

Important note to candidates
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Special items: nil

To be provided by the candidate
Standard items: correction fluid/tape, eraser, ruler, highlighters
Pens (blue/black preferred), pencils (including coloured), sharpener,

To be provided by the supervisor
Formula sheet
This Question/Answer booklet

Materials required/recommended for this section
Working time: fifty minutes
Reading time before commencing work: five minutes

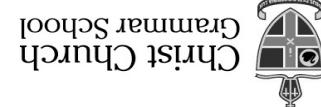
Student name:

SOLUTIONS

Question/Answer booklet

Semester 2 Examination, 2019

Year 1 Mathematics Methods
Mathematics Department



Section One:
UNITS 1 AND 2
METHODS
MATHEMATICS

Calculator-free

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

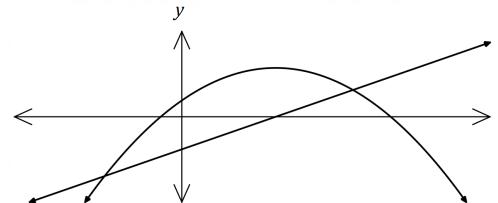
Instructions to candidates

- The rules for the conduct of Christ Church Grammar School assessments are detailed in the Reporting and Assessment Policy. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Question 21

(6 marks)

The graphs of $y = f(x)$ and $y = g(x)$ are shown below where $f(x) = 1 + 4x - 2x^2$ and $g(x) = 2x + k$.



Determine the value(s) of the constant k so that the equation $f(x) = g(x)$ has

- (a) one solution. (5 marks)

Solutions	
g must be a tangent to f : $f'(x) = 4 - 4x$ $= 2$ when $x = \frac{1}{2}$ y-coordinate of point of tangency: $f\left(\frac{1}{2}\right) = 1 + 4\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right)^2 = \frac{5}{2}$ Equation of tangent: $y - \frac{5}{2} = 2\left(x - \frac{1}{2}\right)$ $y = 2x + \frac{3}{2}$ Hence $k = \frac{3}{2} = 1.5$	$1 + 4x - 2x^2 = 2x + k$ $2x^2 - 2x + (k - 1) = 0$ For one solution, $\Delta = 0$ $b^2 - 4ac = 0$ $4 - 4(2)(k - 1) = 0$ $k - 1 = 0.5$ $\therefore k = 1.5$
Specific behaviours	
✓ indicates tangent required ✓ determines x -coordinate of point of tangency ✓ determines y -coordinate of point of tangency ✓ equation of tangent ✓ states correct value of k	✓ equates $f(x)$ and $g(x)$ ✓ generates quadratic equation with RHS being 0 ✓ states discriminant = 0 ✓ sets up equation in k ✓ solves for k

- (b) no solutions. (1 mark)

Solution	
$k > 1.5$	$\Delta < 0$ $4 - 8(k - 1) < 0$ $\therefore k > 1.5$
Specific behaviours	
✓ correct inequality	

