

Question	Marks	Max	Question	Marks	Max
4		5	7	7	7
3		7	6		12
2		5			
1	4	5	8		

No other items may be taken into the examination room. It is **your responsibility** to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Important note to candidates

Special items: nil

To be provided by the candidate Standard items: pens (blue/black preferred), pencils (including coloured), sharpeners, correction fluid/tape, eraser, ruler, highlighters

Formula sheet

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time: fifty minutes
Reading time before commencing work: five minutes

Time allowed for this section

Your Teacher's Name:

Your Name:

Section One:

Calculator-free

UNIT 3 & 4

MATHEMATICS METHODS

Question/Answer booklet

Semester Two Examination, 2020

INDEPENDENT PUBLIC SCHOOL

Exceptional schooling. Exceptional students.



Structure of this paper

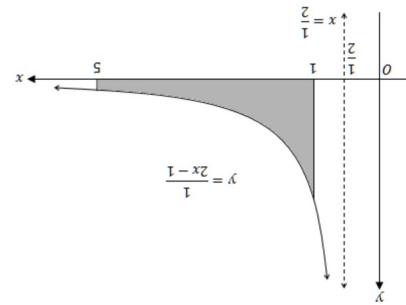
Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	10	10	100	94	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

- ✓ Use integral for area under the curve
- ✓ Use the correct boundary points
- ✓ Determine the correct antiderivative
- ✓ Determine the correct simplified answer as one term.

$$\int_5^1 \frac{1}{2x-1} dx = \left[\frac{1}{2} \ln(2x-1) \right]_5^1 = \frac{1}{2} \ln 9 - \frac{1}{2} \ln 1 = \frac{1}{2} \ln 9$$



Find the area bounded by the curve $y = \frac{1}{2x-1}$, the x -axis and the lines $x=1$ and $x=5$. Leave your answer in the exact simplified form.

(4 marks)

Question 1

Working time: 50 minutes.

- This section has **seven** questions. Answer all questions. Write your answers in the spaces provided.
- Continue writing an answer: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
 - Planning: if you use the spare pages for planning, indicate this clearly at the top of the page.
 - Responses and/or as additional space if required to continue an answer.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

(50 Marks)

Section One: Calculator-free

Question 2

(7 marks)

Suppose that $f(x)$ and $g(x)$ are differentiable functions that satisfy the following properties.

$f(3)$	-2
$g(3)$	3
$f'(3)$	-1
$g'(3)$	0

- (a) Given $h(x)=\frac{f(x)}{g(x)}$, determine the value for $h'(3)$. (2 marks)

$$h'(3)=\frac{f'(3)g(3)-f(3)g'(3)}{[g(3)]^2}=\frac{(-1)(3)-(-2)(0)}{3^2}=\frac{-1}{3}$$

- ✓ Demonstrate the use of quotient rule correctly
- ✓ Calculate the correct answer

- (b) Given $T(x)=f(g(x))$, determine the value for $T'(3)$. (2 marks)

$$T'(3)=f'(g(3)) \times g'(3)=f'(3) \times g'(3)=(-1)(0)=0$$

- ✓ Demonstrate the use of chain rule correctly
- ✓ Calculate the correct answer

- (c) Given $S(x)=\ln(-f(x))$, determine the value for $S'(3)$. (3 marks)

$$S'(3)=\frac{1}{-f(3)} \times (-f'i|_3)=\frac{1}{2}$$

- ✓ Demonstrate the differentiation of natural log correctly
- ✓ Demonstrate the differentiation of natural log correctly with the use of chain rule
- ✓ Calculate the correct answer

Additional working space

Question number: _____

$$\int_{e^2}^e \ln(x) dx = x \ln(x) - x \Big|_{e^2}^e = (2e \ln(2) - 2e) - (e \ln(e) - e) = 2e \ln(2) - 3e$$

$$\int x \ln(x) dx = x^2 \ln(x) - \int x^2 d(\ln(x)) = x^2 \ln(x) - \int x^2 \cdot \frac{1}{x} dx = x^2 \ln(x) - \int x dx = x^2 \ln(x) - \frac{x^2}{2}$$

$$\int x^2 \ln(x) dx = x^2 \cdot \frac{1}{3} x^3 \ln(x) - \int x^2 \cdot \frac{1}{3} x^3 d(\ln(x)) = x^2 \cdot \frac{1}{3} x^3 \ln(x) - \int x^2 \cdot \frac{1}{3} x^3 \cdot \frac{1}{x} dx = x^2 \cdot \frac{1}{3} x^3 \ln(x) - \frac{1}{3} \int x^5 dx = x^2 \cdot \frac{1}{3} x^3 \ln(x) - \frac{1}{18} x^6$$

