CATHOLIC EDUCATION WESTERN AUSTRALIA

ViSN Virtual School Network

2020 Year 11 ViSN Mathematics Specialist Units 1 & 2
Test 6 – Complex numbers & Proof
Section One – Calculator Free

Mr Daniel Comtesse
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Calculator Free:______/14
Calculator Assumed:_____/22

Result: _____/36 ______%

Student Name: Solutions

School: ________________

Time allowed: Section One - 15 minutes
Section Two – 30 minutes

Assessment Date:

Material required/recommended

To be provided by the supervisor
This Question/Answer Paper
SCSA Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Submission Details
Timed Assessments are to be returned to the ViSN teacher by the ViSN mentor (scan completed assessment and email to teacher above) within 24 hours of assessment date (above).

Instructions to Students

1. **ALL** questions should be attempted.

2. Write your answers in the spaces provided in this Question/Answer Booklet.

3. **SHOW ALL YOUR WORKING CLEARLY**. Your working should be sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Correct answers given without supporting reasoning may not be allocated full marks. Incorrect answers given without supporting reasoning cannot be allocated any marks.

4. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.

5. It is recommended that you **do not use pencil**, except in diagrams.

Question 1 **[3, ~~3~~ 2 = ~~6~~ 5 marks]**

(a) Determine the values of the real constants b and c if $z = 1 + 3i$ is a solution of the equation $z^2 + bz + c = 0$.

$\therefore z = 1 - 3i$ is also a solution. ✓conjugate

Hence $(z - 1 - 3i)(z - 1 + 3i) = z^2 + bz + c$

$\therefore c = 10, \ b = -2$

✓c

✓b

(b) Express the real quadratic polynomial $z^2 - 4z + 8$ as a product of its linear factors.

$$z = \frac{4 \pm \sqrt{16 - 4(1)(8)}}{2}$$

$$z = \frac{4 \pm 4i}{2}$$

$z = 2 \pm 2i$ ✓solutions

$\therefore z^2 - 4z + 8 = (z - 2 + 2i)(z - 2 - 2i)$ ✓product

 5

Question 2 2 marks each. []

(a) A set of real numbers is given by $\{\sqrt{2}, 3.\overline{14}, \pi, \sqrt[3]{14}\}$. Clearly show that one of the numbers in the set is rational.

Let $x = 3.\overline{14}$ (1)

$100x = 314.\overline{14}$ (2) ✓ Sets up equations

(2) − (1)

$99x = 311$

$x = \frac{311}{99}$

$\therefore 3.\overline{14} = \frac{311}{99}$ ✓ Proves

(b) Prove that $9.\overline{9} \equiv 10$.

$99.\overline{9}$

Let $x = 9.\overline{9}$ (1) ✓

$10x =$ (2)

(2) − (1)

$9x = 90$

$x = 10$

$\therefore 9.\overline{9} \equiv 10$ ✓

4

Question 3 [] 5 marks

Let z_1 and z_2 be complex numbers such that $2z_1 + 3z_2 = 7$ (1) and $z_1 + iz_2 = 4 + 4i$.

Determine z_1 and z_2 in the form $z = a + bi$, where $a, b \in Z$.

(2) $2z_1 + 2iz_2 = 8 + 8i$

(2) − (1)

$2iz_2 - 3z_2 = 1 + 8i$ ✓ determining equation for z_1 or z_2

$z_2(2i - 3) = 1 + 8i$

$z_2 = \frac{1 + 8i}{-3 + 2i} \times \frac{-3 - 2i}{-3 - 2i}$ ✓ realises denominator

$= \frac{-3 - 26i + 16}{13}$

$z_2 = 1 - 2i$ ✓ z_2

$\therefore z_1 = 4 + 4i - i(1 - 2i)$ ✓ subs. for z_1

$z_1 = 4 + 4i - i - 2$

$z_1 = 2 + 3i$ ✓ z_1

End of Section One

5

Additional working space

Question number: _________

2020 Year 11 ViSN Mathematics Specialist Units 1 & 2
Test 6 – Complex numbers & Proof
Section Two – Calculator Assumed

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Calculator Assumed:_____/ 22

Student Name: ____

School: ____

Time allowed: Section One - 15 minutes
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Assessment Date:

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To be provided by the supervisor
This Question/Answer Paper
SCSA Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: 1 A4 (one sided) page of notes, up to three scientific and/or CAS calculators

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Question 4 [] 3 marks

Show that if n is one more than a multiple of three, then n^2 will also be one more than a multiple of three, where $n \in Z$.

