Name_____   Time: 50 minutes
                                                    Total: 46 marks

1.   [3, 2, 6 marks]

(a)   If $z_1 = 2\,cis\left(\dfrac{\pi}{12}\right)$ and $z_2 = 5\,cis\left(\dfrac{\pi}{6}\right)$, prove that: $z_1 z_2 = 5\sqrt{2}(1+i)$

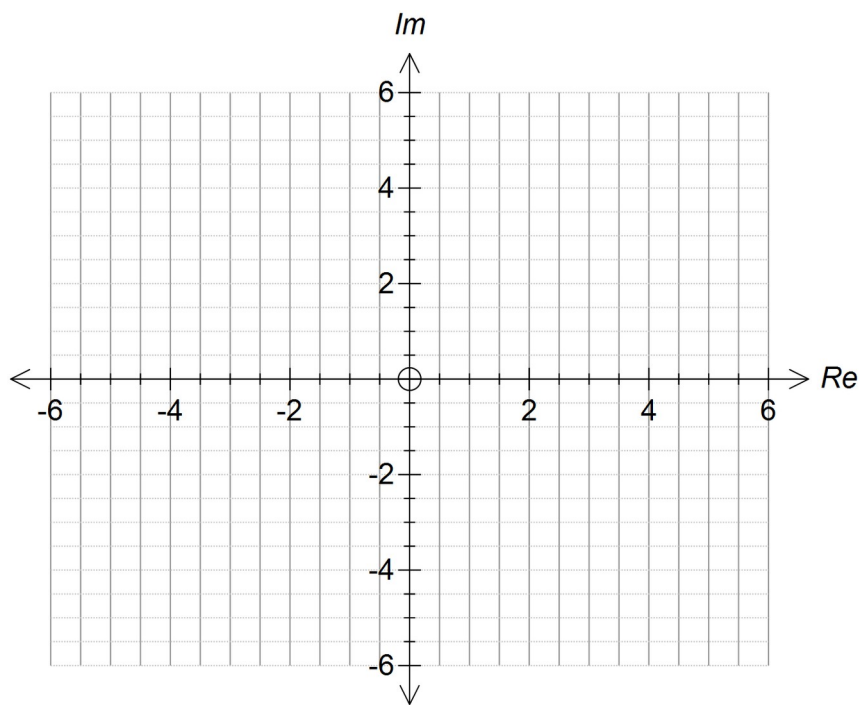(b)   Simplify $\dfrac{3\,cis\left(\dfrac{-\pi}{2}\right) \times 4\,cis\left(\dfrac{2\pi}{3}\right)}{2\,cis\left(\dfrac{5\pi}{4}\right) \times cis\left(\dfrac{-7\pi}{12}\right)}$

(c) Determine z if: $z\bar{z}+2z=\dfrac{1+4i}{4}$

2.   [3, 1, 1, 2, 3 marks]

(a)   Represent the following set on the Argand diagram below.

$$\{Z : \left|Z+4-2i\right| \le 2\}$$

(b) Find

   i)   the minimum possible value of $Im(z)$

   ii)   the maximum possible value of $\left|\Re\left(z\right)\right|$

   iii)   the minimum value of $\left|z\right|$

   iv)   the maximum possible value of $\arg(z)$, leave your answer in trig form.

3   [2, 1, 3 marks]
(a)   Find the remainder when $2x^3 - x^2 + 2$ is divided by $x - 3$

(b)   If $(x-2)$ is a factor of $ax^2 - 12x + 4$ find a.

(c)   The function $f(x) = x^4 - 7x^3 + px^2 + qx - 30$ has $(x-3)$ as a factor but a remainder of -48 is left when $f(x)$ is divided by $(x+1)$. Find p and q.

