Student Name: _____

Methodist Ladies' College Semester 2, 2010

3CD MATHEMATICS

Question/Answer Booklet – Section 1 – Calculators *NOT* allowed – Notes sheets *NOT* allowed

Teacher's Name: **SOLUTIONS**

Time allowed for this paper

Section	Reading	Working
Calculator-free	5 minutes	50 minutes
Calculator-assumed	10 minutes	100 minutes

Materials required/recommended for this paper

Section One (Calculator-free): 40 marks

To be provided by the supervisor
Section One Question/Answer booklet Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Section Two (Calculator-assumed): 80 marks

To be provided by the supervisor
Section Two Question/Answer booklet Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important Note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Instructions to candidates

1. **All** questions should be attempted.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare answer pages may be found at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued (i.e. give the page number).

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

Structure of this paper

Questions	Marks available	Your score
1	5	
2	5	
3	4	
4	4	
5	2	
6	4	
7	7	
8	4	
9	5	
Total:	**40**	
10	5	
11	3	
12	5	
13	8	
14	10	
15	7	
16	8	
17	6	
18	7	
19	5	
20	7	
21	6	
22	3	
Total:	**80**	
Total marks = 120		
		%

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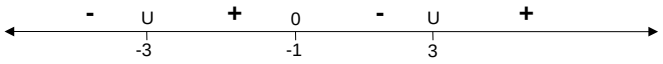
Section One: Calculator-free **(40 Marks)**

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the space provided.

Suggested working time for this section is 50 minutes.

Question 1 **(5 marks)**

Solve $\frac{2x^2+13x+15}{x^2-9} \geq \frac{2x+1}{x+3}$

Solution
$\frac{2x^2+13x+15-(2x+1)(x-3)}{(x-3)(x+3)} \geq 0$ $\frac{18x+18}{(x-3)(x+3)} \geq 0$ Critical values: $x=-1, x=\pm 3$    Solution: $\{x: -3<x\leq -1 \text{ or } x>3, x \in R\}$
Specific behaviours
✓ recognizes common denominator ✓ multiplies by common denominator correctly ✓ simplifies ✓ identifies critical values ✓ correct solution and notation (set notation not necessary)

Question 2 **(5 marks)**

Find the following:

(a) $\int x^2\sqrt{x^3+5}\,dx$ [2]

Solution
$\frac{1}{3}\int (x^3+5)^{\frac{1}{2}}.3x^2\,dx¿\frac{1}{3}(x^3+5)^{\frac{3}{2}}.\frac{2}{3}+c¿\frac{2}{9}(x^3+5)^{\frac{3}{2}}+c$
Specific behaviours
✓ express integral in terms of $\int [f(x)]^n f'(x)\,dx$ ✓ integrates correctly and adds constant

(b) $\int_0^3 e^{-4x}\,dx$ [2]

Solution
$\int_0^3 e^{-4x}\,dx=\left[\frac{-1}{4}e^{-4x}\right]_0^3¿-\frac{1}{4}(e^{-12}-e^0)¿\frac{1}{4}(1-e^{-12})$
Specific behaviours
✓ finds the integrand ✓ substitutes limits of integration and simplifies

(c) $\frac{d}{dx}\int_3^x \frac{8t}{\sqrt{t+9}}\,dt$ [1]

Solution
$\frac{8x}{\sqrt{x+9}}$
Specific behaviours
✓ applies the Fundamental Theorem of Calculus correctly

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Additional working space

Question number(s):_____

Additional working space

Question number(s):_____

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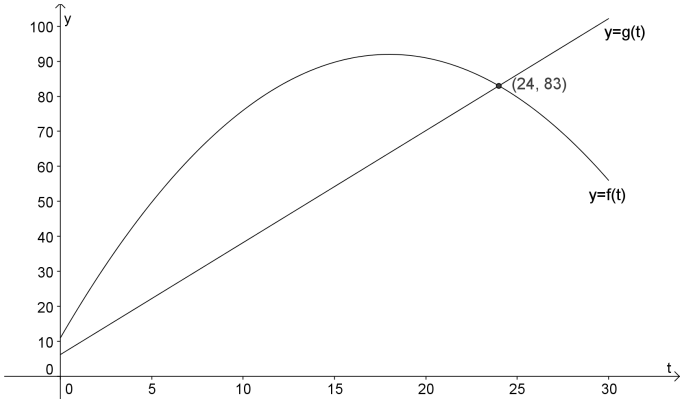
Question 3 **(4 marks)**

The graphs of $f(t) = -0.25t^2 + 9t + 11$ and $g(t) = 3.2t + 6.2$ are shown on the axes below.



(a) Write down an expression for the area enclosed by the two graphs and the vertical axis. [1]

Solution
Area $= \int_0^{24} f(t) - g(t)\,dt$
Specific behaviours
✓ correct expression

(b) If $f(t)$ represents the marginal revenue (in hundreds of dollars) for a product, where t is measured in months and $g(t)$ represents the marginal cost (also in hundreds of dollars) for the product, what does this enclosed area represent? [2]

Solution
Profit for the 24 month period.
Specific behaviours
✓ recognizes that the area represents profit ✓ states correct time period

See next page

(c) Write down an expression for the volume of the solid generated when the part of the curve $y=f(t)$ between $t=5$ and $t=25$ is rotated about the horizontal axis. [1]

