Papers written by
Australian Maths
Software

SEMESTER ONE

MATHEMATICS SPECIALIST

REVISION 1

UNIT 3

2016

SOLUTIONS

Section One

1. (10 marks)

(a)

$$\begin{bmatrix}1 & 1 & 1 & 2\\1 & -1 & 1 & 6\\1 & 0 & 1 & 4\end{bmatrix}$$

$$\begin{bmatrix}1 & 1 & 1 & 2\\0 & 2 & 0 & -4\\0 & 1 & 0 & -2\end{bmatrix} \qquad R_1 - R_2 \qquad R_1 - R_3$$

$$\begin{bmatrix}1 & 1 & 1 & 2\\1 & 2 & 0 & -4\\0 & 0 & 0 & 0\end{bmatrix} \qquad R_2 - 2R_3$$

$0 = 0$

$\therefore$ There are an infinite number of solutions.
None of the planes are parallel.
There are either two or three identical planes or the three planes intersect in a common line.
None of the planes have identical/equivalent equations so the three planes meet in a common line. (4)

(b) Two of the planes are parallel. Therefore there is no intersection.
$x + y + z = 2$ and $-x - y - z = 1 \Leftrightarrow x + y + z = -1$ are parallel (2)

(c)

$$\begin{bmatrix}1 & 1 & 1 & 6\\2 & 1 & -1 & 1\\3 & -1 & -1 & -2\end{bmatrix}$$

$$\begin{bmatrix}1 & 1 & 1 & 2\\0 & 1 & 3 & 11\\0 & 4 & 4 & 20\end{bmatrix} \qquad 2R_1 - R_2 \qquad 3R_1 - R_3$$

$$\begin{bmatrix}1 & 1 & 1 & 2\\0 & 1 & 3 & 11\\0 & 1 & 1 & 5\end{bmatrix} \qquad R_3 \div 4$$

$$\begin{bmatrix}1 & 1 & 1 & 2\\0 & 1 & 3 & 11\\0 & 0 & 2 & 6\end{bmatrix} \qquad R_2 - R_3$$

$z = 3$

$y + 9 = 11 \rightarrow y = 2$

$x + 2 + 3 = 6 \rightarrow x = 1$

The point of intersection is (1, 2, 3) (4)

2. (6 marks)

(a) $f(x) = \sin(|x|)$ ✓✓ (2)

(b) $g(x) = |x^3 + 1|$ ✓✓ (2)

(c) $p(x) = e^{x-1}$

$q(x) = 1 + \ln(x)$ (2)

3. (13 marks)

(a) $z^3 - z^2 - 4 = 0$

Let $P(z) = z^3 - z^2 - 4$

$P(2) = 8 - 4 - 4 = 0$

$\therefore z = 2$ so $z - 2$ is a factor

$$\begin{array}{r} z^2 + z + 2 \\ z-2 \overline{)\, z^3 - z^2 + 0z - 4} \\ -\underline{(z^3 - 2z^2)} \\ z^2 + 0z \\ -\underline{(z^2 - 2z)} \\ 2z - 4 \\ -\underline{(2z - 4)} \\ 0 \end{array}$$

$z = 2$ or $z^2 + z + 2 = 0$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1 - 8}}{2} \qquad \Delta = -7 = 7i^2$$

$$z = \frac{-1 \pm i\sqrt{7}}{2}$$

$$\therefore \; z = \frac{-1 \pm i\sqrt{7}}{2} \text{ or } z = 2$$ (4)

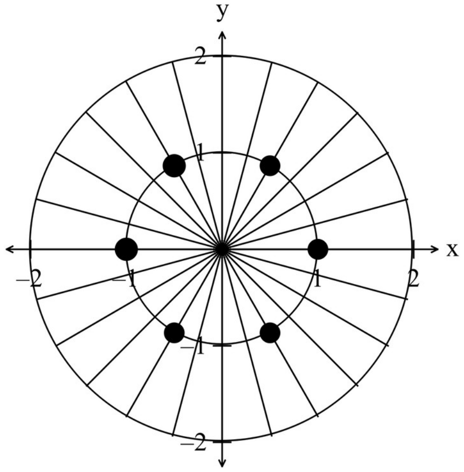
✓✓

(b) $z^3 = 8\,cis(\pi)$ ✓✓

$z^3 = -8$

(3)

(c) (i)



✓✓ -1 per error

(2)

(ii) $z^6 = 1$

$$z^6 = cis(0 + 2n\pi) \quad n \in R$$

$$z = (cis(2n\pi))^{\frac{1}{6}}$$

$$z = cis\left(\frac{2n\pi}{6}\right)$$

$$z = cis\left(\frac{n\pi}{3}\right)$$

$$n = 0, \quad z = cis(0) = 1$$

$$n = 1, \quad z = cis\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{i\sqrt{3}}{2}$$

-1 per error (4)

$$n = 2, \quad z = cis\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$$

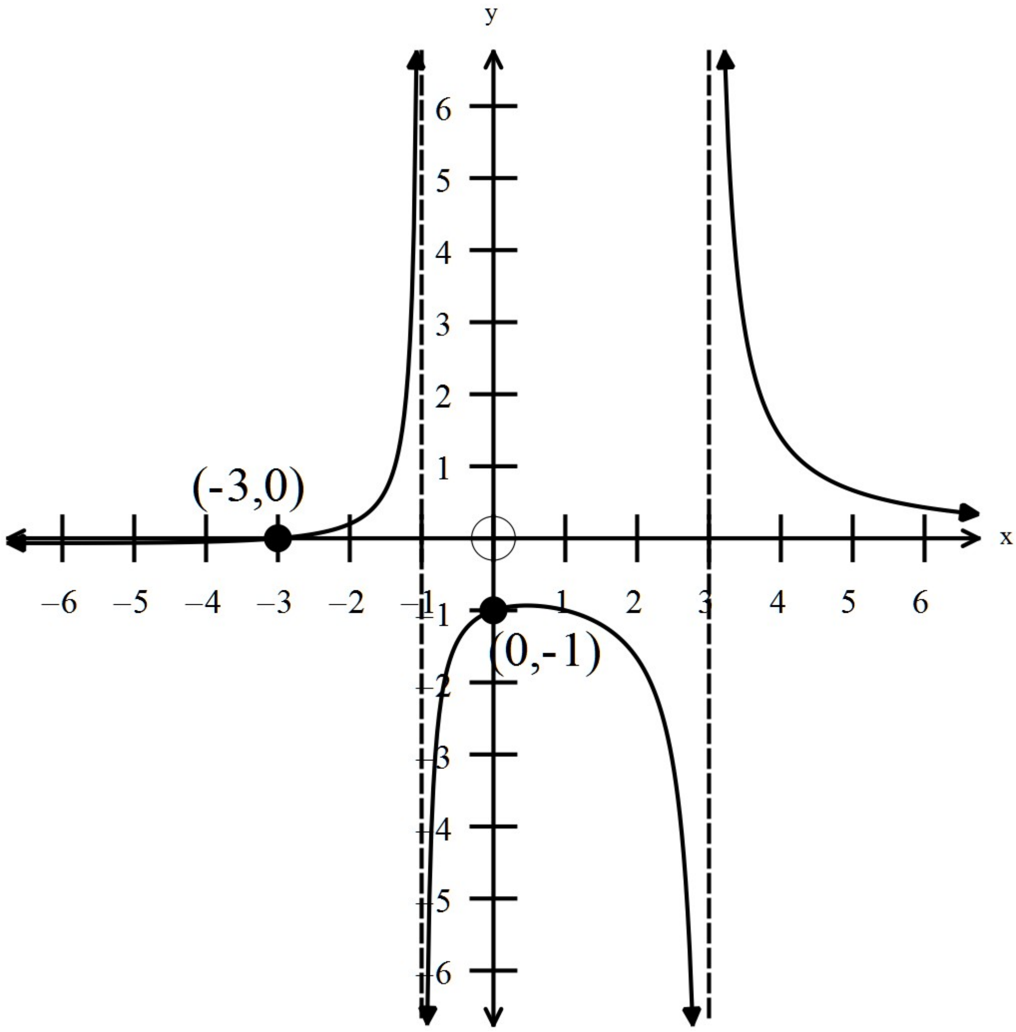
$$n = 3, \quad z = cis(\pi) = -1$$

$$n = -1, \quad z = cis\left(-\frac{\pi}{3}\right) = \frac{1}{2} - \frac{i\sqrt{3}}{2}$$

$$n = -2, \quad z = cis\left(-\frac{2\pi}{3}\right) = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$$

4. (8 marks)

