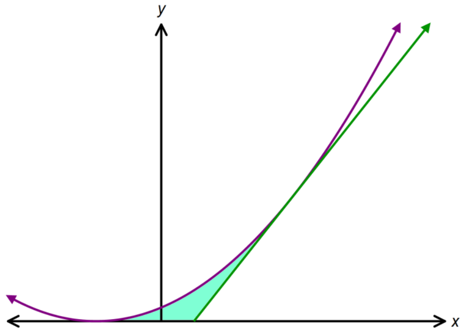
Question Six: **[3, 5 = 8 marks]** **CF**

The curve $y=(x+1)^2$ and the tangent line at $x=2$ are graphed below.



(a) Determine the equation of the tangent to $y=(x+1)^2$ drawn above.

$\frac{dy}{dx}=2(x+1)$ ✓

$x=2 \quad \frac{dy}{dx}=2(2+1)=6$ ✓

$x=2 \quad y=(2+1)^2=9$

$y=6x+c$

$9=6\times2+c$

$c=-3$

$\therefore y=6x-3$ ✓

(b) Hence find the area shaded on the graph above.

$Area=\int_{-1}^{2}(x+1)^2\,dx-\int_{0.5}^{2}6x-3\,dx$ ✓ ✓

$=\left[\frac{(x+1)^3}{3}\right]_{-1}^{2}-\left[3x^2-3x\right]_{0.5}^{2}$ ✓

$=(9+0)-\left(6+\frac{3}{4}\right)$ ✓

$=2\frac{1}{4}\ units^2$ ✓

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Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 =8 marks] **CF**

(a) Calculate $\int\cos\left(\frac{t}{3}\right)dt$

(b) Use your answer to part (a) to evaluate $\int_{\pi}^{2x+1}\cos\left(\frac{t}{3}\right)dt$, in terms of x

(c) Use your answer to part (b) to evaluate $\frac{d}{dx}\left(\int_{\pi}^{2x+1}\cos\left(\frac{t}{3}\right)dt\right)$

(d) Hence evaluate $\frac{d}{dx}\left(\int_{\pi}^{f(x)}\cos\left(\frac{t}{3}\right)dt\right)$

Question Two: [2, 2, 2 = 6 marks] CF

Determine each of the following:

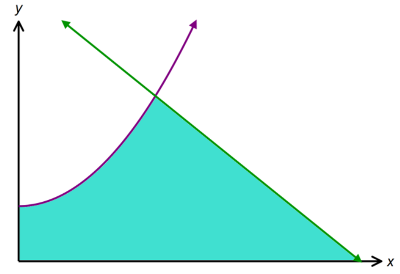
(a) $\int_{-1}^{1} 2x^3\, dx$

(b) $\int_{-1}^{0} e^x\, dx - \int_{1}^{0} e^x\, dx$

(c) $\frac{d}{dx}\left(\int_{-3}^{x^2} \frac{\sqrt{2t-3}}{t+1}\, dt\right)$

Question Five: [1, 2, 4 = 7 marks] CF

The functions $f(x) = x^2 + 2$ and $h(x) = -2x + 10$ are drawn below.



(a) Solve $h(x) = 0$

$-2x + 10 = 0$
$-2x = -10$
$x = 5$ ✓

(b) Solve $f(x) = h(x)$

$x^2 + 2 = -2x + 10$
$x^2 + 2x - 8 = 0$ ✓
$(x+4)(x-2) = 0$
$x = -4,\ x = 2$ ✓

(c) Hence find the area shaded on the graph above.

$Area = \int_0^2 x^2 + 2\, dx + \int_2^5 -2x + 10\, dx$ ✓

$= \left[\frac{x^3}{3} + 2x\right]_0^2 + \left[-x^2 + 10x\right]_2^5$ ✓

$= \left(\frac{8}{3} + 4\right) - (0+0) + (-25+50) - (-4+20)$ ✓

$= 15\frac{2}{3}\ units^2$ ✓

Question Four: [4, 5 = 9 marks] **CF**

Consider the function $f(x) = x^3 + 2x^2 - x - 2$

(a) Determine the roots of the function.

