

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name

Teacher's name

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

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See next page

Question 1 (4 marks)

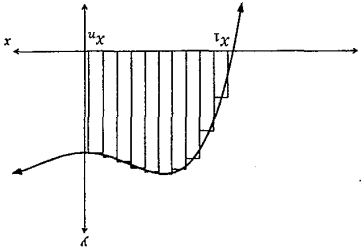
At any point (x, y) on a particular curve $\frac{d^2y}{dx^2}$ is a quadratic function of x .

(a) From the information about $\frac{d^2y}{dx^2}$,

(i) what is the maximum number of stationary points for this original curve? (1 mark)

(ii) what is the maximum number of points of inflection for this original curve? (1 mark)

(b) Part of the curve is shown below. The rectangles can be used to approximate the area under the curve.



(i) What is represented by the expression $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(x_i) \times \Delta x \right)$? (1 mark)

(iii) Simplify the expression in (i) above, using Calculus symbols. (1 mark)

See next page

Question 2

(4 marks)

At any point (x, y) , a particular curve is defined by $\frac{d^2y}{dx^2} = 1 - x^2$.

A tangent drawn to the curve at $(1, 1)$ has equation $y = 2 - x$.

Determine the equation of the curve.

Question 9

(4 marks)

Let the proportion of students at CCGS who are left footed be π . A sample of 400 students at CCGS yielded a confidence interval for π as $0.23 \leq \pi \leq 0.29$.

- (a) How many in this sample are left footed? (1 mark)

$$\text{Point Estimate for } \pi, \hat{\pi} = \frac{0.23 + 0.29}{2}$$

$$\hat{\pi} = \underline{0.26}$$

$$\therefore \text{N}^{\circ} \text{ left footers} = 400 \times 0.26 = \underline{104} \checkmark (\text{student N}^{\circ})$$

- (b) If 50 samples of 400 students each were selected, and the associated confidence intervals for π calculated in the same manner, how many of these confidence intervals would actually contain π ? (3 marks)

Since $n \geq 30$ by (CLT) assume $\hat{\pi}$ is Normally dist.

$$\text{M. Error} = 0.29 - 0.26 = 0.03$$

$$\boxed{Z \times \sqrt{\frac{0.26(1-0.26)}{400}} = 0.03} \checkmark (\text{sets up equation})$$

$$Z = 1.36788$$

$$\text{But } P(-1.36788 \leq Z \leq 1.36788) = 0.82865 \checkmark (\text{Confidence Interval})$$

Hence level of confidence is 82.9%.

ie ~41 of the 50 intervals contain π .

\checkmark (Number of students)

End of questions

See next page

Question 8

(4 marks)

Let the proportion of parents at CCGS that support their son having one hour per night on electronic devices be p . A random sample of n parents (where $n \geq 100$) was selected and 56 indicated that they supported the proposal. Find n if the magnitude of the margin of error for the 99% confidence interval for p is 0.1.

Point estimate $\hat{p} = \frac{56}{n}$ ✓ (pt est.)

Since $n \geq 30$ (by CLT) $\hat{p} \Rightarrow$ Normal Dist.

Margin of error for 99% CI

$$0.1 = 2.576 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \checkmark \quad \left(\text{Equation with ME} \right)$$

Using CRAD $n = -216.29, 62.6, 153.69$

Since $n \geq 100$

$n = 153$ ✓
Accept $\hat{p} = 153$ (Soln 153)

$$\begin{aligned} 152 &\Rightarrow 0.1007882969 \\ 153 &\Rightarrow 0.1003202471 \\ 154 &\Rightarrow 0.09985546279 \end{aligned}$$

See next page

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Question 3

(4 marks)

Differentiate each of the following with respect to x .

(a) $y = \frac{4}{x} - 3\sqrt{x^3}$

(Leave your answer with positive indices)

(2 marks)

(b) $y = \frac{5-x}{5x+1}$

(Simplify your answer)

(2 marks)

See next page

Question 4

(3 marks)

$g(x)$ is a function such that $g(-1) = 4$ and $g'(-1) = 2$.

$f(x)$ is a function such that $f(-1) = f'(-1) = 3$.

