

19. (a)  $A = A_0 e^{0.05t}$   
 $\therefore A = 0.6e^{0.05(10)} = 0.99 \text{ Ha}$   
 (b)  $5 = 0.6e^{0.05t}$   
 $t = 42.4 \text{ hours}$
- (a)  $f'(x) = -\sin x + \cos x$   
 (b) Maximum value occurs when  $f'(x) = 0$   
 $\therefore -\sin x + \cos x = 0 \rightarrow f(x) = \sqrt{2} \text{ when } x = \frac{\pi}{4}$   
 $f''(\frac{\pi}{4}) < 0 \text{ then maximum}$   
 Since  $f''(\frac{\pi}{4}) < 0$  then  $f''(x) = 0$   
 $\therefore -\cos x - \sin x = 0 \rightarrow \tan x = -1$   
 $x = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}$   
 $\therefore \left(\frac{3\pi}{4}, 0\right) \text{ and } \left(\frac{7\pi}{4}, 0\right)$

20. (a)  $f'(x) = -\sin x + \cos x$   
 (b) Maximum value occurs when  $f'(x) = 0$   
 $\therefore -\sin x + \cos x = 0 \rightarrow f(x) = \sqrt{2} \text{ when } x = \frac{\pi}{4}$   
 $f''(\frac{\pi}{4}) < 0 \text{ then maximum}$   
 Since  $f''(\frac{\pi}{4}) < 0$  then  $f''(x) = 0$   
 $\therefore -\cos x - \sin x = 0 \rightarrow \tan x = -1$   
 $x = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}$   
 $\therefore \left(\frac{3\pi}{4}, 0\right) \text{ and } \left(\frac{7\pi}{4}, 0\right)$
- (a)  $f'(x) = -\sin x + \cos x$   
 (b) Maximum value occurs when  $f'(x) = 0$   
 $\therefore -\sin x + \cos x = 0 \rightarrow f(x) = \sqrt{2} \text{ when } x = \frac{\pi}{4}$   
 $f''(\frac{\pi}{4}) < 0 \text{ then maximum}$   
 Since  $f''(\frac{\pi}{4}) < 0$  then  $f''(x) = 0$   
 $\therefore -\cos x - \sin x = 0 \rightarrow \tan x = -1$   
 $x = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}$   
 $\therefore \left(\frac{3\pi}{4}, 0\right) \text{ and } \left(\frac{7\pi}{4}, 0\right)$

21. (a)  $10 \text{ L/sec}$   
 $\frac{10}{20} = 0 \rightarrow t = 200 \text{ secs}$   
 (b)  $F = \int_0^{200} \left(10 - \frac{t}{20}\right) dt$   
 $= 1000 \text{ L}$

22. (a)  $r' = e^{-t}$   
 $r'(4) = e^{-4} = 0.018$   
 and  $r'(5) = e^{-5} = 0.0067$   
 (b)  $r''(t) = -e^{-t}$   
 Since  $-e^{-t} \neq 0$ , then the fire is spreading at its greatest rate at the start of the fire.  
 Or the graph has greatest slope at  $t=0$ .  
 (c)  $A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r$   
 $\therefore \frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = 2\pi r \cdot e^{-t}$   
 $\therefore \frac{dA}{dt} = 2\pi(-e^{-t}) + 0.018 = 0.458 \text{ cm}^2/\text{sec}$   
 When  $t=4$   $\frac{dA}{dt} = 2\pi(-e^{-4}) + 0.018 = 0.458 \text{ cm}^2/\text{sec}$

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## MATHEMATICS METHODS UNIT 3

