John Wollaston Anglican Community School

Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section Two:
Calculator-assumed

SOLUTIONS

WA student number: In figures

In words ____________

Your name ____________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

Question number: _________

Question 21 **(8 marks)**

A squad of 6 cyclists is to be chosen at random from 17 applicants. 3 of applicants live in Tasmania, 6 live in WA and the rest live in Queensland.

(a) Determine the number of different squads that can be chosen. (2 marks)

Solution
$\binom{17}{6} = 12\,376$
Specific behaviours
✓ indicates use of combination formula ✓ correct number

(b) Determine the number of different squads that can be chosen that

(i) include all the Tasmanians. (2 marks)

Solution
$\binom{3}{3}\binom{14}{3} = 1 \times 364$ $= 364$
Specific behaviours
✓ indicates correct method ✓ correct number

(ii) include an equal number of cyclists from each of the states. (2 marks)

Solution
$\binom{3}{2}\binom{6}{2}\binom{8}{2} = 3 \times 15 \times 28$ $= 1\,260$
Specific behaviours
✓ indicates correct method ✓ correct number

(iii) have at least 5 cyclists from Queensland. (2 marks)

Solution
$\binom{8}{5}\binom{9}{1} + \binom{8}{6}\binom{9}{0} = 56 \times 9 + 28 \times 1$ $= 504 + 28$ $= 532$
Specific behaviours
✓ indicates correct method ✓ correct number

End of questions

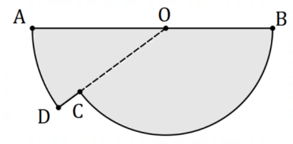
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

Shape $AOBCDA$ below consists of sector BOC of circle centre O joined to sector DOA of a different circle, also centre O. AB is a straight line of length 65 cm, arc AD is 12 cm long and $\angle AOD = 0.32$ radians.



(a) Determine the length OA. (2 marks)

Solution
Let $OA = R$ so that $0.32R = 12$ $R = 37.5$ cm
Specific behaviours
✓ correct use of arc length ✓ correct length

(b) Determine the area of the shape. (3 marks)

Solution
$A_{DOA} = \frac{1}{2} \times 37.5^2 \times 0.32$ $= 225$ Let $OB = r$ $r = 65 - 37.5$ $= 27.5$ $A_{BOC} = \frac{1}{2} \times 27.5^2(\pi - 0.32)$ $= 1067$ $\text{Area} = 225 + 1067$ $= 1\,292\text{ cm}^2$
Specific behaviours
✓ area of sector DOA ✓ radius and angle of sector BOC ✓ area of shape

See next page

Question 10 **(8 marks)**

The height h metres of a particle above level ground is defined as a function of time t seconds as follows:

$$h(t) = 68.75 + 15t - 5t^2, \qquad 0 \le t \le 5.5\,.$$

(a) Determine the height of the particle when

(i) $t = 0$. (1 mark)

(ii) $t = 4.5$. (1 mark)

Solution
$h(0) = 68.75\text{ m}$ $h(4.5) = 35\text{ m}$
Specific behaviours
✓ (i) correct ✓ (ii) correct

(b) Determine the maximum height reached by the particle and the time it reached this height. (2 marks)

Solution
From graph of $h(t)$: Maximum height: $h = 80\text{ m}$ when $t = 1.5\text{ s}$.
Specific behaviours
✓ correct height ✓ correct time

(c) Determine the time(s) that the particle was at a height of 75 m. (2 marks)

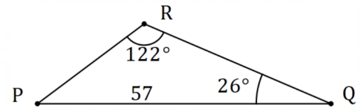
Solution
From graph of $h(t)$: $h = 75$ when $t = 0.5\text{ s}, 2.5\text{ s}$
Specific behaviours
✓ one time ✓ both times

(d) State the range of the function $h(t)$ for the given domain. (2 marks)

