

to ensure

No other items may be used in this section of the examination. It is your responsibility

Important note to students

Council for this examination

Curriculum

and up to three calculators satisfying the conditions set by the

A4 paper,

Special items: drawing instruments, templates, notes on two unfolded sheets of

highlighters

fluid/tape, ruler,

Standard items: pens, pencils, pencil sharpener, eraser, correction

To be provided by the student

Formula Sheet (retained from Section One)

This Question/Answer Booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time for this section:

Reading time before commencing work:

10 minutes

100 minutes

Name of Student: _____

Marking Key: _____

Calculator-assumed**Section Two:****MATHEMATICS 3C**

(This paper is not to be released to take home before 25/6/2012)

Question/Answer Booklet**Semester 1 Examination 2012****(REAP)****Revision Examination Assessment Papers****MATHEMATICS 3C****CALCULATOR-ASSUMED**

Solution	$\int_{\frac{3}{4}}^{\frac{5}{4}} f(x) dx$
Exact value of	$\left(\frac{1}{2} \times \pi \times 2^2 + \left(-\frac{1}{2} \times 2 \times 1 \right) + \left(\frac{1}{2} \times 1 \times 1 \right) \right)$
Specific behaviours	$= 2\pi - \frac{1}{2}$

MATHEMATICS 3C**CALCULATOR-ASSUMED**

that you do not have any unauthorised notes or other items in the examination room.
If you have any unauthorised material with you, hand it to the supervisor **before**
reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	12	12	100	100	
Total			150	100	

Instructions to students

- 1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- 2 **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you

- (ii) the area between the curve and the x-axis from $x = -3$ to $x = 0$ (1)

Solution
Area = $75 + 30 = 105$
Specific behaviours
✓ or X

Question 18 (continued)

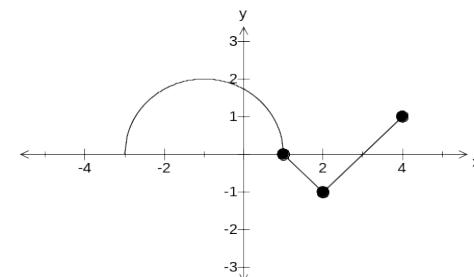
- (iii) $\int_{-3}^2 f(x) dx$ (1)

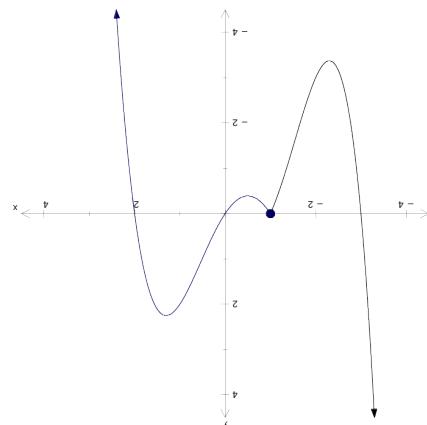
Solution
$-74 - 30 + 50 = -55$
Specific behaviours
✓ or X

- (b) The graph of a function $f(x)$ consists of a semi-circle and two line segments as shown.

$$\int_{-3}^4 f(x) dx$$

Find the exact value of (2)



Question 18(a) For the function $y = f(x)$ below**(6 marks)**

- repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.

3

3

wishes to have marked.

repeat an answer to any question, ensure that you cancel the answer you do not

$$\int_0^1 f(x) dx = \int_0^2 xp(x) dx$$

of the following.

Use the information above and mathematical reasoning to determine the value of each

The area under the curve from $x = -1$ to $x = 2$ is 80 square units.

$$\int_{-1}^2 f(x) dx = 80$$

$$\int_{-1}^1 f(x) dx = 75$$

It is known that

Specific behaviours	
Solution	
$\int_0^1 f(x) dx = -30$ Solving the two equations simultaneously, $p = 30$, $q = 50$ $-p + q = 20$, $p + q = 80$ $b = xp(x)$, $f(x) = xp(x)$, $d = b$	$\int_0^1 f(x) dx = -30$ $\int_0^1 f(x) dx = -30$ $\int_0^1 f(x) dx = -30$

(2)

$$\int_0^1 f(x) dx$$

**Section Two: Calculator-assumed
(100 marks)**

This section has **twelve (12)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

Question 7 (10 marks)

- (a) Emily is a very strong soccer player who has a probability of $\frac{3}{5}$ of scoring a goal with each attempt. She has 15 attempts. Find the probability that the number of goals she scores is less than 7.
(2)

Solution
$P(X < 7) = P(X \leq 6) = B\left(15, 6, \frac{3}{5}\right) = 0.0950$
Specific behaviours
✓ identifies Binomial distribution
✓ correct probability

- (b) Suppose that Y is distributed normally with unknown mean μ and standard deviation σ .

Given that $P(\mu - 2.5 \leq Y \leq \mu + 2.5) = 0.9$, find the value of σ . (2)

Solution
$1.645 = \frac{\mu + 2.5 - \mu}{\sigma} \quad \text{or} \quad -1.645 = \frac{\mu - 2.5 - \mu}{\sigma}$
$\sigma = 15.20$
Specific behaviours
✓ z score of 1.645
✓ correct value for σ

- (c) Alice, Bronwyn and Cathy independently each think of an integer in the set {1, 2, 3, 4, 5, 6, 7}

Find the **probability** that, of the three integers selected,

Specific behaviours
✓ or X

(1)

- (i) all three are greater than 4

(ii)

Question 17 (continued)

(i) If t represents time in years and S represents tooth size, find the value of K .

$S = S_0 e^{kt}$
$0.99S = S_0 e^{1000k}$
$i.e., 0.99 = e^{1000k}$
$K = -0.000\ 01$
Specific behaviours
$S = S_0 e^{-0.000\ 01t}$
Solution
\checkmark solves correctly for k

(ii) In how many years will human tooth size be 90% of their present size?

