

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations.

Standard items: fluid/tape, eraser, ruler, highlighters, pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/ink.

To be provided by the candidate
Formula Sheet (retained from Section One)
This Question/Answer Booklet
To be provided by the supervisor

Materials required/recommended for this section
Working time for section: one hundred minutes
Reading time before commencing work: ten minutes

Your Teacher's name

Your name

MATHEMATICS
METHODS
UNIT 3 & 4
Section Two:
Calculator-assumed

PETH MODERN SCHOOL
Exceptional schooling. Exceptional students.

Perth Modern School



CALCULATOR ASSUMED
METHODS UNIT 3 AND 4

Question/Answer Booklet

Semester Two Examination, 2019

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	14	14	100	105	65
Total				155	100

Question number: _____

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

<p>✓ obtains expression for area in terms of side length</p> <p>✓ uses incremental theorem</p> <p>✓ differentiates correctly</p> <p>✓ determines approx. change in area (no need for units)</p>	<p>Specific behaviours</p>
$A = \frac{1}{2} x^2 \sin 60$ $= \frac{\sqrt{3}}{4} x^2$ $\Delta A \approx \frac{\sqrt{3}}{4} \Delta x$ $= \frac{2\sqrt{3}(5)}{4} \Delta x \approx 0.866$	
<p>Solution</p>	

The area of a triangle can be found by the formula $A = \frac{1}{2} ab \sin C$. Using the incremental formula, determine the approximate change in area of an equilateral triangle, with each side of 5 cm, when each side increases by 0.2 cm.

(4 marks)

Question 8

This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

(105 Marks)

Question 9

(9 marks)

- (a) The probability of Australian Year 12 students obtaining their driver's licence is thought to be around 65%. How large a sample of students should be used to establish this within a margin of error of 5% at the 90% confidence level? (3 marks)

Solution

Specific behaviours

- ✓ uses z percentile for 0.90
- ✓ states an equation for sample size
- ✓ rounds n up

- (b) A survey of 1000 Australians renting houses found that 35% of their income was paid in rent.

- (i) Estimate the standard deviation of the sampling distribution. (2 marks)

Solution

Specific behaviours

- ✓ uses formula
- ✓ states standard deviation

See next page

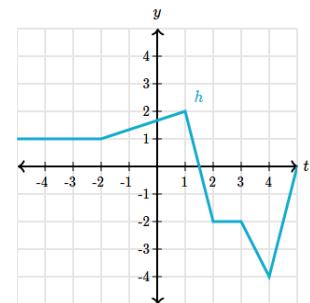
$$\begin{aligned}
 g(x) &= \int_{-2}^x f(t)dt \\
 g'(x) &= f(x) \\
 g''(x) &= f'(x) \\
 f'(x) &= 6x \\
 f(x) &= 3x^2 + c \\
 g(2) &= \int_{-2}^2 (3t^2 + c)dt = 20 = [t^3 + ct]_{-2}^2 = (8 + 2c) - (-8 - 2c) \\
 20 &= 16 + 4c \\
 c &= 1 \\
 f(x) &= 3x^2 + 1
 \end{aligned}$$

Specific behaviours
✓ uses fundamental theorem
✓ obtains expression for second derivative of g
✓ integrates to find f with constant
✓ uses g integral to set up 1 equation in terms of c OR 2 equations in terms of 2 constants
✓ determined function f in terms of x

End of Questions

NOTE: max of 2 marks for interval only

(10 marks)

**Question 20**The graph of $h(x)$ is shown on the right.

(a) Evaluate the following definite integrals

$$(i) \int_{-2}^1 h'(x)dx \quad (2 \text{ marks})$$

Solution

$$\int_{-2}^1 h'(x)dx = [h(x)]_{-2}^1 = h(1) - h(-2) = 2 - 1 = 1$$

Specific behaviours

- ✓ uses fundamental theorem
- ✓ determines integral

$$(ii) \int_2^3 10h'(x)dx \quad (2 \text{ marks})$$

Solution

$$10 \int_2^3 h'(x)dx = 10 [h(x)]_2^3 = 10[h(3) - h(2)] = 10(0) = 0$$

Specific behaviours

- ✓ uses fundamental theorem with factor 10
- ✓ determines integral

$$(iii) \int_{-2}^4 (h(x) + 1)dx \quad (2 \text{ marks})$$

Solution

$$\begin{aligned} \int_{-2}^4 (h(x) + 1)dx &= [h(x) + x]_{-2}^4 = (h(4) + 4) - (h(-2) - 2) \\ &= 0 - 1 + 2 = 1 \end{aligned}$$

