WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2016
TEST 2: Functions

Name: ________________________

Friday 1st April

Time: 50 minutes **Mark /45 = %**

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet in **both** sections of the test.
- You are permitted one A4 page (one side) of notes in the calculator assumed section.

Calculator free section **Suggested time: 30 minutes** **/28**

1. [11 marks]

Two functions *f* and *g* are defined by $f(x) = x^2 - 1$ and $g(x) = \sqrt{9 - x}$

a) Evaluate $g \circ f(\sqrt{6})$

$$= g(5)$$

$$= \sqrt{9 - 5} = \sqrt{4} = 2$$

[2]

b) What is the range of $y = f(x)$ when $x \in \mathbf{R}$?

$$y \in \mathbf{R} \,;\, y \geq -1$$

[1]

c) What is the natural domain of $y = g(x)$

$$9 - x \geq 0$$

$$\Rightarrow x \leq 9; \quad x \in \mathbf{R}$$

[2]

d) Predict the domain and range for $y = g^{-1}(x)$

Domain: $x \geq 0$

Range: $y \leq 9$

[2]

e) Determine $y = f \circ g(x)$, including all domain restrictions

$$f \circ g(x) = 9 - x - 1 = 8 - x$$

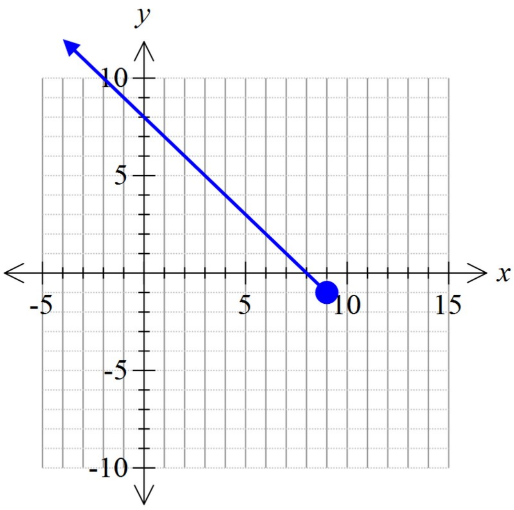
for $x \leq 9$

[2]

Turn over for the last part of question 1

f) Sketch $y = f \circ g(x)$ and clearly indicate the range of this composite function. [2]

Range: $y \geq -1;\ y \in \mathbb{R}$



2. [17 marks]

Consider the rational function $h(x) = \dfrac{2x^2 - 4x}{x^2 + x - 6}$

a) Identify and classify all points of discontinuity [4]

$$h(x) = \frac{2x^2 - 4x}{x^2 + x - 6} = \frac{2x(x-2)}{(x-2)(x+3)} = \frac{2x}{x+3}$$ provided $x \neq 2$

Vertical/infinite discontinuity at $x = -3$

(Singular) point discontinuity at $\left(2, \dfrac{4}{5}\right)$

b) List the asymptotes (horizontal and vertical) [2]

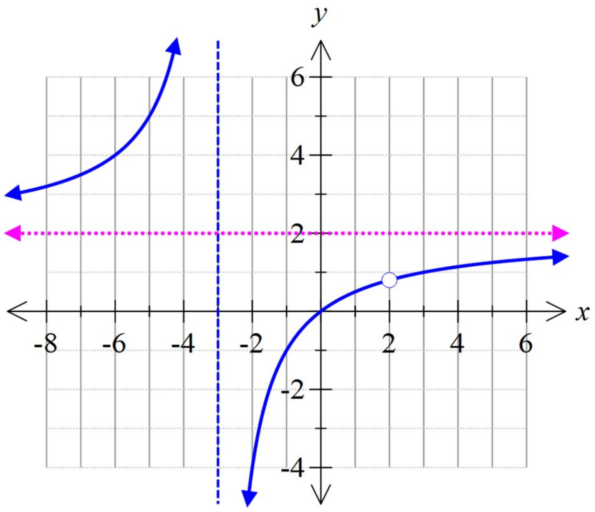
$x = -3$ (vertical)

$y = 2$ (horizontal, from $x \to \pm\infty$)

c) Determine all intercepts [2]

$(0,0)$ is the only intercept

d) Sketch $y = h(x)$ [3]



e) Does $y = h(x)$ possess an inverse function $y = h^{-1}(x)$? How do you know? [2]

Yes; it is a 1 to 1 function over the domain $x \in \mathbf{R}$; $x \neq -3, x \neq 2$

(passes horizontal line test)

f) Show algebraically that $h^{-1}(x) = \dfrac{3x}{2 - x}$ and identify appropriate restrictions on the domain and range. Use a simplified expression for $y = h(x)$ in your calculations. [4]

$y = \dfrac{2x}{x+3}$ has an inverse defined by $x = \dfrac{2y}{y+3}$

$$\Rightarrow xy + 3x = 2y$$

$$2y - xy = 3x$$

$$y(2 - x) = 3x$$

$$\Rightarrow y = \frac{3x}{2 - x}$$

Domain restrictions: $x \neq 2, x \neq \frac{4}{5}$

Range restrictions: $y \neq -3, y \neq 2$

Calculator assumed section **Suggested time: 20 minutes** **/17**

Name: ________________________

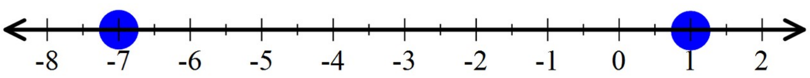
3. [5 marks]

Mark solutions to these equations on the number lines provided.

