Rossmoyne Senior High School

Semester One Examination, 2015

Question/Answer Booklet

MATHEMATICS 3C
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33⅓
Section Two: Calculator-assumed	12	12	100	100	66⅔
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

Determine the area of the region enclosed between the line $y = 6x + 9$ and the curve $y = 3x^2$.

$$3x^2 = 6x + 9$$
$$3x^2 - 6x - 9 = 0$$
$$3(x^2 - 2x - 3) = 0$$
$$3(x - 3)(x + 1) = 0 \Rightarrow x = -1,\ x = 3$$

$$A = \left| \int_{-1}^{3} 3x^2 - 6x - 9 dx \right|$$
$$= \left| \left[x^3 - 3x^2 - 9x \right]_{-1}^{3} \right|$$
$$= \left| (27 - 27 - 27) - (-1 - 3 + 9) \right|$$
$$= \left| -27 - 5 \right|$$
$$= 32 \text{ sq units}$$

Question 2 **(8 marks)**

Determine the following, simplifying where possible.

(a) $\frac{d}{dx}\left(\frac{2x^2 - 1}{1 - 3x}\right)$. (2 marks)

$$\frac{4x(1 - 3x) - (2x^2 - 1)(-3)}{(1 - 3x)^2} = \frac{4x - 6x^2 - 3}{(1 - 3x)^2}$$

(b) $\int \frac{1}{2\sqrt{x}} - \frac{x^3}{5}\,dx$. (2 marks)

$$\sqrt{x} - \frac{x^4}{20} + c$$

(c) $\frac{d}{dx}\left(x^2\sqrt{x+1}\right)$. (2 marks)

$$2x\sqrt{x+1} + x^2 \times \tfrac{1}{2}(x+1)^{-\frac{1}{2}} = 2x\sqrt{x+1} + \frac{x^2}{2\sqrt{x+1}}$$

(d) $\int 3xe^{x^2+1}\,dx$. (2 marks)

$$\tfrac{3}{2}e^{x^2+1} + c$$

See next page

Question 3 **(8 marks)**

Two functions are defined as $f(x) = 2x + 5$ and $g(x) = 2 - \sqrt{x}$.

(a) State $g \circ f(x)$ with its domain and range. (3 marks)

$g \circ f(x) = 2 - \sqrt{2x + 5}$

$D: \ 2x + 5 \geq 0 \ \Rightarrow \ x \geq -2.5$

$R: \ \sqrt{2x + 5} \geq 0 \ \Rightarrow \ y \leq 2$

(b) Determine the domain and range of $f\left(\frac{2}{x}\right)$. (2 marks)

$f\left(\frac{2}{x}\right) = 2\left(\frac{2}{x}\right) + 5$

$D: \ x \neq 0$

$R: \ y \neq 5$

(c) Determine $h(x)$ if $h \circ f(x) = 6x + 5$. (3 marks)

Let $y = f(x) = 2x + 5 \ \Rightarrow \ x = \frac{y - 5}{2}$

$$\begin{aligned} h(y) &= h \circ f(x) \\ &= 6x + 5 \\ &= 6\left(\frac{y - 5}{2}\right) + 5 \\ &= 3y - 10 \end{aligned}$$

$h(x) = 3x - 10$

See next page

Question 4 **(10 marks)**

A function is given by $f(x) = (7 - x)(x - 1)^2$.

(a) Determine the coordinates of the axes intercepts of the graph of $y = f(x)$. (2 marks)

$(0, 7)$, $(1, 0)$ and $(7, 0)$

(b) Determine the coordinates of the stationary points of the graph of $y = f(x)$. (4 marks)

