

Marks	Question	TOTAL
1		
2		
3		
4		
5		
6		
7		

You, hand it to the supervisor before reading any further.

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material if you have any unauthorised material with

Important note to candidates

Special items: nil

Standard items: pens (blue/black preferred), pencil (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate

Formula sheet

This Question/Answer booklet

Materials required/recommended for this section

Working time: fifty minutes

Reading time before commencing work:

five minutes

Time allowed for this section

Teacher's Name:

Name:

Name:

Calculator-free

Section One:

UNIT 1 AND 2

MATHEMATICS METHODS

Question/Answer booklet

Semester Two Examination, 2019



INDEPENDENT PUBLIC SCHOOL

Exceptional schooling. Exceptional students.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	
Section Two: Calculator-assumed			100		
Total				100	

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

✓ equating to 0
✓ calculating a

- b) Determine whether the particle reached a local maximum or minimum displacement at $t=2$, show all working. (2 marks)

Solution
$x''(t)=6at$ $x''(2)=6\left(\frac{-1}{2}\right)(2)$ $\therefore -6$ \therefore the particle reaches a local maximum at $t=2$
✓ double differentiation / sign test ✓ local maximum at $t=2$

End of questions

$\begin{aligned} & 6 = 2x \\ & x + 6 = 3x \\ & x + 3 = 3x - 3 \\ & 2x + 3 - x = 5x - (2x + 3) \end{aligned}$	Solution
--------------------------------------------------------------------------------------------------------------	-----------------

(3 marks)

- b) If x , $2x+3$, and $5x$ are the first three terms of an arithmetic sequence, calculate the value of x .

$\begin{aligned} & 16000 = 0.02 \text{ or } \frac{2}{100} \text{ or } 2 \times 10^{-2} \\ & 800000 = 0.02 \text{ or } \frac{2}{100} \text{ or } 2 \times 10^{-2} \end{aligned}$	Solution
---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-----------------

(2 marks)

$$\text{ii)} \quad 16 \times 10^3 \div (8 \times 10^5)$$

$\begin{aligned} & 2 \times 5 = 10 \\ & \text{Simplifying both terms} \\ & \text{correct answer} \end{aligned}$	Solution
-----------------------------------------------------------------------------------------------------------------	-----------------

(2 marks)

$$\text{i)} \quad \frac{8}{3} \times 25^{0.5}$$

a) Calculate the value of:

Question 1

Working time: 50 minutes.

- In the original answer space where the answer is continued, i.e. give the page number, indicate in the question number of the question that you are continuing to answer at the top of the page.
- Continuing an answer: if you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number, indicate in the question number of the question that you are continuing to answer at the top of the page.
 - Planning: if you use the spare pages for planning, indicate this clearly at the top of the page.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- This section has **seven (7)** questions. Answer all questions. Write your answers in the spaces provided.

(52 Marks)

Section One: Calculator-free

$\begin{aligned} & x(1) = 3at + 6 \\ & x(2) = 0 \text{ since particle is changing direction} \\ & 3a(2)^2 + 6 = 0 \\ & a = -\frac{2}{3} \end{aligned}$	Solution
--------------------------------------------------------------------------------------------------------------------------------------------------------	-----------------

- a) If the particle changes direction when $t=2$, calculate the value of a . (3 marks)

$$x(t) = at^3 + 6t + 8$$

The displacement, x , of a particle is given by:

(5 marks)

Question 20

(3 marks)

a.

b.

c.

- ii) Is checked in at the hotel and had breakfast elsewhere, given that he/she had dinner in the hotel. (2 marks)

$\begin{aligned} & P(C \cup B') = 0.12 \\ & P(C \cap B') = 0.03 \\ & \text{correct denominator} \end{aligned}$	Solution
----------------------------------------------------------------------------------------------------------------	-----------------

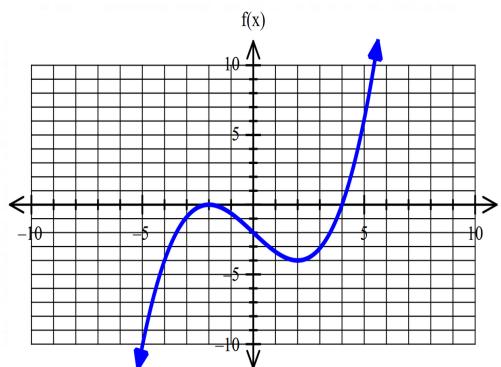
$3=x$

- setting up the equation
- correct working
- correct answer

Question 2

(10 marks)

The graph $f(x)=ax^3 - \frac{3}{2}x + b$ is shown below.



a) Determine the values of the coefficients a and b .

