

SEMESTER TWO

Papers written by
Australian Maths
Software

MATHEMATICS METHODS

REVISION 2

UNITS 3-4

2016

SOLUTIONS

SECTION ONE

1. (6 marks)

(a) (i) $f'(x) = 2e^{2x} \tan(x) + e^{2x} \sec^2(x) = e^{2x} [2\tan(x) + \sec^2(x)]$

(ii) $g(x) = \ln(1+x)(1-x)$

$g(x) = \ln(1+x) + \ln(1-x)$

$g'(x) = \frac{1}{(1+x)} - \frac{1}{(1-x)} = -\frac{2x}{(1-x^2)}$

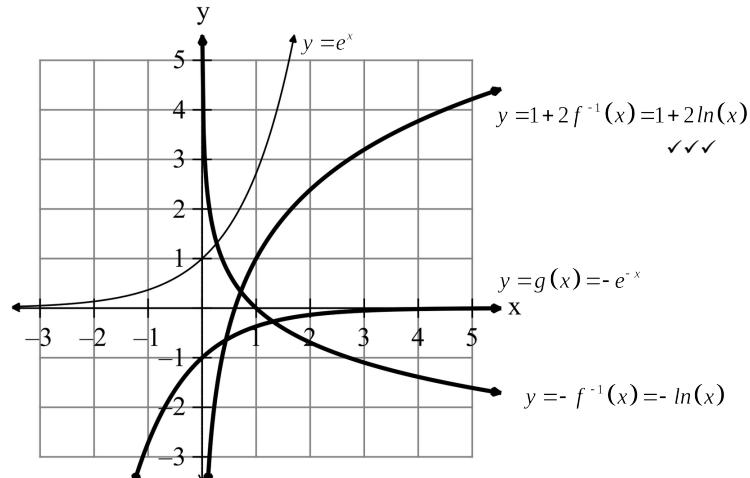
(iii) $h(x) = \frac{\sin(x)}{\cos(3x)}$

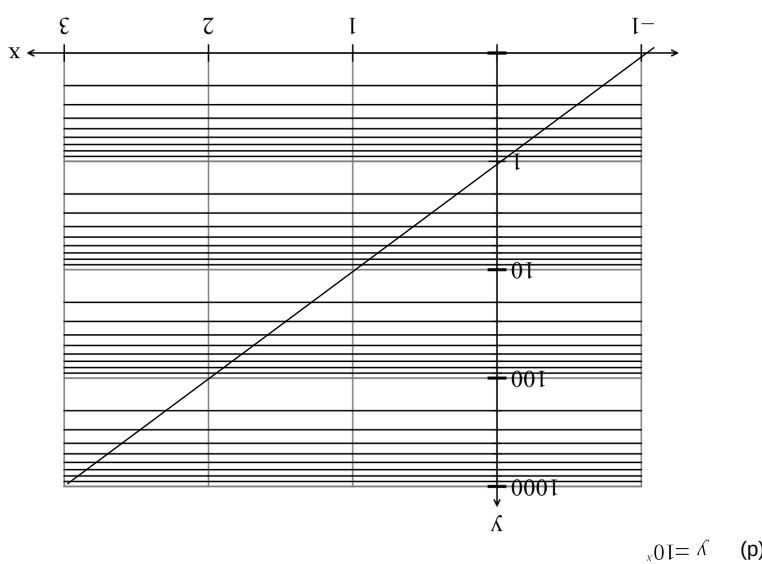
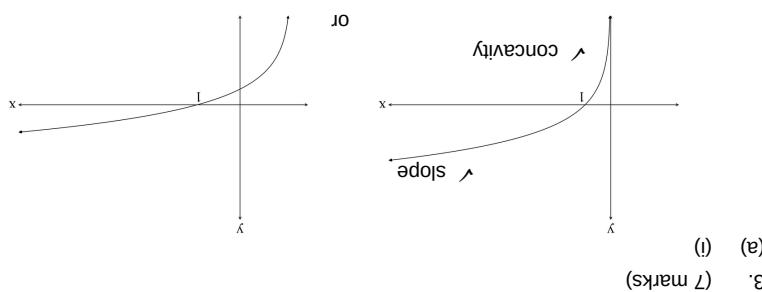
$h'(x) = \frac{\cos(x)\cos(3x) - 3(-\sin(3x))\sin(x)}{\cos^2(3x)} \quad \checkmark \checkmark -1/\text{error}$

$h'(x) = \frac{\cos(x)\cos(3x) + 3\sin(3x)\sin(x)}{\cos^2(3x)}$

2. (16 marks)