$f'(x) = 0$ when

$(-1)(x - 1)^2 + (7 - x)(2)(x - 1) = 0$

$-(x^2 - 2x + 1) + 2(8x - 7 - x^2) = 0$

$-3x^2 + 18x - 15 = 0$

$-3(x^2 - 6x + 5) = 0$

$-3(x - 1)(x - 5) = 0 \Rightarrow x$

$(1, 0)$ ar

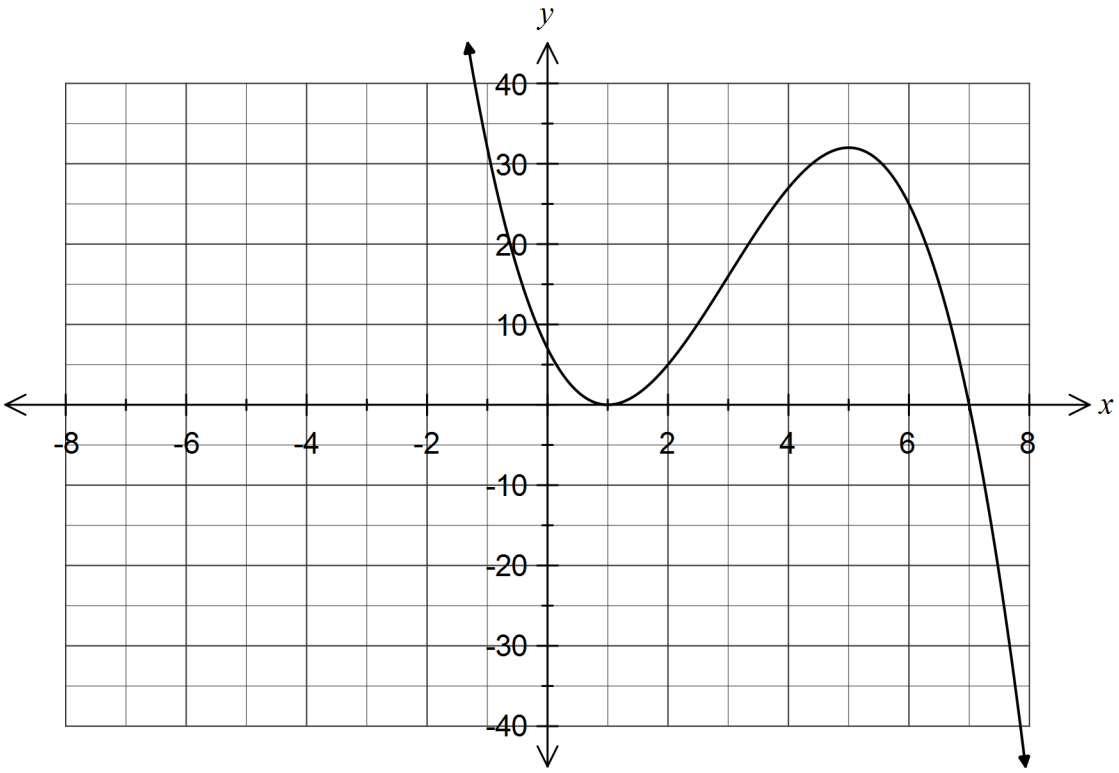
(c) Determine the location of the point of inflection of the graph of $y = f(x)$. (2 marks)

$f''(x) = -6x + 18$

$-6x + 18 = 0 \Rightarrow x = 3$

$(3,\ 16)$

(d) Sketch the graph of $y = f(x)$ on the axes below. (2 marks)



Question 5 **(8 marks)**

(a) Show that $1 - \frac{1}{x-1} - \frac{3}{x+3} = \frac{(x+1)(x-3)}{(x-1)(x+3)}$. (3 marks)

$$1 - \frac{1}{x-1} - \frac{3}{x+3} = \frac{(x-1)(x+3) - (x+3) - 3(x-1)}{(x-1)(x+3)}$$
$$= \frac{x^2 + 2x - 3 - x - 3 - 3x + 3}{(x-1)(x+3)}$$
$$= \frac{x^2 - 2x - 3}{(x-1)(x+3)}$$
$$= \frac{(x+1)(x-3)}{(x-1)(x+3)}$$

(b) Show that the curve $y = 1 - \frac{1}{x-1} - \frac{3}{x+3}$ has a root at (3, 0). (1 mark)

$$y = 1 - \frac{1}{2} - \frac{3}{6} = 0$$

Or

$$x = 3 \Rightarrow \frac{(x+1)(x-3)}{(x-1)(x+3)} = 0 \Rightarrow y = 0$$

See next page

(c) The equation of the tangent to the curve $y = \dfrac{(x+1)(x-3)}{(x-1)(x+3)}$ at the point (3, 0) is $y = ax + b$.
Determine the values of a and b. (4 marks)

