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## Test 4

*Logarithmic Functions*

### Semester One 2018 Year 12 Mathematics Methods Calculator Assumed

**Name:**

Date: 29/06/2018 7.45am

You may have a calculator, a single-sided page of notes and a formula sheet for the test.

**40 Minutes**

**Total \_\_\_\_\_/35 marks**

**Teacher:**

\_\_\_\_\_ **Mr McClelland**

\_\_\_\_\_ **Mrs. Carter**

\_\_\_\_\_ **Mr Gannon**

\_\_\_\_\_ **Ms Cheng**

\_\_\_\_\_ **Mr Staffe**

\_\_\_\_\_ **Mr Strain**

**Questions 1****(7 marks)**

Find the derivatives of the following. Do not simplify your answer.

(a)  $\ln(2x^3 - 3x^2 + 4x - 1)^3$  (2 marks)

(b)  $e^x \ln(x)$  (2 marks)

(c)  $\ln(x) \cos(x) + \frac{\sin(x)}{x}$  (3 marks)

**Question 2****(5 marks)**

(a) Use Polynomial Long division to simplify  $\frac{x^2 - 2x + 5}{x - 3}$ . (3 marks)

(b) Hence find  $\int \frac{x^2 - 2x + 5}{x - 3} dx$ . (2 marks)

**Question 3****(5 marks)**

(a) Find the constants  $a$  and  $b$  given that for  $\{x \in \mathbb{R} : x \neq 2, x \neq -3\}$ .

(3 marks)

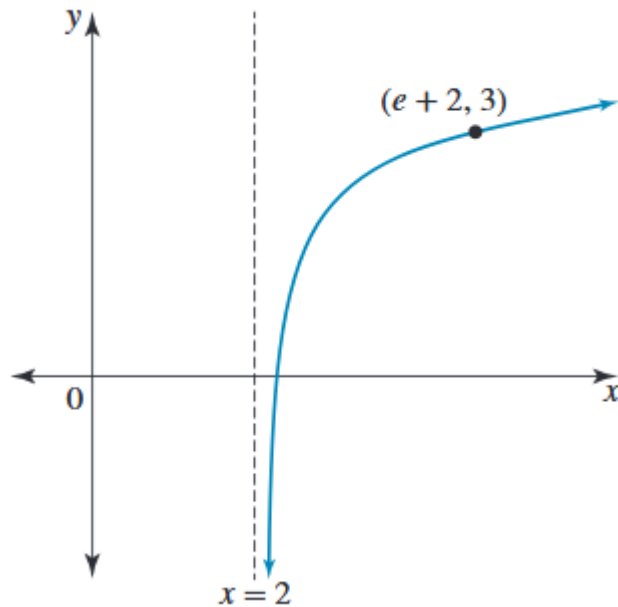
$$\frac{a}{x-2} + \frac{b}{x+3} = \frac{x+8}{x^2+x-6}$$

(b) Hence find  $\int \frac{x+8}{x^2+x-6} dx$ .

(2 marks)

**Question 4****(2 marks)**

The rule for the function shown is  $y = \ln(x - m) + n$ . Find the values of  $m$  and  $n$ .

**Question 5****(3 marks)**

Solve the following equations for  $x$ . Show full algebraic reasoning.

$$3e^{2x} - 5e^x - 2 = 0$$

**Question 6****(5 marks)**

The graph of the function with the rule  $y = 3 \log(x+1) + 2$  intersects the axes at the point  $(a, 0)$  and  $(0, b)$ . Find the exact values of  $a$  and  $b$ . Show full algebraic reasoning.

**Question 7****(8 marks)**

There are two species of insects living in a suburb: the *Asla bibla* and the *Cutus pius*. The number of *Asla bibla* alive at time  $t$  days after 1 January 2000 is given by

$$N_A(t) = 10\,000 + 1000t, 0 \leq t \leq 15$$

The number of *Cutus pius* alive at time  $t$  days after 1 January 2000 is given by

$$N_C(t) = 8000 + 3 \times 2^t, 0 \leq t \leq 15$$

- (a) (i) Show full reasoning that  $N_A(t) = N_C(t)$  if and only if  $t = 3\log_2 10 + \log_2 \left( \frac{2+t}{3} \right)$ .  
(4 marks)

(ii) Solve the value for  $t$  if  $t = 3 \log_2 10 + \log_2 \left( \frac{2+t}{3} \right)$ . (2 marks)

- (b) It is found by observation that the model for *Cutus pius* does not quite work. It is known that the model for the population of *Asla bible* is satisfactory. The form of the model for *Cutus pius* is  $N_C(t) = 8000 + c \times 2^t$ . Find the value of  $c$ , correct to two decimal places, if it is known that  $N_A(15) = N_C(15)$ . (2 marks)

**End of Test**