

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name

Teacher's name

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

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See next page

(4 marks)

Question 1
Differentiate the following with respect to x : (Do not simplify your answers)

(a) $y = 5\sin^2(3x)$ (2 marks)

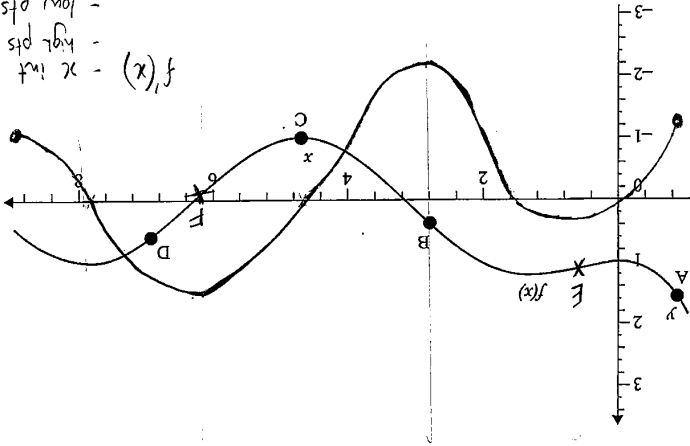
(b) $y = 3x(8x^4 - 2x)^5$ (2 marks)

(10 marks)

CALCULATOR-ASSUMED

Question 9

The graph of a function $f(x)$ is given on the interval $-1 \leq x \leq 9$.



$f'(x)$ - 1st inf ✓
- high pts ✓
- low pts ✓
- general shape ✓

- (a) A, B, C, D are four points on the graph of $f(x)$. Determine whether the first and second derivatives are positive, negative or equal to zero at these points. Record your findings in the table below. (4 marks)

| Point | $f'(x)$ | $f''(x)$ |
|-------|---------|----------|
| A | NEG | POS |
| B | NEG | ZERO |
| C | ZERO | POS |
| D | POS | NEG |

- (b) Indicate on the graph of f above the other two inflection points and label them E and F. (2 marks)
✓ ✓ (or graph)

- (c) Sketch the graph of $f'(x)$ on the same axes of the graph of $f(x)$ above. (4 marks)
(see graph)

End of questions

See next page

Question 2

(3 marks)

Given the function $y = -\frac{1}{\sqrt{4x+3}}$ then $\frac{dy}{dx} = \frac{m}{(4x+3)^n}$

(a) State the values of m and n .

(2 marks)

(b) Determine the instantaneous rate of change of y when $x = 1.5$.

(1 mark)

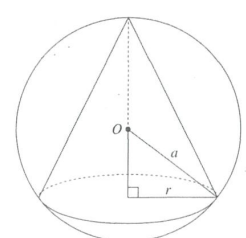
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Question 8

(6 marks)

A cone is inscribed in a sphere of radius a , centred at O . The height of the cone is x and the radius of the base is r , as shown in the diagram.

(a) Show that the volume, V , of the cone is given by $V = \frac{1}{3}\pi(2ax^2 - x^3)$ (2 marks)



$V = \frac{1}{3}\pi r^2 h \Rightarrow r=r$
 $h=x$
 $V = \frac{1}{3}\pi r^2(x)$
 $V = \frac{1}{3}\pi(2ax - x^2)x$
 $V = \frac{1}{3}\pi(2ax^2 - x^3)$

$a^2 = r^2 + (x-a)^2$
 $r^2 = a^2 - (x-a)^2$
 $r^2 = a^2 - x^2 + 2ax - a^2$
 $r^2 = 2ax - x^2$

As Required (Finds r^2)

(b) Find the value of x for which the volume of the cone is a maximum. Verify that your value of x gives the maximum value. (4 marks)

$$\text{Max } \frac{dV}{dx} = 0 \Rightarrow \frac{-(3x^2 - 4ax)\pi}{3} = 0 \quad \checkmark \text{ (Deriv=0)}$$

Solve $3x^2 - 4ax = 0$
 $x = 0, x = \frac{4a}{3} \quad \checkmark (x = \frac{4a}{3})$
 reject $x=0$ as $x = \text{height}$

$$\left. \frac{d^2V}{dx^2} \right|_{x=\frac{4a}{3}} = \frac{-\pi(6(\frac{4a}{3}) - 4a)}{3} \quad \checkmark \text{ (2nd Deriv check)}$$

$$= \frac{-4a\pi}{3} < 0 \quad \therefore \text{Max} \quad \checkmark \text{ (less than 0)}$$

See next page when $x = \frac{4a}{3}$

Question 7

(5 marks)

The function h is defined so that $h(0) = 2$ and $h'(0) = 1$.

