PERTH MODERN SCHOOL
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**Independent Public School**

# Course ___Methods_Test 2_ Year __12________

Student name: _____________ Teacher name: _____________

Date: 30 March

**Task type:** **Response**

**Time allowed for this task: _____45______ mins**

**Number of questions: _____9______**

**Materials required:** Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

**Marks available: __46____ marks**

**Task weighting: __10__%**

**Formula sheet provided: Yes**

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

Q1 (3.2.1-3.2.3) (3 & 3 = 6 marks)

Determine $y$ in terms of $x$ for the following.

a) $\frac{dy}{dx}=5x^3-\frac{2}{x^2}$ given that $y=10$ when $x=2$.

| Solution |
| --- |
| $\frac{dy}{dx}=5x^3-\frac{2}{x^2}=5x^3-2x^{-2}$<br>$y=\frac{5x^4}{4}+2x^{-1}+c$<br>$10=\frac{5(16)}{4}+1+c$<br>$c=-11$<br>$y=\frac{5x^4}{4}+2x^{-1}-11$ |
| **Specific behaviours** |
| ✓ uses negative indices<br>✓ anti-differentiates<br>✓solves for constant |

b) $\frac{dy}{dx}=\frac{50x^2}{\left(5-x^3\right)^5}$ given that $y=100$ when $x=2$.

| Solution |
| --- |

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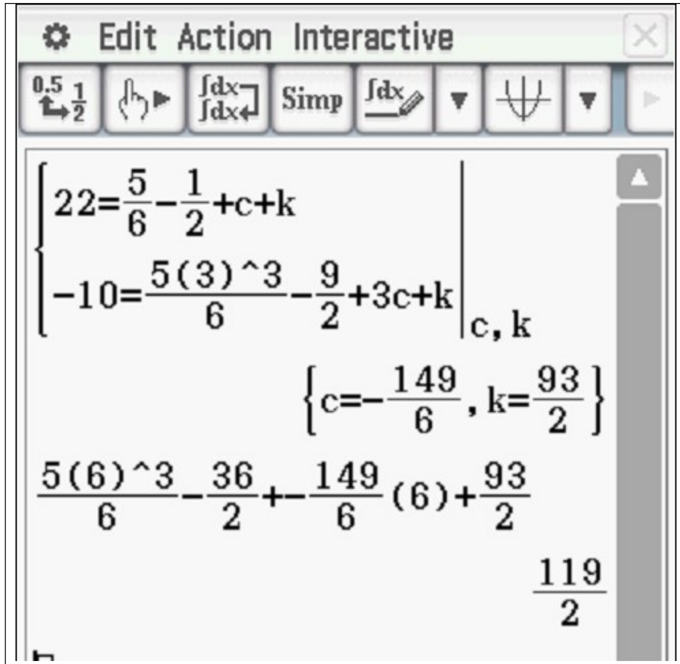
| |
|---|
| $\frac{dy}{dx}=\frac{50x^2}{(5-x^3)^5}$<br>$y=A(5-x^3)^{-4}+c$<br>$y'=-4A(5-x^3)^{-5}(-3x^2)$<br>$50=12A$<br>$A=\frac{25}{6}$<br>$100=\frac{25}{6}(-3)^{-4}+c$<br>$c=\frac{48575}{486}\approx 99.948...$<br>$y=\frac{25}{6}(5-x^3)^{-4}+\frac{48575}{486}$ |
| **Specific behaviours** |
| ✓ recognises that numerator is proportional to derivative of brackets<br>✓ solves for multiplier constant<br>✓solves for added constant, accept approx |

Q2 (3.2.21-3.2.22) (4 marks)

A particle travels along a straight line such that its acceleration at time $t$ seconds is equal to $(5t-1)\,m/s^2$. When $t=1$ the displacement is 22 metres and when $t=3$
The displacement is -10 metres. Determine the displacement when $t=6$.

| **Solution** |
|---|
| $a=(5t-1)$<br>$v=\frac{5t^2}{2}-t+c$<br>$x=\frac{5t^3}{6}-\frac{t^2}{2}+ct+k$ |




| Specific behaviours |
|---|
| ✓ determines velocity function<br>✓ determines displacement function with two constants<br>✓ solves for both constants<br>✓determines displacement at t=6 |





Q9 (3.2.6) (6 marks)

(a) Determine $\frac{d}{dx}\left(x(x+1)^{\frac{1}{3}}\right)$.

| Solution |
|---|
| $\frac{d}{dx}\left(x(x+1)^{\frac{1}{3}}\right)=x\frac{1}{3}(x+1)^{-\frac{2}{3}}+(x+1)^{\frac{1}{3}}$ |
| **Specific behaviours** |
| ✓ uses product rule correctly<br>✓ determines derivative |

