

**TEST 1 – POLAR COORDINATES & COMPLEX NUMBERS**

NAME: \_\_\_\_\_ DATE: \_\_\_\_\_

**[To achieve full marks and to allow assessment of particular outcomes, working and reasoning should be shown.]**

**[A maximum of 2 marks will be deducted for incorrect rounding, units, etc.]**

***This is Resource Rich – 50 minutes for 53 marks:***

1. [1, 1, 1, 1 = 4 marks]

Convert:

a) (4,-6) into polar coordinates with  $-180^\circ < \theta \leq 180^\circ$ .

b)  $(3, -\sqrt{3})$  into **exact** polar coordinates with  $-\pi < \theta \leq \pi$ .

c)  $[4, 35^\circ]$  into Cartesian coordinates.

d)  $[8, -\frac{3}{4}\pi]$  into **exact** Cartesian coordinates.

2. [3, 3 = 6 marks]

Clearly show how you obtain your answers, find:

a) the distance between  $[20, -210^\circ]$  and  $[\sqrt{5}, -50^\circ]$ .

b) the **exact** distance between  $\left[ \frac{\sqrt{5}}{3}, -\frac{2\pi}{3} \right]$  and  $\left[ 10, -\frac{7\pi}{6} \right]$ .

3. [8 marks]

Find the **exact** distance between  $(\sqrt{3k}, \sqrt{k})$  and  $\left[ \sqrt{k}, \frac{5\pi}{6} \right]$ . Draw a diagram.

4. [3, 2 = 5 marks]

a) Find, in **exact** form, the modulus and principal argument of  $-\sqrt{3} + i$ , and hence rewrite  $-\sqrt{3} + i$  in **exact** polar (**cis**) form.

b) Convert  $2 \operatorname{cis} \left( \frac{\pi}{4} \right)$  into **exact** algebraic Cartesian/rectangular form.

5. [3, 3 = 6 marks]

Evaluate, giving answers in **exact** form:

a)  $4 \operatorname{cis} \frac{\pi}{3} \times 2 \operatorname{cis} \frac{3\pi}{4}$

b) 
$$\frac{4 \operatorname{cis} \left( -\frac{5\pi}{6} \right)}{2 \operatorname{cis} \left( \frac{5\pi}{6} \right)}$$

6. [4 marks]

Given  $z = 2 \operatorname{cis} \frac{\pi}{4}$ , express  $z^{-1}$  and  $\bar{z}$  in **exact** polar and rectangular form.

7. [2, 2, 2, 2 = 8 marks]

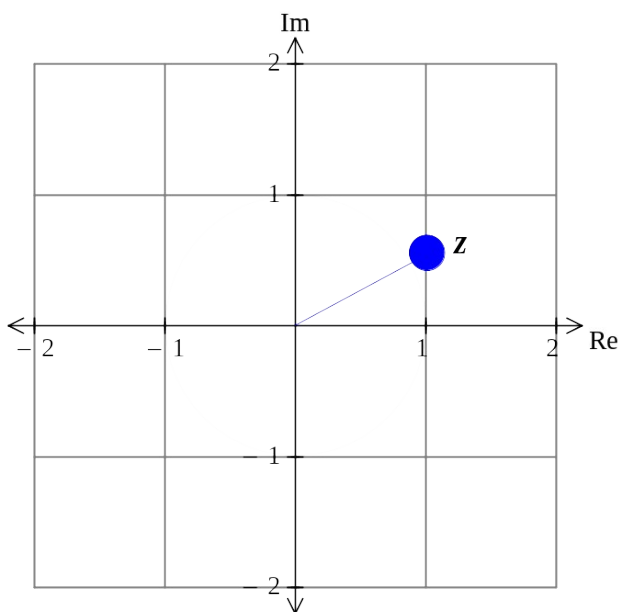
The Argand diagram below shows the point representing the complex number  $z$  where  $|z| > 1$ . Plot on the same diagram, the points representing the complex numbers:

a)  $\bar{z}$

b)  $iz$

c)  $z^2$

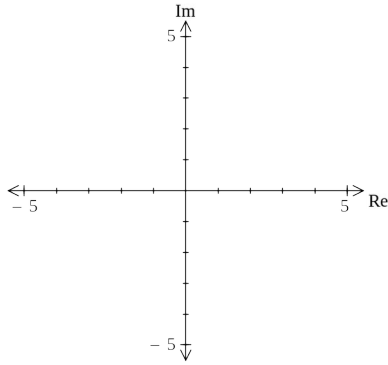
d)  $\frac{1}{z}$



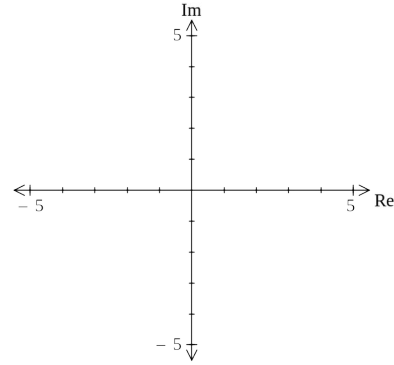
8. [3, 3, 3, 3 = 12 marks]

Sketch on an Argand diagram, the locus of the point  $z = x + iy$ , satisfying each of the following conditions. In each case, give the Cartesian equation or inequality of the locus.

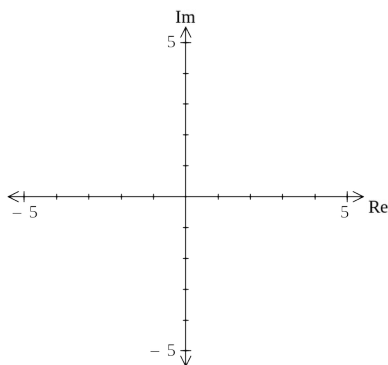
a)  $\operatorname{Re}(z) = -2$



b)  $\operatorname{Im}(z) = \operatorname{Re}(z)$



c)  $\operatorname{Re}(z) + 2\operatorname{Im}(z) > 3$



d)  $\operatorname{Re}(z) \cdot \operatorname{Im}(z) = 1$

