



PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

INDEPENDENT PUBLIC SCHOOL

**Semester Two
Examination, 2018**

Question/Answer booklet

MATHEMATICS SPECIALIST UNITs 3 & 4

**Section One:
Calculator-free**

Your Name

Your Teacher's Name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Total Marks	Awarded Marks
1	4	
2	6	
3	8	
4	6	
5	6	
6	6	
7	9	
8	6	

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	12	12	100	100	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free

(51 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1

(4 marks)

Consider the function $f(z) = az^3 + bz^2 + 50z + 150$ where a & b are real constants.
Given that $(z - 5i)$ & $(z + 3)$ are factors of $f(z)$, determine the values of a & b .

Question 2

(3 & 3 = 6 marks)

Consider the definite integral $\int_0^1 \sqrt{1-x^2} dx$

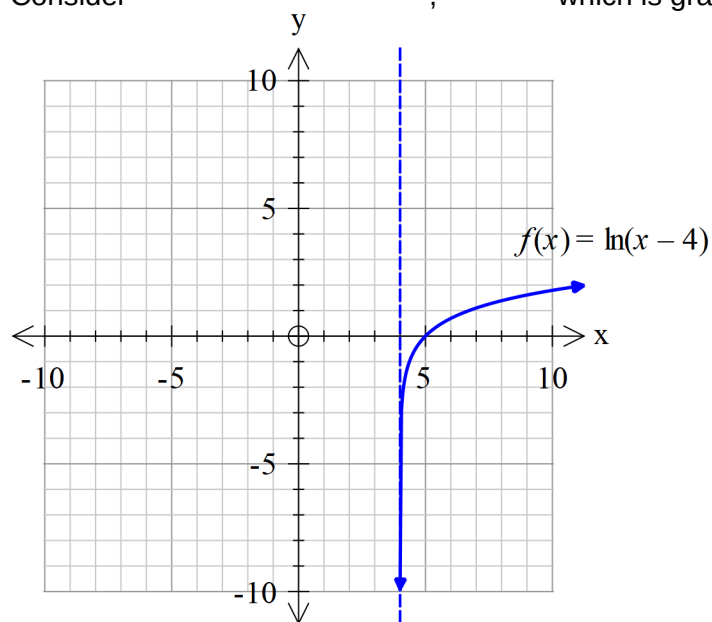
- i) By using the substitution $x = \sin u$ show that $\int_0^1 \sqrt{1-x^2} dx = \int_a^b \cos^2 u du$ and state the values of a & b .

- ii) Hence evaluate $\int_0^1 \sqrt{1-x^2} dx$ exactly.

Question 3

(3, 3 & 2 = 8 marks)

Consider $f(x) = \ln(x - 4)$, $x > 4$ which is graphed below.



a) Sketch the inverse of $f(x)$ on the axes above.

b) Determine the rule for $f^{-1}(x)$ stating the domain and range.

c) Determine the rule for $f \circ f(x)$ and state the largest possible domain for it to exist.

Question 4

(2, 3 & 1= 6 marks)

Consider the functions f & g where $f(x) = \sqrt{x-4}$ and $g(x) = \frac{1}{2x-3}$.

a) Determine the natural domain and range of f .

b) Does $g \circ f(x)$ exist over the natural domain of f ? If not then restrict the domain of f to give the largest possible domain for $g \circ f(x)$ to exist.

c) State the rule for $g \circ f(x)$ and the corresponding range for your answer to (b).

Question 5

(3 & 3 = 6 marks)

Solve for $y(x)$ in the following.

(a) $\frac{dy}{dx} = 3y$ given that y contains the point $(2, 300)$ (3 marks)

(b) $\frac{dy}{dx} = \frac{y-3}{x^5}$ given that y contains the point $\left(\frac{1}{2}, 100\right)$ (3 marks)

