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Anti-Differentiation Techniques

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 3, 3, 3, 3, 3 = 21 marks] **CF**

Anti-differentiate each of the following, showing all working. Leave all answers with positive indices.

(a) $\int \frac{4}{t^2}\, dt$

(b) $\int -\sin 2u \; du$

(c) $\int (4x-5)^3 \; dx$

(d) $\int (e^{-5x} + 2\pi x - \sqrt{x}) \; dx$

(e) $\int \frac{4t^6 - 6t^2}{8t^2}\ dt$

(f) $\int \left(x^2 - 2\right)^3\ dx$

(g) $\int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2}\right) dx$

(h) $\int \left(e^{-2x} + 1\right)\left(e^{3x} - 2\right) dx$

Question Five: [1, 2, 3 = 6 marks] CF

Given that $\int_{-1}^{2} f(x)\,dx = 4$ and $\int_{-1}^{7} f(x)\,dx = 10$, determine:

(a) $2\int_{-1}^{7} f(x)\,dx$

$= 2 \times 10$

$= 20$ ✓

(b) $\int_{7}^{2} f(x)\,dx$

$= \int_{-1}^{7} f(x)dx - \int_{-1}^{2} f(x)dx$ ✓

$= 10 - 4$

$= 6$

$\therefore -6$ ✓

(c) $\int_{-1}^{2} \left(f(x) + x\right)dx$

$= \int_{-1}^{2} f(x)dx + \int_{-1}^{2} x\,dx$ ✓

$= 4 + \left[\frac{x^2}{2}\right]_{-1}^{2}$

$= 4 + \left(\frac{4}{2} - \frac{1}{2}\right)$ ✓

$= 4 + \frac{3}{2}$

$= 5\frac{1}{2}$ ✓

Question Two: [3, 3, 3 = 9 marks] CF

Calculate the following integrals, showing all working.

(a) $\int_{-1}^{2} \left(x^2 - 1\right)dx$

(b) $-2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 3x\,dx$

(c) $\int_{-1}^{3} \left(-e^{4x} + 2\right)dx$

Question Three: [3 marks] CF

The derivative of $f(x)$ is given by $f'(x) = 2e^{2x} + 3x^2$. Given that $f(1) = 4 + e^2$, find an expression for $f(x)$.

Question Four: [6 marks] CF

The gradient function of $f(x)$ is given by $f'(x) = ax^2 + b$. Determine the values of a and b if $f'(-2) = 28,\ \ f(0) = 1$ and $f(1) = 7$.

Question Three: [3 marks] CF

The derivative of $f(x)$ is given by $f'(x) = 2e^{2x} + 3x^2$. Given that $f(1) = 4 + e^2$, find an expression for $f(x)$.

$$f(x) = \int 2e^{2x} + 3x^2 \ dx$$
$$f(x) = e^{2x} + x^3 + c \checkmark$$
$$4 + e^2 = e^2 + 1 + c \checkmark$$
$$c = 3$$
$$f(x) = e^{2x} + x^3 + 3 \checkmark$$

Question Four: [6 marks] CF

The gradient function of $f(x)$ is given by $f'(x) = ax^2 + b$. Determine the values of a and b if $f'(-2) = 28,\ \ f(0) = 1$ and $f(1) = 7$.

$$28 = 4a + b \checkmark$$
$$f(x) = \frac{ax^3}{3} + bx + c \checkmark$$
$$1 = c \checkmark$$
$$7 = \frac{a}{3} + b + 1 \checkmark$$

$$6 = \frac{a}{3} + b$$
$$28 = 4a + b$$
$$22 = \frac{11}{3}a$$
$$\frac{66}{11} = a$$
$$6 = a \checkmark$$
$$28 = 24 + b$$
$$b = 4 \checkmark$$

Question Two: [3, 3, 3 = 9 marks] CF

Calculate the following integrals, showing all working.

(a) $\int_{-1}^{2}\left(x^2-1\right)dx$

$$=\left[\frac{x^3}{3}-x\right]_{-1}^{2}$$
$$=\left(\frac{8}{3}-2\right)-\left(\frac{-1}{3}+1\right)$$
$$=\frac{9}{3}-3$$
$$=3-3$$
$$=0$$

(b) $-2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sin 3x\ dx$

$$=-2\left[\frac{-\cos 3x}{3}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$
$$=-2\left[\frac{-\cos\pi}{3}-\frac{-\cos\frac{\pi}{2}}{3}\right]$$
$$=-2\left(\frac{1}{3}+0\right)$$
$$=\frac{-2}{3}$$

(c) $\int_{-1}^{3}\left(-e^{4x}+2\right)dx$

$$=\left[\frac{-e^{4x}}{4}+2x\right]_{-1}^{3}$$
$$=\left(\frac{-e^{12}}{4}+6\right)-\left(\frac{-e^{-4}}{4}-2\right)$$
$$=\frac{-e^{12}+e^{4}}{4}+8$$

Question Five: [1, 2, 3 = 6 marks] CF

Given that $\int_{-1}^{2} f(x)\ dx=4$ and $\int_{-1}^{7} f(x)\ dx=10$, determine:

(a) $2\int_{-1}^{7} f(x)\ dx$

(b) $\int_{7}^{2} f(x)\ dx$

(c) $\int_{-1}^{2}\left(f(x)+x\right)dx$

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Question One: [2, 2, 2, 3, 3, 3, 3, 3 = 21 marks] **CF**

Anti-differentiate each of the following, showing all working. Leave all answers with positive indices.

(a) $\int \frac{4}{t^2} dt$

$\int 4t^{-2} dt$

$= \frac{4t^{-1}}{-1} + c$

$= \frac{-4}{t} + c$

(b) $\int -\sin 2u \, du$

$= \frac{\cos 2u}{2} + c$

(c) $\int (4x-5)^3 \, dx$

$= \frac{(4x-5)^4}{4 \times 4} + c$

$= \frac{(4x-5)^4}{16} + c$

(d) $\int (e^{-5x} + 2\pi x - \sqrt{x}) \, dx$

$= \frac{e^{-5x}}{-5} + \frac{2\pi x^2}{2} - \frac{2x^{\frac{3}{2}}}{3} + c$

$= \frac{-1}{5e^{5x}} + \frac{2\pi x^2}{2} - \frac{2x^{\frac{3}{2}}}{3} + c$

(e) $\int \frac{4t^6 - 6t^2}{8t^2} \, dt$

$= \int \frac{4t^6}{8t^2} - \frac{6t^2}{8t^2} dt$

$= \int \frac{t^4}{2} - \frac{3}{4} dt$

$= \frac{t^5}{10} - \frac{3t}{4} + c$

(f) $\int (x^2 - 2)^3 \, dx$

$= \int \left(x^6 + 3(x^2)^2(-2) + 3(x^2)(-2)^2 + (-2)^3\right) dx$

$= \int \left(x^6 - 6x^4 + 12x^2 - 8\right) dx$

$= \frac{x^7}{7} - \frac{6x^5}{5} + \frac{12x^3}{3} - 8x + c$

(g) $\int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2}\right) dx$

$= 3\sin\left(\frac{x}{3}\right) + \frac{3(6x)^{\frac{4}{3}}}{8} + c$

(h) $\int (e^{-2x} + 1)(e^{3x} - 2) \, dx$

$= \int \left(e^x - 2e^{-2x} + e^{3x} - 2\right) dx$

$= e^x + \frac{1}{e^{2x}} + \frac{e^{3x}}{3} - 2x + c$