Stage 3 Physics:
Motion and Forces in a Gravitational Field
Student workbook Part 1 – TEACHER'S GUIDE

Contexts:
- Movement in sport, funfairs and play-grounds
- Motion of satellites, planetary motion and the universe
- Structures, bridges and buildings

Plan For Unit.
This workbook will give you an outline of the content to be covered and the related text pages. It is expected that you will follow the workbook and read the related text, experiments and investigations **before** the lesson. It will be assumed that you have read the text and have some introductory knowledge of the work to be covered each lesson; failure to do so may affect your progress in class. Your teacher will then teach you the concepts and show you how to do the examples in the workbook thus ensuring you have exemplars when completing additional questions from the workbook and texts. Your teacher also has all the worked answers to the additional questions in this workbook and you must check your answers when you complete the questions.

In addition to the work set for homework, it is essential that you set up a study plan and regularly review the work covered. This plan should be set up from day one. Regular reviewing not only makes study easy, it ensures good grades.

Wk	Content	Text Reference	Exploring Physics		Assessments
			Problem Sets	Experiments & Investigations	
1	**Students start Monday** 1. *describe and apply the principle of conservation of energy* 2. *resolve, add and subtract vectors in one plane* 3. draw free body diagrams, showing the forces acting on objects, from descriptions of real life situations involving forces acting in one plane 4. explain and apply the concept of centre of mass	CD Pg. 416-419 CD Pg. 421 CD Pg. 420	Set 1: Vectors	Expt: 1.1 Expt: 1.2	
2-3	5. describe and apply the concepts of distance and displacement, speed and velocity, acceleration, energy and momentum in the context of motion in a plane, including the trajectories of projectiles in the absence of air resistance —this will include *applying the relationships*: $v_{av} = \frac{s}{t}$, $v_{av} = \frac{v + u}{2}$, $a = \frac{v - u}{t}$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$ $p = mv$, $\sum p_{before} = \sum p_{after}$, $F\Delta t = mv - mu$ $E_k = \frac{1}{2}mv^2$, $E_p = mg\Delta h$, $W = Fs$, $W = \Delta E$ 6. describe qualitatively the effects of air resistance on projectile motion	Stage 2 Review CD Pg. 422-432 CD Pg. 433-444 Pg. 7-14 Pg. 19-22 Pg. 26-32	Set 2: Projectile Motion	Expt: 2.1 Expt: 2.2 (both needs camera and stroboscope lamp)	

Wk	Content	Text Reference	Exploring Physics		Assessments
			Problem Sets	Experiments & Investigations	
4	7. explain and apply the concepts of centripetal acceleration and centripetal force, as applied to uniform circular motion—this will include *applying the relationships*: $a_c = \frac{v^2}{r}$, resultant $F = ma = \frac{mv^2}{r}$	Pg. 33-51	Set 3: Circular Motion	Expt: 3.1 Invest: 3.2 (home research)	
5	**Week 5 Monday Holidays** 8. describe and interpret the radial gravitational field distribution around a single (point) mass 9. explain and apply Newton's Law of Universal Gravitation and the concept of gravitational acceleration, g, as gravitational field strength—this will include *applying the relationships*: $F_g = G\frac{m_1 m_2}{r^2}$, $g = G\frac{M}{r^2}$	Pg. 58-62 Pg. 54-62		Expt: 4.1 Optional: Expt: 4.2 Expt: 4.3	**Task 6**: Test Projectile motion and circular motion
6	10. explain the conditions for a satellite to remain in a stable circular orbit in a gravitational field, and calculate the parameters of satellites in stable circular orbits—this will include *applying the relationships*: $v_{av} = \frac{s}{t}$, $a_c = \frac{v^2}{r}$, resultant $F = ma = \frac{mv^2}{r}$, $F_g = G\frac{m_1 m_2}{r^2}$, $g = G\frac{M}{r^2}$. 11. describe and explain the impact of satellites and associated technologies on everyday life	Pg. 63-68 Pg. 68-71	Set 4: Gravitation and Satellites		
7-8	12. explain and apply the concept of torque or moment of a force about a point, and the principle of moments, and their application to situations where the applied force is perpendicular to the lever arm—this will include *applying the relationships*: $\tau = rF$ and $\Sigma\tau = 0$. 13. explain and apply the concept of a rigid body in equilibrium—this will include *applying the relationships*: $\Sigma F = 0$, $\tau = rF$ and $\Sigma\tau = 0$ **Unit Test**	Pg. 72-76 Pg. 77-91	Set 5: Moments and Equilibrium	Expt: 5.1 Expt: 5.2	**Task 7**: Test Motion and forces in a gravitational field

Assessment outline: Stage 3 PHYSICS

Outcome 01: Investigating and Communicating in Physics;
Outcome 02: Energy;
Outcome 03: Forces and Fields

Assessment type	Assessment type weightings	Tasks	Content	Outcomes coverage			Weighting %		
				O1	O2	O3	3A	3B	Total
Experiments and investigations (20-40%)	21%	**Task 1:** Practical exam (3A)	Practical exam on 3A experiments and investigations	✓	✓	✓	5		**5**
		Task 2: Research topic	Validation activity on student research (written report)		✓	✓		3	**3**
		Task 3: Extended Investigation	Extended investigation	✓	✓	✓	4	4	**8**
		Task 4: Practical exam (3B)	Practical exam on 3B experiments and investigations	✓	✓	✓		5	**5**
Tests and Examinations (60-80%)	79%	**Task 5**:Validation tests on Assignments, Problem Sets and Homework	Accumulation of validation tests on Assignments, Problem Sets and homework		✓	✓	2	2	**4**
		Task 6: Test Projectile Motion	Test on projectile motion			✓	3		**3**
		Task 7: Test Motion and forces in a gravitational field	Test on Motion and forces in gravitation al field			✓	4		**4**
		Task 8: Test Particles, waves and quanta	Test on Particles, waves and quanta unit		✓		6		**6**
		Task 9: Test electricity and magnetism	Test on Electricity and Magnetism unit		✓			6	**6**
		Task 10: Test Motion and Forces in Electric and Magnetic Field	Test on Motion and Forces in Electric and Magnetic Field unit		✓	✓		6	**6**
		Task 11: Semester One Examination	Examination on 3A	✓	✓	✓	20		**20**
		Task 12:Stage 3 Examination (includes 20% of 3A)	Examination on Stage 3 work	✓	✓	✓	5	25	**30**
						TOTALS	50	50	**100**

Motion and Forces in a Gravitational Field

Central ideas

- Energy, Force, Momentum and Vectors
- Projectile motion
- Circular motion
- Universal gravitation (movement of planets and satellites)

Unit learning contexts

Within the unit content organisers of **motion and forces in a gravitational field,** teachers must present the unit content through one or more contexts, such as the following (this list is not exhaustive): Playground equipment; physics in sport; space travel.

Working in physics

Students are given opportunities to develop their skills related to investigating and communicating scientifically. They plan and conduct investigations to obtain valid and reliable results and are prepared to justify their findings. *Their problem-solving techniques include combinations of concepts and principles. They consider the level of absolute and percentage uncertainty in experimental measurements. This includes the use of error bars when displaying data graphically*.

Motion and forces in a gravitational field

1. *describe and apply the principle of conservation of energy*
2. *resolve, add and subtract vectors in one plane*
3. draw free body diagrams, showing the forces acting on objects, from descriptions of real life situations involving forces acting in one plane
4. explain and apply the concept of centre of mass
5. describe and apply the concepts of distance and displacement, speed and velocity, acceleration, energy and momentum in the context of motion in a plane, including the trajectories of projectiles in the absence of air resistance—this will include *applying the relationships*:

$$v_{av} = \frac{s}{t}, \quad v_{av} = \frac{v+u}{2}, \quad a = \frac{v-u}{t}, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

$$p = mv, \quad \sum p_{before} = \sum p_{after}, \quad F\Delta t = mv - mu$$

$$E_k = \tfrac{1}{2}mv^2, \quad E_p = mg\Delta h, \quad W = Fs, \quad W = \Delta E$$

6. describe qualitatively the effects of air resistance on projectile motion
7. explain and apply the concepts of centripetal acceleration and centripetal force, as applied to uniform circular motion—this will include *applying the relationships*:

$$a_c = \frac{v^2}{r}, \quad \text{resultant } F = ma = \frac{mv^2}{r}$$

8. describe and interpret the radial gravitational field distribution around a single (point) mass
9. explain and apply Newton's Law of Universal Gravitation and the concept of gravitational acceleration, g, as gravitational field strength—this will include *applying the relationships*:

$$F_g = G\frac{m_1m_2}{r^2}, \quad g = G\frac{M}{r^2}$$

10. explain the conditions for a satellite to remain in a stable circular orbit in a gravitational field, and calculate the parameters of satellites in stable circular orbits—this will include *applying the relationships*:

$$v_{av} = \frac{s}{t}, \quad a_c = \frac{v^2}{r}, \quad \text{resultant } F = ma = \frac{mv^2}{r}, \quad F_g = G\frac{m_1m_2}{r^2}, \quad g = G\frac{M}{r^2}.$$

11. describe and explain the impact of satellites and associated technologies on everyday life
12. explain and apply the concept of torque or moment of a force about a point, and the principle of moments, and their application to situations where the applied force is perpendicular to the lever arm—this will include *applying the relationships*: $\tau = rF$ and $\Sigma\tau = 0$.

13. explain and apply the concept of a rigid body in equilibrium—this will include *applying the relationships*: $\Sigma F = 0$, $\tau = rF$ and $\Sigma\tau = 0$

Use of Significant Figures

The correct number of significant figures must be used in your numerical answers. A small proportion of the marks for your assessments and WACE Examination will relate to the correct use of significant figures and units. In general, external examinations have three significant figures however all **estimate** questions will have no more than two significant figures. For large or small numbers you will need to give your answer in scientific notation. For example, 98 344 N to three significant figures is 9.83×10^4 N.

RULES:

1. All non zero digits are significant.
   E.g. 7.92 has 3 significant figures (hence forth called sf)

2. Zeros that fall between two significant digits are significant.
   E.g. 9.002×10^{-4} *has 4 sf.*

3. Zeros to the left of the first non-zero digit are not significant.
   Eg. 0.35 has 2 sf.

4. Zeros at the end of a number and to the right of the decimal point are significant.
   Eg. 0.0710 has 3 sf.

5. Zeros at the end of a number and to the left of the decimal point are not significant unless otherwise indicated. Eg. 3500 has 2 sf. If zeros are significant then scientific notation can be used to show this. *E.g.* 3.500×10^4 *has 4 sf while* 3.50×10^4 *has 3 sf.*

6. When multiplying or dividing, the answer is given with as many significant figures as the measurement with the least significant figures.
   E.g. $1.498\ g \div 6.2 \times 10^{-1}\ L$ *(4 sf ÷ 2 sf so answer must have 2 sf)*
   $= 2.316129\ gL^{-1}$
   $= 2.4\ gL^{-1}$ *(2 sf)*

7. When adding or subtracting, the answer is quoted with as many decimal places as the measurement with the least decimal places.
   E.g. $1.49 \times 10^2\ gL^{-1} + 6.2\ g$
   $= 149 + 6.2\ g$ *(zero decimal places so answer must have zero)*
   $= 155.2\ g$
   $= 155\ g$

8. When rounding an answer, if the leftmost digit to be rounded is:
   - > than 5 then increase the preceding number by 1.
   - < than 5 then decrease the preceding number by 1.
   - = 5 then increase the preceding number by 1.

