



PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester One Examination,
2018

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section One:
Calculator-free

Your Name _____

Your Teacher's Name _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Question	Mark
1		5	
2		6	
3		7	
4		8	

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	95	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free

(50 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1

(8 marks)

Consider the polynomial $f(z) = z^2 + 3z + 3$ where $z = x + iy$

(a) Determine the roots of $f(z) = 0$ and label them α & β (2 marks)

(b) Determine $\overline{\alpha\beta} + \overline{\alpha}\beta$ (3 marks)

(c) Determine $f(\alpha + \beta) + 3$ (3 marks)

Question 2

(7 marks)

Let $w = 2a - bi$ where a & b are real numbers.

- (a) Show that $|w^6| = (4a^2 + b^2)^3$ (2 marks)

- (b) The expression $w + \frac{1}{w}$ can be written in the form $\frac{c + di}{4a^2 + b^2}$, determine expressions for the real constants c & d in terms of a & b . (3 marks)

- (c) Given that the $Arg(w^6) = Arg(w)$ and $0 < Arg(w) < \frac{\pi}{2}$, determine the $Arg(w)$. (2 marks)

Question 3

(7 marks)

Consider the following functions f & g .

$$f(x) = \frac{1}{\sqrt{x-5}} \qquad g(x) = (2x-1)^3$$

(a) State the natural domain and range of f . (2 marks)

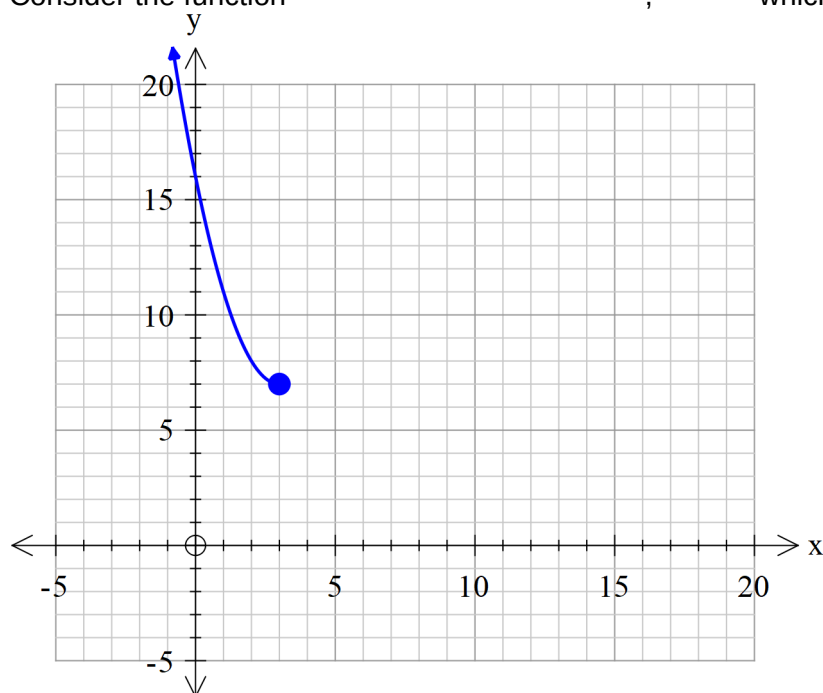
(b) State the natural domain and range of $g \circ f(x)$ (3 marks)

(c) Does $f \circ g(x)$ exist over the natural domain of g ? Explain. (2 marks)

Question 4

(7 marks)

Consider the function $f(x) = x^2 - 6x + 16$, $x \leq 3$ which is plotted on the axes below.