Let $n = 3k+1$ where $k \in \mathbb{Z}$ ✓ defines n

$\therefore n^2 = (3k+1)^2$ ✓ squares

$= 9k^2 + 6k + 1$

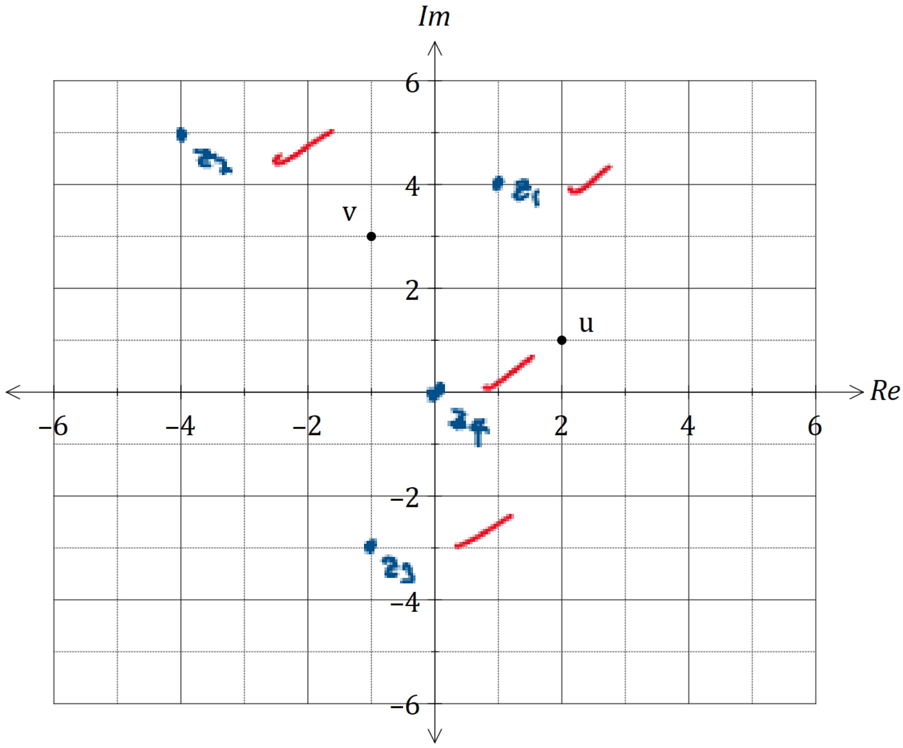
$= 3(3k^2 + 3k) + 1$ ✓ factorises and concludes

which is one more than a multiple of 3. QED

3

Question 5 **[1 mark each]**

The complex numbers u and v are shown in the complex plane below.



Plot and label the following complex numbers:

(a) $z_1 = u + v$ $= 1 + 4i$

(b) $z_2 = 2v - u$ $= -2 + 6i - 2 - i$

(c) $z_3 = \overline{v}$

(d) $z_4 = \overline{u+v} - \overline{u} - \overline{v}$

$= 1 - 4i - (2 - i) - (-1 - 3i)$

$= 0$

4

Question 6 [] 4 marks.

Prove that $\sqrt{7}$ is irrational by contradiction.

Assume $\sqrt{7}$ is rational ✓ Assumption

$\therefore \sqrt{7} = \frac{a}{b}$, $a, b \in \mathbb{Z}$ with no common factors. ✓

$7 = \frac{a^2}{b^2}$

$7b^2 = a^2$

$\therefore$ a is a multiple of 7.

Let $a = 7k$, $k \in \mathbb{Z}$

$\therefore 7b^2 = (7k)^2$

$b^2 = 7k^2$ ✓

which implies b is a multiple of 7.

This is a contradiction as a and b have no common factors.

Hence, $\sqrt{7}$ is irrational QED. ✓

4

Question 7 **[~~2~~ 1, ~~4~~ 5 = 6 marks]**

The sum of the first n terms of the sequence $2+8+14+20+..+(6n-4)$ is $n(3n-1)$.

(a) Show that the statement is true when $n=5$.

$2+8+14+20+26 = 5(3(5)-1)$

$70 = 70$ ✓

(b) Use mathematical induction to prove the statement is true for $n \in Z, n \geq 5$.

Assume true for $n=k$

$\therefore 2+8+14+\ldots+(6k-4) = k(3k-1)$ ✓

For $n=k+1$, LHS:

$2+8+14+\ldots+(6k-4)+(6k+2) = k(3k-1)+6k+2$ ✓

$= 3k^2-k+6k+2$

$= 3k^2+5k+2$ ✓

RHS $= k+1(3k+2)$

$= 3k^2+5k+2$

$=$ LHS ✓

Hence, as it was true for $n=5$ and by induction it is true for $n=k$, $n=k+1$, the statement is true for $n \geq 5$. ✓

6

Question 8 **[5 marks]**

Use mathematical induction to prove that $7^{2n-1}+5$ is always divisible by 12, for $n \in N$.

When $n = 1$

$7^{2(1)-1} + 5 = 12$ which is true

Assume true for $n = k$

$7^{2k-1} + 5 = 12m \quad , \quad m \in \mathbb{Z}$

When $n = k+1$

$LHS = 7^{2k+2-1} + 5$

$= 7^2 \times 7^{2k-1} + 5$

$= 49 \times 7^{2k-1} + 5$

$= 48 \times 7^{2k-1} + 7^{2k-1} + 5$

$= 48 \times 7^{2k-1} + 12m$

$= 12(4 \times 7^{2k-1} + m)$

and concludes.

which is divisible by 12.

Hence, as it was true when $n=1$, and by induction it was true for $n = k$, $n=k+1$, $7^{2n-1}+5$ is always divisible by 12. QED

5

End of Assessment

Additional working space

Question number: __________