1.1 Energy Considerations

(NB this section should have been covered in the 2A/2B course in year 11 but is included here as revision notes required for some questions in the 3A/3B course)

Work is done whenever the point of action of a force is moved through a distance.
The amount of work done equals the value of the force multiplied by the distance moved against the force i.e.

$$\boxed{W = F \times d}$$

units of work are newtons x metres, or **joules**.

Example 1

Calculate the work done by John, who drags a 25 kg bag of salt across the pavers against a frictional force of 55 N for a distance of 12 metres.

Solution 1

$W = F \times d = 55 \times 12 = 6.6 \times 10^2$ joules

Example 2

What work is done by Mary, a 50.0 kg girl who climbs 15.0 m upstairs to bed?

Solution 2

Gravitational force on the girl is mg = 50 x 9.8 = 490 N
$W = F \times d = 490 \times 15 = 73.5$ kilojoules

Energy is a **store of work** that can be used. In the case of humans, energy is stored in the chemicals contained in our food (e.g. carbohydrates, fats). Work can also be stored due to the position of an object e.g. a demolition ball lifted up high can be used to knock down a wall – this the ball has stored gravitational **potential energy** (PE). As it swings and moves fast the E_P becomes converted into **kinetic energy**, or energy of movement.

$$\boxed{E_P = mgh}$$

Other ways in which energy can be stored are: kinetic energy E_K (motion), heat energy, sound energy, nuclear energy, electrical energy and electromagnetic energy.

The **Law of conservation of energy** states that energy can never be created nor destroyed, but only converted from one form to another. In the case of the demolition ball mentioned, the chemical energy in the crane's fuel was converted to potential energy of the ball, which was converted to kinetic energy of the ball and then to heat energy and sound energy as the ball hit the wall.

The fuel, itself was formed millions of years ago when the Sun's electromagnetic energy (light) caused plants to photosynthesise, store chemical energy as starch, which then became incorporated into the fuel when they fossilized.

Kinetic Energy is energy due to movement of an object $\boxed{E_K = \frac{1}{2} mv^2}$

Example 3

A demolition ball of mass 500 kg is lifted to a height of 2.2 m above its normal position and then allowed to strike the wall. With what kinetic energy does it collide with the wall?

Solution 3

Work done = E_P stored = mgh = 500 x 9.8 x 2.2 = 10780 J. This is converted to KE

So E_K of ball = 1.1×10^4 J (2 s.f)

Example 4

In the last, demolition ball question, find the velocity with which the ball strikes the wall.

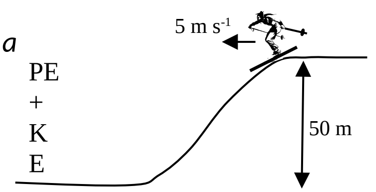
Solution 4

$mgh = \frac{1}{2}mv^2$ (mass cancels out) so $v^2 = 2gh$ $v^2 = 2 \times 9.8 \times 2.2 = 43.12$

So $v = 6.57 \text{ m s}^{-1}$

Example 5

A skier moving at 5.0 m s^{-1} comes over a ski slope which is 50.0 m above a valley. What will her speed be when she reaches the bottom?



Solution 5

Conserving energy: $E_P + E_K$ at top = $E_P + E_K$ at bottom

$m \times 9.8 \times 50 + \frac{1}{2}m \times 5^2 = \frac{1}{2}mv^2$ (m cancels throughout)

$9.8 \times 50 + \frac{1}{2} \times 25 = \frac{1}{2}v^2$ So $v = 31.7 \text{ m s}^{-1}$

Example 6

A set of light in a town run from a hydroelectric generator. Outline the energy conversions in this system.

Solution 6

Electromagnetic radiation from the sun causes seawater to evaporate and rise into the upper atmosphere which gives it E_P. When it falls as rain this water is collected in a dam on a mountain where E_P is stored and converted to E_K as it runs down tubes to the hydroelectric power station. The turbine turns a generator where the E_K is converted to electrical energy.

The electrical energy relayed to the footy lights is converted to heat and light in the spotlights.

These are the energy transformations:

POTENTIAL ENERGY ⇨ **KINETIC ENERGY** ⇨ **KINETIC ENERGY** ⇨ **ELECTRICAL ENERGY** ⇨ **HEAT & LIGHT**

1.2 Power

Two boys, Tom and Jerry, have equal masses of 62 kg and run up the school staircase which is 4.2 m above the ground. The work done by each boy against the gravitational force is mgh
= 62 x 9.8 x 4.2 = 2552 J – they both do the same amount of work. However, Tom runs up in 6.5 seconds while Jerry takes 8.2 seconds.

Each second Tom's body does an amount of work = 2552/6.5 = 393 joules per second while Jerry's body can only do 311 joules of work per second.

We can say that Tom is more **powerful** than Jerry because he can do more work per second. The amount of work done (or energy gained) per second is called **power** and it is measured in J s^{-1} or **Watts** (W).

Power = the rate of doing work:	**P = W/t**

Checkpoint 1

1. If a ball is thrown upwards the main energy conversions **not** applicable are:

   (A) Chemical to E_k (B) E_k to E_p C) Heat to PE (D) E_p to E_k

2. in the previous question a ball thrown upwards at 15.0 ms^{-1} will reach a height of:

   (A) 2.40 m (B) 11.5 m (C) 15.9 m (D) 23.0 m

3. A 25 kg bag of salt is dragged 5 m along the ground against a 30 N frictional force. What are the energy conversions occurring?

   (A) KE to PE (B) PE to heat (C) PE to KE (D) KE to heat

4. The heat generated in the previous question would be:

   (A) 5.0 J (B) 75 J (C) 150 J (D) 750 J

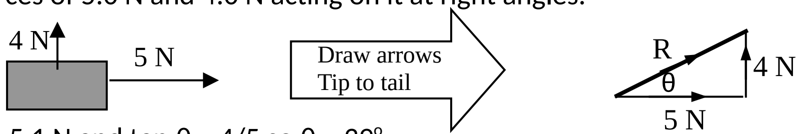
5. An electric motor is able to pump 2.60 kg of water per second up to a tank that is 3.45 m off the ground. The power required to pump the water is about:

   (A) 6.90 W (B) 19.2 W (C) 33 W (D) 88 W

1.3 Vector Quantities

(NB this section should have been covered in the 2A/2B course in year 11 but is included here as revision notes required for some questions in the 3A/3B course)

Vectors are added by constructing a **vector diagram** or triangle.
e.g. a block of wood has forces of 5.0 N and 4.0 N acting on it at right angles.

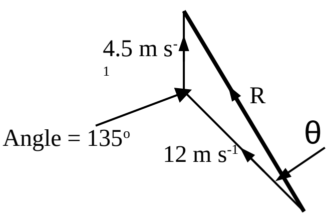


$R^2 = 4.0^2 + 5.0^2$ So R = 5.1 N and tan θ = 4/5 so θ = 39°

Example 7

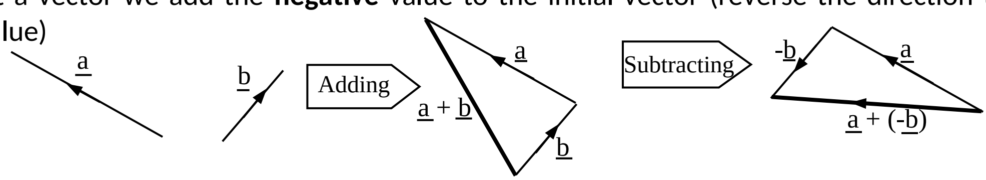
A crow can fly at 4.5 m s^{-1} in still air and heads due north with a wind blowing from the southeast at 12 m s^{-1}. Find the bird's resultant velocity with the wind blowing.

Solution 7



In this case the solution of this triangle requires the use of the cosine rule:
$R^2 = 4.5^2 + 12^2 - 2 \times 4.5 \times 12\cos 135$ R = 7.7 m s^{-1}
To find θ the sine rule must be used: 7.74/sin135 = 4.5/sin θ   sin θ = 4.5sin135/7.74 = 0.150 so θ = 8.7° So R is at 45 – 8.63° from the north = 36.4° west of north.

Whenever a vector changes from one value to another we need to subtract vectors to find the change that has occurred. The value of the change = final vector – initial vector.
To subtract a vector we add the **negative** value to the initial vector (reverse the direction to find the negative value)
e.g.

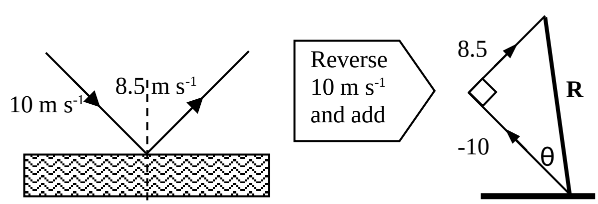


Example 8

A ball moving at 10 ms^{-1} strikes a wall at 45° and bounces off at 45° and a speed of 8.5 m s^{-1}. Find the ball's change in velocity.

Solution 8

The change in velocity (Δv) is given by:
$\Delta v^2 = 10^2 + 8.5^2$
Δv = 13 m s^{-1}
tanθ = 8.5/10 so θ = 40°



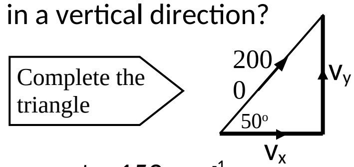
Direction of $\Delta v = 45 + 40 = 85^o$ to wall

Vector Components

The magnitude of a vector acting in one particular direction is called its **component** in that direction. The **horizontal and vertical components** of a vector can be found by completing the other 2 sides of the vector triangle

Example 9

A bullet is fired at 50° to the horizontal and a speed of 200 m s^{-1}.
How fast is it moving along the ground and how fast is the bullet rising in a vertical direction?

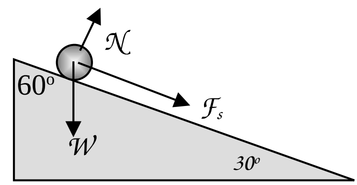


Solution 9

cos 50 = v_x/200 So v_x = 200 cos50 = 128 m s^{-1}
sin 60 = v_y /250 So v_y = 200 sin50 = 153 m s^{-1}
The horizontal component of the rocket = 128 ms^{-1} and its vertical component = 153 m s^{-1}

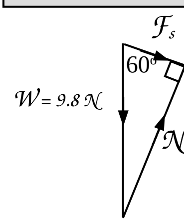
Example 10

What is the net force on a 1.0 kg ball that is rolling down a slope of 30°?



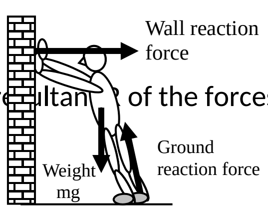
Solution 10

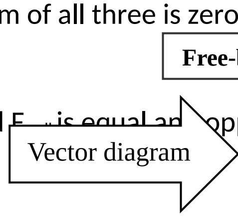
Show all the forces acting (W and N)
W is the weight of the ball
N is the normal reaction force from the surface
F_s is the force down the slope which is the resultant of W and N.
Construct the vector triangle with the forces shown in the correct directions.
F_s = 9.8sin30 = 4.9 N

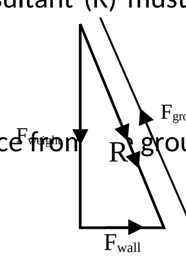


1.4 Free-Body Diagrams

In the diagram above arrowed vectors have been drawn to show the direction of the forces acting (a_p and a_d) and another vector to show the direction of the net force or resultant. These free-body diagrams are essential in solving problems involving vectors. In the above case a_s is the resultant acceleration and so it indicates the direction the object will accelerate. In other cases where an object is at rest all the vectors must add up to zero. E.g. a man leaning against a wall.
As the man is in equilibrium any two of the vectors added to give a resultant (R) must be equal and opposite to the third vector, so the sum of all three is zero.







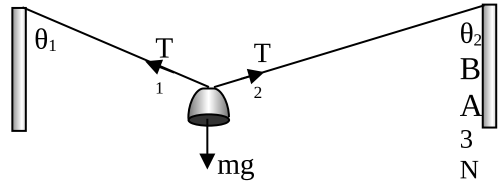
The resultant R of the forces F_{weight} and F_{wall} is equal and opposite to the force from the ground, F_{ground}.

1.5 Equilibrium of Forces

If any object has no acceleration then all the forces acting on it must sum to zero in every direction i.e. $\Sigma F = 0$

e.g. A street lamp is suspended on two wires

The system is in equilibrium so the tensions in the wires must have upward vertical components that balance the weight of the lamp downwards.

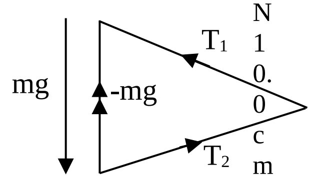


Also, the left horizontal component of T_1 must equal the right horizontal component of T_2 for horizontal equilibrium.

So, vertically: $T_1 \cos\theta_1 + T_2 \cos\theta_2 = mg$

And Horizontally: $T_1 \sin\theta_1 = T_2 \sin\theta_2$

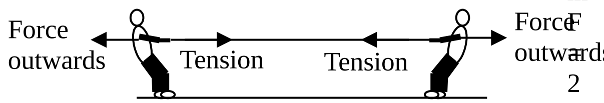
If we draw a clear vector triangle then the sum of T_1 and T_2 will give a resultant that will be vertically upwards that is equal and opposite to the weight of the lamp.



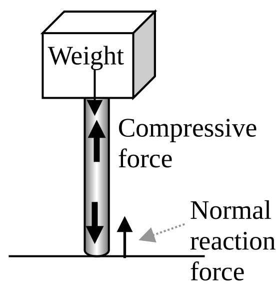
Therefore the net force on the system is mg + (-mg) = zero because no acceleration is occurring.

1.6 Tensile and Compressive forces

When a rod/wire is being stretched it is under tension and when a rod/beam is being squeezed it is under compression.

