Christ Church Grammar School

WA Exams Practice Paper C, 2015

Question/Answer Booklet

MATHEMATICS
METHODS
UNITS 1 AND 2
Section One:
Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words ______________________

Your name ______________________

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	14	14	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free **(52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(7 marks)**

(a) Solve the equation $3(1-2a)-2(a-1)=6$. (2 marks)

(b) The coordinates of three points are A(-2, -6), B(4, -2) and C(4, 2).

(i) If A is the mid-point of C and D, determine the coordinates of D. (2 marks)

(ii) Determine the gradient of the line through A and B. (1 mark)

(iii) Find the equation of the line through C that is perpendicular to the line AB. (2 marks)

See next page

Question 2 **(7 marks)**

(a) Determine the coordinates of all axes intercepts of $y = (x+1)^2 - 4$. (2 marks)

(b) State the coordinates of the turning point of $y = x^2 - 10x - 21$. (2 marks)

(c) Solve

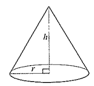
(i) $(2x-5)(x+3) = 0$. (1 mark)

(ii) $x^2 - x = 20$. (2 marks)

See next page

Question 19 **(6 marks)**

A cone has a radius r and perpendicular height h and is such that the sum of the radius and twice the height is 45 cm.



(a) Show that the volume, V, of the cone is given by $V = \frac{\pi}{3}\left(4h^3 - 180h^2 + 2025h\right)$ cm³. (3 marks)

$$V = \frac{1}{3}\pi r^2 h$$

But $r + 2h = 45$

$$V = \frac{\pi}{3}(45 - 2h)^2 h$$

$$V = \frac{\pi}{3}\left(4h^3 - 180h^2 + 2025h\right)$$

(b) Using calculus techniques, find the height that will maximise the volume of the cone, and state this maximum volume, rounded to one decimal place. (3 marks)

$$\frac{dV}{dh} = \pi\left(4h^2 - 120h + 675\right)$$

$$\pi\left(4h^2 - 120h + 675\right) = 0$$

$h = 7.5,\ h = 22.5$

$V(7.5) = 7068.583$

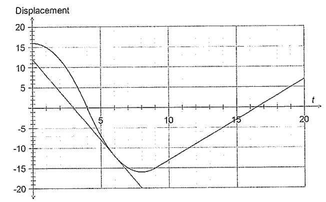
$V(22.5) = 0$

Maximum volume of 7068.6 cm³ when height is 7.5 cm

See next page

Question 20 **(5 marks)**

A small toy train is able to travel backwards and forwards along a straight track built on level ground. The displacement in metres, of the train relative to point A, is shown on the graph below for the interval $0 \le t \le 20$ seconds.



(a) State an interval of time during which the train is moving towards point A. (1 mark)

$0 < t < 4$ or $9 < t < 16.5$

(b) What total distance did the train travel during the 20 second interval? (1 mark)

$16 + 16 + 16 + 7 = 55$ metres

(c) By drawing adding a suitable tangent to the graph above, determine an estimate for the velocity of the train when $t = 6$. (3 marks)

Draw tangent when $t = 6$, as shown.

Gradient: $m \approx -\frac{12}{3} \approx -4$

Estimate is -4 m/s.

End of questions

Question 17 **(7 marks)**

When a capacitor discharges through a resistor, the voltage, V in volts, across the capacitor decays according to the rule $V = 20(0.38)^t$, where t is the time, in seconds, after the discharge began.

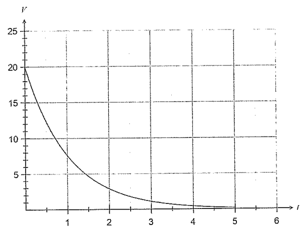
(a) What was the initial voltage across the capacitor? (1 mark)

$\boxed{V_0 = 20 \text{ volts}}$

(b) What was the voltage across the capacitor after four seconds? (1 mark)

$\boxed{V = 20(0.38)^4 = 0.417 \text{ volts}}$

(c) Draw the graph of the voltage against time for $0 \le t \le 5$. (3 marks)



(d) How long, to the nearest millisecond, does it take for the voltage across the capacitor to halve? (2 marks)

$20(0.38)^t = 10 \Rightarrow t = 0.716369$

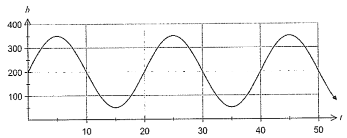
$t = 716$ milliseconds

Question 18 **(9 marks)**

The brightness of a small incandescent light globe, b lumens, t milliseconds after measurements began can be modelled by the function $b(t) = r + p\sin(qt)$.

Initially the brightness was 200 lumens, increasing after 5 milliseconds to a maximum of 350 lumens and then dropping to a minimum brightness of 50 lumens after a further 10 milliseconds.

