SCHOOL

Trial WACE Examination, 2012

Question/Answer Booklet

MATHEMATICS SPECIALIST 3A/3B

SOLUTIONS

Section One: Calculator-free

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

See next page

Section One: Calculator-free **(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(6 marks)**

The complex numbers z and w are given by $z = 8 - 6i$ and $w = 3 + ai$.

(a) Determine $z \times \overline{z}$. (2 marks)

$$\begin{aligned} z \times \overline{z} &= (8 - 6i)(8 + 6i) \\ &= 64 + 36 \\ &= 100 \end{aligned}$$

(b) Find $\text{Im}(w^2)$ if $a = 5$. (2 marks)

$$\begin{aligned} w^2 &= (3 + 5i)(3 + 5i) \\ &= 9 - 25 + 30i \\ \text{Im}(w^2) &= 30 \end{aligned}$$

(c) The value of a if $w^2 = z$. (2 marks)

$$\begin{aligned} z &= 8 - 6i \\ w^2 &= (3 + ai)(3 + ai) \\ &= 9 - a^2 + 6ai \end{aligned}$$

From equating imaginary parts

$$\begin{aligned} 6a &= -6 \\ a &= -1 \end{aligned}$$

See next page

Question 2 **(4 marks)**

Determine the values of a and b if the function

$$f(x)=\begin{cases} a+\dfrac{4}{x} & 0<x<2 \\ bx^2-2 & x\geq 2 \end{cases}$$

is both continuous and differentiable at $x=2$.

Continuous:

$$a+\frac{4}{2}=b\times 2^2-2 \Rightarrow a=4b-4$$

Differentiable:

$$f'(x)=\begin{cases} -\dfrac{4}{x^2} \\ 2bx \end{cases}$$

$$-\frac{4}{2^2}=2\times b\times 2 \Rightarrow b=-\frac{1}{4}$$

$$a=4\times -\frac{1}{4}-4=-5$$

$$a=-5,\ b=-\frac{1}{4}$$

See next page

Question 3 **(9 marks)**

(a) Solve $\log_x 64 = 3$. (1 mark)

$$x^3 = 64 \Rightarrow x = 4$$

(b) Rearrange the equation $11^{2x+3} = 33$ for x. (2 marks)

$$(2x+3)\log 11 = \log 33$$
$$2x + 3 = \frac{\log 33}{\log 11}$$
$$x = \frac{\log 33}{2\log 11} - \frac{3}{2}$$

(c) Solve $2(\log_3 x + 1) = \log_3 25$. (3 marks)

$$\log_3 x + 1 = (0.5)\log_3 25$$
$$\log_3 x + \log_3 3 = \log_3 25^{0.5}$$
$$\log_3 3x = \log_3 5$$
$$x = \frac{5}{3}$$

(d) If $a = \log_5 4$ and $b = \log_5 8$, express the following in terms of a and b:

(i) $\log_5 32$ (1 mark)

$$\log_5 32 = \log_5 (4 \times 8)$$
$$= \log_5 4 + \log_5 8$$
$$= a + b$$

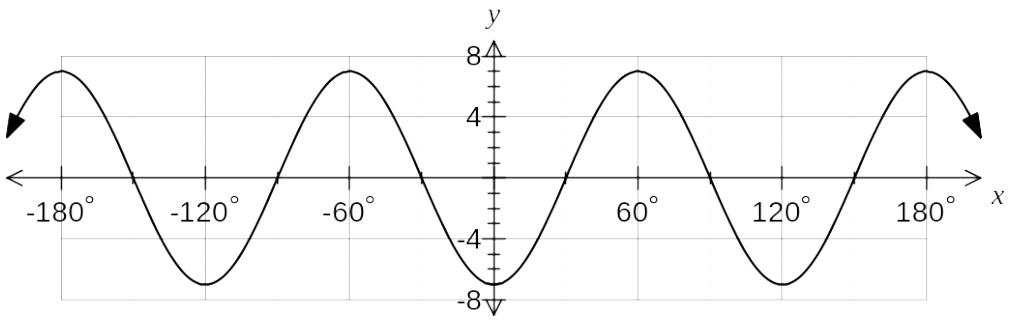
(ii) $\log_5 400$ (2 marks)

$$\log_5 400 = \log_5 (5 \times 4)^2$$
$$= 2(\log_5 5 + \log_5 4)$$
$$= 2(1 + a)$$

See next page

Question 4 **(11 marks)**

(a) The graph of the trigonometric function $y = f(x)$ is shown.



