PERTH MODERN SCHOOL
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Independent Public School

Course Methods Test 1 Year 12

Student name: _______________ Teacher name: _______________

Task type: **Response**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: **_____8______**

Materials required: No Cals allowed at all!

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **40 marks**

Task weighting: **13%**

Formula sheet provided: **no**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2 & 3 = 5 marks)
Determine the equation of the tangent to the following curves at the sated point:

a) $y = 2x^3 - 3x + 1$ at the point $(1, 0)$

Solution
$y = 2x^3 - 3x + 1$ $y' = 6x^2 - 3$ $x = 1, y' = 3$ $y = 3x + c$ $0 = 3 + c$ $c = -3$ $y = 3x - 3$
Specific behaviours
✓ determines gradient ✓ solves for constant of tangent

b) $y = -5x^3 + \frac{1}{x^2}$ at the point $(-1, 6)$

Solution
$y = -5x^3 + \frac{1}{x^2} = -5x^3 + x^{-2}$ $y' = -15x^2 - 2x^{-3}$ $x = -1, y' = -15 + 2 = -13$ $y = -13x + c$ $6 = 13 + c$ $c = -7$ $y = -13x - 7$
Specific behaviours
✓ differentiates correctly ✓ solves for gradient ✓ solves for constant

Q2 (3 & 3 = 6 marks)
Determine the derivatives of the following using the quotient rule and simplify your answer.

a) $f(x) = \dfrac{x+3}{2x^3+2}$

Solution

$$f(x) = \frac{x+3}{2x^3+2}$$

$$f'(x) = \frac{(2x^3+2) - (x+3)6x^2}{(2x^3+2)^2} = \frac{2x^3+2-6x^3-18x^2}{(2x^3+2)^2}$$

$$= \frac{2-4x^3-18x^2}{(2x^3+2)^2}$$

$$= \frac{2(1-2x^3-9x^2}{(2x^3+2)^2}$$

$$= \frac{(1-2x^3-9x^2)}{2(x^3+1)^2}$$

Specific behaviours

✓ correct numerator of quotient rule
✓ correct denominator
✓ simplified to above with factors of 2 taken out

b) $f(x) = \dfrac{3x^2+1}{(5x-1)^3}$

Solution

$$f(x) = \frac{3x^2+1}{(5x-1)^3}$$

$$f'(x) = \frac{(5x-1)^3 6x - (3x^2+1)3(5x-1)^2 5}{(5x-1)^6}$$

$$= \frac{3(5x-1)^2\left[2x(5x-1) - 5(3x^2+1)\right]}{(5x-1)^6}$$

$$= \frac{3(5x-1)^2\left[-2x-5x^2-5\right]}{(5x-1)^6}$$

$$= \frac{-3\left[2x+5x^2+5\right]}{(5x-1)^4}$$

Specific behaviours

✓ uses quotient rule correctly ✓ expands and adds like terms in numerator ✓ simplifies as shown in last line above (-ve may be inside brackets)

Q3 (5 marks)

Determine the coordinates of the stationary points of $f(x)=x^3-3x+2$ using calculus and justify their nature.

Solution
$f(x)=x^3-3x+2$ $f'(x)=3x^2-3=3(x-1)(x+1)$ $f'(x)=0, x=\pm 1$ $f''(x)=6x$ $(1,0)\quad f''(1)=6\therefore local\min$ $(-1,4)\quad f''(-1)=-6\therefore local\max$
Specific behaviours
✓ differentiates function ✓ equates to zero and solves for x values ✓ gives both coordinates for each stationary point ✓ uses sign derivative test with actual values stated ✓states nature of each point

Q4 (1, 2 & 3 = 6 marks)
Consider an object initially at the origin that moves only in a straight line with displacement from origin, x, given by $x=\frac{t^3}{3}-\frac{3t^2}{2}+2t$ at time, t seconds.
Determine:

a) Acceleration at $t=1$ second.

Solution
$v=t^2-3t+2$ $a=2t-3$ $t=1, a=-1$
Specific behaviours
✓ states value (no need for units)

b) The times the object is at rest.

