PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course Methods Year 12 test three 2022

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: **____40_____ mins**

Number of questions: **___6_____**

Materials required: **Upto 3 calculators/classpads allowed**

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, **one page of A4 notes doublesided**

Marks available: **__43__ marks**

Task weighting: **_10__%**

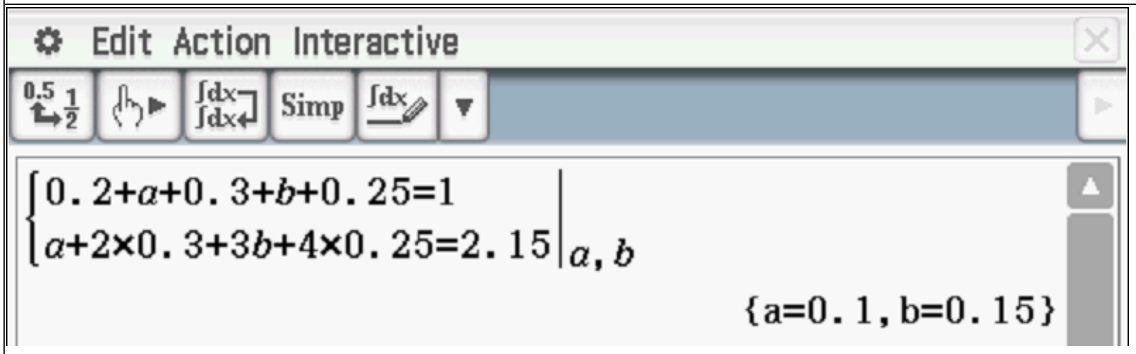
Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3, 3 & 2 = 8 marks)
Consider the discrete random variable X and the table of probabilities below.

X	0	1	2	3	4
$P(X=x)$	0.2	a	0.3	b	0.25

a) Given that the expected value of X is 2.15, determine the values of a & b.

Solution

Specific behaviours
✓ sets up one equation for a & b ✓ sets up two equations for a&b ✓ solves for a&b

b) Determine the standard deviation of X to 3 dp, showing all reasoning.

Solution
$V(x) = (0-2.15)^2 0.2 + (1-2.15)^2 0.1 + (2-2.15)^2 0.3 + (3-2.15)^2 0.2 + (4-2.15)^2 0.25$ $= 2.028$ $std = 1.424$
Specific behaviours
✓ shows a sequence for variance ✓ determines variance ✓ determines std to 3 dp

c) Determine the $E(3X+4)$ and Variance($3X+4$).

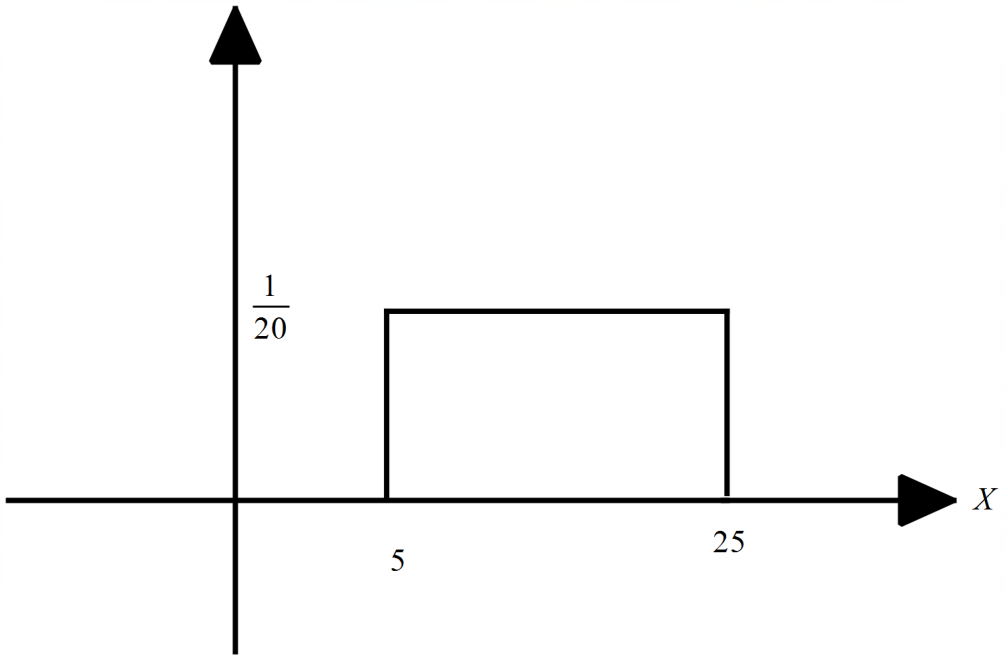
Solution
$E(3X+4) = 10.45$ Variance($9X+4 = 18.2975$).

