Rossmoyne Senior High School

Year 11 Examination, 2014

Question/Answer Booklet

MATHEMATICS:
SPECIALIST 3A/3B
Section Two:
Calculator-assumed

SOLUTIONS

Student Number:	In figures	
	In words	______________________
	Your name	______________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33⅓
Section Two: Calculator-assumed	12	12	100	100	66⅔
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

Section Two: Calculator-assumed **(100 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(8 marks)**

The latitude and longitude of three cities are given in this table, to the nearest degree.

City	*Latitude*	*Longitude*
Jiujiang (China)	30°N	116°E
Perth (Australia)	32°S	116°E
Austin (USA)	30°N	98°W

Assume the radius of the earth is 6350 km.

(a) Calculate the distance between Perth and Jiujiang along their common line of longitude. (2 marks)

$$D = \frac{2\pi \times 6350 \times (30 + 32)}{360}$$
$$= 6871 \text{ km}$$

(b) Calculate the distance between Jiujiang and Austin along their common line of latitude. (3 marks)

$$D = \frac{2\pi \times 6350 \times (360 - (116 + 98))}{360} \times \cos(30)$$
$$= 14013 \text{ km}$$

(c) The town of Forster, on the east coast of Australia, is 3570 km away from Perth and on the same line of latitude. Determine the longitude of Forster, to the nearest degree. (3 marks)

$$3570 = \frac{2\pi \times 6350 \times \theta}{360} \times \cos(32)$$
$$\theta = 37.98^\circ$$

Longitude is $116 + 38 = 154^\circ$ E

Question 9 **(7 marks)**

Two vectors are given by $\mathbf{a} = -3\mathbf{i} + (k-2)\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} + (k+2)\mathbf{j}$.

(a) Determine the value(s) of k if $\mathbf{a}$ and $\mathbf{b}$ are

(i) parallel. (2 marks)

$$\frac{-3}{4} = \frac{k-2}{k+2}$$
$$-3k - 6 = 4k - 8$$
$$7k = 2$$
$$k = \tfrac{2}{7}$$

(ii) perpendicular. (3 marks)

$$\begin{bmatrix} -3 \\ k-2 \end{bmatrix} \bullet \begin{bmatrix} 4 \\ k+2 \end{bmatrix} = 0$$
$$-12 + (k-2)(k+2) = 0$$
$$k^2 = 16$$
$$k = \pm 4$$

(b) If $k = 3$, determine the angle between $\mathbf{a}$ and $\mathbf{b}$ to the nearest degree. (2 marks)

$\mathbf{a} = -3\mathbf{i} + \mathbf{j}$

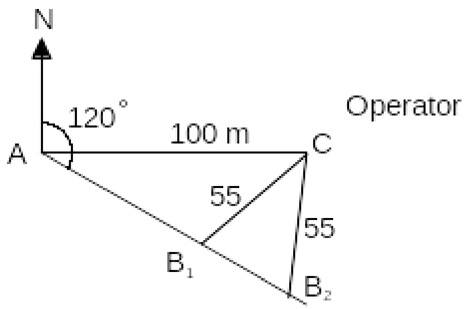
$\mathbf{b} = 4\mathbf{i} + 5\mathbf{j}$

Using CAS, angle is 110°

Question 10 **(7 marks)**

A small radio controlled boat leaves point A on a river bank and heads off at a constant speed on a bearing of 120°. The operator is standing 100 metres due east of point A and notes that after 45 seconds, the boat is 55 m away from her position.

(a) Use the above information to complete the diagram below. (1 mark)



(b) Calculate the two possible distances travelled by the boat in the 45 seconds. (4 marks)

$$\frac{\sin B}{100} = \frac{\sin 30}{55}$$

$B = 65.4^\circ$ or 114.6°, $C = 84.6^\circ$ or 35.4°

$$AC = \sqrt{100^2 + 55^2 - 2(100)(55)\cos C}$$

$AC = 63.7$ or 109.5 metres

At this time, after 45 seconds, the operator turns the boat so that it heads directly towards their position without changing its speed.

