PRESBYTERIAN LADIES' COLLEGE

A COLLEGE OF THE UNITING CHURCH IN AUSTRALIA

MATHEMATICS DEPARTMENT

Year 12 MATHEMATICS SPECIALIST

DATE: 4th December 2015 Name___________________________

Reading Time: 3 minutes

SECTION ONE: CALCULATOR FREE

TOTAL: 27 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet.

WORKING TIME: 25 minutes (maximum)

SECTION TWO: CALCULATOR ASSUMED

TOTAL: 25 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing instruments, templates, up to 3 Calculators,

1 A4 page of notes (one side only), SCSA formula sheet.

WORKING TIME: 20 minutes (minimum)

SECTION 1 Question	Marks available	Marks awarded	SECTION 2 Question	Marks available	Marks awarded
1	6		5	7	
2	6		6	4	
3	10		7	7	
4	5		8	7	
Total	27			25	

Section One: Calculator-free **[27 marks]**

This section has **Four (4)** questions. Answer **all** questions. Write your answers in the spaces provided

Question 1 [6 marks]

Simplify each of the following expressions, writing your answer in exact polar form.

(a) $\left(\sqrt{3}-i\right)^2\left(\sqrt{3}-i\right)$

$$=\left(\sqrt{3}-i\right)^3$$

$$=\left[2cis\left(\frac{-\pi}{6}\right)\right]^3$$ ✓ change to polar form

$$=8cis\left(\frac{-\pi}{2}\right)$$ Applies de Moivre’s theorem

[2]

(b) $3cis\left(\frac{\pi}{4}\right)\times\left[2cis\left(\frac{-\pi}{3}\right)\right]^{-1}$

$$=3cis\left(\frac{\pi}{4}\right)\times\frac{1}{2}cis\left(\frac{\pi}{3}\right)$$ Applies de Moivre’s theorem

$$=\frac{3}{2}cis\left(\frac{7\pi}{12}\right)$$ ✓Simplifies answer

[2]

(c) $\dfrac{1}{\sqrt{2cis\left(\frac{\pi}{2}\right)}}$

$$\left[2cis\left(\frac{\pi}{2}\right)\right]^{\frac{-1}{2}}$$ ✓Able to manipulate index

$$=\frac{\sqrt{2}}{2}cis\left(\frac{-\pi}{4}\right)$$ Applys de Moivre’s theorem

[2]

Question 2 [6 marks]

(a) (i) Find the quotient and the remainder for $\dfrac{z^3 - 2z^2 + 4z - 1}{z^2 - z + 1}$, hence rewrite $z^3 - 2z^2 + 4z - 1$ in the form $H(z)\times(z^2 - z + 1) + R(z)$ [3]

$$\frac{z^3 - 2z^2 + 4z - 1}{z^2 - z + 1}$$

$$z^2 - z + 1\overline{)z^3 - 2z^2 + 4z - 1}$$ ✓ Divides to find H(z)

✓ Finds the remainder is 2z

$$H(z) = z - 1$$

$$(z - 1)\times(z^2 - z + 1) + 2z$$ ✓ rewrites the expression

(ii) Hence, solve $z^3 - 2z^2 + 4z - 1 = 2z$ [3]

