



PERTH MODERN SCHOOL
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Independent Public School

Course Methods Year 12 test three 2022

Student name: _____ Teacher name: _____

Task type: Response

Time allowed for this task: ____40____ mins

Number of questions: ____6____

Materials required: **Upto 3 calculators/classpads allowed**

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, **one page of A4 notes doublesided**

Marks available: ____43____ marks

Task weighting: ____10____%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3, 3 & 2 = 8 marks)

Consider the discrete random variable X and the table of probabilities below.

X	0	1	2	3	4
$P(X = x)$	0.2	a	0.3	b	0.25

- a) Given that the expected value of X is 2.15, determine the values of a & b .

Solution
<p>TI-84 Plus calculator interface showing a system of equations for a and b:</p> $\begin{cases} 0.2 + a + 0.3 + b + 0.25 = 1 \\ a + 2 \times 0.3 + 3b + 4 \times 0.25 = 2.15 \end{cases} \quad a, b$ <p>$\{a=0.1, b=0.15\}$</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ sets up one equation for a & b ✓ sets up two equations for a & b ✓ solves for a & b

- b) Determine the standard deviation of X to 3 dp, showing all reasoning.

Solution
$V(x) = (0 - 2.15)^2 \cdot 0.2 + (1 - 2.15)^2 \cdot 0.1 + (2 - 2.15)^2 \cdot 0.3 + (3 - 2.15)^2 \cdot 0.2 + (4 - 2.15)^2 \cdot 0.25$ $= 2.028$ $std = 1.424$
Specific behaviours
<ul style="list-style-type: none"> ✓ shows a sequence for variance ✓ determines variance ✓ determines std to 3 dp

- c) Determine the $E(3X + 4)$ and $\text{Variance}(3X + 4)$.

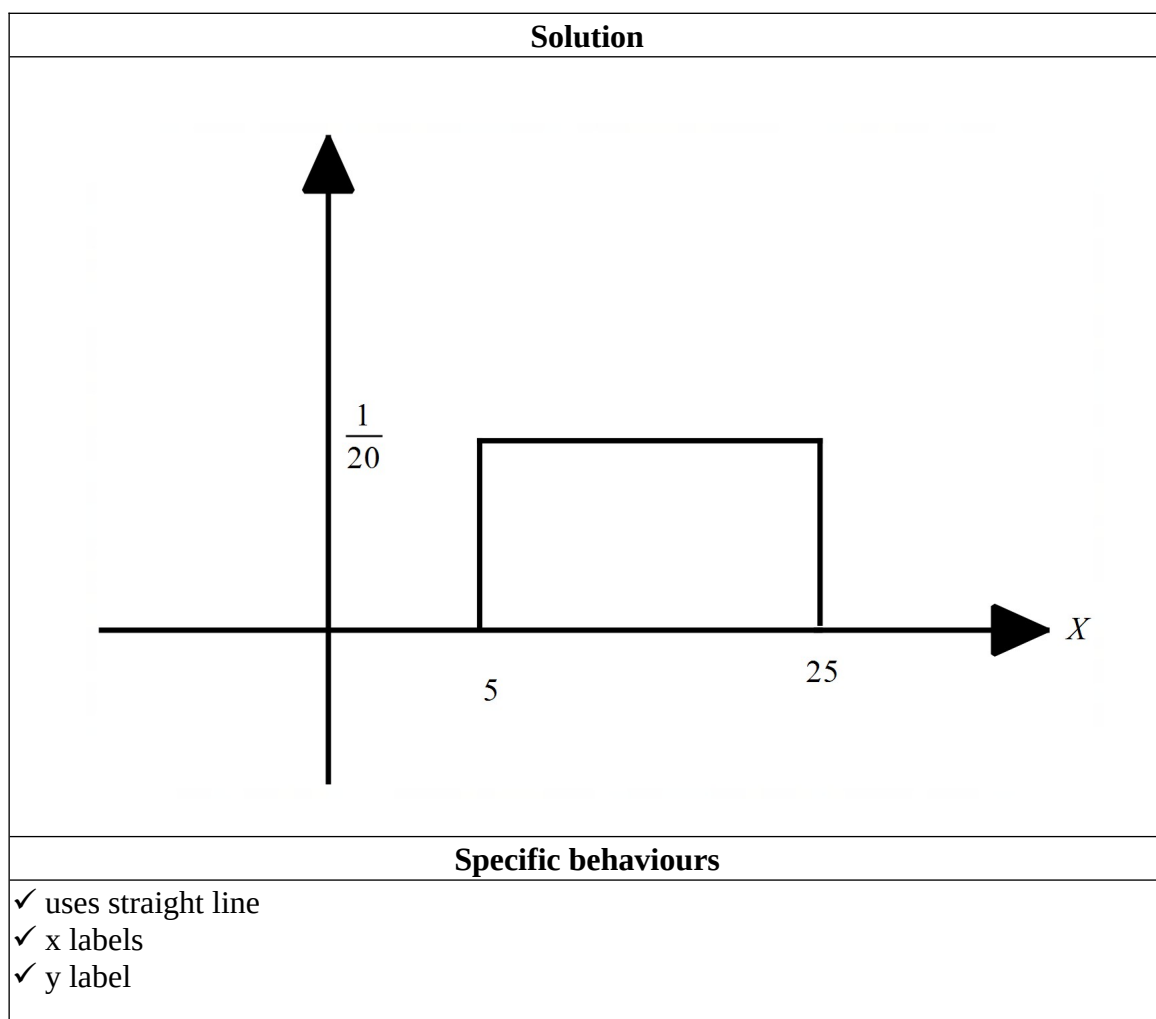
Solution
$E(3X + 4) = 10.45$ $\text{Variance}(3X + 4) = 18.2975$

