

Question	Marks	Max	Question	Marks	Max
3		7	7	6	7
2		9	5		12
1		7	4		8

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

### Important note to candidates

Special items: **nil**

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters  
**To be provided by the candidate**

Formula sheet

This Question/Answer booklet

**To be provided by the supervisor**

### Materials required/recommended for this section

Reading time before commencing work: five minutes  
 Working time: fifty minutes  
**Time allowed for this section**

Reading time before commencing work: five minutes  
 Working time: fifty minutes  
**Time allowed for this section**

Your Teacher's Name:

Your Name:

Calculator-free

Section One:

### MATHEMATICS METHODS

### UNIT 3 & 4

Question/Answer booklet

Semester One Examination, 2021

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	50	33
Section Two: Calculator-assumed	11	11	100	100	67
<b>Total</b>					<b>100</b>

**Additional working space**

Question number: \_\_\_\_\_

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Solutions	Behaviour
$P(x=0) = \binom{4}{3} \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^1 = \frac{81}{64}$	$P(x \geq 1) = 1 - \frac{81}{64} = \frac{625}{64} \approx 0.9765625$
	Recognises $P(x \geq 1)$
	Uses the complementarity event.
	States the correct probability (2 marks) (accept index form)

(c) Determine the probability of the student getting **at least** one night of 8+ hours of sleep during the 4 nights. (3 marks)

<input checked="" type="checkbox"/> <b>Solutions</b>	$E(X) = 4 \cdot 0.4 = 1.6$ $Var(X) = 4 \cdot 0.4 \cdot (0.6) = 0.96$	<b>Behaviour</b>	<input checked="" type="checkbox"/> States the correct $E(X)$ . <input checked="" type="checkbox"/> States the correct $Var(X)$ .
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(b) Determine  $E(X)$  and  $\text{Var}(X)$ . (2 marks)

Solutions	$X \sim Bin(4, 0.4)$	Behaviour	States Binomial	States correct Parameters
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(a) Determine the probability distribution of  $X$ . (2 marks)

Let  $X$  represent the number of nights where the student gets at least 8 hours of sleep during the next 4 nights.

✓ marks)

Working time: 50 minutes.

Responses and/or additional space if required to continue your planning pages are included at the end of this booklet. They can be used for planning your responses or as additional space for planning, if you need to use the space at the top of the page for planning. If you use the space for planning, indicate this clearly at the top of the page. Continuity in answers: if you need to use the space for planning, indicate this clearly at the top of the page. Continuity in answers: if you need to use the space where the answer is continued, i.e. give the page number. Fill in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

This section has **six** questions. Answer **all** questions. Write your answers in the spaces provided.

**Section One: Calculations** (30 marks)

SACCOLATATOR-FREE METHODS

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Question number:

Additional working space

MATHEMATICS METHODS

## Question 2

(9 marks)

- (a)  $F(x) = \frac{1}{e^{f(x)}}$ ;  $f(3) = 0$  and  $f'(3) = -1$ , determine the value for  $F'(3)$ . (3 marks)

Solutions
$F'(3) = \frac{-e^{f(3)}}{(e^{f(3)})^2}$ $(e^{f(3)} \cdot 2f'(3)) = \frac{-e^0 \times f'(3)}{(e^0)^2} = -(-1) = 1$
Behaviour
✓ Demonstrates the use of quotient rule correctly. ✓ Substitutes correct values. ✓ Determines the correct answer.

- (b) Determine the gradient of the line tangent to the graph of  $y = \ln(\sqrt{3x+1})$  at  $x=1$ . (3 marks)

Solutions
$y = \ln \sqrt{3x+1}$
$y' = \frac{1}{2} \frac{3}{3x+1}$
$y'(1) = \frac{3}{8}$
Behaviour
✓ Simplifies $y$ by using log law. ✓ Determines the correct $y'$ ✓ Determines the correct gradient

- (c) Given that  $g(x) = [f(x)]^3$ ,  $f(0) = -\frac{1}{2}$  and  $f'(0) = \frac{8}{3}$ , determine an equation of the line tangent to the graph of  $g(x)$  at  $x=0$ . (3 marks)

Solutions
$g'(0) = 3[f(0)]^2 \times f'(0) = 3\left(-\frac{1}{2}\right)^2 \left(\frac{8}{3}\right) = 2$
$g(0) = [f(0)]^3 = -\frac{1}{8}$
$\frac{y - \left(-\frac{1}{8}\right)}{x - 0} = 2$
$y = 2x - \frac{1}{8}$
Behaviour
See next page

