SOLUTIONS

PERTH COLLEGE

Year 12

Semester Two Examination 2012

Question/Answer booklet

MATHEMATICS 3C/3D
Section One:
Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures

In words ______________________

Your name ______________________

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens(blue/black preferred), pencils(including coloured), sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	$33\frac{1}{3}$
Section Two: Calculator-assumed	13	13	100	100	$66\frac{2}{3}$
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

5. The Formula Sheet is not handed in with your Questions/Answer Booklet.

Question 20 **(9 marks)**

(a) A function is such that $f'(x) = x^2 - 2x - 3$.

(i) State the x-coordinate of the minimum of $f(x)$. (2 marks)

$x^2 - 2x - 3 = 0$

$x = -1,\ x = 3$

$f''(x) = 2x - 2$

$f''(-1) = 2(-1) - 2 = -ve$ $\therefore$ max

$f''(3) = +ve$ $\therefore$ min

i.e. $x = 3$

(ii) Justify that $f(x)$ has a point of inflection when $x = 1$. (2 marks)

$f''(x) = 2x - 2$

$f''(1) = 2(1) - 2$

$= 0$

$+ f'(1) \neq 0$ $\therefore$ p.o.i.

(iii) Find $f(-1) - f(2)$. (2 marks)

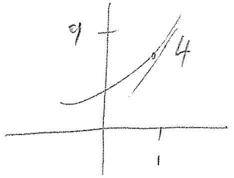
$f(-1) = \frac{5}{3} + c$ ½

$f(2) = -\frac{22}{3} + c$ ½

$\therefore f(-1) - f(2) = 9$

(b) A function $g(x)$ has the properties $g(x) > 0$, $g(1) = 9$ and $g'(1) = 4$.

If $h(x) = \sqrt{g(x)}$, find $h'(1)$. (3 marks)



$h(1) = 3$

$h'(x) = \frac{1}{2}(g(x))^{-\frac{1}{2}} \times g'(x)$

$h'(x) = \dfrac{g'(x)}{2\sqrt{g(x)}}$

$= \dfrac{4}{2\sqrt{9}}$

$= \dfrac{2}{3}$

End of Questions

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Section One: Calculator-free **(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(4 marks)**

For the two independent events A and B, $P(A) = 0.3$ and $P(B) = 0.1$.

Calculate

(a) $P(\overline{B})$ (1 mark)

0.9

(b) $P(A \cap B)$ (1 mark)

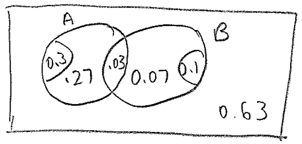
0.03

(c) $P(A \cup \overline{B})$ (1 mark)

0.93

(d) $P(\overline{A} | B)$ (1 mark)

$\dfrac{.07}{0.1}$ or 0.7



Question 2 **(5 marks)**

Solve the system of equations

$$3x + 2y + 6z = 3$$
$$x + 3y + 4z = 9$$
$$2x + 8 = 2z + y$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 6 & 3 \\ 1 & 3 & 4 & 9 \\ 2 & -1 & -2 & -8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & 9 \\ 0 & 7 & 6 & 24 \\ 0 & 7 & 10 & 26 \end{array}\right] \begin{array}{l} R_2 \\ 3R_2 - R_1 \\ 2R_2 - R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & 9 \\ 0 & 7 & 6 & 24 \\ 0 & 0 & 4 & 2 \end{array}\right] \quad R_3^* - R_2^*$$

$4z = 2$

$\therefore z = \frac{1}{2}$

$7y + 6z = 24$

$7y = 24 - 3$

$7y = 21$

$y = 3$

$x + 3y + 4z = 9$

$x + 9 + 2 = 9$

$x = -2$

Soln $(x, y, z) = (-2, 3, \frac{1}{2})$.

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(d) Prove the conjecture in (c). (4 marks)

If x is ODD and y is ODD.

$5xy$ is ODD

$-x^2$ is EVEN

-3 is ODD

$\therefore$ A is ODD.

If x is EVEN and y is ODD.

$5xy$ is EVEN

$-x^2$ is EVEN

-3 is ODD

$\therefore$ A is ODD

If x is ODD and y is EVEN

$5xy$ is EVEN

$-x^2$ is ODD

-3 is EVEN

$\therefore$ A is EVEN

and B is ODD + EVEN = ODD.

