

Question/Answer booklet

Semester Two Examination, 2020

INDEPENDENT PUBLIC SCHOOL  
EXCEPTIONAL SCHOOLDAY, EXCEPTIONAL STUDENTS.  


Question	Mark	Max	Question	Mark	Max
1	6	5	2	5	6
11	6	5	3	7	9
12	11	6	6	7	9
			7		4

**Important note to Candidates**  
Other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material if you have any unauthorised material with you.

Special items: nil

*o be provided by the candidate*

## ormula Sheet

**M**aterials required/recommended for this section  
to be provided by the supervisor

**Time allowed for this section**: five minutes  
**Working time**: reading time before commencing work:

Our Teacher's Name

Our Name

**Calculator-free**  
**Section One:**

UNITS 3 & 4

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	11	11	100	89	65
<b>Total</b>					<b>100</b>

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Solution	$\int_{-1}^0 \sin^2(5x) \cos(5x) dx$	$= \int_{-1}^0 \cos 5x \frac{5 \cos 5x}{1} du$	$= \int_{-1}^0 u^2 du = \left[ \frac{u^3}{3} \right]_{-1}^0 = \frac{0 - (-1)}{3} = \frac{1}{3}$
Uses change of variable			
Obtains new integral with changed limits			
Determines exact value			

(3 marks)

$$\int_{\pi}^{\pi/2} \sin^7(5x) \cos(5x) dx \quad \text{let } u = \sin 5x$$

(3 marks)

$$a) \int \cos^2(3x) dx$$

6 marks)

## Question 1

Working time: 50 minutes.

- spare pages are included at the end of this booklet. They can be used for planning your  
expenses and/or as additional space if required to continue to answer.  
Planning: If you need to use the spare pages for planning, indicate this clearly at the top of the page.  
Continuing an answer: If you need to use the spare space to continue an answer, indicate in the  
original answer space where the answer is continued, i.e. give the page number. Fill in the  
number of the question that you are continuing to answer at the top of the page.

This section has seven (7) questions. Answer all questions. Write your answers in the spaces provided.

**Section One: Calculator-free**

CALCULATOR-FREE MATHEMATICS METHODS 3

MATHEMATICS CALCULATOR-FREE METHODS

**Question 2 (5 marks)**

Consider the function  $P(z) = z^4 + 10z^2 + 9$  where  $z$  is a complex number.

a) Show that  $(z + 3i)$  is a factor of  $P(z)$ .

(2 marks)

Solution
$P(-3i) = (-3i)^4 + 10(-3i)^2 + 9 = 81 - 90 + 9 = 0$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses value of <math>-3i</math></li> <li>✓ evaluates each term and shows sum is zero</li> </ul>

b) Solve for all values for  $P(z) = 0$  in the form  $a + bi$ .

(3 marks)

Solution
$\begin{aligned} P(z) &= (z - 3i)(z + 3i)(z^2 + az + b) \\ &= (z^2 + 9)(z^2 + az + b) \\ z = 0 \quad 9b &= 9 \quad b = 1 \\ z^2 : a &= 0 \\ P(z) &= (z^2 + 9)(z^2 + 1) \\ \text{roots: } &\pm 3i, \pm i \end{aligned}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses conjugate of <math>-3i</math></li> <li>✓ fully factorises</li> <li>✓ states four complex roots</li> </ul>


**Specific behaviours**

a) Determine the values of  $a, b \& c$ . (3 marks)

Given that  $\int 10x^2 - 6x + 2 \, dx = a(x-2)^2 + b(x+1)(x-2) + c(x+1)$

$$\int 10x^2 - 6x + 2 \, dx = \frac{a}{3}(x-2)^3 + \frac{b}{2}(x+1)(x-2)^2 + cx + D$$

**Question 3** (6 marks)**CALCULATOR-FREE**


**Solution**

b) Hence determine the exact value of  $\int 10x^2 - 6x + 2 \, dx$ . (simply) (4 marks)

**Specific behaviours**

$u = 1$   
 $1 = 2A, A = \frac{1}{2}$   
 $u = -1$   
 $1 = 2B, B = \frac{1}{2}$   
 $u = -1$   
 $1 = A(1+u) + B(1-u)$   
 $\frac{1}{1-u} = \frac{1}{1+u} + \frac{B}{1-u} = A + \frac{B}{1+u}$   
 $-2 \int \frac{1-u}{1+u} du = -\left[ \ln(1-u) + \ln(1+u) \right] = -\ln \left| \frac{1-u}{1+u} \right| = -\ln 3$