Questions:

1. How many significant figures in the following?

   a. 125 ___**3**__ b. 0.013 __**2**__ c. 1007 __**4**___ d. 25.10 __**4**___

   e. 3700 __**2**___ f. 0.003 __**1**__ g. 10.030 ___**5**___ h. 630 ___**2**__

2. Convert the following to three significant figures and in scientific notation.

   a. 34 791 $\mathbf{3.48 \times 10^4}$ b. 0.000 034 53 $\mathbf{3.45 \times 10^{-5}}$ c. 3 478 $\mathbf{3.48 \times 10^3}$

   d. 6 953 $\mathbf{6.95 \times 10^3}$ e. 0.005 451 $\mathbf{5.45 \times 10^{-3}}$ f. 0.1955 $\mathbf{1.96 \times 10^{-1}}$

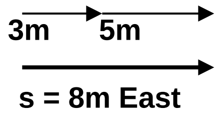
4. Multiply 2.340 by 5.29002 **12.38 (least number of s.f. is 4)**
5. Subtract 0.984 from 5.6 **4.6 (least number of decimal places is 1)**

Outcome 2: Resolve, add and subtract vectors in one plane

Vector Addition   Text Reference: CD Pg. 416-419

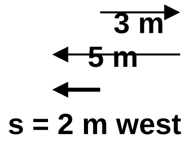
Vectors are represented by arrows. When adding vectors, the tail of the second is added to the head of the first. The resultant vector is found by drawing a resultant arrow from the tail of the first vector to the head of the final vector.

Vectors need a direction. Make one direction positive and the other negative.



Example 1: Karis walks 3m East then 5 m East. Calculate her displacement.

East is positive
s = 3 + 5
s = <u>8 m East</u>

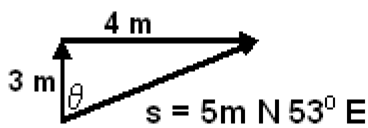


Example 2: Tony runs 3 m East then 5 m West. Calculate his displacement.

East is positive so west is negative
s = 3 + (-5)
s = -2
s = 2m West

Example 3: Michael walks his dog 3 m North then 4m East.
What was his displacement?

$s = \sqrt{(3^2 + 4^2)}$ $\theta = \tan^{-1}(4 \div 3)$
$s = 5m$ $\theta = 53^0$
<u>s = 5m N 53^0 E</u>



In 3A Physics it is important to consider direction when dealing with questions relating to vector quantities such as projectile motion questions. Select one direction as positive and the other is negative.

For example.
You throw a ball up into the air with an initial velocity of 5.45 ms^{-1} and it hits the ground with a final velocity of 7.29 ms^{-1}. How long was it in the air?

One way to do this problem is to find the time up. Then the time down. Then add the two together.

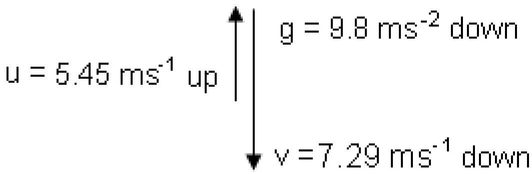
The method above means two equations as shown and a possibility of more errors. It is often better to use one and get the direction correct.

u = 5.45 ms^{-1}   **v = u + gt**
v = 0 ms^{-1}   **0 = 5.45 + 9.8t**
g = 9.8 ms^{-2}   **-5.45 = 9.8 t**
t = 0.556 s up

(we can ignore the negative sign as time is not a vector quantity)

u = 0   **v = u + gt**
v = 7.29 ms^{-1}   **7.29 = 0 + 9.8t**
g = 9.8 ms^{-2}   **t = 0.744 down**

t_T = 0.556 + 0.744
= 1.30 s



Let up be negative and down be positive.

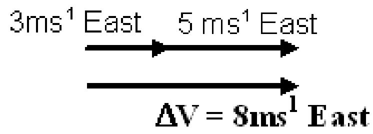
u = -5.45 ms^{-1}   **v = u + gt**
v = +7.29 ms^{-1}   **+7.29 = -5.45 + (9.8 x t)**
g = +9.8 ms^{-2}   **+7.29 + 5.45 = 9.8t**
12.74 = 9.8t
t = 1.30 s

Vector Subtraction   Text Reference: CD Pg. 416-419

When subtracting vectors, you **add the opposite vector**.

Example 1. A ball hits a wall at 5 ms^1 West then rebounds at 3 ms^1 East Find the change in velocity.
The change in velocity is the final velocity subtract the initial velocity.

$\Delta V = v - u$
ΔV = 3ms^1 East – 5ms^1 West *is the same as*
ΔV = 3ms^1 East + (-5ms^1 West) *opposite of 5ms^1 West is 5ms^1 East so*
ΔV = 3ms^1 East + 5 ms^1 East *now draw diagram*



Example 2: A 0.5kg ball is thrown at a wall at 3.5 ms^{-1} North then rebounds at 2.5 ms^{-1} South. Change of velocity took 30.0 ms. Calculate force of wall on ball.
(Wall on ball so need force South)

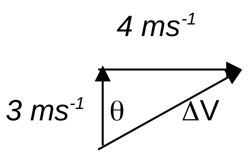
ΔV = 2.5 ms^{-1} South – 3.5 ms^{-1} North
ΔV = 2.5 ms^{-1} South + 3.5 ms^{-1} South
ΔV = 6.0 ms^{-1} South
Now calculate force.

$$F = ma = \frac{m\Delta V}{t} = \frac{0.5 \times 6.0}{30 \times 10^{-3}}$$

F = 100 N South.

Example 3: A bike is travelling 4.0 ms^{-1} West, it then turns a corner to travel 3.0 ms^{-1} North. The change of velocity took 2.0 s. What is the acceleration of the bike?
First find change in velocity.

ΔV = 3.0 ms^{-1} North - 4.0 ms^{-1} West
ΔV = 3.0 ms^{-1} North + (-4.0 ms^{-1} West)
ΔV = 3.0 ms^{-1} North + 4.0 ms^{-1} East *now draw a diagram*



$\Delta V = \sqrt{(3.^2 + 4^2)}$ $\quad \theta = \tan^{-1}(4.0 \div 3.0)$
ΔV = 5.0 ms^{-1} $\quad \theta = 53.1^0$ $\quad \Delta V$ = 5.0 ms^{-1} N 53.1^0 E

Now using time, you can find the acceleration around the corner.

Helpful Hint:

In Stage 2 you used $a = \frac{(v - u)}{t}$ *for acceleration but as the direction has changed (v – u) is NOT (3.0 – 4.0) but the ΔV you calculated above – 5.0 ms^{-1} N 53.1^0 E*

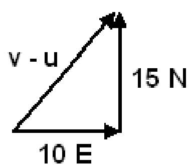
$$\text{Acceleration} = \frac{\Delta V}{t} = \frac{5.0}{2.0}$$

a = 2.5 ms^{-2} N 53.1^0 E *(Acceleration is a vector quantity so must also have a direction)*

Activity:

1. A motorcycle going around a corner is travelling initially at 15.0 ms^{-1} south then, after the corner, the motorcycle is travelling at 10.0 ms^{-1} east. Determine the car's acceleration if the change in velocity takes 1.40 s.

u = 15.0 ms^{-1} S
v = 10.0 ms^{-1} E
t = 1.40 s
v – u = 10 E – 15 S
= 10 E + 15 N



v – u = $\sqrt{(10^2 + 15^2)}$
= 18.0 ms^{-1}

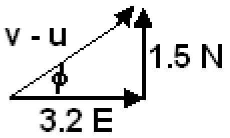
$\theta = \tan^{-1}(15 \div 10)$
= 56.3^0 **v – u = 18.0 ms^{-1} E 56.3^0 N**

$$a = \frac{v - u}{t} = \frac{18.0}{1.40}$$

a = 12.9 ms^{-1} E 56.3^0 N

2. You are walking south along the corridor at 1.50 ms^{-1} when you turn east to go to your class. If the turn took 0.700 s and you are now walking at 3.20 ms^{-1}, what was your acceleration around the corner?

u = 1.50 ms^{-1} S
v = 3.20 ms^{-1} E
t = 0.700 s
v – u = 3.2 E – 1.5 S
= 2.3 E + 1.5 N



$$v - u = \sqrt{(3.2^2 + 1.5^2)} = 3.534\ ms^{-1}$$

$$\theta = \tan^{-1}(1.5 \div 3.2) = 25.1^0$$

v – u = 3.53 ms^{-1} E 25.1^0 N

$$a = \frac{v - u}{t} = \frac{3.534}{0.700}$$

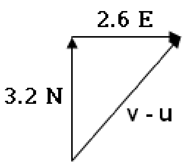
<u>**a = 5.05 ms^{-1} E 25.1^0 N**</u>

<u>Helpful Hint</u>: For the following questions don't forget to change velocity to ms^{-1}.

3. Fern (my ginger cat) is running at 9.36 kmh^{-1} west chasing a mouse. She turns north to follow the mouse and is now running at 11.52 kmh^{-1}. If the turn took 0.400 s, what was Fern's acceleration?

u = 2.60 ms^{-1} W
v = 3.20 ms^{-1} N
t = 0.400 s

v-u = 3.20 N – 2.60 W
= 3.20 N + 2.60 E



$$v - u = \sqrt{(3.2^2 + 2.6^2)} = 4.12\ ms^{-1}$$

$$\phi = \tan^{-1}(2.6 - 3.2) = 39.1^0$$

so v-u = 4.12 ms^{-1} N 39.1^0 E

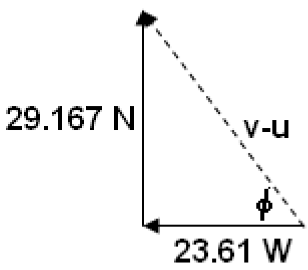
$$a = \frac{v - u}{t} = \frac{3.14}{0.400}$$

<u>**a = 10.3 ms^{-1} N 39.1^0 E**</u>

4. A pitcher throws a baseball due south towards the batter at 105.0 kmh^{-1}. The batter hits the ball foul by striking it due west at 85.0 kmh^{-1}. What is the change in the baseball's velocity?

u = 29.167 ms^{-1} S
v = 23.61 ms^{-1} W

v-u = 23.61 W – 29.167 S
= 23.61 W + 29.167 N



$$v - u = \sqrt{(23.61^2 + 29.167^2)}$$

$$v - u = 37.53\ ms^{-1}$$

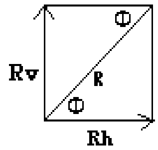
$$\phi = \tan^{-1}(29.167 \div 23.61) = 50.9^0$$

so v – u = <u>37.5 ms^{-1} W 50.9^0 N</u>

Outcome 2: Resolve vectors in one plane

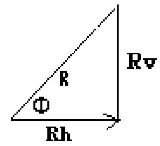
Vector Components   Text Reference: CD Pg. 416-419

A vector can be resolved into a horizontal and a vertical component that are at right angles to each other as shown in the diagram.



Rv = R sin Φ

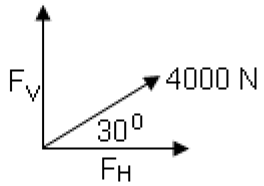
Rh = R cos Φ



Your teacher will work through the following examples with you.

Example 1:

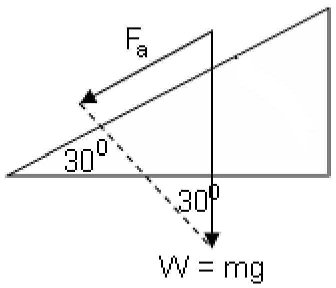
A glider is being pulled by a plane to lift it into the air. The rope has a tension of 4.00 x 10^3 N and makes an angle of 30.0° to the ground. Find the horizontal and vertical components of the force.



