

No other items may be taken into the examination room. It is your responsibility to ensure that it to the supervisor before reading any further.
you do not have any unauthorised material. If you have any unauthorised material with you, hand

Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Standard items: fluid/tape, eraser, ruler, highlighters
Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

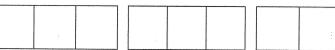
Formula sheet (retained from Section One)

MATERIALS REQUIRED/RECOMMENDED FOR THIS SECTION

Working time: one hundred minutes
Reading time before commencing work: ten minutes

Time allowed for this section

Your name



In words

Student Number: In figures

Calculator-assumed
Section Two:
UNIT 3
METHODS
MATHEMATICS

Question/Answer booklet

Semester One Examination, 2017



SOLUTIONS

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	11	11	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

L

(d) At what rate was the voltage decreasing at the instant it reached 8 volts? (2 marks)

Solution	$V(t) = kV$
Specific behaviours	$= -0.0175 \times 8 = -0.14$
Decreasing at 0.14 volts/s	\checkmark
Uses rate of change	\checkmark

(c) How long did it take for the initial voltage to halve? (2 marks)

Solution	$t = 39.6 \text{ s}$
Specific behaviours	$0.5 = e^{-0.0175t}$
Writes equation	\checkmark
Solves, rounding to 3sf	\checkmark

(b) Determine the value of k . (2 marks)

Solution	$k = -0.0175$
Specific behaviours	$0.6 = 14e^{180k}$
Writes equation	\checkmark
Solves, rounding to 3sf	\checkmark

(a) State the initial voltage between the plates. (1 mark)

Solution	$V_0 = 14 \text{ Volts}$
Specific behaviours	$V_0 = 14 \text{ Volts}$
States value (units not required)	\checkmark
Initial voltage	\checkmark

The voltage between the plates of a discharging capacitor can be modelled by the function $V(t) = 14e^{-kt}$, where V is the voltage in volts, t is the time in seconds and k is a constant. It was observed that after three minutes the voltage between the plates had decreased to 0.6 volts.

Working time: 100 minutes.

This section has eleven (11) questions. Answer all questions. Write your answers in the spaces provided.

Section Two: Calculator-assumed
METHODS UNIT 3
65% (98 Marks)

Question 9 (7 marks)

Working time: 100 minutes.

This section has eleven (11) questions. Answer all questions. Write your answers in the spaces provided.

CALCULATOR-ASSUMED
METHODS UNIT 3
3

Question 10

The gradient function of f is given by $f'(x) = 12x^3 - 24x^2$.

- (a) Show that the graph of $y = f(x)$ has two stationary points.

(11 marks)

(2 marks)

Solution
Require $f'(x) = 0$ $12x^2(x - 2) = 0$ $x = 0, x = 2$ ✓✓
Hence two stationary points
Specific behaviours
✓ equates derivative to zero and factorises ✓ shows two solutions and concludes two stationary points

- (b) Determine the interval(s) for which the graph of the function is concave upward. (3 marks)

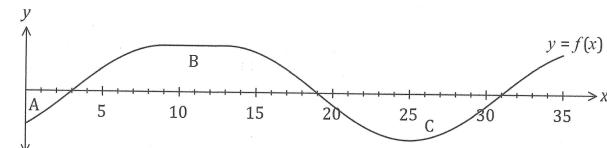
Solution
$f''(x) = 36x^2 - 48x$ $f''(x) > 0 \Rightarrow x < 0, x > \frac{4}{3}$ ✓✓
Specific behaviours
✓ shows condition for concave upwards ✓ uses second derivative ✓ states intervals

- (c) Given that the graph of $y = f(x)$ passes through $(1, 0)$, determine $f(x)$. (2 marks)

Solution
$f(x) = \int f'(x) dx = 3x^4 - 8x^3 + c$ $f(1) = 0 \Rightarrow c = 5$ $f(x) = 3x^4 - 8x^3 + 5$ ✓✓
Specific behaviours
✓ integrates $f'(x)$ ✓ determines constant

Question 19

The graph of $y = f(x)$ is shown below. The areas between the curve and the x -axis for regions A , B and C are 3, 20 and 12 square units respectively.



