Rossmoyne Senior High School

Year 11 Examination, 2015

Question/Answer Booklet

MATHEMATICS SPECIALIST UNITS 1 AND 2
Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures

In words ____

Your name ____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(6 marks)**

The work done, in joules, by a force $\mathbf{F}$ Newtons in changing the displacement of an object $\mathbf{s}$ metres is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$.

(a) Determine the work done by a force of 200 N that moves an object 2.7 m, given that the force acts at an angle of 17º to the direction of movement. (1 mark)

$$200 \times 2.7 \times \cos 17^\circ = 516.4 \text{ J}$$

(b) When an object is moved $0.8\mathbf{i} - 0.6\mathbf{j}$ m by a force of 130 N, the work done is 126 J.

(i) Show that one possible force is $120\mathbf{i} - 50\mathbf{j}$ N. (2 marks)

$$\begin{bmatrix} 0.8 \\ -0.6 \end{bmatrix} \cdot \begin{bmatrix} 120 \\ -50 \end{bmatrix} = 96 + 30 = 126 \text{ J}$$

and

$$\sqrt{120^2 + (-50)^2} = 130 \text{ N}$$

(ii) Another possible force is $x\mathbf{i} + y\mathbf{j}$ N. Determine the values of x and y. (3 marks)

$$\begin{bmatrix} 0.8 \\ -0.6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 126 \Rightarrow 0.8x - 0.6y = 126$$

$$x^2 + y^2 = 130^2$$

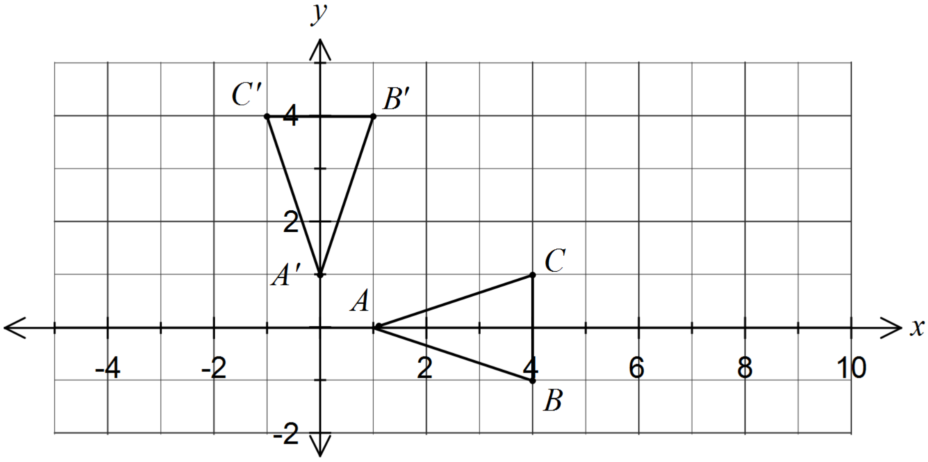
$$\cancel{x = 120, y = -50}$$

or

$$x = 81.6, y = -101.2$$

Question 10 **(7 marks)**

On the axes below, triangle ABC is transformed to $A'B'C'$ by a linear transformation.



(a) State the appropriate transformation matrix. (1 mark)

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

(b) Following a second transformation, $A'(0, 1)$ and $B'(1, 4)$ are transformed to $A''(0, 2)$ and $B''(3, 8)$.

(i) Determine the matrix for this second transformation. (2 marks)

$$T\begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 2 & 8 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 3 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

(ii) Calculate the area of triangle $A''B''C''$. (2 marks)

$\det(T) = 6$

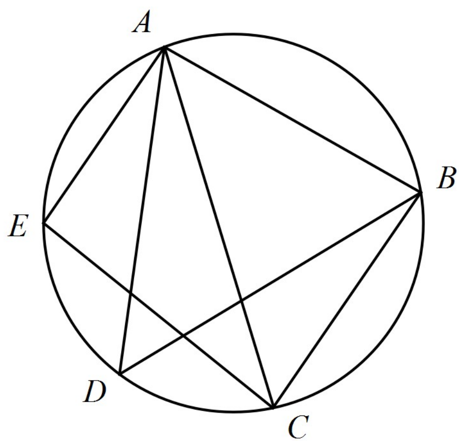
$6 \times 3 = 18$ sq units

(c) Determine the transformation matrix that will transform triangle $A''B''C''$ back to ABC. (2 marks)

$$\left(\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right)^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{3} & 0 \end{bmatrix}$$

