

it to the supervisor before reading any further.
you do not have any unauthorised material. If you have any unauthorised material with you, hand
No other items may be taken into the examination room. It is **your responsibility** to ensure that

Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators approved for use in this examination

Standard items: correction fluid/tape, eraser, ruler, highlighters
pens (blue/black preferred), pencils (including coloured), sharpener,
To be provided by the candidate

Formula sheet (referred from Section One)

This Question/Answer booklet
Materials required/recommended for this section

Number of additional answer sheets used (if applicable):
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Working time:
Reading time before commencing work:
ten minutes

Your name _____

in words _____

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WA student number: in figures _____

Solutions
Semester Two Examination, 2020
Melville Senior High School
Mathematics
Methods
Units 1&2
Calculator-assumed
Section Two:

Question/Answer booklet



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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Supplementary page

Question number: _____

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

(2 marks)

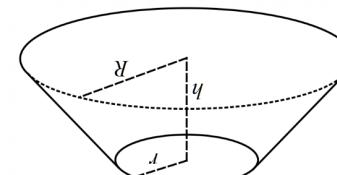
(1 mark)

(1 mark)

(1 mark)

(5 marks)

The frustum shown at right is a truncated right cone. The radius of the larger circle and R is the radius of the smaller circle. R is the distance between the two parallel circles.



The volume of such a solid is $V = \frac{1}{3} \pi h (r^2 + R^2 + rR)$, where r is the radius of the smaller circle and R is the radius of the larger circle and h is the perpendicular distance between the two parallel circles.

The frustum shown at right is a truncated right cone.

Consider frustum F where $r = x$ cm, $R = 3x$ cm, and $h = 36 - x$ cm.

$$\text{(a)} \quad \text{Show that the volume of frustum } F \text{ is } 156\pi x^2 - \frac{3}{3} \pi x^3 \text{ cm}^3.$$

(3 marks)

$$\therefore V = \frac{\pi}{3} (36 - x) \left(x^2 + 3x^2 + x(3x) \right)$$

$$r = x, R = 3x, h = 36 - x$$

Solution

Substitutes r , R and h in terms of x
expresses r , R and h in terms of x
clear steps to obtain final expression

Solution

Derivative is zero when:
 $\frac{dV}{dx} = 312\pi x - 13\pi x^2 = 0$
 $x = 24$
 $V(24) = 29952\pi \approx 94097$
 $V(24) = 94097 \text{ cm}^3 \text{ when } x = 24 \text{ cm.}$
 \checkmark states root of derivative must equal zero
 \checkmark indicates derivative must equal zero
 \checkmark states maximum volume

(b) Use a calculus method to determine the value of x that maximises the volume of frustum
 F and state this maximum volume, rounding your answer to the nearest cm^3 . (4 marks)

(b)

(c)

(d)

<input checked="" type="checkbox"/> correct probability
<input checked="" type="checkbox"/> specific behaviours
$P(A \cup B) = 0.29 + 0.25 = 0.54$
Solution

<input checked="" type="checkbox"/> correct probability
<input checked="" type="checkbox"/> specific behaviours
$P(A \cup B) = 1 - 0.33 = 0.67$
Solution

<input checked="" type="checkbox"/> correct probability
<input checked="" type="checkbox"/> specific behaviours
$P(A \cup B) = 0.52 + 0.25 - 0.13 = 0.64$
Solution

Determine $P(A \cup B)$ when
 For the events A and B, $P(A) = 0.52$ and $P(B) = 0.25$.

Question 9

Working time: 100 minutes.

This section has thirteen questions. Answer all questions. Write your answers in the spaces provided.

The frustum shown at right is a truncated right cone.

The volume of such a solid is $V = \frac{1}{3} \pi h (r^2 + R^2 + rR)$,

Question 10

(6 marks)

The cost, C dollars, for a gigabyte of computer memory between the end of year 2006 ($t=0$) and the end of year 2016 ($t=10$) can be modelled by the equation $C=14.5(0.84)^t$.

- (a) Calculate C at the end of year 2010.

