John Wollaston Anglican Community School

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS
METHODS
UNIT 1
Section Two:
Calculator-assumed

SOLUTIONS

Student number:	In figures							
	In words	_____

Your name   _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

Question number: _____

Section Two: Calculator-assumed   **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9   **(6 marks)**

(a)   The points A and B have coordinates $(4, -6)$ and $(5, 8)$ respectively. If B is the midpoint of A and C, determine the coordinates of C.   (3 marks)

Solution
If $C(x, y)$ then
$\dfrac{4+x}{2} = 5 \Rightarrow x = 6$ and $\dfrac{-6+y}{2} = 8 \Rightarrow y = 22$
$C(6, 22)$

Specific behaviours
✓ writes equations for midpoints
✓ x-coordinate of C
✓ y-coordinate of C

(b)   The points D and E have coordinates $(5p, -q)$ and $(2q, 3p)$ respectively, where p and q are constants. Determine the value of p and the value of q if the midpoint of D and E is at $(21, 17)$.   (3 marks)

Solution
$\dfrac{5p + 2q}{2} = 21$ and $\dfrac{-q + 3p}{2} = 17$
Solve simultaneously CAS to get $p = 10, q = -4$

Specific behaviours
✓ equations for both coordinates of midpoint
✓ value of p
✓ value of q

Supplementary page

Question number: _____

See next page

Question 10   **(8 marks)**

A positive integer less than 10 is chosen at random.

The outcome sets for events B, C and D are $B = \{1, 4, 9\}$, $C = \{1, 6, 9\}$ and $D = \{5, 6, 7\}$.

(a)   List the following sets:

     (i) $C \cap D$.   (1 mark)

Solution
$C \cap D = \{6\}$
Specific behaviours
✓ correct set

     (ii) $B \cup C \cup D$.   (2 marks)

Solution
$B \cup C \cup D = \{1, 4, 5, 6, 7, 9\}$
Specific behaviours
✓ correct set
✓ uses curly braces to define set

     (iii) $(B' \cap C')'$.   (1 mark)

Solution
$(B' \cap C')' = B \cup C = \{1, 4, 6, 9\}$
Specific behaviours
✓ correct set

(b)   Determine

     (i) $n(C \cap D')$.   (1 mark)

Solution
$C \cap D' = \{1, 9\} \Rightarrow n(C \cap D') = 2$
Specific behaviours
✓ correct number

     (ii) $P(B \cap D)$.   (1 mark)

Solution
$B \cap D = \{\,\} \Rightarrow P(B \cap D) = 0$
Specific behaviours
✓ correct probability

     (iii) $P(B' | (C \cup D))$.   (2 marks)

Solution
$P = \dfrac{3}{5}$
Specific behaviours
✓ denominator
✓ numerator

Supplementary page

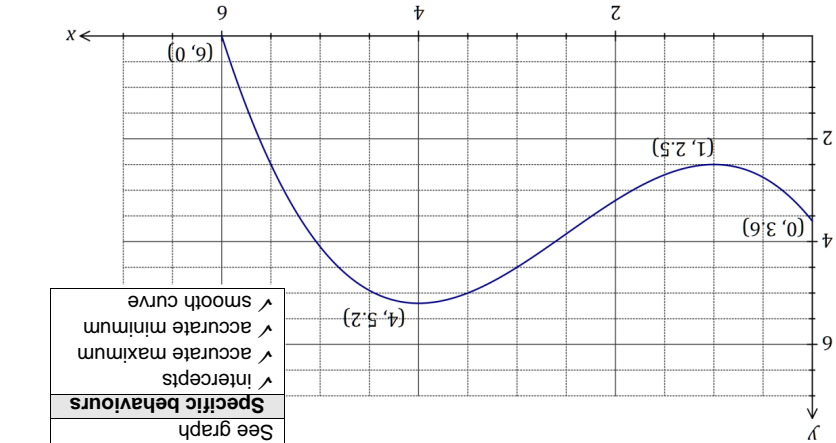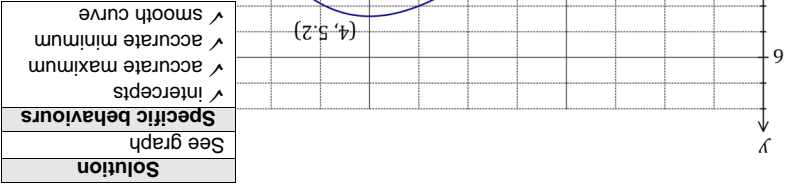Question number: _____

Question 11   **(8 marks)**

Question number: _____

In an experiment, the sound intensity, S, can be modelled by $S(x) = 3.6 - 2.4x + 1.5x^2 - 0.2x^3$, where x is the distance from the sound source in metres and $0 \leq x \leq 6$.

