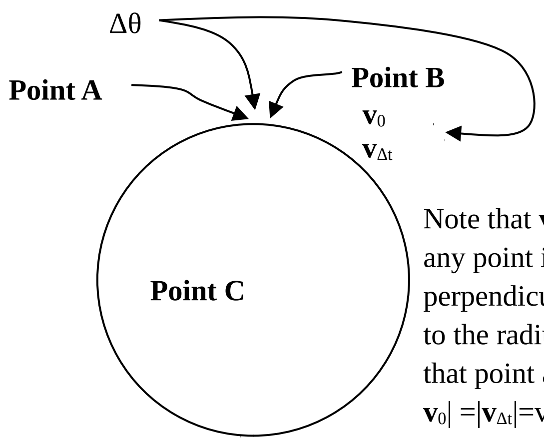
Derivation of $a_c = v^2/r$

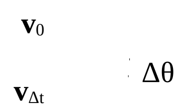
The rate at which an object rotates is often stated in *rpm* or *revolutions per minute*.
In one revolution, the object passes through 360° or 2π radians.
Units of the form *angle per second* could be used to measure rate of rotation.
The “natural” unit for angle is *radians* and these will be used throughout the following.
Therefore, units of *radians per second* will be used to measure rate of rotation.
The rate of rotation, given in *radians per second*, is a rate of change of angle.
It is called angular velocity for which the symbol is ω (the lowercase Greek letter omega).
$\omega=\Delta\theta / \Delta t$
Radians per second is the mathematically preferred unit for measuring angular velocity.

If an object revolves about a point at a radius of r and a speed of v, then one complete revolution will take T seconds where T is called the period of revolution.
In one revolution, the object will travel a distance of $2\pi r$ so $v=2\pi r/T$
The object will have an angular velocity $\omega=2\pi/T$ *radians per second*.
Combining $\omega=\Delta\theta / \Delta t$ and $v=2\pi r / T$ and $\omega=2\pi / T$ gives $\omega=v / r= \Delta\theta / \Delta t$

If an object in circular motion about C travels from Point A to Point B, it will have a change of velocity of Δ**v** where Δ**v** is as sketched below.



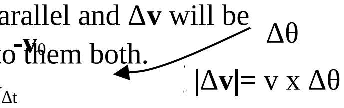
Note that **v** at any point is perpendicular to the radius to that point and $|\mathbf{v}_0| = |\mathbf{v}_{\Delta t}| = v$



If Δθ is small and as $\mathbf{v}_0$ and $\mathbf{v}_{\Delta t}$ are of equal length, v, the magnitude of Δ**v** will be the length of the arc between them.

Another consequence of a small Δθ is that

$\mathbf{v}_0$ and $\mathbf{v}_{\Delta t}$ are parallel and Δ**v** will be perpendicular to them both.



$|\Delta\mathbf{v}| = v \times \Delta\theta$

(an arc length is equal to r x θ)

The magnitude of Δ**v** will be the arc length sketched above. The direction of Δ**v** will be parallel to the radius and towards the centre of the orbit.

$$
\begin{aligned}
\mathbf{a} &= |\Delta\mathbf{v}| / \Delta t \\
&= v \times \Delta\theta / \Delta t \\
&= v \times \omega \\
&= v \times v / r \\
&= v^2 / r \text{ toward the centre of the orbit}
\end{aligned}
$$