

Papers written by
Australian Maths
Software

SEMESTER TWO

YEAR 12

MATHEMATICS METHODS

Units 3-4

REVISION 2

2016

Section Two

(Calculator–assumed)

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 10 minutes

Working time for section: 100 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: drawing instruments, templates, notes on up to two unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section Two.
Formula sheet retained from Section One.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	5	5	50	50	35
Section Two Calculator—assumed	13	13	100	100	65
Total marks				150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

6. (6 marks)

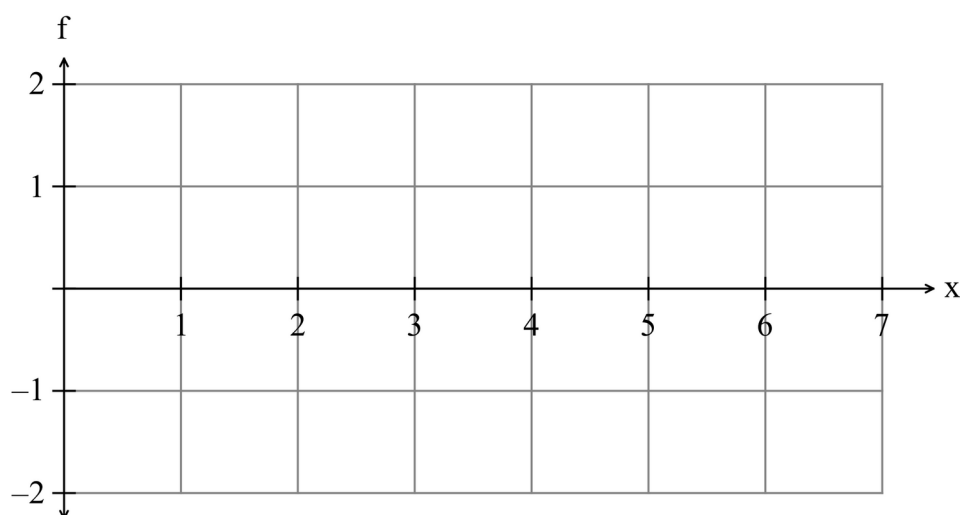
Consider the function $f(x) = \lim_{h \rightarrow 0} \left(\frac{x^h - 1}{h} \right)$ i.e. as $h \rightarrow 0$.

(a) Use your calculator to complete the following chart.

x	$f(x)$
0.5	
1	
2	
3	
4	
7	

(3)

(b) Sketch $y = f(x)$ on the set of axes below.



