PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2020

Question/Answer booklet

12 SPECIALIST MATHS
UNIT 3

Section Two:
Calculator-assumed

Your Name ______________________________________

Your Teacher's Name________________________________

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Questio n	Marks	Max
8	5		17	12	
9	8		18	10	
10	10				
11	9				
12	12				
13	5				
14	9				
15	8				
16	10				

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examinatio n
Section One: Calculator-free	7	7	50	50	34
Section Two: Calculator-assumed	11	11	100	98	66
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **(98 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8 **(5 marks)**

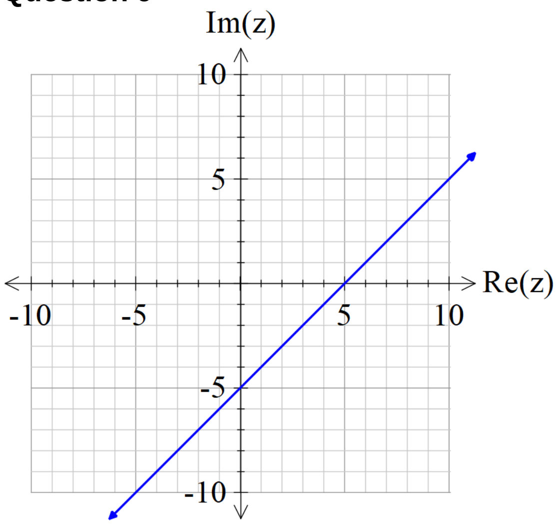
Consider the following system of linear equations with $p \,\&\, q$ are constants.

$$2x - y + pz = 0$$
$$x + 2y - 3z = q$$
$$-3x + 4y - 2z = 12$$

Determine all the values of $p \,\&\, q$ such that:

(i) There will be an unique solution
(ii) There will be infinite solutions
(iii) There will be no solutions

Question 9 **(8 marks)**



Consider the locus of $z = x + iy$ which is drawn above.

(a) If the locus above can be defined by $\mathrm{Im}(z) = a\,\mathrm{Re}(z) + b$, determine the constants $a \,\&\, b$. (2 marks)

(b) Determine the exact minimum value of $|z|$ on the locus above. (3 marks)

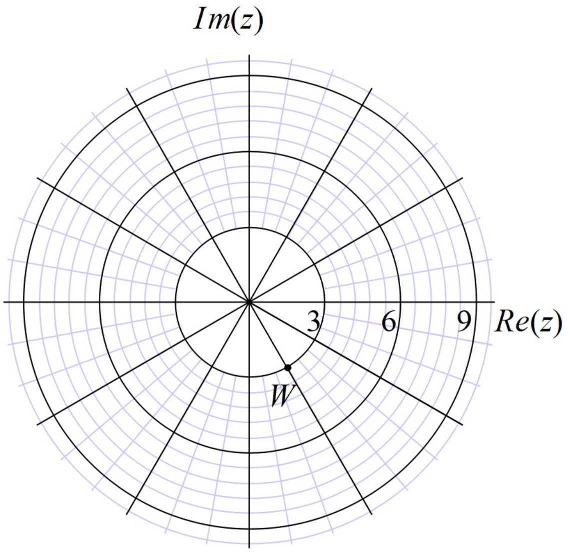
(c) Sketch the new locus of $|z - 5| = |z + 5i|$ on the axes above showing major features. (3 marks)

Question 10 **(10 marks)**

Let $z = \dfrac{-\sqrt{3}+i}{6}$.

(a) Express the complex number z in polar form using the principal argument in radians. (2 marks)

The complex number w is drawn in the complex plane as shown below.



(b) Express the complex number w in polar form using the principal argument (2 marks)

(c) Plot on the axes above, the complex numbers zw, zw^2 & zw^3. (4 marks)

(d) Explain geometrically the transformation effect of multiplying by w. (2 marks)

Question 11 **(9 marks)**

Consider the plane $r.\begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} = 7$.

