



PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2020

Question/Answer booklet

## MATHEMATICS METHODS UNIT 1 AND 2

### Section Two: Calculator-assumed

Name: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

Reading time before commencing work:	Time allowed for this section
Working time:	ten minutes
	one hundred minutes

### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer booklet

Formula sheet (retained from Section One)

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
10		4	18		9
11		10	19		8
12		4	20		4
13		3	21		4
14		5	22		8
15		4	23		6
16		4			
17		12			

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	44	34
Section Two: Calculator-assumed	14	14	100	85	66
<b>Total</b>					100

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

(85 Marks)

This section has **14** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

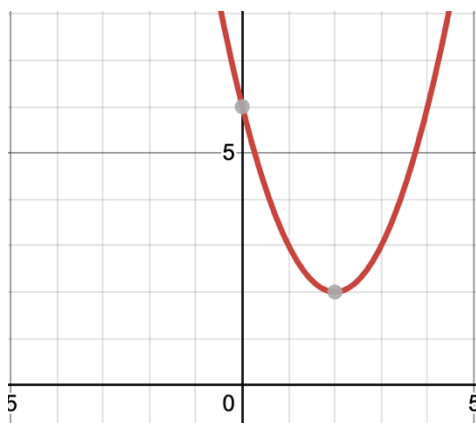
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

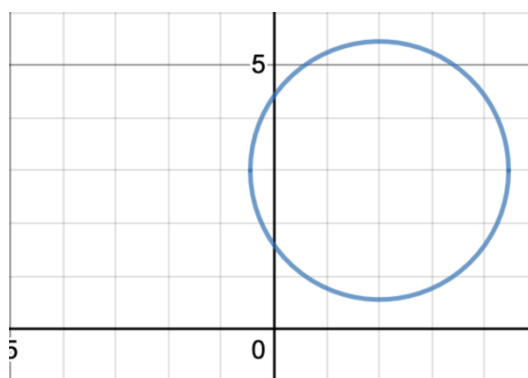
**Question 10 {1.1.28}**

**(4 marks)**

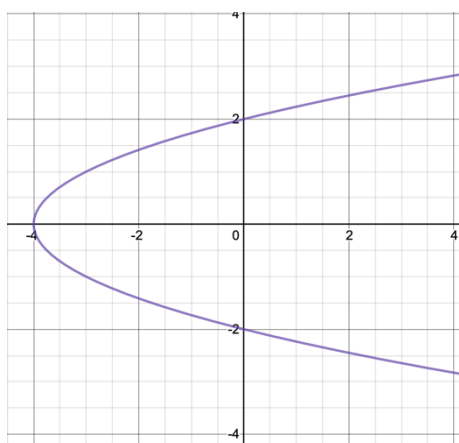
Apply the vertical line test to each of the following graphs and conclude whether the graph represents a relation or a function.



