PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 4 Year 12

Student name: ________________ Teacher name: ________________

Task type: **Response**

Time allowed for this task: **____40_______ mins**

Number of questions: **_____7______**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **_44_____ marks**

Task weighting: **_10___%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3 & 3 = 6 marks)
Solve the following.

a) $\frac{dy}{dx} = \frac{3x - 2}{y(5 - y^2)}$ given that when $x = 1,\ y = 1$.

Solution
$\frac{dy}{dx} = \frac{3x-2}{y(5-y^2)}$ $\int y(5-y^2)dy = \int 3x - 2$ $\frac{5}{2}y^2 - \frac{1}{4}y^4 = \frac{3}{2}x^2 - 2x + c$ $x = 1. y = 1\ c = \frac{11}{4}$
Specific behaviours
P separates variables P integrates all terms P solves for constant

b) $3x^4 \cos(2y)\frac{dy}{dx} = 10$ given that when $x = 5,\ y = \pi$.

Solution
$3x^4\cos(2y)\frac{dy}{dx} = 10$ $\int \cos(2y)dy = \int \frac{10}{3}x^{-4}dx$ $\frac{1}{2}\sin(2y) = -\frac{10}{9}x^{-3} + c$ $x = 5, y = \pi, c = \frac{2}{225}$ (c=+2/225)
Specific behaviours
P separates variables P integrates all terms P solves for constant

Q2 (4 marks)

An iron has a temperature of $54^{\circ}C$ is left in a room, of temperature $18^{\circ}C$, to cool such that the temperature $T^{\circ}C$ at time t minutes is given by $\frac{dT}{dt}=k(T-18)$. After 15 mins the temperature of the iron is $37^{\circ}C$. Determine the time taken for the iron's temperature to drop to $22^{\circ}C$.

Solution

$$\frac{dT}{dt}=k(T-18)$$

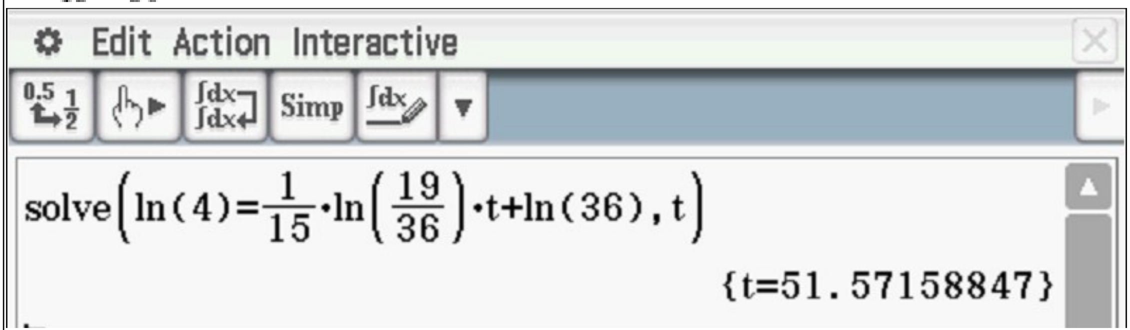
$$\int\frac{dT}{(T-18)}=\int k\,dt$$

$$\ln|T-18|=kt+c$$

$$t=0,\ c=\ln|54-18|=\ln 36$$

$$t=15, T=37, \ln 19=15k+\ln 36$$

$$k=\frac{1}{15}\ln\frac{19}{36}$$



51.57 minutes

Specific behaviours

P separates variables and integrates

Pderives an expression involving both variables

P solves for both constants (may be approx.)

P determines approx time and must give units

Q3 (1, 5 & 2 = 8 marks)

The number N thousands, of bacteria cells living in a petri dish at time t hours is given by

$$\frac{dN}{dt} = 0.30N - 0.05N^2.$$

The initial number of cells was 2 thousand.

a) What is the limiting value of the number of cells as $t \to \infty$?

