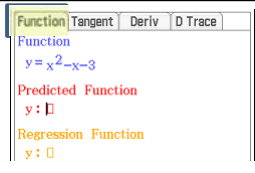



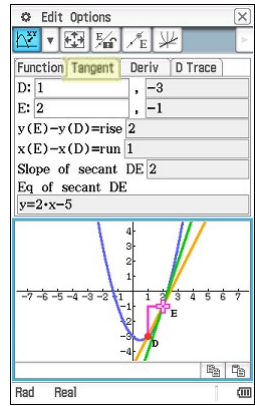


11 Investigation 2 Methods: Out 14 June In class: 21 June

For the in class, you are allowed to bring this take home in with you plus any other notes. You will need your classpad for the in class as well.

Aim: Explore the gradient of a chord or secant and generalise. Visually see the gradient of the tangent as the limiting value as the two points get closer together.

1. Explore the gradient of secant. Go to menu and open **InteractiveDiffCalc**

<p>Define function</p> <ul style="list-style-type: none"> • Open InteractiveDiffCalc • Tap Function tab • Enter $y = x^2 - x - 3$ 	
<p>Gather data on the secant (line DE)</p> <ul style="list-style-type: none"> • Tap Tangent tab • Controls • Left right arrows • Press  to fix value of x_D or x_E • Press  to draw/hide tangent • Press  to switch the fixed point 	

- a) Complete the table of values

D	E	Rise DE	Run DE	Gradient DE	Equation DE
(1, -3)	(2, -1)	2	1	2	$y = 2x - 5$
(1.5,)	(2, -1)				
(2.5,)	(2, -1)				
(1.9,)	(2, -1)				
(1.99,)	(2, -1)				

- b) As D moves closer to point E, what happens to the:

- (i) two lines

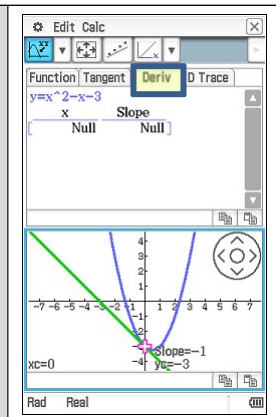
- (ii) the gradient of the secant and the gradient of the tangent?

- c) For the points $(x + h, f(x + h))$ and $(x, f(x))$ write an expression for the gradient of the secant and then simplify. (Hint: use M)

2. Explore the gradient of the tangent to a curve

Explore the tangent using Deriv tab

- Tap the **Deriv** tab
- Press the arrow keys to move the cursor
- Press E or the centre of the on-screen wheel to plot a point
Or press a number key to type an x -value



- a) What does the y -value of the plotted points represent?
- b) Complete the table.

x	Gradient of tangent
0	
1	
2	
3.3	
-1.2	

- c) Predict a function that describes the relationship between the gradient and x -value.
- d) Check your prediction.
Tap **to** enter your prediction for the gradient function. ClassPad will then plot your prediction. (Note: you must have plotted the gradient in at least 4 positions)

3. Explore the D Trace tab

- Tap **D Trace**
- Move the left and right arrow keys.
- Tap y_1, y_2, y_3 to turn on/off different parts of the display.

x	y1	y2	y3
0	-3	-3	-2
0.5	-3.3	-0	2
1	-3	1	2
1.5	-2.3	2	2

Refer to the graph in the screenshot above. What does the graph of

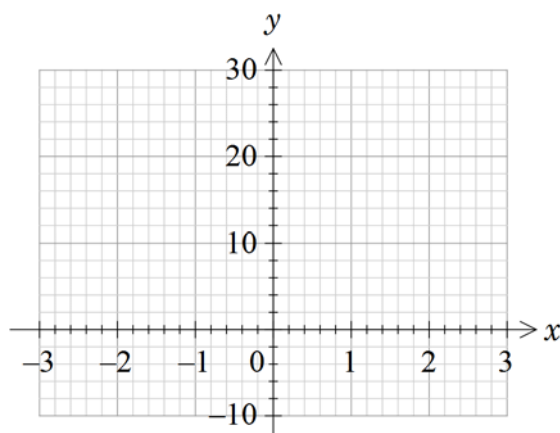
- y_2 (the red line) represent?
- y_3 (the pink line) represent?

4. For each of the following functions:

- sketch the graph, the tangent at the specified point and the gradient function on the grid; and

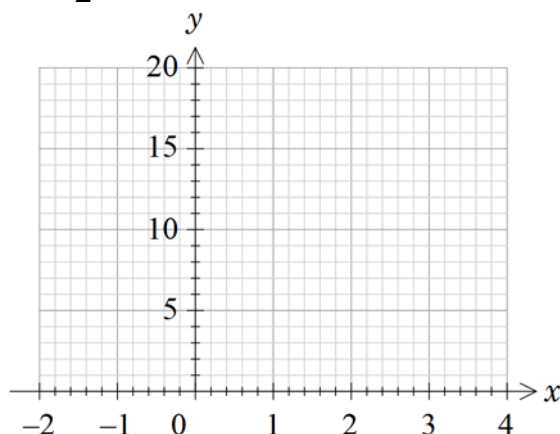
- state the equation of the gradient function, y' . that is $\left(\frac{dy}{dx}\right)$

