**PERTH COLLEGE**

# Year 12

**Semester One Examination 2010**

**Question/Answer booklet**

## MATHEMATICS 3CMAS/3DMAS

# Section Two (calculator - assumed)

Student Name: _____

**TIME ALLOWED FOR THIS PAPER:**

| | |
|---|---|
| Reading time before commencing the paper | 10 minutes |
| Working time for paper | 100 minutes |
| Total Pages: | 20 pages |
| Total Questions | 14 questions |
| Total Marks: | 80 marks |

## Material required/recommended for this section

**To be provided by the supervisor**

Question/answer booklet for Section Two
Formula sheet

**To be provided by the candidate**

*Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler*
*Special items: drawing instruments, templates, notes (two unfolded sheets of A4 paper) and up to three calculators (CAS, graphic or scientific) which satisfy the conditions set by the Curriculum Council for this course.*

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor BEFORE reading any further.

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## STRUCTURE OF THIS PAPER

| Section | Number of questions available | Number of questions to be answered | Suggested working time (minutes) | Marks available |
|---|---|---|---|---|
| Section One: Calculator-free | 6 | 6 | 50 | 40 |
| Section Two: Calculator-assumed | 14 | 14 | 100 | /80 |
| | | | Total | /120 % |

## Instructions to candidates

1. Answer the questions in the spaces provided.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer
    a. Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    b. Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

**Question 1** [ 8 marks]

Consider the three functions given below.

$$y+1=3t^2 \quad z=\frac{1}{1-t} \qquad x+5=z^3$$

**(a)** Find $\frac{dy}{dx}$ in terms of *t*. [5]

**(b)** Find $\frac{d^2y}{dx^2}$ in terms of *t*. (Do not simplify) [3]

**Question 2** [1 + 2 +2 + 2 + 3 = 10 marks]

Three position vectors are given: $A=\begin{pmatrix}0\\2\\3\end{pmatrix}$ $B=\begin{pmatrix}1\\1\\2\end{pmatrix}$ $C=\begin{pmatrix}-1\\-1\\4\end{pmatrix}$

**(a)** Find a unit vector in the direction of *A* with the magnitude of *B*. [1]

**(b)** Find the equation of the line $L_1$ that passes through *A* and *B*. [2]

**(c)** Find the equation of a line $L_2$ that passes through *C* and is perpendicular to $L_1$. [2]

*(Question 2 continued)*

**(d)** Find the equation of the plane $\prod_1 : ax+by+cz=1$, that passes through all three points. [2]

**(e)** Another plane $\prod_2$ is parallel to $\prod_1$ and is 1 unit away from $\prod_1$.

Find an equation for $\prod_2$ of the form $r=m+\lambda n+\mu p$. [3]

*[Hint: Find a point D that lies 1 unit away from A]*

**Question 3** [5 marks]

Find the equation of the tangent to the curve $\sin xy = \sin x + \sin y$ at the point $(0, \pi)$.
Express your answer using exact values.
Show ALL working.

[5]

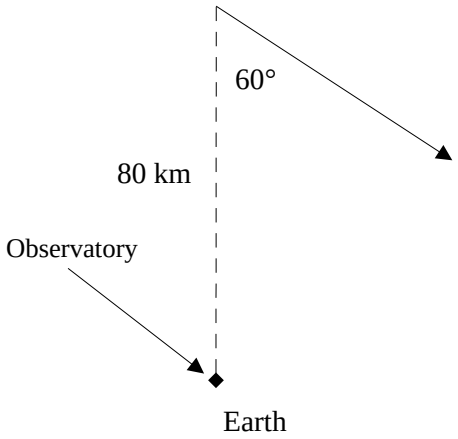
**Question 4** [6 marks]

An observatory is studying the average speed at which shooting stars enter our atmosphere. A shooting star normally starts to glow from 80km above the surface of the Earth.

One shooting star enters the atmosphere at an angle of 60° with the vertical as shown. When the shooting star is 90 km from the observatory, the star is measured to be moving at 1200 $kmh^{-1}$ from the observatory itself. (i.e. relative to the observatory)

Find the linear speed of the shooting star at this instant.

[6]



**Question 5** [4 marks]

The radius of the Moon was measured to be 1 737.4 km, however the digital equipment used to obtain this measurement had an error margin of 0.5%.