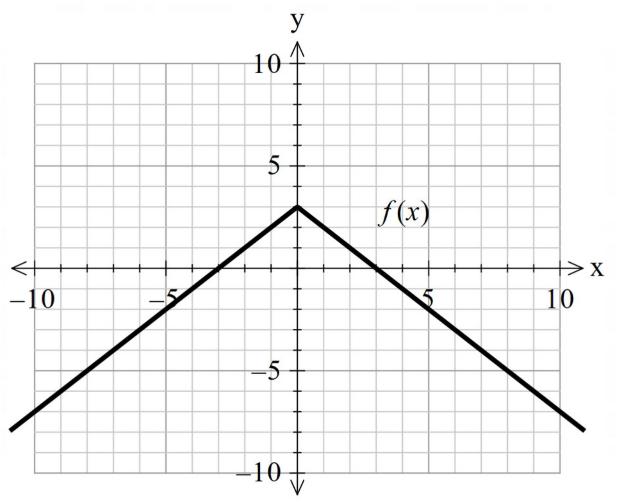
Solution
$v = t^2 - 3t + 2 \quad = (t-1)(t-2) = 0$ $t = 1, 2$
Specific behaviours
✓ equates velocity to zero ✓ states times (no need for units)

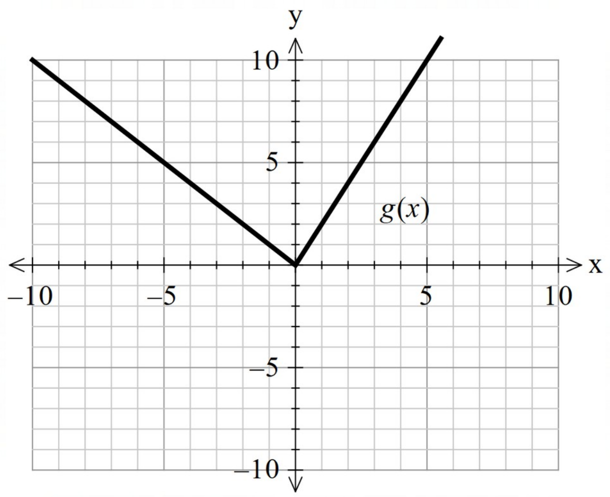
c) The distance travelled in the first 3 seconds.

Solution
$x = \frac{t^3}{3} - \frac{3t^2}{2} + 2t$ $t = 1, x = \frac{5}{6}$ $t = 2, x = \frac{2}{3}$ $t = 3, x = \frac{3}{2}$ $distance = \frac{5}{6} + \frac{5}{6} - \frac{2}{3} + \frac{3}{2} - \frac{2}{3} = \frac{11}{6}$
Specific behaviours
✓ solves for x at t=1 ✓ solves for x at t=2&3 ✓ states distance as one term

Q5 (2, 2 & 2 = 6 marks)

The graphs of f and g are displayed below.





a) Determine the derivative of $f(x)g(x)$ at $x=3$.

Solution
$y=f(x)g(x)$ $y'=f(x)g'(x)+g(x)f'(x)$ $=0+6(-1)=-6$
Specific behaviours
✓ uses product rule ✓ states value

b) Determine the derivative of $\dfrac{f(x)}{g(x)}$ at $x=2$.

Solution
$y=\dfrac{f(x)}{g(x)}$ $y'=\dfrac{gf'-fg'}{g^2}=\dfrac{4(-1)-(1)2}{16}=-\dfrac{6}{16}\,or\,-\dfrac{3}{8}$
Specific behaviours
✓ uses quotient rule ✓ states value (accept -6/16)

