

Question/**A**nswer **B**oeklet

Semester One Examination, 2021

MATHEMATICS

METHODS

ATAR Year 12

Section Two:

Calculator-assumed

Teacher: Miss Hosking

Miss Rowden

Please circle your teacher's name

Student Name: _____

Important note to candidates

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items: drawing instruments, templates, notes on two unruled sheets of A4 paper, and up to three calculators approved for use in this examination

<input type="checkbox"/>	Number of additional answer books used (if applicable):
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This Question/Answer Booklet
Formula Sheet (referred from Section One)

To be provided by the supervisor
MATERIALS required/recommended for this paper

Working time for paper: 100 minutes
Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Time allowed for this paper

Formula Sheet (referred from Section One)
This Question/Answer Booklet
Number of additional answer books used (if applicable):

To be provided by the candidate

Examination paper, and up to three calculators approved for use in this examination



No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Total	Percentage of examination	Marks available (minutes)	Suggested time working to be answered	Number of questions available	Section One:	Calculator free	Section Two:	Calculator assumed	Section Three:	Working time	Marks available	Number of questions available to be answered	Section
100	35	52	50	8	Calculator free	Section One:	Calculator assumed	Section Three:	Working time	Marks available	Number of questions available to be answered	Section	

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The VCE Mathematics Handbook 2021. Sitting this examination implies that you agree to abide by
Information Handbooks 2021. Sitting this examination implies that you agree to abide by

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Instructions to candidates

2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Supplementary pages for the use planning/containing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning, incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, if you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed

This section has thirteen (13) questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 9**(7 marks)**

A hot potato was removed from an oven and placed on a cooling rack. Its temperature T , in degrees Celsius, t minutes after being removed from the oven was modelled by

$$T = 16 + 188e^{kt}.$$

The temperature of the potato halved between $t=0$ and $t=6.8$.

- (a) Determine the value of the constant k . (3 marks)

- (b) The temperature of the potato eventually reached a stable temperature. Determine the time taken for its temperature to first fall to within 4°C of this stable temperature. (2 marks)

- (c) Determine the time at which the potato was cooling at a rate of 4°C per minute. (2 marks)

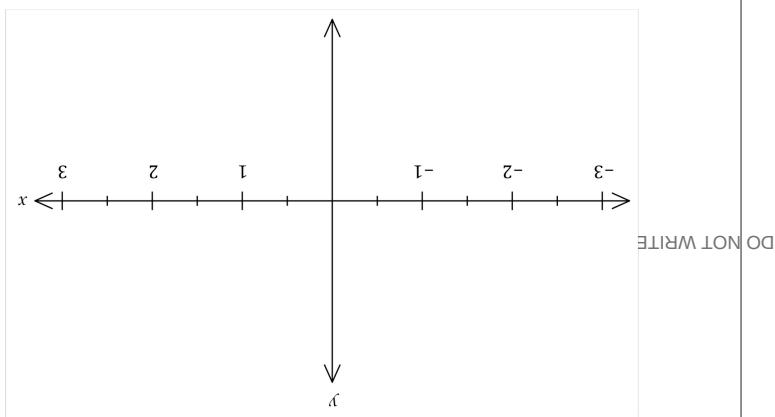
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See next page

Supplementary page

Question number: _____

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Question 10Let $f(x) = 3x^4 + ax^2 + 1$.(a) Sketch the graph of $y = f(x)$ when $a = -24$.
(4 marks)

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See next page

(4 marks)

(b) Show that the graph of $y = f(x)$ will always have a maximum turning point at $x = 0$
if $a < 0$.
(4 marks)

(8 marks)

Question 11

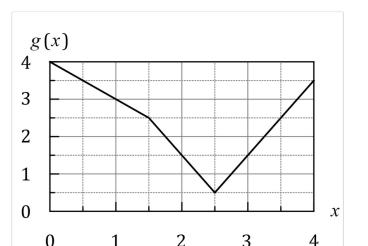
The graph of function g , and a table of values for function f and its derivatives are shown below.

x	1	2	3
$f(x)$	3	1	2
$f'(x)$	1	4	2
$f''(x)$	2	-1	-2

(a) Evaluate $h'(k)$ when

(i) $h(x)=f(g(x))$ and $k=1$.

(3 marks)



(ii) $h(x)=g(x)\div f(x)$ and $k=2$.

(3 marks)

(b) Evaluate $h''(3)$ when $h'(x)=f'(x)\times g'(x)$.

(2 marks)

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Supplementary page

End of questions

Question number: _____

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- (a) If $x = \log_6 4$ and $y = \log_6 9$ then, in terms of x and y , determine:

$$(i) \log_6 36$$

$$(ii) \log_6 \left(\frac{3}{2} \right)$$

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(2 marks)

(3 marks)

$$(iii) \log_6 144b^3$$

Question 21

- (a) Determine the value of the constant a and the value of the constant b that make each of the following statements true, given that $f(x)$ is a polynomial:

$$(i) \int_1^a f(x) dx + \int_2^3 f(x) dx = \int_q^{-3} f(x) dx.$$

$$(ii) \int_0^2 f(x) dx - \int_1^2 f(x) dx = \int_q^d f(x) dx + \int_0^1 f(x) dx.$$

(2 marks)

(2 marks)

See next page

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Question 12 continued

- (b) The loudness L , in decibels, of sound is given by the equation determine:

$$L = 10 \log_{10} \left(\frac{I}{I_0} \right)$$

Where I is the intensity of sound and I_0 is the intensity of the sound just audible to the human ear.

- (i) Find the loudness if the sound is 140 times as intense as I_0 . (2 marks)

- (ii) If the loudness was 28dB find in terms of I_0 intensity of sound. (2 marks)

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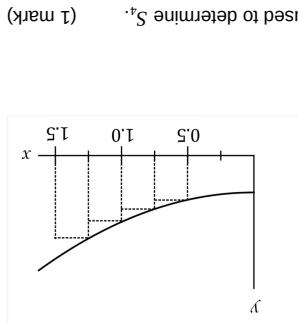
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(8 marks)

MATHEMATICS METHODS

9

CALCULATOR-ASSUMED



Question 13
(8 marks)

The graph of $y = f(x)$ is shown at right with 4 equal width inscribed rectangles. An estimate for the area under the curve between $x = 0.5$ and $x = 1.5$ is required.

The function f is defined as $f(x) = 2x^2 + 7$ and let S_n , the area estimate using n inscribed rectangles can be calculated using

$$S_n = \sum_{i=1}^{n-1} f(x_i) \Delta x$$

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(3 marks)

(b) Calculate the value of S_4 .

(2 marks)

(c) Explain, with reasons, how the value of Δx and the area estimate S_n will change as the number of inscribed rectangles increase.

(d) Determine the limiting value of S_n as $n \rightarrow \infty$.

(2 marks)

16

CALCULATOR-ASSUMED

(8 marks)

QUESTION 20

(a) Determine the value of the constant p and the value of the constant q . (6 marks)

Initially, A has a displacement of 4 cm relative to a fixed point O and is moving with a velocity of 9 cm/s. Two seconds later, A has a displacement of 8.8 cm and a velocity of -3.6 cm/s.

$$a = pr + q$$

Small body A moves in a straight line with acceleration a cm/s 2 at time t s given by

See next page

17

CALCULATOR-ASSUMED

(2 marks)

QUESTION 21

See next page

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(2 marks)

(b) Determine the minimum velocity of A.

See next page

(6 marks)

Question 14

The area A of a regular polygon with n sides of length x is given by

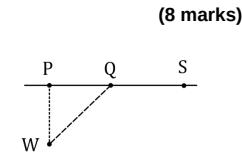
$$A = \frac{n x^2 \cos\left(\frac{\pi}{n}\right)}{4 \sin\left(\frac{\pi}{n}\right)}$$

- (a) Determine the exact area of a regular hexagon with side length 3 cm. (1 mark)
- (b) Simplify the above formula when $n=12$ to obtain a function for the area of a regular dodecagon. (2 marks)
- (c) Use the increments formula to estimate the change in area of a regular dodecagon when its side length increases from 10 cm to 10.3 cm. (3 marks)

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Question 19

An offshore wind turbine W lies 12 km away from the nearest point P on a straight coast. It must be connected to a power storage facility S that lies on the coast 24 km away from P .



Engineers will lay the cable in two straight sections, from W to Q , where Q is a point on the coast x km from P , and then from Q to S .

The cost of installing cable along the coastline is \$1000 per km and offshore is \$2600 per km.

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(a) Determine, to the nearest hundred dollars, the cost of installing the cable when Q lies midway from S to P . (2 marks)

(b) Show that C , the cost in hundreds of dollars, to run the cable from W to Q to S , is given by $C=26\sqrt{x^2+144}-10x+240$. (2 marks)

(c) Use calculus techniques to determine, with justification, the minimum cost of laying the cable from W to S . (4 marks)

(8 marks)

Question 16

The volume, V litres, of fuel in a tank is reduced between $t=0$ and $t=48$ minutes so that

$$\frac{dV}{dt} = -175\pi \sin\left(\frac{\pi t}{48}\right)$$

- (a) Determine, to the nearest litre, the amount of fuel emptied from the tank

- (i) in the first minute.

(3 marks)

- (ii) in the last 7 minutes.

(1 mark)

The tank initially held 18 600 litres of fuel.

- (b) Determine the volume of fuel in the tank 5 minutes after the volume in the tank reached 12 000 litres.

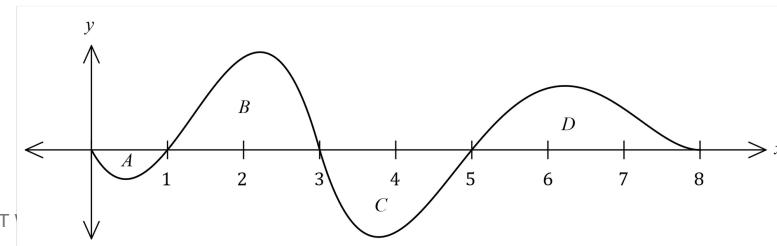
(4 marks)

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Question 17

(7 marks)

Regions A, B, C and D bounded by the curve $y=f(x)$ and the x -axis are shown on this graph:



The areas of A, B, C and D are 5, 31, 27 and 23 square units respectively.

- (a) Determine the value of

$$(i) \int_0^3 f(x) dx.$$

(1 mark)

$$(ii) \int_3^8 4f(x) dx.$$

(2 marks)

$$(iii) \int_1^8 (5-f(x)) dx.$$

(2 marks)

- (b) Explain why $\int_1^5 f'(x) dx = 0$.

(2 marks)