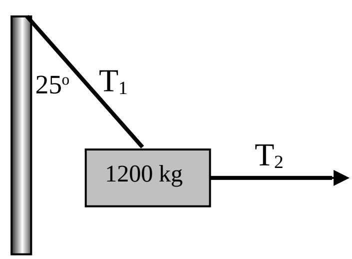


To maintain equilibrium at the ends the tensile force muse be equal and opposite to the applied force and the same is true for a compressive force. Hence, for a rod under tension, the direction of the **tensile force** at each end is **inwards**, whereas for a rod under **compression** the forces at each end are **outwards.**



Example 11

A crane has a rope at its end at 25.0° to the vertical holding a 1200 kg load. Another rope is also attached to the load and pulling it towards the deck of the ship



B A 3 N 5 N 4 N 1 0. 0 c m 8. 4 c m F 2 5 N 2 6. 2 c m F = 0 1 3 N 5 N 4 N 1 0. 0 c m 2 *making this rope horizontal. Calculate the tension in each rope.*

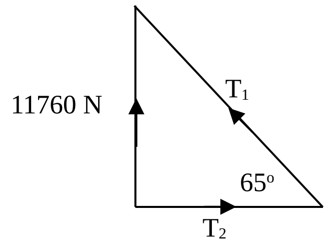
Solution 11 *(Components method)*

Vertically: vertical component of T_1 = weight (mg)

So $T_1\sin65 = 11760 \rightarrow T_1 = 1.298 \times 10^4$ N

Horizontally: horizontal component of $T_1 = T_2$

So $T_2 = T_1\cos65 = 1.298 \times 10^4 \times \cos65 = 5.483 \times 10^3$ N



(Vector triangle method)

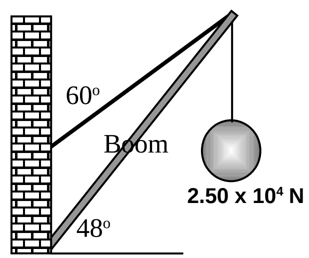
$\sin65 = 11760/T_1$ So $T_1 = 1.298 \times 10^4$

$\tan65 = 11760/T_2$ So $T_2 = 5.483 \times 10^3$ N

- In general, the vector triangle method is a quicker method.

Example 12

A crane boom hangs at an angle of 48° to the ground and holds a load of 2.50 x 10^4 N. A cable is attached to the end of the boom at an angle of 60° to the vertical. Calculate the tension in the cable and the compressive force in the boom.



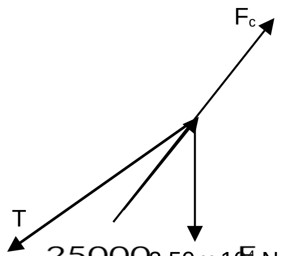
Solution 12

(Using a vector triangle method)

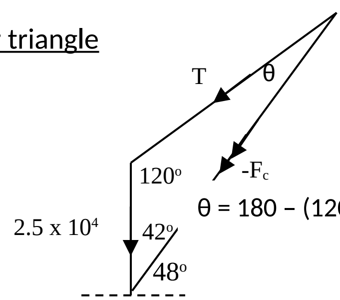
The compressive force in the boom (F_c) must act directly outwards along the boom.

The forces acting are shown in the force diagram.

Triangle of forces



Vector triangle



θ = 180 − (120 + 42) = 18°

Using the sine rule $\frac{25000}{\sin18} = \frac{F_c}{\sin120} = \frac{T}{\sin42}$

From this: F_c in the boom = 7.00 x 10^4 N Tension in wire, T = 5.41 x 10^4 N

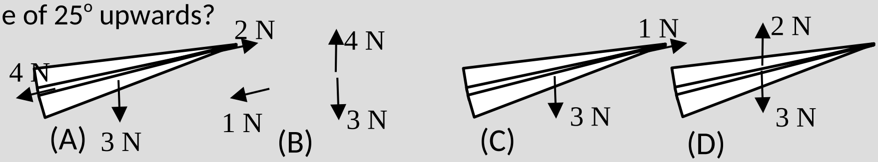
Note: The resultant of the weight plus tension acts downwards, so the compressive force in the boom (F_c) must be equal and opposite to this resultant so all the forces are in equilibrium and there is no

Checkpoint 2

1. Consider the following quantities:
   (i) 25 litres of water; (ii) 16 newtons north; (iii) a speed of 65 ms^{-1} east; (iv) 15 cm
   Which quantities are scalar quantities?

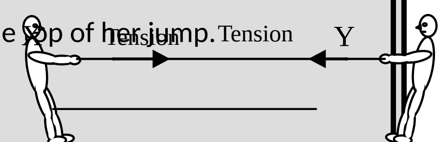
   (A) (i) & (ii) (B) (ii) & (iii) (C) (iii) & (iv) (D) (iv) & (i)

2. Which diagram below possibly show the correct sizes of the forces as it flies at an angle of 25° upwards?



3. A girl gymnast crouches down and then springs up into the air from a gym mat. Which statement about the forces acting on her as she springs is correct?

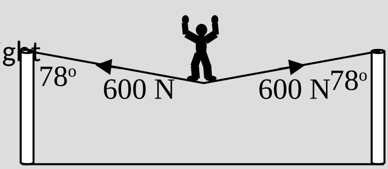
   (A) As action and reaction are equal and opposite, she cannot accelerate upwards.
   (B) The normal reaction from the ground must equal her weight at all times
   (C) As she moves upwards the Earth must recoil backwards
   (D) The acceleration on her body reverses as she reaches the top of her jump.



4. The men shown above are in a tug-of-war contest and are moving at a constant speed of 0.5 m s^{-1} to the right.

   Which statement about the forces acting is NOT true?

   (A) Person Y is pulling with a larger force than person X
   (B) The reaction force on the ground on Y is not vertical
   (C) X has a component of his reaction force acting left
   (D) The tension on the left is equal to the tension on the right

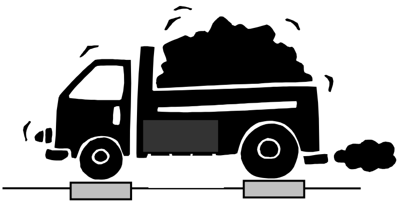


5. A gymnast bounces on a trampoline so that the 2 springs at the sides stretch down to an angle of 78° to the posts. If the force in each spring is 600 N then the force acting upwards on the gymnast will be:

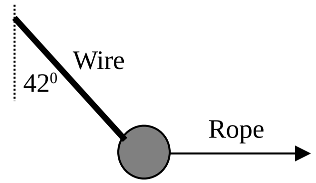
   (A) 125 N (B) 187 N (C) 250 N (D) 590 N

Set 1 Vectors Problems

1 A truck loaded with rocks stops so that it has its front wheels on two different sets of heavyweight weighing scales. The front scales read 11500 N and the rear scales read 28500 N.
What is the mass of the truck?

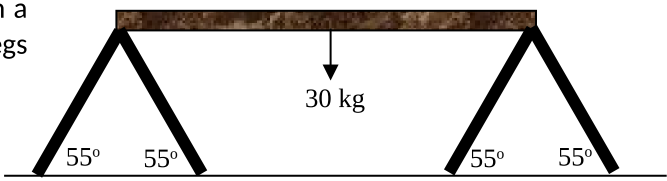


2. A demolition ball has a mass of 500 kg and hangs on a steel wire but is pulled over to the right by a horizontal rope. If the wire is inclined at an angle of 42^0 to the vertical calculate the tension in the wire and the force in the rope required to keep the ball in equilibrium.

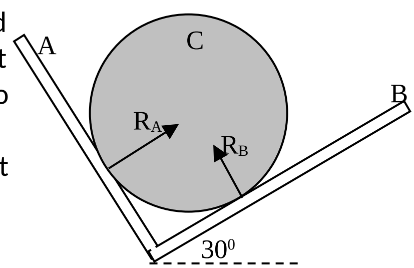


3. A painter's trestle table is constructed from a 30 kg flat sheet of wood supported on 4 legs at an angle of $55°$ to the ground.

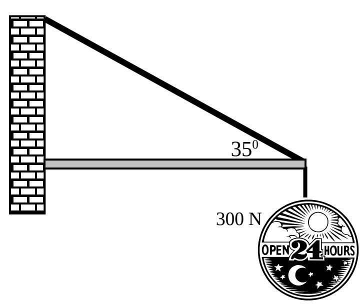
Calculate the force in each supporting leg.



4. A cylindrical roller C is being held for machining in a 'V'-shaped block shown. The two faces of the V are A and B, attached at 90^0 to each other and the right-hand face is inclined at 30^0 to the horizontal.
The normal reaction force from face A is called R_A and that from face B is called R_B.
Calculate the ratio of the forces, R_A / R_B.

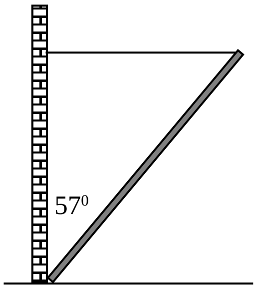


5. A heavy metal shop sign of mass 300 N hangs from a rod of negligible mass and is held horizontal by a cable attached to the wall. The cable fixes to the pole at an angle of 35^0 to the horizontal. If the structure is in

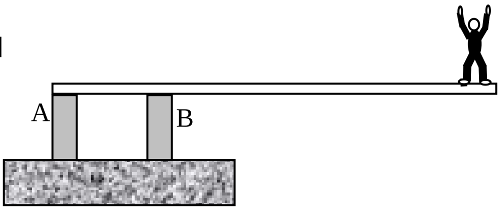


equilibrium, calculate the tension in the cable and the force in the rod.

6. A steel scaffolding pole of mass 110 kg has its bottom end between a wall and the floor. The pole is held at an angle of 57^0 to the vertical by a horizontal wire attached to the pole and a bolt in the wall. Calculate the tension in the wire and the force in the pole.



7. A diving board has a mass of 30 kg and is supported at one end by two blocks A and B.
A boy of mass 60 kg is standing at the other end ready to dive.
If the force in block B is 3.10 kN upwards, what is the force in block A and its direction?



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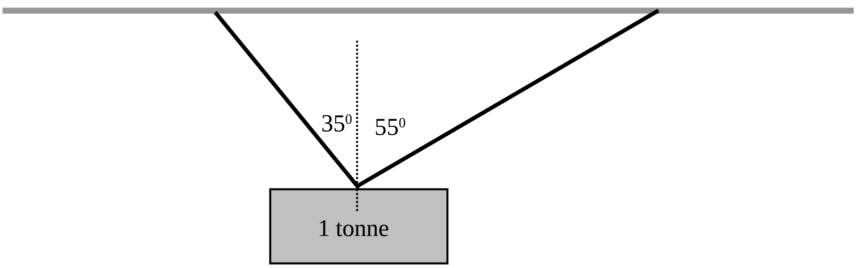
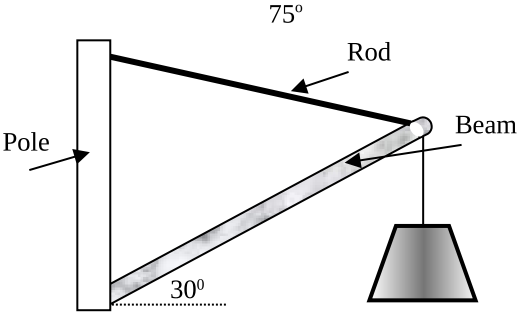
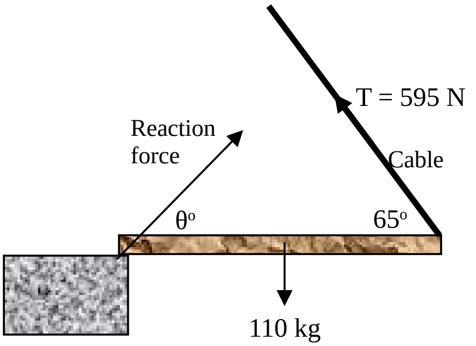


A 1.0 tonne load is supported from a roof by two steel wires. The wire on the left makes an angle with the vertical of 35^0 whilst the right-hand wire makes an angle of 55^0 with the vertical.
Calculate the tensile forces in each wire.

9. A hanging lamp support at a railway station consists of a light wooden beam 1.8 m in length supported by a light rod securing it to an upright pole. The rod meets the pole at angle of 75^0 and the mass of the lamp being supported is 30 kg. Find the forces in the rod and beam and their directions .



10. A viewing point is constructed from a wooden platform suspended over a cliff. The left end stands on a concrete base with the right hand end of the platform held up by a cable attached at 65° to the horizontal. The mass of the platform is 110 kg and the tensile force in the wire supporting it is 595 N. The reaction force of the concrete on the platform is upwards making an angle of θ° to the ground, as shown.

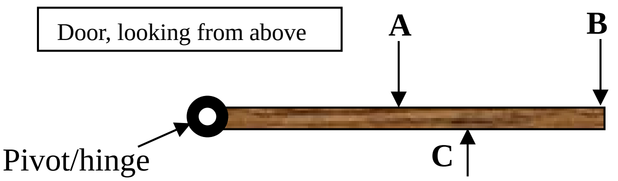


Calculate the magnitude and direction of the reaction force at the left hand end of the platform.

1.7 Torques

Objects that can rotate around a pivot have their own set of principles and equations which are different to objects that move in a straight line. Consider a door rotating about its hinges.

If one person pushes on the door at point B the door will open faster (accelerate) than if another person applied the same force at point A. This is because person B is applying more leverage, or torque than person A.



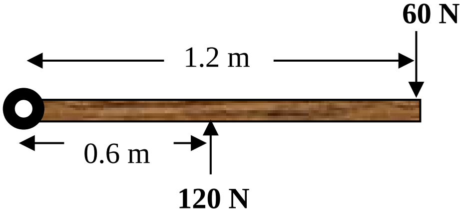
Torque is defined as the product of **force x distance** (F.r) from the pivot, so, although A and B apply the same forces, B exerts twice the torque, because the force is applied at twice the distance. The applied torque at A and B is called "Clockwise Torque" because it would make the door turn in an clockwise direction. To prevent the door from turning an opposite torque must be applied at, say, point C. A force upwards at C would be an anticlockwise torque.

The equivalent of Newton's First Law for rotating objects, then, is:

For equilibrium, the sum of all anticlockwise torques = the sum of all clockwise torques

ΣACT = ΣCT

If a person is pushing at the end of a 1.2 m wide door with a force of 60 N, the value of the clockwise torque is
F.r = 60 x 1.2 = 72 newton metres.



For another person, pushing in the middle of the door, to stop the door from opening they will need to push with **twice** the force, or 120 N, to obtain the same torque
72 = F.r = 120 x 0.6
The person pushing at the end has twice the leverage, and so needs only half the other force.
ΣACT = ΣCT applies if there is no rotation.

Newton's 1st Law must still apply if there is no translational acceleration (i.e. the door does not move up or down. Hence the net force on the door must be zero (ΣF = 0).

This means that there must be **another force** acting at the pivot which must be equal to 60 N and be acting **downwards**. Without this force at the pivot, the left-hand end of the door would move upwards.

Example 13

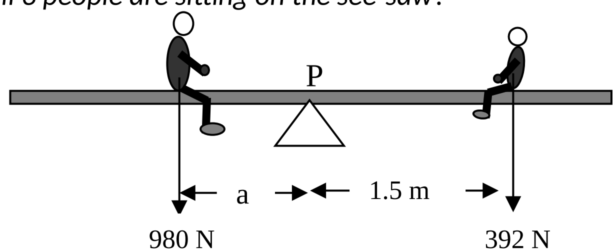
Little Johnny (mass 40 kg) sits on a see-saw 1.5 m from the fulcrum (pivot) and asks his uncle Jim (mass 100 kg) to sit on the other end so the see-saw just balances.

a)Where must uncle Jim sit?

b) If Johnny's brother Jack (mass 30 kg) joins little Johnny and sits 2.0 m from the fulcrum, where must uncle Jim now sit to balance both boys?

c) What is the reaction force at the fulcrum when all 3 people are sitting on the see-saw?

