PERTH MODERN SCHOOL
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Independent Public School

Course Methods test 2 Year 12

Student name: ________________ Teacher name: ________________

Task type: **Response**

Time allowed for this task: **___40________ mins**

Number of questions: **____8_______**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **___41___ marks**

Task weighting: **_10___%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3 & 3 = 6 marks) (3.2.9)
Determine y in terms of x for the following. Show all working.

a) $\frac{dy}{dx}=15x^2+14x$ and $y=13$ when $x=1$.

Solution
$\frac{dy}{dx}=15x^2+14x$ $y=5x^3+7x^2+C$ $13=5+7+C$ $C=1$ $y=5x^3+7x^2+1$
Specific behaviours
✓ anti-diffs terms ✓ introduces an unknown constant and subs to solve ✓ states value of constant

b) $\frac{dy}{dx}=10(2x+1)^4$ and $y=10$ when $x=-1$.

Solution
$\frac{dy}{dx}=10(2x+1)^4$ $y=\frac{10(2x+1)^5}{2(5)}+C=(2x+1)^5+C$ $10=(-1)^5+C$ $C=11$ $y=(2x+1)^5+11$
Specific behaviours
✓ anti-diffs terms ✓ introduces an unknown constant and subs to solve ✓ states value of constant

Q2 (3 & 2 = 5 marks) (3.2.22, 3.2.5)

A car travels in a straight line from the origin, initially at rest, with constant acceleration $4\cos(3t)\,m/s^2$ with t time in seconds.

a) Determine the distance from the origin at $t = \frac{\pi}{3}$ seconds?

Solution
$a = 4\cos(3t)$ $v = \frac{4}{3}\sin(3t) + c$ $t = 0, v = 0$ $0 = 0 + c$ $c = 0$ $x = \frac{-4}{9}\cos(3t) + k$ $t = 0, x = 0$ $0 = \frac{-4}{9} + k$ $k = \frac{4}{9}$ $x = \frac{-4}{9}\cos(3t) + \frac{4}{9}$ $t = \frac{\pi}{3}$ $x = \frac{4}{9} + \frac{4}{9} = \frac{8}{9}m$
Specific behaviours
✓ integrates to find v and shows solving for constant with subs ✓ states the correct rule for x ✓ states exact value for x at required time, no need for units

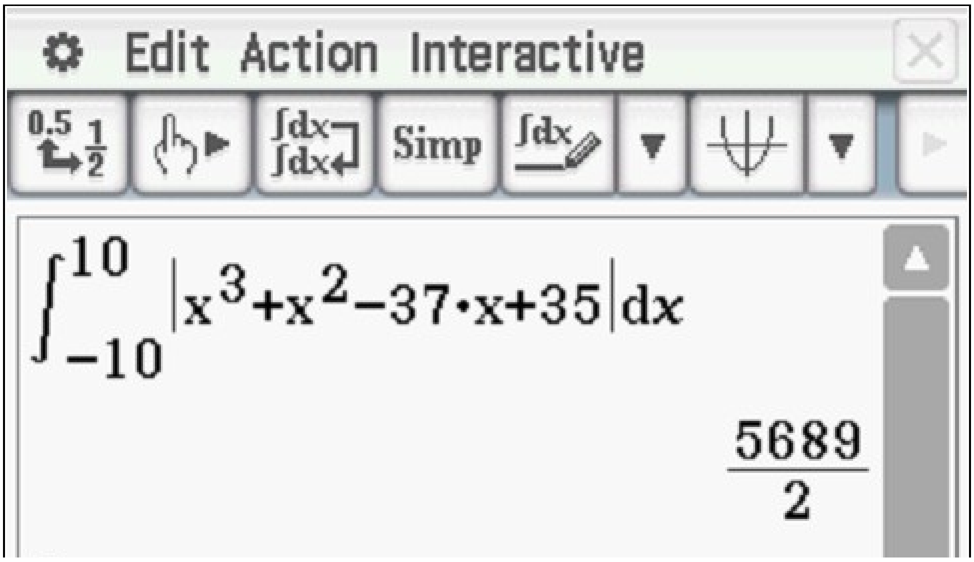
b) What is the velocity of the car at $t = \frac{\pi}{3}$ seconds?

