#### MATHEMATICS METHODS

# 8 AWAM Semester 2 (Units 3 and 4) Examination 2016

**Calculator-free** 

## Marking Key

0 MAWA, 2016

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· the end of week 1 of term 4, 2016

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# CALCULATOR-FREE MARKING KEY

#### MATHEMATICS METHODS SEMESTER 2 (UNITS 3 AND 4) EXAMINATION

#### Section One: Calculator-free

(54 Marks)

Question 1(a)

Solution	
$ \ln m = \frac{3}{2} \Longrightarrow m = e^{\frac{3}{2}} $	
Marking key/mathematical behaviours	Marks
identifies correct base	1
determines correct power	1

#### Question 1(b)

Solution

$$\log[(m+3)m] = 1$$

$$(m+3)m=10^1$$

$$m^2 + 3m - 10 = 0$$
$$(m+5)(m-2) = 0$$

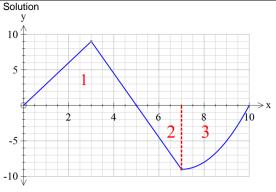
m = -5 or 2 but since m has to be greater than zero, m = 2 is the only solution.

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Marking key/mathematical behaviours	Marks
applies logarithmic rule for a product correctly	1
recognises base 10	1
creates equation with correct trinomial	1
<ul> <li>solves equation correctly giving the correct value of m</li> </ul>	1

Question 2(a)(i)

Question 2(a) (i)		
Solution		
$\frac{dy}{dx} = \frac{(6x^4 - x^3 + e)(4e^x) - (4e^x)(24x^3 - 3x^2)}{(4e^x)^2 + (4e^x)^2}$		
$\left(6x^4 - x^3 + e\right)^2$		
Marking key/mathematical behaviours	Marks	
differentiates the 1st term on numerator correctly	1	
differentiates the 2nd term on numerator correctly	1	
squares factor on denominator	1	

#### Question 10(b)



Area 
$$2 = \frac{1}{2} \times 2 \times 9$$
  
= 9 square units

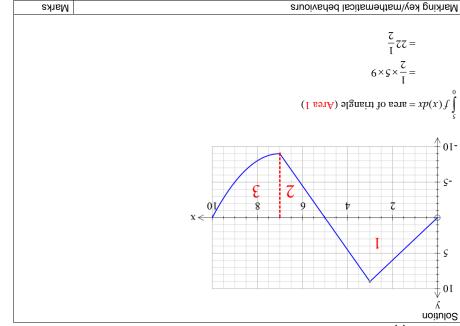
Area 
$$3 = 50 - 22\frac{1}{2} - 9$$
  
=  $18\frac{1}{2}$  square unit

$$\int_{7}^{6} f(x)dx = -\text{Area } 3$$
$$= -18\frac{1}{2}$$

Marking key/mathematical behaviours	Marks
calculates area 2	1
calculates area 3	1
determines integral	1

# Question 10(a)

L



#### Solution Question 2(b)

# $\frac{\partial}{\partial x} = \frac{\partial}{\partial x} \qquad \text{bns} \qquad (x) \sin x + x = \frac{\partial}{\partial x} \iff (x) \cos x - 2x = u \text{ follow}$

differentiates correctly 2nd term

differentiates correctly 1st term

Marking key/mathematical behaviours

 $[(x)\text{mis})\text{ml}\frac{b}{xb} - [(\xi + \varepsilon x\xi)\text{ml}]\frac{b}{xb} =$ 

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 $[(x)\text{nis})\text{nl} - \left(\xi + \varepsilon x \xi\right)\text{nl} \frac{b}{xb} = \frac{xb}{xb}$ 

 $= \frac{15x^2}{15x^3 + 3} - \frac{\cos(x)}{\sin(x)} = \frac{\cos(x)}{\sin(x)}$ 

MATHEMATICS METHODS

Solution

Question 2(a)(ii)

Marking key/mathematical behaviours

$$((x)\operatorname{nis} + x2)\frac{1}{(x)\operatorname{nos}^{-2}x^{9}} = ((x)\operatorname{nis} + x2) \times \frac{1}{x^{9}} = \frac{xb}{xb} \times \frac{\sqrt{b}}{xb} = \frac{\sqrt{b}}{xb}$$

applies correctly logarithmic rule for quotients

$$((x) \operatorname{uis} + xz) - \frac{z}{z} = ((x) \operatorname{uis} + xz) \times \frac{z}{z} = \frac{xp}{xp} \times \frac{yz}{yz} = \frac{xp}{xz}$$

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differentiates correctly to determine 2nd factor in chain rule	•	
differentiates correctly to determine 1st factor in chain rule	•	

L

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Marks

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Marks

**WARKING KEY** 

CALCULATOR-FREE

$$x$$
 for smith of  $\frac{\sqrt{b}}{x}$  sessential  $\frac{\sqrt{b}}{x}$  sessential  $\frac{\sqrt{b}}{x}$ 

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determines integral.

identifies integral as area of correct triangle

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#### MATHEMATICS METHODS SEMESTER 2 (UNITS 3 AND 4) EXAMINATION

# CALCULATOR-FREE MARKING KEY

Question 3(a)

Solution	
Discrete random variable	
Marking key/mathematical behaviours	Marks
determines correct category	1

Question 3(b)

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Solution	
Non-random variable	
Marking key/mathematical behaviours	Marks
determines correct category	1

Question 3(c)

Solution	
Continuous random variable	
Marking key/mathematical behaviours	Marks
determines correct category	1