(b) Using your result from part (a) and without using your classpad determine $\int\frac{x}{3(x+1)^{\frac{2}{3}}}dx$

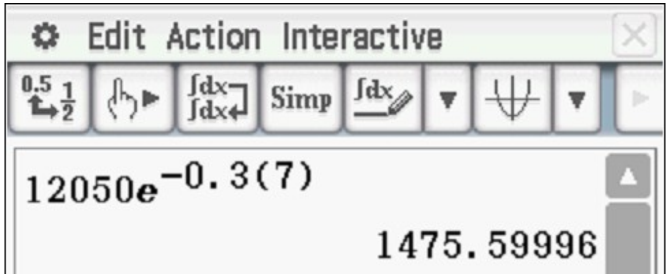
| Solution |
|---|
| $\int\frac{d}{dx}\left(x(x+1)^{\frac{1}{3}}\right)dx=\int x\frac{1}{3}(x+1)^{-\frac{2}{3}}dx+\int(x+1)^{\frac{1}{3}}dx$<br>$\int\frac{x}{3(x+1)^{\frac{2}{3}}}dx=x(x+1)^{\frac{1}{3}}-\frac{3}{4}(x+1)^{\frac{4}{3}}+c$ |
| **Specific behaviours** |
| ✓ Uses linearity of antidifferentiation<br>✓ uses Fundamental Theorem of Calculus<br>✓integrates (x+1)1/3 term correctly<br>✓Determines integral with a constant |

b) $\frac{d^2y}{dx^2}$

| Solution |
|---|
| $3x^2 + 15(1+4e^{2x})^4 8e^{2x}$ |
| **Specific behaviours** |
| ✓ uses chain rule correctly<br>✓ determines derivative |

Q8 (3.1.4) (4 marks)

The instantaneous rate of decline in the number of kangaroos on a particular park is 30% of the population per year. If there were 12 050 kangaroos on the park 3 years ago, how many will be on the park in four years from now

| Solution |
|---|
| $\frac{dN}{dt} = -0.3N$<br>$N = 12050e^{-0.3t}$ |
|  |
| **Specific behaviours** |
| ✓ recognizes exponential decay<br>✓ uses correct model of rule<br>✓ uses correct parameters (both)<br>✓determines final population ( no need to round) |

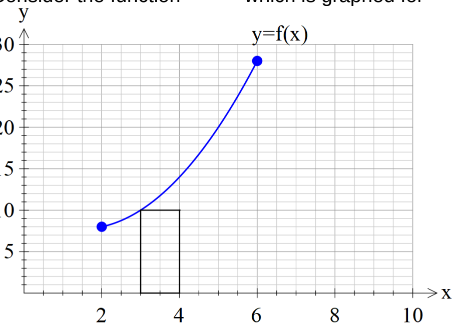






Q3 (3.2.10-3.2.11) (2. 2, 1 & 2 = 7 marks)

Consider the function $f(x)$ which is graphed for $2 \le x \le 6$.




a) By using rectangles of width one unit, as shown above, determine a lower estimate for the area under $f(x)$ for $2 \le x \le 6$.

| Solution |
|---|
| $8\times1 + 10\times1 + 14\times1 + 20\times1 = 52$ accept(50 to 54) |
| **Specific behaviours** |
| ✓ uses y intercepts from the left of each rectangle<br>✓ determines sum of areas |

b) By using rectangles of width one unit, as shown above, determine an upper estimate for the area under $f(x)$ for $2 \le x \le 6$.

| Solution |
|---|
| $10\times1 + 14\times1 + 20\times1 + 28\times1 = 72$ accept (70 to 75) |
| **Specific behaviours** |
| ✓ uses y intercepts from the right of each rectangle<br>✓ determines sum of areas |

c) Determine a better approximation for the area under $f(x)$ for $2 \le x \le 6$.

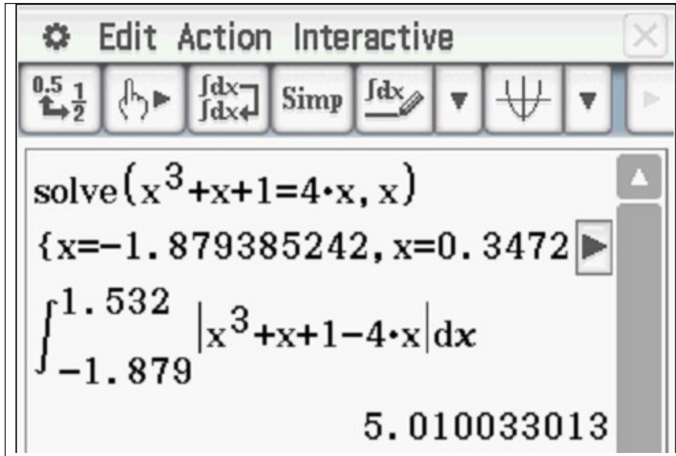
| Solution |
|---|
| $\frac{52+72}{2} = 62$ |
| **Specific behaviours** |
| ✓ determines average |

d) Describe two different methods to improve the approximation for the area under $f(x)$ for $2 \le x \le 6$.

