



PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.

Test 1
Differentiation , applications and Optimisation.
Basic antidifferentiation
Semester One 2018
Year 12 Mathematics Methods
Calculator Free

Date: Friday 16th February 7.45am

You may have a formula sheet for this section of the test.

Total_____/21

20 Minutes

Question 1

(3 marks)

Given that the function f has a rule of the form $f(x) = ax^2 + bx$ and $f(1) = 6$ and $f'(1) = 0$, find the values of a and b .

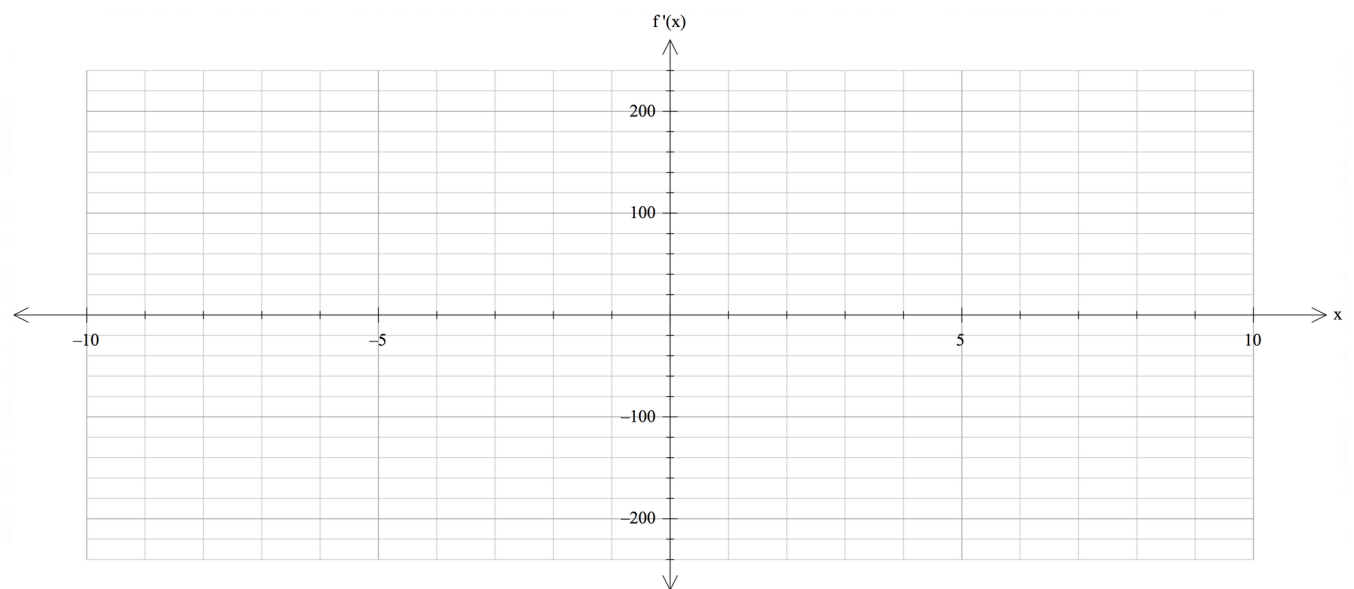
Question 2

(8 marks)

Consider the gradient function $f'(x) = 12x^2 - 120x + 324$

(a) Graph the gradient function

(4 marks)



(b) What kind of feature is at the point $(-5, -225)$ on the graph of $f(x)$?

(2 marks)

(c) What kind of feature is at the point $(-2, -144)$ on the graph of $f(x)$?

(2 marks)

Question 3

(6 marks)

Clearly showing your use of the product, quotient or chain rule differentiate the following.

(YOU MAY LEAVE YOUR ANSWERS IN UNSIMPLIFIED FORM)

a) $10 p^{\frac{1}{2}}$

(2 marks)

b) $\frac{1}{\sqrt{x+2}}$

(1 marks)

c) Consider the function $f(x) = (x-1)^2(x-2) + 1$

(3 marks)

If $f'(x) = (x-1)(ux+v)$, where u and v are constants, use calculus to find the values of u and v .

Question 4

(4 marks)

The time T seconds, for one complete swing of a pendulum of length l m, is given by the rule $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant.

- (a) Determine $\frac{dT}{dl}$, (2 marks)

- (b) Using the formula $\partial T \approx \frac{dT}{dl} \times \partial l$, find the approximate increase in T when l is increased from 1.6 to 1.7. Give the answer in terms of g . (2 marks)



PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.

Test 1
Differentiation , applications and Optimisation.
Basic antidifferentiation
Semester One 2018
Year 12 Mathematics Methods
Calculator Assumed

Date: Friday 16th February 7.45am_

You may have

- a formula sheet
- one page of A4 notes, one side
- a scientific calculator
- a Classpad

Total_____/25

25 minutes

Question 1

(9 marks)

A model train travels on a straight track such that its acceleration after t seconds is given by

$$a(t) = pt - 13 \text{ cm/s}^2, 0 \leq t \leq 10, \text{ where } p \text{ is a constant.}$$

(a) Determine the initial acceleration of the model train.

(1 mark)

The model train has an initial velocity of 5 cm/s . After 2 seconds, it has a displacement of -50 cm .

A further 4 seconds later its displacement is 178 cm .

(b) Determine the value of the constant p . (4 marks)

(c) When is the model train at rest? (2 marks)

(d) How far did the model train travel in the 8th second. (2 marks)

Question 2

(6 marks)

A beverage company has decided to release a new product. “Modmash” is to be **sold** in 375 mL cans that are perfectly cylindrical. {Hint: $1 \text{ mL} = 1 \text{ cm}^3$ }

- (a) If the cans have a base radius of $x \text{ cm}$ show that the surface area of the can, S , is given

by: $S = 2\pi x^2 + \frac{750}{x}$. (2 marks)

- (b) Using calculus methods, and showing full reasoning and justification, find the dimensions of the can that will minimise its surface area

(4 marks)

Question 3

(10 marks)

Let $f(x) = -x^3 + 3x^2 - 2x + 1$.

- (a) Use calculus to locate and classify all the stationary points of $f(x)$ and find any points of inflection. (6 marks)

- (b) On the axes provided sketch the graph of $f(x)$, $-1 \leq x \leq 4$, labelling all key features. (4 marks)