

MATHEMATICS METHODS

MAWA Semester 2 (Units 3 and 4) Examination 2017

Calculator-free

Marking Key

© MAWA, 2017

Licence Agreement

This examination is Copyright but may be freely used within the school that purchases this licence.

- The items that are contained in this examination are to be used solely in the school for which they are purchased.
- They are not to be shared in any manner with a school which has not purchased their own licence.
- The items and the solutions/markings keys are to be kept confidentially and not copied or made available to anyone who is not a teacher at the school. Teachers may give feedback to students in the form of showing them how the work is marked but students are not to retain a copy of the paper or marking guide until the agreed release date stipulated in the purchasing agreement/licence.

The release date for this exam and marking scheme is

- the end of week 1 of term 4, 2017

Section One: Calculator-free (52 Marks)

Question 1(a)

Solution	
$k + 2k + 0.5k + 0.5k + k = 1$ $5k = 1$ $k = \frac{1}{5}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> correctly solves for k. 	1

Question 1(b)

Solution	
<p>No, This is not a discrete probability distribution as $P(5)$ has a negative value and probabilities need to be positive.</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> recognises the properties of discrete probability distributions 	1
<ul style="list-style-type: none"> relates above property back to the question to justify answer 	1

Question 1(c)

Solution	
$\mu = np = 90$ $\therefore \sigma^2 = 6^2 = 36$ $\sigma^2 = np(1-p)$ $36 = 90(1-p) \quad \text{Expand and solve for } p$ $\therefore p = 0.6$ $\frac{6}{10}n = 90 \quad \text{Sub value of } p \text{ and solve for } n.$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> determines variance 	1
<ul style="list-style-type: none"> solves for parameter, p 	1
<ul style="list-style-type: none"> solves for parameter, n 	1

Question 2(a)

Solution	A census involves every member of the population being tested/questioned/investigated
Marking key/mathematical behaviours	
Marks	1

Question 2(b)

Solution	There would be no stoves left to sell as all of them would have broken down.
Marking key/mathematical behaviours	<ul style="list-style-type: none"> indicates that there would be no items left for sale (no marks for cheaper or quicker)
Marks	1

Question 2(c)

Solution	Use the unique serial numbers to select a random sample or similar
Marking key/mathematical behaviours	<ul style="list-style-type: none"> indicates use of a suitable random selection method (based on serial numbers or other method)
Marks	1

Question 2(d)

Solution	Using the list of the serial numbers, select every 400 th stove
Marking key/mathematical behaviours	<ul style="list-style-type: none"> indicates use of a suitable selection method
Marks	1

Question 3(a) (i)

Solution	$f'(x) = \frac{(2 + \cos x) \times 1 - (-\sin x)x}{2 + \cos x + x \sin x} = \frac{(2 + \cos x)^2}{(2 + \cos x)^2}$
Marking key/mathematical behaviours	<ul style="list-style-type: none"> correct use of the quotient rule differentiates correctly
Marks	1 1

Question 3(a) (ii)

Solution	$f'(x) = (3 + 2 \cos 2x)e^{3x + \sin 2x}$
Marking key/mathematical behaviours	<ul style="list-style-type: none"> correct use of the chain rule differentiates correctly
Marks	1 1

Question 3(b)

Solution	
Differentiating both sides: $f''(x)f(x) + f'(x)^2 = 0$	
so $f''(x) = -\frac{f'(x)^2}{f(x)}$	
But $f'(x) = \frac{1}{f(x)}$ and so $f''(x) = -\frac{1}{f(x)^3}$ i.e. $f''(x)f(x)^3 = -1$	
Marking key/mathematical behaviours	Marks
• uses product rule correctly	1
• obtains correct expression for $f''(x)$	1
• replaces $f'(x)$ with $\frac{1}{f(x)}$	1

Question 4(a)(i)

Solution	
Approximately 200 samples are involved	
Marking key/mathematical behaviours	Marks
• States the number of samples (allow 190 to 210)	1

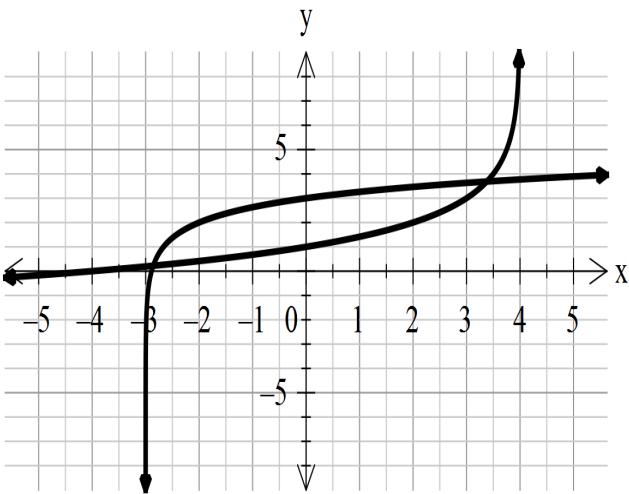
Question 4(a)(ii)

Solution	
Find the mean of the sample proportions, (from the graph) ≈ 0.4	
May use sample proportion as an estimate of the population proportion	
Marking key/mathematical behaviours	Marks
• identifies sample proportion as 0.4 (by reference to the graph or calculation)	1
• uses the sample proportion as an estimate for the population proportion	1

Question 4(b)

Solution	
<ul style="list-style-type: none"> Survey is restricted to listeners of one particular station and therefore not representative of the population Survey is using a self-selection model and this indicates bias Timing may exclude some groups of people Access to a telephone is presumed People could respond more than once Nature of the question means football fans may be more likely to respond 	
Marking key/mathematical behaviours	Marks
• Lists one possibility	1
• Lists a second possibility	1