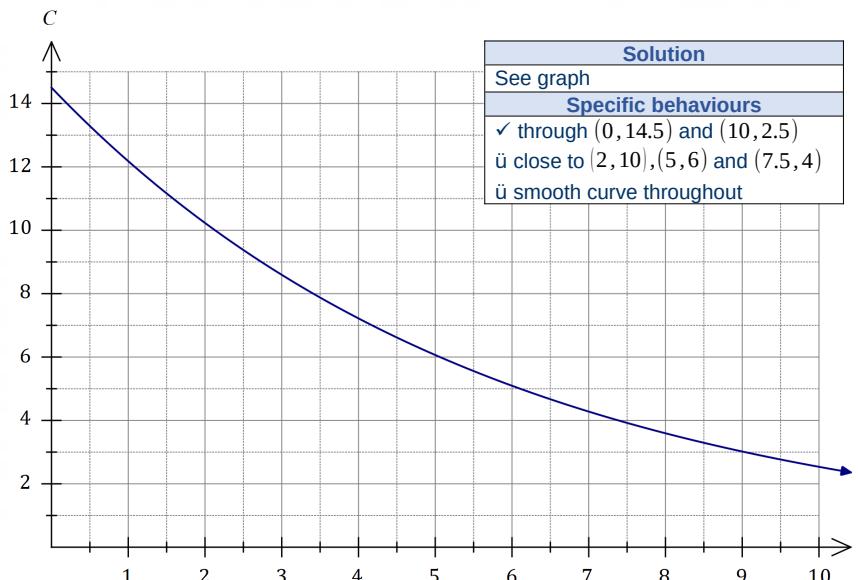
(1 mark)

Solution
$C(4)=\$7.22$
Specific behaviours

✓ correct cost, to nearest cent

- (b) Draw the graph of C against t on the axes below.

(3 marks)



- (c) Assuming that the model continues to be valid, during which year will the cost of computer memory fall below \$1 per gigabyte?

(2 marks)

Solution
$C(t)=1 \Rightarrow t=15.3$
Hence during the year 2006+16=2022
Specific behaviours

✓ correct value of t
 ü correct year

Question 20

(7 marks)

A reader bought 14 different novels, planning to read a selection of them when on holiday.

- (a) Determine the number of different combinations of novels the reader could choose from if they select

(i) six novels.

(1 mark)

Solution
$\binom{14}{6}=3003$ combinations
Specific behaviours

✓ correct number

(ii) five or six novels.

(2 marks)

Solution
$\binom{14}{5}+\binom{14}{6}=2002+3003$ $\therefore 5005$ combinations
Specific behaviours

✓ ways to choose five
 ü correct number

Four of the 14 different novels are by the author Harper.

- (b) The reader makes a random selection of six novels. Determine the probability that

(i) none of the novels selected are by Harper.

(2 marks)

Solution
Must choose from 10 not by Harper: $\binom{10}{6}=210$
$P=\frac{210}{3003}=\frac{10}{143} \approx 0.0699$

Specific behaviours
✓ ways to choose ü correct probability

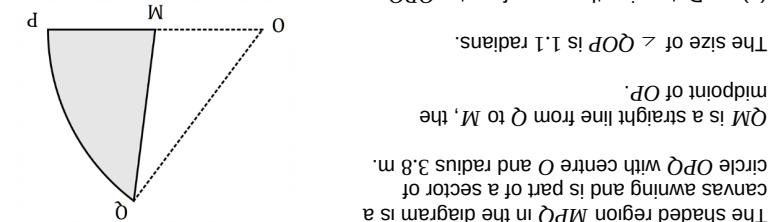
(ii) one of the novels selected is by Harper.

(2 marks)

Solution
$P=\frac{\binom{10}{5}\binom{4}{1}}{3003}=\frac{252 \times 4}{3003}=\frac{48}{143} \approx 0.3357$
Specific behaviours

✓ indicates correct method
 ü correct probability

The shaded region MPQ in the diagram is a canvas awning and is part of a sector of circle OQP with centre O and radius 3.8 m. QM is a straight line from Q to M , the mid-point of OP . The size of $\angle QOP$ is 1.1 radians.



(a) Determine the area of sector OQP .
(1 mark)

✓ correct sector area
$A_{OQP} = 0.5(3.8)^2 \times 1.1 = 7.942 \text{ m}^2$
Solution
✓ indicates use of difference of areas
✓ area of triangle
✓ correct area with units

(b) Determine the area of the canvas awning.
(1 mark)

✓ correct sector area
$A_{OQP} = 0.5(3.8)^2 \times 1.1 = 7.942 \text{ m}^2$
Solution
✓ indicates use of difference of areas
✓ area of triangle
✓ correct area with units

(c) The edge of the canvas is to be reinforced with thin wire. Determine the length of wire required.
Using flat rate depreciation, the value of another machine after 5 years will be \$2 695 and after a further 7 years it will become worthless. The value T_n of this machine after n years can be modelled using $T_n = an + b$, where a and b are constants.

