education equals

Calculator Assumed
Mixed Applications of Discrete and Continuous Random Variables

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2, 2, 3, 3 = 11 marks] **CA**

The average age of women in Australia when they first have a child is approximately normally distributed with a mean age of 28.9 years and a standard deviation of 3.1 years.

(a) Determine the probability that an Australian woman between the ages of 24 years and 30 years has had her first child.

(b) If a woman is less than 32 years in age, what is the probability that she has had her first child at an age older than 28?

(c) What is the age of the youngest woman in top two percentile?

At a local hospital maternity ward, three women are admitted to the ward, each expecting their first child.

(d) What is the probability that the first two are younger than 27 years and the third is older?

(e) What is the probability that exactly two of the three women are older than 28 years?

Question Two: [5 marks] CA

The probability density function of a continuous random variable X is given by:

$$f(x) = \begin{cases} \dfrac{x}{Q}; \; 0 \le x \le P \\ 0 \; ; \; otherwise \end{cases}$$

Calculate the value of P and Q if the mean of the distribution is $\dfrac{20\sqrt{2}}{3}$.

Question Four: **[4, 2 = 6 marks]** **CA**

The mean and standard deviation of a binomial distribution are 20 and 4 respectively.

(a) Calculate the number of trials and the probability of success for this distribution.

$np = 20$ ✓
$np(1-p) = 16$ ✓
$n = 100$ ✓
$p = 0.2$ ✓

(b) Hence describe the shape of the distribution of this discrete random variable.

Due to a low probability, the shape will be positively skewed. ✓✓

Question Five: **[2, 2, 2 = 6 marks]** **CA**

At a local hair dressing salon, customers first have their hair coloured, then washed and finally blow-dried.

Each of these stages can be modelled by continuous random variables, with the following statistics:

	Mean	Standard Deviation
Colouring	35 mins	7 mins
Washing	5 mins	1 min
Blow-Drying	10 mins	3 mins

(a) Calculate the mean of the total time taken to complete this three stage process.

✓ $\frac{35+5+10}{3} = 16.67\text{ min}$ ✓

(b) Calculate the standard deviation of the total time taken to complete this three stage process.

✓ $\sqrt{\frac{49+1+9}{3}} = 4.43\text{min}$ ✓

(c) Hence or otherwise determine the mean and standard deviation of the total time taken to complete the three stage process if the times are recorded in hours.

$\mu = 0.2778 \quad \sigma = 0.035$
✓ ✓

Question Three: **[1, 2, 3, 2, 2, 1, 3, 3 =17 marks]** **CA**

The timing belt in the engine of a certain model of car has been found to be made of inferior material. This make of car has been recalled by the company to replace this belt since imperfections have been found in the original belts.

The average number of imperfections in each belt is 3.4 and can be modelled by:

$$P(X = x) = \frac{e^{-3.4}3.4^x}{x!}\ ; x = 0,1,2,3...$$

(a) Determine the probability that in a randomly selected belt there are 5 imperfections.

(b) Determine the probability that there is at least one imperfection.

(c) Determine the probability that there are less than 2 imperfections given that there is at least 1 imperfection.

(d) Calculate the mean number of imperfections.

Examining the belts more closely, it seems that on average the distance between imperfections is 20 cm and the distance between imperfections can be modelled by the continuous random variable :

$$P(y)=\begin{cases}\dfrac{1}{20}e^{\frac{-x}{20}}\ ;\ x\geq 0\\ 0\ ;\ otherwise\end{cases}$$

(e) Confirm, using calculus techniques (you may use your calculator), that the expected value of this distribution is 20 cm.

(f) Given that 1 cm = 0.4 inches, state the expected value of this distribution in inches.

(g) Determine the probability that on the next car brought in, the distance between two imperfections is:

(i) less than 10 cm.

(ii) less than 30 cm given that it was more than 5 cm.

(h) Of the next 5 cars brought in for inspection, determine the probability that at least 2 cars had the distance between two imperfections of more than 5 cm.

Examining the belts more closely, it seems that on average the distance between imperfections is 20 cm and the distance between imperfections can be modelled by the continuous random variable :

$$P(y)=\begin{cases}\dfrac{1}{20}e^{\frac{-x}{20}}\ ;\ x\geq 0\\ 0\ ;\ otherwise\end{cases}$$

(e) Confirm, using calculus techniques (you may use your calculator), that the expected value of this distribution is 20 cm.

$$\int_0^\infty \frac{xe^{\frac{-x}{20}}}{20}\,dx=20$$

(f) Given that 1 cm = 0.4 inches, state the expected value of this distribution in inches.

