Additional working space

Question number: __________

Latest Printout 27/9/16.

WESLEY COLLEGE

By daring & by doing

Semester Two Examination, 2016

Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 AND 4

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

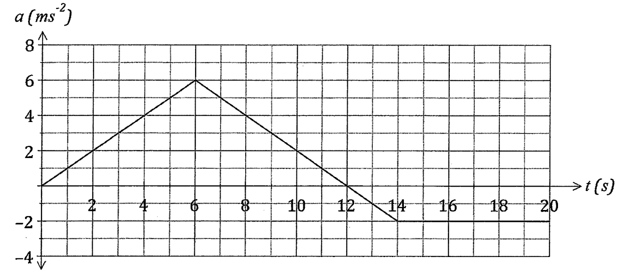
Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

Question 20 **(8 marks)**

A particle, initially stationary and at the origin, moves subject to an acceleration, a ms^{-2}, as shown in the graph below for $0 \le t \le 20$ seconds.



(a) Determine the velocity of the object when

(i) $t = 6$. (1 mark)

$\frac{1}{2} \times 6 \times 6 = 18$ m/s

(ii) $t = 20$. (2 marks)

$2 \times 18 - 2 - 12$

$= 22$ m/s

(b) At what time is the velocity of the body a maximum, and what is the maximum velocity? (2 marks)

$v' = 0$ ie $a = 0$ $\therefore t = 12$ s

$v = 18 + 18$

$= 36$ m/s

(c) Determine the distance of the particle from the origin after 3 seconds. (3 marks)

from graph: $a = t$

$\therefore v = \frac{t^2}{2}$

$\therefore x = \frac{t^3}{6}$

$x(3) = \frac{27}{6} = 4.5$ m

End of questions

Question 19 **(7 marks)**

The moment magnitude scale M_w is used by seismologists to measure the size of earthquakes in terms of the energy released. It was developed to succeed the 1930's-era Richter magnitude scale.

The moment magnitude has no units and is defined as $M_w = \frac{2}{3}\log_{10}(M_0) - 10.7$, where M_0 is the total amount of energy that is transformed during an earthquake, measured in dyn·cm.

(a) On 28 June 2016, an estimated 2.82×10^{21} dyn·cm of energy was transformed during an earthquake near Norseman, WA. Calculate the moment magnitude for this earthquake. (1 mark)

$$M_w = \frac{2}{3}\log(2.82 \times 10^{21}) - 10.7$$
$$= 3.6$$

(b) A few days later, on 8 July 2016, there was another earthquake with moment magnitude 5.2 just north of Norseman. Calculate how much energy was transformed during this earthquake. (2 marks)

$$\frac{2}{3}\log x - 10.7 = 5.2$$
$$x = 7.08 \times 10^{23}$$

(c) Show that an increase of 2 on the moment magnitude scale corresponds to the transformation of 1000 times more energy during an earthquake. (4 marks)

$$M_w = \frac{2}{3}\log(M_0) - 10.7$$
$$\log_{10} M_0 = \frac{3}{2}(M_w + 10.7)$$
$$M_0 = 10^{1.5(M_w + 10.7)}$$
$$M_w = 1 \text{ ie } M_0 = 10^{1.5(11.7)} = 10^{17.55}$$
$$M_w = 3 \text{ ie } M_0 = 10^{1.5(13.7)} = 10^{20.55}$$

Dividing: $10^3 = 1000$ times greater.