(b) Determine the value of a the value of b .
(3 marks)

(c) Given that both machines begin to depreciate at the same time, determine the number of years until the machines have the same value and state what this value is.
(2 marks)

✓ clearly states each value
✓ initial value
✓ annual loss in value
✓ uses cosine rule for QM
✓ length of QM
✓ arc length QP
✓ total length with units

✓ clearly states each value
✓ initial value
✓ annual loss in value
✓ uses cosine rule for QM
✓ length of QM
✓ arc length QP
✓ total length with units

✓ clearly states each value
✓ initial value
✓ annual loss in value
✓ uses cosine rule for QM
✓ length of QM
✓ arc length QP
✓ total length with units

(d) Determine the value of the machine after 5 years.
(1 mark)

✓ correct number
$V_n = 0 \Rightarrow n = 14 \text{ years}$
Solution
✓ specific behaviours

(e) (i) the number of years until the machine has no value.
(1 mark)

✓ correct value
$V_0 = \$2475$
Solution
✓ specific behaviours

(e) (ii) the value of the machine after 5 years.
(1 mark)

✓ correct value
$V_5 = \$2475$
Solution
✓ specific behaviours

(e) (iii) the value of the machine after 5 years.
(1 mark)

✓ correct value
$V_0 = \$2475$
Solution
✓ specific behaviours

(f) Determine the value of an asset is depreciated by a fixed amount each year. Using the flat rate depreciation, the value of an asset is given by $V_{n+1} - V_n = V_0 - 275$, in flat rate depreciation, the value V_0 of a machine in dollars after n years is given by $V_{n+1} = V_n - 275$.

(g) The edge of the canvas is to be reinforced with thin wire. Determine the length of wire required.
(2 marks)

✓ years
✓ value
✓ uses table
✓ specific behaviours
✓ Solution

✓ clearly states each value
✓ initial value
✓ annual loss in value
✓ uses table
✓ specific behaviours
✓ Solution

✓ clearly states each value
✓ initial value
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✓ uses table
✓ specific behaviours
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✓ Solution

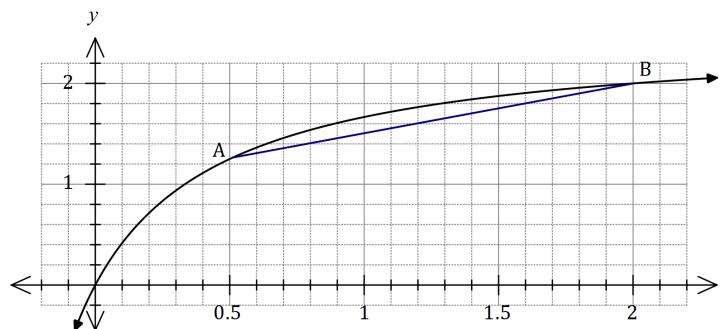
✓ clearly states each value

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(8 marks)

Question 12

Part of the graph of $y=f(x)$ is shown below, where $f(x)=\frac{5x}{2x+1}$.



Points A and B lie on the curve and have x -coordinates of 0.5 and 2 respectively.

- (a) Draw the chord to the curve between A and B on the axes above and determine the gradient of this chord. (3 marks)

Solution
$m = \frac{f[2] - f[0.5]}{2 - 0.5} = \frac{2 - 1.25}{1.5} = 0.5$
Specific behaviours
✓ draws chord on graph ü correct y -values ü correct gradient

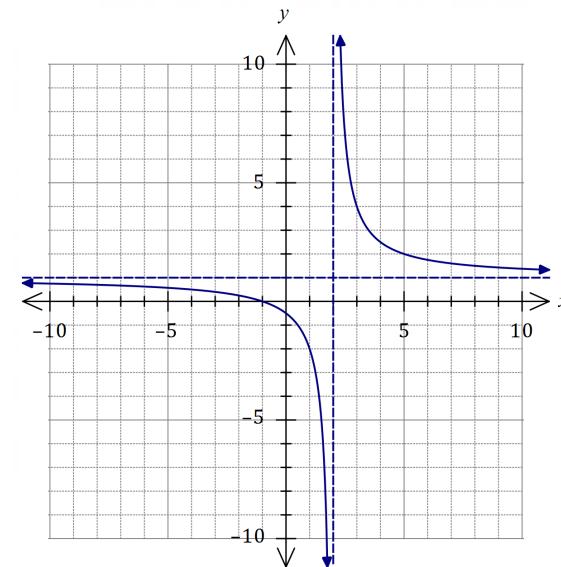
Point C, with an x -coordinate of $0.5+h$, lies on the curve between A and B. The gradient of the chord AC is m_{AC} .

