

Section Two: Calculator-assumed 64% (96 Marks)

This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

- (a) The points  $A$  and  $B$  have coordinates  $(4, -6)$  and  $(5, 8)$  respectively. If  $B$  is the midpoint of  $A$  and  $C$ , determine the coordinates of  $C$ . (2 marks)

- (b)  $x$  and  $y$  are linearly related variables such that the points  $D(5p, -q)$  and  $E(2q, 3p)$  lie on  $y = mx + c$ . Determine the relationship for  $q$  in terms of  $p$ , if:
- (i) the gradient of the line is 2. (2 marks)

- (ii) Hence, determine the value of the  $y$  intercept if  $p = 3$ . (2 marks)

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**Question 10****(8 marks)**

A random sample of 121 passengers arriving at an airport were asked to complete a brief survey. They were asked to categorise their main place of residence as Australia or overseas and the main purpose of their travel as work, holiday or other. It was found that

- half of the 84 passengers who resided overseas were on holiday
- 14 passengers were on holiday and resided in Australia
- of the 27 who were travelling for other reasons, 11 more resided overseas than in Australia.

- (a) Use the above information to complete the two-way table below. (3 marks)

	Work	Holiday	Other	Total
Australia				
Overseas				84
Total			27	121

- (b) If one passenger was selected at random from those surveyed, determine the probability (to 4 decimal places)

- (i) that the main purpose of their travel was work. (1 mark)

- (ii) that they resided overseas, given that the main purpose of their travel was work. (1 mark)

- (iii) that the main purpose of their travel was work, given that they resided in Australia. (1 mark)

- (c) Explain whether the survey indicates that purpose of travel appears to be independent of main place of residence for these passengers. (2 marks)

A positive integer less than 11 is chosen at random.

The outcome sets for events  $O$ ,  $T$  and  $S$  are such that:  
 $O = \{\text{odd numbers}\} = \{1, 3, 5, 7, 9\}$   
 $T = \{\text{triangular numbers}\} = \{1, 3, 6, 10\}$  and  
 $S = \{\text{square numbers}\} = \{1, 4, 9\}.$

(a) List the elements of the following sets:

(i)  $O \cap T$ . (1 mark)

(iii)  $T \cup (O \cap S)$ . (2 marks)

(b) Determine (1 mark)

(i)  $n(O \cap S \cap T)$ . (1 mark)

(ii)  $P(O' \cap (T \cap S))$ . (1 mark)

(iii)  $P(T' | (O \cup S))$ . (2 marks)

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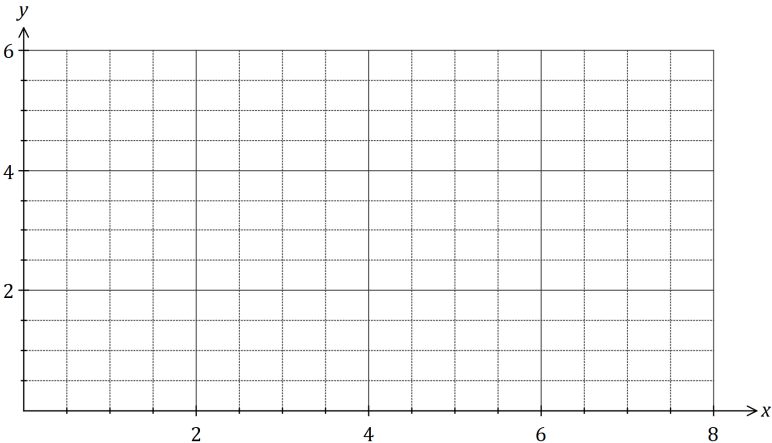
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Question 12 (8 marks)

The distortion of a signal,  $D$ , can be modelled by  $D(x) = 4.55 - 4.5x + 1.95x^2 - 0.2x^3$ , where  $x$  is the distance from the signal source in metres and  $0 \leq x \leq 7$ .

(a) Determine  $D$  when  $x = 1$ . (1 mark)

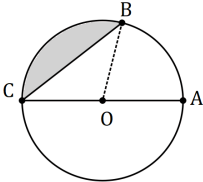
(b) Draw the graph of  $y = D(x)$  on the axes below. (4 marks)



(c) The strength of the signal,  $S$ , is inversely proportional to the distance from the signal source,  $x$ , such that at 1.5 metres from the source, the strength is 1.5. Determine the distances at which the distortion,  $D$ , is equal to the signal strength,  $S$ . (3 marks)

Question 21 (8 marks)

(a) The circle shown has centre  $O$  and diameter  $AC$  of length 60 cm. Determine the shaded area given that  $7 \times \angle AOB = 5 \times \angle BOC$ . (4 marks)



(b) A sector of a circle with radius  $r$  and subtended angle  $\theta$  has a perimeter of 91 cm and an area of  $490 \text{ cm}^2$ . Determine the possible values of  $r$  and  $\theta$  that satisfy these conditions. (4 marks)

(7 marks)

Question 13  
A triangle  $ABC$  has  $a = 36$  cm,  $c = 52$  cm and an area of  $748$  cm<sup>2</sup>.

- (a) Sketch a triangle to show this information. (1 mark)

(7 marks)

Question 20  
A shelf held a collection of 22 different books, of which 5 were encyclopedias, 10 were science fiction and the rest were poetry.

A random selection of 4 books is to be made from the shelf.

- (a) Determine the number of ways (i) this can be done. (1 mark)

- (ii) a selection can be made that will not contain any encyclopedias. (2 marks)

- (b) Determine the probability that (i) the selection will only contain poetry. (2 marks)

- (iii) the selection will contain exactly one poetry book given that it does not contain any encyclopedias. (2 marks)

(7 marks)

Question 13  
A triangle  $ABC$  has  $a = 36$  cm,  $c = 52$  cm and an area of  $748$  cm<sup>2</sup>.

