

it to the supervisor **before** reading any further.
you do not have any unauthorised material. If you have any unauthorised material with you, hand
No other items may be taken into the examination room. It is **your responsibility** to ensure that

Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators approved for use in this examination.

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Formula sheet (retained from Section One)

This Question/Answer booklet

Materials required/recommended for this section

Working time:
ten minutes

Reading time before commencing work:
ten minutes

Time allowed for this section

Your name
in words

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Student number: in figures

Calculator-assumed

Section Two:

UNIT 3

METHODS

MATHEMATICS

SOLUTIONS

Question/Answer booklet

Semester One Examination, 2019



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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Supplementary page

Question number: _____

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Question 10

(8 marks)

The potential difference, V volts, across the terminals of an electrical capacitor t seconds after it begins to discharge through a resistor can be modelled by the equation

$$V = V_0 e^{-kt}$$

V_0 is the initial potential difference and k is a constant that depends on the size of the capacitor and the resistor.

- (a) If $V_0 = 15.8$ volts and $k = 0.013$, determine

- (i) the potential difference across the capacitor 2 minutes after discharge began. (2 marks)

Solution
When $t=120$, $V=3.32$ volts
Specific behaviours
✓ uses correct time ✗ calculates correct voltage

- (ii) the time taken for the potential difference to drop from 10.5 to 7.5 volts. (3 marks)

Solution
When $V=10.5$, $t=31.4$ and when $V=7.5$, $t=57.3$. Hence takes $57.3 - 31.4 = 25.9$ seconds.
Specific behaviours
✓ calculates first time ✗ calculates second time ✗ calculates difference, correct to at least 1 dp

- (iii) the rate of change of V when the potential difference is 5 volts. (1 mark)

Solution
$\frac{dV}{dt} = -kV = -0.013 \times 5 = -0.065$ volts/sec
Specific behaviours
✓ calculates rate

- (b) Another capacitor takes 110 seconds for its maximum potential difference to halve. It is instantly recharged to its maximum every 4 minutes, which is the time required for the potential difference to fall from its maximum to 3.5 volts. Determine the maximum potential difference for this capacitor. (2 marks)

Solution
$e^{-110k} = 0.5 \Rightarrow k = 0.0063$
$3.5 = V_0 e^{-0.0063 \times 240} \Rightarrow V_0 = 15.88$
Specific behaviours
✓ determines k ✗ determines V_0

Question 20

(7 marks)

A small body has displacement $x=0$ when $t=2$ and moves along the x -axis so that its velocity after t seconds is given by

$$v(t) = 60 \cos\left(\frac{\pi t}{12}\right) \text{ cm/s}$$

- (a) Determine an equation for $x(t)$, the displacement of the body after t seconds. (3 marks)

Solution
$x = \frac{60 \times 12}{\pi} \sin\left(\frac{\pi t}{12}\right) + c$
$t=0 \Rightarrow 0 = \frac{720}{\pi} \sin\left(\frac{\pi}{6}\right) + c \Rightarrow c = -\frac{360}{\pi}$
$x = \frac{720}{\pi} \sin\left(\frac{\pi t}{12}\right) - \frac{360}{\pi}$
Specific behaviours
✓ integrates v correctly ✗ attempts to find constant using substitution ✗ correct equation

- (b) Describe, with justification, how the speed of the body is changing when $t=10$. (4 marks)

Solution
$v(10) = 60 \cos\left(\frac{5\pi}{6}\right) = -30\sqrt{3}$
$a = \frac{-60\pi}{12} \sin\left(\frac{\pi t}{12}\right)$
$a(10) = -5\pi \sin\left(\frac{5\pi}{6}\right) = \frac{-5\pi}{2}$
Since the body has a negative velocity and a negative acceleration then its speed is increasing when $t=10$.
Specific behaviours
✓ clearly shows v is negative ✗ expression for a ✗ clearly shows a is negative

An aquarium, with a volume of $50\ 000 \text{ cm}^3$, takes the shape of a rectangular prism with square ends of side x cm and no top. The glass for the four vertical sides costs 0.5 cents per square centimetre and the base costs 0.08 cents per square centimetre. The cost of glue to join the edges of two adjacent pieces of glass is 0.6 cents per centimetre. Assume the glass has negligible thickness and ignore any other costs.

