MATHEMATICS METHODS

8 to 2 noits ministration 1016 AWAM

Calculator-Assumed

Marking Key

Page 1

Section Two: Calculator-assumed

(98 Marks)

Question 10

Solution	
$\frac{dV}{dr} = 2\pi r^2$	
$\frac{\delta r}{r} \approx \frac{1}{2\pi r^2} \times \frac{\delta V}{r}$	
$= \frac{1}{3} \times \frac{3}{2\pi r^3} \times \delta V $ OR	$V = \frac{2}{3} \pi r^3$
$=\frac{1}{3}\times\frac{\delta V}{V}$	$\frac{\delta V}{V} \approx \frac{2\pi r^2 \delta r}{\frac{2\pi r^2}{3\pi r^3}} \delta r = 3\frac{\delta r}{r}$
$=\frac{1}{3}\times\frac{1.5}{100}$	
$=0.005\times100=0.5\%$	$\therefore \frac{\delta r}{r} = \frac{0.015}{3} = 0.005 \times 100 = 0.5\%$
Marking key/mathematical behaviours	Marks

	-	
Marking key/mathematical behaviours		Marks
states the correct volume		1
 uses incremental formula correctly 		1
 writes the incremental formula as ratios 		1
calculates the correct percentage change		1

Question 11(a)

lution

у	1	2	3
P(Y=y)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Markin	g key/mathematical behaviours	Marks
•	calculates both probabilities correctly	1

Question 11(b)(i)

Solution

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

	U. Caracian de la Car
Marking key/mathematical behaviours	Marks
calculates correct probability	1

© MAWA, 2016

This examination is Copyright but may be freely used within the school that purchases this licence.

- The items that are contained in this examination are to be used solely in the school for which they are purchased.
- They are not to be shared in any manner with a school which has not purchased their own licence.
- The items and the solutions/marking keys are to be kept confidentially and not copied or made available
 to anyone who is not a teacher at the school. Teachers may give feedback to students in the form of
 showing them how the work is marked but students are not to retain a copy of the paper or the
 marking guide until the agreed release date stipulated in the purchasing agreement/licence.

Published by The Mathematical Association of WA 12 Cobbler Place, MIRRABOOKA 6061

MATHEMATICS METHODS SEMESTER 1 (UNIT 3) EXAMINATION

calculates correct probability

CALCULATOR-ASSUMED MARKING KEY

MATHEMATICS METHODS SEMESTER 1 (UNIT 3) EXAMINATION

Question 11(b)(ii)

Marking key/mathematical behaviours	Marks
7	
_	
I	
Solution	

Question 11(b)(iii)

l	calculates correct probability
Marks	Marking key/mathematical behaviours
	9 C 7
	$\frac{1}{2} = \frac{1}{1} \times \frac{1}{1}$
	Solution

Question 11(b)(iv)

l	 calculates correct probability
l	 indicates both faces being 1, 2 or 3
Warks	Marking key/mathematical behaviours
	$\frac{2}{81} = \frac{1}{8} \times \frac{1}{8} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$
	() ()

Question 11(b)(v)

l	 calculates correct probability
۱'۱	 gives correct pairings including both possibilities for 1 and 3
Marks	Marking key/mathematical behaviours
	$\frac{9\varepsilon}{\varepsilon I} = \frac{7}{I} \times \frac{7}{I} + 7 \times \frac{9}{I} \times \frac{\varepsilon}{I}$
	Solution Possible pairings are 13 or 31 or 22

Question 22 Solution

Area of triangle $OPR: \frac{h}{2} = (h + h) \frac{h}{2} = 3 + h$
Equation of line OO is $VO = \frac{b}{\sqrt{ax^2 + bx}} \left (xd + \sqrt{axb}) \frac{b}{\sqrt{axb}} \right = VO = VO$
Area under curve from 0 to $h = \int_0^h \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx = \int_0^1 dx dx + \frac{\varepsilon_{xx}}{\varepsilon} \int_0^1 dx dx + \varepsilon$
Height of triangle $A = A + \Delta A = A + \Delta A$
u

B region B	$\frac{\varepsilon}{\varepsilon qp} = \frac{\zeta}{\zeta qq} - \frac{\zeta}{\zeta qq} + \frac{\varepsilon}{\varepsilon qp}$
: A noiger to sert	$\frac{9}{\varepsilon qp} = \left(\frac{7}{\varepsilon qq} + \frac{\xi}{\varepsilon qp}\right) - \frac{7}{\varepsilon qq} + \frac{7}{\varepsilon qp}$
: \widetilde{AOO} elgnsint to sent	$\frac{z}{z^{qq}} = qq \times \frac{z}{q}$
	7 7 7

