

SOLUTIONS

2018
UNIT TEST 5

Christ Church
Grammar School



MATHEMATICS METHODS Year 11

Section One:
Calculator-free

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specific to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Additional working space

Question number: _____

Question 4

Consider the curve:

$$y = 5^x$$

What is the vertical dilation that would have the same transformation effect on this curve as a horizontal translation of 3 units right?

$$y = 5^{x-3} \quad \checkmark \text{ (for giving result of transformation 3 units)}$$

$$= 5^{-3} 5^x = \frac{1}{125} 5^x$$

\therefore vertical dilation of scale factor $\frac{1}{125}$

\checkmark (find answer)

End of questions

Question 1

(3 marks)

Last year a peanut farmer produced 6×10^4 kg of peanuts. The peanuts each weighed an average of 8×10^{-4} kg. The farmer put the peanuts into boxes each containing 20000 peanuts. How many boxes of peanuts did the farmer produce last year? Give your answer in scientific form correct to two significant figures.

$$6 \times 10^4 \div 8 \times 10^{-4}$$

$$= \frac{6}{8} \times 10^8$$

$$= 0.75 \times 10^8$$

$$= 7.5 \times 10^7$$

\checkmark (for calculating number of peanuts)

$$7.5 \times 10^7 \div 20000$$

$$= 7.5 \times 10^7 \div 2 \times 10^4$$

$$= 3.75 \times 10^3$$

$$= 3.8 \times 10^3 \text{ boxes}$$

\checkmark (for calculating number of boxes)

\checkmark (for rounding to 2 sig figs)

See next page

Question 2

(5 marks)

Determine the equation of the other tangent to the curve:

$$y = 1 - 3x + 12x^2 - 8x^3$$

which is parallel to the tangent at (1,2).

$$\frac{dy}{dx} = -3 + 24x - 24x^2 \quad \checkmark \text{ (for } \frac{dy}{dx} \text{)}$$

$$\text{at } x=1 \quad \frac{dy}{dx} = -3 + 24(1) - 24(1)^2$$

$$= -3 \quad \checkmark \text{ (for } \frac{dy}{dx} \text{ at } x=1 \text{)}$$

\therefore parallel lines require gradient must be -3

$$\text{i.e. } -3 + 24x - 24x^2 = -3 \quad \checkmark \text{ (for quadratic expression for gradient)}$$

$$\therefore 24x - 24x^2 = 0$$

$$x - x^2 = 0$$

$$x(x-1) = 0$$

$$x = 0, 1$$

When $x=0, y=1$ so other point is (0,1) \checkmark (for other point with tangent gradient -3)

So tangent is $y = -3x + C$

$$\text{use (0,1)} \quad 1 = -3 \times 0 + C$$

$$C = 1$$

\therefore other tangent is $y = -3x + 1$ \checkmark (for final answer)

See next page

Question 3

(5 marks)

(a) Simplify $16^{-\frac{3}{4}}$ giving your answer as a fraction. (2 marks)

$$16^{-\frac{3}{4}} = (2^4)^{-\frac{3}{4}} \quad \checkmark \text{ (for using base of 2)}$$

$$= 2^{-3}$$

$$= \frac{1}{8} \quad \checkmark \text{ (for final answer)}$$

(Note: $\frac{1}{2^3}$ is also OK)

(b) Simplify the following expression:

$$\frac{2^{2n} \times 9^{2n-1}}{6^{n-1}}$$

(3 marks)

$$= \frac{2^{2n} \times (3^2)^{2n-1}}{(2 \times 3)^{n-1}} \quad \checkmark \text{ (for writing in terms of bases 2 and 3)}$$

$$= \frac{2^{2n} \times 3^{4n-2}}{2^{n-1} \times 3^{n-1}}$$

$$= 2^{2n-(n-1)} \times 3^{(4n-2)-(n-1)} \quad \checkmark \text{ (for writing without denominator)}$$

$$= 2^{n+1} \times 3^{3n-1} \quad \checkmark \text{ (for final answer)}$$

See next page