(c) Determine the minimum possible time that the boat will take to reach the operator. (2 marks)

Maximum speed is $109.5 \div 45 = 2.43$ m/s.

Minimum time is $55 \div 2.43 \approx 22.6$ seconds.

Question 11 **(9 marks)**

$A(2, 3)$, $B(1, -2)$ and $C(-3, 1)$ are the vertices of a triangle.

(a) State the vector $\overrightarrow{AC}$. (1 mark)

$$\overrightarrow{AC} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}$$

(b) Determine the exact value of $|\overrightarrow{BC}|$. (2 marks)

$$\overrightarrow{BC} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$$|\overrightarrow{BC}| = \sqrt{4^2 + 3^2}$$
$$= 5$$

(c) Determine the vector equation of the line

(i) through A parallel to $\overrightarrow{BC}$. (2 marks)

$$\mathbf{r} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

(ii) through B perpendicular to $\overrightarrow{AC}$. (2 marks)

$$\mathbf{r} \bullet \begin{bmatrix} -5 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \bullet \begin{bmatrix} -5 \\ -2 \end{bmatrix}$$

$$\mathbf{r} \bullet \begin{bmatrix} -5 \\ -2 \end{bmatrix} = -1$$

(d) A circle with centre at C passes through (0, 0). Determine the vector equation of this circle. (2 marks)

$$|\overrightarrow{OC}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$\left| \mathbf{r} - \begin{bmatrix} -3 \\ 1 \end{bmatrix} \right| = \sqrt{10}$$

Question 12 **(7 marks)**

Point A has polar coordinates $[4,\ 135^\circ]$ and point B has Cartesian coordinates $\left(-3, -3\sqrt{3}\right)$.

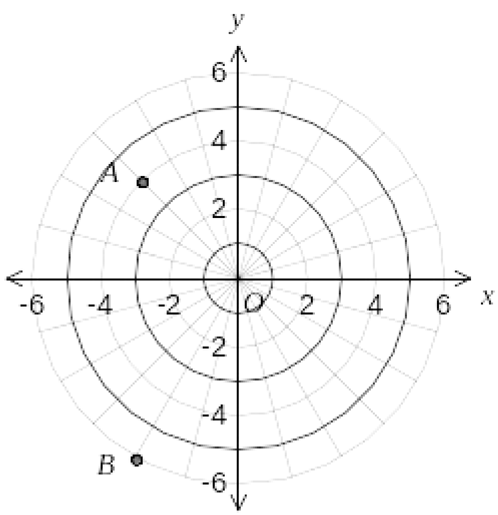
(a) Convert the polar coordinates of point A into exact Cartesian coordinates. (1 mark)

$$\boxed{\left(-2\sqrt{2},\ 2\sqrt{2}\right)}$$

(b) Convert the Cartesian coordinates of point B into polar coordinates $[r,\ \theta]$, where $r \geq 0$ and $0 \leq \theta \leq 360^\circ$. (1 mark)

$$\boxed{[6,\ 240^\circ]}$$

(c) Plot the points A and B on the axes below. (2 marks)



(d) If O is the origin, determine

(i) the size of $\angle AOB$. (1 mark)

$$\boxed{240 - 135 = 105^\circ}$$

(ii) the length AB. (1 mark)

$$\boxed{\begin{aligned} AB &= \sqrt{4^2 + 6^2 - 2(4)(6)\cos 105^\circ} \\ &= 8.03 \text{ units} \end{aligned}}$$

(iii) the area of the triangle AOB. (1 mark)

$$\boxed{0.5(4)(6)\sin 105 = 11.6 \text{ square units}}$$

Question 13 **(10 marks)**

The area of an oil slick around a ship, $A(t)$ in square metres, t minutes after the rupture of the boats fuel tank, was modelled by

$$A(t) = \begin{cases} 6(1.15^t - 1) & 0 \le t \le 30 \\ 650 - 6100(0.9)^t & t > 30 \end{cases}$$

30 minutes after the tank was ruptured, the crew of the ship took steps to stem the fuel leakage.

