PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS
UNIT 1 AND 2
Section One:
Calculator-free

Name: _____

Teacher's Name:_____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	50	33
Section Two: Calculator-assumed	9	9	100	100	67
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **(50 Marks)**

This section has **six (6)** questions. Answer **all** questions. Write your answers in the spaces provided.

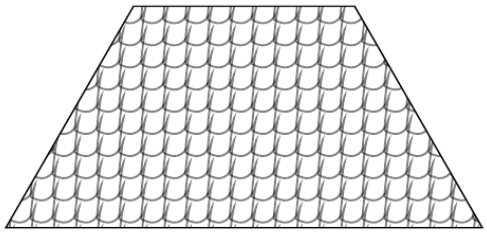
Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 **(5 marks)**

The roof of a heritage building has roof tiles arranged neatly in horizontal rows. A sample diagram is shown below.



There are 28 roof tiles in the top row and each consecutive row has an extra 4 tiles than the row above it. If the bottom row has 96 tiles, show that there is a total of 1116 tiles on the roof of the museum.

Solution
$a_1 = 28\ 96 = 28 + (n-1)(4)$ $a_n = 96\ 96 = 24 + 4n$ $d = 4\ 18 = n$ $S_{18} = \frac{18}{2}(28+96)$ ¿ 1 116 ∴ There are 18 rows of tiles, and a total of 1 116 tiles on the roof.
✔ identifying important info ✔ setting up equation to find n ✔ the value of n ✔ setting up equation to find S_n ✔ showing that there are 1 116 tiles

Question 2 **(5 marks)**

Evaluate the expression below, giving your answer as a simplified fraction.

$$\left(125^{\frac{1}{3}} \times 25^{\frac{1}{2}} + 16^{\frac{3}{4}} \times 64^{\frac{1}{3}} + \frac{1}{49^{\left(-\frac{1}{2}\right)}} \right)^{-\frac{2}{3}}$$

Solution
$= \left((5^3)^{\frac{1}{3}} \times (5^2)^{\frac{1}{2}} + (2^4)^{\frac{3}{4}} \times (2^6)^{\frac{1}{3}} + 7 \right)^{-\frac{2}{3}}$ $= (5 \times 5 + 2^3 \times 2^2 + 7)^{-\frac{2}{3}}$ $= (64)^{-\frac{2}{3}}$ $= \left(64^{\frac{1}{3}} \right)^{-2}$ $= 4^{-2}$ $= \frac{1}{16}$
✔✔ evaluating indices ✔applying BIMDAS ✔evaluated/simplified fraction

Question 3 **(11 marks)**

Given the function $f(x)=2x^2-6x-10$,

(a) state $f(x+h)$. Do not simplify. (2 marks)

Solution
$f(x+h)=2(x+h)^2-6(x+h)-10$
✔✔ (-✔ for every wrong term)

(b) set up and evaluate the difference quotient to find the derivative of the function. (4 marks)

Solution
$\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=¿\dfrac{\lim\limits_{h\to 0}2(x+h)^2-6(x+h)-10-(2x^2-6x-10)}{h}¿$ $¿\dfrac{\lim\limits_{h\to 0}2x^2+4xh+2h^2-6x-6h-10-2x^2+6x+10}{h}$ $¿\dfrac{\lim\limits_{h\to 0}4xh+2h^2-6h}{h}$ $¿\lim\limits_{h\to 0}4x+2h-6$ $¿4x-6(as\ h\to 0)$
✔ setting up difference quotient with limit notation ✔ expanding $f(x+h)$ ✔ cancelling terms ✔ derivative

(c) What is the instantaneous rate of change when $x=-2$? (2 marks)

Solution
$f'(x)=4x-6$ $f'(-2)=-14$

(d) Is the rate of change increasing or decreasing for $x \leftarrow 2$? Justify. (3 marks)

Solution
Stationary point is when $f'(x)=0$ $f'(x)=4x-6$ $0=2(2x-3)$ $\therefore$ Stationary point is when $x=1.5$ Taking a point to the left of $x=-2$, $f'(-3)=-18$. Hence, the rate of change is decreasing for $x \leftarrow 2$.
✔ determining stationary points ✔ rate of change increasing ✔ valid justification