METHODS UNITS 1 AND 2		CALCULATOR-FREE		METHODS UNITS 1 AND 2	
Question 20		Section One: Calculator-free		Section Two: Calculator-assumed	
(a) Show that the probability that $n = 3$ is $\frac{25}{216}$. A fair six-sided die number 1, 2, 3, 4, 5 and 6 is thrown n times until it lands on a 6.	(1 mark)	$P(n=3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$ Solution	Specific behaviours ✓ shows product of three fractions	$P(n=3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$ Solution	Specific behaviours ✓ shows product of three fractions
This section has eight (8) questions. Answer all questions. Write your answers in the spaces provided. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.	(8 marks)	Working time: 50 minutes.			
Determine the gradient of the curve $y = x^2 - 4x - 60$ at the point(s) where it crosses the x -axis. Question 1	(5 marks)	Determine the gradient of the curve $y = x^2 - 4x - 60$ at the point(s) where it crosses the x -axis. Question 1	(1 mark)	$P(n=5) = \frac{5}{6} \times \frac{1}{6} = \frac{625}{7776} \approx 0.0804$ Solution	Specific behaviours ✓ correct probability
Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.				$P(n=5) = \frac{5}{6} \times \frac{1}{6} = \frac{625}{7776} \approx 0.0804$ Solution	Specific behaviours ✓ correct probability
Write an expression in terms of n for the probability that the first 6 is thrown on the n th throw and explain why the probabilities form a geometric sequence. Question 2	(2 marks)	Determine the probability that the first 6 is thrown in 12 or less attempts. Question 2	(2 marks)	$p = \frac{1}{5}^{n-1}$ or The expression takes the form of $a(r)^{n-1}$. Solution	Specific behaviours ✓ correct expression
At $(-6, 0)$ gradient is -16 and at $(10, 0)$ gradient is 16 .		$x = 10, \frac{dy}{dx} = 16$ $x = -6, \frac{dy}{dx} = -16$ $\frac{dy}{dx} = 2x - 4$ $(x+6)(x-10) = 0$ Solution		$p = \frac{1}{5}^{n-1}$ or There exists a common ratio $\left(\frac{1}{5}\right)$. Solution	Specific behaviours ✓ correct expression
$\frac{dy}{dx} = 2x - 4$ $(x+6)(x-10) = 0$ Solution		$x = -6, x = 10$ $\frac{dy}{dx} = 2x - 4$ $x = -6, \frac{dy}{dx} = -16$ $x = 10, \frac{dy}{dx} = 16$ At $(-6, 0)$ gradient is -16 and at $(10, 0)$ gradient is 16 . Solution		$At (-6, 0) \text{ gradient is } -16 \text{ and at } (10, 0) \text{ gradient is } 16.$ Solution	Specific behaviours ✓ factorises quadratics ✓ determines roots ✓ derivative of quadratic ✓ one point and gradient ✓ second point and gradient
(e) Determine the probability that the first 6 is thrown in k or less attempts must be at least 99%. Question 3	(2 marks)	Determine the last value of integer k such that the probability of throwing a 6 in k or less attempts is at least 99%. Question 3	(2 marks)	$0.99 = \frac{1}{6} \left(1 - \left(\frac{5}{6}\right)^k\right) \Leftrightarrow k = 25.3$ Solution	Specific behaviours ✓ solves for n ✓ correct value of k

Question 2

(4 marks)

The line segment between the points $A(-1, -2)$ and $B(-1, 8)$ is the diameter of a circle.

Determine the equation of the circle in the form $x^2 + ax + y^2 + by = c$, where a, b and c are constants.

Solution
Centre: $(-1, \frac{-2+8}{2}) = (-1, 3)$
Radius: $r = 8 - 3 = 5$
Equation: $(x + 1)^2 + (y - 3)^2 = 5^2$
$x^2 + 2x + 1 + y^2 - 6y + 9 = 25$
$x^2 + 2x + y^2 - 6y = 15$
Specific behaviours
✓ centre ✓ radius ✓ factored equation ✓ correct equation

Question 19

(8 marks)

Two events A and B are such that $P(A \cap \bar{B}) = x$, $P(A) = 0.2$ and $P(\bar{A} \cap B) = 0.6$.

- (a) Determine $P(A \cap B)$ when $x = 0.12$.

(2 marks)

Solution
$P(A \cap B) = P(A) - P(A \cap \bar{B})$
$= 0.2 - 0.12$
$= 0.08$
Specific behaviours
✓ use of Venn diagram or other method ✓ correct probability

- (b) Determine an expression for $P(A \cap B)$ in terms of x .

(1 mark)

Solution
$P(A \cap B) = 0.2 - x$
Specific behaviours
✓ correct expression

- (c) Determine the value of x when

(1 mark)

- (i) A and B are mutually exclusive.

Solution
$P(A \cap B) = 0.2 - x = 0 \Rightarrow x = 0.2$
Specific behaviours
✓ correct value

- (ii) A and B are independent.

(2 marks)

Solution
$P(A \cap B) = P(A) \times P(B)$
$0.2 - x = 0.2 \times (0.6 + 0.2 - x)$
$x = 0.05$
Specific behaviours
✓ uses rule for independence ✓ correct value

- (iii) $P(A|B) = 0.04$.

