



TEST 4 – POLAR COORDINATES, COMPLEX NUMBERS & PROOFS

NAME: _____

DATE: _____

[To achieve full marks and to allow assessment of particular outcomes, working and reasoning should be shown.]

[A maximum of 2 marks will be deducted for incorrect rounding, units, etc.]

This is *Resource Free*, no calculator allowed – 50 minutes for 40 marks:

Question 1 [3 marks]

Express $1 - i$ and $1 + \sqrt{3}i$ in polar form and **hence** simplify $(1 + \sqrt{3}i)^5 \div (1 - i)^4$.

Question 2 [2 marks]

Find the **exact** distance between the points A $[6, 25^\circ]$ and B $[10, 145^\circ]$

Question 3 [5marks]

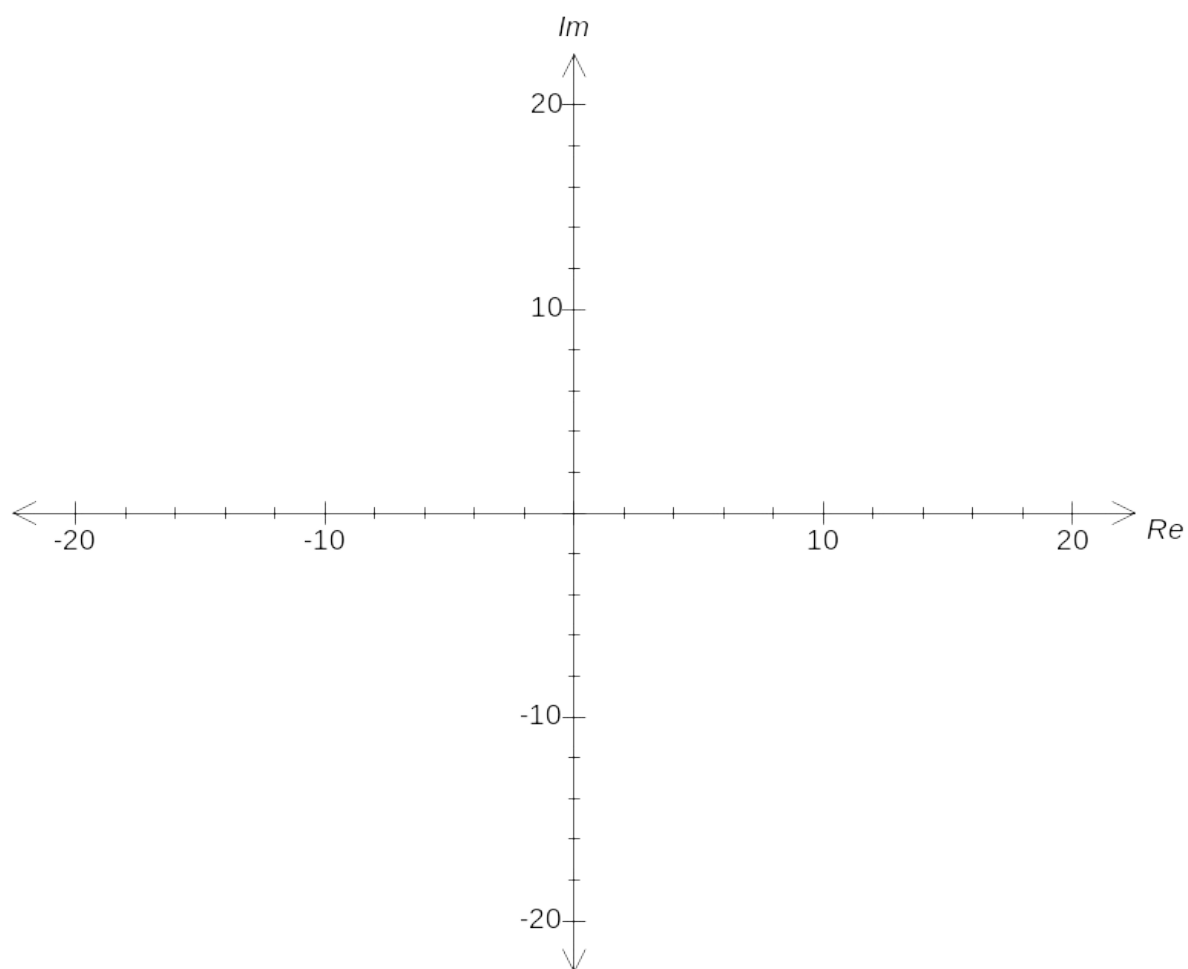
Given $z = 3 - 3i$ indicate on a single Argand Diagram the points representing