F_V = F sin ϕ
= 4000 sin 30
= 2000
F_V = 2.00 x 10^3 N

Example 2:

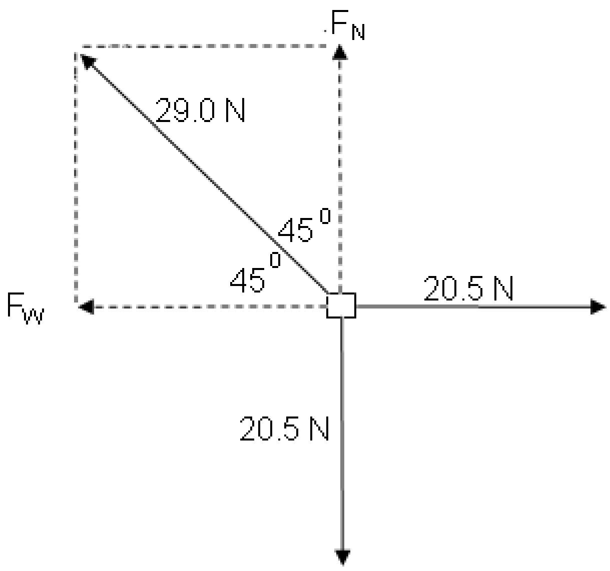
A boy on a skateboard is accelerating down a hill which is inclined at 30.0° to the horizontal. If the combined mass of the boy and skateboard is 45.0 kg, what force is accelerating them down?



F_a = W sin ϕ
= 45 x 9.8 x sin 30
= 220.5
F_a = 2.21 x 10^2 N

Example 3:

Three boys are pulling on a toy. Sam pulls with a force of 20.5 N east, Jim with a force of 20.5 N south and Tom with a force of 29.0 N north-west. Who will pull the toy their way?

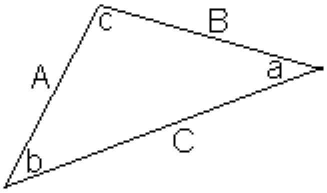


F_N = 29.0 cos 45
= 20.5 N

F_W = 29.0 sin 45
= 20.5 N

As forces north, south, east and west are equal, the toy will not move.

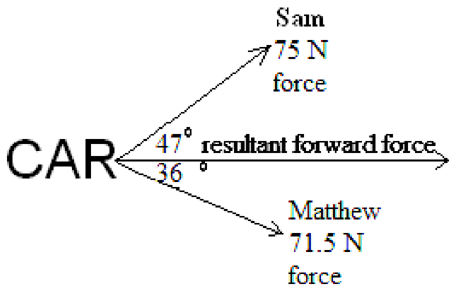
Helpful Hint: Occasionally, you will need to find the resultant and angle when the triangle is not a right-angled triangle. In these instances use the cosine and sine rules.



Cosine rule:
$R^2 = A^2 + B^2 - 2AB \cos c$
Sine rule:
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Questions on Vectors ***Worked answers page 28 and 29 of this workbook.***

1. While working at Coles, Giles needs to move cartons of tomatoes to the Fruit and Vegetable section of the store using a trolley. The handle of the trolley is at an angle of 65.0^0 to the vertical and the horizontal component of the force is 23.02 N. With what force is Giles pulling the cart?
2. Tina is taking a plane trip to New Zealand. The plane is climbing at an angle of 40.0^o to the ground with a constant speed of 3.50×10^2 kmh^{-1}. What is the effective forward speed of the plane?
3. A small van with a mass of 1.50×10^3 kg is parked at the top of a hill that is inclined at 67.0^o to the vertical. The brakes fail. Calculate the force that is now pulling the truck down the hill?
4. Explain why it is easier to pull a heavy roller across a lawn than it is to push it.
5. Sam and Matt are trying to pull Matt's car out of the sand where it has become bogged. Sam is using 75.0 N of force at an angle of 47.0^0 while Matt uses 71.5 N at an angle of 36.0^0 to the resultant forward force. The boys need an overall force of 105 N to pull the car free. Will they be able to do so? You must show calculations to justify your answer.



6. Rohan throws a 340 g ball with a velocity of 6.40 ms^{-1} East at a wall. The ball rebounds returns at 3.80 ms^{-1} West. If the change in velocity occurred in 0.350 s, what force did the wall apply to the ball?
7. A pitcher throws a baseball due south towards the batter at 104.4 kmh^{-1}. The batter hits the ball foul by striking it due west at 86.4 kmh^{-1}. If the ball's change in velocity occurred in 0.200 s, what was the ball's acceleration?

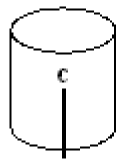
Exploring Physics Stage 3:
- **Experiment 1.1 and 1.2**
- **Set 1: Vectors**

Outcome 4: Explain and apply the concept of centre of mass *(assessed later in the unit)*

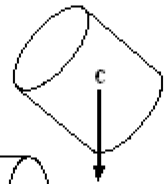
Stability – Centre of Mass (gravity) **Text Reference: CD pg. 420_**

The closer the centre of mass of an object is to the ground and the larger the base, the more stable the object is. Greatest stability comes from low centre of mass and large area over which the centre of mass acts. A large force is then required to move the centre of mass beyond the base .

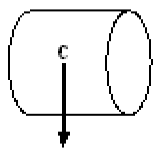
Stable:
Stable position with centre of mass over the base. Slight movement has no overall effect - centre of mass may be raised but object returns to its original position



Unstable:
Centre of mass is over the pivot. Slight movement causes object to move resulting in a new, often lower, centre of mass.



Neutral:
Centre of mass is over the pivot. Slight movement causes object to move to a new position although the centre of mass usually remains at the same height.



<u>Outcome 5</u>: describe and apply the concepts of distance and displacement, speed and velocity, acceleration, energy and momentum in the context of motion in a plane, including the trajectories of projectiles in the absence of air resistance—this will include *applying the relationships*:

$$v_{av} = \frac{s}{t}, \quad v_{av} = \frac{v+u}{2}, \quad a = \frac{v-u}{t}, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

$$p = mv, \quad \sum p_{before} = \sum p_{after}, \quad F\Delta t = mv - mu$$

$$E_k = \tfrac{1}{2}mv^2, \quad E_p = mg\Delta h, \quad W = Fs, \quad W = \Delta E$$

Review of 2A Physics as it relates to 3A Physics Text Reference: CD pg. 422-444

Work together in groups to define the following terms including an example for each term.
The following answers are only suggestions.

Scalar quantity:

Quantity that has magnitude only e.g. time, distance, speed

Vector quantity:

Quantity that has magnitude and direction e.g. displacement, velocity, force

Distance:

Measure of the actual distance travelled or the total path travelled e.g. travelling from Perth to Bunbury

Displacement:

Shortest path between two points e.g. if you run around a circular 400 m track, your distance is 400 m but your displacement is zero if you start and finish at the same position.

Speed:

Total distance travelled divided by the time taken e.g. if you run the 400 m track in 94.1 s, you will have a velocity of 400 ÷ 94.1 = 4.25 ms^{-1}. No direction required as a scalar quantity.

Velocity:

Displacement divided by time taken e.g. on a straight road, a car travelling east, takes 3 minutes to travel 6.0 km then the velocity is 6000 m ÷ (3 x 60) = 33.3 ms^{-1} east.

Acceleration:

Change in velocity divided by time taken. If a car is travelling at 20 ms^{-1} and then increases its velocity to 25 ms^{-1} in 0.50 s, then the acceleration is $\frac{25 - 20}{0.5}$ = 10 ms^{-2}

A Review of Momentum Text Reference: CD pg 438-440

Within a corridor of the school, two different groups of students are throwing balls to each other. One group is using a foam ball, the other group a cricket ball. Both are about the same size but if the cricket ball were to hit a window the window could break however the foam ball, thrown at the same velocity, would not break the window. Why the difference?

This is due to the effect of momentum which is the mass of an object times the velocity of that object

$\rho = mv$ where ρ = momentum in kgms^{-1} ; m = mass in kg ; v = velocity in ms^{-1}

The cricket ball can have a mass of 0.160 kg while the foam ball has a mass of 0.0200 kg. If both are thrown at 15.0 ms^{-1} you can quickly see that there is a large different in the momentum and the cricket ball has a much larger effect on the window.

$\rho_{\text{cricket ball}} = mv$
$= 0.160 \times 15.0$
$= 2.40$ kgms^{-1}

$\rho_{\text{foam ball}} = mv$
$= 0.0200 \times 15.0$
$= 0.300$ kgms^{-1}

A Review of Change in Momentum   Text Reference: CD pg 439

You are leisurely riding home on your bike when it starts to rain. Quickly you increase your velocity to limit the time you get wet. Here you have experienced a change in momentum. The change in anything is the final minus the initial, so change in momentum = final momentum - initial momentum

or $$\Delta\rho = mv - mu$$

Example:
While running in the park you are initially travelling at 3.50 ms^{-1} east. A dog runs out and starts to chase you so you increase your velocity to 6.00 ms^{-1} east. Assuming that your mass is 60.0 kg, what was your change in momentum?

m = 60.0 kg
u = 3.50 ms^{-1}
v = 6.00 ms^{-1}

$\Delta\rho = mv - mu$
$= m(v - u) = 60.0(6.00 - 3.50)$
$= 60.0 \times 2.5$
$\underline{\Delta\rho = 1.50 \times 10^2 \text{ kgms}^{-1} \text{ east}}$

A change in momentum can also include a change in direction. Consider the foam ball hitting the window. The window doesn't break and the foam ball rebounds at the same velocity (assume no loss of energy). While the magnitude of the velocity is unchanged, its direction has changed so it is a different velocity and there has been a change in momentum.

Example:
The 20.0 g foam ball hits the window at 15.0 ms^{-1} north and rebounds at 15.0 ms^{-1} south. What is the change in momentum?

m = 0.020 kg
u = 15.0 ms^{-1} north
v = 15.0 ms^{-1} south
$\Delta V = (v - u)$
$= (15.0 \text{ ms}^{-1} \text{ south} - 15.0 \text{ ms}^{-1} \text{ north})$
$= (15.0 \text{ ms}^{-1} \text{ south} + 15.0 \text{ ms}^{-1} \text{ south})$
$= 30.0 \text{ ms}^{-1}$ south

$\Delta\rho = m(v - u) = m\Delta V$
$= 0.0200 \times 30.0$
$\underline{\Delta\rho = 0.600 \text{ kgms}^{-1} \text{ south}}$

A Review of Newton's Second Law of Motion

Newton's Second Law states that force is equal to the rate of change of momentum,

OR $$F = \frac{(mv - mu)}{t}$$ which is the same as $$F = \frac{m(v - u)}{t}$$

Now we know that change in velocity divided by time, $\frac{(v - u)}{t}$, is acceleration so substituting this into the above equation results in **F = ma**; a simple way of expressing Newton's Second Law.

A Review of Impulse   Text Reference: CD pg 440-441

If you are unlucky enough to have a car accident, you will experience a force on you that could cause injury or death. Car designers try to make cars as safe as possible, so to do this they try to decrease the force acting on you. In a car crash your velocity changes so you undergo a change in momentum.

$$F = \frac{m(v - u)}{t}$$ this can be re-written as

$$Ft = mv - mu$$

Ft is known as impulse (units Ns) and is equal to the change in momentum: Impulse = Ft
(Helpful Hint: As Ft = Δp, then Δp can also have the units Ns.)

Putting it all together

Now back to your car crash. For this situation, your change in momentum can't be altered, that means that the magnitude of the impulse can't be changed. However, it is the force on you that car designers are trying to decrease. How can they do this?

While the impulse (also known as impulsive force) can't be changed, the time of the crash can. If the time over which the crash takes place is increased, the force must be decreased (simple maths). To increase the time of the crash, car designers now design cars that crumple – this increases the time for the car to stop in a crash, thus decreases the force and possibly saving you from injury or death (however it does often mean that a small crash can cause crumpling of your car and can result in a large repair bill – well you can't have it both ways!!!).

Example:
Remember your run in the park when you were initially travelling at 3.50 ms^{-1} east when a dog started to chase you and you increased your velocity to 6.00 ms^{-1} east. Assuming that your mass is 60.0 kg and that the change of momentum happened in 1.50 s, what force was applied to increase your velocity?

