ALL SAINTS' COLLEGE

MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 3 - 2017
Discrete Random Variables and Binomial Distributions
Resource Free

Name: SOLUTIONS **Teacher:** _____

Marks: 22

Time Allowed: 15 minutes

Instructions: You are NOT allowed any Calculators or notes.

You will be supplied with a formula sheet.

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1) [2,2,3 = 7 marks]

a) The table below shows the values taken by a function f(x).

x	-1	0	1	0.5
f(x)	0.2	0.6	0.1	0.1

John argues that f(x) cannot be a probability distribution function of a discrete random variable as x has a negative value. Comment on his answer.

x can be negative. It's f(x) that can't be negative ✓

JOHN is WRONG ✓

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b) The table below shows the values taken by a function f(x).

x	0	0.5	1	1.5	2
f(x)	0.2	0.5	a	b	0.1

i. Under what conditions can f(x) represent the probability distribution of a discrete random variable?

$a+b=0.2$ ✓

$a \geq 0, b \geq 0$ ✓

ii. If f(x) is the pdf of a discrete random variable X, find the values of a and b given that P(X=1) = P(X=1.5).

$a=b$ ✓

$a+b=0.2$

$\Rightarrow a=b=0.1$ ✓✓

2) [3,2,2,3,3 = 11 marks]

A certain tropical plant produces both white and pink orchid flowers. 20% of the flowers are white. The flower colour white forms a binomial distribution. One of these plants has 3 flowers.

[Note: You do not need to simplify your answers to parts b,c and d].

(a) What are the values of n, p and q?

$n=3$ ✓

$p=0.2$ ✓

$q=0.8$ ✓

(b) What is the probability that all flowers are pink?

$(0.8)^3$ ✓✓

(c) What is the probability that at least 1 flower is white?

$1 - P(\text{all pink})$ [or $P(1) + P(2) + P(3)$]

$1 - (0.8)^3$ ✓✓

(d) What is the probability that 2 flowers are pink given that at least 1 of them is pink?

$P(\text{at least 1 pink}) = 1 - P(\text{all white})$

$= 1 - (0.2)^3$ ✓

$P(2 \text{ pink}) = \binom{3}{2}(0.8)^2(0.2)^1$

$= 3(0.8)^2(0.2)^1$ ✓

Then $P(2 \text{ pink} \mid \geq 1 \text{ pink}) = \dfrac{3(0.8)^2(0.2)^1}{1-(0.2)^3}$ ✓✓

[0.387097 approx]

3) [4 marks]

It is know that 2% of all new electrical components produced on a mass production assembly line are defective. The components are packed in boxes of 6. Boxes are 'passed' only if they contain no defective items. What is the probability that a randomly chosen box is rejected?

$1 - P(0 \text{ defective})$ ✓

$n = 6,\ p = 0.02,\ q = 0.98$ ✓

$1 - (0.98)^6$ ✓✓

i.e. very little chance!! ≈ 0.114

5) [4 marks]

A binomial distribution has a mean of 4.8 and a standard deviation of approximately 1.833. Find the number of trials, n, and the probability of success, p.

$np = 4.8$

$\sqrt{npq} = 1.833$

$npq \approx 3.359889$ ✓

$\therefore q = \frac{3.359889}{4.8}$ ✓

$\Rightarrow q = 0.7$

$\therefore p = 0.3$

If $p = 0.7$ ✓

$n = 16$ ✓

6) [6 marks]

The probability that Jordy tosses a coin into a container from 2 metres is 0.2.

a) If she tosses 9 coins, what is the probability of her getting at least two coins into the container?

$P \approx 0.5637924$ (CLASSPAD) ✓

b) How many coins would she need to toss so that the probability of getting at least 1 coin into the container is greater than 0.65?