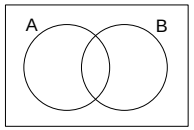
Solution
Volume = $\pi\int_5^{25}[f(t)]^2\,dt$
Specific behaviours
✓ correct expression

Question 4 **(4 marks)**

In a probability experiment, events A and B are such that

$$P(A)=\frac{1}{4},\ P(B|A)=\frac{1}{3} \text{ and } P(\overline{A}\cap\overline{B})=\frac{1}{4}.$$

Find
(a) $P(A\cup B)$ [2]

Solution
$P(A\cup B)=1-P(\overline{A}\cap\overline{B})=\frac{3}{4}$ 
Specific behaviours
✓ uses complement ✓ correct answer

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Additional working space

Question number(s):_____

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Question number(s):_____

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(b) $P(B)$ [2]

Solution
$P(B\|A)=\frac{P(B\cap A)}{P(A)}$ $P(B\cap A)=\frac{1}{3}\times\frac{1}{4}$ ¿ $\frac{1}{12}$ Hence, $P(B)=\frac{1}{12}+\frac{1}{2}=\frac{7}{12}$
Specific behaviours
✓ uses conditional probability rule ✓ correct answer

Question 5 **(2 marks)**

Which of the following statements is true for two events, each with probability greater than 0? Justify your answer.

A: If the events are mutually exclusive, they must be independent.

B: If the events are independent, they must be mutually exclusive.

C: If the events are not mutually exclusive, they must be independent.

D: If the events are not independent, they must be mutually exclusive.

E: If the events are mutually exclusive, they cannot be independent.

Solution
Statement E is true. If events M and N are mutually exclusive, then $P(M \cap N) = 0$ If events M and N are independent, then $P(M \cap N) = P(M).P(N) > 0$ because both $P(M) > 0$ and $P(N) > 0$. Hence, if events M and N are mutually exclusive, then they cannot be independent.
Specific behaviours
✓ identifies E as the only true statement ✓ justifies choice of event E

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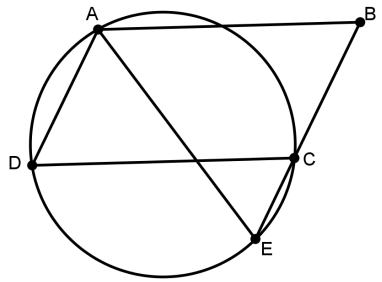
Question 6 **(4 marks)**

In the diagram below, ABCD is a parallelogram.



Prove that ΔABE is isosceles.

Question 9 **(5 marks)**

Determine the equation of the line tangential to the curve $f(x)=\frac{9-x^2}{x}$ at the point where $x=1$.

Solution
$f(x)=\frac{9}{x}-x$ $f'(x)=\frac{-9}{x^2}-1$ $f(1)=8$ and $f'(1)=-10$ Equation of tangent to curve at (1,8) with gradient -10, $y=-10x+c$ (1,8) $8=-10+c$ $c=18$ Hence, $y=-10x+18$

Specific behaviours
✓ correctly differentiates $f(x)$ ✓ determines $f(1)$ and $f'(1)$ ✓ recognizes that $f'(1)$ is the gradient of the tangent line ✓ uses the point (1,8) to determine the vertical intercept ✓ states equation of the tangent

End of questions

Solution
Given: Parallelogram ABCD To Prove: ΔABE is isosceles Proof: ∠AEC = ∠ADC ∠'s subtended by arc AC In parallelogram ABCD, ∠ADC = ∠ABC opposite ∠'s of a parallelogram In ΔABE, ∠AEB = ∠AEC same angle ∴ ∠AEB = ∠ABC Hence, ΔABE is isosceles two angles are congruent

Specific behaviours
✓ Correctly reasons ∠AEC = ∠ADC ✓ Correctly reasons ∠ADC = ∠ABC ✓ Correctly reasons ∠AEB = ∠ABC ✓ Concludes ΔABE is isosceles

Question 7 **(7 marks)**

Consider the following system of equations:

$$3x-2y+z=-7$$
$$-x+2y+2z=11$$
$$ax+bz=4$$

(a) Determine the value of a and of b such that the system of equations has an infinite number of solutions. [2]

Solution
$3x-2y+z=-7$ $-x+2y+2z=11$ $ax+bz=4$ ①+② $2x+3z=4$ $a=2$ and $b=3$

Specific behaviours
✓ solves for a ✓ solves for b

(b) Solve the system when $a=-2$ and $b=1$. [5]

Solution
$-2x+z=4$

See next page

	$2x+3z=4$	④
③+④	$4z=8$ $z=2$	
Substitution gives	$y=3$ and $x=-1$	

Specific behaviours
✓ Eliminates one variable from two pairs of equations
✓ ✓ Evaluates each of the variables correctly

Question 8 **(4 marks)**

Given $f(x)=\sqrt{x}+2$, $g(x)=\frac{1}{x+5}$, and $k(x)=\frac{1}{x}-5$, determine:

(a) $g \circ f(1)$ [2]

Solution
$g \circ f(1)=g(3)=\frac{1}{8}$
Specific behaviours
✓ correctly evaluates $f(1)$
✓ correctly evaluates $g \circ f(1)$

(b) the domain and range of $k \circ f$ [2]

Solution
x $f(x)$ $k(f(x))$
$D_{k \circ f}=D_f=\{x: x \geq 0, x \in R\}$
$R_{k \circ f}=\{y: -5 < y \leq -4.5, y \in R\}$
Specific behaviours
✓ Correctly states domain
Correctly states range

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