11 Investigation 2 Methods: Out 14 June In class: 21 June

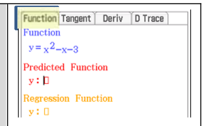
For the in class, you are allowed to bring this take home in with you plus any other notes. You will need your classpad for the in class as well.

Aim: Explore the gradient of a chord or secant and generalise. Visually see the gradient of the tangent as the limiting value as the two points get closer together.

1. Explore the gradient of secant. Go to menu and open **InteractiveDiffCalc**

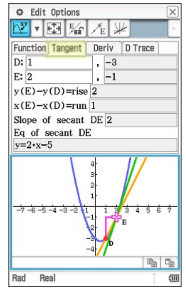
Define function

- Open InteractiveDiffCalc
- Tap Function tab
- Enter $y = x^2 - x - 3$



Gather data on the secant (line DE)

- Tap Tangent tab
- Controls
- Left right arrows
- Press to fix value of x_D or x_E
- Press to draw/hide tangent
- Press to switch the fixed point



a) Complete the table of values

D	E	Rise DE	Run DE	Gradient DE	Equation DE
(1 ,-3)	(2 , - 1)	2	1	2	$y = 2x - 5$
(1.5 ,)	(2 , - 1)				
(2.5 ,)	(2 , - 1)				
(1.9 ,)	(2 , - 1)				
(1.99 ,)	(2 , - 1)				

b) As D moves closer to point E, what happens to the:

(i) two lines

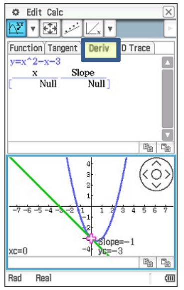
(ii) the gradient of the secant and the gradient of the tangent?

c) For the points $(x+h, f(x+h))$ and $(x, f(x))$ write an expression for the gradient of the secant and then simplify. (Hint: use M)

2. Explore the gradient of the tangent to a curve

Explore the tangent using Deriv tab

- Tap the Deriv tab
- Press the arrow keys to move the cursor
- Press E or the centre of the on-screen wheel to plot a point
  Or press a number key to type an x-value



a) What does the y-value of the plotted points represent?

b) Complete the table.

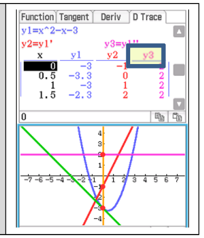
x	Gradient of tangent
0	
1	
2	
3.3	
–1.2	

c) Predict a function that describes the relationship between the gradient and x-value.

d) Check your prediction.
Tap to enter your prediction for the gradient function. ClassPad will then plot your prediction. (Note: you must have plotted the gradient in at least 4 positions)

3. Explore the D Trace tab

- Tap D Trace
- Move the left and right arrow keys.
- Tap to turn on/off different parts of the display.



Refer to the graph in the screenshot above. What does the graph of

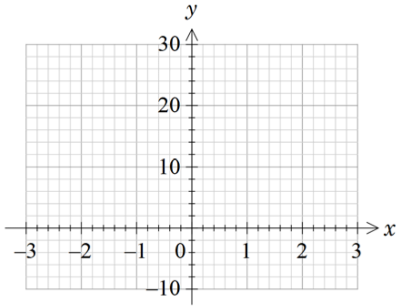
a) y2 (the redline) represent?

b) y3 (the pink line) represent?

4. For each of the following functions:

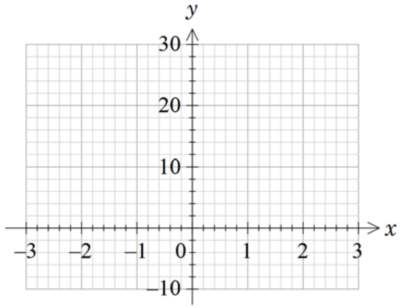
- sketch the graph, the tangent at the specified point and the gradient function on the grid; and
- state the equation of the gradient function, y'. that is ($\frac{dy}{dx}$)