Solution
$\frac{dN}{dt} = 0.30N - 0.05N^2 = N(0.30 - 0.05N)$ $N = \frac{0.3}{0.05} = 6$ 6 thousand
Specific behaviours
P states limiting value with units

b) Using calculus and partial fractions, show every step to express N in terms of t.

Solution

$$\frac{dN}{dt} = 0.30N - 0.05N^2 = N(0.30 - 0.05N)$$

$$\int \frac{dN}{N(0.30 - 0.05N)} = \int dt$$

$$\frac{1}{N(0.30 - 0.05N)} = \frac{a}{N} + \frac{b}{(0.30 - 0.05N)}$$

$$1 = a(0.30 - 0.05N) + bN$$

$$N = 0$$

$$1 = 0.3a,\ a = \frac{10}{3}$$

$$N = 6$$

$$1 = 6b,\ b = \frac{1}{6}$$

$$\frac{10}{3}\ln N - \frac{10}{3}\ln\left|0.30 - 0.05N\right| = t + c \quad Note: N < 6 \therefore 0.30 - 0.05N > 0$$

$$\ln\frac{N}{0.30 - 0.05N} = 0.3t + c$$

$$\frac{N}{0.30 - 0.05N} = ce^{0.3t}$$

$$\frac{0.30 - 0.05N}{N} = ce^{-0.3t}$$

$$0.30 - 0.05N = Nce^{-0.3t}$$

$$N = \frac{0.30}{0.05 + ce^{-0.3t}}$$

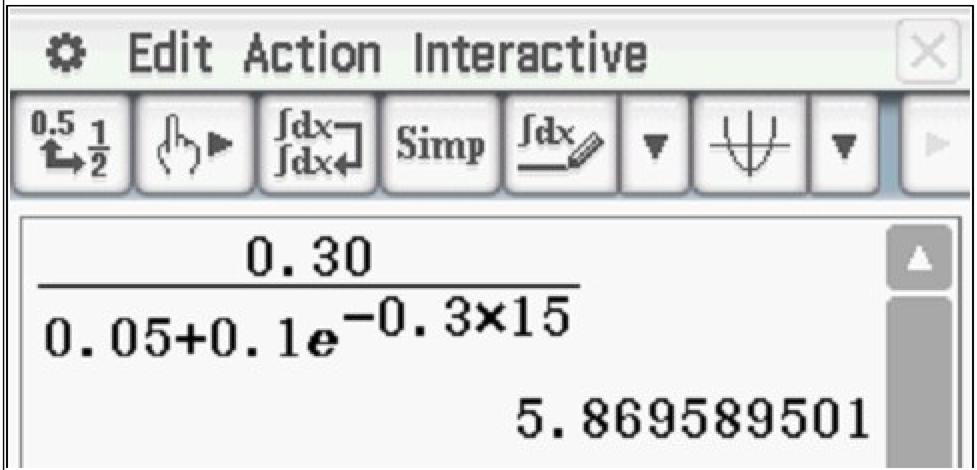
$$\text{solve}\left(2 = \frac{0.3}{0.05 + c}, c\right)$$

$$\{c = 0.1\}$$

Specific behaviours

P separates variables
P uses partial fractions and shows working for constants
P shows why absolute value not needed for log function
P rearranges to make N the subject with a constant
Psolves for constant

c) Determine the number of cells after 15 hours.

