



Christ Church Grammar School

SOLUTIONS

2018

UNIT TEST 2

at most -1 marks for
rounding on whole test
(not including Q 8(a))

at most -1 marks for units
on whole test

MATHEMATICS METHODS Year 11

Section Two:

Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes

Working time for this section: 30 minutes

Marks available: 30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your responsibility** to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specific to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that **you do not use pencil**, except in diagrams.

Question 5

(3 marks)

Consider the quadratic equation:

$$x^2 - 2x + m = 0$$

- (a) Give the value of the discriminant in terms of
- m
- .

(1 mark)

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (-2)^2 - 4m \\ &= 4 - 4m \quad \leftarrow \text{(for giving the value for } \Delta \text{ in terms of } m\right) \end{aligned}$$

- (b) Hence determine the values of
- m
- for which the quadratic equation has two solutions.

(2 marks)

For two solutions, need $\Delta > 0$

$$\begin{aligned} \text{so } 4 - 4m &> 0 \quad \leftarrow \text{(for giving initial inequality)} \\ -4m &> -4 \end{aligned}$$

$$m < 1 \quad \leftarrow \text{(for giving final values of } m\right)$$

Question 6

(2 marks)

If M is the mid-point of XY, find the coordinates of Y when X is (4, -3) and M is (1, -3).

Let Y be (a, b)

so $\left(\frac{a+4}{2}, \frac{b-3}{2}\right) = (1, -3)$ ← (for giving expression for x and y coordinates of Y)

$\frac{a+4}{2} = 1 \Rightarrow a = -2$

$\frac{b-3}{2} = -3$

$b-3 = -6$

$b = -3$

Y is $(-2, -3)$ ← (for giving final coordinates of Y)

(3 marks)

Question 7

A line has equation:

$$y = 5x - 2$$

Find the distance between the x axis intercept of the line and the y axis intercept of the line.

y intercept is $(0, -2)$ ← (for giving y intercept)

for x intercept: $y=0$ $0 = 5x - 2$

$5x = 2$
 $x = 0.4$ $(0.4, 0)$ ← (for giving x intercept)

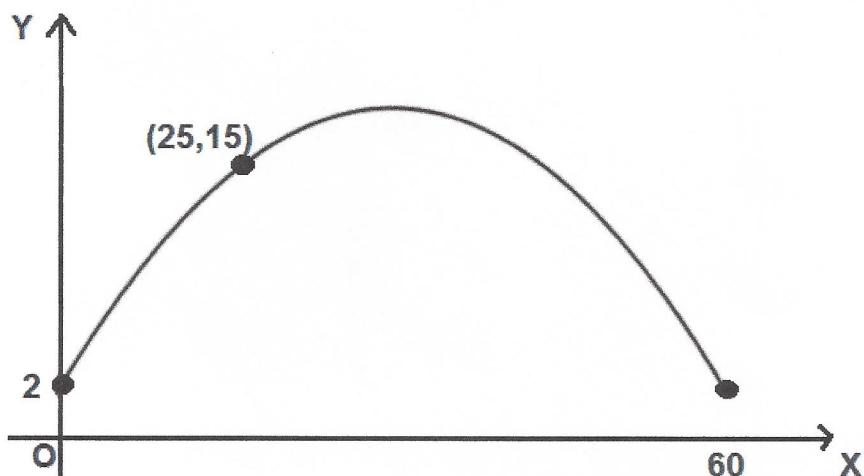
$d = \sqrt{(0.4 - 0)^2 + (0 - -2)^2}$

$= \sqrt{(0.4)^2 + 2^2}$

$= 2.04$ ← (for giving distance)

Question 8

(5 marks)



A cricket ball is thrown by a fielder. It leaves his hand at a height of 2 metres above the ground and the wicketkeeper takes the ball 60 metres away, again at a height of 2 metres. It is known that after the ball has gone 25 metres, it is 15 metres above the ground. The path of the cricket ball follows a quadratic equation of the form:

$$y = ax^2 + bx + c$$

- (a) Calculate the values of a , b and c . You must give your answers correct to 5 decimal places where appropriate. (3 marks)

$$(0, 2) \quad 2 = a \times 0 + b \times 0 + c \Rightarrow c = 2$$

$$(60, 2) \quad 2 = 60^2 a + 60 b + 2$$

$$(25, 15) \quad 15 = 25^2 a + 25 b + 2$$

Solve on Classpad

$$a = -0.01486$$

$$b = 0.89143$$

$$c = 2$$

(for giving expressions involving a , b and c , or other appropriate working out)

two marks
(for giving all 3 values correctly
 a and b must be to S.d.p.)
(one mark for two values correct)

- (b) Evaluate the maximum height that the ball reaches above the ground. (2 marks)

max height when halfway, i.e. $x = 30$ ← (for identifying that max height is in the middle)

$$y = -0.01486 \times 30^2 + 0.89143 \times 30 + 2$$

$$\therefore 15.4 \text{ m}$$

✓ (for calculating max height)

Question 9

(4 marks)

A shopkeeper buys a first crate of eggs at \$1.50 per dozen. He buys another crate, containing three dozen more eggs than the first crate, at \$2.00 per dozen. He sells them all for \$2.50 a dozen and makes \$15 profit. How many dozens were there in the first crate of eggs? Note: there are 12 items in a dozen.

