

Question 3

Let $f(x) = -(x+1)^2(x-3)$.

(a) Use calculus to locate and classify all the stationary points of $f(x)$ and find any points of inflection.

$$f'(x) = -2(x+1)(x-3) - (x+1)^2 \cdot 1$$

$$= -(x+1)(2x-6+x+1)$$

$$= -(x+1)(3x-5) = 0$$

$$x = -1 \text{ or } x = \frac{5}{3}$$

x	$(-\infty, -1)$	-1	$(-\frac{5}{3}, \frac{5}{3})$	$(\frac{5}{3}, +\infty)$
$f'(x)$	-	0	+	-
$f''(x)$	↓	↓	↑	↑
		local min	local max	

$$f(-1) = 0 \text{ local min } (-1, 0)$$

$$f(\frac{5}{3}) = \frac{256}{27} \text{ local max } (\frac{5}{3}, \frac{256}{27}) \approx (1.67, 9.48)$$

point of inflection: $x = 1$

$$f''(x) = -(3x-5) - (x+1) \times 3$$

$$= -3x+5-3x-3$$

$$= -6x+2 = 0$$

$$x = \frac{1}{3}, y = -(\frac{1}{3}+1)^2(\frac{1}{3}-3)$$

$$= \frac{128}{27} (\approx 4.74)$$

$$P.O.I. (\frac{1}{3}, \frac{128}{27})$$

$$f''(-1) = -6(-1)+2 > 0$$

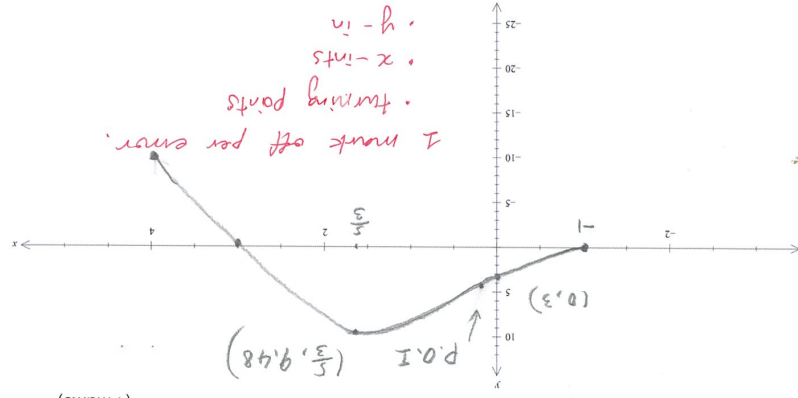
$$f'(-1) = 0, (-1, 0) \text{ local min}$$

$$f''(\frac{5}{3}) = -(3 \times \frac{5}{3} - 5) - (\frac{5}{3} + 1) \times 3 < 0$$

$$\therefore f(\frac{5}{3}) = \frac{256}{27}, (\frac{5}{3}, \frac{256}{27}) \text{ local max}$$

(b) On the axes provided sketch the graph of $f(x)$, $-1 \leq x \leq 4$, labelling all key features.

(4 marks)



- turning points
- x-ints
- y-int
- P.O.I.
- shape
- domain

Test 1

Differentiation, applications and Optimisation.
Basic antidifferentiation
Semester One 2018
Year 12 Mathematics Methods
Calculator Free



PERTH MODERN SCHOOL
exceptional schooling. exceptional students

Name:

CHENG

Date Monday 20th February 7:45am

You may have a formula sheet for this section of the test.

