Semester One Examination, 2017

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________________________

Your name ______________________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	11	11	100	85	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Question 19 **(6 marks)**

a) Determine the quotient and remainder when $6x^3 - 17x^2 - 31x + 17$ is divided by $(3x - 1)$

(4 marks)

Solution
$$\begin{array}{r} 2x^2 - 5x - 12 \\ 3x - 1 \overline{)\, 6x^3 - 17x^2 - 31x + 17} \end{array}$$ $6x^3 - 2x^2$ $0 - 15x^2 - 31x$ $0 - 15x^2 + 5x$ $0 + 0 - 36x + 17$ $0 + 0 - 36x + 12$ $0 + 0 + 0 + 5$
Specific behaviours
✓ uses long division method   ✓ determines squared term of quotient   ✓ determines complete quotient and states this as the quotient   ✓determines and labels remainder

b) Determine the value of the constant a so that $(3x - 1)$ is a factor of $6x^3 - 17x^2 - 31x + a$

(2 marks)

Solution
$f(x) = 6x^3 - 17x^2 - 31x + a$ $f(\frac{1}{3}) = 0$ $6\frac{1}{3}^3 - 17\frac{1}{3}^2 - 31\frac{1}{3} + a = 0$ $-12 + a = 0$ $a = 12$
Specific behaviours
✓ sets up equation with constant   ✓ solves for constant

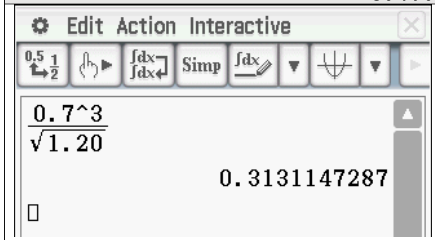
Question 18 **(8 marks)**

a) A quantity z varies partly as directly proportional to x squared plus directly proportional to y cubed. For x=7 and y=11, z = 4189. For x=12 and y=5, z=951. Determine the value of z for x=9 and y=15. (4 marks)

Solution
$z = k_1 x^2 + k_2 y^3$ $4189 = k_1 7^2 + k_2 11^3$ $951 = k_1 12^2 + k_2 5^3$ $k_1 = 4,\ k_2 = 3$ $4(9)^2 + 3(15)^3 = 10449$
Specific behaviours
✓ writes equation for z in terms of two constants ✓ sets up two equations with constants ✓ solves both constants ✓determines final value of z

b) If $w \propto \dfrac{x^3}{\sqrt{y}}$ determine the percentage change in w if x decreases by 30% and y increases by 20%. (4 marks)

Solution



Specific behaviours

✓ uses 0.7 for x
✓ uses 1.20 for y
✓ subs into equation for w
✓states a decrease of 69%

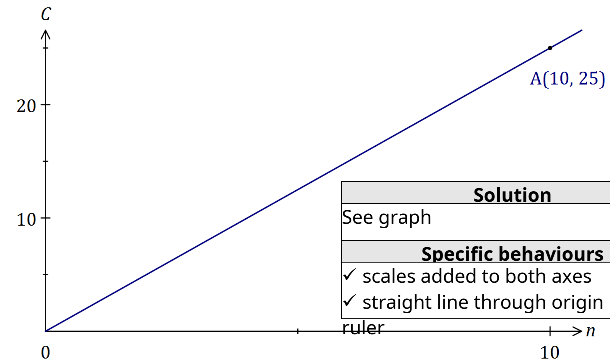
Section Two: Calculator-assumed **65% (85 Marks)**

This section has **eleven (11)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(6 marks)**

(a) The variables C and n are directly proportional to each other, so that when $n=10$, it is known that $C=25$. Sketch a graph of the relationship between C and n on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ scales added to both axes ✓ straight line through origin using ruler

(b) The variables A and n are inversely proportional to each other, so that when $n=10$, it is known that $A=60$.

(i) Write an equation that relates A and n. (2 marks)

Solution
$A \propto \frac{1}{n} \Rightarrow A = \frac{k}{n}$ $k = 10 \times 60 = 600 \Rightarrow A = \frac{600}{n}$
Specific behaviours
✓ correct form of equation ✓ determines constant

(ii) Determine the value of n when $A=15$. (1 mark)

Solution
$n = \frac{600}{15} = 40$
Specific behaviours
✓ states value

Question 9 **(5 marks)**

(a) The volume (V) in litres (L) of a gas, at a fixed temperature and of a certain mass, varies inversely to the pressure (P) in Pascals (Pa).