(a) Determine the area of the oil slick after

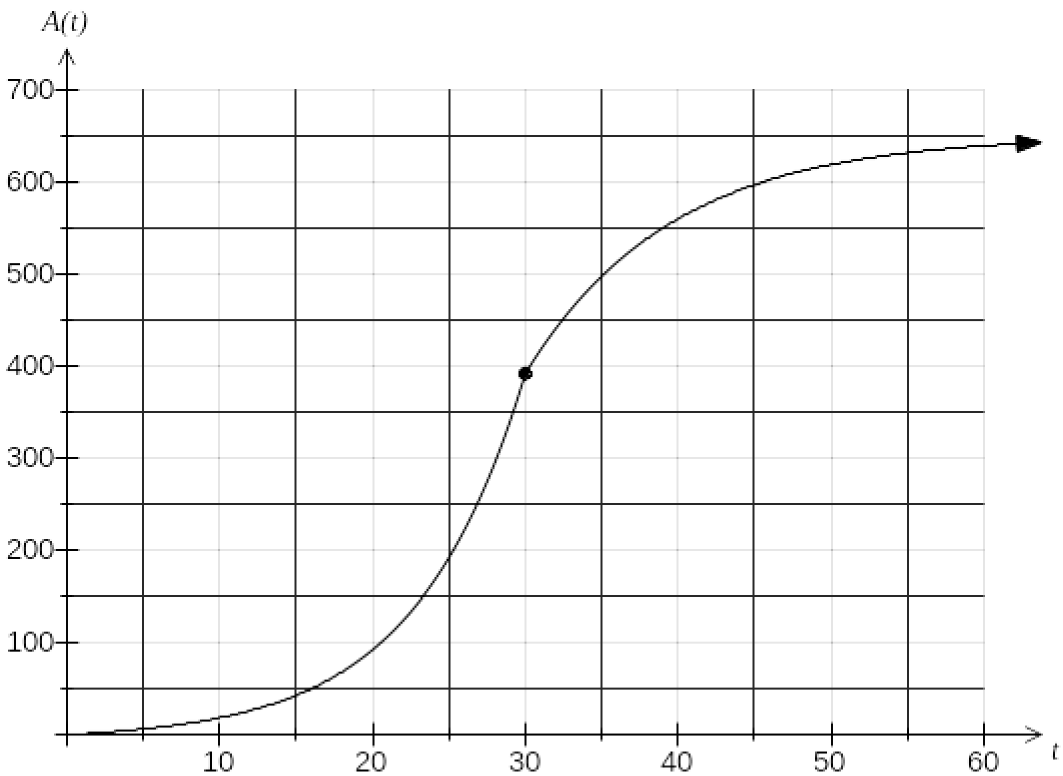
(i) 30 minutes. (1 mark)

391 m^2

(ii) 60 minutes. (1 mark)

639 m^2

(b) Sketch the graph of $A(t)$ on the axes below. (3 marks)



(c) Determine the time taken for the area of the slick to treble in size from 200 m^2 to 600 m^2, giving your answer in minutes to one decimal place. (3 marks)

$$6(1.15^t - 1) = 200$$
$$t = 25.3$$

$$650 - 6100(0.9)^t = 600$$
$$t = 45.6$$

$$\Delta t \approx 20.3 \text{ minutes}$$

(d) Comment on how the size of the oil slick is changing several hours after the initial rupture. (2 marks)

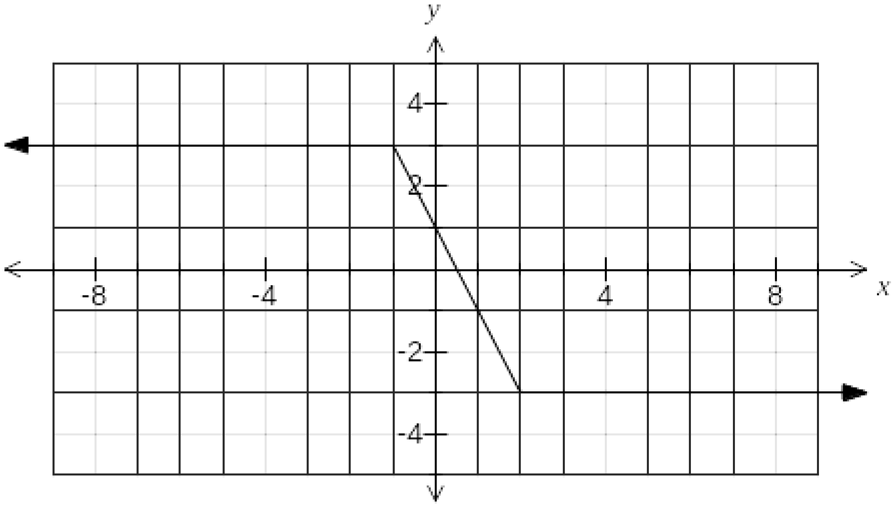
The slick is increasing at a decreasing rate.

