Papers written by Australian Maths Software

SEMESTER ONE

YEAR 12

MATHEMATICS METHODS

REVISION 3

Unit 3

2016

Section Two

(Calculator–assumed)

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 10 minutes
Working time for section: 100 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: drawing instruments, templates, notes on up to two unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section Two.
Formula sheet retained from Section One.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	7	7	50	50	35
Section Two Calculator—assumed	**13**	**13**	**100**	**100**	**65**
			Total marks	150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the Question/Answer booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

20. (5 marks)

A multiple choice test has 20 questions, each of which has five possible answers, only one of which is correct.
Peter has studied the subject and thinks he will know 60% of the work and will have to guess the rest.

(a) How many answers will Peter expect to have to guess? (1)

(b) Assuming Peter does know 60% of the answers and guesses the rest, what is the probability that he gets 80% or more for the test? (4)

END OF SECTION TWO

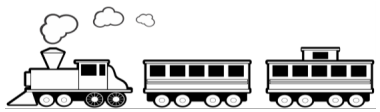
8. (6 marks)

A toy train follows the path defined by $x = t^3 - 12t$ for $t \geq 0$ where x represents metres and t represents time in seconds.

(a) The train starts at $x = 0$. When is the train back at its starting point? (2)

(b) Find an expression for the velocity and use it to find when the train changes direction. (2)

(c) Determine when the acceleration is 12 m/s^2 and find the displacement at that time. (2)



9. (9 marks)

(a) Consider the discrete probability distribution table below.

y	10	20	30	40
$P(Y=y)$	a	0.1	a	0.5

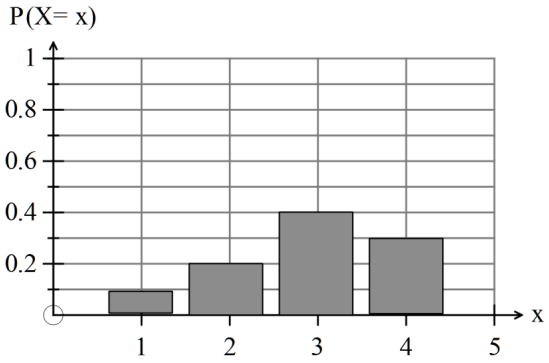
(i) Find the value of a. (1)

(ii) Find $P(x \leq 30)$ (1)

(iii) Determine the expected value and standard deviation of X. (4)

19. (8 marks)

The graph of a discrete probability distribution appears below.



(a) Complete the probability table below: (2)

x	1	2	3	4
$P(X=x)$				

(b) Determine $P(X=2$ or $X>3)$ (2)

(c) If each of the columns were twice as high and did not represent probability, could the appropriate probabilities of 1, 2, 3 or 4 be determined? Explain. (2)

(d) By inspecting the graph, estimate the mean of the distribution. (2)

18. (9 marks)

A very large container is filled with an equal number of white, pink, variegated and red geranium seeds, and thoroughly mixed.
A small handful of seeds are planted in a row.

(a) What is the probability that the geranium plant at the far end of the row will have pink flowers? (2)

(b) Write down the probability that of the first five plants, two of them will be pink flowering? (2)

(c) What is the probability that none of the first five plants will be pink? (2)

(d) What is the probability that at least two of the first five plants will be pink? (2)

(e) Calculate the expected number of pink geraniums in the first five plants. (1)

(b) A discrete probability distribution has an expected value of 10 and a standard deviation of 3.5.

(i) Write down the variance of the distribution. (1)

The variables are all halved.

(ii) Write down the expected value and standard deviation. (2)

10. (4 marks)

A dangerous algae is slowly expanding forming a circular mass threatening to cover one side of a lake.

What is the corresponding increase in area covered by the algae when the radius expands from 100 m to 100.5 m? Use a calculus method to find your answer. (4)

17. (8 marks)

(a) Complete the table showing probabilities for the number of boys in a four child family: (2)

x	0	1	2	3	4
$P(X = x)$					

(b) Determine the probability that given two four child families were staying in a caravan park, that both families had three boys out of four. (2)

(c) (i) In a group of ten four child families, how many of them would you expect to have no boys? (3)

(ii) In such a group of ten four child families, why would you never actually get the result expected in (i)? (1)

16. (8 marks)

The pesticide DDT was widely used until its ban in 1972. The half-life of DDT is approximately 15 years. Half-life is the time it takes for half of the amount of a substance to decay.

(a) If 100 grams of DDT was sprayed on a paddock in 1972, how long will it take to reduce to 25 grams? (1)

Let $A = 100(a)^t$ represent the amount of DDT left where A is in grams of DDT and where t is measured in years. Take $t = 0$ in 1972.

