Revision Examination Assessment Papers (REAP)
Semester 1 Examination 2012

Question/Answer Booklet
(This paper is not to be released to take home before 25/6/2012)

MATHEMATICS: SPECIALIST 3A

Section One:
Calculator-free

Name of Student: ________________________Marking key___________________

Time allowed for this section

Reading time before commencing work: 5 minutes
Working time for this section: 50 minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the student

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items: nil

Important note to students

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	12	12	100	100	
			Total	150	100

Instructions to students

1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2 **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

Section One: Calculator-free (50 marks)

This section has **six (6)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 50 minutes

Question 1 **(10 marks)**

Let P be point with polar coordinates $\left(4, \frac{2\pi}{3}\right)$ and O with co-ordinates (0,0)

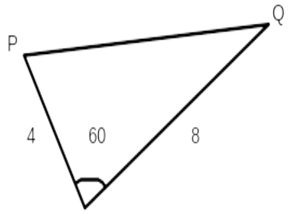
(i) State the **Cartesian** co-ordinates of P. (3)

Solution
x: $\cos 60^0 = \frac{x}{4}$, x = 2
y: $\sin 60^0 = \frac{y}{4}$, $y = 2\sqrt{3}$
Cartesian coordinates of P = $\left(-2, 2\sqrt{3}\right)$
Specific behaviours
✓ x-coordinate of 2 ✓ negative sign for x-coordinate ✓ y-coordinate of $2\sqrt{3}$

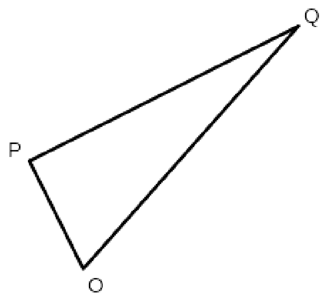
(ii) Q is a point which lies in the first quadrant such that it has Cartesian co-ordinates $\left(4, 4\sqrt{3}\right)$. State the **polar** co-ordinates of Q. (2)

Solution
$r = \sqrt{4^2 + \left(4\sqrt{3}\right)^2} = 8$
$\cos\theta = \frac{4}{8}$, $\theta = 60^0$ or $\tan\theta = \frac{4\sqrt{3}}{4}$, $\theta = 60^0$
Polar coordinates of Q = $\left(8, \frac{\pi}{3}\right)$
Specific behaviours
✓ "r" value of 8 ✓ argument of $\frac{\pi}{3}$ or 60^0

(iii) Hence, calculate the **exact** distance between P and Q. (3)

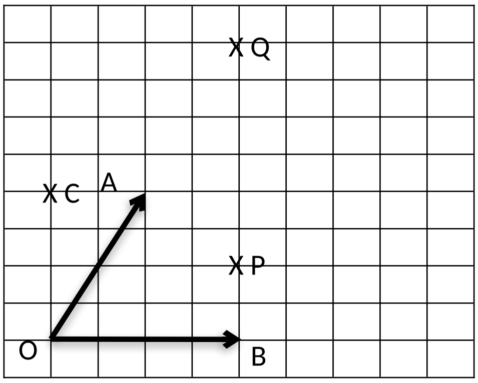
Solution
 $PQ^2 = 4^2 + 8^2 - 2(4)(8)\cos 60^0$ $PQ^2 = 16 + 64 - 64.\frac{1}{2}$ $PQ^2 = 48$ PQ = $\sqrt{48}$ or $4\sqrt{3}$
Specific behaviours
✓ use of Cosine Rule ✓ simplify the equation ✓ correct answer of $\sqrt{48}$ or $4\sqrt{3}$

(iv) Hence, or otherwise state what type of triangle is ΔPOQ ? Justify your answer. (2)

Solution
 $PQ^2 + PO^2 = 48 + 16 = 64 = OQ^2$ By Pythagoras' theorem, triangle is a right-angled triangle with angle P as 90^0
Specific behaviours
✓ state triangle is a right-angled triangle ✓ evidence of Pythagoras Theorem

Question 2 **(10 marks)**

(a)



It is given that $OA = 4$**a** and $OB = 4$**b**.

