SCHOOL

Semester One Examination, 2013

Question/Answer Booklet

MATHEMATICS 3C
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	49	33
Section Two: Calculator-assumed	13	13	100	100	67
			Total	149	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

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Section One: Calculator-free **(49 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

Find the area of the region trapped between the line $y = 2$ and the curve $y = x^2 - 4x + 5$.

$$x^2 - 4x + 5 = 2$$
$$x^2 - 4x + 3 = 0$$
$$(x - 1)(x - 3) = 0$$
$$x = 1,\ 3$$

$$\int_1^3 2 - (x^2 - 4x + 5)\,dx = \int_1^3 4x - x^2 - 3\,dx$$
$$= \left[2x^2 - \frac{x^3}{3} - 3x \right]_1^3$$
$$= \left[18 - \frac{27}{3} - 9 \right] - \left[2 - \frac{1}{3} - 3 \right]$$
$$= 10 - \frac{26}{3}$$
$$= \frac{4}{3} \text{ sq units}$$

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Question 2 **(7 marks)**

(a) Determine $\frac{dy}{dx}$ for each of the following. Do not simplify your answers.

(i) $y = \frac{5x^2}{3x + 2}$ (2 marks)

$$\frac{dy}{dx} = \frac{10x(3x + 2) - 3(5x^2)}{(3x + 2)^2}$$

(ii) $y = (x^2 - 4)^3$ (2 marks)

$$\frac{dy}{dx} = 3 \times 2x(x^2 - 4)^2$$

(b) Find the coordinates of the point on the curve $y = x^3 - 2x^2 - 5x + 1$ where $\frac{d^2y}{dx^2} = 2$. (3 marks)

$$\frac{dy}{dx} = 3x^2 - 4x - 5$$

$$\frac{d^2y}{dx^2} = 6x - 4$$

$$2 = 6x - 4$$

$$x = 1$$

$$y = 1 - 2 - 5 + 1 = -5$$

At the point (1, -5)

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Question 3 **(4 marks)**

Find the global minimum and maximum values of the function $f(x) = \frac{8}{x^2} + 2x$ over the interval $1 \leq x \leq 4$.

$$f'(x) = \frac{-16}{x^3} + 2$$

$$0 = \frac{-16}{x^3} + 2$$

$$x^3 = 8$$

$$x = 2$$

$$f(1) = 10$$

$$f(2) = 6$$

$$f(4) = 8.5$$

Min value is 6 and max value is 10

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Question 4 **(7 marks)**

Let $f(x) = x(x+1)$ and $g(x) = 5x - 1$.

(a) State the domain of $f(x)$. (1 mark)

Domain: All real x

(b) For what value(s) of x does $f \circ f(x) = f(x)$? (3 marks)

$$f(x)(f(x)+1) = f(x)$$
$$(f(x))^2 + f(x) = f(x)$$
$$(f(x))^2 = 0$$
$$f(x) = 0$$
$$x(x+1) = 0$$
$$x = 0,\ x = -1$$

(c) Determine the range of $f \circ g(x)$. (4 marks)

$f \circ g(x) = (5x - 1)(5x - 1 + 1)$

$= 25x^2 - 5x$

$f \circ g(x)$ is parabolic with minimum:

$\frac{dfg(x)}{dx} = 50x - 5$

$= 0$ when $x = 0.1$

$fg(0.1) = 0.25 - 0.5$

$= -0.25$

Range: $y \geq -0.25$

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Question 5 **(9 marks)**

(a) Differentiate $y = (x - 1)(x^2 + 1)^3$ with respect to x, expressing your answer as a product of quadratic factors. (4 marks)

$$\begin{aligned}\frac{dy}{dx} &= (1)(x^2 + 1)^3 + (x - 1)(6x)(x^2 + 1)^2 \\ &= (x^2 + 1)(x^2 + 1)^2 + (6x^2 - 6x)(x^2 + 1)^2 \\ &= (7x^2 - 6x + 1)(x^2 + 1)^2\end{aligned}$$

(b) Find the equation of the tangent to the graph of $y = \frac{e^{x^2-1}}{x^3+2}$ at the point where $x = -1$.