(a) $f(x)=\dfrac{(x+3)}{(x+1)(x-3)}$



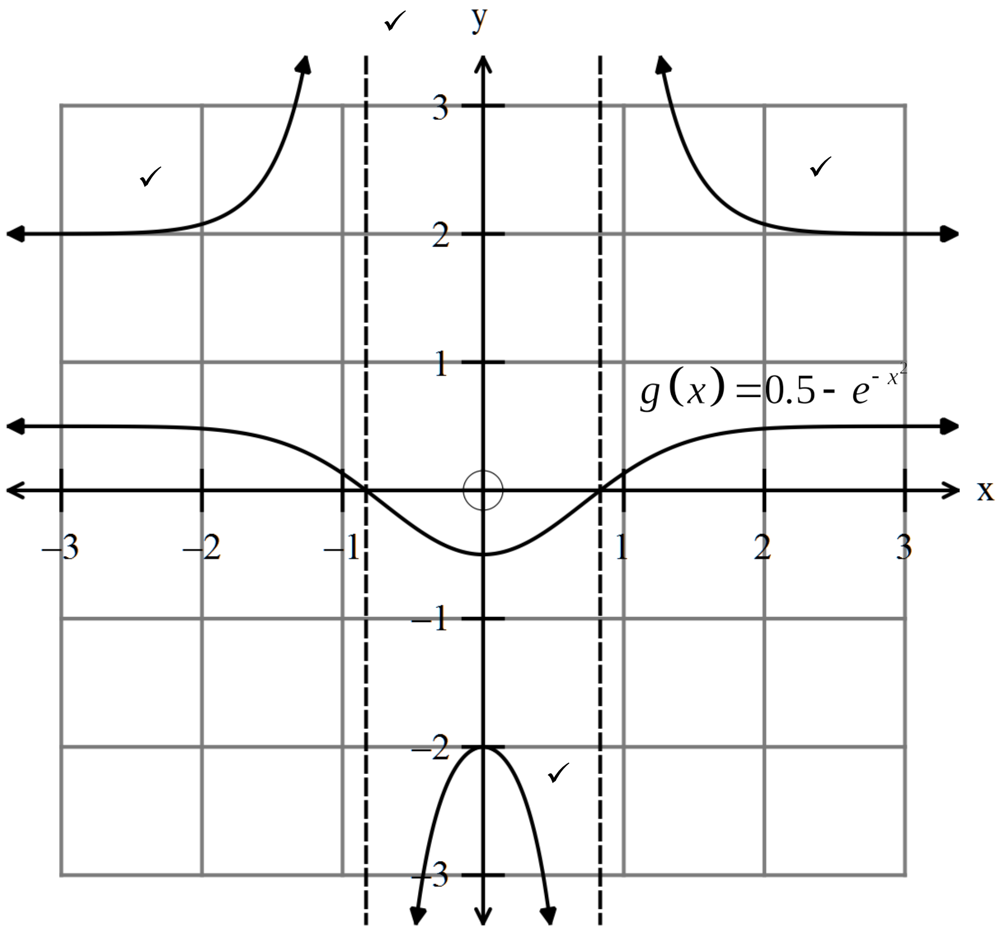
Vertical asymptotes at $x=-1$, $x=3$ ✓ ✓

x intercept at $x=-3$ ✓

y intercept at $y=-1$ ✓

General shape – maximum turning point (w/0 cutting x axis); limits as $x\to\pm\infty$; limits about asymptotes ✓ -1/error (4)

(b)



(4)

5. (13 marks)

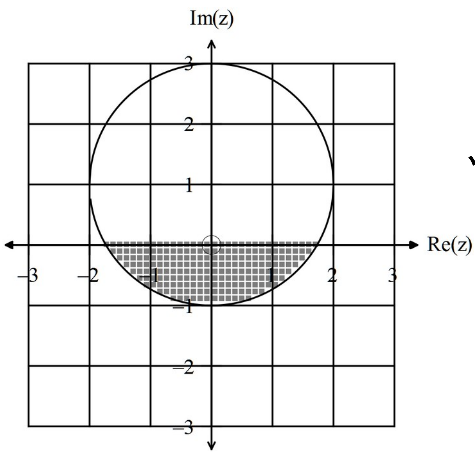
(a) $\left(3\sqrt{3}+\sqrt{2}i\right)\left(2\sqrt{3}-2\sqrt{2}i\right)$.

$= 18 + 2i\sqrt{6} - 6i\sqrt{6} - 4i^2$ ✓✓

$= 22 - 4i\sqrt{6}$ ✓

(3)

(b)



✓✓✓ -1/error

(3)

(c)

$$(1-i)^{10} = \left(\sqrt{2}\,cis\left(-\frac{\pi}{4}\right)\right)^{10}$$

$$= \left(\sqrt{2}\right)^{10} cis\left(-\frac{10\pi}{4}\right)$$

$$= 2^5 cis\left(-\frac{5\pi}{2}\right) \quad \checkmark$$

$$= 32cis\left(-\frac{\pi}{2}\right)$$

$$= 32(0-i)$$

$$(1-i)^{10} = -32i \quad \checkmark$$

(2)

(d) $Re\left(\frac{3-4i}{1+2i}\right) = Re(-1-2i) = -1$

✓ ✓ (2)

(e) (i) $z = cis\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \Rightarrow \bar{z} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ ✓ (1)

