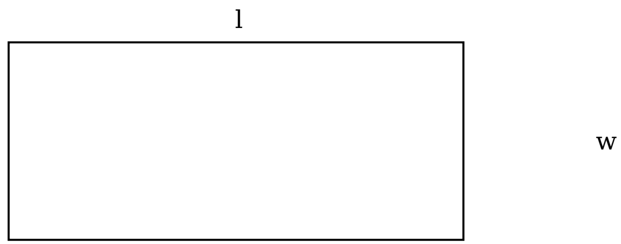
Rectangle of perimeter L m. Find in terms of L:

[a] The maximum area.



$2w + 2l = L \rightarrow l = \frac{L-2w}{2}$

$A = lw = (\frac{L-2w}{2})w$

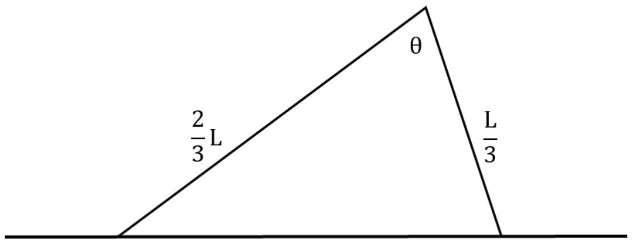
[b] The dimensions.

$A' = \frac{-4x+y}{2} \rightarrow A' = 0 \rightarrow w = \frac{L}{4}$

$l = \frac{L-2(\frac{L}{4})}{2} = \frac{L}{2} \text{ x } \frac{1}{2} = \frac{L}{4}$

A triangle has one side twice as long as the other, the third being replaced with a sufficiently long straight wall.

[a] Determine the maximum area, in terms of L. You don't necessarily have to use calculus techniques but be sure to state your reasoning.



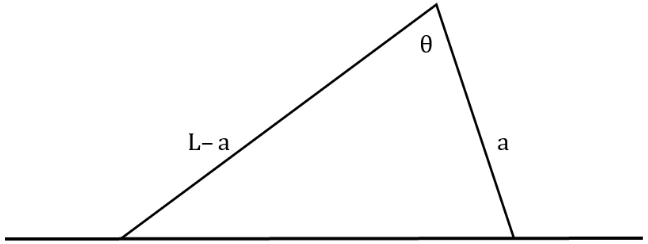
Let 'A' represent area.

$$A = \frac{1}{2}\left(\frac{2}{3}L\right)\left(\frac{L}{3}\right)\sin\theta = \frac{2L^2}{18}\sin\theta = \frac{L^2}{9}\sin\theta$$

$$\frac{dA}{d\theta} = \frac{L^2}{9}\cos\theta \rightarrow A' = 0 \rightarrow \theta = \frac{\pi}{2} \ (0<\theta<\pi)$$

$$A = \frac{L^2}{9}\sin\left(\frac{\pi}{2}\right) = \frac{L^2}{9}$$

[b] Determine what would happen if the sides had no restrictions. Use calculus.



Let 'A' represent area.

$$A = \frac{1}{2}(L-a)(a)\sin\theta = \frac{La-a^2}{2}\sin\theta$$

$$\frac{dA}{d\theta} = \frac{La-a^2}{2}\cos\theta \rightarrow A' = 0 \rightarrow \theta = \frac{\pi}{2} \ (0<\theta<\pi)$$

$$A = \frac{La-a^2}{2}\sin\left(\frac{\pi}{2}\right) = \frac{La-a^2}{2}$$

$$\frac{dA}{da} = \frac{-2a+L}{2} \rightarrow A' = 0 \rightarrow a = \frac{L}{2}$$

$$A = \frac{1}{2}(L-a)(a) = \frac{1}{2}\left(L - \frac{L}{2}\right)\left(\frac{L}{2}\right) = \frac{1}{2}\left(\frac{L}{2}\right)^2 = \frac{1}{2} \text{ x } \frac{L^2}{4} = \frac{L^2}{8}$$