(5 marks)

$$y = \left.\frac{e^{x^2-1}}{x^3+2}\right|_{x=-1}$$
$$= 1$$

$$\frac{dy}{dx} = \left.\frac{2xe^{x^2-1} \times (x^3+2) - 3x^2 \times e^{x^2-1}}{(x^3+2)^2}\right|_{x=-1}$$
$$= \frac{(-2) \times (1) - 3}{1}$$
$$= -5$$

$$y - 1 = -5(x - (-1))$$
$$y = -5x - 4$$

Question 6 **(10 marks)**

(a) If $f'(x) = 20(1-x)^3$, and $f(2) = 5$, determine $f(3)$. (3 marks)

$$f(x) = \frac{20(1-x)^4}{(-1)(4)} + c$$
$$= -5(1-x)^4 + c$$

$$f(2) = -5(1-2)^4 + c = 5$$
$$-5 + c = 5 \Rightarrow c = 10$$

$$f(3) = -5(1-3)^4 + 10$$
$$= -70$$

(b) Evaluate $\int_1^2 x - \frac{3}{x^3}\,dx$. (3 marks)

$$\left[\frac{x^2}{2} + \frac{3}{2x^2}\right]_1^2 = \left(2 + \frac{3}{8}\right) - \left(\frac{1}{2} + \frac{3}{2}\right)$$
$$= \frac{3}{8}$$

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(c) Consider two polynomial functions $g(x)$ and $h(x)$ for which the following is known:

$$\int_1^1 g(x)\,dx = 15 \qquad \int_1^1 (g(x)+h(x))\,dx = 12$$

$$\int_1^2 h(x)\,dx = 2 \qquad \int_1^2 (g(x)+h(x))\,dx = -7$$

Determine:

(i) $\int_1^1 h(x)\,dx$ (1 mark)

$$\int_1^1 (g+h) = \int_1^1 (g) + \int_1^1 (h)$$
$$12 = 15 + \int_1^1 (h)$$
$$\int_1^1 (h) = -3$$

(ii) $\int_1^2 g(x)\,dx$ (3 marks)

$$\int_1^1 (g+h) + \int_1^2 (g+h) = \int_1^2 (g+h)$$
$$12 + \int_1^2 (g+h) = -7$$
$$\int_1^2 (g+h) = -19$$

$$\int_1^2 (h) = \int_1^1 (h) + \int_1^2 (h)$$
$$\int_1^2 (h) = -3 + 2$$
$$= -1$$

$$\int_1^2 (g) + \int_1^2 (h) = \int_1^2 (g+h)$$
$$\int_1^2 (g) + (-1) = -7$$
$$\int_1^2 (g) = -6$$

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Question 7 **(7 marks)**

(a) Write as a single fraction $1+\frac{2}{3x}+\frac{4}{x^2}$ (2 marks)

$$1+\frac{2}{3x}+\frac{4}{x^2}=\frac{3x^2}{3x^2}+\frac{2x}{3x^2}+\frac{12}{3x^2}$$

$$=\frac{3x^2+2x+12}{3x^2}$$

(b)

(i) Show that $\frac{3-2x}{4x^2-4x+1}+\frac{1}{2x-1}$ can be written as $\frac{2}{(1-2x)^2}$. (2 marks)

$$\frac{3-2x}{1-4x+4x^2}+\frac{1}{2x-1}=\frac{3-2x}{(2x-1)^2}+\frac{2x-1}{(2x-1)^2}$$

$$=\frac{2}{(2x-1)^2}$$

(ii) Given that $\int\frac{3-2x}{4x^2-4x+1}+\frac{1}{2x-1}\,dx=\frac{a}{b-cx}+k$, find the values of the **positive** constants a, b and c. (3 marks)

$$\int\frac{3-2x}{4x^2-4x+1}+\frac{1}{2x-1}\,dx=\int 2(2x-1)^{-2}dx$$

$$=\frac{2(2x-1)^{-1}}{-1\times 2}+k$$

$$=\frac{-1}{2x-1}+k$$

$$=\frac{1}{1-2x}+k$$

$a=1$

$b=1$

$c=2$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____

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