(i) Find k, the constant of proportionality, given that when P = 11.5 Pa and V = 2.84 L. (2 marks)

Solution
(i) $V = \frac{k}{P}$ $k = 2.84 \times 11.5$ $k = 32.66$
Specific behaviours
✓ states equation ✓ determines constant

(ii) Describe the effect on V when P is halved. (1 mark)

Solution
The volume doubles.
Specific behaviours
✓ states double

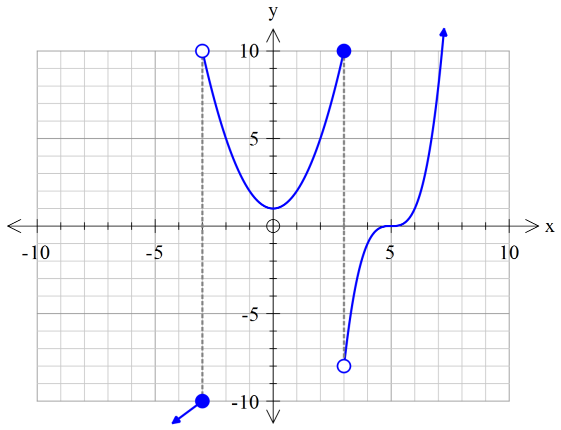
(b) Jan is a real estate agent who earns a commission of 3.25% on the sale of a house. If $c is the commission and $s is the sale price of a house, show clearly *c* is directly proportional to *s*.
(2 marks)

Solution
$c = 0.0325s$ is a linear relationship in the form $y = kx$ where $k = 0.0325$ and is therefore directly proportional.
Specific behaviours
✓ states equation ✓ shows constant for direct proportion

Question 17 **(8 marks)**

Let
$$f(x) = \begin{cases} x - 7 & , x \le -3 \\ x^2 + 1 & , -3 < x \le 3 \\ (x-5)^3 & , x > 3 \end{cases}$$

a) Sketch the function on the axes below. (6 marks)



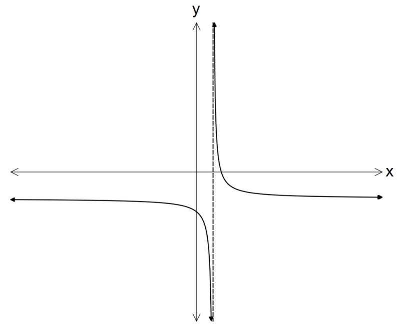
Solution
Specific behaviours
✓ correct circle shaded for x=-3 ✓ correct circle shaded fpr x=3 ✓y intercept shown ✓ x intercept shown ✓correct shape of function at x-3 ✓ all sections have correct shape

b) State the maximal(natural) domain and range. (2marks)

Solution
Domain R Range $R \setminus (-10, 8]$ or $y \le -10, y > -8$
Specific behaviours
✓ correct domain ✓ correct range

Question 10 **(5 marks)**

The reciprocal function $y = f(x)$ shown below has asymptotes at $x = a$ and $y = b$.
The y-intercept is at $(0, c)$.



In terms of a, b and/or c:

(a) determine the equation of the horizontal asymptote for $y = f(2x) + 4$. (1 mark)

Solution
$y = 4 + b$
Specific behaviours
✓ states correct answer

(b) determine the coordinates of the y-intercept of $y = -f\left(\frac{x}{3}\right) - 2$. (2 marks)

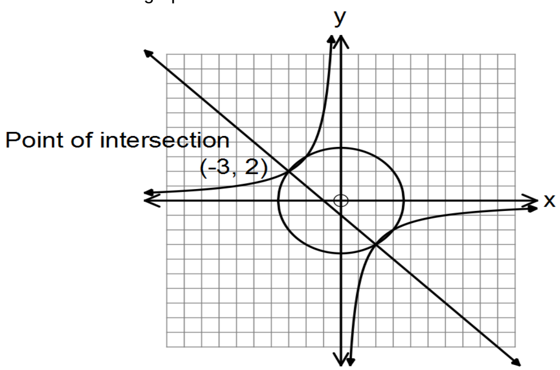
Solution
$(0, -c - 2)$
Specific behaviours
✓ determines y value ✓ gives as coordinates

(c) state the domain of the function $f(x + 3)$. (2 marks)

Solution
$\{x : x \neq a - 3; x \in \mathbb{R}\}$
Specific behaviours
✓ vertical asymptote moves 3 to left ✓ states domain

Question 11 **(11 marks)**

Consider the functions graphed below.



