PERTH MODERN SCHOOL
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Independent Public School

Course Methods test 2 Year 12

Student name: _______________ Teacher name: _______________

Task type: **Response**

Time allowed for this task: **__40______ mins**

Number of questions: **___8_____**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of
A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **__41__ marks**

Task weighting: **_10__%**

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3 & 3 = 6 marks) (3.2.9)

Determine y in terms of x for the following. Show all working.

a) $\frac{dy}{dx} = 15x^2 + 14x$ and $y = 13$ when $x = 1$.

Solution
$\frac{dy}{dx} = 15x^2 + 14x$ $y = 5x^3 + 7x^2 + C$ $13 = 5 + 7 + C$ $C = 1$ $y = 5x^3 + 7x^2 + 1$
Specific behaviours
✓ anti-diffs terms ✓ introduces an unknown constant and subs to solve ✓ states value of constant

b) $\frac{dy}{dx} = 10(2x+1)^4$ and $y = 10$ when $x = -1$.

Solution
$\frac{dy}{dx} = 10(2x+1)^4$ $y = \frac{10(2x+1)^5}{2(5)} + C = (2x+1)^5 + C$ $10 = (-1)^5 + C$ $C = 11$ $y = (2x+1)^5 + 11$
Specific behaviours
✓ anti-diffs terms ✓ introduces an unknown constant and subs to solve ✓ states value of constant

Q2 (3 & 2 = 5 marks) (3.2.22, 3.2.5)

A car travels in a straight line from the origin, initially at rest, with constant acceleration $4\cos(3t)\,m/s^2$ with t time in seconds.

a) Determine the distance from the origin at $t=\frac{\pi}{3}$ seconds?

Solution
$a=4\cos(3t)$ $v=\frac{4}{3}\sin(3t)+c$ $t=0, v=0$ $0=0+c$ $c=0$ $x=\frac{-4}{9}\cos(3t)+k$ $t=0, x=0$ $0=\frac{-4}{9}+k$ $k=\frac{4}{9}$ $x=\frac{-4}{9}\cos(3t)+\frac{4}{9}$ $t=\frac{\pi}{3}$ $x=\frac{4}{9}+\frac{4}{9}=\frac{8}{9}m$
Specific behaviours
✓ integrates to find v and shows solving for constant with subs ✓ states the correct rule for x ✓ states exact value for x at required time, no need for units

b) What is the velocity of the car at $t=\frac{\pi}{3}$ seconds?

Solution

$$v = \frac{4}{3}\sin(3t)$$
$$t = \frac{\pi}{3}$$
$$v = 0$$

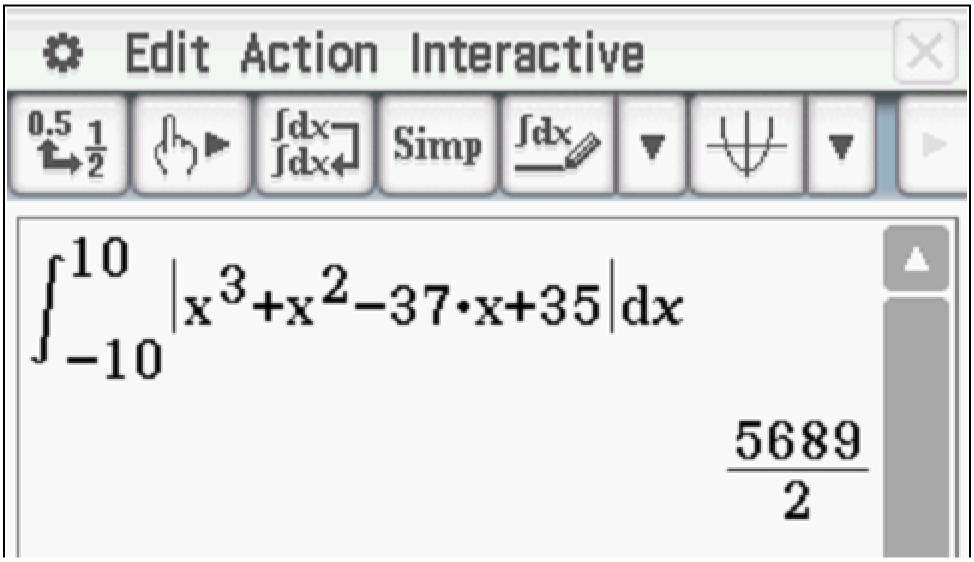
Specific behaviours

✓ subs into v
✓ states velocity, no need for units

Q3 (2 marks) (3.2.19)

Determine the exact area between $y = x^3 + x^2 - 37x + 35$ and the x axis from $x = -10$ to $x = 10$.