See next page

- |   |  |
|---|--|
| <ul style="list-style-type: none"> <li>✓ Equates 1<sup>st</sup> derivative to 0 and solves for <math>\theta</math></li> <li>✓ Uses 2<sup>nd</sup> derivative or otherwise to justify why maximum.</li> <li>✓ Demonstrates the use of product rule correctly for <math>g(x)</math></li> <li>✓ Determines the correct gradient <math>g'(0)=2</math></li> <li>✓ States the correct angle.</li> </ul> | <p>Note: Follow through will not occur if function is easy to differentiate.</p> |
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## Question 3

(7 marks)

A projectile is launched upward from ground level with an initial velocity of  $v_0=100 \text{ m/s}$  and acceleration  $a=-20 \text{ m/s}^2$ .

- (a) Determine the velocity at  $t=4 \text{ s}$  and  $t=6 \text{ s}$ .

(2 marks)

## Solutions

$$v(4)=20 \text{ m/s and } v(6)=-20 \text{ m/s}$$

## Behaviour

- ✓ Demonstrates the correct  $v(4)$
- ✓ Demonstrates the correct  $v(6)$

- (b) Determine the maximum height the projectile will rise and the time when that occurs.

(3 marks)

## Solutions

$$100-20t=0, t=5 \text{ s}$$

$$x(t)=\int v(t) dt = \int 100-20t dt = 100t - 10t^2 + x_0 (\because x_0=0)$$

$$x(t)=100t - 10t^2$$

$$x(5)=500-10(25)=250 \text{ m}$$

## Behaviour

- ✓ Demonstrates the correct time
- ✓ Demonstrates the correct expression for displacement
- ✓ Determines the correct maximum height

- (c) Determine the **speed** of the projectile when it hits the ground.

(2 marks)

## Solutions

$$x(t)=100t - 10t^2 = 0$$

$$t=10 \text{ s}$$

$$v(10)=100-20(10)=-100 \text{ m/s}$$

$$\|v(10)\|=100 \text{ m/s}$$

## Behaviour

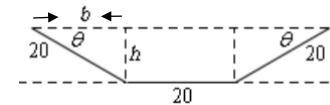
- ✓ Demonstrates the correct time when it hits the ground.
- ✓ States the correct speed

See next page

## Question 6

(7 marks)

A trough for holding water is formed by taking a piece of sheet metal 60 cm wide and folding the 20 cm on either end up as shown below.



- (a) Determine the expression for the base  $b$  and the height  $h$  in terms of  $\theta$ .

(2 marks)

## Solutions

$$b=20 \cos \theta, h=20 \sin \theta$$

## Behaviour

- ✓ Determines the correct  $b$ .
- ✓ Determines the correct  $h$ .

- (b) Determine the angle  $\theta$  that will maximise the amount of water that the trough can hold.

Hint:  $\sin^2 \theta = 1 - \cos^2 \theta$ .

(5 marks)

## Solutions

$$A = 20h + 2 \left( \frac{1}{2}bh \right) = 400 \sin \theta + (20 \cos \theta)(20 \sin \theta) = 400(\sin \theta + \sin \theta \cos \theta)$$

$$\begin{aligned} A'(\theta) &= 400(\cos \theta + \cos^2 \theta - \sin^2 \theta) \\ &= 400(\cos \theta + \cos^2 \theta - (1 - \cos^2 \theta)) \\ &= 400(2\cos^2 \theta + \cos \theta - 1) \\ &= 400(2\cos \theta - 1)(\cos \theta + 1) \end{aligned}$$

$$\begin{aligned} 2\cos \theta - 1 = 0 &\Rightarrow \cos \theta = \frac{1}{2} &\Rightarrow \theta = \frac{\pi}{3} \\ \cos \theta + 1 = 0 &\Rightarrow \cos \theta = -1 &\Rightarrow \theta = \pi \end{aligned}$$

$$\text{Hence } \theta = \frac{\pi}{3}$$

$$A''(\theta) = 400(-\sin \theta - 2\sin 2\theta) < 0,$$

Therefore,  $\theta = \frac{\pi}{3}$  will maximise the amount of water.

- ✓ Sets up the correct expression for the area/volume.
- ✓ Determines the correct 1<sup>st</sup> derivative.