(a) (i) (ii) (iii)





$$(c) \quad \frac{(\log_6 3 + \log_6 2)}{(\log_6 6)^2} = \frac{-\log_6 36}{\log_6 6^2} = \frac{-\log_6 36}{-2(1)} = -\frac{1}{2}$$

$$(ii) \quad x = e^{\ln(2)} \quad \ln(x) = \ln(2) \quad x = 2$$

$$x = 6$$

$x = \pm 6$ but $x > 0$ as a base cannot be negative.

$$(b) \quad (i) \quad \log_x(36) = 2 \\ 36 = x^2$$

END OF SECTION TWO

Should use a sample size of 68 people to have a confidence level of 90% with an error margin of 10%.
 $n \approx 68$

$$n = 67.65$$

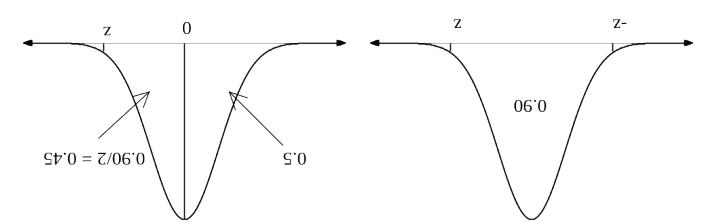
$$0.10 = 1.645 \times \sqrt{\frac{0.25}{n}}$$

Therefore

$$E = z \times s \text{ but } E = 0.10 \\ \text{So with } p = 0.5 \quad sd = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.25}{n}}$$

Use $p = 0.5$ as the maximum value as p is unknown.

$$z = 1.645 \\ P(X < z) = 0.95$$



The confidence limit is 0.16667 ± 0.06668 i.e. $(0.10, 0.23)$

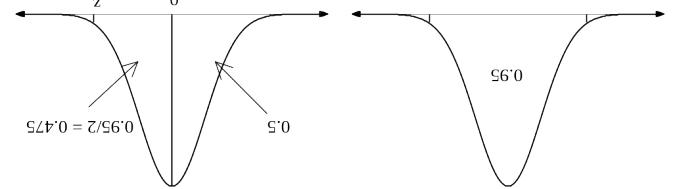
$$E = 0.06668$$

$$E = z \times s = 1.96 \times 0.03402$$

$$s = 0.03402$$

So, 95% confidence level means $z = 1.96$

$$P(X < z) = 0.975$$



$$(iii) \quad sd^4 = 0.03402 \quad sd^4 = \sqrt{0.16 \times 0.83} \\ 120$$

(b) (i) $\frac{d}{dx} (\ln(x-2)) = \frac{1}{x-2}$

(ii) $\int \frac{2}{(x-2)} dx = 2\ln(x-2) \text{ for } x > 2$

4. (7 marks)

(a) $\int \frac{3}{x^2} - 2x + \sqrt{x} dx$
 $= \int 3x^{-2} - 2x + x^{1/2} dx$
 $= -\frac{3}{x} - x^2 + \frac{1}{2}x^{1/2} + c$
 $= -\frac{3}{x} - x^2 + \frac{1}{2\sqrt{x}} + c$

(b) $\int \frac{dx}{(1-4x)^5} = \int (1-4x)^{-5} dx = \frac{(1-4x)^4}{(-4)(-4)} + c = \frac{1}{16(1-4x)^4} + c$

(c) $\int_0^{\pi/2} \sin(2x) + \cos(2x) dx$
 $= \left[-\frac{\cos(2x)}{2} + \frac{\sin(2x)}{2} \right]_0^{\pi/2} \checkmark$
 $= -\frac{1}{2} [\cos(2\pi) - \sin(2\pi)]_0^{\pi/2}$
 $= -\frac{1}{2} [(\cos(\pi) - \sin(\pi)) - (\cos(0) - \sin(0))]$
 $= -\frac{1}{2} [(-1) - (1)]$
 $= 1$

(c) $p = 0.4, q = 0.6 \Rightarrow np = 80 \times 0.4 = 32 > 5$

$nq = 80 \times 0.6 = 48 > 5$ so can use normal distribution.

Mean $= p = 0.4$
 $sd_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.4 \times 0.6}{80}}$
 $sd_{\hat{p}} = 0.05477$

Standardised score (using 19.5 to 25.5)

$$z = \frac{X - \mu}{\sigma}$$

$$z_1 = \frac{19.5 - 0.4}{0.05477} = -2.85284$$

$$z_2 = \frac{25.5 - 0.4}{0.05477} = -1.48348$$

The probability is 0.07.