(a) Determine

(i) $P(X < 6)$.
(1 mark)

Solution
$P(X < 6) = \frac{3}{5} = 0.6$
Specific behaviours
E(X)

(ii) $P(X > 7 | X \geq 2)$.
(4 marks)

Solution
$P(X > 7 X \geq 2) = \frac{5}{4}$
Specific behaviours
P(X ≥ 2)

(b) Calculate the exact value of
 $P(X > 7 | X \geq 2)$.
(2 marks)

Solution
$E(X) = \frac{1+2+4+8+12}{5} = 5.4$
Specific behaviours
E(X)

(i) $E(X)$.
(2 marks)

(a) Show that $C = \frac{100}{x^2} + \frac{90}{x^2} + \frac{600}{x^2}$, where C is the cost, in dollars, to make the aquarium.
Ignore any other costs.

An aquarium, with a volume of $50\ 000 \text{ cm}^3$, takes the shape of a rectangular prism with square ends of side x cm and no top. The glass for the four vertical sides costs 0.5 cents per square centimetre and the base costs 0.08 cents per square centimetre. The cost of glue to join the edges of two adjacent pieces of glass is 0.6 cents per centimetre. Assume the glass has negligible thickness and ignore any other costs.

(i) Determine

(i) $P(X < 6)$.
(1 mark)

Solution
$\sigma_x = \sqrt{\frac{5}{26}} \approx 4.079, \sigma_x^2 = \frac{5}{25} = 16.64$
Standard deviation
Specific behaviours

(ii) $Var(X)$.
(2 marks)

(b) Show use of a calculus method to determine the minimum cost of making the aquarium.
(3 marks)

Solution
$\frac{dC}{dx} = 0 \text{ when } x = 34.48 \text{ cm}$
$\frac{dC}{dx} = \frac{500x^3}{x^4 + 18x^3 - 4500x - 60000}$
Shows marginal cost determines minimum cost, to nearest cent.

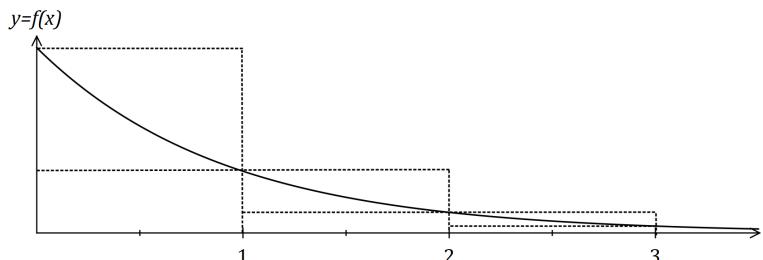
Shows marginal cost determines value of x so that marginal cost is zero

Determines minimum cost to nearest cent.

(5 marks)

Question 12

The function $f(x) = \frac{3}{3^x}$ is shown below.



- (a) Use the sum of the areas of the circumscribed rectangles shown in the diagram to explain why $\int_0^3 f(x) dx < \frac{13}{3}$.

(2 marks)

Solution

$$\text{Sum} = 1 \times \left(3 + 1 + \frac{1}{3}\right) = \frac{13}{3}$$

Since the definite integral represents area under curve, the value of the integral must be less than $\frac{13}{3}$.

Specific behaviours

✓ shows calculation for area overestimate

✗ explains area under curve must be less than overestimate

- (b) Use the average of the sum of the areas of the inscribed rectangles and the sum of the

areas of the circumscribed rectangles shown to determine an estimate for $\int_0^3 f(x) dx$.

(2 marks)

Solution

$$\text{Sum}_2 = 1 \times \left(1 + \frac{1}{3} + \frac{1}{9}\right) = \frac{13}{9}$$

$$\text{Avg} = \frac{13}{3} + \frac{13}{9} = \frac{26}{9} \text{ sq units}$$

Specific behaviours

✓ shows calculation for underestimate

✗ calculates average

- (c) Suggest a modification to the method used in (b) to achieve a better estimate for

$$\int_0^3 f(x) dx.$$

Solution

Use a larger number of narrower rectangles.

