Nuclear binding energy
Solution

1. *Of the two hydrogen isotopes, deuterium, $^{2}_{1}H$, and tritium, $^{3}_{1}H$, which has the highest binding energy per nucleon?*

mass defect = mass of component neutrons and protons - mass of nucleus

mass defect for $^{2}_{1}H$ = {m(1 proton + 1 neutron)} - m($^{2}_{1}H$)

= $1.00728 + 1.00867 - 2.01350$

= 0.00245 u

= $0.00245 \times 1.6606 \times 10^{-27}$ kg

= 4.068×10^{-30} kg

Binding energy = mc^2

= $4.068 \times 10^{-30} \times (3.00 \times 10^{8})^{2}$

= 3.662×10^{-13} J

Binding energy per nucleon $^{2}_{1}H$ = $3.662 \times 10^{-13} / 2$

= 1.831×10^{-13} J

mass diff. for $^{3}_{1}H$ = {m(1 proton + 2 neutrons)} - m($^{3}_{1}H$)

= $1.00728 + 2 \times 1.00867 - 3.01550$

= 0.00912 u

= $0.00912 \times 1.6606 \times 10^{-27}$ kg

= 1.514×10^{-29} kg

Binding energy = mc^2

= $1.514 \times 10^{-29} \times (3.00 \times 10^{8})^{2}$

$= 1.363 \times 10^{-12}$ J

Binding energy per nucleon ${}^{3}_{1}H$ $= 1.363 \times 10^{-12} / 3$

$= 4.54 \times 10^{-13}$ J

Tritium has the higher binding energy per nucleon

2. *A small proportion of all the carbon in living organisms is the radioactive isotope carbon-14. Calculate the binding energy per nucleon of both the carbon-12 and carbon-14 nuclei and state which one is the most stable. The atomic number of carbon is 6.*

mass defect. for ${}^{12}_{6}C$ $= \{m(\text{ 6 proton + 6 neutrons})\} - m({}^{12}_{6}C)$

$= 6 \times 1.00728 + 6 \times 1.00867 - 11.99671$

$= 0.09899$ u

$= 0.09899 \times 1.6606 \times 10^{-27}$ kg

$= 1.644 \times 10^{-28}$ kg

Binding energy $= mc^2$

$= 1.644 \times 10^{-28} \times (3.00 \times 10^{8})^2$

$= 1.4794 \times 10^{-12}$ J

Binding energy per nucleon ${}^{14}_{6}C$ $= 1.4794 \times 10^{-12} / 12$

$= 1.233 \times 10^{-12}$ J

mass defect for ${}^{14}_{6}C$ $= \{m(\text{ 6 proton + 8 neutrons})\} - m({}^{14}_{6}C)$

$= 6 \times 1.00728 + 8 \times 1.00867 - 13.99995$

$= 0.11309$ u

$= 0.11309 \times 1.6606 \times 10^{-27}$ kg

$= 1.878 \times 10^{-28}$ kg

Binding energy = mc^2

= $1.878 \times 10^{-28} \times (3.00 \times 10^8)^2$

= 1.690×10^{-12} J

Binding energy per nucleon ${}^{14}_{6}C$ = $1.690 \times 10^{-12} / 14$

= 1.207×10^{-12} J

3. *One of the simplest fusion reactions is*

$${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{1}e + \text{energy}$$

a. What mass does a single fusion reaction convert to energy?

mass diff. for reaction = $m({}^{2}_{1}H) + m({}^{0}_{+1}e) - 2\,m({}^{1}_{1}H)$

= $2.01350 + 0.000549 - 2 \times 1.00728$

= -0.000511 u

= $-0.000511 \times 1.6606 \times 10^{-27}$ kg

= -8.49×10^{-31} kg

b. What energy does a single fusion reaction release?

energy change = mc^2

= $-8.49 \times 10^{-31} \times (3.00 \times 10^8)^2$

= -7.64×10^{-14} J

energy released = 7.64×10^{-14} J

(note that if the masses in kg are used rounding errors give an answer of 7.1×10^{-14} J)

4. *Uranium-238 undergoes a series of radiocative decays, the first of which is:*

$${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} + \textit{energy}$$

How much energy does each decay release?

mass diff. for reaction	=	$m({}^{234}_{90}\text{Th}) + m({}^{4}_{2}\text{He}) - m({}^{238}_{92}\text{U})$
	=	233.98418 + 4.00150 - 238.00028
	=	-0.0146 u
	=	$-0.0146 \times 1.6606 \times 10^{-27}$ kg
	=	-2.42×10^{-29} kg
energy change	=	mc^2
	=	$-2.42 \times 10^{-29} \times (3.00 \times 10^{8})^2$
	=	-2.182×10^{-12} J
energy released		= 2.182×10^{-12} J

5. *Hydrogen and deuterium fuse to give the isotope* ${}^{3}_{2}\text{He}$.

a. How much energy does a single fusion release?

$${}^{2}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He}$$

mass diff. for reaction	=	$m({}^{3}_{2}\text{He}) - m({}^{2}_{1}\text{H}) - m({}^{1}_{1}\text{H})$
	=	3.01493 + 2.01350 - 1.00728
	=	-0.00585 u
	=	$-0.00585 \times 1.6606 \times 10^{-27}$ kg
	=	-9.71×10^{-30} kg
energy change	=	mc^2
	=	$-9.71 \times 10^{-30} \times (3.00 \times 10^{8})^2$
	=	-8.74×10^{-13} J

energy released = 8.74×10^{-13} J

b. *How much energy does a kilogram of reactant release?*

(2.01350 + 1.00728) x 1.6606 x 10^{-27} kg produces 8.74 x 10^{-13} J

1.000 kg produces 8.74 x 10^{-13} / ((2.01350 + 1.00728) x 1.6606 x 10^{-27}) J

= 1.74 x 10^{14} J

7. *Under certain circumstances, a gamma ray photon may suddenly change into an electron and a positron.*

$${}^{0}_{0}\gamma \rightarrow {}^{0}_{-1}e + {}^{0}_{+1}e$$

Calculate the minimum energy of the photon.

mass diff. for reaction = m(${}^{0}_{+1}e$) + m(${}^{0}_{-1}e$) - m(${}^{0}_{0}\gamma$)

= 0.000549 + 0.000549 - 0.000000

= 0.001098 u

= 0.001098 x 1.6606 x 10^{-27} kg

= 1.823 x 10^{-30} kg

energy change = mc^2

= 1.823 x 10^{-30} x (3.00 x 10^8)^2

= 1.641 x 10^{-13} J

minimum energy of photon = 1.641 x 10^{-13} J

8. *When 3.0000 MeV alpha particles bombard nitrogen 14, oxygen-17 forms and the reaction releases a proton. Calculate the energy this reaction releases.*

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

mass diff. for reaction = $m(^{1}_{1}H) + m(^{17}_{8}O) - m(^{4}_{2}He) - m(^{14}_{7}N)$

= 1.00728 + 16.99474 - 4.00150 - 13.99923

= 0.00129 u

= $0.00129 \times 1.6606 \times 10^{-27}$ kg

= 2.142×10^{-30} kg

energy change = mc^2

= $2.142 \times 10^{-30} \times (3.00 \times 10^{8})^2$

= 1.928×10^{-13} J

input energy of $^{4}_{2}He$ = $3.0000 \times 10^{6} \times 1.60 \times 10^{-19}$ J = 4.80×10^{-13} J

energy released = $4.80 \times 10^{-13} - 1.928 \times 10^{-13}$ J

= 2.872×10^{-13} J