

Perth College

Semester Two Examination, 2017

Question/Answer booklet

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| | If required by your examination administrator, please place your student identification label in this box | |

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| SC | 10 | H | T3 | M |
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Galculator-free

| u 2nA | Your name |
|-------|-----------|
| 17017 | In words |

Time allowed for this section

fifty minutes Working time: five minutes Reading time before commencing work:

Student Number: In figures

To be provided by the supervisor Materials required/recommended for this section

fluid/tape, eraser, ruler, highlighters

This Question/Answer booklet

Formula sheet

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction To be provided by the candidate

Special items:

Important note to candidates

it to the supervisor before reading any further. you do not have any unauthorised material. If you have any unauthorised material with you, hand No other items may be taken into the examination room. It is your responsibility to ensure that

CALCULATOR-ASSUMED

METHODS UNITS 1 AND 2

(8 marks)

(S marks)

Question 21

spown below. A composite solid is made from a cone and a cylinder, both of height h cm and radius au cm, as



The dimensions are such that the sum of h and 9r is 45 cm.

(a) Show that the volume of the solid is given by
$$V = 60\pi r^2 - 12\pi r^3$$
.

$$V = \frac{1}{5} \pi r^2 + 4\pi r^3 + 4\pi r^2 + 4\pi r^3 - 9\pi r^3 - 12\pi r^3$$

$$= \frac{1}{5} \pi r^2 - 3\pi r^3 + 4\pi r^3 - 9\pi r^3 - 12\pi r^3$$

$$= \frac{1}{5} \pi r^2 - 12\pi r^3$$

of the solid, and state this maximum volume. Use calculus techniques to determine the values of r and h that will maximise the volume

E-201-770NB

End of questions

| CAL | LCU | LAT | AP. | ED | EE |
|-----|-----|-----|-----|----|----|
| UAI | LUU | LMI | UK. | гπ | |

2

METHODS UNITS 1 AND 2

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|------------------------------------|-------------------------------------|--|------------------------------|--------------------|---------------------------|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
| | | | | Total | 100 |

Instructions to candidates

SN077-102-1

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this
 examination implies that you agree to abide by these rules.
- 2. Write your answers in this Question/Answer booklet.
- You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- 4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
- 5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- 6. It is recommended that you do not use pencil, except in diagrams.
- 7. The Formula sheet is not to be handed in with your Question/Answer booklet.

CALCULATOR-ASSUMED

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METHODS UNITS 1 AND 2

(d) (i) Find the obtuse $\angle PBQ$, in radians, correct to three decimal places.

(ii) Find the reflex $\angle PAQ$, in radians, correct to three decimal places.

(1 mark)

$$(os (PAB) = \frac{10^2 + 7^2 - 13^2}{2(10)(7)}$$

) 2777 360 (-Cost = answer - Gillo

96-205 = 1.714 x2

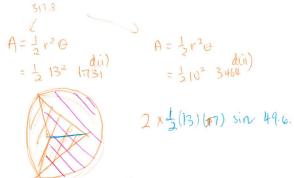
(accept 3.425)

from lovs frounding

(iii) Determine the area common to both circles. Answer to one decimal place.

(3 marks)

$$\frac{1}{2}(13)^{2}(1.731-\sin 1.731)+\frac{1}{2}(10)^{2}(3.428-\sin 3.428)$$



SN077-102-3

32% (25 Marks)

CALCULATOR-FREE

Section One: Calculator-free

This section has eight (8) questions. Answer all questions. Write your answers in the spaces

Working time: 50 minutes.

(7 marks)

(3 wsuks)

Question 1

The tenth term of an arithmetic sequence is 67 and the fourteenth term is 51.

(4 marks) Determine the sum of the first 20 terms of the sequence.

V 008) = 1 ((+-) b) + 902) = = 45 100 = 0 100 = 0 100 = 0 100 = 0 100 = 07081+0=1S 706 +0 =L9

(b) The geometric series x, x^2 , x^3 , x^4 ... has a sum to infinity of 24.

