Papers written by
Australian Maths
Software

YEAR 11

2016

REVISION 1

MATHEMATICS
SPECIALIST
UNITS 1 & 2

SEMESTER TWO
SOLUTIONS

SECTION 1 – Calculator-free

Question 1 **(7 marks)**

(a) (i)

$$y = x + 2 \to x - y = -2$$
$$x + 8 = 2y \to x - 2y = -8$$
$$so$$
$$\begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -8 \end{bmatrix} \quad ✓$$

$$✓ \quad \frac{1}{-1} \times \begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} -2 \\ -8 \end{bmatrix} \quad ✓$$
$$I \times \begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} -4 \\ -6 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} \quad ✓$$

(b)

$$A \times B = \begin{pmatrix} x & 2 \\ -2 & -1 \end{pmatrix} \times \begin{pmatrix} 2 & y \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2x+2 & xy+4 \\ -5 & -2y-2 \end{pmatrix} \quad ✓$$
$$B \times A = \begin{pmatrix} 2 & y \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} x & 2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 2x-2y & 4-y \\ x-4 & 0 \end{pmatrix} \quad ✓$$
$$If \;\; A \times B = B \times A$$
$$0 = -2y - 2 \Rightarrow y = -1$$
$$-5 = x - 4 \Rightarrow x = -1 \quad ✓$$
$$Check$$
$$2x + 2 = 2x - 2y \;\; IFF \;\; 0 = -2 + 2 Yes$$
$$xy + 4 = 4 - y \;\; IFF \;\; 5 = 4 - (-1) Yes$$
$$x = -1, y = -1$$

Question 2 **(6 marks)**

(a) (i)

$$\begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax \\ y \end{bmatrix}$$

Dilates the point (x, y) in the direction of the x axis by a factor of a. ✓

(ii)

$$\begin{bmatrix} 1 & 0 \\ 0 & b \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ by \end{bmatrix}$$

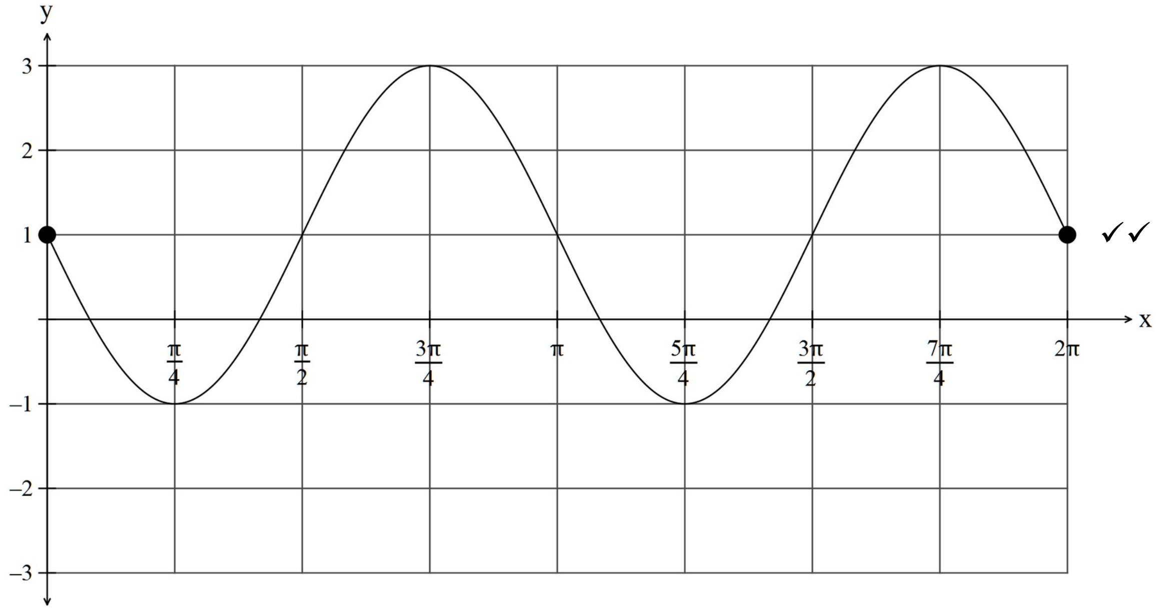
Dilates the point (x, y) in the direction of the y axis by a factor of b. ✓

(b)