(a) Sketch how the brightness varied over the first 50 milliseconds on the axes below. (3 marks)



(b) Explain why $p = 150$, $q = \frac{\pi}{10}$ and $r = 200$. (3 marks)

r is mean of oscillation:
$r = 200$

q adjusts period to 20 ms:
$q = \frac{2\pi}{20} = \frac{\pi}{10}$

p is amplitude of function:
$350 = 200 + p \Rightarrow p = 150$

(c) For what percentage of each cycle is the brightness of the globe less than 90 lumens? (3 marks)

Solve $90 = 200 + 150\sin(\frac{\pi t}{10})$ to get first two solutions of $t = 12.62$ and $t = 17.38$.

$\frac{17.38 - 12.62}{20} \times 100 = 23.8\%$

Question 3 **(5 marks)**

(a) Determine as an exact value $\sin 45° \cdot \cos 45° + \cos 45° \cdot \sin 45°$. (1 mark)

(b) Solve $\cos x = \sqrt{3}\sin x$ for $-\pi \le x \le \pi$. (2 marks)

(c) Determine the coefficient of the x^4 term in the expansion of $(x-3)^6$. (2 marks)

Question 4 **(8 marks)**

(a) Simplify $\left(2\frac{7}{9}\right)^{-\frac{1}{2}}$. (2 marks)

(b) If $a = 5\times10^{2}$ and $b = 8\times10^{6}$ evaluate $a^{2} \div b^{1/3}$. (2 marks)

(c) Solve $25^{x} = 125\sqrt{5}$. (2 marks)

(d) State the equation of the asymptote of the following graphs:

(i) $y = 0.5^{x+2}$. (1 mark)

(ii) $y = 0.5^{x} - 2$. (1 mark)

Question 15 **(8 marks)**

A store accepts credit card payments from customers using American Express, Mastercard or VISA cards. Records indicate that 65% of customers use a credit card, and of these customers, 20% use American Express, 35% Mastercard and the rest VISA. Further analysis shows that the male to female ratio for users of each type of card is 5:3 for American Express, 2:3 for Mastercard and 3:2 for VISA.

(a) Calculate the probability that a randomly selected customer from the records will be a female who uses an American Express credit card. (2 marks)

$$\boxed{0.65\times0.2\times\tfrac{3}{8} = 0.04875}$$

(b) Given that a randomly selected customer used a credit card, what is the probability that they are male? (3 marks)

$$\begin{aligned} P(\text{male}) &= 0.2\times\tfrac{5}{8} + 0.35\times\tfrac{2}{5} + 0.45\times\tfrac{3}{5} \\ &= 0.125 + 0.14 + 0.27 \\ &= 0.535 \end{aligned}$$

(c) What is the probability that a randomly selected female customer who used a credit card used a VISA card? (3 marks)

$$\begin{aligned} P(\text{Female} \mid \text{used card}) &= 1 - 0.535 \\ &= 0.465 \end{aligned}$$

$$\begin{aligned} P(\text{Female and VISA} \mid \text{used card}) &= 0.45\times\tfrac{2}{5} \\ &= 0.18 \end{aligned}$$

$$\begin{aligned} P &= \frac{0.18}{0.465} \\ &= \frac{12}{31} \\ &\approx 0.3871 \end{aligned}$$

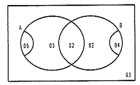
Question 16 **(7 marks)**

Two independent events A and B are such that $P(A\cap B) = 0.2$ and $P(\bar{B}) = 0.6$.

(a) Calculate

(i) P(A) (2 marks)

$$\begin{aligned} P(A\cap B) &= P(A)\times P(B) \\ P(A) &= 0.2 \div 0.4 \\ &= 0.5 \end{aligned}$$



(ii) $P(A\cup B)$ (1 mark)

$$\boxed{0.7}$$

(iii) $P(\bar{B} \mid (\bar{A}\cup\bar{B}))$ (2 marks)

$$\boxed{\frac{0.6}{0.8} = 0.75}$$

(b) A third event, C, is complementary with event A.
What is the maximum possible value of $P(C\cup B)$? (2 marks)

$$\begin{aligned} &P(C) = 1 - P(A) = 0.5 \\ &P(B\cap\bar{A}) = 0.2 \\ &0.5 \le P(C\cup(B\cap\bar{A})) \le 0.7 \end{aligned}$$

Maximum value is 0.7

(ii) Determine the area of the central shaded region. (6 marks)

Area of pentagon:
$5\times10\times\frac{10}{\tan(36)}=688.19\text{ cm}^2$

Area of a semi-circle:
$\frac{\pi\times10^2}{2}=157.08\text{ cm}^2$

Area of semi-circle overlap:
$2\times\frac{1}{2}\times10^2\left(\frac{2\pi}{5}-\sin\left(\frac{2\pi}{5}\right)\right)=30.56\text{ cm}^2$

Required area:
$688.19-5\times(157.08-30.56)=688.19-5\times126.52$
$=55.58\text{ cm}^2$

Question 14 **(7 marks)**

Consider the function $f(x)=136-96x-4x^2+24x^3-4x^4$.

