MATHEMATICS METHODS UNIT 3

Semester One

2019

SOLUTIONS

Calculator-free Solutions

1. (a) $y = -(4x+3)^{-\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{2}{(4x+3)^{\frac{3}{2}}}$ ✓

$m = 2 \quad n = \dfrac{3}{2}$ ✓

(b) $\dfrac{2}{(9)^{\frac{3}{2}}}$

$= \dfrac{2}{27}$ ✓ [3]

2. (a) $f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$4x^2(x-3) = 0$ ✓

$x = 0$ or $x = 3$ ✓

Stationary points (0, 0) and (3, −¿ 27) ✓

(b) $f''(x) = 12x^2 - 24x$ ✓

$f''(0) = 0$ $\therefore$ horizontal point of inflection occurs at (0, 0) ✓

Since $f''(3) > 0$, then minimum occurs at (3, -27) ✓✓

(c) (i) $f''(x) = 12x^2 - 24x$

$12x(x-2) = 0$

$\therefore$ $x = 0$ or $x = 2$ ✓

Point of oblique inflection at (2, −¿16) ✓

(ii) *m* = −¿16

$y = -16x + c$ ✓

$-16 = -16(2) + c$

$c = 16$

Tangent is $y = -16x + 16$ ✓ [11]

3. (a) $y = e^{-12x}$

$\dfrac{dy}{dx} = -\dfrac{12}{e^{12x}}$ ✓✓

(b) $-2\sin x\, e^{\cos x}$ ✓✓ [4]

END OF QUESTIONS

4. (a) The values of $f(x)$ are all positive ✓

$$\frac{5}{15}+\frac{4}{15}+\frac{3}{15}+\frac{2}{15}+\frac{1}{15}=1$$ ✓

$\therefore$ Probability function

(b) $$P(X<3)=\frac{9}{15}$$ ✓

(c) $$\frac{\frac{5}{15}}{\frac{14}{15}}=\frac{5}{14}$$ ✓✓ [5]

5. $$g(x)=-\cos x+\frac{1}{2}\sin 2x+c$$ ✓

$$1=-\cos\left(\frac{\pi}{2}\right)+\frac{1}{2}\sin\pi+c$$ ✓

$$1=c$$

$$g(x)=-\cos x+\frac{1}{2}\sin 2x+1$$ ✓ [3]

6. (a) $$f'(x)=\frac{(1-e^{2x})(-\sin x)-\cos x(-2e^{2x})}{(1-e^{2x})^2}$$ ✓✓

(b) $$\frac{dy}{dx}=6x\sin^4(2x-3)+24x^2\cos(2x-3)\sin^3(2x-3)$$ ✓✓ [4]

7. (a) (i) -3 ✓

(ii) 6+¿2) = 4 ✓✓

(iii) 5 units^2 ✓✓

(b) $$x=2e^{2t}+\frac{2}{3}(t+1)^{\frac{3}{2}}+c$$ ✓

$$1=2+\frac{2}{3}+c$$ ✓

$$c=-\frac{5}{3}$$

$$x=2e^{2t}+\frac{2}{3}(t+1)^{\frac{3}{2}}-\frac{5}{3}$$ ✓ [8]

8. (a) $E(X^2) - [E(X)]^2 = \frac{1}{2}$

$2p + 4p - (2p + 2p)^2 = 6p - 16p^2 = \frac{1}{2}$ ✓

$(32p^2 - 12p + 1) = 0$

$\therefore (8p - 1)(4p - 1) = 0$

$\therefore p = \frac{1}{8}$ or $\frac{1}{4}$ ✓

$\therefore E(X) = \frac{1}{2}$ or 1 ✓✓

(b) (i) 21 ✓

(ii) Standard deviation of $X = 3$

$|2| \times 3 = 6$ ✓✓ [7]

9. (a) $\left[\pi\sqrt{2x+3}\right]_{-1}^{3}$ ✓

$= 3\pi - \pi = 2\pi$ ✓

(b) $y = -\frac{2}{\pi}\cos\frac{\pi x}{2} + x + c$ ✓

$4 = 0 + 1 + c \quad \therefore c = 3$ ✓

Its path is $y = -\frac{2}{\pi}\cos\frac{\pi x}{2} + x + 3$ ✓ [5]

Calculator-assumed Solutions

10. (a) $A = \pi r^2 = \pi(3t + 1)^2$ ✓

(b) $\frac{dr}{dt} =$ 3 cm/s ✓

(c) $\frac{dA}{dt} = 6\pi(3t + 1)|t = 1$

$= 24\pi$ cm^2/s ✓

(d) $\delta A = \frac{dA}{dr} \times \delta r$

$= 2\pi(r)(0.05)$ ✓✓

When $r = 4$, $\delta A = 0.4\pi \approx 1.26$ cm^2 ✓ [6]

