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SEMESTER ONE

REVISION 1

MATHEMATICS METHODS

UNIT 3

2016

SOLUTIONS

SECTION ONE

1. (7 marks)

(a) $y = x^2(2x - 1)$

$\frac{dy}{dx} = 2x(2x - 1) + 2(x^2)$

✓✓ -1/error (2)

$\frac{dy}{dx} = 6x^2 - 2x$

(b) $y = \frac{sin(2x)}{2x}$

$\frac{dy}{dx} = \frac{2(cos(2x)) \times 2x - 2(sin(2x))}{4x^2}$ ✓ ✓

$\frac{dy}{dx} = \frac{4x(cos(2x)) - 2(sin(2x))}{4x^2}$ ✓

(3)

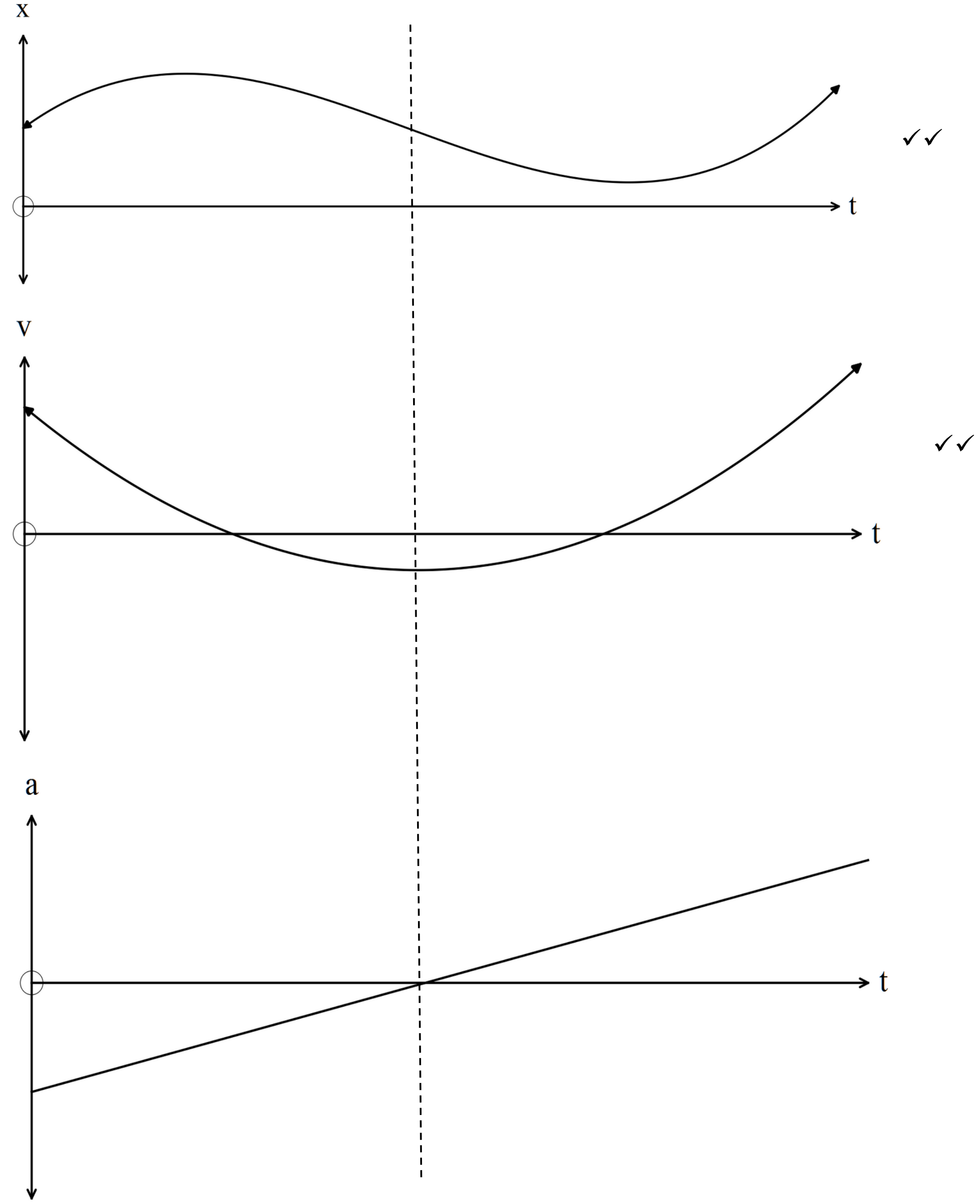
(c) $y = (x + e^x)^4$

$\frac{dy}{dx} = 4(x + e^x)^3 \times (1 + e^x)$ (2)

