ATMAM Unit 3 – Test 3 – 2017

CANNING VALE COLLEGE

Name: Marking Key

Calculator Free Section (No notes or calculators. Formula sheet provided.)

Time allowed – 25 minutes **Marks: ~~20~~ 24**

Question 1 **[2, 2, 3, 2, 2 marks]**

a) If $f(x)=\dfrac{\sin(2\pi x)}{g(x)}$ and $g(x)\neq 0$, find $f'(x)$

$$f'(x)=\frac{g(x)\,2\pi\cos(2\pi x)-\sin(2\pi x)\,g'(x)}{[g(x)]^2}$$

✓ - deriv of $\sin 2\pi x$

✓ - quotient rule used correctly

b) Differentiate $y=4x^2\cos(x^3)$

$$\frac{dy}{dx}=\cos(x^3)(8x)-12x^4\sin(x^3)$$

✓ - each derivative correct

✓ - product rule used correctly

c) Find $\dfrac{d}{dx}(\sin(5-4x))$ and hence find $\int 12\cos(5-4x)\,dx$

$\dfrac{dy}{dx}=-4\cos(5-4x)$ ✓

$\therefore \int 12\cos(5-4x)\,dx$

$=-3\int -4\cos(5-4x)\,dx$ ✓

$=-3(\sin(5-4x))+c$

$=-3\sin(5-4x)+c$ ✓

d) If $f'(x) = 2\cos(5x)$, find $f(x)$

$f(x) = \frac{2\sin 5x}{5} + c$

$\int 2\cos(5x)\,du$

$= \frac{2}{5}\int 5\cos(5x)\,dx$ ✓

$= \frac{2\sin 5x}{5} + c$ ✓

e) $\frac{d}{dx}\left(\int_2^x \tan\theta \; d\theta\right)$

$= \tan x$ ✓

Question 2 **[2 marks]**

Janine drives to work each morning and passes through three traffic intersections with traffic lights. The number, X, of traffic lights that are red when Janine is driving to work is a random variable with probability distribution given by:

x	0	1	2	3
$P(X = x)$	0.1	0.2	0.3	0.4

Janine drives to work on two consecutive days. What is the probability that the number of traffic lights that are red is the same on both days?

P(same on both days) $= (0.1)^2 + (0.2)^2 + (0.3)^2 + (0.4)^2$ ✓

$= 0.01 + 0.04 + 0.09 + 0.16$

$= \underline{0.3}$ ✓

Question 3 **[3 marks]**

The table below describes the probability distribution for a discrete random variable X.

X	0	1	2	3
$P(X = x)$	$0.4p^2$	0.1	0.1	$1 - 0.6p$

Find the value of p

$0.4p^2 + 1 - 0.6p + 0.2 = 1$ ✓

$4p^2 + 10 - 6p + 2 = 10$

$4p^2 - 6p + 2 = 0$

$2p^2 - 3p + 1 = 0$ ✓

$(2p - 1)(p - 1) = 0$

$\underline{p = \frac{1}{2} \text{ or } p = 1}$ ✓

ATMAM Unit 3 – Test 3 – 2017

CANNING VALE COLLEGE

Name:____________________________

Calculator Assumed Section (1 A4 page of notes allowed. Formula sheet provided.)

Time allowed – 30 minutes **Marks: 28**

Question 7 **[4 marks]**

Find the equation of the tangent to the curve with equation $y = 3\sin(2x) - \cos(2x)$, at the point where $x = \frac{\pi}{4}$

$\frac{dy}{dx} = 6\cos(2x) + 2\sin(2x)$ ✓

at $x = \pi/4$, $\frac{dy}{dx} = 2$ ✓

at $x = \pi/4$, $y = 3$ ✓

Eq'n: $y = 2x + c$

$3 = 2\left(\frac{\pi}{4}\right) + c$

$c = -\frac{\pi}{2} + 3$

$y = 2x + 3 - \frac{\pi}{2}$ ✓

Question 8 **[1, 2, 2 marks]**

Left-handed people make up 9½ % of the population. What is the probability that in a randomly selected group of four people:

a) $X \sim B(4, 0.905)$ or $Y \sim B(4, 0.095)$ ✓

b) There are exactly 3 right-handed people?

