9. [2, 2, 5 = 9 marks]
A cuboid has a total surface area of 150 cm² with a square base of side length x cm.

(a) Show that the height, h cm, of the cuboid is given by $h = \frac{75-x^2}{2x}$.

$SA = 2x^2 + 4xh = 150$ ✓

$4xh = 150 - 2x^2$

$h = \frac{150 - 2x^2}{4x}$

$h = \frac{75 - x^2}{2x}$ ✓

(b) Express the volume of the cuboid in terms of x.

$V = x^2 h$

$= x^2 \cdot \frac{(75-x^2)}{2x}$ ✓

$V = \frac{75}{2}x - \frac{x^3}{2}$ ✓

(c) Hence, use calculus to determine its maximum volume as x varies.

$V' = \frac{75}{2} - \frac{3x^2}{2}$ ✓

$V' = 0, \quad \frac{3x^2}{2} = \frac{75}{2}$

$3x^2 = 75$

$x^2 = 25$

$x = \pm 5$ ✓, reject $x = -5$

$V'' = -3x$

$V''(5) < 0 \quad \therefore$ Rel max at $x = 5$ ✓

$V(5) = \frac{75}{2}(5) - \frac{(5)^3}{2}$ ✓

$= 187.5 - 62.5$

$= 125$

$\therefore$ Max volume is 125 cm³ ✓

End of Section Two

APPLECROSS
SENIOR HIGH SCHOOL

Year 12 Mathematics **TEST 0**
METHODS UNIT 3 TERM 4, 2018
Test Date: Thursday, 23 November

Name: Solutions

All working is to be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily so part marks may be awarded if the answer is incorrect. For any question worth more than 2 marks valid working or justification must be shown to be awarded full marks.

		Total	
Section 1		20	%
Section 2		40	
Total		60	

SECTION 1 – Resource Free Working Time: 20 minutes

1. [3, 3 = 6 marks]
Find $\frac{dy}{dx}$ for each of the following using the best method, simplifying the answers.

(a) $y = (2x-1)^2(6-3x^2)$

$\frac{dy}{dx} = 2(2x-1).2.(6-3x^2) + (2x-1)^2(-6x)$ ✓✓

$= (8x-4)(6-3x^2) + (4x^2-4x+1)(-6x)$

$= 48x - 24x^3 - 24 + 12x^2 - 24x^3 + 24x^2 - 6x.$

$= -48x^3 + 36x^2 + 42x - 24$ ✓

(b) $y = \frac{5-4x}{7x+3}$

$\frac{dy}{dx} = \frac{(-4)(7x+3) - (5-4x).7}{(7x+3)^2}$ ✓✓

$= \frac{-28x - 12 - 35 + 28x}{(7x+3)^2}$

$= \frac{-47}{(7x+3)^2}$ ✓

2. [3 marks]

The function $y = x^3 + ax + b$ has a local minimum point at (2, 3).
Use <u>differentiation</u> to find the values of a and b.

$y' = 3x^2 + a$ ✓

$y'(2) = 12 + a = 0$

$a = -12$ ✓

$y = x^3 - 12x + b$

$(2,3) \Rightarrow \quad 3 = 8 - 24 + b$

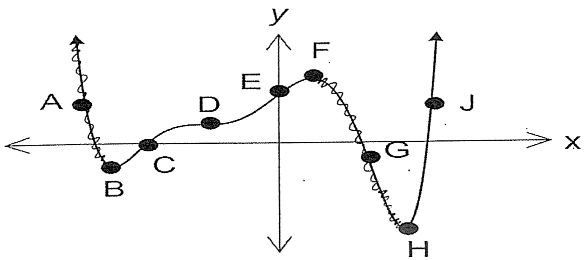
$3 + 16 = b$

$b = 19$ ✓

$\therefore \quad a = -12 \ \& \ b = 19$

3. [1, 1, 1, 1 = 4 marks]

Consider the graph of the function y = f(x). Use the features of this graph to answer the following questions.



(a) List all stationary points. B, D, F, H ✓

(b) State the points of inflection. C, D, E, G ✓

(c) Highlight the sections with a negative value of $\frac{dy}{dx}$. ✓

(d) Which point on this curve has the properties that f(x) > 0 and $f''(x) < 0$?

F ✓

8. [1, 2, 1, 1, 3, 2 = 10 marks]

A bullet is fired upwards. After t seconds the height of the bullet is found from the rule
$H(t) = 150t - 4.9t^2 + 2$ where t is measured in seconds and H in metres.

