Revision Examination Assessment Papers (REAP)
Semester 1 Examination 2012

Question/Answer Booklet
(This paper is not to be released to take home before 25/6/2012)

MATHEMATICS: SPECIALIST 3C

Section One:
Calculator-free

Name of Student: ____________________Marking Key_____________________________

Time allowed for this section

Reading time before commencing work:	5 minutes
Working time for this section:	50 minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the student

Standard items:	pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items:	nil

Important note to students

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	11	11	100	100	
			Total	150	100

Instructions to students

1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2 **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

Section One: Calculator-free **(50 marks)**

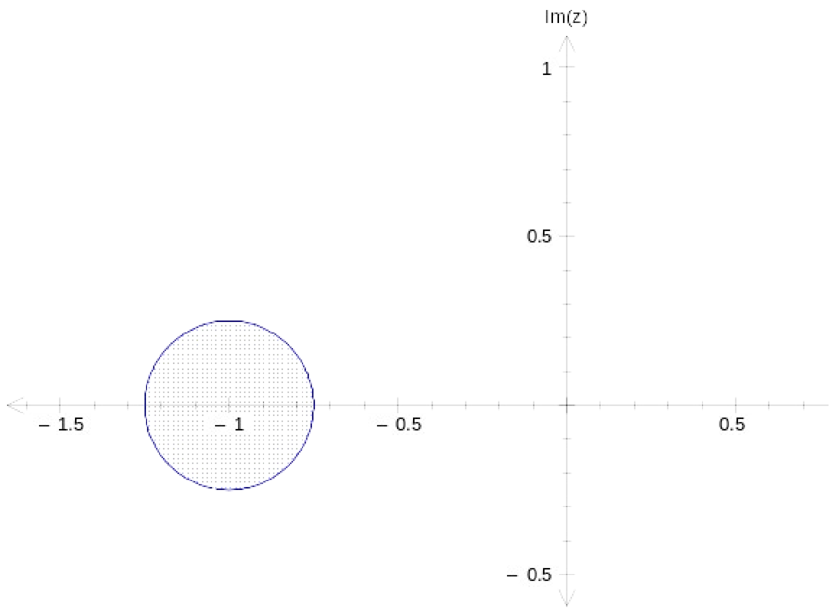
This section has **six (6)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 50 minutes

Question 1 **(9 marks)**

(a) (i) Sketch on the complex plane below the region defined by

$$|z+1| \le \frac{1}{4}$$ (3)



Solution
$\lvert z-(-1+0i)\rvert \le \frac{1}{4}$ Centre = (-1,0) Radius = $\frac{1}{4}$
Specific behaviours
✓ circular disk ✓✓ correct coordinates of centre (-1,0) and radius $\frac{1}{4}$

(ii) Hence, find the minimum value of $|z|$ (1)

Solution
Min $\|z\| = 0.75$
Specific behaviours
✓ or X

Question 1 (continued)

(b) The curve C is defined parametrically by $x = t^2$, $y = t + 2$, $t > 0$. Find the area of the region bounded by the curve, C, the x-axis, $x = 1$ and $x = 4$. (5)

Solution
$x = 1, t = 1$ $x = 4, t = 2$ Area = $\int_1^4 y\ dx$ $x = t^2$, $dx = 2t\ dt$ Or Area = $\int_1^4 \sqrt{x} + 2\ dx$ $= \int_1^2 (t+2)2t\ dt$ $= \left[\frac{2\sqrt{x^3}}{3} + 2x\right]_1^4$ $= \left[\frac{2t^3}{3} + 2t^2\right]_1^2$ $= \left(\frac{16}{3} + 8\right) - \left(\frac{2}{3} + 2\right)$ $= \frac{14}{3} + 6$ $= \frac{14}{3} + 6$ $= \frac{32}{3}$ $= \frac{32}{3}$
Specific behaviours
✓ t values for respective x values ✓ integral as a function of t ✓ integrates each term correctly ✓ substitutes limits of integration ✓ evaluates correctly

Question 2 **(7 marks)**

Determine each of the following integrals.