Question 9(b)

Solution	
	
Marking key/mathematical behaviours	Marks
• asymptote at $x = 4$	1
• y intercept at $(0,1)$	1
• x intercept at $(-4,0)$	1
• graph of standard logarithm function reflected in both axes	1

Question 5

Solution	
Note that: $y = 1 - 4x^2 \Rightarrow y' = -8x$ $\therefore \int \frac{10x}{1 - 4x^2} dx = -\frac{10}{8} \int \frac{1 - 4x^2}{1 - 4x^2} dx$ $= -\frac{10}{8} \ln 1 - 4x^2 + c$ $= -\frac{5}{4} \ln 1 - 4x^2 + c$	
Marking key/mathematical behaviours	
Marks	1
	1
	1
	1
	1
<ul style="list-style-type: none">Recognises $\frac{f'(x)}{f(x)}$Determines (or uses) derivative of denominatorUses constants to achieve numerator of $-8x$Finds integral accurately	

Question 6(a)

Solution	
$S^{-2} = \frac{1}{x + 24y} \log_5 \left(\frac{25}{x + 24y} \right) = \ln_5(x + y)$ (#) Since $S^{-2} = \frac{1}{x + 24y}$ we have $\log_5 \left(\frac{25}{x + 24y} \right) = \ln_5(x + y)$ i.e. $\frac{x + 24y}{25} = x + y$ and hence $y = -24x$	
Marking key/mathematical behaviours	
Marks	1
	1
<ul style="list-style-type: none">Obtains equation (#) or equivalentObtains correct answer	

Question 6(b)(i)

Solution	
If $a > b$ then $\log_a b > 0$ i.e. negative	
Marking key/mathematical behaviours	
Marks	1
	1
<ul style="list-style-type: none">obtains correct answer	

Question 6(b)(iii)

Solution	
If $a \neq b$ then $\log_a b$ and $\log_b a$ have opposite signs (#) and are unequal. So $a = b$ if $\log_a b = \log_b a$	
Marking key/mathematical behaviours	
Marks	1
	1
<ul style="list-style-type: none">deduces that $\log_a b$ and $\log_b a$ have opposite signs if $a \neq b$complete proof correctly	

Question 8(c)

Solution	
(i) From the graph, read the relative frequencies for the parking times (0.30] mins and (30, 60] mins \Rightarrow Pr (at most 60 minutes) = 0.38 + 0.42 = 0.8.	
(ii) relative frequency of vehicles parked between 1 and 1.5hrs = 0.14. $200 \times 0.14 = 28$ vehicles.	
(iii) Answers will vary. One possible answer: The data shows that 80% of the vehicles are parked for at most 60 minutes. Businesses that would tailor to this time length could be a nearby café or a mini mall with a few necessities stores (ie. Grocery store, bakery, pharmacy, clothing shop).	
(iv) Answers will vary. One possible answer: On which day of the week was the data collected?	
Marking key/mathematical behaviours	
Marks	1
	1
	1
	1
	1
<ul style="list-style-type: none">sums the two relative frequencies requiredmultiplies the relative frequency by the total number of parked carslinks to a type of business requiring less than 1 hour of parkinglinks question to day of the week or other plausible variable.	

Question 9(a)

Solution	
Since the graph has a vertical asymptote at $x = -3$, $b = 3$ Since the point $(-2, 2)$ lies on the graph, $2 = \log_a(-2 + 3) + c$ i.e. $c = 2$ Since the point $(0, 3)$ lies on the graph, $3 = \log_a + 2$ i.e. $\log_a(3) = 1$ and hence $a = 3$	
Marking key/mathematical behaviours	
Marks	1, 1, 1
	1
<ul style="list-style-type: none">evaluates a, b and c correctly	

Question 7(a)

Solution $A = -\int_1^4 \left(\frac{-8}{\sqrt{x}} \right) dx$ $\int_1^4 \frac{8}{\sqrt{x}} dx$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none">sets up integral in either form	1

Question 7(b)

Solution $A = \int_1^4 \frac{8}{\sqrt{x}} dx$ $= \left[16\sqrt{x} \right]_1^4$ $= 16(2) - 16$ $= 16 \text{ square units}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none">integrates correctly	1
<ul style="list-style-type: none">correctly evaluates the integral	1

Question 7(c)

Solution $\int_1^a \frac{-8}{\sqrt{x}} dx = -25$ $\Rightarrow -16\sqrt{a} + 16 = -25$ $-16\sqrt{a} = -41$ $a = \frac{41^2}{16^2}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none">correctly states the equation to be solved	1
<ul style="list-style-type: none">solves for a	1

Question 8(a)

Solution $\int_0^c \frac{x^3}{4} dx$ $= \frac{1}{4} \left[\frac{x^4}{4} \right]_0^c$ $\frac{c^4}{16} = 1$ $c^4 = 16$ $c = \pm 2$ <p>As -2 is outside of the lower boundary</p> $c = 2$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none">uses the correct integral	1
<ul style="list-style-type: none">equates the pdf to one and solves for c	1
<ul style="list-style-type: none">Clearly states why $c = -2$ has been rejected	1

Question 8(b)

Solution $E(x) = \int_{-\infty}^{\infty} xw(x) dx$ $= \int_{12}^{32} x \left(\frac{1}{20} \right) dx$ $= \left[\frac{1}{20} \times \frac{x^2}{2} \right]_{12}^{32}$ $= \frac{1}{40} (32^2 - 12^2)$ $= 22$ $\therefore E(x) = 22$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none">Substitutes into the correct formula	1
<ul style="list-style-type: none">Integrates correctly and arrives at the required answer	1