

Rossmoyne Senior High School

Semester One Examination, 2015
Question/Answer Booklet

MATHEMATICS 3C
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures
In words _____
Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33%
Section Two: Calculator-assumed	12	12	100	100	66%
Total			150	100	

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the Year 12 Information Handbook 2015. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Additional working space

Question number: _____

- (a) What is the probability that a randomly selected loaf produced by the bakery
(i) weighs 450 g? (1 mark)

$$P(450 < X < 450) = 0$$

- (ii) is a standard loaf? (1 mark)

$$P(450 \leq X \leq 500) = 0.864$$

- (b) Determine the probability that a randomly selected standard loaf weighs less than 470 g.
(2 marks)

$$\begin{aligned}
 P(X < 470 | 450 < X < 500) \\
 &= \frac{P(450 < X < 470)}{P(450 < X < 500)} \\
 &= 0.3944 \\
 &= 0.8640 \\
 &= 0.4565
 \end{aligned}$$

- (c) In a batch of 250 loaves, how many would be expected to weigh less than a standard loaf? (2 marks)

$$P(X < 450) = 0.1056$$

$$250 \times 0.1056 = 26.4$$

≈ 26 loaves

- (d) The harmful effects of the pollutant are thought to become negligible after the concentration of pollutant first falls below 0.5 parts per million.

(i) Determine how many days, to the nearest day, it will take for this to occur.

$$t = 62.963$$

≈ 63 days

- (ii) At this time, determine the rate that the concentration of pollutant is changing. (2 marks)

$$\begin{aligned}\frac{dC}{dt} &= -kC \\ &= -0.0924(0.5) \\ &= -0.04605\end{aligned}$$

Decreasing at 0.04605 ppm/day.

- What is the most likely number of double tops that a packet of Cherry Tops will contain?

$$X = 3 | X \geq 3) = \frac{0.228511}{0.438621} = 0.521$$

- | | |
|---|---|
| <p>(3 marks)</p> <p>Determine the probability that a randomly chosen peck of Chern Tops contains exactly one defective top.</p> | <p>(1 mark)</p> <p>$P(\text{exactly one defective}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$</p> |
|---|---|

$$= \frac{1}{16} \times 0.15 \times (1 - 0.15)$$

$$= 2.4$$

$$= 16 \times 0.15$$

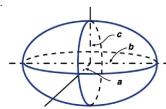
- (a) Is X a discrete or continuous random variable? Justify your answer.
 (2 marks)

$X \sim B(14, \frac{11}{15})$

- Determine the probability that, on any one day of the festival, five or more intervals will be less than seven minutes long.

(9 marks)

Question 10
A general ellipsoid has semi-axes lengths a , b and c , as shown in the diagram below and has volume given by



Consider the ellipsoid where the relationship between the semi-axes lengths is that b is three times a , and that the sum of a and c is 42 cm.

(a) Show that the volume of this ellipsoid is given by $168\pi a^2 \cdot 4\pi a^3$. (2 marks)

$$\begin{aligned} V &= \frac{4\pi abc}{3} \\ &= \frac{4\pi a(3a)(42-a)}{3} \\ &= 168\pi a^2 \cdot 4\pi a^3 \end{aligned}$$

(b) Use calculus to determine the dimensions of the ellipsoid that maximise its volume and state the maximum volume, rounded to three significant figures. (4 marks)

$$\begin{aligned} \frac{dV}{da} &= 336\pi a - 12\pi a^2 \\ 336\pi a - 12\pi a^2 &= 0 \Rightarrow a = 0, a = 28 \\ a &= 28 \\ b &= 84 \\ c &= 14 \\ V &= 43904\pi \\ &\approx 138000 \text{ cm}^3 \quad (3 \text{ sf}) \end{aligned}$$

See next page

(9 marks)

Question 17

(a) Records of a company that has a large workforce indicate that 35 percent of employees take sick leave during any given year.

If the records of five employees are selected at random from the previous year, what is the probability that

- (i) exactly four of the five took sick leave? (2 marks)

$$\begin{aligned} X &= \text{number from group of 5} \\ X &\sim B(5, 0.35) \\ P(X = 4) &= 0.0488 \end{aligned}$$

- (ii) fewer than three took sick leave? (2 marks)

$$P(X \leq 2) = 0.7648$$

- (b) Amongst the 20 management staff of the company, seven of them had taken sick leave during the previous year.

If five of these staff are selected at random, what is the probability that

- (i) two or less took sick leave? (3 marks)

$$\begin{aligned} Y &= \text{number from group of five staff taking sick leave} \\ P(Y \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\ &= \frac{C_0}{20} C_5 + \frac{C_1}{20} C_4 + \frac{C_2}{20} C_3 \\ &= \frac{1}{20} \frac{5!}{0!5!} + \frac{1}{20} \frac{5!}{1!4!} + \frac{1}{20} \frac{5!}{2!3!} \\ &= \frac{1287}{15504} + 6006 \\ &= \frac{12298}{15504} = 0.7932 \end{aligned}$$

- (ii) none of the staff took sick leave, given that two or less took sick leave? (2 marks)

$$\begin{aligned} P(Y = 0 | Y \leq 2) &= \frac{1287}{12298} = \frac{12298}{15504} \\ &= \frac{1287}{15504} = 0.1047 \end{aligned}$$

See next page

(9 marks)

Question 18

(3 marks)

Question 19

(2 marks)

Question 20

(2 marks)

Question 21

(2 marks)

Question 22

(2 marks)

Question 23

(2 marks)

Question 24

(2 marks)

Question 25

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Question 26

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