Solution

Specific behaviours

- ✓ shows two asymptotes
- ✓ shows x intercept
- ✓ correct shape shown only for $x > -2$

Question 10

Solution

b) Sketch $f(x)$ on the axes below showing any intercepts and asymptotes. (3 marks)

Specific behaviours

- ✓ states an un-simplified expression
- ✓ diff in correctly
- ✓ starts a tangent line at point A

Solution

$$f(x) = \frac{x^2}{x+2} - \ln(x+2)$$

$$f(x) = \frac{x}{\ln(x+2)}$$

a) Determine $f(x)$ using the quotient rule. (3 marks)

(6 marks)

Solution

Specific behaviours

- ✓ differentiates curve
- ✓ determines derivative at $x=0$
- ✓ states common point A
- ✓ states gradients both -1 at pt A

As both line and curve have a common point A at $x=0$ and same gradient at this point the line is a tangent.

Solution

$$\frac{d}{dx} (x+1) \cdot (x-1) \cdot (x-d) = 0$$

$$3x^2 - 2 \cdot d \cdot x - 1 |_{x=0}$$

Question 11

(9 marks)

In a general population, 15% of people are left-handed. A class has 32 students and number of who are left-handed is counted. Let X = number of left-handed students in a class of 32 students.

- (a) State and describe the type of distribution of X . (3 marks)

Solution
$X \sim \text{Bin}(32, 0.15)$
Specific behaviours
✓ states Binomial ✓ states n value ✓ states p value

- (b) How many in the class would you expected to be left-handed? (1 mark)

Solution
$\mu = 32 \times 0.15 = 4.8$
Specific behaviours
✓ states mean value

- (c) What is the probability that the number of left-handed students in the class is within one standard deviation of the mean? (3 marks)

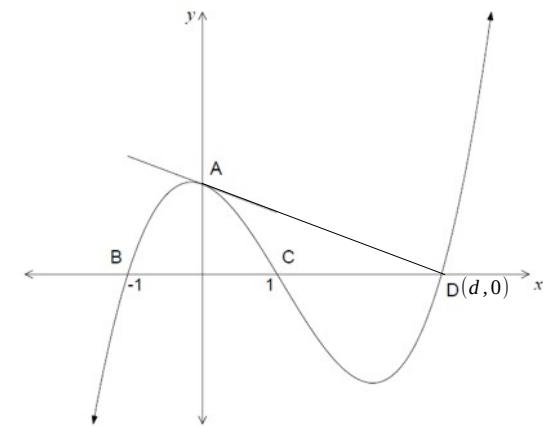
Solution
Specific behaviours
✓ calculates standard deviation

See next page

Question 19

(6 marks)

The diagram below shows the graph of $y=(x+1)(x-1)(x-d)$ with constant d where $d>1$. The graph intercepts the y -axis at the point A.



- (a) Find the equation of line AD in terms of d . (2 marks)