(3 marks)

uses independence law

answer

b) Complete the Venn Diagram to illustrate the information above.

(4 marks)

Solution
<input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> for each correct answer

Solution

$$b = -2$$

substitute(-2, 0)

$$0 = a(-2)^3 - \frac{3}{2}(-2) - \cancel{b}$$

$$0 = -8a + 3 - 2$$

$$8a = 1$$

$$a = \frac{1}{8}$$

finding b

substituting a point into $f(x)$ correctly

correct value for a

c) Determine the probability that this person:

i) stayed at the hotel and had breakfast elsewhere.

(2 marks)

Solution

$$P(C|B) = 0.3 + 0.12$$

$$\cancel{0.42}$$

$$\checkmark 0.3$$

0.12 (-1 mark for additional numbers)

b) State the interval(s) where $f(x)$ is decreasing.

(2 marks)

Solution

<input type="checkbox"/> $y = 0$	<input type="checkbox"/> $y = -4$
Solution	Solution

- d) Determine the value(s) for y where there are exactly two solutions to the equation $y = f(x)$. (2 marks)

<input type="checkbox"/> correct x -value	<input type="checkbox"/> correct y -value
(4, 12)	

- (ii) $-3f(x-2)$
 (iii) $f(|x|+5)$ (2 marks)

<input type="checkbox"/> correct coordinate
(2, 1)

- c) The point $(2, -4)$ lies on the function $f(x)$. State the new coordinates under the following transformations.
- (1 mark)

<input type="checkbox"/> between -2 and 2	<input type="checkbox"/> correct inequality signs
$-2 < x < 2$ OR $(-2, 2)$	

<input type="checkbox"/> $p(D) = 0.4$
<input type="checkbox"/> $0.5 \times p(D) = 0.2$
Solution

- a) What is the probability that a person selected had dinner at the hotel? (2 marks)

- D - had dinner at the hotel.
 C - checked in guests.
 B - had breakfast at the hotel.

Let the following variables represent the probability for the following events:

Crown Towers or had breakfast in the hotel is 0.5, while the probability that they had breakfast and dinner is 0.2. The probability that this person is a checked-in guest and had dinner at one of the restaurants inside the hotel is 0.32. It is thought that a person having breakfast or dinner are independent events.

At the Crown Towers, checked-in guests can choose to have breakfast and dinner at the hotel. Walk-in guests can book to have dinner. On a certain day, a person who resides at

Crown Towers or had dinner at the hotel is selected at random. The probability that this person had breakfast in the hotel is 0.5, while the probability that they had breakfast and

dinner is 0.2. The probability that this person is a checked-in guest and had dinner at one of the restaurants inside the hotel is 0.32. It is thought that a person having breakfast or dinner

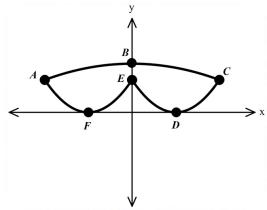
are independent events.

Question 19

Question 3

(5 marks)

The design for the A380 Qantas sleeping eye mask is illustrated below:



The curves that form the outline are modelled by the equations:

$$y = 0.25(x+4)^2$$

$$y = \frac{-1}{32}x^2 + 6$$

$$y = 0.25(x-4)^2$$

- a) Which equation determines the curve ABC? (1 mark)

Solution

$$y = \frac{-1}{32}x^2 + 6$$

 correct equation

- b) Which equation determines the curve AFE? (1 mark)

Solution

$$y = 0.25(x+4)^2$$

 correct equation

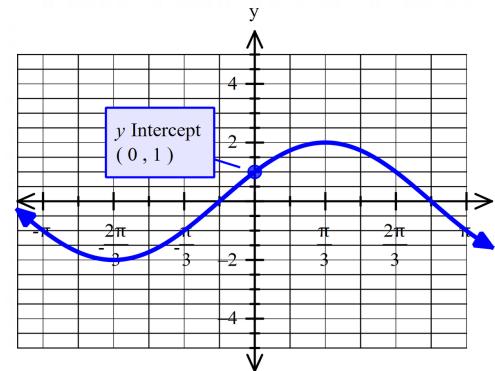
- c) If D lies on the axis of symmetry for curve EDC determine its coordinate. (1 mark)

