WESLEY COLLEGE
SOUTH PERTH

Semester Two Examination 2011

Question/Answer Booklet

MATHEMATICS

3C/3D

Section One:

Calculator-free

Student Name: Solutions

Time allowed for this section

Reading time before commencing work: Five (5) minutes

Working time for this section: Fifty (50) minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question 8 **(4 marks)**

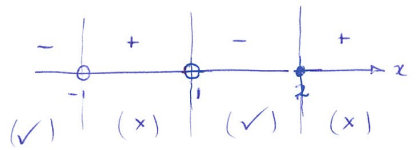
Find the value(s) of x for which

$$\frac{1}{x+1} \leq \frac{1}{x^2-1} \qquad x \neq \pm 1$$

$$\frac{1}{x+1} - \frac{1}{x^2-1} \leq 0$$

$$\frac{(x-1)-1}{(x+1)(x-1)} \leq 0$$

$$\frac{(x-2)}{(x+1)(x-1)} \leq 0 \quad \checkmark$$



(✓) (×) (✓) (×) ✓

$x < -1 \qquad 1 < x \leq 2$ ✓✓

[4]

Question 7 **(6 marks)**

A bag contains 40 beads of the same shape and size.

The ratio of red to green to blue beads is 1 : 3 : 4 and there are no beads of any other colour.

A bead is picked at random, its colour noted and the bead replaced in the bag. This is done ten times

(a) Find an expression for the probability that

(i) five are blue

X = n° of blue balls.
$X \sim Bin(10, \frac{1}{2})$ ✓

$P(X = 5) = {}^{10}C_5 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^5$ ✓

or $= {}^{10}C_5 \times \left(\frac{1}{2}\right)^{10}$

[2]

(ii) at least one is red

y = n° of red balls
$y \sim Bin(10, \frac{1}{8})$ ✓

$P(y \geq 1) = 1 - P(y = 0)$
$= 1 - {}^{10}C_0 \left(\frac{1}{8}\right)^0 \left(\frac{7}{8}\right)^{10}$
$= 1 - \left(\frac{7}{8}\right)^{10}$ ✓

[2]

The experiment is repeated, but this time a bead is picked out and replaced n times

(b) Find in the form $a^n < b$, where a and b are exact fractions, the condition which n must satisfy in order to have at least a 99% chance of picking out at least one red bead.
(You do not need to solve the inequality)

R = n° of reds in n picks. $R \sim Bin(n, \frac{1}{8})$

$P(R \geq 1) > 0.99 \Rightarrow P(R = 0) < 0.01$ ✓
$\left(\frac{7}{8}\right)^n < \frac{1}{100}$ ✓

[2]

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	40	33 1/3
Section Two: Calculator-assumed	13	13	100	80	66 2/3
				120	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2011*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

Section One: Calculator-free **(40 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

The working time for this section is 50 minutes.

Question 1 **(2 marks)**

Show, by counter-example, that the conjecture

$$a > b \quad \Rightarrow \quad (a+1)^2 > (b+1)^2$$

is not true for all integers a and b.

eg/ $a = 0$, $b = -3$ $\Big\}$ $a > b$ ✓

$(0+1)^2 = 1$

$(-3+1)^2 = 4$

$(a+1)^2 < (b+1)^2$ ✓

$\Rightarrow$ Statement not true

or

eg/ $a = 1$, $b = -4$ $\Big\}$ $a > b$

$(1+1)^2 = 4$

$(-4+1)^2 = 9$

$(a+1)^2 < (b+1)^2$

$\Rightarrow$ Statement not true. **[2]**

Question 6 **(4 marks)**

(a) Differentiate $\left(1-\frac{1}{x}\right)^3$

$\frac{d}{dx}\left(1-\frac{1}{x}\right)^3 = 3\left(1-\frac{1}{x}\right)^2 \cdot \frac{1}{x^2}$ ✓ use of chain rule

[1]

(b) The gradient function of a curve is given by $\frac{dy}{dx} = \frac{3}{x^2}\left(1-\frac{1}{x}\right)^2$

