

You do not have any unauthorised material if you have any unauthorised material with you, hand it to the supervisor **before** reading any further.
No other items may be taken into the examination room. It is your responsibility to ensure that

Important note to candidates

Special items:
drawing instruments, templates, notes on two unruled sheets of A4 paper,
and up to three calculators, which can include scientific, graphical and
Computer Algebra System (CAS) calculators, are permitted in this course
examination

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighting markers

Formula sheet (retained from Section One)

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time:
Reading time before commencing work: ten minutes
Working time: one hundred minutes
Number of additional answer books used (if applicable):



Time allowed for this section

Your Teacher's name _____

Your name _____

MATHEMATICS METHODS UNIT 1&2 SECTION TWO: CALCULATOR-ASSUMED

Question/Answer booklet

Semester Two Examination, 2022



Exceptional schooling. Exceptional students.

Question number: _____

Supplementary page

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	12	12	50	51	35
Section Two: Calculator-assumed	13	13	100	100	65
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
13	8	
14	8	
15	6	
16	5	
17	5	
18	11	
19	8	
20	9	
21	8	
22	8	
23	10	
24	9	
25	5	
S2 Total	100	
S2 Wt ($\times 0.65$)	65%	

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Supplementary page

Question number: _____

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This section has thirteen questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes.

The n -th term of a sequence is given by $A_n = 4(0.2)^{n-1}$

(a) State the first three terms of the sequence.

(1 mark)

(b) Write an expression for the sum of n terms.

(1 mark)

Given $y = \sqrt{3} - 2 \sin\left(2x - \frac{\pi}{3}\right)$.

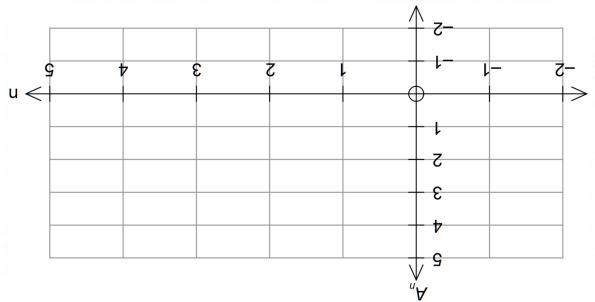
(a) State the amplitude and period.

(2 marks)

(b) Find the exact coordinates of all intercepts of $y = \sqrt{3} - 2 \sin\left(2x - \frac{\pi}{3}\right)$ with the axes for $0 \leq x \leq 2\pi$.

(3 marks)

(d) Sketch the graph of $A_n = 4(0.2)^n$ on the axes below for $n \geq 0$.



(c) Determine the sum to infinity of the series.

(2 marks)

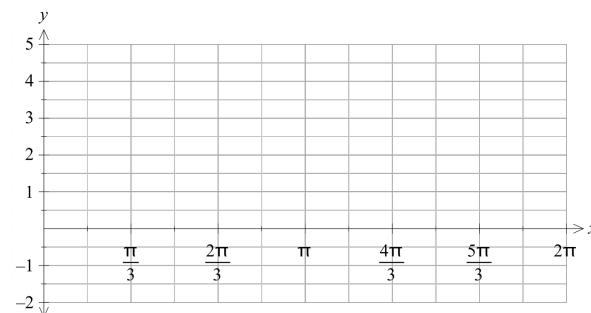
(c) Hence sketch the graph of $y = \sqrt{3} - 2 \sin\left(2x - \frac{\pi}{3}\right)$ for $0 \leq x \leq 2\pi$, showing all features from part (a) and (b).

(3 marks)

See next page

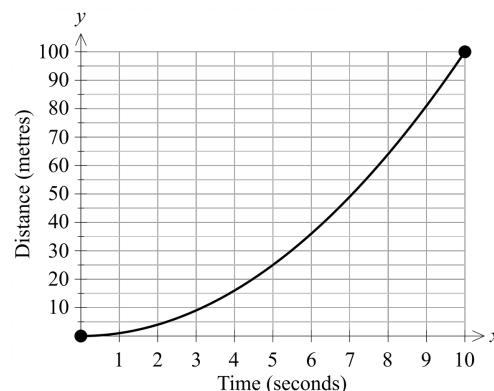
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**Question 14** (2.3.1 – 2.3.5, 2.3.11 - 2.3.16)

(8 marks)

The graph of $y=x^2$ is shown below. The graph is a distance-time graph for an athlete running a 100 m sprint. The horizontal axis represents time in seconds, and the vertical axis the distance in metres.



- (a) Determine the **average** speed of the athlete in the last five seconds.

(2 marks)

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The athlete wants to know their speed at exactly 5 seconds. To help determine this they determine their distance at a number of other times as shown on the graph below.

See next page

Question 24 (2.2.1 – 2.2.5)

(9 marks)

- (a) Consider the following arithmetic sequence: $(x + 5), (37 - x), (x + 13)$

- (i) Determine the value of x .