Solution

$$(4, 0)$$

 correct coordinate

- b) Given that the curve below is represented by
- $y = a\sin(x+b)$
- , determine possible constants for
- a
- and
- b
- . (2 marks)

**Solution**

$$y = 2\sin\left(x + \frac{\pi}{6}\right)$$

-
- correct value for
- a
-
-
- correct value for
- b

- c) A cosine curve
- $y = f(x)$
- has a period of
- 2π
- with a mean line of
- $y=5$
- and a phase shift of
- $\frac{\pi}{3}$
- . Given that
- y
- has a minimum value of 1, find a possible equation of this curve. (3 marks)

Solution

$$y = a\cos\left(x + b\right) + c$$

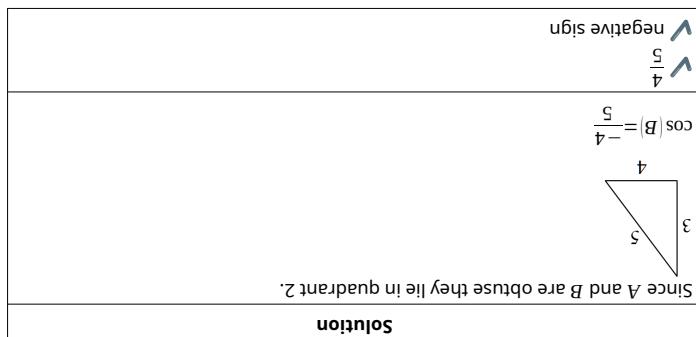
$$y = 4\cos\left(x - \frac{\pi}{3}\right) + 5 \text{ or } y = -4\cos\left(x + \frac{2\pi}{3}\right) + 5$$

-
- correct value for
- a
-
-
- correct value for
- b
-
-
- correct value for
- c

$\sin(n+A) = -\sin(A)$
Solution

(2 marks)

b) $\sin(n+A)$

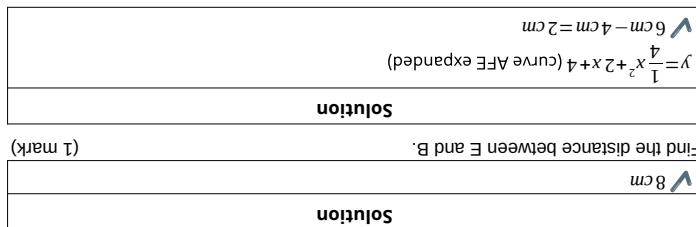


(2 marks)

a) $\cos(B)$

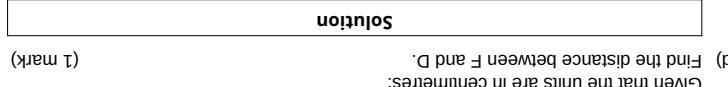
If $\sin(A) = 12/13$ and $\sin(B) = 3/5$ and both A and B are obtuse angles, find:

Question 4 (7 marks)



(1 mark)

e) Find the distance between E and B.



(1 mark)

d) Find the distance between F and D.

Given that the units are in centimetres:

Determine $P(S)$.

$\frac{d}{q} - a$	for each part
ii) b iii) a iv) $-a$	

(1 mark)

iv) $\sin(90-\theta)$

(1 mark)

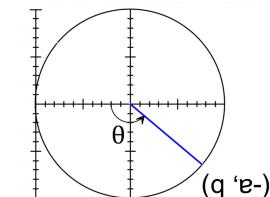
iii) $\cos(180-\theta)$

(1 mark)

ii) $\tan\theta$

(1 mark)

i) $\sin\theta$



a) Given the unit circle below determine the value (in terms of a and/or b) of:

Question 18 (9 marks)

$P(S) = 0.5$	Uses independence law
$P(S \cup T) = P(S) + P(T) - P(S \cap T) = 0.6 + 0.2 - 0.2 \cdot 0.8 = 0.7$	Substitutes for values except P(S)
$P(S \cup T) = P(S) + P(T) - P(S \cap T)$ (addition law)	Uses addition law
$P(S \cap T) = P(S) \times P(T)$ (independent)	Evaluates P(T)
The events S and T are independent.	

b) S and T are independent events such that $P(T) = \frac{5}{1}$ and $P(S \cup T) = \frac{5}{3}$. (4 marks)

