



Perth Modern School
Semester One Examination, 2012

PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

Question/Answer Booklet

MATHEMATICS

3A/3B

Section Two:

Calculator-assumed

Student Name _____

Teachers Name _____

Year _____

Student Number: In figures

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In words

Time allowed for this section

Reading time before commencing work: 10 minutes

Working time for this section: 100 minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	7	7	50	50	
Section Two Calculator-assumed	13	13	100	100	
			Total	150	100

Instructions to students

- 1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- 2 **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- 3 It is recommended that you **do not use pencil**, except in diagrams.

**Section Two: Calculator-assumed
(100 marks)**

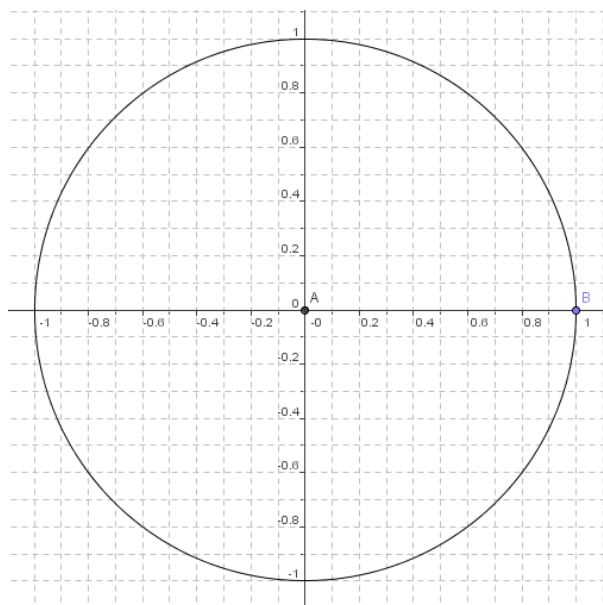
This section has **thirteen (13)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

Question 8

(5 marks)

Demonstrate how to use the unit circle below (there must be evidence of using the unit circle before full marks will be awarded)



(i) to evaluate the value of $2\cos 120^\circ$ (2)

(ii) to find the value(s) of x , to the nearest whole number, such that

$$\sin 2x = 0.9 \quad (3)$$

Question 9

(9 marks)

Consider the following sequence of numbers 2, 5, 2, 9, 2, 13, 2, 17, ...

(i) Write down the next two terms of the sequence, if the pattern continues. (1)

(ii) In this sequence, $T_1 = T_3 = T_5 =$ etc, complete the sentence,

“ If n is an odd number, then

$$T_n = _, T_1 = _ \quad (1)$$

(iii) Write a recursive formula for the sequence 5, 9, 13, 17, ... (2)

(iv) Write down the values of T_{99} and T_{100} for the sequence 2, 5, 2, 9, 2, 13, 2, 17, ... (2)

(v) Which term is 61? (1)

(vi) Calculate the sum of the first 100 terms of the sequence. (2)

Question 10**(6 Marks)**

Four digit numbers may be formed using the digits 0, 2, 4, 6, 8 such that no digit can be used more than once. How many of these 4-digit numbers

(i) may be formed? (1)

(ii) begin with 8? (1)

(iii) end in a 2 or a 4? (1)

(iv) are less than 486? (2)

(v) What is the probability that a randomly picked 4-digit number from this set is more than 8640?
(1)

Question 11**(6 marks)**

- (a) The distribution of heights of a plant is normally distributed with a mean of 72 cm and a standard deviation of 6 cm. If there were 100 plants, estimate the number of plants which are less than 64cm in height.

(2)

- (b) Achievement awards in a national Mathematics competition are allocated as

- High Distinctions to the top 15% of performances
- Distinctions to the next 25%
- Credits to the next 30%
- Participations to the remainder

If the scores were normally distributed with a mean of 75 and a standard deviation of 12, find the minimum score a competitor has to achieve to be awarded a

- (i) Credit (2)

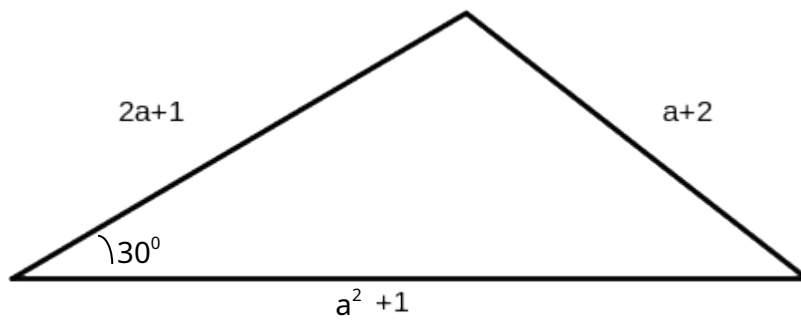
- (ii) Distinction (1)

