Question 6: **[1, 2, 2, 2, 3 = 10 marks]** **CA**

The time patients wait for their appointments at a dentist is uniformly distributed over the interval 2 to 25 minutes.

The waiting times of 15 samples of 30 patients each are recorded.

(a) Calculate the mean waiting time for all of the patients at this dentist.

$$\frac{25-2}{2} = 11.5 + 2 = 13.5 \text{ min}$$

(b) Calculate the standard deviation of the waiting times for all of the patients at this dentist.

$$\sqrt{\frac{(25-2)^2}{12}} = 6.64 \text{ min}$$

$$\therefore \sigma = \frac{6.64}{\sqrt{30}} = 1.2122$$

By Central Limit Theorem:

(c) Define the probability distribution that best models the distribution of the means of the samples.

$X \sim N(13.5, 1.2122^2)$

(d) Find the probability that a random sample has a mean waiting time of less than 15 minutes.

$P(X < 15) = 0.8920$

(e) Calculate the probability that at most two samples had mean waiting times of less than 15 minutes.

$Y \sim Bin(15, 0.8920)$

$P(Y = 2) = 2.27 \times 10^{-11}$

Calculator Assumed
Sample Proportions and the Central Limit Theorem

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [3, 3 = 6 marks] **CA**

(a) Use the fact that a linear transformation of a random variable X of the form $Y = aX + b$ yields $E(Y) = aE(X) + b$ and $Var(Y) = a^2 Var(X)$ to show that $E(\hat{p}) = p$, where $\hat{p}$ and p are the sample proportion and expected probability respectively.

(b) Using the same facts as above, also establish that $Var(\hat{p}) = \frac{p(1-p)}{n}$

Question Two:[1, 3, 1, 2, 2, 2 =11 marks] **CA**

On a Saturday night in Perth, a Random Breath Test (RBT) location is set up on a busy Perth street and 3000 RBTs are conducted by police. 45 drivers were found to have a blood alcohol limit in excess of 0.05.

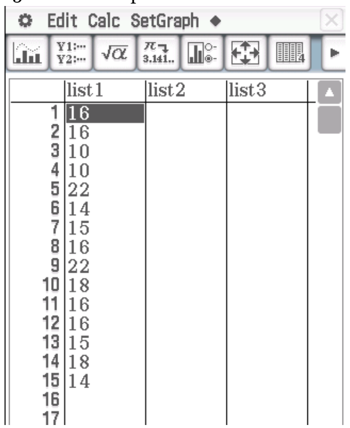
(a) State the sample proportion of drivers with a blood alcohol concentration in excess of the limit of 0.05.

(b) Police conduct further RBTs in similar locations over the next 15 Saturday nights in Perth. Calculate the mean and standard deviation of the proportion of samples if each sample is 3000 drivers.

Police decide to conduct RBTs in this same location on a Sunday morning. They first decide to model the scenario using a binomial random variable X.

(c) If each sample is to be of size 1000 on a Sunday morning, define X.

15 random samples are simulated and the results are given below.



Question 4: **[2, 2, 3 =7 marks]** **CA**

Samples of size 70 are taken of a Bernoulli variable with $p = 0.6$.

(a) State the approximate mean and standard deviation of the samples.

$\mu = 0.6$

$\sigma = \sqrt{\dfrac{0.6 \times 0.4}{70}} = 0.0586$

(b) Use an appropriate distribution to calculate $P(0.55 < \hat{P} < 0.7)$

$X \sim N(0.6, 0.0586^2)$

$P(0.55 < X < 0.7) = 0.7593$

(c) Show the use of a standard normal distribution to obtain the same answer as in (b).

$z_1 = \dfrac{0.55 - 0.6}{0.0586} = -0.8532$

$z_2 = \dfrac{0.7 - 0.6}{0.0586} = 1.7065$

$P(-0.8532 < Z < 1.7065) = 0.7593$

Question 5: **[2 marks]** **CA**

One hundred samples of 10 people are tested for a gene which occurs in 5.5% of the population.

Another one hundred sample of 100 people are tested for the same gene.

If both sets of samples were graphed, which would most look like a normal distribution? Explain your answer.

The second set of samples – the sample size of 100 is much larger than 10 and the sample proportions would better approximate a normal distribution.

Question Three: **[1, 1, 4, 1, 2 = 9 marks]** **CA**

A random sample of three items is selected from a batch of 15 items which contains five defective.

(a) What is p, the proportion of defectives in the batch?

$p = \frac{5}{15} = \frac{1}{3}$ ✓

(b) What are the possible values of the sample proportion $\hat{p}$ of defective in the sample?

$0, \frac{1}{3}, \frac{2}{3}, 1$ ✓

(c) Construct a probability distribution table which summarises the sampling distribution of the sample proportion of defectives in the sample.