Solution 13



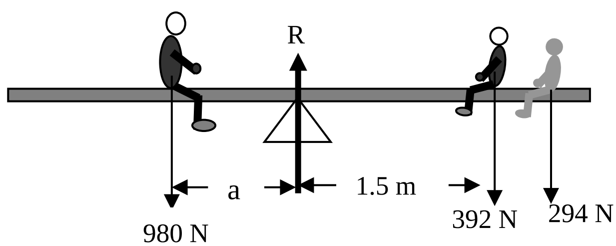
a) (Put all forces in on diagram)

Taking torques about the fulcrum P:

ΣACT = ΣCT

392 x 1.5 = 980a

a = 0.6 m



b) ΣACT = ΣCT

980 x a= (392 x 1.5) + (294 x 2.0) = 1176

a = 1176/980 = 1.2 m

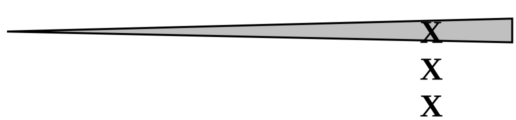
c) $\Sigma F_{down} = \Sigma F_{up}$

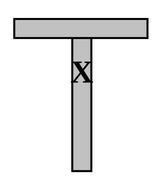
Σ Forces down = 980 + 392 + 294 = 1666 N

Therefore the force from the pivot must be 1666 N upwards.

1.8 Centre of mass

The centre of mass of any object is a point at which all the ACTs equal the CTs i.e. the exact balance point. For a symmetrical object the C of M will be at its geometrical centre but for something like a pool cue the C of M (X) will be closer to one end.



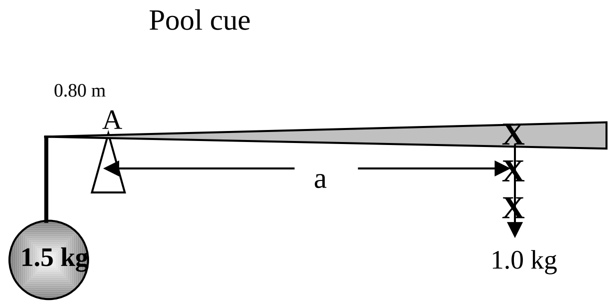


(X shows the centre of mass of the object)

Example 14

A pool cue has a mass of 1000 grams. It is placed on a knife edge positioned 80 cm from its thin end with a mass of 1500 g hanging from its tip.

If the cue is exactly balanced in this position, calculate the position of its centre of mass from the tip.



Solution 14

Taking torques about the fulcrum A

ΣACT = ΣCT

14.7 x 0.80 = 1.0 x 9.8 x a

a = 1.2 m from the pivot, or 2 m from the tip.

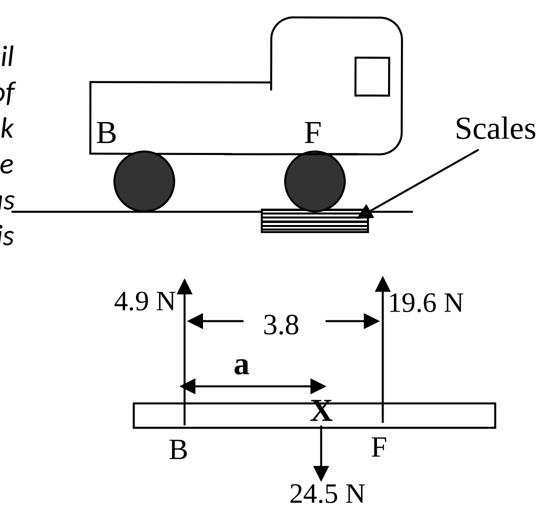
2-Pivot Problems

Some problems involve objects supported on two pivots and the method for calculating forces involves the same principles, except that torques are taken about more than one pivot.

NB. If the system is in equilibrium then the sum of the torques about **any chosen point** must be zero but by choosing a specific point makes the mathematics easier.

Example 15

A truck of mass 2.5 tonne is driven to the Council weighbridge where the force downwards on each set of wheels can be found. When the front wheels of the truck are on the scales the reading is 19.6 kN and with the back wheels on the scales a reading of 4.9 kN was obtained. If the distance between the axles of the truck is 3.8 m, where is the centre of mass of the truck?

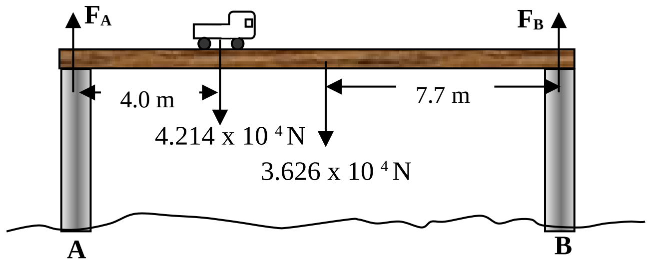


Solution 15

Let the distance of the C of M from B be a.
Taking torques about B: ΣACT = ΣCT
3.8 x 19.6 = 24.5 x a so a = 3.04 m from the rear wheels.

Example 16

A bridge across a river consists of a rectangular beam 15.4 m long of mass 3700 kg held by two supporting poles, one at each end. If a truck of mass 4.3 tonne stopped at a distance of 4.0 m from one end of the bridge, what would be the force in each of the supporting poles?



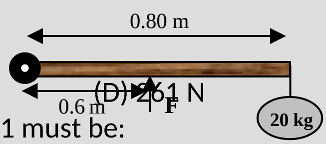
Solution 16

Taking torques about end A:
ΣACT = ΣCT
$(4.0 \times 4.214 \times 10^4) + (7.7 \times 3.626 \times 10^4) = 15.4 \times F_B$
$F_B = 2.908 \times 10^4$ N (for both supporting poles)

Now, taking torques about pole B:
ΣACT = ΣCT
$(7.7 \times 3.626 \times 10^4) + (11.4 \times 4.214 \times 10^4) = 15.4 \times F_A$
$F_A = 4.932 \times 10^4$ N (for both supporting poles)
Check ΣF = 0 : Net force down = $4.214 \times 10^4 + 3.626 \times 10^4 = 7.84 \times 10^4$ N
Net force up = $F_A + F_B = 4.932 \times 10^4 + 2.908 \times 10^4 = 7.84 \times 10^4$ N (same as force down)

Checkpoint 3

1. In this diagram an upward force is needed to balance the 20 kg mass. The value of F is:



   (A) 147 N (B) 196 N (C) 222 N (D) 261 N

2. The magnitude of the reaction force at the hinge in Q1 must be:

   (A) 65 N (B) 109 N (C) 345 N (D) 457 N

3. For a Frisbee is moving at a constant height, speed and rotational frequency,
   (A) Sum of all forces = zero and sum of all torques = zero
   (B) Sum of all forces = zero but sum of all torques is non zero
   (C) Sum of all forces is non zero and sum of all torques = zero
   (D) Sum of all forces is non zero and sum of all torques is non zero

4. A man stands in the back of a ute as it accelerates. The reason he topples over is:
   (A) His normal force becomes larger than his weight force
   (B) The frictional force acting on his feet becomes larger than his clockwise torque
   (C) The frictional force on his feet rotates his body around his centre of mass
   (D) His normal force becomes less than the frictional force on his feet

5. The centre of mass of a half empty beer can will be at:
   (A) The bottom (B) The top (C) The centre (D) Between centre and bottom

Set 2 Torques and Centre of mass

1. Before a footy team gets onto a double-decker bus the driver tells the men to fill up the lower deck first. The driver says that filling the top deck first could be dangerous when driving.
The best scientific explanation for this would be:

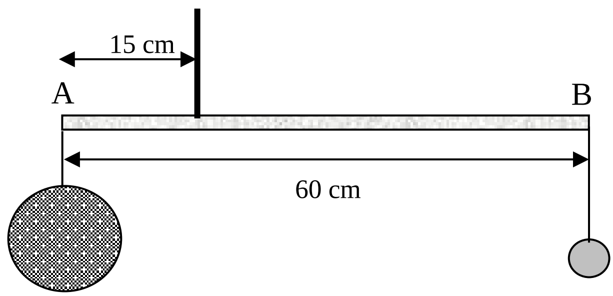
A. Too much weight upstairs could cause the suspension springs to compress too much when going round corners

B. Filling the top deck first could put an uneven distribution of weight on the axles, causing instability

C. With a large weight upstairs the centre of gravity of the bus changes making toppling easier when rounding corners

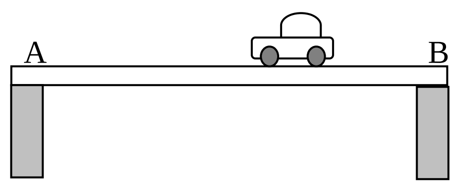
D. As the passengers climb the stairs this can cause an anticlockwise torque to be exerted on the axles, decreasing the lateral stability of the body.

2. A child's mobile hanging from the ceiling comprises two objects suspended from wires attached to a light horizontal rod 60 cm long. At end B hangs a 200 g object whilst a heavier object hangs at end A.
The rod has a mass of 100 g and the mobile balances at a point 15 cm from A.



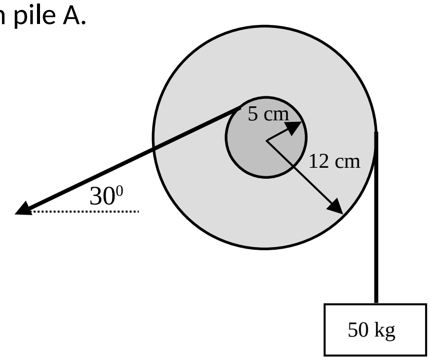
What is the mass of the object at end A?

3. A uniform wooden bridge over a river is 15 m long and has a mass of 800 kg, supported at each end by wooden piles A and B. A car of mass 1100 kg stops 5.0 m from end B of the bridge.

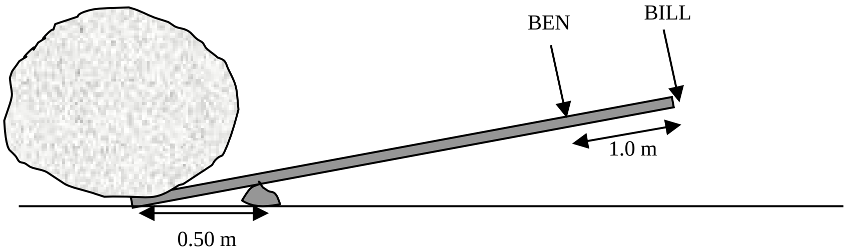


In this situation, calculate the reaction force on the bridge from pile A.

4. The wheel and axle machine shown is used to raise a 50 kg load from the floor by pulling on the left hand rope.
If a man pulls down on the left-hand string at an angle of 30^0, what is the force needed to raise the load?



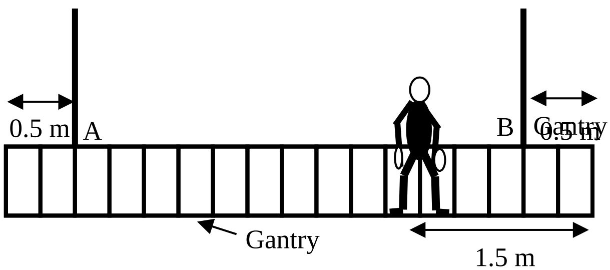
5.



Bill and Ben want to move a 900 kg boulder in their garden. To do this they use a 3.5 m-long steel scaffolding pipe as a lever with a rock as a pivot. One end of the pipe is placed under the boulder whilst the rock is placed 0.50 m from that end. Bill pushes down at the other end of the pipe whilst Ben pushes down with an equal force at a point 1.0 m from Bill.

What minimum force must each man exert to just lift the boulder?

6. A painter stands on a gantry 5.0 m long and mass 80 kg. The gantry is supported by four ropes attached 0.5 m in from the ends and the painter (mass 95 kg) stands 1.5 m from end B.

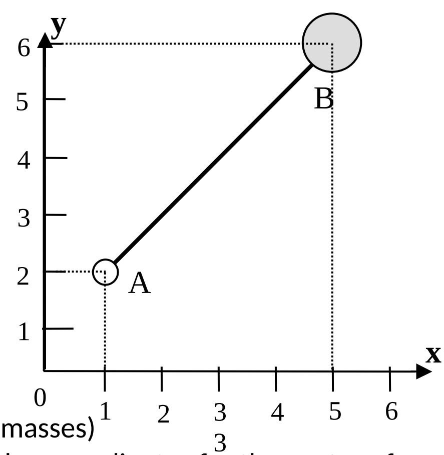


What will the tension in each of the ropes at end A and B be?

7. A sculpture is made from two spheres of metal (A and B), welded onto a thin metal bar. A has a mass of 10 kg and B has a mass of 40 kg.

The sculptor's plan is shown (right), using Cartesian co-ordinates, but he needs to know the exact balance point so he can attach a support to the structure. An equation in a physics book gives the x-co-ordinate for the centre of mass as:

$$x = \frac{\Sigma mx}{\Sigma m}$$

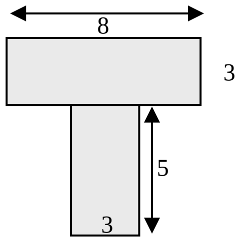


(the sum of torques about point O divided by the sum of masses)

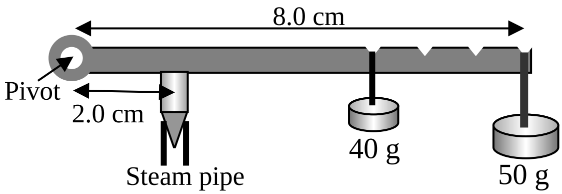
Using this formula in the x and y directions, what will be the co-ordinates for the centre of mass?

8. A ‘T’-shaped bracket is made from two 3.0 cm-wide bars welded together. The lower bar is 5.0 cm long whilst the cross bar is 8.0 cm long.

Determine the position of the centre of mass of the whole bracket.



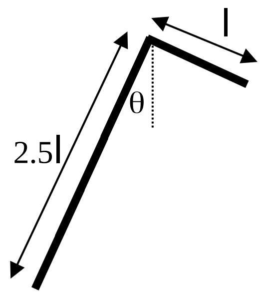
9. A steam pipe pressure limiter operates by the torque from weights acting on a tapered needle pushing into a steam pipe. A 50 g weight site permanently at a distance of 8.0 cm from the pivot but another, 40 g mass can be moved to different positions from the pivot.



Where must this 30 g mass be placed to balance an upward steam force of 2.85 N in the pipe.

10. A piece of wire is bent into a right-angled shape with sides in the ratio of 1:2.5 and then suspended from the ceiling by a string. The shape will come to an equilibrium position with the longer side at an angle of θ to the vertical.