a) z^{-1}

b) z^2

c) iz

d) $z\bar{z}$



Question 4 [2, 3 marks]

a) Express the following in the form (a, b) where (a, b) represents the complex number $a + bi$:

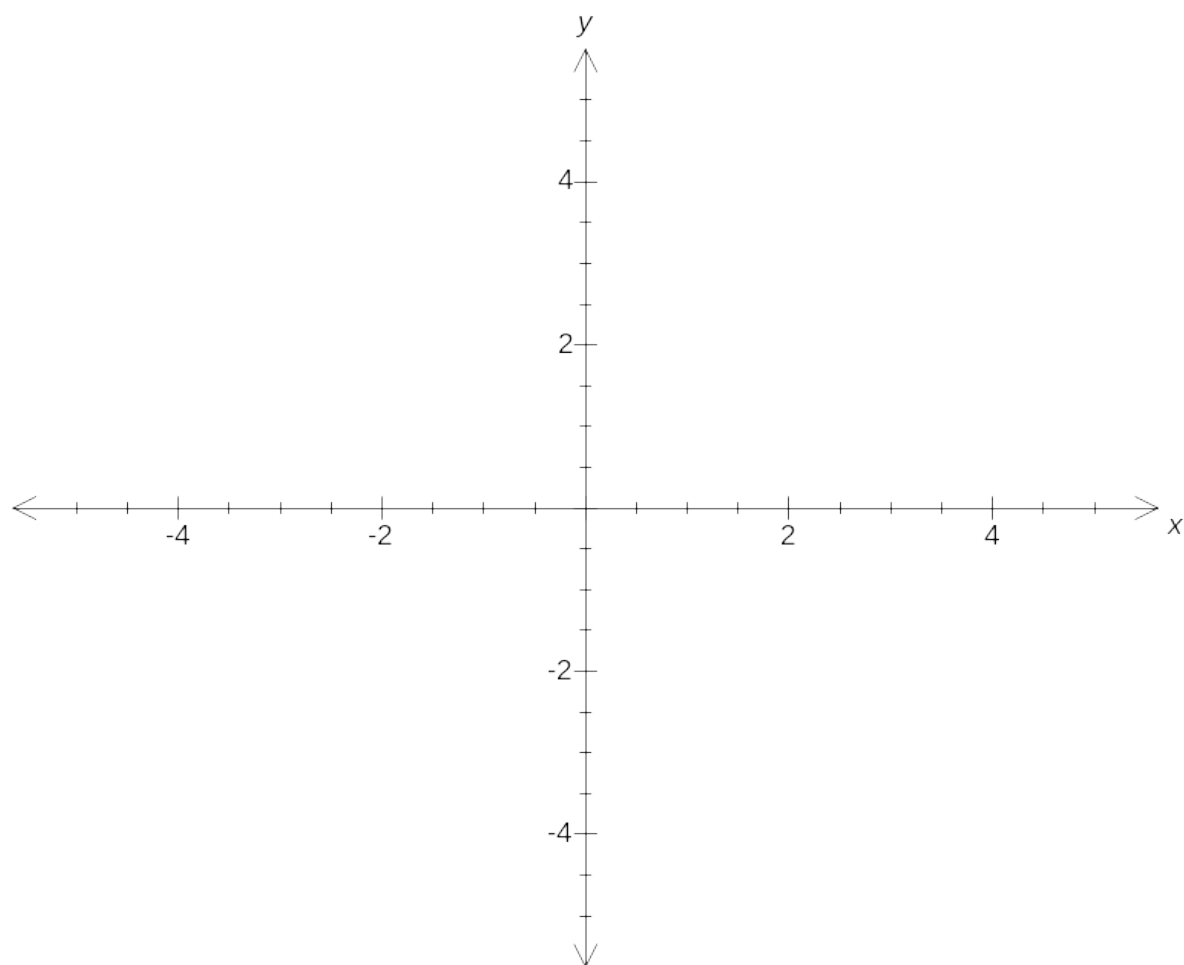
$$e^{2 - 0.5\pi i}$$

b) Using $\cos \theta + i \sin \theta = e^{i\theta}$ and $\cos \theta - i \sin \theta = e^{-i\theta}$ obtain an expression for both $\sin \theta$ and $\cos \theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$.

Hence prove $\sin (2\theta) = 2 \sin \theta \cos \theta$.

Question 5 [2, 2 marks]

a) Draw the graph of $y = |2x + 1| + |2x - 1|$



b) Hence, or otherwise, define y as a piecewise function

Question 6 [3 marks]

