KINGSWAY CHRISTIAN COLLEGE

MATHS DEPARTMENT

Course: Math Methods unit 3

Assessment Task: Test 2

Student Name: Solution Key.

Date: 23rd & 24th March 2017

Assessment Score: ________ / 50

Year Score: ________

Comments: ________

Teacher signature: ________

Parent/ Guardian signature: ________

Comments: ________

Question 2 **(6 marks)**

Determine the maximum and minimum value for $f(x)$ and the value of x at which they occur, for the function $f(x) = 3x^4 - 16x^3 + 18x^2$ over the domain $-1 \le x \le 2$.

$f(-1) = 3 + 16 + 18$
$= 37$

$f(2) = 48 - 128 + 72$
$= -8$

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$\therefore f'(x) = 12x^3 - 48x^2 + 36x$
$= 12x(x^2 - 4x + 3)$
$= 12x(x-3)(x-1)$ ✓

for max/min:
$f'(x) = 0$ ✓ $\Rightarrow x = 0$ or $x = 1$ or $x = 3$

$x = 3 \downarrow$ N.A.

$f(0) = 0 \downarrow f''(0) = 0$ ✓ p.o.i

$f(1) = 5 \downarrow f''(0) < 0$ ✓ local max.

$\therefore$ Min : $(2; -8)$ ✓

Max : $(-1; 37)$ ✓

Question 3 **(7 marks)**

Determine the coordinates of all intercepts, stationary points and points of inflection of the function $y = x\,e^{3x}$.

Justify the nature of the stationary points found using a standard test.

$$\frac{dy}{dx} = e^{3x} + x \cdot e^{3x} \cdot 3$$
$$= e^{3x}(1+3x).$$

for min/max: $\frac{dy}{dx} = 0$

$\therefore e^{3x}(1+3x) = 0$

$\therefore x = -\frac{1}{3}$

$y = -\frac{1}{3}e^{-1}$

$= -\frac{1}{3e}.$

$\therefore \left(-\frac{1}{3}; -\frac{1}{3e}\right).$

and $\left.\frac{d^2y}{dx^2}\right|_{x=-\frac{1}{3}} = 3e^{-1}(2-1)$

$= \frac{3}{e}$

> 0

$\therefore$ min

$$\frac{d^2y}{dx^2} = 3e^{x}(1+3x) + e^{3x} \cdot 3$$
$$= 3e^{3x}(2+3x)$$

for P.O.I $\frac{d^2y}{dx^2} = 0$

$\Rightarrow x = -\frac{2}{3}.$

$\therefore y = \frac{-2}{3}e^{-2}$

$= -\frac{2}{3e^2}$

$\therefore \left(-\frac{2}{3}; -\frac{2}{3e^2}\right)$

and

x intercept: $(0;0).$

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Question 4 **(3 marks)**

Determine the equation of the normal to the curve $y = x(3-x)^2$ at (2,2).

$$\frac{dy}{dx} = 1(3-x)^2 + x(-2)(3-x)$$

$$\therefore \frac{dy}{dx} = (3-x)^2 - 2x(3-x)$$

3

$$\left.\frac{dy}{dx}\right|_{x=2} = (3-2)^2 - 2(2)(3-2)$$
$$= 1-4$$
$$= -3.$$

$\therefore y = -3x + C$

tangent $m = -3$

$\therefore$ normal $m = \frac{1}{3}$.

$y = \frac{1}{3}x + C$

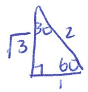
$2 = \frac{1}{3}(2) + C \quad \therefore C = \frac{4}{3}$

$$\therefore y = \frac{1}{3}x + \frac{4}{3}.$$

Question 5 **(5 marks)**

Find the equation of the tangent to the curve $y = 2x + cos2x$ at the point $(\frac{\pi}{3}; \frac{2\pi}{3} - \frac{1}{2})$

$$\therefore \frac{dy}{dx} = 2 - 2\sin 2x.$$

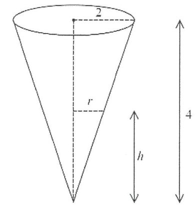


$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{3}} = 2 - 2\sin\frac{2\pi}{3}$$
$$= 2 - 2\frac{\sqrt{3}}{2}$$
$$= 2-\sqrt{3} = m.$$

$\therefore y = mx + C$

Question 9 **(5 marks)**

A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m.



