SEMESTER ONE

MATHEMATICS METHODS UNITS 1

2017

SOLUTIONS

Calculator-free Solutions

1. (a) (i) $P' \cap Q$ ✓
   (ii) $M \cap (L \cup N)$ *or* $(L \cap M) \cup (M \cap N)$ ✓
   (b) (i) $A \cap B \cap C = \{3\}$ ✓
   (ii) $(A \cup B)' = \{8, 9, 10, 14, 15, 16, 18, 19\}$ ✓
   (iii) $n(B \cup C) = 8$ ✓ [5]

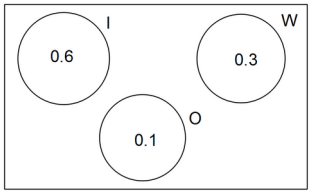
2. (a) (i) $x^2 + y^2 = 13$ ✓✓
   (ii) $y = -\frac{6}{x}$ ✓✓
   (iii) $y = -x - 1$ ✓✓
   (b) $x = -3, -2, 2, 3$ ✓✓
   (c) (i) $(-\frac{3}{2}, 3)$ ✓✓
   (ii) $m = -2$ ✓ [11]

3. $(x-1)^2 + (y-1)^2 = 25$ ✓
   Centre (1, 1)
   $\frac{4+x}{2} = 1 \quad \frac{5+y}{2} = 1$ ✓
   M $(-2, -3)$ ✓✓ [4]

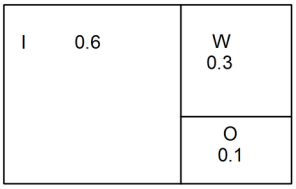
4. (a) $\sin 225°$ ✓
   $= -\sin 45°$
   $= -\frac{\sqrt{2}}{2}$ ✓
   (b) $\tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$ ✓
   $= \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{3}}{1 - \left(\tan\frac{\pi}{4}\right)\left(\tan\frac{\pi}{3}\right)}$ ✓
   $= \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$ ✓
   $= -2 - \sqrt{3}$ ✓ [6]

5. (a) $A = \frac{1}{2}(x)(x+3)(\sin 60)$ ✓
   $= \frac{x}{2}(x+3)\left(\frac{\sqrt{3}}{2}\right)$ ✓
   $= \frac{x}{4}(\sqrt{3}\,x + 3\sqrt{3})$ cm^2
   (b) $x^2 + (x+3)^2 - 2(x)(x+3)(\cos 60) = 49$ ✓
   $x^2 + 3x - 40 = 0$
   $(x+8)(x-5) = 0$ ✓
   $(x = -8$ discard$) \therefore x = 5$ ✓
   (c) $A = \frac{5\sqrt{3}}{4}(5+3)$ ✓
   $10\sqrt{3}$ cm^2 ✓ [7]

6. (a) 60% ✓
(b) 40% ✓
(c) 70% ✓
(d)



Or



✓✓ [5]

7. (a) $4\cos(x) - 2 = -4\sin\left(x - \frac{\pi}{2}\right) - 2$

$a = 4 \quad b = 2 \quad c = -4 \quad d = \frac{\pi}{2}$ or $c = 4\ d = -\frac{\pi}{2}$ ✓✓✓✓

(b) $R_{k(x)} = \{y : -6 \leq y \leq 2, y \in \mathbb{R}\}$ ✓✓

(c) For every x value there is only one y value ✓
Vertical line through graph ✓ [8]

8. (a) $\theta = 210°$ or $330°$ ✓✓

(b) $\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$ ✓

$\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ ✓✓ [5]

Calculator-assumed Solutions

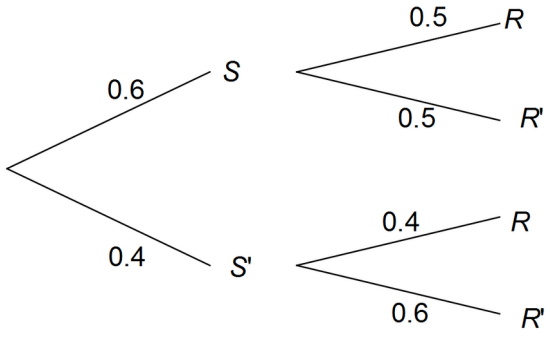
9. *(a)* (i) $V = \frac{k}{P}$

$k = 2.84 \times 11.5$ ✓
$k = 32.66$ ✓

(ii) The volume doubles. ✓

(b) $s = 0.0325c$ is a linear relationship in the form $y = kx$ where $k = 0.0325$ and is therefore directly proportional. ✓✓ [5]

