

ACKNOWLEDGEMENTS

Question 9
Data source: Australian Bureau of Statistics. (n.d.) *CensusAtSchool Australia random sampler*. Retrieved January 23, 2012, from www.cas.abs.gov.au/cgi-local/cassampler.pl.

- Question 13
Map of Lake Letroy by courtesy of the examining panel.
- Question 14
Internet access survey data by courtesy of the examining panel.
- Question 14(c)
Dot frequency graph of internet access for the suburb of Curtin by courtesy of the examining panel.
- Question 14(d)
Dot frequency graph of internet access for two Deakin suburbs by courtesy of the examining panel.
- Question 16(d)
Graph of fish population by courtesy of the examining panel.
- Question 17
Diagram of forest bounded by roads by courtesy of the examining panel.
- Question 18
Diagram of hemispherical paperweight by courtesy of the examining panel.

MATHEMATICS
2A/2B

Calculator-assumed

WACE Examination 2012

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

When examiners design an examination, they develop provisional marking keys that can be reviewed at a marking key ratification meeting and modified as necessary in the light of candidate responses.

Section Two: Calculator-assumed

(100 Marks)

Question 8

(15 marks)

- (a) Charlotte earns \$16 per hour waitressing during weekdays (Monday – Friday). From Monday to Friday, she works a total of 15 hours.

- (i) How much does Charlotte earn for working Monday to Friday? (1 mark)

Solution
$16 \times 15 = \$240$
Specific behaviours
✓ correctly uses the hourly rate to calculate the wage

On Saturdays Charlotte's pay is 10% more than her weekday rate and on Sundays her pay is 20% more than her Saturday rate.

- (ii) How much does Charlotte earn per hour on Sundays? (2 marks)

Solution
$16 \times 1.1 \times 1.2 = \$21.12 / \text{h}$
Specific behaviours
✓ identifies the total percentage increase (i.e. 132% or 1.32) or correctly calculates Saturday's rate
✓ correctly calculates the Sunday rate

- (iii) Determine Charlotte's weekend pay when she works 2 hours on Saturday and 5 hours on Sunday. (2 marks)

Solution
$16 \times 1.1 \times 2 + 21.12 \times 5 = 35.20 + 105.60$ $= \$140.80$
Specific behaviours
✓ calculates one day's wage
✓ calculates the other day's wage or the total wage

- (b) The Earth has a land area of 148 940 000 km².

- (i) Write this number using scientific notation. (1 mark)

Solution
1.4894×10^8
Specific behaviours
✓ correctly converts to scientific notation

- (c) Lochie says he calculated Row 20 to have an answer in the middle column of 177 001. Tayla does not know the numbers used by Lochie in his calculation but she knows Lochie's answer is not a possible result. How might Tayla have used her calculator to work out that Lochie's answer was not possible? (1 mark)

Solution
Tayla uses the square root of the number to show it is not a perfect square
Specific behaviours
✓ correctly reasons Lochie's answer is not a perfect square

- (d) The left column of the pattern above consists of three terms. For example in Row 1 the three terms are 1², 2² and 2². The bases for these three terms are 1, 2 and 2. Within each row a relationship exists between the bases of the first two terms and the base of the third term. What is this relationship? (2 marks)

Solution
Base 1 × Base 2 = Base 3
Specific behaviours
✓ provides an example to demonstrate relationship
✓ correctly generalises relationship

- (e) Write Row 25 of the above number pattern. (2 marks)

Solution
$25^2 + 26^2 + 650^2 = 423\,801 = 651^2$ Row 25
Specific behaviours
✓ correctly completes the left side of the equation
✓ correctly completes all of Row 25

(ii) Australia has a land area of 7 682 300 km². What percentage of the Earth's land area is this? Give your answer to **two (2)** decimal places. (2 marks)

Solution
$\frac{7\,682\,300}{148\,940\,000} \times 100\% = 5.16\%$
Specific behaviours
✓ represents the correct expression for calculating the percentage
✓ correctly rounds percentage to two decimal places

(c) Angelique sells chemical-free beauty products.

A body moisturiser is sold in bottles in two sizes:

375 mL for \$19.70 or 600 mL for \$32

Angelique needs to advise her clients which bottle is better value for money. Use calculations to show which bottle is the better buy. (3 marks)

Solution
$\frac{19\,70}{375} = 5.25\text{ c / mL} \qquad \text{or} \qquad \frac{32\,00}{600} = 5.33\text{ c / mL}$ <p>therefore the 375 mL bottle is the better buy</p>
Specific behaviours
✓ calculates the unit pricing for the 375 mL bottle or cost per millilitre, in cents, for 375 mL bottle
✓ calculates the unit pricing for the 600 mL bottle or cost per millilitre, in cents, for 600 mL bottle
✓ compares the unit pricings and states the best buy

(d) Sonya receives a 22% commission on all she sells.

