Mercedes College

YEAR 12 MATHEMATICS METHODS Test 2 2016

Exponential and Trigonometric Functions

NAME: ______________________ **Date: Tuesday 10th May**

TEACHER: ______________________

Non-calculator section:	33 minutes	33 marks
Calculator section:	17 minutes	17 marks
OVERALL:	50 minutes	50 marks

INSTRUCTIONS:

Show FULL working **Answer all questions on this test paper**

Questions or parts of questions worth more than two marks require working to be shown to receive full marks.

Allowed: Maths Methods WACE formula sheets

TRIG FORMULA: $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\sin 2\theta = 2\sin\theta\cos\theta$

$\lim\limits_{h\to 0}\left(\frac{1-\cos h}{h}\right) = 0$ $\lim\limits_{h\to 0}\left(\frac{\sin h}{h}\right) = 1$

Q1 **(5 marks)**

Determine the equation of the tangent to the curve $y = \frac{\sin x}{x}$ at the point $(\pi, 0)$.

$y = \frac{\sin x}{x}$

$u = \sin x \quad v = x$

$u' = \cos x \quad v' = 1$

$\frac{dy}{dx} = \frac{x\cos x - \sin x}{x^2}$ ✓

$\left.\frac{dy}{dx}\right|_{x=\pi} = \frac{\pi\cos\pi - \sin\pi}{\pi^2}$

$= \frac{\pi(-1) - 0}{\pi^2}$

$= -\frac{\pi}{\pi^2}$

$\frac{dy}{dx} = -\frac{1}{\pi}$ ✓✓

LINEAR

$y = mx + c$

$y = -\frac{1}{\pi}x + c$

subst $(\pi, 0)$

$0 = -\frac{1}{\pi}(\pi) + c$

$c = 1$ ✓

$\boxed{y = -\frac{1}{\pi}x + 1}$

Q2 **(3 + 3 + 3 + 3 = 12 marks)**

Determine $\frac{dy}{dx}$ for each of the following simplifying answers where possible.

(a) $y = e^{x^2-1} + 2\cos(2x-1) + e^3$

$$\frac{dy}{dx} = 2xe^{x^2-1} - 4\sin(2x-1)$$

✓ ✓✓

(b) $y = \sin^3 5x = (\sin 5x)^3$

$$\frac{dy}{dx} = 3(\sin 5x)^2(\cos 5x)\,5$$

$$= 15\cos 5x\,\sin^2 5x$$

✓✓✓

(c) $y = \frac{\cos x}{e^x}$

$u = \cos x \qquad v = e^x$

$u' = -\sin x \qquad v' = e^x$

$$\frac{dy}{dx} = \frac{-\sin x\, e^x - \cos x\, e^x}{e^{2x}}$$

$$= \frac{-e^x(\sin x + \cos x)}{e^{2x}}$$

$$= -\frac{(\sin x + \cos x)}{e^x}$$

✓✓✓

(d) $y = e^{(1-x)}\sin 2x$

$u = e^{1-x} \qquad v = \sin 2x$

$u' = -e^{1-x} \qquad v' = 2\cos 2x$

$$\frac{dy}{dx} = -e^{1-x}\sin 2x + 2\cos 2x\, e^{1-x}$$

$$= e^{1-x}(2\cos 2x - \sin 2x)$$

✓✓✓

Q3 **(4 + 2 + 1 = 7 marks)**
Evaluate the following.

(a) $\int_0^{\frac{\pi}{3}} (\cos\frac{x}{2} - \sin x).\,dx$

$= \left[2\sin\frac{x}{2} + \cos x\right]_0^{\frac{\pi}{3}}$

$= 2\sin\frac{\pi}{6} + \cos\frac{\pi}{3} - 2\sin 0 - \cos 0$

$= 2(\frac{1}{2}) + \frac{1}{2} - 0 - 1$

$= \frac{1}{2}$

(b) $\frac{d}{dx}(\int_3^{x^2} e^{(\sqrt{t}-1)}.\,dt)$ when $x = 2$

$= e^{\sqrt{x^2}-1}(2x)$

$= e^{x-1}(2x)$

at $x = 2$ $= e^1(4)$

$= 4e$

(c) $\lim_{h\to 0}\frac{\sin h}{2h}$

$= \frac{1}{2}\lim_{h\to 0}\frac{\sin h}{h}.$

$= \frac{1}{2}.(1)$

$= \frac{1}{2}$

Q4 **(2 + 2 + 3 + 2 = 9 marks)**

Evaluate the following integrals.

