

Papers written by
Australian Maths
Software

SEMESTER TWO

YEAR 12

MATHEMATICS METHODS

REVISION 3

Units 3-4

2016

**Section One
(Calculator-free)**

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work:

5 minutes

Working time for section:

50 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section One.

A formula sheet which may also be used for Section Two.

Structure of this examination

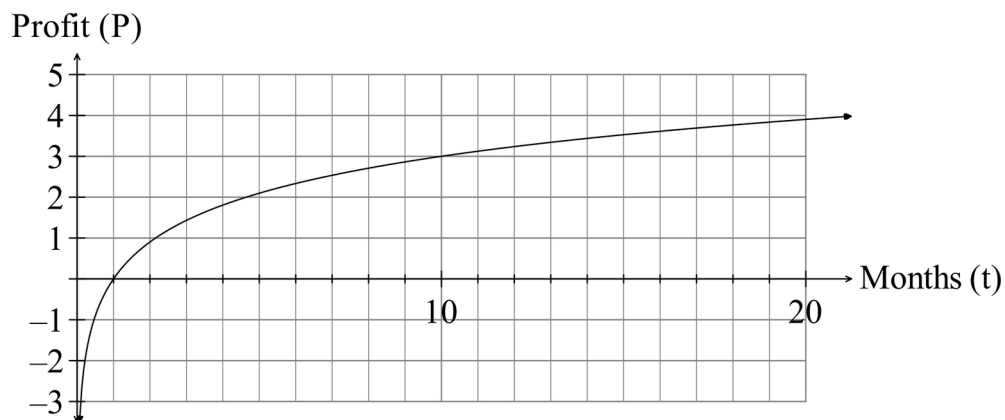
| | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|--|-------------------------------|------------------------------------|------------------------|-----------------|--------------------|
| Section One Calculator—free | 5 | 5 | 50 | 50 | 35 |
| Section Two Calculator— assumed | 13 | 13 | 100 | 100 | 65 |
| Total marks | | | | 150 | 100 |

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

1. (9 marks)

- (a) The profit of a small company is graphed below. The profit is measure in thousands of dollars and the time in months



- (i) When did the company “break even” i.e. start to make a profit? (1)
- (ii) Determine the profit equation. (2)
- (b) Determine x given $\log_x 9 = -2$ for $x \geq 0$. (3)
- (c) Simplify
- $$\frac{(\log_a 2 + \log_a 4) \times (\log_a 3^2)}{2 \log_a 9 \times (\log_a 2 - \log_a 1)}$$
- (3)

2. (14 marks)

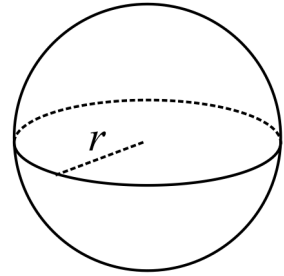
(a) Find the derivative of

(i) $f(x) = \ln(\sqrt{e^{-2x}})$ (2)

(ii) $g(x) = \frac{\ln(x)}{x^2}$ (3)

(iii) $h(x) = (e^x)\cos(2x)$ (2)

- (b) The volume of a sphere is increasing at a rate of $3 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of increase of the radius when $r = 2 \text{ cm}$. (3)



- (c) Consider the functions

$$h(x) = \pi + x^2$$

$$i(x) = \frac{x^3 + x}{x}$$

$$j(x) = \sqrt{x^4} - 1$$

$$k(x) = \sqrt{1 + x^4}$$

$$l(x) = \frac{\pi}{2} - (-x^2)$$

$$m(x) = \left(\frac{x}{2}\right)^2 - 4$$

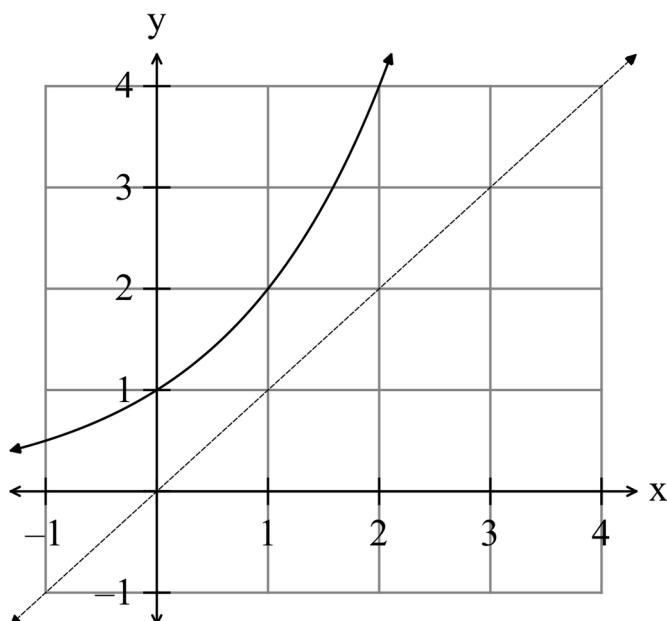
Many of the functions have the same derivative.

- (i) Which of the functions have a different derivative to most of the others? (2)

- (ii) Find the derivative of the functions where the derivative is not equal to $2x$. (2)

3. (7 marks)

(a) The graph of $f(x) = 2^x$ is shown on the set of axes below,



Sketch on the same set of axes

(i) $y = f^{-1}(x)$, the inverse of the function $f(x) = 2^x$. (2)

(ii) $y = f^{-1}(x - 2)$ (3)

(iii) $y = 1 - f^{-1}(x)$. (2)

4. (9 marks)

Evaluate the following

(a) (i) $\int (3y - 5)^{-2} dy$ (2)

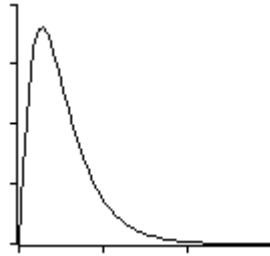
(ii) $\int_{\pi/4}^{\pi/6} \cos^{-2}(x) dx$ (2)

(iii) $\int_2^3 \left(x^2 + x + 1 + \frac{1}{x} \right) dx$ (3)

- (b) If $g'(x) = e^{-x}$ find the expression for $y = g(x)$ given $g(0) = -1$. (2)

5. (11 marks)

- (a) 50 samples of size 20 of a highly skewed distribution population, as shown in the diagram, were taken and the mean of each calculated,



Sketch the shape of the distribution of the set of means of the samples. (2)

- (b) Determine whether or not the function $f(x) = \begin{cases} x^2 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ represents a continuous probability density function. Give your reasons. (3)

- (c) (i) Explain why the mean of the means of many samples of the same size is a good approximation to the population mean. (2)

- (ii) Explain why the sampling distribution (of the sample means) has a very small standard deviation. (2)

- (iii) Give three different types of bias that can be encountered when conducting a survey.
(2)

END OF SECTION ONE