

**Papers written by
Australian Maths
Software**

SEMESTER TWO

YEAR 12

MATHEMATICS SPECIALIST

UNIT 3-4, REVISION TWO

2016

**Section Two
(Calculator–assumed)**

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 10 minutes

Working time for section: 100 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: drawing instruments, templates, notes on up to two unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

To be provided by the supervisor

Question/answer booklet for Section Two.

Formula sheet retained from Section One.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	8	8	50	52	35
Section Two Calculator—assumed	12	12	100	98	65
Total marks				150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answer in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula Sheet is not to be handed in with your Question/Answer booklet.

9. (7 marks)

- (a) Describe fully the intersection of the sphere defined by $\left\| \begin{pmatrix} x - 1 \\ y - 2 \\ z \end{pmatrix} \right\| = 2$ and the plane $y = 3$. (3)

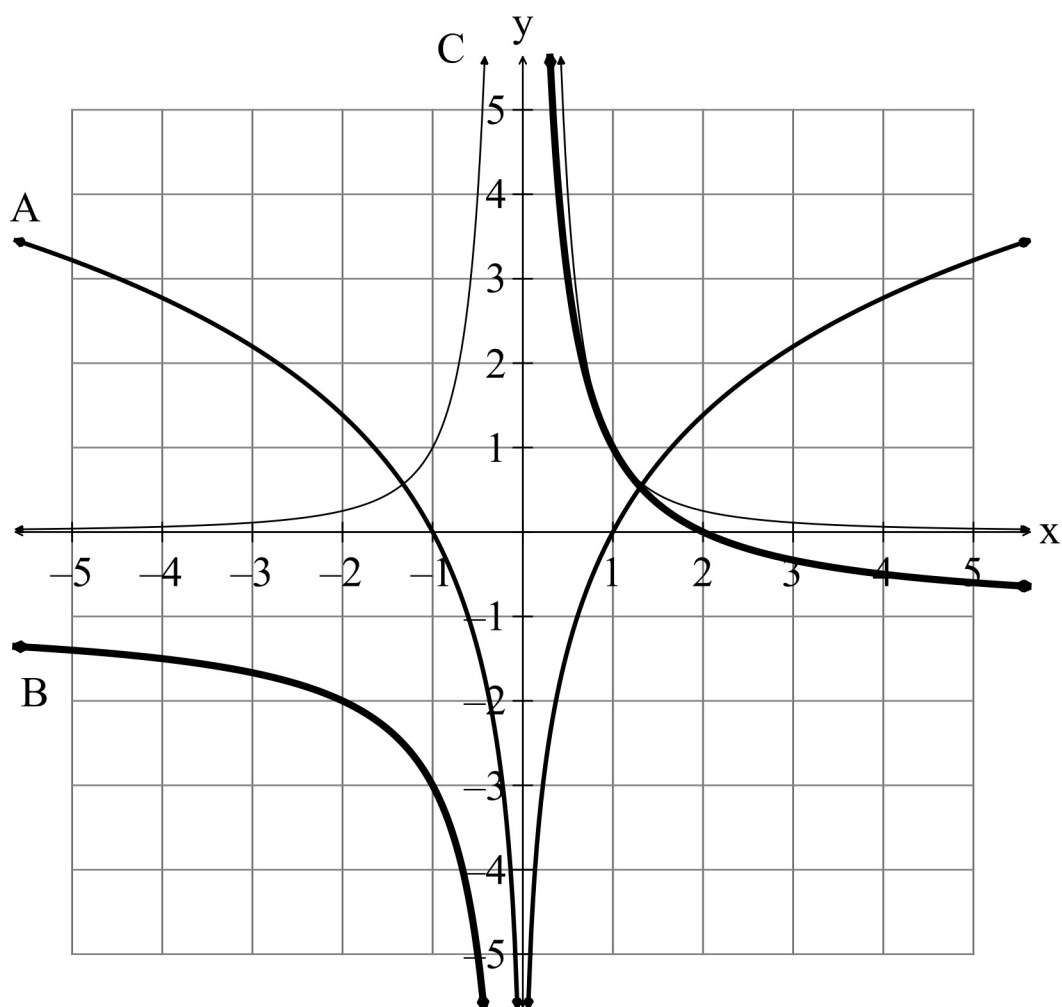
- (b) Determine whether or not the two lines defined below intersect.

$$\mathbf{r}_1(t) = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \text{ and } \mathbf{r}_2(s) = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + s \begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix} \quad (4)$$