- (a) If $f(x) = x \times h(x)$, determine $f'(0)$.

$$f'(x) = 1 \times h(x) + x \cdot h'(x)$$

$$f'(0) = 1 \times h(0) + 0 \times h'(0)$$

$$f'(0) = 2 + 0 \times 1$$

$$f'(0) = 2$$

✓ (Sub in and evaluates)

(2 marks)

(5 marks)

$$u'v - uv'$$

(3 marks)

✓ (Quotient Rule or other for $\frac{1}{h(x)}$ deriv.)

$$g'(x) = h(x) - \left[\frac{h(x)}{h'(x)} \right]^2$$

$$g'(0) = h(0) - \left[\frac{h(0)}{h'(0)} \right]^2$$

$$g'(0) = 1 - \left[\frac{-1}{2} \right]^2$$

$$g'(0) = 1 + \frac{1}{4}$$

$$g'(0) = \frac{5}{4}$$

✓ (correct answer)

See next page

5

Question 3

(4 marks)

A spherical balloon is leaking gas. Use the incremental formula to estimate the percentage change in the radius if its volume decreases from 800 cm^3 to 788 cm^3 .

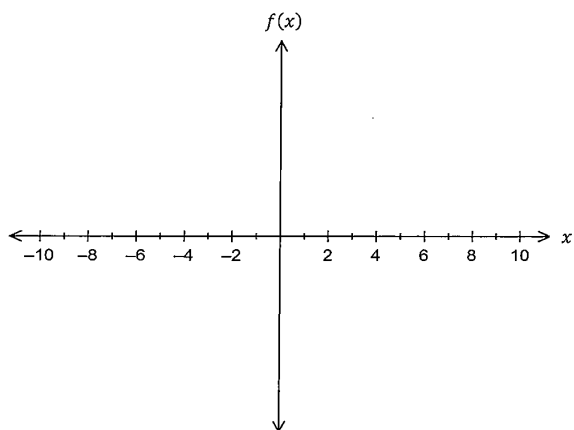
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Question 4

(4 marks)

Sketch a function $y = f(x)$ with all of the following features. Label the critical features clearly.

- $f(-2) = f(6) = f(8) = 0$
- $f'(1) = f'(7) = 0$
- $f''(4) = 0$
- $f''(x) < 0$ for $x < 4$ only



End of questions

Question 6

(3 marks)

If $f(x) = (1 - x^2)^{\frac{3}{2}}$;

(a) determine $f''(x)$.

(1 mark)

Class Pad

$$f''(x) = \frac{6x^2 - 3}{\sqrt{-x^2 + 1}}$$

OR

$$\frac{3(2x^2 - 1)}{\sqrt{-x^2 + 1}}$$

(b) determine the domain of $f''(x)$.

(2 marks)

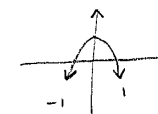
$$\sqrt{-x^2 + 1} > 0$$

$$\therefore 1 - x^2 > 0$$

$$-1 < x < 1$$

✓
lower

✓
upper



See next page

Question 5

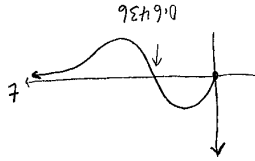
A particle is moving along a straight line that runs in an east-west direction. Its position

function $s(t)$ at time t is given by $s(t) = \frac{t^4+1}{t^2+1}$

(a) Determine the velocity function of the particle. (1 mark)

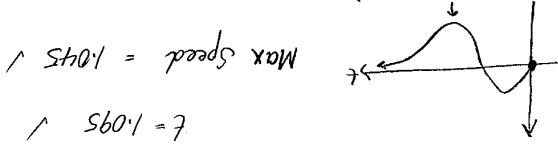
$$v(t) = \frac{-2t^5 - 4t^3 + 2t}{(t^2+1)^2}$$

(b) The particle is moving in an easterly direction when the velocity is positive. Use the graph of the velocity function on your calculator to determine when the particle is moving in a westerly direction. (2 marks)



$$t > 0.6436 //$$

(c) Use the graph of the velocity function to determine the maximum speed of the particle and when it is attained. (2 marks)



$$t = 1.095 //$$

(d) Calculate the position of the particle at the time when the maximum speed is attained. (1 mark)

$$s(1.095) = 0.902 //$$

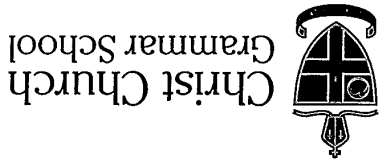
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9

MATHEMATICS METHODS Year 12

Section Two:

Calculator-assumed



Christ Church
Grammar School

2020
TEST 1

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Working time for this section: 30 minutes
Marks available: 30 marks

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This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
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Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations.