Solution



Specific behaviours

✓ writes a correct integral for the area
✓ states exact value

Q4 (2, 2 & 3 = 7 marks) (3.2.18)

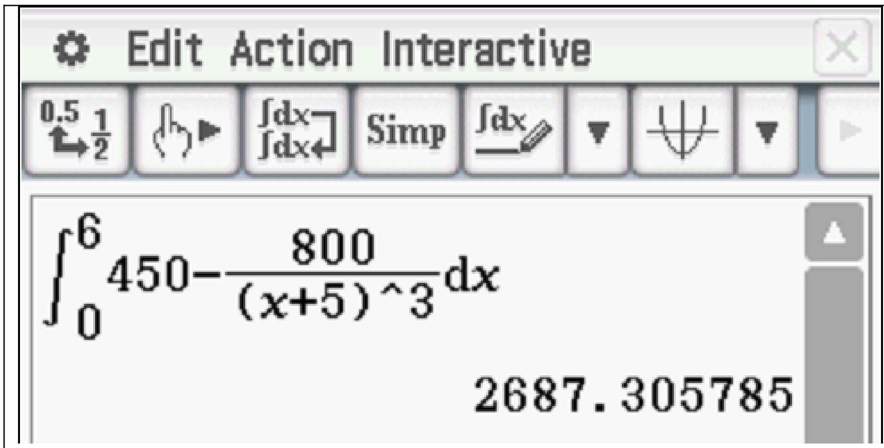
A factory produces electric vehicles. The total number, E, that the company has produced t months after production commenced is such that:

$$\frac{dP}{dt} = 450 - \frac{800}{(t+5)^3}$$

Determine the number produced in

a) The first 6 months

Solution



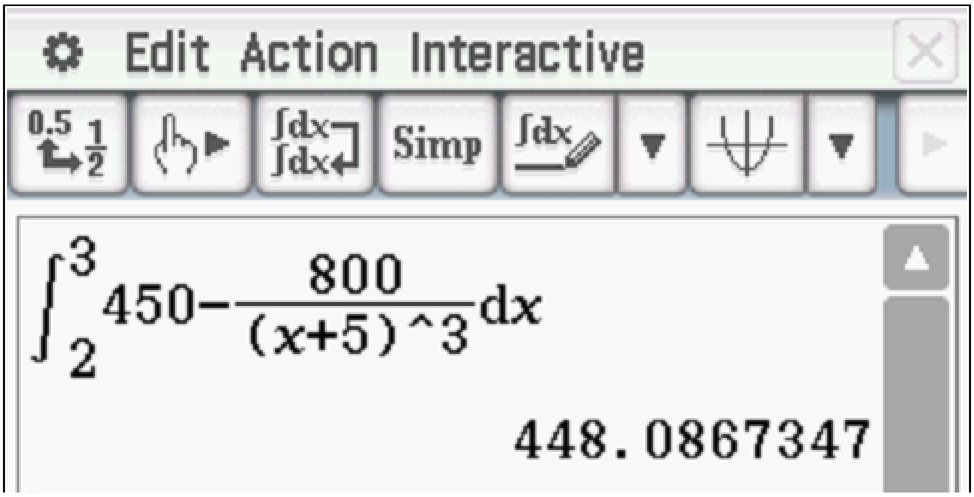
Number is 2687 vehicles

Specific behaviours

✓ writes a correct integral with limits or ant-diff with a constant
✓ states change, accept decimal