(i) Determine the amount of money Sonya earns if she sells \$520 worth of products. (2 marks)

Solution
Commission = 22% of 520 = \$114.40
Specific behaviours
✓ identifies commission as the percentage of the sales
✓ calculates the commission correctly

Question 19

(8 marks)

Consider the first four complete rows of the number pattern given below.

Row 1	$1^2 + 2^2 + 2^2$	=	9	=	3^2
Row 2	$2^2 + 3^2 + 6^2$	=	49	=	7^2
Row 3	$3^2 + 4^2 + 12^2$	=	169	=	13^2
Row 4	$4^2 + 5^2 + 20^2$	=	441	=	21^2
Row 5	$5^2 + 6^2 + 30^2$	=	_____	=	_____
Row 6	_____	=	_____	=	_____

(a) Complete Row 5. (1 mark)

Solution
$5^2 + 6^2 + 30^2 = 961 = 31^2$
Specific behaviours
✓ correctly completes Row 5

(b) Extend the pattern by completing Row 6. (2 marks)

Solution
$6^2 + 7^2 + 42^2 = 1849 = 43^2$
Specific behaviours
✓ correctly completes left hand expression in Row 6
✓ correctly completes middle term and right hand term in Row 6

- (ii) In one week Sonya earns \$92.40 commission. What was the value of her sales in this week? (2 marks)

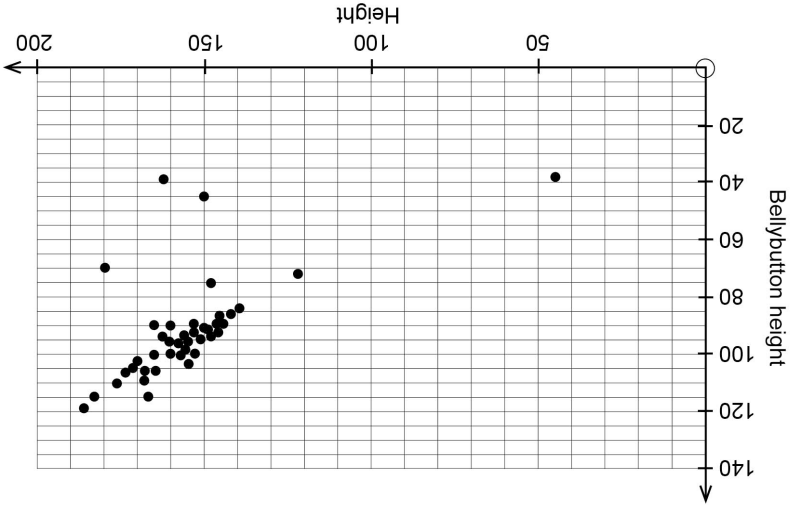
Solution
<p>Commission = 22% of sales</p> <p>92.40 = 22% × sales</p> <p>Sales = 92.40/0.22</p> <p>= \$420</p> <p>Alternative solution</p> <p>22% = 92.40</p> <p>1% = $\frac{92.4}{22} = \\$4.20$</p> <p>100% = $\frac{92.4}{22} \times 100$</p> <p>= \$420</p>
Specific behaviours
<p>✓ substitutes correct information into equation to be solved</p> <p>✓ correctly calculates sales</p> <p>or</p> <p>✓ correctly calculates 1%</p> <p>✓ correctly calculates 100%</p>

- (b) Another solid paperweight, in the shape of a cylinder with radius 3 cm, is made from 60 cm³ of glass. What is the height of this paperweight? (3 marks)

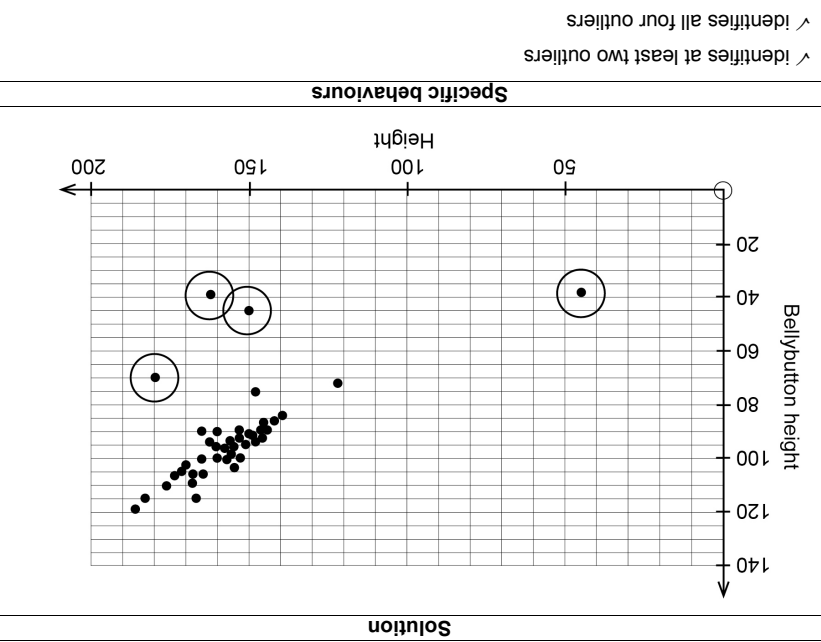
Solution
<p>Volume = $\pi r^2 h$</p> <p>$60 = \pi \times 3^2 \times h$</p> <p>$h = \frac{60}{9\pi}$</p> <p>= 2.12 cm</p>
Specific behaviours
<p>✓ shows use of correct formula</p> <p>✓ substitutes into formula correctly</p> <p>✓ correctly evaluates the required height</p>