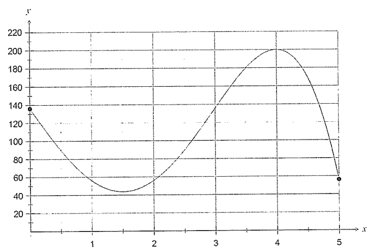
(a) Using calculus techniques, determine the coordinates of all stationary points of the graph of $y=f(x)$ in the interval $0\le x\le5$. (4 marks)

$\frac{dy}{dx}=-16x^3+72x^2-8x-96$

Solve $0=-16x^3+72x^2-8x-96$
$\cancel{x=-1}$, $x=4$, $x=1.5$

Stationary points at $(4,200)$ and $(1.5,43.75)$

(b) Sketch the graph of $y=f(x)$ over the interval $0\le x\le5$. (3 marks)



Question 5 **(8 marks)**

(a) Let $f(x)=2(x+1)(x-1)^2$.

(i) State the coordinates of the y-intercept of the graph of $y=f(x)$. (1 mark)

(ii) State the coordinates of the roots of the graph of $y=f(x)$. (1 mark)

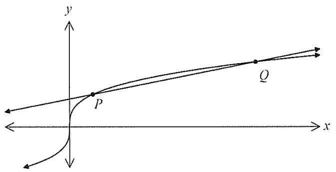
(iii) Determine the range of $f(x)$ over the domain $x\ge1$. (1 mark)

(b) Expand $(x+1)(x+2)(2x-1)$. (2 marks)

(c) Solve $x^3-x^2-10x-8=0$. (3 marks)

Question 6 **(8 marks)**

The graph of $y=\sqrt[3]{x}$ is shown below together with the secant cutting the graph at the points P and Q, where $x=1$ and $x=8$ respectively.



(a) Determine the equation of the secant PQ. (2 marks)

(b) If the x-coordinate of point Q was decreased from 8 towards 1, explain the effect this would have on your answer to (a). (2 marks)

(c) Determine the equation of the tangent to the graph of $y=\sqrt[3]{x}$ at P. (3 marks)

(d) Draw the tangent from (c) on the graph above. (1 mark)

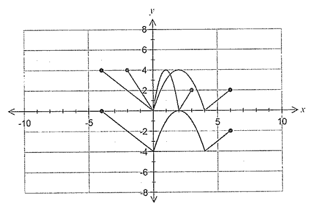
Question 12 **(5 marks)**

(a) State the domain and range of $f(x)=x^4-4$. (2 marks)

$x: x\in\mathbb{R}$

$y: y\geq -4$

(b) The graph of $y=g(x)$ is shown below.



On the same axes, sketch the graph of

(i) $y=g(x)-4$. (1 mark)

(ii) $y=g(2x)$. (2 marks)

Question 13 **(12 marks)**

(a) A thin piece of glass has been cut into the shape of an obtuse-angled triangle with an area of 135.5 cm² and two sides of 21.8 cm and 25.4 cm.

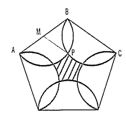
Calculate the length of the third side, correct to 3 significant figures. (4 marks)

$135.5=0.5\times21.8\times25.4\times\sin\theta$
$\therefore\theta=150.7°$ (obtuse solution)

$x^2=21.8^2+25.4^2-2\times21.8\times25.4\times\cos150.7°$
$x=45.67$
$x\approx45.7$ cm to 3sf

(b) The diagram shows five congruent semicircles standing on the inside of a regular pentagon with sides of length 20 cm.

M is the midpoint of the side AB and P is the point of intersection of two semi-circles.



(i) Show that the size of angle $\angle BMP=72°$. (2 marks)

BP bisects $\angle ABC$ so that $\angle MBP=108\div2=54$

ΔMBP is isosceles, so $\angle BMP=180-2\times54=72°$

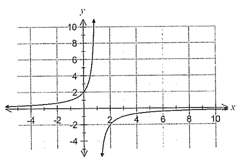
Question 10 **(8 marks)**

(a) Let $f(x)=\frac{2}{1-x}$.

(i) State the equations of the asymptotes of the graph of $y=f(x)$. (2 marks)

$\boxed{x=1 \text{ and } y=0}$

(ii) Sketch the graph of $y=f(x)$. (2 marks)



(b) The two variables h and w are inversely proportional to one another.

