

| Question | Marks | Max | Question | Marks | Max |
|----------|-------|-----|----------|-------|-----|
| 4        | 8     | 8   |          |       |     |
| 3        | 8     | 8   |          |       |     |
| 2        | 6     | 10  |          |       |     |
| 1        | 5     | 8   |          |       |     |
|          | 6     | 13  |          |       |     |

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Important note to Candidates**

Special items: nil

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

**To be provided by the candidate**

Formula sheet

This Question/Answer booklet

Materials required/recommended for this section

Working time: fifty minutes  
Reading time before commencing work: five minutes

Your Teacher's Name:

Your Name:

Calculator-free

Section One:

UNIT 3 & 4

MATHEMATICS METHODS

Question/Answer booklet

Semester Two Examination, 2022



**Structure of this paper**

| Section                            | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|------------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|---------------------------|
| Section One:<br>Calculator-free    | 6                             | 6                                  | 50                     | 49              | 35                        |
| Section Two:<br>Calculator-assumed | 10                            | 10                                 | 100                    | 100             | 65                        |
| <b>Total</b>                       |                               |                                    |                        |                 | <b>100</b>                |

**Instructions to candidates**

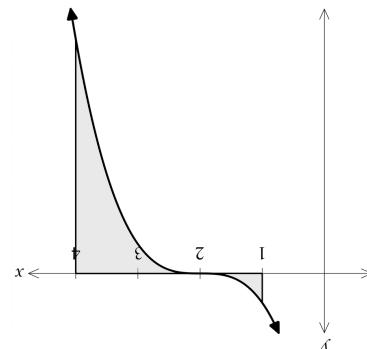
1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

$$k \left[ \frac{8}{(2x-4)^4} \right]_1^2 = 17$$

$$\int_2^1 k|2x-4|^3 dx - \int_2^1 k|2x-4|^3 dx = 17$$

**Solution**

If the area of the shaded region is 17 units<sup>2</sup>, determine the value of  $k$ .



The graph with equation  $y = k|2x-4|^3$  is shown below.

(5 marks)

**Question 1**

Working time: 50 minutes.

Number of the question that you are continuing to answer at the top of the page.

- Continuing an answer: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
- Planning: if you use the spare pages for planning, indicate this clearly at the top of the page.
- Responses and/or as additional space if required to continue an answer.

Space pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

This section has **six** questions. Answer all questions. Write your answers in the spaces provided.

(49 marks)

**Section One: Calculator-free**

$$k([0-2] - [32-0]) = 17 - 34 \quad k = 17$$
$$k = \frac{-1}{2}$$

**Specific behaviours**

- ✓ Writes at least one definite integral to determine area.
- ✓ Recognises that the area from  $x=2$  to  $x=4$  is a signed area, and deals with this correctly.
- ✓ Correctly integrates using chain rule.
- ✓ Correctly substitutes boundaries and simplifies.
- ✓ Determines the value of  $k$ .

**Question 2 (10 marks)**

A random experiment can either result in a success with a probability of  $p$  or a failure.

In an event, this random experiment is conducted twice. Each experiment is independent of the other. Let the number of successes be represented by  $X$ .

- (a) State the distribution, and its parameters, that can be used to model the event described above. (2 marks)

| Solution  |
|---|
| $X \text{ Bin}(2, p)$   |
| Specific behaviours   |
| ✓ Identifies the distribution as binomial.<br>✓ Writes down the correct parameters. . |

The graph below shows of the variance for the distribution in part (a) for various values of  $p$ .

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| Specific behaviours  |  |
|--|--|
| Solution   | States value of $p$ .                    |
| (ii) Hence determine the exact value of the standard deviation, for the value of $p$ in part (i). (1 mark) | $Var(X) = 2 \times 0.5 \times 0.5 = 0.5$ |
|  | $SD(X) = \sqrt{\frac{1}{2}}$             |

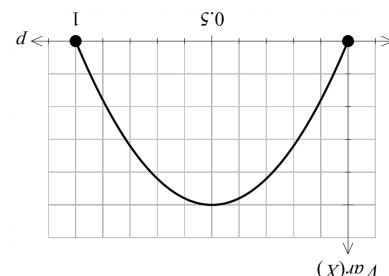
(iii) Hence determine the exact value of the standard deviation, for the value of  $p$  in part (i). (1 mark)

| Specific behaviours |                       |
|---------------------|-----------------------|
| Solution            | States value of $p$ . |
|                     | $p = 0.5$             |

(c) (i) State the value of  $p$  for which the standard deviation of  $X$  is maximised. (1 mark)

| Specific behaviours |  |
|---------------------|--|
| Solution            | Explains why the graph is continuous.  |
|                     | The probability is any value between 0 and 1, and hence the graph is continuous. |

(b) Explain why the graph is continuous, despite the distribution being discrete. (1 mark)



End of questions

P Determines standard deviation

For the distribution in part (a), the probability that at least one of these experiments results in a success is 0.51.

