PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Methods TEST 3 7 June 2019 TIME: 45 minutes working **Calculator Assumed** 44 Marks 6 Questions

Name: ______________________________ Teacher: ______________________________

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 **(5 marks)**

(a) Differentiate $x\sin x$ (2 marks)

Solution
$\dfrac{d}{dx}(x\sin x) = \sin x + x\cos x$
Specific behaviours
✓ uses product rule ✓ obtains derivative

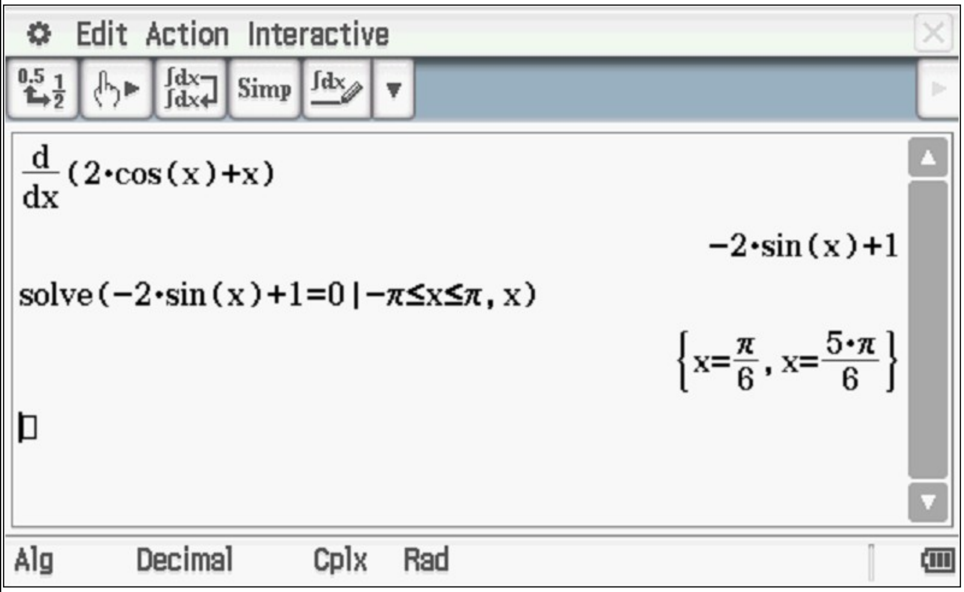
(b) Hence find $\int_0^{\frac{\pi}{2}} x\cos x\,dx$ **using** the result in(a) above. (3 marks)

Solution
$\dfrac{d}{dx}(x\sin x) = \sin x + x\cos x$ $\int \dfrac{d}{dx}(x\sin x)\,dx = -\cos x + \int x\cos x\,dx$ $x\sin x + \cos x + c = \int x\cos x\,dx$ $\int_0^{\frac{\pi}{2}} x\cos x\,dx = [x\sin x + \cos x]_0^{\frac{\pi}{2}} = \left(\dfrac{\pi}{2}\right) - (1)$

Specific behaviours
✓ integrates equation in (a) ✓ uses fundamental theorem ✓ uses limits correctly to obtain exact result

Question 2 **(3 marks)**

Determine the x-coordinates of all points on the graph of $f(x) = 2\cos(x) + x$ for $-\pi \le x \le \pi$ where the tangent line is horizontal. (Justify your answers)

Solution


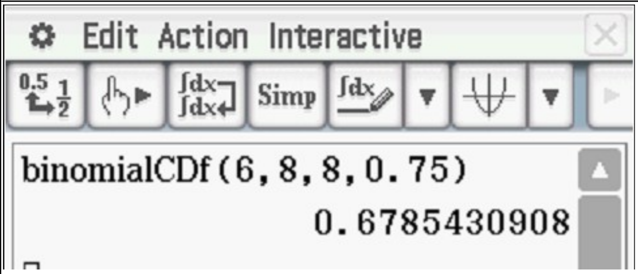
Specific behaviours
✓ differentiates(must be stated) ✓ equates derivative to zero ✓ solves for exact x coordinates within required domain

Question 3 **(7 marks)**

A survey conducted by a local bank shows that 75% of its customers use an ATM at least once a month.