Question 6**(6 marks)**

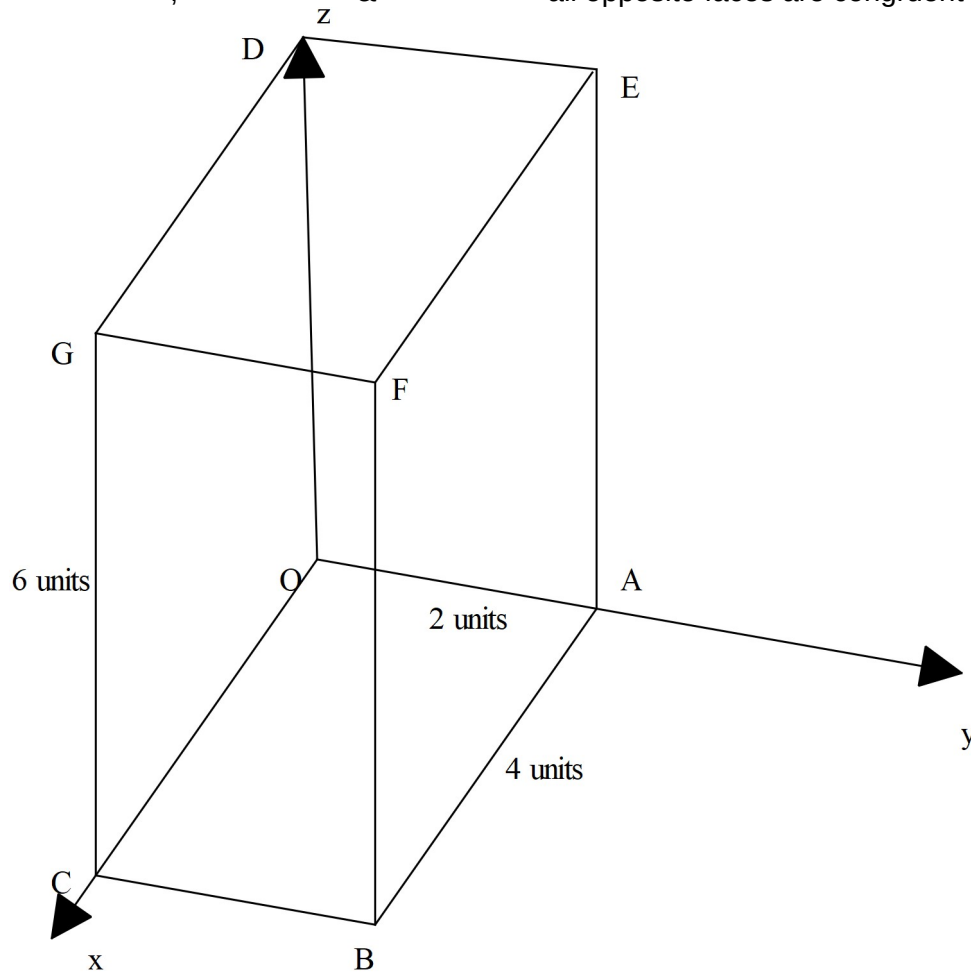
Determine the following integral.

$$\int \frac{8x^2 - 5x + 12}{(x - 1)(x^2 + 4)} dx$$

Question 7

(2, 4 & 3 = 9 marks)

Consider a rectangular box $OABCDEFG$ with points $A(0, 2, 0)$, $B(4, 2, 0)$, $C(4, 0, 0)$, $D(0, 0, 6)$ & $F(4, 2, 6)$ all opposite faces are congruent and parallel.



a) Determine the vectors \overrightarrow{CE} & \overrightarrow{BD} , the diagonals of the rectangular box. (2 marks)

b) Prove that the diagonals \overrightarrow{CE} & \overrightarrow{BD} bisect each other, i.e meet at their midpoints. (4 marks)

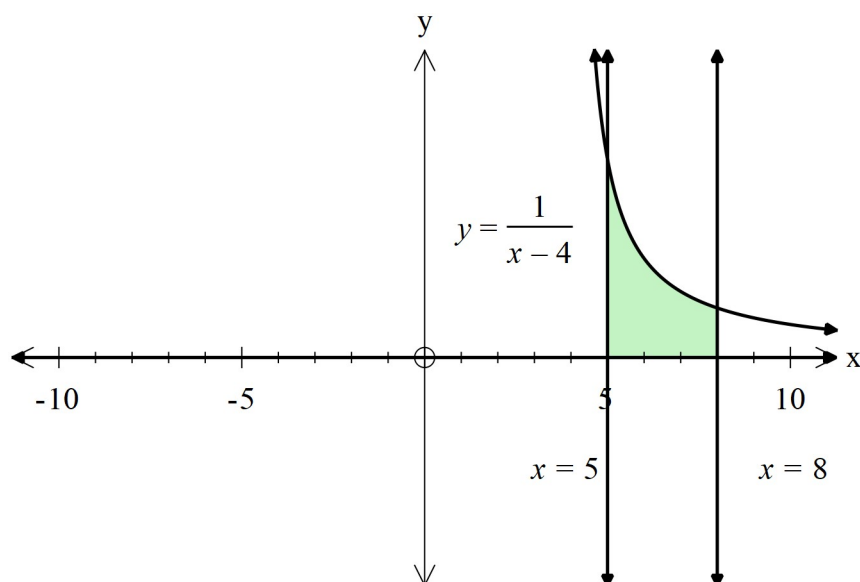
Q7 cont-

- c) Determine the cartesian equation of the plane that contains the points ACD .
(3 marks)

Question 8

(6 marks)

Consider the area enclosed between $y = \frac{1}{x-4}$, the lines $x=5$ & $x=8$ and the x axis.
 If this area is revolved around the y axis, a three dimensional object is formed.
 Determine the volume of this three dimensional object.



Additional working space

Question number:

Additional working space

Question number:

