

No other items may be used in this section of the examination. It is **your responsibility** to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Important note to students

To be provided by the student

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters, nil

Materials required/recommended for this section

Time allowed for this section

Reading time before commencing work: 5 minutes

Working time for this section: 50 minutes

Section One:
Calculator-free

MATHEMATICS 3A



Rossmyne Senior High School
Semester 1 Examination 2012
Question/Answer Booklet

(This paper is not to be released to take home before 25/6/2012)

Solution	As in the table	Specific behaviours	✓✓ I mark for each
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$y = f(x)$	A
$y = f(x - 1)$	C

Structure of this paper

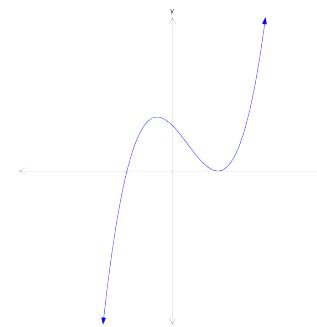
Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	13	13	100	100	
		Total	150	100	

Instructions to students

- 1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- 2 **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- 3 It is recommended that you **do not use pencil**, except in diagrams.

Question 6 (continued)

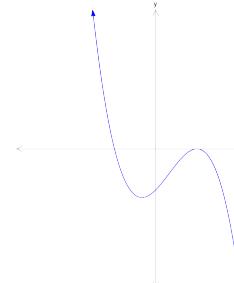
- (b) The graph of $y=f(x)$ has been plotted below



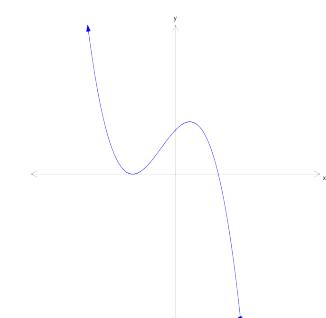
On the next 3 pairs of axes A, B, C are graphs of $y=f(-x)$, $f(x-1)$, $-f(x)$ in some order. Say which corresponds to which graph.

(3)

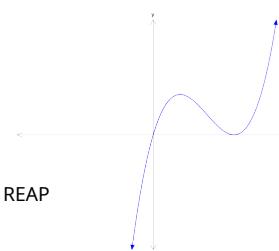
A



B



C



Function	Graph
$y = f(-x)$	B

Solution	$\begin{array}{r} X = 80 \\ - 2 = \frac{X - 100}{10} \\ \hline X = 80 \end{array}$
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- (i) Christopher has a Z-score of -2. What mark did he achieve in the test? (1)

Solution	$\begin{array}{r} \checkmark \text{ or } X \\ Z = \frac{115 - 100}{10} = 1.5 \end{array}$
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- (ii) Jon's mark is 115. What is his Z-score? (1)

The marks in Mr Green's Chemistry test are normally distributed. The mean is 100 and the standard deviation is 10.

Question 1 (5 marks)

- This section has **six (6)** questions. Answer all questions. Write your answers in the spaces provided.
- Working time: 50 minutes
- Section One: Calculator-free
marks) (50)

50

Solution	$\begin{array}{r} \checkmark \text{ solves the two equations simultaneously} \\ \checkmark \text{ solves the two equations simultaneously} \end{array}$
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- (iii) The following linear equations for the mean μ and the standard deviation σ were determined for the distribution of the weights of individual yogurt bars:
- $\mu + \sigma = 35.15$ and $\mu - 2\sigma = 34.7$
- Use the equations to find the mean weight and standard deviation of yogurt bars:
- $\mu + \sigma = 35.15$ and $\mu - 2\sigma = 34.7$
- (iv) Yogurt Muesli Bars: Muesli Bars:
- $\mu + \sigma = 35.15$ and $\mu - 2\sigma = 34.7$
- The following linear equations for the mean μ and the standard deviation σ were determined for the distribution of the weights of individual yogurt bars:
- $\mu + \sigma = 35.15$ and $\mu - 2\sigma = 34.7$