10. (a)



✓✓✓

(b) (i) 0.3 ✓
(ii) 0.24 ✓
(iii) 0.46 ✓✓
(iv) 0.348 or $\frac{16}{46}$ ✓✓ [9]

11. (a) $y = 4 + b$ ✓

(b) $(0,\ -c-2)$ ✓✓

(c) $\{x : x \neq a-3;\ x \in \mathbb{R}\}$ ✓✓ [5]

12. (a) $\sin\theta = -\dfrac{\sqrt{161}}{15}$ ✓

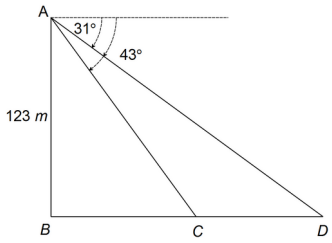
$\cos 2\theta = (\cos\theta)(\cos\theta) - (\sin\theta)(\sin\theta)$ ✓

$= \dfrac{64}{225} - \dfrac{161}{225}$ ✓

$= -\dfrac{97}{225}$ ✓

(b) $AC = \dfrac{123}{\sin 43°}$ ✓

$\angle CAD = 12°$ and $\angle ADC = 31°$ ✓



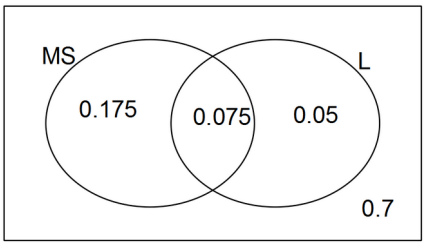
$\dfrac{CD}{\sin 12°} = \dfrac{AC}{\sin 31°}$

$\therefore CD = AC\dfrac{\sin 12°}{\sin 31°}$ ✓

$= \dfrac{123}{\sin 43°} \times \dfrac{\sin 12°}{\sin 31°}$

$= 73$ m ✓ [8]

13. (a)



✓✓✓

(b) 0.075 ✓

(c) 133 ✓

(d) $\dfrac{0.075}{0.125} = 0.6$ ✓✓

(e) $P(MS \cap L) = P(MS) \times P(L)$ to be independent.

$0.075 \neq 0.25 \times 0.125$ ✓

Therefore not independent or $P(M \mid L) \neq P(M) \therefore$ Not independent ✓ [9]

14. (a) (i) 3 ✓

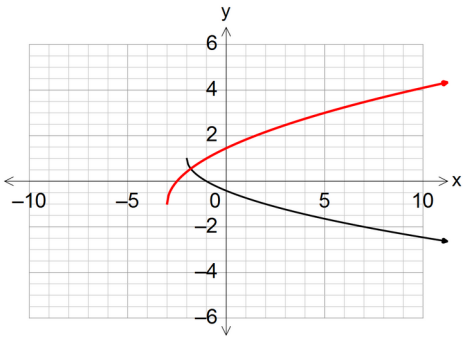
(ii) 0 ✓

(iii) 1 ✓

(b) Domain of $p(x)$ is the same as $m(x)$. For both: $x \geq 0,\ x \in \mathbb{R}$ ✓✓

(c) $q(x) = -\sqrt{x+2} + 1$ ✓✓

(d)



✓✓✓ [10]

15. (a) $-x^2 + 3x + 4 = 2x + q$ ✓

$-x^2 + x + 4 - q = 0$ ✓

$b^2 - 4ac = 0$ for one solution

$\therefore 17 - 4q = 0$ ✓

$\therefore q = \frac{17}{4}$ ✓

(b) $17 - 4q < 0$ ✓

$\therefore q > \frac{17}{4}$ ✓ [6]