(i) Circle all of the equations below that reflect this relationship, where k is a constant. (2 marks)

$h+w=k$ $\quad w=hk$ $\quad \boxed{wh=k}$ $\quad \frac{h}{k}=w$ $\quad \boxed{\frac{k}{w}=h}$ $\quad \frac{h}{w}=k$

(ii) When $h=12.5$, $w=38.8$. If h decreases by 2.8, by how much will w change? (2 marks)

$12.5-2.8=9.7$

$12.5\times 38.8=9.7\times w$
$w=50$

$50-38.8=11.2$

w will increase by 11.2

See next page

Question 11 **(10 marks)**

(a) Determine x if the terms 12, x, 27 form part of a geometric sequence. (2 marks)

$x^2=12\times 27$
$x=\pm 18$

(b) A team of workers is using a pile driver to drive wooden poles 4 metres long into the ground. The first hit of the pile driver drives a pole 50 cm into the ground. The second hit drives the pole another 40 cm into the ground. The third hit drives the pole another 32 cm into the ground and successive distances driven by the pile driver form a geometric sequence.

(i) How much further will the fourth hit drive the pole into the ground? (1 mark)

$40\div 50=0.8$
$32\times 0.8=25.6$ cm

(ii) Determine the total distance the wooden pole has been driven into the ground after 12 hits of the pile driver. (2 marks)

$S_{12}=\frac{50(1-0.8^{12})}{1-0.8}=232.82$ cm

(iii) If the workers continued in this way for some time, what length of the wooden pole will always be left above the ground? Justify your answer. (2 marks)

$S_\infty=\frac{50}{1-0.8}=250$

$400-250=150$ cm

(c) A geometric series has a first term of 80 and a sum to infinity of 800. Determine the minimum number of terms of this sequence required so that their sum exceeds 700. (3 marks)

$800=80\div(1-r) \Rightarrow r=0.9$

$\frac{50(1-0.9^n)}{1-0.9}\geq 700 \Rightarrow n\geq 20$

See next page

Question 7 **(9 marks)**

(a) Determine $f'(-1)$ if $f(x)=\frac{x^3}{6}-\frac{x}{3}$. (2 marks)

(b) Determine $f(1)$ given that $f(2)=5$ and $f'(x)=8x^3-8x+1$. (2 marks)

(c) A curve has equation $y=ax^2+bx+c$. The curve has a turning point at (4, 9) and a gradient of -1 when $x=3$. Determine the values of a, b and c. (5 marks)

End of questions

Section Two: Calculator-assumed (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 (5 marks)

In a random survey of 162 swimmers at a council owned pool, 109 said they swam regularly. 39 males said they swam regularly and 16 fewer males than females were surveyed.

(a) Complete this two-way table using the above information. (2 marks)

	Swam regularly	Did not swim regularly	Total
Female	70	19	89
Male	39	34	73
Total	109	53	162

(b) If one swimmer is selected at random from those surveyed, determine the probability

(i) they swam regularly. (1 mark)

$$\boxed{\frac{109}{162} \approx 0.673}$$

(ii) they swam regularly, given that they were female. (1 mark)

$$\boxed{\frac{79}{89} \approx 0.787}$$

(c) Based on the information in the table, is there any indication that swimming regularly at the council owned pool is independent of the gender of the swimmer? Justify your answer. (1 mark)

No, as from the probabilities calculated in (b) it can be seen that $P(SR) \neq P(SR | F)$.

Question 9 (9 marks)

(a) An arithmetic sequence has third term 28 and eighth term 41.75.

(i) Determine a definition of this sequence in the form $T_n = a + (n-1)d$. (2 marks)

$$d = \frac{41.75 - 28}{8 - 3} = 2.75$$
$$a = 28 - 2 \times 2.75 = 22.5$$
$$T_n = 22.5 + (n-1) \times 2.75$$

(ii) Determine the sum of the first twenty terms of this sequence. (1 mark)

$$S_{20} = \frac{20}{2}(2 \times 22.5 + (20-1) \times 2.75)$$
$$= 972.5$$

(b) A book editor charged clients 65 cents per page plus a flat fee of $120.

