

MATERIALS REQUIRED/RECOMMENDED FOR THIS SECTION	To be provided by the supervisor	To be provided by the student	Standard items:	Special items:	Curriculum	Council for this examination
			Fluid/tape, ruler, pens, pencils, pencil sharpener, eraser, correction fluid	Highlighters A4 paper, drawing instruments, templates, notes on two unfolded sheets of		and up to three calculators satisfying the conditions set by the
			Formula Sheet (retained from Section One)			
			This Question/Answer Booklet			

Name of Student: _____ Marking Key: _____

**Calculator-assumed
Section Two:**

MATHEMATICS 3AB

Question/Answer Booklet

Semester 1 Examination 2012



Important note to students

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	7	7	50	50	
Section Two Calculator-assumed	13	13	100	100	
		Total	150	100	

Instructions to students

- 1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- 2 **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you

repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

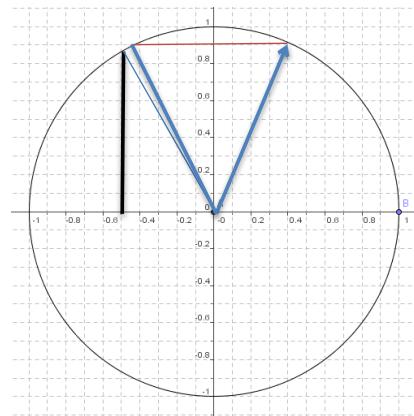
Section Two: Calculator-assumed
(100 marks)

This section has **thirteen (13)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

Question 8 **(5 marks)**

Use the Unit circle below (there must be evidence of using the unit circle before full marks will be awarded)



- (i) to evaluate the value of $2 \cos 120^\circ$ (2)

Solution

From unit circle, $\cos 120^\circ = -0.5$

$$\text{Therefore } 2 \cos 120^\circ = 2 \times (-0.5) = -1$$

Specific behaviours

✓ value for 120°

✓ correct value for $2 \cos 120^\circ$

- (ii) to find the value(s) of x , to the nearest whole number, such that

$$\sin 2x = 0.9 \quad \text{(3)}$$

Solution

Read the two angles such that the y value is 0.9, giving 66° and 114° ($\pm 1^\circ$)

$$x^2 = 5$$

$$x = \sqrt{5}$$

$$x = \pm 2.2$$

Specific behaviours

✓ draws the line $y=x$ as shown on the graph

$$y = \frac{5}{x}$$

✓ reads the x values where $y=x$ intersects with

Solution

- (v) By adding one suitable function on the axes, show how you solve $x^2 = 5$ and estimate the solution(s) from your graph, correct to 1 decimal place.

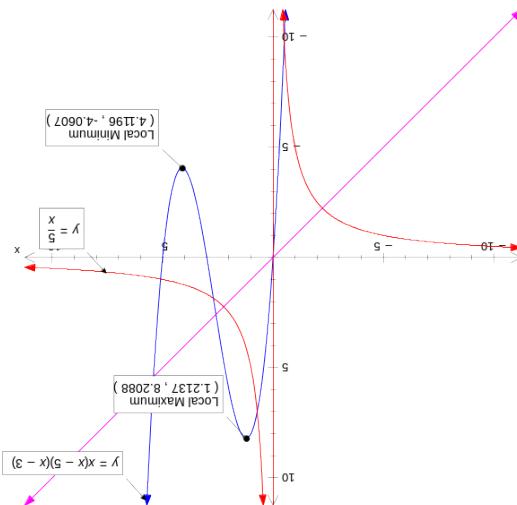
Solution	4 solutions	Specific behaviours	✓ OR X

$$\frac{\tau^X}{\varsigma} = (\Sigma - X)(\Sigma - X)$$

- (iv) Read from the graph the number of solutions to the equation.

Question 20 (continued)

Solution	As displayed in diagram
Specific behaviours	<ul style="list-style-type: none"> ✓✓ 1 mark each for $f(x)$ and $g(x)$ ✓ shows clearly x and y intercepts, turning points
Conclusion	



	Divide both angles by 2 to give $x = 33^\circ$ and 57°	✓ two values for $2x$ ✓ two angles for x
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Question 9

(9 marks)

Consider the following sequence of numbers 2, 5, 2, 9, 2, 13, 2, 17, ...

