MATHEMATICS METHODS

MAWA Semester 1 (Unit 3) Examination 2018 Calculator-free

Marking Key

© MAWA, 2018

Licence Agreement

This examination is Copyright but may be freely used within the school that purchases this licence.

The items that are contained in this examination are to be used solely in the school for which they

- are purchased.

 They are not to be shared in any manner with a school which has not purchased their own licence.
- The items and the solutions/marking keys are to be kept confidentially and not copied or made available to anyone who is not a teacher at the school. Teachers may give feedback to students in the form of showing them how the work is marked but students are not to retain a copy of the paper or marking guide until the agreed release date stipulated in the purchasing agreement/licence.

The release date for this exam and marking scheme is

the end of week 8 of term 2, 2018

CALCULATOR-FREE SEMENATION SEMESTER 1 (UNIT 3) EXAMINATION

MATHEMATICS METHODS

75

@ MAWA 2018

MATHEMATICS METHODS

Section One: Calculator-free

(50 Marks)

Question 1 (a)	(2 marks)
----------------	-----------

2

C	
Solution	
$\frac{d}{dx}(x\cos x) = x(-\sin x) + \cos x$	
$=\cos x - x \sin x$	
Mathematical behaviours	Marks
applies product rule	1
differentiates COS X term	1

Question 1 (b) (2 marks)

Solution	
$\frac{d}{dx}(x^3 + 4\sin x)^5 = 5(x^3 + 4\sin x)^4 \cdot \frac{d}{dx}(x^3 + 4\sin x)$	
$=5(x^3 + 4\sin x)^4(3x^2 + 4\cos x)$	
Mathematical behaviours	Marks
applies the chain rule	1
differentiates sin x term	1

uestion 1 (c)				(3 marks)
		Solution		
$\frac{d}{dx} \left(\frac{e^{-2x}}{4x + 2} \right)$				
$f(x) = e^{-2x}$	$f'(x) = -2e^{-2x}$	g(x) = 4x + 2	g'(x) = 4	
$=\frac{(4x+2)\cdot(-2e^{-x})}{(4x+2)}$	$\frac{e^{-2x}}{2} - e^{-2x} \cdot 4$			
$=\frac{-2(4x+2)(e^{-2x}+2)}{(4x+2)}$	$(e^{x}) - 4e^{-2x}$			
	Mathemati	cal behaviours		Marks
applies chain	rule to obtain $f'(x)$			1
 applies quotie 				1
 correct answer 				1

Question 8 (b) (2 marks)

11

Solution	
8	
$\int f'(x) dx = f(8) - f(6) = 0 - 3 = -3$	
J	
Mathematical behaviours	Marks
applies the Fundamental Theorem	1
evaluates result	1

Question 8 (c) (i) (2 marks)

* * * * * * * * * * * * * * * * * * * *	• •
Solution	
Area $\Delta = 8$	
3 3	
$\therefore \int_{2} f(x) \ dx = -4 \Rightarrow \int_{0} f(x) \ dx = 0$	
∴ one value of m is $m = 3$.	
0	
Also, $\int_0^f f(x) dx = 0$ for any function	
hence, $m = 0$ is another solution.	
From the symmetry of the graph, $m = 6,9,12$	
Hence $m = 0, 3, 6, 9, 12$.	
Mathematical behaviours	Marks
• states $m = 0$ or $m = 3$	1
states all correct values for <i>m</i>	1

Question 8 (c) (ii) (2 marks)

Solution	
$\int_{0}^{4} g(x) dx = \int_{0}^{4} f(x) + 2 dx$ $= \int_{0}^{4} f(x) dx + \int_{0}^{4} dx$	
= (-4) + 2(4-0)	
= 4	
Mathematical behaviours	Marks
• uses linearity to split $g(x)$	1
evaluates sum of integrals	1

OΤ

Question 2 (4 marks)

