



Rossmyrne Senior High School

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS METHODS YEAR 12 (ATMAM)

Section Two: Calculator-assumed

Circle your Teacher's Name: Alvaro Bestall Fraser-Jones Kigodi
 Koulianos Luzuk Murray Tanday

Student number: In figures

 In words _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
 correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
 and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material with you. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

METHODS UNIT 3 12 CALCULATOR-ASSUMED 13 CALCULATOR-ASSUMED METHODS UNIT 3

Question 17
Seeds were planted in rows of five and the number of seeds that germinated in each of the 120 rows are summarised below.

Number of germinating seeds	0	1	2	3	4	5
Number of rows	1	1	3	16	46	53

- (a) Use the results in the table to determine
- (i) the probability that no more than 4 seeds germinated in a randomly selected row. (1 mark)
- (ii) the mean number of seeds that germinated per row. (1 mark)
- (b) Another row of five seeds is planted. Determine the probability that no more than 4 seeds germinate in this row. Assume the number that germinate per row is binomially distributed with the above mean. (2 marks)
- (iii) the probability that in eight randomly chosen rows, exactly six rows have at least 9 seeds germinating in them. (2 marks)
- (iv) the probability that at least 9 seeds germinate in a randomly chosen row. (2 marks)
- (v) Assuming that seeds germinate independently of each other, determine the most likely number of seeds to germinate in a row. (1 mark)

See next page

END-19-1

END-19-1

See next page

METHODS UNIT 3

14

CALCULATOR-ASSUMED

Question 18

9 marks

The graph of $y = f(x)$ is shown below.

METHODS UNIT 3

11

CALCULATOR-ASSUMED

Question 16

7 marks

The graph of $y = f(x)$ is shown below for $-4 \leq x \leq 5$.

The area trapped between the x -axis and the curve for regions R_1 , R_2 , R_3 , R_4 , R_5 and R_6 are 3.5, 5.2, 2.8 and 2.4 square units respectively.

(a) Determine the value of

(i) $\int_{-2}^4 f(x) dx$. (1 mark)

(ii) $\int_5^{-2} f(x) dx$. (2 marks)

(iii) $\int_5^{-2} f(x) - 7 dx$. (2 marks)

(iv) $\int_1^{-1} f(x) dx - \int_1^4 f(x) dx$. (2 marks)

METHODS UNIT 3

2

CALCULATOR-ASSUMED

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total				100	

Instructions to candidates

1.

The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.

Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.

You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.

It is recommended that you do not use pencil, except in diagrams.

6.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.

The Formula sheet is not to be handed in with your Question/Answer booklet.

METHODS UNIT 3

10

CALCULATOR-ASSUMED

7 marks

Question 15

$f(x) = 2 + e^{-0.4x-2}$.

(a) Sketch the graph of $y = f(x)$ on the axes below.

(2 marks)

(b) The line $y = 1 - 0.4x$ is a tangent to the curve $y = f(x)$ at $x = -5$, and it intersects the x-axis at the point $(k, 0)$. Add the line to the graph above and shade the area enclosed by the line, the curve and $x = k$.

(2 marks)

(c) Determine the area enclosed by the line, the curve and $x = k$.

(3 marks)

METHODS UNIT 3

15

CALCULATOR-ASSUMED

15

Key features:

(c) Sketch the graph of $y = A(x)$ on the axes below, indicating and labeling the location of all

(5 marks)

See next page

METHODS UNIT 3

3

CALCULATOR-ASSUMED

65% (98 Marks)

Section Two: Calculator-assumed

This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(5 marks)

Fuel flows into a storage tank that is initially empty at a rate of $\sqrt{4 + 3t}$ litres per minute, where t is the time in minutes and $0 \leq t \leq 100$.

(a) Determine how much fuel is in the tank after 20 minutes.

(2 marks)

(b) If the tank is completely full after 100 minutes, determine the time required for the tank to become one-quarter full.

(3 marks)

See next page

See next page

SN085-135-2

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See next page

CALCULATOR-ASSUMED

4

METHODS UNIT 3

(7 marks)

Question 10
X is a uniform discrete random variable where $x = 2, 3, 5, 7, 11, 13$.

(a) Determine
(i) $P(X \geq 5)$. (1 mark)

(ii) $P(X < 12 \mid X \geq 3)$. (2 marks)

(b) Calculate the exact value of
(i) $E(X)$. (2 marks)

(ii) $\text{Var}(X)$. (2 marks)

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SN085-135-2

See next page

(b)

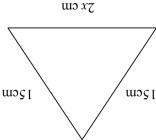
Use calculus methods to find the maximum area of the triangle.

(4 marks)

(a)

Show that $A = x\sqrt{225 - x^2}$.

