Rossmoyne Senior High School

Semester One Examination, 2015

Question/Answer Booklet

MATHEMATICS SPECIALIST 3C

Section One: Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	6	6	50	50	33⅓
Section Two: Calculator-assumed	13	13	100	100	66⅔
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **(50 Marks)**

This section has **six (6)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

(a) Determine $\int x^2 e^{1-2x^3}\,dx$. (3 marks)

$$y = e^{1-2x^3} \Rightarrow y' = -6x^2 e^{1-2x^3}$$

$$\int x^2 e^{1-2x^3}\,dx = -\frac{1}{6} e^{1-2x^3} + c$$

(b) Evaluate $\int_{\pi/4}^{\pi/3} \frac{1}{\cos^2 x}\,dx$. (3 marks)

$$\int_{\pi/4}^{\pi/3} \frac{1}{\cos^2 x}\,dx = \left[\tan x\right]_{\pi/4}^{\pi/3}$$

$$= \tan \pi/3 - \tan \pi/4$$

$$= \sqrt{3} - 1$$

Question 2 **(9 marks)**

The plane Π has equation $x - z = 4$ and the points A and B have coordinates $(-1, -1, 2)$ and $(-2, -5, 6)$ respectively.

(a) Calculate the distance between A and B. (1 mark)

$$\sqrt{1^2 + 4^2 + 4^2} = \sqrt{33}$$

(b) Determine the vector equation of

(i) the plane containing A that is parallel to Π. (2 marks)

$$x - z = -1 + 0 - 2$$
$$x - z = -3$$
$$\mathbf{r} \bullet (\mathbf{i} - \mathbf{k}) = -3$$

(ii) the line passing through A that is perpendicular to Π. (2 marks)

$$\mathbf{r} = -\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - \mathbf{k})$$

See next page

(c) Determine the position vector of the point of intersection of the plane Π and the line that passes through the point B in the direction $2\mathbf{i} + \mathbf{j} - \mathbf{k}$. (4 marks)

$$\mathbf{r} = \begin{bmatrix} -2 \\ -5 \\ 6 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{aligned} x - z &= 4 \\ (2\lambda - 2) - (6 - \lambda) &= 4 \\ 3\lambda &= 12 \\ \lambda &= 4 \end{aligned}$$

$$\mathbf{r} = \begin{bmatrix} -2 \\ -5 \\ 6 \end{bmatrix} + 4 \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$= 6\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

See next page

Question 3 **(11 marks)**

(a) Determine $\frac{dy}{dx}$ when $y = \sin(3\ln(2x))$. (3 marks)

$$\frac{dy}{dx} = \frac{3}{x}\cos(3\ln(2x))$$

(b) Determine $h'(2)$ if $h(x) = \sqrt{\frac{4x+1}{x-1}}$. (4 marks)

$$h'(x) = \frac{1}{2} \times \frac{1}{\sqrt{\frac{4x+1}{x-1}}} \times (4(x-1) - 1(4x+1))$$

$$h'(2) = \frac{1}{2}\sqrt{\frac{1}{9}}(4-9)$$

$$= -\frac{5}{6}$$

See next page

(c) Determine the derivative $f'(x)$ from first principles when $f(x) = 3x + x^2$. (4 marks)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \left(\frac{3(x+h) + (x+h)^2 - (3}{h} \right.$$

$$= \lim_{h \to 0} \left(\frac{3x + 3h + x^2 + 2xh + h}{h} \right.$$

$$= \lim_{h \to 0} \left(\frac{(3 + 2x + h)h}{h} \right)$$

$$= \lim_{h \to 0} (3 + 2x + h)$$

$$= 3 + 2x$$

See next page

Question 4 **(10 marks)**

(a) Let x° be an angle measured in degrees. Determine $\int \cos x^\circ \, dx$ in terms of x°. (2 marks)

$$\begin{aligned}\int \cos x^\circ \, dx &= \int \cos \frac{\pi x}{180} \, dx \\ &= \frac{180}{\pi} \sin \frac{\pi x}{180} \\ &= \frac{180}{\pi} \sin x^\circ + c\end{aligned}$$