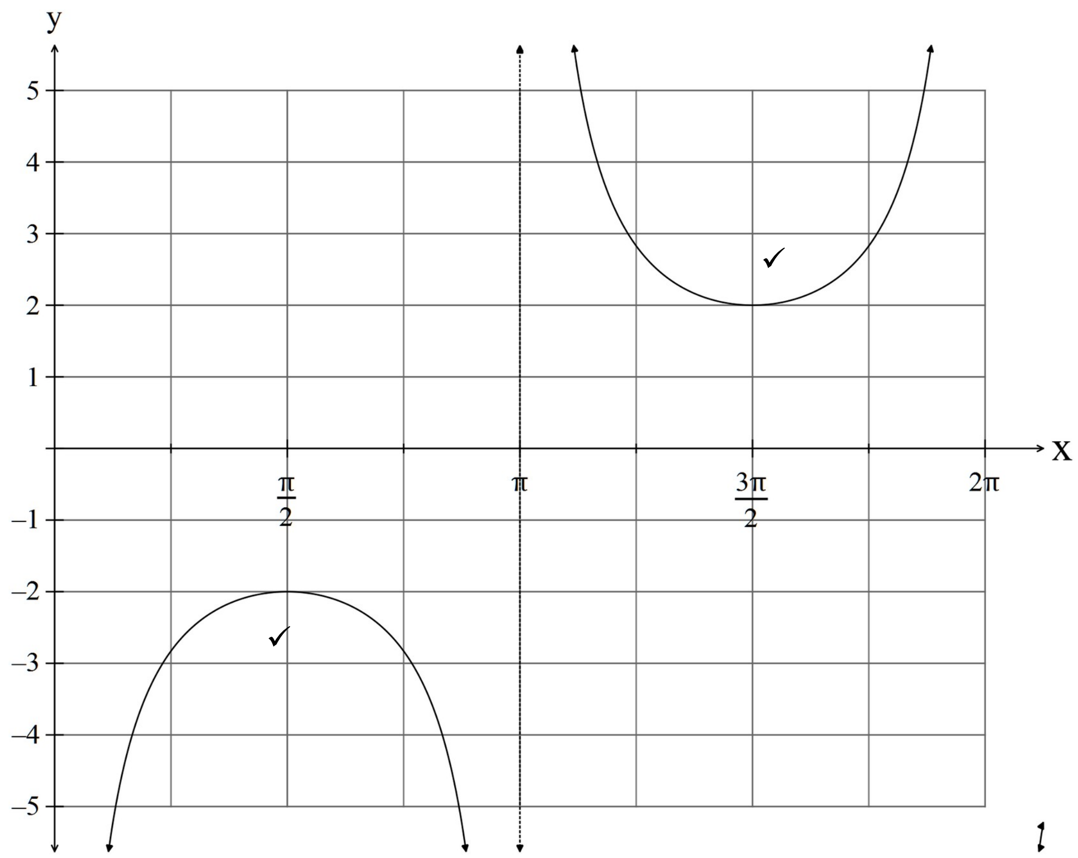
$$\begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2x \\ 3y \end{bmatrix} \quad so \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \quad ✓✓$$

Question 3 **(4 marks)**

(a) $y = 1 - 2\sin(2x)$ on the domain $0 \le x \le 2\pi$.

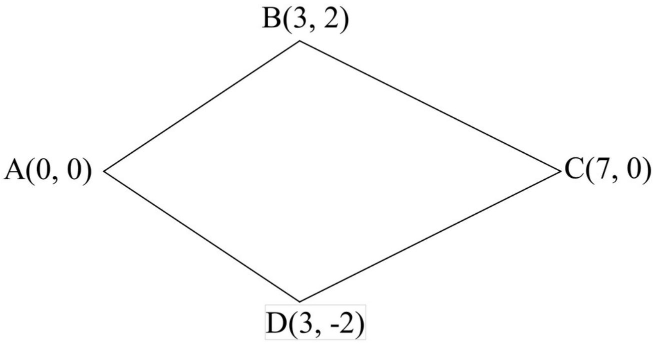


(b) $y = 2\sec\left(x + \frac{\pi}{2}\right)$ on the domain $0 < x < 2\pi$



Question 4 **(8 marks)**

(a)



(i) $\boldsymbol{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\boldsymbol{DC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

$\boldsymbol{AB} \neq \boldsymbol{DC}$ ✓

Therefore $ABCD$ is not a parallelogram.

(ii) $\boldsymbol{AC} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$, $\boldsymbol{DB} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ $\boldsymbol{AC} \bullet \boldsymbol{DB} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 4 \end{pmatrix} = 0$

$\boldsymbol{AC} \neq \boldsymbol{0}$, $\boldsymbol{DB} \neq \boldsymbol{0}$ therefore $\cos\theta = 0$

Therefore $\theta = \frac{\pi}{2}$. The diagonals are perpendicular. ✓

Midpoint of $\boldsymbol{DB} = \boldsymbol{AB} + \frac{1}{2}(\boldsymbol{BD}) = \boldsymbol{AM}$ NB Point A is the origin.

$\boldsymbol{AM} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

$\boldsymbol{AM} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ ✓

$\boldsymbol{AC} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} = k\begin{pmatrix} 3 \\ 0 \end{pmatrix} = k\,\boldsymbol{AM}$ The vectors are collinear. $M \in AC$. ✓

Therefore the midpoint of one diagonal belongs to the other diagonal and they are perpendicular. ✓

Therefore $ABCD$ is a kite,

(b) A(1,1) and $\boldsymbol{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$\boldsymbol{OP} = \boldsymbol{OA} + t\,\boldsymbol{AB}$

$\boldsymbol{OP} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$\boldsymbol{OP} = \begin{pmatrix} 13 \\ -5 \end{pmatrix}$

$P(13,-5)$ ✓

(c) $\boldsymbol{a}\bullet\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos(\theta)$

$$\cos(\theta) = \frac{\begin{pmatrix}2\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-7\end{pmatrix}}{\left|\begin{pmatrix}2\\3\end{pmatrix}\right|\left|\begin{pmatrix}1\\-7\end{pmatrix}\right|} \quad ✓$$

