



Semester One Examination 2011  
Question/answer booklet

YEAR 12 MATHEMATICS  
3C/DMAT

Section One  
(Calculator-Free)

Student Name: \_\_\_\_\_ **SOLUTIONS** \_\_\_\_\_

Circle your teacher's name

S. ROWDEN                      N. EDMUNDS

Time allowed for this section

Reading time before commencing work: 5 minutes  
Working time for section: 50 minutes

Material required/recommended for this section

To be provided by the supervisor  
Question/answer booklet for Section One  
Formula sheet

To be provided by the candidate  
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler  
Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One: Calculator-free	10	10	50 minutes	40
Section Two: Calculator-assumed	15	15	100 minutes	80
Total marks				120

Instructions to candidates

1. Answer the questions in the spaces provided.
2. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
3. **Show all your working clearly.**  
Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.  
**For any question or part question worth more than two marks, valid working or justification is required to receive full marks.**  
If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.

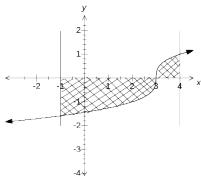
40 Marks

Section One: Calculator-Free

This section has Ten (10) questions. Attempt all questions.

Question 1 (3 marks)

Explain how you would find the area of the region bounded by the x-axis and the equations  $y = 2(x - 2)^2$ ,  $x = -1$  and  $x = 4$ . You are not required to find the area.



Solution
Find the definite integral over the boundary -1 to 4 of the absolute value of the function Or Find the opposite of the definite integral for the boundary -1 to 3 for function plus the definite integral for the boundary 3 to 4 for the function
Specific behaviours
✓ definite integral ✓ correct boundaries ✓ absolute value of function Or ✓ opposite of definite integral of the function between -1 to 3 ✓ definite integral of the function between 3 to 4 ✓ add two areas

Question 5 (5 marks)

Solve the system of equations

$$\begin{aligned}x + y + z &= 9 \\ 3x + y + z &= 1 \\ x + z &= 2y\end{aligned}$$

Question 7 (3 marks)

Determine the value of b for the following, given b is a positive integer:

$$10 = \int_1^b \frac{1}{x^2} dx$$

Solution
$10 = \int_1^b \frac{1}{x^2} dx$ $10 = \left[ -\frac{1}{x} \right]_1^b$ $10 = -\frac{1}{b} - \left( -\frac{1}{1} \right)$ $10 = -\frac{1}{b} + 1$ $9 = -\frac{1}{b}$ $b = -\frac{1}{9}$
Specific behaviours
✓ integrates function ✓ substitutes upper and lower boundaries ✓ solves correctly for b

Solution
$x + y + z = 9$ (1) $3x + y + z = 1$ (2) $x + z = 2y$ (3) $2y - 3(2) = 1$ (4) $2y - 6 = 1$ (5) $2y = 7$ $y = \frac{7}{2}$ (6) $2x + 5z = 26$ $2x + 5\left(\frac{7}{2}\right) = 26$ $2x + \frac{35}{2} = 26$ $2x = 26 - \frac{35}{2}$ $2x = \frac{52}{2} - \frac{35}{2}$ $2x = \frac{17}{2}$ $x = \frac{17}{4}$ $z = 2y - x$ $z = 2\left(\frac{7}{2}\right) - \frac{17}{4}$ $z = 7 - \frac{17}{4}$ $z = \frac{28}{4} - \frac{17}{4}$ $z = \frac{11}{4}$
Specific behaviours
✓ reduces the system of equations to 2 equations with 2 variables ✓ reduces to 1 equation with 1 variable and solves equation ✓ back substitutes to find other 2 variables

Additional working space  
Question number: \_\_\_\_\_

Question 2 (5 marks)  
Differentiate the following: (do not simplify your answers)

(a)  $y = 8x^4 + \frac{2}{x} - \frac{3}{7}$  [1]

Solution
$\frac{dy}{dx} = 32x^3 + 2x^{-1}$
Specific behaviours
✓ correct derivative

