

Papers written by
Australian Maths
Software

SEMESTER TWO

YEAR 12

MATHEMATICS SPECIALIST

UNIT 3-4

REVISION TWO

2016

**Section One
(Calculator-free)**

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work:

5 minutes

Working time for section:

50 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: nil.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section One.

A formula sheet which may also be used for Section Two.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	8	8	50	52	35
Section Two Calculator—assumed	12	12	100	98	65
Total marks				150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answer in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

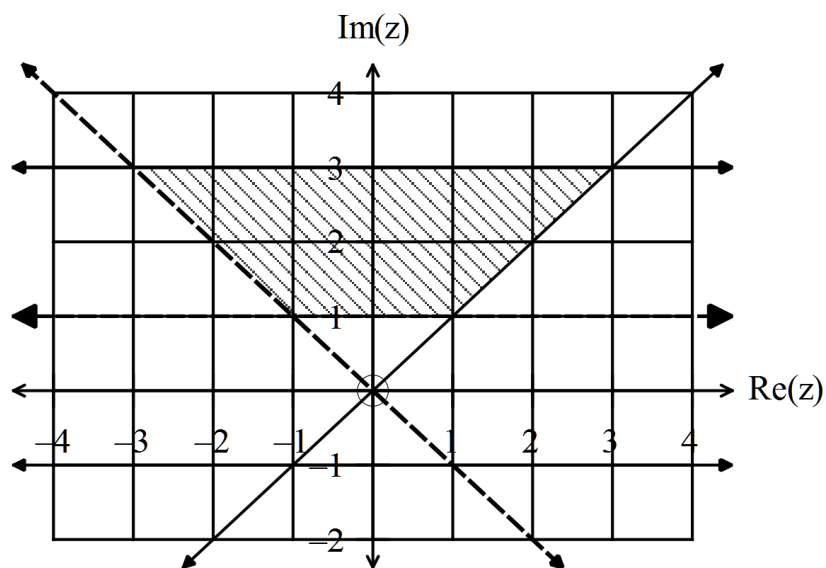
1. (6 marks)

(a) If $z = \frac{1+i}{1-i}$ find

(i) $1 + \frac{1}{z}$ (2)

(ii) $\frac{\bar{z}}{|z|^2}$ (2)

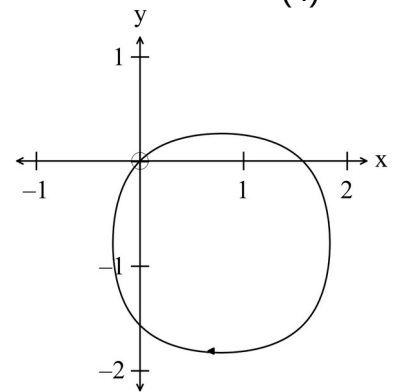
(b) Define the shaded region graphed on the set of axes below. (2)



2. (6 marks)

- (a) Find the expression for $\frac{dy}{dx}$ given the relationship $\sin(x - y) + \cos(x + y) = 1$

(4)



- (b) Hence find $\frac{dy}{dx}$ at the point $\left(\frac{\pi}{2}, 0\right)$.

(2)

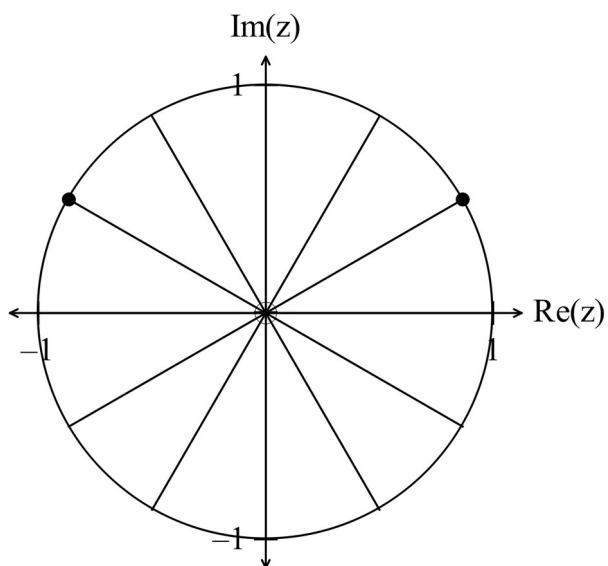
3. (10 marks)

