11. (11 marks)

Discrete random variable A represents the number of accidents each month on a building site. Over time, the average number of accidents per month is determined to be 3. There seems to be no limit to the number of accidents that can occur in any month.

The rule $P(A = a) = \dfrac{e^{-3} \times 3^a}{a!}$ where $a = 1, 2, 3, \ldots$ is used to calculate some values given in the partially completed table below.

a	0	1	2	3	4	5	…
$P(A = a)$	0.05	0.149	0.224	0.224	0.168	0.101	…

(a) Use the rule to determine the probability of exactly 1 accident next month. [1]

Above

(b) Determine the probability that:

(i) there are more than 2 accidents next month. [2]

$P(A>2) = 1 - P(A \leq 2)$

$= 1 - (0.05 + 0.149 + 0.224)$

$= 0.577$

F/T

(ii) there were more than 2 accidents in a month if it is known that there was at least one. [3]

$P(A>2 \mid A \geq 1) = \dfrac{0.577}{1-0.05}$

$= 0.6074$

(c) Determine the probability that during the next year the building site has at least 3 months which are accident free. [3]

X: month's which are accident free

$X \sim B(12, 0.05)$

$P(X \geq 3) = 0.0196$

TRINITY COLLEGE

Mathematics Methods Units 3/4
Test 3 2017

Section 1 Calculator Free

Calculus of Trig Functions, DRVs, Binomial Distributions & Logs

STUDENT'S NAME MARKING KEY

DATE: Thursday 18 May **TIME:** 20 minutes **MARKS:** 24

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (4 marks)

Determine:

(a) $\int \cos(3x) - 4\sin(-x) + 6 \; dx$ [2]

$= \dfrac{\sin 3x}{3} - 4\cos x + 6x + c$

(b) $\int \cos(8x)\cos(3x) + \sin(8x)\sin(3x) \; dx$ [2]

Given $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$= \int \cos(5x)$

$= \dfrac{\sin 5x}{5} + c$

2. (7 marks)

The discrete random variable Z has the probability distribution:

z	1	2	3	4
$P(Z = z)$	a	b	0.3	c

where a, b and c are constants.

The cumulative distribution function for Z is given by the following table:

z	1	2	3	4
$P(Z \le z)$	0.1	0.5	d	1

where d is a constant.

(a) Determine the value of a, b, c and d [5]

$P(Z=1) = P(Z \leq 1)$
$= 0.1$
$\therefore \underline{\underline{a = 0.1}}$ ✓

$P(Z \leq 2) = 0.5$
$\therefore\ 0.5 = 0.1 + b$
$\underline{\underline{b = 0.4}}$ ✓

$c = 1 - (0.1 + 0.4 + 0.3)$
$\underline{\underline{c = 0.2}}$ ✓✓

Allow F/T

$d = a + b + 0.3$
$= 0.1 + 0.4 + 0.3$
$\underline{\underline{d = 0.8}}$ ✓

(b) Given $Y = 3Z + 2$ determine $P(Y \ge 8)$ [2]

y	5	8	11	14
$P(Y=y)$	0.1	0.4	0.3	0.2

$3y + 2 \geqslant 8$
$3y \geqslant 6$
$y \geqslant 2$

✓✓

Allow F/T

$P(Z \geqslant 2) = 0.4 + 0.3 + 0.2$
$= 0.9$

9. (5 marks)

Brandon is the designated penalty taker for his soccer team and is practicing his penalty attempts. Past statistics show that the probability Brandon scores a goal at each penalty attempt is 70%.

Brandon 10 takes ten penalties. Determine the probability that:

(a) he scores all 10 penalties. [1]

$0.7^{10} = 0.0282$ ✓

(b) he scores the first 8 penalties but misses the last two. [2]

$(0.7)^8 (0.3)^2 = 0.0052$ ✓✓

(c) he scores less than 8 penalties if he scores at least five. [2]

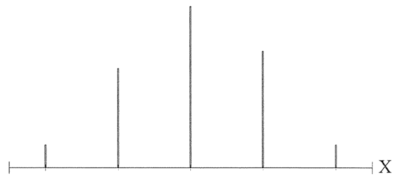
$X \sim B(10, 0.7)$

$P(X < 8 \mid X \geqslant 5) = \dfrac{P(5 \leq X \leq 7)}{P(X \geqslant 5)} = 0.5982$ ✓✓

10. (3 marks)

This graph represents a binomial probability distribution.