(b) Hence determine $\int_{e^2}^e \ln(x) dx$ (4 marks)

- ✓ Obtain the simplified expression
- ✓ Differentiate $\ln(x)$ correctly
- ✓ Use product rule correctly

$$\frac{d}{dx} \ln\left(\frac{x}{x+1}\right) = \frac{1}{\frac{x}{x+1}} \cdot \frac{1}{(x+1)} = \frac{1}{x+1}$$

(a) Determine $\frac{d}{dx} x \ln(x)$ and simplify your answer (3 marks)

Question 3
(7 marks)

Question 3

CALCULATOR-FREE

12

MATHEMATICS METHODS

CALCULATOR-FREE

5

MATHEMATICS METHODS

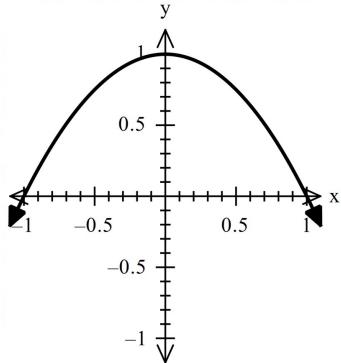
Additional working space

Question number: _____

Question 4

(5 marks)

A rectangle is inscribed with its base on the x -axis and its upper corners on the parabola $y=1-x^2$. Determine the dimensions of such a rectangle with the greatest possible area.



$$A = 2x(1-x^2) = -2x^3 + 2x$$

$$\frac{dA}{dx} = -6x^2 + 2 = 0, x^2 = \frac{1}{3}, x = \frac{-1}{\sqrt{3}} \text{ (reject)} \vee \frac{1}{\sqrt{3}}$$

$$\frac{d^2A}{dx^2} = -12x = \frac{-12}{\sqrt{3}} < 0, \text{ hence local maximum at } x = \frac{1}{\sqrt{3}} \text{ units}$$

$$y = 1 - \frac{1}{3} = \frac{2}{3} \text{ units}$$

$$\text{Dimensions are width} = \frac{2}{\sqrt{3}} \text{ units} \wedge \text{height} = \frac{2}{3} \text{ units}$$

- ✓ Write an equation for area in terms of x
- ✓ Determine first derivative
- ✓ Equate first derivative to 0 and solve for x
- ✓ Use second derivative or otherwise to justify.
- ✓ State dimensions for both x and y .

Additional working space

Question number: _____

- ✓ Differentiate y correctly
- ✓ Calculate $\frac{dy}{dx}$ at $x = -1$
- ✓ Calculate constant c correctly
- ✓ Write the correct equation of the line

$$\text{Hence, } y = -2x - 2$$

$$\text{Substituting } (-1, 0), 0 = -2(-1) + c, c = -2$$

$$y = 1 - 1 = 0$$

$$\text{At } x = -1, \frac{dy}{dx} = -2 + 0 = -2,$$

$$\frac{dy}{dx} = 2x + \frac{\pi}{2} \cos\left(\frac{\pi}{2}x\right)$$

(4 marks)

- (b) Find an equation of the line tangent to the graph of $y = x^2 + \sin\left(\frac{\pi}{2}x\right)$ at $x = -1$.

✓ solves for c (follow through only for (5,0) used)

✓ subs grad into $y = mx + c$

✓ determines gradient

✓ determines height of triangle

OR

✓ uses the negative result of part a as area from 5 to 7

✓ obtains b in terms of a (follow through only for $b = -5a$)

✓ integrates linear rule

✓ solves for a and b

✓ determines height of triangle

✓ uses formula for area of triangle

✓ determines gradient of line is -1 .

$$\text{Therefore, } y = -x + 2$$

$$\text{At } x = 2, y = 0, \text{ substituting } (2, 0), -2 + c = 0, c = 2$$

$$\text{Hence, the gradient of line is } -1.$$

$$\begin{aligned} x &= 2, \frac{dy}{dx} = 1, \\ \frac{dy}{dx} &= x - 1, \end{aligned}$$

(4 marks)

- (a) Find an equation of the line perpendicular to the graph of $y = \ln(x - 1)$ at $x = 2$.

