PERTH COLLEGE

Year 12

Semester One Examination 2011

Question/Answer booklet

MATHEMATICS 3CMAT/3DMAT
Section One (Calculator - free)

Student Name: _____

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section
To be provided by the supervisor
Question/answer booklet for Section One
Formula sheet which may also be used for Section Two

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further

Structure of this paper

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One Calculator-free	**8**	**8**	**50 minutes**	**40**
Section Two Calculator-assumed	12	12	100 minutes	80
			Total marks	120

Instructions to candidates

1. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer
   a. Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   b. Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.

3. It is recommended that you **do not use pencil** except in diagrams

Question 1   **[4 marks]**

Consider the following functions:

$$g(x) = (x - 1)^2 + 1 \qquad \text{and} \qquad h(x) = \sqrt{x - 5}$$

a)   Determine the exact value of $hg(4)$.

[1]

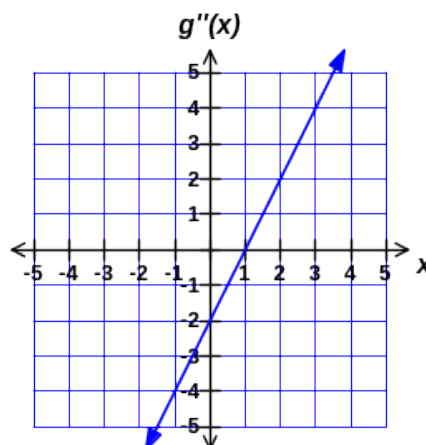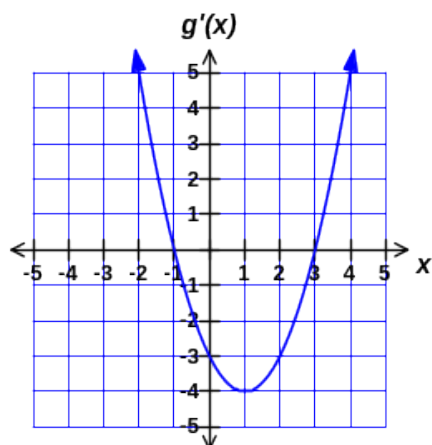b)   State the domain and range of $hg(x)$.

[3]

Question 2   **[5 marks]**

Let $y = g(x)$ be defined as some function of x.
The graph of g has an inflection point at **P**, a local minimum at **M** and a local maximum at **Q**.
Sketches of the graphs of $g'(x)$ and $g''(x)$ are shown below.

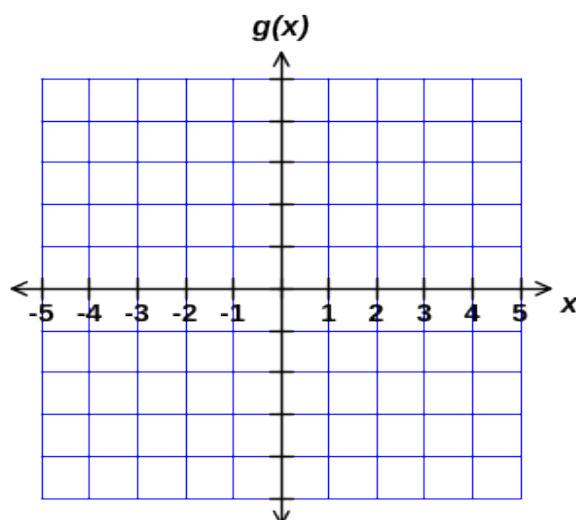

a)   Use the information above to determine:

(i)   the **x-coordinate** of **P**.

[1]

(ii)   the **x-coordinate** of **M**.

[1]

b)   Given that $g(1) = 0$, sketch a possible graph of $g(x)$. On your sketch, show the points **P, M** and **Q**, labeling each one clearly.

[3]

4

Question 3   **[9 marks]**

a)   Differentiate $\dfrac{2p-5}{p^2+3p-1}$ with respect to p. Fully simplify the numerator of your answer.

[3]

b)   Determine the values of c and d given that $\dfrac{d}{dx}\left(4x\sqrt{2x+3}\right) = \dfrac{cx+d}{\sqrt{2x+3}}$.

[4]

c)   Find $\dfrac{dy}{dx}$ given $y = \left(3e^{4x}+2\right)^5$. Fully simplify your answer.

[2]

5

Question 4   **[6 marks]**

The points (-2, 1), (1, -2) and (3, 16) all lie on the parabola $f(x) = ax^2 + bx + c$.

a)   Use this information to form three equations in terms of *a*, *b* and *c*.

[2]

b)   Solve these equations, using the elimination method, to determine the values of *a*, *b* and *c*.