If x is EVEN and y is EVEN

$5xy$ is EVEN

$-x^2$ is EVEN

-3 is ODD

$\therefore$ Only way for A to Be even is if x is ODD + y is even

Question 19 **(10 marks)**

Let $A=5xy-x^2-3$ and $B=x+y$, where x and y are integers.

(a) Evaluate A and B when $x=3$ and $y=2$. (1 mark)

$A = 5(3)(2) - 3^2 - 3$
$= 18$

$B = 3+2$
$= 5$

(b) The parity of an object states whether it is even or odd. Complete these tables for the parity of the product and difference of odd and even numbers. (2 marks)

×	**odd**	**even**
odd	odd	even
even	even	even

−	**odd**	**even**
odd	even	odd
even	odd	even

(c) Examine the parity of A and B for various values of x and y, and hence state a conjecture about the parity of B when A is even. (3 marks)

$x=3,\ y=2$ $A=18$ even $B=5$ ODD

$x=-1,\ y=4$ $A=-24$ $B=3$

$x=2,\ y=5$ $A=43$ ODD $B=7$ ODD

$x=10,\ y=-1$ $A=-153$ $B=-4$

$x=1,\ y=5$ $A=21$ odd $B=6$

@ least 4 examples

Conjecture: When A is even B is ODD.

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Question 4 **(8 marks)**

(a) Determine the maximum and minimum values of the function $y=2x+\frac{27}{x^2}$ over the domain $1 \le x \le 7$. (4 marks)

$\frac{dy}{dx} = 2 + \frac{(-2)27}{x^3}$

$0 = 2 - \frac{54}{x^3}$

$\frac{54}{x^3} = 2$

$x^3 = 27$

$x = 3$

$\frac{d^2y}{dx^2} = \frac{6(27)}{x^4}$ +ve

$\therefore x=3$ is a min

$y(3) = 6 + \frac{27}{9} = 9$

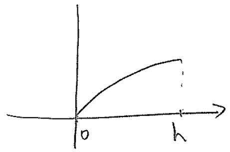
Endpoints : $y(1) = 2 + \frac{27}{1} = 29$

$y(7) = 14 + \frac{27}{49}$

$\therefore$ Min is 9 and max is 29

(b) A drinking glass is shaped by rotating the curve $y=\sqrt{x}$ around the x axis from 0 to h, where h is the height of the glass.

(i) Write an expression in terms of h for the volume of the glass. (2 marks)



$V = \pi \int_0^h x \, dx$

$= \left[\frac{\pi x^2}{2}\right]_0^h = \frac{\pi h^2}{2}$

(ii) Determine the height of one of these drinking glasses if it is to have a volume of 120cm^3. Give your answer in terms of π. (2 marks)

$120 = \frac{\pi h^2}{2}$

$\pi h^2 = 240$

$h^2 = \frac{240}{\pi}$

$h = \sqrt{\frac{240}{\pi}}$ (reject −ve)

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Question 3 **(8 marks)**

(a) Differentiate the following with respect to x. There is no need to simplify your answer.

(i) $y = 2x^3\sqrt{3-x^2}$ (2 marks)

$$\frac{dy}{dx} = 6x^2\sqrt{3-x^2} + \frac{1}{2}(3-x^2)^{-\frac{1}{2}}(-2x)\,2x^3$$

✓ ✓

(ii) $y = \dfrac{1+e^{3x-1}}{2e^{-x^2}}$ (3 marks)

✓ ✓

$$\frac{dy}{dx} = \frac{3e^{3x-1}\,2e^{-x^2} - (-4xe^{-x^2})(1+e^{3x-1})}{(2e^{-2x^2})^2}$$

✓

(b) Simplify $\dfrac{d}{dx}\displaystyle\int_2^{x^2}\left(\frac{t^2}{3}\right)dt$ (3 marks)

$$= \frac{(x^2)^2}{3} \times 2x$$ ✓ ✓

$$= \frac{2x^5}{3}$$ ✓

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Question 18 **(5 marks)**

A continuous random variable X has probability distribution function $f(x) = 0.04$, $14 \le x \le 39$.