3

τ	determines correct result
τ	subs in limits of integration correctly
τ	anti-differentiates integral correctly
τ	states a correct expression using integrals to determine the area
Marks	Mathematical behaviours
	Solution Solution $\frac{\pi}{\varepsilon} = A$ $\frac{\pi}{\varepsilon} \left[x \cos x + \cos x \right] = \frac{1}{\varepsilon} \left[x \cos x + \cos x \right]$ $\frac{\pi}{\varepsilon} \left[x \cos x + \cos x \right] = \frac{1}{\varepsilon} \left[x \cos x + \cos x \right]$ $\frac{\pi}{\varepsilon} = \frac{\pi}{\varepsilon}$
	anitulo2

8102 AWAM ©

• states f(x)

(3 marks)

1

1

CALCULATOR-FREE

SEMESTER 1 (UNIT 3) EXAMINATION

Question 3	(3 marks)
Solution	
$f'(x) = x + \sqrt{3 + 6x}$	
$\therefore f(x) = \frac{x^2}{2} + \frac{(3+6x)^{\frac{3}{2}}}{6} \cdot \frac{2}{3} + c$	
$f(1) = 10 \Rightarrow 10 = \frac{1}{2} + \frac{(3+6(1))^{\frac{3}{2}}}{6} \cdot \frac{2}{3} + c$	
ie $10 = \frac{1}{2} + \frac{9^{\frac{3}{2}}}{9} + c$	
ie $c = 6\frac{1}{2}$	
$\therefore f(x) = \frac{x^2}{2} + \frac{(3+6x)^{\frac{3}{2}}}{9} + 6.5$	
Mathematical behaviours	Marks
anti-differentiates square root term	1

4

Question 4 (a) (1 mark)

• uses anti-derivative and f(1) = 10 to determine C

Solution	
X has a discrete uniform distribution	
Mathematical behaviours	Marks
states that the distribution is uniform	1

Question 4 (b) (1 mark)

Solution	
There are $550-250+1=301$ whole numbers in the interval $250 \le X \le 550$.	
So $P(250 \le X \le 550) = 0.301$	
Mathematical behaviours	Marks
correct answer	1

Question 4 (c) (2 marks)

Solution	
There are $\frac{1000}{7}$ = 142 $\frac{6}{7}$, and so there are 142 whole numbers in the interval 1	$\leq X \leq 1000$
that are divisible by 7.	
So P i.	
Mathematical behaviours	Marks
obtains 142 whole numbers divisible by 7	1
divides by 1000	1

Question 7 (b) (4 marks)

9

• • • • • • • • • • • • • • • • • • • •	,
Solution	
$\frac{dy}{dx} = -\frac{\sin\left(\frac{\pi}{3} - x\right)}{\cos^2\left(\frac{\pi}{3} - x\right)}, \text{ when } x = \frac{2\pi}{3}$	
$=-\frac{\sin\left(\frac{\pi}{3}-\frac{2\pi}{3}\right)}{\cos^2\left(\frac{\pi}{3}-\frac{2\pi}{3}\right)}=-\frac{\sin\left(-\frac{\pi}{3}\right)}{\cos^2\left(-\frac{\pi}{3}\right)}$	
$= -\frac{-\sqrt{3}}{2} \left(-\frac{1}{2}\right)^2 = \frac{\sqrt{3}}{2} \div \frac{1}{4} = 2\sqrt{3}$	
Mathematical behaviours	Marks
correct substitution and subtraction of fractions	1+1
both exact values correct	1
correct simplified answer	1

Question 8 (a) (i) (1 mark)

Solution	
4	
$\int_{0}^{A} f(x) dx = A - B$	
Mathematical behaviours	Marks
determines expression	1

(3 marks) Question 8 (a) (ii)

Solution	
$\int_{0}^{4} 2f(x) dx + \int_{8}^{4} f(x) dx$ $= 2 \int_{0}^{4} f(x) dx - \int_{4}^{8} f(x) dx$ $= 2(A - B) - 2A = -2B$	
Mathematical behaviours	Marks
$\int_{0}^{4} 2f(x) dx = 2(A-B)$ • uses linearity to deduce	1
$\int_{0}^{4} f(x) dx = -\int_{0}^{8} f(x) dx$	1
uses relationship ⁸ sums expressions and simplifies	1

g

Question 6 (c) (2 marks)

