Christ Church Grammar School

2021
TEST 4

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

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See next page

Question 9 **(7 marks)**

The pH (pouvior hydrogene – hydrogen power) of a solution is a measure of its hydrogen ion concentration. It is calculated using the formula: $pH = -log_{10}H^+$, where H^+ is the concentration of (H^+) ions in the solution (moles/litre). Pure water at 22°C has a concentration of 1×10^{-7} moles/litre.

(a) Calculate the pH of water at 22°C. (2 marks)

$ph = -\log_{10} H^+$

$ph = -\log_{10} 1\times10^{-7}$ ✓ [subs 1×10^{-7}]

$= 7$ ✓ [ANSW]

(b) Calculate the concentration of hydrogen ions in a solution with pH of 8.7. (2 marks)

$8.7 = -\log_{10} H^+$ ✓ [subs 8.7]

$10^{-8.7} \approx 2\times10^{-9}$ moles/L ✓ [ANSW]

1.99526×10^{-9}

(c) Solution A has a pH of 9 whereas solution B has a pH of 3. Calculate the ratio of hydrogen ions in solution B to that in solution A in the form $x : 1$. (3 marks)

$3 = -\log_{10} H^+ \Rightarrow H^+ = 10^{-3}$ (B)

$9 = -\log_{10} H^+ \rightarrow H^+ = 10^{-9}$ (A)

✓ obtains both H^+

B : A

$10^{-3} : 10^{-9}$ ✓ [ratio]

1,000,000 : 1 ✓ [ratio $x:1$]

Question 8 **(7 marks)**

Staples sells rulers with lengths normally distributed with a mean of 100 cm and a standard deviation of 0.95cm.

(a) Determine the proportion of rulers that are between 98 cm and 101cm. (1 mark)

$\mu = 100$

$\sigma = 0.95$

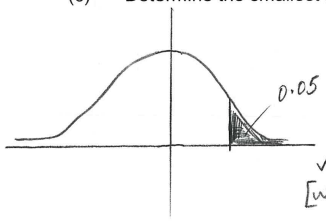
$P(98 < X < 101) = 0.8361$ ✓

(b) If 250 rulers were purchased, how many would you expect to be between 98 cm and 101 cm? (2 marks)

$250 \times 0.8361 = 209.025$ ✓

ie ~ 209 rulers ✓ [Int. value]

(c) Determine the smallest length of the largest 5% of rulers. (to 3dp) (2 marks)

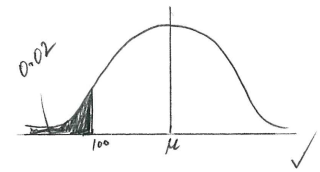


✓ [working]

solve (Norm cdf $(x, \infty, 0.95, 100) = 0.05$)

$x = 101.563$ cm ✓

(d) Staples advertises that the rulers are 1 metre in length. What will the mean of the distribution need to be if only 2% of rulers are to be below 1 metre in length? Assume the standard deviation remains at 0.95cm (2 marks)



✓ [working]

solve (Norm cdf $(-\infty, 100, 0.95, z) = 0.02$)

$\mu = 101.95$ cm ✓

OR $z = -2.053749$

$z = \frac{100 - \mu}{0.95}$

Solve for μ

See next page

Question 1 **(6 marks)**

Solve for x in the following equations, using exact values where necessary:

(a) $5^x = 10^{2-x}$ (3 marks)

(b) $(\ln x)^2 - 10\ln(x) + 24 = 0$ (3 marks)

See next page

Question 2 **(3 marks)**

Determine the equation of the tangent to the curve $y = \ln(\sin x)$ at the point where $x = \frac{\pi}{4}$.

Question 7 **(4 marks)**

A continuous random variable X has pdf:

$$f(x) = ax^2 \ for \ 0 \le x \le b$$

If $P(X \le 1) = \frac{1}{8}$, determine the value of a and b.

$\int_0^1 ax^2\,dx = 1/8$ ✓ (OR CPAO) [S statement]

$\left[\frac{ax^3}{3}\right] = \frac{1}{8}$

$a = 3/8$ ✓ [value]

$\int_0^b \frac{3}{8}x^2\,dx = 1$ ✓ [S statement]

$b = 2$ ✓ [value]

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Question 6 **(6 marks)**

The time that Finn arrives at school for ice hockey training is uniformly distributed between 6:58am and 7:08am. Finn is considered late if he arrives after 7:00am.

(a) Determine the probability that Finn arrives:

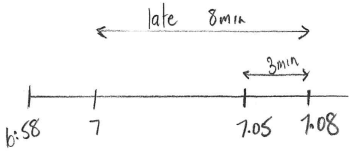
(i) at exactly 7:00am. (1 mark)

0 ✓

(ii) between 7:02am and 7:08am. (1 mark)

6/10 ✓

(iii) after 7:05am, given that he is late. (2 marks)



$\Rightarrow \dfrac{P(X>7.05)}{P(X>7)}$ ✓ (NUMER)

$= \frac{3}{8}$ ✓ (ANSW)

(b) During a term, Finn attends ice hockey training on 30 occasions. Determine the probability that he is late on at least 20 of these occasions. (2 marks)

$Y \sim B(30, 0.8)$ ✓ [BINOMIAL]

$P(X \geq 20) = 0.9744$ ✓ [ANSW]

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Question 3 **(3 marks)**

Consider $y = \ln((2e + x)^3)$. Showing use of the Increments formula, approximate the small change in y, when x changes from $6e$ to $7e$.

