



Semester One Examination, 2022
Question/Answer booklet

MATHEMATICS METHODS
UNIT 3
Section One:
Calculator-free

Your Name: _____
Your Teacher's Name: _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

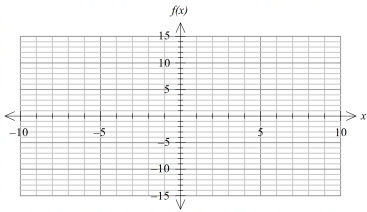
Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/rape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
1		8	5		6
2		10	6		13
3		8			
4		8			



Additional working space

Question number: _____

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Section One: Calculator-free (53 marks)

This section has six questions. Answer all questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 (8 marks)

The function f is defined for $x > 0$ by $f(x) = \frac{e^{4x-1}}{x}$, and $f'(x) = \frac{2(8x^2 - 4x + 1)}{x^2} e^{4x-1}$.

a) Show that $f''(x) = \frac{e^{4x-1}(4x-1)}{x^2}$ (2 marks)

b) Determine the coordinates and nature of all stationary points of $y = f(x)$. Justify your answer. (4 marks)

c) Show that $y = f(x)$ has no points of inflection. (2 marks)

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ii. $P(C \geq 190)$ (2 marks)

i. $P(A = 4)$ (1 mark)

c) Using your answer from part b), show how to calculate the following without the use of a calculator. Do not evaluate your answer.

b) The standard deviation when $p = 0.25$ is 6. Determine the value of n . (3 marks)

b) Hence, evaluate the following in exact form: $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} -\sin 4x \cos^3 4x \, dx$ (4 marks)

a) Determine $\frac{dy}{dx}(\cos^3 4x)$ (2 marks)

(6 marks)

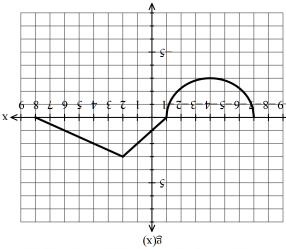
a) The student notes that one of the distributions produced a graph that is skewed to the right (tong tail to the right). Which of the values of p were used to produce this graph? (2 marks)

A student observes the graphs of three binomial distributions with the assigned random variables A , B and C for each graph. For all three distributions, the value of n is constant, but the values of p are 0.25, 0.5 and 0.9 respectively.

(8 marks)

Question 6 (13 marks)

The graph of $g(x)$ is given as below, which consists of a semi-circle for $-7 \leq x \leq -1$ and a triangle for $-1 \leq x \leq 8$.



Given that $f'(x) = \int_{-7}^x g(t) dt$, where $-7 \leq x \leq 8$.

a) Determine the intervals where $f'(x)$ is increasing and decreasing, respectively. (2 marks)

b) Determine the intervals where $f'(x)$ is concave up and concave down, respectively. (2 marks)

c) Determine the value(s) of x when $f'(x)$ reaches any stationary point(s) and the nature of the stationary point(s), if any. (2 marks)

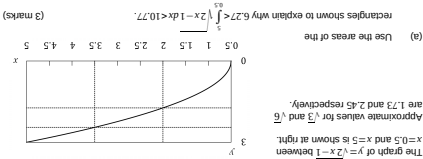
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Question 3

The graph of $y = \sqrt{2-x-1}$ between $x = 0.5$ and $x = 5$ is shown at right.

Approximate values for $\sqrt{3}$ and $\sqrt{6}$ are 1.73 and 2.45 respectively.



(a) Use the areas of the rectangles shown to explain why $6.27 < \int_{0.5}^5 \sqrt{2-x-1} dx < 10.77$. (3 marks)

(b) Evaluate $\int_{0.5}^5 \sqrt{2-x-1} dx$. (3 marks)

(c) Evaluate $\int_5^{6.5} \left(\sqrt{2x-1} - 3 \right) dx$. (2 marks)

f) Sketch the graph of $f(x)$ in the axes provided below, for $-7 \leq x \leq 8$. Label key features. (3 marks)

e) Determine the exact values $f'(8)$. (2 marks)

d) Determine the exact values of $f'(-1)$. (2 marks)

Question 2 (10 marks)

The discrete random variable X has a probability function with $\text{Var}(X) = \frac{14}{9}$

$$P(X = x) = \begin{cases} \frac{x}{k}, & x = 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine the value of k . (2 marks)

Determine:

(b) (i) $P(X < 4 \vee X > 1)$ (2 marks)

(ii) $E(X)$ (2 marks)

(c) A second discrete random variable Y is defined to be $Y = aX + b$. If $E(Y) = 2$ and the standard deviation of Y is $\sqrt{14}$, determine a and b . (4 marks)

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