Your Senior High School

Question/Answer Booklet

Name

_____

MATHEMATICS:SPECIALIST 3AB

Semester One Exam 2010

Section One:
Calculator-free

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for this section: 50 minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Suggested working time (minutes)	Marks available
Section One: Calculator-free	11	11	50	40
Section Two: Calculator-assumed	14	14	100	80
				120

Instructions to candidates

1. Answer the questions according to the following instructions.

   Section One: Write answers in this Question/Answer Booklet. **All** questions should be answered.

   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

   For any question or part question worth more than two marks, valid working or justification is required to receive full marks.

   If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

   It is recommended that you **do not use pencil** except in diagrams.

2. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

MARK ALLOCATION AND RECORDS:

Section	Question	Marks	Awarded
ONE	1	5	
	2	5	
	3	3	
	4	6	
	5	3	
	6	2	
	7	3	
	8	4	
	9	5	
	10	2	
	11	2	
	Penalties	- 1/2/3	
	ONE	**40**	
	TWO	**80**	
TOTAL		**120**	

Penalties	
Rounding (-1)	
Units (-1)	
Notation (-1)	

%

Question 1 [2, 3 marks]

(a) Sketch on the number line, the solution to $|x + 3| < 8$.

(b) Solve for x given that $|2x - 5| = |x + 2|$.

Question 2 [2, 3 marks]

(a) Express the following as A power of 5.

$$\frac{5^6 \div 5^{-6}}{25^2}$$

(b) Determine the value of n.

$(a^3)^n \cdot a^{1+n} = a^{13}$

Question 3 [1, 2 marks]

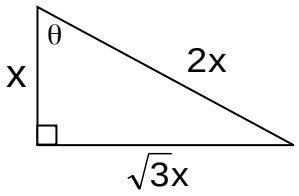
Simplify.

(a) $\log_3 81$

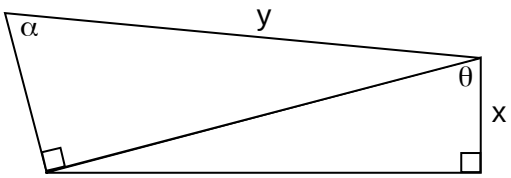
(b) $\log 500 - \log 5 - 2$

Question 4 [2, 4 marks]

(a) Determine the exact value for θ in degrees.



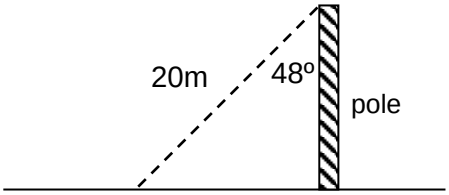
(b) Prove that $\frac{x}{y} = \sin\alpha \cos\theta$



Question 5 [3 marks]

(a) Using any information necessary from the table, determine the height of the vertical pole.

sin 42º = 0·67
sin 48º = 0·74
tan 42º = 0·90



Question 6 [1, 1 marks]

(a) Convert 20° into radians in exact form.

(b) Convert $\frac{7\pi}{12}$ radians into degrees.

Question 7 [3 marks]

A vector has a westerly component of 5 units and a southerly component of 3 units.

If $\tan^{-1}(0{\cdot}6) \approx 31^\circ$, determine the bearing and the exact magnitude of the vector.

Question 8 [2, 2 marks]

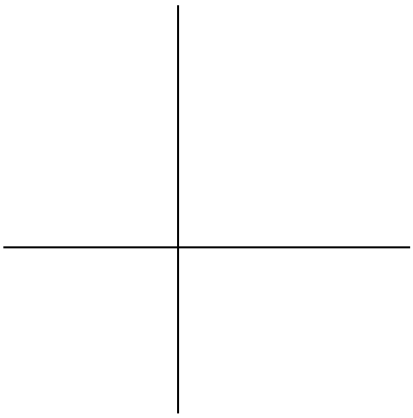
If $\mathbf{a} = 3\mathbf{i} + 5\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j}$ find

(a) $|\mathbf{a} + \mathbf{b}|$

(b) A unit vector in the same direction as $\mathbf{a} - \mathbf{b}$.

Question 9 [5 marks]

Points A and B have position vectors <-1, 10> and <11, 1> respectively. Find the position vector of the point which divides AB internally in the ratio 1:2.



Question 10 [2 marks]

Solve the following equation giving the answer in exact from using logarithms to the base ten.

$7^x = 14$

Question 11 [2 marks]

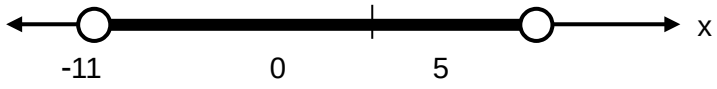
Given that $f(x) = x^2 - x$ and $g(x) = x + 3$, find the value of $g \circ f(-3)$.

MAS 3AB Section One (Calculator free) SOLUTIONS

Question 1 [2, 3 marks]

(a) Sketch on the number line, the solution to $|x + 3| < 8$.

Solution is $-11 < x < 5$.



✓✓

(b) Solve for x given that $|2x - 5| = |x + 2|$.

✓

Let $2x - 5 = x + 2$ or $2x - 5 = -x - 2$

$\therefore x = 7$ ✓ $3x = 3$

$\therefore x = 1$ ✓

Question 2 [2, 3 marks]

(a) Express the following as A power of 5.

$$\frac{5^6 \div 5^{-6}}{25^2} = \frac{5^{12}}{5^4}$$ ✓

$= 5^8$ ✓

(b) Determine the value of n.

