

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

### Important note to candidates

Special items: nil

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters  
**To be provided by the candidate**

Materials required/recommended for this section  
**To be provided by the supervisor**

Working time for this section: fifty minutes  
Reading time before commencing work: five minutes  
**Time allowed for this section**

Your name \_\_\_\_\_  
In words \_\_\_\_\_

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Student Number: in figures

Section One:  
Calculator-free  
**MATHEMATICS 3A/3B**

Question/Answer Booklet

Year 12 Trial WACE Examination, 2013

SCHOOL

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**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	12	12	100	100	67
		<b>Total</b>		150	100

**Additional working space**

Question number: \_\_\_\_\_

**Instructions to candidates**

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil**, except in diagrams.

(1 mark)

$$\frac{1 \times 4 \times 3 \times 2 \times 1 \times 4}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{4}{30} = \frac{2}{15}$$

- (b) The student plays all six songs in a random order. What is the probability that Androids is played first and Blackout is not played last?

(2 marks)

$$1 \times 4 \times 3 \times 2 \times 1 \times 4 = 96 \text{ orders}$$

- Androids must be played first and Blackout must not be played last? (2 marks)

(1 mark)

there are no restrictions?)

(a) In how many different orders can these six songs be played given

One of the songs is called Androids and another is called Blackout.

A student wants to listen to six new songs that they have just bought from an online music store. (4 marks) Question 1

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Working time for this section is 50 minutes.

provided.

This section has **seven** (7) questions. Answer all questions. Write your answers in the spaces provided.

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**Section One: Calculator-free (50 Marks)**

CALCULATOR-FREE

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**Question 2**

- (a) Determine  $\frac{dy}{dx}$  for each of the following.

(i)  $y = \frac{x^3}{6}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{3x^2}{6} \\ &= \frac{x^2}{2}\end{aligned}$$

(1 mark)

(ii)  $y = x^5 - x^3 + x - 7$

(2 marks)

$$\begin{aligned}\frac{dy}{dx} &= 5x^4 - 3x^2 + 1\end{aligned}$$

- (b) Evaluate  $f'(-2)$  when  $f(x) = x^4 - x^3$ .

$$\begin{aligned}f'(x) &= 4x^3 - 3x^2 \\ f'(-2) &= 4(-8) - 3(4) \\ &= -44\end{aligned}$$

(2 marks)

- (c) Determine the equation of the tangent to the curve  $y = \frac{x^2}{2} - 3x + 5$  at the point  $(4, 1)$ .

$$\begin{aligned}\frac{dy}{dx} &= x - 3 \\ \text{At } (4, 1) \quad \frac{dy}{dx} &= 4 - 3 = 1 \\ \text{Tangent is } y - 1 &= 1(x - 4) \\ y &= x - 3\end{aligned}$$

(3 marks)

**Question 7**

Three integers  $a$ ,  $b$  and  $c$ , are derived from two positive whole numbers  $m$  and  $n$ , with  $m > n$ , as shown:

$$a = m^2 - n^2$$

$$b = 2mn$$

$$c = m^2 + n^2$$

- (a) When  $m = 3$  and  $n = 1$ , calculate the values of  $a$ ,  $b$  and  $c$ .

(1 mark)

$$\begin{aligned}a &= 3^2 - 1^2 \\ &= 8 \\ b &= 2 \times 3 \times 1 \\ &= 6 \\ c &= 3^2 + 1^2 \\ &= 10\end{aligned}$$

- (b) Show that the values of  $a$ ,  $b$  and  $c$  calculated in part (a) form a Pythagorean triple of the form  $a^2 + b^2 = c^2$ .

$$\begin{aligned}a^2 + b^2 &= 8^2 + 6^2 \\ &= 64 + 36 \\ &= 100 \\ &= 10^2 \\ &= c^2\end{aligned}$$

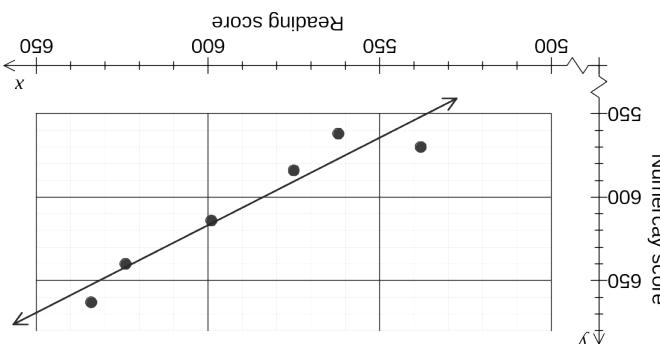
- (c) Prove that the values of  $a$ ,  $b$  and  $c$  are always of the form  $a^2 + b^2 = c^2$ .

(3 marks)

$$\begin{aligned}LHS &= a^2 + b^2 \\ &= (m^2 - n^2)^2 + (2mn)^2 \\ &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 - n^4 \\ &= (m^2 + n^2)^2 \\ &= c^2 \\ &= RHS\end{aligned}$$

## Question 3

(9 marks) Research into reading and numeracy levels at six schools compared achievement using NAPLAN test results. The graph below shows the average reading and numeracy scores for the six schools and the linear regression model  $y = 1.042x - 8.055$  calculated from the data.



