PERTH MODERN SCHOOL
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Independent Public School

Course Methods Test 1 Year 12

Student name: _______________ Teacher name: _______________

Task type: **Response**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **_____6______**

Materials required: No Cals allowed at all!

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper,

Marks available: **34 marks**

Task weighting: **13%**

Formula sheet provided: no but formulae listed on next page.

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
$\frac{d}{dx}e^{ax-b} = ae^{ax-b}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\ln x = \frac{1}{x}$		$\int \frac{1}{x}dx = \ln x + c, \quad x > 0$
$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$		$\int \frac{f'(x)}{f(x)}dx = \ln f(x) + c, \quad f(x) > 0$
$\frac{d}{dx}\sin(ax-b) = a\cos(ax-b)$		$\int \sin(ax-b)\,dx = -\frac{1}{a}\cos(ax-b) + c$
$\frac{d}{dx}\cos(ax-b) = -a\sin(ax-b)$		$\int \cos(ax-b)\,dx = \frac{1}{a}\sin(ax-b) + c$
Product rule	If $y = uv$ then $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$	or If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$
Quotient rule	If $y = \frac{u}{v}$ then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	or If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$
Chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$
Fundamental theorem	$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$	and $\int_a^b f'(x)\,dx = f(b) - f(a)$
Increments formula	$\delta y \approx \frac{dy}{dx} \times \delta x$	
Exponential growth and decay	$\frac{dP}{dt} = kP \Leftrightarrow P = P_0 e^{kt}$	

No calculators allowed!!!

Q1 (2, 2 & 2 = 6 marks)

Determine the gradient function $\frac{dy}{dx}$ for each of the following.

i) $y = x^3 + \frac{1}{x^2}$

c
$y = x^3 + \frac{1}{x^2}$ $y' = 3x^2 - 2x^{-3}$
Specific behaviours
✓ diffs first term   ✓ diffs second term

ii) $y = \frac{8x^4 - 5x}{x}$

c
$y = \frac{8x^4 - 5x}{x} = 8x^3 - 5$ $y' = 24x^2$
Specific behaviours
✓ rearranges y or uses quotient rule   ✓ states derivative

iii) $y = (x^3 - 1)(5 + \sqrt{x})$

c
$y = (x^3 - 1)(5 + \sqrt{x})$ $y' = (x^3 - 1)\frac{1}{2}x^{-\frac{1}{2}} + (5 + \sqrt{x})3x^2$
Specific behaviours
✓ uses product rule   ✓ diffs all terms correctly (no need to simplify)

Q2 (4 marks)

Determine the equation of the tangent to the curve $y=\frac{5x-7}{3x+2}$ at the point $\left(1,\frac{-2}{5}\right)$.

c
$y=\frac{5x-7}{3x+2}$ $y'=\frac{(3x+2)5-(5x-7)3}{(3x+2)^2}=\frac{15x+10-15x+21}{(3x+2)^2}=\frac{31}{(3x+2)^2}$ $x=1, y'=\frac{31}{25}$ $y=\frac{31}{25}x+c$ $-\frac{2}{5}=\frac{31}{25}+c$ $c=\frac{-10}{25}-\frac{31}{25}=-\frac{41}{25}$ $y=\frac{31}{25}x-\frac{41}{25}$
Specific behaviours
✓ uses quotient rule ✓ determines gradient at x=1 ✓ solves for constant of tangent equation ✓ states equation

Q3 (2, 2, 2 & 4= 10 marks)

The table below contains the values of the polynomial function $f(x)$ and its first and second derivatives for $x=0,1,2,3,4,5,6$.
There are no stationary points for non-integer values of x.

x	0	1	2	3	4	5	6
$f(x)$	12	5	-2	-13	-20	-35	-5
$f'(x)$	-4	-12	-5	0	-11	0	15
$f''(x)$	-8	0	2	0	-5	7	10

a) Evaluate $\frac{d}{dx}\left[f(x)\right]^2$ when $x=1$

c
$\frac{d}{dx}\left[f(x)\right]^2 = 2f(x)f'(x)$ $= 2f(1)f'(1) = 10(-12) = -120$
Specific behaviours
✓ uses chain rule ✓ subs correct values Note: no follow through if chain not used

b) Evaluate $\frac{d}{dx}\left[f(2x)\right]$ when $x=3$

c
$\frac{d}{dx}\left[f(2x)\right] = f'(2x)2$ $= f'(6)2 = 30$
Specific behaviours
✓ uses chain rule ✓ subs correct values Note: no follow through if chain not used

c) Evaluate $\frac{d}{dx}\left[\frac{1}{f(x)}\right]$ when $x=2$

c
$\frac{d}{dx}\left[\frac{1}{f(x)}\right] = -f(x)^{-2}f'(x)$ $= -f(2)^{-2}f'(2)$ $= \frac{5}{4}$
Specific behaviours

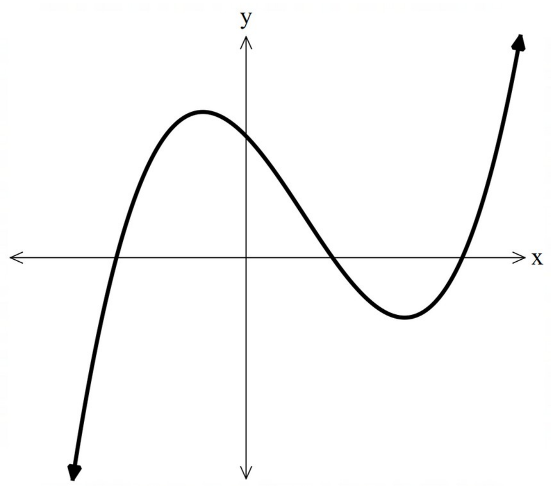
✓ uses chain rule ✓ subs correct values Note: no follow through if chain not used

d) Determine the x-coordinate of any **stationary** points and their nature. Justify your answer.

