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Calculator Free
Differentiation Techniques

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2, 3, 3, 3, 3, 3 = 18 marks] CF

Differentiate each of the following functions with respect to *x*. Do not simplify your answers.

(a) $y = e^{-3x}$

(b) $g(x) = -\cos\left(\frac{x}{2}\right)$

(c) $f(x) = x^2 e^{2x-1}$

(d) $y = \frac{\sin x}{5x - 1}$

(e) $h(x) = \sqrt{x^4 - 2x}$

(f) $y = \sin^2(4x)$

(g) $y = 2f(3x - 1)$

Question Two: [4 marks] CF

Show, using the quotient rule, that $\frac{d}{dx}\tan(x) = 1 + \tan^2 x$.

Question Three: [4 marks] CF

A curve is defined parametrically as $x = 4t$ and $y = t^3 - 1$.

Determine an expression for the rate of change of *y* with respect to *x*, in terms of *x* only. Simplify your answer.

Question Four: [5 marks] CF

Given that $y = e^{x^2 - 1}$, show that $\frac{d^2 y}{dx^2} \times y^{-1} - 2 = 4x^2$

Question Five: [2 marks] CF

Given $f'(g(x)) = e^{0.5x} \cos(2e^{0.5x})$ and $g(x) = e^{0.5x}$, determine $f(x)$.

Question Six: **[5 marks]** **CF**

By using first principles and the limits $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$, establish that $\frac{d}{dx}\sin x = \cos x$.

Remember that $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

Question Seven: [3, 4 = 7 marks] **CF**

(a) Calculate the gradient of the curve $y = \frac{e^{-2x}}{5x}$ at $x = -1$.

(b) Determine the equation of the tangent to the curve $f(x) = -\cos(4x)$ at $x = \frac{\pi}{6}$.

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Question One: [1, 2, 3, 3, 3, 3, 3 = 18 marks] CF

Differentiate each of the following functions with respect to *x*. Do not simplify your answers.

(a) $y = e^{-3x}$

$$\frac{dy}{dx} = -3e^{-3x}$$

(b) $g(x) = -\cos\left(\frac{x}{2}\right)$

$$g'(x) = \frac{1}{2}\sin\left(\frac{x}{2}\right)$$

(c) $f(x) = x^2 e^{2x-1}$

$$f'(x) = 2x(e^{2x-1}) + 2x^2 e^{2x-1}$$

(d) $y = \frac{\sin x}{5x - 1}$

$$\frac{dy}{dx} = \frac{(5x - 1)(\cos x) - (5\sin x)}{(5x - 1)^2}$$

(e) $h(x) = \sqrt{x^4 - 2x}$

$$h'(x) = \frac{1}{2}(x^4 - 2x)^{\frac{-1}{2}}(4x^3 - 2)$$

(f) $y = \sin^2(4x)$

$$y = (\sin(4x))^2$$

$$\frac{dy}{dx} = 2(\sin(4x))(4\cos(4x))$$

(g) $y = 2f(3x - 1)$

$$\frac{dy}{dx} = 2f'(3x - 1)(3)$$

Question Two: [4 marks] **CF**

Show, using the quotient rule, that $\frac{d}{dx}\tan(x) = 1 + \tan^2 x$.

$$y = \tan x = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{dy}{dx} = 1 + \tan^2 x$$

Question Three: **[4 marks]** **CF**

A curve is defined parametrically as $x = 4t$ and $y = t^3 - 1$.

Determine an expression for the rate of change of y with respect to x, in terms of x only. Simplify your answer.

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 3t^2 \times \frac{1}{4}$$

$$\frac{dy}{dx} = \frac{3t^2}{4}$$

$$t = \frac{x}{4}$$

$$\therefore \frac{dy}{dx} = \frac{3\left(\frac{x}{4}\right)^2}{4}$$

$$= \frac{3x^2}{16 \times 4}$$

$$= \frac{3x^2}{64}$$

Question Four: [5 marks] CF

Given that $y = e^{x^2 - 1}$, show that $\frac{d^2y}{dx^2} \times y^{-1} - 2 = 4x^2$

$\frac{dy}{dx} = 2xe^{x^2-1}$ ✓

$\frac{d^2y}{dx^2} = 2e^{x^2-1} + 4x^2e^{x^2-1}$ ✓

$= 2e^{x^2-1}(1 + 2x^2)$ ✓

$\frac{d^2y}{dx^2} \times y^{-1} - 2$ ✓

$= 2e^{x^2-1}(1 + 2x^2) \times \frac{1}{e^{x^2-1}} - 2$

$= 2 + 4x^2 - 2$ ✓

$= 4x^2$

Question Five: [2 marks] CF

Given $f'(g(x)) = e^{0.5x}\cos(2e^{0.5x})$ and $g(x) = e^{0.5x}$, determine $f(x)$.

$f(x) = \sin 2x$

✓✓

Question Six: [5 marks] CF

By using first principles and the limits $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ and $\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = 0$, establish that $\frac{d}{dx} \sin x = \cos x$.

Remember that $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} \checkmark$$
$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \checkmark$$
$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} \checkmark$$
$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h \to 0} \frac{\cos x \sin h}{h} \checkmark$$
$$= 0 + \cos x \checkmark$$
$$= \cos x$$

Question Seven: [3, 4 = 7 marks] **CF**

(a) Calculate the gradient of the curve $y = \dfrac{e^{-2x}}{5x}$ at $x = -1$.

$$\frac{dy}{dx} = \frac{5x(-2e^{-2x}) - 5e^{-2x}}{25x^2}$$

$$\frac{dy}{dx} = \frac{-5(-2e^{2}) - 5e^{2}}{25} = \frac{e^{2}}{5}$$

(b) Determine the equation of the tangent to the curve $f(x) = -\cos(4x)$ at $x = \dfrac{\pi}{6}$.

$$f'(x) = 4\sin(4x)$$

$$f'\left(\frac{\pi}{6}\right) = 4\sin\frac{2\pi}{3} = 2\sqrt{3}$$

$$f\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$y = 2\sqrt{3}x + c$$

$$\frac{1}{2} = 2\sqrt{3}\left(\frac{\pi}{6}\right) + c$$

$$c = \frac{3 - 2\pi\sqrt{3}}{6}$$

$$\therefore y = 2\sqrt{3}x + \frac{3 - 2\pi\sqrt{3}}{6}$$