

ROSSMOYNE SENIOR HIGH SCHOOL

Semester One Examination, 2013

Question/Answer Booklet

MATHEMATICS 3A

Section Two:

Calculator-assumed

SOLUTIONS

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes

Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
Total				150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil**, except in diagrams.

Section Two: Calculator-assumed

(100 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8

(7 marks)

A sample of 179 cats brought to a veterinary clinic over the course of one month were categorised on the basis of breed and gender as follows:

	Persian	Ragdoll	Siamese	Other	Total
Female	19	28	22	18	87
Male	28	17	26	21	92
Total	47	45	48	39	179

(a) Complete the missing entries in the table above. (2 marks)

(b) What is the probability that a cat chosen at random from the sample is

(i) a female Siamese? (1 mark)

$$\frac{22}{179}$$

(ii) a male? (1 mark)

$$\frac{92}{179}$$

(iii) a Persian or a male? (1 mark)

$$\frac{111}{179}$$

(iv) a Ragdoll, given that they are female? (1 mark)

$$\frac{28}{87}$$

Question 9

(6 marks)

The number of page views of a website per day, following a TV advertising campaign, is shown in the table below.

Day (n)	1	2	3	4
Views (T_n)	275 562	91 854	30 618	10 206

Assume that this pattern continues over the next few weeks.

- (a) How many page views are expected on day five?

(1 mark)

$$10206 \div 3 = 3402 \text{ views}$$

- (b) Write a recursive rule for the number of page views per day.

(2 marks)

$$T_{n+1} = T_n \div 3 \quad T_1 = 275562$$

- (c) What is the total number of page views of the website after one week?

(1 mark)

$$S_7 = 413154$$

- (d) Will the total number of page views reach half a million? Justify your answer.

(2 marks)

No. From the rule, $T_{12} = 0.5$ and so after day 12 the rule predicts no-one is viewing the website. At this time $S_{12} \approx 413343$ so 500000 will never be reached.

Question 10

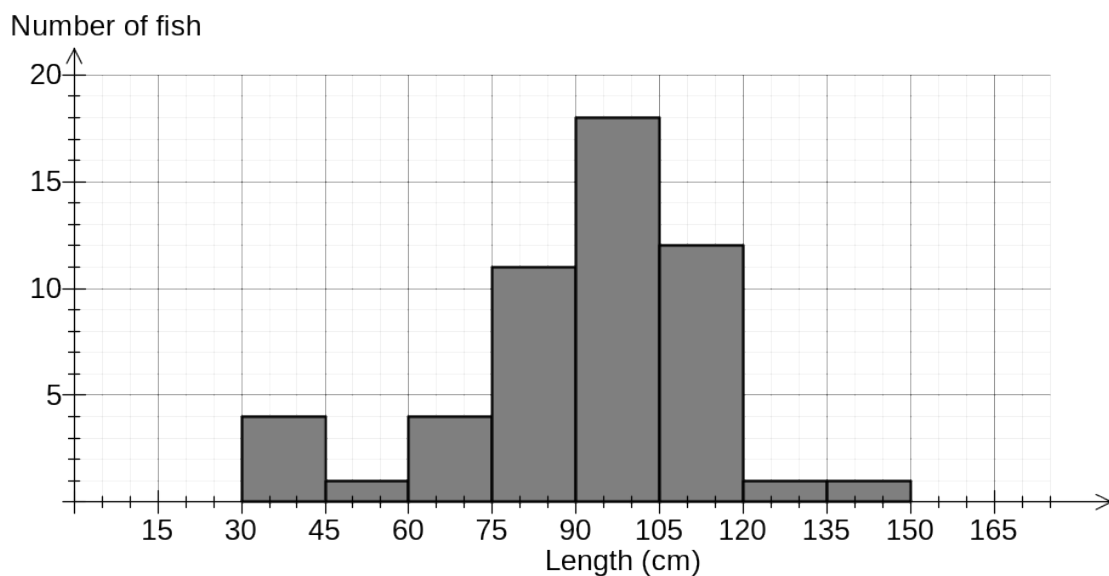
(7 marks)

Recent research into introducing a size limit for catches of longtail tuna in Australian coastal waters included the data below from a group of recreational fishermen.

Length (cm)	Number of fish
$30 < L \leq 45$	4
$45 < L \leq 60$	1
$60 < L \leq 75$	4
$75 < L \leq 90$	11
$90 < L \leq 105$	18
$105 < L \leq 120$	12
$120 < L \leq 135$	1
$135 < L \leq 150$	1
Total	52

- (a) Draw a histogram for the lengths of longtail tuna.