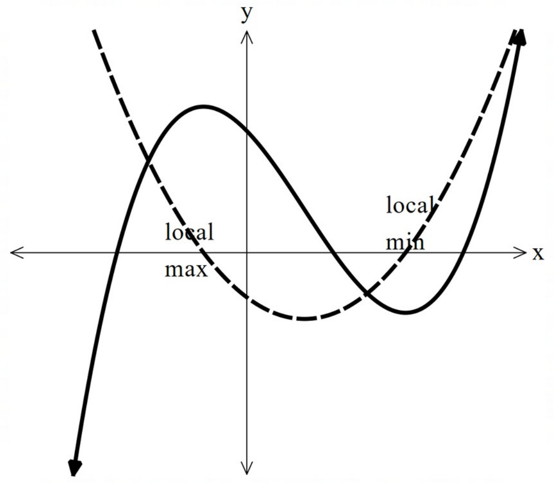
c
$f'(x)=0$ $x=3,5$ $x=3$ $f''(3)=0$ $f''(2)=2\ \&\ f''(4)=-5$ $Hence\ horizontal\ \inf lection$ $x=5$ $f''(5)=7$ $Hence\ local\ \min$
Specific behaviours
✓ states only 2 stationary points only ✓ states nature of both points ✓ states two part argument for inflection (Note may use same first derivatives either side ✓ states argument for local min

Q4 (3 & 3 = 6 marks)

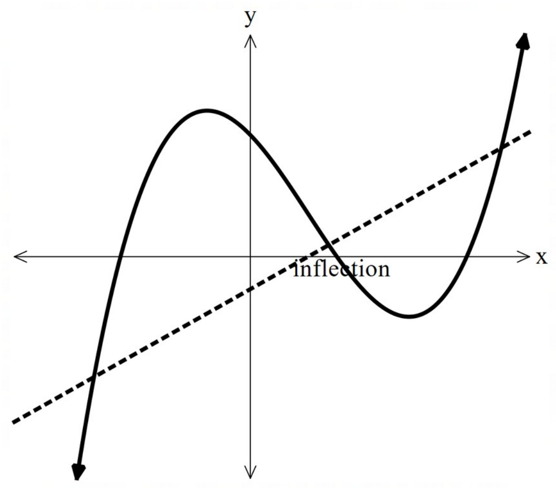
Consider the curve of $y=f(x)$ which is graphed below.



a) Sketch below a graph of the first derivative of $y = f(x)$. Label on this new graph stationary points.

c

Specific behaviours
✓ correct shape ✓ correct positions of x intercepts on new graph ✓ labels nature of x intercepts on new graph (accept old graph)

b) Sketch below a graph of the second derivative of $y = f(x)$. Label on this new graph any inflection points

c


Specific behaviours
✓ correct shape (need not be exact line- but close to it) ✓ correct position of x intercepts on new graph (accept old graph) ✓ labels inflection pt

Q5 (4 marks)

The cost $\$C$ for the production of x thousand units of a certain product is given by $C=(3x+5)^4$, $x>0$.

Determine the value of x for which the **average cost per unit** is a minimum and find this minimum average cost. Justify. (No need to simplify)

c
$C=(3x+5)^4$ $A=\frac{C}{x}=\frac{(3x+5)^4}{x}$ $A'=\frac{x12(3x+5)^3-(3x+5)^4}{x^2}=\frac{(3x+5)^3[9x-5]}{x^2}$ $A'=0,\rightarrow\ x=\frac{5}{9}$ $x=0, 9x-5=-5 \therefore A'<0$ $x=1, 9x-5=4 \therefore A'>0$ $x=\frac{5}{9}, local\ \min$ $\frac{(3x+5)^4}{x}=\frac{\left(\frac{5}{3}+5\right)^4}{\frac{5}{9}}$
Specific behaviours
✓ divides cost by x ✓ uses quotient rule ✓ solves for stationary point ✓ states min av cost, un simplified (no need for units) NOTE max of 1 mark if quotient not used (i.e average cost)

Q6 (4 marks)
Consider a train moving in a straight line. The displacement, x km, from its starting position at time t minutes is given by $x = \frac{t^3}{3} - \frac{3t^2}{2} + 2t$, $t \geq 0$. The train changes direction twice. Determine the distance in km between these two positions on the track.

c
$x = \frac{t^3}{3} - \frac{3t^2}{2} + 2t$ $v = t^2 - 3t + 2 = (t-1)(t-2) = 0$ $t = 1, 2$ $x(1) = \frac{1}{3} - \frac{3}{2} + 2 = \frac{5}{6}$ $x(2) = \frac{8}{3} - 6 + 4 = \frac{2}{3}$ $dis\tan ce = \frac{5}{6} - \frac{2}{3} = \frac{1}{6} km$
Specific behaviours
✓ determines velocity function and equates to zero ✓ solves for x for one rest stop ✓ solves for x for second stop and then subtracts the two ✓ simplifies the distance between and gives units