Question 11 **(8 marks)**

(a) In the diagram below $\angle AEC = 85°$ and $\angle BAC = 38°$. Determine the size of $\angle ADB$.
(3 marks)

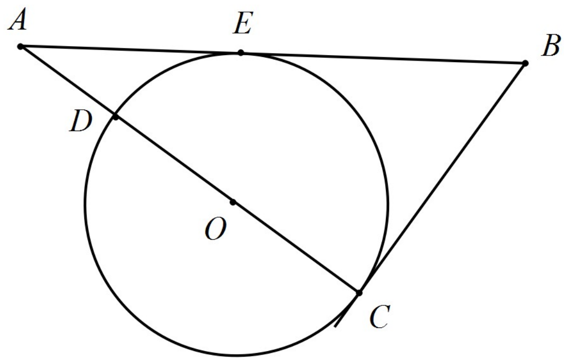


$\angle ABC = 180 - 85 = 95°$

$\angle ACB = 180 - 95 - 38 = 47°$

$\angle ADB = 47°$

(b) In the diagram shown below, not drawn to scale, a circle with centre O has tangents at E and C that meet at B. If the length of BC is 8 cm and the length of AE is 9 cm, determine the length of DC.
(5 marks)



$BE = BC = 8$

$AB = AE + EB = 9 + 8 = 17$

$AC^2 = AB^2 - BC^2$
$= 17^2 - 8^2$
$AC = 15$

$AE^2 = AD \times AC$
$AD = \dfrac{9^2}{15}$
$= 5.4$ cm

$DC = AC - AD$
$= 15 - 5.4 = 9.6$ cm

Question 12 **(9 marks)**

Let $A = \begin{bmatrix} -6 & 4 \\ 5 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$.

(a) Given that $A^{-1} = kB$, determine the value of k. (2 marks)

$$A^{-1} = \begin{bmatrix} 1.5 & 2 \\ 2.5 & 3 \end{bmatrix}$$

$$k = \frac{1}{2}$$

(b) The equations $4y = 6x + 4$ and $5x = 3y$ can be expressed as a matrix equation in the form $AX = C$.

(i) State the matrices X and C. (2 marks)

$$-6x + 4y = 4$$
$$5x - 3y = 0$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

(ii) Write down a matrix equation to determine X in terms of B and C. (2 marks)

$$AX = C$$
$$X = A^{-1}C$$

$$X = \frac{1}{2}BC$$

(c) Determine the matrix D, if $(B - D)B = 2A$. (3 marks)

$$(B - D)BB^{-1} = 2AB^{-1}$$

$$B - D = A^2$$
$$D = B - A^2$$

$$D = \begin{bmatrix} -53 & 40 \\ 50 & -23 \end{bmatrix}$$

Question 13 **(6 marks)**

(a) Determine the angle between the vectors $(-12, 7)$ and $(3, 8)$. (2 marks)

Using CAS, 80.3° (3sf)

(b) Determine the value of a so that the vectors $(7, a)$ and $(10, 4)$ are perpendicular. (2 marks)

