

Semester Two Examination, 2018  
Question/Answer booklet

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MATHEMATICS  
METHODS  
UNITS 1 AND 2  
Section Two:  
Calculator-assumed

Student Name \_\_\_\_\_

**Time allowed for this section**  
Reading time before commencing work: ten minutes  
Working time: one hundred minutes

**Materials required/recommended for this section**  
*To be provided by the supervisor*  
This Question/Answer booklet  
Formula sheet (retained from Section One)

*To be provided by the candidate*  
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper (both sides), and up to three calculators approved for use in this examination

**Important note to candidates**  
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	14	14	100	98	65
Total					100

**Instructions to candidates**

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet, preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that **you do not use pencil**, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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**Question 9** (6 marks)

(a) A sequence is defined by  $T_{n+1} = T_n - 3.7$ ,  $T_1 = 685$ . Determine the value of  $T_{90}$ . (1 mark)

(iii) the sum of the first 90 terms of this sequence. (2 marks)

(b) Another sequence is defined by  $T_n = 685(0.8)^{n-1}$ . Determine the value of  $T_4$ . (1 mark)

(iii) the value that the sum of the first  $n$  terms of the sequence approaches as  $n \rightarrow \infty$ . (2 marks)

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## Question 10

(5 marks)

- (a) Given that the graph of  $y = x^3 - 6x^2 + kx - 4$  has exactly one point at which the gradient is zero, determine the value of  $k$ . (3 marks)

- (b) Calculate the greatest and least values of  $4 - 3x^2 + x^3$ , for  $-2 \leq x \leq 3$ . (2 marks)

## Additional working space

Question number: \_\_\_\_\_

Additional working space

Question number: \_\_\_\_\_

**Question 11** (6 marks)

A mobile phone retailer classified recent sales of 375 phones by the age of customer and if the phone was bought outright or on a plan. A summary of the data is shown in the table below.

	Aged under 30	Aged 30 or over	Total
Bought outright	$p$	94	$r$
Bought on a plan	115	$q$	224
Total			375

(a) Determine the values of  $p$ ,  $q$  and  $r$  shown in the table. (3 marks)

(b) A recent sale is selected at random from those recorded above. Event  $A$  occurs if the customer was aged under 30 and event  $B$  occurs if the phone was bought outright. Determine the following probabilities:

(i)  $P(A \cup B)$ . (1 mark)

(iii)  $P(\bar{A} | \bar{B})$ . (2 marks)

See next page

## Question 12

(11 marks)

- (a) Calculate the area of the minor segment that subtends an arc of  $108^\circ$  in a circle of diameter 130 cm. (2 marks)

- (b) A chord of length 56 cm subtends an angle of  $\frac{\pi}{7}$  at the centre of a circle. Calculate the radius of the circle. (2 marks)

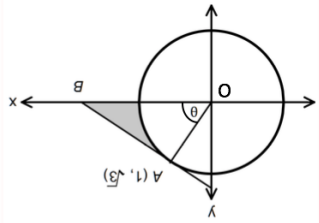
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## Additional working space

Question number: \_\_\_\_\_

Question 12 continued

(c) The point  $A$  with coordinates  $(1, \sqrt{3})$  lies on a circle with centre at the origin,  $O$ . A tangent to the circle is drawn at  $A$  and this intersects with the  $x$ -axis at the point  $B$ . Angle  $AOB = \theta$ .



(i) Calculate  $\theta$  in radians and the length of the radius of the circle. (3 marks)

(iii) Hence, calculate the area, correct to 2 decimal places, of the shaded region. (4 marks)

Question 22

Consider the curve

$$y = \frac{x^2}{8}$$

(a) Determine the equation of the tangent line to this curve at the point where  $x = a$ , giving your answer in terms of  $a$ . (4 marks)

(b) Give the coordinates of the  $x$  intercept for the tangent line in part (a). (1 mark)

End of questions

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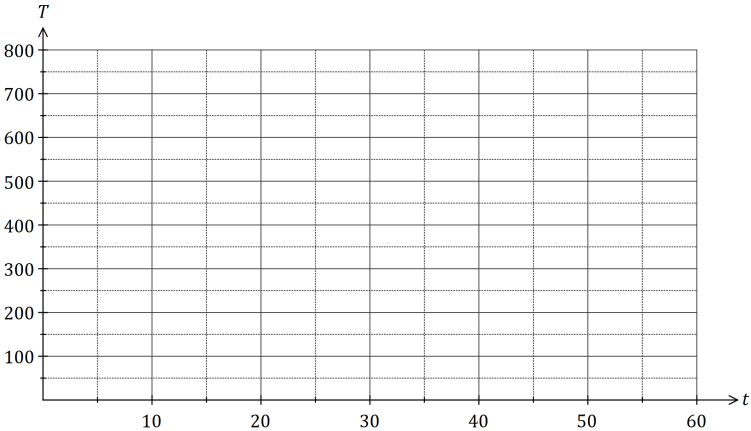
Question 13

(9 marks)

The temperature  $T$  of a cast taken out of an oven cools according to the model  $T = 790(0.95)^t$ , where  $t$  is the time in minutes since the cast was removed from the oven.  $T$  is measured in  $^{\circ}\text{C}$ .