8

τ	anti-differentiates each part correctly	
τ	 exbands brackets correctly 	
Marks	Mathematical behaviours	
	$\partial + \frac{\zeta}{xz} - x\zeta - \frac{\zeta}{xz} = xp x\zeta - \zeta - \zeta = xp x\zeta - \zeta = xp \zeta = x$	
noitulo2		

Question 7 (a) (3 marks)

τ	correct answer
τ	applies chain rule
τ	 correctly differentiates sec x
Marks	Mathematical behaviours
	$\frac{\left(x - \frac{\varepsilon}{x}\right)_{\varepsilon} soo}{\left(x - \frac{\varepsilon}{x}\right) uis} = (t -) \cdot \frac{\left(x - \frac{\varepsilon}{x}\right)_{\varepsilon} soo}{\left(x - \frac{\varepsilon}{x}\right) uis} = \frac{xp}{\sqrt{p}}$
	$(x)_{n} \cdot \frac{((x)n)_{z} \cos}{((x)n) \operatorname{uis}} = \frac{xp}{\lambda p}$
	$\frac{x \operatorname{uis}}{x \operatorname{uis}} = x \operatorname{obs} \frac{xp}{p} \qquad 1 = (x), n \qquad x - \frac{x}{\xi} = (x)n$
	$(x - \frac{\varepsilon}{\iota}) \text{ obs} = \delta$
	noitulo2

8102 AWAM © 8105 TAMAM TO STORE THE STORE THE

CALCULATOR-FREE

SEMESTER 1 (UNIT 3) EXAMINATION

Question 4 (d) (4 marks)

6

Solution

In the interval $1 \le X \le 1000$ there are:

100 whole numbers that are divisible by 10, 40 whole numbers that are divisible by 25, and 20 whole numbers that are divisible by both 10 and 25, (i.e. divisible by $50\,\mathrm{\&}$ So there are 100+40-20=120 whole numbers that are divisible by 10 or 25. and so $P \stackrel{\iota}{\iota}$.

Mathematical behaviours	Marks
correct numbers for divisibility by 10 and by 25	1+1
• uses $\dot{\iota}(A \cup B) = \dot{\iota}(A) + \dot{\iota}(B) - \dot{\iota}(A \cap B)$	1
divides by 1000	1

(2 marks) Question 4 (e)

Solution	
The following numbers have exactly two 3's in their decimal expansion:	
33,133,233,433,,933, 303,313,323,343,,393, and 330,331,332,334,339	
So P i.	
Mathematical behaviours	Marks
 obtains 27 whole numbers with the desired property 	1
divides by 1000	1

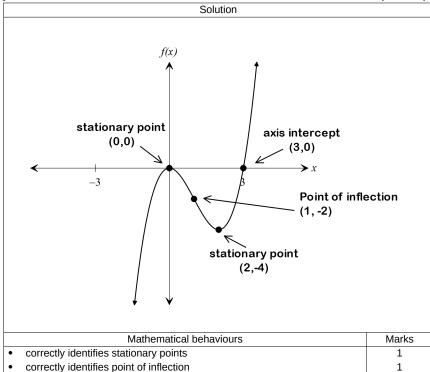
MATHEMATICS METHODS

CALCULATOR-FREE SEMESTER 1 (UNIT 3) EXAMINATION

1

Question 5 (3 marks)

7



Question 6 (a) (2 marks)

accurate sketch of the curve including *x* axis intercepts

£	(=,
Solution	
$\int \frac{1-2x}{x^3} dx = \int x^{-3} - 2x^{-2} dx = \frac{2}{x} - \frac{1}{2x^2} + c$	
Mathematical behaviours	Marks
 splits the fraction into two parts and anti-differentiates x⁻³ states anti-derivative including +c 	1 1

Question 6 (b) (2 marks)

Solution		
$\int \sin\left(x - \frac{\pi}{4}\right) - \cos\pi x dx$	$= -\cos\left(x - \frac{\pi}{4}\right) - \frac{\sin\pi x}{\pi} + c$	
Mathe	ematical behaviours	Marks
anti-differentiates sin or co	os part of expression correctly	1
 states correct solution 		1

© MAWA 2018 © MAWA 2018