(2 marks)

Solution
$P(A B) = \frac{P(A \cap B)}{P(B)}$
$0.04 = \frac{0.2 - x}{0.6 + 0.2 - x}$
$x = 0.175$
Specific behaviours
✓ uses conditional probability rule ✓ correct value

Question 18
(9 marks)

A right circular cone of base radius 10 cm and volume $V \text{ cm}^3$ stands on a horizontal surface. A cylinder of radius x cm and volume $V \text{ cm}^3$ stands inside the cone with its axis coincident with the cone and such that the cylinder touches the curved surface of the cone as shown.

of the cone and such that the cylinder touches the curved surface of the cone as shown.
A right circular cone of base radius 10 cm and volume $V \text{ cm}^3$ stands on a horizontal surface. A cylinder of radius x cm and height 25 cm stands inside the cone with its axis coincident with the cone and such that the cylinder touches the curved surface of the cone as shown.

(7 marks)

(a) Solve the following exponential equations.

$$(i) \quad 25x = \sqrt[5]{125}$$

Solution	
<i>specific behaviours</i>	
$5^2x = 5^{-2.5}$	
$2x = -2.5$	
$x = -1.25 = -\frac{5}{4}$	
<i>correct solution</i>	

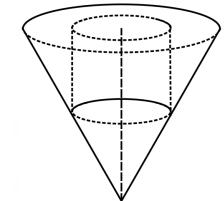
(b) Solve the following exponential equations.

$$(ii) \quad \sqrt[9]{x^2+1} = 3^{x+3}$$

(3 marks)

Solution	
<i>specific behaviours</i>	
$3^{2x^2+2} = 3^{x+3}$	
$2x^2 + 2 = x + 3$	
$x^2 - x - 2 = 0$	
$(x + 1)(x - 2) = 0$	
$x = -1 \text{ or } 2$	
<i>writes LHS in base 3</i>	
<i>simplices index in LHS</i>	
<i>generates quadratic equation</i>	
<i>correct solution</i>	

(3 marks)



Solution	
<i>specific behaviours</i>	
$\frac{dV}{dx} = 50\pi x - 7.5\pi x^2$	
$\frac{dV}{dx} = 0 \text{ when } x = 0, x = \frac{20}{3}$	
$x = 0 \Leftrightarrow V = 0 \text{ (minimum)}$	
$x = \frac{20}{3} \Leftrightarrow V = \frac{27}{27} \approx 1164 \text{ cm}^3 \text{ (maximum)}$	
<i>derivative</i>	
<i>equates derivative to 0</i>	
<i>solves for x</i>	
<i>states maximum volume</i>	

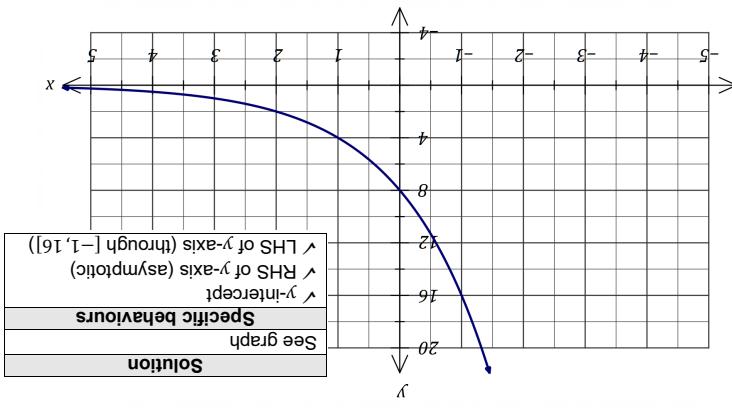
(a) Show that the volume of the cylinder, $V = 25\pi x^2 - 2.5\pi x^3$.

Solution	
<i>specific behaviours</i>	
$V = \pi r^2 h$	
$V = \pi x^2(25 - 2.5x)$	
$= 25\pi x^2 - 2.5\pi x^3$	
<i>relates h in terms of x</i>	
<i>substitutes h into cylinder volume formula</i>	

(b) Given that x can vary, use a calculus method to determine the maximum value of V .

