



PERTH COLLEGE

Year 12

Semester One Examination 2010

Question/Answer booklet

MATHEMATICS 3CMAS/3DMAS

Section One (calculator - free)

Student Name: _____

Time allowed for this section

Reading time before commencing work: 5 minutes

Working time for paper: 50 minutes

Total Pages: 12 pages

Total Questions: 10 questions

Total Marks: 40 marks

Material required/recommended for this section

To be provided by the supervisor

Question/answer booklet for Section One

Formula sheet which may also be used for Section Two

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

STRUCTURE OF THIS PAPER

| Section | Number of questions available | Number of questions to be answered | Suggested working time (minutes) | Marks available |
|---------------------------------|-------------------------------|------------------------------------|----------------------------------|-----------------|
| Section One: Calculator-free | 6 | 6 | 50 | /40 |
| Section Two: Calculator-assumed | 14 | 14 | 100 | 80 |
| | | | Total | /120 % |

Instructions to candidates

1. Answer the questions in the spaces provided.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer
 - a. Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - b. Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.

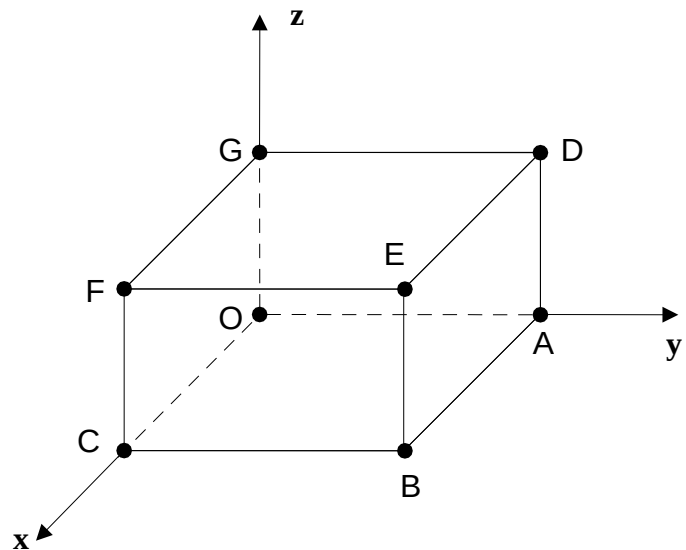
Question 1 [1 + 1 + 1 + 3 = 6 marks]

The figure below shows the cuboid OABCGDEF, with:

$$\overline{OA}=3, \overline{OC}=4 \text{ and } \overline{OG}=2.$$

Given that:

$$i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



express the following in terms of i, j and k .

(a) $\overrightarrow{OE} = i$
[1]

(b) $\overrightarrow{DF} = i$
[1]

(c) $\overrightarrow{AF} = i$
[1]

(d) If M is the mid-point of \overline{DE} and N is the mid-point of \overline{FC} , then find $|MN|$. [3]

Question 2 [2 + 1 + 1 + 5 = 9 marks]

An object moves with constant speed, in ms^{-1} , and its position r_1 from the origin at any time t , is given by:

$$\frac{2-x}{3} = \frac{y+1}{2} = \frac{z}{5}$$

(a) What is the velocity vector of this object? [2]

(b) What is the position vector of this object at $t = 4$? [1]

The position vector of another object is given by: $r_2 = \begin{pmatrix} -8 \\ 29 \\ 20 \end{pmatrix} + t \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$

(c) What is the relative velocity of the second object with respect to the first? [1]

(Question 2 continued)

(d) Show that these two object collide, and find the time and position where this occurs.

[5]

Question 3 [4 marks]

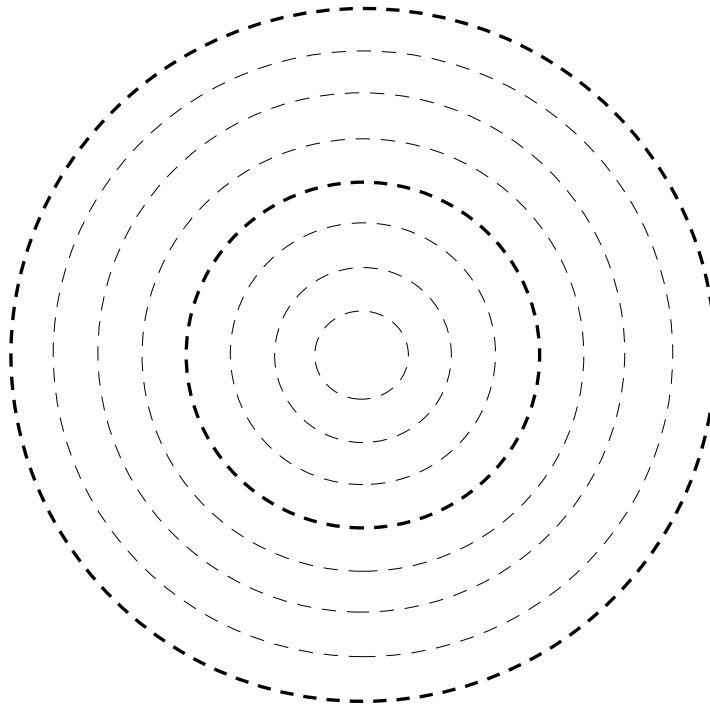
Consider the following two curves:

$$r = \frac{3\pi}{4}$$

$$r = \frac{\theta}{2}$$

- (a) Sketch the graphs of these functions using the axes below for $0 \leq \theta \leq 2\pi$.
Clearly indicate where these curves intersect both axes.

[3]



- (b) Find the coordinates of the point of intersection of these two curves.

[1]

Question 4 [3 + 2 + 2 = 7 marks]

A particle follows an elliptical path described by the parametric equations given below.

$$x(t) = 2 \cos t \quad \text{and} \quad y(t) = \sin t$$

(a) Find the Cartesian equation of the path described by this particle. [3]

(b) Obtain $\frac{dy}{dx}$ in terms of x and y . [2]

(c) Evaluate $\frac{dy}{dx}$ for $t = \frac{\pi}{4}$. [2]

Question 5 [7 marks]

Choose an appropriate trigonometric substitution to determine:

$$\int \frac{\sqrt{4-x^2}}{3} dx \quad [7]$$

Show ALL working.

Question 6 [7 marks]

The curves $\sin x$ and $\cos x$ intersect twice in the interval $0 < x < 2\pi$.
Determine the exact area enclosed by the curves within the two intersections.
Show ALL working.

[7]

END OF SECTION ONE

(Deliberately left blank)

(Deliberately left blank)

(Deliberately left blank)