

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Special items: nil

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpeners, correction fluid/tape, eraser, ruler, highlighters

To be provided by the supervisor
Materials required/recommended for this section

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Your name

In words

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Student Number: In figures

Section One:
Calculator-free

MATHEMATICS
METHODS
UNIT 3

SOLUTIONS

Question/Answer booklet

Semester One Examination, 2017

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	11	11	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (52 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

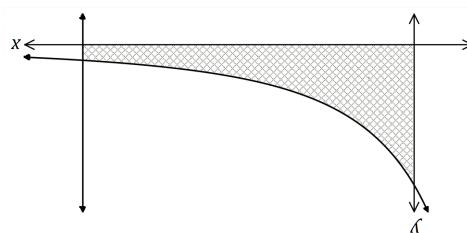
(5 marks)

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✓ simplefies
✓ substitutes bounds
✓ antiderivatives - correct multipliers
✓ antiderivatives - correct power
✓ writes integral
Specific behaviours
$A = \int_{-5}^0 180(2x+5)^{-2} dx = \left[-\frac{180}{2} \times (2x+5)^{-1} \right]_{-5}^0$
Solution



Determine the area of the shaded region, enclosed by the x -? axis, the y -? axis, the line $x=5$ and the curve.

The graph below shows the curve $y = \frac{180}{(2x+5)^2}$ and the line $x=5$.

Question number: _____

Additional working space

Question 2

A small body, initially at the origin, moves in a straight line with acceleration $a(t)=6t-10 \text{ ms}^{-2}$, where t is the time in seconds, $t \geq 0$. When $t=5$, it was observed to have a velocity of 31 ms^{-1} .

- (a) Determine an expression for $v(t)$, the velocity of the body.

(8 marks)

(2 marks)

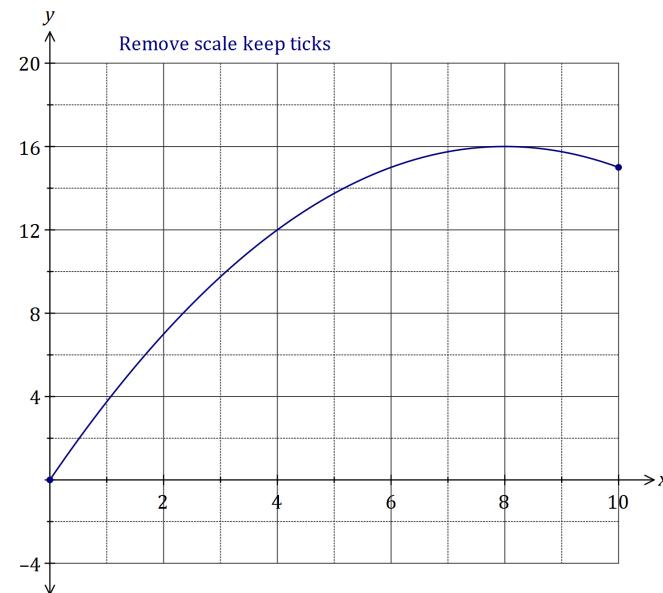
Solution
$v(t)=3t^2-10t+c$ $31=75-50+c \Rightarrow c=6$ $v(t)=3t^2-10t+6$
Specific behaviours
✓ antidifferentiates ✓ evaluates constant and states expression

- (b) Determine the acceleration of the body when $v=19$.

