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Semester One Examination, 2023

Question/Answer booklet

12 SPECIALIST MATHEMATICS
UNIT 3
Section Two:
Calculator-assumed

Your Name ______________________________________

Your Teacher's Name________________________________

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
7			15		
8			16		
9			17		
10			18		
11					
12					
13					
14					

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	50	34
Section Two: Calculator-assumed	12	12	100	97	66
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **(97 Marks)**

This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

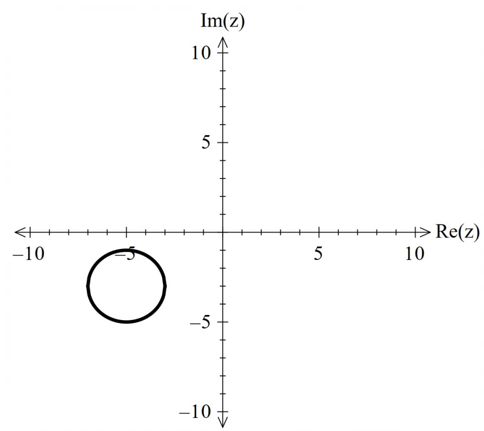
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 7 **(5 marks)**

Consider the locus $|z+5+3i|=2$.

a) Sketch the locus on the Argand Diagram below. (2 marks)

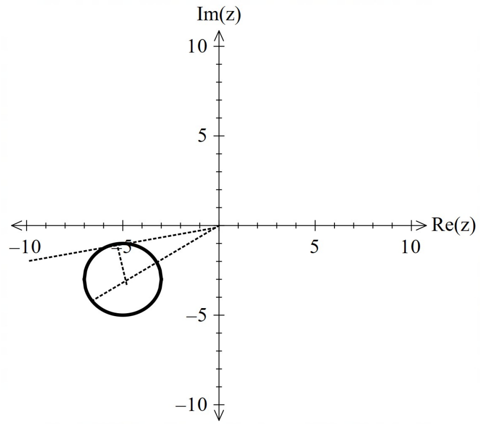
c

Specific behaviours
P centre of circle P radius

See next page

b) Determine the minimum value of: (3 marks)

i) $|z|$

ii) Arg(z)

c

Edit Action Interactive
$\sqrt{5^2+3^2}-2$
3.830951895
$-\sin^{-1}\left(\frac{2}{\sqrt{5^\wedge 2+3^\wedge 2}}\right)-\pi+\tan^{-1}\left(\frac{-3}{-5}\right)$
−2.951278931
$-2.951278931\times180/\pi$
−169.0958269
Specific behaviours
P minimum modulus P argument of centre of circle P minimum argument, radians or degrees

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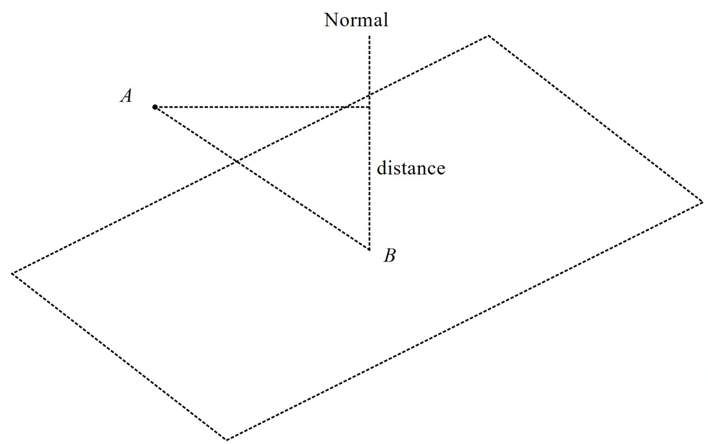
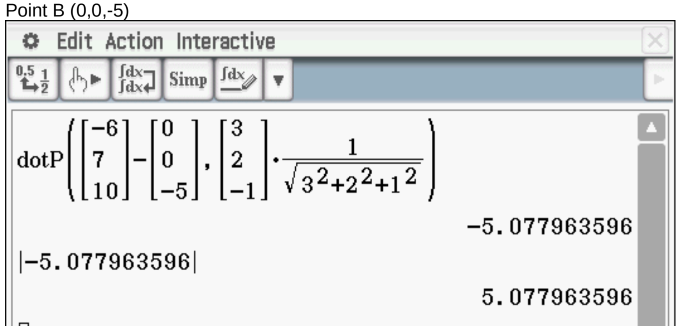
OSBORNE PARK WA 6017