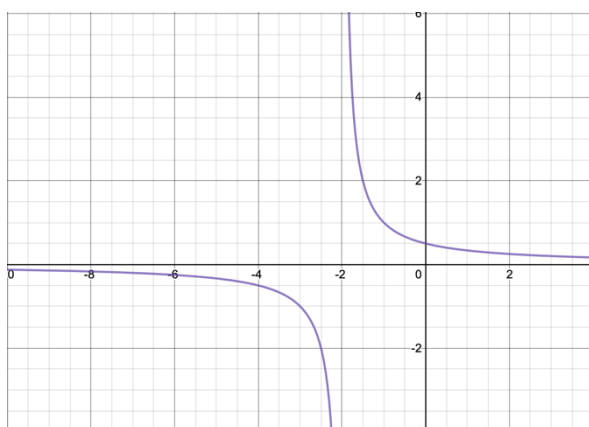
a) \_\_\_\_\_



b) \_\_\_\_\_



c) \_\_\_\_\_



d) \_\_\_\_\_

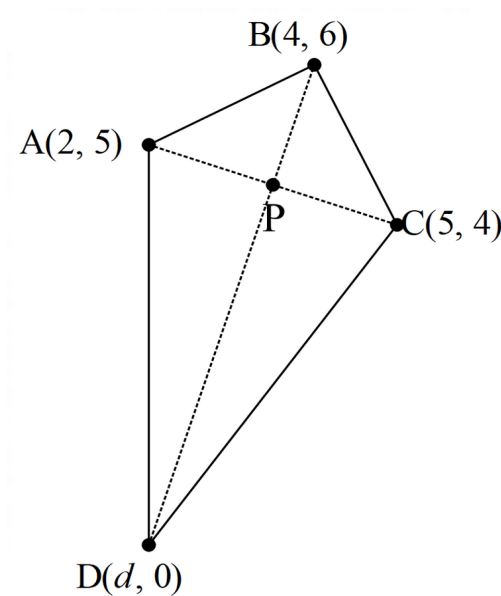
**Question 11 (1.1.1 – 1.1.6)**

**(2, 2, 1, 3, 2 = 10 marks)**

The kite ABCD is graphed below.

The coordinates of the vertices are A(2, 5), B(4, 6), C(5,4) and D( $d$ , 0) as shown below.

NB Kites have diagonals that meet at right angles.



a) Show that  $AB = BC$ .

b) Find the gradient of AC and hence the equation of the diagonal BD.

- c) Determine the value of  $d$ , the  $x$ -coordinate of point D.
- c) Find the midpoint of AC and show it belongs to diagonal BD.
- e) Show how the midpoint of AC could have been used to determine the value of  $d$ .

**Question 12 {1.1.26}****(2, 2 = 4 marks)**

- a) Give the equation of the image of  $y=x^2$  after the following transformation:  
Translate 3 units horizontally left followed by reflection in the  $x$ -axis.

- b) Give the equation of the image of  $y=\frac{1}{x}$  after the following transformation:  
Translate vertically up 2 units then reflect in the  $y$ -axis.

**Question 13 {2.3.4}****(3 marks)**

Determine the gradient of the secant passing through the graph  $y=x^3-2x^2-4$  at the points where  $x=6$  and  $x=9$ .

**Question 14 {2.3.1}****(3, 2 = 5 marks)**

A car's position at time  $t$  seconds is represented by  $x(t)$  metres, where:

$$x(t) = 30t + \frac{30}{t+1} - 30, \text{ for } 0 \leq t \leq 4$$

By first calculating the relevant positions of the car, determine its average velocity in:

a) the first 2 seconds.

b) the last 2 seconds.

## Question 15 {2.3.5, 2.3.8}

(3, 1 = 4 marks)

A balloon's volume at time  $t$  seconds is represented by  $V(t) \text{ cm}^3$ , where:

$$V(t) = -2t^2 + 800, \text{ for } 0 \leq t \leq 20$$

- a) Using the difference quotient, i.e.  $\frac{f(x+h) - f(x)}{h}$ , determine the average rate of change over  $[5, 5+h]$ .

- b) Hence, determine the instantaneous rate of change as  $h \rightarrow 0$ .



## Question 16 {2.3.6}

(4 marks)

Differentiate  $f(x) = 5x^3$  by first principles. i.e.  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

**Question 17 {2.3.19}****(3, 2, 3, 4 = 12 marks)**

A particles displacement (in metres) from the origin with respect to time (in seconds) is given by the following equation:

$$s(t) = \frac{t^3}{3} - 5t^2 + at + b \quad \text{where } 0 \leq t \leq 8s$$

- a) Given that the particle is initially at the origin and has a displacement of 27 metres at 3 seconds, determine the values of a & b.
- b) Determine the particles displacement at 5 seconds to 2 decimal places.
- c) Use calculus techniques to determine when the particle is at rest.
- d) Use calculus techniques to determine the maximum displacement of the particle.

