SEMESTER TWO

MATHEMATICS METHODS UNITS 1 & 2

2019

SOLUTIONS

Calculator-free Solutions

1. (a) $2^{-2} \times a^{-6} \times b^{2}$ ✓

$= \dfrac{b^2}{4a^6}$ ✓

(b) $\dfrac{3^{-3} \times x^9 \times y^{-6}}{9^{-2} \times x^4 \times y^{-4}}$ ✓

$= \dfrac{81 \times x^5}{27y^2}$ ✓

$= \dfrac{3x^5}{y^2}$ ✓ [5]

2. (a) $2^{2-2x} = 2^{3-3x}$ ✓

$\therefore\ 2 - 2x = 3 - 3x \rightarrow x = 1$ ✓

(b) $3^{x^2+1} = 3^{x+3}$ ✓

$\therefore\ x^2 + 1 = x + 3$ ✓

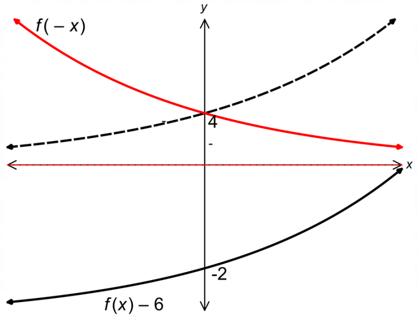
$\therefore\ x^2 - x - 2 = 0 \rightarrow (x-2)(x+1) = 0$ ✓

$\therefore\ x = 2$ or $x = -1$ ✓ [6]

3. (a) (i) $m = 4$ ✓

(ii) $f(-2) = 4a^{-2} = \dfrac{4}{a^2}$ ✓

(b)



✓✓✓✓ [6]

4. (a) (i) $\dfrac{dy}{dx} = 3x^3 + 5x^4$ ✓✓

(ii) $f(x) = \dfrac{x}{3} + \dfrac{2x^2}{\pi}$

$\therefore\ f'(x) = \dfrac{1}{3} + \dfrac{4}{\pi}x$ ✓✓

(b) $-3x^2$ ✓ [5]

5. (a) (i) $y(1) = -4$ and $y(3) = -18$ ✓

$\therefore$ Av rate of change $= \dfrac{-18-(-4)}{2} = -7$ ✓

(ii) $y' = 3t^2 - 10t$ ✓

$\therefore$ $y'(2) = -8$ ✓

(b) $\delta y = (3t^2 - 10t) \times 2$ ✓

$\therefore$ $\delta y = (-7) \times 2 = -14$ ✓

(c) $\dfrac{dy}{dt} = -2 \rightarrow y = -8t + c$ ✓

$\therefore$ $(2, -12) \rightarrow -12 = -16 + c \rightarrow c = 4$ ✓

$\therefore$ $y = -8t + 4$ ✓ [9]

6. (a) (i) $t = 1.5$ ✓

(ii) $t = 0.5$ ✓

(b) $x(t) = a(t-0.5)^2 + 2.5$ ✓

$(0, 2) \rightarrow 2 = a(0.25) + 2.5 \rightarrow a = -2$

$\therefore$ $x(t) = -2(t-0.5)^2 + 2.5$ ✓ [4]

7. (a) $\dfrac{dy}{dx} = 2x - \pi^2 + \dfrac{3}{5}x^2$

$\therefore$ $y = x^2 - \pi^2 x + \dfrac{1}{5}x^3 + c$ ✓✓

(b) $f'(n) = \dfrac{1}{3}n^2 - \dfrac{4}{3}n$ ✓

$\therefore$ $f(n) = \dfrac{1}{9}n^3 - \dfrac{2}{3}n^2 + c$ ✓✓ [5]

8. (a) $T_{n+1} = T_n + 10$ where $T_1 = 100$ ✓

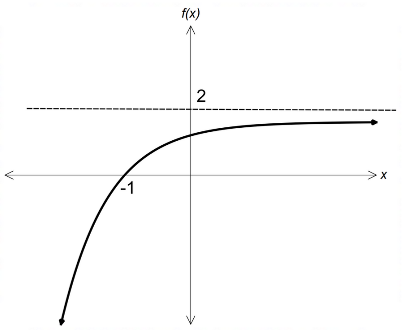
(b) $T_n = 100 + (n-1)(10) = 10n + 90$ ✓