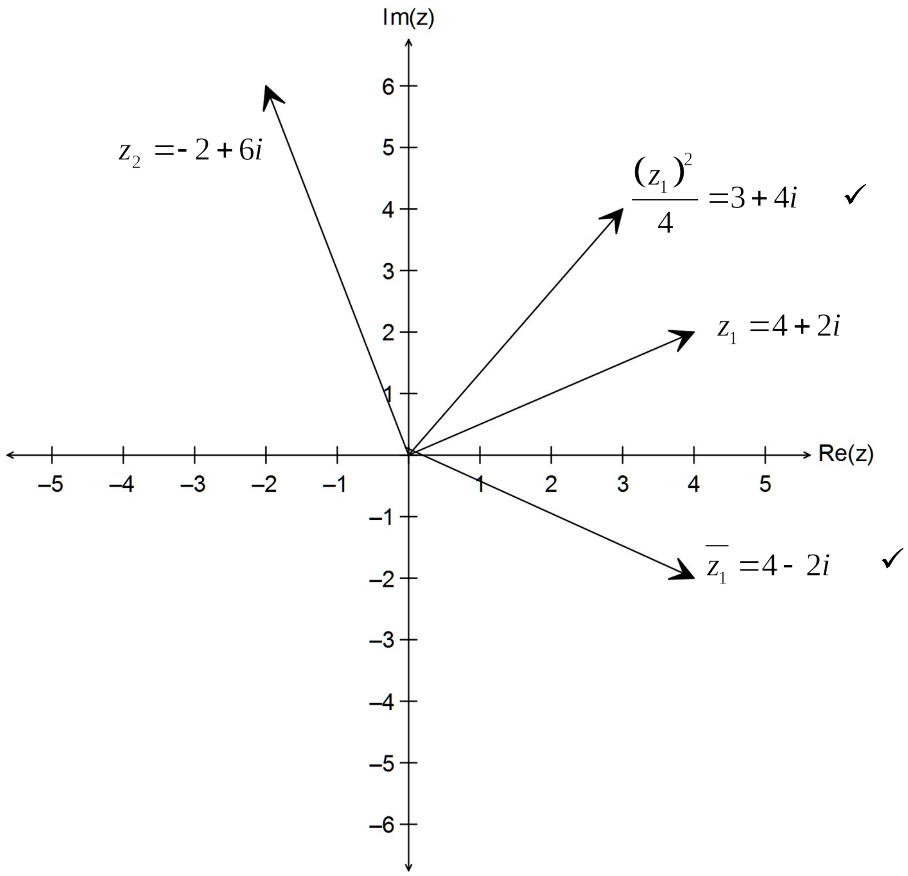
$$\cos(\theta) = \frac{-19}{\sqrt{13}\sqrt{50}}$$

$$\cos(\theta) = \frac{-19}{5\sqrt{26}} \quad ✓$$

Question 5 **(11 marks)**

(a) (i) $\overline{z_1} = 4 - 2i$

(ii) $\frac{(z_1)^2}{4} = \frac{(4+2i)^2}{4} = \frac{16+16i+4i^2}{4} = \frac{12+16i}{4} = 3+4i$ ✓



(b) (i) $|z_1 - z_2| = |4+2i-(-2+6i)|$

$= |6-4i|$ ✓

$= \sqrt{52}$ ✓

$= 2\sqrt{13}$ ✓

(ii) $\dfrac{10}{z_1} - 2z_2 = \dfrac{10}{4+2i} - 2(-2+6i)$

$= \dfrac{5}{2+i} + 4 - 12i$ ✓

$= \dfrac{5}{2+i} \times \dfrac{2-i}{2-i} + 4 - 12i$ ✓

$= \dfrac{5(2-i)}{5} + 4 - 12i$

$= 6 - 13i$ ✓

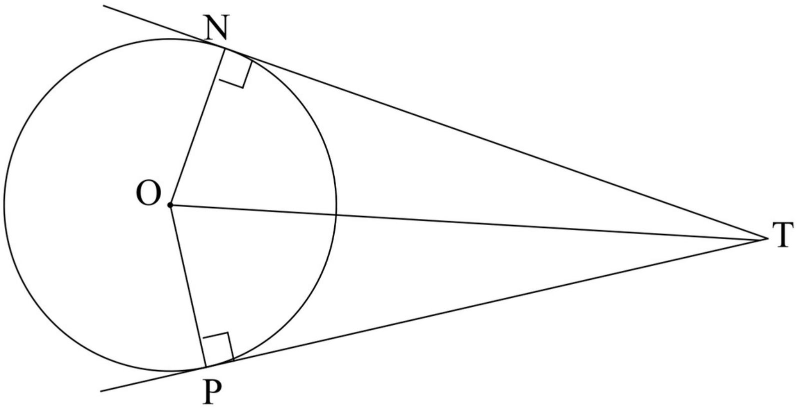
(iii) $z_1 - i\,z_2 = 4 + 2i - i(-2+6i)$

$= 10 + 4i$ ✓

Question 6 **(9 marks)**

(a) The converse is
"*If the opposite angles are supplementary, then the quadrilateral is cyclic.*" ✓

(b) Prove that "*The tangents drawn to a circle from an external point are equal.*"



Let O be the centre of the circle, N and P the points of contact of the tangents and T the point of intersection of the tangents. ✓

Join ON, OP and OT.

