



PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 2

Section One:
Calculator-free

Your Name _____

Your Teacher's Name _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Max	Question	Mark	Max
1		3	6		8
2		6	7		6
3		7	8		4
4		8	9		3
5		7			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	52	36
Section Two: Calculator-assumed	13	13	100	94	64
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free

(52 Marks)

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1

(3 marks)

Prove the following statement:

If a and b are each 1 less than a multiple of 3, then ab is 1 more than a multiple of 3.

Question 2

(6 marks)

Consider the system of simultaneous linear equations:

$$3x - ay = 6 - 6x + 4y = b$$

- a) Write down the matrix A such that the equation

$$A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ b \end{bmatrix}$$

is equivalent to the system of equations above.

(1 mark)

- b) Suppose that A is singular (non-invertible).

- i. Determine the value of a (show working).

(2 marks)

- ii. State the possible number(s) of solutions that the system of equations could have with the value of a you just found.

(2 marks)

- iii. State the number of solutions the system has if a has the value found above and $b = 11$.

(1 mark)

Question 3

(7 marks)

a) Write $3 \cos 5x + 3\sqrt{3}$ in the form $a \sin(bx + \alpha)$.

(3 marks)

b) Hence, solve the equation $3 \cos 5x + 3\sqrt{3} = 0$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

(4 marks)

Question 4

(8 marks)

A 2×2 real matrix A can 'transform' a complex number if we view the complex number as a column vector. That is, for any complex number $z = a + bi$, the matrix A transforms z to $c + di$ where $\begin{bmatrix} c \\ d \end{bmatrix} = A \begin{bmatrix} a \\ b \end{bmatrix}$.

Find the matrix A which (according to this rule) will transform any complex number z to:

a) $3z$

(2 marks)

b) \underline{z}

(2 marks)

c) iz

(2 marks)

d) $i\underline{z}$

(2 marks)

Question 5

(7 marks)

Evaluate the following for complex numbers $z=2+5i$ and $w=1-4i$

a) $z - w$

(2 marks)

b) $z(w + \overline{w})$

(2 marks)

c) $\frac{w}{z}$

(3 marks)

Question 6

(8 marks)

In this question, a *proper* factor is a factor greater than 1.

Assume that a and b are both integers, and consider the following statement:

If ab has no proper square factors, then neither a nor b has a proper square factor.

a) Prove the statement using the method of proof by contradiction. (3 marks)

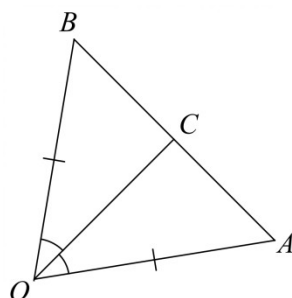
b) Write the converse of the statement. (2 marks)

c) State whether the converse is true or false and prove or disprove it accordingly. (3 marks)

Question 7

(6 marks)

Let O be the origin, let A and B be points such that $OA = OB$, and let C be a point on \underline{AB} such that \underline{OC} bisects $\angle AOB$.



Let $a = \overrightarrow{OA}$, $b = \overrightarrow{OB}$ and $c = \overrightarrow{OC}$.

a) Show that $a \cdot c = b \cdot c$.

(3 marks)

b) **Hence**, prove that \underline{OC} is perpendicular to \underline{AB} .

(3 marks)

Question 8

(4 marks)

Prove the following identity.

$$\frac{\sin 7\theta - \sin 2\theta}{\cos 2\theta + \cos 7\theta} = \tan \tan \frac{5\theta}{2}$$

Question 9

(3 marks)

Let l be a line containing a point P , and let Q be a point not on l . Suppose that \hat{n} is a unit vector perpendicular to the line l . Prove that the perpendicular distance from Q to l is $|\overrightarrow{PQ} \cdot \hat{n}|$.

Additional working space

Question number:

Additional working space

Question number: