

Question/Answer booklet

Semester One Examination, 2023

## MATHEMATICS METHODS



INDEPENDENT PUBLIC SCHOOL  
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### UNIT 3

#### Section Two:

Calculator-assumed

Your Teacher's Name:

Your Name:

Time allowed for this section  
Materials required/recommended for this section  
*To be provided by the supervisor*

Working time:  
Reading time before commencing work: ten minutes  
Formula sheet (retained from Section One)  
This Question/Answer booklet

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

**Important note to candidates**

No other items may be taken into the examination room. It is **your responsibility** to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Question	Max	Marks	Question	Max	Marks	Max
12			13			
11			10			
10			15			
9			12			
8			14			
7			13			
6			11			
5			12			
4			8			

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	51	36
Section Two: Calculator-assumed	10	10	100	90	64
<b>Total</b>					<b>100</b>

## Additional working space

Question number: \_\_\_\_\_

## Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Additional working space

Question number: \_\_\_\_\_

This section has ten questions. Answer all questions. Write your answers in the spaces provided.

Section Two: Calculator-assumed  
(90 Marks)

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Working time: 100 minutes.
- Continuing a question that you are continuing to answer at the top of the page.
  - Answer space where the answer is continued, i.e. give the page number. Fill in the number of the page.
  - Continuing an answer: if you need to use the space for planning, indicate this clearly at the top of the page.
  - Planning: if you use the space for planning, indicate this clearly at the top of the page.

58 mg of a radioisotope with a half-life of 63 hours was injected into a patient before a CT scan. The mass  $M$  of the radioisotope decays continuously so that  $t$  hours after administration, the mass remaining is given by  $M = M_0 e^{-kt}$ , where  $M_0$  and  $k$  are constants.

(a) Determine the value of the constants  $M_0$  and  $k$ . (3 marks)

(b) Determine the mass of the radioisotope that remains in the patient exactly 6 days after their injection. (1 mark)

(c) Eventually, the mass of the remaining radioisotope falls to 2 mg.

(ii) Determine the rate at which the radioisotope is decaying at this time. (2 marks)

See next page

See next page

**Question 8****(6 marks)****Additional working space**

A barrel is filled with 34 balls numbered with the integers 1,2,3,...,33,34, but otherwise identical.

Let the random variable  $X$  be the number on a ball drawn at random from the barrel.

- (a) Explain why  $X$  has a uniform distribution. (1 mark)

- (b) Determine the expected value of  $X$ . (1 mark)

Let the random variable  $Y$  take the value 1 when  $X < 10$  and the value 0 otherwise.

- (c) State the particular name given to two-outcome random variables such as  $Y$ . (1 mark)

- (d) Determine  $P(Y=1)$ . (1 mark)

- (e) Three balls are drawn at random from the barrel. Determine the probability that exactly two of the balls are marked with single digit numbers. (2 marks)

Question number: \_\_\_\_\_

(3 marks)

- (b) Use calculus to determine the cross-sectional area of the inselberg shaded in the figure above.

(4 marks)

- (b) Using calculus determine the value of  $x$  which minimises the area.

(2 marks)

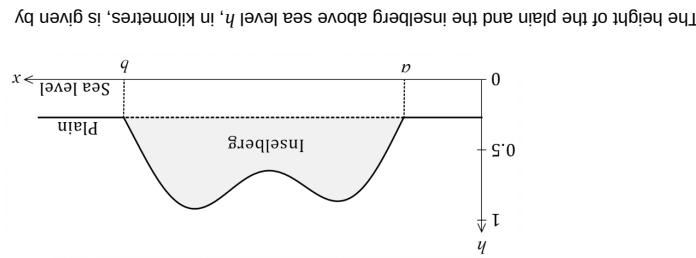
- (a) Determine the value of  $a$  and the value of  $b$ , the  $x$  displacements where the inselberg meets the surrounding plain.

(4 marks)

$$A = x^2 + \frac{900 - 120x + 4x^2}{\pi}$$

- (a) By expressing  $r$  in terms of  $x$ , show that the total area of both shapes is given by:

Before construction, a stencil was made, and the total perimeter of both figures was found to be 60cm.



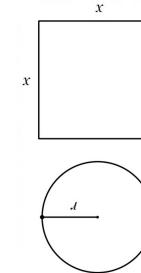
A vertical cross section through the highest point of an inselberg, a mountain range that rises above a surrounding level plain, is shown in the figure below.

(12 marks)

Question 9

(8 marks)

The diagram below shows a logo which includes a circle of radius  $r$  cm and a square with side  $ac$  cm.



