PERTH MODERN SCHOOL
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Independent Public School

Course: Mathematics Specialist Year 11
Test 1 2021

Student name: Marking Key Teacher name: ______________

Date: 19th February 2021

Task type: **Response**

Time allowed for this task: **40 mins**

Number of questions: **Eight**

Materials required:

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates

Marks available: ~~**38**~~ **marks** 34

Task weighting: **10%**

Formula sheet provided: **No**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 {2.3.2} 4 (~~3~~ marks)

Show that $3.4\overline{67}$ is a rational number.

If rational it can be written in the form of $\frac{p}{q}$ with $p, q \in \mathbb{Z}$ $q \neq 0$

Let $n = 3.4\overline{67}$

Now $1000n = 3467.\overline{67}$ ... (1)

$10n = 34.\overline{67}$ ... (2)

(1) - (2) $990n = 3433$

ie $n = \frac{3433}{990}$

Hence $3.4\overline{67}$ is rational

✓ defines rational
✓ sets up equations
✓ solves for n
✓ states rational

Question 2 {1.3.1} 3 (~~6~~ marks)

Consider the statement: "If the river floods, then school will be cancelled."
Assuming this to be a true statement:

(a) Write down the contrapositive of the statement. (1)

If the school is not cancelled then the river is not flooding ✓

(b) Is the contrapositive valid? Explain. (2)

Yes, ✓ Flooding would cause the school to be cancelled hence it is not flooding ✓

Question 3 {1.3.1} **(3 marks)**

The statement 'if a natural number is a multiple of 4 and a multiple of 5 then the natural number is a multiple of 20' is true. Write the contrapositive of the statement and explain whether or not the contrapositive is also true.

If a natural number is not a multiple of 20 then it is not a multiple of 4 or not a multiple of 5 ✓NOT ✓OR

True because, for example, 8 is not a multiple of 20. It is a multiple of 4 but not a multiple of 5. } Explain ✓

Question 4 {1.3.1} **(3 marks)**

You are travelling to a town and come to a point where the road splits into two (ie a fork in the road). There are two people (Person A and Person B) standing at the fork and as you do not know which way to go you need to ask for help. It is known that one of the people will always lie and the other will always tell the truth. You can only ask one question to only one of the people. The question you ask of Person A is "Which road would the other person tell me to take?"

Explain why this question would always give you the correct road to take.

If person A lies then they would say that B will say the wrong road ✓

If person A tells the truth then they would say that peron B will say the wrong road ✓

∴ the answer from person A is always the wrong road (so take the other road) ✓

Question 5 {1.3.1, 2.3.1} **(4 marks)**

Prove that the reciprocal of any irrational number is irrational by proving the contrapositive.

Contrapositive If the reciprocal of a number is rational then the number is rational. ✓

Let n be the number. Its reciprocal is $\frac{1}{n}$

If the reciprocal is $\frac{1}{n}$, it can be written as $\frac{p}{q}$ ✓ with $p, q \in \mathbb{Z}$, $q \neq 0$. By implication $p \neq 0$ o'wise reciprocal will not exist Now $\frac{1}{n} = \frac{p}{q}$ ✓

Hence $n = \frac{q}{p}$ which is rational

Hence the contrapositive is true ⇒ the original is true ✓

Question 6 {2.3.1} **(7 marks)**

a) An arithmetic sequence is a set of numbers which has a first term and a common difference. For the sequence {5, 9, 13, 17, ..} the first term is 5 and the common difference is 4.

(i) Write down the next three terms of this sequence. (1 mark)

21, 25, 29

(ii) Choose any three consecutive terms and show that the sum of these terms is a multiple of 3. (2 marks)

eg $9+13+17 = 39$

$= 3 \times 13$ ✓

ie multiple of 3 ✓

b) Prove that the sum of any three consecutive terms of an arithmetic sequence with first term a and common difference d is always a multiple of three, for $a, d \in \mathbb{N}$. (4 marks)

Let m be the middle term ✓

$\therefore$ the terms are: $m-d$, m, $m+d$ ✓

Sum of terms $= m-d+m+m+d$ ✓

$= 3m$

Which is a multiple of 3 ✓

Alternate Sol^{n}

Three terms are: $a+(n-1)d$, $a+nd$, $a+(n+1)d$ ✓

Sum $= a+(n-1)d + a+nd + a+(n+1)d$ ✓

$= 3a + 3nd$

$= 3(a+nd)$ ✓

Which is a multiple of 3 ✓

Question 7 {2.3.1} **(5 marks)**

Prove by that, for every positive real number x, $\frac{x}{x+1} < \frac{x+1}{x+2}$.

If $\frac{x}{x+1} < \frac{x+1}{x+2}$ then $\frac{x}{x+1} - \frac{x+1}{x+2} < 0$ ✓

Now $\frac{x}{x+1} - \frac{x+1}{x+2} = \frac{x(x+2) - (x+1)^2}{(x+1)(x+2)}$ ✓

$= \frac{x^2+2x - x^2 - 2x - 1}{(x+1)(x+2)}$

$= \frac{-1}{(x+1)(x+2)}$ ✓

Numerator is −ve

Denominator is +ve as $x+1>0$ and $x+2>0$
because x is +ve ✓

Hence $\frac{x}{x+1} - \frac{x+1}{x+2} < 0$

$\therefore \frac{x}{x+1} < \frac{x+1}{x+2}$ ✓

Question 8 {1.3.1, 2.3.1} **(5 marks)**

A proposition states that for any integer n, if $n^2 - 4n - 3$ is even, then n is odd.

(a) Write the contrapositive of this proposition. (1 mark)

If n is even then $n^2 - 4n - 3$ is odd ✓

(b) Use the contrapositive statement to prove the proposition is true. (4 marks)

If n is even then $n = 2p \quad p \in \mathbb{Z}$ ✓

Now $n^2 - 4n - 3$

$= (2p)^2 - 4(2p) - 3$ ✓

$= 4p^2 - 8p - 4 + 1$

$= 2(2p^2 - 4p - 2) + 1$ ✓

which is odd

Hence contrapositive is true and therefore original statement is true ✓

END OF TEST