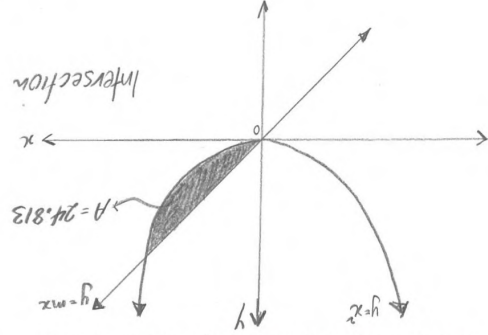


Question 8

(3 marks)

The area enclosed by the line $y = mx$ and the parabola $y = x^2$ is 24.813. Find the value of m , correct to 1 decimal place, where $m > 0$.



$$x^2 = mx$$

$$x^2 - mx = 0$$

$$x(x - m) = 0$$

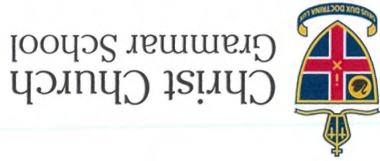
$$\therefore x = 0, m \quad \checkmark \text{ (Intersection)}$$

$$\text{Hence } \int_0^m (mx - x^2) dx = 24.813 \quad \checkmark \text{ (Equation)}$$

$$\text{Using ClassPad } m = 5.3 \quad \checkmark \text{ (m to 1dp)}$$

3

End of questions



Christ Church
Grammar School

2019
TEST 2

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name _____
Teacher's name _____

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

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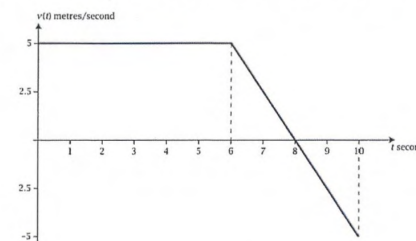
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See next page

Question 7

(7 marks)

A particle moves along a straight line. The velocity – time graph is shown below.



- (a) Find the velocity of the particle when $t = 3$. (1 mark)

$$t=3, \quad v = 5 \text{ m/s} \checkmark$$

- (b) Write an expression for v in terms of t for $6 \leq t \leq 10$. (2 marks)

Equation of line
$$v(t) = -2.5t + 20 \text{ m/s} \checkmark \checkmark$$

- (c) Find the acceleration of the particle when

- (i) $t = 1$ (1 mark)

$$a = 0 \text{ ms}^{-2} \checkmark$$

- (ii) $t = 7$ (1 mark)

$$a = -2.5 \text{ ms}^{-2} \checkmark$$

- (d) Find the change in displacement for $0 \leq t \leq 10$. (2 marks)

Net Area under curve = change in displacement
 $\therefore 5 \times 6 = 30 \text{ m}$ (as triangles cancel out) $\checkmark \checkmark$

See next page

7

Question 6 continued

(b) $\int_2^0 (x - f(x)) dx$

(3 marks)

$$= \int_0^2 x dx - \int_0^2 f(x) dx$$

✓ (Separates Integral)

$$= -\int_2^0 x dx + \int_2^0 f(x) dx$$

$$= -\left[\frac{x^2}{2}\right]_2^0 - 4$$

$$= -\left(\frac{0^2}{2} - 4\right)$$

$$= -2 - 4$$

$$= -6$$

✓ (Correct Answer)

See next page

3

Question 1 (9 marks)

Evaluate each of the following integrals (Leave answers with positive indices):

(a) $\int \frac{\sqrt{x}}{2} - \sqrt[3]{x} dx$

(2 marks)

(b) $\int \frac{x^3 - 1}{x^3 - 4x^2} dx$

(3 marks)

