PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS METHODS UNITS 3&4

Section One: Calculator-free

Your name _____

Your Teacher's name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Max	Question	Mark	Max
1		10	5		7
2		6	6		7
3		7	7		6
4		8			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	34.5
Section Two: Calculator-assumed	11	11	100	97	65.5
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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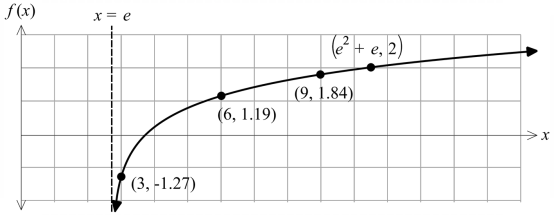
Supplementary page

Question number: _____

Question 7 **(6 marks)**

Part of the graph of $f(x)=\log_a(x-b)$, where $a>1$, is shown below.



(a) State the value of b. (1 mark)

(b) Determine the value of a, given that the curve passes through $(e^2+e,2)$. (2 marks)

(c) Using the **graph**, determine an **approximation** to the following definite integral: (3 marks)

$$\int_3^9 \frac{1}{x-e}\,dx$$

End of questions

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Section One: Calculator-free **34.5% (51 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

See next page

Question 1 **(10 marks)**

(a) Solve $\ln(x-2)=\ln x+2$.

(4 marks)

(b) Find the exact solution of $2^{x-3}=7$ and express your answer in terms of logarithms.

(3 marks)

(c) Show $\log_3 5=\frac{\ln 5}{\ln 3}$.

(3 marks)

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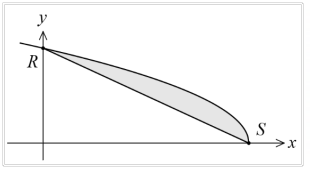
Question 6 **(7 marks)**

The graph of the curve $y=\sqrt{16-x}$ is shown to the right together with the chord RS that joins the points of intersection of the curve with the axes.



(a) Determine the slope of the curve at R. (2 marks)

(b) Determine the area of the shaded region. (5 marks)

See next page

Question 5 **(7 marks)**

The random variable X takes the values 0, 1, 2, 3 only and its probability distribution is shown below.

x	0	1	2	3
$P(X=x)$	a	b	0.05	0.15

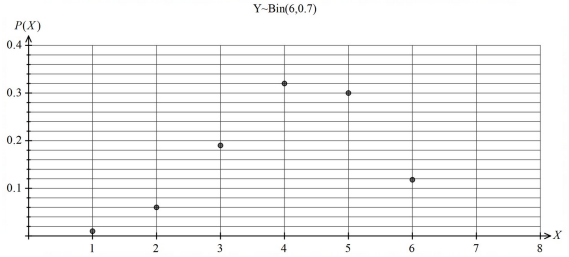
(a) (i) Given that $E(X)=1.05$, find the values of a and b.

(3 marks)

(ii) Does X have a binomial distribution? Justify your answer.

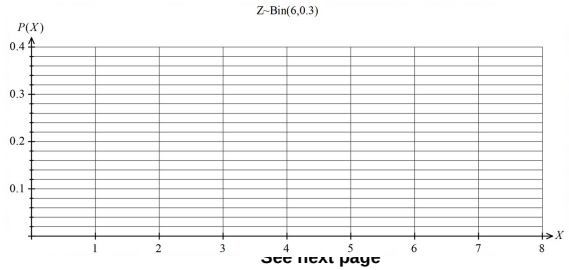
(2 marks)

(b) A binomial distribution for $Y \sim Bin(6, 0.7)$ is shown below.



Draw the graph of $Z \sim Bin(6, 0.3)$ on the axes below.

(2 marks)



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Question 2 **(6 marks)**

(a) Determine $\frac{dy}{dx}$ when

(i) $y = x^3 + \cos(2x - 3)$. (1 mark)

(ii) $y = \int_2^x \ln(t^2 - 3t)\,dt$. (1 mark)

(b) Determine $\frac{d}{dx}(x \ln(3x))$. (2 marks)

(c) Hence, or otherwise, determine $\int (\ln(3x) + 5)\,dx$. (2 marks)

See next page

Question 3 **(7 marks)**

The time, in minutes, that Jake takes to serve a customer at the local supermarket follows a uniform distribution defined over the interval [2,8].

(a) Determine

(i) Jake's expected checkout time.

(1 mark)

(ii) the variance of the time taken to serve a customer.

(2 marks)

(iii) the probability that he will take more than 6 minutes to serve a customer.

(1 mark)

(b) Given Jake has already spent 3 minutes serving a customer, find the probability he will take less than another 4 minutes to finish.

(3 marks)

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Question 4 **(8 marks)**

A tank initially contains 24 L of water. Let $V(t)$ be the volume, in litres, of water in the tank t seconds after it is ruptured, so that

$$V'(t)=\frac{-10t}{t^2+4}, 0\le t\le 20.$$

Determine

(a) $V'(4)$. (1 mark)

(b) $V''(4)$. (3 marks)

(c) $V(4)$. (4 marks)

See next page