PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.

Perth Modern School
End of Year Examination, 2011

Question/Answer Booklet

MATHEMATICS 3C/3D
Section One: Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	40	33
Section Two: Calculator-assumed	12	12	100	80	67
			Total	120	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2011*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

See next page

Section One: Calculator-free **(40 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(4 marks)**

Find the minimum and maximum values of $f(x) = 2x^3 - 3x^2 - 12x + 27$ over the interval $-3 \le x \le 3$.

$f'(x) = 6x^2 - 6x - 12$

$f'(x) = 6(x+1)(x-2) \Rightarrow f'(x) = 0$ when $x = -1,\ x = 2$

$f(-3) = -18$

$f(-1) = 34$ (local max)

$f(2) = 7$ (local min)

$f(3) = 18$

Hence minimum value is -18 and maximum value is 34.

See next page

Question 2 **(5 marks)**

Find $\frac{dy}{dx}$ in terms of x for each of the following.

(a) $y = x(1 + 2e^{3x})$ (2 marks)

$$\frac{dy}{dx} = 1 + 2e^{3x} + 6xe^{3x}$$

(b) $y = \int_1^x t^2 + t - 1 \; dt$ (1 mark)

$$\frac{dy}{dx} = x^2 + x - 1$$

(c) $y = z^3 - z$ and $z = x^2 - 9$ (2 marks)

$$\frac{dy}{dz} = 3z^2 - 1 \quad \text{and} \quad \frac{dz}{dx} = 2x$$

$$\frac{dy}{dx} = \left(3(x^2 - 9)^2 - 1\right) \times 2x$$

$$= 6x(x^2 - 9)^2 - 2x$$

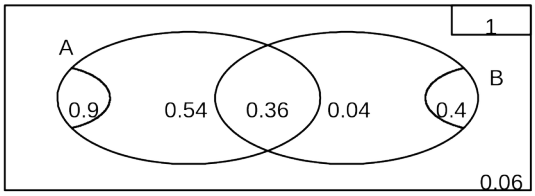
See next page

Question 3 **(5 marks)**

Two independent events A and B are such that $P(A) = 0.9$ and $P(B) = 0.4$.

(a) Find $P(\overline{A \cup B})$. (2 marks)

$P(A \cap B) = 0.9 \times 0.4 = 0.36$



Ans: 0.06

(b) Find $P(\overline{B} \mid \overline{A} \cup B)$. (1 mark)

$$\frac{0.06}{0.46} = \frac{3}{23}$$

(c) Show that $\overline{A}$ and $\overline{B}$ are also independent. (2 marks)

$P(\overline{A}) = 0.1$

$P(\overline{B}) = 0.6$

$P(\overline{A} \cap \overline{B}) = 0.1 \times 0.6$

$= 0.06$

$= P(\overline{A \cup B})$ as above. Hence independent.

Question 4 **(7 marks)**

Two functions are defined as $f(x) = \sqrt{x - 1}$ and $g(x) = \dfrac{1}{x - 1}$.

(a) Evaluate $g \circ f\left(\dfrac{13}{9}\right)$. (2 marks)

$$f\left(\frac{13}{4}\right) = \sqrt{\frac{13}{9} - 1} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$
$$g\left(\frac{2}{3}\right) = \frac{1}{\frac{2}{3} - 1} = 1 \div -\frac{1}{3} = -3$$

(b) Find in simplified form $g \circ g(x)$. (2 marks)

$$g \circ g(x) = \frac{1}{\frac{1}{x - 1} - 1}$$
$$= 1 \div \frac{1 - (x - 1)}{x - 1}$$
$$= \frac{x - 1}{2 - x}$$

(c) Determine the domain of $f(g(x))$. (3 marks)

$$f(g(x)) = \sqrt{\frac{1}{x - 1} - 1}$$

Require that $\dfrac{1}{x - 1} - 1 \geq 0$

$$\frac{1 - (x - 1)}{x - 1} \geq 0$$
$$\frac{2 - x}{x - 1} \geq 0$$

Hence domain is $1 < x \leq 2$.

See next page

Question 5 **(4 marks)**

Solve the system of equations

$$\begin{aligned} c + 2a &= 3 + 4b \\ a + 2b + 2c &= 4 \\ 5a + 3c &= 5 + 2b \end{aligned}$$

$$\begin{aligned} 2a - 4b + c &= 3 \\ a + 2b + 2c &= 4 \\ 5a - 2b + 3c &= 5 \end{aligned}$$

$i + 2ii$

$$4a + 5c = 11$$

$ii + iii$

$$6a + 5c = 9$$

$$-2a = 2$$
$$a = -1$$
$$c = 3$$
$$b = -0.5$$

Question 6 **(5 marks)**

(a) Determine $\int \frac{2e^{-0.2y}}{5}\,dy$. (1 mark)

$$\frac{2}{5(-0.2)}e^{-0.2y} + c$$
$$= -2e^{-0.2y} + c$$

(b) Determine $\int (t-1)(1-2t+t^2)^3\,dt$. (2 marks)

$$\frac{1}{2\times 4}(1-2t+t^2)^4 + c$$
$$= \frac{(1-2t+t^2)^4}{8} + c$$

(c) Evaluate $\int_1^6 \frac{3}{x^2}\,dx$. (2 marks)

$$\left[-\frac{3}{x}\right]_1^6$$
$$= -\frac{1}{2} - (-3)$$
$$= 2.5$$

See next page

Question 7 **(4 marks)**

The region in the first quadrant bounded by $x=0$, $y=0$ and $y=1-\frac{x^2}{9}$ is rotated 360° about the y-axis. If x and y are distances measured in centimetres, find the volume of the solid formed.

When $x=0$, $y=1$

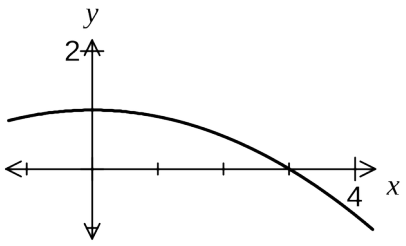
$$x^2=9(1-y)$$

$$V=\int_0^1 \pi x^2\,dy$$

$$=9\pi\int_0^1 (1-y)\,dy$$

$$=9\pi\left[y-\frac{y^2}{2}\right]_0^1$$

$$=\frac{9\pi}{2}\text{ cm}^3$$



Question 8 **(6 marks)**

The variables k and m are both integers such that $m^2 + 3 = 2k$.

(a) Use counter-examples to disprove any two of the three conjectures listed below. (2 marks)

- m can be any even integer.

$m = 2$ then $k = 3.5$ which is not an integer, so statement false.

- m can be any odd integer.

(Statement true, so no counter-examples exist)

- m must be a positive odd integer.

$m = -1$ then $k = 2$. But m is a negative integer, so statement false.

(b) Using the fact that any odd integer can be written in the form $2n + 1$ or otherwise, prove that k is always the sum of three square numbers. (4 marks)

$2k$ is even and hence $m^2 + 3$ must be even.
Since 3 is odd then m^2 must be odd, and so m must also be odd.

$$\begin{aligned} 2k &= m^2 + 3 \\ &= (2n+1)^2 + 3 \\ &= 4n^2 + 4n + 4 \\ k &= 2n^2 + 2n + 2 \\ &= n^2 + n^2 + 2n + 1 + 1 \\ &= n^2 + (n+1)^2 + 1^2 \end{aligned}$$

End of questions

Additional working space

Question number: _____

Published by WA Examination Papers
PO Box 445 Claremont WA 6910