Specific behaviours
✓ determines new mean ✓ determines new Variance

Q2 (3, 2, 1 & 3 = 9 marks)

The number of X minutes late a train arrives at a particular station is a uniform probability distribution from 5 mins to 25 mins.

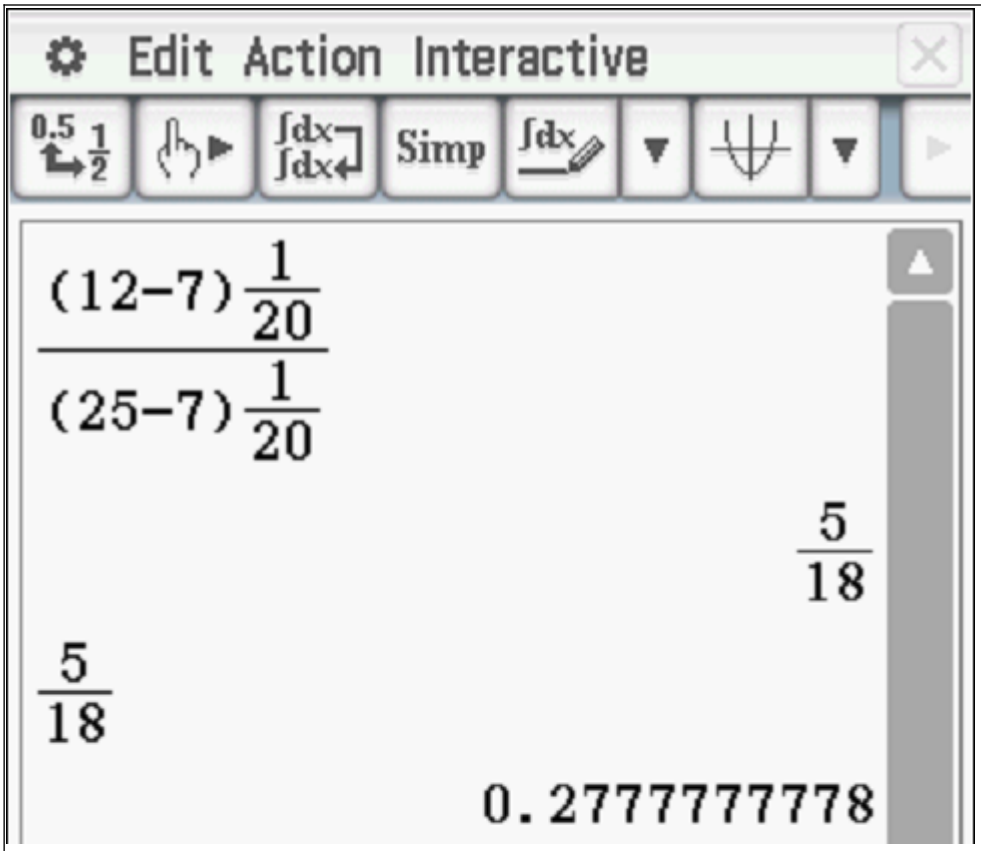
a) Sketch the probability density function for X showing all relevant features and labels.

Solution

Specific behaviours
✓ uses straight line ✓ x labels ✓ y label

Q2 continued

b) Determine the probability that the train will be less than 12 mins late given that it is at least 7 mins late.

