MATHEMATICS METHODS

MAWA Semester 1 (Unit 1) Examination 2015

Calculator-free

Marking Key

Section One: Calculator-free (60 Marks)

Question 1(a)

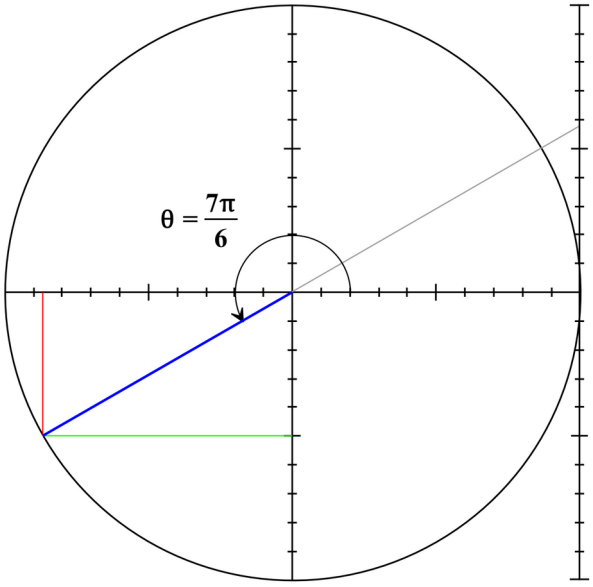
Solution

$\frac{\pi}{6}=30^\circ \Rightarrow \frac{7\pi}{6}=7\times 30^\circ =210^\circ$

Marking key/mathematical behaviours	Marks
• Determines $\frac{7\pi}{6}=210^\circ$	1

Question 1(b)

Solution



$\cos\theta =-\sqrt{1-\sin^2\theta}=-\sqrt{1-\left(-\frac{1}{2}\right)^2}=-\sqrt{\frac{3}{4}}=-\frac{\sqrt{3}}{2}$ by using the right triangle identity or

$\cos\theta =\cos\frac{7\pi}{6}=-\cos\frac{\pi}{6}=-\frac{\sqrt{3}}{2}$ by knowledge of exact values

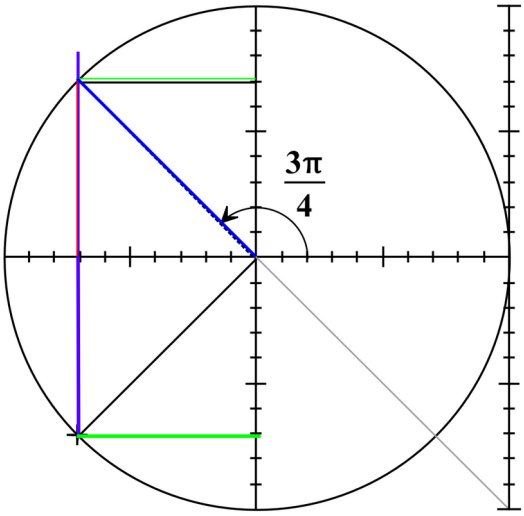
Similarly $\tan\theta =\tan\left(\frac{7\pi}{6}\right)=\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$

Marking key/mathematical behaviours	Marks
• indicates use of $\cos\frac{7\pi}{6}=-\cos\frac{\pi}{6}$ or uses $\sin\theta=-\frac{1}{2}$ meaningfully	1
• states correct exact value of $\cos\frac{7\pi}{6}$ (accept $-\frac{\sqrt{3}}{2}$ or $-\frac{3}{2\sqrt{3}}$)	1
• indicates use of $\tan\frac{7\pi}{6}=\tan\frac{\pi}{6}$	1

• states correct exact value of $\tan\frac{7\pi}{6}$ (accept $\frac{\sqrt{3}}{3}$ or $\frac{3}{3\sqrt{3}}$)	1

Question 1(c)

Solution



From exact values and use of unit circle, $\theta = \frac{3\pi}{4}$ or $-\frac{3\pi}{4}$

Marking key/mathematical behaviours	Marks
• indicates method of determination on diagram	1
• states both correct values of θ	1

Question 2(a)

Solution

$$\frac{x+3}{4} - \frac{x-3}{5} = \frac{x}{2}$$
$$5(x+3) - 4(x-3) = 10x$$
$$5x + 15 - 4x + 12 = 10x$$
$$9x = 27 \Rightarrow x = 3$$

Marking key/mathematical behaviours	Marks
• multiplies the equation by the LCD	1
• expands brackets and simplifies	1
• solves for x	1

Question 2(b)

Solution

$$x^2 + x - 72 = 0$$
$$(x-8)(x+9) = 0$$
$$x = -9 \text{ or } x = 8$$

Marking key/mathematical behaviours	Marks
• factorises trinomial	1
• solves for x	1

Question 2(c)