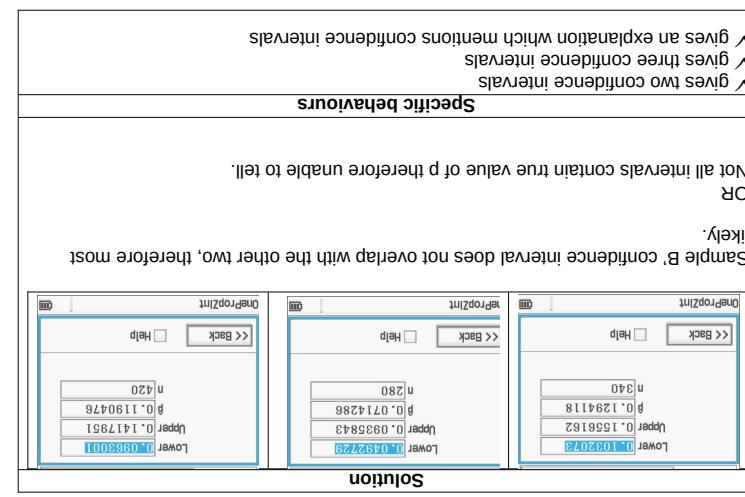
Solution
$A(0, d) D(d, 0)$
$\text{gradient} = \frac{0-d}{d-0} = -1$
$y = -x + c$
$d = c$
$y = -x + d$
Specific behaviours
✓ determines gradient ✓ determines equation

- (b) Show that a line that passes through A and D is tangent to the graph at A. (4 marks)

See next page



- (d) Find the probability that at most half of the class are left-handed. (2 marks)



$$a = 15 \sin(2t + \frac{\pi}{3})$$

The equation $a = 15 \sin(2t + \frac{\pi}{3})$ is used to measure the acceleration, a metres per second squared at time t seconds, of a weight at the end of a spring. Initially the weight is at rest at the origin.

- (a) Find the velocity of the weight after $\frac{\pi}{4}$ seconds. (3 marks)

$$a = 15 \sin(2t + \frac{\pi}{3})$$

$$v = -\frac{15}{2} \cos(2t + \frac{\pi}{3}) + c$$

$$0 = -\frac{15}{2} + c$$

$$c = \frac{15}{4}$$

Specific behaviours

- ✓ integrates sine term
- ✓ solves for constant
- ✓ determines velocity (no need for units)

- (b) Find its displacement from the origin after $\frac{\pi}{2}$ seconds. (3 marks)

Solution

Lower	0.0769161
Upper	0.1230839
\hat{p}	0.1
n	350

0.0769- 0.1231

Margin of error = 0.0231

Specific behaviours

- ✓ states z percentile
- ✓ determines confidence interval
- ✓ determines margin of error
- ✓ rounds to 4 dp

- c) Determine the probability that in a random sample of 350 vehicles, the proportion of polluting vehicles lies between 0.08 and 0.12. (2 marks)

Specific behaviours

- ✓ uses normal with correct parameters
- ✓ states probability (no need to give 4 dp)

- (b) Find its displacement from the origin after $\frac{\pi}{2}$ seconds. (3 marks)

- d) Below are three samples of vehicles, one of them is actually from another country. Which one is most likely not to be in Perth and give reasons. (Continue with a confidence of 85%) (3 marks)

Sample A 44 polluting vehicles out of a sample size of 340	Sample B 20 polluting vehicles out of a sample size of 280	Sample C 50 polluting vehicles out of a sample size of 420
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See next page

See next page

Specific behaviours

Note: 2 marks for answer only

- ✓ determines probability
- ✓ states upper limit
- ✓ states lower limit

Solution

normCDF(29, 41, 4, 35)
0.8663855975

- a) Determine the probability that a bar of chocolate produced, will be usually sent to a store for sale.
- A chocolate factory makes bars of chocolate such that the mass of these bars forms a normal distribution of mean mass of 35 grams and a standard deviation of 4 grams. Only those bars that have a mass within 1.5 standard deviations are usually sent to stores for sale.
- (3 marks)
- Question 13**

Specific behaviours

- ✓ determines displacement (no need for units) NOTE: follow through from (a) above
- ✓ solves for constant
- ✓ integrates cosine term

Solution

$\frac{1}{4} \sin\left(2\left(\frac{\pi}{3}\right)x + \frac{\pi}{3}\right) + \frac{15}{4} \sin\left(\frac{\pi}{3}x\right)$
12.38567675

- a) Calculate the sample proportion of vehicles that are illegally polluting.
- A government agency is investigating the number of motor vehicles in Perth that are producing an illegal amount of pollution. In a sample of 350 motor vehicles, 35 were found to produce an illegal amount of pollution.
- (1 mark)
- Question 18**