(2 marks)

- (ii) State the recursive rule for the sequence.

(2 marks)

- (iii) Find the 10th term of the sequence.

(2 marks)

- (b) The sum of the first three terms of a geometric sequence is 91 and its common ratio is 3, determine the third term.

(3 marks)

See next page

(2 marks)

- (d) Using parts (b) and (c), determine the gradient of the chord in part (b) as $\Delta x \rightarrow 0$.
 Clearly explain how you determined your answer.

where Δx and Δy represent changes in the variables x and y .

$$\frac{\Delta y}{\Delta x} \approx \frac{f(x+h) - f(x)}{h}$$

The gradient of the chords in part (b) can be written as

(2 marks)

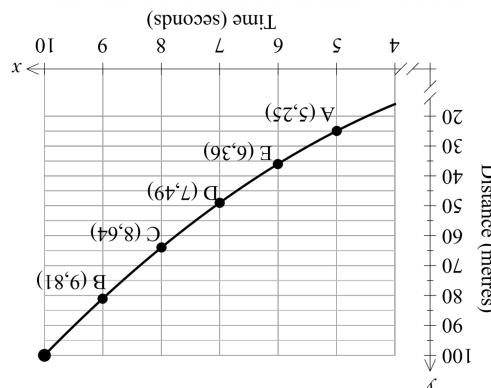
- (c) Show using first principles that the derivative of $y = x^2$ is $y' = 2x$.

(1 mark)

- (b) Complete the table by calculating the gradient of chords AE.

Chord	Gradient	14	13	12	
AB					
AC					
AD					
AE					

The table below shows the gradients of the chords AB and AC.



MATHEMATICS METHODS UNIT 1&2
CALCULATOR-ASSUMED
5

- (b) Use the information in (a) to determine the co-ordinates of A.
 (3 marks)

16 CALCULATOR-ASSUMED
MATHEMATICS METHODS UNIT 1&2

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- (c) Determine the equation of the tangent to the curve $y = \frac{1}{4}x^2 - x$ which passes through B.
 (3 marks)

$$y = \frac{1}{4}x^2 - x$$

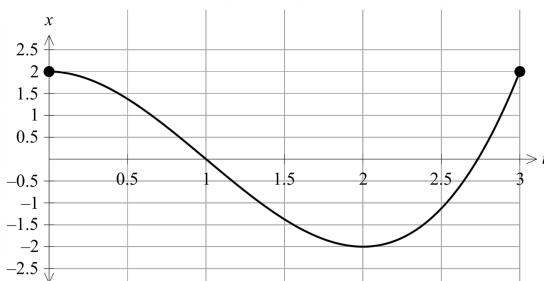
- (e) Hence state the speed of the athlete at exactly 5 seconds. (1 mark)

- Question 15** (2.3.14, 2.3.18, 2.3.19, 2.3.22) (6 marks)

Particle A starts 2 m to the right of the origin, and after t seconds has a velocity of $v=3t^2-6t$.

- (a) Show that the displacement function is $x=t^3-3t^2+2$ (2 marks)

The displacement, x m, of particle A is shown on the position-time graph below.



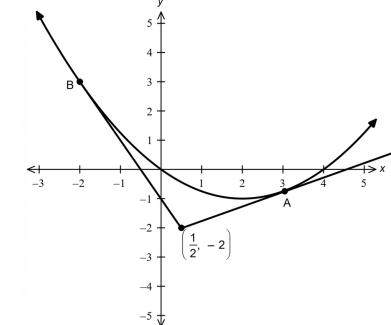
- (b) (i) Show algebraically, that particle A starts at rest, and comes to rest again after 2 seconds. (2 marks)

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- Question 23** (1.1.1 – 1.1.6) (10 marks)

$$y = \frac{1}{4}x^2 - x \text{ from the point } \left(\frac{1}{2}, -2\right).$$

Consider the two tangents to the curve



Let the co-ordinates of A be (x_A, y_A) .

- (a) Show that:

$$\frac{y_A + 2}{x_A - \frac{1}{2}} = \frac{1}{2}x_A - 1.$$

(i)

(2 marks)

$$(ii) \text{ and hence } y_A = \frac{1}{2}x_A^2 - \frac{5}{4}x_A - \frac{3}{2}.$$

(2 marks)

(2 marks)

- (iii) Explain how the gradient of the above graph shows that the particle is travelling to the left for $0 < t < 2$.

(5 marks)

Question 16 (1.3.15, 1.3.16)

$$P(B) = x, P(A) = \frac{4}{7}x \text{ and } P(A \cup B) = 0.75.$$

A and B are two events where

- (a) Determine $P(B)$ so that events A and B are mutually exclusive.

- (b) Determine $P(B)$ so that events A and B are independent.

(2 marks)

(3 marks)

- (b) Determine $P(B)$ so that the events A and B are independent.

