Year 12 Mathematics Specialist 2017

Test Number 2: Functions and Graph Sketching

Resource Free

Name: Solutions **Teacher: DDA**

Marks: 20

Time Allowed: 22 minutes

Instructions: You **ARE NOT** permitted any notes or calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 1 **[3 marks]**

If $f(x)=1+\sqrt{x-2}$ determine the formula for $f^{-1}(x)$ the inverse of $f(x)$, and state its domain and range.

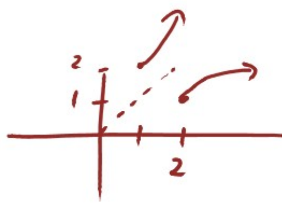
$x = 1+\sqrt{y-2}$

$x-1 = \sqrt{y-2}$

$(x-1)^2 = y-2$

$y = (x-1)^2+2$

$\therefore f^{-1}(x) = (x-1)^2+2$ ✓ ; $x \geq 1$



Domain $= \{x: x \in R;\ x \geq 1\}$ ✓

Range $= \{y: y \in R;\ y \geq 2\}$ ✓

Question 2 **[3 marks]**

State the domain and range of $gof(x)$ if $f(x)=5\sqrt{x}$ and $g(x)=x^2+x$.

$$gof = g(5\sqrt{x}) = (5\sqrt{x})^2 + 5\sqrt{x}$$
$$= 25x + 5\sqrt{x}$$

Domain $= \{x : x \in \mathbb{R};\ x \geq 0\}$

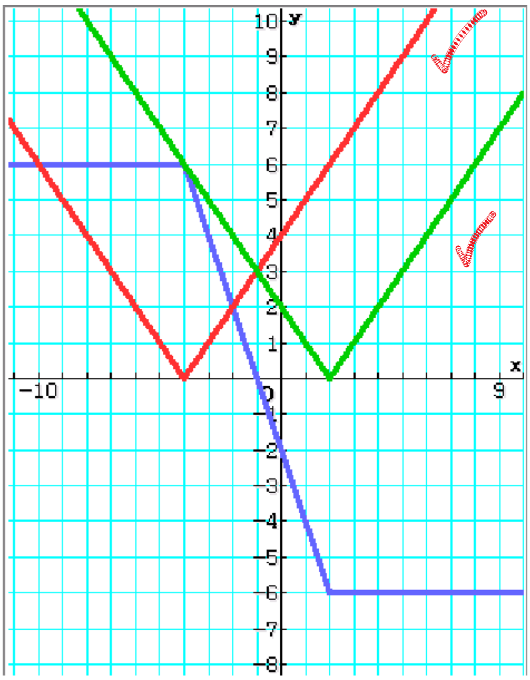
Range $= \{y : y \in \mathbb{R};\ y \geq 0\}$

Question 3 **[2, 2, 2, 1 = 7 marks]**

a) On the graph below accurately draw: $y=|x-2|$ and $y=|x+4|$

b) Using these, or otherwise, draw $y=|x-2|-|x+4|$

c) Express this as a piecewise function $f(x)$.



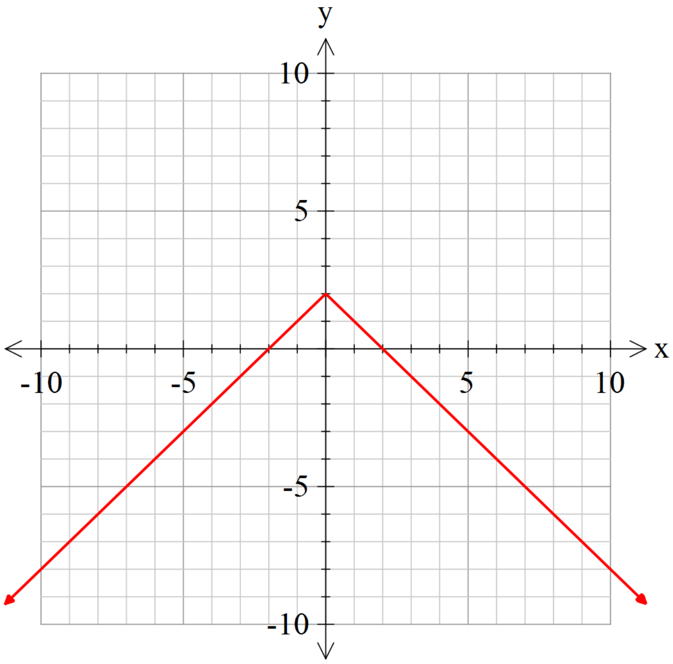
$$f(x)=\begin{cases} 6 & x \leftarrow 4 \\ -2x-2 & -4 \leq x \leq 2 \\ -6 & x>2 \end{cases}$$

