Insert School Logo

Semester Two Examination 2019
Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 & 4

Section One: Calculator–free

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula Sheet

To be provided by the candidate
Standard items: pens(blue/black preferred), pencils(including coloured), sharpener, correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	**9**	**9**	**50**	**52**	**35**
Section Two Calculator—assumed	15	15	100	98	65
					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2019.* Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

   Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

   It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section One: Calculator–free 52 marks

This section has **nine (9)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 50 minutes

Question 1 (4 marks)

Given the following: $\int_0^2 g(x)\,dx = a$

(a) Determine $\int_2^0 3 - 4g(x)\,dx$. (2 marks)

(b) Evaluate a if $g(x) = 2xe^{\ln x}$. (2 marks)

Question 2 (6 marks)

$f(x) = \ln(1 + \sin x)$ where $-\pi < x < \pi$

(a) Determine the coordinates of the stationary point(s) of the graph of the function. (3 marks)

(b) Evaluate $f''(x)$ and hence state the nature of the stationary point(s) found in (a). (3 marks)

Question 3 (3 marks)

The cubic function $h(x) = ax^3 + bx^2 + cx + d$ has a point of inflection at $x = m$.

Show that $m = -\frac{b}{3a}$. (3 marks)

Question 4 (8 marks)

(a) Solve the following for x

(i) $(\ln x)^2 + \ln x^2 - 3 = 0$ (3 marks)

(ii) $\log_2(3x - 4) = 5$ (2 marks)

(b) Find the equation of the tangent to the curve $g(x) = \ln\left(\frac{5}{x^2}\right)$ at the point $x = e$. (3 marks)

Question 5 (8 marks)

(a) If Y is a random variable such that Pr (Y > 7) = *a* and Pr (Y > 10) = *b*, then determine

Pr (X < 7| X < 10) in terms of *a* and *b*. (2 marks)

(b) A 95% confidence interval for the population proportion, *p*, for the number of televisions per Australian household is (1.15, 4.20).

For each of the following statements about this confidence interval, choose True or False. For the statements that are False, explain why they are False. (6 marks)

(i) The probability that *p* is between 1.15 and 4.20 is 0.95.

(ii) 95% of all Australian households have between 1.15 and 4.20 televisions.

(iii) If 100 intervals were calculated in the same way, we expect 95 of them to contain the population proportion.

(iv) If 100 intervals were calculated in the same way, we expect 100 of them to capture the sample proportion.

See Next Page

See Next Page

Question 6 (9 marks)

The velocity of a particle is given by $v = 2 - 4\cos t$ for $0 \leq t \leq 2\pi$ where v is measured in metres per second and t is measured in seconds.

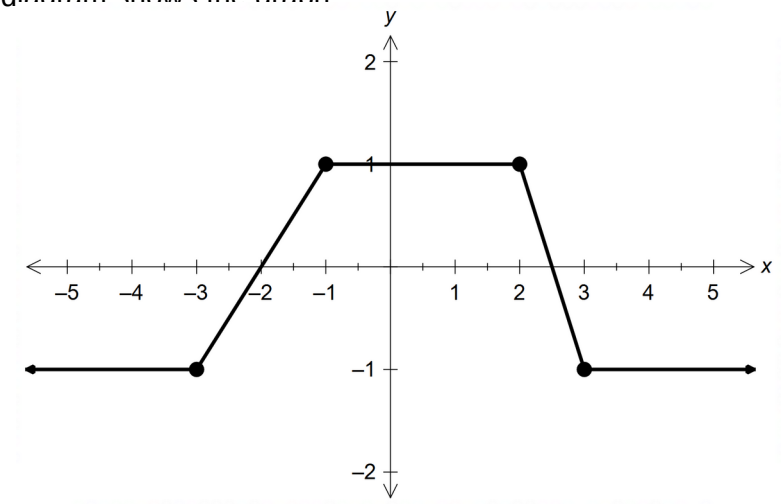
(a) At what times during this period is the particle at rest? (2 marks)

(b) Determine the maximum velocity of the particle during this period. (3 marks)

(c) Calculate the total distance travelled by the particle between $t = 0$ and $t = \pi$. (4 marks)

Question 7 (6 marks)

(a) The diagram shows the graph $y = f(x)$



Determine the value of p, where $p > 0$, so that $\int_{-p}^{p} f(x)\ dx = 0$.
Show your reasoning. (3 marks)

(b) Determine $\int_{0}^{1} \frac{x^2}{x^3 + 1}\ dx$ (3 marks)

Question 8 (4 marks)

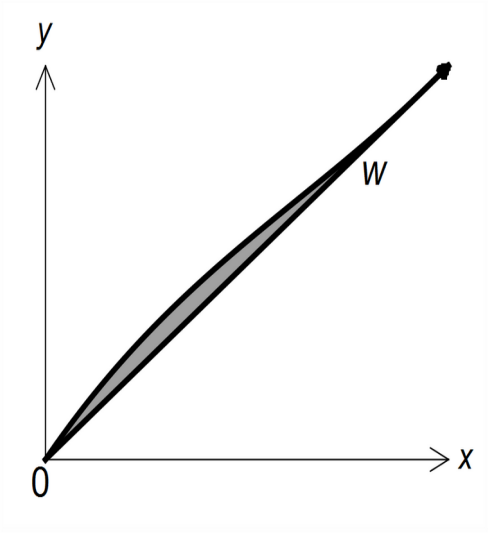
(a) Find b in terms of a for the following: $\log_m b + 2\log_m a = \log_m 5$ (2 marks)

(b) Simplify the following. (2 marks)

$\log_3 45 + \log_7 49$

Question 9 (4 marks)

The graphs of $g(x) = 4x^3 - 4x^2 + 3x$ and $h(x) = 2x$ meet at *O* and at *W.*
Find the area of the shaded region. (4 marks)



End of Section One

Additional working space

Question number(s): ……

Additional working space

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