John Wollaston Anglican Community School

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section One: Calculator-free

SOLUTIONS

WA student number: In figures

In words

Your name

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

Question number: _________

Question 8 **(6 marks)**

Two polynomial functions are defined by $f(x) = (2x-3)(x+2)$ and $g(x) = x^3 + 4x^2 - 4x - 12$.

Determine the coordinates of the point(s) of intersection of $f(x)$ and $g(x)$.

Solution
Expand $f(x)$ $f(x) = (2x-3)(x+2)$ $= 2x^2 + x - 6$ Equate functions: $x^3 + 4x^2 - 4x - 12 = 2x^2 + x - 6$ Equate to zero: $x^3 + 2x^2 - 5x - 6 = 0$ Find root: $x = -1 \Rightarrow -1 + 2 + 5 - 6 = 0$ Start factorising: $x^3 + 2x^2 - 5x - 6 = (x+1)(x^2 + x - 6)$ Complete factorising: $x^3 + 2x^2 - 5x - 6 = (x+1)(x+3)(x-2)$ Coordinates: $f(-1) = (-5)(1) = -5$ $f(-3) = (-9)(-1) = 9$ $f(2) = (1)(4) = 4$ Intersect at $(-1,-5)$, $(-3,9)$ and $(2,4)$.
Specific behaviours
✓ expands quadratic ✓ equate functions and then to zero ✓ finds first root ✓ factors into linear and quadratic ✓ completes factorisation ✓ determines y-coordinates and states coordinates of all points

End of questions

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

Solve the following equations for x.

(a) $(2x+5)(x-4) = 0$. (2 marks)

Solution
$2x + 5 = 0 \Rightarrow x = -\frac{5}{2} = -2.5$ $x - 4 = 0 \Rightarrow x = 4$ $x = -2.5, \quad x = 4$
Specific behaviours
✓ first correct solution ✓ second correct solution

(b) $x^2 - 10x - 11 = 0$. (2 marks)

Solution
$(x-11)(x+1) = 0$ $x = -1, \quad x = 11$
Specific behaviours
✓ indicates correct method ✓ both correct solutions

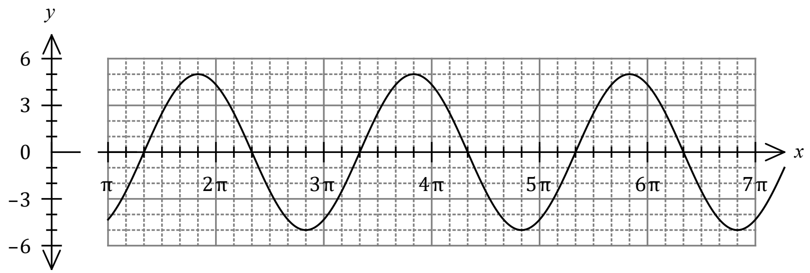
(c) $(x-8)^2 - 100 = 0$. (2 marks)

Solution
$(x-8)^2 = 10^2$ $x - 8 = \pm 10$ $x = 18, \quad x = -2$
Specific behaviours
✓ indicates correct method ✓ both correct solutions

See next page

Question 2 **(6 marks)**

(a) The graph of $y = a\sin(x + b)$ is shown below, where a and b are positive constants.



Determine the value of a and the least value of b. (2 marks)

Solution
$a = 5, \quad b = \dfrac{2\pi}{3}$
Specific behaviours
✓ amplitude a ✓ least value of phase shift b

(b) Let $f(x) = 4\tan\left(x - \dfrac{\pi}{6}\right)$.

Determine the zeros of the graph of $y = f(x)$ for $0 \le x \le 2\pi$. (2 marks)

Solution
$x - \dfrac{\pi}{6} = 0, \pi \Rightarrow x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$
Specific behaviours
✓ locates one zero ✓ locates second zero

(c) Let $g(x) = 3 + \cos\left(\dfrac{x}{2}\right)$.

