

[2]

(a) $\log_8 64 = ?$

Solve each of the following equations, showing all working.

2. [5 marks]

[2]

(b)
$$\frac{\log_{\frac{3}{2}} 5}{\log 135 - \log 5}$$

[2]

(a) $3 \log_2 6 - \log_2 27$

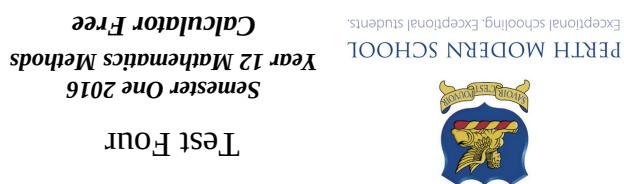
Evaluate each of the following showing full working:

1. [4 marks]

- Complete all questions
- Show all necessary working
- Total Marks = 24
- 24 minutes

Teacher:	Mr Staffe
	Mrs Carter
	Mr Roohi
	Ms Chene

Name:



(b) $5e^{2x} = 100$ (Leave answer in terms of Logs)

[3]

3. [7 marks]

Differentiate each of the following with respect to x , using the appropriate rule and showing full working:

(a) $f(x) = e^{bx} \ln(x)$

[2]

(b) $g(x) = \ln\left(\frac{x^2}{\sqrt{x-1}}\right)$

[2]

(c) $y = \ln(\sin(3x))$

[3]

[3]

$$\int \frac{3x^2 - 1}{9} dx$$

(d) Calculate the following definite integral, simplifying your answer using logarithmic laws.

[2]

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} x^2 dx$$

(c)

[2]

$$\int_{\sin x}^{\cos x} dx$$

(q)

[1]

$$\int \frac{2x^2 - 1}{2} dx$$

Determine each of the following anti-derivatives, simplifying your answer where possible:

4. [8 marks]



Test Four
Semester One 2016
Year 12 Mathematics Methods
Calculator Assumed

Name:

- Complete all questions
- Show all necessary working
- Total Marks = 26
- 26 minutes

1. [6 marks]

The mass M , in grams, of a radioactive substance after t years is given by:

$$M = 13.8 - \ln(t + 43.1)$$

- (a) State the initial mass of the substance. [1]
- (b) Determine an expression for the instantaneous rate of change of the mass with respect to time. [1]
- (c) Calculate the decrease in mass in the 100th year. [2]
- (d) Calculate the average decrease in the mass over 100 years. [2]

Teacher:

- Mr Staffe
 Mrs. Carter
 Mr Roohi
 Ms Cheng

This scenario can be best modelled by an exponential probability density function which is given by

$$p(x) = ke^{-kx}; x \geq 0 \text{ where } \frac{1}{k} \text{ is the mean time between serving customers.}$$

- (b) Hence state the probability density function for X , where X represents the time between serving each customer.

$$p(x) = \frac{1}{6} e^{-\frac{x}{6}} \quad \checkmark \quad \checkmark$$

- (c) Show, using calculus methods, that your answer to part (b) does in fact represent a continuous probability density function.

$$\begin{aligned} & \int_0^{\infty} \frac{1}{6} e^{-\frac{x}{6}} dx \quad \checkmark \\ &= \left[-e^{-\frac{x}{6}} \right]_0^{\infty} \quad \checkmark \\ &= 0 - (-1) \\ &= 1 \quad \checkmark \end{aligned}$$

- (d) State the expected value of the distribution.

$$\begin{aligned} & \int_0^{\infty} x \frac{1}{6} e^{-\frac{x}{6}} dx \\ &= 6 \quad \checkmark \end{aligned}$$

Calculate the probability that the next customer will be served:

- (e) 5 minutes or less after the previous one.

$$\begin{aligned} & \int_0^5 \frac{1}{6} e^{-\frac{x}{6}} dx \\ &= 0.5654 \quad \checkmark \end{aligned}$$

- (f) between 5 and 7 minutes after the previous one.

$$\begin{aligned} & \int_5^7 \frac{1}{6} e^{-\frac{x}{6}} dx \\ &= 0.1232 \quad \checkmark \end{aligned}$$

- (g) less than 8 minutes after the previous one given that it took longer than 5 minutes.

$$\begin{aligned} & \int_5^8 \frac{1}{6} e^{-\frac{x}{6}} dx \quad \checkmark \\ &= \frac{0.1710}{0.5654} = 0.3024 \quad \checkmark \end{aligned}$$

[2]

- (a) Sketch the probability distribution function for T .


(b) Calculate the mean time taken to download a movie.

$$\mu = \frac{24 - 18}{2} = 3 + 18 = 21 \text{ min}$$

(c) 75% of the time it takes less than k minutes to download a movie. Calculate the value of k .

$$k = 22.5$$

(d) Calculate $P(T > 20 | T < 23)$

$$\frac{P(20 < T < 23)}{P(T < 23)} = \frac{6}{9} = \frac{2}{3}$$

(e) 75% of the time it takes less than k minutes to download a movie. Calculate the value of k .

$$k = 20.75$$

(f) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(g) Calculate $P(T > 20 | T < 23)$

$$\text{Calculate } P(T > 20 | T < 23) \text{ Calculate the value of } k.$$

(h) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(i) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(j) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(k) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(l) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(m) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(n) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(o) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(p) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(q) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(r) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(s) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(t) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(u) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(v) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(w) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(x) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(y) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

(z) Calculate $P(T > 20 | T < 23)$

$$75\% \text{ of the time it takes less than } k \text{ minutes to download a movie. Calculate the value of } k.$$

三

2.

3. [12 marks]

During December, a local shopping centre has a gift wrapping stall which is open 12 hours a day. On average, the gift wrapping stall serves 120 customers per day.

- (a) What is the average length of time between serving each customer at this gift wrapping stall?
Give your answer in minutes. [2]

This scenario can be best modelled by an exponential probability density function which is given by

$$p(x) = ke^{-kx}; x \geq 0 \text{ where } \frac{1}{k} \text{ is the mean time between serving customers.}$$

- (b) Hence state the probability density function for X, where X represents the time between serving each customer. [2]

- (c) Show, using calculus methods, that your answer to part (b) does in fact represent a continuous probability density function. [3]

- (d) State the expected value of the distribution. [1]



Test Three

Semester One 2016
Year 12 Mathematics Methods
Calculator Assumed

Name: **Mark King-Gyde**

Teacher:

- Mr Staffe
- Mrs. Carter
- Mr Bertram
- Mr Roohi
- Ms Cheng

- Complete all questions
- Show all necessary working
- Total Marks = 26
- 26 minutes

1. [6 marks]

The mass M, in grams, of a radioactive substance after t years is given by:

$$M = 13.8 - \ln(t + 43.1)$$

- (a) State the initial mass of the substance.

$$M = 13.8 - \ln 43.1 = 10.04g \quad \checkmark$$

- (b) Determine an expression for the instantaneous rate of change of the mass with respect to time.

$$\frac{dM}{dt} = \frac{-1}{t + 43.1} \quad \checkmark$$

- (c) Calculate the decrease in mass in the 100th year.

$$M = [13.8 - \ln(100 + 43.1)] - [13.8 - \ln(99 + 43.1)] \quad \checkmark$$

$$M = -0.00701g$$

$$\text{A decrease of } 0.00701g \quad \checkmark$$

- (d) Calculate the average decrease in the mass over 100 years.

$$\frac{M(100) - M(0)}{100} = -0.012g/\text{year} \quad \checkmark$$

- [2] (g) less than 8 minutes after the previous one given that it took longer than 5 minutes.

- [1] (f) between 5 and 7 minutes after the previous one.

- [1] (e) 5 minutes or less after the previous one.
Calculate the probability that the next customer will be served:

$$\begin{aligned} &= 2 \ln \left(\frac{5}{11} \right) \\ &= 2(\ln 11 - \ln 5) \\ &= 2 \ln |1| - 2 \ln |5| \\ &= \boxed{2 \ln |3x - 1|} \end{aligned}$$

$$\int_{\frac{1}{9}}^{\frac{1}{3}} \frac{3x-1}{x^2} dx$$

laws.

- (d) Calculate each of the following definite integrals, simplifying your answers using logarithmic

$$= \boxed{\ln e^x - 2 + C}$$

$$(c) \int_{e^{-x}}^{e^x} \frac{2}{x} dx$$



Test Three
Semester One 2016
Year 12 Mathematics Methods
Calculator Free

Name: **Mark King-Gyde**

- Total Marks = 24
- 24 minutes

1. Evaluate each of the following showing full working:

$$(a) \quad 3\log_2 6 - \log_2 27$$

$$\begin{aligned} &= \log_2 216 - \log_2 27 \quad \checkmark \\ &= \log_2 8 \\ &= \log_2 2^3 \\ &= 3\log_2 2 \\ &= 3 \quad \checkmark \end{aligned}$$

$$(b) \quad \frac{\log 135 - \log 5}{\log 3^2}$$

$$\begin{aligned} &= \frac{\log 27}{2\log 3} \quad \checkmark \\ &= \frac{\log 3^3}{2\log 3} \\ &= \frac{3\log 3}{2\log 3} \\ &= \frac{3}{2} \quad \checkmark \end{aligned}$$

2. Solve each of the following equations, showing all working.

$$(a) \quad \log_y 64 = 2$$

$$\begin{aligned} y^2 &= 64 \quad \checkmark \\ y &= 8 \quad (y > 0) \quad \checkmark \end{aligned}$$

Teacher:

- Mr Staffe
- Mrs. Carter
- Mr Bertram
- Mr Roohi
- Ms Cheng

$$(b) \quad 5e^{2-x} = 100$$

$$\begin{aligned} e^{2-x} &= 20 \quad \checkmark \\ (2-x)\ln e &= \ln 20 \\ 2-x &= \ln 20 \quad \checkmark \\ x &= 2 - \ln 20 \quad \checkmark \end{aligned}$$

3. [7 marks]

Simplify or Evaluate the following integrals as appropriate

$$(a) \quad f(x) = e^{1-x} \ln(x)$$

$$f'(x) = -e^{1-x} \ln(x) + \frac{e^{1-x}}{x} \quad \checkmark$$

$$(b) \quad g(x) = \ln\left(\frac{x^2}{\sqrt{x-1}}\right)$$

$$\begin{aligned} g(x) &= \ln(x^2) - \frac{1}{2}\ln(x-1) \quad \checkmark \\ g'(x) &= \frac{2}{x} - \frac{1}{2(x-1)} \quad \checkmark \end{aligned}$$

$$(c) \quad y = \ln(\sin(3x))$$

$$\frac{dy}{dx} = \frac{3\cos 3x}{\sin 3x} \quad \checkmark$$

4. [8 marks]

Determine each of the following anti-derivatives, simplifying your answer where possible:

$$(a) \quad \int \frac{2}{2x-1} dx$$

$$= \ln|2x-1| + c \quad \checkmark$$

$$(b) \quad \int \frac{\sin x}{\cos x} dx$$

$$= -\ln|\cos x| + c \quad \checkmark$$