10. (10 marks)

Given $f(x) = x^2$, $g(x) = \frac{1}{x}$, $h(x) = \ln(x)$, $j(x) = 2x - 1$ and $k(x) = \sqrt{x}$,

(a) identify the function of functions graphed below. (6)

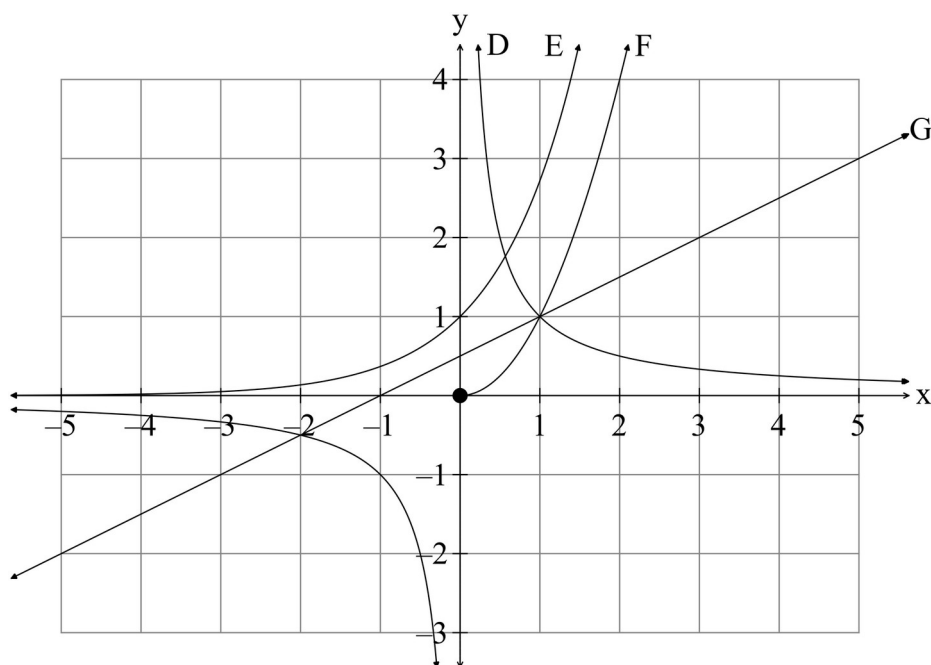


A (2)

B (2)

C (2)

- (b) Find the equations of each of the graphs below given that each is one the inverse of one the listed functions..



D (1)

E (1)

F (1)

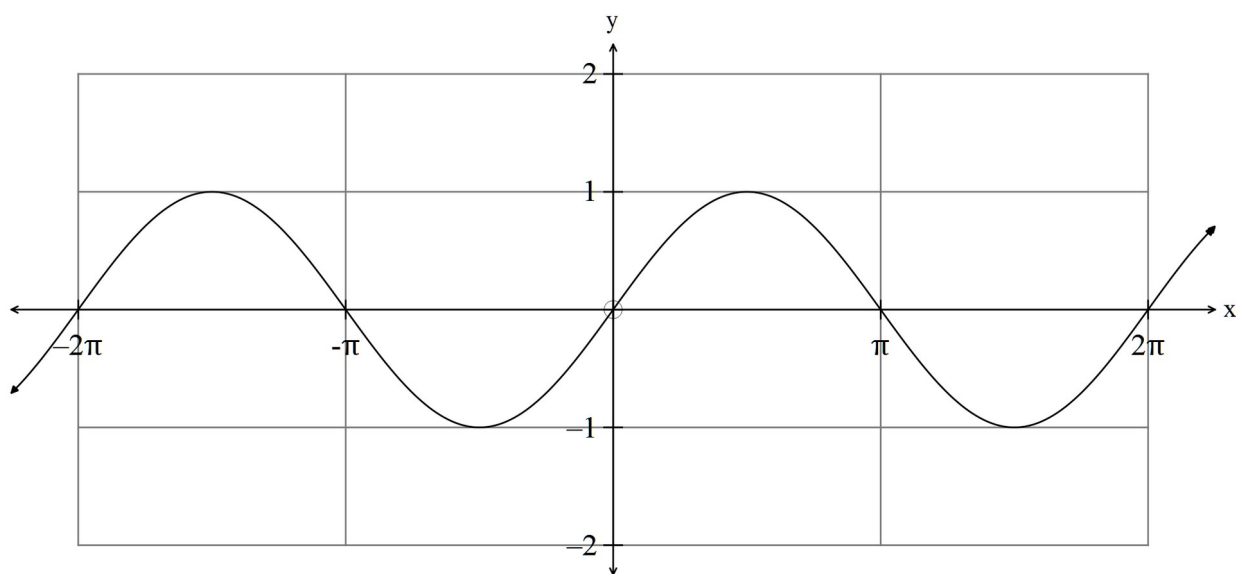
G (1)

11. (7 marks)

(a) Solve the equation $|x| - 1 = 1 - 2|x - 1|$ (5)

(b) The function $g(x) = \sin(x)$ is sketched on the set of axes below.