(a) Find the height of the bullet after 5 seconds.

$H(5) = 750 - 4.9(25) + 2$

$= 629.5 \text{ m}$ ✓

(b) Determine the average speed of the bullet during the fifth second.
Indicate your method.

$\frac{H(5) - H(4)}{5 - 4} = \frac{629.5 - [600 - (4.9)16 + 2]}{1}$ ✓

$= 629.5 - 523.6$

$= 105.9 \text{ m/s}$ ✓

The speed of the bullet is the instantaneous rate of change of the height of the bullet.

(c) Find a rule for the speed of the bullet at any time t.

$H'(t) = 150 - 9.8t$ ✓

(d) Find the speed of the bullet after 5 seconds.

$H'(5) = 150 - 9.8(5)$

$= 101 \text{ m/s}$ ✓

(e) Find the maximum height of the bullet, to the nearest metre.
Indicate your method.

$H'(t) = 0, \quad 150 = 9.8t$

$t \approx 15.31 \text{ s}$ ✓

$H''(t) = -9.8$

$H''(15.31) < 0 \quad \therefore$ Rel max at $t = 15.31 \text{ s}$ ✓

$H(15.31) = 150(15.31) - 4.9(15.31)^2 + 2$

≈ 1149.96

$= 1150 \text{ m}$ ✓

(f) Determine the bullet's speed as it hits the ground, on the way down, to 2 decimal places.

$H(t) = 0, \quad 150t - 4.9t^2 + 2 = 0$ ✓

$t = -0.01 \ \& \ 30.62557243$

$\therefore H'(30.62\ldots) = -150.13 \text{ m/s}$

$\therefore$ Speed of the bullet is 150.13 m/s ✓

7. [2, 4, 3, 2 = 10 marks]

(a) The curve $y = (x+2)(x^2 - 11x + 37)$ cuts the x-axis at (-2, 0).
Is this the only place the curve cuts the x-axis? Justify your answer.

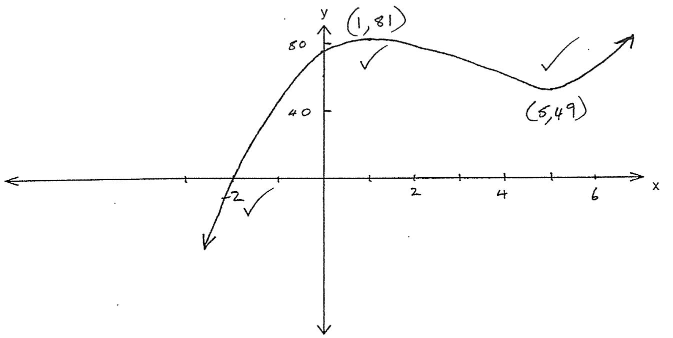
$x^2 - 11x + 37 = 0$ + $x + 2 = 0$
No real solution $x = -2$
$\therefore$ curve cuts the x-axis at only $(-2, 0)$

(b) Find the coordinates and nature of any stationary points on the curve.

$y' = 1(x^2 - 11x + 37) + (x+2)(2x - 11)$
$= x^2 - 11x + 37 + 2x^2 - 11x + 4x - 22$
$= 3x^2 - 18x + 15$
$y' = 0,\quad 3(x^2 - 6x + 5) = 0$
$3(x-5)(x-1) = 0$
$x = 1, 5$

$y'' = 6x - 18$
$y''(1) < 0 \quad \therefore$ Rel max at $(1, 81)$
$y''(5) > 0 \quad \therefore$ Rel min at $(5, 49)$

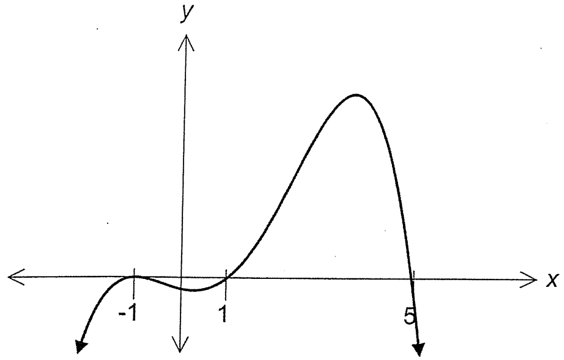
(c) Hence sketch the curve indicating **clearly** the intercepts with the axes and the coordinates of all stationary points.



(d) Determine the greatest and least values of $(x+2)(x^2 - 11x + 37)$ for values of x in the interval $-2 \le x \le 8$.