In Δ ONT and Δ OPT

(i) ONT = OPT = 90° Tangents are perpendicular to radius at the point of contact.

(ii) ON = OP Equal radii

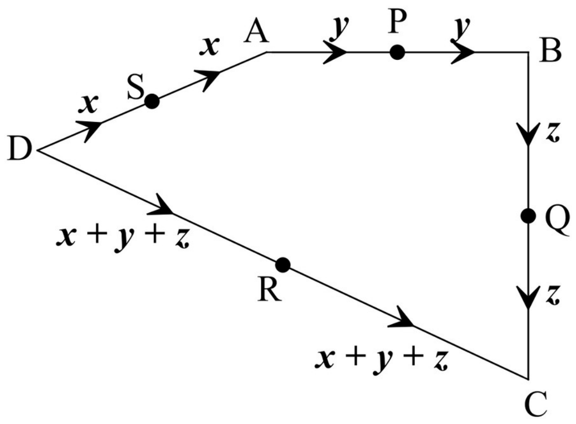
(iii) OT = OT Same line ✓

$\therefore \ \Delta ONT \simeq \Delta OPT$ ✓ Right angle, hypotenuse, side

Therefore NT = PT ✓

Therefore "*The tangents drawn to a circle from an external point are equal.*"

(c) Prove the following using vectors:
"*If the midpoints of the adjacent sides of a quadrilateral are joined, then the resulting quadrilateral is a parallelogram*."



$Let\ \boldsymbol{DA} = 2\boldsymbol{x},\ \ \boldsymbol{AB} = 2\boldsymbol{y},\ \ \boldsymbol{BC} = 2\boldsymbol{z}$

$\boldsymbol{DC} = \boldsymbol{DA} + \boldsymbol{AB} + \boldsymbol{BC}$

$\boldsymbol{DC} = 2\boldsymbol{x} + 2\boldsymbol{y} + 2\boldsymbol{z}$

$\boldsymbol{DR} = \boldsymbol{RC} = \boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z}$ ✓

$\boldsymbol{RS} = -\boldsymbol{x} - \boldsymbol{y} - \boldsymbol{z} + \boldsymbol{x}$ ✓

$\boldsymbol{RS} = -\boldsymbol{y} - \boldsymbol{z}$

$\boldsymbol{QP} = -\boldsymbol{z} - \boldsymbol{y}$

$\boldsymbol{QP} = \boldsymbol{RS}$ ✓

Equal and parallel sides. ✓

Therefore RSPQ is a parallelogram

Question 7 **(7 marks)**

(a) $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

Let $y = 2x$

$\sin(x+2x) = \sin(x)\cos(2x) + \cos(x)\sin(2x)$ ✓

$\sin(3x) = \sin(x)\left(1 - 2\sin^2(x)\right) + \cos(x)\left(2\sin(x)\cos(x)\right)$

$\sin(3x) = \sin(x) - 2\sin^3(x) + 2\sin(x)\left(\cos^2(x)\right)$

$\sin(3x) = \sin(x) - 2\sin^3(x) + 2\sin(x)\left(1 - \sin^2(x)\right)$ ✓

$\sin(3x) = \sin(x) - 2\sin^3(x) + 2\sin(x) - 2\sin^3(x)$

$\sin(3x) = 3\sin(x) - 4\sin^3(x)$ ✓

(b) Solve $\tan^2\left(x+\frac{\pi}{4}\right)=1$ for $-\pi\le\theta\le\pi$.

$\tan^2\left(x+\frac{\pi}{4}\right)=1$

$\tan\left(x+\frac{\pi}{4}\right)=\pm1$ ✓

$x+\frac{\pi}{4}=\frac{\pi}{4}\pm n\pi$ ✓ *or* $x+\frac{\pi}{4}=-\frac{\pi}{4}\pm n\pi$

$x=\pm n\pi$ $x=-\frac{\pi}{2}\pm n\pi$

$x=0,\ \pm\pi$ ✓ $x=-\frac{\pi}{2},\frac{\pi}{2}$ ✓

END OF SECTION ONE

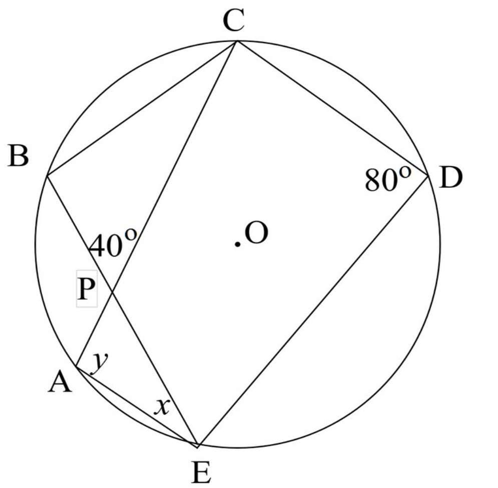
SECTION 2 – Calculator-assumed

Question 8 **(5 marks)**

(a) $3 \times 5 = 4 \times x \Rightarrow x = \frac{15}{4} = 3.75$ ✓

When two chords intersect each other inside a circle, the products of their segments are equal. ✓

(b)



$\angle EBC = 100^o$ The opposite angles in a cyclic quadrilateral are supplementary.

