

Mathematics 3C/3D
Semester One Exam, 2010



Name: _____

Teacher: _____

Solutions

Section Two: Calculator-assumed

TIME ALLOWED FOR THIS PAPER

Reading time before commencing work: 10 minutes

Working time for this section: 100 minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items:

drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks
Section One	8	8	50	40
Section Two: Calculator-assumed	13	13	100	80
				120

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.

Section Two: Calculator-assumed (80 Marks)

This section has **fifteen (15)** questions. Answer all questions.
Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.

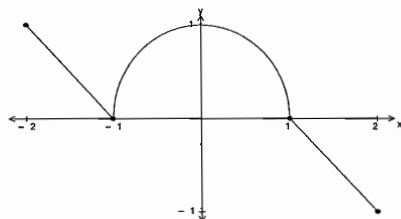
Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Suggested working time for this section is 100 minutes.

STRUCTURE OF THE PAPER

Question	Marks	Total
9	5	
10	4	
11	13	
12	3	
13	5	
14	5	
15	6	
16	7	
17	3	
18	4	
19	7	
20	4	
21	5	
22	5	
23	4	
TOTAL	80	

Question 9. [5 marks]



$y = f(x)$ is shown above. Note: $f(x) = 0$ has solutions of $x = -1, 1$

Solve for real values of x .

$$(a) \quad f(x^2) = -1 \quad x^2 = 2 \quad (1) \quad [2]$$

$$x = \sqrt{2} = 1.41 \quad (1)$$

$$(b) \quad [f(x)]^2 - 1 = 0 \quad y = 1 \quad x = -2, 0 \quad (1) \quad [3]$$

$$[f(x)]^2 = 1 \quad y = -1 \quad x = 2 \quad (1)$$

$$f(x) = \pm 1$$

Question 10. [4 marks]

A new packing machine is being introduced into a sugar mill. It was found that the machine packs 500 gram bags of sugar with a normally distributed with a mean 514g and standard deviation 12 grams. A batch of 1000 bags was packed on a given day.

(a) How many of these bags would you expect to contain:

$$(i) \quad \text{less than 500 grams of sugar,} \quad [1]$$

$$P(X < 500) = N \text{ CDF}(-\infty, 500, 12, 514)$$

$$= 0.1216 \quad \therefore \text{no less than 500} \approx 122.$$

$$(ii) \quad \text{between 500 and 520 grams?} \quad [1]$$

$$P(500 < X < 520) = 0.5697$$

$$\text{No between 500 and 520} \approx 570$$

(b) Calculate the 75-percentile for the entire population.

$$P(X < x) = 0.75 \quad [2]$$

$$x = 522 \quad (1) \quad \text{g for units.}$$

Question 23 [4 marks]

The velocity of a particle v cm/s, as it moves from rest along a straight line is given by $v = 8\sqrt{x}$ where x is its distance from the origin.

Show that if δx and δv denote corresponding small increases in x and v , then

$$\delta v \approx \frac{32 \delta x}{v}$$

Hence find the approximate change in the velocity of the particle when x increases from 36 to 37 cm.

$$\frac{dv}{dx} = 8 \cdot \frac{1}{2} x^{-1/2} \quad (1)$$

$$= \frac{4}{\sqrt{x}} \cdot \frac{8}{8} \quad (1)$$

$$= \frac{32}{8\sqrt{x}}$$

$$= \frac{32}{v} \quad (1)$$

$$\delta v \approx \frac{32}{v} \delta x$$

$$\text{When } x = 36 \quad v = 48 \quad \delta x \approx 1$$

$$\delta v \approx \frac{32 \times 1}{48}$$

$$\approx 0.67 \text{ or } 2/3 \text{ cm/sec} \quad (1)$$

Question 21 [5 marks]

In a certain population, 20% of the people have blue eyes.
(a) Find the probability that in a random sample of 10 people:

(i) exactly 2 have blue eyes.
 $P(X=2) = \text{Binomial PDF}(2, 10, 0.2)$

(ii) at most 4 have blue eyes.
 $P(X \leq 4) = \text{Binomial CDF}(4, 10, 0.2)$

(iii) at least 3 have an eye colour other than blue.
 $P(\text{other than blue}) = 0.8$

(b) What is the size of the smallest sample of people for which the probability that 3 or more have blue eyes, is at least 0.2.
 $P(X \geq 3) = \text{Binomial CDF}(3, 10, 0.2)$

Trial and guess
 $n=6, n=7, n=8, n=9$

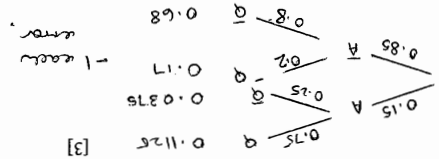
$\therefore P > 0.2 \quad n = 8, \text{ trials}$

Question 22 [5 marks]

In a certain large company 15% of the employees have attended a health and safety training course. Of the employees who have attended the training course, 75% are qualified to perform first aid, whereas of the employees who have not attended the training course only 20% are qualified to perform first aid.