$$\frac{z^3 - 2z^2 + 4z - 1}{z^2 + z + 1} = \frac{2z}{z^2 + z + 1}$$

$$(z - 1)(z^2 - z + 1) = 0$$

$$z = 1$$ ✓ solves for z = 1

$$z^2 - z + 1 = 0$$ ✓✓ solves for the other two solutions

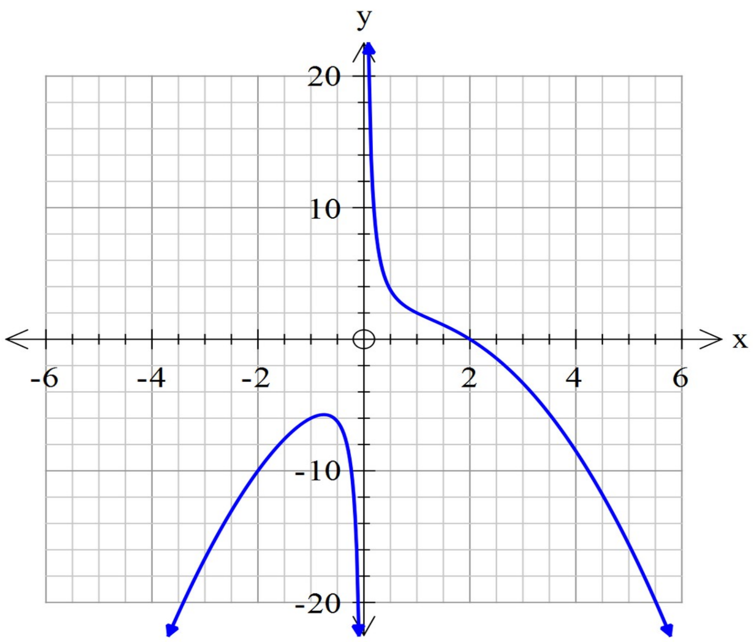
$$\left(z - \frac{1}{2}\right)^2 - \frac{1}{4} + 1 = 0$$

$$z = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

$$z = 1 \quad or \quad \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

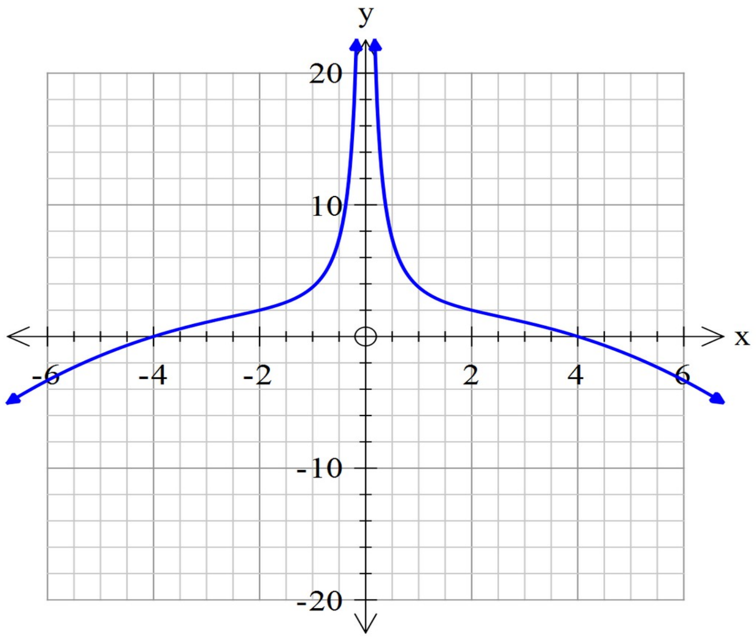
Question 3   [10 marks]

Given the graph of $y = f(x)$ is given as follows;