(a) $\int (3x+1)\, e^{3x^2+2x-1}\, dx$ (2)

Solution
$=\frac{1}{2}\int (6x+2)e^{3x^2+2x-1}dx$ $=\frac{1}{2}e^{3x^2+2x-1}+c$
Specific behaviours
✓ express as $\int f'(x)e^{f(x)}dx = e^{f(x)}+c$ ✓ correct answer

(b) $\int \left(1-e^{\cos x}\right)\sin x\, dx$ (2)

Solution
$\int \sin x - \sin x\, e^{\cos x}dx$ $=-\cos x + e^{\cos x}+c$
Specific behaviours
✓ expands and integrates correctly ✓ recognises $\int f'(x)e^{f(x)}dx = e^{f(x)}+c$

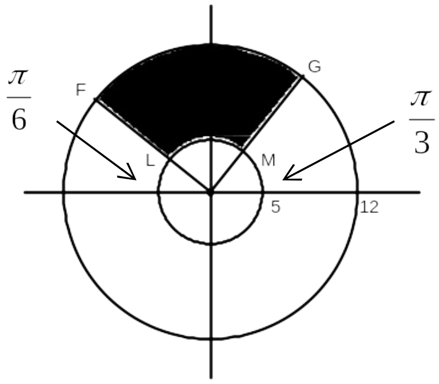
(c) $\int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{5-4\cos x}\, dx$ (3)

Solution
$=\int \frac{\sin x}{5-4\cos x}dx$ $=\frac{1}{4}\int \frac{4\sin x}{5-4\cos x}dx$ $=\frac{1}{4}\ln\left\|5-\cos x\right\|+c$
Specific behaviours
✓ recognises $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ ✓gives logarithmic solution

✓ correct coefficient of $\frac{1}{4}$ and plus "c"

Question 3 **(6 marks)**

(a) Use **polar inequalities** to describe the region bounded by minor arcs LM and FG and the straight lines, FL and GM. (2)

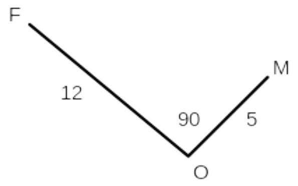


Solution
$5 \le r \le 12$ and $\frac{\pi}{3} \le \theta \le \frac{5\pi}{6}$
Specific behaviours
✓ inequalities with both radii ✓ inequalities with bot angles in polar notation

(b) If the graph of $r = k\theta$, k > 0 passes through M, find a possible value for k. (2)

Solution
Polar coordinates of M = $\left(5, \frac{\pi}{3}\right)$ i.e. $5 = k \cdot \frac{\pi}{3}$ $k = \frac{15}{\pi}$
Specific behaviours
✓ polar coordinates of M ✓ solves for "k" correctly

(c) Find the distance between F and M. (2)

Solution
 $\left\|FM\right\| = 13$
Specific behaviours
✓ recognises triangle FOM is a right angled triangle ✓ FM = 13 units (by Pythagoras theorem)

Question 4 **(9 marks)**

The line L has equation **r** = 3**i** + 2**j** + **k** + λ (**i** + 2**j** + **k**) where λ is real parameter

(i) Find the acute angle θ between the x – y plane and the line L. (3)

Solution
$(\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k})\cdot(\underset{\sim}{i}+\underset{\sim}{j})=\left|(\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k})\right|\left|(\underset{\sim}{i}+\underset{\sim}{j})\right|\cos\theta$ $1+2=\sqrt{6}\sqrt{2}\cos\theta$ $\dfrac{3}{2\sqrt{3}}=\cos\theta$ $\cos\theta=\dfrac{\sqrt{3}}{2}$ $\theta=30^0$
Specific behaviours
✓ vector for x-y plane as $\underset{\sim}{i}+\underset{\sim}{j}$ ✓ uses $\underset{\sim}{a}\cdot\underset{\sim}{b}=\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert\cos\theta$ ✓ correctly finds θ

(ii) Show that the point A with coordinates (7, -4, 3) lies on the line which passes through (3,2,1) and is parallel to the vector 2**i** - 3**j** + **k**. (3)

Solution
Vector equation of second line is $\underset{\sim}{r}=(3,2,1)+\lambda(2,-3,1)$ $(7,-4,3)=(3,2,1)+\lambda(2,-3,1)$ Or $4\underset{\sim}{i}-6\underset{\sim}{j}+2\underset{\sim}{k}=\lambda(2\underset{\sim}{i}-3\underset{\sim}{j}+\underset{\sim}{k})$ Equating coefficients: $7=3+2\lambda$, $-4=2-3\lambda$, $3=1+\lambda$ $\lambda=2$ As λ is unique, point A lies on second line
Specific behaviours
✓ vector equation of second line ✓✓ solves for unique value of λ and concludes point A lies on line

(iii) Find the value of m such that 2**i** + **j** + m**k** is perpendicular to the vector 2**i** – 3**j** + **k**. (3)

Solution
$(2\underset{\sim}{i}+\underset{\sim}{j}+m\underset{\sim}{k})\cdot(2\underset{\sim}{i}-3\underset{\sim}{j}+\underset{\sim}{k})=0$ $4-3+m=0$ $m=-1$
Specific behaviours
✓ dot product for perpendicular vectors

✓ equation involving m ✓ solves for m correctly

Question 5 **(10 marks)**

(a) Express each of the following in polar form such that r > 0, $-\pi \le \theta \le \pi$.

