Applecross Senior High School

Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

In words ______________________

Your name ______________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

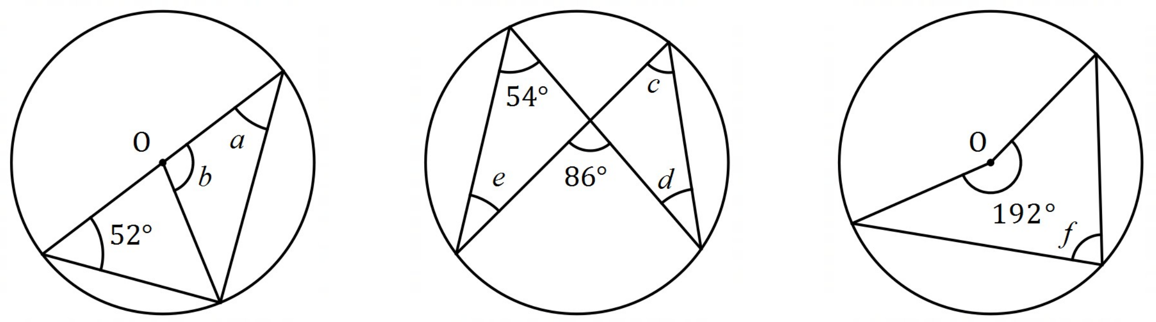
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

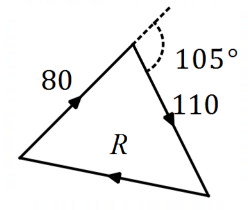
Determine the size of the angles marked a, b, c, d, e and f shown in the circles below. Where marked, O is the centre of the circle.



Solution
$a = 90° - 52° = 38°$ $b = 2 \times 52° = 104°$ $c = 54°$ $d = 86° - 54° = 32°$ $e = 32°$ $f = \frac{1}{2}(360° - 192°) = 84°$
Specific behaviours
✓ each correct angle

Question 10 **(5 marks)**

Three forces act on an object so that it remains in equilibrium. Two of the forces have magnitudes of 80 N and 110 N and the angle between their directions is $105°$. Determine the magnitude of the third force and the angle its direction makes with the smaller force.

Solution
 $R^2 = 80^2 + 110^2 - 2(80)(110)\cos 75° R = 118.1\,\mathrm{N}$ $\frac{\sin\theta}{110} = \frac{\sin 75°}{118.1} \theta = 64.1°$ Hence angle between directions is $180 - 64 = 116°$.
Specific behaviours
✓ diagram showing vector sum is zero □ uses cosine rule to solve triangle □ magnitude of resultant □ uses sine rule □ direction with smaller force

Question 11 **(8 marks)**

(a) An art gallery plans to display a single painting on each of the three walls in a room. Determine how many arrangements of paintings are possible in the room if they have a selection of 24 different paintings to choose from. (2 marks)

Solution
${}^{24}P_3 = 24 \times 23 \times 22 = 12\,144$
Specific behaviours
✓ indicates method ✓ correct number of arrangements

(b) In another room, the gallery plan to hang 8 different paintings in a row. If 2 of the paintings are by the artist Marr, determine the number of different arrangements of paintings that are possible when

(i) the paintings by Marr must be at the ends. (2 marks)

Solution
$2 \times 6! = 1\,440$
Specific behaviours
✓ uses $6!$ ✓ correct number of arrangements

(ii) the paintings by Marr must be next to each other. (2 marks)

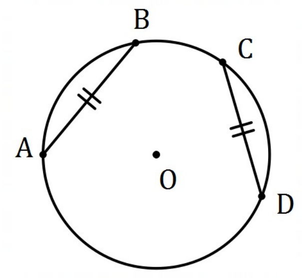
Solution
$2! \times 7! = 10\,080$
Specific behaviours
✓ groups Marr together ✓ correct number of arrangements

(iii) the paintings by Marr must be apart and not at the ends. (2 marks)

Solution
6 non-Marr leave 5 spaces to hang Marr in between (N_N_N_N_N_N): $n = 6! \times 5 \times 4 = 14\,400$
Specific behaviours
✓ indicates method ✓ correct number of arrangements

Question 12 **(8 marks)**

(a) Prove that chords of equal length subtend equal angles at the centre of a circle. (3 marks)



Solution
$AB=DC$ (given) $OA=OB=OC=OD=r$ (all radii) Hence $\Delta OAB\equiv\Delta OCD$ (SSS) Hence $\angle AOB=\angle COD$ - chords of equal length subtend equal angles at the centre.
Specific behaviours
✓ establishes congruency of sides ✓ establishes congruency of triangles ✓ concludes equal angles

