PERTH MODERN SCHOOL
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Semester 1 Examination 2012

Question/Answer Booklet

MATHEMATICS   3CD

Section Two:
Calculator-assumed

Name of Student: _____ _____

 Name of Teacher :   _____

Time allowed for this section
Reading time before commencing work: 10 minutes
Working time for this section: 100 minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the student
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler,
                                        highlighters
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
                                        and up to three calculators satisfying the conditions set by the Curriculum
                                        Council for this examination

Important note to students

No other items may be used in this section of the examination. It is **your** responsibility to ensure
that you do not have any unauthorised notes or other items in the examination room.
If you have any unauthorised material with you, hand it to the supervisor **before**
reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	12	12	100	100	
			Total	150	100

Instructions to students

1   Write your answers in the spaces provided in this Question/Answer Booklet.
       Spare pages are included at the end of this booklet. They can be used for
       planning your responses and/or as additional space if required to continue an
       answer. If you need to use the space to continue an answer, indicate in the
       original answer space where the answer is continued. i.e. give the page number.
       Fill in the number of the question(s) that you are continuing to answer at the top
       of the page.

2   **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3   It is recommended that you **do not use pencil**, except in diagrams.

Section Two: Calculator-assumed
(100 marks)

This section has **twelve (12)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

_____

Question 7　　　　　　　　　　　　　　　　　　　　　　　　　**(10 marks)**

(a)　The radius of a circular oil slick is increasing at a rate of 0.4 m s^{-1}. Find the rate at which the area of the oil slick is increasing when the radius is 50 m.

(4)

(b)　A sink is formed by the rotation of the curve $y = \dfrac{x^3}{8}$, for $y > 0$, around the Y axis. If the depth of the sink is 8cm, how many cubic centimetres of water would it hold?

(6)

Question 8 **(7 marks)**

(a) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth 'y' of   fluid in the tank 't' hours after the valve is opened is given by

$$y = 6\left(1 - \frac{t}{12}\right)^2$$ metres.

(i) Find the rate $\dfrac{dy}{dt}$ (m/hour) at which the tank is draining at time, *t*.   (2)

(ii)   When is the fluid in the tank falling fastest and slowest?

What are the values of $\dfrac{dy}{dt}$ at these times?   (2)

Question 8 (continued)

(b)   If the volume of a cylinder is given by $V = 2\pi r^3$, find the appropriate percentage

change in V when r changes by $\dfrac{1}{2}$ %

(3)

Question 9   **(10 marks)**

(a)   Give two reasons why the following cannot be a probability distribution. (2)

x	3	1	2	3	5	0
P(X=x)	0.0	0.1	0.4	0.1	0.2	0.3

(b)   The probability distribution of x where random variable, X is the sum of the uppermost numbers when two fair die are rolled is tabulated below.

x	2	3	4	5	6	7	8	9	10	11	12
P(X=x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Find

(i) $P(X > 3)$ (2)

(ii) $P(X < 10 | X > 3)$ (2)

(iii)   If event A is $X > 3$ and event B is $X < 10$, are these two events independent? Justify your answer.
                (4)

Question 10   **(7 marks)**

(a)   The function *f(x)* is differentiable for all $x \varepsilon R$ and satisfies the conditions