Sketch $y = -2g(|x|)$ on the set of axes below. (2)



12. (8 marks)

(a) Solve $z^5 - 1 = 0$ leaving your answers in $\text{cis}(\theta)$ form.

(5)

(a) Find the area enclosed between the functions $f(x) = x(x - 3)(x + 3)$ and $g(x) = 9 - x^2$.

(3)

13. (9 marks)

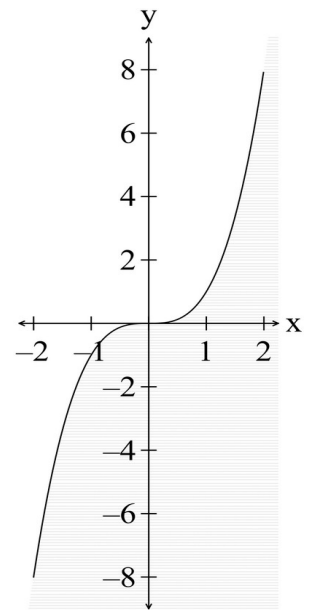
- (a) Consider the function $f(x) = x^3$ for $-2 \leq x \leq 2$.

By finding the volume when the function is rotated about

(i) the y axis.

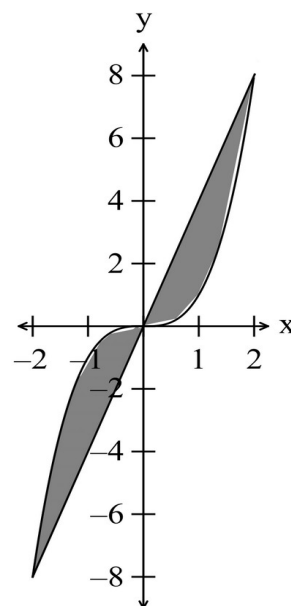
(ii) the x axis

Explain the reason for the difference in the generated volume. (6)



- (b) Consider the two functions $f(x) = x^3$ and $y = 4x$ on $[-2, 2]$.

Write down the expression that needs to be calculated
To find the volume generated when the shaded area is
rotated about the y axis. (3)



14. (12 marks)

(a) Given the velocity of a particle is equal to $\sqrt{4 - 2x}$ cm s⁻¹
find

(i) the acceleration when $t = 2$. (2)

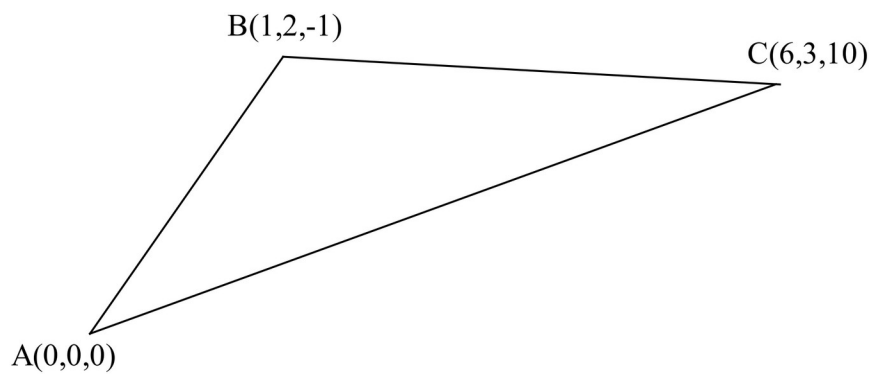
(ii) the displacement when $v = 3$. (2)

- (b) Given $\frac{dy}{dx} = 2xy - y$, find y in terms of x given that the point $(1, e)$ belongs to the function. (4)

- (c) Given the volume of a sphere is increasing at a rate of 2 cm^3 per second, find the rate of change of the radius at $r = 10 \text{ cm}$. (4)

15. (7 marks)

- (a) The points $A(0,0,0)$, $B(1,2,-1)$ and $C(6,3,10)$ as shown in the diagram below.