(i) Determine a recursive rule for the amount, a_n , the editor charges to edit a book of n pages, where a_n is in dollars. (2 marks)

$$a_n = a_{n-1} + 0.65$$
$$a_0 = 120$$

(ii) The editor charged a client less than $220 to edit a draft manuscript. Determine the maximum number of pages the draft contained. (1 mark)

$$\boxed{120 + n \times 0.65 \leq 220 \Rightarrow n \leq 153}$$

(c) Determine T_{10} of the arithmetic sequence where $T_1 = x - 3$, $T_2 = 2x + 1$ and $T_3 = 4x - 1$. (3 marks)

$$d = 4x - 1 - (2x + 1) = 2x + 1 - (x - 3) \Rightarrow x = 6$$
$$d = 10$$
$$a = 6 - 3 = 3$$
$$T_{10} = 3 + 9 \times 10$$
$$= 93$$

Question 7 **(9 marks)**

(a) Determine $f'(-1)$ if $f(x)=\frac{x^3}{6}-\frac{x}{3}$. (2 marks)

$$f'(x)=\frac{x^2}{2}-\frac{1}{3}$$
$$f'(-1)=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$$

(b) Determine $f(1)$ given that $f(2)=5$ and $f'(x)=8x^3-8x+1$. (2 marks)

$$f(x)=2x^4-4x^2+x+c$$
$$5=32-16+2+c \Rightarrow c=-13$$
$$f(1)=2-4+1-13$$
$$=-14$$

(c) A curve has equation $y=ax^2+bx+c$. The curve has a turning point at (4, 9) and a gradient of -1 when $x=3$. Determine the values of a, b and c. (5 marks)

$\frac{dy}{dx}=2ax+b$
When $x=3$, $y'=-1$
When $x=4$, $y'=0$
$6a+b=-1$
$8a+b=0$
Subtract to get $2a=1$
$\therefore a=0.5$
$\therefore b=-4$
Use (4, 9) to find c
$9=0.5(4)^2-4(4)+c$
$\therefore c=17$

End of questions

Christ Church Grammar School

WA Exams Practice Paper C, 2015

Question/Answer Booklet

MATHEMATICS METHODS UNITS 1 AND 2
Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures [ ][ ][ ][ ][ ][ ][ ][ ]

In words ______________________

Your name ______________________

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Question 5 **(8 marks)**

(a) Let $f(x)=2(x+1)(x-1)^2$.

(i) State the coordinates of the y-intercept of the graph of $y=f(x)$. (1 mark)

(0, 2)

(ii) State the coordinates of the roots of the graph of $y=f(x)$. (1 mark)

(-1, 0) and (1, 0)

(iii) Determine the range of $f(x)$ over the domain $x\geq 1$. (1 mark)

Root at (1, 0) is also a minimum turning point.

Hence $y\geq 0$.

(b) Expand $(x+1)(x+2)(2x-1)$. (2 marks)

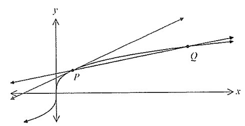
$(2x-1)(x^2+3x+2)=2x^3+5x^2+x-2$

(c) Solve $x^3-x^2-10x-8=0$. (3 marks)

$$f(-1)=0 \Rightarrow (x+1)(x-2x-8)=0$$
$$(x+1)(x+2)(x-4)=0$$
$$x=-1,\ x=-2,\ x=4$$

Question 6 **(8 marks)**

The graph of $y=\sqrt[3]{x}$ is shown below together with the secant cutting the graph at the points P and Q, where $x=1$ and $x=8$ respectively.



(a) Determine the equation of the secant PQ. (2 marks)

$$m=\frac{\sqrt[3]{8}-\sqrt[3]{1}}{8-1}=\frac{1}{7}$$
$$y-1=\tfrac{1}{7}(x-1)$$
$$y=\tfrac{1}{7}x+\tfrac{6}{7}$$

(b) If the x-coordinate of point Q was decreased from 8 towards 1, explain the effect this would have on your answer to (a). (2 marks)

Gradient of secant would increase and y-intercept would decrease.

(c) Determine the equation of the tangent to the graph of $y=\sqrt[3]{x}$ at P. (3 marks)

$$\frac{dy}{dx}=\tfrac{1}{3}x^{-\frac{2}{3}}\Big|_{x=1}=\frac{1}{3}$$
$$y-1=\tfrac{1}{3}(x-1)$$
$$y=\tfrac{1}{3}x+\tfrac{2}{3}$$

(d) Draw the tangent from (c) on the graph above. (1 mark)

Question 3 **(5 marks)**

(a) Determine as an exact value $\sin 45° \cdot \cos 45° + \cos 45° \cdot \sin 45°$. (1 mark)

$$\boxed{\begin{aligned}\sin 45° \cdot \cos 45° + \cos 45° \cdot \sin 45° &= \sin(90°)\\ &= 1\end{aligned}}$$

(b) Solve $\cos x = \sqrt{3}\sin x$ for $-\pi \le x \le \pi$. (2 marks)

$$\boxed{\begin{aligned}\frac{1}{\sqrt{3}} &= \frac{\sin x}{\cos x}\\ \tan x &= \frac{1}{\sqrt{3}}\\ x &= -\frac{5\pi}{6},\ \frac{\pi}{6}\end{aligned}}$$