Sketch the graph of

(a)   (i) $y = f\left|\dfrac{x}{2}\right|$



✓ removes the graph x<0

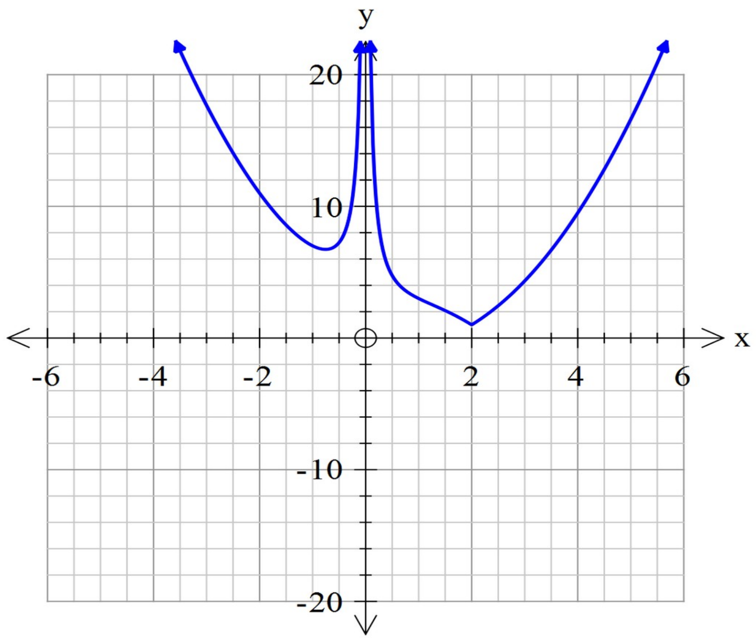
✓ mirrors the graph x>0 over the y-axis

✓ dilates the graph by a scale factor of 2 along the x-axis

[3]

(ii) Sketch the graph of $y = |f(x)| + 1$.

[3]

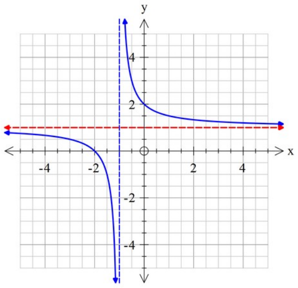


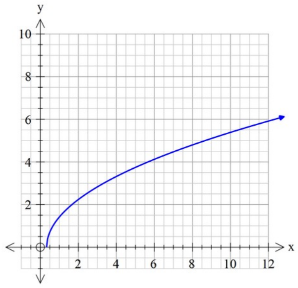
✓ reflects the part of the graph y<0 over the x-axis for x>0

✓ reflects the part of the graph y<0 over the x-axis for x<0

✓ translates the graph 1 unit up

(b) Given that $g(x)=\sqrt{3x-1}$ and $h(x)=\frac{x+2}{x+1}$, find the domain and range of the composite function $goh(x)$ [4]





Domain

Range

$x>-1$ $x\le\frac{-5}{2}$	$\ge\frac{1}{3}$ $\ne 1$	$y\ge 0$ $y\ne\sqrt{2}$

$\{x\in\Re; x\le\frac{-5}{2},\quad x>-1\}$

$\{y\in\Re; y\ge 0,\quad y\ne\sqrt{2}\}$

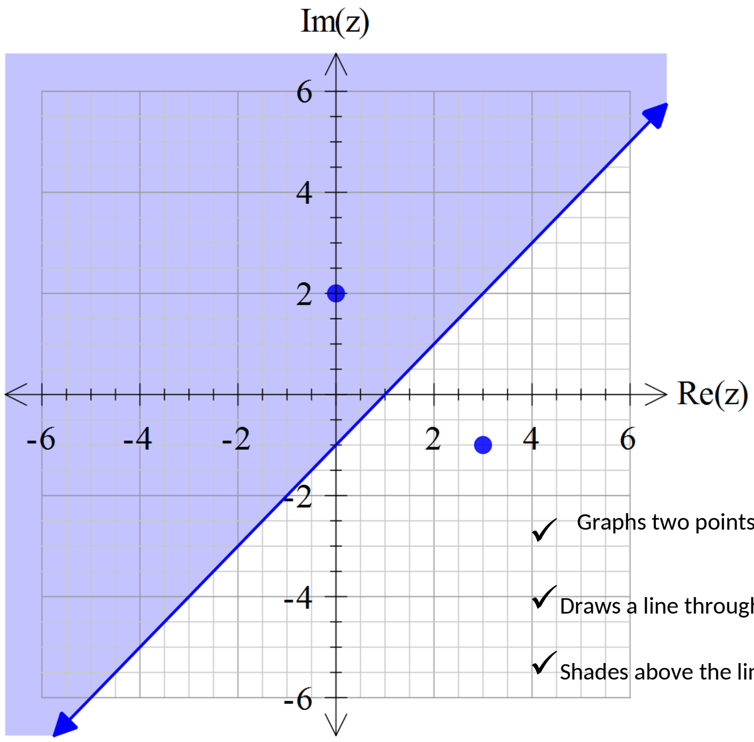
✓ ✓ Both domain restrictions (1 mark each)

