TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 2   2018

Section 1   Calculator Free
Applications of Calculus

STUDENT'S NAME
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DATE: Thursday 5 April   **TIME:** 30 minutes   **MARKS:** 33

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1.   (4 marks)

Determine the equation of the tangent to the curve $y \sin x = x$ at the point $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

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8.   (6 marks)

A continuous function $f(x)$ is increasing on the interval $0 < x < 3$ and decreasing on the interval $3 < x < 6$. Some of its values are given in the table below.

x	0	1	2	3	4	5	6
$f(x)$	5	16	27	32	25	0	-49

The function $F(x)$ is defined, for $0 \le x \le 6$, by $F(x) = \displaystyle\int_0^x f(t)\,dt$.

(a)   At which value of x in the interval $0 \le x \le 6$ is $F(x)$ greatest? Justify your answer.   [2]

(b)   At which value of x in the interval $0 \le x \le 6$ is $F'(x)$ greatest? Justify your answer.   [2]

(c)   Use the values of $f(x)$ in the table to show that $48 \le F(3) \le 75$.   [2]

2. (9 marks)

(a) Determine each of the following (do not simplify)

(i) $\dfrac{d}{dx}\dfrac{x^2}{e^{\sin 3x}}$

[3]

(ii) $\dfrac{d}{dx}e^{-x}(\sin 2x - \tan 2x)$

[3]

(b) Given $f(x)=\int_x^1 (3-t)^{\frac{5}{2}}\,dt$ determine $f'(-1)$.

[3]

7. (4 marks)

Two of the fission products of an explosion are found to decay according to the laws

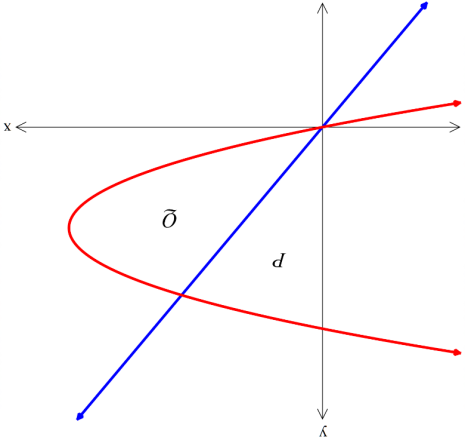$$\dfrac{dM_1}{dt}=-k_1 M_1 \qquad \text{where } e^{-k_1}=\dfrac{1}{4}$$

$$\dfrac{dM_2}{dt}=-k_2 M_2 \qquad \text{where } e^{-k_2}=\dfrac{1}{2}$$

If the initial ratio $\dfrac{M_1}{M_2}=3$ what is the ratio after 6 days?

6. (6 marks)

In the graph shown below, Q is the area enclosed by the graphs of $y = x$ and $x = 6y - y^2$. P is the area bounded by the two graphs and the y-axis

Calculate

(a) the size of area Q [3]

(b) the size of area P [3]

3. (12 marks)

(a) Determine each of the following

(i) $\int \left(e^x + e^{-x}\right)^2 dx$ [3]

(ii) $\int 3e^{1-6x} + e \, dx$ [3]

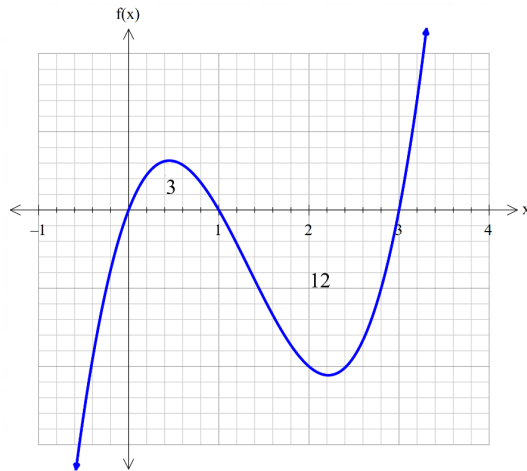(b) (i) determine $\frac{d}{dx} x\cos 2x$ [3]

(ii) use the result of (i) to determine $\int 2x \sin 2x \, dx$ [3]

4.   (8 marks)

The graph of y = f(x) is shown below. The size of the area of the two parts enclosed between the curve and the x-axis is shown on the graph.

Determine

(a) $\int_0^3 f(x)\,dx$ [1]

(b) $\int_0^3 |f(x)|\,dx$ [1]

(c) $\int_1^0 f(x)\,dx$ [2]

(d) $\int_1^3 (2f(x)+3)\,dx$ [4]

Mathematics Methods Units 3,4
Test 2   2018

Section 2   Calculator Assumed
Applications of Calculus

STUDENT'S NAME _____

DATE: Thursday 5 April   TIME: 20 minutes   MARKS: 22

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.
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5.   (6 marks)

Scientists are studying a population of endangered small mammals in a protected environment. They conclude the population is increasing at a rate of given by $B'(t) = 5.2e^{0.4t}$ where t is the number of weeks since the study began.

(a)   What is the change in the population in the fourth week?   [3]

(b)   When the study began there were 500 of these mammals. The study will conclude when the population reaches 2000. When will this occur?   [3]