PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Year 12 Test Four 2022

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: **_____40_____ mins**

Number of questions: **_____6_____**

Materials required: Upto 3 Calculators with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, No notes allowed
A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **__40____ marks**

Task weighting: **_10___%**

Formula sheet provided: **Yes**

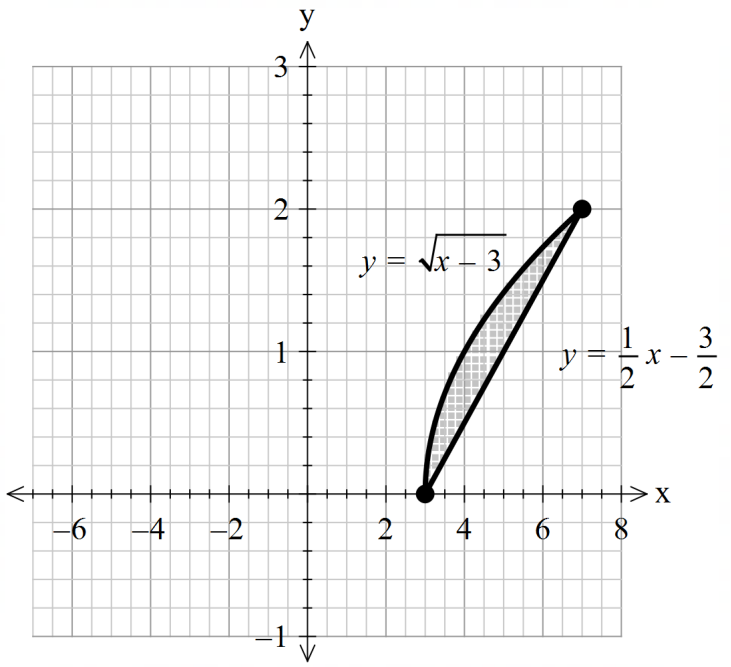
Note: All part questions worth more than 2 marks require working to obtain full marks.

No notes allowed

No notes allowed

Q1 (5 marks) (4.1.6)

Determine the volume of the solid formed by rotating the area enclosed between $y = \sqrt{x - 3}$ & $y = \frac{1}{2}x - \frac{3}{2}$ about the y axis, as shown below.



Q2 (5, 3 & 2= 10 marks) (4.1.4)

a) By using integration and partial fractions, show how to derive $N = \frac{a}{b + Ce^{-at}}$ from the differential equation $\frac{dN}{dt} = aN - bN^2$ ($a, b > 0$) and c is a constant.

Q2 continued

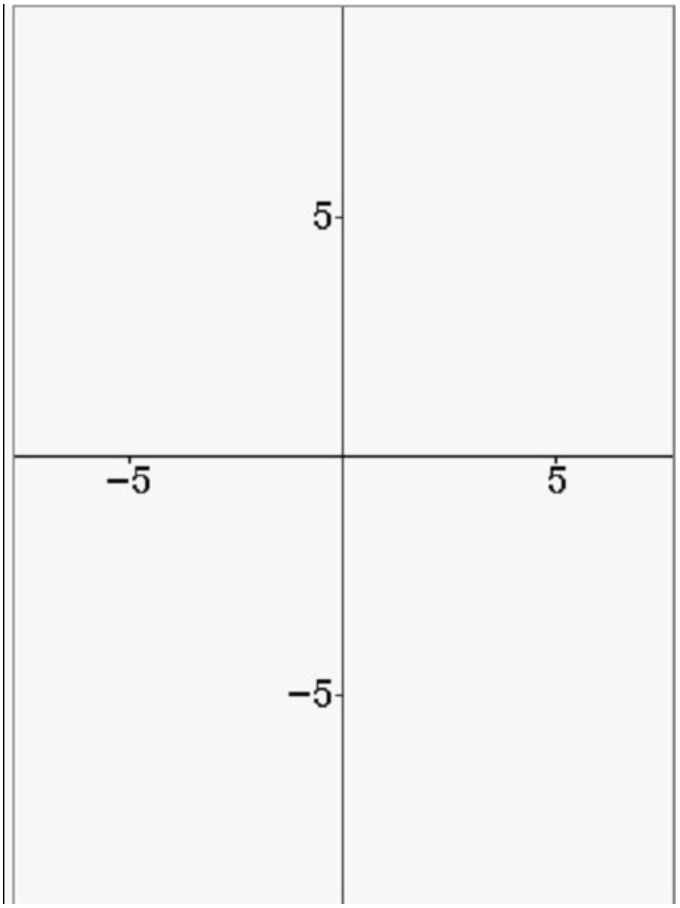
b) Let N equal the number of kangaroos living in a habitat after t years and $\frac{dN}{dt} = \frac{1}{5}N - \frac{1}{12500}N^2$.

If initially there are 50 kangaroos, determine the number in 10 years time.

c) Determine the size of the population at the maximum growth rate.

Q3 (3, 2 & 3 = 8 marks) (4.2.5)

a) Sketch the slope field on the axes below for $\frac{dy}{dx} = 3^x$



b) Show the solution curve on the axes above that passes through point (2,2).

c) Determine in cartesian form the solution curve for b above **without using a classpad**.
Hint – use logarithmic differentiation. Show all working.

Q4 (5 marks) (4.2.1)

Determine expressions in terms of $x \& y$ only for $\frac{dy}{dx} \& \frac{d^2y}{dx^2}$ in terms of $x, y \& y'$ only, using the following equation $x^3y^2 = 5 - xy$

Q5 (3 & 3 = 6 marks) (4.2.7)

Consider a particle that is moving with SHM such that $\ddot{x} = -9x$ with a maximum speed of 12 m/s.

a) Determine the exact speed when the particle is half of an amplitude from the origin.

b) Determine the percentage of the time that the particle is more than half an amplitude from the centre.

Q6 (4 & 2 = 6 marks) (4.2.7)

The motion of a bullet through a wall is modelled by the equation $a = -25(v+75)^2, \quad v > 0$ where $a\ m/s^2$ is its acceleration and $v\ m/s$ its velocity t seconds after impact. Initially at impact the speed is 300 m/s and is at the origin ($x = 0$ metres)

a) Determine x in terms of v only.

Q6 continued-

b) Determine how far the bullet penetrates the wall before coming to rest to the nearest mm.