Determine angle θ.



1.9 Momentum

(NB this section should have been covered in the 2A/2B course in year 11 but is included here as revision notes required for some questions in the 3A/3B course)

Momentum = mass x velocity (p = mv)

In all cases of collisions momentum remains constant i.e.

<u>Total momentum before the collision = Total momentum after the collision</u>

$$M_1 \times u_1 + M_2 \times u_2 = M_1 \times v_1 + M_2 \times v_2$$

Elastic Collisions are defined as ones where all the kinetic energy is conserved – these can only occur realistically where colliding objects do not actually touch (magnets, atoms).

All collisions in the "big" world involve a conversion of kinetic energy into heat during the collision. We know that low speed atomic collisions are an example of elastic collisions because a sealed cylinder of gas will maintain its pressure indefinitely. If E_K were lost the pressure would decrease with time.

1.10 Impulse

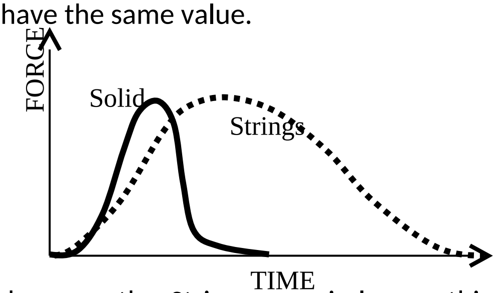
Impulse occurs when one object strikes another, e.g. a bat hits a ball.

From Newton's 2nd Law: $F = \Delta p/\Delta t$ so $F \times \Delta t = \Delta p$

Force multiplied by the time of contact ($F \times \Delta t$) is defined as impulse.

This means that **impulse is equal to momentum change** – they have the same value.

Suppose we had one bat made of solid wood and another made with stretchy strings. If the ball was hit with each racquet, using the same force, then the force / time graphs for each would look as shown on the right. The larger impulse given by the stringed racquet would equate to more momentum change and therefore a greater final velocity.



A measure of force x time is the **area under the graph** so, because the Strings area is larger, this indicates more impulse and therefore more momentum change.

Checkpoint 4

1. A large piece of plasticine is thrown downwards at the ground at a speed of 5.0 ms^{-1} and as it impacts it spreads out into a flat pancake with no rebound speed. Which statement about this situation is correct:

   (A) The speed of the plasticine changes but the velocity doesn't

   (B) The momentum of the plasticine remains constant in this collision

   (C) The total momentum of the system is zero after the collision

   (D) The total momentum of the plasticine and earth remains constant

2. A 100 gram ball is thrown northwards at a wall with a speed of 5.0 m s^{-1} and it rebounds with the same speed. What is the change of momentum of the ball

   (A) 1.0 kg m s^{-1} south (B) 0.5 kg m s^{-1} south (C) Zero (D) 0.5 kg m s^{-1} north

3. During target practice a 10 g bullet moving at 200 m s^{-1} strikes a can of mass 200 g sitting on a wall. The bullet lodges inside the can as it recoils. The recoil velocity of the can with bullet will be about
   (A) 5.4 m s^{-1} (B) 9.5 m s^{-1} (C) 27 m s^{-1} (D) 200 m s^{-1}

4. When parachutists land they are taught to roll on the ground. This is because:
   (A) There is less rotational momentum on the feet in rolling

   (B) The impact time is lengthened to decrease the impulse

   (C) The frictional force from the ground is greater to reduce the horizontal velocity

   (D) The time of impact is increased, thus reducing the decelerating force

5. A hammer of mass 250 g is swung at 8.5 m s^{-1} at some concrete to break it up. If the velocity of the hammer after the blow is 0.5 m s^{-1} in the same direction what impulse is delivered to the concrete approximately?
   (A) 2 N s (B) 4 N s (C) 6 N s (D) 8 N s

1.11 Accelerated motion equations

(NB this section should have been covered in the 2A/2B course in year 11 but is included here as revision notes required for some questions in the 3A/3B course)

Acceleration a = $\dfrac{\text{change in velocity}}{\text{time taken}}$

where u = initial velocity, v = final velocity

Rearranging this equation gives: $\boxed{v = u + at}$ 1

Acceleration is measured in units: metres per second squared, m/s^2(written as ms^{-2})

The acceleration of any object falling down onto the Earth is 9.8 m s^{-2} (called 'g').

Suppose the values of u, v, s and t are given in a question - then another equation is required.

Displacement of an object will be equal to the average velocity ($\frac{u+v}{2}$) x time, so another equation to be used is

$$\boxed{s = t\left(\frac{u+v}{2}\right)} \quad 2$$

Another equation can be formed by substituting v = u + at into

This gives: $s = t\left(\dfrac{u+u+at}{2}\right) \Rightarrow s = \dfrac{2ut}{2} + \dfrac{at^2}{2}$ or $\boxed{s = ut + \frac{1}{2}at^2}$ 3

A fourth equation can be obtained by squaring both sides of v = u + at:

$v^2 = (u + at)^2 \Rightarrow u^2 + 2uat + a^2t^2$ Factorising: $v^2 = u^2 + 2a(ut + ½at^2) = u^2 + 2as$

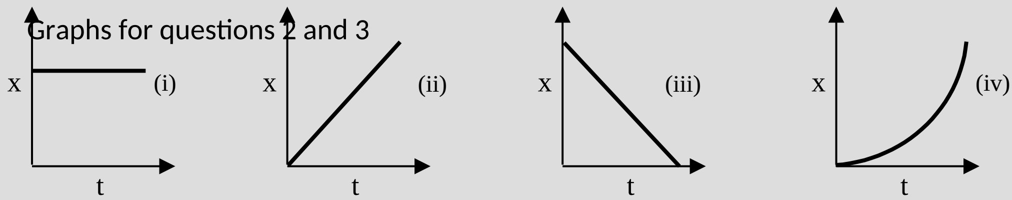
So $\boxed{v^2 = u^2 + 2as}$ 4

Checkpoint 5

1. A car moves with an average speed of 15 m s^{-1}. The distance it travels in 1 hour is:

(A) 0.54 Km (B) 5.4 km (C) 54 km (D) 540 km

Graphs for questions 2 and 3



1. If the variable x being plotted in the graphs is displacement, which graphs represent an object moving at constant speed?

(A) (i) and (ii) (B) (ii) and (iii) (C) (iii) and (iv) (D) (iv) and (i)

2. If the variable x being plotted in the graphs is velocity, which graphs represent an object moving at constant acceleration?

(A) (i) and (ii) (B) (ii) and (iii) (C) (iii) and (iv) (D) (iv) and (i)

3. A car moving at 5.00 ms $^{-1}$ accelerated to 15.0 m s^{-1} in 1.25 s. The acceleration of the car was:

(A) 2.00 m s^{-2} (B) 4.00 m s^{-2} (C) 6.00 m s^{-2} (D) 8.00 m s^{-2}

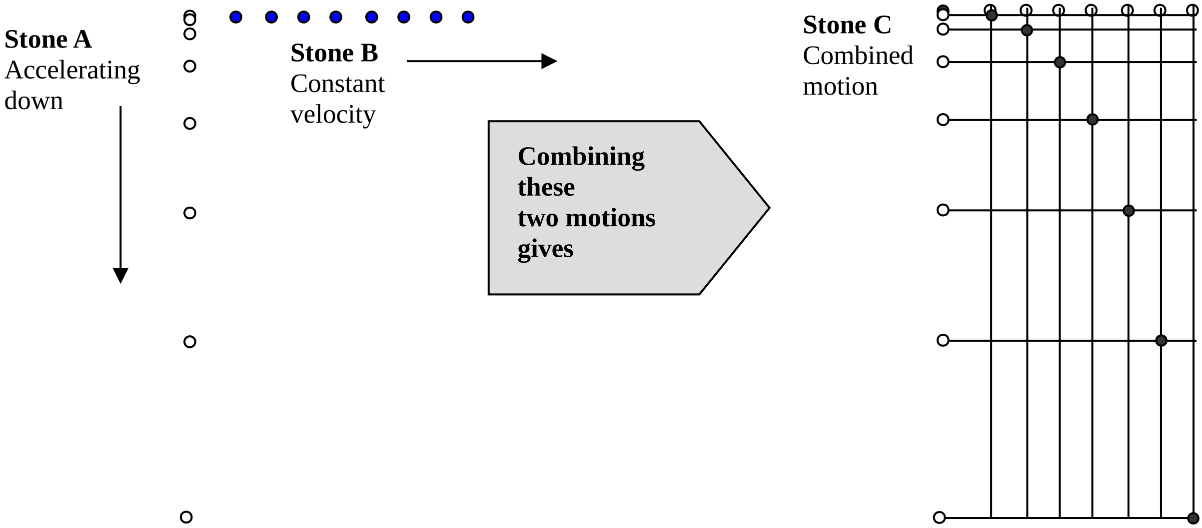
5. The displacement of the car in Q4 over the 1.25 s whilst accelerating was:

(A) 11.25 m (B) 12.5 m (C) 25.0 m (D)27.5 ms

1.12 2-dimensional Motion

Suppose a rock was thrown horizontally from the top of a cliff onto the beach below. Gravity would only have an effect on the **vertical** motion of the stone, causing it to accelerate downwards. The horizontal motion of the stone, however would not be affected by gravity, so its horizontal velocity would remain constant throughout its motion.

With 2-D motion we must always consider the vertical and horizontal motions as being **totally separate.** The only link between these two directions is **time.** Below is a displacement plot of the motion of stone A with time falling directly downward under gravity (stone A) and another stone B which is moving horizontally with no downward force acting on it.



Stone C is projected horizontally from the top of a cliff and the stone C graph shows the total x and y displacement added together: It accelerates downwards **and** moves horizontally at a constant rate **at the same time.**

The resultant path taken by the stone is **parabolic**, as can be seen from the following proof.

Horizontal motion is given by: $s_H = u_H t$ or $t = s_H/u_H$ (1)

Vertical motion is given by: $s_V = u_V t + ½gt^2$ (2)

Substituting for t from (1): $s_V = u_V\left(\frac{s_H}{u_H}\right) + \frac{g}{2}\left(\frac{s_H}{u_H}\right)^2$

Initially at $t = 0$ $u_V = 0$

So $s_V = \frac{g}{2u_H^2}(s_H)^2$ i.e. vertical displacement depends on s_H squared.

This has the form of $y = kx^2$ which is a **parabola.**

Example 17

A boy throws a rock horizontally at 10 ms^{-1} from the top of a cliff 50 m high onto the beach below.

a) What is the time of flight?

b) Where does the stone land?

c) What is the stone's impact velocity?

Solution 17

a) $u = 0 \text{ m s}^{-1}$

$a = 9.8 \text{ m s}^{-2}$

$t = ?$

$s = 50 \text{ m}$

$s = ut + ½at^2$

$50 = 0 + 4.9t^2$

$t^2 = 50/4.9$

$t = 3.19 \text{ s}$

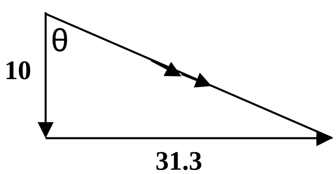
b) Horizontally the stone is moving constantly at 10 ms^{-1} for a time of 3.19 s

So s = v x t = 10 x 3.19 = 31.9 m from the base of the cliff.

c) Vertically $v_H = u_H + at = 0 + 9.8 \times 3.19 = 31.3 \text{ m s}^{-1}$

Horizontally $v_H = 10$ constantly

Adding these two vectors will give the resultant velocity:



Using Pythagoras' theorem:

$v^2 = 31.3^2 + 10^2 = 1080$

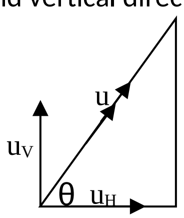
$v = 32.9 \text{ m s}^{-1}$

$\tan θ = 31.3/10 = 3.13$

$θ = 72.3°$ to the vertical

1.13 Projectile motion

When an object is projected at an angle to the ground it will possess two different initial velocities in the horizontal and vertical directions – the vertical and horizontal components: u_H and u_V.



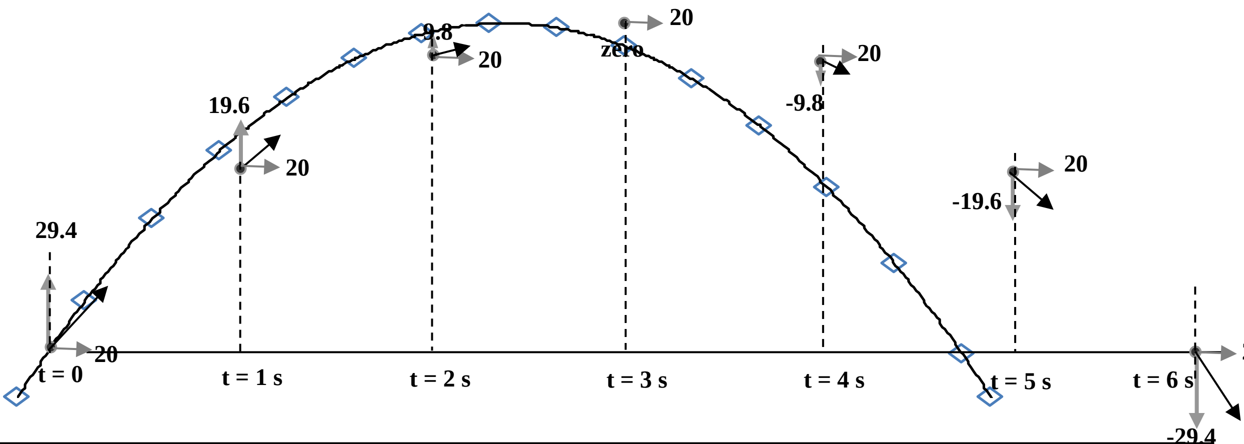
From the vector triangle

$u_V = u\sin θ$ and

$u_H = u\cos θ$

As in the last 2-D example u_H will again remain constant throughout the flight and u_V only will be affected by gravity.

a



Graph showing Motion of a projectile projected with a vertical velocity of 29.4 ms^{-1} and a horizontal velocity of 20 m s^{-1}

Example 18

An arrow is fired into the air at a speed of 25 m s^{-1} at an angle of 60^0 to the horizontal.

a) How high will the arrow go? b)What is the arrow's speed after 3.0 s?

b) Where will the arrow land?

