**Calculator Assumed**
**Discrete Random Variables – Mixed Applications 2**

Time: 45 minutes
Total Marks: 45
Your Score: / 45

**Question One: [2, 2, 2, 2, 3 = 11 marks]** **CA**

During strawberry picking season, 1 in every 20 strawberries will have been nibbled by a snail.

Breanna has been strawberry picking and has randomly picked 50 strawberries.

Determine the probability that:

(a) exactly 20 strawberries have been nibbled by a snail.

(b) less than half have been nibbled by a snail.

(c) at least 80% have been nibbled by a snail.

(d) less than 10 have been nibbled by a snail given that at least 5 have been were nibbled by a snail.

On another strawberry farm, the chances of having at most 1 strawberry nibbled by a snail out of 20 picked is 0.9118.

(e) What is the probability of an individual strawberry being nibbled by a snail?

**Question Two: [1, 2, 3 = 6 marks] CA**

A discrete random variable has the probability function $P(\mathrm{X})=\dfrac{e^{-\lambda}\lambda^{x}}{x!}$ $x=0,1,2,3...$

where $\lambda=3$

Determine:

(a) $P(\mathrm{X}=2)$

(b) $P(X>1)$

(c) $P(X<2\,|\,X>0)$

**Question Five: [1, 2, 2, 2, 2 = 9 marks] CA**

Kali is interested in whether her toddler is left or right handed. Each day she plays a game with him where she places a crayon in front of him and asks him to pick it up. She places it in front of him 3 times each day and records what he does. Kali does this experiment for 21 days.

The number of times he picks it up with his right hand, $x$, is recorded below.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Frequency | 11 | 6 | k | 2 |

(a) Calculate the value of $k$.

$k = 2$ ✓

(b) Calculate the mean number of times Kali's son uses his right hand.

$\bar{x} = 0.7619$ ✓✓

(c) What is the probability that the next time Kali places the crayon in front of her son he will pick it up with his right hand?

$$E(X) = 0\times\frac{11}{21} + 1\times\frac{6}{21} + 2\times\frac{2}{21} + 3\times\frac{2}{21}$$ ✓

$$E(X) = 0.7619$$ ✓

Kali thinks that her son's actions each day can be modelled by a binomial distribution.

(d) Using a suitable binomial distribution, how many days over the next three weeks can Kali expect her son to only use his right hand?

$X \sim Bin(21, 0.7619)$ ✓

$E(X) = 21\times 0.7619 = 16$ ✓

(e) Is Kali correct about a binomial distribution being an appropriate approximation in this situation? Explain your answer.

No she is not correct because Kali's original data is positively skewed and this binomial distribution is negatively skewed.

✓✓





**Question Three: [7, 2, 1 = 10 marks] CA**

In one game of Oz Lotto, players choose seven different numbers from 1 to 45 inclusive.

For one game of Oz Lotto, let $Y$ be the number of correct numbers guessed.

(a) Define the probability distribution for $Y$ using a table of values.

(b) How many numbers can a player expect to get correct?

(c) What is the most likely number of correct guesses from the 7 chosen?

**Question Four: [2, 3, 2, 2 = 9 marks] CA**

In a MasterChef challenge, participants are to randomly choose a coloured apron from a box.

The challenge will be different this time and instead of there being an equal number of each colour, there are 6 red and 4 blue aprons in the box.

During rehearsal the 5 producers of the show each take turns choosing an apron and then putting it back in the box.

(a) What is the probability that exactly 3 of the producers end up with a red apron?

(b) What is the probability that at least two producers chose a red apron given that at least 1 chose a blue apron?

It's time to film the actual challenge and the 10 participants take it in turns to select an apron until there are none left in the box. The first five participants choose their aprons before the commercial break.

(c) What is the probability that all five chose a red apron?

(d) What is the probability that the first two are red and the next three are blue?





**Question Four: [2, 3, 2, 2 = 9 marks] CA**

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During rehearsal the 5 producers of the show each take turns choosing an apron and then putting it back in the box.

(a) What is the probability that exactly 3 of the producers end up with a red apron?

$X \sim Bin(5, 0.6)$ ✓

$P(X=3) = 0.3456$ ✓

(b) What is the probability that at least two producers chose a red apron given that at least 1 chose a blue apron?

$P(X \geq 2 \mid X \leq 4)$ ✓

$= \dfrac{P(2 \leq X \leq 4)}{P(X \leq 4)}$

$= \dfrac{0.8352}{0.92224}$ ✓

$= 0.9056$ ✓

It's time to film the actual challenge and the 10 participants take it in turns to select an apron until there are none left in the box. The first five participants choose their aprons before the commercial break.