(d) (i) $\hat{p} = \frac{20}{120} = 0.16^{\frac{1}{1}}$

(ii) $\hat{p} = 0.16^{\frac{1}{1}}$
 $Var_{\hat{p}} = \frac{\hat{p}(1-\hat{p})}{n}$
 $Var_{\hat{p}} = \frac{0.16 \times 0.83}{120}$
 $Var_{\hat{p}} \approx 0.00116$

END OF SECTION ONE

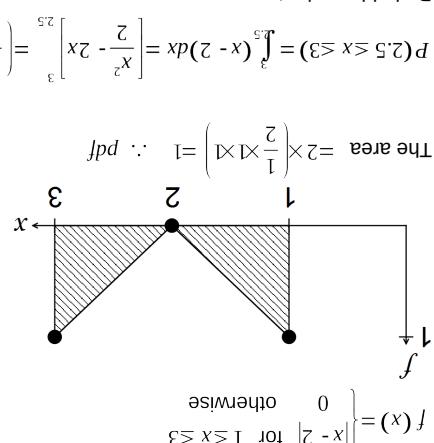
$$\begin{aligned} F(x) &= \frac{6}{(x-2)} \\ &= \frac{6}{1} \times (x-2) \\ &= \frac{6}{1} [x-2] \\ &\text{(iv)} \quad F(x) = \int_3^x \frac{6}{1} dx \end{aligned}$$

$$\text{(iii)} \quad P(x \geq 3 | x \leq 7) = \frac{5}{1} \quad \checkmark$$

$$\text{(ii)} \quad P(3 \leq x \leq 5) = \frac{6}{2}$$

$$\text{(c)} \quad \text{(d)} \quad a = \frac{6}{1}$$

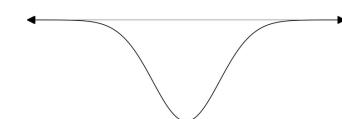
Probably easier to use areas.



- (a) (i) Not a probability density function as the probabilities do not add to 1. \checkmark
- (ii) Yes, a probability density function as the probabilities add to 1. \checkmark

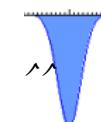
5. (14 marks)

$$u = \$1800 \text{ 000}, v = \$150 \text{ 000}$$



17. (11 marks)

- (a) (i) Bias occurs because buyers are there possibly to buy the specials or they may be in a financial position where they need to buy goods on special. \checkmark
- (ii) Assuming the survey only needs to speak to Woolworth's customers, you could randomly sample the people on the electoral role, then ask if they are Woolworth's customers. If so, you have a sample point. If not, try someone else. \checkmark
- (iii) Bases only needs to speak to Woolworth's customers to buy goods on special. \checkmark



18. (22 marks)

- (a) $P(x > 200000) = 0.09121121973 \approx 0.09 \quad \checkmark$
- (b) $P(x > 200000 | x > 165000) = \frac{P(x > 200000)}{P(x > 165000)} = \frac{0.0912112197}{0.8413447461} \approx 0.1084$
- (c) $P(1200000 < x < 1900000) = 0.7474757912 \approx 0.75 \quad \checkmark$
- (d) $P(x > 2000000) = 0.09121121973 \approx 0.09 \quad \checkmark$
- $P(\text{she had sales worth over \$2 000 000 in two of the months and sales worth less than \$2 000 000 in the other two months}) = C^2 (0.09121121973)^2 (1 - 0.09121121973)^2 \approx 0.04122623 \quad \checkmark$
- $\approx 0.04122623 \quad \checkmark$
- $\text{The area} = 2 \times \left[\frac{1}{2} x(1-x) \right] = 1 \quad \therefore \quad p.d.f$
- $P(\text{she had sales worth over \$2 000 000 in two of the months and sales worth less than \$2 000 000 in the other two months}) = C^2 (0.09121121973)^2 (1 - 0.09121121973)^2 \approx 0.04122623 \quad \checkmark$
- $\approx 0.04122623 \quad \checkmark$
- $\text{Probabilty easier to use areas.}$

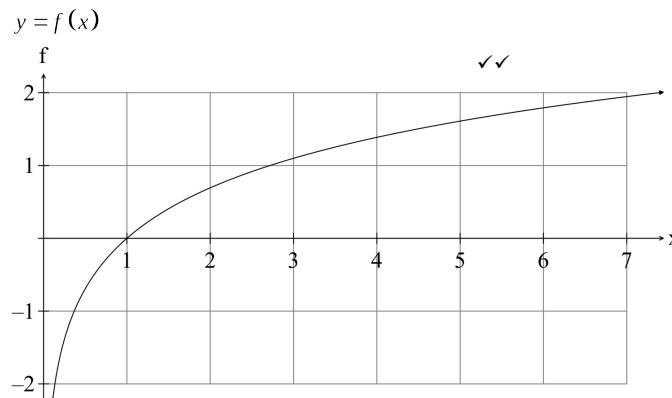
6. (6 marks)