(a) Describe in words the association between the variables  $x$  (reading score) and  $y$  (numeracy score) for these six schools. (2 marks)

Strong, positive linear association.

(b) Consider Pearson's correlation coefficient  $r$  between the variables  $x$  and  $y$ .  
(1 mark)

(i) Place a cross on the scale below for your estimate of  $r$ . (1 mark)

(ii) Would you expect  $r$  to increase or decrease if the data point (538, 570) was removed from the data set? Justify your answer. (2 marks)

(iii) One of the data points is at (562, 562). Would you predict that a seventh school with a reading score of 562 will also have a numeracy score of 562? Justify your answer. (2 marks)

No. Increase. The remaining five points lie much closer to a straight line.

(c) One of the data points is at (562, 562). Would you predict that a seventh school with a reading score of 562 will also have a numeracy score of 562? Justify your answer. (2 marks)

(d) Another data point is at (634, 663). Write down a calculation to determine the residual for the linear regression model at this point, but do not evaluate it. (2 marks)

$$(663) - (1.042 \times 634 - 8.055)$$

## Question 6

(7 marks) Use calculus techniques to determine the maximum area of the plane figure. (3 marks)

(a) The area of a plane figure is given by  $A = 12t - 3t^2 + 11$ , for  $0 \leq t \leq 3.5$ .

$$\frac{dA}{dt} = 12 - 6t$$

$$12 - 6t = 0 \text{ when } t = 2$$

$$A(2) = 12(2) - 3(2)^2 + 11$$

$$= 24 - 12 + 11$$

$$= 23 \text{ sq units}$$

$$\frac{dy}{dx} = 2ax(1-x) - (ax^2 + 2)$$

$$\text{When } x = 1, \frac{dy}{dx} = 0$$

$$0 = 2a(1-1) - (a+2)$$

$$0 = -a - 2$$

$$a = -2$$

Determine the value of  $a$ .

(b) The function  $y = (ax^2 + 2)(1-x)$  has a minimum turning point when  $x = 1$ .

(4 marks)

**Question 4**(a) Solve for  $x$ :

(i)  $\sqrt[3]{x} = 100$

$$\begin{aligned}x &= 100^3 \\&= 1\,000\,000\end{aligned}$$

(1 mark)

(ii)  $x^2 + 3x - 40 = 0$

$$\begin{aligned}(x + 8)(x - 5) &= 0 \\x &= -8, 5\end{aligned}$$

(2 marks)

(iii)  $9^{2x} = 27$

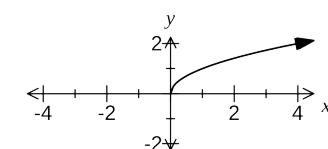
$$\begin{aligned}3^{4x} &= 3^3 \\4x &= 3 \\x &= \frac{3}{4}\end{aligned}$$

(2 marks)

(b) Find the coordinates of the point of intersection of the lines  $2x + 3y = 11$  and  $5x + 4y = 10$ .

(4 marks)

$$\begin{aligned}10x + 15y &= 55 \\10x + 8y &= 20 \\7y &= 35 \\y &= 5 \\2x + 15 &= 11 \\2x &= -4 \\x &= -2\end{aligned}$$

**Question 5**(a) The graph of  $y = \sqrt{x}$  is shown at right.Describe how to sketch the following graphs using the graph of  $y = \sqrt{x}$ .

(i)  $y = \sqrt{3x}$

Dilate horizontally by a scale factor of  $\frac{1}{3}$ 

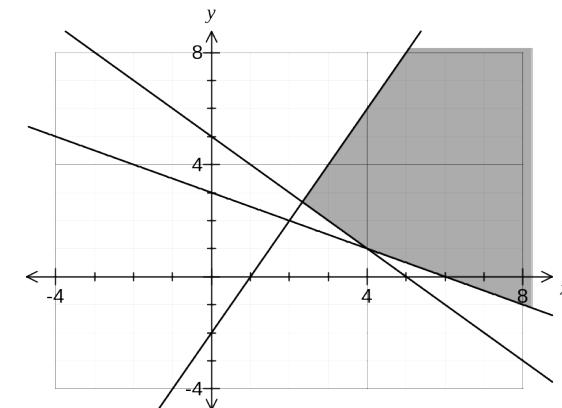
(1 mark)

(ii)  $y = -\sqrt{x+5}$

Reflect in the  $x$ -axis and translate 5 units to the left.

(2 marks)

(b) The shaded area on the graph below is the intersection of the half-planes of two inequalities.

One of the inequalities is  $y \leq 2x - 2$ .

(i) Write down the second inequality shown on the graph.

(2 marks)

$$x + y \geq 5$$

(ii) Add the inequality  $x + 2y \geq 6$  to the graph and clearly shade the resulting region satisfied by all three inequalities.

(3 marks)