PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2023

Question/Answer booklet

12 SPECIALIST MATHEMATICS

Section One:
Calculator-free

Your Name

Your Teacher's Name

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Max	Question	Mark	Max
1			5		
2			6		
3					
4					

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	50	34
Section Two: Calculator-assumed	12	12	100	97	66
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free **(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 **(8 marks)**

Consider the functions $f(x)=\dfrac{1}{x-3}$ and $g(x)=\sqrt{2x+5}$.

a) Determine the natural domain and range of $g(x)$. (2 marks)

c
$d_g : x \geq \dfrac{-5}{2}$ $r_g : y \geq 0$
Specific behaviours
P states domain P states range

b) Does $f \circ g(x)$ exist over the natural domain of $g(x)$? Explain. (3 marks)

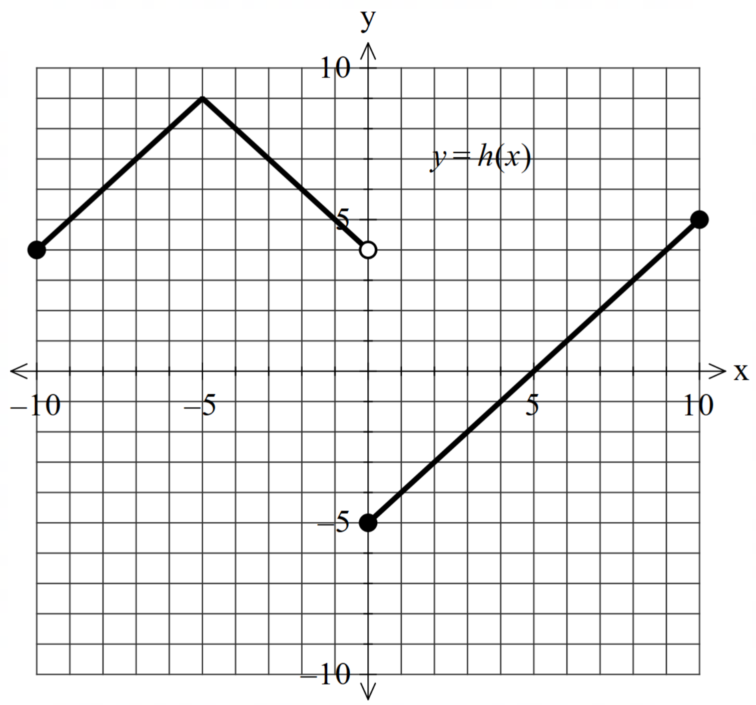
c
$f \circ g(x)$ $r_g \subseteq d_f$? $y \geq 0 \not\subset x \neq 3$ Therefore does not exist over natural domain.
Specific behaviours
P states condition needed to exist P states required domain and range P concludes that does not exist.

c) Determine $f \circ f(x)$ and its natural domain. (3 marks)

c
$f \circ f(x) = \dfrac{1}{\dfrac{1}{x-3} - 3} = \dfrac{x-3}{10-3x}$ $d: x \neq 3, x \neq \dfrac{10}{3}$
Specific behaviours
P obtains an expression for function   P simplifies as shown above   P states domain

Question 2 (8 marks)

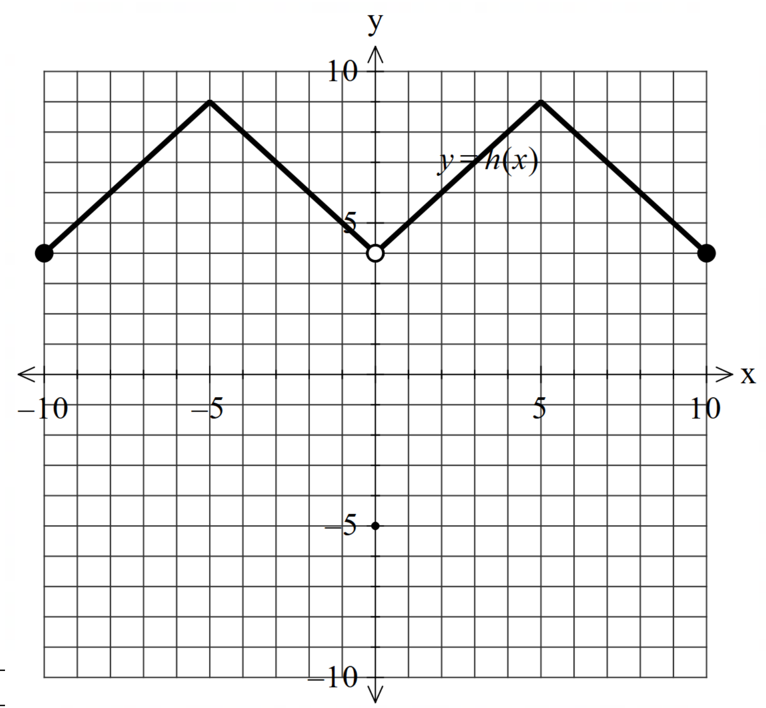
Consider then function $h(x)$ which is graphed below.



