**PERTH MODERN SCHOOL**

**UNIT 3C/3D MAS – 2011**

**TEST 1 – POLAR COORDINATES & COMPLEX NUMBERS**

**NAME: SOLUTIONS** **DATE: 9/10 February, 2011**

**[To achieve full marks and to allow assessment of particular outcomes, working and reasoning should be shown.]**
**[A maximum of 2 marks will be deducted for incorrect rounding, units, etc.]**

**This is *Resource Rich – 50 minutes for 53 marks*:**

1. [1, 1, 1, 1 = 4 marks]
   Convert:
   a) (4,-6) into polar coordinates with $-180^\circ < \theta \le 180^\circ$.

      toPol([4,-6]) ⇒ [7.21, -56.3°] ✓

   b) $(3,-\sqrt{3})$ into ***exact*** polar coordinates with $-\pi < \theta \le \pi$.

      toPol($[3,-\sqrt{3}]$) ⇒ $[2\sqrt{3}, -\frac{\pi}{6}]$ ✓

   c) $[4,35^\circ]$ into Cartesian coordinates.

      toRect($[4,\angle 35^\circ]$) ⇒ (3.28, 2.29) ✓

   d) $[8,-\frac{3}{4}\pi]$ into ***exact*** Cartesian coordinates.

      toRect($[8,\angle -\frac{3}{4}\pi]$) ⇒ $(-4\sqrt{2}, -4\sqrt{2})$ ✓

2. [3, 3 = 6 marks]
Clearly show how you obtain your answers, find:
a) the distance between [20,-210°] and [ $\sqrt{5}$ ,-50°].

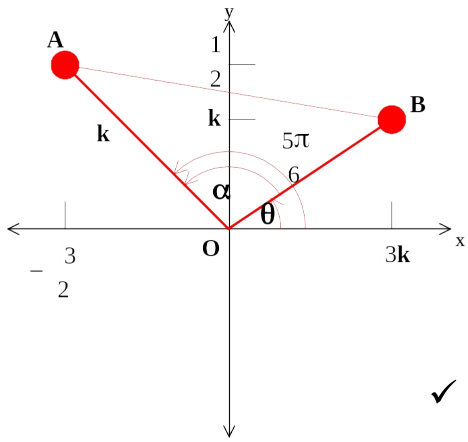
toRect([20,∠(-210°)]) ⇒ (-17.32, -10) ✓
toRect([$\sqrt{5}$ ,∠(-50°)]) ⇒ (1.44, -1.71) ✓
norm([-17.32, -10] – [1.44, -1.71]) ⇒ 22.11 ✓

b) the ***exact*** distance between $\left[\frac{\sqrt{5}}{3}, -\frac{2\pi}{3}\right]$ and $\left[10, -\frac{7\pi}{6}\right]$.

toRect([$\frac{\sqrt{5}}{3}$,∠( $-\frac{2\pi}{3}$)]) ⇒ ( $-\frac{\sqrt{5}}{6}$, $-\frac{\sqrt{15}}{6}$ ) ✓
toRect([10,∠( $-\frac{7\pi}{6}$)]) ⇒ (-5 $\sqrt{3}$ , 5) ✓
norm([ $-\frac{\sqrt{5}}{6}$ , $-\frac{\sqrt{15}}{6}$] – [-5 $\sqrt{3}$ , 5]) ⇒ $\frac{\sqrt{905}}{3}$ ✓

3. [8 marks]

Find the ***exact*** distance between ( $\sqrt{3k}$ , $\sqrt{\mathbf{k}}$ ) and $\left[\sqrt{\mathbf{k}}, \frac{5\pi}{6}\right]$ . Draw a diagram.




$$OB = \sqrt{3\mathbf{k} + \mathbf{k}}$$
$$= 2\sqrt{\mathbf{k}} \quad ✓$$
$$\tan\theta = \frac{\sqrt{\mathbf{k}}}{\sqrt{3\mathbf{k}}} \quad ✓$$
$$= \frac{\sqrt{3}}{3}$$
$$\theta = \frac{\pi}{6} \quad ✓$$
$$\therefore \alpha = \frac{2\pi}{3} \quad ✓$$

