

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material if you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Important note to candidates

Special items: n/a

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the supervisor
Materials required/recommended for this paper
Formula Sheet
This Question/Answer Booklet
Number of additional answer books used (if applicable):

Time allowed for this paper
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Please circle your teacher's name
Teacher: Miss Long Miss Rowden Ms Stone

Student Name: _____

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MATHEMATICS
METHODS
ATAR Year 12
Section One:
Calculator-free

Question/Answer Booklet
Semester Two Examination, 2020



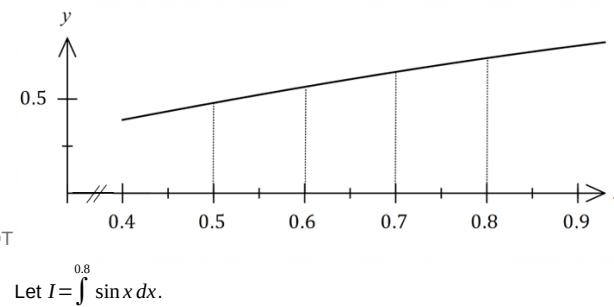
Structure of this paper

Section	Number of questions available	Number of questions to be answered	Suggested working time (minutes)	Marks available	Percentage of examination
Section One: Calculator free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

- The rules for the conduct of the ATAR course examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet.
- You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
- Supplementary pages for the use planning/continuing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

DO NOT



x	y
0.4	0.39
0.5	0.48
0.6	0.56
0.7	0.64
0.8	0.72
0.9	0.78

- (a) By using the information shown and considering sums of the form $\sum_i f(x_i) \delta x_i$ explain why $I < 0.192$.

Solution	(3 marks)
With $\delta x = 0.1$, $x_1 = 0.6$, $x_2 = 0.7$ and $x_3 = 0.8$ then $\sum_i f(x_i) \delta x_i = 0.1(0.56 + 0.64 + 0.72) \triangleq 0.1(1.92) \triangleq 0.192$ Hence I must be less than this value as it is the area of circumscribed rectangles that overestimate the area under the curve.	
Specific behaviours	
✓ indicates x -ordinates for circumscribed rectangles ✗ shows sum of $f(x_i) \delta x_i$ ✗ explains inequality	

- (b) In a similar manner to (a), determine the lower estimate, L , for the value of I . That is, the value of L for $I > L$.

Solution	(2 marks)
With $\delta x = 0.1$, $x_1 = 0.5$, $x_2 = 0.6$ and $x_3 = 0.7$ then $L = \sum_i f(x_i) \delta x_i = 0.1(0.48 + 0.56 + 0.64) \triangleq 0.1(1.68) \triangleq 0.168$	
Specific behaviours	
✓ indicates x -ordinates for inscribed rectangles ✗ value of L	

- (c) Use your previous answers to determine a numerical estimate for I and explain whether your estimate is smaller or larger than the exact value of I .

Solution
$I = \frac{0.168 + 0.192}{2} = 0.18$
This value slightly underestimates the exact value of I as the curve is concave downwards.
Specific behaviours
✓ correctly averages values ✗ states underestimate with reason

End of questions

Section One: Calculator-free
35% (52 Marks)

This section has eight (8) questions. Answer all questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

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(7 marks)

Question 11

(a) Determine an expression for $f'(x)$ when

(2 marks)

$$(i) \quad f(x) = \ln(1 - \cos 3x).$$

Solution

$f(x) = 3 \sin 3x$

$\frac{d}{dx} f(x) = 3 \sin 3x \cdot 3 \cdot \cos 3x$

$\frac{d}{dx} f(x) = 9 \sin 3x \cos 3x$

$\frac{d}{dx} f(x) = \frac{9}{2} \sin 6x$

(3 marks)

$$(ii) \quad f(x) = e^{\sin x}(5 - 2x)^3.$$

Solution

$N.B. \text{ Simplifies to } (19 - 10x)(5 - 2x)^2 e^{\sin x}$

$f(x) = 5e^{\sin x}(5 - 2x)^3 + e^{\sin x} \cdot 3 \cdot (-2)(5 - 2x)^2$

$f(x) = 5e^{\sin x}(5 - 2x)^3 - 6e^{\sin x}(5 - 2x)^2$

Specific behaviours

$\frac{d}{dx} e^{\sin x} = e^{\sin x} \cos x$

$\frac{d}{dx} (5 - 2x)^3 = 3(5 - 2x)^2 \cdot (-2)$

$\frac{d}{dx} (5 - 2x)^2 = 2(5 - 2x) \cdot (-2)$

(2 marks)

Determine the value(s) of x for which $\frac{dA}{dx} = 0$.

$$\frac{dA}{dx} = \frac{d}{dx} \int_x^0 (8 - 2t^2) dt = 8 - 2x^2$$

Solution

$\therefore 2x^2 = 8 \Rightarrow x^2 = 4 \Rightarrow x = \pm\sqrt{2}$

Specific behaviours

$\checkmark \text{ expression for } A(x)$

$\checkmark \text{ correct value(s)}$

$\checkmark \text{ next page}$

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(7 marks)

The graph and a table of values for $y = f(x)$ is shown below, where $f(x) = \sin x$.