d) Use your graph to find the values of x which satisfy: $|x-2|-|x+4| \geq 3$

$x \leq -2.5$

Question 4 **[2 marks]**

Sketch the graph $y = -|x| + 2$



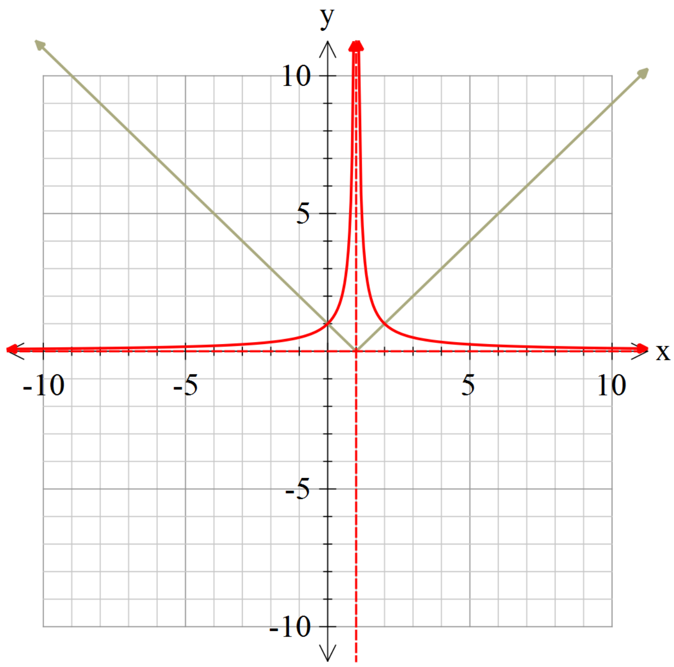
✓ inverted

✓ ↑2

assuming otherwise O.K.

Question 5 **[2, 1 = 3 marks]**

Sketch the graph of $f(x) = \dfrac{1}{|x-1|}$. Write the domain of $f(x)$.



✓✓

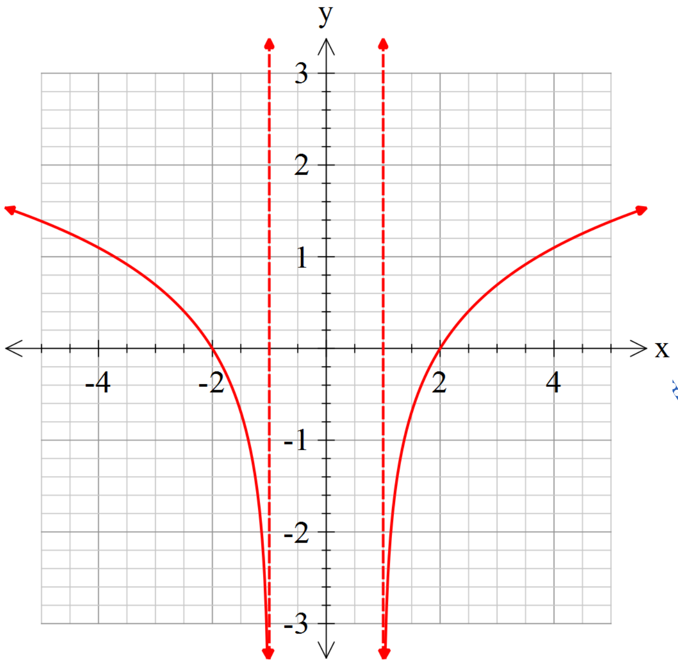
Must show

- (1,1) & (-1,1)
- Asymptote
- Behaviour $x \to 1$
- Behaviour $x \to \pm\infty$

Domain $= \{x : x \in R;\ x \neq 1\}$ ✓

Question 6 **[2 marks]**

Sketch the graph of $y = f(|x|)$ given that $f(x) = \ln(x-1)$ as shown on the graph below.



must be sketched by student

Year 12 Mathematics Specialist 2017

Test Number 2: Functions and Graph Sketching

Resource Rich

Name: _______Solutions___________________________ **Teacher: DDA**

Marks: 20

Time Allowed: 23 minutes

Instructions: You are permitted 1 A4 pages of notes and your calculators. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 7 **[1 mark]**

Circle all of the choices A-E which are true of the following statement.

