



PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.

Test Three
Semester One 2016
Year 12 Mathematics Methods
Calculator Free

Teacher:

_____ Mr Staffe

_____ Mrs. Carter

_____ Mr Bertram

_____ Mr Roohi

_____ Ms Cheng

Name:

- Complete all questions
- Show all necessary working
- Total Marks = 27
- 27 minutes

1. [6 marks]

Differentiate each of the following functions with respect to x . **Do not simplify** your answers.

(a) $g(x) = -\cos\left(\frac{x}{2}\right)$ [2]

(b) $y = \frac{\sin x}{5x - 1}$ [2]

(c) $y = \sin^2(4x)$ [2]

2. [4 marks]

Show, using the quotient rule, that $\frac{d}{dx} \tan(x) = 1 + \tan^2 x$.

3. [9 marks]

Simplify or Evaluate the following integrals as appropriate

(a) $\int -\sin 2u \, du$ [3]

(b) $\int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} \right) dx$ [3]

(c) Evaluate $-2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 3x \, dx$ [3]

4. [8 marks]

(a) Simplify $\int \cos\left(\frac{t}{3}\right) dt$ [2]

(b) Use your answer to part (a) to evaluate $\int_{-\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt$, in terms of x [2]

(c) Use your answer to part (b) to evaluate $\frac{d}{dx} \left(\int_{\pi}^{2x+1} \cos \left(\frac{t}{3} \right) dt \right)$ [2]

(d) Hence evaluate $\frac{d}{dx} \left(\int_{\pi}^{f(x)} \cos \left(\frac{t}{3} \right) dt \right)$ [2]



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Name:

- **Complete all questions**
- **Show all necessary working**
- **Total Marks = 23**
- **23 minutes**

1. [6 marks]

Determine, with reasoning, whether each of the following represent a discrete random variable.

(a)

x	0	1	2	3	4
$P(X=x)$	0.3	0.1	0.4	0.05	0.15

(b)

x	-2	-1	1	3	5
$P(X=x)$	0.2	0	-0.3	0.6	0.5

(c) $P(X = x) = \left(\frac{1}{2}\right)^x ; x = 1, 2, 3, 4, \dots$

2. [8 marks]

Each of the following represent discrete probability functions. Determine the value of k for each.

(a)

x	0	1	2	3	4
$P(X=x)$	0.2	k	0.3	0.1	0.15

[2]

(b) $P(x) = \frac{x}{2k-1}; x = 1, 2, 3, 4, 5, 6$

[3]

(c)

x	1	2	3	5	7
$P(X=x)$	$2k$	k	k	$5k$	$6k$

[3]

3. [9 marks]

Studies in Britain have recorded that 1 in 100 eight year-old children need at least one tooth removed caused by sugary drinks and severe tooth decay.

A typical primary school class of 24 eight year-olds are investigated for the need to remove at least one tooth.

Determine the probability of:

- (a) 2 students needing at least one tooth removed. [1]
- (b) No students requiring the removal of any teeth. [1]
- (c) At least one student requiring the removal of at least one tooth. [2]
- (d) Less than 4 students requiring the removal of at least one tooth given that at least one student required tooth removal. [2]

Of the thirteen year-olds in Britain requiring tooth removal, the probability of them requiring any one tooth out of their 32 permanent teeth removed, independent of the others is 5%.

- (e) Calculate the probability of a permanent tooth in a thirteen year - old needing removal.

[3]



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Teacher:

_____ Mr Staffe

_____ Mrs. Carter

_____ Mr Bertram

_____ Mr Roohi

_____ Ms Cheng

Name: **Mark King-Gyde**

- *Total Marks = 27*
- *27 minutes*

1. [6 marks]

Differentiate each of the following functions with respect to x . Do not simplify your answers.

