



MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 (7 marks)

Differentiate with respect to x . Simplify your answers. (IN FULLY FACTORISED FORM)

(a) $\frac{2x+1}{5-x} \quad \frac{v}{u} \quad \frac{u'v - uv'}{u^2} = \frac{\frac{v^2}{u^2}}{2(5-x) - (2x+1)(-1)}$ (2 marks)

$$= \frac{10 - 2x + 2x + 1}{(5-x)^2}$$

$$= \frac{(5-x)^2}{(5-x)^2}$$

(b) $\frac{(2x^4 + 2)(4 - x)^8}{u^4 + uv'}$

$= 8x^3(4-x)^8 + (2x^4 + 2) \cdot 8(4-x)^7(-1)$ (3 marks)

$= 8x^3(4-x)^8 - 16x^4(4-x)^7 - 16(4-x)^7$

$= 8(4-x)^7 [x^3(4-x) - (2x^4 + 2)]$ ✓

$= 8(4-x)^7 (4x^3 - x^4 - 2x^4 - 2)$

$= 8(4-x)^7 (-3x^4 + 4x^3 - 2)$ ✓

$$= 3 \cos(3x) + 8 \sin(2x)$$

(c) $\sin(3x) - 4 \cos 2x$

(2 marks)

See next page

7

Question 2

(2 marks)

the value of x , in terms of a and b .

Find where the equation $y = 2ax^2 + b^2x$ has a derivative equal to zero given that a and b are positive constants.

$$If \quad y = 2ax^2 + b^2x$$

$$\frac{dy}{dx} = 4ax + b^2$$

$$0 = 4ax + b^2 \quad \checkmark$$

$$4ax = -b^2$$

$$\therefore x = \frac{-b^2}{4a} \quad \checkmark$$

See next page

2

Question 9

(6 marks)

A particle is in rectilinear motion and its velocity, v , at any time t seconds is given by

$$v = \cos(2t) \text{ ms}^{-1}$$

- (i) Determine an expression for $\frac{dv}{dt}$, the acceleration of the particle. (1 mark)

$$\frac{dv}{dt} = -2\sin 2t \quad \checkmark$$

- (ii) What is the velocity and the acceleration of the particle when $t = \pi$? (2 marks)

$$t = \pi, \quad v = \cos 2\pi = 1 \quad \checkmark$$

$$\frac{dv}{dt} = -2\sin 2\pi = 0 \quad \checkmark$$

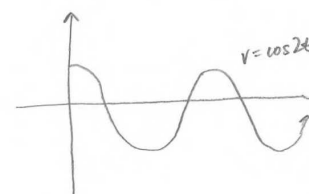
- (iii) What feature of the ~~nature~~ ^{velocity} is indicated by the value of the acceleration when $t = \pi$. (1 mark)

velocity
not changing
NOT OK!
2

$\frac{dv}{dt} = 0$ indicates that $t = \pi$, gives local max/min

Max value of $v = \cos 2t$ is 1 \therefore Max @ $t = \pi$ \checkmark

- (iv) During a particular second, the acceleration increases from -1.8 ms^{-2} to 1.5 ms^{-2} . Describe the velocity of the particle during this second. (1 mark)



Acc is gradient of velocity
Gradient '-' before min
'+' after min

During this second, v decreasing until min
then v increases. \checkmark

End of questions

6

Question 7

(2,2 marks)

A particle moves such that its displacement from the origin O , at time t seconds second, is x meters, where:

$$x(t) = \frac{2t^4 - t^3 - 28t^2}{t+4} + 5 \quad \text{for } t \geq 0.$$

Determine:

(i) the distance travelled by the particle in the first 5 seconds.

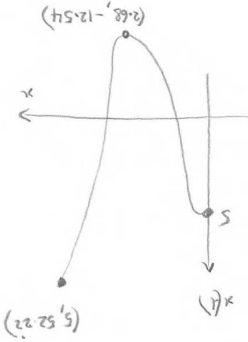
$$\text{When } t = 5 \quad |x(5)| = 52.2$$

$$\therefore \text{Dist} = 2 \times 12.54 + 52.22 + 5$$

$$\therefore \text{Dist} = 82.3 \text{ m.}$$

(iii) the acceleration of the particle when $t = 5$.

$$x''(5) = 42.35 \text{ ms}^{-2} \quad \text{on cp.}$$



(3 marks)

Use calculus to approximate the small change in the side length of a cube (correct to 4 decimal places) when its area changes from 486cm^2 to 487cm^2 .

Let cube length = x

$$\therefore A = 6x^2$$

$$\frac{dA}{dx} \sim \frac{\delta A}{\delta x}$$

$$\frac{dA}{dx} = 12x$$

$$dA = 12x \cdot dx$$

$$\delta x = \frac{dA}{12x}$$

$$\delta x = \frac{1}{108}$$

$$\delta x = 0.0093$$

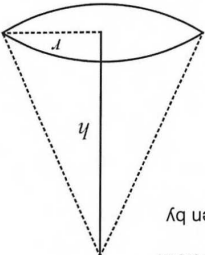
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Question 3

(2, 4 marks)

A tent in the shape of a cone is to be pitched. A bamboo frame is needed for the circumference of the base and the height of the cone. 8 metres of bamboo is to be used for the framework, represented by the solid lines in the diagram below.



