Additional Working Space

Question Number:_____

HALE SCHOOL

Semester 2 Examination 2016

Question/answer booklet

Circle your teacher's initials

JIB MAW MPC SWA VMU

Mathematics Methods Units 3 & 4

Section One
(Calculator Free)

Your name: M. Key

Time allowed for this section

Reading time before commencing work: five minutes
Working time for paper: fifty minutes

52

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction tape/fluid, eraser, ruler, highlighters

Special items: nil.

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	**8**	**8**	**50**	**52**	**35**
Section Two: Calculator-assumed	13	13	100	103	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
     Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

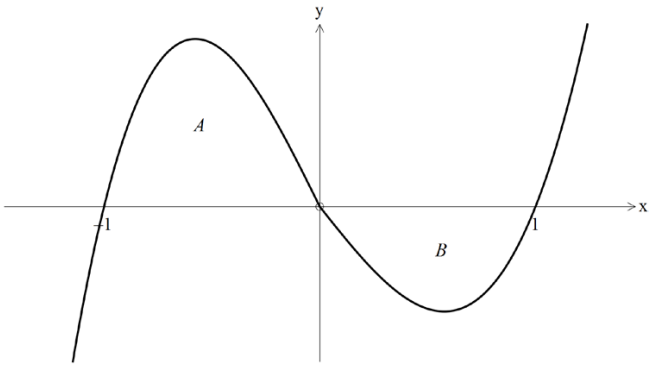
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Additional Working Space

Question Number:_____

Question 8 **(6 marks)**

Part of the graph of $y = f(x)$ is shown below. The areas of the bounded regions A and B are 9 and 5 square units respectively.



a) Evaluate $\int_{-1}^{1} f(x)\,dx$ (2 marks)

$9 + (-5) = 4$ ✓adds signed areas ✓correct answer

b) Evaluate $\int_{-1}^{1} |f(x)|\,dx$ (1 mark)

$9 + 5 = 14$ ✓correct answer

c) Evaluate $\int_{-1}^{1} 3 - f(x)\,dx$ (3 marks)

$= \int_{-1}^{1} 3\,dx - \int_{-1}^{1} f(x)\,dx$ ✓identifies the two components of the integral

$= 6 - (9 - 5)$ ✓determines integrals correctly

$= 2$ ✓correct answer

END OF SECTION ONE

Question 1 **(8 marks)**

Differentiate (simplifying and leaving answers with positive indices where appropriate):

a) $y = \frac{3 + x^4}{x^2}$. (2 marks)

$y = \frac{3}{x^2} + x^2$ ✓simplifies first

$\frac{dy}{dx} = -\frac{6}{x^3} + 2x$ ✓correct answer

b) $f(x) = x^3.\sin(3x)$. Factorise your answer. (3 marks)

$f'(x) = 3x^2 \sin 3x + 3x^3 \cos 3x$ ✓✓differentiates using product rule correctly

$= 3x^2(\sin 3x + x \cos 3x)$ ✓expresses as a product

c) $F(x) = \int_{1}^{2x} t^3 e^t\,dt$ (3 marks)

$F'(x) = (2x)^3 e^{2x} (2)$ ✓ ✓

$= 16x^3 e^{2x}$ ✓correct simplified answer

Question 2 **(5 marks)**

a) Determine $\frac{d}{dx}\left(e^{-x}(x-2)\right)$. (2 marks)

$= -e^{-x}(x-2) + e^{-x}$ ✓uses product rule

✓differentiates product correctly

$= e^{-x}(3-x)$

$= \frac{3-x}{e^{x}}$

b) Hence, or otherwise, evaluate exactly $\int_0^1 \frac{3-x}{e^x}\,dx$. (3 marks)

From (a) $\frac{d}{dx}\left(e^{-x}(x-2)\right) = \frac{3-x}{e^x}$ ✓understands and uses result from part (a)

Hence $\int_0^1 \frac{3-x}{e^x}\,dx = \left[e^{-x}(x-2)\right]_0^1$ ✓integrates correctly

$= e^{-1}(-1) - e^{0}(-2)$

$= 2 - \frac{1}{e}$ ✓correct answer

Question 7 **(7 marks)**

a) Solve $2[\log_2(x)]^2 - 9\log_2(x) + 4 = 0$ giving your answer(s) exactly. (4 marks)

i.e. $(2\log_2 x - 1)(\log_2 x - 4) = 0$ ✓factorises correctly

$\therefore \log_2 x = \frac{1}{2}$ or $\log_2 x = 4$ ✓uses null factor law correctly

