Revision Examination Assessment Papers (REAP)
Semester 1 Examination 2012

Question/Answer Booklet
(This paper is not to be released to take home before 25/6/2012)

MATHEMATICS 3C

Section Two:
Calculator-assumed

Solutions

Name of Student: ______________________

Time allowed for this section

Reading time before commencing work:	10 minutes
Working time for this section:	100 minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the student

Standard items:	pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items:	drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

Important note to students

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section Two: Calculator-assumed **(100 marks)**

This section has **twelve (12)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

Question 7 **(10 marks)**

(a) Emily is a very strong soccer player who has a probability of $\frac{3}{5}$ of scoring a goal with each attempt. She has 15 attempts. Find the probability that the number of goals she scores is less than 7. (2)

$P(X<7) = P(X \leq 6)$

$= B \sim (15, 6, \frac{3}{5})$ ✓

$= 0.0950$ ✓

(b) Suppose that Y is distributed normally with unknown mean μ and standard deviation σ. Given that $P(\mu - 25 \leq Y \leq \mu + 25) = 0.9$, find the value of σ. (2)

$1.645 = \frac{\mu + 25 - \mu}{\sigma}$ or $-1.645 = \frac{\mu - 25 - \mu}{\sigma}$ ✓

$\sigma = 15.20$ ✓

Question 7 (continued)

(6)

(c) The West Coast Eagles have a squad of 29 players. Only 22 players are selected to form a team. Cox and Shuey are members of the squad. How many different teams are possible if

(i) all players are available?

$\binom{29}{22} = 1560780$ ✓

(ii) Cox must be included?

$\overset{c}{\binom{1}{1}}\binom{28}{21} = 1184040$ ✓

(iii) Shuey is injured and cannot play?

$\overset{S}{\binom{1}{0}}\binom{28}{22} = 376740$ ✓

(iv) Cox will not be included but Shuey must play?

$\overset{c}{\binom{1}{0}}\overset{S}{\binom{1}{1}}\binom{27}{21} = 296010$ ✓

(v) Cox and Shuey must be included in the team?

$\overset{c}{\binom{1}{1}}\overset{S}{\binom{1}{1}}\binom{27}{20} = 888030$ ✓

(vi) Cox and Shuey will not be in the team together?

All possibilities − both together = Not both together at same time

$= 1560780 - 888030$

$= 672750$ ✓

Question 8 **(7 marks)**

Question 8 **(7 marks)**

(a) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth 'y' of fluid in the tank 't' hours after the valve is opened is given by $y = 6\left(1 - \frac{t}{12}\right)^2$ metres.

(i) Find the rate $\frac{dy}{dt}$ m/hour at which the tank is draining at time, t. (2)

$$\frac{dy}{dt} = 12\left(1 - \frac{t}{12}\right)^1\left(-\frac{1}{12}\right) \quad ✓$$

$$= -\left(1 - \frac{t}{12}\right) \quad ✓$$

or $\frac{t}{12} - 1$

(ii) When is the fluid in the tank falling fastest and slowest?

What are the values of $\frac{dy}{dt}$ at these times? (2)

Slowest when $t = 12$ ✓, $\frac{dy}{dt} = 0$ ✓ $\left(\frac{1}{2}\text{ each}\right)$

Fastest when $t = 0$ ✓, $\frac{dy}{dt} = -1$ ✓

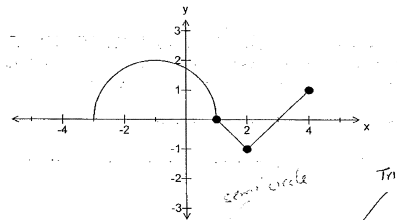
Question 18 (continued)

(iii) $\int_{-3}^{2} f(x)\, dx$ (1)

$-75 - 30 + 30 = -55$ ✓

(b) The graph of a function $f(x)$ consists of a semi- circle and two line segments as shown.