[4]

Question 5   **[5 marks]**

Determine each of the following indefinite integrals. Express your answers with positive indices and fully simplify where necessary.

a) $\displaystyle\int (x^3 - 2x - 4)\, dx$

[1]

b) $\displaystyle\int 3x^2 (4x^3 - 5)^7\, dx$
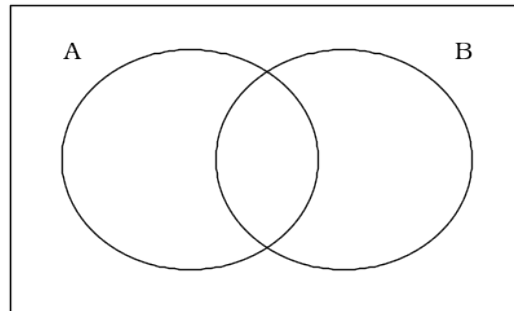
[2]

c) $\displaystyle\int \frac{2x^3 - 4x}{\sqrt{x}}\, dx$

[2]

Question 6   **[5 marks]**

a)   For the events A and B represented in the Venn diagram below,

     $P(A \cap B) = 0.4,$ $P(A) = 0.6$ and $P(A \mid B) = 0.8.$

     Determine

     (i)   P(B)

                                                  [2]

     (ii) $P\left(\overline{A \cup B}\right)$

                                                  [1]

b)   Consider another two events C and D such that $P\left(\overline{C} \cap \overline{D}\right) = 0.6$, P (C) = 0.4 and P (D) = 0.2. Determine whether or not C and D are mutually exclusive events and justify your answer mathematically.

                                                  [2]

Question 7   **[2 marks]**

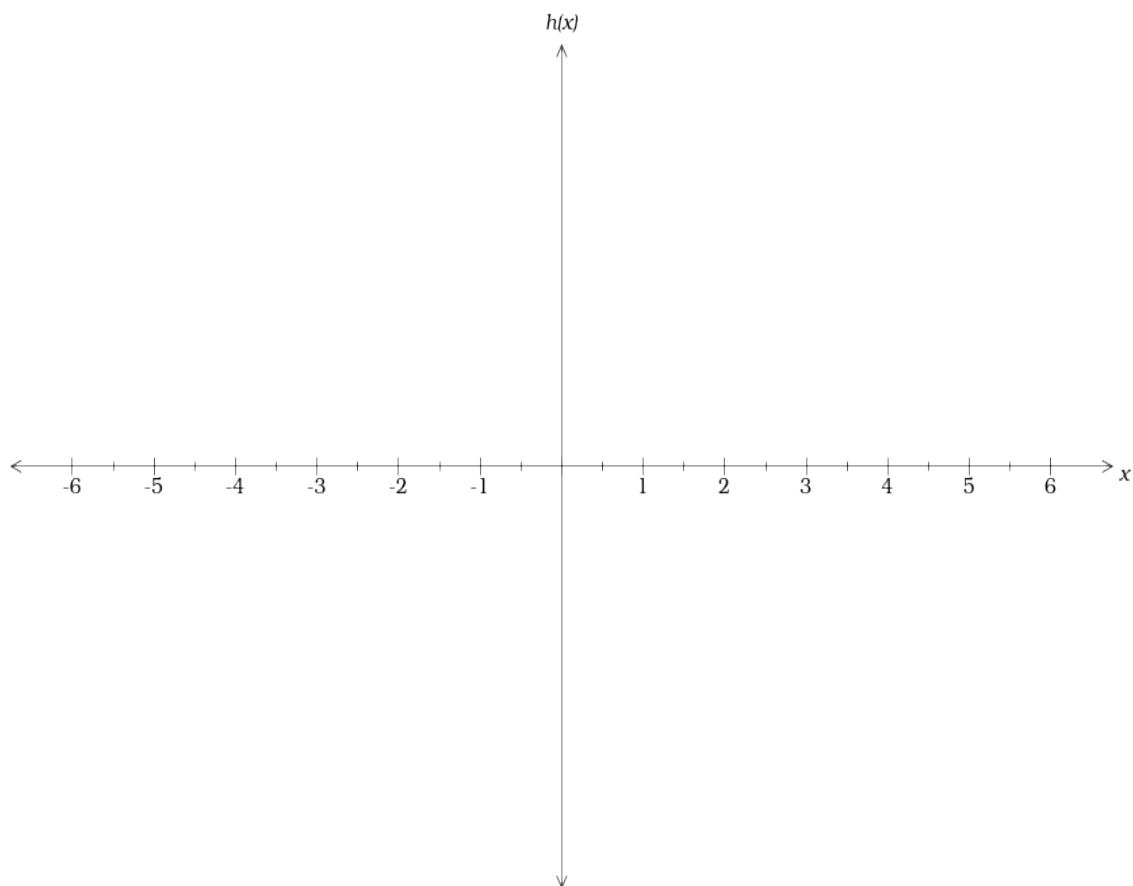The point (3, $2e^2$) lies on the graph of $f(x) = 2e^{x-1}$.

If the following transformations are applied to $f(x)$ **in succession**, what would be the co-ordinates of the resulting location of this point?

- Reflection about the *x*-axis
- Horizontal translation of 5 units right
- Vertical translation of 4 units down
- Reflection about the *y*-axis

Question 8   **[4 marks]**

Draw a neat sketch of a function $y = h(x)$ which satisfies **all** of the following conditions:

- $h(-4) = h'(-4) = 0$
- $h'(-1) = h''(-1) = 0$
- $h''(-3) = 0$
- $h'(x) < 0$ <u>only</u> for $x < -4$

$h(x)$

-6   -5   -4   -3   -2   -1   1   2   3   4   5   6 x

END OF SECTION ONE

10

EXTRA PAGE FOR WORKING
Clearly number any questions you do here.

Your Name: _____

Your Teacher (please circle)

Mrs Macnaughtan   **Mr Rippon**   **Mrs Yap**

	Question	Marks Available	Your Mark
	1	4	
	2	5	
	3	9	
	4	6	
	5	5	
	6	5	
	7	2	
	8	4	
	TOTAL SECTION 1	40	
	TOTAL SECTION 2	80	
	OVERALL	120	