PENRHOS COLLEGE

STRIVE FOR THE HIGHEST

Penrhos College
Semester 2 Examination, 2010

Question/Answer Booklet

MATHEMATICS SPECIALIST: 3C/3DMAS

Section Two: Calculator-assumed

Student Name:_____

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for section: 100 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section Two. Candidates may use the removable formula sheet from Section One

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special items: drawing instruments, templates, notes on up to two unfolded sheets of A4 paper, and up to three calculators, CAS, graphic or scientific, which satisfy the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Instructions to candidates

1. **All** questions should be attempted.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare answer pages may be found at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued (i.e. give the page number).

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

See next page

Section Two: Calculator-assumed **(80 Marks)**

This section has **eleven (11)** questions. Answer **all** questions. Write your answers in the space provided.

Suggested working time for this section is 100 minutes.

Question 8 **(6 marks)**

(a) Express in polar form the cube roots of $\frac{-\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$. [3]

(b) An equilateral triangle is to have its vertices on an Argand diagram on a circle, centre (0,0), radius 2. One vertex represents a complex number with argument $\frac{\pi}{6}$. Find, in polar form, the numbers represented by the vertices, and the Cartesian equation which has these three numbers for roots.. [3]

DO NOT WRITE IN THIS AREA

Question 9 **(8 marks)**

(a) Let $A=\begin{bmatrix}-4 & 20 & 2\\ -3 & 15 & -3\\ 7 & -17 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3 & 5\\ 1 & 1 & 1\\ 3 & -4 & 0\end{bmatrix}$.

Determine $C=A\times B$. [1]

(b) Tickets to a concert cost \$2 for children, \$3 for teenagers and \$5 for adults. 570 people attended the concert and the total ticket receipts were \$1950. The ratio of teenagers to children attending was 3 to 4.

Use your answer to (a) to determine how many children, teenagers and adults attended the concert. [7]

Question 10 **(9 marks)**

Water flows into and leaks out of a container such that $\frac{dx}{dt} = -k(2x-1)$, where the depth of the water is x metres at time t seconds. At $t = 0$, the depth is 0.75 m and is decreasing at a rate of 0.01 ms^{-1}.

(a) Show that $k = 0.02$. [2]

(b) Determine the time at which the depth will be 0.55 m. [7]

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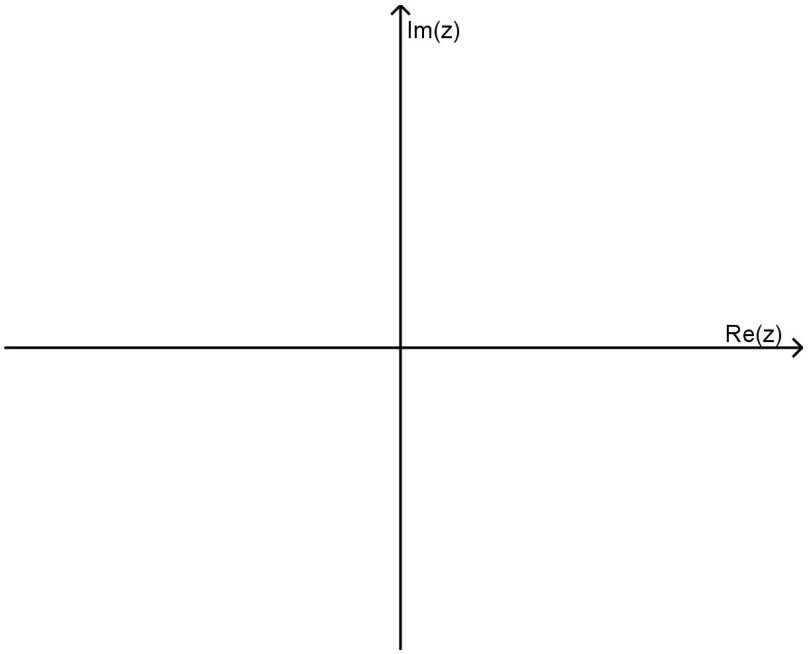
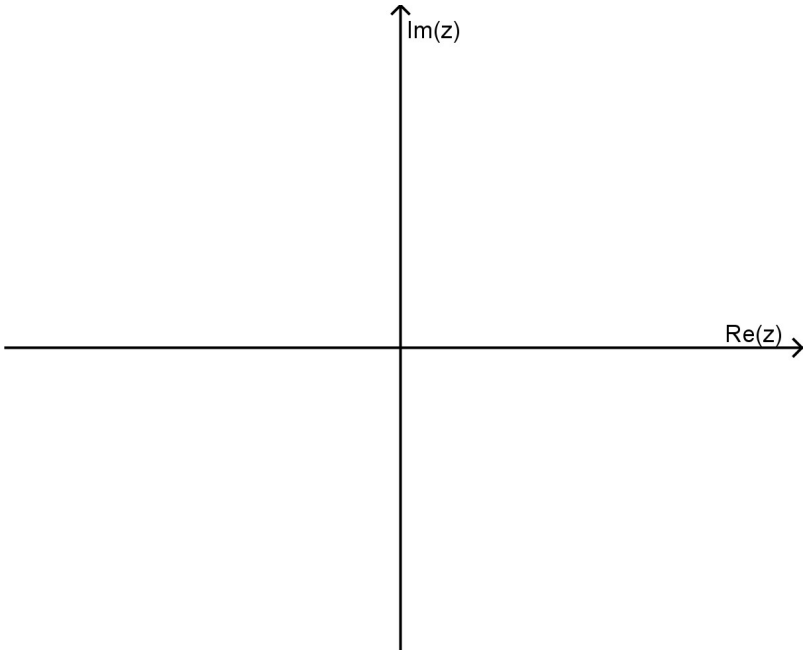
Question 11 **(10 marks)**

(a) Sketch in the complex plane the graph of $\arg\left(\frac{1}{z}\right)=\frac{3\pi}{4}$. [3]



(b) (i) Sketch the locus represented by $|z-2i|\leq 4$. [2]



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(ii) Hence, find the exact greatest and least values of $¿z-3+4i\vee¿$ given that $|z-2i|\leq 4$. [3]

(c) The complex number $x+iy$ is such that $arg(x+iy)=\theta$, where $x>0$, $y>0$. Find, in terms of θ and π, the values of

(i) $arg(-x+iy)$ [1]

(ii) $arg(-y+ix)$ [1]

See next page

Question 12 **(6 marks)**

The table below shows the details of a population of kangaroos in a region of Western Australia in 2000.