Specific behaviours
<ul style="list-style-type: none"> ✓ determines new mean ✓ determines new Variance

Q2 (3, 2, 1 & 3 = 9 marks)

The number of X minutes late a train arrives at a particular station is a uniform probability distribution from 5 mins to 25 mins.

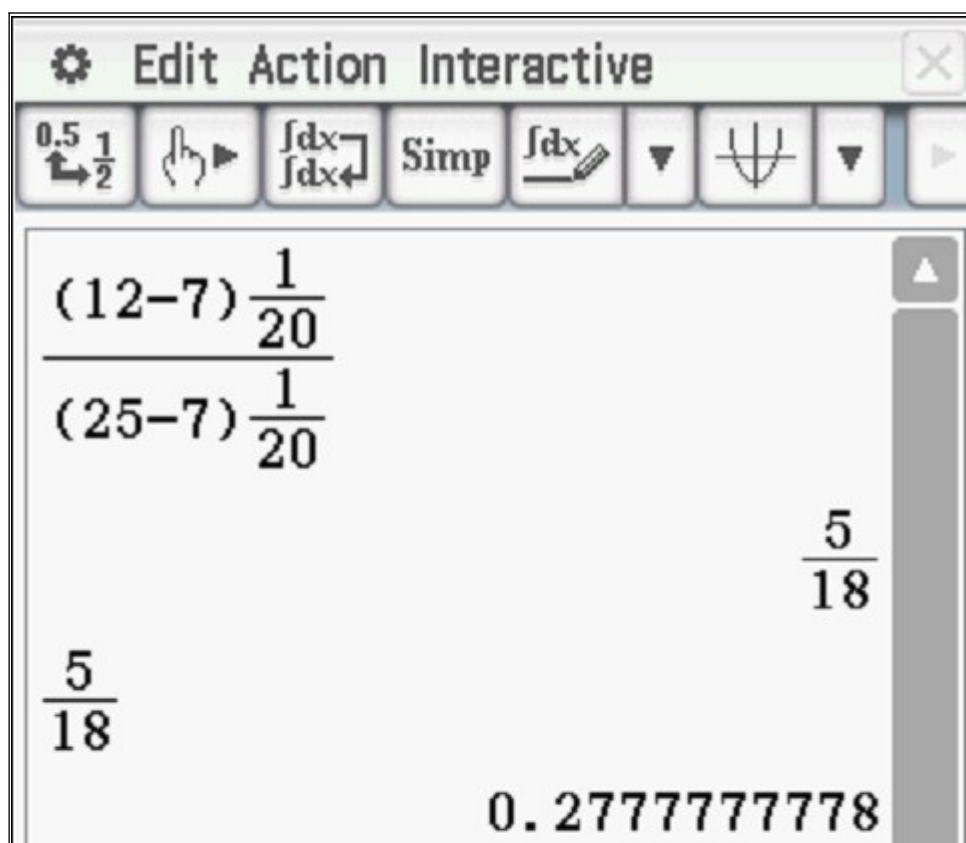
- a) Sketch the probability density function for X showing all relevant features and labels.



