PERTH MODERN SCHOOL
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Independent Public School

Course Methods Test 2 Year 12

Student name: ________________ Teacher name: ________________

Task type: **Response**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **_____6______**

Materials required: Upto three calculators/classpads

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper,

Marks available: **41 marks**

Task weighting: **13%**

Formula sheet provided: **no but formulae listed on next page.**

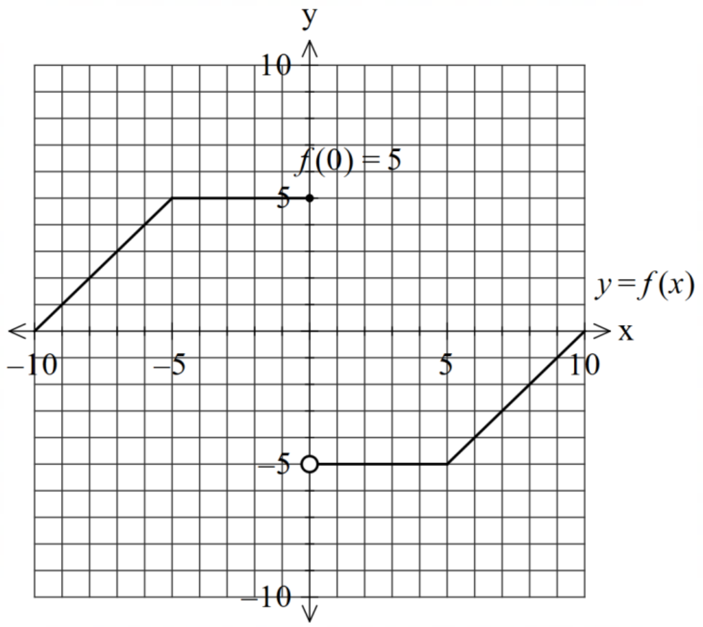
Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
$\frac{d}{dx}e^{ax-b} = ae^{ax-b}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\ln x = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln x + c, \quad x > 0$
$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$		$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c, \quad f(x) > 0$
$\frac{d}{dx}\sin(ax-b) = a\cos(ax-b)$		$\int \sin(ax-b)\,dx = -\frac{1}{a}\cos(ax-b) + c$
$\frac{d}{dx}\cos(ax-b) = -a\sin(ax-b)$		$\int \cos(ax-b)\,dx = \frac{1}{a}\sin(ax-b) + c$
Product rule	If $y = uv$ then $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$	or If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$
Quotient rule	If $y = \frac{u}{v}$ then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	or If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$
Chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$
Fundamental theorem	$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$	and $\int_a^b f'(x)\,dx = f(b) - f(a)$
Increments formula	$\delta y \approx \frac{dy}{dx} \times \delta x$	
Exponential growth and decay	$\frac{dP}{dt} = kP \Leftrightarrow P = P_0 e^{kt}$	

Q1 (2, 3, 2 ,2 & 3 =12 marks)

Consider the function $y = f(x)$ which is graphed below.



a) $\int_{-10}^{10} f(x)\,dx$.

b) $\int_{-3}^{3} f(x) - 4\,dx$.

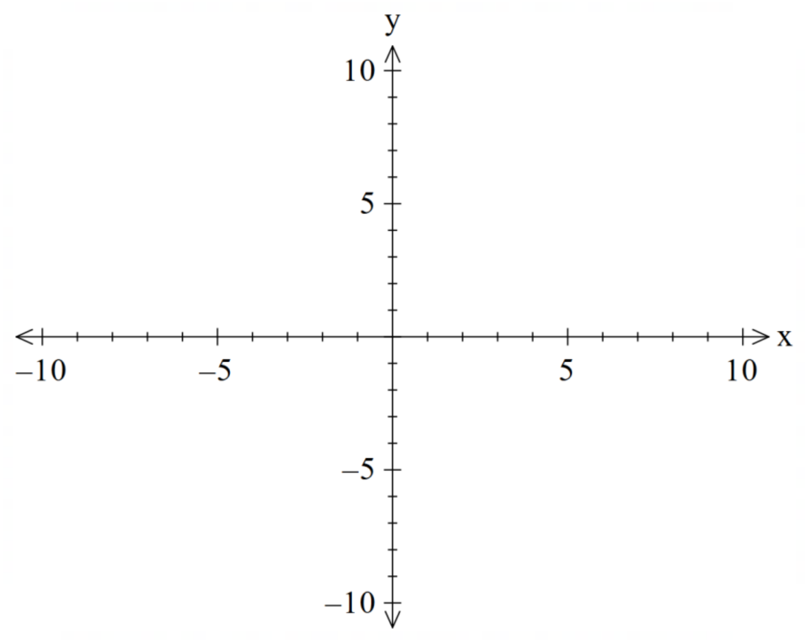
c) $\frac{d}{dt}\int_{t}^{6} f(x)\,dx$ when $t = 8$.

d) $\int_{-9}^{-6} f'(x)\,dx$.

e) $\frac{d}{dt}\int_{5}^{t^2} f(x)\,dx$ in terms of t for $0 < t < 2$.