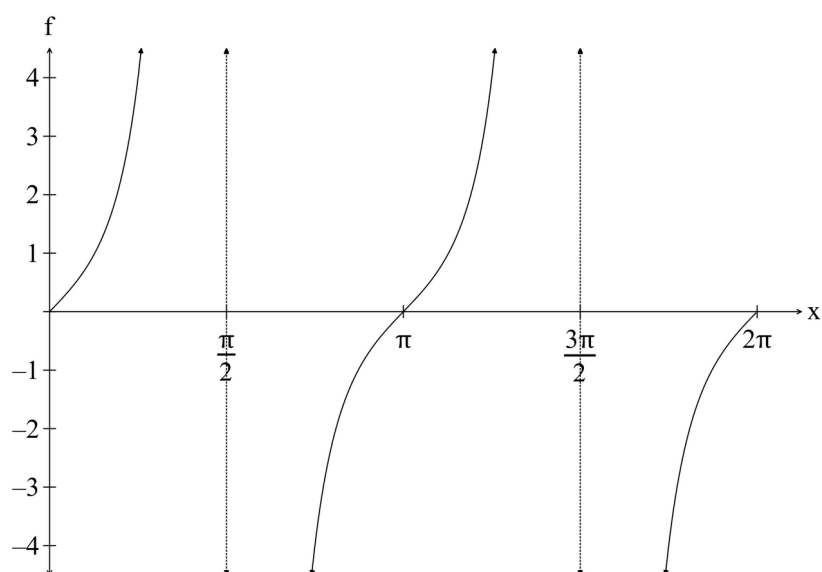
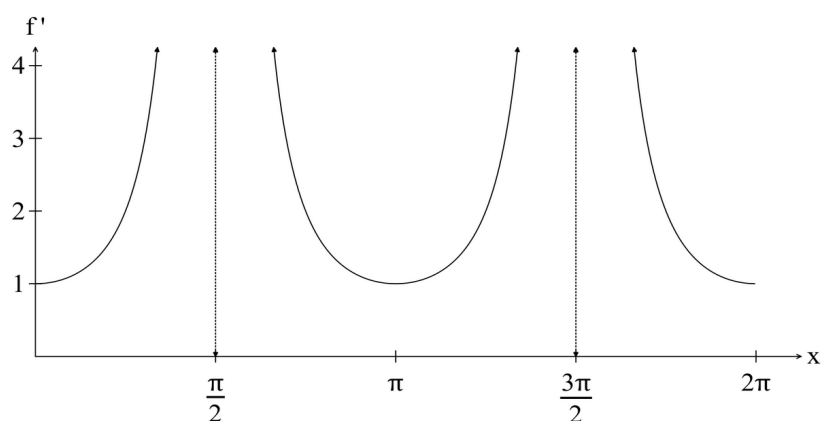
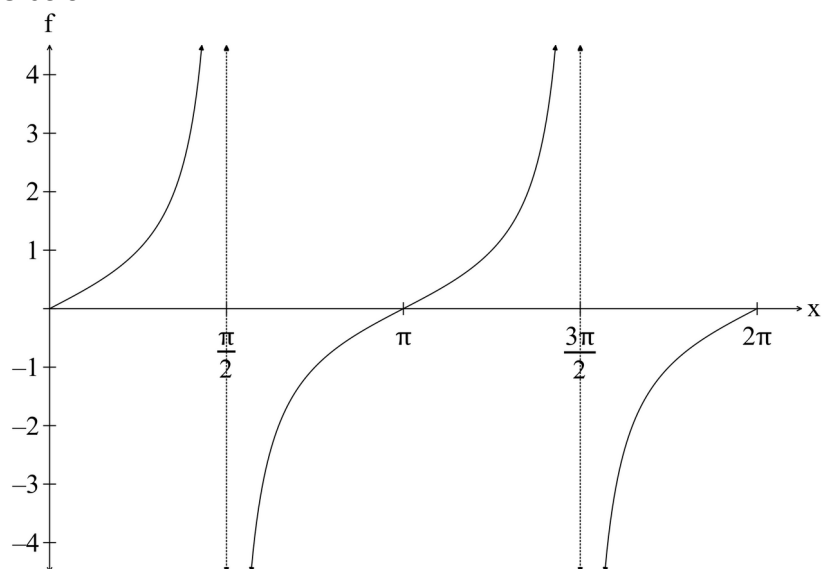
(2)

(c) Write down the equation for the function you have drawn.

(2)

7. (9 marks)

The function $f'(x) = \sec^2(x)$, $y = f(x)$ and $y = f''(x)$ are graphed on the sets of axes below.



(a) Find the equation of the functions

(i) $y = f(x)$ (2)

(ii) $y = f''(x)$. (3)

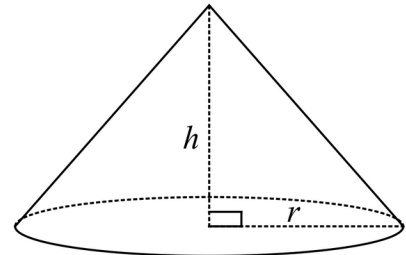
(b) Discuss fully the relationship between the functions f , f' and f'' .
In your discussion you should mention turning points, points of inflection, concavity, undefined points etc. (4)

8. (7 marks)

- (a) A cone is such that $r = \frac{4}{3}h$ where r is the base radius and h is the height.

The height of the cone is increasing at a rate of 10 cm per minute.

