

Semester One Examination 2017 Question/Answer Booklet

MATHEMATICS METHODS UNIT 3

Section One: Calculator-free

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula Sheet

To be provided by the candidate

Standard items: pens(blue/black preferred), pencils(including coloured), sharpener,
correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

| | Number of questions available | Number of questions to be attempted | Suggested working time (minutes) | Marks available | % |
|--|-------------------------------|-------------------------------------|----------------------------------|-----------------|-----------|
| Section One Calculator—free | 8 | 8 | 50 | 52 | 35 |
| Section Two Calculator—assumed | 14 | 14 | 100 | 98 | 65 |
| | | | | 150 | 100 |

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2017*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section One: Calculator-free**52 marks**

This section has **eight (8)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 50 minutes

Question 1 (7 marks)

- (a) The line $4x + y - 7 = 0$ is parallel to the tangent to the curve $y = 2x^3 + 3x^2 + kx + 1$ at $x = -1$. Determine the value of k . (2 marks)

- (b) Let $g'(x) = h'(x) + 7$, $g(0) = 3$ and $h(0) = -1$. Determine $g(x)$ in terms of $h(x)$. (2 marks)

- (c) Find $f'(x)$ when $x = \frac{\pi}{6}$, given that $f(x) = x \cos x$. (3 marks)

Question 2 (7 marks)

The discrete random variable X has a mean of 0.3, a variance of 0.61 and the following probability distribution.

| | | | |
|------------|-----|-----|-----|
| X | -1 | 0 | 1 |
| $P(X = x)$ | a | b | 0.5 |

(a) (i) Find a and b . (2 marks)

(ii) Find $P(X = 1 \mid X \geq 0)$. (1 mark)

(b) The random variable X is transformed to the random variable Y according to the equation $Y = 2X - 0.1$

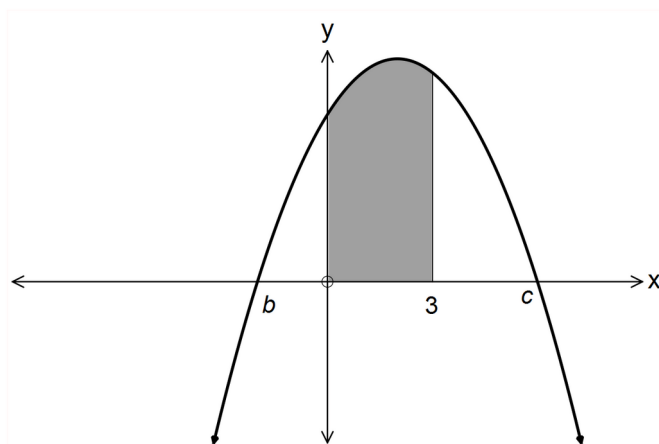
(i) Determine the expected value and the variance of the random variable Y . (2 marks)

(ii) Evaluate $P(Y < 1.9)$. (1 mark)

(c) Explain the relationship between the standard deviation of X and the standard deviation of Y . (1 mark)

Question 3 (4 marks)

The graph of the function $y = -x^2 + ax + 12$ is shown below. If b and c are the x -intercepts of the function and the shaded area is 45 units², show that $a = 4$ and find the values of b and c .
(4 marks)



Question 4 (3 marks)

Given that $y = x e^x$, find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$

Hence, suggest a formula for $\frac{d^n y}{dx^n}$ where n is a positive integer. (3 marks)

Question 5 (7 marks)

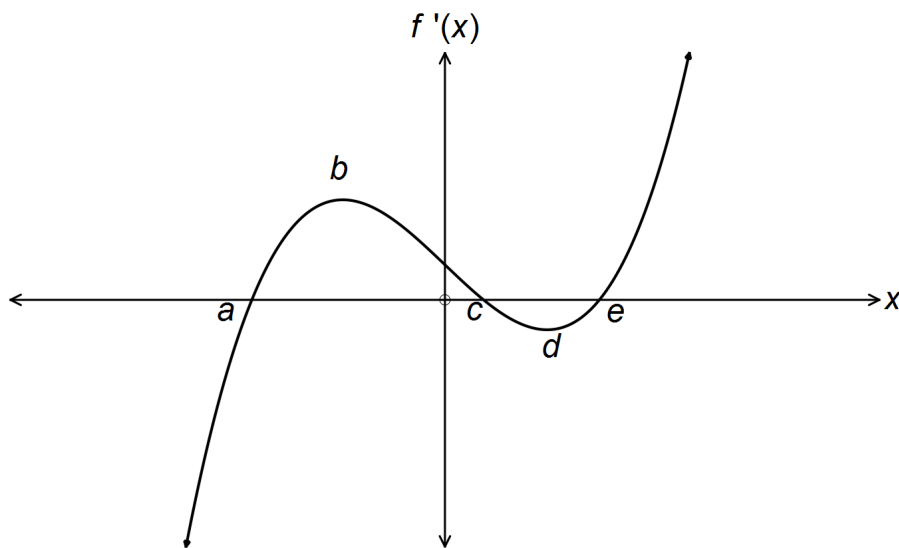
- (a) If $f(t) = e^{3\pi + \cos(2t)}$, find $f'(t)$. (2 marks)