- (d) (i) Show that  $(1-p)^2=0.49$  (2 marks)

**Solution**

$$\begin{aligned} P(X=0) &= 1 - 0.51 \\ (1-p)^2 &= 0.49 \end{aligned}$$

**Specific behaviours**

P Uses complement to  $P(X \geq 1)$ .

P Shows that  $P(X=0)=(1-p)^2$ .

- (ii) Hence show that the value of  $p$  is 0.3. (1 mark)

**Solution**

$$\begin{aligned} 1-p &= 0.7 \\ p &= 0.3 \end{aligned}$$

**Specific behaviours**

P Correctly square roots 0.49 and determines  $p$ .

- (e) Determine  $E(X)$  for this distribution. (1 mark)

**Solution**

$$E[X]=0.6$$

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ü smooth curve, concave up throughout

- (c) Given that  $\log_6(x+6)=\frac{\ln(x+6)}{\ln(6)}$ , determine the value of  $x$  where the slopes of  $y=f(x)$  and line  $L$  are the same. (2 marks)

**Solution**

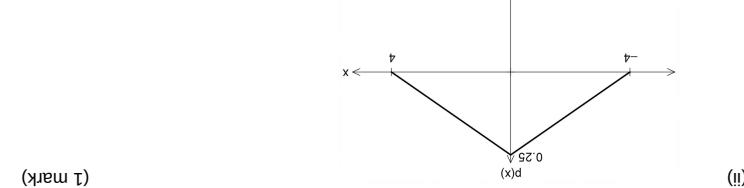
$$\begin{aligned} f(x) &= -2 \log_6(x+6) - 1 = \frac{-2}{\ln(6)} \ln(x+6) - 1 \\ f'(x) &= \frac{-2}{\ln(6)} \times \frac{1}{x+6} = \frac{-2}{\ln(6)} \times \frac{1}{x+6} = \frac{-2}{5} \rightarrow x = \frac{5}{\ln(6)} - 6 \end{aligned}$$

**Specific behaviours**

P correctly differentiates  $f$

ü solves for  $x$ -coordinate and fully simplified

Note- no follow through if mistake makes solving too easy



(1 mark)

(1 mark)

$$f(x) = \frac{x+2}{x}, x = 0, 1, 2.$$

Justify each answer.

- (a) Determine whether the following represent or do not represent a probability distribution.

- (b) Sketch the graph of  $y=f(x)$  on the axes below.

- (c) Calculate two points on curve (Must state paired coordinates).

P Explains using a reflection.

Specific behaviours

Question 3 (10 marks)

The distribution of  $Y$  is a reflection of  $X$ .

Solution

(1 mark)

- (d) Explain why  $\text{Var}(Y)=\text{Var}(X)$ .

Given that  $\text{Var}(X)=0.42$ .A second random variable  $Y$  is defined as  $Y \sim B(2, 0.7)$ .

P Correctly determines expected value.

Specific behaviours

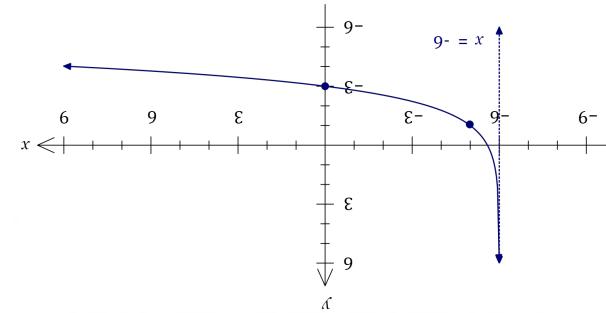
Question 6

The graph of  $y=f(x)$  intersects line  $L$  with equation  $5y+2x+15=0$  when  $x=0$  and  $x=-5$ .

- ü asymptote, correct curvature nearby  
ü through two points from (b)

Specific behaviours

Solution



(3 marks)

- (b) Sketch the graph of  $y=f(x)$  on the axes below.