$n = ?$

$P(x \geq 1) > 0.65$

$1 - P(x = 0) > 0.65$

$P(x = 0) < 0.35$ ✓

$\binom{n}{0}(0.2)^0(0.8)^n < 0.35$ ✓

$n \ln 0.8 < \ln 0.35$

$\Rightarrow n > \frac{\ln 0.35}{\ln 0.8}$

$n > 4.7$ ✓

$\therefore n = 5$ coins ✓✓

4) [4,2,6 = 12 marks]

Three marbles are drawn one at a time from a bag containing 5 blue marbles and 7 green marbles. The marble is replaced after each draw. Find:

a) The probability distribution for the random variable X, the number of blue marbles drawn,

3S 2S 1S 0S

x	0	1	2	3
P(X=x)	0.1985	0.4253	0.3038	0.0723

✓ ✓ ✓ ✓

b) The probability that all three marbles were the same colour,

$\left(\frac{5}{12}\right)^3 + \left(\frac{7}{12}\right)^3$

$= \frac{13}{48}$ ✓✓ (−1 for ANY decimal EXCEPT $0.27083\overline{3}$ if '=' sign used)

c) The probability that at most 1 green marble was drawn given that at least one marble was blue.

P(at least 1 BLUE) = 1 − P(0 BLUE)
≈ 1 − 0.1985
≈ 0.8015 ✓✓

P(at most 1 green | ≥1 blue)
$\approx \frac{0.0723 + 0.3038}{0.8015}$ ✓✓
≈ 0.4692 ✓✓ [EXACT: $\frac{434}{1145}$]

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ALL SAINTS' COLLEGE

MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 3 - 2017
Discrete Random Variables and Binomial Distributions
Resource Rich

Name: ANSWERS **Teacher:** _____

Marks: ~~40~~ 45

Time Allowed: 30 minutes

Instructions: You are allowed a ClassPad and scientific calculator but NO notes.
You will be supplied with a formula sheet.

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1. [2,2,2,3 = 9 marks]

A probability distribution is defined by the following table:

x	0	1	2	3	4
P(X=x)	0.1	0.4	k 0.11	3k 0.33	0.06

a) Find the value of k

0.56 + 4k = 1 ✓
⇒ 4k = 0.44
∴ k = 0.11 ✓

b) Find the P(X>2)

P(x = 3) + P(x = 4)
= 0.39 ✓✓

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c) Find E[X]

$E[X] = (0\times0.1 + 1\times0.4 + 2\times0.11 + 3\times0.33 + 4\times0.06)$

$= 1.85$ ✓✓

d) Find the standard deviation of X

$E[(X^2)] = (0^2\times0.1 + 1^2\times0.4 + 2^2\times0.11 + 3^2\times0.33 + 4^2\times0.06)$

$= 4.77$ ✓

Hence $SD(X) = \sqrt{4.77 - (1.85)^2}$ ✓

≈ 1.161 ✓

2) [1,2,3 = 6 marks]

a) What is the probability of guessing the month in which a person is born?

$\frac{1}{12}$ ✓

b) What is the percentage probability of correctly guessing the month of birth of 3 students form a group of 10 students?

$\binom{10}{3}\left(\frac{1}{12}\right)^3\left(\frac{11}{12}\right)^7 \approx 0.03777$ ✓

$\therefore$ 3.78% ✓

c) If the probability of correctly guessing the birthday of 5 students from a group of n students is approximately 0.1595 what is the value of n?

$\binom{n}{5}\left(\frac{1}{12}\right)^5\left(\frac{11}{12}\right)^{n-5} \approx 0.1595$ ✓

From CALC guess and check

$n = 47$ ✓✓

3) [2,3,3 = 8 marks]

A certain binomial experiment has 15 trials. The probability of success in any trial is 0.315. The random variable X is the number of successes. Calculate the probability of

a) X=11

$p \approx 0.0009106$ ✓✓

b) X is at most 9

$P(x \leq 9) \approx 0.9946$ ✓✓✓

c) X is between 5 and 8 inclusive.

$P(x \leq 8) - P(x \leq 4)$

$= 0.9787892 - 0.4644469$

$= 0.5143423$ ✓✓✓