12 State the first term.

 $\chi = 7h7 - h7$ (iii) Determine the value of x. $\int \psi = \frac{\lambda}{\lambda - 1}$

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METHODS UNITS 1 AND 2

CALCULATOR-ASSUMED

Question 20

apart and the circles intersect at P and Q. Two circles of radii 10 cm and 13 cm have centres at A and B respectively. The centres are 7 cm

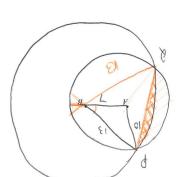
(5 marks)

(11 marks)

Sketch a diagram of the two circles and clearly show triangle ABP.

I triangle Velreles

(z warks)



Show that $\angle PBA = 49.6^{\circ}$, when rounded to one decimal place.

 $\sqrt{\frac{01-\xi1+2L}{7}}=(9)507$

 $\frac{1}{8} \frac{1}{20}$

(S marks)

Determine the length of the chord PQ to the nearest millimetre.

1 7.66500 X EIX EIX 7 - EI + EI = 80

PQ = 19.8 cm

(1811) [MIS (B)] = OR WISK = DI See next page

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METHODS UNITS 1 AND 2

Question 2

(5 marks)

Determine f'(x) if

(i) $f(x) = 8x^5 - x + 1$.

(1 mark)

(ii) $f(x) = (3x+5)^2$.

(2 marks)

$$f(x) = 9n^2 + 30x + 25 \checkmark$$

$$f'(x) = 18x + 30$$

The area of an oil spill, at time t hours, is given by $A(t) = 0.25t^2 + 0.5t + 0.75 \text{ m}^2$. Determine the instantaneous rate of change of area of the spill when t = 8 hours.

(2 marks)

$$A'(t) = 0.5t + 0.5$$

 $A'(8) = 0.5(8) + 0.5$
 $= 4.5 \text{ m}^2/\text{h} \sqrt{}$

CALCULATOR-ASSUMED

13

METHODS UNITS 1 AND 2

Question 19

(8 marks)

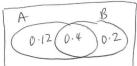
Events A and B occur at random and it is known that P(B) = 0.6 and $P(A \cup B) = 0.72$.

Determine P(A) when

A and B are mutually exclusive.

(1 mark)

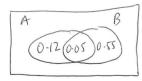
 $P(A \cap B) = 0.4.$



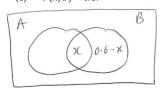
(1 mark)

 $P(\bar{A} \cap B) = 0.55.$

(1 mark)

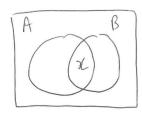


(d) P(A/B) = 0.3.



(2 marks)

(e) A and B are independent.



$$P(A) \times P(B) = P(A \cap B)$$

$$(1 \cdot 2 + X) \times 0 \cdot 6 = X$$

$$\chi = 0.18 \vee$$

$$P(A) = 0.3 \vee$$

SN077-102-3

(g marks)

CALCULATOR-FREE

Question 3

Determine the values of the constants a, b and c. The graph of $y=\alpha x^3+bx+c$ has a stationary point at (2, 29) and a gradient of 18 when x=1.

$$7+(z)+z+z(z)z-=bi$$

$$\uparrow + i = 9$$

$$q+(z-)z=8i$$

$$\uparrow z-=0$$

$$\uparrow b-=8i$$

$$\uparrow q+z = 0$$

$$\uparrow q+z = 0$$

$$\uparrow q+z = 0$$

(S warks)

(b) Determine the coordinates of any other stationary points.

7+8h+ 91- = 67

V E-=J

See next page

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METHODS UNITS 1 AND 2

CALCULATOR-ASSUMED

of five letters.