Solution

$v = \frac{4}{3}\sin(3t)$ $t = \frac{\pi}{3}$ $v = 0$
Specific behaviours
✓ subs into v ✓ states velocity, no need for units

Q3 (2 marks) (3.2.19)

Determine the exact area between $y = x^3 + x^2 - 37x + 35$ and the x axis from $x = -10$ to $x = 10$.

Solution

Specific behaviours
✓ writes a correct integral for the area ✓ states exact value

Q4 (2, 2 & 3 = 7 marks) (3.2.18)

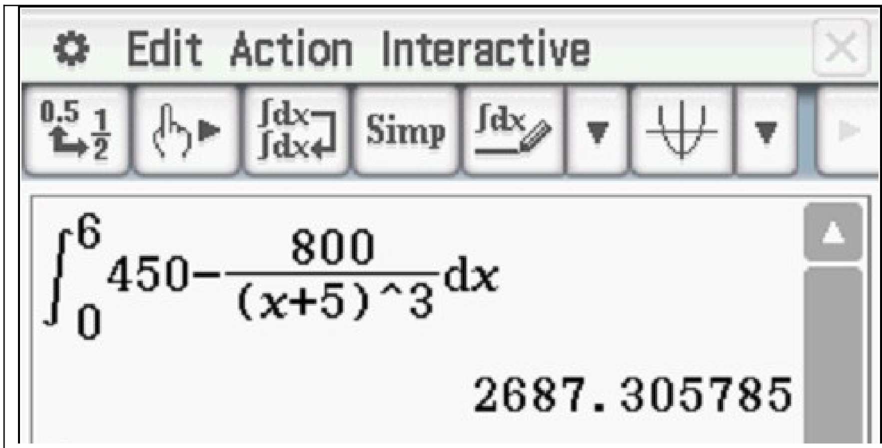
A factory produces electric vehicles. The total number, E, that the company has produced t months after production commenced is such that:

$$\frac{dP}{dt} = 450 - \frac{800}{(t+5)^3}$$

Determine the number produced in

a) The first 6 months

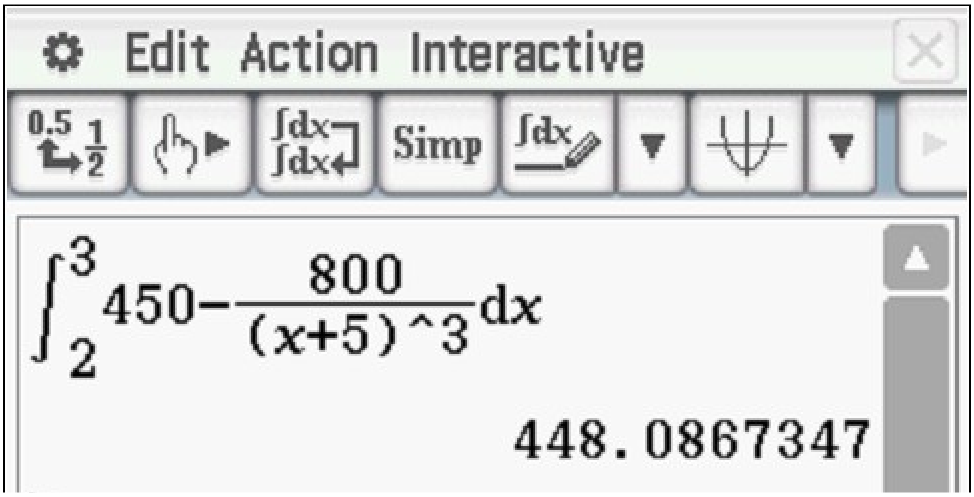
Solution



Number is 2687 vehicles

Specific behaviours
✓ writes a correct integral with limits or ant-diff with a constant ✓ states change, accept decimal