Total /21

20 Minutes

Question 1

(3 marks)

Given that the function f has a rule of the form $f(x) = ax^2 + bx$ and $f(1) = 6$ and $f'(1) = 0$, find the values of a and b .

$$\begin{aligned} f(1) &= a + b = 6 \\ f'(1) &= 2ax + b = 2a + b = 0 \end{aligned}$$

$$\begin{aligned} a &= -6 \\ b &= 12 \end{aligned}$$

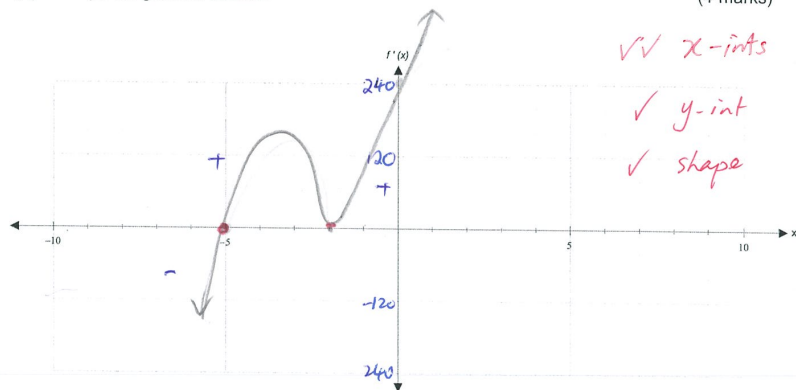
Question 2

(8 marks)

Consider the gradient function $f'(x) = 12(x+2)^2(x+5)$.

(a) Graph the gradient function

(4 marks)

(b) What kind of feature is at the point $(-5, -225)$ on the graph of $f(x)$?

(2 marks)

Handwritten notes: $(-\infty, -5)$ and $(-5, -2)$ are both decreasing (\downarrow). At $x = -5$, there is a local minimum (T.P. \checkmark). \therefore local min. \checkmark min. T.P.

(c) What kind of feature is at the point $(-2, -144)$ on the graph of $f(x)$?

(2 marks)

Handwritten notes: $(-5, -2)$ and $(-2, +\infty)$ are both increasing (\uparrow). At $x = -2$, the gradient is zero and the curve is horizontal. \therefore point of inflection. \checkmark

Question 2

(6 marks)

A beverage company has decided to release a new product. "Modmash" is to be sold in 375 mL cans that are perfectly cylindrical. {Hint: 1 mL = 1 cm³}(a) If the cans have a base radius of x cm show that the surface area of the can, S , is given

$$S = 2\pi x^2 + \frac{750}{x}$$

(2 marks)

$$\begin{aligned} S &= 2\pi x^2 + 2\pi x h \\ &= 2\pi x^2 + 2\pi x \times \frac{375}{\pi x^2} \quad \checkmark \\ &= 2\pi x^2 + \frac{750}{x} \quad \checkmark \end{aligned}$$

$$h = \frac{375}{\pi x^2} \quad \text{1 mark}$$

(b) Using calculus methods, and showing full reasoning and justification, find the dimensions of the can that will minimise its surface area

(4 marks)

$$S(x) = 2\pi x^2 + 750x^{-1}$$

$$S'(x) = 4\pi x - 750x^{-2} = 0 \quad \checkmark$$

$$x \approx 3.90796 \text{ cm.} \quad \checkmark$$

$$x \in (-\infty, 3.90796) \cup (3.90796, +\infty)$$

$$S'(x)$$

-

+

$$S(x)$$

 \downarrow \uparrow

(Explain why min.)

Hence, $S(x)$ reaches min at $x = 3.90796$ cm.

$$h = \frac{375}{\pi \times 3.90796^2} = 7.82 \text{ cm.} \quad \checkmark$$

$$= 3.91 \text{ cm}$$

OR $(S.A)'' = 37.70 > 0 \therefore \text{MIN}$

The model train has an initial velocity of 5 cm/s. After 2 seconds, it has a displacement of -50 cm. A further 4 seconds later its displacement is 178 cm. i.e. $t = 6$

$$\frac{1}{8}$$

(4 marks)

(b) Determine the value of the constant p .

$$d(t) = \frac{p}{3} - 13t^2 + 5t + d$$

$$d(t=2) = \frac{p}{8} - \frac{13 \times 4}{2} + 10 + d = -50$$

$$d(t=6) = \frac{p \times 6^3}{3} - \frac{13 \times 36}{2} + 6 \times 5 + d = 178$$