Additional working space

Question number: __________

Question 8 **(4 marks)**

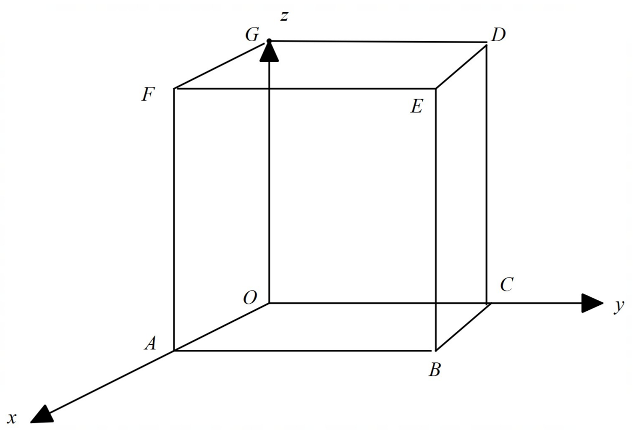
Consider the plane $3x+2y-z=5$ and the point A $(-6,7,10)$.
Determine the distance of point A from the plane.

c
 $d=\left\|AB.\hat{n}\right\|$ Point B (0,0,-5) 
Specific behaviours
P uses normal vector OR line parallel to normal P uses AB OR solves for line meeting plane P uses dot product

P determines approx. distance, no need for units

Question 9 **(9 marks)**

Consider the rectangular box with vertices A(5,0,0), B(5,4,0),C(0,4,0), D(0,4,7), E(5,4,7),F(5,0,7)&G(0,0,7) and the origin.



a) If point H divides the diagonal $\overline{AD}$ in the ratio 3:2, determine the position vector OH.
(2 marks)

$$\overrightarrow{AD} = \begin{pmatrix}0\\4\\7\end{pmatrix} - \begin{pmatrix}5\\0\\0\end{pmatrix} = \begin{pmatrix}-5\\4\\7\end{pmatrix}$$

$$\overrightarrow{OH} = \overrightarrow{OA} + \frac{3}{5}\overrightarrow{AD} = \begin{pmatrix}5\\0\\0\end{pmatrix} + \frac{3}{5}\begin{pmatrix}-5\\4\\7\end{pmatrix} = \begin{pmatrix}2\\ \frac{12}{5}\\ \frac{21}{5}\end{pmatrix}$$

Specific behaviours
P uses correct ratio P states position vector

Additional working space

Question number: ________

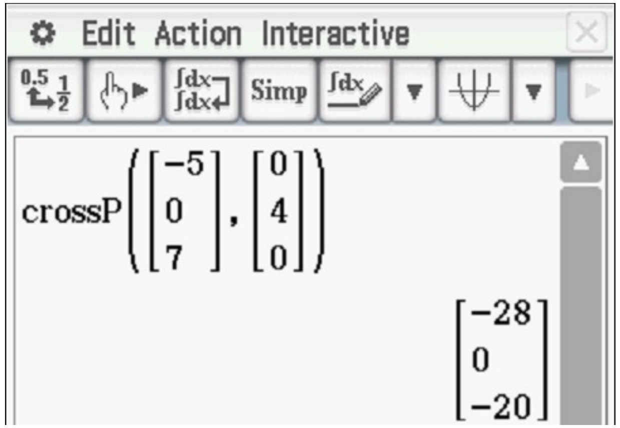
Additional working space

Question number: ________

b) Determine the cartesian equation of the plane that contains the points A, G & B.
(4 marks)

$\overrightarrow{AG} = \begin{pmatrix} -5 \\ 0 \\ 7 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$



$r \cdot \begin{pmatrix} -28 \\ 0 \\ -20 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -28 \\ 0 \\ -20 \end{pmatrix} = -140$

$-28x - 20z = -140$

$7x + 5z = 35$

Specific behaviours
P determines two vectors in plane
P uses cross product
P determines vector equation
P determines cartesian equation (no need to simplify)

See next page

c) Prove that the diagonals of the box above, bisect each other using vectors.