f'(x) < 0 where *x < 2*

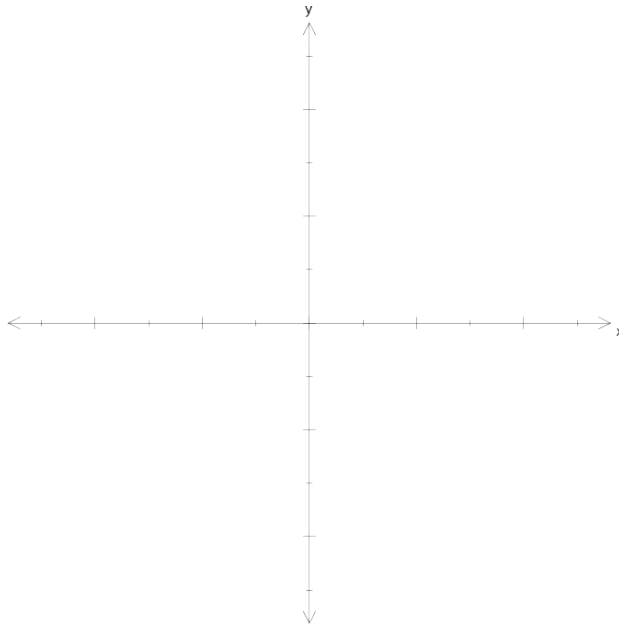*f'(x) = 0* where *x = 2*

f'(x)= 0 where *x = 4*

f'(x) > 0 where *2 < x < 4*

f'(x) > 0 where *x > 4*

(i)   Draw a sketch of this function *f(x)*.   (3)
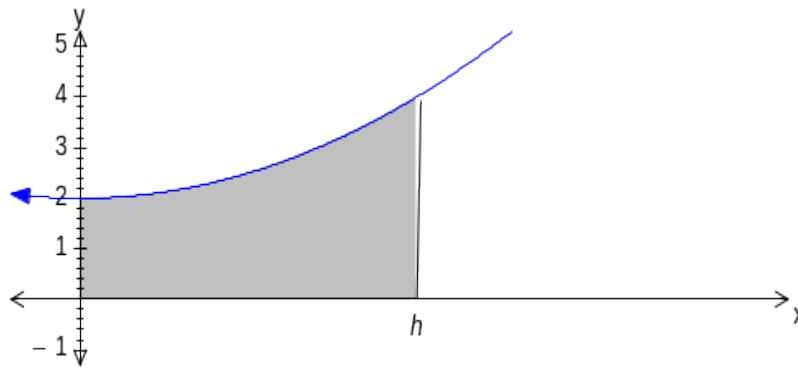

(ii)   State whether the following statement is true or false.
"The graph *f(x)* has a stationary point of inflection where *x=4"*.   (1)

(b)   If $\displaystyle\int_0^a f(x)\, dx = a$, find $\displaystyle 2\int_0^{5a}\left[f\left(\frac{x}{5}\right) + 3\right] dx$ (3)

Question 11   **(7 marks)**

A section of the function $y = 0.5x^2 + 2$ is graphed below, along with a shaded region
enclosed by the function, the axes and the line $x = h$.

(a)   Show that the volume of the solid generated when the shaded region

is rotated about the x-axis is given by V = $\pi[0.05h^5 + \frac{2}{3}h^3 + 4h]$.

(2)

(b)   If h increases at the rate of 0.5 units per second, find an expression,
in terms of h, for the *rate of change* of the volume of the solid generated
when the shaded region is rotated about the x-axis.

(2)

(c)   Use the incremental formula, $\delta V \approx \dfrac{dV}{dh}\delta h$,

to estimate the change in volume when h increases from 3 to 3.01.

(3)

Question 12   **(9 marks)**

(a)   A company produces fruit balls coated in either dark chocolate or milk
        chocolate. A large number of these fruit balls are placed in a box. Twenty per
        cent of the fruit balls in the box are coated with dark chocolate.

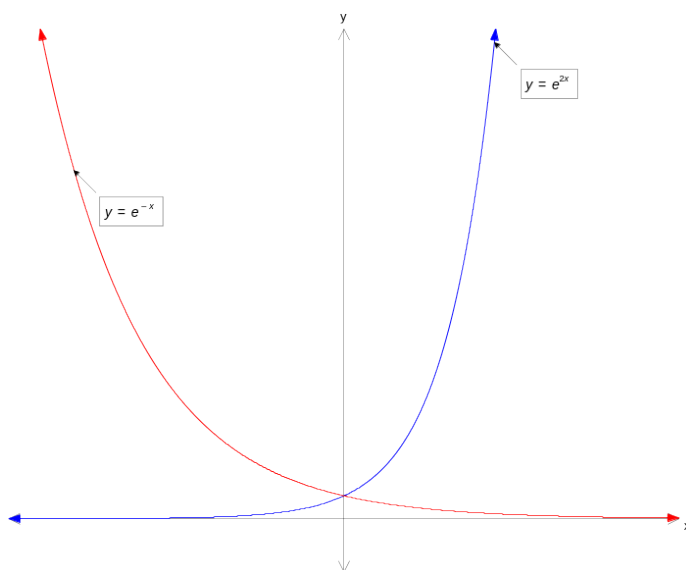   (i)   Calculate $C_4^{10}(0.2)^4(0.8)^6$ (1)

        (ii)   A random sample of ten fruit balls is taken from the box.
                Explain the meaning of $C_4^{10}(0.2)^4(0.8)^6$ with respect to this sample.

