Christ Church Grammar School

2019
TEST 5

MATHEMATICS METHODS Year 11

MARKING KEY

Time and marks available:

Calculator-Free

Working time for this section:	30 minutes
Marks available:	**30 marks**

Calculator-Assumed

Working time for this section:	10 minutes
Marks available:	**10 marks**

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

Calculator-Free Section (30 marks)

Question 1 (5 marks)

(a) Evaluate $32^{-0.6}$ giving your answer as a fraction. (2 marks)

Solution
$32^{-\frac{3}{5}} = \left(2^5\right)^{-\frac{3}{5}} = 2^{-3} = \dfrac{1}{8}$
Specific behaviours
✓ expresses the given expression using the correct rational index ✓ obtains the correct exact value as a fraction

2.1.1, 2.1.2

(b) Given that $\dfrac{\sqrt{a^3} \times b^{-2}}{\sqrt[3]{ab^9}} = \dfrac{a^m}{b^n}$ determine the values for m, n. (3 marks)

Solution
$\dfrac{\sqrt{a^3} \times b^{-2}}{\sqrt[3]{ab^9}} = \dfrac{a^{\frac{3}{2}} \times b^{-2}}{a^{\frac{1}{3}} \times b^3} = \dfrac{a^{\frac{3}{2}-\frac{1}{3}}}{b^5} = \dfrac{a^{\frac{7}{6}}}{b^5}$ Hence $m = \dfrac{7}{6},\ n = 5$.
Specific behaviours
✓ expresses the given expression using the correct rational indices ✓ applies index laws correctly ✓ states the correct values for m, n.

2.1.1, 2.1.2

Question 2 (3 marks)

The mass of the sun is approximately 2×10^{30} kg whilst the mass of the earth is approximately 6×10^{24} kg.

Determine the ratio of the mass of the sun to the mass of the earth, in the form *n* : 1, where *n* is written in scientific notation correct to 3 significant figures.

Solution
Ratio of mass of sun to the earth $= \dfrac{2\times10^{30}}{6\times10^{24}} = \dfrac{1}{3}\times10^{6} = 0.3333...\times10^{6}$ $= 3.33\times10^{5}$ (3 sig. figures)
Specific behaviours
✓ forms the expression for the ratio of masses correctly. ✓ writes the ratio using 10^5 as the power of 10 for scientific form ✓ writes the ratio 3.33 as the number between 1 and 10 for scientific form

2.1.3

(b) By using Calculus methods, determine the height of the rectangular prism (correct to the nearest 0.1 cm) which maximises the volume. (4 marks)

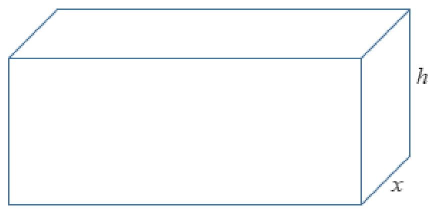
Solution
Require $\dfrac{dV}{dx}=0$ for a maximum value. $\dfrac{dV}{dx} = 160x-60x^2 = 0$ Solving using CAS: $x=0$ or $x=\dfrac{8}{3}$ Optimum height $h=20-5(2.33..) = 6.7cm$ (nearest 0.1 cm)
Specific behaviours
✓ states that the derivative must equal zero for a maximum ✓ differentiates the volume function correctly ✓ solves the equation $V'(x)=0$ correctly for the value of x ✓ writes a conclusion stating the height correct to the nearest 0.1 cm

2.3.21

Question 9 **(6 marks)**

An 80 cm long wire frame is used to make the 12 edges of a rectangular prism. This prism is to have the length of the base equal to four times the width of the base.

Let $x =$ the width of the base of the rectangular (cm)
$h =$ the height of the rectangular prism (cm)



(a) Show that the volume of the prism formed is given by $V = 80x^2 - 20x^3$. (2 marks)

Solution
Total wire length $4(x)+4(4x)+4(h)=80$ *i.e.* $20x+4h=80$ *i.e.* $h=20-5x$ Volume $V = (4x)(x)(h) = 4x^2(20-5x)$ $\therefore\ V = 80x^2-20x^3$
Specific behaviours
✓ obtains the expression for h in terms of x correctly   ✓ forms the volume expression in terms of x correctly

2.3.21

Question 3 **(5 marks)**

Consider the graphs of $f(x)=2^x$ and $g(x)=4\left(2^{-x}\right)$.

