PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Test 2 Year 12

Student name: _______________ Teacher name: _______________

Task type: **Response/Investigation**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: 7

Materials required: Upto 3 classpads/calculators

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper SINGLE SIDED, and up to three calculators approved for use in the WACE examinations

Marks available: **40 marks**

Task weighting: **13%**

Formula sheet provided: no but formulae stated on page 2

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = \lvert z\rvert = \sqrt{a^2+b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan\theta = \frac{b}{a}, \quad -\pi < \theta \le \pi$
$\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$	$\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = \lvert z\rvert^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{\lvert z\rvert^2}$
$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$	$\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$
Polar form	
$z = a + bi = r(\cos\theta + i\sin\theta) = r\,cis\,\theta$	$\bar{z} = r\,cis\,(-\theta)$
$z_1 z_2 = r_1 r_2\,cis\,(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,cis\,(\theta_1 - \theta_2)$
$cis(\theta_1 + \theta_2) = cis\,\theta_1\,cis\,\theta_2$	$cis(-\theta) = \frac{1}{cis\,\theta}$
De Moivre's theorem	
$z^n = \lvert z\rvert^n\,cis\,(n\theta)$	$(cis\,\theta)^n = \cos n\theta + i\sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\frac{\theta + 2\pi k}{q} + i\sin\frac{\theta+2\pi k}{q}\right)$, for k an integer	

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Length of arc $= r\theta$
$a^2 = b^2 + c^2 - 2bc\cos A$	Area of segment $= \frac{1}{2}r^2(\theta - \sin\theta)$
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	Area of sector $= \frac{1}{2}r^2\theta$
Identities	
$\cos^2 x + \sin^2 x = 1$	$1 + \tan^2 x = \sec^2 x$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	$\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$
$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$	$\sin 2x = 2\sin x\cos x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$	$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$
$\cos A\cos B = \frac{1}{2}(\cos(A - B) + \cos(A + B))$	$\sin A\cos B = \frac{1}{2}(\sin(A + B) + \sin(A - B))$
$\sin A\sin B = \frac{1}{2}(\cos(A - B) - \cos(A + B))$	$\cos A\sin B = \frac{1}{2}(\sin(A + B) - \sin(A - B))$

Q1 (3 marks)

Consider the inequality $|3x - 7| \le a$ which is only true for $b \le x \le 9$ where $a \& b$ are constants. Determine the values of $a \& b$.

Q2 (4 marks)

Consider two ships $A \& B$ moving with constant velocities and the following noted position vectors.

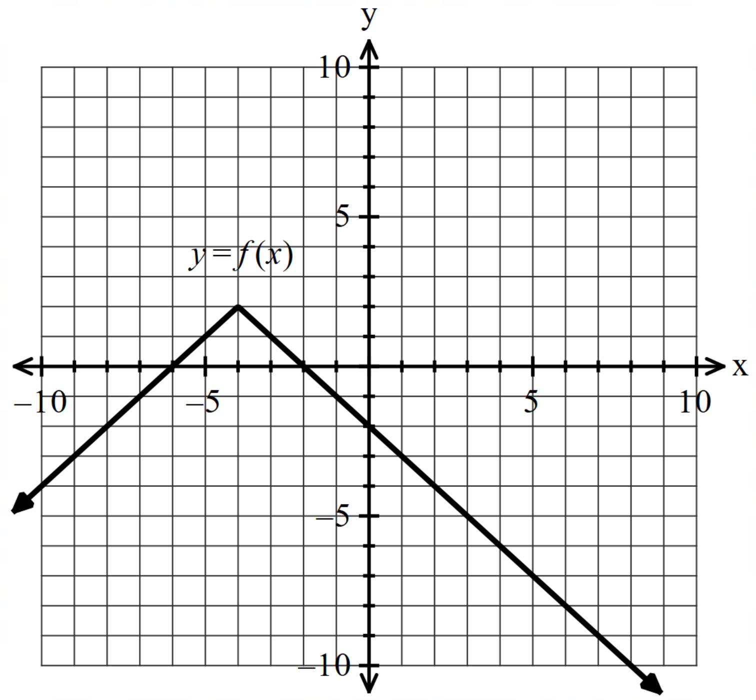
$$r_A = \begin{pmatrix} 2 \\ -18 \end{pmatrix} km \quad v_A = \begin{pmatrix} 2 \\ 7 \end{pmatrix} km/h \quad at\ 11:00am$$

$$r_B = \begin{pmatrix} -6 \\ 44 \end{pmatrix} km \quad v_B = \begin{pmatrix} 4 \\ -6 \end{pmatrix} km/h \quad at\ 11:30am$$