(a) Calculate

(i) $P(21 < X < 22.5)$. (1 mark)

$1.5 \times 0.04 = 0.06$ ✓

(ii) $P(X < 29 \mid X > 25)$. (2 marks)

$$\frac{4 \times 0.04}{14 \times 0.04} = \frac{2}{7}$$ ✓ ✓

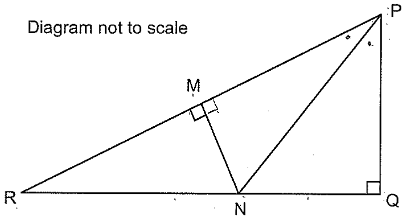
(b) If $P(20 < X < k \mid X < k) = 0.75$, find the value of k. (2 marks)

$$\frac{(k-20) \times \cancel{0.04}}{(k-14) \times \cancel{0.04}} = 0.75$$ ✓

$\therefore k = 38$ ✓

Question 17 **(5 marks)**

In the diagram, PQR is a right-angled triangle with $\angle PQR = 90°$ and M is the midpoint of PR. N is the point where the perpendicular to PR at M meets QR.



(a) Prove that ΔPNM is congruent with ΔRNM. (2 marks)

RTP $\Delta PNM \cong \Delta RNM$

$\overline{NM} \cong \overline{NM}$ (common) ½

$\angle PMN \cong \angle RMN$ (Both 90°). ½

$\overline{PM} = \overline{RM}$ (given M is midpoint) ½

$\therefore \Delta PNM \cong \Delta RNM$ (RHS) ½

(b) If PN bisects $\angle QPR$, show that the ratio of the areas of $\Delta PQN : \Delta PQR$ is 1:3. (3 marks)

$\Delta PMN \cong \Delta PQN$

As $\overline{PN} \cong \overline{PN}$ (common side

$\angle MPN \cong \angle QPN$ (Given)

$\angle PMN \cong \angle PQN$ (Both 90°)

So AAS

$\therefore$ All three Δ's congruent

$\therefore$ Area ΔPQN is $\frac{1}{3}$ area ΔPQR

$\therefore$ Ratio of area :

$\Delta PQN : \Delta PQR$ is 1 : 3

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Question 5 **(9 marks)**

Let $f(x) = \frac{1}{1-x}$ and $g(x) = e^{2x}$.

(a) Determine the domain of $f(g(x))$. (2 marks)

$e^{2x} \neq 1$

$\therefore x \neq 0$

i.e. $\{x \in \mathbb{R} : x \neq 0\}$

(b) Determine the range of $g(f(x))$. (3 marks)

e^{2x}

$\frac{1}{1-x}$

$\mathbb{R}$

subst

$\neq 1 \rightarrow \neq 0$

$\neq 1$ and > 0

$\therefore \{y \in \mathbb{R} : y > 0 \cup y \neq 1\}$

(c) Solve $f(x) \geq -x+1$ (4 marks)

$$\frac{1}{1-x} \geq -x+1$$

$$\frac{1}{1-x} + x - 1 \geq 0 \checkmark$$

$$\frac{1 + x(1-x) - 1(1-x)}{1-x} \geq 0$$

$$\frac{1 + x - x^2 - 1 + x}{1-x} \geq 0$$

$$\frac{2x - x^2}{1-x} \geq 0 \checkmark$$

CP: $x = 0$, $x = 2$, $x = 1$

$-\quad+\quad-\quad+$

$0 \quad 1 \quad 2$

$\therefore 0 \leq x < 1$, ✓
$x \geq 2$ ✓

See next page

Question 6 **(9 marks)**

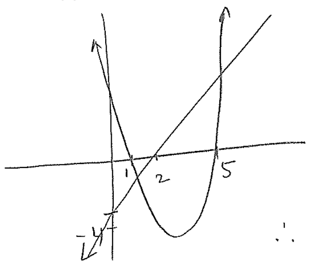
(a) Determine $\int x(3x^2+6x)^4 + (3x^2+6x)^4 \, dx$ (3 marks)

$= \int (x+1)(3x^2+6x)^4 \, dx$ ✓

$= \frac{1}{6}\int 6(x+1)(3x^2+6x)^4 \, dx$ ✓

$= \dfrac{(3x^2+6x)^5}{30} + c$ ✓

(b) Calculate the area bounded by the functions $f(x) = (x-2)^2 - 3$ and $g(x) = 2x - 4$. (6 marks)



