PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Specialist TEST 3 Monday 1 July 2019 TIME: 45 minutes working Classpads allowed One page of notes 42 marks 6 Questions

Name:________________________________

Teacher: _____________________________

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3 & 3 = 6 marks)

a) Solve for the following system of linear equations **without using a classpad**.

$$x + 2y - z = 3$$
$$2x + 3y + 2z = -1$$
$$3x + 7y - 2z = 6$$

Q1 - continued

b) Determine the values of $m \,\&\, p$ such that $\begin{aligned} x+2y-z&=3 \\ 2x+3y+2z&=m \\ 3x+py-2z&=6 \end{aligned}$ such that the system has

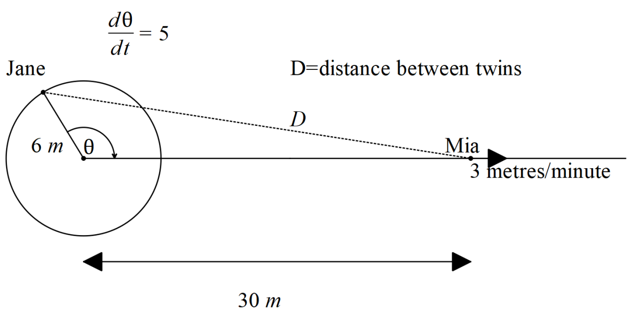
(i) Infinite solutions

(ii) No solutions

Q6 (4 marks)

Given that $x \,\&\, y$ are functions of t, show that $\dfrac{d^2y}{dx^2}=\dfrac{\dfrac{dx}{dt}\dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}\dfrac{d^2x}{dt^2}}{\left(\dfrac{dx}{dt}\right)^3}$

Q5 (2 & 5 = 7 marks)
Consider two rides at a circus, one is a merry go round and the other is a train on a straight line. Two twins decide to each try one of the two rides, Jane sits on the merry go round with a constant angular speed of $\frac{d\theta}{dt}=5$ radians/minute moving in a clockwise direction and radius 6 metres and Mia sits on a train moving at 3 metres/minute away from the merry go round. See the diagram below.



a) Determine the distance between Jane and Mia when $\theta = \frac{2\pi}{3}$ and the train is 30 metres from the centre of the merry go round.

b) Determine the time rate of change of this distance at the point defined in (a) above. (metres/minute)

Q2 (2 & 3 = 5 marks)
If $r = \begin{pmatrix} t^2 - 1 \\ 5 - t \end{pmatrix}$, determine

a) $\left|\frac{dr}{dt}\right|$

b) $\frac{d|r|}{dt}$ (no need to simplify)

Q3 (2, 3 & 3 = 8 marks)
An object is initially at the origin with initial speed of $\begin{pmatrix} 3 \\ 7 \end{pmatrix} m/s$ and an acceleration given by $a = \begin{pmatrix} 3e^t \\ 2 - \sin t \end{pmatrix} m/s^2$ at time t seconds.
Obtain an expression for the:

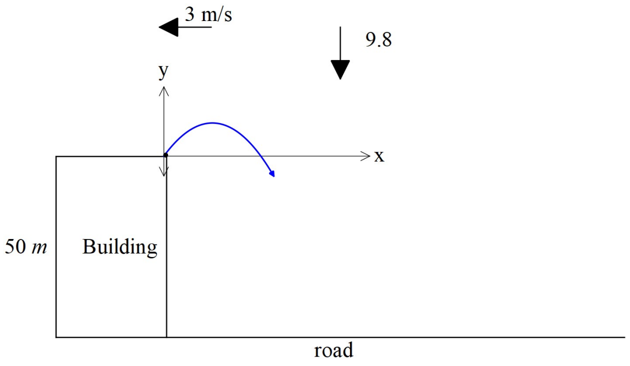
a) Velocity at time t.

b) Position vector r at time t.

c) Is the velocity ever perpendicular to the acceleration? Explain and if necessary solve for t values (if any).

Q4 (3, 3, 3 & 3 = 12 marks)

Consider a cannon ball that is projected from the top of a building with speed V at an angle θ to the surface of the roof. There is a constant cross wind of 3 metres per second acting against the ball and the acceleration due to gravity is $9.8m/s^2$ down as shown in the diagram below. (Note- let the origin be at the top of the building on the edge)



a) Given that the acceleration is given by $\ddot{r} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} m/s^2$ show using vector calculus that the velocity $\dot{r} = \begin{pmatrix} V\cos\theta - 3 \\ V\sin\theta - 9.8t \end{pmatrix} m/s$.

b) Determine the cartesian equation of the path of the cannon ball in terms of $V \,\&\, \theta$. Show your working.

Q4 continued

c) Given that a point on the cartesian path has been measured as $(7.4, 1.1)$ metres and the initial speed V of the ball from the cannon is 12 m/s, determine the initial angle θ of the ball when projected into the air.

d) If $V = 25m/s$ and $\theta = 45$ and a cross wind of 3 m/s as in the diagram on last page, determine how far from the foot of the building that the cannon ball lands on the road.