Christ Church Grammar School

2021
TEST 1

MATHEMATICS METHODS Year 11

Section One:
Calculator-free

Your name Solutions

Teacher's name _____

Time and marks available for this section

Reading time for this section:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments,

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

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2. Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable or gel pens.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than one mark, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(5 marks)**

(a) Convert 80° to radians. (1 mark)

$\frac{80}{4} \times \frac{\pi}{180_{9}}$

$= \frac{4\pi}{9}$

✓ Converts to radians

(b) Convert $\frac{7\pi}{18}$ radians to degrees. (1 mark)

$\frac{7\pi}{18} \times \frac{180^{10}}{\pi}$

$= 70°$

✓ Converts to degrees

(c) Find the exact value of the following.

(i) $\tan(60)$ (1 mark)

$= \sqrt{3}$ allow $\frac{1}{\sqrt{3}}$

✓ Correct exact value

(ii) $\cos(120)$ (1 mark)

$= -\frac{1}{2}$

✓ As above

(iii) $\sin(-\frac{\pi}{6})$ (1 mark)

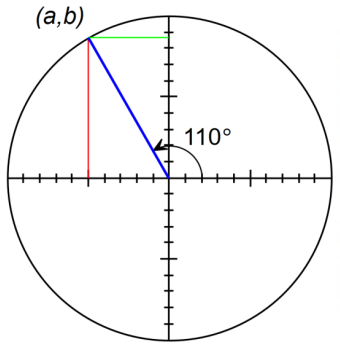
$= -\frac{1}{2}$

✓ As above

See next page

Question 2 **(2 marks)**

Information has been added to the unit circle below.



Determine the following trigonometric ratios in terms of a and b.

(a) $\sin(70)°$ (1 mark)

b

✓ Correct variable

(b) $\tan(250)°$ (1 mark)

$\frac{-b}{a}$

✓ Correct expression

See next page

Additional working space

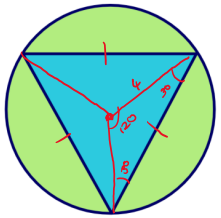
Question number: _____

Additional working space

Question number: _____

Question 3 **(3 marks)**

A design for a pendant is an equilateral triangle inside a circle of radius 4 cm. Show that the area of the pendant, excluding the triangle, is $16\pi - 12\sqrt{3}$ cm^2.



$\theta = \frac{2\pi}{3}$

$A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}(4)^2\left(\frac{2\pi}{3} - \sin\left(\frac{2\pi}{3}\right)\right)$

$= 8\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$3 \times A = 3 \times \left(8\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)\right)$

$= 24\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= 16\pi - 12\sqrt{3}$

$\therefore$ total area is $(16\pi - 12\sqrt{3})$ cm^2

$\theta = 120°$

$A = \frac{\theta}{360}\pi r^2 - \frac{1}{2}r^2\sin\theta$

$= \frac{120}{360}\pi(4)^2 - \frac{1}{2}(4)^2\sin(120)$

$= \frac{1}{3}\pi(16) - 8\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{16\pi}{3} - 4\sqrt{3}$

$3A = 3\left(\frac{16\pi}{3} - 4\sqrt{3}\right)$

$= (16\pi - 12\sqrt{3})$ cm^2

✓ Identifies θ

✓ Calculates one segment

✓ Multiplies by 3.

See next page

Question 4 **(5 marks)**

A windshield wiper blade on a car window has a length of 36 cm and rotates through an angle of $\frac{17\pi}{18}$. See the diagram below.

(a) Determine the exact distance the far end of the wiper blade would travel. (2 marks)

$L = r\theta$
$= 36\left(\frac{17\pi}{18}\right)$
$= 34\pi$ cm

$L = \frac{\theta}{360} 2\pi r$
$= \frac{170}{360} \times 2\pi(36)$
$= 34\pi$ cm

$\frac{17\pi}{18} \times \frac{180}{\pi} = 170°$

✓ Uses formula
✓ Exact solution

The closest part of the wiper blade does not clean the window as shown in the diagram above.

(b) If the distance from start of the wiper blade to the smaller arc is 6 cm, calculate the exact area of windshield that is cleaned. (3 marks)

Area $= \frac{1}{2}(36)^2\left(\frac{17\pi}{18}\right)$
$= 612\pi$ cm^2
Area $= \frac{1}{2}(6)^2\left(\frac{17\pi}{18}\right)$
$= 17\pi$
$612\pi - 17\pi = 595\pi$ cm^2

✓ total area

Area $= \frac{170}{360}\pi(36)^2$
$= 612\pi$ cm^2

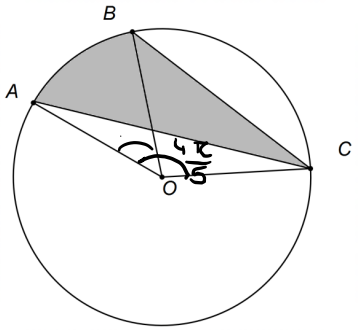
Area $= \frac{170}{360}\pi(6)^2$
$= 17\pi$ cm^2

✓ Calculates area when $r = 36$
✓ Area when $r = 6$

End of questions

Question 9 **(4 marks)**

Determine the area of the shaded region ABC in the circle below with the centre O and radius 24 cm, given that $\angle AOB = \frac{\pi}{5}$ and $\angle BOC = \frac{3\pi}{5}$. Give your answer to the nearest square centimetre.