Q2 (4 marks)
Sketch a continuous function **showing the x coordinates and labelling** of all special features on the axes below that meet the following requirements.

$f(-4) = 0 = f(3)$
$f(0) = -7$
$f'(-4) = 0 = f'(1)$
$f''(1) > 0,\ f''(-4) < 0$
Has **exactly** two stationary points.



Q3 (3 marks)

Consider a balloon whose volume V , litres, varies with time, t seconds, such that $\frac{dV}{dt}=\frac{-100t^2}{\left(2t^3+5\right)^2}$.

If the balloon fully deflates after 12 seconds, determine the initial volume. Full reasoning must be shown for full marks.

Q4 (2, 2 & 3 = 7 marks)

An object's displacement, x metres at t seconds, from the origin is $x=5e^{-3t}\cos\left(5t\right)$ metres.

a) Determine the velocity function at time t seconds.

b) Determine the first two times that the object changes direction.

Q4 continued-

c) Determine the distance travelled in the first 1.5 seconds.

Q5 (2 & 4 = 6 marks)

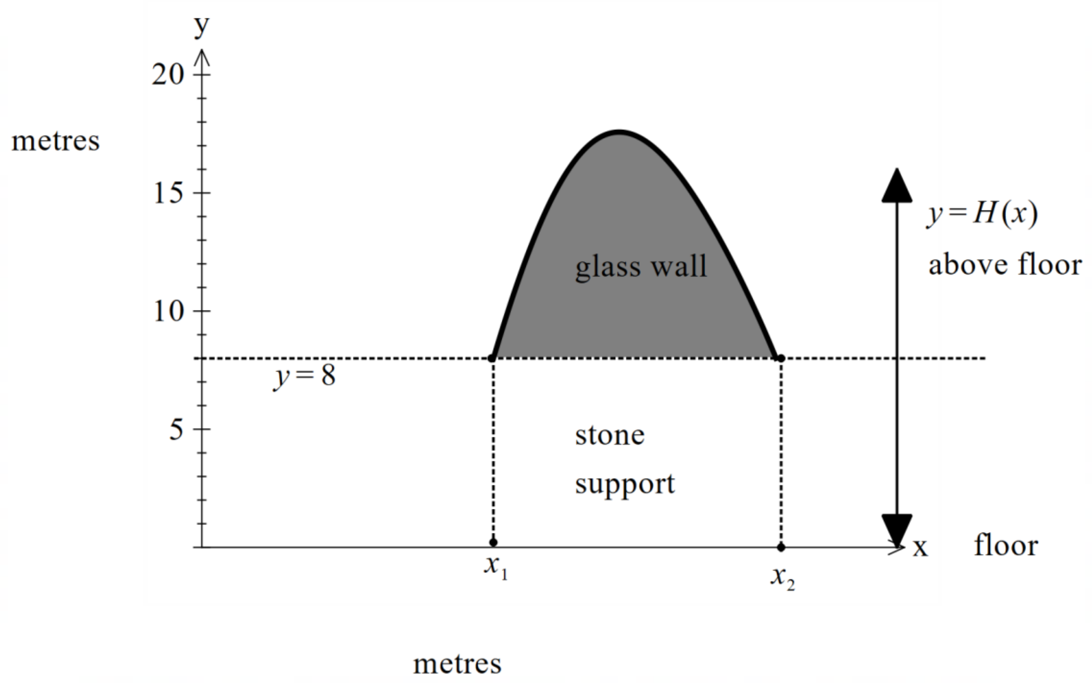
a) Determine $\frac{d}{dx}\left(3x\cos\frac{\pi x}{6}\right)$ **without the use of a classpad**. Full reasoning must be given.

b) Hence show how to determine $\int \frac{\pi}{6}x\sin\frac{\pi x}{6}\,dx$ **without the use of a classpad**. Full reasoning must be given **using** the result from part a.

Q6 (2, 4 & 3 = 9 marks)

Consider a glass wall with the height $H(x)$ metres **above floor** at x metres along the floor according to

$H(x) = 17 - (2x-9)^2 - \cos(2x - \frac{3\pi}{2})$. The glass wall sits on a stone support of height 8 metres.



a) Determine the values x_1 & x_2 to the nearest cm.

b) Using calculus, determine the maximum height of the wall. Justify.

Q6 continued

c) If the wall is 5 cm thick determine the volume of glass with units, needed to make the wall.

End of test.