

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material if you have any unauthorised material with you, hand it to the supervisors before reading any further.

Important note to candidates

Special items: nil

Standard items: fluid/tape, eraser, ruler, highlighters
To be provided by the candidate pens (blue/black preferred), pencils (including coloured), sharpener, correction

Materials required/recommended for this section
This Question/Answer Booklet
To be provided by the supervisor
Working time: fifty (50) minutes
Reading time before commencing work: five (5) minutes

Teacher: Al Friday White
Student Name: *Solutions*

Section One:
Calculator-free

UNITS 3 & 4
METHODS
MATHEMATICS

C 0 1 1 E 6 E
SECTION

Question/Answer booklet

Semester Two Examination, 2020



METHODS UNITS 3&4

16

CALCULATOR ASSUMED

Supplementary page

Question number: _____

1

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	36%
Section Two: Calculator-assumed	12	12	100	93	64%
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your response to the specific questions asked and to follow any instructions that are specified to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- Do not place the Formula Sheet inside your Question/Answer booklet. It will be collected separately.

See next page

Markers Use Only		
Question	Max.	Mark
1	7	
2	6	
3	5	
4	7	
5	7	
6	7	
7	7	
8	6	
Section One Total	52	
Section One %		
Section Two Total	92	
Section Two %		
Overall Deductions	Units	
	Rounding	
	Notation	
Total	144	
Overall %		

Question 20

(6 marks)

A student was set the task of determining the proportion of people in their suburb who use public transport at least once a week.

- (a) Briefly discuss the main source of bias in each of the following sampling methods.
- (i) The student invites people via social media to respond to their survey. (1 mark)

eg. { Volunteer sampling : some of population self Selection sampling : have zero chance of selection.

eg. People who respond may not live in suburb : not representative of suburb population. ✓ indicates one source/type of BIAS and WHY not.

- (ii) The student asks everyone she meets until she has a large enough sample. (1 mark)

eg. Convenience sampling - no regard for need of sample to represent population

eg. May only be close to where student lives, not represent whole suburb. ✓ indicates one source/type of BIAS and WHY not.

- (b) The student noted that 39 out of all those sampled said they used public transport at least once a week and went on to construct the confidence interval (0.49, 0.81). Determine the level of confidence of this interval. (4 marks)

$$\hat{p} = \frac{0.49 + 0.81}{2} \\ = 0.65$$

$$M.E = 0.81 - 0.65 \\ = 0.16$$

$$\frac{39}{n} = 0.65 \\ n = 60$$

$$\therefore 0.16 = z \sqrt{\frac{0.65(0.35)}{60}} \\ z = 2.598$$

$$P(-2.598 < z < 2.598) = 0.9906$$

level of confidence 99.06%.

✓ level of confidence

End of Questions

Question 2

(6 marks)

The continuous random variable X takes values in the interval 3 to 8 and has cumulative distribution function $F(x)$ where

$$F(x) = P(X \leq x) = \begin{cases} 0 & x < 3 \\ \frac{x-3}{5} & 3 \leq x \leq 8 \\ 1 & x > 8. \end{cases}$$

(a) Determine

(i) $P(X \leq 4.5)$.

$$\begin{aligned} F(4.5) &= \frac{4.5-3}{5} \\ &= \frac{1.5}{5} \\ &= 0.3 \end{aligned} \quad (1 \text{ mark})$$

✓ correct probability

(ii) the value of k , if $P(X > k) = 0.75$.

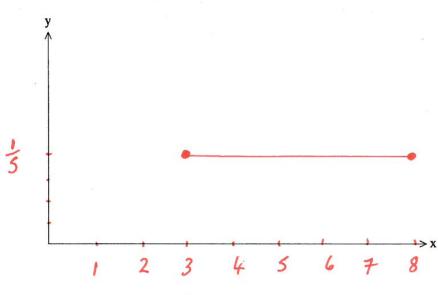
$$\begin{aligned} P(X \leq k) &= 0.25 \\ \frac{k-3}{5} &= 0.25 \\ k &= 4.25 \end{aligned} \quad \begin{aligned} &\checkmark \text{ indicates use} \\ &\text{of } P(X \leq k) \\ &\checkmark \text{ correct value} \\ &\text{of } k. \end{aligned} \quad (2 \text{ marks})$$

