Revision Examination Assessment Papers (REAP)
Semester 1 Examination 2012

Question/Answer Booklet
(This paper is not to be released to take home before 25/6/2012)

MATHEMATICS: SPECIALIST 3A

Section Two:
Calculator-assumed

Name of Student: _____Marking Key_____

Time allowed for this section
Reading time before commencing work: 10 minutes
Working time for this section: 100 minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the student
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

Important note to students
No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	12	12	100	100	
			Total	150	100

Instructions to students

1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2 **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

4 You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

Section Two: Calculator-assumed **(100 marks)**

This section has **twelve (12)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

Question 7 **(10 marks)**

(a) If $\underline{a} = 6\underline{i} - 4\underline{j},\ \underline{b} = 3\underline{i} + 4\underline{j},\ \underline{c} = 2\underline{i} + 5\underline{j}$

(i) Determine $|\underline{c} - \underline{b}|$ Leave your answer as a surd (2)

Solution
$\lvert -\underline{i} + \underline{j}\rvert = \sqrt{2}$
Specific behaviours
✓ simplify $\underline{c} - \underline{b}$ to $-\underline{i} + \underline{j}$ ✓ determines magnitude of $\sqrt{2}$

(ii) Determine $2\underline{b} - 3\underline{a} + \underline{c}$ (2)

Solution
$6\underline{i} + 8\underline{j} - 18\underline{i} + 12\underline{j} + 2\underline{i} + 5\underline{j}$ $= -10\underline{i} + 25\underline{j}$
Specific behaviours
✓ correct expression for $2\underline{b} - 3\underline{a} + \underline{c}$ ✓ simplifies to correct answer

(iii) Determine a vector, $\underline{v}$ (in **exact** simplest form) in the direction of $\underline{a}$ such that $|\underline{v}| = 5$. (3)

Solution
$\lvert\underline{a}\rvert = \sqrt{52} = 2\sqrt{13}$ Unit vector $\underline{a} = \dfrac{6\underline{i} - 4\underline{j}}{2\sqrt{13}}$ $\therefore \underline{v} = \dfrac{6\underline{i} - 4\underline{j}}{2\sqrt{13}} \cdot 5$

$\underline{v} = \frac{5}{2\sqrt{13}} \cdot 2(3\underline{i} - 2\underline{j})$ $\underline{v} = \frac{5}{\sqrt{13}} \cdot (3\underline{i} - 2\underline{j})$ Or $\underline{v} = \frac{15\underline{i}}{\sqrt{13}} - \frac{10\underline{j}}{\sqrt{13}}$
Specific behaviours
✓ magnitude of $\underline{a}$ ✓ unit vector of $\underline{a}$ times 5 ✓ simplified answer

(b) Find the value of k if $\underline{p}$ and $\underline{q}$ are parallel vectors.

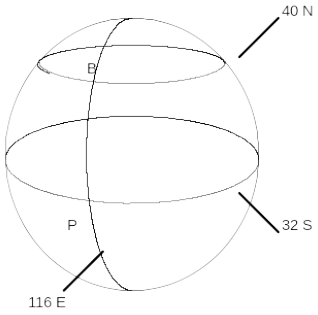
$$\underline{p} = \sqrt{2}\begin{pmatrix} k \\ -3 \end{pmatrix}, \quad \underline{q} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(3)

Solution
$\sqrt{2}k\underline{i} - 3\sqrt{2}\underline{j} = m(2\underline{i} + \underline{j})$ $-3\sqrt{2} = m$ $\sqrt{2}k = 2m$ $\sqrt{2}k = 2(-3\sqrt{2})$ $k = -6$
Specific behaviours

Question 8 **(8 marks)**

(a) Beijing in China is located at 40°N, 116°E. Perth in Western Australia is located at 32°S, 116°E. Karen's direct flight from Beijing to Perth departed at 11.00pm on Sunday. If the plane had an average flying speed of 900km/h, how long does the flight take? Express your answer in hours and minutes.
Take the radius of the earth to be 6360km. (4)

Solution
 Arc BP = $\frac{72}{360} \times 2 \times \pi \times 6360$ = 7992.21 km Time taken to reach Perth = $\frac{7992.21}{900}$ = 8.88 hr = 8 hr 53 min
Specific behaviours
✓ angle of 72^0 ✓ arc length BP ✓ time in hour ✓ time in hour and mins

(b) Determine the equation for each of the following graphs by selecting from the list below.