Specific behaviours

Solution

invNorm("C", 0.85, 1, 0)
-1.439531471

- b) Determine an 85% confidence interval of the true proportion of vehicles illegally polluting to four decimal places and state the margin of error.
- (4 marks)
- Question 19**

Specific behaviours

Solution

$35/350 = 0.10$

- a) Calculate the sample proportion of vehicles that are illegally polluting.
- (1 mark)
- Question 20**

- A government agency is investigating the number of motor vehicles in Perth that are producing an illegal amount of pollution. In a sample of 350 motor vehicles, 35 were found to produce an illegal amount of pollution.
- (10 marks)
- Methods Unit 3 & 4**
- CALCULATOR-ASSUMED**
22

In December each year, the person who's job is to check the mass of bars is on holiday and no one replaces their role. The bars of chocolate are packed in boxes of 50.

Let X = number of bars suitable for sale in a box of 50 in December.

- b) State the distribution for X giving the mean and standard deviation. (3 marks)

Solution
$X \sim \text{Bin}(50, 0.866)$
$\mu = 43.3$
$\sigma = 2.410$
Specific behaviours
<ul style="list-style-type: none"> ✓ states binomial ✓ states mean ✓ states standard deviation

- c) Determine the probability that in a given box of 50 that at least 40 are suitable for sale in December. (2 marks)

Solution
<p>The calculator screen shows the command <code>binomialCDF(40, 50, 50, 0.866)</code> and the result <code>0.9359394825</code>.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ uses correct parameters ✓ states probability

See next page

Edit Action Interactive
$\begin{cases} 21a=15 \\ 76a+b=65 \end{cases} _{a,b}$ <p>{$a=0.7142857143, b=10.71428571$}</p> $\begin{cases} 21a=-15 \\ 76a+b=65 \end{cases} _{a,b}$ <p>{$a=-0.7142857143, b=119.2857143$}</p>
<input type="button" value="Alg"/> <input type="button" value="Decimal"/> <input type="button" value="Cplx"/> <input type="button" value="Rad"/>
Specific behaviours
<ul style="list-style-type: none"> ✓ sets up one equation for a only ✓ sets up second equation with a and b ✓ solves for one pair of values ✓ solves for second pair of values <p>Max of 3 out of 4 marks if multiplying by a is not first</p>

See next page

<ul style="list-style-type: none"> ✓ solves for other variable ✓ equates to zero and solves for exact value of one variable (no need to simplify) ✓ differentiates ✓ obtains expression in terms of one variable only 	
Specific behaviours	
$\begin{aligned} S &= 2x + y = 2x + 36x \\ y &= \frac{36}{x} \\ x^2 &= 18 \\ x &= \sqrt{18} = 3\sqrt{2} \\ \frac{dy}{dx} &= 2 - 36x^{-2} = 0 \\ 2 &= 36x^{-2} \\ x^2 &= 18 \\ x &= \sqrt{18} = 3\sqrt{2} \\ y &= \frac{36}{x} \\ xy &= 36 \end{aligned}$	Solution

- (a) Determine two positive numbers, x and y , whose product is 36 and such that $2x + y$ is a minimum.

For the following questions, show **using calculus** how to determine an optimal solution.

(8 marks)

Question 14

<ul style="list-style-type: none"> ✓ reasonable random method of selecting boxes. ✓ does not accept every third box etc as not random 	
Specific behaviours	
Use a random number generator for selecting which order of boxes will be examined.	Solution

- d) It is decided to employ a part time worker in December to select boxes of 50 at random to check the number of bars that are suitable for sale. State a method for randomly selecting these boxes for inspection.

(1 mark)

	Solution

- (c) Determine the value(s) of a and b .
- The school has decided to scale the results down to mean of 65 and standard deviation of 15, using the transformation $Y = aX + b$ where a and b are constants and X is the scaled percentage scores.