$x = -2$ gives the least value of 0.
$x = 8$ gives the greatest value of 130.

4. [7 marks]
The graph below shows the graph of $y = f'(x)$ for a function $y = f(x)$.
Find the values of x for which the graph of $y = f(x)$ has a stationary point and state the nature of each stationary point.



Stationary points at $x = -1, 1, 5$

$x < -1,\ f'(x) < 0$
$x > -1,\ f'(x) < 0$ $\therefore$ Horizontal point of inflection at $x = -1$.

$x < 1,\ f'(x) < 0$
$x > 1,\ f'(x) > 0$ $\therefore$ Rel. min at $x = 1$.

$x < 5,\ f'(x) > 0$
$x > 5,\ f'(x) < 0$ Rel max at $x = 5$

✓✓✓ Showing a test.
✓✓✓ Correct interpretation

End of Section One

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Year 12 Mathematics — TEST 0
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____ 40

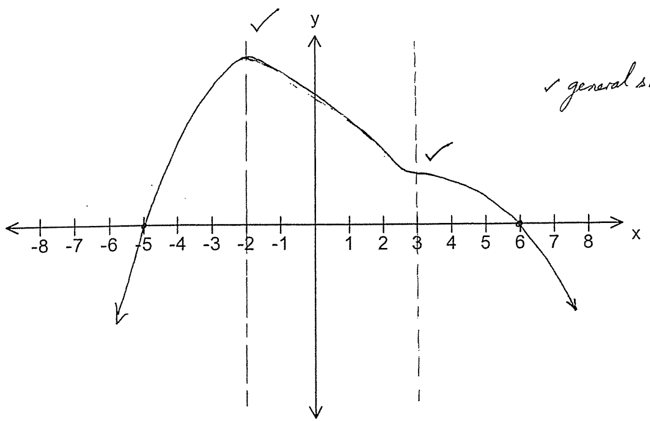
SECTION 2 – Resource Rich — Working Time: 40 minutes

5. [3 marks]

On the axes below, sketch a possible graph satisfying all cases:

- **the function has roots at -5 and 6**
- **there are stationary points at x = -2 and at x = 3**
- **for x <-2 the gradient is positive**
- **for -2 < x < 3 and for x > 3 the gradient is negative**

✓ ✓



✓ general shape.

6. [5, 2 = 7 marks]
Consider the curve whose equation is $y = (4x^2 - 1)^5$.

(a) Use **calculus methods** to determine the nature and location of all stationary points.

$\frac{dy}{dx} = 5(4x^2-1)^4 . (8x)$ ✓

$\frac{dy}{dx} = 0, \quad 40x(4x^2-1)^4 = 0$

$x = 0$ or $4x^2 = 1$

$x^2 = \frac{1}{4}$

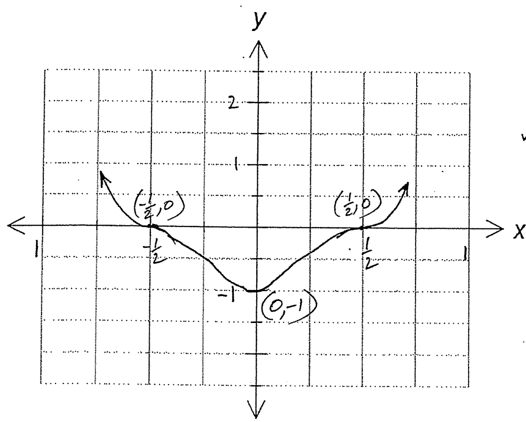
$x = \pm\frac{1}{2}$ ✓

$x < -\frac{1}{2}, \ \frac{dy}{dx} < 0$
$x > -\frac{1}{2}, \ \frac{dy}{dx} < 0$ $\quad\therefore$ Horizontal point of inflection at $(-\frac{1}{2}, 0)$ ✓

$x < 0, \ \frac{dy}{dx} < 0$
$x > 0, \ \frac{dy}{dx} > 0$ $\quad \therefore$ Rel min at $(0, -1)$ ✓

$x < \frac{1}{2}, \ \frac{dy}{dx} > 0$
$x > \frac{1}{2}, \ \frac{dy}{dx} > 0$ $\quad\therefore$ Horizontal point of inflection at $(\frac{1}{2}, 0)$ ✓

(b) Hence, draw a neat sketch of the curve of the function on the set of axes below. Label the significant points with their coordinates.



✓✓ accuracy.