$(7, a)\cdot(10, 4) = 0$

$70 + 4a = 0 \Rightarrow a = -\dfrac{35}{2}$

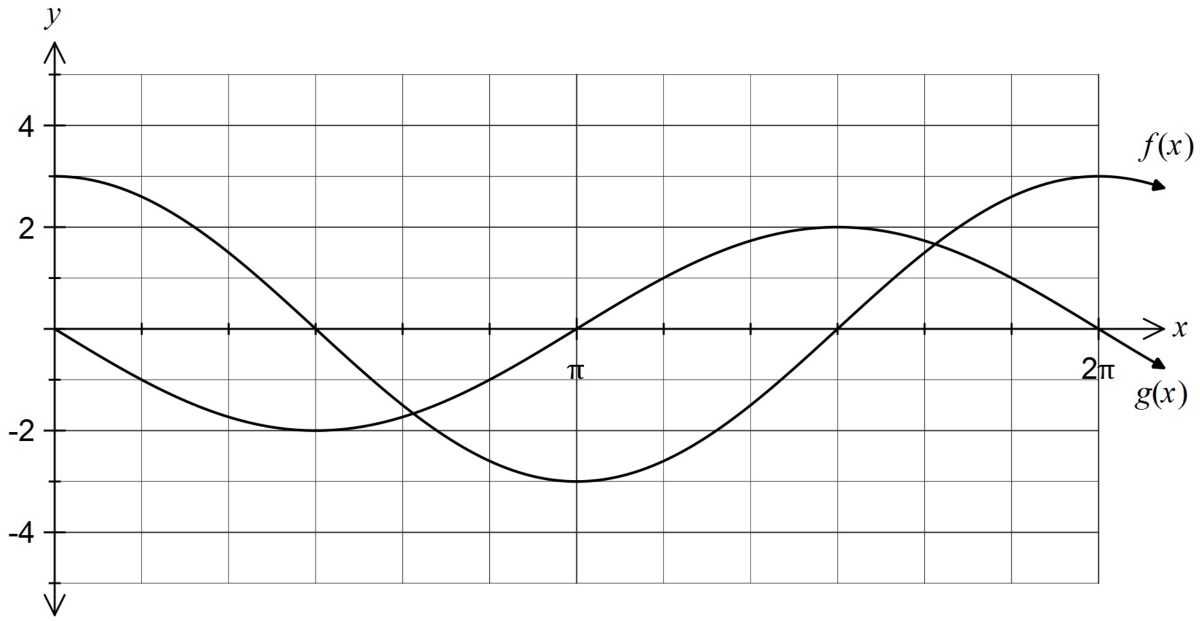
(c) Determine the exact scalar projection of $(3, -5)$ on $(-8, 4)$. (2 marks)

$$\frac{(3, -5)\cdot(-8, 4)}{|(-8, 4)|}$$

$$= -\frac{11\sqrt{5}}{5}$$

Question 14 **(9 marks)**

(a) The graphs of $y=f(x)$ and $y=g(x)$ are shown below for $0 \le x \le 2\pi$.



(i) If $f(x)=a\cos x$ and $g(x)=b\sin x$, state the values of a and b. (1 mark)

$a=3$

$b=-2$

(ii) If $h(x)=f(x)+g(x)$ express $h(x)$ in the form $R\cos(x+\alpha)$. (3 marks)

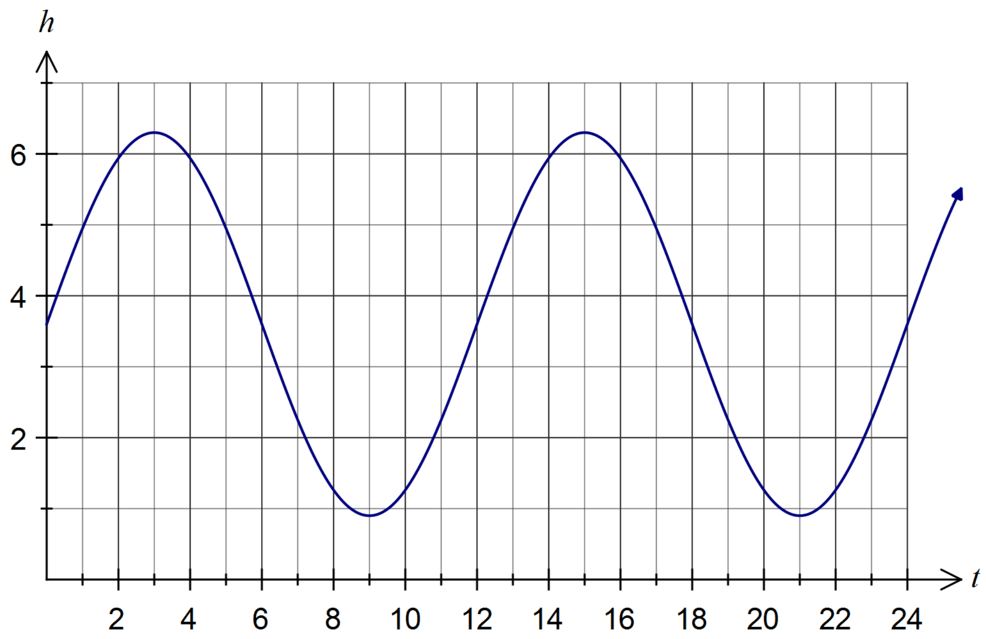
$R=\sqrt{3^2+2^2}=\sqrt{13}$

$\alpha=\tan^{-1}\left(\frac{2}{3}\right)\approx 0.588$

$h(x)=\sqrt{13}\cos(x+0.588)$

(b) The clearance, h metres, under a bridge spanning a river estuary varies with the time since midnight, t hours, and is given by $h = 3.6 + 2.7\sin\left(\frac{\pi t}{6}\right)$.

