

Question	Marks	Max	Question	Marks	Max
14	8		13	9	20
12	10	19	11	8	
10	4	18	8	8	
9	6	17	10	10	
8	8	16	8	8	
	6	15		8	

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination.

To be provided by the candidate Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the supervisor Formula sheet (referred from Section One)

MATERIALS REQUIRED/RECOMMENDED FOR THIS SECTION This Question/Answer booklet

Time allowed for this section Working time: one hundred minutes
Reading time before commencing work: ten minutes

Your Teacher's Name:

Your Name:

Section Two:
Calculator-assumed



INDEPENDENT PUBLIC SCHOOL
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MATHEMATICS METHODS

UNIT 3

Question/Answer booklet

Semester One Examination, 2019

PERTH MODERN SCHOOL

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MATHEMATICS METHODS

2CALCULATOR-ASSUMED

CALCULATOR-ASSUMED

23MATHEMATICS METHODS

Additional working space

Question number: _____

Instructions to candidates

Structure of this paper

CALCULATOR-ASSUMED MATHEMATICS METHODS

22CALCULATOR-ASSUMED

Additional working space

MATHEMATICS METHODS

Question number:

Section One: Calculator-assumed

(104 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8

(6 marks)

Consider the following table.

x	1	2	3	4	5
$P(X \leq x)$	0.1	0.4	0.7	0.9	1

- a) Complete the probabilities in the table below

(2 marks)

x	1	2	3	4	5
$P(X=x)$	0.1				

- b) Determine $P(x \geq 4)$

(1 mark)

- c) Determine $P(x > 2 \vee x < 4)$ (simplify)

(3 marks)

Additional working space

Question number: _____

(4 marks)

- (c) Determine $\frac{dA}{dt}$ when $t=1$ second as an exact value. (no need to simplify)

The radius, r , varies with time, t seconds, by the model $r = \sqrt{5t^2 + 1}$.

(2 marks)

- (b) What is the meaning of your answer in (a) above?

(4 marks)

$$\theta = \frac{x}{r}$$

(d)

- Determine the acceleration of the spot of light when $\theta = \frac{\pi}{6}$ radians.

(2 marks)

- (a) Determine $\frac{dA}{dr}$ when $r = 5$ metres.

A liquid is spilled onto a floor forming a circle of radius r metres. The surface area, A , square metres, of the split liquid is given by $A = \int_0^r 15e^{\frac{x}{2}} dx$.

(8 marks)

Question 9

(3 marks)

$$\left(\text{Hint-use } \frac{dx}{dt} = \frac{d\theta}{dt} \frac{dx}{d\theta} \text{ with } \frac{d\theta}{dt} = 4\pi \right)$$

$$\frac{dx}{dt} = \frac{d\theta}{dt} \frac{dx}{d\theta} \quad \text{with } \frac{d\theta}{dt} = 4\pi$$

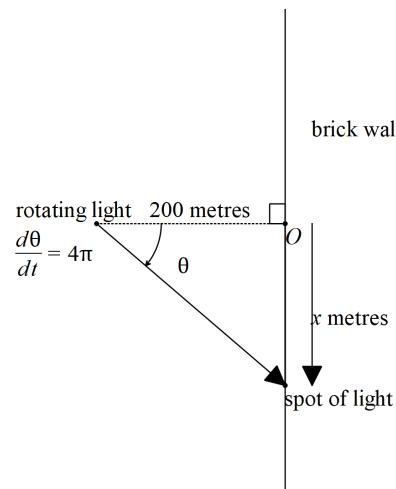
(c)

- Determine the velocity of the spot of light, $\frac{dt}{dx}$ in metres/second, when $\theta = \frac{\pi}{6}$ radians.

Q20 continued

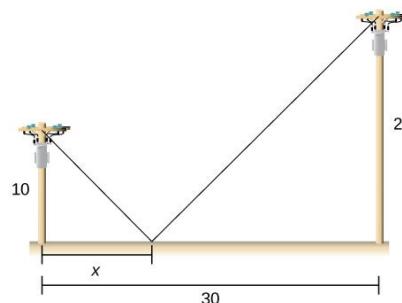
Question 20 (11 marks)

On a tarmac of a wide airfield is a rotating light that has two complete revolutions per second. (4π radians/second) The light is placed 200 metres in front of a long brick wall as shown in the diagram below. As the light is shone against the wall, the spot of light can be seen racing across the wall. Let x = the displacement of the spot of light from the point closest to the light, point O, on the wall.

