SHENTON COLLEGE

ATMAM Mathematics Methods

Test 1 Calculator Free

Name: ....

Teacher: Friday Smith

Time Allowed : 25 minutes

Marks	/27

Materials allowed: Formula Sheet.

Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given as exact values.
Marks may not be awarded for untidy or poorly arranged work.

1 Differentiate each of the following with respect to x, clearly showing use of the appropriate rules. Do not simplify your answers.

a) $y = \frac{5x^3}{4} + 2x^2 - \frac{7}{x}$

$\frac{dy}{dx} = \frac{15x^2}{4} + 4x + \frac{7}{x^2}$

✓ polynomial terms

✓ reciprocal

b) $y = (3x^2 - 1)(5 - 2x)$ (2, 2)

$\frac{dy}{dx} = (6x)(5-2x) + (3x^2-1)(-2)$

✓ product rule

✓ correct differentiation.

c) $y = \sqrt{e^{3x} + 2}$

$\frac{dy}{dx} = \frac{1}{2}(e^{3x}+2)^{-\frac{1}{2}}(3e^{3x})$

or $= \frac{1}{2\sqrt{e^{3x}+2}}(3e^{3x})$

✓ chain on $\sqrt{\ }$

✓ chain on power of e

d) $y = \frac{\cos(x + \frac{\pi}{4})}{x^2}$ (2, 2)

$\frac{dy}{dx} = \frac{-x^2\sin(x+\frac{\pi}{4}) - 2x\cos(x+\frac{\pi}{4})}{x^4}$

✓ quotient rule

✓ cos → - sin

2 Given that $t = \sin 3w$ and $w = v^2 - 1$, find $\frac{dt}{dv}$ using the chain rule. Give your answer in terms of v. (3)

$$\frac{dt}{dv} = \frac{dt}{dw}\frac{dw}{dv} \qquad \frac{dt}{dw} = 3\cos 3w$$

$$\frac{dw}{dv} = 2v$$

✓ correct chain

✓ individual derivatives correct.

$$\frac{dt}{dv} = 6v\cos 3w$$

$$= 6v\cos(3v^2 - 3)$$

✓ in terms of v.

3 Consider the function $f(x) = 2x^3 + 12x^2 + 18x - 3$.

a) Use calculus to determine the location of all stationary points. (4)

$$f'(x) = 6x^2 + 24x + 18 \qquad -2 + 12 - 18 - 3 = -11$$

$$-54 + 108 - 54 - 3 = -3$$

$$0 = 6x^2 + 24x + 18$$

$$= x^2 + 4x + 3$$

$$= (x+3)(x+1) \Rightarrow x = -3 \text{ or } x = -1$$

Stationary points at $(-1, -11)$ and $(-3, -3)$

✓ derivative

✓ $f'(x) = 0$

✓ factorise & solve

✓ coordinates

b) Use the second derivative to determine the nature of those stationary points. (2)

$$f''(x) = 12x + 24$$

$$f''(-1) = 12 \Rightarrow \text{minimum}$$

$$f''(-3) = -12 \Rightarrow \text{maximum.}$$

$(-1, -11)$ is a minimum $\quad (-3, -3)$ is a maximum

✓ $f''(x)$

✓ sub & interpret

c) Show how the point where the concavity of the function changes can be located by using both of the derivatives you found in part a). (2)

$$f''(x) = 0$$

$$\Rightarrow 12x + 24 = 0$$

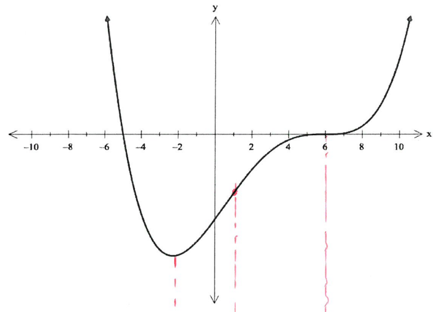
$$x = -2$$

$\therefore (-2, -7)$ is a point of inflection

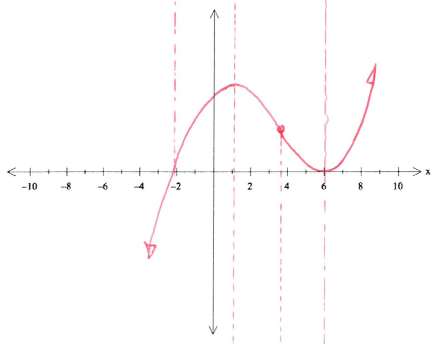
✓ $f''(x) = 0$

✓ Solve for x.

5 For the function shown below:



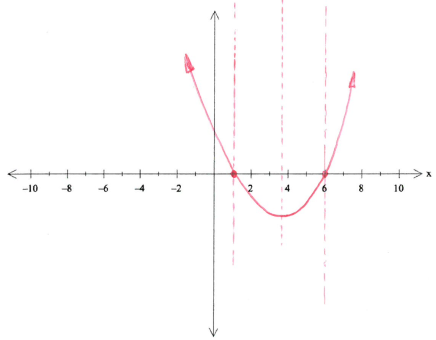
a) Sketch the first derivative, clearly indicating the relationship between relevant points. (2)



✓ stationary → roots

✓ inflection → max

b) and the second derivative, again clearly showing relevant relationships. (2)



✓✓ as above.