Determine $R'(-1)$ where $R(x) = f(x) \times (g(x))^2$

End of questions

Question 7

(9 marks)

The number of snow leopards in Siberia has been decreasing at a rate proportional to the number present from 1993. At the beginning of 1993 there were 440 snow leopards in Siberia. That is $\frac{dw}{dt} = -kw$ where k is the constant of proportionality and t is the number of years since 1993.

- (a) Show clearly that $W = W_0 e^{-kt}$ satisfies the above equation. (2 marks)

$$\frac{dw}{dt} = (-k) W_0 e^{-kt} \quad \checkmark \quad (\text{differentiates})$$

$$\frac{dw}{dt} = -k W \quad \checkmark \quad (\text{Sub } W \text{ for } W_0 e^{-kt})$$

By the beginning of 2000 there were only 356 snow leopards in Siberia.

- (b) Determine:

(1 mark)

- (i) the value of W_0 .

$$W_0 = \underline{440} \quad \checkmark \quad (\text{Given})$$

- (ii) the value of the constant of proportionality, correct to three decimal places.

(2 marks)

$$356 = 440 e^{-7k} \quad \checkmark \quad (\text{Equation})$$

$$k = \underline{0.030} \quad \checkmark \quad (k \text{ to 3dp})$$

Hence, or otherwise,

- (c) determine the expected number of snow leopards at the beginning of 2010.

$$W = 440 e^{-0.030(17)} \quad \checkmark \quad (\text{Equation}) \quad (2 \text{ marks})$$

$$W \approx \underline{264} \text{ snow leopards} \quad \checkmark \quad (\text{Accept } \underline{263})$$

- (d) determine during which year the number of snow leopards will first fall below 300.

(2 marks)

$$300 = 440 e^{-0.03(t)}$$

$$t = 12.8$$

$$\checkmark$$

$$(t\text{-value})$$

$$\therefore \text{During } \underline{2005} \quad \checkmark \quad (\text{yr } 2005)$$

See next page

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Question 6

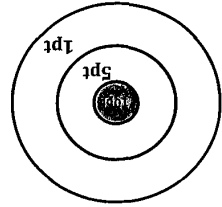
(6 marks)

At Andre's gun shooting gallery a target is made up of three concentric circular regions. Andre is very accurate when he shoots he never records a miss. Points are scored for hitting various parts of the target.
The innermost region (Bulls-eye) scores 10 points. The next outer region scores 5 points and the outer region scores 1 point.
The random variable X represents the number of points Andre scores.
Part of the probability distribution for X is given below.

x	1	5	10
$P(X = x)$	0.6	0.3	0.1

(a) Calculate $P(X = 10)$.

$P(X=10) = 0.1$ ✓



Andre fires two bullets at the target. Assume that each shot is independent of the other.

(b) Calculate the probabilities of the following events:

(i) The first bullet scores 5 and the second scores 1.

$0.3 \times 0.6 = 0.18$ ✓

(ii) The first bullet scores 5 or the second scores 1.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.18$ ✓

$= 0.72$ ✓

(iii) The second bullet scores 5 given the first scored 1.

$\frac{0.18}{0.3} = 0.6$ ✓

(iv) Andre scores a total of 6 points.
5 + 1 or 1 + 5

$2 \times 0.18 = 0.36$ ✓

See next page

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Additional working space

Question number: _____

Question 5

(7 marks)

Consider the graph of $f(x) = x(x-1)(x+2)$.

(a) Determine, correct to two decimal places where necessary, the coordinates of:

- (i) the
- x
- intercepts of
- $f(x)$
- . (1 mark)

$$\underline{(-2, 0) \quad (0, 0) \quad (1, 0)} \quad \checkmark \quad \left(\begin{array}{c} \text{MUST HAVE} \\ \text{ALL} \end{array} \right)$$

- (ii) the point(s) of inflection of
- $f(x)$
- . (1 mark)

$$\underline{(-0.33, 0.74)} \quad \checkmark \quad (\text{MUST BE TO 2dp})$$

(b) Determine, the x -value(s), correct to two decimal places where necessary, where:

- (i)
- $f(x) > 0$
- . (2 marks)

$$\underline{-2 < x < 0} \quad \checkmark \quad \text{or} \quad \underline{x > 1} \quad \checkmark$$

- (ii)
- $f'(x) < 0$
- . (2 marks)

$$\underline{-1.22} \quad \checkmark \quad < x < \underline{0.55} \quad \checkmark$$

- (iii)
- $f''(x) > 0$
- . (1 mark)

$$\underline{x > -0.33} \quad \checkmark$$

See next page

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See next page



Christ Church
Grammar School

2019
TEST 5

MATHEMATICS METHODS Year 12

Section Two:

Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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See next page

Christ Church
Grammar School2019
TEST 5**MATHEMATICS METHODS Year 12****Section Two:****Calculator-assumed**Your name MARK - KING (2)

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section**To be provided by the supervisor**

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Formula Sheet (retained from Section One)

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Question 4

(3 marks)

$g(x)$ is a function such that $g(-1) = 4$ and $g'(-1) = 2$.
 $f(x)$ is a function such that $f(-1) = f'(-1) = 3$.
 $R(x) = f(x) \times (g(x))^2$ where $R(x) = f(x) \times (g(x))^2$
 $u'v + uv'$

$$R'(x) = f'(x)(g(x))^2 + f(x) \cdot 2g(x) \cdot g'(x)$$

(Product Rule)

$$= 3 \times 4^2 + 3 \times 2 \times (4) \cdot (2)$$

(substitution)

$$= 48 + 48$$

(value)

End of questions

3

Question 5

(7 marks)

Consider the graph of $f(x) = x(x - 1)(x + 2)$.

(a) Determine, correct to two decimal places where necessary, the coordinates of:

(i) the x-intercepts of $f(x)$. (1 mark)

(ii) the point(s) of inflection of $f(x)$. (1 mark)

(b) Determine, the x-value(s), correct to two decimal places where necessary, where:

(i) $f(x) > 0$. (2 marks)

(ii) $f'(x) < 0$. (2 marks)

(iii) $f''(x) > 0$. (1 mark)

See next page

Question 6

(6 marks)

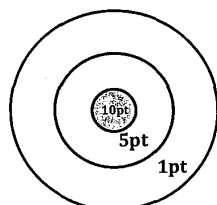
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The innermost region (Bullseye) scores 10 points. The next outer region scores 5 points and the outer region scores 1 point.

The random variable X represents the number of points Andre scores.

Part of the probability distribution for X is given below.

x	1	5	10
$P(X = x)$	0.6	0.3	



(a) Calculate $P(X = 10)$.

(1 mark)

Andre fires two bullets at the target. Assume that each shot is independent of the other.

(b) Calculate the probabilities of the following events:

(i) The first bullet scores 5 and the second scores 1.

(1 mark)

(ii) The first bullet scores 5 or the second scores 1.

(2 marks)

(iii) The second bullet scores 5 given the first scored 1.

(1 mark)

(iv) Andre scores a total of 6 points.

(1 mark)

See next page

Question 3

(4 marks)

Differentiate each of the following with respect to x .

(a) $y = \frac{4}{x} - 3\sqrt{x^3}$

(Leave your answer with positive indices)

(2 marks)

$$y = 4x^{-1} - 3x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = -4x^{-2} - 3 \times \frac{3}{2} x^{\frac{1}{2}} \quad \checkmark \left(\frac{dy}{dx} \right)$$

$$\frac{dy}{dx} = -\frac{4}{x^2} - \frac{9\sqrt{x}}{2} \quad \checkmark \left(\frac{dy}{dx} + \text{positive indices} \right)$$

(b) $y = \frac{5-x}{5x+1}$

$$\frac{u'v - uv'}{v^2} \quad (\text{Simplify your answer})$$

(2 marks)

$$\frac{dy}{dx} = \frac{-1(5x+1) - (5-x)(5)}{(5x+1)^2} \quad \checkmark \left(\frac{dy}{dx} \right)$$

$$= \frac{-1 - 5x - 25 + 5x}{(5x+1)^2}$$

$$= \frac{-26}{(5x+1)^2} \quad \checkmark \quad (\text{simplified})$$

See next page

(4 marks)

Question 2

At any point (x, y) , a particular curve is defined by $\frac{dy}{dx} = 1 - x^2$.
A tangent drawn to the curve at $(1, 1)$ has equation $y = 2 - x$.

