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Mathematics Methods Unit 3

Calculator Assumed Applications of Ami-Differentiation 1

Time: 45 minutes Total Marks: 45 Your Score: / 45



Question One: [3 marks] CA

The area under the curve  $f(x) = 4e^{kx}$  over the domain  $f(x) = 4e^{kx}$  over the domain  $f(x) = 4e^{kx}$ 

Determine the value of k.

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# Mathematics Methods Unit 3

Question Two:

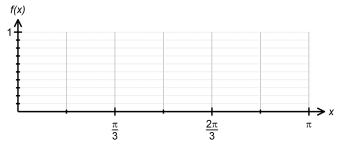
CA

$$f(x) = \sin\left(\frac{x}{2}\right)$$

Consider the function

$$0 \le x \le \pi$$

Sketch f(x) over the domain  $0 \le x \le \pi$ 



Draw rectangles on your graph that can be used to overestimate the area

under over the domain , where

 $0 \le x \le \pi$ 

Hence approximate the area under the curve over the domain

Calculate the margin of error between your answer in part (c) and the exact  $0 \le x \le \pi$ value of the area under the curve over the domain

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(e) Calculate the distance travelled in the third second.
(b) Calculate the change in displacement in the first second.
(c) Calculate when the speed of the particle is $4 \text{ m/s}$ .
(b) Determine an expression for the velocity of the particle.
(a) Determine the initial acceleration of the particle.
, where $t$ is time in seconds and $$ is $ms^{\text{-}z}.$ The initial velocity of the particle is -4 m/s.
The acceleration of a particle moving in rectilinear motion is given by $a(t) = -4\cos(2t) + 12t$
Question Three: $[1, 2, 2, 2, 2 = 9 \text{ marks}]$ CA
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(e) Calculate the distance travelled in t	
<ul><li>(d) Calculate the change in displaceme</li></ul>	
(c) Calculate when the speed of the par	
(b) Determine an expression for the ve	
(a) Determine the initial acceleration o	
of the particle is -4 m/s.	
$a_1(1) = 4.008(2t) + 12t$ s in sin is in set $t$ is time in set $t$ is $t$ in set $t$ is $t$ in set $t$ in $t$	
The acceleration of a particle moving in re	
Question Three: $[1, 2, 2, 2, 2 = 9 \text{ mark}]$	
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#### Mathematics Methods Unit 3

Question Four: [2, 2, 3 = 7 marks] CA

 $120 - 0.5x + 0.01x^2$ 

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The marginal cost of producing  $\boldsymbol{x}$  units of a certain product is dollars per unit.

- (a) Determine the extra cost associated with producing the 31st item.
- (b) Find the increase in cost if the production level is increased from 200 units to 500 units.

(c) The marginal revenue from producing and selling x units of a certain product  $x+2x^2$  is . Determine the profit function if the profit from producing 10 items is \$38.33.

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$$f'(x) = 2ax$$

$$f'(1) = 1$$

$$2a = 1 \checkmark$$

$$a = \frac{1}{2} \checkmark$$

$$\int_{-1}^{2} \frac{1}{2} x^{2} + b \, dx = 10.5$$

$$\left[\frac{x^{3}}{6} + bx\right]_{-1}^{2} = 10.5 \checkmark$$

$$\frac{8}{6} + 2b + \frac{1}{6} - b = 10.5$$

$$\frac{9}{6} + b = 10.5 \checkmark$$

$$b = 9 \checkmark$$

 $f(1) = \frac{1}{2} + 9 = 9.5$ 

9.5 = 1 + cc = 8.5

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### Mathematics Methods Unit 3

Question Five: **CA** [4 marks]

the domain  $x \ge x \ge 0$ Calculate the area enclosed between the two curves  $y = \cos x$   $y = 3\sin(2x)$ 

Draw a sketch to support your solution.

#### ΚŊ Question Six:[4 marks]

2stinu ∂ si wol9d The area of the shaded region of xq uis  $p = \lambda$ 

Determine the values of a and b.

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 $0 = xb \ xd \text{ nis } p$ 

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Question Seven: [8 marks]

stinu 3.01 si  $2 \ge x \ge 1$ and the xaxis over the domain The area bounded by the curve q + zxv = (x)

K)

.  $\sin x = x$  is  $\sin x = x$  is  $\sin x = x + c$  The equation of the tangent to

Determine the values of a, b and c.

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# Mathematics Methods Unit 3

Question Seven: [8 marks] CA

$$f(x) = ax^2 + b$$

The area bounded by the curve and the x axis over the domain

$$-1 \le x \le 2$$
 is 10.5 units<sup>2</sup>.

o 
$$f(x)$$
  $x = 1$   $y = x + c$ 

The equation of the tangent to

Determine the values of *a*, *b* and *c*.

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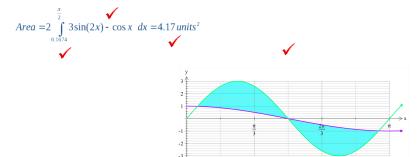
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# Mathematics Methods Unit 3

#### Question Five: [4 marks] CA

 $y = 3\sin(2x)$ Calculate the area enclosed between the two curves and  $0 \le x \le \pi$ the domain

Draw a sketch to support your solution.