Five different letters are selected from the eleven in the word COMRADESHIP. The order in (8 marks) Question 18

15

selection RAMOC, and so on. which the letters are selected is not important, so that the selection COMRA is the same as the

Determine the number of different selections

179h = 1(1)

of five letters that contain one vowel and four consonants.

 \sim $Ohl = \begin{pmatrix} t \\ L \end{pmatrix} \begin{pmatrix} t \\ t \end{pmatrix}$

Determine the probability that a random selection of five different letters

includes the letters M and R.

$$\left(\begin{array}{c} \varepsilon_{H} \\ \overline{\Sigma} \end{array}\right) = \left(\begin{array}{c} \varepsilon_{H} \\ \overline{\varepsilon} \end{array}\right) \left($$

(2 marks)

(S marks)

(5 marks)

(2 marks)

includes at least one vowel.

$$\frac{1}{1} \frac{1}{1} \frac{1}{1} = \frac{1}{1} \frac{$$

it probas

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METHODS UNITS 1 AND 2

Question 4

SN077-102-1

(7 marks)

(a) Evaluate $x^{2a} \div x^b$ when x = 16, a = 1.5 and b = 3.5.

(3 marks)

$$\frac{16^{3}}{16^{3.5}} \checkmark = \frac{1}{16^{0.5}} \checkmark = \frac{1}{4} \checkmark$$

The first two terms of a geometric sequence are 1.5×10^{-2} and 3×10^{-5} respectively. Calculate the fifth term of the sequence, giving your answer in scientific notation. (4 marks)

$$\Gamma = \frac{3 \times 10^{-5}}{1.5 \times 10^{-2}} = 2 \times 10^{-3} \checkmark$$

CALCULATOR-ASSUMED

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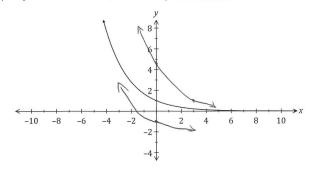
METHODS UNITS 1 AND 2

Question 17

SN077-102-3

(8 marks)

The graph of $y = a^x$ is shown below, where a is a positive constant.



On the same axes, sketch and label the graphs of

(i) $y = a^{x-3}$. goes through (3,1) \checkmark shape

(2 marks)

(ii) $y = a^x - 2$. y int (0,-1)asymptote 4=-2V

(2 marks)

The graph of $y = a^{x-3}$ intersects the graph of $y = 1.2^x$ when x = 2.1.

Determine, giving your answers to 3 significant figures,

(i) the y-coordinate of the point of intersection.

(1 mark)

y=1.2^{2.1} ≈ 1.47 ✓

Ray (21,1.47)

the value of the constant a. (HINT: don't use any previously rounded solution)

$$1.2^{2.1} = a^{2.1-3}$$
 $\alpha = 0.653$

See next page Condon gives (a)
See next page

CALCULATOR-FREE

(1 marks)

(5 marks)

2 noiteauD

Solve the following equations for x:

(a) $2\cos x = 1$, $0 \le x \le 360^\circ$.

 $\frac{7}{1} = 750$

x= 300° x

(5 marks)

(2+x) Z = (5-x) & $\frac{1}{2} = \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$ (d)

++x2 = b-x8

1 51 = 2

(3 marks)

 $0 = 3\xi - {}^{2}(\xi - x\zeta)$ (a)

 $9 = 2 - \times 7$ $9 = 2 \left(2 - \times 7 \right)$

 $1 - \frac{1}{2} = \frac{1}{2} =$

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METHODS UNITS 1 AND 2

CALCULATOR-ASSUMED

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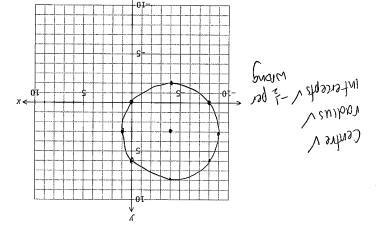
(e marks)

(z wsrks)

(a) The variables x and y are related by $(x+4)^2+(y-3)^2=25$.