$\therefore x = \sqrt{2}$ or $x = 16$

✓$x = \sqrt{2}$ ✓$x = 16$

b) Express y in terms of x if $2\log_e x + 1 = \frac{\log_e 3y}{2}$. (3 marks)

i.e. $4\ln x + 2 = \ln 3y$

$\therefore 4\ln x + \ln e^2 = \ln 3y$

i.e. $\ln(x^4 e^2) = \ln 3y$ ✓uses log laws correctly

$\therefore x^4 e^2 = 3y$ ✓equates equivalent sides

$\therefore y = \frac{x^4 e^2}{3}$ ✓solves for y

Question 6 **(5 marks)**

Given that $\log_{10} 2 = x$ and $\log_{10} 3 = y$, express each of the following in terms of x and y.

a) $\log_{10} 6$ (1 mark)

$= \log 2 + \log 3$

$= x + y$ ✓ans

b) $\log_{10} 0.6$ (1 mark)

$= \log\left(\frac{6}{10}\right)$

$= \log 6 - \log 10$

$= x + y - 1$ ✓ans

c) $\log_{10} 45$ (3 marks)

$= \log(9 \div 2 \times 10)$ ✓identifies 45 as $9/2 \times 10$

$= \log 9 - \log 2 + \log 10$

$= 2\log 3 - \log 2 + \log 10$ ✓$\log 9 = 2\log 3$

$= 2y - x + 1$ ✓ans

Question 3 **(7 marks)**

Differentiate the following with respect to x, simplifying your answers.

a) $y = (1 - \ln x)^3$ (2 marks)

$\frac{dy}{dx} = 3(1-\ln x)^2 \cdot \left(-\frac{1}{x}\right)$ ✓uses chain rule to differentiate

$= \frac{-3(1-\ln x)^2}{x}$ ✓simplifies answer

b) $y = \log_2(x)$ (2 marks)

$\therefore y = \frac{\ln x}{\ln 2}$ using change of base rule ✓c.o.b.

$\frac{dy}{dx} = \frac{1}{\ln 2} \times \frac{1}{x} = \frac{1}{x \ln 2}$

✓differentiates correctly

c) $y = \ln\left(\frac{x^3}{7-4x}\right)$ (3 marks)

$y = 3\ln x - \ln(7-4x)$ ✓uses log laws to simplify

$\therefore \frac{dy}{dx} = \frac{3}{x} - \frac{-4}{7-4x}$ ✓✓differentiates each term correctly

$= \frac{3}{x} + \frac{4}{7-4x}$

Question 4 **(8 marks)**

A biased die with six faces is rolled. The discrete random variable X represents the score on the uppermost face. The probability distribution of X is shown in the table below.

x	1	2	3	4	5	6
$P(X = x)$	a	a	a	b	b	0.3

a) Given that $E(X) = 4.2$ find the value of a and the value of b. (5 marks)

$3a + 2b + 0.3 = 1$ ✓$\Sigma p = 1$ $E(X) = 4.2$ ✓calc $E(x)$ correctly

i.e. $3a + 2b = 0.7$ ① $\therefore a + 2a + 3a + 4b + 5b + 1.8 = 4.2$

$\therefore 6a + 9b = 2.4$ ②

$2 \times$ ① $6a + 4b = 1.4$ ③ ✓valid method shown

$6a + 9b = 2.4$ ②

② − ③ $5b = 1 \Rightarrow b = 0.2$ ✓solves for b

sub in ① $3a + 0.4 = 0.7 \Rightarrow a = 0.1$ ✓solves for a

b) Given $E(X^2) = 20.4$, determine $\text{Var}(5 - 10X)$. (3 marks)

$\text{Var}(5 - 10X) = (-10)^2 \text{Var}(X)$ ✓correct change of scale

$= 100[20.4 - 4.2^2]$ ✓subs in correctly

$= 100[20.4 - 17.64]$

$= 100[2.76]$

$= 276$ ✓ans

Question 5 **(6 marks)**

A biased die with five faces is rolled. The discrete random variable D represents the score which is on the uppermost face.

The cumulative distribution function of D is shown in the table below.

d	1	2	3	4	5
$P(D \leq d)$	$\frac{1}{10}$	$\frac{2}{10}$	$3k$	$4k$	$5k$

a) Calculate the value of k. (1 mark)

$5k = 1 \Rightarrow k = \frac{1}{5}$ or $\frac{2}{10}$ ✓ans

b) Give the probability distribution of D. (3 marks)

d	1	2	3	4	5
$P(D = d)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$\frac{2}{10}$

✓tabulates
✓ $P(D = 3)$ correct
✓ all correct

This die is rolled twice and the two scores are added.

c) Calculate the probability that the sum of the two scores equals 3. (2 marks)

i.e. $P(1,2) + P(2,1) = \frac{1}{10} \times \frac{1}{10} + \frac{1}{10} \times \frac{1}{10}$

✓identifies 2,1 and 1,2

$= \frac{2}{100}$ or $\frac{1}{50}$ or 0.02

✓determines prob correctly