PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS METHODS
UNITS 3&4

Section Two: Calculator-assumed

In words　　　　　_____

Your name　　　　_____

Time allowed for this section
Reading time before commencing work:　　ten minutes
Working time:　　　　　　　　　　　　　one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items:　　pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items:　　drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
8		8	14		9
9		6	15		8
10		7	16		8
11		7	17		9
12		8	18		19
13		8			

See next page

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	34.5
Section Two: Calculator-assumed	11	11	100	97	65.5
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65.5% (97 Marks)**

This section has **eleven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(8 marks)**

The manager of a local bank wants to know what proportion of their customers never use cash and has asked two clerks to collect sample data by standing in the bank foyer for an hour before lunch on a Friday morning and questioning as many people as they can.

(a)   Identify and explain two possible sources of bias with this sampling procedure.   (4 marks)

(b)   Briefly describe a sampling procedure that the manager could use in order to minimise all sources of bias.   (2 marks)

From the 150 responses obtained using a reliable sampling procedure, the manager was presented with the confidence interval $(0.1652, 0.2748)$ for the proportion of their customers who never use cash.

(c)   Determine the number of customers in the sample who said they never use cash.
                                                                                                                                    (2 marks)

Question 9 **(6 marks)**

The loudness, measured in decibels, of a sound is given by:

$L = 10 \ln\left(\dfrac{I}{I_0}\right)$ where I is the intensity measured in $watt/m^2$. I_0 is 10^{-12} which is barely audible.

(a) Ear damage occurs if the intensity of a sound is greater than $8.1 \times 10^{-9}\, watt/m^2$.
What is the maximum loudness of a sound to prevent damage?

(2 marks)

(b) If the loudness of a sound at a concert is 110 decibels, find the intensity of the sound.
Give your answer in scientific notation rounded to 2 decimal points.
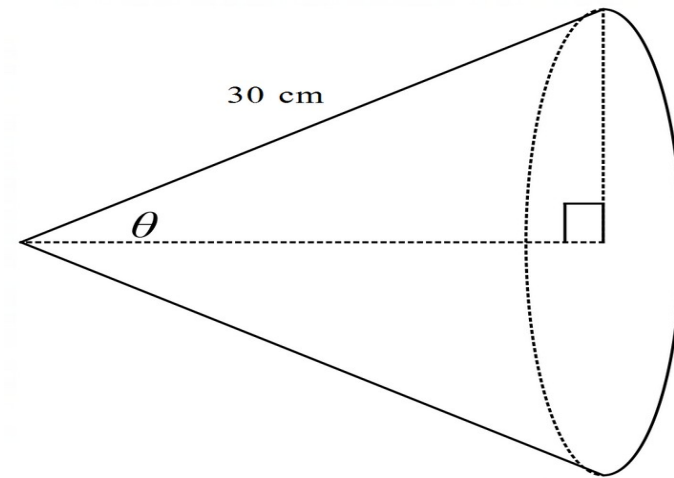
(2 marks)

(c) If the intensity of a sound is tripled, what effect does this have on the loudness?
Justify your answer.

(2 marks)

Question 10 **(7 marks)**

A line segment 30 cm in length forms an angle of θ with the x-axis. A cone is formed by rotating this segment around the x-axis.

(a) Show the volume of this cone can be written as: $V = 9000\pi \sin^2\theta (\cos\theta)$.

(1 mark)

(b) Using calculus methods find the maximum possible volume of this cone.

(4 marks)

See next page

(c) If the size of the angle decreases by 0.02, use the incremental formula to find the corresponding decrease in the volume of the cone when $\theta = \dfrac{\pi}{6}$.

(2 marks)

Question 11 **(7 marks)**

The air pressure, P kPa, inside the tyre of a motor vehicle t seconds after it was punctured can be modelled by the equation $P = a + 122\,e^{kt}$, where a and k are constants.

The initial pressure in the tyre was 220 kPa and after 8.5 seconds it had dropped to 142 kPa.

