

Important note to Candidates
it to the supervisor before reading any further.
No other items may be taken into the examination room. It is your responsibility to ensure that
you do not have any unauthorised material. If you have any unauthorised material with you, hand
it to the supervisor before reading any further.

Special items: and up to three calculators approved for use in this examination
drawing instruments, templates, notes on two unruled sheets of A4 paper,

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters,
To be provided by the candidate

Materials required/recommended for this section
Formula sheet (referred from Section One)
This Question/Answer booklet
To be provided by the supervisor

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Teacher name (circle one): Ali Friday Smith

Your name: SALUTON

Calculator-assumed
Section Two:
UNIT 3
METHODS
MATHEMATICS

Question/Answer booklet

Semester One Examination, 2019

SHENTON
C 0 1 1 E 6 E



Supplementary page
Question number: _____

CALCULATOR-ASSUMED

METHODS UNIT 3

20

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available |
|---------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|
| Section One: Calculator-free | 8 | 8 | 50 | 51 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 96 |

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

| Markers use only | | |
|------------------|---------|------|
| Question | Maximum | Mark |
| 9 | 5 | |
| 10 | 4 | |
| 11 | 5 | |
| 12 | 7 | |
| 13 | 4 | |
| 14 | 8 | |
| 15 | 5 | |
| 16 | 5 | |
| 17 | 6 | |
| 18 | 9 | |
| 19 | 7 | |
| 20 | 7 | |
| 21 | 7 | |
| 22 | 7 | |
| 23 | 8 | |
| S2 Total | 96 | |

See next page

Question 23

(8 marks)

An aquarium, with a volume of $80\ 000 \text{ cm}^3$, takes the shape of a rectangular prism with square ends of side $x \text{ cm}$ and no top. The glass for the base costs 0.05 cents per square cm and for the four vertical sides costs 0.08 cents per square cm. The cost of glue to join the edges of two adjacent pieces of glass is 0.6 cents per cm. Assume the glass has negligible thickness and ignore any other costs.

- (a) Show that $C = \frac{x^2}{625} + \frac{9x}{250} + \frac{168}{x} + \frac{960}{x^2}$, where C is the cost, in dollars, to make the aquarium. (4 marks)

$$\begin{aligned} x^2y &= 80\ 000 \\ y &= \frac{80\ 000}{x^2} \end{aligned}$$

✓ expression
for other side (y)

$$\begin{aligned} \text{Cost} &= 0.05xy + 0.08(2x^2) + 0.08(2xy) + 6x(0.6) + 2y(0.6) \\ (\text{C}) &= 0.05\left(\frac{80\ 000}{x}\right) + 0.16x^2 + 0.08\left(\frac{160\ 000}{x}\right) + 3.6x + \frac{96\ 000}{x^2} \end{aligned}$$

✓ Cost of
Glass
Shown

$$= \frac{4000}{x} + \frac{4}{25}x^2 + \frac{12800}{x} + \frac{18}{5}x + \frac{96\ 000}{x^2}$$

✓ Cost of
edges
shown

$$\text{Cost} = \frac{960}{x^2} + \frac{168}{x} + \frac{9x}{250} + \frac{x^2}{625}$$

(not
simplified
OK)

✓ Shows cost in \$.
clear How they get this.

- (b) Show use of a calculus method to determine the minimum cost of making the aquarium. (4 marks)

$$\frac{dc}{dx} = \frac{4x^4 + 45x^3 - 210\ 000x - 2\ 400\ 000}{1250x^3}$$

✓ Shows
 $\frac{dc}{dx}$

$$\frac{dc}{dx} = 0 \quad \text{when } x = 37.49 \text{ cm}$$

✓ $\frac{dc}{dx} = 0$
 $x =$

$$\frac{d^2c}{dx^2} \Big|_{x=37.49} > 0 \quad \therefore \text{minimum cost at } x = 37.49 \text{ cm}$$

✓ checks
 $\frac{d^2c}{dx^2} > 0$
 $x = 37.49$

$$C(37.49) = \$8.76$$

End of Examination

See next page

✓ Min. cost
to nearest cent

Question 10

(4 marks)

Given that $\int_2^6 \frac{f(x)}{3} dx = 4$,

(a) evaluate $\int_2^6 f(x) dx$

$$= 3 \int_2^6 \frac{f(x)}{3} dx$$

$$= 3(4)$$

$$= 12$$

(1 mark)

✓ determines
correct $\int_2^6 f(x) dx$

(b) evaluate $\int_2^6 \frac{3f(x)-1}{2} dx$

(3 marks)

$$= \frac{3}{2} \int_2^6 f(x) dx - \int_2^6 \frac{1}{2} dx$$

✓ shows use
of linearity
and additivity

$$= \frac{3}{2}(12) - \left[\frac{1}{2}x \right]_2^6$$

✓ antiderivatives
 $\frac{1}{2}$

$$= \frac{3}{2}(12) - (3-1)$$

$$= 16$$

✓ determines
correct integral

Suppose it is known that 66% of all seeds planted will germinate and that seeds are now planted in rows of 16.

