MATHEMATICS METHODS

MAWA Semester 1 (Unit 3) Examination 2020
Calculator-free

Marking Key

The release date for this exam and marking scheme is

- **June 12th the end of week 7 of term 2, 2020**

Section One: Calculator-free (50 Marks)

Question 1(a) (2 marks)

Solution

$$f(x) = (3 + x^3)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(3 + x^{3)})^{\frac{-1}{2}}(3x^2)$$

$$= \frac{3x^2}{2\sqrt{3 + x^3}}$$

Mathematical behaviours	Mark
• applies chain rule	1
• obtains correct result	1

Question 1(b) (2 marks)

Solution

$$z = t^2\cos(2t - 1)$$

$$\frac{dz}{dt} = \cos(2t - 1) \times 2t + t^2 \times (-2)\sin(2t - 1))$$

$$= 2t\cos(2t - 1) - 2t^2\sin(2t - 1)$$

Mathematical behaviours	Marks
• differentiates cos term correctly	1
• applies product rule and states result	1

Question 1(c) (3 marks)

Solution

$$y = 5\sin(4x + 3)$$

$$\frac{dy}{dx} = 5\cos(4x + 3\ ^2 + 16 \times (5\sin(4x + 3))^2$$

$$= 400\cos^2(4x + 3) + 400\sin^2(4x + 3)$$

$$= 400(\cos^2(4x + 3) + \sin^2(4x + 3)) \qquad ...(*)$$

$$= 400 \qquad \because \cos^2(4x + 3) + \sin^2(4x + 3) = 1$$

Mathematical behaviours	Marks
• differentiates correctly	1
• substitutes and simplifies to (*)	1
• evaluates correctly, stating Pythagorean identity	1

Question 2(a) **(2 marks)**

Solution
$f(x)=0 \Rightarrow x^3 - 12x = 0$ $\Rightarrow x(x^2 - 1) = 0$ $\Rightarrow x = 0, \pm\sqrt{12}$

Mathematical behaviours	Marks
• equates function to 0 and obtains $x = 0$ • states $x = \pm\sqrt{12}$	1 1

Question 2(b) **(4 marks)**

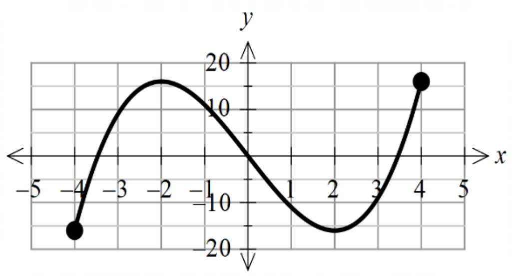
Solution
$f(x) = x^3 - 12x$ $f'(x) = 3x^2 - 12 = 0 \Rightarrow x = \pm 2$ $\quad f(2) = -16, \quad f''(2) = 12 > 0 \Rightarrow$ min $f''(x) = 6x$ $\quad f(-2) = 16, \quad f''(-2) = -12 \Rightarrow$ max $f''(x) = 0 \Rightarrow x = 0,\ f(0) = 0 \Rightarrow$ point of inflection

Mathematical behaviours	Marks
• differentiates, equates to 0 and solves	1
• obtains correct y values of the stationary points	1
• uses second derivative test (or sign test) to determine nature of stationary points	1
• locates point of inflection	1

Question 2(c) **(1 mark)**

Solution
$f(x) = x^3 - 12x$ $f(-4) = -64 + 48 = -16$ $f(4) = 64 - 48 = 16$ $\therefore$ maximum is 16 since $f(-2)$ is also 16

Mathematical behaviours	Marks
• determines $f(4)$ and concludes maximum	1

Solution


Mathematical behaviours	Marks
• plots zeros at 0 and such that $-4 < x < -3$ and $3 < x < 4$	1 1

• plots stationary points and point of inflection accurately • obtains correct shape for the graph, scale and end points	1

Question 2(d) **(3 marks)**

Question 3(a) **(1 mark)**

Solution	
$x = 2$, $\frac{dc}{dx} = 2(8+1)^{\frac{1}{2}} = 6$	
Mathematical behaviours	Mark
• states correct answer	1

Question 3(b) **(4 marks)**

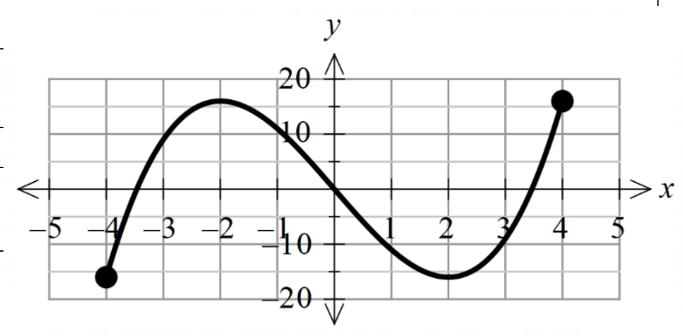
Solution	
$\int_0^2 x(2x^2+1)^{\frac{1}{2}} dx$ $= \frac{1}{4}\int_0^2 4x(2x^2+1)^{\frac{1}{2}} dx$ $= \frac{1}{4}\left[(2x^2+1)^{\frac{3}{2}} \cdot \frac{2}{3}\right]_0^2$ $= \frac{1}{6}(27 - 1)$ $= \frac{13}{3}$	
Mathematical behaviours	Marks
• states the change as $\int_0^2 x(2x^2+1)^{\frac{1}{2}} dx$ • anti-differentiates correctly • substitutes correct limits of integration • determines correct answer	1 1 1 1