b) The third month

Solution



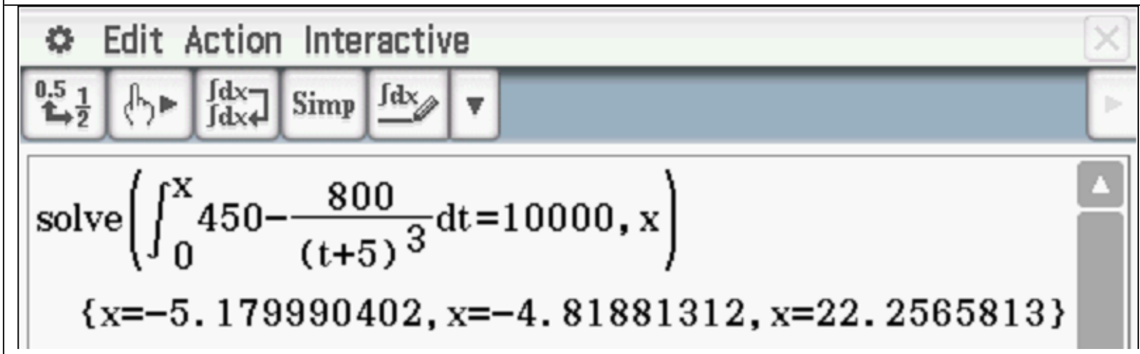
Specific behaviours

✓ writes a correct integral with limits or ant-diff with a constant
✓ states change, accept decimal

Determine the minimum number of months required to produce:

c) 10000 vehicles.

Solution



Need at least 22.27 OR 23 months

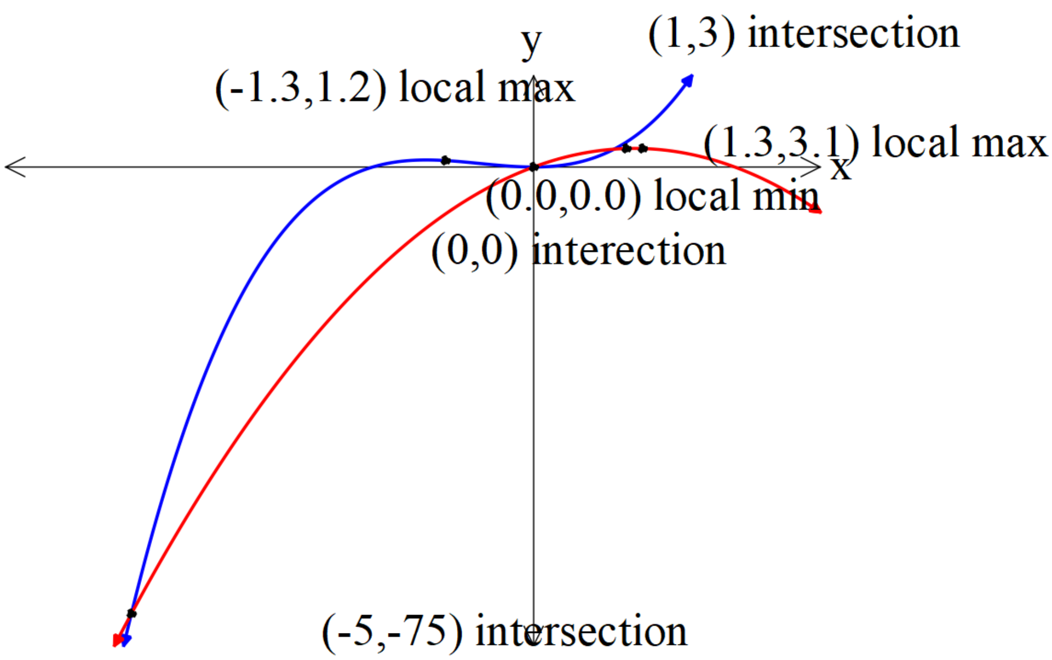
Specific behaviours

✓ sets up an integral with unknown upper limit
✓ solves for a decimal number of months
✓ states number of months

Q5 (5 & 3 = 8 marks) (3.2.20)

a) On the axes below, sketch the following graphs: $y=x^3+2x^2$ and $y=5x-2x^2$. Indicate on your sketch coordinates (one decimal place) of any stationary points and label their nature and of any points where the graphs intersect each other.

