PENRHOS COLLEGE
STRIVE FOR THE HIGHEST

Semester 1 Examination, 2012

Question/Answer Booklet

MATHEMATICS
3C/3D (Year 12)
Section Two:
Calculator-assumed

Your name: ______________________

Please circle your teacher's name: S Ebert T Hosking S Rowden

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time for paper:	one hundred minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items:	pens, pencils, pencil sharpener, eraser, correction/tape fluid, ruler, highlighters
Special items: paper, Curriculum	drawing instruments, templates, notes on two unfolded sheets of A4 and up to three calculators satisfying the conditions set by the Council for this course.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	8	8	50	50
Section Two: Calculator-assumed	13	13	100	100
				150

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

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Section Two: Calculator-assumed (100 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time for this section is 100 minutes.

Question 9 (4 marks)

Consider the function $f(x) = x^3 + \boldsymbol{a}x^2 + 2x + \boldsymbol{b}$ where ***a*** and ***b*** are constants.

(a) Find an expression for the gradient of the curve.

[1]

(b) Given that the tangents at A(0, ***b***) and B(2, 5) are parallel, find the value of ***a*** and ***b***.

[3]

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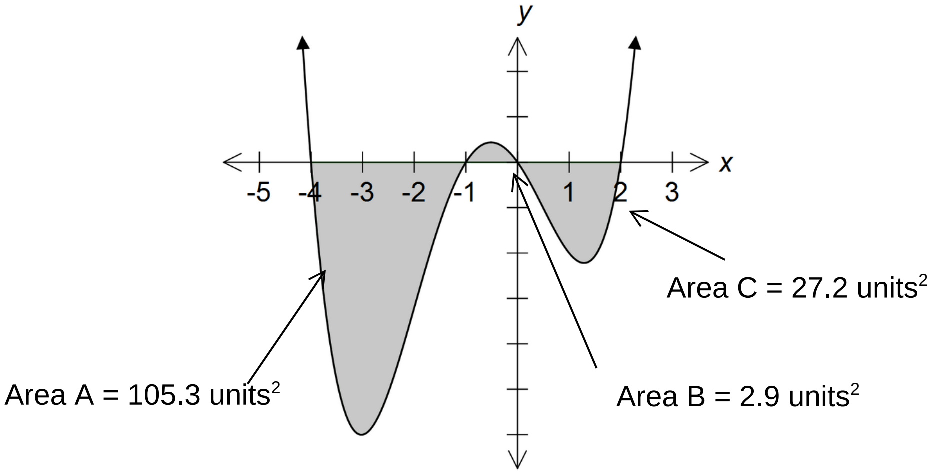
Question 10 **(13 marks)**

(a) The function $f(x)$ is shown below with the areas given in square units for the shaded region.



Determine the value of

(i) $\int_{-4}^{-1} f(x)\, dx$

[1]

(ii) $\int_{-4}^{2} f(x)\, dx$

[2]

(iii) $\int_{-1}^{0} (2f(x)+3)\, dx$

[3]

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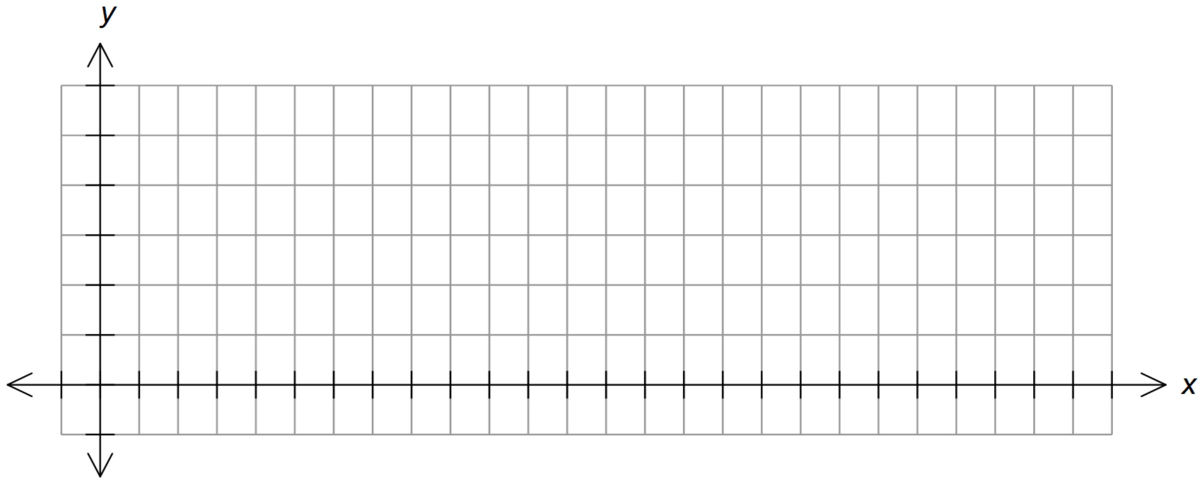
Question 10 continued