$\frac{12}{13}$

✓ $\frac{12}{13}$
✓ negative sign

c) $\sin(A-B)$

(3 marks)

Solution

$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\frac{12}{13}\left(-\frac{4}{5}\right) - \left(-\frac{5}{13}\right)\left(\frac{3}{5}\right)$$

$$-\frac{33}{65}$$

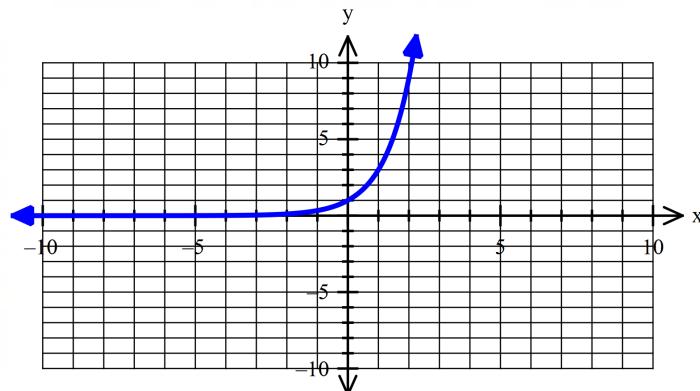
✓ use the compound formula
✓ use correct signs for cosines
✓ evaluate

Question 5

(10 marks)

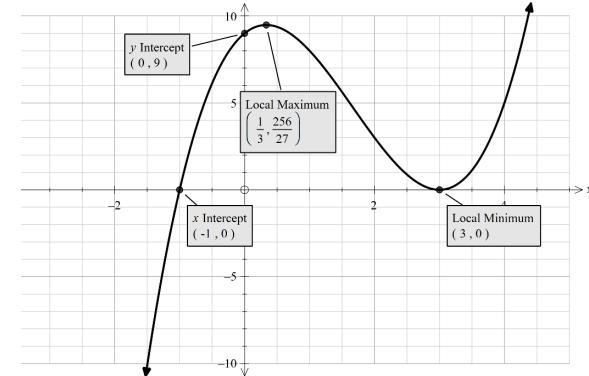
a) Sketch the graph of $y=3^x$ on the axes below.

(2 marks)



Solution

c) Sketch the graph of $f(x)$ showing all important features of the graph. (3 marks)



✓ shape
✓ maximum
✓ intercepts

Question 17

a) A bag contains five similar sized cards, each with a different digit on it. The digits are 2, 3, 4, 5 and 6. Three cards are removed at random from the bag and placed next to each other to form a number.

(i) How many different numbers can be made? (1 mark)

Solution

$$5 \times 4 \times 3 = 60$$

✓ correct number

(ii) What is the probability that the number does not contain the digit 4? (1 mark)

Solution

$$\frac{4 \times 3 \times 2}{60} = \frac{2}{5}$$

✓ correct probability

(iii) What is the probability that the number is a multiple of 5, given that the first digit chosen is an even number? (2 marks)

Solution

$$\frac{(1 \times 3 \times 1) \times 3}{(1 \times 4 \times 3) \times 3} = \frac{1}{4}$$

✓ correct numerator

✓ correct denominator

Solution

$$(3^x - 3)(3^x + 2) = 0$$

$$3^x - 3 = 0 \quad \text{Let } y = 3^x$$

$$y - 2(y - 1) = 0$$

$$y - 3y + 2 = 0$$

$$\boxed{(y-2)(y-1)=0}$$

Recognising the quadratic equation in terms of 3^x :
Substituting $y = 3^x$:
Correct factorising

- c) Show that the equation $9^x - 3^{x+1} = -2$ can be written as $|y-1|(y-2) = 0$. (3 marks)

Solution

$$f(x) = x^3 - 5x^2 + 3x + 9$$

$$c = 9$$

$$3 = 8 - 20 + 6 + c$$

$$f(x) = x^3 - 5x^2 + 3x + c$$

$$f(x) = x^3 - 10x + 3$$

Horizontal translation one unit to the left
Horizontal dilation with a scale factor of $\frac{1}{2}$

- ii) the graph of $y = 3^x$ onto the graph of $y = 3^{2x}$. (1 mark)