As t increases, the size of the slick is tending to 650 m^2 because $6100(0.9)^t \to 0$

Question 14 **(11 marks)**

Consider the function $f(x) = |x - 2| - |x + 1|$.

(a) Draw the graph of $y = f(x)$ on the axes below. (3 marks)



(b) Write a piecewise definition of $f(x)$. (3 marks)

$$f(x) = \begin{cases} 3 & x \le -1 \\ 1 - 2x & -1 < x < 2 \\ -3 & x \ge 2 \end{cases}$$

Let $g(x) = |x + a| - |x + b|$, where $b > a$.

(c) For which values of x is $g(x)$ constant? (2 marks)

$x \ge -a$ or $x \le -b$

(d) Write a piecewise definition of $g(x)$. (3 marks)

$$g(x) = \begin{cases} b - a & x \le -b \\ -2x - a - b & -b < x < -a \\ a - b & x \ge -a \end{cases}$$

Question 15 **(6 marks)**

A function is defined as $f(x)=\begin{cases} a(1-x)+\dfrac{e^x}{4} & x\le 0 \\ \dfrac{b}{x+2} & x>0 \end{cases}$.

Determine the constants a and b if $f'(x)$ is continuous and differentiable everywhere.

For continuity at $x=0 \Rightarrow \; a+\frac{1}{4}=\frac{b}{2}$ ...(1)

$$f'(x)=\begin{cases} -a+\dfrac{e^x}{4} & x\le 0 \\ \dfrac{-b}{(x+2)^2} & x>0 \end{cases}$$

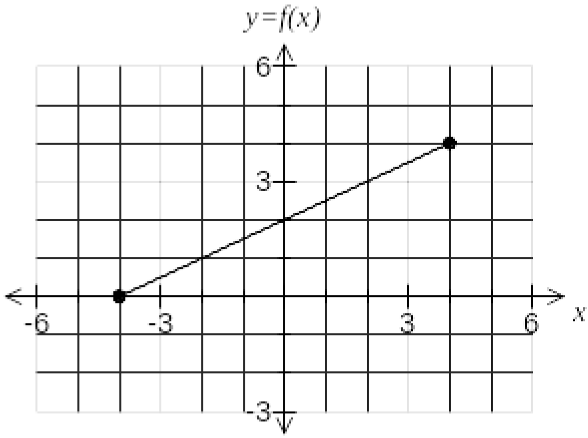
For differentiablity at $x=0 \Rightarrow \; -a+\frac{1}{4}=-\frac{b}{4}$ ...(2)

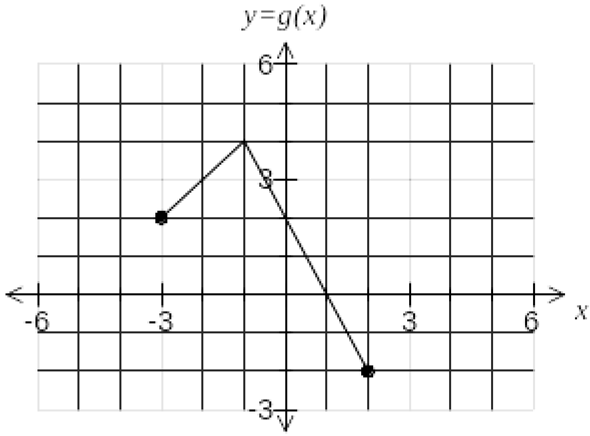
Solving (1) and (2) simultaneously:

$a=\frac{3}{4},\; b=2$

Question 16 **(10 marks)**

The graphs of $y = f(x)$ and $y = g(x)$ are shown below over their respective domains.