Find the rate of change of the volume when the height is equal to 20 cm. (4)

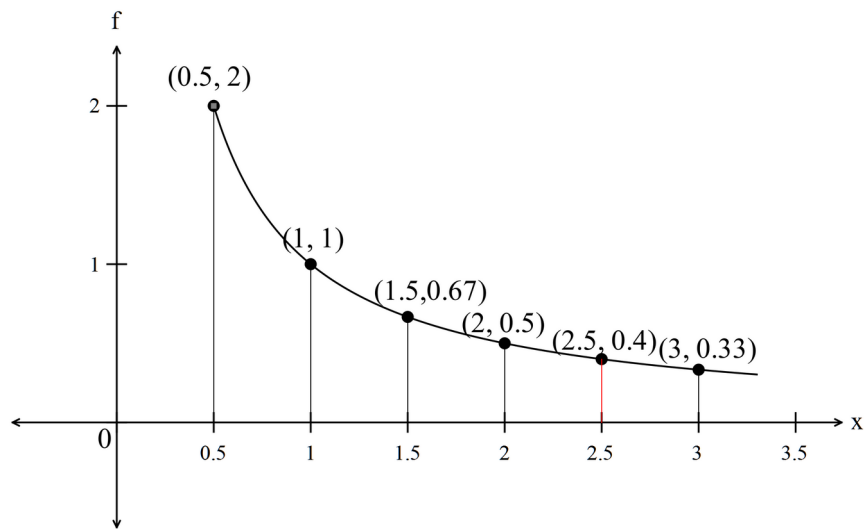


$$V_{\text{cone}} = \frac{\pi}{3} r^2 h$$

- (b) Use a calculus method to find the increase in the volume in (a) when the height changes from 15 cm to 15.01 cm. (3)

9. (4 marks)

Consider the diagram below:



- (a) Use rectangles to determine the lower estimate of the area between the function $y = f(x)$ and the x axis on the domain $[0.5, 3]$. (3)

- (b) Given the function is $f(x) = \frac{1}{x}$ determine the area under the curve on the domain $[0.5, 3]$. (1)

10. (4 marks)

Determine the area between the functions $f(x) = x^3 - 4x - 2$ and $g(x) = -x$.

Show your working.

(4)

11. (6 marks)

Given $a = 4 \text{ ms}^{-2}$, $v_0 = 2 \text{ ms}^{-1}$, $x_0 = 1 \text{ m}$ defined for $t \geq 0$ find

(a) the velocity and displacement at $t = 2 \text{ s}$. (4)

(b) the distance travelled for $0 \leq t \leq 4$. (2)

12. (6 marks)

(a) Find $\frac{d}{dx} \int_1^x (\tan(2t) - 2) dt$ (2)

(b) (i) Find $f'(t)$ given $f(t) = \cos(\ln(t))$ (1)

(ii) Hence find $\int_1^e -\frac{\sin(\ln(t))}{t} dt$. Show all working. (3)

13. (6 marks)

In 1990 the mountain pygmy possum population was estimated at 2635 individuals.
In 2010, the population was estimated at 2075.

- (a) What is the annual rate of decrease of the mountain pygmy possum population from 1990 to 2010? (3)

In 2010, steps were taken to protect the possums from predators i.e. the cull of feral cats and red foxes and further habitat destruction was prevented where possible.

In 2015, the population of mountain pygmy possums was estimated to be 1830.

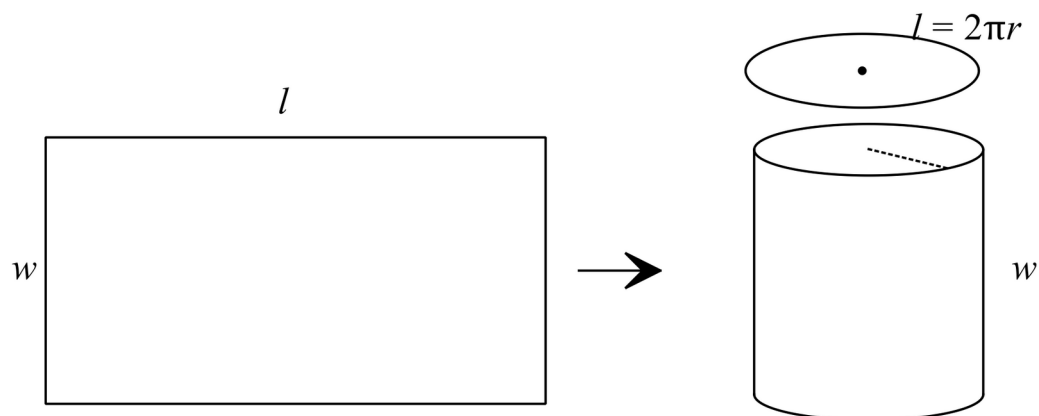
- (b) Compare the population in 2015 to what the population was expected to be and decide if the “steps taken” were effective. (3)



