MATHEMATICS METHODS

MAWA Semester 1 (Unit 3) Examination 2019
Calculator-free

Marking Key

The release date for this exam and marking scheme is 14th June.

Section One: Calculator-free (50 Marks)

Question 1(a) (2 marks)

Solution	
$Let\ f(x) = xe^{3x}$ $f'(x) = e^{3x} + x3e^{3x} = e^{3x} + 3xe^{3x}$	
Mathematical behaviours	Mark
• applies product rule	1
• differentiates exponential correctly	1

Question 1(b) (3 marks)

Solution	
$\frac{d}{dx}\left(\frac{\cos x}{x^3}\right) = \frac{x^3(-\sin x) - \cos x(3x^2)}{(x^3)^2} = \frac{-x^2(x\sin x + 3\cos x)}{x^6} = -\frac{(x\sin x + 3\cos x)}{x^4}$	
Mathematical behaviours	Marks
• applies quotient rule	1
• differentiates $\cos x$ correctly	1
• simplifies result	1

Question 1(c) (3 marks)

Solution	
$g(u) = \sqrt{u} \Rightarrow \frac{dg}{du} = \frac{1}{2}u^{-\frac{1}{2}}$ $\quad u = 2 - 3x^2 \Rightarrow \frac{du}{dx} = -6x$ $\Rightarrow \frac{dg}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \times -6x = \frac{-3x}{\sqrt{2-3x^2}}$	
Mathematical behaviours	Marks
• states $\frac{dg}{du}$	1
• states $\frac{du}{dx}$	1
• states $\frac{dg}{dx}$ in terms of x.	1

Question 1(d) (3 marks)

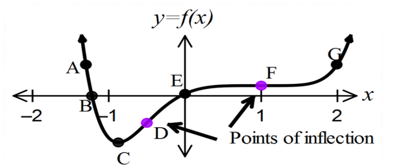
Solution	
$x(t) = 3\sin 2t \Rightarrow v(t) = 3\times 2\cos 2t$ $v(t) = 0 \Rightarrow \cos 2t = 0$ $ie\ 2t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{4}s$	
Mathematical behaviours	Marks
• differentiates to obtain $v(t)$	1
• equates $v(t) = 0$	1 1

• determines t value	

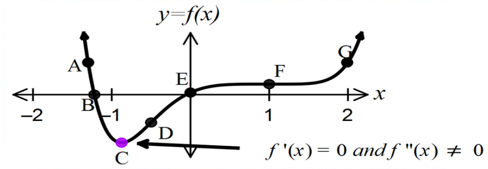
Question 2 (a) **(4 marks)**

Solution

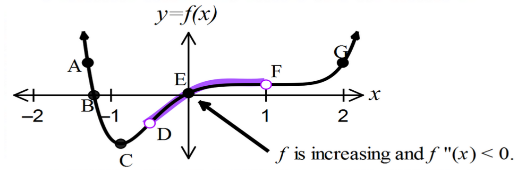
(i)



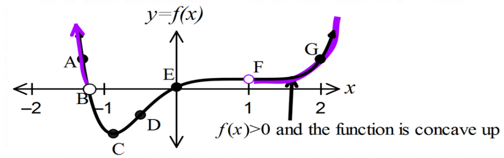
(ii)



(iii)



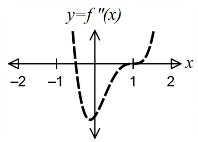
(iv)



Mathematical behaviours	Mark
(i)	
• states D and F	1
(ii)	
• states C	1
(iii)	
• states E	1
(iv)	
• states A and G	1

Question 2(b) **(1 mark)**

Solution



Mathematical behaviours	Marks

• circles the 2nd graph	1

Question 3 **(4 marks)**

Solution

Area = $\int_0^{2\pi}\left[\sin x - x(x-2\pi)\right]dx$

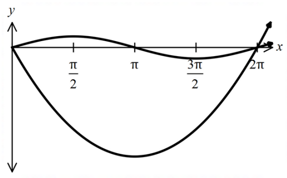
$= \int_0^{2\pi}(\sin x - x^2 + 2\pi x)\,dx$