Ratio of region A to region B:
$$\frac{\frac{2h^3}{4h^3}}{1:2} = 1:2$$

l	calculates ratio of region A noigen of region B	•
l	$oldsymbol{a}$ bns $oldsymbol{A}$ noiger to sers senimreteb	•
l	\mathfrak{R}_{OO} bns \mathfrak{R}_{TO} salgnsit to ssars sanimratab	•
l	${\it \Omega}O$ anil to notatione sanimistab	•
l	determines area under curve	•
l	h to smret in RAO elgasit to theight semimreteb	•
Marks	ng key/mathematical behaviours	Marki

Marks 1

1

1

Question (2(u)
Solution
$\frac{dy}{dx} = -4axe^{x^2}$
dx
$0 = -4axe^{x^2}$
x = 0
when $x = 0$,
$y = a - 2ae^0$
y = -a
stationary point at $(0,-a)$
Marking key/mathematical behaviours

Question 12(b)

Solution

$$\frac{d^2y}{dx^2} = -4ax(2xe^{x^2}) - 4ae^{x^2}$$
$$\frac{d^2y}{dx^2} = -4a$$

· equates to zero and solves

• substitutes to determine y-coordinate

Since a is a positive constant the second derivative is negative.

• determines the derivative using the chain rule

lt is a maximum

it is a maximum					
Marking key/mathematical behaviours					
 determines the first and second parts of the second derivative using the product rule and chain rule 	1,1				
 determines the value of the second derivative when x=0 	1				
states the nature of the stationary point	1				

Question 13(a)

Solution				
dv				
$\frac{dy}{dx} = x\cos(x) + \sin(x)$				
Marking key/mathematical behaviours Marks				
correctly differentiates using the product rule	1,1			

MATHEMATICS METHODS SEMESTER 1 (UNIT 3) EXAMINATION Question 21 (a)

Solution

Define $P(t)=1500 \cdot e^{0.07 \cdot t}$ done P(3)1850. 51709

Marking key/mathematical behaviours

• writes the function for the population
• determines the population when t=31

Question 21 (b)

Solution	
solve(P(t)=2000,t)	
{t=4.109743892}	
During 2014	
During 2014	
Marking key/mathematical behaviours	Mark
	Mark

Question 21(c)

(4001.011 = 1(0)	
Solution	
P(6)	
2282.942333	
Define $Q(t)=P(6) \cdot e^{-0.05 \cdot t}$	
done	
solve(1500=Q(t),t)	
{t=8.4}	
During May 2024	
Madina langa tha an atha a bahania	Manta
Marking key/mathematical behaviours	Marks
 determines the population at the start of 2016 	1
 states an equation for the new population 	1
 equates this new equation to 1500 and solves for t 	1
 states the month and year corresponding to this value of t 	1
ctates the mental year conceptioning to the value of t	

Page 13

Marks

Question 13(b) SEMESTER 1 (UNIT 3) EXAMINATION MATHEMATICS METHODS

integrates sin(x) correctly

rearranges correctly

replaces the LHS by y

Marking key/mathematical behaviours

 $x p(x) uis \int -(x) uis x = x p(x) soox \int$

 $3 + (x)\cos + (x)\operatorname{nis} x =$

 $xb(x)\text{mis} + xb(x)\text{soox} \quad \int = \frac{\sqrt{b}}{xb} \int \frac{dy}{dx}$

integrates both sides of the derivative obtained in part (a)

MARKING KEY CALCULATOR-ASSUMED

SEMESTER 1 (UNIT 3) EXAMINATION **MATHEMATICS METHODS**

Question 20 (c)

· ·	
l	 integrates v(t) to obtain general rule for x(t)
Marks	Marking key/mathematical behaviours
	$m09.9\xi =$
	$\chi(\Sigma) = e^{4} - 20 + 2$
	$\zeta = \mathfrak{I}$
	$\mathfrak{Z} = \mathfrak{C}_0 + \mathfrak{C}$
	$\sigma = 101 - 100$
	$ib(01 - {}^{12}95) = (i)x$
	Polution

uses the initial conditions to determine c

calculates the displacement at t=2

Question 14(a)

Solution

	1 '1 '1 '1		 calculates correct probability for each score 							
I	Narks		ng key/mathematical behaviours							
	₽ 0.0	91.0	91.0	91.0	26.0	91.0	(x = X)d			
	Þ١	l l	6	8	9	t	x			
L								Solution		

Question 14(b)