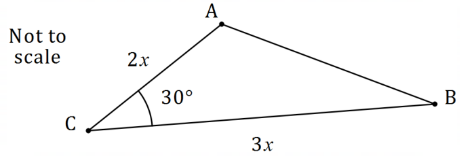
Solution
Range of h: $0 \le h \le 80$
Specific behaviours
✓ upper limit ✓ lower limit, correct inequality

Question 20 **(8 marks)**

(a) Determine the area of triangle PQR when $\angle PQR = 26°$, $\angle PRQ = 122°$ and $PQ = 57$ cm. (4 marks)

Solution
 $\dfrac{q}{\sin 26°} = \dfrac{57}{\sin 122°} \Rightarrow q = 29.46$ $\text{Area} = \dfrac{1}{2}(57)(29.46)\sin(32°)$ $= 445\text{ cm}^2$
Specific behaviours
✓ sketch of triangle ✓ correct use of sine rule ✓ length of second side ✓ correct area

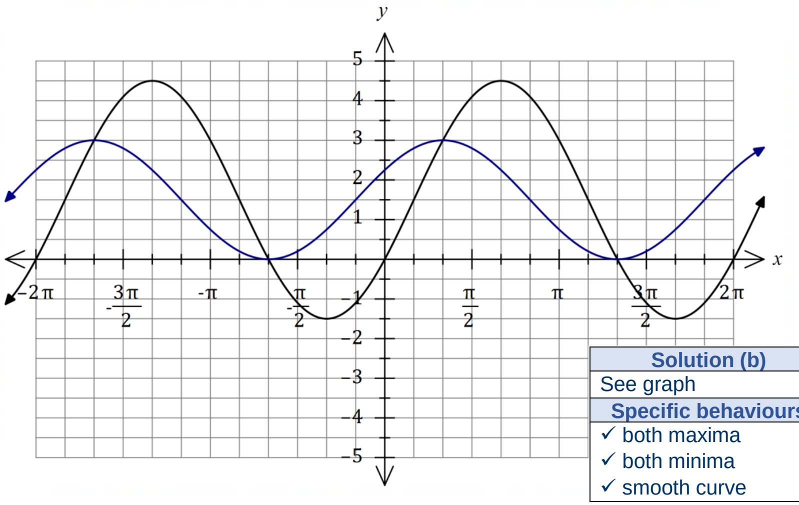
(b) The area of triangle ABC is 96 cm^2, $\angle ACB = 30°$ and $2BC = 3AC$ as shown in the diagram. Determine the length of AB. (4 marks)



Solution
$\dfrac{1}{2}(2x)(3x)\sin(30°) = 96$ $x = 8$ $AB^2 = 16^2 + 24^2 - 2(16)(24)\cos(30°)$ $AB = 12.92\text{ cm}$
Specific behaviours
✓ area equation ✓ value of x ✓ cosine rule ✓ length of AB

Question 19 **(8 marks)**

The graph of $y = a + b\sin(x - c)$ is drawn below, where a, b and c are positive constants.



Solution (b)
See graph
Specific behaviours
✓ both maxima ✓ both minima ✓ smooth curve

(a) Determine the value of a, the value of b and the value of c, where $c < \pi$. (3 marks)

Solution
$a = 1.5, \quad b = 3, \quad c = \frac{\pi}{6}$
Specific behaviours
✓ value of a ✓ value of b ✓ value of c

(b) On the same axes, draw the graph of $y = a + \frac{b}{2}\sin(x + c)$. (3 marks)

(c) Solve $b\sin(x - c) = \frac{b}{2}\sin(x + c)$ for $-\pi \le x \le \pi$. (2 marks)

Solution
Using intersection of graphs: $x = -\frac{2\pi}{3}, \quad x = \frac{\pi}{3}$
Specific behaviours
✓ a correct solution, anywhere ✓ two solutions as given

Question 11 **(6 marks)**

Two events are such that $P(X) = 0.2$, $P(Y) = 0.5$ and $P(Y|X) = 0.1$.

