PERTH MODERN SCHOOL
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**Independent Public School**

# Course Methods test 3 Year 12

Student name: _______________ Teacher name: _______________

**Task type:** **Response**

**Time allowed for this task:** **___40_______ mins**

**Number of questions:** **___8______**

**Materials required:** Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

**Marks available:** **___46___ marks**

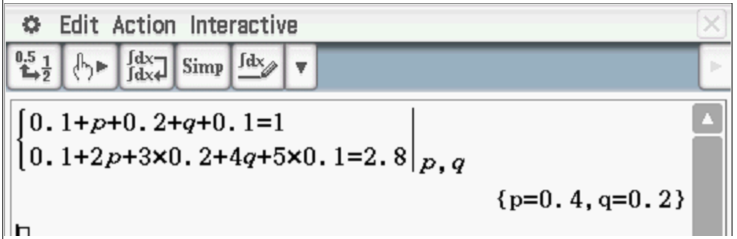
**Task weighting:** **_10__%**

**Formula sheet provided: Yes**

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

Q1 (3 marks)
The expected value of the discrete probability distribution given below is 2.8. Determine the values of $p \& q$ and hence determine Var( $X$ ), the variance of $X$ .

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| P( $X = x$ ) | 0.1 | $p$ | 0.2 | $q$ | 0.1 |

| Solution |
|---|
|  |
| **Specific behaviours** |
| ✓ sets up one equation with p&q<br>✓ sets up two equations with p&q<br>✓ solves for both p&q<br>(Note: max 2 marks if no working shown) |



Q2 (9 marks)
A students wishes to play a gambling game on mufti day involving throwing two regular fair dice, each numbered 1 to 6. To play the game the student must pay \$2 for each throw of two dice. If they score a double i.e two 1s, two 2s etc they win \$6. If they throw a total of 7 they win \$11 and anything else they receive nothing.
Let \$ $X$ equal the profit a player receives on a single play.

a) Describe the random variable $X$ . (1 mark)

| Solution |
|---|
| Discrete random variable |
| **Specific behaviours** |
| ✓ states discrete |





| |
|---|
| ✓ solves for a & b to two decimal places **(must round)** |

Q8 (5 marks)

Consider a continuous random variable, $X$, that has the following probability density function.

$$f(x)=\begin{cases} ae^{-bx} & ,0\le x\le 5 \\ 0 & ,elsewhere \end{cases}$$ with $a\,\&\,b$ being constants.

a) Determine the cumulative distribution function, $P(X\le x)$, in terms of $a\,\&\,b$. (2 marks)

| Solution |
|---|
| $P(X\le x)=\int_0^x ae^{-bu}du$ <br> $=\left[\frac{-a}{b}e^{-bu}\right]_0^x=\frac{-a}{b}(e^{-bx}-1)$ |
| **Specific behaviours** |
| ✓ integrates with correct limits (no need to change variables) <br> ✓ states cumulative function |

b) Given that $P(X\le 3)=0.7$ solve for approximate values of $a\,\&\,b$ to two decimal places. (3 marks)

| Solution |
|---|
| Edit Action Interactive <br> $\begin{cases} e^{-5b}=1-\frac{b}{a} \\ \frac{-a}{b}(e^{-3b}-1)=0.7 \end{cases}\Bigg\vert_{a,b}$ <br> $\{a=0.2995948184, b=0.1742122177\}$ <br> a=0.30 & b=0.17 |
| **Specific behaviours** |
| ✓ sets up equation using total prob of one with x=5 <br> ✓ sets up second equation at x=3 |





b) Complete the following table for $X$. (3 marks)

**Solution**

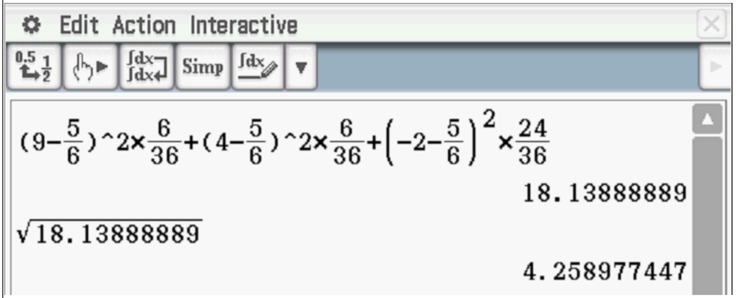
| $x$ | \$9 | \$4 | -\$2 |
|---|---|---|---|
| P($X=x$) | $\frac{6}{36}$ | $\frac{6}{36}$ | $\frac{24}{36}$ |

| Specific behaviours |
|---|
| ✓ correct values for x <br> ✓ at least 1 probs correct <br> ✓ all probs correct |

c) Determine the expected profit by a player on a single game. (3 marks)

| Solution |
|---|
| $9\times\frac{6}{36}+4\times\frac{6}{36}-2\times\frac{24}{36}$ $\quad\frac{5}{6}$ <br> $\frac{5}{6}$ <br> $0.8333333333$ |
| **Specific behaviours** |
| ✓ multiplies x by prob <br> ✓ shows sum of products <br> ✓ states expected profit, approx. or exact (no need for units) <br> (Note two marks for answer only) |

d) Determine the standard deviation of $X$. (3 marks)

| Solution |
|---|
|  |
| **Specific behaviours** |
| ✓ shows calculation<br>✓ determines variance<br>✓states standard deviation<br>Note: Answer only two marks- third mark refers to shown working |