(Check other methods)

Section One: Calculator-free **35% (52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(6 marks)**

A particle leaves the origin when $t = 1$ and moves in a straight line with velocity at any time t seconds, where $t \geq 1$, given by

$$v(t) = \frac{t^2}{4} + \frac{4}{t} - \frac{7}{4} \text{ ms}^{-1}$$

(a) Determine the time when the acceleration of the particle is zero. (2 marks)

$$a(t) = \frac{2t}{4} - \frac{4}{t^2}$$
$$= \frac{t}{2} - \frac{4}{t^2}$$
$$a(t) = 0 \Rightarrow \frac{t}{2} - \frac{4}{t^2} = 0$$
$$t^3 = 8$$
$$t = 2s$$

(b) Determine the exact displacement of the particle from the origin when $t = 4$. (4 marks)

$$x(t) = \int \frac{t^2}{4} + \frac{4}{t} - \frac{7}{4}\, dt$$
$$= \frac{t^3}{12} + 4\ln t - \frac{7t}{4} + c$$

Now $x(1) = 0$ ie $\frac{1}{12} + 0 - \frac{7}{4} + c = 0$

$$\frac{1-21}{12} + c = 0$$
$$c = \frac{20}{12} = \frac{5}{3}$$

$$\text{So } x(4) = \frac{4^3}{12} + 4\ln 4 - \frac{7.4}{4} + \frac{5}{3}$$
$$= 4\ln 4 + \frac{16+5-21}{3}$$
$$= 4\ln 4$$

Question 2 **(7 marks)**

(a) Calculate $f'(0)$ when $f(x) = e^{2x}(1+5x)^3$. (3 marks)

$f'(x) = 2e^{2x}(1+5x)^3 + e^{2x} \cdot 3(1+5x)^2 \cdot 5$

$f'(0) = 2e^0(1+0)^3 + e^0 \cdot 3(1+0)^2 \cdot 5$

$= 2 + 1 \cdot 15$

$= 17$

(b) Determine $\frac{d}{dx}\int_x^5 \sqrt{t^2+1}\,dt$. (2 marks)

ie $-\frac{d}{dx}\int_5^x \sqrt{t^2+1}\,dt$

$= -\sqrt{x^2+1}$

(c) Given $f'(x) = (1-2x)^4$ and $f(1) = -1$, determine $f(x)$. (2 marks)

$f(x) = \int (1-2x)^4\,dx$

$= \frac{(1-2x)^5}{-10} + c$

$f(1) = \frac{(-1)^5}{-10} + c = -1$

$c = -1\frac{1}{10}$

$\therefore f(x) = \frac{(1-2x)^5}{-10} - 1\frac{1}{10}$

Question 18 **(7 marks)**

From a random sample of n people, it was found that 54 of them subscribe to a streaming music service. A symmetric confidence interval for the true population proportion who subscribe is $0.1842 < p < 0.2958$.

(a) Determine the value of n, by first finding the mid-point of the interval. (3 marks)

$\frac{0.1842 + 0.2958}{2} = 0.24$

So $\frac{54}{n} = 0.24 \Rightarrow n = 225$

(b) Determine the confidence level of the interval. (4 marks)

Std error $\sqrt{\frac{0.24(0.76)}{225}} = 0.02847$

So $0.24 + z \times 0.02847 = 0.2958$

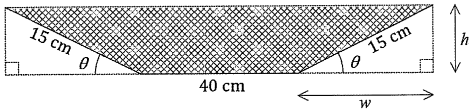
$z = 1.95996$

~ 1.96

$\therefore$ a 95% C.I.

Question 17 **(7 marks)**

A trough for holding water is to be formed by taking a length of metal sheet 70 cm wide and folding 15 cm on either end, up through an angle of θ. The following diagram shows the cross-section of the trough with the cross-sectional area, A, shaded.