b) The third month

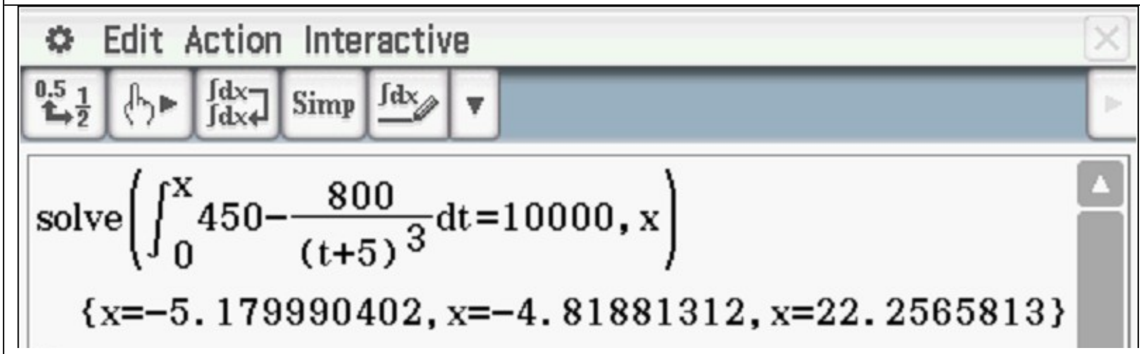
Solution


Specific behaviours
✓ writes a correct integral with limits or ant-diff with a constant ✓ states change, accept decimal

Determine the minimum number of months required to produce:

c) 10000 vehicles.

Solution



Need at least 22.27 OR 23 months

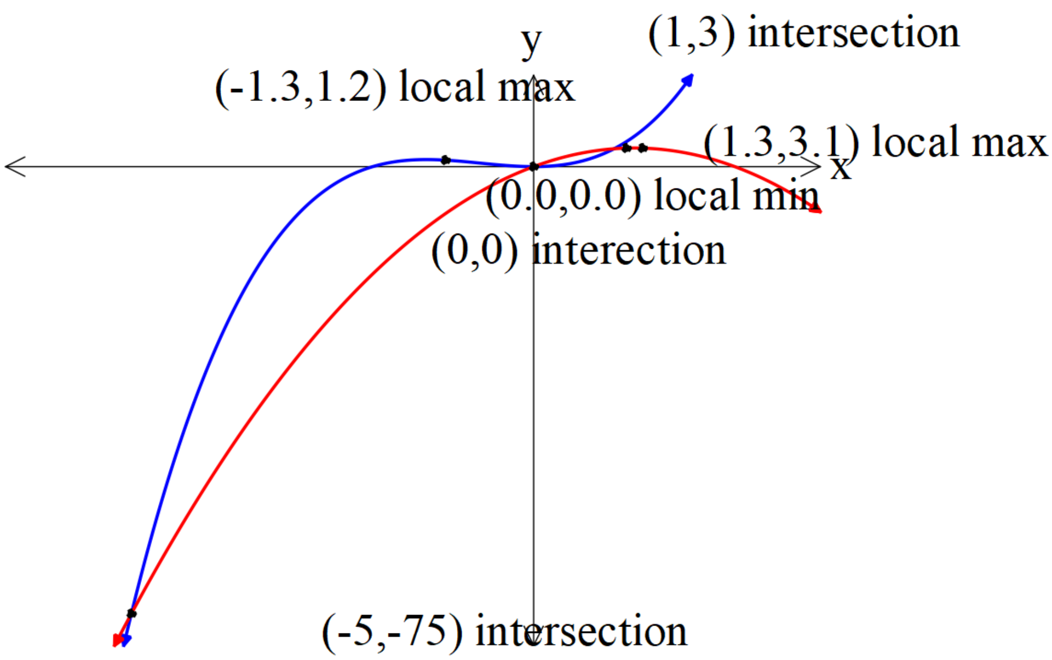
Specific behaviours

✓ sets up an integral with unknown upper limit
✓ solves for a decimal number of months
✓ states number of months

Q5 (5 & 3 = 8 marks) (3.2.20)

a) On the axes below, sketch the following graphs: $y = x^3 + 2x^2$ and $y = 5x - 2x^2$. Indicate on your sketch coordinates (one decimal place) of any stationary points and label their nature and of any points where the graphs intersect each other.

