PERTH MODERN SCHOOL
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Independent Public School

Course Methods Test 3 Year 12

Student name: ______________ Teacher name: ______________

Task type: **Response/Investigation**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **____6_______**

Materials required: No classpads

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper

Marks available: **__38____ marks**

Task weighting: **_14___%**

Formula sheet provided: No but some formulae given on page 2

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Logarithms

$x = \log_a b \Leftrightarrow a^x = b$	$a^{\log_a b} = b$ and $\log_a(a^b) = b$
$\log_a mn = \log_a m + \log_a n$	$\log_a \frac{m}{n} = \log_a m - \log_a n$
$\log_a(m^k) = k\log_a m$	$\log_e x = \ln x$

$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln x + c, \quad x > 0$
$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c, \quad f(x) > 0$

Product rule	If $y = uv$ then $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$	or	If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$
Quotient rule	If $y = \frac{u}{v}$ then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	or	If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$
Chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or	If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$
Fundamental theorem	$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$	and	$\int_a^b f'(x)\,dx = f(b) - f(a)$

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End of test
Working out space

$$P(X \le x) = \begin{cases} \int_0^x \frac{t}{20}\,dt, 0 \le x \le 4 \\ \frac{2}{5} + \int_4^x \frac{-t}{30} + \frac{1}{3}\,dt, 0 \le x \le 4 \end{cases}$$

$$P(X \le x) = \begin{cases} \frac{x^2}{40}, \; 0 \le x \le 4 \\ \frac{2}{5} + \left[\frac{-t^2}{60} + \frac{t}{3}\right]_4^x, 4 \le x \le 10 \end{cases}$$

$$P(X \le x) = \begin{cases} \frac{x^2}{40}, \; 0 \le x \le 4 \\ \frac{2}{5} - \frac{x^2}{60} + \frac{x}{3} + \frac{16}{60} - \frac{4}{3}, 4 \le x \le 10 \end{cases}$$

Specific behaviours
✓ defines cumulative in integral form and in two parts ✓ evaluates function for 0<x<4 ✓ evaluates function for 4<x<10, no need to simplify

Q1 (2 & 2 = 4 marks)
Express each of the following as a single logarithm.

a) $\log_a b + 3\log_a(ab) - 4\log_a b$.

c
$\log_a b + 3\log_a(ab) - 4\log_a b$ $\log_a b + \log_a(ab)^3 - \log_a b^4$ $\log_a a^3$
Specific behaviours
✓ uses log laws ✓ expresses as one log statement (Do not accept 3)

b) $5 + 3\log c - \log(c^3) + \log b$.

c
$5 + 3\log c - \log(c^3) + \log b$ $\log_5 5^5 + \log_5 c^3 - \log_5(c^3) + \log_5 b$ $\log_5 5^5 b$
Specific behaviours
✓ changes 5 into a log statement ✓ expresses as one log statement

Q2 (2 & 2 = 4 marks)
Solve each of the following, giving your answer in exact form.

a) $2^{2x} - 12(2^x) + 32 = 0$

c
$2^{2x} - 12(2^x) + 32 = 0$ $w = 2^x$ $w^2 - 12w + 32 = 0$ $(w-4)(w-8) = 0$ $w = 4, 8$ $x = 2, 3$
Specific behaviours
✓ uses quadratic expression ✓ solves for both x values

b) $7^x + 3(7^{x+2}) = 31$

c
$7^x + 3(7^{x+2}) = 31$ $7^x(1+3(7)^2) = 31$ $7^x = \frac{31}{148}$ $x = \frac{\log \frac{31}{148}}{\log 7}$ or $\log_7 \frac{31}{148}$
Specific behaviours
✓ factorises ✓ solves in log form

Q3 (1, 3 & 3 = 7 marks)

The Richter scale, R, of an earthquake of intensity I is given by $R = \log_{10}\left(\frac{I}{I_o}\right)$ where I_o is a minimum intensity level used for comparison.