(b) Points P and Q lie on a circle of radius 23.3 cm so that $PQ=21$ cm. Determine

(i) the distance of chord PQ from the centre of the circle. (3 marks)

Solution
Let midpoint of chord be M. Then $O\,M^2=r^2-P\,M^2 OM=\sqrt{23.3^2-10.5^2}$ ¿ $20.8\,\text{cm}$
Specific behaviours
✓ uses/defines midpoint or sketch diagram ✓ indicates correct method ✓ correct distance

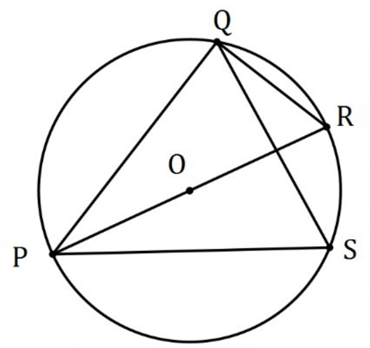
(ii) the angle subtended by chord PQ at the centre of the circle. (2 marks)

Solution
Let $\theta=\angle POM$ (half angle required). Then $\theta=\sin^{-1}\left(\frac{10.5}{23.3}\right)$ ¿ $26.78°\angle POQ=2\theta\approx 53.6°$
Specific behaviours
✓ indicates correct method ✓ correct angle

Question 13 **(7 marks)**

(a) The diagram shows points P, Q, R and S that lie on the circumference of a circle centre O. PR is a diameter and the size of $\angle QPR = 27°$.

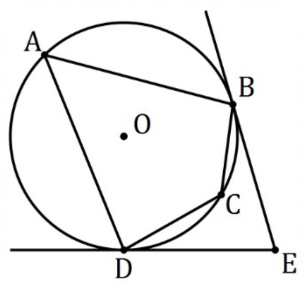
Determine, with reasons, the size of $\angle PSQ$. (3 marks)



Solution
$\angle PQR = 90°$ (angle in semicircle) $\angle PRQ = 180° - 90° - 27° = 63°$ (angle sum in triangle) $\angle PSQ = \angle PRQ = 63°$ (angles on same arc)
Specific behaviours
✓ uses angle in semicircle ✓ uses angle sum in triangle ✓ correct size of angle, with reason

(b) In the diagram shown, A, B, C and D are points on the circumference of a circle with centre O. Tangents to the circle at B and D intersect at E.

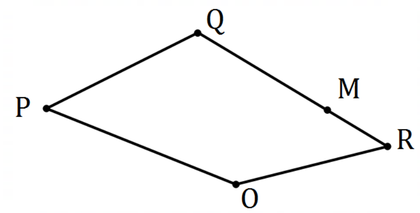
Determine, with justification, the size of $\angle BCD$ when $\angle BED = 72°$. (4 marks)



Solution
$\angle OBE = \angle ODE = 90°$ (radius-tangent angle) $\angle BOD = 360° - 180° - 72° = 108°$ (angle sum of quadrilateral $BODE$) $\angle BAD = \frac{1}{2}(108°) = 54°$ (centre-circumference angles) $\angle BCD = 180° - 54° = 126°$ (opposite angles in cyclic quadrilateral)
Specific behaviours
✓ uses radius-tangent angle ✓ correct $\angle BOD$ ✓ uses angle at centre-circumference ✓ correct angle

Question 14 **(8 marks)**

In quadrilateral $OPQR$ shown below, M lies on QR so that $\left|\overrightarrow{QM}\right| = 3 \vee \overrightarrow{MR} \vee ¿$.



(a) If $\overrightarrow{OP} = p, \overrightarrow{OQ} = q$ and $\vec{¿} = r$, express the following in terms of p, q and/or r.

(i) $\overrightarrow{PR}$. (1 mark)

Solution
$\overrightarrow{PR} = r - p$
Specific behaviours
✓ correct expression

(ii) $\overrightarrow{RM}$. (2 marks)

Solution
$\overrightarrow{RM} = \frac{1}{4}\overrightarrow{RQ} = \frac{1}{4}(q - r)$
Specific behaviours
✓ uses correct vector notation
□ correct expression

(iii) $\overrightarrow{PM}$. (2 marks)

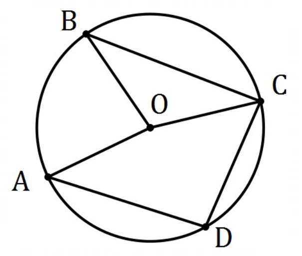
Solution
$\overrightarrow{PM} = \overrightarrow{PR} + \overrightarrow{RM} ¿ r - p + \frac{1}{4}(q - r)$ $¿ \frac{3}{4}r + \frac{1}{4}q - p$
Specific behaviours
✓ indicates suitable vector sum
□ correct expression