a) $y = x^4$ with tangent at $x = 2$



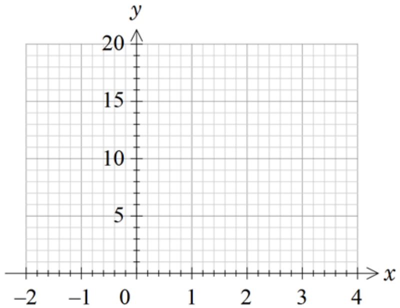
Prediction for y':

b) $y = x^4 + 10$ with tangent at $x = 2$



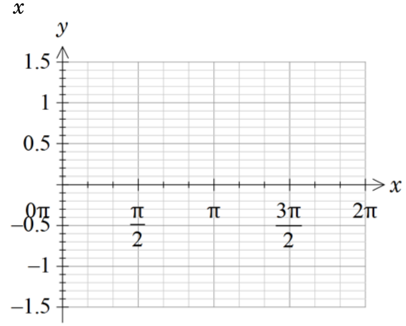
Prediction for y':

c) $y = 2^x$ with tangent at $x = 3$



Prediction for y':

d) $y = \sin x$ with tangent at $x = \frac{\pi}{3}$



Prediction for y':

5. The screenshot reproduces the steps in this activity done algebraically.

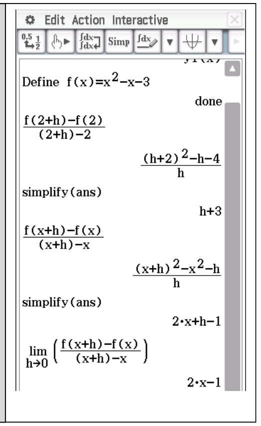
1. Define the function
2. Write and expression for the gradient of a secant between $(2, f(2))$ and $(2+h, f(2+h))$

   and simplify

   As $h \to 0$ the secant approaches the tangent at $x = 2$. (In this case the gradient of the tangent will be 3)
3. Write and expression for the gradient of a secant between $(x, f(x))$ and $(x+h, f(x+h))$

   and simplify

   As $h \to 0$ the secant approaches the tangent at any point



a) Reproduce the screenshot as shown.

b) Edit the function definition to complete the table.

Function	Gradient of tangent at $x = 2$	Gradient of secant between $(x, f(x))$ and $(x+h, f(x+h))$	Gradient function
$x^2 - x - 3$	3	$2x + h - 1$	$2x - 1$
x^3			
x^4			
$5x^3$			
$\sqrt{x}$ or $x^{\frac{1}{2}}$			
$\sin x$			
$\cos x$			
2^x			

6. EXTENSION

Predict the gradient functions for

a) $y = x^n$

b) $y = ax^n$

c) $y = x^n + x^m$

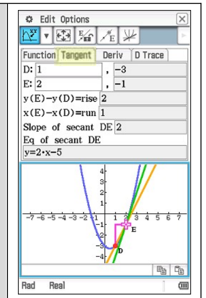
Learning notes

The strength in the InteractiveDiffcalc app is being able to follow the first principles approach visually. By dynamically moving the points closer together students can see the secant getting closer to the tangent.

It also supports the development of the idea of the gradient function and taking time to understand the plotting of the gradient at specific points is a neat way of encouraging students to develop this concept.

Tangent tab

- Controls
- Left right arrows
- Press to fix value of x_D or x_E
- Press to draw/hide tangent
- Press to switch the fixed point

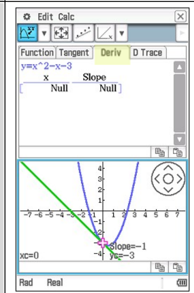


Deriv tab

- Press the arrow keys to move the cursor
- Press E or the centre of the on-screen wheel to plot a point

  Or press a number key to type an x-value
- Press to calculate a regression

  see pull down menu for types of regression
- Press to enter prediction for the gradient function



D Trace tab

- Move the left and right arrow keys.
- Tap to cycle through different displays