(c) What is the probability that all five chose a red apron?

$\dfrac{6}{10} \times \dfrac{5}{9} \times \dfrac{4}{8} \times \dfrac{3}{7} \times \dfrac{2}{6} = 0.0238$ ✓ ✓

(d) What is the probability that the first two are red and the next three are blue?

$\dfrac{6}{10} \times \dfrac{5}{9} \times \dfrac{4}{8} \times \dfrac{4}{7} \times \dfrac{3}{6} = 0.0476$ ✓ ✓

**Question Three: [7, 2, 1 = 10 marks]** **CA**

In one game of Oz Lotto, players choose seven different numbers from 1 to 45 inclusive.

For one game of Oz Lotto, let $Y$ be the number of correct numbers guessed.

(a) Define the probability distribution for $Y$ using a table of values.

| $y$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(Y=y)$ | $\frac{^7C_0\,^{38}C_7}{^{45}C_7}$ | $\frac{^7C_1\,^{38}C_6}{^{45}C_7}$ | $\frac{^7C_2\,^{38}C_5}{^{45}C_7}$ | $\frac{^7C_3\,^{38}C_4}{^{45}C_7}$ | $\frac{^7C_4\,^{38}C_3}{^{45}C_7}$ | $\frac{^7C_5\,^{38}C_2}{^{45}C_7}$ | $\frac{^7C_6\,^{38}C_1}{^{45}C_7}$ | $\frac{^7C_7\,^{38}C_0}{^{45}C_7}$ |

| $y$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(Y=y)$ | 0.2781 | 0.4258 | 0.2323 | 0.0569 | 0.0065 | 0.0003 | 0.000005 | ~0 |

(b) How many numbers can a player expect to get correct?

$E(Y)=1.08863$

$\therefore 1$

(c) What is the most likely number of correct guesses from the 7 chosen?

1 correct number because it has the highest probability





**Question Five: [1, 2, 2, 2, 2 = 9 marks]** **CA**

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| Frequency | 11 | 6 | k | 2 |

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(b) Calculate the mean number of times Kali's son uses his right hand.

(c) What is the probability that the next time Kali places the crayon in front of her son he will pick it up with his right hand?

Kali thinks that her son's actions each day can be modelled by a binomial distribution.

(d) Using a suitable binomial distribution, how many days over the next three weeks can Kali expect her son to only use his right hand?

(e) Is Kali correct about a binomial distribution being an appropriate approximation in this situation? Explain your answer.

**SOLUTIONS**
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Breanna has been strawberry picking and has randomly picked 50 strawberries.

Determine the probability that:

(a) exactly 20 strawberries have been nibbled by a snail.

$X \sim Bin(50, 0.25)$ ✓
$P(X = 20) = 0.0077$ ✓

(b) less than half have been nibbled by a snail.

$P(0 \le X \le 24) = 0.999877$ ✓ ✓

(c) at least 80% have been nibbled by a snail.

$P(40 \le X \le 50) = 5.204 \times 10^{-16} \approx 0$ ✓ ✓

(d) less than 10 have been nibbled by a snail given that at least 5 have been were nibbled by a snail.

$$= P(X < 10 \mid X \ge 5) \checkmark$$
$$= \frac{P(5 \le X \le 9)}{P(X \ge 5)}$$
$$= \frac{0.1616}{0.9979}$$
$$= 0.1619 \checkmark$$





On another strawberry farm, the chances of having at most 1 strawberry nibbled by a snail out of 20 picked is 0.9118.

(e) What is the probability of an individual strawberry being nibbled by a snail?

$Y \sim Bin(20, p)$ ✓
$P(Y \le 1) = 0.9118$
${}^{20}C_0 (p)^0 (1-p)^{20} + {}^{20}C_1 (p)^1 (1-p)^{19} = 0.9118$ ✓
$p = 0.025$ ✓

**Question Two: [1, 2, 3 = 6 marks] CA**

A discrete random variable has the probability function $P(X) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ $x = 0, 1, 2, 3...$

where $\lambda = 3$

Determine:

(a) $P(X = 2)$

$P(X = 2) = 0.2240$ ✓

(b) $P(X > 1)$

$$1 - P(X \le 1) \checkmark$$
$$= 1 - P(X = 0) - P(X = 1)$$
$$= 0.8 \checkmark$$

(c) $P(X < 2 \mid X > 0)$

$$= \frac{P(X = 1)}{1 - P(X = 0)} \checkmark$$
$$= \frac{0.1494}{0.9502} \checkmark$$
$$= 0.1572 \checkmark$$