# 3D Vectors

Questions are taken from VCE Secondary Papers

## 2009

**Question 3**

Resolve the vector $5\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ into two vector components, one which is parallel to the vector $-2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and one which is perpendicular to it.

## Answers

3. parallel: $2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ perpendicular: $3\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$

## 2008

**Question 8**

The coordinates of three points are $A(1, 0, 5)$, $B(-1, 2, 4)$ and $C(3, 5, 2)$.

**a.** Express the vector $\overrightarrow{AB}$ in the form $x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$.

**b.** Find the coordinates of the point $D$ such that $ABCD$ is a parallelogram.

**c.** Prove that $ABCD$ is a rectangle.

## Answers

8. a. $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$
   b. $(5, 3, 3)$
   c. Show ONE pair of adjacent sides are at right angles OR show that diagonals have the same length.

## 2007

**Question 4**

An aircraft approaching an airport with velocity $\underset{\sim}{v} = 30\underset{\sim}{i} - 40\underset{\sim}{j} - 4\underset{\sim}{k}$ is observed on the control tower radar screen at time $t = 0$ seconds. Ten seconds later it passes over a navigation beacon with position vector $-500\underset{\sim}{i} + 2500\underset{\sim}{j}$ relative to the base of the control tower, at an altitude of 200 metres.

Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be horizontal orthogonal unit vectors and let $\underset{\sim}{k}$ be a unit vector in the vertical direction. Displacement components are measured in metres.

**a.** Show that the position vector of the aircraft relative to the base of the control tower at time $t$ is given by

$$\underset{\sim}{r}(t) = (30t - 800)\underset{\sim}{i} + (2900 - 40t)\underset{\sim}{j} + (240 - 4t)\underset{\sim}{k}.$$

**b.** When does the aircraft land and how far (correct to the nearest metre) from the base of the control tower is the point of landing?

**c.** At what angle from the runway, correct to the nearest tenth of a degree, does the aircraft land?

**d.** At what time, correct to the nearest second, is the aircraft closest to the base of the control tower?

e. What distance does the aircraft travel from the time it is observed on the radar screen to the time it lands? Give your answer correct to the nearest metre.

## Answers

4. a. $\underset{\sim}{\mathrm{r}}(t) = 30t\underset{\sim}{\mathrm{i}} - 40t\underset{\sim}{\mathrm{j}} - 4t\underset{\sim}{\mathrm{k}} + \underset{\sim}{\mathrm{c}}$, $-500\underset{\sim}{\mathrm{i}} + 2500\underset{\sim}{\mathrm{j}} + 200\underset{\sim}{\mathrm{k}} = 300\underset{\sim}{\mathrm{i}} - 400\underset{\sim}{\mathrm{j}} - 40\underset{\sim}{\mathrm{k}} + \underset{\sim}{\mathrm{c}}$

b. $240 - 4t = 0 \Rightarrow t = 60$, $\underset{\sim}{\mathrm{r}}(60) = 1000\underset{\sim}{\mathrm{i}} + 500\underset{\sim}{\mathrm{j}}$, Distance $= \sqrt{1000^2 + 500^2} = 1118$ m.

c. $\sin\theta = \dfrac{4}{\sqrt{30^2 + 40^2 + 4^2}}$ or equivalent gives $\theta = 4.6°$

d. $\underset{\sim}{\mathrm{r}} \cdot \underset{\sim}{\mathrm{v}} = 0 \Rightarrow 30(30t - 800) - 40(2900 - 40t) - 4(240 - 4t) = 0$, which gives $t = 56$ seconds.

e. $D = \sqrt{30^2 + 40^2 + 4^2} \times 60$, or other method, gives $D = 3010$ m