Question Seven: [3, 4 = 7 marks] **CF**

(a) Calculate the gradient of the curve $y=\frac{e^{-2x}}{5x}$ at $x=-1$.

$$\frac{dy}{dx}=\frac{5x(-2e^{-2x})-5e^{-2x}}{25x^2}$$
$$\frac{dy}{dx}=\frac{-5(-2e^2)-5e^2}{25}=\frac{e^2}{5}$$

(b) Determine the equation of the tangent to the curve $f(x)=-\cos(4x)$ at $x=\frac{\pi}{6}$.

$$f'(x)=4\sin(4x)$$
$$f'\left(\frac{\pi}{6}\right)=4\sin\frac{2\pi}{3}=2\sqrt{3}$$
$$f\left(\frac{\pi}{6}\right)=\frac{1}{2}$$
$$y=2\sqrt{3}x+c$$
$$\frac{1}{2}=2\sqrt{3}\left(\frac{\pi}{6}\right)+c$$
$$c=\frac{3-2\pi\sqrt{3}}{6}$$
$$\therefore y=2\sqrt{3}x+\frac{3-2\pi\sqrt{3}}{6}$$

education equals

Calculator Free
Differentiation Techniques

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2, 3, 3, 3, 3, 3 = 18 marks] **CF**

Differentiate each of the following functions with respect to x. Do not simplify your answers.

(a) $y=e^{-3x}$

(b) $g(x)=-\cos\left(\frac{x}{2}\right)$

(c) $f(x)=x^2e^{2x-1}$

(d) $y=\frac{\sin x}{5x-1}$

(e) $h(x)=\sqrt{x^4-2x}$

(f) $y=\sin^2(4x)$

(g) $y=2f(3x-1)$

Question Six: [5 marks] **CF**

By using first principles and the limits $\lim\limits_{\theta\to0}\dfrac{\sin\theta}{\theta}=1$ and $\lim\limits_{\theta\to0}\dfrac{\cos\theta-1}{\theta}=0$, establish that $\dfrac{d}{dx}\sin x=\cos x$.

Remember that $\sin(A+B)=\sin A\cos B+\cos A\sin B$.

$$=\lim_{h\to0}\frac{\sin(x+h)-\sin x}{h}\ ✓$$
$$=\lim_{h\to0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}\ ✓$$
$$=\lim_{h\to0}\frac{\sin x(\cos h-1)+\cos x\sin h}{h}\ ✓$$
$$=\lim_{h\to0}\frac{\sin x(\cos h-1)}{h}+\lim_{h\to0}\frac{\cos x\sin h}{h}\ ✓$$
$$=0+\cos x\ ✓$$
$$=\cos x$$

Question Four: **[5 marks]** **CF**

Given that $y = e^{x^2-1}$, show that $\frac{d^2y}{dx^2} \times y^{-1} - 2 = 4x^2$

$\frac{dy}{dx} = 2xe^{x^2-1}$ ✓

$\frac{d^2y}{dx^2} = 2e^{x^2-1} + 4x^2e^{x^2-1}$ ✓

$= 2e^{x^2-1}(1+2x^2)$ ✓

$\frac{d^2y}{dx^2} \times y^{-1} - 2$ ✓

$= 2e^{x^2-1}(1+2x^2) \times \frac{1}{e^{x^2-1}} - 2$

$= 2 + 4x^2 - 2$ ✓

$= 4x^2$

Question Five: **[2 marks]** **CF**

Given $f'(g(x)) = e^{0.5x}\cos(2e^{0.5x})$ and $g(x) = e^{0.5x}$, determine $f(x)$.

$f(x) = \sin 2x$

✓✓

Question Two: **[4 marks]** **CF**

Show, using the quotient rule, that $\frac{d}{dx}\tan(x) = 1 + \tan^2 x$.

Question Three: **[4 marks]** **CF**

A curve is defined parametrically as $x = 4t$ and $y = t^3 - 1$.

Determine an expression for the rate of change of y with respect to x, in terms of x only.
Simplify your answer.

Question Four: **[5 marks]** **CF**

Given that $y = e^{x^2-1}$, show that $\frac{d^2y}{dx^2} \times y^{-1} - 2 = 4x^2$

Question Five: **[2 marks]** **CF**

Given $f'(g(x)) = e^{0.5x}\cos(2e^{0.5x})$ and $g(x) = e^{0.5x}$, determine $f(x)$.

Question Two: **[4 marks]** **CF**

Show, using the quotient rule, that $\frac{d}{dx}\tan(x) = 1 + \tan^2 x$.

$$y = \tan x = \frac{\sin x}{\cos x} \checkmark$$
$$\frac{dy}{dx} = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2 x} \checkmark$$
$$\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \checkmark$$
$$\frac{dy}{dx} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} \checkmark$$
$$\frac{dy}{dx} = 1 + \tan^2 x$$

Question Three: **[4 marks]** **CF**

A curve is defined parametrically as $x = 4t$ and $y = t^3 - 1$.

Determine an expression for the rate of change of y with respect to x, in terms of x only. Simplify your answer.

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$
$$\frac{dy}{dx} = 3t^2 \times \frac{1}{4} \checkmark \checkmark$$
$$\frac{dy}{dx} = \frac{3t^2}{4}$$
$$t = \frac{x}{4} \checkmark$$
$$\therefore \frac{dy}{dx} = \frac{3\left(\frac{x}{4}\right)^2}{4}$$
$$= \frac{3x^2}{16 \times 4}$$
$$= \frac{3x^2}{64} \checkmark$$

(e) $h(x) = \sqrt{x^4 - 2x}$

$h'(x) = \frac{1}{2}(x^4 - 2x)^{\frac{-1}{2}}(4x^3 - 2)$

(f) $y = \sin^2(4x)$

$y = (\sin(4x))^2$

$\frac{dy}{dx} = 2(\sin(4x))(4\cos(4x))$

(g) $y = 2f(3x-1)$

$\frac{dy}{dx} = 2f'(3x-1)(3)$

Question Six: [5 marks] **CF**

By using first principles and the limits $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$, establish that $\frac{d}{dx}\sin x = \cos x$.

Remember that $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

Question Seven: [3, 4 = 7 marks] **CF**

(a) Calculate the gradient of the curve $y = \dfrac{e^{-2x}}{5x}$ at $x = -1$.

(b) Determine the equation of the tangent to the curve $f(x) = -\cos(4x)$ at $x = \dfrac{\pi}{6}$.

education equals

SOLUTIONS
Calculator Free
Differentiation Techniques

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2, 3, 3, 3, 3, 3 = 18 marks] **CF**

Differentiate each of the following functions with respect to x. Do not simplify your answers.

(a) $y = e^{-3x}$

$$\frac{dy}{dx} = -3e^{-3x}$$

(b) $g(x) = -\cos\left(\dfrac{x}{2}\right)$

$$g'(x) = \frac{1}{2}\sin\left(\frac{x}{2}\right)$$

(c) $f(x) = x^2 e^{2x-1}$

$$f'(x) = 2x(e^{2x-1}) + 2x^2 e^{2x-1}$$

(d) $y = \dfrac{\sin x}{5x-1}$

$$\frac{dy}{dx} = \frac{(5x-1)(\cos x) - (5\sin x)}{(5x-1)^2}$$