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Rossmoyne Senior High School

WA Exams Practice Paper B, 2015

Question/Answer Booklet

MATHEMATICS METHODS UNITS 1 AND 2
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Question 20 **(10 marks)**

A function is given by $f(x)=1+24x-30x^2+16x^3-3x^4$.

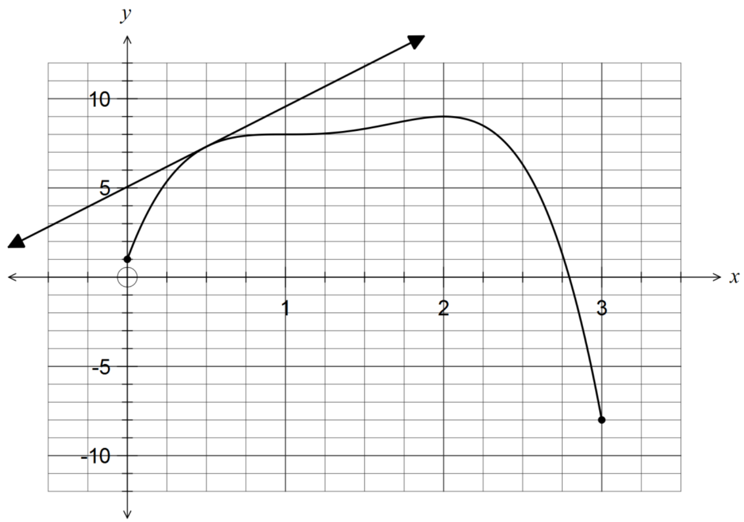
(a) Use calculus techniques to determine the coordinates of all stationary points of the function. (3 marks)

$f'(x)=24-60x+48x^2-12x^3$

$f'(x)=0$ when $x=1,\ x=2$

Stat pts at $(1,\ 8)$ and $(2,\ 9)$

(b) Sketch the graph of $y=f(x)$ for $0\le x\le 3$ on the axes below. (4 marks)



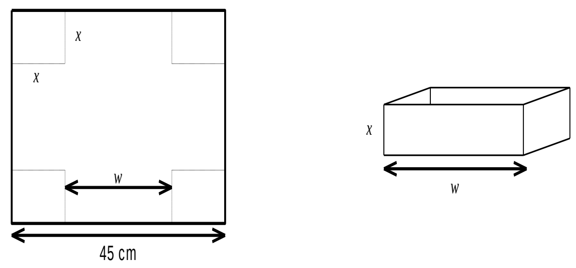
(c) Determine the equation of the tangent to the curve $y=f(x)$ when $x=0.5$ and draw the tangent on the graph in part (c). (3 marks)

$$y=\frac{9x}{2}+\frac{81}{16}$$
$$=4.5x+5.0625$$

End of questions

Question 19 **(7 marks)**

A square sheet of metal has sides of length 45 cm. An open box, with a square base of side w cm, is made by cutting squares with sides of x cm out of the corners of the metal sheet and folding up the sides.



(a) Explain why $w = 45 - 2x$. (1 mark)

Width of box is width of sheet (45 cm) less two corners ($2x$).

(b) Show that the volume of the open box is given by $V = 4x^3 - 180x^2 + 2025x$ cm³. (2 marks)

$$\begin{aligned} V &= LWH \\ &= w \cdot w \cdot x \\ &= (45-2x)(45-2x)x \\ &= 4x^3 - 180x^2 + 2025x \end{aligned}$$

(c) Using calculus techniques, determine the dimensions of the open box that has the maximum possible volume and state what this volume is. (4 marks)

$$\frac{dV}{dx} = 12x^2 - 360x + 2025$$

$$0 = 3x^2 - 180x + 2025 \Rightarrow x = 7.5, \ \cancel{x = 22.5}$$

$$w = 45 - 2(7.5) = 30$$

$V_{max} = 6750$ cm³ when box is 30 by 30 by 7.5 cm

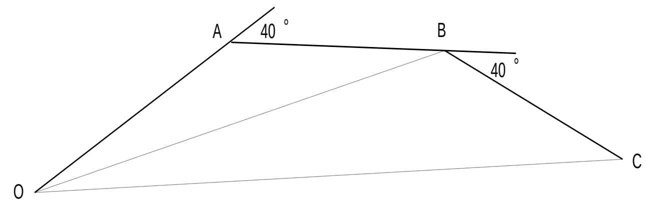
Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

The diagram below shows the path of a student who was walking on a level playing field. The student left O and walked for 40m to A, where they turned 40º to their right and then walked on for another 35m to B. At B, they turned another 40º to their right and walked 30m to C, where they stopped.



