Christ Church Grammar School

2021
TEST 5

# MATHEMATICS METHODS Year 12

**Section One:**
**Calculator-free**

Your name _____

Teacher's name _____

## Time and marks available for this section

| | |
|---|---|
| Reading time before commencing work: | 2 minutes |
| Working time for this section: | 15 minutes |
| Marks available: | 15 marks |

## Materials required/recommended for this section

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This Question/Answer Booklet
Formula Sheet

***To be provided by the candidate***
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Special items: nil

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See next page



**Question 9** **(6 marks)**

A metal worker is required to cut a circular cylinder from a solid sphere of metal of radius 5cm. The diagram shows a cross section of the sphere and cylinder.

Let $V$ be the volume of the sphere.

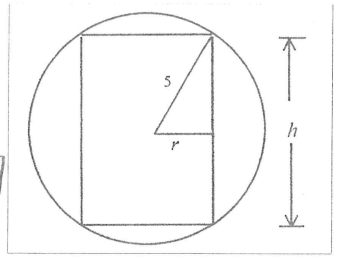
(a) Show that $r = \frac{1}{2}\sqrt{100 - h^2}$ (2 marks)

$5^2 = r^2 + (h/2)^2$ ✓ [Pythag]

$r^2 = 25 - \frac{h^2}{4}$

$r^2 = \frac{100 - h^2}{4}$ ✓ [Re-arranges]

$r = \frac{1}{2}\sqrt{100 - h^2}$




(b) Find the value of $h$ that maximises the volume of the cylinder. Justify the optimisation. (4 marks)

$V = \pi r^2 h$

$V = \pi\left(\frac{100 - h^2}{4}\right)h$ ✓ [Subs $r^2$]

$\frac{dV}{dh} = \frac{1}{4}\pi(100 - 3h^2)$ ✓ [Shows $\frac{dV}{dh}$]

$\frac{dV}{dh} = 0$

$h = \frac{10}{\sqrt{3}}$ (5.77) ✓ [Finds h value]

Justify

$\frac{d^2V}{dh^2} = \frac{1}{4}\pi(-6h)$

$\left.\frac{d^2V}{dh^2}\right|_{h=\frac{10}{\sqrt{3}}} < 0$ ✓ $\left[\frac{d^2V}{dh^2} < 0\right]$

$\therefore h = \frac{10}{\sqrt{3}}$ is MAX.

6

End of questions

**Question 8** **(3 marks)**

In the domain $0 \leq x \leq \pi$, find, correct to 3 decimal places, the coordinates of the position on the curve $y = 3sinx - sin^3x$ where the slope of the curve is $\frac{3}{8}$.

$\frac{dy}{dx} = -3\cos x \cdot \sin^2 x + 3\cos x$ ✓ [$dy/dx$]

$\frac{3}{8} = -3\cos x \cdot \sin^2 x + 3\cos x$

$x = 1.047$ ✓ [x-value]

$\therefore y = 1.948$ ✓ [y-value] OR 1.949

[-1 if no 3dp]

if all $x$ decimal places used.

3





**Question 1** **(5 marks)**

(a) To investigate people's attitudes to control of gun ownership, a TV station conducts a phone-in poll, where people are asked to telephone one number if they are in favour of tighter gun control, and another if they are against. Is this an appropriate method of choosing a random sample? Give reasons for your answer. (2 marks)

(b) In a certain school, 35% of the students travel on the school bus. A group of 100 students were selected in a random sample, and 42 of them travel on the school bus. In this context,

(i) describe the population. (1 mark)

(ii) determine the value of the population proportion $p$. (1 mark)

(iii) determine the value of the sample proportion $\hat{p}$. (1 mark)

**Question 2** **(6 marks)**

(a) Determine the anti-derivative of $(2x+\frac{1}{x})(2x-\frac{1}{x})$ (3 marks)

(b) $y = x + 1$ is a tangent to the curve $y = ax + b\sin(x)$ at the point $\left(\frac{\pi}{2}, 1+\frac{\pi}{2}\right)$.

Determine the values of $a$ and $b$. (3 marks)





**Question 7** **(4 marks)**

Find the value(s) of $k$ for which $y = e^{kx}$ is a solution of the equation

$$2\times\frac{d^2y}{dx^2}-\frac{dy}{dx}-3y=0$$

Must show 1st & 2nd derivatives

✓ $\frac{dy}{dx} = ke^{kx}$

✓ $\frac{d^2y}{dx^2} = k^2e^{kx}$

$2k^2e^{kx} - ke^{kx} - 3e^{kx} = 0$ ✓ [sub in]

$e^{kx}(2k^2-k-3) = 0$

$e^{kx}(k+1)(2k-3) = 0$

$e^{kx} = 0$ ↓ No Soln ; $k=-1$ or $k=\frac{3}{2}$ ✓ [Both k-values]

4

**Question 6** **(6 marks)**

The amount $R$, in grams, of a radioactive substance $X$ remaining at time $t$ years is given by $R = 100e^{-0.01t}, t \geq 0$.