(c) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(3 marks)



(3 marks)

(d) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(e) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(f) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(g) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(h) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(i) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(j) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(k) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(l) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(m) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(n) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(o) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(p) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(q) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(r) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(s) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(t) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(u) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(v) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(w) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(x) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(y) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(z) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(aa) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

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(gg) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(hh) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(ii) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(jj) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(kk) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(4 marks)

(ee) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

(4 marks)

(ff) Sketch the graph of $y = 2^{(3-x)}$ on the axes below.

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(7 marks)

Question 4

- (a) A small body A is moving along a straight line so that at any time t seconds, its displacement relative to a fixed point O on the line is given by $x = 2t^3 - 9t^2 + 1$ cm.

- (i) Determine the velocity of A when $t = 1$.

(2 marks)

Solution	
$v = \frac{dx}{dt} = 6t^2 - 18t$	
$v(10) = 6(1)^2 - 18(1)$ = -12 cm/s	
Specific behaviours	
✓ expression for velocity ✓ correct velocity	

- (ii) Determine the displacement of A relative to O at the instant(s) that it is stationary.

(3 marks)

Solution	
$6t^2 - 18t = 0$	
$6t(t - 3) = 0$	
$t = 0, t = 3$	
$x(0) = 1$ cm, $x(3) = -26$ cm	
Specific behaviours	
✓ factorises velocity ✓ one correct displacement ✓ both correct displacement	

- (b) A small body B has velocity given by $v = 6t^2 - 4t - 2$ cm/s and when $t = 2$ it has a displacement of 6 cm relative to O .

- Determine an expression for the displacement of B relative to O at any time t . (2 marks)

Solution	
$\frac{dx}{dt} = 6t^2 - 4t - 2$	
$x = 2t^3 - 2t^2 - 2t + c$	
$c = 6 - (16 - 8 - 4) = 2$	
$x = 2t^3 - 2t^2 - 2t + 2$	
Specific behaviours	
✓ antidifferentiates ✓ correct expression	

- (c) Use a calculus method to determine the exact coordinates of the local minimum of the graph of $y = f(x)$. (3 marks)

Solution	
$f'(x) = 3x^2 + 6x - 9$	
$f'(x) = 0 \Rightarrow x = -3, 1$	
$f(1) = -32$	
Local minimum at $(1, -32)$	
Specific behaviours	
✓ shows $f'(x)$ ✓ shows $f'(x) = 0$ and solutions ✓ correct coordinates	

- (d) Determine the coordinates of the point where the tangent to $y = f(x)$ at $(0, -27)$ intersects the curve $y = f(x)$, other than at the point of tangency. (3 marks)

Solution	
$f'(0) = -9$	
Tangent: $y = -9x - 27$	
$x^3 + 3x^2 - 9x - 27 = -9x - 27$	
$x = 0, x = -3$	
Intersects at $(-3, 0)$	
Specific behaviours	
✓ equation of tangent ✓ equates tangent to curve and solves ✓ correct coordinates	

(2 marks)

Solution

Determine the minimum possible value of each of the constants.

Specific behaviours

- ✓ value of a
- ✓ value of b

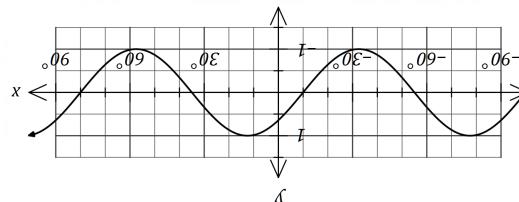
Solution

Period of $90^\circ \Rightarrow a = 360^\circ \div 90^\circ = 4$

$$y = \sin(4(x + 10)) = \sin(4x + 40^\circ)$$

$$a = 4, \quad b = 40^\circ$$

(2 marks)



