

it to the supervisor **before** reading any further.
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material with you, and

© 2018 WA Exam Papers. Kennedy Baptist College has a non-exclusive licence to copy and communicate this document for non-commercial educational use within the school. No other copying, communication or use is permitted without the express written permission of WA Exam Papers. NS245-125-2.

Important note to candidates

Special items: nil

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate
This Question/Answer booklet

Materials required/recommended for this section

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Your name _____
In words _____

Student number: In figures _____
Calculator-free
Section One:
UNITS 3 AND 4
METHODS
MATHEMATICS
Question/Answer booklet

Semester Two Examination, 2018
SOLUTIONS



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Supplementary page

Question number: _____

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Question 2

- (a) Simplify
- $\log_2(16) \div \log_5(125^2)$
- .

Solution
$\frac{\log_2 2^4}{\log_5 5^6} = \frac{4}{6} = \frac{2}{3}$
Specific behaviours

- ✓ expresses as powers of log bases
- ✗ simplifies

- (b) Solve the equation
- $\ln(4-x) + \ln 2 = 2 \ln x$
- .

(3 marks)

Solution
$\ln(8-2x) = \ln x^2$
$x^2 + 2x - 8 = 0 \quad (x+4)(x-2)=0$
$x = -4 \text{ or } x = 2$
But from equation, $0 < x < 4$ $\therefore x = 2$
Specific behaviours

- ✓ writes both sides as single logs
- ✗ factorises quadratic
- ✗ identifies just one solution

Question 7

(6 marks)

The time, t years, to repay a loan of \$57000 at 8.4% interest with monthly repayments of x dollars can be approximated by

$$t = 12 \ln \left(\frac{x}{x-400} \right), x > 400$$

- (a) Determine the time to repay the loan when the monthly repayment is \$600, simplifying your answer. (1 mark)

Solution
$t = 12 \ln \left(\frac{600}{200} \right) = 12 \ln 3 \text{ years}$
Specific behaviours

- ✓ substitutes and simplifies

- (b) Use the increments formula to estimate the time saved in repaying the loan if the monthly repayment of \$600 is increased by 5%. (5 marks)

Solution
$t = 12 \ln x - 12 \ln(x-400)$
$\frac{dt}{dx} = \frac{12}{x} - \frac{12}{x-400}$
$\left. \frac{dt}{dx} \right _{x=600} = \frac{12}{600} - \frac{12}{200} = \frac{-24}{600}$
$\delta x = 600 \times 0.05$
$\delta t \approx -\frac{24}{600} \times 600 \times 0.05 \approx -1.2$
Time saved is 1.2 years
Specific behaviours

- ✗ uses log laws to simplify t
- ✗ correct derivative
- ✗ evaluates derivative
- ✓ indicates value of δx

Question 4

The random variable X has probability density function

$$f(x) = \begin{cases} k\left(\frac{x}{4}-1\right)^3, & 4 \leq x \leq 12 \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Determine the value of the constant k .

Solution
$k \int_4^{12} \left(\frac{x}{4}-1\right)^3 dx = 1$
$\int_4^{12} \left(\frac{x}{4}-1\right)^3 dx = \left[\left(\frac{x}{4}-1\right)^4\right]_4^{12}$
$\cancel{2^4} - \cancel{0^4} = 16$
$16k = 1 \Rightarrow k = \frac{1}{16}$
Specific behaviours
✓ writes integral with correct limits ■ integrates correctly ■ equates integral to 1 ■ correct value of

(4 marks)

- (b) Write down the cumulative distribution function $F(t) = P(X \leq t)$ for $4 \leq t \leq 12$ and hence determine $P(X \leq 8)$.

(2 marks)

Solution
$F(t) = \frac{1}{16} \int_4^t \left(\frac{x}{4}-1\right)^3 dx = \frac{1}{16} \left(\frac{t}{4}-1\right)^4$
$F(8) = \frac{1}{16}$
Specific behaviours
✓ correct $F(t)$ ■ correct probability

SN245-125-2

Question 5

- (a) Determine the anti-derivative of $\frac{\cos(3x)}{5+\sin(3x)}$.

(6 marks)

(2 marks)

Solution
$f(x) = \frac{1}{3} \int \frac{3\cos 3x}{5+\sin 3x} dx$
$\cancel{1} \ln(5+\sin 3x) + C$
Specific behaviours
✓ writes in form $f'(x) \div f(x)$ ■ correct integral and constant

- (b) Determine $f'(x)$ when $f(x) = 2x \ln(5x)$.

(2 marks)

Solution
$f'(x) = 2 \times \ln(5x) + 2x \times \frac{5}{5x}$
$\cancel{2} \ln(5x) + 2$
Specific behaviours
✓ uses product rule correctly ■ differentiates log term correctly

- (b) Write down the cumulative distribution function $F(t) = P(X \leq t)$ for $4 \leq t \leq 12$ and hence determine $P(X \leq 8)$.

(2 marks)

- (c) Evaluate $\int_{0.2}^1 (2 \ln(5x) + 2) dx$.

(2 marks)

Solution
$[2x \ln(5x)]_{0.2}^1$
$\cancel{2} \ln 5 - 0.4 \ln 1 \cancel{2} \ln 5$
Specific behaviours
✓ antiderivative ■ evaluates correctly

See next page

SN245-125-2

See next page