(i) Sketch the graph of the clearance against time on the axes below. (3 marks)



(ii) Determine the percentage of any 24-hour period during which the clearance under the bridge is no more than two metres. (2 marks)

$h \le 2 \Rightarrow 7.21 \le t \le 10.79$

$\frac{10.79 - 7.21}{12} \times 100 = 29.8\%$

Question 15 **(8 marks)**

(a) A committee of eight people is to be selected from 10 junior, 14 adult and 11 senior nominations from the members of a club. Determine the number of ways the committee can be selected if

(i) there are no restrictions. (1 mark)

$$^{35}C_8 = 23\ 535\ 820$$

(ii) there must be five adults and more seniors than juniors. (3 marks)

$$^{14}C_5 \times \left(^{11}C_3 \times ^{10}C_0 + ^{11}C_2 \times ^{10}C_1 \right) = 2002 \times (165 \times 1 + 55 \times 10)$$
$$= 1\ 431\ 430$$

(b) Six books are to be selected for promotion in a newsletter from a choice of nine crime, seven fantasy and six romance novels. Determine the number of selections that include three fantasy or three romance novels. (4 marks)

$$n(3F) = {}^{7}C_3 \times {}^{15}C_3 = 15\ 925$$

$$n(3R) = {}^{6}C_3 \times {}^{16}C_3 = 11\ 200$$

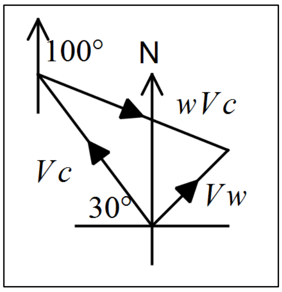
$$n(3F \cap 3R) = {}^{7}C_3 \times {}^{6}C_3 = 700$$

$$n(3F \cup 3R) = 15925 + 11200 - 700 = 26\ 425$$

Question 16 **(7 marks)**

A cyclist pedals at a speed of 25 km/h along a road on a bearing of 300º. Relative to the cyclist, the wind appears to be blowing from 280º with a speed of 30 km/h.

(a) Sketch a labelled diagram to show the relationship between the velocities of the cyclist, the wind and the wind relative to the cyclist. (2 marks)



$Vw = Vc + wVc$

(b) Express the velocities of the cyclist and the wind relative to the cyclist in the component form, rounding coefficients to two decimal places. (2 marks)

$$Vc = 25\begin{bmatrix}\cos(150)\\ \sin(150)\end{bmatrix} \approx \begin{bmatrix}-21.65\\ 12.50\end{bmatrix}$$

$$wVc = 30\begin{bmatrix}\cos(-10)\\ \sin(-10)\end{bmatrix} \approx \begin{bmatrix}29.54\\ -5.21\end{bmatrix}$$

(c) Determine the true speed of the wind and the bearing from which it is blowing. (3 marks)

$$Vw = Vc + wVc$$
$$= \begin{bmatrix}-21.65\\ 12.50\end{bmatrix} + \begin{bmatrix}29.54\\ -5.21\end{bmatrix}$$
$$= \begin{bmatrix}7.89\\ 7.29\end{bmatrix}$$

$|Vw| = 10.7$ km/h
at 42.7° from x-axis

Bearing is 047.3°

Or using CAS to solve triangle

$Vw = 10.7$

at angle of 107.3° to Vc

Bearing is 047.3°

Question 17 **(6 marks)**

(a) Show how to express $5.\overline{25}$ as a rational number. (2 marks)

Let $x = 5.\overline{25}$

$$100x = 525.2525252525...$$
$$x = 5.252525252525...$$
$$99x = 520$$
$$x = \frac{520}{99} \Rightarrow x \text{ is rational}$$

(b) Prove by contradiction that $\sqrt[3]{4}$ is an irrational number. (4 marks)

Suppose that the cube root of 4 is rational, so that

$\sqrt[3]{4} = \frac{a}{b}$, where a and b are integers with no common factors.