(8 marks)

$$5a + b = 0, b = -5a$$

$$\int_7^5 a t + b dt = -\frac{2}{5}$$

$$\int_7^5 a t - 5a dt = \left[\frac{a t^2}{2} - 5at \right]_7^5 = \left[\frac{49a}{2} - 35a \right] - \left[\frac{25a}{2} - 25a \right] = ? 2a = -\frac{2}{5} ?$$

$$b = \frac{4}{25}$$

$$a = -\frac{4}{5}$$

Question 5

Question 6

(12 marks)

The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$P(X=x)$	$\frac{1}{5}$	a	$\frac{1}{10}$	a	$\frac{1}{5}$

where a is a constant.

- (a) Determine the value of a .
marks)

(2)

$$2a + \frac{1}{5} + \frac{1}{10} + \frac{1}{5} = 1, a = \frac{1}{4}$$

- ✓ Use the sum of probability equals 1
- ✓ Calculate the correct value for a .

- (b) Determine $E[X]$.

(2 marks)

$$E[X] = (-1)\left(\frac{1}{5}\right) + 1\left(\frac{1}{10}\right) + 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{5}\right) = 1$$

- ✓ Set up a correct equation for mean
- ✓ Calculate the correct answer

- (c) Determine $\text{Var}[X]$.

(3 marks)

$$\begin{aligned} \text{Var}[X] &= (-1-1)^2\left(\frac{1}{5}\right) + (0-1)^2\left(\frac{1}{4}\right) + (1-1)^2\left(\frac{1}{10}\right) + (2-1)^2\left(\frac{1}{4}\right) + (3-1)^2\left(\frac{1}{5}\right) \\ &\quad \textcolor{brown}{\cancel{\frac{4}{5}}} + \frac{1}{4} + \frac{1}{4} + \frac{4}{5} \\ &\quad \textcolor{brown}{\cancel{\frac{21}{10}}} \sqrt{2.1} \end{aligned}$$

- ✓ Set up an equation for variance
- ✓ Calculate the correct answer

The random variable $Y = 6 - 2X$

- (d) Determine $\text{Var}[Y]$.

(2 marks)

$$\text{Var}[Y] = (-2)^2(2.1) = 8.4$$

- ✓ Use $(-2)^2 \sqrt{4}$
- ✓ Calculate the correct answer

- (e) Calculate $P(X \geq Y)$.

(3 marks)

$$X \geq 6 - 2X, X \geq 2$$

See next page

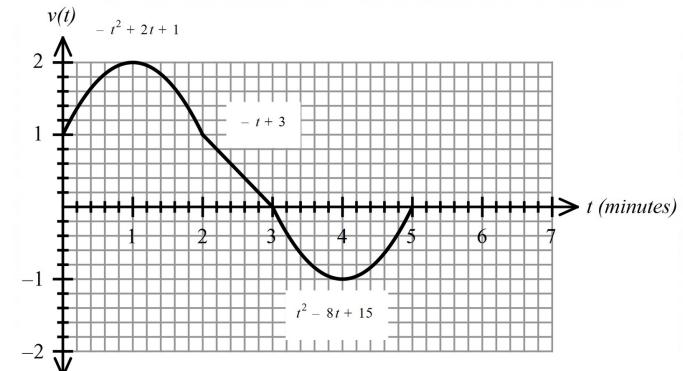
$$P(X \geq 2) = P(X=2) + P(X=3) = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$$

- ✓ Recognise $P(X \geq Y) = P(X \geq 2)$
- ✓ Use cumulative probability correctly
- ✓ Calculate the correct answer

Question 7

(7 marks)

The following diagram shows the instantaneous velocity $v(t) \text{ m/min}$ of a moving object during the first 5 minutes, where t is in minutes.



$$v(t) = \begin{cases} -t^2 + 2t + 1, & 0 \leq t < 2 \\ -t + 3, & 2 \leq t < 3 \\ t^2 - 8t + 15, & 3 \leq t < 5 \\ g(t), & 5 \leq t \leq 7 \end{cases}$$

- (a) Determine $\int_0^5 v(t) dt$.

(3 marks)

$$\int_0^5 v(t) dt = 2 + \frac{1}{2} = \frac{5}{2}$$

- ✓ recognizes that two parabolas are congruent and negate each other
- ✓ determines area of rectangle and triangle
- ✓ determines net area
- OR
- ✓ writes definite integral sum
- ✓ anti-diffs and subs correct limits
- ✓ determines sum

- (b) Determine a formula for a linear function $g(t) = at + b$, $5 \leq t \leq 7$, given that the object returns to the origin at the end of 7 minutes, that is, $\int_0^7 v(t) dt = 0$.

(4 marks)

$$t=5, v(t) = [5]^2 - [8](5) + 15 = 0$$

See next page