PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

**Semester One Examination, 2023**

**Question/Answer booklet**

# 12 SPECIALIST MATHEMATICS
# UNIT 3
## Section Two:
## Calculator-assumed

Your Name _____

Your Teacher's Name_____

### Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

### Materials required/recommended for this section
***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet (retained from Section One)

***To be provided by the candidate***
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

### Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

| Question | Marks | Max | Question | Marks | Max |
|---|---|---|---|---|---|
| 7 | | | 15 | | |
| 8 | | | 16 | | |
| 9 | | | 17 | | |
| 10 | | | 18 | | |
| 11 | | | | | |
| 12 | | | | | |
| 13 | | | | | |
| 14 | | | | | |

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 6 | 6 | 50 | 50 | 34 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 97 | 66 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

**See next page**

**Section Two: Calculator-assumed** **(97 Marks)**

This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

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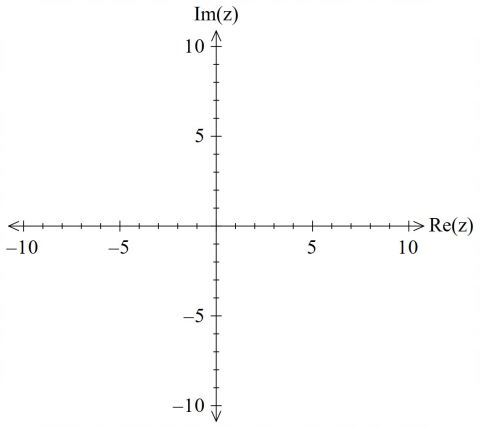
**Question 7** **(5 marks)**

Consider the locus $|z+5+3i|=2$.

a) Sketch the locus on the Argand Diagram below. (2 marks)




b) Determine the minimum value of: (3 marks)

i) $|z|$

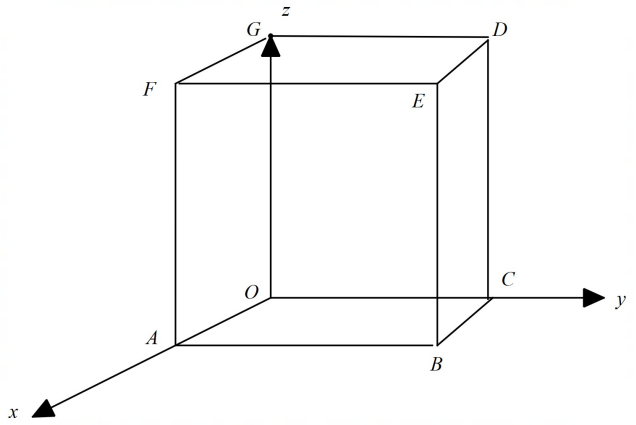
ii) $\text{Arg}(z)$

**Question 8** **(4 marks)**

Consider the plane $3x + 2y - z = 5$ and the point A$(-6, 7, 10)$.
Determine the distance of point A from the plane.

**Question 9** **(9 marks)**

Consider the rectangular box with vertices A(5,0,0), B(5,4,0),C(0,4,0), D(0,4,7), E(5,4,7),F(5,0,7)&G(0,0,7) and the origin.

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**Additional working space**

Question number: _____

**Additional working space**

Question number: _____



**Q9 continued-**

a) If point H divides the diagonal $\overline{AD}$ in the ratio 3:2, determine the position vector $OH$.
(2 marks)

b) Determine the cartesian equation of the plane that contains the points A, G & B.
(4 marks)

c) Prove that the diagonals of the box above, bisect each other using vectors.
(3 marks)

See next page 

**Question 10** **(6 marks)**

Consider the complex numbers $P, Q, R \& W$.

$|P| = 5$, Arg $(P) = \frac{3\pi}{4}$

$\overline{Q} = (1 - i)P$, $R = \frac{7}{iP^2}$

$W = \frac{\sqrt{5}PQ}{(\sqrt{3} - i)R}$

a) Determine the exact value of $|W|$ (3 marks)

b) Determine the exact value of Arg $(W)$. (3 marks)

See next page



**Additional working space**

Question number: _____

**End of questions**

Q18 continued



**Question 11** **(6 marks)**

Consider the sphere $\left| r - \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \right| = \alpha$, where $\alpha$ is a positive constant, and the line $r = \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$.