(c) $T_8 = 10(8) + 90 = 170$ km ✓✓

(d) $S_{12} = 6(200 + 110) = 6(310)$ ✓

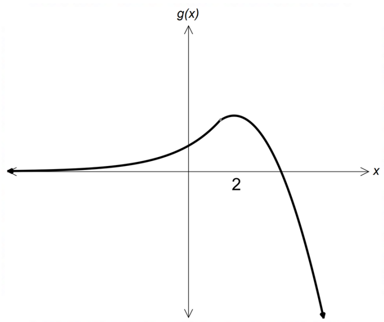
$\therefore = 1860$ km ✓ [6]

9. (a)



✓✓✓

(b)



✓✓✓ [6]

21. (a) $g'(x) = 4x^3 - 16x$ ✓

$g'(1) = -12$ ✓

$\therefore \quad y = -12x + c$

$(1, 9) \rightarrow 9 = -12(1) + c \rightarrow c = 21$ ✓

$\therefore \quad y = -12x + 21$ ✓

(b) (1.45, 3.61) and (-3.45, 62.4) ✓✓ [6]

22. (a)

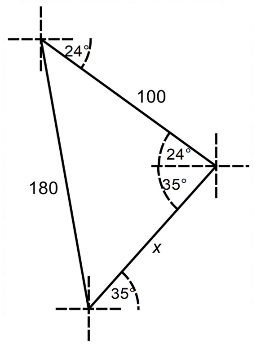
$$\frac{(\sin a\cos 45° + \sin 45°\cos a)(\cos a\cos 45° - \sin 45°\sin a)}{(\sin a - \cos a)(\sin a + \cos a)}$$ ✓

$$= \frac{\left(\frac{\sqrt{2}}{2}\sin a + \frac{\sqrt{2}}{2}\cos a\right)\left(\frac{\sqrt{2}}{2}\cos a - \frac{\sqrt{2}}{2}\sin a\right)}{(\sin a - \cos a)(\sin a + \cos a)}$$ ✓

$$= \frac{\frac{\sqrt{2}}{2}(\sin a + \cos a)\frac{\sqrt{2}}{2}(\cos a - \sin a)}{(\sin a - \cos a)(\sin a + \cos a)}$$ ✓

$$= \left(\frac{2}{4}\right)(-1) = -\frac{1}{2}$$ ✓

(b)



✓

$180^2 = 100^2 + x^2 - 2x(100)\cos 59°$ ✓✓

$\therefore \quad x = 210$ km ✓ [8]

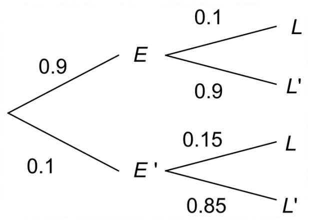
23.

$$\left(\sin x + \frac{1}{\sin x}\right)^3$$

$$= \sin^3 x + 3(\sin^2 x)\left(\frac{1}{\sin x}\right) + 3(\sin x)\left(\frac{1}{\sin x}\right)^2 + \left(\frac{1}{\sin x}\right)^3$$ ✓✓

$$= \frac{1}{\sin^3 x}(\sin^6 x + 3\sin^4 x + 3\sin^2 x + 1)$$ ✓ [3]

18. (a)



(b) (i) $P(L) = 0.9 \times 0.1 + 0.1 \times 0.15 = 0.105$ ✓✓ ✓✓

(ii) $P(L'|E') = 0.85$ ✓

(iii) $P(E|L) = \dfrac{0{\cdot}9 \times 0{\cdot}1}{0{\cdot}105}$ ✓

$= 0.857$ ✓ [7]

19. $a = 0.5$ ✓

$1.5T = 2\pi \rightarrow T = \dfrac{4\pi}{3}$ ✓

$\therefore \quad b = \dfrac{3}{2}$ ✓

$c = \dfrac{\pi}{4}$ ✓

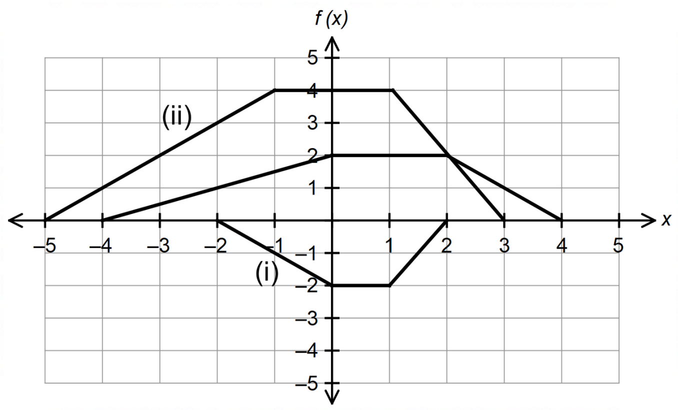
$d = -2$ ✓

$e = -1$ ✓ [6]