(3 marks) Sketch the graph of this relationship, showing all key features.

10



(1 mark) (ii) How does the vertical line test indicate that y is not a function of x?

cut the graph more than once v

(b) State the domain and range of the function $f(x) = 4 - \sqrt{x} + 3$.

domain 271-3 V

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METHODS UNITS 1 AND 2

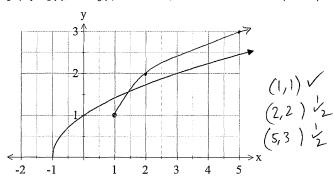
Question 6

(5 marks)

(a) The graph of y = f(x) is shown below, where $f(x) = \sqrt{x+1}$.

Add the graph y = g(x), where $g(x) = \sqrt{x-1} + 1$, to the axes.

(2 marks)



(b) Using first principles, find the value of the gradient of the curve $f(x) = x^3$ at the point where x = 5. (3 marks)

$$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$1 \Rightarrow 0$$
 $\frac{\chi^3 + 3\chi^2 h + 3\chi h^2 + h^3 - \chi^3}{h}$

$$\lim_{h \to 0} 3x^2h + 3xh^2 + h^3$$

$$\lim_{h \to 0} 3x^2 + 3zh + h^2$$

$$= 3 (s)^2$$

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See next page

CALCULATOR-ASSUMED

9

METHODS UNITS 1 AND 2

Question 15

(8 marks)

(2 marks)

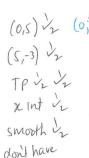
(1 mark)

(2 marks)

A particle is moving along a straight line so that its displacement, x metres, from a fixed point θ after t seconds is given by

$$x = 5 - \frac{18t}{5} + \frac{12t^2}{5} - \frac{2t^3}{5}.$$

 Sketch the displacement of the particle on the axes below for 0 ≤ t ≤ 5, labelling key points.



2 2 2 3 4 (5, -3)

to write (b) in coord but has to be on Correct

point

Determine the velocity of the particle when t = 0.5. $\frac{dx}{dt} = \frac{-3.6}{5} + \frac{4.8t}{24t} - \frac{2t^2}{6t^2}$ $\frac{dx}{dt} = \frac{-18}{5} + \frac{24t}{5} - \frac{6t^2}{5}$

at
$$t=0.5$$
 $\frac{dx}{dt}=-1.5$ m/s $\sqrt{}$

(c) For how long during the first five seconds is the graph decreasing?

$$3 < t < 1$$
 3 seconds $\sqrt{}$

d) Determine the time(s) when the velocity of the particle is -3.6 m/s.

$$-3.6 = \frac{-18}{5} + \frac{24t}{5} - \frac{6t^2}{7}$$

$$t = 0 \checkmark \qquad t = 4 \checkmark$$

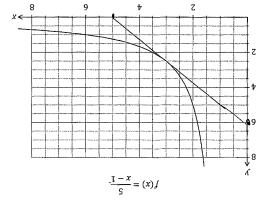
SN077-102-3

CALCULATOR-FREE

Question 7

(e marks)

The graph of the function y = f(x) is shown below, where



estimate the value of ∫'(3). (a) Draw the tangent to the graph at x=3 so that it cuts both axes, and use the tangent to

tangent V tangent yanu behveen s and T V (anggest cuts xanis between it and b

Mse for gradient V

(3 warks) Calculate the average rate of change of the function as \boldsymbol{x} increases from 3 to 3.5.

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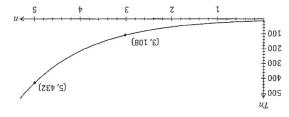
METHODS UNITS 1 AND 2

(8 marks)

CALCULATOR-ASSUMED

Question 14

months, is shown in the exponential graph below. The number of followers of a social media influencer, counted at the start of five successive



The number of followers (T_n) at the start of month n can be modelled by the recursive equation

 $T_{n+1} = rT_n$, $T_1 = a$.