(2 marks)



A triangle has dimensions as shown below:

METHODS UNIT 3
Question 19

(6 marks)

16

CALCULATOR-ASSUMED

(a)

Determine an equation for $x(t)$, the displacement of the body after t seconds. (3 marks)

$$v(t) = 10 \sin\left(\frac{2\pi}{3}t\right) \text{ cm/s}$$

after t seconds is given by

A small body has displacement $x = 0$ when $t = 0$ and moves along the x -axis so that its velocity

METHODS UNIT 3
Question 14
(7 marks)

9

CALCULATOR-ASSUMED

Question 11

(8 marks)

The potential difference, V volts, across the terminals of an electrical capacitor t seconds after it begins to discharge through a resistor can be modelled by the equation

$$V = V_0 e^{-kt}$$

V_0 is the initial potential difference and k is a constant that depends on the size of the capacitor and the resistor.

- (a) If $V_0 = 22.6$ volts and $k = 0.018$, determine
- (i) the potential difference across the capacitor 4 minutes after discharge began. (2 marks)
- (ii) the time taken for the potential difference to drop from 17.5 to 12.5 volts. (3 marks)

- (iii) the rate of change of V when the potential difference is 20 volts. (1 mark)

- (b) Another capacitor takes 99 seconds for its maximum potential difference to halve. It is instantly recharged to its maximum every 3 minutes, which is the time required for the potential difference to fall from its maximum to 1.8 volts. Determine the maximum potential difference for this capacitor. (2 marks)

- (d) Use the increments formula to determine the approximate volume of gas produced in the 5 seconds following $t = 2$. (3 marks)

- (c) Use the increments formula to determine the approximate change in r between 30 and 33 seconds after production began. (3 marks)

- (b) Calculate the rate that gas is being produced after 2 minutes. (1 mark)

- (a) State the maximum rate that gas can be produced at. (1 mark)

Question 13
A manufacturing process begins and the rate at which it produces gas after t minutes ($t \geq 0$) is modelled by
 $r(t) = 45(1 - e^{-0.4t})$ m^3/minute

(8 marks)

Question 20

(7 marks)

- (a) Given that $f(t) = \sin\left(3t + \frac{\pi}{2}\right)$ and $F(x) = \int_x^{40} f(t) dt$, determine the exact value of

(i) $F\left(\frac{\pi}{2}\right)$.

(1 mark)

(b) $F'\left(\frac{\pi}{2}\right)$.

(2 marks)

- (b) Given that $G(x) = \int_x^1 \theta(t) dt$, $\frac{d^2G}{dx^2} = 4 + 3\sqrt{x}$ and $G(4) = 56$, determine $g(1)$. (4 marks)

$P(Y = y)$				
Y (\$)	0			
Goals scored	$x = 0$	$x = 1$	$2 \leq x \leq 3$	$x \geq 4$
				500
				0.1806

(b) Complete the probability distribution table for Y . (3 marks)

The random variable Y is the bonus each player is paid after a match, depending on the number of goals the team scored. For four or more goals \$500 is paid, for two or three goals \$250 is paid and for one goal \$100 is paid. No bonus is paid if no goals are scored.

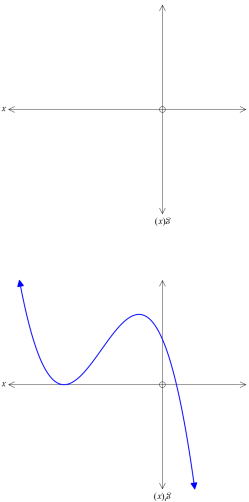
(a) Show that the probability that the team scores at least one goal in a match is $P(X \geq 1) = 0.8892$. (2 marks)

$P(X = x) = \frac{2 \cdot 2^x \cdot e^{-2.2}}{x!}$ for $x = 0, 1, 2, 3, \dots$ to infinity

The random variable X is the number of goals scored by a team in a soccer match, where

Question 21 (12 marks)

METHODS UNIT 3 18 CALCULATOR-ASSUMED



(b) The graph of a gradient function is shown below. On the set of axes provided sketch a possible graph of its antiderivative. (3 marks)

METHODS UNIT 3 7 CALCULATOR-ASSUMED

- (c) Calculate
- (i) the mean bonus paid per match. (2 marks)
- (ii) the standard deviation of the bonus paid per match. (2 marks)
- (d) The owner of the team plans to increase the current bonuses by \$50 next season (so that the players will get a bonus of \$50 even when no goals are scored) and then further raise them by 12% the following season. Determine the mean and standard deviation of the bonus paid per match after both changes are implemented. (3 marks)

- METHODS UNIT 3 6 CALCULATOR-ASSUMED
- Question 12 (6 marks)
- (a) Draw a graph that satisfies all the conditions listed below. Label the critical features clearly. (3 marks)
- $f(-2) = f(6) = f(8) = 0$
 $f'(4) = 0$ and $f''(x) < 0$ for $x < 4$ only
 $f'(1) = f'(7) = 0$

