PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course Specialist Test 4 Year 12

Student name: ______________ Teacher name: ______________

Task type: **Response**

Time allowed for this task: ____40_______ **mins**

Number of questions: _____7______

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **_44_____ marks**

Task weighting: **_10___%**

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3 & 3 = 6 marks)
Solve the following.

a) $\frac{dy}{dx}=\frac{3x-2}{y(5-y^2)}$ given that when $x=1,\ y=1$.

Solution
$\frac{dy}{dx}=\frac{3x-2}{y(5-y^2)}$ $\int y(5-y^2)dy=\int 3x-2$ $\frac{5}{2}y^2-\frac{1}{4}y^4=\frac{3}{2}x^2-2x+c$ $x=1.y=1\ c=\frac{11}{4}$
Specific behaviours
P separates variables P integrates all terms P solves for constant

b) $3x^4\cos(2y)\frac{dy}{dx}=10$ given that when $x=5,\ y=\pi$.

Solution
$3x^4\cos(2y)\frac{dy}{dx}=10$ $\int\cos(2y)dy=\int\frac{10}{3}x^{-4}dx$ $\frac{1}{2}\sin(2y)=-\frac{10}{9}x^{-3}+c$ $x=5,\ y=\pi,\ c=\frac{2}{225}$ (c=+2/225)
Specific behaviours
P separates variables P integrates all terms P solves for constant

Specific behaviours
P determines a confidence interval for population mean using sample given. P looks to see if 23.4 lies in interval or discusses that not every interval contains u P states that fault is likely as 23.4 lies outside interval OR no inference possible

$\text{invNormCDf}(\text{"C"}, 0.95, 1, 0)$

-1.959963985

$\text{solve}\left(1.96 \cdot \frac{3.2}{\sqrt{n}} = 0.35, n\right)$

$\{n = 321.1264\}$

Sample size = 322

Specific behaviours
P states correct z score (must show) P sets up equation for n P rounds up

c) On a particular day the operator of a machine that makes jelly beans is suspected of being faulty. A sample of 200 jelly beans had a sample standard deviation of 3.8 mg with a total mass of 5.4 grams. Present a mathematical argument to either support or to dismiss such a claim.

Solution

$\bar{x} \sim N\left(\frac{5400}{200}, \left[\frac{3.8}{\sqrt{200}}\right]^2\right)$

95% *confidence interval*

Lower 26.962761
Upper 27.037239
$\bar{x}$ 27
n 200

23.4 does not lie in interval hence there is evidence that machine is faulty and a different population mean is likely.
OR
As not every confidence interval contains the true value of mean, no inference can be made on one confidence interval

Q2 (4 marks)

An iron has a temperature of $54\ C$ is left in a room, of temperature $18\ C$, to cool such that the temperature $T\ C$ at time t minutes is given by $\frac{dT}{dt} = k(T - 18)$. After 15 mins the temperature of the iron is $37\ C$. Determine the time taken for the iron's temperature to drop to $22\ C$.

Solution

$$\frac{dT}{dt} = k(T - 18)$$

$$\int \frac{dT}{(T-18)} = \int k\, dt$$

$$\ln|T - 18| = kt + c$$

$$t = 0, c = \ln|54 - 18| = \ln 36$$

$$t = 15, T = 37, \ln 19 = 15k + \ln 36$$

$$k = \frac{1}{15}\ln\frac{19}{36}$$

Edit Action Interactive

Simp

$\text{solve}\left(\ln(4) = \frac{1}{15} \cdot \ln\left(\frac{19}{36}\right) \cdot t + \ln(36), t\right)$

$\{t = 51.57158847\}$

51.57 minutes

Specific behaviours
P separates variables and integrates Pderives an expression involving both variables P solves for both constants (may be approx.) P determines approx time and must give units

Q3 (1, 5 & 2 = 8 marks)

The number N thousands, of bacteria cells living in a petri dish at time t hours is given by

$$\frac{dN}{dt}=0.30N-0.05N^2$$.

The initial number of cells was 2 thousand.

a) What is the limiting value of the number of cells as $t\to\infty$?