(a) Determine A in terms of w and h. (1 mark)

$A = 40h + wh$

(b) Show that $A = 600\sin\theta + 225\sin\theta\cos\theta$. (2 marks)

$W = 15\cos\theta \quad h = 15\sin\theta$

$\therefore A = 40 \cdot 15\sin\theta + 15\cos\theta \cdot 15\sin\theta$

$= 600\sin\theta + 225\sin\theta\cos\theta$

(c) Use calculus to determine the maximum possible cross-sectional area. (4 marks)

$\frac{dA}{dx} = 225(\cos^2 x - 225\sin x)^2 + 600\cos x$

At max, $\frac{dA}{d\theta} = 0$ ie $\theta = 1.26^c$ or 72.23°

$A(1.26) = 636.8 \text{ cm}^2$

Check $\frac{d^2A}{dx^2} = -900\cos x\sin x - 600\sin x$

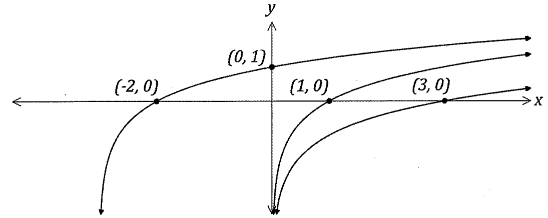
$\left.\frac{d^2A}{dx^2}\right|_{x=1.26} = -833.3 < 0 \therefore$ max.

See next page

Question 3 **(7 marks)**

(a) The function f is defined by $f(x) = \log_a x$, $x > 0$, where a is a constant, $a > 1$.

The graphs shown below have equations, not in order:
$y = f(x)$, $y = f(x+b)$ and $y = f(x) + c$, where b and c are constants.



Determine the values of the constants a, b and c. (4 marks)

$y = f(x+b)$ must pass thru $(-2, 0)$

so $b = 3$

$(0,1) \Rightarrow \log_a(0+3) = 1 \quad \therefore a = 3$

$y = f(x)$ goes thru $(1, 0)$

so $y = f(x) + c$ goes thru $(3, 0)$

ie $\log_3 3 + c = 0$

$c = -1$

(b) Determine

(i) the equation of the asymptote of the graph of $y = \log_e(x-3) - 2$. (1 mark)

$x = 3$

(ii) the coordinates of the y-intercept of the graph of $y = \log_2(x+8) - 5$. (2 marks)

Y int $\Rightarrow x = 0$ ie $y = \log_2 8 - 5$

$= 3 - 5$

$= -2$

ie $(0, -2)$

See next page

Question 4 **(8 marks)**

A curve has equation $y = 2x^5 - 5x^4 + 10$.

(a) Point A lies on the curve at $(-1, 3)$. Use the increments formula $\delta y \approx \frac{dy}{dx} \times \delta x$ to estimate the y-ordinate of point B that has an x-ordinate of -0.99. (4 marks)

$\frac{dy}{dx} = 10x^4 - 20x^3$

$\left.\frac{dy}{dx}\right|_{x=-1} = 10 + 20 = 30$

$\therefore \delta y \approx 30 \times 0.01$

$= 0.3$

$\therefore$ estimate for $y = 3 + 0.3 = 3.3$

(b) Point C also lies on the curve, at $(2, -6)$. Verify that C is a stationary point and determine its nature. (4 marks)

$\frac{dy}{dx} = 10x^3(x-2)$

$\frac{dy}{dx} = 0 \Rightarrow 10x^3(x-2) = 0$

$x = 0, 2$

$\frac{d^2y}{dx^2} = 40x^3 - 60x^2$

$\left.\frac{d^2y}{dx^2}\right|_{x=2} = 320 - 240 > 0 \therefore$ min

$\therefore (2, -6)$ is a minimum.

(b) The stationery company that supplies pens to the conference centre claim that no more than 3 in 50 pens fail to write. Use your previous working to comment on the validity of this claim. (2 marks)

$\frac{3}{50} = 0.06$

0.06 is within CI

$\therefore$ valid claim.

(c) Comment on how the margin of error would change in (a) (ii) if

(i) the quality of the pens had been better. (1 mark)

decrease as P would be smaller

(ii) the required level of confidence decreased. (1 mark)

Z will decrease

So ME will decrease

Question 16 **(9 marks)**

The management at a conference centre was concerned about the quality of the free pens that it provided in its meeting rooms. A staff member tested a random sample of 150 pens and found that 18 of them fail to write.

