Question/Answer Booklet

MATHEMATICS 3CD
Section Two
(Calculator Assumed)

Circle your teacher's initials

GJ   JIB   BAH

Your name _____

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 90 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section Two.
Formula sheet.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.
Special items: drawing instruments, templates, notes on two unfolded sheets of A4
                    paper, and up to three calculators satisfying the conditions set by the
                    Curriculum Council for this course.

Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that
you do not have any unauthorised notes or other items of a non-personal nature in the
examination room. If you have any unauthorised material with you, hand it to the supervisor
before reading any further.

Structure of this examination

	Number of questions	Working time (minutes)	Marks available
Section 1 Calculator Free	6	50	40
This Section (Section 2) Calculator Assumed	**10**	**90**	**70**
		Total marks	110

Instructions to candidates

1. The rules for the conduct of WACE external examinations are detailed in the booklet *WACE Examinations Handbook*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions in the spaces provided.

3. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

4. Show all working clearly. Any question, or part question, worth more than 2 marks requires valid working or justification to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

Question 7 **(8 marks)**

a) A discrete random variable X can take on values 0 , 1 or 2:

$$P(X=x) = \binom{5}{x} \ldots \quad \text{for x = 0, 1 or 2}$$

P (X= x) =   for x = 0 , 1 or 2

   i) Show, in the form of a table, the probability distribution for X

[3]

   ii) Find P(X < 1 | X ≤ 2)

[1]

b) A continuous random variable Y has the triangular probability density function *f:*

$$f(y)= \begin{cases} 0 & \text{for } y<0 \\ b(10-y) & \text{for } 0\leq y\leq 10 \end{cases}$$

   i) Find the constant **b.**

[2]

   ii) Find $P(Y\geq 1))$

[2]

Question 8 **(8 marks)**

3 boarding housemasters and 8 day house masters are to sit in the front row at assembly.

a)   In how many ways can they be arranged in a row

    (i) without restriction?

[1]

    (ii) if a boarding housemaster must be on each end of the row?

[2]

    (iii) if the boarding housemasters must sit together?

[2]

b)   Find the probability that the Buntine housemaster must *not* sit next to the Parry housemaster.

[3]

Question 9 **(5 marks)**

A ladder 7m long rests against a vertical wall and is standing on flat ground. The bottom of the ladder is being pulled along the ground and towards the wall at a steady rate of 0.1 m/s. How fast is the top sliding up the wall when the bottom is 2m out of the wall?

Question 10   **(11 marks)**

The foreman in a brick factory knows that the sand used for casting is too dry 5% of the time, and too wet 2% of the time.
He also knows that defects occur 0.5% of the time when the sand is perfect, 5% when sand is too dry, and 25% of the time when the sand is too wet.

a)   Represent the above information using a probability tree and showing all relevant probabilities.

[3]

b) Find the probability that a randomly selected casting

   (i) was defective if we know that the sand was wet.

[2]

   (ii) was defective and the sand was wet.

[2]

   (iii) was defective

[2]

c)	Find the probability that a randomly selected casting, which was found to be defective, had sand which was too wet.

[2]

Question 11 **(3 marks)**

Consider the equation $y = 8x^3$. When x increases from 25 to 25.1, show

that the approximate increase in y obtained using $\delta y = \dfrac{dy}{dx} \bullet \delta x$ is 1500.

Question 12   **(7 marks)**

Packets of soap powder are labelled as weighing 850 grams. However, the actual weights of the packets are normally distributed with a mean of 870 grams and standard deviation of 30 grams.

a)   Find the probability that a randomly chosen packet weighs less than the labelled weight.

                                                             [1]

b)   A consumer group is lobbying to have the following legislation introduced: if any packet weighs less than the labelled weight by more than 30 grams then it is deemed unacceptable and the manufacturers should be prosecuted.

       What percentage of those packets weighing less than the labelled weight would be deemed unacceptable?

                                                             [3]

c)   The company claims that 95% of its packets lie within a range of weights that it would consider acceptable. Find to the nearest whole number of grams the range of weights that the company would consider acceptable.

