



MATHEMATICS DEPARTMENT
MATHEMATICAL METHODS YEAR 12 – TEST 1

DATE: 2nd December 2015

Name:

Mark Aluwipatti

Reading Time: 3 minutes

SECTION ONE: CALCULATOR FREE

WORKING TIME: Maximum 25 minutes

TOTAL:

24 marks

EQUIPMENT:

pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet
(provided)

SECTION TWO: CALCULATOR ASSUMED

WORKING TIME: Minimum 25 minutes

TOTAL:

26 marks

EQUIPMENT:

pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing
instruments, templates, up to 3 calculators, formula sheet (provided) one A4
page of notes (one side only)

Question	Marks available	Marks awarded	Question	Marks available	Marks awarded
1	6	3	4	4	4
2	4	4	5	8	8
3	7	7	6	7	7
4	7	7	Sect 2 Total	26	
Sect 1 Total	24		TOTAL	50	

Question 1

(6 marks)

Find the antiderivative of each of the following

a) $\frac{4}{(2x-5)^3} = 4(2x-5)^{-3}$

(3 marks)

$$\int 4(2x-5)^{-3} = \frac{4(2x-5)^{-2}}{-2 \times 2} \quad \checkmark$$

$$= -(2x-5)^{-2} \quad \checkmark$$

$$= \frac{-1}{(2x-5)^2} \quad \checkmark$$

(b) $(10x+5)(x^2+x-3)^4$

(3 marks)

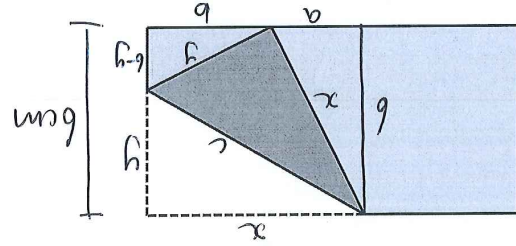
$$\int (10x+5)(x^2+x-3)^4$$

$$= \frac{(10x+5)(x^2+x-3)^5}{5(2x+1)} \quad \checkmark$$

$$= (x^2+x-3)^5 \quad \checkmark$$

(7 marks)

A rectangular piece of paper measures 12 cm by 6 cm. One corner of the sheet of paper is folded up to just reach the opposite side as shown below.



What is the minimum length of the resulting crease in the paper?

End of Test

∴ minimum length of crease is 7.79 cm

From C.A.S. min at (6.36, 7.79)

$$c = \sqrt{x^2 + \left(\frac{6}{x(x - \sqrt{x^2 - 36})}\right)^2}$$

$$y = \frac{6}{x(x - \sqrt{x^2 - 36})} \quad \text{From C.A.S.}$$

$$x = \sqrt{x^2 - 36} + \sqrt{y^2 + (6 - y)^2}$$

$$x = a + b$$

$$a = \sqrt{x^2 - 6^2} \quad b = \sqrt{y^2 - (6 - y)^2}$$

$$x^2 + y^2 = c^2$$

Question 2

(4 marks)

Let $f(x) = (x+3)(1-x^2)^5$.

The derivative of $f(x)$ can be written in the form $f'(x) = (1-x^2)^4(ax^2 + bx + c)$.

Determine the value of a , b and c .

$$f'(x) = 1(1-x^2)^5 + (x+3) \times 5(1-x^2)^4 \times (-2x)$$

$$= (1-x^2)^5 + (-10x)(x+3)(1-x^2)^4$$

$$= (1-x^2)^5 + (-10x^2 - 30x)(1-x^2)^4$$

$$= (1-x^2)^4 [(1-x^2) + (-10x^2 - 30x)]$$

$$= (1-x^2)^4 (-11x^2 - 30x + 1)$$

$$a = -11$$

$$b = -30$$

$$c = 1$$

-1 per error in working.

