PERTH MODERN SCHOOL
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Independent Public School

Mathematics Specialist Year 11

Student name: _____ Teacher name: _____

Date: Friday 14 May 2021

Task type:	**Response**
Time allowed:	**40 minutes**
Number of questions:	**8**
Materials required:	Calculator with CAS capability (to be provided by the student)
Standard items:	Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	Drawing instruments, templates, notes on two unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations
Marks available:	**40 marks**
Task weighting:	**10%**
Formula sheet provided:	**Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1

Let p and q be positive integers. Prove by contradiction that

$$\frac{p}{q}+\frac{q}{p}\geq 2$$

Assume that p & q are positive integers such that $\frac{p}{q}+\frac{q}{p}<2$ ✓ assumption

$$\Rightarrow \frac{p^2+q^2}{pq}<2$$

$$\Rightarrow p^2+q^2<2pq \quad \text{since } pq>0$$

$$\Rightarrow p^2+q^2-2pq<0$$

$$\Rightarrow (p-q)^2<0$$ ✓ simplification

This is a contradiction since $(p-q)^2$ cannot be negative ✓ conclusion

$$\therefore \frac{p}{q}+\frac{q}{p}\geq 2$$

Question 2

A high school student council has 7 boys and 6 girls. A yearbook committee of 7 people is to be appointed from the student council.

a) In how many different ways can the committee be selected if all the members are available?

$^{13}C_7 = 1716$ ✓ states value

b) In how many different ways can the committee be selected if it is to have more girls than boys?

girls	boys
6	1
5	2
4	3

✓ indicates correct selections

$^{6}C_{6} \times {}^{7}C_{1} + {}^{6}C_{5} \times {}^{7}C_{2} + {}^{6}C_{4} \times {}^{7}C_{3}$

$= 1 \times 7 + 6 \times 21 + 15 \times 35$

$= 658$ ✓ states answer

Question 3

Anna, Belinda and Charles belong to a team of 8 students.

a) How many selections of 4 students would include both Anna and Belinda but not Charles?

$^{2}C_{2} \times {}^{5}C_{2} = 10$ ✓ correct answer

✓ correct selections

A team photo is about to be taken with all eight students standing next to each other.

b) Determine the number of arrangements in which;

a. Anna is standing at one end and Belinda and Charles are standing next to each other

A BC O_1 O_2 O_3 O_4 O_5

fixed

BC or CB

"others"

$6! \times 2 \times 2 = 2880$

✓ correct setup

BC or CB

either end

✓✓ states answer

b. Anna and Belinda are standing next to each other and Charles is standing next to them

O_1 O_2 O_3 O_4 O_5

"others"

✓ correct setup

ABC

ACB

CAB

BCA

CBA

CAB

✓ four ways for AB to be together

$6! \times 4 = 2880$

✓ states answer

Question 4

a) Seven teams play 23 volleyball games. Prove that there is some pair of teams that play each other more than once. (2 marks)

$^7C_2 = 21$ ✓ calculates selections

There are 21 ways for 2 teams to be selected out of 7 to compete

Since 23 games are played, some pair of teams must play each other at least twice ✓ Explains conclusion

b) How many of the integers from 1 to 220 inclusive are divisible by 2, 5 or 11? (5 marks)

multiples of 2 : 110
multiples of 5 : 44
multiples of 11 : 20
} ✓

multiples of 10 (2 & 5) : 22
multiples of 22 (2 & 11) : 10
multiples of 55 (5 & 11) : 4
} ✓ obtains relevant multiples

multiples of 110 (2, 5 & 11) : 2 ✓

Using the inclusion - exclusion principle, the required number of integers is

$110 + 44 + 20 - 22 - 10 - 4 + 2 = 140$ ✓ correct answer

✓ use of inclusion - exclusion principle to obtain correct answer

Question 5

Let $a=3i-j$, $b=2i+j$ and $c=4i+yj$

a) Find a unit vector in the same direction as b (1 mark)

$|\underset{\sim}{b}| = \sqrt{5}$

$\hat{\underset{\sim}{b}} = \frac{1}{\sqrt{5}}(2\underset{\sim}{i} + \underset{\sim}{j})$ ✓ correct vector

$= \frac{2}{\sqrt{5}}\underset{\sim}{i} + \frac{1}{\sqrt{5}}\underset{\sim}{j}$

b) A vector in the same direction as b but equal in magnitude to a (2 marks)

$|\underset{\sim}{a}| = \sqrt{10}$ ✓ states magnitude

required vector is $\sqrt{10}\left(\frac{1}{\sqrt{5}}(2\underset{\sim}{i} + \underset{\sim}{j})\right)$

$= \sqrt{2}(2\underset{\sim}{i} + \underset{\sim}{j})$ ✓ correct vector

$= 2\sqrt{2}\underset{\sim}{i} + \sqrt{2}\underset{\sim}{j}$

c) Solve for y if a and c are perpendicular (2 marks)

