MATHEMATICS METHODS

MAWA Semester 1 (Unit 3) Examination 2018
Calculator-free

Marking Key

The release date for this exam and marking scheme is

- **the end of week 8 of term 2, 2018**

Section One: Calculator-free (50 Marks)

Question 1 (a) **(2 marks)**

Solution

$$\frac{d}{dx}(x\cos x) = x(-\sin x) + \cos x$$
$$= \cos x - x\sin x$$

Mathematical behaviours	Marks
• applies product rule	1
• differentiates $\cos x$ term	1

Question 1 (b) **(2 marks)**

Solution

$$\frac{d}{dx}(x^3+4\sin x)^5 = 5(x^3+4\sin x)^4 \cdot \frac{d}{dx}(x^3+4\sin x)$$
$$= 5(x^3+4\sin x)^4(3x^2+4\cos x)$$

Mathematical behaviours	Marks
• applies the chain rule	1
• differentiates $\sin x$ term	1

Question 1 (c) **(3 marks)**

Solution

$$\frac{d}{dx}\left(\frac{e^{-2x}}{4x+2}\right)$$

$f(x) = e^{-2x}$ $\quad f'(x) = -2e^{-2x}$ $\quad g(x) = 4x+2$ $\quad g'(x) = 4$

$$= \frac{(4x+2)\cdot(-2e^{-2x}) - e^{-2x}\cdot 4}{(4x+2)^2}$$
$$= \frac{-2(4x+2)(e^{-2x}) - 4e^{-2x}}{(4x+2)^2}$$

Mathematical behaviours	Marks
• applies chain rule to obtain $f'(x)$	1
• applies quotient rule	1
• correct answer	1

Question 8 (b) **(2 marks)**

Solution

$$\int_6^8 f'(x)\,dx = f(8) - f(6) = 0 - 3 = -3$$

Mathematical behaviours	Marks
• applies the Fundamental Theorem	1
• evaluates result	1

Question 8 (c) (i) **(2 marks)**

Solution

Area $\Delta = 8$

$\therefore \int_2^3 f(x)\,dx = -4 \Rightarrow \int_0^3 f(x)\,dx = 0$

$\therefore$ one value of m is $m = 3$.

Also, $\int_0^0 f(x)\,dx = 0$ for any function

hence, $m = 0$ is another solution.

From the symmetry of the graph, $m = 6, 9, 12$

Hence $m = 0, 3, 6, 9, 12$.

Mathematical behaviours	Marks
• states $m = 0$ or $m = 3$	1
• states all correct values for m	1

Question 8 (c) (ii) **(2 marks)**

Solution

$$\int_0^4 g(x)\,dx = \int_0^4 f(x) + 2]\,dx$$
$$= \int_0^4 f(x)\,dx + \int_0^4 2\,dx$$
$$= (-4) + 2(4-0)$$
$$= 4$$

Mathematical behaviours	Marks
• uses linearity to split $g(x)$	1
• evaluates sum of integrals	1

Question 2 **(4 marks)**

Solution

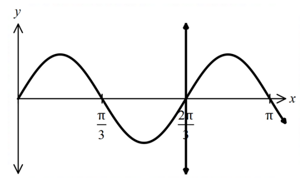
$$A = 2\int_0^{\frac{\pi}{3}} \sin 3x\, dx$$

$$= 2\left[\frac{-\cos 3x}{3}\right]_0^{\frac{\pi}{3}}$$

$$= \frac{2}{3}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{2}{3}\left[1+1\right]$$

$$= \frac{4}{3}$$



Mathematical behaviours	Marks
• states a correct expression using integrals to determine the area	1
• anti-differentiates integral correctly	1
• subs in limits of integration correctly	1
• determines correct result	1

Question 3 **(3 marks)**

Solution

$$f'(x) = x + \sqrt{3+6x}$$

$$\therefore f(x) = \frac{x^2}{2} + \frac{(3+6x)^{\frac{3}{2}}}{6}.\frac{2}{3} + c$$

$$f(1) = 10 \Rightarrow 10 = \frac{1}{2} + \frac{(3+6(1))^{\frac{3}{2}}}{6}.\frac{2}{3} + c$$

ie $10 = \frac{1}{2} + \frac{9^{\frac{3}{2}}}{9} + c$

ie $c = 6\frac{1}{2}$

$$\therefore f(x) = \frac{x^2}{2} + \frac{(3+6x)^{\frac{3}{2}}}{9} + 6.5$$

Mathematical behaviours	Marks
• anti-differentiates square root term	1
• uses anti-derivative and $f(1) = 10$ to determine c	1
• states $f(x)$	1

Question 4 (a) **(1 mark)**

Solution

X has a discrete uniform distribution

Mathematical behaviours	Marks
• states that the distribution is uniform	1

Question 4 (b) **(1 mark)**

Solution

There are $550 - 250 + 1 = 301$ whole numbers in the interval $250 \le X \le 550$.
So $P(250 \le X \le 550) = 0.301$

Mathematical behaviours	Marks
• correct answer	1

Question 4 (c) **(2 marks)**

Solution

There are $\frac{1000}{7} = 142\frac{6}{7}$, and so there are 142 whole numbers in the interval $1 \le X \le 1000$ that are divisible by 7.

So P¿.