(a) If p is the true proportion of pens that fail to write and $\hat{p}$ is the corresponding sample proportion, use the above sample to determine

(i) $\hat{p}$. (1 mark)

$\frac{18}{150} = \frac{3}{25} = 0.12$

(ii) the approximate margin of error for a 98% confidence interval for p. (3 marks)

$98\% \Rightarrow z = 2.326$

$ME = \sqrt{\frac{0.12(0.88)}{150}}$

$= 0.02653$

$E = 2.2326 \times 0.02653$

$= 0.0617$

(iii) an approximate 98% confidence interval for p. (1 mark)

$0.0583 < p < 0.1817$

(Use One-Prop z int * Stats/Calc)

Question 5 **(8 marks)**

(a) Determine the coordinates of the root of the graph of $y = \log_3(2x+1) - 2$. (3 marks)

$y=0$ ie $\log_3(2x+1) = 2$

$2x+1 = 9$

$x = 4$

$\therefore$ coords are $(4, 0)$

(b) Solve $\log_6 x + \log_6(x-5) = 2$ (2 marks)

3 marks

$\log_6 x(x-5) = \log_6 36$

$\therefore x^2 - 5x - 36 = 0$

$(x-9)(x+4) = 0$

$x = 9$ or $\cancel{-4}$

$\therefore x = 9$.

(c) If $\log_3 x + \log_3 y - 2 = \log_3 M$, determine an expression for M in terms of x and y. (3 marks)

2 marks

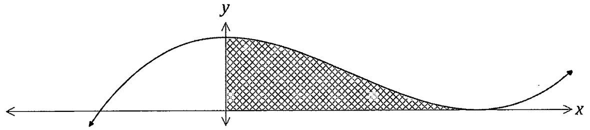
$\log_3 x + \log_3 y - \log_3 9 = \log_3 M$

$\log_3\left(\frac{xy}{9}\right) = \log_3 M$

$\therefore M = \left(\frac{xy}{9}\right)$

Question 6 **(8 marks)**

The diagram below shows the curve $y = x^3 - 3x^2 + k$, where k is a constant. The curve has a turning point on the x-axis.



(a) Determine the value of k. (3 marks)

$\frac{dy}{dx} = 3x^2 - 6x$

T.P.'s $\Rightarrow \frac{dy}{dx} = 0$ ie $3x(x-2) = 0$

$x = 0, 2$

$(2,0) \Rightarrow 2^3 - 3.2^2 + k = 0$

$k = 4$

(b) Determine the set of values of x for which $\frac{dy}{dx}$ is increasing. (2 marks)

$\frac{d^2y}{dx^2} = 6x - 6$

$\frac{dy}{dx}$ increasing for $\frac{d^2y}{dx^2} > 0$ ie $x > 1$

(c) Calculate the area of the shaded region. (3 marks)

$\int_0^2 x^3 - 3x^2 + 4 \, dx$

$= \left[\frac{x^4}{4} - x^3 + 4x \right]_0^2$

$= 4 - 8 + 8$

$= 4$ sq units

See next page

Question 15 **(8 marks)**

An analysis of the number of dogs registered by each household within a suburb resulted in the following information:

Number of dogs registered	0	1	2	3 or more
Percentage of households	21	44	27	8

(a) A council worker selects households at random to visit. What is the probability that the first five households visited all have at least one dog registered? (2 marks)

$44 + 27 + 8 = 79$

$(0.79)^5 = 0.3077$

(b) A random sample of 40 households within the suburb is selected.

