



Course Specialist Test 4 Year 12

Student name: \_\_\_\_\_ Teacher name: \_\_\_\_\_

Task type: Response

Time allowed for this task: 40 mins

Number of questions: 7

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: 44 marks

Task weighting: 10 %

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3 &amp; 3 = 6 marks)

Solve the following.

a)  $\frac{dy}{dx} = \frac{3x-2}{y(5-y^2)}$  given that when  $x=1, y=1$ .

b)  $3x^4 \cos(2y) \frac{dy}{dx} = 10$  given that when  $x=5, y=\pi$ .

Q2 (4 marks)

An iron has a temperature of  $54^\circ\text{C}$  is left in a room, of temperature  $18^\circ\text{C}$ , to cool such that the

temperature  $T^\circ\text{C}$  at time  $t$  minutes is given by  $\frac{dT}{dt} = k(T - 18)$ . After 15 mins the temperature of the iron is  $37^\circ\text{C}$ . Determine the time taken for the iron's temperature to drop to  $22^\circ\text{C}$ .

Q7 (2, 3 &amp; 3 = 8 marks)

A lolly company makes jelly beans where the mass of one jelly bean is normally distributed with a mean of 23.4 mg and a standard deviation of 3.2 mg. (Note: 1g=1000mg)

- a) Determine the probability to two decimal places that the total mass of 85 jelly beans is more than two grams.

- b) Given that the probability that the mean mass of a jelly bean differs from the population mean by more than 0.35 mg is 5%, determine  $n$ , the number of jelly beans that need to be sampled.

- c) On a particular day the operator of a machine that makes jelly beans is suspected of being faulty. A sample of 200 jelly beans had a sample standard deviation of 3.8 mg with a total mass of 5.4 grams. Present a mathematical argument to either support or to dismiss such a claim.

Q5 (2, 2 & 3 = 7 marks)  
Consider an object that is moving with Simple Harmonic Motion such that  $\ddot{x} = -9x$  with  $x, t$  in metres and seconds respectively. At  $t = 0$ ,  $x = 7$  metres and is a rest.  
a) Determine a rule for  $x$  in terms of  $t$ .

b) Determine the exact speed when  $x = 3$  metres.

c) Determine the percentage of the time, to one decimal place, that the object is less than 3 metres from the mean position,  $x = 0$ .

Q6 (4 marks)  
Consider an object that is initially at the origin and at rest such that its acceleration is given by  $\frac{dv}{dt} = \frac{v}{1+v^2} m/s^2$  where  $v$  equals the speed in  $m/s$  at  $t$  seconds . Determine the exact speed when its displacement from the origin is  $\ln(3)$  metres.

Q3 (1, 5 & 2 = 8 marks)  
The number  $N$  thousands, of bacteria cells living in a petri dish at time  $t$  hours is given by  $\frac{dN}{dt} = 0.30N - 0.05N^2$ .  
The initial number of cells was 2 thousand.  
a) What is the limiting value of the number of cells as  $t \rightarrow \infty$  ?

b) Using calculus and partial fractions, show every step to express  $N$  in terms of  $t$ .

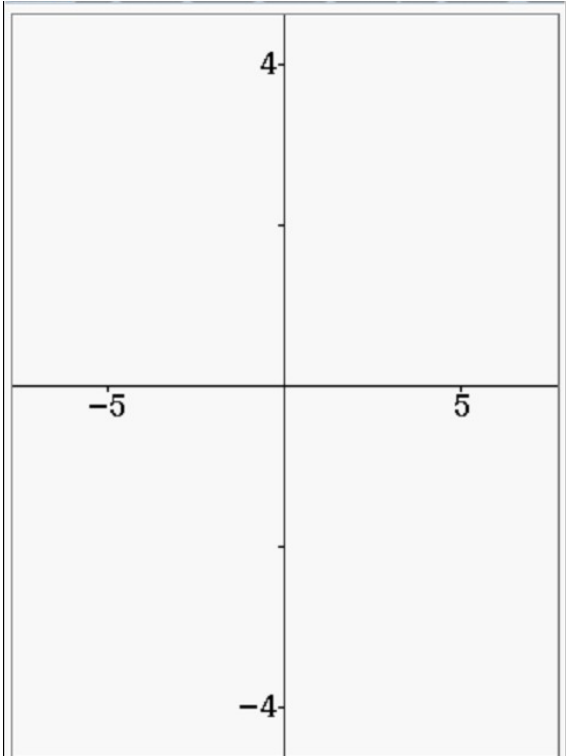
Q3-cont

c) Determine the number of cells after 15 hours.

Q4 (3, 2 & 2 = 7 marks)

Consider the slope field  $\frac{dy}{dx} = (x - 3)(x + 2)$

a) Sketch this field on the axes below.



b) Draw the solution curve, axes above, that contains the point (1,1).

c) Determine the equation of the solution curve that contains (1,1).