$(x-2)^2 - 3 = 2x - 4$

$x^2 - 4x + 4 - 3 = 2x - 4$ ✓

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 5, \ x = 1$ ✓

$\therefore \int_1^5 (2x-4) - \left[(x-2)^2 - 3\right] dx$ ✓

$= \int_1^5 2x - 4 - (x^2 - 4x + 4 - 3) \, dx$

$= \int_1^5 -x^2 + 6x - 5 \, dx$

$= \left[\frac{-x^3}{3} + \frac{6x^2}{2} - 5x\right]_1^5$ ✓

$= \frac{-125}{3} + 75 - 25 - \left(-\frac{1}{3} + 3 - 5\right)$ ✓

$= \frac{-124}{3} + 52$ ✓

$= 10\frac{2}{3}$

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(d) The objective function is changed to $Q = ax + 30y$.

What is the minimum possible value of the constant a, given that the minimum value of Q still occurs at the same corner point? (3 marks)

want min Q C(30, 15) < min Q (42, 13)

$30a + 30(15) < 42a + 30(13)$ ✓✓

$12a > 60$

$a > 5$ ✓

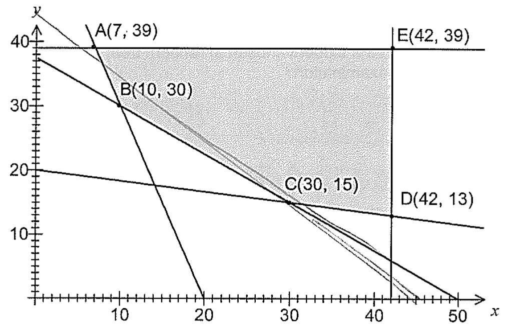
(e) An additional constraint $x + y \geq 45$ is imposed. How does this additional constraint affect the minimum value of Q in the feasible region? Give a reason to support your answer (2 marks)

No affect ✓

as C lies on the line $x + y = 45$. ✓

Question 16 **(10 marks)**

The feasible region of a linear programming problem is shown below.



The objective function is $Q = 15x + 30y$.

(a) Determine the inequality satisfied by x and y that corresponds to the edge AB of the feasible region. (2 marks)

line : $y = -\frac{9}{3}x + c$

$30 = -3(10) + c$

$c = 60$

$\therefore y = -3x + 60$ ✓

$y + 3x = 60$

$\therefore$ Inequality is $3x + y \geq 60$ ✓

(b) Determine the maximum value of Q in the feasible region. (1 mark)

Occurs @ E

$Q = 15(42) + 30(39)$

$= 1020$ ✓

(c) Determine the minimum value of Q in the feasible region. (2 marks)

Occurs @ C ✓, $Q = 15(30) + 30(15)$

$= 900$ ✓

A 1275
B 1050
C 900
D. 1020

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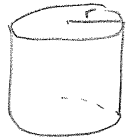
Question 7 **(7 marks)**

A closed cylindrical can of radius r cm has a volume of 250π cm^3.

(a) Show that the total surface area, A cm^2, of this can is given by $A = \frac{500\pi}{r} + 2\pi r^2$. (2 marks)



$V = \pi r^2 h$

$250\pi = \pi r^2 h \Rightarrow h = \frac{250}{r^2}$

$A = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi \cancel{r}\left(\frac{250}{r\cancel{^2}}\right)$

$= 2\pi r^2 + \frac{500\pi}{r}$

□

(b) Determine the minimum possible surface area of the can, in terms of π and the radius and height required to achieve this optimum area. (5 marks)

$\frac{dA}{dr} = 4\pi r - \frac{500\pi}{r^2}$

$-\frac{500\pi}{r^2} + 4\pi r = 0$

$4\cancel{\pi} r = \frac{500\cancel{\pi}}{r^2}$

$r^3 = \frac{500}{4}$

$r^3 = 125$

$r = 5$

$\frac{d^2A}{dr^2} = 4\pi + \frac{500\pi}{r^3}$

always +ve

$\therefore$ min

$@ r = 5, h = \frac{250}{25} = 10$

$\therefore A = \frac{500\pi}{5} + 2\pi(25)$

$= 100\pi + 50\pi$

$A = 150\pi$

i.e. $\boxed{\begin{array}{l} r = 5 \\ h = 10 \\ A = 150\pi \end{array}}$

End of Questions

Additional working space

Question number(s): ____________

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Question 15 **(9 marks)**

At the end of a technology course, all students sat a practical and a theory examination, with 20% achieving a distinction in the practical examination, 3% of students achieving distinctions in both examinations and 76% achieving no distinction in either examination.

