SHENTON COLLEGE

Year 11 Mathematics Methods (AEMAM)

Test 5 2016

Calculator Free

Time Allowed: 20 minutes

Marks	/ 25

Name: Marking Key

Circle Your Teachers Name: McRae Friday Mackenzie

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1. [5,2 marks]

(a) Show use of calculus methods to determine the coordinates and nature of any stationary points of the function $f(x) = 3x^2 - x^3$.

$f(x) = 3x^2 - x^3$

$f'(x) = 6x - 3x^2$ ✓ correct derivative

Stationary when $f'(x) = 0$ ✓ set $f'(x) = 0$

$6x - 3x^2 = 0$ ✓ Test for nature

$3x(2 - x) = 0$ ✓ $(0,0)$ local min

$x = 0$ or $x = 2$ ✓ $(2,4)$ local max

$f(0) = 0$ $f(2) = 4$

Test

x	-1	0	0.1
$f'(x)$	$-$	0	$+$

x	1	2	3
$f'(x)$	$+$	0	$-$

Minimum Turning Point at $(0,0)$ and Max T.P at $(2,4)$

(b) Determine the minimum and maximum values of $f(x)$ if $-2 \le x \le 3$

$f(-2) = 20$ Min value 0 ✓ Determines min

$f(3) = 0$ Max value 20 ✓ Determines max

2. [2,3 marks]

Determine the antiderivative of:

(i) $\frac{dy}{dx} = 3x^3 + 4$

$y = \frac{3}{4}x^4 + 4x + c$ ✓ ✓

works out $y =$

(ii) $\frac{dy}{dx} = \frac{9x^3 - 8x^4}{x^2}$

$\frac{dy}{dx} = 9x - 8x^2$ ✓ Simplifies

$y = \frac{9x^2}{2} - \frac{8x^3}{3} + c$ ✓ ✓ works out $y =$

* (−1) if No 'c' ONLY once over whole paper.

3. [3 marks]
The function $y = x^3 + ax + b$ has a local minimum point at (2,3). Use differentiation to find the values of a and b.

$\frac{dy}{dx} = 3x^2 + a$

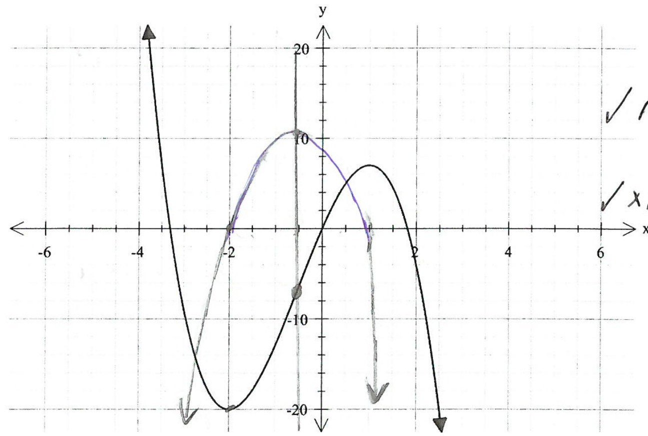
Min when $3x^2 + a = 0$ at $x = 2$
$12 + a = 0$
$a = -12$

✓ recognizes that $3x^2 + a = 0$ at $x = 2$

at (2,3) $3 = 8 - 12(2) + b$
ie $b = 19$

✓ $a = -12$
✓ $b = 19$

4. [3,2 marks]
Below is a graph of y = f(x)



✓ ∩ max
$x = -0.5$
✓ x intercepts at $x = -2$ and $x = 1$

a) State the value(s) of x for which:

i) f ' (x) < 0 $x < -2$ and $x > 1$ Both ✓

ii) f ' (x) = 0 $x = 2$ and $x = 1$ Both ✓

iii) f ' (x) > 0 $-2 < x < 1$ ✓

b) On the grid above, draw a possible graph of y = f ' (x)

10. [1,1,3,4 marks]

The displacement s (in metres) at time t (in seconds) of a particle moving in a horizontal straight line is given by:

$s(t) = (t - 3)(2t + 3)(t - 6)$

Determine

(a) The initial displacement of the particle.

$s(0) = 54$ m ✓ initial displacement

(b) the displacement of the particle when t=4.

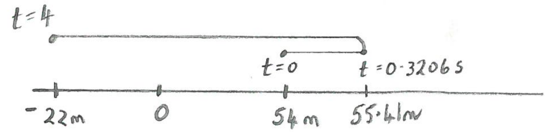
$s(4) = -22$ m ✓ s(4)

(c) When the particle changes direction, using calculus.

$s'(t) = 0$ when $t = 0.3206$ s
and $t = 4.68$ s

✓ $s'(t) = 0$
✓ $t =$
✓ $t =$

(d) The total distance travelled in the first four seconds (to the nearest metre).



Total distance $= 2(55.41 - 54) + (54 + 22)$ ✓ ✓ ✓
$= 79$ m to the nearest m ✓

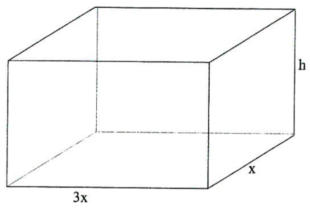
Clear process shown
✓ tot dist

OR $(55.41 + 22) + (55.41 - 54)$ ✓ ✓
$= \cancel{77}.41 + 1.41$
$= 78.82$ ✓
$= 79$ m to nearest m ✓

End of Section 2

9. [2,1,3 marks]

A piece of wire, 300cm long is used to make the 12 edges of the frame of a rectangular box. The length of the rectangular frame is three times that of the width of the frame, x cm.