- (i) Write down the next two terms of the sequence, if the pattern continues. (1)

Solution
2,21
Specific behaviours
✓ or X

- (ii) In this sequence, $T_1=T_3=T_5=$ etc, complete the sentence,

" If n is an odd number, then

$$T_n = \underline{\hspace{2cm}}, T_1 = \underline{\hspace{2cm}} \quad (1)$$

Solution
$T_n = 2, T_1 = 2$
Specific behaviours
✓ or X

- (iii) Write a recursive formula for the sequence 5, 9, 13, 17, ... (2)

Solution
$T_{n+1} = T_n + 4, T_1 = 5$
Specific behaviours
✓✓ or X

- (iv) Write down the values of T_{99} and T_{100} . (2)

Solution
$T_{99} = 2, T_{100} = 201$
Specific behaviours
✓✓ or X

- (v) Which term is 61? (1)

Solution
From CAS, T_{30} is 61
Specific behaviours

Question 20

(9 marks)

The point $P(1,k)$ lies on the curve with equation $f(x) = (x^2 - 6x)(x - 2) + 3x$

- (i) Find the value of k . (1)

Solution
$k=8$
Specific behaviours
✓ or X

- (ii) Express $f(x)$ in a completely factorised form. (2)

Solution
$f(x) = x(x^2 - 8x + 15) = x(x - 5)(x - 3)$
Specific behaviours
✓ expand ✓ factorised form

- (iii) On the axes below sketch the curves with the following equations

$$f(x) = (x^2 - 6x)(x - 2) + 3x \quad \text{and} \quad g(x) = \frac{5}{x}, \text{ showing clearly where they meet the axes.} \quad (3)$$

$\sum_{n=0}^{50} 2 = 100$	Solution	$5 + 7 + 9 + \dots + 201 = 5150$ $\sum_{n=0}^{50} \text{of the first } n \text{ terms} = 5150$ $\sum_{n=0}^{50} 2 = 100$
$5 + 7 + 9 + \dots + 201 = 5150$ $\sum_{n=0}^{50} \text{of the first } n \text{ terms} = 5150$	Specific behaviours	$5 + 7 + 9 + \dots + 201 = 5150$ $\sum_{n=0}^{50} 2 = 100$
\checkmark sum of the first 100 terms \checkmark sum of the fifty 2's and sum of 5,7,9,...,201 \checkmark sum of the first 100 terms		

- (vi) Calculate the sum of the first 100 terms of the sequence.

✓ OR X

Specific behaviours		✓✓✓✓ 1 mark each
(iV)	F	
(III)	F	
(II)	F	

Question 10

(6 Marks)

Four digit numbers may be formed using the digits 0, 2, 4, 6, 8 such that no digit can be used more than once. How many of these 4-digit numbers

(i) may be formed? (1)

Solution	
$5! = 120$	
Specific behaviours	

(ii) begin with 8? (1)

Solution	
$1 \times 4 \times 3 \times 2 = 24$	
Specific behaviours	

(iii) end in a 2 or a 4? (1)

Solution	
$1 \times 4 \times 3 \times 2 \times 2 = 48$	
Specific behaviours	

(iv) are less than 486? (2)

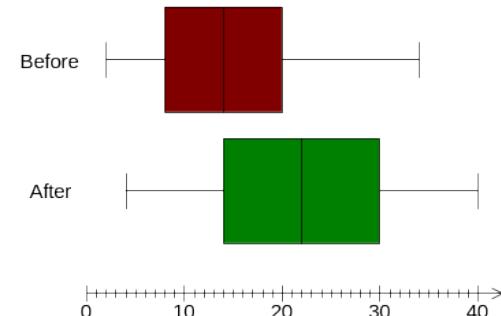
Solution	
400's: 5 200's: 6 Therefore total is 11	
Specific behaviours	

(v) What is the probability that a randomly picked 4-digit number from this set is more than 8640? (1)

Solution	
Only 8642	

Question 19 (continued)

(b) The test results for a group of 45 students involved in a reading program are displayed below in the form of a pair of box plots. The data collected shows the reading speed of the students both before and after the administration of the program.

