WATP

SEMESTER TWO

MATHEMATICS
METHODS
UNITS 1 and 2

2020

SOLUTIONS

Calculator-free Solutions

1. (a) $x = 3$ or -3 ✓✓

(b) $f(x) = x^3 + 3x^2 - 9x - 27$

$f'(x) = 3x^2 + 6x - 9 = 0$ ✓

$3(x+3)(x-1) = 0$

$x = -3$ or 1 ✓

x		-3		1	
$f(x)$	↑	-	↓	-	↑
$f'(x)$	+	0	-	0	+

✓

(- 3, 0) Maximum ✓

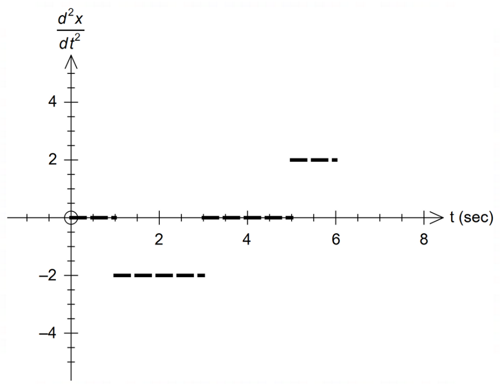
$(1, -32)$ Minimum ✓

(c) $g(x) = f(x+1) = (x+1)^3 + 3(x+1)^2 - 9(x+1) - 27$ ✓ [8]

2. (a) $0 < t < 1$ or $3 < t < 5$ ✓✓

(b) 6 seconds (<0) 2 seconds (>0) ✓

(c)



✓✓ [5]

3. (a) (i) $10^{x+5} = 10^8$

$x = 3$ ✓

(ii) $x^2 - 1 = 0$ ✓

$x = \pm 1$ ✓

(b) $4 \times 48 \times 81 = 2^2 \times 2^4 \times 3 \times 3^4$ ✓

$= 2^6 \times 3^5$

$= 2(2 \times 3)^5$ ✓

$k = 5$ ✓

(c) 6×10^3 ✓✓ [8]

4. (a) 2π ✓

(b) $(-\pi, -3)\ (0, 3)\ (\pi, -3)\ (2\pi, 3)$ ✓✓

(c) $x = \pi$ or $x = -\pi$ ✓

(d) $-\pi < x < 0$ or $\pi < x < 2\pi$ ✓✓ [6]

5. (a) $2x - 3 - 4x = 4x - 3 - (2x - 3)$ ✓

$-2x - 3 = 2x$

$x = -\frac{3}{4}$ ✓

(b) $T_{n+1} = T_n - 1.5 \quad T_1 = -3$ ✓✓ [4]

6. (a) $m = \dfrac{\beta^2 - \alpha^2}{\beta - \alpha} = \dfrac{(\beta + \alpha)(\beta - \alpha)}{\beta - \alpha}$ ✓✓

$\therefore m = \alpha + \beta$

$y = (\alpha + \beta)x + b \;\; | \left(\alpha,\ \alpha^2\right)$ ✓

$\alpha^2 - \alpha^2 - \alpha\beta = b$ ✓

$\alpha\beta = -b$

(b) (i) $y = 2x + 8$ ✓

(ii) $A(-2,\ 4) \quad B(4,\ 16)$ ✓

$d_{AB} = \sqrt{(-2-4)^2 + (4-16)^2}$

$d = \sqrt{180}$ ✓

$= 6\sqrt{5}$ units

(iii) $\dfrac{-2 + x}{2} = -0.25$

$x = 1.5$ ✓

$y = 1.5^2 = 2.25$ (or $\dfrac{4 + y}{2} = 3.125$ $y = 2.25$) ✓ [9]

20. (a) (i) $3x^2 - 6x + 3 = 12$

$x = -1$ or 3

$(-1,\ -3)$ $(3,\ 13)$ ✓✓

(ii) $y = 3x + 4$ $m = 3$

$3x^2 - 6x + 3 = 3$

$x = 0$ or $x = 2$

$(0,\ 4)$ $(2,\ 6)$ ✓✓

(iii) $3x^2 - 6x + 3 = 27$

$x = -2$ or $x = 4$

$(-2,\ -22)$ $(4,\ 32)$ ✓✓

(b) $(-2)^2 + a(-2) + b = 0$

$\therefore 2a - b = 4$

And $a + b = -1$

$\therefore a = 1$ and $b = -2$ ✓✓

$f(x) = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 2x + c \;\; | (2, 3)$ ✓