Solution

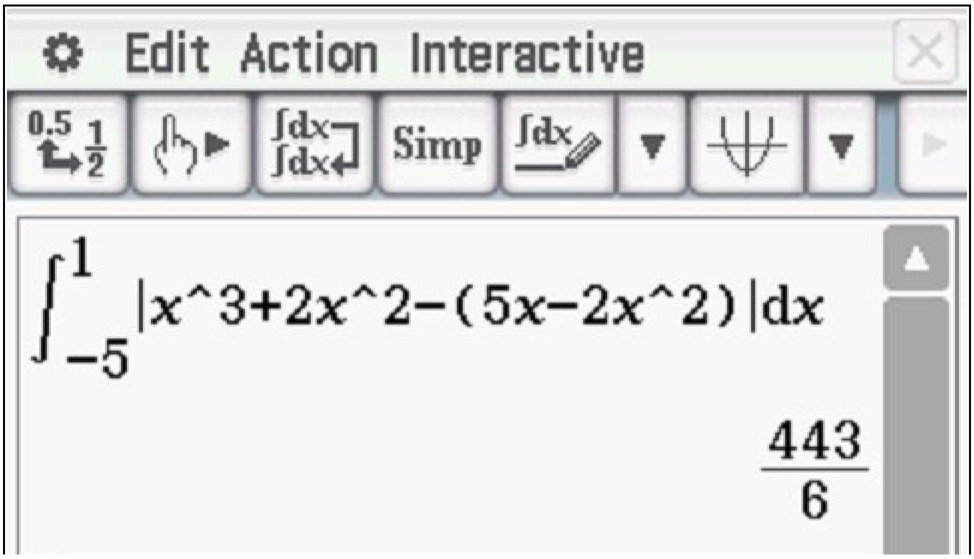


Specific behaviours

✓ states coordinates 1dp local max on both graphs
✓ states coordinates 1 dp local min on cubic (must label min)
✓ shows that graphs intersect at three points
✓ states coordinates of 3 points of intersection
✓sketches correct shape of both graphs

NOTE: follow through does not apply if mistake makes easier!

b) Determine the exact area between $y = x^3 + 2x^2$ and $y = 5x - 2x^2$.

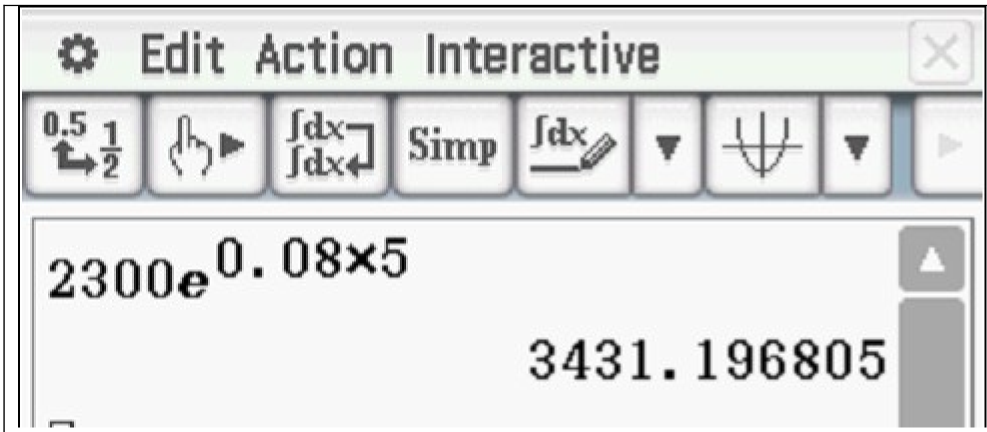
Solution

Specific behaviours
✓ sets up correct integral(s) ✓uses correct limits for integrals ✓ states exact area

Q6 (2 & 2 = 4 marks) (3.1.3, 3.1.4)

The number of kangaroos, N in a particular site that have developed disease W are increasing such that $\frac{dN}{dt} = 0.08N$ with t the time in years. There are initially 2300 kangaroos.

a) Determine the number of kangaroos with disease W in 5 years' time.

