

MATHEMATICS METHODS

MAWA Semester 2 (Units 3 and 4) Examination 2017

Calculator-free

Marking Key

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The release date for this exam and marking scheme is

- **the end of week 1 of term 4, 2017**

Section One: Calculator-free

(52 Marks)

Question 1(a)

<p>Solution</p> $k + 2k + 0.5k + 0.5k + k = 1$ $5k = 1$ $k = \frac{1}{5}$	
<p>Marking key/mathematical behaviours</p>	<p>Marks</p>
<ul style="list-style-type: none"> correctly solves for k. 	<p>1</p>

Question 1(b)

<p>Solution</p> <p>No, This is not a discrete probability distribution as $P(5)$ has a negative value and probabilities need to be positive.</p>	
<p>Marking key/mathematical behaviours</p>	<p>Marks</p>
<ul style="list-style-type: none"> recognises the properties of discrete probability distributions relates above property back to the question to justify answer 	<p>1 1</p>

Question 1(c)

<p>Solution</p> $\mu = np = 90$ $\therefore \sigma^2 = 6^2 = 36$ $\sigma^2 = np(1-p)$ $36 = 90(1-p) \quad \text{Expand and solve for } p$ $\therefore p = 0.6$ $\frac{6}{10}n = 90 \quad \text{Sub value of } p \text{ and solve for } n.$	
<p>Marking key/mathematical behaviours</p>	<p>Marks</p>
<ul style="list-style-type: none"> determines variance solves for parameter, p solves for parameter, n 	<p>1 1 1</p>

Question 2(a)

Solution A census involves every member of the population being tested/questioned/investigated	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> indicates the need to include every member of the population 	1

Question 2(b)

Solution There would be no stoves left to sell as all of them would have broken down.	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> indicates that there would be no items left for sale (no marks for cheaper or quicker) 	1

Question 2(c)

Solution Use the unique serial numbers to select a random sample or similar	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> indicates use of a suitable random selection method (based on serial numbers or other method) 	1

Question 2(d)

Solution Using the list of the serial numbers, select every 400 th stove	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> indicates use of a suitable selection method 	1

Question 3(a) (i)

Solution $f'(x) = \frac{(2 + \cos x) \times 1 - (-\sin x)x}{(2 + \cos x)^2} = \frac{2 + \cos x + x \sin x}{(2 + \cos x)^2}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> correct use of the quotient rule 	1
<ul style="list-style-type: none"> differentiates correctly 	1

Question 3(a) (ii)

Solution $f'(x) = (3 + 2 \cos 2x)e^{3x + \sin 2x}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> correct use of the chain rule 	1
<ul style="list-style-type: none"> differentiates correctly 	1

Question 3(b)

<p>Solution</p> <p>Differentiating both sides: $f''(x)f(x) + f'(x)^2 = 0$</p> <p>so $f''(x) = -\frac{f'(x)^2}{f(x)}$</p> <p>But $f'(x) = \frac{1}{f(x)}$ and so $f''(x) = -\frac{1}{f(x)^3}$ i.e. $f''(x)f(x)^3 = -1$</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> uses product rule correctly 	1
<ul style="list-style-type: none"> obtains correct expression for $f''(x)$ 	1
<ul style="list-style-type: none"> replaces $f'(x)$ with $\frac{1}{f(x)}$ 	1

Question 4(a)(i)

<p>Solution</p> <p>Approximately 200 samples are involved</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> States the number of samples (allow 190 to 210) 	1

Question 4(a)(ii)

<p>Solution</p> <p>Find the mean of the sample proportions, (from the graph) ≈ 0.4</p> <p>May use sample proportion as an estimate of the population proportion</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> identifies sample proportion as 0.4 (by reference to the graph or calculation) 	1
<ul style="list-style-type: none"> uses the sample proportion as an estimate for the population proportion 	1

Question 4(b)