Question 4 **(7 marks)**

Given the equation $2^{x+1}+2^{3-x}=17, x \in R$,

(a) use a suitable substitution to rewrite the equation above as a quadratic equation, (3 marks)

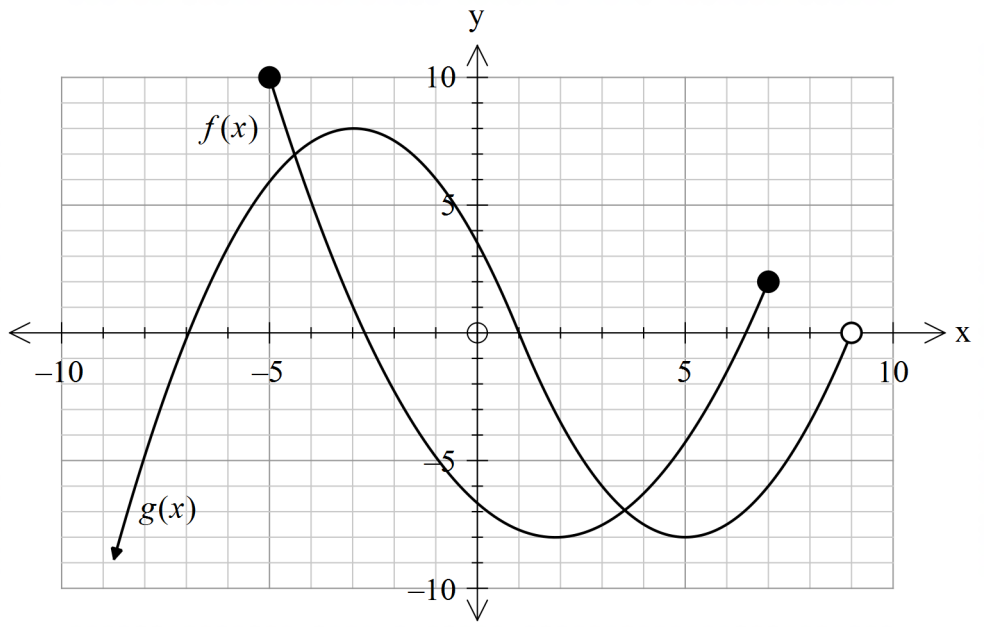
Solution
$2^{x+1}+2^{3-x}=17$ $2^x \times 2+2^3 \times 2^{-x}=17 \qquad let: 2^x = y$ $2y+\frac{8}{y}=17$ $2y^2-17y+8=0$
✔ using appropriate substitution ✔✔ quadratic equation (-✔ for every wrong term)

(b) hence, determine the solution(s) to the exponential equation. (4 marks)

Solution
$2y^2-17y+8=0$ $(2y-1)(y-8)=0$ $\therefore y=\frac{1}{2} \vee 8.$ $2^x=\frac{1}{2} \rightarrow x=-1$ $2^x=8 \rightarrow x=3$
✔ factorising quadratic ✔ y values ✔✔ for solutions to the exponential equation

Question 5 **(11 marks)**

The graphs of the functions $f(x)$ and $g(x)$ are shown below.



(a) State the domain of $g(x)$. (1 mark)

Solution
$\{x \in R : x < 9\}$

(b) State the range of $f(x)$. (1 mark)

Solution
$\{f(x) \in R : -8 \leq f(x) \leq 10\}$

(c) Which function(s), if any, have a point of inflection? (1 mark)

Solution
$g(x)$

(d) Describe how the graph of $y = -g(x)$ may be obtained from the graph of $y = g(x)$ and hence state the coordinates of the turning point(s) of the graph $y = -g(x)$. (3 marks)

Solution
Reflection about the x axis.

Turning points: $(-3,8)\rightarrow(-3,-8)$ and $(5,-8)\rightarrow(5,8)$
✔ identifying transformation ✔✔ correct turning points

(e) Describe how the graph of $y = f(x+1)$ may be obtained from the graph of $y = f(x)$ and hence state the coordinates of the endpoints of the graph $y = f(x+1)$. (3 marks)

Solution
Horizontal translation of 1 unit to the left. End points: $(-5,10) \rightarrow (-6,10)$ and $(7,2) \rightarrow (6,2)$
✔ identifying transformation ✔✔ correct end points

(f) State the approximate solution(s) to the equation $f(x) = g(x)$. (2 marks)