Question 9 (8 marks)

Fifty students were randomly chosen from those who completed the 2011 CensusAtSchool survey. Their heights and bellybutton heights are plotted below.



(a) Circle the **four (4)** outliers, then ignore them for the rest of the question. (2 marks)

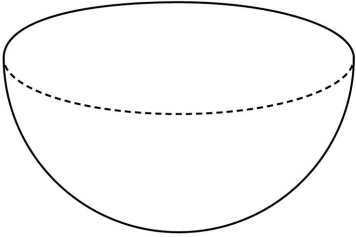


✓ identifies at least two outliers
✓ identifies all four outliers

(c) Determine, to the nearest metre, the distance between K and M. (3 marks)

Solution	
$\tan 23.6 = \left(\frac{KM}{1120} \right) \quad \text{or} \quad \frac{KM}{1120} = \frac{1752}{765.66}$	
$KM = 1120 \times \tan 23.6 \quad \text{or} \quad \frac{KM}{1120} = 489.47$	
$KM = 489.47$	
$= 489 \text{ m}$	
✓ states a correct trigonometric ratio to solve for KM or uses proportion ✓ correctly reorganises ratio to find KM ✓ correctly calculates the length of KM to the nearest metre	

Question 18 (7 marks)



A solid metal paperweight is in the shape of a hemisphere (half of a sphere) with radius 3 cm.

(a) Determine the surface area of the paperweight. (4 marks)

Solution	
$\text{Surface area} = 2\pi r^2 + \pi r^2$	
$= 3 \times \pi \times 3^2$	
$= 27\pi$	
$= 84.8 \text{ cm}^2$	
Specific behaviours	
✓ shows use of correct formula for the curved surface of the hemisphere ✓ includes area of circular base in surface area ✓ shows correct substitution into formula ✓ correctly evaluates the surface area	

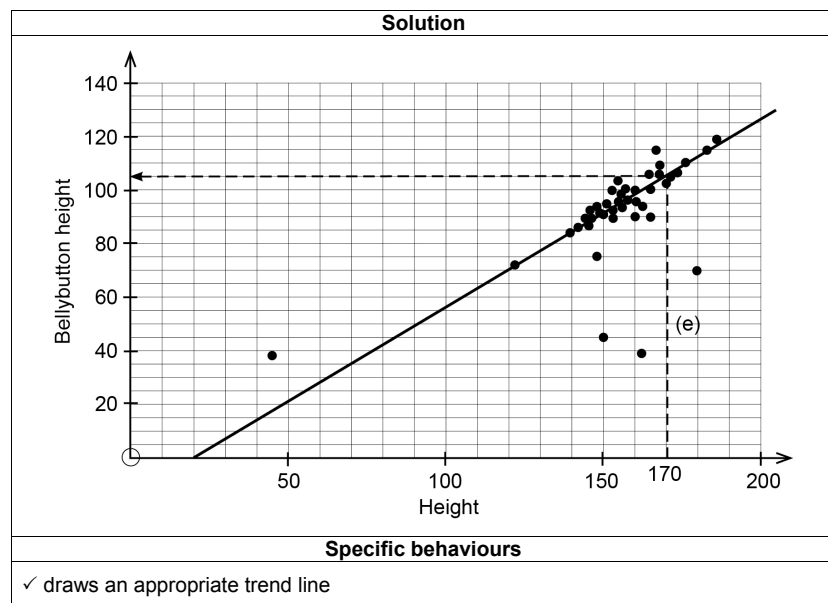
- (b) What is the approximate difference between someone's height and the height of their bellybutton? (1 mark)

Solution
60 cm (55 cm – 65 cm)
Specific behaviours
✓ calculates the approximate difference

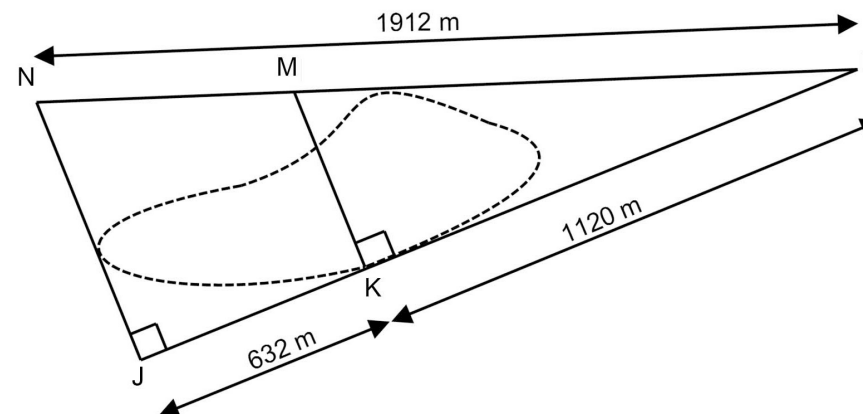
- (c) Describe the trend in this data. (1 mark)