(a) Determine $\int \tan(x) dx$ put $u = \cos(x)$ (3)(b) Determine $\int \frac{\ln(x) dx}{x}$ put $u = \ln(x)$ (3)(c) Evaluate $\int_0^{\pi} \sin(3x) \times e^{\cos(3x)} dx$ put $u = \cos(3x)$ (4)

4. (3 marks)

Two of the six roots of an equation are sketched below.
Find the equation and the other roots.

(3)



5. (6 marks)

- (a) Prove that the equations $z^3 - 2z^2 - 3z + 10 = 0$ and $z^3 - 7z^2 + 17z - 15 = 0$ have two common roots. Determine the common roots. (4)

- (b) Determine the equation that has the roots $z = i$, $z = 2 - i$ and $z = 3 + i$. (2)

6. (6 marks)

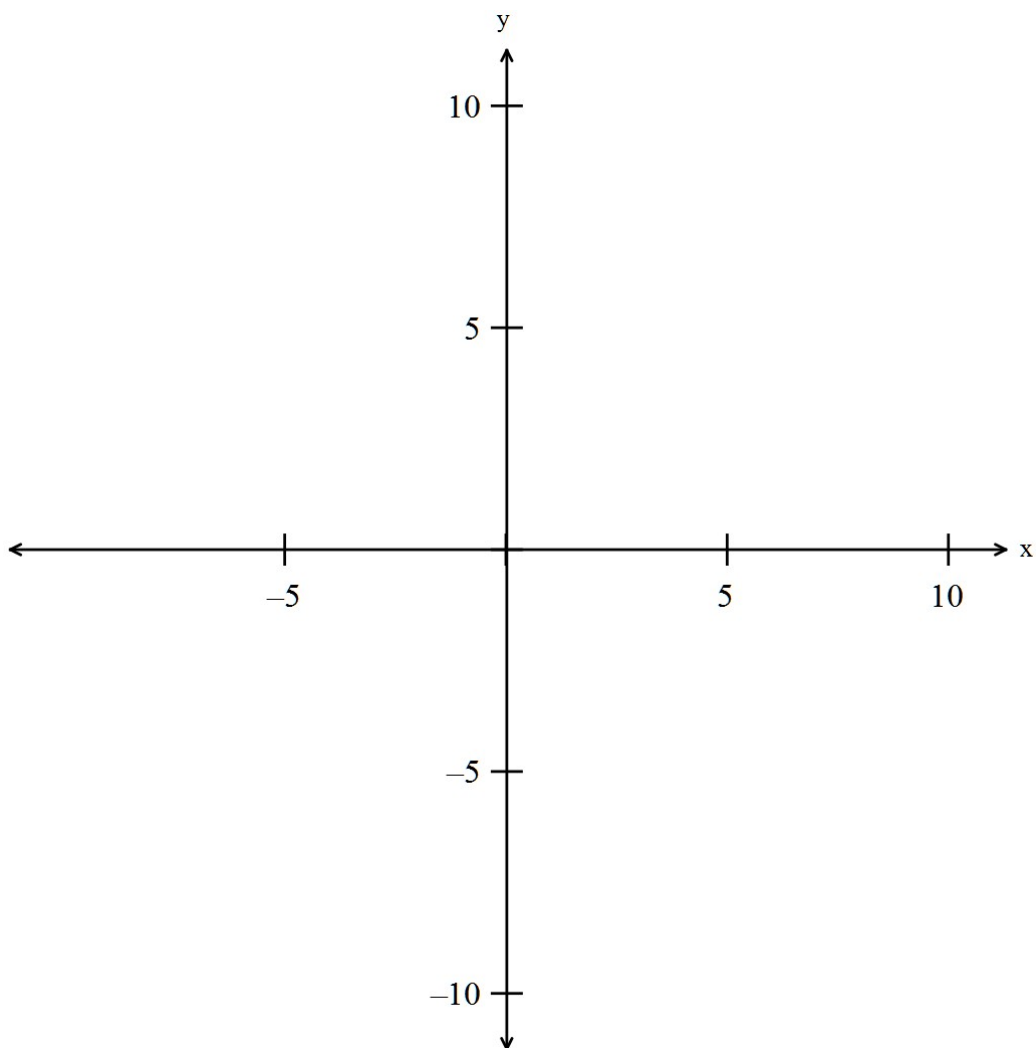
(a) The function $f(x) = \frac{2x^2 - 2x + 1}{(x - 1)} = \frac{1}{x - 1} + 2x$.