Q2 continued

- b) Determine the probability that the train will be less than 12 mins late given that it is at least 7 mins late.

Solution



Specific behaviours

- ✓ correct denominator
- ✓ correct numerator

c) Determine the mean number of minutes late.

Solution

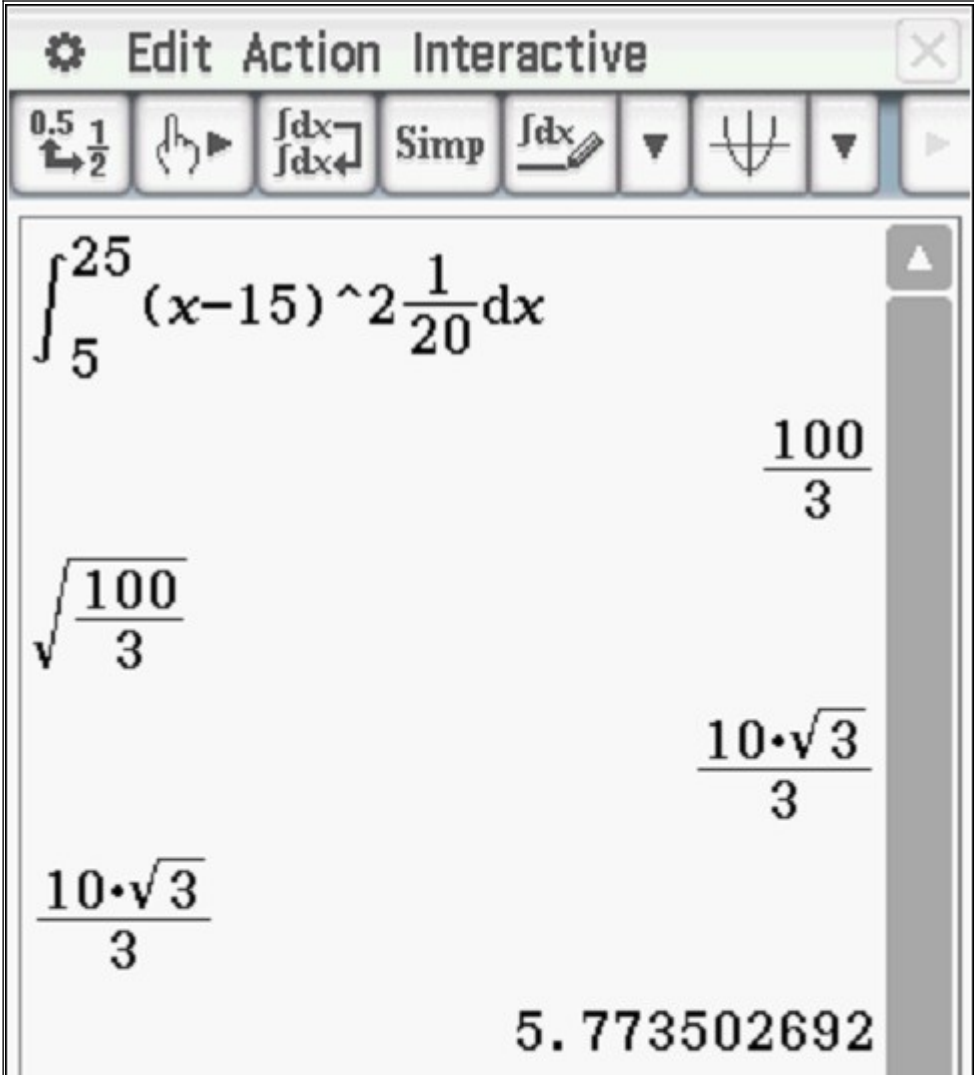
Mean = 15 mins

Specific behaviours

- ✓ states midpoint

d) Determine the standard deviation of X showing all reasoning.

