

Western Australian Certificate of Education

Examination, 2015

Question/Answer Booklet

MATHEMATICS

3C/3D

Section One:

Calculator-free

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Student Number:

In figures

In words

Time allowed for this section

Reading time before commencing work: five minutes

Working time for section: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33⅓
Section Two: Calculator-assumed	13	13	100	100	66⅔
Total					100

Instructions to candidates

- The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Additional working space

Question number: _____

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33⅓% (50 Marks)

Section One: Calculator-free

This section has eight (8) questions. Answer all questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 (6 marks)

The waiting times at a Perth railway station vary between five and 13 minutes and are distributed uniformly.

- (a) Jim has observed that 60% of the time the train arrives before T minutes. Determine the value of T . (3 marks)

- (b) Jim has been waiting at the station for eight minutes. What is the probability that he waits less than 10 minutes? (3 marks)

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Additional working space

Question number: _____

Question 2

(6 marks)

A small car company produces three different types of cars, models 'A', 'B' and 'C'. Model A is a two-door coupé, model B is a four-door sedan while model C is a six-wheel SUV with four doors. All three models use the same type of doors and wheels. Each coupé costs \$10 000 to make, each sedan \$30 000 and each SUV \$50 000. The company only has 72 doors in stock, 106 wheels and \$680 000 for production costs. The company notices that it is possible to use all available resources to produce a certain number of each type of vehicle.

Let x = the number of coupés, y = the number of sedans and z = the number of SUV's.

- (a)
- Write the above information as three simultaneous equations in terms of x , y and z .
(2 marks)

- (b)
- Solve the simultaneous equations from part (a) to determine the number of each type of car that can be produced.
(4 marks)

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Additional working space

Question number: _____

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(5 marks)

(2 marks)

(3 marks)

(5 marks)

Question 3

(a) Determine the derivative of e^{e^x} .

(b) Use your result from part (a) to determine $\int_1^0 e^{e^{1/x}} dx$.

Question 4

Solve the following inequality.

$$x - 2 \leq \frac{4 - x}{x + 1}$$

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Question 5

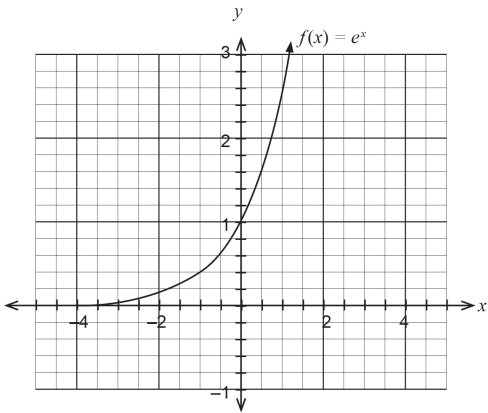
(8 marks)

The functions f and g are defined as follows:

$f(x) = e^x, x \in R,$
 $g(x) = 2x - 3, x \in R.$

- (a)
- Determine the function $f \circ g(x)$ and state its domain and range.
- (3 marks)

The function $f(x)$ is drawn on the axes below.



- (b)
- Sketch $f \circ g(x)$ on the axes above.
- (2 marks)

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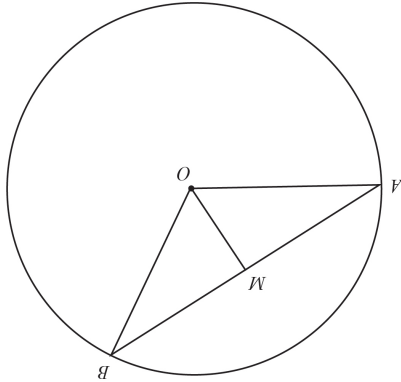
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- (b)
- Hence, or otherwise, prove that if two chords of a circle are of the same length, then both chords are equidistant from the centre.
- (3 marks)

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End of questions

Consider the chord \overline{AB} in a circle with centre O , as in the diagram below.



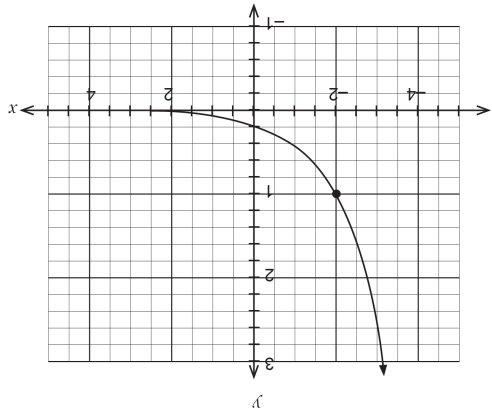
- (a) Prove that if M is the midpoint of the chord \overline{AB} , then the line segment \overline{OM} is perpendicular to \overline{AB} . (3 marks)

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The function h is defined as $h(x) = -x + a$, $x \in R$ where a is a constant integer. The function $f \circ h(x)$ is drawn below and includes the point $(-2, 1)$.



- (c) Determine the value of a . (1 mark)

- (d) On the axes above, sketch the function $y = -f \circ h(x) + 2$. (2 marks)

See next page

Question 6

(5 marks)

The function $f(x)$ has the following properties:

- $f(x)$ is defined for all real numbers
- $\int_{-x}^x f(t) dt = f(x)$
- $f(0) = 1$.

- (a) Determine a function $f(x)$ that satisfies all of the above properties. (3 marks)
(Hint: consider the derivative of $f(x)$.)

- (b) Is the function $f(x)$ above unique? Justify your answer. (2 marks)

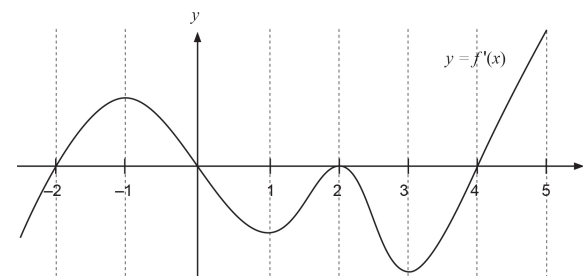
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Question 7

(9 marks)

The figure below shows the graph of the derivative f' of a function f .

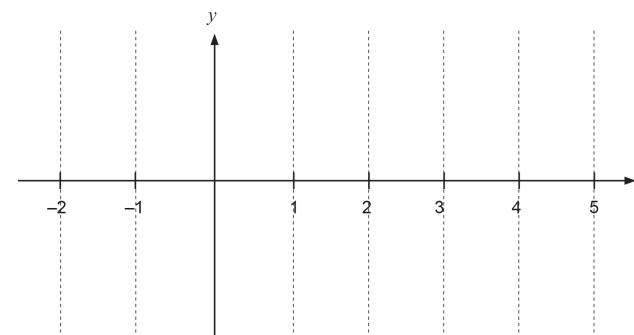


- (a) For what values of x does f have a local maximum or minimum? (2 marks)

- (b) For what values of x does f have an inflection point? (2 marks)

- (c) Does f have a horizontal point of inflection? Explain. (2 marks)

- (d) On the axis below, sketch the graph of f'' . (3 marks)



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