PERTH MODERN SCHOOL
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Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two: Calculator-assumed

Your Name _____

Your Teacher's Name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
9			16		
10			17		
11			18		
12			19		
13			20		
14			21		
15					

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	14	14	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **(97 Marks)**

This section has **14** questions. Answer **all** questions. Write your answers in the spaces provided.

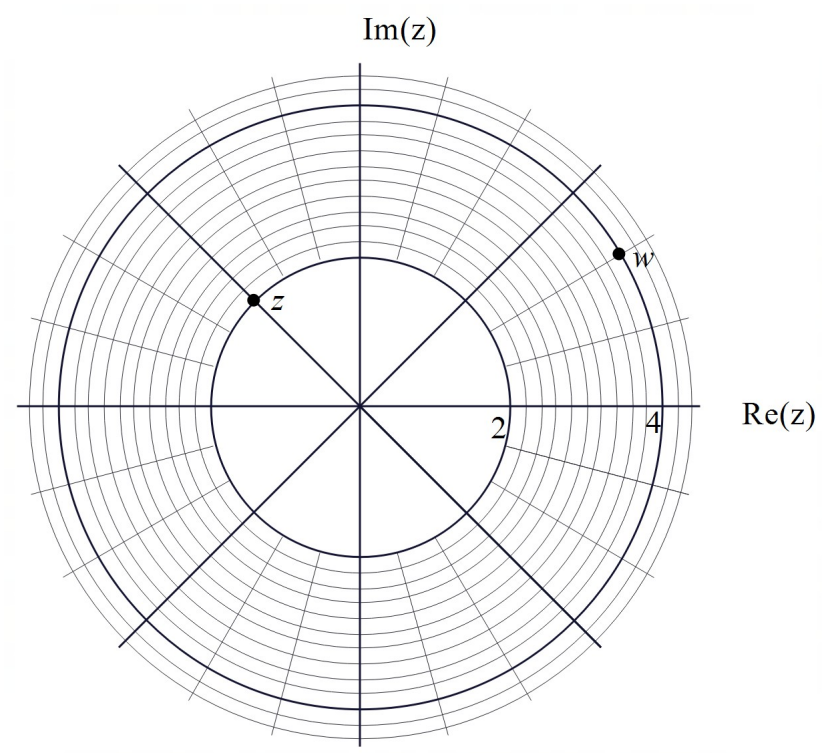
Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(7 marks)**

Consider the complex numbers $z \& w$ plotted on the Argand plane below.



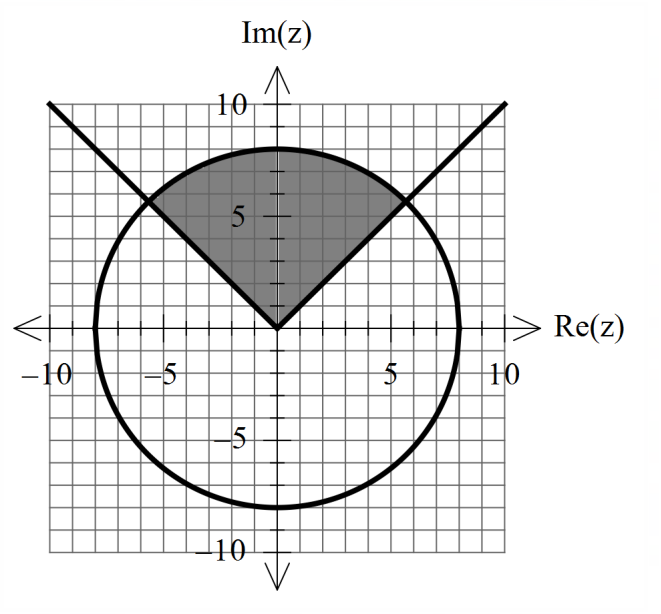
a) Express z in polar form with principal argument. (2 marks)

b) Express w in cartesian form. (2 marks)

c) Plot $iz \& -iw$ on the axes above. (3 marks)

Question 10 **(10 marks)**

Consider the region shaded in the Argand plane below.