(substitution)

$$\therefore \begin{cases} p = 12 \\ d = -50 \end{cases}$$

(2 marks)

$$V(t) = 12t^2 - 13t + 5 = 6t^2 - 13t + 5 = 0$$

$$(2t-1)(3t-5) = 0$$

$$\therefore t = \frac{1}{2} \text{ sec or } \frac{5}{3} \text{ sec}$$

(d) How far did the model train travel during the 8th second? (2 marks)

$$d(t=8) - d(t=7)$$

$$= \left(12 \times (8)^3 - \frac{13 \times (8)^2}{2} + 5 \times 8 - 50 \right) - \left(12 \times 7^3 - \frac{13 \times 7^2}{2} + 5 \times 7 - 50 \right) = 245.5 \text{ cm}$$

(6 marks)

Clearly showing your use of the product, quotient or chain rule differentiate the following.

a) $10p(1-p)^9$

$$\frac{dy}{dx} = 10(1-p)^9 + 10p \times 9(1-p)^8 \times (-1) = 10(1-p)^9 - 90p(1-p)^8$$

(2 marks)

b) $\frac{1}{\sqrt{x+2}}$

$$\frac{dy}{dx} = -\frac{1}{2} (x+2)^{-\frac{3}{2}} = -\frac{1}{2} (x+2)^{-\frac{3}{2}}$$

(2 marks)

c) Consider the function $f(x) = (x-1)^2(x-2) + 1$

If $f'(x) = (x-1)(ux+v)$, where u and v are constants, use calculus to find the values of u and v .

$$f'(x) = 2(x-1)(x-2) + (x-1)^2 = (x-1)(2x-4) + (x-1)^2 = (x-1)(2x-4+x-1) = (x-1)(3x-5)$$

$$\therefore u=3, v=-5$$

Question 4

(4 marks)

The time T seconds, for one complete swing of a pendulum of length l m, is given by the rule $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant.

- (a) Determine $\frac{dT}{dl}$. $T = 2\pi\left(\frac{l}{g}\right)^{\frac{1}{2}} = 2\pi g^{-\frac{1}{2}} l^{\frac{1}{2}}$ (2 marks)

$$\frac{dT}{dl} = \frac{1}{2} \times 2\pi \times \left(\frac{l}{g}\right)^{-\frac{1}{2}} \times \left(\frac{1}{g}\right) = \pi \times \frac{\sqrt{g}}{\sqrt{l}} \times \frac{1}{g} = \frac{\pi}{\sqrt{gl}}$$

- (b) Using the formula $\partial T \approx \frac{dT}{dl} \times \partial l$, find the approximate increase in T when l is increased from 1.6 to 1.7. Give the answer in terms of g . (2 marks)

$$\delta T = \frac{\pi}{\sqrt{1.6g}} \times 0.1 \text{ sec}$$

$$= \frac{\pi}{\sqrt{1.6} \times \sqrt{10} \times \sqrt{g}} \times \frac{1}{10}$$

$$= \frac{\pi \times \sqrt{10}}{0.4 \times 10 \times \sqrt{g} \times 10}$$

$$= \frac{\pi \sqrt{10}}{40 \sqrt{g}}$$

$$= \frac{\pi \sqrt{10}}{40 \sqrt{g}} \text{ sec.}$$



PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.

Test 1

Differentiation, applications and Optimisation.

Basic antidifferentiation

Semester One 2018

Year 12 Mathematics Methods

Calculator Assumed

Name: CHENG

Date: 11 February 7.45am

You may have

- a formula sheet
- one page of A4 notes, one side
- a scientific calculator
- a classpad

Teacher:

_____ Mr McClelland
_____ Mrs. Carter
_____ Mr Gannon
_____ Ms Cheng
_____ Mr Staffe
_____ Mr Strain

Total: _____/24

25 minutes

Question 1

(9 marks)

A model train travels on a straight track such that its acceleration after t seconds is given by $a(t) = pt - 13 \text{ cm/s}^2$, $0 \leq t \leq 10$, where p is a constant.

- (a) Determine the initial acceleration of the model train.

(1 mark)

$$a(0) = p - 13 \text{ cm/s}^2$$