(a) Determine the fall in temperature of the cast during the first 6 minutes. (2 marks)

(b) Graph the temperature of the cast against time on the axes below. (4 marks)



(c) State the name of this type of function. (1 mark)

(d) The temperature of the cast falls to room temperature of  $20^{\circ}\text{C}$ .

(i) Determine the time taken for the cast to reach room temperature. (1 mark)

(ii) Comment on the usefulness of the model for large values of  $t$ . (1 mark)

Question 21 continued

(c) Find the probability that a randomly chosen seventeen year old who has taken a driver's license test

(i) passed on the first attempt. (1 mark)

(ii) failed on the first attempt or is male. (1 mark)

(iii) failed on the first attempt, given that he is a male. (2 marks)



Question 14

Parallelogram  $PQRS$  has side  $QR = 24$  cm, side  $RS = 39$  cm and an area of  $460\text{ cm}^2$ . Determine the lengths of the diagonals of  $PQRS$ .

(4 marks)

Question 21

53% of seventeen year olds who take their driver's license test are male, 49% of seventeen year old males pass on the first attempt, while 58% of seventeen year old females pass on their first attempt.

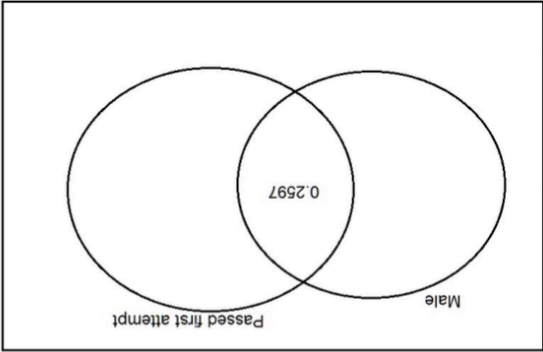
(8 marks)

- (a) From the population of seventeen year olds who have taken their driving test a random person is selected. Show that the probability that this person is a male who passed the test first time is 0.2597.

(1 mark)

- (b) Complete the Venn diagram below to represent the relevant probabilities for seventeen year olds who have taken their driving test.

(3 marks)



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## Question 15

(8 marks)

A council took a random sample of 125 and 172 properties from suburbs  $P$  and  $Q$  respectively. A total of 36 of the properties in the sample were in arrears with their rates, of which 21 of these properties were in suburb  $Q$ . 'In arrears' means that payment of rates is overdue.

- (a) Council officers wanted to randomly choose 4 of the properties that were in arrears. How many different selections of properties are possible? (1 mark)

- (b) Determine the probability that one randomly chosen property from the sample

- (i) is not in arrears and is in suburb  $Q$ . (2 marks)

- (ii) is in suburb  $P$  given that it is in arrears. (2 marks)

- (c) Justifying your answer with conditional probabilities and rounding the conditional probabilities to 2 decimal places in your analysis, comment on whether being in arrears with rates is independent of the suburb the property is in. (3 marks)

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## Question 20

(8 marks)

A pyramid with a rectangular base of length  $L$  and width  $w$  has perpendicular height  $h$ . The length of the base is three times its width and the sum of the width, length and height is 180 cm.

- (a) Calculate the length, height and volume of the pyramid when  $w = 15$  cm. (2 marks)

- (b) Show that the volume of the pyramid is given by  $V = 180w^2 - 4w^3$ . (2 marks)

- (c) Use calculus to determine the maximum volume of the pyramid. (4 marks)

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(6 marks)

Question 16  
A function is defined by  $f(x) = \frac{6}{x^4} - \frac{2x^3}{3}$ .