Solution

$$f(x) = (x-3)(3x-1)$$

$$c = 9$$

$$3 = 8 - 20 + 6 + c$$

$$f(x) = x^3 - 5x^2 + 3x + c$$

$$f(x) = x^3 - 10x + 3$$

Antiderivative
Substituting coordinate $(2, 3)$ to find c .
Finding $f(x)$

Solution

$$f(-1) = 0$$

$$f(-1) = (-1)^2 - 5(-1)^2 + 3(-1) + 9$$

$$x^2 - 5x^2 + 3x + 9 \div (x+1) = x^2 - 6x + 9$$

$$\therefore (x+1) \text{ is a factor}$$

$$?x-3?$$

$$\therefore (x+1) \text{ is a factor of } (x-3)^2$$

$$\text{Finding the second factor } (x-3)^2$$

- b) Show clearly that the function can be expressed in the form $(x+1)(x-3)^2$. (2 marks)

Solution

$$f(x) = x^3 - 5x^2 + 3x + 9$$

$$c = 9$$

$$3 = 8 - 20 + 6 + c$$

$$f(x) = x^3 - 5x^2 + 3x + c$$

$$f(x) = x^3 - 10x + 3$$

Antiderivative
Substituting coordinate $(2, 3)$ to find c .
Finding $f(x)$

- a) Find an equation of the curve, giving your answer as a polynomial in simplest form. (3 marks)

- i) the graph of $y = 3^x$ onto the graph of $y = 3^{2x}$, (2 marks)

Solution

$$f(x) = (x-3)(3x-1)$$

$$c = 9$$

$$3 = 8 - 20 + 6 + c$$

$$f(x) = x^3 - 5x^2 + 3x + c$$

$$f(x) = x^3 - 10x + 3$$

Curve is given by
A curve with the equation $f(x)$ passes through the point $(2, 3)$. The gradient function of this curve is given by

Question 16

show expansion

- d) Solve the exponential equation in part c) giving approximate solution(s) where necessary. (2 marks)

Solution
$y=1$ and $y=2$ $3^x=1$ and $3^x=2$ $x=0$ ✓ and $x=0.6$ ✓
✓ $x=0$ ✓ accept a solution between (0.5 – 0.8) (ft as they are reading of their graph in part a))

- c) Use differentiation to find the maximum possible area of ABCD. (4 marks)

d) Solution
Max/Min when $\frac{dA}{dx}=0$ $\frac{dA}{dx}=63-2x$ Hence when $x=31.5$ $Area=63 \times 31.5 - 31.5^2 - 440$ $= 552.25$ sq cm
(need to show it is the max using either the sign test or second derivative)
✓ Finds derivative ✓ Puts derivative =0 and solves for x ✓ Finds area ✓ Shows that this is max – sign test or second derivative

Question 6

(8 marks)

- a) Simplify

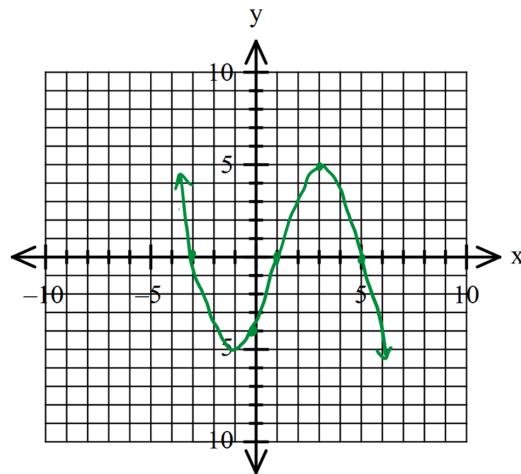
(i) $\frac{d}{dx} [5x^3 - 4x^{-1} + 7].$ (1 mark)

Solution
✓ $15x^2 + 4x^{-2}$

(ii) $\lim_{h \rightarrow 0} \frac{3(x+h)^2 - 3x^2}{h}.$ (1 mark)