Determine the probability that

(a) both events occur. (2 marks)

Solution
$P(Y\|X) = \frac{P(X \cap Y)}{P(X)} \Rightarrow P(X \cap Y) = 0.2 \times 0.1 = 0.02$
Specific behaviours
✓ indicates use of conditional formula ✓ correct probability

(b) at least one event occurs. (2 marks)

Solution
$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$ $= 0.2 + 0.5 - 0.02$ $= 0.68$
Specific behaviours
✓ indicates use of rule ✓ correct probability

(c) neither event occurs. (1 mark)

Solution
$P(\bar{X} \cap \bar{Y}) = 1 - P(X \cup Y)$ $= 1 - 0.68$ $= 0.32$
Specific behaviours
✓ correct probability

(d) X occurs given that Y has occurred. (1 mark)

Solution
$P(X\|Y) = \frac{P(X \cap Y)}{P(Y)}$ $= \frac{0.02}{0.5}$ $= 0.04$
Specific behaviours
✓ correct probability

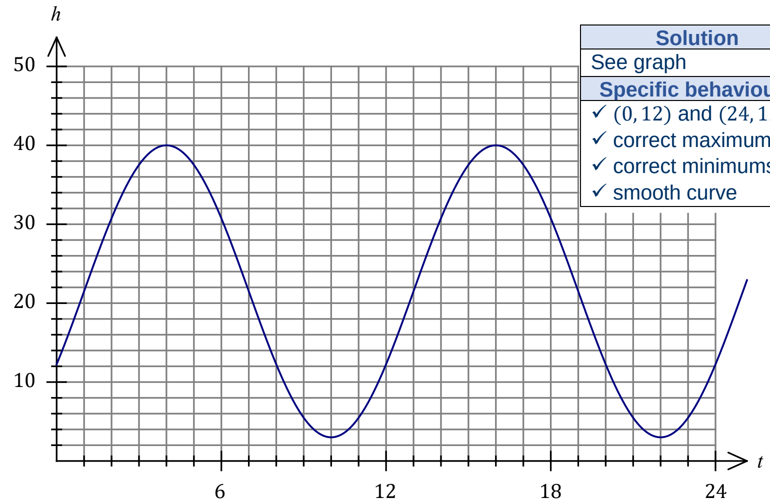
Question 12 **(8 marks)**

The height above ground level, h m, of a seat on a steadily rotating Ferris wheel t minutes after the wheel begins to move is given by $h = 21.5 - 18.5\cos\left(\frac{\pi t}{6} + \frac{\pi}{3}\right)$.

(a) Determine the initial height of the seat. (1 mark)

Solution
$h(0) = 12.25$ m
Specific behaviours
✓ correct height

(b) Graph the height of the seat against time on the axes below. (4 marks)



Solution
See graph
Specific behaviours
✓ $(0, 12)$ and $(24, 12)$
✓ correct maximums
✓ correct minimums
✓ smooth curve

(c) Determine

(i) the maximum height above ground reached by the seat. (1 mark)

Solution
$h_{MAX} = 40$ m
Specific behaviours
✓ correct height

(ii) the time taken, to the nearest second, for the seat to first reach a height of 4 m above ground level. (2 marks)

Solution
$h = 4 \Rightarrow t = 9.37$
$0.37 \times 60 = 22$
$t = 9$ m 22 s $(562$ s$)$
Specific behaviours
✓ time as decimal
✓ time to nearest second

(d) The attendance of Cleo at the next work social is independent of the attendance of anyone else. Determine the probability that none of the three named people attend the next work social. (3 marks)

Solution
$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$ $= 1 - 0.8$ $= 0.2$ Since event C is independent: $P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A} \cap \bar{B}) \times P(\bar{C})$ $= 0.2 \times (1 - 0.85)$ $= 0.03$
Specific behaviours
✓ $P(\bar{A} \cap \bar{B})$
✓ uses independence rule
✓ correct probability

Question 18 **(9 marks)**

The probabilities that Alf, Bess and Cleo will attend the next work social are $P(A) = 0.7$, $P(B) = 0.55$ and $P(C) = 0.85$ respectively. It is also known that $P(A \cap B) = 0.45$.