Consider the limit of the function as $x \rightarrow 0$ and as $x \rightarrow \pm\infty$.

$$\lim_{x \rightarrow 0} \left(\frac{1}{x - 1} + 2x \right) \rightarrow \frac{1}{x - 1}$$

$$\lim_{x \rightarrow \pm\infty} \left(\frac{1}{x - 1} + 2x \right) \rightarrow 2x$$

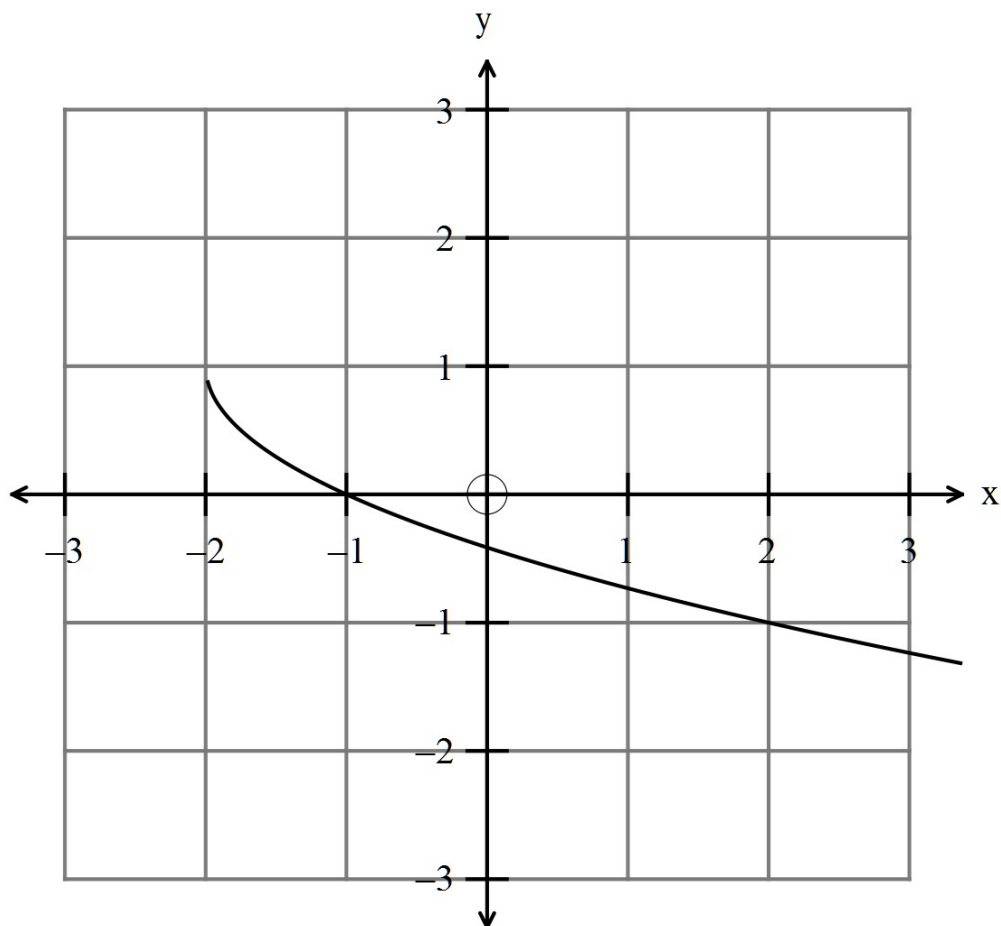
Use these limits to sketch the graph of the function $f(x) = \frac{2x^2 - 2x + 1}{(x - 1)}$. (4)