a) Determine R for an earthquake with intensity $10000I_o$.

c
$R = \log_{10}\left(\frac{10000I_o}{I_o}\right) = 4$
Specific behaviours
✓ states answer

b) An earthquake measuring 5 on the Richter scale is how many times as intense as that of one measuring 4 on the Richter scale?

c

c
$E(X) = \int_0^4 \frac{x^2}{20}dx + \int_4^{10} \frac{-x^2}{30} + \frac{x}{3}dx$ $\left[\frac{x^3}{60}\right]_0^4 + \left[\frac{-x^3}{90} + \frac{x^2}{6}\right]_4^{10}$ $\frac{64}{60} - \frac{1000}{90} + \frac{100}{6} + \frac{64}{90} - \frac{16}{6} = \frac{14}{3}$
Specific behaviours
✓ sets up integral in two parts ✓ evaluates one part integral without simplifying ✓ evaluates second part integral without simplifying

b) Derive the cumulative probability function $P(X \le x)$ for $0 \le x \le 10$.

c

OR $\frac{1}{2}\times10\times\frac{1}{5}-\frac{1}{2}\times2\times\frac{1}{10}-\frac{1}{2}\times3\times\frac{1}{10}$
Specific behaviours
✓ determines area from x=2 to 4 OR uses two triangles 0 to2 and 7 to 10 ✓ determines area from x=2 to x=4 OR subtracts the area of two triangles above from 1 ✓ adds to give simplified total area

ii) $P(X>3\mid X<5)$.(No need to simplify)

c
$P(X>3\mid X<5)=\frac{P(3<X<5)}{P(X<5)}$ $=\frac{\int_3^4 \frac{x}{20}dx+\int_4^5\left(\frac{-x}{30}+\frac{1}{3}\right)dx}{\frac{1}{2}(4)\frac{1}{5}+\int_4^5\left(\frac{-x}{30}+\frac{1}{3}\right)dx}$ $=\frac{\left[\frac{x^2}{40}\right]_3^4+\left[\frac{-x^2}{60}+\frac{20x}{60}\right]_4^5}{\frac{2}{5}+\left[\frac{-x^2}{60}+\frac{20x}{60}\right]_4^5}=\frac{\frac{7}{40}+\frac{11}{60}}{\frac{2}{5}+\frac{11}{60}}=\frac{43}{70}$ OR use triangles $\frac{1-\frac{1}{2}\times3\times\frac{3}{20}-\frac{1}{2}\times5\times\frac{5}{30}}{1-\frac{1}{2}\times5\times\frac{5}{30}}(accept)$ $OR\ \frac{\frac{43}{120}}{\frac{7}{12}}$ Note- Height of triangle above must = $\frac{5}{30}$ for full marks (do not accept approx.)
Specific behaviours
✓ writes a conditional prob statement (or directly implied) ✓ evaluates denominator/area (un simplified but evaluated) ✓ evaluates numerator/area (un simplified but evaluated)

iii) E(X) i.e the mean. (No need to simplify)

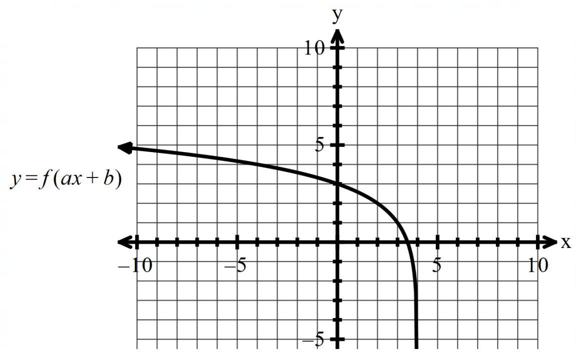
$5=\log_{10}\left(\frac{I}{I_o}\right)$ $10^5 I_o=I_5$ $10^4 I_o=I_4$ $\frac{I_5}{I_4}=10$
Specific behaviours
✓ converts log statement into index form ✓ divides both intensities ✓ states ratio (1 mark for answer only)

c) If an earthquake registers x on the Richter scale and a second earthquake registers $x+4$ on the Richter scale, how many more times as intense is the second earthquake?

