PRACTICE EXAM QUESTIONS (II)

[6, 5, 12 = 23 marks]

1.1 For the vectors $\mathbf{a} = \mathbf{OA} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = \mathbf{OB} = 5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, find:

(a) the angle between $\mathbf{a}$ and $\mathbf{b}$

(b) a vector equation of the line passing through C (1, 2, –3) and D, the midpoint of AB

(c) the points where the above line intersects the sphere $|\mathbf{r} - \langle 3, -4, 1\rangle| = 10$.

PRACTICE EXAM QUESTIONS (II) – SOLUTIONS

(a) $|\mathbf{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$ ✓

$|\mathbf{b}| = \sqrt{5^2 + 1^2 + 3^2} = \sqrt{35}$ ✓

$\mathbf{a} \cdot \mathbf{b} = \langle 2, -3, 4\rangle \ \langle 5, 1, -3\rangle$

$= 10 - 3 - 12$

$= -5$ ✓

$\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

$= \dfrac{-5}{\sqrt{29}\sqrt{35}}$ ✓

$\theta = 99^\circ$ ✓

The angle between the vectors is 99° ✓

(b) $\mathbf{OD} = \frac{1}{2}(\mathbf{OA} + \mathbf{OB})$ ◄----------

$= \frac{1}{2}(\langle 2, -3, 4\rangle + \langle 5, 1, -3\rangle)$

$= \frac{1}{2}\langle 7, -2, 1\rangle$ ✓

$= \langle 3.5, -1, 0.5\rangle$ ✓

If A and B are the end points of a line and D is the midpoint of AB, then the position vector of D is given by

$\mathbf{OD} = \frac{1}{2}(\mathbf{OA} + \mathbf{OB})$

$\mathbf{CD} = \mathbf{OD} - \mathbf{OC}$

$= \langle 3.5, -1, 0.5\rangle - \langle 1, 2, -3\rangle$ ✓

$= \langle 2.5, -3, 3.5\rangle$ ✓

Using point C (1, 2, −3) on the line

$\mathbf{r} = \langle 1, 2, -3\rangle + t\langle 2.5, -3, 3.5\rangle$ ✓----------

(c) $|\mathbf{r} - \langle 3, -4, 1\rangle| = 10$

$|\langle x, y, z\rangle - \langle 3, -4, 1\rangle| = 10$ ✓

$|\langle x - 3, y + 4, z - 1\rangle| = 10$ ✓

$(x-3)^2 + (y+4)^2 + (z-1)^2 = 100$ ✓

$\mathbf{r} = \langle 1 + 2.5t, 2 - 3t, -3 + 3.5t\rangle$ ✓ component form of the line

$x = 1 + 2.5t, \quad y = 2 - 3t, \quad z = -3 + 3.5t$ ✓ parametric equations

sub into sphere equation

$(1 + 2.5t - 3)^2 + (2 - 3t + 4)^2 + (-3 + 3.5t - 1)^2 = 100$ ✓

$(2.5t - 2)^2 + (6 - 3t)^2 + (-4 + 3.5t)^2 = 100$ ✓

$6.25t^2 - 10t + 4 + 36 - 36t + 9t^2 + 16 - 28t + 12.25t^2 = 100$ ✓

$27.5t^2 - 74t - 44 = 0$ ✓

$t = 3.192$ or -0.501 by using the quadratic formula or graphics calculator ✓

if $t = 3.192$, intersection point is (9.0, −7.6, 8.2) ✓

if $t = -0.501$, intersection point is (−0.3, 3.5, −4.8) ✓