Question 9 **(7 marks)**

Determine all turning points and points of inflection of the function $f(x) = 2x^3 - 3x^2 - 12x + 20$, and use these to sketch its graph.

Solution

If $f(x) = 2x^3 - 3x^2 - 12x + 20$, then $f'(x) = 6x^2 - 6x - 12$ and $f''(x) = 12x - 6$

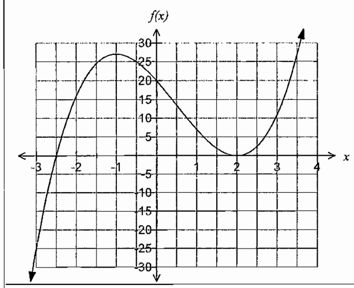
$6x^2 - 6x - 12 = 0 \Rightarrow 6(x-2)(x+1) = 0$

So the critical points occur at $x = 2$ and $x = -1$.

$12x - 6 = 0 \Rightarrow x = \frac{1}{2}$, where the point of inflection will be found.

Now $f(2) = 0,\ f(-1) = 27$ *and* $f(\frac{1}{2}) = \frac{27}{2}$, $f(0) = 20$

So the graph is



Specific behaviours

✓ determines $f'(x)$
✓ determines $f''(x)$
✓ finds critical points
✓ finds the point of inflection
✓ graph passes through the correct y-intercept
✓ graph passes through appropriate range of x values for intercept, i.e. (–3 to –2)
✓ correct shape of graph

Section One: Calculator-free **(40 Marks)**

Question 1 **(4 marks)**

Determine the domain and range of $f(g(x))$, given that $f(x) = \sqrt{x}$ and $g(x) = 4 - 2^x$

Solution

$f(g(x)) = f(4 - 2^x)$
$= \sqrt{4 - 2^x}$

Domain: We need $4 - 2^x \geq 0$, i.e. $2^x \leq 4$, i.e. $x \leq 2$.
Range: $0 \leq y < 2$

✓ determines $f(g(x))$ correctly
✓ correctly identifies requirement that $4 - 2^x \geq 0$
✓ correctly states domain
✓ correctly states range

Question 2 **(4 marks)**

Differentiate the following, without simplifying:

(a) $y = e^{2x-x^2}$ (2 marks)

Solution

Derivative: $(2-2x)e^{2x-x^2}$

Specific behaviours

✓ differentiates $(2x - 2x^2)$ part
✓ e^{2x-x^2} remains in solution

(b) $y = \frac{5x}{x^2+4}$ (2 marks)

Solution

Derivative: $\frac{(x^2+4)5 - (2x)(5x)}{(x^2+4)^2}$

Specific behaviours

✓ applies quotient rule correctly
✓ correctly differentiates each part

or

Solution

$y = 5x(x^2+4)^{-1}$ so
$y' = 5(x^2+4)^{-1} + 5x(-2x)(x^2+4)^{-2}$

Specific behaviours

✓ correctly applies chain rule

Question 3 **(3 marks)**

The probabilities of two events A and B are given by: $P(A) = 0.6$ and $P(B) = 0.3$
Calculate $P(A \cup B)$, given that A and B are independent.

Solution

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

and $P(A \cap B) = P(A) \times P(B)$ by independence

So $P(A \cup B) = 0.6 + 0.3 - 0.6 \times 0.3 = 0.72$

Specific behaviours

✓ selects appropriate rule from formula sheet
✓ uses multiplication rule for independence
✓ substitutes and calculates probability

Question 4 **(5 marks)**

Find the maximum and minimum values over the interval $1 \leq x \leq 4$ of the function

$$f(x) = x + \frac{4}{x^2}$$

Solution

The function is continuous and differentiable in the interval $1 \leq x \leq 4$ and so the extreme values occur at the end points or at critical points.