- (iii) High Distinction (1)

Question 12

(6 marks)

Given the triangle below,



- (i) Find a simplified algebraic expression for the area of the triangle, given that

$$\sin 30^\circ = \frac{1}{2}. \quad (2)$$

- (ii) If the area is 17.5 units^2 , determine the value of 'a'. (2)

- (iii) Hence, state the lengths of the triangle. (2)

Question 13**(8 marks)**

(a) A group of fifteen students travelling to Tasmania have their bags weighed at the airport

check-in counter. The weights of the bags in kilograms are listed in order as follows;

m, 9, n, 10, 15, 18, 21, p, 25, 29, 35, q, 38, r, 43.

The following statistics of this set of data are known.

$\bar{x} = 24$, Median = 21, Interquartile range = 27.

There are two modes of which 10 is one of them.

The minimum score is 2 less than the smaller mode.

Determine the values of m, n, p, q, r.

(5)

Question 13 (continued)

- (b) In collecting data, biased samples and poor methods of collection can lead to invalid inferences. Consider the following method to determine the extent of unemployment in a local community.

“A team of data collectors phoned five randomly selected households on every twentieth page of the local telephone directory during regular office hours”.

- (i) List two ways which might suggest this data collection method cause bias? (2)

- (ii) Suggest how this bias could be avoided. (1)

Question 14
marks)

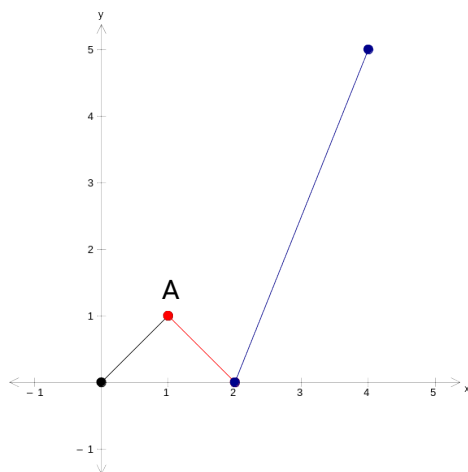
(10

- (a) The graph of $f(x)$ is dilated parallel to the x -axis by a scale factor of 2, followed by a translation of 4 units in the positive direction of the x -axis. What is the equation of the final graph obtained?

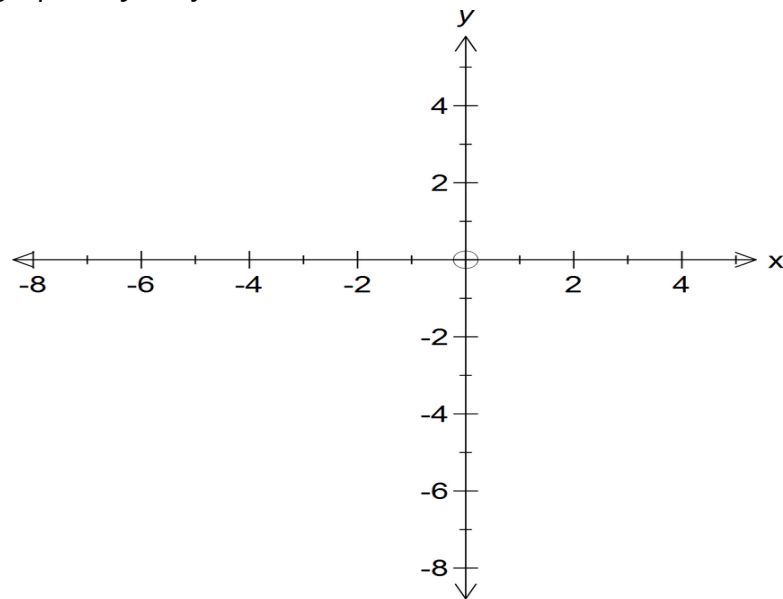
(2)

- (b) The function $y=x^3$ is transformed to $y=-2(x+5)^3$. Describe the transformation in order. (3)

- (c) The graph of $y=f(x)$ is given below.



