Test 4

Logarithmic Functions & Continuous Random Variables

Semester One 2018

Year 12 Mathematics Methods

Calculator Assumed

PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students

Teacher:

_____ Mr McClelland

_____ Mrs. Carter

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_____ Ms Cheng

_____ Mr Staffe

_____ Mr Strain

Name:

Date: ____________ 7.45am

You may have a calculator, a single-sided page of notes and a formula sheet for this section of the test.

Total__________/ marks 40 Minutes

Questions 1 **(7 marks)**

Find the derivatives of the following. Do not simplify your answer.

(a) $\ln(2x^3 - 3x^2 + 4x - 1)^3$ (2 marks)

$$= \frac{3(2x^3-3x^2+4x-1)^2 \times (6x^2-6x+4)}{(2x^3-3x^2+4x-1)^3}$$ ✓ (chain Rule) ✓ $\left(\frac{d}{dx}\ln u = \frac{1}{u}\right)$

(b) $e^x \ln(x)$ (2 marks)

$$= e^x \ln x + e^x \frac{1}{x}$$ ✓ ✓ (product rule)

(c) $\ln(x)\cos(x) + \frac{\sin(x)}{x}$ (3 marks)

$$= \frac{1}{x}\cos x + \ln x(-\sin x) + \frac{\cos x\cdot(x) - \sin(x)}{x^2}$$ ✓ ✓

Question 2 **(~~4~~ 5 marks)**

(a) Use Polynomial Long division to simplify $\frac{x^2 - 2x + 5}{x - 3}$. (~~2~~ 3 marks)

$$\begin{array}{r} x + 1 \\ x-3 \overline{) x^2 - 2x + 5} \\ x^2 - 3x \\ \hline x + 5 \\ x - 3 \\ \hline 8 \end{array}$$

$$\frac{x^2 - 2x + 5}{x - 3} = (x+1) + \frac{8}{x-3}$$

(b) Hence find $\int \frac{x^2 - 2x + 5}{x - 3}\, dx$. (2 marks)

$$= \int (x+1)\, dx + \int \frac{8}{x-3}\, dx$$

$$= \frac{x^2}{2} + x + 8\ln(x-3) + C$$

-1 for missing "C"

Question 3 **(5 marks)**

(a) Find the constants a and b given that for $\{x \in \mathbb{R}: x \neq 2, x \neq -3\}$. (3 marks)

$$\frac{a}{x-2} + \frac{b}{x+3} = \frac{x+8}{x^2+x-6}$$

$ax + 3a + bx - 2b$

$$\frac{a(x+3)}{(x-2)(x+3)} + \frac{b(x-2)}{(x-2)(x+3)} = \frac{x+8}{(x-2)(x+3)}$$

x coeff. → $a + b = 1$ ①

number coeff → $3a - 2b = 8$ ②

$2a + 2b = 2$ ③

② + ③ $= 5a = 10$

$a = 2$.

$b = -1$.

(b) Hence find $\int \frac{x+8}{x^2+x-6}\, dx$. (2 marks)

$$\int \frac{x+8}{x^2+x-6}\, dx = \int \frac{2}{x-2}\, dx - \int \frac{1}{x+3}\, dx$$

$$= 2\ln(x-2) - \ln(x+3) + C$$

$$= \ln\frac{(x-2)^2}{x+3} \; ?$$

-1 for missing "C"

ft previous "C"

2

Solve t

(ii) ~~Plot the graphs of~~ $\cancel{y = x \text{ and }} y = 3log_2 10 + log_2\left(\frac{2+t}{3}\right)$, ~~and find the coordinates of the point of intersection~~. (2 marks)

$(12.21, \boxed{12.21})$ ✓✓

(b) It is found by observation that the model for *Cutus pius* does not quite work. It is known that the model for the population of *Asla bible* is satisfactory. The form of the model for *Cutus pius* is $N_C(t) = 8000 + c \times 2^t$. Find the value of c, correct to two decimal places, if it is known that $N_A(15) = N_C(15)$. (2 marks)

$8000 + c \times 2^{15} = 10\,000 + 1000 \times 15$ ✓

$c \times 2^{15} = 17000$

$\therefore c = 0.52$ ✓

Question 6 (5 marks)

The graph of the function with the rule $y = 3\log_2(x + 1) + 2$ intersects the axes at the points $(a,0)$ and $(0,b)$.