(1 mark)

Specific behaviours

✓ sensible modification

Suppose it is known that 87% of all seeds planted will germinate and that seeds are now planted in rows of 20.

- (c) Assuming that seeds germinate independently of each other, determine

- (i) the most likely number of seeds to germinate in a row.

(1 mark)

Solution
18 seeds
Specific behaviours
✓ correct number

- (ii) the probability that no more than 16 seeds germinate in a randomly chosen row.

(2 marks)

Solution
$W \sim B(20, 0.87)$
$P(W \leq 16) = 0.2573$
Specific behaviours
✓ states distribution
✗ correct probability

- (iii) the probability that in six randomly chosen rows, exactly three rows have no more than 16 seeds germinating in them.

(2 marks)

Solution
$V \sim B(6, 0.2573)$
$P(V=3) = 0.1396$
Specific behaviours
✓ states distribution
✗ correct probability

$$r(t) = 20(1 - e^{-0.25t}) \text{ m}^3/\text{minute}$$

Number of germinating seeds	
Number of rows	1
0	1
1	2
2	3
3	4
4	5
5	6
6	18
18	33
33	22
22	

$$P(X \geq 4) = \frac{55}{80} = 0.6875$$

Specific behaviours	
✓ correct rate (exact or at least 1dp)	
$r(1) = 20(1 - e^{-1}) = 12.64 \text{ m}^3/\text{minute}$	
Solution	
Seconds after production began.	

Specific behaviours	
✓ correct r' (1)	
$\frac{1}{e^{0.25}} \approx 0.130 \text{ m}^3/\text{minute}$	
$dr \approx \frac{dt}{dt} \times r' \approx 5e^{-0.25t} \times \frac{5}{2} \times \frac{1}{e^{0.25t}}$	
Solution	
seconds after production began.	

Specific behaviours	
✓ uses correct t and dt	
✓ correct use of increments formula	
correct estimate (at least 2dp)	
Solution	

Number of rows	1	0	6	2	3	18	33	22
Number of germinating seeds	0	1	2	3	4	5		

Question 14

(12 marks)

The random variable X is the number of goals scored by a team in a soccer match, where

$$P(X=x) = \frac{2.2^x e^{-2.2}}{x!} \text{ for } x=0, 1, 2, 3, \dots \text{ to infinity}$$

- (a) Determine the probability that the team scores at least one goal in a match. (2 marks)

Solution
$P(X=0) = 0.1108$
$P(X>0) = 1 - 0.1108 = 0.8892$
Specific behaviours
✓ $P(X=0)$ correct probability

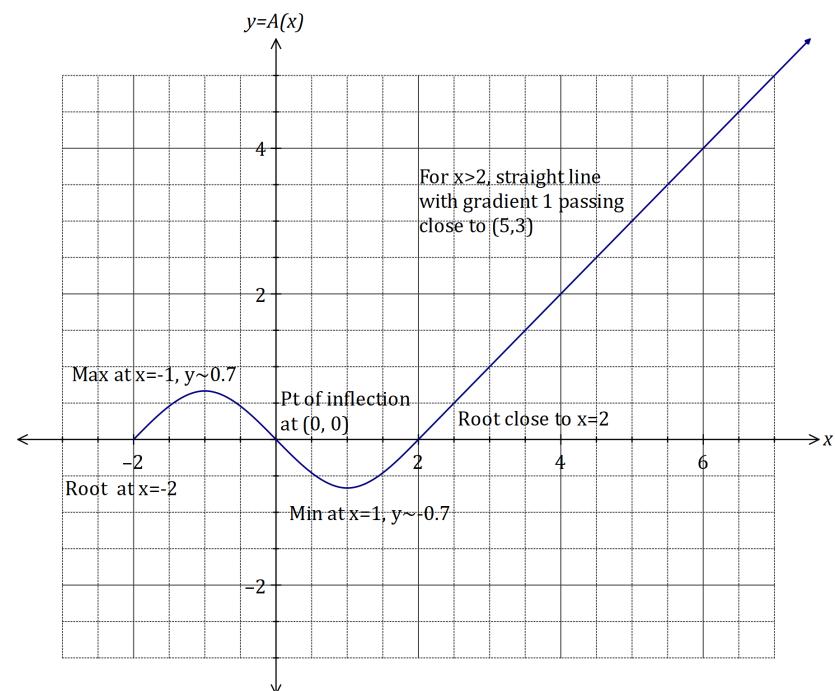
The random variable Y is the bonus each player is paid after a match, depending on the number of goals the team scored. For four or more goals \$500 is paid, for two or three goals \$250 is paid and for one goal \$100 is paid. No bonus is paid if no goals are scored.

