Name: _____

Thursday 3rd March

Time: 55 minutes   **Mark**   **/45 =**   **%**

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet in **both** sections of the test.
- You are permitted one A4 page (one side) of notes in the calculator assumed section.

Calculator free section   **Suggested time: 30 minutes**   **/25**

1. [7 marks]

   A complex polynomial $P(z)$ is defined by $P(z) = z^3 - 2iz^2 - 9z + k$.

   When $P(z)$ is divided by $(z - 1)$, there is a remainder of $-8 + 16i$.

   a) Show that $k = 18i$

   [2]

   b) Use synthetic substitution to evaluate $P(2i)$

   [2]

   c) Determine all solutions to $P(z) = 0$

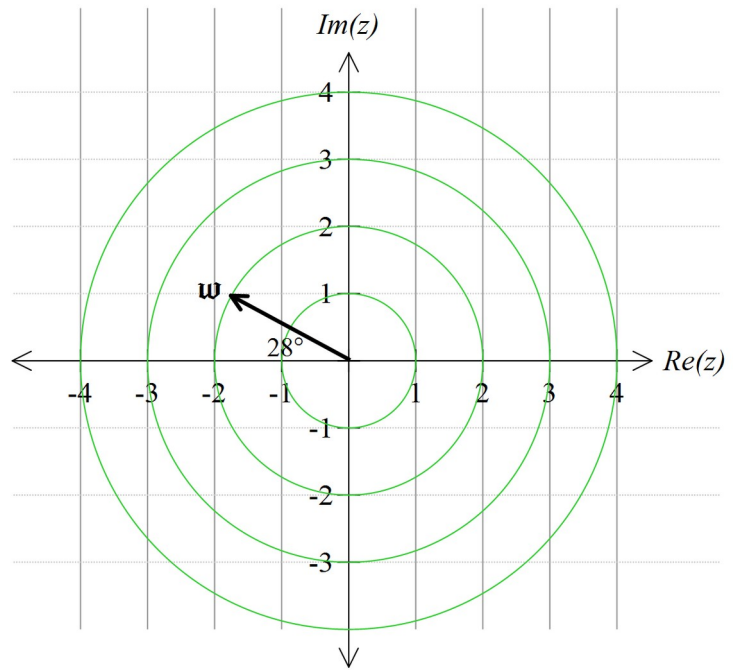   [2]

   d) Write $P(z)$ as a product of linear factors

   [1]

2. [6 marks – 1 each]

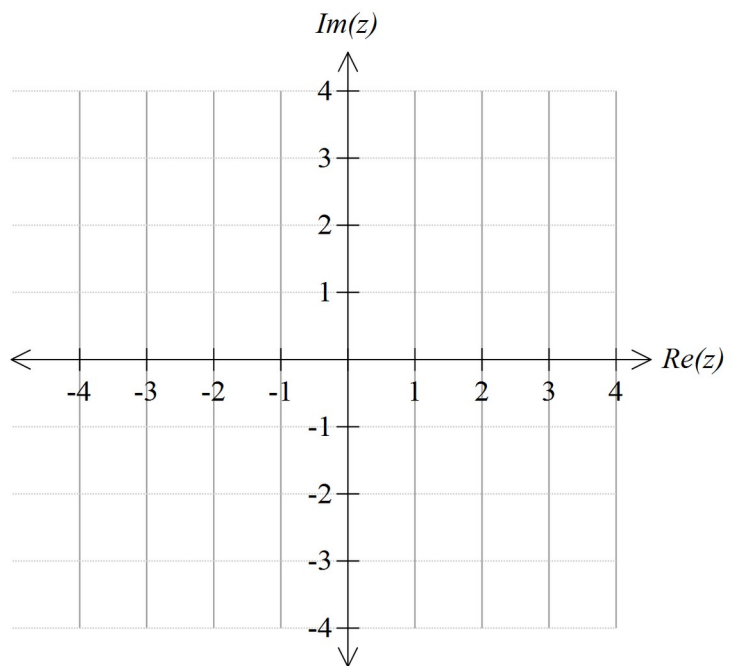A complex number ω is defined by the vector shown.

