(-1) no + c

SHENTON COLLEGE

Year 12 Mathematics Methods (ATMAM)

Test 2 2017

Calculator Free

Time Allowed: 25 minutes

Marks	/ 27

Name: Marking Key

Circle Your Teachers Name: Mrs Friday Mr Smith

Question 1 [3,3,2] 8

Determine the following:

(a) $\int(4x^3 + 2\sqrt[3]{x} - \frac{4}{x^3})dx$

$= x^4 + \frac{3x^{4/3}}{2} + \frac{2}{x^2} + c$

✓✓✓ each antiderivative

(b) $\int(\frac{e^{2x}+e^{-3x}}{e^x})dx$

$= \int e^x + e^{-4x}\, dx$ ✓ separate terms

$= e^x - \frac{e^{-4x}}{4} + c$ ✓✓ each antiderivative

(c) $\int 2sin3x + \cos(4x + \pi)dx$

$= -\frac{2}{3}\cos 3x + \frac{\sin(4x+\pi)}{4} + c$ ✓✓ each antiderivative

Question 2 [3,3] 6

Evaluate

(a) $\int_2^6 \frac{1}{\sqrt{2x-3}}dx$

$= \int_2^6 (2x-3)^{-\frac{1}{2}}dx$

$= \left[(2x-3)^{\frac{1}{2}}\right]_2^6$ ✓ correct antideriv

$= 9^{\frac{1}{2}} - 1^{\frac{1}{2}}$ ✓ correct interp of limits

$= 2$ ✓ evaluate

(b) $\int_0^{\frac{\pi}{3}}(cos3\theta + sin3\theta)d\theta$

$= \left[\frac{\sin 3\theta}{3} - \frac{\cos 3\theta}{3}\right]_0^{\frac{\pi}{3}}$

$= \left(\frac{\sin\pi}{3} - \frac{\cos\pi}{3}\right) - \left(\frac{\sin 0}{3} - \frac{\cos 0}{3}\right)$

$= \frac{1}{3} - \left(-\frac{1}{3}\right)$

$= \frac{2}{3}$

✓ correct antider.

✓ correct application of limits

✓ evaluate

Question 3 [1,3]

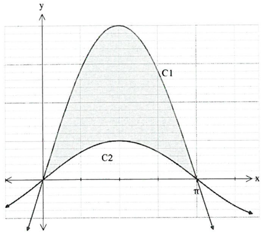
The illustrated curves are the graphs of

$y = sinx \text{ and } y = 4sinx.$

(a) Identify each curve

$C_1 \quad y = 4\sin x$ Both correct ✓

$C_2 \quad y = \sin x$



(b) Determine the shaded area.

$\int_0^{\pi} (4\sin x - \sin x)\,dx$ ✓ correct definite integral

$= \int_0^{\pi} (3\sin x)\,dx$

$= [-3\cos x]_0^{\pi}$

$= -3\cos\pi - (-3\cos 0)$ ✓ evaluate limits

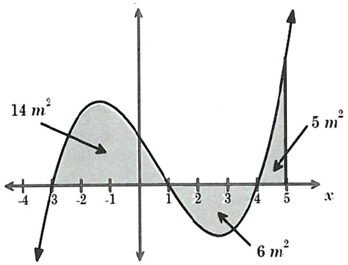
$= 3 + 3$

$= 6$ ✓ Area

Question 4 [1,1,2]

For the graph of y = h(x) to the right the areas between the curve and the x-axis are shown.

Use this to state the value of the following integrals.



(a) $\int_{-3}^{5} h(x)dx = 14 - 6 + 5$

$= 13$ ✓ correct

(b) $\int_{5}^{4} h(x)dx = -\int_4^5 h(x)\,dx$

$= -5$ ✓ correct

(c) $\int_{-3}^{1} [h(x) + 2]dx = \int_{-3}^{1} h(x)\,dx + \int_{-3}^{1} 2\,dx$

$= 14 + [2x]_{-3}^{1}$

$= 14 + (2 - (-6))$ ✓ correct $\int_{-3}^{1} 2\,dx$ evaluate.

$= 22$ ✓ correct

Question 9 [2,1]

Consider the function $f(x) = (x-4)(x+1)(2x+7)$

(a) Write down a sum of integrals which when evaluated could be used to determine the area trapped by $f(x)$ and the $x-axis$.

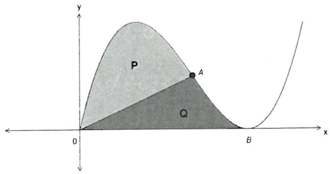
$\int_{-3.5}^{-1} f(x)\,dx + \int_{4}^{-1} f(x)\,dx$ ✓✓ correct integrals

(b) Calculate the area.

240.89 ✓ area

Question 10 [2,3,2]

The diagram below shows part of the curve $y = x(x-3)^2$, which passes through the point of inflection at A and touches the x-axis at B.



(a) Locate the coordinates of the points A and B.

