6 Popular sports clothing company Gallopinter™ manufacture the red-and-white striped jumpers sold to fans of the local Floreat Fuchsiaphobes rugby team. They need to pay particular attention to the quality of the red dye used, so the dye is mixed using an automated mixing machine. The mass of powder added to 1L of solvent is normally distributed with a mean of 0.4g and standard deviation of 0.02g

If the amount of red powder is more than 1.2 standard deviations below the mean, the colour looks pink, which considered abhorrent by the extremely prejudiced Fuchsiaphobe fans. On the other hand, if the concentration of the red dye is more than 0.43g/L, the red dye leeches out in the wash, causing the white stripes to be stained pink, which also upsets the fans.
What is the probability that…

a) A randomly selected jumper will look pink before it is washed. (1)

0.1151

b) A randomly selected jumper that has been washed does not look pink. (2)

✓ both tails
✓ complement

$P(\text{not pink}) = 0.8181$

c) A randomly selected jumper will look pink after it has been washed, given that the dye concentration was at least 0.37g/L. (3)

$P(\text{conc} > 0.37) = 0.9332$ ✓ condition

$0.9332 - 0.8181 = 0.1151$ ✓ intersection

$P = \frac{0.1151}{0.9332}$

$= 0.1233$ ✓ prob.

SHENTON COLLEGE

ATMAM Mathematics Methods

Test 3 Calculator Free

Name: Solutions

Teacher: Friday Smith

Time Allowed : 25 minutes

Marks /33

Materials allowed: Formula Sheet.

All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

1 Evaluate the following logarithms

a) $\log 1000$

3

b) $\log_7 \frac{1}{49}$ (1, 1)

-2

c) $\log_{27} 3$

$\frac{1}{3}$

d) $3^{\left(\frac{\ln 4}{\ln 3}\right)}$ (1, 2)

$= 3^{\log_3 4}$ ✓ change of base

$= 4$ ✓

or

$3^{\frac{\ln 4}{\ln 3}} = x$

$\ln 3^{\frac{\ln 4}{\ln 3}} = \ln x$ ✓ log and log laws

$\frac{\ln 4}{\ln 3} \ln 3 = \ln x$

$\ln 4 = \ln x$

$4 = x$ ✓

2 Express the following as single logarithms.

a) $4\log_5 x - 3\log_5 \frac{1}{y}$

$= \log_5 x^4 + \log_5 y^3$ ✓ power law

$= \log_5 (x^4 y^3)$ ✓

b) $\log_3 y^2 - \log_3 x + 4$ (2, 2)

$= \log_3 y^2 - \log_3 x + \log_3 81$ ✓ 4 as $\log_3$

$= \log_3 \left(\frac{81y^2}{x}\right)$ ✓

3 If $p = \log_2 5$ and $q = \log_2 3$, express the following in terms of p and q.

a) $\log_2 1.8$

b) $\log_2 60$

(2, 2)

$= \log_2 \frac{3^2}{5}$

$= 2q - p$

$= \log_2 (2^2 \times 5 \times 3)$ ✓ rewrite

$= 2 + p + q$ ✓ $p \& q$

4 Use natural logarithms to solve the following equations. Express your answers using the fewest logs possible.

a) $3^{2x} = 5^{x+1}$

(4)

$2x \ln 3 = (x+1) \ln 5$ ✓ logs

$2x \ln 3 - x \ln 5 = \ln 5$

$x(2\ln 3 - \ln 5) = \ln 5$ ✓ factorise

$x = \frac{\ln 5}{2\ln 3 - \ln 5}$ ✓ $x =$

$x = \frac{\ln 5}{\ln 1.8}$ ✓ fewest logs

b) $2^{x+3} - 21 = 2^x$

(4)

$2^3 2^x - 2^x = 21$ ✓ index law

$2^x(2^3 - 1) = 21$ ✓ factorise

$2^x = 3$

$x \ln 2 = \ln 3$ ✓ logs

$x = \frac{\ln 3}{\ln 2}$ ✓ $x =$

4 A normally distributed random variable has 15% of its values above 70 and 40% of its values below 50. (5)

By calculating the standard z scores associated with the tail probabilities above, then writing equations for their equivalent z scores using the given boundaries, determine the values of the mean and standard deviation to one decimal place.

15% right tail $z \approx 1.0364$ ✓

40% left tail $z \approx -0.2533$ ✓

$\frac{70 - \bar{x}}{\sigma} = 1.0364$

$\frac{50 - \bar{x}}{\sigma} = -0.2533$ ✓ equations

$\bar{x} \approx 53.9$ ✓

$\sigma \approx 15.5$ ✓

5 Fabrics treated with water proofing spray are able to repel a proportion of incidental water such as rain. The proportion of water absorbed by a fabric which has been protected by x coats of water proofing spray can be modelled by the equation $W = W_0 e^{kx}$, where W_0 is the water absorbed by an unprotected sample of the fabric.

a) If each coat of a particular brand of spray reduces the water absorption to 60% of its previous value, find

(i) The value of k for that brand, correct to four decimal places. (1)

$0.6 = e^k$

$\ln 0.6 = k$

$k \approx -0.5108$

(ii) Determine the minimum number of coats to reduce the absorption to less than $0.01W_0$. Show use of natural logarithms in your working. (3)

$0.01 = e^{-0.5108x}$ ✓

$\ln 0.01 = -0.5108x$

$x = 9.0152$ ✓

=> Minimum number of coats is 10 ✓

d) Determine $P(x < 4 \mid x > 1.8)$ (2)

$= \frac{2.2 \times 0.2}{3.2 \times 0.2}$ $\quad 0.44$

$= \frac{11}{16}$ $\quad 0.64$

0.6875

e) Determine the value t such that $P(x < t \mid 3 < x) = \frac{3}{8}$ (3)

$\frac{0.2(t-3)}{0.2(5-3)} = \frac{3}{8}$

$\frac{t-3}{2} = \frac{3}{8}$

$t - 3 = \frac{3}{4}$

$t = \frac{15}{4}$ or 3.75

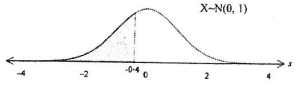
✓ conditional statement

✓ $= \frac{3}{8}$

✓ t.