- (a) Evaluate

$$\text{(i)} \quad \int_0^{31} f(x) dx.$$

Solution
$\int_0^{31} f(x) dx = (-3) + 20 + (-12) = 5$ ✓✓
Specific behaviours
✓ sums signed areas

$$\text{(ii)} \quad \int_{19}^0 f(x) dx.$$

Solution
$\int_{19}^0 f(x) dx = -\int_{19}^{19} f(x) dx = -((-3) + 20) = -17$ ✓✓
Specific behaviours
✓ reverses limits and negates ✓ sums signed areas

$$\text{(iii)} \quad \int_3^{31} 2 - 3f(x) dx.$$

Solution
$\int_3^{31} 2 - 3f(x) dx = \int_3^{31} 2 dx - 3 \int_3^{31} f(x) dx = 56 - 3(8) = 32$ ✓✓
Specific behaviours
✓ splits integral and takes difference ✓ rectangle ✓ function

It is also known that $A(31) = 0$, where $A(x) = \int_{10}^x f(t) dt$.

- (b) Evaluate

$$\text{(i)} \quad A(19).$$

Solution
$A(19) + \int_{19}^{31} f(t) dt = 0 \Rightarrow A(19) = 12$ ✓✓
Specific behaviours
✓ states area of region C

$$\text{(ii)} \quad A(0).$$

Solution
$A(3) = -(20 - 12) = -8$ ✓✓ $A(0) = -8 - (-3) = -5$ ✓✓
Specific behaviours
✓ deduces $A(3)$ ✓ deduces $A(0)$

11
7+4

8

Solution	Solution
See graph	See graph
Specific behaviours	Specific behaviours
minimum	minimum

- ✓ smooth curve
- ✓ points of inflection
- ✓ roots
- ✓ minimum

and

roots

at

and

Question 11

(7 marks)

- (a) Four random variables W , X , Y and Z are defined below. State, with reasons, whether the distribution of the random variable is Bernoulli, binomial, uniform or none of these.
- (4 marks)

The dice referred to is a cube with faces numbered with the integers 1, 2, 3, 4, 5 and 6.

- (i) W is the number of throws of a dice until a six is scored.

Solution
Neither - distribution is geometric
✓ answer with reason

- (ii) X is the score when a dice is thrown.

Solution
Uniform - all outcomes are equally likely
✓ answer with reason

- (iii) Y is the number of odd numbers showing when a dice is thrown.

Solution
Bernoulli - two complementary outcomes
✓ answer with reason

- (iv) Z is the total of the scores when two dice are thrown.

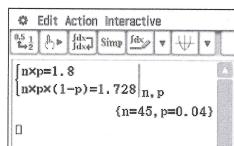
Solution
Neither - distribution is triangular
✓ answer with reason

- (b) Pegs produced by a manufacturer are known to be defective with probability p , independently of each other. The pegs are sold in bags of n for \$4.95. The random variable X is the number of faulty pegs in a bag.

If $E(X) = 1.8$ and $\text{Var}(X) = 1.728$, determine n and p .

(3 marks)

Solution
$np = 1.8, np(1-p) = 1.728$
$\therefore 1-p = \frac{1.728}{1.8} = 0.96$
$p = 0.04$
$n = \frac{1.8}{0.04} = 45$
Specific behaviours
✓ writes equations for mean and variance
✓ solves for p
✓ solves for n



See next page

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7

- (e) Let Y be a Bernoulli random variable with parameter $p = P(A)$. Determine the mean and standard deviation of Y .
- (2 marks)

Solution
Y is a Bernoulli rv, so $\bar{Y} = p = \frac{5}{42} \approx 0.119$ ✓
$\sigma_Y = \sqrt{(p(1-p))} = \sqrt{\frac{5}{42} \times \frac{37}{42}} = \frac{\sqrt{185}}{42} \approx 0.324$ ✓
Specific behaviours
✓ indicates Bernoulli rv and states mean
✓ states sd

