# KINGSWAY CHRISTIAN COLLEGE

**MATHS DEPARTMENT5**

**Course:** Mathematics Methods Year 12

**Assessment Task:** Test 5 – Discreet Random Variables and The Binomial Distribution

**Student Name:** Sol Key.

**Date:** 10th August 2017

**Assessment Score:** ________ / 45

**Year Score:** ________

**Comments:** ________

**Teacher signature:** ________

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**Comments:** ________

**METHODS YEAR 12 Test 5 2017**

**Discreet Random variables and Distributions**

**Calculator Allowed** **Time: 45 mins** **Marks: / 45**

**Calculators are allowed for this test, but no notes. Please show work out where needed.**

**Question 1** **(3,4,3 = 10 marks)**

The discrete random variable $X$ can only take the values 0, 1, 2, 3, 4, 5. The probability distribution of $X$ is given by the following

$$P(X=0)=P(X=1)=P(X=2)=a$$
$$P(X=3)=P(X=4)=P(X=5)=b \qquad \text{where } a \text{ and } b \text{ are constants.}$$
$$P(X\geq 2)=3P(X<2)$$

(a) Determine the values of $a$ and $b$.

$a+3b = 3(2a)$ ✓ and $3a+3b=1$. ✓

3

Solve $\left\{\begin{array}{l} a+3b=6a \\ 3a+3b=1 \end{array} ;\ a,b \right\}$ $\rightarrow$ $a=\frac{1}{8}$ ✓

$b=\frac{5}{24}$.

(b) Show that the expectation of $X$ is $\frac{23}{8}$ and determine the exact variance of $X$.

$E(X) = (0\times\frac{1}{8}) + (1\times\frac{1}{8}) + (2\times\frac{1}{8}) + (3\times\frac{5}{24}) + (4\times\frac{5}{24}) + (5\times\frac{5}{24})$

$= \frac{3}{8} + \frac{60}{24}$ ✓ any working.

$= \frac{23}{8}.$ ✓

4

$V(X) = (0^2\times\frac{1}{8}) + (1^2\times\frac{1}{8}) + (2^2\times\frac{1}{8}) + (3^2\times\frac{5}{24}) + (4^2\times\frac{5}{24})$

$- \left(\frac{23}{8}\right)^2$ ✓ any working.

$= \frac{533}{192}.$ ✓

**Question 6** **(3,2,2,2 = 9 marks)**

A manufacturer of chocolate produces 3 times as many soft centred chocolates as hard centred ones. The chocolates are randomly packed in boxes of 20.

*Let the Discreet Random Variable X = the number of hard centred chocolates per box.*

(a) Find the probability that in a box there are

(i) an equal number of soft centred and hard centred chocolates

$X \sim Bi(20;\ 0{,}25)$ ✓

$P(X=10) = 0{,}00992$ ✓

(ii) at least one hard centred chocolate.

$P(X\geq 1) = 1 - P(X=0)$ ✓

$1 - 0{,}003171.$

$= 0{,}997$ ✓

(iii) fewer than 5 hard centred chocolates.

$P(X<5) = P(X\leq 4)$ ✓

$= 0{,}41484$

$\approx 0{,}4148$ ✓

(b) A random sample of 5 boxes is taken from the production line. Use your answer from question (iii), to find the probability that exactly 3 of the boxes contain fewer than 5 hard centred chocolates.

*Let the Discreet Random Variable Y= the number of boxes that contain fewer than 5 hard centred chocolates.*

$Y \sim Bi(5;\ 0{,}41484)$ ✓

$\therefore P(Y=3) = 0{,}24445$

$\approx 0{,}2445$ ✓

**Question 5** **(1,3,1,2 = 7 marks)**

A Study found that 80 per cent of people exhibiting common influenza symptoms recovered without taking any medication. A random sample of 30 people who had developed influenza symptoms was taken.

Let X denote the number of people in this sample who recovered without taking any medication.

(a) State why X is classified as discrete and not continuous?

Discrete as X is an integer ✓ 1

∴ something we "count" and do not "measure"

(b) State the probability distribution of X and the mean and standard deviation of this distribution.

$X \sim Bi(n; p)$ ✓ $\quad \mu = 30 \times 0{,}8 = 24$

$\therefore X \sim Bi(30; 0{,}8)$ ✓ $\quad \sigma = \sqrt{npq}$ ✓

3 $\quad = \sqrt{30 \times 0{,}8 \times 0{,}2}$

$= 2{,}191$

(c) What is the probability, correct to three decimal places that

(i) Exactly 25 people recovered without any medication?