(b)  $y = \sqrt{x^2 + \frac{1}{x^2}}$  [2]

Solution
$y = (x^2 + x^{-2})^{\frac{1}{2}}$
$\frac{dy}{dx} = \frac{1}{2}(x^2 + x^{-2})^{\frac{1}{2}-1}(2x + 2x^{-3})$
Specific behaviours
✓ $M(x)^{n-1}$
✓ $f'(x)$

(c)  $y = \frac{2x+1}{(3x+2)^2}$  [2]

Solution
$\frac{dy}{dx} = \frac{2(3x+2)^2 - (2x+1)2(3x+2)2}{(3x+2)^4}$
$= \frac{2(3x+2)^2 - 4(2x+1)(3x+2)}{(3x+2)^4}$
Specific behaviours
✓ applies quotient rule correctly
✓ applies chain rule correctly

Question 8 (3 marks)

Solve for  $y$  the inequality  $y - \frac{1}{y} > \frac{1}{y+1}$

Solution
$\frac{1}{y} - \frac{1}{y+1} > 0$
$\frac{2}{y(y+1)} > 0$
$\frac{y-1}{y} > 0$ or $y < -1$
$y < -1$ or $y > 1$
Specific behaviours
✓ simplifies inequality
✓ identifies critical regions and states correct answer

Question 5 (3 marks)

Find the equation of the tangent to the curve  $y = \frac{3x+2}{1}$  at  $(-1, -1)$ .

Solution
$\frac{dy}{dx} = -3(3x+2)^{-2}$
$\frac{dy}{dx} = -3$
$x = -1$
$\frac{dy}{dx} = -3(-1) = 3$
$y = 4$
$y = -3x + 4$
Specific behaviours
✓ correct derivative
✓ determines correctly the y intercept and hence states equation

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✓ applies chain rule
Specific behaviours
$\int_1^2 x^2 \left( \frac{1}{3} + x^2 \right) dx = \frac{1}{3} \left( \frac{1}{3} + x^2 \right) = \frac{1}{9} + \frac{1}{3}x^2$
Solution

(d)  $\int_2^3 (3 - \sqrt{x}) dx$

[2]

✓ raises the power of the function by one ✓ divides by both raised power and derivative of $3x - 2$
Specific behaviours
$\int (3x - 2)^2 dx = \frac{1}{3} (3x - 2)^3 + C = \frac{1}{3} (9x^2 - 12x + 4) + C = 3x^2 - 4x + \frac{4}{3}C$
Solution

(e)  $\int \frac{1}{(3x - 2)^2} dx$

Question 4 Continued

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✓ uses the chain rule ✓ correctly uses the Fundamental Theorem of Calculus
Specific behaviours
$\frac{d}{dx} \int_0^x (6 - 2x^2) dx = 6 - 2x^2$
Solution

Find  $\frac{d}{dx} \int_0^x t(6 - t) dt$

Question 9 (2 marks)

Question 3 (4 marks)

In a probability experiment, events  $M$  and  $N$  are such that

$P(M) = \frac{1}{4}$ ,  $P(N|M) = \frac{1}{3}$  and  $P(M \cap N) = \frac{1}{4}$

Find

(a)  $P(M \cup N)$

[2]

Solution
$P(M \cup N) = 1 - P(M \cap N) = 1 - \frac{1}{4} = \frac{3}{4}$
Specific behaviours
✓ identifies $P(M \cap N)$ as the complement of $P(M \cup N)$ ✓ correct

(b)  $P(N)$

[2]

Solution
$P(M \cap N) = P(M) + P(N) - P(M \cup N) = P(M) + P(N) - P(M \cap N) \times P(M)$ $\frac{1}{4} = \frac{1}{4} + P(N) - \frac{1}{4} \times \frac{1}{4}$ $P(N) = \frac{7}{12}$
Specific behaviours
✓ determines $P(M \cap N)$ ✓ uses addition rule to correctly determine $P(N)$ ✓

SEE NEXT PAGE

Additional working space

Question number: \_\_\_\_\_