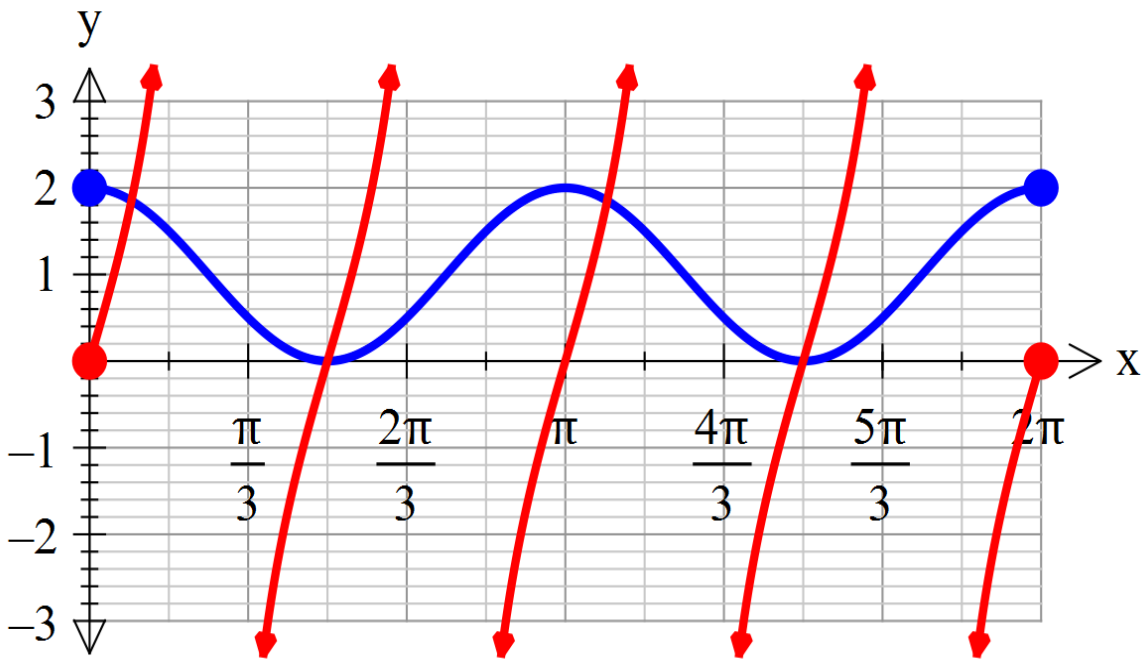
Solution
$\left(4\frac{1}{3},7\right) \wedge \left(3\frac{1}{2},-7\right)$

Question 6 **(12 marks)**

Given the following equations:

$$y_1 = 2\cos 2x, y_2 = 3\tan 2x, 0 \le x \le 2\pi.$$

(a) Sketch on a single set of axes the graph of y_1 and y_2. (4 marks)



Solution
See graph above.
For each function: ✔ shape ✔ accuracy

(b) Show that the coordinates of the points of intersection between the graphs of y_1 and y_2 are solutions of the equation $2\sin^2 2x + 3\sin 2x - 2 = 0$. (4 marks)

Solution
$2\cos 2x = 3\tan 2x$ $2\cos 2x = \dfrac{3\sin 2x}{\cos 2x}$ $2\cos^2 2x = 3\sin 2x$ 2¿ $2\sin^2 2x + 3\sin 2x - 2 = 0$
✔ rewriting tan in terms of sin and cos ✔ multiplying both sides by cos ✔ using Pythagorean identity ✔ required equation

(c) Hence find the x coordinates of the points of intersection between the graphs of y_1 and y_2 over the domain $[0, 2\pi]$. (4 marks)

Solution
$let \sin 2x = y$ $2y^2 + 3y - 2 = 0$ $(y+2)(2y-1) = 0 \rightarrow y = -2 \vee \frac{1}{2}$ $\rightarrow \sin 2x = -2 \; NO\ SOLUTIONS$ $\rightarrow \sin 2x = \frac{1}{2}$ $2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$
✔️ factorizing quadratic and finding two solutions ✔️ indicating that there are no solutions for $\sin 2x = -2$ ✔️✔️ finding the 4 solutions (-✔️ for every incorrect answer)

END OF SECTION 1

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____