                                                             [3]

Question 13 **(8 marks)**

A particle moves in a straight line such that its position s(t) metres from a fixed point O at time t seconds is given by the equation s(t) = $18t - 3t^2$, where $t \geq 0$.

Find:

a) when and where its velocity is zero

[3]

b) the acceleration at this time

[2]

c) the distance travelled in the first 4 seconds

[3]

End of examination

Question 14

(10 marks)

The Nibbly's Nuts Company produces two different bags of fruit & nut mixes. One is sold under the company name (Nibbly's Nuts) whilst the other is sold as a 'budget' brand.

Each 1 kg bag of fruit & nut mix contains different quantities of the two ingredients, nuts and dried fruit. The table below shows these mixtures along with the cost and sell price information.

Let x represent the number of 1 kg bags of the Nibbly's Nuts mix, and Let y represent the number of 1 kg bags of the Budget mix.

	Nuts	**Dried fruit**	**Cost per kg**	**Sell per kg**
Nibbly's Nuts (x)	600g	400g	$1.80	$2.90
Budget (y)	800g	200g	$1.50	$2.70

Each week the company has a maximum amount of 1600 kg of nuts and 920kg of dried fruit available to be processed into the 1 kg bag mixes.
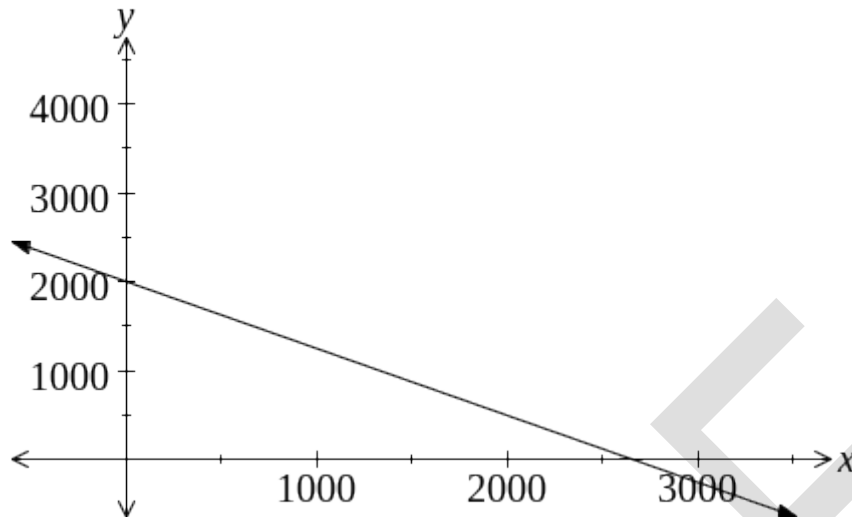
The company also plans to make at least twice as many packets of the Nibbly's Nuts mix as the Budget mix.

a) Write down the three inequalities (other than $x \geq 0$ and $y \geq 0$) that represent the constraints in the above situation.

[2]

b) One of these constraints is already on the graph below. Add the other constraints and shade in the feasible region.

[3]

See next page

c) How many 1 kg bags of each mix should the Nibbly's Nuts Company produce in order to maximise profits and what is this maximum profit?

[3]

d) By how much should the Nibbly's Nuts Company increase the sell price of the Budget mix before production numbers change from those found in (c)?

[2]

End of examination

Question 15 (5 marks)

a) Sketch the gradient function graph of $y = f(x)$ on the same set of axes provided.

