PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Mathematics Specialist
Unit 3

TEST 3

Student name: _____ **Teacher name:** _____

Time allowed for this task:	***45 minutes***, in class, under test conditions Calculator-Assumed
Materials required:	
Standard items:	Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters, SCSA Formula Sheet. Classpad Calculator and Scientific Calculator.
Special items:	Drawing instruments, templates
Marks available:	***44 marks***
Task weighting:	**8%**

Question 1 **(7 marks)**

The points A and B have position vectors $3\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ respectively.

(a) Determine a vector equation for the straight line passing through A and B (2 marks)

(b) Write your answer to (a) in its parametric equivalent and hence, or otherwise, express the Cartesian equation of the line in the form $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$. (3 marks)

(c) Determine a unit vector parallel to the straight line in (a). (2 marks)

Question 2 **(9 marks)**

A plane Π contains the two lines $\mathbf{r} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ and

$$\mathbf{r} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu(-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$$

(a) Write down a vector equation of the plane Π. (1 mark)

(b) The point $8\mathbf{i} + 2\mathbf{j} + c\mathbf{k}$ lies in the plane Π. Determine the value of the constant c. (3 marks)

(c) The vector $a\mathbf{i} + b\mathbf{j} + \mathbf{k}$ is perpendicular to the plane Π. Determine the values of the constants $\boldsymbol{a}$ and $\boldsymbol{b}$. (3 marks)

(d) State the equation of the plane Π in the form $\boldsymbol{r} \cdot \boldsymbol{n} = k$. (2 marks)

Question 3. **(5 marks)**

(a)

(i) Find the Cartesian equation of a sphere with centre (1, -2, 3) and radius 5. (2marks)

(ii) Hence write the vector equation of this sphere. (1mark)

(b) Find the radius and centre of a sphere with the equation: (2marks)

$$x^2 + y^2 + z^2 - 6x + 8y + 4z + 4 = 0$$

Question 4 **(9 marks)**

A particle P, begins from a point $10\underset{\sim}{j}$ m and continues with constant velocity $2\underset{\sim}{i} - \underset{\sim}{j}\ ms^{-1}$. One second later another particle, starts at the point $2\underset{\sim}{i} + 15\underset{\sim}{j}$ m and moves with constant velocity $2\underset{\sim}{i} - 5\underset{\sim}{j}\ ms^{-1}$.

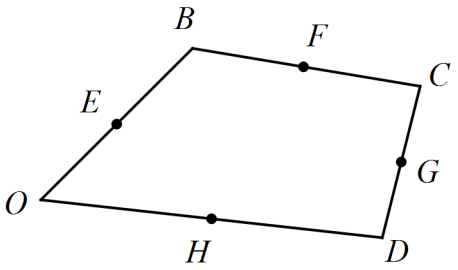
(a) Show that the particles collide. (5 marks)

(b) Find the Cartesian equations of their paths. (2 marks)

(c) Find the Cartesian coordinates for the point of collision. (2 marks)

Question 5 **(7 marks)**

In the diagram below, E, F, G and H are midpoints of the sides of the quadrilateral $OBCD$.



Let $\overrightarrow{OB} = 2b$ $\overrightarrow{OC} = 2c$ *and* $\overrightarrow{OD} = 2d$.

(a) Show that $\overrightarrow{OF} = b + c$. (2 marks)

(b) Determine $\overrightarrow{OG}$ in terms of ***b, c, and d*** (2 marks)

(c) Prove that $EFGH$ is a parallelogram. (3 marks)

Question 6 **(7 marks)**

Use the vector product (cross product) to find the area of the triangle with vertices

A(-1,3,2), **B**(3,5,1) and **C**(1,6,-2)