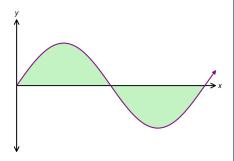


#### Question Six: [4 marks] CA

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The area of the shaded region of below is 6 units2.

Determine the values of *a* and *b*.



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Mathematics Methods Unit 3

# SOLUTIONS Calculator Assumed Applications of Anti-Differentiation 1

Time: 45 minutes Total Marks: 45 Your Score: \ 45



Question One: [3 marks] CA

The area under the curve  $f(x) = 4e^{bx} \qquad \text{over the domain } 0 \le x \le 10$ 

Determine the value of k.

$$\int_{0}^{10} 4e^{kx} dx = \frac{40}{3} \left( -e^{-3} + 1 \right)$$

$$\frac{de^{10k}}{\sqrt{4e^{kx}}} \int_{10}^{10} = \frac{40}{40} \left(-e^{-3} + 1\right)$$

$$\sqrt{(1+\epsilon^{-6})^{-4}} = \frac{40}{8} = \frac{40}{3}$$

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Mathematics Methods Unit 3

Question Four: [z, z, 3 = 7 marks]

The marginal cost of producing x units of a certain product is dollars per unit.

(a) Determine the extra cost associated with producing the  $31^{st}$  item.

$$C.(30) = 2114$$
  $C.(30) = 150 - 0.2(30) + 0.01(30)^2$ 

(b) Find the increased in cost if the production level is increased from 200 units to 500 units.

002 
$$\xi = xb^{-x}x10.0 + x2.0 - 021 \int_{00z}^{00z}$$

(c) The marginal revenue from producing and selling x units of a certain product  $x + 2 \chi^2$ 

. Determine the profit function if the profit from producing of

CA.

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$$P'(x) = x + 2x^{2} - (120 - 0.5x + 0.01x^{2})$$

$$P'(x) = x + 2x^{2} - (120 - 0.5x + 0.01x^{2})$$

$$P'(x) = \frac{1.99x^{2}}{3} + \frac{2x}{4} - \frac{120x + c}{4}$$

$$P'(x) = \frac{1.99x^{2}}{3} + \frac{3x^{2}}{4} - \frac{120x + c}{4}$$

$$P'(x) = \frac{1.99x^{2}}{3} + \frac{3x^{2}}{4} - \frac{120x + 5}{4}$$

$$P'(x) = \frac{1.99x^{2}}{4} + \frac{3x^{2}}{4} - \frac{120x + 5}{4}$$

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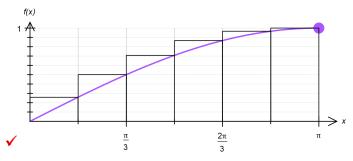
#### Mathematics Methods Unit 3

Question Two: [2, 2, 3, 3 = 10 marks] CA

$$f(x) = \sin\left(\frac{x}{2}\right)$$

Consider the function

(a) Sketch over the domain  $0 \le x \le \pi$ 



(b) Draw rectangles on your graph that can be used to overestimate the area

$$f(x) \qquad 0 \le x \le \pi \qquad \delta x = \frac{\pi}{6}$$
 under over the domain , where

$$0 \le x \le \pi$$

(c) Hence approximate the area under the curve over the domain

$$Area = \frac{\pi}{6} \left( \sin \left( \frac{\pi}{12} \right) + \sin \left( \frac{\pi}{6} \right) + \sin \left( \frac{\pi}{4} \right) + \sin \left( \frac{\pi}{3} \right) + \sin \left( \frac{5\pi}{12} \right) + \sin \left( \frac{\pi}{2} \right) \right)$$

$$Area = 2.25 units^{2}$$

(d) Calculate the margin of error between your answer in part (c) and the exact  $0 \le \chi \le \pi$  value of the area under the curve over the domain .

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$$\int_{0}^{\pi} \sin\left(\frac{x}{2}\right) dx = 2$$
 2.25 - 2 = 0.25

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#### Mathematics Methods Unit 3

### Question Three: [1, 2, 2, 2, 2 = 9 marks] CA

The acceleration of a particle moving in rectilinear motion is given by  $a(t) = -4\cos(2t) + 12t$  a(t), where t is time in seconds and is ms $^{-2}$ . The initial velocity of the particle is -4 m/s.

(a) Determine the initial acceleration of the particle.

$$a(0) = -4ms^{-2}$$

(b) Determine an expression for the velocity of the particle.

$$v(t) = \int -4\cos(2t) + 12t \ dt$$

$$v(t) = -2\sin(2t) + 6t^2 + c$$

$$-4 = -2\sin(0) + 6(0)^2 + c$$

$$c = -4$$

$$v(t) = -2\sin(2t) + 6t^2 - 4$$

(c) Calculate when the speed of the particle is 4 m/s.

$$|v(t)| = 4$$
  $\checkmark$   
 $t = 0s, 0.543s, 1.24s$   $\checkmark$ 

(d) Calculate the change in displacement in the first second.

$$\int_{0}^{1} v(t) dt = -3.42m$$

(e) Calculate the distance travelled in the third second.

$$\int_{2}^{3} v(t) dt = 35.62m$$