** km, whilst the plane is at 40**j** + 5**k** km.

a. Determine the expression for the position of the plane from the car at time t hours after 1030 hrs.

[3]

A neutral stationary observer notices that the plane is travelling approximately in a north-easterly direction.

b. If it is known that the plane is closest to the car at exactly 1200 hrs, determine the value(s) of the constant k correct to 0.001.

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[6]

2. b. If it is known that the plane is closest to the car at exactly 1200 hrs, determine the value(s) of the constant k correct to 0.001.

[6]

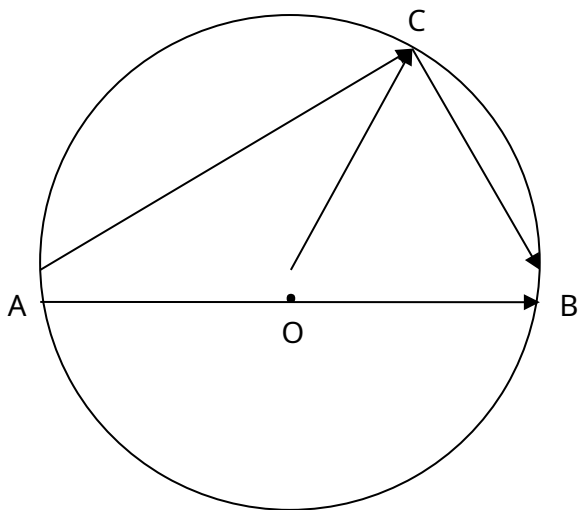
c. Give the distance of the closest approach correct to the nearest metre.

[2]

In questions 3 and 4, each must be proved using vector methods.

3. A circle is given with centre O. Points A and B form a diameter of the circle, with point C being any point of the semi-circle.

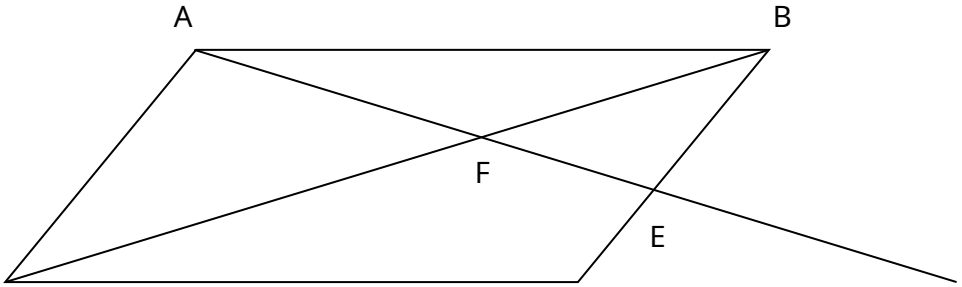
Prove that s$\angle$ ACB = 90^o.



Hint : Let $\overrightarrow{OB}$ = **b** and $\overrightarrow{OC}$ = **c**

[6]

4. Parallelogram OABC is shown, where $\overrightarrow{OA}$ = **a**, $\overrightarrow{OC}$ = **c**. $\overrightarrow{OC}$ is extended to determine point D such that $3\overrightarrow{CD} = 2\overrightarrow{OC}$. $\overrightarrow{AD}$ intersects $\overrightarrow{BC}$ at point E whilst $\overrightarrow{OB}$ intersects $\overrightarrow{AD}$ at point F.



O C D

a. Express $\overrightarrow{OD}$ in terms of $\overrightarrow{OC}$.

[1]

Let $\overrightarrow{AE} = k\overrightarrow{AD}$ and $\overrightarrow{BE} = m\overrightarrow{BC}$, where k, m are real constants.

b. Write an expression for $\overrightarrow{AD}$ in terms of vectors **a** and **c** and the constant k.

[2]

c. Given that $\overrightarrow{AE} = k\overrightarrow{AD}$ and $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$, obtain TWO expressions for the vector $\overrightarrow{AE}$, and hence prove that k = m = 0.6

[5]

4. d. Prove that $\overrightarrow{AF} = \frac{3}{8}\overrightarrow{AD}$ and $\overrightarrow{OF} = \frac{5}{8}\overrightarrow{OB}$.

[6]

End of Assessment Task

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