

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Materials required/recommended for this section

Working time for this section: one hundred minutes
Reading time before commencing work: ten minutes

Time allowed for this section

Your name _____

In words _____

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SOLUTIONS

MATHEMATICS 3C/3D
Section Two:
Calculator-assumed

Question/Answer Booklet

Year 12 Trial WACE Examination, 2013

SCHOOL

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
Total				150	100

Additional working space

Question number: _____

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil**, except in diagrams.

(2 marks)

$$2 \int_0^2 f(x)dx - \int_0^2 3dx = 2 \times 9 - 2 \times 3 = 12$$

(c) $\int_2^0 f(x) - 3 dx$

(1 mark)

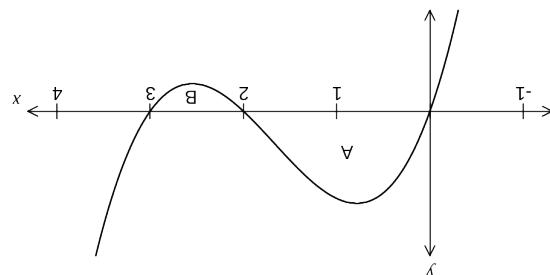
$$9 - 1 = 8$$

(d) $x \int_3^0 f(x) dx$

(1 mark)

$$-1$$

(a) Evaluate $\int_3^2 f(x) dx$



Part of the graph of $y = f(x)$ is shown below. The areas of regions A and B, bounded by the

(4 marks)

Question 9

Working time for this section is 100 minutes.

This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

Section Two: Calculator-assumed (100 Marks)

Question number: _____

MATHEMATICS 3C/3D

(6 marks)

Question 10

Let $f(x) = \sqrt{3x+3}$ and $g(x) = 2^x - 3$.

- (a) Determine a simplified expression for $f(g(x))$ and hence state the domain of $f(g(x))$.
(3 marks)

$$\begin{aligned} f(g(x)) &= \sqrt{3(2^x - 3) + 3} \\ &= \sqrt{3 \cdot 2^x - 6} \\ 3 \cdot 2^x - 6 &\geq 0 \\ x &\geq 1 \end{aligned}$$

- (b) Write down an expression for $g(f(x))$ and determine the range of this composite function.
(3 marks)

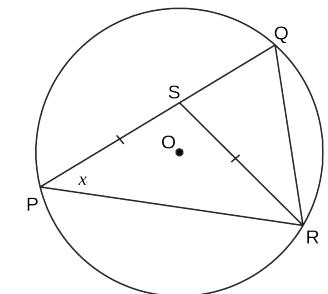
$$\begin{aligned} g(f(x)) &= 2^{\sqrt{3x+3}} - 3 \\ \text{Since } f(x) \geq 0 \text{ then} \\ y &\geq 2^0 - 3 \\ y &\geq -2 \end{aligned}$$

(6 marks)

Question 21

The three vertices of $\triangle PQR$ lie on a circle with centre O .

Point S lies on PQ such that the lengths PS and RS are equal and $\angle QPR = x$.



- (a) Explain why $\angle QOR = 2x$.
(1 mark)

The angle standing on a common chord (QR) at the centre of circle (QOR) is twice angle on circumference (QPR).

- (b) Prove that $QROS$ is a cyclic quadrilateral.
(3 marks)

$\angle PRS = x$ ($\triangle PRS$ is isosceles).

$\angle QSR = 2x$ (external angle of triangle ($\triangle PRS$) is sum of opposite interior angles of triangle ($\angle SPR$ and $\angle SRP$)).

Note that $\angle QSR = \angle QOR = 2x$.

Hence $QROS$ is a cyclic quadrilateral. (Angles standing on a common chord (QR) on the circumference of a circle are equal).

- (c) Calculate the area of $\triangle QRS$ given the area of $\triangle PRS$ is 30 cm^2 and the ratio of lengths $QS : QP = 2 : 5$.
(2 marks)

Ratio of areas of $\triangle QRS : \triangle PRS = 2 : 3$

$$\text{Area of } \triangle QRS = \frac{2}{3} \times 30 = 20 \text{ cm}^2$$

(7 marks)

Question 12

A sports medical unit performs urine tests to detect the presence of a certain steroid. It is known that for 89% of samples containing the steroid, the test indicates its presence but for 2% of samples free of the steroid, the test falsely indicates its presence.

At a large event, the unit suspect that 8% of the athletes are using the steroid and plan to randomly sample urine from 25% of the athletes.