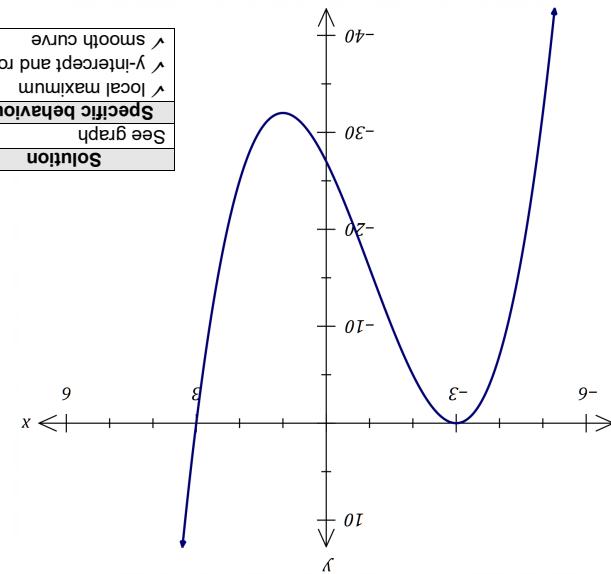
(b)

The graph of $y = \sin(ax + b)$ is shown below, where a and b are positive constants.

Solution

Specific behaviours

- ✓ smooth curve
- ✓ y-intercept and root
- ✓ local maximum
- ✓ graph

(b) Sketch the graph of $y = f(x)$ on the axes below.

(3 marks)

(4 marks)

Solution

$x = 90^\circ$

$$\sin x = 1$$

$$2 \left(\cos x \left(\frac{1}{2} \right) + \sin x \left(\frac{\sqrt{3}}{2} \right) \right) = \sqrt{3} + \cos x$$

$$2(\cos x \cos 60^\circ + \sin x \sin 60^\circ) = \sqrt{3} + \cos x$$

Specific behaviours

- ✓ uses angle difference identity
- ✓ substitutes exact values
- ✓ simplifies equation
- ✓ correct solution in degrees

(iii) $2 \cos(x - 60^\circ) = \sqrt{3} + \cos x, 0^\circ \leq x \leq 360^\circ$.

(ii) $\tan(2x) = -\sqrt{3}, 0^\circ \leq x \leq \pi$.

(i) $\tan(2x) = -\sqrt{3}, 0^\circ \leq x \leq \pi$.

(2 marks)

Solution

$f(x) = (x - 3)(x + 3)^2$

$$= x^3 + 3x^2 - 9x - 27$$

Specific behaviours

- ✓ writes in factored form
- ✓ expands
- ✓ states all three values

$a = 3, \quad b = -9, \quad c = -27$

(3 marks)

(a) Determine the value of a , the value of b and the value of c .

(3 marks)

Solution

$$2x = \frac{3}{\pi}, \quad \frac{3}{\pi} \text{ and } \frac{6}{\pi}$$

$$x = \frac{3}{2\pi}, \quad \frac{3}{\pi}$$

Specific behaviours

- ✓ one correct solution in radians
- ✓ both correct solutions in radians

(2 marks)

The graph of $y = f(x)$ has the following features:

The graph of $y = f(x)$ has the following features:

The function f is defined by $f(x) = x^3 + ax^2 + bx + c$, where a , b and c are constants.

(a) Solve the following equations.

(b) Passes through $(0, -27)$ and $(3, 0)$

(c) Has a local maximum at $(-3, 0)$

(d) Determine the value of a , the value of b and the value of c .

(3 marks)

(8 marks)

(12 marks)

Question 17

METHODS UNITS 1 AND 2

CALCULATOR-FREE

7

METHODS UNITS 1 AND 2

CALCULATOR-ASSUMED

12

Question 6

- (a) Expand
- $(x - 3)^3$
- .