(ii) $z^2 = cis\left(\frac{2\pi}{3}\right)^2 = cis\left(\frac{4\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ ✓ (1)

(ii) $\frac{1}{z^2} = \frac{1}{\left(cis\left(\frac{2\pi}{3}\right)\right)^2} = \frac{1}{cis\left(\frac{4\pi}{3}\right)} = cis\left(-\frac{4\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ ✓ (1)

END OF SECTION ONE

Section Two

6. (6 marks)

(a) $\mathbf{v}(t) = (2\cos(2t))\mathbf{i} + (-\sin(t))\mathbf{j}$

$\mathbf{r}(t) = \int ((2\cos(2t))\mathbf{i} + (-\sin(t))\mathbf{j})\,dt$

$\mathbf{r}(t) = (\sin(2t))\mathbf{i} + (\cos(t))\mathbf{j} + \mathbf{c}$

$\mathbf{r}(\pi) = -\mathbf{j}$ so

$-\mathbf{j} = (\sin(2\pi))\mathbf{i} + (\cos(\pi))\mathbf{j} + \mathbf{c}$

$-\mathbf{j} = -\mathbf{j} + \mathbf{c} \Rightarrow \mathbf{c} = \mathbf{0}$

$\therefore \mathbf{r}(t) = (\sin(2t))\mathbf{i} + (\cos(t))\mathbf{j}$

$\mathbf{a}(t) = (-4\sin(2t))\mathbf{i} + (-\cos(t))\mathbf{j}$

(3)

(b) $\mathbf{r}\left(\frac{3\pi}{2}\right) = (\sin(3\pi))\mathbf{i} + \left(\cos\left(\frac{3\pi}{2}\right)\right)\mathbf{j} = 0\mathbf{i} + 0\mathbf{j} = \mathbf{0}$ ✓

$\mathbf{v}\left(\frac{3\pi}{2}\right) = (2\cos(3\pi))\mathbf{i} + \left(-\sin\left(\frac{3\pi}{2}\right)\right)\mathbf{j} = -2\mathbf{i} + \mathbf{j}$ ✓ (2)

(c) Show that $4\mathbf{r}(t) + \mathbf{a}(t) = 3\cos(t)\mathbf{j}$.

$$4\mathbf{r}(t) + \mathbf{a}(t) = 4\begin{pmatrix} \sin(2t) \\ \cos(t) \end{pmatrix} + \begin{pmatrix} -4\sin(2t) \\ -\cos(t) \end{pmatrix}$$

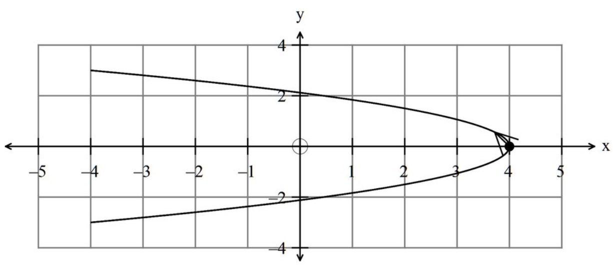
$$= \begin{pmatrix} 0 \\ 3\cos(t) \end{pmatrix}$$ ✓ (1)

$$= (3\cos(t))\mathbf{j}$$

7. (20 marks)

(a) $\mathbf{r}(0) = (4\cos(0))\mathbf{i} + (3\sin(0))\mathbf{j} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ ✓

$\mathbf{r}(0^+) = (4\cos(0^+))\mathbf{i} + (3\sin(0^+))\mathbf{j} = \begin{pmatrix} 4^- \\ 0^+ \end{pmatrix}$ ✓

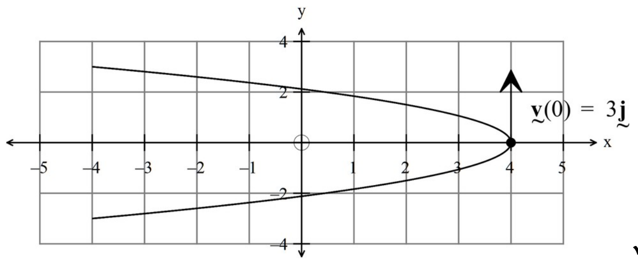


✓

(3)

(b) $\boldsymbol{v}(t)=(-8\sin(2t))\boldsymbol{i} + (3\cos(t))\boldsymbol{j}$ ✓✓

$\boldsymbol{v}(0)=3\boldsymbol{j}$ ✓



✓

(4)

(c) $\boldsymbol{a}(t)=(-16\cos(2t))\boldsymbol{i} + (-3\sin(t))\boldsymbol{j}$ ✓✓ (2)

(d) $\boldsymbol{a}(t)=\begin{pmatrix}16\\-3\end{pmatrix} \Rightarrow t=\dfrac{\pi}{2}$ ✓

