PERTH MODERN SCHOOL
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Semester 1 Examination 2012

Question/Answer Booklet

MATHEMATICS 3CD

Section One:
Calculator-free

Name of Student: ____________________Marking key______________________

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for this section: 50 minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the student
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items: nil

Important note to students
No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	12	12	100	100	
			Total	150	100

Instructions to students

1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2 **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

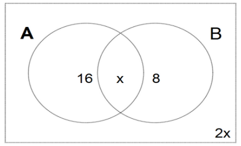
Section One: Calculator-free **(50 marks)**

This section has **six (6)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 50 minutes

Question 1 **(8 marks)**

(a) Given the following Venn Diagram showing events A and B



Determine x if

(i) A and B are mutually exclusive (1)

(ii) A and B are independent. (The condition from (i) does not necessarily hold)

(3)

Solution
(i) A and B are mutually exclusive if $A \cap B = 0$ So x = 0

(c) (ii) $\int_{-3}^{2}(4f(x)+3)\,dx$ (2)

Solution
$=\int_{-3}^{2}4f(x)dx+\int_{-3}^{2}3\,dx$ $=4(-12)+[3x]_{-3}^{2}$ $=(-48)+[6-(-9)]$ $=(-48)+15$ $=-33$
Specific behaviours
✓✓

Specific behaviours
✓ expands $(1+3x^2)^3$ correctly ✓ integrates each term correctly

(b) Determine $\int 3x^3(2x^4-5)^8\,dx$ (2)

Solution
$=\frac{3}{8}\int 8x^3(2x^4-5)^8\,dx$ $=\frac{3}{8}\left[\frac{(2x^4-5)^9}{9}\right]+c$ $=\left[\frac{(2x^4-5)^9}{24}\right]+c$
Specific behaviours
✓ integrates to obtain $k(2x^4-5)^9$ ✓ simplify k to $\frac{1}{24}$

(c) $f(x)$ is defined such that $\int_{-3}^{6} f(x)\,dx = 24$ and $\int_{2}^{6} f(x)\,dx = 36$

Find

(i) $\int_{-3}^{2} f(x)dx$. (2)

Solution
$\int_{-3}^{2} f(x)dx = \int_{-3}^{6} f(x)dx - \int_{2}^{6} f(x)dx$ $\int_{-3}^{2} f(x)dx = 24 - 36 = -12$
Specific behaviours
✓ ✓ or X

Question 6 (continued)

$P(A \mid B) = P(A)$ $\therefore \frac{x}{x+8} = \frac{16+x}{24+3x}$ $\frac{3x}{3x+24} = \frac{16+x}{24+3x}$ $3x = 16 + x$ $x = 8$ (ii)
Specific behaviours
✓ factorises denominator ✓ critical values of -3,-1,1 ✓✓ correct intervals

(b) The functions $f(x)$ and $g(x)$ are defined as follows

$f(x) = x^2 - 4$ and $g(x) = \sqrt{x-5}$

(i) Determine expressions for $f\left[g(x)\right]$ and $g\left[f(x)\right]$. (2)

Solution
$f\left[g(x)\right] = x - 9$ $g\left[f(x)\right] = \sqrt{x^2-9}$
Specific behaviours
✓✓ I mark for each

(ii) Determine the range of $f\left[g(x)\right]$. (1)

Solution
$R_{f(g(x))} = \{y : y \geq -4, y \in \mathbb{R}\}$
Specific behaviours
✓ or X

(iii) Determine the domain of $g\left[f(x)\right]$. (1)

Solution
$D_{g(f(x))} = \{x : x \geq 3 \text{ or } x \leq -3, x \in \mathbb{R}\}$
Specific behaviours
✓ or X

Question 5 (continued)

(b) Events *A* and *B* are such $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\overline{A \cup B}) = \frac{1}{4}$

(i) Show that event *A* and *B* are **NOT** mutually exclusive. (3)

Solution	
$P(A \cup B) = 1 - \frac{1}{4} = \frac{3}{4}$	$P(A \cap B) = \frac{1}{2} + \frac{7}{12} - \frac{3}{4} = \frac{1}{3}$
OR	
$P(A) + P(B) = \frac{1}{2} + \frac{7}{12} = \frac{13}{12}$	$P(A \cap B) \neq 0$
As $P(A \cup B) \neq P(A) + P(B)$	
∴ A and B are NOT mutually exclusive	∴ A and B are NOT mutually exclusive
Specific behaviours	
✓ $P(A \cup B)$ value ✓ $P(A) + P(B)$ value ✓ as they are not equal, concludes A and B are not M.E.	