$y = \sqrt{x - 2} + 1$

$y = 1 + \sqrt{x + 2}$

$y = 2e^{-x}$

$y = e^{-x} + 2$

$y = -\ln 2x$

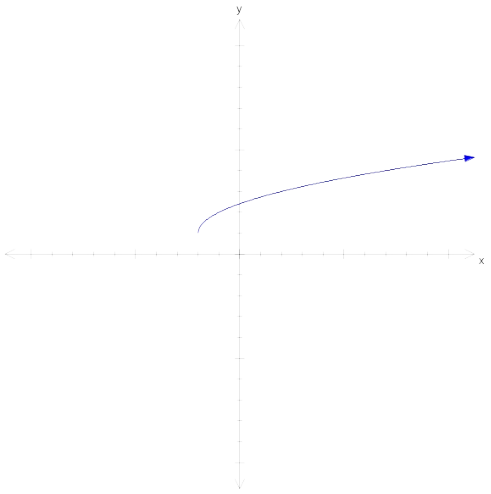
$y = -\ln \frac{1}{2}x$

$y = \frac{1}{x - 1} - 2$

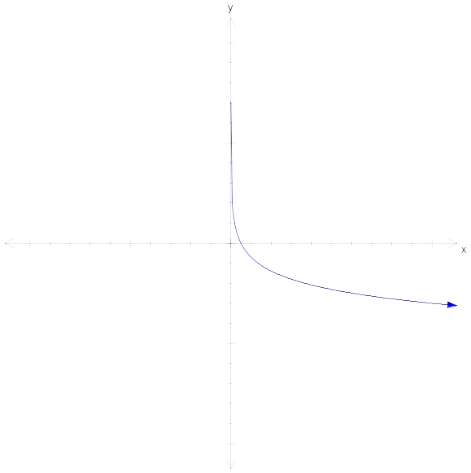
$y = \frac{1}{x + 1} - 2$

(4)

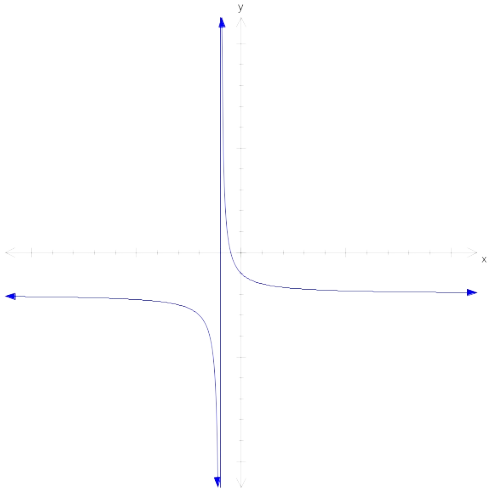
(i)



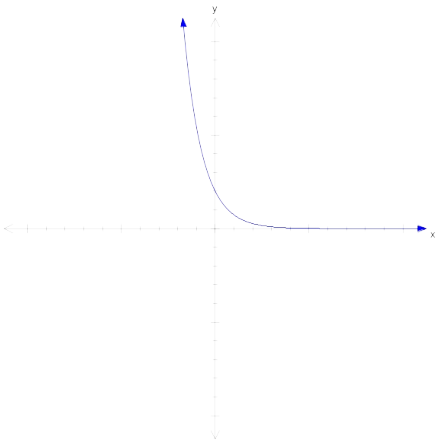
(ii)



(iii)



(iv)



Solution
Graph (i) - $y = 1 + \sqrt{x+2}$ Graph (ii) – $y = -\ln 2x$ Graph (iii) – $y = \frac{1}{x+1} - 2$ Graph (iv) - $y = 2e^{-x}$
Specific behaviours
✓✓✓✓ for each correct answer

Question 9 **(10 marks)**

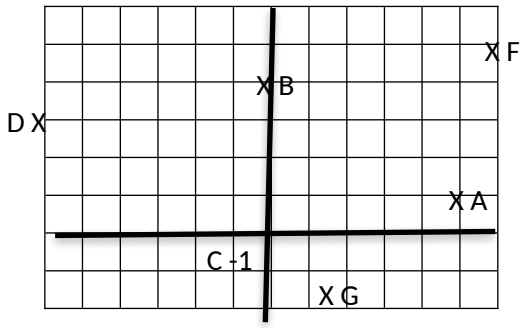
(a) Point A has position vector $k\underline{i} - \underline{j}$. Point B has position vector $6\underline{i} - k\underline{j}$.

