PRESBYTERIAN LADIES' COLLEGE
A COLLEGE OF THE UNITING CHURCH IN AUSTRALIA

MATHEMATICS DEPARTMENT

Year 12 MATHEMATICS SPECIALIST

TEST 4: DIFFERENTIATION AND DIFFERENTIAL EQUATIONS

DATE: 28th June 2016 Name___________________________

Reading Time: 3 minutes

SECTION ONE: CALCULATOR FREE

TOTAL: 33 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet.

WORKING TIME: 30 minutes (maximum)

SECTION TWO: CALCULATOR ASSUMED

TOTAL: 25 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing instruments, templates, up to 3 Calculators,

1 A4 page of notes (one side only), SCSA formula sheet.

WORKING TIME: 20 minutes (minimum)

SECTION 1 Question	Marks available	Marks awarded	SECTION 2 Question	Marks available	Marks awarded
1	6		5	8	
2	6		6	8	
3	11		7	9	
4	10				
Total	33			25	

Section One: Calculator-free **[33 marks]**

This section has **four (4)** questions. Answer **all** questions. Write your answers in the spaces provided.

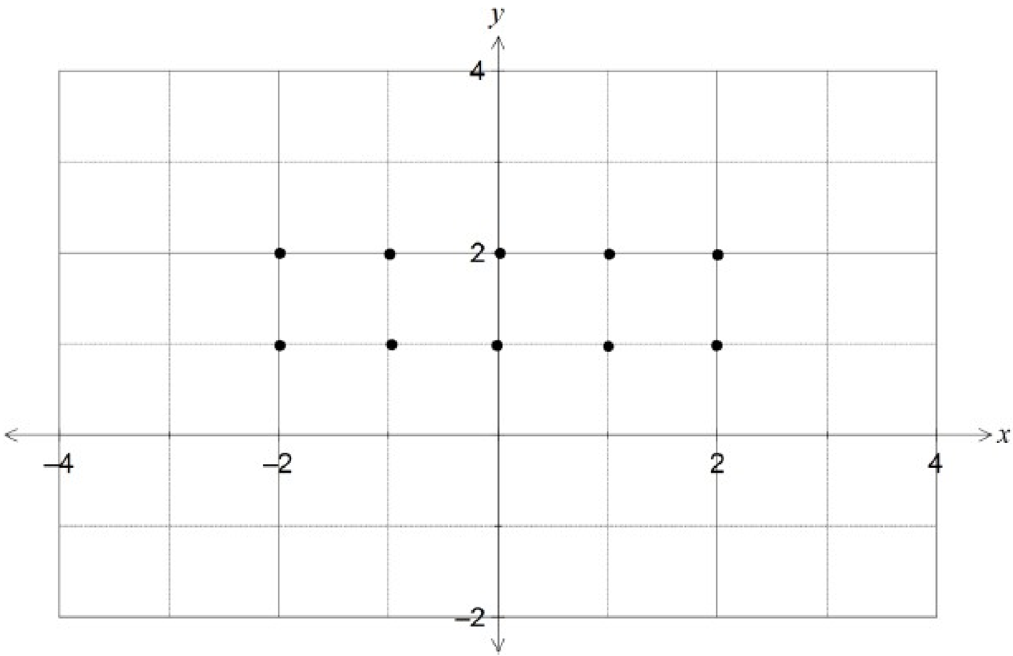
Question 1 [6 marks]

A first order differential equation is given by $\frac{dy}{dx} = xy$

(a) Use the equation to complete the table below. [2]

x	-2	-1	0	1	2	3
y	2	2	2	2	2	3
$\frac{dy}{dx}$						

(b) Create a slope field on the 10 points on the graph below. [2]



(c) Find the solution that passes through the point given by $x=1$ and $y=1$. [2]

Question 2 [6 marks]

A function is defined parametrically by the equations $x(t)=t^2+2t$ and $y(t)=t^3-9t$

(a) Find $\frac{dy}{dx}$ in terms of t [2]

(b) By finding the second derivative, $\frac{d^2y}{dx^2}$ in terms of t, show that there are no points of inflection on this curve. [4]

Question 3 [11 marks]

The equation of a curve in the plane is $x^2+3y^2+2xy=12$.

