Perth Modern School
End of Year Examination, 2012

Question/Answer Booklet

PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.

MATHEMATICS 3C/3D
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

In words ______________________________

Your name ______________________________

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Published by WA Examination Papers
PO Box 445 Claremont WA 6910

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

Additional working space

Question number: _________

Additional working space

Question number: __________

Section One: Calculator-free **(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(4 marks)**

For the two independent events A and B, $P(A) = 0.3$ and $P(B) = 0.1$.

Calculate

(a) $P(\overline{B})$ (1 mark)

$\boxed{0.9}$

(b) $P(A \cap B)$ (1 mark)

$\boxed{0.1 \times 0.3 = 0.03}$

(c) $P(A \cup \overline{B})$ (1 mark)

$\boxed{0.3 + 0.63 = 0.93}$

(d) $P(\overline{A} \mid B)$ (1 mark)

$\boxed{0.7}$

See next page

Question 2 **(5 marks)**

Solve the system of equations

$$3x + 2y + 6z = 3$$
$$x + 3y + 4z = 9$$
$$2x + 8 = 2z + y$$

$$\begin{aligned}
3x + 9y + 12z &= 27 \\
3x + 2y + 6z &= 3 \\
7y + 6z &= 24 \\[1em]
2x + 6y + 8z &= 18 \\
2x - y - 2z &= -8 \\
7y + 10z &= 26 \\[1em]
4z &= 2 \\
z &= 0.5 \\[1em]
7y &= 24 - 6(0.5) \\
y &= 3 \\[1em]
x &= 9 - 3(3) - 4(0.5) \\
x &= -2
\end{aligned}$$

Question 7 **(7 marks)**

A closed cylindrical can of radius r cm has a volume of 250π cm^3.

(a) Show that the total surface area, A cm^2, of this can is given by $A = \dfrac{500\pi}{r} + 2\pi r^2$. (2 marks)

$$\begin{aligned}
V &= \pi r^2 h \\
250\pi &= \pi r^2 h \Rightarrow h = \frac{250}{r^2} \\[1em]
A &= 2\pi r^2 + 2\pi r h \\
&= 2\pi r^2 + 2\pi r \frac{250}{r^2} \\
&= \frac{500\pi}{r} + 2\pi r^2
\end{aligned}$$

(b) Determine the minimum possible surface area of the can and the radius and height required to achieve this optimum area. (5 marks)

$$\begin{aligned}
\frac{dA}{dr} &= -\frac{500\pi}{r^2} + 4\pi r \\
-\frac{500\pi}{r^2} + 4\pi r &= 0 \\
r^3 = 125 &\Rightarrow r = 5 \text{ cm} \\
h = \frac{250}{5^2} &= 10 \text{ cm} \\
A &= \frac{500\pi}{5} + 2\pi \times 5^2 \\
A &= 150\pi
\end{aligned}$$

Question 6 **(9 marks)**

(a) Determine $\int x(3x^2+6x)^4 + (3x^2+6x)^4 \, dx$ (3 marks)

$$\begin{aligned}
&= \int (x+1)(3x^2+6x)^4 dx \\
&= \frac{1}{6}\int (6x+6)(3x^2+6x)^4 dx \\
&= \frac{(3x^2+6x)^5}{6\times 5} + c \\
&= \frac{(3x^2+6x)^5}{30} + c
\end{aligned}$$

(b) Calculate the area bounded by the functions $f(x)=(x-2)^2-3$ and $g(x)=2x-4$.
(6 marks)

Solve $f(x)=g(x)$

$$\begin{aligned}
x^2-4x+4-3-2x+4&=0 \\
x^2-6x+5&=0 \\
(x-1)(x-5)&=0 \\
x=1,\ x&=5
\end{aligned}$$

Integrate to find area

$$\int_1^5 g(x)-f(x)dx$$

$$\begin{aligned}
&=-\int_1^5 x^2-6x+5dx \\
&=-\left[\frac{x^3}{3}-3x^2+5x\right]_1^5 \\
&=-\left(\frac{125}{3}-75+25\right)+\left(\frac{1}{3}-3+5\right) \\
&=10\tfrac{2}{3} \text{ square units}
\end{aligned}$$

Question 3 **(8 marks)**

(a) Differentiate the following with respect to x. There is no need to simplify your answer.

