

MATHEMATICS

METHODS

UNIT 1

Section One:

Calculator-free

Student number: In figures

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

Question number: _____

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

(5 marks)
(2 marks)

Solution
$10t + 5 - 3t + 12 = 0$ $7t + 17 = 0$ $t = -\frac{17}{7}$
Specific behaviours
✓ expands and simplifies correctly ✓ solves for t

(a) Solve $5(2t + 1) - 3(t - 4) = 0$ for t .

(3 marks)

(b) Solve $\frac{a-5}{7} - \frac{4a}{3} = 0$ for a .

Solution
$\frac{a-5}{7} = \frac{4a}{3} \Leftrightarrow 3(a-5) = 28a$ $28a = 3a - 15$ $25a = -15$ $a = -\frac{15}{25} = -\frac{3}{5}$
Specific behaviours
✓ cross-multiplies ✓ expands and simplifies ✓ solves for a

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(7 marks)

(a) The graph of the relation $y^2 = x$ passes through the points $(16, a)$ and $(b, -5)$. Determine the values of a and b .

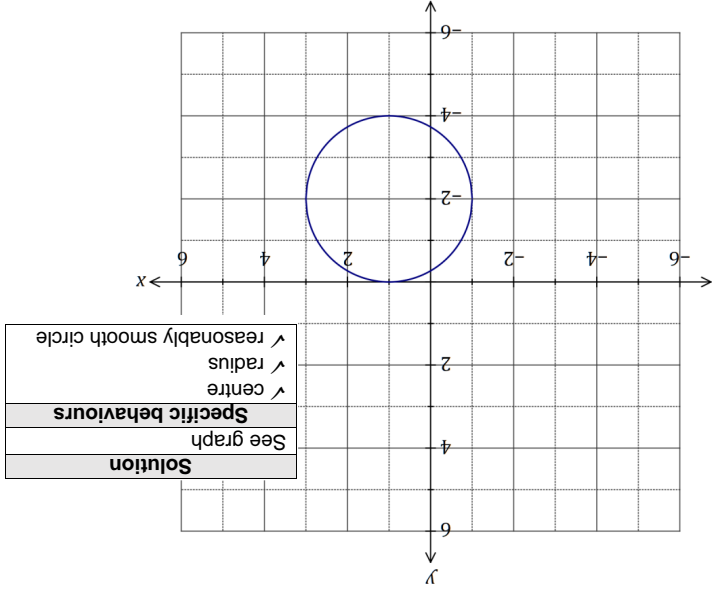
(3 marks)

Solution
$a^2 = 16 \Rightarrow a = 4, a = -4$ $(-5)^2 = b \Rightarrow b = 25$
Specific behaviours
✓ one value of a ✓ both values of a ✓ value of b

(b) Another relation is defined by $(x - 1)^2 + (y + 2)^2 = 4$.

(3 marks)

(i) Sketch the graph of this relation on the axes below.



Solution
See graph
Specific behaviours
✓ centre ✓ radius ✓ reasonably smooth circle

Solution
A vertical line can be drawn that intersects the circle more than once, and thus shows a relation.
Specific behaviours
✓ uses vertical line test

(1 mark)

(ii) What feature of the graph indicates that a relation rather than a function is shown?

End of questions

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Question 2

Solve the following equations.

(a) $6x^2 = 3x$.

Solution
$3x(2x - 1) = 0$
$x = 0, \quad x = \frac{1}{2}$
Specific behaviours
<ul style="list-style-type: none"> ✓ equates to zero and factorises ✓ solutions

(b) $x(x + 2) = 24$.

Solution
$x^2 + 2x - 24 = 0$
$(x + 6)(x - 4) = 0$
$x = -6, \quad x = 4$
Specific behaviours
<ul style="list-style-type: none"> ✓ expands and equates to zero ✓ factorises ✓ solutions

(5 marks)

(2 marks)

(3 marks)

Question 7

- (a) The tenth row of Pascal's triangle begins with the numbers 1, 10, 45, 120, 210, 252 and so on.

- (i) State the value of
- $\binom{10}{4}$
- .

Solution
$\binom{10}{4} = 210$
Specific behaviours
✓ correct value

(1 mark)

- (ii) Deduce the value of
- $\binom{11}{3}$
- .

Solution
$\binom{11}{3} = \binom{10}{2} + \binom{10}{3} = 45 + 120 = 165$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates use of terms in previous row ✓ correct value

(2 marks)

- (iii) Calculate the sum of all the terms in the ninth row of Pascal's triangle.

(2 marks)

Solution
Sum = 2^9 = 512
Specific behaviours
<ul style="list-style-type: none"> ✓ expresses as power of 2 ✓ evaluates

- (b) The coefficient of the x^2 term in $x^3 + 5x^2 - 3x + 1$ is 5. Determine the coefficient of the x^2 term in the expansion of

- (i)
- $(3x - 5)^2$
- .