(a) Transformations to the graph of $f(x)=2^x$ are required to obtain the graph of $g(x)=4\left(2^{-x}\right)$. Suppose that the following transformations to $f(x)=2^x$ were considered:

Transformation A	Reflect about $y=0$
Transformation B	Reflect about $x=0$
Transformation C	Translate 2 units LEFT
Transformation D	Translate 2 units RIGHT.

Using ONLY transformations A, B, C, D, which transformations (and in the correct order) must be applied to $f(x)$ in order to obtain the graph of $g(x)$? (3 marks)

Solution
$g(x)=4\left(2^{-x}\right)= 2^2\times 2^{-x} = 2^{-x+2}$ $2^x \xrightarrow{C} 2^{x+2} \xrightarrow{B} 2^{-x+2}$ i.e. Apply C then B.   OR alternatively $2^x \xrightarrow{B} 2^{-x} \xrightarrow{D} 2^{-(x-2)}$ i.e. Apply B then D.
Specific behaviours
✓ applies index laws to express $g(x)$ as a power of 2   ✓ identifies two correct transformations   ✓ states the correct order of the transformations

2.1.1, 2.1.4, 2.1.5

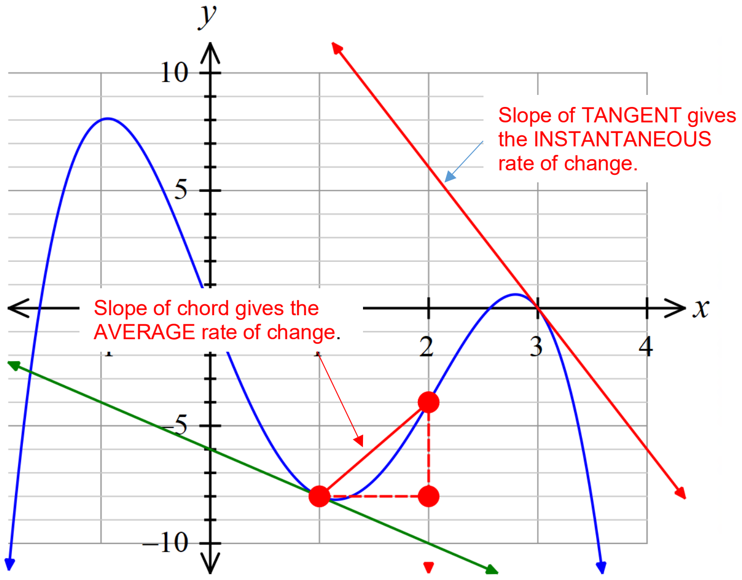
(b) Determine the exact solution to the equation $g(x)=\sqrt{8}$. (2 marks)

Solution
$4(2^{-x}) = \sqrt{8} = \sqrt{2^3}\ \therefore\ 2^{2-x} = 2^{\frac{3}{2}}$ $\therefore\ 2-x = \frac{3}{2}$ $\therefore\ x = \frac{1}{2}$
Specific behaviours
✓ expresses both sides correctly as powers of 2   ✓ solves correctly for x

2.1.7

Question 4 **(6 marks)**

The graph of $y = f(x)$ is shown below.



By drawing appropriate lines/labelling on the above graph, explain how the:

(a) average rate of change of $y = f(x)$ from $x = 1$ to $x = 2$ is measured. (2 marks)

Solution
Average rate of change is measured by the SLOPE of the CHORD connecting the points on the graph between $x = 1$ and $x = 2$.
Specific behaviours
✓ indicates the chord connecting the points $(1,-8)$ and $(2,-4)$
✓ indicates the SLOPE of the chord measures the average rate of change

2.3.4

(b) instantaneous rate of change of $y = f(x)$ at $x = 3$ is measured. (2 marks)

Solution
Instantaneous rate of change is measured by the SLOPE of the TANGENT drawn to the curve at $x = 3$.
Specific behaviours
✓ indicates the tangent to the curve at $(3,0)$
✓ indicates the SLOPE of the tangent measures instantaneous rate of change

2.3.9

Calculator-Assumed Section **(10 marks)**

Question 8 **(4 marks)**

Data is collected on the growth of bacteria in an organism is tabulated below.

Time t minutes	10	20	30	40
Bacteria $B(t)$	41	66	108	176

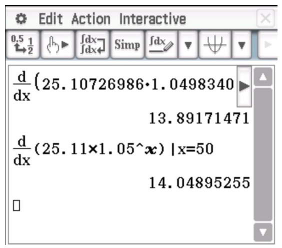
It was suggested that an exponential model of the form $B(t) = c(k)^t$ be used to model this growth.

