PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS METHODS
UNIT 3

Section One:
Calculator-free

Your Name: ______________________________

Your Teacher's Name: ______________________________

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Mark	Max
1		5	5		8
2		7	6		10
3		7	7		6
4		5			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	48	33
Section Two: Calculator-assumed	12	12	100	98	67
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free **(48 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 **(5 marks)**

Jamie and Catherine are racing on roller skates. They race along a long, straight track, and whoever has gone the farthest after 5 seconds wins a prize.
If Jamie can skate at a velocity of $f(t)=5+2t$ m/s and Catherine can skate at a velocity of $g(t)=t^2+2$ m/sec, what is the outcome of this race?

Jamie: $\int_0^5 5+2t\,dt=\left[5t+t^2\right]_0^5=50\,m$

Catherine $\int_0^5 t^2+2\,dt=\left[\frac{t^3}{3}+2t\right]_0^5=\frac{125}{3}+\frac{30}{3}=\frac{155}{3}=51\frac{2}{3}\,m$

Therefore, Catherine wins.

✔ Obtain the correct antiderivative for Jamie’s velocity

✔ Calculate the correct displacement for Jamie

✔ Obtain the correct antiderivative for Catherine’s velocity

✔ Calculate the correct displacement for Catherine

✔ State the outcome and correctly supported by calculus

✔

Question 2 **(7 marks)**

Suppose that $f(x)$ and $g(x)$ are differentiable functions and that $h(x)=f(x)g(x)$. You are given the following table of values.

x	-1
$h(x)$	-9
$g(x)$	9
$f'(x)$	-2
$h'(x)$	-20

(a) Determine the value for $f(-1)$ and hence determine $\frac{d}{dx}[f(x)h(x)]$ when $x=-1$.

(3 marks)

$$f(-1)=\frac{h(-1)}{g(-1)}=-1$$

$$\frac{d}{dx}[f(x)h(x)]=f(x)h'(x)+f'(x)h(x)=(-1)(-20)+(-2)(-9)=38$$

✔ Determine the correct value for $f(-1)$

✔ Uses product rule

✔ States derivative at $x=-1$

(b) Determine the value for $\frac{d}{dx}[h(-1)]^2$ (2 marks)

$$\frac{d}{dx}[h(-1)]^2=2h(-1)h'(-1)=2(-9)(-20)=360$$

✔ Demonstrate the use of chain rule

✔ Determine the correct value

(c) Determine the value for $T'(-1)$, given that $T(x)=f(f(x))$. (2 marks)

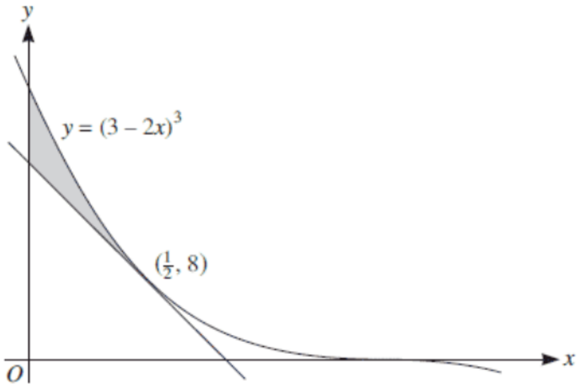
$$T'(-1)=f'(f(-1))\times f'(-1)=f'(-1)\times f'(-1)=(-2)(-2)=4$$

✔ Demonstrate the use of chain rule

✔ Determine the correct value

Question 3 **(7 marks)**

The diagram below shows the curve $y=(3-2x)^3$ and the tangent line to the curve at $\left(\frac{1}{2},8\right)$.



(a) Find the equation of this tangent, giving your answer in the form $y=mx+c$. (3 marks)

$$y'=3(3-2x)^2(-2)=-6(3-2x)^2$$

$$x=\frac{1}{2},\ y'=-24=m$$

$$8=-24\left(\frac{1}{2}\right)+c,\ c=20$$

$$y=-24x+20$$

✔ Determine the gradient $m=-24$

✔ Substitute the coordinate to solve the correct value for c

✔ Give the equation of the tangent line

(b) Find the area of the shaded region. (4 marks)

$$\int_0^{\frac{1}{2}} (3-2x)^3-(-24x+20)\,dx=¿¿$$

✔ Use the integration of difference of the two functions

✔

✔ Use the correct boundary points

✔ Determine the correct expression for the antiderivative

✔ Determine the correct area

Question 4 **(5 marks)**

The total cost, \$$C$ to manufacture x items at a factory is given by the rule $C=(2x+16)^3$.
Determine the minimum value of the **average cost per unit**. Justify

$$C = (2x+16)^3$$

$$Av = \frac{(2x+16)^3}{x}$$

$$\frac{dAv}{dx} = \frac{x3(2x+16)^2 2 - (2x+16)^3}{x^2} = \frac{(2x+16)^2\left[6x - (2x+16)\right]}{x^2} = \frac{(2x+7)^2\left[4x - 16\right]}{x^2}$$

$$x = 4$$

$$x = 3 \quad (4(3) - 16) < 0$$

$$x = 5 \quad (4(5) - 16) > 0$$

$$\therefore local \quad \min$$

$$Min\ Av\ \cos t = \$\frac{24^3}{4}$$

- ✔ Demonstrate the expression for average in terms of x
- ✔ Determine the derivative using quotient rule
- ✔ Equating derivative to 0 and solve for x
- ✔ Use the sign test or 2nd derivative to reason why minimum is reached
- ✔ Calculate average cost in index form, un-simplified