Solution

	
Specific behaviours	
<ul style="list-style-type: none"> ✓ sets up integral ✓ determines variance ✓ determines stdev 	

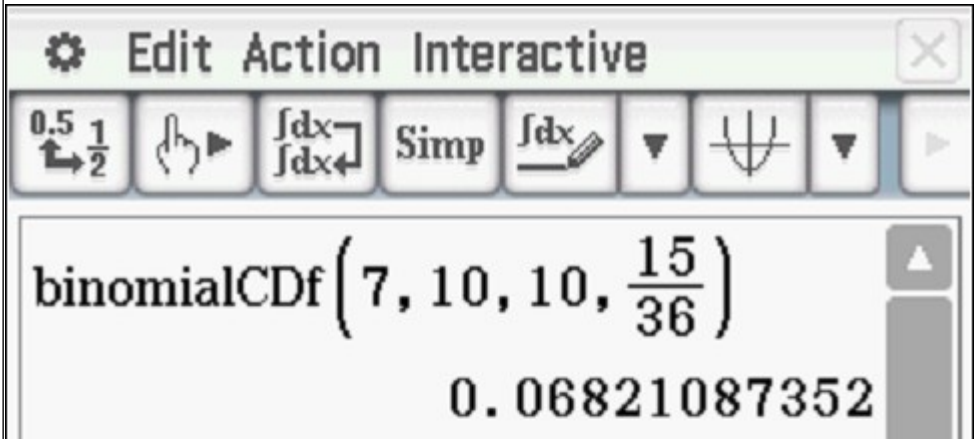
Q3 (3, 3 & 2 = 8 marks)

Consider a game where two ordinary dice are thrown into the air and then land and the sum of the two top numbers is added. If the sum is a prime number (2,3,5,7,...) etc then this is considered a win.

a) Determine the probability of a win. Show reasoning.

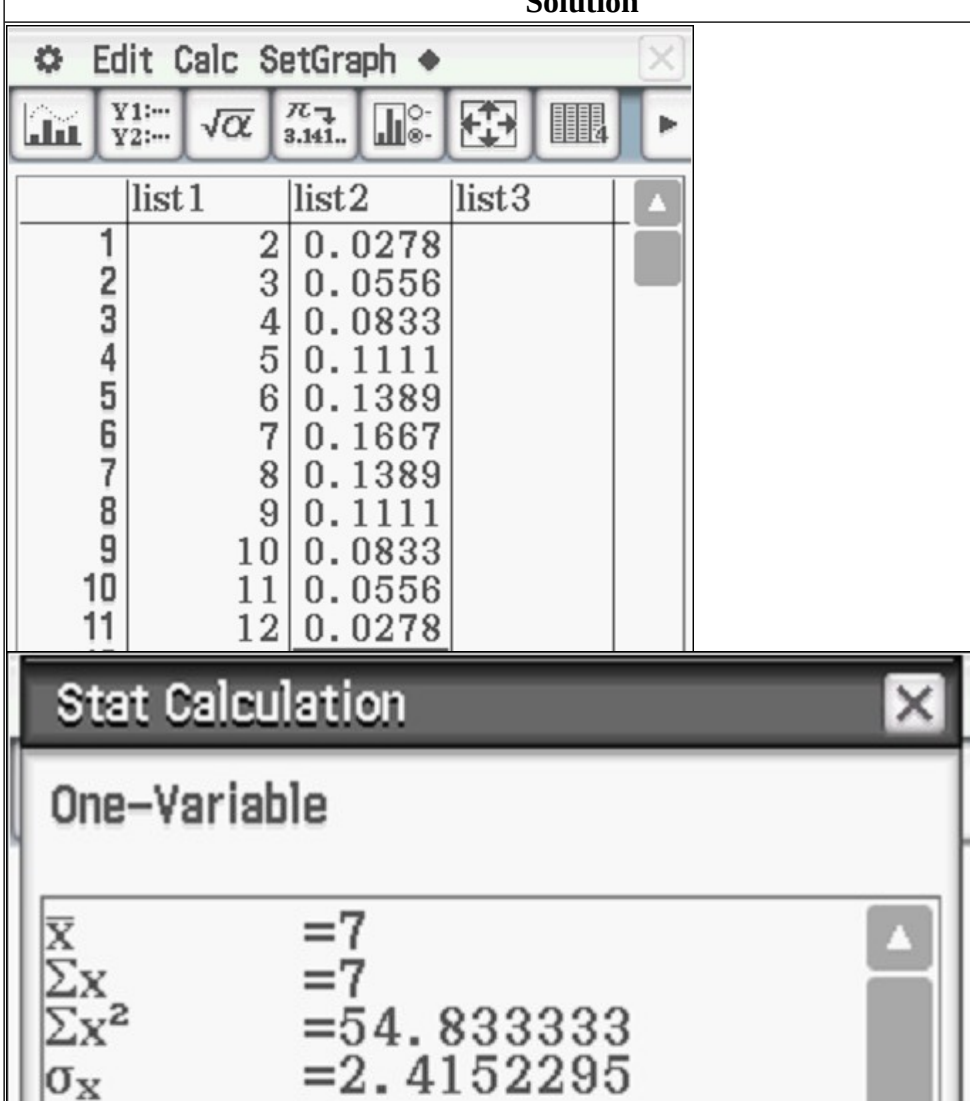
Solution						
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12
Prime numbers 2,3,5,7,11 $\Pr(\text{prime}) = 15/36$						
Specific behaviours						
✓ shows sample space ✓ number of favourable outcomes shown ✓ states probability						

- b) If this game was played 10 times, determine the probability that a win occurs at least 7 times. Show all reasoning.

Solution	
$X \sim B(10, \frac{15}{36})$ $P(X \geq 7)$ 	
Specific behaviours	
✓ states Binomial with parameters ✓ uses cumulative expression ✓ states prob	

- a) Let X = sum of the top numbers of both dice. Determine the mean and standard deviation for X .

Solution



The calculator screen displays a list of data in the following table:

	list 1	list 2	list 3
1		2	0.0278
2		3	0.0556
3		4	0.0833
4		5	0.1111
5		6	0.1389
6		7	0.1667
7		8	0.1389
8		9	0.1111
9	10	0.0833	
10	11	0.0556	
11	12	0.0278	

The Stat Calculation window shows the following results for One-Variable statistics:

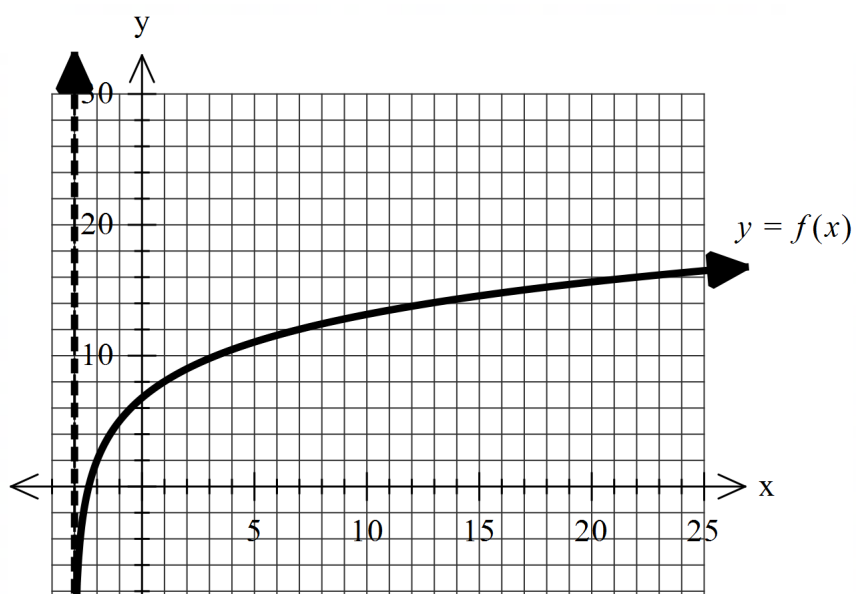
\bar{X}	=7
$\sum X$	=7
$\sum X^2$	=54.833333
σ_X	=2.4152295

Specific behaviours

- ✓ states mean of 7
- ✓ states stdev of 2.415

Q4) (5 marks)

Consider $f(x) = r \log_5(x+p) + q$ where r, p & q are constants.



Using the graph above and given that the following points

$(22,16)$ & $(2,9)$ lie on the curve $y = f(x)$, determine the values of r, p & q .

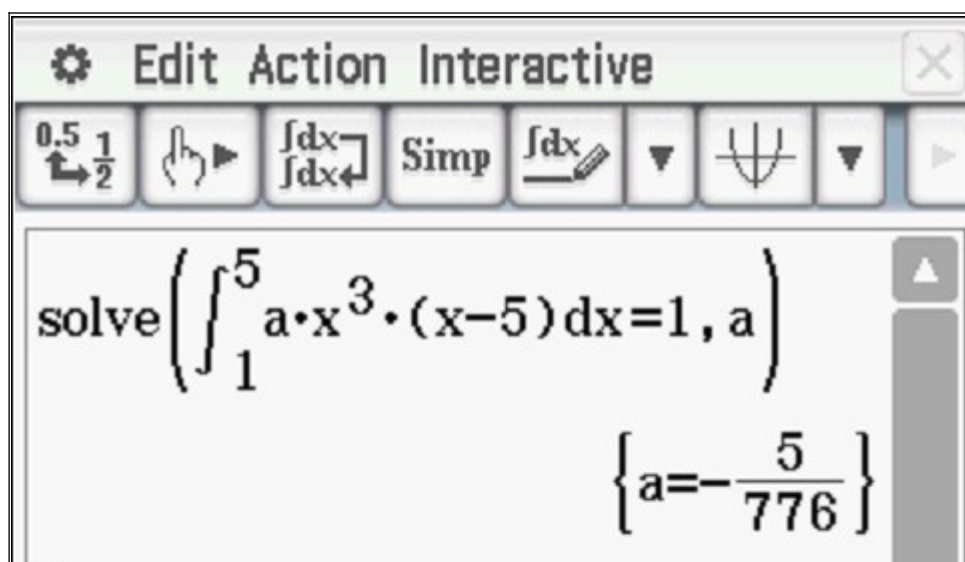
Solution
$f(x) = r \log_5(x + p) + q$ Asymptote $x = -3$, $p = 3$ $(2,9)$ $9 = r \log_5(5) + q = r + q$ $(22,16)$ $16 = r \log_5(25) + q = 2r + q$ $16 - 9 = r$ $r = 7$ $16 = 14 + q$ $q = 2$
Specific behaviours
✓ identifies asymptote at $x = -3$ ✓ determines p ✓ sets up one equation with two unknowns ✓ sets up two equations with two unknowns ✓ states r & q

Q5 (2, 2 & 2 = 6 marks)

Consider the probability density function $f(x) = ax^3(x - 5)$, $1 \leq x \leq 5$ and zero for all other values of x .

a) Show that $a = \frac{-5}{776}$.

Solution



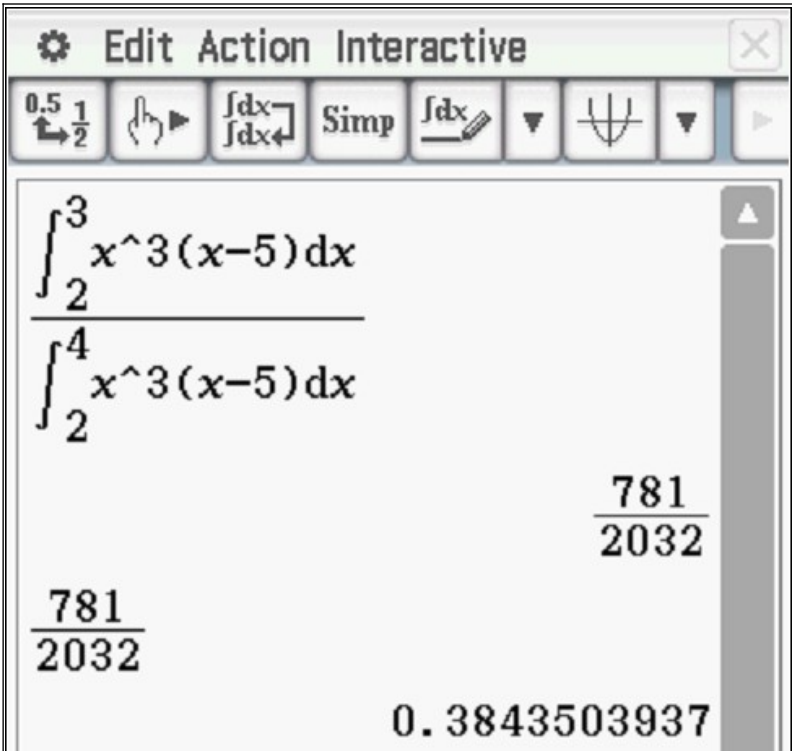
Specific behaviours

- ✓ sets up integral for total area =1
- ✓ shows equation to solve for a

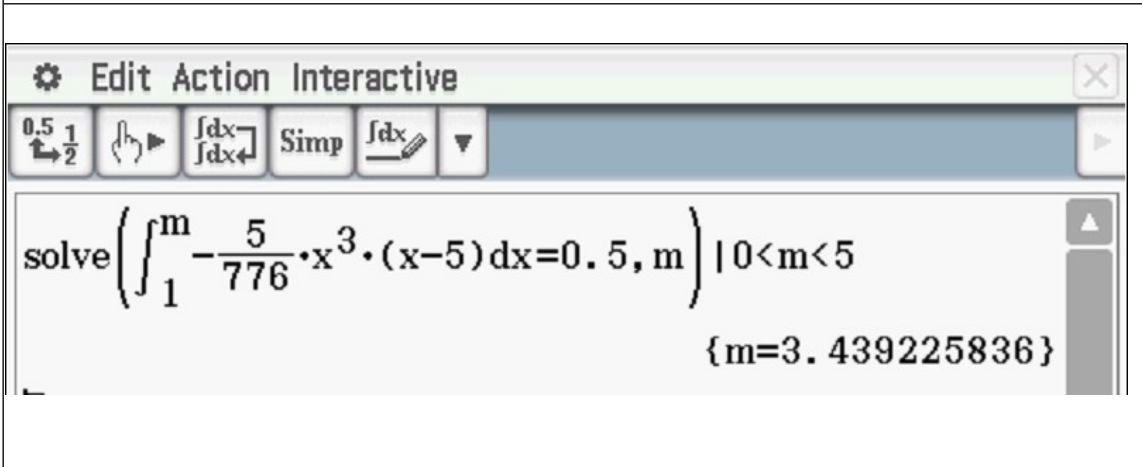
b) Determine the probability $\Pr(X \leq 3 | 2 \leq X \leq 4)$ for the above function.

Solution

$$\Pr(X \leq 3 | 2 \leq X \leq 4) = \frac{\Pr(2 \leq X \leq 3)}{\Pr(2 \leq X \leq 4)}$$

	
Specific behaviours	
<ul style="list-style-type: none"> ✓ numerator ✓ denominator 	

c) Determine the median.

Solution	
	
Specific behaviours	
<ul style="list-style-type: none"> ✓ sets up equation for median ✓ states one median only 	

Q6 (2 & 5 = 7 marks)

- a) Show **without the use of a classpad** how to $\frac{d}{dx} [5x \ln(3x+1)]$.

Solution
$\frac{d}{dx} [5x \ln(3x+1)] = 5x \frac{3}{3x+1} + 5 \ln(3x+1)$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses product rule ✓ obtains correct expression

- b) Using (a) above and **without the use of a classpad**, show how to evaluate $\int_0^1 3 \ln(3x+1) dx$.

Hint-use $\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$

Solution
$\frac{d}{dx} [5x \ln(3x+1)] = 5x \frac{3}{3x+1} + 5 \ln(3x+1)$ $\int \frac{d}{dx} [5x \ln(3x+1)] = 5 \int \frac{3x}{3x+1} dx + 5 \int \ln(3x+1) dx$ $[5x \ln(3x+1)]_0^1 = 5 \int \left(1 - \frac{1}{3x+1}\right) dx + 5 \int \ln(3x+1) dx$ $5 \ln 4 = 5 \left[x - \frac{1}{3} \ln(3x+1) \right]_0^1 + 5 \int \ln(3x+1) dx$ $5 \ln 4 = 5 \left(1 - \frac{1}{3} \ln 4\right) + 5 \int \ln(3x+1) dx$ $3 \ln 4 = 3 \left(1 - \frac{1}{3} \ln 4\right) + 3 \int \ln(3x+1) dx$ $3 \int \ln(3x+1) dx = 3 \ln 4 - 3 + \ln 4 = 4 \ln 4 - 3$ <p>Note- zero marks if answer given without any working!</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ integrates expression from part a ✓ uses FTC ✓ changes $\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$ and shows integration of these two terms (or uses $u=3x+1$) du

- ✓ evaluates $x=0$ and $x=1$ showing both values for two terms after integration
- ✓ changes factor to give required definite integral(no need to simplify)