$P(x = 3) = 0.2817$ or $P(y = 1) = 0.2817$ ✓

c) There are more left-handed than right-handed people?

$P(x < 2) = 0.0032$ ✓ ✓ or $P(y > 2)$

0.003185

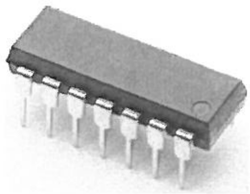
d) They are all left-handed, given that there are more left-handed people than right-handed people in the group?

$P(x=0 \mid x<2) = \frac{8.14 \times 10^{-5}}{3.185 \times 10^{-3}}$ ✓ or $P(y=4 \mid y>2)$

$= 0.0256$ ✓

(0.02557200)

Question 9 [4, 2 marks]



It is known that 5% of a batch of computer chips are defective. A sample of twenty chips is randomly selected from this batch.

a) Determine the probability that there: $X \sim B(20, 0.05)$ ✓

(i) are no more than 2 defective chips in this sample.

$P(x \leq 2) = 0.9245$ ✓

(ii) is at least one defective chip in this sample.

$P(x \geq 1) = 0.64$ ✓

(iii) is no more than 2 defective chips in this sample, if it is known that there is at least 1 defective chip in this sample.

$$P(x \leq 2 \mid x \geq 1) = \frac{P(1 \leq x \leq 2)}{P(x \geq 1)} \checkmark$$

$$= \frac{0.566}{0.64}$$

$$= 0.8823 \checkmark$$

b) Determine the expected number of defective chips in a sample of 1000 chips and its associated standard deviation.

$E(x) = 1$ ✓

$SD = \sqrt{np(1-p)}$

$= \sqrt{0.95}$

$= 0.9747$ ✓

Question 10 [6 marks]

For the discrete probability distribution shown below investigate the possible values of the mean and the variance. (Do not use STAT menu)

x	1	2	3	4	5
$P(X = x)$	0.1	0.1	0.1	a	a
				0.35	0.35

$a = 0.35$ ✓

$E(x) = 0.6 + 9a$ ✓

$= 3.75$ ✓

$SD = \sqrt{1^2(0.1) + 2^2(0.1) + 3^2(0.1) + 4^2(0.35) + 5^2(0.35) - (3.75)^2}$ ✓✓

$= 1.299$ ✓

Question 11 **[3, 2, 2 marks]**

A particle moves along a straight line so that its acceleration a in m/s^2 at time t seconds is given by:

$$a = -\frac{3\pi^2}{4}\cos\left(\frac{\pi t}{2}\right)$$

Initial velocity is 0 m/s^2. Initial displacement is 3 metres to the right of the origin.

Determine:

a) The maximum velocity of the particle, and the time at which this first occurs. *(Show some reasoning for full marks)*

max v at $a = 0$

$-\frac{3\pi^2}{4}\cos\left(\frac{\pi t}{2}\right) = 0$ ✓

max at $t = 3$ ✓

$v = -\frac{3\pi}{2}\sin\left(\frac{\pi t}{2}\right)$

at $t = 3$, $v = \frac{3\pi}{2}$ ✓

4.712

acc = 0
Solve to get t=3
sub into v

b) An expression for the displacement of the particle at time t

$d = \int -\frac{3\pi}{2}\sin\left(\frac{\pi t}{2}\right) dt$

$= 3\cos\left(\frac{\pi t}{2}\right) + c$ ✓

at $t = 0$ $d = 3$

$\therefore c = 0$ ✓

disp $= 3\cos\left(\frac{\pi t}{2}\right)$

c) The total distance travelled by the particle before returning to its initial position.

Returns at $t = 4$ ✓

Distance $= \int_0^4 \left|-\frac{3\pi}{2}\sin\left(\frac{\pi t}{2}\right)\right| dx$

$= 12$ ✓