(a) Determine $P(A \cup B)$. (2 marks)

Solution
$P(A \cup B) = 0.7 + 0.55 - 0.45$ $= 0.8$
Specific behaviours
✓ uses probability rule ✓ correct probability

(b) Describe, in the context of this question, the event $(A \cap \bar{B}) \cup (\bar{A} \cap B)$ and calculate the probability that it happens. (3 marks)

Solution
The event means that either Alf or Bess but not both attend the next social. $P(A \cap \bar{B}) = 0.7 - 0.45 = 0.25$ $P(\bar{A} \cap B) = 0.55 - 0.45 = 0.1$ $P = 0.25 + 0.1 = 0.35$
Specific behaviours
✓ description ✓ one correct part probability ✓ correct answer

(c) State, with justification, whether events A and B are mutually exclusive. (1 mark)

Solution
No, since $P(A \cap B) \neq 0$.
Specific behaviours
✓ uses probability rule for ME events

Question 13 **(6 marks)**

The graph $y = f(x)$, where $f(x) = x^2 + bx + c$ has a turning point at $(2, -7)$.

(a) State the equation of the line of symmetry for the graph of $y = f(x)$. (1 mark)

Solution
$x = 2$
Specific behaviours
✓ correct equation

(b) Determine the value of the constant b and the value of the constant c. (3 marks)

Solution
$f(x) = (x-2)^2 - 7$ $= x^2 - 4x + 4 - 7$ $b = -4$ $c = -3$
Specific behaviours
✓ writes $f(x)$ in squared form ✓ value of b ✓ value of c

(c) The graph of $y = f(x)$ is translated 3 units to the left and 2 units upwards. Determine the equation of the resulting curve. (2 marks)

Solution
New turning point at $(2-3, -7+2) = (-1, -5)$. Equation is $y = (x+1)^2 - 5 = x^2 + 2x - 4$
Specific behaviours
✓ identifies new turning point ✓ correct equation (either form)

Question 14 **(9 marks)**

When a random sample of 173 people from a university were classified according to whether they had a driver's licence (event D) and whether they wore spectacles (event S), it was observed that $n(D) = 140$, $n(S) = 53$ and $n(S \cap \bar{D}) = 10$.

(a) Determine

(i) $n(\bar{S})$. (1 mark)

Solution
$n(\bar{S}) = 173 - 53 = 120$
Specific behaviours
✓ correct number

(ii) $n(D \cap S)$. (1 mark)

Solution
$n(D \cap S) = 53 - 10 = 43$
Specific behaviours
✓ correct number

(b) Determine the probability that a randomly chosen person from the sample

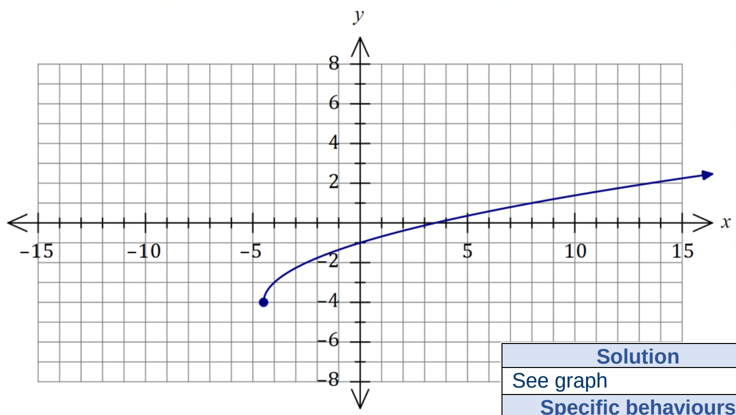
(i) does not have a driver's licence. (2 marks)