(3 marks)

c
Let P equal midpoint of diagonal AD Let Q equal midpoint of diagonal GB $\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AD} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -5 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ 2 \\ \frac{7}{2} \end{pmatrix}$ $\overrightarrow{OQ} = \overrightarrow{OG} + \frac{1}{2}\overrightarrow{GB} = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ 2 \\ \frac{7}{2} \end{pmatrix}$ $\overrightarrow{OP} = \overrightarrow{OQ}$ $\therefore P = Q$
Specific behaviours
P defines midpoints of two diagonals P determines position vectors of both midpoints P shows that such position vectors are identical

Q18 continued

End of questions

Solution	Specific behaviours
$0<k<1$	✓ Determines lower boundary. ✓ Determines upper boundary. [do not penalize ≤ instead of ¿]

(ii) Does the equation $|f(x)|=k$ ever have 3 solutions? (1 marks)

Solution	Specific behaviours
No	✓ States no.

(c) Determine the solutions to $f(-|x|)=2$. (2 marks)

Solution	Specific behaviours
$x=-2$ $x=2$	✓ States $x=-2$. ✓ States $x=2$. [Award at most 1 FT mark if answer of 'no solutions' given and is consistent with graph.]

Question 10 **(6 marks)**

Consider the complex numbers $P, Q, R \& W$.

$|P|=5$, Arg $(P)=\frac{3\pi}{4}$

$\overline{Q}=(1-i)P$, $R=\frac{7}{iP^2}$

$W=\frac{\sqrt{5}PQ}{(\sqrt{3}-i)R}$

$|W|=\frac{\sqrt{5}|P||Q|}{2|R|}$

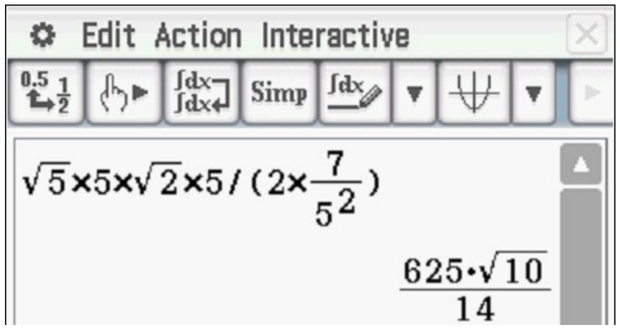
$Arg(W)=Arg(P)+Arg(Q)+\frac{\pi}{6}-Arg(R)$

a) Determine the exact value of $|W|$ (3 marks)

c

$W=\frac{\sqrt{5}PQ}{(\sqrt{3}-i)R}$

$|W|=\frac{\sqrt{5}|P||Q|}{2|R|}$



Specific behaviours

P determines modulus of R
P determines modulus of given values (brackets)
P determines exact modulus of W

b) Determine the exact value of Arg (W) in Principle form.
(3 marks)

c
$Arg(R) = -\frac{\pi}{2} - 2Arg(P) = \frac{-\pi}{2} - \frac{3\pi}{2} = -2\pi = 0$ $Arg(W) = Arg(P) + Arg(Q) + \frac{\pi}{6} - Arg(R)$ $= \frac{3\pi}{4} - \frac{\pi}{2} + \frac{\pi}{6} - 0 = \frac{5\pi}{12}$
Specific behaviours
P determines arg(R) P determines arg (Q) P determines arg(W) in principle form

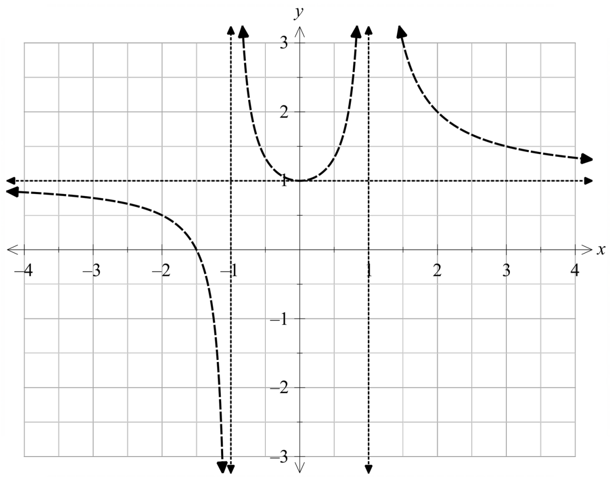
Question 11 **(6 marks)**

Consider the sphere $\left| r - \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \right| = \alpha$, where α is a positive constant, and the line $r = \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$.

