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Test One

**Semester One 2018
UNIT 1 METHODS**

Calculator Assumed 40 minutes /45 marks
Scientific Calculator, ClassPad, Formula Sheet and
One page one side of A4 notes is permitted

Name: Solutions

Place a tick in the box next to your Mathematics teachers name:

- | | |
|--------------------------|------------|
| <input type="checkbox"/> | Mr Strain |
| <input type="checkbox"/> | Ms Sindel |
| <input type="checkbox"/> | Ms Rimando |
| <input type="checkbox"/> | Mr Gannon |
| <input type="checkbox"/> | Mr Young |
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| <input type="checkbox"/> | Ms Enslly |

Question 1

(2, 2, 2 = 6 marks)

Consider the following points, A (x_1, y_1) and B (x_2, y_2).

- i) Determine the exact distance from point A to B.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(12 - 4)^2 + (20 - 9)^2} \checkmark \\ &= \sqrt{9 + 256} \\ &= \sqrt{265} \approx 16.28 \checkmark \end{aligned}$$

- ii) Determine the midpoint between points A and B.

$$\begin{aligned} M &= \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) \\ &= \left(\frac{4 + 20}{2}, \frac{9 + 20}{2} \right) \checkmark \\ &= (12, 10\frac{1}{2}) \checkmark \end{aligned}$$

- iii) If point B was the midpoint of points A and point C. Determine the coordinates of point C.

$$A(4, 9) \quad B(20, 12) \quad C(x, y)$$

$$(20, 12) = \left(\frac{4 + x}{2}, \frac{9 + y}{2} \right) \checkmark$$

$$\begin{aligned} \text{So} \quad 20 &= \frac{4 + x}{2} & 12 &= \frac{9 + y}{2} \\ 40 &= 4 + x & 24 &= 9 + y \\ x &= 36 & y &= 15 \end{aligned}$$

$$\therefore \text{Point C is } (36, 15) \checkmark$$

Question 2

(2, 2, 1, 3 = 8 marks)

Determine the equation of a line that passes through the point $(-10, 3)$ and :

i) passes through the point $(5, -7)$.

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= m \\ \frac{-7 - 3}{5 - (-10)} &= m \\ \frac{-10}{15} &= m \\ m &= -\frac{2}{3} \end{aligned}$$

$$y = mx + c$$

$$3 = -\frac{2}{3}(-10) + c$$

$$3 = \frac{20}{3} + c$$

$$c = 3 - \frac{20}{3}$$

$$c = -\frac{10}{3}$$

$$y = -\frac{2}{3}x - \frac{10}{3}$$

ii) is parallel to the line $y = -5x + 11$.

$$m = -5$$

$$y = mx + c$$

$$3 = -5(-10) + c$$

$$3 = 50 + c$$

$$c = -47$$

$$y = -5x - 47$$

iii) is parallel to the y axis.

$$x = -10$$

iv) is perpendicular to the line $x - 4y = 9$.

$$-4y = -x + 9$$

$$y = \frac{1}{4}x - \frac{9}{4}$$

$$m_1 = \frac{1}{4}$$

$$m_1 \cdot m_2 = -1$$

$$\frac{1}{4} \cdot m_2 = -1$$

$$m_2 = -4$$

$$y = mx + c$$

$$3 = -4(-10) + c$$

$$3 = 40 + c$$

$$c = -37$$

$$y = -4x - 37$$

Question 3

(2, 3, 3 = 8 marks)

Consider the line $5x + my = 21$, where m is a constant.

- i) In terms of m , determine the y intercept.

$$my = -5x + 21 \checkmark$$

$$y = \frac{-5x}{m} + \frac{21}{m} \checkmark$$

$$\therefore y \text{ intercept is } (0, \frac{21}{m}) \checkmark$$

- ii) In terms of m , determine the midpoint of the x and y intercepts.

$$\text{At } y=0 \quad 5x = 21 \checkmark$$

$$x = \frac{21}{5} \checkmark$$

$$\therefore x \text{ intercept is } (\frac{21}{5}, 0) \checkmark$$

$$\text{Midpoint} = (\frac{0 + \frac{21}{5}}{2}, \frac{\frac{21}{m} + 0}{2}) \checkmark$$

$$= (\frac{21}{10}, \frac{21}{2m}) \checkmark$$

- iii) Determine the value of m so that the line will never cross $y = 7x$.