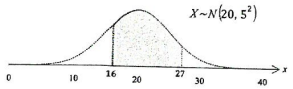
working

2 Determine the probability indicated by the shaded region on the following normal probability distributions.

a) 

(1)

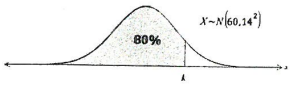
0.3446

b) 

(1)

0.7074

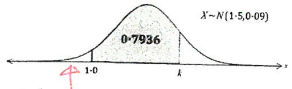
3 Determine the value of k in for the normal distributions shown below.

a) 

(1)

$k = 71.78$

(71.7827)

b) 

(2)

Left tail = 0.0477904 ✓

total P = 0.8413904

$k = 1.80$

✓

5 Find $\frac{dy}{dx}$ for each of the following functions.

a) $y = e^{\ln x^2}$ (2)

$y = x^2$ ✓ rewrite

$\frac{dy}{dx} = 2x$ ✓ differentiate

b) $y = \ln\left(\frac{x+1}{(x-3)^2}\right)$ (2)

$y = \ln(x+1) - 2\ln(x-3)$ ✓ rewrite

$\frac{dy}{dx} = \frac{1}{x+1} - \frac{2}{x-3}$ ✓ differentiate

Not simplified as before

c) $y = \sin x \ln x$ (2)

$\frac{dy}{dx} = \cos x \ln x + \frac{\sin x}{x}$

✓ product rule.

✓ diff. trig & ln correctly.

d) $y = \log_5(5x - 5)$ (2)

$5^y = 5x - 5$

$y \ln 5 = \ln(5x - 5)$ ✓ terms of ln

$y = \frac{1}{\ln 5} \ln(5x - 5)$

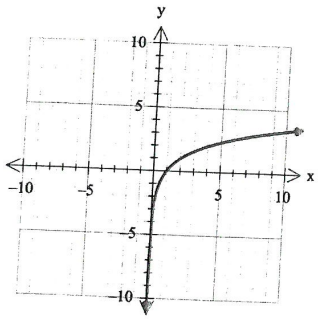
$\frac{dy}{dx} = \frac{1}{\ln 5} \frac{5}{5x - 5}$ ✓ differentiate

$= \frac{1}{(x-1)\ln 5}$

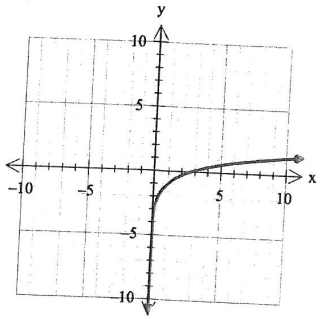
if given they need to

6 Match the graphs below with the appropriate logarithmic function. (4)

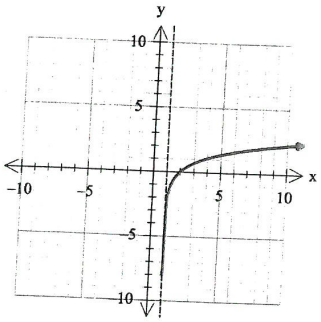
A



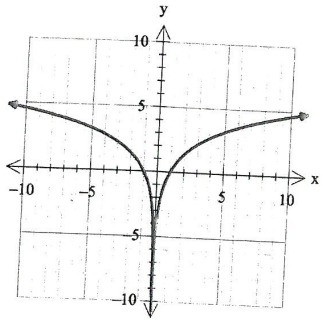
B



C



D



D ..... $y = \ln x^2$

C ..... $y = \ln(x - 1)$

B ..... $y = \ln x - 1$

A ..... $y = \log_2 x$

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ATMAM Mathematics Methods

Test 3 Calculator Assumed

Name: .....

Teacher: Friday Smith

Time Allowed : 25 minutes

Marks /31

Materials allowed: Classpad, Formula Sheet.

All necessary working and reasoning must be shown for full marks.
Where appropriate, values should be given to two decimal places, except for probabilities which should be given to four decimal places.
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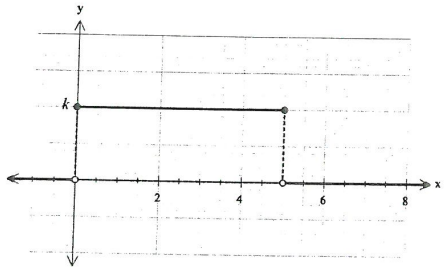
1 A uniform random variable can take any value between 0 and 5, with a PDF as shown in the graph below.



a) Determine the value of k. 0.2 (1)

b) Determine $P(3 < x \leq 4.5)$ 0.3 (1)

c) Determine $P(3 \leq x < 10)$ 0.4 (1)