#### MATHEMATICS METHODS SEMESTER 2 (UNITS 3 AND 4) EXAMINATION

# CALCULATOR-FREE MARKING KEY

#### Question 9(c)(i)

Solution

$n_2$ is larger than $n_1$	
To increase confidence a larger interval is required for a stable sample size. Increasing the standard error and thus the interval can remain the same.	g n reduces
Marking key/mathematical behaviours	Marks
<ul> <li>states n₂ is larger with reason</li> </ul>	1

#### Question 9(c)(ii)

Solution

$$E_1 = 1 \times \sqrt{\frac{m(1-m)}{n_1}}$$

$$E_2 = 1.5 \times \sqrt{\frac{m(1-m)}{n_2}}$$

Same interval so  $E_1 = E_2$ 

states correct reason

$$\sqrt{\frac{m(1-m)}{n_1}} = 1.5 \times \sqrt{\frac{m(1-m)}{n_2}}$$

$$\frac{m(1-m)}{n_1} = (1.5)^2 \frac{m(1-m)}{n_2}$$

$$\frac{n_2}{n_1} = 2.25$$

$$n_2 = 2.25n_1$$

Marking key/mathematical behaviours	Marks
equates E <sub>1</sub> and E <sub>2</sub>	1
squares both sides	1
states relationship	1

Marks

Marks

# Question 9(a)(ii)

### Question 5

Solution 4 noiteauD

Marking key/mathematical behaviour
$f = q \text{ no } \frac{1}{\mu} = q \iff 0 = (\xi - q \hbar)(1 - q \hbar)$
$0 = \xi + q \partial \mathbf{I} - ^{2} q \partial \mathbf{I}$
$\frac{91}{\xi} = \left(\frac{\xi}{\xi}\right) = (d-1)d$
Polition

• calculates the value of k evaluates integral correctly integrates correctly

 $\lambda = \frac{12}{5} = \lambda \iff 1 = \left[\frac{1}{5} - \frac{1}{5}\right]$ 

 sets up integral and equates to one Marking key/mathematical behaviours

$$16p^{-} - 16p + 3 = 0$$

$$(4p - 1)(4p - 3) = 0 \Rightarrow p = \frac{1}{4} \text{ or } p = \frac{3}{4}$$
Marking key/mathematical behaviours

derives quadratic equation	•	
sets up equation using variance of a Bernoulli distribution	•	

- factorises trinomial
- solves correctly for p

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#### SEMESTER 2 (UNITS 3 AND 4) EXAMINATION MATHEMATICS METHODS

estates interval	i
• simplifies €	ı
• simplifies square root	ı
<ul> <li>substitutes values for z, n and p</li> </ul>	l
Marking key/mathematical behaviours	Магкѕ
95% CI is (0.12,0.28)	
80.0 =	
$\frac{2}{2} \times 2 = \frac{2}{3}$	
$\frac{1}{\sqrt{2800}} \sqrt{\times 2} = \frac{1}{\sqrt{2800}}$	
$\mathbb{E} = \mathbb{Z} \times \sqrt{\frac{\frac{1}{5}(1-\frac{1}{5})}{100}}$	
olution	

#### Question 9(b)

l	<ul> <li>states confidence interval.</li> </ul>
l	∃ sənimıətəb •
Marks	Marking key/mathematical behaviours
	$(\overline{\frac{(m-1)m}{n}} + m, \overline{\frac{(m-1)m}{n}} - m) \text{ si ID } \%89$
	$\underbrace{\frac{(m-1)m}{n}}_{l} \bigvee \times I = \mathcal{I}$
	Solution

# MATHEMATICS METHODS SEMESTER 2 (UNITS 3 AND 4) EXAMINATION

# CALCULATOR-FREE MARKING KEY

#### MATHEMATICS METHODS SEMESTER 2 (UNITS 3 AND 4) EXAMINATION

Question 6(a)

aucstion o(u)		
Solution		
Function is valid for $x \ge -3$		
Marking key/mathematical behaviours	Marks	
correctly states the values of x for which the function is valid	1	

Question 6(b)

Solution	
$\frac{dy}{dx} = \frac{2}{2x+6} = 4 \Rightarrow \frac{2x+6}{2} = \frac{1}{4} \Rightarrow x+3 = \frac{1}{4} \Rightarrow x = -2.75$	
Marking key/mathematical behaviours	
differentiates correctly	1
solves equation correctly	1

Question 7(a)

Question r(a)						
Solution						
у	0	1	2	3	4	
P(Y = y)	0	k	4 <i>k</i>	9k	16k	
Marking key/mathematical behaviours					Marks	
correctly completes two values				1		
correctly completes 4 values					1	

Question 7(b)

Solution 
$$k+4k+9k+16k=1$$

$$30k=1 \implies k=\frac{1}{30}$$
Marking key/mathematical behaviours

• sums probabilities equal to one
• correctly solves equation for  $k$ 

1

#### **Question 8**

Solution
$f(x) = \int f'(x)  dx$
$= \int 2xe^{3x^2-1} dx$
$= \frac{1}{3}e^{3x^2 - 1} + c$
since $f(0) = 0$ :
$0 = \frac{1}{3}e^{-1} + c$
$c = -\frac{1}{3e}$
$1_{3x^2-1}$ 1

Marking key/mathematical behaviours	
determines indefinite integral	1
<ul> <li>substitutes initial conditions to calculate the constant c</li> </ul>	1
• states f(x)	1

#### Question 9 (a)(i)

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Solution	
$\hat{p} = \frac{20}{100} = \frac{1}{5}$	
Marking key/mathematical behaviours	Marks
determines the proportion	1