Mathematical behaviours	Marks
• obtains 142 whole numbers divisible by 7	1
• divides by 1000	1

Question 7 (b) **(4 marks)**

Solution

$$\frac{dy}{dx} = -\frac{\sin\left(\frac{\pi}{3} - x\right)}{\cos^2\left(\frac{\pi}{3} - x\right)}, \text{ when } x = \frac{2\pi}{3}$$

$$= -\frac{\sin\left(\frac{\pi}{3} - \frac{2\pi}{3}\right)}{\cos^2\left(\frac{\pi}{3} - \frac{2\pi}{3}\right)} = -\frac{\sin\left(-\frac{\pi}{3}\right)}{\cos^2\left(-\frac{\pi}{3}\right)}$$

$$= -\frac{\frac{-\sqrt{3}}{2}}{\left(-\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} \div \frac{1}{4} = 2\sqrt{3}$$

Mathematical behaviours	Marks
• correct substitution and subtraction of fractions	1+1
• both exact values correct	1
• correct simplified answer	1

Question 8 (a) (i) **(1 mark)**

Solution

$$\int_0^4 f(x)\ dx = A - B$$

Mathematical behaviours	Marks
• determines expression	1

Question 8 (a) (ii) **(3 marks)**

Solution

$$\int_0^4 2f(x)\ dx + \int_8^4 f(x)\ dx$$

$$= 2\int_0^4 f(x)\ dx - \int_4^8 f(x)\ dx$$

$$= 2(A-B) - 2A = -2B$$

Mathematical behaviours	Marks
• uses linearity to deduce $\int_0^4 2f(x)\ dx = 2(A-B)$	1
• uses relationship $\int_8^4 f(x)\ dx = -\int_4^8 f(x)\ dx$	1
• sums expressions and simplifies	1

Question 6 (c) **(2 marks)**

Solution	
$\int \left(e^x - \frac{1}{e^x}\right)^2 dx = \int e^{2x} - 2 + e^{-2x}\ dx = \frac{e^{2x}}{2} - 2x - \frac{e^{-2x}}{2} + c$	
Mathematical behaviours	Marks
• expands brackets correctly • anti-differentiates each part correctly	1 1

Question 7 (a) **(3 marks)**

Solution	
$y = \sec(\frac{\pi}{3} - x)$ $u(x) = \frac{\pi}{3} - x \qquad u'(x) = -1 \qquad \frac{d}{dx}\sec x = \frac{\sin x}{\cos^2 x}$ $\frac{dy}{dx} = \frac{\sin(u(x))}{\cos^2(u(x))} \cdot u'(x)$ $\frac{dy}{dx} = \frac{\sin\left(\frac{\pi}{3} - x\right)}{\cos^2\left(\frac{\pi}{3} - x\right)} \cdot (-1) = -\frac{\sin\left(\frac{\pi}{3} - x\right)}{\cos^2\left(\frac{\pi}{3} - x\right)}$	
Mathematical behaviours	Marks
• correctly differentiates $sec\,x$ • applies chain rule • correct answer	1 1 1

Question 4 (d) **(4 marks)**

Solution
In the interval $1 \le X \le 1000$there are: 100 whole numbers that are divisible by 10, 40 whole numbers that are divisible by 25, and 20 whole numbers that are divisible by both 10 and 25,(i.e. divisible by 50¿ So there are $100+40-20=120$ whole numbers that are divisible by 10 or 25 . and so P¿.

Mathematical behaviours	Marks
• correct numbers for divisibility by 10 and by 25	1+1
• uses ¿$(A \cup B)=$¿$(A)+$¿$(B)-$¿$(A \cap B)$	1
• divides by 1000	1

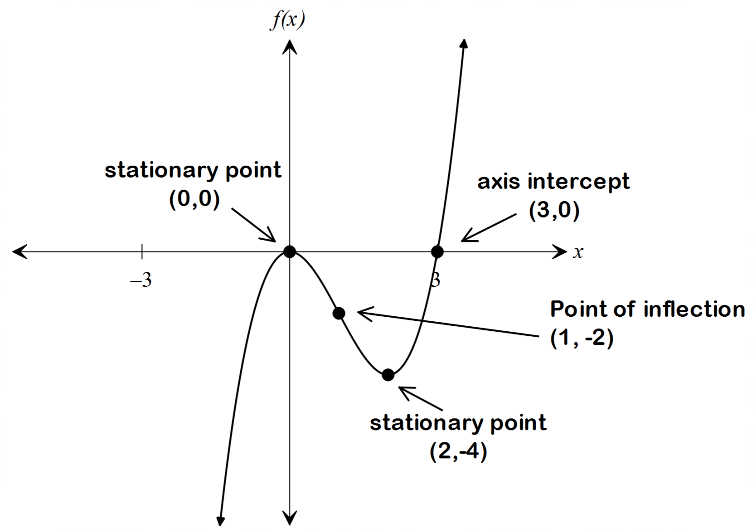
Question 4 (e) **(2 marks)**

Solution
The following numbers have exactly two 3's in their decimal expansion: $33, 133, 233, 433, \ldots, 933$, $303, 313, 323, 343, \ldots, 393$, and $330, 331, 332, 334, \ldots 339$ So P¿.

Mathematical behaviours	Marks
• obtains 27 whole numbers with the desired property	1
• divides by 1000	1

Question 5 **(3 marks)**

Solution



Mathematical behaviours	Marks
• correctly identifies stationary points	1
• correctly identifies point of inflection	1
• accurate sketch of the curve including x axis intercepts	1

Question 6 (a) **(2 marks)**

Solution
$\int \frac{1-2x}{x^3}\,dx = \int x^{-3} - 2x^{-2}\,dx = \frac{2}{x} - \frac{1}{2x^2} + c$

Mathematical behaviours	Marks
• splits the fraction into two parts and anti-differentiates x^{-3}	1
• states anti-derivative including $+c$	1

Question 6 (b) **(2 marks)**

Solution
$\int \sin\left(x - \frac{\pi}{4}\right) - \cos \pi x \quad dx = -\cos\left(x - \frac{\pi}{4}\right) - \frac{\sin \pi x}{\pi} + c$

Mathematical behaviours	Marks
• anti-differentiates sin or cos part of expression correctly	1
• states correct solution	1