Find the exact value of $\int_{-3}^{4} f(x)\, dx$ (2)



semi circle

Triangle below x-axis

Triangle above x-axis

$$\int_{-3}^{4} f(x)\,dx = \frac{1}{2}(\pi \times 2^2) + \left(-\frac{1}{2}(2)(1)\right) + \frac{1}{2}(1)(1)$$

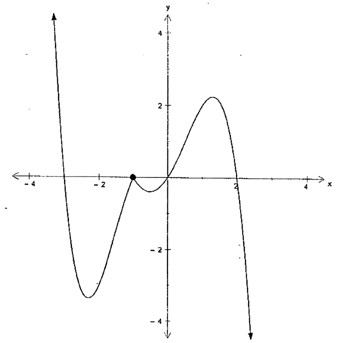
$$= 2\pi - 1 + \frac{1}{2}$$

$$= 2\pi - \frac{1}{2}$$

✓ ✓

Question 18 **(6 marks)**

(a) For the function $y = f(x)$ below



It is known that

$\int_{-3}^{-1} f(x)\,dx = -75$,

$\int_{-1}^{2} f(x)\,dx = 20$

The area under the curve from $x=-1$ to $x=2$ is 80 square units.

Use the information above and mathematical reasoning to determine the value of each of the following.

(i) $\int_{-1}^{0} f(x)\,dx$ (2)

Let $\int_{-1}^{0} f(x) = p$, $\int_{0}^{2} f(x)\,dx = q$

$-p + q = 20$, $p + q = 80$ ✓

Solving simultaneously , $p = 30$, $q = 50$

$\therefore \int_{-1}^{0} f(x)\,dx = -30$ ✓

(ii) the area between the curve and the x-axis from $x=-3$ to $x=0$ (1)

Area = $75 + 30 = 105$ square units ✓

Question 8 (continued)

(b) If the volume of a cylinder is given by $V = 2\pi r^3$, find the appropriate percentage change in V when r changes by $\frac{1}{2}$% (3)

$\frac{\delta r}{r} = \frac{0.5}{100} = \frac{5}{1000}$

$\frac{dV}{dr} = 6\pi r^2$ ✓

$\frac{\delta V}{\delta r} \simeq \frac{dV}{dr}$

ie $\delta V = \frac{dV}{dr} \times \delta r$

$\therefore \frac{\delta V}{V} = \frac{dV}{dr} \times \delta r \times \frac{1}{V}$

$= 6\pi r^2 \times \frac{\delta r}{2\pi r^3}$ ✓

$= \frac{3\,\delta r}{r}$

$= 3\left(\frac{5}{1000}\right)$

$= \frac{15}{1000}$

$= 1.5\%$ ✓

Question 9 (10 marks)

(a) Give two reasons why the following cannot be a probability distribution. (2)

x	3	1	2	3	5	0
P(X=x)	0.0	0.1	0.4	0.1	0.2	0.3

Different probabilities for same value of x ie $P(x=3)=0.0$ and $P(x=3)=0.1$ ✓

Sum of all probabilities is greater than 1 (1.1) ✓

(b) The probability distribution of *x* where random variable, X is the sum of the uppermost numbers when two fair die are rolled is tabulated below.

x	2	3	4	5	6	7	8	9	10	11	12
P(X=x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Find

(i) $P(X>3)$ $=\frac{33}{36}=\frac{11}{12}$ ✓✓ (2)

(ii) $P(X<10|X>3)$ $=\frac{P(3<x<10)}{P(x>3)}$ (2)

$=\frac{\frac{27}{36}}{\frac{11}{12}}$ ✓✓ $=\frac{27}{36}\times\frac{12}{11}=\frac{9}{11}$

(iii) If event A is $X>3$ and event B is $X<10$, are these two events independent? Justify your answer. (4)

$P(A\cap B)=\frac{27}{36}$ ✓

$P(A)=\frac{33}{36}$, $P(B)=\frac{30}{36}$ ✓

Now $\frac{11}{12}\times\frac{\cancel{30}^{5}}{\cancel{36}_{6}}=\frac{55}{72}\neq\frac{27}{36}$ ✓

ie $P(A)\times P(B)\neq P(A\cap B)$ ✓

∴ A & B are not independent events

Question 17 (continued)