Question 4(a) **(2 marks)**

Solution	
$k + 3k + 5k + 4k = 1 \Rightarrow k = \frac{1}{13}$	
Mathematical behaviours	Marks
• states the sum of probabilities is 1 • deduces k value	1 1

Question 4(b) **(2 marks)**

$P(X > 2) = 1 - \frac{1}{13} = \frac{12}{13}$



Mathematical behaviours	Marks
• states an expression to calculate required probability • determines probability	1 1

Question 4(c) **(2 marks)**

Solution
$P(X \le 5 \mid X > 2) = \dfrac{\frac{8}{13}}{\frac{12}{13}} = \dfrac{8}{12} = \dfrac{2}{3}$

Mathematical behaviours	Marks
• writes fraction with the correct denominator • obtains simplified result	1 1

Question 5(a) **(4 marks)**

Solution
(i) $\int_0^{2\pi} 2\sin(4x)\,dx$ $= \left[\dfrac{-2\cos(4x)}{4}\right]_0^{2\pi}$ $= -\dfrac{1}{2}\left[\cos 8\pi - \cos 0\right]$ $= 0$ (ii) $\int \dfrac{x+\sqrt{x}}{x}\,dx$ $= \int 1 + x^{\frac{-1}{2}}\,dx$ $= x + 2\sqrt{x} + c$

Mathematical behaviours	Marks
(i) • states anti-derivative • evaluates result (ii) • rewrites fraction as sum of two functions • anit-differentiates including c	 1 1 1 1

Question 5(b) **(3 marks)**

Solution	
(i) $\int_1^6 f(x)\,dx = -\int_4^1 f(x)\,dx + \int_4^6 f(x)\,dx$ $= -1 + 4$ $= 3$ (ii) $\int_4^1 (2f(x)+1)\,dx = 2\int_4^1 f(x)\,dx + \int_4^1 1\,dx$ $= 2(1) + [x]_4^1$ $= 2 + (1 - 4)$ $= -1$	
Mathematical behaviours	**Marks**
(i) • applies linearity of integrals, swaps bounds of integration and determines the correct result (ii) • applies linearity of integrals correctly • integrates correctly and calculates the result	 1 1 1

Question 5(c) **(5 marks)**

Solution
$$y=\frac{1}{e^{2x}+1}=\left(e^{2x}+1\right)^{-1}$$ $$\frac{dy}{dx}=\frac{-2e^{2x}}{\left(e^{2x}+1\right)^{2}}=-2\left(\frac{e^{x}}{\left(e^{2x}+1\right)}\right)^{2}$$ $$\int\frac{dy}{dx}\,dx=\int-2\left(\frac{e^{x}}{\left(e^{2x}+1\right)}\right)^{2}dx$$ $$ie\quad y+C=-2\int\left(\frac{e^{x}}{\left(e^{2x}+1\right)}\right)^{2}dx$$ $$ie\ \frac{1}{e^{2x}+1}+C=-2\int\left(\frac{e^{x}}{\left(e^{2x}+1\right)}\right)^{2}dx$$ $$ie\left(\frac{-1}{2}\right)\frac{1}{e^{2x}+1}+C=\int\left(\frac{e^{x}}{\left(e^{2x}+1\right)}\right)^{2}dx\Rightarrow A=\frac{-1}{2}$$

Mathematical behaviours	Marks
• applies the chain rule to the derivative	1
• differentiates e^{2x} correctly	1
• recognises application of the Fundamental Theorem	1
• factors out -2 and re-writes fraction involving e^{2x} in numerator and denominator as one fraction squared	1
• multiplies both sides of expression by $-\frac{1}{2}$ to obtain desired result	1

Question 6(a) **(3 marks)**

Solution
$$y=\frac{8x}{(x-1)^{2}}\Rightarrow\frac{dy}{dx}=\frac{(x-1)^{2}8-8x\times(2)(x-1)}{(x-1)^{4}}$$ $$=\frac{8(x-1)-16x}{(x-1)^{3}}\Rightarrow c=1,\ d=-1$$ $$=\frac{-8x-8}{(x-1)^{3}}\Rightarrow a=b=-8.$$

Mathematical behaviours	Marks
• applies quotient rule	1
• differentiates both parts correctly and states the value of c and d	1
• simplifies result and states value of a and b	1

Question 6(b) **(2 marks)**

Solution	
Parallel to x axis $\Rightarrow \frac{dy}{dx}=0 \Rightarrow \frac{-8x-8}{(x-1)^3}=0 \Rightarrow x=-1 \Rightarrow y=\frac{-8}{4}=-2.$ the coordinates of B are $(-1,-2)$ So	
Mathematical behaviours	**Marks**
• equates derivative to 0 and solves • states co-ordinates of B	1 1

Question 7(a) **(2 marks)**

Solution	
It is the area between the two curves from $x = 0$ to $x = \pi$.	
Mathematical behaviours	**Marks**
• states it is the area between the two given curves • states the area is from $x=0$ to $x=\pi$	1 1

Question 7(b) **(3 marks)**

Solution	
$\int \sin x - xe^{-x^2}\,dx = -\cos x - \left[-\frac{1}{2}\int -2xe^{-x^2}\,dx\right]$ $= -\cos x + \frac{1}{2}e^{-x^2} + c$	
Mathematical behaviours	**Marks**
• anti-differentiates $\sin x$ correctly • anti-differentiates xe^{-x^2} correctly • includes constant of integration	1 1 1