> *Helpful Hint:*
> *As you are dealing with velocity which is a vector quantity. You need to consider direction. This formula is true when all velocities are in the same direction. If final velocity is in the opposite direction then you* ***add*** *velocities. Likewise be careful of force.*

m = 60.0 kg
u = 3.50 ms^{-1} E
v = 6.00 ms^{-1} E
t = 1.50 s

$$F = \frac{m(v - u)}{t}$$

$$F = \frac{60(6.00 - 3.50)}{1.50}$$

$\underline{F = 1.00 \times 10^2 \text{ N East}}$

Activity:
While cycling along the road you apply the brakes to stop in 8.70 m to avoid hitting a car backing out of a driveway. You are moving at a velocity of 36.0 kmh^{-1} and the total mass of your bicycle and yourself is 76.0 kg. Calculate the force exerted by the brakes, the change in momentum and then the impulse you experience.
(Helpful Hint:: remember that vector quantities have magnitude and direction.)

s = 8.7 m
u = 36 kmh^{-1}
= 10 ms^{-1}
m = 76 kg

$v^2 = u^2 + 2as$
$0 = 10^2 + (2 \times a \times 8.7)$
$0 = 100 + 17.4a$
$a = -\frac{100}{17.4}$
$a = 5.74\ ms^{-2}$

$\Delta\rho = m(v - u)$
$= 76 \times (0 - 10)$
$= 76 \times -10$
$\Delta\rho = -760\ kgms^{-1}$
$\Delta\rho$ = 760 $kgms^{-1}$ in opposite direction to initial motion

F = ma
= 76 x -5.747
= - 437 N
= 437 N in opposite direction to initial motion

Impulse = $\Delta\rho$
= 760 $kgms^{-1}$ in opposite direction to initial motion

Activity:
A 1.50 tonne car is travelling at 72.0 kmh^{-1}, experiences a force of 1.00×10^4 N for 0.800 s when the brakes are applied. What is its final velocity?

u = 72 kmh^{-1}
= 20 ms^{-1}
t = 0.8 s
v = ?
F = -10 000 N
(this is part students forget as acting in opposite direction)

$Ft = m(v - u)$
$-10\ 000 \times 0.8 = 1.5 \times 10^3 (v - 20)$
$-8\ 000 = 1.5 \times 10^{-3} v - 30\ 000$
$-8\ 000 + 30\ 000 = 1.5 \times 10^{-3} v$
$22\ 000 = 1.5 \times 10^3 v$
$v = 14.6\ ms^{-1}$ x 3.6
$\underline{v = 52.8\ kmh^{-1}}$

Conservation Of Momentum

If you fire a high-speed rifle there is recoil, as the bullet goes forward, the gun moves back. The law of conservation of momentum states that; *in the absence of an external force, the momentum of a system remains unchanged.* Collisions are an example of conservation of momentum.

There are two types of collisions, **elastic** where the objects bounce apart and **inelastic,** where they stick together. In both cases:

net momentum after collision = net momentum before collision

$$\Sigma\rho_{after} = \Sigma\rho_{before}$$

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

Outcome 1: Describe and apply the principle of conservation of energy

Work Text Reference pg. 433-438

In everyday speech, 'work' has a variety of meanings. In science, however, 'work' has a clearly defined definition in Physics. Work is done only when a force (in Newtons) makes an object move over a distance (in metres). So when, for instance, you push a lawn mower over a lawn you make it move and you do work. If you push on something and do not move it, or hold a heavy object stationary, no work is done. Therefore:

Work = force x displacement

$$W = Fs$$

The unit of work is the joule (J). One joule of work is done if a force of 1 Newton moves an object through a displacement of 1 metre in the direction of the force.

Example:

If we were to push the family car along the driveway into the garage, a distance of 12.0 m, by applying a constant horizontal force of 2.50×10^2 N, the work done is:

s = 12.0 m	W = Fs
F = 250 N	W = 12 x 250
	W = 3000 = 3.00×10^3 J

Activity:

a. Two boys are using a force of 1.00×10^2 N to move a bed 3.00 m across a bedroom. What is the work done?

W = Fs

= 100 x 3

W = 3.00×10^2 J

b. What work is required to lift a 5.00 kg bag of potatoes from the floor to a shelf 1.50 m above the ground?

W = Fs = mgs

W = 5 x 9.8 x 1.5

W = 73.5 J

Energy Text Reference pg. 433-438

Work and energy are related. Energy can be defined as *the capacity to do work*. Whenever work is done, some energy is transferred from one object to another. Another way to define the energy of an object is defined as *the amount of work that the object could do if it changed its state to some specified zero energy state*. The meaning of this will become easier to understand when particular types of energy are described. The unit of energy is the same as that of work, namely the **joule (J).**

Kinetic Energy

Kinetic energy is the energy an object has because it is moving. A moving car has kinetic energy whereas a stationary car does not.

The kinetic energy is defined as the amount of work it could do in coming to rest. The faster an object moves, the greater its kinetic energy. Also the greater the mass of the object, the greater its kinetic energy.

$$E_k = \frac{1}{2} mv^2$$

E_k = kinetic energy (J)
m = mass (kg)
v = velocity (ms^{-1})

<u>Example:</u>
A car of mass 1.50 tonne is moving at 20.0 ms^{-1}. What is its kinetic energy?

$$\begin{aligned} E_k &= \tfrac{1}{2} mv^2 \\ &= 0.5 \times 1500 \times (20)^2 \\ &= \underline{3.00 \times 10^5 \text{ J}} \end{aligned}$$

<u>Activity:</u>
A man is running at 5.00 ms^1 and using 937 J of energy, what is his mass?

$$\begin{aligned} \mathbf{E_k} &= \mathbf{\tfrac{1}{2} mv^2} \\ \mathbf{937} &= \mathbf{0.5 \times m \times 5^2} \\ \mathbf{937} &= \mathbf{12.5\ m} \\ \mathbf{m} &= \mathbf{74.96} \\ \underline{\mathbf{m}} &\underline{\mathbf{= 75.0\ kg}} \end{aligned}$$

Gravitational Potential Energy

A car at the top of a hill has energy because of its position. If allowed to roll down the hill, the car could do work because it could exert a force on another object and hence move it. We call the energy an object has because of its height above Earth, its **gravitational potential energy**.

$$E_p = mgh$$

E_p = potential energy (J)
m = mass (kg)
g = acceleration due to gravity (ms^{-2})
h = height above earth (m)

<u>Examples:</u>
What is the potential energy of a small car of mass 1.00 x 10^3 kg at the top of a hill of height 50.0 m?

$$\begin{aligned} E_p &= mgh \\ &= 1000 \times 9.8 \times 50 \\ &= \underline{4.90 \times 10^5 \text{ J}} \end{aligned}$$

<u>Activity:</u>
A ball is thrown into the air. Just before it starts its downward journey (velocity equal to zero) it is at a height of 2.50 m above the ground. If it has a mass of 0.500 kg, what is its potential energy?

$$\begin{aligned} \mathbf{E_p} &= \mathbf{mgh} \\ &= \mathbf{0.5 \times 9.8 \times 2.5} \\ &= \mathbf{12.25} \\ \underline{\mathbf{E_p}} &\underline{\mathbf{= 13.0\ J}} \end{aligned}$$

Law Of Conservation Of Energy Text Reference pg. 433-438

The amount of energy within a system is constant in all conversions and this is stated in the **Law of Conservation of Energy**: *Energy is neither created nor destroyed, but can be readily converted from one form to another.*

Energy Transformations Between Kinetic And Potential Energy

A car at the top of a hill has potential energy. If it starts to roll it loses potential energy and starts to gain potential energy. By the time it reaches the bottom of the hill it has lost all its potential energy and gained it as kinetic energy (for this section we will ignore other forms of energy involved e.g. heat from friction).

$$E_{p\ lost} = E_{k\ gained}$$
$$mgh = \tfrac{1}{2} mv^2 \qquad \text{masses cancel so}$$
$$gh = \tfrac{1}{2} v^2$$

<u>Example:</u>
A ball is thrown into the air with an initial velocity of 15.0 ms^{-1}. What is the maximum height it can reach?

$$gh = \tfrac{1}{2} v^2$$
$$h = \frac{0.5 \times (15)^2}{9.8}$$
$$\underline{h = 11.5\ m}$$

<u>Activity:</u>
A girl is on a diving board 3.00 m above the water below. With what velocity will she hit the water when she dives?

$$\mathbf{gh = 0.5\ v^2}$$
$$\mathbf{9.8 \times 3.00 = 0.5\ v^2}$$
$$\mathbf{29.4 = 0.5\ v^2}$$
$$\mathbf{v = \sqrt{(29.4 \times 2)}}$$
$$\mathbf{\underline{v = 7.67\ ms^{-1}}}$$

Outcome 3: Draw free body diagrams, showing the forces acting on objects, from descriptions of real life situations involving forces acting in one plane

Free body diagrams   Text Reference: CD pg. 421

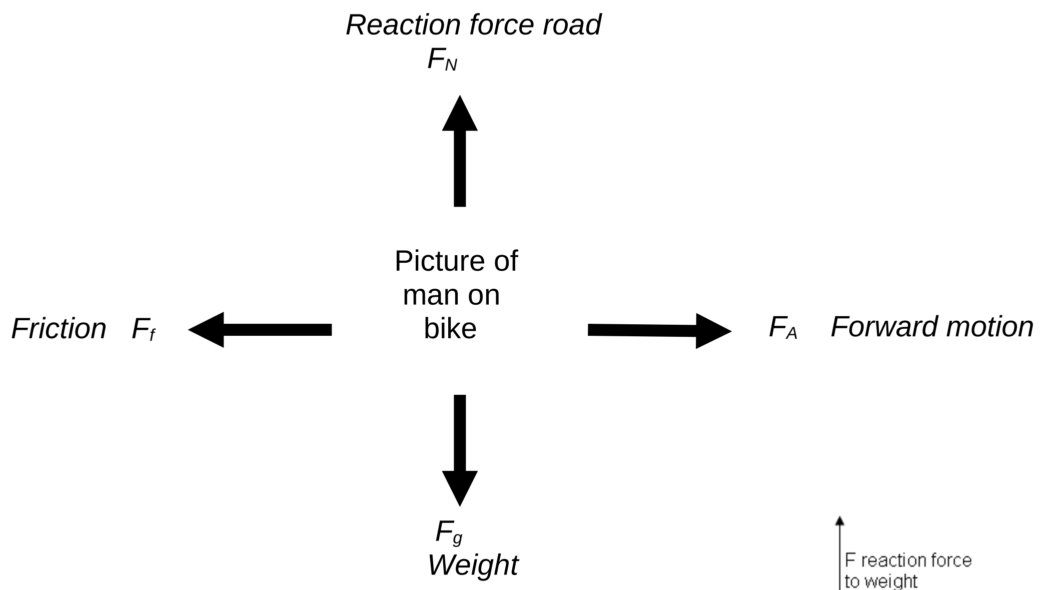
In solving problems, it is sometimes useful to illustrate on a diagram all the forces that are acting on the object. This type of diagram is called a **free body diagram**. Arrows will show the direction of the force and the length will show the size of the force relative to other forces shown on the diagram.

Questions to ask yourself about the situation – *youtube*
http://www.youtube.com/watch?v=BuPfDI7TyL0

Basically:

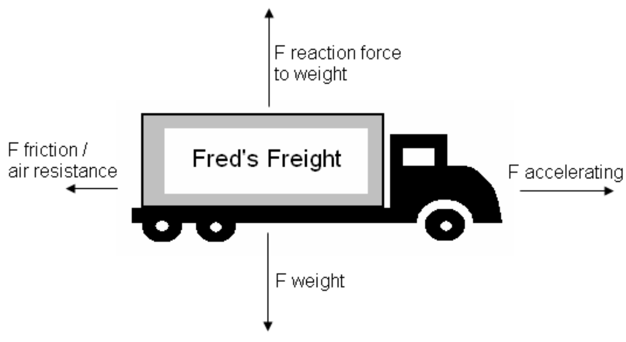
1. Is there gravity? This will be downwards. F_g
2. Is the object on a surface? Upward normal force for gravity. F_N
   Generally these two forces are the same length unless the object is ascending or descending.
3. Is something pushing or pulling. This is the forward action force. F_A
4. Is there friction. This is the force that retards the motion. F_f
5. Is the object accelerating. Positive acceleration then F_A arrow larger than friction, negative acceleration then F_A arrow is shorter than friction.