(a) State the equation for:

(i) $f(x)$, the circle with centre at the origin. (2 marks)

Solution
$x^2 + y^2 = 13$
Specific behaviours
✓ uses squared terms ✓ uses correct radius

(ii) $g(x)$, the hyperbola. (2 marks)

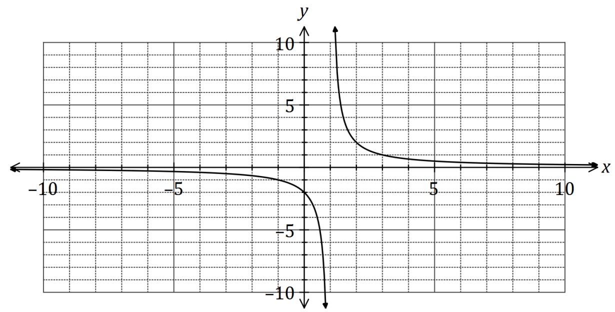
Solution
$y = -\frac{6}{x}$
Specific behaviours
✓ uses correct format ✓ use correct constant

(iii) $h(x)$, the straight line. (2 marks)

Solution
$y = -x - 1$
Specific behaviours
✓ correct gradient ✓ correct constant

Question 16 **(8 marks)**

The graph of the function is defined by $f(x) = ¿\frac{a}{x+b}$ is shown below.



(a) Determine the values of a and b. (2 marks)

Solution
Using vertical asymptote, $b=-1$ Using y-intercept, $a=2$
Specific behaviours
✓ value of a, ✓ value of b

(b) State the domain and range of $f(x)$. (2 marks)

Solution
$D_f = \{x : x \in R, x \neq 1\}$ $R_f = \{y : y \in R, y \neq 0\}$
Specific behaviours
✓ indicates $x \neq 1$, ✓ indicates $y \neq 0$

(c) Determine the equations of the asymptotes of the graph of $y = f(2x)$. (2 marks)

Solution
Vertical asymptote: $x = \frac{1}{2}$, horizontal asymptote: $y=0$

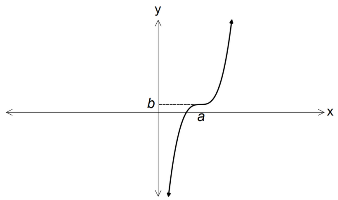
(d) Describe the transformation required on the graph of $y = f(x)$ to obtain the graph of

(i) $y = f(x+8)$. (1 mark)

Solution
Translate 8 units to the left.
Specific behaviours
✓ description

(ii) $y = \frac{1}{2} f(x)$. (1 mark)

Solution
Dilate vertically by scale factor $\frac{1}{2}$



Solution
$y = (x - a)^3 + b$
Specific behaviours
✓ correct expression

(b) Hence, solve the equation $f(x) = g(x)$. There are four solutions. (2 marks)

Solution
$x = -3, -2, 2, 3$
Specific behaviours
✓ states two correct solns ✓ states four correct solns

(c) The graph $g(x)$ undergoes the following transformation $g(2x) + 1$.

(i) State the coordinates of the point (–3, 2) after this transformation has occurred. (2 marks)

Solution
$\left(-\frac{3}{2}, 3\right)$
Specific behaviours
✓ correct x ✓ correct y

(ii) The graph $h(x)$ undergoes the same transformation namely $h(2x) + 1$. State the gradient of the transformed function. (1 mark)

Solution
m = –2
Specific behaviours
✓ states correct value

Question 12 **(7 marks)**

(a) Consider the following sets of ordered pairs:
$f = \{ (1,2), (2, 3), (3,4) \}$ $h = \{ (-1, 4), (0, 3), (1,2) \}$

(i) Find $f(2)$. (1 mark)

Solution
3
Specific behaviours
✓ states correct value

(ii) Find a such that $h(a) = 3$. (1 mark)

Solution
0
Specific behaviours
✓ correct value

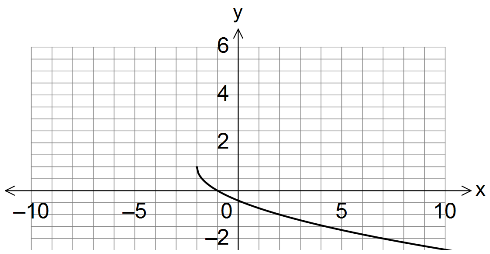
(iii) Find t such that $f(t) = h(t)$. (1 mark)

Solution
1
Specific behaviours
✓ correct value

(b) Compare the domain of $p(x) = (\sqrt{x})^2$ and $m(x) = \sqrt{x}$. (2 marks)

Solution
Domain of $p(x)$ is the same as $m(x)$. For both: $x \geq 0, x \in \mathbb{R}$
Specific behaviours
✓ states the same ✓ shows x greater than or equal to zero

(c) The function, $q(x)$ below, is a transformation of $y = \sqrt{x}$.
State the equation of the function, $q(x)$, below. (2 marks)



