9. (3, 2, 3, 1 = 9 marks)

A particle's is moving with rectilinear motion and its position can be modelled by the function $v(t) = 3t^2 - 12t + 9$ for $0 \le t \le 4$, where v is measured in metres/seconds and t is measured in seconds.

a) Determine when the velocity of the particle is maximised.

$v(t) = 3(t-3)(t-1)$ ✓

Reasoning

Working ✓

$t = 0, t = 4$ ✓

b) If the particle is initially at the origin determine an expression for the displacement.

$$x(t) = t^3 - 6t^2 + 9t + c$$
$$x(0) = 0 \quad \therefore c = 0$$ ✓
$$\therefore x(t) = t^3 - 6t^2 + 9t$$ ✓

c) Determine the total distance travelled in the first 3 seconds.

$v(t) = 0 \quad @ \quad t = 1 \quad \& \quad t = 3$ ✓

$x(0) = 0$

$x(1) = 4$ ✓

$x(3) = 0$

$\therefore$ dist $= 8$ m ✓

d) Determine the change in displacement in the 2nd second.

-2 units ✓

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Time:	25 minutes
Total Marks:	24 marks

Year 12 Test 2

Thursday 29th April 2021

Resource Free

ClassPad calculators are Not permitted.

Formulae Sheet is Permitted.

Name:

1. (2 , 2 = 4 marks)

Differentiate the following with respect to x. Do not simplify.

a) $3x^2e^x$

$6xe^x + 3x^2e^x$

✓ ✓

b) $3e^{2x^3+1}$

$3e^{2x^3+1} \cdot 6x^2$

✓ ✓

2. (2, 2 , 2, 1 = 7 marks)

-1 mark for $2 \times$ missing c

a) Evaluate the following $\int \frac{\sqrt{x}+x}{x}\,dx$.

$$\int x^{-\frac{1}{2}}+1\,dx$$
$$=2x^{\frac{1}{2}}+x+c$$

b) Find Q in terms of p given that $\frac{dQ}{dp}=4-\frac{6}{p^3}$ and $Q=-3$ when $p=1$.

$$\int 4-6p^{-3}dx$$
$$=4p+3p^{-2}+c$$

$$Q(1)=-3$$
$$\therefore c=-10$$
$$\therefore Q=4p+\frac{3}{p^2}-10$$

c) $\int 2x^3e^{x^4}\,dx$

$$\frac{1}{2}\int 4x^3e^{x^4}dx$$
$$\frac{1}{2}e^{x^4}+c$$

d) $\frac{d}{dx}\int_{-2}^{x}\frac{t^2+3}{\pi-\sqrt{t}}\,dt$

$$\frac{x^2+3}{\pi-\sqrt{x}}$$

7. (1, 2 = 3 marks)

A population changes such that $\frac{dP}{dt}=-0.12P$, where t is in years.

a) Is the the population growing or decaying?

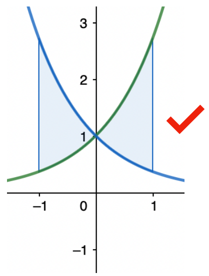
Decaying

b) If the population is 120 000 after 8 years. Calculate (to the nearest 1000) the original population.

$$P=P_0\cdot e^{-0.12t}$$
$$120000=P_0\cdot e^{-0.12(8)}$$
$$P_0=313000$$

8. (4 marks)

Given $f(x)=e^x$ and $g(x)=e^{-x}$ find the **exact** area of the regions enclosed by the two functions, $x=-1$ and $x=1$. Show the use of a sketch in your solution.



$$\text{Area } =2\int_0^1 e^x-e^{-x}dx$$
$$=2\left[e^x+e^{-x}\right]_0^1$$
$$=2\left[\left(e^1+e^{-1}\right)-(1+1)\right]$$
$$=2\left(e^1-\frac{1}{e}-2\right)$$
$$=2e+2e^{-1}-4$$

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Time:	20 minutes
Total Marks:	20 marks

Year 12 Test 2

Thursday 29th April 2021

Resource Assumed

ClassPad calculators are permitted.
Formulae Sheet is Permitted.

Name:

6. (2, 2 = 4 marks)

An imaginary radioactive isotope Coraronium decays at a rate of $\frac{dA}{dt} = -0.14A$ where A (kg) is the amount of Coraronium remaining and t is in years.

a) If 2 kg of Coraronium exists originally, determine how much will remain after 10 years.