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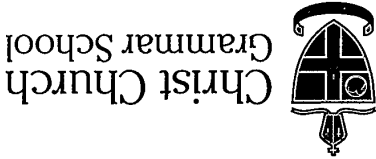
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See next page



2020
TEST 1

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

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Teacher's name _____

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MATHEMATICS METHODS Year 123CALCULATOR-ASSUMED

Question 5(6 marks)

A particle is moving along a straight line that runs in an east-west direction. Its position function $s(t)$ at time t is given by $s(t) = \frac{t^2+1}{t^4+1}$

(a) Determine the velocity function of the particle. (1 mark)

(b) The particle is moving in an easterly direction when the velocity is positive. Use the graph of the velocity function on your calculator to determine when the particle is moving in a westerly direction. (2 marks)

(c) Use the graph of the velocity function to determine the maximum speed of the particle and when it is attained. (2 marks)

(d) Calculate the position of the particle at the time when the maximum speed is attained. (1 mark)

See next page

Question 6

(3 marks)

If $f(x) = (1 - x^2)^{\frac{3}{2}}$, then determine,

(a) $f''(x)$.

(1 mark)

(b) the domain of $f''(x)$.

(2 marks)

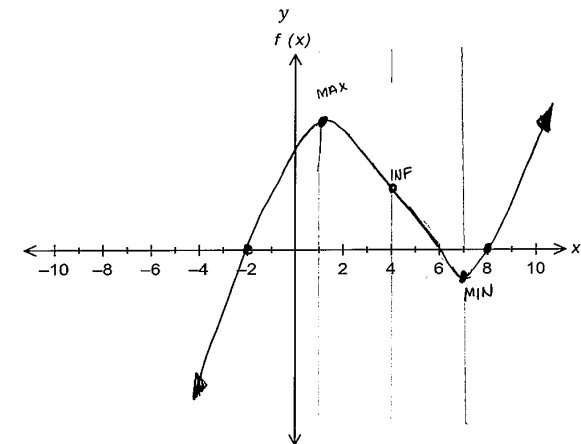
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Question 4

(4 marks)

Sketch a function $y = f(x)$ with all of the following features. Label the critical features clearly.

- $f(-2) = f(6) = f(8) = 0$
- $f'(1) = f'(7) = 0$
- $f''(4) = 0$
- $f''(x) < 0$ for $x < 4$ only



- ✓ (Roots)
- ✓ (Inflection)
- ✓ (Max)
- ✓ (Min)

End of questions

Question 3

A spherical balloon is leaking gas. Use the incremental formula to estimate the percentage change in the radius if its volume decreases from 800cm^3 to 788cm^3 .

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$\therefore \frac{dv}{dr} = 4\pi r^2$$

$$\left(\text{uses } \frac{dv}{dr} \times \delta r \right) \checkmark$$

$$\frac{\delta v}{\delta r} \approx \frac{dv}{dr} \times \delta r$$

$$\left(\text{Sub for } \delta v, \delta r \right) \checkmark$$

$$\frac{\delta v}{\delta r} \approx \frac{800}{\frac{1}{3}}$$

$$\therefore \frac{\delta r}{\delta v} \approx \frac{-12}{2400}$$

$$\frac{\delta r}{\delta v} \approx -\frac{1}{200}$$

$$\frac{\delta r}{\delta v} \approx -0.005$$

$\approx 0.5\%$ reduction

(4 marks)

See next page

4

$$\boxed{\text{Also } \delta v = -12 \text{ cm}^3}$$

$$\frac{\delta v}{\delta r} \approx \frac{dv}{dr}$$

$$\delta v \approx \frac{dv}{dr} \times \delta r$$

$$\delta r \approx \frac{\delta v}{\frac{dv}{dr}}$$

\Rightarrow Isolate $\frac{\delta r}{\delta v}$

$$\delta v \approx 4\pi r^2 \delta r$$

$$\therefore \frac{\delta r}{\delta v} \approx \frac{1}{4\pi r^2}$$

Question 7

The function h is defined so that $h(0) = 2$ and $h'(0) = 1$.

