



PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

**Perth Modern School  
End of Year Examination, 2012**

**Question/Answer Booklet**

**MATHEMATICS 3C/3D**

**Section One:  
Calculator-free**

**SOLUTIONS**

Student Number: In figures

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In words

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Your name

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**Time allowed for this section**

Reading time before commencing work: five minutes

Working time for this section: fifty minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer Booklet

Formula Sheet

***To be provided by the candidate***

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
Total				150	100

## Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil**, except in diagrams.

**Section One: Calculator-free****(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

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**Question 1****(4 marks)**

For the two independent events  $A$  and  $B$ ,  $P(A) = 0.3$  and  $P(B) = 0.1$ .

Calculate

(a)  $P(\bar{B})$

(1 mark)

0.9

(b)  $P(A \cap B)$

(1 mark)

$0.1 \times 0.3 = 0.03$

(c)  $P(A \cup \bar{B})$

(1 mark)

$0.3 + 0.63 = 0.93$

(d)  $P(\bar{A} | B)$

(1 mark)

0.7

**Question 2****(5 marks)**

Solve the system of equations

$$3x + 2y + 6z = 3$$

$$x + 3y + 4z = 9$$

$$2x + 8 = 2z + y$$

$$3x + 9y + 12z = 27$$

$$3x + 2y + 6z = 3$$

$$7y + 6z = 24$$

$$2x + 6y + 8z = 18$$

$$2x - y - 2z = -8$$

$$7y + 10z = 26$$

$$4z = 2$$

$$z = 0.5$$

$$7y = 24 - 6(0.5)$$

$$y = 3$$

$$x = 9 - 3(3) - 4(0.5)$$

$$x = -2$$

Question 3

(8 marks)

(a) Differentiate the following with respect to  $x$ . There is no need to simplify your answer.

(i)  $y = 2x^3 \sqrt{3 - x^2}$  (2 marks)

$$\frac{dy}{dx} = 6x^2 \sqrt{3 - x^2} + 2x^3 \left(\frac{1}{2}\right)(-2x)(3 - x^2)^{-0.5}$$

(ii)  $y = \frac{1 + e^{3x-1}}{2e^{-x^2}}$  (3 marks)

$$y = \frac{1}{2}(e^{x^2} + e^{3x-1+x^2})$$

$$\frac{dy}{dx} = \frac{1}{2}(2xe^{x^2} + (3 + 2x)e^{3x-1+x^2})$$

or using quotient rule

$$\frac{dy}{dx} = \frac{(3e^{3x-1})(2e^{-x^2}) - (1 + e^{3x-1})(-4xe^{-x^2})}{(2e^{-2x^2})^2}$$

(b) Simplify  $\frac{d}{dx} \int_2^{x^2} \left(\frac{t^2}{3}\right) dt$  (3 marks)

$$\frac{dy}{dx} = \frac{(x^2)^2}{3} \times 2x$$

$$= \frac{2x^5}{3}$$

**Question 4****(8 marks)**

A function is defined by  $f(x) = 6x^2 - 2x^3$ .

- (a) Find the coordinates of the turning points of  $f(x)$  and state their nature. (3 marks)

$$f'(x) = 12x - 6x^2$$

$$= 0 \text{ when } x = 0, x = 2$$

$$(0, 0) \text{ is a minimum and } (2, 8) \text{ is a maximum.}$$

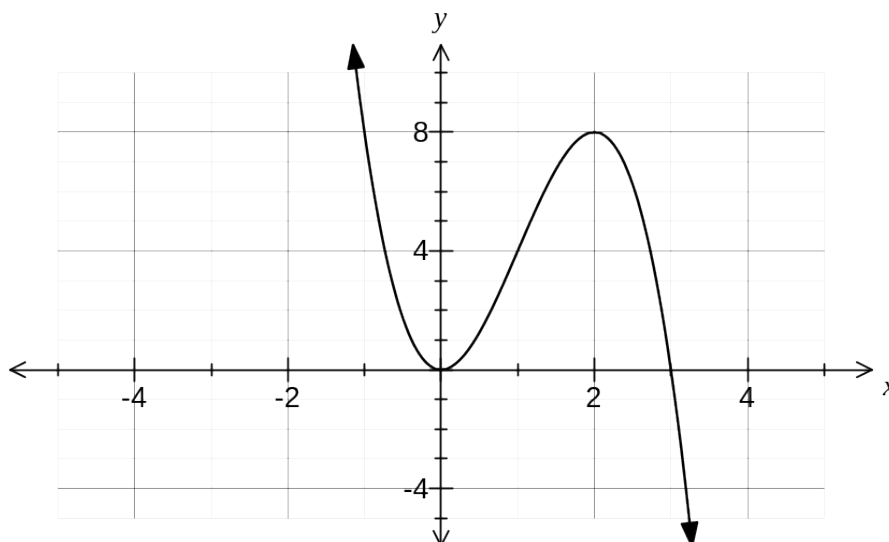
- (b) Find the coordinates of the point of inflection of  $f(x)$ . (1 mark)

$$f''(x) = 12 - 12x$$

$$= 0 \text{ when } x = 1$$

$$\text{At } (1, 4)$$

- (c) Sketch the graph of  $y = f(x)$ . (3 marks)



- (d) What is the maximum value of  $f(x)$  in the interval  $-2 \leq x \leq 4$ ? (1 mark)

$$f(-2) = 6(4) - 2(-8) = 40$$

Max value is 40.