Solution



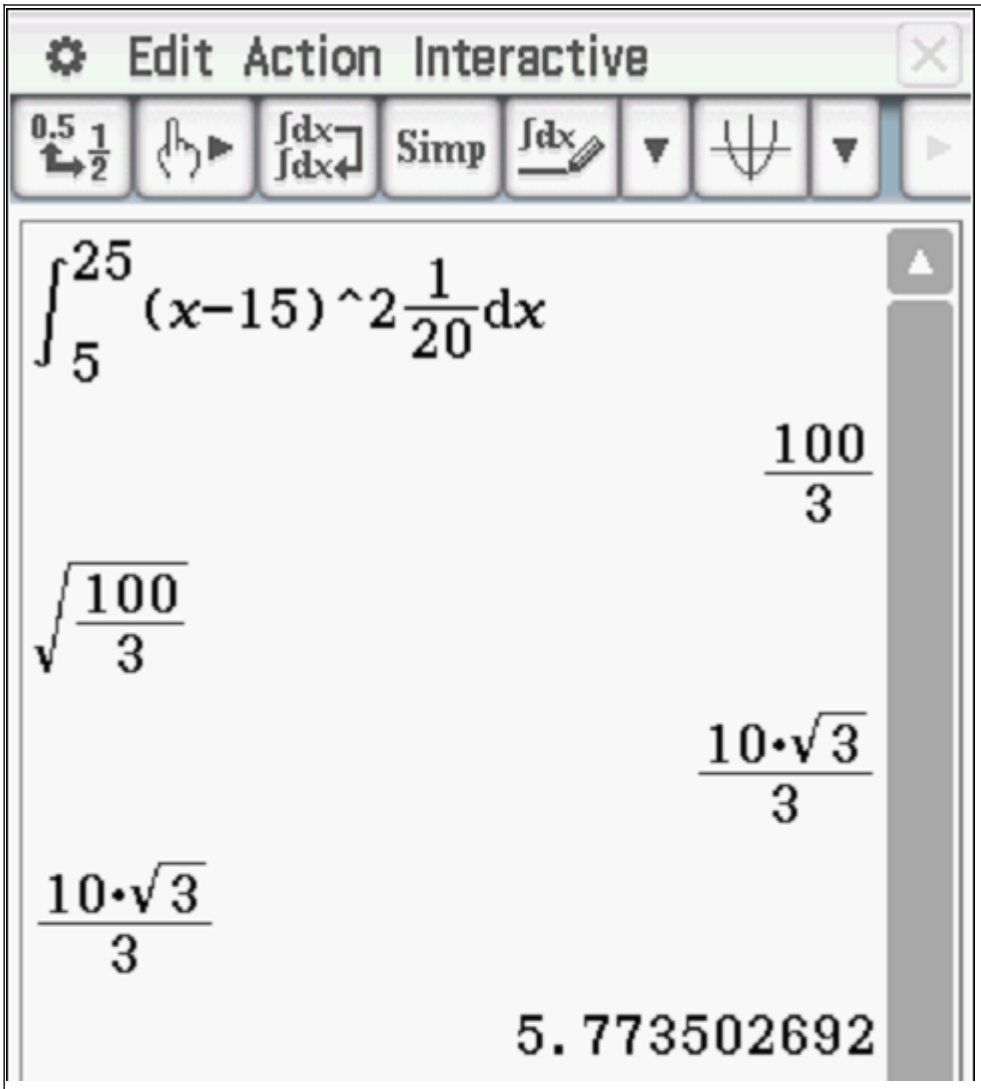
Specific behaviours
✓ correct denominator ✓ correct numerator

c) Determine the mean number of minutes late.

Solution
Mean = 15 mins
Specific behaviours
✓ states midpoint

d) Determine the standard deviation of X showing all reasoning.

