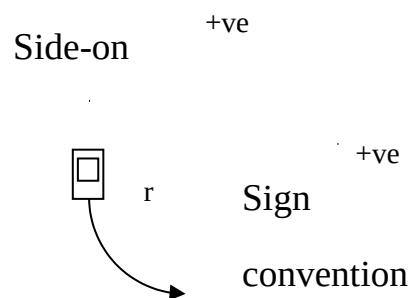
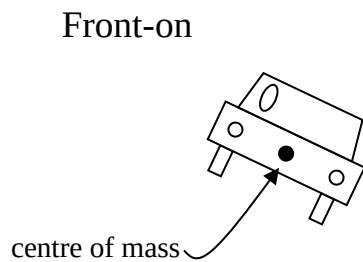


### 3 solutions to Question 6 of Circular Motion Booklet P13



$\theta$

|   |   |
|---|---|
| <p>Solution 1: <math>F_f</math> is down the slope</p>   | <p>Solution 2: <math>F_f</math> is up the slope</p>   |
| <p>Horz: <math>N_h + F_{f:h} = F_{c:h}</math></p> <p>Vert: <math>W_v + N_v + F_{f:v} = 0</math></p>   | <p>Horz: <math>N_h + F_{f:h} = F_{c:h}</math></p> <p>Vert: <math>W_v + N_v + F_{f:v} = 0</math></p> |
| <p>Solution 3: <math>F_f</math> is zero</p>   |   |
| <p>Horz: <math>N_h + F_{f:h} = F_{c:h}</math> <math>F_{f:h} = F_{c:h}</math></p> <p>Vert: <math>W_v + N_v + F_{f:v} = 0</math> <math>F_{f:v} = 0</math></p> |   |

|  |  |
|--|--|
|  | <p>There are three solutions to the combination of W (the weight force), N (the normal reaction force) and <math>F_f</math> (the frictional force) that will give <math>F_c</math> (the resultant force) that is horizontal and towards the centre of circular path around which the car is turning.</p> <p>In all cases:</p> <p><math>\Sigma F = 0</math> in the vertical direction</p> |
|--|--|

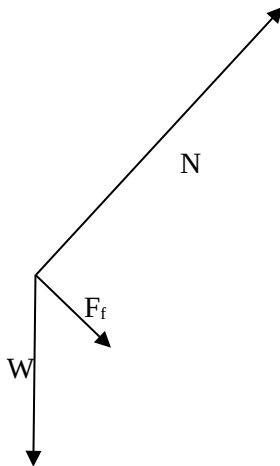
**Note that the equations are the same in all cases!**

$\Sigma F = F_c$  in the horizontal direction

## Do we need to draw these sum of the forces diagrams?

It is not always necessary to draw a sum of the forces diagram, instead a free body diagram can be the starting point. Consider the free body diagram where  $F_f$  is down the slope.

Free body diagram for Solution 1:  
 $F_f$  is down the slope



We know that the sum of these forces must be the centripetal acceleration, i.e.

$$\Sigma F = mv^2/r$$

As this the centripetal acceleration is in the horizontal plane, we can use a approach to solve for the forces that involves resolution into horizontal and vertical components.

### Horizontal

$$\Sigma F = mv^2/r$$

### Vertical

$$\Sigma F = 0$$

Over the next few pages, I have set out approaches to solving problems represented by the scenarios on the first page of these notes, using resolution into horizontal and vertical components

## Calculation of values of the forces

### Vertical

$$\begin{aligned} |W_v| &= |m \times g| \\ |N_v| &= |N \times \cos\theta| \\ |F_{f,v}| &= |F_f \times \sin\theta| \\ F_{c,v} &= 0 \end{aligned}$$

### Horizontal

$$\begin{aligned} |W_h| &= 0 \\ |N_h| &= |N \times \sin\theta| \\ |F_{f,h}| &= |F_f \times \cos\theta| \\ |F_{c,h}| &= |m \times v^2 / r| \end{aligned}$$

