PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course _____Specialist_____ Year ___12_____

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: **____40_____ mins**

Number of questions: **_____7_____**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **___38___ marks**

Task weighting: **_10___%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2, 2 & 3 = 7 marks) (3.1.1 to 3.1.3)

If $z = 3 - 4i$ & $w = -1 + 2i$ determine the following.

a) $w\overline{z}$

b) $\frac{z}{w}$

c) $\frac{1}{z} - \frac{1}{w}$

Q2 (3 marks) (3.1.2)

Determine all possible pairs of real numbers $a \& b$ such that $\frac{19 - 33i}{a + 2i} = 1 + bi$

Q3 (2 & 3 = 5 marks) (3.1.13- 3.1.15)

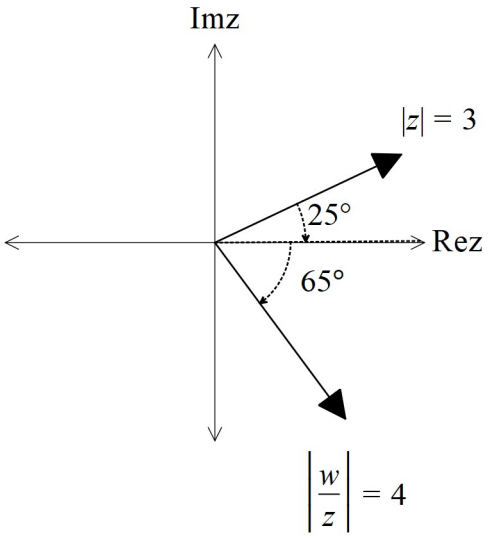
Consider the function $f(x) = x^3 - 5x^2 + 9x - 45$.

a) Determine the remainder of $f(x)$ when divided by $x - 5$.

b) Show that $x - 3i$ is a factor of $f(x)$ and hence determine all linear factors.

Q4 (3 marks) (3.1.9)

Determine the complex number w in the form $rcis\theta$ with $r \geq 0$ & $-180 < \theta \leq 180$.



Q5 (2, 2, 3 & 3 = 10 marks) (3.1.10)

Consider the following set of complex numbers z such that $|z - 5 - 3i| = 4$.
Determine the following.

a) Minimum value of $|z|$. (exact)

b) Maximum value of $|\overline{z}|$. (exact)

c) Maximum value of Arg(z) in radians to two decimal places.

d) Maximum value of $|z + 3|$ (exact)

Q6 (3 & 3 = 6 marks) (3.1.6)

Let $p, q \& s$ be complex numbers such that

$$|p| = 5 \quad Arg(p) = \frac{\pi}{6} \quad \overline{q} = 1 - i$$

$$s = \frac{p^5}{(3 + 3i)q}$$

a) Determine the exact value of Arg(s) in principal form (i.e $-\pi < Arg(s) \leq \pi$)

b) Determine the exact value of $|s|$

Q7 (4 marks) (3.1.10)

Sketch the locus of complex numbers that satisfy **both** of the following $|z+2i| = |z-3| + \sqrt{13}$ **AND** $|z+2i| \leq \sqrt{13} + 5$ in the Argand diagram below.