$A_{AC} = \frac{1}{2}(24)^2\left(\frac{4\pi}{5} - \sin\left(\frac{4\pi}{5}\right)\right)$
$= 554.54$

$A_{BC} = \frac{1}{2}(24)^2\left(\frac{3\pi}{5} - \sin\left(\frac{3\pi}{5}\right)\right)$
$= 268.96$

Shaded area $= 285.58$
≈ 286 cm^2

✓ Calculates area of segment AC
✓ Calculates area of segment BC
✓ Calculates difference
✓ Nearest cm^2

End of questions

Question 8 **(7 marks)**

An arc of length 15 cm on a circle of radius 8 cm subtends an angle of θ at the centre.

(a) Determine the exact size of θ in radians. (2 marks)

$15 = 8\theta$

$\theta = \frac{15}{8}$ rad

✓ Substitutes into $L = r\theta$

✓ Solves for θ

(b) Calculate the area of the sector which has θ as the included angle. (2 marks)

$A = \frac{1}{2}(8)^2\left(\frac{15}{8}\right)$

$= 60 \text{ cm}^2$

✓ Substitutes into sector formula

✓ Simplifies

(c) Show that the area of a sector with angle θ, with radius r and arc length a can be found by calculating $\frac{ra}{2}$. (3 marks)

$a = r\theta$

$\theta = \frac{a}{r}$

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2\left(\frac{a}{r}\right)$

$= \frac{1}{2}ra$

$= \frac{ra}{2}$

✓ rearranges arc length to make θ subject.

✓ Subs θ into sector formula

✓ simplifies

See next page

Additional working space

Question number: _____

Additional working space

Question number: _____

Question 6 continued

(c) Find the bearing of tower C from tower A. (3 marks)

$\theta = \cos^{-1}\left(\frac{8^2-5^2-5.96^2}{-2(5)(5.96)}\right)$

$= 93.35°$

$112 + 93.35$

bearing $= 205.35°$

allow $205°$

$\frac{\sin\theta}{8} = \frac{\sin(48)}{5.96}$ ✓applies sine or cosine rule

$\theta = \sin^{-1}\left(\frac{8\times\sin(48)}{5.96}\right)$ ✓correct angle

$= 85.96°$

$\theta = 180 - 85.96$

$= 94.04°$ ✓Calculates bearing.

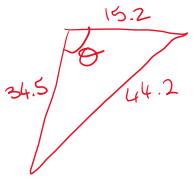
Must be $94.04°$

$112 + 94.04 = 206.04°$

Answer may be different due to rounding allow $205°$ to $206.04°$ but must show 5.96 or 6 being used in working.

Question 7 **(3 marks)**

Determine the size of the largest angle in a triangle with sides 15.2 cm, 34.5 cm, and 44.2 cm. Give your answer in radians to 1 decimal place.



✓Substitutes correctly into cosine rule

$\theta = \cos^{-1}\left(\frac{44.2^2-34.5^2-15.2^2}{-2(34.5)(15.2)}\right)$ ✓identifies correct angle

$= 120.5°$

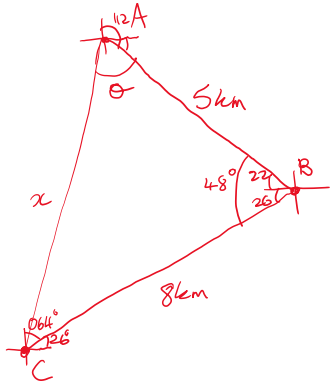
≈ 2.1 rad ✓Calculates angle.

See next page

Question 6 **(8 marks)**

Heather, a ranger for the Shire of Harvey, is in an observation tower at A. She recorded a bush fire at B, 5 km away on a bearing of 112°. John, a second ranger is in a tower at C and observed the fire 8 km away on a bearing of 064°.

(a) Draw a labelled diagram of the situation. (2 marks)



✓ lengths included

✓ bearings included.

(b) Find the direct distance of tower C from tower A. (3 marks)

$x^2 = 8^2 + 5^2 - 2(8)(5)\cos(48)$

$= \sqrt{35.47}$

$= 5.96$ km

allow 6 km & 5.956 km

✓ Applies cosine rule

✓ correct substitution

✓ Correct length

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2021
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MATHEMATICS METHODS Year 11

Section Two:
Calculator-assumed

Your name Solutions

Teacher's name _____

Time and marks available for this section

Reading time for this section:	3 minutes
Working time for this section:	30 minutes
Marks available:	26 marks

Materials required/recommended for this section

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Formula sheet (retained from Section One)

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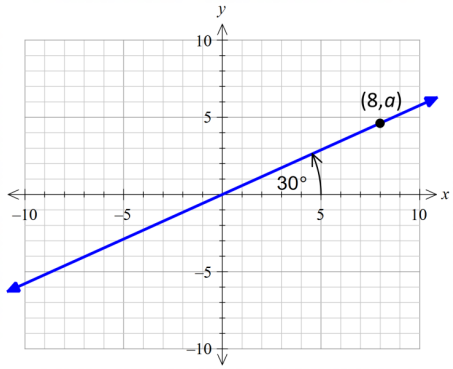
Question 5 **(4 marks)**

The line shown below passes through the point $(0,0)$. Use the graph below to answer the questions that follow.



(a) Determine the gradient of the line as an exact value. (2 marks)

$\tan 30 = \frac{\sqrt{3}}{3}$

also accept $\frac{1}{\sqrt{3}}$

✓ applies tan to angle of inclination

✓ exact value

(b) The point $(8, a)$ lies on the line. Determine the value of a as an exact value. (2 marks)

$a = \frac{\sqrt{3}}{3}(8)$

$a = \frac{8\sqrt{3}}{3}$

also allow $a = \frac{8}{\sqrt{3}}$

✓ Substitutes into formula

✓ Correct value of a