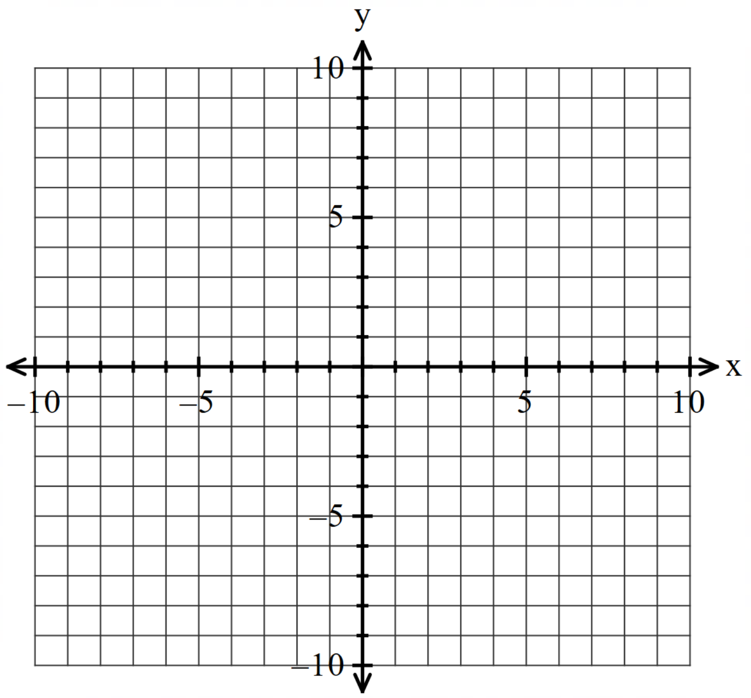
Determine if the ships will collide and if they do, determine the time and position of this collision.

Q3 (2, 3 & 3 = 8 marks)

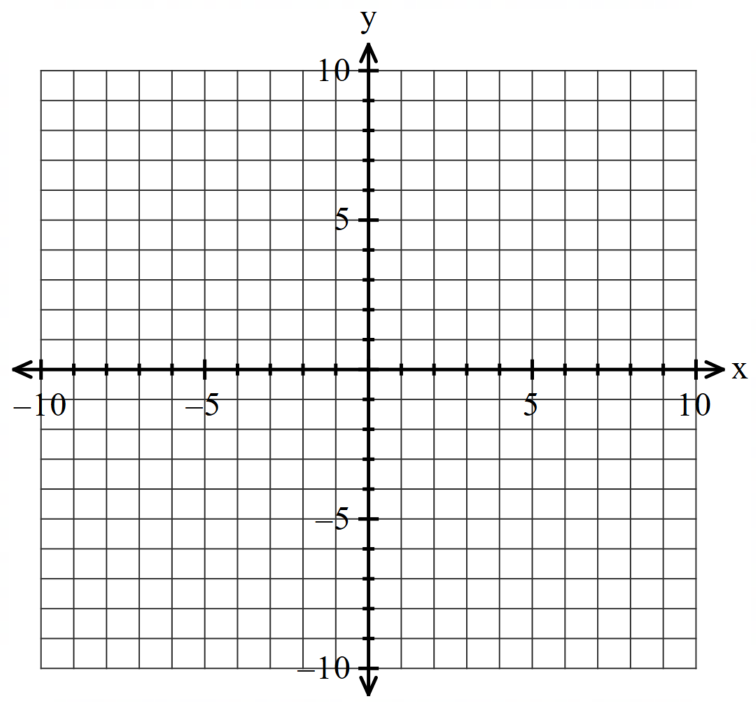
Consider the function $y = f(x)$ below.



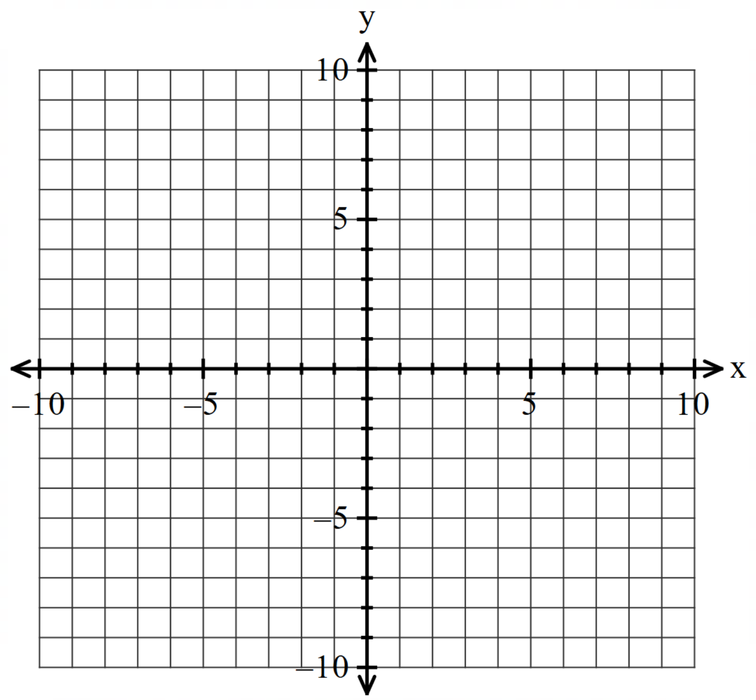
a) Sketch $|f(x)|$ on the axes below.



b) Sketch $f(-|x|)$ on the axes below.