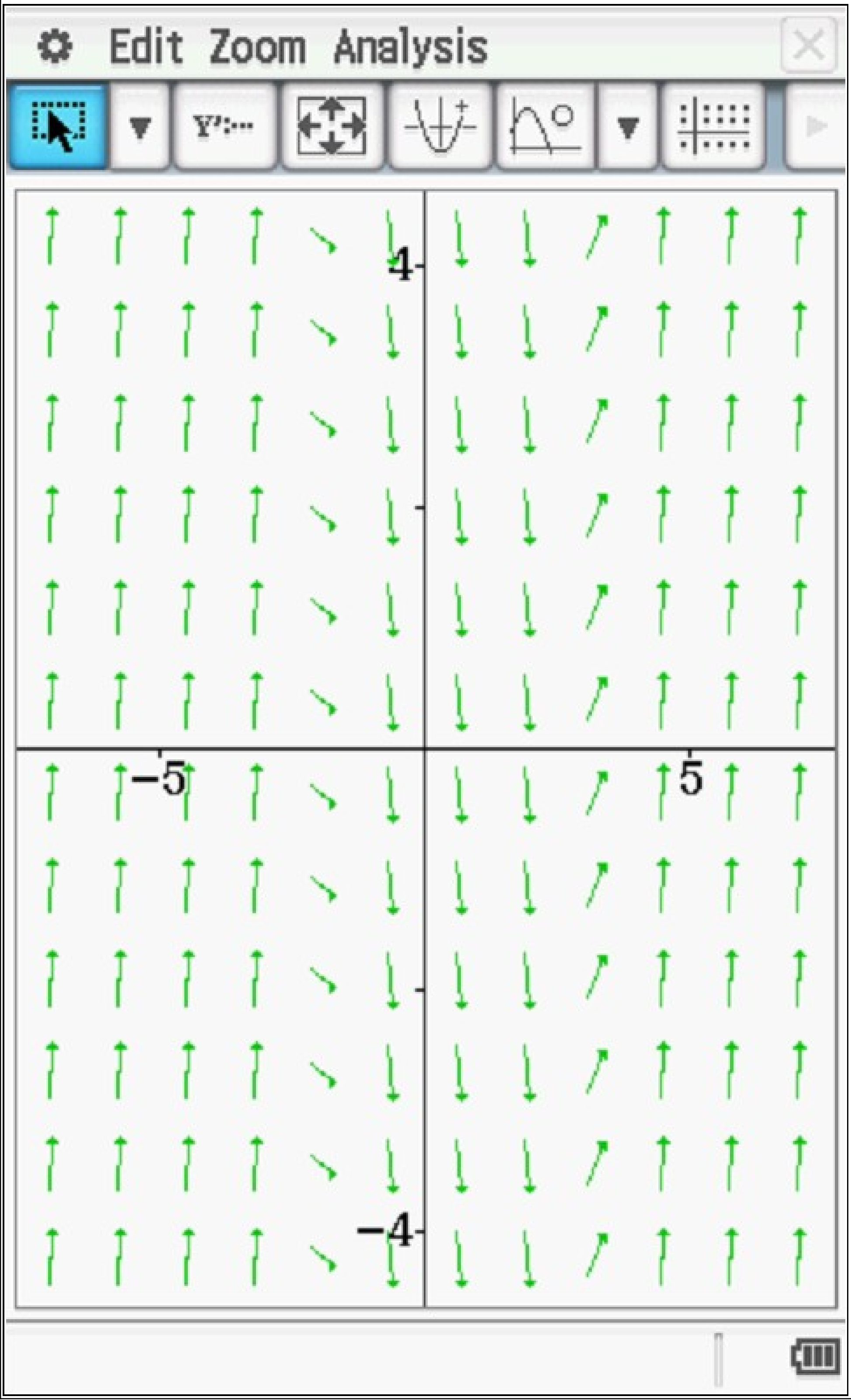
Solution
 Approximately 5870 cells
Specific behaviours
P subs t=15 into rule from part b P determines quantity with units of thousands or 5870 cells (Note –max –1 for units for entire question)

Q4 (3, 2 & 2 = 7 marks)

Consider the slope field $\frac{dy}{dx} = (x-3)(x+2)$

a) Sketch this field on the axes below.