$f'(x) = 1 - \frac{8}{x^3} = 0$ when $x = 2$ and $f(2) = 2 + \frac{4}{2^2} = 3$

Also $f(1) = 1 + \frac{4}{1^2} = 5$ and $f(4) = 4 + \frac{4}{4^2} = 4\frac{1}{4}$

So $f_{max} = 5$ and $f_{min} = 3$.

Specific behaviours

✓ correctly differentiates
✓ solves $f'(x) = 0$
✓ evaluates $f(2)$
✓ evaluates $f(1)$ and $f(4)$
✓ states maximum and minimum

Question 5 **(4 marks)**

Solve for x in the equation

$$\frac{3}{x}+\frac{4x}{1+2x}=2$$

Solution
$\frac{3(1+2x)+4x^2}{x(1+2x)}=2$ $3(1+2x)+4x^2=2x(1+2x)$ $3+6x+4x^2=2x+4x^2$ $3+4x=0$, $x=-\frac{3}{4}$
Specific behaviours
✓ recognises common denominator ✓ multiplies by common denominator correctly ✓ simplifies ✓ states correct solution

Question 6 **(4 marks)**

Determine the following integrals:

(a) $\int \frac{x^2-1}{(x^3-3x)^2}\,dx$ (2 marks)

Solution
$=\frac{1}{3}\int\frac{3x^2-3}{(x^3-3x)^2}dx=\frac{1}{3}\int(x^3-3x)^{-2}(3x^2-3)\,dx$ $=\frac{1}{3}\frac{(x^3-3x)^{-1}}{(-1)}+C \quad =-\frac{1}{3(x^3-3x)}+C$
Specific behaviours
✓ expresses integral in terms of $\int[f(x)]^n f'(x)\,dx$ ✓ integrates correctly and adds constant

(b) $\int_0^5 e^{-2x}\,dx$ (2 marks)

Solution
$\int_0^5 e^{-2x}dx=\left(-\frac{1}{2}e^{-2x}\right)\Big\vert_{x=0}^{x=5}$ $=\frac{1}{2}\left(-e^{-10}+e^{0}\right)=\frac{1}{2}\left(1-e^{-10}\right)$
Specific behaviours
✓ finds the integrand ✓ substitutes limits of integration and simplifies

Question 7 **(5 marks)**

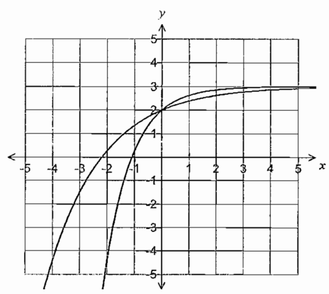
Solve the system of equations

$$\begin{aligned}x+3y+z&=2\\2x+5y+3z&=11\\4x+3y+2z&=16\end{aligned}$$

Solution
For example: $-y+z=7$ Eq2 - 2Eq1 → Eq2 $-9y-2z=8$ Eq3 - 4Eq1 → Eq3 $11z=55$ 9Eq2 - Eq3 So $z=5$ Substitution gives $y=-2$ and $x=3$
Specific behaviours
✓✓ eliminates one variable from two pairs of equations ✓✓✓ evaluates each of the variables correctly

Question 8 **(4 marks)**

The graph of $y=ae^{bx}+c$ is shown below. The graph passes through the point $(0, 2)$, and $y\to 3$ as $x\to\infty$



(a) Is b positive or negative? Justify your answer. (1 mark)

Solution
Since $y\to 3$ as $x\to\infty$, $e^{bx}\to 0$ as $x\to\infty$. So b must be negative.
Specific behaviours
✓ gives logical argument as to why b is negative

(b) Evaluate a and c. (2 marks)

Solution
Since $y\to 3$ as $x\to\infty$, $c = 3$. Since $y(0)=ae^0+c=a+c=2$, $a=-1$.
Specific behaviours
✓ evaluates c ✓ evaluates a

(c) Sketch on the same axes the graph of $y=ae^{2bx}+c$. (1 mark)

Solution
See graph above.
Specific behaviours
✓ draws graph with correct shape for $x > 0$ and $x < 0$, relative to the original graph