- (a) Sketch a triangle to show this information. (1 mark)

If  $\angle B$  is an obtuse angle in the triangle

- (b) Determine the size of  $\angle B$ . (2 marks)

- (c) Show that  $b \approx 79$  cm. (2 marks)

- (d) Show that  $\angle C \approx 32^\circ$ . (2 marks)

(7 marks)

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- (b) Determine the probability that (i) the selection will only contain poetry. (2 marks)

- (iii) the selection will contain exactly one poetry book given that it does not contain any encyclopedias. (2 marks)

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A random selection of 4 books is to be made from the shelf.

- (a) Determine the number of ways (i) this can be done. (1 mark)

- (ii) a selection can be made that will not contain any encyclopedias. (2 marks)

- (b) Determine the probability that (i) the selection will only contain poetry. (2 marks)

- (iii) the selection will contain exactly one poetry book given that it does not contain any encyclopedias. (2 marks)

## Question 14

(8 marks)

Two events,  $A$  and  $B$ , have probabilities  $P(A) = 0.4$  and  $P(B) = 0.65$ .

(a) Determine  $P(A \cap B)$  in each of the following cases:

(i)  $A$  and  $B$  are independent.

(1 mark)

(ii)  $P(A \cup B) = 0.8$ .

(2 marks)

(iii)  $P(A|(A \cup B)) = \frac{4}{9}$ .

(3 marks)

(b) Is it possible that  $A$  and  $B$  are mutually exclusive events? Explain your answer. (2 marks)

## Question 19

(6 marks)

Let  $p = \cos \frac{13\pi}{18}$  and  $q = \sin \frac{7\pi}{36}$ .

Give your answers to the following in terms of  $p$  and/or  $q$ .

(a) Write down an expression for

(i)  $\sin \frac{29\pi}{36}$

(1 mark)

(ii)  $\cos \frac{5\pi}{18}$

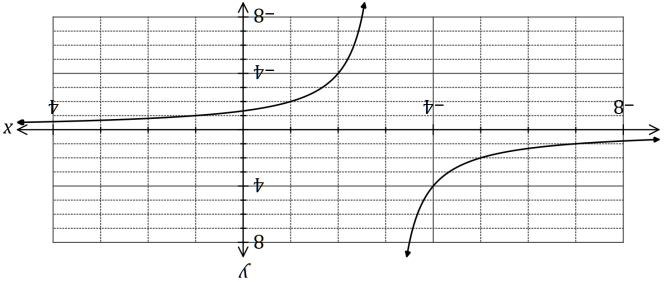
(1 mark)

(b) Using your understanding of the unit circle, determine all other values of  $\theta$ , within two revolutions whereby  $\cos \theta = p$ . Give your answers in degrees. (4 marks)

(7 marks)

Question 15

The graph of  $y = f(x)$  is shown below where  $f(x) = \frac{-a}{x-b}$ .



- (a) The hyperbola shown above has two asymptotes. State their equations. (2 marks)

- (b) State the values of constants  $a$  and  $b$ . (2 marks)

- (c) Describe how to transform the graph of  $y = f(x)$  to obtain the graph of  $y = f(x) + 1$  and state the domain and range of the transformed function. (3 marks)

See next page

(9 marks)

- (a) The equation of the axis of symmetry for the graph of  $y = 3x^2 + 6x + 7$  is  $x = k$ . Determine the value of  $k$ , using a method that does not refer to the graph of the parabola. (2 marks)

- (b) A parabola has a turning point at  $(6, -5)$  and passes through the point  $(-2, -37)$ .  
(i) Determine the equation of the function. (2 marks)

- (iii) Show that the equation has no real zeroes (i.e. no real roots) (2 marks)

- (c) Determine the value of the discriminant for the quadratic equation  $16x^2 - 24x + 9 = 0$  and use it to explain how many solutions the equation  $(x + 1)(16x^2 - 24x + 9) = 0$  will have. (3 marks)

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**Question 16****(7 marks)**

An examination consisted of two papers, one of which was much harder than the other. 12% of candidates gained a distinction in the first paper (event  $A$ ) and 4% gained a distinction in the second paper (event  $B$ ) whilst 87% did not gain a distinction in either paper.

- (a) Using an appropriate diagram, determine the probability that a randomly chosen candidate

(i) gained a distinction in both papers.

**(3 marks)**

(ii) gained a distinction in one paper but not the other.

**(1 mark)**

(iii) gained a distinction in the second paper given that they gained a distinction in the first.

**(1 mark)**

- (b) State, with justification, whether events  $A$  and  $B$  are independent.

**(2 marks)**

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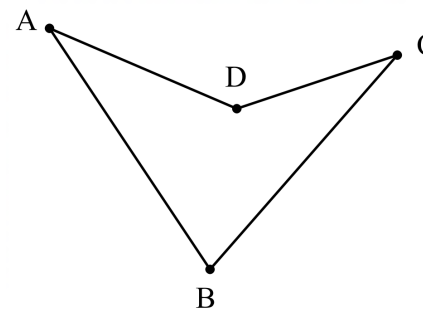
**Question 17****(8 marks)**

- (a)  $A$  and  $B$  are two points on a coastline, and  $C$  is a point at sea. The points  $A$  and  $B$  are 1070m apart. The angles  $CAB$  and  $CBA$  have magnitudes of  $74^\circ$  and  $69^\circ$  respectively. Find the distance from  $C$  to  $A$  to the nearest metre.

**(3 marks)**

- (b) Determine the area of the quadrilateral shown below given that  $\angle BDC = \angle ADC$

$AB = 252\text{m}$  and  $AD = BD = CD = 174\text{m}$ .

**(5 marks)**

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