(3 marks)

(1 mark)

108 x 12 2 13 2 1 Use the graph to determine the values of τ and a.

1 LZ = D ハてニ」

Assuming the growth rate continues,

how many followers are expected at the start of month 12?

196255

at the start of which month will the number of followers first exceed 750 000?

16th month V

(3 marks) will they have less than 2 000 followers? started to lose 25% of their followers each month. After how many months from this time When the number of followers reached 1 million, the influencer fell out of favour and

~ (SL.0) 0000001 = 0007

See next page

5-201-110NS

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CALCULATOR-FREE

Question 8

(7 marks)

(a) Determine the coefficient of the a^3 term in the expansion of $(2a-3)^4$.

(3 marks)

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} (2\alpha)^3 (-3)$$

$$4 \times 2^3 \times -3 = -96$$

(b) Consider the equation $x^3 - 4x^2 - 11x + 30 = 0$.

(i) Show that x = 2 is a solution of the equation.

(1 mark)

$$2^{3}-4(2)^{2}-11(2)+30=0$$

(ii) Determine all other solutions

(3 marks)

CALCULATOR-ASSUMED

7

METHODS UNITS 1 AND 2

Question 13

(6 marks)

The quadratic function $f(x) = ax^2 + bx + c$ passes through P(3, -10) and has roots at x = -5 and x = 8.

(a) Determine the values of the constants a, b and c.

(3 marks)

$$f(x) = a (x+5) (x-8) \sqrt{10}$$

$$f(x) = a (x^2 - 3x - 40)$$

$$-10 = a (3^2 - 3(3) - 40)$$

$$-10 = -40a$$

$$a = 0.25 \sqrt{10}$$

$$b = -0.75 \sqrt{10}$$

$$c = -10 \sqrt{10}$$

(b) State the y-intercept of the graph y = -5f(x).

(1 mark

$$y=-5[0.25x^2-0.75x-10]$$
 yint (-10).
 $(0,50)$ or $C=50$

(c) State the roots of the graph y = f(2x).

(2 marks)

ш

Additional working space

Question number:

CALCULATOR-FREE

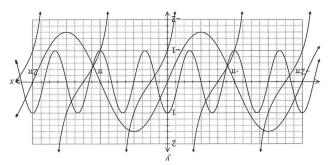
METHODS UNITS 1 AND 2

CALCULATOR-ASSUMED

(7 marks)

Question 12

(a) The graphs of $y = a \sin x$, $y = \cos(bx)$ and $y = \tan(x + c)$ are shown below.



(3 marks)

(S marks)

Determine the values of the constants a, b and c.

hours, after midnight. For this day, determine in the sew t be setter and the depth in metres and t was the time, in the time, in $t = 9.5 + 3.2 \cos(0.5t - 0.4)$, where $t = 0.5 + 3.2 \cos(0.5t - 0.4)$ (b) One day, the depth of water in a tidal basin was modelled (in radians) by

(i) the depth of water at 4.30 am. (5 marks)

179.8=(h.0-5.4×5.0)502 2.8+ 5.6=7 45.4=7

(ii) the first time in the afternoon that the depth of water was 7 m.

(3) 07:9 7 md81:9 1 32 48:81 = + E (4.0-75.0) SOO 2.8 + 5.6=[

See next page

S-S01-170NS

METHODS UNITS 1 AND 2

12

| Markers use only | | | | | |
|--------------------|---------|------|--|--|--|
| Question | Maximum | Mark | | | |
| 1 | 7 | | | | |
| 2 | 5 | | | | |
| 3 | 8 | | | | |
| 4 | 7 | | | | |
| 5 | 7 | | | | |
| 6 | 5 | _ | | | |
| 7 | 6 | | | | |
| 8 | 7 | | | | |
| S1 Total | 52 | _ | | | |
| S1 Wt (×0.6731) | 35% | _ | | | |
| S2 Wt | 65% | | | | |
| Total | 100% | | | | |

CALCULATOR-ASSUMED

5

METHODS UNITS 1 AND 2

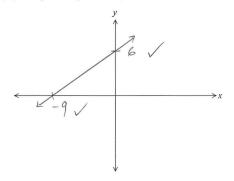
Question 11

(6 marks)

Line L_1 has equation 3y - 2x = 18.

a) Sketch the graph of L_1 , showing all intercepts.