- (b) Calculate m_{AC} for the values of h shown in the table below, recording the gradients in the table to 3 decimal places. (3 marks)

h	1	0.5	0.1	0.05	0.01
m_{AC}	0.625	0.833	1.136	1.190	1.238

Solution
See table
Specific behaviours
✓ one correct gradient ü at least three correct gradients ü all correct gradients

- (ii) Draw the graph of $y=1+\frac{a}{x+b}$ on the axes below, clearly indicating any asymptotes. (3 marks)



Solution
See graph
Specific behaviours
✓ both asymptotes ü LHS, smooth curve, through (1, -2) ü RHS, smooth curve, through (3, 4)

Question 18 (a) Point A(11, -5) lies on the circumference of a circle with centre (-4, 3). Determine the equation of the circle.

(3 marks) (9 marks)

Solution

Using given point: $15^2 + (-8)^2 = k \Rightarrow k = 289$ ($\text{?} 17^2$)

Equation: $(x+4)^2 + (y-3)^2 = k$

As $h \rightarrow 0$ then $m_{AC} \rightarrow 1.25$. This is the gradient of $y=f(x)$ at the point A.

Solution

Specific behaviors

✓ limiting value

✓ states gradient at the point A

(c) Determine a limiting value for m_{AC} as h becomes very close to 0 and state what feature of the graph of $y=f(x)$ this value represents. (2 marks)

Solution

Using given point: $15^2 + (-8)^2 = k \Rightarrow k = 289$ ($\text{?} 17^2$)

Equation: $(x+4)^2 + (y-3)^2 = k$

Specific behaviors

✓ forms equation using centre and constant point

✓ correct equation (any form)

Solution

$$\begin{aligned} -2 &= 1 + \frac{1+b}{a}, 4 = 1 + \frac{3+b}{a} \\ -2 = 1 + b, 4 = 1 + b \end{aligned}$$

(d) Determine the value of each of the integer constants a and b . (3 marks)

(b) The graph of $y=1+\frac{x+b}{a}$ passes through the points (1, -2) and (3, 4).

Specific behaviors
✓ uses points to form two equations
✓ solves simultaneously

Solve simultaneously for $a=3, b=-2$.

$$-2 = 1 + \frac{1+b}{a}, 4 = 1 + \frac{3+b}{a}$$

Specific behaviors
✓ uses simultaneous equations
✓ both values correct

Question 13

(7 marks)

A set of 175 undergraduates were asked to choose their electives for the following year.
85 chose calculus, 58 chose statistics and 67 chose neither calculus nor statistics.

- (a) Determine how many of the undergraduates chose both calculus and statistics. (2 marks)

Solution
$n(C \cup S) = 175 - 67 = 108$
$n(C \cap S) = 85 + 58 - 108 = 35$
Hence 35 chose both electives.

Specific behaviours
✓ indicates union of sets ü correct number

- (b) Determine the probability that a randomly chosen undergraduate from the set chose

- (i) statistics.

Solution
$P(S) = \frac{58}{175} \approx 0.3314$
✓ correct probability

- (ii) statistics but not calculus.

Solution
$P(S \cap \bar{C}) = \frac{58 - 35}{175} = \frac{23}{175} \approx 0.1314$
✓ correct probability

- (iii) statistics given that they chose calculus.