$\boldsymbol{r}\left(\dfrac{\pi}{2}\right)=(4\cos(\pi))\boldsymbol{i} + \left(3\sin\left(\dfrac{\pi}{2}\right)\right)\boldsymbol{j}$

$\boldsymbol{r}\left(\dfrac{\pi}{2}\right)=-4\boldsymbol{i} + 3\boldsymbol{j}$ ✓

(2)

(e) $\boldsymbol{a}(t)=(-16\cos(2t))\boldsymbol{i} + (-3\sin(t))\boldsymbol{j}$

$\boldsymbol{r}(t)=(4\cos(2t))\boldsymbol{i} + (3\sin(t))\boldsymbol{j}.$

$\boldsymbol{a}(t)\neq k\,\boldsymbol{r}(t)$ ✓

as $-1\times3=-3$ *but* $-1\times4\neq-16$ ✓ (2)

(f) If $\boldsymbol{a}(t)=\boldsymbol{0}$ then $\boldsymbol{a}(t)=(-16\cos(2t))\boldsymbol{i} + (-3\sin(t))\boldsymbol{j}=\boldsymbol{0}$

$x=-16\cos(2t)=0$ and $y=(-3\sin(t))=0$

$2t=\dfrac{\pi}{2}+k\pi$ $t=0,\pi,2\pi$

$t=\dfrac{\pi}{4}+\dfrac{k\pi}{2}$

$t=\dfrac{\pi}{4},\dfrac{3\pi}{4},\dfrac{5\pi}{4},\dfrac{7\pi}{4}$

The x and y coordinates are never zero at the same time so $\boldsymbol{a}(t)\neq\boldsymbol{0}$ (4)

(g) $\boldsymbol{r}(t) = (4\cos(2t))\boldsymbol{i} + (3\sin(t))\boldsymbol{j}$

$x = (4\cos(2t))$ $\quad\quad y = (3\sin(t))$

$x = 4(1 - 2\sin^2(t))$ $\quad\quad \sin(t) = \dfrac{y}{3}$

$$x = 4\left(1 - \frac{2y^2}{9}\right)$$

(3)

8. (10 marks)

(a) $A(3, 4, 0)$, $B(4, -3, 0)$ and $C(0, 0, 5)$.

$$\boldsymbol{AB} = \begin{pmatrix} 1 \\ -7 \\ 0 \end{pmatrix}, \ \boldsymbol{AC} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$$

$$\boldsymbol{r}(t) = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ -7 \\ 0 \end{pmatrix} + t\begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$$

There are alternative answers as they can also use ***BC*** as a direction vector and then any of the points A,B or C for the equation.

(2)

(b) $x^2 + y^2 + z^2 = 25$ ✓✓ (2)

(c) (i) $\boldsymbol{r}_{ball}(t) = \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$

Hits the ground at $2 - t = 0$ i.e. $t = 2$ seconds ✓ (2)

(ii) $\boldsymbol{r}_{Paul}(t) = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3.5 \\ -1 \end{pmatrix}$

$$\boldsymbol{r}_{Paul}(2) = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 3.5 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \\ 0 \end{pmatrix}$$

$$\boldsymbol{r}_{ball}(2) = \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \\ 0 \end{pmatrix}$$

Both Paul and the ball are at the same point $Q(5,11,0)$ at ground level when $t = 2$, so Paul catches the ball. (3)

(ii) $2\times\left|\begin{pmatrix}1\\3.5\\-1\end{pmatrix}\right| = 2\times\sqrt{1+12.25+1} = 7.55$ m (1)

9. (6 marks)

(a) (i) $\boldsymbol{MN} = \begin{pmatrix}-4\\-6\\-8\end{pmatrix}$

$r(t) = \begin{pmatrix}2\\1\\3\end{pmatrix} + t\begin{pmatrix}-4\\-6\\-8\end{pmatrix}$ (2)

(ii) $x = 2-4t \quad y = 1-6t \quad z = 3-8t$

$t = \dfrac{2-x}{4} \quad t = \dfrac{1-y}{6} \quad t = \dfrac{3-z}{8}$

$so \quad \dfrac{2-x}{4} = \dfrac{1-y}{6} = \dfrac{3-z}{8}$

(2)

(b) $L_1:\ \boldsymbol{r}_1(t) = \begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}-1\\0\\-1\end{pmatrix} \Rightarrow \begin{matrix}x = 1-t\\ y = 2\\ z = 3-t\end{matrix}$

$L_2:\ \boldsymbol{r}_2(s) = \begin{pmatrix}0\\2\\2\end{pmatrix} + s\begin{pmatrix}0\\0\\2\end{pmatrix} \Rightarrow \begin{matrix}x = 0\\ y = 2\\ z = 2+2s\end{matrix}$

It can be seen that $y = 2$. If $x = 0 \Rightarrow t = 1 \quad i.e. \quad z = 2 \Rightarrow (0,2,2)$

If $z = 2$ then $s = 0$ which does not contradict the x and y values and gives $(0,2,2)$. ✓ logic

Yes, the lines intersect, and do so at $(0,2,2)$.