Question 17 (2.1.4 - 2.1.7)

(5 marks)

The population of a city, Allocated, increased from 1 358 000 at the beginning of 1990 to 3 246 000 at the end of 2020.

- (a) Show clearly that the annual compound growth rate of the population is a percentage is 2.9%.

(3 marks)

See next page

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(8 marks)

- (a) Express 108° as an exact radian measure and hence determine the length of an arc of a circle that subtends an angle of 108° at the centre of a circle of radius 15 cm. (2 marks)

(2 marks)

- (c) The lengths of the sides of a triangle with an area of 190 cm^2 are in the ratio 4 : 5 : 8. Determine the length of the shortest side of the triangle. (4 marks)

- (b) Determine the area of a segment of a circle that subtends an angle of 54° at the centre of a circle of radius 46 cm. (2 marks)

- (iii) Express 108° as an exact radian measure and hence determine the length of an arc of a circle that subtends an angle of 108° at the centre of a circle of radius 15 cm. (2 marks)

- (b) Determine the area of a segment of a circle that subtends an angle of 54° at the centre of a circle of radius 46 cm. (2 marks)

Question 16 (1.3.15, 1.3.16)

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Question 17 (2.1.4 - 2.1.7)

- (a) Show clearly that the annual compound growth rate of the population is a percentage is 2.9%.

- (b) If the same annual growth rate of the population is maintained, determine the population of Alicetown at the beginning of 2050. (2 marks)

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See next page

Question 21 (2.2.1 – 2.2.6)

(8 marks)

- (a) A large outdoor amphitheatre has 17 seats in the first row, 20 in the second row and so on, where each row has three more seats than the previous row. There are 36 rows of seats in the amphitheatre.

- (i) Determine the number of seats in the last row of the amphitheatre. (2 marks)

- (ii) Determine the total number of seats in the last 20 rows of the amphitheatre. (3 marks)

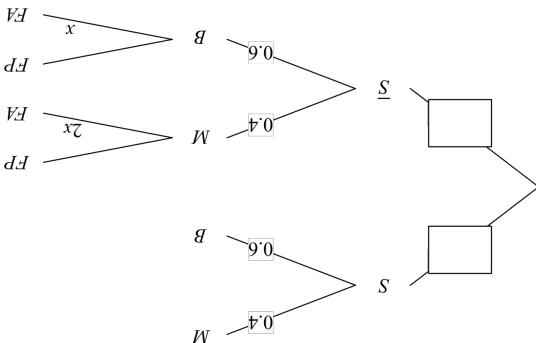
- (b) The sum of the first and second terms of a geometric series is 20, and the sum of the second and third terms of the series is 60. Determine the sum of the first three terms of the series. (3 marks)

See next page

(a) Given that a survey by Google found that only 25% of passwords are considered strong (S), complete the following tree diagram showing the authentication methods and their associated probabilities.

, completing the following tree diagram showing the authentication methods and their associated probabilities.

(b) A user's phone only has a fingerprint scanner. State the value of x in this case. (1 mark)



(2 marks)

Out of 200 randomly selected users who used the app, it was found that 63 had to scan their face.

(c) Using this information show that $x = 0.3$. (2 marks)

(1 mark)

(d) was an adult, given that the patient was known to have required further treatment. (2 marks)

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(3 marks)

(b) required no further treatment. (2 marks)

(2 marks)

(c) was a child or required further treatment. (2 marks)

(d) was an adult, given that the patient was known to have required further treatment. (2 marks)

- (d) (i) Using the tree diagram, or otherwise, show that the probability the website asks a user to enter a code sent to their mobile phone is independent of whether their password is strong or not. (2 marks)

- (ii) Hence, explain why events B and S are independent or not. (1 mark)

An analysis of the strong passwords used by users found the following:

- 42 % consisted of at least one capital letter (C)
- 78 % consisted of at least one number (N)
- All the strong passwords used contain either capital letter(s) or number(s) or both.

- (e) (i) Describe the passwords in the set $C \cap N$. (1 mark)

- (ii) Determine the percentage of passwords in $C \cap N$, and explain why the two sets are not mutually exclusive. (3 marks)

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- Question 19** (1.1.14, 1.1.24, 1.1.25) (8 marks)
In an industrial process to create hydrogen, an engineer observed that the higher the power density P W/cm², the shorter the cathode lifetime T in hours, so that when $P=1.4$, $T=120$ and when $P=2.4$, $T=75$.

The relationship between the variables is of the form $T = \frac{a}{bP+0.2}$, where a and b are constants.

- (a) Determine the value of a and the value of b . (3 marks)

- (b) If the process is only possible for $0.4 \leq P \leq 3.9$, state the corresponding range of T . (2 marks)

- (c) Graph the relationship on the axes below over the domain $0.4 \leq P \leq 3.9$. (3 marks)