(a) $g(x) = -\cos\left(\frac{x}{2}\right)$ $g'(x) = \frac{1}{2}\sin\left(\frac{x}{2}\right)$

(b) $y = \frac{\sin x}{5x - 1}$

$$\frac{dy}{dx} = \frac{(5x - 1)(\cos x) - (5 \sin x)}{(5x - 1)^2}$$

(c) $y = \sin^2(4x)$

$$y = (\sin(4x))^2$$

$$\frac{dy}{dx} = 2(\sin(4x))(4\cos(4x))$$

2. [4 marks]

Show, using the quotient rule, that $\frac{d}{dx} \tan(x) = 1 + \tan^2 x$.

$$y = \tan x = \frac{\sin x}{\cos x} \quad \checkmark$$

$$\frac{dy}{dx} = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2 x} \quad \checkmark$$

$$\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \quad \checkmark$$

$$\frac{dy}{dx} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} \quad \checkmark$$

$$\frac{dy}{dx} = 1 + \tan^2 x$$

3. [9 marks]

Simplify or Evaluate the following integrals as appropriate

$$(a) \quad \int -\sin 2u \quad \checkmark = \frac{\cos 2u}{2} + c \quad \checkmark$$

$$(b) \quad \int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} \right) dx = 3\sin\left(\frac{x}{3}\right) + \frac{3(6x)^{\frac{4}{3}}}{8} + c \quad \checkmark$$

$$(c) \quad -2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 3x \, dx$$

$$= -2 \left[\frac{-\cos 3x}{3} \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \quad \checkmark$$

$$= -2 \left[\frac{-\cos \pi}{3} - \frac{-\cos \frac{\pi}{2}}{3} \right] \quad \checkmark$$

$$= -2 \left(\frac{1}{3} + 0 \right)$$

$$= -\frac{2}{3} \quad \checkmark$$

4. [8 marks]

$$(a) \quad \text{Calculate } \int \cos\left(\frac{t}{3}\right) dt$$

\checkmark

$$= 3\sin \frac{t}{3} + c \quad \checkmark$$

- (b) Use your answer to part (a) to evaluate $\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt$, in terms of x

$$\begin{aligned} &= \left[3\sin \frac{t}{3} + c \right]_{\pi}^{2x+1} \\ &= \left(3\sin \frac{2x+1}{3} + c \right) - \left(3\sin \frac{\pi}{3} + c \right) \\ &= 3\sin \frac{2x+1}{3} - \frac{3\sqrt{3}}{2} \quad \checkmark \end{aligned}$$

- (c) Use your answer to part (b) to evaluate $\frac{d}{dx} \left(\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt \right)$

$$\begin{aligned} &\frac{d}{dx} \left(3\sin \frac{2x+1}{3} - \frac{3\sqrt{3}}{2} \right) \\ &= 3\cos \frac{2x+1}{3} \times 2 \quad \checkmark \\ &= 6\cos \frac{2x+1}{3} \quad \checkmark \end{aligned}$$

- (d) Hence evaluate $\frac{d}{dx} \left(\int_{\pi}^{f(x)} \cos\left(\frac{t}{3}\right) dt \right)$
- $$= \cos\left(\frac{f(x)}{3}\right) \times f'(x) \quad \checkmark$$



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Name: **Mark King-Gyde**

- Complete all questions
- Show all necessary working
- Total Marks = 36
- 36 minutes

1. [6 marks]

Determine, with reasoning, whether each of the following represent a discrete random variable.

(a)

x	0	1	2	3	4
$P(X=x)$	0.3	0.1	0.4	0.05	0.15

Yes this table does represent a DRV, all probabilities add to 1 and there are no negative values. ✓✓

(b)

x	-2	-1	1	3	5
$P(X=x)$	0.2	0	-0.3	0.6	0.5

No, this table does not represent a DRV. Despite all the probability values adding to 1, as one is negative, this cannot represent a DRV. ✓✓

(c) $P(X = x) = \left(\frac{1}{2}\right)^x ; x = 1, 2, 3, 4, \dots$

The sequence of probabilities is: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$ ✓
 $S_{\infty} = \frac{0.5}{1 - 0.5} = 1$

Therefore all probabilities will add to 1, and none are negative. ✓

2. [8 marks]

(a)

x	0	1	2	3	4
$P(X=x)$	0.2	k	0.3	0.1	0.15

$$k = 1 - 0.2 - 0.3 - 0.1 - 0.15$$

$$k = 0.25 \quad \checkmark \checkmark$$

(b) $P(x) = \frac{x}{2k-1}; x = 1, 2, 3, 4, 5, 6$

$$1 = \frac{1+2+3+4+5+6}{2k-1} \quad \checkmark$$

$$1 = \frac{21}{2k-1} \quad \checkmark$$

$$k = 11 \quad \checkmark$$

(c)

x	1	2	3	5	7
$P(X=x)$	$2k$	k	k	$5k$	$6k$

$$2k + k + k + 5k + 6k = 1 \quad \checkmark \checkmark$$

$$k = \frac{1}{15} \quad \checkmark$$

3. [9 marks]

Studies in Britain have recorded that 1 in 100 eight year-old children need at least one tooth removed caused by sugary drinks and severe tooth decay.