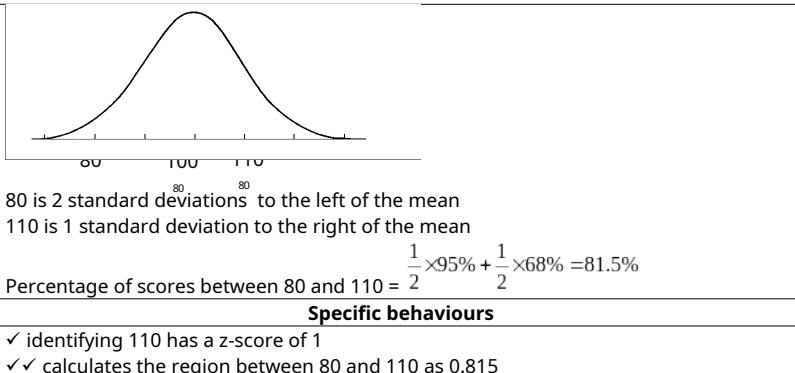
Specific behaviours	
✓ or X	

- (iii) What percentage of marks lie between 80 and 110? (3)

You may assume the following:

- 68% of marks have Z-scores between -1 and 1
- 95% of marks have Z-scores between -2 and 2
- 99.7% of marks have Z-scores between -3 and 3

Solution



Question 2 (8 marks)

- (a) Jonathon used the 'capture-recapture' technique to estimate the number of yabbies living in a dam.

- * He caught, tagged and released 20 yabbies.
- * Later he caught 36 yabbies at random from the same dam.
- * He found that 8 of these 36 yabbies had been tagged.

Estimate the total number of yabbies living in this dam. (3)

Solution

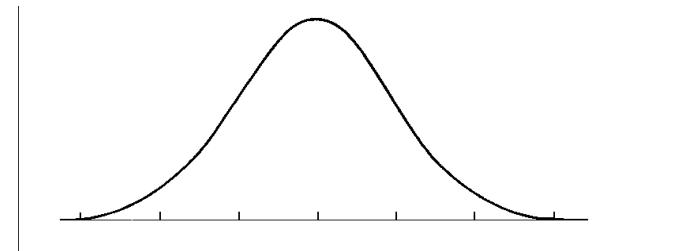
Let population be x

$$\frac{20}{x} = \frac{8}{36}$$

Question 6 (8 Marks)

- (a) The weight (W) in grams of individual Yoghurt Muesli Bars in a batch was measured to investigate their weight distribution.

- (i) Using the normal distribution curve below with mean μ and standard deviation σ illustrate the meaning of $P(w \geq \mu + 2\sigma) = 0.1587$ (1)



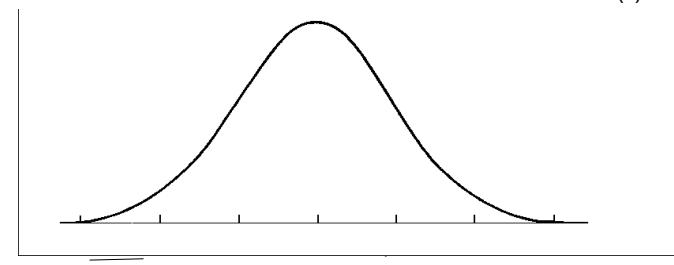
Solution

As shown on diagram

Specific behaviours

✓ or X

- (ii) Using the normal distribution curve below, illustrate the meaning of $P(w \geq \mu + \sigma) = 0.1587$ (2)



Solution

As shown on diagram

Specific behaviours

✓ $\mu + \sigma$
✓ shaded region

✓ correct values
Specific behaviours
$x = 0, x = 1$
$x(x - 1) = 0$

(c) Evaluate $\left(\frac{1}{2}\right)^{\frac{3}{5}}$

(d) Solve the equation, showing all working steps

$$\begin{aligned} 3x + 1 &= 5 \\ 3x &= 4 \\ x &= \frac{4}{3} \end{aligned}$$

Question 5 (continued)

✓ correct answer for x
Specific behaviours
$3x + 1 = 5$
$3x = 4$

(2)

(d) Solve the equation, showing all working steps

✓ correct answer of 4
Specific behaviours
$(2^{\frac{3}{5}})^{\frac{5}{3}} = 2^2 = 4$
$(2^{\frac{3}{5}})^{\frac{5}{3}} = 2^2 = 4$