Calculate the probability that

- (a) a randomly chosen athlete who does not use the steroid is selected for a urine test and the test falsely indicates its presence. (1 mark)

$$0.25 \times 0.02 = 0.005$$

- (b) the test indicates the presence of the steroid in a urine sample from a randomly chosen athlete. (3 marks)

$$\begin{aligned} P(S \cap I) &= 0.08 \times 0.89 \\ &= 0.0712 \end{aligned}$$

$$\begin{aligned} P(\bar{S} \cap I) &= 0.92 \times 0.02 \\ &= 0.0184 \end{aligned}$$

$$\begin{aligned} P(I) &= 0.0712 + 0.0184 \\ &= 0.0896 \end{aligned}$$

- (c) the urine of a randomly chosen athlete contains the steroid, given that the test does not indicate its presence. (3 marks)

$$\begin{aligned} P(\bar{I}) &= 1 - 0.0896 \\ &= 0.9104 \end{aligned}$$

$$\begin{aligned} P(S \cap \bar{I}) &= 0.08 \times 0.11 \\ &= 0.0088 \end{aligned}$$

$$\begin{aligned} P(S | \bar{I}) &= \frac{0.0088}{0.9104} \\ &\approx 0.0097 \quad (4 \text{ dp}) \end{aligned}$$

See next page

- (d) Find the time at which the displacement of the object is a maximum. (3 marks)

Maximum displacement when $v = 0$

$$v(12) = 75$$

$$75 - 5a = 0$$

$$a = 15$$

$$v(12 + 15) = 0$$

$$t = 27 \text{ seconds}$$

See next page

(2 marks)

- (c) Using the formula $\frac{dV}{dt} = \frac{dh}{dt} \times 9\pi r^2$, find the approximate increase in the volume of water in the cone as the depth of water increases from 6 cm to 6.05 cm.

$$\frac{dV}{dt} = \frac{4}{9\pi r^2} \times 0.05$$

$$= 12.7 \text{ cm}^3 \text{ per second}$$

- (c) Determine the displacement of the object when $t = 6$. (3 marks)

$$x = \int v dt$$

$$x = \int 10t dt$$

$$x = 5t^2$$

$$x(6) = 5 \times 6^2$$

$$x(6) = 180 \text{ m}$$

$$Between t = 0 \text{ and } t = 6 \quad d = 180.$$

- (c) Determine the displacement of the object when $t = 6$. (3 marks)

(3 marks)

- (b) An inverted cone with the above proportions is being filled with water at a rate of 200 cm^3 per second. At what rate is the depth of water in the cone increasing at the time when the depth is 6 cm?

$$\frac{dV}{dt} = dh \times \frac{dV}{dh}$$

$$200 = \frac{9\pi h^2}{4} \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = 200 \div \frac{9\pi h^2}{4}$$

$$\approx 0.786 \text{ cm per second}$$

(1 mark)

$$v(10) = 80 \text{ m/s}$$

$$\text{Maximum velocity when } d = 0$$

- (b) Find the maximum velocity of the body. (1 mark)

(3 marks)

- (a) Find the velocity of the object when $t = 6$ and when $t = 30$. (3 marks)

$$v(0) = 0$$

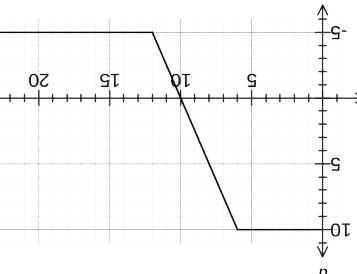
$$v(6) = v(0) + 6 \times 10 = 60 \text{ m/s}$$

$$v(10) = v(6) + 0.5 \times 4 \times 10 = 80$$

$$v(12) = v(10) - 0.5 \times 2 \times 5 = 75$$

$$v(30) = v(12) - 5 \times 18 = -15 \text{ m/s}$$

(3 marks)



(2 marks)

- (a) Show that the volume of the cone is given by $V = \frac{3\pi h^3}{4}$. (2 marks)

$$V = \frac{1}{3}\pi r^2 h$$

$$r = \frac{3h}{2} \Rightarrow h = \frac{2}{3}r$$

$$h = \frac{1}{3} \times \left(\frac{3h}{2}\right)^2 h$$

$$h = \frac{4}{3\pi h^3}$$

(7 marks)

- Question 13
CALCULATOR-ASSUMED (7 marks)

MATHEMATICS 3C/3D

- The acceleration, a ms⁻², of an object moving in a straight line for an interval of 30 seconds is shown in the graph below. When $t = 0$, both the displacement, x m and velocity, v ms⁻¹ of the object are zero.

- Question 19
CALCULATOR-ASSUMED (10 marks)
- The acceleration, a ms⁻², of an object moving in a straight line for an interval of 30 seconds is shown in the graph below. When $t = 0$, both the displacement, x m and velocity, v ms⁻¹ of the object are zero.

Question 14

When the batteries of an analogue alarm clock fail, both hands of the clock stop in a random position.

