Name:

Calculator Free Section (No notes or calculators. Formula sheet provided.)

Time allowed – 25 minutes

Question 1   [2, 3, 3, 3, 4   marks]

a)   Evaluate $\displaystyle\int_{-1}^{2} 2(x+1)^3 \, dx$

b)   Find the area enclosed by the curve $y = x^2 - 1$ and $y = 7 - x^2$

c)   Evaluate $f\,'(0)$ for $f(x) = xe^{3x}$

d) Find $\dfrac{d}{dx}\displaystyle\int_{4}^{e^x} e^{2t} + 3t\ dt$

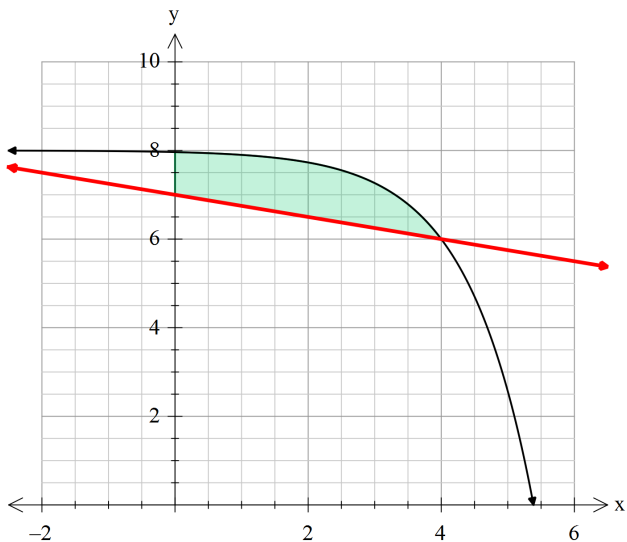e) Show that $\displaystyle\int \dfrac{e^{5x} + 3 + e^x}{4e^{3x}} dx \;=\; \dfrac{e^{5x} - 2 - e^x}{8e^{3x}} + c$

Question 2   [4 marks]

Find the x-coordinate of all stationary points of the function $f(x) = \dfrac{(x-1)^2}{e^x}$

Question 3   [4 marks]

The graphs of $y = 8 - 2e^{x-4}$ and $y = -\frac{1}{4}x + 5$

intersect at x=4 for x≥0 as shown. Determine the

exact area between $y = 8 - 2e^{x-4}$ and

$y = -\frac{1}{4}x + 5$ and the y-axis.

Name:_____

Calculator Assumed Section (1 A4 page of notes allowed. Formula sheet provided.)

Time allowed – 30 minutes

Question 4   [3 marks]

Water flows into a tank at a rate given by: $W'(t) = \dfrac{1}{75}(20t - t^2 + 600)$ where W'(t) is measured in

L/hour and t is in hours. Initially, there are 200 L of water in the tank. How many litres of water are in

the tank after 24 hours?

Question 5   [3 marks]

A part of the function $g(x) = x^2 - 4$ is shown. The area of the region

marked A is the same as the area of the region marked B. Evaluate the

exact value of *a*
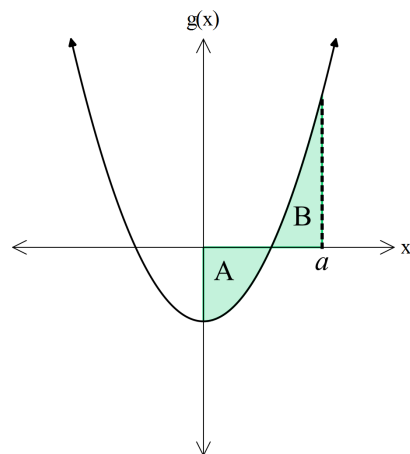
Question 6   [2, 4, 3 marks]

An object moves along the *x*-axis with acceleration $a = (3t - 2)\, m/s^2$. Initially it is at the origin and moving with speed $7.5\, m/s$ in a *negative* direction.

(a) Find an expression for velocity in terms of *t*.

(b) When and where does the object change direction?

(c) How far does the object travel in the first 5 seconds?

Question 7   [1, 2, 3 marks]

Certain medical tests require the patient to be injected with a solution containing 0.5 micrograms (µg) of the radioactive substance Technetium-99. This material decays according to the rule:

$T = T_0 e^{-0.1155t}$ where t is the time in hours from injection.

a) What is the value of T_0?

b) What is the half-life of Technetium-99?

c) How long is the amount of Technetium-99 left in the patient's system less than 1% of the initial amount? Give your answer to the nearest hour.

Question 8   [5, 1 marks]

A train is travelling at 40metrers per second when the brakes are applied. The velocity of the train is given by the equation

$v = 40 - 0.4x^2$, where t represents the time in seconds after the brakes are applied.

The area under a velocity-time graph gives the total distance travelled for a particular time period.

Velocity-time graph
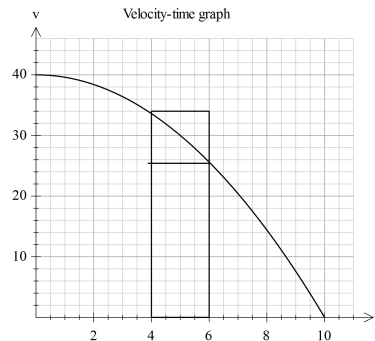
a) Complete the tables below and estimate the distance travelled by the train during the first six seconds by calculating the mean of the areas of the circumscribed and inscribed rectangles. (The rectangles for the 4–6 seconds interval are shown on the grid above.)

Time (t)	0	2	4	6
Velocity (V)		38.4		25.6

Rectangle	0-2	2-4	4-6	Total
Circumscribed area			67.2	
Inscribed area			51.2	

Estimated total distance travelled = _____

b) Describe how you could better estimate the distance travelled by the train during the first six seconds than by the method used in part (a).