(a)   Determine S when $x = 3$.   (1 mark)

Solution
$S(3) = 4.5$

Specific behaviours
✓ correct value

(b)   Draw the graph of $y = S(x)$ on the axes below.   (4 marks)

Solution
See graph

Specific behaviours
✓ intercepts
✓ accurate maximum
✓ accurate minimum
✓ smooth curve

Graph showing points $(0, 3.6)$, $(1, 2.5)$, $(4, 5.2)$, $(6, 0)$

(c)   Determine the equation of the straight line L that passes through the x-intercept and the y-intercept of the graph of $y = S(x)$.   (2 marks)

Solution
$(0, 3.6)$ & $(6, 0) \Rightarrow m = -3.6 \div 6 = -0.6$
$y = -0.6x + 3.6$

Specific behaviours
✓ gradient
✓ y-intercept and equation

(d)   Determine the coordinates of the point of intersection of L with the graph of $y = S(x)$ where $x > 0$ and $y > 0$.   (1 mark)

Solution
$(1.5, 2.7)$

Specific behaviours
✓ correct coordinates

Question 12   **(8 marks)**

A random sample of 121 passengers arriving at an airport were asked to complete a brief survey. They were asked to categorise their main place of residence as Australia or overseas and the main purpose of their travel as work, holiday or other. It was found that

- half of the 84 passengers who resided overseas were on holiday
- 14 passengers were on holiday and resided in Australia
- of the 27 who were travelling for other reasons, 11 more resided overseas than in Australia.

(a)   Use the above information to complete the two-way table below.   (3 marks)

	Work	Holiday	Other	Total
Australia	15	14	8	37
Overseas	23	42	19	84
Total	38	56	27	121

Solution
See table
Specific behaviours
✓ holiday column, ✓ other column, ✓ table correct

(b)   If one passenger was selected at random from those surveyed, determine the probability that

(i)   the main purpose of their travel was work.   (1 mark)

Solution
$\dfrac{38}{121} \approx 0.314$
Specific behaviours
✓ correct probability

(ii)   they resided overseas, given that the main purpose of their travel was work.   (1 mark)

Solution
$\dfrac{23}{38} \approx 0.605$
Specific behaviours
✓ correct probability

(iii)   the main purpose of their travel was work, given that they resided in Australia.   (1 mark)

Solution
$\dfrac{15}{37} \approx 0.405$
Specific behaviours
✓ correct probability

(c)   Explain whether the survey indicates that purpose of travel appears to be independent of main place of residence for these passengers.   (2 marks)

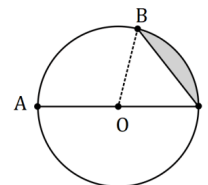| Solution |
|---|
| Purpose of travel is NOT independent of residence, as $P(T_W) = 0.314$ and $P(T_W|R_A) = 0.405$ but for independence these should be the same. |
| **Specific behaviours** |
| ✓ indicates not independent |
| ✓ reasoning compares appropriate probabilities |

Question 21   **(8 marks)**

(a)   The circle shown has centre O and diameter AC of length 50 cm. Determine the shaded area given that $2 \times \angle AOB = 3 \times \angle BOC$.   (4 marks)

Solution
$\angle AOB = \dfrac{3}{2} \angle BOC$
$\angle BOC + \dfrac{3}{2} \angle BOC = \pi \Rightarrow \angle BOC = \dfrac{2\pi}{5} = 72°$
$A = \dfrac{1}{2}(25)^2 \left(\dfrac{2\pi}{5} - \sin \dfrac{2\pi}{5} \right)$
$= 95.5 \text{ cm}^2$
Specific behaviours
✓ equation using angles
✓ correct angle for segment
✓ substitutes correctly into formula
✓ correct area

(b)   A sector of a circle has a perimeter of 112 cm and an area of 735 cm². Determine the radius of the circle.   (4 marks)