The clockwise angle from 12 at which the hour hand stops is uniformly distributed between 0° and 360° with a mean of 180° and a standard deviation of 104° .

- (a) What is the probability that the hour hand of an analogue alarm clock stops at an angle between 120° and 210° ? (1 mark)

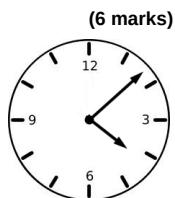
$$\frac{210 - 120}{360} = \frac{1}{4}$$

- (b) Five analogue alarm clocks stop independently of each other. Determine the probability that none of their hour hands stop at an angle between 120° and 210° . (2 marks)

$$\left(\frac{3}{4}\right)^5 = \frac{243}{1024} \approx 0.2373$$

- (c) 100 analogue alarm clocks stop independently of each other. Use the Central Limit Theorem to estimate the probability that the mean of the 100 angles at which the hour hands stop is less than 160° . (3 marks)

$$\begin{aligned} \bar{X} &\sim N(180, \frac{104^2}{100}) \\ &\sim N(180, 10.4^2) \\ P(\bar{X} < 160) &= 0.0272 \end{aligned}$$



(6 marks)

- (c) One week Botan reduces the amount he charges and so his profit per job decreases. What would his profit per job have to drop below so that it would not be worth him working? (4 marks)

$$\begin{aligned} P(a, b) &= 2 \times 24 \times a + 3 \times k \times b \\ &= 48a + 3kb \end{aligned}$$

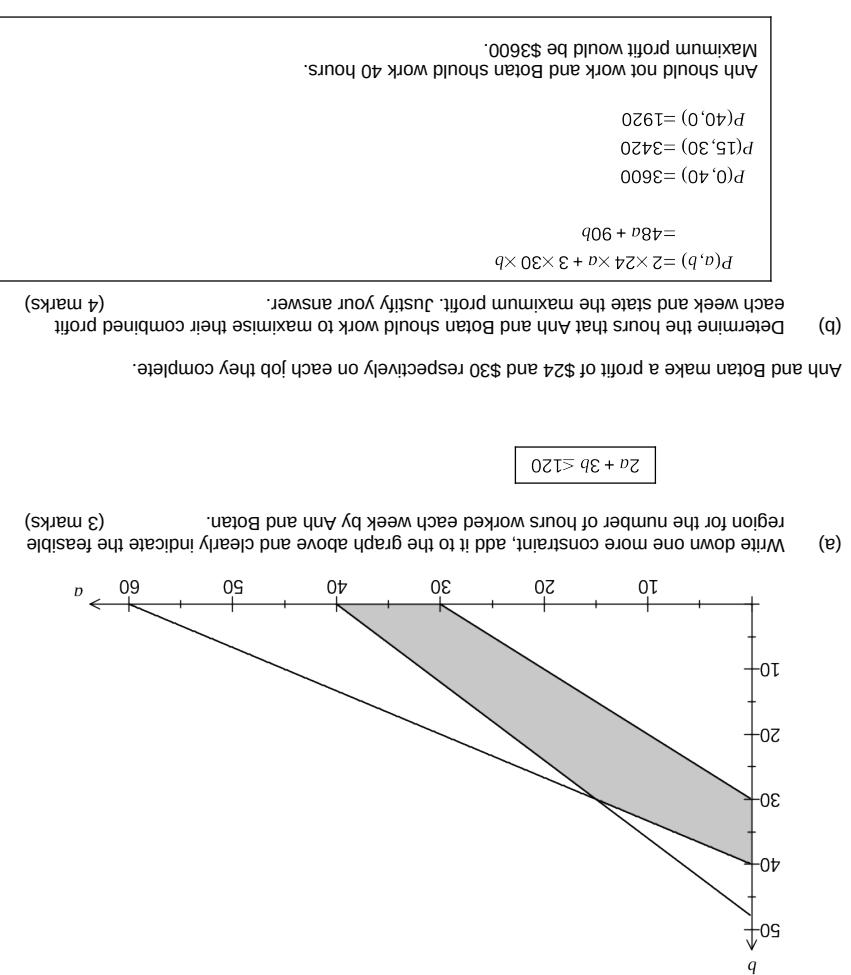
$$\begin{aligned} P(15, 30) &< P(40, 0) \\ 720 + 90k &< 1920 \\ 90k &< 1200 \\ k &< 13.\bar{3} \end{aligned}$$

His profit per job would need to drop below \$13.33 to the nearest cent.

