

| Question | Mark | Max | Question | Mark | Max |
|----------|------|-----|----------|------|-----|
| 1        | 8    | 5   | 2        | 8    | 6   |
| 3        |      | 8   | 4        |      | 7   |
| 5        |      | 9   | 6        |      | 9   |
| 7        |      | 10  | 8        |      | 10  |

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

#### Important note to candidates

Special items: **nil**

To be provided by the candidate: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/lap, eraser, ruler, highlighting pens.

Formula sheet

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time: fifty minutes  
Reading time before commencing work: five minutes

Time allowed for this section

Your Teacher's Name \_\_\_\_\_

Your Name \_\_\_\_\_

Section Done: \_\_\_\_\_  
Calculator-free

## 12 SPECIALIST MATHEMATICS

Question/Answer booklet

Semester One Examination, 2023

**Structure of this paper**

| Section                            | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|------------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|---------------------------|
| Section One:<br>Calculator-free    | 6                             | 6                                  | 50                     | 50              | 34                        |
| Section Two:<br>Calculator-assumed | 12                            | 12                                 | 100                    | 97              | 66                        |
| <b>Total</b>                       |                               |                                    |                        |                 | <b>100</b>                |

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

(3 marks)

c) Determine  $f \circ f(x)$  and its natural domain.

(3 marks)

b) Does  $f \circ g(x)$  exist over the natural domain of  $g(x)$ ? Explain.

(2 marks)

a) Determine the natural domain and range of  $g(x)$ .

Consider the functions  $f(x) = \frac{x-3}{1}$  and  $g(x) = \sqrt{2x+5}$ .

(8 marks)

### Question 1

Working time: 50 minutes.

- This section has **seven (7)** questions. Answer all questions. Write your answers in the spaces provided.
- Continuuing an answer: If you need to use the space for planning, indicate this clearly at the top of the page.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Responses and/or as additional space if required to continue an answer.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

provided.

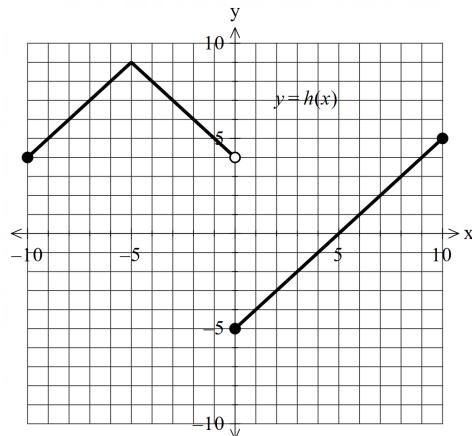
(50 Marks)

Section One: Calculator-free

**Question 2**

(8 marks)

Consider the function  $y = h(x)$  which is graphed below.

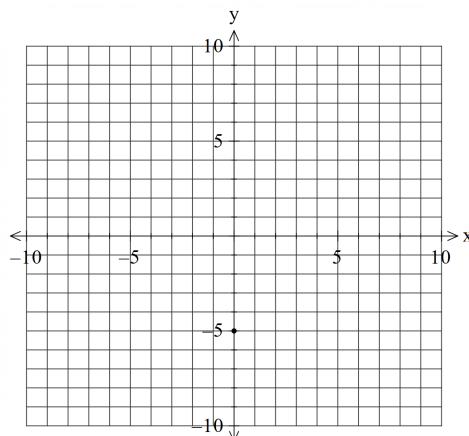


- a) Solve for  $|h(x)| = 5$ .

(2 marks)

- b) Sketch  $y = h(-|x|)$  on the axes below.

(3 marks)

**See next page**

(2 marks)

- a) Show that none of these planes are parallel.

$$3x - 5y + 2z = -36$$

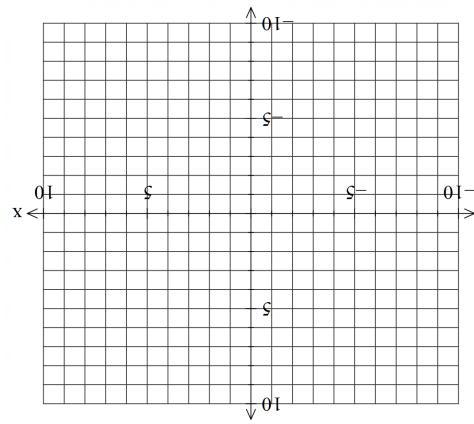
$$2x - 3y + 5z = -26$$

$$x + 2y - 3z = 10$$

Consider the following planes:

(8 marks)

**Question 3**



(3 marks)

- c) Sketch  $y = \frac{h(x)}{l}$  on the axes below.