Solution
$(x - 3)^3 = x^3 + 3x^2(-3) + 3x(-3)^2 + (-3)^3$ $= x^3 - 9x^2 + 27x - 27$
Specific behaviours
✓ correct coefficients ✓ correct expansion

- (b) Hence, or otherwise, determine the equation of the tangent to the curve
- $y = (x - 3)^3$
- at the point where
- $x = -1$
- . (4 marks)

Solution
$\frac{dy}{dx} = 3x^2 - 18x + 27$
When $x = -1$ $y = (-4)^3 = -64$
$\frac{dy}{dx} = 3 + 18 + 27 = 48$
Hence equation of tangent is $y + 64 = 48(x + 1)$ thus $y = 48x - 16$
Specific behaviours
✓ derivative ✓ y -coordinate ✓ gradient ✓ equation of tangent (any form)

(6 marks)
(2 marks)**Question 16**

(7 marks)

When a patient takes a painkilling drug A, the probability that they experience some side effects is known to be 0.1.

- (a) A doctor prescribes drug A to two unrelated patients. Determine the probability that

- (i) neither patient experiences some side effects. (1 mark)

Solution
$P = (0.9)^2 = 0.81$
Specific behaviours
✓ correct probability

- (ii) one patient experiences some side effects and the other does not. (2 marks)

Solution
$P = 0.1 \times 0.9 \times 2$ $= 0.18$
Specific behaviours
✓ calculates $p(1 - p)$ ✓ doubles to obtain correct probability

Other painkilling drugs are available. Of those who take drug A, 88% of patients who suffer some side effects will switch to another drug whereas no patient who has no side effects will switch.

- (b) The doctor prescribes drug A to a patient. Determine the probability that the patient does not switch to another drug. (2 marks)

Solution
$P = 0.9 + 0.1 \times 0.12$ $= 0.9 + 0.012$ $= 0.912$
Specific behaviours
✓ probability of side effect and does not switch ✓ correct probability

- (c) The doctor prescribes drug A to three unrelated patients. Determine the probability that at least one of these patients switch to another drug. (2 marks)

Solution
$P(\text{none}) = 0.912^3$ ≈ 0.7586
$P = 1 - 0.7586$ ≈ 0.2414
Specific behaviours
✓ probability none switch ✓ correct probability

Question 8

(7 marks)

An arithmetic sequence has a recursive definition given by $T_{n+1} = T_n + d$, $T_1 = a$. It has fourth term of 50 and tenth term of 20.

- (a) Determine the value of the constant a and the constant d .

(2 marks)

Solution
$(10 - 4)d = 20 - 50$
$6d = -30$
$d = -5$
$a = 50 - 3(-5) = 65$
Specific behaviours
✓ value of d
✓ value of a

- (b) Determine T_{2019} .

(2 marks)

Solution
$T_n = 65 + (n - 1)(-5)$
$T_{301} = 65 + 2018(-5)$
$= -10\ 025$
Specific behaviours
✓ indicates rule for general term
✓ correct value

- (c) The sum of the first m terms of the sequence is 350. Determine the value(s) of the integer constant m .

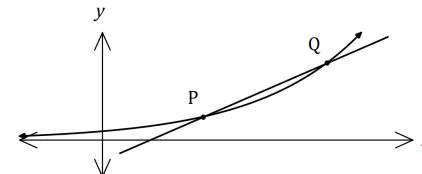
(3 marks)

Solution
$350 = \frac{m}{2}(2(65) + (m - 1)(-5))$
$700 = m(130 - 5m + 5)$
$5m^2 - 135m + 700 = 0$
$m^2 - 27m + 140 = 0$
$(m - 7)(m - 20) = 0$
$m = 7, \quad m = 20$
Specific behaviours
✓ substitutes into sum formula
✓ simplifies and equates quadratic to zero
✓ both correct solutions

Question 14

(7 marks)

The graph of $y = f(x)$ is shown below, where $f(x) = 4^x$, together with the secant to the curve through the points P and Q .



P has coordinates $(1, 4)$ and Q has coordinates $(1 + h, f(1 + h))$ where $0 < h \leq 1$.