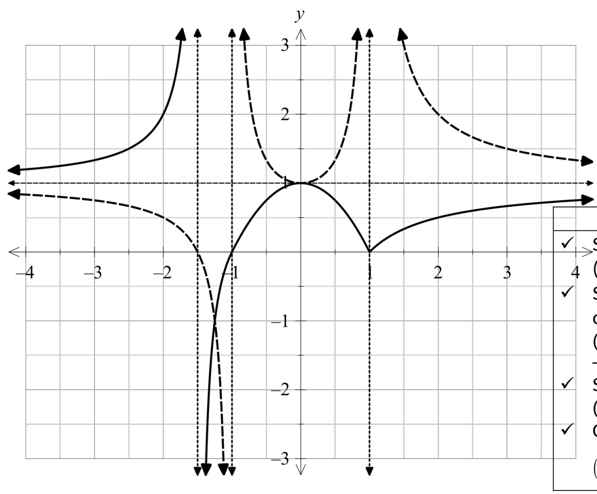
Determine all possible values of α such that:
- i) There is only one point of contact between sphere and line.
- ii) There are two points of contact between sphere and line.
- iii) There are no points of contact between sphere and line.

Question 18 **(9 marks)**

The graph of $y = \frac{1}{f(x)}$ is shown with a dotted curve on the axes below.



(a) On the same axes draw the graph of $f(x)$. (4 marks)



Specific behaviours
✓ Section $x \leftarrow 1.5$ drawn correctly. (shape and asymptotic behaviour)
✓ Section $-1.5 < x < 1$ drawn correctly. (shape and asymptotic behaviour – accept if correct from -1 to 1)
✓ Section $x > 1$ drawn correctly. (shape and asymptotic behaviour)
✓ Curve passes through at least 2 of $(-2, 2)$, $(0, 1)$ and $\left(2, \frac{1}{2}\right)$.

(b) (i) The equation $|f(x)| = k$ has 4 solutions for what range of values of k? (2 marks)

End of questions

$¿(2)\|4c\| \le 11 \Rightarrow -\frac{11}{4} \le c \le \frac{11}{4}$ *however* (1) *is a smaller range, hence* $\frac{-5}{2} \le c \le \frac{5}{2}$	✓ Determines range of c. [Award at most 3/5 if final range for c is $\frac{-11}{4} \le c \le \frac{11}{4}$ and no calculation included for beam to hit small mirror.]

See next page

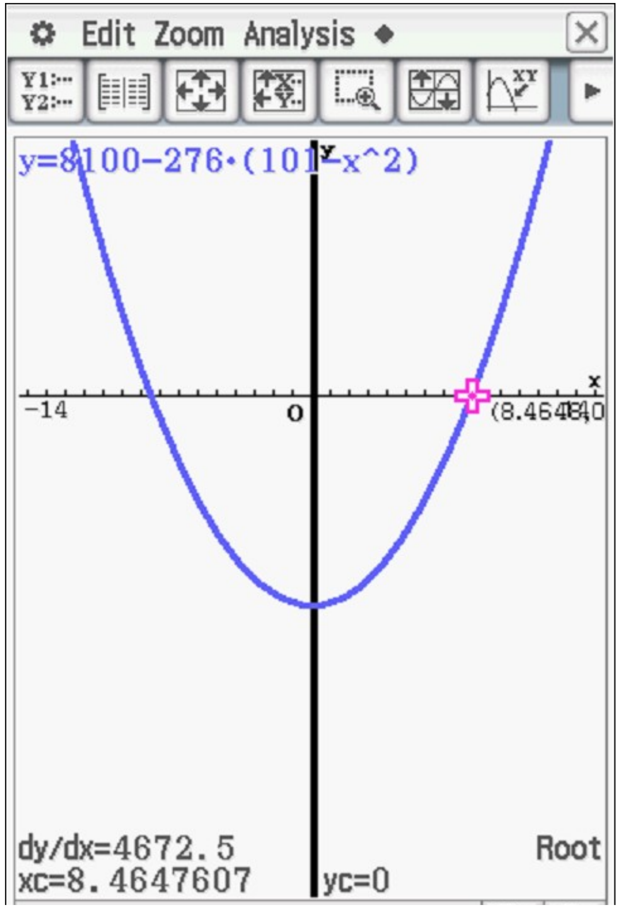
$$\left| \begin{pmatrix} 7+\lambda \\ -1-8\lambda \\ 3+2\lambda \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \right| = \alpha$$

