PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 2 Year 12

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: **____40_____ mins**

Number of questions: **_____7_____**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **_41_____ marks**

Task weighting: **_10___%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2, 2 & 3 = 7 marks) (3.2.1-3.2.3)

Consider the functions $f(x)=\frac{1}{x-2}$ and $g(x)=\sqrt{x}$.

a) State the natural domain and range of $f(x)$.

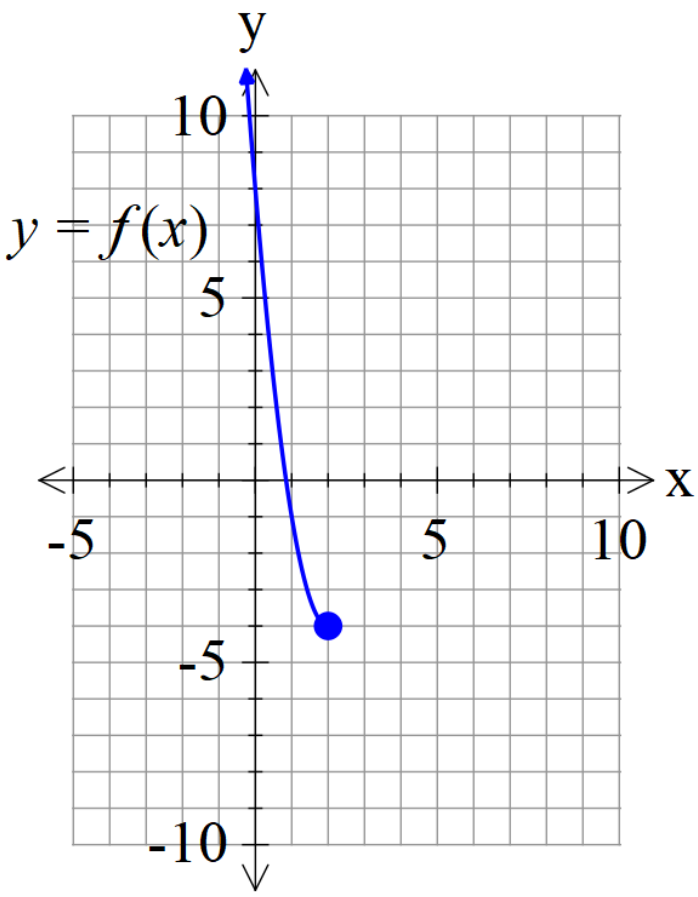
b) Does $g \circ f(x)$ exist over the natural domain of $f(x)$? Explain.

c) State the rule and natural domain and range of $f \circ g(x)$.

Q2 (3, 3, 1 &2 = 9 marks) (3.2.4)

Consider the function $f(x)=3x^2-12x+8$ with domain $x \leq 2$.

a) Sketch the inverse function on the axes below.



b) Determine the inverse function $f^{-1}(x)$ stating its domain. (Show all working)

c) Determine $f \circ f^{-1}(x)$

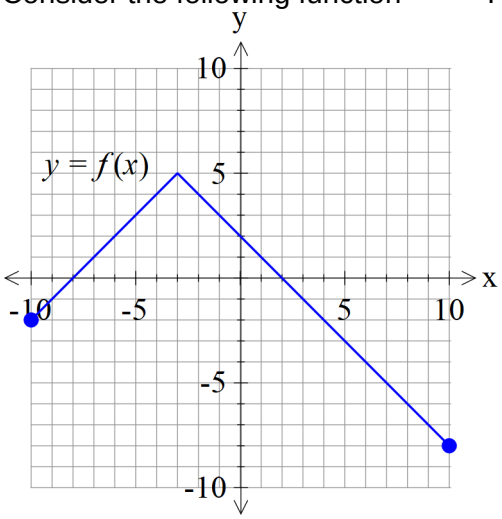
d) Determine when $f(x) = f^{-1}(x)$ exactly.

Q3 (3 marks) (3.2.6)

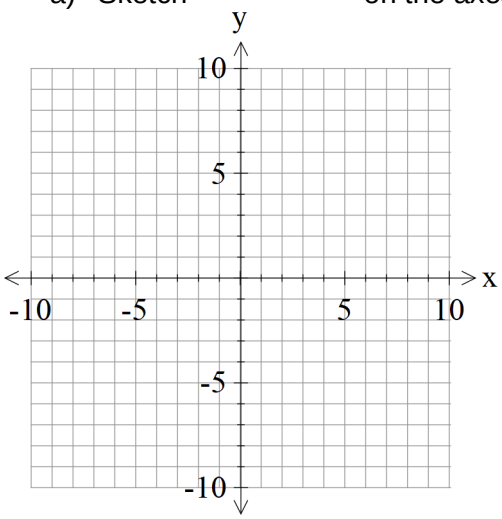
Consider the inequality $\left|\frac{3}{2}x+b\right| \leq 4.5$ is **only true** for $4 \leq x \leq 10$ with b a constant.
Determine the value of b.

Q4 (3 & 3 = 6 marks) (3.2.7)

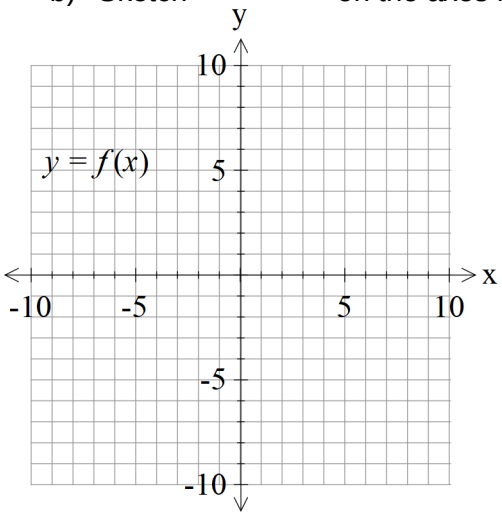
Consider the following function $f(x)$.



a) Sketch $y = f(-|x|)$ on the axes below.



b) Sketch $y = \frac{1}{f(x)}$ on the axes below.



Q5 (3 & 3 = 6 marks) (3.3.3-3.3.6)

Consider two rockets A & B, moving with constant velocities such that at time $t = 0$ hours their positions and velocities are as follows:

$$r_A = \begin{pmatrix} -2 \\ 3 \\ 7 \end{pmatrix} km \quad r_B = \begin{pmatrix} 6 \\ -2 \\ -1 \end{pmatrix} km$$

$$v_A = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} km/h \quad v_B = \begin{pmatrix} -2 \\ 5 \\ 2 \end{pmatrix} km/h$$

a) Determine the time and distance of their closest approach.

b) Given that the rockets leave smoke trails that stay in the air for a long period of time, determine if the smoke trails cross at all and if they do, the point in space. Justify.

Q6 (6 marks) (3.3.4, 3.3.6)

Consider the line $r = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ -2 \end{pmatrix}$ and the sphere $\left| r - \begin{pmatrix} 6 \\ \beta \\ -7 \end{pmatrix} \right| = 5$ with β a constant.

Determine the value(s) of β, to one decimal place, such that:

a) The line is a tangent to sphere.
b) The line meets the sphere in two places.
c) The line misses the sphere completely.

Q7 (4 marks) (3.1.4)

The solutions to the complex equation $z^n = k$ are plotted in the complex plane. (n is an integer & k is a complex constant). Exactly **four** of the solutions are plotted in the second quadrant, $\frac{\pi}{2} < Arg(z) < \pi$, and **no more**. Of these four solutions, the smallest argument is $\frac{7\pi}{12}$.

Determine all possible values of n.