7. (9 marks)

A recent poll by ReachTEL, commissioned by prominent Perth business figures, sampled 10478 voters asking them the following question:

Who of the following do you think would make the better premier?

4574 of the respondents indicated they preferred Colin Barnett over Mark McGowan.

(a) Determine a sample proportion, $\hat{p}$, for those who preferred Colin Barnett. [1]

$$\hat{p} = \frac{4574}{10478} = 0.4365$$

(b) Determine a 99% confidence interval for the sample proportion who preferred Colin Barnett. [2]

$$0.4241 < \hat{p} < 0.4490$$

(c) Population figures from the Australian Bureau of Statistics currently state that WA has a population of 2.6039 million people. Determine the minimum number of people, that we can be 99% confident, who will vote for Colin Barnett. [2]

$$0.4241 \times 2.6039 = 1104192.909$$

$$\approx 1100\,000 \text{ people}$$

(d) Using $\hat{p}$ as an estimate for the true proportion of WA voters who prefer Colin Barnett, determine how many people need to be sampled to achieve a margin of error of 1%. [2]

$$E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \implies n = 16319.6$$

$$\approx 16320$$

where $E = 0.01$

$\hat{p} = 0.4365$

(e) The original 10478 voters were also asked the following question:

Would you be more of less likely to vote for the Liberal party if Colin Barnett were replaced as leader?

The majority of voters stated their vote would remain unchanged.
Without knowing how many people indicated this, determine the maximum possible margin of ~~standard~~ error for a 99% confidence interval. [2]

Max error when $\hat{p} = 0.5$

$$\therefore E_{max} = 2.5758\sqrt{\frac{0.5(1-0.5)}{10478}}$$

$$= 0.0126$$

$$\approx 1.26\ \%$$

TRINITY COLLEGE

Mathematics Methods Year 12
Test 6 2016

Section 1 Calculator Free

Semester 2 Test

STUDENT'S NAME Solutions

DATE: Friday 2 September **TIME:** 20 minutes **MARKS**: 21

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, Formula page

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Determine the derivative of each of the following. Do not simplify your answers.

(a) $y = (x-3)^2 \ln x^2$ [2]

$$\frac{dy}{dx} = (x-3)^2 \frac{2x}{x^2} + \ln(x^2) \cdot 2(x-3)$$

(b) $y = \ln\sqrt{\frac{(x^5-2x)^3}{2x^2}}$ [3]

$$= \frac{1}{2}\left[3\ln(x^5-2x) - \ln 2x^2\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{3(5x^4-2)}{x^5-2x} - \frac{4x}{2x^2}\right]$$

2. (6 marks)

Determine each of the following:

(a) $\int \frac{x^2+5x^4}{2x^3}\,dx$ [3]

$$= \int \frac{x^2}{2x^3} + \frac{5x^4}{2x^3} \quad dx$$

$$= \int \frac{1}{2x} + \frac{5x}{2} \quad dx$$

$$= \frac{1}{2}\ln|x| + \frac{5x^2}{4} + c$$

(b) $\int \frac{5x^5-5x^2}{(x^3-1)^2}\,dx$ [3]

$$= \int \frac{5x^2\cancel{(x^3-1)}}{(x^3-1)^{\cancel{2}}} \quad dx$$

$$= \int \frac{5x^2}{x^3-1}$$

$f(x) = x^3 - 1$

$f'(x) = 3x^2$

$$= \frac{5}{3}\int \frac{3x^2}{x^3-1} \quad dx$$

$$= \frac{5}{3}\ln|x^3-1| + c$$

For oranges classified as **large**, the quantity of juice obtained from each orange is a normally distributed random variable with a mean of 74 mL and a standard deviation of 9 mL.

(d) What is the probability that a randomly selected large orange produces less than 85 mL of juice, given that it produces more than 74 mL of juice? [3]

$O \sim N(74, 9^2)$

$$P(O \le 85 \mid O > 74)$$

$$= \frac{P(74 < O \le 85)}{P(O > 74)}$$

$$= \frac{0.3892}{0.5} = 0.7784$$

Mani also grows lemons, which are sold to a food factory. When a truckload of lemons arrives at the food factory, the manager randomly selects and weighs four lemons from the load. If one or more of these lemons is underweight, the lead is rejected. Otherwise it is accepted.

It is known that 3% of Mani's lemons are underweight.