Solution



Specific behaviours
✓ sets up integral ✓ determines variance ✓ determines stdev

Q3 (3, 3 & 2 = 8 marks)
Consider a game where two ordinary dice are thrown into the air and then land and the sum of the two top numbers is added. If the sum is a prime number $(2,3,5,7....)$ etc then this is considered a win.

a) Determine the probability of a win. Show reasoning.

Solution

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Prime numbers 2,3,5,7,11
Pr(prime)=15/36

Specific behaviours

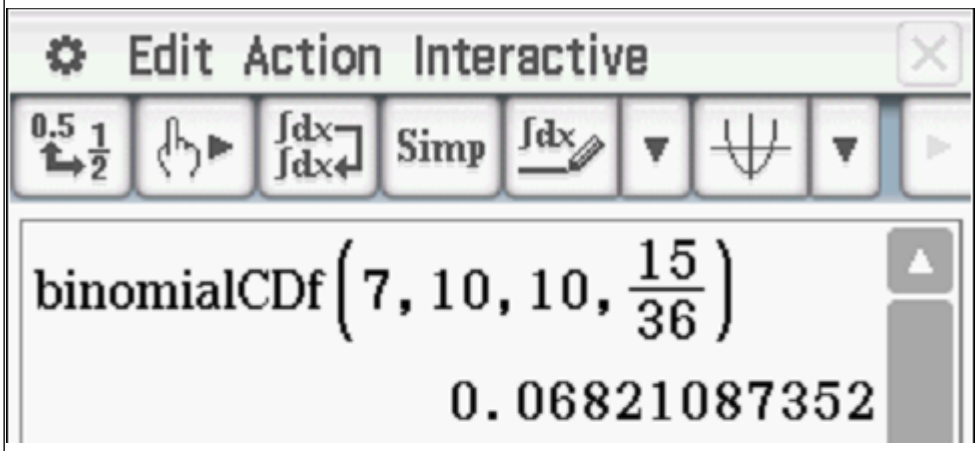
✓ shows sample space
✓ number of favourable outcomes shown
✓ states probability

b) If this game was played 10 times, determine the probability that a win occurs at least 7 times. Show all reasoning.