- (b) Use the quotient rule to differentiate $y = \frac{\sin^2 4x}{\cos x^2}$. (Do not simplify your answer.) (2 marks)

- (c) Find the gradient of the tangent line to the curve of $y = x \cos x$ at the point where $x = \frac{\pi}{6}$. (3 marks)

Question 6 (5 marks)

Consider the gradient function $y = f'(x)$ below. The local maximum is at $x = b$ and the local minimum is at $x = d$. The roots are at $x = a, c, e$.



- (a) At which values of x are the points of inflection on the function $f(x)$? (1 mark)
- (b) At which value(s) of x is/are the local minima on the function $f(x)$? (1 mark)
- (c) State the concavity at c on the function $f(x)$. (1 mark)
- (d) Draw the graph of $f(x)$ on the same set of axes. (2 marks)

Question 7 (11 marks)

Consider the equation of the curve $y = 3x^5 - 5x^3$.

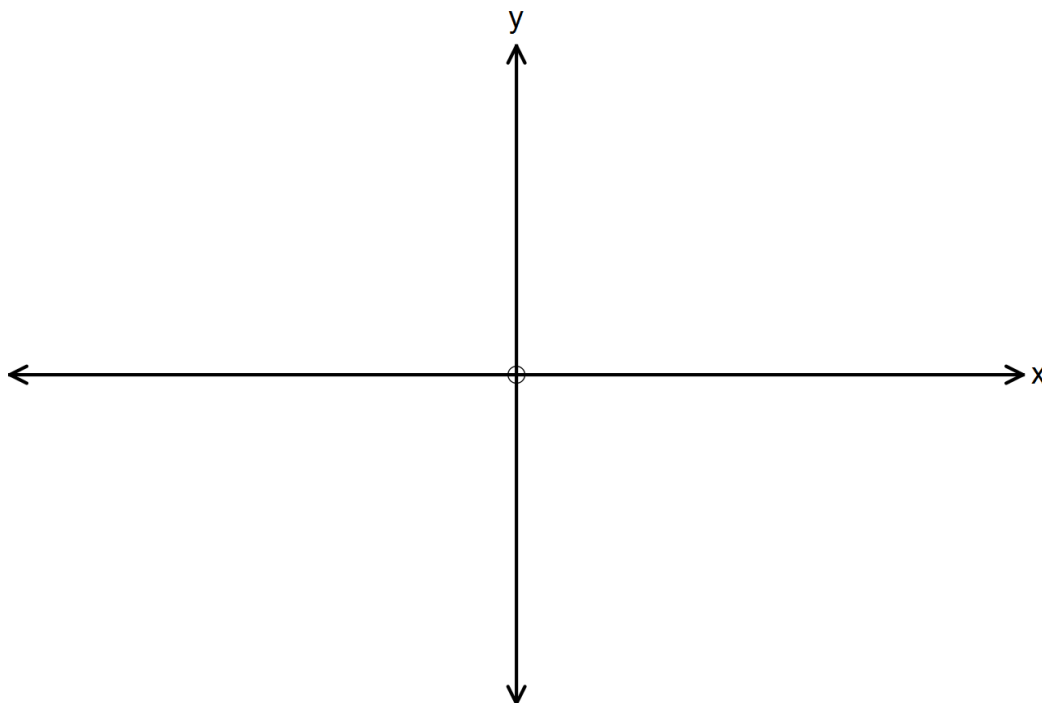
- (a)** Use calculus to find the coordinates of the stationary points on the curve and determine the nature of these points. (5 marks)

- (b)** Determine the x – value(s) of the inflection point(s) and identify their nature. (3 marks)

Question 7 (Continued)

(c) Sketch the graph on the axis below. Label the stationary points.

(3 marks)



Question 8 (8 marks)

(a) State the function $g(x)$ if $g'(x) = 6x e^{3x^2 + 2} + \cos x$. (2 mark)

(b) Determine the antiderivative of $\frac{4x^3 - \sqrt{x} + 12}{x^3}$. (2 marks)

(c) Determine $\int 3x^2 \sqrt{x^3 + 4} \, dx$. (2 marks)

(d) The gradient of a curve is given by $\frac{dy}{dx} = 1 - 6\sin(3x)$.
The curve passes through the point $(0, 7)$. What is the equation of the curve? (2 marks)

End of Section One

Additional working space

Question number(s):

Additional working space

Question number(s):

WATP acknowledges the permission of School Curriculum and Assessment Authority in providing instructions to students.