- (f) Determine the probability that A occurs no more than twice in ten random selections of four letters from those in the word LOGARITHM.
- (2 marks)

Solution
$W \sim B\left(10, \frac{5}{42}\right)$
$P(W \leq 2) = 0.8933$ ✓
Specific behaviours
✓ indicates binomial distribution with parameters
✓ calculates probability

6 +

4

See next page

SN001-095-4

= 10

Question 13

(9 marks)

75% of the avocados produced by a farm are known to be first grade, the rest being second grade. Trays of 24 avocados are filled at random in a packing shed and sent to market.

Let the random variable X be the number of first grade avocados in a single tray.

- (a) Explain why X is a discrete random variable, and identify its probability distribution.

Solution	
X is a DRV as it can only take integer values from 0 to 24 ✓	
X follows a binomial distribution: $X \sim B(24, 0.75)$ ✓	
Specific behaviours	
✓ explanation using discrete values	
✓ identifies binomial, with parameters	

(2 marks)

- (b) Calculate the mean and standard deviation of X .

Solution	
$\bar{X} = 24 \times 0.75 = 18$ ✓	
$\sigma_x = \sqrt{18 \times 0.25} = \frac{3\sqrt{2}}{2} \approx 2.12$ ✓	
Specific behaviours	
✓ mean, ✓ standard deviation	

(2 marks)

- (c) Determine the probability that a randomly chosen tray contains

- (i) 18 first grade avocados.

Solution	
$P(X = 18) = 0.1853$ ✓	
Specific behaviours	
✓ probability	

(1 mark)

- (ii) more than 15 but less than 20 first grade avocados.

Solution	
$P(16 \leq X \leq 19) = 0.6320$ ✓	
Specific behaviours	
✓ uses correct bounds	
✓ probability	

(2 marks)

- (d) In a random sample of 1000 trays, how many trays are likely to have fewer first grade than second grade avocados.

(2 marks)

Solution	
$P(X \leq 11) = 0.0021$	
$0.0021 \times 1000 \approx 2$ trays ✓	
Specific behaviours	
✓ identifies upper bound and calculates probability	
✓ calculates whole number of trays	

See next page

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9

- (d) Calculate the change of displacement of P during the third second.

(2 marks)

Solution	
$\Delta x = \int_2^3 v dt = \frac{31}{35} \approx 0.886$ m ✓	
Specific behaviours	
✓ uses correct bounds	
✓ integrates to find change in displacement	

- (e) Determine the maximum speed of P during the first three seconds and the time when this occurs.

(2 marks)

Solution	
Observe $ v $ decreases then increases: $ v(0) = 3, v(3) \approx 0.92$	
Hence maximum speed is 3 ms^{-1} . ✓	
Specific behaviours	
✓ examines v at endpoints	
✓ determines maximum speed	

- (f) Calculate the total distance travelled by P during the first three seconds.

(2 marks)

Solution	
$d = \int_0^3 v dt$ or $d = -\int_0^{0.5} v dt + \int_{0.5}^3 v dt$	
$d = \frac{16}{7} \approx 2.286$ m ✓	
Specific behaviours	
✓ uses integral(s) to determine distance	
✓ evaluates distance	

6+

6

≈ 12

See next page

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(1 mark)

Solution	Specific behaviours	Valid suggestion
Solution	Specific behaviours	Use a larger number of thinner rectangles.
(c) Suggest one change to the above procedure to improve the accuracy of the estimate.	Specific behaviours	Use a larger number of thinner rectangles.