۱٬۱٬۱		correctly completes distribution table						
Marks		arking key/mathematical behaviours						
1 0.0	91.0	91.0	91.0	26.0	91.0	P(Y = y)		
Þ	ı	1 -	Z -	7 -	9–	K		
		I	1		1	ı	Solution	

Question 14(c)

l	seol s si ti tedt setste	•
l	03 yd səilqirinm	•
l	correctly calculates expected value	•
Marks	g key/mathematical behaviours	Markin
	The sum of $y \times P(Y = \gamma) = -2.40$ cents a loss of \$2.40 For 50 games = -240 cents which is a loss of \$2.40	
	U	Solutio

L	determines the distance travelled
l	4 bns 0 as stimil adt saitifiabi •
L	 recognises the need for absolute value
Marks	Marking key/mathematical behaviours
	m 20.8492 =
	69.2492 + 20.1 + 20.4 = TSIG
	96.2462 = (4)x
	$\delta 0.1 - = (08.0)x$
	$\varepsilon = (0)x$
	ЯО
	m 20.8492 =
	$tp\left 0I - {}_{1z}\partial Z\right \int_{0}^{t} = (t)x$
	Solution
	Juestion 20 (d)

MATHEMATICS METHODS SEMESTER 1 (UNIT 3) EXAMINATION

CALCULATOR-ASSUMED MARKING KEY

CALCULATOR-ASSUMED MARKING KEY

Question 15

Solution	
$\frac{d}{dx} \int_{a}^{x} (f(t) + e^{t}) dt - 2 \int_{0}^{x} \frac{d}{dt} (f(t) + e^{2t}) dt = 2$	
$f(x) + e^x - 2(f(x) + e^{2x} - f(0) - 1) = 2$	
$-f(x) + e^x - 2e^{2x} + 4 = 2$	
$f(x) = 2 + e^x - 2e^{2x}$	
Marking key/mathematical behaviours	Marks

Marking key/mathematical behaviours	
applies the fundamental theorem to first integral	1
evaluates second integral	1
expands brackets correctly	1
• substitutes $f(0)$	1
 rearranges for f(x) correctly 	1

Question 16

adestion to	
Solution	
$\int_0^k (\sqrt{k} - \sqrt{x})^2 dx$	
$c^k = \frac{1}{c^k}$	
$= \int_0^k k - 2\sqrt{k} x^{\frac{1}{2}} + x dx$	
$= \left[kx - \frac{4}{3} \sqrt{k} x^{\frac{3}{2}} + \frac{x^2}{2} \right]^k$	
$= k^2 - \frac{4}{3}k^{\frac{3}{2}}k^{\frac{3}{2}} + \frac{k^2}{2} - 0$	
$=\frac{k^2}{6}$	
Marking key/mathematical behaviours	Marks

Marking key/mathematical behaviours		
correctly expands brackets	1	
correctly integrates	1	
 correctly substitutes limits 	1	
 correctly simplifies 	1	

Question 19(c) Solution Area =

MATHEMATICS METHODS

Area =
$$\int_0^{\frac{\pi}{3}} f(x) dx = \left[\frac{\sin x}{1 + \cos x} \right]_0^{\frac{\pi}{3}}$$
$$= \frac{\frac{\sqrt{3}}{2}}{\frac{2}{3}}$$
$$= \frac{\sqrt{3}}{3}$$

SEMESTER 1 (UNIT 3) EXAMINATION

3	
Marking key/mathematical behaviours	
correctly uses part (c) for the integral	1
evaluates the integral	1
simplifies solution	1

Question 20(a)

Solution $solve(0=2\cdot e^{2\cdot t}-10,t)$ {t=0.8047189562} Marking key/mathematical behaviours Marks • recognises that the particle is at rest when v = 01 1 • Solves for t.

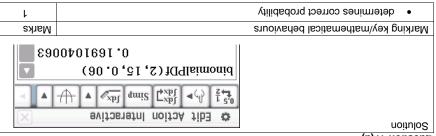
Question 20(b)

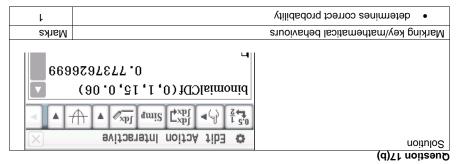
Solution

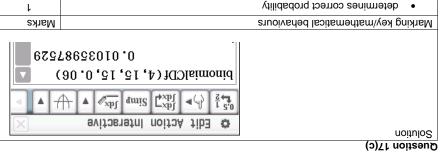
$$a(t) = \frac{dv}{dt}$$
$$= 4e^{2t}$$
$$a(0) = 4(1)$$
$$= 4 \text{ m/s}^2$$