c) Determine the derivative of $f(g(x))$ at $x=-1$

Solution
$y=f(g(x))$ $y'=f'(g(x))g'(x)=f'(1)(-1)=1$
Specific behaviours

✓ uses chain rule ✓ states value

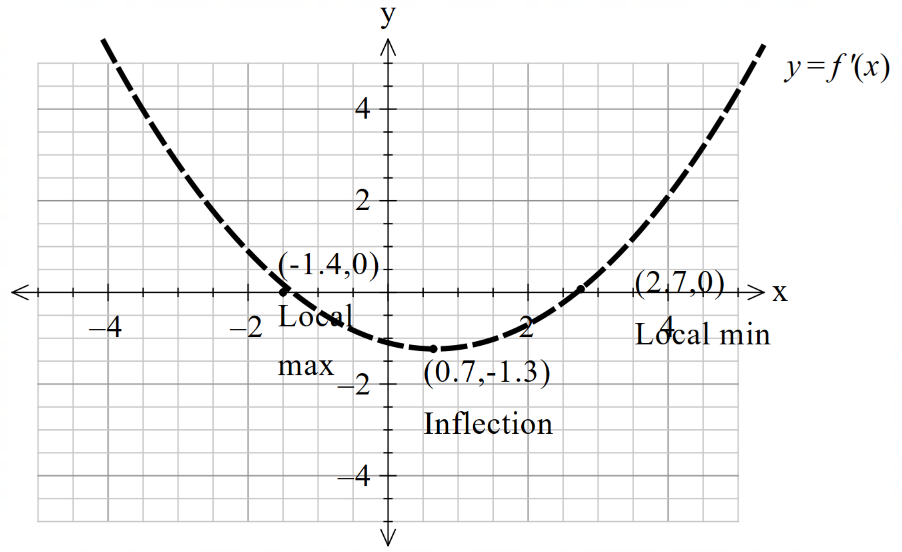
Q6 (3 marks)

If $q = \frac{5}{t^{\frac{3}{2}}}$ use differentiation to determine the approximate percentage change in q when t increases by 3%.

Solution
$q = \frac{5}{t^{\frac{3}{2}}} = 5t^{\frac{-3}{2}}$ $\frac{\Delta q}{q} \approx \frac{\frac{-15}{2} t^{\frac{-5}{2}} \Delta t}{5t^{\frac{-3}{2}}} = \frac{-3}{2} \frac{\Delta t}{t} = -4.5\%$
Specific behaviours
✓ uses small change formula correctly ✓ derives an expression for % change of q ✓ states value as negative or decrease

Q7 (5 marks)

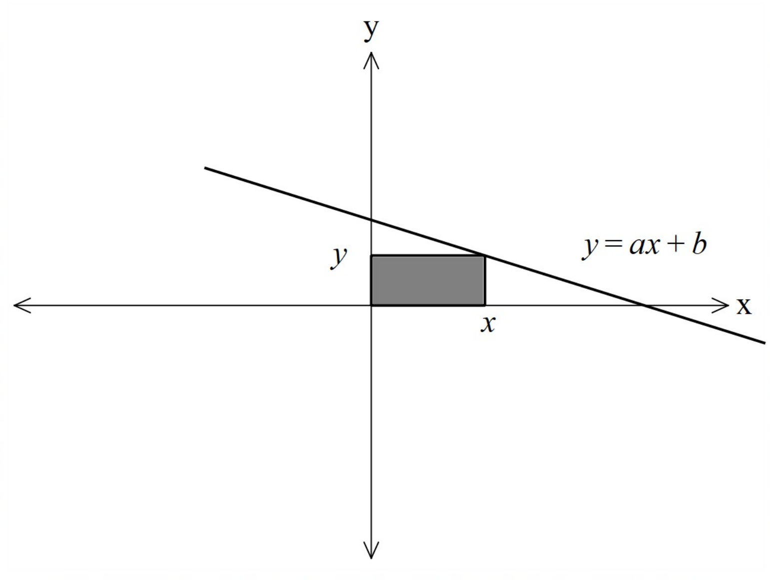
Consider the function $f(x)$ as graphed below. On the axes below sketch the function $y = f'(x)$ and on this graph label and show the coordinates and nature of all important features of $f(x)$.



Solution
Specific behaviours

✓ shape being all concave up ✓ All 3 points given as approx. cords (allow variance for y coord of inflection) ✓ local min labelled on derivative graph correctly ✓ local max labelled on derivative graph correctly ✓inflection labelled on derivative graph correctly

Q8 (4 marks)
A rectangle has one vertex at the origin, another on the positive x-axis, another on the positive y-axis and a fourth on the line $y = ax + b$ where $a \& b$ are constants.



The greatest area occurs when $x = 8$ units with an area of 32 sq units**. Using calculus**, determine the values of the constants $a \& b$.

Solution
$A = xy = x(ax + b)$ $A = ax^2 + bx$ $A' = 2ax + b$ $2a(8) + b = 0$ $b = -16a$ $32 = 8(8a - 16a) = 8(-8a)$ $a = -\frac{1}{2}, b = 8$
Specific behaviours
✓ sets up an expression for area in terms of x

✓ diffs and equates to zero ✓ uses optimal x value to derive one equation for a & b ✓ solves for a & b Note: max of 1 mark if calculus not used