

PERTH MODERN SCHOOL



PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

Independent Public School

Trial WACE Examination, 2012

Question/Answer Booklet

MATHEMATICS SPECIALIST 3C/3D

Section One: Calculator-free

SOLUTIONS

Student Number: In figures

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In words

Your name

Teacher

Time allowed for this section

Reading time before commencing work: five minutes

Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|---------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|--------------------|
| Section One: Calculator-free | 7 | 7 | 50 | 50 | 33 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 100 | 67 |
| Total | | | | 150 | 100 |

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil**, except in diagrams.

Section One: Calculator-free

(50 Marks)

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

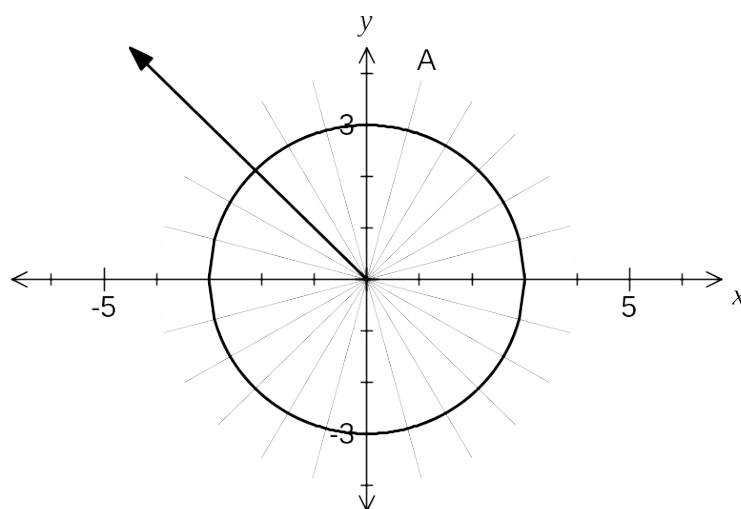
Working time for this section is 50 minutes.

Question 1

(5 marks)

- (a) Sketch the polar graphs of $r = 3$ and $\theta = \frac{3\pi}{4}$ on the axes below.

(2 marks)



- (b) A is the point with polar coordinates $\left(4, \frac{5\pi}{12}\right)$. Find the exact distance between A and the point of intersection of the graphs of $r = 3$ and $\theta = \frac{3\pi}{4}$.

(3 marks)

$$\begin{aligned} d^2 &= 3^2 + 4^2 - 2 \times 3 \times 4 \times \cos\left(\frac{3\pi}{4} - \frac{5\pi}{12}\right) \\ d^2 &= 9 + 16 - 24 \cos \frac{\pi}{3} \\ d^2 &= 13 \\ d &= \sqrt{13} \end{aligned}$$

Question 2

(6 marks)

- (a) Determine $\int 12xe^{x^2} (e^{x^2} + 1)^2 dx$.

(2 marks)

$$2(e^{x^2} + 1)^3 + c$$

- (b) Evaluate $\int_{0.2}^1 \frac{9}{x^2} \sqrt{1 + \frac{3}{x}} dx$, using the substitution $u = 1 + \frac{3}{x}$ or otherwise.

(4 marks)

$$\begin{aligned} du &= -\frac{3}{x^2} dx \\ \text{When } x &= 1, u = 4 \text{ and when } x = 0.2, u = 16. \\ \int_{16}^4 -3u^{\frac{1}{2}} du \\ &= \left[-2u^{\frac{3}{2}} \right]_{16}^4 \\ &= [-2(8)] - [-2(64)] \\ &= -16 + 128 \\ &= 112 \end{aligned}$$

Question 3

(5 marks)

Determine a unit vector perpendicular to the plane containing the lines $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

Let a perpendicular vector to plane be $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + \mathbf{k}$ (assuming \mathbf{k} coefficient is not 0). Then:

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0 \Rightarrow x + 2y = 3$$

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0 \Rightarrow x + y = 1$$

$$x + 2y - (x + y) = 3 - 1 \Rightarrow y = 2$$

$$x = 1 - 2 = -1$$

Hence $\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and so required unit vector is

$$\pm \frac{1}{\sqrt{6}}(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

Question 4

(11 marks)

(a) Find $\frac{dy}{dx}$ for the following and simplify:

(i) $y = \log_{16}(x^2)$.

(3 marks)

$$\begin{aligned} y &= \frac{\ln x^2}{\ln 16} \\ y &= \frac{2 \ln x}{4 \ln 2} \\ \frac{dy}{dx} &= \frac{1}{2x \ln 2} \end{aligned}$$

(ii) $y = \cos^3(e^{(1-x^2)})$.

(3 marks)

$$\begin{aligned} \frac{dy}{dx} &= 3(-2x)(-\sin(e^{(1-x^2)}))\cos^2(e^{(1-x^2)}) \cdot e^{(1-x^2)} \\ &= 6x \sin(e^{(1-x^2)}) \cos^2(e^{(1-x^2)}) \cdot e^{(1-x^2)} \end{aligned}$$

(b) Find the equation of the tangent to the curve $y = \sqrt{x+y}$ when $y = 6$.