Number of defective items in sample	0	1	2	3
$\hat{p}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$P(\hat{P} = \hat{p})$	$\frac{\binom{5}{0}\binom{10}{3}}{\binom{15}{3}} = \frac{120}{455}$ ✓	$\frac{\binom{5}{1}\binom{10}{2}}{\binom{15}{3}} = \frac{225}{455}$ ✓	$\frac{\binom{5}{2}\binom{10}{1}}{\binom{15}{3}} = \frac{100}{455}$ ✓	$\frac{\binom{5}{3}\binom{10}{0}}{\binom{15}{3}} = \frac{10}{455}$ ✓

(d) Use you sampling distribution to determine the probability that the proportion of defectives is less than 0.4.

$\frac{120 + 225}{455} = \frac{345}{455}$ ✓

(e) Find $P(\hat{P} < 0.4 \mid \hat{P} > 0)$

✓

$\frac{225}{455} \div \frac{335}{455} = \frac{225}{335}$ ✓

(d) Calculate the sample proportions from the simulations given above.

(e) Calculate the mean and standard deviation of the sample proportions.

(f) Comment on your answers to part (b) and (f).

Question Three: [1, 1, 4, 1, 2 = 8 marks] **CA**

A random sample of three items is selected from a batch of 15 items which contains five defective.

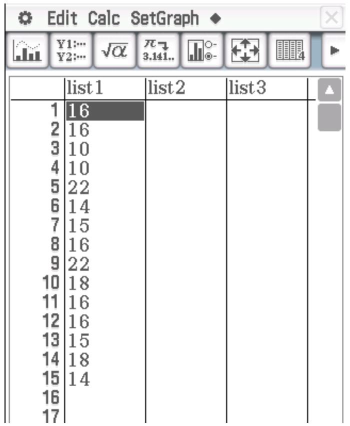
(a) What is p, the proportion of defectives in the batch?

(b) What are the possible values of the sample proportion $\hat{p}$ of defective in the sample?

(c) Construct a probability distribution table which summarises the sampling distribution of the sample proportion of defectives in the sample.

(d) Use you sampling distribution to determine the probability that the proportion of defectives is less than 0.4.

(e) Find $P(\hat{p} < 0.4 \mid \hat{p} > 0)$



(d) Calculate the sample proportions from the simulations given above.

0.016, 0.016, 0.01, 0.01, 0.022, 0.014, 0.015, 0.016, 0.022, 0.018, 0.016, 0.016, 0.015, 0.018, 0.014

(e) Calculate the mean and standard deviation of the sample proportions.

$\mu = 0.0159$
$\sigma = 0.003284$

(f) Comment on your answers to part (b) and (e).

The mean and standard deviation of the proportion of samples in (b) is very close to the mean and standard deviation of the sample proportions.

Question Two: [1, 3, 1, 2, 2, 2 =11 marks] **CA**

On a Saturday night in Perth, a Random Breath Test (RBT) location is set up on a busy Perth street and 3000 RBTs are conducted by police. 45 drivers were found to have a blood alcohol limit in excess of 0.05.

(a) State the sample proportion of drivers with a blood alcohol concentration in excess of the limit of 0.05.

$$=\frac{45}{3000}=0.015$$ ✓

(b) Police conduct further RBTs in similar locations over the next 15 Saturday nights in Perth. Calculate the mean and standard deviation of the proportion of samples if each sample is 3000 drivers.

$E(\hat{p})=0.015$ ✓

$Var(\hat{p})=\frac{0.015\times0.995}{3000}=0.000004975$ ✓

$StDev=0.00223$ ✓

Police decide to conduct RBTs in this same location on a Sunday morning. They first decide to model the scenario using a binomial random variable X.

(c) If each sample is to be of size 1000 on a Sunday morning, define X.

$X \sim Bin(1000, 0.015)$

✓

15 random samples are simulated and the results are given below.

Question 4: **[2, 2, 3 =7 marks]** **CA**

Samples of size 70 are taken of a Bernoulli variable with $p=0.6$.

(a) State the approximate mean and standard deviation of the samples.

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(c) Show the use of a standard normal distribution to obtain the same answer as in (b).

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(e) Calculate the probability that at most two samples had mean waiting times of less than 15 minutes.

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SOLUTIONS
Calculator Assumed
Mixed Applications of Discrete and Continuous Random Variables

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [3, 3 = 6 marks] **CA**

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$$\hat{P} = \frac{X}{n}$$

$$E(X) = np$$

$$E(\hat{P}) = E\left(\frac{X}{n}\right) = \frac{1}{n}E(X) = \frac{1}{n} \times np = p$$

(b) Using the same facts as above, also establish that $Var(\hat{p}) = \dfrac{p(1-p)}{n}$

$$Var(X) = np(1-p)$$

$$Var(\hat{P}) = Var\left(\frac{X}{n}\right)$$

$$= \left(\frac{1}{n}\right)^2 Var(X)$$

$$= \frac{1}{n^2} np(1-p)$$

$$= \frac{p(1-p)}{n}$$