A function can be identified as one-to-one if, for all values in the domain,

A $\frac{dy}{dx} = 0$. **B** $\frac{dy}{dx} > 0$. **C** $\frac{d^2y}{dx^2} > 0$. need both

D $\frac{d^2y}{dx^2} < 0$. **E** $\frac{dy}{dx} < 0$.

Question 8 **[1, 2 = 3 mark]**

Is the function $f(x) = (x-1)^3 + x^2$ one-to-one?

Justify your answer.

$f'(x) = 3(x-1)^2 + 2x = 3(x^2-2x+1) + 2x$

$= 3x^2 - 6x + 3 + 2x$

$f'(x) = 3x^2 - 4x + 3$

$\Delta = b^2 - 4ac$ ✓ justification for *

$= 16 - 4(9)$

$= 16 - 36$

$= -20$

$\therefore$ no real solns.

$a > 0 \;\therefore$ ✓

$\therefore f'(x) > 0$ always* $\quad \therefore f(x)$ is one-to-one ✓

Question 9 **[1, 1, 3, 3, 2 = 10 marks]**

Given $f(x) = \dfrac{x^2 - 2x + 3}{x - 1}$

a) Find the following:

i) Intercepts: $(0,-3)$ no x-intercepts.

ii) Vertical asymptotes: when $x-1=0$ i.e. $x=1$

iii) Behaviour of $f(x)$ as $x \to \pm\infty$ (including any oblique asymptotes):

$$\begin{array}{r|l} & x-1 \\ \hline x-1 & x^2-2x+3 \\ & x^2-x \\ \hline & -x+3 \\ & -x+1 \\ \hline & 2 \end{array}$$

or

$\text{propFrac}\left(\frac{x^\wedge 2-2x+3}{x-1}\right)$

$x+\frac{2}{x-1}-1$

$$\frac{x^2-2x+3}{x-1} = x-1+\frac{2}{x-1}$$

$\therefore$ As $x \to \pm\infty$, $f(x) \to x-1$ ie $\nearrow \infty$ $\searrow -\infty$

Oblique asymptote at $y = x-1$

iv) Stationary points (accurate to 1 d.p.):

$\text{diff}\left(\frac{x^\wedge 2-2x+3}{x-1}\right)$

$\frac{x^2-2\cdot x-1}{(x-1)^2}$

solve(

$\{x=-\sqrt{2}+1, x=\sqrt{2}+1\}$

$\frac{x^\wedge 2-2x+3}{x-1} \mid x=-\sqrt{2}+1$

$\frac{-\sqrt{2}\cdot\left((\sqrt{2}-1)^2+2\cdot(\sqrt{2}-1)+3\right)}{2}$

simplify(

$-2\cdot\sqrt{2}$

$\frac{x^\wedge 2-2x+3}{x-1} \mid x=\sqrt{2}+1$

$\frac{\sqrt{2}\cdot\left((\sqrt{2}+1)^2-2\cdot(\sqrt{2}+1)+3\right)}{2}$

simplify(

$2\cdot\sqrt{2}$

when $f'(x)=0$

$f'(x) = \frac{x^2-2x-1}{(x-1)^2} = 0$ ✓

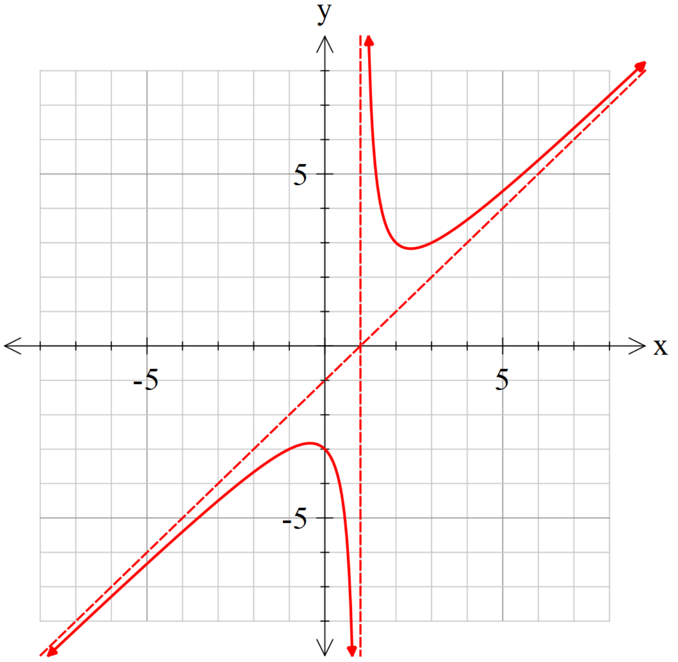
$\Rightarrow x = 1 \pm \sqrt{2}$

pts: $(1+\sqrt{2}, 2\sqrt{2})$ ✓ or $(2.4, 2.8)$

Approx $(1-\sqrt{2}, -2\sqrt{2})$ ✓ $(-0.4, -2.8)$

$f'(x)$ needed

b) Hence, sketch the graph of $f(x) = \dfrac{x^2 - 2x + 3}{x - 1}$.



