8. (6 marks)

A continuous function $f(x)$ is increasing on the interval $0 < x < 3$ and decreasing on the interval $3 < x < 6$. Some of its values are given in the table below.

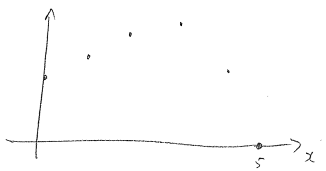
x	0	1	2	3	4	5	6
$f(x)$	5	16	27	32	25	0	– 49

The function $F(X)$ is defined, for $0 \le x \le 6$, by $F(x) = \int_0^x f(t)dt$.

(a) At which value of x in the interval $0 \le x \le 6$ is $F(X)$ greatest? Justify your answer. [2]

$x = 5$

AREA UNDER FUNCTION INCREASES UNTIL $x = 5$



(b) At which value of x in the interval $0 \le x \le 6$ is $F'(x)$ greatest? Justify your answer. [2]

$F'(x) = f(x)$

IT IS AN INCREASING FUNCTION UP TO $x = 3$.

$x = 3$ MAX VALUE

(c) Use the values of $f(x)$ in the table to show that $48 \le F(3) \le 75$. [2]

UNDER ESTIMATE $\le F(3) \le$ OVER ESTIMATE

$5 + 16 + 27 \le F(3) \le 16 + 27 + 32$

$48 \le F(3) \le 75$

TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 2 2018

Section 1 Calculator Free

Applications of Calculus

STUDENT'S NAME SOLUTIONS

DATE: Thursday 5 April **TIME:** 30 minutes **MARKS:** 33

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (4 marks)

Determine the equation of the tangent to the curve $y \sin x = x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$y = \frac{x}{\sin x}$

$y' = \frac{\sin x - x\cos x}{\sin^2 x}$

$y'\big|_{x=\frac{\pi}{2}} = \frac{1 - 0}{1^2}$

$= 1$

$y = 1x + c$

$\left(\frac{\pi}{2}, \frac{\pi}{2}\right) \quad \frac{\pi}{2} = \frac{\pi}{2} + c \quad \therefore c = 0$

$y = x$

2. (9 marks)

(a) Determine each of the following (do not simplify)

(i) $\frac{d}{dx}\frac{x^2}{e^{\sin 3x}}$ $= \frac{2x e^{\sin 3x} - x^2 . 3\cos 3x e^{\sin 3x}}{e^{2\sin 3x}}$ [3]

(ii) $\frac{d}{dx}e^{-x}(\sin 2x - \tan 2x)$ [3]

$= -e^{-x}(\sin 2x - \tan 2x) + e^{-x}\left(2\cos 2x - \frac{2}{\cos^2 2x}\right)$

(b) Given $f(x) = \int_x^1 (3-t)^{\frac{5}{2}}\, dt$ determine $f'(-1)$. [3]

$f'(x) = -\frac{d}{dx}\int_1^x (3-t)^{\frac{5}{2}}\, dt$

$= -(3-x)^{\frac{5}{2}}$

$f'(-1) = -(4)^{\frac{5}{2}}$

$= -32$

7. (4 marks)

Two of the fission products of an explosion are found to decay according to the laws

$$\frac{dM_1}{dt} = -k_1 M_1 \quad \text{where } e^{-k_1} = \frac{1}{4}$$

$$\frac{dM_2}{dt} = -k_2 M_2 \quad \text{where } e^{-k_2} = \frac{1}{2}$$

If the initial ratio $\frac{M_1}{M_2} = 3$ what is the ratio after 6 days?

$M_1 = (M_1)_0 e^{-k_1 t}$ $M_2 = (M_2)_0 e^{-k_2 t}$

$M_1 = (M_1)_0 \left(\frac{1}{4}\right)^t$ $= (M_2)_0 \left(\frac{1}{2}\right)^t$

AFTER 6 DAYS

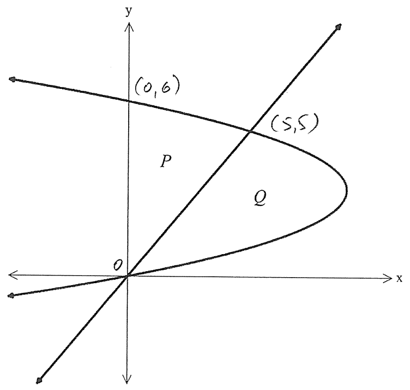
$\frac{M_1}{M_2} = \left(\frac{M_1}{M_2}\right)_0 \frac{\left(\frac{1}{4}\right)^6}{\left(\frac{1}{2}\right)^6}$