(1 mark)

Solution
$9x^2 \dots$ Coefficient is 9
Specific behaviours
✓ correct value

- (ii)
- $(2x + 2)^4$
- .

(3 marks)

Solution
Required term is $(2x)^2(2)^2 \times {}^4C_2 = 4x^2 \times 4 \times 6 = 96x^2$
Coefficient is 96
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates elements of required term ✓ indicates use of 4C_2 and/or Pascals triangle ✓ correct value

(6 marks)

(1 mark)

Solution
$f(12) = \sqrt{36} = 6$
Specific behaviours
✓ correct value

A function is defined by $f(x) = \sqrt{3x}$.

(a) Calculate $f(12)$.

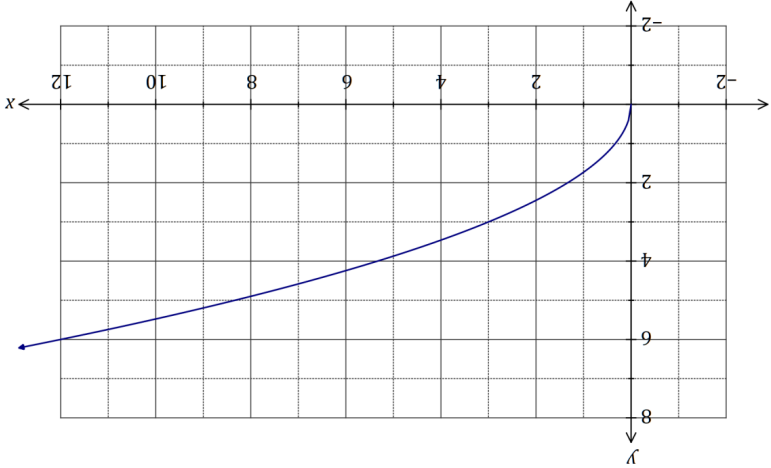
(b) State the domain and range of $f(x)$.

(2 marks)

Solution
$D_f: x \geq 0, R_f: y \geq 0$
Specific behaviours
✓ domain, ✓ range

(3 marks)

(c) Sketch the graph of $y = f(x)$ on the axes below.



Solution
See graph
Specific behaviours
✓ starts at (0,0) ✓ passes through (3, 3) and (12, 6) ✓ smooth curve

See next page

Question 3

A function is defined by $f(x) = \sqrt{3x}$.

(a) Calculate $f(12)$.

Solution
$f(12) = \sqrt{36} = 6$
Specific behaviours
✓ correct value

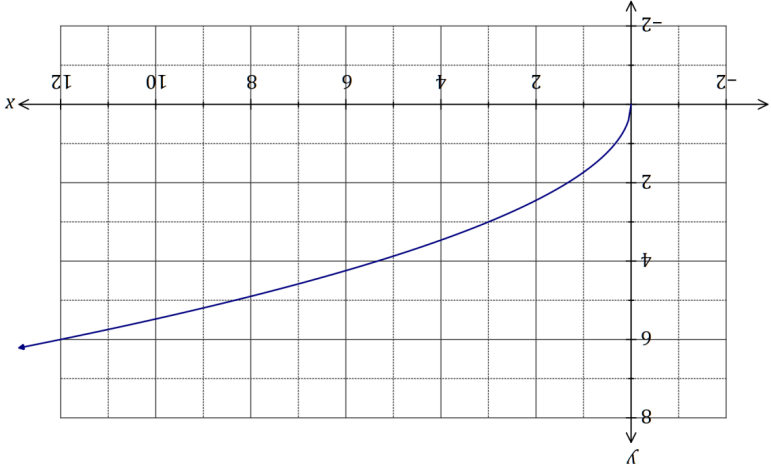
(b) State the domain and range of $f(x)$.

(2 marks)

Solution
$D_f: x \geq 0, R_f: y \geq 0$
Specific behaviours
✓ domain, ✓ range

(3 marks)

(c) Sketch the graph of $y = f(x)$ on the axes below.



Solution
See graph
Specific behaviours
✓ starts at (0,0) ✓ passes through (3, 3) and (12, 6) ✓ smooth curve

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Question 6

(a) Solve the equation $\sqrt{3} \tan(x) - 3 = 0$ for $0 \leq x \leq 2\pi$.

(3 marks)

(9 marks)

Solution
$\frac{3}{\tan x} = \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$ $\frac{\pi}{4\pi} = \frac{\pi}{4\pi}$ $x = \frac{3}{3}$
Specific behaviours
✓ expression for $\tan x$ ✓ one solution ✓ second solution (penalise once for use of degrees)

(b) A function has a period of k and is defined by $f(x) = 4 \cos(2x)$.