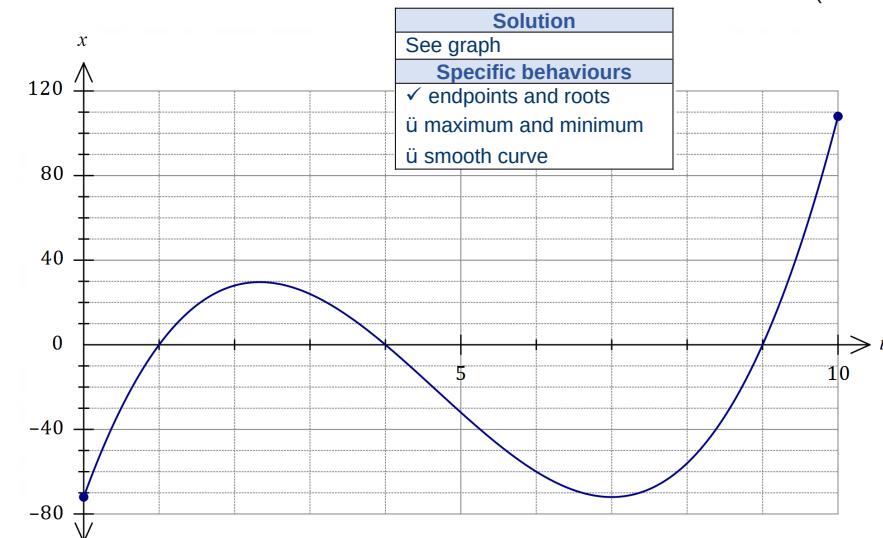
Solution
$P(S C) = \frac{35}{85} = \frac{7}{17} \approx 0.4118$
✓ correct probability

- (c) Use your answers above to explain whether the choice of statistics and calculus electives is independent for these undergraduates. (2 marks)

Solution
Choice is not independent, as $P(S) \neq P(S C)$.
(Undergraduates are more likely to choose statistics if they have chosen calculus.)

Specific behaviours
✓ states not independent ü explanation using existing probabilities

- (c) Use the axes below to sketch the displacement of the body over the given domain. (3 marks)



- (d) State the number of times the body passed through O and determine the minimum speed and maximum speed of the body as it passed through this point. (3 marks)

Solution
Passes through O when $t = 1, 4, 9$ s - on 3 occasions.
$v(1) = 48$, $v(4) = -30$, $v(9) = 80$
Hence minimum speed is 30 cm/s and maximum speed is 80 cm/s.

Specific behaviours
✓ correct number of times
ü minimum speed
ü maximum speed

(2 marks)

(1 mark)

(3 marks)

(3 marks)

(6 marks)

METHODS UNITS 1&2

9

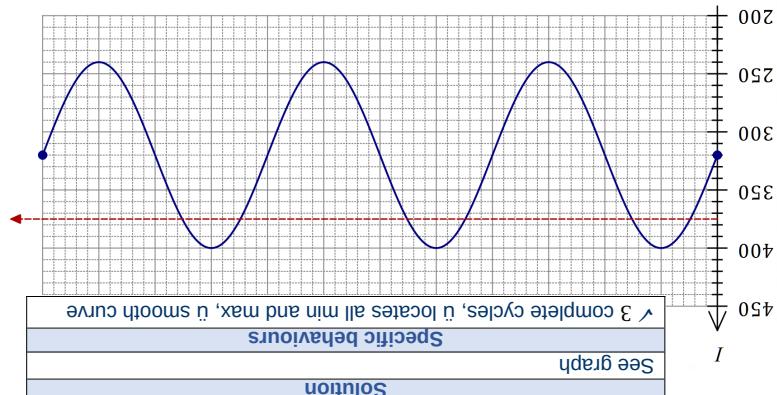
CALCULATOR-ASSUMED

Question 14

When an alternating current is used to power a light globe, the intensity of light emitted from the globe, I lumens, varies with time t milliseconds and can be modelled by the formula

$$I = 320 + 80 \sin\left(\frac{\pi t}{10}\right).$$

(a) Draw the graph of I against t on the axes below for $0 \leq t \leq 60$. (3 marks)



(a) Obtain an expression for the velocity $v(t)$ in the form $v(t) = (at+b)(ct+d)$, where a, b, c and d are integer constants. (3 marks)

(a) a, b, c and d are integer constants.

$$x(t) = 2t^3 - 28t^2 + 98t - 72, \quad 0 \leq t \leq 10.$$

(a) Obtain an expression for the velocity $v(t)$ in the form $v(t) = (at+b)(ct+d)$, where a, b, c and d are integer constants.