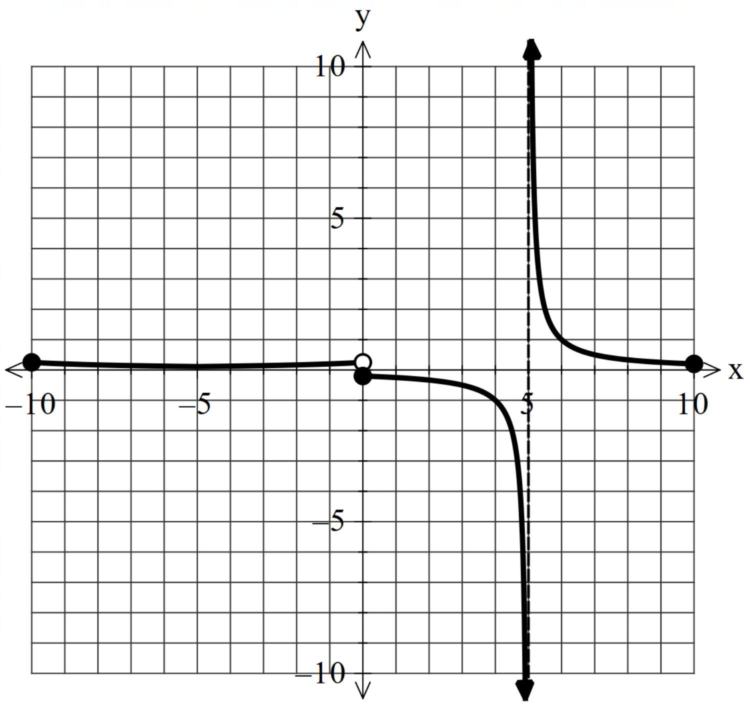
a) Solve for $|h(x)| = 5$. (2 marks)

c
$x = -9, -1, 0, 10$
Specific behaviours
P states two correct values of x   P states all values only

b) Sketch $y = h(-|x|)$ on the axes below. (3 marks)

c

P uses left side to give new right side P open hole on y axis at y=4 P shape

c) Sketch $y = \dfrac{1}{h(x)}$ on the axes below. (3 marks)

c

Specific behaviours
P asymptote at x=5 P correct limits for y intercept , left open and right closed P correct shape

Question 3 **(8 marks)**

Consider the following planes:

$$x + 2y - 3z = 10$$
$$2x - 3y + 5z = -26$$
$$3x - 5y + 2z = -36$$

a) Show that none of these planes are parallel. (2 marks)

c
Three normal vectors $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$ are not parallel to each other.

Specific behaviours
P states all 3 normal vectors P states that the vectors are not parallel

b) Solve the system of simultaneous equations. (3 marks)

c
$\begin{bmatrix} 1 & 2 & -3 & 10 \\ 2 & -3 & 5 & -26 \\ 3 & -5 & 2 & -36 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & -3 & 10 \\ 0 & 7 & -11 & 46 \\ 0 & 11 & -11 & 66 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & -3 & 10 \\ 0 & 7 & -11 & 46 \\ 0 & -4 & 0 & -20 \end{bmatrix}$ $-4y = -20, y = 5$ $35 - 11z = 46, z = -1$ $x + 10 + 3 = 10, x = -3$ $soln \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$
Specific behaviours
P eliminates one variable from an equation P eliminates two variables from the one equation P states all 3 variables

c) Consider the system of equations below with $p \& q$ being constants. (3 marks)

$$x + 2y - 3z = p$$
$$2x - 3y + qz = -26$$
$$3x - 5y + 2z = -36$$

Determine all possible values of $p \& q$ such that there are:

i) No solutions.
ii) Infinite solutions.
iii) A unique solution.