(3 marks)

Solution
$3t^2-10t+6=19$ $3t^2-10t-13=0$ $(3t-13)(t+1)=0$ $a=6 \times \frac{13}{3}-10=16 \text{ m/s}^2$
Specific behaviours

- ✓ uses $v=19$ to obtain quadratic equal to zero
- ✓ solves quadratic for t (+ve only)
- ✓ determines a



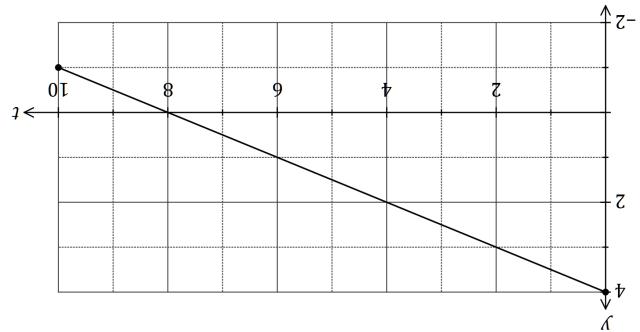
(3 marks)

- (c) Determine the velocity of the body as it passes through the origin for the last time.

Solution
$x(t) = t^3 - 5t^2 + 6t$
$0 = t(t-2)(t-3)$
$t = 3$
$v(3) = 27 - 30 + 6 = 3 \text{ m/s}$
Specific behaviours
antidifferentiates to obtain displacement equation
solves for last t
determines v
indicates area calculation
correct estimate

(2 marks)

(a) Use the graph to determine an estimate for $\int_2^0 f(t) dt$.



The graph of $y = f(t)$ is shown below over the interval $0 \leq t \leq 10$.

(6 marks)

Question 8

Solution
See graph
specific behaviours
starts at (0, 0) and includes (2, 7) from (a)
maximum at (8, 16)
endpoint close to (10, 15)
smooth parabolic shape

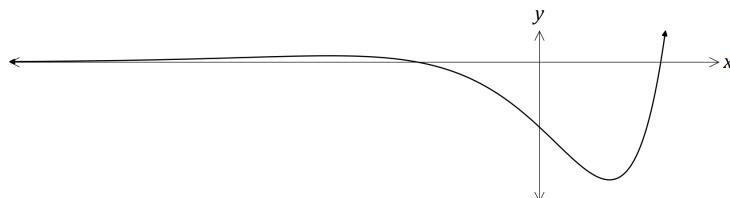
$$F(x) = \int_x^0 f(t) dt.$$

- (b) On the axes below, sketch the graph of $y = F(x)$ for $0 \leq x \leq 10$, where

Question 3

(6 marks)

The graph of $y=f(x)$ is shown below, where $f(x)=e^x(x^2-3)$.



- (a) Show that $f'(x)=e^x(x^2+2x-3)$.

(1 mark)

Solution
$f'(x)=e^x(x^2-3)+e^x(2x)=e^x(x^2+2x-3)$
Specific behaviours
✓ indicates use of product rule

- (b) Determine the x -coordinates of the stationary points of $f(x)$.

(2 marks)

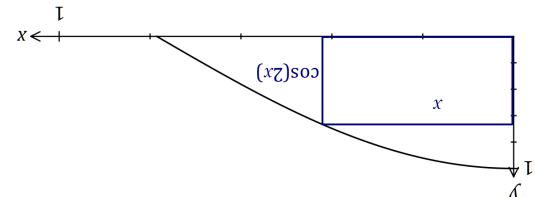
Solution
$e^x(x^2+2x-3)=0(x+3)(x-1)=0$
Specific behaviours
✓ factorises
✓ states x -values

Solution
$f''(-3) = 9 - 12 - 1 = -4 \Rightarrow \text{Local maximum when } x = -3$
$f''(1) = 1 + 4 - 1 = 4 \Rightarrow \text{Local minimum when } x = 1$

(3 marks)

- (c) Given that $f(x) = e^x(x^2 + 4x - 1)$, use the second derivative to justify that one of the stationary points is a local minimum and that the other is a local maximum.

- (a) Sketch a possible rectangle on the graph above and explain why the perimeter of the rectangle is given by the function $P(x) = 2x + 2\cos 2x$. (2 marks)



right corner on the curve shown below, $y = \cos 2x$, $0 \leq x \leq \frac{\pi}{4}$.