16. (a) $100^2 = 80^2 + 80^2 - 2(80)(80)\cos A$

$\angle A = 77.3644°$ or $\angle A = 1.3503^R$ ✓

Area $= \frac{1}{2}(80)(80)\sin 77.3644°$ ✓

$= 3122.4993 \text{ cm}^2$ ✓

(b) $\angle BCA = \angle ABC = 0.8957^R$ ✓

Area of sector $= \frac{1}{2}(50^2)(0.89566)$

$= 1119.56 \text{ cm}^2$ ✓

Area of small sector $= \frac{1}{2}(30^2)(1.3503)$

$= 607.635 \text{ cm}^2$ ✓

Area of shaded section $= 3122.4993 - (2)(1119.58) - (607.635)$

$= 275.704$

$= 276 \text{ cm}^2$ ✓ [7]

17. (a) (i) $8 \times 7 \times 6 \times 5 \times 4 = 6720$ ✓✓

(ii) $\frac{1}{6720} = 0.00015$ ✓

(b) (i) ${}^4\mathbf{C}_0\, a^4 + {}^4\mathbf{C}_1\, a^3 b + {}^4\mathbf{C}_2\, a^2 b^2 + {}^4\mathbf{C}_3\, ab^3 + {}^4\mathbf{C}_4\, b^4$ ✓✓

(ii) ${}^7\mathbf{C}_3\,(2x)^4(-y)^3 = -560x^4y^3$ ✓✓

(c) (i) ${}^5\mathbf{C}_3 \times {}^6\mathbf{C}_2 = 150$ ✓

(ii) $({}^6\mathbf{C}_4 \times {}^5\mathbf{C}_1) + ({}^6\mathbf{C}_5 \times {}^5\mathbf{C}_0) = 81$ ✓✓ [10]

18. (a) $\tan 122° = -1.6$

$y = -1.6x + c$ ✓

$5.2 = (-2)(-1.6) + c$

$c = 2$

$y = -1.6x + 2$ ✓

(b) $2\left(x^2 - \frac{3}{2}x - \frac{1}{2}\right) = y$

$2\left(\left(x - \frac{3}{4}\right)^2 - \frac{17}{16}\right) = y$ ✓

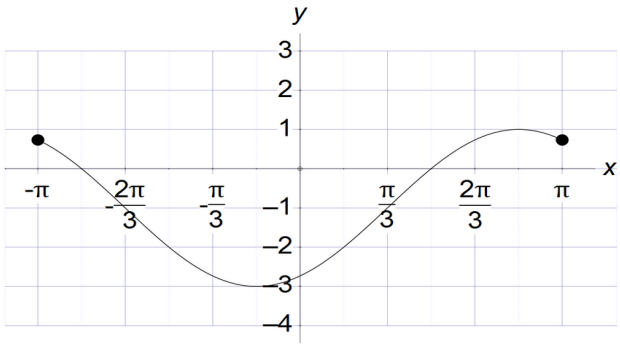
$2\left(x - \frac{3}{4}\right)^2 - \frac{17}{8} = y$ ✓

Turning Point $\left(\frac{3}{4}, -\frac{17}{8}\right)$ ✓

(c)	(i)	1 m	✓	
	(ii)	5.5m	✓	
	(iii)	1.77 m or 28.23 m away	✓✓	[9]

19. (a) $y = 2\sin\left(x - \frac{\pi}{3}\right) - 1$ ✓✓

(b)



✓✓✓

(c) $x = -\frac{2\pi}{3}$ or $\frac{\pi}{3}$ ✓ [6]

20. (a)

	H	H'	
G	0.4	0.3	0.7
G'	0.2	0.1	0.3
	0.6	0.4	1

✓✓✓

(b) (i) 0.3 ✓

(ii) 0.6 ✓

(iii) 0.1 ✓

(iv) 0.9 ✓

(v) $\frac{2}{3}$ ✓✓

(c) Not mutually exclusive: $P(G \cap H) = 0.4$ Should be 0 ✓ [10]

21. (a) $C\left(\frac{3}{2}, \frac{5}{2}\right)$ $r = 3$ ✓✓

(b) (i) $y = -(x + a)(x - b)^2$ ✓

(ii) $y = x(x + a)(x - b)$ ✓

(iii) $y = (x - a)^3 + b$ ✓ [5]

End of Questions