See next page

Question 1 continued

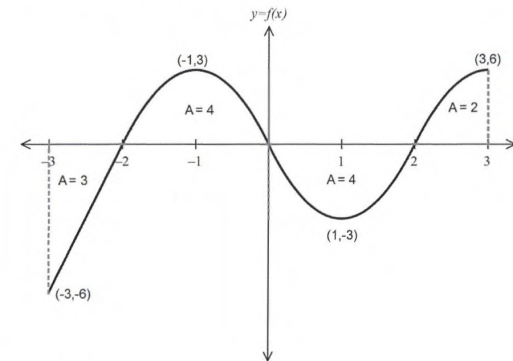
- (c) Find the exact area between the curve $y = x^2 - 3$ and the x -axis. Include a diagram as part of your solution. (4 marks)

See next page

Question 6

(7 marks)

The graph of the function $f(x)$ is shown below for $-3 \leq x \leq 3$. The areas, (A), enclosed between the graph, the x -axis and the lines $x = -3$ and $x = 3$ are marked in the appropriate regions.



Determine:

- (a) (i) the value of $\int_{-3}^2 f(x) dx$ (2 marks)

$$\int_{-3}^2 f(x) dx = -3 + 4 - 4 = -3 \quad \checkmark \checkmark$$

- (ii) the area enclosed between the graph of $f(x)$ and the x axis, from $x = -3$ to $x = 2$. (2 marks)

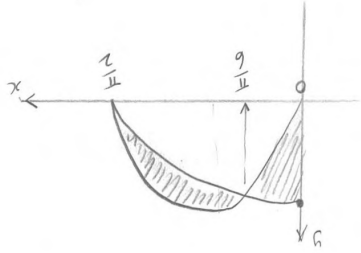
$$\text{Area} = 3 + 4 + 4 = 11 \text{ units}^2 \quad \checkmark \checkmark$$

See next page

4

Question 5 (3 marks)

Calculate the area of the region enclosed by the curves $y = \cos(x)$ and $y = \sin(2x)$ for $0 \leq x \leq \frac{\pi}{2}$. Include a sketch as part of your solution.



(sketch)

$$\text{Area} = \int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \cos(x) - \sin(2x) \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin(2x) - \cos(x) \, dx$$

(Integral)

$$\text{Area} = \frac{1}{4} + \frac{1}{4}$$

$$\text{Area} = \frac{1}{2} \text{ units}^2$$

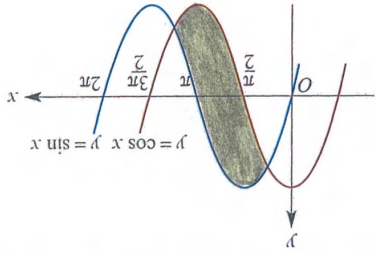
(Answer)

See next page

3

Question 2 (3 marks)

Find the exact area of the shaded region below.

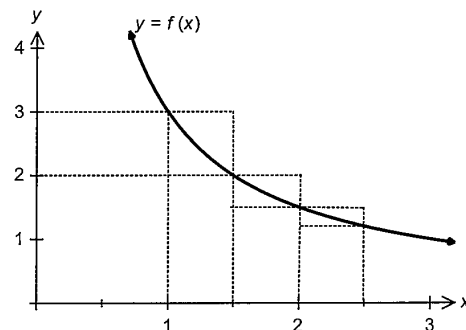


See next page

Question 3

(3 marks)

Consider the graph of $f(x)$, and the table of values of $f(x)$ for certain x values.



x	1	1.5	2	2.5
$f(x)$	3	2	1.5	1.2

By considering the rectangles drawn on the graph, calculate the value of p and q , such that $p < \int_1^{2.5} f(x) dx < q$, where p and q represent the under and over estimates respectively.

End of questions

Question 4

(10 marks)

The acceleration of an object undergoing rectilinear motion is given by $a = 3t + 5 \text{ m s}^{-2}$. The object has an initial velocity of 20 m/s and begins its motion at a displacement of -10 m .

