Christ Church Grammar School

2021
TEST 3

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name_____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

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See next page

Additional working space

Question number: _____

Question 1 **(6 marks)**

Evaluate the following.

(a) $\int_1^2 \frac{d}{dx}(x^2e^{2x})\,dx$ (2 marks)

(b) $\frac{d}{dx}(\sin(2x) + e^{4x^2})$ (2 marks)

(c) $\int -2xe^{2x^2}dx$ (2 marks)

Question 2 **(3 marks)**

Given $\frac{d}{dx}(xe^x - e^x) = xe^x$, determine exactly $\int_0^1 (xe^x + x^2)dx$.

See next page

Additional working space

Question number: _____

See next page

Question 9 **(6 marks)**

It is known that 5% of cars manufactured in the Bitsaremissin assembly line will have some kind of defect.

(a) If a random sample of 8 cars is selected for testing, find that probability that

(i) no cars have a defect. (1 mark)

$0.95^8 = 0.6634$ ✓

(ii) exactly 3 cars have a defect. (1 mark)

CPAD 0.0054165 ✓

(iii) no cars have a defect, given that less than 5 cars have a defect. (2 marks)

$\frac{0.6634}{0.9999846}$ ✓ $\neq 0.6634$ ✓

OR $\frac{0.6634}{1}$

(b) What is the largest number of cars that can be selected in a random sample such that the probability of there being at least 1 defective car is less than 20%? (2 marks)

None defective > 0.8

$0.95^n > 0.8$ [equ] ✓

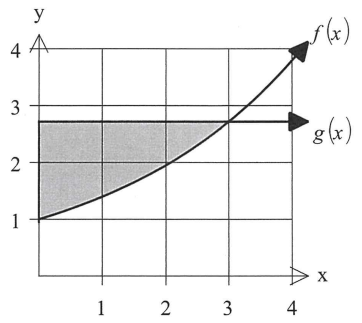
$n < 4.350345$

$n = 4$ ✓ [ANSN]

End of questions

Question 3 **(3 marks)**

The functions $f(x) = e^{\frac{x}{3}}$ and $g(x) = e$ are graphed below, intersecting at $(3, e)$.



Determine the area bound by the two curves and the y-axis.

See next page

Question 4 **(3 marks)**

Determine the x coordinates of all stationary points of the function:

$$f(x) = \int_0^{4x} e^{t^2} - e\, dt$$

End of questions

Question 8 continued

(c) Given that $SD(X) = 1.6$, determine

(i) $SD(3X)$ (1 mark)

$3 \times 1.6 = 4.8$ ✓

(ii) $Var(3X)$ (1 mark)

$4.8^2 = 23.04$ ✓

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See next page

Question 8 **(9 marks)**

Consider the discrete probability distribution shown below.

x	-1	0	1	2	3
$P(X = x)$	0.25	0.3	0.1	p	q

(a) Determine:

i) $P(X > -1)$ (1 mark)

0.75 ✓

ii) $P(X = -1 \mid X \leq 1)$ (2 marks)

$\frac{0.25}{0.65} = 0.3846$ ✓[DENOM] ✓[ANSW]

(b) Given that $E(X) = 0.85$ determine:

i) the values of p and q. (3 marks)

$\sum P(X=x) = 1 \quad \therefore p+q = 0.35 \quad -①$ ✓[Eqn ①]

$-0.25 + 0 + 0.1 + 2p + 3q = 0.85$

$2p + 3q = 1 \quad -②$ Solve on CPad. ✓[Eqn ②]

$p = 0.05$, $q = 0.3$ ✓[MUST HAVE BOTH]

ii) $E(2X - 1)$ (1 mark)

$= 2(0.85) - 1$

$= 0.7$ ✓[ANSW]

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See next page

Additional working space

Question number: _____

Question 7 **(4 marks)**

The concentration, C mg/KL, of a chemical in the CCGS pool, at time t weeks is given by $C = 0.2(1 + 8t)e^{-t}$, for $0 \leq t \leq 8$. Find:

(a) the exact value of t when the instantaneous rate of change of C with respect to t is 0. (2 marks)

$\frac{-(8t-7)e^{-t}}{5} = 0$ ✓ [Eqn]

$t = \frac{7}{8}$ weeks ✓ [Answ]

(b) the exact maximum concentration of the chemical and state when this occurs. (2 marks)

EXACT MAX $C = \frac{8}{5}e^{-7/8}$ mg/KL (/AD ✓ [MAX]

0.666979

When $t = \frac{7}{8}$ weeks ✓ [t-value]

[-1 overall if incorrect units]

Question 6 **(5 marks)**

A population grows continuously such that $\frac{dP}{dt} = 0.09P$, where P is the size of the population t years after observation commenced. When observation commenced, the size of the population was 350 000.