A rectangle has its base on the x -axis, its lower left corner at $(0, 0)$ and its upper

(6 marks)

Solution
$p(x) = 2 - 4 \sin 2x$ $p(x) = 0$ when $\sin 2x = \frac{1}{2}$ $x = \frac{\pi}{12}$
\checkmark derivative \checkmark solves for x within domain \checkmark equates to zero and obtains trig equation \checkmark determines P_{MAX}

- (b) Determine the largest perimeter of the rectangle. (4 marks)

(b) Determine the largest perimeter of the rectangle.

Specific behaviours
$p(x) = 2 - 4 \sin 2x$ $p(x) = 0$ when $\sin 2x = \frac{1}{2}$ $x = \frac{\pi}{12}$

Question 4(a) Determine $\frac{d}{dx} \underline{u}$.

(8 marks)

(3 marks)

Solution
$\frac{d}{dx} \underline{u}$
Specific behaviours
✓ obtains u' ✓ obtains uv' ✓ uses correct form of quotient rule (simplification not required)

(ii) Determine $C(6)$.

(3 marks)

Solution
$\Delta C = \int_1^6 3\sqrt{\square} \, dx$
$C(6) = C(1) + \Delta C = 10 + 38 = 48$
Specific behaviours
✓ antidifferentiates ✓ evaluates total change ✓ correct value

(b) Determine $\frac{d}{dx} (2x \sin(3x))$.

(2 marks)

Solution
$\frac{d}{dx} (2x \sin(3x)) = 2 \sin(3x) + 2x \cdot 3 \cos(3x) = 2 \sin(3x) + 6x \cos(3x)$
Specific behaviours
✓ applies product rule ✓ differentiates correctly (simplification not required)

(7 marks)	Question 6
(3 marks)	(3 marks)
(3 marks)	(3 marks)

(a)

- (c) Use your answer from (b) to determine $\int 6 \cos(3x) dx$.

(7 marks)	Question 6
(3 marks)	(3 marks)

(1 mark)	Solution
Reason	The increment in x from 1 to 6 is not small.
Specific behaviours	\checkmark uses formula to calculate increment

- (i) Explain why the increments formula would not yield an approximate value for $C(6)$.
- (ii) The function C is such that $C(1)=10$ and $C(x)=3\sqrt{x}$.

(1 mark)	Solution
Reason	The increment in x from 1 to 6 is not small.
Specific behaviours	\checkmark identifies values of x and δx
(1.05) $\approx -2 + 0.1 \approx -1.9$	$\delta y \approx \sqrt{x}$
$x=1, \delta x=0.05$	$y=f(x) \Leftrightarrow \delta y \approx f'(x) \delta x$
$f(1.05)\approx -2 + 0.1 \approx -1.9$	\checkmark calculates approximation
(3 marks)	(3 marks)

(3 marks)	Solution
(3 marks)	(3 marks)

(3 marks)	Solution
(3 marks)	(3 marks)

Question 5**(6 marks)**

The table below shows the probability distribution for a random variable X .

It is known that $E(X)=1.7$ and $\text{Var}(X)=1.41$.

x	0	1	2	3
$P(X=x)$	a	$a+b$	b	$2a$

- (a) Determine the values of the constants a and b .

(4 marks)

Solution
$4a+2b=1$ $0(a)+1(a+b)+2(b)+3(2a)=1.7$
$7a+3b=1.7$ $6a+3b=1.5$
$a=0.2, b=0.1$

Specific behaviours
✓ equation using sum of probabilities
✓ equation using expected value
✓ determines a
✓ determines b

See next page

- (b) Determine

(i) $E(3-2X)$.

(1 mark)

Solution
$3-2(1.7)=-0.4$
Specific behaviours
✓ states value

- (ii) $\text{Var}(3-2X)$.

(1 mark)

Solution
$(-2)^2(1.41)=5.64$
Specific behaviours
✓ states value

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