(a) What percentage of employees in the company are not qualified to perform first aid?

	Q	A	Total
A = Attends	11.25	3.75	15
Q = Qualified	17	68	85
Total	28.25	71.75	100%

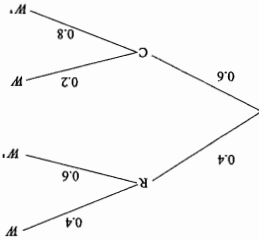


(b) A randomly chosen employee is found to be qualified to perform first aid. What is the probability that she attended the training course?

$P(A|Q) = \frac{P(A \cap Q)}{P(Q)}$
 $= \frac{11.25}{28.25} \approx 39.8\%$

Question 11. [13 marks]

On a Saturday afternoon, Frances always goes to either the races(R) or the casino(C).
Which ever of these options she chooses, she either wins or doesn't win.
Her probabilities for her choices are shown below.

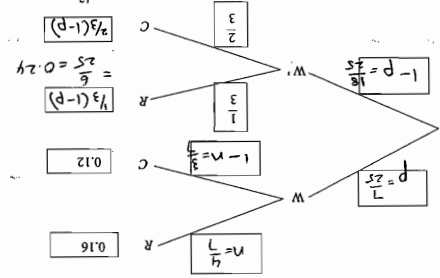


(a) $P(W|R) = \frac{P(W \cap R)}{P(R)} = \frac{0.4}{0.24} = \frac{40}{24}$

(b) $P(W) = 0.16 + 0.12 = 0.28$

(c) $P(R|W) = \frac{P(R \cap W)}{P(W)} = \frac{0.16}{0.28} = \frac{28}{16}$

(d) Complete the diagram below.



(e) Are the events R and W independent? Explain.

$P(R \cap W) = P(R) \times P(W)$
 $= (0.16 + 0.24) \times 0.28$
 $= 0.112$
 $P(W|R) = 0.16$
 $0.112 \neq 0.16 \therefore \text{not independent.}$

3 for correct completion of tree diagram
-1 each incorrect ending.
[2]

Question 12. [3 marks]

A bacterial culture grows according to the rule $\frac{dc}{dt} = 0.03C$, where C is the size of the culture and t is time measured in days. How long will it take for the culture to double in size?

$$C = C_0 e^{0.03t} \quad (1)$$

$$C = 2C_0 \quad \text{i.e.} \quad 2 = e^{0.03t} \quad (1)$$

$$t = 23.1 \quad (1)$$

$\approx 24 \text{ days}$ - if incorrect
or no units

Question 13. [5 marks]

A committee of 6 people is to be chosen to edit the school magazine. 10 girls and 5 boys apply, and the school decides to appoint 4 girls and 2 boys.

(a) How many different committees can be formed?

[3 marks]

$$\binom{10}{4} \times \binom{5}{2} = 2100 \quad (1)$$

(b) Two of the girls who applied are the twins Sarah and Samantha. Assuming that all possible committees have the same probability of being formed, what is the probability that both Sarah and Samantha are selected?

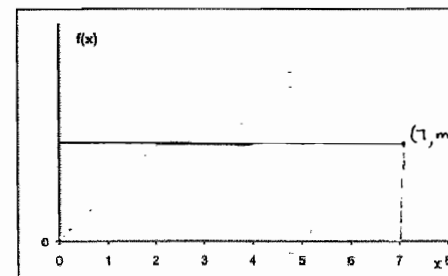
[2 marks]

$$\frac{\binom{2}{2} \binom{5}{2} \binom{8}{2}}{2100} = \frac{2}{15} \quad (1)$$

$\approx \frac{280}{2100}$

Question 20 [4 marks]

The probability density function for a continuous random variable, X , is given by the graph below.



(a) Find the exact value of m .

[1]

$$7m = 1$$

$$m = \frac{1}{7}$$

(b) Find the probability that X is less than 3.

[1]

$$P(X < 3) = \frac{3}{7}$$

(c) Given that X is less than 3, what is the probability that X is less than 2?