4   [6 marks]

Find two complex numbers, *w* and *z*, in Cartesian form, such that

$$iz + 2w = 3 \text{ and } z - (1 + i) = -2 \text{ where } i = \sqrt{-1}.$$
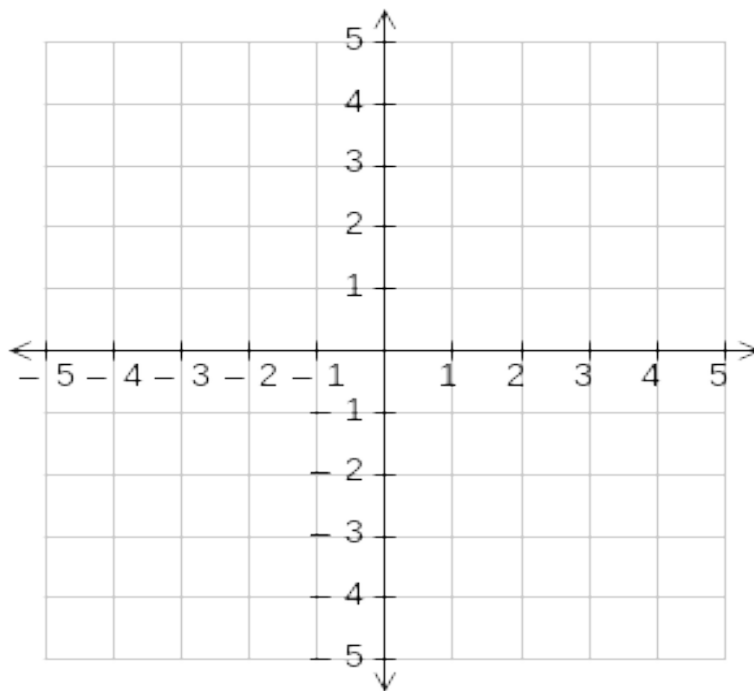
5   [5 Marks]

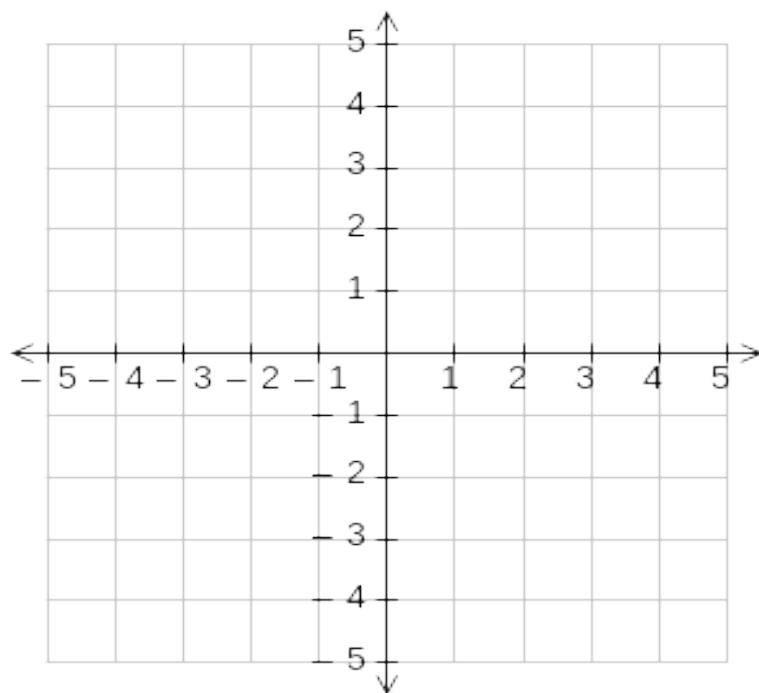Use de Moivre's Theorem to prove that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$

6   [2+4+2= 8 marks]
Draw separate sketches of the following sets of points in the complex plane.

(a) $\{z : |z-2+3i| = 1\}$

(b) $\{z : |z+2-i| < |z-2+3i|\}$

(c) $\{z : \bar{z} = iz\}$