✓✓ Both range restrictions (1 mark each)

Question 4 [5 marks]

(a) On an Argand diagram sketch the loci of points and that satisfy the following condition;

$$|z-2i|\le|z-3+i|$$



Graphs two points

Draws a line through the midpoint, perpendicular

Shades above the line

[3]

(b) Give the equation of the locus in Cartesian form.

$$|z-2i| = |z-3+i|$$
$$x^2 + (y-2)^2 = (x-3)^2 + (y+1)^2$$
$$-4y + 4 = -6x + 9 + 2y + 1$$
$$-6y = -6x + 6$$
$$y \geq x - 1$$

Sets up Cartesian equation simplified

Simplifies with correct inequality

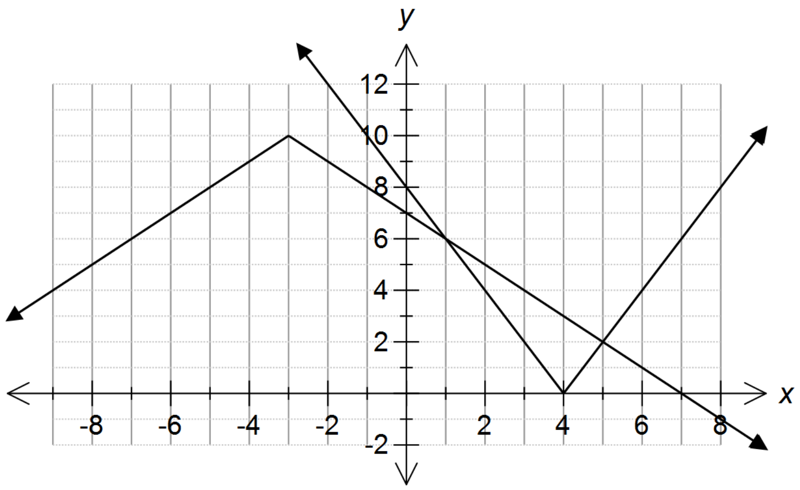
[2]

NAME:____

Section Two: Calculator-assumed **[25 marks]**

This section has **four (4)** questions. Answer **all** questions. Write your answers in the spaces provided

Question 5 [7 marks]



(a) Use the diagram above to solve for x in the following.

(i) $-|x+3|+10=7$ [1]

✓ Both x values given

$x=0 \; or \; -6$

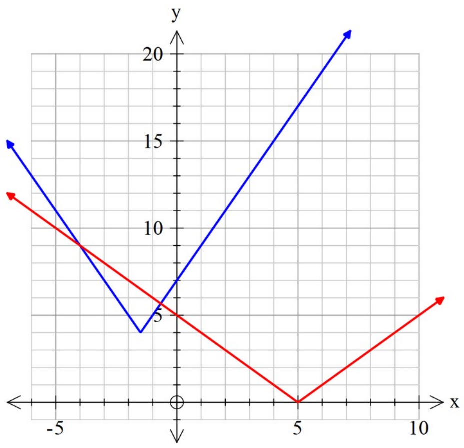
(ii) $-|x+3|+10 \geq |2x-8|$

✓ Correct x –values

✓ Correct domain [2]

$1 \leq x \leq 5$

(b) Solve the following algebraically $4+|3+2x|>|x-5|$ [4]