(b) Determine $f(x)$, the probability density function of X , and use the axes below to sketch the graph of $y = f(x)$.

$$\begin{aligned} \frac{d}{dx}\left(\frac{1}{5}x - \frac{3}{5}\right) &= f(x) = F'(x) = \begin{cases} \frac{1}{5} & 3 \leq x \leq 8 \\ 0 & \text{otherwise.} \end{cases} \quad (3 \text{ marks}) \end{aligned}$$

✓ $f(x)$

✓ draws
 $f(x) = \frac{1}{5}$
 between
 endpoints



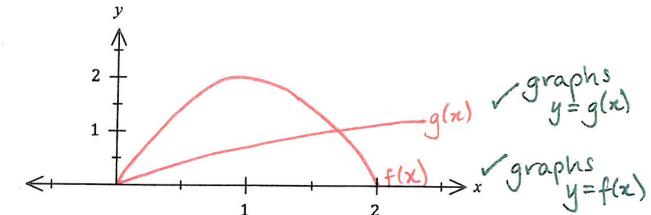
See next page

Question 18

(8 marks)

Let $f(x) = 2 - 2(x-1)^2$ and $g(x) = \ln(x+1)$.(a) Sketch the graphs of $y = f(x)$ and $y = g(x)$ for $x \geq 0$ on the axes below.

(2 marks)

(b) Show that $\frac{d}{dx}((x+1)\ln(x+1) - (x+1)) = \ln(x+1)$.

(2 marks)

$$\begin{aligned} &= (x+1)\left(\frac{1}{x+1}\right) + \ln(x+1)(1) - 1 \\ &= \frac{x+1}{x+1} + \ln(x+1) - 1 \\ &= 1 + \ln(x+1) - 1 \\ &= \ln(x+1) \end{aligned}$$

✓ uses product rule correctly

✓ differentiates
 $\ln(x+1)$ term
 correctly

(c) Show that the area of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$, and the straight line $x = 1$ is exactly $\frac{7}{3} - 2\ln 2$ square units.

(4 marks)

(HINT: use your answer in part (b).)

$$\begin{aligned} A &= \int_0^1 f(x) - g(x) \, dx \\ &= \int_0^1 2 - 2(x-1)^2 - \ln(x+1) \, dx \quad \begin{aligned} &\checkmark \text{ writes correct} \\ &\text{integral for area} \end{aligned} \\ &= \left[2x - \frac{2(x-1)^3}{3} - ((x+1)\ln(x+1) - (x+1)) \right]_0^1 \\ &\quad \text{or} \\ &= \left[-\frac{2x^3}{3} + 2x^2 - ((x+1)\ln(x+1) - (x+1)) \right]_0^1 \\ &= (2 - 0 - 2\ln 2 + 2) - (0 + \frac{2}{3} - \ln 1 + 1) \\ &= 2 - 2\ln 2 + 2 - \frac{2}{3} - 1 \\ &= \frac{7}{3} - 2\ln 2 \end{aligned}$$

✓ show correct
 substitution and
 simplification*if used calc to antiderive
 area, max 3 marks.

See next page

Question 4

(7 marks)

- (a) Determine an expression for $f'(x)$ for each of the following functions.
DO NOT SIMPLIFY YOUR ANSWERS.

(i) $f(x) = \ln(1 - \cos 3x)$.

(2 marks)

$$\begin{aligned} f'(x) &= \frac{1}{1 - \cos 3x} \cdot 3 \sin 3x \\ &= \frac{3 \sin 3x}{1 - \cos 3x} \end{aligned}$$

✓ numerator
or $\frac{d}{dx}(1 - \cos 3x)$
✓ denominator.

(ii) $f(x) = e^{5x}(5 - 2x)^3$.

(3 marks)

$$\begin{aligned} f'(x) &= e^{5x} \cdot 3(5-2x)^2(-2) + (5-2x)^3 5e^{5x} \sqrt{\frac{d}{dx} e^{5x}} \\ &\quad \times 5e^{5x} \\ &\quad \checkmark \frac{d}{dx}(5-2x)^3 \\ &\quad \checkmark \text{demonstrates use of product rule} \end{aligned}$$

- (b) For the positive number x , let $A(x) = \int_0^x (8 - 2t^2) dt$.

Determine the value(s) of x for which $\frac{dA}{dx} = 0$.