$= \left[-\cos x - \frac{x^3}{3} + \pi x^2\right]_0^{2\pi}$

$= \left[-\cos 2\pi - \frac{(2\pi)^3}{3} + \pi(2\pi)^2\right] - \left[-\cos 0\right]$

$= -1 - \frac{8\pi^3}{3} + 4\pi^3 + 1$

$= \frac{4\pi^3}{3}$



Mathematical behaviours	Marks
• states a correct expression using integrals to determine the area	1
• anti-differentiates each part correctly	1
• substitutes in limits of integration	1
• evaluates result	1

• rearranges to get required result	1

Question 7(c) **(3 marks)**

Solution

$\int_0^{\frac{\pi}{6}}(\sin x\cos x + 2)\,dx = \left[\frac{1}{2}\sin^2 x + 2x\right]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\left[\left(\frac{1}{2}\right)^2 - 0^2\right] + 2\left[\frac{\pi}{6} - 0\right]$

$= \frac{1}{8} + \frac{\pi}{3}$

Mathematical behaviours	Marks
• recognises $\sin^2 x$ term is to be involved	1
• states correct integral and bounds of integration	1
• substitutes bounds of integration and simplifies	1

• states the sum of the area of the three rectangles and simplifies correctly	1

Question 6(b) **(2 marks)**

Solution	
Using trapeziums is equivalent to averaging the results from part (a) i.e. Estimated area under $f(x)=\sin x$ from $x=0$ to $x=\frac{\pi}{2}$ is $\left[\frac{\pi}{6}\left(\frac{3+\sqrt{3}}{2}\right)+\frac{\pi}{6}\left(\frac{1+\sqrt{3}}{2}\right)\right]\div 2$ $=\left[\frac{\pi}{6}\left(\frac{4}{2}+\frac{2\sqrt{3}}{2}\right)\right]\div 2$ $=\left[\frac{\pi}{6}\left(2+\sqrt{3}\right)\right]\div 2$ $=\frac{\pi}{6}\left(\frac{2+\sqrt{3}}{2}\right)$	
Mathematical behaviours	Marks
• determines the average of the two areas obtained in part (a) • simplifies to deduce the required result	1 1

Question 7(a) **(1 mark)**

Solution	
$y=\sin^2 x$ $\frac{dy}{dx}=2\sin x\cos x$	
Mathematical behaviours	Mark
• States correct answer	1

Question 7(b) **(3 marks)**

Solution	
$y=\sin^2 x$ $\frac{dy}{dx}=2\sin x\cos x$ $\int\frac{dy}{dx}dx=\int 2\sin x\cos x\,dx$ $y=\int 2\sin x\cos x\,dx+c$ $\sin^2 x=\int 2\sin x\cos x\,dx+c$ $\int\sin x\cos x\,dx=\frac{1}{2}\sin^2 x+c$	
Mathematical behaviours	Marks
• integrates both sides of equation • applies fundamental theorem	1 1

Question 4(a) **(2 marks)**

Solution	
$\int\left(2e^{2x}-\frac{3}{\sqrt{x}}\right)dx=\int\left(2e^{2x}-3x^{\frac{-1}{2}}\right)dx$ $=2\frac{e^{2x}}{2}-3\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ $=e^{2x}-6\sqrt{x}+c$	
Mathematical behaviours	Marks
• anti-differentiates the exponential function correctly • anti-differentiates the square root function correctly	1 1

Question 4(b) **(2 marks)**

Solution	
$\int_0^1(3-2x)^2\,dx$ $=\left[\frac{(3-2x)^3}{3\times(-2)}\right]_0^1$ $=\frac{-1}{6}\left(1^3-3^3\right)$ $=\frac{26}{6}$ $=\frac{13}{3}$	
Mathematical behaviours	Marks
• aniti-differentiates correctly • substitutes limits of integration and evaluates	1 1