(a) Determine an expression for P in terms of t. (1 mark)

$P_t = 350\,000\, e^{0.09t}$ ✓

(b) How long will it take for the population to reach 1 000 000? (2 marks)

$1\,000\,000 = 350\,000\, e^{0.09t}$ ✓ [Equ]

$t = 11.66469$ yrs

$\therefore t > 11.665$ ✓ [Must be $1\,000\,000^+$]

(c) A second population is increasing at the same rate as that from part (a), however the initial population is unknown. How long will it take for this population to double in size. (2 marks)

$2P_0 = P_0 e^{0.09t}$ ✓ [Equ]

$2 = e^{0.09t}$

$t = 7.70163$ yrs

$\therefore t > 7.702$ yrs. ✓ [Answ]

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MATHEMATICS METHODS Year 12
Section Two: Calculator-assumed

Your name _____

Teacher's name _____

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See next page

Question 5 **(6 marks)**

$X \sim B\,[n, p]$, with $Var(X) = 1.5$ and $E(X) = 2$. (i.e. A BINOMIAL DISTRIBUTION)

(a) Determine the value of n and p. (4 marks)

[Var equ] ✓ [E(X) equ] ✓

$VAR(X): \quad np(1-p) = 1.5$, $E(X): \quad np = 2$

$2(1-p) = 1.5$

$p = 0.25$ ✓ $\quad n = 8$ ✓

(b) Determine an expression for $P(X \geq 1)$. Do not simplify. (2 marks)

$= 1 - 0.75^8$ ✓✓ (0.899887)

OR $\sum_{x=1}^{8} {}^8C_x (0.25)^x (0.75)^{8-x}$

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See next page

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Question 5 **(6 marks)**

Given that $X \sim B\,[n, p]$, (i.e. a Binomial Distribution) with $Var(X) = 1.5$ and $E(X) = 2$.

(a) Determine the value of n and p. (4 marks)

(b) Determine an expression for $P(X \geq 1)$. Do not simplify your answer. (2 marks)

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Question 6 **(5 marks)**

A population grows continuously such that $\frac{dP}{dt} = 0.09P$, where P is the size of the population t years after observation commenced. When observation commenced, the size of the population was $350\,000$.

(a) Determine an expression for P in terms of t. (1 mark)

(b) How long will it take for the population to reach $1\,000\,000$? (2 marks)

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Question 4 **(3 marks)**

Determine the x coordinates of all stationary points of the function:

$$f(x) = \int_0^{4x} e^{t^2} - e\, dt$$

$f'(x) = [e^{(4x)^2} - e][4]$ ✓ [FTC]

$f'(x) = 4(e^{16x^2} - e)$

Stat pt $f'(x) = 0$

$4(e^{16x^2} - e) = 0$ ✓ [$f'(x) = 0$]

$e^{16x^2} - e^1 = 0$

$e^{16x^2} = e^1$

$\therefore 16x^2 = 1$

$x^2 = \frac{1}{16}$

$x = \pm\frac{1}{4}$ ✓ [Both ANSW]

3

End of questions

Question 7 **(4 marks)**

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(a) the exact value of t when the instantaneous rate of change of C with respect to t is 0. (2 marks)

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Consider the discrete probability distribution shown below.

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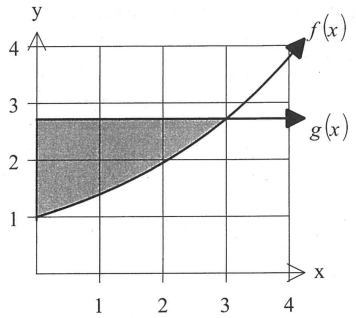

Determine the area bound by the two curves and the y-axis.

$\int_0^3 e - e^{x/3}\,dx$ ✓ [correct integral]

$= \left[ex - 3e^{x/3}\right]_0^3$

$= 3e - 3e^{1} - (e \times 0 - 3e^{0})$ ✓ [sub values]

$= 3e - 3e + 3e^{0}$

$= 3 \text{ units}^2$ ✓ [ANSW]

3

Question 2

Given $\frac{d}{dx}(xe^x - e^x) = xe^x$, determine exactly $\int_0^1 (xe^x + x^2)dx$. (3 marks)

$\therefore \int xe^x dx = xe^x - e^x + C$

So $\int_0^1 xe^x dx + \int_0^1 x^2 dx$ ✓ [Splits Integral]

$= \left[xe^x - e^x\right]_0^1 + \left[\frac{x^3}{3}\right]_0^1$ ✓ [uses given info]

$= e^1 - e^1 - [0 - e^0] + \frac{1}{3} - 0$

$= 1 + \frac{1}{3}$

$= 1\frac{1}{3}$ ✓ (ANSW)

3

Question 8 continued

(c) Given that $SD(X) = 1.6$, determine

(i) $SD(3X)$ (1 mark)

(ii) $Var(3X)$ (1 mark)

Question 9 **(6 marks)**

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(b) What is the largest number of cars that can be selected in a random sample such that the probability of there being at least 1 defective car is less than 20%? (2 marks)

End of questions

Question 1 **(6 marks)**

Evaluate the following.

(a) $\int_1^2 \frac{d}{dx} x^2 e^{2x}\, dx$ (2 marks)

$= \left[x^2 e^{2x}\right]_1^2$ ✓ [Recognises FTC]

$= 4e^4 - e^2$ ✓ [ANSW]

OR $= e^2(4e^2 - 1)$

OR $= e^2(2e-1)(2e+1)$

(b) $\frac{d}{dx}(\sin(2x) + e^{4x^2})$ (2 marks)

$= 2\cos(2x) + 8x e^{4x^2}$

✓ ✓ [Each part]

(c) $\int -2xe^{2x^2} dx$ (2 marks)

$= -\frac{1}{2}\int 4x e^{2x^2} dx$

$= -\frac{1}{2} e^{2x^2} + c$ ✓ (ANSW) ✓ (+C)

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Additional working space

Question number: _____

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Calculator-free

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