✓ $\therefore \ y = 100^o$ Two angles at the circumference subtended by the same arc are equal.

$\angle APE = 40^o$ Vertically opposite angles.

$x = 180^o - 40^o - 100^o$ Angles in a triangle add to 180°. ✓ reasons

✓ $x = 40^o$

Question 9 **(11 marks)**

(a) (i) 0.9 (units to the turning point) ✓✓

(ii) As p increases, the graph moves to the left and vv. ✓✓

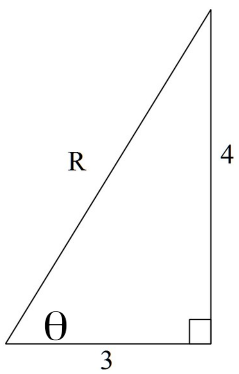
(iii) $R\cos(x - p) = 3\underline{\cos(x)} + 4\underline{\underline{\sin(x)}}$

$R\cos(x - p) = R\underline{\cos(x)}\cos(p) + R\underline{\underline{\sin(x)}}\sin(p)$ ✓

$\therefore R\cos(p) = 3 \quad R\sin(p) = 4$

$\cos(p) = \frac{3}{R} \qquad \sin(p) = \frac{4}{R}$ ✓

Pythagoras $R^2 = 3^2 + 4^2 \rightarrow R = 5$



$\tan(\theta) = \frac{4}{3}$

$\theta = 0.927$

$\therefore \ R = 5 \quad and \quad \theta = 0.927$

✓ ✓

(iv) $3\cos(x)+4\sin(x)=2.5$

$5\cos(x-0.927295218)=2.5$ ✓

$\cos(x-0.927295218)=\frac{1}{2}$

$x=-0.12$ or $x=1.97$ $(2dp)$
✓ ✓

(b) $y=1-\tan(x)$ ✓✓

Question 10 **(7 marks)**

(a) $\dfrac{(1+2i)^2(1-2i)^2}{1+i}=\dfrac{(1-4i^2)^2}{(1+i)}\times\dfrac{1-i}{1-i}$ ✓ ✓

$=\dfrac{25(1-i)}{1-i^2}$

$=\dfrac{25(1-i)}{2}$ ✓

(b) $\sqrt{3+4i}=x+iy$

$3+4i=x^2+2xiy+i^2y^2$

$=(x^2-y^2)+i(2xy)$

Equating real and complex coefficients ✓

$3=x^2-y^2$ and $4=2xy\Rightarrow y=\dfrac{2}{x}$

$3=x^2-\dfrac{4}{x^2}$

$3x^2=x^4-4$

$0=x^4-3x^2-4$ ✓

$0=(x^2-4)(x^2+1)$

$x=\pm2$ as $x^2\neq-1$ as x is real.

If $x=2,\ y=\dfrac{2}{2}=1$ If $x=-2,\ y=\dfrac{2}{2}=-1$ ✓

Therefore $\sqrt{3+4i}=2+iy$ or $\sqrt{3+4i}=-2-iy$

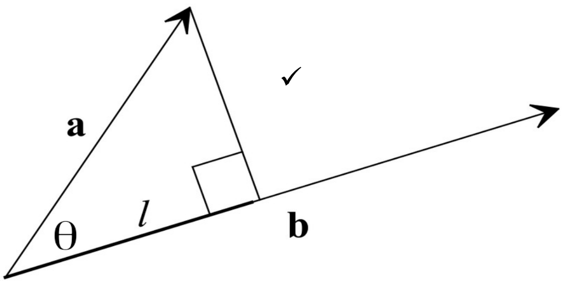
but $\sqrt{3+4i}>0$

so $\sqrt{3+4i}=2+iy$ only ✓

Question 11 **(12 marks)**

(a) (i) $|\mathbf{PQ}| = \left|\begin{pmatrix} -3 \\ -1 \end{pmatrix}\right| = \sqrt{10}$ ✓

(ii) $\mathbf{PQ} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} = \boldsymbol{b}, \quad \boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$



✓

$\cos(\theta) = \dfrac{l}{|\boldsymbol{a}|} \Rightarrow l = |\boldsymbol{a}|\cos(\theta)$

$\boldsymbol{a} \bullet \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos(\theta)$

$|\boldsymbol{a}|\cos(\theta) = \dfrac{\boldsymbol{a} \bullet \boldsymbol{b}}{|\boldsymbol{b}|}$ ✓

so $l = \dfrac{\boldsymbol{a} \bullet \boldsymbol{b}}{|\boldsymbol{b}|} = \dfrac{\begin{pmatrix} 1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ -1 \end{pmatrix}}{\sqrt{10}}$

$l = -\dfrac{4}{\sqrt{10}}$ ✓

(b) $\begin{pmatrix} 90 \\ 350^o \end{pmatrix} + \begin{pmatrix} 0 \\ 110 \end{pmatrix} + \begin{pmatrix} 130 \\ 012^o \end{pmatrix}$