Solution
increasing
Specific behaviours
✓ identifies the trend

- (d) Draw a trend line for this data. (1 mark)



Question 17 (8 marks)



The diagram above shows an area of forest bounded by roads JKL, JN and NML. It is known that JK = 632 m, KL = 1120 m, NL = 1912 m and that angles LJN and LKM are both 90°.

- (a) Determine the length of road JN. (2 marks)

Solution
$\overline{JN}^2 = 1912^2 - (632 + 1120)^2$ $= 765.66 \text{ m}$ $= 766 \text{ m}$
Specific behaviours
✓ correctly shows use of Pythagoras' Theorem ✓ correctly states the length of the road

- (b) Use trigonometry to determine the size of angle JLN. (3 marks)

Solution
$\cos \theta = \left(\frac{JL}{LN} \right) \quad \sin \theta = \left(\frac{JN}{LN} \right) \quad \tan \theta = \left(\frac{JN}{JL} \right)$ $\cos \theta = \left(\frac{1752}{1912} \right) \quad \text{or} \quad \sin \theta = \left(\frac{765.66}{1912} \right) \quad \text{or} \quad \tan \theta = \left(\frac{765.66}{1752} \right)$ $\theta = 23.6^\circ \quad \theta = 23.6^\circ \quad \theta = 23.6^\circ$
Specific behaviours
✓ states the trigonometric ratio equation correctly ✓ substitutes values into the equation ✓ correctly calculates the size of the angle

(e)

Show use of your trend line to predict the bellybutton height for someone who is 170 cm tall and state your prediction.

(2 marks)

Solution	
105 cm	
Specific behaviours	
✓ shows use of trend line for height of 170 cm ✓ correctly states predicted value from their trend line	

Why would your prediction in Part (e) be considered reliable?

(1 mark)

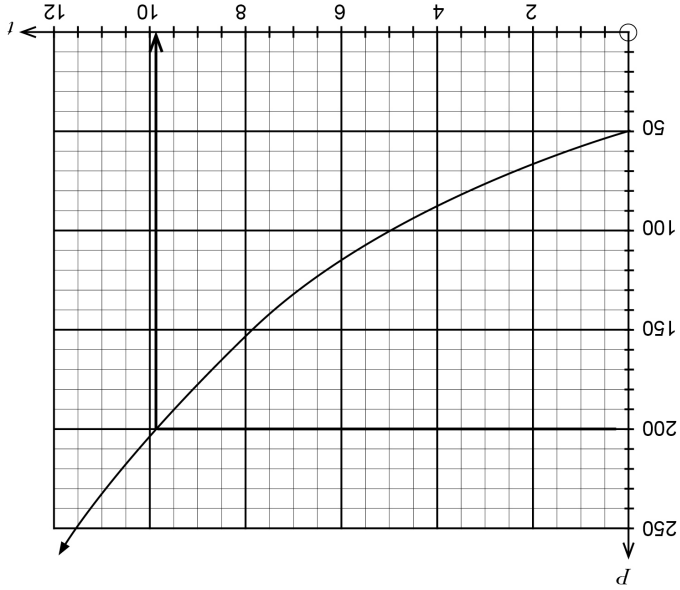
Solution	
The prediction is reliable as it requires interpolation or as most dots are near the trend line.	
Specific behaviours	
✓ identifies prediction as interpolation or a similar valid explanation ✓ makes reference to the strength of the relationship or	

(f)

(d)

A graph of the fish population is shown below. Use the graph to determine during which month the population of the fish first exceeded 200. Indicate clearly your working on the graph and state your solution.

(2 marks)



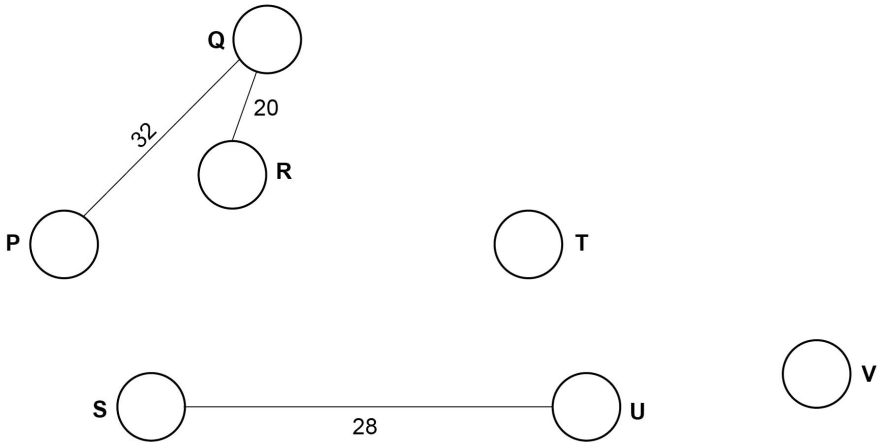
Solution	
First exceeds in 10 th month	
Specific behaviours	
✓ uses graph to determine time when first exceeds 200 ✓ correctly states the 10 th month	

