
Practice 2 Semester Two Examination, 2016

Question/Answer Booklet

SPECIALIST UNITS 3 AND 4

**Section One:
Calculator-free**

If required by your examination administrator, please place
your student identification label in this box

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	101	65
Total				153	100

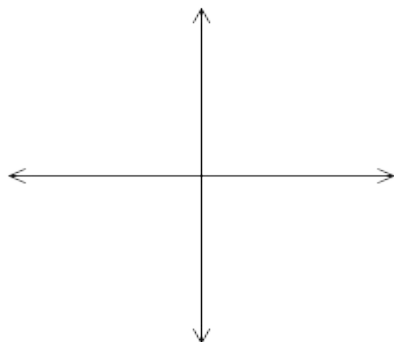
Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

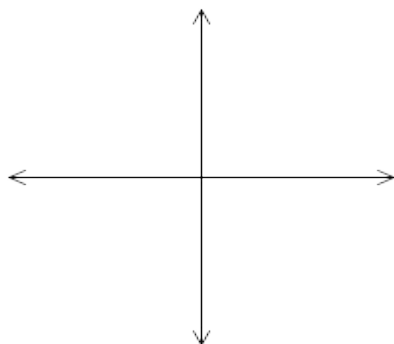
1. [8 marks]

Sketch the locus of the point z defined by:

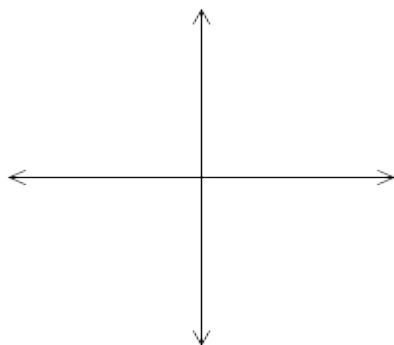
(a) $\operatorname{Im}(z) = |\operatorname{Re}(z)|$ [2]



(b) $\operatorname{Im}(z) = |z|$ [3]



(c) $z + \bar{z} = 1$ [3]



2. [7 marks]

Solve the following equations, giving your answer in exact Cartesian form.

(a) $z^2 + 2\pi z + 5\pi^2 = 0$ [3]

(b) $z^3 = -e^3$ [4]

3. [8 marks]

Consider $f(x) = \frac{1}{1 + \ln(x)}$.

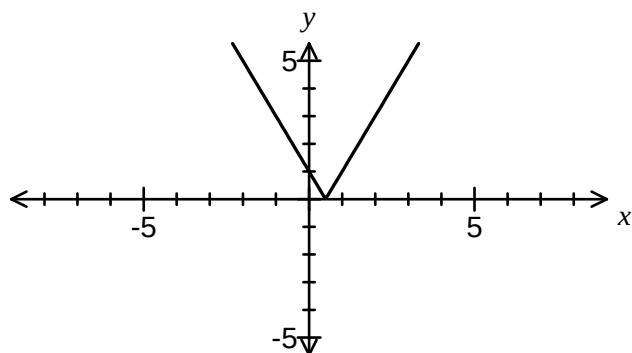
(a) State the natural domain for f . [2]

(b) Find the range corresponding to the natural domain for f . [2]

(c) Determine with reasons if f^{-1} exists. Give the rule for f^{-1} if it exists. [4]

4. [10 marks]

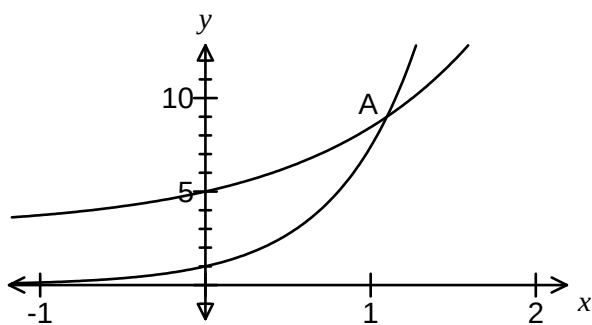
The graph of $f(x) = |2x - 1|$ is shown below.