$= \begin{pmatrix} -90\,cos(80^o) \\ 90\,sin(80^o) \end{pmatrix} + \begin{pmatrix} 0 \\ 110 \end{pmatrix} + \begin{pmatrix} 130\,cos(78^o) \\ 130\,sin(78^o) \end{pmatrix}$ ✓

$= \begin{pmatrix} 11.40018382 \\ 325.7918859 \end{pmatrix}$

$\left|\begin{pmatrix} 11.40018382 \\ 325.7918859 \end{pmatrix}\right| = \sqrt{106\,270.3}$

≈ 326 ✓

$tan(\theta) = \dfrac{325.7918859}{11.40018382}$

$\theta \approx 88^o$ ✓

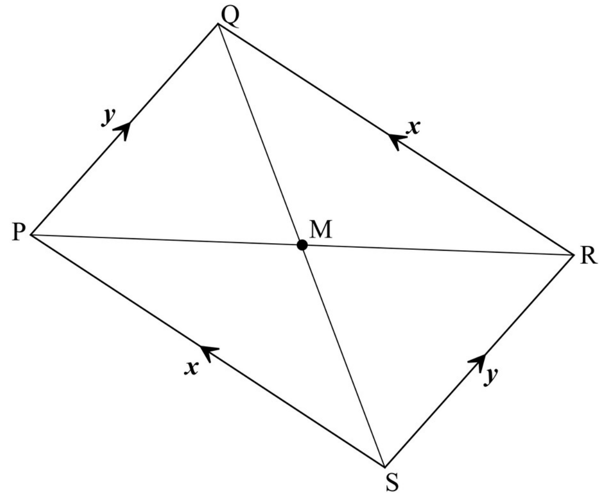
The magnitude is 326 N and the direction is 088^o.
They will not have to use the tractor. ✓

.

(c) Use a vector proof to show that

"The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides."

Use the diagram below:



$\boldsymbol{PQ} = \boldsymbol{SR} = \boldsymbol{y}$

$\boldsymbol{SP} = \boldsymbol{RQ} = \boldsymbol{x}$

$\boldsymbol{PR} = \boldsymbol{y} - \boldsymbol{x}$ ✓

$\boldsymbol{SQ} = \boldsymbol{y} + \boldsymbol{x}$ ✓

$\boldsymbol{PR} \bullet \boldsymbol{PR} = |\boldsymbol{PR}| \times |\boldsymbol{PR}| \times \cos(0^{\circ}) = |\boldsymbol{PR}|^2 = PR^2$ ✓

Likewise $\boldsymbol{SQ} \bullet \boldsymbol{SQ} = SQ^2$

$$PR^2 + SQ^2 = (\boldsymbol{y} - \boldsymbol{x})^2 + (\boldsymbol{y} + \boldsymbol{x})^2$$

$$= \boldsymbol{y} \bullet \boldsymbol{y} - \cancel{\boldsymbol{y} \bullet \boldsymbol{x}} - \cancel{\boldsymbol{x} \bullet \boldsymbol{y}} + \boldsymbol{x} \bullet \boldsymbol{x} + \boldsymbol{y} \bullet \boldsymbol{y} + \cancel{\boldsymbol{y} \bullet \boldsymbol{x}} + \cancel{\boldsymbol{x} \bullet \boldsymbol{y}} + \boldsymbol{x} \bullet \boldsymbol{x} \quad ✓$$

$$= 2\boldsymbol{y} \bullet \boldsymbol{y} + 2\boldsymbol{x} \bullet \boldsymbol{x}$$

$$= 2|\boldsymbol{y}|^2 + 2|\boldsymbol{x}|^2$$

$$= PQ^2 + SR^2 + SP^2 + RQ^2 \quad ✓$$

$\therefore\ PR^2 + SQ^2 = PQ^2 + SR^2 + SP^2 + RQ^2$

$\therefore$ "The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides."

Question 12 **(9 marks)**

(a) $(z - 3)^2 + 1 = 0$

$(z-3)^2 = -1 = i^2$ ✓

$z - 3 = \pm i$

$z = 3 \pm i$ ✓

(b) (i) $\sqrt{-25} = \sqrt{25i^2} = 5i$ ✓

(ii) $\frac{3+2i}{4-3i} = \frac{3+2i}{4-3i} \times \underset{✓}{\frac{4+3i}{4+3i}} = \underset{✓}{\frac{12+8i+9i-6i^2}{16+9}} = \underset{✓}{\frac{1}{25}(18+17i)}$

(c) $\boldsymbol{x} = \begin{pmatrix} 3cos(60^0) \\ 3sin(60^0) \end{pmatrix} = \begin{pmatrix} 1.5 \\ 1.5\sqrt{3} \end{pmatrix}$ ✓

$\boldsymbol{y} = \begin{pmatrix} 2cos(45^0) \\ 2sin(45^0) \end{pmatrix} = \begin{pmatrix} 2/\sqrt{2} \\ 2/\sqrt{2} \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix}$ ✓

$\boldsymbol{x} \cdot \boldsymbol{y} = \begin{pmatrix} 1.5 \\ 1.5\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix}$

$= 1.5\sqrt{2} + 1.5\sqrt{6}$ ✓

Question 13 **(6 marks)**

(a) "*If Donna does not go to the library, then it is not Tuesday*." ✓

(b) Therefore Big Boy has four legs. ✓

(c) Use a proof by contradiction to prove that there are infinitely many prime numbers.