(c) Assuming that seeds germinate independently of each other, determine

(i) the most likely number of seeds to germinate in a row. (1 mark)

$$N=16 \quad p=0.66$$

11 seeds (from GRAPH)

✓ correct
number
of
seeds

(ii) the probability that at least 9 seeds germinate in a randomly chosen row. (2 marks)

$$X \sim B(16, 0.66)$$

$$P(X \geq 9) = 0.8609$$

✓ states
distribution

✓ correct
probability
stated

(iii) the probability that in eight randomly chosen rows, exactly six rows have at least 9 seeds germinating in them. (2 marks)

$$X \sim B(8, 0.8609)$$

$$P(X=6) = 0.2206$$

✓ correct
parameters

✓ correct
probability

Question 12X is a uniform discrete random variable where $x = \{2, 3, 5, 7, 11, 13\}$.

(a) Determine

(i) $P(X \geq 5)$.

$$\begin{aligned} P(X \geq 5) \\ = \frac{4}{6} \\ = \frac{2}{3} \end{aligned}$$

(7 marks)

(1 mark)

✓ determines probability

(ii) $P(X < 12 | X \geq 3)$.

$$\begin{aligned} P(X < 12 | X \geq 3) &= \frac{P(X=3) + P(X=5) + P(X=7) + P(X=11)}{P(X \geq 3)} \\ &= \frac{\frac{1}{6}}{\frac{5}{6}} \\ &= \frac{1}{5} \end{aligned}$$

✓ $P(X \geq 3)$
determined

✓ determines probability

(b) Calculate the value of

(i) $E(X)$.

$$\begin{aligned} E(X) &= \frac{1}{6}[2+3+5+7+11+13] \\ &= \frac{41}{6} \quad (6.83) \end{aligned}$$

(2 marks)

✓ expression
for $E(X)$

✓ $E(X)$

(ii) $\text{Var}(X)$.

$$\begin{aligned} \sigma_x^2 &= \frac{1}{6}(4+9+25+49+121+169) - \left(\frac{41}{6}\right)^2 \\ &= \frac{377}{6} - \frac{1681}{36} \\ &= \frac{581}{36} \\ &= 16.138 \end{aligned}$$

✓ expression
for σ_x^2

✓ $\text{Var}(X)$

Question 20

(7 marks)

(a) For each of the situations below, decide whether the answer could be obtained using a binomial distribution, a Bernoulli distribution, a discrete uniform distribution or none of those distributions.

(i) A discrete random variable, X is such that $P(X=x)=k$ for $x = \{5, 10, 15, 20\}$

Discrete Uniform distribution

✓ correct responses

(ii) It is known that 40 students out of a group of 120 students are international. Two students, both international, are selected without replacement. Assume the selection is random.

None of the distributions

✓

(iii) A fair eight-sided die is rolled 100 times and the probability of obtaining more than 6 odd scores is calculated.

Binomial distribution

✓

(iv) In a very large population of students, 23% are known to be international students. 25 students are selected and the probability that exactly 9 students are international is calculated. Assume the selection is random.

Binomial distribution

✓ (1mark)

(b) Which of the following functions, $f(x)$ could represent a discrete probability function for the random variable, X ? Justify your answer.

(i) $f(x) = \frac{x}{6}$, where $x = -1, 1, 2, 4$.

| | | | | |
|--------|----------------|---------------|---------------|---------------|
| x | -1 | 1 | 2 | 4 |
| $f(x)$ | $-\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{2}{6}$ | $\frac{4}{6}$ |

Not a probability function
 $f(-1) < 0$

✓ No WITH reason

| | | | | |
|--------|------|------|------|-----|
| x | 4 | 6 | 8 | 10 |
| $f(x)$ | 0.05 | 0.30 | 0.25 | 0.4 |

(2 marks)

Probability Function
 $f(x) \geq 0$ for all values of x .
 $\sum f(x) = 1$

✓ Yes with 1 reason
✓ 2 reasons supplied

Concave down.
Which is true
Value of $f''(x)$ is
less than 0.