c
$x+4=\log\left(\frac{I_{x+4}}{I_o}\right)$ $I_{x+4}=I_o10^{x+4}$ $I_x=I_o10^x$ $\frac{I_{x+4}}{I_x}=10^4$
Specific behaviours
✓ converts log statement into index form ✓ divides both intensities ✓ states ratio (1 mark for answer only)

Q4 (3 marks)

Consider the function $f(x)=\log_2 x$ which undergoes a transformation $f(ax+b)$ where $a\,\&\,b$ are constants. The graph $y=f(ax+b)$ is plotted below, determine the values of $a\,\&\,b$ showing reasoning.



c
$f(b) = 3 = \log_2 b = 3$
$b = 8$
$f(4a+8) = undefined$
$4a + 8 = 0$
$a = -2$
Specific behaviours
✓ sets up equation to solve for b
✓ sets up equation to solve for a
✓ states values of a & b (max 1 mark for answer only)

Q5 (3 & 5 = 8 marks)

Consider the function $g(x) = (x^2+3)\ln(x^3+3x)$.

a) Determine $g'(x)$.(Simplify)

c
$g(x) = (x^2+3)\ln(x^3+3x)$
$g' = (x^2+3)\dfrac{3x^2+3}{x^3+3x} + 2x\ln(x^3+3x)$
$= 3x + \dfrac{3}{x} + 2x\ln(x^3+3x)$
Specific behaviours
✓ uses product rule
✓ correct derivatives
✓ simplifies

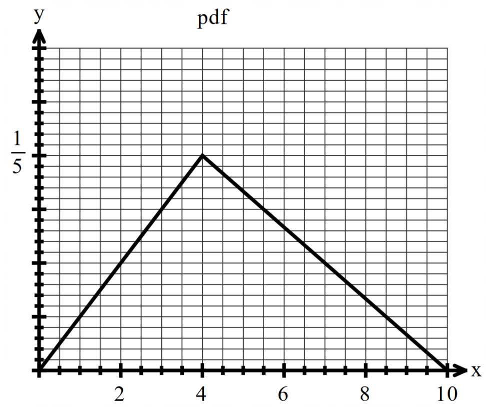
b) Use the result from part a to determine $\int 2x\ln(x^3+x)\,dx$.

c

$\int \frac{d}{dx}\left[(x^2+3)\ln(x^3+3x)\right]dx = \int (3x + \frac{3}{x})dx + \int 2x\ln(x^3+3x)dx$
$(x^2+3)\ln(x^3+3x) = \dfrac{3x^2}{2} + 3\ln x + \int 2x\ln(x^3+3x)dx$
$\int 2x\ln(x^3+3x)dx = (x^2+3)\ln(x^3+3x) - \dfrac{3x^2}{2} - 3\ln x + c$
Specific behaviours
✓ shows the integration of all terms in derivative statement from part a
✓ uses FTC
✓ uses natural log in integration of one term
✓ integrates all required terms
✓ adds a constant at end

Q6 (3, 3, 3 & 3 = 12 marks)

Consider the continuous random variable X and its probability density function which is graphed below.



a) Determine the following exactly.

i) $P(2 < X < 7)$.(Simplify)

c
$\int_2^4 \dfrac{x}{20}dx + \int_4^7 \dfrac{-x}{30} + \dfrac{1}{3}dx$
$\left[\dfrac{x^2}{40}\right]_2^4 + \left[\dfrac{-x^2}{60} + \dfrac{20x}{60}\right]_4^7$
$\dfrac{12}{40} + \dfrac{27}{60} = \dfrac{3}{10} + \dfrac{9}{20} = \dfrac{3}{4}$