**Working out space**





PERTH MODERN SCHOOL
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**Independent Public School**

# Course Specialist Test 1 Year 12

Student name: ______________ Teacher name: ______________

| | |
|---|---|
| **Task type:** | **Response/Investigation** |
| **Reading time for this test :** | **5 mins** |
| **Working time allowed for this task:** | **40 mins** |
| **Number of questions:** | **7** |
| **Materials required:** | No cals allowed!! |
| Standard items: | Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters |
| Special items: | Drawing instruments, templates, NO notes allowed! |
| **Marks available:** | **41 marks** |
| **Task weighting:** | **13%** |
| **Formula sheet provided:** | **no, but formulae stated on page 2** |

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

## Useful formulae

### Complex numbers

| Cartesian form | |
|---|---|
| $z = a + bi$ | $\bar{z} = a - bi$ |
| $\text{Mod}(z) = \lvert z\rvert = \sqrt{a^2+b^2} = r$ | $\text{Arg}(z) = \theta, \quad \tan\theta = \frac{b}{a}, \quad -\pi < \theta \le \pi$ |
| $\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ | $\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$ |
| $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ | $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$ |
| $z\bar{z} = \lvert z\rvert^2$ | $z^{-1} = \frac{1}{z} = \frac{\bar{z}}{\lvert z\rvert^2}$ |
| $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ | $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$ |
| **Polar form** | |
| $z = a + bi = r(\cos\theta + i\sin\theta) = r\,cis\,\theta$ | $\bar{z} = r\,cis(-\theta)$ |
| $z_1 z_2 = r_1 r_2\,cis(\theta_1 + \theta_2)$ | $\frac{z_1}{z_2} = \frac{r_1}{r_2}\,cis(\theta_1 - \theta_2)$ |
| $cis(\theta_1 + \theta_2) = cis\,\theta_1\,cis\,\theta_2$ | $cis(-\theta) = \frac{1}{cis\,\theta}$ |
| **De Moivre's theorem** | |
| $z^n = \lvert z\rvert^n cis(n\theta)$ | $(cis\,\theta)^n = \cos n\theta + i\sin n\theta$ |
| $z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\frac{\theta+2\pi k}{q} + i\sin\frac{\theta+2\pi k}{q}\right)$, for $k$ an integer | |

$$(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$





Q7 (4 marks)

Consider the roots of the equation $z^n = a$ with $z$ being a complex variable with $a$ as a complex constant and $n$ being an integer $n > 3$. A root is defined to be in the first quadrant if the Argument lies in $0 < Arg(z) < \frac{\pi}{2}$.

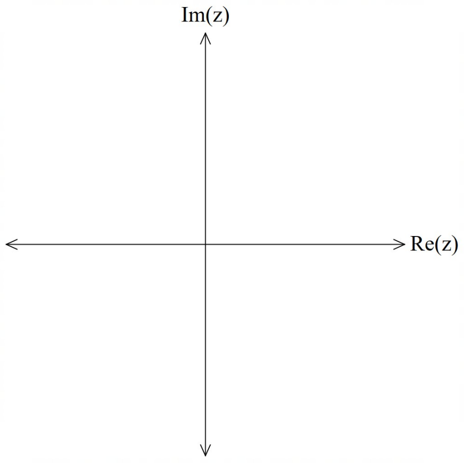
Determine **all** the allowable values of $n$ such that there will be **exactly** 3 roots in the first quadrant and the smallest argument of these 3 roots will be $\frac{\pi}{10}$.

**(Note: answers without working will receive zero marks)**

Q6 (2, 1, 2 & 2 = 7 marks)

Consider the locus of complex numbers $z$ that satisfy $\left|z - \sqrt{3} + i\right| = 2$.

a) Sketch the locus on the axes below.




b) State the maximum value of $|z|$

c) State the minimum value of $Arg(z)$ such that $Arg(z) > Minimum$.

d) State the maximum value of $Arg(z)$ such that $Arg(z) < Maximum$.





**No cals allowed!!**

Q1 (2, 2, 2 & 2 = 8 marks)

If $z = 5 - 4i$ and $w = 2 + 3i$ determine the following:

a) $zw$

b) $\frac{1}{w}$

c) $\frac{\overline{z}}{w}$

d) $z^2\overline{w}$

Q2 (2 & 3 = 5 marks)

a) Determine the complex roots of $3z^2 + z + 2 = 0$ .

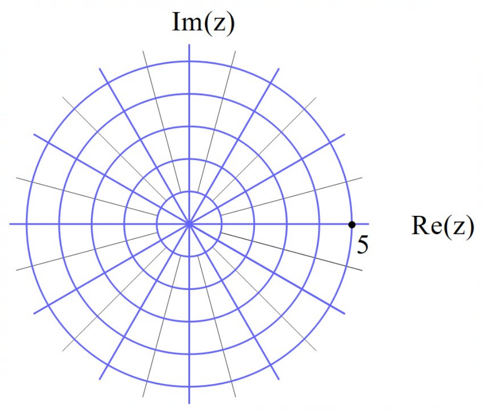
b) Use the quadratic equation to prove that if a quadratic equation with real coefficients has any non-real roots then it must have two complex roots and they must be conjugates of each other.

Q3 (4 marks)

Determine all possible real number pairs $a \,\&\, b$ such that $\dfrac{37+9i}{5+ai} = b - i$.

Q4 (2, 2, 2 & 2 = 8 marks)

Consider the complex number $z = \sqrt{3} + i$.




Plot the following on the axes above.

a) $z$

b) $iz$

c) $(1+i)z$





d) $\dfrac{z}{(1+i)}$

Q5 (5 marks)

Consider the polynomial $f(z) = az^4 + bz^3 + cz^2 + dz + e$ where $a, b, c, d \,\&\, e$ are real numbers.

Given that $f(2+i) = 0 = f(5-2i)$

and $f(0) = -290$

Determine the values of $a, b, c, d \,\&\, e$.

**(Note: answers without working will receive zero marks)**