(b) If O is the origin and points P, Q and R have coordinates $(-2, 39), (28, -14)$ and $(32, -18)$ respectively, determine the distance PM. (3 marks)

Solution
$\overrightarrow{PM} = \frac{3}{4}\begin{pmatrix} 32 \\ -18 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 28 \\ -14 \end{pmatrix} - \begin{pmatrix} -2 \\ 39 \end{pmatrix} ¿ \begin{pmatrix} 33 \\ -56 \end{pmatrix}$ $\left\| \begin{pmatrix} 33 \\ -56 \end{pmatrix} \right\| = 65$
Specific behaviours
✓ substitutes into expression for $\overrightarrow{PM}$
□ $\overrightarrow{PM}$
□ correct magnitude

Question 15 **(8 marks)**

(a) The vertices of quadrilateral $ABCD$ lie on the circumference of a circle centre O shown below. Given that $\angle ADC = 95°$ and $\angle AOB = 84°$, determine with reasoning the size of angle BCO. (4 marks)



Solution
$\angle AOC = 2 \times \angle ADC = 190°$ (angles at centre-circumference) $\angle BOC = \angle AOC - \angle AOB = 190° - 84° = 106°$ (adjacent angles) $\angle BCO = \frac{1}{2}(180° - \angle BOC)$ (isosceles triangle) $\angle BCO = \frac{1}{2}(180° - 106°) = 37°$
Specific behaviours
✓ uses angles at centre-circumference uses adjacent angles

 uses isosceles triangles

(b) The vertices of triangle ABC lie on the circumference of a circle. Given that $AB = 10$ cm, $AC = 7$ cm and $BC = 6$ cm, prove by contradiction that AB is not a diameter of the circle. (4 marks)

Solution
Assume that AB is a diameter of the circle, so that the angle in a semicircle theorem implies that ΔABC must be right angled at C. If ΔABC is right angled, then Pythagoras' theorem implies that $AC^2 + BC^2 = AB^2$. But $AC^2 + BC^2 = 7^2 + 6^2 = 49 + 36 = 85$ and $AB^2 = 10^2 = 100$. This result contradicts our assumption that AB is a diameter and so AB cannot be a diameter of the circle.
Specific behaviours
✓ states assumption and uses angle in semicircle theorem uses Pythagoras' theorem to state relationship between side lengths shows relationship is false

Question 16 **(7 marks)**

(a) A calculator can generate random integers between 10 and 25. Use the pigeonhole principle to explain why 49 random integers should be generated to be certain that at least 4 of them are the same. (3 marks)

Solution
There are 16 pigeonholes (integers from 10 to 25) and each random integer produced is a pigeon. By the pigeonhole principle: If only 48 integers are produced, there will be at least $\lceil 48 \div 16 \rceil = 3$ pigeons in at least one pigeonhole, but if 49 integers are produced then there will be at least $\lceil 49 \div 16 \rceil = 4$ pigeons in at least one pigeonhole. Hence 49 integers should be produced to be certain that at least 4 of them are the same.
Specific behaviours
✓ defines pigeonholes ✓ shows 48 insufficient ✓ shows 49 sufficient

(b) 16 customers bought a total of 130 items from a supermarket. Given that each customer bought at least one item, show that at least two of the customers bought the same number of items. (4 marks)

Solution
Assume that each customer bought a different number of items. Then the minimum number of items bought would be: $$1+2+3+\ldots+15+16=136$$ But the number of items bought (130) was less than this minimum, which contradicts the assumption made. Hence at least two customers bought the same number of items.
Specific behaviours
✓ states assumption ✓ uses assumption to calculate minimum ✓ states contradiction ✓ summary statement

Question 17 **(9 marks)**

(a) Determine the scalar product of

(i) $3.5i+6.5j$ and $8i-2j$. (1 mark)

Solution
$3.5\times 8+6.5(-2)=15$
Specific behaviours
✓ correct value

(ii) two vectors with directions $60°$ apart that have magnitudes of 15 and 18. (1 mark)

Solution
$15\times 18\times \cos 60°=135$
Specific behaviours
✓ correct value

(b) Given that $|a|=3$ and $|b|=7$ simplify $(a+b)\cdot(a+b)+a\cdot(a-2b)$. (3 marks)

