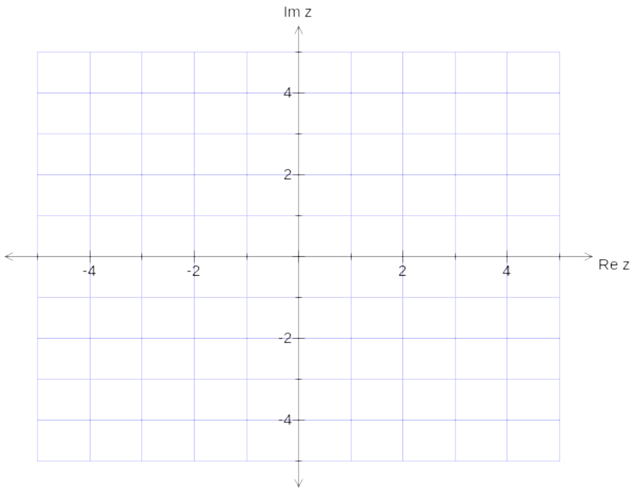
CHURCHLANDS SENIOR HIGH SCHOOL
MATHEMATICS SPECIALIST 3, 4 TEST ONE 2017
NON-Calculator Section
Chapters 1, 2,

Name__________________________________ **Time: 50 minutes**
Total: 49 marks

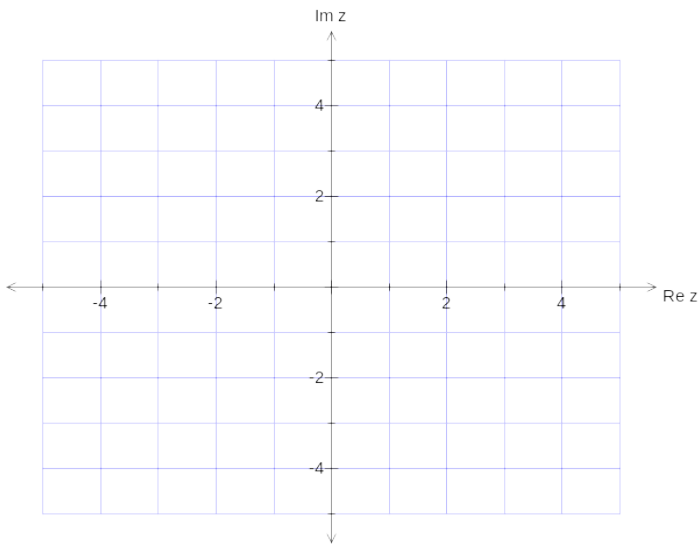
1.[12 marks: 3,3,3,3]
Describe and sketch each of the following subsets of the complex plane.

a) $\{z : |z+1-i| \le 3\}$



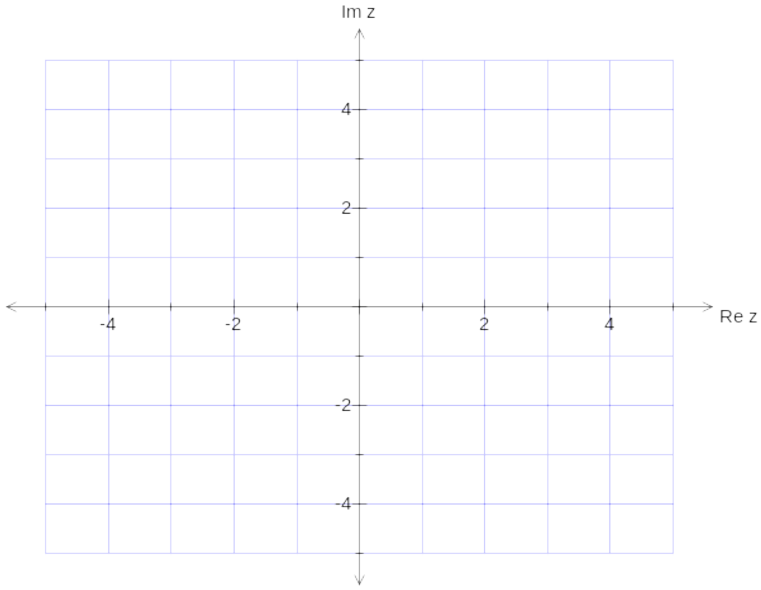
Description :

b) $\{z : |z+2-i| = |z-1+2i|\}$



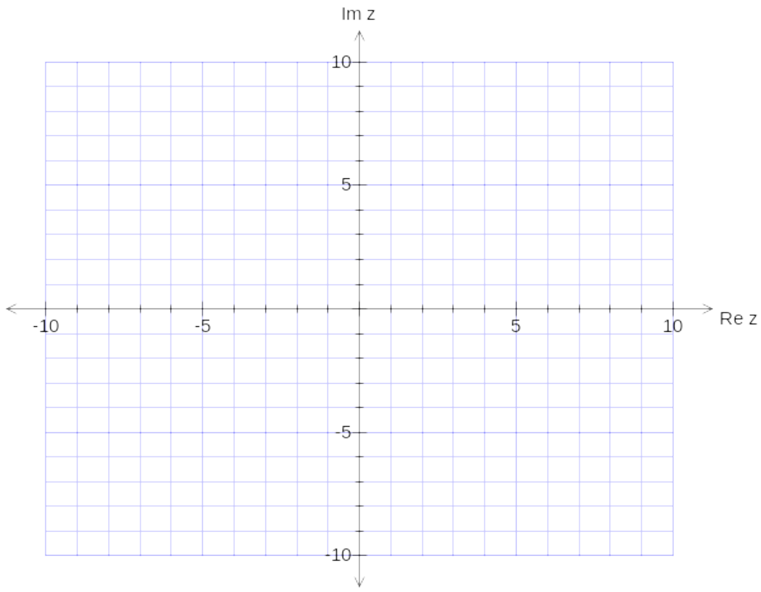
Description:

c) $\{z : z\,\overline{z} = 4\}$



Description:

d) $\{z : Re z - Im z \leq 9\}$

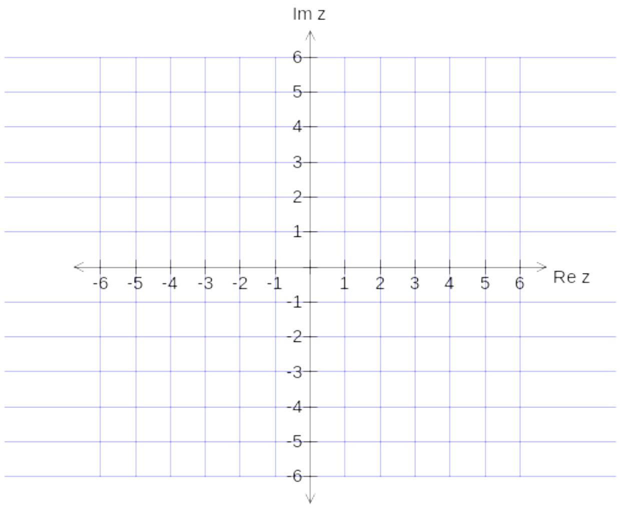


Description:

2. [6 marks: 2,1,1,1,1]

Sketch on the complex plane below the region defined by

$$|z-i+1|=4$$



Hence, find

i) the maximum value of $|z|$

ii) the minimum value of $|z|$

iii) the maximum value of Re(z)

iv) the minimum value of Im(z)

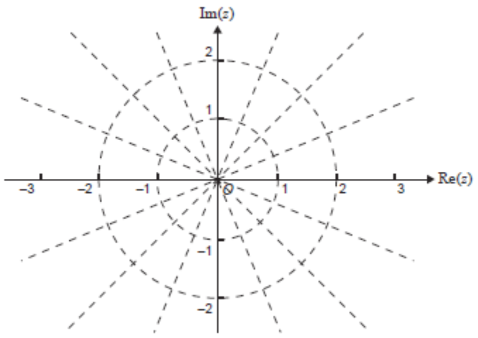
3. [7marks: 1,2,4]

i) Find the remainder when x^3-4x^2+7x-6 is divided by $x-2$.

ii) When x^3-x^2+cx-3 is divided by $x-3$, the remainder is 30. Find c.

iii) When $3x^3 - ax^2 - bx + 1$ is divided by $x - 2$, the remainder is 15. If $x - 1$ is a factor of the given polynomial, find the values of a and b.

4. [2 marks]
Plot the roots of $z^8 = 1$ on the Argand diagram below.



5. [4 marks:1,2,1]

Let $\beta = 1 - i\sqrt{3}$.
i) Express β in polar form.

ii) Express β^5 in polar form.

iii) Hence express β^5 in the form $x + iy$.

6. [12 marks:3, 3, 6 marks]

(a) If $z_1 = 3\,cis\left(\frac{4\pi}{3}\right)$ and $z_2 = \frac{1}{2}\,cis\left(\frac{\pi}{6}\right)$, prove that: $\frac{z_1}{z_2} = -3(\sqrt{3} + i)$

b) Simplify $\dfrac{\left(3\,cis\,\frac{\pi}{3}\right)\left(4\,cis\,\frac{\pi}{2}\right)}{6\,cis\,\frac{\pi}{4}}$ giving your answer in the form $rcis\theta$.

c)
Find all $z = x + iy$, given that $z\bar{z} + 3z = \bar{z} + 4i$

7. [6 marks]
If $z = cis\theta$ and by using De Moivre's theorem together with knowledge of the binomial expansion to find z^3, show that $\cos 3\theta = \cos^3\theta - 3cos\theta\sin^2\theta$ and $\sin 3\theta = 3\cos^2\theta sin\theta - \sin^3\theta$.