Solution 18

Vertical velocity = 25sin60 = 21.65 m s^{-1} Horizontal velocity = 25cos60 = 12.5 m s^{-1}

a) At top of flight v = 0, using $v^2 = u^2 + 2as$ $0 = 21.65^2 + 2(-9.8)s$ So $\mathbf{s_{max}}$ = 23.9 m

b) Vertically

u = 21.65 m s^{-1} v = u + at

v = ? v = 21.65 + (– 9.8) x 3.0

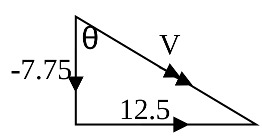
a = -9.8 m s^{-2} v = -7.75 (downwards)

t = 3.0

Horizontally u = 12.5, constant

From the vector triangle $v^2 = 12.5^2 + (-7.75)^2$

v = 14.7 m s^{-1} Tanθ = 12.5/7.75 So = 58.2° to the vertical



c) To find the range firstly find the time of flight, in this example $s_{vertical} = 0$

Vertically

Using $s = ut + ½at^2$

$0 = 21.65t - 4.9t^2$ t = 2.10 s

Horizontally u = 12.5 m s^{-1}, constant

s = ut = 12.5 x 2.10 = 26.3 m

Example 19

WA cricketers can win a $10,000 prize if they hit the ING sign which is 67 m away from the wicket and suspended on stands with its top edge 6.0 m up in the air. A batsman hits the ball with a velocity of 30 m s^{-1} at an angle of 65°.

Calculate whether the ball hits the ING sign.

Solution 19

Vertical velocity = 30sin65 = 27.19 m s^{-1}

Horizontal velocity = 30cos65 = 12.68 m s^{-1}

Horizontally

Time to travel 67 m to the sign = 67/12.68 = 5.28 s

Vertically

u = 27.19 m s^{-1}

a = -9.8 m s^{-2}

t = 5.52 s

s = ?

Using $s = ut + ½at^2$

s = 27.19 x 5.28 – 0.5 x (-9.8) x $(5.28)^2$

= + 7.06 m (+ means above the ground)

The ball passes over the sign, missing it by 1.06 m

Example 20

A female physicist darts-player has perfected the skill of throwing her dart at an angle of exactly 35° to the horizontal. The dartboard is suspended on a wall 2.5 m away with its center at a height of 1.6 m from the floor. If the woman throws the dart from a height of 1.3 m, how fast must she throw it if she is to score a bulls-eye?

Solution 20

Vertical velocity = usin35

Horizontal velocity = ucos35

Horizontally

Time of flight = 2.5/ucos35 = 3.05/u (this will be substituted into the vertical equation)

Vertically

u = usin35 = 0.5736u m s^{-1}

a = -9.8 m s^{-2}

t = 3.05/u s

s = 1.6 – 1.3 = 0.3 m

Using $s = ut + ½at^2$

0.3 = 0.5736 x 3.05 + ½ (-9.8)$(3.05/u)^2$

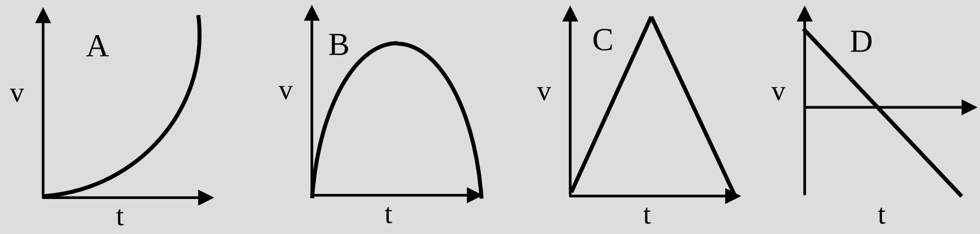
0.3 = 1.749 – $45.58/u^2$

u^2= -45.58/(-1.449)

u = 5.61 m s^{-1}

Checkpoint 6

1. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1}. Which of the graphs below shows the velocity of the ball with time?



2. What happens to the total kinetic energy of a projectile as it travels and lands if we neglect air resistance?

   (A) E_P all gets converted to E_K at the top of its flight

   (B) The total E_K throughout the flight remains constant

   (C) The total E_K plus E_P throughout the flight remains constant

   (D) The total E_P throughout the flight remains constant

3. Considering the horizontal and vertical motions of a projectile that is launched at an angle, which one factor is constant for both of these motions at all times?

   (A) Displacement (B) Time (C) Acceleration (D) Velocity

4. If a stone is thrown horizontally outwards at 5.0 ms^{-1} from a tower 100 m high, the position it lands at will have a horizontal displacement for the base of the tower of about:

   (A) 23 m (B) 36 m (C) 66 m (D) 100 m

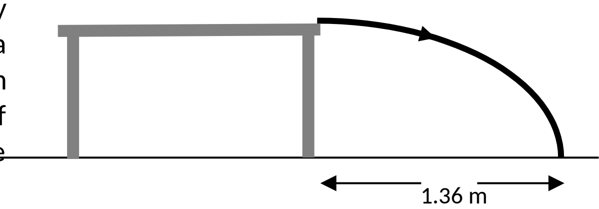
5. In the previous question, the angle the stone makes with the ground when it lands will be about:

   (A) 12° (B) 36° (C) 71° (D) 84°

Set 3 Projectiles Questions

1. A stone is thrown horizontally at 15 m s^{-1} from the top of a cliff which is 30 m above the sea.
   a) How far out at sea does it land?
   b) How fast must the stone be thrown if it is to land on top of a floating piece of wood 25 m from the base of the cliff
   c) How long will it take to strike the wood?
   d) With what velocity will it strike?

2. You are wiping the surface of the main dining table at home after a meal, when quite accidentally you strike a fork that then slides off the table at a horizontal speed of 3.70 m s^{-1}. If the fork lands on the ground, 1.36 m horizontally from the edge of the table, how high off the ground is the top of the table?

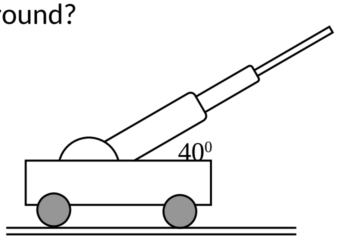


3. A Channel 7 helicopter is ascending vertically at 4.00 m s^{-1} whilst travelling horizontally forwards at 16.0 m s^{-1}. When the helicopter is 110 m above the ground the news cameraman accidentally drops his camera out of the helicopter.

   a) How long does it take for the camera to reach the ground?
   b) What was the maximum height of the camera above the ground?

c) How far forward does the camera travel before landing on the ground?

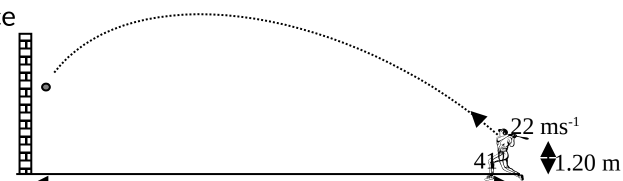
4. Big Bertha was the name of a huge cannon used by the Germans in the First World War to fire across the Channel to England.
This gun was mounted on a train and could be moved to different positions. The shell's muzzle velocity was 750 kmh^{-1}.



a) If the gun were angled at 40^0 to the horizontal what would be the shell's velocity measured along the ground?
b) What would be the maximum height that the shell reached above the ground?
c) What is the range of the gun at 40^0?
d) What would be the gun's maximum range?
e) What would be the velocity and angle of the shell 5.0 seconds after firing?

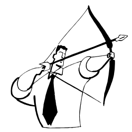
(Ignore air resistance in these cases.)

5. In training, a baseball batter hits a ball from a height of 1.20 m off the ground at 22 m s^{-1} at an angle of 41^0 towards a vertical metal fence that is 35 m away.



a) How far from the ground does his ball strike the fence?
b) At what angle does it strike the fence?

6. Jonathon, a world class archer, is shown preparing to shoot an arrow. At the instant of release, the 0.050 kg arrow has a velocity of 28.5 m s^{-1} at an angle of 30^0 to the ground. The arrow is 1.40 m from the ground when released.



(a) Calculate the vertical and horizontal components of the arrow's initial velocity.

(b) How far away from Jonathon does the arrow hit the ground?

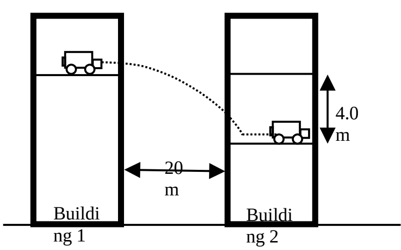
(c) (No calculations are required for the following)

(i) Sketch the trajectory of the arrow on a graph. Label this i) on the graph

(ii) Sketch on the graph the trajectory of the arrow showing how it would be changed if you allowed for air resistance. Label this as ii) on the graph and explain why the trajectory changes in this way.

(iii) Sketch on the graph the path of the arrow if it is fired at an angle of 40^0 rather than 30^0 (neglecting air resistance). Label this as iii) on the graph.

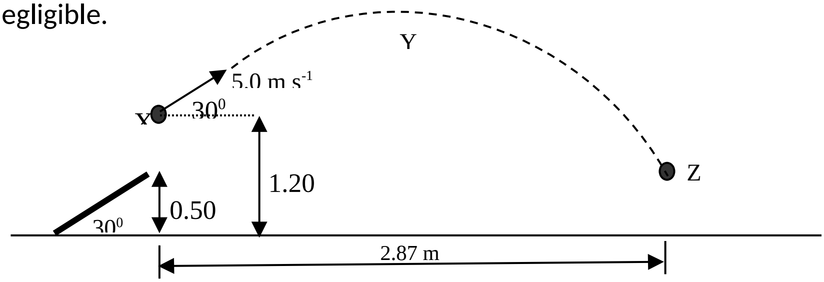
7. In the movie Car Escape, Taylor and Jones drove their sports car across a horizontal car park in building 1 and landed it in the car park of building 2, one floor lower. Building 2 is 20 metres from building 1, as shown in the diagram. The floor where the car lands is 4.0 m below the floor from which it started in building 1. In the following questions, treat the car as a point particle and assume that air resistance is negligible.



a) Calculate the minimum speed, at which the car should leave building 1 in order to land in the car park of building 2. Show your working.

b) In order to be sure of landing in the car park of building 2, Taylor and Jones, in fact, left building 2 at a speed of 25 m s^{-1}. Calculate the magnitude of the velocity of the car just prior to landing in the car park of building 2.

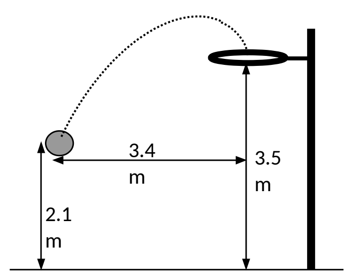
8. A skateboarder rides up a ramp as shown in the diagram below. At the instant the skateboard leaves the ramp, the centre of mass of the skateboard and rider is 1.20 m above the ground (as shown) and is initially moving with a speed of 5.00 m s^{-1}, at an angle of 30.0° above the horizontal. The parabola XYZ is the path of the centre of mass of the skateboard and rider. Assume that air resistance is negligible.



a) Calculate the height above the ground of the centre of mass at the highest point of the motion (Y).

b) When the skateboard touches the ground the centre of mass has moved has moved to point Z, as shown above. Calculate the time that the skateboarder is airborne.

9. A game of netball is being played in the gymnasium. A goalie shoots for a goal as described in the diagram. The ball takes 1.1 s to travel the trajectory shown.



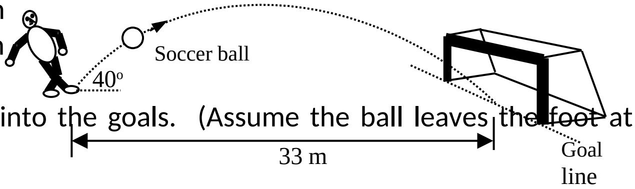
(a) What is the launch speed of the ball?

(b) At what angle to the horizontal was the ball launched?

(c) Throughout the flight the ball's horizontal component remains constant.
What does this indicate about the atmospheric conditions that exist within the gym?

(d) The following weekend the same goalie is required to launch a ball in the same situation as before but this time she is playing on an outside court where there is a horizontal crosswind of 2.0 m s^{-1} blowing. (This is wind that blows at 90˚ to the direction of the throw). Calculate the adjustment to the horizontal component and the vertical component of the throw's velocity that would need to be made, in order to successfully score a goal.

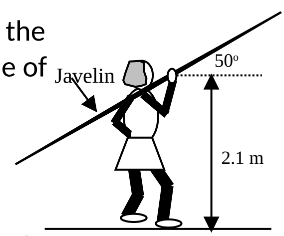
10. A soccer player kicks a ball from ground level at an angle of 40.0° to the horizontal in an attempt to score a goal. The goals are 33.0 m away.



The ball lands on the goal line and bounces into the goals. (Assume the ball leaves the foot at ground level).

a) With what velocity does the ball leave the player's boot? (ignore air resistance)
b) What is the time of flight of the ball?
c) What is the highest point above the ground that the ball reaches?
d) At what horizontal distance from the kick off point does the ball reach its maximum height?

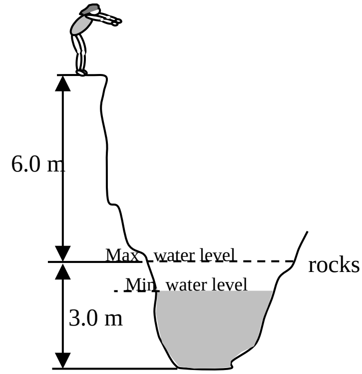
11. Deanne, a world class javelin thrower, is shown preparing to throw. At the instant of release, the 0.600 kg javelin has a velocity of 27.5 m s^{-1} at an angle of 50.0° to the ground.



The javelin is 2.10 m from the ground when released.

a) How long is it before the javelin reaches its maximum height?
b) What height above ground level does the javelin reach?
c) How far away from Deanne horizontally does the javelin hit the ground?
d) (i) Sketch the trajectory of the javelin on the graph below. Label this i) on the graph.
(ii) Sketch on the graph the trajectory of the javelin showing how it would be changed if you allowed for air resistance. Label this ii) on the graph.
Explain why the trajectory changes in this way.

12. A diver wishes to make a difficult high dive from a cliff top into the water 6 metres below (see diagram). She always pushes off with zero vertical velocity. One problem is that the water is only deep enough to dive into when a large wave is directly below her. She must hit the water at the instant when the water level is at its maximum if she is to avoid injury.



(a) How long before the large wave arrives should she jump from the cliff top?

Another problem is that if she pushes off with too little horizontal speed she will land on the base of the cliff that she is standing on but if she pushes off with too much horizontal speed she will land on some rocks on the other side of the water. She needs to travel 2.5 metres in the horizontal direction during her dive to land safely in the water.

(b) What horizontal speed should she push off with?

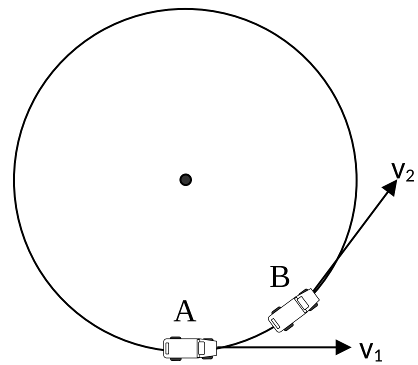
(c) Calculate her speed and direction when she hits the water.

