MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 4 - 2016
Integration and Logarithms
Resource Rich - SOLUTIONS

1 $\int (4x+7)^3\,dx = \dfrac{(4x+7)^4}{4\times 4} + c$

$= \frac{1}{16}(4x+7)^4$

⊛ B **[1 mark]**

2 Area between $x = -10$ and $x = -6$ is $\int_{-10}^{-6} g(x) - f(x)dx$

Area between $x = -6$ and $x = 0$ is $\int_{-6}^{0} f(x) - g(x)dx$

Total area = $\int_{-10}^{-6} g(x) - f(x)dx + \int_{-6}^{0} f(x) - g(x)dx$

⊛ D **[1 mark]**

3 $\int_0^4 (3\sqrt{x}+x)dx = \int_0^4 (3x^{\frac{1}{2}} + x)dx$

$$= \left[\frac{3\times 2x^{\frac{3}{2}}}{3} + \frac{x^2}{2}\right]_0^4$$

$$= \left[2x^{\frac{3}{2}} + \frac{x^2}{2}\right]_0^4$$

$$= \left(2\times 4^{\frac{3}{2}} + \frac{4^2}{2}\right) - (0+0)$$

$= 2 \times 2^3 + 8$

$= 24$

⊛ A **[1 mark]**

4 $\dfrac{d}{dx}e^{x^3+6x} = 3(x^2+2)\ e^{x^3+6x}$

So $\int 3(x^2+2)e^{x^3+6x}dx = e^{x^3+6x} + c$

$3\int (x^2+2)e^{x^3+6x}dx = e^{x^3+6x} + c$

So $\int (x^2+2)e^{x^3+6x}dx = \dfrac{1}{3}\int 3(x^2+2)e^{x^3+6x}dx$

$$= \frac{1}{3}e^{x^3+6x} + c$$

⊛ C **[1 mark]**

5 Total change = $\int_a^b R'(t)dt$

$= \int_0^5 10e^{0.2t}\,dt$

$= \left[\frac{10e^{0.2t}}{0.2}\right]_0^5$

$= \left[50e^{0.2t}\right]_0^5$

$= 50e^1 - 50e^0$

$= 85.914\ldots$

B **[1 mark]**

6 B **[1 mark]**

7 B **[1 mark]**

8 A **[1 mark]**

9 A **[1 mark]**

10 D **[1 mark]**

11 a $\int(3x^4 + 2x)dx = \int 3x^4\,dx + \int 2x\,dx$

$= \frac{3x^5}{5} + \frac{2x^2}{2} + c$

$= \frac{3x^5}{5} + x^2 + c$ **[1 mark]**

b $\int(4 - 3x)^2\,dx = \int(16 - 24x + 9x^2)dx$

$= 16x - \frac{24x^2}{2} + \frac{9x^3}{3} + c$

$= 16x - 12x^2 + 3x^3 + c$ $= \frac{(3x - 4)^3}{9} + c$

[1 mark]

c $\int\frac{3x^2 - 4x + 7}{\sqrt{x}}dx = \int(3x^2 - 4x + 7)x^{-\frac{1}{2}}\,dx$

$= \int(3x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 7x^{-\frac{1}{2}})\,dx$

$= \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{7x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{2 \times 3x^{\frac{5}{2}}}{5} - \frac{2 \times 4x^{\frac{3}{2}}}{3} + 14x^{\frac{1}{2}} + c$

$= \frac{6x^2\sqrt{x}}{5} - \frac{8x\sqrt{x}}{3} + 14\sqrt{x} + c$ **[2 marks]**

12 $\frac{dy}{dx} = 14x - 4$

$y = 7x^2 - 4x + c$ **[1 mark]**

$y = 8$ when $x = -1$, so $8 = 7 \times (-1)^2 - 4 \times -1 + c$

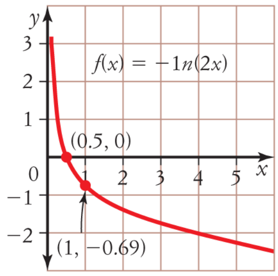
$8 = 11 + c$

$c = -3$ **[1 mark]**

$y = 7x^2 - 4x - 3$ **[1 mark]**

13 $\frac{dy}{dx} = \frac{2}{2x + 1}$ **[2 marks]**

13?