- (c) Calculate two points on curve (Must state paired coordinates).

Using  $(0, -3)$ :  $-3=k \log_6(6)-1 \rightarrow k=-2$ Using  $(-5, -1)$ :  $-1=k \log_6(1)+c \rightarrow c=-1$ 

$$x=0, y=\frac{5}{-10+15}=-3, x=-5, y=\frac{5}{-10+15}=-1$$

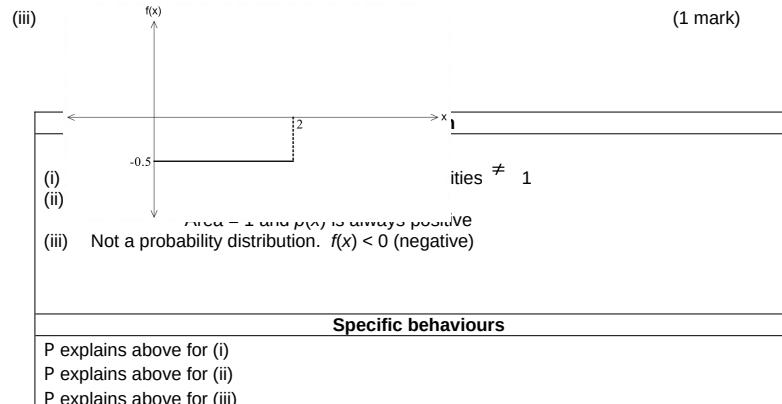
- (a) Determine the value of the constant  $C$  and the value of the constant  $k$ .

(3 marks)

- The graph of  $y=f(x)$  intersects line  $L$  with equation  $5y+2x+15=0$  when  $x=0$  and  $x=-5$ .

Let  $f(x)=k \log_6(x+6)+c$ , where  $k$  and  $c$  are constants.

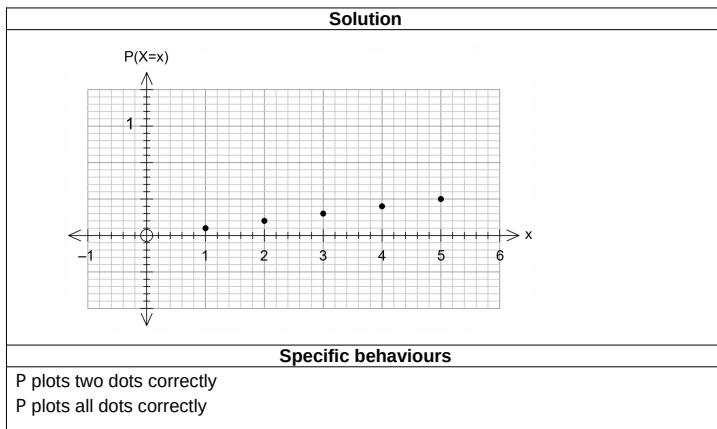
(8 marks)



- b) A probability distribution of a random variable  $X$  is given by

$$P(X=x) = \frac{x}{15} \text{ where } x = 1, 2, 3, 4, 5$$

- (i) Graph the probability function on the axes below. (2 marks)



See Next Page

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- (b) The current,  $I$  amps, flowing through component B reaches a peak very quickly and then declines as time goes on, as modelled by  $I(t) = \frac{2+ln t}{4t}$ . Determine, in simplest form, the maximum current that flows through this component. (4 marks)

| <b>Solution</b>   |
|---|
| $I'(t) = \frac{\left(\frac{1}{t}\right)(4t) - (2+\ln t) 4 }{(4t)^2} \cdot \frac{4-4(2+\ln t)}{4 \times 4t^2} \cdot \frac{-1-\ln t}{4t^2}$ |
| $I'(t) = 0 \Rightarrow \ln t = -1 \Rightarrow t = e^{-1}$   |
| $I(e^{-1}) = \frac{2-1}{4e^{-1}} = \frac{e}{4}$   |
| Maximum current is $\frac{e}{4}$ amps.  |
| <b>Specific behaviours</b>  |
| P uses quotient rule  |
| ü obtains correct derivative and equates to zero  |
| ü obtains root of derivative  |
| ü calculates maximum current in simplified form   |