(a) Show that for all points on the curve $(3y+x)\frac{dy}{dx}=-x-y$. [4]

(b) Find the equation of the tangent to the curve at the point $(0, 2)$. [3]

(c) At what points on the curve is the tangent parallel to the *y*-axis? [4]

Question 4 [10 marks]

The volume *V* of blood flowing through an artery in unit time can be modelled by the formula $V = \mathbf{kr}^4$, where *r* is the radius of the artery and *k* is a constant.

(a) What is the effect on the volume of blood flow if the radius of the artery is halved?

[2]

(b) Use the incremental formula to estimate the percentage decrease in the radius of a

partially clogged artery that will produce a 10% decrease in the flow of blood.

[5]

(c) Show that the incremental formula gives a physically absurd estimate for the change

in *V* resulting from a halving of the radius of the artery. Explain why this

estimate is so poor compared to the true answer found in (a).

[3]

YEAR 12 MATHS SPECIALIST TEST 4 2016

NAME:____

Section Two: Calculator-assumed **[25 marks]**

This section has **three (3)** questions. Answer **all** questions. Write your answers in the spaces provided

---

Question 5 [8 marks]

The needle in a sewing machine moves vertically with simple harmonic motion, and the distance between the highest and lowest positions of the tip is 8 mm.

The height of the tip of the needle above its mid-point position t seconds after it starts to move is $x(t)$ mm, where $x(t)$ satisfies the differential equation

$$\frac{d^2x}{dt^2} = -16\pi^2 x.$$

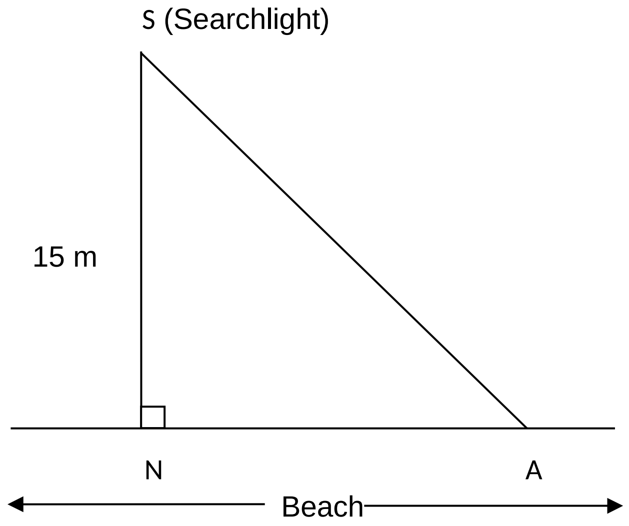
(a) Determine $x(t)$, given that the needle starts at its highest point. [3]

(b) How long does it take for the needle to return to its highest point? [2]

(c) How far does the tip travel in the first 0.3 seconds? [3]

Question 6 [8 marks]

A searchlight S is just above sea level and is revolving in the horizontal plane. The searchlight is located 15 metres out to sea from the nearest point N on a straight beach. S and N are in the same horizontal plane and the searchlight rotates at 2 revolutions per minute.



Determine the rate at which the beam of light is moving along the beach when:

(a) the beam illuminates the beach at a point A such that the angle SAN is 30° [6]

(b) the beam illuminates at a point B on the beach 39 metres from S. [2]

Question 7 [9 marks]

The expected uptake of a new model of smart phone in a country, currently with one million models in use, can be modelled by the logistic equation $\frac{dx}{dt}=\frac{x(20-x)}{250}$, where x is the total number of models in millions and t is the time in weeks.

(a) Express x as a function of t in the form $x=\frac{a}{1+be^{-ct}}$ where a, b and c are positive constants. [5]

(b) Calculate

(i) the expected number of models in use after 30 weeks. [1]

(ii) the week during which the number of models in use is increasing at the greatest rate. [3]

END OF QUESTIONS