(a) If $f(x) = x \times h(x)$, then determine $f'(0)$.

(2 marks)

(b) If $g(x) = h(x) - \frac{1}{x}$, then determine $g'(0)$.

(3 marks)

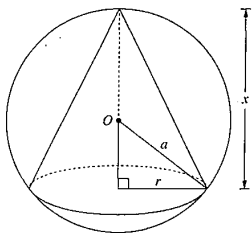
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Question 8

(6 marks)

A cone is inscribed in a sphere of radius a , centred at O . The height of the cone is x and the radius of the base is r , as shown in the diagram.

- (a) Show that the volume, V , of the cone is given by $V = \frac{1}{3}\pi(2ax^2 - x^3)$ (2 marks)



- (b) Find the value of x for which the volume of the cone is a maximum. Verify that your value of x gives the maximum value. (4 marks)

See next page

Question 2

(3 marks)

Given the function $y = -\frac{1}{\sqrt{4x+3}}$ then $\frac{dy}{dx} = \frac{m}{(4x+3)^n}$

- (a) State the values of m and n . (2 marks)

$$y = -(4x+3)^{-1/2} \quad \therefore \frac{dy}{dx} = \frac{1}{2}(4x+3)^{-3/2} \times (4)$$

$$\frac{dy}{dx} = \frac{2}{(4x+3)^{3/2}}$$

$$\therefore m = 2 \quad \checkmark \quad (m \text{ value})$$

$$n = \frac{3}{2} \quad \checkmark \quad (n \text{ value})$$

- (b) Determine the instantaneous rate of change of y when $x = 1.5$. (1 mark)

$$\left. \frac{dy}{dx} \right|_{x=1.5} = \frac{2}{(4 \times 1.5 + 3)^{3/2}}$$

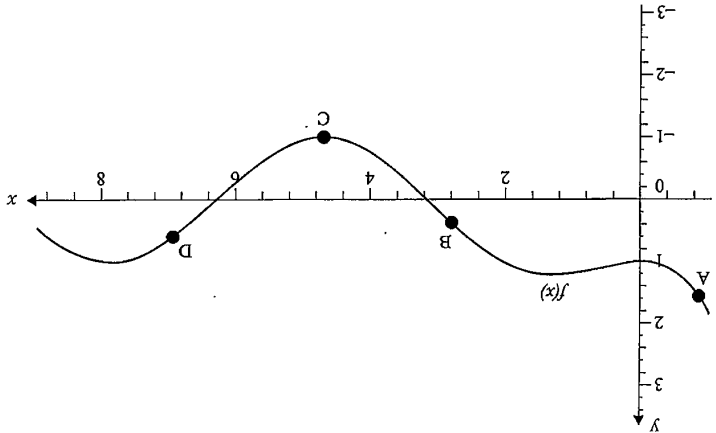
$$= \frac{2}{9^{3/2}}$$

$$= \frac{2}{27} \quad \checkmark \quad (\text{value})$$

See next page

Question 9 (10 marks)

The graph of a function $f(x)$ is given on the interval $-1 \leq x \leq 9$.



- (a) A, B, C, D are four points on the graph of $f(x)$. Determine whether the first and second derivatives are positive, negative or equal to zero at these points. Record your findings in the table below. (4 marks)

| Point | $f'(x)$ | $f''(x)$ |
|-------|---------|----------|
| A | | |
| B | | |
| C | | |
| D | | |

- (b) Indicate on the graph of f above the other two inflection points and label them E and F. (2 marks)

- (c) Sketch the graph of $f'(x)$ on the same axes of the graph of $f(x)$ above. (4 marks)

End of questions

Question 1 (4 marks)

Differentiate the following with respect to x : (Do not simplify your answers)

- (a) $y = 5\sin^2(3x)$ (2 marks)

$$\frac{dy}{dx} = 5(2) \sin(3x) \cos(3x) \times 3$$

$$\frac{dy}{dx} = 30 \sin(3x) \cos(3x)$$

(b) $y = 3x(8x^4 - 2x)^5$ (2 marks)

$$\frac{dy}{dx} = 3(8x^4 - 2x)^5 + 3x(5)(8x^4 - 2x)^4(32x^3 - 2)$$

take 1mk off for each error until zero

take 1mk off for each error until zero

See next page

4



Christ Church
Grammar School

2020
TEST 1

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name → Solutions ←

Teacher's name _____

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MATHEMATICS METHODS Year 12

2

CALCULATOR-FREE

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