✓ ✓

2. (9 marks)

(a)



(b) The displacement graph is a cubic polynomial. ✓ (1)

(c) At $a(t)=0$ on the acceleration graph, there is a point of inflection on the displacement graph. ✓ (1)

(d) At $a(t)=0$, the velocity graph has a turning point. ✓

In this case the velocity goes from decreasing to increasing so the velocity graph has a minimum turning point. (1)

(e) At $v(t)=0$, then the particle changes direction on the displacement graph. The velocity goes from positive to negative or from negative to positive. ✓ (2)

On the displacement graph, the first time $v(t)=0$ the displacement had been increasing and the particle turned around and began decreasing.

The second time $v(t)=0$, the displacement had been decreasing and the particle turned around and began increasing.

3. (6 marks)

(a) (i) $\int\sqrt{2x+1}\,dx=\frac{2\sqrt{(2x+1)^3}}{3\times 2}+c=\frac{\sqrt{(2x+1)^3}}{3}+c$ ✓ ✓ (2)

(ii) $\int 1+x-e^{-x}\,dx=x+\frac{x^2}{2}+e^{-x}+c$ ✓ ✓ (2)

(b) $\frac{dy}{dx}=2x+3x^2-x^{1/2}$

$y=x^2+x^3-\frac{2x^{3/2}}{3}+c$

$(1,4)$ belongs to the function

$4=1+1-\frac{2}{3}+c$

$c=2\frac{2}{3}$

$y=x^2+x^3-\frac{2x^{3/2}}{3}+\frac{8}{3}$

(2)

4. (7 marks)

(a) $\int_2^4 (x^2-2x+3)dx=\left[\frac{x^3}{3}-x^2+3x\right]_2^4$

$=\left(\frac{64}{3}-16+12\right)-\left(\frac{8}{3}-4+6\right)$

$=\frac{56}{3}-4-2$

$=12\frac{2}{3}$

(b) $\int_{\pi/2}^{\pi}\left(\sin(x)-\cos(x)\right)dx=\left[-\cos(x)-\sin(x)\right]_{\pi/2}^{\pi}$

$$=-\left(\left(\cos(\pi)+\sin(\pi)\right)-\left(\cos\left(\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{2}\right)\right)\right)$$
$$=-\left(-1+0-(0+1)\right)$$
$$=2$$

(2)

(c) $\int_0^1 \sqrt{e^x}\,dx=\int_0^1 e^{x/2}\,dx=2\left[e^{x/2}\right]_0^1=2\left(e^{1/2}-1\right)=2\left(\sqrt{e}-1\right)$ (2)

✓ ✓

5. (5 marks)

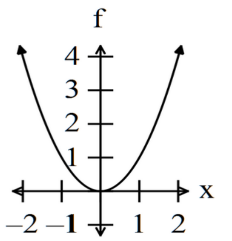
(a) (i) $\int_{2}^{2}\left(x^3\right)dx=0$ ✓ (1)

(ii) $\int_0^2\left(x^3\right)dx=\frac{1}{4}\left[x^4\right]_0^2=4$ ✓ (1)

(iii) Area = 2 $\int_0^2\left(x^3\right)dx=2\times\frac{1}{4}\left[x^4\right]_0^2=8$ units^2 ✓ (1)

(b) (i) $2\int_0^2 x^2dx=\int_{2}^{2}x^2dx$

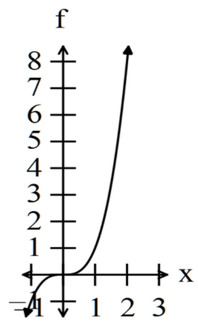
True as the function is
(a) symmetrical about the y axis
(b) not below the x axis.