Reading speed

NOTE : IN THE EXAM the before and After was swapped around accidentally.

For each of the following, answer True (T) or False (F):- (4)

(i) For the "after" data, the median has increased by 8 and the interquartile range has increased by 4.

(ii) Both distributions have the same median.

(iii) The 'after' set of scores is more evenly distributed than the 'before' set of scores.

(iv) The range of both sets of scores is the same.

Solution	
(i)	F

$\therefore P(>8640) = \frac{1}{120}$	Specific behaviours	✓ or X
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Question 11

(6 marks)

- (a) The distribution of heights of a plant is normally distributed with a mean of 72 cm and a standard deviation of 6 cm. If there were 100 plants, estimate the number of plants which are less than 64cm in height.

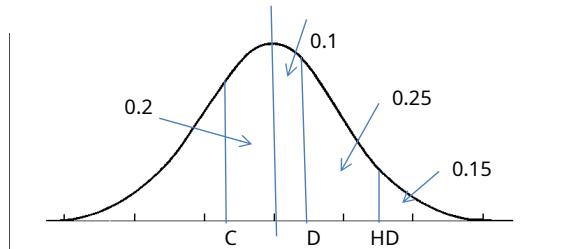
(2)

Solution
$P(X < 64) = 0.0912$
Number of plants = $0.0912 \times 100 = 9$
Specific behaviours
✓ probability
✓ Number

- (b) Achievement awards in a national Mathematics competition are allocated as

- High Distinctions to the top 15% of performances
- Distinctions to the next 25%
- Credits to the next 30%
- Participations to the remainder

If the scores were normally distributed with a mean of 75 and a standard deviation of 12, find the minimum score a competitor has to achieve to be awarded a



(i) Credit

(2)

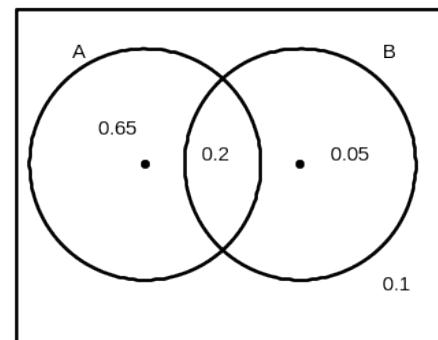
Solution
$P(X < \text{Credit}) = 0.3$
$C = 68.7$
Minimum score for a Credit is 68.7
Specific behaviours

Question 19

(9 marks)

- (a) A and B are events such that $P(A \cup B)=0.9$, $P(A \cap B)=0.2$, $P(A|B)=0.8$

- (i) Represent the diagram in the Venn diagram below. (3)



Solution
As shown in diagram
Specific behaviours

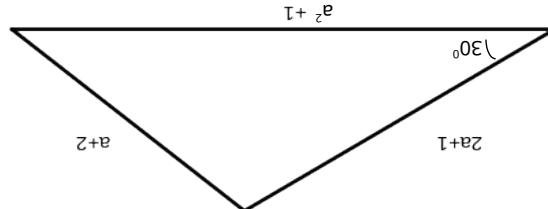
- ✓ 0.65
- ✓ 0.05
- ✓ 0.1

- (ii) Hence, find $P(A)$ (1)

Solution
0.85
Specific behaviours

- (iii) Hence find $P(\bar{B})$ (1)

Solution
0.75
Specific behaviours



Given the triangle below,

(6 marks)

Question 12

✓ or X
Specific behaviours
Minimum score for a High Distinction is 87
$HD = 87$
$P(X > HD) = 0.15$
Solution

(1)

(iii) High Distinction

✓ minimum score for a Distinction
Specific behaviours
Minimum score for Distinction is 78
$D = 78$
$P(X > D) = 0.40$
Solution

(1)