Use a binomial distribution with $n = 40$, together with relevant information from the table in each case, to determine the probability that the sample contains:

(i) exactly 6 households with no dogs registered. (2 marks)

$X \sim B(40, 0.21)$

$P(X=6) = 0.1088$

(ii) no more than 15 households with at least two dogs registered. (2 marks)

$X \sim B(40, 0.35)$

$P(X \leq 15) = 0.6946$

(c) A random sample of 25 households within the city is to be selected. If X is the number of households in the sample that have exactly one dog registered, determine the mean and variance of X. (2 marks)

$n = 25 \quad p = 0.44$

$\bar{X} = 25 \times 0.44 = 11$

$Var = 11(1 - 0.44) = 6.16$

See next page

Question 14 **(8 marks)**

The random variable X denotes the number of hours that a business telephone line is in use per nine hour working day.

The probability density function of X is given by $f(x) = \begin{cases} \frac{(x-a)^2+b}{k} & 0 \le x \le 9 \\ 0 & \text{otherwise} \end{cases}$,

where a, b and k are constants.

(a) If $a = 15$ and $b = 3$, determine the value of k. (2 marks)

$$\int_0^9 \frac{(x-15)^2+3}{k}\, dx = 1$$

$k = 1080$ (ClassPad)

(b) Let $a = 16$, $b = 1$ and $k = 1260$.

(i) The business is open for work for 308 days per year. On how many of these days can the business expect the phone line to be in use for more than eight hours? (2 marks)

$$\int_8^9 \frac{(x-16)^2+1}{1260}\, dx = 0.0455$$

$\therefore 0.0455 \times 308 = 14$ days

(ii) Determine, correct to two decimal places, the mean and variance of X. (4 marks)

$$E(X) = \int_0^9 x \cdot \frac{(x-16)^2+1}{1260}\, dx$$

$= 3.39$

$$V(X) = \int_0^9 (x-3.39)^2\, \frac{(x-16)^2+1}{1260}\, dx$$

$= 5.78$

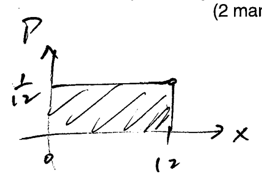
Question 7 **(8 marks)**

The Perth sight-seeing bus departs the Elizabeth Quay station every 12 minutes. If a person arrives at the station at a random time to catch the bus their waiting time, *X*, until the next bus departs is a uniformly distributed random variable.

(a) Determine

(i) the probability density function of *X* and sketch the associated probability density function. (2 marks)

$y = \frac{1}{12}$, $0 \le x \le 12$



(ii) the probability that the person has to wait at least 8 minutes. (1 marks)

$\frac{4}{12} = \frac{1}{3}$

(iii) the probability that the person has to wait at least 8 minutes given that she has been waiting at least 6 minutes. (2 marks)

$\frac{4}{6} = \frac{2}{3}$

(b) Someone catches the bus on 3 consecutive days, determine the probability that they have to wait at least 8 minutes on 2 of those days. (3 marks)

$X \sim B(3, \frac{1}{3})$

$P(X=2) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)$

$= 3 \times \frac{1}{9} \times \frac{2}{3}$

$= \frac{2}{9}$

End of questions

Additional working space

Question number: _________

Question 13 **(7 marks)**

A hardware store sells wooden stakes, of nominal length 1.8 metres, to be used for supporting newly planted trees. The length, X metres, of the stakes can be modelled by a normal distribution with mean 1.85 and standard deviation σ.

(a) If $\sigma = 0.035$, determine

$X \sim N(1.85, 0.035)$

(i) the probability that a randomly chosen stake is shorter than 1.8 metres. (1 mark)

$P(X < 1.8) = 0.0766$

(ii) the probability that a randomly chosen stake is longer than 1.79 m given that it is shorter than 1.8 metres. (2 marks)

$$\frac{P(1.79 < X < 1.8)}{P(X < 1.8)}$$

$$= \frac{0.0333}{0.0766} = 0.435$$

(iii) the value of k, if the longest 15% of stakes exceed k metres in length. (1 mark)

$P(X > k) = 0.15$

ie $k = 1.886$

(b) A large number of stakes were measured and it was found that 97% of them were longer than their nominal length. Show how to use this information to deduce that the value of σ is 0.027 when rounded to three decimal places. (3 marks)