(ii) Hence find $P(A \cap B)$. (2)

Solution
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\frac{3}{4} = \frac{1}{2} + \frac{7}{12} - P(A \cap B)$ $P(A \cap B) = \frac{4}{12} = \frac{1}{3}$
Specific behaviours
✓ uses $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ✓ correct value for $P(A \cap B)$

Question 6 **(8 marks)**

(a) Determine $\int (1+3x^2)^3 dx$ (2)

Solution
$(1+3x^2)^3 = 1 + 9x^2 + 27x^4 + 27x^6$ $\int 1 + 9x^2 + 27x^4 + 27x^6 \, dx = x + 3x^3 + \frac{27}{5}x^5 + \frac{27}{7}x^7 + c$

Question 5 **(10 marks)**

(a) A spherical balloon is being deflated in such a way that the volume is decreasing at a constant rate of 120cm^3/sec. At time t (seconds), the radius of the balloon is r cm.

Find the rate of change of the surface area when the volume is $36\pi^4$ cm^3. (5)

Solution
When $\frac{4\pi r^3}{3} = 36\pi^4$, $r = 3\pi$ $\frac{dA}{dr} = 8\pi r$, $\frac{dV}{dr} = 4\pi r^2$, $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$, $\frac{dr}{dt} = \frac{1}{4\pi r^2} \times \frac{(-120)}{1} = \frac{-30}{\pi r^2}$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$, $\frac{dA}{dt} = 8\pi r \times \frac{-30}{\pi r^2} = \frac{-240}{r} = \frac{-240}{3\pi} = \frac{-80}{\pi}$ cm^2/sec
Specific behaviours
✓ $r = 3\pi$ ✓ $\frac{dV}{dr} = 4\pi r^2$ ✓ $\frac{dA}{dr} = 8\pi r$ ✓ $\frac{dr}{dt} = \frac{-30}{\pi r^2}$ ✓ $\frac{dA}{dt} = \frac{-80}{\pi}$

Question 2 **(9 marks)**

(a) Differentiate the following with respect to x.

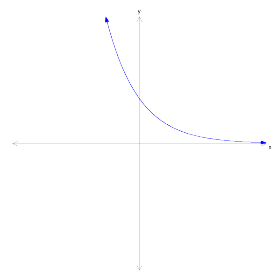
(i) $f(x) = \frac{-x}{x^2+1}$ (express in simplest form) (3)

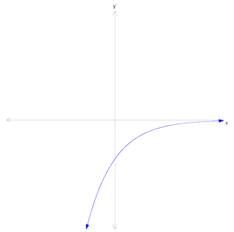
Solution
$f'(x) = \frac{(x^2+1)(-1)-(-x)(2x)}{(x^2+1)^2}$ $f'(x) = \frac{x^2-1}{(x^2+1)^2}$
Specific behaviours
✓✓ uses quotient rule correctly ✓ simplifies to $\frac{x^2-1}{(x^2+1)^2}$

(ii) $g(x) = (x+1)^2 e^{x^2}$ (do not simplify) (2)

Solution
$g'(x) = e^{x^2} \cdot 2(x+1) + (x+1)^2 \cdot e^{x^2} \cdot 2x$
Specific behaviours
✓✓ uses product rule and differentiate correctly

(b) Sketch the graph of the derivative function for on the axes below. (2)





Solution
As shown on graph above
Specific behaviours
✓ shape ✓ all below x-axis

✓ Equates first derivative to zero , factorises and solve for x values ✓ second derivative test for max and min ✓ states $x = \frac{-2}{3}$ when volume is max

Question 4 **(8 marks)**

The volume of a certain rectangular box is given by the equation
$f(x) = x^3 - 5x^2 - 8x + 48$.