(2)

✓ correct values
Specific behaviours
$x(x - 1) = 0$
$x = 0, x = 1$

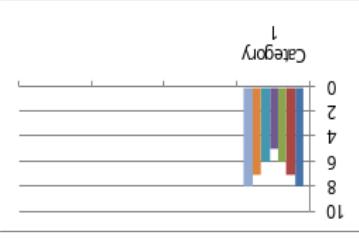
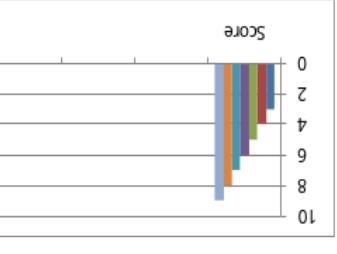
(2)

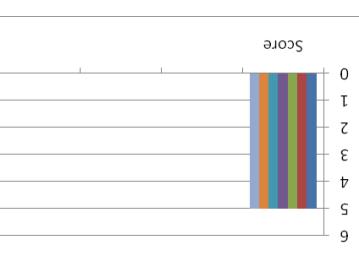
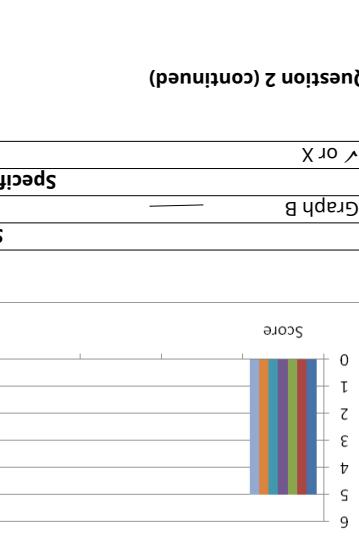
(iii) x if $g(x) = 0$

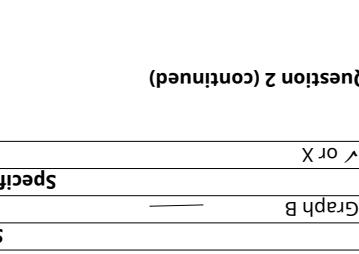
✓ expands correctly
Specific behaviours
$(2x+1)^2 - (2x+1) = 4x^2 + 2x$
$x = 20 \times 36 = 90$

Total number of yabbies is 90
 $x = 20 \times 36 = 90$
 $x = 8$
 $x = 36$
✓ rearrange and simplify x to 90

(b) Which of the following frequency histograms shows data that could be normally distributed?

- (i) (A)

 (B)


- (ii) (C)

 (D)


- (iii) (E)


- (c) Radar checks were carried out on the speed driven by drivers on two days, on a stretch of Spencer Road. The results are tabled below.

Days	Mean	Standard deviation	Number of drivers
Wednesday	60	10	100
Thursday	70	5	100

On which day would you expect there to be more drivers exceeding 85km/h?

Explain your answer.

(2)

Solution

Wednesday because $85 = 60 + 2.5 \sigma$ while Thursday $85 = 70 + 3 \sigma$

Specific behaviours

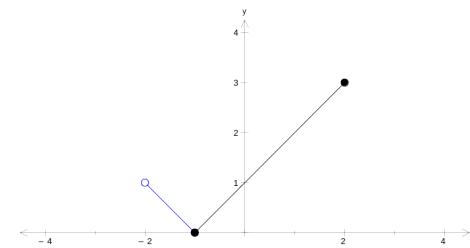
- ✓ Wed
- ✓ valid reason

Question 5
marks)

(12

(3)

- (a) State the domain and range (in set notation) for the function $y = f(x)$ drawn below.