(a) Determine:

(i) the velocity at $t = 3$ seconds. (3 marks)

$$v(t) = \frac{3t^2}{2} + 5t + c$$

$$\text{but } v(0) = 20$$

$$\therefore 20 = c$$

$$v(t) = \frac{3t^2}{2} + 5t + 20$$

$$v(3) = 48.5 \text{ m/s}$$

(ii) the displacement at $t = 3$ seconds. (3 marks)

$$x(t) = \int \left(\frac{3t^2}{2} + 5t + 20 \right) dt \Rightarrow \frac{t^3}{2} + \frac{5t^2}{2} + 20t - 10$$

$$\therefore x(3) = 86 \text{ m}$$

(b) Show that the object is never at rest. (2 marks)

$$\text{At rest if } v = 0$$

$$\text{but } \frac{3t^2}{2} + 5t + 20 \neq 0, \forall t$$

$$\text{or } \Delta < 0$$

$$\Delta = -95$$

Hence, or otherwise,

(c) determine the distance travelled by the object in the first three seconds. (2 marks)

$$\begin{array}{l} \text{At } t=0 \text{ object @ } -10 \text{ m} \\ t=3 \text{ object @ } 86 \text{ m} \end{array} \left. \begin{array}{l} \text{No turns} \\ \text{between} \end{array} \right\}$$

$$\therefore \text{Dist in 1st 3 sec} = 96 \text{ m}$$

See next page

Additional working space

Question number: _____

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See next page



Christ Church
Grammar School

2019
TEST 2

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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Christ Church
Grammar School

2019
UNIT TEST 2

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section

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Additional working space

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Question 4

(10 marks)

The acceleration of an object undergoing rectilinear motion is given by $a = 3t + 5 \text{ ms}^{-2}$. The object has an initial velocity of 20 m/s and begins its motion at a displacement of -10 m .

(a) Determine:

(i) the velocity at $t = 3$ seconds.

(3 marks)

(ii) the displacement at $t = 3$ seconds.

(3 marks)

(b) Show that the object is never at rest.

(2 marks)

(c) Hence, or otherwise, determine the distance travelled by the object in the first three seconds.

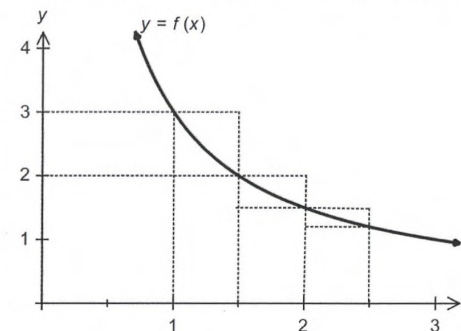
(2 marks)

See next page

Question 3

(3 marks)

Consider the graph of $f(x)$, and the table of values of $f(x)$ for certain x values.



x	1	1.5	2	2.5
$f(x)$	3	2	1.5	1.2

By considering the rectangles drawn on the graph, calculate the value of p and q , where:

$$p < \int_1^{2.5} f(x) dx < q$$

$$p = 0.5 [2 + 1.5 + 1.2]$$

$$p = \frac{1}{2} (4.7)$$

$$p = 2.35$$

$$q = 0.5 [3 + 2 + 1.5]$$

$$q = \frac{1}{2} (6.5)$$

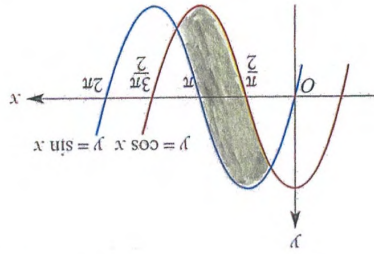
$$q = 3.25$$

1mk for one correct
3mks for both with working

End of questions

Question 2

Find the exact area of the shaded region below.