(a) Determine the values for the constants c and k each correct to 0.01. (2 marks)

Solution
Using CAS Satistics Application: $B(t) = ab^x$ From CAS: $a = 25.11,\ b = 1.05$ i.e., $B(t) = 25.11(1.05)^t$ $\therefore\ c = 25.11,\ k = 1.05$ (2 d.p.)
Specific behaviours
✓ determines the value of c ✓ determines the value of k

2.1.6

(b) Using this model, make a prediction for the rate of growth in the bacteria at $t = 50$ minutes. (2 marks)

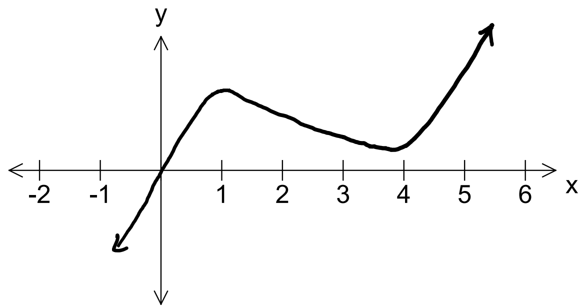
Solution
Rate of growth at $t = 50$ is given by $B'(50)$. Using CAS: $B'(50) = 14.04$ or $B'(50) = 13.89$ i.e. bacteria are increasing at a rate of 14 per minute after 50 minutes. 
Specific behaviours
✓ states that the derivative value at $t = 50$ is required ✓ evaluates the derivative correctly using CAS

2.3.16

Question 7 (3 marks)

On the axes provided below, sketch a possible graph $y = f(x)$ satisfying the following requirements:

- The curve only cuts the *x*-axis at the origin
- $\frac{dy}{dx} = 0$ at $x = 1$ and $x = 4$
- $\frac{dy}{dx} < 0$ only for $1 < x < 4$



Specific behaviours
✓ curve cuts the x-axis at the origin only ✓ turning points located at x = 1 and x = 4 ✓ curve is decreasing between x = 1 and x = 4

2.3.20

(c) Using the graph determine the value for $f'(1)$. (1 mark)

Solution
Require the slope of the tangent at $x = 1$. From the graph $f'(1) = -2$.
Specific behaviours
✓ determines the slope of the tangent correctly (allow tolerance ± 0.5)

2.3.8, 2.3.10, 2.3.16

(d) Using the graph, solve the inequality $f'(x) < 0$ for the domain $x < 2$. (1 mark)

Solution
This requires where the graph of $y = f(x)$ is DECREASING where $x < 2$. From the graph this is where $-0.9 < x < 1.2$.
Specific behaviours
✓ determines the interval of x values correctly (allow tolerance ± 0.1)

2.3.13

Question 5 **(4 marks)**

Consider the function $g(x) = 2\sqrt{x} - x^3$.
Determine the equation of the tangent to the curve at $x = 1$.

Solution
$g(1) = 1 \ \therefore (1,1)$ $g'(x) = \frac{1}{\sqrt{x}} - 3x^2 \quad \therefore g'(1) = -2$ Equation of tangent is given by: $y - (1) = -2(x-1)$ i.e. $y = -2x + 3$
Specific behaviours
✓ determines $g(1)$ correctly ✓ differentiates to determine $g'(x)$ correctly ✓ determines $g'(1)$ correctly ✓ forms the equation for the tangent correctly

2.3.14, 2.3.15, 2.3.17

Question 6 **(4 marks)**

Consider the function $f(x) = \frac{1}{1+x}$.

Using the definition that $f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$, determine from first principles the value for $f'(2)$.

Solution
$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$ $= \lim_{h\to 0}\frac{\frac{1}{1+(2+h)} - \frac{1}{1+2}}{h}$ $= \lim_{h\to 0}\frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h\to 0}\frac{3-(3+h)}{3(3+h)(h)} \quad \ldots (A)$ $= \lim_{h\to 0}\frac{-h}{3(3+h)(h)}$ $= \lim_{h\to 0}\frac{-1}{3(3+h)} = -\frac{1}{9}$
Specific behaviours
✓ forms the correct expression for $f(2+h)$ ✓ performs algebra correctly to obtain expression (A) ✓ cancels the common factor h since $h \neq 0$ ✓ obtain the correct value for $f'(2)$

2.3.5, 2.3.6