<p>Solution</p> <ul style="list-style-type: none"> Survey is restricted to listeners of one particular station and therefore not representative of the population Survey is using a self-selection model and this indicates bias Timing may exclude some groups of people Access to a telephone is presumed People could respond more than once Nature of the question means football fans may be more likely to respond 	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> Lists one possibility 	1
<ul style="list-style-type: none"> Lists a second possibility 	1

Question 5

<p>Solution</p> <p>Note that:</p> $y = 1 - 4x^2 \Rightarrow y' = -8x$ $\therefore \int \frac{10x}{1-4x^2} dx = -\frac{10}{8} \int \frac{-8x}{1-4x^2} dx$ $= -\frac{5}{4} \ln 1-4x^2 + c$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> Recognises $\frac{f'(x)}{f(x)}$ 	1
<ul style="list-style-type: none"> Determines (or uses) derivative of denominator 	1
<ul style="list-style-type: none"> Uses constants to achieve numerator of $-8x$ 	1
<ul style="list-style-type: none"> Finds integral accurately 	1

Question 6(a)

<p>Solution</p> <p>Since $5^{-2} = \frac{1}{25}$ we have $\log_5 \left(\frac{x+24y}{25} \right) = \ln_5(x+y) \quad (\#)$</p> <p>i.e. $\frac{x+24y}{25} = x+y$ and hence $y = -24x$</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> Obtains equation (#) or equivalent 	1
<ul style="list-style-type: none"> Obtains correct answer 	1

Question 6(b)(i)

<p>Solution</p> <p>If $a > b$ then $\log_a b < 0$ i.e. negative</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> obtains correct answer 	1

Question 6(b)(ii)

<p>Solution</p> <p>If $a \neq b$ then $\log_a b$ and $\log_b a$ have opposite signs (#) and are unequal.</p> <p>So $a = b$ if $\log_a b = \log_b a$</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> deduces that $\log_a b$ and $\log_b a$ have opposite signs if $a \neq b$ 	1
<ul style="list-style-type: none"> complete proof correctly 	1

Question 7(a)

<p>Solution</p> $A = - \int_1^4 \left(\frac{-8}{\sqrt{x}} \right) dx$ $= \int_1^4 \frac{8}{\sqrt{x}} dx$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> sets up integral in either form 	1

Question 7(b)

<p>Solution</p> $A = \int_1^4 \frac{8}{\sqrt{x}} dx$ $= [16\sqrt{x}]_1^4$ $= 16(2) - 16$ $= 16 \text{ square units}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> integrates correctly 	1
<ul style="list-style-type: none"> correctly evaluates the integral 	1

Question 7(c)

<p>Solution</p> $\int_1^a \frac{-8}{\sqrt{x}} dx = -25$ $\Rightarrow -16\sqrt{a} + 16 = -25$ $-16\sqrt{a} = -41$ $a = \frac{41^2}{16^2}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> correctly states the equation to be solved 	1
<ul style="list-style-type: none"> solves for a 	1

Question 8(a)

<p>Solution</p> $\int_0^c \frac{x^3}{4} dx$ $= \frac{1}{4} \left[\frac{x^4}{4} \right]_0^c$ $\frac{c^4}{16} = 1$ $c^4 = 16$ $c = \pm 2$ <p>As -2 is outside of the lower boundary</p> $c = 2$	
Marking key/mathematical behaviours	Marks
• uses the correct integral	1
• equates the pdf to one and solves for c	1
• Clearly states why $c = -2$ has been rejected	1

Question 8(b)

<p>Solution</p> $E(x) = \int_{-\infty}^{\infty} xw(x) dx$ $= \int_{12}^{32} x \left(\frac{1}{20} \right) dx$ $= \left[\frac{1}{20} \times \frac{x^2}{2} \right]_{12}^{32}$ $= \frac{1}{40} (32^2 - 12^2)$ $= 22$ $\therefore E(x) = 22$	
Marking key/mathematical behaviours	Marks
• Substitutes into the correct formula	1
• Integrates correctly and arrives at the required answer	1

