



Perth College

Semester Two Examination, 2016

place your student identification label in this box

If required by your examination administrator, please

Question/Answer Booklet

METHEMATICS
METHODS
UNITS 1 AND 2

Section One: Calculator-free

| ., | |
|------------|-----------------|
| Your name | |
| ln words | |
| sənugit nl | Student Number: |

Time allowed for this section
Reading time before commencing work:

Working time for section:

Iffy minutes

To be provided by the supervisor

This Question/Answer Booklet Formula Sheet

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To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction
fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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 METHODS UNITS 1 AND 2
 16
 CALCULATOR-ASSUMED

 Question 20
 (9 marks)

(a) Show that the equation of the tangent to the curve $y = \frac{x + x^3}{2}$ at the point where x = 2 is 13x - 2y = 16. (4 marks)

(b) The line with equation y = 5x + c is a tangent to the curve $y = x^3 + 3x^2 - 4x - 12$. (5 marks)

End of questions

| METHODS | LIMITS 4 | AND 2 |
|------------|----------|-------|
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2

CALCULATOR-FREE

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|------------------------------------|-------------------------------------|--|------------------------------|--------------------|--------------------|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
| | | | Total | 150 | 100 |

Instructions to candidates

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- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
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See next page

CALCULATOR-ASSUMED

METHODS UNITS 1 AND 2

Question 19

(7 marks)

- A sequence is defined by $T_{n+1} = T_n 7$, $T_1 = 111$.
 - (i) Determine T_{20} .

(1 mark)

(ii) The sum of the first 40 terms, S_{40} .

(1 mark)

(iii) The value of n that maximises S_n .

(2 marks)



A geometric sequence with $T_2 = 87.5$ has a sum to infinity of 800. Determine all possible values of T_1 for this sequence. (3 marks)

$$800 = \frac{\alpha}{1-r} \qquad \alpha r = 87.5$$

Solve

$$\alpha = 100$$
 $\alpha = 700$ $r = 0.125$

$$r = 0.875$$



| 7 | GNA | ŀ | STINU | METHODS | |
|---|------------|---|-------|---------|--|
| | | | | | |

32% (25 Marks)

(1 mark)

CALCULATOR-FREE

a marker pen.

(q)

Section One: Calculator-free

This section has eight (8) questions. Answer all questions. Write your answers in the spaces

3

Working time for this section is 50 minutes.

(4 marks) Question 1

table. Some of the marker pens are permanent and the rest are non-permanent. A box contains a total of 500 marker and highlighter pens of various colours, as shown in the

| | Jnc | DIOO | | |
|------------|------|-----------|-------|----------------------|
| Green | Pink | Yellow | Black | Type of pen |
| 24 | 04 | 83 | 99 | Permanent marker |
| 12 | 24 | 49 | 97 | Non-permanent marker |
| P 9 | 97 | 09 | 0 | Highlighter |

A pen is selected at random from the box. Determine the probability that it is

(1 mark) a yellow pen.

(1 mark) a yellow pen or a marker pen. (c)

(J wark) a green pen, given that it is a highlighter. (p)

See next page

CALCULATOR-ASSUMED ヤレ METHODS UNITS 1 AND 2

(8 marks) Question 18

(a) Two students are to be chosen from a class of 18.

(1 mark) Determine how many different pairs of students may be chosen.

(S marks)

18 5 = 153

that this student is included in the pair chosen? One of the students in the class is the oldest in the school. What is the probability

remainder that do not. Four cans are to be selected at random from the box. A box contains 13 cans of soup, four of which have tomato as an ingredient and the

(1 mark) Calculate how many different selections of four cans can be made from the box.

Determine the probability that none of the four cans will have tomato as an

(2 marks)

(S warks) number of cans with and without tomato as an ingredient. Determine the probability that in the selection of four cans, there will be an equal

CALCULATOR-FREE

Question 2

(8 marks)

(a) Determine f'(x) when

(i)
$$f(x) = 3$$
.
 $f'(x) = 0$

(1 mark)

$$f'(x) = 0$$

$$f(x) = 5x^2 - 4x.$$

(1 mark)

$$f'(x) = 10 x - 4$$

(2 marks)

$$f(x) = \frac{x^3 - 5x}{x}.$$

$$f(x) = x^2 - 5$$

$$f'(x) = \lambda x$$

(b) Simplify $\lim_{h\to 0} \frac{(x+h)^4 - x^4}{h}$

(1 mark)

(3 marks)

$$\frac{dy}{dx} = 10x^4 - 12x^3 \checkmark$$

Calculate the gradient of the curve $y = 2x^5 - 3x^4$ where x = -1.

$$QX = -1$$
 $\frac{dy}{dx} = 10 + 12$

See next page

CALCULATOR-ASSUMED

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METHODS UNITS 1 AND 2

Question 17

(8 marks)

(2 marks)

The perimeter of a sector of a circle, of radius r cm and central angle θ radians, is 60 cm.