Question 3

(7 marks)

Let A, B, C, D, E, F and G be points on the graph of a continuous function $f(x)$. The table below shows the information about the sign of $f(x)$, $f'(x)$ and $f''(x)$ at these points.

Point	A	B	C	D	E	F	G
x	-4	-3	-1	0	1	2	4
$f(x)$	+	0	-	0	+	+	+
$f'(x)$	-	-	0	+	+	0	+
$f''(x)$	+	+	+	0	-	0	+

There are no other points at which $f(x)$, $f'(x)$ or $f''(x)$ are equal to zero.

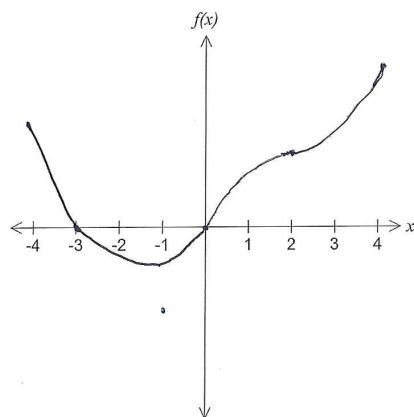
- (a) Which point is a local minimum? (1 mark)

C ✓

- (b) Describe the nature of the graph at point F. (2 marks)

Horizontal point of inflexion. ✓

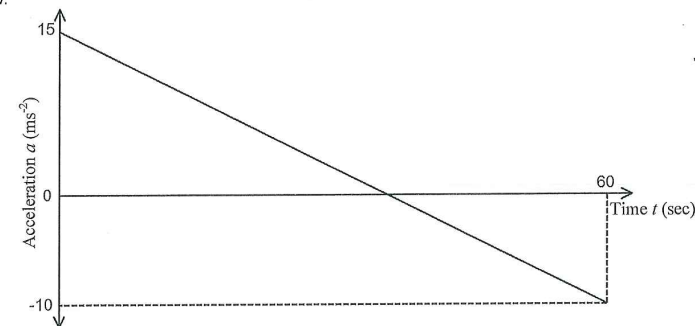
- (c) Sketch the function on the axes below. (4 marks)



Question 7

(8 marks)

A jet plane travels horizontally along a straight path for one minute, starting at time $t = 0$, where t is measured in seconds. The acceleration, a , measured in ms^{-2} , of the jet plane is given by the straight line graph below.



- (a) Find an expression for the acceleration of the jet plane during this time, in terms of t . (2 marks)

$$m = \frac{-25}{60} \quad a = -\frac{5}{12}t + 15 \quad \checkmark$$

$$= -\frac{5}{12}t \quad \checkmark$$

- (b) Given that when $t = 0$ the jet plane is travelling at 125 ms^{-1} , find its maximum velocity in ms^{-1} during the minute that follows. (3 marks)

$$\frac{-5}{12}t + 15 = 0 \quad \checkmark \quad \text{or} \quad V = -\frac{5t^2}{24} + 15t + C \quad \checkmark \quad \text{max at } (36, 395)$$

$$t = 36 \quad \checkmark \quad V = -\frac{5t^2}{24} + 15t + 125 \quad \checkmark \quad \therefore \text{max velocity of } 395 \text{ m/s} \quad \checkmark$$

- (c) Given that the jet plane breaks the sound barrier at 295 ms^{-1} , find out for how long the jet plane is travelling greater than this speed. (3 marks)

$$295 = -\frac{5x^2}{24} + 15x + 125 \quad \checkmark$$

$$x_1 = 14.091098 \quad x_2 = 57.908902 \quad \checkmark$$

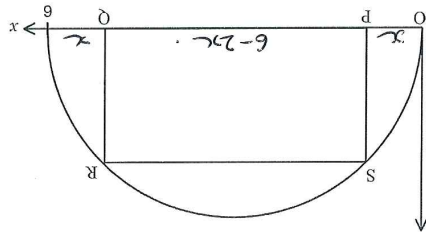
$$x_2 - x_1 = 43.817804$$

\therefore The plane is breaking the sound barrier for 43.82 seconds. ✓

(7 marks)

Consider the graph of the semicircle given by $f(x) = \sqrt{6x - x^2}$, for $0 \leq x \leq 6$.

