

# PERTH MODERN SCHOOL

## UNIT 3CD MAS – 2014

### TEST 1

#### POLAR COORDINATES, COMPLEX NUMBERS & VECTORS

NAME: \_\_\_\_\_

DATE: Thurs. 13<sup>th</sup> Feb.

Total: 43 marks

Time: 45 min.

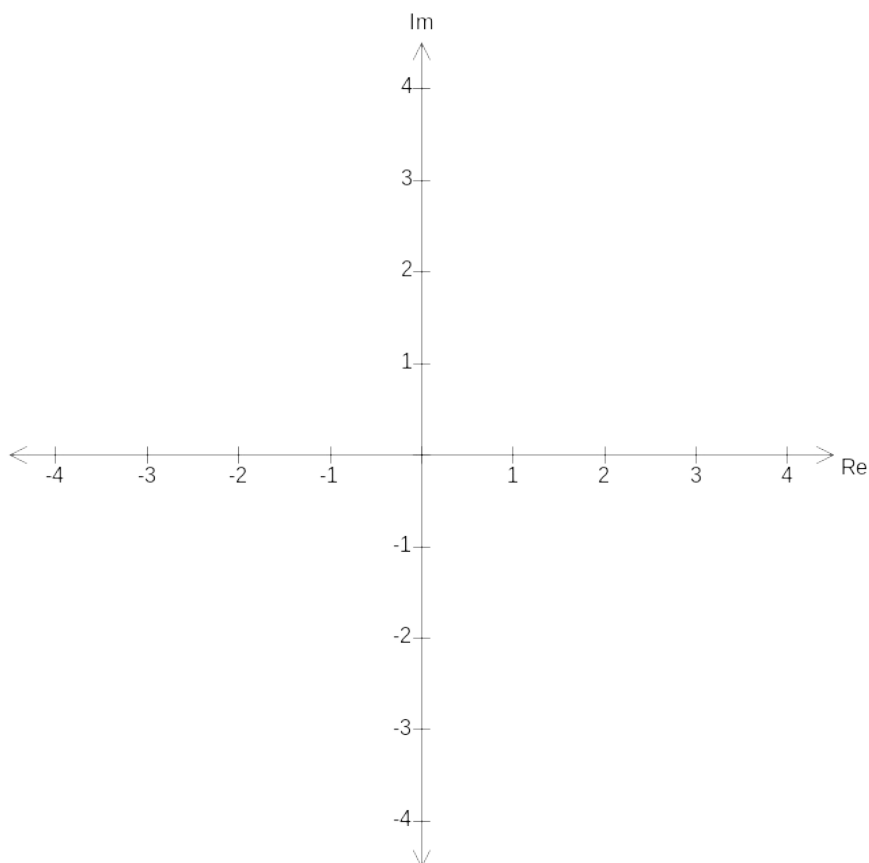
1.  $Z$  is a complex number. Sketch the region given by

[5]

$$\operatorname{Re}(Z) < 1 \quad \text{and} \quad \operatorname{Im}(Z) > -2$$

$$\text{and} \quad 1 < |Z| < 3$$

$$\text{and} \quad -\frac{5\pi}{12} \leq \operatorname{Arg} Z \leq \frac{2\pi}{3}$$



2. Express  $Z = -1 - \sqrt{3}i$  in polar form.

[2]

3. If  $Z_1 = 5\text{cis}\frac{\pi}{6}$  and  $Z_2 = 2\text{cis}\frac{\pi}{12}$ , then prove  $Z_1Z_2 = 5\sqrt{2}(1 + i)$

[4]

4. Find  $Z$  if  $Z\bar{Z} + 2Z = \frac{1}{4} + i$

[6]

5. Consider the 3D position vectors

[8]

$$\overrightarrow{OA} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$$

Determine:

a) the length of  $\overrightarrow{AB}$ .

b)  $\angle AOB$  to the nearest degree.

c) the vector equation of the line, in parametric form, through the points A and B.