Determine the equation of the curve.

$$\frac{dy}{dx} = x - \frac{x^3}{3} + c_1 \quad \checkmark \quad (1^{st} \text{ derivative} + c)$$

$$\frac{dy}{dx} = -1 \quad \text{when } x=1 \Rightarrow c_1 = -1^2 \quad \checkmark \quad (c_1 \text{ value})$$

$$\therefore y = \frac{x^2}{2} - \frac{x^4}{12} - \frac{5x}{3} + c_2$$

$$\text{Sub in } (1, 1) \quad 1 = \frac{1}{2} - \frac{1}{12} - \frac{5(1)}{3} + c_2$$

$$\therefore c_2 = \frac{27}{12} \quad \checkmark \quad (c_2 \text{ value})$$

$$y = \frac{x^2}{2} - \frac{x^4}{12} - \frac{5x}{3} + \frac{27}{12} \quad \checkmark \quad (\text{equation})$$

See next page

4

Question 7 (9 marks)

The number of snow leopards in Siberia has been decreasing at a rate proportional to the number present from 1993. At the beginning of 1993 there were 440 snow leopards in Siberia. That is $\frac{dw}{dt} = -kw$ where k is the constant of proportionality and t is the number of years since 1993.

(a) Show clearly that $W = W_0 e^{-kt}$ satisfies the above equation. (2 marks)

(b) Determine:

(i) the value of W_0 . (1 mark)

(ii) the value of the constant of proportionality, correct to three decimal places. (2 marks)

Hence, or otherwise,

(c) determine the expected number of snow leopards at the beginning of 2010. (2 marks)

(d) determine during which year the number of snow leopards will first fall below 300. (2 marks)

See next page

Question 8

(4 marks)

Let the proportion of parents at CCGS that support their son having one hour per night on electronic devices be \hat{p} . A random sample of n parents (where $n \geq 100$) was selected and 56 indicated that they supported the proposal. Find n if the magnitude of the margin of error for the 99% confidence interval for \hat{p} is 0.1.

Question 1

(4 marks)

At any point (x, y) on a particular curve $\frac{d^2y}{dx^2}$ is a quadratic function of x .

(a) From the information about $\frac{d^2y}{dx^2}$,

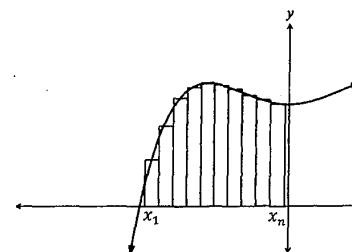
(i) what is the maximum number of stationary points for this original curve? (1 mark)

3 (original quartic) ✓

(ii) what is the maximum number of points of inflection for this original curve? (1 mark)

2 (original quartic) ✓

(b) Part of the curve is shown below. The rectangles can be used to approximate the area under the curve.



(i) What is represented by the expression $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(x_i) \times \Delta x \right)$? (1 mark)

The estimated area under the curve ✓

(ii) Simplify the expression in (i) above, using Calculus symbols. (1 mark)

$\int_{x_1}^{x_n} f(x) dx$ (must have x_n) ✓

MATHEMATICS METHODS Year 122CALCULATOR-FREE

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See next page

MATHEMATICS METHODS Year 127CALCULATOR-ASSUMED

Question 9 (4 marks)

Let the proportion of students at CCGS who are left footed be π . A sample of 400 students at CCGS yielded a confidence interval for π as $0.23 \leq \pi \leq 0.29$.

(a) How many in this sample are left footed? (1 mark)

(b) If 50 samples of 400 students each were selected, and the associated confidence intervals for π calculated in the same manner, how many of these confidence intervals would actually contain π ? (3 marks)

End of questions

Additional working space

Question number: _____



Christ Church
Grammar School

2019
TEST 5

MATHEMATICS METHODS Year 12

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Calculator-free

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Teacher's name _____

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