

**Papers written by
Australian Maths
Software**

**SEMESTER TWO
YEAR 11**

**MATHEMATICS SPECIALIST
Units 1 & 2
2016**

REVISION 1

**Section Two
(Calculator–assumed)**

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 10 minutes

Working time for section: 100 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: drawing instruments, templates, notes on up to two unfolded sheets of A4 paper, and up to three calculators approved for use in examinations.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

To be provided by the supervisor

Question/answer booklet for Section Two.

Formula sheet retained from Section One.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	7	7	50	52	35
Section Two Calculator—assumed	12	12	100	98	65
Total marks				150	

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answer in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula Sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

98 marks

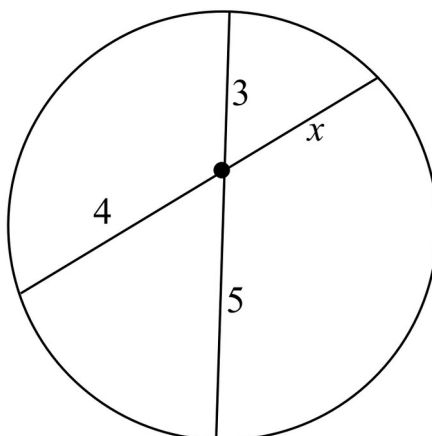
This section has **twelve (12)** questions. Attempt **all** questions.
Working time: 100 minutes

Question 8

(5 marks)

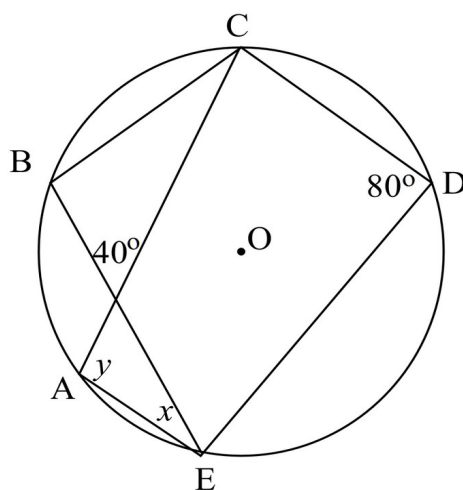
(a) Solve for x , giving reasons.

(2)



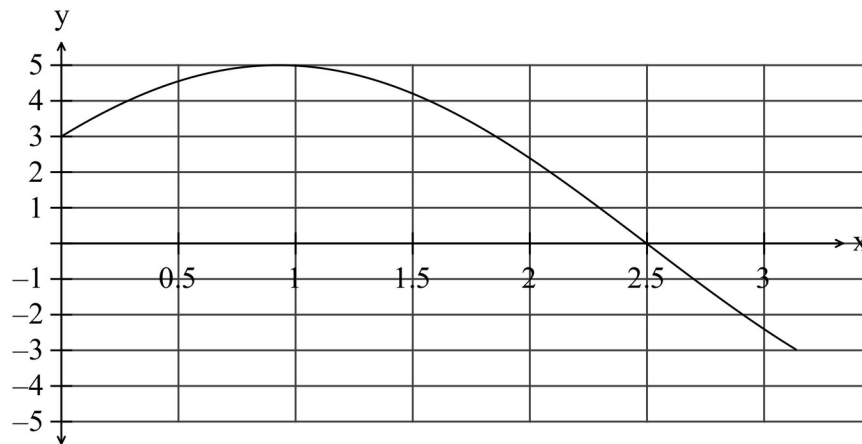
(b) Solve for x and y , giving reasons.

(3)



Question 9**(11 marks)**

- (a) (i) Use the graph of $y = 5\cos(x + p)$ to estimate the value of p . (1)

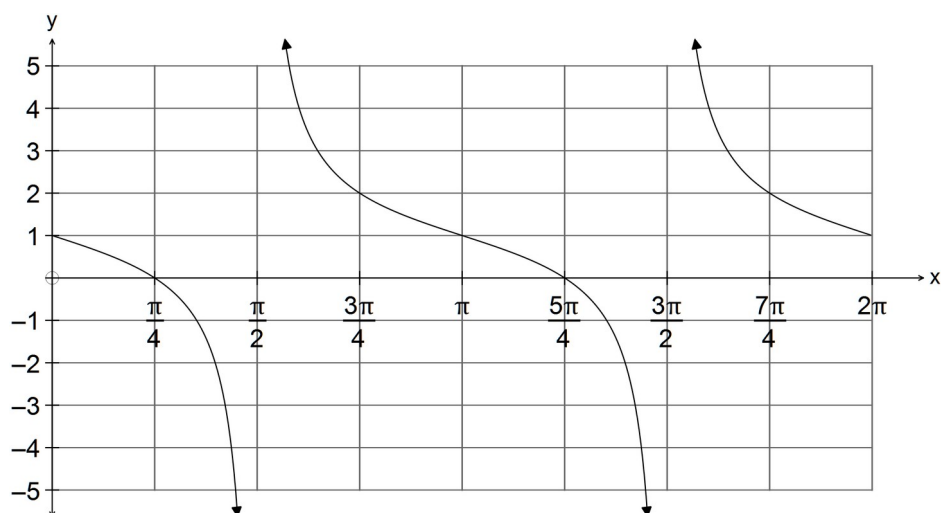


- (ii) Comment on the effect on the graph of changing the value of p . (1)