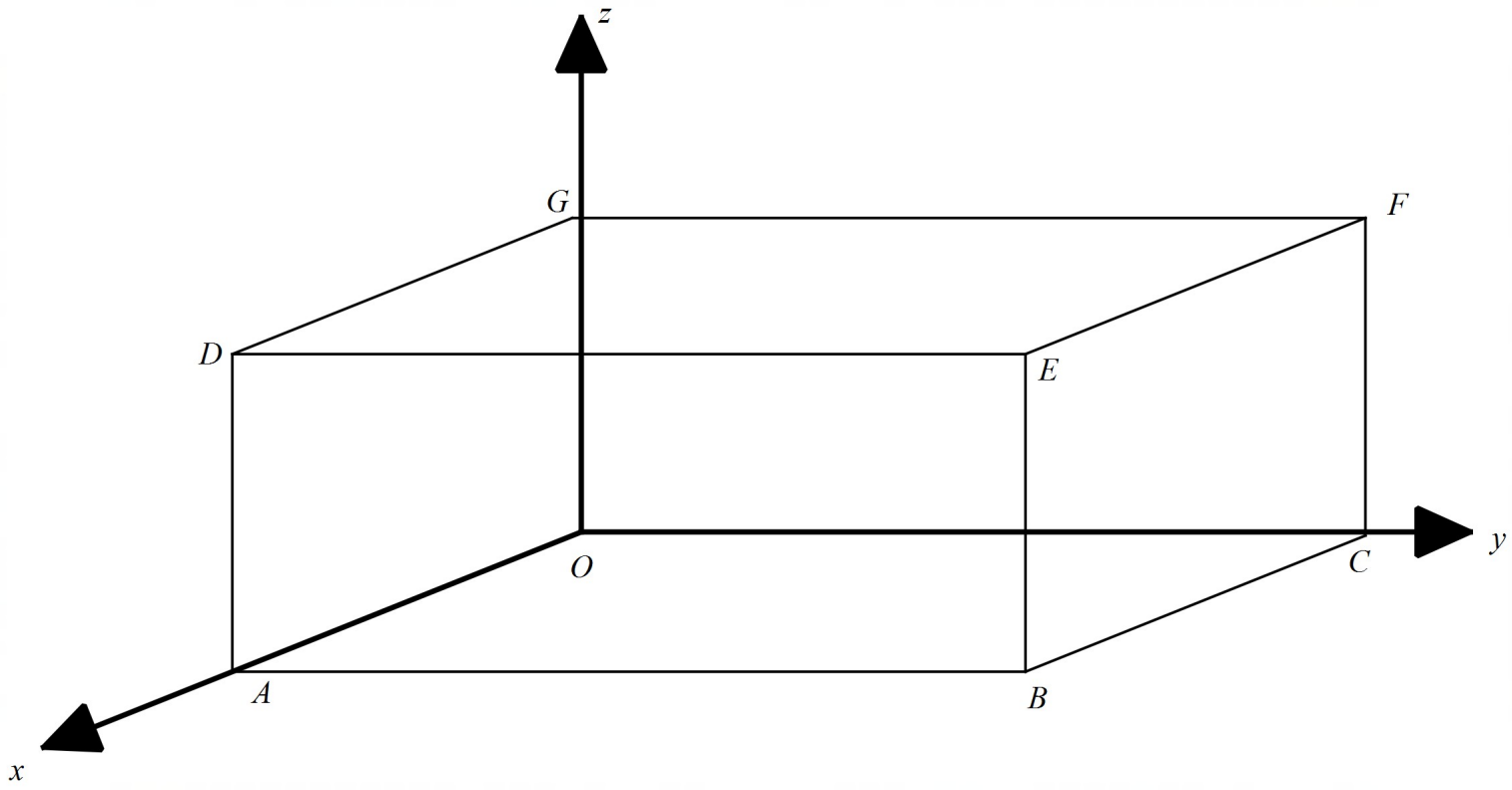
a) In terms of z, describe the region of complex numbers shaded above. (4 marks)

b) i) Sketch the locus w such that $|w - 3 + 3i| = 2$ on the Argand plane above. (3 marks)

ii) Determine the maximum value of Arg(w) (3 marks)