$$m = 7 \checkmark$$

$$5x + my = 21$$

$$my = -5x + 21$$

$$y = \frac{-5x}{m} + \frac{21}{m}$$

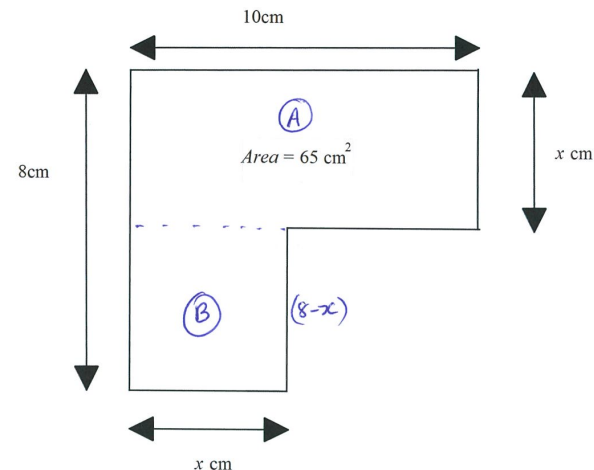
$$\frac{1}{5} = 7 \checkmark$$

$$m = \frac{-5}{7} \checkmark$$

Question 8

(4 marks)

Determine the value of x for the shape below.



$$\text{Area A} = 10x$$

$$\text{Area B} = (8-x)x$$

$$= -x^2 + 8x \checkmark$$

Total Area

$$65 = -x^2 + 8x + 10x$$

$$0 = -x^2 + 18x - 65 \checkmark$$

$$= x^2 - 18x + 65$$

$$= (x-5)(x-13) \checkmark$$

$$\therefore x = 5 \text{ or } 13$$

As $13 > 10$, 13 is not a solution

$$\therefore x = 5 \text{ cm} \checkmark$$

End of test

Question 7

(2, 2 = 4 marks)

For each of the following write down the equation of a parabola that satisfies the following:

(No need to simplify)

- i) A quadratic with intercepts (4,0) and (-7,0) with a y intercept of (0, -56).

$$y = a(x - 4)(x + 7)$$

$$\text{At } (0, -56) \quad -56 = a(-4)(7)$$

$$a = -28a$$

$$y = 2(x - 4)(x + 7)$$

- ii) A quadratic with a maximum turning point (7,1) and an x intercept (10,0).

$$\text{TP } (7,1)$$

$$y = a(x - 7)^2 + 1$$

$$0 = a(3)^2 + 1$$

$$a = -\frac{1}{9}$$

$$a = -\frac{1}{9}$$

$$y = -\frac{1}{9}(x - 7)^2 + 1$$

Question 4

(1, 1, 2 = 4 marks)

Jessica needs to hire a car for a number of days. The hire car company has two options from which she can choose.

Budget: \$15 per day plus \$0.25 per km travelled

Deluxe: \$42 per day for unlimited travel

- i) Jessica will hire the car for n days and drive a total of x km.

- a) Find an expression for the cost, $\$C$, in terms of n for the Deluxe option.

$$C = 42n$$

- b) Find an expression for the cost, $\$C$, in terms of n and x , for the Budget option.

$$C = 15n + 0.25x$$

- ii) If Jessica plans to drive a total of 600 km, find the maximum number of days for which she can hire the car so that it is cheaper for her to take the Deluxe option.

$$\text{If } x = 600$$

$$42n < 15n + 0.25(600)$$

$$27n < 150$$

$$n < 5.55$$

\therefore 5 days it is cheaper to take the Deluxe option

Question 5

(2, 2, 2 = 6 marks)

Factorise the following expressions:

i) $4x^2y - 12xy^4$

$4xy(x-3y^3)$

ii) $x^3 - 3x^2 - 9x + 27$

$= x^2(x-3) - 9(x-3)$

$= (x^2-9)(x-3)$

$= (x-3)(x+3)(x-3) = (x-3)^2(x+3)$

iii) $18x^2 + 33x - 30$

$= 3(3x-2)(2x+5)$

Question 6

(5 marks)

On the axes below, sketch the parabola $y = -2(x+3)^2 + 6$ showing all major features such as line of symmetry, turning points, x intercepts (if any).

