



PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2020

Question/Answer booklet

SPECIALIST MATHS UNIT 3

Section One: Calculator-free

Your Name _____

Your Teacher's Name _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Max	Question	Mark	Max
1	6		5	5	
2	10		6	8	
3	9		7	9	
4	3				

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	10	10	100	100	67
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free**(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1**(6 marks)**

Consider the polynomial $P(z) = z^4 - z^3 + 3z^2 - 5z - 10$.

(a) Determine $P(-1)$ (1 mark)

(b) Show that $(z - \sqrt{5}i)$ is a factor of $P(z)$. (2 marks)

(c) Determine all the roots to $P(z) = 0$ (3 marks)

Question 2

(10 marks)

Consider the functions $g(x) = x^2$ & $f(x) = \frac{1}{\sqrt{x+5}}$.

(a) Determine the natural domain and range of $f(x)$. (2 marks)

(b) Determine the rule for $g \circ f(x)$ and state its natural domain and range. (3 marks)

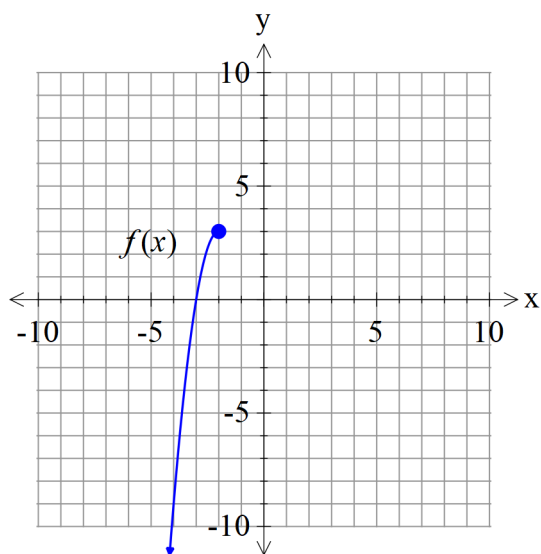
(c) Determine the rule and natural domain for $f \circ f(x)$. Explain why the composite exists. (3 marks)

(d) Does $g \circ f(x) = \frac{1}{x+5}$? Justify. (2 marks)

Question 3

(9 marks)

Consider the function $f(x)$ which is drawn below and defined only for $x \leq -2$.



(a) Sketch $y = f^{-1}(x)$ on the axes above.

(2 marks)

(b) Given that $f(x) = -3x^2 - 12x - 9, x \leq -2$, determine the rule for $y = f^{-1}(x)$ and state the domain and range. (4 marks)

(c) Determine the exact solution(s) to $f(x) = f^{-1}(x)$ if any.

(3 marks)

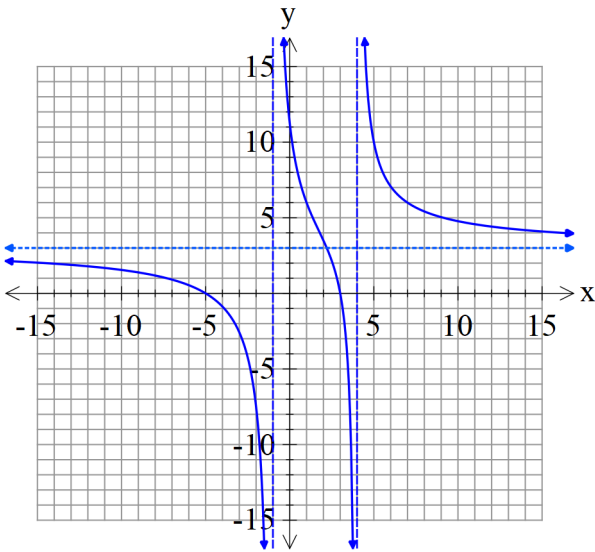
Question 4

(3 marks)

Consider the complex equation $z^n = 1 + i$ for any positive integer $n \geq 3$. The n roots are designated $z_0, z_1, z_2, \dots, z_{n-1}$.
Let $p = z_0 \times z_1 \times z_2 \times \dots \times z_{n-1}$, determine $|p|$ for any positive integer $n \geq 3$. Explain.