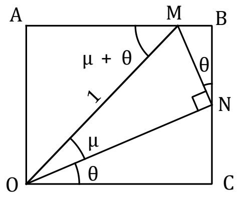
Determine the coordinates of the minimum of the graph of $y = g(x)$ for $0 \le x \le 4\pi$. (2 marks)

Solution
Minimum of $y = \cos x$ when $x = \pi$, but period doubled and so now when $x = 2\pi$. Hence minimum at $(2\pi, 3 - 1) = (2\pi, 2)$.
Specific behaviours
✓ correct x-coordinate ✓ correct y-coordinate

Question 7 **(7 marks)**

Consider rectangle $OABC$ that contains the right triangle OMN as shown.

Let the length of $OM = 1$,
$\angle NOC = \angle MNB = \theta$,
$\angle MON = \mu$ and
$\angle AMO = \mu + \theta$.



(a) Explain why $OC = \cos\mu\cos\theta$. (2 marks)

Solution
In triangle OMN, $ON = \cos\mu$. Hence, in triangle ONC, $OC = ON\cos\theta = \cos\mu\cos\theta$.
Specific behaviours
✓ uses ΔOMN for length of ON ✓ uses ΔONC to obtain result

(b) Determine expressions for the lengths of BM and AM and hence prove the angle sum identity $\cos(\mu + \theta) = \cos\mu\cos\theta - \sin\mu\sin\theta$. (3 marks)

Solution
$MB = MN\sin\mu$ $= \sin\theta\sin\mu$ $AM = \cos(\mu + \theta)$ Because $OABC$ is a rectangle then $AM = OC - MB$ $\cos(\mu + \theta) = \cos\mu\cos\theta - \sin\theta\sin\mu$
Specific behaviours
✓ length of MB ✓ length of AM ✓ uses congruent sides of rectangle to complete proof

(c) Use the identity from part (b) to show that $\cos\left(x + \dfrac{\pi}{2}\right) = -\sin x$. (2 marks)

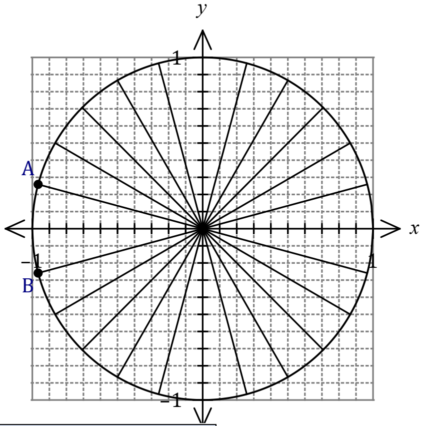
Solution
$\cos\left(x + \dfrac{\pi}{2}\right) = \cos x\cos\dfrac{\pi}{2} - \sin x\sin\dfrac{\pi}{2}$ $= \cos x \times 0 - \sin x \times 1$ $= -\sin x$
Specific behaviours
✓ expands using identity ✓ clearly shows both known values and simplifies

Question 6 **(6 marks)**

(a) A unit circle is shown.

Mark on the circumference of the circle the points A and B so that rays drawn from the origin to each point make clockwise angles of $165°$ and $\frac{13\pi}{12}$ from the positive x-axis respectively.

Hence estimate the value of $\cos 165°$ and the value of $\sin\left(\frac{13\pi}{12}\right)$.



(3 marks)

Solution
See graph for points. $\cos 165° = x$, where $-0.98 \le x \le 0.95$ $\sin\left(\frac{13\pi}{12}\right) = y, -0.28 \le y \le -0.24$
Specific behaviours
✓ both points located correctly ✓ value of cosine within range ✓ value of sine within range

(b) Solve the equation $3\tan(2x - 10°) = \sqrt{3}$ for $0° \le x \le 180°$. (3 marks)

Solution
$\tan(2x - 10°) = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ $2x - 10° = 30°, 210°$ $2x = 40°, 220°$ $x = 20°, 110°$
Specific behaviours
✓ eliminates tan from equation ✓ one correct solution ✓ second correct solution

Question 3 **(7 marks)**

Consider the function $f(x) = \frac{a}{x+b}$, where a and b are constants. The graph of $y = f(x)$ has an asymptote with equation $x = 1$ and passes through the point $(3, -1)$.