Solution
$a\cdot a+2a\cdot b+b\cdot b+a\cdot a-2a\cdot b \text{¿} 2|a|^2+|b|^2$ $\text{¿} 2\times 3^2+7^2=67$
Specific behaviours
✓ expands using scalar products □ simplifies using magnitudes □ correct value

(c) The position vectors of points P, Q and R are $\begin{pmatrix}3\\-2\end{pmatrix}, \begin{pmatrix}-2\\-1\end{pmatrix}$ and $\begin{pmatrix}-5\\3\end{pmatrix}$. Show use of a vector method to determine the size of angle PQR. (4 marks)

Solution
$\overrightarrow{QR}=\begin{pmatrix}-5\\3\end{pmatrix}-\begin{pmatrix}-2\\-1\end{pmatrix}=\begin{pmatrix}-3\\4\end{pmatrix}$ $\overrightarrow{QP}=\begin{pmatrix}3\\-2\end{pmatrix}-\begin{pmatrix}-2\\-1\end{pmatrix}=\begin{pmatrix}5\\-1\end{pmatrix}$ $\cos\angle PQR=\dfrac{\begin{pmatrix}-3\\4\end{pmatrix}\cdot\begin{pmatrix}5\\-1\end{pmatrix}}{5\times\sqrt{26}}=\dfrac{-19}{5\sqrt{26}}$ $\angle PQR=138°$
Specific behaviours
✓ vectors $\overrightarrow{QR}$ and $\overrightarrow{QP}$ □ shows magnitudes

□ shows scalar product

Question 18 **(8 marks)**

A school yearbook is produced by a committee of 3 teachers and 8 students. 5 teachers and 17 students have nominated for the committee.

(a) Determine how many different committees could be formed from the nominations. (2 marks)

Solution
$\binom{5}{3}\binom{17}{8} = 10 \times 24310 = 243\,100$
Specific behaviours
✓ chooses teachers and students separately ✓ correct number

(b) The student nominations include two sets of twins. Determine how many different committees could be chosen that include at least one set of twins. (4 marks)

Solution
Choose students with at least one set of twins (Set A, Set B, Others): $n = \binom{2}{2}\binom{2}{2}\binom{13}{4} + \left[\binom{2}{2}\binom{2}{1} + \binom{2}{1}\binom{2}{2}\right]\binom{13}{5} + \left[\binom{2}{2}\binom{2}{0} + \binom{2}{0}\binom{2}{2}\right]\binom{13}{6} ¿ 715 + 5148 + 3432$ $¿ 9295$ Ways to choose whole committee: $\binom{5}{3} \times 9295 = 92\,950$
Specific behaviours
✓ indicates isolation of cases ✓ uses systematic approach ✓ correct ways to choose students ✓ correct number of committees

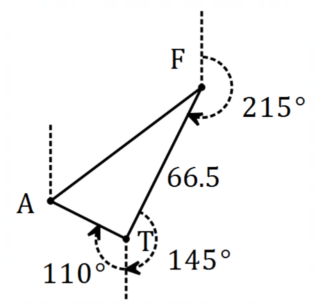
(c) Suppose one of the teachers in the committee will be appointed as treasurer and one of the students will be appointed as secretary. Determine how many different committees can be formed with this structure. (2 marks)

Solution
Select a teacher and others, select a student and others: $\binom{5}{1}\binom{4}{2} \times \binom{17}{1}\binom{16}{7} = 30 \times 194480 ¿ 5\,834\,400$
Specific behaviours
✓ indicates correct method ✓ correct number

Question 19 **(8 marks)**

Oil platform T lies 66.5 km away from another oil platform F on a bearing of $215\,°$. A steady current of 4.5 km per hour flows between the platforms on a bearing of $100\,°$. A small boat at F, with a cruising speed of 12 km per hour, needs to arrive at T by 4 pm.

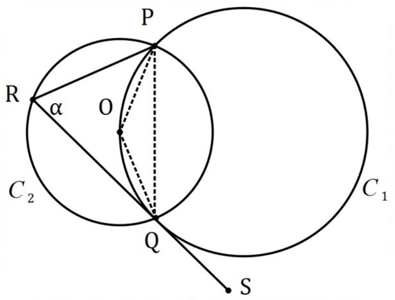
Determine the bearing that the boat should steer and the latest time it should depart from F.