(i) If the height of the box is $(4 - x)$ units, determine an algebraic expression for the area of the base of the box. (3)

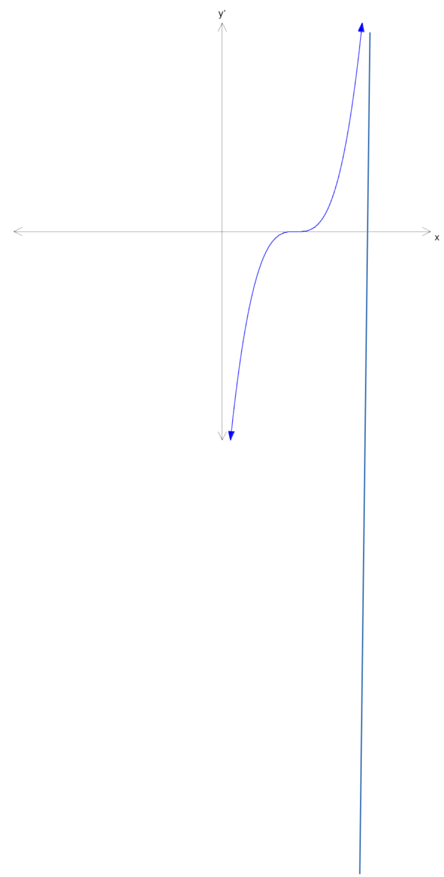
Solution
Area of base = $(x^3 - 5x^2 - 8x + 48) \div (-x + 4) = -x^2 + x + 12$
Specific behaviours
✓✓ uses Long Division ✓ correct answer of $-x^2 + x + 12$

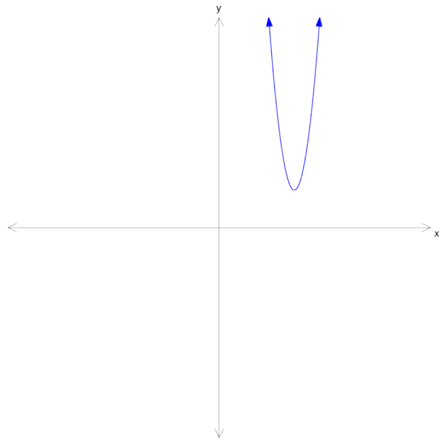
(ii) Calculate the value of *x* for which the volume is a maximum. (5)

Solution
$\frac{dV}{dx} = 3x^2 - 10x - 8$ $\frac{d^2V}{dx^2} = 6x - 10$ $3x^2 - 10x - 8 = 0$ $(3x+2)(x-4) = 0$ $x = \frac{-2}{3}$ or $x = 4$ When $x = 4$, $\frac{d^2V}{dx^2} > 0 \Rightarrow Min$ When $x = \frac{-2}{3}$, $\frac{d^2V}{dx^2} < 0 \Rightarrow Max$ Value of x when volume is a maximum is $x = \frac{-2}{3}$
Specific behaviours
✓ $\frac{dV}{dx} = 3x^2 - 10x - 8$ ✓ $\frac{d^2V}{dx^2} = 6x - 10$

Question 2 (continued)

(c) Given the derivative function, sketch the graph of the function. (2)





Solution
As shown above
Specific behaviours
✓ shape ✓ turning point and within boundary

Question 3 **(7 marks)**

(a) It is claimed that the tangent to the curve $y = x^3 - 2x^2 - 4x + 3$ at the point where *x*=1 passes through the point (3, 8). Is this claim valid? Justify your answer. (5)

Solution
$\frac{dy}{dx} = 3x^2 - 4x - 4$ $\left.\frac{dy}{dx}\right\|_{(1,-2)} = 3 - 4 - 4 = -5$ Equation of tangent at (1,-2) is $y - (-2) = -5(x - 1)$ $y + 2 = -5x + 5$ $y = -5x + 3$ Substitute (3, 8) into equation $8 = -5(3) + 3$ $8 = -12$ X Claim is not valid as the tangent at (1,-2) to the curve does not pass through (3, 8)
Specific behaviours
✓y = -2 when x = 1 ✓ gradient function ✓ gradient at (1,-2) ✓ equation of tangent ✓ substitute (3, 8) and states claim is not valid

(b) Two identical coins are tossed together, and the outcome is recorded. After a large number of trials it is observed that the probability that both coins land showing heads is 0.36.

What is the probability that both coins land showing tails? (2)

Solution
$P(\text{2 heads}) = 0.36 \Rightarrow P(\text{1 head}) = 0.6$ P(1 Tail) = 0.4 P(2Tails)=0.4X0.4 = 0.16
Specific behaviours
✓ probability of 1 tail ✓ correct answer of 0.16