$$\sqrt{(9+\lambda)^2 + (4+8\lambda)^2 + (2+2\lambda)^2} = \alpha$$

$$81 + 18\lambda + \lambda^2 + 16 + 64\lambda + 64\lambda^2 + 4 + 8\lambda + 4\lambda^2 = \alpha^2$$

$$69\lambda^2 + 90\lambda + 101 - \alpha^2 = 0$$

$$\Delta = 8100 - 276(101 - \alpha^2)$$



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$one\ point\quad \alpha=8.465$ $two\ points\quad \alpha>8.465$ $no\ points\quad 0\le\alpha<8.465\quad or\ 0<\alpha<8.465$
Specific behaviours
P subs line into sphere vector equation   P determines a quadratic equation with $\lambda\ \&\ \alpha$ only   P states an expression for determinant in terms of α only   P states value for one point   P states interval of values for two points   Pstates interval of values for no solns   MAX OF 5 MARKS if students did not note that a is a positive constant

Question 12 (6 marks)

Particles A and B are moving with constant velocities and have initial positions $\begin{pmatrix}-8\\2\\10\end{pmatrix}$ m and $\begin{pmatrix}7\\7\\-15\end{pmatrix}$ m respectively. 2 seconds later A is at $\begin{pmatrix}0\\-2\\4\end{pmatrix}$ m.

(a) Determine the velocity of A. (1 mark)

Solution
$v_A=\frac{1}{2}\left(\begin{pmatrix}0\\-2\\4\end{pmatrix}-\begin{pmatrix}-8\\2\\10\end{pmatrix}\right)=\begin{pmatrix}4\\-2\\-3\end{pmatrix}$
Specific behaviours
ü correct velocity

The velocity of B is $\begin{pmatrix}1\\-3\\2\end{pmatrix}$ m/s.

(b) Show that the paths of A and B cross, state the position vector of this point, and explain whether the particles collide. (5 marks)

Solution
$r_A(t)=\begin{pmatrix}-8\\2\\10\end{pmatrix}+t\begin{pmatrix}4\\-2\\-3\end{pmatrix},\ r_B(s)=\begin{pmatrix}7\\7\\-15\end{pmatrix}+s\begin{pmatrix}1\\-3\\2\end{pmatrix}$ For paths to cross we require $r_A=r_B$. Equating i and j coefficients and solving simultaneously:

The laser beam is then reflected with direction $d=-i-6j+k$.

(b) Determine the position vector of, B, the point where the beam hits the larger mirror. (3 marks)

Solution	Specific behaviours
$Laser\ Path: r=-2i+12j+2k+\mu(-i-6$ $ai+0j+bk=(-2-\mu)i+(12-6\mu)j+(2+$ $j\ components: \mu=2$ $\overrightarrow{OB}=-4i+4k$	✓ Determines vector equation of line. ✓ Solves for parameter, or recognises connection with parameter in part (a). ✓ Determines position vector of point where beam hits the mirror.

A second beam is fired from the origin with a direction of $d_1=ai+6j+ck$. When it hits the smaller mirror, it is then reflected with direction of $d_2=ai-6j+ck$. You may assume that the speed of the beam does not change.

There are laser beams from the origin which after being reflected in the small mirror do not hit the larger mirror.

(c) Determine the range of values of a and c, that ensure the beams **are reflected** in the larger mirror. (5 marks)

Solution	Specific behaviours
¿ $part(a): \lambda=2$	
$Hits\ small\ mirror\ at: 2ai+12j+2ck$	✓ Determines location where the beam hits the small mirror.
$\lvert 2a\rvert\le5\Rightarrow-\frac{5}{2}\le a\le\frac{5}{2}$ $\lvert 2c\rvert\le5\Rightarrow-\frac{5}{2}\le c\le\frac{5}{2}\ (1)$	✓ Determines range of a and c so beam hits the small mirror.
$Hits\ large\ mirror\ at:$ $r=2ai+12j+2ck+2(ai-6j+ck)$ $r=4ai+4ck\ (2)$	✓ Uses $\lambda=2$, and determines location where the beam hits the larger mirror.
$\lvert 4a\rvert\le6\Rightarrow-\frac{3}{2}\le a\le\frac{3}{2}$	✓ Determines range of a.