(c) Use calculus to determine and justify the maximum height of the inselberg above the surrounding plain.  
(7 marks)

(d) Calculate the Standard deviation of  $X$ .  
(2 marks)

(e) The factory owner decides to introduce a points scheme using the rule  $Y = 20 - 5X$ .  
Find the points Jake will expect to receive and state the variance of his points score.  
(Give your answers to the nearest unit)

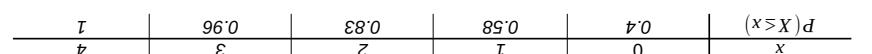
(2 marks)

Question 14  
(10 marks)

- cumulative probability distribution
- Table 14 shows the cumulative probability distribution of the number  $X$  of damaged tyres on a random day at Jake's works in a tyre factory. The number  $X$  of damaged tyres that Jake produces follows a discrete uniform distribution.
- (a) Use the quotient rule to show that  $P(X \leq x) = \frac{e^{0.5x}}{3} - \frac{e^{0.5(x-1)}}{2}$ .
- (b) Calculate  $P(X_1 \vee X_4)$ .
- (c) Determine the expected value of  $X$  and interpret your answer in the context of this question.

(i) more than two damaged tyres.  
(ii) three damaged tyres.  
(iii) four damaged tyres.

- (a) Determine the probability that Jake produces:



- Question 14  
(10 marks)
- Question 10  
(3 marks)
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- (c) The height  $h$  of a plant, initially 9 cm, is changing at a rate given by  $\frac{dh}{dt} = \frac{2t}{e^{0.5t}}$  cm/day, for  $t \geq 0$ .

- (i) Determine an equation to model the height of the plant as a function of time and hence determine its height after 7 days. (3 marks)

A Mathematics teacher asks each of her 20 students to write down the fraction they obtained from rolling the two dice.

- (d) Determine the probability that at least half of the class obtained a fraction where the numerator is greater than or equal to the denominator. (3 marks)

- (ii) According to the model, what height will the plant never exceed? (1 mark)

- (e) The teacher asks each student one-by-one what fraction they obtained. The teacher dismisses the class when *exactly* 5 students have a fraction of less than 1.

Determine the probability that the teacher will need to ask *exactly* ten students before dismissing the class. (3 marks)

Question 13

(10 marks)

A Mathematics teacher gives a student two fair six-sided dice. One die is coloured red and the other is coloured blue.

The student rolls the two dice, and then writes down the following fraction:

$$\text{Fraction } (F) = \frac{\text{Number on the red die}}{\text{Number on the blue die}}$$

Between  $t=0$  and  $t=12$  hours, water is pumped into the tank at the rate of

$$W(t) = 55 - 15\cos\left(\frac{\pi t}{3}\right) \text{ litres/hour}$$

Initially a water tank contains 300 Litres.

Question 11

(12 marks)

(2 marks)

(b) Is the level of water rising or falling when  $t=6$ ?

(2 marks)

(2 marks)

(c) How many litres are in the tank at  $t=12$ ?

(3 marks)

Friction F	$F < 1$	$1 \leq F < 2$	$2 \leq F < 3$	$3 \leq F < 6$	$F = 6$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{9}$

(d) Complete the table above, providing the missing probabilities. (2 marks)

(2 marks)

(c) Determine the probability that a student obtains a fraction that is at least 2, given that the fraction is less than 3. (3 marks)

The teacher draws up the following table on the board.

(2 marks)

(d) When is the amount of water in the tank at a minimum? (3 marks)

(2 marks)

(2 marks)

(3 marks)

**Question 12**

The Lupu Bridge in Shanghai was the longest steel arch bridge when it opened in 2003.

The main arch can be represented by the graph of the function

$$f(x) = 500 - 200 \left( e^{\frac{x}{400}} + e^{-\frac{x}{400}} \right)$$

where  $x$  is the distance, in metres, from the middle, and  $f(x)$  is the distance, in metres, above the Hangpu River.

The bridge is symmetrical about the vertical axis.



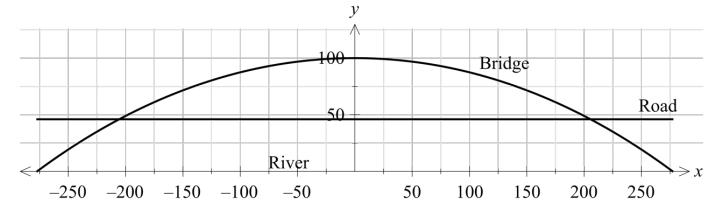
(13 marks)

- (a) Determine  $f'(x)$  writing your answer in the form  $f'(x) = a \left( e^{\frac{x}{400}} - e^{-\frac{x}{400}} \right)$ , where  $a$  is a rational number.

(2 marks)

- (b) Using calculus, verify that the **maximum** height of the bridge is 100 m. (4 marks)

The graph below shows the Lupu Bridge and the road, which is 46 m above the river.



- (c) Determine, correct to the nearest 100  $\text{m}^2$ , the cross-sectional area between the road, the bridge and water. (4 marks)

An observation deck is positioned at the top of the bridge. To access the deck, visitors need to climb the arch. The distance,  $D$ , travelled along the arch is given by

$$D(t) = \int_0^t \sqrt{\frac{1}{2} + \frac{e^{\frac{x}{200}}}{4} + \frac{e^{-\frac{x}{200}}}{4}} dx$$

where  $D$  is measured in metres, and  $t$  is measured in seconds.

- (d) Determine the speed,  $s = \frac{dD}{dt}$  of the visitors, when they are 2 minutes into their ascent. (3 marks)