(1 mark)

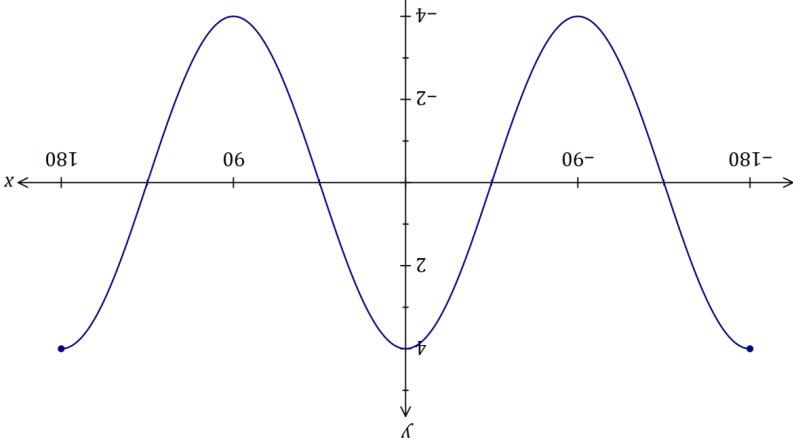
Solution
(i) $k = \frac{2}{2\pi} = \pi$ or $k = \frac{2}{360} = 180^\circ$ (ii) Amplitude is 4
Specific behaviours
✓ period (either unit) ✓ amplitude

(1 mark)

(ii) State the amplitude of $f(x)$.

(iii) Sketch the graph of $y = f(x)$ over the domain $-k \leq x \leq k$.

(4 marks)



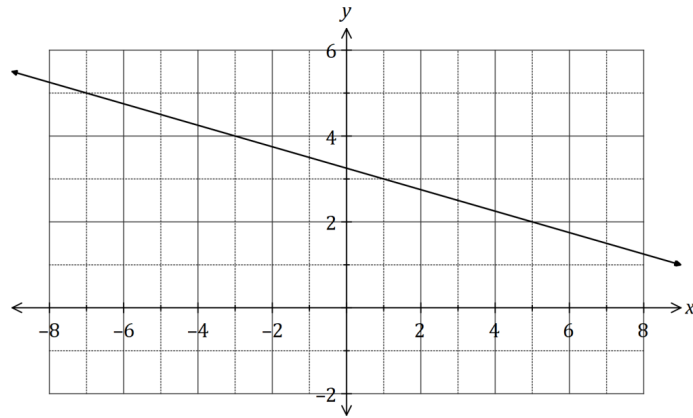
Solution
See graph (student choice of radians or degrees)
Specific behaviours
✓ amplitude correct, with scale indicated ✓ axes intercepts, with scale indicated ✓ two complete cycles (no penalty for slightly exceeding domain ✓ smooth curve

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Question 4

(5 marks)

The graph of the line L_1 is shown below.



- (a) Determine the equation of L_1 .

Solution
$m = -\frac{1}{4}$ $y - 3 = -\frac{1}{4}(x - 1)$ $y = -\frac{1}{4}x + \frac{13}{4}$
Specific behaviours
<ul style="list-style-type: none"> ✓ gradient ✓ correct equation (any form)

(2 marks)

Two points are located at $A(-10, 5)$ and $B(6, 29)$.

- (b) Line L_2 is perpendicular to L_1 and passes through the mid-point of A and B . Determine the equation of L_2 .

Solution
$M(-2, 17)$ $m = -1 \div \left(-\frac{1}{4}\right) = 4$ $y - 17 = 4(x - (-2))$ $y = 4x + 25$
Specific behaviours
<ul style="list-style-type: none"> ✓ coordinates of midpoint ✓ perpendicular gradient ✓ equation of line (any form)

(3 marks)

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Question 5

(6 marks)

- (a) Expand and simplify $(x + 2)(2x - 5)(x - 2)$.

(2 marks)

Solution
$(x + 2)(2x - 5)(x - 2) = (2x - 5)(x^2 - 4)$ $= 2x^3 - 5x^2 - 8x + 20$
Specific behaviours
<ul style="list-style-type: none"> ✓ expands one pair of terms ✓ simplified expansion

- (b) One solution to the equation $x^3 + 36 = 5x^2 + 12x$ is $x = 2$. Determine all other solutions.

(4 marks)

Solution
$x^3 - 5x^2 - 12x + 36 = 0$ $(x - 2)(x^2 + ax - 18) = 0$ $-2 + a = -5 \Rightarrow a = -3$ $(x - 2)(x^2 - 3x - 18) = 0$ $(x - 2)(x - 6)(x + 3) = 0$ <p>Other solutions: $x = 6, \quad x = -3$</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ equates to zero and identifies $(x - 2)$ as a factor ✓ factors out quadratic expression ✓ identifies value of a ✓ factors quadratic and states other two solutions

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