(a) Show that $\theta = \frac{60}{r} - 2$.

l+2r=60 & l=r0

2.
$$r\theta + 2r = 60$$

$$r\theta = 60 - 2r$$

$$\theta = 60 - 2r$$

$$\theta = \frac{60}{7} - 2$$
Show that the area of the sector is given by $30r - r^2$.

(2 marks)

$$A = \frac{1}{2}r^{2}\theta$$

$$= \frac{1}{2}r^{2}\left(\frac{60}{r} - 2\right) \checkmark$$

$$= \frac{1}{2}r \times 60 - r^{2}$$

$$= \frac{30r - r^{2}}{r^{2}}$$

Use calculus to determine the maximum area of the sector and state the values of r and θ that achieve this maximum. (4 marks)

$$\frac{dA}{dr} = 30 - 2r \sqrt{2}$$

$$30 - 2r = 0$$

$$r = 15 \sqrt{4} \quad 0 = \frac{60}{15} - 2$$

$$Shape \qquad = 2 \sqrt{2}$$

(8/4)9 - (8)9 + (A)9 = (c) $P(A \cup B)$. (2 marks) (1 mark) (q) (a) $P(A \cap B)$. (1 mark) A and B are independent events such that $P(A) = \frac{1}{\epsilon}$ and $P(B) = \frac{1}{\epsilon}$. Determine (4 marks) Question 3 METHODS UNITS 1 AND 2 9 CALCULATOR-FREE

/ Loop Gung : 0961-M(850.1) 28 = 005 prison population and the model no longer applied. In what year did this occur? (1 mark) (d) When R first exceeded 500, steps were taken to address the exponential growth in the (000, tornor of) sumarind coop 488 = 195.818 × 0997 :- 000000997 1 LOTIS'E18 = K=82 (1,038)35 (3 marks) the US at this time, to the nearest 1 000. The population of the US was 266 million in 1995. Determine the number of prisoners in 1980 0261 (3 marks) (b) Draw the graph of the imprisonment rate for 1960 $\le n \le 2000$ on the axes below. adord 000001/smosud 818 2 : 48,575 = on (880'1)58 (1 mark) (a) Calculate the imprisonment rate in the year 2000. $R = 85(1.038)^{n-1960}$ years 1960 and 2000, can be modelled by the following equation, where n is the year. The imprisonment rate R, in number of prisoners per 100 000 people, in the US between the

15

(8 marks)

CALCULATOR-ASSUMED

See next page

500

300

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009

Question 16

METHODS UNITS 1 AND 2

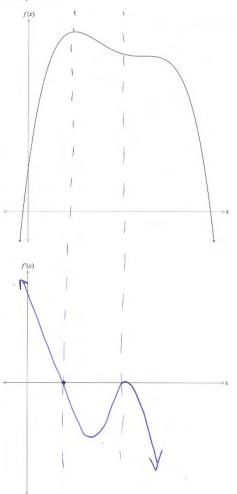
6

CALCULATOR-FREE

Question 4

(4 marks)

The graph of y = f(x) is drawn below. Use this to draw a possible graph of y = f'(x) on the axes provided.



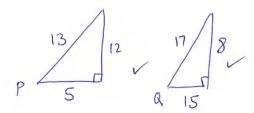
See next page

CALCULATOR-ASSUMED

11

METHODS UNITS 1 AND 2

(b) P and Q are acute angles with $\sin P = \frac{12}{13}$ and $\cos Q = \frac{15}{17}$. Determine the **exact** value of $\cos(P - Q)$. (4 marks