13. In a football game, a place kicker kicks a football 36 m from the goalposts, and the ball must clear the crossbar which is 3.1 m from the ground, as shown in the diagram.

When kicked, the ball leaves the foot at 20 m s^{-1} at an angle of 53° to the horizontal.
How long does it take the ball to travel the distance to the goalposts?
How far above or below the crossbar is the ball when it passes through the goal posts?

1.14 Circular Motion

Consider a car moving around in a circle at constant speed from point A to point B.
At point A the inertia of the car would carry it tangentially in the direction of the arrow but at point B it is moving in a different direction i.e. the speed is the same but the velocity has changed by an amount Δv. $\Delta v = v_2 + (-v_1)$ which can be found from a vector subtraction:



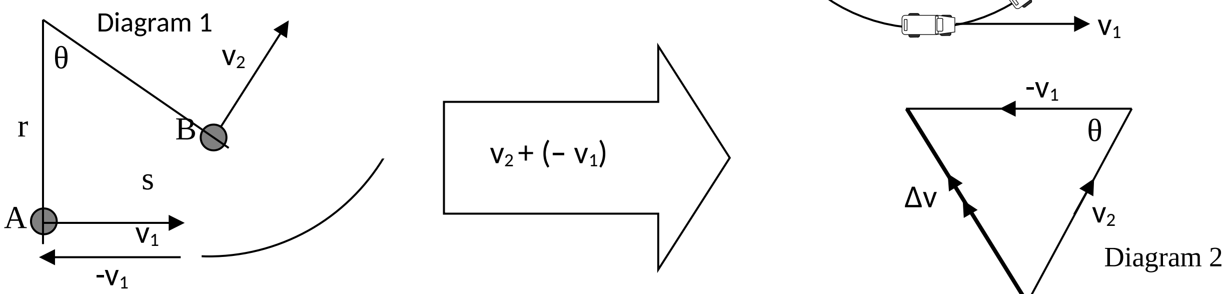


Diagram 1: For small angles $\tan\theta = s/r$

Diagram 2: $\tan\theta = \Delta v/-v_1$ (length of v_1 = length v_2)

So $$\frac{\Delta v}{-v_1} = \frac{s}{r} \qquad \Delta v = \frac{vs}{r}$$

Acceleration $= \dfrac{\Delta v}{\Delta t}$ (Δt = time to go from A to B)

So $$a_c = \frac{vs}{r\Delta t} = \frac{v}{r}\left(\frac{s}{\Delta t}\right) \qquad \text{but} \qquad \frac{s}{\Delta t} = v$$

so centripetal acceleration is $$a_c = \frac{v^2}{r}$$

The direction of a_c is the direction of the change in velocity vector (Δv) which is towards the centre of the circle.
For the acceleration to act towards the centre there must also be a force towards the centre as $F = ma$. This inwards force is called the centripetal force Fc.

In the case of the car, the friction of the tyres on the road provides this centripetal force. If the car were on ice then it could not move in a circle when the steering wheel was turned because the inertia would cause the car to carry on going in a straight line in the same direction as v_1.

Other systems where objects move in a circle must also have a force that pushes or attracts the object towards the centre e.g.

System	Source of Centripetal force
Ball on a string	Tension force of the string
Moon in orbit	Gravitational force
Electron in orbit	Electrostatic force
Circling ice-skater	Normal reaction force of ice
Racing car turning on a banked track	Normal reaction force of track

Example 21

A car of mass 1200 kg, moving at 72 km h^{-1} goes round a bend of radius 100 m

a) What frictional force is exerted by the tyres

b) If the maximum frictional force the tyres can provide is 7.2 kN, what is the minimum radius of curvature that the car can go round safely at 72 km h^{-1} safely without banking

Solution 21

a) 72 km h^{-1} = 72/3.6 = 20 m s^{-1}

$a_c = v^2/r = 20^2/100 = 4.0$ m s^{-2}

Centripetal force from tyres $F_c = ma_c = 1200 \times 4 = 4.8$ kN

b) $F_{max} = 7.2 \times 10^3 = 1200a_c$

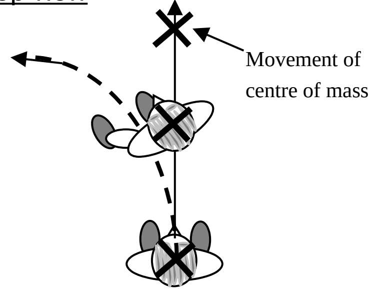
So $a_c = 7200/1200 = 6.0$ m s^{-2}

$6.0 = v^2/r = 20^2/r$ So r = 400/6 = 66.7 m

Normal Force

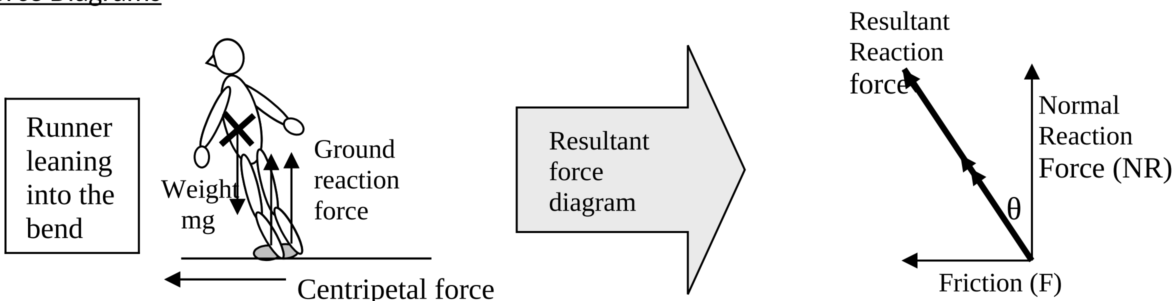
When sprinters run round a bend they have to **lean in** otherwise their inertia will cause them to topple outwards. Actually they only **appear** to move outwards **-** the runner's centre of mass will tend to carry on moving in a straight line because friction is the only force acting. This causes their feet to move inwards.

Top view



By leaning towards the centre of the circle the runner would tend to topple inwards if it were not for the fact that the feet are also moving inwards. This produces a state of **dynamic equilibrium**.

Force Diagrams



NB If the line of action of the resultant reaction force passes through the centre of mass of the runner then a dynamic equilibrium occurs and the runner will not topple. Zero toppling can only prevail if the angle of lean is correct.

To determine the angle: $\tan\theta = F/NR$

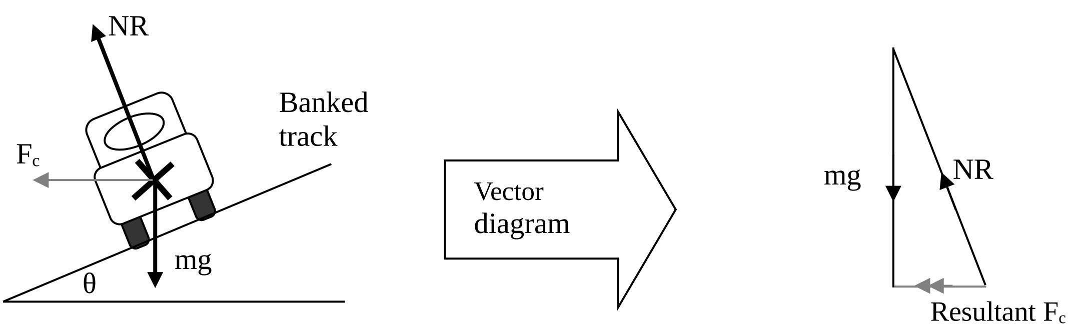
Vertical equilibrium: $NR = mg$

Horizontally: friction = centripetal force $= \dfrac{mv^2}{r}$

So $\tan\theta = \dfrac{\frac{mv^2}{r}}{mg}$ $\boxed{\tan\theta = \dfrac{v^2}{rg}}$

Curved bends in roads are often banked so that a component of the normal force from the road provides the inwards force. This means that the moving vehicles are less dependent on the friction of their tyres.

The equation for banking is the same as before:



Example 22

A cyclist and her bike have a combined mass of 110 kg. She wants to negotiate a bend in a flat track where the radius is 50 m.

a) If the maximum frictional force from the tyres is 600 N, at what angle must she lean to get round the bend safely?

b) If the track were waterlogged, what angle of banking would allow the cyclist to go round the bend safely at a speed of 12 m s^{-1} without relying on the grip from her tyres?

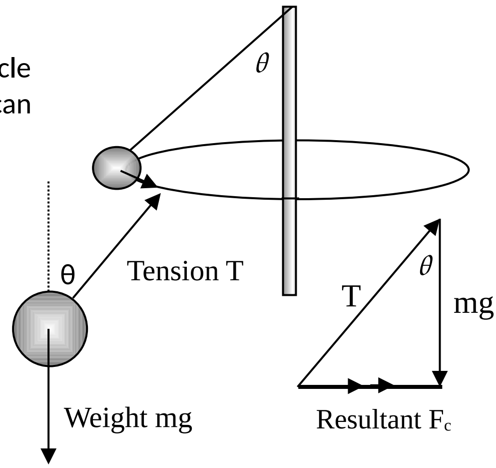
Solution 22

a) $\tan\theta = 600/(110 \times 9.8) \quad \Rightarrow \theta \quad = 29^0$

b) $\tan\theta = \dfrac{v^2}{rg} = \dfrac{12^2}{50 \times 9.8} \quad \Rightarrow \theta \quad = 16.4^0$

The Conical Pendulum

A ball hanging on a string and going round in a horizontal circle is called a **conical pendulum.** One example of where this can be seen is in athletics hammer-throwing events.



The two forces acting on the ball are shown:

The sum of the 2 forces acting is equal to the centripetal force F_c – found from the vector triangle.

F_c is equal to the horizontal component of the string's tension, which provides the centripetal force to accelerate the ball inwards towards the centre of the circle. $\quad F_c = T \sin \theta$

There is no net vertical force so: $mg = T \cos \theta$

Example 23

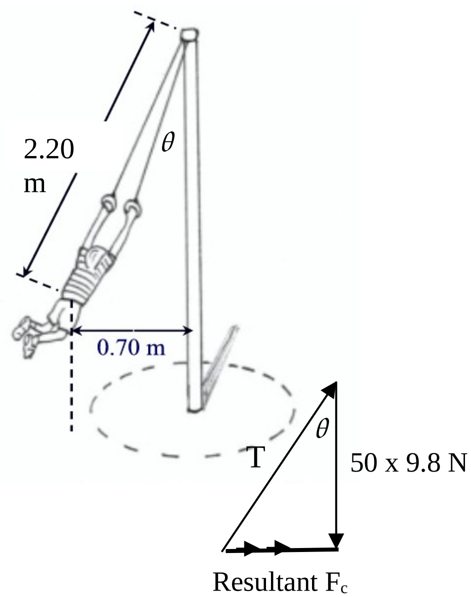
A boy of mass 50 kg swings round a maypole in a circle of radius 0.70 m. Suspended on the two ropes, his centre of mass is 2.20 m from the top of the pole.

Find:

a) The tension in the string

b) The centripetal acceleration

c) The time for the boy to go round once.



Solution 23

a) From the dimensions triangle, θ at the top is given by:

so $\theta = 18.6^0$

From the vectors triangle: Vertically: T cos18.6 = 50 x 9.8

So T = 517 N (252 N each rope)

b) Centripetal force $F_c = T \sin\theta = 517 \sin 18.6 = 164$ N

Centripetal acceleration $a_c = F_c/m = 164/50 = 3.29$ m s^{-2}

$a_c = \dfrac{v^2}{r}$ $v^2 = a_c r = 3.29 \times 0.7 = 2.30$ So v = 1.52 m s^{-1}

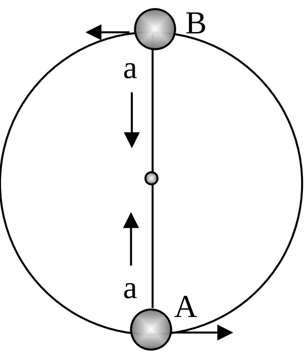
$v = \dfrac{2\pi r}{t}$ so $t = \dfrac{2\pi r}{v} = \dfrac{2\pi \times 0.7}{1.29}$ = 2.90 s

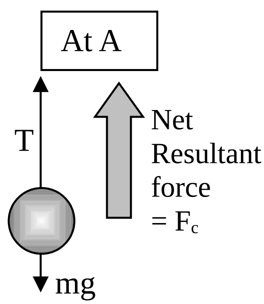
Vertical Circles

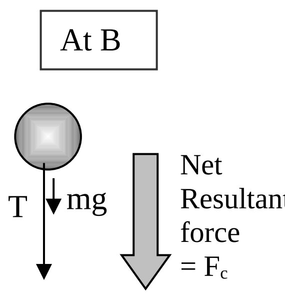
Consider the motion of a ball on a string whirling in a vertical circle and the forces acting:

At point A and point B the net force on the ball is making it accelerate towards the centre of the circle.

At A the ball is accelerating upwards and at B its acceleration is downwards.





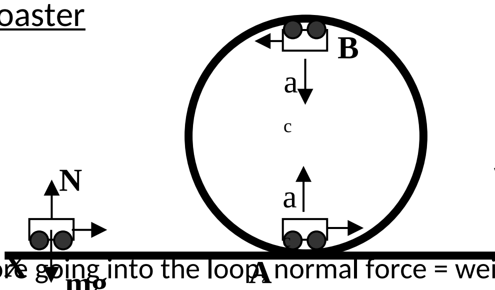


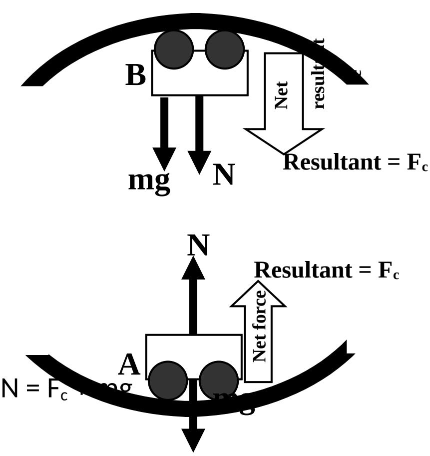
At A: $F_c = T - mg$ So $T = F_c + mg$ (tension > centripetal force)

At B $F_c = T + mg$ So $T = F_c - mg$ (tension < centripetal force)

NB If $F_c = mg$ then tension can go slack drop to zero.

Roller Coaster





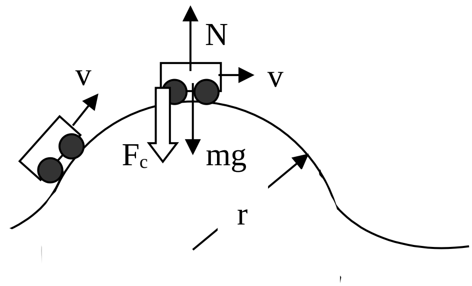
At X, before going into the loop normal force = weight.