Solution



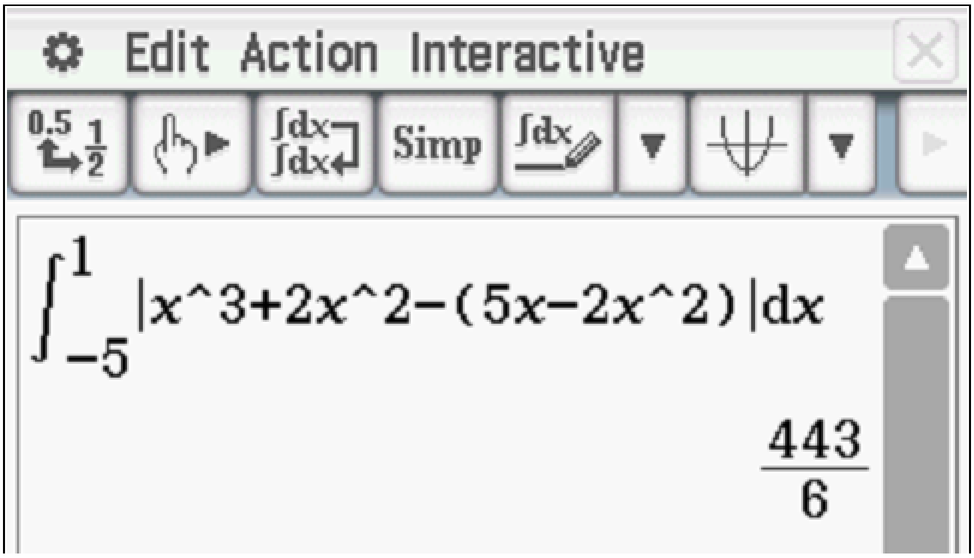
Specific behaviours

✓ states coordinates 1dp local max on both graphs
✓ states coordinates 1 dp local min on cubic (must label min)
✓ shows that graphs intersect at three points
✓ states coordinates of 3 points of intersection
✓sketches correct shape of both graphs

NOTE: follow through does not apply if mistake makes easier!

b) Determine the exact area between $y = x^3 + 2x^2$ and $y = 5x - 2x^2$.

Solution



Specific behaviours

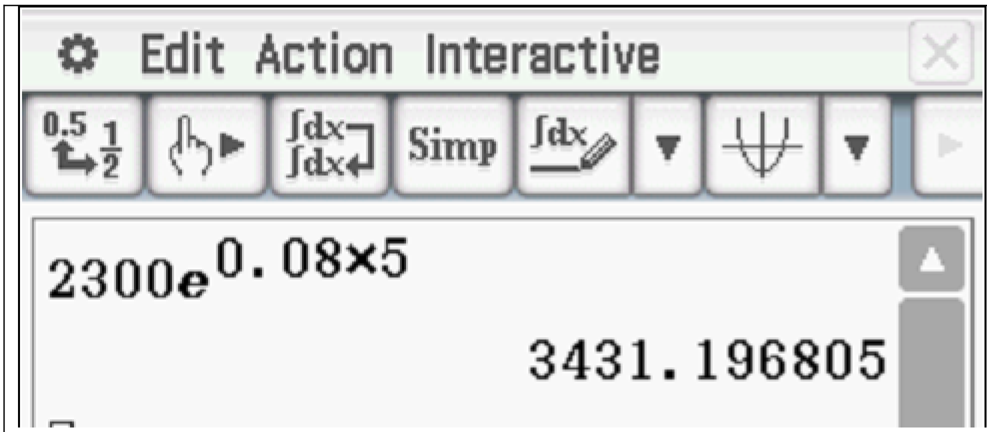
✓ sets up correct integral(s)
✓uses correct limits for integrals
✓ states exact area

Q6 (2 & 2 = 4 marks) (3.1.3, 3.1.4)

The number of kangaroos, N in a particular site that have developed disease W are increasing such that $\frac{dN}{dt} = 0.08N$ with t the time in years. There are initially 2300 kangaroos.

a) Determine the number of kangaroos with disease W in 5 years' time.

