



WESLEY COLLEGE

By daring & by doing

**YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2017
QUESTIONS OF REVIEW 4:
Vector Calculus, Equations and Applications**

Name: _____

Wednesday 31st May

Time: 35 minutes

Mark

/35

Calculator allowed.

1. [7 marks – 2, 1, 2 and 2]

A particle is moving through a 3 dimensional space with velocity given by the vector

$$v(t) = 6t\mathbf{i} + (8t - 5)\mathbf{j} + 3\mathbf{k}$$

- a) Determine $r(t)$ given that the particle started at $(0, -3, 0)$
- b) Write down an expression for the acceleration vector $a(t)$
- c) Decide if, and when, the acceleration is perpendicular to the direction of motion
- d) Calculate the distance travelled in the first 5 seconds of motion.

2. [10 marks – 4, 1, 1, 2 and 2]

A child's model train is moving on a track with position given by

$$\vec{r}_C = 2 \sin\left(\frac{\pi}{6}t\right) \vec{i} + \left(2 - 2 \cos\left(\frac{\pi}{6}t\right)\right) \vec{j}$$

a) Describe its motion in terms of:

shape of the track

direction of travel

period of motion

{Hints: Zoom initialize, set $t_{\max} \approx 20$ }

b) Determine a Cartesian equation to represent the shape of the track.

c) Specify $\vec{v}(t)$, the velocity vector

d) How far does the train travel in 24 seconds?

e) Calculate the maximum and minimum values of the train's speed.

3. [9 marks – 3, 3 and 3]

- (a) Complete the indicated elementary row operations and bring the augmented matrix to echelon form:

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 2 & k & 3 & k-1 \\ 3 & 2 & k+3 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} R_1 & & & \\ R_2 - 2R_1 & & & \\ R_3 - 3R_1 & & & \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} R_1 & & & \\ R_2 & & & \\ kR_3 - 2R_2 & & & \end{array} \right]$$

- (b) Use this echelon matrix to solve
$$\begin{cases} x + 2z = 0 \\ 2x + 4y + 3z = 3 \\ 3x + 2y + 7z = 1 \end{cases}$$

- (c) For which values of k will
$$\begin{cases} x + 2z = 0 \\ 2x + ky + 3z = k - 1 \\ 3x + 2y + (k + 3)z = 1 \end{cases}$$
 have:

(i) no solutions

(ii) a unique solution

4. [9 marks – 3, 2, 1 and 3]

a) Use elementary row operations to determine the number of solutions to the system of

equations represented by the augmented matrix
$$\left[\begin{array}{ccc|c} 0 & -2 & -1 & -6 \\ 2 & 0 & -3 & 14 \\ 1 & 3 & 0 & 16 \end{array} \right]$$

Given that $\vec{a} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\vec{a} \times \vec{v} = \begin{bmatrix} -6 \\ 14 \\ 16 \end{bmatrix}$

b) Explain why the system
$$\begin{cases} -2y - z = -6 \\ 2x - 3z = 14 \\ x + 3y = 16 \end{cases}$$
 represents this situation

c) Write down an equation to represent $\vec{a} \cdot \vec{v} = -10$

d) Determine \vec{v}