### Semester One

2018

### SOLUTIONS

10. (a)  $y = (ex)(e^x)$   
 $\frac{dy}{dx} = e^x(ex) + (ex)(e^x) = ex(e^x + e)$
11. (a)  $K = \frac{1}{2} \sin \theta - \frac{d\theta}{dK} = \frac{1}{2} \cos \theta = \frac{1}{2} \cos \theta \text{ where } r = 4$   
 $6K = 8\cos \frac{\theta}{4} \times 0.95 \sin \theta \Rightarrow 6K = 8\cos \theta \times (0.95\sin \theta)$   
 $6K = 0.6896 \times 0.95 \sin \theta \Rightarrow 6K = 0.6456 \sin \theta$
12. (a)  $g(x) = 0.8986 - 0.1853 \times 100 = 8.3\%$   
 $\text{Slope of } g(x) = -0.1853$   
 $\frac{dy}{dx} = 0.8986 - 0.2x + c$   
 $g(x) = 0.8986 - 0.2x + c$   
 $g(0) = 0.8986 - 0.2(0) + c = 0.8986 + c$   
 $g(1) = 0.8986 - 0.2(1) + c = 0.8986 - 0.2 + c = 0.6986 + c$   
 $g(2) = 0.8986 - 0.2(2) + c = 0.8986 - 0.4 + c = 0.4986 + c$   
 $g(3) = 0.8986 - 0.2(3) + c = 0.8986 - 0.6 + c = 0.2986 + c$   
 $g(4) = 0.8986 - 0.2(4) + c = 0.8986 - 0.8 + c = 0.0986 + c$
13. (a)  $f(x) = 4 - \cos \frac{x}{2} + c$   
 $f(\frac{\pi}{2}) = 4 - \cos \frac{\pi}{2} + c = 4 - 2\sqrt{2} \rightarrow c = 4$
14. (a)  $0 = 4 + \frac{1}{4} + (5 + 4) = 3$   
 $-5 + \frac{1}{4} + 7 = -1 + (28 - 14) = 13$   
 $\int_0^3 f(x) dx = 2(4) = 8$
15. (a)  $P(X \geq 25) = 0.1047$   
 $P(X \leq 25) = 0.8953$   
 $P(X \leq 25) = 0.0026$   
 $P(Y \geq 10) = 0.97$   
 $P(Y \leq 10) = 0.03$   
 $n = 19$   
 $Y \sim B(19, 0.03)$   
 $\text{Using trial and error}$   
 $\therefore P(Y \geq 10) \approx 0.99 \rightarrow n = 19$   
 $\therefore Y \sim B(19, 0.02)$   
 $\text{where } Y \text{ is the number who graduated out of } n$
16. (a)  $AK = 0.163$   
 $AK = \frac{1}{2} \sin 0.95\theta - \sin \frac{\theta}{4}$   
 $AK = 0.689$
17. (a)  $\text{Error} = \frac{0.8986 - 0.1853}{0.8986 - 0.1853} \times 100 = 8.3\%$   
 $\text{Slope of } g(x) = -0.1853$   
 $\frac{dy}{dx} = 0.8986 - 0.2x + c$   
 $g(x) = 0.8986 - 0.2x + c$   
 $g(0) = 0.8986 - 0.2(0) + c = 0.8986 + c$   
 $g(1) = 0.8986 - 0.2(1) + c = 0.8986 - 0.2 + c = 0.6986 + c$   
 $g(2) = 0.8986 - 0.2(2) + c = 0.8986 - 0.4 + c = 0.4986 + c$   
 $g(3) = 0.8986 - 0.2(3) + c = 0.8986 - 0.6 + c = 0.2986 + c$   
 $g(4) = 0.8986 - 0.2(4) + c = 0.8986 - 0.8 + c = 0.0986 + c$
18. (a)  $0.957 = 0.0987$   
 $0.957 = 0.0026$   
 $0.957 = 0.00196$   
 $0.957 = 0.00196$
19. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
20. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
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21. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
22. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
23. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
24. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
25. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
26. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
27. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
28. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
29. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
30. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
31. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
32. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
33. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
34. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
35. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
36. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
37. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
38. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
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 $2 = 0$
39. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
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40. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
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 $2 = 0$
41. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
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42. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
43. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
44. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
45. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
46. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
47. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
48. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
49. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
50. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
51. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
52. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
53. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
54. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
55. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
56. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
57. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
58. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
59. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$   
 $-\cos \pi + \cos 0 + \sin \pi - \sin 0 = 0$   
 $-(-1) + 1 + 0 - 0 = 0$   
 $1 + 1 = 0$   
 $2 = 0$
60. (a)  $\int_0^{\pi} (\sin x + \cos x) dx = 0$   
 $\int_0^{\pi} \sin x dx + \int_0^{\pi} \cos x dx = 0$   
 $-\cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi} = 0$

### Calculator-free Solutions

- |     |   |     |
|-----|---|-----|
| 1.  | $f'(x) = 0$ when $\frac{1-x}{e^x} = 0$  | ✓   |
|     | $\therefore x = 1$  | ✓   |
|     | $f''(x) = \frac{(e^x)(-1) - e^x(1-x)}{e^{2x}}$  | ✓   |
|     | Since $f''(1) < 0$ , then maximum   | [4] |
| 2.  | $A = \int_0^k e^{3-x} dx$   | ✓   |
|     | $\therefore \left[ -e^{3-x} \right]_0^k = (-e^{3-k}) - (-e^{3-0})$  | ✓   |
|     | $= -e^{3-k} + e^3$  | ✓   |
|     | If $A = e^3(1 - e^{-k}) = e^3$ then $e^{-k} = 0$ , which is impossible.                                   | ✓   |
|     | If $A = e^3(1 - e^{-k}) = e^3 - 1$  | ✓   |
|     | $\therefore k = 3$  | ✓   |
|     |   | [6] |
| 3.  | (a) Stationary point occurs when $3x^2 - kx = 0$  | ✓   |
|     | $M'(6) = 108 - 6k = 0$  | ✓   |
|     | $\therefore k = 18$   | ✓   |
| (b) | $M(x) = x^3 - 9x^2 + c$   | ✓   |
|     | Since $M(6) = 1 \rightarrow 216 - 324 + c = 1 \rightarrow c = 109$  | ✓   |
|     | $M(x) = x^3 - 9x^2 + 109$   | ✓   |
|     | Vertical intercept is 109   | ✓   |
|     |   | [6] |
| 4.  | (a) $e^t = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}$                                 | ✓   |
|     | $= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} = \frac{217}{24}$                                     | ✓   |
| (b) | $\frac{d}{dx} \left[ 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \right]$ | ✓   |
|     | $= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$                                      | ✓   |
|     | $= \frac{1}{1!} + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \dots = e^x$                           | ✓   |
|     |   | [4] |