Determine all possible values of $\alpha$ such that:

i) There is only one point of contact between sphere and line.

ii) There are two points of contact between sphere and line.

iii) There are no points of contact between sphere and line.

See next page 

**Q11 continued.**

**Question 12 (6 marks)**

Particles $A$ and $B$ are moving with constant velocities and have initial positions $\begin{pmatrix}-8\\2\\10\end{pmatrix}$ m and $\begin{pmatrix}7\\7\\-15\end{pmatrix}$ m respectively. 2 seconds later $A$ is at $\begin{pmatrix}0\\-2\\4\end{pmatrix}$ m.

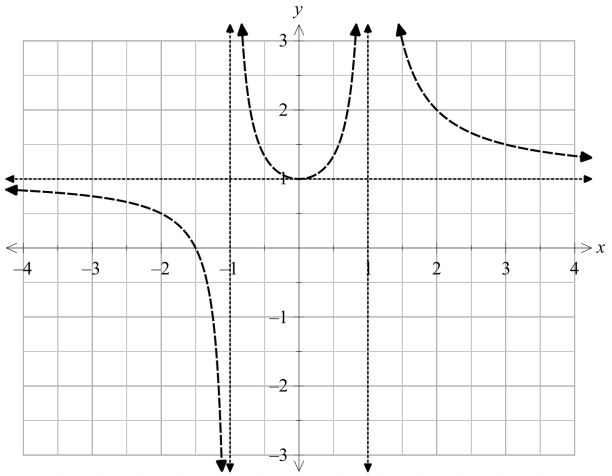
(a) Determine the velocity of $A$. (1 mark)

See next page



**Question 18** **(9 marks)**

The graph of $y=\frac{1}{f(x)}$ is shown with a dotted curve on the axes below.




(a) On the same axes draw the graph of $f(x)$. (4 marks)

(b) (i) The equation $|f(x)|=k$ has 4 solutions for what range of values of $k$? (2 marks)

(ii) Does the equation $|f(x)|=k$ ever have exactly 3 solutions? (1 marks)

(c) Determine the solutions to $f(-|x|)=2$. (2 marks)

**End of questions**

The laser beam is then reflected with direction $d = -i - 6j + k$.

(b) Determine the position vector of, $B$, the point where the beam hits the larger mirror. (3 marks)

A second beam is fired from the origin with a direction of $d_1 = ai + 6j + ck$. When it hits the smaller mirror, it is then reflected with direction of $d_2 = ai - 6j + ck$. You may assume that the speed of the beam does not change.

There are laser beams from the origin which after being reflected in the small mirror do not hit the larger mirror.

(c) Determine the range of values of $a$ and $c$, that ensure the beams **are reflected** in the larger mirror. (5 marks)



Q12 cont-

The velocity of $B$ is $\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ m/s.

(b) Show that the paths of $A$ and $B$ cross, state the position vector of this point, and explain whether the particles collide. (5 marks)

**See next page** 

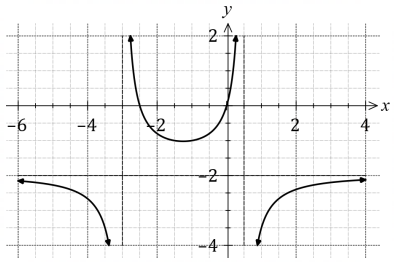
**Question 13** **(8 marks)**

(a) Determine the equations of all asymptotes of the graph of $y=f(x)$ when

(i) $f(x)=\dfrac{2+5x^2}{2x(1-3x)}$. (2 marks)

(ii) $f(x)=\dfrac{x^2-5}{x+6}$. (2 marks)

(b) The graph of $y=g(x)$ is shown in the diagram, together with its three asymptotes.




The defining rule is given by

$$g(x)=\frac{ax(2x+b)}{(x+c)(d-2x)}$$

*where* $a, b, c$ and $d$ are positive integer constants.