Solution

$X \sim B(10, \frac{15}{36})$

$P(X \geq 7)$

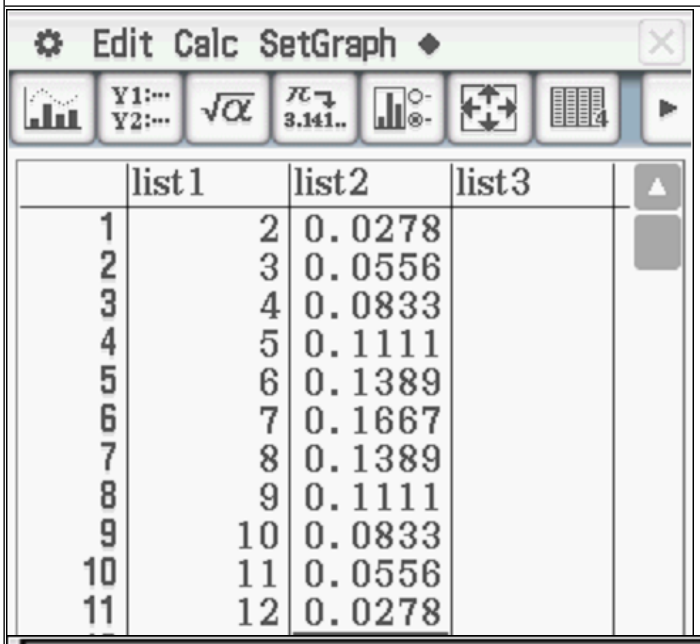


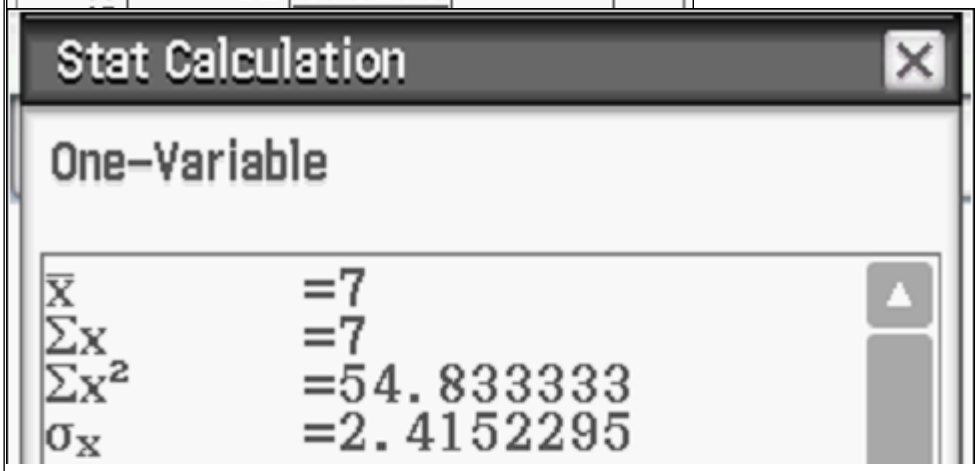
Specific behaviours

✓ states Binomial with parameters
✓ uses cumulative expression
✓ states prob

a) Let X = sum of the top numbers of both dice. Determine the mean and standard deviation for X.

Solution



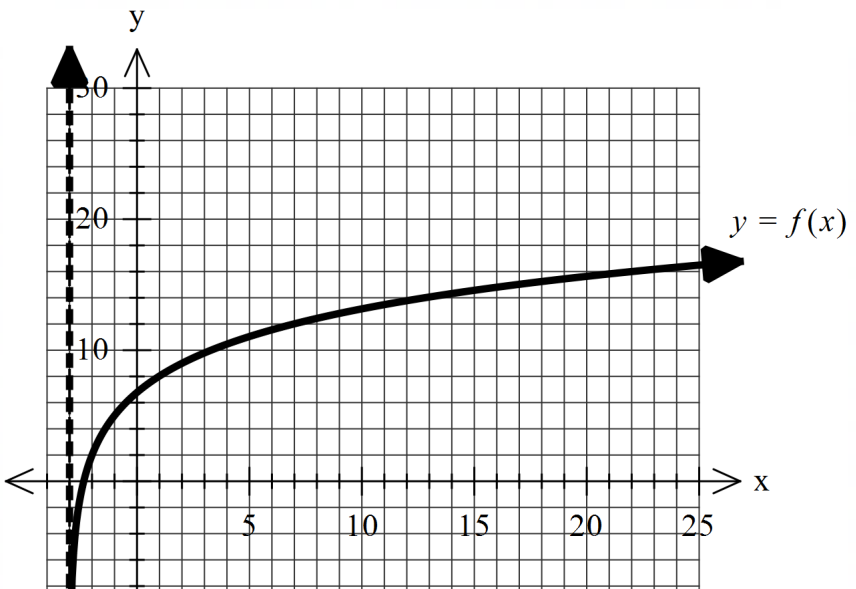


Specific behaviours

✓ states mean of 7
✓ states stdev 0f 2.415

Q4) (5 marks)

Consider $f(x) = r\log_5(x+p)+q$ where $r, p \& q$ are constants.



Using the graph above and given that the following points $(22,16)\&(2,9)$ lie on the curve $y=f(x)$, determine the values of $r, p \& q$.