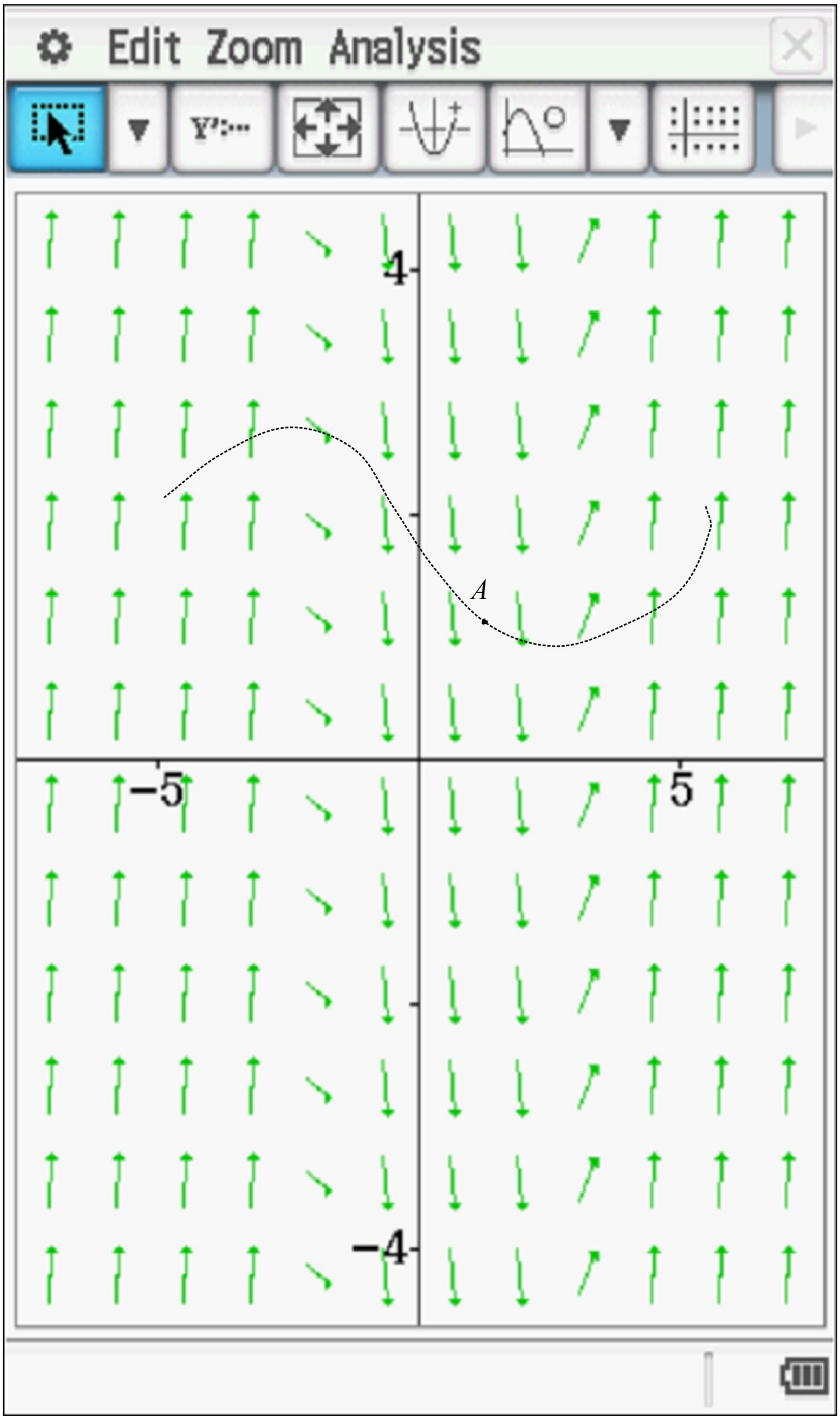
Solution



Specific behaviours
P shows horizontal grads at x=-2 P shows horizontal grads at x=3 P pattern at far left and right

b) Draw the solution curve, axes above, that contains the point (1,1).