Question 10

(7 marks)

The table below shows travel times (in minutes) along lines of a rail network connecting stations at P, Q, R, S, T, U and V. The diagram (not drawn to scale) shows the positions of the stations.

P	-						
Q	32	-					
R	28	20	-				
S	23	-	-	-			
T	-	34	18	21	-		
U	-	-	-	28	16	-	
V	-	64	-	-	29	23	-
	P	Q	R	S	T	U	V



Question 16

(6 marks)

Scientists were trying to increase the population of a rare species of fish, so they placed 50 fish in a small lake and monitored the population monthly. They discovered that the population of the fish, P , increased according to the rule:

$P = 50 \times 1.15^t$ where t was the time in months after the fish were placed in the lake.

- (a)
- What was the population of the fish after 18 months?
- (1 mark)

Solution
$P = 50 \times 1.15^{18}$ $= 618.8$ $= 618$ fish Accept 619 or 620, as the formula is modelling the real situation.
Specific behaviours
✓ correctly calculates the population

- (b)
- By what percentage did the population of the fish increase each month?
- (1 mark)

Solution
15%
Specific behaviours
✓ correctly states the rate of monthly change

- (c)
- Complete the recursive rule representing the growth in population of the fish.

$P_{n+1} =$ (2 marks)

Solution
$P_{n+1} = 1.15 \times P_n, \quad P_0 = 50$
Specific behaviours
✓ correctly states the initial population ✓ correctly states the recursive rule

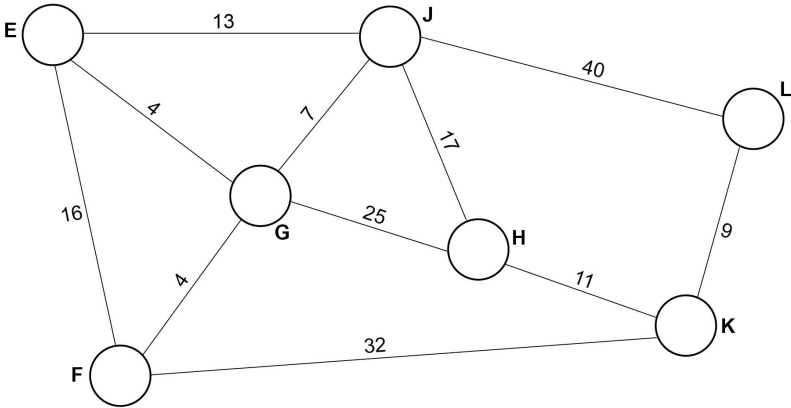
(a) Complete the diagram by drawing all rail lines and labelling with the travel times. (2 marks)

Solution
✓ correctly draws and labels at least four lines ✓ correctly draws and labels all lines

(d) For the result of choosing the student required in Part (c), rank the following events in decreasing order from most likely to least likely:

A: the student is a girl B: the student is a Year 12 boy C: the student is not from Year 8 D: the student is not from Year 11 E: the student is from either Year 9 or Year 10.
✓ correctly orders all five events ✓ correctly has C as first and B as last
Specific behaviours
C, D, A, E, B
Solution

- (b) A second rail network connecting stations E, F, G, H, J, K and L, with travel times between stations given in minutes, is shown below.



- (i) Explain, with reasons, whether a maintenance worker could travel all lines of this second network without going along any line more than once. (2 marks)

Solution
The network has four odd vertices (E, F, H and K) therefore the network is not traversable, hence the maintenance worker could not travel all lines of this network without going along any line more than once.
Specific behaviours
✓ determines there are more than two odd vertices ✓ concludes whether the network is traversable based on the number of odd vertices determined

Question 15 (5 marks)

Boolham High School has students enrolled in Years 8, 9, 10, 11 and 12. One-fifth of the students are in Year 8, 23% of the students are in Year 9, 11% of the students are in Year 10 and the probability that a randomly chosen student is in Year 11 is 0.26. Within each year group the number of boys is equal to the number of girls.

