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**School Curriculum and Standards Authority**

## ATAR course examination, 2018

## Question/Answer booklet

# MATHEMATICS METHODS

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

## Section One: Calculator-free

Student number: In figures | | | | | | | | |

In words _____

_____

### Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

### Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet

***To be provided by the candidate***
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Ref: 18-051
MAM-S1

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 11 | 11 | 100 | 99 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2018*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**See next page**







Supplementary page

Question number: _____

Supplementary page

Question number: _____



DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF



**Section One: Calculator-free** **35% (52 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

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**Question 1** **(9 marks)**

A bag contains 1 red marble and 4 green marbles. A single marble is drawn from the bag. The random variable $Y$ is defined as the number of green marbles drawn from the bag.

(a) Complete the probability distribution for $Y$ shown below. (2 marks)

| $y$ | 0 | 1 |
|---|---|---|
| $P(Y = y)$ | | |

(b) State the distribution of $Y$. (1 mark)

(c) Determine the mean and standard deviation of the distribution. (2 marks)

The above process is repeated five times, with the marble being replaced every time. The random variable $X$ is defined as the number of green marbles drawn from the bag in five attempts.

(d) State the distribution of $X$, including its parameters. (2 marks)

(e) Evaluate the probability of selecting exactly two green marbles. (2 marks)

**See next page**

**Question 2** **(6 marks)**

For a set of data values that are normally distributed, approximately 68% of the values will lie within one standard deviation of the mean, approximately 95% of the values will lie within two standard deviations of the mean and approximately 99.7% of the values will lie within three standard deviations of the mean.

If the heights of a large group of women are normally distributed with a mean $\mu$ = 163 cm and standard deviation $\sigma$ = 7 cm, use the above information to answer the following questions:

(a) A statistician says that almost all of the women have heights in the range 142 cm to 184 cm. Comment on her statement. (2 marks)

(b) Approximately what percentage of women in the group has a height greater than 170 cm? (2 marks)

(c) Approximately 2.5% of the women are shorter than what height? (2 marks)

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Supplementary page

Question number: _____

Supplementary page

Question number: _____



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**Question 3** **(12 marks)**

(a) Differentiate $(2x^3 + 1)^5$. (2 marks)

(b) Given $g'(x) = e^{2x}\sin(3x)$, determine a simplified value for the rate of change of $g'(x)$ when $x = \frac{\pi}{2}$. (3 marks)

(c) Determine the following:

(i) $\int 3\cos(2x)\,dx$. (2 marks)

(ii) $\int_0^1 \frac{3x+1}{3x^2+2x+1}\,dx$. (3 marks)

**See next page**

**Question 3** (continued)

(d) If $f'(x) = e^{-2x}$, find the expression for $y = f(x)$, given $f(0) = -2$. (2 marks)





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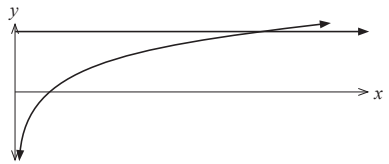
The graphs of the functions $f(x) = 5$ and $g(x) = \ln(x)$ are shown below.




(c) Determine the exact area enclosed between the $x$ axis, the $y$ axis and the functions $f(x)$ and $g(x)$. (4 marks)

**Question 7** **(10 marks)**

(a) Determine a simplified expression for $\frac{d}{dx}(x\ln(x))$. (2 marks)

(b) Use your answer from part (a) to show that $\int \ln(x)\,dx = x\ln(x) - x + c$, where $c$ is a constant. (4 marks)

See next page





**Question 4** **(4 marks)**

Ten shop owners in a coastal resort were asked how many extra staff they intended to hire for the next holiday season. Their responses are shown below:

3, 0, 2, 1, 2, 1, 1, 0, 2, 1

If $N$ = number of additional staff,

(a) complete the probability distribution of $N$ below. (2 marks)

| $n$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P ($N = n$) | | | | |

(b) what is the mean number of staff the shop owners intend to hire? (2 marks)

**Question 5** **(3 marks)**

A 95% confidence interval for a population proportion based on a sample size of 200 has width $w$. What sample size is required to obtain a 95% confidence interval of width $\frac{w}{3}$?

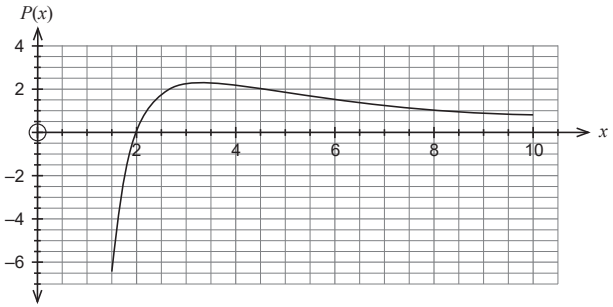
See next page

**Question 6** **(8 marks)**

A company manufactures and sells an item for $\$x$. The profit, $\$P$, made by the company per item sold is dependent on the selling price and can be modelled by the function:

$$P(x) = \frac{50\ln\left(\frac{x}{2}\right)}{x^2} \text{ where } 1.5 \leq x \leq 10$$

The graph of $P(x)$ is shown below:




(a) Describe how the profit per item sold varies as the selling price changes. (3 marks)







(b) Determine the exact price that should be charged for the item if the company wishes to maximise the profit per item sold. (5 marks)