Then $4 = \frac{a^3}{b^3} \Rightarrow a^3 = 4b^3 = 2(2b^3)$

Hence a must be even $\Rightarrow a = 2n,\ n \in \mathbb{Z}$

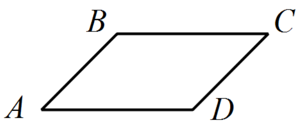
$4b^3 = (2n)^3 = 8n^3$

$b^3 = 2n^3 \Rightarrow b$ must be even.

But if both a and b are even, this contradicts that they have no common factors. Hence cube root of 4 is irrational.

Question 18 **(9 marks)**

(a) Figure $ABCD$ is a parallelogram. Let $\overrightarrow{AB}=$¿ **b** and $\overrightarrow{AD}=$¿ **d**. Prove that the diagonals AC and BD are perpendicular only when |**b**| ¿ |**d**|. (4 marks)



$$\mathbf{AC} = \mathbf{b} + \mathbf{d}$$
$$\mathbf{BD} = \mathbf{d} - \mathbf{b}$$

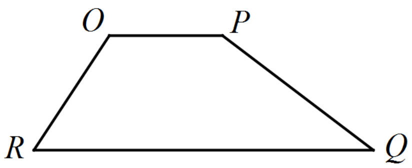
Perpendicular when:

$$\mathbf{AC} \cdot \mathbf{BD} = 0$$
$$(\mathbf{b}+\mathbf{d}) \cdot (\mathbf{d}-\mathbf{b}) = 0$$

$$\mathbf{b}\cdot\mathbf{d} - \mathbf{b}\cdot\mathbf{b} + \mathbf{d}\cdot\mathbf{d} - \mathbf{d}\cdot\mathbf{b} = 0$$

$$|\mathbf{d}|^2 = |\mathbf{b}|^2 \Rightarrow |\mathbf{d}| = |\mathbf{b}|$$

(b) Figure $OPQR$ is a trapezium, with OP parallel to RQ and $RQ = 3OP$. If M is the point of intersection of OQ and PR, $\overrightarrow{OP}=$¿ **p**, $\vec{¿}=$¿ **r**, $\overrightarrow{OM}=\lambda\overrightarrow{OQ}$ and $\overrightarrow{RM}=\mu\overrightarrow{RP}$ show that $\overrightarrow{OM}=\frac{1}{4}r+\frac{3}{4}p$. (5 marks)



$$\overrightarrow{OM} = \lambda\overrightarrow{OQ} = \overrightarrow{OR} + \mu\overrightarrow{RP}$$

$$\lambda(\mathbf{r}+3\mathbf{p}) = \mathbf{r} + \mu(\mathbf{p}-\mathbf{r})$$

Equate $\mathbf{r}$ coeffs: $\lambda = 1-\mu$

Equate $\mathbf{p}$ coeffs: $3\lambda = \mu$

$$\lambda = \tfrac{1}{4}$$

$$\overrightarrow{OM} = \tfrac{1}{4}\overrightarrow{OQ} = \tfrac{1}{4}(\mathbf{r}+3\mathbf{p})$$

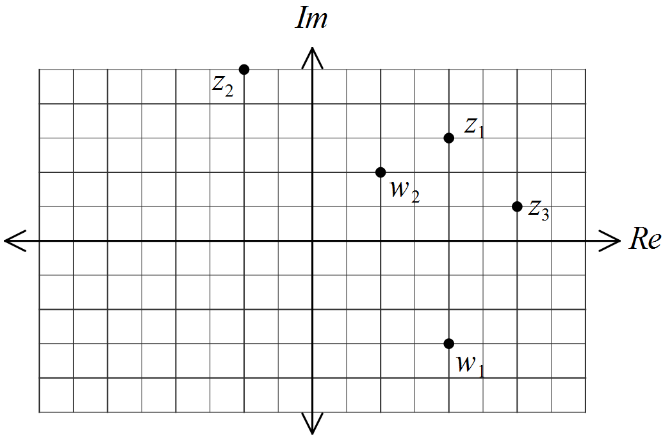
Question 19 **(8 marks)**

(a) Solve $2(z-3)^2+2=0$. (2 marks)

$(z-3)^2=-1=i^2$

$z=3\pm i$

(b) The complex numbers w_1 and w_2 are shown in the Argand plane below.



