Test 7 : Wednesday 7th September

Interval Estimates for Proportions

This assessment contributes 7% towards the final year mark.

40 minutes are allocated for this test.

Calculators are required.

No notes of ANY nature are permitted.

Full marks may not be awarded to correct answers unless sufficient justification is given.

Name :

40 minutes

Total = 38

Do NOT turn over this page until you are instructed to do so.

THIS PAGE HAS BEEN LEFT INTENTIONALLY BLANK

Question 1 **(3 marks)**

A law firm has 16 partners, 28 secretaries, 27 paralegals and 29 associates. How many people from each category do you need for a stratified sample of 20 people?

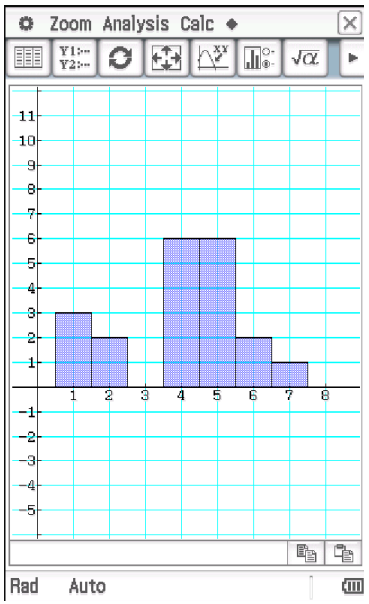
Question 2 **(3 marks)**

James, a Year 12 student, needs to construct a survey of which modes of transport are used by students at his school for an assignment. He decides to hand out his survey to other students in Year 12. Explain why this method will result in a sample that is not representative of the population and provide an alternative method for James to use.

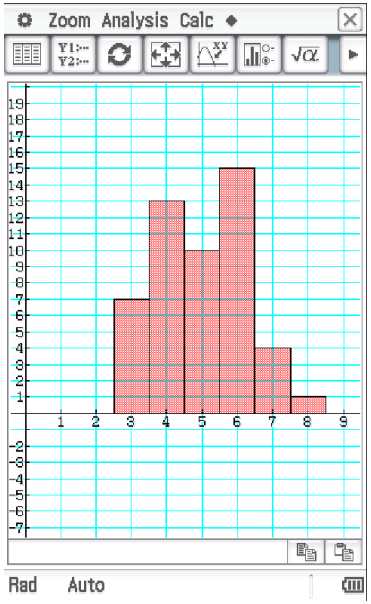
Question 3 **(11 marks)**

The two graphs on the calculator screen shots below display the results of two different simulations measuring the number of heads shown when a coin is tossed 10 times.

Simulation 1:



Simulation 2:



(a) State the probability distribution and the parameters for this situation.

(2 marks)

(b) What is the theoretical expected value and standard deviation for this situation?

(2 marks)

Question 6 **(5 marks)**

A business analyst for Wesfarmers wanted to evaluate the number of Target stores with 100% compliance in their marketing strategy. His survey of 150 store managers provided a confidence interval of $0.36 \leq p \leq 0.54$. Calculate the point estimate for the proportion of compliant stores in this sample and the level of confidence that the analyst can have in this interval.

End of Test

Question 5 **(7 marks)**

A random sample of 150 investment bankers found that 65 had slept less than 30 hours that week as they were working so hard.

(a) Determine a 90% confidence interval for the population proportion p.

(2 marks)

(b) If the true proportion of investment bankers who had slept less than 30 hours that week is 35%, what is the probability that the sample proportion is at most 0.4 in a new sample of 360 bankers?

(3 marks)

(c) Another 11 surveys of investment bankers with sample size 100 were conducted and for each a 90% confidence interval for p was calculated. Calculate the probability that less than 8 of the intervals included the true value for p.

(2 marks)

Question 3 (Continued)

(c) In Simulation 1 the mean value is 4 and the standard deviation is 1.70 while in Simulation 2 the mean value is 4.98 and the standard deviation is 1.27. By comparing these values with the answers to part (b) and the shape of the two histograms, explain how these simulations illustrate an important principle when collecting samples.

(3 marks)

(d) A third simulation is conducted with 100 trials of tossing a coin 10 times. If $\hat{p}$ represents the proportion of trials containing at least 6 heads, determine a probability density distribution and its parameters for $\hat{p}$.

(4 marks)

Question 4 **(9 marks)**

A random sample of 600 people found that 144 of them owned dogs.

(a) Determine the sample proportion, $\hat{p}$, of those who owned dogs.

(1 mark)

(b) Use the survey results to estimate the standard deviation of $\hat{p}$.

(2 marks)

(c) Another survey is to be taken. Using a 99% confidence interval and the sample proportion from the initial survey, estimate the sample size required to ensure the margin of error is at most 0.04.

(4 marks)

Question 4 (continued)

A 95% confidence interval for the population proportion is calculated from the original random sample and found to be $0.2058 \leq \hat{p} \leq 0.2742$.

(d) If a new sample of 360 people at the beach is taken and 135 are found to own dogs, what does this indicate?

(2 marks)