(1)

(ii) $\int_0^1 x^3dx=\int_1^2 x^3dx$

False as the graph has different y values on the different domain. The area below the curve is different on the same base as the height of the curve changes.



(1)

6. (8 marks)

(a) If $F(x) = x^3 - x^2$

(i) $F'(x) = 3x^2 - 2x$ ✓ (1)

(ii) $\int_0^p F'(x)dx = \left[x^3 - x^2\right]_0^p = p^3 - p^2$ ✓✓ (2)

(b) $F(x) = \int^x t^3\,dt$

$F'(x) = x^3$ ✓✓ (2)

(c) $\frac{d}{dx}\left(\int^{3x} cos(2y)dy\right) = 3cos(6x)$ (3)

✓ ✓ ✓

7. (8 marks)

Given $f(x) = e^x$, $g(x) = cos(x)$ and $h(x) = -x$

(a) (i) $y = h(g(x)) = h(cos(x)) = -cos(x)$ ✓ (1)

(ii) show $\frac{d}{dx}(h(g(x))) = g\left(\frac{\pi}{2} - x\right)$

$\frac{d}{dx}(h(g(x))) = -(-sin(x)) = sin(x)$ ✓

$g\left(\frac{\pi}{2} - x\right) = cos\left(\frac{\pi}{2} - x\right) = sin(x)$ ✓

$\therefore \frac{d}{dx}(h(g(x))) = g\left(\frac{\pi}{2} - x\right)$ (2)

(b) (i) $y = f(h(x)) = f(-x) = e^{-x}$ ✓ (1)

(ii) $\frac{d}{dx}(f(h(x))) = -e^{-x}$ ✓✓ (2)

(c) $g(f(x)) = g(e^x) = cos(e^x)$ ✓

$g(f(0)) = cos(e^0) = cos(1)$ ✓ (2)

END OF SECTION ONE

SECTION TWO

8. (4 marks)

$V = x^3$

$\frac{dV}{dx} = 3x^2$

$\delta V \approx \frac{dV}{dx} \times \delta x$

$\delta V \approx 3x^2 \times \delta x$

$At\ \delta x = -0.1\ cm,\ x = 1\ cm$

$\delta V \approx 3 \times 1^2 \times (-0.1) = -0.3$

$\delta V \approx -0.3 cm^3$

The decrease in volume when the side has melted to 9 mm is 0.3 cm^3 (4)

9. (10 marks)

(a) (i) Not a probability density function as the total is only 0.9. ✓ (1)

(ii) Not a probability density function as you can't have a negative probability. ✓ (1)

(ii) **Is** a probability density function as the total is 1. ✓ (1)

(iv) Not a probability density function as you can't have a probability greater than 1. ✓ (1)

(b) (i)

y	1	2	3	4	5	6
$P(Y = y)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(1)

(ii) $P(y \le 4) = \frac{4}{6}$ ✓ (1)

(iii) $E(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = \frac{21}{6} = 3.5$ ✓

$Var(X) = E(X^2) - (E(X))^2$

$E(X^2) = 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6} = 15\frac{1}{6}$

$Var(X) = 15\frac{1}{6} - 3.5^2 = 2.91\dot{6}$

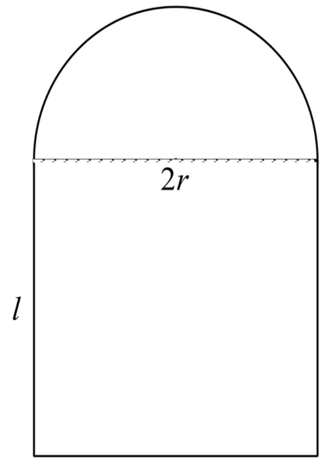
$Sd(X) = 1.707825$

$Sd(X) \approx 1.7$

(4)

10. (8 marks)

(a) $P = \pi r + 2r + 2l$ (1)



(b) $A = \dfrac{\pi r^2}{2} + 2rl$

But $P = 4$

$4 = \pi r + 2r + 2l \Rightarrow l = \dfrac{4 - \pi r - 2r}{2}$

$\therefore A = \dfrac{\pi r^2}{2} + 2r\left(\dfrac{4 - \pi r - 2r}{2}\right)$

$A = \dfrac{\pi r^2}{2} + 4r - \pi r^2 - 2r^2$

$A = 4r - \dfrac{\pi r^2}{2} - 2r^2$

(2)

