WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER TWO 2016
TEST 3: Derivatives and Integrals

Name: _____

Monday 15th August Time: 45 minutes Mark /40
Section 1 – Calculator free 20 marks

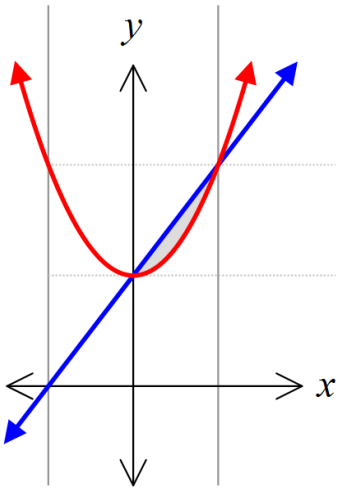
1. [4 marks]

A curve is defined by the equation $ye^{x}+xy^{2}=1$

Use implicit differentiation to determine the equation of the normal drawn at the point $(0,1)$.

2. [3 marks]

Determine the area enclosed by the curves $y=x+1$ and $y=x^{2}+1$, as shown.



3. [13 marks – 4, 1, 4 and 4]

Determine each of the integrals given

(a) $\int_0^2 3\pi x^2 (x^3 - 1)\,dx$ by using a suitable substitution

(b) $\int \sin 3x \cos^2 3x\,dx$

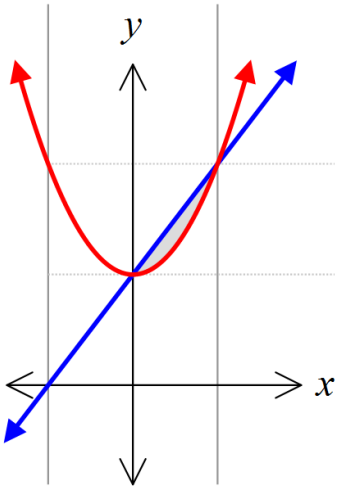
(c) $\int \sin^4 x\,dx$

(d) $\int \frac{dx}{x^2 - 1}$ in the form $\ln A + c$

Section 2 – Calculator assumed 20 marks

4. [3 marks]

Determine the volume generated when the region enclosed by the curves $y = x + 1$ and $y = x^2 + 1$, as shown, is rotated around the *x*-axis.



5. [5 marks – 2, 2 and 1]

Fluid flow through a narrow pipe has been modelled by the equation $F = kr^{\frac{3}{2}}$ where *F* is the flow, *k* a constant and *r* the radius.

(a) How will the rate of flow change if the radius is increased by 44%?

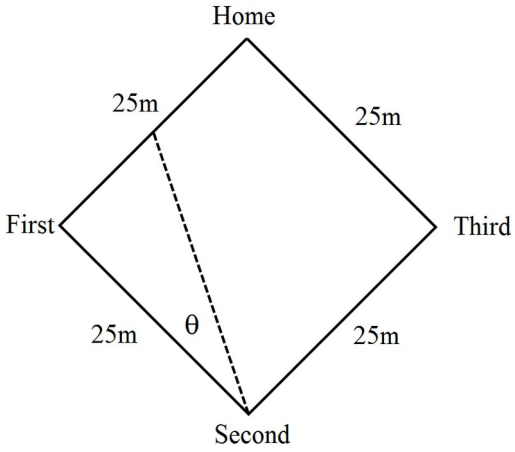
(b) Use the incremental technique to estimate the change of radius that produces a 10% decrease in flow.

(c) Is the incremental technique appropriate in (a)? Explain.

6. [7 marks – 4 and 3]

A baseball diamond consists of a 25 metre square.

A batter goes from the home plate and runs directly towards first base at 8 metres per second.



(a) How fast is his distance from second base changing when he is half-way to first base?

(b) How is angle θ changing at the same instant?

7. [5 marks]

Calculate the exact volume generated when the region contained between $y = 2$ and $y = \frac{-10}{x^2 - 9}$ for $-3 < x < 3$ is rotated about the *y*-axis.