(3 marks)

See next page

3

$$Area = \frac{14\sqrt{2}}{2} = 7\sqrt{2} \text{ square units.} \quad \checkmark \text{ (Correct Area)}$$

MUST KNOW EXACT VALUES!

$$= \left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) \right] = \left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right] = 2\sqrt{2}$$

$$\therefore \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx = \left[-\cos x - \sin x \right]_{\pi/4}^{5\pi/4} = \left[-\cos\left(\frac{5\pi}{4}\right) - \sin\left(\frac{5\pi}{4}\right) - \left(-\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right) \right) \right]$$

$$\cos x = \sin x \text{ when } x = \frac{\pi}{4}, \frac{5\pi}{4}$$

(Differentiates)

(Sets up integral)

See next page

Question 5

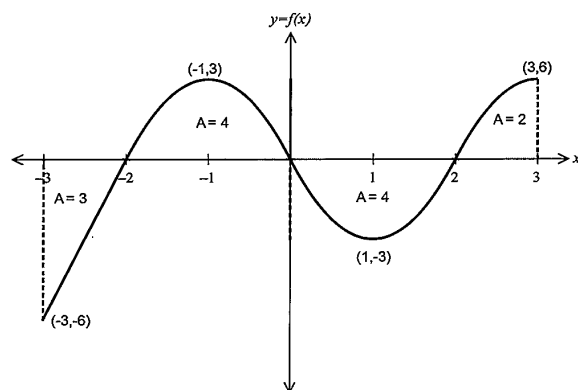
(3 marks)

Calculate the area of the regions enclosed by the curves $y = \cos(x)$ and $y = \sin(2x)$ for $0 \leq x \leq \frac{\pi}{2}$. Include a sketch as part of your solution.

Question 6

(7 marks)

The graph of the function $f(x)$ is shown below for $-3 \leq x \leq 3$. The areas, (A), enclosed between the graph, the x -axis and the lines $x = -3$ and $x = 3$ are marked in the appropriate regions.



Determine:

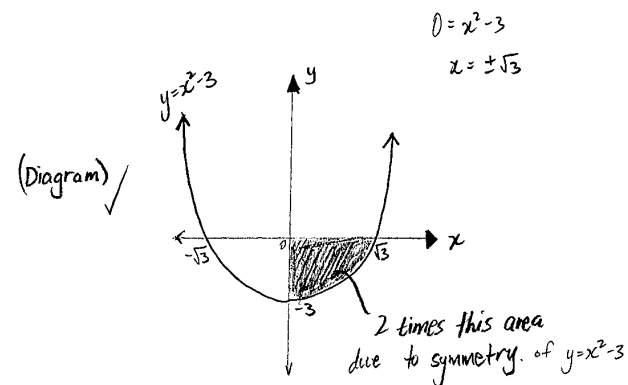
- (a) (i) the value of $\int_{-3}^2 f(x) dx$ (2 marks)

- (ii) the area enclosed between the graph of $f(x)$ and the x axis, from $x = -3$ to $x = 2$. (2 marks)

See next page

Question 1 continued

- (c) Find the exact area between the curve $y = x^2 - 3$ and the x -axis. Include a diagram as part of your solution. (4 marks)



(Correct Integral) ✓

(Integrates Correct) ✓

$$\therefore 2 \times \int_0^{\sqrt{3}} x^2 - 3 dx$$

$$= \left[\frac{x^3}{3} - 3x \right]_0^{\sqrt{3}}$$

$$= 2 \times \left(\frac{3\sqrt{3}}{3} - \frac{3\sqrt{3}}{1} - 0 \right)$$

$$= 2 \times \left(-\frac{6\sqrt{3}}{3} \right)$$

$$\therefore \text{Area} = 4\sqrt{3} \text{ units}^2$$

$$\text{or } \int_{-\sqrt{3}}^{\sqrt{3}} x^2 - 3 dx$$

$$= \left[\frac{x^3}{3} - \frac{3x}{1} \right]_{-\sqrt{3}}^{\sqrt{3}}$$

$$= \left[\frac{x^3 - 9x}{3} \right]_{-\sqrt{3}}^{\sqrt{3}}$$

$$= \frac{3\sqrt{3} - 9\sqrt{3}}{3} - \left(\frac{-3\sqrt{3} + 9\sqrt{3}}{3} \right)$$

$$= \frac{6\sqrt{3} - 18\sqrt{3}}{3}$$

$$= \frac{-12\sqrt{3}}{3}$$

$$\therefore \text{Area} = 4\sqrt{3} \text{ units}^2$$

See next page

4

Question 1

Evaluate each of the following integrals (Leave answers with positive indices):