Irrespective of which of the above 3 situations is involved, the formulae relating these are:

$$\text{Horizontal:} \quad N_h + F_{f,h} = F_{c,h}$$

$$\text{Vertical:} \quad W_v + N_v + F_{f,v} = 0$$

### A non-general solution with the P18 Tricky Activity as an example.

To solve problems where  $F_f$  is not known, e.g., the Tricky Activity on P18,

- Assume that  $F_f$  is down the slope (Solution 1, the first diagram in the table).
- Write down the formulae relating the actual forces  $N$ ,  $F_f$ , and  $W$  and the resultant force  $F_c$  in the horizontal and vertical directions.
- Substitute **values** for those forces.
  - Ensure that a sign convention is assigned
  - Ensure correct signs are given to the forces to allow scalar addition of the forces.
- Simultaneously solve for  $N$  or  $F_f$ .  
If  $F_f$  is positive, negative or zero, the Solution is 1, 2 or 3 respectively.
- Use  $N$  to find  $F_f$  or vice versa

$$\begin{aligned} \text{Horizontal:} \quad N_h + F_{f,h} &= F_{c,h} \Rightarrow |N| \times \sin\theta + |F_f| \times \cos\theta = mv^2/r \\ &\Rightarrow |N| \times \sin 30^\circ + |F_f| \times \cos 30^\circ = 2000 \times (110/3.6)^2/50 \\ &\Rightarrow 0.5|N| + 0.86603|F_f| = 3.7356 \times 10^4 \quad \text{I} \\ \text{Vertical:} \quad W_v + N_v + F_{f,v} &= 0 \Rightarrow -|m \times g| + |N| \times \cos\theta - |F_f| \times \sin\theta = 0 \\ &\Rightarrow -9.8 \times 2000 + \cos 30^\circ |N| - |F_f| \times \sin 30^\circ = 0 \\ &\Rightarrow -1.96 \times 10^4 + 0.86603|N| - 0.5|F_f| = 0 \quad \text{II} \\ \text{From I} \quad |F_f| &= (3.7356 \times 10^4 - 0.5|N|) / 0.86603 \quad \text{III} \end{aligned}$$

Substitute III into II

$$\begin{aligned} \Rightarrow -1.96 \times 10^4 + 0.86603|N| - 0.5(3.7356 \times 10^4 - 0.5|N|) / 0.86603 &= 0 \\ \Rightarrow |N| (0.86603 + 0.5 \times 0.5 / 0.86603) - 0.5 \times 3.7356 \times 10^4 / 0.86603 - 1.96 \times 10^4 &= 0 \\ \Rightarrow 1.1547|N| - 4.1667 \times 10^4 &= 0 \\ = |N| &= 4.1667 \times 10^4 / 1.1547 = 3.56 \times 10^4 \text{ N} \end{aligned}$$

substitute for  $|N|$  in equation III

$$|F_f| = (3.7356 \times 10^4 - 0.5 \times 3.56 \times 10^4) / 0.86603 = 2.25 \times 10^4 \text{ N}$$

## A general solution with the P18 Tricky Activity as an example.

To solve problems where  $F_f$  is not known, e.g., the Tricky Activity on P18,

- Assume that  $F_f$  is down the slope (Solution 1, the first diagram in the table).
- Write down the formulae relating the actual forces  $N$ ,  $F_f$ , and  $W$  and the resultant force  $F_c$  in the horizontal and vertical directions.
- Substitute **equations** for those forces.
  - Ensure that a sign convention is assigned
  - Ensure correct signs are given to the forces to allow scalar addition of the forces.
- Simultaneously solve for  $N$  or  $F_f$ .  
If  $F_f$  is positive, negative or zero, the Solution is 1, 2 or 3 respectively.
- Use  $N$  to find  $F_f$  or vice versa