Question 5

(5 marks)



The function $f(x) = \frac{ax^2 + bx + c}{x^2 + px + q}$ is drawn to the left where a, b, c, p & q are all integers.

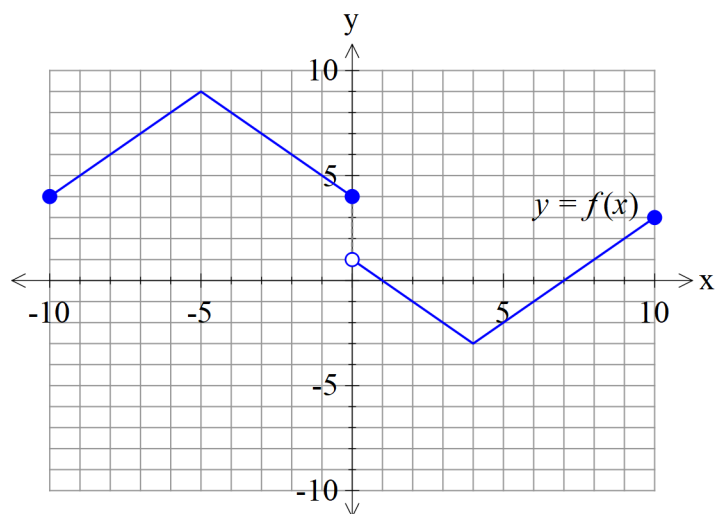
Complete the following table.

a	b	c	p	q

Question 6

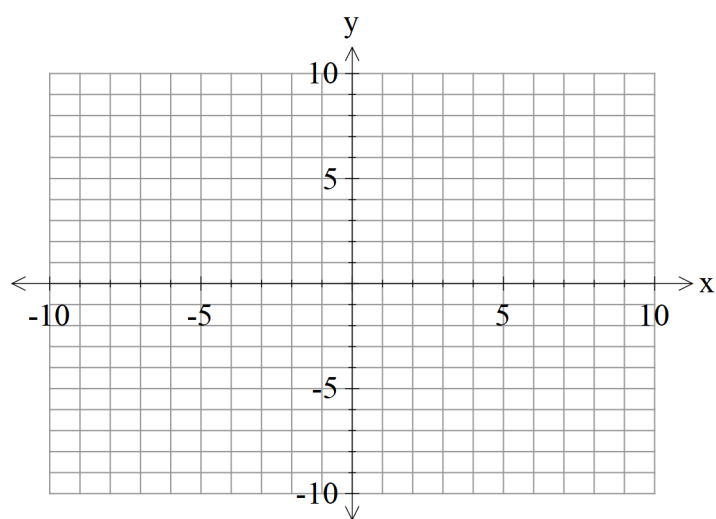
(8 marks)

Consider the function $f(x)$ which is drawn below and is defined for $-10 \leq x \leq 10$.

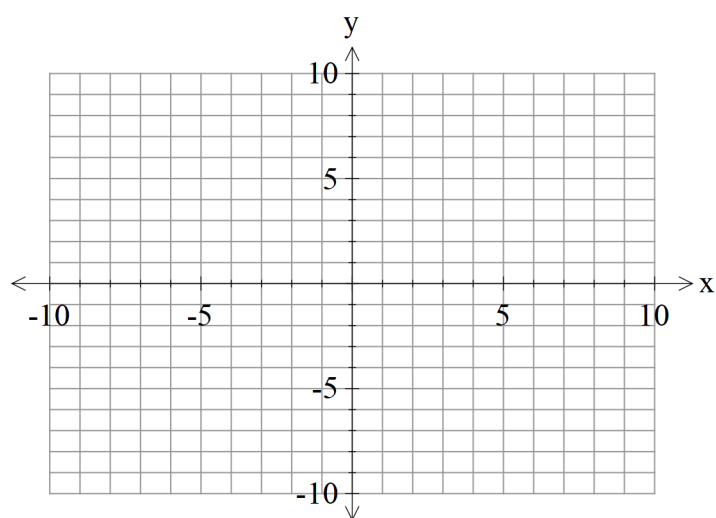


(a) Sketch $y = f(-|x|)$ on the axes below.