Marking key/mathematical behaviours	
determines the derivative of the velocity function	1
 determines the acceleration when t = 0. 	1

Question 17(a)







l		 determines correct probability
l	11	 correctly uses complementary even
Marks		Marking key/mathematical behaviours
2012	807408.0	
V	1-binomialPDf(0, 15, 0.06)	
A	▼ ▼ State of the	
\times	Selit Action Interactive	P(at least 1)=1 - P(0)
	1 1 1 1 1 1 1 1	Solution
		h)אַן uoiteəu)

Question 19(a)

1'1		 correctly calculates the values of y 					
Marks	Marking key/mathematical behaviours						
1.02	Mean						
31.1	23.0	32	.0	82.0	lle area	Upper rectang	
68.0	96.0	82	.0	92.0	lle area	Lower rectangle area	
Total	$\frac{2}{\pi} - \frac{\epsilon}{\pi}$	$\frac{\pi}{\varepsilon}$ –	9 11	$\frac{\pi}{6} - 0$	θĮ	Rectangle	
						_	
	l	79.0	1 9:0	3.0	Λ		
	$\frac{7}{\pi}$	$\frac{\pi}{\varepsilon}$	$\frac{9}{\pi}$	0	х	1	

Question 19(b)

correctly calculates totals
 correctly calculates mean

correctly calculates areas of upper and lower rectangles

ı	 uses the given information correctly and obtains solution
ı	 expands brackets and simplifies
l	 correctly differentiates using the quotient rule
Marks	Marking key/mathematical behaviours
	notition $\frac{(x \operatorname{mis} -) x \operatorname{mis} - (x \operatorname{soo})(x \operatorname{soo} + 1)}{\sum_{x = 0}^{\infty} \frac{(x \operatorname{soo} + 1)}{x \operatorname{mis} + x \operatorname{soo} + x \operatorname{soo}}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{soo})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{soo})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{soo})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{soo})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{soo})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{soo})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{soo})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{soo})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{mis})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{mis})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{mis})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{mis})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}{\sum_{x = 0}^{\infty} \frac{(1 + x \operatorname{mis})}{x \operatorname{soo} + 1}} = \frac{\left(\frac{x \operatorname{mis}}{x \operatorname{soo} + 1}\right) \frac{b}{xb}}$

1,1 1

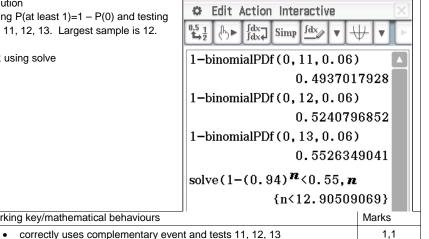
1

Question 17(e)

Solution
Using P(at least 1)=1 – P(0) and testing
n = 11, 12, 13. Largest sample is 12.
OR using solve

Marking key/mathematical behaviours

· determines correct sample size



Question 18(a)

Solution In triangle, height = $2\cos\theta$ and base = $2\sin\theta$ $V = \text{area of trapezium } \times 8$ $= \frac{2\cos\theta}{2} \times (2 + 2 + 2 \times 2\sin\theta) \times 8$ $=\cos\theta\times(4+4\sin\theta)\times8$ $=32\cos\theta(1+\sin\theta)$

Marking key/mathematical behaviours	
identifies height and base of triangle	1
 uses suitable formula for area of base 	1
simplifies and factorises result	1

Question 18(b)	
Solution	
solve $\left(10=32\cdot\cos(\theta)\cdot(1+\sin(\theta)), \theta, 0, 0, \frac{\pi}{2}\right)$	
$\{\theta=1.412913449\}$	
Marking key/mathematical behaviours	Marks
 recognises to equate the volume equation to 10 	1
solves for theta	1

MATHEMATICS METHODS

states the capacity

SEMESTER 1 (UNIT 3) EXAMINATION

Question 18(c)	
Solution	
$V'(\theta) = -32\sin\theta(1+\sin\theta) + 32\cos\theta\cos\theta$	
For max: $V'(\theta) = 0 \Rightarrow \theta = 0.52$	
$V''(0.52) = -83.14 \Rightarrow \text{maximum}$	
$V(0.52) = 41.57 \text{ m}^3$	
Maximum capacity is 41570 kL	
Marking key/mathematical behaviours	Marks
 determines the first part of the derivative using the product rule 	1
 determines the second part of the derivative using the product rule 	1
 equates derivative to zero and solves for theta 	1
justifies maximum	1
determines the volume	1