$Laser\ Path: r=\lambda(-i+6j+k)$ $ai+12j+bk=\lambda(-i+6j+k)$ $j\ components: 12=6\lambda \Rightarrow \lambda=2$ $\overrightarrow{OA}=-2i+12j+2k$	✓ Determines vector equation of line. ✓ Recognises j component is 12 (or substitutes into equation for plane) ✓ Solves for parameter. ✓ Determines position vector of point where beam hits the mirror.

See next page

$$4t-8=s+7, 2-2t=7-3s \Rightarrow t=5, s=5$$

Check $\underset{\sim}{k}$ coefficients are equal with these values of t and s:

$$t=5 \Rightarrow 10-3(5)=-5, s=5 \Rightarrow -15+2(5)=-5$$

Because $\underset{\sim}{r}_A(5)=\underset{\sim}{r}_B(5)=\begin{pmatrix}12\\-8\\-5\end{pmatrix}$, their paths cross at this point and because both particles reach this point at the same time they collide.

Specific behaviours

ü indicates equations for both paths
ü forms two equations using same or different time parameters (both will work here)
ü solves equations and checks third coefficient
ü correct position vector
ü explains why paths cross and whether particles collide

Question 13 (8 marks)

(a) Determine the equations of all asymptotes of the graph of $y=f(x)$ when

(i) $f(x)=\dfrac{2+5x^2}{2x(1-3x)}$. (2 marks)

Solution

$$f(x)=\frac{5x^2+2}{-6x^2+2x}, \lim_{x\to\pm\infty} f(x)=\frac{-5}{6}$$

Asymptotes: $x=0, x=1/3, y=-5/6$.

Specific behaviours

ü horizontal asymptote
ü vertical asymptotes

(ii) $f(x)=\dfrac{x^2-5}{x+6}$. (2 marks)

Solution

$$f(x)=\frac{x^2-5}{x+6}=x-6+\frac{31}{x+6}$$

Asymptotes: $x=-6$, $y=x-6$. Or $y=x$

Specific behaviours

ü oblique asymptote

See next page

ü vertical asymptote

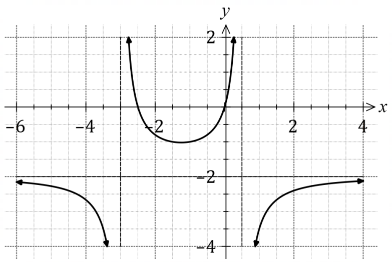
(b) The graph of $y=g(x)$ is shown in the diagram, together with its three asymptotes.

The defining rule is given by

$$g(x)=\frac{ax(2x+b)}{(x+c)(d-2x)}$$

where a, b, c and d are positive integer constants.

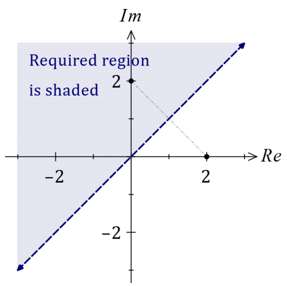
Determine, with brief reasons, the value of a, b, c and d. (4 marks)



Solution
Asymptote $y=-2 \rightarrow 2a/-2=-2 \rightarrow a=2$.
Root at $(-2.5, 0) \rightarrow b=5$.
Asymptote $x=-3 \rightarrow c=3$.
Asymptote $x=0.5 \rightarrow d=1$.
Specific behaviours
üüüü each value with appropriate reason
Max 2 marks if no reasons given with all correct values

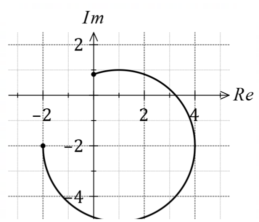
Question 14 **(9 marks)**

(a) Draw the subset of the complex plane determined by $|z-2i|<¿z-2\vee¿$ on the axes below. (3 marks)



Solution
See diagram
Specific behaviours
ü indicates points in plane
ü draws perp' bisector with dotted line
ü shades correct region

(b) The circular arc in the diagram represents the locus of a complex number z.



