

SCHOOL

Trial WACE Examination, 2011

Question/Answer Booklet

**MATHEMATICS
SPECIALIST 3A/3B**

SOLUTIONS

**Section One:
Calculator-free**

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	5	5	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
Total				150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2011*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil**, except in diagrams.

Section One: Calculator-free

(40 Marks)

This section has **five (5)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1

(8 marks)

Solve each of the following for x .

(a) $\log_x 16 = 2$

(2 marks)

$x^2 = 16$ $x = 4 \quad (+ve \text{ soln only})$
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(b) $5^{x-1} = 10$

(3 marks)

$(x - 1) \log 5 = \log 10$ $x - 1 = \frac{1}{\log 5}$ $x = \frac{1}{\log 5} + 1$
--

(c) $\log_7 x - 2 = \log_7 2$

(3 marks)

$\log_7 x - \log_7 49 = \log_7 2$ $\log_7 \frac{x}{49} = \log_7 2$ $\frac{x}{49} = 2$ $x = 98$
--

Question 2

(10 marks)

Determine $\frac{dy}{dx}$ for each of the following, simplifying your answers:

(a) $y = x^2 e^{1-2x}$

(3 marks)

$$\begin{aligned}\frac{dy}{dx} &= 2x(e^{1-2x}) + x^2(-2)(e^{1-2x}) \\ &= 2x(1-2x)(e^{1-2x})\end{aligned}$$

(b) $y = \frac{x}{\ln x}$

(3 marks)

$$\begin{aligned}\frac{dy}{dx} &= \frac{1 \times \ln x - x \times \frac{1}{x}}{(\ln x)^2} \\ &= \frac{\ln x - 1}{(\ln x)^2}\end{aligned}$$

(c) $y = \ln \left(\frac{\sqrt{x+1}}{1-x} \right)$

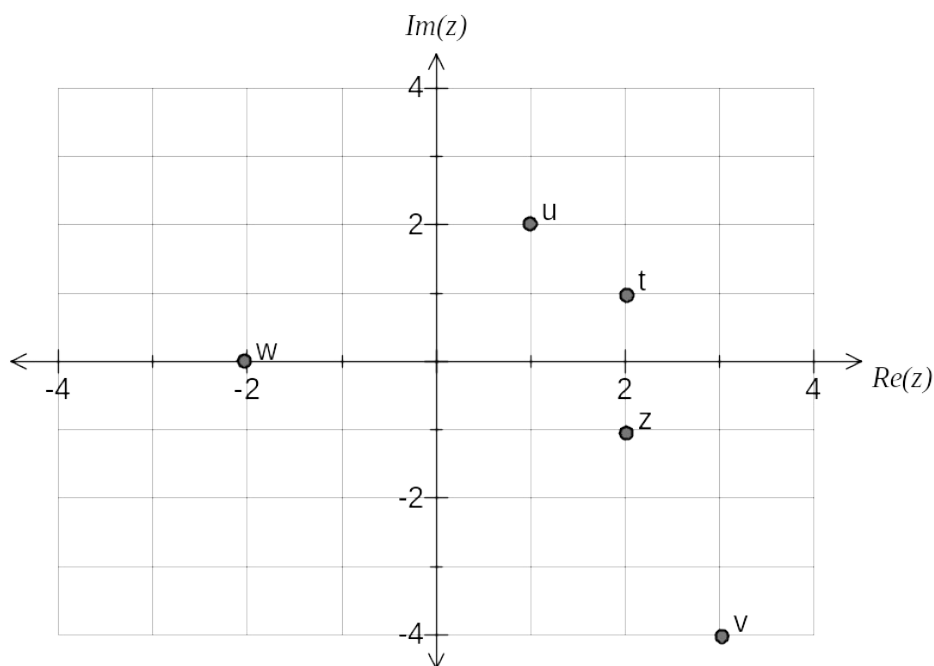
(4 marks)

$$\begin{aligned} y &= \frac{1}{2} \ln(x+1) - \ln(1-x) \\ \frac{dy}{dx} &= \frac{1}{2} \times \frac{1}{x+1} - \frac{-1}{1-x} \\ &= \frac{1}{2(x+1)} - \frac{1}{x-1} \end{aligned}$$

Question 3

(12 marks)

The Argand diagram below shows the complex number z .