**Solution**


**Specific behaviours**


**Solution**

Using the substitution  $u = \sin x$ , evaluate the integral  $\int_{-\pi/2}^{\pi/2} \cos x \, dx$ . (simply)

**Question 7** (6 marks)**CALCULATOR-FREE**

- c) Explain why  $\int_{-2}^2 \frac{10x^2 - 6x + 2}{(x+1)(x-2)^2} dx$  does not exist. (2 marks)

Solution
Discontinuous at $x=-1,2$ which is within interval
Specific behaviours
✓ states discontinuous ✓ states x values

**Question 4 (3,3 & 1 = 7 marks)**

Consider the following functions:

$$f(x) = e^{x+1}$$

$$g(x) = \frac{1}{\sqrt{x-2}}$$

$$h(x) = (x+3)^3$$

- a) Determine  $f^{-1}(x)$  and its domain. (3 marks)

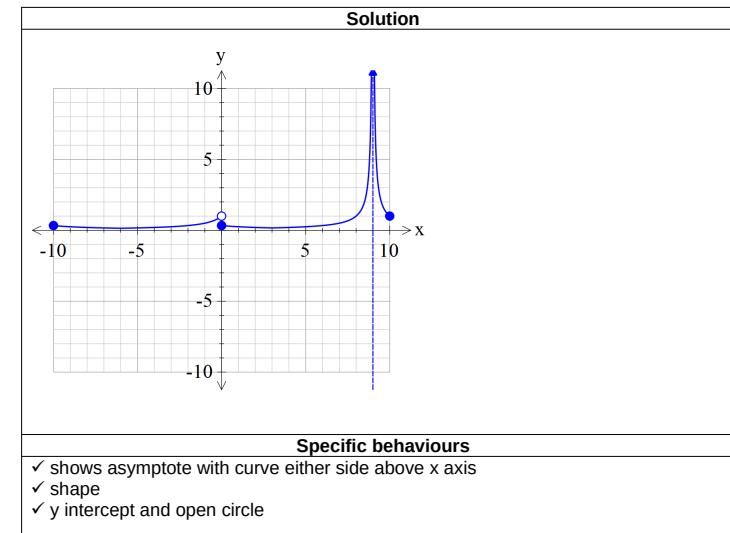
Solution
$x = e^{y+1}$ $\ln x = y + 1$ $f^{-1}(x) = \ln x - 1$ domain: $x > 0$
Specific behaviours
✓ shows method for determining inverse ✓ states inverse rule ✓ states domain

- b) Determine  $g \circ h(x)$  and its domain. (3 marks)

Solution
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See next page

d)  $y = \frac{1}{|f(x)|}$  (3 marks)



See next page

Solution
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a) Plot  $y = g^{-1}(x)$  on the axes above showing all major features. (3 marks)

Consider the function  $g(x)$  which is plotted below.

Question 5 (6 marks)

Solution
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c) Determine the solution(s), if any for  $f \circ h(x) = -1$ , explaining. (1 mark)

$$f \circ h(x) = e^{(x+3)^2} < -1$$

- ✓ states both parts of domain
- ✓ solves for BOTH limit values for  $x$
- ✓ states rule

#### Specific behaviours

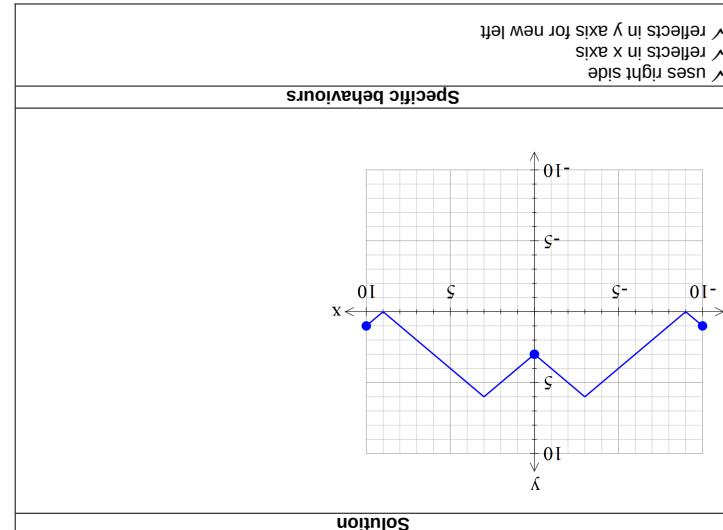
$$g \circ h(x) = \frac{\sqrt{x+3} - 2}{x}$$