$$\cos (P-Q) = \cos P \cos Q + \sin P \sin Q$$

$$= \frac{5}{13} \times \frac{15}{17} + \frac{12}{13} \times \frac{8}{17}$$

$$= \frac{171}{221}$$

The equation $x^3 - x^2 - 14x + 24 = 0$ has x = 2 as a solution. Determine all other 1 9/15 9/1=x 1 1 TIS 1 TI = XE Sin (3x+2) = 1 O £ 3x £ 3TT (3 marks) $\exists x \ge x \ge 0$ $\exists x = x \text{ sin } x \le x \le x \le \pi$. - | pr error or missing 696, 098, 088, 08 = x (ii) $2 \cos x = \sqrt{3}$, $0 \le x \le 720^\circ$. (3 marks) 1 = x 1 7c7-8= 1+x 1 x-1(28)= 1+x8 (3 marks) $x^{-1}6 = x^{+x}$ (i) (a) Solve the following equations for x: (13 marks) Question 5 METHODS UNITS 1 AND 2 1 CALCULATOR-FREE

(4 marks) solutions to the equation.

$$\frac{0}{n^{e+\kappa}e^{1}} - \frac{0}{\kappa^{e-\kappa}}$$

$$\frac{0}{n^{e+\kappa}e^{1}} - \frac{1}{\kappa^{e-\kappa}}$$

$$\frac{\pi^{e-\kappa}}{\pi^{e-\kappa}}$$

$$\frac{\pi^{e+\kappa}n_{1} - \pi^{e-\kappa}}{\pi^{e-\kappa}}(e^{-\kappa})$$

$$\frac{\pi^{e+\kappa}n_{1} - \pi^{e-\kappa}}{\pi^{e-\kappa}}(e^{-\kappa})$$

$$\frac{\pi^{e+\kappa}n_{1} - \pi^{e-\kappa}}{\pi^{e-\kappa}}(e^{-\kappa})$$

$$\frac{\pi^{e+\kappa}n_{1} - \pi^{e-\kappa}}{\pi^{e-\kappa}}(e^{-\kappa})$$

See next page

CALCULATOR-ASSUMED

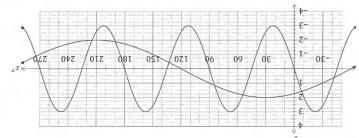
METHODS UNITS 1 AND 2

(10 marks)

(a) The graphs of $f(x) = a \sin(bx)$ and $g(x) = c \cos(x+d)$, where x is in degrees, are shown Question 15

10





(4 marks)

Determine the values of the constants a,b,c and d.

$$(08-x) = 9000 (x-30)$$

$$(08-x) = 9000 (x-30)$$

$$(08-x) = 9000 (x-30)$$

(2 marks) (ii) Use the graph to solve, to the nearest degree, f(x) = g(x), $0^{\circ} \le x \le 180^{\circ}$.

CALCULATOR-FREE

Question 6

(5 marks)

The expression $(2x-1)^3$ can be expanded to give $8x^3 + ax^2 + 6x - 1$. Show that the

$$(2x-1)^{3} = (2x)^{3} + 3(2x)^{2}(-1) + 3(2x)(-1)^{2} + (-1)^{3}$$

$$= 8x^{3} - 12x^{2} + 6x - 1$$

Term:
$$3(2x)^2(-1)$$
 gives ax^2
 $3x4x^2(-1)$
= $-12x^2$ i.e. $a = -12$

Using the result from (a), or otherwise, determine f(x) if $f'(x) = (2x - 1)^3$ and f(1) = 5.

See next page

CALCULATOR-ASSUMED

METHODS UNITS 1 AND 2

Question 14

(10 marks)

The function f is given by $f(x) = x^3 - 3x + 2$.

Show that the graph of y = f(x) has two roots and state their coordinates (2 marks)

$$x^{3}-3x+2=0$$

 $x=1,-2$
ie. $(1,0)$ $(-2,0)$

Use calculus techniques to determine the coordinates of all stationary points of the graph of y=f(x) and use the sign test to determine the nature of these points. (5 marks

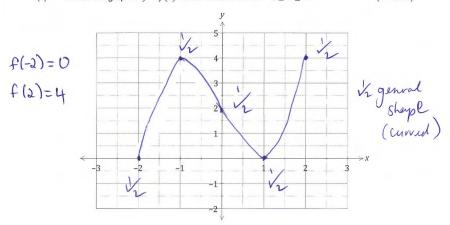
$$f'(x) = 3x^2 - 3$$

 $3x^2 - 3 = 0$
 $x^2 = 1$
 $x = \frac{1}{2} | V$

: (1,0) is a min : (-1,4) is a max

Sketch the graph of y = f(x) on the axes below for $-2 \le x \le 2$.