|                        |                                      |   |     |
|------------------------|--------------------------------------|---|-----|
| Horizontal:            | $N_h + F_{fh} = F_{c,h} \Rightarrow$ | $ N  \times \sin\theta +  F_f  \times \cos\theta = mv^2/r$  | I   |
| Vertical:              | $W_v + N_v + F_{f,v} = 0$            | $\Rightarrow - m \times g  +  N  \times \cos\theta -  F_f  \times \sin\theta = 0$                           | II  |
| From I                 | $ F_f $                              | $= (mv^2/r -  N  \times \sin\theta) / \cos\theta$   | III |
| Substitute III into II | 0                                    | $= - m \times g  +  N  \times \cos\theta - \sin\theta \times (mv^2/r -  N  \times \sin\theta) / \cos\theta$ |     |
|                        |                                      | $=  N  (\cos\theta + \sin^2\theta / \cos\theta) -  m \times g  - (\sin\theta / \cos\theta) \times mv^2/r$   |     |
| $\Leftrightarrow$      | $ N $                                | $= ( m \times g  + (\sin\theta / \cos\theta) \times mv^2/r) / (\cos\theta + \sin^2\theta / \cos\theta)$     |     |
|                        |                                      | $= ( m \times g  + (\sin\theta / \cos\theta) \times mv^2/r) / ((\cos^2\theta + \sin^2\theta) / \cos\theta)$ |     |
|                        |                                      | $= ( m \times g  + (\sin\theta / \cos\theta) \times mv^2/r) \times \cos\theta$                              |     |
|                        |                                      | $= \sin\theta \times mv^2/r +  m \cdot g \cdot \cos\theta $   |     |
|                        |                                      | $= \sin 30^\circ \times 2000 \times (110/3.6)^2 / 50 + 2000 \times 9.8 \times \cos 30^\circ$                |     |
|                        |                                      | $= 3.56 \times 10^4 \text{ N}$  |     |

Now substitute the value calculated for  $N$  into equation III to find  $F_f$ .

or find  $F_f$  first

$$\begin{aligned}
 \text{From I} \quad |N| &= (mv^2/r - |F_f| \times \cos\theta) / \sin\theta & \text{IIIa} \\
 \text{Substitute III into II} \quad 0 &= -|m \times g| + \cos\theta \times (mv^2/r - |F_f| \times \cos\theta) / \sin\theta - |F_f| \times \sin\theta \\
 &= |F_f| \times -(\sin\theta + \cos^2\theta / \sin\theta) - |m \times g| + (\cos\theta / \sin\theta) \times mv^2/r \\
 \Leftrightarrow \quad |F_f| &= (|m \times g| - (\cos\theta / \sin\theta) \times mv^2/r) / -(\sin\theta + \cos^2\theta / \sin\theta) \\
 &= (|m \times g| - (\cos\theta / \sin\theta) \times mv^2/r) / -((\sin^2\theta + \cos^2\theta) / \sin\theta) \\
 &= (|m \times g| - (\cos\theta / \sin\theta) \times mv^2/r) / -(1 / \sin\theta) \\
 &= (|m \times g| - (\cos\theta / \sin\theta) \times mv^2/r) \times -\sin\theta \\
 &= \cos\theta \times mv^2/r - |m.g.\sin\theta| \\
 &= \cos 30^\circ \times 2000 \times (110/3.6)^2 / 50 - 2000 \times 9.8 \times \sin 30^\circ \\
 &= 2.25 \times 10^4 \text{ N}
 \end{aligned}$$

Now substitute the value calculated for  $F_f$  into equation IIIa to find  $N$ .

## A Revaluation of the general solution

We derived two simple equations after much algebra:

$$\begin{aligned}
 |N| &= \sin\theta \times mv^2/r + |m.g.\cos\theta| \\
 |F_f| &= \cos\theta \times mv^2/r - |m.g.\sin\theta|
 \end{aligned}$$

This is simply resolving into components parallel to and perpendicular to the slope.

Question: Would not that have been simpler in the first place?

Answer: Yes, but who would have thought to do it?

So, as 3AB Physics students, how should we proceed?

The approach to solving such problems that we are least likely to forget is to always resolve into horizontal and vertical components and substitute in numerical values early in the manipulation to simplify the algebra.

That approach works in a very wide set of situations and, although it can result in significant complexity, careful setting out can address that difficulty.