Question 11 **(8 marks)**

Consider a rectangular prism $OABCDEFG$,as shown below, with $A(5,0,0), C(0,7,0) \& G(0,0,4)$



a) Prove that the diagonals $\overline{AF} \& \overline{GB}$ bisect each other using vector methods. (4 marks)

Q11 continued-

b) Determine the exact vector equation of a sphere that goes through all vertices of the rectangular prism $OABCDEFG$. (4 marks)

Question 12 **(9 marks)**

Consider rockets $A \& B$ that are ignited at the same times from the following positions and constant velocities. (At time $t = 0$)

$$r_A = \begin{pmatrix} 12 \\ -18 \\ 20 \end{pmatrix} km \quad , r_B = \begin{pmatrix} -35 \\ 22 \\ -8 \end{pmatrix} km$$

$$v_A = \begin{pmatrix} -7 \\ 14 \\ 2 \end{pmatrix} km/h \quad , v_B = \begin{pmatrix} 15 \\ -7 \\ 5 \end{pmatrix} km/h$$

a) Prove using vector methods that the two rockets do not meet. (3 marks)

b) Determine the closest approach between the two rockets **using vector methods**. (4 marks)

Q12 cont-

c) At time $t = 1$ hour, rocket A will change its velocity so that it will collide with rocket B at time $t = 3$ hours. Determine this new constant velocity of rocket A to 2 decimal places.
(4 marks)

Question 13 **(6 marks)**

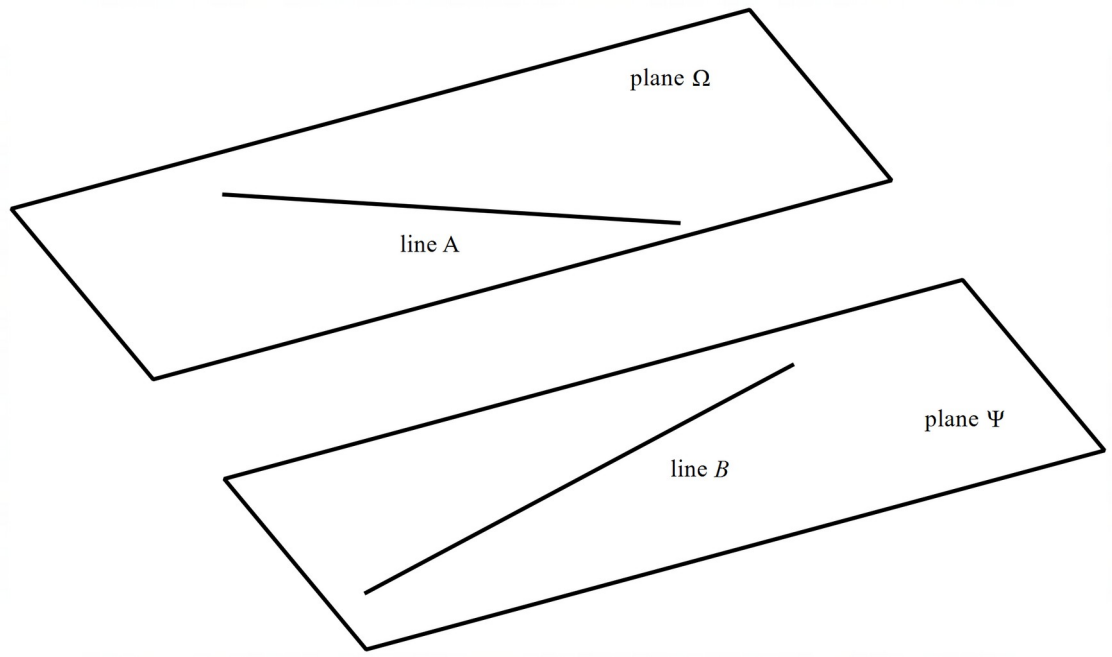
Consider the line $r = \begin{pmatrix} -5 \\ 1 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and the point $A(11,-5,7)$.

a) Determine the distance of point A to the line using vector **dot** product. (3 marks)

b) Determine the distance of point A to the line using vector **cross** product. (3 marks)