(a) Add the function $g(x) = \frac{|x + 2|}{2}$ to the graph. (1 mark)

(b) Solve $f(x) - g(x) \geq 0$. (3 marks)

The graphs of $y = e^{2x}$ and $y = 2e^x + 3$ intersect at the point A, shown on the graph below.



(c) Show that the x -coordinate of A is $\log_e 3$. (3 marks)

- (d) Determine the exact area, in simplest form, of the region bounded by the two curves and the y-axis. (3 marks)

5.

(7 marks)

Two complex numbers are given by $z = 2\text{cis}\frac{\pi}{3}$ and $w = \sqrt{3} - i$.

- (a) Determine $\arg \frac{z}{w}$. (2 marks)

- (b) Evaluate $\left| w \times \overline{w} \times z \right|$. (3 marks)

- (c) Find the complex number u given that $\frac{z \times u}{2} = \text{cis}\left(\frac{3\pi}{4}\right)$. (2 marks)

6. [8 marks]

(a) Determine $\int x^2 [1 - \sqrt{1 + x^3}] dx$ [4]

(b) Determine $\int \frac{1}{e^x (1 - e^{-x})} dx$ [2]

Show clearly that

(c) $\int e^{ix} dx = -i e^{ix} + c$ [2]

7. [4 marks]

(a) Determine $\frac{d}{dx} [\ln (\cos^2 2x)]$ [2]

Hence,

(b) determine $\int \tan 2x \, dx$ [2]

Practice 2 Semester Two Examination, 2016

Question/Answer Booklet

SPECIALIST UNITS 3 AND 4

Section Two:
Calculator-assumed

If required by your examination administrator, please place
your student identification label in this box

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes

Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

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Structure of this paper

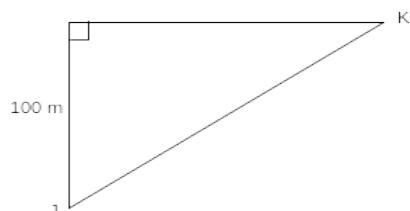
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8. [10 marks]

A kite K is constantly at a height of 100 m. The string is being let out at J to keep the kite at that height. At time $t = 0$, the distance between K and the kite flyer at J is 120 m. K is being blown in a horizontal direction with a uniform speed of 10 ms^{-1} .



Determine the rate, five seconds later, at which:

(a) the angle of elevation of the kite is changing.

[6]

(b) the distance between K and J is changing.

[4]

9 [13 marks]

(a) (i) Express $(1 + i) \times 2 \operatorname{cis} \left(\frac{\pi}{3} \right)$ in exact cis form. [2]

(ii) Express $(1 + i) \times 2 \operatorname{cis} \left(\frac{\pi}{3} \right)$ in exact Cartesian form. [2]

(iii) Hence, or otherwise, find $\cos \left(\frac{7\pi}{12} \right)$ in exact form. [2]

(b) The polynomial $f(z) = az^4 + bz^3 + cz^2 + dz + e$, where a, b, c, d and e are real, has roots $(2 - i)$ and $2i$.

(i) Give another two complex roots of $f(z)$. [2]

(ii) Find a, b, c, d and e if $f(1) = 20$. [5]

10. [7 marks]

Consider the curve

$$y = \sin 2x - \pi \cos^2 x$$

(a) Determine the gradient of the curve at the point of intersection with the y - axis. [5]

(b) Determine the equation of the line which is perpendicular to the curve above at the point of intersection of the curve with the y - axis. [2]

11. [7 marks]