(3 marks)



- (b) Calculate an estimate for the mean and standard deviation of the lengths of longtail tuna.

(2 marks)

$$\bar{x} = 91.44 \text{ cm}$$

$$sd = 22.45 \text{ cm}$$

- (c) If the original data was available for all 52 longtail tuna, so that the median length could be determined, would you expect it to be the same, smaller than, or greater than the mean length? Justify your answer with reference to the histogram.

(2 marks)

Median would be **greater than** mean, because of the large number of lengths in the LH column of the histogram (negative skew) when compared to the otherwise fairly symmetrical histogram. This group act as an outlier, pulling the mean towards it and away from the

Question 11

(7 marks)

Let the universal set $U = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$.

Two subsets, R and Q, are such that $R = \{ 2, 3, 5, 7 \}$ and $Q = \{ 1, 3, 5, 7, 9 \}$.

(a) Determine

(i) $R \cap Q$

$\{ 3, 5, 7 \}$

(1 mark)

(ii) $n(R \cup Q)$

6

(1 mark)

(b) If a number is chosen at random from the universal set, determine

(i) $P(R \cap \bar{Q})$

$\frac{1}{9}$

(1 mark)

(ii) $P(Q | \bar{R})$

$\frac{2}{5}$

(2 marks)

(c) A third subset A is such that $A \cap Q = \emptyset$, $n(A \cap R) = 1$ and $n(A) = 2$. List all such possible subsets for A. (2 marks)

$A = \{ 2, 4 \}$ or $\{ 2, 6 \}$ or $\{ 2, 8 \}$

Question 12

(6 marks)

A machine fills bags of sugar so that the weights of the bags are normally distributed with a mean of 526 g and a standard deviation of 11 g.

(a) Determine the probability that a randomly selected bag weighs

(i) more than 545 g

(1 mark)

$$P(X > 545) = 0.042$$

(ii) between 520 and 530 g.

(1 mark)

$$P(520 < X < 530) = 0.349$$

(b) 5% of the bags weigh less than w grams. Find the value of w , correct to three significant figures. (2 marks)

$$\begin{aligned} P(X < w) &= 0.05 \\ w &= 507.906 \approx 508 \text{ g to 3sf} \end{aligned}$$

(c) The sugar is packed onto pallets which can hold 440 bags. How many bags on a full pallet would be expected to weigh less than 500 g? (2 marks)

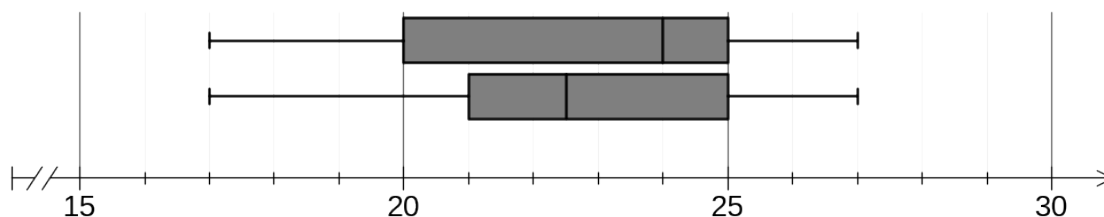
$$\begin{aligned} P(X < 500) &= 0.00905 \\ 0.00905 \times 440 &= 3.98 \approx 4 \text{ bags} \end{aligned}$$

Question 13

(10 marks)

37 students in a school sat a Chemistry test that had a maximum possible score of 40.

The scores of the 19 students in Class M are summarised in this boxplot.



- (a) State the interquartile range for the scores of Class M. (1 mark)

5 marks

- (b) One student in Class M scored 27 out of 40. If they had scored an extra five marks, would their score be considered an outlier for the class? Justify your answer. (2 marks)

$1.5 \times 5 = 7.5$
 $25 + 7.5 = 32.5$
 Score of $27 + 5 = 32$ is below $1.5 \times \text{IQR}$ above Q_3 , so not outlier.

The scores of the remaining students, in Class P, are listed below in ascending order.

17	18	19	20	21	21
22	22	22	23	24	25
25	25	26	26	26	27

- (c) Construct a boxplot for these 18 scores on the above diagram, next to that for Class M. (3 marks)

- (d) Explain which class

- (i) performed better in the test (2 marks)

Class M - had a higher median of 24 compared to 22.5 for Class P.

- (ii) had the least skewed marks (2 marks)

Class P - It's quartiles are fairly equal in width, but Class M has positive skew - width of upper half much less than width of lower half of boxplot.

Question 14

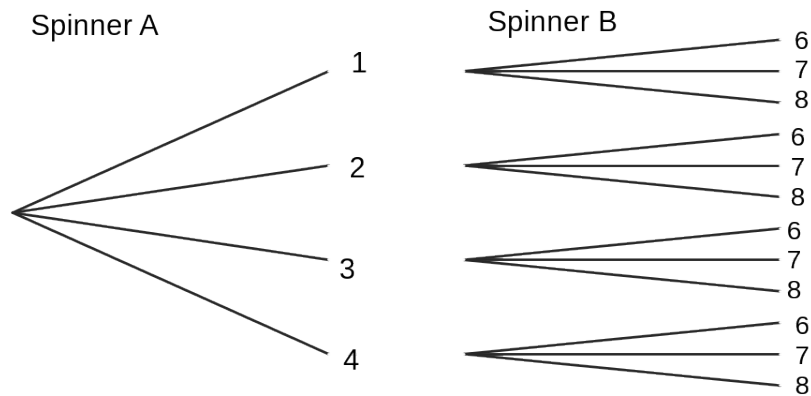
(8 marks)

Two spinners, A and B, are marked with both odd and even numbers.

Spinner A has sides marked 1, 2, 3 and 4 and each score has an equal chance of occurring.

Spinner B has sides marked 6, 7 and 8 and again, each score has an equal chance of occurring.

- (a) Draw a tree diagram to show all the ways spinner A and spinner B could stop. (3 marks)



- (b) Determine the probability that

- (i) both spinners stop on even numbers.

(1 mark)

$$\frac{4}{12}$$

- (ii) one spinner stops on an even and the other stops on an odd number.

(1 mark)

$$\frac{6}{12}$$

- (iii) spinner A or spinner B stops on an even number.

(1 mark)

$$\frac{10}{12}$$

- (iv) both spinners stop on an odd number, given that at least one spinner has stopped on an odd number. (2 marks)

$$\frac{2}{8}$$

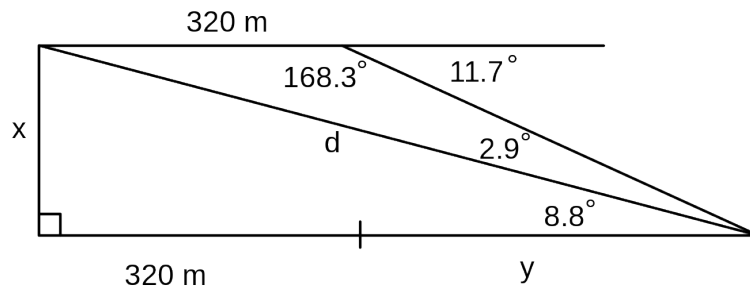
Question 15

(6 marks)

A remotely controlled drone flies at a constant speed of 40 m s^{-1} above level ground and at a steady height, directly towards a tower.

At one point during the flight, the operator of the drone noted that the angle of depression to the base of the tower was 8.8° . Eight seconds later, this angle had increased to 11.7° .

Calculate, to the nearest metre, the height the drone was flying at.



$$\frac{d}{\sin 168.3} = \frac{320}{\sin 2.9}$$

$$d = 1282.6$$

$$x = 1282.6 \times \sin 8.8$$

$$x = 196.2 \approx 196 \text{ m}$$

Question 16

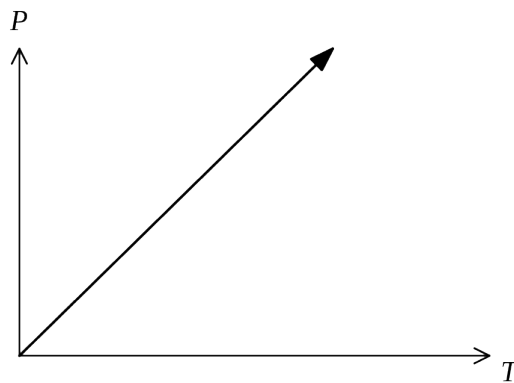
(7 marks)

- (a) In a science experiment, quantity P was observed to be related to the another quantity T by the equation $P = \frac{T}{255}$.

