



Course — Mathematics Methods — Year — 11 —

Student name: Mark Inghide Teacher name: _____

Date: 21 September 2020

Task type: Response

Time allowed for this task: 45 mins

Number of questions: 7

Materials required: This assessment is calculator-free

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper (double sided)

Marks available: 44 marks

Task weighting: 16 %

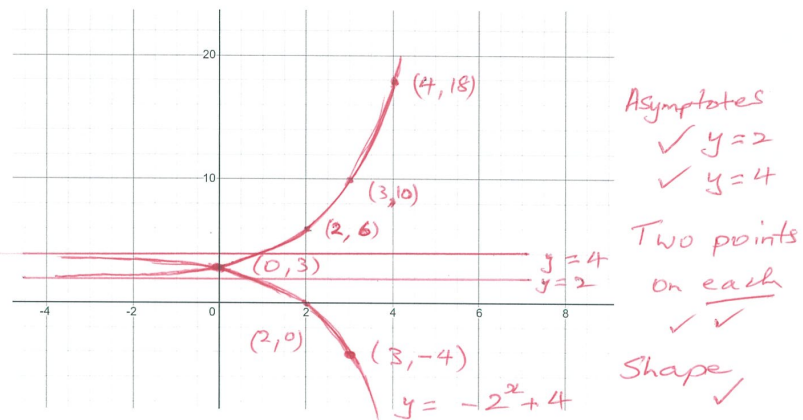
Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 (2.1.1- 2.1.7)

[5+1+4 = 10 marks]

- (a) Sketch the graphs of $y = 2^x + 2$ and $y = -2^x + 4$ on the axes below, showing important features of each graph.



- (b) Using your graph (or otherwise), find the intersection point of these two functions.

From the graph, intersection is $(0, 3)$ ✓

or $2^x + 2 = -2^x + 4$ or

$\Rightarrow 2^x + 2^x = 2$

$\Rightarrow 2^{x+1} = 2^1 \Rightarrow x+1 = 1 \Rightarrow x=0$ ✓

- (c) Solve for x : $9^{2x-1} = 243$

$9^{2x-1} = 243$ $9 = 3^2$ $243 = 3^5$ ✓

Thus $3^{2(2x-1)} = 3^5$ ✓

Equate indices

$4x - 2 = 5$ ✓

$\therefore 4x = 7$

$x = 7/4$ ✓

Question 2 (2.3.1, 2.3.4, 2.3.5)

[4+2 = 6 marks]

(a) For the function $f(x) = 3x^2$, use *first principles* to find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ and hence show that $f'(x) = 6x$

$$f(x) = 3x^2 \quad f(x+h) = 3(x+h)^2$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h}$$

$$= \lim_{h \rightarrow 0} (6x + 3h)$$

$$= 6x \text{ as } h \rightarrow 0$$

(b) Briefly describe what $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ represents on a graph of $f(x)$.

Instantaneous rate of change of $f(x)$ at x . Or gradient of tangent to $f(x)$ at point x .

Question 3 (2.3.7, 2.3.13 - 2.3.17)

[4+4 = 8 marks]

The curve with the equation $y = (x+1)(x-2)(x-5)$ cuts the x -axis at the points $A(-1, 0)$, $B(2, 0)$ and $C(5, 0)$. The expanded equation is $y = x^3 - 6x^2 + 3x + 10$

(a) Find $\frac{dy}{dx}$ and hence show that the *tangents* to the curve at points A and C are parallel.

$$\frac{dy}{dx} = 3x^2 - 12x + 3$$

$$\text{At } A, x = -1 \rightarrow \frac{dy}{dx} = 3 + 12 + 3 = 18$$

$$\text{At } C, x = 5 \rightarrow \frac{dy}{dx} = 3(25) - 12(5) + 3 = 18$$

Tangents have the same gradient

\therefore tangents are parallel.

- (b) Find the equation of the tangent to the curve at the point C and find the point (x, y) where the tangent crosses the y -axis.

$$y = 18x + c \quad \checkmark \text{ at } (5, 0) \text{ Substitute } (x, y)$$

$$0 = 18(5) + c \quad \checkmark$$

$$\Rightarrow c = -90$$

$$y = 18x - 90 \quad \checkmark$$

$$y\text{-intercept} = (0, -90) \quad \checkmark$$

Question 4 (2.3.8 - 2.3.11)

[3+3 = 6 marks]

A jet pilot follows a flight path defined by $f(x) = x^3 - 9x^2 + 15x - 8$.