(2)

10. (9 marks)

(a) $\boldsymbol{AB} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$, $\boldsymbol{AC} = \begin{pmatrix} 2 \\ -4 \\ -6 \end{pmatrix}$

$Area_{\Delta} = \frac{1}{2} a \times b \times sin(C) \quad and \quad |\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}| \, sin(\theta)$

$Area_{\Delta} = \frac{1}{2}|\boldsymbol{AB} \times \boldsymbol{AC}|$

$= \frac{1}{2}\left|\begin{pmatrix} -28 \\ -2 \\ -8 \end{pmatrix}\right|$

$= \frac{1}{2}\sqrt{784 + 4 + 64}$

$Area_{\Delta} = 14.6 \ units^2$

(4)

(b) $AB : AC = |\boldsymbol{AB}| : |\boldsymbol{AC}| = \sqrt{21} : \sqrt{56} = \frac{\sqrt{7 \times 3}}{\sqrt{7 \times 8}} = \sqrt{3} : 2\sqrt{2}$ (2)

(c) $P(1.5, 3, 1) \quad Q(2, 0, 0)$

$\boldsymbol{PQ} = \begin{pmatrix} 0.5 \\ -3 \\ -1 \end{pmatrix} \quad \boldsymbol{BC} = \begin{pmatrix} 1 \\ -6 \\ -2 \end{pmatrix}$

$\boldsymbol{BC} = 2\begin{pmatrix} 0.5 \\ -3 \\ -1 \end{pmatrix}$

$\therefore \ 2\,\boldsymbol{PQ} = \boldsymbol{BC}$

Therefore PQ is parallel to BC. (3)

11. (13 marks)

(a) $$\frac{(xy)^3}{\sqrt{z}} = \frac{\left(cis\left(\frac{\pi}{4}\right)\right)^3 (1-i)^3}{\left(1+\sqrt{3}i\right)^{1/2}}$$

$$= \frac{\left(cis\left(\frac{3\pi}{4}\right)\right)\left(\left(\sqrt{2}\right)^3 cis\left(-\frac{3\pi}{4}\right)\right)}{\left(2cis\left(\frac{\pi}{3}\right)\right)^{1/2}}$$

$$= \frac{2\cancel{\sqrt{2}}\left(cis\left(\frac{3\pi}{4}\right)\right)\left(cis\left(-\frac{3\pi}{4}\right)\right)}{\left(\cancel{\sqrt{2}}\,cis\left(\frac{\pi}{6}\right)\right)}$$

$$= 2\left(cis\left(\cancel{\frac{3\pi}{4}} - \cancel{\frac{3\pi}{4}} - \frac{\pi}{6}\right)\right)$$

$$= 2\left(cis\left(-\frac{\pi}{6}\right)\right)$$

$$= 2\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$$

$$= \sqrt{3} - i$$

(4)

(b) $\left\{z: 1 \le |z| \le 2 \;\cap\; -\frac{3\pi}{4} \le arg(z) \le \frac{\pi}{4}\right\}$ ✓ correct inequalities (3)

(c) (i) Given $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ show that $\overline{z_1 z_2} = \overline{z_1} \times \overline{z_2}$.

$$\overline{z_1} \times \overline{z_2} = \overline{(x_1 + iy_1)} \times \overline{(x_2 + iy_2)}$$

$$= (x_1 - iy_1) \times (x_2 - iy_2)$$

$$= x_1 x_2 + i^2 y_1 y_2 + i(-x_1 y_2 - x_2 y_1)$$

$$= x_1 x_2 - y_1 y_2 - i(x_1 y_2 + x_2 y_1)$$

$$\overline{z_1 z_2} = \overline{(x_1 + iy_1)(x_2 + iy_2)}$$

$$= \overline{x_1 x_2 + i^2 y_1 y_2 + ix_1 y_2 + ix_2 y_1}$$

$$= \overline{x_1 x_2 - y_1 y_2 + i(x_1 y_2 + x_2 y_1)}$$

$$= x_1 x_2 - y_1 y_2 - i(x_1 y_2 + x_2 y_1)$$

$$= \overline{z_1} \times \overline{z_2}$$

(3)