(a) What is the probability that a student chosen at random from the course achieved a distinction in the theory examination? (~~4~~ 3 marks)

	T_D	$\overline{T}_D$	
P_D	0.03	0.17	0.2
$\overline{P}_D$	0.04	0.76	0.8
	0.07	0.93	1

✓✓

$P(T_D) = 0.07$ ✓

(b) Are the events 'achieving a distinction in the practical examination' and 'achieving a distinction in the theory examination' independent? Explain your answer. (~~2~~ 3 marks)

To be in $P(T_D) \times P(P_D) = P(T_D \cap P_D)$

Here $P(T_D) = 0.07$

$P(P_D) = 0.2$ ✓

$P(T_D \cap P_D) = 0.03$

$0.2 \times 0.07 \neq 0.03$ ✓

$\therefore$ Not independent ✓

(c) In a group of 14 students who took the course, three achieved a distinction in the practical examination. If five students are selected at random from this group, what is the probability that at least two of them achieved a distinction in the practical examination? (3 marks)

$$\frac{{}^3C_2\,{}^{11}C_3 + {}^3C_3\,{}^{11}C_2}{{}^{14}C_5}$$ ✓ ✓ ✓

$$= \frac{495 + 55}{2002}$$

$$= \frac{25}{91} \simeq 0.2747$$

Question 14 **(6 marks)**

A spherical snowball is melting at a rate of 18 litres per hour.

At the instant the volume of the snowball is 4 000 cm^3, calculate

(a) the rate of change of radius of the snowball, in cm per minute. (4 marks)

Given: $\frac{dV}{dt} = -18\ l/hr$ $18\,l/ = -18000\ mL/hr.$
$\therefore = -300\ mL/min$

Want: $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$ $V = \frac{4}{3}\pi r^3$ When $V = 4000$, $r = 9.847$

$\frac{dV}{dr} = 4\pi r^2$ ✓

$= -300 \times \frac{1}{4\pi(9.847)^2}$ ✓

$= -0.246\ cm/min$ ✓

(b) the rate at which the surface area of the snowball is decreasing, in cm^2 per minute. (2 marks)

$SA = 4\pi r^2$

Want $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ ✓

$= 8\pi(9.847) \times -0.246$

$= -60.93\ cm^2/min$ ✓

i.e. decreasing @ $60.9\ cm^2/min$

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Additional working space

Question number(s): ____________

SECTION 1

Question	Available Marks	Your Mark
1	4 ✓	
2	5 ✓	
3	8 ✓	
4	8 ✓	
5	9 ✓	
6	9 ✓	
7	7 ✓	
TOTAL	50 ✓	

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(b) A worker at the pottery took 150 of the defective mugs, filled them with soil and then planted four seeds in each. After 14 days, the number of seeds which germinated in each of the mugs was noted, with these results:

Number of germinating seeds	0	1	2	3	4
Number of mugs	1	9	16	57	67
	0.006	0.06	0.106	0.38	0.446

(i) What is the mean number of seeds germinating per mug? (1 mark)

$\bar{x} = 3.2$ ✓

(ii) Show that ~~What~~ is the probability of a ~~one~~ seed germinating is 0.8. (1 mark)

$E(X) = np$

$3.2 = 4p$

$p = 0.8$ ✓

(iii) Use an associated binomial distribution to calculate the theoretical frequency distribution for the number of seeds germinating in the 150 mugs and comment on how well your distribution models the observed results above. (3 marks)

$X \sim Bin(4, 0.8)$

	Expected.	Actual
$P(X=0) = 0.0016$	$\times 150 =$ 0	1
$P(X=1) = 0.0256$	4	9
$P(X=2) = 0.1536$	23	16
$P(X=3) = 0.4096$	61	57
$P(X=4) = 0.4096$ ✓	61 ✓	67

The theoretical results are a reasonably close match to the observed results, suggesting that the binomial model is appropriate. ✓

(Accept, not exact ∴ not appropriate)

Question 13 **(12 marks)**

(a) A pottery produces souvenir coffee mugs, of which it is known that 5% are defective.