$20\times 0.4=8\,inches$

(g) Determine the probability that on the next car brought in, the distance between two imperfections is:

(i) less than 10 cm.

$$\int_0^{10}\frac{e^{\frac{-x}{20}}}{20}\,dx=0.3935$$

(ii) less than 30 cm given that it was more than 5 cm.

$$\frac{\int_5^{30}\frac{e^{\frac{-x}{20}}}{20}\,dx}{\int_5^{\infty}\frac{e^{\frac{-x}{20}}}{20}\,dx}=\frac{0.5557}{0.7788}=0.7135$$

(h) Of the next 5 cars brought in for inspection, determine the probability that at least 2 cars had the distance between two imperfections of more than 5 cm.

$Y\sim Bin(5,0.7788)$

$P(Y\geq 2)=0.9901$

Question Three: [1, 2, 3, 2, 2, 1, 3, 3 =17 marks] CA

The timing belt in the engine of a certain model of car has been found to be made of inferior material. This make of car has been recalled by the company to replace this belt since imperfections have been found in the original belts.

The average number of imperfections in each belt is 3.4 and can be modelled by:

$$P(X=x)=\frac{e^{-3.4}3.4^x}{x!}\ ; x=0,1,2,3\ldots$$

(a) Determine the probability that in a randomly selected belt there are 5 imperfections.

$P(X=5)=0.1264$ ✓

(b) Determine the probability that there is at least one imperfection.

$1-P(X=0)$ ✓
$=1-0.03337$
$=0.9666$ ✓

(c) Determine the probability that there are less than 2 imperfections given that there is at least 1 imperfection.

$P(X<2\,|\,X\geq 1)$
$=\dfrac{P(X=1)}{P(X\geq 1)}$ ✓
$=\dfrac{0.1135}{0.9666}$ ✓
$=0.1174$ ✓

(d) Calculate the mean number of imperfections.

As stated in the question, the mean is 3.4

✓ ✓

Question Four: [4, 2 = 6 marks] CA

The mean and standard deviation of a binomial distribution are 20 and 4 respectively.

(a) Calculate the number of trials and the probability of success for this distribution.

(b) Hence describe the shape of the distribution of this discrete random variable.

Question Five: [2, 2, 2 = 6 marks] CA

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(a) Calculate the mean of the total time taken to complete this three stage process.

(b) Calculate the standard deviation of the total time taken to complete this three stage process.

(c) Hence or otherwise determine the mean and standard deviation of the total time taken to complete the three stage process if the times are recorded in hours.

SOLUTIONS
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(a) Determine the probability that an Australian woman between the ages of 24 years and 30 years has had her first child.

$X \sim N(28.9, 3.1^2)$
$P(24 < X < 30) = 0.5817$ ✓

(b) If a woman is less than 32 years in age, what is the probability that she has had her first child at an age older than 28?

$P(X > 28 \mid X < 32)$ ✓
$\dfrac{P(28 < X < 32)}{P(X < 32)} = \dfrac{0.4556}{0.8413} = 0.5415$ ✓

(c) What is the age of the youngest woman in top two percentile?

$P(X < k) = 0.98$ ✓
$k = 35.27\ years$ ✓

At a local hospital maternity ward, three women are admitted to the ward, each expecting their first child.

(d) What is the probability that the first two are younger than 27 years and the third is older?

$P(X < 27) = 0.2600$ ✓
$= 0.26 \times 0.26 \times 0.74$ ✓
$= 0.0500$ ✓

(e) What is the probability that exactly two of the three women are older than 28 years?

$P(X > 28) = 0.6142$ ✓
$Y \sim Bin(3, 0.6142)$ ✓
$P(Y = 2) = 0.4366$ ✓

Question Two: [5 marks] **CA**

The probability density function of a continuous random variable X is given by:

$$f(x) = \begin{cases} \dfrac{x}{Q} ; 0 \le x \le P \\ 0 ; otherwise \end{cases}$$

Calculate the value of P and Q if the mean of the distribution is $\dfrac{20\sqrt{2}}{3}$.

$\int_0^P \dfrac{x}{Q}\, dx = 1$

$\left[\dfrac{x^2}{2Q}\right]_0^P = 1$

$\dfrac{P^2}{2Q} = 1$ ✓

$\int_0^P \dfrac{x^2}{Q}\, dx = \dfrac{20\sqrt{2}}{3}$ ✓

$\left[\dfrac{x^3}{3Q}\right]_0^P = \dfrac{20\sqrt{2}}{3}$

$\dfrac{P^3}{3Q} = \dfrac{20\sqrt{2}}{3}$ ✓

$P = 10\sqrt{2}$ ✓

$Q = 100$ ✓