Solution



Specific behaviours
P shape of solution curve P shows pt A labelled on curve

c) Determine the equation of the solution curve that contains (1,1).

Solution
$\frac{dy}{dx} = x^2 - x - 6$ $y = \frac{x^3}{3} - \frac{x^2}{2} - 6x + c$ $1 = \frac{1}{3} - \frac{1}{2} - 6 + c$ $c = \frac{43}{6}$
Specific behaviours
P integrates x terms P solves for constant

Q5 (2, 2 & 3 = 7 marks)

Consider an object that is moving with Simple Harmonic Motion such that $\ddot{x} = -9x$ with x, t in metres and seconds respectively. At $t = 0$, $x = 7$ metres and is a rest.

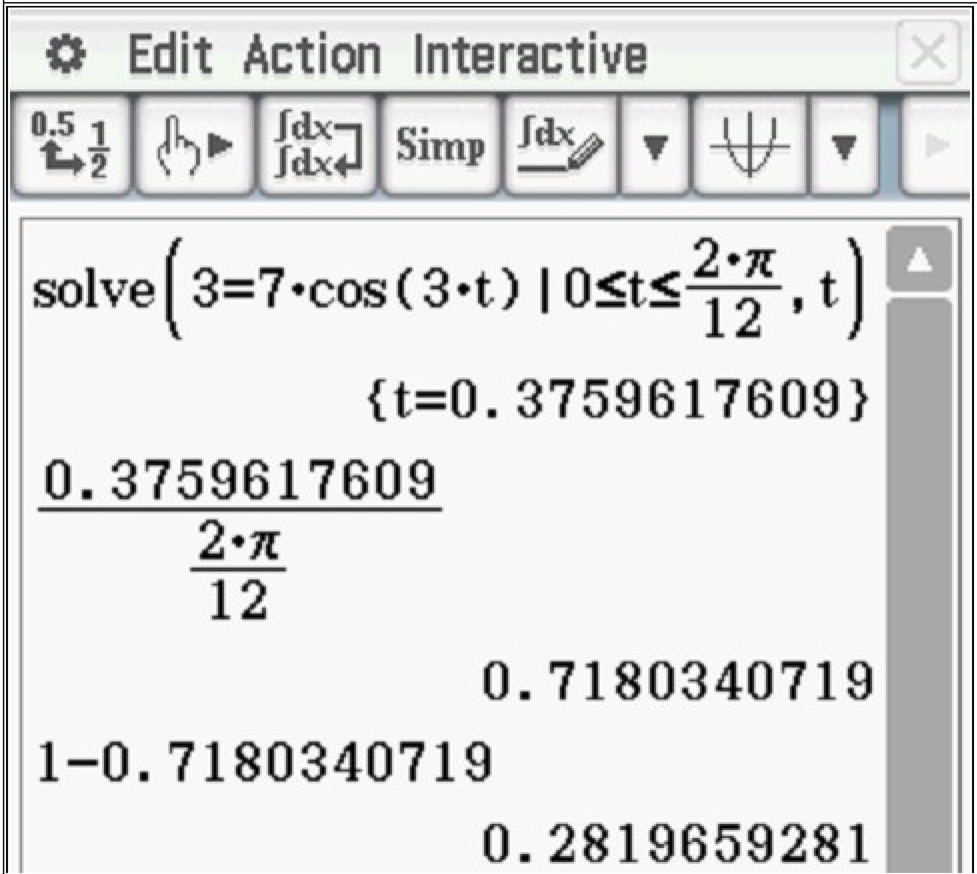
a) Determine a rule for x in terms of t.

Solution
$x = 7\cos 3t$
Specific behaviours
P uses an appropriate trig function P states all constants

b) Determine the exact speed when $x = 3$ metres.