Let's consider Samuel who is riding his bike along a straight road at a constant velocity. For this particular situation consider the following forces that are acting on Samuel: *weight, reaction force from road, forward motion* and *frictional forces (including air resistance).* As his velocity is constant, forces up must equal forces down and forces right must equal forces left:

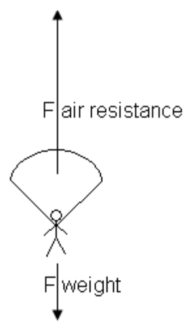


Activity:
A driver places a large heavy box in the middle of the tray of a delivery truck. The box is not tied down.

a. On the diagram show all the major forces acting on the box as the truck accelerates. Use labelled arrows to show their direction and related size of each force.



b. On the right, draw a skydiver who has just opened his parachute.



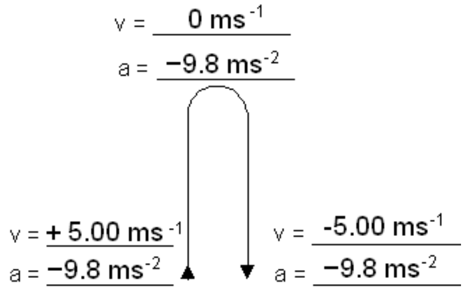
Power Point on Free Body Diagrams.
http://www.wisc-online.com/objects/index_tj.asp?objID=TP1502

Outcome 5: describe and apply the concepts of distance and displacement, speed and velocity, acceleration, energy and momentum in the context of motion in a plane, including the trajectories of projectiles in the absence of air resistance—this will include *applying the relationships*:

$$v_{av} = \frac{s}{t}, \quad v_{av} = \frac{v+u}{2}, \quad a = \frac{v-u}{t}, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

Flying through the air - Projectiles

Vertical projectiles



A ball is thrown into the air at 5.0 ms^{-1} and then returns to the same height it was thrown from. Complete the following diagram to show the velocity and acceleration at the start, middle and end of the throw assuming no air resistance.

We know from stage 2 that all objects will fall to earth regardless of their size and mass (assuming no air resistance) at 9.8 ms^{-2} due to the pull of gravity. If we throw an object into the air, it will take the same time to reach the top, as it will to return to its starting position. Newton's equations of motion can be used to solve vertical projectile problems.

NOTE: When an object moves up and down, when dealing with displacement, velocity and acceleration, a direction must be indicted. Choose one direction as positive (I will be using up), then values in the other direction must be negative.

Your teacher will work through these problems with you.

Example 1:
During a car accident a piece of the car is projected vertically upwards from the ground with an initial velocity of 20.0 ms^{-1}. Neglecting air resistance find

a. maximum height the piece reaches

$g = -9.8\ ms^{-2}$

$u_v = +20.0\ ms^{-1}$
$v_v = 0$

Note the use of subscript "v" for vertical Get into the habit of using "v" or "h"

$v_v^2 = u_v^2 + 2gs$
$0 = (20)^2 + (2 \times -9.8 \times s)$
$0 = 400 - 19.6s$
$-400 = -19.6s$
$400 = 19.6s$
$\underline{s = 20.4\ m}$

(as positive then indicates upwards)

b. total time in the air after returning

$u = +20.0\ ms^{-1}$
$v = -20.0\ ms^{-1}$
(as returns to same position but in different direction

$g = -9.8\ ms^{-2}$

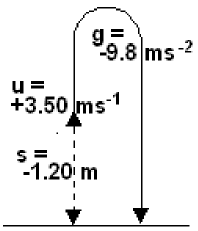
$t = \frac{v-u}{g} = \frac{-20 - 20}{-9.8}$

$\underline{t = 4.08\ s}$

Example 2:
A ball, which is held 1.20 m above the ground, is thrown into the air at 3.50 ms^{-1} then returns to hit the ground.

a. With what velocity will it hit the ground?

$u_v = 3.50\ ms^{-1}$
$s_v = -1.20\ m$
$g = -9.8\ ms^{-2}$



$v_v^2 = u_v^2 + 2gs$
$= (3.5)^2 + (2 \times -9.8 \times -1.2)$
$= 12.25 + 23.52$
$= 35.77$
$v_v = \sqrt{35.77}$
$\underline{v_v = 5.98\ ms^{-1}\ \text{down}}$

b. How long is it in the air?

$v_v = u_v + gt$
$-5.98 = +3.50 + (-9.8t)$
$-9.48 = -9.8t$
$\underline{t = 0.967\ s}$

Activity:

1. A kangaroo jumps to a vertical height of 2.80 m. How long was it in the air before returning to earth?

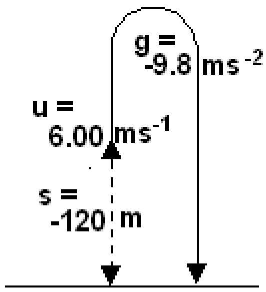
for journey down
$g = -9.8\ ms^{-2}$
$s = -2.80$ m
$u = 0$

$$s = u_v t + \tfrac{1}{2}gt^2$$
$$-2.80 = 0 + (-4.9t^2)$$
$$-2.80 = -4.9t^2$$
$$2.80 = 4.9t^2$$
$$t^2_{\ down} = 0.571$$
$$t_{\ down} = 0.7556\ s$$

total t = 0.7556 x 2

<u>t = 1.51 s</u>

2. A helicopter is ascending vertically with a velocity of 6.0 ms^{-1}. At a height of 1.20 x 10^2 m above the earth, a package is dropped from a window. How much time does it take for the package to reach the ground?



If you have a calculator with a solver function the best way to do this is:

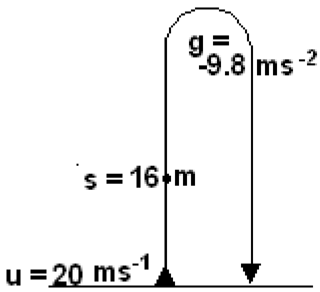
$$s_v = u_v t + 1/2\ gt^2$$
$$-120 = 6t - 4.9t^2$$
$$-4.9t^2 + 6t + 120 = 0$$
$$\underline{t = 5.60\ s}$$

OR

$$v_v^2 = u_v^2 + 2gs$$
$$= 6^2 + (2 \times -9.8 \times -120)$$
$$= 2388$$
$$v_v = 48.867\ ms^{-1}\ \text{down}$$

$$v_v = u_v + gt$$
$$-48.867 = 6.0 + (-9.8t)$$
$$-54.867 = -9.8t$$
$$\underline{t = 5.60\ s}$$

3. A toy rocket is fired vertically upwards from the ground at an initial velocity of 2.00 ms^{-1}.
   a. How fast is it moving when it reaches a height of 16.0 m on the way up?



Be careful with sign as the rocket will be 16.0 m above the ground on two separate occasions.

$$v_v^2 = u_v^2 + 2gs$$
$$= 20^2 + (2 \times -9.8 \times 16)$$
$$= 400 - 313.6$$
$$= 86.4$$
$$\underline{v_v = 9.30\ ms^{-1}\ \text{up}}$$

   b. How long is required to reach this height?

$u_v = 20.0\ ms^{-1}$

$v_v = 9.30\ ms^{-1}$

$g = -9.8\ ms^{-2}$

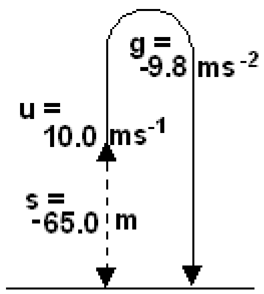
$s = 16$ m

$$t = \frac{v - u}{g}$$

$$t = \frac{9.30 - 20}{-9.8} = \frac{-10.7}{-9.8}$$

$$\underline{t = 1.09\ s}$$

4. A stone is thrown vertically upwards with an initial velocity of 10.0 ms^{-1} from the edge of a cliff 65.0 m high.
   a. What is its velocity just before hitting the ground below?



$v_v^2 = u_v^2 + 2gs$
$= 10^2 + (2 \times -9.8 \times -65\_$
$= 1374$
$\underline{v_v = 37.1\ ms^{-1}\ \text{down}}$

   b. How much later does it reach the bottom of the cliff?

$v_v = u_v + gt$
$-37.068 = 10 – 9.8t$
$-47.068 = -9.8t$
$t = 4.80\ s$

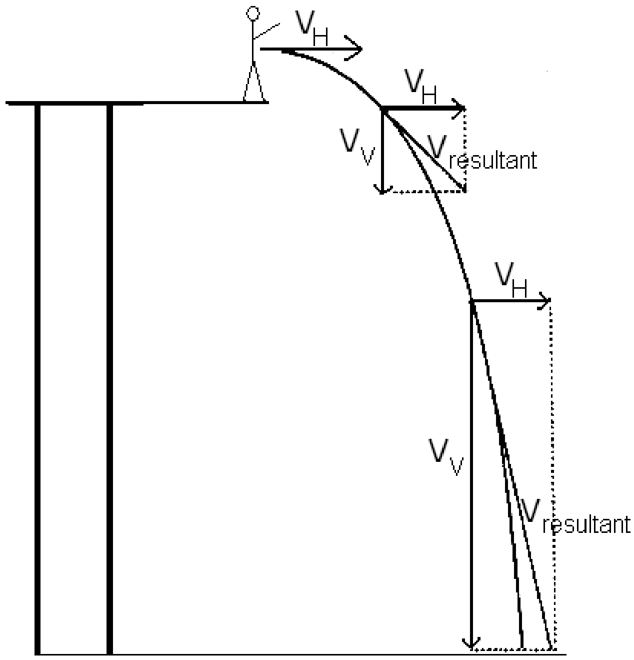
Projectiles In General

A projectile is something that is projected into the air such that it lands a distance from where it started. There are two main types of projectile motion:

Horizontal Projectiles
Text Reference pg. 26-32

These are projectiles where the initial velocity is horizontal to the ground and there is no initial vertical velocity. For example, a diver diving off a diving board.

The diagram shows the resultant velocity formed from the horizontal and components of the initial velocity.



NOTE: The time taken for a horizontal projectile to land is the same whether dropped or projected horizontally, as the acceleration is the same in both cases (acceleration due to gravity). The difference, is the horizontal displacement (range) they travel, and this depends on the value of the horizontal velocity.

Your teacher will work through these examples with you.

Example 1:
A boy throws a rock into the sea with a horizontal velocity of 5.00 ms^{-1}. The height of the cliff is 49.0 m and the rock is thrown 1.00 m above the top of the cliff. How far from the face of the cliff did the rock land?

Helpful Hint: Don't forget that up is positive and down is negative.

1. find time

$s_v = -49 + -1.0$
$= -50$ m
$u_v = 0$
$u_H = 5.00\ ms^{-1}$
$g = -9.8\ ms^{-2}$

$s_v = u_v t + 1/2gt^2$
$-50 = 0 + (-4.9t^2)$
$t^2 = 10.20$ s
$t = 3.1943$ s
(don't round yet!)

2. find horizontal displacement; s_H

$s_H = u_H t$
$= 5.00 \times 3.1943$
$= 15.972$
$\underline{s_H = 16.0\text{ m}}$

You should now see the important of using the subscripts V and H for horizontal and vertical displacement and velocity. If you don't you could easily mix up which displacement and velocity values to use in your calculations.

Example 2:
A boy jumps horizontally from a diving tower 1.00 m above the water and lands 2.50 m away from the edge of the tower.
a. How long was he in the air?
b. What was his horizontal take-off velocity?

