EXTENDED PIECE OF WORK #3

Time Allowed: 60 minutes Total Marks: 31

Throughout this investigation you are required to state conclusions with sufficient evidence to justify them.

When writing rules use notations such as: n + 1, $\mathbf{F}_n$, f_{n+1}, $\mathbf{T}_{a+b}$, t_{n+2} etc

PART A

Triangle Matrices

The triangle sequence is:

1	3	6	10	15	21	28	36	45	55	…
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	

In this part you are going to explore triangle matrices.

If t_n, t_{n+1}, and t_{n+2} are consecutive triangle numbers then a triangle matrix is defined by

$$\mathbf{T}_n = \begin{bmatrix} t_n & t_{n+1} \\ t_{n+1} & t_{n+2} \end{bmatrix}$$

The first five triangle matrices will be:

$$T_1 = \begin{bmatrix} 1 & 3 \\ 3 & 6 \end{bmatrix}, T_2 = \begin{bmatrix} 3 & 6 \\ 6 & 10 \end{bmatrix}, T_3 = \begin{bmatrix} 6 & 10 \\ 10 & 15 \end{bmatrix}, T_4 = \begin{bmatrix} 10 & 15 \\ 15 & 21 \end{bmatrix}, T_5 = \begin{bmatrix} 15 & 21 \\ 21 & 28 \end{bmatrix}$$

Using the above definitions, investigate the following. You will need to show some examples to support your conclusions.

1. The addition of two consecutive triangle matrices. [4 marks]

2. The value of the determinant of the triangle matrices [3 marks]

3. The squaring of triangle matrices [4 marks]

4. The product of two consecutive triangle matrices [4 marks]

TOTAL 15 marks

PART B

Fibonacci Matrices

The Fibonacci sequence is:

0	1	1	2	3	5	8	13	21	34	55	89	…
f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	

In this part you are going to explore Fibonacci matrices.

If f_n, f_{n+1}, and f_{n+2} are consecutive Fibonacci numbers then a Fibonacci matrix is defined by

$$\mathbf{F}_n = \begin{bmatrix} f_n & f_{n+1} \\ f_{n+1} & f_{n+2} \end{bmatrix}$$

The first five Fibonacci matrices will be:

$$F_1 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, F_3 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, F_4 = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}, F_5 = \begin{bmatrix} 3 & 5 \\ 5 & 8 \end{bmatrix}$$

Using the above definitions, investigate the following. You will need to show some examples to support your conclusions.

1. The addition of two consecutive Fibonacci matrices. [3 marks]

2. The product of two consecutive Fibonacci matrices [3 marks]

3. The product of two non-consecutive Fibonacci matrices [3 marks]

4. The value of the determinant of Fibonacci matrices [3 marks]

5. Solutions of simultaneous equations of the form

$$\begin{aligned} f_n x + f_{n+1} y &= f_m \\ f_{n+1} x + f_{n+2} y &= f_{m+1} \end{aligned}$$

such that $m \geq n + 1$.

eg $f_4 x + f_5 y = f_9$ gives $2x + 3y = 21$
$f_5 x + f_6 y = f_{10}$ $3x + 5y = 34$

or

$f_6 x + f_7 y = f_7$ gives $5x + 8y = 8$
$f_7 x + f_8 y = f_8$ $8x + 13y = 13$

but not

$f_9 x + f_{10} y = f_5$ gives $21x + 34y = 3$
$f_{10} x + f_{11} y = f_6$ $34x + 55y = 5$

[4 marks]

TOTAL 16 marks