(2 marks)



(b) Determine the equation of line L_2 that is parallel to L_1 and passes through the point with coordinates (-5,-6). (2 marks)

coordinates (-5, -6).

$$3y-2x=18$$

 $y=\frac{2}{3}x+6$
 $y=\frac{2}{3}x+C$ $y=\frac{2}{3}x-\frac{8}{3}$
 $-6=\frac{2}{3}(-5)+C$ $y=\frac{2}{3}x-\frac{8}{3}$
 $C=-\frac{8}{3}$ $C=\frac{1}{3}$

(c) Determine the equation of line L_3 that is perpendicular to L_1 and has the same y-intercept as L_1 . (2 marks

$$y = -\frac{3}{2} \times + 6$$

SN077-102-3



Perth College

Semester Two Examination, 2017

Question/Answer booklet

| | If required by your examination administrator, please place your student identification label in this box |
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S GNA 1 STINU **WETHODS NATHEMATICS**

Calculator-assumed Section Two:

| | Amen anox | · 27 oly 2017 | |
|-----------------|------------|---------------|---|
| | In words | INAH | |
| Student Number: | ln figures | | Γ |

Time allowed for this section

one hundred minutes Working time: Reading time before commencing work: ten minutes

To be provided by the supervisor Materials required/recommended for this section

Formula sheet (retained from Section One) This Question/Answer booklet

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fluid/tape, eraser, ruler, highlighters

and up to three calculators approved for use in this examination drawing instruments, templates, notes on two unfolded sheets of A4 paper, Special items:

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> (8 marks) Ot noitesup METHODS UNITS 1 AND 2 CALCULATOR-ASSUMED

canteen. 72 had bought neither, 128 had bought a snack and 48 had bought both. A group of 240 students were asked whether they had bought a drink or a snack from the school

(a) Determine the number of students who only bought a drink. (S marks)

104 = (5 10 €) N ch2

(b) Determine the probability that a randomly chosen student from the group had bought

(1 mark) a snack or a drink.

(1 mark) ouly a snack.

(2 marks) (iii) a snack given that they had bought a drink.

independent? Justify your answer. For this group of students, are the events buying a snack and buying a drink

(5 marks)

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|-----|-----|------|-------|------|

2

METHODS UNITS 1 AND 2

Structure of this paper

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CALCULATOR-ASSUMED

3

METHODS UNITS 1 AND 2

Section Two: Calculator-assumed

65% (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(6 marks)

(a) The tangent to the curve $y = 10 + 2x - x^2$ at (2,10) intersects the x-axis at (a,0). Determine the value of a. (3 marks)

$$y'=2-2x$$
 $0=-2x+14$
 $y'=-2$ $x=7$
 $y=-2x+C$ $a=7$
 $10=-2(2)+C$
 $c=14$
 $y=-2x+14$

(b) If $f'(x) = 1 + x - x^3$ and f(1) = 0, determine f(3).

(3 marks)

$$f(x) = x + \frac{x^{2}}{2} - \frac{x^{4}}{4} + C$$

$$0 = 1 + \frac{1^{2}}{2} - \frac{1^{4}}{4} + C$$

$$C = -\frac{5}{4} - \frac{1 \cdot 25}{4}$$

$$f(3) = 3 + \frac{3^{2}}{2} - \frac{3^{4}}{4} - \frac{5}{4}$$

$$= -14 \checkmark$$

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