



**PERTH MODERN SCHOOL**  
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**Independent Public School**

## Course Specialist Year 12 Test One 2022

Student name: \_\_\_\_\_ Teacher name: \_\_\_\_\_

**Task type:** Response

**Time allowed for this task:** \_\_\_\_40\_\_\_\_ mins

**Number of questions:** \_\_\_\_8\_\_\_\_

**Materials required:** Calculator with CAS capability (to be provided by the student)

**Standard items:** Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

**Special items:** Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

**Marks available:** \_\_\_\_42\_\_\_\_ marks

**Task weighting:** \_\_\_\_10\_\_\_\_%

**Formula sheet provided:** Yes/No

**Note:** All part questions worth more than 2 marks require working to obtain full marks.

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Q1 (2, 3 & 3 = 8 marks) (3.1.1- 3.1.6)

Let  $z = 5 - 3i$  and  $w = 7 - i$ .

Simplify the following.

a)  $z^2 w$

b)  $\frac{1}{w}$

c)  $\frac{z}{w}$  (simplify)

Q2 (3 marks) (3.1.1-3.1.3)

Determine all possible real number pairs  $a, b$  such that  $\frac{101 + 47i}{a - 5i} = 6 + bi$

Q3 (3 marks) (3.1.13-3.1.15)

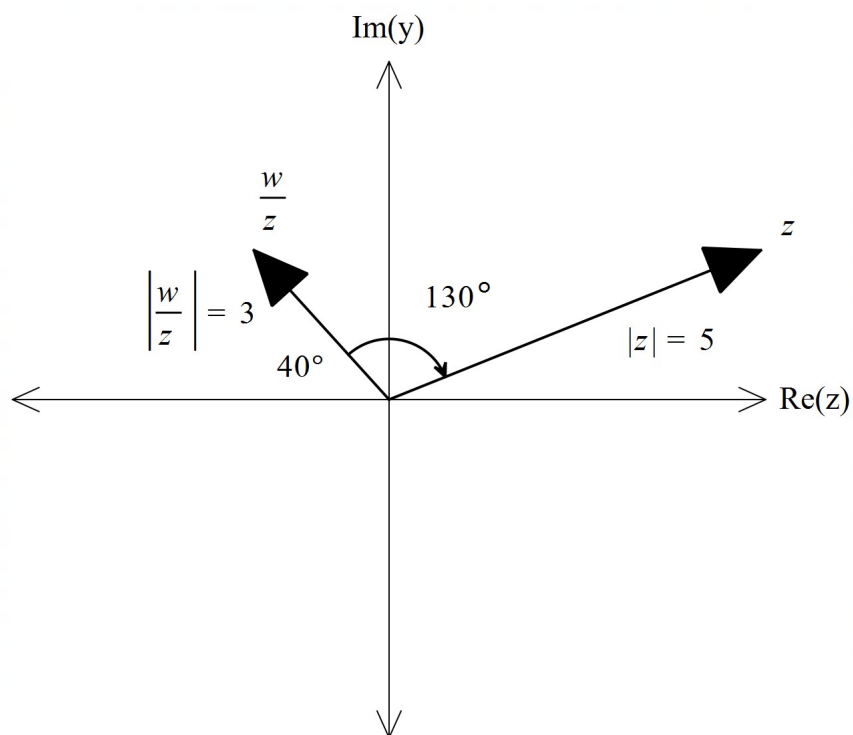
Consider the polynomial  $f(z) = z^3 + bz^2 + cz + d$  where  $b, c$  &  $d$  are real numbers.

Given that  $f(3) = 0$  and  $f(2 - 5i) = 0$  determine the values of  $b, c$  &  $d$ .

Q4 (3 marks) (3.1.8-3.1.10)

Using the diagram below determine the complex number  $w$  in exact cartesian form.

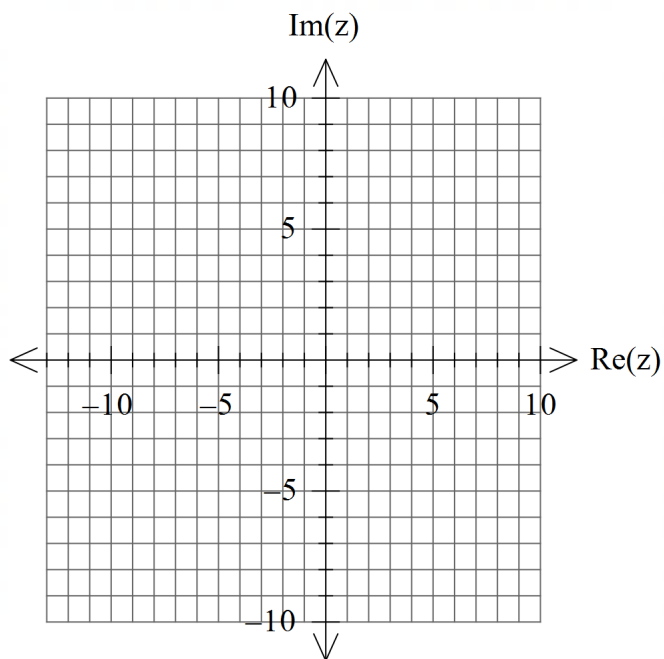
(Note: Not drawn to scale)