Use trigonometry to show that when the student reached C, the straight line distance back to O was close to 90m.

$OB = \sqrt{40^2 + 35^2 - 2(40)(35)\cos 140°} = 70.498m$

$A\hat{B}O = \sin^{-1}\left(40 \times \frac{\sin 140°}{70.498}\right) = 21.39°$

$O\hat{B}C = 180 - 40 - 21.39 = 118.61°$

$OC = \sqrt{70.498^2 + 30^2 - 2(70.498)(30)\cos 118.61°} = 88.856m$

Hence distance is just under 90m.

Question 9 **(6 marks)**

The pressure, P, in an air bubble varies inversely with the volume, V, of the bubble.

It is known that $P = 2.4$ kPa when $V = 5$ cm^3.

(a) Find the value of the constant k in the equation $P = \frac{k}{V}$. (1 mark)

$$\boxed{\begin{aligned} 2.4 &= \frac{k}{5} \\ 2.4 \times 5 &= k \\ k &= 12 \end{aligned}}$$

(b) Determine

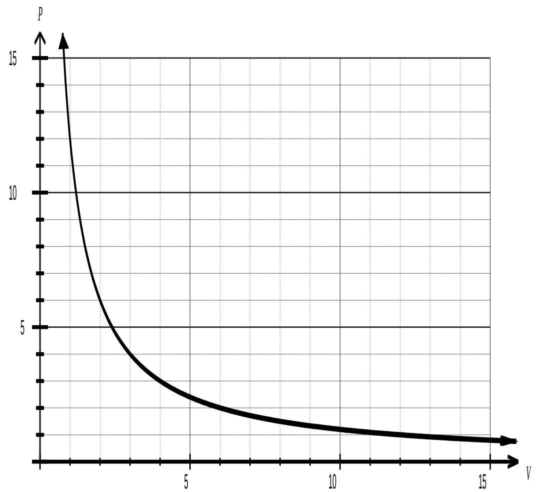
(i) the value of P when $V = 2.5$ cm^3. (1 mark)

$$\boxed{\begin{aligned} P &= \frac{12}{2.5} \\ &= 4.8 \text{ kPa} \end{aligned}}$$

(ii) the value of V when $P = 10$ kPa. (1 mark)

$$\boxed{\begin{aligned} 10 &= \frac{12}{V} \\ V &= 1.2 \text{ cm}^3 \end{aligned}}$$

(c) On the axes below, draw a graph to show how P varies with V. (3 marks)



Question 18 **(8 marks)**

The initial area of a lupin crop, A, in square metres, infested by cowpea aphids was 230 m^2. One week later the area infested had increased to 270 m^2.

(a) Assuming that the area infested is increasing exponentially, determine

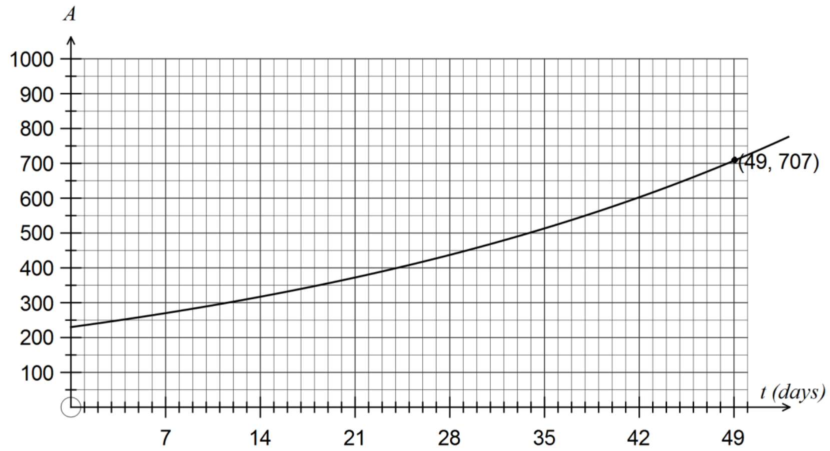
(i) the daily percentage growth rate, rounded to two decimal places. (2 marks)

$$\boxed{\begin{gathered} r^7 = 270 \div 230 \ \Rightarrow\ r = 1.0232 \\ \text{Growth rate is 2.32\% per day.} \end{gathered}}$$