(2 marks)

$$\begin{aligned} \frac{dA}{dx} &= \frac{d}{dx} \int_0^x (8 - 2t^2) dt \\ &= 8 - 2x^2 \quad \checkmark \text{expression for } \frac{dA}{dx} A'(x) \\ \therefore 8 - 2x^2 &= 0 \\ 8 &= 2x^2 \\ 2^3 &= 2x^2 \\ x^2 &= 3 \\ x &= \sqrt{3} \quad \checkmark \text{correct value of } x. \end{aligned}$$

- (d) Describe how two factors affect the closeness of the approximate distribution in (c) to the true distribution of proportions. (2 marks)

- large sample size

✓ indicates large sample size

- p close to 0.5

✓ indicates p close to 0.5

✓ solves and
✓ requires fewer steps

✓ factors out x

log equation
✓ writes as

(3 marks)

✓ correct value
log laws
✓ denotes舍去
✓ correct舍去

(2 marks)

$$\begin{aligned} \log_a(x^2) &= \log_a(x^{2/5}) \\ 2\log_a x + \frac{2}{5}\log_a x &= \log_a x^{2/5} \\ 2(0.8) + \frac{2}{5}(0.8) &= 2.5(0.8) \\ x &= x \end{aligned}$$

(b) Given that $\log_a x = 0.8$, determine the value of $\log_a(x^2\sqrt{x})$.

(2 marks)

$$\begin{aligned} \log 5 + \log 4 - \log \frac{1}{2} &= \log \frac{125}{4} \\ \log 1000 &= \log 125 \\ 3 &= 3 \end{aligned}$$

(a) Simplify $3 \log 5 + \log 4 - \log \frac{1}{2}$.

(7 marks)

- (a) Use a discrete probability distribution to determine the probability that the number of people in one sample who have blue eyes is less than 7%.
- Random samples of 165 people are taken from a large population. It is known that 8% of the population have blue eyes.
- in one sample who have blue eyes is less than 7%.

- (b) Those with blue eyes is less than 7% in exactly half of these samples.
- Ten consecutive random samples are taken. Determine the probability that the number of those with blue eyes is less than 7% in exactly half of these samples.
- $P(Y=5) = 0.1271$ ✓ correct probability
- $Y \sim B(10, 0.3241)$ ✓ define binomial distribution (2 marks)
- alternative solution: $10C_5 \times (0.3241)^5 \times (1-0.3241)^5$ ✓ method clearly shown
- A large number of random samples of 165 people are taken. The proportion of blue eyed people calculated for each sample and the distribution of these sample proportions analysed.
- (c) Describe the continuous probability distribution that these sample proportions approximate, including any parameters.

- $X \sim N(0.08, 0.0211^2)$ ✓ normal distribution
- $\sigma = 0.0211$ ✓ correct σ , σ^2
- $p = 0.08$ ✓ correct mean
- $\mu = \frac{0.08 \times 0.165}{0.08 \times 0.165} = 0.1271$ ✓ answer
- including any parameters.

- $P(X < 11.55) = P(X \leq 11) \text{ or } P(X \leq 12)$ ✓ correct probability
- $= 11.55 \%$ of 165 = 7% of 165
- $X \sim B(165, 0.08)$ ✓ define binomial distribution (3 marks)
- in one sample who have blue eyes is less than 7%.
- those with blue eyes is less than 7% in exactly half of these samples.

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- alternative solution: $10C_5 \times (0.3241)^5 \times (1-0.3241)^5$ ✓ method clearly shown

Question 6

The discrete random variable X is defined by

$$P(X = x) = \begin{cases} \frac{2x+k}{3} & x = 0, 1 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Determine the value of the constant k .

$$\begin{aligned} \frac{2(0)+k}{3} + \frac{2(1)+k}{3} &= 1 && \checkmark \text{ forms equation} = 1 \\ \frac{2k+2}{3} &= 1 \\ 2k+2 &= 3 \\ 2k &= 1 \\ k &= \frac{1}{2} && \checkmark \text{ correct value.} \end{aligned}$$

- (b) Determine

(i) $P(X = 0)$.

$$\begin{aligned} P(X = 0) &= \frac{2(0)+\frac{1}{2}}{3} && (1 \text{ mark}) \\ &= \frac{1}{6} && \checkmark \text{ correct probability.} \end{aligned}$$

