Mercedes College

YEAR 12 MATHEMATICS METHODS Test 3 2016

Logarithms

NAME: SOLUTIONS **Date: Wednesday 29 June 2016**

TEACHER: _____

Calculator section:	**15 minutes (max)**	**11 marks**
Non-Calculator section:		**34 marks**
OVERALL:	**45 minutes**	**45 marks**

INSTRUCTIONS:
Show FULL working Answer all questions on this test paper
Questions or parts of questions worth more than two marks require working to be shown to receive full marks.
Allowed: Maths Methods WACE formula sheets, 3 calculators, 1 A4 page of notes

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Question 1 [3 + 2 = 5 marks]

a. Accurately plot the graph $y = \log(x - 3)$ on the axes below, clearly detailing the coordinates of any axis intercepts and the equations of any asymptotes.

b. The equation for the function shown below is $y = a \times \log(x + b)$
What are the values of a and b ?

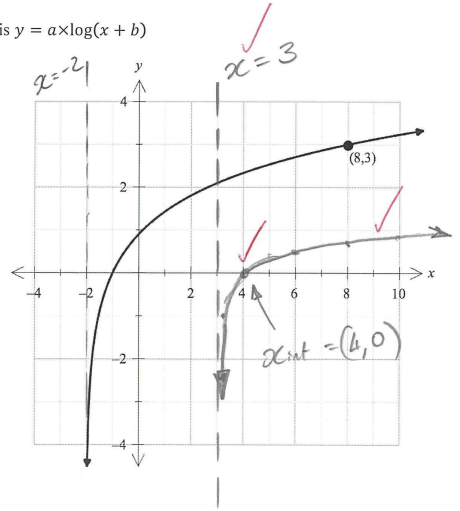
Moved 2 left
$\therefore b = 2$ ✓

$y = a \times \log(x+2)$

Sub $(8, 3)$

$3 = a \times \log(8+10)$

$\Rightarrow a = 3$ ✓



5

Question 2 **[2 + 2 + 2 = 6 marks]**

The intensity of sound is measured in decibels. As a consequence of the sensitivity of the human ear, this scale is logarithmic, which allows sound intensities across a wide spectrum (from almost inaudible to ear-splittingly loud). Decibels are measured using the equation below:

$$D = 10\log\left(\frac{I}{I_n}\right)$$

where D = Decibel level (dB)
I = Intensity of sound in watts per square metre (W/m^2)
$I_n = 1\times10^{-12}$ W/m^2 (this is the intensity of the least audible sound a human can hear)

a. Calculate the decibel level for

(i) normal conversation, which has a sound intensity of $I = 1\times10^{-6}$ W/m^2.

$D = 10 \times \log\left(\frac{1\times10^{-6}}{1\times10^{-12}}\right) = 60$ dB ✓

(ii) the kerb-side of a busy road, with a sound intensity of $I = 1\times10^{-4}$ W/m^2.

$D = 10 \times \log\left(\frac{1\times10^{-4}}{1\times10^{-12}}\right) = 80$ dB ✓

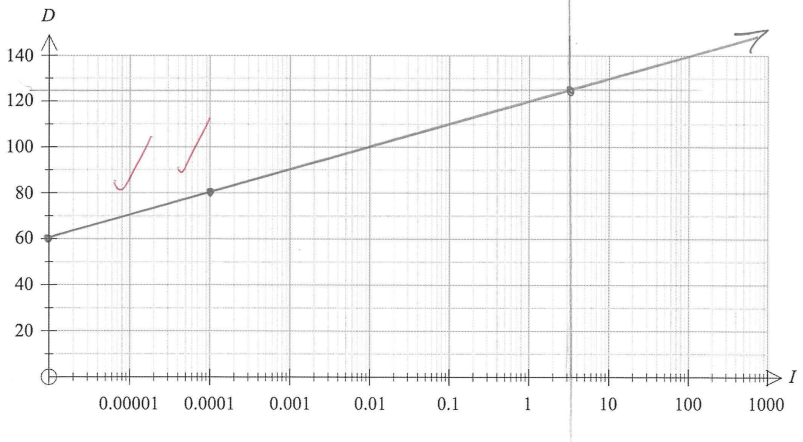
b. Calculate the sound intensity (I) that corresponds to the pain threshold of 125 dB.

$125 = 10\times\log\left(\frac{I}{1\times10^{-12}}\right)$ ✓

$\sqrt{10}$ $\therefore I = 3.16$ w/m^2 ✓

6

c. Represent the above three points on the logarithmic graph paper, using them to plot the relationship between I and D



End of calculator section – go back and check your working
Raise your hand when you are ready to go to the non-calculator section
At this stage you may work on both papers (without a calculator or notes)

Question 8 **[4 + 3 = 7 marks]**

a. Calculate the equation of the tangent to the curve $y = \ln x$ at the point $(e^2, 2)$.

$y' = \frac{1}{x}$

@ $x = e^2$, $y' = \frac{1}{e^2}$ ✓

$y = mx + c$

$y = \frac{1}{e^2}x + c$

Sub $(e^2, 2)$

$2 = \frac{1}{e^2} \times e^2 + c$ ✓

$c = 1$ ✓

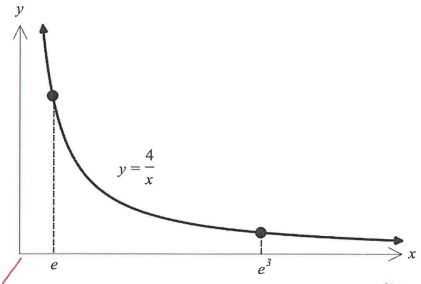
$\therefore y = \frac{1}{e^2}x + 1$ ✓

$y = \frac{x}{e^2} + 1$ ✓

b. Evaluate the area contained between the function $y = \frac{4}{x}$ and the x-axis from an x-value of e to an x-value of e^3.