(c) Determine the coefficient of the x^4 term in the expansion of $(x-3)^6$. (2 marks)

$$\boxed{\begin{aligned}&\ldots + \binom{6}{4}(x)^4(-3)^2 + \ldots\\ &\ldots + 15x^4 \times 9 \ldots\\ &\ldots + 135x^4 + \ldots\\ &\text{Coefficient is 135}\end{aligned}}$$

Question 4 **(8 marks)**

(a) Simplify $\left(2\frac{7}{9}\right)^{-\frac{1}{2}}$. (2 marks)

$$\boxed{\left(\frac{25}{9}\right)^{-\frac{1}{2}} = \left(\frac{9}{25}\right)^{\frac{1}{2}} = \frac{3}{5}}$$

(b) If $a = 5 \times 10^2$ and $b = 8 \times 10^6$ evaluate $a^2 \div b^{1/3}$. (2 marks)

$$\boxed{\begin{aligned}\frac{5^2 \times 10^{2\times 2}}{8^{1/3} \times \left(10^6\right)^{1/3}} &= \frac{25 \times 10^4}{2 \times 10^2}\\ &= 12.5 \times 10^2\\ &= 1250\end{aligned}}$$

(c) Solve $25^x = 125\sqrt{5}$. (2 marks)

$$\boxed{\begin{aligned}5^{2x} &= 5^3 \times 5^{0.5}\\ 5^{2x} &= 5^{3.5}\\ 2x &= 3.5\\ x &= 1.75\end{aligned}}$$

(d) State the equation of the asymptote of the following graphs:

(i) $y = 0.5^{x+2}$. (1 mark)

$$\boxed{y = 0}$$

(ii) $y = 0.5^x - 2$. (1 mark)

$$\boxed{y = -2}$$

Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

In a random survey of 162 swimmers at a council owned pool, 109 said they swam regularly. 39 males said they swam regularly and 16 fewer males than females were surveyed.

(a) Complete this two-way table using the above information. (2 marks)

	Swam regularly	Did not swim regularly	Total
Female			
Male			
Total	109		162

(b) If one swimmer is selected at random from those surveyed, determine the probability

(i) they swam regularly. (1 mark)

(ii) they swam regularly, given that they were female. (1 mark)

(c) Based on the information in the table, is there any indication that swimming regularly at the council owned pool is independent of the gender of the swimmer? Justify your answer. (1 mark)

Question 9 **(9 marks)**

(a) An arithmetic sequence has third term 28 and eighth term 41.75.

(i) Determine a definition of this sequence in the form $T_n = a + (n-1)d$. (2 marks)

(ii) Determine the sum of the first twenty terms of this sequence. (1 mark)

(b) A book editor charged clients 65 cents per page plus a flat fee of $120.

(i) Determine a recursive rule for the amount, a_n, the editor charges to edit a book of n pages, where a_n is in dollars. (2 marks)

(ii) The editor charged a client less than $220 to edit a draft manuscript. Determine the maximum number of pages the draft contained. (1 mark)

(c) Determine T_{10} of the arithmetic sequence where $T_1 = x-3$, $T_2 = 2x+1$ and $T_3 = 4x-1$. (3 marks)

Section One: Calculator-free **(52 Marks)**

This section has **seven (7)** questions. Answer all questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(7 marks)**

(a) Solve the equation $3(1-2a)-2(a-1)=6$. (2 marks)

$$\boxed{\begin{aligned} 3(1-2a)-2(a-1)&=6\\ 3-6a-2a+2&=6\\ -8a&=1\\ a&=-\tfrac{1}{8}\end{aligned}}$$

(b) The coordinates of three points are A(-2, -6), B(4, -2) and C(4, 2).

(i) If A is the mid-point of C and D, determine the coordinates of D. (2 marks)

D(-2-6, -6-8)

D(-8, -14)

(ii) Determine the gradient of the line through A and B. (1 mark)

$$\boxed{\frac{-2--6}{4--2}=\frac{4}{6}=\frac{2}{3}}$$

(iii) Find the equation of the line through C that is perpendicular to the line AB. (2 marks)

Perpendicular gradient is $-\dfrac{3}{2}$

$y=-\dfrac{3}{2}x+c$

$2=-\dfrac{3}{2}\times 4+c$

$c=8$

$y=-\frac{3}{2}x+8$

Question 2 **(7 marks)**

(a) Determine the coordinates of all axes intercepts of $y=(x+1)^2-4$. (2 marks)

$y=x^2+2x-3$

$=(x-1)(x+3)$

(0, −3), (1, 0), (−3, 0)