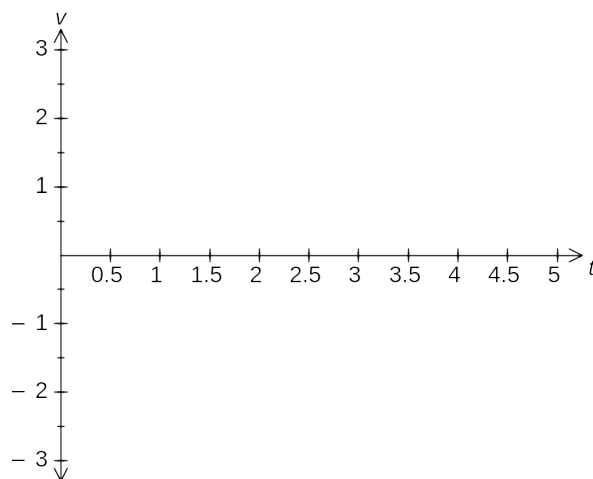
A complex number, z , is defined by $z = \frac{2(a - i)}{1 - i} + \frac{3 - i}{1 + i}$, where a is real.

Find the value of a for which this point lies on the curve with equation $|z| = 5$. [7]

12. [12 marks]

A particle P starts off from a fixed point O with velocity $v = -5e^{-t} + 2 \text{ ms}^{-1}$.

- (a) Sketch the graph of v against t for $0 \leq t \leq 5$ seconds. Indicate clearly the main features of this curve. [3]



- (b) Describe the motion of P for large values of t . [1]

- (c) When does P change its initial direction of motion? [1]

(d) Find when P returns to O. Show your reasoning.

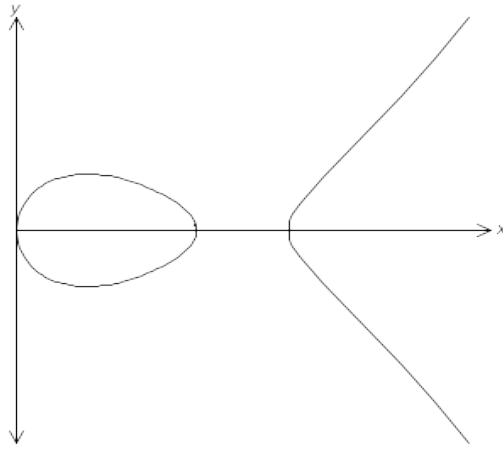
[4]

(e) Find the distance travelled in the first 5 seconds.

[3]

13. [8 marks]

The graph of $y^2 = x^3 - 5x^2 + 6x$ is shown below.

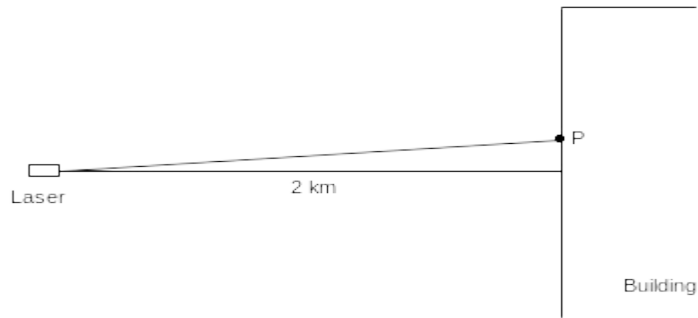


- (a) Use an algebraic method to find the roots of the curve. [4]
- (b) The region trapped by the curve $y = \sqrt{x^3 - 5x^2 + 6x}$, the x -axis and the lines $x = 0$ and $x = 4$, is rotated 2π radians about the x -axis. Use an analytical method to find the exact volume of the solid generated. [4]

14. [6 marks]

A laser beam is to be targeted at point P on the side of a tall building. The horizontal distance between the laser and P is 2 km. The angle of elevation of P from the laser is 2° . Use the method involving small increments to find how far the laser is off the target if the angle of elevation is mistakenly set at 2.01° .

[6]



15. [5 marks]

A line and a plane are given by $\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \lambda(-2\mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} \cdot (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) = 33$.

