Calculator Assumed
Discrete Random Variables – Mixed Applications 1

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [3, 1, 1, 2, 2 = 9 marks] **CA**

At the local school fete, Daniel plays a game where he gambles on the roll of two dice.

Each time the two dice are rolled, he places a bet. The sum of the uppermost faces are noted and the prizes are awarded as follows:

- $0 if the sum is 7
- $1 if the sum is even
- $4 if the sum is 3 or 5
- $6 if the sum is 9 or 11

Let Y represent the prizes offered.

(a) Represent the probability density function Y in a table below.

Daniel bets $1 for every roll of the two dice.

(b) What is the probability Daniel makes a loss?

(c) What is the probability Daniel breaks even?

(d) What is the probability that Daniel makes a profit given that he didn't make a loss?

(e) What are Daniel's expected winnings?

Question Two: **[2, 2, 2, 2 = 8 marks]** **CA**

A fair six-sided dice is rolled 12 times.

(a) Determine the probability that the first three and the last three rolls are all sixes, while the others are not.

(b) Determine the probability that every second roll is a six, while the others are not.

(c) Determine the probability that only half of the rolls give a six.

(d) Determine the probability that at most 8 of the rolls give a six.

Question Five: **[2, 3, 3 = 8 marks]** **CA**

A mother and father are both carriers of a gene that can produce offspring with a serious blood disorder.

When both parents are carriers of the gene (but do not themselves suffer from the disorder) the chance of having a child who will have the disorder is 1 in 4. The chance of this same couple having a child who is a carrier of the gene, but does not suffer from the disorder, is 1 in 2.

This couple has three children.

(a) What is the probability that only the first child inherits the blood disorder?

$=(0.25)^1(0.75)^2$ ✓

$=0.140625$ ✓

(b) What is the probability that one child has the blood disorder and the other two are carriers?

$=(0.25)^1(0.5)^2\times 3$ ✓ ✓

$=0.1875$ ✓

(c) How many of the children would you expect to be carriers? Note that a child with the disorder is also a carrier.

$Y \sim Bin(3, 0.75)$ ✓

$E(Y) = 3\times 0.75 = 2.25$ ✓ ✓

question again, this time using miles, the likelihood of accident remains at 0.0001875.

Question Three: [5, 1, 2 = 8 marks] CA

Five cards are drawn randomly from a standard pack of 52 cards. Let X represent the number of diamond cards drawn.

(a) Describe the probability distribution for X in a table below.

(b) What is the most likely number of diamonds to be drawn in this selection of 5 cards?

(c) What is the expected number of diamonds to be drawn in this selection of 5 cards?

Question Four: **[1, 2, 2, 4, 3 = 12 marks]** **CA**

In the testing being undertaken for Google's driverless cars, there have been 0.6 minor accidents for every 160 000 km of travel.

(a) If a typical testing journey is 50 km, what is the likelihood of an accident occurring?

(b) In 5 successive typical testing journeys, what is the probability that the first three result in an accident the last two do not?

(c) In 15 successive typical testing journeys, what is the probability that exactly 2 will result in an accident?

(d) In 15 successive typical testing journeys:

(i) what is the expected number of accidents?

(ii) what is the standard deviation of the number of accidents?

(e) If the typical testing journey was recorded in terms of miles, rather than kilometres, explain what effect, if any, would this have on the mean number of accidents recorded in 15 typical testing journeys? (1 mile = 1.6 km)

Question Four: **[1, 2, 2, 4, 3 = 12 marks]** **CA**

In the testing being undertaken for Google's driverless cars, there have been 0.6 minor accidents for every 160 000 km of travel.