$s_v = -1.00$ m
$u_v = 0\ ms^{-1}$
$s_H = 2.50$ m
$u_H = ?$
$g = -9.8\ ms^{-2}$

a. Find time

$s_v = u_v t + ½gt^2$
$-1.00 = 0 + (-4.9t^2)$
$t^2 = \frac{-1.0}{-4.9} = 0.2041$
$t = 0.4518$ s
$\underline{t = 0.452\text{ s}}$

b. Find u_H

$u_H = \frac{s_H}{t} = \frac{2.50}{0.4518}$
$\underline{v_H = 5.53\ ms^{-1}}$

Example 3:
Two murderers heave a body horizontally off a 60.0 m high cliff top at 1.20 ms^{-1}, hoping that it will land in bushes 5.00 m from the base of the cliff. Determine if they succeed and if not, how far away from the bush does the body land.

$s_v = -60.0$ m
$u_v = 0$
$s_H = ?$
$u_H = 1.20\ ms^{-1}$

$s_v = u_v t + ½\ gt^2$
$-60.0 = 0 + (-4.9t^2)$
$t^2 = \frac{-60}{-4.9} = 12.2449$
$t = 3.49927$ s

$s_H = u_H \times t$
$= 1.20 \times 3.49927$
$s_H = 4.20$ m

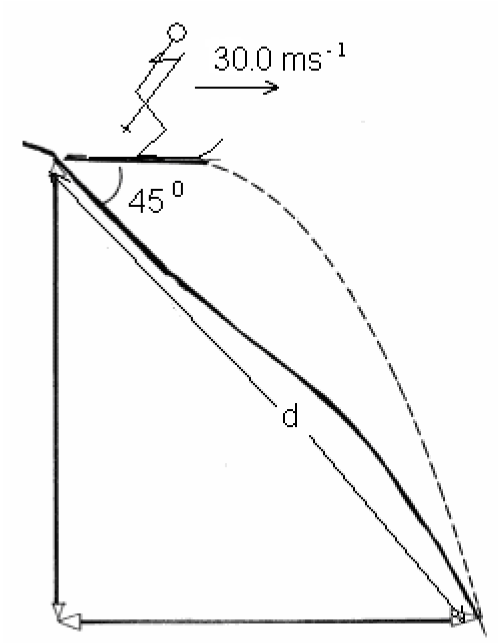
s_H **required = 5.00 m**

As the horizontal distance is 4.20 m and 5.00 m is required to hide the body, then the murderers did not succeed.

Helpful Hint: Don't round in the middle of calculations such as in the time above. Leave your rounding to the final answer.

Helpful Hint: Don't forget to put a final written answer is required such as in example (3) otherwise you will lose marks.

Example 4: (a much harder question)
A ski jumper competing in the winter Olympics travels down a slope and leaves the ski track moving in the horizontal direction with a speed of 30.0 m s^{-1}. The landing incline below the take off point slopes downwards at 45° to the horizontal. (Assume no air resistance or friction.)



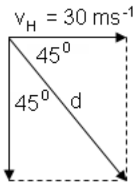
a. If the skier lands on the slope, calculate the skier's displac down the slope.
b. How long is the skier in the air before landing?
c. What is the vertical component of velocity just before the instant lands?

This type of question is not common but in case there is one in the WACE examinations you need to know how to answer them.

Up is positive, down is negative

a. Vertical
$s_v = u_v t + ½ gt^2$
but $u_v = 0$
so $s_v = -4.9t^2$

Horizontal
$s_H = u_H t$
$u_H = 30.0$
so $u_H = 30t$



from the diagram on the left showing the vector components of "d"

$s_V = d \sin 45$
but s_v **also equals** $-4.9t^2$
so $-4.9t^2 = d \sin 45$ **(equation 1)**

$s_H = d \cos 45$
but also equals 30t
$30t = d \cos 45$
re-arranging for t
$t = \frac{d x \cos 45}{30}$ **(equation 2)**

substitute equation 2 into equation 1 and simplify

$-4.9 \left(\frac{d x \cos 45}{30}\right)^2 = d \sin 45$

$-4.9 (d \times 0.02357)^2 = d \times 0.7071$

"d" cancels so

$-4.9 \times d^2 \times 5.55555 \times 10^{-4} = d \times 0.7071$
$d \times 2.72222 \times 10^{-3} = 0.7071$
<u>d = 260 m</u>

b.

$s_H = d \cos 45$
$= 259.751 \times \cos 45$
$= 183.67$ m

$t = \frac{s_H}{u_H} = \frac{183.67}{30}$

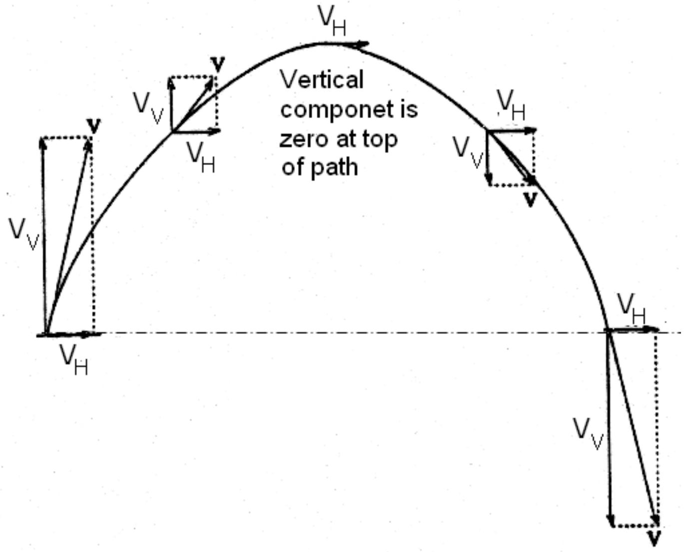
<u>t = 6.12 s</u>

c.

$v_v = u_v + gt$
$= 0 + (-9.8 \times 6.12233)$
$= 59.9988$
$v_v = -60.0$ ms^{-1}

<u>v_v = 60.0 ms^{-1} down</u>

Oblique Projectiles
Text Reference pg. 26-32



These are projectiles where the initial velocity is not horizontal to the ground but is at some angle e.g. an athlete doing the high jump.

As the diagrams shows, projectile motion consists of

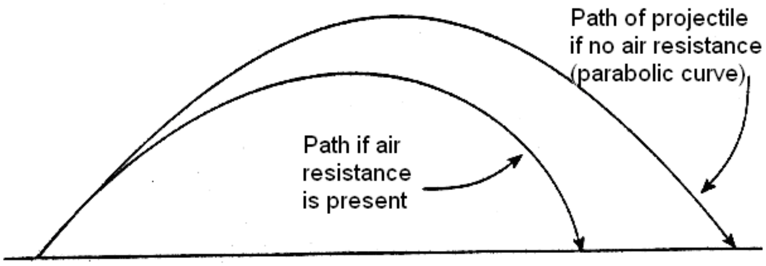
a. a horizontal component with constant velocity (unchanged throughout flight)
b. a vertical component with constant downward acceleration (9.8 ms^{-2})

The combination of these two components produces the characteristic curve associated with projectile motion.

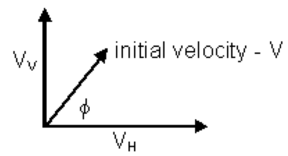
1. The path of an oblique projectile is a parabolic curve (if no air resistance)

Outcome 6: describe qualitatively the effects of air resistance on projectile motion

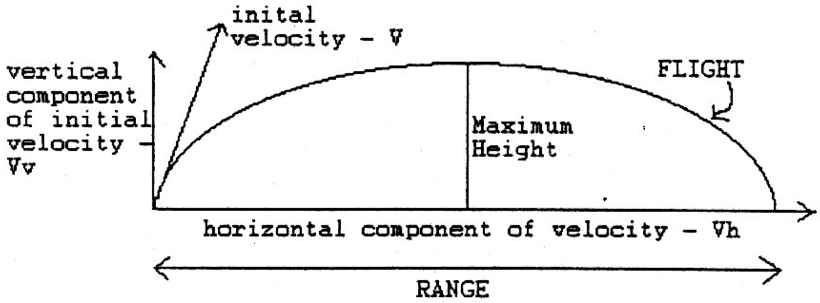
2. If the object is acted on by air resistance, the parabolic curve is changed such that there is a gradual rise and a steeper fall, it also reaches its maximum height more quickly than it takes to fall back to the ground. The speed with which it returns to the ground is also less than the launch speed.

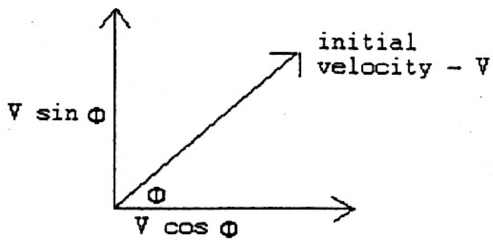


3. Initial velocity has vertical and horizontal components:
   vertical velocity $V_V = v \sin \phi$
   horizontal velocity $V_H = v \cos \phi$



4. Range is the horizontal displacement and the flight is the path the projectile takes.





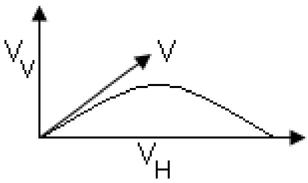
5. Maximum range when initial velocity is at 45^0. Depending on situation, air resistance and height above ground (and other factors), other angles can be preferred. For example, the angle of projection in cricket is often small, this is because the bowler requires the ball to have as large a horizontal velocity as possible.

6. Horizontal component constant (if no air resistance) and depends only on initial horizontal velocity.
7. Vertical component decreases to top of flight (where it is zero) then increases to bottom of flight. If take off and landing at same height, initial and final velocities are the same. Vertical component depends on both initial velocity and acceleration due to gravity.
8. Direction is important for displacement, velocity and acceleration. Use one direction as positive and one as negative – your teacher will mostly use down as positive (as gravity always acts down).

Your teacher will work through the following examples with you.

<u>Examples:</u>

1. An archer fires an arrow at 20.0 ms^{-1} into the air at an angle of 40.0^0 to the horizontal.
   a. How long was it in the air before it hit the target which was at the same height as the arrow was fired?



$V_V = u_v = 20 \times \sin 40$
$= 12.86\ ms^{-1}$
$v_v = -12.86\ ms^{-1}$
$V_H = u_H = 20 \times \cos 40$
$= 15.32\ ms^{-1}$
$g = -9.8\ ms^{-2}$

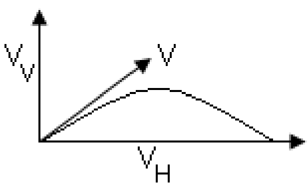
$$t = \frac{v - u}{g} = \frac{-12.86 - 12.86}{-9.8}$$

<u>t = 2.62 s</u>

   b. How far did it travel before hitting the target at the same height it was fired from?

$s_H = u_h \times t$
$= 15.32 \times 2.62$
<u>$s_H = 40.2$ m</u>

2. Bruce kicks a ball at an angle of 30.0^0 from the horizontal with an initial speed of 15.0 ms^{-1}. Find how far the ball travels horizontally before hitting the ground.



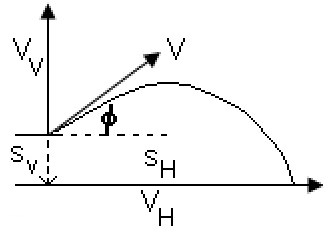
$V_V = u_v = 15 \times \sin 30$
$= 7.50\ ms^{-1}$
$v_v = -7.50\ ms^{-1}$
$V_H = u_H = 15 \times \cos 30$
$= 12.990\ ms^{-1}$
$g = -9.8\ ms^{-2}$

$$t = \frac{v - u}{g} = \frac{-7.50 - 7.50}{-9.8}$$

t = 1.5306 s

$s_H = u_H \times t$
$= 12.990 \times 1.5306$
<u>$s_H = 19.9$ m</u>

3. An archer fires an arrow at an angle of 35.0^0 to the horizontal with an initial velocity of 25.0 ms^{-1}. If the arrow is 1.30 m above the ground when fired, how far from the archer will it hit the ground?