| Solution |
|---|
| Use rectangles of smaller widths<br>Use calculus with an accurate rule for function<br>Model parabolas for the top of each rectangle and then integrate<br>(Note: Trapezium method is the same as averaging upper & lower rectangles therefore do NOT accept) |
| **Specific behaviours** |
| ✓ at least one appropriate method<br>✓ at least two appropriate methods |








| |
|---|
| Area = 5.01 sq units |
| **Specific behaviours** |
| ✓ determines points of intersection<br>✓ uses integral with difference between functions OR sets up integral from<br>✓ uses integrals with absolute values<br>✓determines area no need to round to 2 dp |

Q7 (3.2.16) (2 & 2 = 4 marks)

Consider $y = \int_0^x t^3 + 3\left(1+4e^{2t}\right)^5 dt$

Determine.

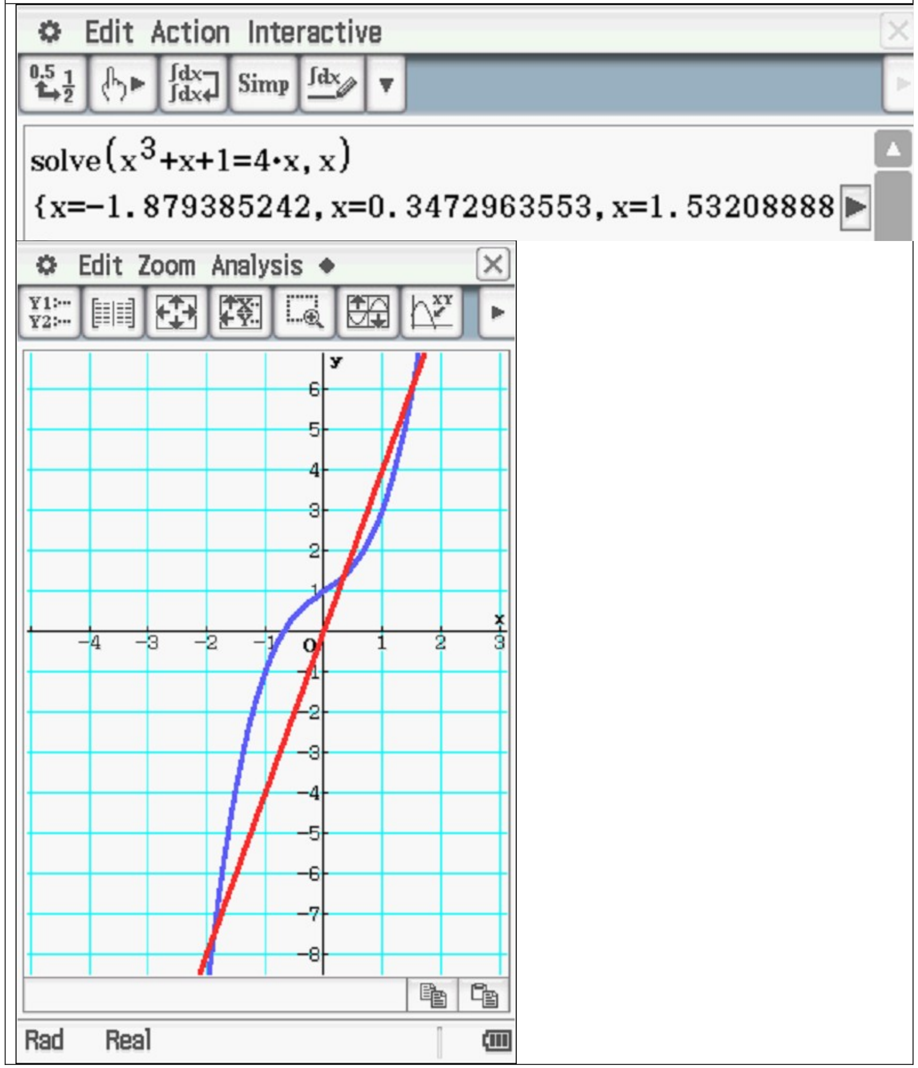
a) $\frac{dy}{dx}$

| Solution |
|---|
| $\frac{d}{dx}\int_0^x t^3 + 3\left(1+4e^{2t}\right)^5 dt = x^3 + 3(1+4e^{2x})^5$ |
| **Specific behaviours** |
| ✓ uses fundamental theorem<br>✓ determines derivative in terms of x |

