

Semester One Examination 2017 Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula Sheet

To be provided by the candidate

Standard items: pens(blue/black preferred), pencils(including coloured), sharpener,
correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One Calculator—free	7	7	50 minutes	50
Section Two Calculator—assumed	12	12	100 minutes	100
				150

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2017*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section One: Calculator–free

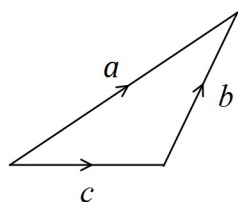
50 marks

This section has **seven (7)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes

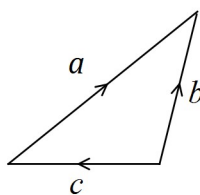
Question 1 (6 marks)

- (a) Determine vector \mathbf{c} in terms of vectors \mathbf{a} and \mathbf{b} for each diagram below. (3 marks)

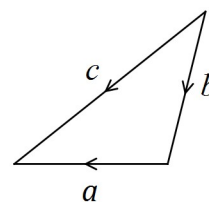


$\mathbf{c} =$

$\mathbf{c} =$



$\mathbf{c} =$



- (b) Draw a vector diagram containing vectors \mathbf{a} , \mathbf{b} and \mathbf{c} such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$ (1 mark)

- (c) Given that $(\mathbf{a} + \mathbf{b}) \cdot \mathbf{c} = 0$, circle the correct statement(s) from the list below. (2 marks)
Neither \mathbf{a} nor $\mathbf{b} = \mathbf{0}$.

- I \mathbf{a} is perpendicular to \mathbf{c}
- II \mathbf{b} is perpendicular to \mathbf{c}
- III neither \mathbf{a} nor \mathbf{b} is perpendicular to \mathbf{c}
- IV \mathbf{c} is perpendicular to $\mathbf{a} + \mathbf{b}$

Question 2 (8 marks)

There are 50 houses on Venn Street, and they are all occupied. 30 of these houses are occupied by married couples, of which 20 have children, 14 have pets and 6 have neither children nor pets.

- (a) Show how to use the inclusion-exclusion principle for two-sets to determine the number of houses on Venn St that contain married couples that have both children and pets. (3 marks)

It is also known that 27 houses have children living in them, 27 houses have pets, and 5 houses are occupied by single individuals with no children and no pets.

- (b) Show how to use the inclusion-exclusion principle for three-sets to determine the number of houses on Venn St that have both children and pets. (3 marks)

- (c) What is the minimum number of houses that must be chosen to ensure that there is at least one house that contains a married couple with both children and pets?
State the name of the principle used to determine the answer. (2 marks)

Question 3 (5 marks)

The angle between \mathbf{a} and $(\mathbf{a} - \mathbf{b})$ is θ , with $|\mathbf{a}| = 2$ and $|\mathbf{b}| = 3$.

- (a) Draw a clearly labelled sketch of the vectors \mathbf{a} , \mathbf{b} and $(\mathbf{a} - \mathbf{b})$, including the location of θ . (2 marks)

- (b) Given that $\mathbf{a} \cdot \mathbf{b} = 5$, determine the value of $(\mathbf{a} - \mathbf{b}) \cdot (2\mathbf{b} - \mathbf{a})$. (3 marks)

Question 4 (13 marks)

Consider the portion of Pascal's triangle shown below.

					1		2		1					
					1		3		3		1			
				1		4		6		4		1		
			1		5		10		10		5		1	
		1		6		15		20		15		6		1
	1		7		21		35		35		21		7	
1		8		28		56		70		56		28		8

(a) Evaluate.

(i) ${}^6C_3 =$ (1 mark)

(ii) $\frac{7!}{4! \times 3!} =$ (1 mark)

(b) State the value of x in each case below.