If $|AB| = 5$, find the value(s) of *k*. (4)

Solution
$AB = (6-k)\underline{i} + (k+1)\underline{j}$ $\|AB\| = \sqrt{(6-k)^2 + (k+1)^2}$ $(6-k)^2 + (k+1)^2 = 25$ $k = 2, k = 3$
Specific behaviours
✓ express AB in ***i*** **–** ***j*** component correctly ✓ for $(6-k)^2 + (k+1)^2 = 25$ ✓✓ solves for the two "k" values

(b) Let A=(5,1), B=(0,4), C=(-1,0)

(Hint: Use the grid below to help you find the points)



Find

(i) D such that $AB = CD$ (2)

Solution
D = (-6, 3)
Specific behaviours
✓ for x-coordinate ✓ for y- coordinate

(ii) F such that $AF = -BC$ (2)

Solution
F = (6, 5)
Specific behaviours
✓ for x-coordinate ✓ for y-coordinate

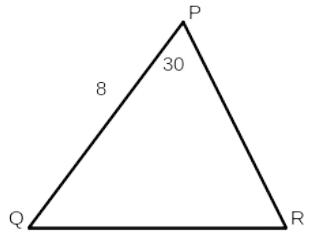
(iii) G such that $AB = 2GC$ (2)

Solution
G = (1.5,-1.5)
Specific behaviours
✓ for x-coordinate ✓ for y-coordinate

Question 10 **(10 marks)**

The area of ΔPQR is 16cm^2. PQ=8cm, $\angle$RPQ=30°.

(i) Find the length of PR. (2)

Solution
 $16 = \frac{1}{2} \times 8 \times PR \times \sin 30^0$ PR = 8 cm
Specific behaviours
✓ uses the Area of non right triangle formula ✓ correct answer for PR

(ii) Hence, find the size of $\angle$PQR. (2)

Solution
As ΔPQR is an isosceles triangle $\angle PQR = \frac{180^0 - 30^0}{2} = 75^0$
Specific behaviours
✓ identifying triangle is isosceles ✓ size of $\angle PQR$

(iii) Using the SINE rule and $\sin 75^\circ = \frac{1+\sqrt{3}}{2\sqrt{2}}$, show that $QR = 4\sqrt{2}\left(\sqrt{3} - 1\right)$
Show all working steps. (6)

Solution
$\frac{QR}{\sin 30^0} = \frac{8}{\sin 75^0}$

$$QR = \frac{8 \times \sin 30^0}{\sin 75^0}$$

$$QR = \frac{8 \times \frac{1}{2}}{\frac{1+\sqrt{3}}{2\sqrt{2}}}$$

$$QR = 4 \times \frac{2\sqrt{2}}{1+\sqrt{3}}$$

$$QR = \frac{8\sqrt{2}}{1+\sqrt{3}}$$

$$QR = \frac{8\sqrt{2}}{1+\sqrt{3}} \times \frac{(1-\sqrt{3})}{(1-\sqrt{3})}$$

$$QR = \frac{8\sqrt{2}(1-\sqrt{3})}{1-3}$$

$$QR = \frac{8\sqrt{2}(1-\sqrt{3})}{-2}$$

$$QR = -4\sqrt{2}(1-\sqrt{3})$$

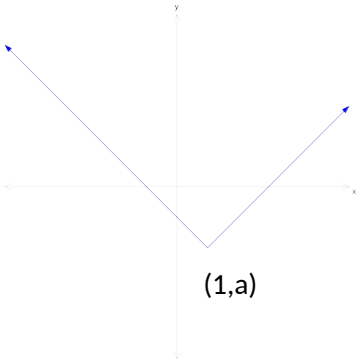
$$QR = 4\sqrt{2}(\sqrt{3}-1)$$

Specific behaviours

✓ expression for the Sine Rule
✓ substitute expression for sin 750
✓ simplify expression
✓ multiply by conjugate of denominator
✓✓ simplify and gets to $4\sqrt{2}(\sqrt{3}-1)$

Question 11 **(9 marks)**

The diagram shows the graph of $y=f(x)$, $x \in R$.