Use the calculus of small changes to obtain the approximate percentage of error obtained when we estimate the surface area of the Moon.

[4]

**Question 6** [3 + 4 = 7 marks]

Determine:

**(a)** $\int \cos^3 at\, dt$ where $a$ is a constant. [3]

**(b)** $\int \frac{x+1}{\sqrt{x-1}}\, dx$ using the substitution $u^2 = x - 1$ [4]

**Question 7** [5 marks]

Given that $y = \frac{1+\sqrt{x}}{1-\sqrt{x}}$, show that $\frac{dy}{dx} = \frac{a}{x^b\left(1-\sqrt{x}\right)^c}$, and give the values of $a$, $b$ and $c$.

Show ALL working.

[5]

**Question 8** [ 2 + 2 + 3 = 7 marks]

A particle starts from rest at $t = 0$, and its acceleration is given by $a = \sqrt{1+4t}$ ms$^{-1}$.

**(a)** Find an expression for the velocity of the particle in terms of $t$. [2]

**(b)** Find and expression of the displacement of the particle in terms of $t$. [2]

**(c)** Find the displacement and the acceleration of the particle when its velocity is $\frac{7}{6}$ ms$^{-1}$. [3]

**Question 9** [2 + 2 = 4 marks]

**(a)** Find the acute angle between the planes $2x+3y-5z=10$ and $-x+2y+4z=4$. [2]

**(b)** Find the value of $k$ so that the vector $\begin{pmatrix} k \\ 1+k \\ 1-k \end{pmatrix}$ belongs to the plane $r \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 5$. [2]

**Question 10** [3 + 3 = 6 marks]

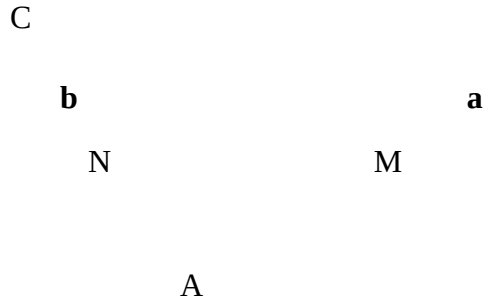
A particle moves with velocity $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ kmh$^{-1}$ and passes through $\begin{pmatrix} 10 \\ -40 \\ 40 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -20 \\ 10 \end{pmatrix}$ at 6 am and 8 am respectively.

**(a)** Find the values of $a$, $b$ and $c$. [3]

**(b)** Find where and when the particle crosses the $x$-$y$ plane. [3]

**Question 11** [4 marks]

In $\Delta ABC$ shown, the points $M$ and $N$ divide the segments $\overline{AB}$ and $\overline{AC}$ respectively in the ratio $1:3$. Let $\overrightarrow{AB}=a$ and $\overrightarrow{AC}=b$.




**(a)** Find an expression for $\overrightarrow{BC}$ and $\overrightarrow{MN}$ in terms of $a$ and $b$. [2]

**(b)** Prove that $\overrightarrow{BC}=4\overrightarrow{MN}$ [2]

**Question 12** [4 marks]

Consider the following expression: $\lim_{h \to 0} \dfrac{\cos^2(3x+3h) - \cos^2\left(\dfrac{3\pi}{4}\right)}{h}$

**(a)** Identify the function being differentiated. [1]

**(b)** Identify the point at which the derivative is being evaluated. [1]

**(c)** Obtain the derivative of this function and then evaluate it at the point found in (b). [2]

**Question 13** [2 + 3 = 5 marks]

Determine $\frac{dy}{dx}$ for each function given below.

**(a)** $y = \frac{\cos x}{\sin x}$ [2]

**(b)** $y = \int_0^{3x^2} \frac{2}{t}\,dt$ [3]

**Question 14** [3 + 2 = 5 marks]

The motion of a particle is described by the equation: $\frac{d^2x}{dt^2} + 4\pi^2 x = 0$

**(a)** Given that the particle begins at the origin, with positive velocity and has a maximum velocity of 8π m/sec, determine the displacement of the particle at any time *t*. [3]

**(b)** Find the amplitude and the period of its acceleration. [2]

**END OF PAPER**

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