(9 marks)

(a) $\int \frac{x}{2} - \sqrt[3]{x} \, dx$

(2 marks)

$$\int 2x^{\frac{1}{2}} - x^{\frac{1}{3}} \, dx$$

$$= 2x^{\frac{1}{2} + 1} \cdot \frac{1}{\frac{1}{2} + 1} - \frac{x^{\frac{1}{3} + 1}}{\frac{1}{3} + 1} + C$$

(Integrates correctly)

$$= 4x^{\frac{3}{2}} - 3x^{\frac{4}{3}} + C$$

(positive indices)

(b) $\int \frac{x^3 - 1}{x^3 - 4x} \, dx$

(3 marks)

$$= \frac{1}{4} \int \frac{4x^{\frac{3}{3}} - 4}{4x^{\frac{3}{3}} - 4} \, dx$$

Adjusts numerator

$$\text{If } u = (x^{\frac{4}{3}} - 4x)^{-2} \quad \frac{du}{dx} = -2(x^{\frac{4}{3}} - 4x)^{-3} (4x^{\frac{1}{3}} - 4)$$

$$= \frac{1}{4} \times \frac{1}{-2} (x^{\frac{4}{3}} - 4x)^{-2} + C$$

$$= \frac{1}{4} \times \frac{1}{-2} \frac{(x^{\frac{4}{3}} - 4x)^{-2}}{(4x^{\frac{1}{3}} - 4)^{-3}}$$

$$= -\frac{1}{8} \frac{(x^{\frac{4}{3}} - 4x)^{-2}}{(4x^{\frac{1}{3}} - 4)^{-3}} + C$$

(positive indices)

See next page

5

(b) $\int_0^2 (x - f(x)) \, dx$

(3 marks)

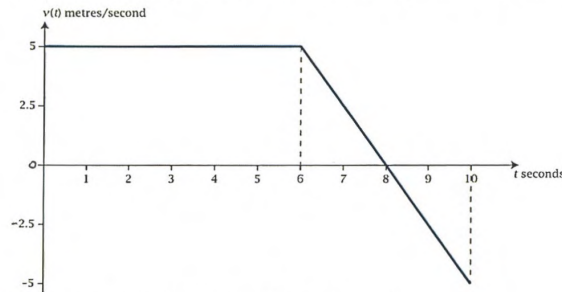
Question 6 continued

See next page

Question 7

(7 marks)

A particle moves along a straight line. The velocity – time graph is shown below.



(a) Find the velocity of the particle when $t = 3$. (1 mark)

(b) Write an expression for v in terms of t for $6 \leq t \leq 10$. (2 marks)

(c) Find the acceleration of the particle when

(i) $t = 1$ (1 mark)

(i) $t = 7$ (1 mark)

(d) Find the change in displacement for $0 \leq t \leq 10$. (2 marks)

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2019
UNIT TEST 2

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Student name _____
SOLUTIONS

Teacher name _____

Time and marks available for this section
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MATHEMATICS METHODS Year 12 **8** **CALCULATOR-ASSUMED**
Question 8 **(3 marks)**

The area enclosed by the line $y = mx$ and the parabola $y = x^2$ is 24.813. Find the value of m , correct to 1 decimal place, where $m > 0$.

End of questions

MATHEMATICS METHODS Year 12 9

CALCULATOR-ASSUMED

Additional working space

Question number: _____

MATHEMATICS METHODS Year 12 10

CALCULATOR-ASSUMED

Additional working space

Question number: _____