Solution
$2r + r\theta = 112$
$\dfrac{1}{2}r^2\theta = 735$
Solving simultaneously gives
$r = 21, \theta = \dfrac{10}{3}$ or $r = 35, \theta = \dfrac{6}{5}$
Hence $r = 21$ or $r = 35$ cm
Specific behaviours
✓ equation for perimeter
✓ equation for area
✓ solution of equations
✓ states both values of r

End of questions

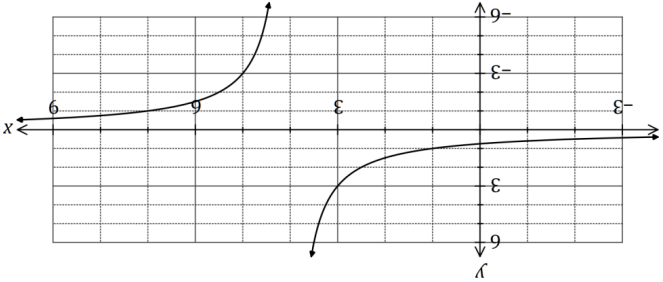## Question 13 (8 marks)

The graph of $y = f(x)$ is shown below where $f(x) = \dfrac{c}{x-a}$.

(a) State the value of the constant a and the value of the constant c.   (3 marks)

Solution
$a = 4$
Using (3,3) ⇒ 3 = c ÷ 1 ⇒ c = 3

Specific behaviours
✓ value of a
✓ uses point on curve
✓ value of c

(b) The hyperbola shown above has two asymptotes. State their equations.   (2 marks)

Solution
$x = 4$, $y = 0$

Specific behaviours
✓ vertical asymptote
✓ horizontal asymptote

(c) Describe how to transform the graph of $y = f(x)$ to obtain the graph of $y = f(x-3)$ and state the domain and range of the transformed function.   (3 marks)

Solution
Translate the graph 3 unit to the right.
Domain: $x \neq 7$
Range: $y \neq 0$

Specific behaviours
✓ transformation
✓ domain
✓ range

Question 20 (8 marks)

A shelf held a collection of 18 different books, of which 7 were cookbooks, 3 were dictionaries and the rest were travel guides.

A random selection of 5 books is to be made from the shelf.

(a) Determine the number of ways

(i) this can be done.   (2 marks)

Solution
$\binom{18}{5} = 8\,568$

Specific behaviours
✓ indicates use of combinations
✓ correct number

(ii) a selection can be made that will not contain any dictionaries.   (2 marks)

Solution
$\binom{15}{5} \times \binom{3}{0} = 3\,003$

Specific behaviours
✓ correct number to choose from
✓ correct number

(b) Determine the probability that

(i) the selection will only contain cookbooks.   (2 marks)

Solution
$\binom{7}{5} \times \binom{3}{0} \times \binom{8}{0} = 21$
$p = \dfrac{21}{8568} = \dfrac{1}{408} \approx 0.002245$

Specific behaviours
✓ correct number for selection
✓ correct probability (no need to simplify)

(ii) the selection will contain exactly one cookbook given that it does not contain any dictionaries.   (2 marks)

Solution
$\binom{7}{1} \times \binom{8}{4} \times \binom{3}{0} = 490$
$p = \dfrac{490}{3003} = \dfrac{70}{429} \approx 0.16317$

Specific behaviours
✓ correct number for selection
✓ correct probability (no need to simplify)

Question 14   **(8 marks)**

Two events, A and B, have probabilities $P(A) = 0.4$ and $P(B) = 0.65$.

(a)   Determine $P(A \cap B)$ in each of the following cases:

    (i) A and B are independent.   **(1 mark)**

Solution
$P(A) \times P(B) = 0.4 \times 0.65$
$= 0.26 = \dfrac{13}{50}$
Specific behaviours
✓ correct probability

    (ii) $P(A \cup B) = 0.8$.   **(2 marks)**

Solution
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 0.4 + 0.65 - 0.8$
$= 0.25 = \dfrac{1}{4}$
Specific behaviours
✓ indicates use of appropriate rule
✓ correct probability

    (iii) $P\big(A|(A \cup B)\big) = \dfrac{4}{9}$.   **(3 marks)**

Solution
$x = P(A \cap B)$
$P(A \cup B) = 0.4 - x + 0.65 - x = 1.05 - 2x$
$P\big(A|(A \cup B)\big) = \dfrac{0.4}{1.05 - x} = \dfrac{4}{9}$
$x = P(A \cap B) = 0.15 = \dfrac{3}{20}$
Specific behaviours
✓ indicates use of appropriate rules
✓ forms equation
✓ correct probability