(ii) $E(3X - 1)$.

$$\begin{array}{c|cc} x & (1-p) & p \\ \hline P(X=x) & \frac{1}{6} & \frac{5}{6} \end{array}$$

Bernoulli

(iii) $\text{Var}(3X - 1)$.

$$\begin{aligned} \text{Var}(X) &= p(1-p) && (2 \text{ marks}) \\ &= \frac{5}{6} \left(\frac{1}{6}\right) \\ &= \frac{5}{36}. \end{aligned}$$

$$\begin{aligned} \text{Var}(3X-1) &= 3^2 \left(\frac{5}{36}\right) && \checkmark \text{ Var}(3X-1) \\ &= \frac{45}{36} \end{aligned}$$

See next page

(7 marks)

Question 15

The probability density function for a continuous random variable T is given by:

$$f(t) = \begin{cases} at(t-3) & 0 \leq t \leq 2 \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Show use of calculus to determine the value of the constant a .

(4 marks)

$$\begin{aligned} \text{or } \int_0^2 at(t-3) dt &= 1 && \checkmark \text{ correct integral with limits } = 1 \\ \int_0^2 at^2 - 3at dt &= 1 \\ a \left[\frac{t^3}{3} - \frac{3t^2}{2} \right]_0^2 &= 1 && \checkmark \text{ antiderivative} \\ \text{or } \left[\frac{at^3}{3} - \frac{3at^2}{2} \right]_0^2 &= 1 \\ \left(\frac{8a}{3} - \frac{12a}{2} \right) - 0 &= 1 && \checkmark \text{ substitution} \\ a = -0.3 \text{ or } -\frac{3}{10} & && \checkmark a = \end{aligned}$$

- (b) Determine $P(1 \leq T \leq 2)$

$$\begin{aligned} \int_1^2 -0.3t^2 + 0.9t dt & && \checkmark \text{ correct definite integral with } a \text{ from part a).} \\ = 0.65 \text{ or } \frac{13}{20} & && \checkmark \text{ probability} \end{aligned}$$

- (c) If $E(T) = \frac{6}{5}$ determine the variance of T .

$$\begin{aligned} \sigma^2 &= \int_0^2 \left(t - \frac{6}{5}\right)^2 \cdot f(t) dt && \checkmark \text{ correct integral} \\ &= 0.24 \text{ or } \frac{6}{25} && \checkmark \sigma^2 \end{aligned}$$

- (d) Find the median of T .

$$\begin{aligned} \int_0^K -0.3t^2 + 0.9t dt &= 0.5 && \checkmark \text{ correct integral} \\ K = 1.238 & && = 0.5 \\ \checkmark \text{ answer rounds to } 1.24 & && \end{aligned}$$

See next page

With reason
✓ *reverse mark*

See next page

slightly overmarks area as curve is towards upwards.

$$\frac{0.426 + 0.654}{2} = 0.54$$

two eschmarks

(c) Use your previous answers to determine a numerical estimate for I . And explain whether your estimate is smaller or larger than the exact value of I . (2 marks)

$$= 0.654$$

correct sum

$$= 0.2(3.27)$$

$$= 0.2[0.68 + 1.03 + 1.56]$$

$$A = 0.2[f(0.6) + f(0.8) + f(1)]$$

(b) In a similar manner to (a), determine the best estimate for the value of the constant U , where $I < U$. (2 marks)

$$A \text{ is based on undetermined coefficients which result in a straight line fit to the data.}$$

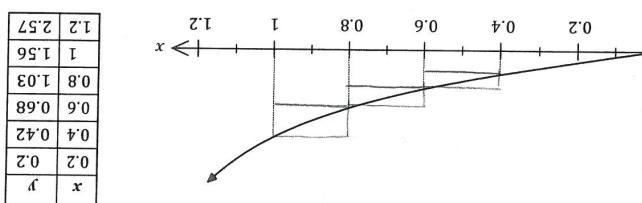
$$= 0.426$$

correct sum

$$= 0.2(2.13)$$

$$A = 0.2[f(0.4) + f(0.6) + f(0.8)]$$

(a) By using the information shown and considering sums of the form $\sum f(x_i)dx_i$, explain why $I > 0.426$. (3 marks)



The graph and a table of values for $y = f(x)$ is shown below, where $f(x) = \tan x$.

$$\text{Let } I = \int_1^4 \tan x \, dx.$$

(7 marks)

Question 7

CALCULATOR FREE

9

METHODS UNITS 3&4

(7 marks)

Question 14
(6 marks)

8

CALCULATOR ASSUMED

The voltage (in volts) generated by a circuit at time t seconds is given by $V(t) = 6e^{0.2t} \cos(3t)$ for $0 \leq t \leq 4$.