$AB^2 = OA^2 + OB^2 – 2(OA)(OB) \cos\alpha$ ✓
$= k + 4k – 2(\sqrt{\mathbf{k}})(2\sqrt{\mathbf{k}})(-½)$
$= 7k$ ✓
∴ AB = $\sqrt{7\mathbf{k}}$
∴ The distance between the points is $\sqrt{7\mathbf{k}}$ units. ✓

✓

4. [3, 2 = 5 marks]

a) Find, in ***exact*** form, the modulus and principal argument of $-\sqrt{3} + i$, and hence rewrite $-\sqrt{3} + i$ in ***exact*** polar (***cis***) form.

Modulus $= \sqrt{3 + 1} = 2$ ✓

Principal argument $= \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \frac{5\pi}{6}$ ✓

$\therefore\ -\sqrt{3} + i = 2 \text{ cis } \frac{5\pi}{6}$ ✓

b) Convert $2\ cis\left(\frac{\pi}{4}\right)$ into ***exact*** algebraic Cartesian/rectangular form.

$2 \text{ cis}\left(\frac{\pi}{4}\right) = 2\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right]$ ✓

$= \sqrt{2} + i\sqrt{2}$ ✓

5. [3, 3 = 6 marks]

Evaluate, giving answers in ***exact*** form:

a) $4\ cis\frac{\pi}{3} \times 2\ cis\frac{3\pi}{4} = 8 \text{ cis}\left(\frac{\pi}{3} + \frac{3\pi}{4}\right)$ ✓

$= 8 \text{ cis}\frac{13\pi}{12}$ ✓

$= 8 \text{ cis}\left(-\frac{11\pi}{12}\right)$ ✓

b) $\dfrac{4\ cis\left(-\frac{5\pi}{6}\right)}{2\ cis\left(\frac{5\pi}{6}\right)} = 2 \text{ cis}\left(-\frac{5\pi}{6} - \frac{5\pi}{6}\right)$ ✓

$= 2 \text{ cis}\left(-\frac{5\pi}{3}\right)$ ✓

$= 2 \text{ cis}\left(\frac{\pi}{3}\right)$ ✓

6. [4 marks]

Given $z = 2\,cis\,\dfrac{\pi}{4}$, express $z^{-1}$ and $\bar{z}$ in ***exact*** polar and rectangular form.

$$z^{-1} = \left[\, 2\,\text{cis}\,\frac{\pi}{4} \right]^{-1}$$

$$= \tfrac{1}{2}\,\text{cis}\left( -\frac{\pi}{4} \right) \quad \leftarrow \text{Polar form} \quad \checkmark$$

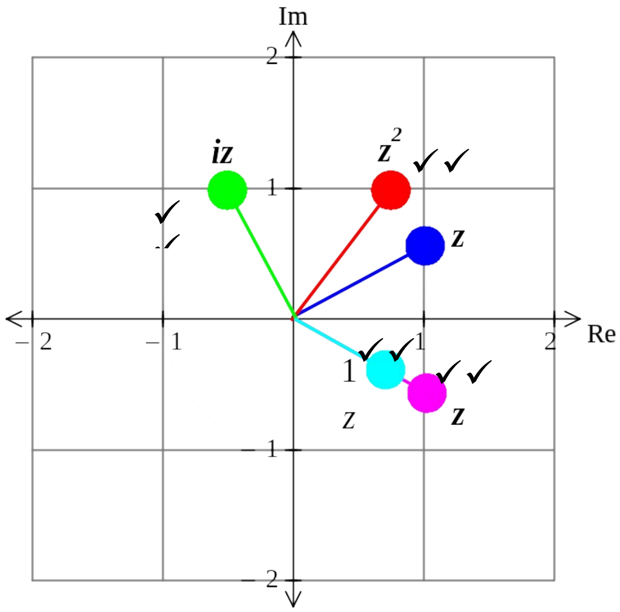
$$= \frac{\sqrt{2}}{4}(1 - \mathbf{i}) \quad \leftarrow \text{Rectangular form} \quad \checkmark$$