(a) Find the probability that in a random sample of 8 customers, **at least 75%** of them use an ATM machine at least once a month. (2 marks)

Solution



Specific behaviours

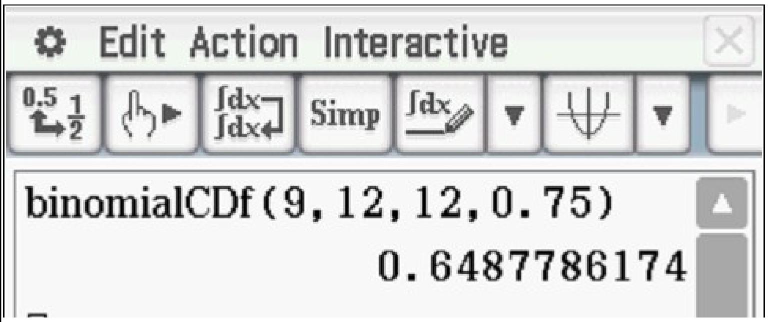
✓ uses binomial parameters and at least 6 successes out of 8

✓ states probability

(b) If the random variable X follows a binomial distribution with n=12 and p=0.75, what is the mean of this distribution and what is P¿X≥mean¿? (3 marks)

Solution

$\mu = 12 \times 0.75 = 9$



Specific behaviours

✓ calculates mean

✓ uses binomial parameters

✓ states probability

(c) If the sample size became very large what would you expect P($X \geq$ mean) to approach? Briefly explain your answer. (2 marks)

Solution
As sample size becomes larger, the distribution becomes more symmetrical about the mean, approaching a probability of 0.5.
Specific behaviours
✓ states approaching 0.5 ✓ describes the ideal shape of distribution as sample size becomes very large

Question 4 **(10 marks)**

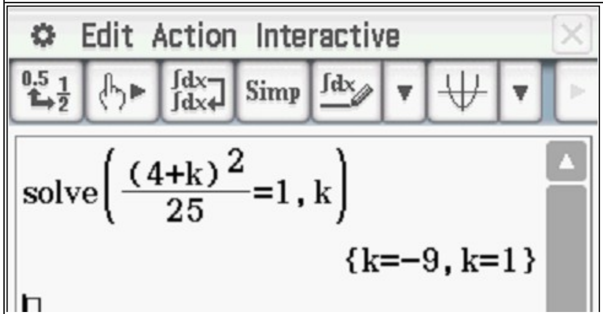
The discrete random variable X can only take the values 2, 3 or 4. For these values the cumulative distribution function is defined by

$$P(X \leq x) = \frac{(x+k)^2}{25}$$

for $x = 2, 3 \wedge 4$, where $k$is a positive constant integer.

(a) Find the value for k. (3 marks)

Solution



K equals 1 as k is positive.

Specific behaviours

✓ uses $P(x \le 4) = 1$

✓ sets up equation for k

✓ solves for k and states only a positive value.

(b) Complete the following table for X. (3 marks)

Solution

X	2	3	4
$P(X \le x)$	$\frac{9}{25}$	$\frac{16}{25}$	1
$P(X = x)$	$\frac{9}{25}$	$\frac{7}{25}$	$\frac{9}{25}$

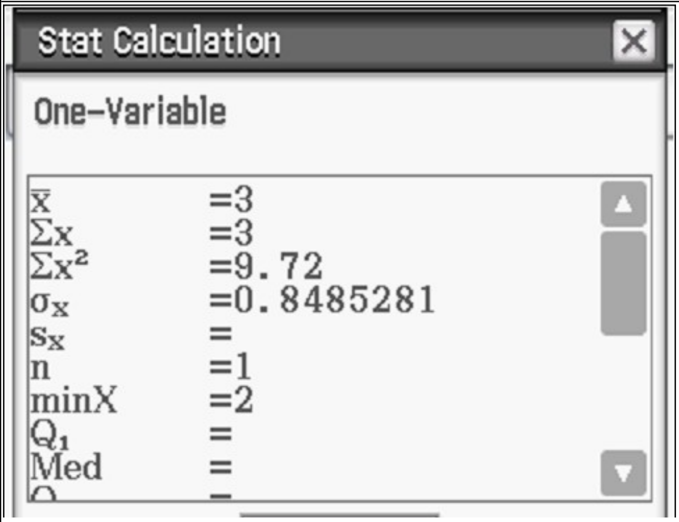
Specific behaviours

✓ $P(x \le 4) = 1$

✓ sum of second row equals one

✓ all entries correct

(c) Hence find $E(X)$ and $SD(X)$. (2 marks)