Q2 continued-

Working out space.

Q3 continued-

b) Solve the system of simultaneous equations.

(3 marks)

End of section one

Working out space.

c) Consider the system of equations below with  $p$  &  $q$  being constants.

(3 marks)

$$x + 2y - 3z = p$$

$$2x - 3y + qz = -26$$

$$3x - 5y + 2z = -36$$

Determine all possible values of  $p$  &  $q$  such that there are:

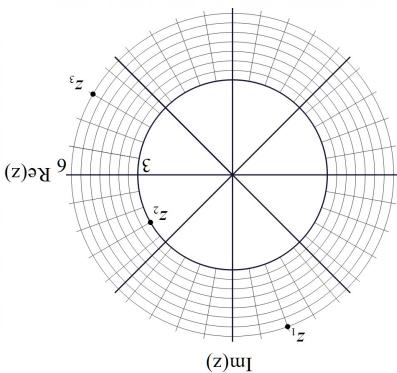
- i) No solutions.
- ii) Infinite solutions.
- iii) A unique solution.

See next page

See next page

Question 4  
(7 marks)

Consider the three complex numbers plotted below in the Argand diagram.



(c) Determine, in polar form,  $z_C$ .

Let  $C$  be the complex number  $z_C$ , and  $B$  be the complex number  $z_B$ .

The rays  $MC$  and  $MB$  form angles of  $\frac{\pi}{3}$  with the positive direction of the real axis.

(d) Explain why  $z_C = z_C \operatorname{cis} \left( -\frac{\pi}{n} \right)$ .

(2 marks)

a) Determine the complex number  $z_2$  in exact cartesian form.  
(2 marks)

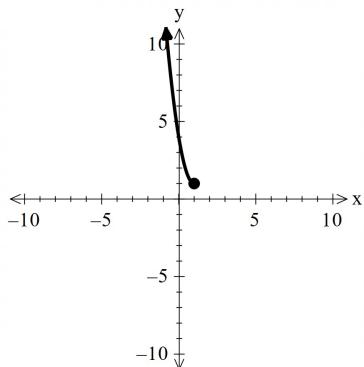
b) Plot the complex number  $z_1 \times z_3$  on the axes above.  
(2 marks)

c) State the modulus and argument of  $\sqrt[3]{z_1 z_3}$ .  
(3 marks)

**Question 5**

(10 marks)

Consider the function  $f(x) = 3x^2 - 6x + 4, x \leq 1$  which is graphed below.



- a) On the axes above, plot  $f^{-1}(x)$ .

(2 marks)

- b) Determine the rule for  $f^{-1}(x)$  and state its domain.

(3 marks)

Consider the function  $g(x) = ax^3$  where  $a$  is a positive constant.

- c) Does  $g(x)$  have an inverse function? Explain.

(2 marks)

- d) Determine the  $x$  values in terms of  $a$  for where  $g(x) = g^{-1}(x)$ .

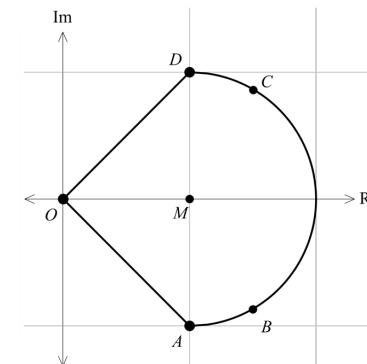
(3 marks)

See next page

**Question 6**

(9 marks)

The Argand diagram below shows a right-angled triangle  $AOD$ , with semicircle  $ABCD$  centred at  $M$ .



- (a) Given  $A$  represents the complex number  $1-i$ , determine the complex number representing  $D$ .

(1 mark)

- (b) State the locus of points that define semicircle  $ABCD$ .

(3 marks)

See next page