- (a) What is the probability that a randomly chosen student at Boolham High School is in Year 12? (1 mark)

Solution
$1 - (0.2 + 0.23 + 0.11 + 0.26) = 0.2$
Specific behaviours
✓ correctly calculates the probability

- (b) There are 1200 students enrolled at the school. How many of these students are enrolled in Year 11? (1 mark)

Solution
$0.26 \times 1200 = 312$
Specific behaviours
✓ correctly calculates the expected value

- (c) One student is to be randomly chosen to represent the school at an Arbor Day ceremony. From which year group is the student most likely to be chosen? (1 mark)

Solution
Year 11
Specific behaviours
✓ correctly states the most likely year group

(ii) What is the quickest travel time between stations E and L, and what route gives this quickest time? (3 marks)

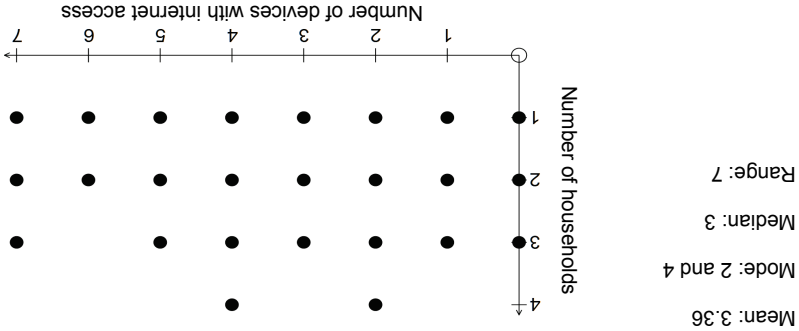
Solution

Quickest travel time = 48 minutes
Quickest route = EGJHKL

Specific behaviours

- ✓ shows use of network to calculate shortest path
- ✓ states quickest time
- ✓ states quickest path

Some statistics for the suburb of Deakin are shown below. A dot frequency graph is also shown. Devices with internet access



(d) Compare the number of devices with internet access for the two suburbs by describing **two (2)** ways in which these data sets differ. (2 marks)

Solution

The most noticeable difference is in the dot plots: The data from Deakin is uniformly distributed whereas the data from Curtin has the score 3 occurring 68% of the time. Their means are close but Deakin's is larger. Deakin has two modes whereas Curtin only has one mode. Curtin has a larger range.

✓ One mark each for stating any two of:
The data from Deakin evenly distributed while the data from Curtin is not;
The data from Curtin has a larger range;
The data from Deakin has two modes whereas the data from Curtin only has one mode;
The data from Curtin has a smaller mean than the data from Deakin.

Specific behaviours

(e) Write down **one (1)** advantage of using the median, rather than the mean, to describe the 'average' of a data set. (1 mark)

Solution

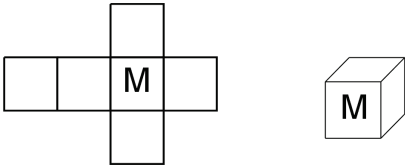
The mean can be affected by outliers making it appear smaller or larger than expected. The median is less influenced by outliers
The median is simpler to find than the mean.

Specific behaviours

- ✓ states an advantage of using the median

Question 11 (10 marks)

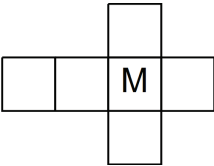
A six-sided die has one side marked with the letter 'M'. The other five sides are each randomly labelled with one of the letters M, N and P. For example, if there was a one-half chance of rolling an 'M' then one possibility is the die was labelled M, M, M, N, N, P. The net of the original die with just the one permanent letter 'M' is drawn below.



- (a) If the die is labelled with 3 Ms, 2 Ns and 1 P, what is the probability of rolling an N? (1 mark)

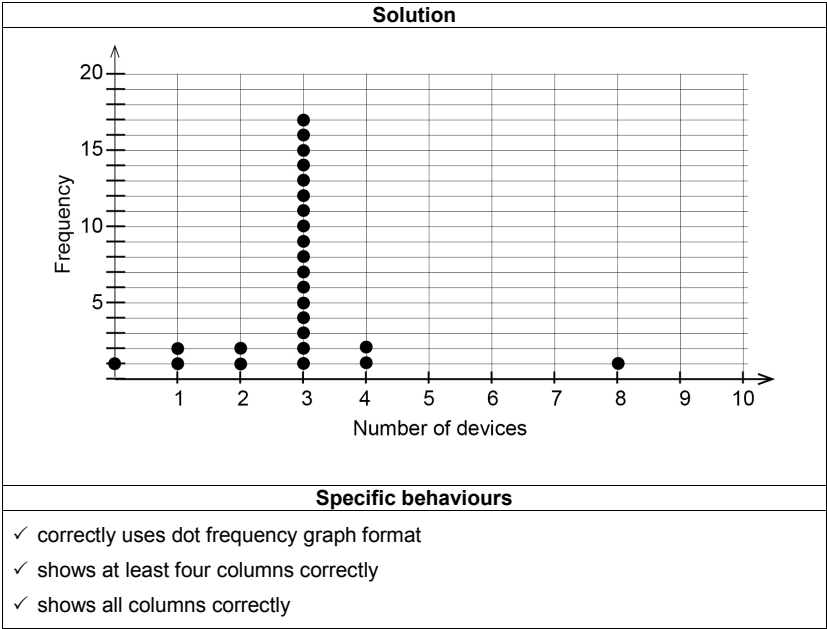
Solution
$\frac{2}{6} = \frac{1}{3}$ (Accept either)
Specific behaviours
✓ correctly states the probability

- (b) Label the net of the die to show there is a two-thirds chance of rolling an N. (1 mark)

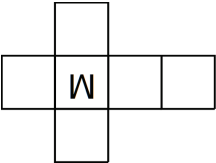


Solution				
		N		
N	N	M	P	
		N		
i.e. 4 Ns, 1 P and 1 M.				
Specific behaviours				
✓ correctly labels the net to include 4 Ns				

- (c) Draw a dot frequency graph representing these data for the suburb of Curtin on the grid below. (3 marks)



(c) Label the net of the die to show there is a one-third chance of rolling a P and a greater chance of rolling an N than an M. (2 marks)



Solution
<div><div><div><div>N</div><div>M</div><div>P</div></div><div><div>N</div><div>N</div><div>M</div></div><div><div>N</div></div></div></div> <p>i.e. Any solution where there are 2 Ps and 3 Ns.</p>
Specific behaviours
✓ labels two sides with Ps ✓ labels three sides with Ns

Question 14 (11 marks)
A survey was conducted in two suburbs, Deakin and Curtin, regarding the number of devices in each household with internet access. Twenty-five households in each suburb were randomly selected.

Number of devices in household with internet access	Deakin frequency	Curtin frequency
0	3	1
1	3	2
2	4	2
3	3	17
4	4	2
5	3	0
6	2	0
7	3	0
8	0	1

(a) State **one (1)** way in which these data may have been collected. (1 mark)

Solution
Households randomly selected from the two suburbs by: assigning a number for each and randomly selecting a number; randomly listing all household addresses and choosing every 50 th etc.
Specific behaviours
✓ correctly describes method that relates to the random nature the households were selected

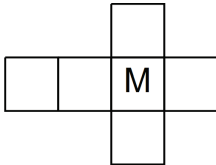
(b) Determine the mean, mode, median and range of the number of devices with internet access for the suburb of Curtin. (4 marks)

Solution
Mean = 2.92 Mode = 3 Median = 3 Range = 8
Specific behaviours
✓ ✓ ✓ correctly states each statistic

(d) The die is rolled 12 times. The results are below.

Letter	Frequency
M	1
N	7
P	4

(i) Based on the results in the table above, what would be the most likely labelling of the die? (1 mark)



Solution					
<p>A net of a cube with six faces. The central square is labeled 'M'. It is surrounded by five other squares: one above labeled 'P', one below labeled 'N', one to the left labeled 'N', one to the right labeled 'P', and one attached to the right square of the 'M' square labeled 'N'.</p>					
Specific behaviours					
✓ correctly labels the net with 1M, 2Ps and 3Ns					

(ii) Is it possible that the die was labelled M, M, M, M, N, P? Give a reason to support your answer. (1 mark)

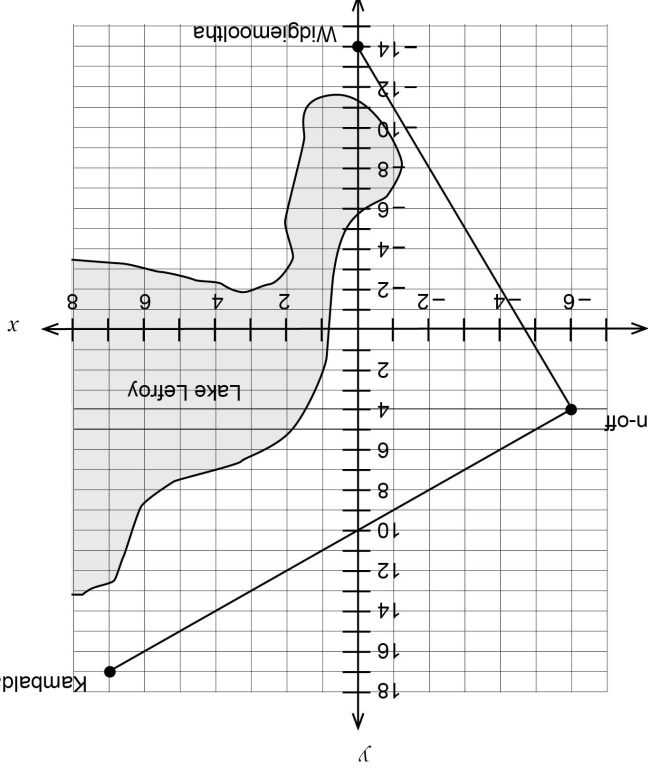
Solution	
It is possible, as M, N and P appear on the die.	
Specific behaviours	
✓ gives a reason linked to the possibility	