(i) ${}^8C_x = 70$ (1 mark)

(ii) ${}^xC_4 = 15$ (1 mark)

(iii) ${}^xC_2 = {}^xC_5$ (1 mark)

(iv) ${}^8C_{x-2} = {}^8C_{x+2}$ (1 mark)

Question 4 (Continued)

- (c) An academic team of 5 members is to be composed from 3 mathematicians, 3 physicists and 2 chemists.
- (i) How many different teams of 5 academics can be assembled if there are no other restrictions? (1 mark)
- (ii) How many different teams of 5 academics can be assembled if it must contain at least 2 mathematicians? (3 marks)
- (d) The 5 academics from (c) line up for a team photo. How many different arrangements are possible if:
- (i) no other restrictions apply? (1 mark)
- (ii) the team has two mathematicians, and they must not stand next to each other? (2 marks)

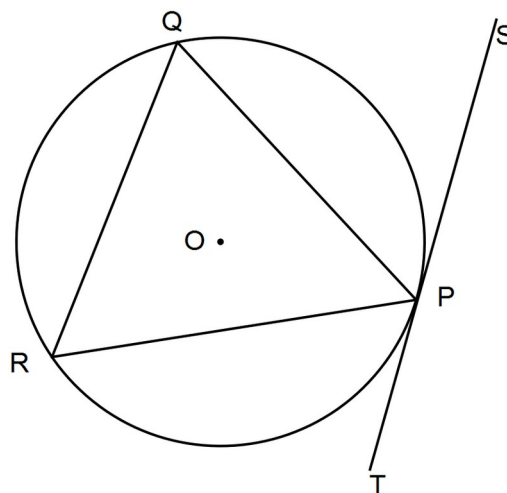
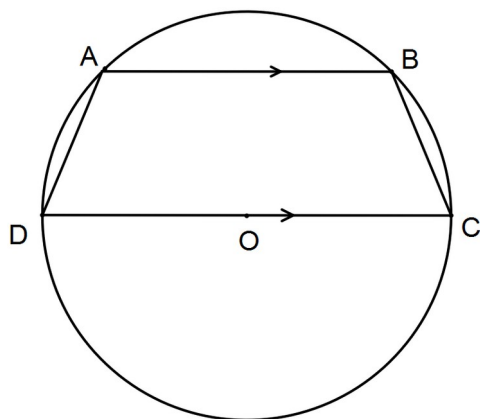
Question 5 (6 marks)

(a) If $2\mathbf{a} + \mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{a} - \mathbf{b} = 5\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, then determine $\mathbf{a} + \mathbf{b}$. (4 marks)

(b) Obtain a unit vector normal to $2\mathbf{i} - 3\mathbf{j}$. (2 marks)

Question 6 (7 marks)

In the diagrams below, $\angle AOB = 70^\circ$, $\angle POQ = 100^\circ$ and $\angle ROP = 140^\circ$ as shown.



(a) Determine the size of $\angle OAB$, $\angle ACD$ and $\angle ABD$. (3 marks)

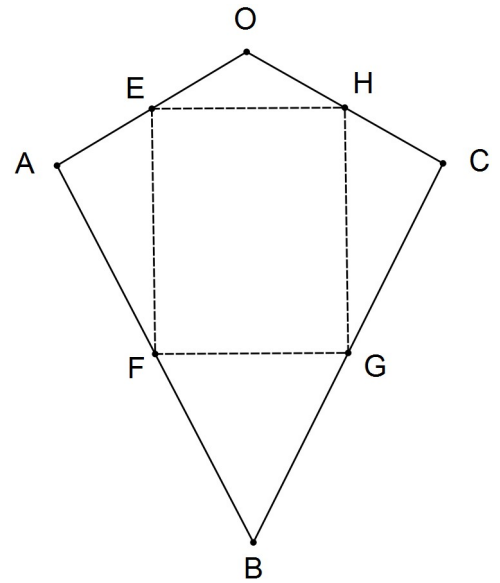
(b) Determine the size of $\angle QPR$ providing reasons for your answer. (2 marks)

(c) Determine the size of $\angle SPQ$ and state the name of the theorem used. (2 marks)

Question 7 (5 marks)

Quadrilateral OABC shown has F, G, H and E as midpoints of AB, BC, CO and OA respectively.

Let $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OC} = \underline{c}$ and $\overrightarrow{OB} = \underline{b}$



(a) Determine \overrightarrow{OF} and \overrightarrow{OG} in terms of \underline{a} , \underline{c} and/or \underline{b} . (2 marks)

(b) Prove that quadrilateral FGHE is a parallelogram.

(3 marks)

End of Section One

Additional working space

Question number(s):

Additional working space

Question number(s):

WATP acknowledges the permission of School Curriculum and Assessment Authority in providing instructions to students.