$$f''(x) = 12x^2 - 6x$$

$$at x = 0 \text{ and } x = \frac{1}{2}$$

$$f''(x) < 0$$

$$0 < x < \frac{1}{2} \quad f''(x) > 0$$

$$x < 0 \quad f''(x) < 0$$

$$\therefore \text{Concave down } 0 < x < \frac{1}{2}$$

From which, points of inflection
are $x = 0$ and $x = \frac{1}{2}$.

$$f(x) = x^4 - x^3$$

$$f'(x) = 4x^3 - 3x^2$$

$$f''(x) = 12x^2 - 6x$$

$$f''(x) = 0 \quad \text{for points of inflection}$$

$$0 = 12x^2 - 6x$$

$$0 = x(12x - 6)$$

$$0 = x \quad \text{or} \quad 12x - 6 = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{1}{2}$$

$$\therefore \text{High point of inflection}$$

Use your calculator to sketch the graph of $f(x) = x^4 - x^3$. Use calculus to justify the type and location of any points of inflection and with justification state any intervals where the curve is concave down.

Question 13

Shows sum
of differences
of areas.

$$(iv) \int_0^5 xf(x) dx + \int_0^5 xp(x) dx$$

$$= (0-0) + (0-5)f =$$

$$= (16-5) + (16-16)f =$$

$$= 11$$

Shows sum
of differences
of areas.

$$(2 \text{ marks})$$

$$\int_{-5}^0 f(x) dx =$$

$$= \int_{-5}^0 xf(x) dx$$

Shows sum
of differences
of areas.

$$(2 \text{ marks})$$

$$\int_{-3}^2 xf(x) dx - \int_{-3}^2 f(x) dx$$

$$= -39 + 18 - [4x]_{-3}^2$$

$$= -39 + 18 - 4x$$

$$= -21$$

Shows sum
of differences
of areas.

$$(2 \text{ marks})$$

$$\int_{-5}^2 f(x) dx =$$

$$= -27 - 39 + 18$$

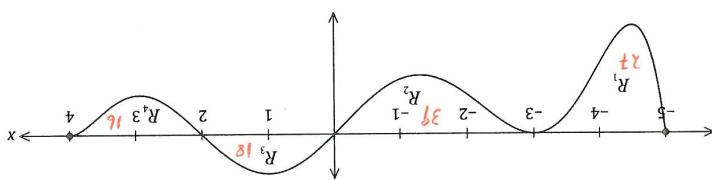
$$= -64$$

Shows sum
of differences
of areas.

$$(1 \text{ mark})$$

$$\int_4^2 f(x) dx = -16$$

The area trapped between the x -axis and the curve for regions R_1, R_2, R_3 and R_4 are 27, 39, 18 and 16 square units respectively.



The graph of $y = f(x)$ is shown below for $-5 \leq x \leq 4$.

Question 19

Question 14

(8 marks)

An insurance company offers a 'death and disability' policy that pays \$50 000 if you die or \$10 000 if you are permanently disabled. It makes no other payouts. The company charges a premium (the cost for this policy) of \$1000 per year for this benefit. Ignore all other costs incurred by the company.

The death rate per year is estimated to be 1 in every 100 people and another 2 out of every 100 people suffer a permanent disability.

Let the random variable $\$X$ denote the amount of profit earned in a year by the insurance company from a typical policy.

- (a) Complete the table below

| Outcome | Death | Permanent Disability | No payout |
|-------------------------|-----------|----------------------|-----------|
| Profit earned (x) | -\$49 000 | -\$9 000 | \$1000 |
| Probability ($X = x$) | 0.01 | 0.02 | 0.97 |

(3 marks)

- (b) How much profit can the insurance company expect to make, on average, from each policy that it sells?

$$\begin{aligned} E(X) &= -49000(0.01) + (-9000)(0.02) + 1000(0.97) \\ &= \$300 \end{aligned}$$

✓ uses
E(X)
formula
correctly
✓ calculates
E(X)

- (c) Determine the standard deviation of the company's yearly profit from a typical policy.

