

Popular sports clothing company Gallopint<sup>TM</sup> manufacture the red-and-white striped jumpers sold to fans of the local Floreat Fuchsiaphobes rugby team. They need to pay particular attention to the quality of the red dye used, so the dye is mixed using an automated mixing machine. The mass of powder added to 1L of solvent is normally distributed with a mean of 0.4g and standard deviation of 0.02g

If the amount of red powder is more than 1.2 standard deviations below the mean, the colour looks pink, which considered abhorrent by the extremely prejudiced Fuchsiaphobe fans. On the other hand, if the concentration of the red dye is more than 0.43g/L, the red dye leeches out in the wash, causing the white stripes to be stained pink, which also upsets the fans. What is the probability that...

- a) A randomly selected jumper will look pink before it is washed. (1)

- b) A randomly selected jumper that has been washed does not look pink. (2)

- c) A randomly selected jumper will look pink after it has been washed, given that the dye concentration was at least 0.37g/L. (3)



SHENTON  
COLLEGE

Test 3

Calculator Assumed

ATMAM Mathematics Methods

Name: .....

Teacher: Friday Smith

Time Allowed : 25 minutes

Marks

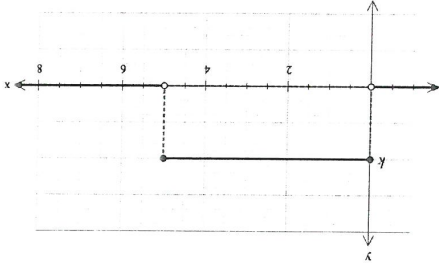
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Materials allowed: Caspad, Formula Sheet.

All necessary working and reasoning must be shown for full marks.

Where appropriate, values should be given to two decimal places, except for probabilities which should be given to four decimal places.  
Marks may not be awarded for untidy or poorly arranged work.

A uniform random variable can take any value between 0 and 5, with a PDF as shown in the graph below.



- a) Determine the value of k. (1)

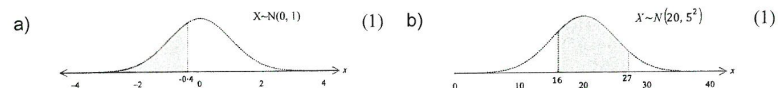
- b) Determine  $P(3 < x \leq 4.5)$  (1)

- c) Determine  $P(3 \leq x < 10)$  (1)

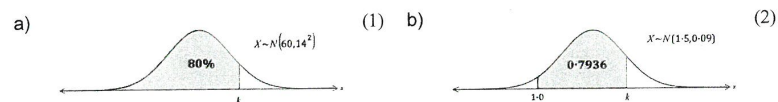
d) Determine  $P(x < 4 \mid x > 1.8)$  (2)

e) Determine the value  $t$  such that  $P(x < t \mid 3 < x) = \frac{3}{8}$  (3)

2 Determine the probability indicated by the shaded region on the following normal probability distributions.



3 Determine the value of  $k$  in for the normal distributions shown below.



4 A normally distributed random variable has 15% of its values above 70 and 40% of its values below 50. (5)

By calculating the standard  $z$  scores associated with the tail probabilities above, then writing equations for their equivalent  $z$  scores using the given boundaries, determine the values of the mean and standard deviation to one decimal place.

5 Fabrics treated with water proofing spray are able to repel a proportion of incidental water such as rain. The proportion of water absorbed by a fabric which has been protected by  $x$  coats of water proofing spray can be modelled by the equation  $W = W_0 e^{kx}$ , where  $W_0$  is the water absorbed by an unprotected sample of the fabric.

a) If each coat of a particular brand of spray reduces the water absorption to 60% of its previous value, find  
(i) The value of  $k$  for that brand, correct to four decimal places. (1)

(ii) Determine the minimum number of coats to reduce the absorption to less than  $0.01W_0$ . Show use of natural logarithms in your working. (3)