Solution

$$D_x = \{x : -2 < x \leq 2, x \in \mathbb{R}\}$$

$$R_x = \{y : 0 \leq y \leq 3, y \in \mathbb{R}\}$$

Specific behaviours

- ✓ domain
- ✓ range
- ✓ use of correct notation to describe the sets

- (b) Given that $g(x) = x^2 - x$ find

(i) $g(-2)$

(1)

Solution

$$f(-2) = 6$$

Specific behaviours

- ✓ or X

(ii) $g(2x+1)$

(2)

Solution

$2(x - 1)^2 + k = 0$ has two real roots with opposite signs

(2)

$\wedge -k > 2$	Solution
$\therefore k < -2$	Specific behaviours
$\wedge k < -2$	Points A and B as indicated on diagram
$4 = 2(x-1)^2$	or X
(1)	

(c) Indicate on the graph where you would read off the values for x if

(1)

$\wedge k < -2$	Solution
$\wedge -k > 2$	Specific behaviours
$Any\ combination\ such\ that\ the\ sum\ is\ 320\ cm\ and\ range\ from\ 150\ to\ 170\ cm$	Points A and B as indicated on diagram
Possible heights are 150 cm and 170 cm, 155 cm and 165 cm, 151 and 169 cm	Specific behaviours
Mean of the two absent students is 160	Use your graph to solve $2x^2 - 4x + 6 = 0$. Justify your answer.

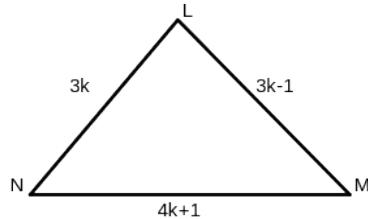
(d) Use your graph to solve $2x^2 - 4x + 6 = 0$. Justify your answer.

(3)

$2x^2 - 4x + 2 + 4 = 0$	Solution
$2(x^2 - 2x + 1) + 4 = 0$	Specific behaviours
$2(x - 1)^2 = -4$	As the graph does not intersect the line $y = -4$, there is no solution
$2(x^2 - 2x + 1) + 4 = 0$	Parabola does not cut horizontal line of $y = -4$
$2(x - 1)^2 = -4$	states No solution

Question 3**(7 marks)**

$\triangle LMN$ is drawn with $LN = 3k$ units, $MN = (4k+1)$ units and $LM = (3k-1)$ units with $k > 0$.



- (i) Which side of $\triangle LMN$ is the longest side? Justify your answer **algebraically**. (3)

Solution

$$\begin{aligned} 4k &> 3k \\ 4k + 1 &> 3k - 1 \\ \text{And } 3k - 1 &< 3k \\ \therefore 3k - 1 &< 3k < 4k + 1 \\ \text{MN is the longest side} \end{aligned}$$

Specific behaviours

- ✓✓ algebraic reasoning
- ✓ determines MN is the longest side

- (ii) If $\triangle LMN$ is a right-angled triangle calculate the value(s) of k . (4)

Solution

$$\begin{aligned} \text{By Pythagoras theorem, } (3k)^2 + (3k-1)^2 &= (4k+1)^2 \\ 9k^2 + 9k^2 - 6k + 1 &= 16k^2 + 8k + 1 \\ 2k^2 - 14k + 1 &= 0 \\ 2k(k-7) &= 0 \\ k = 7 \text{ as } k = 0 \text{ is not possible} \end{aligned}$$

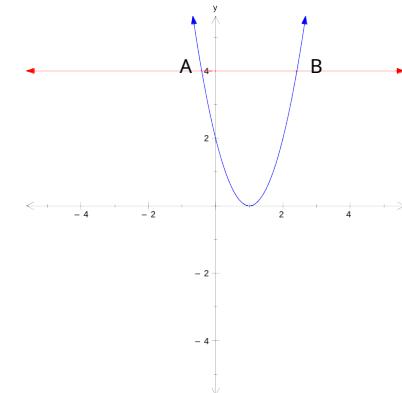
Specific behaviours

- ✓ equation using Pythagoras' Theorem
- ✓✓ simplify and factorise
- ✓ correct answer of 7 for "k"

(4)

(10 marks)**Question 4****(10**

- (a) Sketch the graph of $f(x) = 2(x-1)^2$. Show all intercepts. (2)

**Solution**

As shown in diagram above

Specific behaviours

- ✓ parabola shape
- ✓ (0,2), (1,0)

- (b) Use the graph to find the value(s) of k for which

- (i) $f(x) = k$ has 1 root (1)

Solution	
$k = 0$	
Specific behaviours	

- (ii) $f(x) = k$ has real roots (1)

Solution	
$k > 0$	
Specific behaviours	