(i) State the amplitude of $f(x)$. (1 mark)

7

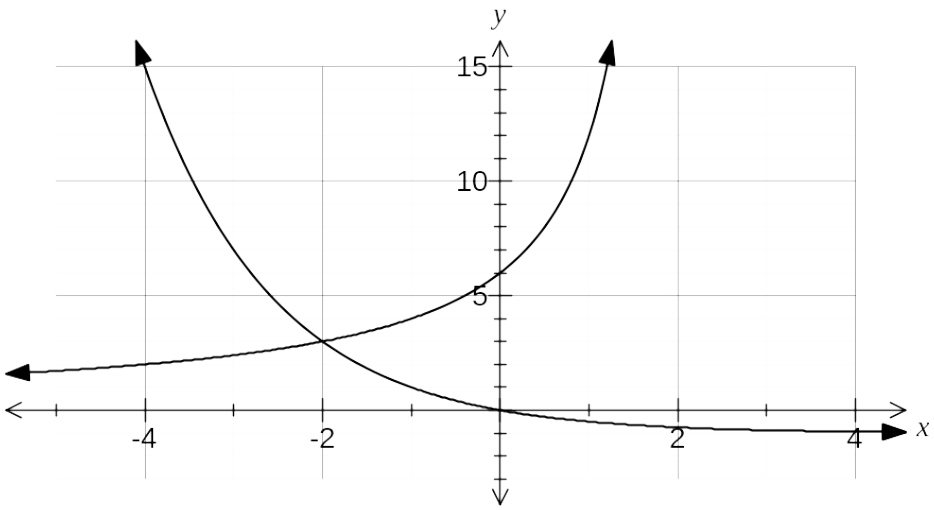
(ii) State the period of $f(x)$. (1 mark)

120°

(iii) State the equation of the trigonometric function $f(x)$. (2 marks)

$y = -7\cos(3x)$

(b) The graphs of $g(x) = \frac{a}{b - x}$ and $h(x) = 4^{cx} + d$, where a, b, c and d are all non-zero constants are shown below, intersecting at (-2, 3).



(i) Explain why $g(x)$ cannot pass through the origin. (2 marks)

Because $g(0) = \frac{a}{b}$, but a is non-zero and so $g(0) \neq 0$.

(ii) Find the values of a, b, c and d. (5 marks)

$g(x)$:

$x = 0 \Rightarrow 6 = \frac{a}{b} \Rightarrow a = 6b$

$x = -2 \Rightarrow 3 = \frac{a}{b + 2}$

$3b + 6 = 6b$

$b = 2$

$a = 12$

$h(x)$:

$x = 0 \Rightarrow 0 = 1 + d \Rightarrow d = -1$

$x = -2 \Rightarrow 3 = 4^{-2c} - 1$

$4 = 4^{-2c}$

$-2c = 1$

$c = -0.5$

$a = 12,\ b = 2,\ c = -0.5,\ d = -1$

See next page

Question 5 **(8 marks)**

Differentiate the following, simplifying where possible.

(a) $y = \dfrac{\log_e(4 - x^3)}{12}$ (2 marks)

$$\frac{dy}{dx} = \frac{1}{12} \times \frac{-3x^2}{4 - x^3}$$
$$= \frac{x^2}{4(x^3 - 4)}$$

(b) $y = \dfrac{3x}{2x+1} - \dfrac{1}{2x+1}$ (3 marks)

$$y = \frac{3x - 1}{2x+1}$$
$$\frac{dy}{dx} = \frac{3(2x+1) - 2(3x - 1)}{(2x+1)^2}$$
$$= \frac{6x + 3 - 6x + 2}{(2x+1)^2}$$
$$= \frac{5}{(2x+1)^2}$$