Solution

<p>Question 15</p> <p>rectangle ABCD is enclosed within another rectangle PQRS so that the sides AB and PQ are parallel and 2 cm apart. The vertical distance between sides CD and RS is 3 cm and the horizontal distances between sides BC and QR and sides CD and RS are both 4 cm.</p> <p>(iii) $\int (2x-1)^2 dx$ (2 marks)</p> <p>Solution</p> <p>$\sqrt{4x^2 - 4x + 1}$ (expanding $(2x-1)^2$)</p> <p>$\frac{3}{4x^3 - 2x^2 + x + c}$ (must include $+c$ for the second mark and SIMPLIFIED)</p> <p>rectangle ABCD is enclosed within another rectangle PQRS so that the sides AB and PQ are parallel and 2 cm apart. The vertical distance between sides CD and RS is 3 cm and the horizontal distances between sides BC and QR and sides CD and RS are both 4 cm.</p> <p>Let $PQ = x$ and $PS = y$. The perimeter of PQRS is 120 cm.</p> <p>Solution</p> <p>$P_{(rect)} = 2L + 2W$</p> <p>$\therefore 120 = 2x + 2y$</p> <p>Explains</p> <p>$2x + 2y = 120$ (1 mark)</p> <p>b) Sketch the graph of a function that satisfies all of the conditions stated below. (You do not need to determine the equation of such a function) (4 marks)</p> <p>• The function intersects the x-axis at $(-3, 0)$, $(1, 0)$ and $(5, 0)$ only.</p> <p>• The function intercepts the y-axis at $(0, -4)$.</p> <p>• The gradient of the function is zero for $x = -1$ and $x = 3$.</p> <p>• For $-1 < x < 3$ the gradient is always positive.</p> <p>• For $x \rightarrow 1$ and $x \rightarrow 3$ the gradient is always negative.</p> <p>Solution</p> <p>$AB = x - 8$ $AD = y - 5$</p> <p>$\therefore \text{Area} = (x-8)(55-x) = 63x - x^2 - 440$</p> <p>$\rightarrow AD = 60 - x - 5 = 55 - x$</p> <p>From (a) $y = 60 - x$</p> <p>Explains</p> <p>$\sqrt{\text{Length and width in terms of } x \text{ and } y}$</p> <p>$\sqrt{\text{substitutes for } y}$</p>	<p>b) Show that the area of rectangle ABCD $= 63x - x^2 - 440$ (3 marks)</p> <p>Solution</p> <p>$\therefore \text{Area} = (x-8)(55-x) = 63x - x^2 - 440$</p> <p>$\rightarrow AD = 60 - x - 5 = 55 - x$</p> <p>From (a) $y = 60 - x$</p> <p>Explains</p> <p>$\sqrt{\text{Length and width in terms of } x \text{ and } y}$</p> <p>$\sqrt{\text{substitutes for } y}$</p>
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Solution

- ✓ correct x and y intercepts
- ✓ maximum point at $x=3$ and a minimum point at $x=-1$
- ✓ For $-1 < x < 3$ the gradient is always positive.
- ✓ For $x < -1$ and $x > 3$ the gradient is always negative.

Question 14
(6 marks)

The town council has been trying to control the wild rabbit population after it blew out of control before 2010. Their observations of the population since control measures were put in place in 2010 have been recorded below.

Year	2010	2012	2014
Population	60 000	48 600	39 366

- a) Determine an equation in the form $P=a(b)^t$ that models the population of the species for t years after 2010. (3 marks)

Solution
$b^2 = \frac{48600}{60000}$
$b^2 = 0.81$
$b = 0.9$
$P(t) = 60000 \times 0.9^t$
✓ determining b^2 ratio
✓ determining b
✓ determining equation

- b) By what percentage is the population decreasing each year? (1 mark)

Solution
10%
✓ correct answer (allow ft)

Question 7
(5 marks)

By using a suitable binomial expansion, calculate $(1.5)^5$.

Solution

$$\begin{aligned} &\text{Red: } i \left(1 + \frac{1}{2}\right)^5 \quad \checkmark \text{ express as a binomial} \\ &\text{Red: } i 1^5 + 5(1)^4 \left(\frac{1}{2}\right) + 10(1)^3 \left(\frac{1}{2}\right)^2 + 10(1)^2 \left(\frac{1}{2}\right)^3 + 5(1) \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5 \quad \checkmark \text{ Identifies correct} \end{aligned}$$

- c) What will be the population in 2020? Give your answer in scientific notation to 2 decimal places. (2 marks)

Solution
$P(10) = 20920.7\dots$
$P(10) = 2.09 \times 10^4$
✓ correct answer (allow ft)

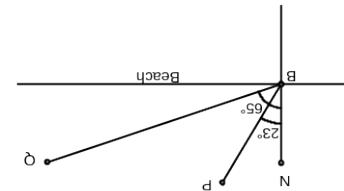
Question 13

(5 marks)

A shark is sighted at P, 800m from the beach on a bearing of 023° from B. A few minutes later it is sighted at Q, 1100m from B on a bearing of 065° heading back towards P.