Solution

Specific behaviours
✓ states mean ✓ states standard deviation

(d) Calculate $Var(3-2X)$ giving your answer to two decimal places. (2 marks)

Solution
$Var(3-2X)=2^2 Var(X)=4\times(0.8485)^2=2.8798\approx 2.88$
Specific behaviours
✓ multiplies old variance by positive 4

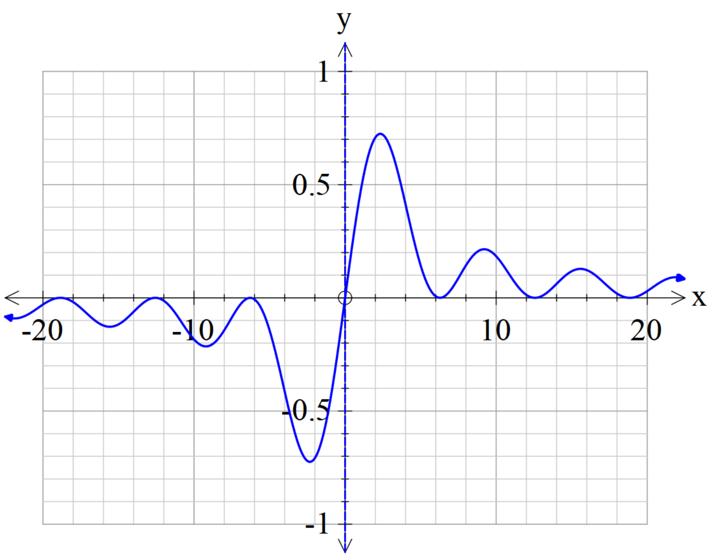
✓ rounds to 2 decimal places (only pay this if working is shown for new variance)

Question 5 **(8 marks)**

Consider the function $f(x) = \dfrac{1 - \cos x}{x}$ where x is in radians.

a) Sketch $f(x)$ on the axes below for $-20 \le x \le 20$ on the axes below. Clearly label undefined points (if any).

(3 marks)

Solution

Specific behaviours
✓ shape

✓ open hole at origin or stated undefined at origin ✓ accuracy with intercepts (within 0.1)

b) As x approaches zero from the positive side, state the value that $f(x)$ approaches.
(1 mark)

Solution
Approaches zero
Specific behaviours
✓ states approaching zero

c) As x approaches zero from the negative side, state the value that $f(x)$ approaches.
(1 mark)

Solution
Approaches zero
Specific behaviours
✓ states approaching zero

d) Use the above to define a value for $f(x)$ as x approaches zero, that is the following limit $\lim_{x \to 0} \frac{1 - \cos x}{x}$.
(1 mark)

Solution
equals zero
Specific behaviours
✓ states equals zero

It can be shown that $\frac{d}{dx}(\cos x) = -\cos x \lim_{h\to 0}\frac{1-\cos h}{h} - \sin x \lim_{h\to 0}\frac{\sin h}{h}$.

e) Using the fact that $\lim_{h\to 0}\frac{\sin h}{h} = 1$ and the above results, show that $\frac{d}{dx}(\cos x) = -\sin x$. (2 marks)

Solution
$\frac{d}{dx}(\cos x) = -\cos x \lim_{h\to 0}\frac{1-\cos h}{h} - \sin x \lim_{h\to 0}\frac{\sin h}{h}$ $= -\cos x(0) - \sin x(1)$ $= -\sin x$
Specific behaviours
✓ uses values of both limits ✓ shows that derivative simplifies to required result

Question 6 **(11 marks)**

A game is played by throwing two standard six-sided dice into the air once. The sum of the uppermost numbers are added together and if the sum is greater than 8 the player wins $5.