<ul style="list-style-type: none"> ✓ squares standard deviation ✓ multiples by positive four only. 	
Specific behaviours	
$Var = 2^2 \times 21^2 = 1764$	Solution

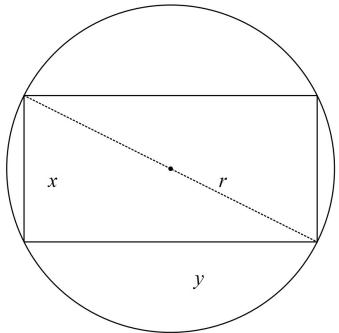
- (b) $Var(10 - 2X)$

<ul style="list-style-type: none"> ✓ states new expected value 	
Specific behaviours	
$76 - 6 = 70$	Solution

- (a) $E(X - 6)$

- A school has analysed the exam scores for all Year 12 Methods students and found that the mean was 76 with a standard deviation of 21. Let X be the exam scores out of a 100. Determine the following.

- (b) Find the area of the largest rectangle of lengths x & y that can be enclosed in a circle of radius r .
(4 marks)

**Solution**

$$\begin{aligned}x^2 + y^2 &= 4r^2 \\y &= \sqrt{4r^2 - x^2} \\A &= xy = x\sqrt{4r^2 - x^2}\end{aligned}$$

$$\frac{dA}{dx} = x \frac{1}{2}(4r^2 - x^2)^{-\frac{1}{2}}(-2x) + \sqrt{4r^2 - x^2} = 0$$

$$\frac{x^2}{\sqrt{4r^2 - x^2}} = \sqrt{4r^2 - x^2}$$

$$x^2 = 4r^2 - x^2$$

$$2x^2 = 4r^2$$

$$x = \sqrt{2}r$$

$$y = \sqrt{4r^2 - 2r^2} = \sqrt{2}r$$

$$A = xy = 2r^2$$

Specific behaviours

- ✓ obtains y in terms of x and r (or vice-versa)
- ✓ differentiates area using the product rule
- ✓ equates to zero and solves for x in terms of r
- ✓ determines other variable in terms of r and states area in terms of r .

See next page

- (d) Total distance travelled in the first 12 seconds.
(2 marks)

Solution
<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> Edit Action Interactive <div style="border: 1px solid #ccc; padding: 2px; display: flex; justify-content: space-around; gap: 10px;"> 0.5 $\frac{1}{2}$ \int_0^1 \int_0^x \int_x^1 \int_a^b \int_b^a \int_{-1}^1 \int_{-x}^x \int_{-b}^b \int_{-a}^a </div> </div> <div style="border: 1px solid #ccc; padding: 10px; font-family: monospace; background-color: #f0f0f0;"> $\int_0^{12} \left \left(\frac{t}{3} - 4 \right) \cdot \frac{1}{3} \right dt$ </div> <div style="text-align: right; margin-top: 10px;">8</div>
Specific behaviours
<ul style="list-style-type: none"> ✓ uses appropriate integral or number line diagram ✓ states distance of 8 metres (no need for units)

See next page

(3)

- A game consists of a wheel divided into sixteen equal sectors numbered 1 through 16. You may place a \$2 bet on any number. If your number comes up, you get \$20 back. That is, you win \$18 after getting your \$2 back. If you lose then you have lost \$2.
- (a) Let X be the amount of profit won on a bet by the player. Find the amount the player expects to profit on a bet, i.e., $E(X)$
- (b) Let X be the standard deviation of profit won on a bet by the player. Find the amount the player expects to lose on a bet, i.e., explain what this means.
- (c) Marks

Question 15

		See next page								
		Solution								
		(b) Determine the standard deviation of X , amount of profit won on a bet by the player (3 marks)								
		<ul style="list-style-type: none"> ✓ states mean ✓ shows calculation of mean ✓ explains that the player expects to lose on a play 								
		Solution								
<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>$p(x)$</th> <th>$\\$18$</th> <th>$-\\2</th> </tr> </thead> <tbody> <tr> <td>$\frac{1}{16}$</td> <td>$\frac{1}{16}$</td> <td>$\frac{1}{16}$</td> <td>$\frac{15}{16}$</td> </tr> </tbody> </table>		x	$p(x)$	$\$18$	$-\$2$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{15}{16}$	$18 \times \frac{1}{16} + -2 \times \frac{15}{16}$
x	$p(x)$	$\$18$	$-\$2$							
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{15}{16}$							
		Solution								
		-0.75								