Solution
$\frac{dN}{dt}=0.30N-0.05N^2=N(0.30-0.05N)$ $N=\frac{0.3}{0.05}=6$ 6 thousand
Specific behaviours
P states limiting value with units

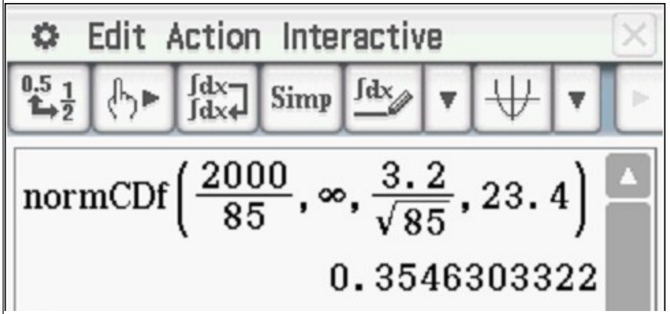
b) Using calculus and partial fractions, show every step to express N in terms of t.

Solution

Q7 (2, 3 & 3 = 8 marks)

A lolly company makes jelly beans where the mass of one jelly bean is normally distributed with a mean of 23.4 mg and a standard deviation of 3.2 mg. (Note: 1g=1000mg)

a) Determine the probability to two decimal places that the total mass of 85 jelly beans is more than two grams.

Solution
$\bar{x}\sim N\left(23.4,\left[\frac{3.2}{\sqrt{85}}\right]^2\right)$ $P\left(\bar{x}>\frac{2000}{85}\right)$  Prob=0.35
Specific behaviours
P uses correct parameters P determines prob to 2 dp

b) Given that the probability that the mean mass of a jelly bean differs from the population mean by more than 0.35 mg is 5%, determine n, the number of jelly beans that need to be sampled.

Solution

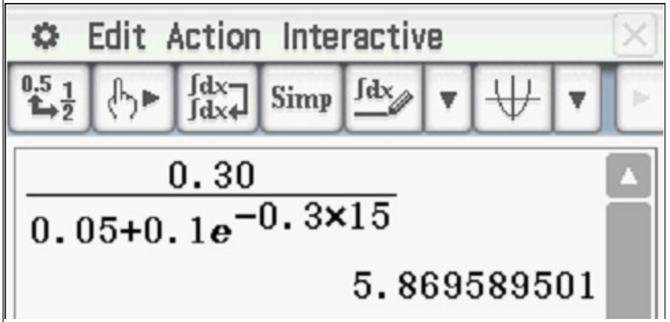
Consider an object that is initially at the origin and at rest such that its acceleration is given by $\frac{dv}{dt}=\frac{1+v^3}{v}\,m/s^2$ where v equals the speed in m/s at t seconds . Determine the exact speed when its displacement from the origin is $\ln(3)$ metres.

Solution
$\frac{dv}{dt}=\frac{1+v^3}{v}$ $v\frac{dv}{dx}=\frac{1+v^3}{v}$ $\int\frac{v^2}{1+v^3}dv=\int dx$ $\frac{1}{3}\ln\lvert 1+v^3\rvert=x+c$ $v>0,1+v^3=ce^{3x}$ $x=0,v=0,c=1$ $1+v^3=e^{3x}=e^{\ln 3^3}=27$ $v=\sqrt[3]{26}$
Specific behaviours
P separates variables and uses appropriate form of acceleration P integrates and uses a constant P solves for constant P determines exact speed

$\frac{dN}{dt}=0.30N-0.05N^2=N(0.30-0.05N)$ $\int\frac{dN}{N(0.30-0.05N)}=\int dt$ $\frac{1}{N(0.30-0.05N)}=\frac{a}{N}+\frac{b}{(0.30-0.05N)}$ $1=a(0.30-0.05N)+bN$ $N=0$ $1=0.3a,\ a=\frac{10}{3}$ $N=6$ $1=6b,\ b=\frac{1}{6}$ $\frac{10}{3}\ln N-\frac{10}{3}\ln\lvert 0.30-0.05N\rvert=t+c \quad Note: N<6\ \therefore 0.30-0.05N>0$ $\ln\frac{N}{0.30-0.05N}=0.3t+c$ $\frac{N}{0.30-0.05N}=ce^{0.3t}$ $\frac{0.30-0.05N}{N}=ce^{-0.3t}$ $0.30-0.05N=Nce^{-0.3t}$ $N=\frac{0.30}{0.05+ce^{-0.3t}}$ $\text{solve}\left(2=\frac{0.3}{0.05+c},c\right)$ $\{c=0.1\}$
Specific behaviours
P separates variables P uses partial fractions and shows working for constants P shows why absolute value not needed for log function P rearranges to make N the subject with a constant Psolves for constant

c) Determine the number of cells after 15 hours.