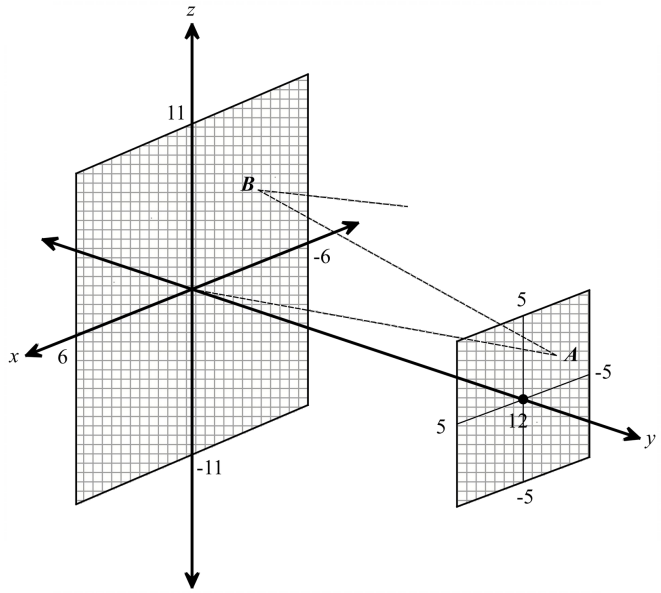
Determine, with brief reasons, the value of $a, b, c$ and $d$. (4 marks)





**Question 17** **(12 marks)**

Two parallel mirrors are shown in the diagram below. The larger mirror passes through the origin and is coincident with the $xz$ plane, and the smaller mirror is in the plane $y=12$.




A laser beam is fired through a small hole at the origin. The dotted line shows one such beam. The beam then hits the mirror at $y=12$ and is reflected back towards the larger mirror.

The laser beam is pointed with direction $d=-i+6j+k$.

(a) Determine the position vector of, $A$, the point where the beam hits the smaller mirror. (4 marks)

**Q16 cont-**

(d) Determine the exact polar form of $z_1+z_2$. (3 marks)

$z_1+z_2$, $z_3+z_4$ and $z_5+z_6$ are roots of $z^3=k\,cis\left(\frac{11\pi}{m}\right)$.

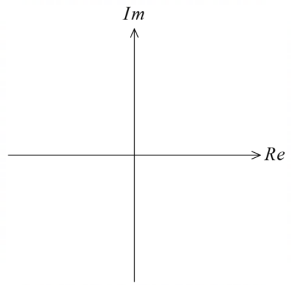
(d) Determine the values of $k$ and $m$. (3 marks)





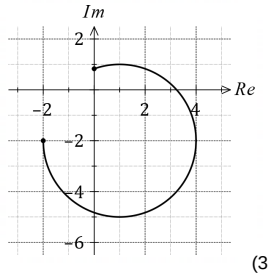
**Question 14** **(9 marks)**

(a) Draw the subset of the complex plane determined by $|z-2i|<¿\,z-2\vee¿$ on the axes below. (3 marks)




(b) The circular arc in the diagram represents the locus of a complex number $z$.

Without using $\Re(z)$ or $\Im(z)$, write equations or inequalities in terms of $z$ for the indicated locus.




(3 marks)

(c) Describe the subset, or sketch, of the complex plane determined by $|z-5i|+|z+5|=5\sqrt{2}$.

(3 marks)

**Question 15** **(8 marks)**

(a) Determine all solutions to the equation $z^3+27i=0$ in exact polar form. (3 marks)

(b) Consider the seventh roots of unity expressed in polar form $r\,cis\,\theta$.

(i) Determine the roots for which $-\pi<\theta\leq\frac{\pi}{2}$. (2 marks)

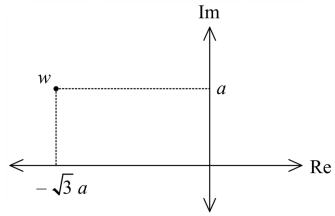
(ii) Use all seven roots to show that $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)=-\frac{1}{2}$. (3 marks)





**Question 16** **(15 marks)**

The complex number $w$ has been plotted on the Argand diagram below.




(a) Express $w$ in Cartesian form. (1 mark)

(b) Express $w$ in polar form. (3 marks)

(c) The complex number $z_1$ is a root of $z^6=w$, with the smallest positive argument.

(i) Given that $a=32$, determine $z_1$ in polar form. (3 marks)

(ii) Determine the remaining roots in polar form. Label the roots as $z_2, z_3, z_4, z_5$ and $z_6$ moving in an anticlockwise direction from the positive real axis. (2 marks)