Age (years)	0 – 2	2 – 4	4 – 6	6 - 8	8 - 10
Initial population	1200	1400	1600	810	425
Breeding Rate	0	0.1	3.5	2.5	0.5
Survival Rate	0.4	0.5	0.7	0.2	0

(a) Write down the Leslie matrix, L, for this population. [1]

(b) What is the total population in 2006? [1]

(c) Find the percentage growth rate between the 3rd and 4th generation. [2]

Due to complaints from local property owners, it is decided that culling is necessary.

(d) To reduce the population growth, 10 % of kangaroos aged between 4 and 8 years are culled at the start of every second year. What will the population be in 2016? [2]

Question 13 **(7 marks)**

The lines l_1 and l_2 have equations

$$r=\begin{pmatrix}3\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\4\end{pmatrix} \text{ and } r=\begin{pmatrix}1\\-1\\1\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$$

respectively, where λ and μ are parameters.

(a) Find the acute angle between l_2 and the line joining the points P$(1,-1,1)$ and Q $(2,-1,-4)$, giving your answer correct to the nearest degree. [2]

(b) Determine the position vector of the point R that lies on the line joining P$(1,-1,1)$ and Q$(2,-1,-4)$ such that PR : RQ = 1 : 2. [3]

(c) Find an equation of the plane Π through Q$(2,-1,-4)$ and perpendicular to l_1, in the form $r\cdot n=\rho$. [2]

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Question 14 **(7 marks)**

The path of a particle is defined by the parametric equations

$$x = 120t - 4t^2, \quad y = 60t - 6t^2.$$

The path crosses the $x-$¿axis at $t = a$ and at $t = b$.

(a) Determine the value of a and b. [2]

(b) Determine the value of $\frac{dy}{dx}$ at $t = a$ and at $t = b$ [5]

Question 15 **(6 marks)**

(a) Use de Moivre's Theorem to show that $\cos n\theta = \frac{1}{2}\left(z^n + z^{-n}\right)$ and

$\sin n\theta = \frac{1}{2i}\left(z^n - z^{-n}\right)$, where $z = \cos\theta + i\sin\theta$. [3]

(b) Use (a) to prove the identity, $\sin 3\theta \cos 2\theta = \frac{1}{2}\left(\sin 5\theta + \sin\theta\right)$ [3]

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Question 16 **(5 marks)**

Use mathematical induction to show that

$$n! > 2^n$$

for all positive integers $n \geq 4$.

Note: $n! = 1 \times 2 \times 3 \times \ldots \times n$

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Question 17 **(10 marks)**

A rocket ship leaves space station A which is located at $\begin{pmatrix} -20 \\ 40 \\ 20 \end{pmatrix}$ km at 9 am with a constant velocity of $\begin{pmatrix} 60 \\ 120 \\ 360 \end{pmatrix}$ km h^{-1}. It is supposed to reach the neighbouring space station B, which is located at $\begin{pmatrix} 80 \\ 160 \\ 2020 \end{pmatrix}$ km.

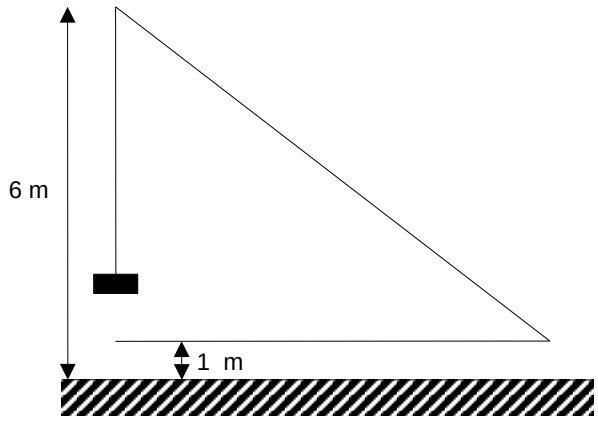
(a) Determine whether or not this rocket ship reaches space station B. If not, find the closest distance between the rocket ship and space station B and when this occurs to the nearest minute. [6]

A second rocket ship is also launched from space station B at 9 am with constant velocity and is aimed to collide with the first rocket at exactly 1 pm.

(b) Determine the velocity of the second rocket ship that will ensure collision takes place at the required time. [4]

Question 18 **(6 marks)**

A weight W is attached to a rope 16 m long that passes over a pulley at point P, 6 m above the ground. The other end of the rope is attached to a truck at a point A, 1 m above the ground, as shown in the diagram.



(a) Show that $y=\sqrt{25+x^2}-11$ represents the distance in metres the weight is above point B, given x m represents the horizontal distance from point B to the truck. [2]

(b) If the truck moves away at the rate of 3 ms^{-1}, how fast is the weight rising when it is 2 m above the ground? [4]

Additional working space

Question number(s):_____

Additional working space

Question number(s):_____

Additional working space

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