$4+|3+2x|=|x-5|$

$4-3-2x=-x+5$

$1-x=5$

$x=-4$

$4+3+2x=-x+5$

$3x=-2$

$x=-\frac{2}{3}$

$-4>x>-\frac{2}{3}$

✓ Draws a graph and identifies the critical points of x=-1.5 and x=5

✓ Finds the correct linear equations of each relevant function

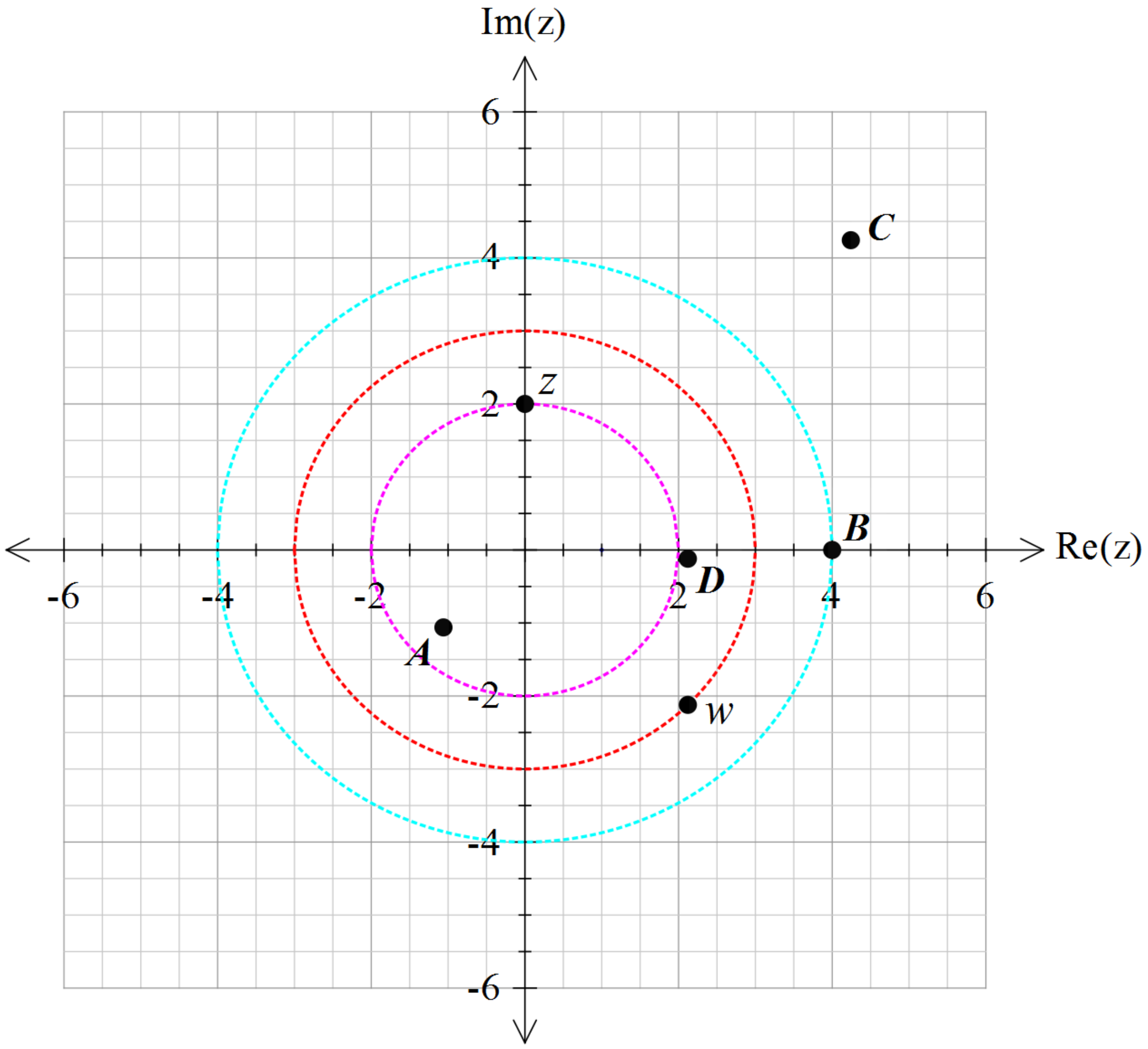
✓ Solves for the 2 intersections

✓ Writes the inequality correctly

Question 6 [4 marks]

Given the position of *z* and *w* on the Argand diagram below. Label the points A, B, C and D using the following options.