(e) (i) Determine the probability that a particular load of lemons will be rejected. [3]

$L \sim B(4, 0.03)$ L: # lemons underweight from 4

$P(L \geq 1) = 0.1147$

(ii) Suppose that instead of selecting only four lemons, n lemons are selected at random from a particular load.
Determine the smallest integer value of n such that the probability of at least one lemon being underweight exceeds 0.5. [3]

$L \sim B(n, 0.03)$ L: # lemons underweight from n

$P(L \geq 1) \geq 0.5$

$P(L < 1) \leq 0.5$

$P(L = 0) \leq 0.5$

$\Rightarrow$ ${}^{n}C_0 (0.03)^0 (0.97)^n \leq 0.5$

$0.97^n \leq 0.5$

$\Rightarrow$ $n \geq 22.75$

$\therefore$ $n = 23$

6. (21 marks)

Mani is a fruit grower. After his oranges have been picked, they are sorted by a machine, according to size. Oranges classifies as **medium** are sold to fruit shops and the remainder are made into orange juice.

The distribution of the diameter, in centimetres, of medium oranges is modelled by a continuous random variable, X , with probability density function

$$f(x)=\begin{cases}\frac{3}{4}(x-6)^2(8-x) & 6\le x\le 8\\ 0 & \textit{otherwise}\end{cases}$$

(a) Determine the probability that a randomly selected medium orange has a diameter greater than 7 cm. [2]

$$P(X \geq 7) = \int_7^8 f(x)\, dx$$

$$= 0.6875$$

(b) Mani randomly selects three medium oranges.
Determine the probability that exactly one of the oranges has a diameter greater than 7 cm. [2]

Y : # oranges diameter > 7 from 3

$Y \sim B(3, 0.6875)$ $\qquad P(Y=1) = 0.2014$

(c) Determine, in centimetres, the:

(i) mean diameter of medium oranges [2]

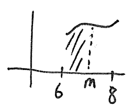
$$\mu = E(X) = \int_6^8 x f(x)\, dx = 7.2 \text{ cm}$$

(ii) standard deviation of medium oranges. [3]

$$\sigma^2 = \int_6^8 (x-7.2)^2 f(x)\, dx = 0.16$$

$$\therefore \sigma = 0.4$$

(iii) median of medium oranges. [3]



$$\text{Med}(X): \int_6^m f(x)\, dx = 0.5$$

$$\Rightarrow m = 7.23,\ 8.49$$

$\therefore$ median is 7.23

3. (5 marks)

A random variable X has the cumulative distribution function with rule:

$$F(x)=\begin{cases}0 & x<0\\ \frac{x^2}{16} & 0\le x\le 4\\ 1 & x>4\end{cases}$$

Calculate:

(a) $P(X \geq 2) = 1 - P(X<2)$ [2]

$$= 1 - \frac{2^2}{16}$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

(b) $P(X \geq 2 \mid X < 3) = \dfrac{P(2 \leq X < 3)}{P(X<3)}$ [2]

$$= \frac{\frac{3^2}{16} - \frac{2^2}{16}}{\frac{3^2}{16}}$$

$$= \frac{5}{9}$$

(c) $P(X = 2 \mid X > 3)$ [1]

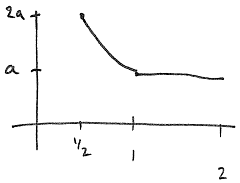
$$= 0$$

4. (5 marks)

The function f is a probability density function with rule

$$f(x)=\begin{cases} \frac{a}{x} & \frac{1}{2}\le x\le 1 \\ a & 1<x<2 \\ 0 & otherwise \end{cases}$$

Determine the value of a.



$$\int_{1/2}^{1} \frac{a}{x}\,dx + \int_{1}^{2} a\,dx = 1$$

$$\Big[a\ln|x|\Big]_{1/2}^{1} + a = 1$$

$$a\ln|1| - a\ln\tfrac{1}{2} + a = 1$$

$$0 + a\ln 2 + a = 1$$

$$a(\ln 2 + 1) = 1$$

$$a = \frac{1}{\ln 2 + 1}$$

TRINITY COLLEGE

Mathematics Methods Year 12
Test 6 2016

Section 2 Calculator Assumed

Semester 2 Test

STUDENT'S NAME ________________________________

DATE: Friday 2 September **TIME:** 30 minutes **MARKS**: 35

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, Formula page (retain from Section 1)

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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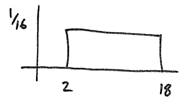
5. (5 marks)

The time (in seconds) that it takes a student to complete a puzzle is a continuous uniform random variable X. It takes students between 2 and 18 seconds to complete the puzzle.

(a) Determine the probability distribution function for X. [2]

$$f(x)=\begin{cases} \frac{1}{16} & 2\le x\le 18 \\ 0 & elsewhere \end{cases}$$



(b) Determine the probability that a student takes less than 12 seconds to complete the puzzle. [1]

$$P(X\le 12) = \frac{10}{16} = \frac{5}{8}$$

(c) Determine the probability that a student takes between 8 and 10 seconds to complete the puzzle, given that he takes less than 12 seconds. [2]

$$P(8\le X\le 10 \mid X\le 12) = \frac{2/16}{10/16}$$

$$= \frac{2}{10}$$

$$= \frac{1}{5}$$