$$(x+3)^2 - 2 > 0$$

$$x + 3 = \pm\sqrt{2}$$

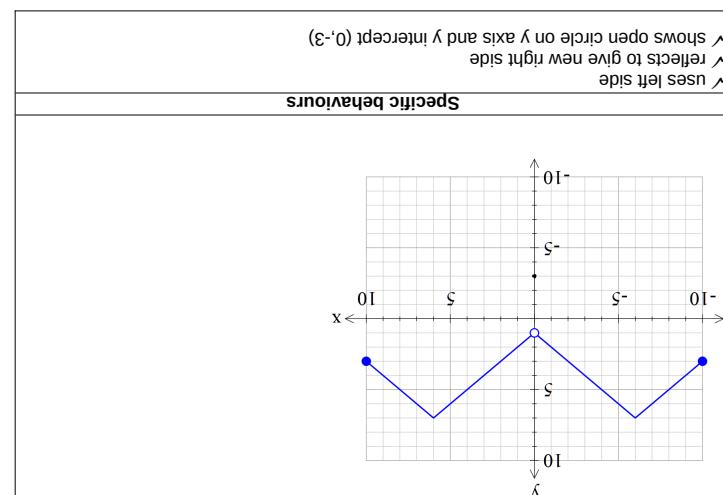
$$x < \sqrt{2} - 3, x > -\sqrt{2} - 3$$

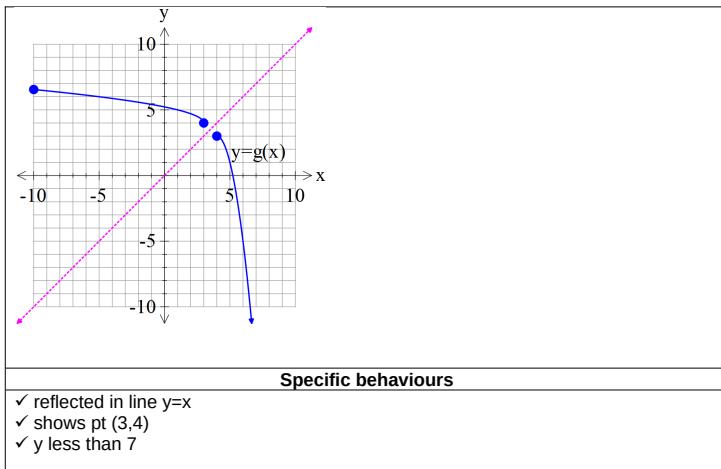
Solution
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(3 marks)

$$y = f(|x|)$$





- b) Given that  $g(x) = -2(x-4)^2 + 3, x \geq 4$ , determine the defining rule for  $g^{-1}(x)$  and its domain. (3 marks)

**Solution**

$$g(x) = -2(x-4)^2 + 3, x \geq 4$$

$$x = -2(y-4)^2 + 3, x \leq 3$$

$$\frac{x-3}{-2} = (y-4)^2$$

$$\sqrt{\frac{3-x}{2}} = \pm(y-4)$$

$$y = 4 + \sqrt{\frac{3-x}{2}} = f^{-1}(x)$$

domain:  $x \leq 3$

**Specific behaviours**

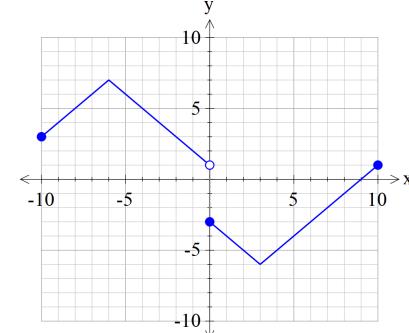
- ✓ rearranges x and y
- ✓ states rule
- ✓ states domain

See next page

(11 marks)

**Question 6**

Consider the function below.

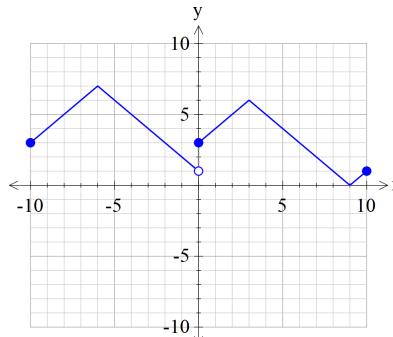


Sketch the following functions showing all major features.

a)  $y = |f(x)|$

(2 marks)

**Solution**



**Specific behaviours**

- ✓ shows left is unchanged
- ✓ reflects right in x axis

b)  $y = f(-|x|)$

(3 marks)

**Solution**

See next page