(a) On the same diagram plot and label the four complex numbers given by

(8 marks)

$$\begin{aligned}
 t &= \bar{z} \\
 &= 2 + i \\
 u &= i z \\
 &= i(2 - i) = 1 + 2i \\
 v &= z^2 \\
 &= (2 - i)(2 - i) = 4 - 4i + 1 = 3 - 4i \\
 w &= \frac{-4 + 2i}{z} \\
 &= \frac{(-4 + 2i)(2 + i)}{(2 - i)(2 + i)} = \frac{-8 - 4i + 4i - 2}{5} = -2
 \end{aligned}$$

- (b) Find both complex solutions to the equation $x^2 - 6x + 10 = 0$. (4 marks)

$$\begin{aligned}x &= \frac{6 \pm \sqrt{36 - 40}}{2} \\&= \frac{6 \pm 2i}{2} \\&= 3 \pm i\end{aligned}$$

Question 4**(12 marks)**

The three points P, Q and T have position vectors $2\mathbf{i} - 3\mathbf{j}$, $6\mathbf{i} + 9\mathbf{j}$ and $x\mathbf{i} + 2\mathbf{j}$ respectively.

(a) Find the vector equation of the line passing through P and Q.

(3 marks)

$$\begin{aligned} \vec{PQ} &= \begin{bmatrix} 6 \\ 9 \end{bmatrix} - \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix} \\ \mathbf{r} &= \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ 12 \end{bmatrix} \end{aligned}$$

(b) The vector equation of the circle with diameter PQ.

(4 marks)

$$\begin{aligned} &\text{Centre when } \lambda = 0.5 \\ &\begin{bmatrix} 2 \\ -3 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \\ &\text{Radius} = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10} \\ &\left| \mathbf{r} - \begin{bmatrix} 4 \\ 3 \end{bmatrix} \right| = 2\sqrt{10} \end{aligned}$$

- (c) Determine the value of x if the angle between OT and PQ is 45° , where O is the origin.
(5 marks)

Direction of PQ is $(1,3)$

$$\cos 45^\circ = \frac{(x, 2) \cdot (1, 3)}{|(x, 2)| \times |(1, 3)|}$$

$$\frac{1}{\sqrt{2}} = \frac{x + 6}{\sqrt{x^2 + 4} \sqrt{10}}$$

$$10(x^2 + 4) = 2(x + 6)^2$$

$$5x^2 + 20 = (x + 6)^2$$

$$4x^2 - 12x - 16 = 0$$

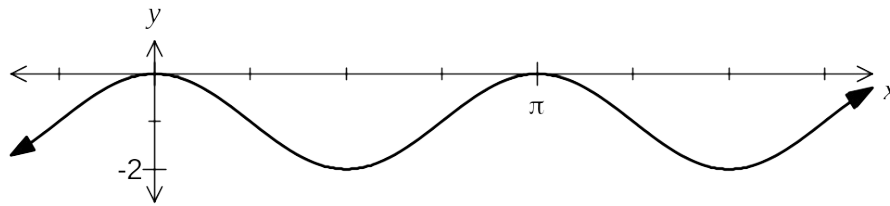
$$(x - 4)(x + 1) = 0$$

$$x = 4 \text{ or } x = -1$$

Question 5

(8 marks)

The graph of a trigonometric function $y = f(x)$ is shown below.



(a) Clearly circle all of the functions listed below that could be $f(x)$.

(5 marks)

$$y = -2 \sin^2 x$$

$$y = \cos(-2x) - 1$$

$$y = \cos(2x) - 1$$

$$y = \cos\left(\frac{\pi}{2} + 2x\right) - 1$$

$$y = -2 \cos^2 x$$

$$y = \cos(2(x + \pi)) - 1$$

$$y = \sin\left(2x + \frac{\pi}{2}\right) - 1$$

$$y = \sin(2x) - 1$$

$$y = \sin\left(\frac{\pi}{2} - 2x\right) - 1$$

(b) Solve $f(x) = -\frac{1}{2}$ for $0 \leq x \leq \pi$.

(3 marks)

$$\begin{aligned} -2 \sin^2 x &= -\frac{1}{2} \\ \sin^2 x &= \frac{1}{4} \\ \sin x &= \pm \frac{1}{2} \\ x &= \frac{\pi}{6} \quad \text{or} \quad x = \frac{5\pi}{6} \end{aligned}$$

Additional working space

Question number: _____

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