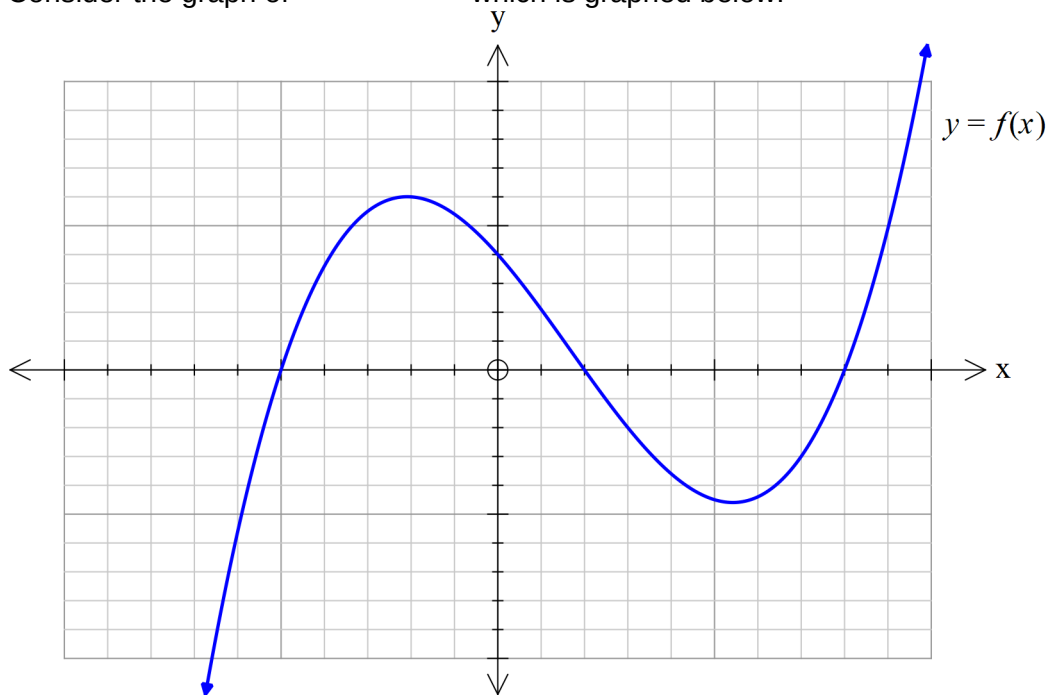
- (a) Sketch the inverse function $f^{-1}(x)$ on the axes above. (3 marks)

- (b) Determine the rule for the inverse function $f^{-1}(x)$ and state the domain and range. (4 marks)

Question 5

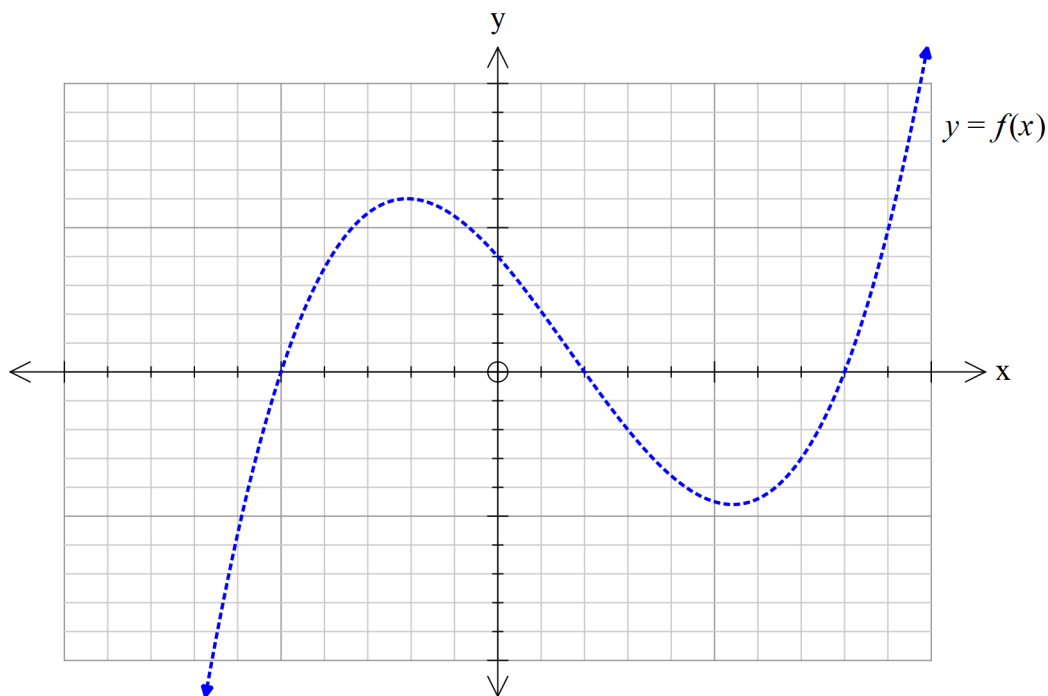
(5 marks)

Consider the graph of $y = f(x)$ which is graphed below.