Question 14
(6 marks)

METHODS UNITS 3&4

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The voltage (in volts) generated by a circuit at time t seconds is given by $V(t) = 6e^{0.2t} \cos(3t)$ for $0 \leq t \leq 4$.

METHODS UNITS 3&4

9

CALCULATOR ASSUMED

Question 14
(6 marks)

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METHODS UNITS 3&4

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METHODS UNITS 3&4

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Question 8

(6 marks)

The acceleration at time t seconds of a small body travelling in a straight line is given by

$$a(t) = \frac{-12}{\sqrt{4t+5}} \text{ cm/s}^2, \quad t \geq 0.$$

When $t = 1$ the body was at the origin and 4 seconds later its displacement was 2 cm.

$$v(t) = \int \frac{-12}{(4t+5)^{\frac{1}{2}}} dt$$

$$= -12 \cdot \frac{(4t+5)^{\frac{1}{2}}}{\frac{1}{2} \cdot (4)} + C$$

$$= -6(4t+5)^{\frac{1}{2}} + C$$

$t=1 \quad x=0$
change in displacement
 $\Delta x = 2$.

✓ antiderivative
of $a(t)$

$$\Delta x = \int_1^{4+1} v(t) dt$$

$$= \int_1^5 -6(4t+5)^{\frac{1}{2}} + C dt$$

✓ integral
for Δx

$$= \left[-6 \cdot \frac{(4t+5)^{\frac{3}{2}}}{\frac{3}{2} \cdot (4)} + Ct \right]_1^5$$

✓ antiderivative
of $v(t)$

$$= \left[-6(4t+5)^{\frac{3}{2}} + Ct \right]_1^5$$

$$= -(25)^{\frac{3}{2}} + 5C + 9^{\frac{3}{2}} - C$$

$$= -5^3 + 3^3 + 4C$$

$$\Delta x = -98 + 4C$$

✓ simplifies
equation for C

$$\Delta x = 2 \quad \therefore 4C - 98 = 2$$

$$4C = 100$$

$$C = 25$$

✓ uses Δx to
determine value
of C

$$v(11) = -6(4(11)+5)^{\frac{1}{2}} + 25$$

$$= -6(49)^{\frac{1}{2}} + 25$$

$$= -42 + 25$$

$$= -17 \text{ cm/s} \quad \checkmark \text{ correct velocity.}$$

UNITS

Question 13

(8 marks)

The heights of girls H in a large study of 3-year-old children are normally distributed with a mean of 94.5 cm and a standard deviation of 3.15 cm.

- (a) Determine the probability that a randomly selected girl from the study has a height

- (i) that rounds to 93 cm, to the nearest cm.

(2 marks)

$$P(92.5 < H < 93.5) = 0.1127$$

✓ 9.5 < x < 93.5

✓ correct p to
at least 4 dp.

- (ii) of at least 90 cm given that they are shorter than 94.5 cm.

(2 marks)

$$P(H \geq 90 / H < 94.5) = P(90 \leq H < 94.5)$$

✓ indicates
correct use of
conditional prob.

$$= \frac{0.4234}{0.5}$$

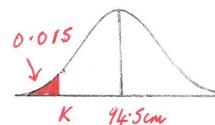
✓ correct
prob.

$$= 0.8469$$

(unit)

- (b) The shortest 1.5% of girls were classified as unusually short. Determine the greatest height of a girl to be classified in this manner.

(1 mark)

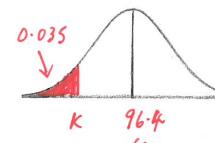


$$P(X < K) = 0.015$$

$$K = 87.66 \text{ cm}$$

✓ correct
height.