(a) Show that the height, h, of the rectangular box is given by, $h = 75 - 4x$.

$300 = 4(3x) + 4x + 4h$ ✓ correct $= 300$

$4h = 300 - 16x$

$h = 75 - 4x$ ✓ process correct to show $h =$

(b) Show that the volume , V, of the box is given by $V = 225x^2 - 12x^3$

$V = L \cdot w \cdot h$

$= (3x)(x)(75 - 4x)$ ✓ correct $l \times w \times h$ shown

$= 225x^2 - 12x^3$

(c) Use a calculus method to determine the dimensions of the frame that will maximize the volume of the box.

$V = 225x^2 - 12x^3$

Max when $\frac{dV}{dx} = 0$ (ie. $-36x^2 + 450x = 0$)

$x = 0$ or $x = 12 \cdot 5$

✓ Process indicated $\frac{dV}{dx} = 0$ to obtain $x = 12 \cdot 5$

Check Max

x	12	12·5	13
$\frac{dV}{dx}$	+	0	−

Max when width = 12·5 cm

length = 37·5 cm

height = 25 cm

✓✓ other Dimensions correct.

5. [3,2 marks]

(a) Determine the rule for the curve that passes through (1,-1) with a gradient function $f'(x) = 6(1 - x^2)$.

$f'(x) = 6(1 - x^2) = 6 - 6x^2$

$f(x) = 6x - 2x^3 + c$ ✓✓ correct antiderivative

at (1,-1) $-1 = 6 - 2 + c$

$c = -5$

$\therefore f(x) = 6x - 2x^3 - 5$ ✓ correct value of 'c' to complete rule

(b) Find the equation of the tangent to the curve at the point (2,-9)

$f'(x) = 6 - 6x^2$

$f'(2) = -18$ ✓ gradient at $x = 2$

Eqn of tangent:

$y = -18x + b$

at (2,-9) $-9 = -36 + b$

$b = 27$

$y = -18x + 27$ ✓ correct equation

End of Section 1

SHENTON COLLEGE

Year 11 Mathematics Methods (AEMAM)
Test 5 2016
Calculator Assumed

Time Allowed: 30 minutes

Marks	/ 32

Name: Marking Key

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6. [5 marks]

Given that $A = x^2y$ and $x + y = 10$, where $x > 0$, use a calculus method to determine the maximum value of A and the corresponding values of x and y. Give exact answers.

$x + y = 10$
$y = 10 - x$
$A = x^2(10-x)$
$= 10x^2 - x^3$ ✓ equation for A in terms of x

$\frac{dA}{dx} = 20x - 3x^2$

Min/Max when $20x - 3x^2 = 0$ ✓ correct $\frac{dA}{dx} = 0$

$x = 0$ or $x = 6\frac{2}{3}$ ✓ correct x

check Max

x	6	$6\frac{2}{3}$	7
$\frac{dA}{dx}$	+	0	−

Max at $(6\frac{2}{3}, 3\frac{1}{3})$ ✓ correct exact value of y

✓ Max Value of A $A = \left(\frac{20}{3}\right)^2\left(\frac{10}{3}\right) = \frac{4000}{27}$

7. [4 marks]

The total cost of producing x blankets per day is $\frac{1}{4}x^2 + 8x + 20$ *dollars* and each blanket may be sold at $\left(23 - \frac{1}{2}x\right)$ *dollars*.

Use a calculus method to determine how many blankets should be produced each day to maximise the total profit.

Profit $= S.P - C.P$ ✓ each blanket

$= (23 - \frac{1}{2}x)x - (\frac{1}{4}x^2 + 8x + 20)$ ✓ equation for $P =$

$\frac{dP}{dx} = -1.5x + 15$

Max when $-1.5x + 15 = 0$
$x = 10$ ✓ correct $\frac{dP}{dx} = 0$ or state $\frac{dP}{dx} = 0$

Check max

x	9	10	11
$\frac{dP}{dx}$	+	0	−

Max when 10 blankets sold each day ✓ word answer

8. [1,2,1,2,2 marks] * (−1 overall for units for Qn 8/9/10 once (−1) but not again)

A bullet is fired upwards. After t seconds the height of the bullet is found from the rule

$H(t) = 150t - 4.9t^2 + 2$ where t is measured in seconds and H in metres.

(a) Find the height of the bullet after 5 seconds.

$H(5) = 629.5$ m ✓ height

(b) Determine the average speed of the bullet during the fifth second. Indicate your method.

Av Speed $= \frac{H(5) - H(4)}{5 - 4}$ ✓ method
$= 629.5 - 523.6$
$= 105.9$ m/s ✓ av speed

The speed of the bullet is the instantaneous rate of change of the height of the bullet.

(c) Find the speed of the bullet after 5 seconds.

$H'(t) = -9.8t + 150$
$H'(5) = 101$ m/s ✓ speed

(d) Find the maximum height of the bullet, to the nearest metre.

Max Height when $H'(t) = 0$
$t = 15.31$ s ✓ correct time if shown
$H(t) = 1150$ m ✓ correct height with correct accuracy.

(e) Determine the bullet's speed as it hits the ground, on the way down correct to two decimal places.

Hits ground when $H(t) = 0$
$t = 30.63$ s ✓ $t =$
$H'(t) = -150.13$ ✓ correct speed

$\therefore$ speed on way down 150.13 m/s