**PRESBYTERIAN LADIES' COLLEGE**

**A COLLEGE OF THE UNITING CHURCH IN AUSTRALIA**

**MATHEMATICS DEPARTMENT**

**Year 12 SPECIALIST MATHEMATICS**

**TEST 3:**        **INTEGRATION**

DATE:  6th April 2016                        Name_____

**Reading Time:** 3 minutes

**SECTION ONE: CALCULATOR FREE**

TOTAL:                35 marks

EQUIPMENT:        Pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet.

WORKING TIME:        35 minutes (maximum)

**SECTION TWO: CALCULATOR ASSUMED**

TOTAL:                19 marks

EQUIPMENT:        Pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing instruments, templates, up to 3 Calculators,

                1 A4 page of notes (one side only), SCSA formula sheet.

WORKING TIME:        20 minutes (minimum)

| SECTION 1 Question | Marks available | Marks awarded | SECTION 2 Question | Marks available | Marks awarded |
|---|---|---|---|---|---|
| 1 | 10 | | 6 | 5 | |
| 2 | 8 | | 7 | 6 | |
| 3 | 4 | | 8 | 8 | |
| 4 | 6 | | | | |
| 5 | 7 | | | | |
| | | | | | |
| Total | 35 | | | 19 | |

**Calculator-free**                                                                                    **[35 marks]**

This paper has **Five (5)** questions. Answer **all** questions. Write your answers in the spaces provided

**Question 1     [10 marks]**

Determine the following indefinite integrals.

a)  $\int \dfrac{x^2 + 2x - 5}{x^2}\, dx$                                                                          **[3]**

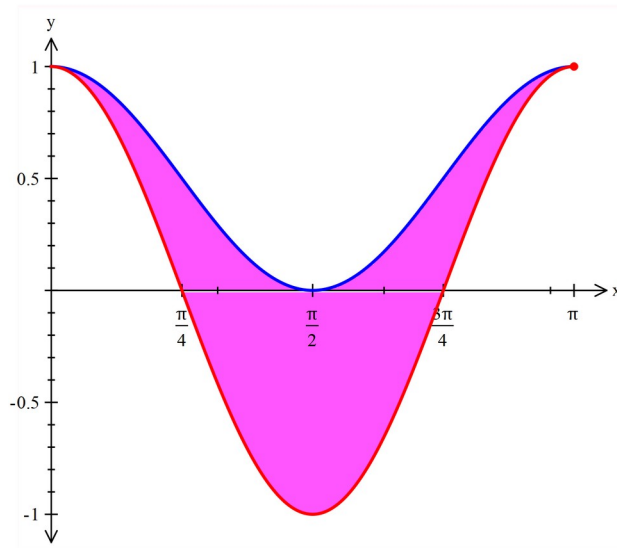b)  $\int \sin^3(2x)\cos(2x)\, dx$                                                                          **[3]**

c)  $\int x\sqrt{x - 1}\, dx$                                                                          **[4]**

**Question 2     [8 marks]**

Evaluate:

**a)**   $\int_0^{\frac{\pi}{4}} \cos 2x \; dx$                                                                                    **[3]**

**b)**   $\int_0^1 \frac{x-1}{x^2+3x+2} \; dx$                                                                          **[5]**

## Question 3    [4 marks]

The curves below are $y = \cos(2x)$ and $y = \cos^2 x$. Determine the area of the shaded region.
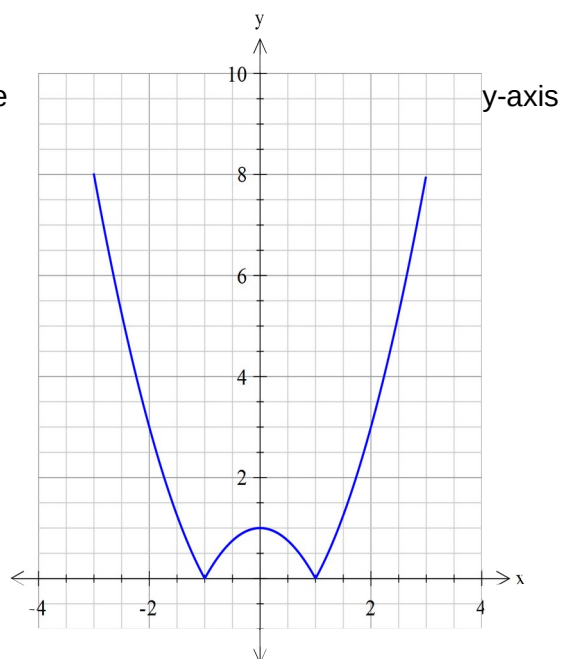
## Question 4    [6 marks]

A glass is formed by rotating the function $f(x) = |x^2 - 1|$ about the y-axis
as shown in the diagram.

**a)** Complete the middle line for the piecewise function for
$f(x) = |x^2 - 1|$.

$$f(x) = \begin{cases} x^2 - 1 & ; \ x \leq -1 \\ \\ x^2 - 1 & ; \ x \geq 1 \end{cases}$$



**[1]**

**b)** Given the height of the glass is 8 cm determine the volume of the glass. **[5]**

## Question 5    [7 marks]

Find $\displaystyle\int^{\sqrt{3}} \sqrt{4 - x^2}\, dx$ using the substitution $x = 2\sin\theta$.

**NAME:……………………………………………………………………………**

**Calculator Allowed**                    **20 minutes**                              **[19 marks]**

This paper has **Three (3)** questions. Answer **all** questions. Write your answers in the spaces provided

**Question 6    [5 marks]**
Determine the value of p in the system of linear equations below such that there is

    a)  no solution

$$x - 2y - 3z = 11$$
$$2x - y + z = 5$$
$$3x + 3y + pz = -6$$

**[3]**

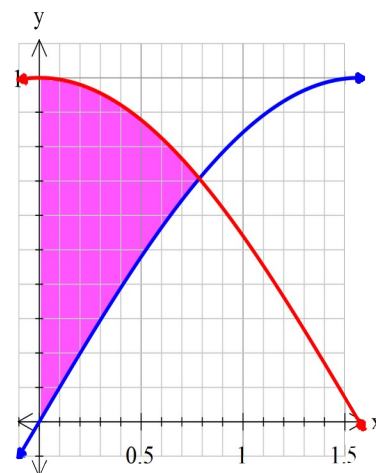    b)  a unique solution                                                                                  **[1]**

    **c)**  infinitely many solutions                                                                    **[1]**

## Question 7    [6 marks]

The graph at the right show the curves $y = \cos x$ and $y = \sin x$.



a) Prove that the intersection of $y = \cos x$ and $y = \sin x$, is $x = \dfrac{\pi}{4}$
for the domain $0 \le x \le \dfrac{\pi}{2}$.

[1]

b) Determine the exact area of the region (shaded) which is bounded by the *y*-axis and the curves $y = \cos x$ and $y = \sin x$.

[2]

c) Determine the volume of revolution obtained when this area is rotated about the *x*-axis.        [3]

**Question 8    [8 marks]**

a) Use the identity $\cos 2\theta = 2\cos^2 \theta - 1$ to prove that $\cos\left(\frac{1}{2}x\right) = \sqrt{\frac{1+\cos x}{2}}$, $0 \le x \le \pi$.  [2]

b) Find a similar expression for $\sin\left(\frac{1}{2}x\right)$, $0 \le x \le \pi$.  [2]

c) Hence show that $\int_0^{\pi} \left(\sqrt{1+\cos x} + \sqrt{1-\cos x}\right) dx = 2\sqrt{2}$  [4]

**END OF QUESTIONS**