$= 3 . \frac{\left(\frac{1}{4}\right)^6}{\left(\frac{1}{2}\right)^6}$

$= \frac{3}{64}$

6. (6 marks)

In the graph shown below, Q is the area enclosed by the graphs of $y = x$ and $x = 6y - y^2$. P is the area bounded by the two graphs and the y-axis



Calculate

(a) the size of area Q [3]

$y = 6y - y^2$

$0 = 5y - y^2$

$0 = y(5-y)$ $\quad y = 0, 5$

$$AREA_Q = \int_0^5 6y - y^2 - y \, dy$$

$$= \frac{125}{6}$$

(b) the size of area P [3]

$$AREA_P = \int_0^6 6y - y^2 \, dy - \frac{125}{6}$$

$$= 36 - \frac{125}{6}$$

$$= \frac{91}{6}$$

3. (12 marks)

(a) Determine each of the following

(i) $\int (e^x + e^{-x})^2 \, dx$ [3]

$$= \int e^{2x} + 2 + e^{-2x} \, dx$$

$$= \frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} + c$$

(ii) $\int 3e^{1-6x} + e \, dx$ [3]

$$= \frac{3}{-6} \int -6 e^{1-6x} \, dx + \int e \, dx$$

$$= -\frac{1}{2} e^{1-6x} + ex + c$$

(b) (i) determine $\frac{d}{dx} x \cos 2x$ [3]

$$= \cos 2x - 2x \sin 2x$$

(ii) use the result of (ii) to determine $\int 2x \sin 2x \, dx$ [3]

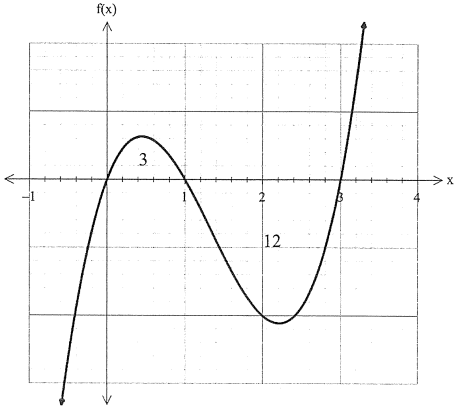
$$\int (\cos 2x - 2x \sin 2x) \, dx = x \cos 2x$$

$$\int \cos 2x \, dx - x \cos 2x = \int 2x \sin 2x \, dx$$

$$\frac{\sin 2x}{2} - x \cos 2x + c = \int 2x \sin 2x \, dx$$

4. (8 marks)

The graph of y = f(x) is shown below. The size of the area of the two parts enclosed between the curve and the x-axis is shown on the graph.



Determine

(a) $\int_0^3 f(x)\,dx$ $\quad -9$ [1]

(b) $\int_0^3 |f(x)|\,dx$ $\quad 15$ [1]

(c) $\int_1^0 f(x)\,dx$ $\quad = -\int_0^1 f(x)\,dx$ [2]

$= -3$

(d) $\int_1^3 (2f(x)+3)\,dx$ $\quad = 2\int_1^3 f(x) + \int_1^3 3\,dx$ [4]

$= 2(-12) + \left[3x\right]_1^3$

$= -24 + 9 - 3$

$= -18$

TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 2 2018

Section 2 Calculator Assumed
Applications of Calculus

STUDENT'S NAME ______________________

DATE: Thursday 5 April **TIME:** 20 minutes **MARKS:** 22

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

5. (6 marks)

Scientists are studying a population of endangered small mammals in a protected environment. They conclude the population is increasing at a rate of given by $B'(t) = 5.2e^{0.4t}$ where t is the number of weeks since the study began.

(a) What is the change in the population in the fourth week? [3]

NET CHANGE $= \int_3^4 5.2e^{0.4t}\,dt$

$= 21$ MAMMALS

(b) When the study began there were 500 of these mammals. The study will conclude when the population reaches 2000. When will this occur? [3]

$\int_0^x 5.2e^{0.4t}\,dt = 1500$

$x = 11.9$

ie DURING 12^{TH} WEEK