

MATHEMATICS METHODS
Semester Two Examination, 2021
Question/Answer Booklet



ATAR Year 12
Section Two:
Calculator-assumed

Please circle your teacher's name
Teacher: Miss Hosking Miss Rowden

Time allowed for this paper:
Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Student Name: _____

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Materials required/recommended for this paper
This Question/Answer Booklet
Formula Sheet (retained from Section One)

Number of additional answer booklets used (if applicable): _____

To be provided by the candidate
Standard items:
Pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters
Special items:
Drawing instruments, templates, notes on two unfolded sheets of A4
paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Supplementary page

Question number: _____

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Question number: _____

STRUCTURE OF THIS PAPER

Section	Number of questions available	Number of questions to be answered	Suggested working time (minutes)	Marks	Percentage of examination	Total
Section One: Calculator free	8	8	50	51	35	65
Section Two: Calculator-assumed	13	13	100	97	97	100
Total						100

INSTRUCTIONS TO CANDIDATES

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THE RULES FOR THE CANDIDATES OF THE ATAR USE THESE EXAMINATIONS ARE DETAILED IN THE YEAR 22 INFORMATION HANDBOOK 2020. STICKING THIS EXAMINATION IMPLIES THAT YOU AGREE TO ADOBE BY THESE RULES.

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2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Supplementary pages for the use of planning/containing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

This section has thirteen (13) questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 9 (7 marks)

A capacitor in a circuit starts to discharge. The voltage V across the capacitor after t milliseconds is changing at a rate given by

$$\frac{dV}{dt} = \frac{-156}{(3t+2)^2}, t \geq 0.$$

- (a) Calculate the initial rate of change of voltage. (1 mark)

- (b) Determine the change in voltage during the fourth millisecond. (3 marks)

- (c) Given that the initial voltage across the capacitor was 25 volts, determine the time for the voltage to fall to 1 volt. (3 marks)

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Supplementary page

Question number: _____

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Question 10**Question 10**

- An online employment survey on a public internet forum attracted 72 responses from health workers, of whom 18 said that they were employed on a casual basis.
- (a) Use the survey data to construct a 95% confidence interval for the proportion of health workers employed on a casual basis.

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- (b) Assuming the survey was reliable, determine the sample size required to conduct a follow-up survey so that a 99% confidence interval for the population proportion of health workers employed on a causal basis will have a margin of error close to 0.07. (3 marks)

- (c) Identify and explain a possible source of bias that may arise from this type of survey. (2 marks)

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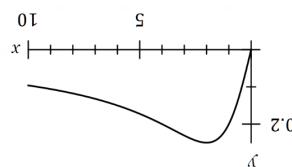
Question 21

The graph of $y = f(x)$ is shown, where

$$f(x) = \frac{4+x^2}{x}, x \geq 0.$$

- (a) Determine the area bounded by the graph of f and the line $y = \frac{20}{x}$. (3 marks)

$f(x)$ is concave down for $0 < x < 2\sqrt{3}$.



- (b) Determine the values of the slope m for which R exists.
The line $y = mx$ and the graph of f enclose a finite region R .

- (c) Determine the area of R in terms of m .

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(2 marks)

End of questions

End of questions

(8 marks)

Question 11

A factory makes identical plastic key fobs in four different colours. 15% are red, 20% are green, 25% are blue and the remainder orange. The key fobs are randomly packed into boxes of 120.

Quality control at the factory randomly sample several boxes from the production line daily and record, amongst other things, the proportion of orange key fobs in each box.

- (a) Describe the continuous probability distribution that the sample proportion of orange key fobs will approximate over time, including any parameters. (4 marks)

- (b) Calculate an approximation for the probability that the proportion of orange key fobs in a randomly chosen box is at least 35%. (2 marks)

- (c) Briefly explain why the distribution in part (a) is an approximation and state the key factor that determines the closeness of the approximation. (2 marks)

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(7 marks)

Question 20

A popcorn container of capacity 660 mL is made from paper and has the shape of an open inverted cone of radius r and height h .

Determine the least area of paper required to make the container.

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Question 12

(7 marks)

(a)

A company packages salt in jars marked with a net weight of 225 g. The weight of salt in the jars is normally distributed with a mean of 231.5 g and a standard deviation of 3.9 g.

weight.

Determine the probability that a randomly selected jar contains less than the marked

(b)

What is the probability that a randomly selected jar containing less than the marked

weight contains less than 223 g of salt?

(2 marks)

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- (c) The company has decided that no more than 1 in 300 jars should contain less than the marked weight of salt. To achieve this, they will pack more salt in each jar and hence increase the mean of the distribution whilst maintaining the existing standard deviation. Determine the minimum increase in the mean required.