(3 marks)

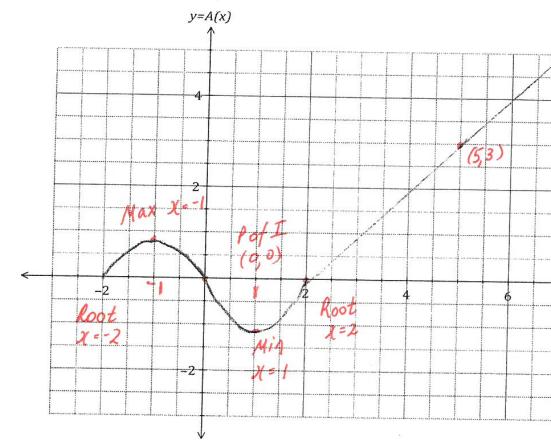
$$\begin{aligned} \sigma_X^2 &= (-49000)^2(0.01) + (-9000)^2(0.02) + (1000)^2(0.97) \\ &\quad - (300)^2 \\ &= 26,510,000 \end{aligned}$$

✓ variance
correct
✓ st. dev.
determined

See next page

- (c) Sketch the graph of $y = A(x)$ on the axes below, indicating and labelling the location of all key features.

(5 marks)

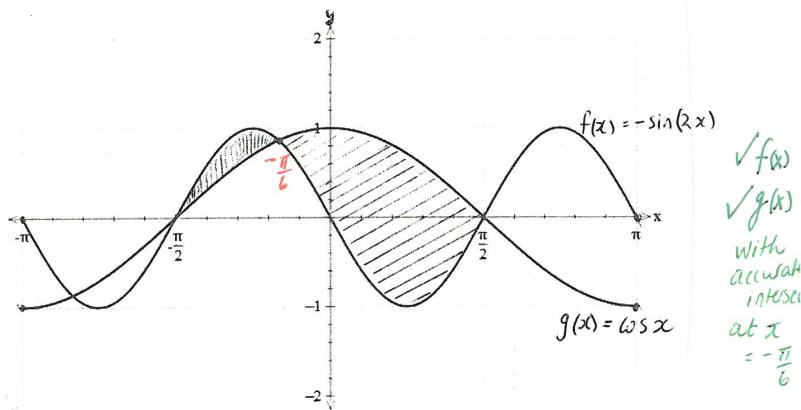


✓ labels p. of I
✓ labels roots
✓ $-2 < x < 0$
Max labelled
✓ $0 < x < 2$
Min labelled
✓ line indicated
going close to (5, 3)

See next page

Question 16Let $f(x) = -\sin(2x)$ and $g(x) = \cos x$

- (a) Sketch the graph of
- $f(x)$
- and
- $g(x)$
- $-\pi \leq x \leq \pi$
- on the axes below. (2 marks)



- (b) Without using absolute values, write an expression to determine the area of the region enclosed by the curves between
- $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
- . (2 marks)

$$A = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} -\sin(2x) - \cos x \, dx + \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x + \sin(2x) \, dx$$

$\checkmark \int$ with correct bounds

$\checkmark \int$ with correct bounds

- (c) Calculate the area of the region in (b). (1 mark)

$$A = \left[\frac{1}{2} \cos(2x) + \sin x \right]_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} + \left[\sin x + \frac{1}{2} \cos(2x) \right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$= 2.5$ square units

\checkmark area determined

Question 17

A local producer grows and sells figs. The costs involved amount to \$50 plus 80c per fig.

The producer estimates that if she charges \$ r for each fig, she will sell n figs where

$$r = 3.5 - 0.01n$$

- (a) If \$
- P
- is the total profit from selling
- n
- figs, show that
- $P = 2.7n - 0.01n^2 - 50$
- . (2 marks)

$$\text{Cost} = 50 + 0.8n$$

$$\text{Revenue} = (3.5 - 0.01n)n$$

$$\begin{aligned}\text{Profit} &= 3.5n - 0.01n^2 - 50 - 0.8n \\ &= 2.7n - 0.01n^2 - 50\end{aligned}$$

\checkmark provides cost and revenue expressions
 \checkmark shows profit =

- (b) If the producer charges \$2.50 per fig, how many will she sell
- and
- what will the marginal profit be? (3 marks)

$$3.5 - 0.01n = 2.5$$

$$n = 100$$

Sells 100 figs.

$$P = 2.7n - 0.01n^2 - 50$$

$$\frac{dP}{dn} = 2.7 - 0.02n$$

$$\frac{dP}{dn}|_{n=100} = 0.7$$

$$\text{Marginal Profit} = \$0.70$$

\checkmark no of figs sold determined
 \checkmark uses df/dn for Marginal Profit

\checkmark determines marginal profit

- (c) Explain what the marginal profit means in this context. (1 mark)

The profit received when selling one extra fig after n figs have already been sold

\checkmark suitable explanation