(iii) Distinction

✓ minimum score for Credit
✓ correct probability

✓✓✓ 1 mark each
Specific behaviours

(i) Find a simplified algebraic expression for the area of the triangle, given that

$$\sin 30^\circ = \frac{1}{2} \quad (2)$$

Solution
$\begin{aligned} \text{Area} &= \frac{1}{2} \times (2a+1) \times (a^2 + 1) \times \sin 30^\circ \\ &= \frac{(2a+1) \times (a^2 + 1)}{4} \end{aligned}$

Specific behaviours

- ✓ area of non-right triangle formula with the correct sides
- ✓ correct expression

(ii) If the area is 17.5 units², determine the value of 'a'. (2)

Solution
$\begin{aligned} \frac{(2a+1) \times (a^2 + 1)}{4} &= 35 \\ a &= 3 \end{aligned}$

Specific behaviours

- ✓ equates area expression to 17.5
- ✓ solves correctly for "a"

(iii) Hence, state the lengths of the triangle. (2)

Solution
Lengths are 7, 5, 10

Specific behaviours

- ✓ substitutes a = 3 into each of the three sides
- ✓ three correct lengths

Question 18

(8 marks)

The final result of the students in Mrs Bond's classes of 3CDMAT and 3CDMAS are summarised in the frequency table below.

Mark in percent	3CDMAT	Cumulative frequency	3CDMAS	Cumulative frequency
0 - 19	2	2	1	1
20 - 39	2	4	3	4
40 - 59	5	C	A	7
60 - 79	D	20	9	B
80 - 99	7	27	5	21

(a) Find the values of A, B C and D (2)

Solution
$A = 3, B = 16, C = 9, D = 11$

Specific behaviours

- ✓✓ for all four correct answers

(b) Calculate the mean and standard deviation of the 3CDMAS marks (2)

Solution
$\bar{x} = 62.83, \sigma = 22.54$

Specific behaviours

- ✓✓ 1 mark each

(c) If a student from one of these classes is picked at random, what is the probability that the student

(i) received a mark greater than or equal to 60 ? (1)

(ii) was in the 3CDMAT class? (1)

(iii) was in the 3CD MAS class and received a mark ≥ 80 ? (1)

(iv) was in the 3CDMAS class if the student received a mark between 60 and 79 inclusive ? (1)

Solution
(i) $\frac{2}{3}$ (ii) $\frac{27}{48} = \frac{9}{16}$ (iii) $\frac{5}{48}$ (iv) $\frac{9}{20}$

Solution	- during office hours, some unemployed people will be contacted but so too will home-carers, retired individuals, people running a business from home, etc - not all people are listed on the telephone directory
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- (2) bias?
 (i) List two ways which might suggest this data collection method causes

"A team of data collectors phoned five randomly selected households on every twelfth page of the local telephone directory during regular office hours".

- (b) In collecting data, biased samples and poor methods of collection can lead to unemployment in a local community.
 invalid inferences. Consider the following method to determine the extent of

Question 13 (continued)

Solution	p = 21 q = 37 n = 10 m = 8 r = 41 <i>Specific behaviours</i> VVVV 1 mark each
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- (5) Determine the values of m, n, p, q, r.

The minimum score is 2 less than the smaller mode.

There are two modes of which 10 is one of them.

$$\bar{x} = 24, \text{Median} = 21, \text{Interquartile range} = 27.$$

The following statistics of this set of data are known.

$$m, 9, n, 10, 15, 18, 21, p, 25, 29, 35, q, 38, r, 43.$$

follows:

- (a) A group of fifteen students travelling to Tasmania have their bags weighed at the airport check-in counter. The weights of the bags in kilograms are listed in order as follows:
 m, 9, 10, 15, 18, 21, p, 25, 29, 35, q, 38, r, 43.

Question 13 (8 marks)

<i>Specific behaviours</i>	✓ or X
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- Thus those interviewed will not provide the right kind of data to make supportable inferences
Specific behaviours
✓✓ 2 valid reasons

(ii) Suggest how this bias could be avoided. (1)

Solution
- Door knock after regular working hours
- Ring after regular working hours to enable more people to be reached
- Survey form
Specific behaviours
✓ any 1 valid suggestion

- (iii) Josh decides to reduce his social life activities in order to pay an extra \$200 a month from the 13th payment onwards. Values have been rounded to the nearest ten cents.