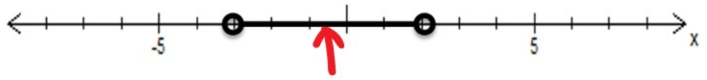
Required:

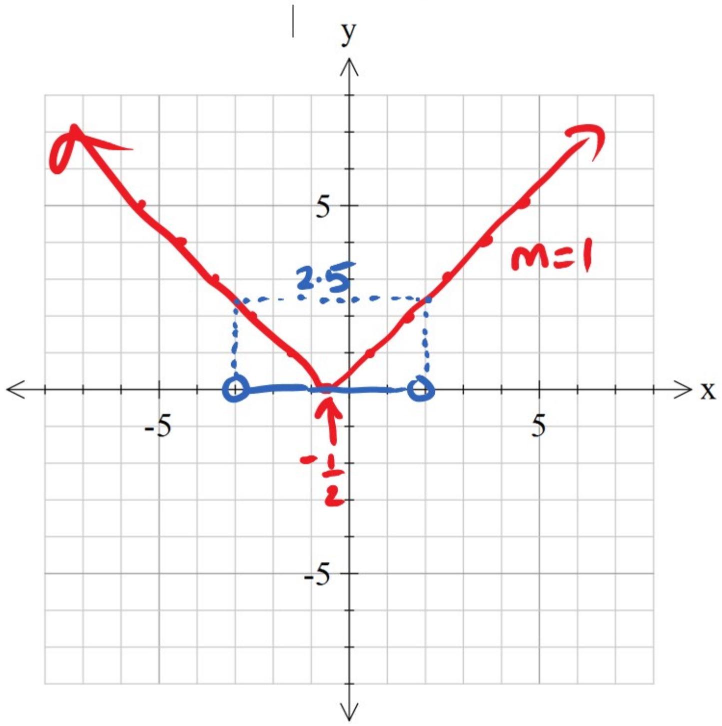
- y-int
- 2 asymptotes
- TPs
- Sketched in with arrows

Question 10 **[3 marks]**

If the number line drawn below represents the solution to the equation $|x+k|-2 \blacksquare p$, where $\blacksquare$ represents an inequality symbol find the values of p and k and also determine which symbol $\blacksquare$ represents.

$|x+k| \blacksquare p+2$





$k = \frac{1}{2}$

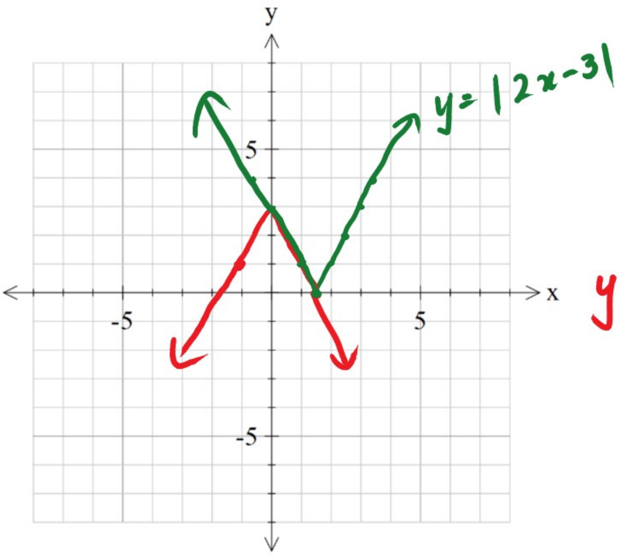
$p = 0.5$

$\blacksquare$ is $<$

Question 11 **[3 marks]**

Given that $a|x|+b=|2x-3|$ is true for $0 \le x \le 1.5$ only, what are the values of a and b?

You may wish to use the grid below.



$y = -|2x| + 3$

$= -2|x| + 3$

$a = -2$

$b = 3$