(ii) a formula for A in terms of t, the number of days since observations began. (2 marks)

$$\boxed{A = 230(1.0232)^t}$$

(b) Sketch the graph of the area infected against time for the first 7 weeks on the axes below. (3 marks)



(c) If no measures were taken to control the spread of cowpea aphids, after how many days will more than 1000m^2 of the crop be infested? (1 mark)

$$\boxed{230(1.0232)^t = 1000 \ \Rightarrow\ t = 64.2 \text{ days}}$$

Question 17 **(10 marks)**

(a) The value of an investment, $\$V$, after n whole years in an account paying R% simple interest each year, is given by $V = 5250 + 250(n-1)$.

(i) What was the initial value of the investment? (1 mark)

$\boxed{\$5000}$

(ii) After how many years did the value of the investment reach $6500? (1 mark)

$\boxed{6 \text{ years}}$

(iii) Determine the simple interest rate. (1 mark)

$$\boxed{\frac{250}{5000} \times 100 = 5\% \text{ pa}}$$

(b) An arithmetic sequence has an 9th term of 267 and a 14th term of 237.

(i) The sequence is defined by the rule $T_n = a + (n-1)d$. Determine the values of a and d. (2 marks)

$$d = \frac{237 - 267}{14 - 9} = -6$$

$$a = 267 - 8(-6) = 315$$

(ii) Write a recursive rule for this sequence. (2 marks)

$$\boxed{T_{n+1} = T_n - 6, \quad T_1 = 315}$$

(iii) Calculate T_{50}. (1 mark)

$$\boxed{T_{50} = 21}$$

(iv) If $T_1 + T_2 + ... + T_n = 0$, determine the value of n. (2 marks)

$$\frac{n}{2}\big(2a - 6(n-1)\big) = 0 \Rightarrow n = 0, \ n = 106$$

Solution: $n = 106$

Question 10 **(9 marks)**

A small ball is dropped vertically from a height of 4 metres onto the ground below. The ball rebounds upwards such that the height of each bounce is 80% of the height of the previous bounce.

(a) Determine the height reached by the ball after the first bounce. (1 mark)

$$\boxed{4 \times 0.8 = 3.2 \text{ m}}$$

(b) The height, in metres, reached by the ball after the n^{th} bounce is given by the formula $T_n = ar^{n-1}$. State the values of a and r. (2 marks)

$$a = 3.2$$
$$r = 0.8$$

(c) Determine which bounce is the first to have a height of less than 5 cm. Justify your answer. (2 marks)

$$T_{19} = 0.0576$$
$$T_{20} = 0.0461$$

So bounce 20 is the first less than 5cm.

(d) Determine the total distance travelled by the ball at the instant it hits the ground for the fourth time. (2 marks)

$$4 + 2 \times S_3 = 4 + 2 \times 7.808$$
$$= 19.616 \text{ m}$$

(e) Determine the total distance travelled by the ball until it ceases to bounce. (2 marks)

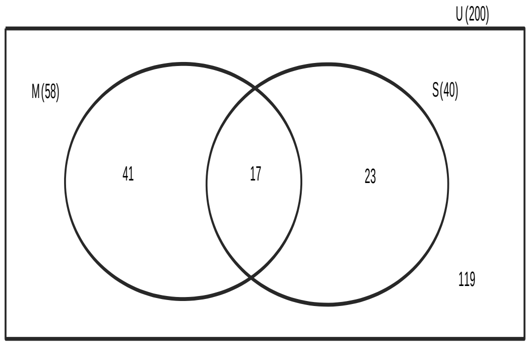
$$4 + 2S_\infty = 4 + 2 \times \frac{3.2}{1 - 0.8}$$
$$= 4 + 2 \times 16$$
$$= 36 \text{ m}$$

Question 11 **(5 marks)**

Two subsets, M and S, belong to a universal set of 200 students. Students belonging to subset M have attended a math revision seminar and students belonging to subset S have attended a science revision seminar.

It is known that $n(M)=58$, $n(S)=40$ and $n(M\cup S)=81$.