(1 mark)

Solution	Specific behaviours	Values 1st col, Values 2nd col, Values 3rd col
Solution	Specific behaviours	Sums
Solution	Specific behaviours	Estimate = $\frac{94.8 + 122.95}{2} \approx 108.9 \text{ m}$
(c) Suggest one change to the above procedure to improve the accuracy of the estimate.	Specific behaviours	Estimate = $\frac{94.8 + 122.95}{2} = 122.95$

(1 mark)

Solution	Specific behaviours	See table (my have slightly different values if using exact values of $v(t)$ rather than those from (a))
Interval	0 - 2.5	2.5 - 5
Inscribed area	30.0	32.3
Circumscribed area	32.3	33.25
(c) Suggest one change to the above procedure to improve the accuracy of the estimate.	Specific behaviours	24.15

(1 mark)

(b)	Specific behaviours	The rectangles for the 7.5 to 10 second interval are shown on the graph.
(b)	Specific behaviours	Complete the following table and hence estimate the distance travelled by the car during the first ten seconds by calculating the mean of the sums of the inscribed areas and the circumscribed areas, using four rectangles of width 2.5 seconds.

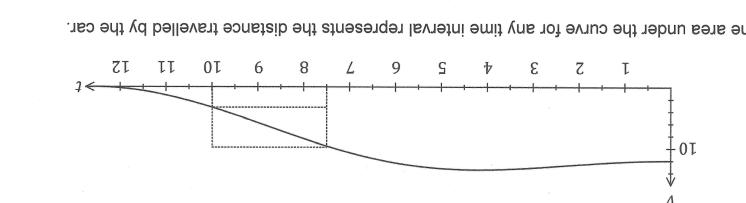
(5 marks)

Solution	Specific behaviours	See table, rounding
t	0	2.5
$v(t)$	12.00	12.92
t	5	7.5
$v(t)$	13.30	9.66
t	10	12.5
$v(t)$	9.66	3.34

(2 marks)

(a)	Specific behaviours	Complete the table below, rounding to two decimal places.
(a)	Specific behaviours	The area under the curve for any time interval represents the distance travelled by the car.

(2 marks)



The speed, in metres per second, of a car approaching a stop sign is shown in the graph below and can be modelled by the equation $v(t) = 6(t + \cos(0.25t) + \sin(0.25t))$, where t represents the time in seconds.

(8 marks)

Question 14

CALCULATOR-ASSUMED

METHODS UNIT 3

12 CALCULATOR-ASSUMED

METHODS UNIT 3

Question 16

CALCULATOR-ASSUMED

METHODS UNIT 3

12 METHODS UNIT 3

METHODS UNIT 3

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CALCULATOR-ASSUMED

Question 15

(10 marks)

A slot machine is programmed to operate at random, making various payouts after patrons pay \$2 and press a start button. The random variable X is the amount of the payout, in dollars, in one play of the machine. Each payout can be assumed to be independent of other payouts.

The probability, P , that the machine makes a certain payout, x , is shown in the table below.

Payout (\$)	0	1	2	5	10	20	50	100
Probability $P(X = x)$	0.25	0.45	0.2125	0.0625	0.0125	0.005	0.005	0.0025

(a) Determine the probability that

- (i) in one play of the machine, a payout of more than \$1 is made. (1 mark)

Solution
$P(X > 1) = 1 - (0.25 + 0.45) = 0.3$ ✓
Specific behaviours

- (ii) in ten plays of the machine, it makes a payout of \$5 no more than once. (2 marks)

Solution
$Y \sim B(10, 0.0625)$ ✓✓
Specific behaviours

✓ indicates binomial distribution
✓ calculates probability

- (iii) in five plays of the machine, the second payout of \$1 occurs on the fifth play. (3 marks)

Solution
First payout in one of four plays: $W \sim B(4, 0.45)$ ✓ $P(W = 1) = 0.2995$ ✓
Second payout: $P = 0.2995 \times 0.45 = 0.1348$ ✓✓
Specific behaviours

✓ uses first and second event
✓ calculates P for first event
✓ calculates P for both events

(b) Calculate the mean and standard deviation of X . (2 marks)

Solution
$\bar{X} = 1.9125, \sigma_X = 6.321$
Specific behaviours

✓ mean
✓ sd

(c) In the long run, what percentage of the player's money is returned to them? (2 marks)

Solution
$\frac{1.9125}{2} \times 100 = 95.625\%$ ✓✓
Specific behaviours

✓ uses mean and payment
✓ calculates percentage