SHENTON COLLEGE

2019 YEAR 12 MATHEMATICS: METHODS
Test 3 (Continuous Random Variables, Normal Distribution, Logarithms)

NAME:_____

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Calculator-Assumed	Formula sheet provided	Working time: 25 minutes	Marks: 37 marks

QUESTION 7

[4 marks –2, 2]

Calculate the exact value of a in each of the following probability density functions of continuous random variables.

a) $p(x) = \begin{cases} ax^2 & 1 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$

b) $p(x) = \begin{cases} 3e^{-2x} & 0 \le x \le a \\ 0 & x < 0 \end{cases}$

QUESTION 8

[3 marks – 1, 2]

A continuous random variable X, as the probability density function given by

$$p(x) = \begin{cases} \frac{1}{2}\cos x & -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$

Calculate the following probabilities correct to **four decimal places.**

a) $P(X > \frac{\pi}{3})$	b) $P(X < \frac{\pi}{4} \mid X > -\frac{\pi}{6})$

QUESTION 9

[10 marks – 2, 2, 2, 2, 2]

A continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}(2x+1) & 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

a) Calculate the mean of X.

b) Calculate the standard deviation of X.

c) Calculate the median of X.

d) State the cumulative distribution function, $F(x)$.

e) Show how you would use the cumulative distribution function to calculate $P(1.2 < X < 1.7)$.

QUESTION 10

[3 marks – 1, 2]

The heights of 50 Year 12 students are displayed in the table below.

Height (cm) x	**Frequency**
$140 \le x < 150$	2
$150 \le x < 160$	10
$160 \le x < 170$	19
$170 \le x < 180$	15
$180 \le x < 190$	3
$190 \le x < 200$	1

Use the data in the table to calculate the following probabilities.

a) $P(160 < X < 180)$

b) $P(X < 150 | X < 170)$

QUESTION 11

[5 marks – 2, 1, 2]

Each note on a piano keyboard is one semi-tone apart. The ratio of frequencies between each semitone is 5.946%.

This means that if one note has a frequency of f_1 and another higher note has a frequency of f_2, then

$$1.05946^x = \frac{f_2}{f_1}$$

where x the number of semitones between the two notes.

a) Apply logarithms of base ten to both sides of the above equation and hence obtain a rule for x in terms of f_1 and f_2.

Middle C has a frequency of 261.63 Hz.

b) The next C on the keyboard, which is an octave higher, has a frequency of 523.25 Hz. Show the use of your formula from part a) to verify that there are 12 semitones in an octave.

c) An interval between two notes is called a "perfect fifth" if they are 7 semi-tones apart. Calculate the frequency of the note that is a perfect fifth higher than middle C.