PERTH MODERN SCHOOL

UNIT 3CD MAS – 2014

TEST 1

POLAR COORDINATES, COMPLEX NUMBERS & VECTORS

NAME: _____ **DATE: Thurs. 13th Feb.**

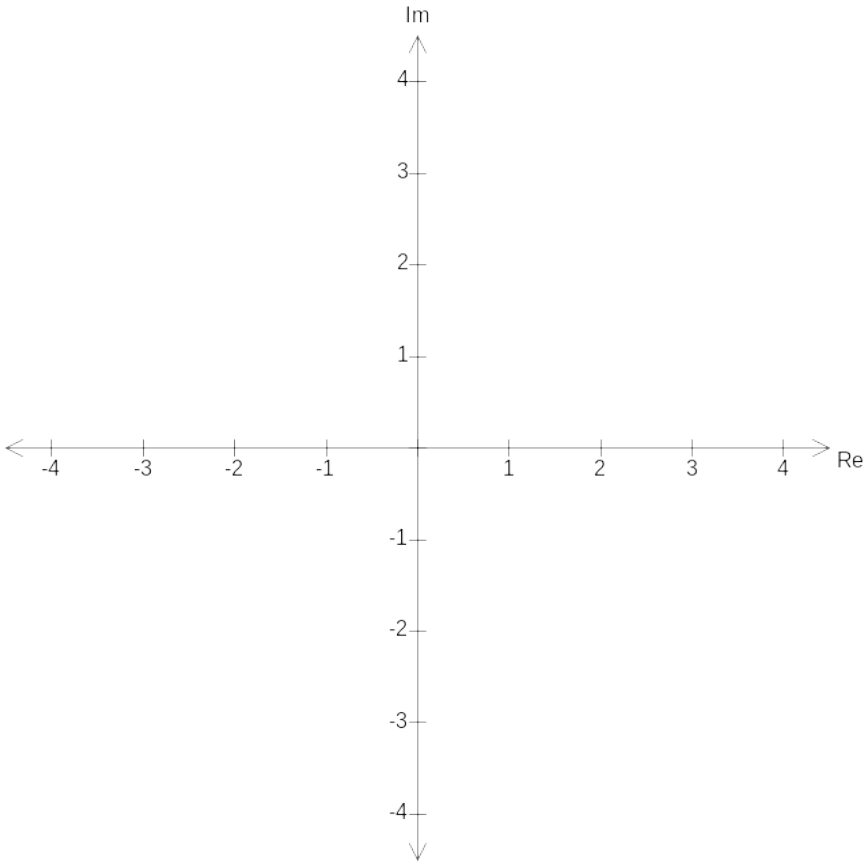
Total: 43 marks **Time: 45 min.**

1. Z is a complex number. Sketch the region given by **[5]**

$Re(Z) < 1$ and $Im(Z) > -2$

and $1 < |Z| < 3$

and $-\frac{5\pi}{12} \leq ArgZ \leq \frac{2\pi}{3}$



2. Express $Z = -1 - \sqrt{3}\,i$ in polar form. **[2]**

3. If $Z_1 = 5\text{cis}\frac{\pi}{6}$ and $Z_2 = 2\text{cis}\frac{\pi}{12}$, then prove $Z_1 Z_2 = 5\sqrt{2}\,(1 + i)$ **[4]**

4. Find Z if $Z\overline{Z} + 2Z = \frac{1}{4} + i$ **[6]**

5. Consider the 3D position vectors **[8]**

$\overrightarrow{OA} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ *and* $\overrightarrow{OB} = 5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

Determine:

a) the length of $\overrightarrow{AB}$.

b) *$\angle AOB$ to the nearest degree.*

c) the vector equation of the line, in parametric form, through the points A and B.

6. **[8]**

A has the rectangular coordinates $\left(-1, \sqrt{3}\right)$ *and B has polar coordinates* $\left(4, \frac{5\pi}{4}\right)$.

a) What are the exact polar coordinates of A? (1)

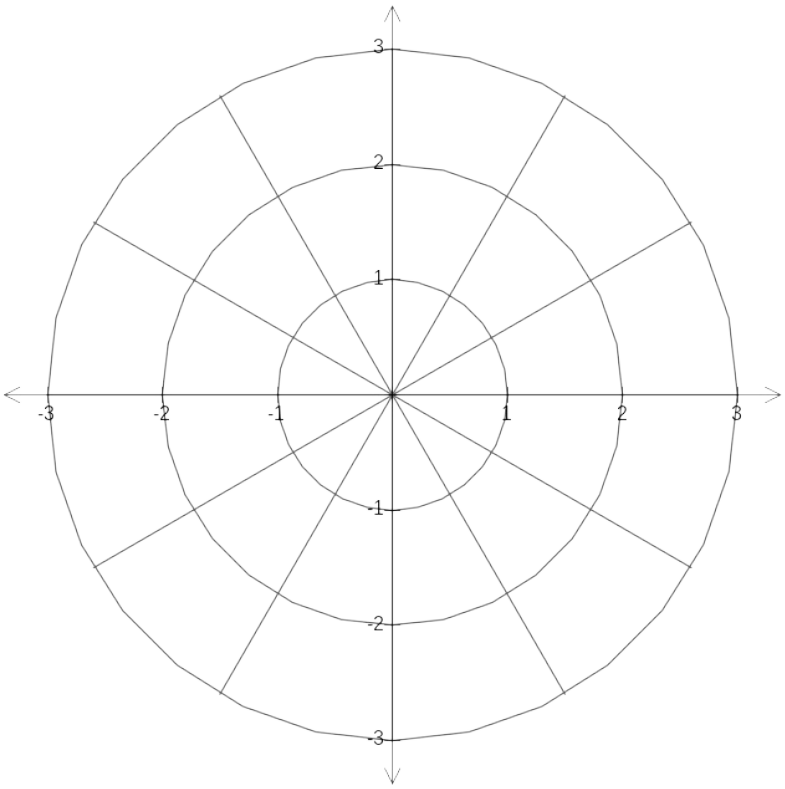
b) What are the exact rectangular coordinates of B? (2)

c) The graph of the polar equation $\mathbf{r} = \mathbf{k}\theta$ passes through the point B.

If $\mathbf{k} > 0$, determine the value of **k**.

Then, on the axes below, sketch the graph of $\mathbf{r} = \mathbf{k}\theta$ *for* $0 \leq \theta \leq \pi$.,

showing important features. (5)



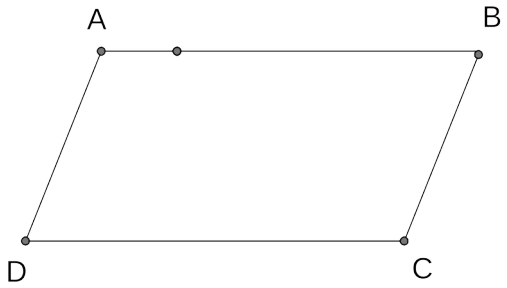
7. **[10]**

ABCD is a parallelogram with points E and F such that $\overrightarrow{AE} : \overrightarrow{EB} = 1 : 4$ *and* $\overrightarrow{BF} : \overrightarrow{FC} = 3 : 1$.

$\overrightarrow{ED}$ *and* $\overrightarrow{AF}$ *intersect each other at G.*

Let $\overrightarrow{AB} = a$ ***and*** $\overrightarrow{AD} = d$.

a) Complete the diagram below with the information given above. (2)



b) Determine the ratios in which $\overrightarrow{AF}$ *and* $\overrightarrow{ED}$ intersect each other, if the intersection point is at G. (8)