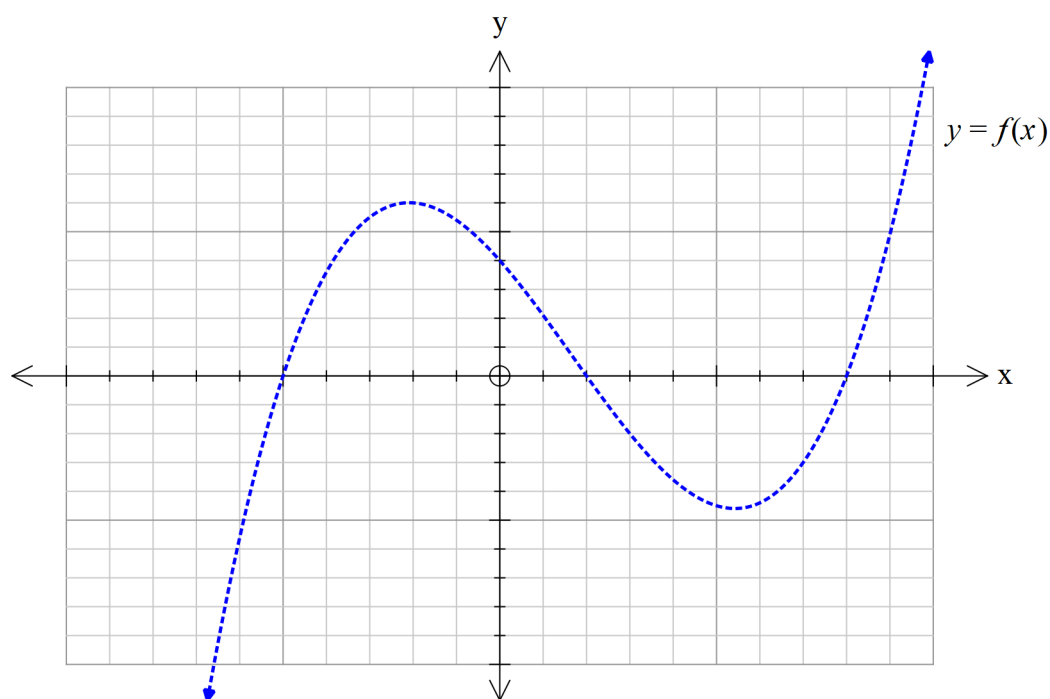
(a) On the axes below, sketch $y = f(|x|)$

(2 marks)



- (b) On the axes below, sketch $y = -|f(|x|)|$

(3 marks)

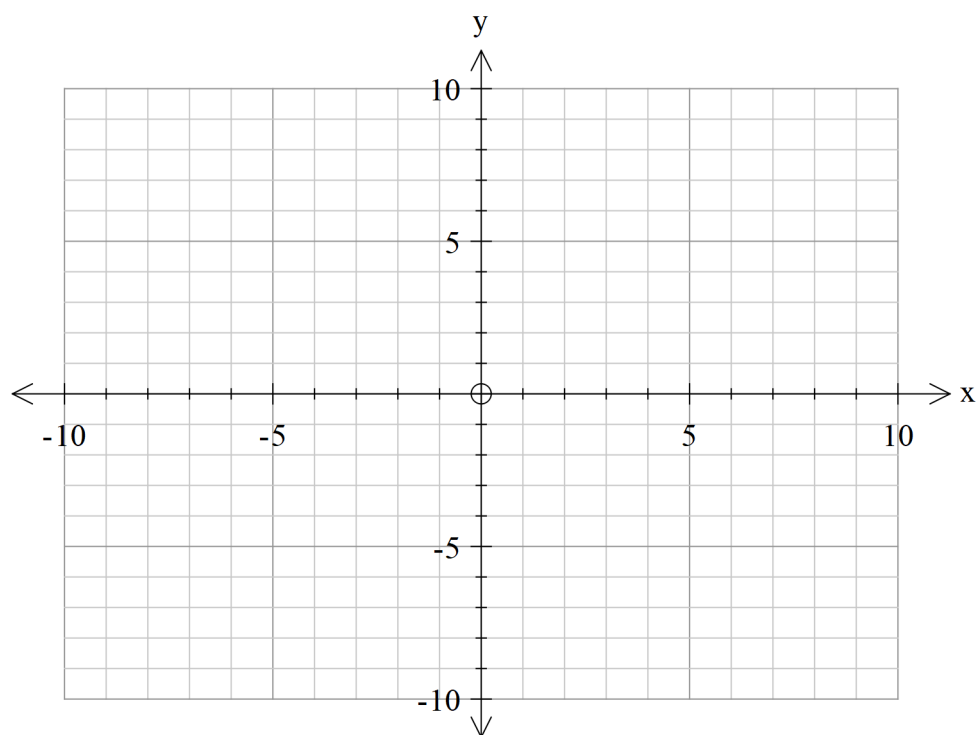


Question 6

(6 marks)

$$f(x) = \frac{2(x^2 - 4)}{(x^2 + x - 12)}$$

Sketch the graph of $f(x)$ on the axes below.