Plot and label the complex numbers given by

(i) $z_1=\overline{w}_1$. (1 mark)

(ii) $z_2=w_2-w_1$. (1 mark)

(iii) $z_3=\overline{w_1+w_2}$. (1 mark)

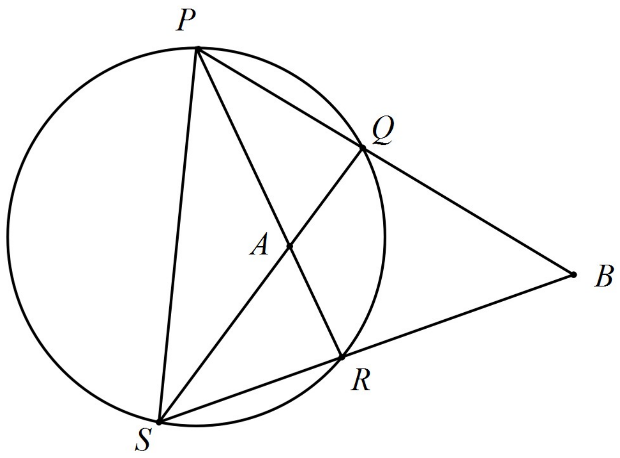
(c) One solution of the quadratic equation $x^2+bx+c=0$ is $x=3-2i$. Determine the values of the real coefficients b and c. (3 marks)

$(x-(3-2i))\times(x-(3+2i))=x^2-6x+13$

$b=-6,\ c=13$

Question 20 **(8 marks)**

The points P, Q, R and S lie on a circle of radius r. PR and QS meet at A. PQ and SR are produced to meet at B, and $AQBR$ is a cyclic quadrilateral.



(a) Prove that BS is perpendicular to PR. (6 marks)

$\angle PQS = 180^\circ - \angle BQS$ (angle on straight line)
$\angle PRS = 180^\circ - \angle PRB$ (angle on straight line)

$\angle PQS = \angle PRS$ (stand on same arc PS)

$180^\circ - \angle BQS = 180^\circ - \angle PRB \Rightarrow \angle BQS = \angle PRB$

$\angle BQS + \angle PRB = 180$ (Opp angles in cyclic quad)

Hence $2\angle PRB = 180 \Rightarrow \angle PRB = 90^\circ$, that is, BS is perpendicular to PR.

(b) Prove that the length of PS is $2r$. (2 marks)

$\angle PRS = \angle PRB = 90^\circ$

Hence PS must be diameter of circle (Angle in semi-circle)

Length of PS is twice radius: $PS = 2r$

Question 21 **(7 marks)**

Let $P(n) = 10^n + 18n - 1$.

(a) If $P(1) = 9a$ and $P(2) = 9b$, evaluate a and b. (2 marks)

$P(1) = 27 = 9 \times 3 \Rightarrow a = 3$

$P(2) = 135 = 9 \times 15 \Rightarrow b = 15$

(b) Prove by induction that $P(n)$ is always a multiple of nine when n is a positive integer. (5 marks)

$P(1) = 9(3)$

When $n = k$

$P(k) = 10^k + 18k - 1 = 9M,\ M \in \mathbb{Z}$

When $n = k + 1$

$$
\begin{aligned}
P(k+1) &= 10^{k+1} + 18(k+1) - 1 \\
&= 10.10^k + 18k + 18 - 1 \\
&= 10^k + 18k - 1 + 18 + 9.10^k \\
&= 9M + 9(2 + 10^k) \\
&= 9(M + 2 + 10^k)
\end{aligned}
$$

Thus, as $P(1)$ is true and the truth of $P(k)$ implies the truth of $P(k+1)$, then $P(n)$ is true for all positive integers n.

End of questions

Additional working space

Question number: _________

Additional working space

Question number: _________

Additional working space

Question number: _________

Published by WA Examination Papers
PO Box 445 Claremont WA 6910