Q3 (7 marks)
A factory produces toy cars. The probability that any toy car being defective is 0.15. If 20 toy cars are selected at random, let $X$ equal the number of defective cars out of 20.

a) Describe the distribution $X$. ( 2 marks)

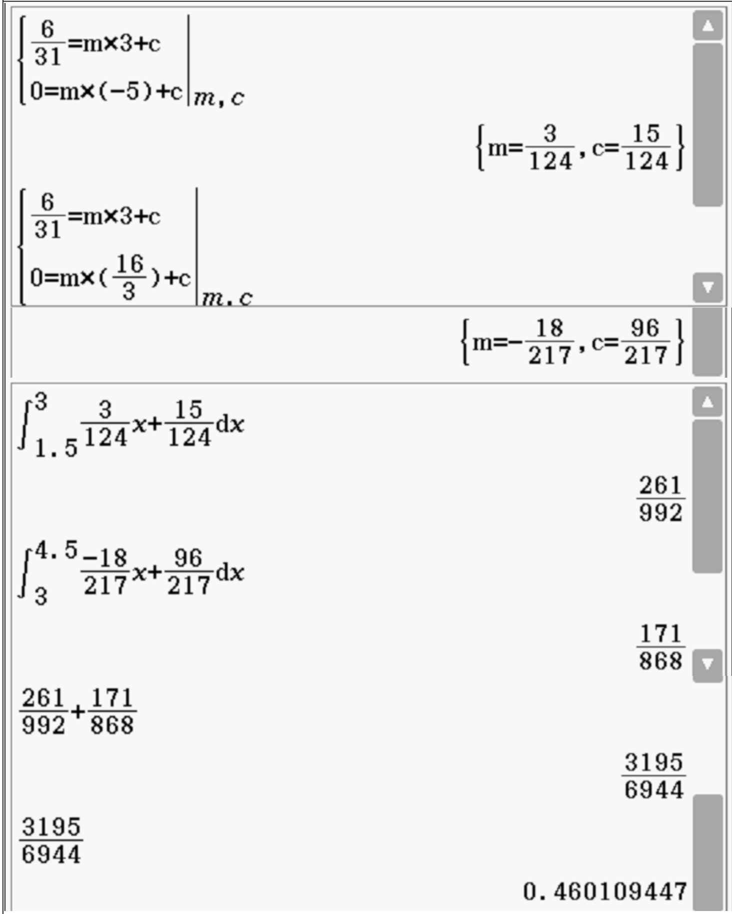
| Solution |
|---|
| $X \sim Bin(20, 0.15)$ |
| **Specific behaviours** |
| ✓ states Binomial<br>✓ states n & p |

b) Determine that probability that exactly 4 cars will be defective. (2 marks)

| Solution |
|---|

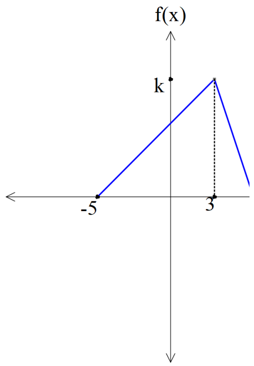




| Solution |
|---|
|  |
| **Prob = 0.4601** |
| **Specific behaviours** |
| ✓ determines equation of one side<br>✓ determines equations of both sides<br>✓ states integrals with correct limits for total area<br>✓ states approx. area to 4 decimal places (accept exact) |

Q7 (6 marks)

Consider the continuous random variable $X$ and its probability density function shown below.