A $(2,2)$ ✓ Point of Inflection

B $(3,0)$ ✓ root

(b) Find area of the region labelled P. Indicate the integral you used.

OA is $y = x$ ✓ $y=x$ identified

$\int_0^2 (x(x-3)^2 - x)\,dx$ ✓ correct integral

$= 4$ ✓ Area

(c) Find the area of the region labelled Q.

$\int_0^2 x\,dx + \int_2^3 x(x-3)^2\,dx$ ✓ integral sum

$= 2.75$ ✓ Area

$\int_0^3 x(x-3)^2\,dx - P.$

End of test

Question 5 [5]

The function $y = f(x)$ passes through the point (0,-1). A tangent to $f(x)$ has a gradient of 3 at that point. $f''(x) = 80(2x-1)^3$. Determine the function $f(x)$.

$f'(x) = \int 80(2x-1)^3\,dx$

$= \frac{80(2x-1)^4}{2 \cdot 4} + c$

$f'(x) = 10(2x-1)^4 + c$ ✓ correct $f'(x)+c$

$x=0$, $f'(x)=3$: $3 = 10(-1)^4 + c$ ✓ evaluates c

$c = -7$

$f(x) = \int 10(2x-1)^4 - 7\,dx$

$= (2x-1)^5 - 7x + c$ ✓ correct $f(x)+c$

$(0,-1)$: $f(x) = (-1)^5 + c$

$-1 = (-1)^5 + c$

$c = 0$ ✓ evaluates c

$\therefore f(x) = (2x-1)^5 - 7x.$ ✓ $f(x)$

SHENTON COLLEGE

Year 12 Mathematics Methods (ATMAM)

Test 2 2017
Calculator Assumed
Time Allowed: 25 minutes

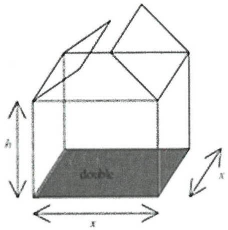
Marks / 27

Name: Marking Key

Circle Your Teachers Name: Mrs Friday Mr Smith

Question 6 [1,2,3,1]

A manufacturer produces cardboard boxes that have a square base. The top of each box consists of a double flap that opens as shown. The base of the box has a double layer of cardboard for strength. Each box must have a volume of 12 cubic metres.



(a) Show that the area of cardboard required is given by $C = 3x^2 + 4xh$

$C = 2(\text{Area Base}) + \text{Area lid} + 4(\text{Area side})$

$= 2x^2 + x^2 + 4xh$

$= 3x^2 + 4xh$ ✓ demonstrates where C comes from.

(b) Express C as a function of x only.

$V = x^2 h$

$12 = x^2 h$ ✓ uses V correctly to obtain h

$h = \frac{12}{x^2}$

$\therefore C = 3x^2 + \frac{48}{x}$ ✓ correct C as a function of x only

(c) Use calculus to determine what dimensions will minimise the amount of cardboard used.

For Min $C'(x) = 0 \therefore x = 2$ States $C'(x)=0$ for Min ✓

$C''(2) > 0 \therefore$ min Checks that it is a min ✓

Min when $x = 2m$ and $h = 3m$ Dimensions ✓

(d) What is the minimum area of cardboard used?

$C = 36\ m^2$ ✓ correct Area

Question 7 [4]

Use calculus to estimate the percentage change in y for $y = 2x^3$ when x decreases by 2%

$y = 2x^3$ $\delta x = -0.02x$ ✓ identifies incremental change

$\frac{dy}{dx} = 6x^2$

$\delta y \approx \frac{dy}{dx} . \delta x$ ✓ use of $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\approx 6x^2 . (-0.02x)$

$\frac{\delta y}{y} = \frac{{}^{3}\cancel{6x^2}(-0.02\cancel{x})}{\cancel{2x^3}}$ ✓ compares $\frac{\delta y}{y}$

$= -0.06$

$\therefore$ 6% decrease ✓ correct % change

Question 8 [1,2,3]

The cost of producing x items of a product is given by $\$[5x + 2000e^{-0.01x}]$. Each item is sold for \$24.90.

(a) Write an equation to describe $R(x)$, the revenue from selling the product .

$R(x) = 24.90x$ ✓ correct

(b) Write an equation for $P(x)$, the profit function.

$P(x) = 24.90x - (5x + 2000e^{-0.01x})$ ✓ uses $R(x) - C(x)$

$= 19.90x - 2000e^{-0.01x}$ ✓ correct expression (not nec. simplified)

(c) Demonstrate the use of calculus to find the profit associated with the sale of the 501st item at the point in production where 500 items are produced.

$\frac{dP}{dx} = 19.9 + 20e^{-0.01x}$ $\delta x = 1$ ✓ $\frac{dP}{dx}$

$\delta C \approx \left.\frac{dP}{dx}\right|_{x=500} . \delta x$ ✓ use of incremental concept $x = 500$

≈ 20.03

\$20.03 profit with sale of 501st item ✓ Profit correct