



**PERTH MODERN SCHOOL**  
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**Independent Public School**

## Course Specialist Year 12 Test Four 2022

Student name: \_\_\_\_\_ Teacher name: \_\_\_\_\_

**Task type:** Response

**Time allowed for this task:** \_\_\_\_40\_\_\_\_ mins

**Number of questions:** \_\_\_\_6\_\_\_\_

**Materials required:** Upto 3 Calculators with CAS capability (to be provided by the student)

**Standard items:** Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

**Special items:** Drawing instruments, templates, No notes allowed  
A4 paper, and up to three calculators approved for use in the WACE examinations

**Marks available:** \_\_\_\_40\_\_\_\_ marks

**Task weighting:** \_\_\_\_10\_\_\_\_%

**Formula sheet provided:** Yes

**Note:** All part questions worth more than 2 marks require working to obtain full marks.

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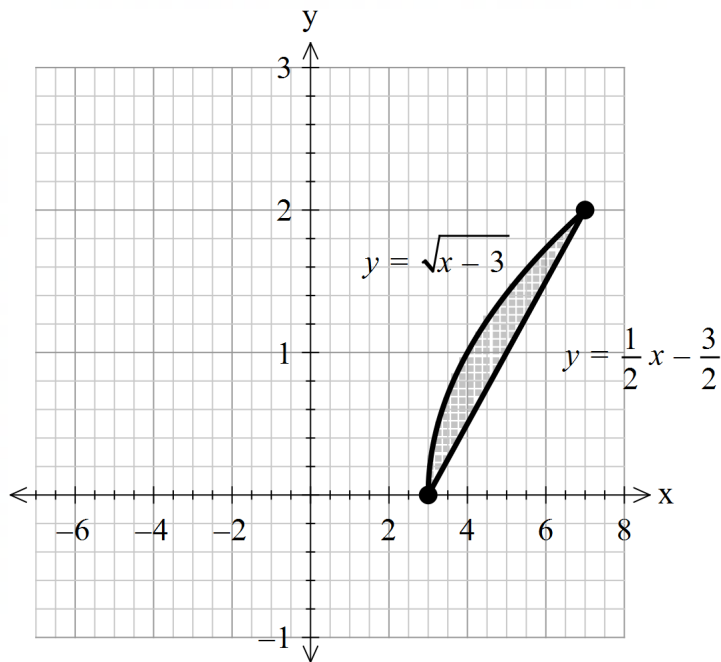
## No notes allowed

# No notes allowed

Q1 (5 marks) (4.1.6)

Determine the volume of the solid formed by rotating the area enclosed between

$y = \sqrt{x-3}$  &  $y = \frac{1}{2}x - \frac{3}{2}$  about the y axis, as shown below.



Q2 (5, 3 &amp; 2= 10 marks) (4.1.4)

- a) By using integration and partial fractions, show how to derive  $N = \frac{a}{b + Ce^{-at}}$  from the differential equation  $\frac{dN}{dt} = aN - bN^2$  ( $a, b > 0$ ) and  $c$  is a constant.

Q2 continued

- b) Let  $N$  equal the number of kangaroos living in a habitat after  $t$  years and

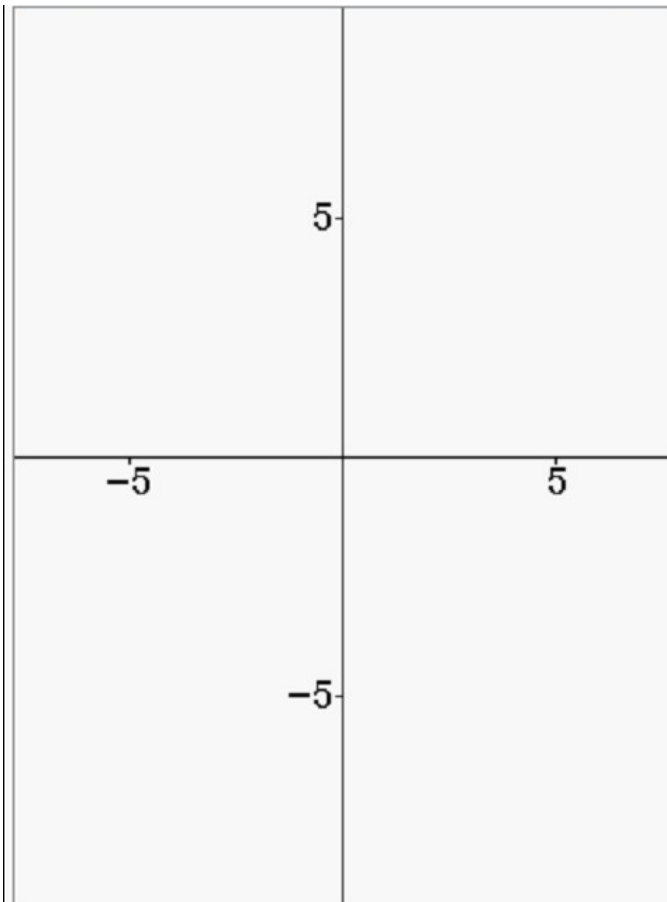
$$\frac{dN}{dt} = \frac{1}{5}N - \frac{1}{12500}N^2.$$

If initially there are 50 kangaroos, determine the number in 10 years time.

- c) Determine the size of the population at the maximum growth rate.

Q3 (3, 2 &amp; 3 = 8 marks) (4.2.5)

- a) Sketch the slope field on the axes below for  $\frac{dy}{dx} = 3^x$



- b) Show the solution curve on the axes above that passes through point (2,2).
- c) Determine in cartesian form the solution curve for b above **without using a classpad**.  
Hint – use logarithmic differentiation. Show all working.

Q4 (5 marks) (4.2.1)

Determine expressions in terms of  $x$  &  $y$  only for  $\frac{dy}{dx}$  &  $\frac{d^2y}{dx^2}$  in terms of  $x, y$  &  $y'$  only, using the following equation  $x^3y^2 = 5 - xy$

Consider a particle that is moving with SHM such that  $\ddot{x} = -9x$  with a maximum speed of 12 m/s.

a) Determine the exact speed when the particle is half of an amplitude from the origin.

- b) Determine the percentage of the time that the particle is more than half an amplitude from the centre.

The motion of a bullet through a wall is modelled by the equation  $a = -25(v + 75)^2$ ,  $v > 0$  where  $a \text{ m/s}^2$  is its acceleration and  $v \text{ m/s}$  its velocity  $t$  seconds after impact. Initially at impact the speed is 300 m/s and is at the origin ( $x = 0$  metres)

- a) Determine  $x$  in terms of  $v$  only.

Q6 continued-

- b) Determine how far the bullet penetrates the wall before coming to rest to the nearest mm.