Question 4(c) **(2 marks)**

Solution	
$F(x)=\int_x^1\frac{dt}{1+\sqrt{1-t}}$ $=-\int_1^x\frac{dt}{1+\sqrt{1-t}}$ $\therefore F'(x)=-\frac{1}{1+\sqrt{1-x}}$	
Mathematical behaviours	Marks
• uses the relationship $\int_x^1\frac{dt}{1+\sqrt{1-t}}=-\int_1^x\frac{dt}{1+\sqrt{1-t}}$ • applies Fundamental Theorem of Calculus	1 1

Question 4(d) **(3 marks)**

Solution	
$\int_{-m}^{m}(m^3 - x^3)\,dx = 1250$ $\left[m^3x - \frac{x^4}{4}\right]_{-m}^{m} = 1250$ $\left[m^4 - \frac{m^4}{4}\right] - \left[-m^4 - \frac{m^4}{4}\right] = 1250$ $\frac{3m^4}{4} + \frac{5m^4}{4} = 1250$ $\frac{8m^4}{4} = 1250$ $2m^4 = 1250$ $m^4 = 625$ $m = \pm 5$	
Mathematical behaviours	**Marks**
• anti-differentiates integral correctly	1
• substitutes in limits of integration correctly and simplifies to obtain correct expression on the LHS	1
• determines correct answers for m.	1

Question 5(a) **(3 marks)**

Solution	
Bernoulli distribution with $\mu = \frac{1}{36},\ \sigma^2 = \frac{1}{36} \times \frac{35}{36} = \frac{35}{36^2}$	
Mathematical behaviours	**Marks**
• states Bernoulli	1
• states mean	1
• states variance	1

Question 5(b) **(3 marks)**

Solution	
This represents a Binomial with $n = 15$ and $p = \frac{1}{36}$.	
Mathematical behaviours	**Marks**
• states Binomial	1
• states n	1
• states p	1

Question 5(c) **(1 mark)**

Solution	
$W \sim Bin\left(15, \frac{1}{36}\right)$ $P(W = 1) = {}^{15}C_1 \left(\frac{1}{36}\right)^1 \left(\frac{35}{36}\right)^{14} = 15\left(\frac{1}{36}\right)^1 \left(\frac{35}{36}\right)^{14}$	
Mathematical behaviours	**Marks**
• states correct expression	1

Question 5(d) **(3 marks)**

Solution	
$Let\ Z \sim Bin\left(30, \frac{1}{36}\right)$ $P(Z = 2 \mid Z \geq 1) = \frac{P(Z = 2)}{P(Z \geq 1)} = \frac{P(Z = 2)}{1 - P(Z = 0)} = \frac{{}^{30}C_2 \left(\frac{1}{36}\right)^2 \left(\frac{35}{36}\right)^{28}}{1 - {}^{30}C_0 \left(\frac{1}{36}\right)^0 \left(\frac{35}{36}\right)^{30}}$	
Mathematical behaviours	**Marks**
• recognises the situation involves a binomial $(30, \frac{1}{36})$ and conditional probability	1
• states correct expression for numerator	1
• states correct expression for denominator	1

Question 6(a) **(1 mark)**

Solution	
(i) Under-estimated Area = $\left(\frac{\pi}{6} \times \frac{1}{2}\right) + \left(\frac{\pi}{6} \times \frac{\sqrt{3}}{2}\right)$ $= \frac{\pi}{6}\left(\frac{1 + \sqrt{3}}{2}\right)$ (ii) Over-estimated Area = $\left(\frac{\pi}{6} \times \frac{1}{2}\right) + \left(\frac{\pi}{6} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{\pi}{6} \times 1\right)$ $= \frac{\pi}{6}\left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 1\right)$ $= \frac{\pi}{6}\left(\frac{3 + \sqrt{3}}{2}\right)$	
Mathematical behaviours	**Marks**
(i) • states the sum of the area of the two rectangles and simplifies correctly (ii)	1