✓ indicates derivative of $x(t)$ required
✓ factors into required form
✓ correct derivative
✓ correct derivative of $x(t)$ required
✓ correct velocity

(b) Determine the initial velocity of the body. (1 mark)

✓ correct velocity
✓ correct behaviours
✓ Solution

(ii) the displacement of the body at the instant(s) that it is stationary. (3 marks)

✓ both correct, with units
✓ one correct displacement
✓ times when stationary
✓ correct behaviours

$$\begin{aligned} x(7) &= -72 \text{ cm} \\ x\left(\frac{3}{7}\right) &= \frac{800}{27} \approx 29.63 \text{ cm} \\ v(t) = 0 &\Leftrightarrow t = \frac{3}{7}, t = 7 \end{aligned}$$

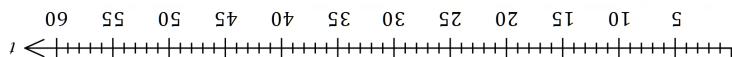
(c) Determine the percentage of each cycle that the intensity of light exceeds 375 lumens. (2 marks)

✓ indicates interval in ms
✓ specific behaviours
$\frac{20}{5.174} \times 100 \approx 26\%$
Solution

(c) Determine the percentage of each cycle that the intensity of light exceeds 375 lumens. (2 marks)

✓ correct period
✓ specific behaviours
Period is 20 milliseconds

(b) State the period of I . (1 mark)



(b) State the period of I . (1 mark)

(c) Determine the displacement of the body at the instant(s) that it is stationary. (3 marks)

(d) Determine the initial velocity of the body. (1 mark)

✓ correct velocity
✓ correct behaviours
✓ Solution

(e) the displacement of the body at the instant(s) that it is stationary. (3 marks)

✓ both correct, with units
✓ one correct displacement
✓ times when stationary
✓ correct behaviours

$$\begin{aligned} x(7) &= -72 \text{ cm} \\ x\left(\frac{3}{7}\right) &= \frac{800}{27} \approx 29.63 \text{ cm} \\ v(t) = 0 &\Leftrightarrow t = \frac{3}{7}, t = 7 \end{aligned}$$

A small body is moving in a straight line. Relative to a fixed point O , it has a displacement of x cm at time t seconds given by

cm at time t seconds given by

(7 marks)

Question 15

A farmer was treating a large area of land for an invasive weed. The area treated on the first day was 275 m^2 . Over the following months more resources were utilised so that the area treated each day was 7.5% more than the previous day.

- (a) Determine the area treated on the 28
- th
- day.

(2 marks)

Solution
$T_{28} = 275(1.075)^{(28-1)} \approx 1938 \text{ m}^2$
Specific behaviours
✓ indicates use of general term formula ü correct area

The cost of the treatment was 35.8 cents per square metre.

- (b) On which day did the cost of the days treatment first exceed \$10 000?

(3 marks)

Solution
$C_n = 0.358 \times 275(1.075)^{(n-1)}$ $\approx 98.45(1.075)^{(n-1)}$
$98.45(1.075)^{(n-1)} \geq 10000$ $n \geq 65$
On day 65.
Specific behaviours
ü adjusts sequence ✓ indicates equation/inequality to solve ü correct day

- (c) Determine, to the nearest ten dollars, the total cost of the first 15 days of treatment.

(2 marks)

Solution
$S_{15} = \frac{98.45(1 - 1.075^{15})}{1 - 1.075} \approx \2570
Specific behaviours
✓ indicates use of sum formula ü total cost, rounded as required

Question 16

(8 marks)

A farm grows two varieties of apples - Fuji and Gala. 42% of all apples are grown in orchard A, 36% in orchard B and the remainder in orchard C. The proportion of Fiji apples that are grown in orchards A, B and C are 25%, 30% and 35% respectively. After harvesting, the farm stores all the apples together in a large silo before using them to make apple juice.

- (a) Determine the probability that an apple chosen at random from the silo is

- (i) a Fuji grown in orchard C.

(2 marks)

Solution
$P(C) = 1 - 0.42 - 0.36 = 0.22$
$P(C \cap F) = 0.22 \times 0.35 \approx 0.077$

Specific behaviours

- ✓ proportion grown in C
- ü correct probability

- (ii) a Gala.

(3 marks)

Solution
$P(A \cap G) = 0.42 \times 0.75 = 0.315$
$P(B \cap G) = 0.36 \times 0.7 = 0.252$
$P(C \cap G) = 0.22 \times 0.65 = 0.143$
$P(G) = 0.315 + 0.252 + 0.143 \approx 0.71$
Specific behaviours
✓ at least one correct proportion ü all correct proportions ü correct probability

- (b) Given that an apple selected at random is a Fuji, determine the probability that it was grown in orchard A.

(3 marks)

Solution
$P(F) = 1 - 0.71 = 0.29$
$P(A \cap F) = 0.42 \times 0.25 = 0.105$
$P(A F) = \frac{0.105}{0.29} = \frac{21}{58} \approx 0.362$
Specific behaviours
✓ $P(F)$ ü $P(A \cap F)$ ü correct probability