$c = \dfrac{7}{3}$ The equation of the curve is $f(x) = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 2x + \dfrac{7}{3}$. ✓ [10]

End of Questions

19. (a) Roots at $(0,\ 0)\quad(2,\ 0)\quad(-2,\ 0)$ ✓

(b) $f'(x) = 4x^3 - 8x = 4x(x^2 - 2) = 0$ (for stationary points) ✓

Stationary points at $(0,\ 0)\ (\sqrt{2},\ -4)\ (-\sqrt{2},\ -4)$ ✓

$f''(x) = 12x^2 - 8 \qquad f''(0) < 0 \therefore$ Max ✓

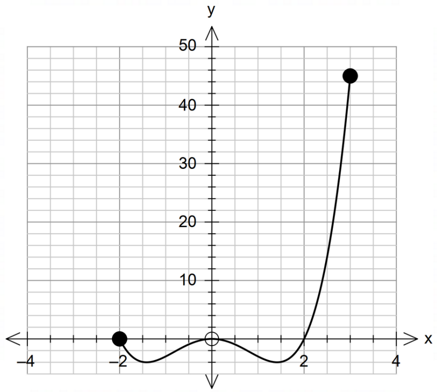
$f''(-\sqrt{2}) > 0 \therefore$ Min ✓

$f''(\sqrt{2}) > 0 \therefore$ Min ✓

Or Sign table:

x		$-\sqrt{2}$		0		$\sqrt{2}$	
f(x)	↓	—	↑	—	↓	—	↑
f′(x)	-	0 Min	+	0 Max	-	0 Min	+

(c)



✓✓✓ [9]

7. (a)

	Location A	Location B	Totals
Buses left late	15	**40**	55
Buses left on time	**25**	20	**45**
Totals	40	**60**	100

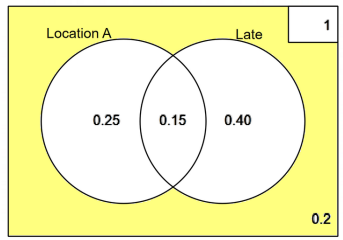
✓✓

(b) (i) The events are whether a bus leaves from Location A or not and whether a bus left late or not. ✓

(ii) Probability of leaving from Location A = 0.4
Probability of leaving late = 0.55
P(Leaving from Location A and leaving late) = 0.15 ✓
0.4 x 0.55 = 0.22 and not 0.15, ✓
therefore the events are dependent. ✓

(iii) $\dfrac{0.25}{0.4} = 0.625$ or $\dfrac{25}{40}$ ✓

(c) (i)



✓✓

(ii) Shading ✓ [10]

Calculator-assumed Solutions

8. (a) $V(10) = 2500$ litres ✓

(b) 60 mins ✓

(c) $V'(t) = 2t - 120$ ✓

$V'(20) = -80$ litres per minute ✓

(d) $V_2(t) = 7200\left(1 - \frac{t}{60}\right)^2$

$V'(t) = 4t - 240$ ✓

This is $2(2t - 120)$ $\therefore$ double the rate ✓ [6]

9. (a) $T_{n+1} = T_n + 2575 \quad T_1 = 52000$ ✓✓

(b) $A_n = 52000(1.04)^{n-1},\ n \geq 1$ ✓✓

(c) \$74 012.21 ✓

(d) During the 13th year (83253>82900) ✓✓

(e) Skye: \$1 548 460. 09 ✓

Indy: \$1 529 250.00 ✓

Skye's total earnings exceeds Indy's total earnings

by \$19 210.09 ✓ [10]

10. (a) $V = \pi x^2(2h)$ and $x^2 = R^2 - h^2$ ✓✓

$V = \pi\left(R^2 - h^2\right)(2h)$

$\therefore V = 2\pi h\left(R^2 - h^2\right)$ ✓

(b) $V'(h) = -6\pi h^2 + 2\pi R^2 = 0$ (for max/min) ✓

$h^2 = \frac{2\pi R^2}{6\pi} = \frac{R^2}{3}$ ✓

$h = \frac{\mp\sqrt{3}R}{3}$ Discard negative h

18. (a) (i) $\frac{1-t}{t+1} = \frac{2-5t}{1-t} = r$ ✓

$6t^2 + t - 1 = 0$

$t = \frac{1}{3}$ or $-\frac{1}{2}$ ✓

For $t = \frac{1}{3} \quad r = \frac{1 - \frac{1}{3}}{\frac{1}{3} + 1} = \frac{1}{2}$ ✓

For $t = -\frac{1}{2} \quad r = \frac{1 + \frac{1}{2}}{-\frac{1}{2} + 1} = 3$ Discard ✓