Determine:

a) the probability of winning $5 in one game. (2 marks)

Solution
1 **2** **3** **4** **5** **6**

1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	5	7	8	9	10	11
6	7	8	9	10	11	12

$$P(sum > 8) = \frac{10}{36} = \frac{5}{18}$$

Specific behaviours

✓ recognises that there are 36 outcomes

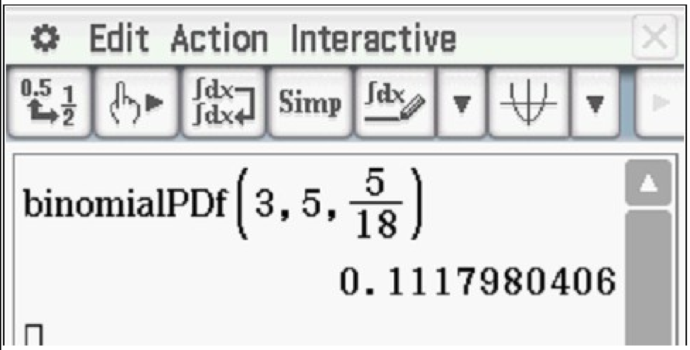
✓ states prob (no need to simplify)

b) the probability of winning exactly $15 in 5 games. (3 marks)

Solution

$$X \sim B\left(5, \frac{5}{18}\right)$$

$$P(x = 3)$$



Specific behaviours

✓ states Binomial

✓ uses parameters

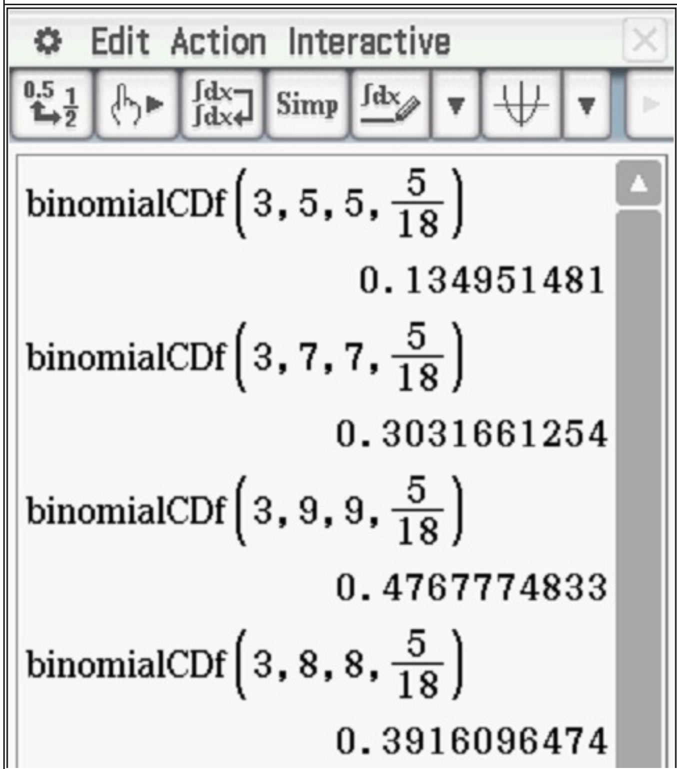
✓ states prob

c) the probability of winning at least $15 in at most 5 games. (3 marks)

(assume that $P(n=3)=\frac{1}{3}=P(n=4)=P(n=5)$

Solution
$P(n=3)\,P(x=3)+P(n=4)\,P(x\geq 3)+P(n=5)\,P(x\geq 3)$
$\frac{1}{3}0.02143347051+\frac{1}{3}0.06787265661+\frac{1}{3}0.134951481$
0.07475253604
Specific behaviours
✓ examines 3 games with correct parmeters binomialCDf
✓ examines 4 and 5 games and cumulative values
✓ states final prob

d) the minimum number of games to be played so that the probability of winning at least \$15 is greater than 0.47. (Justify) (3 marks)

Solution
 Min number of games is 9
Specific behaviours
✓ uses cumulative Binomial with correct parameters ✓ shows at least 3 sets of trials ✓ demonstrates that 9 games is the minimum