14. (6 marks)

The rectangle below is such that $l + w = 10$.

The rectangle is wrapped around to form a cylinder as shown in the diagram below.



(a) Express the volume of the cylinder in terms of r .

(2)

- (b) Determine the dimensions of the rectangle that give a cylinder of maximum volume. Show working. (4)

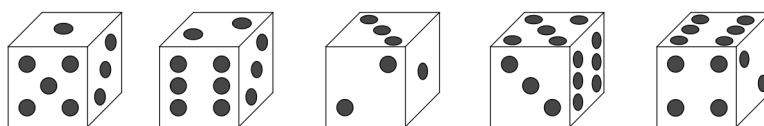
15. (4 marks)

(a) Five six-sided dice are rolled.

Find the probability of obtaining

(i) exactly two 3s. (2)

(ii) at least two 3s. (2)



16. (9 marks)

- (a) Three dice are rolled. Y is defined as the number of sixes rolled. $Y = \{0, 1, 2, 3\}$

Complete the following table. (2)

y	0	1	2	3
$P(Y = y)$				

- (b) Consider the following probability density function.

x	0	1	2	3
$P(X = x)$	0.1	0.2	0.3	0.4

- (i) Find $P(X \geq 2)$ (1)

- (ii) Find the mean and the variance of the distribution. (4)

Each x is doubled then increased by one.

- (iii) Find the mean and variance of the transformed scores. (2)

17. (11 marks)

A real estate agent sells an average of \$1 800 000 worth of houses a month. Her monthly sales are normally distributed with a standard deviation of \$150 000.

- (a) What is the probability that next month the agent will sell \$2 000 000 worth of housing? (2)
- (b) The real estate manager was aware that the agent had sold at least \$1 650 000 of housing last month, but what is the probability she had in fact sold \$2 000 000 worth of housing? (3)
- (c) What is the probability that the agent sold between \$1 200 000 and 1 900 000 in any given month? (2)

- (d) The agent worked March, April, May and June. What is the probability that she had sales worth over \$2 000 000 in two of the months and sales worth less than \$2 000 000 in the other two months? (4)



18. (22 marks)

- (a) One hundred samples of random numbers between 0 and 30 each of size 20, is produced. Sketch a histogram of the means of the samples to illustrate the sampling distribution. (2)
- (b) (i) A survey was conducted in the foyer of the local Woolworth's about the price of goods that happened to be on special that week inside the store. Comment on any possible bias while conducting the survey. (2)
- (ii) Suggest an alternative means of conducting the survey in (i) to ensure impartiality. (2)
- (c) The probability of a seventeen year old boy having a part-time job while at school is 0.4.
In a sample of 80 seventeen year old boys, what is the probability that between 20 and 25 of them have a part-time job? (4)

- (d) Twenty out of a surveyed 120 people wanted more roundabouts on suburban roads to slow down the traffic.
- (i) Use this result to estimate the probability of a randomly chosen person living in the district wants more roundabout to slow down the traffic. (1)
- (ii) Find an estimate of the variance of the sampling distribution. (2)
- (iii) Find the 95% confidence interval for the probability estimate. (4)

- (e) Perth City Council considered having free parking in the city from 7pm over the two weeks when the Perth International Artist's Festival was held to encourage people into the city. It was not known what proportion of the population would consider this viable.

What sample size should be surveyed to determine the proportion of people in favour of free parking during the Perth International Artists' Festival using an error margin of 10% and a confidence level of 90%?

(5)