- (c) The heights of boys in the study are normally distributed with mean of 96.4 cm and the shortest 3.5% of boys, with a height less than 90.2 cm, were classified as unusually short. Demonstrate use of the standard normal distribution to determine the standard deviation of the boys' heights.



$$P(Z < K) = 0.035$$

$$K = -1.8119$$

✓ indicates use
of z-score
to obtain
 $K = -1.8119$

$$\therefore \frac{90.2 - 96.4}{\sigma} = -1.8119$$

$$\sigma = 3.42 \text{ cm}$$

✓ forms equation
for σ

(unit)
Demonstrate
for
3 marks

✓ correct σ

If value for
 σ given and not
rounded correctly - wrong
unless full value shown
and then rounded incorrectly

See next page

Supplementary page

Question number: _____

Question 11

(7 marks)

In a sample of 1 325 university students, 64% said that they never look at their phone while driving.

- (a) Show how to use the figures from this sample to construct the 95% confidence interval for the proportion of university students who never look at their phone while driving. (3 marks)

$$\hat{p} = 0.64$$

$$s = \sqrt{\frac{(0.64)(0.36)}{1325}} \\ = 0.0132$$

✓ standard deviation correct

$$CI = \hat{p} \pm 1.96(s) \\ = 0.64 \pm (1.96)(0.0132) \\ = (0.6142, 0.6658)$$

✓ show correct use of formula for C.I.

(0.614, 0.666)
✓ correct C.I. to at least 3 dp

- (b) According to a newspaper article, "70% of university students never look at their phone while driving". Explain whether the interval from (a) supports this claim. (2 marks)

Interval does not support claim as 0.7 does not lie within the interval.

Careful with interpretations of confidence interval
- Not required here anyway!

✓ not supported stated
✓ interval does not include 0.7.

- (c) Another source claims that "the majority of university students never look at their phone while driving". Explain whether the interval from (a) supports this claim. (2 marks)

Does support this claim as both lower and upper bound of interval are greater than 0.5

✓ states claim supported

✓ states bounds > 0.5.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	36%
Section Two: Calculator-assumed	12	12	100	93	64%
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your response to the specific questions asked and to follow any instructions that are specified to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- Do not place the Formula Sheet inside your Question/Answer booklet. It will be collected separately.

Markers Use Only		
Question	Max.	Mark
9	6	
10	7	
11	7	
12	8	
13	8	
14	6	
15	10	
16	10	
17	8	
18	8	
19	8	
20	6	
Section Two Total	92	
Section Two %		

See next page

Section Two: Calculator-assumed

(93 Marks)

This section has twelve questions. Answer all questions. Write your answers in the spaces provided.
Working time: 100 minutes.

Question 9

(6 marks)

- (a) Function f is defined by $f(x) = 3 \log_6(x+6) - 2$ over its natural domain. Determine
(i) the value of the y -intercept of the graph of $y = f(x)$.

$$(0, 1)$$

$$y=1$$

✓ correct value

- (ii) the equation of the asymptote of the graph of $y = f(x)$.

$$x = -6$$

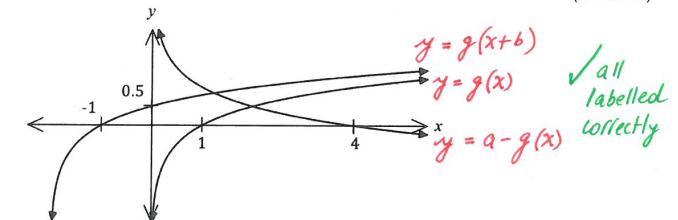
✓ correct EQUATION

- (b) Function g is defined by $g(x) = \log_n x$ over its natural domain, where n is a constant greater than 1.

The graphs shown below have equations $y = g(x)$, $y = a - g(x)$ and $y = g(x+b)$, where a and b are constants.

- (i) Label each graph with the appropriate equation from those listed above.
(ii) Determine the value of b and hence determine the value of n and a .

label graphs with equations
 $y =$



$$\text{At } (-1, 0) \quad g(-1) = \log_n (-1+b) = 0 \\ n^0 = -1+b \\ b = 2$$

✓ correct value of b

$$\text{At } (0, 0.5) \quad \log_n (0+b) = 0.5 \\ n^{0.5} = 2 \\ n = 4$$

✓ value of n

$$\text{At } (4, 0) \quad a - \log_4 4 = 0 \\ a = 1$$

✓ value of a

See next page