MATHEMATICS METHODS

MAWA Semester 2 (Unit 3&4) Examination 2019
Calculator-free

Marking Key

The release date for this exam and marking scheme is

- **the end of week 1 of term 4, Fri October 18th 2019**

Section One: Calculator-free (50 Marks)

Question 1 (a) (3 marks)

Solution	
$f'(x) = -2x . e^{-x^2}\sqrt{2x-5} + \frac{1}{2}(2x-5)^{\frac{-1}{2}} .2. e^{-x^2} ¿ - e^{-x^2}\left(2x\sqrt{2x-5} - \frac{1}{\sqrt{2x-5}}\right)$	
Mathematical behaviours	**Marks**
• uses product rule correctly • differentiates e^{-x^2} correctly • differentiates $\sqrt{2x-5}$correctly	1 1 1

Question 1 (b) (3 marks)

Solution	
Let $u = x^2 + 16$.(*) Then $\frac{du}{dx} = 2x$, and so $g(x) = \int \frac{xdx}{x^2+16} = \int \frac{du}{2u} = \frac{1}{2}\ln u + c = \frac{1}{2}\ln(x^2+16) + c$(**) Since $g(0) = \ln 5 \ln 5 = \frac{1}{2}\ln 16 + c \ln 5 = \ln 4 + c c = \ln 5 - \ln 4 c = \ln\frac{5}{4} g(x) = \frac{1}{2}\ln(x^2+16) + \ln\frac{5}{4}$	
Mathematical behaviours	**Marks**
• makes substitution (*) • integrates correctly (**) (use of rule $\int\left[\frac{f'(x)}{f(x)}\right]dx = \ln\lvert f(x)\rvert + c$ to integrate correctly – award both marks) • evaluates integration constant correctly	1 1 1

Question 2 **(3 marks)**

Solution	
$z\sqrt{\frac{p(1-p)}{n_1}}=kz\sqrt{\frac{p(1-p)}{n_2}}\sqrt{\frac{p(1-p)}{3n_2}}=k\sqrt{\frac{p(1-p)}{n_2}}\frac{p(1-p)}{3n_2}=k^2\frac{p(1-p)}{n_2}\frac{n_2}{3n_2}=k^2k^2=\frac{1}{3}k=\frac{1}{\sqrt{3}}$	
Mathematical behaviours	**Marks**
• uses the formula for margin of error to compare each sample	1
• simplifies equation by squaring and dividing	1
• re-arranges equation to determine the value of k	1

Question 3(a) **(2 marks)**

Solution

$P(x=1)=k\log_e e^1=k$

$P(x=2)=k\log_e e^2=2k$

$P(x=a)=k\log_e e^a=ak$

x	1	2	a
$P(x)$	k	$2k$	ak

Mathematical behaviours	Marks
• uses log laws to find probability, P(2).	1
• uses log laws to find probability, P(3).	1

Question 3(b) **(2 marks)**

Solution

$$k+2k+ak=1$$
$$3k+ak=1$$
$$a=\frac{1-3k}{k}$$

Mathematical behaviours	Marks
• sums probabilities and equates to 1	1
• rearranges formula to express a in terms of k	1

Question 3(c) **(3 marks)**

Solution	
$k=\frac{1}{3} \Rightarrow a=0$ $E(X)=1\times k+2\times 2k$ $=5k$ $k=\frac{1}{3} \Rightarrow E(X)=\frac{5}{3}$	
Mathematical behaviours	**Marks**
• determines value of a	1
• substitutes into the expected value formula	1
• states expected value	1

Question 4(a) **(3 marks)**

Solution	
$2^x=3^{x-1}$ $ie\ x\log 2=(x-1)\log 3$ $ie\ x\log 2-x\log 3=-\log 3$ $ie\ x(\log 2-\log 3)=-\log 3$ $ie\ x=\dfrac{\log 3}{\log 3-\log 2}$	
Mathematical behaviours	**Marks**
• rewrites equation by taking logarithms of each side and applying log laws	1
• rearranges equation to isolate x	1
• solves for x	1

Question 4(b) **(4 marks)**

Solution	
$\log_{10}(x+2)+\log_{10}(2x-3)=2\log_{10}x$ $ie\ \log_{10}(x+2)(2x-3)=\log_{10}x^2$ $ie\ (x+2)(2x-3)=x^2$ $ie\ 2x^2+x-6=x^2$ $ie\ x^2+x-6=0$ $ie\ (x+3)(x-2)=0$ $ie\ x=-3,2$ $2x-3>0 \Rightarrow x=2$	
Mathematical behaviours	**Marks**
• uses $\log$ laws to simplify both sides of equation • obtains quadratic equation	1 1 1

• simplifies quadratic and solves • solves for x, justifying answer	1

Question 4(c) **(4 marks)**

Solution
$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos x}{\sin x}\,dx = \left[\ln \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $= \ln \sin(\frac{\pi}{3}) - \ln \sin \frac{\pi}{6}$ $= \ln \frac{\sqrt{3}}{2} - \ln \frac{1}{2}$ $= \ln \sqrt{3} - \ln 2 - (\ln 1 - \ln 2)$ $= \ln \sqrt{3}$ $= \frac{\ln 3}{2}$ $\therefore a = 0.5,\ b = 3$