(a) Find the initial amount of $X$. (1 mark)

Initial value $= 100e^{-0.01(0)}$

$= 100\,g$ ✓

(b) Find the continuous rate of decay for $X$ as a percentage. (1 mark)

$X$ is $0.01 \times 100$ $= 1\%$ per year ✓

(c) Find the amount of $X$ that has decayed after 100 years. (2 marks)

$t = 100$ amount remaining is $R = 100e^{-0.01(100)}$

$= 36.79\,g$ ✓[Remain]

$\therefore 100 - 36.79 = 63.2\,g$ decayed ✓[Decay]

(d) Find how long it will take for the amount of $X$ to be halved. (i.e. the half life of $X$) (2 marks)

When $X$ is halved $R = 50g$

$100e^{-0.01t} = 50$

$e^{-0.01t} = \frac{1}{2}$ ✓ 69.31471806

$t = 69.3$ yrs ✓

6

See next page

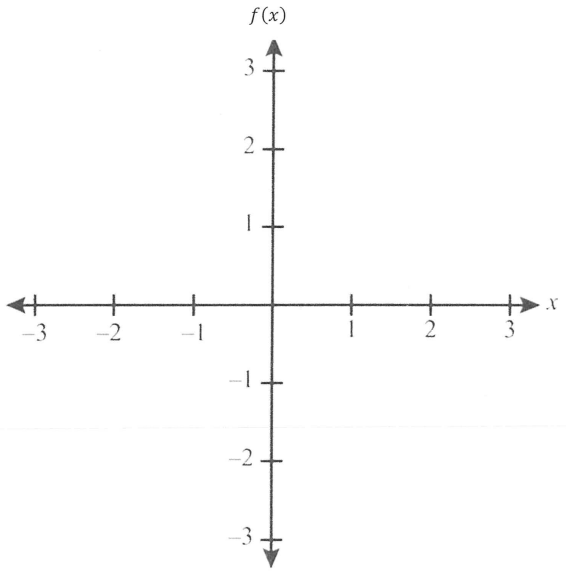


**Question 3** **(4 marks)**

Sketch, on the axes below, the following function with ***all*** of the properties listed.

$y = f(x)$ such that:

- $f(-1) = 0$
- $f'(0) = 0$
- $f(x) > 0 \; for \; x > -1$
- $f(x) < 0 \; for \; x < -1$
- $f'(x) < 0 \; for \; x > 0$




**End of questions**

**Additional working space**

Question number: _____



**Question 5** **(4 marks)**

Let the proportion of parents in a college that support a four-day school week be $\pi$. A random sample of 200 parents was selected and 78 indicated that they support the proposal. Find the level of confidence for a confidence interval for $\pi$ with an error of $\pm 0.1$.

$\hat{\pi} = \frac{78}{200} \Rightarrow 0.39$ ✓ [Proportion]

As $n > 30$ by CLT $\hat{\pi} \sim$ Normal

$z_c \times \sqrt{\frac{0.39(1-0.39)}{200}} = 0.1$

$z_c = 2.89946$ ✓ [z-value]

$\therefore P(-2.89946 \leq z \leq 2.89946)$ ✓ [$\pm z_c$]

$= 0.99627$

$\therefore$ Level of Confidence is 99.6% ✓ [C.I]

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See next page

**Question 4** **(7 marks)**

To predict the number of people in Australia having a certain disease, a random sample of 1600 people was tested, in which 56 were found to have the disease.

(a) Determine the sample proportion, $\hat{p}$, of people having the disease. (1 mark)

$\frac{56}{1600} = 0.035$ ✓

(b) Determine the 90% confidence interval for the proportion of the population that have the disease. (1 marks)

$0.027744 \leq p \leq 0.04256$ ✓ [Chad]

(c) Another random sample is taken. A 90% confidence interval is wanted with a margin of error of at most 0.5%. Determine the minimum sample size required. (3 marks)

90% ⇒ $k = 1.645$

$0.5\% = 0.005$ ✓ [ME]

$\therefore 0.005 = 1.645\sqrt{\frac{0.035(1-0.035)}{n}}$ ✓ [EQU]

$n = 3655.84$

$n = 3656$ ✓ [Round correctly]

(d) A survey in an isolated town, with a population of 12030, finds that 2503 people have a disease. Compare these findings with that of the population. (2 marks)

$\frac{2503}{12030} = 0.2081$ ✓ [proportion]

Not consistent. Bias due to town being isolated... etc ✓ [Explanation]

See next page

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# MATHEMATICS METHODS Year 12

**Section Two:**
**Calculator-assumed**

Your name _____

Teacher's name _____

**Time and marks available for this section**

| | |
|---|---|
| Reading time before commencing work: | 3 minutes |
| Working time for this section: | 30 minutes |
| Marks available: | 30 marks |

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MATHEMATICS METHODS Year 12 2 CALCULATOR-ASSUMED

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2021
TEST 5

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**Section Two:**
**Calculator-assumed**

Your name _____

Teacher's name _____

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Working time for this section: 30 minutes
Marks available: 30 marks

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(b) Determine the 90% confidence interval for the proportion of the population that have the disease. (1 mark)

(c) Another random sample is taken. A 90% confidence interval is wanted with a margin of error of at most 0.5%. Determine the minimum sample size required. (3 marks)

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See next page

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Let the proportion of parents in a college that support a four-day school week be $\pi$. A random sample of 200 parents was selected and 78 indicated that they support the proposal. Calculate the level of confidence for a confidence interval for $\pi$ with an error of $\pm 0.1$.