(5 marks)

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2} \left(1 + \frac{dy}{dx} \right) \frac{1}{\sqrt{x+y}} \\ \text{When } y = 6, 6 &= \sqrt{x+6} \Rightarrow x = 30 \\ \frac{dy}{dx} &= \frac{1}{2} \left(1 + \frac{dy}{dx} \right) \frac{1}{6} \\ 12 \frac{dy}{dx} &= 1 + \frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{1}{11} \\ \text{Tangent is } y - 6 &= \frac{1}{11}(x - 30) \Rightarrow 11y - x = 36 \end{aligned}$$

Question 5

(7 marks)

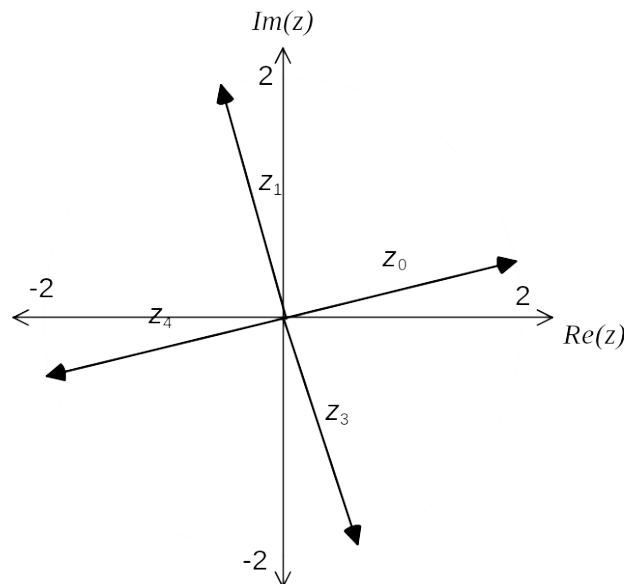
(a) Find all complex solutions of the equation $z^4 = 8 + 8\sqrt{3}i$.

(5 marks)

$$\begin{aligned} z^4 &= 16 \operatorname{cis} \left(\frac{\pi}{3} \right) \\ z_0 &= \sqrt[4]{16} \operatorname{cis} \left(\frac{\pi}{3} \times \frac{1}{4} \right) \\ &= 2 \operatorname{cis} \left(\frac{\pi}{12} \right) \\ z_1 &= 2 \operatorname{cis} \left(\frac{7\pi}{12} \right) \\ z_2 &= 2 \operatorname{cis} \left(-\frac{5\pi}{12} \right) \\ z_3 &= 2 \operatorname{cis} \left(-\frac{11\pi}{12} \right) \end{aligned}$$

(b) Sketch the solutions from (a) on the complex plane.

(2 marks)



Question 6

(9 marks)

- (a) Find the value(s) of a for which the matrix $\begin{bmatrix} a & 3 \\ 1 & a+2 \end{bmatrix}$ is singular. (2 marks)

Singular if
 $a(a+2) - 1 \times 3 = 0$
 $a^2 + 2a - 3 = 0$
 $(a+3)(a-1) = 0$
 $a = -3, a = 1$

- (b) A system of linear equations is given by

$$\begin{aligned} a + 4b + 2c &= 9 \\ 2a + 3b &= 13 + c \\ 3c + 3a + 9 &= 2b \end{aligned}$$

The system can be solved using the matrix equation $Q = P^{-1}R$. Write down suitable matrices for P , Q and R , but **do not solve** the system. (3 marks)

$$P = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & -1 \\ 3 & -2 & 3 \end{bmatrix} \quad Q = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad R = \begin{bmatrix} 9 \\ 13 \\ -9 \end{bmatrix}$$

- (c) Find matrix A if $AB + 2B = 4I$, where I is the identity matrix and $B = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$. (4 marks)

$$\begin{aligned} (A + 2I)B &= 4I \\ A &= 4B^{-1} - 2I \\ A &= 4 \times \frac{1}{4} \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ A &= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \end{aligned}$$

Question 7

(7 marks)

- (a) Show that $\frac{1+e^a}{1+e^{-a}} = e^a$, where a is a constant.

(2 marks)

$$\begin{aligned} \frac{1+e^a}{1+e^{-a}} &= \frac{e^a}{e^a} \times \frac{1+e^a}{1+e^{-a}} \\ &= \frac{e^a(1+e^a)}{e^a+1} \\ &= e^a \end{aligned}$$

- (b) Show that $\int_{-3}^3 \frac{e^{kx}}{1+e^{kx}} dx = 3$, where k is a constant.

(5 marks)

$$\begin{aligned} \frac{1}{k} \int_{-3}^3 \frac{ke^{kx}}{1+e^{kx}} dx &= \frac{1}{k} \left[\ln|1+e^{kx}| \right]_{-3}^3 \\ &= \frac{1}{k} \left(\ln(1+e^{3k}) - \ln(1+e^{-3k}) \right) \\ &= \frac{1}{k} \ln \left(\frac{1+e^{3k}}{1+e^{-3k}} \right) \\ &= \frac{1}{k} \ln(e^{3k}) \\ &= \frac{1}{k} \times 3k \\ &= 3 \end{aligned}$$

Additional working space

Question number: _____

Additional working space

Question number: _____

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