(ii) $z = x + iy \quad x = ? \quad y = ?$

$$z(1+i)+\bar{z}(1-i)+2z = 10-2i$$
$$(x+iy)(1+i)+(x-iy)(1-i)+2x+2iy = 10-2i$$
$$(x-y)+i(x+y)+(x-y)+i(-x-y)+2x+2iy = 10-2i$$
$$(x-y+x-y+2x)+i(x+y-y-x+2y) = 10-2i$$
$$(4x-2y)+i(2y) = 10-2i$$
$$Im: 2y = -2$$
$$y = -1$$
$$Re: 4x-2y = 10$$
$$4x+2 = 10$$
$$x = 2, \qquad y = -1$$

(3)

12. (6 marks)

(a)
$$\boldsymbol{a}(t) = -9.8\,\boldsymbol{j}$$
$$\boldsymbol{v}(t) = \int -9.8\,\boldsymbol{j}\,dt = -9.8t\,\boldsymbol{j} + \boldsymbol{c}_1$$
$$\boldsymbol{v}(0) = 20cos(60^0)\boldsymbol{i} + 20sin(60^0)\boldsymbol{j} = 10\boldsymbol{i} + 10\sqrt{3}\,\boldsymbol{j} \Rightarrow \boldsymbol{c}_1 = 10\boldsymbol{i} + 10\sqrt{3}\,\boldsymbol{j}$$
$$\boldsymbol{v}(t) = 10\boldsymbol{i} + \left(10\sqrt{3} - 9.8t\right)\boldsymbol{j}$$

$$\boldsymbol{r}(t) = \int 10\boldsymbol{i} + \left(10\sqrt{3} - 9.8t\right)\boldsymbol{j}\,dt = 10t\,\boldsymbol{i} + \left(10\sqrt{3}t - 4.9t^2\right)\boldsymbol{j} + \boldsymbol{c}_2$$
$$\boldsymbol{r}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \boldsymbol{c}_2 = \boldsymbol{j}$$
$$\boldsymbol{r}(t) = 10t\,\boldsymbol{i} + \left(10\sqrt{3}t - 4.9t^2 + 1\right)\boldsymbol{j}$$
$$If \quad 10t = 50,\ t = 5 \quad and \quad h = 10\sqrt{3}t - 4.9t^2 + 1$$
$$At \quad t = 5,\ h = -34.9\ m$$

This means the ball is not in flight for five seconds so Tom could not have kicked the ball through the window.

(4)

(b)
$$h = 10\sqrt{3}t - 4.9t^2 + 1$$
$$At \quad t = 3,\ h = 8.9\,m$$

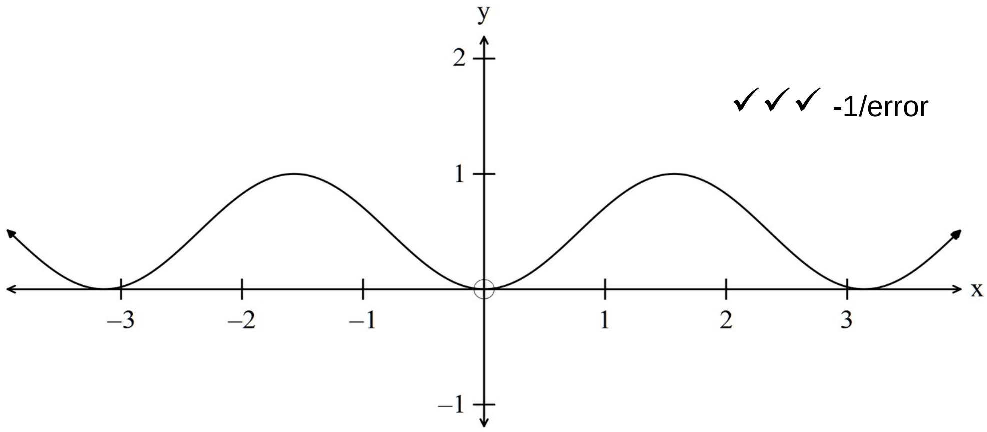
The ball was still in flight so the deputy may have seen it.

(1)

13. (14 marks)

(a) (i) $y = g\left(f(x)\right) = g\left(sin(x)\right) = \left(sin(x)\right)^2$ (1)

