Rossmoyne Senior High School

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNITS 1&2

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Circle your Teacher's Name:

Mrs Bestall | Mr Buckland | Mrs Fraser-Jones
Mr Gibbon | Ms Goh/Mr Freer | Ms Leonard
Mr Luzuk | Mr Ng | Mrs Murray

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
9	5	
10	8	
11	9	
12	8	
13	8	
14	8	
15	5	
16	5	
17	6	
18	9	
19	10	
20	9	
21	7	
S2 Total	97	
S2 Wt (×0.6633)	65%	

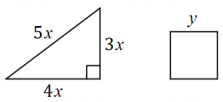
See next page

Supplementary page

Question number: _____

Question 21 **(7 marks)**

A length of wire 72 cm long is cut into two pieces. One piece is bent into a right triangle with sides of length $3x, 4x$ and $5x$ cm and the other piece is bent into a square of side y cm.



(a) Show that the combined area of the triangle and square in terms of x is $A = 15x^2 - 108\,x + 324$ (3 marks)

(b) Use a calculus method to determine the value of x that minimises this combined area and state this minimum area. (4 marks)

End of Questions

Section Two: Calculator-assumed **65% (97 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

Sector POQ subtends an angle of $80°$ in a circle with centre O and radius r.

(a) Express $80°$ as an exact and simplified radian measure. (1 mark)

The area of sector POQ is 50π cm^2.

(b) Determine the radius of the circle. (2 marks)

(c) Determine the area of the minor segment bounded by arc PQ and chord PQ. (2 marks)

See next page

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Question 10 **(8 marks)**

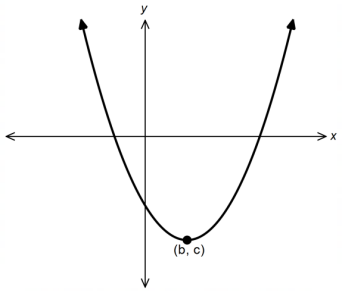
The graphs of the following equations are drawn below.

(i) $y = 2^x - 2$ **(ii)** $y = -x(1-x)^2$ **(iii)** $y = (1-x)^3 + 1$

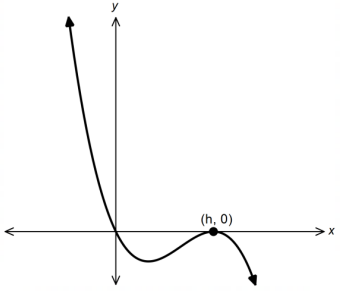
(iv) $y = 2 - 2^x$ **(v)** $y = (1-x)^2 - 3$ **(vi)** $y = (x+1)^2 - 4$

a, *b*, *c*, *d*, e, *f*, *g* and *h* are arbitrary constants, *x* and *y* are variables.

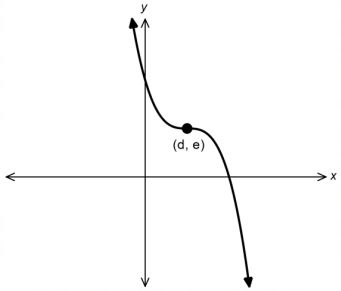
A



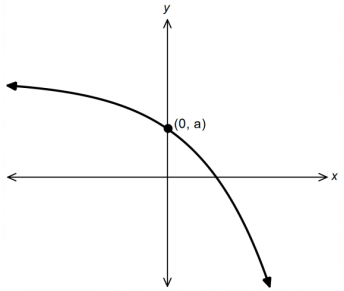
B



C



D



(d) Determine the length of time during the first 3 seconds for which $h_C > h_A > h_B$, correct to 3 decimal places. (3 marks)

Question 20 **(9 marks)**

Three small weights A, B and C, each attached to a spring, are oscillating vertically above level ground. The height, h cm, above the ground of each weight at time t seconds, $t \geq 0$, is given by

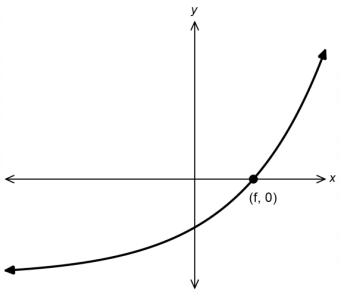
$$h_A = 16\cos\left(\frac{3\pi t}{4}\right) + 20, \qquad h_B = 12\sin\left(\frac{3\pi t}{4}\right) + 25, \qquad h_C = 12\cos\left(\frac{5\pi t}{4}\right) + 20.$$