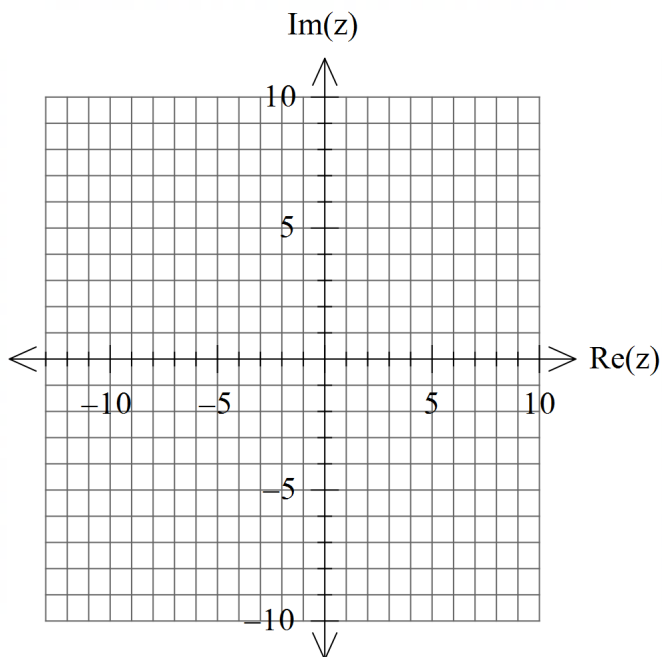
Q5 (3 &amp; 3= 6 marks) (3.1.10)

Sketch the locus for the following labelling important features and points.

a)  $|z - 3 + 7i| \geq |z - 5|$



b)  $|z + 3 + 7i| = |z - 5| + \sqrt{113}$



Q6 (2 & 4 = 6 marks) (3.1.10)

Consider the set of points  $z$  in the complex plane such that  $|z - 2 - 3i| = 5$ .

a) Determine the maximum value of  $|z|$ .

b) Determine the maximum value of  $\text{Arg}(z + 12)$ .

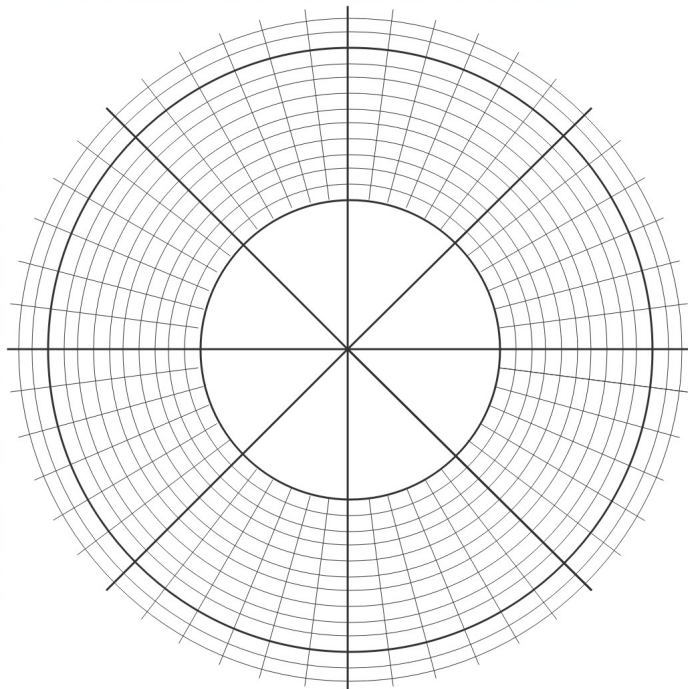
Q7 (4 marks) (3.1.7)

Using De Moivre's Theorem, derive an expression for  $\sin(4\theta)$  in terms of  $\cos(\theta)$  &  $\sin(\theta)$ .

Q8 (4, 2 & 3 = 9 marks) (3.1.11-3.1.12)

a) Solve for all the roots  $z^6 = 1 - i$  in polar form  $z = r\text{cis}\theta$  with  $-\pi < \theta \leq \pi$ .

b) Plot these roots on the complex plane below.



c) Adjacent points can be joined by lines to form a polygon. Determine the exact area of this polygon.

Working out space

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