Question 8

End of questions

Solution

$v(t) = -18(3t+1)^{\frac{2}{3}} + 70$

$v(5) = -18(3(5)+1)^{\frac{2}{3}} + 70 = -72 + 70 = -2 \text{ cm/s}$

Specific behaviours

$\checkmark \text{ antiderivative of } v(t)$

$\checkmark \text{ equation for } t=1$

$\checkmark \text{ value for } t=5$

$\checkmark \text{ solves simultaneously to determine value of } c$

$\checkmark \text{ correct velocity}$

$c = 490c = 70$

$22 = -4(3 \times 8 + 1)^{\frac{2}{3}} + 8c + k$

$0 = -4(3 \times 1 + 1)^{\frac{2}{3}} + 1c + k$

$x(t) = -18(3t+1)^{\frac{2}{3}} + ct + k$

$v(t) = -27(3t+1)^{\frac{2}{3}} dt = \frac{1}{3} \cdot 27(3t+1)^{\frac{1}{3}} + C$

(7 marks)

Question 2

The discrete random variable X is defined by

$$P(X=x) = \begin{cases} \frac{x+b}{5x+2} & x=0,1 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Determine the value of the constant b .

Solution

$$\frac{b}{2} + \frac{1+b}{7} = 17b + 2 + 2b = 14b = \frac{4}{3}$$

Specific behaviours

- ✓ forms equation using $x=0$ and $x=1$
- correct value

(2 marks)

- (b) Determine

- (i) $P(X=0)$.

Solution

$$P(X=0) = \frac{4}{3} \div 2 = \frac{2}{3}$$

Specific behaviours

- ✓ correct probability

(1 mark)

- (ii) $E(7X-3)$.

Solution

$$E(X) = p = P(X=1) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$E(7X-3) = 7 \times \frac{1}{3} - 3 = \frac{-2}{3}$$

Specific behaviours

- ✓ indicates $E(X)$
- correct value

(2 marks)

- (iii) $\text{Var}(7X-3)$.

Solution

$$\text{Var}(X) = p(1-p) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

$$\text{Var}(7X-3) = 7^2 \times \frac{2}{9} = \frac{98}{9}$$

Specific behaviours

- ✓ indicates $\text{Var}(X)$
- correct value

(2 marks)

See next page

(6 marks)

Question 7

The acceleration at time t seconds of a small body travelling in a straight line is given by

$$a(t) = \frac{-27}{\sqrt{3t+1}} \text{ cm/s}^2, t \geq 0.$$

When $t=1$ the body was at the origin and 7 seconds later its displacement was 22 cm.

Determine the velocity of the body when $t=5$.

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Solution

$$v(t) = \int -27(3t+1)^{-\frac{1}{2}} dt = \frac{1}{2} \times 3(-18)(3t+1)^{\frac{1}{2}} + c$$

$$= -18(3t+1)^{\frac{1}{2}} + c$$

$$\Delta x = \int_1^{1+7} -18(3t+1)^{\frac{1}{2}} + c dt = \left[\frac{-18}{2} (3t+1)^{\frac{3}{2}} + ct \right]_1^8$$

$$= \left[-4(3t+1)^{\frac{3}{2}} + ct \right]_1^8 = [-4(125)+8c] - [-4(27)+c]$$

$$= 7c - 468$$

But $\Delta x = 22$

$$7c - 468 = 22 \Rightarrow 7c = 490 \Rightarrow c = 70$$

$$v(5) = -18(3(5)+1)^{\frac{1}{2}} + 70 = -72 + 70 = -2 \text{ cm/s}$$

Specific behaviours

- ✓ antiderivative of $a(t)$
- integral for change in displacement Δx
- antiderivative of $v(t)$
- simplifies equation for c
- uses given Δx to determine value of c
- correct velocity

See next page

(1 mark)

Solution
$t \rightarrow \infty, P \rightarrow 10 \text{ psi}$
Specific behaviours
✓ correct pressure

(iii) the minimum pressure in the tank for $t > 0$.

