# Rossmoyne Senior High School

## Semester One Examination, 2022

## Question/Answer booklet

# MATHEMATICS METHODS UNIT 3

## Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures | | | | | | | | |

In words _____

_____

**Circle your Teacher's Name:**

| | | |
|---|---|---|
| Mrs Alvaro | Mrs Bestall | Mrs Fraser-Jones |
| Mr Gibbon | Mrs Greenaway | Mr Koulianos |
| Mr Luzuk | Mrs Murray | Mr Tanday |

### Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): ☐

### Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet
Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 55 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 95 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

| Markers use only | | |
|---|---|---|
| Question | Maximum | Mark |
| 1 | 6 | |
| 2 | 7 | |
| 3 | 9 | |
| 4 | 10 | |
| 5 | 8 | |
| 6 | 9 | |
| 7 | 6 | |
| S1 Total | 55 | |
| S1 Wt (×0.6731) | 35% | |
| S2 Wt | 65% | |
| Total | 100% | |

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

**Section One: Calculator-free** **35% (55 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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**Question 1** **(6 marks)**

(a) Determine $f'(-2)$ when $f(x) = 2(3x + 5)^3$. (3 marks)

(b) Determine $g(2)$ when $g'(x) = 12e^{3x-3}$ and $g(1) = 7$. (3 marks)



**See next page**

**Question 2** **(7 marks)**

Let $f(x) = 15 - 4x - 6x^2 - 4x^3 - x^4$.

(a) The curve $y = f(x)$ cuts the horizontal axis at $x = 1$. State, with reasons, whether the function is increasing, decreasing or neither at this point. (2 marks)

(b) Determine $f''(0)$ and use this value to describe the concavity of the curve $y = f(x)$ where it crosses the vertical axis. (2 marks)

(c) Does the curve $y = f(x)$ have any points of inflection? If it does, determine the coordinates of their location. If not, justify your answer. (3 marks)

**See next page**









Supplementary page

Question number: _____

Supplementary page

Question number: _____







**Question 3** **(9 marks)**

The function $f$ is defined for $x > 0$ by $f(x) = \frac{e^{3x-2}}{x}$, and $f''(x) = \frac{(9x^2 - 6x + 2)e^{3x-2}}{x^3}$.

(a) Determine the coordinates and nature of all stationary points of the graph of $y = f(x)$. Justify your answer. (6 marks)

(b) Show that the graph of $y = f(x)$ has no points of inflection. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF



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**Question 4** **(10 marks)**

The discrete random variable $X$ has a probability function with $\text{Var}(X) = \frac{14}{9}$.

$$P(X = x) = \begin{cases} \frac{x}{k}, & x = 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that $k = 15$. (2 marks)

Determine:

(b) (i) $P(X < 4 \mid X > 1)$ (2 marks)

(ii) $E(X)$ (2 marks)

(c) A second discrete random variable $Y$ is defined to be $Y = aX + b$.
If $E(Y) = 2$ and the standard deviation of $Y$ is $\sqrt{14}$, determine $a$ and $b$. (4 marks)





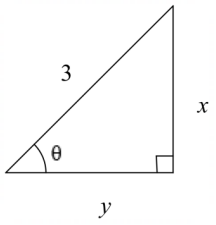




(b) Determine the value of $\theta$ that will maximise the area of the triangle. (4 marks)

**Question 7** **(6 marks)**

Given $\cos(2x) = \cos^2 x - \sin^2 x$ and the diagram below;




(a) Show that the area of the triangle is given by $A(\theta) = \frac{9}{2}\sin\theta\cos\theta$. (2 marks)



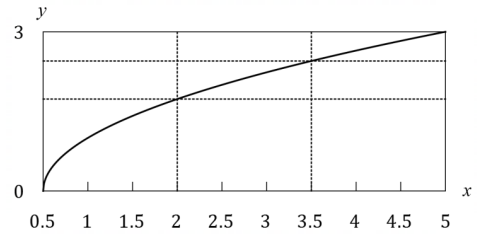




**Question 5** **(8 marks)**

The graph of $y = \sqrt{2x-1}$ between $x = 0.5$ and $x = 5$ is shown at right.

Approximate values for $\sqrt{3}$ and $\sqrt{6}$ are 1.73 and 2.45 respectively.




(a) Use the areas of the rectangles shown to explain why $6.27 < \int_{0.5}^{5} \sqrt{2x-1}\,dx < 10.77$. (3 marks)

(b) Evaluate $\int_{0.5}^{5} \sqrt{2x-1}\,dx$. (3 marks)

(c) Evaluate $\int_{0.5}^{5} \left(\sqrt{2x-1} - 3\right) dx$. (2 marks)

**Question 6** **(9 marks)**

Let $f(x) = e^{-3x}(\cos 3x + \sin 3x)$.

(a) Determine $f'(x)$, simplifying your answer. (3 marks)

(b) Hence, show that

$$\int \left(e^{-3x} \sin 3x\right) dx = -\frac{1}{6} e^{-3x}(\cos 3x + \sin 3x) + c,$$

where $c$ is a constant. (3 marks)









(c) The graph of $y = e^{-3x} \sin 3x$ is shown below. Determine the area of the region $R$, bounded by the curve and the $x$-axis. (3 marks)