PERTH MODERN SCHOOL
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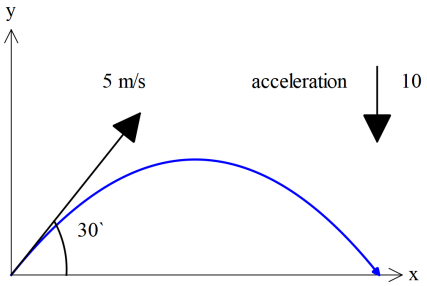
Independent Public School

Year 12 Specialist
TEST 4
27 July 2018
TIME: 50 minutes working
NO Classpads NOR calculators allowed!
50

Marks 7 Questions

Name:_____ Teacher: _____

Q1 (2, 3, 3 & 2 = 10 marks)



A particle is projected at the origin with an initial speed of $5m/s$ at 30° to the horizontal. The particle experiences a constant downward acceleration of $10m/s^2$.
Determine

i) the initial velocity of the particle in $i - j$ form.

ii) the position vector, $r(t)$, t seconds after projection.

iii) the cartesian equation of the path.

iv) the range, that is the distance along the x axis when the particle lands.

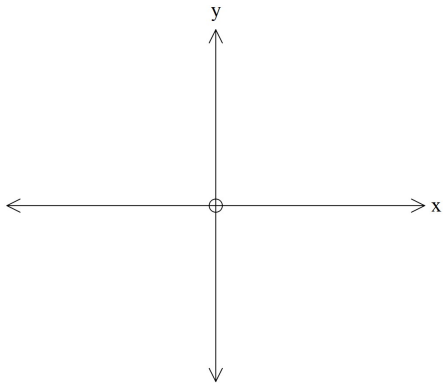
Q2 (2, 3, 3 & 3 = 11 marks)

An object moves such that its position vector, r metres, at time t seconds is given by

$$r = \begin{pmatrix} 3\sin(4\pi t) \\ -3\cos(4\pi t) \end{pmatrix}$$

i) Determine the cartesian equation of the path of the object and the period of the motion.

ii) Sketch the cartesian path giving the initial position and direction.



iii) Show that the velocity is always perpendicular to the position vector.

iv) Show that the acceleration is directly proportional to the position vector, stating the constant of proportionality (i.e $\ddot{r} = -k r$ where k is a constant)

Q7 (4 marks)

By using an appropriate substitution **and** integration, show that

$$\int \frac{\sin x}{1 - \cos^2 x}\,dx = \frac{1}{2}\ln\left(\frac{\cos x - 1}{\cos x + 1}\right) + c$$

Q3 (3 & 3 = 6 marks)

Consider the curve $x^2 = \cos(y)$. In terms of x & y determine an expression for

i) $\dfrac{dy}{dx}$

ii) $\dfrac{d^2y}{dx^2}$

Q4 (3 & 3 = 6 marks)

Show every step in evaluating the following integrals.

i) $\int (5x+1)(3x-2)^7\,dx$ with substitution $u = 3x - 2$

ii) $\int \sin^3(2x)\cos^4(2x)\,dx$

Q5 (4 & 3 = 7 marks)
Consider the curve described parametrically by

$x = 5\cos t$
$y = -3\sin t$ from $t = 0$ to $t = \frac{\pi}{2}$

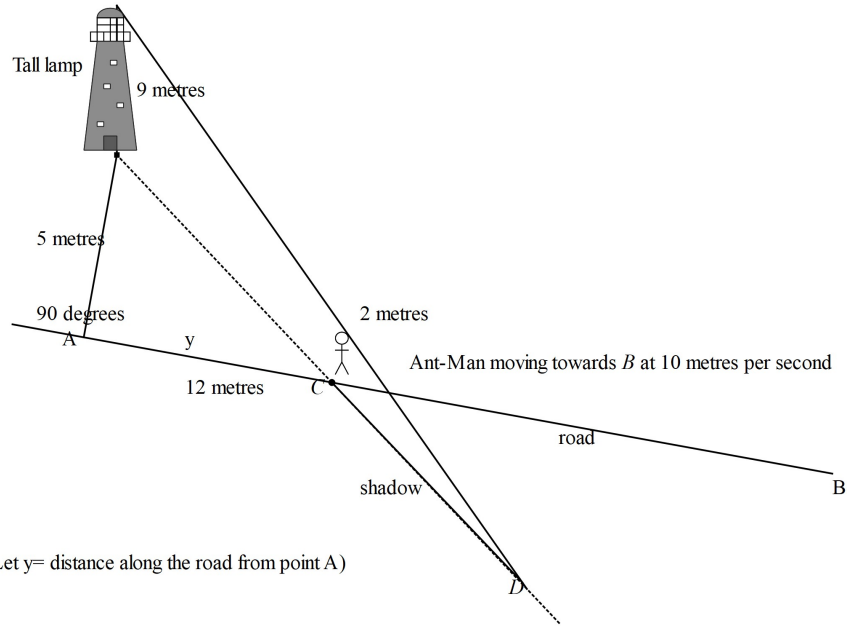
If this curve is revolved around the x axis a three dimensional shape is formed.

i) Show that the volume of this three dimensional shape is $\int_0^{\frac{\pi}{2}} 45\pi \sin^3 t \, dt$
(Hint- consider direction of integration)

ii) Evaluate this integral to determine the exact volume.

Q6 (6 marks)

Consider the Ant-Man walking along a road AB towards point B at an incredible constant speed of $10 m/s$. The height of the Ant-Man is 2 metres. Let point A be the closest point of the base of the Tall lamp from the road, i.e 5 metres and the height of the Tall lamp being 9 metres.



(Hint- Let y= distance along the road from point A)

Determine at the point where the Ant-Man is **12 metres along the road** from point A, the time rate of change of the length of the shadow CD.