(a) $\int (e^x + \cos x + \sin x).\, dx$

$= e^x + \sin x - \cos x + c$

(b) $\int \frac{2}{e^{3x}} .\, dx$

$= \int 2e^{-3x}\, dx$

$= -\frac{2e^{-3x}}{3} + c$

(c) $\int \frac{\sin x \cos^3 x}{2} .\, dx$

$= \left(\frac{1}{-4}\right) \frac{1}{2} \int \boxed{-4 \sin x \cos^3 x\, dx}$

$= -\frac{1}{8} \cos^4 x + c$

$y = (\cos x)^4$

$\frac{dy}{dx} = -4 (\cos x)^3 \sin x$

$= -4 \sin x \cos^3 x$

(d) $\int 4 \sin x \cos x .\, dx$

$= 2 \int 2 \sin x \cos x \cdot dx$

$= 2 \int \sin(2x) \cdot dx$

$= \frac{2(-\cos 2x)}{2} + c$

$= -\cos 2x + c.$

END OF SECTION 1

SOLUTIONS

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Allowed: Maths Methods WACE formula sheets, 3 calculators, 1 A4 page of notes

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Q5 (4 marks)

A curve passes through the point $(\frac{\pi}{2}, \pi - 2)$ and has a gradient function given by $\frac{dy}{dx} = 1 - 2\cos x$. Determine the equation of the original curve.

$y = x - 2\sin x + c$ ✓

sub $(\frac{\pi}{2}, \pi - 2)$ $\pi - 2 = \frac{\pi}{2} - 2\sin(\frac{\pi}{2}) + c$ ✓

$\pi - 2 = \frac{\pi}{2} - 2(1) + c$

$\frac{\pi}{2} = c$ ✓

$y = x - 2\sin x + \frac{\pi}{2}$ ✓

Q6 **(1 + 2 + 1 + 2 + 4 = 10 marks)**

The mass of a drug remaining in the bloodstream of a patient is changing according to the rule $\frac{dM}{dt} = -0.12M$, where M is the mass of drug remaining t hours after the initial dose of 60 milligrams was administered.

(a) **Circle** the response below that best describes the type of relationship between M and t.

EXPONENTIAL GROWTH

EXPONENTIAL DECAY

✓

(b) Write down an equation for M in terms of t.

$$M = 60e^{-0.12t}$$

✓✓

(c) Determine the mass of drug remaining in the bloodstream after one day.

$$M\Big|_{t=24} = 60e^{-0.12(24)}$$

✓

$$= \underline{3.37\text{mg}}$$

(d) Determine, to the nearest hour, the time taken for less than one percent of the initial dose to remain in the bloodstream of the patient.

$$0.01 = e^{-0.12t}$$

$$t = 38.376 \text{ hrs.}$$

$$\approx 38 \text{ hrs.}$$

✓✓

(e) At what rate is the mass of the drug in the bloodstream changing

(i) after 12 hours?

$$\frac{dm}{dt} = -0.12\left(60e^{-0.12\times 12}\right)$$

$$= -1.706 \text{ mg/hr}$$

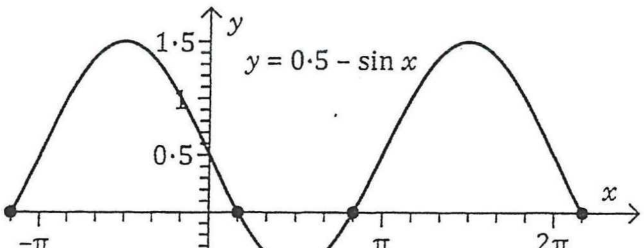
(ii) when 25mg of the drug remains?

$$\frac{dm}{dt} = -0.12 \times 25$$

$$= -3 \text{ mg/hr.}$$

Q7 **(3 marks)**

A section of the graph of the function $y = 0.5 - \sin x$ is shown below. Calculate the **enclosed area** between the function stated and the x axis as shown in the diagram.



$$\int_{-\frac{7\pi}{6}}^{\frac{13\pi}{6}} |0.5 - \sin x| \, dx$$

$$= 8.338 \text{ units}^2$$

END OF SECTION 2