[2]

[4 marks]

Domain $(x : x > 0\}$, Range **R** **[2]**

14 Since $\int \frac{2x}{x^2 + 1} dx = \log_e(x^2+1) + c$

then $\int \frac{6x}{x^2 + 1} dx = 3\int \frac{2x}{x^2 + 1} dx = 3\log_e(x^2 + 1) + c$ **[2 marks]**

15 $\frac{1}{\log_3(60)} + \frac{1}{\log_4(60)} + \frac{1}{\log_5(60)}$

$= \frac{1}{\frac{\log_{10}(60)}{\log_{10}(3)}} + \frac{1}{\frac{\log_{10}(60)}{\log_{10}(4)}} + \frac{1}{\frac{\log_{10}(60)}{\log_{10}(5)}}$ **[1 mark]**

$= \frac{\log_{10}(3)}{\log_{10}(60)} + \frac{\log_{10}(4)}{\log_{10}(60)} + \frac{\log_{10}(5)}{\log_{10}(60)}$ **[1 mark]**

$= \frac{\log_{10}(3) + \log_{10}(4) + \log_{10}(5)}{\log_{10}(60)}$

$= \frac{\log_{10}(3 \times 4 \times 5)}{\log_{10}(60)}$

$= \frac{\log_{10}(60)}{\log_{10}(60)}$ **[1 mark]**

$= 1$ **[1 mark]**

16 $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 6x^2 - 4x - 3$

$\frac{1}{2}v^2 = \int(6x^2 - 4x - 3)dx$ **[1 mark]**

$= 2x^3 - 2x^2 - 3x + c$

When $x = 0$, $v = 3$

$\frac{1}{2}(3)^2 = 2(0)^3 - 2(0)^2 - 3(0) + c$

$c = \frac{9}{2}$

$\frac{1}{2}v^2 = 2x^3 - 2x^2 - 3x + \frac{9}{2}$

$v^2 = 4x^3 - 4x^2 - 6x + 9$

$v = \pm\sqrt{4x^3 - 4x^2 - 6x + 9}$ **[1 mark]**

The condition $v = 3$ when $x = 0$ is satisfied by:

$v = \sqrt{4x^3 - 4x^2 - 6x + 9}$ **[1 mark]**

When $x = 2$,

$v = \sqrt{4(2)^3 - 4(2)^2 - 6(2) + 9}$

$= \sqrt{13}$

When $x = 2$, $v = \sqrt{13}$ **[1 mark]**

17 a $r = \frac{1}{3}\ln\left(\frac{31\,800}{10\,000}\right)$

≈ 0.3856 **[2 marks]**

Therefore, the annual growth rate is 0.3856 or 38.56%.

b $0.3856 = \frac{1}{7}\ln\left(\frac{A}{10\,000}\right)$

$0.3856 \times 7 = \ln\left(\frac{A}{10\,000}\right)$

$e^{(0.3856 \times 7)} = \frac{A}{10\,000}$

$10\,000e^{(0.3856 \times 7)} = A$

$A = \$148\,678.33$ **[3 marks]**

c $0.3856 = \frac{1}{t}\ln\left(\frac{50\,000}{10\,000}\right)$

$0.3856 = \frac{1}{t}\ln(50)$

$0.3856t = \ln(50)$

$t = \frac{\ln(50)}{0.3856}$

$t = 10.1453$ **[3 marks]**

Therefore, it will take 10.1453 years after the year 2009 for the $10 000 investment to grow to $500 000.