If $f(x) = |x - 1| - 2$,

(i) Find the value of a and the value of b. (2)

Solution
f(0) = b ---- b = -1 f(1) = a ---- a = -2
Specific behaviours
✓✓ for each "a" and "b" value

(ii) Solve **algebraically** the value of x for which $f(x) = 5x$ (3)

Solution
$-(x-1)-2=5x$ $-x+1-2=5x$ $-1=6x$ $x=-\frac{1}{6}$
Specific behaviours
✓ correct piece of f(x) used i.e. –(x-1) - 2 ✓ expand and rearrange ✓ solve correctly for value of x

(iii) Determine the solution set for $|x-1|-2=x-3$ (2)

Solution
$\{x : x \geq 1, x \in \mathbb{R}\}$
Specific behaviours
✓ for $\geq$ ✓ for value of 1

(iv) Find the value of k for which $|x-1|-2=x+k$ has no solution. (2)

Solution
$k < -3$
Specific behaviours
✓ for < ✓ for (-3)

Question 12 **(8 marks)**

(a) The population, P, of cane toads in Australia has been growing at a rate proportional to P, such that $\frac{dP}{dt} = kP$ where *k* is a positive constant. There were 102 cane toads brought to Australia in 1935. Seventy six years later, in 2011, it is estimated that there are 243 million cane toads in Australia.

(i) Find the value of 'k' to 4 decimal places. (2)

Solution
$P = 102e^{kt}$ In 2011, t = 76 $243 \times 10^6 = 102e^{76k}$ Using CAS $k = 0.1932$ to 4 decimal places
Specific behaviours
✓ $243 \times 10^6 = 102e^{76k}$ ✓ correct k value to 4 decimal places

(ii) If the population continues to grow at this rate, how many cane toads will there be in Australia in 2035? (2)

Solution
In 2035, t = 100 P= 2.5 x 10^{10} or P = 25071 million
Specific behaviours
✓ t = 100 ✓ correct value for P

(b) In 2005 there were 13.8million registered motor vehicles in Australia. The number of registered motor vehicles is increasing at a rate of 2.3%₀ per year.

(i) Write an expression to represent the number (in millions) of registered motor vehicles, N, if t represents the number of years after 2005. (2)

Solution
$N = 13.8(1.023)^t$
Specific behaviours
✓ exponential equation, y = k. a^x ✓ k= 13.8, a = 1.023

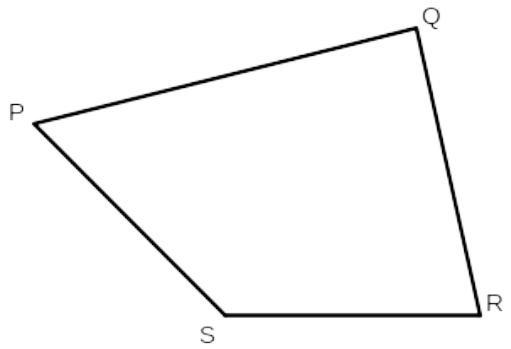
(ii) Hence determine the number of registered vehicles in Australia in 2011.

(2)

Solution
In 2011, t = 6 $N = 13.8(1.023)^6$ N= 15.8 million
Specific behaviours
✓ for t = 6 ✓ for correct value of N

Question 13 **(6 marks)**

In the diagram $PQ = 2\underline{b}$, $PS = 4\underline{a}$ and $SR = 2\underline{a} + \underline{b}$



(a) Express as simply as possible, in terms of $\underline{a}$ and/or $\underline{b}$ (2)

(i) SQ

Solution
$SQ = -4\underline{a} + 2\underline{b}$
Specific behaviours
✓ correct answer

(ii) QR

Solution
$QR = 6\underline{a} - \underline{b}$
Specific behaviours
✓ correct answer

(b) If $PT = hPR$, express PT in terms of $h, \underline{a}$ and $\underline{b}$ (1)

Solution
$PT = h(6\underline{a} + \underline{b})$
Specific behaviours
✓ correct answer