$\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$

$\begin{pmatrix}3\\-1\end{pmatrix} \cdot \begin{pmatrix}4\\y\end{pmatrix} = 0$ ✓ uses dot product

$12 - y = 0$

$y = 12$ ✓ correct value for y

d) Solve for y if a and c are parallel (3 marks)

$\begin{pmatrix} 3 \\ -1 \end{pmatrix} = k\begin{pmatrix} 4 \\ y \end{pmatrix}$ ✓ express one vector as a scalar multiple of the other

$4k = 3$

$k = \frac{3}{4}$ ✓ solve for k

$ky = -1$

$y = -\frac{1}{k} = -\frac{4}{3}$ ✓ states y

Accept working involving scalar product

e) Solve for y if the angle between a and c is 45^0. Make sure you clearly show the use of the scalar product (3 marks)

$\underset{\sim}{a}\ \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos 45^\circ$

$12 - y = \sqrt{10} \times \sqrt{16 + y^2} \times \frac{1}{\sqrt{2}}$ ✓ uses scalar product

$12 - y = \sqrt{5} \times \sqrt{16 + y^2}$

CAS solve : $y = 2$ or $y = -8$

✓ ✓ correct values

Question 6

Let A and B be the points defined by the position vectors $a = 4i + j$ and $b = i - j$ respectively. Find;

a) The component of a that is parallel to b (2 marks)

$$\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\,\underset{\sim}{b} = \frac{3}{2}(\underset{\sim}{i} - \underset{\sim}{j})$$

✓ finds signed length

vector projection

✓

$$= \frac{3}{2}\underset{\sim}{i} - \frac{3}{2}\underset{\sim}{j}$$

b) The component of a that is perpendicular to b (2 marks)

$$\underset{\sim}{a} - \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\,\underset{\sim}{b} = 4\underset{\sim}{i} + \underset{\sim}{j} - \frac{3}{2}(\underset{\sim}{i} - \underset{\sim}{j})$$

✓ ⊥ component

$$= \frac{1}{2}(8\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{i} + 3\underset{\sim}{j})$$

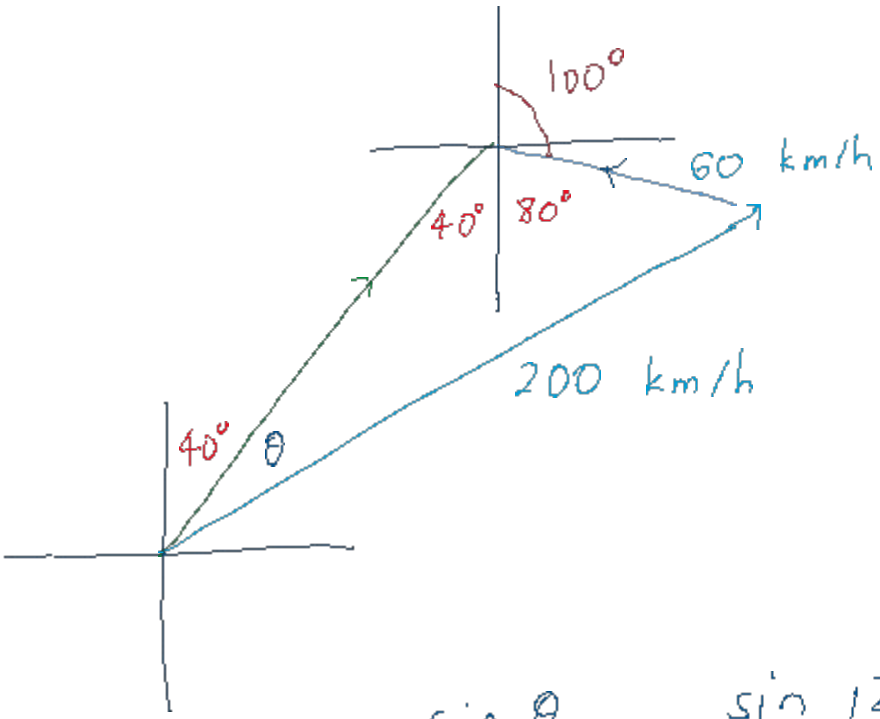
$$= \frac{1}{2}(5\underset{\sim}{i} + 5\underset{\sim}{j})$$

✓ correct vector

$$= \frac{5}{2}\underset{\sim}{i} + \frac{5}{2}\underset{\sim}{j}$$

Question 7

A helicopter can maintain a speed of 200 km/h in still air. What bearing should the pilot set if they wish to fly 380 km in a direction of 040^0? The instrument readout shows that the wind is blowing at 60 km/h from 100^0.



✓ relevant diagram

$$\frac{\sin\theta}{60} = \frac{\sin 120^\circ}{200}$$

✓ applies sine rule

CAS solve $\theta \simeq 15^\circ$ or 165°

✓ correct angle

↑ reject

Bearing is 055°

✓ correct bearing

accept alternative working

(Additional working space)