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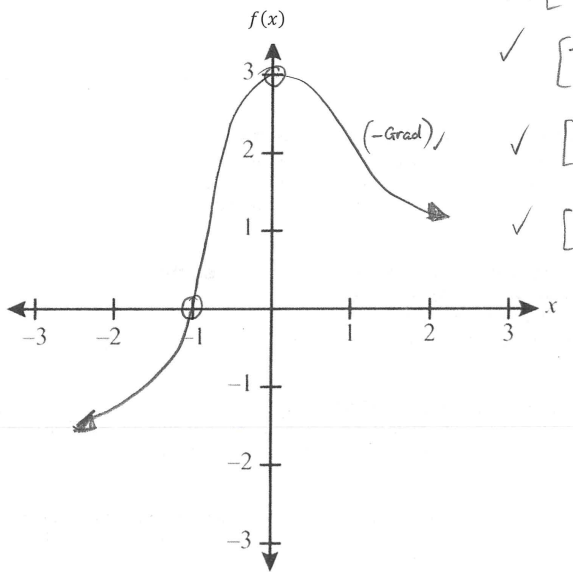


✓ [x-int]
✓ [TP @ x=0]
✓ [f'(x)<0, x>0]
✓ [General shape]

4

**Question 2** **(6 marks)**

(a) Determine the anti-derivative of $(2x+\frac{1}{x})(2x-\frac{1}{x})$ (3 marks)

$\int 4x^2 - x^{-2}\,dx$

$= \frac{4x^3}{3} + x^{-1} + c$

$= \frac{4x^3}{3} + \frac{1}{x} + c$

✓ $x^3$ term
✓ $\frac{1}{x}$ term
✓ $+c$

(b) $y = x + 1$ is a tangent to the curve $y = ax + b\sin(x)$ at the point $\left(\frac{\pi}{2}, 1+\frac{\pi}{2}\right)$.

Determine the values of $a$ and $b$. (3 marks)

$\frac{dy}{dx} = a + b\cos x$ ✓ $\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx} = 1$ when $x = \frac{\pi}{2}$ $\Rightarrow$ $1 = a + b.\cos\frac{\pi}{2}$

$\therefore a = 1$ ✓ (a value)

Also $(\pi/2, 1+\pi/2)$ on curve

$1+\frac{\pi}{2} = \frac{\pi a}{2} + b\sin(\pi/2)$

$1+\frac{\pi}{2} = \frac{\pi}{2} + b$

$\therefore b = 1$ ✓ (b-value)

/6





**Question 6** **(6 marks)**

The amount $R$, in grams, of a radioactive substance $X$ remaining at time $t$ years is given by $R = 100e^{-0.01t}$, $t \geq 0$.

(a) Determine the initial amount of $X$. (1 mark)

(b) Determine the continuous rate of decay for $X$ as a percentage. (1 mark)

(c) Determine the amount of $X$ that has decayed after 100 years. (2 marks)

(d) Determine the time it will take for the amount of $X$ to be halved.
(i.e. the half life of $X$)
(2 marks)

**Question 7** **(4 marks)**

Determine the value(s) of $k$ for which $y = e^{kx}$ is a solution of the equation:

$$2 \times \frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 0$$





**Question 1** **(5 marks)**

(a) To investigate people's attitudes to control of gun ownership, a TV station conducts a phone-in poll, where people are asked to telephone one number if they are in favour of tighter gun control, and another if they are against. Is this an appropriate method of choosing a random sample? Give reasons for your answer. (2 marks)

No, Self selection bias ✓ [NO]

Only interested parties call in

May call several times ✓ [Reason must incl BIAS]

(b) In a certain school, 35% of the students travel on the school bus. A group of 100 students were selected in a random sample, and 42 of them travel on the school bus. In this example:

(i) What is the population? (1 mark)

All students at the school ✓

(ii) What is the value of the population proportion $p$ ? (1 mark)

0·35 ✓

(iii) What is the value of the sample proportion $\hat{p}$ ? (1 mark)

0·42 ✓



5

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Let $V$ be the volume of the sphere.

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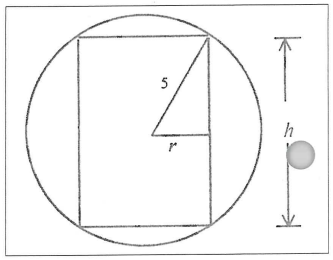


(b) Calculate the value of $h$ that maximises the volume of the cylinder. Justify the optimisation. (4 marks)

**End of questions**

Christ Church Grammar School

2021
TEST 5

# MATHEMATICS METHODS Year 12

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**Calculator-free**

Your name - Solutions -

Teacher's name _____

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