(a) State which two weights are oscillating with the same amplitude, and state what this common amplitude is. (1 mark)

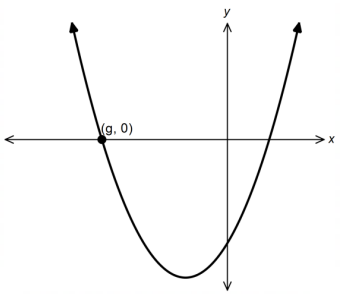
(b) State which two weights are oscillating with the same period, and state what this common period is. (2 marks)

(c) State which of the weights reaches furthest above the ground, state this height and find the time at which it first reaches this position. (3 marks)

E



F



(a) Match the equations with the sketches by writing **(i)** to **(vi)** corresponding to the equation in the table below. (3 marks)

A	B	C	D	E	F

Hence,

(b) determine the values of a, b, c, d, e, f, g and h. (5 marks)

a	b	c	d	e	f	g	h

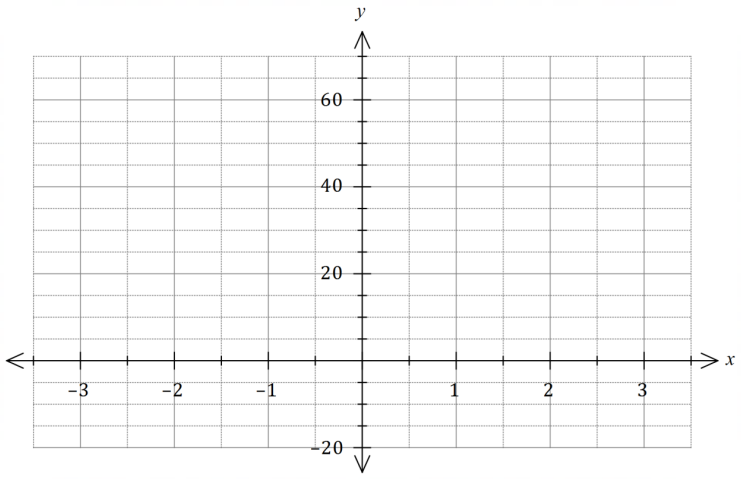
Question 11 **(9 marks)**

A function is defined by $f(x) = x^4 - 6x^2 + 8x + 13$.

(a) Use calculus to determine the coordinates of all stationary points of the graph $y = f(x)$, and then use the sign test to determine their nature.

(5 marks)

(b) Sketch the graph of $y = f(x)$ on the axes below for $-3 \leq x \leq 3$. (4 marks)



(c) Add the line $y = 2x + 3$ to the graph of the hyperbola and state the number of points of intersection it will have with the hyperbola. (2 marks)

(d) The line $y = mx + 3$ is tangential to the hyperbola, where m is a constant. Use an algebraic method to determine all possible values of m. (4 marks)

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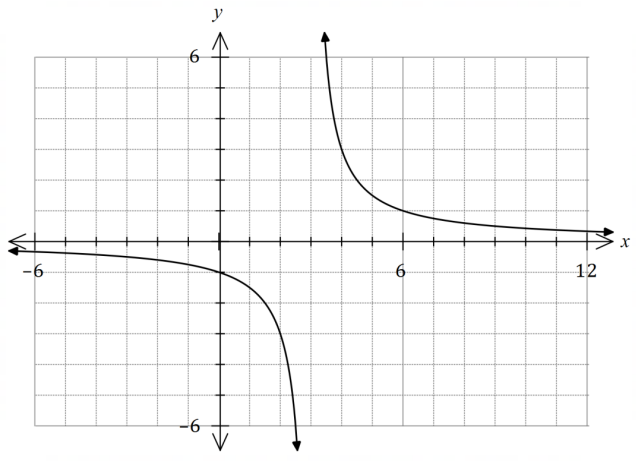
Question 19 **(10 marks)**

The graph of the hyperbola $y = \frac{a}{x+b}$ is shown below, where a and b are constants.



(a) State the equations of all asymptotes of the hyperbola. (2 marks)

(b) Determine the value of a and the value of b. (2 marks)

Question 12 **(8 marks)**

Data from repairs to 405 smartphones showed that 274 of them were Android. The type of repair was classified as battery or other, and of the 136 smartphones that required battery repairs, 98 were Android.