Solution
$n(\bar{D}) = 173 - 140 = 33$ $P(\bar{D}) = \dfrac{33}{173} \approx 0.191$
Specific behaviours
✓ numerator ✓ denominator

(ii) wears spectacles given that they have a driver's licence. (2 marks)

Solution
$P(S\|D) = \dfrac{n(D \cap S)}{n(D)} = \dfrac{43}{140} \approx 0.307$
Specific behaviours
✓ numerator ✓ denominator

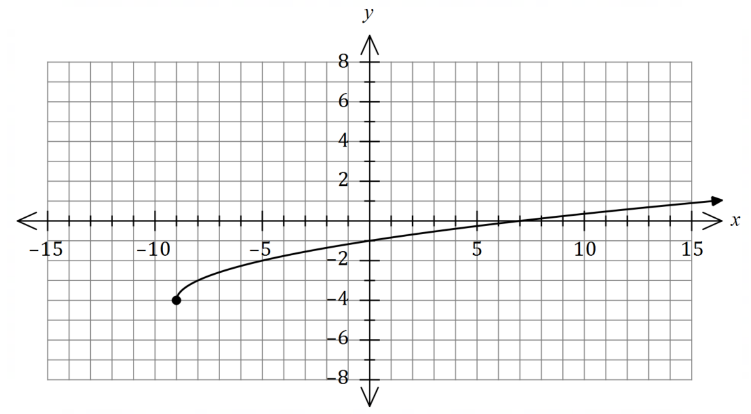
(c) Draw the graph of $y = f(2x)$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ endpoint at $(-4.5, -4)$ ✓ thru' $(0, -1)$ and $(3.5, 0)$ ✓ smooth curve

Question 17 **(8 marks)**

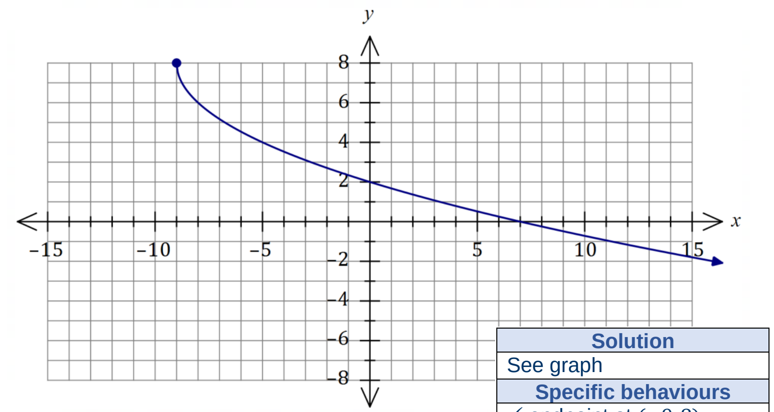
The graph of $y = f(x)$ is drawn below, where $f(x) = \sqrt{x+a} + b$.



(a) Determine the value of the constant a and the value of the constant b. (2 marks)

Solution
$a = 9, \quad b = -4$
Specific behaviours
✓ value of a
✓ value of b

(b) Draw the graph of $y = -2f(x)$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ endpoint at $(-9, 8)$
✓ thru' $(0,2)$ and $(7,0)$
✓ smooth curve

(c) Does the sample provide any indication of possible independence of events S and D? Justify your answer. (3 marks)

Solution
Yes, since $P(S) = \frac{53}{173} \approx 0.306$ and $P(S\|D) \approx 0.307$, it can be seen that the probability that a person wears spectacles barely changes given that they also have a driver's licence. Hence the events are likely to be independent.
Specific behaviours
✓ calculates $P(S)$
✓ compares with $P(S\|D)$
✓ explains why independence indicated