(b)   Is it possible that A and B are mutually exclusive events? Explain your answer.   **(2 marks)**

Solution
No. Since $P(A \cap B) = 0$ for mutually exclusive events, then $P(A \cup B) = P(A) + P(B) = 1.05 > 1$ - impossible.
Specific behaviours
✓ states no
✓ explains using $P(A \cap B) = 0$

Question 19   **(8 marks)**

(a)   The equation of the axis of symmetry for the graph of $y = 3x^2 + 6x + 7$ is $x = k$.
Determine the value of k, using a method that does not refer to the graph of the parabola.   **(2 marks)**

Solution	
$x = -\dfrac{6}{2 \times 3} = -1$	$y = 3(x + 1)^2 + c$
$k = -1$	$\therefore k = -1$
Specific behaviours	
✓ uses $x = -b \div (2a)$ or partially completes the square	
✓ value of k	

(b)   A parabola with equation $y = ax^2 + bx + c$ has a turning point at $(6, -5)$ and passes through the point $(-2, -37)$. Determine the value of a, the value of b and the value of c.   **(3 marks)**

Solution
$y = a(x - 6)^2 - 5$
$-37 = a(-2 - 6)^2 - 5 \Rightarrow a = -0.5$
$y = -0.5(x - 6)^2 - 5$
$= -0.5x^2 + 6x - 23$
$a = -0.5, \quad b = 6, \quad c = -23$
Specific behaviours
✓ correctly writes in turning point form
✓ solves for a using point
✓ expands and states all values

(c)   Determine the value of the discriminant for the quadratic equation $16x^2 - 24x + 9 = 0$ and use it to explain how many solutions the equation $(x + 1)(16x^2 - 24x + 9) = 0$ will have.   **(3 marks)**

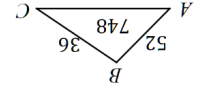| Solution |
|---|
| $d = (-24)^2 - 4(16)(9) = 0$ |
| When $d = 0$, quadratic will have one solution. |
| Hence equation will have two solutions - one from linear factor and one from quadratic factor. |
| **Specific behaviours** |
| ✓ value of discriminant |
| ✓ uses discriminant to say quadratic will have one solution |
| ✓ explains why equation has two solutions |

Question 15   (7 marks)

An **obtuse** angled triangle ABC has $a = 36$ cm, $c = 52$ cm and an area of 748 cm².

(a) Sketch a triangle to show this information.   (1 mark)

Solution

Specific behaviours
✓ triangle with all information

(b) Determine the size of ∠B.   (2 marks)

Solution
$748 = \frac{1}{2}(52)(36)\sin B$
$B = 126.95 \approx 127°$

Specific behaviours
✓ substitutes into area equation
✓ correct (obtuse) angle

(c) Show that $b \approx 79$ cm.   (2 marks)

Solution
$b^2 = 52^2 + 36^2 - 2(52)(36)\cos 126.95$
$b = 79.06 \approx 79$ cm

Specific behaviours
✓ uses appropriate equation that includes b
✓ substitutes correctly and solves to at least 1 dp

(d) Show that ∠$C \approx 32°$.   (2 marks)

Solution
$\dfrac{79.06}{\sin 126.95} = \dfrac{52}{\sin C}$
$\angle C = 31.71 \approx 32°$

Specific behaviours
✓ uses appropriate equation that includes C
✓ substitutes correctly and solves to at least 1 dp

Question 18   (6 marks)

Let $a = \sin 50°$ and $b = \cos 100°$.

Give your answers to the following in terms of a and/or b.