- (b) Complete the probability distribution table for Y . (3 marks)

Goals scored	$x=0$	$x=1$	$2 \leq x \leq 3$	$x \geq 4$
$y(\$)$	0	100	250	500
$P(Y=y)$	0.1108	0.2438	0.4648	0.1806

Solution
$P(Y=100) = P(X=1) = 0.2438$
$P(Y=250) = 1 - 0.1108 - 0.2438 - 0.1806 = 0.4648$
Specific behaviours
✓ missing y values ■ $P(Y=0)$ and $P(Y=100)$ ■ $P(Y=250)$

- (c) Sketch the graph of $y=A(x)$ on the axes below, indicating and labelling the location of all key features. (5 marks)



Solution
See graph
Specific behaviours
✓ Labelled point of inflection at origin ■ Labelled roots, as indicated ■ Curve $-2 < x < 0$ with labelled maximum ■ Curve $0 < x < 2$ with labelled minimum ■ Straight line, as indicated

(2 marks)

- (i) the mean bonus paid per match.

Solution
$V = 0 + 24.38 + 116.20 + 90.30 \text{ or } \230.88
Specific behaviours
expression
mean

(2 marks)

- (ii) the standard deviation of the bonuses paid per match.

Solution
$\sigma_V^2 = 2332.40 \text{ or } \152.75
specific behaviours
variance
standard deviation

- (d) The owner of the team plans to increase the current bonuses by \$50 next season (so that the players will get a bonus of \$50 even when no goals are scored) and then further raise them by 12% the following season. Determine the mean and standard deviation of the bonuses paid per match after both changes are implemented. (3 marks)

Solution
$Z = V + 50 \times 1.12$
$Z = (230.88 + 50) \times 1.12 = \314.59
$\sigma_Z^2 = 152.75 \times 1.12 = \171.08
specific behaviours
new mean
new standard deviation

(9 marks)

- (c) Calculate

(9 marks)

- (b) Using the graph of
- $y = f(x)$
- or otherwise, explain why
- $A(5) = 3$
- . (2 marks)

✓ shows use of and integral

Specific behaviours

From the graph $\int_5^2 f(x) dx = 1 \times 3 = 3$, and hence $A(5) = 0 + 3 = 3$.

$$A(5) = A(2) + \int_5^2 f(x) dx.$$

Solution

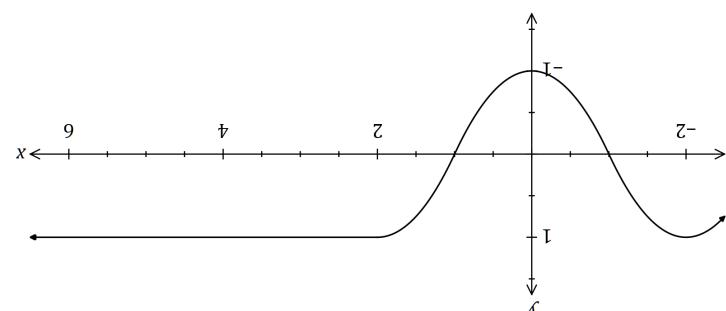
- (b) Using the graph of
- $y = f(x)$
- or otherwise, explain why
- $A(5) = 3$
- . (2 marks)

it is also known that the $A(2) = 0$.

Solution
At $x = -1$ there is a maximum
At $x = 1$ there is a minimum
location of maximum
location of minimum

- (a) Use the graph of $y = f(x)$ to identify all the turning points of the graph of $y = A(x)$, starting the x -coordinate and nature of each point. (2 marks)

$$\text{Let } A(x) \text{ be defined by the integral } A(x) = \int_x^{-2} f(t) dt \text{ for } x \geq -2.$$



The graph of $y = f(x)$ is shown below.