c
$\begin{bmatrix} 1 & 2 & -3 & p \\ 2 & -3 & q & -26 \\ 3 & -5 & 2 & -36 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & -3 & 10 \\ 0 & 7 & -6-q & 2p+26 \\ 0 & 11 & -11 & 3p+36 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & -3 & 10 \\ 0 & 7 & -16-q & 2p+26 \\ 0 & 0 & 11-11q & p+34 \end{bmatrix}$ $no\ so\ln s, q = 1 \& p \neq -34$ $\inf inite\ q = 1 \& p = 34$ $unique\ q \neq 1$
Specific behaviours
P obtains a line with two zeros   P states values for uniqueness   P state values for infinite and no solns

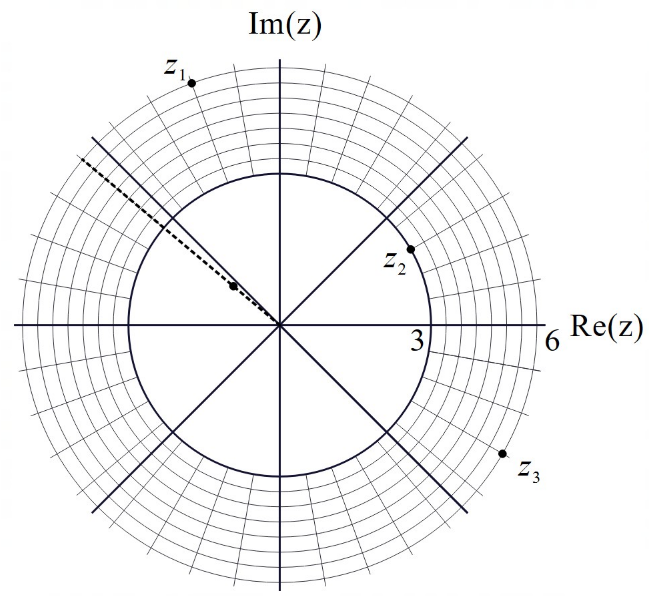
Question 4 (7 marks)

Consider the three complex numbers plotted below in the Argand diagram.

a) Determine the complex number z_2 in exact cartesian form. (2 marks)

$3cis\frac{3\pi}{18} = 3(\frac{\sqrt{3}}{2} + i\frac{1}{2})$
Specific behaviours
P converts to polar   P converts to exact cartesian

b) Plot the complex number $z_1 \times (z_3)^{-1}$ on the axes above. (2 marks)

c

Specific behaviours
P correct argument P modulus of near 1 unit

c) State the modulus and argument of $\dfrac{(1+i)z_2}{\sqrt{3}\,\overline{z}_1 z_3}$. (3 marks)

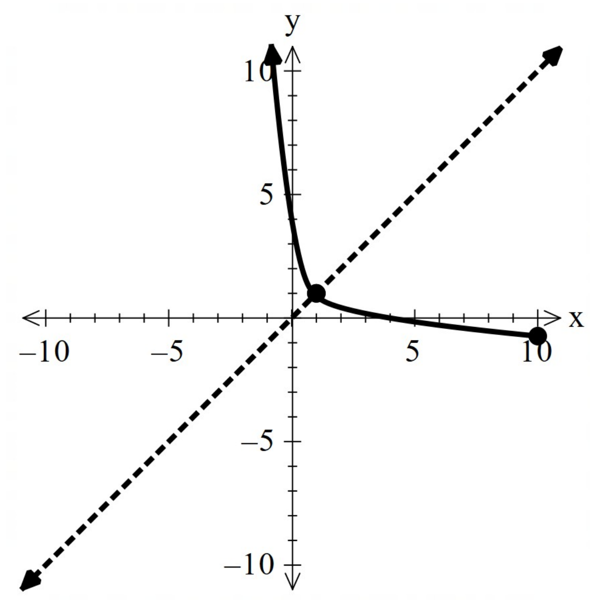
$\dfrac{(1+i)z_2}{\sqrt{3}\,\overline{z}_1 z_3} = \dfrac{\sqrt{2}cis\dfrac{\pi}{4}3cis\dfrac{\pi}{6}}{\sqrt{3}(6)cis\dfrac{-11\pi}{18}6cis\left(\dfrac{-\pi}{6}\right)}$ $\dfrac{3\sqrt{2}}{36\sqrt{3}}cis\left(\dfrac{\pi}{4}+\dfrac{\pi}{6}+\dfrac{11\pi}{18}+\dfrac{\pi}{6}\right)$
Specific behaviours

P converts to polar form for each value P states modulus, un-simplified P states argument un-simplified

Question 5 (10 marks)

Consider the function $f(x) = 3x^2 - 6x + 4, \quad x \le 1$ which is graphed below.