Suppose there are not infinitely many prime numbers.

i.e. there is a finite number of prime numbers $p_1, p_2, p_3, ..., p_n$
Consider the number $x = p_1 \times p_2 \times p_3 \times ... \times p_n + 1$. ✓

x leaves a remainder of 1 when it is divided by any of the prime numbers since each of them divides evenly into $p_1 \times p_2 \times p_3 \times ... \times p_n$ ✓

i.e. x is not divisible by any prime number and leaves a remainder of 1.

This means x is also a prime number. ✓

Contradiction.

Therefore our assumption that there are not infinitely many prime numbers is incorrect. ✓

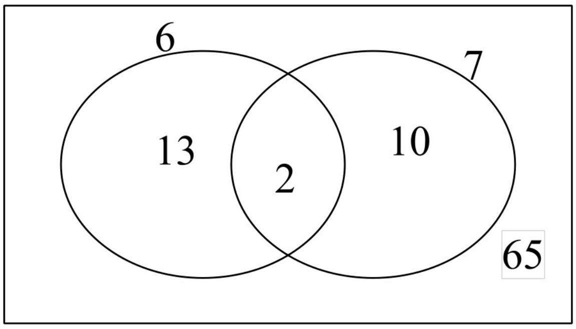
Therefore, there are infinitely many prime numbers.

Question 14 **(9 marks)**

(a) Each initial can take be one of the 26 English letters, meaning there are
26 x 26 = 676 possible values for the first and last. ✓
If the first 676 Concert Hall patrons have different combinations of the first and second letter, the 677th person must match someone. ✓
Therefore if 872 people turn up, there will be at least two people in the hall who have the same first and last initial in their names.

(b) 3! = 6 ✓

(c)



There are 15 multiples of 6 in 90.
There are 12 multiples of 7 in 90. ✓
There are 2 multiples of 42 in 90. ✓
So 90 – 25 = 65 ✓

(d) Prove that $\binom{n-1}{r-1} + \binom{n-1}{r} = \binom{n}{r}$

$$\binom{n-1}{r-1} + \binom{n-1}{r} = \frac{(n-1)!}{(n-r)!(r-1)!} + \frac{(n-1)!}{(n-r-1)!(r)!} \quad ✓$$

$$=(n-1)!\left(\frac{1}{(n-r)(n-r-1)!(r-1)!} + \frac{1}{(n-r-1)!r(r-1)!}\right)$$

$$=\frac{(n-1)!}{(n-r-1)!(r-1)!}\left(\frac{1}{(n-r)} + \frac{1}{r}\right) \quad ✓$$

$$=\frac{(n-1)!}{(n-r-1)!(r-1)!}\left(\frac{r+(n-r)}{(n-r)r}\right)$$

$$=\frac{n(n-1)!}{(n-r)(n-r-1)!r(r-1)!}$$

$$=\frac{n!}{(n-r)!r!} \quad ✓$$

$$=\binom{n}{r}$$

$$\therefore \binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

Question 15 **(6 marks)**

(a) $At\ t = 0,\quad T = 28 - 10\sin\left(\frac{2\pi}{365}(-80.5)\right)$

$T = 37.82$

$T \approx 38^o$ ✓

(b) Using calculator, minimum at t =171.75, about 172 days.

January	31	
February	28	
March	31	90
April	30	121
May	31	151 to here ✓
June	22	173

The minimum expected temperature occurs about June 22 ✓

(c) Intersection at t = 104.4 and t = 239.1

June	30	203
July	31	234

90+14 = 104 April 14-15 ✓

234 +5 =239 August 5 ✓

The temperature is expected to be less than 24° between approximately April 15 and August 5.

(c) At T = 32, t = 56.594 and t =286.906 ✓

$$\frac{56.594 + (365 - 286.906)}{365} \times 100 = 36.9\%$$ ✓

About 37% of days have temperatures greater than 32°.

Question 16 **(4 marks)**

Prove $\dfrac{\sin(2x)}{1+\cos(2x)} = \dfrac{1-\cos(2x)}{\sin(2x)}$

$$\frac{\sin(2x)}{1+\cos(2x)} = \frac{2\sin(x)\cos(x)}{1+2\cos^2(x)-1} \quad \checkmark$$

$$= \frac{2\sin(x)\,\cancel{\cos(x)}}{2\cos(x)\,\cancel{\cos(x)}}$$

$$= \frac{\sin(x)}{\cos(x)} \times \frac{2\sin(x)}{2\sin(x)} \quad \checkmark$$

$$= \frac{2\sin^2(x)}{\sin(2x)} \quad \checkmark \qquad but\cos(2x) = 1-2\sin^2(x)$$

$$= \frac{2\sin^2(x)-1+1}{\sin(2x)} \quad \checkmark$$

$$\frac{\sin(2x)}{1+\cos(2x)} = \frac{1-\cos(2x)}{\sin(2x)}$$

Question 17 **(14 marks)**

(a) (i)

$$\begin{pmatrix} \cos(45^o) & -\sin(45^o) \\ \sin(45^o) & \cos(45^o) \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \underset{\checkmark}{\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}}\underset{\checkmark}{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$= \begin{pmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \checkmark$$