(b) A group of anthropologists found that human tooth size is continuing to decrease, such that $\frac{dS}{dt}=kS$
In Northern Europeans, for example, it has been found that tooth size has reduced 1% in the last 1000 years and this trend is expected to continue

(i) If *t* represents time in years and *S* represents tooth size, find the value of k. (2)

$S=S_0e^{kt}$

$0.99S_0=S_0e^{1000k}$ ✓

$0.99=e^{1000k}$

$k=-0.00001$ ✓

(ii) In how many years will human tooth size be 90% of their present size? (2)

$0.9S_0=S_0e^{-0.00001t}$

$0.9=e^{-0.00001t}$ ✓

$t=10536$ years ✓

(iii) What will be our descendant's tooth size 20 000 years from now? (1)

(as a percentage of our present tooth size)

$S=S_0e^{-0.00001\times20000}$

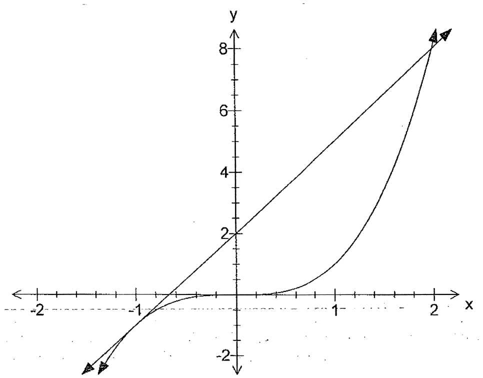
$S=0.8187S_0$

∴ $\approx 82\%$ ✓

Question 17 **(9 marks)**

(a) Find the area of the region bounded by the curves, $y = x^3$, $y = 3x + 2$, in the diagram.

Find the area of the region showing all working steps. (4)



Intersection between $y = x^3$ and $y = 3x+2$
is when $x^3 = 3x+2$
$x^3 - 3x - 2 = 0$
i.e. at $x = -1$ and $x = 2$ ✓

Area of region $= \int_{-1}^{2} (3x+2) - x^3 \, dx$ ✓

$= \left[\frac{3x^2}{2} + 2x - \frac{x^4}{4} \right]_{-1}^{2}$

$= (6 + 4 - 4) - \left(\frac{3}{2} - 2 - \frac{1}{4}\right)$

$= 6 - \left(-\frac{3}{4}\right)$

$= 6\frac{3}{4}$ units^2 ✓✓

Question 10 **(7 marks)**

(a) The function *f(x)* is differentiable for all $x \varepsilon R$ and satisfies the conditions

$f'(x) < 0$ where $x < 2$

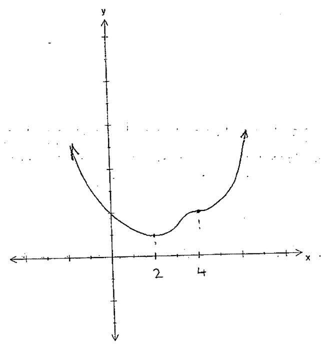
$f'(x) = 0$ where $x = 2$

$f'(x) = 0$ where $x = 4$

$f'(x) > 0$ where $2 < x < 4$

$f'(x) > 0$ where $x > 4$

(i) Draw a sketch of this function *f(x)*. (3)



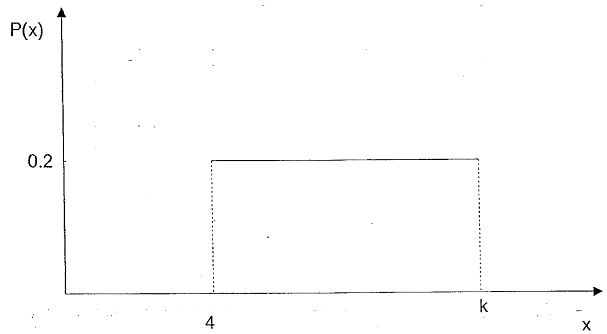
✓ shape
✓ Turns at $x = 2$
✓ pt of inflection at $x = 4$

(ii) State whether the following statement is true or false.
"The graph *f(x)* has a stationary (horizontal) point of inflection where *x=4*". (1)

True ✓

Question 10 (continued)

b) A uniform probability distribution is given below:



The equation of the horizontal line is $y = 0.2$.