Question 8(c)

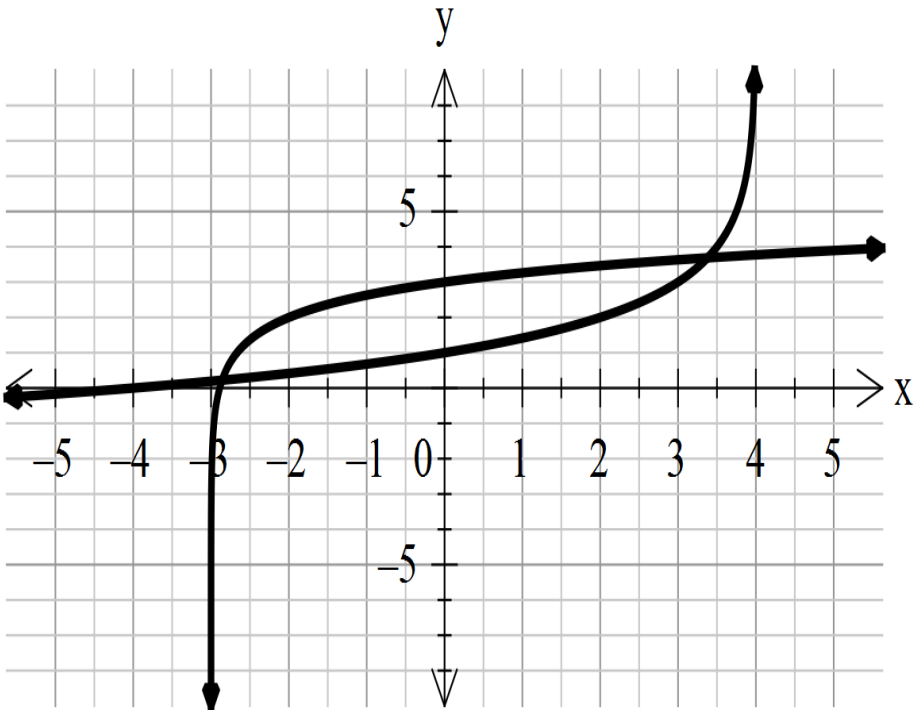
<p>Solution</p> <p>(i) From the graph, read the relative frequencies for the parking times (0,30] mins and (30, 60] mins → Pr (at most 60 minutes) = 0.38 + 0.42 = 0.8.</p> <p>(ii) relative frequency of vehicles parked between 1 and 1.5hrs = 0.14. 200 × 0.14 = 28 vehicles.</p> <p>(iii) Answers will vary. One possible answer: The data shows that 80% of the vehicles are parked for at most 60 minutes. Businesses that would tailor to this time length could be a nearby café or a mini mall with a few necessities stores (ie. Grocery store, bakery, pharmacy, clothing shop).</p> <p>(iv) Answers will vary. One possible answer: On which day of the week was the data collected?</p>	
Marking key/mathematical behaviours	Marks
• sums the two relative frequencies required	1
• multiplies the relative frequency by the total number of parked cars	1
• links to a type of business requiring less than 1 hour of parking	1
• links question to day of the week or other plausible variable.	1

Question 9(a)

<p>Solution</p> <p>Since the graph has a vertical asymptote at $x = -3$, $b = 3$</p> <p>Since the point $(-2,2)$ lies on the graph, $2 = \log_a(-2+3) + c$, i.e. $c = 2$</p> <p>Since the point $(0,3)$ lies on the graph, $3 = \log_a + 2$, i.e. $\log_a(3) = 1$ and hence $a = 3$</p>	
Marking key/mathematical behaviours	Marks
• evaluates a , b and c correctly	1, 1, 1

Question 9(b)

Solution



Marking key/mathematical behaviours	Marks
• asymptote at $x = 4$	1
• y intercept at $(0,1)$	1
• x intercept at $(-4,0)$	1
• graph of standard logarithm function reflected in both axes	1