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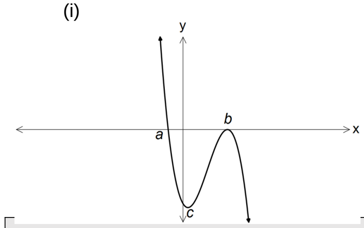
Question 15 **(5 marks)**

(a) State the centre, C, and the radius, r, of the circle given by
$$(x-3)^2+(y-5)^2-36=0$$
(2 marks)

Solution
Centre (3,5) radius 6
Specific behaviours
✓ states centre ✓ states radius

(b) Write a possible equation, in terms of a, b and/or c, which are positive constants, for each graph shown below.
(3 marks)

(i)



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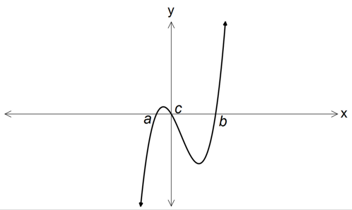
Solution
$y = -(x+a)(x-b)^2$
Specific behaviours
✓ correct expression

(ii)



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Solution
$y = x(x+a)(x-b)$
Specific behaviours
✓ correct expression

(b) One of the solutions to the equation $2x^3+21x^2+cx-495=0$ is $x=5$. Determine the value of c and all other solutions. (3 marks)

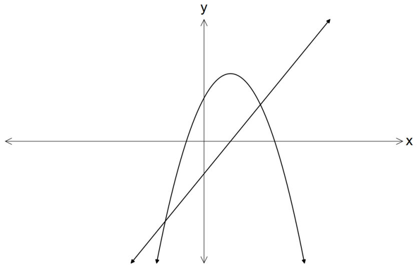
Solution
Using CAS, when $x=5$, $5c+280=0 \Rightarrow c=-56$ Use CAS to solve $2x^3+21x^2-56x-495=0$ $x=-11$, $x=-4.5$ and $x=5$
Specific behaviours
✓ substitutes $x=5$ ✓ determines c ✓ states other two solutions

Solution
$q(x) = -\sqrt{x+2}+1$
Specific behaviours
✓ uses negative sign in front of square root ✓ correct function

Question 13 **(6 marks)**

The following functions are shown below:

$m(x) = -x^2 + 3x + 4$ and $n(x) = 2x + q$, where q is a constant.



For what value(s) of q does the equation $m(x) = n(x)$ produce:

(a) one solution? (4 marks)

Solution
$-x^2 + 3x + 4 = 2x + q$ $-x^2 + x + 4 - q = 0$ $b^2 - 4ac = 0$ for one solution $\therefore 17 - 4q = 0$ $\therefore q = \frac{17}{4}$
Specific behaviours
✓ equates y values   ✓ determines discriminant of quadratic equation   ✓ equates to zero   ✓solves for q

(b) no real solutions? (2 marks)

Solution
$17 - 4q < 0$ $\therefore q > \frac{17}{4}$
Specific behaviours
✓ states discriminant is less than zero   ✓ solves inequality

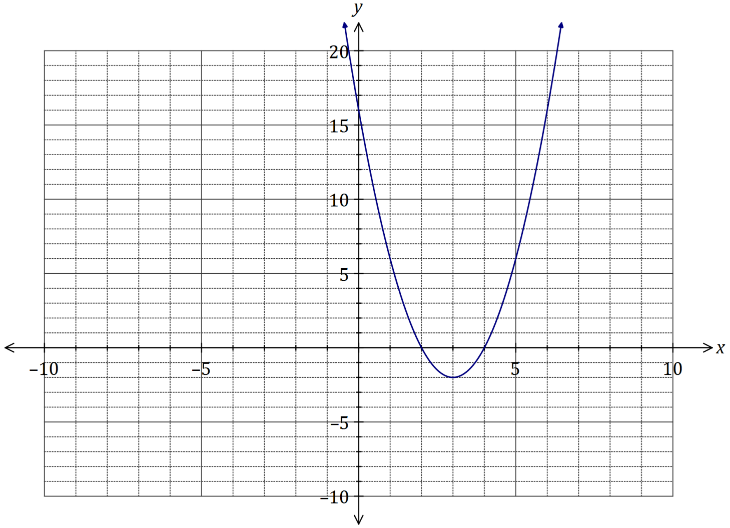
Question 14 **(8 marks)**

(a) The graph of $y = 2x^2 + bx + 16$ has a line of symmetry with equation $x = 3$.

(i) Determine the value of b. (2 marks)

Solution
$x = \frac{-b}{2a} \Rightarrow b = -3 \times 2 \times 2 = -12$
Specific behaviours
✓ uses line of symmetry   ✓ value of b

(ii) Draw the graph of the parabola on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ turning point   ✓ three axes intercepts   ✓ smooth curve