SEMESTER TWO 2017

YEAR 12, Units 3 & 4

MATHEMATICS METHODS

Section Two – Booklet 2
(Calculator–assumed)

Name: Marking Key

Teacher: **MAW** **VMU** **MPC** **AGC**

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: ten minutes

Working time for section: one hundred minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: drawing instruments, templates, notes on up to two unfolded sheet of A4 paper, and up to three calculators approved for use in the ATAR examinations.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section Two.
Formula sheet retained from Section One.

Max -1 for lack of rounding and lack of/or incorrect units.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	9	9	50	50	35
Section Two Calculator—assumed	**12**	**12**	**100**	**86**	**65**
			Total marks	136	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2017.* Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the Question/Answer booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

Spare Working Page

Spare Working Page

Question 10 **(2+3 = 5 marks)**

Gold-198 is a radioactive antineoplastic that is used in the treatment of some cancers. It has a half-life of 2.7 days. This means that every 2.7 days, one half of the gold-198 decays to form an isotope that is no longer radioactive.

This decay can be represented by the equation $A = A_0 e^{kt}$

Where A = amount of gold-198 present after t days, and
A_0 = initial amount of gold-198 initially present.

a) Determine the value of k correct to three decimal places.

$\frac{1}{2} = e^{2.7k}$ ✓ Equation to solve

$k = \frac{\ln(\frac{1}{2})}{2.7}$

$k = -0.257$ (3dp) ✓ correct solution

2

b) A patient is administered a dose of 27 grams of gold-198. During which day after administering the dose, is there less than 2 grams in the patient's system?

Solve $2 = 27e^{-0.257t}$ ✓ Equation to solve

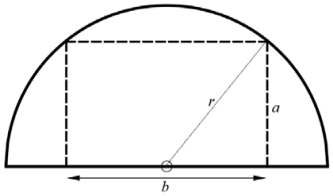
$t = 10.14$ (2dp) ✓ solution

∴ During the 11th day ✓ Answer in context of the question.

3

Question 11 **(3+6 = 9 marks)**

A rectangle is inscribed in a semicircle, as shown in the diagram below.



Where:
- r is the radius of the semicircle, which is 8cm.
- a is the height of the inscribed rectangle.
- b is the width of the inscribed rectangle

a) Show the area of the inscribed rectangle is given by: $A = b\sqrt{64 - \frac{b^2}{4}}$

$\therefore r^2 = a^2 + \left(\frac{b}{2}\right)^2$ ✓ Application of Pythagoras

$r^2 = a^2 + \frac{b^2}{4}$

$\therefore 64 = a^2 + \frac{b^2}{4}$

$a^2 = 64 - \frac{b^2}{4}$

$a = \sqrt{64 - \frac{b^2}{4}}$ ✓ Getting a in terms of b

$A = a \times b$

$= \left(\sqrt{64 - \frac{b^2}{4}}\right) \times b$

$\therefore A = b\sqrt{64 - \frac{b^2}{4}}$ ✓ Correct substitution.

3

b) Use calculus methods to calculate the dimensions of the inscribed rectangle of maximum area.

$\frac{dA}{db} = \frac{128 - b^2}{\sqrt{256 - b^2}}$ ✓ correct 1st derivative

Solve $0 = 128 - b^2$ ✓ Solve derivative to 0.

$b = \pm\sqrt{128}$

Use $b = \sqrt{128}$

$a = \sqrt{32}$

Check Nature: ✓ correct 2nd derivative test

$\left.\frac{d^2A}{db^2}\right|_{b=\sqrt{128}} = -2$

Since $\frac{d^2A}{db^2} < 0$

$\therefore$ Local Max ✓ decision

6

$\therefore$ Dimensions are $a = \sqrt{32}$ cm ✓

$b = \sqrt{128}$ cm ✓ Dimensions.

Spare Working Page

Question 14 **(2+3+2+3 = 10 marks)**

The stylus for the Surface Pro uses a single AAAA battery. These batteries have an operational life which is normally distributed with a mean of 85 hours and standard deviation of 10 hours.

a) Determine the probability that a randomly chosen stylus will fail, due to its battery, before 100 hours of use.

$B \sim N(85, 10^2)$ ✓ Distribution

$P(B < 100) = 0.9332$ (4dp) ✓ Correct answer. [2]

Samples of size n, where $n > 50$, stylus batteries are selected and the proportion with a life exceeding 100 hours is recorded.

b) For $n = 120$ describe the sampling distribution of the proportion of stylus batteries with an operational life exceeding 100 hours.