(3 marks)



See next page

CALCULATOR-FREE

CALCULATOR-ASSUMED

METHODS UNITS 1 AND 2

Question 13

(7 marks)

Question 7

The first three terms, in order, of a geometric sequence are x-5, x-1 and 2x+4.

(S marks)

(a) Explain why (x-1)(1-x) = (1-x)(1-x)

 $\sqrt{\frac{\xi T}{3}} = \frac{1}{17} \text{ mos and swar adminor}$ $\sqrt{\frac{\xi T}{3}} = \frac{1}{17} \text{ mos and swar adminor}$

(7+rp)(S-x)=(1-x)(1-x): molynum sson

(3 marks)

x to (s) all e value (s) of x.

$$(\xi+x)(L-x) = 0$$

$$|\xi-x\eta-_{\tau}x = 0$$

$$07-x01-x\eta+_{\tau}x\xi = 1+x\xi-_{\tau}x$$

(c) Determine all possible values for the fourth term of the sequence.

(6 таткя)

In triangle PQR, PR = 50 cm, QR = 30 cm and $\angle QPR = 25^\circ$.

(a) Sketch two possible triangles that PQR could represent. (Your diagrams do not need to be to scale).

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(b) Given that $\triangle PQR$ is greater than 75° determine

(5 marks)

(i) the size of $\angle PQR$.

$$\frac{1}{\sqrt{\frac{817.819}{5048}}} = \frac{95}{948}$$

(S marks)

(ii) the area of triangle PQR.

See next page

10

CALCULATOR-FREE

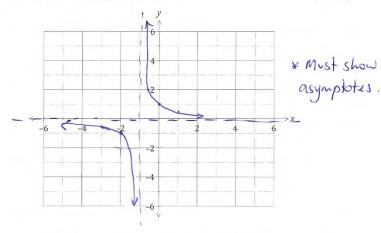
Question 8

(7 marks)

Let $f(x) = \frac{1}{x+1}, x \neq -1.$

(a) Sketch the graph of y = f(x) on the axes below.

(3 marks)



(b) Evaluate $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ to determine the slope of f(x) when x=2. (4 marks)

$$\lim_{h\to 0} \frac{1}{x+h+1} - \frac{1}{x+1} \qquad (P \times 2)$$

$$= \lim_{h\to 0} \frac{1}{h+3} - \frac{1}{3}$$

$$= \lim_{h\to 0} \frac{3 - (h+3)}{3(h+3)} = h$$

$$= \lim_{h\to 0} \frac{-k}{3k(h+3)}$$

$$= -\frac{1}{3(3)}$$

$$= -\frac{1}{3}$$
End of questions

CALCULATOR-ASSUMED

METHODS UNITS 1 AND 2

- (c) The other function is $g(x) = cx^2 + dx + e$.
 - (i) Determine the values of the constants c, d and e, given that g(x) has a maximum at (-3,5). (3 marks)

$$g(x) = a(x+3)^{2} + 5$$
Subst (-5,3) $3 = a(-2)^{2} + 5$

$$3 = 4a + 5$$

$$a = -\frac{1}{2}$$

$$9(x) = -\frac{1}{2}(x^{2} + 6x + 9) + 5$$

$$= -\frac{1}{2}x^{2} - 3x - \frac{9}{2} + 5$$

$$= -\frac{1}{2}x^{2} - 3x + \frac{1}{2} \quad \therefore c = -\frac{1}{2}, d = \frac{3}{2}, e = \frac{1}{2}$$

(ii) State coordinates of the turning point of the graph of y = g(x - 7). (1 mark)

(iii) State the range of the function y = -g(x).

(1 mark)

CALCULATOR-FREE

u

Additional working space

Question number: __

CALCULATOR-ASSUMED

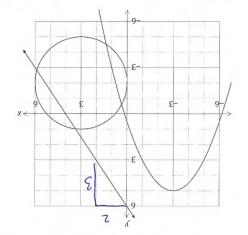
METHODS UNITS 1 AND 2

(10 marks)

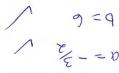
(3 marks)

Question 12

The graph of two functions and a circle of radius 3 units are shown.



One function is f(x) = ax + b. Determine the values of the constants a and b. (2 marks)



(b) The relation can be written in the form $x^2 + px + y^2 + qy + r = 0$.

afants p, q and r.

