PERTH MODERN SCHOOL

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Independent Public School

Course ______12 Methods______ Year __12___

Student name: _______________ Teacher name: _______________

Task type: **Response/Investigation**

Time allowed for this task: ___40______ mins

Number of questions: ____7_____

Materials required: No calculators nor classpads

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of
A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: __40__ marks

Task weighting: _10__%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2, 3 & 3 = 8 marks) (3.1.7-3.1.8)

Determine $\frac{dy}{dx}$ for each of the following.(No need to simplify)

a) $y = \frac{3}{x}$

Solution
$y = \frac{3}{x} = 3x^{-1}$ $y' = -3x^{-2} = \frac{-3}{x^2}$
Specific behaviours
✓ correct coefficient ✓ correct power (no need for positive power)

b) $y = (3x^2 + 4x)(5x - 1)$

Solution
$y = (3x^2 + 4x)(5x - 1)$ $y' = (3x^2 + 4x)5 + (5x - 1)(6x + 4) \rightarrow$ *full marks* $= 15x^2 + 20x + 30x^2 + 14x - 4$ $= 45x^2 + 34x - 4$
Specific behaviours
✓ uses product rule ✓ one correct product ✓ two correct products (no need to simplify)

c) $y = \frac{x + 1}{5 - x^2}$

Solution

$y = \frac{x+1}{5-x^2}$ $y' = \frac{(5-x^2)-(x+1)(-2x)}{(5-x^2)^2} \rightarrow$ *full marks* $= \frac{5+x^2+2x}{(5-x^2)^2}$
Specific behaviours
✓ uses quotient rule ✓ correct denominator ✓ correct numerator(no need to simplify)

Q2 (2 & 3 = 5 marks) (3.1.8)

Consider $f(x) = (4x-2)^5$.

a) Determine $f'(0)$

Solution
$f(x) = (4x-2)^5$ $f'(x) = 5(7x-2)^4 4$ $f'(0) = 20(16) = 320$
Specific behaviours
✓ uses chain rule ✓ evaluates derivative

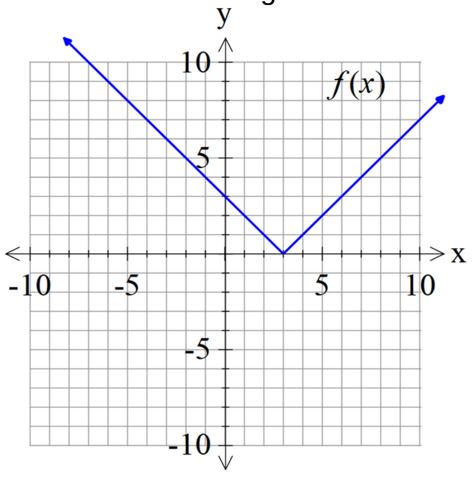
b) Determine the equation of the tangent at $x = 0$

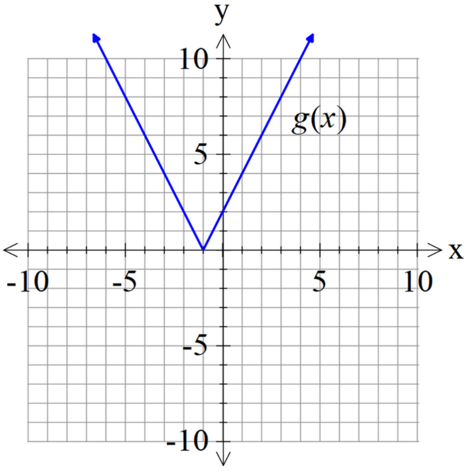
Solution
$f(0) = (-2)^5 = -32$ $y = mx + c = 320x + c$ $c = -32$ $y = 320x - 32$
Specific behaviours

✓ solves for y value at x=0 ✓ solves for constant ✓ states tangent equation

Q3 (1, 1, 3 & 3 = 8 marks) (3.1.7-3.1.8, 3.1.15)

Consider the following functions f & g .





a) Determine the derivative of $f(x)$ when $x=-2$

Solution
Gradient = -1
Specific behaviours
✓ states gradient

b) Determine the derivative of $3g(x)$ when $x=0$

Solution
Gradient = 6
Specific behaviours
✓ states gradient

c) Determine the derivative of $f(x)g(x)$ when $x=0$.

Solution
$\frac{d}{dx}f(x)g(x)=f(x)g'(x)+f'(x)g(x)$ $=3(2)+-1(2)=4$
Specific behaviours
✓ uses product rule ✓ uses correct values for all variables

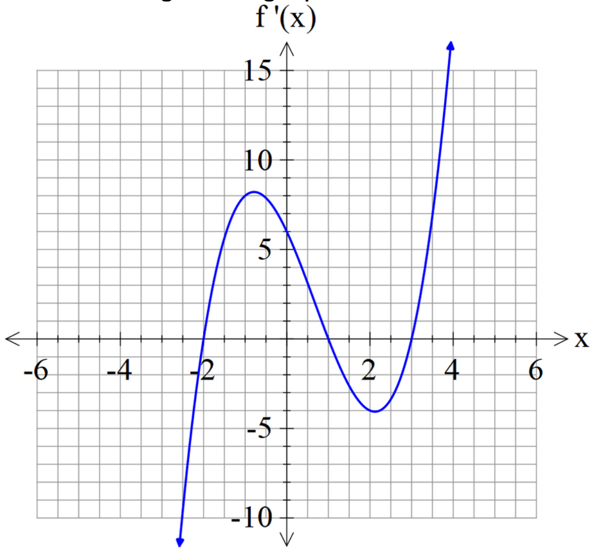
✓ states final value

d) Determine the derivative of $f(g(x))$ when $x = 0$.