Solution

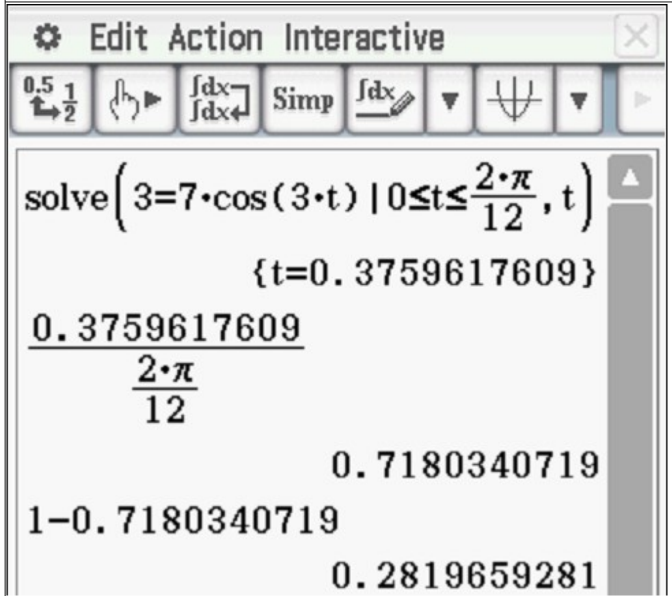


Approximately 5870 cells

Specific behaviours
P subs t=15 into rule from part b
P determines quantity with units of thousands or 5870 cells
(Note –max –1 for units for entire question)

c) Determine the percentage of the time, to one decimal place, that the object is less than 3 metres from the mean position, $x = 0$.

Solution



Specific behaviours
P solves for times at x=3 in a cycle or part cycle
P determines an interval time and then divides by total length of cycle or part cycle
P determines percentage

Q6 (4 marks)

c) Determine the equation of the solution curve that contains (1,1).

Solution
$\frac{dy}{dx} = x^2 - x - 6$ $y = \frac{x^3}{3} - \frac{x^2}{2} - 6x + c$ $1 = \frac{1}{3} - \frac{1}{2} - 6 + c$ $c = \frac{43}{6}$
Specific behaviours
P integrates x terms P solves for constant

Q5 (2, 2 & 3 = 7 marks)

Consider an object that is moving with Simple Harmonic Motion such that $\ddot{x} = -9x$ with x, t in metres and seconds respectively. At $t = 0$, $x = 7$ metres and is a rest.

a) Determine a rule for x in terms of t.

Solution
$x = 7\cos 3t$
Specific behaviours
P uses an appropriate trig function P states all constants

b) Determine the exact speed when $x = 3$ metres.

