

Derivation of $a_c = v^2/r$

The rate at which an object rotates is often stated in *rpm* or *revolutions per minute*.

In one revolution, the object passes through 360° or 2π radians.

Units of the form *angle per second* could be used to measure rate of rotation.

The “natural” unit for angle is *radians* and these will be used throughout the following.

Therefore, units of *radians per second* will be used to measure rate of rotation.

The rate of rotation, given in *radians per second*, is a rate of change of angle.

It is called angular velocity for which the symbol is ω (the lowercase Greek letter omega).

$$\omega = \Delta\theta / \Delta t$$

Radians per second is the mathematically preferred unit for measuring angular velocity.

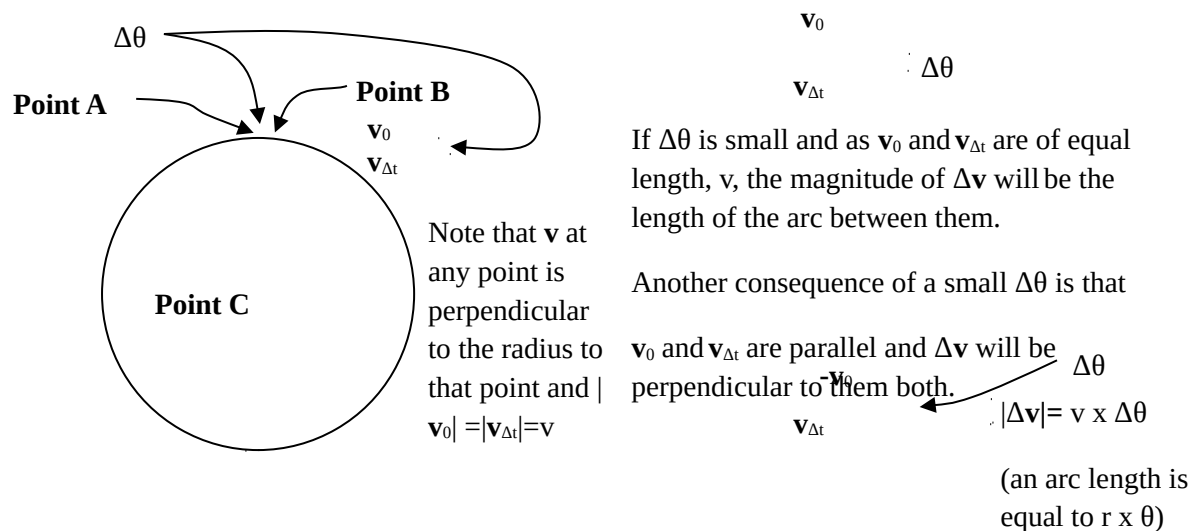
If an object revolves about a point at a radius of r and a speed of v , then one complete revolution will take T seconds where T is called the period of revolution.

In one revolution, the object will travel a distance of $2\pi r$ so $v = 2\pi r / T$

The object will have an angular velocity $\omega = 2\pi / T$ *radians per second*.

Combining $\omega = \Delta\theta / \Delta t$ and $v = 2\pi r / T$ and $\omega = 2\pi / T$ gives $\omega = v / r = \Delta\theta / \Delta t$

If an object in circular motion about C travels from Point A to Point B, it will have a change of velocity of $\Delta\mathbf{v}$ where $\Delta\mathbf{v}$ is as sketched below.



The magnitude of $\Delta\mathbf{v}$ will be the arc length sketched above. The direction of $\Delta\mathbf{v}$ will be parallel to the radius and towards the centre of the orbit.

$$\begin{aligned} \mathbf{a} &= |\Delta\mathbf{v}| / \Delta t \\ &= v \times \Delta\theta / \Delta t \\ &= v \times \omega \\ &= v \times v / r \\ &= v^2 / r \text{ toward the centre of the orbit} \end{aligned}$$