$P(z > k) = 0.97$

$k = -1.881$

So $\frac{1.8 - 1.85}{\sigma} = -1.881$

$\sigma = 0.0266$

Question 12 **(8 marks)**

A box contains a large number of packets of buttons. The number of buttons in a packet may be modelled by the random variable X, with the probability distribution shown below. It is also known that $E(X) = 6.25$.

x	3 or fewer	4	5	6	7	8	9 or more
$P(X = x)$	0	0.05	a	b	0.25	0.15	0

(a) Two packets are randomly chosen from the box. Determine the probability that there are at least 15 buttons altogether in the two packets. (2 marks)

7+8, 8+7, 8+8

$P = 0.25 \times 0.15 + 0.15 \times 0.25 + 0.15 \times 0.15$

$= 0.0975$

(b) Determine the values of a and b. (3 marks)

$a + b = 0.55$

$E(x) \Rightarrow 5a + 6b + 4(0.05) + 7(0.25) + 8(0.15) = 6.25$

Solving $a = 0.2$ $b = 0.35$

(c) Calculate $\mathrm{Var}(X)$. (1 mark)

$\sigma = 1.1875$ (Classpad.)

(d) As part of a fundraiser, patrons pay 75 cents to select a packet at random and then win back 10 cents for each button in the packet. If the random variable W represents the net gain per game for a patron in cents, determine the mean and variance of W. (2 marks)

$E(W) = 10 \times 6.25 - 75 = -12.5$

$\mathrm{Var}(W) = 10^2 \times 1.1875 = 118.75$

See next page

Latest printout 27/9/16

WESLEY COLLEGE

By daring & by doing

Semester Two Examination, 2016

Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 AND 4

Section Two: Calculator-assumed

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In words ____

Your name ____

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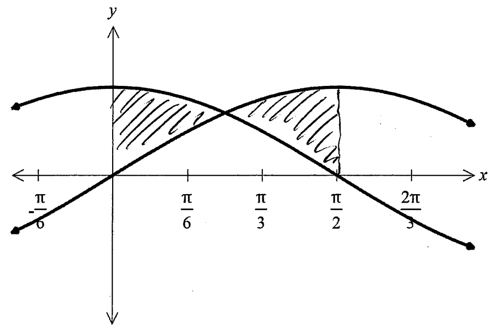
(c) Determine the exact area of the region bounded by:

$y = \sin x$, $y = \cos x$, $x = 0$ and $x = \frac{\pi}{2}$ (4 marks)

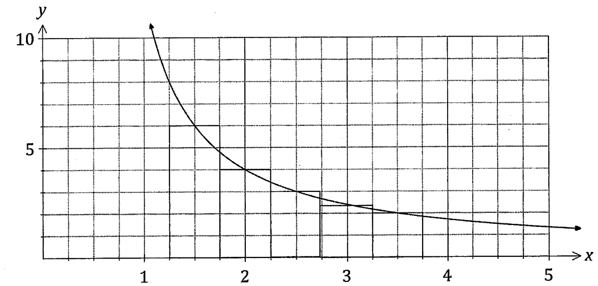


$$\text{Area} = 2\int_0^{\pi/4} \cos x - \sin x \, dx$$

$$= 2(\sqrt{2}-1) \quad \text{(Classpad)}$$

Question 11 **(10 marks)**

(a) The graph below shows the curve $y = f(x)$, where $f(x) = \frac{12}{2x-1}$.