(a) Show that the volume V , of the tent in terms of its radius r , is given by

$$V = \frac{8}{3}\pi r^2 - \frac{3}{2}\pi r^3$$

$$2\pi r + l = 8$$

$$\therefore V = \frac{3}{8}\pi r^2 \times l$$

$$V = \frac{3}{8}\pi r^2 (8 - 2\pi r)$$

$$V = \frac{3}{8}\pi r^2 - \frac{3}{2}\pi r^3$$

(b) Determine the radius of the tent that will maximise the volume, leaving your answer in terms of π . You are **not** required to prove it is a maximum.

$$\frac{dV}{dr} = \frac{3}{4}\pi r - 2\pi r^2$$

$$\frac{dV}{dr} = 0$$

$$\therefore \frac{3}{4}\pi r = 2\pi r^2$$

$$\frac{3}{4} = 2r$$

$$\frac{3}{8}\pi \times \frac{1}{2} = r$$

$$\frac{3\pi}{8} \text{ m} = r$$

End of Questions

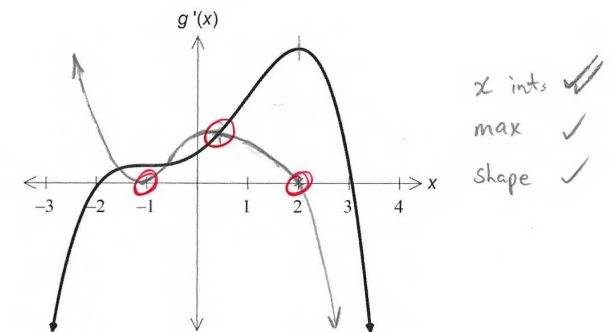
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End of questions

Question 6

(4 marks)

The graph of $y = g'(x)$ is sketched below.
On the same axes, sketch $y = g''(x)$.



See next page

4

Question 5

Consider the functions $f(x) = ax^3 + \frac{x}{b}$ with $f'(1) = 9$ and $f''(1) = 6$. Determine the values of a and b .

$$f'(x) = 3ax^2 - \frac{1}{b}x^{-2}$$

$$9 = 3a - \frac{1}{b} \quad \text{--- (1)}$$

$$f''(x) = 6ax + \frac{2}{b}x^{-3}$$

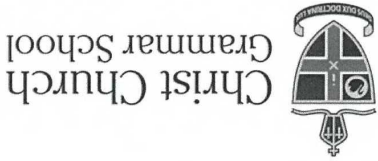
$$6 = 6a + \frac{2}{b} \quad \text{--- (2)}$$

Solve (1) & (2) simultaneously

$$a = 2, \quad b = -3$$

See next page

5



MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Student name _____
Teacher name _____

Solutions

Time and marks available for this section
Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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See next page

Question 4

(8 marks)

- (a) Using the rule $f(x+h) = f(x) + h \times f'(x)$ for the function $f(x) = \frac{1}{\sqrt{x}}$ (4 marks) find an approximation for $\frac{1}{\sqrt{1.05}}$ as x increases from 1 to 1.05. Your answer should be expressed as a fraction.

$$\begin{aligned}
 f(x+h) &= x^{-1/2} + 0.05 \times (-1/2)x^{-3/2} \\
 &= 1 + 0.05 \times (-1/2)(1) \\
 &= 1 + \frac{1}{20} \times (-1/2) \\
 &= 1 - \frac{1}{40} \\
 &= \frac{39}{40} \quad \checkmark \quad 0.975
 \end{aligned}$$

$x=1$
 $h=0.05$ ✓

- (b) For the function $f(x) = \sin(2x)$ find: (1,3 marks)

- (i) the instantaneous rate of change when $x = \frac{\pi}{8}$.

$$f'\left(\frac{\pi}{8}\right) = \sqrt{2} \quad \checkmark$$

- (ii) the average rate of change in terms of π over the interval $\frac{\pi}{6} \leq x \leq \frac{\pi}{4}$.

$$\begin{aligned}
 x &= \frac{\pi}{6}, \quad y = \frac{\sqrt{3}}{2} \\
 x &= \frac{\pi}{4}, \quad y = 1
 \end{aligned}$$

Ave Rate of Change

$$\begin{aligned}
 \frac{y_2 - y_1}{x_2 - x_1} &= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{\pi}{4} - \frac{\pi}{6}} \\
 &= \frac{6(2 - \sqrt{3})}{\pi} \\
 &= (0.512) \quad \checkmark
 \end{aligned}$$

See next page

8