At A the truck is accelerating upwards and so $F_c = N - mg \Rightarrow N = F_c + mg$

i.e. the normal reaction on passengers is greater than their usual weight and they will feel heavier.

At B: $F_c = N + mg$ and so $N = F_c - mg$
The normal reaction on the passenger is less than their usual weight and they will feel lighter but as long as N is greater than zero the seat will be pressing on the passenger and they will stay in their seat.

<u>Speed Humps</u>



When the car goes over the curved speed hump the normal reaction becomes less than the weight of the car i.e. there will be a net force downwards (F_c) causing it to move towards the centre of the circle. $F_c = mg - N$
So: $N = mg - F_c$ or $N = mg - mv^2/r$
For the car to stay on the road N must be greater than zero so $mg > mv^2/r$ or $g > v^2/r$
e.g. for a 5 m radius speed hump the maximum velocity for the car to stay in contact with the road is given by $v = \sqrt{5.0 \times 9.8}$ $= 7.0\ m\ s^{-1}$

<u>Example 24</u>

A girl has a yo-yo of mass 100 g which she swings in a vertical circle of radius 80 cm.

a) *If the yo-yo rotates 2 times per second, calculate the tension in the string at the top and bottom of the swing.*
b) *If the string has a breaking strain of 18.0 N, what is the maximum rate of rotation allowable?*

<u>Solution 24</u>

Period = 0.5 s so $v = \frac{2\pi \times 0.8}{0.5}$ $= 10.1\ m\ s^{-1}$

a) $F_c =$

$$F_c = \frac{mv^2}{r} = \frac{0.100 \times 10.1^2}{0.8}$$

<u>At top</u> Resultant $F_c = T + mg$ so $T = F_c - mg = 12.63 - 0.100 \times 9.8 = 11.6\ N$

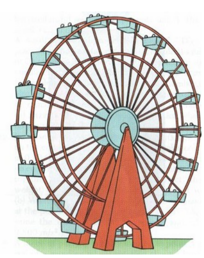
<u>At bottom</u> Resultant $F_c = T - mg$ so $T = F_c + mg$ $T = 12.63 + 0.98 = 13.6\ N$

b) $F_c = \frac{mv^2}{r}$ $18 = \frac{0.1v^2}{0.8}$ So $v_{max} = 12.0\ m\ s^{-1}$

$t = \frac{2\pi \times 0.80}{12}$ $= 0.419\ s$

Frequency = 1/time period = 1/0.419 = 2.39 times per second.

<u>Example 25</u>



The Big Wheel at a showground has a radius of 7.5 m and rotates once every 10 seconds.

What would the apparent weight be for a 65 kg boy when he is

a) at the top of the rotation?

b) at the bottom of the rotation?

Solution 25

Checkpoint 7

1. A car moves clockwise around a circular track of radius 150 m at a constant speed of 12.0 m s^{-1}. Which statement is true?

   (A) The velocity change of the car in 1 revolution is zero

   (B) Without the friction on the tyres the car would accelerate outwards

   (C) The driver's shoulder is being pushed outwards by centrifugal force

   (D) The net force on the car is constantly to the left of the driver.

2. What is the approximate size of the centripetal acceleration in the last question?

   (A) 9.8 m s^{-2} (B) 4.9 m s^{-2} (C) 1.0 m s^{-2} (D)0.08 m s^{-2}

3. Mathematically, why does the angle of lean have to change as a cyclist goes round a bend faster?

   (A) Leaning more gives the tyres more friction

   (B) In the vector triangle the vertical force becomes greater

   (C) In the vector triangle the vertical force becomes less

   (D) In the vector triangle the horizontal force becomes more

4. A 60 kg skateboarder rolls down a circular 'pipe' so, at the bottom he is moving at 4 m s^{-1}. At the bottom of the curve he will feel:

   (A) Lighter as the vertical reaction force becomes larger than his weight

   (B) Heavier as the vertical reaction force becomes larger than his weight

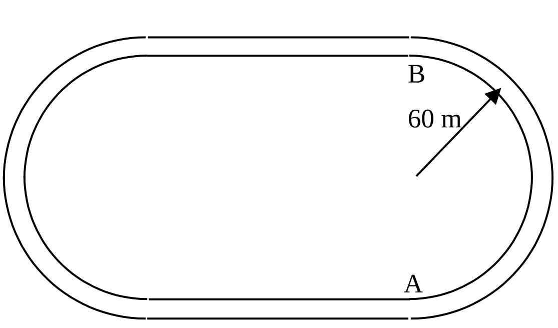
   (C) Lighter as the vertical reaction force becomes smaller than his weight

   (D) Heavier as the vertical reaction force becomes smaller than his weight

5. In the previous question the radius of the pipe is 3.50 m. The upward force on the skateboarder at the bottom of the curve will be about:

   (A) Zero (B) 274 N (C) 588 N (D)860 N

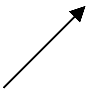
Set 4 Circular Motion



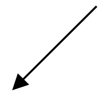
1. In an 800-metre race a girl, running at a constant speed goes into a bend of radius 60 m.
The girl enters the bend at point A and exits at B 31.4 s later.
If the bend has a radius of 60 m, what was the rate of change of the girl's velocity between entering and leaving the bend?

2. An aeroplane flying due north at 720 km h^{-1} suddenly changes its direction to north-west whilst maintaining constant speed. Which vector below correctly indicates the direction of the change in velocity of the aeroplane, if any?

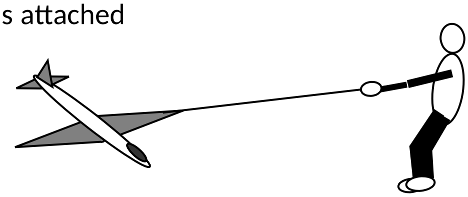
A. No change

B. 

C. 

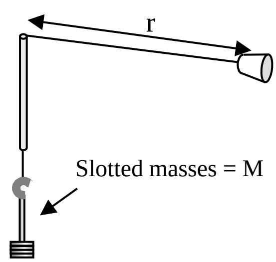
D. 

3. A boy has a 220 g model aeroplane with an engine which is attached to a fishing line so it will fly round in a horizontal circle.
The 'plane is flown at a speed of 25 m s^{-1} on a line of length 45 m.
What is the tension in the line?



4. The maximum frictional force on a car's tyres on a dry road is 10 kN which reduces to 1.2 kN on the same road when it is icy. On the dry road this car can drive round a circular road of radius 50 m at a maximum speed of 20 ms^{-1} without sliding.
What is the maximum speed for the same car to travel around the same bend on an icy road?

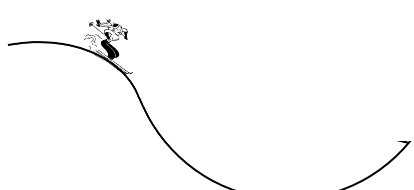
5. In a class physics experiment, a rubber bung is whirled round in a circle on a fishing line.
The slotted masses will remain in a stable position at certain speed of rotation of the bung.
If the radius (r) is held constant but the time of rotation (t) is varied, different values on M are required.



In this case the relationship between t and M (where k is a constant) is given by:

A. $M^2 = kt$ B. $M = k/t$ C. $M = kt^2$ D

6. Using the same apparatus as in the previous question (Q. 5), the ma the radius of rotation (r) was changed. Different values of t were obt



Which of the following plots would result in a straight-line graph?

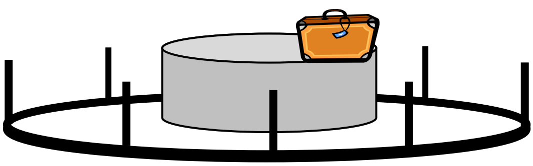
A. $\sqrt{t}$ versus r   B. t versus r   C. t^2 versus r   D. $1/t^2$ versus r

7. 70 kg skier goes down a hill on the ski fields into a small circular valley with a radius of 110 m.

If the skier reaches the bottom of the valley at a speed of 30 m s^{-1}, what will be the reaction force upwards on her feet at the bottom?

8. A local council decides to install a speed hump in a car park so cars travelling at speeds over 30 km h^{-1} cannot stay in contact with the road.
What would need to be the radius of this speed hump?.

9.



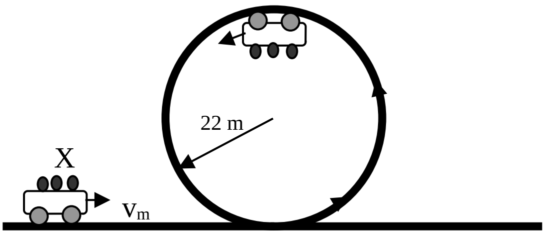
A woman leaves her case on a roundabout in a playground at a distance of 2.0 m from the axis of rotation. Children spin the roundabout faster and faster. The case has a mass of 400 g and the maximum frictional force that can be exerted by the surface on the bag is 3.0 N.
At what rotational speed will the bag slide off?

10. On the roundabout mentioned in the previous question (Q. 9) a girl of mass 50 kg stands on the foot-plate 2.8 m from the axis of rotation. When the roundabout is rotating once every 6.0 s the girl finds she must lean inwards at an angle to maintain her balance.
The correct explanation for this is:

A. Her feet have a centrifugal force acting outwards on them which causes her body to pivot at an angle
B. The force on her upper body is greater than that on her feet and hence her head moves inwards
C. The frictional force on her feet and her normal reaction force combine to form a total reaction vector angled inwards
D. The torque exerted on her upper body must be counterbalanced by an equal torque about her centre of mass

11. In the last question (Q. 10) at what angle the girl must lean at (measured to the vertical) in order to maintain her equilibrium?

12. A roller-coaster car goes into a loop at a speed sufficient to prevent the passengers falling out at the top whilst they are upside down.

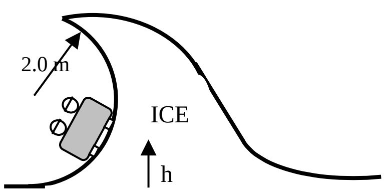


If the radius of the loop is 22 m, what must be the minimum velocity (v_m) of the car as it enters the loop?

13. Because the Earth is rotating, the effective value of 'g' is slightly different to that on a non-rotating Earth (call the rotating value g'). The correct explanation of this is:

A. The Earth's spin tends to throw objects off its surface, which causes a tangential vector and makes g' larger than g
B. As when standing in an accelerating lift, the gravitational force (g') seems less than g due to the centripetal acceleration of the surface supporting the object
C. Due to rotation, the radius of the Earth is slightly different at the equator, which causes the gravitational pull to be slightly more than normal
D. The inertial mass of a person on a moving surface means their body will be "left behind" compared with their feet. This produces an effect where g' is slightly greater than g.

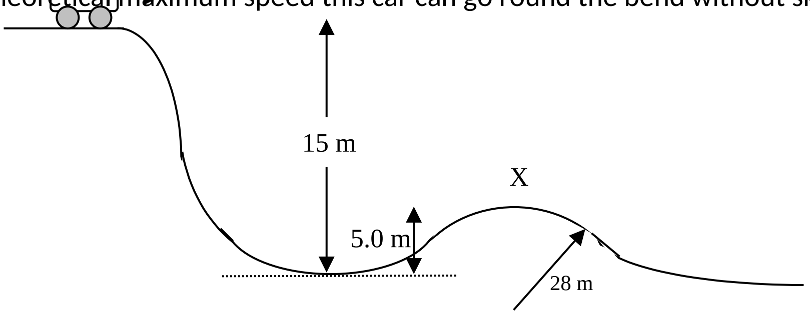
14. A toboggan cannot rely on the frictional force of ice to produce centripetal acceleration round bends. The bend on an Austrian toboggan run has a radius of 40 m. The ice at the bend has a curved surface of radius 2.0 m.



If the toboggan goes round this bend on the run at a speed of 19.8 ms^{-1}, to what vertical height (h) must the toboggan rise up the ice surface above the ground?

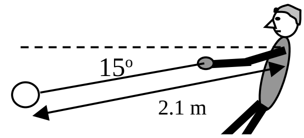
15. On the Indianapolis 500 racetrack one bend has a radius of 350 m and a banking angle of 42.5° to the horizontal. If the maximum frictional force a car of mass 1200 kg can obtain from its tyres is 9 kN, what is the theoretical maximum speed this car can go round the bend without skidding?

16.



A loaded roller-coaster car just moves towards the beginning of a 15 m downward slope leading to a curved, convex part of the track which has a radius of 28 m.
Point X is at the top of the convex track 5.0 m above the lowest part of the downward slope. When the car is at point X gravity appears to be less inside the car due to its motion. Neglecting frictional losses, what is the value of this apparent gravitational acceleration in the car at this point?

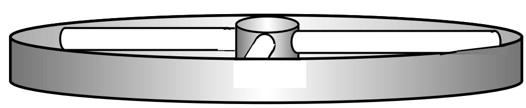
17. A hammer thrower swings his 4.0 kg hammer around in a circle so the wire attached to it makes an angle of 15° to the horizontal.
If the effective distance of the hammer to the thrower's shoulder is 2.1 m then what is the tension in the wire?



18. In the previous question (Q. 17), what would be the time taken for the hammer thrower to rotate one complete turn with the hammer?

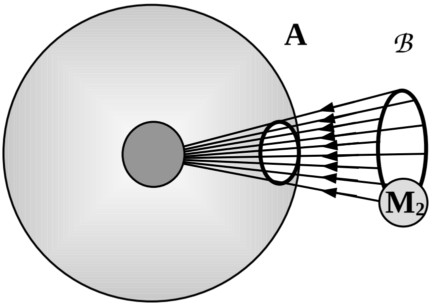
19. A yo-yo has a mass of 120 g and swings at the end of a string 65 cm in length. A girl swings the yo-yo up in the air in a vertical circle with a speed of 3.5 m s^{-1}.
What would be the tension in the yo-yo string when it is at the bottom of its circular swing?

20. A space station rotates in space so as to simulate gravity in the region of its outer ring which has a diameter of 80 m.



At what rate (revolutions per minute) must the space station rotate to give a value for artificial gravity in the outer ring of 8.9 m s^{-2}?

1.15 Gravitation



Sir Isaac Newton was the first scientist to talk about the idea that any mass will attract any other mass with a force called **Gravity**. He imagined that this force exerted itself through field lines, which were indicated by the way one object moved towards the other object e.g. an apple falling to the earth in a straight line. All of these field lines converge towards the centre of the Earth and get spaced out more as they get further from the Earth.