Use the five centred rectangles shown to estimate the shaded area under the curve from $x = 1.25$ to $x = 3.75$. (3 marks)

x	1.5	2	2.5	3	3.5
y	6	4	3	2.4	2

Area $= \frac{1}{2}(6+4+3+2.4+2)$

$= 8.7$ units^2

(b) Given $\int_a^b h(x)\,dx = k$ and $h(x)$ is a polynomial, determine the following in terms of the constants a, b and k:

(i) $\int_a^b 3\,h(x)\,dx$. $= 3k$ (1 mark)

(ii) $\int_a^b 2 - h(x)\,dx$. $= \int_a^b 2\,dx - \int_a^b h(x)\,dx$ (2 marks)

$= [2x]_a^b - k$

$= 2b - 2a - k$

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

Zebra mussels are an invasive species of shellfish recently discovered in some North American waterways. The mussel density, D, in shellfish per square metre, observed in a power station water supply pipe t days after a colony began, was modelled by the following equation, where k is a positive constant:

$$D = 200e^{kt}$$

(a) What was the mussel density in the colony when observations began? (1 mark)

200

The mussel density was observed to double every eight days.

(b) Determine the value of k, rounded to four decimal places. (2 marks)

$e^{8k} = 2$

$k = 0.0866$

(c) The water supply pipe was seriously compromised when the mussel density reached 85 thousand shellfish per square metre. After how many days from the commencement of observations did this happen? (2 marks)

$200\,e^{0.0866t} = 85000$

$t = 69.886$

$\therefore$ 70 days.

Question 9 **(7 marks)**

The speeds of 250 vehicles, on a section of freeway undergoing roadworks with a speed limit of 60 kmh^{-1}, had a mean and standard deviation of 56.9 kmh^{-1} and 3.6 kmh^{-1} respectively. A summary of the data is shown in the table below.

Speed (x kmh^{-1})	$45 \le x < 50$	$50 \le x < 55$	$55 \le x < 60$	$60 \le x < 65$	$65 \le x < 70$
Relative frequency	0.024	0.272	0.504	0.188	0.012

(a) Use the table of relative frequencies to estimate the probability that the next vehicle to pass the roadworks

(i) was not exceeding the speed limit. (1 mark)

$0.024 + 0.272 + 0.504$

$= 0.8$

(ii) had a speed of less than 65 kmh^{-1}, given they were exceeding the speed limit. (2 marks)

$\dfrac{0.188}{1-0.8} = 0.94$

(b) Subsequent tests on the measuring equipment discovered that it had been wrongly calibrated. The correct speed of each vehicle, v, could be calculated from the measured speed, x, by increasing x by 6% and then adding 1.7.

(i) Calculate the adjusted mean and standard deviation of the vehicle speeds. (2 marks)

Mean $= 1.06 \times 56.9 + 1.7 = 62.01$ km/h

St dev $= 1.06 \times 3.6 = 3.82$ km/h

(ii) Determine the correct proportion of vehicles that were speeding. (2 marks)

$1.06x + 1.7 = 60$

$x = 55$

So $0.504 + 0.188 + 0.012$

$= 0.704$

Question 10 **(7 marks)**

A student planned to investigate what proportion of the 1260 students at their school had access to more than one computer at home.

(a) The student thought of the following three ways to select a sample from the population. Briefly discuss the main source of bias in each method.

(i) Wait at the bus-bay after school and ask the first 50 students who show up. (1 mark)

Bus students only – biased

(ii) Advertise the survey in a whole school assembly and ask the first 50 students who volunteer to stay behind. (1 mark)

Self-selected samples – biased

(iii) Select and ask every 100th student from the school roll. (1 mark)

Only 13 students – sample too small.

(b) Assuming that 80% of students had access to more than one computer at home, the student carried out 100 simulations in which a sample proportion was calculated from a random sample of 64 students.

(i) Explain why it is reasonable to expect that the distribution of the sample proportions would approximate normality. (2 marks)

$n = 64 > 30$ i.e. large

and $np = 51.2$ + $n(1-p) = 12.8$

are both > 10

(ii) Determine the mean and standard deviation of the normal distribution that the sample proportions would approximate. (2 marks)

$\bar{x} = 0.8$

$s = \sqrt{\dfrac{0.8(0.2)}{64}} = 0.05$