(2 marks)

Solution
$-0.05t = 10 - 10e^{-0.05t} \Rightarrow 10e^{-0.05t} = 10 - 0.05t$
$10 + 60e^{-0.05t} = 40 \Rightarrow e^{-0.05t} = 0.5$
Specific behaviours
✓ correct time
✓ simplifies equation to $e^{-0.05t} = k$

(i) the time taken for the pressure in the tank to fall to 40 psi.

(b) Determine

Solution
$P(t) = 10 + 60e^{-0.05t}$
Specific behaviours
✓ correctly integrates $P'(t)$
✓ correct expression for $P(t)$

(a) Determine an expression for the pressure P in the tank at any time $t \geq 0$.

time in minutes since it began to empty.

The rate of change of pressure in an air tank is given by $P'(t) = -3e^{-0.05t}$, where t is the

(5 marks)

Question 3

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(a) Simplify $3 \log 20 - \log 4 + \log 5$.
(2 marks)

(3 marks)

(b) Given that $\log_a x = 0.82$, determine the value of $\log_a \sqrt{x}$.
(2 marks)(c) Determine the solution to the equation $2^x = 5^{2-x}$ in the form $x = \frac{\log a}{\log b}$.
(2 marks)

Solution
$3x \log 2 - 2 \log 5 = 2 - x \log 5$
$3x \log 2 + x \log 5 = 2 \log 5$
$x(3 \log 2 + \log 5) = 2 \log 5$
$x = \frac{\log 5}{3 \log 2 + \log 5}$

Solution
$\log_a \sqrt{x} = \log_a x - \log_a 0.5 = 0.5 \log_a x - \log_a 0.5$
$\log_a x = \log_a \sqrt{x} - \log_a 0.5$
$\log_a x = \log_a \sqrt{a}$

Solution
$\log 10^4 = 4$
$\log 20^3 - \log 4 + \log 5 = \log(8000 \div 4 \times 5)$
Specific behaviours
✓ expresses as single log
✓ simplifies to number

(a) Simplify $3 \log 20 - \log 4 + \log 5$.
(2 marks)

(7 marks)

(6 marks)

Question 4

The continuous random variable X takes values in the interval 3 to 8 and has cumulative distribution function $F(x)$ where

$$F(x) = P(X \leq x) = \begin{cases} 0 & x < 3 \\ \frac{x-3}{5} & 3 \leq x \leq 8 \\ 1 & x > 8 \end{cases}$$

(a) Determine

(i) $P(X \leq 4.5)$.

(1 mark)

Solution

$$P(X \leq 4.5) = \frac{4.5-3}{5} = \frac{1.5}{5} = \frac{3}{10} = 0.3$$

Specific behaviours

✓ correct probability

(ii) the value of k , if $P(X > k) = 0.75$.

(2 marks)

Solution

$$P(X \leq k) = 1 - P(X > k) = 1 - 0.75 = 0.25$$

$$\frac{k-3}{5} = 0.25 \Rightarrow k = 4.25$$

Specific behaviours✓ indicates $P(X \leq k)$

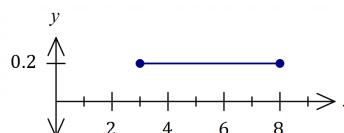
█ correct value

(b) Determine $f(x)$, the probability density function of X , and sketch the graph of $y=f(x)$.

(3 marks)

Solution

$$f(x) = F'(x) = \begin{cases} \frac{1}{5} & 3 \leq x \leq 8 \\ 0 & \text{elsewhere} \end{cases}$$

**Specific behaviours**✓ $f(x)$ (no penalty for just $f(x) = \frac{1}{5}$ if sketch correct)█ draws $y=0.2$ between endpoints

See next page

(7 marks)

Question 5

The function f is defined by $f(x) = \frac{x^2 - 7}{4-x}, x \neq 4$.

The second derivative of f is $f''(x) = 18(4-x)^{-3}$.

Determine the coordinates and nature of all stationary points of the graph of $y=f(x)$.

Solution

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$$f'(x) = 0 \Rightarrow 8x - 2x^2 + x^2 - 7 = 0 \quad (x^2 - 8x + 7) = 0 \quad (x-1)(x-7) = 0$$

$$x = 1, 7$$

$$f'(1) = \frac{18}{3^3} > 0 \Rightarrow \text{Min}, f'(7) = \frac{18}{-3^3} < 0 \Rightarrow \text{Max}$$

$$f(1) = \frac{-6}{3} = -2, f(7) = \frac{42}{-3} = -14$$

$f(x)$ has a minimum at $(1, -2)$ and a maximum at $(7, -14)$.

Specific behaviours

✓ indicates correct use of quotient rule

█ correct $f'(x)$

█ equates numerator to zero

█ determines x -coordinates of stationary points

█ indicates correct use of second derivative for nature

█ correct minimum

█ correct maximum

See next page