Solution
$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x) = f'(2)2 = -2$
Specific behaviours
✓ uses chain rule and is demonstrated ✓ uses correct value for derivative of f ✓ states final value

Q4 (2, 3 & 2 = 7marks) (3.1.13 – 3.1.17)

The following is the graph of $f'(x)$, the derivative of $f(x)$.



a) State the x values of all stationary points of $f(x)$.

Solution
-2, 1 & 3
Specific behaviours
✓ states one correct x value ✓ states all three values

b) State the nature of each stationary point above and justify.

Solution
-2, local min as $f'' > 0$ 1, local max as $f'' < 0$ 3, local min as $f'' > 0$
Specific behaviours
✓ states nature of at least two stationary points ✓ states reason using first or second derivatives for at least two pts ✓ states nature and reason for all three stationary points

c) State approximate x value for an infection point(s) and explain why.

Solution
Near -1 & 2 as $f'' = 0$
Specific behaviours
✓ states near x values ✓ states reason using second derivative

Q5 (3 & 2 = 5 marks) (3.1.12)
The displacement of a body from the origin O, at time t seconds, is x metres where

$$x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 1$$

a) Determine the time(s) that the velocity is zero metres/second.

Solution
$x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 1$ $v = t^2 - 5t + 6 = (t-2)(t-3)$ $t = 2, 3$
Specific behaviours
✓ differentiates ✓ equates velocity to zero and factorises/quadratic formula ✓ states both t values

b) Determine when the acceleration is zero.

Solution
$x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 1$ $v = t^2 - 5t + 6$ $a = 2t - 5 = 0$ $t = \frac{5}{2}$
Specific behaviours
✓ differentiates velocity ✓ solves for t value

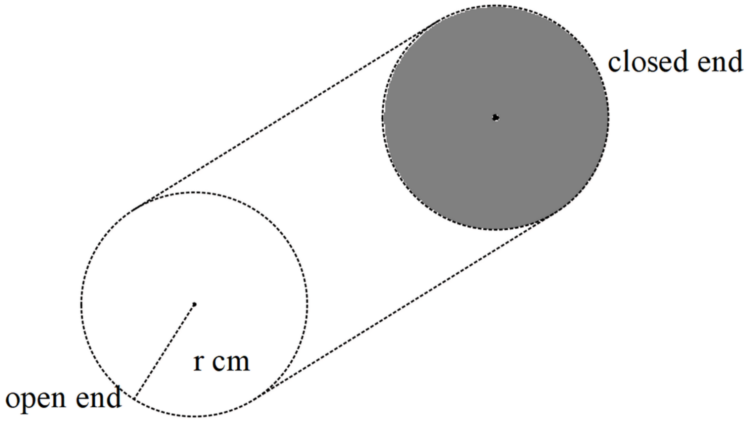
Q6 (3 marks) (3.1.10)

The period T of a swinging pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{10}}$.
Using the increments formula, determine the approximate percentage change in T if l changes by 3%

Solution
$T = 2\pi\sqrt{\frac{l}{10}} = \frac{2\pi}{\sqrt{10}} l^{\frac{1}{2}}$ $\Delta T \approx \frac{\pi}{\sqrt{10}} l^{-\frac{1}{2}} \Delta l$ $\frac{\Delta T}{T} \approx \frac{\frac{\pi}{\sqrt{10}} l^{-\frac{1}{2}} \Delta l}{2\pi\sqrt{\frac{l}{10}}} = \frac{\Delta l}{2l} = \frac{3}{2}\%$
Specific behaviours
✓ uses increments formula ✓ obtains expression for approx. change in T ✓ obtains % change

.Q7 (4 marks) (3.1.16)

Consider a cylindrical container that has an open end. The surface area of the container is $50cm^2$. Determine the exact value of the radius of the closed end that maximises the volume. (Justify)



Total surface area 50cm^2

Solution
$2\pi rh+\pi r^2=50$ $h=\dfrac{50-\pi r^2}{2\pi r}$ $V=\pi r^2\left(\dfrac{50-\pi r^2}{2\pi r}\right)=\dfrac{r}{2}\left(50-\pi r^2\right)=\dfrac{50r-\pi r^3}{2}$ $\dfrac{dV}{dr}=\dfrac{50-3\pi r^2}{2}$ $50-3\pi r^2=0$ $r=\sqrt{\dfrac{50}{3\pi}}$ $\dfrac{d^2V}{dr^2}=-3\pi r<0\therefore local\ \max$
Specific behaviours
✓ obtains constraint equation containing h & r   ✓ obtains expression for V in terms of one variable only   ✓ obtains derivative and equates to zero   ✓ obtains optimal value and confirms with second derivative