If $\int \frac{4}{3c}\,dx$ is incorrect,

$\Rightarrow$ only 1 mk FT



Area $= \int_e^{e^3} \frac{4}{x}\,dx$ ✓

$= [4 \ln|x|]_e^{e^3}$

"Absolute values" Only penalise −1 if not already penalised.

$= 4\ln(e^3) - 4\ln e$ ✓

$= 12 - 4$

$= 8$ units^2 ✓

7

End of non-calculator section – go back and check your working

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Question 3 **[2 marks]**

a. Write $\log_2 64 = 6$ as an exponential statement:

$2^6 = 64$ ✓

b. Write $3^x = 7$ as a logarithmic statement:

$\log_3 7 = x$ ✓

2

Question 4 **[1 + 1 + 2 + 1 = 5 marks]**
Evaluate the following:

a. $\log_2 32$

Since $2^5 = 32$

$\log_2 32 = 5$ ✓

b. $\log_3 \frac{1}{9} = \log_3 (3^{-2})$

$= -2$ ✓

c. $5 + 3\ln e^2$

$= 5 + 3 \times 2 \ln e$ ✓

$= 5 + 3 \times 2$

$= 11$ ✓

d. $3^{\log_3 5}$

$= 5$ ✓

OR Let $3^{\log_3 5} = x$

Write as log statement

$\log_3 x = \log_3 5$

$\therefore x = 5$

[Since $a^{\log_a b} = b$]

5

7

Question 5 **[2 + 2 = 4 marks]**

Express each of the following as a single logarithm:

a. $4\log a - 2\log b + \log c^3$

$= \log a^4 + \log c^3 - \log b^2$ ✓

$= \log\left(\frac{a^4 c^3}{b^2}\right)$ ✓

✓ for $\frac{\log a^4 + \log c^3}{\log b^2}$

b. $\log_7 xy - 2 + \log_7 10$

$= \log_7 xy - 2\times\log_7 7 + \log_7 10$ ✓✓

$= \log_7\left(\frac{10xy}{49}\right)$ ✓

4

Question 6 **[2 + 3 + 4 = 9 marks]**

Solve using your knowledge of logarithms, giving solutions as exact values in simplest form.

a. $2(5^x) = 12$

$5^x = 6$

$x\log 5 = \log 6$ ✓

$x = \frac{\log 6}{\log 5}$ ✓

b. $3^{x+1} = 4^{2x}$

$(x+1)\log 3 = 2x\log 4$ ✓

Must be in a bracket.

$\log 3 = 2x\log 4 - x\log 3$

$= x(2\log 4 - \log 3)$ ✓

$= x(\log 16 - \log 3)$

$\therefore x = \frac{\log 3}{\log 16 - \log 3}$ OR $\frac{\log 3}{\log(16/3)}$ ✓

Do not penalise final mark if left in terms of a negative.

c. $e^{2x} - 5(e^x) = 14$

Let $y = e^{2x}$ ✓

$y^2 - 5y - 14 = 0$

$(y-7)(y+2) = 0$ ✓

$y = 7$ or -2

$\therefore e^x = 7$ or $e^x = -2$ ✓ NOT possible.

Solve $e^x = 7$

$x\ln e = \ln 7$

$x = \ln 7$ ✓

9

No marks for $\ln(e^{2x}) - \ln(5e^x) = \ln(14)$

Question 7 **[3 + 4 = 7 marks]**

a. Calculate $\frac{d}{dx}$ for the following:

(i) $\ln(3-4x)$

$\frac{d}{dx}\left(\ln(3-4x)\right) = \frac{-4}{3-4x}$ ✓

$\frac{\left(\frac{1}{2}\right)(2x^3+1)^{-\frac{1}{2}}\times 6x^2}{(2x^3+1)^{\frac{1}{2}}}$ ✓ OR

$= \frac{3x^2}{2x^3+1}$

(ii) $\ln\left(\sqrt{2x^3+1}\right) = \frac{1}{2}\ln(2x^3+1)$

$\frac{d}{dx}\left(\frac{1}{2}\ln(2x^3+1)\right)$

$= \frac{1}{2}\times\frac{6x^2}{2x^3+1}$ ✓

$= \frac{3x^2}{2x^3+1}$ ✓

b. Evaluate the following integrals:

(i) $\int \frac{12x^2}{7-x^3}dx$

$= -4\times\int\frac{-3x^2}{7-x^3}dx$ ✓

$= -4\ln|7-x^3| + c$ ✓

(ii) $\int 2\tan(4x+1)dx$

$= \int\frac{2\sin(4x+1)}{\cos(4x+1)}dx$

$= \left(-\frac{1}{2}\right)\times\int\frac{-4\sin(4x+1)}{\cos(4x+1)}dx$ ✓

$= -\frac{1}{2}\ln|\cos(4x+1)| + c$ ✓

-1 once if + c missing

-1 once if missing absolute value signs.

-1 once if missing "dx" from integral.

7