Find the exact values of a and b.

when $x = 0$ y-int

$y = 3\log_{10}(1) + 2$

$= \log_{10} 1^3 + 2(\log_{10} 2)$

$= \log_{10}(1 \times 4)$ ✓

$= \log_{10} 4$

$= 2\log_2 2$

$= 2 \Rightarrow (0, 2)$

$\therefore b = 2$ ✓

when $y = 0$: x-int

$0 = 3\log_{10}(x+1) + 2$ ✓

$-2 = 3\log_{10}(x+1)$

$-\frac{2}{3} = \log_{10}(x+1)$

$\therefore 10^{-\frac{2}{3}} - 1 = x \Rightarrow a = 10^{-2/3} - 1$

-0.7846

$100^{-\frac{1}{3}} - 1$

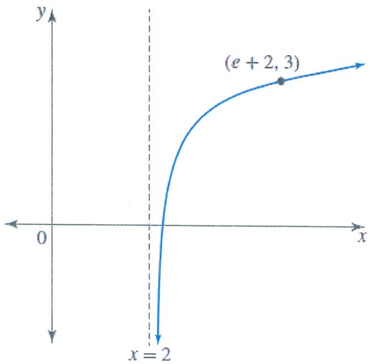
$\frac{1}{100^{\frac{1}{3}}} - 1$

$\frac{1}{10^{\frac{2}{3}}} - 1$

4

Question 4 **(2 marks)**

The rule for the function shown is $y = ln\,(x - m) + n$. Find the values of m and n.



$m = 2.$

$\ln(e+2-2) + n = 3$

$\ln e + n = 3$

$\therefore n = 2.$

Question 5 **(3 marks)**

Solve the following equations for x. Show full algebraic reasoning.

$$3e^{2x} - 5e^{x} - 2 = 0$$

$3\times(e^x)^2 - 5(e^x) - 2 = 0$ $\quad$ $1x \quad -2$ / $3x \quad 1$

$(e^x - 2)(3e^x + 1) = 0$

$e^x = 2 \quad \therefore x = \ln 2.$

$e^x = -\frac{1}{3}$ (reject).

$\therefore x = \ln 2.$

Question 7 **(8 marks)**

There are two species of insects living in a suburb: the *Asla bibla* and the *Cutus pius*. The number of *Ala bibla* alive at time t days after 1 January 2000 is given by

$$N_A(t) = 10\,000 + 1000t, \qquad 0 \le t \le 15$$

The number of *Cutus pius* alive at time t days after 1 January 2000 is given by

$$N_C(t) = 8000 + 3 \times 2^t, \qquad 0 \le t \le 15$$

(a) (i) Show that $N_A(t) = N_C(t)$ if and only if $t = 3log_2 10 + log_2\left(\frac{2+t}{3}\right)$. (4 marks)

$10000 + 1000t = 8000 + 3\times 2^t$

$2000 + 1000t = 3\times 2^t$

$\frac{2000+1000t}{3} = 2^t$ ✓ (expression of 2^t)

$\log\left(\frac{2000+1000t}{3}\right) = \log 2^t$

$\log\left(1000 \times \frac{2+t}{3}\right) = t\log 2$ (factorising)

$\frac{\log 1000 + \log\frac{2+t}{3}}{\log 2} = t$ ✓ (expression of t in terms of $\log$).

$\frac{\log 10^3}{\log 2} + \frac{\log\frac{2+t}{3}}{\log 2} = t$ ✓ (Apply $\log(A\times B) = \log A + \log B$.

$\frac{3\log 10}{\log 2} + \frac{\log\frac{2+t}{3}}{\log 2} = t$

$3\log_2 10 + \log_2\frac{2+t}{3} = t$ ✓ (simplify using change-of-base)