(b) (i) Sketch $y = \sqrt{x}$ and $4y = x$ on the axes below.

[2]



(ii) Find the intersection(s) between $y = \sqrt{x}$ and $4y = x$.

[2]

(iii) $x = c$ divides the region bounded between $y = \sqrt{x}$ and $4y = x$ into two regions of equal area. State an equation involving the use of calculus that represents the given situation.
Hence determine c to 1 decimal place.

[3]

Question 11 **(12 marks)**

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A factory produces 2 types of Year 12 leavers' jacket. The factory has enough cloth available to produce 1000 jackets per week. Jacket type A makes a profit of $30 while type B makes a profit of $45. The factory has a minimum weekly contract for 150 type A jackets and 200 of type B. Facilities for screen printing the jackets are limited to 30 hours per week. This equipment can screen print 60 per hour of type A and 20 per hour of type B.

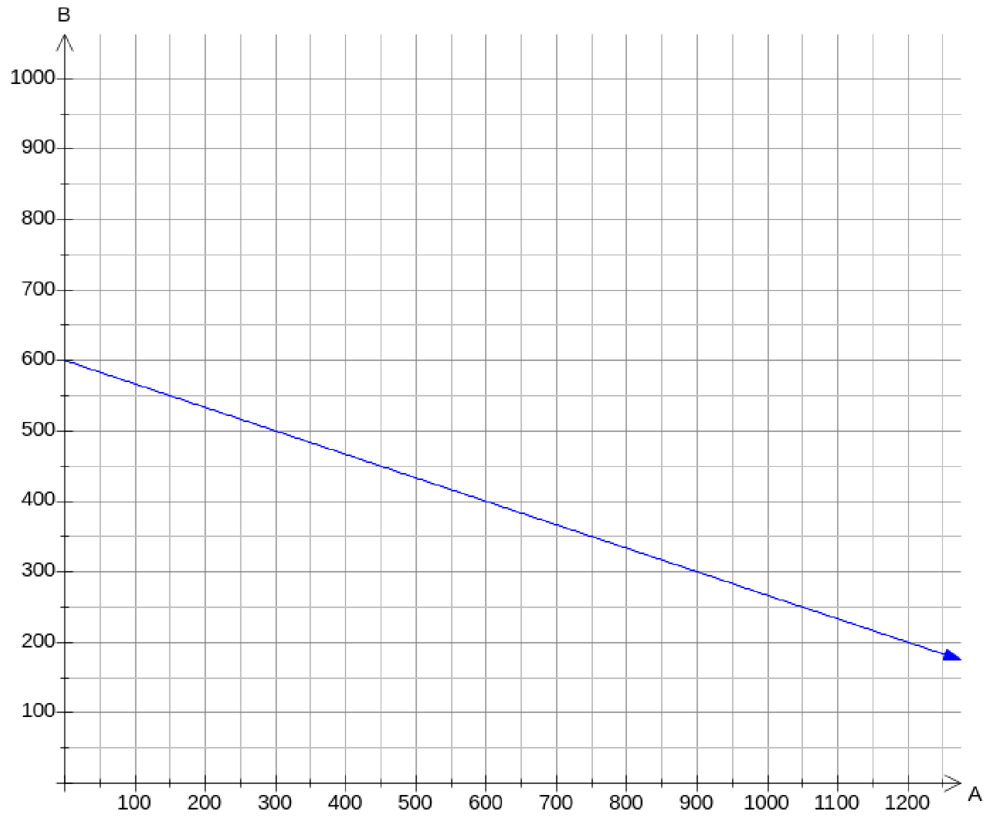
Let A be the number produced per week of type A jacket and B be the number produced per week of type B jacket.