Solution
 $\angle T = 360° - 110° - 145° = 105°$ If journey takes t hours, then $AF = 12t$ and $AT = 4.5t$. $\angle F$ using sine rule: $\dfrac{\sin F}{4.5t} = \dfrac{\sin 105°}{12t} \Rightarrow \sin F = \dfrac{4.5 \sin 105°}{12}$ Hence $\angle F = 21.24°$ and $\angle A = 180° - 20.24° - 105° = 53.76°$. Bearing to steer: $215° + 21.24° \approx 236°$ Distance AF using sine rule: $\dfrac{AF}{\sin 105°} = \dfrac{66.5}{\sin 53.76°} \Rightarrow AF = 79.64$ $t = 79.64 \div 12 = 6.636\text{ h} = 6\text{ h }38\text{ m}$ Hence steer on bearing $236°$ and leave before $09:22\text{ am}$.
Specific behaviours
✓ sketch diagram □ angle at T □ equation using sine rule □ solves angles in triangle □ solves for second side in triangle □ journey time in hours □ correct time to leave

Question 20 **(8 marks)**

Circles C_1 and C_2 intersect at points P and Q. C_1 passes through O, the centre of C_2. R lies on C_2 so that line segment RS is tangential to C_1 at Q. Let $\angle PRQ = \alpha$.

(a) Sketch a diagram to show the above information. (3 marks)

Solution

Specific behaviours
✓ C_1 passing through centre of C_2 □ tangent PQS □ marks angle

(b) Determine $\angle POQ$ in terms of α. (1 mark)

Solution
$\angle POQ = 2\alpha$ (angle centre-circumference C_2)
Specific behaviours
✓ correct expression

(c) Explain why $\angle PQS = 2\alpha$. (1 mark)

Solution
$\angle PQS = \angle POQ$ (alternate angles C_1)
Specific behaviours
✓ states equal to $\angle POQ$ using circle theorem

(d) Prove that $PQ = QR$. (3 marks)

Solution
$\angle RPQ = 2\alpha - \alpha = \alpha$ (exterior - interior sum in ΔRPQ) $\angle RPQ = \alpha = \angle PRQ \Rightarrow \Delta RPQ$ is isosceles Hence $PQ = QR$.
Specific behaviours
✓ deduces $\angle RPQ$ with reason □ states ΔRPQ isosceles □ deduces lengths equal

Question 21 **(8 marks)**

Particle A, initially at the point with position vector $42i - 25j$ cm, moves with a constant velocity of $-8i + 15j$ cm/s. Particle B is stationary at the point with position vector $-35i + 11j$.

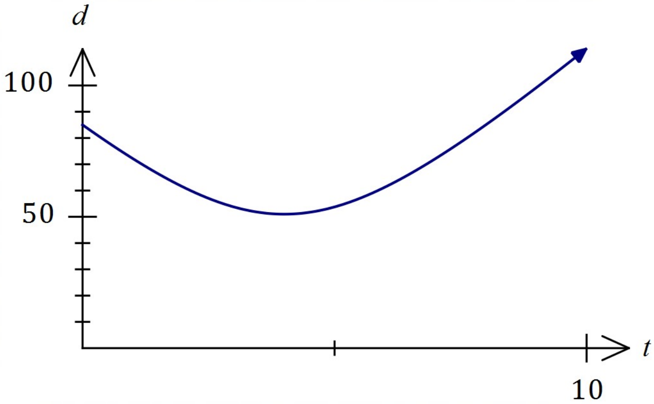
(a) Determine the initial distance of A from B. (2 marks)

Solution
$$\overrightarrow{AB} = \begin{pmatrix} -35 \\ 11 \end{pmatrix} - \begin{pmatrix} 42 \\ -25 \end{pmatrix} = \begin{pmatrix} -77 \\ 36 \end{pmatrix}$$ $$¿\overrightarrow{AB}∨¿85\text{ cm}$$
Specific behaviours
✓ vector $\overrightarrow{AB}$ correct distance

(b) Determine an expression for the distance d between A and B after t seconds. (3 marks)

Solution
$$\overrightarrow{AB} = \begin{pmatrix} -35 \\ 11 \end{pmatrix} - \left[\begin{pmatrix} 42 \\ -25 \end{pmatrix} + t \begin{pmatrix} -8 \\ 15 \end{pmatrix} \right] ¿ \begin{pmatrix} 8t - 77 \\ 36 - 15t \end{pmatrix}$$ $$d = \sqrt{(8t-77)^2 + (36-15t)^2} ¿ 17\sqrt{t^2 - 8t + 25}$$
Specific behaviours
✓ position vector for A at time t ✓ vector $\overrightarrow{AB}$ distance expression (no need to simplify)

(c) Sketch a graph of d against t and hence determine the time that minimises d and state what this minimum distance is. (3 marks)



Solution
Minimum when $t = 4$ s   Minimum distance is 51 cm
Specific behaviours
✓ sketch graph   time   minimum distance

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: _________

Supplementary page

Question number: _________

Supplementary page

Question number: _________

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