$\therefore P(X=25) = 0{,}172.$ ✓ 1

(ii) At least 24 but no more than 28 recovered without any medication?

$P(24 \leq X \leq 28) = 0{,}596.$ ✓ ✓ 2

(also accept 0,597).

(c) Determine the exact probability that the sum of two independent observations from this distribution exceeds 7.

All combinations:

$5,5 \longrightarrow \left(\frac{5}{24}\right)^2$

$5,4$ or $(4,5) \longrightarrow 2 \times \left(\frac{5}{24}\right)^2$

$5,3$ or $(3,5) \longrightarrow 2 \times \left(\frac{5}{24}\right)^2$

$4,4 \longrightarrow \left(\frac{5}{24}\right)^2$

✓ 3

$\therefore P(\text{sum} > 7) = 6 \times \left(\frac{5}{24}\right)^2$ ✓

$= \frac{150}{576} = \frac{25}{96}$ ✓

**Question 2** **(3,2,2,3= 10 marks)**

On a long train journey, a statistician is invited by a gambler to play a dice game. The game uses two ordinary dice which the statistician is to throw.

If the total score is 12, the statistician is paid \$6 by the gambler. If the total score is 8, the statistician is paid \$3 by the gambler. However, if both or either dice show a 1, the statistician pays the gambler \$2. Otherwise, no money changes hands.

Let \$ $X$ be the amount paid to the statistician by the gambler.

(a) Complete the table below.

✓✓✓ (−1) per mistake

| $x$ | $-2$ | 0 | 3 | 6 |
|---|---|---|---|---|
| $P(X=x)$ | $\frac{11}{36}$ | $\frac{19}{36}$ | $\frac{5}{36}$ | $\frac{1}{36}$. |

(b) Explain why the table in part (a) describes a probability distribution for the discrete random variable $X$.

$\sum p(x) = 1$ and $0 \le p \le 1$. ✓ ✓ (2)

(c) Show that, if the statistician played the game 100 times, his expected loss would be \$2.78, to the nearest cent.

$E(X) = \left(-2 \times \frac{11}{36}\right) + \left(0 \times \frac{15}{36}\right) + \left(3 \times \frac{5}{36}\right) + \left(6 \times \frac{1}{36}\right)$

$= -0{,}02\dot{7}$. ✓ (2)

∴ In 100 games he would lose $100 \times 0{,}02\dot{7}$

$= -2{,}\dot{7}$ which is a loss of

\$2,78. ✓ (2 dp).

(d) Find the amount, \$$a$, that the \$6 would have to be changed to in order to make the game unbiased.

For the game to be unbiased: $E(X) = 0$. ✓

∴ $\left(-2 \times \frac{11}{36}\right) + \left(0 \times \frac{15}{36}\right) + \left(3 \times \frac{5}{36}\right) + \left(a \times \frac{1}{36}\right) = 0$. ✓ (working)

∴ solve$\left(-\frac{22}{36} + \frac{15}{36} + \frac{a}{36} = 0, a\right)$ (3)

∴ $a = 7$ ✓

**Question 3** **(3 marks)**

Given that $X \sim B(15, p)$ find the value of $p$ such that $P(X > 13) = 0.4$

Show your working

• realize that it is $\geq 14$

∴ $P(X > 13) = P(X \geq 14)$ ✓ (3)

$= P(X = 14) + P(X = 15)$

∴ $0{,}4 = \binom{15}{14} p^{14} (1-p)^1 + \binom{15}{15} p^{15} (1-p)^0$ • any working ✓

∴ solve$\left(0{,}4 = 15p^{14}(1-p) + p^{15}, p\right) \mid 0 \le p \le 1$

∴ $p = 0{,}869698$

$p \approx 0{,}87$. ✓ • correct answer

**Question 4** **(2,4 = 6 marks)**

In a school of 480 students, 25% said they barracked for the Dockers.

(a) State why " Supported the Dockers" is a Binomial random variable in this context.

Independent trials. ✓

Success | Failure. ✓ (∴ Bernouli trials)

25% | 75%. (2)

(b) Determine $\mu$ and $\sigma$.

$n = 480$ (4)

$p = 0{,}25$

∴ $\mu = np$ ✓

$= 480 \times 0{,}25$

$\mu = 120$ ✓

$\sigma = \sqrt{npq}$ ✓

$= \sqrt{480 \times 0{,}25 \times 0{,}75}$

$= \sqrt{90}$

$\sigma \approx 9{,}49$. ✓