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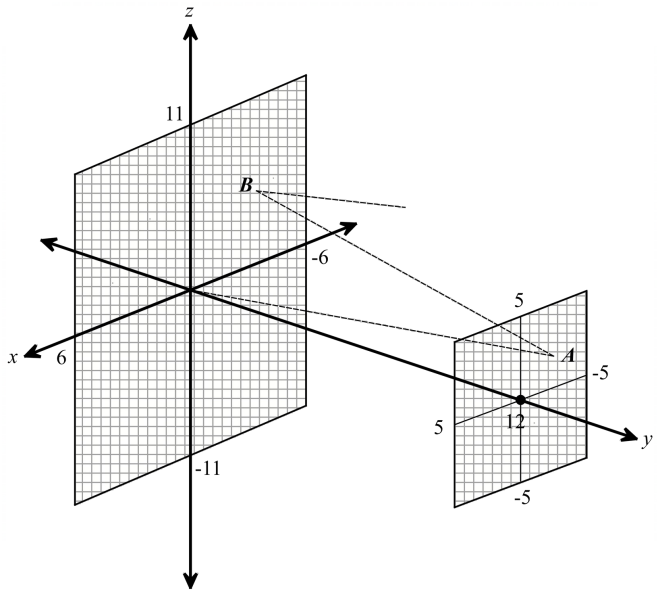
Solution	Specific behaviours
$(z_1+z_2)^3=(2\sqrt{3})^3 cis\frac{33\pi}{36}$	✓ Uses De Moivre's Theorem.
$k=24\sqrt{3}$ $m=12$	✓ Determines k. ✓ Determines m.

Question 17 **(12 marks)**

Two parallel mirrors are shown in the diagram below. The larger mirror passes through the origin and is coincident with the xz plane, and the smaller mirror is in the plane $y=12$.



A laser beam is fired through a small hole at the origin. The dotted line shows one such beam. The beam then hits the mirror at $y=12$ and is reflected back towards the larger mirror.

The laser beam is pointed with direction $d=-i+6j+k$.

(a) Determine the position vector of, A, the point where the beam hits the smaller mirror. (4 marks)

Solution	Specific behaviours

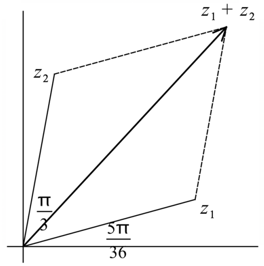
See next page

$z_1=w^{\frac{1}{6}}$ $z_1=64^{\frac{1}{6}}cis\frac{5\pi}{36}$ $z_1=2\,cis\frac{5\pi}{36}$	✓ Indicates that $z_1=w^{\frac{1}{6}}$. ✓ Uses De Moivre's Theorem. ✓ Determines z_1 in polar form.

(ii) Determine the remaining roots in polar form. Label the roots as z_2, z_3, z_4, z_5 and z_6 moving in an anticlockwise direction from the positive real axis. (2 marks)

	Specific behaviours
$z_2=2\,cis\frac{17\pi}{36}, z_3=2\,cis\frac{29\pi}{36}$ $z_4=2\,cis\left(\frac{-31\pi}{36}\right), z_5=2\,cis\left(\frac{-19\pi}{36}\right)$ $z_6=2\,cis\left(\frac{-7\pi}{36}\right)$	✓ Adds $\frac{\pi}{3}$ to argument, to determine at least two further solutions. ✓ Determines at least four solutions.

(d) Determine the exact polar form of z_1+z_2. (3 marks)

Solution	Specific behaviours
	✓ Uses vector addition of complex numbers.
$\lvert z_1+z_2\rvert=\sqrt{2^2+2^2-8\cos\frac{2\pi}{3}}=2\sqrt{3}$ $arg(z_1+z_2)=¿\frac{5\pi}{36}+\frac{\pi}{6}=\frac{11\pi}{36}¿$ $z_1+z_2=2\sqrt{3}\,cis\frac{11\pi}{36}$	✓ Determines modulus of z_1+z_2. ✓ Determines argument of z_1+z_2.

z_1+z_2, z_3+z_4 and z_5+z_6 are roots of $z^3=k\,cis\left(\frac{11\pi}{m}\right)$.

(e) Determine the values of k and m. (3 marks)

Without using $\Re(z)$ or $\Im(z)$, write equations or inequalities in terms of z for the indicated locus.

