Hale School
2011

Question/Answer Booklet

MATHEMATICS 3CD
SEMESTER 2
Section Two
(Calculator Assumed)

Circle your teacher's initials

GJ JIB BAH VMU

Booklet 2 of 3

Your name _____

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section

To be provided by the supervisor
Two Question/Answer Booklets
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

		Number of questions	Working time (minutes)	Marks available
Booklet 1 (Section 1)	Calculator Free	8	50	40
Booklet 2 This Booklet (Section 2)	**Calculator Assumed**	**7**	**100**	**40**
Booklet 3 (Section 2)		5		40
			Total marks	120

Instructions to candidates

1. The rules for the conduct of WACE external examinations are detailed in the booklet *WACE Examinations Handbook*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions in the spaces provided.

3. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

4. Show all working clearly. Any question, or part question, worth more than 2 marks requires valid working or justification to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

See next page

Spare page

Question 9 (7 marks)

A bank is considering the passwords that are allowed for their customers to enter personal accounts on the bank's website.

a) Currently, the password is composed of 4 different characters chosen from the 26 lower case letters of the alphabet and the 10 digits, 0, 1, 2, . . . ,9.

The bank's IT manager has calculated that the number of available passwords is 36 x 35 x 34 x 33 = 1413720.

By what factor will the number of available passwords increase if,

i) the letters used can be upper or lower case? [2]

ii) repetition is allowed (letters can only be lower case) [2]

b) The bank decides to introduce a new password system which has more structure and is therefore easy to remember. 3 different upper case letters will be followed by a number of different digits excluding 0.

How many digits are required to ensure that there are at least 10^9 passwords available? [3]

See next page

Question 10 (7 marks)

The weights of a supply of ball bearings are normally distributed with a mean weight of 0.62 N and standard deviation of 0.01 N.

a) Find the probability that any one ball bearing has a weight of between 0.61 N and 0.62 N. [1]

b) If 10 ball bearings are selected at random find the probability that at least 4 of them have a weight between 0.61 N and 0.62 N [2]

c) If a sample of 100 ball bearings are measured and recorded to the nearest 0.01 N, find the number of ball bearings that you would expect to be measured as 0.61 N. [2]

d) If a sample of 100 ball bearings is taken, find the probability that the mean value of the sample is less than 0.618 N. [2]

See next page

Question 20 (8 marks)

Consider the functions $f(x) = 1 + \sqrt{x - 2}$ and $g(x) = \frac{1}{x - 5}$

a) Write down the natural domain and corresponding range for f(x). [2]

b) Find i) $g \circ f(6)$ [1]

ii) x such that $f \circ f(x) = 4$ [2]

c) State the domain and range of $g \circ f(x)$ [3]

Question 19 (8 marks)

A continuous random variable, X has the following probability density function;

$$f(x) = \begin{cases} Ae^{-kx} & 0 \le x \le 5 \\ 0 & otherwise \end{cases}$$

a) Determine a relationship between A and k to ensure that f(x) is a probability density function. [2]

b) Find a relationship between A and k given that the median of the probability distribution is 2. [2]

c) Determine the values of A and k [2]

d) Find $P(X \le 4 | X \ge 2)$ [2]

See next page

Question 11 (3 marks)

Two normal six sided dice are thrown and the total of the uppermost faces recorded. This is repeated a number of times.

Find the probability of getting

i) a score of at least 11 on the first throw, [1]

ii) a score of at least 11 on exactly 2 of the first 3 throws. [2]

See next page

Question 12 (6 marks)

a) Write down, in the correct order, the transformations that are needed to change the graph of $y = 2e^{x-1}$ into the graph of $y = e^{0.5x+3}$. [3]

b) Find the equation of the new graph when the graph of the function $y = 4 - 5e^{3(x-4)}$ is subject to the following sequence of transformations, in the order shown; [3]

- Dilation of factor 6 horizontally
- Translation of 12 units to the left
- Reflection in the x – axis

See next page

Question 18 (8 marks)

The temperature of a metal bar at time t minutes after it is taken out of a fire is given by $T = A + Be^{-kt}$ where, T is the temperature of the rod and A, B and k are positive constants.

a) Show that $\frac{dT}{dt} = -k(T - A)$ [2]

b) Given that the initial temperature of the rod is 300˚C and the temperature falls towards a lowest value of 30˚C, determine the values of A and B. [2]

c) After 5 minutes the temperature of the metal bar has fallen to 250˚C. Use this fact and the answers from part b) to determine

i) the value of k accurate to 4 decimal places. [2]

ii) the time it takes, to the nearest minute, for the temperature of the metal bar to fall to a value of 100˚C. [2]

See next page

Question 17 (7 marks)

a) If A and B are independent, $P(A|B)=0.8$ and $P(B|A)=0.4$, find

i) P(A) [1]

ii) P(A∪B) [2]