(a) If a typical testing journey is 50 km, what is the likelihood of an accident occurring?

$$\frac{160000}{50} = 3200$$

$$0.6 \div 3200 = 0.0001875$$

(b) In 5 successive typical testing journeys, what is the probability that the first three result in an accident the last two do not?

$$= (0.0001875)^3 (0.9998125)^2$$

$$= 6.5893 \times 10^{-12}$$

(c) In 15 successive typical testing journeys, what is the probability that exactly 2 will result in an accident?

$$X \sim Bin(15, 0.0001875)$$

$$P(X = 2) = 0.0000036824$$

(d) In 15 successive typical testing journeys:

(i) what is the expected number of accidents?

$$E(X) = 15 \times 0.0001875 = 0.0028125$$

(ii) what is the standard deviation of the number of accidents?

$$\sqrt{Var(X)} = \sqrt{15 \times 0.0001875 \times 0.9998125}$$

$$= 0.0530$$

(e) If the typical testing journey was recorded in terms of miles, rather than kilometres, explain what effect, if any, would this have on the mean number of accidents recorded in 15 typical testing journeys? (1 mile = 1.6 km)

This would have no effect on the number of accidents recorded.

#Although there is a change of scale in the units from kilometres to miles, the unit of length has no impact on the proportion of accidents recorded. If we begin the

Question Three: [5, 1, 2 = 8 marks] CA

Five cards are drawn randomly from a standard pack of 52 cards. Let X represent the number of diamond cards drawn.

(a) Describe the probability distribution for X in a table below.

x	0	1	2	3	4	5
$P(X=x)$	${}^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5$ $=0.2373$	${}^5C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^4$ $=0.3955$	0.2637	0.0879	0.0146	0.0009766

(b) What is the most likely number of diamonds to be drawn in this selection of 5 cards?

1 diamond card

(c) What is the expected number of diamonds to be drawn in this selection of 5 cards?

$E(X)=5\times0.25=1.25$

Question Five: [2, 3, 3 = 8 marks] CA

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When both parents are carriers of the gene (but do not themselves suffer from the disorder) the chance of having a child who will have the disorder is 1 in 4. The chance of this same couple having a child who is a carrier of the gene, but does not suffer from the disorder, is 1 in 2.

This couple has three children.

(a) What is the probability that only the first child inherits the blood disorder?

(b) What is the probability that one child has the blood disorder and the other two are carriers?

(c) How many of the children would you expect to be carriers? Note that a child with the disorder is also a carrier.

SOLUTIONS
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- \$0 if the sum is 7
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- \$4 if the sum is 3 or 5
- \$6 if the sum is 9 or 11

Let Y represent the prizes offered.

(a) Represent the probability density function Y in a table below.

y	0	1	4	6
$P(Y=y)$	$\frac{6}{36}$	$\frac{18}{36}$	$\frac{6}{36}$	$\frac{6}{36}$

Daniel bets \$1 for every roll of the two dice.

(b) What is the probability Daniel makes a loss?

$P(Y=0)=\frac{6}{36}$ ✓

(c) What is the probability Daniel breaks even?

$P(Y=1)=\frac{18}{36}$ ✓

(d) What is the probability that Daniel makes a profit given that he didn't make a loss?

$$P(Y\ge 4\mid Y\ge 1)=\frac{\frac{12}{36}}{\frac{30}{36}}=\frac{12}{30}$$ ✓ ✓

(e) What are Daniel's expected winnings?

$E(Y)=-1\times\frac{6}{36}+0\times\frac{18}{36}+3\times\frac{6}{36}+5\times\frac{6}{36}$ ✓

$E(Y)=\$1.17$ ✓

Question Two: [2, 2, 2, 2 = 8 marks] **CA**

A fair six-sided dice is rolled 12 times.

(a) Determine the probability that the first three and the last three rolls are all sixes, while the others are not.

✓ $\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^6=0.000007178$ ✓

(b) Determine the probability that every second roll is a six, while the others are not.

$\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^6=0.000007178$ ✓ ✓

(c) Determine the probability that only half of the rolls give a six.

$X\sim Bin(12,\frac{1}{6})$ ✓

$P(X=6)=0.006632$ ✓

(d) Determine the probability that at most 8 of the rolls give a six.

$P(X\le 8)=0.9999866$ ✓ ✓