- Question 15** (9 marks)
- Ah and Botan run a part-time business updating websites, working for at least 30 hours a week between them. Ahn can complete two jobs in an hour and Botan can complete three jobs in an hour. They have decided to spend no more than \$480 each week on resources and take on no more than 120 jobs.
- Four of the boundaried representing these constraints (including $a \geq 0$ and $b \geq 0$) are shown on the graph below, where a represents the weekly hours worked by Ahn and b the weekly hours worked by Botan.
-
- (a) Define the probability distribution of X . (2 marks)
- Let the discrete random variable X represent the number of calls who will be satisfied with their response to their inquiry using the existing call centre system.
- Feedback to a customer service call centre indicates that 46% of customers are currently satisfied with the responses to their inquiry. The centre management want to improve this percentage and plan to replace the existing system with a new system.
- Feedback to a customer service call centre indicates that 46% of customers are currently satisfied with the responses to their inquiry. The centre management want to improve this system.
- (b) Calculate the mean and standard deviation of X . (2 marks)
- $$X \sim B(200, 0.46)$$
- (c) What is the probability that more than half of the next 200 callers will be satisfied with the response to their inquiry using the existing call centre system? (2 marks)
- $$\begin{aligned} P(X > 100) &= \sum_{x=101}^{200} \binom{200}{x} (0.46)^x (0.54)^{200-x} \\ &\approx 0.114 \end{aligned}$$
- (d) After implementing the new system, customer feedback indicates that 101 out of the next 200 callers were satisfied with the response to their inquiry. Is this strong evidence that the new system is an improvement over the existing system? (3 marks)
- No, there is not strong evidence of an improvement.

From (c) we know that there is an 11.4% chance that 101 or more customers will be satisfied by the existing system.

Although this does suggest an improving system, a probability of more than 1 in 10 does not count as strong evidence.

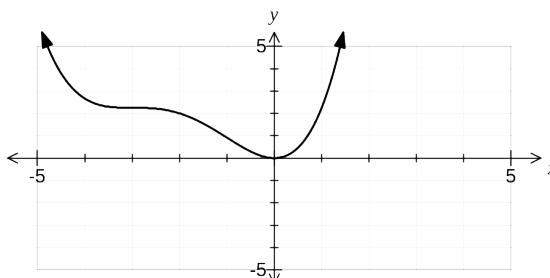


(9 marks)

- Question 16**
- The graph of the continuous function $y = f(x)$ has a local minimum at $(0, 0)$, a horizontal point of inflection at $(-3, 2.25)$ and no other stationary points. The graph also has a point of inflection at $(-1, 1)$.

- (a) Sketch a possible graph of $y = f(x)$ on the axes below.

(3 marks)



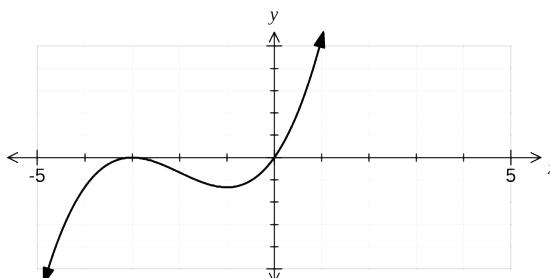
- (b) Determine the coordinates of the roots of the graph of $y = f'(x)$.

(2 marks)

At $(-3, 0)$ and $(0, 0)$.

- (c) Sketch a possible graph of $y = f'(x)$ on the axes below.

(2 marks)



- (d) Will $f''(0)$ be negative, zero or positive? Justify your answer.

(2 marks)

Positive.

Gradient of $y = f'(x)$ is clearly positive on the graph above.

See next page

(8 marks)

- Question 17**
- An internet service provider plans to sample the volume of content downloaded per day by customers subscribing to their ADSL20 plan. From recent research, the company knew that the standard deviation of the volume of downloads per customer was 1.4 GB.

- (a) Determine how large a sample the company should take in order to be 90% confident that the mean volume of downloads per customer calculated from their sample is within 0.25 GB of the true population mean.

$$n = \left(\frac{1.645 \times 1.4}{0.25} \right)^2 \\ = 84.9$$

Take a sample of at least 85 customers

- (b) A random selection of 25 subscribers was made and the total volume downloaded by these customers over a 24 hour period was 120GB. Calculate a 95% confidence interval for the mean volume of content downloaded per day by a customer.

(4 marks)

$$\bar{x} = 120 \div 25 \\ = 4.8$$

$$z \frac{\sigma}{\sqrt{n}} = 1.96 \times 1.4 \div \sqrt{25} \\ = 0.5488$$

$$4.8 \pm 0.5488 = (4.2512, 5.3488)$$

- (c) If the company repeated the random sampling process and subsequent 95% confidence interval calculations from part (b) a total of 40 times, how many of the intervals calculated would you expect to contain the true population mean? Justify your answer.

(2 marks)

38 of the intervals.

The 95% level of confidence refers to the 95% probability that the population mean is contained in the interval.

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