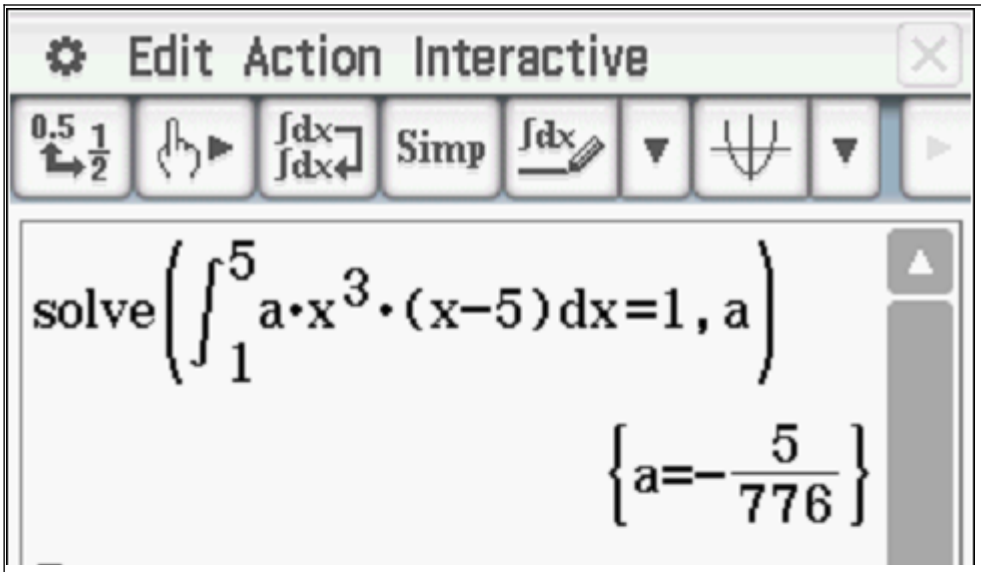
Solution
$f(x)=r\log_5(x+p)+q$ Asymptote x=-3, p=3 $(2,9)$ $9=r\log_5(5)+q=r+q$ $(22,16)$ $16=r\log_5(25)+q=2r+q$ $16-9=r$ $r=7$ $16=14+q$ $q=2$
Specific behaviours
✓ identifies asymptote at x=-3 ✓ determines p ✓ sets up one equation with two unknowns ✓ sets up two equations with two unknowns ✓states r & q

Q5 (2, 2 & 2 = 6 marks)

Consider the probability density function $f(x)=ax^3(x-5) \quad ,1\le x\le 5$ and zero for all other values of x.

a) Show that $a=\frac{-5}{776}$.

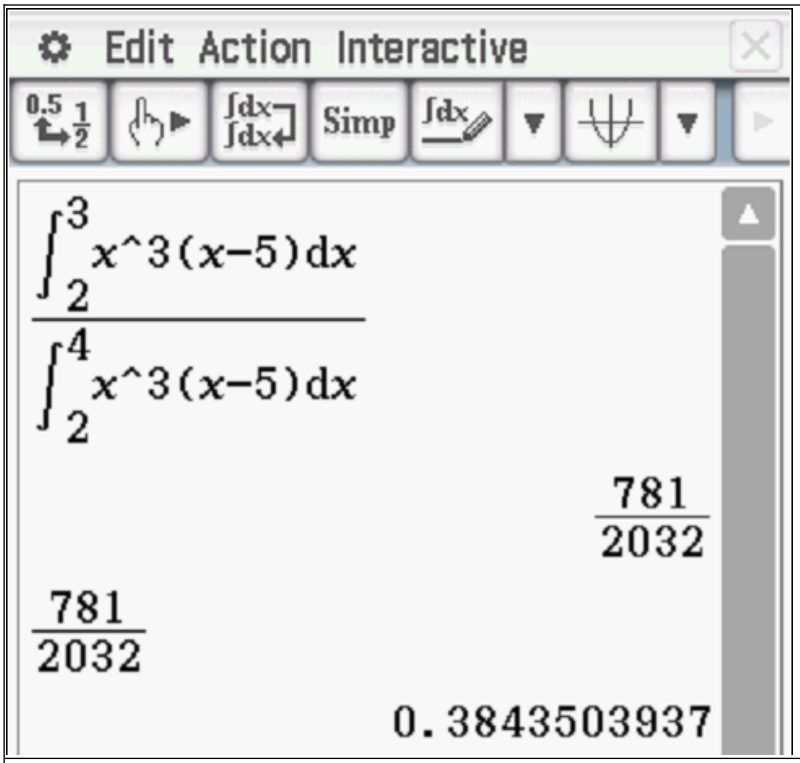
Solution



Specific behaviours
✓ sets up integral for total area =1 ✓ shows equation to solve for a

b) Determine the probability $\Pr(X \leq 3 \mid 2 \leq X \leq 4)$ for the above function.