- (iii) Determine the values of R and p given $3\cos(x) + 4\sin(x) = R\cos(x - p)$
for $0 \leq p \leq \frac{\pi}{2}$ and $R > 0$. (4)

- (iv) Hence solve $3\cos(x) + 4\sin(x) = 2.5$ for x , correct to two decimal places, on the domain $-\pi \leq x \leq \pi$. (3)

- (b) Write down the equation of the function graphed below. (2)



Question 10**(7 marks)**

(a) Simplify $\frac{(1+2i)^2(1-2i)^2}{1+i}$. (3)

(b) Find x and y if $\sqrt{3+4i} = x+iy$ where x and y are real numbers (4)

HINT: Square both sides.

Question 11**(12 marks)**

(a) Given the points $P(5, -1)$ and $Q(2, -2)$,

(i) determine $|\mathbf{PQ}|$. (1)

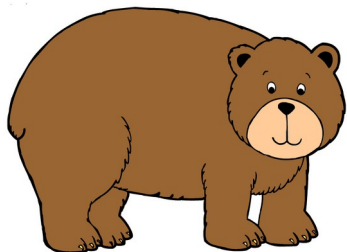
(ii) find the scalar projection of $\mathbf{i} + \mathbf{j}$ on \mathbf{PQ} . (2)

- (b) A bear in a zoo has been anaesthetised so it can be moved to have its annual health check. They have got it onto a plastic sheet and want to drag it to the truck.

Three men exert forces as listed below. If the combined force is 300 N, they can manage the bear without having get the tractor to drag him.

$$\begin{pmatrix} 90 \\ 350^\circ \end{pmatrix} + \begin{pmatrix} 0 \\ 110 \end{pmatrix} + \begin{pmatrix} 130 \\ 012^\circ \end{pmatrix} \quad \begin{array}{l} \text{Direction} \\ \text{of travel} \end{array} \uparrow$$

Determine the magnitude and direction of the resultant force in the direction of travel and decide if they will have to use the tractor. (4)



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- (c) Use a vector proof to show that

“The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides.” (5)

Question 12**(9 marks)**

- (a) Solve the following equation $(z - 3)^2 + 1 = 0$. Show all working. (2)

- (b) Express the complex numbers below in the form $a + bi$.

(i) $\sqrt{-25}$ (1)

(ii) $\frac{3 + 2i}{4 - 3i}$ (3)

- (c) The vectors \mathbf{x} and \mathbf{y} are defined below.
- \mathbf{x} has a magnitude of 3 and makes an angle of 60° with the positive x axis.
- \mathbf{y} has a magnitude of 2 and makes an angle of 45° with the positive x axis.
- Determine $\mathbf{x} \cdot \mathbf{y}$. (3)

Question 13**(6 marks)**

- (a) Write the contrapositive of the statement "*If today is Tuesday, then Donna will go to the library.*"

(1)

- (b) Write the conclusion to the following deductive argument
"*All cats have 4 legs.*
Big Boy is a cat."

(1)

- (c) Use a proof by contradiction to prove that there are infinitely many prime numbers. (4)

Question 14**(9 marks)**

- (a) Perth Concert Hall's auditorium has a total capacity of 1731. At the recent WASO concert, there were 872 people in attendance.

Show that there will be at least two people in the hall who have the same first and last initial in their names. (2)

Paul has one red bin for rubbish, 2 green bins for garden waste and 2 yellow bins for recycling. Paul keeps his bins lined up on the back fence behind his garden shed.

- (b) In how many ways can Paul line up his bins with identically coloured bins are adjacent? (1)



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- (c) How many of the numbers from 1, 2, 3,90 are not multiples of 6 or 7? (3)

- (d) Prove that $\binom{n-1}{r-1} + \binom{n-1}{r} = \binom{n}{r}$ (3)

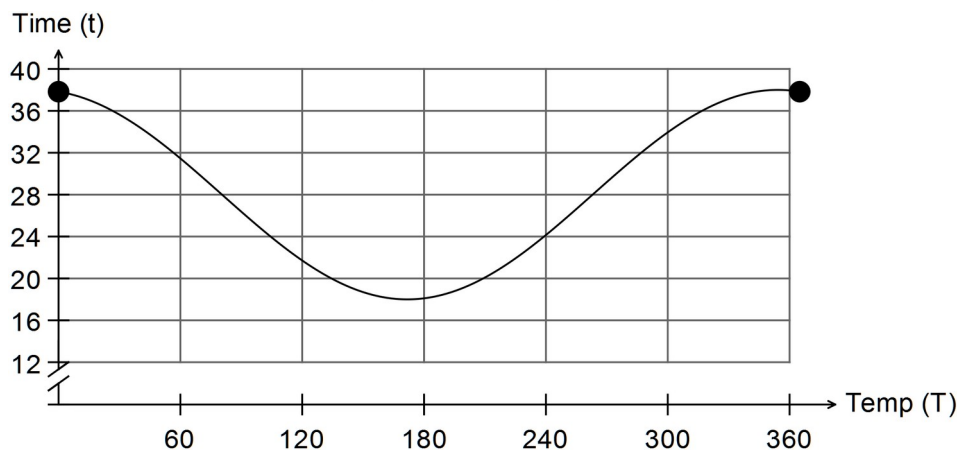
Question 15**(6 marks)**

The average temperature in Perth over one year can be approximated by the function

$$T = 28 - 10 \sin\left(\frac{2\pi}{365}(t - 80.5)\right) \text{ where } T \text{ is degrees C and } t \text{ in days taken from January}$$

1st, when $t = 0$. NB Assume there are 365 days in the year.

