



WESLEY COLLEGE

By doing & by doing

Name: Solutions

Calculus, trigonometry and DRV's

Test 3

YEAR 12 MATHEMATICS METHODS

Marks: /45

Time allowed: 50 mins

Calculator Free (20 marks)

1. [2 marks]

Determine if each of the $p(x)$ as described are discrete probability functions. Justify your answer in either case.

a)

x	0	1	2	5
$P(X=x)$	-0.1	0.1	0.4	0.6

No, negative probability of -0.1 i.e. $P(0) = -0.1$

[1]

b)

x	-3	-2	1	4
$P(X=x)$	0.1	0.3	0.2	0.4

Yes, probabilities sum to 1.

[1]

2. [3 marks]

Given a binomial variable has a mean of 12 and a standard deviation of $\sqrt{8}$, find p , the probability of success and n , the number of trials.

$$np = 12 \text{ and } np(1-p) = 8$$

$$12(1-p) = 8 \quad (1)$$

$$1-p = \frac{2}{3}$$

$$\therefore p = \frac{1}{3} \quad (1)$$

$$\therefore n = \frac{12}{\frac{1}{3}} = 36 \quad (1)$$

1.

5

3. [10 marks]

Determine:

$$\begin{aligned} \text{a) } \frac{d}{dx} \cos^5(3x) &= 5 \cos^4(3x) (-\sin(3x) \cdot 3) \\ &= -15 \sin(3x) \cos^4(3x) \end{aligned}$$

[2]

$$\begin{aligned} \text{b) } \frac{d}{dx} e^{2x+1} \tan(5x) &= 2 e^{2x+1} \tan(5x) + e^{2x+1} \frac{5}{\cos^2(5x)} \\ &= e^{2x+1} \left(2 \tan(5x) + \frac{5}{\cos^2(5x)} \right) \end{aligned}$$

[2]

$$\begin{aligned} \text{c) } \int \frac{\sin(5x)}{4} dx &= \frac{1}{4} \left(-\frac{\cos(5x)}{5} \right) \\ &= -\frac{1}{20} \cos(5x) + C \end{aligned}$$

[2]

$$\text{d) } \int \cos(x) \sin^3(x) dx = \frac{1}{4} \sin^4(x) + C$$

[2]

$$\begin{aligned} \text{e) } \frac{d}{dx} \int_0^{x^3} \cos(3t) dt &= \cos(3x^3) \cdot 3x^2 \\ &= 3x^2 \cos(3x^3) \end{aligned}$$

[2]

2.

10

8. [8 marks]

A soldier fires shots at a target at distances ranging from 25 m to 90 m. The probability of him hitting the target with a single shot is p . When firing from a distance of d m, $p = \frac{3}{200}(90-d)$.

Each shot is fired independently.

The soldier fires 10 shots from a distance of 40 m.

a) Determine the probability that:

(i) Exactly 6 shots hit the target.

$$d = 40 \Rightarrow p = 0.75 \quad (1)$$

$$X \sim B(10, 0.75) \quad (1)$$

$$P(X=6) = 0.146 \quad (1)$$

[3]

(ii) At least 8 shots hit the target.

$$P(X \geq 8) = 0.5256 \quad (1)$$

[2]

The soldier fires 20 shots from a distance of x m.

b) Determine to the nearest integer, the value of x if the soldier has an 80% chance of hitting the target at least once.

$$X \sim B(20, p)$$

$$P(X \geq 1) = 0.8$$

$$\text{i.e. } 1 - P(X=0) = 0.8$$

$$P(X=0) = 0.2$$

$$\text{i.e. } (1-p)^{20} = 0.2 \quad (1)$$

$$\therefore p = 0.0773 \quad (1)$$

$$\text{So } 0.0773 = \frac{3}{200}(90-x)$$

$$\therefore x = 84.85$$

$$\therefore x = 85 \text{ m (nearest int.)}$$

[3]

7.