Q6 (3.2.20) (4 marks)

Determine to two decimal places the area between the curves $y = x^3 + x + 1$ and $y = 4x$.
(Hint- Sketch the curves first on your classpad)

**Solution**





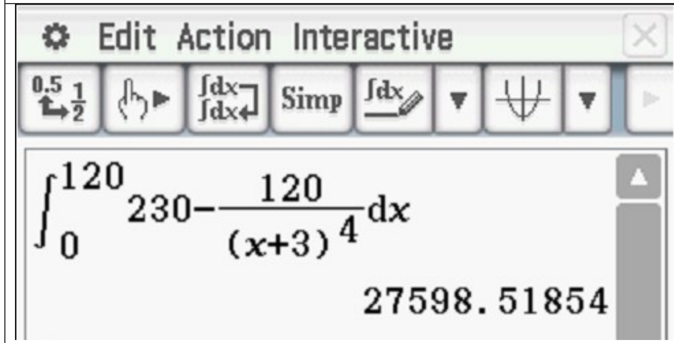



Q4 (3.2.18-3.2.17) (3 & 2 = 5 marks)

An oil tank is drained of oil such that if $V\ kL$ of oil in the tank $t$ seconds after draining commences is described by $\frac{dV}{dt} = 230 - \frac{120}{(t+3)^4}$.
The initially full tank is emptied in 2 mins.

a) How much oil was in the full tank? (nearest kL)

**Solution**



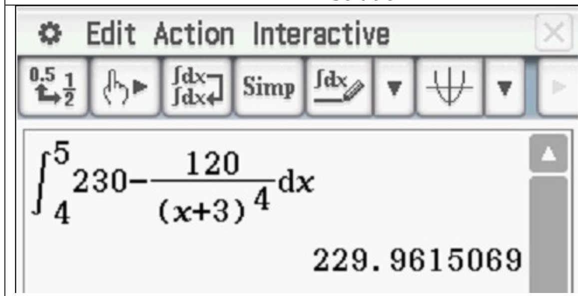

27599 KL

**Specific behaviours**

✓ uses an integral OR anti-differentiates using 0 to 120 seconds
✓ determines change
✓ rounds change to nearest KL (no need to state units)

b) How much oil was drained from the tank in the fifth second, nearest kL.

**Solution**




230 KL

**Specific behaviours**

✓ sets up integral with correct limits OR uses antiderivative with correct limits
✓ states units with answer ( no need for nearest KL)

Q5 (3.2.11-3.2.14) (2, 2 & 2 = 6 marks)

Consider a function $f(x)$ which is only defined for $-5 \le x \le 7$ with

$f(-5)=0=f(0)=f(7)$

$f(-4)=8$

$f(-1)=11$

$\int_{-5}^{0} f(x)dx = 22$

$\int_{-5}^{7} f(x)dx = -43$

It is known that $f(x) \ge 0$ for $-5 \le x \le 0$ and $f(x) \le 0$ for $0 < x \le 7$.
Determine.

a) $\int_{-4}^{-1} f'(x)dx$

| Solution |
| --- |
| $\int_{-4}^{-1} f'(x)dx = \left[f(x)\right]_{-4}^{-1} = f(-1) - f(-4)$<br>$= 11 - 8 = 3$ |
| **Specific behaviours** |
| ✓ uses fundamental theorem<br>✓ evaluates integral |

b) $\int_{0}^{7} f(x)dx$

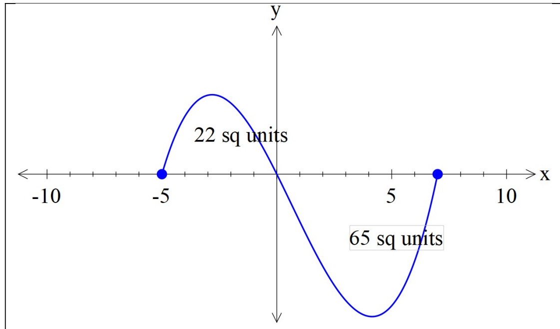
| Solution |
| --- |
| $\int_{-5}^{7} f(x)dx = \int_{-5}^{0} f(x)dx + \int_{0}^{7} f(x)dx$<br>$-43 = 22 + \int_{0}^{7} f(x)dx$<br>$\int_{0}^{7} f(x)dx = -65$ |
| **Specific behaviours** |
| ✓ uses linearity principle<br>✓ solves for required integral |

c) The area between $y = f(x)$ and the x axes for $-5 \le x \le 7$.

| Solution |
| --- |
| |








Area= 22 + 65=87 sq units

| Specific behaviours |
| --- |
| ✓ uses absolute value of each integral<br>✓ determines area |