$x = 1\ is\ a\ factor$ ✓

$$\begin{array}{r} x^2 + 3x + 2 \\ x-1 \overline{\smash{)}\, x^3 + 2x^2 - x - 2} \\ \underline{x^3 - x^2} \\ 3x^2 - x \\ \underline{3x^2 - 3x} \\ 2x - 2 \\ \underline{2x - 2} \\ 0 \end{array}$$

✓

$f(x) = (x-1)(x^2 + 3x + 2)$

$f(x) = (x-1)(x+2)(x+1)$ ✓

$roots = (1,0)\ (-2,0)\ (-1,0)$ ✓

(b) Hence determine the area bounded by the curve and the x – axis.

$= \int_{-2}^{-1} f(x)\,dx + \left|\int_{-1}^{1} f(x)\,dx\right|$ ✓

✓

$= \left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} - 2x\right]_{-2}^{-1} + \left|\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{1}\right|$

$= \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right) - \left(4 - \frac{16}{3} - 2 + 4\right) + \left|\left(\frac{1}{4} + \frac{2}{3} - \frac{1}{2} - 2\right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right)\right|$ ✓

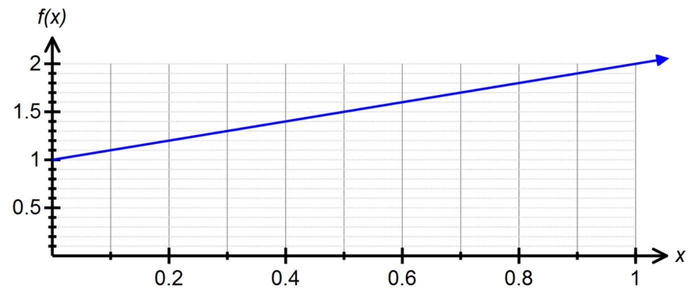
$= \frac{-1}{4} - 4 + \frac{14}{3} + \left|\frac{4}{3} - 4\right|$

$= \frac{5}{12} + 2\frac{2}{3}$ ✓

$= 3\frac{1}{12}\ units^2$ ✓

Question Three: [2, 3, 2 = 7 marks] **CF**

Consider the function $f(x)$ drawn below over the domain $0 \le x \le 1$



(a) Draw rectangles on your graph that can be used to underestimate the area under $f(x)$ over the domain $0 \le x \le 1$, where $\delta x = 0.2$.

(b) Show that $\sum_5 f(x_5)\delta x_5 = \frac{7}{5}\ units^2$

(c) Use the graph of $f(x)$ above to calculate $\int_0^1 f(x)\,dx$

Question Four: [4, 5 = 9 marks] **CF**

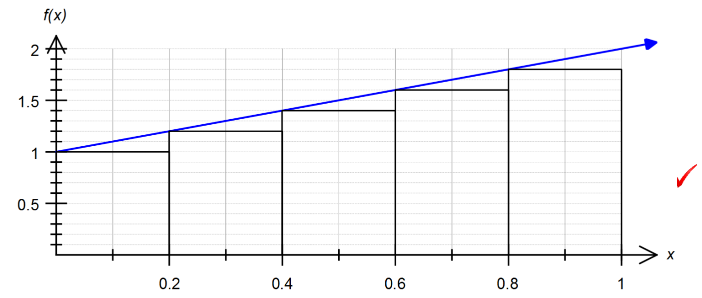
Consider the function $f(x)=x^3+2x^2-x-2$

(a) Determine the roots of the function.

(b) Hence determine the area bounded by the curve and the $x-$axis.

Question Three: [2, 3, 2 = 7 marks] **CF**

Consider the function $f(x)$ drawn below over the domain $0 \le x \le 1$



(a) Draw rectangles on your graph that can be used to underestimate the area under $f(x)$ over the domain $0 \le x \le 1$, where $\delta x = 0.2$.