(a) Find the position vector of the point of intersection of the line and plane. (2 marks)

(b) Find the acute angle between the line and plane. (3 marks)

16. [10 marks]

A body, A, has an initial position of $\begin{bmatrix} -7 \\ 21 \\ 6 \end{bmatrix}$ metres and is moving with a constant velocity of $\begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix}$ metres per second.

(a) A second body, B, is moving with constant velocity of $\begin{bmatrix} 8 \\ 1 \\ -1 \end{bmatrix}$ metres per second and collides with body A after six seconds.

Determine the initial distance apart of body A and body B. (4 marks)

- (b) A third body, C, is initially located at $\begin{bmatrix} 5 \\ -10 \\ 1 \end{bmatrix}$ metres and is also moving with a constant velocity $\begin{bmatrix} 2 \\ y \\ -3 \end{bmatrix}$. After five seconds, the distance between bodies A and C is a minimum.

Find the value of y for which the speed of C is also a minimum.

(6 marks)

17. [8 marks]

A particle moves in rectilinear motion such that its displacement, in centimetres, at any time t is given as:

$$x = 2\cos\frac{\pi t}{2} - 2\cos\left(1 + \frac{\pi t}{2}\right)$$

(a) Show clearly that the particle is undergoing Simple Harmonic Motion. [2]

Hence, or otherwise determine:

(b) when the particle is at rest for the first time after it started, correct to two decimal places. [2]

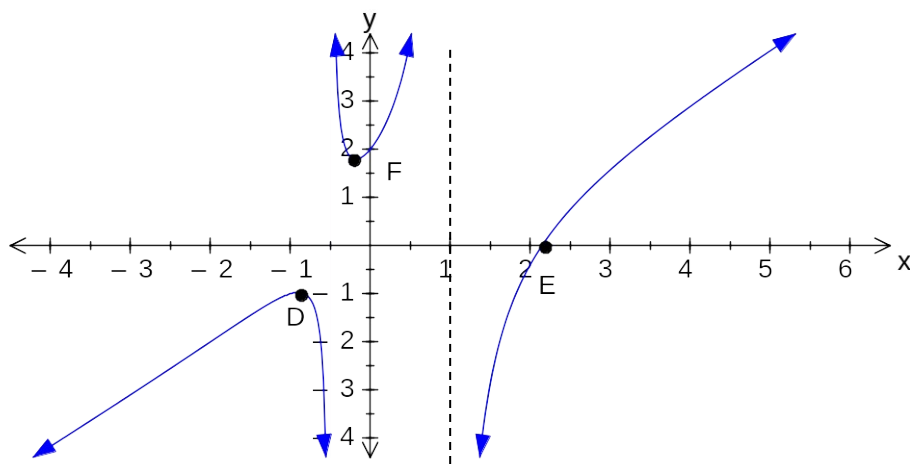
(c) the maximum velocity of the particle, correct to two decimal places. [2]

(d) the exact acceleration of the particle when it is 2 centimetres from the origin for the first time. [2]

18. [6 marks]

The following graph is of the function;

$$f(x) = \frac{2x^2}{2x + A} - \frac{C}{x - B}$$



Determine;

(a) the values of A, B and C.

[3]

Hence,

(b) find the coordinates of D, F (extrema) and E (root), correct to two decimal places. [3]

19. [8 marks]

The temperature in a restaurant cool room is set to 4°C. One day, the refrigerator unit was turned back on after the temperature in the cool room had risen to 27°C due to cleaning and maintenance work. After 15 minutes, the temperature in the cool room had dropped to 11°C, with the temperature, T , falling according to the model

$$\frac{dT}{dt} = k(T - 4)$$

where t is the time in minutes since the refrigerator unit was turned back on.

(a) Find the value of k and express T as a function of t . (5 marks)

(b) If the temperature continues to fall in this way, how long before the temperature in the cool room registers 4°C, to the nearest degree? (3 marks)