Solution
$\Pr(X \leq 3 \mid 2 \leq X \leq 4) = \frac{\Pr(2 \leq X \leq 3)}{\Pr(2 \leq X \leq 4)}$

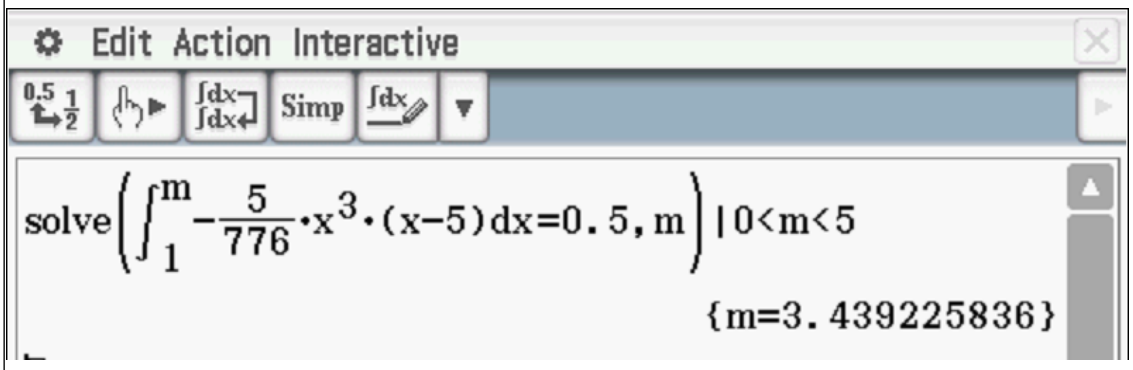


Specific behaviours

✓ numerator
✓ denominator

c) Determine the median.

Solution



Specific behaviours

✓ sets up equation for median
✓ states one median only

Q6 (2 & 5 = 7 marks)

a) Show **without the use of a classpad** how to $\frac{d}{dx}\left[5x\ln(3x+1)\right]$.

Solution
$\frac{d}{dx}\left[5x\ln(3x+1)\right] = 5x\frac{3}{3x+1} + 5\ln(3x+1)$
Specific behaviours
✓ uses product rule ✓ obtains correct expression

b) Using (a) above and **without the use of a classpad**, show how to evaluate $\int_0^1 3\ln(3x+1)dx$.

Hint-use $\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$

Solution
$\frac{d}{dx}\left[5x\ln(3x+1)\right] = 5x\frac{3}{3x+1} + 5\ln(3x+1)$ $\int_0^1 \frac{d}{dx}\left[5x\ln(3x+1)\right] = 5\int_0^1 \frac{3x}{3x+1}dx + 5\int_0^1 \ln(3x+1)dx$ $\left[5x\ln(3x+1)\right]_0^1 = 5\int_0^1 \left(1 - \frac{1}{3x+1}\right)dx + 5\int_0^1 \ln(3x+1)dx$ $5\ln 4 = 5\left[x - \frac{1}{3}\ln(3x+1)\right]_0^1 + 5\int_0^1 \ln(3x+1)dx$ $5\ln 4 = 5(1 - \frac{1}{3}\ln 4) + 5\int_0^1 \ln(3x+1)dx$ $3\ln 4 = 3(1 - \frac{1}{3}\ln 4) + 3\int_0^1 \ln(3x+1)dx$ $3\int_0^1 \ln(3x+1)dx = 3\ln 4 - 3 + \ln 4 = 4\ln 4 - 3$ **Note- zero marks if answer given without any working!**
Specific behaviours
✓ integrates expression from part a ✓ uses FTC ✓ changes $\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$ and shows integration of these two terms (or uses u=3x+1)du

✓ evaluates x=0 and x=1 showing both values for two terms after integration ✓changes factor to give required definite integral(no need to simplify)