(a)

x	$f(x)$
0.5	-0.69
1	0
2	1.69
3	1.099
4	1.386
7	1.946

✓✓✓ -1/error

(b)

(c) $y = \ln(x)$

7. (9 marks)

(a) (i) $f(x) = \tan(\alpha x)$ $(c = 0)$

(ii) $f'(x) = \sec^2(x) = (\cos(x))^{-2}$

$f''(x) = -2(\cos(x))^{-3}(-\sin(x))$ ✓

$f''(x) = \frac{2\sin(x)}{\cos^3(x)}$

15. (4 marks)

(a) $B\left(5, \frac{1}{6}\right)$

$P(X=2) = {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 = 0.1607510288 \approx 0.16$ ✓✓

(b) $P(2 \text{ or } 3 \text{ or } 4 \text{ or } 5) = 0.196244856 \approx 0.20$ ✓✓

16. (9 marks)

(a)

y	0	1	2	3
$P(Y=y)$	0.5787	0.3472	0.0694	0.0046

✓✓ -1/error

(b) (i) $P(X \geq 2) = 0.3 + 0.4 = 0.7$ ✓

(ii) $E(X) = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.4 = 2 \quad \therefore \mu = 2$ ✓✓

$Var(X) = E(X^2) - \mu^2$

$Var(X) = 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.3 + 3^2 \times 0.4 - 2^2$

$Var(X) = 1 \quad \checkmark$

(iii) $\mu_{2X+1} = 2(2) + 1 = 5$

Only multiplying changes the variance, not adding.

$Var(2X+1) = 2^2 = 4$



- (a) $t = 0 \quad 1990 \quad P = 2635(r)^{20}$
 $r = 0.98812497$
- The annual rate of decrease of the mountain pygmy possum population from 1990 to 2010 was 1.7875 %
- Given the actual population in 2015 was 1830, the culling was not effective. The population is a lot less than expected.

For $\frac{\pi}{2} < x < \pi$ function f is decreasing so concave up and $f'' > 0$.

At $x = 0, \pi, 2\pi$, f has turning points so $f'' = 0$.

f , has turning points so f'' has x intercepts.

At the points of inflection in f , f , has a turning point, $f'' = 0$, the concavity of f changes so the gradient of f , changes from increasing to decreasing and vice versa.

$$\begin{aligned} V &= \frac{3}{4}\pi r^2 h \quad \text{but } r = \frac{3}{4}h \\ dV/dh &= 10cm/min \end{aligned}$$

(a) (7 marks)

$$\begin{aligned} dV/dh &=? \quad \text{at } h = 20\text{cm} \\ dV/dh &= ? \end{aligned}$$

$$V = \frac{3}{4}\pi r^2 h$$

$$V = \frac{27}{16}\pi h^3$$

$$\begin{aligned} dV/dh &= \frac{81}{16}\pi h^2 \\ dh/dt &= \frac{9}{16\pi}h^2 \times 10 \\ dt &= \frac{9}{16\pi}h^2 \end{aligned}$$

$$At \quad h = 20 \quad \frac{dV}{dt} = \frac{9}{16\pi} \times 20^2 \times 10$$

$$\frac{dV}{dt} = 22340.21\text{cm}^3/\text{min}$$

When the height is 20 cm, the rate of change of volume is 22 340.21 cm³/min.

- (b) There are no turning points in f so $f' \neq 0$.

f is concave up for all $f' > 0$ and concave down for all $f' < 0$.

f , has a positive gradient at all defined points,

f is concave up for all $f' > 0$ and concave down for all $f' < 0$.

f is always positive.

For $0 < x < \frac{\pi}{2}$ function f , is increasing so concave up and $f'' > 0$.

f , has turning points so f'' has x intercepts.

At the points of inflection in f , f , has a turning point, $f'' = 0$, the concavity of f changes so the gradient of f , changes from increasing to decreasing and vice versa.