$w+z$ $\quad wz$ $\quad \frac{-1}{2}w$ $\quad z\bar{z}$ $\quad \frac{w}{z}$ $\quad w^{-2}$ $\quad z^2$



A $\frac{w}{z}$ [1]

B $z\bar{z}$ [1]

✓✓ ✓ ✓ 1 mark each correct answer

C wz [1]

D $w+z$ [1]

Question 7 [7 marks]

(a) Represent on the Argand diagram provided below, the loci of points, that satisfy the following conditions;

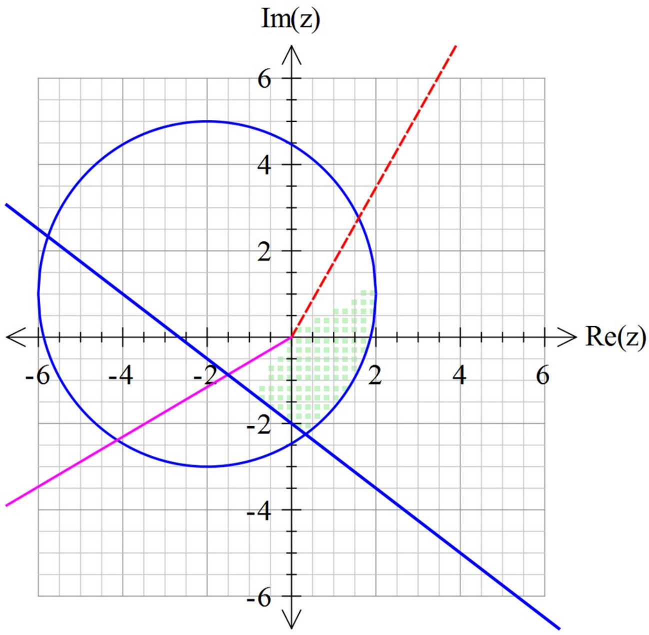
$|z+2-i| \leq 4$, $\frac{-5\pi}{6} \leq \arg(z) < \frac{\pi}{3}$ and $4\,\mathrm{Im}(z) + 3\,\mathrm{Re}(z) + 8 \geq 0$

✓ Circle drawn correctly

✓ Arg drawn correctly

✓ Line drawn correctly

✓ Shades correctly



[4]

(b) Given that $|z+2-i| \leq 4$, state the minimum and maximum value of $|z|$.

$$|z+2-i| \leq 4$$

$$\sqrt{2^2+1^2}$$

¿$\sqrt{5}$

$$\min|z| = 0$$

$$\max|z| = 4+\sqrt{5}$$

✓ Finds radius

✓ States min correctly

✓ States max correctly

[3]

Question 8 [7 marks]

(a) Using your CAS calculator (or otherwise) find all the solutions to $z^5 = 512\left(\sqrt{3} - i\right)$ in exact polar form, where $z = r\left(\cos\theta + i\sin\theta\right)$, $-\pi < \theta \le \pi$ and $r \ge 0$.

[4]

$$z^5 = 512\left(\sqrt{3} - i\right)$$

$$z^5 = 1024cis\left(\frac{-\pi}{6}\right)$$

$$z_0 = 4cis\left(\frac{-\pi}{30}\right)$$

$$z_1 = 4cis\left(\frac{11\pi}{30}\right)$$

$$z_2 = 4cis\left(\frac{23\pi}{30}\right)$$

$$z_3 = 4cis\left(\frac{-25\pi}{30}\right)$$

$$z_4 = 4cis\left(\frac{-13\pi}{30}\right)$$

✓ Changes to polar form

✓ applies De Moivre's theorem and gives first solution

✓ Identifies they need to add 2PI/5

✓ Gives other 3 solutions

(b) Draw the solutions from (a) on the complex plane below. Show all major features.

[3]

✓ Correct placement and magnitude of first point z

✓ All other points magnitude of 4

✓ Other solutions are distributed evenly 2PI/5 angle apart