(ii) Area of triangle ABC $= \dfrac{1}{2} \times 1 \times 1 = \dfrac{1}{2}$ ✓

$$\left|\begin{pmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\right| = \left|-1/2 - 1/2\right| = 1$$

Therefore the areas are the same ✓

The area of triangle A"B"C"D is 0.5 units^2. ✓

(iii) $\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}^{-1} = \frac{1}{-1}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

✓✓ -1/error

(b) (i) $\begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$ ✓✓ -1/error

(ii) $\left|\begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}\right| = |(-6-1)| = 7$ ✓

✓

Therefore quadrilateral $A'B'C'D'$ has an area 7 times the size of quadrilateral $ABCD$.

(c) (i) $\begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix}^{-1}$ does not exist because the determinant is equal to 0. ✓

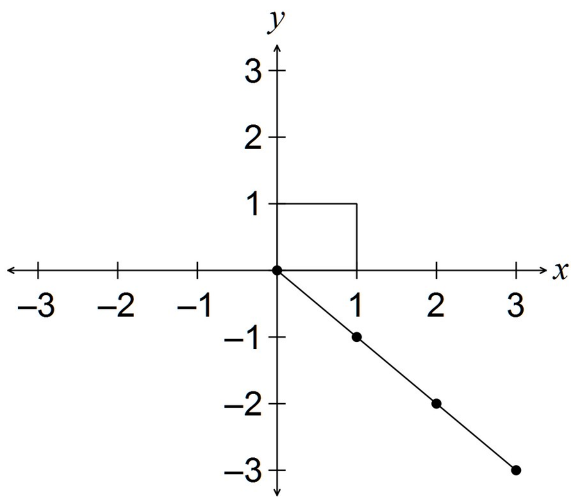
This means the matrix to transform quadrilateral A'B'C'D' back to quadrilateral ABCD cannot be found. ✓

(ii) $\begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x+y \\ -2x-y \end{pmatrix}$ ✓

This means $y = -x$ and that all four points have been collapsed to a line.

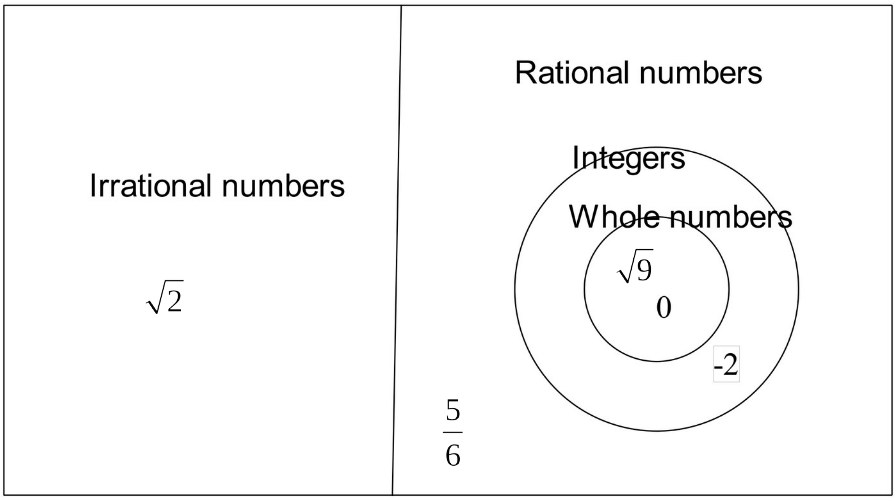
We cannot de-collapse the line to produce the quadrilateral. ✓

For example (0,0), (1,0), (1,1), (0,1) (0,0), (2, -2), (3, -3),(1,-1)
which is graphed below:



Question 18 **(8 marks)**

(a) (i)



✓✓ -1/error

(ii) A transcendental number does not fit into the set of numbers above.
For example π, e

(b) Prove using mathematical induction that $11^n - 6$ is divisible by 5.

Test for $n = 1$.

$11^1 - 6 \rightarrow 5$

So divisible by 5 for $n = 1$. ✓

Assume valid for n i.e. that $11^n - 1 = 5k$ ✓

Test for $"n" = "n+1"$

$11^n - 6 \rightarrow 11^{n+1} - 6$

$11^{n+1} - 6 = 11^n 11 - 6$

$= (5k+1)11 - 1$ ✓

$= 11 \times 5k + 11 - 1$

$= 11 \times 5k + 10$ which is a multiple of 5. ✓

So, if valid for *n*, then valid for *n*+1.

✓

Valid for *n* = 1, so valid for *n* = 2 etc.

Therefore $11^n - 6$ is divisible by 5.

Question 19 **(7 marks)**

(a) $$\begin{pmatrix} \cos(270^o) & -\sin(270^o) \\ \sin(270^o) & \cos(270^o) \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$$ ✓ ✓

$$=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$$ ✓

$$=\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix}$$ ✓

(b) (i) $$\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}=\begin{pmatrix} 3 \\ 3 \end{pmatrix}$$ ✓ Therefore P''(3, 3)

(ii) $$\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix}^{-1}=\frac{1}{6}\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$$ ✓

End of solutions