(a) Write down an expression for

(i) $\sin 130°$.   (1 mark)

Solution
$\sin 130° = \sin 50° = a$

Specific behaviours
✓ correct expression

(ii) $\cos 80°$.   (1 mark)

Solution
$\cos 80° = -\cos 100° = -b$

Specific behaviours
✓ correct expression

(b) Determine an expression for $\cos 130°$.   (3 marks)

Solution
$\cos^2 130° + \sin^2 130° = 1$
$\cos^2 130° = 1 - a^2$
$\cos 130° = \mp\sqrt{1 - a^2}$
but $\cos 130° < 0 \Rightarrow \cos 130° = -\sqrt{1 - a^2}$

Specific behaviours
✓ indicates use of $\sin^2 \theta + \cos^2 \theta = 1$
✓ expression for $\cos^2 130°$
✓ correct expression

(c) Determine an expression for tan 130°.   (1 mark)

Solution
$\tan 130° = \dfrac{\sin 130°}{\cos 130°} = \dfrac{a}{-\sqrt{1 - a^2}}$

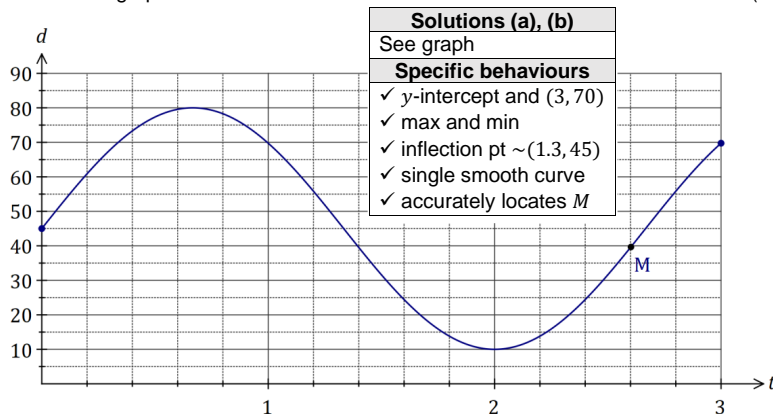| Specific behaviours |
| --- |
| ✓ correct expression |

Question 16   **(8 marks)**

A small weight, attached to the bottom of a spring, oscillated up and down. The distance, d cm, of the weight from the top of the spring after t seconds can be modelled by

$$d = 45 + 35 \sin\left(\frac{3\pi t}{4}\right)$$

(a)   Sketch the graph on the axes below for $0 \le t \le 3$.   (4 marks)

Solutions (a), (b)
See graph
Specific behaviours
✓ y-intercept and $(3, 70)$
✓ max and min
✓ inflection pt $\sim(1.3, 45)$
✓ single smooth curve
✓ accurately locates M

(b)   Mark on your graph point M, where the weight is 40 cm from the top of the spring and moving downwards.   (1 mark)

(c)   Determine

(i)   the maximum distance of the weight from the top of the spring.   (1 mark)

Solution
80 cm
Specific behaviours
✓ correct distance

(ii)   the time taken for the weight to first return to its initial position.   (1 mark)

Solution
$t = \frac{4}{3} = 1.\overline{3}$ s
Specific behaviours
✓ correct time

(iii)   the distance moved by the weight between $t = 1$ and $t = 2$.   (1 mark)

Solution
$d(1) - d(2) = 69.75 - 10$
$= 59.75$ cm
Specific behaviours
✓ correct distance

Question 17   **(7 marks)**

An examination consisted of two papers, one of which was much harder than the other. 15% of candidates gained a distinction in the first paper (event A) and 5% gained a distinction in the second paper (event B) whilst 82% did not gain a distinction in either paper.

(a)   Determine the probability that a randomly chosen candidate

(i)   gained a distinction in both papers.   (2 marks)

Solution
$P(A \cup B) = 1 - 0.82 = 0.18$
$P(A \cap B) = 0.15 + 0.05 - 0.18 = 0.02$
Specific behaviours
✓ calculates $P(A \cup B)$
✓ correct probability

(ii)   gained a distinction in one paper but not the other.   (2 marks)

Solution
$P(A \cap \bar{B}) + P(\bar{A} \cap B) = (0.15 - 0.02) + (0.05 - 0.02)$
$= 0.16$
Specific behaviours
✓ indicates correct method
✓ correct probability

(iii)   gained a distinction in the first paper given that they gained a distinction in the second.   (1 mark)

Solution
$P(A\|B) = \dfrac{0.02}{0.05} = 0.4$
Specific behaviours
✓ correct probability

(b)   State, with justification, whether events A and B are independent.   (2 marks)

Solution
Events are NOT independent since $P(A) = 0.15$ and $P(A\|B) = 0.4$ and so it can be seen that $P(A) \neq P(A\|B)$.
Specific behaviours
✓ states not independent
✓ justification