$V_v = u_v = 25 \times \sin 35$
$= 14.339\ ms^{-1}$
$V_H = u_H = 25 \cos 35$
$= 20.479\ ms^{-1}$
$g = -9.8\ ms^{-2}$
$s_v = -1.30$ m

the easies way is to find time using $s_v = u_vt + ½\ gt^2$ but you need a calculator with solver.
$-1.30 = 14.339t - 4.9t^2$
$-4.9t^2 + 14.339t + 1.30$
$t = 3.01$ s

or find v_v
$v_v^2 = u_v^2 + 2gs$
$= 14.339^2 + (2 \times -9.8 \times -1.3)$
$= 231.09$
$v_v = 15.20\ ms^{-1}$ down

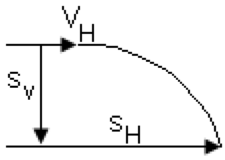
$v_v = u_v + gt$
$-15.2 = 14.339 + (-9.8t)$
$t = 3.01$ s

then either way
$s_H = u_H \times t$
$= 20.479 \times 3.01$
<u>$s_H = 61.7$ m</u>

<u>Activity</u>: Answer the following questions on projectiles yourself.

1. An archer fires an arrow at 35.0 ms^{-1} horizontal to the ground. If it lands 17.3 m away,
   a. how high was the arrow above the ground?
   b. at what velocity and did the arrow hit the ground?

a.



$V_H = u_H = 35.0\ ms^{-1}$
$u_v = 0\ ms^{-1}$
$s_H = 17.3$ m
$g = -9.8\ ms^{-2}$

$$t = \frac{s_H}{v_H} = \frac{17.3}{35.0} = 0.494286$$

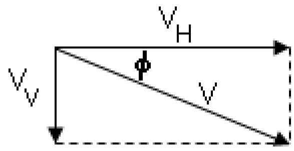
$s_v = u_vt + ½\ gt^2$
$= 0 + (-4.9 \times 0.494286^2)$
$s_v = 1.20$ m

<u>therefore arrow was 1.20 m above the ground</u>.

b. find v_v

$v_v = u_v + gt$
$= 0 + (-9.8 \times 0.494286)$
$= 4.844\ ms^{-1}$

draw vectors

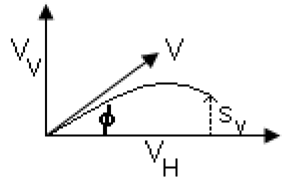


find velocity and angle

$V = \sqrt{(35^2 + 4.844^2)}$
$= 35.3\ ms^{-1}$
$\phi = \tan^{-1} (4.844 \div 35)$
$= 7.88^0$

<u>$V = 35.3\ ms^{-1}$, 7.88^0 below the horizontal.</u>

2. A cricket ball is thrown at an angle of 20.0^0 to the horizontal and reaches a maximum height of 15.0 m above its release point. How fast was it thrown?



$V_v = u_v = ?$
$v_v = 0$
$s_v = +15.0$ m
$g = -9.8\ ms^{-2}$
$\phi = 20.0^0$

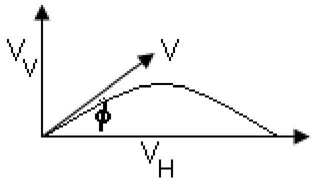
$v_v^2 = u_v^2 + 2gs$
$0 = u_v^2 + (2 \times -9.8 \times 15)$
$0 = u_v^2 - 294$
$u_v = 17.15\ ms^{-1}$ up

$u_v = V \sin 20$

$$V = \frac{17.15}{\sin 20} = 50.14$$

<u>therefore it was thrown at 50.1 ms^{-1}</u>

3. A stuntman is planning a stunt. A car is to take off from a ramp, which is angled at 15.0^0 to the horizontal to land on another ramp also angled at 15.0^0 to the horizontal (in the opposite direction), in-between the two ramps are a number of cars. The car's maximum take-off velocity is 90.0 kmh^{-1} which it can reach as it starts up the first ramp. The distance between the end of the first ramp and the start of the second ramp is exactly 32.0 m. (Assume no friction between car and ramps). Using your knowledge of projectiles, calculate if the stunt is safe to do.



$V_v = u_v$ = 25 sin 15
= 6.470 ms^{-1}
v_v = -6.470 ms^{-1}
$V_H = u_H$ = 25 cos 15
= 24.148 ms^{-1}
g = -9.8 ms^{-2}

V = 90.0 kmh^{-1}
= 25.0 ms^{-1}

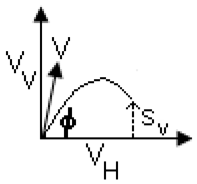
s_H to find = ?
s_H to compare = 32.0 m

$$t = \frac{v - u}{g} = \frac{-6.470 - 6.470}{-9.8}$$

t = 1.3204 s

$s_H = u_H \times t$
= 24.148 x 1.3204
= 31.89 m
<u>As need 32.0 m, the stunt is not safe to do.</u>

4. A high jumper leaves the ground at an angle of 70.0^0 to the horizontal and clears the bar, which is set at just below 1.55 m. She then lands on a mat which is 0.500 m above the ground on the other side of the bar.
   a. What is the size of the vertical velocity the jumper must have to just clear the bar?



$V_v = u_v$ = ?
v_v = 0 ms^{-1}
s_v = +1.55
g = -9.8 ms^{-2}
<u>u_v = 5.51 ms^{-1}</u>

$v_v^2 = u_v^2 + 2gs$
0 = u_v + (2 x -9.8 x 1.55)
0 = u_v^2 – 30.38
u_v^2 = 30.38

   b. How long is the high jumper in the air?

u_v = 5.512 ms^{-1}
s_v = 0.500 m
g = -9.8 ms^{-2}

easiest to do with a calculator with solver
$s_v = u_vt + ½ gt^2$
0.5 = 5.512t – 4.9t^2
-4.9t^2 + 5.512t – 0.5 = 0
t = 0.09934 or 1.0272
first time is for 0.5 m up, second time is total time so <u>t = 1.03 s</u>

OR find time up then time down.

time up
u_v = 5.51 ms^{-1}
v_v = 0
g = -9.8 ms^{-2}

$$t = \frac{v - u}{-9.8} = \frac{0 - 5.51}{-9.8}$$

t up = 0.56224 s

time down
u_v = 0
s_v = -1.55 + 0.5
= -1.05 m
g = -9.8 ms^{-2}

$s_v = u_vt + ½ gt^2$
-1.05 = 0 – 4.9t^2
1.05 = 4.9t^2
t^2 = 0.2143
t down = 0.4629

t total = 0.56224 + 0.4629
= 1.025

<u>t = 1.03 s</u>

Activity: More Questions On Vectors And Projectiles.   **answers on pages**

1. A cyclist, travelling east at 20.0 km h^{-1}, changes her velocity to 25.0 km h^{-1} south.
   a. What is her change in velocity?
   b. If the change occurred in 5.00 seconds, what was her acceleration around the corner?

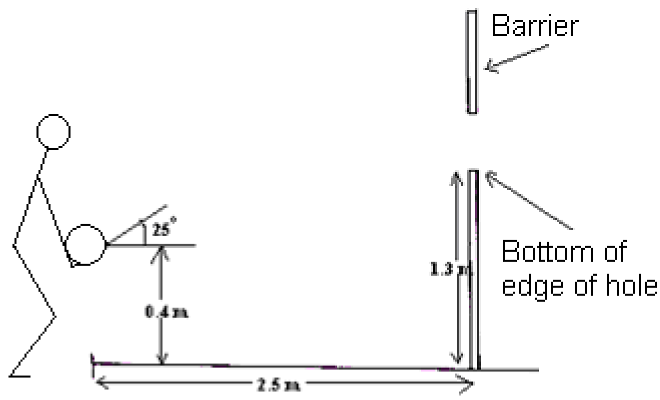
2. A ball is thrown up into the air at 4.50 ms^{-1} and then lands on the ground. If it is thrown from a height of 1.30 m above the ground, what is the ball's maximum height and how long is it in the air?

3. An aeroplane is to make a 'drop' of a survival parcel to people 5.00 x 10^2 m below while it is travelling horizontally at 1.00 x 10^2 ms^{-1}. Find,
   a. the time it takes for the parcel to reach the ground;
   b. the horizontal distance the parcel travels once released from the plane;
   c. the horizontal velocity of the parcel as it hits the ground
   d. the vertical velocity of the parcel as it hits the ground;
   e. use the information from answers to (c) and (d) to find the final velocity it hits the ground.

4. An arrow is fired into the air at an angle of 3.05^0 and an initial velocity of 55.0 ms^{-1}. If it lands at the same height as it was fired, how far will it travel (find the range)? How could the archer increase the distance travelled if the arrow is fired at its maximum initial velocity?

5. A projectile is launched from the top of a cliff, 1.00 x 10^2 m above the ground directly below. The angle of evaluation is 30.0^0 above the horizontal. Determine the total time of its flight and the horizontal distance travelled if its initial velocity is 50.0 ms^{-1}.

6. ***This final one is hard but a similar one appeared in the 2002 TEE. It is uncertain where this type of question will continue in the new WACE examinations.***
   A child attending a school fete is keen to win a prize in a throwing contest. To win a prize he must throw a ball underarm through a hole in a barrier and hit a target on the other side. He stands 2.50 m in front of the hole, the bottom edge of which is 1.30 m above the ground. If he releases the ball at an angle of 25.0^o to the horizontal from 0.40 m above the ground, at what speed must he throw the ball so it just enters the hole clearing the bottom edge?

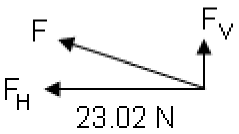


Exploring Physics:
- **Set 2: Projectile Motion**
- **Experiment 2.1 and 2.2 (need camera and stroboscope lamp)**

Task 6: Test Projectile motion and circular motion

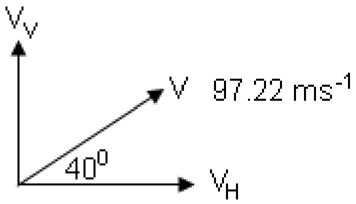
Questions on Vectors from page 10.

1. While working at Coles, Giles needs to move cartons of tomatoes to the Fruit and Vegetable section of the store using a trolley. The handle of the trolley is at an angle of 65.0^0 to the vertical and the horizontal component of the force is 23.02 N. With what force is Giles pulling the cart?



F_H = F cos 25
23.02 = F cos 25
$F = \dfrac{23.02}{\cos 25}$ **<u>F = 25.4 N</u>**

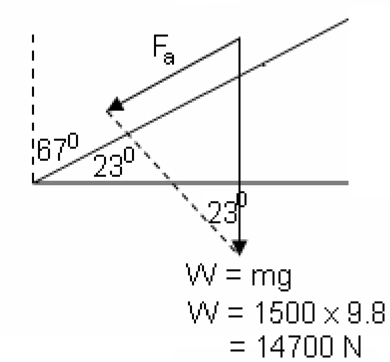
2. Tina is taking a plane trip to New Zealand. The plane is climbing at an angle of 40.0^o to the ground with a constant speed of 3.50 x 10^2 kmh^{-1}. What is the effective forward speed of the plane?



v = 350 kmh^{-1}
= 97.22 ms^{-1}

V_H = V cos 40
= 97.22 cos 40
= 74.4748
<u>V_H = 74.5 ms^{-1}</u>

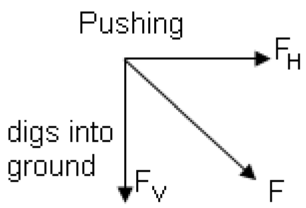
3. A small van with a mass of 1.50 x 10^3 kg is parked at the top of a hill that is inclined at 67.0^o to the vertical. The brakes fail. Calculate the force that is now pulling the truck down the hill?