a) $y = x^4$ with tangent at $x = 2$



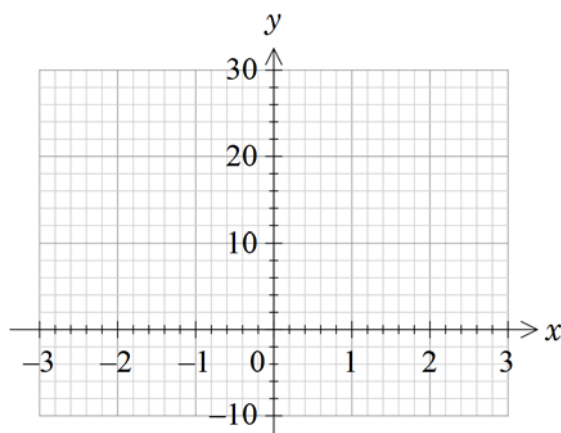
Prediction for y' :

c) $y = 2^x$ with tangent at $x = 3$



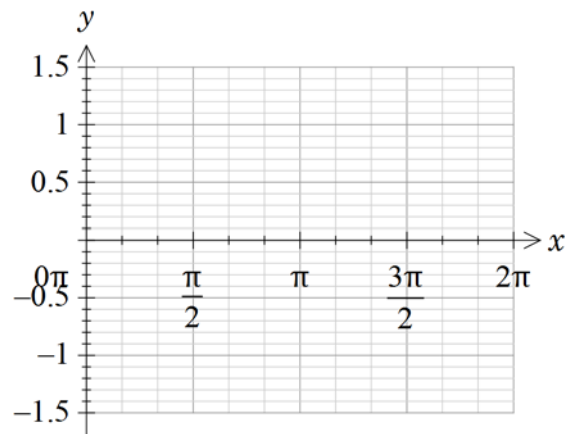
Prediction for y' :

b) $y = x^4 + 10$ with tangent at $x = 2$



d) Prediction for y' :

e) $y = \sin x$ with tangent at $x = \frac{\pi}{3}$



Prediction for y' :

5. The screenshot reproduces the steps in this activity done algebraically.

1. Define the function
2. Write an expression for the gradient of a secant between $(2, f(2))$ and $(2 + h, f(2 + h))$
and simplify
As $h \rightarrow 0$ the secant approaches the tangent at $x = 2$. (In this case the gradient of the tangent will be 3)
3. Write an expression for the gradient of a secant between $(x, f(x))$ and $(x + h, f(x + h))$
and simplify
As $h \rightarrow 0$ the secant approaches the tangent at any point

The screenshot shows a CASIO calculator interface with the following steps:

- Define $f(x) = x^2 - x - 3$
- Calculate the gradient of a secant between $(2, f(2))$ and $(2 + h, f(2 + h))$:

$$\frac{f(2+h) - f(2)}{(2+h) - 2} = \frac{(h+2)^2 - h - 4}{h}$$
- Simplify the expression: $h + 3$
- Calculate the gradient of a secant between $(x, f(x))$ and $(x + h, f(x + h))$:

$$\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{(x+h)^2 - x^2 - h}{h}$$
- Simplify the expression: $2 \cdot x + h - 1$
- Find the limit as $h \rightarrow 0$:

$$\lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{(x+h) - x} \right) = 2 \cdot x - 1$$

- a) Reproduce the screenshot as shown.
- b) Edit the function definition to complete the table.

Function	Gradient of tangent at $x = 2$	Gradient of secant between $(x, f(x))$ and $(x + h, f(x + h))$	Gradient function
$x^2 - x - 3$	3	$2x + h - 1$	$2x - 1$
x^3			
x^4			
$5x^3$			
\sqrt{x} or $x^{\frac{1}{2}}$			
$\sin x$			
$\cos x$			
2^x			

6. EXTENSION




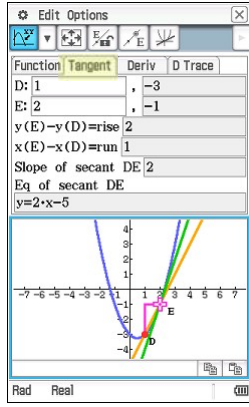


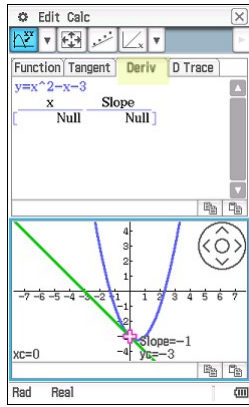

Predict the gradient functions for

- a) $y = x^n$
- b) $y = ax^n$
- c) $y = x^n + x^m$

Learning notes

The strength in the InteractiveDiffcalc app is being able to follow the first principles approach visually. By dynamically moving the points closer together students can see the secant getting closer to the tangent.

It also supports the development of the idea of the gradient function and taking time to understand the plotting of the gradient at specific points is a neat way of encouraging students to develop this concept.

<p>Tangent tab</p> <ul style="list-style-type: none"> Controls Left right arrows Press  to fix value of x_D or x_E Press  to draw/hide tangent Press  to switch the fixed point 	
<p>Deriv tab</p> <ul style="list-style-type: none"> Press the arrow keys to move the cursor Press E or the centre of the on-screen wheel to plot a point Or press a number key to type an x-value Press  to calculate a regression see pull down menu for types of regression Press  to enter prediction for the gradient function 	
<p>D Trace tab</p> <ul style="list-style-type: none"> Move the left and right arrow keys. Tap  to cycle through different displays 	