ALL SAINTS' COLLEGE

Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

Section One: Calculator-free

QUESTIONS

WA student number: In figures

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

End of questions

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

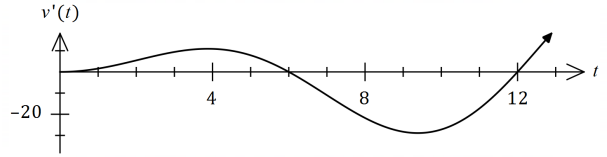
Supplementary page

Question number: _____

Question 8 **(7 marks)**

(a) Determine an expression for $\frac{d}{dt}\left(6t\cos\left(\frac{\pi t}{6}\right)\right)$. (2 marks)

The volume of water in a tank, v litres, is changing at a rate given by $v'(t)=\pi t\sin\left(\frac{\pi t}{6}\right)$, where t is the time in hours. The rate of change is shown in the graph below.



(b) Using the result from part (a) or otherwise, determine the change in volume of water in the tank between $t=0$ and $t=12$ hours. (5 marks)

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

Determine the area bounded by the line $y=-2x$ and the parabola $y=x^2-6x$.

Question 2 **(5 marks)**

A curve, defined for $x>0$, passes through the point $P(2,3)$ and its gradient is given by

$$\frac{dy}{dx}=6x^2-\frac{4}{x^2}-23$$

(a) Verify that P is a stationary point, determine the value of the second derivative at P and hence describe the nature of the stationary point. (3 marks)

(b) Determine the equation of the curve. (2 marks)

Question 7 **(8 marks)**

Initially, particle P is stationary and at the origin. Particle P moves in a straight line so that at time t seconds its acceleration a cms^{-2} is given by $a=8-3\sqrt{t}$ where $t\geq 0$.

(a) Determine the speed of P after 1 second. (3 marks)

(b) Determine the speed of P when it returns to the origin. (5 marks)

Question 6 **(5 marks)**

The graph of $y=f(x)$ has a stationary point at $(-1,2)$ and $f'(x)=ax^2+4x+6$, where a is a constant.

Determine the interval over which $f'(x)>0$ and $f''(x)<0$.

Question 3 **(7 marks)**

A bag contains 40 counters, 15 marked with 0 and the remainder marked with 1. The random variable X is the number on a randomly selected counter from the bag.

(a) Explain why X is a Bernoulli random variable and determine the mean and variance of X. (3 marks)

Each of the 32 students in a class randomly select a counter from the bag, note the number on the counter and then replace it back in the bag. The random variable Y is the number of students in the class who select a counter marked with 0.

(b) Define the distribution of Y and determine the mean and variance of Y. (3 marks)

(c) Explain why it is important that the students replace their counters for the distribution of Y in part (b) to be valid. (1 mark)

Question 4 **(8 marks)**

Determine

(a) $f'(x)$ when $f(x)=\sqrt{4x-3}$. (2 marks)

(b) $\frac{d}{d\theta}\left(\theta^3 e^{4\theta}\right)$ when $\theta=2$. (3 marks)

(c) $f'\left(\frac{\pi}{4}\right)$ when $f(t)=\frac{1+\cos t}{\sin t}$. (3 marks)

Question 5 **(7 marks)**

Functions f and g are such that

$$f(4)=2, f'(x)=18(3x-10)^{-2}$$
$$g(-4)=2, g'(x)=18(3x+10)^{-2}$$

(a) Determine $f(6)$. (3 marks)

(b) Use the increments formula to determine an approximation for $g(-3.98)$. (3 marks)

(c) Briefly discuss whether using the information given about f and the increments formula would yield a reasonable approximation for $f(6)$. (1 mark)