

MATHEMATICS
METHODS
UNITS 1 AND 2
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

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In words

Your name

Time allowed for this section
Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total				150	100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Additional working space

Question number: _____

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1

(a) Solve the equation $3(1 - 2a) - 2(a - 1) = 6$. (2 marks)

$$3(1 - 2a) - 2(a - 1) = 6$$
$$3 - 6a - 2a + 2 = 6$$
$$-8a = 1$$
$$a = -\frac{1}{8}$$

(b) The coordinates of three points are A(-2, -6), B(4, -2) and C(4, 2). (2 marks)

(i) If A is the mid-point of C and D, determine the coordinates of D.

$$D(-2-6, -6-8)$$
$$D(-8, -14)$$

(iii) Determine the gradient of the line through A and B. (1 mark)

$$\frac{-2 - -6}{4 - -2} = \frac{6}{6} = 1$$

(iiii) Find the equation of the line through C that is perpendicular to the line AB. (2 marks)

Perpendicular gradient is $-\frac{3}{2}$

$$y = -\frac{3}{2}x + c$$
$$2 = -\frac{3}{2} \times 4 + c$$
$$c = 8$$
$$y = -\frac{3}{2}x + 8$$

See next page

Question number: _____

Additional working space

Question 2

(7 marks)

- (a) Determine the coordinates of all axes intercepts of $y = (x+1)^2 - 4$. (2 marks)

$$\begin{aligned} y &= x^2 + 2x - 3 \\ &= (x-1)(x+3) \\ (0, -3), (1, 0), (-3, 0) \end{aligned}$$

- (b) State the coordinates of the turning point of $y = x^2 - 10x - 21$. (2 marks)

$$\begin{aligned} y &= (x-5)^2 - 46 \\ (5, -46) \end{aligned}$$

- (c) Solve

(i) $(2x-5)(x+3) = 0$. (1 mark)

$$x = \frac{5}{2}, x = -3$$

(ii) $x^2 - x = 20$. (2 marks)

$$\begin{aligned} (x+4)(x-5) &= 0 \\ x &= -4, x = 5 \end{aligned}$$

Question 7

(9 marks)

- (a) Determine $f'(-1)$ if $f(x) = \frac{x^3}{6} - \frac{x}{3}$. (2 marks)

$$\begin{aligned} f'(x) &= \frac{x^2}{2} - \frac{1}{3} \\ f'(-1) &= \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \end{aligned}$$

- (b) Determine $f(1)$ given that $f(2) = 5$ and $f'(x) = 8x^3 - 8x + 1$. (2 marks)

$$\begin{aligned} f(x) &= 2x^4 - 4x^2 + x + c \\ 5 &= 32 - 16 + 2 + c \Rightarrow c = -13 \\ f(1) &= 2 - 4 + 1 - 13 \\ &= -14 \end{aligned}$$

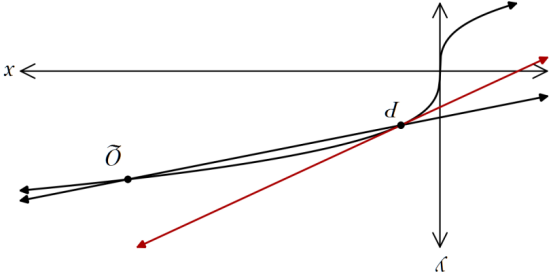
- (c) A curve has equation $y = ax^2 + bx + c$. The curve has a turning point at (4, 9) and a gradient of -1 when $x = 3$. Determine the values of a , b and c . (5 marks)

$$\begin{aligned} \frac{dy}{dx} &= 2ax + b \\ \text{When } x &= 3, y' = -1 \\ \text{When } x &= 4, y' = 0 \\ 6a + b &= -1 \\ 8a + b &= 0 \\ \text{Subtract to get } 2a &= 1 \\ \therefore a &= 0.5 \\ \therefore b &= -4 \\ \text{Use } (4, 9) \text{ to find } c \\ 9 &= 0.5(4)^2 - 4(4) + c \\ \therefore c &= 17 \end{aligned}$$

Question 6

(8 marks)

The graph of $y = \sqrt[3]{x}$ is shown below together with the secant cutting the graph at the points P and Q , where $x = 1$ and $x = 8$ respectively.



(a) Determine the equation of the secant PQ .