Solution



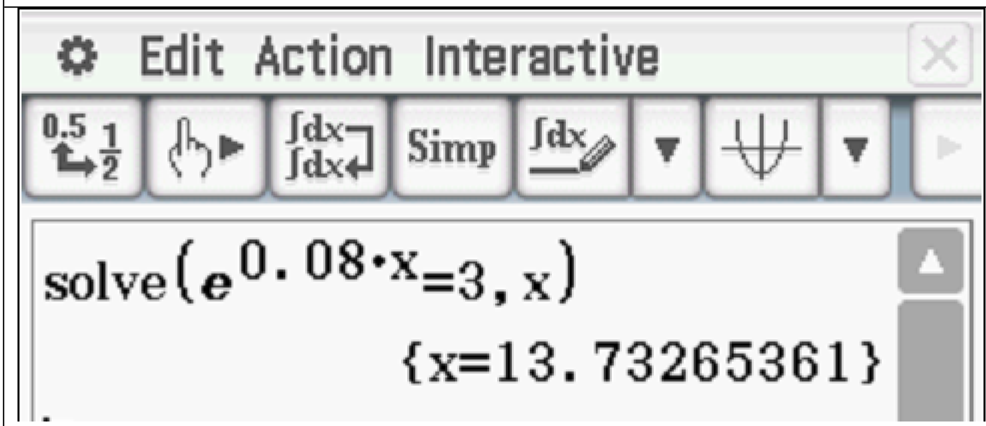
Number of kangaroos 3431

Specific behaviours

✓ uses an exponential function
✓ states number, accept decimal

b) Determine the time taken (years in one decimal place) to triple the number with the disease.

Solution



Years = 13.7

Specific behaviours

✓ sets up an equation
✓ states number of years to one decimal place

Q7 (4 marks) (3.2.16)

Consider the function $G(x) = \int_1^x f(t)\,dt$ such that $G''(x) = \dfrac{3}{4x^{\frac{5}{2}}}$ and $G(4) = \dfrac{79}{2}$.

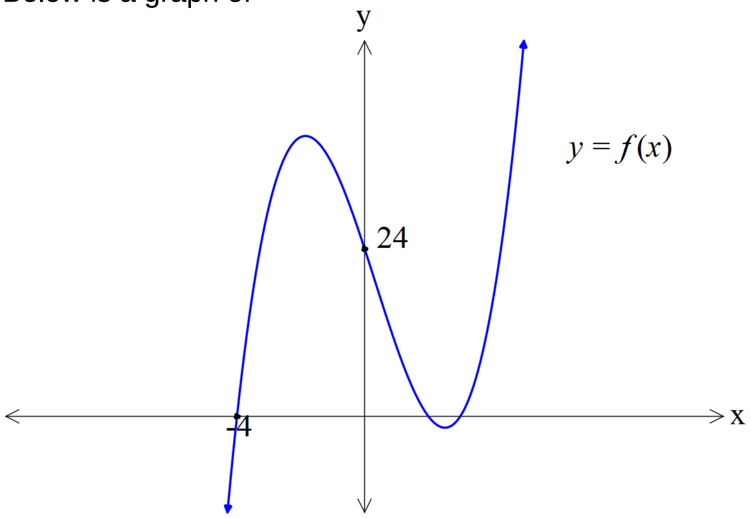
Determine the rule for the function $f(x)$.

Solution
$G(x) = \int_1^x f(t)\,dt$ $G'(x) = f(x)$ $G''(x) = f'(x)$ $f'(x) = \frac{3}{4}x^{\frac{-5}{2}}$ $f(x) = \frac{-2}{3}\left(\frac{3}{4}\right)x^{\frac{-3}{2}} + c = \frac{-1}{2}x^{\frac{-3}{2}} + c$ $\int_1^4 f(t)\,dt = \frac{79}{2} = \left[x^{\frac{-1}{2}} + cx\right]_1^4 = \left(\frac{1}{2} + 4c\right) - (1 + c) = 3c - \frac{1}{2}$ $c = \frac{40}{3}$ $f(x) = \frac{-1}{2x^{\frac{3}{2}}} + \frac{40}{3}$
Specific behaviours
✓ uses fundamental theorem to express $G'' = f'$ ✓ integrates to express f in terms of x and a constant ✓ uses definite integral to set up equation for constant ✓ solves for constant and express f in terms of x in full.

