Section One (calculator-free) 40 marks

This section has **eight (8)** questions. Attempt **all** questions.

Suggested working time: **50 minutes**

The following exact value table may be useful to answer questions in this examination.

	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Question 1: [7 marks – 1, 2, 1, 1, 1, 1]
Differentiate each of the following, simplifying fully:

a) $y = 3x^5 + 4x^3 - \dfrac{8}{x^2}$

$\dfrac{dy}{dx} = 15x^4 + 12x^2 + \dfrac{16}{x^3}$ ✓

b) $y = \dfrac{(x-1)(x+1)}{x^4 + 4}$ $\dfrac{dy}{dx} = 15x^4 + 12x^2 + \dfrac{16}{x^3}$

$y = \dfrac{x^2 - 1}{x^4 + 4}$

$\dfrac{dy}{dx} = \dfrac{2x(x^4 + 4) - 4x^3(x^2 - 1)}{(x^4 + 4)^2}$ ✓

$\dfrac{dy}{dx} = \dfrac{2x^5 + 8x - 4x^5 + 4x^3}{(x^4 + 4)^2}$

$\dfrac{dy}{dx} = \dfrac{-2x^5 + 4x^3 + 8x}{(x^4 + 4)^2}$ ✓

c) $y = e^x \ln x$

$\dfrac{dy}{dx} = e^x \ln x + \dfrac{e^x}{x}$ ✓

d) $y = (\ln(x^4))^2$

$\dfrac{dy}{dx} = \dfrac{2\ln x^4 \times 4x^3}{x^4}$

$\dfrac{dy}{dx} = \dfrac{8 \ln x^4}{x}$ ✓

Question 1 cont…

e) $y = \cos(x^2 - 4)$

$\frac{dy}{dx} = -2x \sin(x^2 - 4)$ ✓

f) $y = x \sin x$

$\frac{dy}{dx} = \sin x + x \cos x$ ✓

Question 2: [3 marks]

Determine $\frac{dy}{dx}$ in terms of t for: $x = 6t^4$, $y = \frac{1}{t+2}$.

$\frac{dx}{dt} = 24t^3$ $\frac{dy}{dt} = -\frac{1}{(t+2)^2}$ ✓ **Differentiation**

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = -\frac{1}{(t+2)^2} \times \frac{1}{24t^3}$$ ✓

$$\frac{dy}{dx} = -\frac{1}{24t^3(t+2)^2}$$ ✓

$$\frac{dy}{dx} = -\frac{1}{25t^5 + 96t^4 + 96t^3}$$

[3]

Question 3: [7 marks – 1, 1, 1, 1, 1, 2]

Integrate the following:

a) $\int 6x^2 - 7\, dx$

$y = 2x^3 - 7x + c$ ✓

b) $\int (2x - 7)^4\, dx$

$y = \frac{(2x-7)^5}{10} + c$ ✓

c) $\int \sqrt{3 - x}\, dx$

$y = \frac{-2(3-x)^{\frac{3}{2}}}{3} + c$ ✓

d) $\int \sin 6x\, dx$

$y = \frac{-\cos 6x}{6} + c$ ✓

e) $\int \frac{8x}{x^2 + 3}\, dx$

$y = 4 \ln |x^2 + 3| + c$ ✓

f) $\int 2 \tan x\, dx$

$\int \frac{2 \sin x}{\cos x}\, dx$ ✓

$y = -2 \ln |\cos x| + c$ ✓

Question 4: [2 marks]

Determine a general solution to the differential equation: $\frac{dy}{dx} = \frac{4x - 3}{y + 1}$. Simplify your solution fully.

$$\int (y + 1)\, dy = \int 4x - 3\, dx \quad ✓$$

$$\frac{1}{2}y^2 + y = 2x^2 - 3x + c \quad ✓$$

[2]

Question 5: [7 marks]

Determine z if: $z\bar{z} + 2z = \dfrac{1 + 4i}{4}$.

Let: $z = a + bi$
then: $\bar{z} = a - bi$

$$z\bar{z} + 2z = \frac{1 + 4i}{4}$$

$$(a + bi)(a - bi) + 2(a + bi) = \frac{1}{4} + i \quad ✓$$

$$a^2 + b^2 + 2a + 2bi - \frac{1}{4} - i = 0 \quad ✓$$

Im: $2b = 1$

$$\therefore b = \frac{1}{2} \quad ✓$$

Real: $a^2 + b^2 + 2a - \frac{1}{4} = 0$ ✓

$$a^2 + \left(\frac{1}{2}\right)^2 + 2a - \frac{1}{4} = 0$$

$$a^2 + \frac{1}{4} + 2a - \frac{1}{4} = 0$$

$$a^2 + 2a = 0$$

$$a(a + 2) = 0$$

$$a = 0 \text{ or } -2$$
✓ ✓

$$\therefore z = \frac{1}{2}i \quad \text{or} \quad z = -2 + \frac{1}{2}i \quad ✓$$

(- $\frac{1}{2}$ each mistake)

[7]

Question 6: [3 marks]

If $z_1 = 2 \text{ cis}\left(\frac{\pi}{12}\right)$ and $z_2 = 5 \text{ cis}\left(\frac{\pi}{6}\right)$, prove that: $z_1 z_2 = 5\sqrt{2}\ (1 + i)$

$$z_1 z_2 = 2 \text{ cis}\left(\frac{\pi}{12}\right) \times 5 \text{ cis}\left(\frac{\pi}{6}\right)$$

$$= 10 \text{ cis}\left(\frac{\pi}{12} + \frac{\pi}{6}\right)$$

$$= 10 \text{ cis}\left(\frac{\pi}{4}\right) \quad ✓$$

$$= 10\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$$

$$= 10\left(\frac{\sqrt{2}}{2}\right) + i\frac{\sqrt{2}}{2}) \quad ✓$$

$$= 5\sqrt{2}\ (1 + i) \quad ✓$$

$\therefore$ Proved

[3]

Question 7: [5 marks]

Determine the gradient of the curve defined by the parametric equations: $x = 2 \sin t$ and $y = 7 \cos 3t$, at the point where $t = \frac{\pi}{6}$.

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= -21 \sin 3t \times \frac{1}{2 \cos t} \quad ✓✓$$

$$= \frac{-21 \sin\left(\frac{\pi}{2}\right)}{2 \cos\left(\frac{\pi}{6}\right)} \quad ✓$$

$$= \frac{-21 \times 1}{2 \times \frac{\sqrt{3}}{2}}) \quad ✓$$

$$= \frac{-21}{\sqrt{3}}$$

$$= -7\sqrt{3} \quad ✓$$

[5]

Question 8: [6 marks]

Determine the coordinates of the points on the graph of: $5x^2 + y^2 - 20x + 3y = 8$, where the tangent to the curve is horizontal.

$5x^2 + y^2 - 20x + 3y = 8$

$10x + 2y\frac{dy}{dx} - 20 + 3\frac{dy}{dx} = 0$ ✓

$(2y + 3)\ \frac{dy}{dx} = 20 - 10x$

$\frac{dy}{dx} = \frac{20 - 10x}{2y + 3}$ ✓

$\frac{dy}{dx} = 0 \qquad 0 = \frac{20 - 10x}{2y + 3}$ ✓

$10x = 20$

$\therefore x = 2$ ✓

$5(2)^2 + y^2 - 20(2) + 3y = 8$

$y^2 + 3y - 28 = 0$

$(y + 7)(y - 4) = 0$

$y = -7, 4$ ✓

$\therefore (2,\ -7)\ ,\ (2,\ 4)$ ✓

[6]

END OF SECTION ONE