terms and powers and uses correct coefficients
 $\textcolor{red}{1} + \frac{5}{2} + \frac{10}{4} + \frac{8}{10} + \frac{16}{5} + \frac{32}{1}$ expands and simplifies
 $\textcolor{red}{32} + 80 + 80 + 40 + 10 + 1$ evaluates fraction out of 32
 $\textcolor{red}{243}$ correct answer
 $\textcolor{red}{**DO NOT ACCEPT } \frac{3}{5}$. Students must use an appropriate binomial expansion.
 $\textcolor{red}{Award 0 marks if they evaluate } \frac{3}{5}$.

a) What is the size of angle PBQ?	Solution	(1 mark)
		correct



b) How far is it from P to Q?	Solution	(2 marks)
		correctly states cosine rule $PQ = \sqrt{BP^2 + BQ^2 - 2 \times BP \times BQ \cos \angle PBQ}$ $\textcolor{red}{?} \sqrt{800^2 + 1100^2 - 2 \times 800 \times 1100 \cos 42^\circ}$ $= 736\text{km}$

c) What is the area of the search region?	Solution	(2 marks)
		correctly states area rule $\textcolor{red}{Area} = \frac{1}{2} \times BP \times BQ \times \sin \angle PBQ$ $= 29447.5\text{d km}$

END OF SECTION

(97 Marks)

Question 8

Section Two: Calculator-assumed

(7 marks)

- a) Are the straight lines given by $3x+4y=12$ and $y=0.75x+1.25$ parallel, perpendicular or neither? Justify your answer. (2 marks)

Solution
Gradient of $3x+4y=12$ is -0.75 Gradient of $y=0.75x+1.25$ is 0.75 . Neither parallel (gradients not equal) nor perpendicular (gradients not negative reciprocals) ie $m_1 \neq m_2$ and $m_1 \times m_2 \neq -1$
<input checked="" type="checkbox"/> finds gradients <input checked="" type="checkbox"/> Compares and states neither

- b) Determine the equation of the straight line perpendicular to the line $y=8-\frac{1}{3}x$ and passing through the point $(2, 1)$. (3 marks)

Solution
Req'd $m=3$ General Form $y=3x + c$ Thro' $(2,1) \rightarrow c = -5$ Eq ⁿ is $y = 3x - 5$
<input checked="" type="checkbox"/> finds gradient <input checked="" type="checkbox"/> calculates c <input checked="" type="checkbox"/> determines eq ⁿ

- c) The point B(2, -3) is the midpoint of the line between the point A(1, -1) and point C. What are the coordinates of C? (2 marks)

Solution
$C(x,y) \rightarrow \frac{1+x}{2}=2 \quad \frac{-1+y}{2}=-3$ ie $x=3$ ie $y=-5$ Therefore C(3, -5)

- uses mid pt formula
 states coordinates of C

- e) Determine the total mass of recycled paper produced between 1998 and 2008, inclusive. Assume the increase in production continues to increase in the same way for at least that long. (3 marks)

Solution																		
$a=400 \quad d=35 \quad T_1=1998 \quad T_{11}=2008$ $n=11$ $S_{11}=\frac{11}{2}((2 \times 400)+(11-1) \times 35)$ $S_{11}=6325$ \therefore total mass is 6 325 000 tonnes																		
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<input checked="" type="checkbox"/> identifies $n=11$ <input checked="" type="checkbox"/> $S_{11}=6325$ <input checked="" type="checkbox"/> correct units (000 tonnes)																		

- f) During which year did the total amount produced since 1998 first exceed fourteen million tonnes? (2 marks)

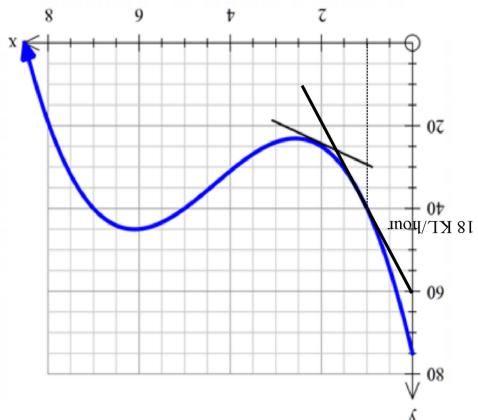
Solution																					
$a=400 \quad d=35 \quad S_n=14000$ $n=\underline{15}$ $14000=\frac{n}{2}((2 \times 400)+(n-1) \times 35)$ $n=19.4$ i.e. in the 20 th year \therefore during 2017																					
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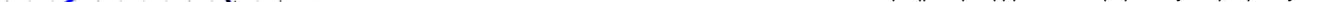
Question 9 (6 marks)

The volume of water in a storage tank changes with time as shown in the graph below. The volume is in kilolitres and the time is in hours from noon.