Question 5 **(8 marks)**

(a) Consider a cubic polynomial $y = Ax^3 + 6x^2 - Bx$, where A and B are unknown constants. Determine the values of A and B,so that the graph of y has a maximum value at $x=-1$ and an inflection point at $x=1$. (4 marks)

$$y' = 3Ax^2 + 12x - B$$

$$y'' = 6Ax + 12$$

$$\{y' = 3Ax^2 + 12x - B = 0 \; y'' = 6Ax + 12 = 0\}$$

$$A = -2$$
$$0 = 3(-2) - 12 - B$$
$$B = -18$$

- ✔ Determine the 1st derivative
- ✔ Determine the 2nd derivative
- ✔ Equating both to 0
- ✔ Determine the correct values for for A & B

b) Find the point (x, y) on the graph of $f(x) = \sqrt{x-2}$ where the tangent line is perpendicular to the line $4x + y = 1$. (4 marks)

$$f(x) = \sqrt{x-2}$$

$$f'(x) = \frac{1}{2}(x-2)^{\frac{-1}{2}} = \frac{1}{2\sqrt{x-2}}$$

$$4x + y = 1 \quad m = -4 \quad perpendicular = \frac{1}{4}$$

$$\frac{1}{2\sqrt{x-2}} = \frac{1}{4} \quad x = 6$$

$$(6, 2)$$

- ✔ Determine the perpendicular gradient to be $\frac{1}{4}$
- ✔ Determine the expression for $f'(x)$
- ✔ Equating $f'(x)$ to $\frac{1}{4}$ and solve for x
- ✔ State correct coordinates

Question 6 **(10 marks)**

A discrete random variable X has the probability function

$$P(X=x)=\begin{cases} k(1-x)^2 & x=-1,0,1 \text{ and } 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k=\frac{1}{6}$ (3 marks)

$$k(1-(-1))^2+k(1-0)^2+k(1-1)^2+k(1-2)^2=1$$
$$4k+1k+0k+1k=1$$
$$6k=1$$
$$k=\frac{1}{6}$$

✔ Substituting values for x

✔ Equating the probability sum to 1

✔ Simplify and solve for $k=\frac{1}{6}$

(b) Determine $E(X)$. (2 marks)

x	-1	0	1	2
$P(X=x)$	$\frac{4}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$

$$E(X)=-1\times\frac{4}{6}+0\times\frac{1}{6}+1\times 0+2\times\frac{1}{6}=\frac{-1}{3}$$

✔ Determine individual probability for each x value

✔ Determine the correct value for $E(X)$

(c) Show that $E(X^2)=\frac{4}{3}$ (2 marks)

x	-1	0	1	2
x^2	1	0	1	4
$P(X=x)$	$\frac{4}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$

$$E(X^2)=1\times\frac{4}{6}+0\times\frac{1}{6}+1\times 0+4\times\frac{1}{6}=\frac{4}{3}$$

✔ Determine correct values for x^2

✔ Demonstrate the equation to calculate $E(X^2)$

(d) Find $Var(1-3X)$ (3 marks)

$$Var(X)=\sum(x-\mu)^2 p$$

$$=\left(-1+\frac{1}{3}\right)^2\frac{4}{6}+\left(0+\frac{1}{3}\right)^2\frac{1}{6}+\left(1+\frac{1}{3}\right)^2 0+\left(2+\frac{1}{3}\right)^2\frac{1}{6}$$ (SCSA prefferred method)

OR $Var(X)=E(X^2)-[E(X)]^2=\frac{4}{3}-\left(\frac{1}{3}\right)^2=\frac{11}{9}$

$$Var(1-3X)=9\times\frac{11}{9}=11$$

- ✔ Determine the correct value for $Var(X)$ (maybe unsimplified)
- ✔ Recognise the scalar as 9
- ✔ Determine the correct value for $Var(1-3X)$

Question 7 **(6 marks)**

Let $f(x)$ be a non-zero function such that $f'(x)=[f(x)]^2$.

(a) Determine an expression for $\frac{d}{dx}\frac{1}{f(x)}$. (3 marks)

$$\frac{d}{dx}\frac{1}{f(x)}=\frac{-f'(x)}{[f(x)]^2}=\frac{-[f(x)]^2}{[f(x)]^2}=-1$$

- ✔ Differentiate using quotient rule
- ✔ Substitute either numerator or denominator using the equation given
- ✔ Determine the expression as -1

(b) Determine an expression for $f(x)$ using the result from (a), or otherwise, given $f(0)=\frac{1}{2}$. (3 marks)

$$\int \frac{d}{dx}\frac{1}{f(x)}=-x+c$$

$$-0+c=2, c=2$$

$$\frac{1}{f(x)}=-x+2$$

$$f(x)=\frac{1}{-x+2}$$

✔️ Integrate the result from (a) using the F.T.C

✔️ Substitute to solve for c, and hence determine the expression for $\frac{1}{f(x)}$

✔️ Determine the expression for $f(x)$