21. In triangle $h = \sqrt{15^2 - x^2}$ therefore A= $\frac{1}{2}bh = x(\sqrt{15^2 - x^2})$ ✓

$A'(x) = \frac{225 - 2x^2}{\sqrt{225 - x^2}} = 0$ ✓

$x = \pm\frac{15\sqrt{2}}{2} \approx 10.6066$ (Discard negative value for x.) ✓

$A''(10.6066) = -3.999 \quad \therefore A'' < 0$ therefore maximum ✓

Maximum area is 112.5 cm^2 ✓ [5]

22. (a) $v(t) = \int \cos t - 4\sin(2t) = \sin t + 2\cos 2t + c$ ✓

$\sin 0 + 2\cos 0 + c = 2 \quad \therefore c = 0$

$v(t) = \sin t + 2\cos 2t = 0$ ✓

The particle changes direction when $t = 1.003$ s or $t = 2.139$ s ✓

(b) $\int_0^{\pi} |\sin t + 2\cos 2t| \, dt = 3.476$ m^2 ✓✓ [5]

23. (a)

$g(0)$	$g(0.2)$	$g(0.4)$	$g(0.6)$	$g(0.8)$	$g(1)$
1	0.96	0.85	0.70	0.53	0.37

✓

Area from left = 0.2 (1 + 0.96 + 0.85 + 0.70 + 0.53) = 0.808 ✓

Area from right = 0.2 (0.96 + 0.85 + 0.70 + 0.53 + 0.37) = 0.682 ✓

Average = 0.745 = 0.75 units^2 ✓

(b) As the width of the rectangle tends to 0,
the more accurate will be the area. ✓ [5]

24. (a) $t = 110$ years

$A = e^{-0.047069} = 0.95402$ grams ✓

Therefore 4.598 % has decayed. ✓

(b) $0.5 = e^{-0.0004279t}$ ✓

$t = 1619.88 \approx 1620$ years ✓

(c) $0.5 = e^{k \times 3.8}$

$\therefore k = -0.1824$ ✓

$A = 10e^{-0.1824(15)}$ ✓

0.648 mg of radon remains ✓ [7]

✓✓ [8]

19. (a) (i) Each washing machine is an independent trial. ✓
There are two outcomes: Success (a washing machine works) and Failure (a washing machine is broken.) ✓

(ii) E(X) = p = 0.2 ✓
Var (X) = p(1 −¿ p) = 0.2 x 0.8 = 0.16
Standard deviation = 0.4 ✓

(b) (i) This means that the first mobile phone was selected and found to be defective. (0 non-defective mobile phones were found) ✓
P(X = 0) = 0.03 ✓

(ii) This means that the first two mobile phones selected were found to be non-defective but the third mobile phone selected was defective. ✓
P(X = 2) = 0.97 x 0.97 x 0.03 = 0.028227 ✓

(iii) 0.97^5 x 0.03 = 0.02576 ✓✓

(iv) 1 - [P(0) + P(1) + P(2)] = 0.91267 ✓✓ [12]

20. (a) $y = x(x-1)(x-3)$
Graph cuts the x- axis at 0, 1 and 3. ✓

$$\int_0^1 \left(x^3 - 4x^2 + 3x\right)\,dx = \frac{5}{12}$$ ✓

$$-\int_1^3 \left(x^3 - 4x^2 + 3x\right)dx = \frac{8}{3}$$ ✓

Area = $\frac{37}{12} = 3\frac{1}{12}$

(b) $h'(x) = 4e^{2x^2+x} + c$ ✓

$h'(0) = 1 \therefore 1 = 4e^0 + c \quad c = -3$ ✓

$h'(1) = 4e^3 - 3$ which is the gradient at $x = 1$ ✓

(c) Graphs intersect at x = 0.6349, 2.5067 ✓

$$\int_0^{0.6349} \cos 2x - \frac{1}{2}\sin x \;\; dx + \int_{0.06349}^{2.5067} \frac{1}{2}\sin x - \cos 2x \; dx + \int_{2.5067}^{\pi} \cos 2x - \frac{1}{2}\sin x \; dx$$

= 2.5203 units^2 ✓✓ [9]

Or: $\int_0^{\pi} \left| \cos 2x - \frac{1}{2}\sin x \right| dx$ = 2.5203 units^2

11. (a) Let P(X = 0) = k then $k + k + 2k + \frac{3}{2}(2k) + 3k + 4k = 1$

$k = \frac{1}{14}$ ✓

X	0	1	2	3	4	5
P(X = x)	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{3}{14}$	$\frac{4}{14}$
Frequency	23	23	46	69	69	92