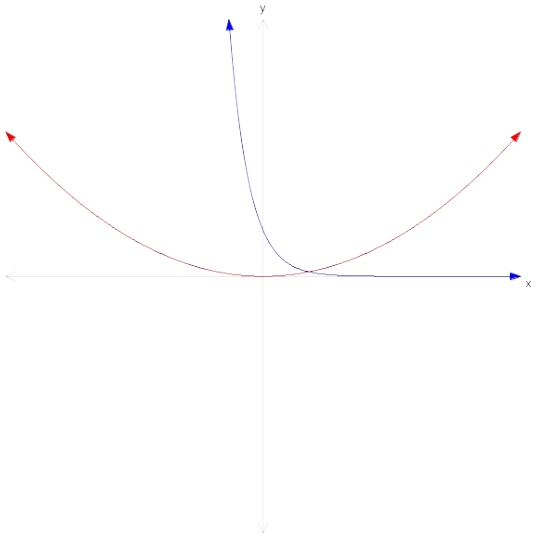
(c) Given that $4ST = SQ$, calculate the value of *h*. (3)

Solution

$4(-4\underline{a}+6h\underline{a}+h\underline{b})=-4\underline{a}+2\underline{b}$ $-16\underline{a}+24h\underline{a}+4h\underline{b}=-4\underline{a}+2\underline{b}$ Equating coefficients: $4h=2 \therefore h=\frac{1}{2}$ Or $-16+24h=-4 \therefore h=\frac{1}{2}$
Specific behaviours
✓ replace equation by $4(-4\underline{a}+6h\underline{a}+h\underline{b})=-4\underline{a}+2\underline{b}$ ✓ simplify ✓ solve correctly for value of h

Question 14 **(11 marks)**

The diagram shows the graph of $f(x)=a^x$ and $g(x)=bx^2$. The graph $f(x)$, intersects $g(x)$ at $\left(1,\frac{1}{10}\right)$



(a) Calculate the value of a and b. (2)

Solution
$a=\frac{1}{10}$, $b=\frac{1}{10}$
Specific behaviours
✓✓ correct vale for "a" and "b"

(b) Write down the equation of the inverse function, $f^{-1}(x)$. (3)

Solution
$\ln y=-x\ln 10$ $f^{-1}(x)=\frac{-\ln x}{\ln 10}$ OR $\log y=x\log\frac{1}{10}$ $\log y=-x$ $f^{-1}(x)=-\log x$ OR using CAS Solve($x=0.1^y$,y) results in $y=\frac{-\ln x}{\ln 10}$
Specific behaviours
✓ take the "log" of both sides of equation ✓✓ re arrange and correct answer for the inverse function

(c) Why does the function *g(x)* not have an inverse function? (1)

Solution
g(x) is a many-to-one function OR For a function to have an inverse function, it must be a one-to-one function OR g(1)=0.1 and g(-1) = 0.1, hence g(x) cannot have an inverse function
Specific behaviours
✓ for anyone of the above reasons

(d) Write down two ways in which the domain of *g(x)* could be restricted in order that *g(x)* can have an inverse function. (2)

Solution
(i) $D_{g(x)} = \{x : x \in [0, \infty)\}$ or $D_{g(x)} = \{x : x \geq 0, x \in \mathbb{R}\}$ (ii) $D_{g(x)} = \{x : x \in (-\infty, 0]\}$ or $D_{g(x)} = \{x : x \leq 0, x \in \mathbb{R}\}$
Specific behaviours
✓✓ for each of the two ways

(e) Determine the *x*-values for which

(i) $f^{-1}(x) > 0$ (2)

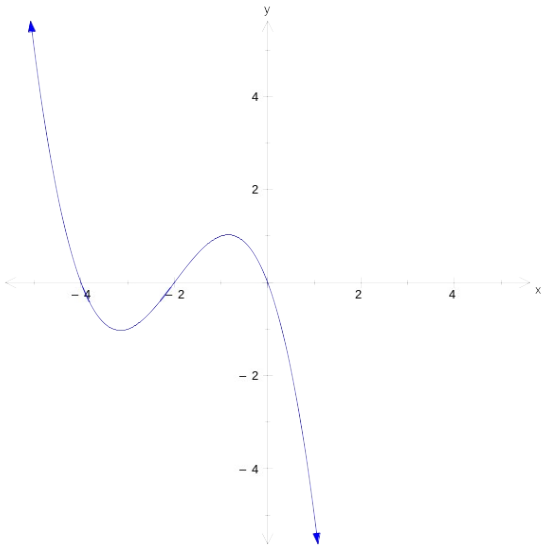
Solution
$0 < x < 1$
Specific behaviours
✓✓ correct interval