a) On the axes above, plot $f^{-1}(x)$. (2 marks)

c

Specific behaviours
P point (1,1) shown on inverse P appears to be reflected in line y=x

b) Determine the rule for $f^{-1}(x)$ and state its domain. (3 marks)

c

$f(x) = 3x^2 - 6x + 4$ $x = 3y^2 - 6y + 4$ $3y^2 - 6y + 4 - x = 0$ $y = \frac{6 \pm \sqrt{36 - 4(3)(4 - x)}}{6}$ $y = \frac{6 \pm \sqrt{12x - 12}}{6} = \frac{6 \pm 2\sqrt{3(x-1)}}{6}$ $as\ y \le 1$ $f^{-1}(x) = \frac{3 - \sqrt{3(x-1)}}{3}$ $d : x \ge 1$
Specific behaviours
P swaps x and y and solves for y P discards positive value and chooses negative P states domain and rule

Consider the function $g(x) = ax^3$ where a is a positive constant.

c) Does $g(x)$ have an inverse function? Explain. (2 marks)

c
As function is a one to one it does have an inverse
Specific behaviours
P states yes with a reason P states that function is one to one

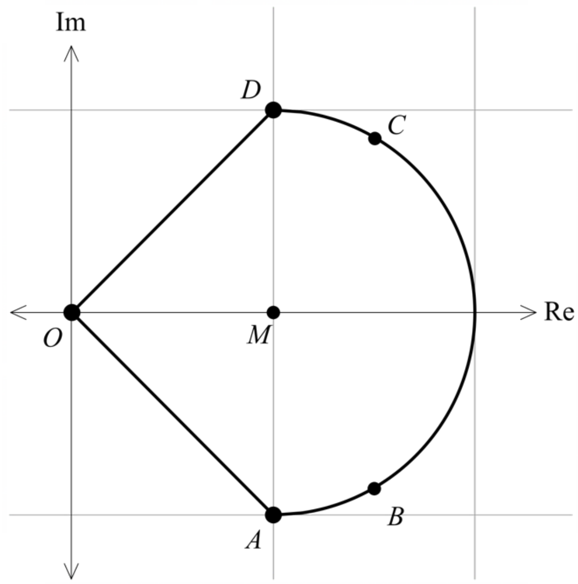
d) Determine the x values in terms of a for where $g(x) = g^{-1}(x)$. (3 marks)

c
$ax^3 = x$ $ax^3 - x = 0$ $x(ax^2 - 1) = 0$ $x = 0, \pm\frac{1}{\sqrt{a}}$
Specific behaviours

P equates to x P states one value in terms of a P states two values in terms of a and zero

Question 6 **(9 marks)**

The Argand diagram below shows a right-angled triangle AOD, with semicircle $ABCD$ centred at M.



(a) Given A represents the complex number $1-i$, determine the complex number representing D. (1 mark)

c
$1+i$
Specific behaviours
PStates D in rectangular form.

(b) State the locus of points that define semicircle $ABCD$. (3 marks)

$\lvert z-1 \rvert = 1 \cap -\frac{\pi}{2} \le arg(z-1) \le \frac{\pi}{2}$
Specific behaviours
P Translates by loci right 1 unit. ✓ States locus representing circle. P States locus restricting the argument.

The rays MC and MB form angles of $\frac{\pi}{3}$ with the positive direction of the real axis.

Let C be the complex number z_C, and B be the complex number z_B.

(c) Determine, in polar form, z_C. (3 marks)

$\angle OMC=\frac{2\pi}{3} \Rightarrow arg\, z_C=\frac{\pi}{6}$ $\lvert z_C\rvert=\sqrt{1^2+1^2-2\cos\frac{2\pi}{3}}=\sqrt{3}$ $Hence\ z_C=\sqrt{3}\,cis\frac{\pi}{6}$
Specific behaviours
✓ Determines $\angle OMC$.
✓ Uses cosine rule to determine the modulus.
P Determines argument of z_C and states z_C in polar form.

(d) Explain why $\overline{z_C}=z_C\,cis\left(\frac{-\pi}{3}\right)$. (2 marks)

Solution	Specific behaviours
$\overline{z_C}\ is\ the\ same\ as\ z_B \wedge ¿$ $z_B\ is\ z_C\ rotated\ \frac{\pi}{3}\ clockwise$ $Hence\ \overline{z_C}=z_C\,cis\left(\frac{-\pi}{3}\right)$	✓ States that $\overline{z_C}=z_B$. ✓ States that z_B is z_C rotated $\frac{\pi}{3}$ clockwise.

End of section one

Working out space.

Working out space.