Question 14 **(7 marks)**

Consider the plane Ω which contains the line A, $r_A = \begin{pmatrix} -1 \\ 5 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} -11 \\ 7 \\ 2 \end{pmatrix}$ and the **parallel plane** Ψ which contains the line B, $r_B = \begin{pmatrix} 15 \\ -10 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ -14 \\ 9 \end{pmatrix}$ as shown in the diagram below, (not drawn to scale).



a) Determine the cartesian equation of plane Ω (3 marks)

b) Determine the distance between the two planes. (4 marks)

Question 15 **(6 marks)**

Let $z = rcis\theta$ where $0 < \theta < \frac{\pi}{2}$.

a) Determine an expression for $\frac{\sqrt{3}+i}{\bar{z}(1+i)}$ in terms of $r \& \theta$ only. Simplify. (3 marks)

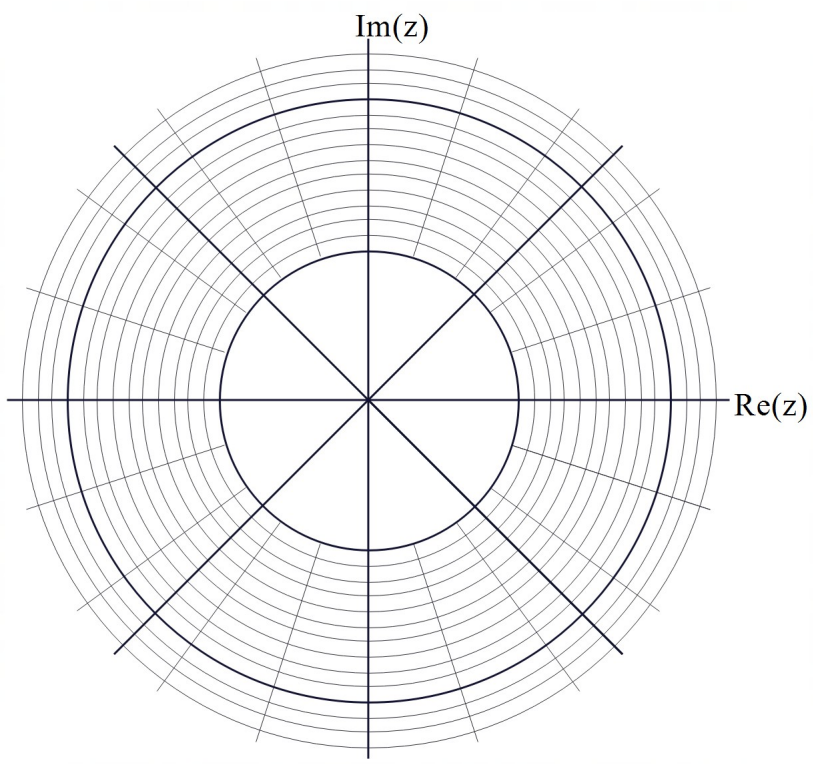
b) Determine an expression for $Arg(z+r)$ in terms of θ only. (3 marks)

Question 16 **(9 marks)**

a) Determine the roots to $z^5 = -16i$ in the form $z = rcis\theta$ with $-\pi < \theta \leq \pi$. (4 marks)

Q16 continued-

b) Plot the roots from part a on the axes below. (2 marks)



c) The roots above form a polygon, determine the perimeter of this polygon.

(3 marks)

Question 17 **(3 marks)**

Consider the locus of points that satisfy $\underset{\sim}{r}.(2i+3j+5k-\underset{\sim}{r})=0$. Describe this locus identifying all major features.

Question 18 **(7 marks)**

Consider the line $r=\begin{pmatrix}-1\\ \alpha\\ 5\end{pmatrix}+\lambda\begin{pmatrix}7\\ 0\\ -2\end{pmatrix}$, α is a constant and the sphere $\left|r-\begin{pmatrix}-2\\ 3\\ -11\end{pmatrix}\right|=20$.
Determine all possible values of α using **vector methods** such that:

i) The line is a tangent to the sphere.
ii) The line passes through the sphere.
iii) The line misses the sphere completely.

Q18 continued-

Question 19 **(4 marks)**

Simplify $\left(\frac{1+\cos 2\theta + i\sin 2\theta}{1+\cos 2\theta - i\sin 2\theta}\right)^n$ showing all reasoning.