(a) Determine

(i) $f(2)$. (1 mark)

$\boxed{3}$

(ii) $(f \circ g)(2)$. (2 marks)

$g(2) = -2,\ f(-2) = 1$

$(f \circ g)(2) = 1$

(b) Determine

(i) the range of $g(x)$. (1 mark)

$\boxed{y : -2 \le y \le 4}$

(ii) the domain for which $g \circ f(x)$ is defined. (2 marks)

Range of $f(x)$ must be restricted to be within domain of $g(x)$:

$x : -4 \le x \le 0$

(c) The defining rule for $(f \circ g)(x) = \begin{cases} ax + b & -3 \le x \le -1 \\ cx + d & -1 < x \le 2 \end{cases}$.

Determine the values of $a,\ b,\ c$ and d. (4 marks)

$$f(x) = 2 + 0.5x, \quad -4 \le x < 4$$

$$g(x) = \begin{cases} x + 5 & -3 \le x \le -1 \\ 2 - 2x & -1 < x \le 2 \end{cases}$$

$$f \circ g(x) = \begin{cases} 2 + 0.5(x+5) & -3 \le x \le -1 \\ 2 + 0.5(2 - 2x) & -1 < x \le 2 \end{cases}$$

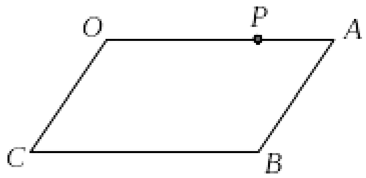
$$= \begin{cases} 0.5x + 4.5 & -3 \le x \le -1 \\ 3 - x & -1 < x \le 2 \end{cases}$$

$a = 0.5$
$b = 4.5$
$c = -1$
$d = 3$

Question 17 **(8 marks)**

$OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

P is the point on side OA such that $OP : PA = 3:1$.



(a) Express in terms of $\mathbf{a}$ and $\mathbf{c}$:

(i) $\overrightarrow{OB}$. (1 mark)

$\boxed{\overrightarrow{OB} = \mathbf{a} + \mathbf{c}}$

(ii) $\overrightarrow{OP}$. (1 mark)

$\boxed{\overrightarrow{OP} = \frac{3}{4}\mathbf{a}}$

(iii) $\overrightarrow{CP}$. (1 mark)

$\boxed{\overrightarrow{CP} = \frac{3}{4}\mathbf{a} - \mathbf{c}}$

N is the point on CP such that O, N and B are collinear.

(b) If $\overrightarrow{ON} = k \cdot \overrightarrow{OB}$ and $\overrightarrow{CN} = h \cdot \overrightarrow{CP}$, use the fact that $\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN}$ to determine the values of h and k. (5 marks)

$$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN}$$
$$k \cdot \overrightarrow{OB} = \mathbf{c} + h \cdot \overrightarrow{CP}$$
$$k(\mathbf{a} + \mathbf{c}) = \mathbf{c} + h(\tfrac{3}{4}\mathbf{a} - \mathbf{c})$$

Consider $\mathbf{a}$ coefficients: $k = \frac{3}{4}h$

Consider $\mathbf{c}$ coefficients: $k = 1 - h$

Solving simultaneously gives: $k = \frac{3}{7}$, $h = \frac{4}{7}$

Question 18 **(7 marks)**

Consider the number patterns below.

n	$A(n)$	$B(n)$	$C(n) = A(n) \times B(n)$
10	$\left(1+\frac{2}{10}\right)^{10} \approx 6.192$	$\left(1-\frac{2}{10}\right)^{10} \approx 0.107$	0.665
100	$\left(1+\frac{2}{100}\right)^{100} \approx 7.245$	$\left(1-\frac{2}{100}\right)^{100} \approx 0.133$	0.961
1000	$\left(1+\frac{2}{1000}\right)^{1000} \approx 7.374$	$\left(1-\frac{2}{1000}\right)^{1000} \approx 0.135$	0.996