(i) In a box of 24 mugs, what is the probability that there are at least 4 defectives? (2 marks)

$X \sim Bin(24, 0.05)$ ✓

$P(X \geq 4) = 0.02978$ (5dp). ✓

(ii) In a box of 12 mugs, what is the probability that there are no defectives? (1 mark)

$Y \sim Bin(12, 0.05)$ ✓

$P(Y = 0) = 0.54036$ (5dp) ✓

(iii) What is the probability that in 10 boxes, each containing 12 mugs, that either two or three of the boxes contain no defectives? (2 marks)

$W \sim Bin(10, 0.54036)$ ✓

$P(2 \leq W \leq 3) = 0.10824$ (5dp). ✓

(iv) The pottery decides to pack n mugs per box for wholesale clients, so that the chance of there being at least one defective mug in a box is no more than 50%. Find the largest value of n. (2 marks)

Want n st $P(X \geq 1) \leq 0.5$

$n = 13$ $\quad P(X \geq 1) = 0.487$ ✓

$\therefore n = 13$ ✓

See next page

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SOLUTIONS

PERTH COLLEGE

Year 12

Semester Two Examination 2012

Question/Answer booklet

MATHEMATICS 3C/3D

Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures

In words ______________

Your name ______________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens(blue/black preferred), pencils(including coloured), sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	$33\frac{1}{3}$
Section Two: Calculator-assumed	13	13	100	100	$66\frac{2}{3}$
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil**, except in diagrams.
5. The Formula Sheet is not handed in with your question/answer booklet.

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Question 12 **(6 marks)**

A body is moving in a straight line with velocity, v m/s, given by $v = 2t^2 - 19t + 30$, where t is the time, in seconds, since the body first passed through a fixed point P.

(a) Show that the body is stationary twice and find the distance travelled by the body between these two instants. (3 marks)

$0 = 2t^2 - 19t + 30$ ✓

$t = 2,\ 7.5$ ✓ $\therefore$ Stationary Twice

$$\int_2^{7.5} |(2t^2 - 19t + 30)|\, dt$$

$\simeq 55.46$ m (2dp) ✓

(b) At what other time(s), if any, does the body again pass through the fixed point P? (3 marks)

$s(t) = \frac{2t^3}{3} - \frac{19t^2}{2} + 30t + c$ ✓

Assume $s(0) = 0 \Rightarrow c = 0$

$\therefore \frac{2t^3}{3} - \frac{19t^2}{2} + 30t = 0$

$t = 0,\ 4.72,\ 9.53$ (2dp)

$\therefore$ Other times are 4.72 s ✓
& 9.53 s ✓

Question 11 **(7 marks)**

On the basis of the results obtained from a random sample of 81 bags produced by a mill, the 95% confidence interval for the mean weight of flour in a bag is found to be (514.56 g, 520.44 g).

(a) Find the value of $\bar{x}$, the mean weight of the sample. (1 mark)

$$\frac{514.56 + 520.44}{2} = 517.5g$$

(b) Find the value of σ, the standard deviation of the normal population from which the sample is drawn. (2 marks)

$$\bar{x} + \frac{z\sigma}{\sqrt{n}} = 520.44$$

$$517.5 + \frac{1.96\sigma}{\sqrt{81}} = 520.44$$

$$\sigma = \frac{26.46g}{1.96} = 13.5g$$

(c) Calculate the 99% confidence interval for the mean weight of flour in a bag. (2 marks)

$$\bar{x} - \frac{z\sigma}{\sqrt{n}} \le \mu \le \bar{x} + \frac{z\sigma}{\sqrt{n}}$$

$$517.5 - \frac{2.576 \times 13.5}{\sqrt{81}} \le \mu \le 517.5 + \frac{2.576 \times 13.5}{\sqrt{81}}$$

$$513.636 \le \mu \le 521.364$$

(d) Using the sample mean from (a) as the best estimate for the population mean, what is the probability that the sample mean of a larger sample of 225 bags is less than 516 g? (2 marks)

$\bar{x} \simeq 517.5$ $\quad S_{\bar{x}} = \frac{13.5}{\sqrt{225}} = 0.9$

$\bar{X} \sim N(517.5, 0.9^2)$

$P(\bar{X} < 516) = 0.0478$ (4dp)

