



NAME: \_\_\_\_\_

DATE: Tues 17<sup>th</sup> Feb.

Total: 45 marks

Time: 50 min.

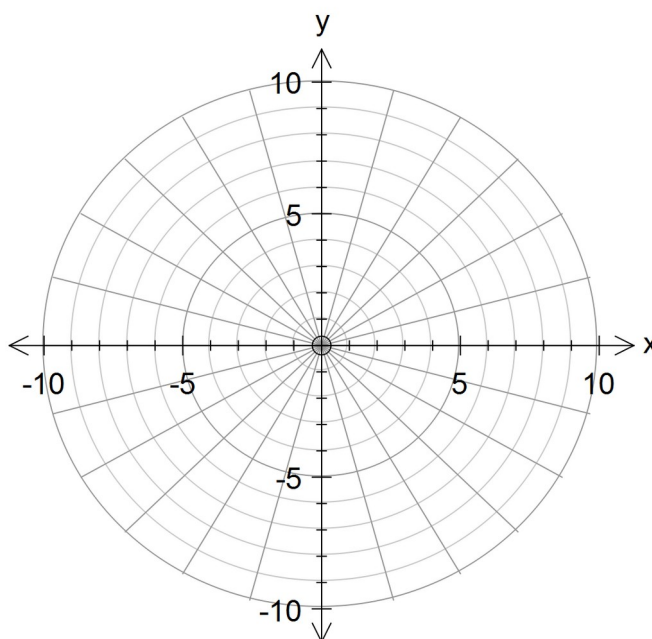
Question 1

(5 marks)

- (i) Point P has polar coordinates  $(4, 17^\circ)$  and it lies on the line  $y = -x + 5$ .  
Point Q also lies on the line and is 12cm away from P.

Find the polar coordinates of Q  $[r, \theta]$  where  $90^\circ < \theta < 180^\circ$ .

(3 marks)



- (ii) Sketch the graph of  $r = \frac{4}{\pi} \theta$  on the axes above and hence state  
where it intersects  $r = 4$

(2 marks)

**Question 2****(9 marks)**

On a 3D computer game, Chris, a keen cyclist leaves from *position*  $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$  metres is travelling at  $(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$  m/s while his mate Dave leaves from position  $(a\mathbf{i} + \mathbf{j} + b\mathbf{k})$  metres running at  $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$  m/s.

- (i) Although they do not collide, their paths do intersect at the point with coordinates  $(a, 1, b)$ .

Determine the values of  $a$  and  $b$ .

**(4 marks)**

- (ii) Find the acute angle between these two paths.

**(2 marks)**

(iii) Hence, determine the perpendicular distance from the point (1,-1,2) to (3 marks)

$$r = \begin{pmatrix} \frac{7}{3} \\ 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

**Question 3****(5 marks)**

- (a) The vectors  $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$  and  $\mathbf{i} + 2\mathbf{j} + a\mathbf{k}$  are perpendicular.

Determine the value of  $a$ .

(1marks)

- (b) Determine whether the two lines

$$\mathbf{r} = 8\mathbf{i} - \mathbf{j} - 8\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{k}) \text{ and } \mathbf{r} = \mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \text{ intersect.}$$

(4 marks)

If they do intersect, state the position vector of their point of intersection.

If they do not intersect, justify your answer.

**Question 4****(7 marks)**

- (a) If  $z = 3 - 4j$ , determine the reciprocal,  $\frac{1}{z}$

(2 marks)

- (b) Let the non-zero complex number  $z = a + bi$ . Show that  $\frac{1}{a + bi} = \frac{\bar{z}}{|z|^2}$

(3 marks)

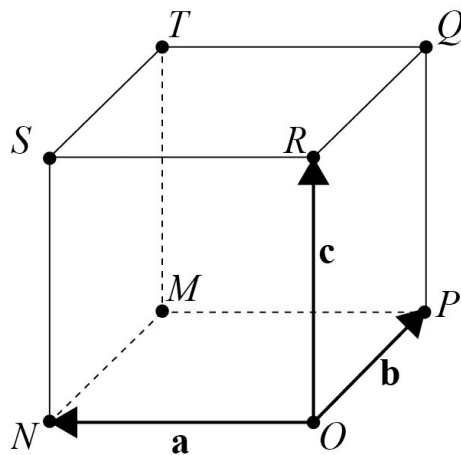
- (c) Describe the geometrical relationship between any non-zero complex number and its reciprocal.

(2marks)

### Question 5

(5 marks)

Let  $MNOPQRST$  be a rectangular prism with sides  $\overrightarrow{ON}$ ,  $\overrightarrow{OP}$  and  $\overrightarrow{OR}$  denoted by the vectors,  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$  respectively, as shown in the diagram below.



Suppose that **A** is the midpoint of  $\overrightarrow{MN}$ , **B** is the midpoint of  $\overrightarrow{MT}$ , **C** is the midpoint of  $\overrightarrow{QR}$  and **D** is the midpoint of  $\overrightarrow{OR}$ .

- (a) Express  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$ ,  $\overrightarrow{OC}$  and  $\overrightarrow{OD}$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$ . (2marks)

- (b) Prove that the quadrilateral ***ABCD*** is a parallelogram. (3 marks)

**Question 6** (5 marks)

- (a) Change the complex equation  $|Z - i| = |Z - 1|$  into its Cartesian equivalent. (3 marks)

- (b) Hence identify, the locus of all points  $Z$  satisfying the equation in (a). (2 marks)

**Question 7****(5 marks)**

An equilateral triangle has vertices  $A$ ,  $B$  and  $C$ , where  $A$  is the point  $\sqrt{3} - \mathbf{j}$  in the Argand plane.

The circumcircle is drawn that passes through vertices  $A$ ,  $B$ , and  $C$  and has a centre inside the triangle called the circumcentre.

The circumcentre is located at the origin.

Find the complex numbers  $z_1$  and  $z_2$  corresponding to the vertices  $B$  and  $C$ , expressing your answers in exact Cartesian form.



**Question 8****(4 marks)**

Find two numbers which have a product of 2 and a sum of 2.