A rectangle PQRS is drawn with upper vertices R and S on the graph of $f(x)$, and PQ on the x-axis, as shown in the following diagram.



Let $OP = x$.

(a) Explain why an equation for the area of the rectangle can be written as

(3 marks)

$$\begin{aligned} \text{length of rectangle (PQ)} &= 6 - 2x \\ \text{height of rectangle (PS)} &= \sqrt{6x - x^2} \end{aligned}$$

$$\therefore \text{Area of rectangle} = \text{length} \times \text{height} = (6 - 2x)\sqrt{6x - x^2}$$

(2 marks)

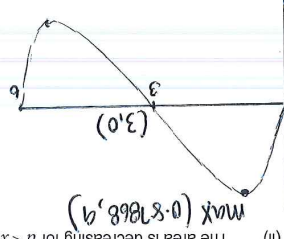
$$A' = \frac{4x^2 - 24x + 18}{\sqrt{-x^2 + 6x}}$$

$$A'(2) = -4.95$$

(2 marks)

$$a = 0.87868$$

$$b = 3$$



(iii) The area is decreasing for $a < x < b$. Find the value of a and of b .

Question 6

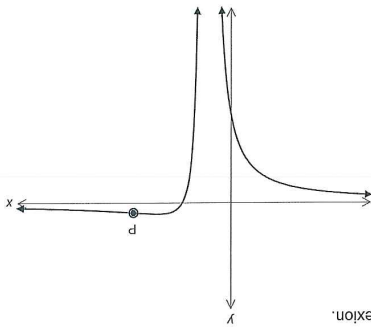
Question 4

(7 marks)

Consider the function $y = \frac{x-2}{(x-1)^2}$, $x \neq 1$.

A sketch of part of the graph of h is given below.

The point P is a point of inflexion.



(a) Find $h'(x)$, writing your answer in the form $\frac{a-x}{(x-1)^n}$

where a and n are constants to be determined.

(4 marks)

$$y' = \frac{(x-1)^2 \times 1 - (x-2) \times 2(x-1)}{(x-1)^4}$$

$$= \frac{(x-1)[(x-1) - 2(x-2)]}{(x-1)^4}$$

$$= \frac{(x-1)(3-x)}{(x-1)^4}$$

$$= \frac{3-x}{(x-1)^3}$$

$$a = 3, n = 3$$

symmetry in x-axis

Question 4 continued...

- (b) Given that $h''(x) = \frac{2x-8}{(x-1)^4}$, calculate the coordinates of P. (3 marks)

$$0 = \frac{2x-8}{(x-1)^4} \quad \checkmark$$

$$0 = 2x - 8$$

$$8 = 2x$$

$$x = 4 \quad \checkmark$$

$$y = \frac{4-2}{(4-1)^2}$$

$$= \frac{2}{9}$$

$$P = \left(4, \frac{2}{9}\right) \quad \checkmark$$

End of Section 1

Section 2 Calculator Assumed.

Name: _____

Question 5

(4 marks)

Given that $y = x^{\frac{1}{3}}$, use $x = 1000$ and the increments formula $\delta y = \frac{dy}{dx} \delta x$ to determine an appropriate value for $\sqrt[3]{1006}$.

$$y = x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{1}{3} x^{-\frac{2}{3}} \quad \checkmark \quad \delta x = 6$$

$$\delta y = \frac{dy}{dx} \times \delta x$$

$$= \frac{1}{3} (1000)^{-\frac{2}{3}} \times 6 \quad \checkmark$$

$$= 0.02 \quad \checkmark$$

$$\therefore \sqrt[3]{1006} \approx 10.02 \quad \checkmark$$