Q8 (5 marks) (3.1.15)

Consider the function $f(x) = ax^3 + bx^2 + cx + d$ where $a, b, c \,\&\, d$ are constants.

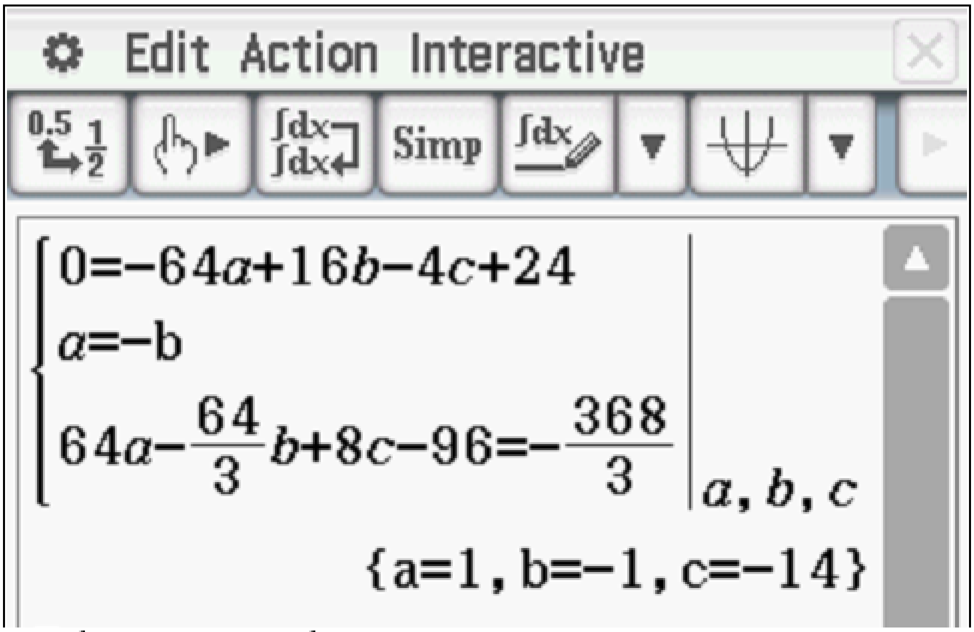
Below is a graph of $f(x)$



There is an x intercept at $x = -4$, y intercept at $y = 24$ and $\int_{-4}^{0} f(x)\,dx = \frac{368}{3}$.

There is an inflection point at $x = \frac{1}{3}$.

Determine the exact values of $a, b, c \,\&\, d$.

Solution
$f(x) = ax^3 + bx^2 + cx + d$ $24 = d$ $0 = -64a + 16b - 4c + 24$ $f'(x) = 3ax^2 + 2bx + c$ $f''(x) = 6ax + 2b$ $0 = 6a\frac{1}{3} + 2b$ $a = -b$ $\int_{-4}^{0} ax^3 + bx^2 + cx + d\,dx = \left[\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + 24x\right]_{-4}^{0} = 0 - \left(64a - \frac{64b}{3} + 8c - 96\right) = \frac{368}{3}$ $64a - \frac{64b}{3} + 8c - 96 = -\frac{368}{3}$  $a = 1, b = -1, c = -14 \,\&\, d = 24$ OR solving without classpad Eq 1 times 2

$0 = -128a + 32b - 8c + 48$ $\frac{-368}{3} = 64a - \frac{64}{3}b + 8c - 96$ add $\frac{-368}{3} = -64a + \frac{32}{3}b - 48$ $a = -b$ $\frac{-368}{3} = 64b + \frac{32}{3}b - 48$ $\left(\frac{-368}{3} + 48\right) = \frac{224}{3}b$ $\frac{-224}{3} = \frac{224}{3}b$ $b = -1$ $a = 1$ $a = 1, b = -1, c = -14 \& d = 24$
Specific behaviours
✓ solves for d ✓ derives $a = -b$ using inflection point ✓ sets up linear equation using x intercept ✓ uses definite integral and then integrates and sets ups a linear equation for unknowns ✓states values for all 4 unknowns NOTE: follow through does not apply if mistake makes easier!