 <p><b>PERTH MODERN SCHOOL</b> Exceptional schooling. Exceptional students. Independent Public School</p>	<p>Year 12 Specialist TEST 3 Monday 1 July 2019 TIME: 45 minutes working Classpads allowed One page of notes 42 marks 6 Questions</p>
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Name: \_\_\_\_\_

Teacher: \_\_\_\_\_

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

Q1 ( 3 & 3 = 6 marks)

- a) Solve for the following system of linear equations **without using a classpad**.

$$x + 2y - z = 3$$

$$2x + 3y + 2z = -1$$

$$3x + 7y - 2z = 6$$

Q1 - continued

$$x + 2y - z = 3$$

$$2x + 3y + 2z = m$$

- b) Determine the values of  $m$  &  $p$  such that  $3x + py - 2z = 6$  such that the system has
- (i) Infinite solutions
  - (ii) No solutions

Q2 (2 & 3 = 5 marks)

If  $r = \begin{pmatrix} t^2 - 1 \\ 5 - t \end{pmatrix}$ , determine

a)  $\left| \frac{dr}{dt} \right|$

b)  $\frac{d|r|}{dt}$  (no need to simplify)

Q3 (2, 3 & 3 = 8 marks)

An object is initially at the origin with initial speed of  $\begin{pmatrix} 3 \\ 7 \end{pmatrix} m/s$  and an acceleration given by

$a = \begin{pmatrix} 3e^t \\ 2 - \sin t \end{pmatrix} m/s^2$  at time  $t$  seconds.

Obtain an expression for the:

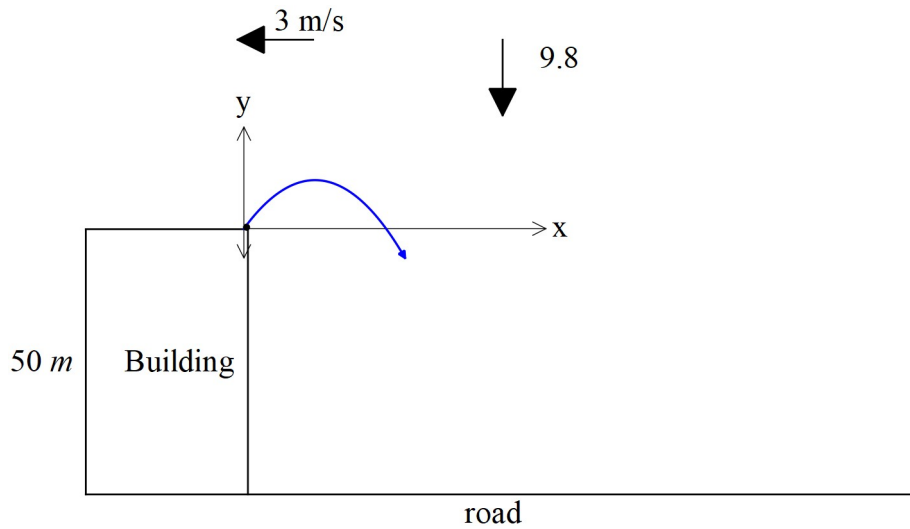
a) Velocity at time  $t$ .

b) Position vector  $r$  at time  $t$ .

c) Is the velocity ever perpendicular to the acceleration? Explain and if necessary solve for  $t$  values (if any).

Q4 (3, 3, 3 & 3 = 12 marks)

Consider a cannon ball that is projected from the top of a building with speed  $V$  at an angle  $\theta$  to the surface of the roof. There is a constant cross wind of 3 metres per second acting against the ball and the acceleration due to gravity is  $9.8 \text{ m/s}^2$  down as shown in the diagram below. (Note- let the origin be at the top of the building on the edge)



- a) Given that the acceleration is given by  $\ddot{\mathbf{r}} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \text{ m/s}^2$  show using vector calculus that the velocity  $\dot{\mathbf{r}} = \begin{pmatrix} V \cos \theta - 3 \\ V \sin \theta - 9.8t \end{pmatrix} \text{ m/s}$ .

- b) Determine the cartesian equation of the path of the cannon ball in terms of  $V$  &  $\theta$ . Show your working.

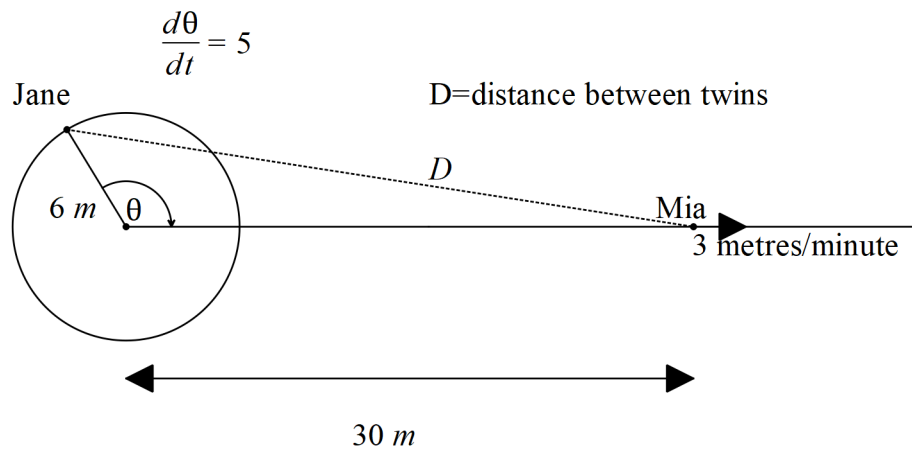
Q4 continued

- c) Given that a point on the cartesian path has been measured as  $(7.4, 1.1)$  metres and the initial speed  $V$  of the ball from the cannon is  $12 \text{ m/s}$ , determine the initial angle  $\theta$  of the ball when projected into the air.
- d) If  $V = 25 \text{ m/s}$  and  $\theta = 45^\circ$  and a cross wind of  $3 \text{ m/s}$  as in the diagram on last page, determine how far from the foot of the building that the cannon ball lands on the road.

Q5 (2 & 5 = 7 marks)

Consider two rides at a circus, one is a merry go round and the other is a train on a straight line. Two twins decide to each try one of the two rides, Jane sits on the merry go round with a constant angular

speed of  $\frac{d\theta}{dt} = 5$  radians/minute moving in a clockwise direction and radius 6 metres and Mia sits on a train moving at 3 metres/minute away from the merry go round. See the diagram below.



- a) Determine the distance between Jane and Mia when  $\theta = \frac{2\pi}{3}$  and the train is 30 metres from the centre of the merry go round.
- b) Determine the time rate of change of this distance at the point defined in (a) above. (metres/minute)

Q6 (4 marks)

$$\frac{d^2 y}{dx^2} = \frac{\frac{dx}{dt} \frac{d^2 y}{dt^2} - \frac{dy}{dt} \frac{d^2 x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

Given that  $x$  &  $y$  are functions of  $t$ , show that