(i) ~~a)~~ Find the value of k

$(k-4)0{\cdot}2 = 1$

$k-4 = 5$

$\therefore k = 9$ ✓

(ii) ~~b)~~ Find $P(x \le 8 \mid x \ge 5)$

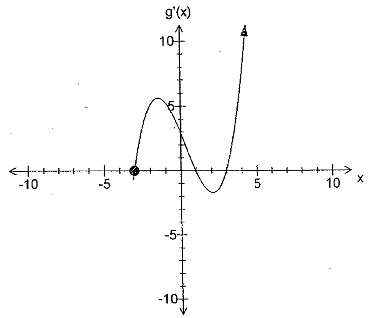
$P(x \le 8 \mid x \geq 5) = \dfrac{P(5 \le x \le 8)}{P(5 \le x \le 9)}$ ✓

$= \dfrac{3(0{\cdot}2)}{4(0{\cdot}2)}$

$= \dfrac{3}{4}$ ✓

Question 16 **(7 marks)**

The graph of g'(x) is given below.



(a) What can be said about the gradient of the function g(x) between x =-3 to x= 1? (1)

Gradient is positive ✓

(b) When does the function, g(x) have a negative gradient? ~~(1)~~ 2

$1 < x < 3$ ✓✓

(c) State an equation for the tangent to the graph of g(x) at x = 3. ~~(2)~~ 1

$y = k$ where k is a constant ✓

(d) Find the value of x at which $g(x)$ has a relative maximum for $-3 \le x \le 4$ (1)

at $x = 1$ ✓

(e) Find the x-coordinate of each point of inflection of the graph of $g(x)$ for $-3 \le x \le 4$ (2)

at $x = -1{\cdot}5$ and $x = 2$
✓ ✓

Question 15 **(11 marks)**

Nuts and Bolts Company manufactures 120mm bolts which are normally distributed with a mean length of 120mm and a standard deviation of 1mm. Only bolts which are between 118.6mm and 121.4mm pass inspection and are packaged as 120mm bolts.

(a) Find the probability of a randomly selected bolt being an acceptable length. (2)

$X \sim N(120, 1^2)$. Find $P(118.6 \le X \le 121.4)$ ✓

$= 0.838487$ ✓

(b) Find the expected number of acceptable bolts in a batch of 100 000 (1)

$0.838487 \times 100000 = 83849$

(c) Is this a reasonable outcome for the company? Justify your answer. (2)

% of acceptable bolts $= \frac{83849}{100000} \times 100\% = 83.85\%$

% of unacceptable bolts = 16.15% which is too high, too much waste

∴ outcome not reasonable ✓

(d) A new quality controller suggests adjusting the settings on the machines so that the standard deviation becomes 0.85mm and that only the shortest 5% and the longest 5% of the bolts are rejected.

(i) Find the new minimum and maximum acceptable lengths correct to the nearest 0.1mm. (3)

$\bar{x} = 120$, $\sigma = 0.85$ $P(a < X < b) = 0.9$ ✓

Using Calc InV NormCD $a = 118.6$ ✓

$b = 121.4$ ✓

(ii) Do the packages contain bolts that are more consistent in length? (1)

Range of size of bolts is the same at 118.6mm to 121.4mm ✓

∴ in terms of consistency the both have the same range

(iii) Is the manufacturer better off? Justify. (2)

Yes, as wastage is reduced from 16.15% to 10% ✓

ie 6150 more bolts will be accepted. ✓

Question 11 **(7 marks)**

(a) The function $y = e^{x(x-1)(x+1)}$ is transformed to $y = -e^{x(1-x^2)}$.

Describe the transformation in order. (3)

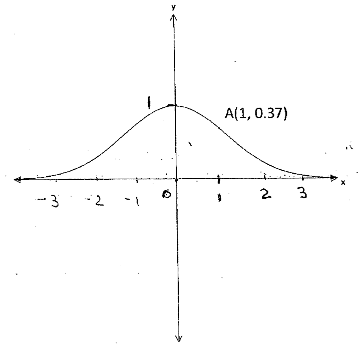
$= -e^{x(1-x)^2}$ $x(1-x)(1+x)$ $-x(x-1)(x+1)$ $= -e^{-x(x^2-1)}$ ✓

or $y = e^{x^3 - x} \longrightarrow y = -e^{-x^3 + x}$

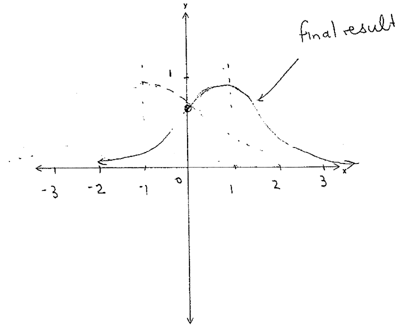
Reflected in the y-axis followed by a reflection in the x-axis ✓

ie $y = e^{x^3 - x} \rightarrow y = -e^{-(x^3 - x)}$ ✓

(b) The curve C has equation $y = e^{-x^2}$ and is drawn below



(i) Sketch the graph of $y = f(-x+1)$. (2)



✓ shape

✓ passes through (1, 1)

Question 11 (continued)

(ii) State the coordinates of A if the curve is transformed to $y = -f\left(\frac{1}{2}x\right) + 2$ (2)

$(1, 0.37) \longrightarrow (2, 0.37) \longrightarrow (2, -0.37) \longrightarrow (2, 1.63)$

✓✓ −1 for each error

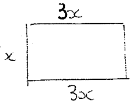
Dilation s.f. 2 in direction of x-axis followed by a reflection in x-axis followed by a translation 2 unit vertically upwards.

Question 14 **(9 marks)**

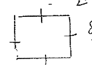
A piece of wire 8cm long is cut into two unequal parts. One part is used to form a rectangle that has a length three times its width. The other part of the wire is used to form a square.

(i) If the width of the rectangle is x units, determine an equation that will give the sum of the areas of the rectangle and the square in terms of x. (4)

8cm $\longrightarrow$ $8x$ and $8-8x$



3x, x, 3x, 3x ✓



$\frac{8-8x}{4} = 2-2x$ ← length of each side ✓

$A = 3x^2 + (2-2x)^2$

$= 3x^2 + 4 - 8x + 4x^2$

$= 7x^2 - 8x + 4$ ✓✓

(ii) Using Calculus, find the length of each part of the wire when the sum of the areas is a minimum. (5)

$A = 7x^2 - 8x + 4$

$\frac{dA}{dx} = 14x - 8$ ✓

Let $\frac{dA}{dx} = 0$ in order to minimise

$0 = 14x - 8$

$8 = 14x$

$x = \frac{8}{14} = \frac{4}{7}$ ✓

$\frac{d^2A}{dx^2} = 14 \Rightarrow min$ ✓

Lengths of each part of wire are $8x$ and $8-8x$

ie $8\left(\frac{4}{7}\right) = \frac{32}{7} = 4\frac{4}{7}$ cm and $3\frac{3}{7}$ cm

✓ ✓

Question 13 (continued)

(d) Adam's little brother, Brodie joins in this business venture. The probability that any one of Brodie's painted garden gnomes is Regular is 0.8. He wants to ensure that the probability that he paints at least two Superior is at least 0.9. Calculate the minimum number of garden gnomes that Brodie would need to paint to achieve this aim. (3)

$P(R) = 0.8, \quad P(S) = 0.2$

He wants $P(X \geq 2) \geq 0.9$ ✓

$B \sim (n, 0.2)$. Need to find n so that $P(X \geq 2) \geq 0.9$

ie $P(X \leq 1) \leq 0.1$ ✓

ie ${}^nC_0 (0.2)^0 (0.8)^n + {}^nC_1 (0.2)^1 (0.8)^{n-1} \leq 0.1$

Using Calc $n = 17.95$

$\therefore$ minimum number is 18. ✓

Question 12 **(9 marks)**

(a) A company produces fruit balls coated in either dark chocolate or milk chocolate. A large number of these fruit balls are placed in a box. Twenty per cent of the fruit balls in the box are coated with dark chocolate.

(i) Calculate $C_4^{10}(0.2)^4(0.8)^6$ (1)

$= 0.08808$ ✓

(ii) A random sample of ten fruit balls is taken from the box.
Explain the meaning of $C_4^{10}(0.2)^4(0.8)^6$ with respect to this sample. (2)

In a sample of 10 fruit balls, the probability of picking exactly 4 coated in dark chocolate is approximately 0.0881 //

(b) (i) Find n given that $C_0^n(0.2)^0(0.8)^n = 0.167\,772\,16$ (1)

using calculator $n = 8$

${}^nC_0 = 1$

$(0.2)^0 = 1$

$\therefore$ need to solve $(0.8)^n = 0.16777216$

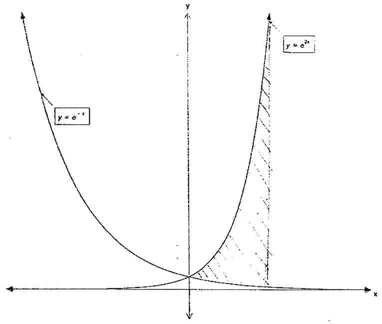
$n = 8$ ✓

(ii) Explain the meaning of your answer to part (b) with respect to the fruit balls. (2)

The probability of picking no dark chocolate fruit ball from 8 is 0.16777216 ✓ ✓

Question 12 (continued)

(c) The curve $y = e^{2x}$ and $y = e^{-x}$ intersect at the point (0, 1) as shown in the diagram.



Find the area enclosed by the curves and the line $x=2$.
Leave your answer in terms of 'e'.

(3)

Required area $= \int_0^2 e^{2x}\,dx - \int_0^2 e^{-x}\,dx$ ✓

$= \left[\frac{e^{2x}}{2}\right]_0^2 - \left(\left[-e^{-x}\right]_0^2\right)$ ✓

$= \frac{e^4}{2} - \frac{e^0}{2} - \left(-e^{-2} - (-e^0)\right)$

$= \frac{e^4}{2} - \frac{1}{2} - \left(-e^{-2} + 1\right)$

$= \frac{e^4}{2} - \frac{1}{2} + e^{-2} - 1$

$= \frac{e^4}{2} + e^{-2} - \frac{3}{2}$ ✓

$= \frac{e^4}{2} + \frac{1}{e^2} - \frac{3}{2}$

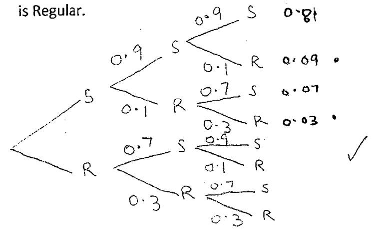
use graphics calculator in standard mode.

Question 13 (8 marks)

Adam paints garden gnomes to sell. He sends the garden gnomes to his father (a qualified quality controller) in the order of completion, who classifies them as either 'Superior' or 'Regular', depending on the quality of their finish.

If the garden gnome is Superior, then the probability that the next garden gnome is superior is 0.9.
If the garden gnome is Regular, then the probability that the next garden gnome is superior is 0.7.

(a) If the first garden gnome inspected is Superior, find the probability that the third gnome is Regular. (2)



✓

$P(SSR) + P(SRR) = (0{\cdot}9) \times (0{\cdot}1) + (0{\cdot}1) \times (0{\cdot}3) = 0{\cdot}12$ ✓

(b) If the first garden gnome inspected is Superior, find the probability that the next three gnomes are Superior. (1)

$(0{\cdot}9)(0{\cdot}9)(0{\cdot}9) = 0{\cdot}729$ ✓

(c) A group of 3 consecutive garden gnomes is inspected and the first is a Regular. It is also found that of these three gnomes,

P(no Superior) = 0.09
P(1 Superior) = 0.28
P(2 Superior) = 0.63

Find the expected number of these gnomes that will be Superior. (2)

$E(X) = 0(0{\cdot}09) + 1(0{\cdot}28) + 2(0{\cdot}63) = 1{\cdot}54$ ✓ ✓