(a) Complete the missing entries in the table below. (3 marks)

	Battery	Other	Total
Android			
Not Android			
Total	136		405

(b) Determine the probability that a randomly selected smartphone from those repaired

(i) did not require a battery repair. (1 marks)

(ii) was an Android smartphone or required battery repairs. (2 marks)

(iii) did not require a battery repair given that it was an Android smartphone. (2 marks)

Question 13 **(8 marks)**

An aeroplane takes off from an airport situated at an altitude of 150 metres above sea level and climbs 450 metres during the first minute of flight. In each subsequent minute, its rate of climb reduces by 4%.

(a) Determine the **increase in altitude** of the aeroplane during the second minute. (1 mark)

(b) Determine the actual **altitude** of the aeroplane at the end of 2 minutes. (1 mark)

(c) Deduce a rule in simplified form for the **altitude** A_n of the aeroplane at the end of the n^{th} minute. (3 marks)

(d) Determine the altitude of the aeroplane at the end of 12 minutes. (1 mark)

(e) Determine the maximum altitude the aeroplane can reach. (2 mark)

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Question 18 **(9 marks)**

A random selection of 4 paint brushes is made from a collection of 16 different brushes, 9 of which are flat and the remainder round.

(a) Show that the probability the selection contains all round brushes is $\frac{1}{52}$. (3 marks)

(b) Determine the probability that the selection contains

(i) all flat brushes. (2 marks)

(ii) at least one round brush. (2 marks)

(iii) at least one round brush and at least one flat brush. (2 marks)

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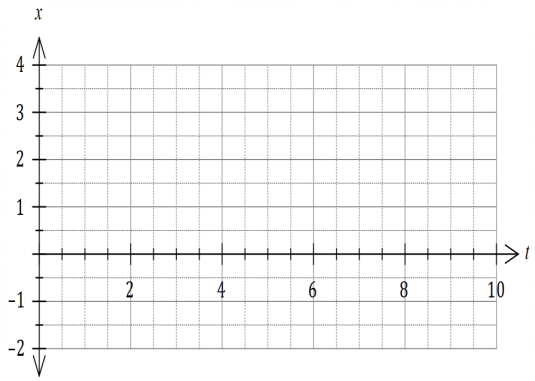
Question 17 **(6 marks)**

Particle P is moving along the x-axis so that its displacement, in cm, at time t seconds, $t \geq 0$, is given by $x = 2.7 + 0.6t - 0.1t^2$.

(a) Sketch the displacement-time graph of particle P on the axes below. (3 marks)



(b) Determine the velocity of particle P at the instant it reaches the origin. (3 marks)

Question 14 **(8 marks)**

Two events S and T are such that $P(S) = 0.46$ and $P(T) = 0.35$.

Determine the following probabilities.

(a) $P(\overline{S \cup T})$ when S and T are mutually exclusive. (2 marks)

(b) $P(S \cup T)$ when $P(\overline{S} \cap T) = 0.22$. (2 marks)

(c) $P(S \cap \overline{T})$ when S and T are independent. (2 marks)

(d) $P(T|S)$ when $P(S|T) = 0.6$. (2 marks)

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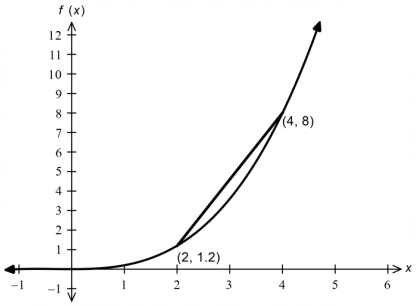
Question 15 **(5 marks)**

Consider the graph below:



(a) Determine the average rate of change of the function $y = f(x)$ between $x = 2$ and $x = 4$. (2 marks)

The following table shows points on the curve from the graph above of $y = f(x)$.

x	2	2.01	2.1	3
y	1.2	1.216	1.37	3.6

(b) Use all the information in the table above to demonstrate how to use the difference quotient $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to show that the instantaneous rate of change of the function $y = f(x)$ at $x = 2$ is 1.6. (3 marks)

Question 16 **(5 marks)**

The sum of the first n terms of a sequence is given by $S_n = 4n^2 + 7n$.

(a) Determine S_4. (1 mark)

(b) Determine T_4, where T_n is the n^{th} term of the sequence. (1 mark)

(c) Determine a simplified rule for the n^{th} term of the sequence. (3 marks)

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