- (i) Sketch the graph of $y=3 - f(x)$ on the axes below.

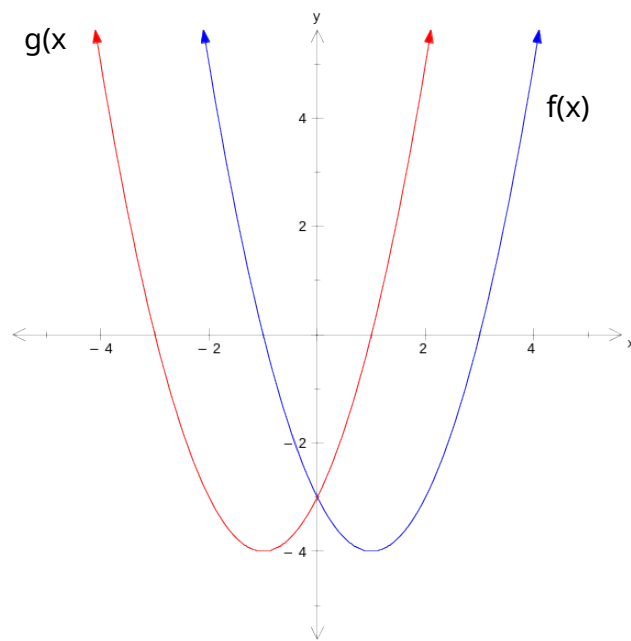


(2)

Question 14 (continued)

- (ii) Write the coordinates of A if $y = -f(-x+3)$. (2)

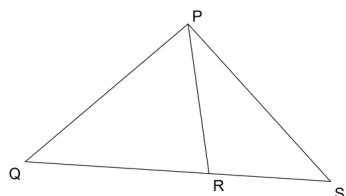
- (d) Describe how the graph of $f(x)$ can be used to obtain the graph of $g(x)$.
(1)



Question 15

(7 marks)

In the given diagram, $PR = 8$ cm, $QR = 5$ cm and the sizes of angles PRQ and PSR are 60° and 40° respectively. Calculate



(a) the exact length of PQ

(2)

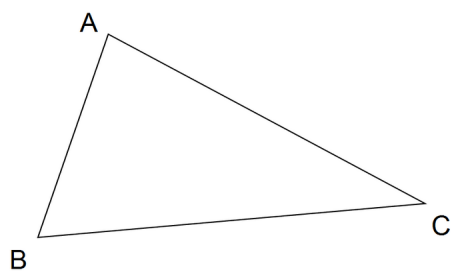
(b) the length of PS correct to 1 decimal place

(2)

c) Triangle ABC is a non-right angled triangle. Given angle $A = 30^\circ$, $a = 6$ and $b = 9$.

(3)

Find the value(s) of angle B .



Question 16**(8 marks)**

- (a) The following table shows statistics of the results of the five mathematics tests for John, Ryan, Julie and Gary.

	Mean	Standard deviation	Range
John	13.6	3.8	7
Ryan	13.9	4.2	7
Julie	13.8	2.7	7
Gary	14.0	3.4	7

Which student has the most consistent results? State a reason why you chose this student. (2)

- (b) Mrs Kendall decided to survey a sample of 10% of the students at her school. The school enrolment is shown in the table below.

Year	7	8	9	10	11	12	Total
Number of students	225	232	233	230	150	130	1200

She surveyed the same number of students in each year group. Explain how would the numbers of students surveyed in Year 10 and Year 11 have changed if she had chosen to use a stratified sample based on year groups?

(3)

Question 16 (continued)

- (c) Solve the indicial equation showing all working steps (3)

$$2 \times 3^{2x+3} = 6 \times 9^{4x}$$

Question 17**(9 marks)**

Josh bought a second hand Hyundai for \$4500. He borrowed the full sum of \$4500 from NO Frills Finance Company and agreed to make a minimum payment of \$300 a month. For a year he pays the minimum monthly payment. At the end of the year, he checks his statement from the Finance company and is surprised that he still owes \$1683.50. He created a spreadsheet to check whether the amount owing is correct.