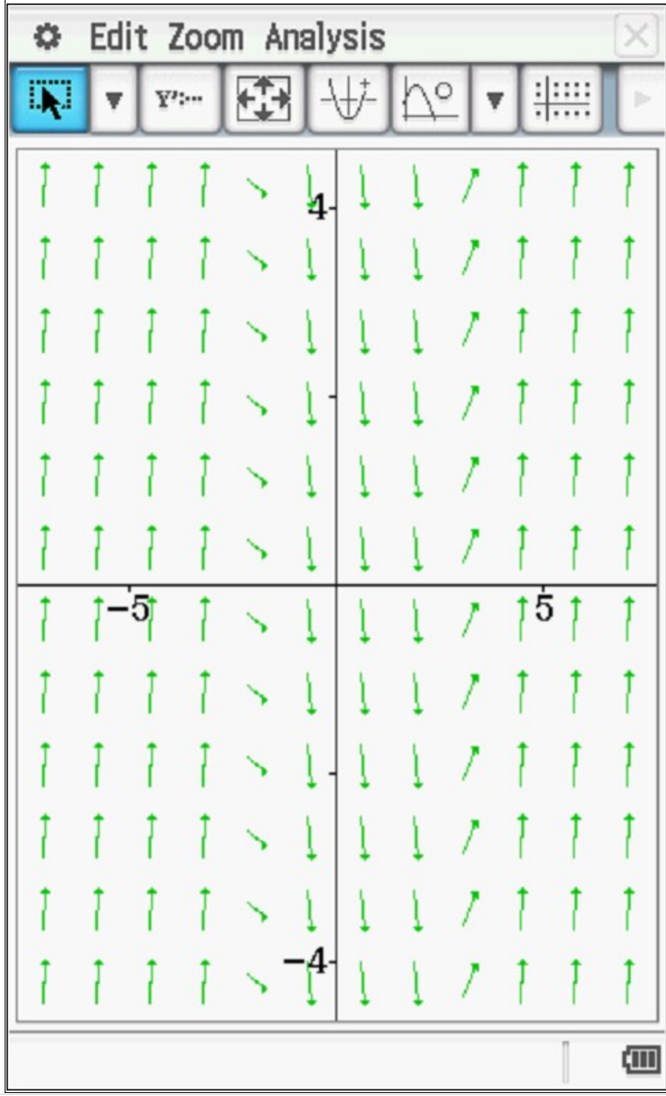
Solution
$v^2 = n^2(A^2 - x^2)$ $= 9(49 - 9)$ $v = \sqrt{360}$
Specific behaviours
P uses appropriate rule P states exact speed, ignore units

Q4 (3, 2 & 2 = 7 marks)

Consider the slope field $\frac{dy}{dx} = (x-3)(x+2)$

a) Sketch this field on the axes below.

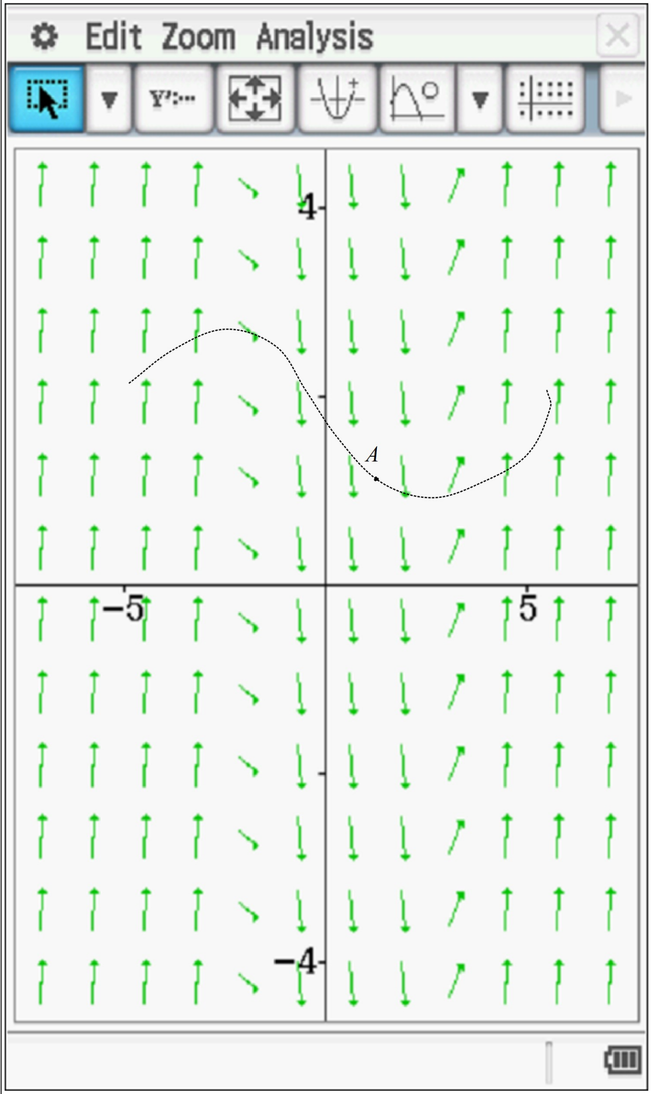
Solution



Specific behaviours
P shows horizontal grads at x=-2 P shows horizontal grads at x=3 P pattern at far left and right

b) Draw the solution curve, axes above, that contains the point (1,1).

Solution



Specific behaviours
P shape of solution curve P shows pt A labelled on curve