(3 marks)

Solution
$\lvert z-(1-2i)\rvert=3\cap\left(\frac{-3\pi}{4}\le arg\,z\le\frac{\pi}{2}\right)$
Other possibilities $-\pi\le arg(z-(1-2i))\le\pi-\cos^{-1}\left(\frac{1}{3}\right)=1.9106$
$-\pi\le arg(z+2i)\le\frac{\pi}{2}$
Specific behaviours
ü indicates correct centre and radius
ü lower bound for principal argument
ü Upper bound for principal argument

(c) Describe the subset , or sketch,of the complex plane determined by $\lvert z-5i\rvert+\lvert z+5\rvert=5\sqrt{2}$.

(3 marks)

Solution
Distance between $5i$ and -5 in complex plane is $5\sqrt{2}$.
Hence z must lie on the line segment between $5i$ and -5 inclusive in the complex plane.
Alternatively, when $z=x+iy$ then locus is $y=5-x, 0\le x\le 5$.
Specific behaviours
ü indicates or sketches a line
ü indicates or sketches is a line segment
ü indicates or sketches labelled correct end points of line segment

Question 15 (8 marks)

(a) Determine all solutions to the equation $z^3+27i=0$ in exact polar form. (3 marks)

Solution
$z^3=27\,cis\left(\frac{-\pi}{2}\right)\Rightarrow z=3\,cis\left(\frac{-\pi+4n\pi}{6}\right),n=-1,0,1$
$z_1=3\,cis\left(\frac{-5\pi}{6}\right),z_2=3\,cis\left(\frac{-\pi}{6}\right),z_3=3\,cis\left(\frac{\pi}{2}\right)$
Specific behaviours
ü expresses $27i$ in polar form
ü states one correct solution
ü states all correct solutions

(b) Consider the seventh roots of unity expressed in polar form $r\,cis\,\theta$.

(i) Determine the roots for which $-\pi<\theta\leftarrow\frac{\pi}{2}$. (2 marks)

Solution
$z^7=1=cis(2n\pi)\Rightarrow z=cis\left(\frac{2n\pi}{7}\right)$ where $n\in Z$.
Hence
$z_1=cis\left(\frac{-6\pi}{7}\right),z_2=cis\left(\frac{-4\pi}{7}\right)$.
Specific behaviours
ü general expression for roots
ü correct roots

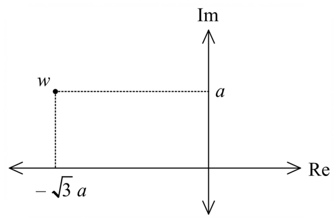
(ii) Use all seven roots to show that $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)=-\frac{1}{2}$. (3 marks)

Solution
The seven roots are given by $z=cis\left(\frac{2n\pi}{7}\right),n=-3,-2,\ldots,2,3$, and the sum of these roots, and hence their real parts, will be 0:
$\cos(0)+\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)+\cos\left(\frac{-2\pi}{7}\right)+\cos\left(\frac{-4\pi}{7}\right)+\cos\left(\frac{-6\pi}{7}\right)=0$
But $\cos(-\theta)=\cos(\theta)$ and $\cos(0)=1$. Hence
$1+2\cos\left(\frac{2\pi}{7}\right)+2\cos\left(\frac{4\pi}{7}\right)+2\cos\left(\frac{6\pi}{7}\right)=0$

$\therefore\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)=\frac{-1}{2}$
Specific behaviours
ü uses sum of real parts of all roots is 0
ü uses $\cos(-\theta)=\cos(\theta)$ and known values
ü sufficient explanation throughout and simplifies to obtain required result

Question 16 (15 marks)

The complex number w has been plotted on the Argand diagram below.



(a) Express w in Cartesian form. (1 mark)

Solution	Specific behaviours
$w=-\sqrt{3}a+ai$	✓ Writes w in Cartesian form.

(b) Express w in polar form. (3 marks)

Solution	Specific behaviours
$\lvert w\rvert=\sqrt{3a^2+a^2}=2a$	✓ Determines modulus.
$arg(w)=\frac{5\pi}{6}$	✓ Determines argument.
$w=2a\,cis\,\frac{5\pi}{6}$	✓ Writes in polar form.

(c) The complex number z_1 is a root of $z^6=w$, with the smallest positive argument.

(i) Given that $a=32$, determine z_1 in polar form. (3 marks)

Solution	Specific behaviours