Month	Previous Balance	Interest	Balance Due	Payment	Balance after payment
1	4500.00	90.00	4590.00	300	4290.00
2	4290.00	85.80	4375.80	300	4075.80
3	4075.80	81.52	4157.32	300	3857.32
4	3857.32	77.15	3934.46	300	3634.46
5	3634.46	72.69	3707.15	300	3407.15
6	3407.15	68.14	3475.29	300	3175.29
7	3175.29	63.51	3238.80	300	2938.80
8	2938.80	58.78	2997.58	300	2697.58
9	2697.58	53.95	2751.53	300	2451.53
10	2451.53	49.03	2500.56	300	2200.56
11	2200.56	44.01	2244.57	300	1944.57
12	1944.57	38.89	1983.46	300	1683.46

- (i) Calculate the annual rate of interest Josh is paying. (1)
- (ii) If Josh continues to pay the monthly payment of \$300, it will take him a total of 19 months to pay off his car. What would be the total amount of his payments? (2)
- (iii) Josh decides to reduce his social life activities in order to pay an extra \$200 a month from the 13th payment onwards. Complete the table of calculation below. (2)

Month	Previous Balance	Interest	Balance Due	Payment	Balance after payment
13	1683.46		1717.13	500	1217.13

14	1217.13	24.34		500	741.47
15	741.47	14.83	756.30	500	

Question 17 (continued)

(iv) How long does it take Josh to repay the loan? (1)

(v) What is the total amount that Josh will pay over the period with the increased repayment? (2)

(vi) How much money does Josh save by paying in this way? (1)

Question 18

(8 marks)

The final result of the students in Mrs Bond's classes of 3CDMAT and 3CDMAS are summarised in the frequency table below.

Mark as a percentage	3CDMAT	Cumulative frequency	3CDMAS	Cumulative frequency
0 – 19	2	2	1	1
20 – 39	2	4	3	4
40 – 59	5	C	A	7
60 – 79	D	20	9	B
80 – 99	7	27	5	21

(a) Find the values of A, B C and D (2)

(b) Calculate the mean and standard deviation of the 3CDMAS marks (2)

(c) If a student from one of these classes is picked at random, what is the probability that the student

(i) received a mark greater than or equal to 60 ? (1)

(ii) was in the 3CDMAT class? (1)

(iii) was in the 3CD MAS class and received a mark ≥ 80 ? (1)

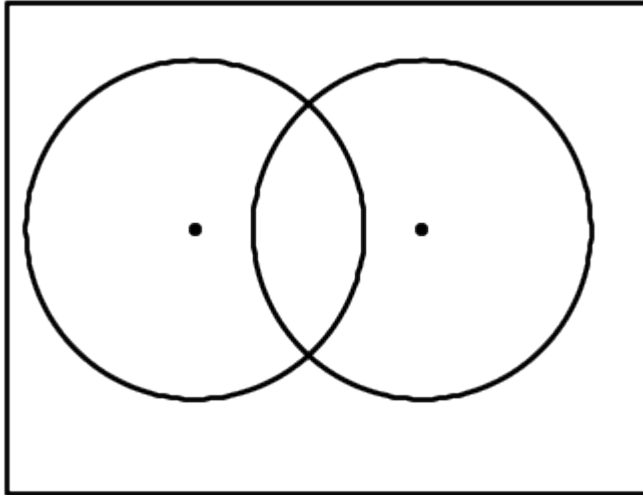
(iv) was in the 3CDMAS class if the student received a mark between 60 and 79 inclusive ? (1)

Question 19

(9 marks)

(a) A and B are events such that $P(A \cup B) = 0.9$, $P(A \cap B) = 0.2$, $P(A|B) = 0.8$

(i) Represent the diagram in the Venn diagram below. (3)

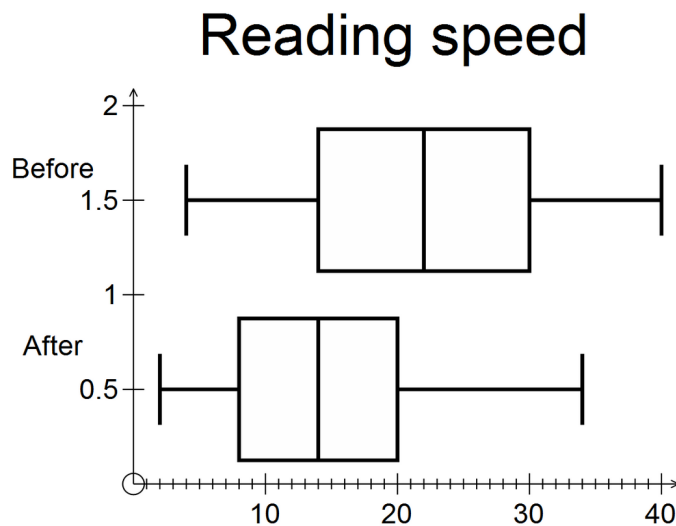


(ii) Hence, find $P(A)$ (1)

(iii) Hence find $P(\bar{B})$ (1)

Question 19 (continued)

- (b) The test results for a group of 45 students involved in a reading program are displayed below in the form of a pair of box plots. The data collected shows the reading speed of the students both before and after the administration of the program.