- (a) Complete the second column in the table below, rounding values to 4 decimal places where necessary. (4 marks)

h	$\frac{f(1 + h) - f(1)}{h}$
1	12
0.1	5.9479
0.01	5.5838
0.001	5.5490

Solution
See table
Specific behaviours
✓ one correct value
✓ three correct values
✓ all correct
✓ last 3 all to 4 dp

- (b) Name the feature of the graph above that the values you calculated in part (a) represent. (1 mark)

Solution
Values are gradient of secant PQ as Q moves closer to P .
Specific behaviours
✓ indicates gradient of secant

- (c) Determine an estimate, correct to 3 decimal places, for the value that $\frac{f(1 + h) - f(1)}{h}$ approaches as h becomes closer and closer to 0 and state what this value represents. (2 marks)

Solution
Value approaches 5.545 (3 dp).
Value is gradient of curve at P .
Specific behaviours
✓ correct value
✓ states value approaches gradient at point

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of Christ Church Grammar School assessments are detailed in the Reporting and Assessment Policy. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

- (c) State the range of
- $C(x)$
- .

(1 mark)

Solution
$4 \leq C(x) \leq 12$
Specific behaviours

- (d) When the manufacturer uses process Z, the cost in dollars per litre
- $K(x)$
- is modelled by

$$K(x) = 10.5 - \frac{x}{6}, \quad 5 \leq x \leq 45.$$

- (i) Add this function
- $K(x)$
- to the graph.

(1 mark)

Solution
See graph for line.
Specific behaviours

- (ii) determine the production quantities for which process X is cheaper than process Z. (2 marks)

Solution
Process X is cheaper than Z for $15 < x < 33$ litres.
Specific behaviours

(a) Determine the cost per litre when 35 L is made.

$$C(x) = \frac{240}{x+15}, \quad 5 \leq x \leq 45.$$

(b) the total cost of making 17 L of the chemical.

Solution
✓ correct cost per litre
Specific behaviours
✓ correct total cost
✓ cost per litre

(1 mark)

(c) Working time: 100 minutes.

Solution
✓ correct cost per litre
Specific behaviours
✓ correct value
✓ the total cost of making 17 L of the chemical.

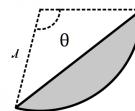
(2 marks)

(d) Convert 108° to an exact radian measure.

Solution
✓ correct value
Specific behaviours
✓ correct behaviour
(a) Convert 108° to an exact radian measure.

(e) 6 marks)

(f) A segment of a circle of radius 28 cm is shown below, where $\theta = 108^\circ$.



(g) Determine the area of the segment.

Solution
✓ indicates correct use of formula
Specific behaviours
✓ correct area
(f) Determine the area of the segment.

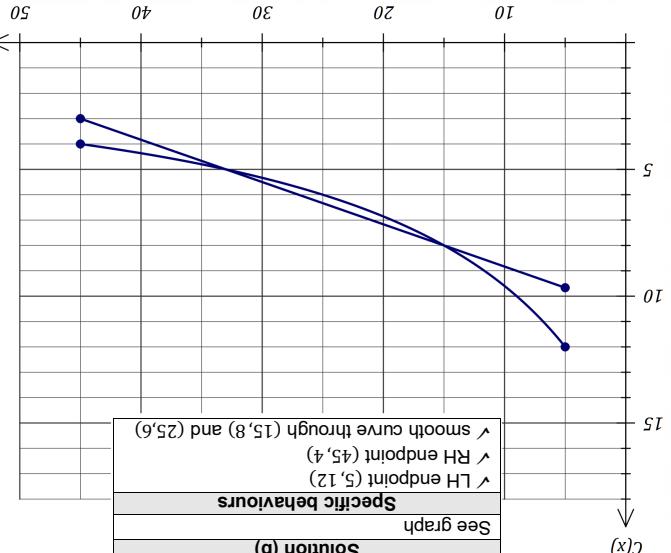
(g) (ii) Determine the perimeter of the segment. (3 marks)

Solution
✓ arc length
✓ uses cosine rule (or $C = 2rsin(\frac{\theta}{2})$) to calculate chord length
✓ correct perimeter
(g) (iii) Determine the perimeter of the segment.

(h) (i) Determine the area of the segment. (2 marks)

Solution
✓ smooth curve through (15, 8) and (25, 6)
✓ LH endpoint (45, 12)
✓ RH endpoint (45, 4)
(h) (i) Determine the area of the segment.