- (i) Describe the relationship between P and T . (1 mark)

P is directly proportional to T

- (ii) Sketch the relationship between P and T . (2 marks)

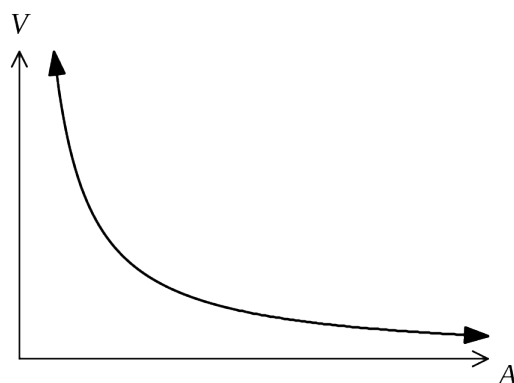


- (b) In another experiment, the quantity V was observed to be related to the another quantity A by the equation $VA = 110$.

- (i) Calculate A when $V = 215$. (2 marks)

$$\begin{aligned} A &= \frac{110}{V} \\ &= \frac{110}{215} \\ &= 0.512 \end{aligned}$$

- (ii) Sketch the relationship between V and A . (2 marks)



Question 17

(12 marks)

Two functions are given by $f(x) = ax^2 + 2$ and $g(x) = x^3 + bx^2 + x + 3$, where a and b are constants.

$f(x)$ has a root at $x = -2$ and $g(x)$ passes through the point $(2, -3)$.

- (a) Show that $a = -\frac{1}{2}$.

(2 marks)

$$\begin{aligned} 0 &= a(-2)^2 + 2 \\ 4a &= -2 \\ a &= -\frac{1}{2} \end{aligned}$$

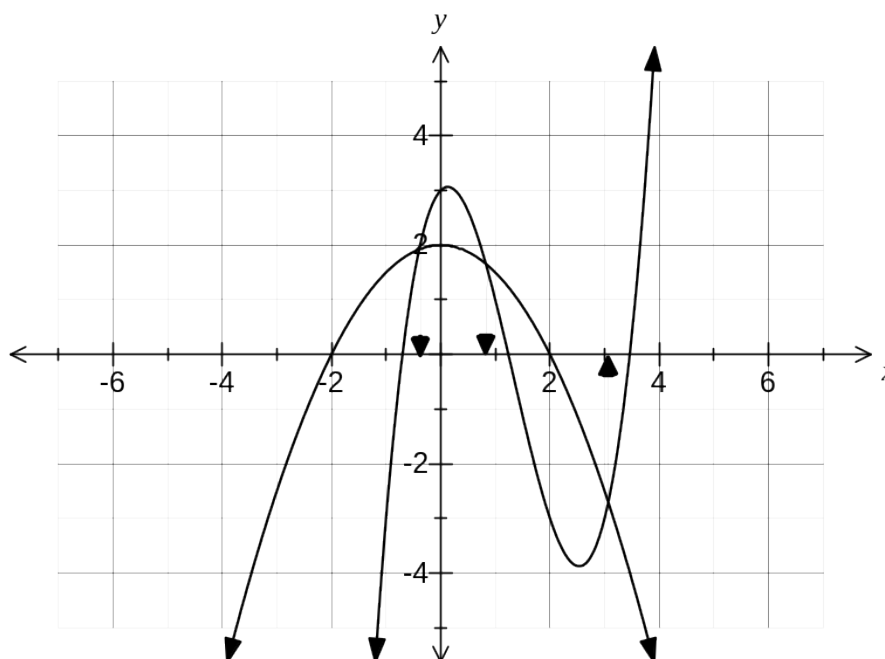
- (b) Show that $b = -4$.

(2 marks)

$$\begin{aligned} -3 &= 2^3 + b(2)^2 + 2 + 3 \\ -3 &= 8 + 4b + 5 \\ 4b &= -16 \\ b &= -4 \end{aligned}$$

- (c) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the axes below.

(5 marks)



- (d) Solve $f(x) = g(x)$, giving your solution(s) rounded to two decimal places.

(2 marks)

$$x = -0.39, x = 0.83, x = 3.07$$

- (e) Show how your graph can be used to obtain the solutions to (d).

(1 mark)

See graph - must clearly indicate x -coordinate only.

Question 18**(5 marks)**

Using the digits 1, 2, 3, 4 and 5, with no digit used more than once, how many integers between 1000 and 9999 can be made if

(a) there are no other restrictions?

