

Additional working space

Question number(s): _____



CHRIST CHURCH GRAMMAR SCHOOL

Place student identification here:

MATHEMATICS 3CMAT Semester One Examination 2010

Section One: Calculator-free Question/answer booklet

Time allowed for this section

Reading time before commencing work: 5 minutes
Working time for paper: 40 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section One, containing a formula sheet which may also be used for
Section Two

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One Calculator—free	7	7	40 minutes	35
Section Two Calculator—assumed	13	13	80 minutes	65
Total marks				100

Instructions to candidates

1. Answer the questions in the spaces provided.
2. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.

Question 20 [4 marks]

An aptitude test for surgeons requires a series of tasks to be performed quickly and efficiently. Results have shown that the times for the tests are normally distributed with a mean of 145 seconds and a standard deviation of 28 seconds.

- (a) In order to pass the test, it must be completed within 130 seconds. What percentage of surgeons pass the test?

[2 marks]

- (b) If 200 surgeons sit the test this month, how many are expected to record a time over 3 minutes?

[1 mark]

- (c) The slowest 20% of surgeons must sit an intense week long training program before they re-sit the test. What time must be beaten to ensure that a surgeon does not have to do the training program?

[1 mark]

END OF SECTION 2

See next page

Section One: Calculator-free 35 marks

This section has seven (7) questions. Attempt all questions.

Question 1 [3 marks]

Evaluate the following exactly:

$$\int_4^2 3x - 1 .dx$$

MATHEMATICS 3CMAT

Question 19 [6 marks]

A recent study by the company, Roof Cover, revealed that 6% of all roof insulations completed in 2009 were found to be defective, and further inspection and re-installation were required to ensure the roofs were safe.

(a) If twenty homes were chosen at random, what is the probability that:

(i) exactly three homes had to have their roof insulation re-installed, [1 mark]

(ii) at least three homes had to have their roof insulation re-installed, [1 mark]

(iii) the first three homes inspected had to have their roof insulation re-installed, [1 mark]

(b) How many homes need to be tested to ensure that the probability of having at least one home needing their insulation re-installed is at least 0.9? [1 mark]

(c) If fifty homes are chosen at random

(i) what is the mean number of homes that need their insulation re-installed? [1 mark]

(iii) what is the standard deviation for the number of homes that need their insulation re-installed? [1 mark]

If $f(x) = 2 + x$ and $g(x) = x^2$, then find the following:

(a) $(f \circ g)(x)$

(b) the range of $(f \circ g)(x)$

(c) if $h(x) = \sqrt{x - 6}$, find:
(i) the domain for $(h \circ f)(x)$ [2 marks]

(ii) $(h \circ f)(2)$ [1 mark]

Question 3 [5 marks]

Simplify the following:

$$\frac{x^3 - 5x^2 - 14x}{x^2 - 2x - 3} \div \frac{x^2 - 4}{2x - 6}$$

Question 4 [7 marks]

Differentiate each of the following with respect to x. Do not simplify after the first line of your solution.

(a) $y = -\frac{x}{4}$

[1 mark]

(b) $y = 2 + x^2 - \pi + e^3$

[1 mark]

(c) $y = \sqrt{x} - \frac{1}{x^2}$

[2 marks]

(d) $y = \frac{3\sqrt{2x+1}}{x^3 - 5}$

[3 marks]

See next page

Question 18 [6 marks]

In the morning Julie drives through three sets of traffic lights to go from home to work. The probability of getting a green light at each of the first two sets is 0.2, and 0.4 at the final set. The only way she can get to work in under ten minutes is if she gets a green at each set of lights. It may be assumed that each set of lights operates independently of the other sets of lights. Warning lights are set up before each intersection and Julie is a very safety conscious driver. Hence she will only proceed through the lights if they are green.

- (a) What is the probability that Julie gets exactly two green lights on the way to work?

[2 marks]

- (b) What is the probability that Julie will get to work in under ten minutes?

[1 mark]

- (c) What is the probability that at least one of the lights will be green?

[1 mark]

If Julie does not get one green light, the probability that her trip to work exceeds 15 minutes is 0.6. If she only gets one green light, the probability that her trip to work exceeds 15 minutes is 0.2. With 2 or more green lights, she will always arrive in under 15 minutes.

On one day she is slow to wake up and leaves home with only 15 minutes to spare.

- (d) What is the probability that she will be late to work?

[2 marks]

Question 17 [5 marks]

A box has 6 chocolates inside of it. There are two turkish delights and four coconut creams. Three chocolates are selected from the bag, one after the other, and eaten after each selection. Draw a probability distribution table, for the random variable X, the number of coconut creams selected in this process.

[4 marks]

Question 5 [5 marks]

A and B are events such that $P(\bar{A}) = 0.3$, $P(B) = 0.3$ and $P(A \cap B) = 0.9$. Find :

(a) $P(A \cap B)$

[2 marks]

(b) $P(A|B)$

[1 mark]

(c) Are the events A and B independent? Justify your answer.

[1 mark]

(d) Are the events A and B mutually exclusive? Justify your answer.

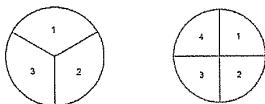
[1 mark]

(b) Find the mean value of X.

[1 mark]

Question 6 [4 marks]

Two spinners shown below are used in a game where the sum of the result on the two spinners is recorded.



- (a) List the sample space when both spinners have been spun.

[1 mark]

- (b) Determine the probability of each of the following:

- (i) the sum of the two spinners is 5,

[1 mark]

- (ii) the sum is even,

[1 mark]

- (iii) the value on the spinner shown on the left, is higher than the value of the other spinner.