$\hat{p}$ will be normally distributed $u = 1 - 0.9332 = 0.0668$

and std dev $= \sqrt{\frac{0.9332 \times (1-0.9332)}{120}} = \sqrt{\frac{0.0623}{120}}$ [3]

✓Normal ✓u ✓σ $\sigma^2 = 0.02279$

$\therefore$ Distribution $\sim N\left(0.0668, \frac{0.0623}{120}\right)$

c) For $n = 120$, calculate the probability that a randomly exceeding chosen sample has a sample proportion of stylus batteries with an operational life exceeding 100 hours of between 0.02 and 0.03.

$P(0.02 < \hat{p} < 0.03) = 0.03317$ (5dp) ✓correct answer [2]

✓probability statement.

d) Determine the possible values of n if the standard deviation of the sampling distribution, $n > 50$, of the proportion of stylus batteries with an operational life exceeding 100 hours is not to exceed 0.01.

Solve $\sqrt{\frac{0.9332 \times (1-0.9332)}{n}} \leq 0.01$ ✓Equation to solve

$n \geq 623.44$ ✓Solution to Equation [3]

$\therefore$ n needs to be greater than or equal to 624. ✓Correct answer Correct rounding.

End of Questions for Booklet 2

Question 12 **(1+2+2+3+3=11 marks)**

A newspaper delivery vehicle sets out each morning at 5:00am to deliver newspapers to his customers. He has two delivery routes, A and B, he can take to deliver newspapers to his customers. All newspapers should be delivered by 6:00am.

When the delivery vehicle uses route A, the delivery times can be assumed to be normally distributed with a mean of 55 minutes and a standard deviation of 10 minutes.
When the delivery vehicle uses route B, the delivery times can be assumed to be normally distributed with a mean of μ minutes and a standard deviation of 12 minutes.

a) For delivery route A, calculate the probability the deliveries take longer than 60 minutes.

$A \sim N(55, 10^2)$ $P(A > 60) = 0.3085$ (4dp) ✓Correct answer. [1]

b) For delivery route B, the probability the delivery takes less than 60 minutes is 0.7. Determine the value of μ.

$P(B < 60) = 0.7$ ✓ Probability Statement.

from ClassPad

$\mu = 53.7072$ (4dp) ✓Correct answer. [2]

c) Which delivery route should the driver choose? Provide reasoning.

Route A $\rightarrow N(55, 10^2)$ $P(A > 60) = 0.3085$

Route B $\rightarrow N(53.71, 12^2)$ $P(B > 60) = 0.3$ ✓Reasoning

B is better route – less chance of taking more than 1 hour
– Lower mean time. ✓correct decision based on their reasoning

* Decision with no reasoning – zero. [2]

Question 12 continued

d) Given the delivery driver took route A and was late completing the deliveries, calculate the probability deliveries were completed by 6:10am.

$P(A<70 \mid A>60) = \frac{P(60<A<70)}{P(A>60)}$ ✓ Probability Statement

$= \frac{0.24173}{0.30854}$ ✓ ✓ } correct probability values.

$= 0.7835$ (4dp)

3

e) On five consecutive days the delivery driver takes route B.
Determine the probability:

i) he completes all newspaper deliveries before 6:00am on all five days.

$P(\text{All deliveries before 6am}) = 0.75$

$= 0.16807$ ✓ correct answer.

1

ii) he completes the deliveries before 6:00am on exactly 3 of the 5 days.

$X \sim B(5, 0.7)$

$P(\text{Late on 2 days}) = \binom{5}{3}(0.7)^3(0.3)^2$

$= 0.3087$ ✓✓ correct answer.

2

Question 13 **(5 marks)**

Sketch the function $y = f(x)$ given the following features.

- $f(-2) = f(4) = 0$ ✓
- $\frac{dy}{dx} = 0$ at $x = -2$ and $x = 2$ ✓
- $\frac{dy}{dx} < 0$ for $-2 < x < 2$ and $x < -2$ ✓
- $\frac{d^2y}{dx^2} = 0$ at $x = -2$ and $x = -1$ ✓
- $\frac{d^2y}{dx^2} > 0$ for $x < -2$ and $x > -1$ ✓

one for each of the features indicated/labelled correctly

5

Clearly label all key features.