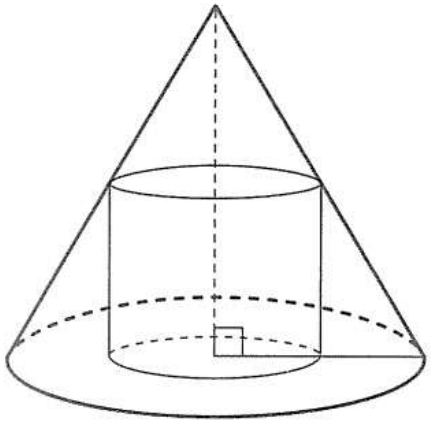
10. (5 marks)

A right circular cone has a radius of 18 cm and a height of 12 cm. Determine the volume of the largest cylinder which will fit inside the cone.



TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 1 2017

Section 1 Calculator Free

Differentiation, Applications of Differentiation, Anti Differentiation

STUDENT'S NAME _____

DATE: Thursday 2 March **TIME:** 33 minutes **MARKS**: 33

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Given $y = x + \sqrt{x^2 - 4}$, show that $(x-4)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

2. (5 marks)

Use calculus to determine the % error in the volume of a spherical hot air balloon of diameter 32 metres if no allowance was made for the stretching of the material resulting in a 3% error in the diameter.

3. (10 marks)

Determine each of the following.

(a) $\int \frac{2x - x^5}{3x^4} dx$ [3]

(b) $\int \frac{2}{\sqrt{1-2x}} dx$ [3]

(c) $\int_{-1}^{2} (x-2)^2 dx$ [4]

9. (4 marks)

Determine an expression for $f(x)$ if $f'(x) = x^2 + x + k$ for all x and $f(0) = -2$ and $f(-1) = 0$

7. (4 marks)

The duration of one vibration of a pendulum of length l is given by $t = \pi\sqrt{\frac{l}{1.1}}$, where t is measured in seconds and l is measured in centimetres. Given that a pendulum of length 97.8 cm vibrates once a second, use calculus to determine the approximate change in time of one vibration if the pendulum is lengthened to a metre.

8. (4 marks)

During the course of an epidemic, the proportion of the population infected t months after the Epidemic began is given by $p = \frac{t^2}{5(1+t^2)^2}$.

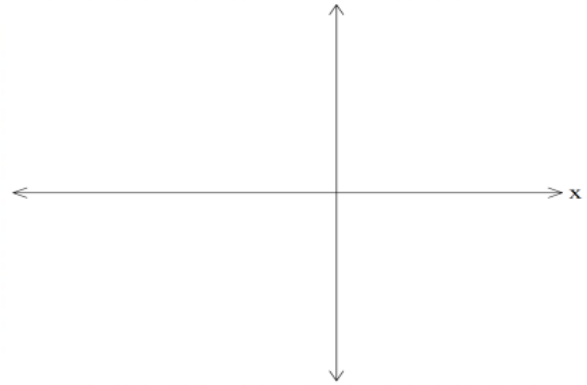
(a) Determine the maximum proportion of the population that becomes infected. [2]

(b) Determine the time at which the proportion infected is increasing most rapidly. [2]

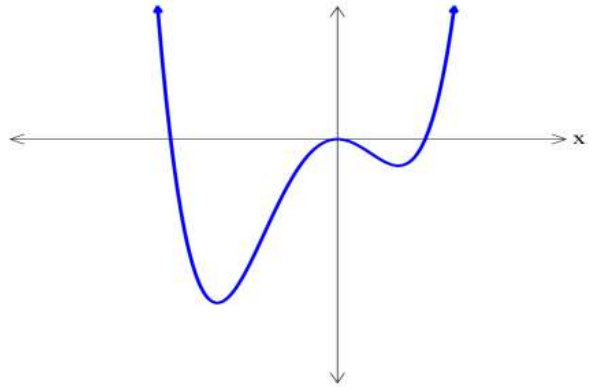
4. (6 marks)

Given the sketch of $y = f'(x)$, sketch $y = f(x)$ and $y = f''(x)$ below.

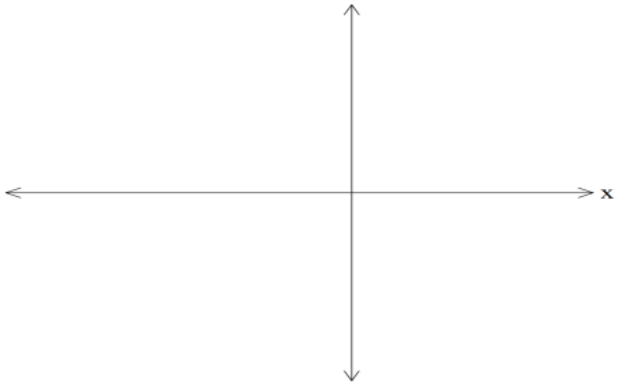
(a) $y = f(x)$ [3]



$y = f'(x)$



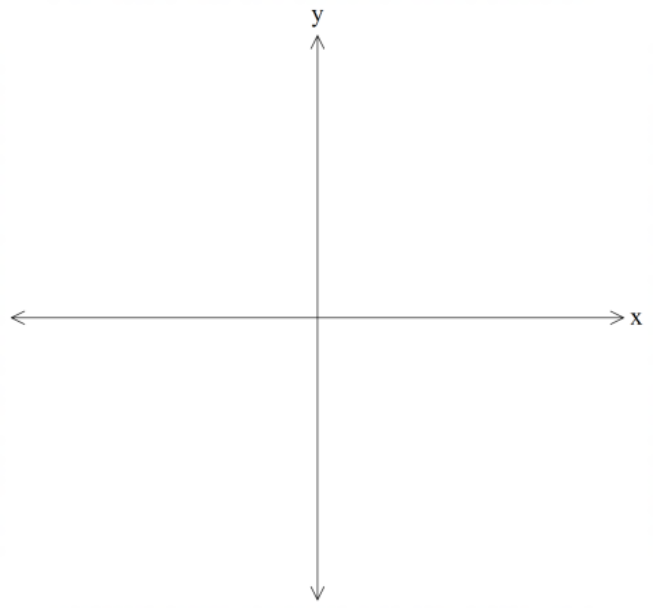
(b) $y = f''(x)$ [3]



5. (6 marks)

By determining each of the following
- Stationary points
- Points of inflection
- Axis intercepts
- Values of y for $x \to \pm\infty$

sketch $y = -x^3 - 3x^2 + 4$ on the axes below.



TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 1 2017

Section 2 Calculator Assumed

Differentiation, Applications of Differentiation, Anti Differentiation

STUDENT'S NAME _____

DATE: Thursday 2 March **TIME:** 21 minutes **MARKS**: 21

INSTRUCTIONS:

Standard Items:	Pens, pencils, drawing templates, eraser
Special Items:	Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

6. (4 marks)

The point $(2, b)$ lies on $y = \dfrac{a + 4x}{3x + 5}$ and the gradient at that point is 8. Determine a and b.