$A = 2e^{-0.14t}$

$A(10) = 0.49\ kg$

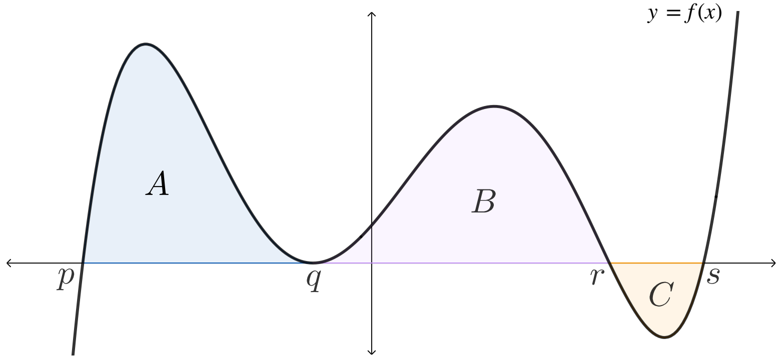
b) Determine the half life of Coraronium, that is the time it takes for the radioactive isotope to be reduced to 50%.

$\frac{1}{2} = e^{-0.14t}$

$t = 4.95$ years

3. (1, 1, 2 = 4 marks)

The three regions between the curve $y = f(x)$ and the x-axis have areas of A, B, and C units² as shown below. Determine the following definite integrals.



a) $\int_p^s f(x)\,dx$

$A + B - C$

b) $\int_p^s 2f(x)\,dx$

$2(A + B - C)$

c) $\int_r^p f(x) + 1\,dx$

$-A - B + p - r$

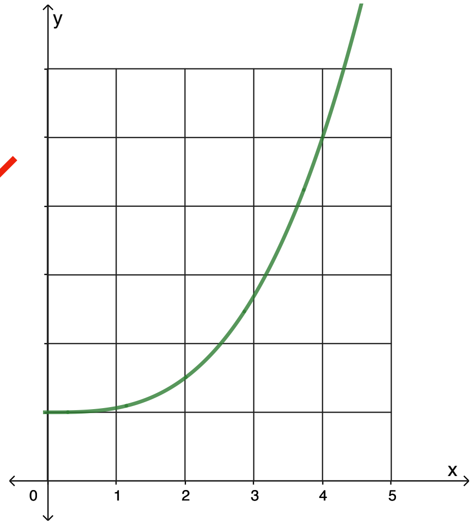
4. (2 marks)

The function $f(x) = x^3 + 1$ is shown below.

a) Using the under-estimate with widths of 1 unit, approximate the area under $f(x)$ for $1 \leq x \leq 3$. Show all working.

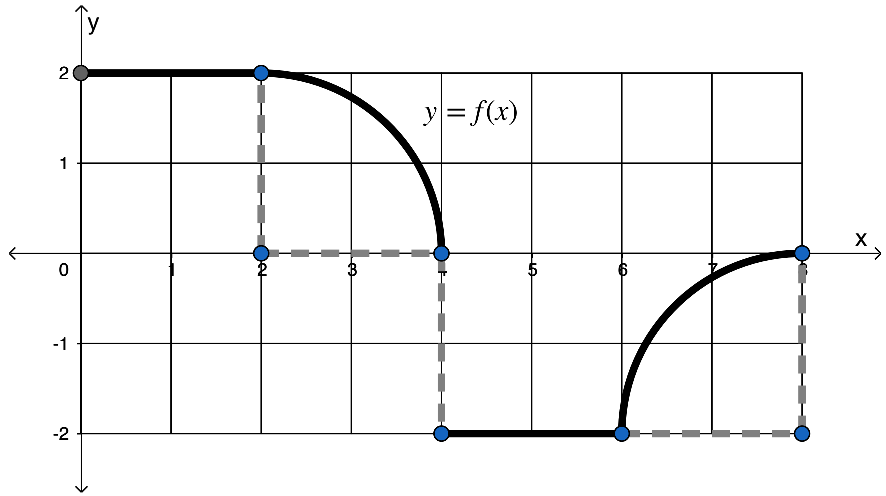
$(1)(1^3+1)+(1)(2^3+1) = 2+9$

$= 11$ units ^{2}



5. (2, 2, 3 = 7 marks)

The function $f(x)$ is shown below.



a) Use the graph above to determine the following in exactly.

i. $\int_0^4 f(x)\,dx$

$\pi + 4$

ii. $\int_4^8 f(x)\,dx$

$-8+\pi$

iii. If $\int_k^8 f(x)\,dx = 0$, solve for k.

$2(2-k)+\pi = 8-\pi$

$4-2k+\pi = 8-7)$

$\frac{2\pi - 4}{2} = k$

$k = \pi - 2$

$k = 8$