(b) State the coordinates of the turning point of $y=x^2-10x-21$. (2 marks)

$y=(x-5)^2-46$

(5, -46)

(c) Solve

(i) $(2x-5)(x+3)=0$. (1 mark)

$$\boxed{x=\tfrac{5}{2},\ x=-3}$$

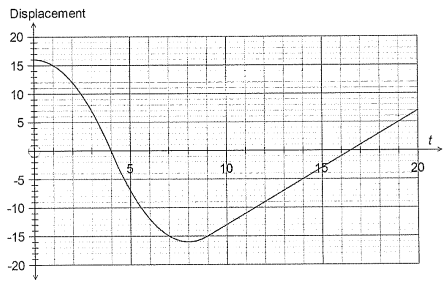
(ii) $x^2-x=20$. (2 marks)

$(x+4)(x-5)=0$

$x=-4,\ x=5$

Question 20 **(5 marks)**

A small toy train is able to travel backwards and forwards along a straight track built on level ground. The displacement in metres, of the train relative to point A, is shown on the graph below for the interval $0 \le t \le 20$ seconds.



(a) State an interval of time during which the train is moving towards point A. (1 mark)

(b) What total distance did the train travel during the 20 second interval? (1 mark)

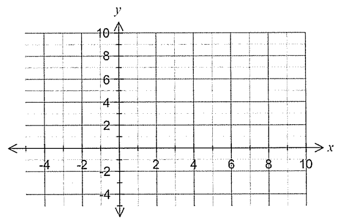
(c) By drawing adding a suitable tangent to the graph above, determine an estimate for the velocity of the train when $t = 6$. (3 marks)

Question 10 **(8 marks)**

(a) Let $f(x) = \dfrac{2}{1-x}$.

(i) State the equations of the asymptotes of the graph of $y = f(x)$. (2 marks)

(ii) Sketch the graph of $y = f(x)$. (2 marks)



(b) The two variables h and w are inversely proportional to one another.

(i) Circle all of the equations below that reflect this relationship, where k is a constant. (2 marks)

$h+w=k$ $\quad w=hk$ $\quad wh=k$ $\quad \dfrac{h}{k}=w$ $\quad \dfrac{k}{w}=h$ $\quad \dfrac{h}{w}=k$

(ii) When $h=12.5$, $w=38.8$. If h decreases by 2.8, by how much will w change? (2 marks)

Question 11 **(10 marks)**

(a) Determine x if the terms 12, x, 27 form part of a geometric sequence. (2 marks)

(b) A team of workers is using a pile driver to drive wooden poles 4 metres long into the ground. The first hit of the pile driver drives a pole 50 cm into the ground. The second hit drives the pole another 40 cm into the ground. The third hit drives the pole another 32 cm into the ground and successive distances driven by the pile driver form a geometric sequence.

(i) How much further will the fourth hit drive the pole into the ground? (1 mark)

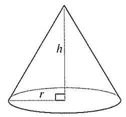
(ii) Determine the total distance the wooden pole has been driven into the ground after 12 hits of the pile driver. (2 marks)

(iii) If the workers continued in this way for some time, what length of the wooden pole will always be left above the ground? Justify your answer. (2 marks)

(c) A geometric series has a first term of 80 and a sum to infinity of 800. Determine the minimum number of terms of this sequence required so that their sum exceeds 700. (3 marks)

Question 19 **(6 marks)**

A cone has a radius r and perpendicular height h and is such that the sum of the radius and twice the height is 45 cm.



(a) Show that the volume, V, of the cone is given by $V = \frac{\pi}{3}\left(4h^3 - 180h^2 + 2025h\right)$ cm³. (3 marks)

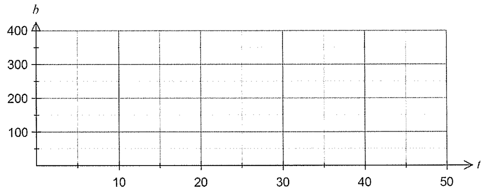
(b) Using calculus techniques, find the height that will maximise the volume of the cone, and state this maximum volume, rounded to one decimal place. (3 marks)

Question 18 **(9 marks)**

The brightness of a small incandescent light globe, b lumens, t milliseconds after measurements began can be modelled by the function $b(t) = r + p\sin(qt)$.

Initially the brightness was 200 lumens, increasing after 5 milliseconds to a maximum of 350 lumens and then dropping to a minimum brightness of 50 lumens after a further 10 milliseconds.