(b) Determine $\int \frac{2 \ln x}{x} \, dx$. (3 marks)

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$\begin{aligned}\int \frac{2 \ln x}{x} \, dx &= \int 2u \, du \\ &= u^2 + c \\ &= (\ln x)^2 + c\end{aligned}$$

See next page

(c) Use the substitution $u = x - 3$ to evaluate $\int_4^7 5x\sqrt{x-3}\,dx$. (5 marks)

$u = x - 3 \Rightarrow du = dx$

$x = 4,\ u = 1 \quad x = 7,\ u = 4$

$$\int_4^7 5x\sqrt{x-3}\,dx = 5\int_1^4 (u+3)u^{1/2}\,du$$

$$= 5\int_1^4 u^{3/2} + 3u^{1/2}\,du$$

$$= \left[2u^{5/2} + 10u^{3/2}\right]_1^4$$

$$= \left[2\times 2^5 + 10\times 2^3\right] - \left[2 + 10\right]$$

$$= 64 + 80 - 12$$

$$= 132$$

See next page

Question 5 **(9 marks)**

Let $z = 2\sqrt{3} - 2i$ and $w = -\sqrt{2} + \sqrt{2}i$.

(a) Express z^3 in polar form. (3 marks)

$$z = 4cis\left(-\frac{\pi}{6}\right)$$
$$z^3 = 4^3 cis\left(-\frac{3\pi}{6}\right)$$
$$= 64cis\left(-\frac{\pi}{2}\right)$$

(b) Express $\frac{\overline{z}}{2\overline{w}}$ in polar form. (3 marks)

$$\overline{z} = 4cis\left(\frac{\pi}{6}\right)$$

$$\overline{w} = 2cis\left(-\frac{3\pi}{4}\right)$$

$$\frac{\overline{z}}{2\overline{w}} = cis\left(\frac{\pi}{6} - -\frac{3\pi}{4}\right)$$
$$= cis\left(\frac{11\pi}{12}\right)$$

See next page

(c) Calculate zw and show that $\overline{zw} = \overline{z} \times \overline{w}$. (3 marks)

$$zw = 4cis\left(-\frac{\pi}{6}\right) \times 2cis\left(\frac{3\pi}{4}\right)$$
$$= 8cis\left(-\frac{\pi}{6} + \frac{3\pi}{4}\right) = 8cis\left(\frac{7\pi}{12}\right)$$

$$\overline{z} \times \overline{w} = 4cis\left(\frac{\pi}{6}\right) \times 2cis\left(-\frac{3\pi}{4}\right)$$
$$= 8cis\left(\frac{\pi}{6} - \frac{3\pi}{4}\right)$$
$$= 8cis\left(-\frac{7\pi}{12}\right)$$
$$= \overline{zw}$$

Hence $\overline{zw} = \overline{z} \times \overline{w}$

See next page

Question 6 **(5 marks)**

(a) Determine

(i) $\frac{d}{dx}\ln x^n$. (1 mark)

$$\frac{1}{x^n}\frac{d}{dx}x^n = \frac{1}{x^n}nx^{n-1}$$
$$= \frac{n}{x}$$

(ii) $\frac{d}{dx}n\ln x$. (1 mark)

$$n\frac{1}{x} = \frac{n}{x}$$

(b) Use the results from (a) to prove that $\ln x^n = n\ln x$. (3 marks)

Since $\frac{d}{dx}\ln x^n = \frac{d}{dx}n\ln x$

then $\ln x^n = n\ln x + c$.

When $x = 1$, $\ln 1 = n\ln 1 + c \Rightarrow c = 0$.

Hence $\ln x^n = n\ln x$.

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____

Published by WA Examination Papers
PO Box 445 Claremont WA 6910