Question 5

(9 marks)

Let  $f(x) = \frac{1}{1-x}$  and  $g(x) = e^{2x}$ .

(a) Determine the domain of  $f(g(x))$ .

(2 marks)

$$\begin{array}{l} 1 - e^{2x} \neq 0 \\ x \neq 0 \end{array}$$

(b) Determine the range of  $g(f(x))$ .

(3 marks)

$$\begin{array}{l} g(f(x)) = e^{\frac{2}{1-x}} \Rightarrow y > 0 \\ \text{But } \frac{2}{1-x} \neq 0 \Rightarrow y \neq 1 \\ \text{Hence range: } y > 0, y \neq 1 \end{array}$$

(c) Solve  $f(x) \geq 3 - 2x$ .

(4 marks)

$$\begin{array}{l} \frac{1}{1-x} - 3 - 2x \geq 0 \\ \frac{1 - (3 - 5x + 2x^2)}{1-x} \geq 0 \\ \frac{2x^2 - 5x + 2}{x-1} \geq 0 \\ \frac{(2x-1)(x-2)}{x-1} \geq 0 \\ \text{Critical points when } x = \frac{1}{2}, 1 \text{ and } 2 \\ \frac{1}{2} \leq x < 1 \text{ or } x \geq 2 \end{array}$$

Question 6

(9 marks)

- (a) Determine  $\int x(3x^2 + 6x)^4 + (3x^2 + 6x)^4 dx$

(3 marks)

$$\begin{aligned} &= \int (x+1)(3x^2 + 6x)^4 dx \\ &= \frac{1}{6} \int (6x + 6)(3x^2 + 6x)^4 dx \\ &= \frac{(3x^2 + 6x)^5}{6 \times 5} + c \\ &= \frac{(3x^2 + 6x)^5}{30} + c \end{aligned}$$

- (b) Calculate the area bounded by the functions  $f(x) = (x - 2)^2 - 3$  and  $g(x) = 2x - 4$ .

(6 marks)

$$\begin{aligned} &\text{Solve } f(x) = g(x) \\ &x^2 - 4x + 4 - 3 = 2x - 4 \\ &x^2 - 6x + 5 = 0 \\ &(x - 1)(x - 5) = 0 \\ &x = 1, x = 5 \end{aligned}$$

Integrate to find area

$$\begin{aligned} &\int_1^5 g(x) - f(x) dx \\ &= \int_1^5 x^2 - 6x + 5 dx \\ &= \left[ \frac{x^3}{3} - 3x^2 + 5x \right]_1^5 \\ &= \left( \frac{125}{3} - 75 + 25 \right) + \left( \frac{1}{3} - 3 + 5 \right) \\ &= 10\frac{2}{3} \text{ square units} \end{aligned}$$



Question 7

(7 marks)

A closed cylindrical can of radius  $r$  cm has a volume of  $250\pi$  cm<sup>3</sup>.

- (a) Show that the total surface area,  $A$  cm<sup>2</sup>, of this can is given by  $A = \frac{500\pi}{r} + 2\pi r^2$ .

(2 marks)

$$\begin{aligned} V &= \pi r^2 h \\ 250\pi &= \pi r^2 h \Rightarrow h = \frac{250}{r^2} \\ A &= 2\pi r^2 + 2\pi r h \\ &= 2\pi r^2 + 2\pi r \frac{250}{r^2} \\ &= \frac{500\pi}{r} + 2\pi r^2 \end{aligned}$$

- (b) Determine the minimum possible surface area of the can and the radius and height required to achieve this optimum area.

(5 marks)

$$\begin{aligned} \frac{dA}{dr} &= -\frac{500\pi}{r^2} + 4\pi r \\ -\frac{500\pi}{r^2} + 4\pi r &= 0 \\ r^3 &= 125 \Rightarrow r = 5 \text{ cm} \\ h &= \frac{250}{5^2} = 10 \text{ cm} \\ A &= \frac{500\pi}{5} + 2\pi \times 5^2 \\ A &= 150\pi \end{aligned}$$

**Additional working space**

Question number: \_\_\_\_\_

**Additional working space**

Question number: \_\_\_\_\_

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