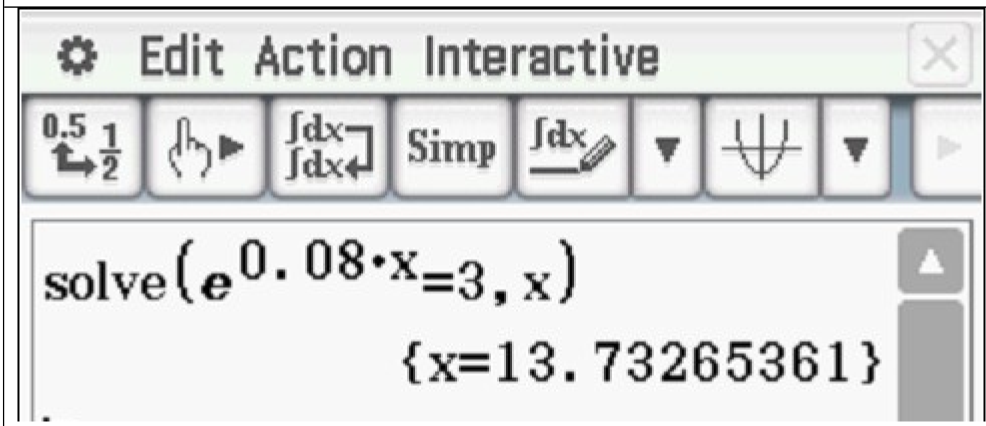
Solution



Number of kangaroos 3431

Specific behaviours
✓ uses an exponential function ✓ states number, accept decimal

b) Determine the time taken (years in one decimal place) to triple the number with the disease.

Solution
 Years = 13.7
Specific behaviours
✓ sets up an equation ✓ states number of years to one decimal place

Q7 (4 marks) (3.2.16)

Consider the function $G(x)=\int^{x} f(t)dt$ such that $G''(x)=\dfrac{3}{4x^{\frac{5}{2}}}$ and $G(4)=\dfrac{79}{2}$.

Determine the rule for the function $f(x)$.

Solution

$$G(x)=\int^{x} f(t)dt$$

$$G'(x)=f(x)$$

$$G''(x)=f'(x)$$

$$f'(x)=\frac{3}{4}x^{\frac{-5}{2}}$$

$$f(x)=\frac{-2}{3}\left(\frac{3}{4}\right)x^{\frac{-3}{2}}+c=\frac{-1}{2}x^{\frac{-3}{2}}+c$$

$$\int^{4} f(t)dt=\frac{79}{2}=\left[x^{\frac{-1}{2}}+cx\right]_{1}^{4}=\left(\frac{1}{2}+4c\right)-(1+c)=3c-\frac{1}{2}$$

$$c=\frac{40}{3}$$

$$f(x)=\frac{-1}{2x^{\frac{3}{2}}}+\frac{40}{3}$$

Specific behaviours

✓ uses fundamental theorem to express $G''=f'$

✓ integrates to express f in terms of x and a constant

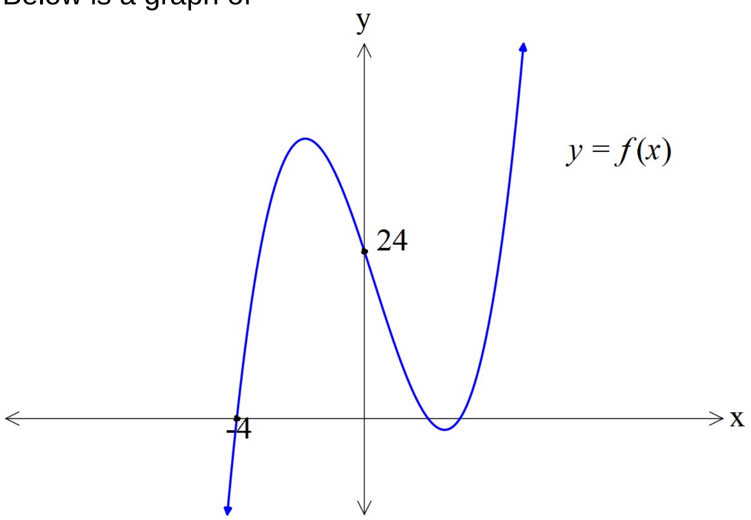
✓ uses definite integral to set up equation for constant

✓ solves for constant and express f in terms of x in full.