(i) If $BP = 2\underline{a} - \underline{b}$, mark the point *P* on the grid. (2)

Solution
$OP = OB + BP$ $= 4\underline{b} + 2\underline{a} - \underline{b}$ $= 2\underline{a} + 3\underline{b}$
Specific behaviours
✓ calculate OP correctly ✓ point P correctly marked on grid

(ii) *Q* is the point of intersection of *OA* and *BP* produced. Mark the point *Q* clearly on the grid. Hence state the value of *n* given that $OQ = nOA$. (2)

Solution
n = 2
Specific behaviours
✓ point Q correctly marked on grid ✓ correct value of n

(iii) If $\vec{CA} = \frac{1}{2}\vec{OB}$, mark the point *C* on the grid. (1)

Solution
$CA = \frac{1}{2} \cdot 4\underline{b} = 2\underline{b}$ $OC = OA + AC$ $= 4\underline{a} - 2\underline{b}$
Specific behaviours
✓ point C correctly marked on grid

(iv) If $OA = 2\underline{i} + 4\underline{j}$, express BC in $\underline{i} - \underline{j}$ component. (1)

Solution
From diagram $BC = -4\underline{i} + 4\underline{j}$
Specific behaviours
✓ correct expression for BC

(b) Solve the equation. (4)

$$2^{x+1} + 7 = \frac{2^2}{2^x}$$

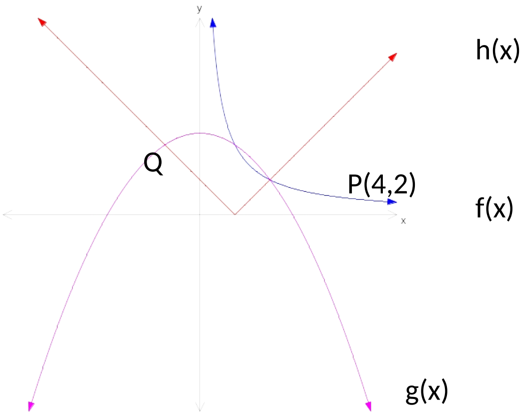
Solution
$2^x \cdot 2^{x+1} + 2^x \cdot 7 = 4$ $2^{2x}.2 + 7.2^x = 4$ $2\left(2^{2x}\right) + 7\left(2^x\right) - 4 = 0$ Let $y = 2^x$ $2y^2 + 7y - 4 = 0$ $(2y - 1)(y + 4) = 0$ $y = \frac{1}{2}$ or $y = -4$ i.e. $2^x = \frac{1}{2}$ $x = -1$ or $2^x = -4$ (not possible)
Specific behaviours
✓ multiply both sides by 2^x ✓ expresses equation as a quadratic equal to zero ✓ factorises correctly ✓ finds a solution which is $x = -1$

Question 3 **(9 marks)**

The graphs of *f(x)*, *g(x)* and *h(x)* are drawn below such that

- *f(x)* is a reciprocal function, $x>0$.
- *g(x)* is a quadratic function such that $6y = -x^2+28$
- *h(x)* is the absolute value function $h(x) = |x - m|$, where *m* is a constant

The point *P*(4,2) is the point of intersection of the three graphs.



(a) Determine the equation of *f(x)* (2)

Solution
$f(x)=\frac{8}{x}$
Specific behaviours
✓ general equation of a reciprocal function
✓ substitutes (4,2) to get k = 8 , hence $f(x)=\frac{8}{x}$

(b) Show that the value of *m* is 2. (3)

Solution
Solve $y=x-m$ and $y=\frac{8}{x}$ simultaneously i.e. $x-m=\frac{8}{x}$ $x^2-mx=8$ But (4,2) lies on f(x) and h(x) i.e. $4^2-4m=8$ $16-8=4m$ $m=2$ OR $2=\|4-m\|$ i.e. $m=2$ or $m=6$ From diagram, $m=2$ because the corner point is to the left of P
Specific behaviours
✓ use $y=x-m$ and $y=\frac{8}{x}$
✓ sub (4,2) into equation
✓ solve for m correctly

(c) Calculate the co-ordinates of *Q*. (4)

Solution
$y=\|x-m\|$ $-x+2=-\frac{x^2}{6}+\frac{28}{6}$ $-6x+12=-x^2+28$ $x^2-6x-16=0$ $(x+2)(x-8)=0$ $x=-2, x=8$ For Q, $x=-2$, y = 4 Coordinates of Q = (-2, 4)
Specific behaviours
✓ equate h(x) and g(x)
✓ solve for x
✓ ignore x = 8 as Q lies in 2nd quadrant
✓ coordinates of Q as (-2, 4)

Question 4 **(6 marks)**

If $f(x) = (x-1)^2 - 4,\ x \le 1$ and $g(x) = \sqrt{x+4},\ x \ge -4$

(i) Find *f*(-2). (1)

Solution
$f(-2) = 5$
Specific behaviours
✓ correct answer of 5

(ii) Determine and simplify $g\left[f(x)\right]$. (3)