**Question 10**

(6 marks)

Two power poles need to be joined using a wire that is also connected to the ground, as shown below. The two poles are 10 and 20 metres high, and are separated by 30 metres.



- (a) Determine an expression for the length of wire needed in terms of x metres. (2 marks)
(No need to simplify)

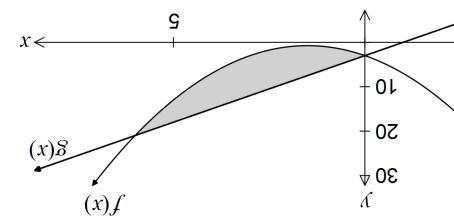
- (b) Using calculus, show how to determine the value of x to minimize the length of wire required. Determine this length to the nearest centimetre.
(Use of a classpad is required) (4 marks)

Question 19

Consider the following 3 equations, where m is a positive real constant:

$$\begin{aligned}f(x) &= x^2 - 3x + 4 \\g(x) &= 3x + 4 \\h(x) &= mx + 4\end{aligned}$$

The graphs of $y=f(x)$ and $y=g(x)$ are shown on the axes below.



a) State the integral required to calculate the shaded area. (2 marks)

b) Show that the graphs of $y=f(x)$ and $y=h(x)$ intersect when $x=0$ and $x=m+3$. (2 marks)

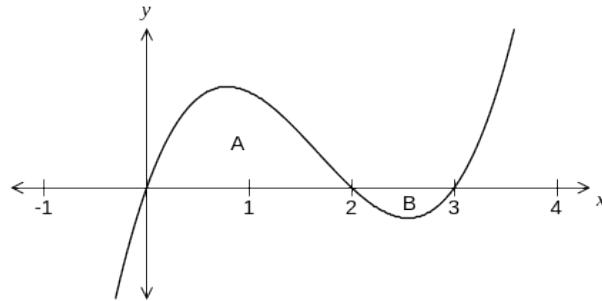
c) The area trapped between the graphs of $y=f(x)$ and $y=h(x)$ is 36 square units. Determine the value of m . (4 marks)

Question 11**(4 marks)**

Part of the graph of $y=f(x)$ is shown below. The areas of regions A and B, bounded by the curve and the x -axis, are 16 and 2 square units respectively.

c) What is the average cost of producing 100 items?

(2 marks)



Evaluate:

a) $\int_2^3 f(x)dx$ (1 mark)

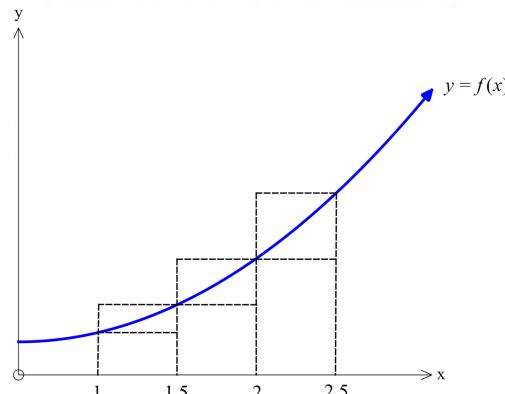
b) $\int_0^3 f(x)dx$ (1 mark)

c) $\int_0^2 f(x)-3dx$ (2 marks)

- Question 12** (10 marks)
- A train on a monorail moves with velocity, v metres per second, at time t seconds, in a straight line, with constant acceleration, a metres per second squared, given by:
- (a) Sketch the graph of f , on the same axes as the graph of f above. (5 marks)
- The train begins at the origin and at rest.
- $$a = 5 \sin \left(3t + \frac{\pi}{2} \right)$$
- Question 18** (8 marks)
- A hovercraft company has established that for selling x units, their revenue function, in dollars, can be given by $R(x) = 3x^3 + 19x^2 + 4x$, and their cost function, in dollars, can be given by $C(x) = 3x^3 + 20x^2 - 96x - 80000$. (Profit = Revenue - Cost)
- Using calculus methods, determine the number of units, x , to maximise the profit. (4 marks)
- (b) The first time that the train begins to decelerate. (3 marks)
- (c) An expression for the displacement of the train from the origin. (3 marks)
- (d) The maximum distance that the train is from the origin. (2 marks)
- (e) Determine the selling price per unit to establish the maximum profit in (a) above. (2 marks)
- Question 19** (10 marks)
- b) Indicate on the graph of f above the inflection point and label as E .
- c) Sketch the graph of f' , on the same axes as the graph of f above. (5 marks)
- MATHEMATICS METHODS 16 CALCULATOR-ASSUMED 9 CALCULATOR-ASSUMED