A weather balloon is released and allowed to float vertically upwards from the ground into the atmosphere. The height increase after 1 minute is 50 metres. Thereafter, its height

a)	the volume of water in the tank after seven and a half hours.	(1 mark)	Use the graph to estimate:
b)	the average rate of change of volume from the fourth to seventh hour.	(2 marks)	Correct answer
c)	the earliest time, to the nearest quarter of an hour, at which the instantaneous rate of decrease of volume of water is 5 litres per second.	(3 marks)	Calculates Ave
d)	the earliest time, to the nearest quarter of an hour, at which the instantaneous rate of increase of volume of water is 5 litres per second.	(3 marks)	Calculates Ave



a) State why the sequence of production of recycled paper could be described as arithmetic:


Year	T _n	400	435	470	505	540	k
Tonnes	T _n	1998	1999	2000	2001	2002	2003

The production of recycled paper in the country of Georgia has been increasing for some years. Since products begian the amount produced, in thousands of tonnes, over the last five years.

increase during each minute is 95% of the height of the height increase during the previous minute.

- a) Find the height increase during the 9th minute. (1 mark)

Solution		
<input checked="" type="checkbox"/> $a_{n+1}=0.95 \cdot a_n$		
n	a_n	Σa_n
5	40.725	226.22
6	38.689	264.91
7	36.755	301.66
8	34.917	336.58
9	33.171	369.75

✓ 33.17m

- b) What is the height of the balloon after 9 minutes? (2 marks)

Solution		
<input checked="" type="checkbox"/> $a_{n+1}=0.95 \cdot a_n$		
n	a_n	Σa_n
5	40.725	226.22
6	38.689	264.91
7	36.755	301.66
8	34.917	336.58
9	33.171	369.75

✓ 369.75m

- c) After how many minutes does the height of the balloon first exceed 650m? (1 mark)

Solution		
✓ After 21 minutes		
n	a_n	Σa_n
16	23.165	559.87
17	22.447	582.85
18	20.900	602.75
19	19.481	622.65
20	18.168	641.51
21	17.924	659.44

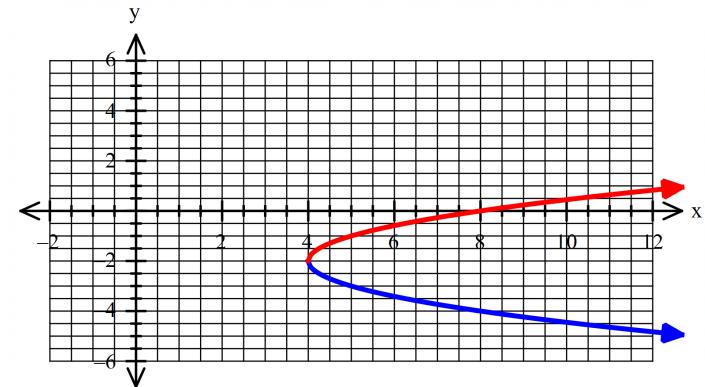
- d) Does the balloon ever reach an altitude of 1 km? Justify your answer. (2 marks)

Solution		
No as Sum to infinity = 1 km. Balloon approaches an altitude of 1 km but never reaches 1 km.		
✓ No		
✓ Justification		

Question 11

(8 marks)

- a) Sketch the function $f(x) = -\sqrt{x-4} - 2$ on the axes below. (2 marks)



- b) Transform the function $f(x)$ by a reflection in the x -axis and a vertical translation of 4 units in the negative direction of the y -axis and sketch the new function, $g(x)$, on the axes above. (2 marks)

Solution		
See red function		
✓ vertex (4, -2)		
✓ shape		

- c) Write an equation which represents the combined relationship of the two graphs drawn. (2 marks)

Solution		
$x = y+2 ^2 + 4$		
✓ $ y+2 ^2$		
✓ +4		

- d) Does your equation in part (c) represent a function? Explain your answer. (2 marks)

Solution		
No, the equation fails the vertical line test as it has multiple values of y for the same value of x .		
✓ No		
✓ Justification		

Question 12

(11 marks)