Mathematical behaviours	Marks
• anti-differentiates to obtain $\ln$ expression • substitutes exact values and evaluates expression • uses $\log$ laws to simplify expression • states the value of a and b	1 1 1 1

Question 5 **(4 marks)**

Solution
$\frac{d}{dx}\left(\int_0^x f(t)dt\right) = \frac{d}{dx}\left(\left[f(x)\right]^2\right)$ $f(x) = 2f(x).f'(x) f'(x) = \frac{1}{2}$ $f(x) = \frac{x}{2} + c \int_0^0 f(t)\,dt = \left[f(0)\right]^2 \therefore 0 = f(0) f(x) = \frac{x}{2}$

Mathematical behaviours	Marks
• uses Fundamental Theorem of Calculus • uses chain rule to differentiate $\left[f(x)\right]^2$ • determines $f(x)$ • determines $f(x)$ and shows how to calculate the constant, c.	1 1 1 1

Question 6(a) **(3 marks)**

Solution
$v = 4 When t = 0 ¿ a = 0 When t = 5 ¿ a = 2 \therefore a = \frac{2}{5} t v = \frac{1}{5} t^2 + c When t = 0 ¿ v = 4 \therefore v = \frac{1}{5} t^2 + 4$ $When t = 5 v = \frac{1}{5}(25) + 4 v = 9 m . s^{-1}$

Mathematical behaviours	Marks
• determines the acceleration equation	1
• anti-differentiates to find the velocity equation	1
• states the correct velocity at 5 seconds	1

Question 6(b) **(3 marks)**

Solution
$x = \left(\frac{1}{5}\right)\frac{t^3}{3} + 4t + c Let x = 0 , when t = 0 \therefore c = 0 x = \frac{1}{15} t^3 + 4t When t = 5 x = \frac{1}{15}(125) + 20$ $x = \frac{25}{3} + 20$ $x = \frac{85}{3} \vee 28.33 m$

Mathematical behaviours	Marks
• anti-differentiates velocity equation to find displacement equation	1
• substitutes for t = 5 seconds	1
• gives correct distance travelled	1

Question 7 **(3 marks)**

Solution
$\int_0^{\ln 2} e^{-2x}\,dx ¿ \left[\frac{e^{-2x}}{-2}\right]_0^{\ln 2} ¿ \frac{e^{-2\ln 2}}{-2} - \frac{e^0}{-2} ¿ \frac{e^{\ln\left(\frac{1}{4}\right)}}{-2} - \frac{1}{-2} ¿ -\frac{1}{8}+\frac{1}{2} ¿ \frac{3}{8}$

Mathematical behaviours	Marks
• anti-differentiates exponential function	1
• substitutes correctly	1
• simplifies correctly	1

Question 8 **(4 marks)**

Solution
$V=\frac{4}{3}\pi r^3$,and so $\frac{dV}{dr}=4\pi r^2$ By the increments formula, $\delta V \cong \frac{dV}{dr}\delta r=4\pi r^2 \delta r$ (*) So $\frac{\delta V}{V} \cong \frac{4\pi r^2}{\frac{4}{3}\pi r^3}\delta r=3\frac{\delta r}{r}$ (**) Now $\frac{\delta V}{V}=\frac{-12}{800}=-0.015$ So $\frac{\delta r}{r} \cong \frac{-0.015}{3}=-0.005$ So the percentage change in the radius r is a decreases of 0.5 %.

Mathematical behaviours	Marks
• differentiates correctly	1
• finds approximation (*)	1
• evaluates $\frac{\delta V}{V}$	1
• obtains correct answer	1

Question 9(a) **(2 marks)**

Solution
$Area\,of\;triangle=1$ So k (y-intercept) = $\frac{1}{3}$ Gradient = $\frac{-1}{3}\div 6$ $f(t)=\frac{-1}{18}t+\frac{1}{3}$ for $0 \le t \le 6$

Mathematical behaviours	Marks
• determines k value	1
• states the probability density function	1

Question 9(b) **(2 marks)**

Solution
$\int_0^t \frac{-1}{18}t+\frac{1}{3}dt¿\frac{-t^2}{36}+\frac{1}{3}t$ i.e. F(t) = ¿

Mathematical behaviours	Marks
• integrates f(t)	1
• states the cumulative distribution function	1

Question 9(c) **(2 marks)**

Solution
$P(t<1)=\frac{11}{36}$ $P(t<3)=\frac{27}{36}$ $P(t>1)=\frac{25}{36}$ $P(t>1\cap t<3)=\frac{16}{36}$ $P(t<3 \vee t>1)=\frac{16}{25}$

Mathematical behaviours	Marks
• determines the intersection of probabilities for t > 1 and t < 3	1
• calculates the conditional probability	1