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Section Two: Calculator-assumed **(100 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

(a) The curve $y = e^x$ is translated 1 unit in the direction of the positive x-axis followed by a dilation in the direction of the positive y-axis by a factor of 3. State the exact coordinates of the y-intercept of the transformed curve. (2 marks)

$y = 3e^{x-1}$

$\therefore$ @ $x=0$ $y = 3e^{-1}$ $\quad \therefore (0, \frac{3}{e})$

or

$(-1, \frac{1}{e}) \rightarrow (0, \frac{1}{e}) \rightarrow (0, \frac{3}{e})$

OR

(b) State a sequence of transformations that would transform the graph of $y = e^{2(x+1)}$ into the graph of $y = 2 - e^x$. (3 marks)

$y = -e^x + 2$

$y = e^{2(x+1)}$

1. Translate one unit right] undoes $e^{2(x+1)}$
2. Dilate horizontally factor 2] to give e^x
3. Reflect in x-axis
4. Translate 2 units up

-1 per error (or omission)

Question 9 **(7 marks)**

Atmospheric pressure, P (kPa), decreases approximately exponentially with increasing height h (m), above sea level according to the relationship $\frac{dP}{dh} = kP$, where k is a constant.

Atmospheric pressure at sea level is 101.3 kPa, and halves with every 5 800 m increase in height.

(a) Find the value of k, rounded to four significant figures. (2 marks)

$$\frac{dP}{dh} = kP$$

$$\therefore P = P_0 e^{kh}$$

$$0.5 = e^{5800k}$$ ✓

$$k = -0.0001195 \quad (4 \text{ sf})$$ ✓

(b) Calculate the atmospheric pressure at the top of a mountain of height 3 785 m. (2 marks)

$$P = 101.3e^{-0.0001195(3785)}$$

$$= 64.44 \text{ kPa} \quad (2dp)$$

(c) Use the increments formula to find the approximate change in pressure as a climber descends 250 m from the top of a mountain of height 3 785 m. (3 marks)

$$\frac{dP}{dh} \simeq \frac{\delta P}{\delta h} \text{ for small changes}$$

$$\frac{dP}{dh} = -0.0001195 \times 101.3e^{-0.0001195h}$$

$$\therefore \delta P \simeq -0.0001195 \times 101.3e^{-0.0001195h} \, \delta h$$ ✓

$$= -0.0001195 \times 101.3e^{-0.0001195h} \times -250$$ ✓

$$= 1.925$$ ✓

ie Change in pressure is $\simeq 1.92k$ kPa increase

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Question 10 **(9 marks)**

(a) Even numbers are to be formed using some, or all, of the digits 5, 6, 7, 8 and 9.

(i) How many even numbers can be formed in this way, if repetition of digits is not allowed? (3 marks)

1 digit + 2 digit + 3 digit + 4 digit + 5 digit

$$2 + 4\times2 + 4\times3\times2 + 4\times3\times2\times2 + 4\times3\times2\times1\times2$$ ✓✓

$$= 2 + 8 + 24 + 48 + 48$$

$$= 130$$ ✓

(ii) What fraction of the numbers in (i) start with a 9? (2 marks)

2-dig + 3 dig + 4 dig + 5 dig

$$1\times2 + 1\times3\times2 + 1\times3\times2\times2 + 1\times3\times2\times1\times2$$

$$= 2 + 6 + 12 + 12$$ ✓

$$= 32$$

ie. $32/130$ ✓

(b) The journey time for a driver between two depots is normally distributed with mean of 55 minutes and standard deviation of 4.5 minutes.

(i) If the driver makes four journeys every day, for five days a week, and for 48 weeks each year, how many of these journeys take less than an hour? (2 marks)

$$4\times5\times48 = 960$$

$$X \sim N(55, 4.5^2)$$

$$P(X < 60) = 0.86674$$ ✓

$$\therefore 960 \times 0.86674 \simeq 832 \text{ journeys}$$ ✓

(ii) What is the probability that a journey takes at least an hour, given that it takes less than 65 minutes? (2 marks)

$$P(X > 60 / X < 65)$$

$$= \frac{P(60 < X < 65)}{P(X < 65)}$$

$$= \frac{0.12012}{0.98687} = 0.1217 \quad (4dp).$$