$(a^3)^n \cdot a^{1+n} = a^{13}$

$a^{3n} \cdot a^{1+n} = a^{13}$ ✓

$\Rightarrow$ $3n + 1 + n = 13$ ✓

$n = 3$ ✓

Question 3 [1, 2 marks]

Simplify.

(a) $\log_3 81 = \log_3 3^4$

$= 4 \log_3 3$

$= 4$ ✓

(b) $\log 500 - \log 5 - 2$

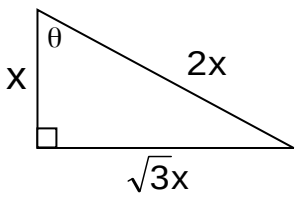
$= \log 100 + \log 5 - \log 5 - 2$ ✓

$= 2 - 2$

$= 0$ ✓

Question 4 [2, 4 marks]

(a) Determine the exact value for θ in degrees.



$$\text{Sin } \theta = \frac{\sqrt{3}x}{2x}$$

i.e. $\text{Sin } \theta = \frac{\sqrt{3}}{2}$ ✓

$\theta = 60^\circ$ ✓

Question 4 [2, 4 marks]

(b) Prove that $\frac{x}{y} = \sin\alpha \cos\theta$

$\frac{x}{h} = \cos\theta$

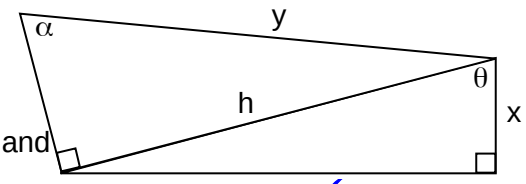
$h = \frac{x}{\cos\theta}$ ------ ①

✓

$\frac{h}{y} = \sin\alpha$

$h = y \,.\, \sin\alpha$ ------ ②

✓



Equating ① and ②

$\frac{x}{\cos\theta} = y \,.\, \sin\alpha$ ✓

$\therefore \quad \frac{x}{y} = \sin\alpha \,.\, \cos\theta$ as required. ✓

Question 5 [3 marks]

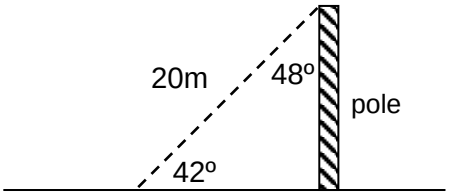
(a) Using any information necessary from the table, determine the height of the vertical pole.

sin 42º = 0·67
sin 48º = 0·74
tan 42º = 0·90

$\frac{p}{20} = \sin 42^\circ$ ✓

$p = 20 \times 0{\cdot}67$ ✓

$\therefore \quad p = 13{\cdot}4$ m ✓



Question 6 [1, 1 marks]

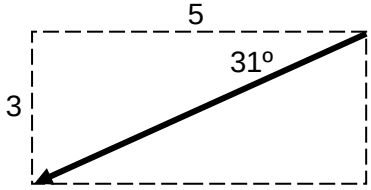
(a) Convert 20º into radians in exact form. $\frac{\pi}{9}$ radians ✓

(b) Convert $\frac{7\pi}{12}$ radians into degrees. 105º ✓

Question 7 [3 marks]

A vector has a westerly component of 5 units and a southerly component of 3 units.

If $\tan^{-1}(0{\cdot}6) \approx 31^\circ$, determine the bearing and the exact magnitude of the vector.



✓

Magnitude = $\sqrt{34}$ ✓

Bearing = 270º - 31º

= 239º ✓

Question 8 [2, 2 marks]

If **a** = 3**i** + 5**j** and **b** = 2**i** - **j** find

(a) $|\mathbf{a} + \mathbf{b}| = |5\mathbf{i} + 4\mathbf{j}|$ ✓

$= \sqrt{41}$ ✓

(b) A unit vector in the same direction as $\mathbf{a} - \mathbf{b}$.

$\mathbf{a} - \mathbf{b} = \mathbf{i} + 6\mathbf{j}$

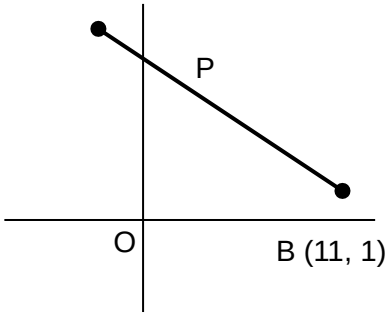
$\therefore \mathbf{u} = \frac{1}{\sqrt{37}}(\mathbf{i} + 6\mathbf{j})$ ✓ ✓

Question 9 [5 marks]

Points A and B have position vectors <-1, 10> and <11, 1> respectively. Find the position vector of the point which divides AB internally in the ratio 1:2.

$\vec{AP} : \vec{PB} = 1 : 2$

A (-1, 10)



$\vec{AB}$ = <11, 1> - <-1, 10>

= <12, -9> ✓

Now $\vec{OP} = \vec{OA} + \vec{AP}$ ✓

= <-1, 10> + $\frac{1}{3}$ <12, -9> ✓ ✓

= <3, 7>

i.e. The position vector for P = $3\mathbf{i} + 7\mathbf{j}$. ✓

Question 10 [2 marks]

Solve the following equation giving the answer in exact from using logarithms to the base ten.

$7^x = 14$ $\quad x \log 7 = \log 14$ ✓

$\therefore \quad x = \frac{\log 14}{\log 7}$ ✓

Question 11 [2 marks]

Given that $f(x) = x^2 - x$ and $g(x) = x + 3$, find the value of g o f(-3).

g o f(x) = $x^2 - x + 3$ ✓

g o f(-3) = $(-3)^2 - (-3) + 3$

= 15 ✓