(a)   Determine the value of a and the value of k.   (3 marks)

(b)   Determine

(i)   the pressure in the tyre after 5 seconds.   (1 mark)

(ii)   the time taken for the pressure in the tyre to fall to 99 kPa.   (1 mark)

(c)   Given that the pressure was falling at a rate of 8 kPa per second after 5 seconds, use the increments formula to estimate the pressure in the tyre after 5.1 seconds.   (2 marks)

Question 12 **(8 marks)**

Alan works from home every Tuesday and always starts work after his digital clock first shows $8\!:\!10$ am and before it shows $8\!:\!30$ am.

The probability density function for T, the time in minutes after $8\!:\!10$ that he starts work, is $f(t)$ and is displayed at right.
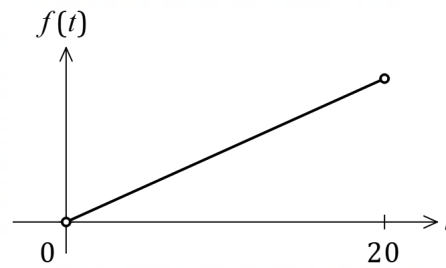
(a) Write the defining rule for the probability density function $f(t)$. (2 marks)

(b) Determine the probability that on a randomly chosen Tuesday, Alan starts work after his clock first shows $8\!:\!23$ am. (2 marks)

(c) Determine the mean and standard deviation of T. (4 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 13 **(8 marks)**

The graph of $y=f(x)$ and a table of values for the function f are shown below.

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
y	0.96	1.38	1.68	1.94	2.16	2.42	2.64	2.84	3.03
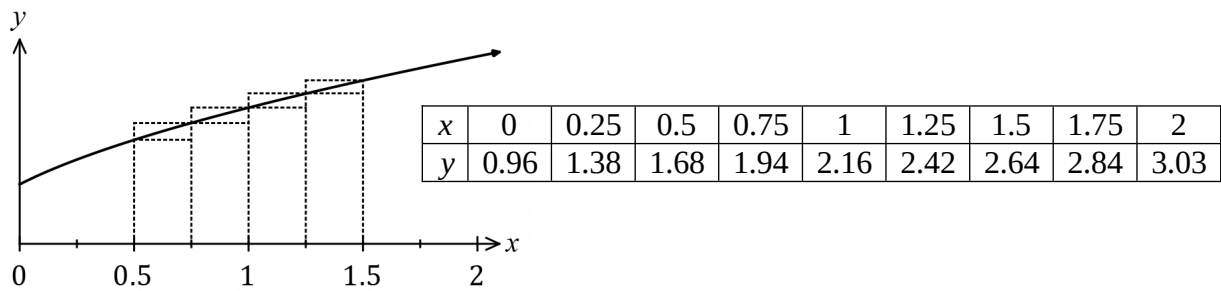
(a) By considering the areas of the rectangles shown, demonstrate and explain why 2.17 is a reasonable estimate for $\int_{0.5}^{1.5} f(x)\,dx$. (3 marks)

(b) Determine, with justification, estimates for

(i) $\int_{0.5}^{1.5} 4f(x)\,dx.$ (1 mark)

(ii) $\int_{0.5}^{1.5} f(x)+4\,dx.$ (2 marks)

(iii) $\int_{0}^{1} f\left(\frac{x}{2}\right)dx.$ (2 marks)

See next page

Question 14 **(9 marks)**

The quantity of juice, in ml, that can be extracted by a machine from different sizes of oranges follows a normal distribution as shown in the table below.

	Mean	Variance
Small	60	36
Large	80	σ^2

(a) What is the probability that more than 65 ml of juice can be extracted from one small orange?

(2 marks)

It is known that 5% of large oranges produce more than 95 ml of juice.
(b) Calculate the value of σ. (3 marks)

If I buy 4 small oranges
(c) (i) Determine the probability that at least three of the oranges produce more than 65 ml of juice. (2 marks)

(ii) Find the probability that at least three oranges produce more than 65ml given that at least one of the four produces more than 65 ml. (2 marks)

See next page

Question 15 **(8 marks)**

Repair tasks undertaken by technical staff who work at an IT company are assigned minor, major or critical status. Over the long term, 4 % of the tasks have been critical, 24 % major and the remainder minor.