Complete the table of calculation below. (2)

Month	Previous Balance	Interest	Balance Due	Payment	Balance after payment
13	1683.50	33.70	1717.20	500	1217.20
14	1217.20	24.30	1241.50	500	741.50
15	741.50	14.80	756.30	500	256.30

Solution
As shown in table
Specific behaviours
✓✓ for the three correct values

Question 17 (continued)

(iv) How long does it take Josh to repay the loan? (1)

Solution
16 months
Specific behaviours
✓ or X

(v) What is the total amount that Josh will pay over the period with the increased repayment ? (2)

Solution
$12 \times 300 + (3 \times 500) + 256.30 + (0.02 \times 256.30) = \5361.40
Specific behaviours
✓✓ calculation and correct answer

(vi) How much money does Josh save by paying in this way? (1)

Solution
$\$5403.50 - \$5361.40 = \$42.10$

Question 14
marks

- (a) The graph of $f(x)$ is dilated parallel to the x -axis by a scale factor of 2, followed by a translation of 4 units in the positive direction of the x -axis. What is the equation of the final graph obtained?

$$f\left(\frac{1}{2}(x-4)\right) = f\left(\frac{1}{2}x-2\right)$$

Solution				
Specific behaviours				
1	4500	90	4590	300
2	4290	85.80	4375.80	300
3	4075.80	81.50	4157.30	300
4	3857.30	77.10	3934.50	300
5	3634.50	72.70	3707.20	300
6	3407.20	68.10	3475.30	300
7	3175.30	63.50	3238.80	300
8	2938.80	58.80	2997.60	300
9	2697.60	53.90	2751.50	300
10	2451.50	49.10	2500.60	300
11	2200.60	44.00	2244.60	300
12	1944.60	38.90	1983.50	300

Solution				
Specific behaviours				
(b)	The function $y=x^3$ is transformed to $y=-2(x+5)^3$. Describe the transformation in order.			
(c)	The function $y=x^3$ is transformed to $y=-2(x+5)^3$. Describe the transformation in correct order.			
				✓ transformations in correct order
				✓ translation 5 left, dilation 2 parallel to y -axis and reflection in the x -axis

(2)

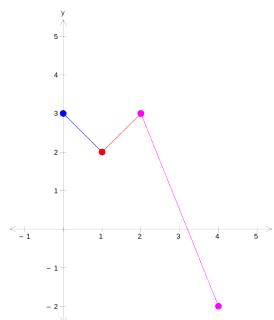
\$1683.50. He created a spreadsheet to check whether the amount owing is correct.

Josh bought a second hand Hyundai for \$4500. He borrowed the full sum of \$4500 from NO Frills Finance Company and agreed to make a minimum monthly payment of \$300 a month. For a year he pays the minimum monthly payment. At the end of the year, he checks his statement from the Finance company and is surprised that he still owes \$1683.50. He created a spreadsheet to check whether the amount owing is correct.

(c) The graph of $y=f(x)$ is given below

- (i) Sketch the graph of $y=3 - f(x)$ on the axes below.

(2)



Solution
As shown above
Specific behaviours
<ul style="list-style-type: none"> ✓ reflected shape ✓ all correct points $(0,3)$, $(1,2)$, $(4,-2)$

Question 14 (continued)

- (ii) Write the coordinates of A if $y = -f(-x+3)$.

(2)

Solution
$(1,1) \rightarrow (-1,1) \rightarrow (2,1) \rightarrow (2,-1)$
Coordinates of A = $(2,-1)$
Specific behaviours
<ul style="list-style-type: none"> ✓ working ✓ correct coordinates of A

- (d) Describe how the graph of $f(x)$ can be used to obtain the graph of $g(x)$.