(a) Use this information to complete all regions of the Venn diagram below. (2 marks)



(b) If a student is selected at random from the group, determine

(i) $P(\bar{M}\cup S)$ (1 mark)

$$\boxed{\frac{17+23+119}{200}=\frac{159}{200}}$$

(ii) $P(\bar{M}\mid\bar{S})$ (1 mark)

$$\boxed{\frac{119}{41+119}=\frac{119}{160}}$$

(c) A sample of six students who attended a science revision seminar is to be selected for a follow up survey. Determine how many different samples can be selected. (1 mark)

$$\boxed{\binom{40}{6}=3838380}$$

Question 16 **(8 marks)**

The events A and B have the properties $P(A)=\frac{3}{8}$ and $P(A\cup B)=\frac{1}{2}$.

(a) Determine $P(B)$ in each of these cases:

(i) If A and B are mutually exclusive. (1 mark)

$$\begin{aligned} P(B)&=\tfrac{1}{2}-\tfrac{3}{8}\\ &=\tfrac{1}{8}\end{aligned}$$

(ii) If $P(A\cap B)=\frac{3}{40}$. (2 marks)

$$\begin{aligned} P(A\cup B)&=P(A)+P(B)-P(A\cap B)\\ \tfrac{1}{2}&=\tfrac{3}{8}+P(B)-\tfrac{3}{40}\\ P(B)&=\tfrac{20-15+3}{40}\\ &=\tfrac{8}{40}\\ &=\tfrac{1}{5}\end{aligned}$$

(iii) If $P(B\mid A)=\frac{1}{6}$. (3 marks)

$$\begin{aligned} P(B\cap\bar{A})&=\tfrac{1}{8}\\ x&=P(B)\\ \\ P(A\cap B)&=x-\tfrac{1}{8}\\ P(B\mid A)&=\left(x-\tfrac{1}{8}\right)\div\tfrac{3}{8}\\ \tfrac{1}{6}\times\tfrac{3}{8}&=x-\tfrac{1}{8}\\ x=P(B)&=\tfrac{3}{16}\end{aligned}$$

(b) For the case where $P(A\cap B)=\frac{3}{40}$, are A and B independent? Justify your answer. (2 marks)

Yes, as $P(A)\times P(B)=P(A\cap B)$

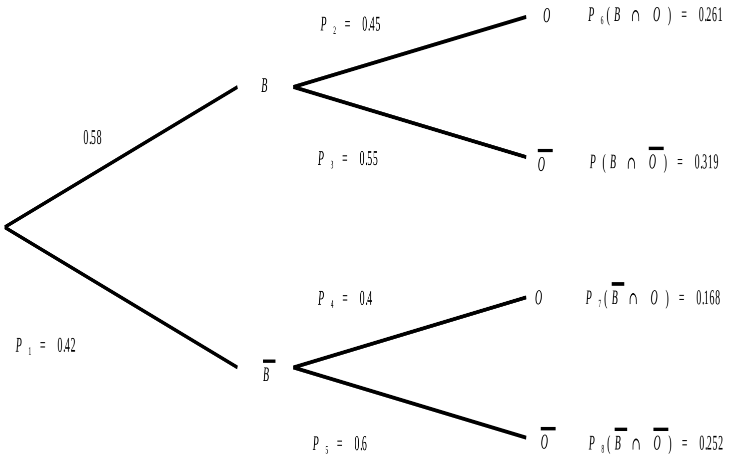
$$\tfrac{3}{8}\times\tfrac{1}{5}=\tfrac{3}{40}$$

Question 15 **(8 marks)**

The clinical records of a large eye hospital indicate that

- 58% of patients are blue eyed (set B)
- 42.9% of patients belong to the blood group O (set O)
- 31.9% of patients are blue eyed and do not belong to blood group O

(a) Use this information to complete the probabilities P_1 to P_8 in the tree diagram below. (4 marks)



(b) What is the probability that a randomly selected patient will

(i) belong to blood group O and have blue eyes? (1 mark)

$\boxed{0.261}$

(ii) have blue eyes or belong to blood group O? (1 mark)

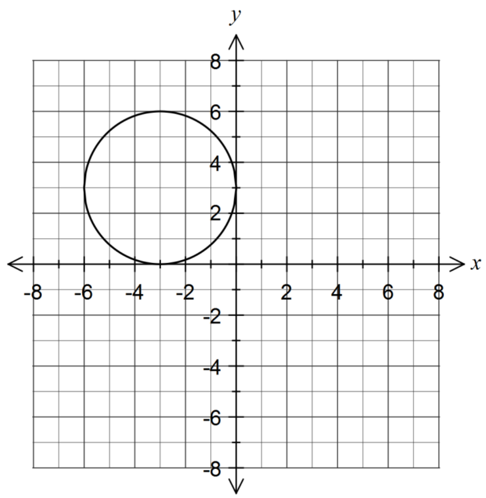
$\boxed{1 - 0.252 = 0.748}$

(iii) not have blue eyes, given they do not belong to blood group O? (2 marks)

$$\frac{0.252}{0.319 + 0.252} = \frac{0.252}{0.571}$$
$$= 0.4413$$

Question 12 **(7 marks)**

(a) Sketch the graph of $(x+3)^2 + (y-3)^2 = 3^2$. (3 marks)