(3 marks)

Question 19

(8 marks)

7

An electronic device is powered by an AAA battery that will always last for a minimum of 12 hours. The random variable T is the number of hours exceeding 12 for which the device will continue to operate, and it has probability density function f shown below:

$$f(t) = \begin{cases} \frac{kt}{4}, & 0 \leq t \leq 4 \\ k, & 4 < t \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

(a)

Determine the value of the constant k .

(3 marks)

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(2 marks)

- (c) Given that $P(T > a) = 0.865$, determine the value of the constant a .

(3 marks)

(7 marks)

Question 13

A small body starts from rest at point A and moves in a straight line until it reaches point B , where it is again stationary.

The acceleration of the body t seconds after leaving A is a m/s 2 , where $a=0.12t-0.006t^2$.

Determine

- (a) the time taken for the body to travel from A to B .

(3 marks)

- (b) the distance from A to B .

(2 marks)

- (c) the maximum velocity of the body between A and B .

(2 marks)

See next page

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(6 marks)

Question 18

A player throws a regular tetrahedral die whose faces are numbered 1, 2, 3 and 4. If the player throws a three, the die is thrown a second time, and in this case the score is the sum of 3 and the second number; otherwise, the score is the number obtained. The player has no more than two throws. Let X be the random variable denoting the player's score.

- (a) Write down the probability distribution of X .

(3 marks)

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- (b) Determine the mean and standard deviation of X .

(2 marks)

- (c) Determine $P(X=4 \vee X \geq E[X])$.

(1 mark)

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MATHEMATICS METHODS CALCULATOR-ASSUMED Question 14 MATHEMATICS METHODS

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CALCULATOR-ASSUMED

(8 marks)

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(3 marks)

(a) Determine an expression for the constant k in the form $a \ln(b)$ and hence show that its value is approximately 0.00422.

(3 marks)

The level can be modelled by equation $C = C_0 e^{-rt}$, where t is the number of years from the start of the year 1960.

(8 marks)

(a) Construct a 99 % confidence interval for the proportion of all people in the city who trust the newspaper and hence comment on the validity of the newspaper's claim.

The local newspaper in a large city claimed that over 75 % of the city's population trusted them. To check this claim, a research group took a random sample of 625 people in the city and found that 450 of them trusted the newspaper.

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(2 marks)

(b) Determine the value of the constant C_0 .

(1 mark)

(c) Calculate the level of atmospheric carbon dioxide at the start of the year 1995.

(2 marks)

(d)

Determine the rate at which the level of atmospheric carbon dioxide was increasing at the start of the year 1995.

(4 marks)

(b) The research group carried out the same sampling task in different city, from which the 95 % confidence interval (0.448, 0.516) was constructed. Determine the number of people in this sample who trusted their local newspaper.

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CALCULATOR-ASSUMED

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(9 marks)

Question 15

A person drives to work n times each month and on any one journey, the probability that they arrive late for work is p .

- (a) When $n=16$ and $p=0.14$ determine the probability that

(i) they are late for work exactly twice in a month.

(2 marks)

(ii) they are late for work at least once in a month.

(1 mark)

(iii) they are never late for work in at least one of three consecutive months.

(3 marks)

- (b) Determine n and p when the mean and variance of the number of times the person is late for work each month is 3.2 and 2.688 respectively.

(3 marks)

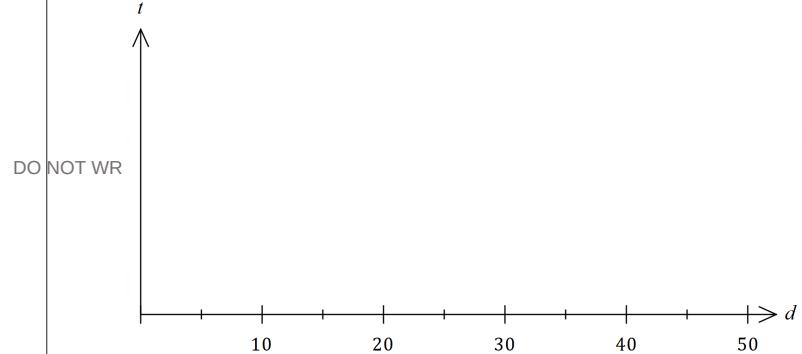
(7 marks)

Question 16

The time, t seconds, for a trained rat to pick a bead out of a container and drop it into a small hole when the distance of the bead container from the hole was d cm can be modelled by the relationship $t=0.6+0.55\ln(d)$ for $d \geq 1$.

- (a) Sketch the graph of t as a function of d for $1 \leq d \leq 50$ cm.

(3 marks)



- (b) Determine the extra time taken by the rat to move a bead when the distance of the bead container from the hole increases from 20 cm to 60 cm.

(1 mark)

- (c) Use the relationship to show that if the distance of the bead container from the hole increases from x cm to $3x$ cm, the change in time is constant.

(3 marks)