Question 13**(9 marks)**

Consider the function $f(x)$ shown graphed below. The table gives the value of the function at the given x values.



x	1	1.5	2	2.5
$f(x)$	1.8	2.6	3.8	5.4

- a) By considering the areas of the rectangles shown, demonstrate and explain why

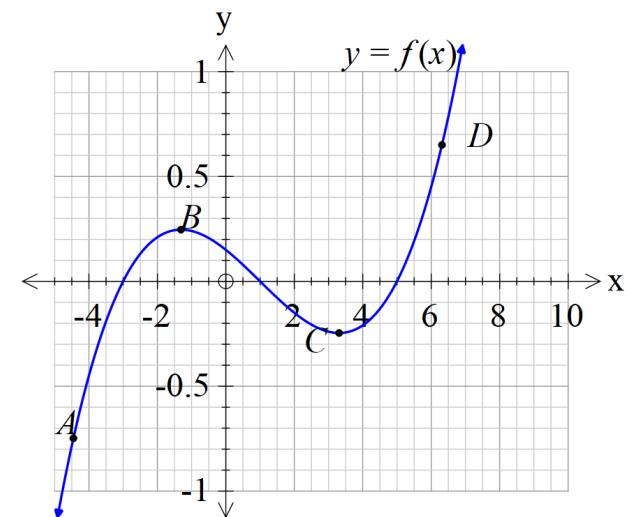
$$4.1 < \int_1^{2.5} f(x) dx < 5.9.$$

(3 marks)

See next page

Question 17**(10 marks)**

The graph of a function f is given below.



- a) A, B, C, D are points on the graph of f . Determine whether the first and second derivatives are positive, negative or equal to zero at these points. Record your findings in the table below.

(4 marks)

Point	f'	f''
A		
B		
C		
D		

See next page

c)

- Find the net change in profit if the number of items sold changes from 80 to 160 items. (3 marks)

- c) State two ways in which you could determine a more accurate value for $\int_3^1 2f(x) dx$ (2 marks)

b)

- Hence determine the profit from selling 80 items. (2 marks)

marks)

- a) Given that the company incurs a loss of \$70 if no items are sold, find an expression for P in terms of x . (3 marks)

- b) Use the table values to determine the best estimate possible for $\int_3^1 2f(x) dx$ (4 marks)

- The marginal profit from the sale of x_{th} item is given by $P(x) = 0.0015x^2 + 1.6x - 4.8$, where $P(x)$ is the profit from selling x items.

Question 16

$f(x)$	0	1	1.5	1.8	2.6	3.8	5.4	7.3	9.6
x	3.5	3	2.5	2	1.5	1	0.5	1	0

Consider the table of further values of $f(x)$ given below.

Question 14**(8 marks)**

A realtor's sales history over any month can be represented by the following probability distribution:

Number of houses sold in a month	0	1	2	3	4
Probability	0.15	0.4	0.3	0.1	0.05

The realtor is paid \$1000 every month with a bonus of \$1500 for every house sold up to three houses and a special bonus of \$1800 if four or more houses are sold in a month.

Let $X = \text{the monthly earning of the realtor}$.

Determine:

(a) The expected monthly earning of the realtor, $E(X)$.

(4 marks)

(b) The standard deviation of X .

(2 marks)

(c) Variance ($5X - 3$).

(2 marks)

Question 15**(8 marks)**

Consider a fair die with the numbers $\{1, 2, 3, 4, 5, 6\}$. The random variable X is defined as the number of times of the thrown die showing an even number on top.

(a) If you throw the die 11 times,

(i) Determine the probability that you will end up with more even numbers than odd numbers.
(2 marks)

(ii) Determine the probability, to 4 decimal places, that there are in total, even number of times that the die shows an even number.
(3 marks)

(b) If you would like to throw at least three times an even number. Find the minimum number of times that you need to throw the die for which the probability of three or more even numbers is at least 85%.

(3 marks)