WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2016
PRACTICE TEST 1: Complex Numbers

Name: ________________________

Time: 60 minutes Mark /55

Mostly calculator free – scientific OK.

1. [7 marks]

a) Complete the table:

	Rectangular form	Polar form
z	$6+6i$	
w		$2cis\left(\frac{\pi}{6}\right)$

[2]

b) Determine:

(i) $\frac{iz}{w}$

(ii) w^3

(iii) $\bar{z}+w$

[5]

2. [7 marks]

Given that $8\text{cis}\left(-\frac{5\pi}{6}\right)$ is one of the cube roots of a complex number *z*, determine:

a) $|z|$

[1]

b) $\text{Arg}(z)$ such that $-\pi < \text{Arg}(z) \leq \pi$

[2]

c) The other two cube roots of *z*

[2]

d) A solution for *w* in the equation $w^9 = z$

[2]

3. [12 marks]

For $f(z) = z^4 - 4z^3 + 9z^2 - 16z + 20$, determine:

a) $f(1)$

[1]

b) the remainder when $f(z)$ is divided by $(x - 1)$

[1]

c) $q(z)$ if $\frac{f(z)}{z-1} = q(z) + \frac{k}{z-1}$ [1]

d) $f(2i)$ [1]

e) two solutions to $f(z) = 0$ [2]

f) all solutions to $f(z) = 0$ [4]

g) $f(z)$ in a fully factored form (as a product of 4 linear terms) [2]

4. [14 marks]

a) Convert both $1 + \sqrt{3}\ i$ and $1 + i$ into polar form.

[2]

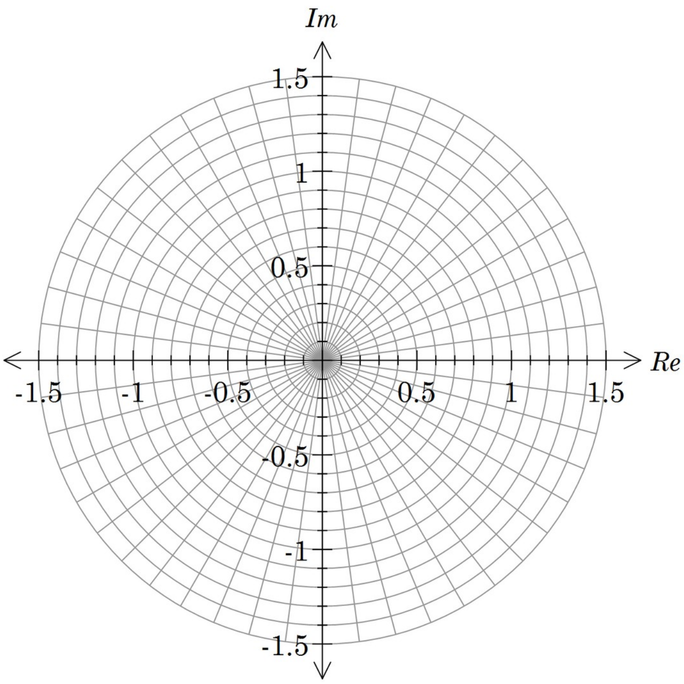
b) Hence express $z = \dfrac{1 + \sqrt{3}\ i}{1 + i}$ in the form $r \text{ cis } \theta$.

[2]

c) Determine the smallest positive integer *n* for which z^n is purely imaginary and state the value of z^n for this particular *n* value.

[3]

d) Solve for w given that $2w^4 = 1 + \sqrt{3}\ i$ and plot your solutions on the Argand diagram below.



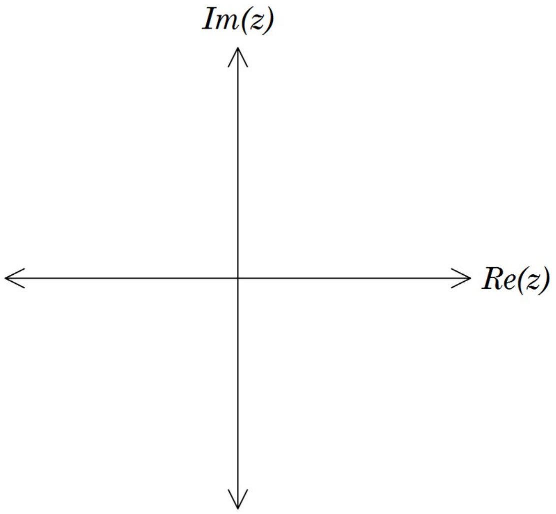
[4]

e) If w_n represents the n^{th} solution to the equation in d) moving anticlockwise around the Argand diagram, explain the geometric meaning of $|w_n - w_{n-1}|$ and determine its value.