Consider the sketch $T = 28 - 10 \sin\left(\frac{2\pi}{365}(t - 80.5)\right)$ on the set of axes below:



(a) Find the temperature expected on January 1st. (1)

(b) Find the date of the minimum expected temperature. (1)

(c) Between what dates is the average temperature expected to be less than 24? (2)

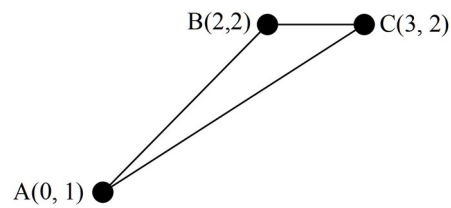
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- (d) What percentage of the year has the average temperature greater than 32? (2)

Question 16**(4 marks)**

Prove $\frac{\sin(2x)}{1 + \cos(2x)} = \frac{1 - \cos(2x)}{\sin(2x)}$ (4)

Question 17**(14 marks)**

- (a) ABC is a triangle. The triangle is reflected about the x axis then rotated 45° about the origin in an anti-clockwise direction to form triangle $A''B''C''$.

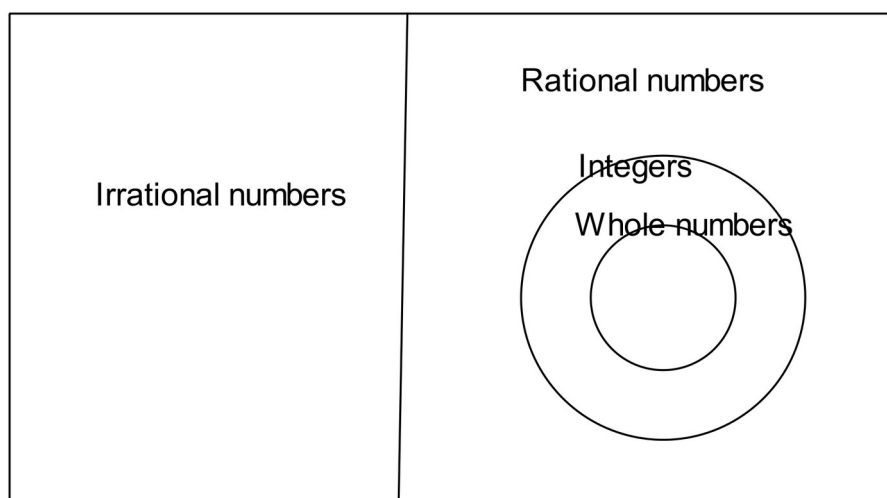


- (i) Write down the matrix that transforms the coordinates A, B and C into the coordinates A'' , B'' and C'' . (3)
- (ii) Find the area of the triangle ABC and hence determine the area of the triangle $A''B''C''$ using the matrix. (3)
- (iii) Determine the matrix that will transform triangle $A''B''C''$ back into triangle ABC. (2)

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- (b) A linear transformation given by $(x, y) \rightarrow (2x + y, x - 3y)$ transforms quadrilateral ABCD into the quadrilateral A'B'C'D'.
- (i) Write down the corresponding transformational matrix. (2)
- (ii) Determine how much bigger in area quadrilateral A'B'C'D' is than quadrilateral ABCD. (2)
- (c) A linear transformation using the matrix $\begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix}$ transforms quadrilateral ABCD into the quadrilateral A'B'C'D'.
- (i) Explain why a matrix to transform quadrilateral A'B'C'D' back to quadrilateral ABCD cannot be found. (1)
- (ii) Explain with reference to the transformed shape. (1)
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Question 18**(8 marks)**

- (a) Given the set of different numbers in the diagram below



- (i) place each of the numbers $-2, \frac{5}{6}, \sqrt{9}, \sqrt{2}, 0$ in the appropriate place on the diagram. (2)

- (ii) Give an example of a number that does not fit into this diagram. (1)

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- (b) Prove using mathematical induction that $11^n - 6$ is divisible by 5. (5)

Question 19**(7 marks)**

Two transformations are defined as

M is a transformation using the matrix $\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ and

R is a rotation of 270° anti-clockwise about the origin.

(a) Find the transformational matrix M followed by R . (4)

(b) (i) Determine the image of $P(1,2)$ under M followed by R . (2)

(ii) Find the matrix that takes the image P' back to $P(1,2)$. (1)

END OF SECTION TWO
