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Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2020

Question/Answer booklet

MATHEMATICS METHODS

Section One: Calculator-free

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

WA student number: In figures | | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	10	10	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2020: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Supplementary page

Question number: _____

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Supplementary page

Question number: _____

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Section One: Calculator-free **35% (51 Marks)**

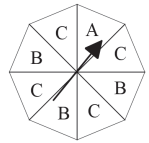
This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 **(6 marks)**

Ashley and Xavier are playing a board game that requires them to use the spinner shown below.



The player spins the arrowhead and the result is where the arrowhead is pointing when it stops moving. The above diagram is showing a result of A.

(a) If the spinner is spun three times, what is the probability that B is never a result? (1 mark)

Let the random variable X be defined as the number of times B is the result when the spinner is spun three times.

(b) Complete the table below showing the probability distribution of X. (3 marks)

x	0	1	2	3
$P(X = x)$				

(c) Determine the mean and variance of the above distribution. (2 marks)

See next page

Question 2 **(4 marks)**

If $h(x) = \dfrac{e^{-x}}{\cos x}$, then evaluate $h'(\pi)$.

See next page

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Supplementary page

Question number: _____

Supplementary page

Question number: _____

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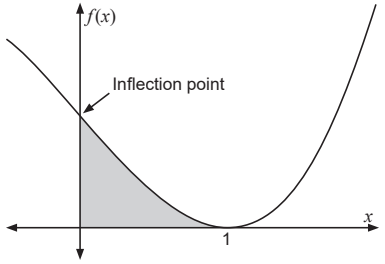
Question 3 **(7 marks)**

The graph of the cubic function $f(x) = ax^3 + bx^2 + cx + d$ is shown below. A turning point is located at (1, 0) and the shaded region shown on the graph has an area of $\frac{3}{2}$ units^2.

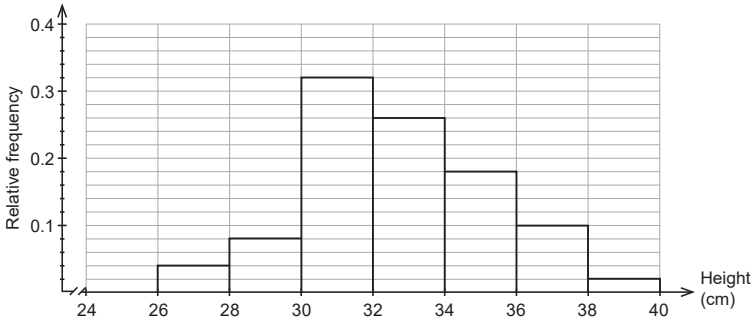


Use the above information to determine the values of a, b, c and d.

See next page

Question 4 **(9 marks)**

The heights reached by a species of small plant at maturity are measured by a team of biologists. The results are shown in the histogram of relative frequencies below.



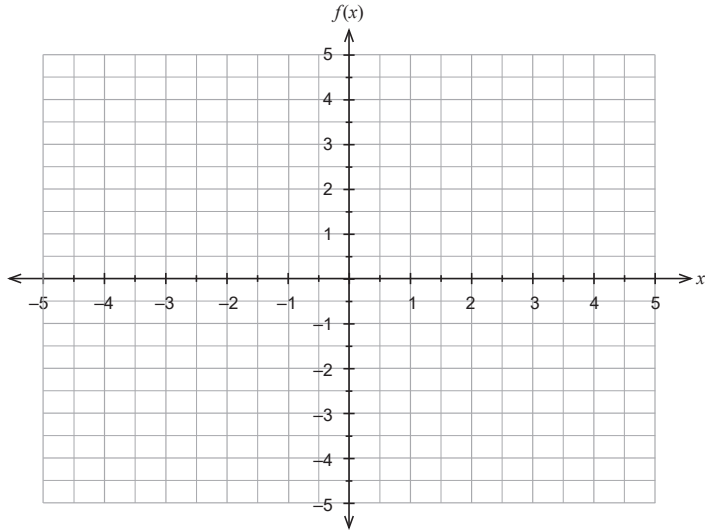
(a) Determine the probability that a mature plant of this species reaches no higher than 30 cm. (1 mark)

(b) If a mature plant reaches a height of at least 32 cm, what is the probability that its height reaches above 38 cm? (2 marks)

See next page

(d) Sketch the function f on the axes below, labelling clearly all intercepts, the turning point and point(s) of inflection. Some approximate values of the natural logarithmic function provided in the table below may be helpful. (4 marks)

x	1	2	3	4
$\ln(x)$	0	0.7	1.1	1.4



End of section

Question 7 **(13 marks)**

Consider the function $f(x) = e^{2x} - 4e^{x}$.

(a) Determine the coordinates of the x-intercept(s) of f. You may wish to consider the factorised version of $f: f(x) = e^{x}(e^{x} - 4)$. (3 marks)

(b) Show that there is only one turning point on the graph of f, which is located at $(\ln(2), -4)$. (3 marks)

(c) Determine the coordinates of the point(s) of inflection of f. (3 marks)

Another team of biologists is studying the mature heights of a species of hedge. The height, h metres, has a probability density function, $d(h)$, as given below.

$$d(h) = \begin{cases} \frac{h-1}{5} & \text{for} \quad 1 \le h \le 2 \\ kh^2 & \text{for} \quad 2 < h \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(c) What percentage of hedges from this study reaches a mature height less than 2 m? (3 marks)

(d) Determine the value of k. (3 marks)

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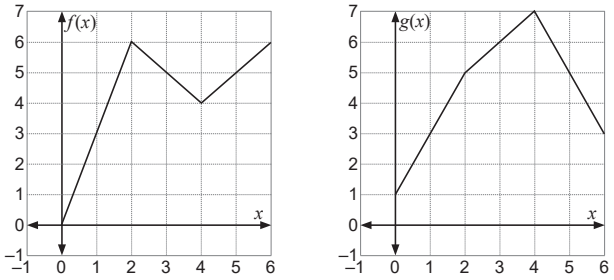
Question 5 **(5 marks)**

The graphs of the functions f and g are displayed below.



(a) Evaluate the derivative of $f(x)$ at $x = 3$. (1 mark)

(b) Evaluate the derivative of $f(x)g(x)$ at $x = 5$. (2 marks)

(c) Evaluate the derivative of $f(g(x))$ at $x = 1$. (2 marks)

Question 6 **(7 marks)**

Consider the function $f(x) = \ln(x)$. The function $g(x) = f(x) + a$ is a vertical translation of f by a units.

(a) Express the function $g(x) = \ln(4x)$ in terms of a vertical translation of f (i.e. in the form $g(x) = f(x) + a$), stating the number of units that f is translated. (2 marks)

The function $h(x) = cf(x)$ is a vertical dilation of f by a scale factor of c.

(b) Express the function $h(x) = \ln(\sqrt{x})$ in terms of a vertical dilation of f, stating the scale factor. (2 marks)

The function $p(x) = f(bx)$ is a horizontal dilation of f by a scale factor of $\frac{1}{b}$.

(c) Express the function $p(x) = \ln(x) + 4$ in terms of a horizontal dilation of f, stating the scale factor. (3 marks)