$$\bar{z} = 2\,\text{cis}\left( -\frac{\pi}{4} \right) \quad \leftarrow \text{Polar form} \quad \checkmark$$

$$= \sqrt{2}(1 - \mathbf{i}) \quad \leftarrow \text{Rectangular form} \quad \checkmark$$

7. [2, 2, 2, 2 = 8 marks]
The Argand diagram below shows the point representing the complex number $z$ where $|\,z\,| > 1$. Plot on the same diagram, the points representing the complex numbers:

a) $\bar{z}$ b) $iz$ c) $z^2$ d) $\dfrac{1}{z}$




If $z = 1 + \tfrac{1}{2}i$

$\Rightarrow \bar{z} = 1 - \tfrac{1}{2}i$

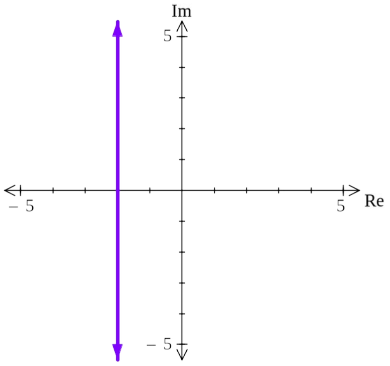
$iz = -\tfrac{1}{2} + i$

$z^2 = \tfrac{3}{4} + i$

$\dfrac{1}{z} = \dfrac{4}{5}\left(1 - \tfrac{1}{2}i\right)$

8. [3, 3, 3, 3 = 12 marks]
Sketch on an Argand diagram, the locus of the point $z = x + iy$, satisfying each of the following conditions. In each case, give the Cartesian equation or inequality of the locus.

a) $\text{Re}(z) = -2 \Rightarrow x = -2$ ✓

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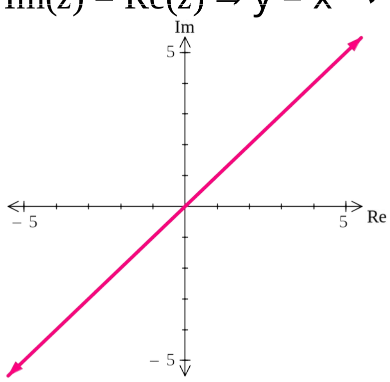
b) $\text{Im}(z) = \text{Re}(z) \Rightarrow y = x$ ✓

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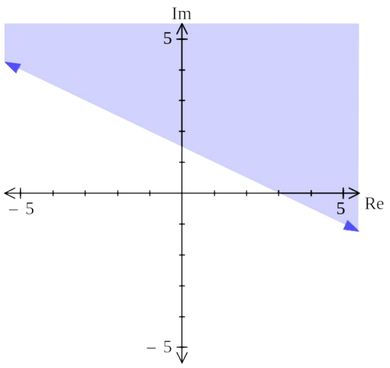
c) $\text{Re}(z) + 2\text{Im}(z) > 3 \Rightarrow x + 2y > 3$ ✓

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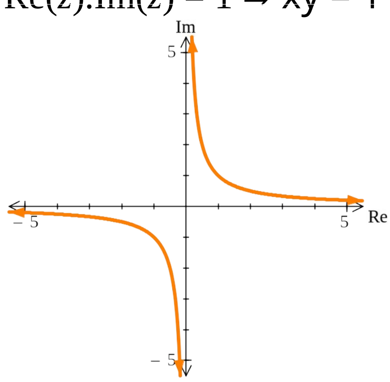
d) $\text{Re}(z).\text{Im}(z) = 1 \Rightarrow xy = 1$ ✓

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