(ii) $f(x) - 1 = g(x)$ (1)

Solution
$x = 0$
Specific behaviours
✓ or X

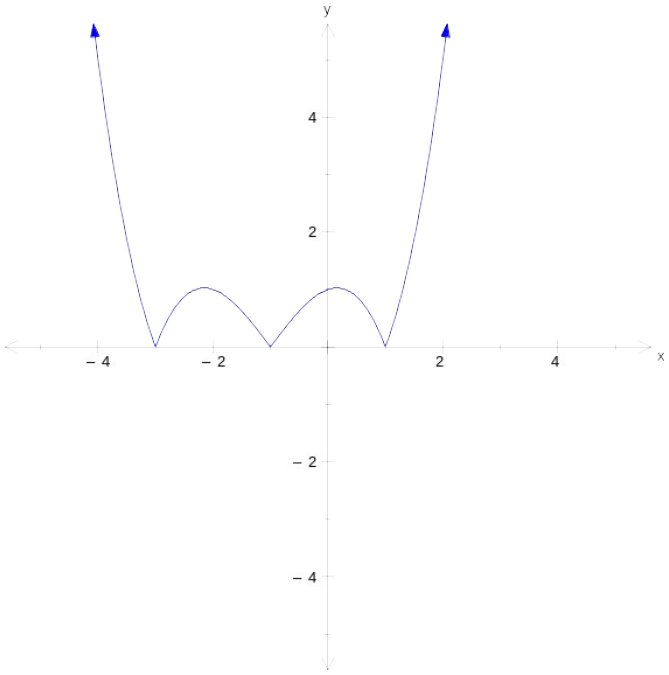
Question 15 **(6 marks)**

The diagram shows the graph of *y=f(x)*



(i) Sketch the graph of

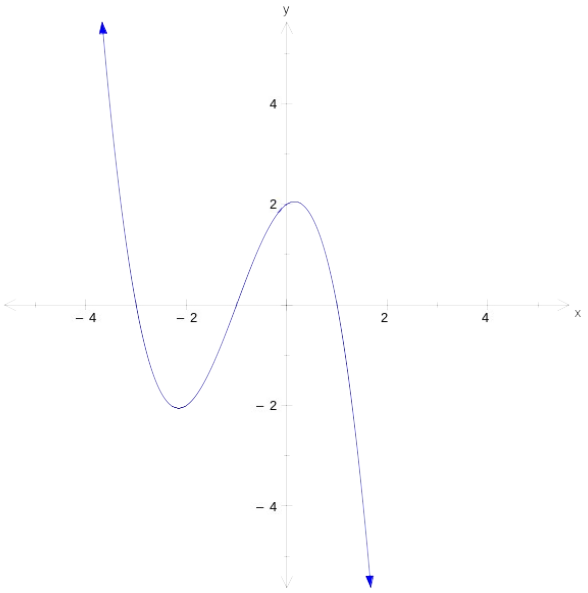
$y = |f(x - 1)|$ showing clearly the coordinates of the turning points and the intersections with the axes. (3)



Solution

As shown in diagram above
Specific behaviours
✓ shape ✓✓ points (-3,0),(-2,1),(-1,0),(0,1),(1,0)

(ii) *f(x)* has been transformed and the transformed graph is shown below.



State the equation of the transformed graph in terms of *f(x)*. (3)
Show how the point (-3, -1) is transformed to (-2, -2)

Solution
$y = 2f(x-1)$ $(-3,-1) \rightarrow (-2,-1) \rightarrow (-2,-2)$
Specific behaviours
✓✓ for equation $y = 2f(x-1)$ ✓ for $(-3,-1) \rightarrow (-2,-1) \rightarrow (-2,-2)$

Question 16 **(6 marks)**

(a) Find a simple expression for e^{2lnx} (1)

Solution
Let $y = e^{2\ln x}$ $y = e^{\ln x^2}$ $\ln y = \ln x^2 \ln e$ $y = x^2$ i.e. $e^{2\ln x} = x^2$ OR Using CAS simplify $\left(e^{2\ln x}\right)$ results in x^2
Specific behaviours
✓ or X

(b) Solve the equation graphically $2e^x = 4x + 3$.