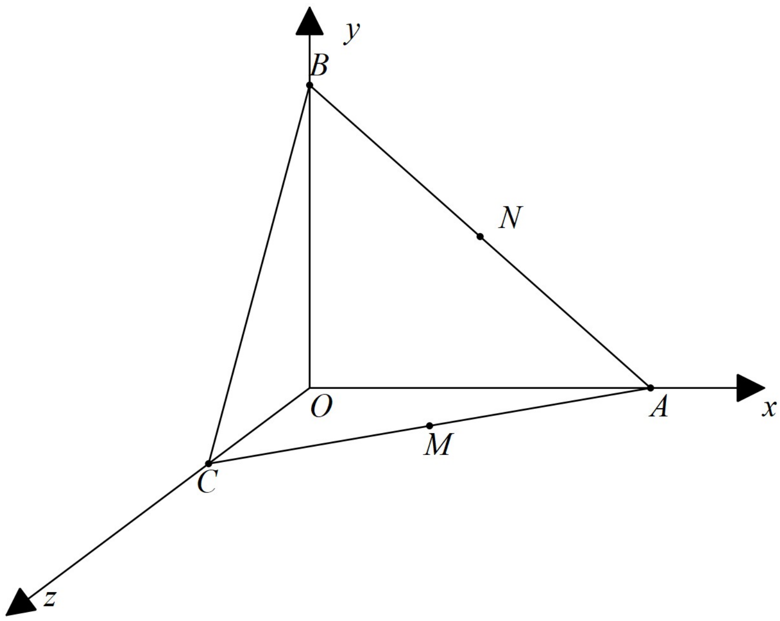
(a) Determine the vector equation of a line that passes through Point A $(3, 1, -7)$ and is perpendicular to the plane above. (2 marks)

(b) Hence or otherwise, determine the distance of point A from the plane above to one decimal place. (3 marks)

(c) Consider the sphere $\left| r - \begin{pmatrix} 6 \\ -3 \\ 11 \end{pmatrix} \right| = \alpha$ where α is a real constant. Determine the value(s) of α to two decimal places so that the line $r = \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -8 \\ 3 \end{pmatrix}$ is a tangent to the sphere. (4 marks)

Question 12 **(12 marks)**

Consider the plane ABC shown below with the following points $A(3,0,0), B(0,5,0) \& C(0,0,2)$.



Let $M \ \& \ N$ be the midpoints of $AC \ \& \ AB$ respectively.

(a) Determine the position vectors $OM \ \& \ ON$ (2 marks)

(b) Using vector methods, show that $\overline{BM} \ \& \ \overline{CN}$ trisect each other, that is divide each other in the ratio $2:1$. (4 marks)

Q12 cont

(c) Determine using **vector methods**, the area of the face ABC (3 marks)

(d) Determine the cartesian equation of the plane ABC. (3 marks)

Question 13 **(5 marks)**

Consider the plane Π $5x - 7y + 3z = 9$, which is parallel to a second plane Ω. Given that point $S(-11,5,1)$ is a point on plane Ω, determine the distance of point S from the plane Π to two decimal places.

Question 14 **(9 marks)**

Particle A started to move with constant velocity $\begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} km/h$ at 11:30am, at 1pm the particle was at position $(8,8,10)km$.

(a) Determine the position of the particle A at 11:30am. (2 marks)

Particle B left $(1,11,-2)km$ at 1pm, moving with constant velocity $\begin{pmatrix} 7 \\ -6 \\ 3 \end{pmatrix} km/h$.

(b) Determine the distance between the two particles at 2pm that day. (3 marks)
(Two decimal places)

(c) Determine the closest distance between the two particles, if they maintain their constant velocities, and the time it occurs. (Two decimal places) (4 marks)

Q14c continued

Question 15 **(8 marks)**

A particle moves with acceleration $\ddot{r} = \begin{pmatrix} 3\sin t \\ -20\cos(2t) + 2 \end{pmatrix} m/s^2$ at time t seconds.