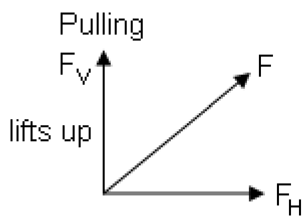


F_a = W sin 23
= 14700 x sin 23
= 5743.75
<u>F_a = 5.74 x 10^3 N</u>

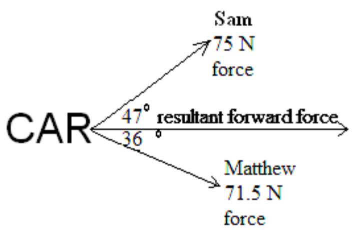
4. Explain why it is easier to pull a heavy roller across a lawn than it is to push it.

Force has horizontal and vertical components. For both pushing and pulling, horizontal component is the same so not difference. when pushing, the vertical component is downwards so the roller digs more into the ground. when pulling, the vertical component is upwards creating a lifting force so easier to get over the ground.





5. Sam and Matt are trying to pull Matt's car out of the sand where it has become bogged. Sam is using 75.0 N of force at an angle of 47.0^0 while Matt uses 71.5 N at an angle of 36.0^0 to the resultant forward force. The boys need an overall force of 105.0 N to pull the car free. Will they be able to do so? You must show calculations to justify your answer.



Sam
F_H = 75 cos 47
= 51.1499 N

Matt
F_H = 71.5 cos 36
= 57.8447 N

F_T = 51.1499 + 57.8447 = 109.00 N
Greater than 105 N so car will pull free.

6. Rohan throws a 340 g ball with a velocity of 6.40 ms^{-1} East at a wall. The ball rebounds returns at 3.80 ms^{-1} West. If the change in velocity occurred in 0.350 s, what force did the wall apply to the ball?

$v - u = 3.8\ W - 6.4\ E$
$= 3.8\ W + 6.4\ W$
$= 10.2\ ms^{-1}\ W$
$m = 0.34\ kg$
$t = 0.350\ s$

$F = m \dfrac{(v - u)}{t}$

$= 0.34 \times \dfrac{10.2}{0.350}$

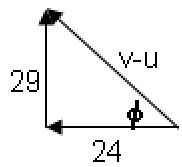
<u>F = 9.91 N West</u>

7. A pitcher throws a baseball due south towards the batter at 104.4 kmh^{-1}. The batter hits the ball foul by striking it due west at 86.4 kmh^{-1}. If the ball's change in velocity occurred in 0.200 seconds, what was the ball's acceleration?

$u = 104.4\ kmh^{-1}$
$= 29\ ms^{-1}\ S$
$v = 86.4\ kmh^{-1}$
$= 24\ ms^{-1}\ W$

$v - u = 24\ W - 29\ S$

$= 24\ W + 29\ N$



$\Delta V = \sqrt{(29^2 + 24^2)}$
$= 37.64\ ms^{-1}$
$\phi = \tan^{-1}(29 \div 25)$
$= 50.4^0$

$a = \dfrac{v - u}{t} = \dfrac{37.64}{0.200}$
$= 188.2$

<u>$a = 188\ ms^{-2}$ W 50.4^0 N</u>

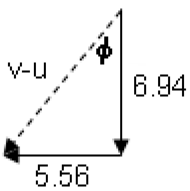
Answers for Questions on vectors and projectiles from page 27 to 31

<u>Activity</u>: More Questions On Vectors And Projectiles. **answers on pages**

1. A cyclist, travelling east at 20.0 km h^{-1}, changes her velocity to 25.0 km h^{-1} south.
   a. What is her change in velocity?
   b. If the change occurred in 5.00 seconds, what was her acceleration around the corner?

a.

$u = 20\ kmh^{-1}\ E$
$= 5.56\ ms^{-1}\ E$
$v = 25\ kmh^{-1}\ S$
$= 6.94\ ms^{-1}\ S$
$v\text{-}u = 6.94\ S - 5.56\ E$
$= 6.94\ S + 5.56\ W$



$v\text{-}u = \sqrt{(5.56^2 + 6.94^2)}$
$= 8.89\ ms^{-1}$

$\phi = \tan^{-1}(5.56 \div 6.94)$
$= 38.7^0$

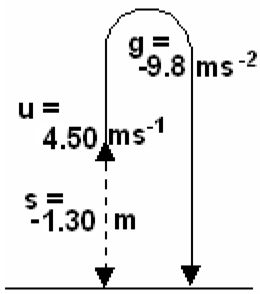
<u>$v\text{-}u = 8.89\ ms^{-1}$ S 38.7^0 W</u>

b.

$a = \dfrac{v - u}{t} = \dfrac{8.89}{5} = 1.778$

<u>$a = 1.78\ ms^{-2}$ S 38.7^0 W</u>

2. A ball is thrown up into the air at 4.50 ms^{-1} and then lands on the ground. If it is thrown from a height of 1.30 m above the ground, what is the ball's maximum height.



$u_v = 4.50\ ms^{-1}$
$v_v = 0$
$s_v = -1.30\ m$
$g = -9.8\ ms^{-2}$

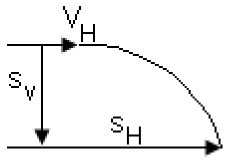
$v_v^2 = u_v^2 + 2gs$
$0 = 4.5^2 + (2 \times -9.8 \times s)$
$0 = 20.25 - 19.6s$
$20.25 = 19.6\ s$
$s = 1.03\ m$

but already 1.30 m above ground so

max height is 1.03 + 1.30
= <u>2.33 m up</u>

3. An aeroplane is to make a 'drop' of a survival parcel to people 5.00 x 10^2 m below while it is travelling horizontally at 1.00 x 10^2 ms^{-1}. Find,

a. the time it takes for the parcel to reach the ground;
b. the horizontal distance the parcel travels once released from the plane;
c. the vertical velocity of the parcel as it hits the ground;
d. the velocity with which it hits the ground (don't forget direction).

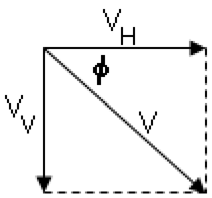


$V_H = 100\ ms^{-1}$
$u_v = 0\ ms^{-1}$
$S_v = -500\ m$
$g = -9.8\ ms^{-2}$

a. **$s_v = u_v t + ½\,gt^2$**
$-500 = 0 - 4.9t^2$
$500 = 4.9t^2$
$t^2 = 102.04$
$\underline{t = 10.1\ s}$

b. **$s_H = v_H \times t$**
$= 100 \times 10.1$
$\underline{s_H = 1.01 \times 10^3\ m}$

c. **$v_v = u_v + gt$**
$= 0 + (-9.8 \times 10.1)$
$= -98.98$
$\underline{v_v = 99.0\ ms^{-1}\ \text{down}}$

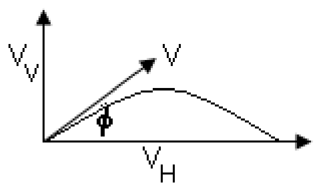


$V_H = 100\ ms^{-1}$
$V_v = 98.98\ ms^{-1}$

d. **$V = \sqrt{(100^2 + 98.98^2)}$**
$V = 141\ ms^{-1}$
$\phi = \tan^{-1}(98.98 \div 100)$
$= 44.7^0$

$\underline{V = 141\ ms^{-1}\ 44.7^0\ \text{down from the horizontal}}$

4. An arrow is fired into the air at an angle of 35.0^0 and an initial velocity of $55.0\ ms^{-1}$. If it lands at the same height as it was fired, how far will it travel (find the range)? How could the archer increase the distance travelled if the arrow is fired at its maximum initial velocity?



$V_v = u_v = 55 \sin 35$
$= 31.547\ ms^{-1}$
$V_H = u_H = 55 \cos 35$
$= 45.05\ ms^{-1}$
$g = -9.8\ ms^{-2}$

$$t = \frac{v - u}{g} = \frac{-31.547 - 31.547}{-9.8}$$

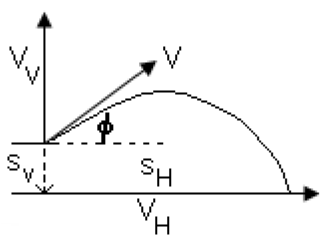
$t = 6.44\ s$

$s_H = u_H \times t$
$= 45.05 \times 6.44$
$\underline{s_H = 2.90 \times 10^2\ m}$

To travel further, use 45^0 instead of 35^0

Helpful hint:
Don't forget significant figures. If you had given answer as 290 m, you would only have had 2 sf.

5. A projectile is launched from the top of a cliff, 1.00×10^2 m above the ground directly below. The angle of evaluation is 30.0^0 above the horizontal. If the time of flight is 7.74 s and the projectile lands 335 m from the take-off point, determine the initial velocity of the projectile and the maximum height reached.



$V_H = u_H = V \cos 30$
$g = -9.8\ ms^{-1}$
$s_v = -100\ m$
$s_H = 335\ m$
$t = 7.74\ s$

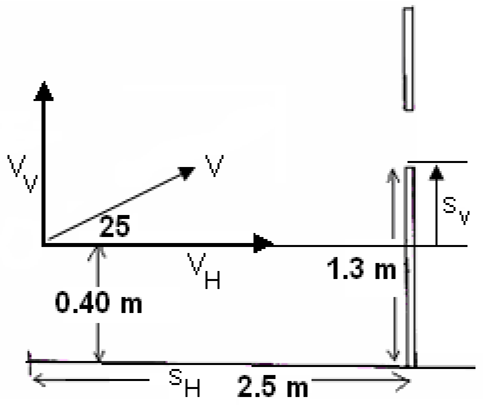
$$u_H = \frac{s_H}{t} = \frac{335}{7.74} = 43.28$$

$V_H = u_H = V \cos 30$
$43.28 = V \cos 30$
$\underline{V = 50.0\ ms^{-1}}$

$V_v = u_v = V \sin 30$
$= 50 \sin 30$
$= 25.0\ ms^{-1}$
$v_v = 0\ ms^{-1}$

$v_v^2 = u_v^2 + 2gs$
$0 = 25^2 + (2 \times -9.8 \times s)$
$s = 625 \div 9.8 = 63.7755\ m$
but projectile already 100 m above ground so
$s = 100 + 63.7755$
$\underline{\text{height above ground} = 164\ m}$

6. This final one is hard but similar ones have appeared in the TEE. It is uncertain whether this type of question will continue in the new WACE examinations.



A child attending a school fete is keen to win a prize in a throwing contest. To win a prize he must throw a ball underarm through a hole in a barrier and hit a target on the other side. He stands 2.50 m in front of the hole, the bottom edge of which is 1.30 m above the ground. If he releases the ball at an angle of 25.0° to the horizontal from 0.40 m above the ground, at what speed must he throw the ball so it just enters the hole clearing the bottom edge?

s_v = 1.3 – 0.4 = 0.90 m
u_v = V sin 25
u_H = V cos 25
s_H = 2.50 m

Horizontal:

$$t = \frac{s_H}{u_H} = \frac{2.5}{V\cos 25} \quad (1)$$

Vertical

$$s_v = u_v t + \tfrac{1}{2} g t^2$$

$$0.9 = (V \sin 25)t - 4.9t^2 \quad (2)$$

substitute (1) into (2)

$$0.9 = V \sin 25 \times \frac{2.5}{V\cos 25} - 4.9\left(\frac{2.5}{V\cos 25}\right)^2$$

$$0.9 = \left(\frac{V\sin 25 x 2.5}{V\cos 25}\right) - \left(\frac{4.9x6.25}{V^2 0.8214}\right)$$

$$0.9 = 1.1658 - \frac{37.2839}{V^2}$$

$$-0.2658 = -\frac{37.2839}{V^2}$$

$$V = \sqrt{\frac{37.2839}{0.2658}}$$

<u>V = 11.8 ms^{-1}</u>