Express your answers to 2 decimal places. (2)

Solution
$x = -0.42$ and $x = 1.51$
Specific behaviours
✓✓ correct answers for x

(c) Solve $3 \times 9^{x+1} = 81^{2x}$ (3)

Solution
$3 \times 3^{2x+2} = 3^{4 \times 2x}$ $3^{2x+3} = 3^{8x}$ $2x + 3 = 8x$ $x = \frac{1}{2}$
Specific behaviours
✓ expresses as powers of 3 ✓ equate $2x + 3 = 8x$ ✓ solves correctly for x

Question 17 **(7 marks)**

M is the mid-point of line segment AB.

If OA, OB and OM are $\underline{a}$, $\underline{b}$, $\underline{m}$ respectively

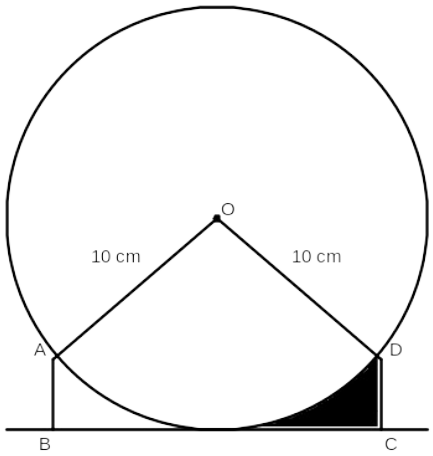
(a) Show that $\underline{m} = \frac{1}{2}(\underline{a} + \underline{b})$ (4)

Solution
$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$ $-\underline{a} + \underline{m} = \frac{1}{2}(-\underline{a} + \underline{b})$ $\underline{m} = -\frac{1}{2}\underline{a} + \underline{a} + \frac{1}{2}\underline{b}$ $\underline{m} = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$ $\underline{m} = \frac{1}{2}(\underline{a} + \underline{b})$
Specific behaviours
✓ $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$ ✓✓✓ correctly shows each line with no shortcut as it is a "show"

(b) Hence, or otherwise state the coordinates of *S* if *S* divides the line segment joining F(1,4) to G(6,9) in the ratio 1:1. (3)

Solution
S is the mid point of FG $\therefore \underline{s} = \frac{1}{2}(\underline{i} + 4\underline{j} + 6\underline{i} + 9\underline{j})$ $\underline{s} = (3.5\underline{i} + 6.5\underline{j})$ $\underline{s} = \frac{1}{2}(7\underline{i} + 13\underline{j})$ $\therefore \underline{s} = (3.5\underline{i} + 6.5\underline{j})$ Coordinates of S = (3.5, 6.5) Or $\overrightarrow{OS} = \overrightarrow{OF} + \overrightarrow{FS}$ $= \underline{i} + 4\underline{j} + \frac{1}{2}(-\underline{i} - 4\underline{j} + 6\underline{i} + 9\underline{j})$ $= 3.5\underline{i} + 6.5\underline{j}$
Specific behaviours
✓ identifies that S is the mid-point of FG hence use result from (a) ✓✓ simplifies and correct coordinates for S

Question 18 **(9 marks)**



The diagram shows a circle of radius 10cm, centre O and a tangent BC of length 16cm, AB=DC, calculate

(i) $\angle$ AOD in radians, to 2 decimal places. (4)

Solution
AD = BC Let M be mid -point of AD $\sin\angle AOM = \frac{8}{10}$ $\angle AOM = 53.13^0$ $\therefore \angle AOD = 106.26^0$ = 1.85^R
Specific behaviours
✓ AM = 8 ✓ angle AOM ✓ angle AOD in degrees ✓ angle AOD in radians to two decimal places

(ii) the area of the shaded region. (5)

Solution
Area of OABCD = 64 + (0.5 x 16 x 6) = 112 Area of sector AOD = $\pi \times 10^2 \times \frac{106.26^0}{360}$ = 92.73 or 92.5 if used 1.85 radians $\therefore$ Shaded area = $\frac{112 - 92.73}{2}$ = 9.64 cm^2 If used 1.85 radians shaded area = 9.75 cm^2
Specific behaviours
✓ area of sector AOD ✓✓ area of OABCD ✓✓ shaded area