See next page

Solutions	$P(X \geq 2) = 1 - 0.1 = 0.9$
Behaviour	Recognises $P(X \geq 2)$ .
^	Determines the correct probability.

(c) Determine  $P(3X+2 \geq 8)$ . (2 marks)

Solutions	$E(X) = 1 \cdot 0.1 + 2 \cdot 0.4 + 3 \cdot 0.3 + 4 \cdot 0.2 = 2.6$
Behaviour	Uses the formula for $E(X)$ .
^	Determines the correct value.

(b) Determine  $E(X)$ . (2 marks)

Solutions	$d=0.8$ , $c=0.2$ , $b=0.4$ , $a=0.1$
Behaviour	Sets up one equation correctly
^	Sets up two equations correctly
(Maximum 2 marks for answers only)	Solves for one correct value

(a) Determine the values for  $a$ ,  $b$ ,  $c$  and  $d$ . (4 marks)

$X$	1	2	3	$d$	4
	0.1	0.5			1

Where  $d$  is a constant.The cumulative distribution function  $C(x) = P(X \leq x)$  of  $X$  is given in the following table.

$X$	1	2	3	$c$

Where  $a$ ,  $b$  and  $c$  are constants.The discrete random variable  $X$  has a probability distribution as follows.**Question 4** (8 marks)

Solutions	$\frac{d}{dx}(\ln x)^2 = \frac{2 \ln(x)}{x}$
Behaviour	$\int \frac{x}{\ln x} dx = \frac{\ln(x)^2}{2} - \frac{1}{2} \ln(\ln x)^2 + C$
^	$\int \frac{x}{\ln(x)^2} dx = \frac{1}{2} \ln(\ln x)^2 + C$
NOTE- no follow through if F.T.C is not used	$\int \frac{x}{\ln(x)^2} dx = \frac{x}{2 \ln(x)} - \frac{1}{2} + C$
(Maximum 8 marks)	Demonstrates the use of F.T.C.

## Question 5

(12 marks)

(a)  $\frac{d}{dx}(\ln x)^2$ .

(2 marks)

**Solutions**

$$\frac{d}{dx}(\ln x)^2 = 2 \ln(x) \left( \frac{1}{x} \right) = \frac{2 \ln(x)}{x}$$

**Behaviour**

- ✓ Demonstrates the use of chain rule.
- ✓ Determines the correct derivative.

Consider the function  $f(x) = \frac{\ln(x)}{x}$ , for  $x > 0$ .

(b) Determine the coordinate of the turning point of  $f(x)$ .

(3 marks)

**Solutions**

$$\frac{d}{dx} \frac{\ln(x)}{x} = \frac{\frac{1}{x}(x) - 1\ln(x)}{x^2} = \frac{1 - \ln(x)}{x^2} = 0$$

$$\ln(x) = 1, x = e$$

$$f(x) = \frac{\ln(e)}{e} = \frac{1}{e}$$

$$T.P\left(e, \frac{1}{e}\right)$$

**Behaviour**

- ✓ Determines the correct 1<sup>st</sup> derivative.
- ✓ Equates 1<sup>st</sup> derivative to 0 and solve for  $x$ .
- ✓ States the correct T.P.

(12 marks)

(c) Determine the coordinate of the point(s) of inflection of  $f(x)$ .

(3 marks)

**Solutions**

$$\frac{d}{dx} \frac{1 - \ln(x)}{x^2} = \frac{-\frac{1}{x}(x^2) - (1 - \ln(x))(2x)}{x^4} = \frac{2\ln(x) - 3}{x^3} = 0$$

$$\ln(x) = \frac{3}{2}, x = e^{\frac{3}{2}}$$

$$f(x) = \frac{\ln(e^{\frac{3}{2}})}{e^{\frac{3}{2}}} = \frac{3}{2e^{\frac{3}{2}}}$$

$$P.O.I\left(e^{\frac{3}{2}}, \frac{3}{2e^{\frac{3}{2}}}\right)$$

**Behaviour**

- ✓ Determines the correct 2<sup>nd</sup> derivative.
- ✓ Equates 2<sup>nd</sup> derivative to 0 and solve for  $x$ .
- ✓ States the correct P.O.I.

(d) Determine the value for  $a$ , so that the area of the region enclosed by  $f(x)$ ,  $x$ -axis and  $x=a$  is exactly  $\frac{1}{2}$ .

(4 marks)