*****Note Before and After should be swapped*****

For each of the following, answer True (T) or False (F):- (4)

- (i) For the "after " data, the median has increased by 8 and the interquartile range has increased by 4.
- (ii) Both distributions have the same median.
- (iii) The 'after' set of scores is more evenly distributed than the 'before' set of scores.
- (iv) The range of both sets of scores is the same.

Question 20**(9 marks)**

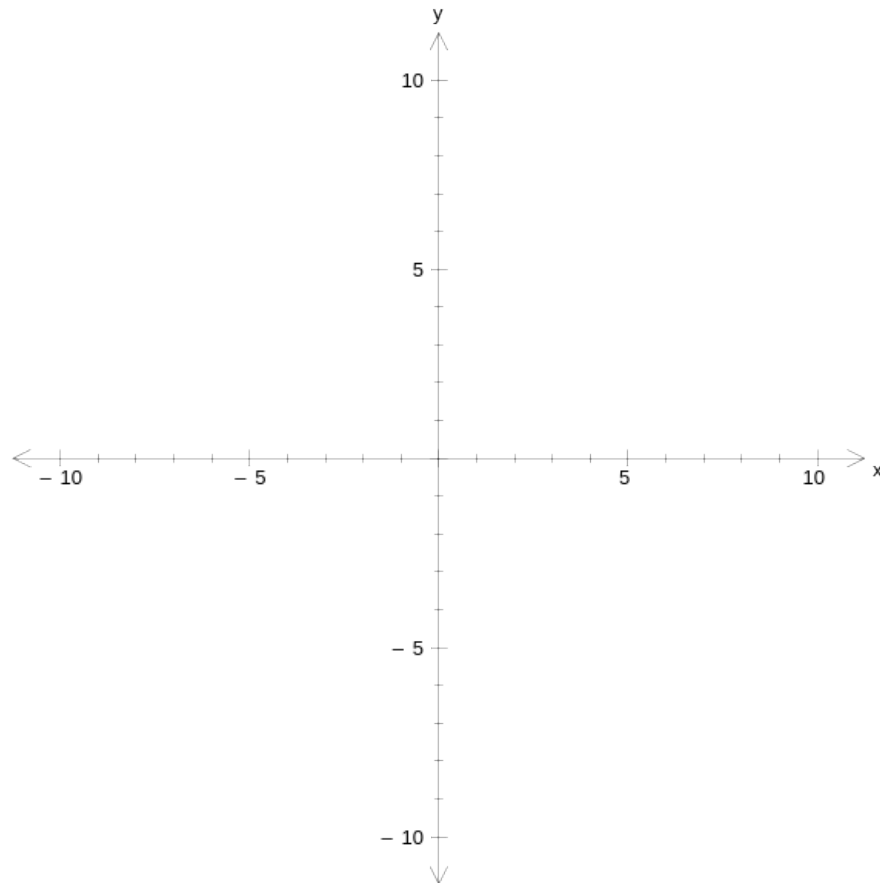
The point $P(1,k)$ lies on the curve with equation $f(x) = (x^2 - 6x)(x - 2) + 3x$

(i) Find the value of k . (1)

(ii) Express $f(x)$ in a completely factorised form. (2)

(iii) On the axes below sketch the curves with the following equations

$f(x) = (x^2 - 6x)(x - 2) + 3x$ and $g(x) = \frac{5}{x}$, showing clearly where they meet the axes. (3)



Question 20 (continued)

- (iv) Read from the graph the **number** of solutions to the equation. (1)

$$(x - 5)(x - 3) = \frac{5}{x^2}$$

- (v) By adding **one** suitable function on the axes, show how you solve $x^2 = 5$ and estimate the solution(s) from your graph, correct to 1 decimal place.
(2)