(a) Calculate e^{-2}, rounding your answer to four decimal places. (1 mark)

$\boxed{0.1353}$

(b) Write a formula for $A(n)$ and for $B(n)$ in terms of n, where n is a positive integer. (2 marks)

$$A(n) = \left(1+\frac{2}{n}\right)^{n}$$

$$B(n) = \left(1-\frac{2}{n}\right)^{n}$$

(c) Determine the values of $A(10000)$, $B(10000)$ and $C(10000)$. (2 marks)

$$A(10000) = 7.3876$$

$$B(10000) = 0.1353$$

$$C(10000) = 0.9996$$

(d) Determine the exact limiting values of $A(n)$, $B(n)$ and $C(n)$ as $n \to \infty$. (2 marks)

$$A(n) \to e^{2}$$

$$B(n) \to e^{-2}$$

$$C(n) \to 1$$

Question 19 **(10 marks)**

At noon, a jet fighter flying at a constant altitude and at position $140\mathbf{i} - 115\mathbf{j}$ km, is given instructions to refuel in mid-air from a tanker aircraft flying at the same altitude and at position $235\mathbf{i} + 120\mathbf{j}$ km.

The jet fighter is told to fly at a constant velocity of 1 150 km/h in order to intercept the tanker aircraft, which is flying with a constant velocity given by $320\mathbf{i} + 410\mathbf{j}$ km/h.

Suppose the velocity vector the jet fighter needs to maintain for interception is $x\mathbf{i} + y\mathbf{j}$ km/h.

(a) Explain why $x^2 + y^2 = 1322500$. (1 mark)

By considering the speed of the jet,

$$|x\mathbf{i} + y\mathbf{j}| = 1150 \Rightarrow x^2 + y^2 = 1322500$$

(b) Determine a vector ${}_j\mathbf{r}_t$ for the initial position of the jet relative to the tanker. (1 mark)

$$\begin{bmatrix} 140 \\ -115 \end{bmatrix} - \begin{bmatrix} 235 \\ 120 \end{bmatrix} = \begin{bmatrix} -95 \\ -235 \end{bmatrix}$$

(c) Determine a vector ${}_j\mathbf{v}_t$ for the velocity of the jet relative to the tanker. (1 mark)

$$\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 320 \\ 410 \end{bmatrix} = \begin{bmatrix} x - 320 \\ y - 410 \end{bmatrix}$$

(d) State a relationship between ${}_j\mathbf{r}_t$ and ${}_j\mathbf{v}_t$ that must hold if the jet is to intercept the tanker after t hours. (1 mark)

$${}_j\mathbf{r}_t = t \cdot {}_j\mathbf{v}_t$$

(e) Determine the values of x and y. (4 marks)

$$t\begin{bmatrix} x-320 \\ y-410 \end{bmatrix} = \begin{bmatrix} -95 \\ -235 \end{bmatrix} \Rightarrow t = \frac{-95}{x-320} = \frac{-235}{y-410}$$

also, $x^2 + y^2 = 1150^2$

Solve simultaneously to get

$x = 295.07,\ y = -1111.50$ or $x = 560.25,\ y = 1004.30$

$x = 295.07 \Rightarrow t = -0.1544$, so discard this solution set.

Hence $x = 560.25,\ y = 1004.30$.

(f) Calculate the position vector of the jet at the instant it intercepts the tanker, giving coefficients to the nearest km. (2 marks)

$$x = 560.25 \Rightarrow t = 0.3954$$

$$\begin{bmatrix} 235 \\ 120 \end{bmatrix} + 0.3954\begin{bmatrix} 320 \\ 410 \end{bmatrix} = \begin{bmatrix} 361.5 \\ 282.1 \end{bmatrix}$$

$$\approx \begin{bmatrix} 362 \\ 282 \end{bmatrix} \text{ to nearest km.}$$

End of questions

Additional working space

Question number: _________

Additional working space

Question number: _________

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