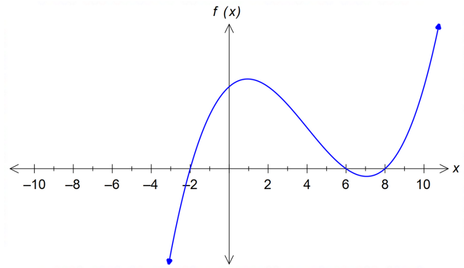
✓✓✓

(b) mode = 5 ✓

mean = $\Sigma\, x.P(X = x) = \frac{23}{7} \approx 3.2857$ ✓

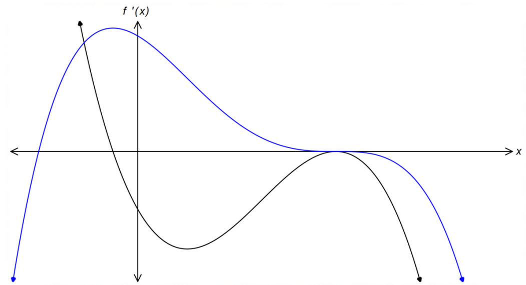
(c) $\frac{5}{14}$ ✓ [7]

12. (a)



✓✓✓

(b)



✓✓

(c) $F'(x) = 2(2x+3)\sin(2x+3)$ ✓ [6]

13. (a) $f'(t) = \frac{2}{50}f(t) = 0.04\ f(t)$

$\therefore\ k = 0.04$ ✓

$2A = Ae^{0.04t}$ ✓

$t = 17.3$ days ✓

(b) (i) $\int_0^8 2e^{2t-7}\ dt = 8103.083\ m^2$ ✓

(ii) $\int_9^{10} 2e^{2t-7}\ dt = 382\ 539.25\ m^2$ ✓

(iii) The exponential growth of the area becomes too large too quickly for the model to be realistic. ✓ [6]

14. (a) $s'(t) = v(t) = -3t^2 + 27 = 0$ ✓

$\therefore\ t = 3$ (discard -3) ✓

The maximum distance from O occurs at $t = 3$ seconds and then the particle turns back towards O.

(b) $s(t) = 27t - t^3 = 0$

$\therefore\ t = 3\sqrt{3}$ (discard $t = 0$ and $-3\sqrt{3}$) ✓

$\therefore\ v(3\sqrt{3}) = -54$ ✓

Therefore speed when the particle returns to O is 54 cm/s. ✓ [5]

15. $\left[x^2\sqrt{1-x^2}\right]_{\frac{1}{2}}^{t} = t^2\sqrt{1-t^2} - \frac{1}{4}\sqrt{\frac{3}{4}}$ ✓✓

$A(t) = t^2\sqrt{1-t^2} - \frac{\sqrt{3}}{8}$ ✓ [3]

16. (a) Profit = Revenue −¿ Cost

$C = 50 + 0.8n$ and $R = n(3.5 - 0.01n)$ ✓

$P = 3.5n - 0.01n^2 - 50 - 0.8n = 2.7n - 0.01n^2 - 50$ ✓

(b) $2.5 = 3.5 - 0.01n$ ✓

$\therefore\ n = 100$ He will sell 100 figs ✓

$P'(n) = 2.7 - 0.02n$ where $n = 100$ ✓

The marginal profit is 0.7 = 70 cents ✓ [6]

17. (a) (i)

Y	0	1	2	3
P(Y= y)	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{15}{28}$	$\frac{5}{28}$

✓✓

(ii) $E(Y) = \frac{105}{56} \approx 1.875$ ✓

$Var(Y) = \frac{225}{448} \approx 0.50$ ✓

(iii) $\frac{15}{56}$ ✓

(b) (i) $X \sim Bin(10,\ 0.7)$

$P(X = 7) = 0.2668$ ✓

(ii) $X \sim Bin(26,\ 0.7)$

$P(X \le 13) = 0.0255$ ✓

(c) $E(X) = np = 12$ ✓

$Var(x) = npq = np(1-p) = 9$ ✓

$n = 48$ and $p = \frac{1}{4}$ ✓ [10]

18. (a) The amount of water cannot be a negative amount. If no water flows in or out the functions can equal zero. ✓✓

(b) $f(t) - w(t)$ megalitres ✓

(c) $\int_{t_0}^{t_1} f(t)\ dt$ ✓

(d) $\int_0^{30}\left(10 - \frac{1}{2}t - 2\sin\ 2\pi t\right)dt = 75$ ✓

75 megalitres of water. ✓

(e)