(b) Find the value of a. (2)

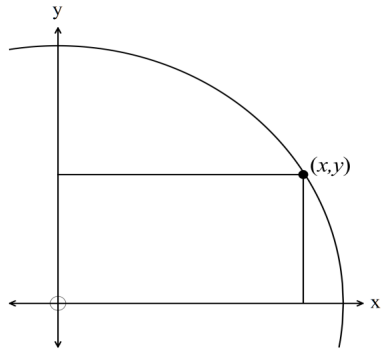
(c) Confirm your answer to (a) using the equation $A = 100(a)^t$. (1)

(d) If the DDT reached acceptable levels to be almost harmless to humans when the 100 grams reduced to 1 gram, how long would this take to happen? (2)

(e) How much of the DDT will remain in the paddock in 2016? (2)

11. (8 marks)

A rectangle is bounded by the x axis, y axis and the circle $x^2 + y^2 = 100$ as shown in the diagram below.



(a) Find an expression for the area of the rectangle in terms of x. (2)

(b) Show <u>using calculus</u> that the area of the rectangle is maximised when $x = y$. (6)

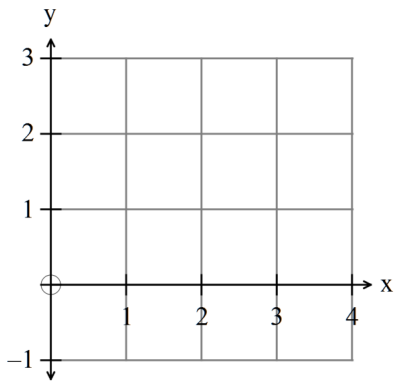
12. (7 marks)

(a) Given $y = \lim_{h \to 0} \frac{a^h - 1}{h}$

use your calculator to find the ordered pairs (x, y) where $x = a$.

$\left(\frac{1}{2}, \_\_\_\_\_\right), (1, \_\_\_\_\_), (2, \_\_\_\_\_), (3, \_\_\_\_\_), (4, \_\_\_\_\_).$ (4)

(b) (i) Graph the set of orders pairs below. (2)



(ii) Find an approximation for the value of a when $y = 1$. (1)

(c) Find the acceleration when the velocity of the toy is 4 m/s. (2)

(d) Find the distance travelled in the first three seconds. (3)

(iv) Use integration to determine the difference between your estimate and the area determined using integration correct to 2 decimal places. (2)

15. (9 marks)

The velocity of a popular toy is $v = \sqrt{1+t}$ m/s for $t \geq 0$.

(a) Find the expression for the acceleration in terms of t and use the expression to explain why a is always positive. (2)

(b) Find the expression for the displacement given the initial displacement is $\frac{1}{3}m$. (2)

13. (5 marks)

(a) Find $\int_2^5 \sqrt{1+2x^3}\,dx$ correct to two decimal places. (2)

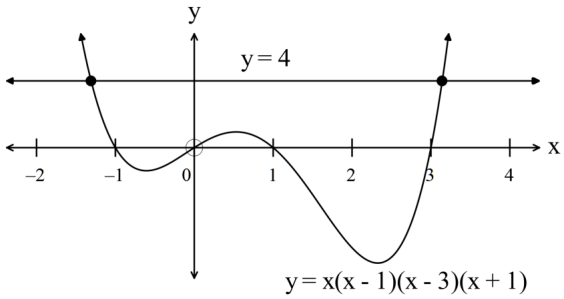
(b) If $f(x) = \int \frac{cos(3x)}{2} dx$

(i) find the expression for $y = f(x)$ given $f(\pi) = \frac{5}{6}$. (2)

(ii) hence find $f\left(\frac{\pi}{2}\right)$. (1)

14. (14 marks)

Consider the graph.

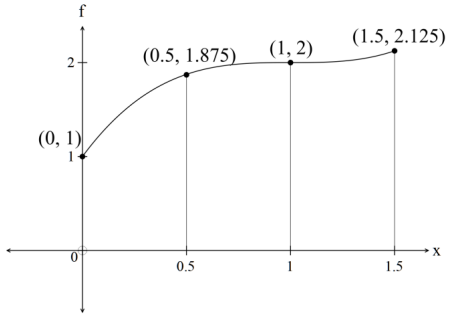


(a) (i) Find the points of intersection of the two functions correct to two decimal Places. (2)

(ii) Write down the expression for finding the area between the two functions. (2)

(ii) Calculate the area between the functions. (2)

(b) Consider the graph of the function $f(x) = (x-1)^3 + 2$ shown in the diagram below:



(i) Use rectangles from below to estimate the area between the function $y = f(x)$ and the x axis on the domain $0 \le x \le 1.5$. (2)

(ii) Use rectangles from above to estimate the area between the function $y = f(x)$ and the x axis on the domain $0 \le x \le 1.5$. (2)

(iii) Explain why you expect the average of the answers to (i) and (ii) to give you a reasonable estimate of the correct answer.
Calculate the average of the estimates. (2)