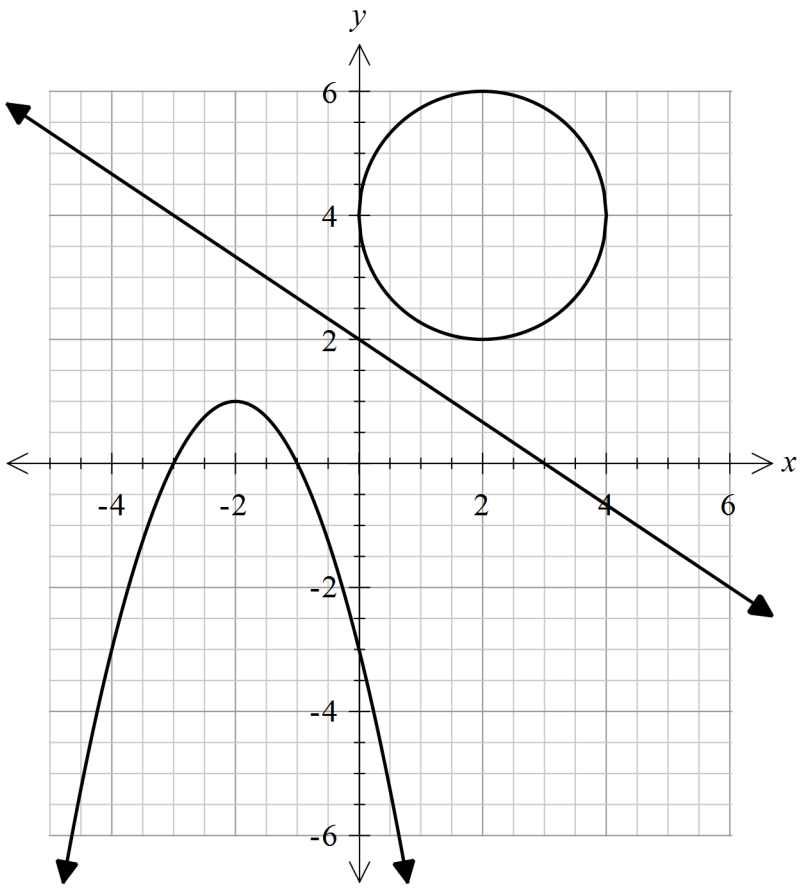
Solution

$$x^2 - 4x + 1 = 0$$
$$(x-2)^2 - 4 + 1 = 0$$
$$(x-2)^2 = 3$$
$$x - 2 = \pm\sqrt{3}$$
$$x = 2 \pm \sqrt{3}$$

Marking key/mathematical behaviours	Marks
• completes the square	1
• equates $(x-2)^2 = 3$	1
• takes square root	1
• solves for x	1

Question 3

Solution



Marking key/mathematical behaviours	Marks
• sketches $2x + 3y = 6$ accurately, showing x and y intercepts	1+1
• sketches $y = -x^2 - 4x - 3$ reflecting the correct turning point, orientation and intercepts	1+1+1
• sketches $(x-2)^2 + (y-4)^2 = 4$ with correct radius and centre	2

Question 4

Solution

Graph A: $y = -\sqrt{x+3}$

Graph B: $y = \dfrac{-3}{x+1}$

Marking key/mathematical behaviours	Marks
• Graph A	
o correct horizontal translation	1
o recognition of reflection in x - axis	1
• Graph C	
o correct horizontal translation	1
o correct dilation factor	1
o recognition of reflection in x - axis	1

Question 5(a)

Solution

(i) $P(X \cup Y) = 0.9 \Rightarrow x = P(X \cap Y) = 0.3 \therefore P(X) = 0.7$

(ii) From part (i), $P(X) = 0.7$ and $P(Y) = 0.5$

$P(X) \times P(Y) = 0.35 \neq 0.3$

$\therefore X$ and Y are not independent

Marking key/mathematical behaviours	Marks
(i)	
• determines $x = 0.3$	1
• determines correct value for $P(X)$	1
(ii)	
• determines $P(X) \times P(Y)$	1
• shows that $P(X) \times P(Y) \neq P(X \cap Y)$	1
• concludes that the two events are not independent	1

Question 5(b)

Solution

(i) $P(X\ |Y)=\dfrac{P(X\cap Y)}{P(Y)}\Rightarrow \dfrac{2}{7}=\dfrac{x}{0.2+x}$

$\therefore \quad 0.4+2x=7x$

i.e. $\quad 0.4=5x$

i.e. $\quad 0.08=x \qquad \therefore P(X)=0.48$

(ii) From part (i) $P(X)=0.48$ and so $P(X\cup Y)=0.48+0.2=0.68$

$P\left(\overline{X\cup Y}\right)=1-\ P(X\cup Y)=1-\ 0.68=0.32$

Marking key/mathematical behaviours	Marks
(i)	
• applies the conditional probability formula	1
• substitutes correctly	1
• multiplies correctly and simplifies	1
• determines correct value for $P(X)$	1
(ii)	
• determines $P(X\cup Y)$	1
• applies complimentary property and arrives at the correct result	1

Question 6(a)

Solution

Has the form $y = a\tan bx + c$

Period = 2π hence $b = \frac{1}{2}$

Vertical translation 1 unit up, hence $c = 1$.