$$y = 1 - (x-1)^{-1} - 3(x+3)^{-1}$$

$$\frac{dy}{dx} = 0 + (x-1)^{-2} + 3(x+3)^{-2}$$

$$= \frac{1}{(x-1)^2} + \frac{3}{(x+3)^2}$$

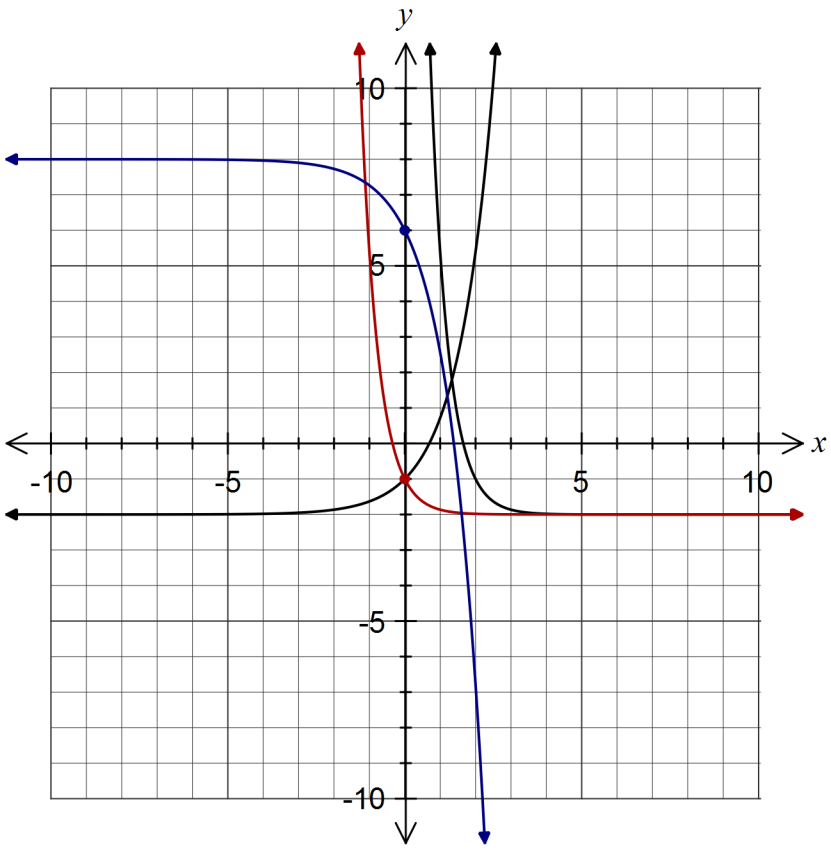
$$x = 3, \quad \frac{dy}{dx} = \frac{1}{4} + \frac{3}{36} = \frac{1}{3}$$

$$y - 0 = \frac{1}{3}(x-3)$$

$$y = \tfrac{1}{3}x - 1 \Rightarrow a = \tfrac{1}{3}, \; b = -1$$

Question 6 **(8 marks)**

The graph of the functions $f(x) = e^x - 2$ and $g(x) = f(a - 2x)$ are shown below.



(a) Determine the value of the constant a. (3 marks)

$(0, -1) \rightarrow (2, -1)$

$g(x) = e^{a-2x} - 2$

$-1 = e^{a-2(2)} - 2$

$e^{a-4} = 1$

$a = 4$

(b) On the same axes, sketch the graphs of

(i) $y = g(x + 2)$. (2 marks)

(ii) $y = 4 - 2f(x)$. (3 marks)

See next page

Question 7 **(3 marks)**

The derivatives of the sequence $1,\ \binom{n}{1}(-2x)^1,\ \binom{n}{2}(-2x)^2,\ \binom{n}{3}(-2x)^3,\ \ldots,\ \binom{n}{n}(-2x)^n$ are $0,\ \binom{n}{1}(-2),\ \binom{n}{2}(-4)(-2x)^1,\ \binom{n}{3}(-6)(-2x)^2,\ \ldots,\ \binom{n}{n}(-2n)(-2x)^{n-1}$.

When n is a positive even integer, the sum of the series

$$1+\binom{n}{1}(-2x)^1+\binom{n}{2}(-2x)^2+\binom{n}{3}(-2x)^3+\ldots+\binom{n}{n}(-2x)^n=(2x-1)^n$$

.

Show that when n is a positive even integer, the sum of the series of derivatives

$$0+\binom{n}{1}(-2)+\binom{n}{2}(-4)(-2x)^1+\binom{n}{3}(-6)(-2x)^2+\ldots+\binom{n}{n}(-2n)(-2x)^{n-1}=\frac{2n}{2x-1}(2x-1)^n$$

.

If sum of series is $(2x-1)^n$, then sum of series of derivatives will be $\frac{d}{dx}(2x-1)^n$.

$$\begin{aligned}\frac{d}{dx}(2x-1)^n&=2n(2x-1)^{n-1}\\&=2n(2x-1)^n(2x-1)^{-1}\\&=\frac{2n}{2x-1}(2x-1)^n\end{aligned}$$

End of questions

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