a) Determine the exact value of $k$. (2 marks)

| Solution |
|---|
| $\text{solve}\left(\frac{1}{2}\cdot\left(5+\frac{16}{3}\right)\cdot k=1, k\right)$ $\left\{k=\frac{6}{31}\right\}$ |
| **Specific behaviours** |
| ✓ uses total area of one<br>✓ solves for exact value of k |

b) Determine Prob $(1.5 < X < 4.5)$ (4 marks)





| |
|---|
| binomialPDf(4, 20, 0.15)<br>0.1821216721 |
| **Specific behaviours** |
| ✓ uses correct parameters<br>✓ states prob |

c) Determine the probability that at least 4 cars will be defective given that we know at least 2 cars are defective. (3 marks)

| Solution |
|---|
| $P(X \geq 4 \mid X \geq 2) = \frac{P(X \geq 4)}{P(X \geq 2)}$<br>binomialCDf(2, 20, 20, 0.15)<br>0.8244421239<br>binomialCDf(4, 20, 20, 0.15)<br>0.3522748258<br>$\frac{0.3522748258}{0.8244421239}$<br>0.4272887272 |
| **Specific behaviours** |
| ✓ uses conditional prob reasoning<br>✓ shows numerator and denominator values<br>✓ states prob |

Q4 (4 marks)

Sound loudness, $L$ dB, is measured by comparing the intensity of the sound, $I$, with the intensity of a sound that is just detectable by the human ear, $I_O$.

$L = 10\log_{10}\left(\frac{I}{I_O}\right)$.

a) If the noise level in a room was 65 dB, express the intensity of sound in this room in terms of . $I_O$. ( 1 mark)

| Solution |
|---|
| $65 = 10\log_{10}\left(\frac{I}{I_O}\right)$<br>$\left(\frac{I}{I_O}\right) = 10^{6.5}$<br>$I = I_O 10^{6.5}$ |
| **Specific behaviours** |
| ✓ states expression |

b) How many times is the intensity of a 105 dB noise level that of the intensity of a 35 dB noise level? (3 marks)

| Solution |
|---|
| $L = 10\log_{10}\left(\frac{I}{I_O}\right)$<br>$\frac{I}{I_O} = 10^{\frac{L}{10}}$<br>$\frac{10^{10.5}}{10^{3.5}} = 10^{7}$ |
| **Specific behaviours** |
| ✓ uses index form<br>✓ shows the powers of 10 for both levels<br>✓ states simplified ratio |



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Q6 (6 marks)

a) Determine $\frac{d}{dx}\left(x^3 \ln x\right)$. (simplify) (3 marks)

| Solution |
|---|
| $\frac{d}{dx}\left(x^3 \ln x\right) = x^3 \frac{1}{x} + 3x^2 \ln x$<br>$= x^2 + 3x^2 \ln x$ |
| **Specific behaviours** |
| ✓ shows use of product rule<br>✓ at least one product correct<br>✓ states simplified derivative |

b) Using your result in a) above and **NOT using your classpad** determine $\int 10x^2 \ln x \, dx$. Show all working. (3 marks)

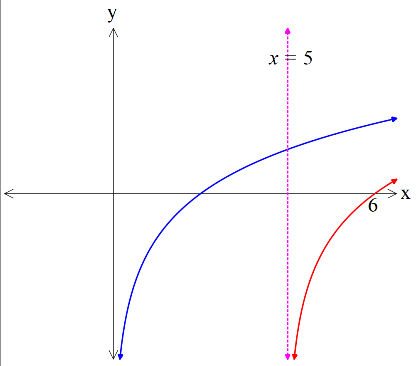
| Solution |
|---|
| $\frac{d}{dx}\left(x^3 \ln x\right) = x^2 + 3x^2 \ln x$<br>$x^3 \ln x = \int x^2 dx + 3\int x^2 \ln x dx$<br>$\int x^2 \ln x dx = \frac{1}{3}\left(x^3 \ln x - \frac{x^3}{3}\right) + c$<br>$\int 10x^2 \ln x dx = \frac{10}{3}\left(x^3 \ln x - \frac{x^3}{3}\right) + c$ |
| **Specific behaviours** |
| ✓ uses linearity (integrates exp in (a) above)<br>✓ integrates squared term and adds a constant<br>✓ obtains exp for required integral (no need to factorise)<br>(zero marks for answer only- from classpad) |

Q5 (5 marks)

Below is a graph of $y = \log_a x$ where $a$ is a positive constant.

a) Sketch on the axes above $y = \log_a(x-5)$ labelling major features. (2 marks)

**Solution**




(Not drawn to scale)

**Specific behaviours**

✓ labels new x intercept at x=6 AND appears to translate to the right
✓ shows a dotted line as asymptote and labels equation

b) Determine the values of $a, b \,\&\, c$ given that $y = \log_a(x+b) + c$ contains points $(-1,-1) \,\&\, (0,5)$ and has a vertical asymptote at $x = -2$. ( 3 marks)

**Solution**

$asymptote\ b = 2$
$-1 = \log 1 + c$
$c = -1$
$5 = \log_a 2 - 1$
$6 = \log_a 2$
$a^6 = 2$
$a = 2^{\frac{1}{6}}$

**Specific behaviours**

✓ states value of $b$
✓ sets up equations containing $a \,\&\, c$





✓ states exact values of $a \,\&\, c$