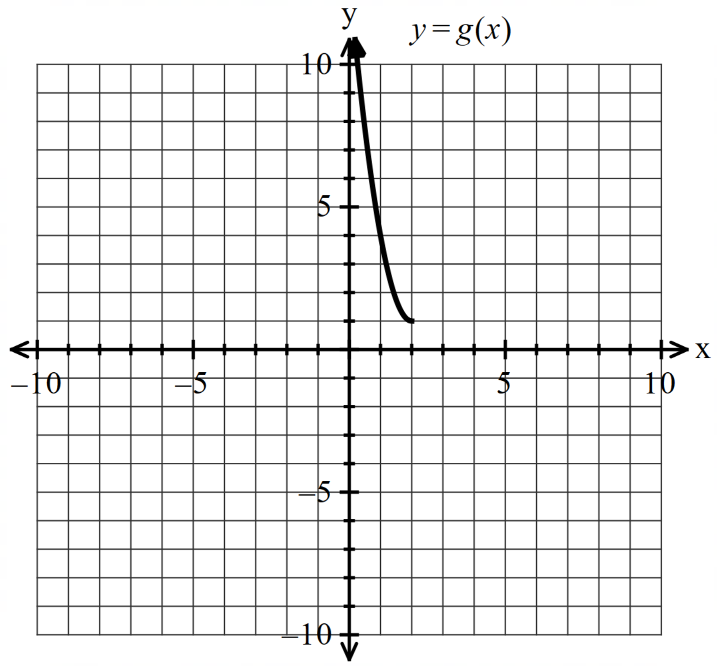


c) Sketch $\frac{1}{f(x)}$ on the axes below.



Q4 (2, 3, 1 & 3 =9 marks)

Consider $g(x) = 3x^2 - 12x + 13$ for $x \leq 2$ which is plotted below.



a) Sketch $g^{-1}(x)$ on the axes above.

b) Determine the rule for $g^{-1}(x)$ showing full working and the domain.

Q4 cont-

c) Determine $g^{-1} \circ g(x)$.

d) Determine all value(s) of x such that $g(x) = g^{-1}(x)$. Show reasoning for full marks.

Q5 (5 marks)

Consider two moving objects $P \& Q$ which at time $t = 0$ seconds have the following positions and constant velocities.

$$r_P = \begin{pmatrix} -8 \\ 7 \end{pmatrix} m \quad v_P = \begin{pmatrix} 5 \\ -4 \end{pmatrix} m/s$$

$$r_Q = \begin{pmatrix} 11 \\ -6 \end{pmatrix} m \quad v_Q = \begin{pmatrix} -3 \\ 10 \end{pmatrix} m/s$$

Determine the minimum distance between them **using vectors** and the time that this occurs.

Q6 (5 marks)

Consider the line $r = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and the circle $\left| r - \begin{pmatrix} 3 \\ \alpha \end{pmatrix} \right| = 9$ where α is a constant.

Determine all possible values of α such that:

i) The line will be a tangent to the circle.

ii) The line crosses the circle at two points.

iii) The line will never meet the circle.

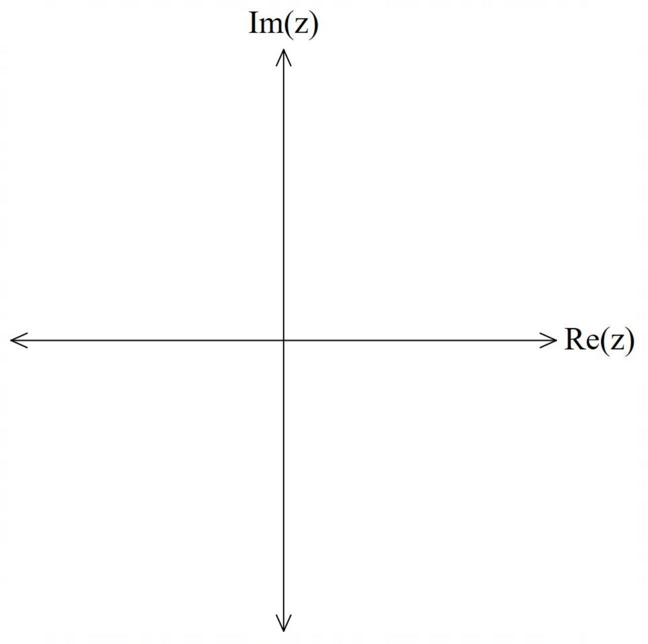
Q7 (2 & 4 = 6 marks)

Consider complex numbers $z \, \& \, w$. It is known that:

$|z| = 10, \; Arg(z) = \theta$ where $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$

$w = z + k$ such that $Arg(w) = \pi - \theta$ where $\text{Im}(k) = 0, \; k > 0$

a) Represent this information on the Argand Diagram below.



b) Determine a **simplified** expression for k in terms of θ. Justify your answer.

Working out space

Working out space