Working out space

PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 3 Year 12

Student name: ________________ Teacher name: ________________

Task type: **Response**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **_____6______**

Materials required:

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Upto 3 calculators/classpads allowed

Special items: Drawing instruments, templates
No notes allowed

Marks available: **___39___ marks**

Task weighting: **_14___%**

Formula sheet provided: no but formulae given on page 2

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)}\,dx = \ln\lvert f(x)\rvert + c$
$\frac{d}{dx}\sin f(x) = f'(x)\cos f(x)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\cos f(x) = -f'(x)\sin f(x)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\tan f(x) = f'(x)\sec^2 f(x) = \frac{f'(x)}{\cos^2 f(x)}$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

Volumes of solids of revolution	
About the x-axis	$V = \pi\int_a^b [f(x)]^2 dx$
About the y-axis	$V = \pi\int_c^d [f(y)]^2 dy$

Prism	$V = Ah$, where A is the area of the cross section	
Pyramid	$V = \frac{1}{3}Ah$, where A is the area of the base	
Cylinder	$V = \pi r^2 h$	$TSA = 2\pi rh + 2\pi r^2$
Cone	$V = \frac{1}{3}\pi r^2 h$	$TSA = \pi rs + \pi r^2$, where s is the slant height
Sphere	$V = \frac{4}{3}\pi r^3$	$TSA = 4\pi r^2$

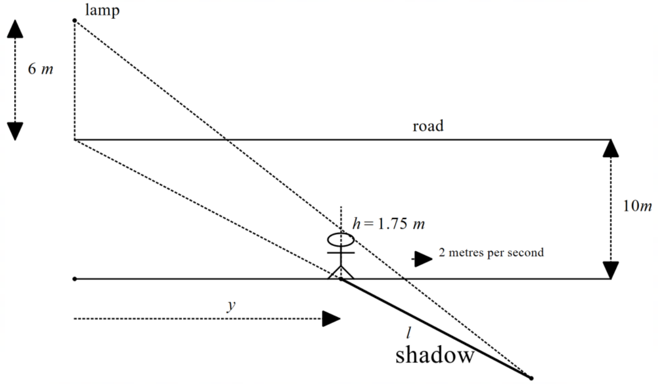
Identities	
$\cos^2 x + \sin^2 x = 1$	$1 + \tan^2 x = \sec^2 x$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	$\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$
$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$	$\sin 2x = 2\sin x\cos x$

Q5 cont-

c) Determine the time taken for the maximum growth rate.

Q6 (5 marks)

Consider a woman of height 1.75 m, travelling at 2 m/s along the edge of a road of width 10 m (See direction below). A lamp of height 6 m on the other side of the road, casts a shadow of the woman of length, l, as shown below. Determine the **exact** time rate of change of the length of the shadow when $y = 20$ m.



Q5 (5, 2 & 2 = 9 marks)
At time $t = 0$ years, 26 kangaroos are placed in an isolated habitat such that the number of kangaroos, N can be modelled by the differential equation $\frac{dN}{dt} = \frac{N}{300}(100 - N)$.

a) Using separation of variables and partial fractions determine $N(t)$ **without** the use of a classpad.

b) Determine the limiting value of the population of kangaroos.

Continued on next page.

No notes allowed

Q1 (2, 3 & 3 = 8 marks)
An object starts from rest at the origin and moves with a velocity $v = \begin{pmatrix} -5\sin 2t \\ 3\sin t \end{pmatrix}$ m/s at time t seconds.
Determine the following.

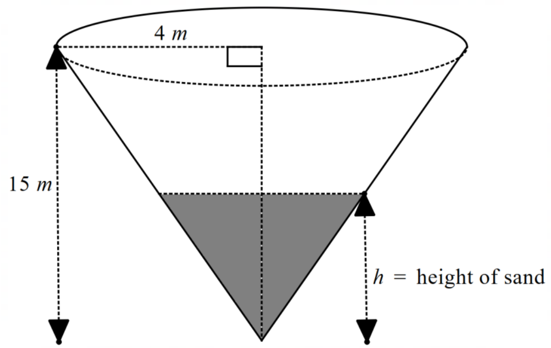
a) Acceleration at time t.

b) The cartesian equation of the path of the object. (Do not simplify)

c) Determine to the nearest second the first time for $t > 0$ that the acceleration and velocity are perpendicular.

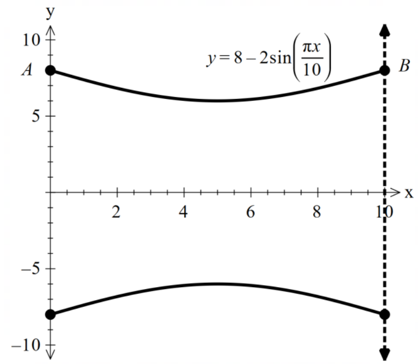
Q2 (5 marks)
If $\frac{dy}{dx} = xy^2$ find an expression for $\frac{d^2y}{dx^2}$ in terms of x & y.

Q3 (6 marks)
Sand is poured into a gigantic metal cone of height 15 m and a radius of 4 m at a rate of 120 cubic metres per minute, as shown below.



Determine the time rate of change, metres per minute, of the height, h metres, of the sand when the height is 5 m.

Q4 (6 marks)
A water pipe of length 10 metres can be modelled by a cross-section AB where $y = 8 - 2\sin\left(\frac{\pi x}{10}\right)$, $0 \le x \le 10$ and this curve is revolved about the x axis.



Determine the volume of water that this length of pipe will hold. Show all working **without** the use of a classpad. (Simplify)