(c) $A = 4r - \dfrac{\pi r^2}{2} - 2r^2$

Maximum area occurs when $A'(r) = 0$ and $A''(r) < 0$

$A'(r) = 4 - \pi r - 4r$

$A''(r) = -\pi - 4$

If $A'(r) = 0$ then $0 = 4 - \pi r - 4r \Rightarrow r = \dfrac{4}{\pi + 4}$

$r = 0.5600991535$

$r \approx 0.56$

$A''\left(\dfrac{4}{\pi + 4}\right) = -\pi - 4 < 0$ so maximum

At $r = 0.56,\ A = 1.1202\, m^2$

The maximum area of the window is 1.12 m^2. (5)

11. (6 marks)

(a) $t = 2s$ ✓ (1)

(b) $a > 0 \ \forall t \ st \ \ t \geq 0.$

$x = (t-2)^2 + 2$

$v = \dfrac{dx}{dt} = 2(t-2)$

$a = \dfrac{d^2x}{dt^2} = 2$ which is always positive!

(2)

(c) $v = 2(t-2)$

At $t = 3,\ v = 2$ m/s. ✓ (1)

(d) Distance travelled for $1 \leq t \leq 4 = ?$

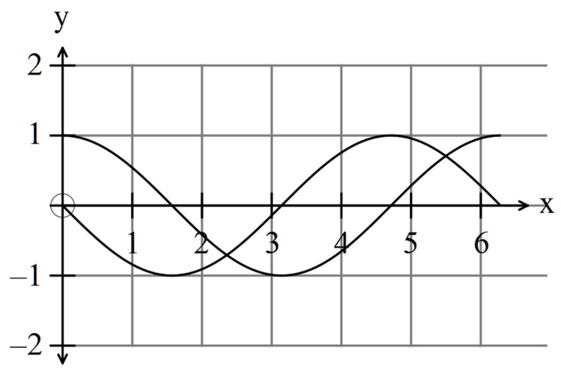
$x(1) = 3,\ x(2) = 2,\ x(4) = 6$

Distance travelled $= 1 + 4 = 5m$

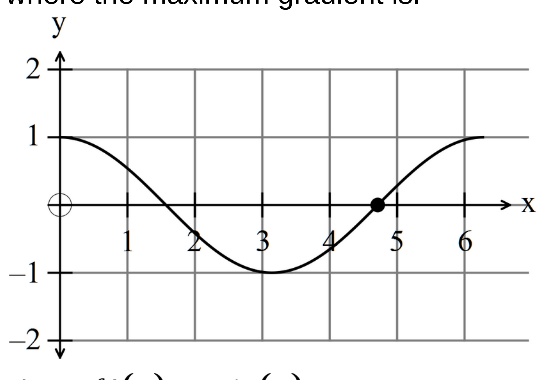
(2)

12. (7 marks)

(a) (i) (3)



(ii) Point shows where the maximum gradient is. (1)



(b) If $f(x) = cos(x)$, then $f'(x) = -sin(x)$ ✓ (1)

(c) $\lim\limits_{h \to 0}\left(\dfrac{e^h - 1}{h}\right) = 1$ ✓✓ (2)

13. (5 marks)

(a) $\int_0^{\pi/8} \left(\frac{sin(2x)}{1+2x} \right) dx = 0.0981$ ✓✓ (2)

(b) (i) $f(x) = e^x \times sin(x) \Rightarrow f'(x) = e^x \times sin(x) + e^x \times cos(x)$

$= e^x (sin(x) + cos(x))$

(1)

(ii) Hence

$y = \int e^x (sin(x) + cos(x)) dx = e^x \times sin(x) + c$

$At\ (0,1)\ 1 = 0 + c$

$\therefore\ y = e^x sin(x) + 1$

(2)

14. (9 marks)

(a) $v = -3t + 6\, m/s$

$x = \int (-3t + 6)\, dt$

$x = -\frac{3t^2}{2} + 6t + c$

$At\ t = 0, x = 2 \Rightarrow c = 2$

$x = -\frac{3t^2}{2} + 6t + 2$

(2)

