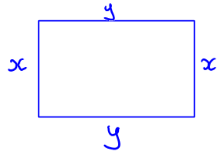
9. [6 marks]

The owner of a garden centre wishes to fence a rectangular area of 360 m^2. She wishes to fence three sides with fencing that costs \$5/m and the fourth side with fencing costing \$11/m.
Show the use of calculus to find the dimensions of the rectangular area that will minimise her fencing costs.



$\text{Area} = 360\,m^2$

$\Rightarrow xy = 360$

$y = \frac{360}{x}$

$\text{Cost} = 5(x+2y) + 11x$ (1) or $C = 5(2x+y) + 11y$

$= 16x + 10y$ $\quad$ $= 10x + 16y$

$= 16x + \frac{3600}{x}$ (1) $\quad$ $= 10x + \frac{5760}{x}$

For stationary points, $\frac{dC}{dx} = 0$ $\quad$ $\frac{dC}{dx} = \frac{10x^2 - 5760}{x^2}$

$\frac{16x^2 - 3600}{x^2} = 0$ (1) for $\frac{dC}{dx}$ $\quad$ $\frac{dC}{dx} = 0$ when $x = \pm 24$

$\vdots$

$x = -15$ or $x = 15$ (1)

As $x > 0$, $x = 15$

6

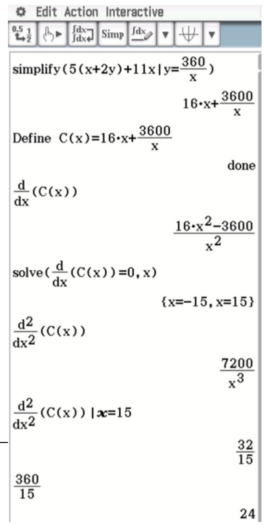
$\frac{d^2C}{dx^2} = \frac{7200}{x^3}$

If $x = 15$, $\frac{d^2C}{dx^2} = \frac{32}{15} > 0$ (1) for justifying $x = 15$ minimises the cost

$\therefore x = 15$ minimises the cost

If $x = 15$, $y = \frac{360}{15} = 24$

So, 15m by 24m will minimise the cost (1)


Edit Action Interactive

simplify$(5(x+2y)+11x|y=\frac{360}{x})$

$16\cdot x+\frac{3600}{x}$

Define $C(x)=16\cdot x+\frac{3600}{x}$

done

$\frac{d}{dx}(C(x))$

$\frac{16\cdot x^2-3600}{x^2}$

solve$(\frac{d}{dx}(C(x))=0,x)$

$\{x=-15,x=15\}$

$\frac{d^2}{dx^2}(C(x))$

$\frac{7200}{x^3}$

$\frac{d^2}{dx^2}(C(x))|x=15$

$\frac{32}{15}$

$\frac{360}{15}$

24

End of Calculator-Assumed Section

6

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Total Time:	25 minutes
Marks:	22 marks
Total Marks:	40

Methods 3&4

Review Response Test 1

(Wed Mar 31st)

Resource Free

ClassPad calculators are NOT permitted.
Formulae sheet is permitted.

Name: ANSWERS

1. [1, 2 & 2 = 5 marks]

Find the following indefinite integrals.

(a) $\int 4\sqrt{x}\, dx$

$= \int 4x^{1/2}\, dx$

$= 4 \cdot \frac{2}{3} x^{3/2} + c$

$= \frac{8}{3} x^{3/2} + c$ (1)

(b) $\int (3x-2)^3\, dx$

$= \frac{1}{3} \int 3(3x-2)^3\, dx$

$= \frac{1}{3} \cdot \frac{1}{4} (3x-2)^4 + c$

$= \frac{1}{12} (3x-2)^4 + c$ (1)

5

(c) $\int (x^2+2)^2\, dx$

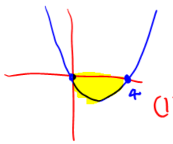
(1) for '+c' in at least 2 of the 3 parts

$= \int x^4 + 4x^2 + 4\, dx$ (1)

$= \frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x + c$ (1)

2. [4 marks]

Find the area bounded between the graph of $y = 3x(x-4)$ and the x-axis.



(1) sketch

$\text{Area} = \int_4^0 3x(x-4)\, dx$ (1) or $-\int_0^4 3x(x-4)\, dx$

$= \int_4^0 3x^2 - 12x\, dx$

$= \left[x^3 - 6x^2 + c \right]_4^0$ (1) antiderivative

$= (0) - (64 - 96)$

$= 32$ square units (1)

4

9

8. [2, 2 & 1 = 5 marks]

(a) Find the coordinates of the points where the curve $y = \frac{3x^2}{2x+1}$ cuts the line $y = 2x - 1$.

They intersect at $(-1, -3)$ (1)

and $(1, 1)$ (1)

(b) Find the gradient of curve $y = \frac{3x^2}{2x+1}$ at each point where it cuts the line $y = 2x - 1$.

$\frac{dy}{dx} = \frac{6x^2 + 6x}{(2x+1)^2}$ (1)

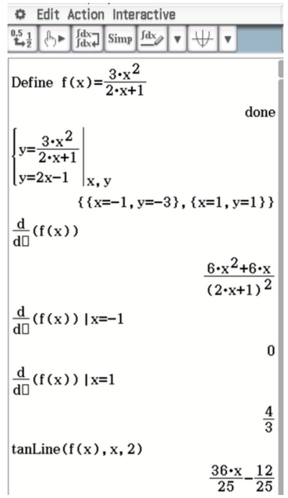
When $x = -1$, $\frac{dy}{dx} = 0$ $\Rightarrow$ Gradient at $(-1, -3)$ is zero

5

When $x = 1$, $\frac{dy}{dx} = \frac{4}{3}$ $\Rightarrow$ Gradient at $(1, 1)$ is $\frac{4}{3}$

(1) for correct gradient at both points
Allow for errors from (a)

(c) Find the equation of the tangent to the curve $y = \frac{3x^2}{2x+1}$ at the point with x-coordinate of 2.



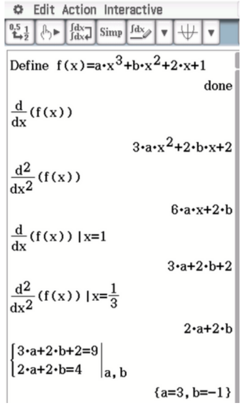
Equation of tangent at point with x-coordinate of 2 is

$y = \frac{36x}{25} - \frac{12}{25}$ (1)

5

6. [4 marks]

Given that $f(x) = ax^3 + bx^2 + 2x + 1$, $f'(1) = 9$ and $f''\left(\frac{1}{3}\right) = 4$, find the value of the constants a and b.



$f'(x) = 3ax^2 + 2bx + 2$ (1)

$f''(x) = 6ax + 2b$ (1)

As $f'(1) = 9$, $9 = 3a + 2b + 2$

As $f''\left(\frac{1}{3}\right) = 4$, $2a + 2b = 4$ (1)

Solving simultaneously gives $a = 3$ and $b = -1$ (1)

4

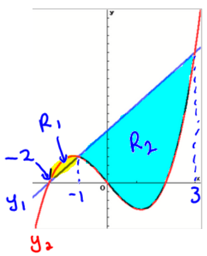
7. [3 marks]

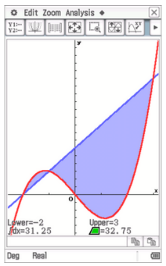
Showing the use of definite integrals (without absolute value), find the area enclosed between the graphs of $y_1 = 3x + 6$ and $y_2 = x(x+2)(x-2)$

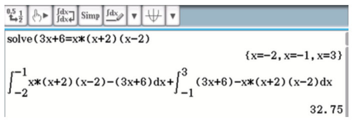
$y_1 = y_2$ when $3x + 6 = x(x+2)(x-3)$

$x = -2, -1, 3$

(1) for points of intersection







3

Area = Area of R_1 + Area of R_2

$= \int_{-2}^{-1} y_2 - y_1 \, dx + \int_{-1}^{3} y_1 - y_2 \, dx$

$= \int_{-2}^{-1} x(x+2)(x-2) - (3x+6)\,dx + \int_{-1}^{3} (3x+6) - x(x+2)(x-2)\,dx$ (1) definite integrals to find area

$= 32.75$ square units (1)

7

3. [3 marks]

Find the equation of the tangent to the curve $y = \frac{2x-1}{x-1}$ at the point (2, 3) giving your answer in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{(2)(x-1) - (1)(2x-1)}{(x-1)^2}$ (1) correctly differentiated

$= \frac{2x - 2 - 2x + 1}{(x-1)^2}$

$= -\frac{1}{(x-1)^2}$

When $x = 2$, $y = 3$ and $\frac{dy}{dx} = -\frac{1}{(1)^2}$

$= -1$ (1)

So, equation of tangent is

$y - 3 = -(x-2)$

$y = -x + 5$ (1)

3

4. [4 marks]

Find the x-coordinates of the points on the graph of $y = x^2(2x+3)$ where the gradient is 12.

$y = 2x^3 + 3x^2$

$\frac{dy}{dx} = 6x^2 + 6x$ (1)

Gradient $= 12$ when $6x^2 + 6x = 12$

$6x^2 + 6x - 12 = 0$ (1)

$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$ (1) method

$x = -2$ or $x = 1$ (1) both required

So gradient $= 12$ when $x = -2$ or $x = 1$

4

7

5. [6 marks]

Use calculus techniques to determine the coordinates, and their nature, of any stationary points on the curve with equation $y = 2x + \frac{18}{x}$.

$y = 2x + 18x^{-1}$

$\frac{dy}{dx} = 2 - 18x^{-2}$ (1)

For stationary points, $\frac{dy}{dx} = 0$

$2 - \frac{18}{x^2} = 0$

$2x^2 - 18 = 0$

$2(x^2 - 9) = 0$

$2(x-3)(x+3) = 0$

$x = 3$ or $x = -3$ (1)

$\frac{d^2y}{dx^2} = 36x^{-3}$

$= \frac{36}{x^3}$

(0,1,2) for use of calculus to find nature of stationary points via 1st or 2nd derivative test

$\frac{\ }{6}$

If $x = 3$, $\frac{d^2y}{dx^2} = \frac{36}{27}$ and $y = 6 + 6$

$= 12$

> 0

$\Rightarrow$ local min at $(3, 12)$

If $x = -3$, $\frac{d^2y}{dx^2} = -\frac{36}{27}$ and $y = -6 - 6$

$= -12$

< 0

$\Rightarrow$ local max at $(-3, -12)$

(1) for nature of each stationary point

(1) found corresponding y-coordinates correctly for each x-coordinate

End of Calculator-Free Section

6

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Total Time:	20 minutes
Marks:	18 marks

Methods 3&4

Review Response Test 1

(Wed Mar 31st)

Resource Assumed

ClassPad calculators ARE permitted.
Formulae sheets are permitted.

Name: ANSWERS