Solution
$v^2 = n^2\left(A^2 - x^2\right)$ $= 9(49 - 9)$ $v = \sqrt{360}$
Specific behaviours
P uses appropriate rule P states exact speed, ignore units

c) Determine the percentage of the time, to one decimal place, that the object is less than 3 metres from the mean position, $x = 0$.

Solution

Specific behaviours
P solves for times at x=3 in a cycle or part cycle P determines an interval time and then divides by total length of cycle or part cycle P determines percentage

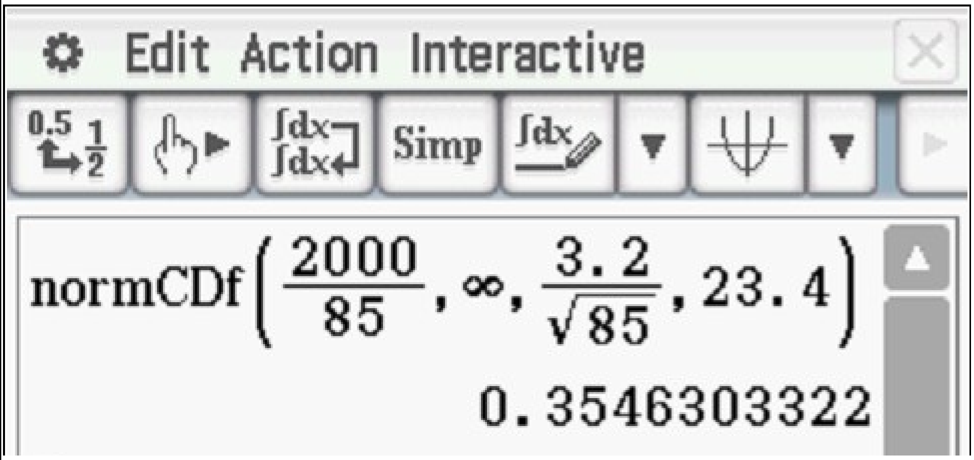
Q6 (4 marks)

Consider an object that is initially at the origin and at rest such that its acceleration is given by $\frac{dv}{dt}=\frac{1+v^3}{v}m/s^2$ where v equals the speed in m/s at t seconds . Determine the exact speed when its displacement from the origin is $\ln(3)$ metres.

Solution
$\frac{dv}{dt}=\frac{1+v^3}{v}$ $v\frac{dv}{dx}=\frac{1+v^3}{v}$ $\int\frac{v^2}{1+v^3}dv=\int dx$ $\frac{1}{3}\ln\left\|1+v^3\right\|=x+c$ $v>0, 1+v^3=ce^{3x}$ $x=0, v=0, c=1$ $1+v^3=e^{3x}=e^{\ln 3^3}=27$ $v=\sqrt[3]{26}$
Specific behaviours
P separates variables and uses appropriate form of acceleration P integrates and uses a constant P solves for constant P determines exact speed

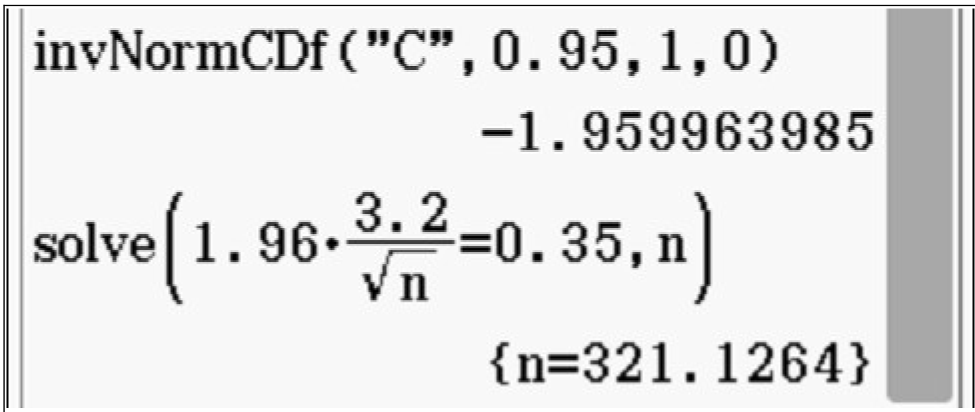
Q7 (2, 3 & 3 = 8 marks)
A lolly company makes jelly beans where the mass of one jelly bean is normally distributed with a mean of 23.4 mg and a standard deviation of 3.2 mg. (Note: 1g=1000mg)

a) Determine the probability to two decimal places that the total mass of 85 jelly beans is more than two grams.