(b) How much further does Jaime have to drive when Lake Lefroy has water in it compared with when it is dry? (4 marks)

Solution	
$WT = \sqrt{18^2 + 6^2}$ $= 18.97 \text{ km}$	<p>A right-angled triangle with a vertical side of 18, a horizontal side of 6, and a hypotenuse of 18.97. A right-angle symbol is at the bottom-right vertex.</p>
$TK = \sqrt{13^2 + 13^2}$ $= 18.38 \text{ km}$	<p>A right-angled triangle with a vertical side of 13, a horizontal side of 13, and a hypotenuse of 18.38. A right-angle symbol is at the bottom-right vertex.</p>
$WK = \sqrt{31^2 + 7^2}$ $= 31.78 \text{ km}$	<p>A right-angled triangle with a vertical side of 31, a horizontal side of 7, and a hypotenuse of 31.78. A right-angle symbol is at the bottom-right vertex.</p>
$WT + TK = 37.35 \text{ km}$ Therefore Jaime travels 5.57 km further	
Specific behaviours	
✓✓✓ correctly calculates the distance for each leg, WT , TK and WK ✓ correctly calculates the difference	

(5 marks)

map below. One unit on the map represents 1 km.



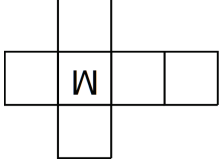
- State the coordinates of the three points plotted, that is, Widgeiemooltha, the turn-off and Kambalda East. (1 mark)

Solution	
Widgiemoooltha (0, -14), turn-off(-6, 4), Kambalda East (7, 17)	Specific behaviours
✓ correctly states coordinates of all three points	

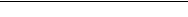
The same die from Part (d) is rolled 1200 times, with the results below:

Letter	Frequency
M	405
N	600
P	195

What would be the most likely labelling of the die?



	M		
N	M	N	N
	P		

<p>✓ correctly labels the net with 3 Ns</p> <p>✓ correctly labels the net with 1 P</p>			
<p>Solution</p>		<p>Specific behaviours</p>	

- (j)

In which labelling of the die, from Parts (d) and (e), would you be more confident? Why? (2 marks)

Solution	Answer from Part (e) as there has been many more trials.
Specific behaviours	✓ correctly states Part (e) ✓ gives valid reason

Question 12

(10 marks)

The Brown Taxi company charge a flag fall of \$3 and a rate of \$2 per kilometre travelled. They use the formula: $F = 3 + 2d$, where d is the distance travelled in kilometres, to calculate the fare charged F (in dollars). For example, a taxi ride of 5 km would cost \$13.

- (a) Matthew catches a Brown Taxi from Perth to home, a distance of 25 km. Calculate the fare Matthew was charged. (1 mark)

Solution
$F = 3 + 2(25)$ $= \$53$
Specific behaviours
✓ correctly evaluates formula or correctly calculates fare

- (b) Peta is charged \$41 for catching a Brown Taxi home. What distance did she travel in the taxi? (2 marks)

Solution
$41 = 3 + 2d$ $d = 19 \text{ km}$
Specific behaviours
✓ correctly substitutes into formula ✓ correctly solves for distance

The Orange Taxi company charge a flag fall of \$6 and \$1.50 per kilometre travelled.

- (c) Write a formula for the fare charged by the Orange Taxi company in terms of d (the distance travelled). (1 mark)

$F =$

Solution
$F = 6 + 1.5d$
Specific behaviours
✓ correctly states the formula

- (d) Madeline is deciding whether to catch a Brown Taxi or Orange Taxi to her home. She wants to choose the cheaper company. She needs to travel 10 km. Which company should she choose? Justify your answer with calculations. (2 marks)

Solution
$\begin{array}{ll} \text{Orange} = 6 + 1.5(10) & \text{Brown} = 3 + 2(10) \\ = \$21 & = \$23 \end{array}$ <p>Therefore choose Orange Taxi</p>
Specific behaviours
✓ correctly calculates cost of travelling 10 km in both Orange and Brown Taxis ✓ states 'choose Orange Taxi'

- (e) Determine the distance travelled that will result in the fares for the Orange and Brown Taxis being equal. (2 marks)

Solution
$3 + 2d = 6 + 1.5d$ $0.5d = 3$ $d = 6$ <p>Therefore 6 km</p>
Specific behaviours
✓ sets up linear equation to solve or graphs the functions or correctly uses 'guess and check' ✓ solves linear equation or reads point of intersection

- (f) Emily wants to give her mother some advice about which taxi company to choose so that her fare is always the cheapest. What advice should Emily give her mother? (2 marks)

Solution
<p>Brown Taxis are cheaper up to 6 km. At 6 km they are equal in price. More than 6 km and Orange Taxis are cheaper.</p>
Specific behaviours
✓ states Brown Taxis are cheaper for less than 6 km ✓ states Orange Taxis are cheaper for more than 6 km