(1)

Specific behaviours
As indicated above

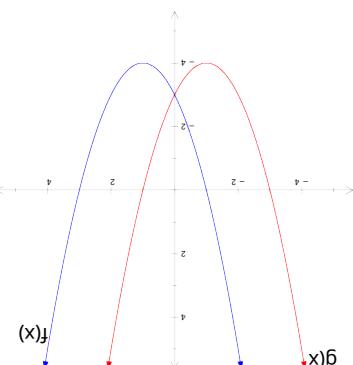
Question 16 (continued)

- (c) Solve the indicial equation showing all working steps

(3)

$$2 \times 3^{2x+3} = 6 \times 9^{4x}$$

Solution
$2 \times 3^{2x+3} = 2 \times 3^{1+8x}$
Equating indices: $2x + 3 = 1 + 8x$
$x = \frac{1}{3}$
Specific behaviours
<ul style="list-style-type: none"> ✓ expresses as powers of 3 ✓ equates indices ✓ solves correctly for x



- (a) The following table shows statistics of the results of the five mathematics tests for John, Ryan, Julie and Gary.
- | | Mean | Standard deviation | Range | (8 marks) |
|-------|------|--------------------|-------|-----------|
| Gary | 14.0 | 3.4 | 7 | |
| Julie | 13.8 | 2.7 | 7 | |
| Ryan | 13.9 | 4.2 | 7 | |
| John | 13.6 | 3.8 | 7 | |
- Which student has the most consistent results? State a reason why you choose this student. (2)

Gary	14.0	3.4	7
Julie	13.8	2.7	7
Ryan	13.9	4.2	7
John	13.6	3.8	7

Solution	Reflection about the y -axis Or $f(x)$ is translated 2 units left Julie because she has the smallest spread of marks (standard deviation)
Specific behaviours	✓ or X

- (b) Mrs Kendall decided to survey a sample of 10% of the students at her school. The school enrollment is shown in the table below.
- | Year | 7 | 8 | 9 | 10 | 11 | 12 | Total |
|--------------------|-----|-----|-----|-----|-----|-----|-------|
| Number of students | 225 | 232 | 233 | 230 | 150 | 130 | 1200 |
- She surveyed the same number of students in each year group. Explain how she had chosen to use a stratified sample based on year groups?
- Would the numbers of students surveyed in Year 10 and Year 11 have changed if i.e. $10\% \times 1200 = 120$ Same number for each group
Year 10: $10\% \text{ of } 230 = 23$ i.e. increase of 3 ✓
Year 11: $10\% \text{ of } 150 = 15$ i.e. decrease of 5 ✓
- (3)

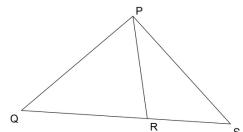
Same number for each group	Solution
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i.e. $10\% \times \frac{6}{1200} = 20$

✓ Year 10: $10\% \text{ of } 230 = 23$ i.e. increase of 3 ✓
✓ Year 11: $10\% \text{ of } 150 = 15$ i.e. decrease of 5 ✓

Question 15**(7 marks)**

- a) In the given diagram, PR = 8 cm, QR = 5 cm and the sizes of angles PRQ and PSR are 60° and 40° respectively. Calculate



(i) the exact length of PQ

(2)

Solution
$c^2 = 5^2 + 8^2 - 2(5)(8)\cos 60^\circ$
$= 49$
$c = 7$

Specific behaviours

- ✓ Correct use of Cosine rule
- ✓ Correct value for PQ

(ii) the length of PS correct to 1 decimal place

(2)

Solution
$\frac{x}{\sin 120^\circ} = \frac{8}{\sin 40^\circ}$
$x = 10.8$

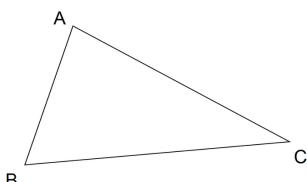
Specific behaviours

- ✓ correct use of sine rule
- ✓ correct answer to 1 decimal place

- a) Triangle ABC is a non-right angled triangle. Given angle A = 30° , a = 6 and b = 9.

(3)

Find the value(s) of angle B.



(2)

Solution
$\frac{\sin x}{9} = \frac{\sin 30^\circ}{6}$
$\sin x = 0.75$
$x = 48.6 \text{ or } 131.4$

Specific behaviours

- ✓ correct use of sine rule
- ✓ correct answers