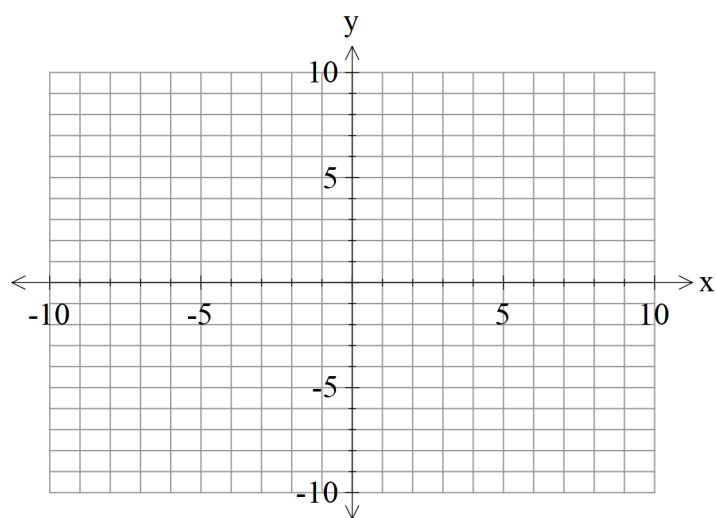
(2 marks)



- (b) Sketch $y = |f(|x|)|$ on the axes below. (3 marks)



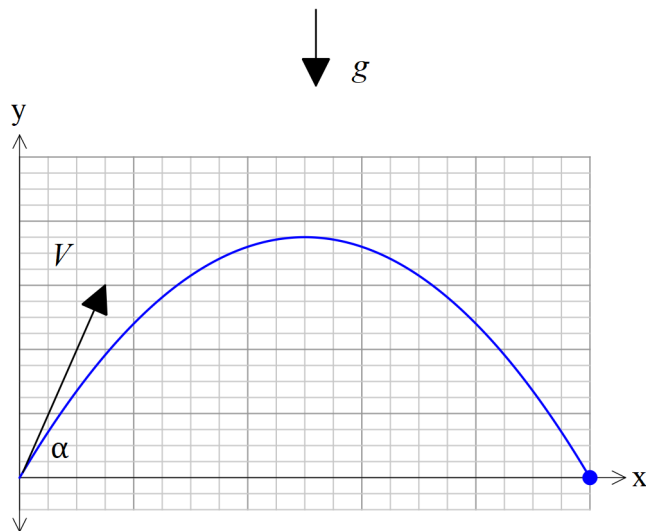
- (c) Sketch $y = \frac{1}{f(x)}$ on the axes below. (3 marks)



Question 7

(9 marks)

Consider a projectile that has an initial speed, $V \text{ m/s}$, at an angle of α with the horizontal that moves with an acceleration of $\ddot{\mathbf{r}} = \begin{pmatrix} 0 \\ -g \end{pmatrix} \text{ m/s}^2$ where g is a constant.



- a) If the projectile begins at the origin, show that a time, $t \text{ s}$, and using vector calculus that the velocity vector is given by: (2 marks)

$$\dot{\mathbf{r}} = \begin{pmatrix} V \cos \alpha \\ V \sin \alpha - gt \end{pmatrix} \text{ m/s}$$

- b) In terms of V, g & α derive the cartesian equation of the projectile. (4 marks)

Q7 cont

- c) Given that $V = \sqrt{5}$ & $g = 10$ derive the following equation.
 $x^2 \tan^2 \alpha - x \tan \alpha + x^2 + y = 0$

(3 marks)