(b) Show that $\sum_5 f(x_5)\delta x_5 = \frac{7}{5}\ units^2$

$\sum_5 f(x_5)\partial x_5 = 0.2\times1+0.2\times1.2+0.2\times1.4+0.2\times1.6+0.2\times1.6$ ✓

$=0.2(1+1.2+1.4+1.6+1.8)$ ✓

$=\frac{1}{5}\times7$ ✓

$=\frac{7}{5}units^2$

(c) Use the graph of $f(x)$ above to calculate $\int_0^1 f(x)\,dx$

$=\frac{1(1+2)}{2}=\frac{3}{2}$ ✓

✓

Question Two: [2, 2, 2 = 6 marks] CF

Determine each of the following:

(a) $\int_{-1}^{1} 2x^3 \, dx$

$= \left[\frac{2x^4}{4}\right]_{-1}^{1}$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

(b) $\int_{-1}^{0} e^x \, dx - \int_{1}^{0} e^x \, dx$

$= \int_{-1}^{0} e^x \, dx + \int_{0}^{1} e^x \, dx$

$= \int_{-1}^{1} e^x \, dx$

$= \left[e^x\right]_{-1}^{1}$

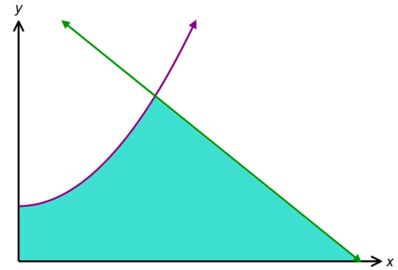
$= e^1 - e^{-1}$

(c) $\frac{d}{dx}\left(\int_{-3}^{x^2} \frac{\sqrt{2t-3}}{t+1} \, dt\right)$

$= \frac{\sqrt{2x^2-3}}{x^2+1} \times 2x$

Question Five: [1, 2, 4 = 7 marks] CF

The functions $f(x) = x^2 + 2$ and $h(x) = -2x + 10$ are drawn below.



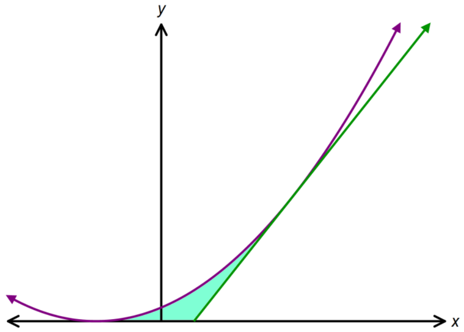
(a) Solve $h(x) = 0$

(b) Solve $f(x) = h(x)$

(c) Hence find the area shaded on the graph above.

Question Six: **[3, 5 = 8 marks]** **CF**

The curve $y=(x+1)^2$ and the tangent line at $x=2$ are graphed below.



(a) Determine the equation of the tangent to $y=(x+1)^2$ drawn above.

(b) Hence find the area shaded on the graph above.

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Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 =8 marks] **CF**

(a) Calculate $\int \cos\left(\frac{t}{3}\right)dt$

$$=3\sin\frac{t}{3}+c$$

(b) Use your answer to part (a) to evaluate $\int_{\pi}^{2x+1}\cos\left(\frac{t}{3}\right)dt$, in terms of x

$$=\left[3\sin\frac{t}{3}+c\right]_{\pi}^{2x+1}$$
$$=\left(3\sin\frac{2x+1}{3}+c\right)-\left(3\sin\frac{\pi}{3}+c\right)$$
$$=3\sin\frac{2x+1}{3}-\frac{3\sqrt{3}}{2}$$

(c) Use your answer to part (b) to evaluate $\frac{d}{dx}\left(\int_{\pi}^{2x+1}\cos\left(\frac{t}{3}\right)dt\right)$

$$\frac{d}{dx}\left(3\sin\frac{2x+1}{3}-\frac{3\sqrt{3}}{2}\right)$$
$$=3\cos\frac{2x+1}{3}\times 2$$
$$=6\cos\frac{2x+1}{3}$$

(d) Hence evaluate $\frac{d}{dx}\left(\int_{\pi}^{f(x)}\cos\left(\frac{t}{3}\right)dt\right)$

$$=\cos\left(\frac{f(x)}{3}\right)\times f'(x)$$