(a) Determine four inequalities from the information given.

[3]

(b) Complete the graph below using your inequalities and shade the feasible region. The line relating to the screen printing constraint has been given.

[3]



Question 11 continued

(c) Determine how many of type A and B jackets the factory should produce per week to maximise the profit and state the maximum profit.

[3]

(d) By what percentage can the profit on jacket A change by before the solution in part (c) is no longer unique.

[3]

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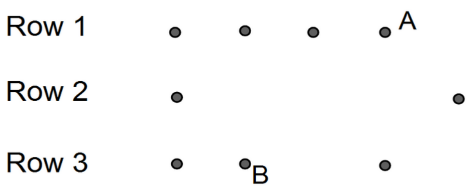
Question 12 **(7 marks)**

Each of the dots in the diagram below can be used to represent a vertex of a triangle.



(a) How many triangles can be formed

(i) using dots from Row 1 and Row 2?

[2]

(ii) if each vertex must come from a different row?

[2]

(b) If each triangle formed can only come from two rows at a time what is the probability that a randomly selected triangle has dot A as one vertex of the triangle given that B is also a vertex of the triangle?

[3]

Question 13 **(6 marks)**

In the first five seconds of inflation, the relationship between the radius (r cm) and time (t sec) of a spherical party balloon are related by the formula:

$$r = -t(t - 10)$$

(a) Show that the relationship between volume (V cm^3) and time is given by

$$V = \frac{4\pi(10t - t^2)^3}{3}$$

[1]

(b) Determine the exact volume of the balloon 3 seconds after first being inflated. [1]

(c) Determine the rate the volume is changing when t = 2 seconds. [1]

(d) Determine the approximate change using the increments formula in volume as t increases from 3 to 3.01 sec. [3]

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Question 14 **(10 marks)**

A worker in the city finishes work at 5:30 pm and drives home. Depending on the conditions he drives home with no lights on or the parking lights on or the headlights on. If fact, for 80% of his journeys home he turns on no lights, for 5% of his journey he only turns on his parking lights, but for the rest, he turns his headlights on. If he uses his parking lights there is a 0.5 chance that he will leave them on overnight. This is certain to flatten his battery. If he uses the headlights there is only a 2% chance that he will leave them on overnight. If he leaves either of the lights on overnight the battery will flat in the morning.

(a) Draw a probability tree diagram to represent the given information.

[2]

(b) What is the probability that on the next journey home from work he uses his lights and turns them off?

[2]

(c) Because he left his lights on, he had a flat battery on Tuesday morning what is the probability that on Monday night he left his parking lights on?

[3]

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Question 14 continued

(d) The driver decided that, in future, whenever he turned on his lights to drive home it would be his headlights on full. Would this decision reduce the chance of a flat battery? If so, by what factor?

[3]

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Question 15 **(6 marks)**

Two competing cyclist are riding with constant speed. At 12 midday cyclist X is 40 metres north of a judge and is riding east at 9m/s, while cyclist Y is 70 metres east of the judge and is riding north at 7m/s.

(a) Show diagrammatically this situation (a scale diagram is not required). [1]

(b) If the distance between the cyclist t seconds later is **D** metres, show that

$$\mathbf{D}^2 = 6500 - 1820t + 130t^2$$

[2]

(c) Determine the time the cyclists are closest together and determine the minimum distance between them. [3]

Question 16 **(8 marks)**

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A particle is moving under rectilinear motion with velocity $v(t) = -2t + 9t^2$ m/s. Answer the following questions for the movement of the particle over the time interval $0 \le t \le 6$.

(a) If the particle was initially 2 m to the right of the origin, what is the displacement from the origin after 2 seconds?

[2]

(b) How far did the particle travel in the first 2 seconds?

[2]

(c) What was the average speed during the 5^{th} second?

[2]

(d) For what subset(s) of the given time interval is the acceleration negative?