Determine the values of the constants p,q and r.

$$h = 1 (h = b (9 - cd))$$

$$0 = b - \epsilon I + hh + zh + x9 - z c$$

$$b = h + hh + zh + b + x9 - z x$$

$$z = z(e + h) + z(e - x)$$

| Question | Marks Available | Marks Obtained |
|----------------|-----------------|----------------|
| 1 | 4 | |
| 2 | 8 | |
| 3 | 4 | |
| 4 | 4 | |
| 5 | 13 | |
| 6 | 5 | |
| 7 | 7 | |
| 8 | 7 | |
| TOTAL | 52 | |
| Weighted Score | 35 | - |

CALCULATOR-ASSUMED

5

METHODS UNITS 1 AND 2

Question 11

(8 marks)

Records show that of the 1756 washing machines sold by a retailer during 2015, 464 were deluxe models and the rest were standard. Of all the machines sold, 42 were returned and 31 of these returned machines were standard models.

Determine how many of the standard models were not returned.

(2 marks)

| Deluxe | Standard | TOTAL |
|--------|-----------|-------------------|
| 11 | 31 | 42 |
| 453 | 1261 | 1714 |
| 4.64 | 1292 | 1756 |
| | 11 453 | 11 31 453 1261 |

1261

Calculate, to three decimal places, the probability that a randomly chosen machine from those sold

was a standard model.

(1 mark)

$$\frac{1292}{1756} = 0.736$$

was returned.

(1 mark)

was returned given that it was a deluxe model.

(2 marks)

Is there any indication that the likelihood of a machine being returned is independent of the model type? Explain your answer.

P(Del 1 Ret) = 1/756
= 0.006 1/2
To 3 dp these are the same :- likely to be (0.00632).
independent.

OR 1/756 \$ 0.00632 see next page

i. Not independent



Perth College

Semester Two Examination, 2016

Question/Answer Booklet

place your student identification label in this box

If required by your examination administrator, please

METHEMATICS METHODS UNITS 1 AND 2

Section Two: Calculator-assumed

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|------------|-----------------------|
| Your name | |
| In words | |
| sənugij ul | Student Number: |

one hundred minutes

Time allowed for this section
Reading time before commencing work: ten minutes

Working time for section:

Materials required/recommended for this section

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Formula Sheet (retained from Section One)

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CALCULATOR-ASSUMED

METHODS UNITS 1 AND 2

Question 10 (8 marks)

A walking club is planning a charity walk from Perth to Esperance. Food and camping supplies are to be set up at each overnight campsite in advance, using a vehicle based in Perth that is just large enough to carry enough for one campsite.

To leave the supplies at the first campaile, the vehicle must travel 40 km. For the second and third campailes, the vehicle must travel 100 km and 160 km respectively, and this pattern continues.

(a) Determine the distances the vehicle will travel to set up campaites four and five. (1 mark)

220, 280 km / R/W

(b) Determine, in simplified form, a rule for the distance, d km, that the vehicle will have to travel to set up campaite n.

09 - 4 09 = 71 MM OH

The vehicle can travel a maximum of 700 km on one tank of fuel. Determine the number of the furthest campaite the vehicle can leave supplies at, using no more than one tank of

100L=08-409

4130mmo) 2461 :

(d) If fuel costs 128 cents per litre and the fuel consumption of the vehicle is 9.5 litres per 100 km, determine the total fuel cost to set up the first 20 campsites.

75'8871\$ = 27'1×5'b × 271

| METHODS UNITS 1 AND | 2 |
|---------------------|---|

2

CALCULATOR-ASSUMED

Structure of this paper

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See next page

CALCULATOR-ASSUMED

METHODS UNITS 1 AND 2

Section Two: Calculator-assumed

65% (98 Marks)

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

3

Working time for this section is 100 minutes.

Question 9

(6 marks)

(a) Determine the equation of the straight line that passes through the point (8,11) and is perpendicular to the line with equation 2x + 5y = 1. (3 marks)

$$3y = 1 - dx$$

$$y = \frac{1 - 2x}{5}$$

$$y = \frac{5}{2}x + c$$

$$11 = \frac{5}{2}(8) + c$$

$$c = -9$$

$$y = \frac{5}{2}x - 9$$

(b) Calculate and use the discriminant to determine the number of solutions to the equation $9x^2 - 24x + 16 = 0$. (3 marks)

$$\Delta = b^2 - 4ac$$

$$= (24)^2 - 4(9)(16)$$
= 0

One soln.