Common ratio = $\frac{1}{2}$ when $t = \frac{1}{3}$

(ii) $S_\infty = \frac{a}{1-r}$ when $a = t + 1 = \frac{4}{3}$ ✓

$= \frac{8}{3}$ ✓

(b) (i) $S_8 = 8^2 - 2(8) = 48$ ✓

(ii) $S_7 = 7^2 - 14 = 35$

$T_8 = S_8 - S_7 = 13$ ✓✓

(iii) $S_{15} - S_{12} = \left(15^2 - 30\right) - \left(12^2 - 24\right)$ ✓

$= 75$ ✓

(c) $T_1 = 27 - 6(1 + 1) = 15$

$T_2 = 27 - 6(2 + 1) = 9$

$a = 15 \quad d = -6$ ✓✓

$S_{20} = -840$ ✓ [14]

17. (a) $2 = r^{12}$

$r = 1.0595$ ✓

$\therefore$ 5.95% per hour is the rate of growth

(b) $9000 = 1000(1.0595)^t$ ✓

t = 38.016

Approximately 38 hours. ✓

(c) $P = 9000\left(\frac{2}{3}\right)^8$ ✓

P = 351 bacteria left ✓

Their expectations were not accurate. The anti-biotic killed more than a third of the bacteria per hour. ✓

OR $9000(r)^8 = 1$

r = 0. 3204 The antibiotic killed 68% per hour which is more than $\frac{2}{3}$. [6]

$\therefore h = \frac{\sqrt{3}R}{3}$ ✓ [6]

11. (a) $T_2 = T_1 + T_3 - 1$ ✓
$T_2 = x + y - 1$

(b) $T_3 = T_2 + T_4 - 1$
$y = x + y - 1 + T_4 - 1$ ✓
$T_4 = 2 - x$ ✓

Sum = $x + x + y - 1 + y + 2 - x = x + 2y + 1$ ✓

(c) No common difference nor common ratio ✓✓ [6]

12. (a) F ✓
(b) F ✓
(c) F ✓
(d) T ✓
(e) F ✓ [5]

13. (a) $\lim\limits_{h \to 0} \dfrac{(2x + 2h + 3)^2 - (2x + 3)^2}{h}$ ✓✓

(b) $y = x^2 \therefore \dfrac{dy}{dx} = 2x$ when $x = 3$ ✓
$2(3) = 6$ ✓

(c) $\dfrac{dy}{dx} = 6x - 2 = 4 \;\therefore x = 1$ ✓
$y = 3(1)^2 - 2(1) + 1 = 2$ $y = 2$ ✓

(d) $\dfrac{17}{2}$ ✓ [7]

14. (a) $y = 3 + 2^x$ D ✓
(b) $y = 2^x$ C ✓
(c) $y = 2^x - 3$ A ✓
(d) $y = \left(\dfrac{1}{2}\right)^x$ B ✓ [4]

15. (a) $l = r\theta$
$\dfrac{63\pi}{20} = (2.7)\theta$ ✓
$\theta = \dfrac{7\pi}{6}$ $\dfrac{2\pi}{12} = 1$ hour $\therefore$ this has taken 7 hours. ✓✓

(b) $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(2.7)^2\left(\dfrac{\pi}{3}\right)$ ✓
$= \dfrac{243\pi}{200}\, m^2$ ✓ [5]

16. (a) $t = 0 \therefore x(0) = 0\ m$ ✓

(b) $x(t) = t^3 - 9t^2 + 16t$
$v(t) = 3t^2 - 16t + 16$ ✓
$v(0) = 16$ m/s ✓
The particle is initially travelling to the right of the origin. ✓

(c) $(t - 4)(3t - 4) = 0$
$t = 4$ or $t = \dfrac{4}{3}$ First changes direction at $\dfrac{4}{3}$ s. ✓

(d) $a(t) = 6t - 16$ ✓
$a(3) = 2\ m.s^{-2}$ ✓

(e) $v(3) = -5m.s$ ✓
Velocity is negative and acceleration is positive therefore the particle is slowing down. ✓

(f) First turns at $\dfrac{4}{3}$ s. $x\left(\dfrac{4}{3}\right) = \dfrac{256}{27}\ m$ $(9.48\ m)$ ✓
Turns again at 4 s. $x(4) = 0$ ✓
$x(5) = 5\ m$
$\therefore$ Total distance = $2\left(\dfrac{256}{27}\right) + 5 = \dfrac{647}{27}\ m$ (or $23.96m$) ✓ [12]