(i) $y=2x^3\sqrt{3-x^2}$ (2 marks)

$$\frac{dy}{dx}=6x^2\sqrt{3-x^2}+2x^3\left(\tfrac{1}{2}\right)(-2x)(3-x^2)^{-0.5}$$

(ii) $y=\dfrac{1+e^{3x-1}}{2e^{-x^2}}$ (3 marks)

$$y=\frac{1}{2}\left(e^{x^2}+e^{3x-1+x^2}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(2xe^{x^2}+(3+2x)e^{3x-1+x^2}\right)$$

or using quotient rule

$$\frac{dy}{dx}=\frac{\left(3e^{3x-1}\right)\left(2e^{-x^2}\right)-\left(1+e^{3x-1}\right)\left(-4xe^{-x^2}\right)}{\left(2e^{-2x^2}\right)^2}$$

(b) Simplify $\dfrac{d}{dx}\displaystyle\int_2^{x^2}\left(\frac{t^2}{3}\right)dt$ (3 marks)

$$\begin{aligned}
\frac{dy}{dx}&=\frac{\left(x^2\right)^2}{3}\times 2x \\
&=\frac{2x^5}{3}
\end{aligned}$$

Question 4 **(8 marks)**

A function is defined by $f(x) = 6x^2 - 2x^3$.

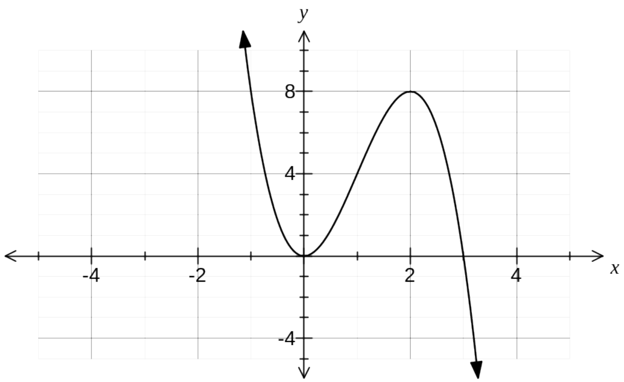
(a) Find the coordinates of the turning points of $f(x)$ and state their nature. (3 marks)

$f'(x) = 12x - 6x^2$
$= 0$ when $x = 0,\ x = 2$
(0, 0) is a minimum and (2, 8) is a maximum.

(b) Find the coordinates of the point of inflection of $f(x)$. (1 mark)

$f''(x) = 12 - 12x$
$= 0$ when $x = 1$
At (1, 4)

(c) Sketch the graph of $y = f(x)$. (3 marks)



(d) What is the maximum value of $f(x)$ in the interval $-2 \le x \le 4$? (1 mark)

$f(-2) = 6(4) - 2(-8) = 40$

Max value is 40.

Question 5 **(9 marks)**

Let $f(x) = \dfrac{1}{1-x}$ and $g(x) = e^{2x}$.

(a) Determine the domain of $f(g(x))$. (2 marks)

$1 - e^{2x} \ne 0$
$x \ne 0$

(b) Determine the range of $g(f(x))$. (3 marks)

$g(f(x)) = e^{\frac{2}{1-x}} \Rightarrow y > 0$

But $\dfrac{2}{1-x} \ne 0 \Rightarrow y \ne 1$

Hence range: $y > 0,\ y \ne 1$

(c) Solve $f(x) \ge 3 - 2x$. (4 marks)

$$\frac{1}{1-x} - 3 - 2x \ge 0$$
$$\frac{1-(3-5x+2x^2)}{1-x} \ge 0$$
$$\frac{2x^2-5x+2}{x-1} \ge 0$$
$$\frac{(2x-1)(x-2)}{x-1} \ge 0$$

Critical points when $x = \frac{1}{2}, 1$ and 2

$\frac{1}{2} \le x < 1$ or $x \ge 2$