(a) Proof that the volume of the tank is given by the following formula: $V(h) = \frac{1}{12}\pi h^3$ (1 mark)

$V = \frac{1}{3}\pi r^2 h.$ and $r = \frac{h}{2}.$

$= \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 \times h$

$V = \frac{1}{12}\pi h^3$

1

(b) If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.
Answer to the nearest cm/min. (4 marks)

$\therefore \frac{dV}{dh} = \frac{3}{12}\pi h^2$

$= \frac{1}{4}\pi h^2 = \frac{\pi h^2}{4}$

$\frac{dV}{dt} = 2m^3/min$

$\frac{dh}{dt} = ?$ $h = 3m.$

4

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$
$$= \frac{4}{\pi h^2} \times 2$$
$$= \frac{8}{\pi h^2}$$

$$\frac{dh}{dt} = \frac{8}{\pi(3)^2} = 0{,}28\,m/min \text{ (or) } 28cm/min$$

Question 8 **(4 marks)**

Use derivatives to find the approximate change in the radius of a spherical balloon corresponding to a change in its volume from 200 cm^3 to 195 cm^3. Answer to 4 decimal places.

$V = \frac{4}{3}\pi r^3$ $\qquad \delta V = -5.$

4

$\frac{dV}{dr} = 4\pi r^2$

for incremental changes: $\frac{\delta V}{\delta r} \approx \frac{dV}{dr}$

$V = \frac{4}{3}\pi r^3$

$200 = \frac{4}{3}\pi r^3$

$r^3 = \frac{150}{\pi}$

$r^2 = \left(\frac{150}{\pi}\right)^{\frac{2}{3}}$

$\frac{\delta V}{\delta r} \approx 4\pi r^2 \quad r^2 = \left(\frac{150}{\pi}\right)^{\frac{2}{3}}.$

$\frac{\delta r}{\delta V} \approx \frac{1}{4\pi r^2}.$

$\delta r \approx \frac{1}{4\pi r^2} \times \delta V$

$\approx \frac{1}{4\pi\left(\frac{150}{\pi}\right)^{\frac{2}{3}}} \times -5.$

$\delta r \approx \underline{-0.0302}$ cm.

(Q5)

$\therefore \frac{2\pi}{3} - \frac{1}{2} = (2-\sqrt{3})\frac{\pi}{3} + C$

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$\cancel{\frac{2\pi}{3}} - \frac{1}{2} - \cancel{\frac{2\pi}{3}} + \frac{\sqrt{3}\pi}{3} = C$

$\therefore C = \frac{\sqrt{3}\pi}{3} - \frac{1}{2}.$

$\therefore y = (2-\sqrt{3})x + \frac{\sqrt{3}\pi}{3} - \frac{1}{2}.$

Math Methods Unit 3 Test 2 2017 Differentiation

Name Sol key.

Resource Assumed **Time: 25 minutes** **Marks: / 23**

CAS calculator and a formula sheet are allowed for this section

Question 6 **(5 marks)**

A cylindrical can is to be made to hold 1 000 cm^3 of oil. Find the dimensions that will minimise the amount of the metal to make the can. Assume the can is made with a lid.

$V = \pi r^2 h$ $SA = 2\pi r^2 + 2\pi r h$.

$\therefore 1000 = \pi r^2 h$

$\therefore h = \dfrac{1000}{\pi r^2}$ $\therefore SA = 2\pi r^2 + 2\pi r\left(\dfrac{1000}{\pi r^2}\right)$

$\therefore SA = 2\pi r^2 + \dfrac{2000}{r}$.

for min $SA' = 0$.

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$SA' = 4\pi r - \dfrac{2000}{r^2}$

for min : $4\pi r - \dfrac{2000}{r^2} = 0$

$\therefore r = \sqrt[3]{\dfrac{500}{\pi}}$

$= 5.42$ cm. (calculator)

and $h = \dfrac{1000}{\pi r^2} = \dfrac{1000}{\pi (5.42)^2}$

$= 10.84$ cm.

Question 7 **(9 marks)**

The cost in dollars of producing x items is given by: $C(x) = (3000 + 5x)$.

The revenue per item sold is given by $\$(40 - 0.02x)$.

(a) State the revenue function $R(x)$ for the number of x items sold. (1 mark)

$R(x) = x(40 - 0.02x)$ 1

(b) Give an expression for the profit function $P(x)$. (1 mark) 1

$P(x) = R(x) - C(x)$

$= x(40 - 0.02x) - (3000 + 5x)$

$\therefore P(x) = -0.02x^2 + 35x - 3000$.

(c) Determine how many items are needed to make a maximum profit and state the maximum profit. (3 marks)

for max $P'(x) = 0$. 3

$\therefore P'(x) = -0.04x + 35 = 0$

$\therefore x = 875$.

$\therefore$ Max Profit $= -0.02(875)^2 + 35(875) - 3000$

$= \$12\,312.50$

(d) Explain clearly if a loss occurred and when it occurred. (2 marks)

a loss occured when $P(x) < 0$. 2

(solve$(-0.02x^2 + 35x - 3000 < 0, x)$).

$\therefore 0 \le x \le 90$ or $x \geq 1660$.

(e) Determine the marginal profit of the 250th item sold. (2 marks)

$\therefore$ 249 are sold already.

$\therefore P'(x) = -0.04x + 35$

$P'(249) = \$25.04$ 2