Solution
$\bar{X} \sim N\left(23.4, \left[\frac{3.2}{\sqrt{85}}\right]^2\right)$ $P\left(\bar{X} > \frac{2000}{85}\right)$    Prob=0.35
Specific behaviours
P uses correct parameters   P determines prob to 2 dp

b) Given that the probability that the mean mass of a jelly bean differs from the population mean by more than 0.35 mg is 5%, determine n, the number of jelly beans that need to be sampled.

Solution



Sample size = 322

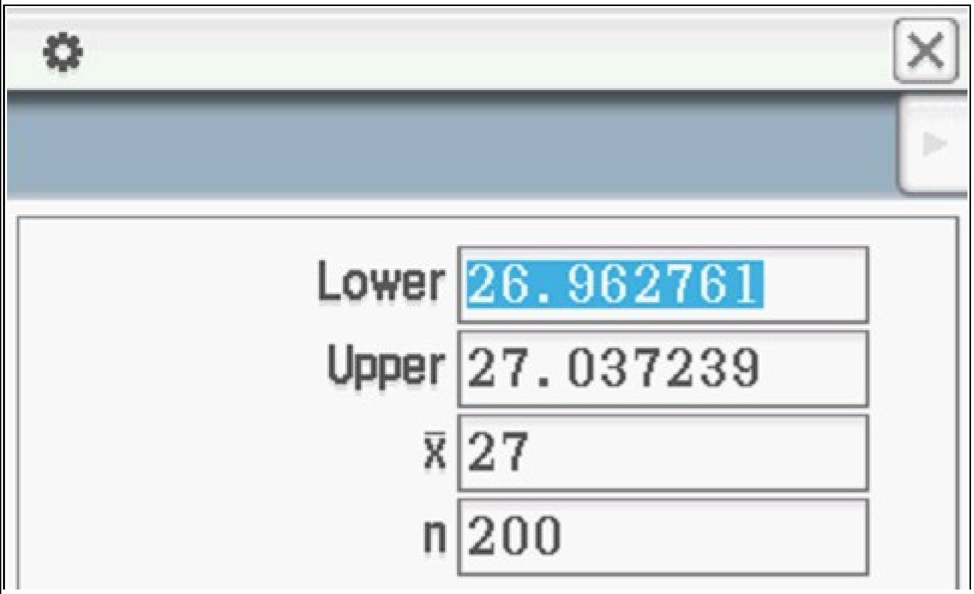
Specific behaviours
P states correct z score (must show)
P sets up equation for n
P rounds up

c) On a particular day the operator of a machine that makes jelly beans is suspected of being faulty. A sample of 200 jelly beans had a sample standard deviation of 3.8 mg with a total mass of 5.4 grams. Present a mathematical argument to either support or to dismiss such a claim.

Solution

$$\bar{X} \sim N\left(\frac{5400}{200}, \left[\frac{3.8}{\sqrt{200}}\right]^2\right)$$

95% confidence interval



23.4 does not lie in interval hence there is evidence that machine is faulty and a different population mean is likely.

OR

As not every confidence interval contains the true value of mean, no inference can be made on one confidence interval

Specific behaviours
P determines a confidence interval for population mean using sample given. P looks to see if 23.4 lies in interval or discusses that not every interval contains u P states that fault is likely as 23.4 lies outside interval OR no inference possible