

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

### Important note to candidates

Special items: nil

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

### To be provided by the candidate

Formula sheet

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time:

Reading time before commencing work: five minutes

Working time:

Reading time before commencing work: five minutes

Time allowed for this section

In words

Your name

Student number: In figures

In words


# SOLUTIONS

MATHEMATICS  
METHODS  
UNITS 1 AND 2  
Section One:

Calculator-free

Section One:

Calculator-free

Question/Answer booklet

Semester Two Examination, 2019



**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					<b>100</b>

**Instructions to candidates**

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free****35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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**Question 1****(4 marks)**

The line segment between the points  $A(-2, -3)$  and  $B(-2, 5)$  is the diameter of a circle.

Determine the equation of circle in the form  $x^2+ax+y^2+by=c$ , where  $a, b$  and  $c$  are constants.

**Solution**

Centre: $\left(-2, \frac{-3+\sqrt{5}}{2}\right) = (-2, 1)$
Radius: $r = 5 - 1 = 4$
Equation: $(x+2)^2 + (y-1)^2 = 4^2$
<b>Specific behaviours</b>
$x^2 + 4x + 4 + y^2 - 2y + 1 = 16x^2 + 4x + y^2 - 2y = 1$
centre
radius
factored equation
correct equation

**Question 2****(5 marks)**

Determine the gradient of the curve  $y=x^2-3x-40$  at the point(s) where it crosses the  $x$ -axis.

**Solution**

$$(x+5)(x-8)=0 \Rightarrow x=-5, x=8$$

$$\frac{dy}{dx} = 2x - 3$$

$$x=-5, \frac{dy}{dx} = -13$$

$$x=8, \frac{dy}{dx} = 13$$

At  $(-5, 0)$  gradient is  $-13$  and at  $(8, 0)$  gradient is  $13$ .

**Specific behaviours**

- factorises quadratic
- determines roots
- derivative of quadratic
- one point and gradient
- second point and gradient

(2 marks)

(8 marks)

<b>Solution</b>	$(3a+2\sqrt{ })$
<b>Specific behaviors</b>	Indicates use of difference of squares
<b>Correct simplification</b>	$\checkmark$

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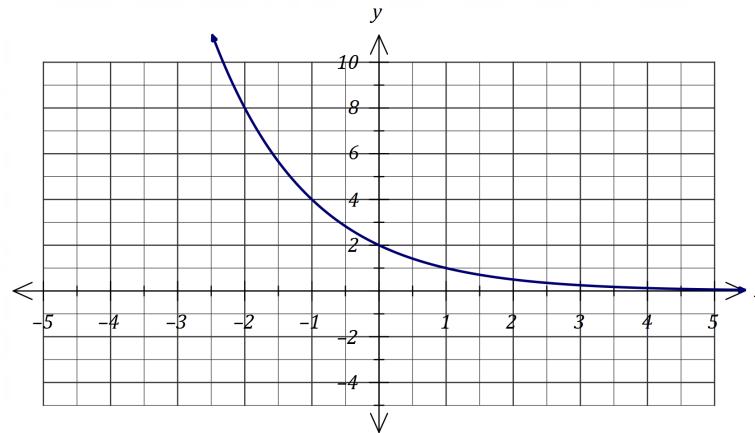
(c) Sketch the graph of  $y = 2^{1-x}$  on the axes below. (3 marks)

$[2^3]^x = 2^{0.5} \times 2^{-5}$	$2^3x = -2^{-4.5}$	$3x = -4.5$	$x = -1.5$
<b>Solution</b>			
<b>Specific behaviours</b>			
<b>Whites B and D2 as powers of 2</b>			
<b>Simplices RHS</b>			
<b>Correct solution</b>			

(b) Solve the equation  $8x = \square$  for  $x$ . (3 marks)

<b>Specific behaviours</b>	Indicates use of difference of squares	✓
<b>Solution</b>	$3a^2 + 2\sqrt{ }$	✓

- RHS of  $y$ -axis (asymptotic)
- LHS of  $y$ -axis (through  $[-2, 8]$ )



$$200 = \frac{m}{2} [2(48) + (m-1)(-4)] \quad 400 = m(96 - 4m + 4) \quad 4m^2 - 1$$

$$m=5, m=20$$

#### Specific behaviours

- substitutes into sum formula
- simplifies and equates quadratic to zero
- both correct solutions

(c) The sum of the first  $m$  terms of the sequence is 200. Determine the value(s) of the integer constant  $m$ .

(a)

Determine the velocity of  $A$  when  $t = 3$ .

(2 marks)

<b>Solution</b>
$v = \frac{dx}{dt} = 3t^2 - 6t$
$v(3) = 3(3^2 - 6 3) = 9 \text{ cm/s}$
<b>Specific behaviours</b>
<b>correct velocity</b>
<b>expressions for velocity</b>
<b>correct</b>

(2 marks)

(b)

Determine the displacement of  $A$  relative to  $O$  at the instant(s) that it is stationary.

(3 marks)

<b>Solution</b>
$3t^2 - 6t = 0 \Rightarrow t = 0, t = 2$
$x(0) = 5 \text{ cm}, x(2) = 1 \text{ cm}$
<b>Specific behaviours</b>
<b>factories velocity</b>
<b>one correct displacement</b>
<b>both correct displacements</b>

(2 marks)

(b) Determine  $T_{2019}$ .

(a)

Determine the displacement of  $A$  relative to  $O$  at the instant(s) that it is stationary.

(3 marks)

<b>Solution</b>
$(10 - 3)d = 12 - 40$
$d = -4$
$a = 40 - 2(-4) = 48$
<b>Specific behaviours</b>
<b>value of <math>a</math></b>
<b>value of <math>d</math></b>
<b>correct</b>

(a) Determine the value of the constant  $a$  and the constant  $d$ .

(2 marks)

Determine the value of the constant  $a$  and the constant  $d$ .

(2 marks)

<b>Solution</b>
$T_{2019} = 48 + (n-1)(-4)$
$T_{2019} = 48 + 2018(-4)$
$T_{2019} = -8024$
<b>Specific behaviours</b>
<b>indicates rule for general term</b>
<b>correct value</b>

(2 marks)

(c) The sum of the first  $m$  terms of the sequence is 200. Determine the value(s) of the integer

(3 marks)

METHODS UNITS 1 AND 2

CALCULATOR-FREE

Question 4

(7 marks)

Question 8

(7 marks)

7

CALCULATOR-FREE

METHODS UNITS 1 AND 2

CALCULATOR-FREE

Question 4

(7 marks)

Question 8

(7 marks)

CALCULATOR-FREE

METHODS UNITS 1 AND 2

CALCULATOR-FREE

Question 4

(7 marks)

Question 8

(7 marks)

CALCULATOR-FREE

METHODS UNITS 1 AND 2

Small body  $B$  has velocity given by  $v=6t^2+2t-3$  cm/s and when  $t=2$  it has a displacement of 3 cm relative to  $O$ .

- (c) Determine an expression for the displacement of  $B$  relative to  $O$  at any time  $t$ . (2 marks)

Solution
$\frac{dx}{dt} = 6t^2 + 2t - 3$
$x = 2t^3 + t^2 - 3t + c$
$c = 3 - (16 + 4 - 6) = -11$
$x = 2t^3 + t^2 - 3t - 11$
Specific behaviours
✓ antiderivatives
✓ correct expression

**Question 7**

(6 marks)

Determine the coordinates of all stationary points of the curve  $y=x^4+2x^2-8x+9$ .

Solution
$\frac{dy}{dx} = 4x^3 + 4x - 8$
$4x^3 + 4x - 8 = 0$
$x^3 + x - 2 = 0$
By inspection, $x=1$ is a solution.
$x^3 + x - 2 = (x-1)(x^2 + ax + 2)$
From $x^2$ coefficient: $-1+a=0 \Rightarrow a=1$
$x^2 + x + 2 = 0$
$b^2 - 4ac = 1 - 4(1)(2) = -7 \Rightarrow \text{No solutions}$
$y = 1 + 2 - 8 + 9 = 4$
Hence just one stationary point at $(1, 4)$ .
Specific behaviours
✓ derivative
✓ equates derivative to 0
✓ one solution by inspection
✓ factorises derivative
✓ indicates quadratic factor has no roots
✓ states coordinates of stationary point

Specific behaviours	
$y = -4x - 7$	or
$y - 1 = -4(x + 2)$	
Hence equation of tangent is	
$\frac{dy}{dx} = -32 + 48 - 24 + 4 = -4$	
$y = (-1)^4 = 1$	
When $x = -2$	
$\frac{dy}{dx} = -4x^3 + 12x^2 + 12x + 4$	
Solution	

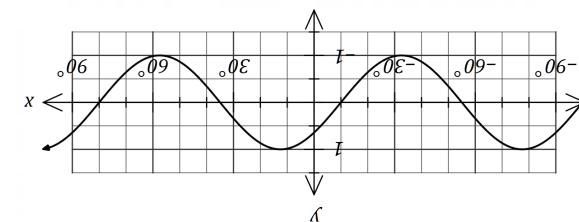
- (c) Hence, or otherwise, determine the equation of the tangent to the curve  $y = |x + 1|^4$  at the point where  $x = -2$ . (4 marks)

$ x+1 ^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$	correct expansion
	uses Pascal's triangle for coefficients
	specific behaviours
Solution	

- (b) Expand  $(x + 1)^4$ . (2 marks)

$a=4, b=40$	value of $a$
$y = \sin \sin(4(x + 10)) = \sin \sin(4x + 40)$	specific behaviours
Period of $90^\circ \Rightarrow a = 360^\circ \div 90^\circ = 4$	solution
	value of $b$

Determine the minimum possible value of each of the constants. (2 marks)



- (b) The graph of  $y = \sin \sin(ax + b)$  is shown below, where  $a$  and  $b$  are positive constants. (7 marks)

$ 42  = 6$	correct value
	specific behaviours
Solution	

- derivative
- $y$ -coordinate
- gradient
- equation of tangent (any form)

**Question 6**

(8 marks)

(a) Solve the following equations.

(i)  $\tan \tan(2x) = \sqrt{\square}, 0 \leq x \leq \pi.$

(2 marks)

Solution
$2x = \frac{\pi}{3}, \frac{4\pi}{3}$
$x = \frac{\pi}{6}, \frac{2\pi}{3}$
Specific behaviours
<input checked="" type="checkbox"/> one correct solution
<input checked="" type="checkbox"/> both correct solutions

(ii)  $2 \cos \cos(x - 60^\circ) = \sqrt{\square}, 0^\circ \leq x \leq 360^\circ.$

(4 marks)

Solution
$2(\cos \cos x \cos \cos 60^\circ + \sin \sin x \sin \sin 60^\circ) = \sqrt{\square}$
Specific behaviours
<input checked="" type="checkbox"/> uses angle difference identity
<input checked="" type="checkbox"/> substitutes exact values
<input checked="" type="checkbox"/> simplifies equation
<input checked="" type="checkbox"/> correct solution