Find the vector equation of the plane ABC.

(2)

- (b) Given the simplified system of equations.

$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 2p & 1-q \end{bmatrix}$$

Find the values of p and q if

- (i) there is no solution. (2)

- (ii) there are an infinite number of solutions. (2)

- (iii) there is exactly one solution. (1)

16. (8 marks)

- (a) Thirty samples of size 16 are taken from a population that has a mean of 15 and a standard deviation of 3.

What is the expected mean and standard deviation of the sampling distribution?

(3)

- (b) A sampling distribution, formed from sets of size 40, has a variance of 10 and a mean of 20.

Determine the mean and the variance of the population.

(2)

- (c) The mean height of 17 year old boys is 182 cm in Australia with a standard deviation of 10 cm.

What is the probability that the average height of 9 boys selected randomly from your class will be over 185 cm?

(3)

17. (7 marks)

- (a) A sample of size 100 had a mean of 10 and a standard deviation of 2.

Find the 99% confidence limits for the mean of the population.

(3)

- (b) Given the 95% confidence limits for the population mean are [8.5, 14.5]

Determine

- (i) the mean of the population.

- (ii) the standard deviation of the population..

(4)

18. (8 marks)

The number of deaths from Avian (bird) Influenza in Cambodia was reported to the World Health Organisation (WHO).

Over the years 2010-2015 the number of deaths from Avian Influenza can be

determined by the equation $N = \frac{8}{1 + 128.866e^{-3.529t}}$ where t is in years where $t = 0$ in 2010.

(a) Determine the number of deaths in Cambodia from Avian Influenza in 2012. (2)

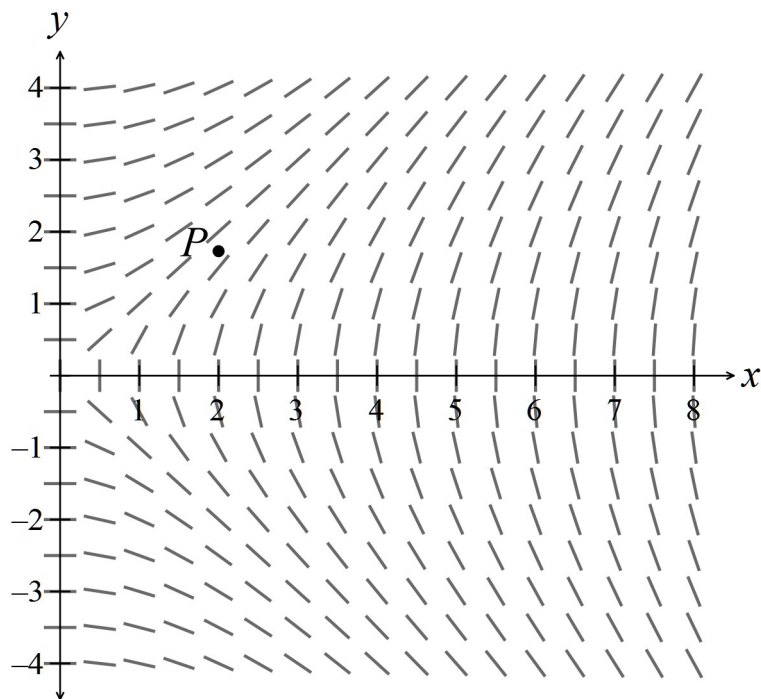
(b) (i) Find the rate of increase of cases of Avian Influenza in 2015. (3)

(ii) Is the number of cases increasing or decreasing in 2013? (1)

(c) Sketch the shape of the curve (2)

19. (4 marks)

Consider the direction graph below.



- (a) Use the direction graph to help you sketch the relationship that passes through the point $P(2, \sqrt{3})$. (2)

- (b) Comment on the shape of the graph that passes through the point $(4, y)$ as y goes from -4 to 4. (2)

(2)

(1)

20. (11 marks)

(a) Simplify
$$\frac{\left(\operatorname{cis}\left(\frac{3\pi}{4}\right) \right)^{-4} \times \left(\frac{1+i}{1-i} \right)^2}{\sqrt{\operatorname{cis}(2\pi)}}$$
 (4)

(b) Prove that $\cos^4(\theta)\sin^3(\theta) = -\frac{1}{64}\sin(7\theta) - \frac{1}{64}\sin(5\theta) + \frac{3}{64}\sin(3\theta) + \frac{3}{64}\sin(\theta)$ (7)

END OF SECTION TWO