A typical primary school class of 24 eight year-olds are investigated for the need to remove at least one tooth.

Determine the probability of:

- (a) 2 students needing at least one tooth removed.

$$X \sim \text{Bin}\left(24, \frac{1}{100}\right)$$

$$P(X = 2) = 0.02213 \quad \checkmark$$

- (b) No students requiring the removal of any teeth.

$$P(X = 0) = 0.7857 \quad \checkmark$$

- (c) At least one student requiring the removal of at least one tooth.

$$P(X \geq 1) = 0.2143 \quad \checkmark$$

- (d) Less than 4 students requiring the removal of at least one tooth given that at least one student required tooth removal.

$$P(X < 4 | X \geq 1) = \frac{P(1 \leq X \leq 3)}{P(X \geq 1)} = \frac{0.2142}{0.2143} = 0.9995 \quad \checkmark$$

Of the thirteen year-olds in Britain requiring tooth removal, the probability of them requiring just one tooth out of their 32 permanent teeth removed is 5%.

- (e) Calculate the probability of a permanent tooth in a thirteen year - old needing removal.

$$Y \sim \text{Bin}(32, 0.05) \quad \checkmark$$

$$P(Y = 1) = 0.3263 \quad \checkmark$$

\checkmark

4. [13 marks]

Phoebe and Katelyn are facing a multiple choice assessment for their least favourite subject.

Marks for this test will be awarded in the following way: 4 marks will be awarded for a correct answer, 0 marks will be awarded for not attempting a question and 2 marks will be deducted for an incorrect answer.

This assessment contains 20 questions, each with four alternative answers.

Katelyn starts reading the test and is certain she knows 6 of the answers.

- (a) If Katelyn attempts all questions, what is the chance she'll answer 15 out of 20 correctly?

$$X \sim \text{Bin}(14, 0.25) \quad \checkmark$$

$$P(X = 9) = 0.00181 \quad \checkmark$$

- (b) If Katelyn attempts all questions, what is the most likely number of questions she'll answer correctly?

$$P(X = 3) = 0.2402 \quad \checkmark$$

This is the highest probability in the above defined distribution.

Therefore Katelyn is most likely to guess three correctly. Hence she is likely to answer 9 questions correctly, with the 6 she already knows. ✓

Phoebe starts reading the test and is certain she knows 10 of the answers.

She has two strategies to employ, detailed below.

Strategy A: Answer the 10 questions she knows for certain and guess the other 10.

Strategy B: Answer the 10 questions she knows for sure and guess 6 of the other 10 questions (thus not attempting 4 questions).

- (c) Calculate the expected number of correct questions if Phoebe uses strategy A.

$$Y \sim \text{Bin}(10, 0.25) \quad \checkmark$$

$$E(Y) = 10 \times 0.25 = 2.5$$

Phoebe is therefore expected to answer 12 or 13 answers correctly. ✓

- (d) Calculate the expected number of correct questions if Phoebe uses strategy B.

$$W \sim \text{Bin}(6, 0.25)$$

$$E(W) = 6 \times 0.25 = 1.5 \quad \checkmark$$

Phoebe is therefore expected to answer 11 or 12 answers correctly. ✓

- (e) Hence calculate the number of marks she can expect to be awarded with each strategy.

$$\text{Strategy A: } 12 \times 4 - 2 \times 8 = 48 - 16 = 32 \quad \checkmark \quad \text{OR } 13 \times 4 - 2 \times 7 = 52 - 14 = 38 \quad \checkmark$$

$$\text{Strategy B: } 12 \times 4 - 2 \times 4 = 48 - 8 = 40 \quad \checkmark \quad \text{OR } 11 \times 4 - 2 \times 5 = 44 - 10 = 34 \quad \checkmark$$

- (f) Which strategy should Phoebe use?

Phoebe will earn more marks using Strategy B. ✓