



**PERTH MODERN SCHOOL**

Exceptional schooling. Exceptional students.

Independent Public School

Year 12 Methods

TEST 1

Friday 22 February 2019

TIME: 45 minutes working

One-page notes allowed

**Calculator Assumed**

39 marks 7 Questions

Name: \_\_\_\_\_ Teacher: \_\_\_\_\_

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

**Question 1**

**(6 marks)**

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	1	-2	-1
2	2	-1	1	0
3	1	-2	2	1

(a) Define  $h(x) = \frac{f(x)}{g(x)}$ , use the table to find the value for  $h'(2)$ . (3 marks)

(b) Define  $I(x) = [g(x)]^5$ , use the table to find the value for  $I'(1)$ . (3 marks)

**Question 2****(3 marks)**

Find the equation of the line tangent to the function  $y = (3x^2 - 2)^3$  at the point  $(2, 2)$ . Give your answer in the gradient-intercept form.

**Question 3****(3 marks)**

If  $\frac{dy}{dx} = (5x + 3)^3$ , and  $y = 50$  when  $x = 1$ , determine the expression of  $y$  in terms of  $x$ .

**Question 4****(7 marks)**

A company is purchasing a type of thin sheet metal required to make a closed cylindrical container with a capacity of  $4000\pi \text{ cm}^3$ . Let the radius of the cylindrical base be  $r$  and the height be  $h$ .

- (a) Show that the surface area of the cylinder can be expressed as  $2\pi r^2 + \frac{8000\pi}{r}$ .

**(3 marks)**

- (b) Using calculus, determine the least area of metal required to make a closed cylindrical container from thin sheet metal in order that it will have a capacity of  $4000\pi \text{ cm}^3$ .

(Work to one decimal place)

**(4 marks)**

**Question 5****(6 marks)**

A share portfolio, initially worth \$26 000, has a value of  $f$  dollars after  $t$  months, and begins with a negative rate of growth. The rate of growth remains negative until after 20 months ( $t=20$ ) when the value of the portfolio is momentarily stationary and then continues with negative growth for the life of the investment. The value of the portfolio,  $f(t)$  after  $t$  months can be modelled by the following model,  $f(t) = -2t^3 + bt^2 + ct + d$ ,  $0 \leq t \leq 37$  months where  $b, c$  &  $d$  are constants.

Determine the values of the constants  $b, c$  &  $d$ .

**Question 6****(8 marks)**

The volume,  $V$  in cubic metres and radius  $R$  metres, of a spherical balloon are changing with time,

$t$  seconds.  $V = \frac{4\pi R^3}{3}$ . The radius of the balloon at any time is given by  $R = 2t(t+3)^3$ .

Determine the following:

a) The value of  $\frac{dR}{dt}$  when  $t = 1$ . (3 marks)

b) The value of  $\frac{dV}{dt}$  when  $t = 1$ . (3 marks)

Consider the volume of the balloon at  $t = 1$ .

c) Use the incremental formula to estimate the change in volume 0.1 seconds later (i.e.  $t = 1.1$ ) (2 marks)

**Question 7****(6 marks)**

The position of a train on a straight mono rail,  $x$  metres at time  $t$  seconds, is modelled by the following formula for the velocity,  $v$  in metres/second,  $v = pt^2 - 12t + q$  where  $p$  &  $q$  are constants.

The deceleration of the train is  $8ms^{-2}$  when  $t = 1$ . The train has a position  $x = \frac{4}{3}$  when  $t = 2$  and is initially at the origin ( $x = 0$ ).

a) Determine the values of the constants  $p$  &  $q$ .

**(4 marks)**

b) Determine the position of the train when the acceleration is  $12ms^{-2}$ .

**(2 marks)**

Supplementary Page

Question Number: \_\_\_\_\_

Supplementary Page

Question Number: \_\_\_\_\_