[3]

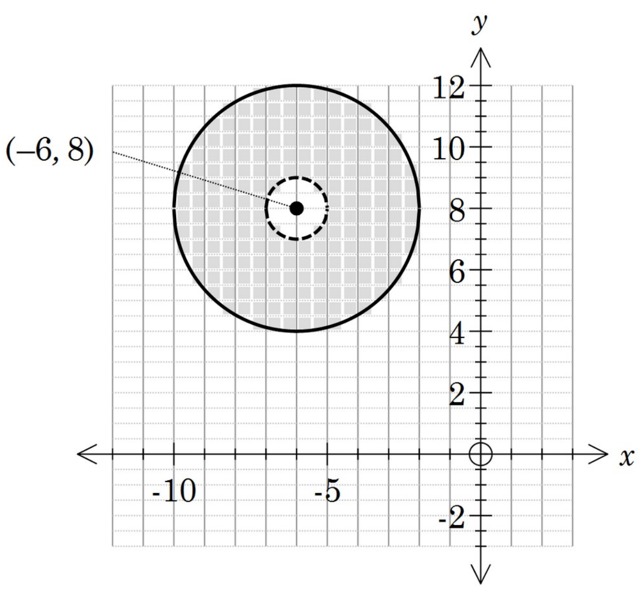
5. [8 marks]

a) Sketch neatly the locus of z where $|z - 2i| = |z + 4|$, then determine the Cartesian equation of the resulting sketch.



[3]

b) Write an inequality that describes the locus of z shown below.



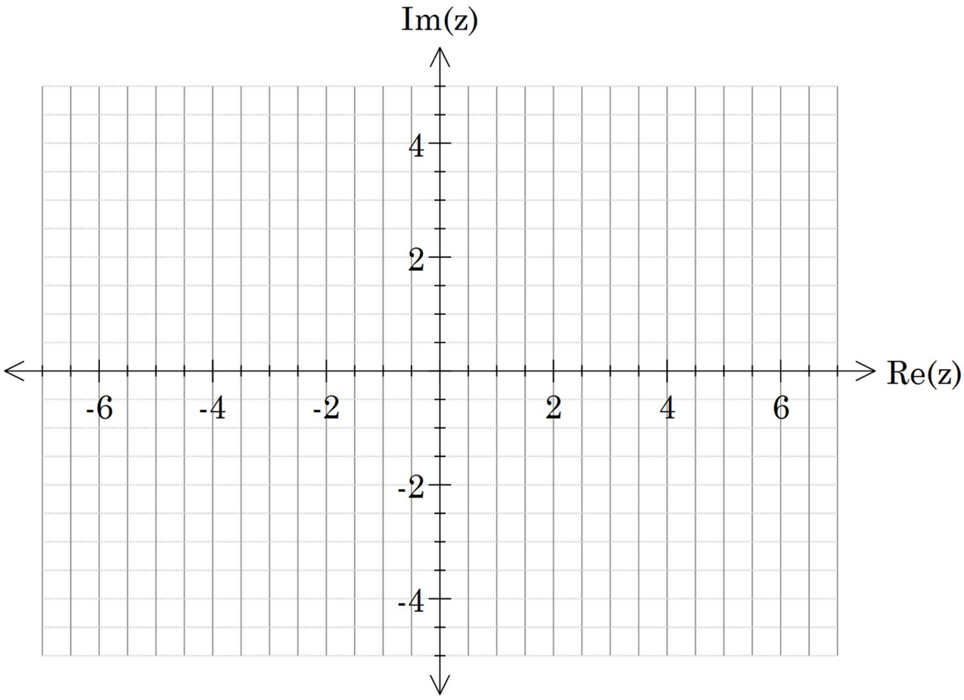
[3]

c) For the locus shown in b) determine the minimum value of $|z|$.

6. [7 marks]

a) Sketch the region in the Argand diagram below that simultaneously satisfies the inequalities:

$$\mathrm{Im}(z+i) \geq 2 \quad \cap \quad |z| \leq \arg(z), \;\; 0 \leq \arg(z) \leq 2\pi$$



[5]

b) For $z = a + bi$, prove that $\dfrac{1}{z} = \dfrac{\overline{z}}{|z^2|}$

[2]