[1 mark]

Question 16 [7 marks]

A scientific expedition to Central Australia consists of 8 geologists, 7 biologists and 5 archaeologists. A special unit is chosen to do some research on Uluru. In how many different ways can this unit be chosen if it is to consist of 3 geologists, 3 biologists and 2 archaeologists and

- (a) there are no other restrictions,

[1 mark]

- (b) Bevan, the biologist, has had some heated arguments with Geoff, the geologist, and they refuse to work with each other?

[2 marks]

What is the probability that

- (c) Bevan is selected in the special unit, with his wife Belinda, who is an archaeologist,

[2 marks]

- (d) Geoff is selected given that Bevan is not in the unit, but it does include Belinda?

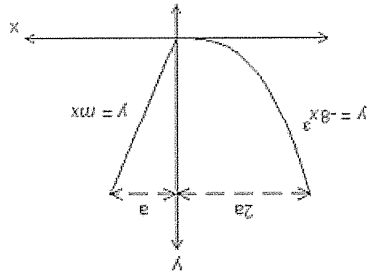
[2 marks]

Question 7 [6 marks]

See next page

Question 15 [6 marks]

The cross section of the blade of a paddle is shown below:



The blade is determined by the area enclosed by the curved edge $y = -8x^3$ and the straight edges, $y = mx$ and $y = 0.512$.

(a) Find the value of m and a .

[4 marks]

(b) Calculate the area of the blade (the area of the enclosed region).

[2 marks]

(d) $\int (e^x - e^{-x})^2 dx$

(d)

(c) $\int 4e^{x+1} dx$

(c)

(b) $\int (2x + 3)^5 dx$

(b)

(a) $\int \frac{3 + 2x}{x^3} dx$

(a)

Find the following indefinite integrals:

[2 marks]

[1 mark]

[1 mark]

[2 marks]

Additional working space

Question number(s): _____

Question 14 [6 marks]

- (a) If $y = kx^3$ for some constant k , use the incremental formula to estimate the percentage change in x required to yield a 15% increase in y .

[3 marks]

- (b) A company sells goods such that its revenue, in dollars, from selling x items is given by the equation,

$$R(x) = 5x(20x - x^2)$$

- (i) Determine the marginal revenue, when $x = 10$.

[2 marks]

- (ii) What does this represent?

[1 mark]

Question 13 [5 marks]

A new laptop which is worth \$2 350 will be worth just \$200 five years later. Assuming that an any instant the value of the laptop is depreciating at a rate proportional to the value,

(a) what is the value of the laptop three years after it was worth \$2 350,

[3 marks]

(b) what is the average annual rate of decrease over the four year period since the laptop was worth \$2 350?

[2 marks]

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MATHEMATICS 3CMAT Semester One Examination 2010

Section Two: Calculator-assumed Question/answer booklet

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 80 minutes

Material required/recommended for this section

To be provided by the supervisor

Question/answer booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: drawing instruments, templates, notes on ONE unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum

Council for this course.

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Question 12 [5 marks]

A curve contains the point (4, 6) and the gradient of the curve at any point is given by

$$\frac{dy}{dx} = 4 - x$$

- (a) Find the equation of the curve.

[2 marks]

- (b) How many solutions are there to the intersection of the graph in (a) and the graph of $y = 7$?
[1 mark]

- (c) If $x = 5p^2 + p$ in (a), find $\frac{dy}{dp}$.

[2 marks]

Question 11 [6 marks]

Consider the curve, $f(x) = \frac{x}{9} - 2x$

Determine the equation(s) of the tangent(s) to the curve at the point where its gradient is equal to -3.

65 marks

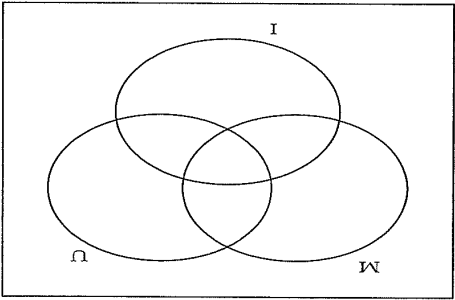
Section Two: Calculator-assumed

This section has thirteen (13) questions. Attempt all questions.

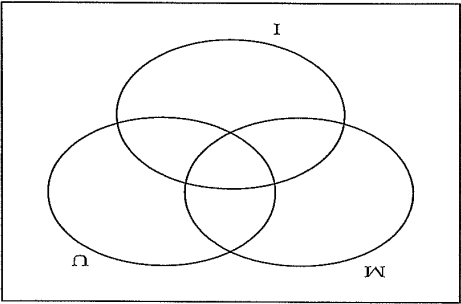
Question 8 [2 marks]

Shade the regions described in each of the Venn diagrams below:

(a) $M \cap (U \cup I)$



(b) $(M \cup I)'$



Question 9 [3 marks]

(a) Evaluate $\int_{-1}^3 (2x + 1).dx$

[1 mark]

(b) Comment on whether the following statement is true or false. The integral in (i) gives the area between the line $y = 2x + 1$ and the x axis, from $x = -1$ to $x = 3$.

[2 marks]



Question 10 [4 marks]

Organisers of the Barracks to the Beach swimming race are assuming that they will get 2 000 entrants if the entry fee is \$10. If the entry fee is increased by 50 cents, they predict they will lose 25 competitors. Before they take any entrants they must raise \$24 000 to cover costs for running the event. Let x represent each 50 cent increase.

(a) Show that the revenue can be expressed as $20\,000 + 750x - 12.5x^2$.

[2 marks]

(b) Find the expression for profit, in terms of x .

[1 mark]

(c) How many entries are required to achieve the maximum profit?

[1 mark]