Newton linked the spacing of the lines to the strength of the field so at point A on the diagram, the field strength is larger than at point B because they are concentrated into a smaller area. If B is twice as far away from the Earth's centre as point A then area B will be ¼ of area A from similar triangles (radius of B is 2x radius A and area = πr^2). Hence the **Inverse Square Law** was formulated stating that gravitational force reduces in the ratio of the square of the distance from the centre of mass $\left(F_g \propto \frac{1}{d^2}\right)$

The direction of the field line indicates the direction in which another object will move. In the case of electric or magnetic force, the lines can go inwards (attractive) or outwards from the object (repulsive). Gravitational fields can only be attractive.
The gravitational force will also depend on how large the mass of the Earth is (M_1) and how large the mass of the object is (M_2). The proportional constant throughout the Universe is given the symbol 'G'.

Newton's Universal gravitational formula is: $\boxed{F = \frac{GM_1M_2}{r^2}}$

The value of G has been found to be 6.67×10^{-11} N m^2 kg^{-2}.
From this formula, Newton could "weigh the Earth".

e.g. At the Earth's surface, the radius is 6.37×10^6 m from the centre, a 1.0 kg mass has a weight of 9.8 newtons.
Using this equation: $9.8 = \frac{6.67 \times 10^{-11} \times M_e \times 1.0}{(6.37 \times 10^6)^2}$

Earth's mass, $M_e = 5.96 \times 10^{24}$ kg.

Gravitational force is extremely small: The attraction between two 1.0 kg masses one metre apart would be: $\frac{6.67 \times 10^{-11} \times 1.0 \times 1.0}{(1.0)^2} = 6.67 \times 10^{-11}$ N
This force would not be noticeable.

Example 27

A meteorite is moving at a height of 5000 km above the earth's surface. What would the value of the gravitational field strength (g') be at this height?

Solution 27

Gravitational force on the meteorite is $\frac{GM_1M_2}{r^2} = M_2g$

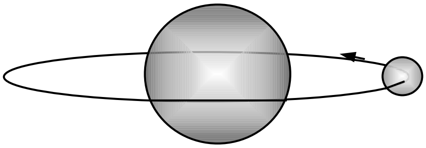
Distance from Earth's centre = $6.37 \times 10^6 + 5.00 \times 10^6 = 1.137 \times 10^7$ m

So $g' = \frac{6.67 \times 10^{-11} \times 5.96 \times 10^{24}}{(1.137 \times 10^7)^2}$ = 3.07 newtons per kilogram (or m s^{-2})

Alternative (Proportion) Method

The force of gravity varies inversely with the square of the distance from the Earth, so if the gravitational field strength on the earth's surface is 9.8 N kg^{-1} then it will be proportionally less 5000 km away by a factor of $\left(\frac{6.37 \times 10^6}{1.137 \times 10^7}\right)^2$ which gives $\left(\frac{6.37 \times 10^6}{1.137 \times 10^7}\right)^2 \times 9.8 = 3.07\,N\,kg^{-1}$

Satellites



The force providing centripetal acceleration and pulling a satellite into a circular orbit equals the gravitational attraction, so we can equate the two forces: $\frac{GM_1M_2}{r^2} = \frac{M_2v^2}{r}$

This gives: $v^2 = \frac{GM}{r}$ from which we can obtain the velocity of a satellite.

Example 28

How fast must a satellite be moving if it is to stay orbiting at a height of 5000 km above the Earth's surface?

Solution 28 $v^2 = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.37 \times 10^6 + 5.0 \times 10^6)}$ v = 5.92×10^3 ms^{-1}

1.16 Kepler's Law

Johannes Kepler derived an empirical law from knowing the radii of the orbits of planets (R) and their times to orbit the Sun (T). He found the complex relationship: $T^2 \propto R^3$ i.e. for any planet, the period of revolution squared divided by the orbital radius cubed was a constant. Why this empirical relationship should be true was a complete mystery until Newton proved it by using his gravitational force equations:

$\frac{GM}{r} = v^2$ But $v = \frac{2\pi R}{T}$ so $\frac{GM}{R} = \left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2R^2}{T^2}$

Rearranging gives: $GMT^2 = 4\pi^2R^3$ which conforms to **Kepler's Law** $T^2 \alpha R^3$ or T^2/R^3 = constant.

1.17 Geostationary Satellites

A satellite must orbit around the centre of mass of the Earth. This can occur in a polar orbit or an equatorial orbit. A polar orbiting satellite travels north to south and for photographic purposes can scan the whole Earth, whereas an equatorial satellite with a period of 24 hours can remain above the same point on the Earth. This **geostationary** orbit is a useful one for radio or TV transmissions from one country to another. Satellites are always launched as close to the equator as possible so as to utilise the tangential velocity of the ground (about 450 m s^{-1} at the equator). Launching to the east gives a 'slingshot' effect to the initial rocket and saves fuel.

Example 29

Calculate the height above the Earth at which a geostationary satellite must remain.

Solution 29

Time period of a geostationary satellite = (24 x 3600) seconds = 8.64×10^4 s

Using Kepler's Law equation: $GMT^2 = 4\pi^2R^3$

$6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times (8.64 \times 10^4)^2 = 4\pi^2R^3$ So R = 4.225×10^7m from the centre,

or height = $4.225 \times 10^7 - 6.37 \times 10 = 3.588 \times 10^7$ m above the surface (35,880 km).

1.18 Uses of Satellites

Since the first earth-orbiting satellite, Sputnik, was launched by USSR in 1957, satellites have had an enormous effect on our society. Sputnik could only receive and send weak analogue radio signals and was very primitive. Nowadays geostationary satellites can be used to provide continuous communication coverage for areas fairly close to the equator but for countries such as Russia low orbiting satellites are used, as they are less expensive to launch and give a stronger signal. A single satellite cannot be used there as they only give coverage for about 10 minutes per day, so several are needed to give continuous reception of signals.

The first geostationary communications satellite was launched by USA in 1964 to transmit TV pictures of the Tokyo Olympics. The first 3-axis stabilised geostationary satellite was launched in 1974, which used a gyroscope to maintain constant altitude and orientation for communication.

Today satellites are used for such things as:

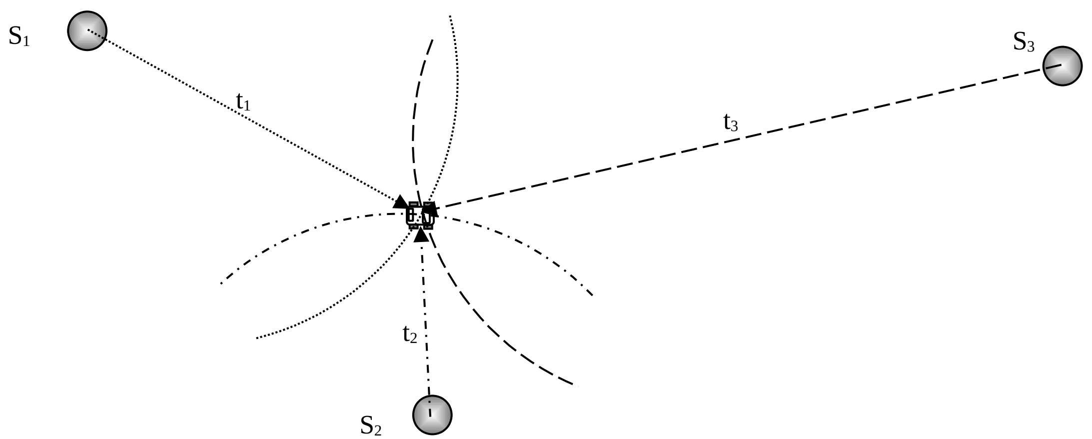
Astronomy (e.g. The Hubble Space Telescope)
Mobile telephone communication
Navigation, tracking and GPS
Weapons guidance
International TV, Internet and radio transmission

Atmospheric studies and weather prediction

Some other uses are:

Search and rescue missions
National security from terrorists
Agricultural studies
Disaster management
Geology, hydrology and mining information
Marine surveillance
Mapping.

Satellites can receive information (TV mobile phone and Internet signals) transmitted on the gigahertz wavebands and retransmit them to a different location. In this way telephone calls and satellite TV stations can be sent and received, using a parabolic microwave aerial. GPS systems calculate the position of a transmitter by finding the time for a signal to travel to four satellites orbiting above the earth, knowing the speed of light accurately. Each satellite must have an atomic clock onboard to make timing calculations accurate. Triangulation between 3 satellites allows the calculation of the x, y and z co-ordinates of a transceiver (e.g GPS system in a car). The 4th satellite is used to correct for errors in the atomic clocks of the other satellites.



For example: If the time for the signal to be transmitted from satellite S_1 and return is 360 µs then the car's distance from this satellite is $360 \times 10^{-6} \times 2.998 \times 10^{8}$ m = 108 km.

Checkpoint 8

1. Which of these forces of nature is the weakest?
   (A) Gravitational force (B) Electric force (C) Magnetic force (D) Frictional force

2. Which statement about the moon orbiting the earth is false?

   (A) The moon is constantly falling towards the earth

   (B) The moon still has weight even though it is a long way away

   (C) Centripetal force on the moon equals the gravitational force on it

   (D) The moon is weightless as it is a long way away

3. Which statement is true about the weight of an astronaut in space?

   (A) His weight changes on each planet where he lands

   (B) His mass and weight both vary with the gravitational field strength

   (C) His mass will depend on the size of the planet he is on

   (D) On a more massive planet he will weigh less

4. Due to the speed of rotation of the earth it is calculated that a person will appear to weigh less at the equator than the poles. The percentage difference is about:

   (A) 0.1% (B) 0.3% (C) 2.0% (D) 5.0%

5. If a satellite is orbiting the earth and it then moves to another orbit further away, which is true?

   (A) Its potential energy increases and so it slows down

   (B) Its kinetic energy increases and so it speeds up

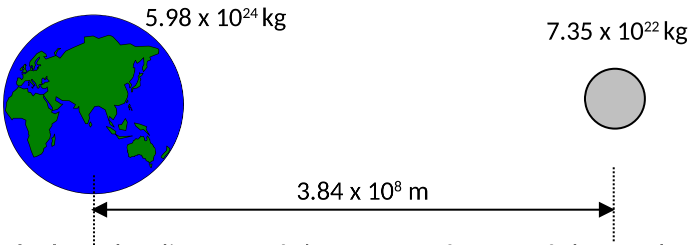
   (C) Its E_P changes to E_K and so it remains at a constant speed

   (D) Work is done against gravity, giving it a greater speed

Set 5 Gravitation

1. A satellite of mass 50 kg is in orbit around the Earth 1270 km above its surface. What force does the satellite exert upon the Earth?

2. The Earth and the Moon both rotate about their common centre of mass.



Using the data given, calculate the distance of the centre of mass of the Earth-Moon system from the Earth's surface?

3. A communications satellite is positioned 400 km above the Earth's surface. Calculate its orbital speed around the Earth?

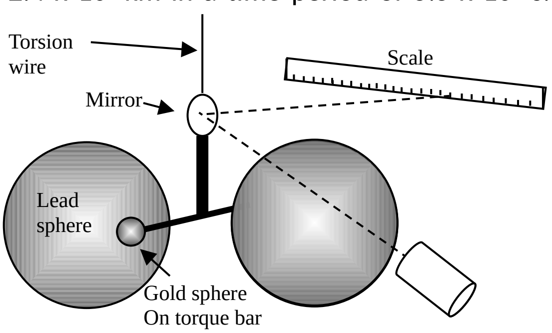
. The record for high jump on the Earth is 2.45 m currently. If the same record-hlder jumped on the surface of the moon, what height is he likely to achieve?

5. Neptune's mass is 17 times that of the Earth and it has a radius of 2.27×10^4 km. Calculate a value for the gravitational field (g) on Neptune's surface.

6. A planet orbits around a neutron star at a radius of 2.4×10^9 km in a time period of 5.5×10^9 s. Calculate the mass of the neutron star.

7. scientist called Boys devised a method for finding the universal gravitational constant (G) by measuring the attractive force between two gold spheres and two lead spheres.

Two gold spheres of mass 500g were suspended on the end of a torque bar attached to a mirror.

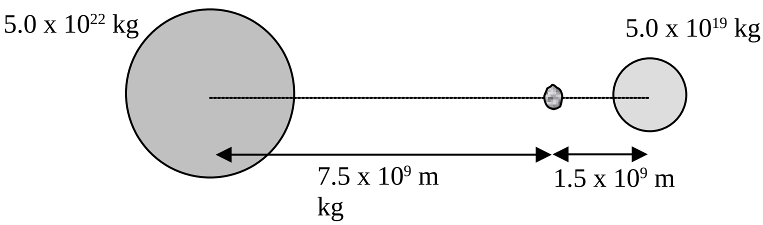


When two 10 kg lead spheres were brought near to the gold spheres the force of attraction between the masses was determined from the angular deflection of the torque bar. A beam of light reflecting from the mirror onto a glass scale gave a value of the turning torque.

In one set-up, lead spheres of radius 6.00 cm and gold spheres of radius 1.80 cm were used. Each lead sphere was positioned so its edge was exactly 1.0 mm away from the edge of a gold sphere. A force of attraction of 5.00×10^{-8} N was recorded between each lead and gold sphere.
From these results, calculate a value for 'G' (in N $m^2 kg^{-2}$).

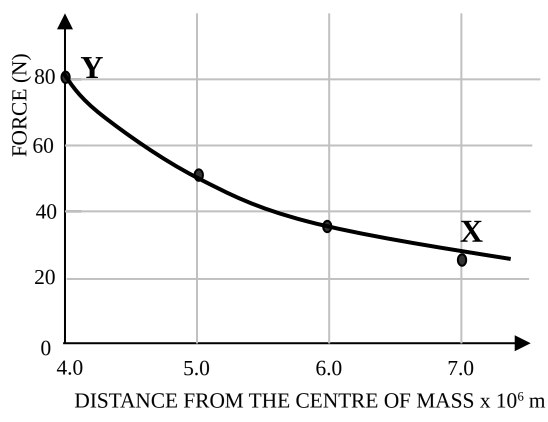
8. Referring to the previous question (Q. 7), the distance between the centres of masses of the gold spheres on the torque bar was 7.55 cm. The value for the torsional constant of the torsion wire was 9.2 μN m per degree (ie a torque of 9.2×10^{-6} Nm will turn the bar through an angle of 1°).
What angle would the mirror have turned through after the lead spheres had been placed in position 1 mm from the gold spheres?

9. A meteor of mass 500 kg is 7.5×10^9 m from the centre of a planet (X), 1.5×10^9 m from the centre of its moon (Y)and positioned along the planetary axis between the two planets.
The planet has a mass of 5.0×10^{22} kg and the moon has a mass of 5.0×10^{19} kg.



What would be the resultant gravitational force on the meteor from the 2 planets?

10. The graph shows the gravitational force acting on an 1100 kg asteroid as it approaches a planet.
The force is measured in newtons and the distance is measured from the centre of mass of the planet as it moves from point X that is 7.0 x 10^6 m away to point Y, 4.0 x 10^6 m away.
As the asteroid moves from X to Y the gravitational force on it increases, causing it to gain velocity.



Make an estimate of energy change from the graph and hence a value for the change in velocity of the asteroid.