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7. [3 marks]

Suppose that 5% of all items coming off a production line are defective. Assume the manufacturer packages his items in boxes of six and guarantees "double your money back" if more than two items in a box are defective. On what percentage of the boxes will the manufacturer have to pay double money back?

$$X \sim B(6, 0.05) \quad (1)$$

$$P(X > 2) = 0.0022 \quad (1)$$

$$[\text{BinomialCDF}(3, 6, 0.05)]$$

∴ manufacturer will have to pay back double money 0.22% of the time. ⁽¹⁾

6.

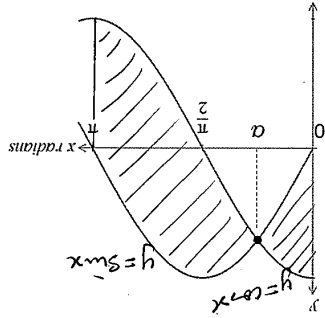
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4. [5 marks]

Determine the area trapped between the functions $y = \sin(x)$, $y = \cos(x)$, $x = 0$ and $x = \pi$.

Hint: First, determine α .

$$\alpha = \frac{\pi}{4}$$



(1)

$$\text{Area} = \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) dx \quad (1)$$

$$= \left[\sin x + \cos x \right]_0^{\pi/4} + \left[-\cos x - \sin x \right]_{\pi/4}^{\pi} \quad (1)$$

$$= \left(\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) \right) - \left(\cos(\pi) + \sin(\pi) \right) - \left(-\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right) \right)$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) - (-1 + 0) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1 - 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \quad (1)$$

$$= \sqrt{2} - 1 + 1 + \sqrt{2}$$

$$= 2\sqrt{2} \text{ sq units.} \quad (1)$$

3.

5



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Calculator Section

(25 marks)

5. [6 marks]

A company produces fruit sweets coated with either dark chocolate or milk chocolate. A large number of these fruit sweets are placed in a box. Twenty percent of the sweets in the box are coated with dark chocolate.

- a) A random sample of ten sweets is taken from the box, explain the meaning of the calculation ${}^{10}C_4 (0.2)^4 (0.8)^6$ with respect to this sample?

The probability of randomly selecting 4 dark ⁽¹⁾
chocolates from a sample of 10 sweets
from the box. (1)
[2]

- b) (i) Find n given that ${}^nC_4 (0.2)^4 (0.8)^n = 0.16777$

ie $0.8^n = 0.16777$ ⁽¹⁾
Solving $n = 8$ (nearest integer) ⁽¹⁾

[2]

- (ii) Explain the meaning of your answer from b) (i) with respect to the fruit sweets.

The probability of getting no dark ⁽¹⁾
chocolates when 8 sweets are chosen. (1)

[2]

4.

6

6. [8 marks]

The random variable X has probability distribution:

x	1	3	5	7	9
$P(X=x)$	0.2	p	0.2	q	0.15

Given that $E(X) = 4.5$, determine:

- a) The value of p and q .

$$E(X) = 4.5 \quad \text{and} \quad 0.55 + p + q = 1$$

$$\downarrow \quad \text{ie} \quad p + q = 0.45 \quad (1)$$

$$\text{and} \quad 1(0.2) + 3p + 5(0.2) + 7q + 9(0.15) = 4.5$$

$$3p + 7q = 1.95 \quad (2)$$

Solve (1) + (2) simultaneously

$$p = 0.3$$

$$q = 0.15$$

- b) $P(4 < x \leq 7) \quad 0.2 + 0.15 = 0.35$

[3]

[1]

Given that $E(X^2) = 27.4$, determine:

$$\text{c) } \text{Var}(X) = \sum(x^2) - E(x)^2$$

$$= 27.4 - 4.5^2$$

$$= 7.15$$

[2]

$$\text{d) } E(19 - 4X) = -4(4.5) + 19$$

$$= 1$$

[1]

$$\text{e) } \text{Var}(19 - 4X) = 4^2 (7.15)$$

$$= 114.4$$

[1]

5.

8