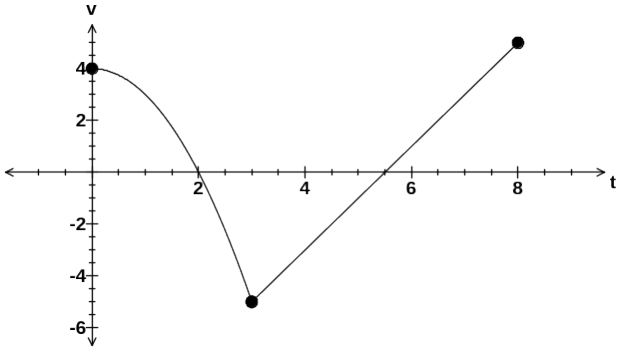
iii) $P(\bar{A}\cap\bar{B})$ [1]

b) If $P(A|B)=0.8$, $P(B|A)=0.4$ and $P(A\cap B)=0.2$ find P(A∪B) [3]

See next page

Question 13 (8 marks)

The velocity-time graph for the motion of a particle, P is shown in the diagram below. V is measured in ms^{-1} and t is measured in seconds.



The formula for the velocity at time t is given by

$$v(t)=\begin{cases} a-t^2 & for \quad 0\le t\le 3 \\ 2t-b & for \quad 3<t\le 8 \end{cases}$$

a) Determine the values of a and b. [2]

b) Find the acceleration of the particle at time t = 2s. [2]

c) Find the distance travelled in the 8 seconds shown. [2]

d) Find the average velocity during the first 8 seconds. [2]

See next page

Question 14 (5 marks)

The waiting times at a doctor's surgery are distributed with a mean value of μ and a standard deviation of σ .

a) The waiting times of 200 patients were recorded and found to have a mean value of 25 minutes with a standard deviation of 8 minutes.

Find a 95% confidence interval for the value of μ, accurate to 2 decimal places. [2]

b) In another sample of 200 patients the mean value was $\bar{x}$ and the standard deviation s.

From these observations a 95% confidence interval was found to be $20.45 \leq \mu \leq 21.95$. Find the values of $\bar{x}$ and s. [3]

b) If the profit on each type A basket remains as \$12, by how much does the profit on each type B basket need to rise so that there is more than one option for producing maximum profit. State the options available. [3]

c) If each type A basket gives a profit of \$m and each type B basket produces a profit of \$n, where m and n are positive constants, find conditions on m and n that will ensure that producing 30 type A baskets and 50 type B baskets is the only way to maximise the profit. [3]

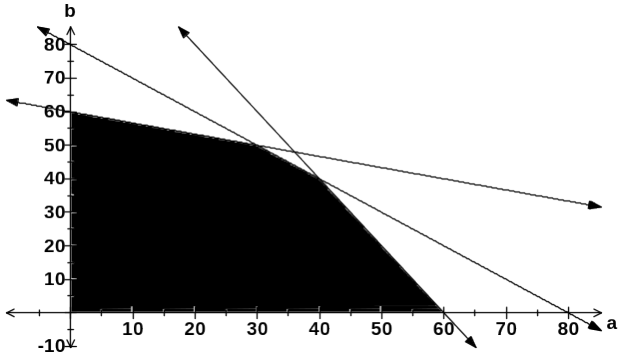
Question 16 (9 marks)

A gourmet delicatessen produces two types of gift basket, A and B.

The following inequalities describe the constraints of production where a is the number of baskets of type A and b is the number of baskets of type B produced in a week.

$$a + b \leq 80$$
$$2a + b \leq 120 \qquad a \geq 0$$
$$a + 3b \leq 180 \qquad b \geq 0$$

The graph below shows the lines equating to the inequalities above.



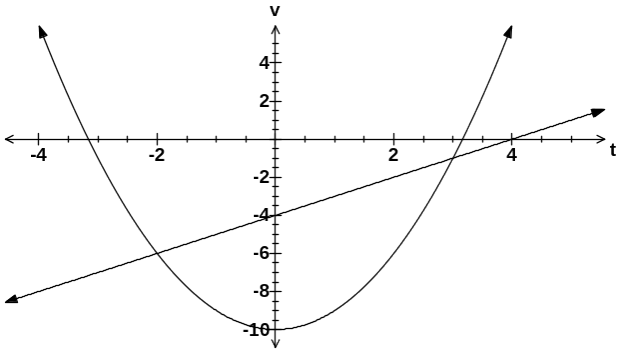
If each type A basket gives a profit of $12 and each type B basket produces a profit of $10,

a) Find the number of each type of basket that should be produced for maximum profit. Show your working. [3]

See next page

Question 15 (4 marks)

The diagram below shows the graph of the curve $y = x^2 - 10$ and the line y = x – 4.



The area trapped between the curve and the line is rotated through 360˚ about the x – axis.

i) Write down an integral calculation to determine the volume generated. [3]

ii) Find the **exact** volume. [1]

End of Booklet 2

Spare page

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