	Year 12 Specialist
PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	TEST 5 20 Aug 2018 TIME: 50 minutes working One page notes allowed Classpads allowed. 43 Marks 9 Questions

Name:_____ Teacher: _____

Q1 (2 & 2 = 4 marks)
Determine the general solution for the following.

a) $$5y\frac{dy}{dx} = 1 - 7x$$

b) $$\frac{dy}{dx} = \frac{x(1-3x)}{\sin y}$$

Q2 (4 marks)

A hot item, initially at $315^\circ C$, is placed in a room with temperature $21^\circ C$ and left to cool, the temperature $T^\circ C$ of the item t minutes later is given by the differential equation

$$\frac{dT}{dt} = -3(T - 21)$$

Determine how long it will take for the temperature of the item to cool to $100^\circ C$ to the nearest second.

Q3 (2, 4 & 3 = 9 marks)

The logistical growth model is given by the following differential equation.

$$\frac{dy}{dx} = ay - by^2$$

where $a \& b$ are positive constants and $y > 0$

   a) State the y value where the gradient will be zero and hence give the limiting value of y.

   b) Using separation of variables and partial fractions, derive the logistical formula

$$y = \frac{a}{b + Ce^{-ax}}$$

where C is a constant. Show all steps without the use of a classpad.

Q3 continued

c) Given that the Population P of a group of Kangaroos at t years (initially 285 kangaroos) can
be modelled by the logistical growth model $\dfrac{dP}{dt} = \dfrac{1}{4}P - \dfrac{1}{13780}P^2$, determine the time
taken for the population to reach 2000 kangaroos. Use your result from (b)

Q4 (4 marks)
A slope field is plotted below showing a particular line of force through point A(1,0). At point A the
slope field is 6.
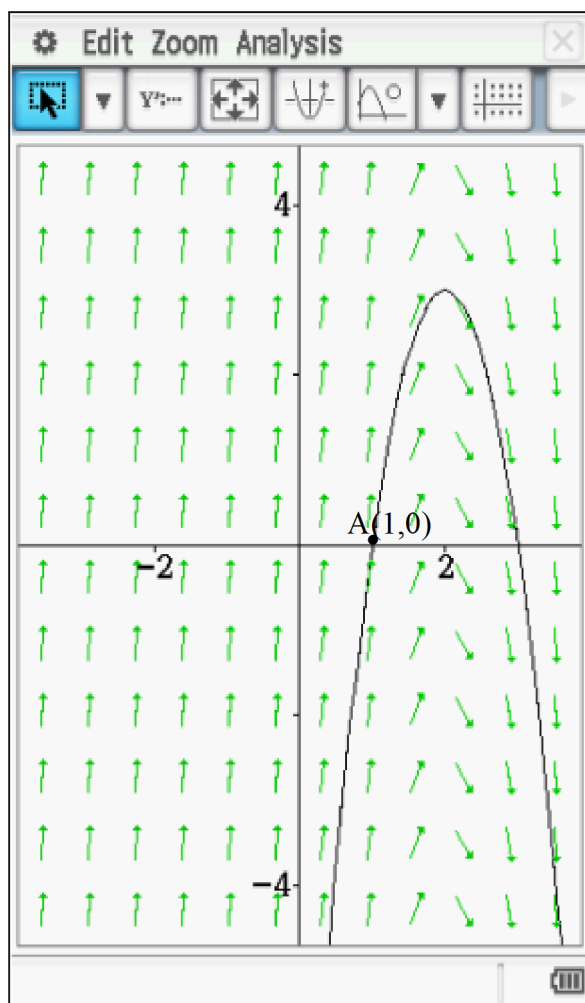
Given that the slopes are horizontal at x=2 and
that the lines of force are parabolic.
Determine the equation of the line of force
through point A and the slope field in terms of x.

Q5 (4 marks)

The Ant-Man is moving in a straight line so that his speed, v metres per second, at displacement x metres form the origin at time t seconds can be described by the following acceleration. The Ant-Man's speed is zero when x=1 metre from the origin.

$$\frac{dv}{dt} = x\left(5 + 3x^2\right)^5$$

Determine the approximate Ant-Man's speed when $x = 5$ metres.

Q6 (4 marks)

A particle is undergoing Simple Harmonic Motion and can be described by $\ddot{x} = -36x$.
Determine what percentage of the time that the particle is **less than three quarters** of the maximum distance from the origin.

Q7 (3 & 2 = 5 marks)

An object is undergoing SHM $\ddot{x} = -4x$ and is initially at rest with $x = 15$ units but with a positive initial acceleration.

Determine.

a) An expression for x in terms of time, t.

b) The distance travelled in the first 10 seconds.

Q8 (3 & 3 = 6 marks)

An object's displacement, x metres at time, t seconds is described by

$$x = 7\cos(3t) - 5\sin(3t)$$

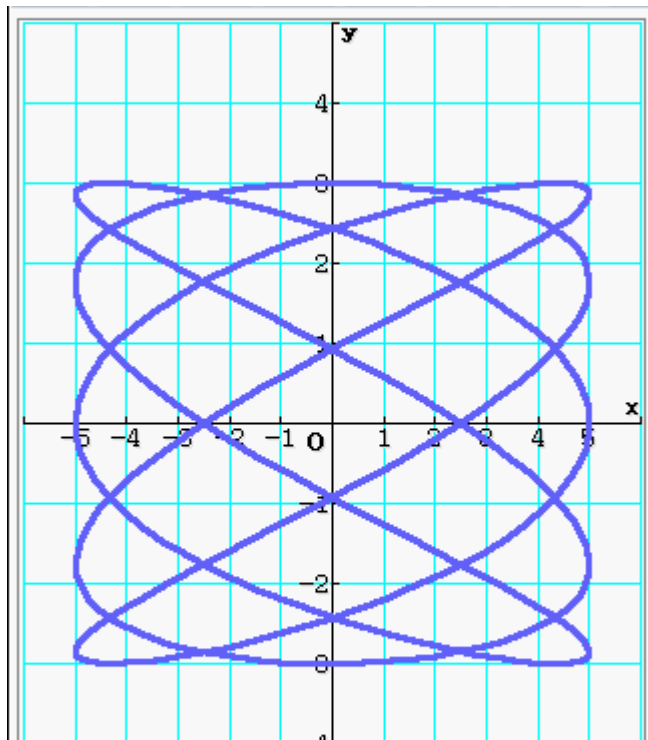a) Show that the motion is Simple Harmonic.

b) Determine the Amplitude and the **exact** speed when $x = 4$ metres.

Q9 (3 marks)

The Iron Man completes a race following a unique race track so that his position vector in metres

at time t seconds is given by $r = \begin{pmatrix} 5\cos\dfrac{2\pi}{3}t \\ 3\sin\dfrac{2\pi}{5}t \end{pmatrix}$ .metres

The motion is graphed as follows.

Determine the time taken to complete one circuit of the race track and the length of this cicuit.