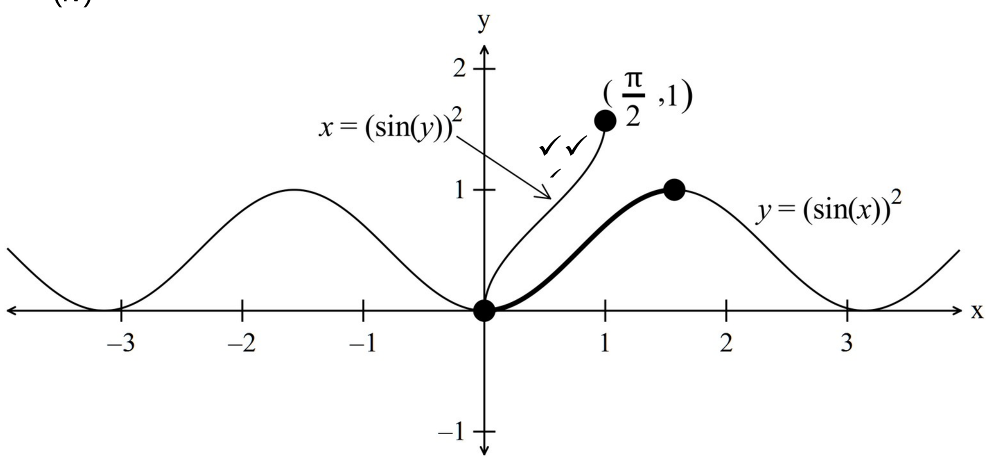
(ii)



(3)

(iii) $a = \frac{\pi}{2}$ ✓✓ (2)

(iv)



(2)

(b) (i) $h(x) = \sqrt{x}$

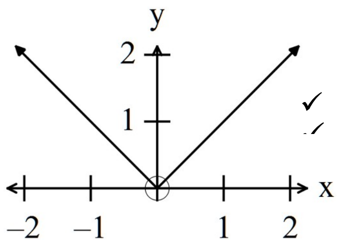
$h^{-1}(x) = x^2$ for $x \geq 0$ ✓ *must have restricted domain* (1)

(ii) $h(x) \geq 0$ ✓ (1)

(c) $\mathrm{g}\left(\mathrm{h}(\mathrm{x})\right) = \mathrm{g}\left(\sqrt{\mathrm{x}}\right) = \mathrm{x}$ ✓

Defined on $[0, 2\pi]$ ✓ (2)

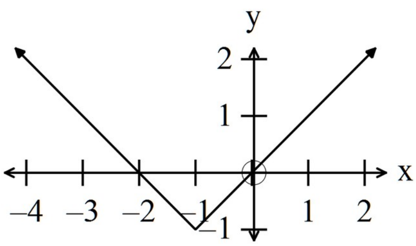
(d) (i)



✓ (1)

(ii) $y = |x|$ has no inverse because it is not a one to one function
i.e. for every x there is not a unique y. ✓ (1)

14. (7 marks)
(a)



✓

$y = |x+1| - 1$

so $x < -2$ or $x > 0$ ✓ (2)

(b) Solve $|1+x| - 1 = |x-1|$

$$y = |1+x| - 1 = \begin{cases} x & \text{for } x \geq -1 \\ -x-2 & \text{for } x < -1 \end{cases}$$ ✓

$$y = |x-1| = \begin{cases} x-1 & \text{for } x \geq 1 \\ 1-x & \text{for } x < 1 \end{cases}$$ ✓

For $x \geq 1$
$x = x - 1$ No solution ✓

For $-1 < x < 1$

$$x = 1 - x$$
$$2x = 1$$
$$x = \frac{1}{2}$$ ✓

For $x < -1$
$-x - 2 = -x + 1$
$-2 = 1$ No solution ✓

$\therefore \ x = \frac{1}{2}$ only

(5)

15. (4 marks)

✓✓

$$y = \frac{x^2}{(x-1)(x+3)}$$

16. (5 marks)

(i)
$$\begin{aligned}
(1-i) &= 1-i & &= 1-i \\
(1-i)^2 &= -2i & &= -2i \\
(1-i)^3 &= -2-2i & &= -2(1+i) \\
(1-i)^4 &= -4 & &= -4 \\
(1-i)^5 &= -4+4i & &= -4(1-i) \\
(1-i)^6 &= 8i & &= 4(2i) \\
(1-i)^7 &= 8+8i & &= 8(1+i) \\
(1-i)^8 &= 16 & &= 16 \\
(1-i)^9 &= 16-16i & &= 16(1-i) \\
(1-i)^{10} &= -32i & &= -32i \checkmark\checkmark
\end{aligned}$$
(2)

(ii) Every fourth result seems to be connected.

Starting with $n=1$, then $n=5$, the pattern seems to be a multiple of $(1-i)$.
Starting with $n=2$, then $n=6$, the pattern seems to be a multiple of i.
Starting with $n=3$, then $n=7$, the pattern seems to be a multiple of i.
Starting with $n=4$, then $n=8$, the pattern seems to be a real multiple of 4.

✓ *for any one of these up to 3 marks*

The coefficients are a pattern of powers of 2.

A lot more analysis is needed, but this is sufficient for 3 marks.

(3)

END OF SECTION TWO