[Care should be taken with the x values of critical points, but the 'heights' of the derivatives are not unique, use whatever makes your sketch easier to draw.]

4 A loaf of bread is removed from an oven where it has been baking at 170°C and placed in a room where the ambient temperature is 20°C. As the loaf cools, the **difference in temperature** between the bread and its surroundings can be modelled by the equation $T = T_0e^{-0.02t}$ where T_0 is the difference in temperature immediately after the bread is removed from the oven and t is the time in minutes since the bread was removed.

a) What is the value of T_0? (1)

150°C

b) How long does it take for the bread to cool to a temperature of 50°C? (1)

$30 = 150e^{-0.02t}$

$t = 80.47$ minutes

c) Write an expression for the rate at which the bread is cooling. (1)

$\frac{dT}{dt} = -3e^{-0.02t}$

d) How long after being removed from the oven is temperature of the bread is changing at a rate of $-1°$ per minute? (2)

$-1 = -3e^{-0.02t}$ ✓ equation

$t = 54.93$ minutes ✓ solve

e) How long will it take for the bread to cool to 20°C? (1)

It will never cool to 20°C.

It will approach 20°C and the ΔT will become infinitesimally small, but never zero.

4 Find $f'(1)$ *and* $f''(1)$ for the function $f(x) = 4e^{x^2-1}$ (4)

$f'(x) = 8xe^{x^2-1}$ ✓ $f'(x)$

$f'(1) = 8$ ✓ product on $f''(x)$

$f''(x) = 8e^{x^2-1} + 8x(2xe^{x^2-1})$ ✓✓ substitute & simplify

$f''(1) = 24$

5 Differentiate the function $f(x) = (x+1)^2$ using the first principles limit $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. (4)

$f(x) = x^2 + 2x + 1$

$\Rightarrow f(x+h) = (x+h)^2 + 2(x+h) + 1$

$= x^2 + 2xh + h^2 + 2x + 2h + 1$

$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h + 1 - x^2 - 2x - 1}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2 + 2h}{h}$ ✓ limit notation correct throughout

$= \lim_{h\to 0} \frac{h(2x + h + 2)}{h}$ ✓ substitute $x+h$

$= \lim_{h\to 0} 2x + 2 + h$ ✓ factorise h & cancel

$= 2x + 2$ ✓ evaluate limit

ATMAM Mathematics Methods

Test 1 Calculator Assumed

SHENTON COLLEGE

Name: ....

Teacher: Friday Smith

Time Allowed : 25 minutes

Marks	/24

Materials allowed: Classpad, calculator, formula sheet.

Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given to two decimal places.
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1 Find the value of p (to 2 decimal places) if the function $y = e^{x^2-p}$ has a gradient of $2e$ when $x = 3$. (3)

$\frac{dy}{dx} = 2xe^{x^2-p}$ ✓ $\frac{dy}{dx}$

$2e = 6e^{9-p}$ Solve on CAS ✓ substitute ✓ p.

$p = 9.10$

2 a) Find the gradient of the normal to the function $f(x) = \cos^3 x$ at the point where $x = \frac{\pi}{6}$. (2)

$f'(\frac{\pi}{6}) = \frac{-9}{8}$ from CAS ✓ $f'(\frac{\pi}{6})$

$\therefore$ gradient of normal $= \frac{8}{9}$ ✓ -ve reciprocal

b) Hence find where this normal crosses the y-axis. (2)

Generate "normal" from CAS
(Calculation → Line → Normal)
and read off y-intercept.

$(0, 0.18)$

3 The height of the tide in an estuary can be modelled by the equation $H(t) = 3.5\cos\frac{\pi t}{6}$, where H is the height in metres and t is the time since midnight, measured in hours on the domain $0 \le t \le 24$.

a) What is the difference in height between the highest tide point and the lowest tide point? (1)

7m

b) What time(s) of the day is the height of the tide decreasing at its fastest rate? (2)

Can be observed from graph,
$t = 3$ and $t = 15$
=> 3am & 3pm.
[$H''(t) = 0$ and $H'(t) < 0$ if using calculus]

c) Show how you would use calculus to determine what time(s) of the day the height is increasing at a rate of 1.75m per hour. (4)

$H'(t) = -\frac{3.5\pi}{6}\sin\frac{\pi t}{6}$ ✓ differentiate

$1.75 = -\frac{3.5\pi}{6}\sin\frac{\pi t}{6}$ ✓ $=1.75$

$t = 8.42$ $t = 20.42$ ✓ solve for t

$t = 9.58$ $t = 21.58$ ✓ time of day

Times 8:25 am & 8:25 pm
9:35 am & 9:35 pm