



Christ Church
Grammar School

MATHEMATICS 3C/3D
Calculator-assumed

Your name _____

Time allowed for this section

Working time for this section:

forty-five (45) minutes

Materials required/recommended for this section

To be provided by the supervisor

Nil

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: up to three calculators approved for use in the WACE examinations

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MATHEMATICS 3C/3D	3	CALCULATOR-ASSUMED
		(45 marks)
Answer all questions.		
Working time: 45 minutes.		
Question 1		
(3 marks)		
<p>By considering the expansion of $(a - b)^2$, prove that the sum of squares of two real numbers is always greater than or equal to twice their product.</p> <p><i>Hint: Let the two real numbers be a and b.</i></p>		

Question 2

(4 marks)

The following pairs of fractions produce the same result if they are added together as when they are multiplied together.

$$\frac{7}{2} \text{ and } \frac{7}{5}$$

$$\frac{11}{4} \text{ and } \frac{11}{7}$$

$$\frac{21}{11} \text{ and } \frac{21}{10}$$

$$\frac{13}{5} \text{ and } \frac{13}{8}$$

$$\frac{19}{7} \text{ and } \frac{19}{12}$$

$$\frac{72}{55} \text{ and } \frac{72}{17}$$

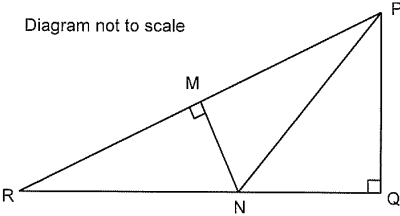
These pairs of fractions are all in the form $\frac{k}{m}$ and $\frac{k}{n}$

- (a) State the relationship that is shown between the numerator k and the denominators m and n . (1 mark)
- (b) For any pair of fractions $\frac{k}{m}$ and $\frac{k}{n}$ where k has this relationship with m and n , prove that $\frac{k}{m} \times \frac{k}{n}$ will produce the same result as $\frac{k}{m} + \frac{k}{n}$ (3 marks)

Question 7

(7 marks)

In the diagram below, PQR is a right-angled triangle with $\angle PQR = 90^\circ$ and M is the midpoint of PR . N is the point where the perpendicular to PR at M meets QR .

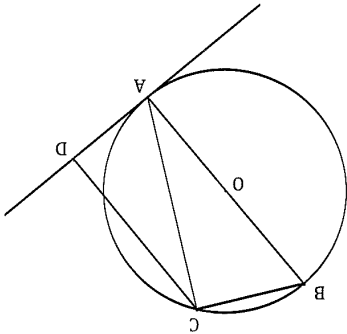


- (a) Prove that $\triangle PNM$ is congruent with $\triangle RNM$. (3 marks)
- (b) If PN bisects $\angle QPR$, prove that $\triangle MPN$ is congruent with $\triangle QPN$. (2 marks)
- (c) By considering the results for (a) and (b), determine the ratio of the area of $\triangle PQN$: area of $\triangle PQR$. (2 marks)

End of questions

Question 6

In the diagram below (not drawn to scale), AOB is the diameter of the circle, AC is a chord of the circle, AD is a tangent to the circle at A and CD is perpendicular to AD .



- (a) Prove that $\triangle ABC$ is similar to $\triangle CAD$. (3 marks)

- (b) Hence show that $AC^2 = AB \times CD$. (1 mark)

- (c) Determine the radius of the circle when $AC = 15$ cm and $AD = 12$ cm. (2 marks)

Question 3

- (a) Write down the values of $m^2 + 7$ for $m = 1, 3, 5, 7$ and 9 . (2 marks)

- (b) By looking at your result for (a), state the largest integer, p , that $m^2 + 7$ is always divisible by, when m is a positive odd integer. (1 mark)

- (c) Prove that $m^2 + 7$ is always divisible by p when m is a positive odd integer. Hint: Let $m = 2k + 1$, where k is an integer. (4 marks)

Question 4

(4 marks)

Consider the following conjecture:

"Every prime number greater than 3 is one more or less than a multiple of 6."

- (a) Show that the conjecture is true for three different prime numbers. (1 mark)

- (b) By considering that any counting number can be written in the form $6n + p$, where n is an integer and p is an integer between 0 and 5, prove the above conjecture.

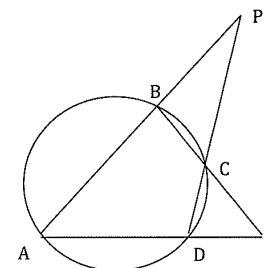
Hint: Consider each of $6n + p$, for $p = 0, 1, 2, 3, 4, 5$. (3 marks)

Question 5

(4 marks)

Consider the diagram below (not drawn to scale) which shows a cyclic quadrilateral ABCD. The sides of the quadrilateral have been extended and these lines meet at the points P and R as shown.

$$\angle ARB = 60^\circ \text{ and } \angle BCP = 40^\circ$$



- (a) Find $\angle ABC$. (2 marks)

- (b) Hence find $\angle BAD$. (2 marks)