[3]
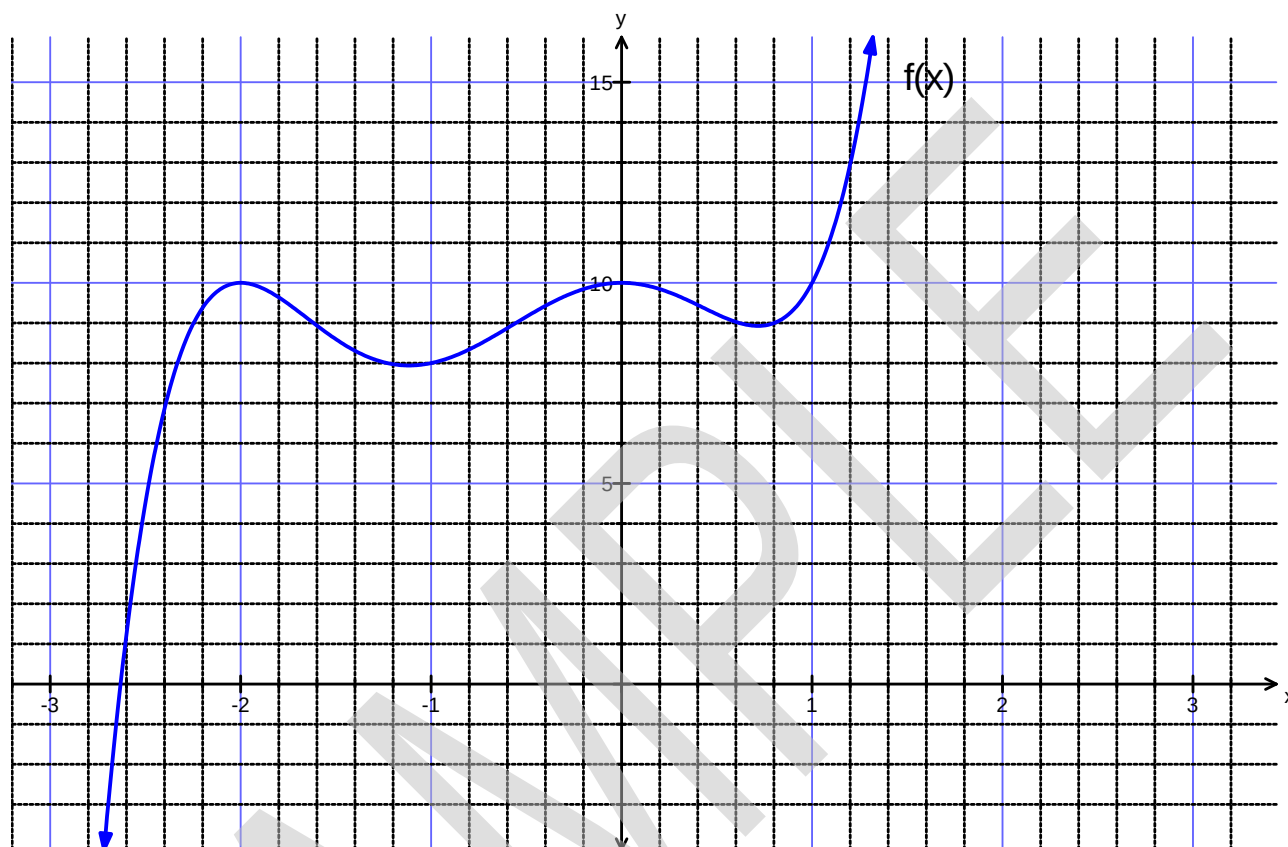
b) Give the exact coordinates of the points of inflection of the function $y = 2x^5 + 7x^3 - 12$

[2]

See next page

Question 16 **(5 marks)**

The car trip from Perth to Northam takes about 100 minutes. A random variable X is the number of minutes (in excess of 100) which it takes to make the trip from Perth to Northam. The probability distribution is modelled by

$$f(x) = \begin{cases} \dfrac{1}{20}\left(1-\dfrac{x}{20}\right); & 0 \leq x \leq 20 \\[2mm] \dfrac{1}{20}\left(1+\dfrac{x}{20}\right); & -20 \leq x < 0 \\[2mm] 0; & \text{otherwise} \end{cases}$$

a) Show that f is a probability function

[2]

b) What is the significance of the negative values of x?

[1]

c) Determine the probability that the trip takes more than 95 minutes given that it took less than 100 minutes.

[2]