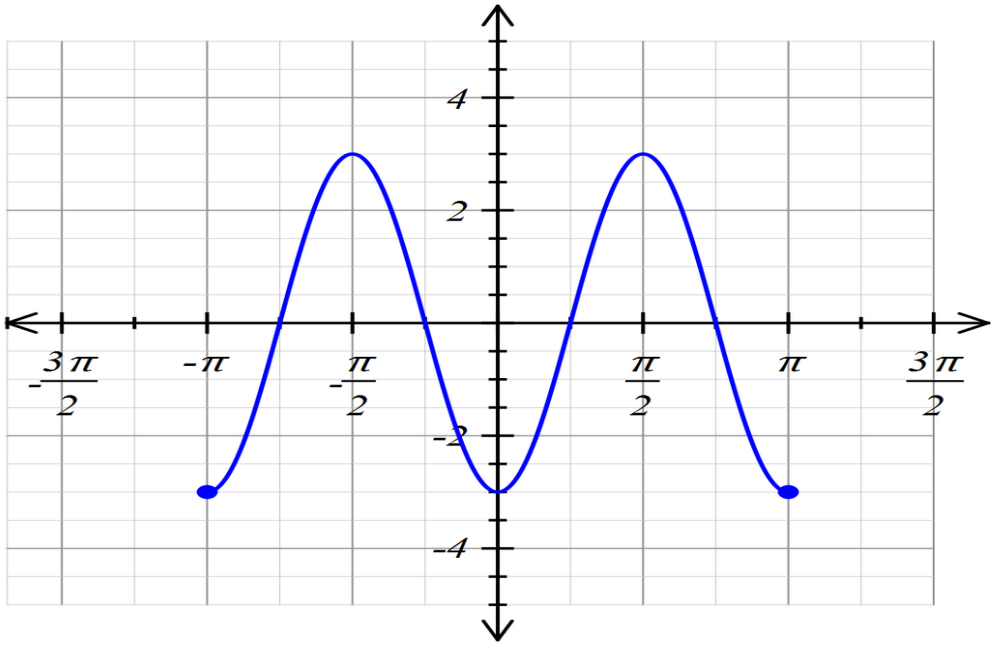
Dilation parallel to y axis, scale factor = 3. Hence, $a = 3$

$\therefore y = 3\tan\frac{x}{2} + 1$

Marking key/mathematical behaviours	Marks
• determines the period and hence b	1
• identifies vertical translation and determines $c = 1$	1
• determines a	1
• states the correct equation	1

Question 6(b)

Solution



Marking key/mathematical behaviours	Marks
• graph is drawn over the correct domain	1
• graph is a cosine curve with the correct amplitude	1
• graph has the correct period	1
• phase shift is correct	1
• graph is accurate passing through (0,-3) and has smooth turning points	1

Question 7 (a)

Solution

$$\left(\alpha+\frac{\beta}{2}\right)^4=\alpha^4+4\alpha^3.\frac{\beta}{2}+6\alpha^2\left(\frac{\beta}{2}\right)^2+4\alpha\left(\frac{\beta}{2}\right)^3+\left(\frac{\beta}{2}\right)^4$$

Marking key/mathematical behaviours	Marks
• uses the correct binomial coefficients	1
• each term has the correct powers for α and β	1
• uses $\frac{\beta}{2}$ correctly in each term	1

Question 7(b)

Solution

The mouse has six choices to enter and five choices to leave by another door.

So, $6\times5=30$

Marking key/mathematical behaviours	Marks
• states the correct result	1

Question 7(c)

Solution

(i) $\binom{7}{2}=\frac{7\times6}{1\times2}=21$

(ii) There is only one card with an even number on it.(2)
There are six other numbers that the 2 can be combined with to give an even product.

$\therefore P(\text{even product})=\frac{6}{21}$

(iii) For the product to be prime, one of the cards must be ONE(1)
Any of the other six cards can be combined with 1 to give a prime product

$\therefore P(\text{prime product})=\frac{6}{21}$

(iv) There are only 4 possible combinations of two of the numbers that sum to a prime number. 1+2, 2+3, 2+5, 2+11.

$\therefore P(\text{prime sum})=\frac{4}{21}$

Marking key/mathematical behaviours	Marks
(i)	
• states correct sample space	1
(ii)	
• determines that there are 6 pairs of numbers (listing or logic) that have an even product and determines the correct probability of an even product	1+1
(iii)	
• indicates that the only way that the product can be prime is if one of the cards has a one on it.	1
• determines the correct probability of a prime product	1
(iv)	
• provides some form of exhaustive listing of the sum of two numbers	1
• determines that there are only 4 possibilities of prime sums	1
• determines the correct probability of the sum being prime	1