- (a) Is the gradient of the flight path positive (going up) or negative (down) at the point $(2, -6)$? Explain your answer.

$$f(x) = x^3 - 9x^2 + 15x - 8$$

$$f'(x) = 3x^2 - 18x + 15 \quad \checkmark \text{ Substitute } x = 2$$

$$= 12 - 36 + 15$$

$$= -9 \quad \checkmark$$

Negative gradient shows that the flight path is downwards at $(2, -6)$ (or $x = 2$)

- (b) At what x -values on the curve $f(x)$ is the tangent parallel to the line $y = 3$?

$$y = 3 \Rightarrow y' = 0 \quad \checkmark$$

$$\therefore \text{Solve } f'(x) = 3x^2 - 18x + 15 = 0$$

$$= 3(x^2 - 6x + 5)$$

$$= 3(x-5)(x-1) \Rightarrow x = 5 \text{ or } 1 \quad \checkmark \text{ both}$$

Question 5 (2.3.3 - 2.3.7, 2.3.22)

[4 marks]

Find y in terms of x if $\frac{dy}{dx} = 3x^2 - 2x - 6$ and the function $f(x)$ passes through the point $(2, 4)$.

$$f'(x) = 3x^2 - 2x - 6$$

$$f(x) = \int (3x^2 - 2x - 6) dx \quad \checkmark \text{ Write Anti-derivative}$$

$$= \frac{3x^3}{3} - \frac{2x^2}{2} - 6x + c \quad \checkmark \text{ Add } c$$

$$= x^3 - x^2 - 6x + c \quad \checkmark \text{ Substitute}$$

$$\text{At } (2, 4), \Rightarrow 4 = 2^3 - 2^2 - 6(2) + c \Rightarrow c = 8 \quad \checkmark$$

$$\therefore y = x^3 - x^2 - 6x + 8 \quad \checkmark \text{ State answer}$$

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Question 6 (2.3.10)

[4 marks]

A section of roller coaster has been constructed using the function:

$$f(x) = x^3 + 3x^2 - 4$$

An amusement park photographer is taking "action shots" near the roller coaster where the gradient is equal to -3 ("negative 3"). In terms of x -values, where is the photographer working? Explain your answer with suitable working.

$$f(x) = x^3 + 3x^2 - 4$$

$$f'(x) = 3x^2 + 6x \quad \checkmark$$

$$\text{Set } f'(x) = -3 \quad \checkmark$$

$$\therefore 3x^2 + 6x = -3$$

$$\text{or } 3x^2 + 6x + 3 = 0 \quad \checkmark$$

$$\therefore 3(x^2 + 2x + 1) = 0$$

$$\therefore 3(x+1)^2 = 0 \Rightarrow x = -1 \quad \checkmark \text{ (check)}$$

$$f''(x) = 6x + 6$$

$$\text{at } x = -1, f''(x) = 0$$

$$\text{Hence point of inflection}$$

$$\therefore x = -1 \quad \checkmark \text{ Ans}$$

Question 7 (2.3.19, 2.3.22)

[3+3 = 6 marks]

A function $V(t)$ for which $V'(t) = 4t + k$, (where k is a constant), has a turning point at $(1, -2)$. Find:

- (a) The value of k

$$V'(t) = 4t + k = 0 \text{ at } (1, -2) \Rightarrow k = -4 \quad \checkmark$$

$$\therefore V(t) = \frac{4t^2}{2} + kt + c = 2t^2 + kt + c$$

- (b) The value of $V(t)$ when $t = 4$

$$V(t) = 2t^2 - 4t + c \quad \checkmark \text{ Subs } (1, -2)$$

$$-2 = 2 - 4 + c \Rightarrow c = 0 \quad \checkmark$$

$$V(t) = 2t^2 - 4t$$

$$\text{when } t = 4, V(t) = 2(16) - 4(4) = 16 \quad \checkmark$$

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END OF ASSESSMENT