[2]

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Question 17 **(8 marks)**

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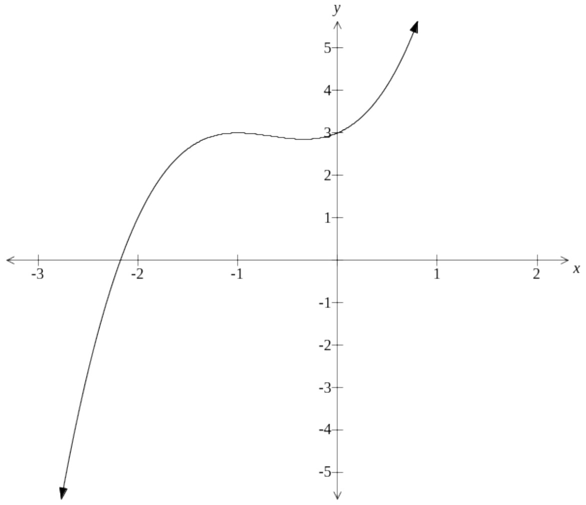
The graph of $y = x^3 + 2x^2 + x + 3$ is shown.



(a) Use the second derivative to show that a possible point of inflection exists at $x = -\frac{2}{3}$.

[2]

(b) Use a sign test to verify that the point where $x = -\frac{2}{3}$ is, in fact, a point of inflection.

[2]

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Question 17 continued

(c) Calculate the equation of the tangent to the curve drawn at the point of inflection.

[2]

(d) A conjecture is made that a tangent drawn through a point of inflection will go through at least one turning point. Use a counter-example to show that this conjecture is false.

[2]

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Question 18 **(8 marks)**

X and Y are two events where 3P(X) = 2P(Y) and P(X ∪ Y) = $\frac{2}{3}$.

(a) If P(Y) = *p* determine the value of *p* given X and Y are mutually exclusive. [2]

(b) If P(Y) = 0.6 determine whether the events X and Y are independent. [4]

(c) If P(Y) = *p* determine the value of *p* given X and Y are independent. [2]

Question 19 **(5 marks)**

(a) Use any method to determine the derivative of $\frac{1}{x}$.

[1]

The expression

$$\frac{f(x+h)-f(x)}{h}$$

is called the Newton quotient and is used to determine the derivative of a function.

(b) Use the Newton quotient to determine the derivative of $\frac{1}{x}$. Simplify your answer.

[2]

(c) If h is approaching 0, does your answer in (b) equal your answer in (a)? Explain.

[2]

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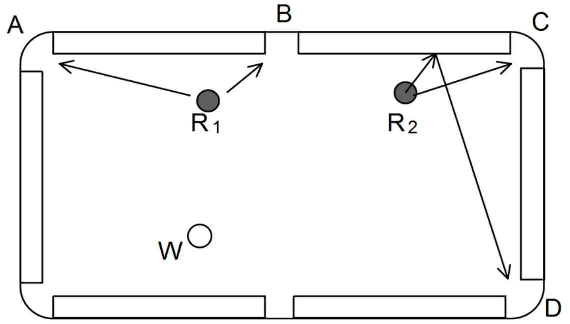
Question 20 **(7 marks)**

In the game of billiards, one of the scoring shots is a "cannon". The player hits the white ball, W, with the cue and it cannons into one of the red balls, R1 or R2. The player's objective is to sink a red ball in one of the pockets A, B, C or D.



The player is about to attempt a cannon. She considers only the four pockets A, B, C or D for this shot as indicated on the diagram.

The probability that she will attempt the shots A, B, C or D is in the ratio 7 : 6 : 5 : 2.

(a) List these probabilities. [1]

If she attempts one of these shots, the respective probabilities of sinking a red are:

$$\frac{5}{6}, \frac{5}{6}, \frac{3}{5}, \frac{2}{5}$$

Determine the probability that

(b) she will sink a red in B. [1]

(c) she will not sink a red. [2]

(d) she attempted shot D given she didn't sink a red. [3]

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Question 21 **(6 marks)**

Given $f(x) = \dfrac{1}{x}$, $g(x) = 2^x$ and $h(x) = 2x + 1$

(a) Use composite function notation to describe:

(i) 2^{-x} [1]

(ii) x [1]

(iii) $2^{x+1} + 1$ [1]

(b) (i) Determine $h \circ f(x)$ [1]

(ii) Determine the domain and range of $h \circ f(x)$ [2]

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Additional working space

Question number(s):______________

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End of Section Two

Additional working space

Question number(s):______________

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