At $x = 0, \pi, 2\pi$, f has turning points so $f'' = 0$.

f , has turning points so f'' has x intercepts.

$$\begin{aligned} dV/dh &= 10cm/min \\ dh/dt &=? \end{aligned}$$

(a) (7 marks)

$$V = 10\pi r^2 - 2\pi^2 r^3$$

$$V = \pi r^2 \times (10 - 2\pi r)$$

$$dV/dr = 20\pi r - 6\pi^2 r^2$$

$$dV/dr = 20\pi r - 12\pi^2 r$$

$$\begin{aligned} dV/dr &= 0, \quad 0 = 20\pi r - 6\pi^2 r^2 \\ r &= \frac{10}{3\pi} \end{aligned}$$

$$\begin{aligned} \max \text{ or min?} \\ r = \frac{10}{3\pi} \\ At \quad r = \frac{10}{3\pi} \quad V = 10\pi r^2 - 2\pi^2 r^3 \\ V = 10\pi \left(\frac{10}{3\pi}\right)^2 - 2\pi^2 \left(\frac{10}{3\pi}\right)^3 \\ V = \frac{1000}{27}\pi - \frac{2000}{81}\pi^2 \end{aligned}$$

$$\begin{aligned} w = 10 - 2\pi r = 10 - 2\pi \left(\frac{10}{3\pi}\right) = \frac{30}{3} = 10 \\ \text{Need dimensions of rectangle.} \end{aligned}$$

$$\begin{aligned} l = 10 - w = 10 - 6\frac{3}{2} = 6 \\ l = 6 \end{aligned}$$

$$(b) V = \frac{16\pi}{27} h^3$$

$$\frac{dV}{dh} = \frac{16\pi}{9} h^2$$

$$\frac{dV}{dh} \approx \frac{\delta V}{\delta r}$$

$$\delta V \approx \frac{16\pi}{9} h^2 \times \delta r$$

At $h = 2$ and $\delta r = 0.01$

$$\delta V \approx \frac{16\pi}{9} \times 15^2 \times 0.01$$

$$\delta V \approx 4\pi m^3 \quad (=12.77m^3)$$

9. (4 marks)

$$(a) \text{Area} = 0.5 \times 1 + 0.5 \times 0.67 + 0.5 \times 0.5 + 0.5 \times 0.4 + 0.5 \times 0.33 \\ = 0.5 \times [1 + 0.67 + 0.5 + 0.4 + 0.33] \\ = 1.45 \text{ units}^2$$

$$(b) \int_{0.5}^3 \frac{1}{x} dx \approx 1.79$$

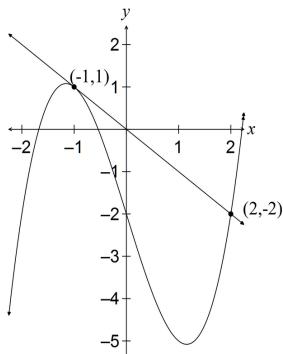
10. (4 marks)

The points of intersection are $(-1,1)$ and $(2,-2)$.

The area between the curves is found by

$$A = \int_1^2 (-x - (x^3 - 4x - 2)) dx$$

$$A = 6.75 \text{ m}^2$$



11. (6 marks)

$$(a) a = 4 \text{ ms}^{-2}, v_0 = 2 \text{ ms}^{-1}, x_0 = 1 \text{ m}$$

$$v = \int 4dt = 4t + c_1$$

$$\text{At } t = 0, v_0 = 2 \text{ ms}^{-1} \rightarrow c_1 = 2$$

$$\therefore v = 4t + 2$$

$$x = \int (4t + 2) dt = 2t^2 + 2t + c_2$$

$$\text{At } t = 0, x_0 = 1 \text{ m} \rightarrow c_2 = 1$$

$$\therefore x = 2t^2 + 2t + 1$$

$$\text{At } t = 2, v = 4(2) + 2 = 10 \text{ ms}^{-1}$$

$$x = 2(2)^2 + 2(2) + 1 = 13 \text{ m}$$

$$(b) v = 0 \text{ at } 4t + 2 = 0 \quad t = -0.5 \text{ but } t \geq 0.$$

There are no changes of direction on the defined domain. 0/2 if not checked.

$$\text{At } t = 0, x = 1$$

$$\text{At } t = 4, x = 41$$

The distance travelled is 40 m. ✓ ✓

12. (6 marks)

$$(a) \frac{d}{dx} \int_1^x (\tan(2t) - 2) dt = \tan(2x) - 2 \quad \checkmark \checkmark$$

$$(b) (i) f(t) = \cos(\ln(t))$$

$$f'(t) = (-\sin(\ln(t))) \times \frac{1}{t} = \frac{-\sin(\ln(t))}{t}$$

$$(ii) \int_1^e -\frac{\sin(\ln(t))}{t} dt = [\cos(\ln(t))]_1^e$$

$$= \cos(\ln(e)) - \cos(\ln(1))$$

$$= \cos(1) - \cos(0)$$

$$= \cos(1) - 1$$