6.

[8]

A has the rectangular coordinates  $(-1, \sqrt{3})$  and B has polar coordinates  $\left(4, \frac{5\pi}{4}\right)$ .

a) What are the exact polar coordinates of A? (1)

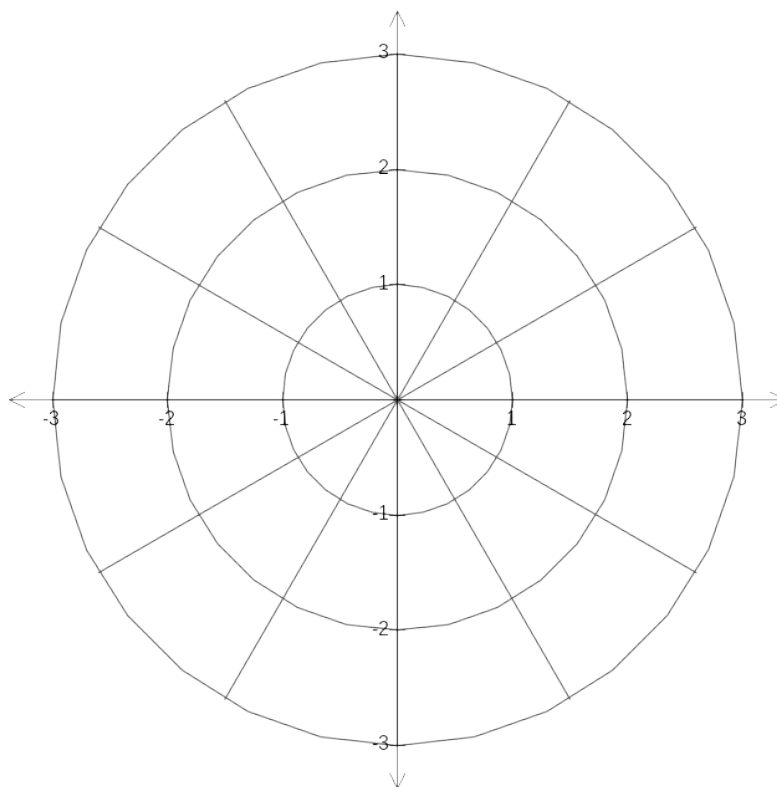
b) What are the exact rectangular coordinates of B? (2)

c) The graph of the polar equation  $r = k\theta$  passes through the point B.

If  $k > 0$ , determine the value of  $k$ .

Then, on the axes below, sketch the graph of  $r = k\theta$  for  $0 \leq \theta \leq \pi$ ,

showing important features. (5)



7.

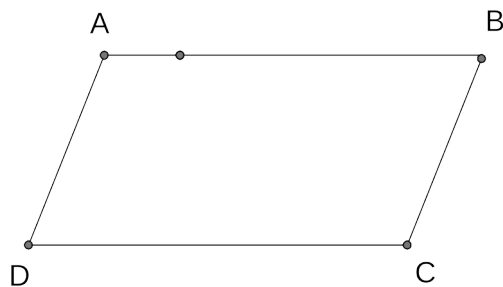
[10]

$ABCD$  is a parallelogram with points  $E$  and  $F$  such that  $\overrightarrow{AE} : \overrightarrow{EB} = 1 : 4$  and  $\overrightarrow{BF} : \overrightarrow{FC} = 3 : 1$ .

$\overrightarrow{ED}$  and  $\overrightarrow{AF}$  intersect each other at  $G$ .

Let  $\overrightarrow{AB} = a$  and  $\overrightarrow{AD} = d$ .

a) Complete the diagram below with the information given above. (2)



- b) Determine the ratios in which  $\overrightarrow{AF}$  and  $\overrightarrow{ED}$  intersect each other, if the intersection point is at G. (8)