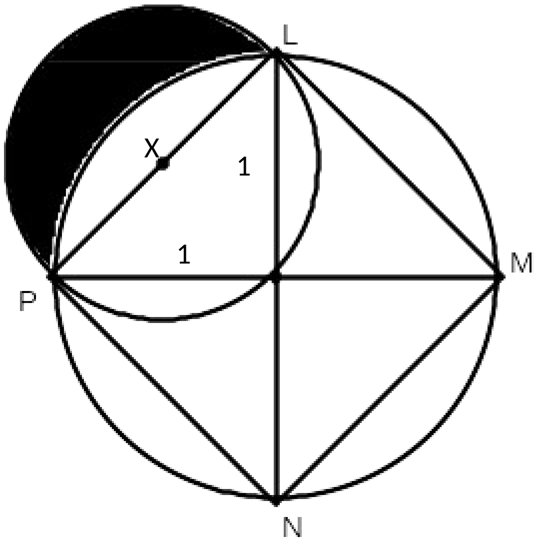
Solution
$g\left(x^2 - 2x + 1 - 4\right)$ $= \sqrt{\left(x^2 - 2x - 3 + 4\right)}$ $= \sqrt{\left(x^2 - 2x + 1\right)}$ $= \sqrt{(x-1)^2}$ $= \lvert x - 1 \rvert$ $= 1 - x$ for $x \le 1$
Specific behaviours
✓ determines the correct function composition ✓ simplifies correctly ✓ correct answer of $1 - x$ for $x \le 1$

(iii) State the natural domain and corresponding range of $g\left[f(x)\right]$. (2)

Solution
$D_{gof} = \{x : x \le 1, x \in \mathbb{R}\}$ $R_{gof} = \{y : y \ge 0, y \in \mathbb{R}\}$
Specific behaviours
✓ correct domain ✓ correct range

Question 5 **(6 marks)**

In the diagram below, LMNP is a square whose diagonals are each 2cm long. MP and LP are diameters of the bigger and smaller circles respectively. Find the perimeter of the shaded region, expressing your answer in surd form.



Solution
Note: The diagonals of a square bisect each other at right angles Let X be the centre of smaller circle LP = $\sqrt{2}$ PX=LX = $\frac{\sqrt{2}}{2}$ Semi- circle of smaller circle: LP = $\frac{1}{2}\times 2\pi \times \frac{\sqrt{2}}{2}$ = $\frac{\sqrt{2}\pi}{2}$ Arc LP of bigger circle = $\frac{90}{360}\times 2\pi \times 1 = \frac{\pi}{2}$ Perimeter of shaded region = $\frac{\sqrt{2}\pi}{2}+\frac{\pi}{2}=\frac{\pi}{2}\left(\sqrt{2}+1\right)$
Specific behaviours
✓ length of LP ✓✓ semi –circle LP ✓✓ arc LP ✓ perimeter of shade region

Question 6 **(9 marks)**

(a) Show that. (3)

$$1+4\left(\frac{\log 3}{\log 4}\right)\left(\frac{\log \frac{1}{2}}{\log 9}\right)=0$$

Solution
LHS = $1+\frac{4\log 3}{2\log 2}\cdot\frac{-\log 2}{2\log 3}$ = $1+\frac{-4}{4}$ = 1 – 1 = 0
Specific behaviours
✓✓ express $\log 4$ as $2\log 2$, $\log 9$ as $2\log 3$, $\log\frac{1}{2}$ as $-\log 2$ ✓ simplify to zero

(b) Solve for *x*. (3)

$$x\log 7=\log\frac{5}{7}+x\log 5$$

Solution
$x\log 7-x\log 5=\log\frac{5}{7}$ $x\log\frac{7}{5}=-1\left(\log\frac{7}{5}\right)$ $x=-1$ OR $x\log 7=\log 5-\log 7+x\log 5$ $x(\log 7-\log 5)=-(\log 7-\log 5)$ $x=-1$
Specific behaviours
✓ express $\log\frac{5}{7}=\log 5-\log 7$ ✓ group LIKE terms ✓ simplify to $x=-1$

(c) If $2^{5.322}=40$ determine $\log_2 80$. (3)

Show all calculations. (Hint: Let $x=\log_2 80$)

Solution
$x=\log_2 80$ $2^x=80$ $2^x=2\times 40$ $\frac{2^x}{2}=40$ $2^{x-1}=40$ i.e. $x-1=5.322$ $x=6.322$
Specific behaviours
✓ express "log" equation as an exponential equation ✓ simplify to $2^{x-1}=40$ and equate $x-1=5.322$ ✓ correct answer of $x=6.322$