Let 1st crate have x dozens
so 2nd crate has $x+3$ dozens

$$\text{Profit} = 15$$

$$\text{So } \underline{\text{Revenue}} - \underline{\text{Costs}} = \underline{15} = 2.5(x+x+3) - (1.5x + 2(x+3))$$

profit expression

$$15 = 2.5(2x+3) - 1.5x - 2(x+3)$$

$$15 = 1.5x + 1.5$$

$$1.5(x+1) = 15$$

$$x+1 = \frac{15}{1.5} = 10$$

$$x = 9$$

so 9 dozens in first crate

Marking key:

- ✓ revenue determined in terms of ' x '
- ✓ cost determined in terms of ' x '
- ✓ expression for profit is correct
- ✓ correct number of dozens calculated

Question 10

(4 marks)

A line with equation $3x - 2y = 12$ intersects a second line at the point where $x = 2$. The second line is perpendicular to the first line. Determine the equation of the second line.

$$3x - 2y = 12$$

$$-2y = 12 - 3x$$

$$\frac{-2y}{-2} = \frac{12 - 3x}{-2}$$

$$y = \frac{3}{2}x - 6$$

$$\text{so gradient} = \frac{3}{2}$$

(for determining
gradient of
first line)

\therefore gradient of perpendicular line is $-\frac{2}{3}$

(for calculating
gradient of
second line)

$$\therefore y = -\frac{2}{3}x + C$$

$$\text{when } x=2 \quad 3x^2 - 2y = 12$$

$$-2y = 6$$

$$y = -3$$

point is $(2, -3)$

(for calculating
the intersection
point)

use $(2, -3)$

$$-3 = -\frac{2}{3} \times 2 + C$$

$$C = -3 + \frac{4}{3} = -\frac{9}{3} + \frac{4}{3} = -\frac{5}{3}$$

$$\therefore \text{line is } y = -\frac{2}{3}x - \frac{5}{3}$$

(for giving equation
of second line)

Question 11

(4 marks)

- (a) Give the **domain** and **range** of the following function:

$$f(x) = 2\sqrt{x-5} \quad (2 \text{ marks})$$

domain = $\{x \in \mathbb{R} : x \geq 5\}$ — (for giving domain)
 $x \geq 5$ is OK

range = $\{y \in \mathbb{R} : y \geq 0\}$ — (for giving range)
 $y \geq 0$ is OK

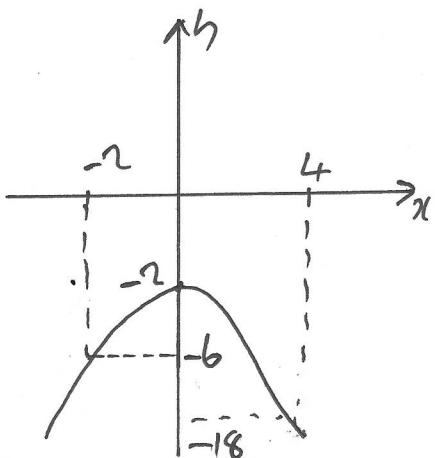
- (b) Consider the following function:

$$f(x) = -x^2 - 2$$

with the domain $\{x \in \mathbb{R} : -2 \leq x \leq 4\}$.

Give the **range** for this function for the given domain.

(2 marks)



max y value in domain is -2
min y value in domain is -18
range is

$\{y \in \mathbb{R} : -18 \leq y \leq -2\}$ —
(for giving the range
 $-18 \leq y \leq 2$ is OK)

Question 12

(5 marks)

The current A (in amperes) that flows in an electric appliance is inversely proportional to the resistance R (in ohms). The current is 3 amperes when the resistance is 80 ohms.

- (a) Determine the current when the resistance is 100 ohms.

(2 marks)

$$A = \frac{k}{R}$$

$$3 = \frac{k}{80}$$

$$k = 240$$

$$A = \frac{240}{R} \quad (\text{for giving } A \text{ in terms of } R) \quad = 2.4 \text{ amperes} \quad (\text{for calculating current})$$

$$\text{When } R = 100$$

$$A = \frac{240}{100}$$

- (b) If the current starts at a certain value, determine the percentage increase in the resistance required to reduce the current to 80% of the starting value. (3 marks)

Let starting values be A and R

$$\text{so } R = \frac{240}{A}$$

Let new values be A₁ and R₁

$$\text{so } R_1 = \frac{240}{A_1}$$

$$\text{but } A_1 = 0.8A$$

$$\text{so } R_1 = \frac{240}{0.8A} = \frac{300}{A} = \frac{300}{240} \times \frac{240}{A} \\ = 1.25R$$

✓✓ (2 marks for clear and logical intermediate steps and working out)

$\therefore 25\%$ increase
in resistance

✓ (for giving final percentage increase)

Additional working space

Question number: _____

Additional working space

Question number: _____