See next page

Question 7

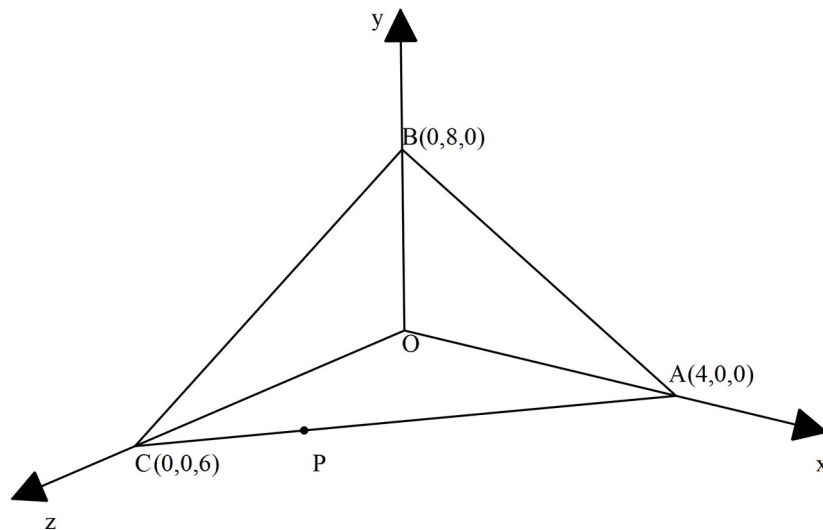
(5 marks)

A triangular prism OACB is shown below with O as the origin and points A, B & C have

$$\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$$

respective position vectors

Point P lies on the line \overline{CA} in the ratio 1:2.



- (a) Determine the vector equation of the line that passes through points B & P .
(2 marks)

- (b) Determine the cartesian equation of the plane that contains points A, B & C .
(3 marks)

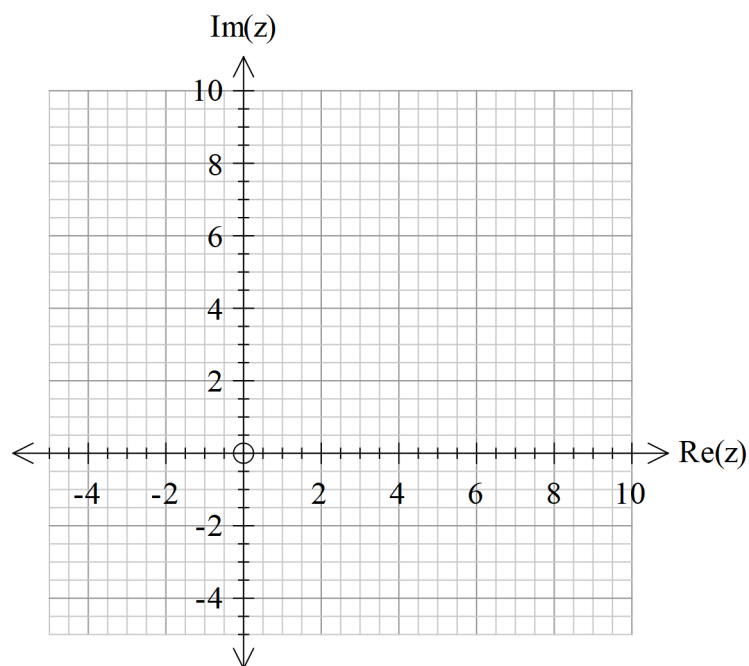
Question 8

(5 marks)

Consider a circle in the complex plane where the centre is given by $3 + 4i$ and a radius of 5 units. Let P be a point on this circle where $P = rcis\theta$ with $|P| \neq 0$ and $\text{Arg}(P) = \tan^{-1} 3$

- (a) Determine P in exact cartesian form $x + iy$. (3 marks)

- (b) Sketch this circle and point P in the complex plane below showing all major features. (2 marks)



Additional working space

Question number: _____

Additional working space

Question number: _____

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Acknowledgements