CALCULATOR-ASSUMED

- (c) Calculate

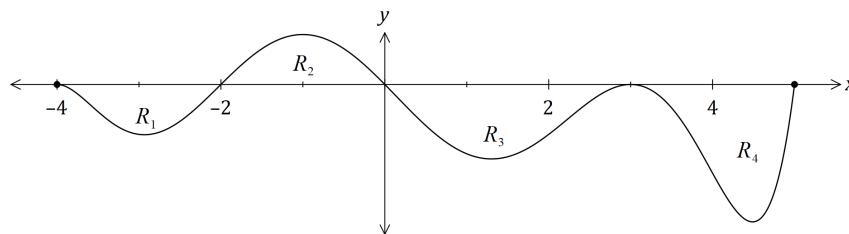
- (b) Calculate

- (a) Calculate

(7 marks)

Question 15

The graph of $y=f(x)$ is shown below for $-4 \leq x \leq 5$.



The area trapped between the x -axis and the curve for regions R_1, R_2, R_3 and R_4 are 21, 25, 43 and 32 square units respectively.

(a) Determine the value of

$$(i) \int_0^3 f(x) dx.$$

Solution
-43
Specific behaviours ✓ correct value

(1 mark)

$$(ii) \int_{-2}^5 f(x) dx.$$

Solution
$25 - 43 - 32 = -50$
Specific behaviours ✓ shows sum of signed areas ✗ correct value

(2 marks)

$$(iii) \int_{-2}^3 |2-f(x)| dx.$$

Solution
$2 \times 5 - (25 - 43) = 10 - (-18) = 28$
Specific behaviours ✓ area of rectangle ✗ correct value

(2 marks)

$$(iv) \int_{-4}^0 f(x) dx + \int_0^5 f'(x) dx.$$

Solution
$-21 + 25 + [0 - 0] = 4$
Specific behaviours ✓ shows second integral is zero ✗ correct value

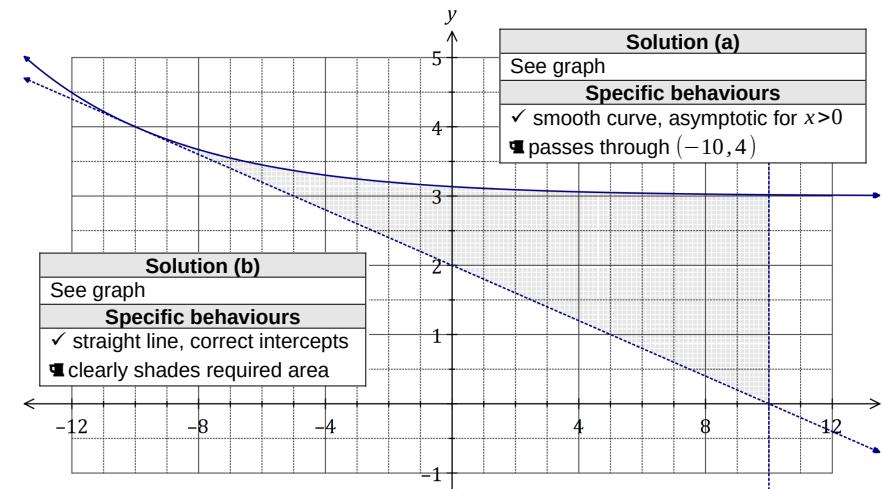
(2 marks)

Question 16

Let $f(x) = 3 + e^{-0.2x-2}$.

(a) Sketch the graph of $y=f(x)$ on the axes below.

(2 marks)



(b) The line $y=2-0.2x$ is tangential to the curve $y=f(x)$ at $x=-10$, and it intersects the x -axis at the point $(k, 0)$. Add the line to the graph above and shade the area enclosed by the line, the curve and $x=k$.

(2 marks)

(c) Determine the area enclosed by the line, the curve and $x=k$.

(3 marks)

Solution
$2 - 0.2k = 0 \Rightarrow k = 10$
$A = \int_{-10}^{10} (3 + e^{-0.2x-2}) - (2 - 0.2x) dx$
$\therefore 25 - 5e^{-4} \approx 24.9 \text{ sq units}$
Specific behaviours ✓ indicates value of k ✗ writes integral using difference of functions ✗ evaluates integral