[2]

$$P(X < 2 | X < 3) = \frac{\frac{2}{7}}{\frac{3}{7}} \quad (1)$$

$$= \frac{2}{3} \quad (1)$$

Question 19 [7 marks]

There are 28 players in a football squad but only 21 are to be selected to form a team. John and Guy are members of the squad. How many different teams are possible (do not simplify) if

(a) all players are available? [1]

$$\binom{28}{21}$$

(b) John must be included? [1]

$$\binom{1}{1} \binom{27}{20}$$

(c) Guy is injured and cannot play? [1]

$$\binom{27}{21}$$

(d) John will not be included but Guy must play? [2]

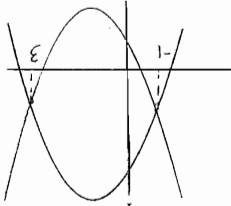
$$\binom{1}{1} \binom{26}{20}$$

(e) John and Guy must be included in the team? [2]

$$\binom{2}{2} \binom{26}{19}$$

Question 14. [5 marks]

Calculate the area between the two functions $f(x) = -x^2 + 2x + 7$ and $g(x) = 2x^2 - 4x - 2$ using calculus techniques.



$$\begin{aligned}
 A &= \int_{-1}^3 f(x) - g(x) \, dx \quad \text{1 for intersect points} \\
 &= \int_{-1}^3 (-x^2 + 2x + 7) - (2x^2 - 4x - 2) \, dx \quad \text{1 for correct operation of functions} \\
 &= \int_{-1}^3 (-3x^2 + 6x + 9) \, dx \quad \text{2 correct integral} \\
 &= -x^3 + 3x^2 + 9x \Big|_{-1}^3 \\
 &= 32 \text{ units}^2 \quad \text{1}
 \end{aligned}$$

Question 15. [6 marks]

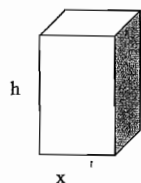
The point $(1, b)$ lies on the curve $y = \frac{a+5x}{2x-5}$ and the gradient at that point is $-\frac{3}{2}$. Determine the values of a and b .

$$b = \frac{a+5}{2} \quad (1)$$

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{5(2x-5) - 2x(a+5x)}{(2x-5)^2} \quad (2) \\
 -\frac{3}{2} &= \frac{10x - 25 - 2ax - 10x^2}{(2x-5)^2} \quad (1) \\
 a &= 4 \quad (1) \\
 b &= -\frac{3}{2} \quad (1)
 \end{aligned}$$

Question 16. [7 marks]

An open water tank has a square base of x metres and a height of h metres. The total surface area of sheet metal used for its construction is 27 metres².



- (a) Find an expression for h in terms of x .

[2]

$$SA = 27 = 4hx + x^2 \quad (1)$$

$$\frac{27 - x^2}{4x} = h \quad (1)$$

- (b) Show that the volume, V , of the tank equals $\left(\frac{27x}{4} - \frac{x^3}{4}\right) \text{m}^3$.

[2]

$$\begin{aligned} V &= x^2 h \\ &= \frac{27 - x^2}{4x} \cdot x^2 \quad (1) \\ &= \frac{27x}{4} - \frac{x^3}{4} \quad (1) \end{aligned}$$

- (c) Hence calculate the maximum volume possible for this tank using calculus techniques.

[3]

$$\frac{dV}{dx} = \frac{27}{4} - \frac{3x^2}{4} \quad (1)$$

$$\text{max or min } \frac{dV}{dx} = 0$$

$$27 - 3x^2 = 0$$

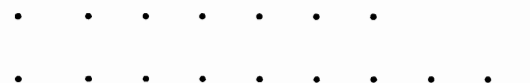
$$x = \pm 3 \quad \text{Reject } -3 \quad (1)$$

$$V = \frac{27}{2} \text{ or } 13.5 \text{ m}^3 \quad (1)$$

must include
units

Question 17. [3 marks]

In an array of dots, there are 7 in the top row and 10 in the bottom row:



Quadrilaterals are formed by selecting 4 dots as vertices, 2 in the top row and 2 in the bottom row.

How many different quadrilaterals are possible?

$$\binom{7}{2} \times \binom{10}{2} = 945$$

Question 18 [4 marks]

An electrical store has 10 lamps left in its storeroom. Four of the lamps have defective wiring and should not be used. A new store assistant randomly selects three of the lamps for a customer. Let X be the number of defective lamps purchased by the customer. Find the probability distribution for X .

$$n = 10 \quad 4 \text{ defective} \quad 6 \text{ ok.} \\ SS \quad \binom{10}{3}$$

X	0	1	2	3
$P(X)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{3}$

$$\text{or } \frac{20}{120} \quad \frac{60}{120} \quad \frac{36}{120} \quad \frac{4}{120}$$

1 mark each entry in
the table.
fractions do not need to
have been simplified