(a) Sketch how the brightness varied over the first 50 milliseconds on the axes below. (3 marks)



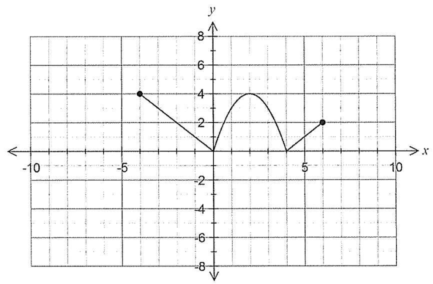
(b) Explain why $p = 150$, $q = \frac{\pi}{10}$ and $r = 200$. (3 marks)

(c) For what percentage of each cycle is the brightness of the globe less than 90 lumens? (3 marks)

Question 12 **(5 marks)**

(a) State the domain and range of $f(x) = x^4 - 4$. (2 marks)

(b) The graph of $y = g(x)$ is shown below.



On the same axes, sketch the graph of

(i) $y = g(x) - 4$. (1 mark)

(ii) $y = g(2x)$. (2 marks)

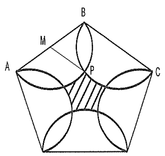
Question 13 **(12 marks)**

(a) A thin piece of glass has been cut into the shape of an obtuse-angled triangle with an area of 135.5 cm^2 and two sides of 21.8 cm and 25.4 cm.

Calculate the length of the third side, correct to 3 significant figures. (4 marks)

(b) The diagram shows five congruent semicircles standing on the inside of a regular pentagon with sides of length 20 cm.

M is the midpoint of the side AB and P is the point of intersection of two semi-circles.



(i) Show that the size of angle $\angle BMP = 72°$. (2 marks)

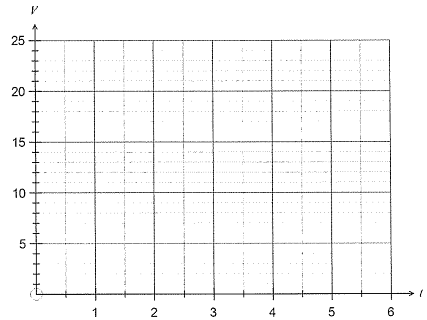
Question 17 **(7 marks)**

When a capacitor discharges through a resistor, the voltage, V in volts, across the capacitor decays according to the rule $V = 20(0.38)^t$, where t is the time, in seconds, after the discharge began.

(a) What was the initial voltage across the capacitor? (1 mark)

(b) What was the voltage across the capacitor after four seconds? (1 mark)

(c) Draw the graph of the voltage against time for $0 \le t \le 5$. (3 marks)



(d) How long, to the nearest millisecond, does it take for the voltage across the capacitor to halve? (2 marks)

Question 16 **(7 marks)**

Two independent events A and B are such that $P(A \cap B) = 0.2$ and $P(\bar{B}) = 0.6$.

(a) Calculate

(i) $P(A)$ (2 marks)

(ii) $P(A \cup B)$ (1 mark)

(iii) $P(\bar{B} \mid (\bar{A} \cup \bar{B}))$ (2 marks)

(b) A third event, C, is complementary with event A.
What is the maximum possible value of $P(C \cup B)$? (2 marks)

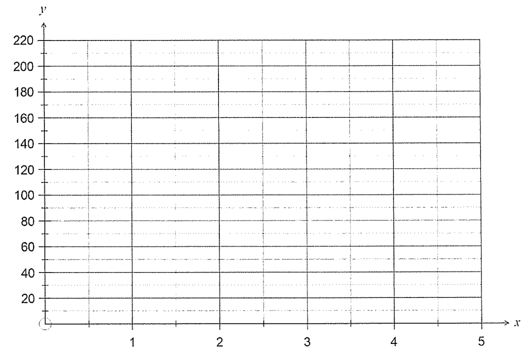
(ii) Determine the area of the central shaded region. (6 marks)

Question 14 **(7 marks)**

Consider the function $f(x) = 136 - 96x - 4x^2 + 24x^3 - 4x^4$.

(a) Using calculus techniques, determine the coordinates of all stationary points of the graph of $y = f(x)$ in the interval $0 \le x \le 5$. (4 marks)

(b) Sketch the graph of $y = f(x)$ over the interval $0 \le x \le 5$. (3 marks)



Question 15 **(8 marks)**

A store accepts credit card payments from customers using American Express, Mastercard or VISA cards. Records indicate that 65% of customers use a credit card, and of these customers, 20% use American Express, 35% Mastercard and the rest VISA. Further analysis shows that the male to female ratio for users of each type of card is 5:3 for American Express, 2:3 for Mastercard and 3:2 for VISA.

(a) Calculate the probability that a randomly selected customer from the records will be a female who uses an American Express credit card. (2 marks)

(b) Given that a randomly selected customer used a credit card, what is the probability that they are male? (3 marks)

(c) What is the probability that a randomly selected female customer who used a credit card used a VISA card? (3 marks)