(b) State two functions that combine to form the graph of $(y-2)^2 = x+3$. (2 marks)

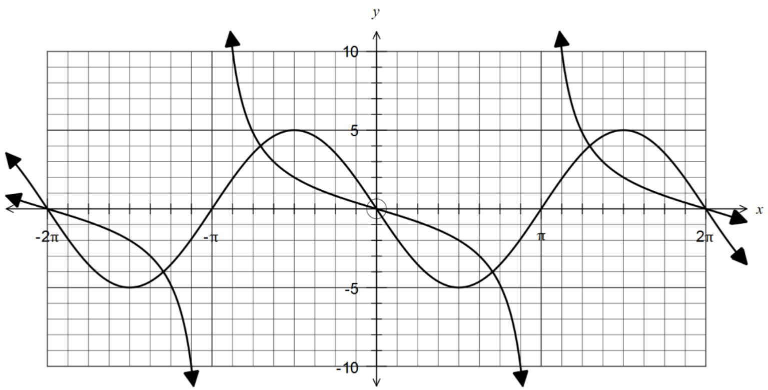
$$y = 2 + \sqrt{x+3}$$
$$y = 2 - \sqrt{x+3}$$

(c) Determine the coordinates of the points of intersection of the line $y + 16 = 7x$ and the circle given by $x^2 + y^2 + 4x + 10y + 4 = 0$. (2 marks)

Graph or solve simultaneously to get (1, -9) and (2, -2).

Question 13 **(9 marks)**

The function $f(x) = a\tan(bx)$ has been graphed below.



(a) Determine the values of the constants a and b. (3 marks)

Period of $\tan x$ is π, $f(x)$ is 2π, so $b = \frac{1}{2}$.

$$f(\tfrac{\pi}{2}) = a\tan(\tfrac{1}{2}\cdot\tfrac{\pi}{2})$$
$$-2 = a\tan(\tfrac{\pi}{4})$$
$$a = -2$$

$a = -2,\ b = \frac{1}{2}$

(b) On the same axes, sketch the graph of $y = 5\cos\left(x + \frac{\pi}{2}\right)$. (3 marks)

(c) State the number of solutions to the equation $5\cos\left(x + \frac{\pi}{2}\right) = f(x)$ over the domain $-\pi \le x \le \pi$. (1 mark)

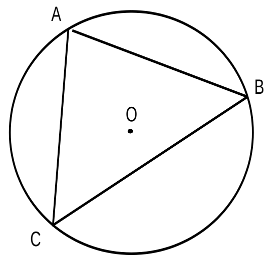
3 solutions

(d) Solve $5\cos\left(x + \frac{\pi}{2}\right) = f(x)$, $\pi < x < 2\pi$, giving your answer(s) correct to three decimal places. (2 marks)

$x = 4.069$

Question 14 **(6 marks)**

A triangle is inscribed in a circle, centre O, with minor arcs AB, BC and CA having lengths 5π, 8π and 5π cm respectively.



(a) Show that the radius of the circle is 9 cm. (1 mark)

$$C = 2\pi r$$
$$18\pi = 2\pi r$$
$$r = 9 \text{ cm}$$

(b) Show that $\angle CAB = 80°$. (3 marks)

$$\angle AOB = \frac{5}{18} \times 360° = 100°$$

$$\angle OAB = \frac{180° - 100°}{2} \text{ (isosceles triangle)} = 40°$$

$$\angle BAC = 2 \times 40° = 80°$$

(c) Determine the area of triangle ABC. (2 marks)

$$AB^2 = 9^2 + 9^2 - 2 \times 9 \times 9 \times \cos 100°$$
$$AB = 13.789$$

$$Area = \frac{1}{2} \times 13.789 \times 13.789 \times \sin 80°$$
$$= 93.624$$
$$\approx 93.6 \text{ cm}^2$$