18

$$M = \frac{2}{3}\log_{10}\left(\frac{E}{10^{4.4}}\right)$$

$$7.6 = \frac{2}{3}\log_{10}\left(\frac{E}{10^{4.4}}\right)$$

$$\frac{7.6 \times 3}{2} = \log_{10}\left(\frac{E}{10^{4.4}}\right)$$ **[1 mark]**

$$10^{\left(\frac{7.6 \times 3}{2}\right)} = \frac{E}{10^{4.4}}$$

$$10^{\left(\frac{7.6 \times 3}{2}\right)} \times 10^{4.4} = E$$

$E = 6.3 \times 10^{15}$ joules **[2 marks]**

19 a i $n = 100$, $t = -5\log_e\left(\frac{1000 - 100}{999 \times 100}\right)$

$t = 23.5$ days

ii $n = 200$, $t = -5\log_e\left(\frac{1000 - 200}{999 \times 200}\right)$

$t = 27.6$ days

iii $n = 998$, $t = -5\log_e\left(\frac{1000 - 998}{999 \times 998}\right)$

$t = 65.6$ days

iv $n = 999$, $t = -5\log_e\left(\frac{1000 - 999}{999 \times 999}\right)$

$t = 69.1$ days **[4 marks]**

b $t = 35$, $35 = -5\log_e\left(\frac{1000 - n}{999n}\right)$ **[1 mark]**

$$\frac{35}{-5} = \log_e\left(\frac{1000 - n}{999n}\right)$$

$$-7 = \log_e\left(\frac{1000 - n}{999n}\right)$$

$$e^{-7} = \frac{1000 - n}{999n}$$

$e^{-7} \times 999n = 1000 - n$

$999e^{-7}n + n = 1000$

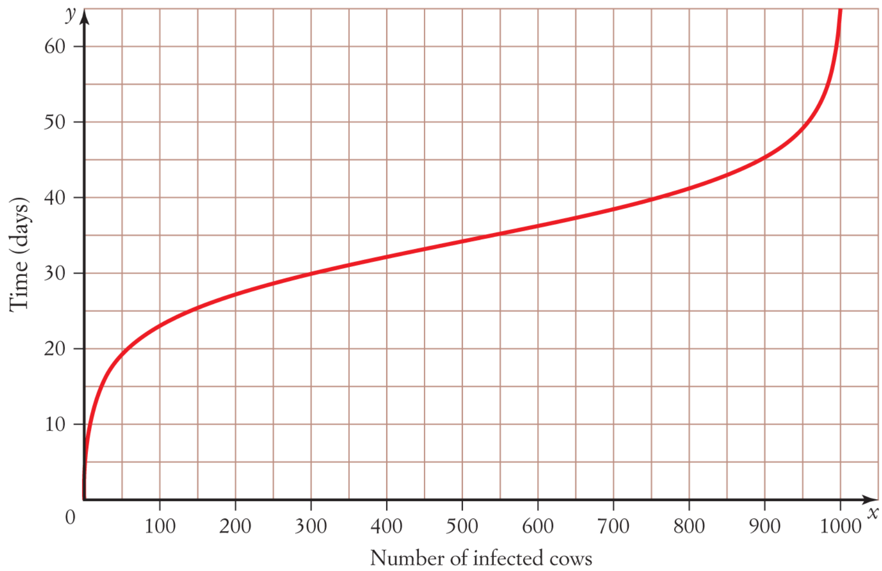
$n(999e^{-7} + 1) = 1000$ **[1 mark]**

$$n = \frac{1000}{999e^{-7} + 1}$$

$n = 523.3$ **[1 mark]**

After 35 days there are 523 cows infected. The 0.3 means that cow number 524 has also contracted the disease partially. Therefore, there will be 524 cows infected after 35 days.
[1 mark]

c



[2 marks]

d This contagious disease can spread very rapidly at first and then very slowly as nearly all of the population has become infected. It is called a *logistic growth model*.

[1 mark]