(b) $a = -3\ m/s^2$ ✓ (1)

(c) $2 = -\frac{3t^2}{2} + 6t + 2 \Rightarrow t = 4\ s$ (2)

✓ ✓

(d) Changes direction when $v = 0\ i.e.\ at\ t = 2\ s$ ✓

$x = 8\ m$ ✓ (2)

(e) $At\ t = 2\ s, x = 8$

$At\ t = 3\ s, x = 6.5.$ ✓

The distance travelled in the second is 1.5 m. ✓ (2)

15. (13 marks)

(a) (i) $\int_0^{\pi} sin(x)dx = 2$ ✓ (1)

(ii) $\int_0^{\pi/2} sin(x)dx = 1$ An estimate cab be made because the graph is symmetrical. ✓ (1)

(iii) Area = 8 $units^2$ ✓ (1)

(iv) $\int_0^{4\pi} sin(x)dx = 0$ ✓ (1)

(b) (i) Estimate from below

$Area = 1\times0.5 + 1\times0.33 + 1\times0.25$
$= 1.08$ ✓✓

Estimate from above

$Area = 1\times1 + 1\times0.5 + 1\times0.33$
$= 1.83$ ✓✓ (4)

(ii) $\int_1^4 \frac{1}{x} dx = \int_1^4 x^{-1} dx = \left[\frac{x^0}{0}\right]_1^4$ ✓✓

Conventional methods do not work as you cannot divide by zero. ✓
(3)

(iii) $\int_1^4 \frac{1}{x} dx = 1.386 \quad (3dp)$ ✓✓ (2)

16. (8 marks)

(a) $2000 \quad t = 0 \quad P = 400$
$2008 \quad t = 8 \quad P = 550$
$550 = 400(r)^8$
$r = 1.040609622$

The annual rate of increase of the population of numbats was 4.06%. (3)

(b) $2016 \quad t = 16 \quad P = ?$
$P = 400(1.040609622)^{16}$
$P = 756.25$
The expected population in 2016 is 756 numbats. (2)

(c) $2016 \quad t = 0 \quad P = 756$
$2020 \quad t = 4 \quad P = 780$
$780 = 756(r)^4$
$r = 1.0078$
The rate of increase has dropped from 4.05% to 0.78%.
It is possible that predators had found a way in. (3)

17. (10 marks)

(a) (i)

House	Hawke	Howard	Gillard	Turnbull
P(House)	0.15	0.25	0.35	0.25

✓

(2)

(ii) 0.15 + 0.35 = 0.5 ✓ ✓ (2)

(b) (i)

House	Hawke	Howard	Gillard	Turnbull
P(House)	0.25	0.25	0.25	0.25

✓

(2)

(ii) A subset of a parent population with elements selected at random can produce a different distribution to that of the parent population. ✓ ✓

(2)

(iii) P(the student is assigned to Howard or Hawke house) = 0.5 ✓ (1)

(iv) P(the student is not assigned to Turnball house) = 0.75 ✓ (1)

18. (9 marks)

(a) 0.00243 ✓ (1)

(b) $P(x \geq 3) = 0.83692$ ✓✓ (2)

(c) $n = 4 \quad P(x = 2) = 0.2646$ ✓✓ (2)

(d) $\mu = np = 5 \times 0.7 = 3.5$ ✓✓ (2)

(e) $\delta = \sqrt{npq} = \sqrt{5 \times 0.7 \times 0.3} = 1.025$ ✓✓ (2)

19. (6 marks)

(a)

x	HH	HT or TH	TT
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

✓✓

(2)

(b) (i) 0.0985 ✓ (1)

(ii) 0.8363 ✓ (1)

(iii) 0.9937 ✓ ✓ (2)

20. (5 marks)

Four Apple MacBooks Three ASUS

$p = 0.7$ $p = 0.8$ ✓

$P(x = 3) \cap P(y = 3)$

$= 0.4116 \times 0.512$ ✓✓

$= 0.2107392$

The probability that 3 Apple MacBooks and 3 ASUS are being used is 0.21 (5)

END OF SECTION TWO