



PERTH MODERN SCHOOL

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Test 2
Area Under Curve, F.T.O.C. Exponential Functions
Semester One 2018
Year 12 Mathematics Methods
Calculator Assumed

Name:Date: Friday 16th March 7.45am

You may have a formula sheet for this section of the test.

Teacher:

_____ Mr McClelland

_____ Mrs. Carter

_____ Mr Gannon

_____ Ms Cheng

_____ Mr Staffe

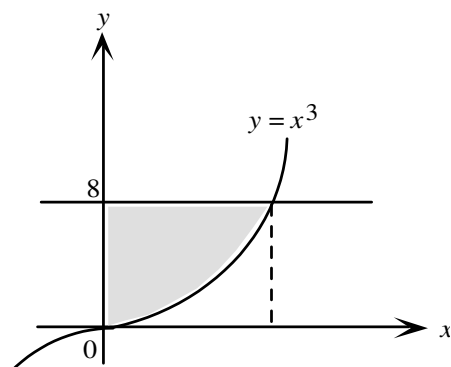
_____ Mr Strain

Total _____/40

45 minutes +5 minutes READING

Question 1**(2 marks)**

The graphs with equations $y = x^3$ and $y = 8$ are shown. Write an expression that shows what the area of the shaded region is equal to:



Question 2**(5 marks)**

(a) Calculate $f'(0)$ when $f(x) = e^{2x}(1+5x)^3$.

(3 marks)

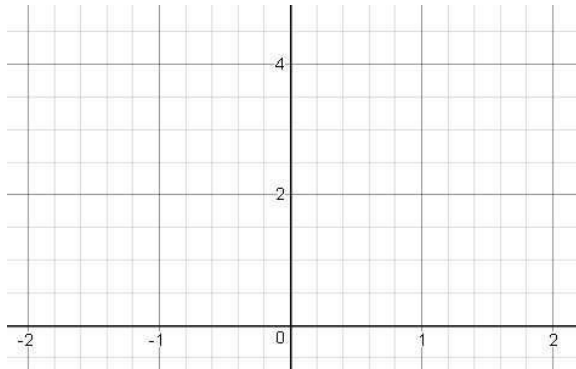
(b) Determine $\frac{d}{dx} \int_x^5 \sqrt{t^2+1} dt$.

(2 marks)

Question 3**(4 marks)**

Show how to calculate the area of the region enclosed by the curves with equations $y = x^2 + 1$ and $y = 4 - x^2$ and the lines $x = -1$ and $x = 1$.

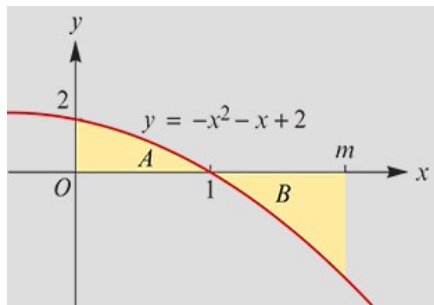
Draw a sketch to help show your solution. Show your working.



Question 4**(4 marks)**

The graph of $y = -x^2 - x + 2$ is shown.

Find the value of m such that A and B have the same area.

**Question 5****(4 marks)**

Given $\frac{dy}{dx} = ae^{-x} + 2$ and that when $x = 0$, $\frac{dy}{dx} = 5$ and $y = 1$,

Find the value of y when $x = 2$.

Question 6**(8 marks)**

A group of biologists has decided that colonies of a native Australian animal are in danger if their populations are less than 1000. One such colony had a population of 2300 at the start of 2011. The population was growing continuously such that $P = P_0 e^{0.065t}$ where P is the number of animals in the colony t years after the start of 2011.

- (a) Determine, to the nearest 10 animals, the population of the colony at the start of 2014. (2 marks)
- (b) Determine the rate of change of the colony's population when $t = 2.5$ years. (2 marks)
- (c) At the beginning of 2017, a disease caused the colony's population to decrease continuously at the rate of 8.25 % of the population per year. If this rate continues, when will the colony become "in danger"? Give your answer to the nearest month. (4 marks)

Question 7**(9 marks)**

(a) What is the sign of $f(x) = x^3 - 6x^2 + 12x - 8$ from $x=0$ to $x=2$?

(1 mark)

(b) What is the sign of $f(x) = x^3 - 6x^2 + 12x - 8$ from $x=2$ to $x=4$?

(1 mark)

(c) Find $\int_0^4 (x^3 - 6x^2 + 12x - 8) dx$.

(2 mark)

(d) Find $\int_0^2 (x^3 - 6x^2 + 12x - 8) dx$.

(2 mark)

(e) What is the area between $f(x) = x^3 - 6x^2 + 12x - 8$ and the x -axis from $x=0$ to $x=4$?

(2 marks)

(f) Explain why the answers to (c) and (e) are different.

(1 mark)

Question 8**(4 marks)**

The population of mice in a closed habitat is known to increase according to the function:

$P'(t) = \frac{t}{3} + 6$, where $P'(t)$ is measured in hundreds of mice per month and t is measured in months. The measurement of the population commences at $t=0$,

- (a) What is the total change in the population in the first 3 months after measuring commenced? (2 marks)

- (b) How long will it take for the increase in the population of mice to reach 4200? (2 marks)