(1 mark)

$$5 \times 4 \times 3 \times 2 = 120$$

(b) the number must begin with the digit 5?

(1 mark)

$$1 \times 4 \times 3 \times 2 = 24$$

(c) the number must be even?

(1 mark)

$$2 \times 4 \times 3 \times 2 = 48$$

(d) the number must begin with the digit 5 **and** be even?

(1 mark)

$$1 \times 2 \times 3 \times 2 = 12$$

(e) the number must begin with the digit 5 **or** be even?

(1 mark)

$$24 + 48 - 12 = 60$$

Question 19

(11 marks)

A loan of \$2300 is to be repaid by monthly payments of \$220. Interest, of 9.6% pa of the balance, is added at the end of each month, just before the repayment is made. The table below shows the balance of the loan at the start and end of each month, together with the monthly interest and repayment for the first few months.

Month	Balance (Start of month)	Interest	Repayment	Balance (End of month)
n	T_n	I_n	R_n	T_{n+1}
1	2300.00	18.40	220.00	2098.40
2	2098.40	16.79	220.00	1895.19
3	1895.19	15.16	220.00	1690.35

- (a) Write a recursive rule to determine the amount of the loan T_n at the start of month n .

(2 marks)

$$T_{n+1} = T_n \times 1.008 - 220$$

$$T_1 = 2300$$

- (b) At the end of which month is the loan repaid in full, and what is the amount of the final repayment?

(2 marks)

Month 11.

Final repayment is \$211.53

- (c) How much interest is paid over the life of this loan?

(2 marks)

$$10 \times 220 + 211.53 - 2300 = \$111.53$$

- (d) Show that the total interest over the life of the loan approximately doubles when the repayment is halved to \$110 per month.

(3 marks)

Now takes 23 months, with final payment of \$106.99.
Total interest is $22 \times 110 + 106.99 - 2300 = \226.99 , which is just over double previous figure.

- (e) If the repayment was repeatedly halved, would you expect the interest to roughly double each time? Explain your answer.

(2 marks)

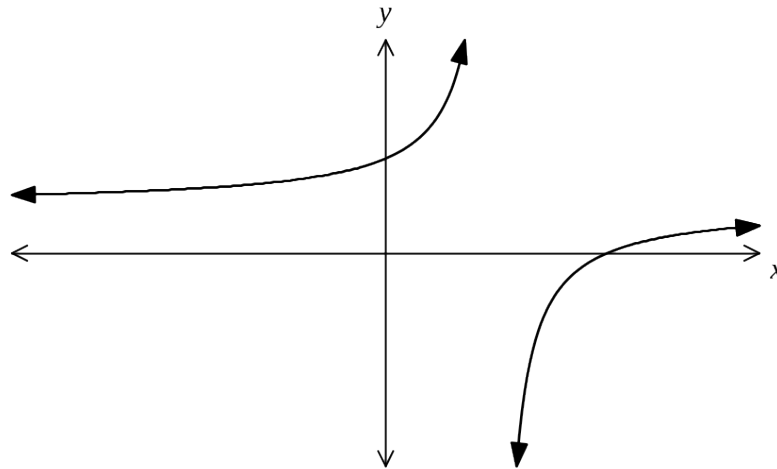
No. Once the repayment has been halved three more times, it is less than the first month's interest, and so the loan would never be paid off, with interest accruing for ever.

Question 20

(8 marks)

The graph of the function $y = f(x)$ is shown, with a root at $(4, 0)$ and y -intercept at $(0, 2)$.

The hyperbola has asymptotes with equations $x = 2$ and $y = 1$.



Determine the

- (a) coordinates of the root of the graph $y = f(x - 2)$.

(2 marks)

Was $(4, 0)$
Becomes $(6, 0)$

- (b) equation of the horizontal asymptote for the graph of $y = f(x - 2) + 3$.

(2 marks)

Was $y = 1$
Becomes $y = 1 + 3 \Rightarrow y = 4$

- (c) equation of the vertical asymptote for the graph of $y = f(2(x + 3))$.

(2 marks)

Was $x = 2$
Becomes $x = \frac{1}{2} \times 2 - 3 \Rightarrow x = -2$

- (d) coordinates of the y -intercept of the graph $y = 3 - f(x + 4)$.

(2 marks)

Need to consider known root at $(4, 0)$
Becomes $(4, 0) \rightarrow (0, 0) \rightarrow (0, 3)$

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