

Working out space

Year 12 Methods TEST 7 June 2019 TIME: 45 minutes working Calculator Assumed 44 Marks 6 Questions	 PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School
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Name: _____ Teacher: _____

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 (5 marks)
(a) Differentiate $x \sin x$ (2 marks)

(b) Hence find $\int_{\frac{\pi}{2}}^0 x \cos x \, dx$ using the result in(a) above. (3 marks)

Question 2**(3 marks)**

Determine the x -coordinates of all points on the graph of $f(x) = 2\cos(x) + x$ for $-\pi \leq x \leq \pi$ where the tangent line is horizontal. (Justify your answers)

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Question 6

(11 marks)

A game is played by throwing two standard six-sided dice into the air once. The sum of the uppermost numbers are added together and if the sum is greater than 8 the player wins \$5.

Determine:

a) the probability of winning \$5 in one game.

(2 marks)

b) the probability of winning exactly \$15 in 5 games.

(3 marks)

c) the probability of winning at least \$15 in at most 5 games.

(3 marks)

d) the minimum number of games to be played so that the probability of winning at least \$15 is greater than 0.47. (Justify)

(3 marks)

Question 3

(7 marks)

A survey conducted by a local bank shows that 75% of its customers use an ATM at least once a month.

(a) Find the probability that in a random sample of 8 customers, at least 75% of them use an ATM machine at least once a month.

(2 marks)

(b) If the random variable X follows a binomial distribution with $n=12$ and $p=0.75$, what is the mean of this distribution and what is $P(X \geq \text{mean})$?

(3 marks)

(c) If the sample size became very large what would you expect $P(X \geq \text{mean})$ to approach?

Briefly explain your answer.

(2 marks)

Question 4
(10 marks)

The discrete random variable X can only take the values 2, 3 or 4. For these values the cumulative distribution function is defined by

$$P(X \leq x) = \frac{(x+k)^2}{25}$$

for $x=2,3,4$, where k is a positive constant integer.

(a) Find the value for k .
 (3 marks)

(b) Complete the following table for X .
 (3 marks)

X	2	3	4
$P(X \leq x)$			
$P(X = x)$			

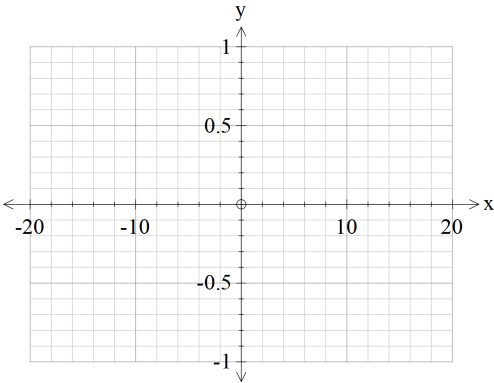
(c) Hence find $E(X)$ and $SD(X)$.
 (2 marks)

(d) Calculate $Var(3-2X)$ giving your answer to 2 decimal places.
 (2 marks)

Question 5
(8 marks)

Consider the function $f(x) = \frac{1 - \cos x}{x}$ where x is in radians.

a) Sketch $f(x)$ on the axes below for $-20 \leq x \leq 20$ on the axes below. Clearly label undefined points (if any).
 (3 marks)



b) As x approaches zero from the positive side, state the value that $f(x)$ approaches.
 (1 mark)

c) As x approaches zero from the negative side, state the value that $f(x)$ approaches.
 (1 mark)

d) Use the above to define a value for $f(x)$ as x approaches zero, that is the following limit $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$.
 (1 mark)

It can be shown that $\frac{d}{dx}(\cos x) = -\cos x \lim_{h \rightarrow 0} \frac{1 - \cosh}{h} - \sin x \lim_{h \rightarrow 0} \frac{\sinh}{h}$.

e) Using the fact that $\lim_{h \rightarrow 0} \frac{\sinh}{h} = 1$ and the above results, show that $\frac{d}{dx}(\cos x) = -\sin x$.
 (2 marks)