20. (a) (i) $x = \pm 4$ ✓

(ii) 2 ✓

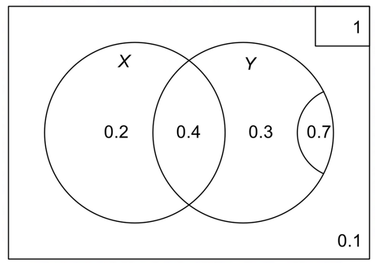
(b)



✓✓✓✓ [6]

Calculator-Assumed Solutions

10. (a)



(b) (i) 0.6 ✓✓ ✓

(ii) $\dfrac{0.6}{0.9} = \dfrac{2}{3}$ ✓✓

(c) $P(X|Y) = \dfrac{0.4}{0.7}$ and $P(X) = 0.6$ ✓

$\therefore$ not independent since $P(X) \neq P(X|Y)$ ✓

or $P(X) \times P(Y) \neq P(X \cap Y)$ [7]

11. (a) $| r | < 1$ ✓

(b) $ar = 20$ and $\dfrac{a}{1-r} = 125$ ✓

$\therefore \quad a = 25$ and $r = 0.8$ or $a = 100$ and $r = 0.2$ ✓✓

(c) $S_{10} = \dfrac{25(1-0.8^{10})}{1-0.8} = 111.6$ ✓

or $S_{10} = \dfrac{100(1-0.2^{10})}{1-0.2} = 125.0$ ✓ [6]

12. (a) (i) (c, 0) , (g, 0) , (k, 0) ✓✓

(ii) (e, f) , (i, j) ✓✓

(iii) (a, b) ✓

(b) a ≤ x < e , g < x < i , k < x ≤ m ✓✓

(c) e < x < g , i < x < k ✓✓

(d) k = 2 ✓ [10]

13. (a) 0.98 means reducing by 2% ✓

(b) $N(12) = 3500(0{\cdot}98)^{12} = 2747$ rats ✓✓

(c) $3500(0.98)^t = 2000$ ✓

$\therefore \quad t = 27.7 \rightarrow$ April 2021 ✓

(d) $3200 = 2000r^{12} \rightarrow r = 1.04$ ✓

$\therefore \quad R(t) = 2000(1.04)^t$ ✓

(e) $3500(0.98)^t = 2000(1.04)^t$ ✓✓

$\therefore \quad t = 9.4$

$\therefore$ October 2019 ✓ [10]

14. (a) 10L ✓

(b) $W'(t) = 0.8t^3 - 1.8t^2 - t$ ✓

$\therefore$ $0.8t^3 - 1.8t^2 - t = 0$ when $t = 2.7$ ✓

$\therefore$ During the third minute. ✓

(c) (i) W(2.7) = 5.2 L ✓

(ii) W(4) = 14.8 L ✓ [6]

15. (a) (i) -15 m ✓

(ii) $v(t) = t^2 - 2t - 4$ ✓✓

(b) $v(3) = -1$ ✓

$\therefore$ Speed = 1 m/s ✓

(c) At rest when $v(t) = 0$

$\therefore$ $t^2 - 2t - 4 = 0$ ✓

$\therefore$ $t = 3.2$ s ✓

(d) $x(0) = -3$

$x(3.2) = -15.12$ ✓

$x(5) = -6.33$ ✓

$\therefore$ Distance travelled = 12.12 + 8.79 = 20.91 m ✓ [10]

16. (a) $c = 5$ ✓

(b) $(-2, -3) \rightarrow -3 = -8 + 4a - 2b + 5$

$\therefore$ $4a - 2b = 0$ ✓

$\frac{dy}{dx} = 3x^2 + 2ax + b = 0$ when $x = -2$

$\therefore$ $4a - b = 12$ ✓

$\therefore$ $a = 6$ and $b = 12$ ✓✓

(c) $y = x^3 + 6x^2 + 12x + 5$

$\therefore$ $x^3 + 6x^2 + 12x + 5 = 0$ when $x = -0.558$ ✓✓ [7]

17. (a) $\frac{dy}{dx} = 15x^4 - 30ax^2 + 15a^2$

$\therefore$ $y = \frac{15x^5}{5} - \frac{30ax^3}{3} + 15a^2x + c$ ✓✓

$\therefore$ $(0, 0) \rightarrow c = 0$ ✓

$\therefore$ $y = 3x^5 - 10ax^3 + 15a^2x$

(b) $\frac{dy}{dx} = 15(x^2 - a)^2 = 0$ when stationary ✓

$\therefore$ $(x^2 - a)^2 = 0$

$\therefore$ $x^2 = a$ ✓

Since two stationary points $a > 0$ ✓ [6]