Question 20 **(4 marks)**

Consider $z = 2 - 2\sqrt{3}i$.

a) Show that $(z^n) - (\overline{z})^n = -2^{2n+1} i \sin\left(\frac{n\pi}{3}\right)$ where n is a positive integer. (3 marks)

b) Determine the positive integer values of n such that $(z^n) - (\overline{z})^n = 0$ (1 mark)

Question 21 **(9 marks)**

Consider the following system of linear equations.

$$x + y - 2z = -1$$
$$x + 3y + z = 0$$
$$-2x - 4y + z = 1$$

a) Show **without** the use of a classpad, that there are infinite solutions. (3 marks)

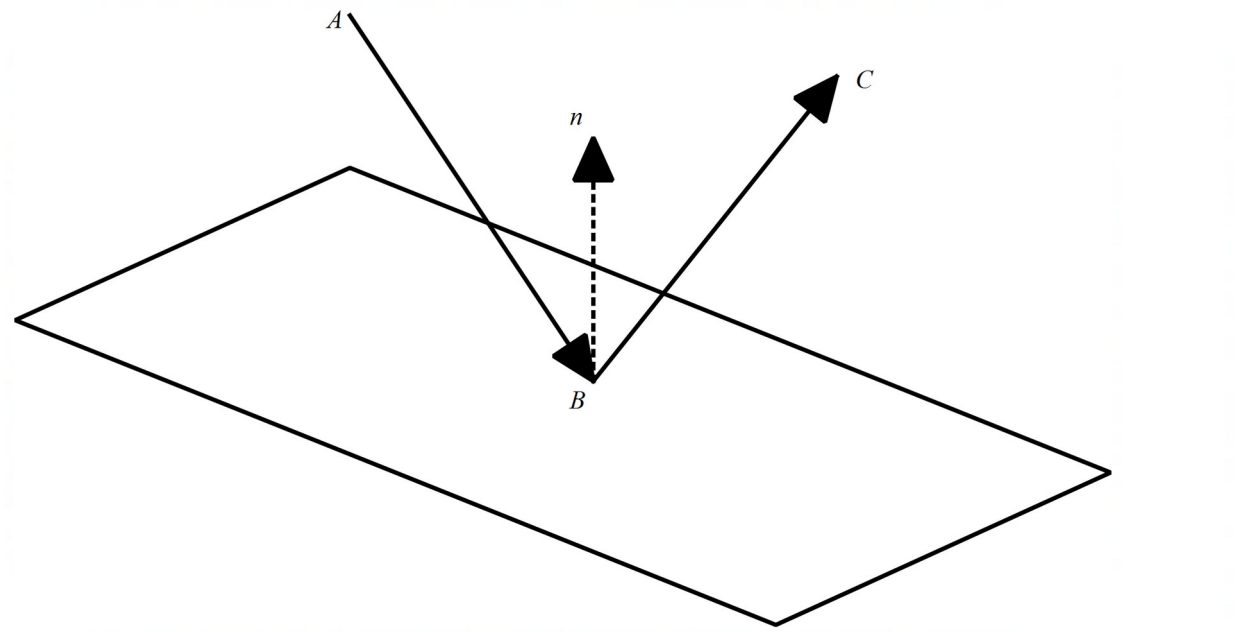
b) Give a geometric interpretation to the solution above. (1 mark)

c) Determine a vector equation for all solutions. (3 marks)

d) If there is the restriction $-3 \leq z \leq 5$, determine the range of values for $x \& y$. (2 marks)

Question 22 **(8 marks)**

Consider a projectile fired from a toy gun which moves at a constant velocity $\begin{pmatrix} -1 \\ 5 \\ -7 \end{pmatrix} m/s$ and rebounds off a plastic flat board with its speed unchanged. See diagram below.



Not drawn to scale

The projectile moves in the direction AB and rebounds in the direction BC with the same speed. The flat board has the equation $r.\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = 5$. The angle of the incoming path AB and the normal $\underset{\sim}{n}$ is equal to the angle of the outcoming path BC and the normal. Both paths AB & BC and the normal exist in the same plane.

Determine the velocity of the reflected projectile and the angle with the above normal to 2 decimal places.

Q22 continued-

Working out space

Working out space

Working out space