Add these vectors to the diagram:

a) ω^2

b) $\overline{\omega^2}$

c) $\sqrt{\omega}$

d) $i\,\omega$

e) $\dfrac{1}{\omega}$

f) $-\omega$

3. [4 marks – 2 each]

a) Sketch the region which satisfies the inequality $|z + 2| \le |z - 2 - 2i|$

b) This region can also be represented by an inequality of the form $a\,\mathrm{Re}(z) + b\,\mathrm{Im}(z) \le 2$. What are the values of *a* and *b*?
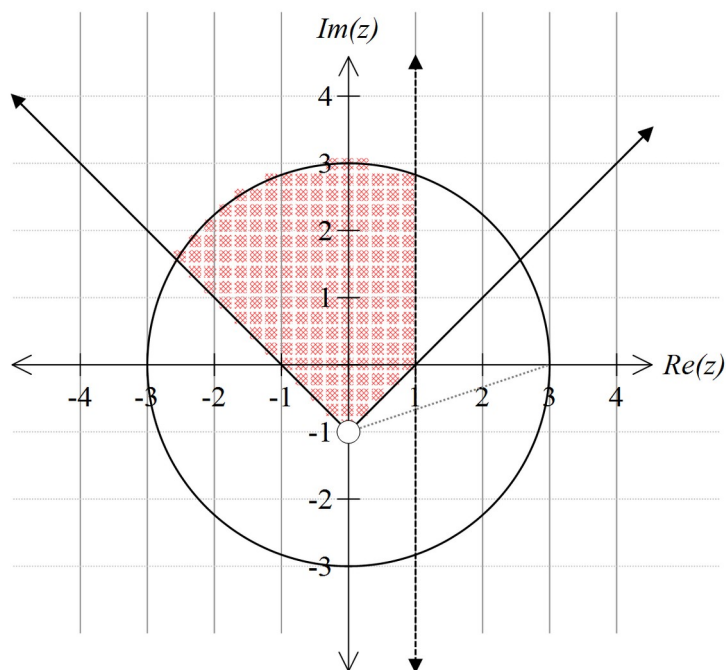
4. [4 marks]

   Determine all solutions to $z^4 = 2 - 2\sqrt{3}\,i$.

5. [4 marks]

   Write inequalities needed to define the
   shaded region:

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6. [8 marks]

Complex numbers μ and v are defined by $\mu = -\sqrt{2} - \sqrt{2}\,i$ and $v = 2\text{cis}\left(\dfrac{5\pi}{6}\right)$.

a) Write μ in polar (cis) form

[1]

b) Convert v to rectangular form

[1]

c) Calculate v^2 in polar form

[2]

d) Calculate $\mu \times v$ in both polar and rectangular forms

[2]

e) Show clearly how to use your result in (d) to find an exact value for $\sin\left(\dfrac{\pi}{12}\right)$
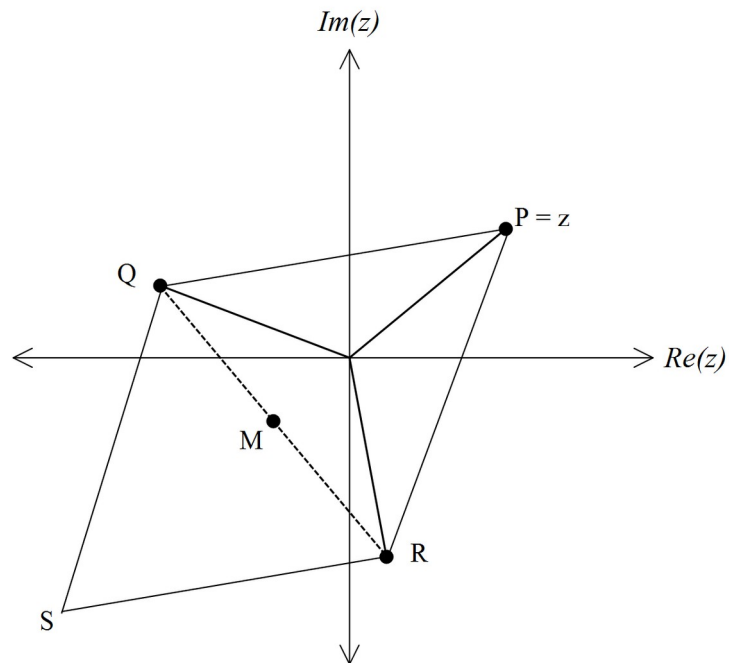
[2]

7. [5 marks]

$$2|z - 3| = |z + 3i|$$

Identify the centre and the radius of the circle in the complex plane with equation

8. [7 marks]

Two complex numbers z and ω are defined by $z = r\operatorname{cis}\theta$ and $\omega = \operatorname{cis}\left(\dfrac{2\pi}{3}\right)$.

On this Argand diagram, P represents z, Q is the product ωz, R is $\overline{\omega} z$ and M the mid-point of QR

a) Write ωz and $\overline{\omega} z$ in polar form

[2]

b) Evaluate $\omega + \overline{\omega}$ and hence, or otherwise, show that M represents the complex number $-\dfrac{1}{2}z$

[3]

c) S is chosen so that $QPRS$ is a parallelogram. Which number is represented by S?