(b) The function $g(x) = 1 - \sqrt{x+2}$ is sketched on the set of axes below.

Sketch $y = \frac{1}{g(x)}$ on the set of axes below.

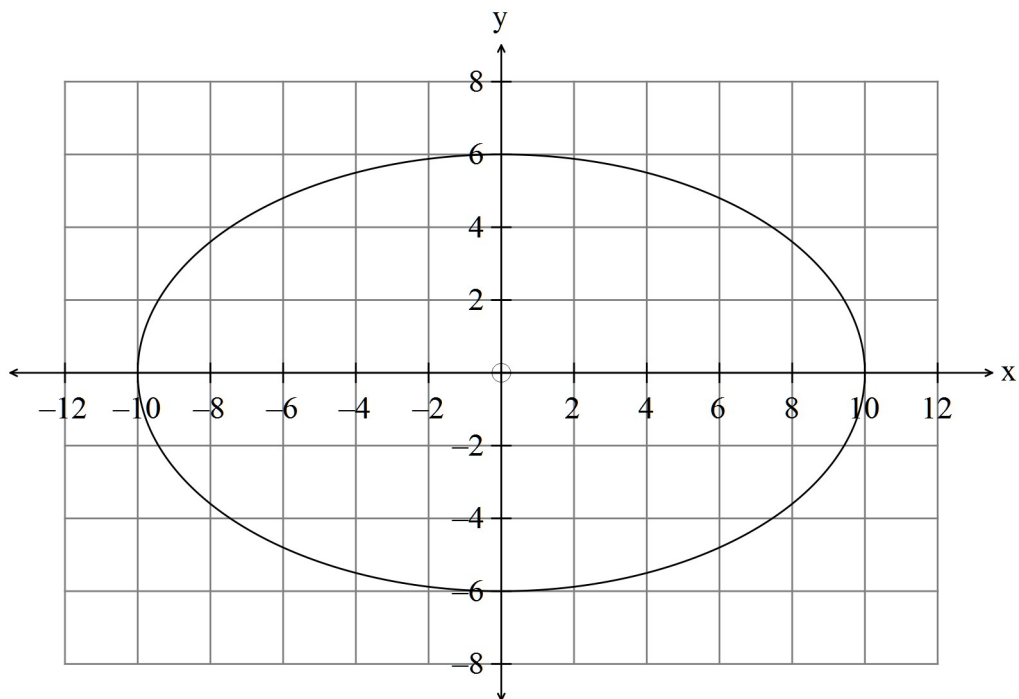
(2)



7. (11 marks)

The position vector of a particle travelling in an ellipse as shown on the graph below is given by $\mathbf{r}(t) = (10\cos(t))\mathbf{i} + (6\sin(t))\mathbf{j}$ for any time t .

Measurements are in metres and time in seconds.



(a) Find an expression for the speed of the particle. (2)

(b) Use the expression found in (i) to determine the maximum and minimum speeds of the particle. (2)

- (c) Determine where on the particle the maximum and minimum speeds occur. (2)

- (d) Find the acceleration at $t = \frac{\pi}{4}$ and sketch the acceleration vector on the diagram at $t = \frac{\pi}{4}$. (3)

- (e) Find the Cartesian equation of the path of the particle. (2)

8 (4 marks)

Determine $\int \frac{-3dx}{(x-2)(x+1)}$

HINT: Use partial fractions.

(4)

END OF SECTION ONE