Find the equation of this curve given it passes through the point $(1, 0)$

$y = \int \frac{3}{x^2}\left(1-\frac{1}{x}\right)^2 dx$

$= \left(1-\frac{1}{x}\right)^3 + c$ ✓

$0 = (0)^3 + c \Rightarrow c = 0$ ✓

$\therefore y = \left(1-\frac{1}{x}\right)^3$ ✓

[3]

Question 5 **(7 marks)**

Given $f(x) = x^2 - 2, \quad x \in \mathbb{R}$, and $g(x) = \sqrt{2-x}, \quad x \leq 2$

(a) Find and simplify an expression for $f \circ g(x)$

$f \circ g(x) = \left(\sqrt{2-x}\right)^2 - 2$ ✓

$= 2 - x - 2$ ✓

$= -x.$ ✓

[2]

(b) State the range of $f \circ g(x)$

$y \geq -2.$

[1]

(c) State the domain and range of $g \circ f(x)$

$g \circ f(x) = \sqrt{2 - (x^2 - 2)}$

$= \sqrt{4 - x^2}$ ✓

Domain $-2 \leq x \leq 2$ ✓

Range $0 \leq y \leq 2$ ✓

[3]

(d) Find an unsimplified expression for $f \circ f(x)$

$f \circ f(x) = (x^2 - 2)^2 - 2$

[1]

Question 2 **(5 marks)**

The velocity *v(t)* in metres per second at time *t* seconds of an object moving in a straight line is given by: $v(t) = 3t^2 - 10t$ where $0 \leq t \leq 5$

(a) Find *x(t)*, the displacement at time *t* given $x(0) = 0$

$x(t) = \int (3t^2 - 10t)\,dt = t^3 - 5t^2 + c$ ✓

$x(0) = 0 \Rightarrow c = 0$

$\therefore \quad x(t) = t^3 - 5t^2$ ✓

[2]

(b) At what time, in the given interval, does the object return to its starting point?

$x(t) = t^3 - 5t^2 = 0$

$t^2(t-5) = 0$

$t = 0 \quad t = 5$ ie when $t = 5$ ✓

[1]

(c) At what time, in the given interval, is the object furthest from its starting point?

when $v(t) = 0$ ✓

$3t^2 - 10t = 0$

$t(3t - 10) = 0$

$\Rightarrow t = 0, \quad t = \frac{10}{3}.$ ie when $t = \frac{10}{3}$ ✓

[2]

Question 3 **(4 marks)**

Find the equation of the tangent to the curve $y=(x+3)^2 e^{-x}$ at the point with coordinates $(0,9)$.

$u = (x+3)^2 \qquad u' = 2(x+3)$ ✓

$v = e^{-x} \qquad v' = -e^{-x}$ ✓

$\frac{dy}{dx} = -e^{-x}(x+3)^2 + 2e^{-x}(x+3) \Big|_{x=0} = -1\times 9 + 2\times 3 = -3$ ✓

$y = mx + c$

$9 = 0 + c \Rightarrow c = 9$

$\therefore y = -3x + 9.$ ✓

[4]

Question 4 **(8 marks)**

(a) Find $\frac{dy}{dx}$;

(You do not need to perform more than the most obvious algebraic simplifications)

(i) $y = \frac{e^{\frac{x}{2}}}{(1-3x)^4}$

$u = e^{x/2} \qquad u' = \frac{1}{2}e^{x/2}$ ✓

$v = (1-3x)^4 \qquad v' = -12(1-3x)^3$ ✓

$\frac{dy}{dx} = \frac{\frac{1}{2}e^{x/2}(1-3x)^4 + 12e^{x/2}(1-3x)^3}{(1-3x)^8}$ ✓

[3]

(ii) $y = \int_0^{5x} \frac{t}{3\sqrt{1-t^2}}\,dt$

$\frac{dy}{dx} = 5 \cdot \frac{5x}{3\sqrt{1-25x^2}}$

✓ substitution
✓ $\times$ by $\frac{d}{dx}(5x)$.

[2]

(b) Evaluate $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$= \left[2e^{\sqrt{x}}\right]_0^1$

$= 2e - 2.$

$\frac{d}{dx}e^{\sqrt{x}} = \frac{1}{2\sqrt{x}}e^{\sqrt{x}}$

✓ $e^{\sqrt{x}}$ idea
✓ details
✓ result.

[3]