(i) $\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$ (2)

Solution
$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$ $\tan\theta = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3} \quad \theta = \frac{\pi}{3}$ $= cis\frac{\pi}{3}$
Specific behaviours
✓ "r" value of 1 ✓ argument of $\frac{\pi}{3}$

(ii) $\left(\frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2}\right)$ (2)

Solution
$r = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = 1$ $\tan\theta = \frac{\sqrt{2}}{2} \times \frac{2}{\sqrt{2}} = 1 \qquad \theta = \frac{\pi}{4}$ As point lies in the fourth quadrant, $\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = cis\left(-\frac{\pi}{4}\right)$
Specific behaviours
✓ "r" value of 1 and argument of $\frac{\pi}{4}$

✓ negative $\frac{\pi}{4}$

(iii) $\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)$ (2)

Solution
$= cis\left(\frac{\pi}{3}\right)cis\left(-\frac{\pi}{4}\right) = cis\left(\frac{\pi}{12}\right)$
Specific behaviours
✓ add arguments ✓ correct answer of $cis\left(\frac{\pi}{12}\right)$

Question 5 (continued)

(b) Simplify and express in exact rectangular form. (2)

$$\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)$$

Solution
$\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{4}-i\frac{\sqrt{2}}{4}+i\frac{\sqrt{6}}{4}+\frac{\sqrt{6}}{4}$ $=\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)+\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)i$
Specific behaviours
✓ real part ✓ imaginary part

(c) **Hence**, find the exact value of $\sin\frac{\pi}{12}$ and $\cos\frac{\pi}{12}$ (2)

Solution
$\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}=\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)+\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)i$ $\cos\frac{\pi}{12}=\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)$

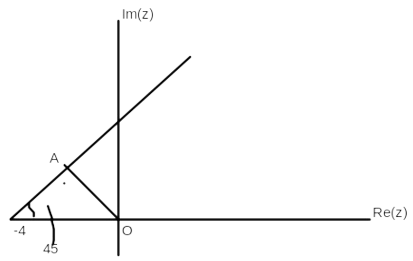
$\sin\frac{\pi}{12} = \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)$
Specific behaviours
✓ ✓ for each of $\cos\frac{\pi}{12}$ and $\sin\frac{\pi}{12}$

Question 6 **(9 marks)**

(a) If z is a complex number such that arg $(z + 4) = \frac{\pi}{4}$, find the least value of $|z|$.

(3)

Solution



Least value is OA. OA is perpendicular to line, $\sin 45^0 = \frac{OA}{4}$, $OA = 4\times\frac{1}{\sqrt{2}}$, $OA = 2\sqrt{2}$

Least value of $|z|$ is $2\sqrt{2}$

Specific behaviours

✓ represent z on Argand diagram

✓ identify OA as the least value of $|z|$

✓ solves for OA as $2\sqrt{2}$

(b) A complex number z satisfies the equation $z + 2|\bar{z}| = 2 + i$, find the complex number.
Hint: let $z = x + yi$.

(6)

Solution

$\bar{z} = x - yi$, $|\bar{z}| = \sqrt{x^2 + y^2}$

$x + yi + 2\sqrt{x^2 + y^2} = 2 + i$

Equating Imaginary: $y = 1$

Equating Real: $x + 2\sqrt{x^2 + y^2} = 2$

$2\sqrt{x^2 + y^2} = 2 - x$

Square both sides: $4(x^2 + y^2) = (2 - x)^2$

$4x^2 + 4 = 4 - 4x + x^2$

$3x^2 + 4x = 0$

$x = 0$ or $x = \frac{-4}{3}$

$\therefore z = i$ or $z = \frac{-4}{3} + i$

Specific behaviours

✓ replaces “z” equation in terms of x and y ✓ equates REAL component to get equation $x + 2\sqrt{x^2 + y^2} = 2$ ✓ equates imaginary component to get value of y = 1 ✓ solves correctly for value of x ✓✓ states the two values for z