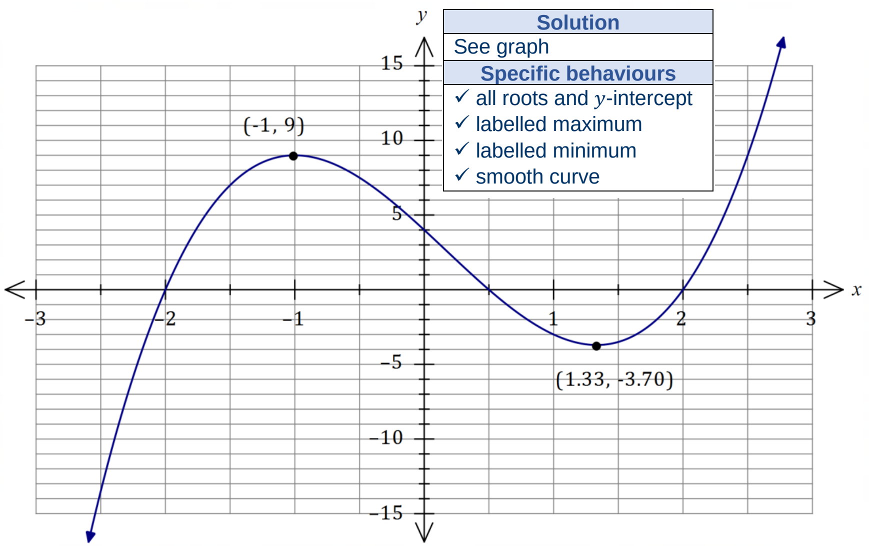
Question 15 **(8 marks)**

A polynomial of degree 3 passes through the points with coordinates $(0, 4), (-2, 0), (2, 0)$ and $(0.5, 0)$.

(a) Determine the equation of the polynomial in expanded form. (4 marks)

Solution
Using roots: $y = a(x+2)(x-2)(x-0.5)$ Use 4th point: $x = 0 \Rightarrow 4 = a(2)(-2)(-0.5)$ $a = 2$ Expand: $y = 2(x+2)(x-2)(x-0.5)$ $= 2x^3 - x^2 - 8x + 4$
Specific behaviours
✓ factored form using roots   ✓ substitutes fourth point   ✓ correct value of a ✓ correct expanded form

(b) Draw the graph of the polynomial on the axes below, indicating the coordinates of all turning points. (4 marks)



Solution
See graph
Specific behaviours
✓ all roots and y-intercept   ✓ labelled maximum   ✓ labelled minimum   ✓ smooth curve

Question 16 **(7 marks)**

Bag A contains 6 red and 4 blue counters. Bag B contains 3 red and 5 blue counters.

(a) A counter is randomly drawn from bag A, replaced and then a second counter randomly drawn from the same bag. Determine the probability that the second counter drawn is blue. (1 mark)

Solution
$P(B) = \frac{4}{10}$
Specific behaviours
✓ correct probability

(b) A counter is randomly drawn from bag B, **not** replaced and then a second counter is randomly drawn from the same bag. Determine the probability that the second counter drawn is red. (3 marks)

Solution
$P(RR) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$ $P(BR) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$ $P(RR \cup BR) = \frac{6+15}{56} = \frac{21}{56} \left(= \frac{3}{8} = 0.375\right)$
Specific behaviours
✓ uses tree or indicates correct method   ✓ either branch correct   ✓ correct probability

(c) A counter is randomly drawn from bag A, its colour noted and then placed in bag B. A second counter is then randomly drawn from bag B. Determine the probability that this counter is the same colour as the first counter drawn. (3 marks)

Solution
$P(RR) = \frac{6}{10} \times \frac{4}{9} = \frac{24}{90}$ $P(BB) = \frac{4}{10} \times \frac{6}{9} = \frac{24}{90}$ $P(RR \cup BB) = \frac{24+24}{90} = \frac{48}{90} \left(= \frac{8}{15} = 0.5\overline{3}\right)$
Specific behaviours
✓ uses tree or indicates correct method   ✓ either branch correct   ✓ correct probability