- |   |  |
|---|--|
| 16. (a) $2\cos x \sin x + \cos x = 0$<br>$\therefore \cos x = 0 \text{ or } \sin x = -\frac{1}{2}$<br>$\therefore x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$<br>(b) $\left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right), \left(\frac{7\pi}{6}, -\frac{\sqrt{3}}{2}\right), \left(\frac{11\pi}{6}, -\frac{\sqrt{3}}{2}\right)$ | <span style="color: green;">✓✓</span>  |
| $\int_{\frac{\pi}{6}}^{\frac{7\pi}{6}} (-\cos x - \sin 2x) dx$  | <span style="color: green;">✓✓</span>  |
| (c) $A = \frac{1}{2} \times 2.25 \text{ units}^2$   | <span style="color: green;">✓</span>   |
| 17. (a) $\frac{x+1}{x-1} = 0 \rightarrow x = -1$<br>$\frac{dy}{dx} = \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$  | <span style="color: green;">✓</span>   |
| (b) (i) $\frac{dy}{dx} \neq 0$ for any value of $x$ .<br>(ii) True since $\frac{d^2y}{dx^2} = \frac{4}{(x-1)^3}$  | <span style="color: green;">✓</span>   |
| (c) (i) $\frac{d^2y}{dx^2} \neq 0$<br>(ii) True since $\frac{d^3y}{dx^3} \neq 0$ for any value of $x$ .   | <span style="color: green;">✓</span>   |
| 18. (a) $v(0) = 3$<br>(b) Stationary when $2t^2 - 5t + 3 = 0$<br>$\therefore t = 1 \text{ and } 1.5$<br>(c) (i) $a = 4t - 5$<br>(ii) $v$ is a minimum when $a = 0 \rightarrow a = 1.25$   | <span style="color: green;">✓</span><br><span style="color: green;">✓✓</span><br><span style="color: green;">✓</span><br><span style="color: green;">✓✓</span> |
| (d) $x = \frac{2}{3}t^3 - \frac{5}{2}t^2 + 3t + c$<br>$\text{Since } x(0) = 0 \rightarrow c = 0$<br>$x(t) = \frac{2}{3}t^3 - \frac{5}{2}t^2 + 3t$<br>$\therefore x(3) = 4.5$  | <span style="color: green;">✓</span><br><span style="color: green;">✓</span><br><span style="color: green;">✓</span>   |
| $\text{Distance} = \int_0^3  v(t)  dt$<br>(e) $= 4.58$<br>(f) Distance travelled $\neq$ Displacement  | <span style="color: green;">✓</span><br><span style="color: green;">✓</span><br><span style="color: green;">✓</span>   |

$$\begin{aligned} V(Y) &= V(\lambda A(X)) = \lambda^2 V(A(X)) = 20 \\ \text{and } V(A(X)) &= \int_{-\pi}^{\pi} \sin^2(x) dx = \frac{1}{2} \int_{-\pi}^{\pi} (1 + \cos(2x)) dx = \frac{1}{2} (\pi) = \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned}
 (c) \quad E(X) &= 0.1 + 0.1 + 0.1 + 0.1 + 0.1 = 0.5 \\
 (d) \quad V\text{ariance} &= (0.1 - 0.5)^2 + (0.2 - 0.5)^2 + (0.3 - 0.5)^2 + (0.4 - 0.5)^2 + (0.5 - 0.5)^2 = 0.15
 \end{aligned}$$

(a) Since PDF  $f(x) = 0.8 + a$  is 1  $\rightarrow a = -\frac{3}{4}$

(b)  $f(0) = -\frac{3}{4}$   
 $\therefore$  Not PDF  
 $f(x)$  is not greater than or equal to 0 for all x

(c) Since  $P(X > 2 | X \leq 2) = \frac{P(X > 2)}{P(X \leq 2)}$   
 $\therefore$  Since  $P(X > 2) = 0.8 + a$

$$\begin{aligned}
 & \text{(a) } \mathbb{P}(X = 1) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \\
 & \text{and } 0 \leq f(x) \leq 1 \text{ for all } x \\
 & \text{PDF} \\
 & \text{(b) } \mathbb{P}(X = 1) = 1 \\
 & \text{and } 0 \leq f(x) \leq 0 \text{ for all } x \\
 & \text{PDF}
 \end{aligned}$$

Mathematics Methods Unit 3 Solutions