(a) Assuming that the long-term proportions are correct, determine the smallest sample size required so that the width of a 90 % confidence interval for the proportion of minor tasks is less than 0.093. (3 marks)

At the end of one month, a manager suspects that the proportion of major tasks has changed and so she takes a random sample of 250 tasks from the last month, of which 45 were major.

(b) Use this sample to construct a 95 % confidence interval for the proportion of major tasks. (3 marks)

(c) Does your confidence interval in part (b) support the managers suspicions? Justify your answer. (2 marks)

Question 16 **(8 marks)**

The acceleration a m/s^2 of a train moving in a straight line at time t seconds is given by

$$a = k + \frac{3}{8}\cos\left(\frac{\pi t}{12}\right), 0 \leq t \leq 60.$$

Initially, the train was at an origin O and moving with a velocity of 2 m/s.

(a) Determine the velocity of the train after 12 seconds when the constant $k = 0.8$. (3 marks)

After 24 seconds the displacement of the train relative to the origin O was 120 m.

(b) Determine the value of the constant k and hence calculate, to the nearest metre, the displacement of the train after 40 seconds. (5 marks)

Question 17 **(9 marks)**

Members of a toy library may take home up to 5 toys per visit. The following frequency table shows the number of toys borrowed by a random sample of 80 members.

Toys borrowed	0	1	2	3	4	5
Frequency	0	10	26	28	12	4

You may assume that relative frequencies obtained from the above data are reliable point estimates of probabilities and that the number of toys borrowed by any two members are independent.

(a) Determine the probability that a member borrows fewer than 5 toys, given that they borrowed at least 3 toys. (2 marks)

(b) Determine the probability that at least 3 of the next 5 borrowers take home an even number of toys. (3 marks)

(c) Show that the mean of the random variable X, the number of toys borrowed by a member, is 2.675. (2 marks)

Observations indicate that members spend 7 minutes at the library plus 3 minutes per toy chosen.

(d)   Determine the mean and standard deviation of the random variable T, the time in minutes spent by members at the toy library.   (2 marks)

Question 18 **(19 marks)**

A researcher wishes to estimate, with 95% confidence, the proportion of people who do not have a land line phone.
She decides to survey people outside her local mobile phone store every morning for a week around lunchtime and asks each person whether they have a landline phone.

(a)   Identify and explain two sources of bias in the proposed sampling method.

(4 marks)

She interviewed 560 people, of these 420 did not have a land line phone.

(b)   Calculate a 95% confidence interval of the true proportion p.
        of people who do not have a land line.

(3 marks)

(c)   Which of the statements below is a valid interpretation of this confidence interval?

(1 mark)

(i)   There is a 95% chance that the true value of p lies within the interval.

(ii)   If many different samples of size 560 were selected and based on each sample, a confidence interval was constructed, 95% of the time the true value of p would lie within the interval.

(iii)   If many different random samples of size 560 were selected and based on each sample, a confidence interval was constructed, in the long term 95% of the confidence intervals would contain the true value of p.

(d) A previous study shows 60% of those interviewed did not have a land line. The researcher wishes to be accurate to within 2% of the true proportion.
Find the minimum sample size required.

(2 marks)

(e) The results from the previous studies were lost. The researcher needs to produce some evidence to justify a minimum sample size for a 95% confidence level.
What assumption does the researcher need to make and what effect if any would this have on the minimum sample size required to be accurate within 2% of the true population?

(2 marks)

(f) The researcher was not pleased with her results and decided to complete another study.
85 of those surveyed, stated they had a landline.
A confidence interval for those having a landline was created: $0.1267 < p < 0.2132$.

(i) Find the value for n,

(2 marks)

(ii) The interval has $a\%$ confidence level. Find the value of a,

(3 marks)

(iii) The researcher decides because of time constraints to either reduce the confidence level or decrease the sample size. What effects would these have on the margin of error?

(2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Supplementary page

Question number: _____