Initially the particle is at the origin with velocity $v = \begin{pmatrix} 5 \\ 0 \end{pmatrix} m/s$

(a) Determine the velocity function at time t seconds. (2 marks)

(b) Determine the first two times that the particle is moving parallel to the x axis. (3 marks)
(2 decimal places)

(c) Determine the **exact** distance of the particle from the origin at time $t = \pi$ seconds.
(3 marks)

Question 16 **(10 marks)**

Consider the following motion defined by $r = \begin{pmatrix} 3\cos(2t + \frac{\pi}{4}) \\ -3\sin(2t + \frac{\pi}{4}) \end{pmatrix} m$ at time t seconds.

(a) Describe the motion. (2 marks)

(b) Determine the initial velocity and acceleration. (3 marks)

(c) Determine the time(s) that the velocity is perpendicular to the acceleration. Justify. (3 marks)

(d) Determine the exact distance travelled in the first 10 seconds. (2 marks)

Question 17 **(12 marks)**

At midday two rockets, A & B were observed moving in the sky above moving with constant velocities. Their positions and velocities were recorded as below at midday. They appear to have been moving for a number of hours and will continue to do so for many more.

$$r_A = \begin{pmatrix} 9 \\ -3 \\ 4 \end{pmatrix} km \quad , v_A = \begin{pmatrix} 7 \\ -2 \\ 5 \end{pmatrix} km/h$$

$$r_B = \begin{pmatrix} 5 \\ 8 \\ -5 \end{pmatrix} km \quad , v_B = \begin{pmatrix} 11 \\ 4 \\ -3 \end{pmatrix} km/h$$

Let t = number of hours from midday.

(a) Determine for Rocket A, the position vector from the origin at time t hours. (2 marks)

(b) Determine the cartesian equation for the path of Rocket A. (2 marks)

(c) Show that the rockets will not collide after midday. (2 marks)

Q17 continue-

(d) Determine the times **after** midday that the rockets are less than 60 km apart.

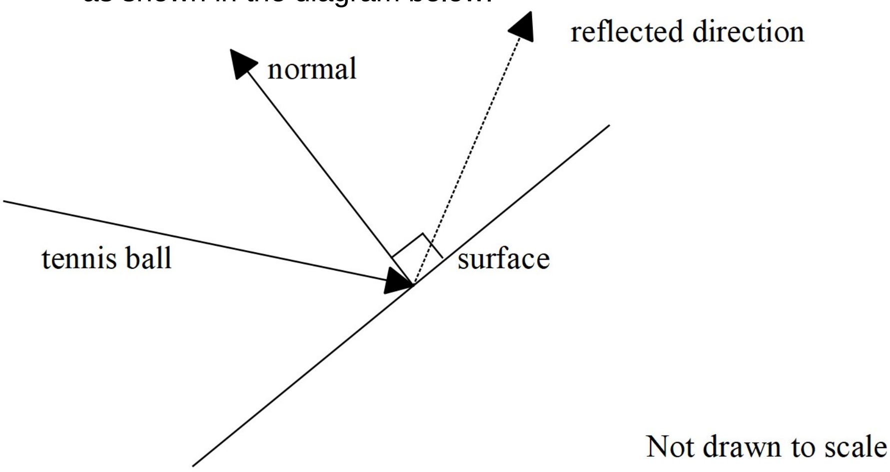
(3 marks)

(e) Determine the closest approach **from midday** and the time that this occurs.

(3 marks)

Question 18 **(10 marks)**

Consider a tennis ball moving with velocity $\begin{pmatrix} -2 \\ -7 \\ -3 \end{pmatrix} m/s$ that hits a surface with a normal vector of $\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$ as shown in the diagram below.



(a) Determine the angle between the velocity vector and the normal vector to two decimal places in degrees. (2 marks)

Let the unit vector $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ be parallel to the reflected direction of the tennis ball. This vector is in the same plane as the velocity and normal vectors above.

(b) Given that the tennis ball is reflected such that the angle with the normal equals that of the incident acute angle with the normal. Show that $\alpha + 4\beta - 5\gamma = 1.905$ when rounded to three decimal places. (3 marks)

Q18 continue-

(c) Derive another two independent equations for $\alpha, \beta \,\&\, \gamma$. (3 marks)

(e) Solve for $\alpha, \beta \,\&\, \gamma$ to two decimal places. (2 marks)

Working out space

Working out space