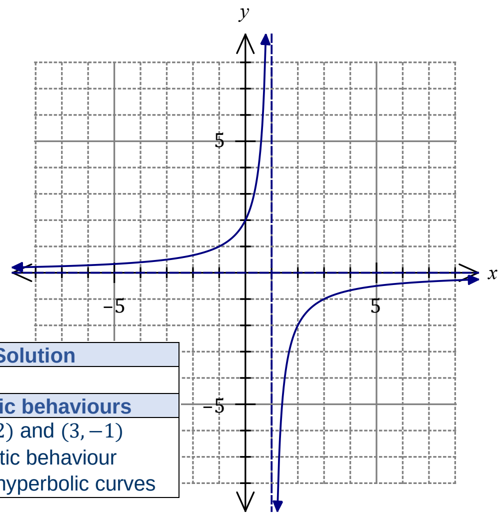
(a) Determine the value of a and the value of b. (3 marks)

Solution
Using asymptote, $1 + b = 0 \Rightarrow b = -1$. Using point: $-1 = \frac{a}{3-1}$ $a = -2$
Specific behaviours
✓ value of b ✓ forms equation using point ✓ calculates value of a

(b) State the equation of the other asymptote of the graph of $y = f(x)$. (1 mark)

Solution
$y = 0$
Specific behaviours
✓ correct equation

(c) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ thru' $(0, 2)$ and $(3, -1)$ ✓ asymptotic behaviour ✓ smooth hyperbolic curves

Question 4 **(7 marks)**

The straight line L has equation $3x - 2y = 1$.

(a) Write the equation of L in the form $y = mx + c$ to show that its gradient is 1.5. (1 mark)

Solution
$2y = 3x - 1 \Rightarrow y = \frac{3}{2}x - \frac{1}{2} \Rightarrow m = \frac{3}{2} = 1.5$
Specific behaviours
✓ correct values of m and c

Line L_1 is parallel to L and passes through the point $(2, -3)$.

Line L_2 is perpendicular to L and passes through the point $(9, 1)$.

(b) Determine the point of intersection of L_1 and L_2. (6 marks)

Solution
$L_1: (y - -3) = \frac{3}{2}(x - 2) \Rightarrow y = \frac{3}{2}x - 6$ $L_2: (y - 1) = -\frac{2}{3}(x - 9) \Rightarrow y = -\frac{2}{3}x + 7$ $\frac{3}{2}x - 6 = -\frac{2}{3}x + 7$ $\left(\frac{3}{2} + \frac{2}{3}\right)x = 13$ $\frac{13}{6}x = 13$ $x = 6$ $y = \frac{3}{2}(6) - 6 = 3$ Lines intersect at $(6, 3)$.
Specific behaviours
✓ equation of L_1 ✓ gradient of L_2 ✓ equation of L_2 ✓ equates lines and groups like terms   ✓ solves for x ✓ solves for y and states point of intersection

Question 5 **(7 marks)**

(a) Determine the number of possible combinations when three students must be chosen from a small class of six. (2 marks)

Solution
1   1 1   1 2 1   1 3 3 1   1 4 6 4 1   1 5 10 10 5 1   1 6 15 20 15 6 1   There are ${}^6C_3 = 20$ combinations.
Specific behaviours
✓ indicates use of formula or Pascals triangle   ✓ correct number

(b) Determine the coefficient of the x^3 term in the expansion of

(i) $(2x + 3)^3$. (2 marks)

Solution
$\binom{3}{0}(2x)^3(3)^0 = 8x^3$ Coefficient is 8.
Specific behaviours
✓ indicates method   ✓ clearly states coefficient

(ii) $(3x - 10)^6$. (3 marks)

Solution
$\binom{6}{3}(3x)^3(-10)^3 = 20 \times 27x^3 \times -1000$ $= -540\,000x^3$ Coefficient is $-540\,000$.
Specific behaviours
✓ indicates use of combination from (a) as part of expansion   ✓ indicates two other parts for required expansion   ✓ expands factors, showing correct coefficient