Q8 (5 marks) (3.1.15)

Consider the function $f(x)=ax^3+bx^2+cx+d$ where $a,b,c\,\&\,d$ are constants.

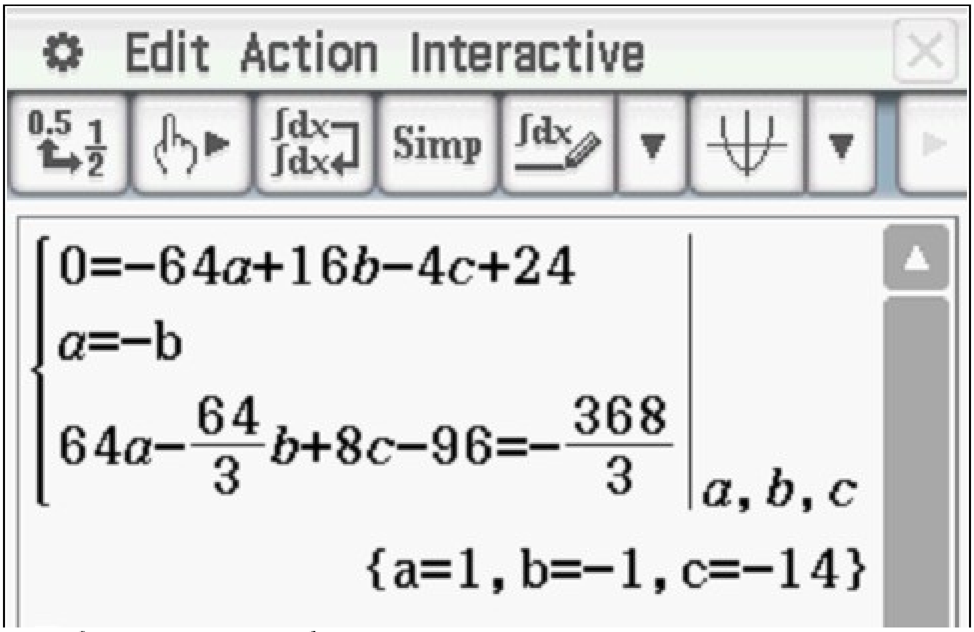
Below is a graph of $f(x)$



There is an x intercept at $x=-4$, y intercept at $y=24$ and $\int_{-4}^{0} f(x)\,dx=\frac{368}{3}$.

There is an inflection point at $x=\frac{1}{3}$.

Determine the exact values of $a, b, c\ \&\ d$.

Solution
$f(x)=ax^3+bx^2+cx+d$ $24=d$ $0=-64a+16b-4c+24$ $f'(x)=3ax^2+2bx+c$ $f''(x)=6ax+2b$ $0=6a\frac{1}{3}+2b$ $a=-b$ $\int_{-4}^{0} ax^3+bx^2+cx+d\,dx=\left[\frac{ax^4}{4}+\frac{bx^3}{3}+\frac{cx^2}{2}+24x\right]_{-4}^{0}=0-\left(64a-\frac{64b}{3}+8c-96\right)=\frac{368}{3}$ $64a-\frac{64b}{3}+8c-96=-\frac{368}{3}$  $a=1, b=-1, c=-14\ \&\ d=24$ OR solving without classpad Eq 1 times 2

$0 = -128a + 32b - 8c + 48$ $\frac{-368}{3} = 64a - \frac{64}{3}b + 8c - 96$ add $\frac{-368}{3} = -64a + \frac{32}{3}b - 48$ $a = -b$ $\frac{-368}{3} = 64b + \frac{32}{3}b - 48$ $\left(\frac{-368}{3} + 48\right) = \frac{224}{3}b$ $\frac{-224}{3} = \frac{224}{3}b$ $b = -1$ $a = 1$ $a = 1, b = -1, c = -14 \,\&\, d = 24$
Specific behaviours
✓ solves for d ✓ derives $a = -b$ using inflection point ✓ sets up linear equation using x intercept ✓ uses definite integral and then integrates and sets ups a linear equation for unknowns ✓states values for all 4 unknowns NOTE: follow through does not apply if mistake makes easier!