(2 marks)

$$m = \frac{\sqrt[3]{8} - \sqrt[3]{1}}{8 - 1} = \frac{\frac{2}{1} - \frac{1}{1}}{7}$$
$$y - 1 = \frac{2}{7}(x - 1)$$
$$y = \frac{2}{7}x + \frac{5}{7}$$

(b) If the x -coordinate of point Q was decreased from 8 towards 1, explain the effect this would have on your answer to (a).

(2 marks)

Gradient of secant would increase and y -intercept would decrease.

(c) Determine the equation of the tangent to the graph of $y = \sqrt[3]{x}$ at P .

(3 marks)

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} \bigg|_{x=1} = \frac{1}{3}$$
$$y - 1 = \frac{1}{3}(x - 1)$$
$$y = \frac{1}{3}x + \frac{2}{3}$$

(d) Draw the tangent from (c) on the graph above.

(1 mark)

See next page

Question 3

(5 marks)

(a) Determine as an exact value $\sin 45^\circ \cdot \cos 45^\circ + \cos 45^\circ \cdot \sin 45^\circ$.

(1 mark)

$$\sin 45^\circ \cdot \cos 45^\circ + \cos 45^\circ \cdot \sin 45^\circ = \sin(90^\circ)$$
$$= 1$$

(b) Solve $\cos x = \sqrt{3} \sin x$ for $-\pi \leq x \leq \pi$.

(2 marks)

$$\frac{1}{\sin x} = \frac{\sqrt{3}}{\cos x}$$
$$\tan x = \frac{1}{\sqrt{3}}$$
$$x = -\frac{5\pi}{6}, \frac{\pi}{6}$$

(c) Determine the coefficient of the x^4 term in the expansion of $(x - 3)^6$.

(2 marks)

$$\dots + \binom{6}{4}(x)^4(-3)^2 + \dots$$
$$\dots + 15x^4 \times 9 \dots$$
$$\dots + 135x^4 + \dots$$

Coefficient is 135

See next page

Question 4

(8 marks)

(a) Simplify $\left(2\frac{7}{9}\right)^{-\frac{1}{2}}$.

(2 marks)

$$\left(\frac{25}{9}\right)^{-\frac{1}{2}} = \left(\frac{9}{25}\right)^{\frac{1}{2}} = \frac{3}{5}$$

(b) If $a = 5 \times 10^2$ and $b = 8 \times 10^6$ evaluate $a^2 \div b^{1/3}$.

(2 marks)

$$\begin{aligned} \frac{5^2 \times 10^{2 \times 2}}{8^{1/3} \times (10^6)^{1/3}} &= \frac{25 \times 10^4}{2 \times 10^2} \\ &= 12.5 \times 10^2 \\ &= 1250 \end{aligned}$$

(c) Solve $25^x = 125\sqrt{5}$.

(2 marks)

$$\begin{aligned} 5^{2x} &= 5^3 \times 5^{0.5} \\ 5^{2x} &= 5^{3.5} \\ 2x &= 3.5 \\ x &= 1.75 \end{aligned}$$

(d) State the equation of the asymptote of the following graphs:

(i) $y = 0.5^{x+2}$.

(1 mark)

$$y = 0$$

(ii) $y = 0.5^x - 2$.

(1 mark)

$$y = -2$$

See next page

Question 5

(8 marks)

(a) Let $f(x) = 2(x+1)(x-1)^2$.

(i) State the coordinates of the y -intercept of the graph of $y = f(x)$.

(1 mark)

$$(0, 2)$$

(ii) State the coordinates of the roots of the graph of $y = f(x)$.

(1 mark)

$$(-1, 0) \text{ and } (1, 0)$$

(iii) Determine the range of $f(x)$ over the domain $x \geq 1$.

(1 mark)

Root at $(1, 0)$ is also a minimum turning point.
Hence $y \geq 0$.

(b) Expand $(x+1)(x+2)(2x-1)$.

(2 marks)

$$(2x-1)(x^2+3x+2) = 2x^3+5x^2+x-2$$

(c) Solve $x^3 - x^2 - 10x - 8 = 0$.

(3 marks)

$$\begin{aligned} f(-1) = 0 &\Rightarrow (x+1)(x-2x-8) = 0 \\ (x+1)(x+2)(x-4) &= 0 \\ x = -1, x = -2, x = 4 \end{aligned}$$

See next page