**Question 18 {2.3.22}****(3, 6 = 9 marks)**

Given that  $f(x)$  runs through the point (2,4) and  $f'(x)=3x^2-3$ :

a) Determine  $f(x)$

b) Using calculus techniques find the location and nature of any stationary points.

**Question 19 {2.3.22}****(8 marks)**

Anika has her own private plane that she uses to travel from Perth to Melbourne. The amount of fuel she consumes on the journey and therefore the cost of her flight,  $C$  (\$), is dependent on the speed at which she flies,  $x$  (km/h).

The relationship between the overall cost of the flight is modelled by the equation below.

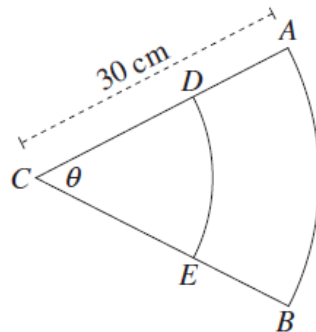
$$C = \frac{x^2}{200} - 8x + 20400 \quad \text{where } 550 \leq x \leq 900$$

Use calculus techniques to determine both the **optimal speed** to travel in order to minimise the the **cost of her flight**.

## Question 20 {1.2.6}

(4 marks)

The region ABC is a sector of a circle with radius 30cm, centred at C. The angle in the sector is  $\theta$ . The arc DE lies on a circle also centred at C, as shown in the diagram.

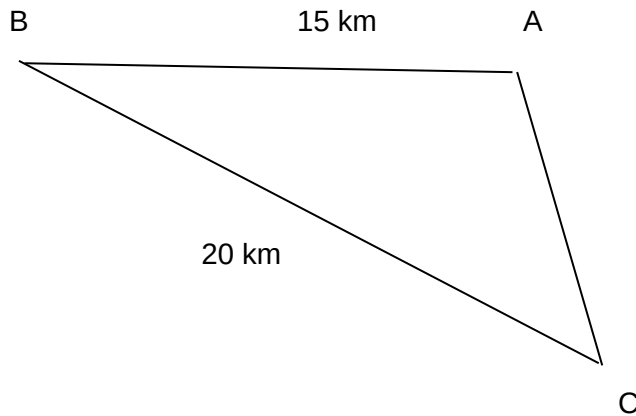


The arc DE divides the sector ABC into two regions of equal area.  
Find the length of the interval CD.

## Question 21 {1.2.4}

(4 marks)

A ground search for a lost hiker is being organised using three camping sites in a national park as bases. It is known that the hiker is within the triangular area formed with the three campsites as vertices. Campsite A is 15km due east from campsite B. Campsite C is on a bearing of  $170^\circ$  from campsite A (hint :  $\angle BAC = 100^\circ$ ). Campsite B is 20km from campsite C.  
Note: The diagram is not to scale.



Calculate the area of the search to the nearest square kilometre.







## Question 23 (1.3.7, 1.3.11, 1.3.14, 1.3.16)

(1, 1, 1, 1, 1, 1 = 6 marks)

a) Events A and B are such that  $P(A)=0.4$ ,  $P(B)=0.7$  and  $P(\overline{A} \cap B)=0.4$ . Determine:

i.  $P(A \cap \overline{B})$

ii.  $P(\overline{A} \cap \overline{B})$

b) Given  $P(D)=\frac{2}{3}$ ,  $P(C|D)=\frac{3}{5}$  and  $P(C|\overline{D})=\frac{1}{5}$ . Determine:

i.  $P(C \cap D)$

ii.  $P(C \cap \overline{D})$

iii.  $P(C)$

iv.  $P(\overline{D}|C)$

**END OF SECTION**  
**Working out space**



Working out space

Working out space



**Additional working space**

Question number: \_\_\_\_\_





**Additional working space**

Question number: \_\_\_\_\_

**Additional working space**

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