Question 4 **(3 marks)**

Determine the following:

(a) $\frac{d}{dx}(\ln(x^2 - 5x))$ (1 mark)

(b) $\int \frac{5x^2}{x^3+10} dx$ (2 marks)

End of questions

Question 5 **(6 marks)**

A continuous random variable X has pdf:

$$f(x) = \begin{cases} 0.0228x^2 + 0.01 \ \ for \ \ 0 \le x \le 5 \\ 0 \ for \ all \ other \ values \ of \ x. \end{cases}$$

(a) Determine $P(1 < X < 3)$ (1 mark)

$\int_1^3 0.0228x^2 + 0.01 \, dx = 0.2176$ ✓ $(\frac{136}{625})$

(b) Determine $Var(X)$ (3 marks)

$E(X) = \int_0^5 x \cdot f(x) \, dx = 3.6875$ $(\frac{59}{16})$ ✓

$VAR(X) = \int_0^5 f(x) \times [x - \frac{59}{16}]^2 \, dx$ ✓ [correct formula]

$= 1.069$ $(\frac{821}{768})$ ✓

(c) Determine the cumulative distribution function for the random variable X. (2 marks)

0.0076

$$P(X \le x) = \begin{cases} 0 \ for \ x < 0 \\ \frac{19x^3}{2500} + \frac{x}{100} \ for \ 0 \le x \le 5 \ \checkmark \\ 1 \ for \ x > 5 \end{cases}$$ ✓ [must have 1]

$\int_0^5 f(x) \, dx$

6

See next page

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MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

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Marks available:	30 marks

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Special items: drawing instruments, templates, and up to three calculators approved for use in this assessment

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Question 4 **(3 marks)**

Determine the following:

(a) $\frac{d}{dx}(ln\,(x^2-5x))$ (1 mark)

$= \frac{2x-5}{x^2-5x}$ ✓

(b) $\int \frac{5x^2}{x^3+10}dx$ Assume $x^3+10 > 10$ (2 marks)

$= \frac{5}{3}\int \frac{3x^2}{x^3+10}dx$

$= \frac{5}{3}\ln|x^3+10|+C$

✓ ✓ [No need for abs value]

-1 if no +C

3

End of questions

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(b) Determine Var(X). (3 marks)

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Question 3 **(3 marks)**

Consider $y = \ln(2e + x)^3$. Showing use of the Increments formula, approximate the small change in y, when x changes from $6e$ to $7e$.

$\frac{dy}{dx} \approx \frac{\delta y}{\delta x}$

$\delta x = 7e - 6e$

$\delta x = e$

$\frac{dy}{dx} = \frac{1}{(2e+x)^3} \times \frac{3(2e+x)^2 \times 1}{1}$

$\frac{\delta y}{\delta x} = \frac{3}{(2e+x)}$ ✓ [Diff]

$\delta y = \frac{3}{(2e+x)} \times e$

$\delta y = \frac{3e}{2e+6e}$ ✓ [subs $x = 6e$, $\delta x = e$]

$\delta y = \frac{3}{8}$ ✓ [ANSW]

3

Question 2 **(3 marks)**

Determine the equation of the tangent to the curve $y = \ln(sinx)$ at the point where $x = \frac{\pi}{4}$.

$\frac{dy}{dx}\Big|_{x=\frac{\pi}{4}}$ $\frac{1}{sinx} \cdot cos\,x$

$\therefore \frac{cos\frac{\pi}{4}}{sin\frac{\pi}{4}} \Rightarrow \frac{\sqrt{2}/2}{\sqrt{2}/2}$

$m = 1$ ✓ [gradient]

$\therefore y = 1x + c \Rightarrow Sub\ (\frac{\pi}{4}, ln\frac{\sqrt{2}}{2})$

$ln\frac{\sqrt{2}}{2} = \frac{\pi}{4} + c$

$c = ln\frac{\sqrt{2}}{2} - \frac{\pi}{4}$ ✓ [c-value]

$\therefore Equ\ y = x + ln\frac{\sqrt{2}}{2} - \frac{\pi}{4}$ ✓ [Equ]

OR $y = x - \frac{1}{2}ln(2) - \frac{\pi}{4}$

OR $y = x - ln\sqrt{2} - \frac{\pi}{4}$

3

Question 7 **(4 marks)**

A continuous random variable X has pdf:

$$f(x) = ax^2 \ for\ 0 \le x \le b$$

If $P(X \le 1) = \frac{1}{8}$, then determine the value of a and b.

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Staples sells rulers with lengths normally distributed with a mean of 100 cm and a standard deviation of 0.95 cm.

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Question 1 **(6 marks)**

Solve for x in the following equations, using exact values where necessary:

(a) $5^x = 10^{2-x}$ (3 marks)

$\log 5^x = \log 10^{2-x}$ ✓ [log both sides]

$x \log 5 = (2-x)\log 10$ → $\log 10 = 1$ ✓ [uses log laws]

$x \log 5 + x = 2$

$x(\log 5 + 1) = 2$

OR $\frac{\ln 100}{\ln 50}$ $\boxed{x = \frac{2}{\log 5 + 1}}$ ✓ [isolates x]

(b) $(\ln x)^2 - 10(\ln x) + 24 = 0$ (3 marks)

let $p = \ln x$

$\therefore p^2 - 10p + 24 = 0$ ✓ [forms quad]

$(p-6)(p-4) = 0$

$p = 6$ OR $p = 4$ ✓ [solves quadratic]

$\ln x = 6$ OR $\ln x = 4$

$\therefore \boxed{x = e^6}$ OR $\boxed{x = e^4}$ ✓ [Both sols]

See next page

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Additional working space

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