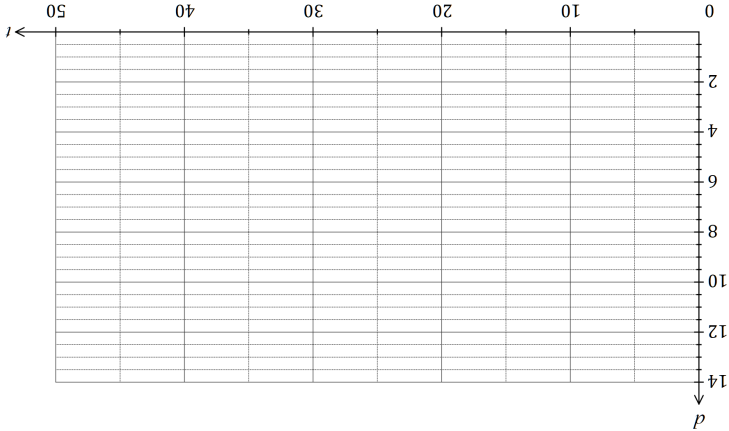
(a) Use the derivative  $f'(x)$  to determine the coordinates of all stationary points of the function. (3 marks)

(8 marks)

Question 19  
The height,  $h$  metres, above level ground of a seat on a steadily rotating giant viewing wheel at the Perth Show,  $t$  seconds after observations began was given by

$$h = 5.5 \sin\left(\frac{\pi t}{15}\right) + 7, \quad t \geq 0.$$

(a) Draw the graph of the height of the seat against time on the axes below. (4 marks)



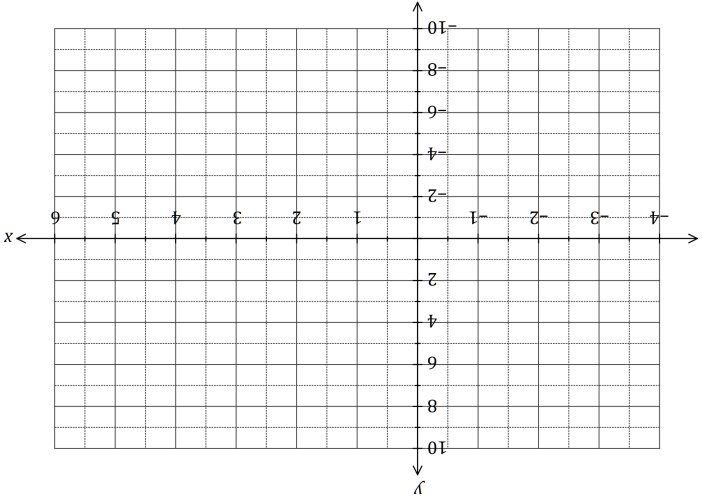
(b) How long did the Ferris wheel take to complete one revolution? (1 mark)

(c) At what time, when the seat was rising, did it first reach a height of 5 metres? (1 mark)

(d) Determine the change in height of the seat between  $t = 95$  and  $t = 96$ , giving your answer rounded to the nearest cm. (2 marks)

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(b) Sketch the graph of  $y = f(x)$  on the axes below. (3 marks)

**Question 17****(8 marks)**

John has two, initially empty, water containers, which are being filled with water. The amount of water added to container *A* each minute follows an arithmetic sequence, with 3 mL poured in during the first minute and 6 mL poured in during the second minute. The amount of water added to container *B* each minute follows a geometric sequence, with 2 mL poured in during the first minute and 2.2 mL poured in during the second minute.

- (a) The amount of water poured into container *B* during the  $n^{\text{th}}$  minute is given by  $a(r)^{n-1}$ . State the value of the constants  $a$  and  $r$ . (2 marks)

- (b) How long does it take to fill container *A* with 360 mL of water? (2 marks)

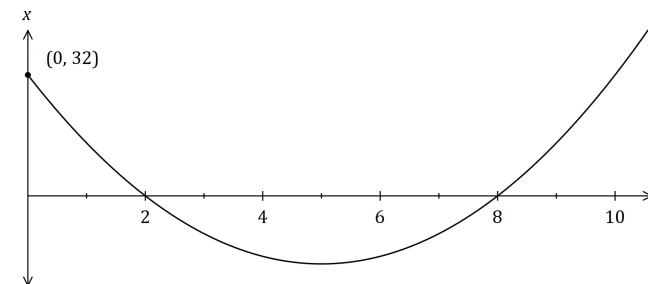
- (c) John measures the amount of water in each of the two containers at the end of each minute. He finds that container *B* first holds more than container *A* at the end of minute  $m$ . Calculate the value of  $m$ . (4 marks)

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**Question 18****(6 marks)**

A small body moves in a straight line so that its displacement  $x$  from a fixed point  $O$  after  $t$  seconds is given by  $x = at^2 + bt + c$  metres.

The position-time graph of the body is shown below.



- (a) Determine the values of the constants  $a$ ,  $b$  and  $c$ . (3 marks)

- (b) Determine the displacement of the body when its velocity is  $24 \text{ ms}^{-1}$ . (3 marks)

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