In (b), clearly explain clearly how to use distance considerations in determining the solution.

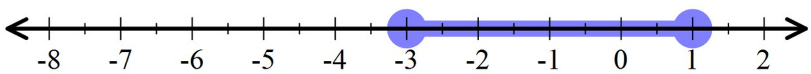
a) $|x+3|=4$

[1]



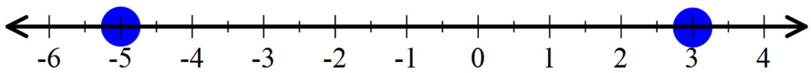
b) $|x+3|+|x-1|=4$

Sum of the distance of *x* from 3 plus distance from 1 should be 4



[3]

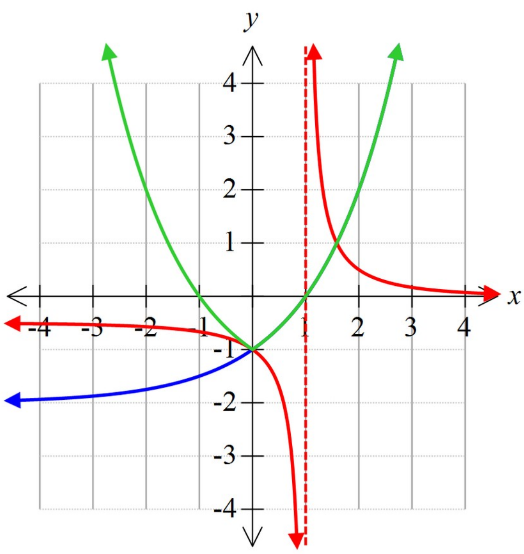
c) $|x+3|+|x-1|=8$



[1]

4. [5 marks]

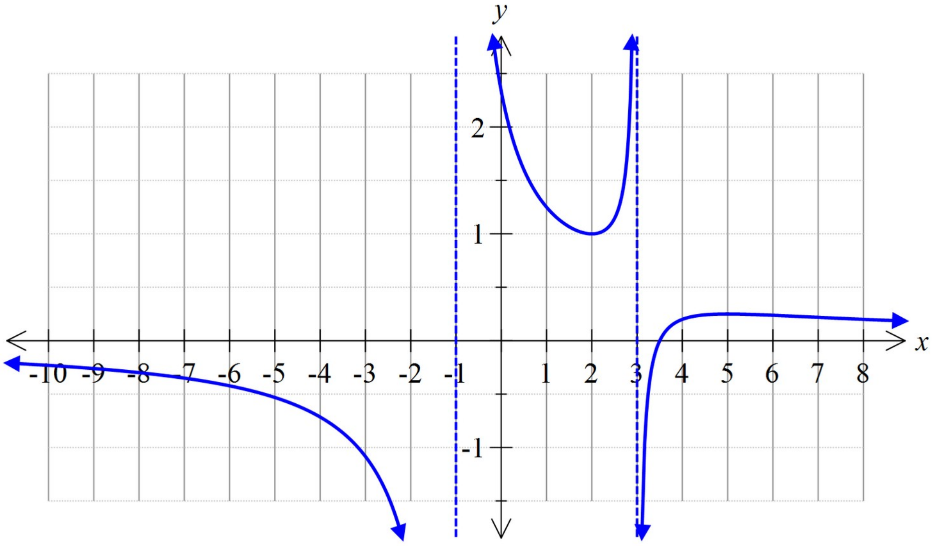
The graph of $y=f(x)$ is shown. Add the graphs of $y=\frac{1}{f(x)}$ and $y=f(|x|)$.



5. [7 marks]

This graph represents a function of the form

$$y = f(x) = \frac{ax + b}{x^2 + cx + d}$$



The vertical asymptotes are as shown, the *x*-intercept is (3.5, 0) and one turning point is at $(2,1)$.

(a) Determine the values of the constants *a*, *b*, *c* and *d*.

Vertical asymptotes $\Rightarrow x^2 + cx + d = (x+1)(x-3) = x^2 - 2x - 3$

$$c = -2,\ d = -3$$

$$(3.5, 0) \Rightarrow 3.5a + b = 0$$

$$(2,1) \Rightarrow \frac{2a + b}{4 - 4 - 3} = 1 \quad \Rightarrow 2a + b = -3$$

Solve simultaneously: $a = 2,\ b = -7$

[5]

(b) What is the range of $y = f(x)$?

Other turning point is $\left(5, \frac{1}{4}\right)$

Range is $\left\{y \le \frac{1}{4}\right\} \cup \{y \ge 1\}$ or $\mathbf{R} \mid 0.25 < x < 1$

[2]