| Solution  |   | Specific behaviours |    |   |   |    |    | P determines at least two correct probs |   | P all probs correct |  |
|---|---|---------------------|----|---|---|----|----|---|---|---------------------|--|
|   |   |                     |    |   |   |    |    |   |   |                     |  |
| P(X ≤ 1)  | 1 |                     | 15 | 3 | 6 | 15 | 10 | 15                                      | 1 |                     |  |
| X   | 1 | 2                   | 3  | 4 | 5 |    |    |   |   |                     |  |
| State the cumulative probability distribution for X.<br>(2 marks) |   |                     |    |   |   |    |    |   |   |                     |  |
| P numerator<br>P denominator                                      |   |                     |    |   |   |    |    |   |   |                     |  |
| Specific behaviours   |   |                     |    |   |   |    |    |   |   |                     |  |
| $\frac{9}{15} = \frac{3}{5}$                                      |   |                     |    |   |   |    |    |   |   |                     |  |
| Solution  |   |                     |    |   |   |    |    |   |   |                     |  |

|   |          |                              |                     |                   |
|---|----------|------------------------------|---------------------|-------------------|
| (ii) Determine the probability of obtaining a value of $X$ less than 3. | Solution | $\frac{3}{15} = \frac{1}{5}$ | Specific behaviours | P determines prob |
|---|----------|------------------------------|---------------------|-------------------|

|   |
|---|
| <p><b>5</b> (8 marks)</p> <p>P correct result</p> <p><b>SPECIFIC BEHAVIOURS</b></p> <p><math>x e^{2x} = 3e^6</math></p> <p><b>Solution</b></p>  |
| <p>d) t seconds after the circuit is turned on.</p> <p>events A and B form part of an electronic circuit, and properties of these components are determined in simple form, the increase in temperature of this component during the first second.</p> <p>rate of change of temperature, <math>T^\circ\text{C}</math>, of component A is given by <math>\frac{dT}{dt} = \frac{3t^2 + 8}{18t}</math>.</p> <p>rate of change of temperature, <math>T^\circ\text{C}</math>, of component A is given by <math>\frac{dT}{dt} = \frac{3t^2 + 8}{18t}</math>.</p> <p>writes integral to evaluate total change</p> <p>substitutes limits of integration</p> <p>correct increase, simplified (also accept <math>\ln(343)</math>)</p> |
| <p><b>SPECIFIC BEHAVIOURS</b></p> <p><math>A = \int_1^0 18t \frac{3t^2 + 8}{dt} dt = 3 \int_1^0 [56 - \ln(8)] dt = 3[56t - t \ln(8)]_1^0</math></p> <p><b>Solution</b></p>  |
| <p>writes integral to evaluate total change</p> <p>substitutes limits of integration</p> <p>correct increase, simplified (also accept <math>\ln(343)</math>)</p>  |
| <p><b>P SPECIFIC BEHAVIOURS</b></p> <p>writes integral to evaluate total change</p> <p>substitutes limits of integration</p> <p>correct increase, simplified (also accept <math>\ln(343)</math>)</p>  |

|                     |                          |
|---------------------|--------------------------|
|                     | P correct result         |
| Specific behaviours |                          |
| Solution            |                          |
| (1 mark)            | $\int_1^t \cos(t+1) dt.$ |

**Question 4**

(8 marks)

Determine the following:

(a)  $\int 6 e^{2x-3} dx.$

(1 mark)

| <b>Solution</b>  |
|--|
| $3e^{2x-3} + c$  |
| <b>Specific behaviours</b>                             |
| P correct antiderivative, with constant of integration |

(b)  $\int_0^{\frac{\pi}{8}} \sin(4x) dx.$

(2 marks)

| <b>Solution</b>  |
|--|
| $\left[ -\frac{1}{4} \cos(4x) \right]_0^{\pi/8} = 0 - \left( -\frac{1}{4} \right) = \frac{1}{4}$ |
| <b>Specific behaviours</b>   |
| P correct antiderivative<br>ü correct value  |

(c)  $f\left(\frac{\pi}{6}\right)$  when  $f(x) = \frac{\cos(3x)}{2+\sin(x)}.$

(3 marks)

| <b>Solution</b>   |
|---|
| $f'(x) = \frac{-3\sin(3x)(2+\sin(x)) - \cos(3x)\cos(x)}{(2+\sin(x))^2} f\left(\frac{\pi}{6}\right) = \frac{-3(2+0.5)-0}{(2+0.5)^2} \textcolor{red}{i} - \frac{3}{2.5} = \frac{-6}{5}$ |
| <b>Specific behaviours</b>  |
| P correctly uses quotient rule<br>ü correctly differentiates all trig terms<br>ü correctly evaluates  |