PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 1 Year 12

Student name: _______________ Teacher name: _______________

Task type: **Response/Investigation**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **7**

Materials required: No cals allowed!!

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, NO notes allowed!

Marks available: **41 marks**

Task weighting: **13%**

Formula sheet provided: no, but formulae stated on page 2

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = \lvert z\rvert = \sqrt{a^2+b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan\theta = \frac{b}{a}, \quad -\pi < \theta \le \pi$
$\lvert z_1 z_2\rvert = \lvert z_1\rvert \lvert z_2\rvert$	$\left\lvert \frac{z_1}{z_2} \right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = \lvert z\rvert^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{\lvert z\rvert^2}$
$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$	$\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$
Polar form	
$z = a + bi = r(\cos\theta + i\sin\theta) = r\,cis\,\theta$	$\bar{z} = r\,cis(-\theta)$
$z_1 z_2 = r_1 r_2\,cis(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,cis(\theta_1 - \theta_2)$
$cis(\theta_1 + \theta_2) = cis\,\theta_1\,cis\,\theta_2$	$cis(-\theta) = \frac{1}{cis\,\theta}$
De Moivre's theorem	
$z^n = \lvert z\rvert^n cis(n\theta)$	$(cis\,\theta)^n = \cos n\theta + i\sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\frac{\theta + 2\pi k}{q} + i\sin\frac{\theta+2\pi k}{q}\right)$, for k an integer	

$$(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

No cals allowed!!

Q1 (2, 2, 2 & 2 = 8 marks)

If $z = 5 - 4i$ and $w = 2 + 3i$ determine the following:

a) zw

b) $\frac{1}{w}$

c) $\frac{\overline{z}}{w}$

d) $z^2\overline{w}$

Q2 (2 & 3 = 5 marks)

a) Determine the complex roots of $3z^2 + z + 2 = 0$.

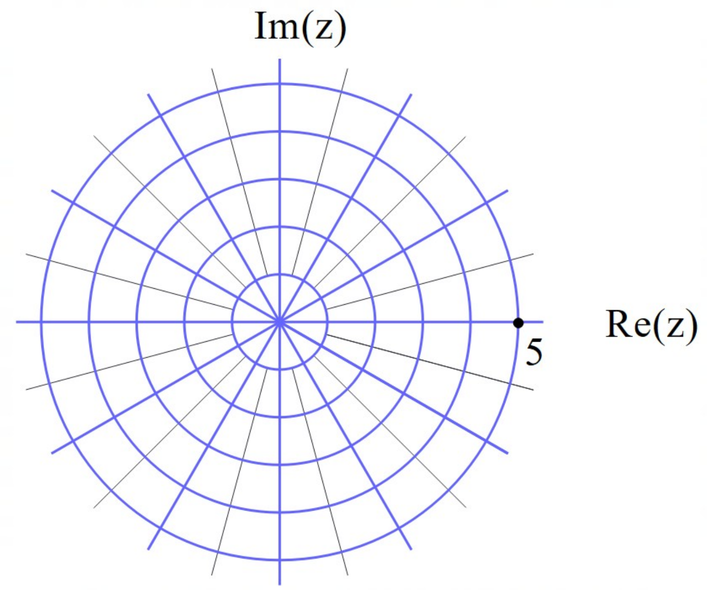
b) Use the quadratic equation to prove that if a quadratic equation with real coefficients has any non-real roots then it must have two complex roots and they must be conjugates of each other.

Q3 (4 marks)

Determine all possible real number pairs $a \,\&\, b$ such that $\dfrac{31-29i}{a+2i}=3+bi$.

Q4 (2, 2, 2 & 2 = 8 marks)

Consider the complex number $z=\sqrt{3}+i$.



Plot the following on the axes above.

a) z

b) iz

c) $(1+i)z$

d) $\frac{z}{(1+i)}$

Q5 (5 marks)

Consider the polynomial $f(z) = az^4 + bz^3 + cz^2 + dz + e$ where $a, b, c, d \, \& \, e$ are real numbers.

Given that $f(1+i) = 0 = f(2-3i)$

and $f(0) = 52$

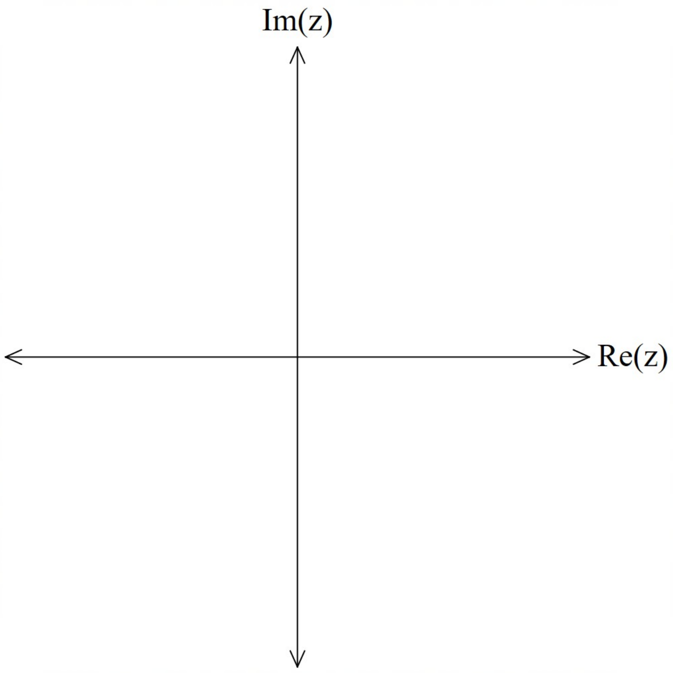
Determine the values of $a, b, c, d \, \& \, e$.

(Note: answers without working will receive zero marks)

Q6 (2, 1, 2 & 2 = 7 marks)

Consider the locus of complex numbers z that satisfy $\left|z-1-\sqrt{3}i\right|=2$.

a) Sketch the locus on the axes below.



b) State the maximum value of $|z|$

c) State the minimum value of $Arg(z)$ such that $Arg(z) > Minimum$.

d) State the maximum value of $Arg(z)$ such that $Arg(z) < Maximum$.

Q7 (4 marks)

In the following simultaneous equations, $a \& b$ are real numbers.

$a^3 = 3ab^2 - 13\sqrt{2}$

$b^3 = 3a^2 b - \sqrt{5}$

In order to determine the value of $a^2 + b^2$ from these equations, it is useful to consider the complex expansion for $(a + bi)^3$. Hence or otherwise, determine the exact value of $a^2 + b^2$.

(Note: answers without working will receive zero marks)

Working out space