                                                                              (2)

(b)   (i)   Find *n* given that $C_0^n(0.2)^0(0.8)^n$ =0.167 772 16   (1)

(ii)   Explain the meaning of your answer to part (b) with respect to the fruit
balls.   (2)

Question 12 (continued)

(c)   The curve $y = e^{2x}$ and $y = e^{-x}$ intersect at the point (0, 1) as shown in the
diagram.

Find the area enclosed by the curves and the line *x=2.*
Leave your answer in terms of '*e*'.

(3)

Question 13   **(8 marks)**

Adam paints garden gnomes to sell. He sends the garden gnomes to his father (a qualified quality controller) in the order of completion, who classifies them as either 'Superior' or 'Regular', depending on the quality of their finish.

 If the garden gnome is Superior, then the probability that the next garden gnome is superior is 0.9. If the garden gnome is Regular, then the probability that the next garden gnome is superior is 0.7.

(a)   If the first garden gnome inspected is Superior, find the probability that the third gnome
        is Regular.   (2)

(b)   If the first garden gnome inspected is Superior, find the probability that the next three
        gnomes are Superior.   (1)

(c)   A group of 3 consecutive garden gnomes is inspected and the first is a Regular. It is also
        found that of these three gnomes,

        P(no Superior) = 0.09
        P(1 Superior) = 0.28
        P(2 Superior) = 0.63

        Find the expected number of these gnomes that will be Superior.   (2)

Question 13 (continued)

(d)   Adam's little brother, Brodie joins in this business venture. The probability that any one of
Brodie's painted garden gnomes is Regular is 0.8. He wants to ensure that the
probability that he paints at least two Superior is at least 0.9. Calculate the
minimum number of garden
gnomes that Brodie would need to paint to achieve this aim.   (3)

Question 14   **(8 marks)**

A piece of wire 8cm long is cut into two unequal parts. One part is used to form a rectangle that has a length three times its width. The other part of the wire is used to form a square.

(i)   If the width of the rectangle is x units, determine an equation that will give the sum of the areas of the rectangle and the square in terms of x.

(3)

(ii)   Using Calculus, find the length of each part of the wire when the sum of the areas is a minimum.   (5)

Question 15   **(11 marks)**

Nuts and Bolts Company manufactures 120mm bolts which are normally distributed with a mean length of 120mm and a standard deviation of 1mm. Only bolts which are between 118.6mm and 121.4mm pass inspection and are packaged as 120mm bolts.

(a)   Find the probability of a randomly selected bolt being an acceptable length.

(2)

(b)   Find the expected number of acceptable bolts in a batch of 100 000   (1)

(c)   Is this a reasonable outcome for the company? Justify your answer.   (2)

(d)   A new quality controller suggests adjusting the settings on the machines so that the

standard deviation becomes 0.85mm and that only the shortest 5% and the longest 5% of the bolts are rejected.

(i)   Find the new minimum and maximum acceptable lengths
correct to the nearest 0.1mm.   (3)

(ii)   Do the packages contain bolts that are more consistent in length?

(1)

(iii)   Is the manufacturer better off? Justify.   (2)