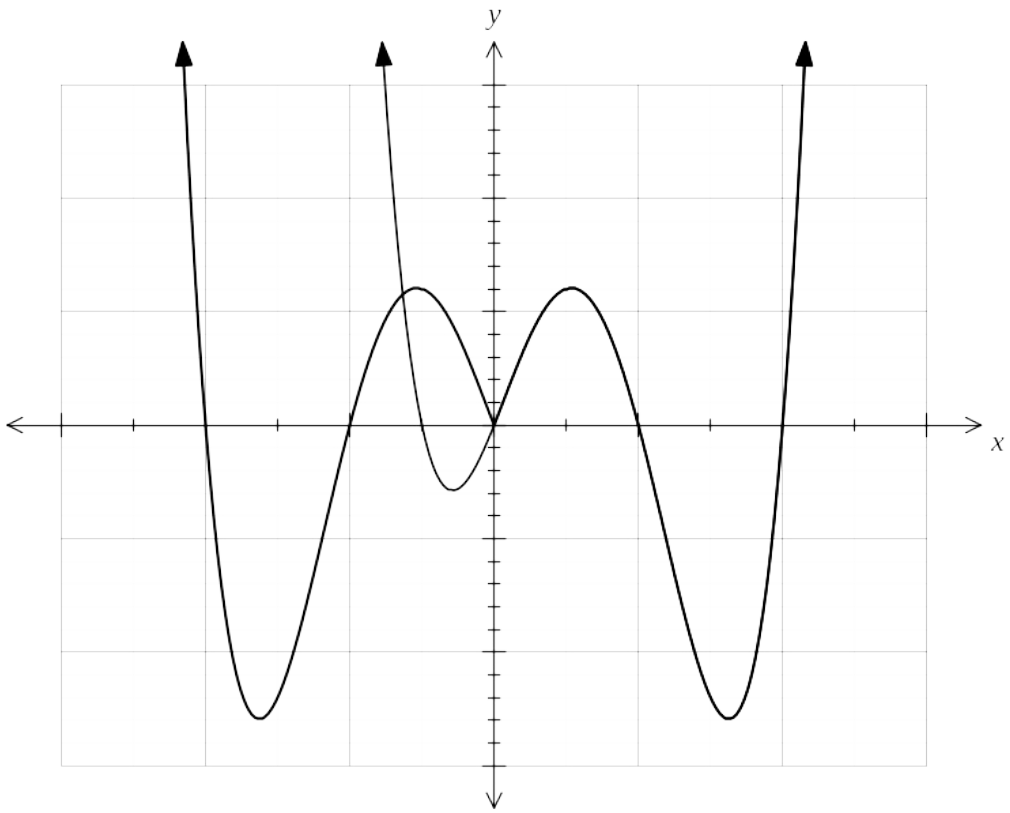
(c) $y = \left(e^{\sqrt{x}} + 5\right)^4$ (3 marks)

$$\frac{dy}{dx} = 4 \times \left(\frac{1}{2\sqrt{x}} e^{\sqrt{x}}\right)\left(e^{\sqrt{x}} + 5\right)^3$$
$$= \frac{2e^{\sqrt{x}}}{\sqrt{x}}\left(e^{\sqrt{x}} + 5\right)^3$$

See next page

Question 6 **(4 marks)**

The graph of $y = f(x)$ is shown.



(a) Draw the graph of $y = f(|x|)$ on the same axes. (2 marks)

(b) Briefly explain how to obtain the graph of $y = f(|x|)$ from $y = f(x)$. (2 marks)

Reflect the part of $y = f(x)$ in the 1st and 4th quadrants in the y-axis.

See next page

Question 7 **(8 marks)**

(a) Show that $\tan(\theta)+\tan(2\theta)=\dfrac{\tan^3(\theta)-3\tan(\theta)}{\tan^2(\theta)-1}$. (3 marks)

$$\tan(\theta)+\tan(2\theta)=\tan(\theta)+\frac{2\tan(\theta)}{1-\tan^2(\theta)}$$
$$=\frac{\tan(\theta)\left(1-\tan^2(\theta)\right)+2\tan(\theta)}{1-\tan^2(\theta)}$$
$$=\frac{3\tan(\theta)-\tan^3(\theta)}{1-\tan^2(\theta)}$$
$$=\frac{\tan^3(\theta)-3\tan(\theta)}{\tan^2(\theta)-1}$$

(b) Solve the equation $\tan(\theta)+\tan(2\theta)=0$ for $0\le\theta\le\pi$. (5 marks)

$$\frac{\tan^3(\theta)-3\tan(\theta)}{\tan^2(\theta)-1}=0$$
$$\tan^3(\theta)-3\tan(\theta)=0$$
$$\tan(\theta)\left(\tan^2(\theta)-3\right)=0$$
$$\tan(\theta)\left(\tan(\theta)+\sqrt{3}\right)\left(\tan(\theta)-\sqrt{3}\right)=0$$

$\tan(\theta)=0 \Rightarrow \theta=0,\ \pi$

$\tan(\theta)+\sqrt{3}=0 \Rightarrow \theta=\frac{2\pi}{3}$

$\tan(\theta)-\sqrt{3}=0 \Rightarrow \theta=\frac{\pi}{3}$

Solutions are $\theta=0,\ \frac{\pi}{3},\ \frac{2\pi}{3},\ \pi$

End of questions

Additional working space

Question number: _____

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