(j) Graph the cost per litre over the given domain on the axes below. (3 marks)



(k) Graph the cost per litre over the given domain on the axes below. (3 marks)

Solution
✓ correct total cost
✓ cost per litre
✓ correct cost per litre
(j) Graph the cost per litre over the given domain on the axes below.

(3 marks)

(l) the cost per filter when 35 L is made.

Solution
✓ correct value
✓ correct cost per litre
✓ correct behaviour
(k) the cost per filter when 35 L is made.

(2 marks)

(m) This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

Solution
✓ correct value
✓ correct behaviour
✓ correct cost per litre
✓ cost per litre

(1 mark)

(n) When a manufacturer makes x litres of a chemical using process X , the cost in dollars per litre

(10 marks)

(o) varies according to the rule

Solution
✓ correct value
✓ correct behaviour
✓ correct cost per litre
✓ cost per litre

(p) $C(x) = \frac{240}{x+15}$, $5 \leq x \leq 45$.

Question 10

(7 marks)

A drone is flying in a straight line and at a constant height h m above a level pitch towards a thin goal post. It maintains a constant speed of 4.5 ms^{-1} .

Initially, the angle of depression from the drone to the base of the post is 8° . Exactly 3 seconds later this angle has increased to 10° .

- (a) Sketch a diagram to show the two angles of depression from the drone to the base of the post. (1 mark)

Solution	
	x
Specific behaviours	
<input checked="" type="checkbox"/> sketch with angles	

- (b) Determine, showing all working, the value of h and calculate the time after leaving its initial position that the drone will collide with the post. (6 marks)

Solution	
$d = 4.5 \times 3 = 13.5$	
$\tan 8^\circ = \frac{h}{x + 13.5}, \tan 10^\circ = \frac{h}{x}$	
$(x + 13.5) \tan 8^\circ = x \tan 10^\circ \Rightarrow x = 53.018$	
$h = 53.018 \times \tan 10^\circ = 9.35 \text{ m}$	
$t = \frac{13.5 + 53}{4.5} = 14.8 \text{ s}$	
Specific behaviours	
<input checked="" type="checkbox"/> calculates distance travelled <input checked="" type="checkbox"/> writes equation using trig <input checked="" type="checkbox"/> writes second equation using trig <input checked="" type="checkbox"/> solves equations <input checked="" type="checkbox"/> states h <input checked="" type="checkbox"/> states time	

Question 11

(8 marks)

From a random survey of telephone usage in 320 households it was found that 48 households had access to a mobile phone but not a landline, 268 households had access to a landline and 188 more households had access to a mobile phone than did not.

- (a) Complete the missing entries in the table below. (3 marks)

	Mobile	No mobile	Total
Landline	206	62	268
No landline	48	4	52
Total	254	66	320

Solution	
See table	
$x + (x + 188) = 320 \Rightarrow x = 66$	
Specific behaviours	
<input checked="" type="checkbox"/> totals column; <input checked="" type="checkbox"/> totals row; <input checked="" type="checkbox"/> rest of table	

- (b) If one household is randomly selected from those surveyed, determine the probability that

- (i) it had access to a landline. (1 mark)

Solution	
$P(L) = 268 \div 320 \approx 0.8375$	
Specific behaviours <input checked="" type="checkbox"/> correct probability	

- (ii) it had no access to a mobile phone given that it had access to a landline. (1 mark)

Solution	
$P(\bar{M} L) = 62 \div 258 \approx 0.2313$	
Specific behaviours <input checked="" type="checkbox"/> correct probability	

- (iii) it had access to a landline given that it no access to a mobile phone. (1 mark)

Solution	
$P(L \bar{M}) = 62 \div 66 \approx 0.9394$	
Specific behaviours <input checked="" type="checkbox"/> correct probability	

- (c) Comment on the possible independence of households having access to a mobile phone and households having access to a landline. Justify your comment. (2 marks)

Solution	
The events are not independent as $P(L) \neq P(L \bar{M})$ (or since $P(L) \times P(M) \neq P(L \cap M)$)	
Specific behaviours <input checked="" type="checkbox"/> states not independent <input checked="" type="checkbox"/> justifies by comparing probabilities	