Question 16   **(7 marks)**

The graph of g'(x) is given below.
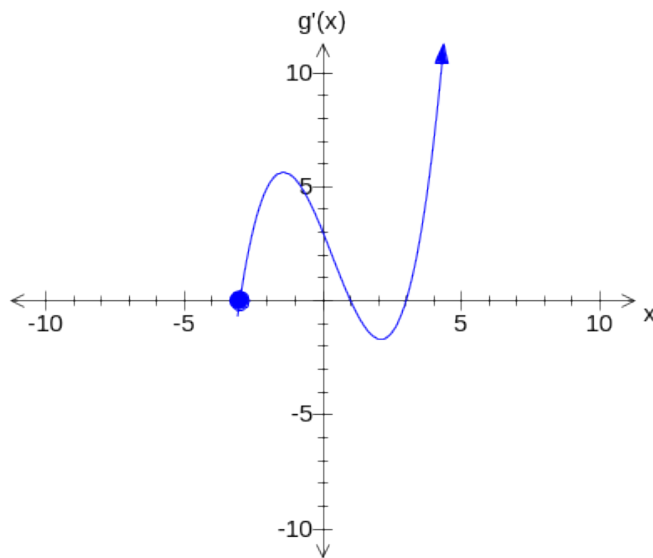
(a)   What can be said about the gradient of the function g(x) between x =-3 to x= 1?
(1)

(b)   When does the function, g(x) have a negative gradient?   (1)

(c)   State an equation for the tangent to the graph of g(x) at x = 3.   (2)

(d)   Find the value of x at which $g(x)$ has a relative maximum for $-3 \leq x \leq 4$ (1)

(e)   Find the x-coordinate of each point of inflection of the graph of $g(x)$ for
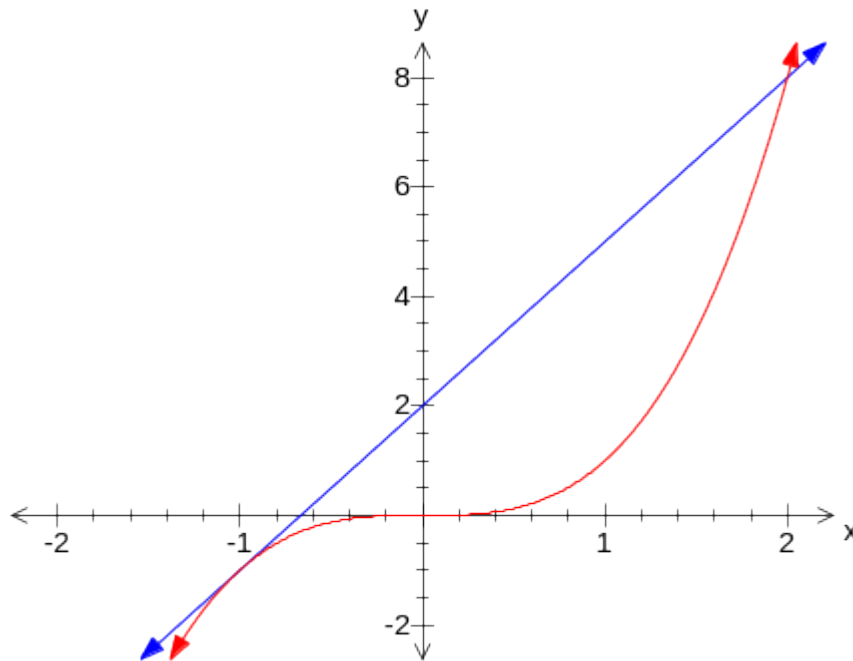$-3 \leq x \leq 4$
        (2)

Question 17 **(9 marks)**

(a)   Shade the region, R, bounded by the curves, $y = x^3$, $y = 3x + 2$, and $x = 0$ in the diagram.

Find the area of the region R, showing all working steps. (4)

Question 17 (continued)

(b)   A group of anthropologists found that human tooth size is continuing to decrease, such that

$$\frac{dS}{dt} = kS$$

In Northern Europeans, for example, tooth size reduction now has a rate of 1% per 1000 years.

   (i)   If t represents time in years and S represents tooth size, find the value of k.

                                                                                      (2)

   (ii)   In how many years will human tooth size be 90% of their present size?
                  (2)

   (iii)   What will be our descendant's tooth size 20 000 years from now?   (1)

              (as a percentage of our present tooth size)

Question 18 **(7 marks)**

A particle is moving in rectilinear motion with acceleration a at any time t, in m s^{-2}, given as

$a = 6t - 1$

Initially the particle is at the origin with a velocity of -2 m/s.

Determine:

(a)   the velocity of the particle at any time t.   [1]

(b)   when the particle is again at the origin.   [2]

(c)   the minimum velocity of the particle.   [2]

(d)   the total distance travelled by the particle in the first three seconds. [2]