



**PERTH COLLEGE**  
**Year 12**  
**Semester One Examination 2011**  
**Question/Answer booklet**

**MATHEMATICS**  
**SPECIALIST 3CD**

**Section One:**  
**Calculator – free**

Student Name: \_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: 5 minutes  
Working time for paper: 50 minutes

**Material required/recommended for this section**

**To be provided by the supervisor**

Question/answer booklet for Section One  
Formula sheet

**To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

	Number of questions available	Number of questions to be attempted	Working Time (minutes)	Marks available
<b>Section One Calculator-free</b>	<b>5</b>	<b>5</b>	<b>50 minutes</b>	<b>40</b>
Section Two Calculator-assumed	11	11	100 minutes	80
<b>Total marks</b>				<b>120</b>

**Instructions to candidates**

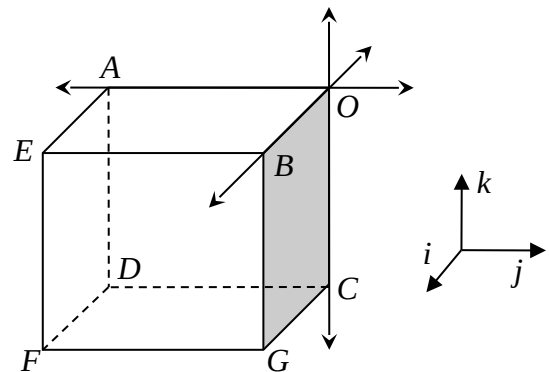
1. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer
  - a. Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - b. Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
2. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.
3. It is recommended that you **do not use pencil**, except in diagrams

**Question 1** [1 + 2 + 2 = 5 marks]

In the diagram shown,  $OAEBCDFG$  is a cube with the origin at  $O$ .

Let  $i$ ,  $j$  and  $k$  be the unit vectors along the  $x$ ,  $y$  and  $z$  axes respectively, as shown.

Let  $OA=a$ ,  $OB=b$  and  $OC=c$ .



(a) Find an expression for  $a$ ,  $b$  and  $c$  in terms of  $i$ ,  $j$  and/or  $k$ , given that  $|a|=|b|=|c|=2$  [1]

(b) Find  $OE$  and  $OF$  in terms of  $i$ ,  $j$  and  $k$ . [2]

(c) Prove that  $FE$  and  $AB$  are perpendicular. [2]

**Question 2** [2 + 1 + 1 + 2 = 6 marks]

A plane  $\Pi$  is such that it is parallel to  $i+j$ , perpendicular to  $k$ , and passes through  $-(i+j+k)$ .

(a) Find the vector equation of the plane  $\Pi$ . [2]

(b) Find the normal equation of the plane  $\Pi$ . [1]

(c) Find the Cartesian equation of the plane  $\Pi$ . [1]

(d) Find the point of intersection of the plane  $\Pi$  and the line  $r = (2\lambda + 1)(i + j) - \lambda k$  [2]

**Question 3** [3 + 2 + 3 + 2 + 3 = 13 marks]

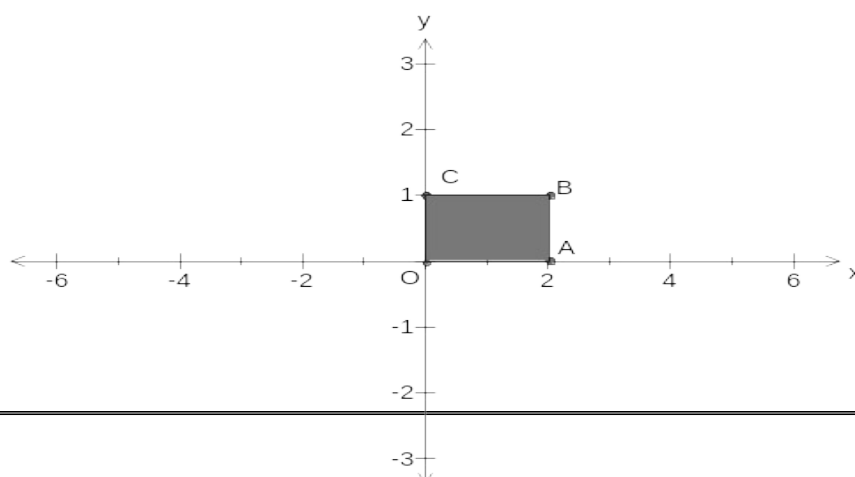
Consider the following transformation matrices:  $T_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$   $T_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$   $T_3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

These transformation are performed in the order  $T_1, T_2$  and then  $T_3$ .

(a) Describe the type of transformation performed by each matrix. [3]

(b) Find the matrix  $T_4$  which will perform all three transformations in a single step. [2]

(c) Rectangle OABC is shown below. Find and sketch rectangle O'A'B'C' obtained using the transformation matrix found in (b). [3]



**(Question 3 continued)**

- (d) The matrix below contains the vertices O'M'N'P' of a shape transformed using  $T_4$ .  
Find the original vertices OMNP. [2]

$$\begin{bmatrix} 0 & -3 & 1 & 4 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

- (e) The transformation matrix  $T_4$  is applied to the line  $y=3-x$ .  
Prove that the new line obtained is perpendicular to the original line. [3]

**Question 4** [1 + 2 + 3 + 2 + 3 = 11 marks]

Find an expression for  $\frac{dy}{dx}$  for each case below. Do not simplify your answers.

(a)  $y = x^2 \sin x^2$  [1]

(b)  $y = \frac{x^2 \ln x^2}{e^{x^2}}$  [2]

(c)  $y = (2x)^{3x}$  (Hint: apply the natural logarithm on both sides) [3]

**(Question 4 continued)**

**(d)**  $y = \ln t^2$  ,  $x = e^t$  (leave your answer in terms of  $x$ ) [2]

**(e)**  $x \sin y - y \sin x = \pi$  [3]



**Question 5** [5 marks]

Perform the following integration.

*(Hint: Refer to formula sheet provided for a suitable trigonometric identity)*

$$\int (\cos x)^3 dx$$

[5]

**END OF SECTION ONE**

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