The height of the last column is 0.053



(a) State the value of n. [1]

$n = 4$ ✓

(b) State the probability of success for this binomial distribution, correct to 2 decimal places. [2]

${}^4C_4\, p^4 (1-p)^0 = 0.053$ ✓
$p = 0.4798$ ✓
$p = 0.48$

8. (9 marks)

The discrete random variable X has the probability function:

$$P(X=x)=\begin{cases} k(1-x)^2 & x=-1,0,1 \text{ and } 2 \\ 0 & \text{otherwise} \end{cases} \quad \text{where } k \text{ is a constant}$$

(a) Show that $k=\frac{1}{6}$ [3]

x	-1	0	1	2
$P(X=x$	$4k$	k	0	k

✓

$4k+k+k = 1$ ✓

$6k = 6$ ✓

$k = \frac{1}{6}$

(c) Determine $E(X)$ [2]

$E(X) = -\frac{1}{3}$ ✓✓

(d) Show that $E(X^2)=\frac{4}{3}$ [3]

$E(X^2) = (-1)^2(4k) + (2)^2(k)$ ✓

$= 8k$

$= \frac{8}{6}$ ✓✓

$= \frac{4}{3}$

(e) Hence or otherwise determine $\mathrm{Var}(1-3X)$ [2]

$Var(1-3X) = 9 \times Var(X)$

$= 9 \times \frac{11}{9}$ ✓✓

$= 11$

3. (5 marks)

Given that $p=\log_5 2$ and $q=\log_5 6$, express each of the following in terms of p and q.

(a) $\log_5 12$ [2]

$= \log_5(2\times 6)$

$= \log_5 2 + \log_5 6$ ✓✓

$= p+q$

(b) $\log_5 15$ [3]

$= \log_5\left(\frac{5\times 6}{2}\right)$ ✓✓✓

$= \log_5 5 + \log_5 6 - \log_5 2$

$= 1+q-p.$

4. (4 marks)

Solve $4-\log x=\log 100x$

$4 = \log 100x + \log x$

$4 = \log 100x^2$ ✓

$10^4 = 100x^2$ ✓

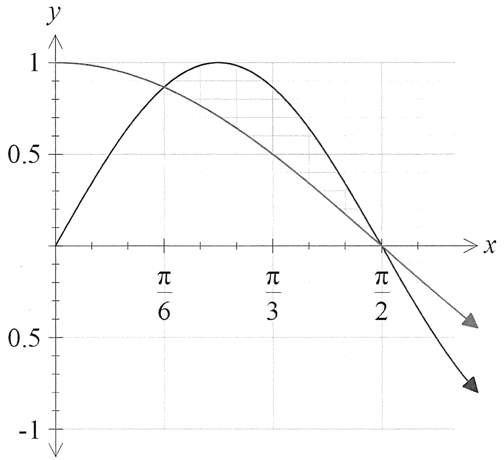
$x = \pm 10$ ✓

$x = 10$

$x \neq -10$ ✓

5. (4 marks)

Determine the area bound by the curves $y = \sin 2x$ and $y = \cos x$ over the domain $\frac{\pi}{6} \le x \le \frac{\pi}{2}$.



$$= \int_{\pi/6}^{\pi/2} \sin 2x - \cos x \, dx \quad ✓$$

$$= \left[\frac{-\cos 2x}{2} - \sin x \right]_{\pi/6}^{\pi/2} \quad ✓$$

$$= \left[-\frac{1}{2}\cos \pi - \sin \frac{\pi}{2} \right] - \left[-\frac{1}{2}\cos \frac{\pi}{3} - \sin \frac{\pi}{6} \right]$$

$$= \left(-\frac{1}{2} \times (-1) - 1 \right) - \left(-\frac{1}{2} \times \frac{1}{2} - \frac{1}{2} \right)$$

$$= \frac{1}{4} \text{ units}^2 \quad ✓✓$$

TRINITY COLLEGE

Mathematics Methods Units 3/4
Test 3 2017

Section 2 Calculator Assumed

Calculus of Trig Functions, DRVs, Binomial Distributions & Logs

STUDENT'S NAME MARKING KEY

DATE: Thursday 18 May **TIME:** 30 minutes **MARKS**: 34

INSTRUCTIONS:

Standard Items:	Pens, pencils, drawing templates, eraser
Special Items:	Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

6. (3 marks)

A particular binomial distribution consists of n trials and the probability of a successful outcome on each trial is p. If the experiment has an expected value of 36 and a standard deviation of 3, determine the values of n and p.

$E(X) = np = 36$ $\quad Var(X) = np(1-p)$, $Var(X) = 9$

$36(1-p) = 9$ ✓ $\quad p = \frac{3}{4}$ ✓

$(1-p) = \frac{1}{4}$ $\quad n = 48$ ✓

7. (3 marks)

A Bernoulli random variable B has a probability of success of 0.2.

(a) Give a situation which could be modelled by this Bernoulli random variable. [1]

Any valid solution. ✓

(b) Determine $E(B)$ and $Var(B)$. [2]

$E(B) = 0.2$ ✓

$Var(B) = 1(0.2)(0.8)$
$= 0.16$ ✓