OABC is a rhombus with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

Use vector methods to prove the diagonals of a rhombus are perpendicular to each other

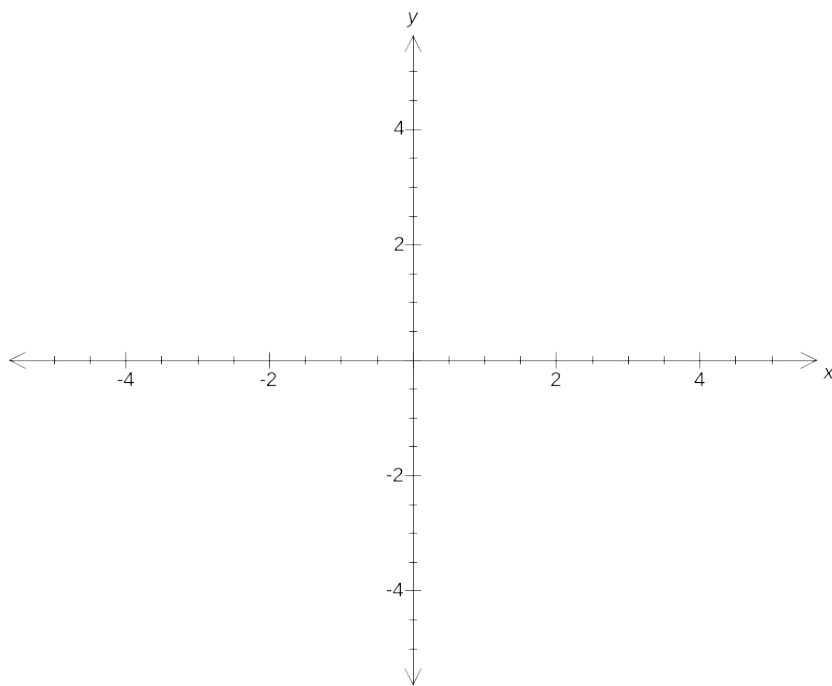
Question 7 [4 marks]

Prove by exhaustion that for a positive integer n then $n(n + 1)(n + 2)$ is a multiple of 3.

Question 8 [2, 2, 2 marks]

a) Find the set of values for x for which $1 - x > 2|x + 1|$.

b) Graph the function $y = 1 - x - 2|x + 1|$.



c) **Explain** how your graph can be used to solve part a).

Question 9 [3 marks]

Determine exact values for all the roots of $z^3 = -8 + 8\sqrt{3}i$

Question 10 [5 marks]

Find all complex numbers Z satisfying $\frac{1}{z} + \frac{2}{z} = 1 + i$