(3)

- (a) Let X be the amount of profit won on a bet by the player. Find the amount the player expects to profit on a bet, i.e., explain what this means.
- (b) Let X be the standard deviation of profit won on a bet by the player. Find the amount the player expects to lose on a bet, i.e.,

marks)

(8 marks)

- (c) See next page

		See next page
		Solution
		<ul style="list-style-type: none"> ✓ examines first 12 secs only ✓ states initial speed of $4/3$ (no need for units) must be positive
		Initial speed of $4/3 \text{ m/s}$
<p>The graph shows a curve starting at $t=0$ with a vertical asymptote at $t=1$. The curve passes through points $(1, 0)$, $(2, 2)$, $(3, 4)$, $(4, 5)$, $(5, 6)$, $(6, 7)$, $(7, 8)$, $(8, 9)$, $(9, 10)$, $(10, 11)$, $(11, 12)$, and $(12, 13)$. A horizontal dashed line is drawn at $y=2$.</p>		$y = \frac{4}{3}t^2$
		Solution

- (c) Maximum speed of the particle and the time when this occurs. (2 marks)

		See next page
		Solution
		<ul style="list-style-type: none"> ✓ integrates velocity over sixth second or uses displacement for 5 & 6 sec ✓ determines displacement
		$\int_6^9 \left(\frac{1}{t} - 4 \right) \cdot \frac{1}{3} dt$
<p>The graph shows a curve starting at $t=6$ with a vertical asymptote at $t=7$. The curve passes through points $(6, 0)$, $(7, 2)$, $(8, 4)$, $(9, 5)$, and $(10, 6)$. A horizontal dashed line is drawn at $v=2$.</p>		$v = \frac{1}{3t} - \frac{4}{3}$
		Solution

Edit Action Interactive

0.5 1 $\frac{1}{2}$ $\int \frac{d}{dx}$ $\int dx$ Simp $\int dx$ ∇ ∇

$$(18+0.75)^2 \times \frac{1}{16} + (-2+0.75)^2 \times \frac{15}{16}$$

$$23.4375$$

$$\sqrt{23.4375}$$

$$4.841229183$$

Specific behaviours

- ✓ shows calculation
- ✓ determines variance (uses square root in calculation)
- ✓ determines standard deviation

- (c) How much should you win for the game to be fair? ($E(x)=0$) (2 marks)

Solution

Edit Action Interactive

0.5 1 $\int \frac{d}{dx}$ $\int dx$ Simp $\int dx$ ∇ ∇

$$\text{solve}\left(w \cdot \frac{1}{16} - 2 \cdot \frac{15}{16} = 0, w\right)$$

$$\{w=30\}$$

WIN \$30 i.e. profit of \$30 Or you get back \$32

Specific behaviours

- ✓ sets up equation for profit on win
- ✓ states winning profit or cash back

See next page

(8 marks)

Question 16The displacement, x , in centimetres of a particle from the point O in a straight line is given by

$$x(t) = \frac{1}{2} \left(\frac{t}{3} - 4 \right)^2 - 2 \quad \text{for } 0 \leq t \leq 12, \text{ where } t \text{ is measured in seconds.}$$

Calculate the:

- (a) Time(s) that the particle is at rest. (2 marks)

Solution

Edit Action Interactive

0.5 1 $\int \frac{d}{dx}$ $\int dx$ Simp $\int dx$ ∇ ∇

$$\text{solve}\left(\left(\frac{t}{3}-4\right) \cdot \frac{1}{3} = 0, t\right)$$

$$\{t=12\}$$

Specific behaviours

- ✓ differentiates
- ✓ solves for t (no need for units)

- (b) Displacement of the particle during the sixth second. (2 marks)

Solution

See next page