$S = S_0 e^{kt}$
$0.9 = e^{-0.000\ 01t}$
$t = 10\ 536\ \text{years}$
Specific behaviours
$0.9 = e^{-0.000\ 01t}$
\checkmark solves correctly for t
Solution

(iii) What will be our descendants tooth size 20 000 years from now? (1)

$S = S_0 e^{-0.000\ 01 \times 20000}$
$S = 0.818750$
Solution

(as a percentage of our present tooth size)

Question 7 (continued)

- (c) (ii) all three integers are greater than 5

(1)

- (iii) the least integer is 5.

(1)

- (iv) the three integers are different given that the least integer selected is 5.

(2)

- (v) the sum of the three integers is more than 15.

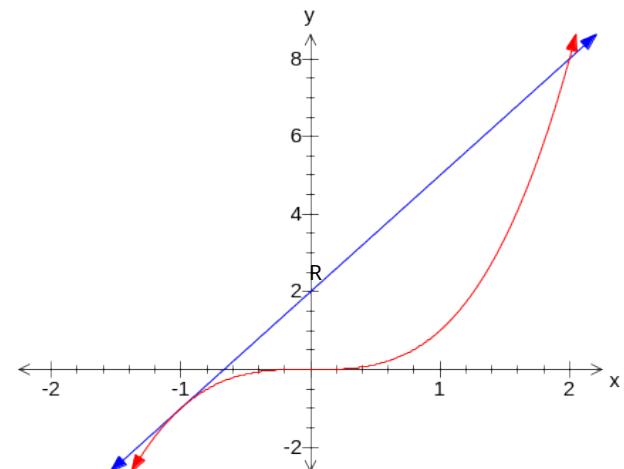
(1)

Solution	
(i)	$\frac{27}{343}$ ✓
(ii)	$\frac{8}{343}$ ✓
(iii)	$\frac{27}{343} - \frac{8}{343} = \frac{19}{343}$ ✓
(iv)	$\frac{6}{19}$ ✓✓
(v)	$\frac{56}{343}$ ✓
Specific behaviours	
As above	

Question 17**(9 marks)**

- (a) Shade the region, R bounded by the curves
- $y = x^3$
- ,
- $y = 3x + 2$
- , and
- $x = 0$
- in the diagram.

Find the area of the region R, showing all working steps. (4)



Solution	
Area of region R =	$\int_{-1}^0 (3x + 2 - x^3) dx$
	$= \left[\frac{3x^2}{2} + 2x - \frac{x^4}{4} \right]_{-1}^0$
	$= \frac{3}{4} \text{ units}^2$
Specific behaviours	
✓ correct shading of R	
✓ integrates each term correctly	
✓ correct answer	

Solution	
Specific behaviours	Fastest when $t = 0$, $\frac{dy}{dt} = -1$
Specific behaviours	Slowest when $t = 12$, $\frac{dy}{dt} = 0$
Values of $\frac{dy}{dt}$	times

(2)

- (ii) When is the fluid in the tank draining fastest and slowest?
 What are the values of $\frac{dy}{dt}$ at these times?

Solution	
Specific behaviours	$\frac{dy}{dt} = -\left(1 - \frac{t}{12}\right)$
Specific behaviours	$\frac{dy}{dt}$

(2)

- (i) Find the rate $\frac{dy}{dt}$ m/hour at which the tank is draining at time, t .

Solution	
Specific behaviours	$y = 6\left(1 - \frac{t}{12}\right)^2$ metres.
Specific behaviours	depth y of fluid in the tank, t hours after the valve is opened by given by
1 mark each	$x = -1.5$ and $x = 2$

(2)

- (a) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth y of fluid in the tank, t hours after the valve is opened by given by

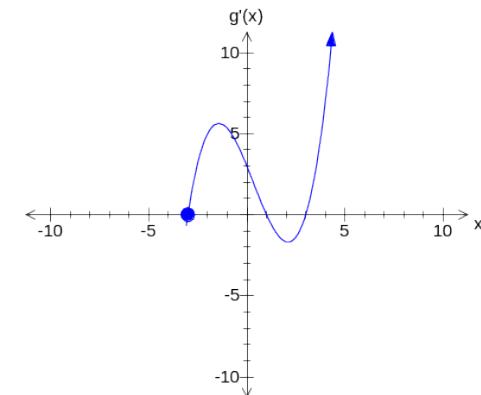
Question 8 (7 marks)

Question 8 (continued)

- (b) If the volume of a cylinder is given by $V = 2\pi r^3$, find the appropriate percentage

change in V when r changes by $\frac{1}{2}\%$
(3)

Solution
$\frac{\delta r}{r} = \frac{5}{1000}$
$\frac{dV}{dr} = 6\pi r^2$
$\delta V = \frac{dV}{dr} \times \delta r$
$\therefore \frac{\delta V}{V} = \frac{6\pi r^2}{2\pi r^3} \times \delta r$
$\frac{\delta V}{V} = 3 \times \frac{\delta r}{r}$
$\therefore \frac{\delta V}{V} = 3 \times \frac{5}{1000} \times 100\% = 1.5\%$
Specific behaviours
$\frac{dV}{dr} = 6\pi r^2$
✓ $\frac{dV}{dr} = 6\pi r^2 > 0$
✓ $\frac{\delta V}{V} = \frac{6\pi r^2}{2\pi r^3} \times \delta r = 3 \times \frac{\delta r}{r}$
✓ correct answer of 1.5 %



- (a) What can be said about the gradient of the function $g(x)$ between $x = -3$ to $x = 1$?
(1)

Solution
Gradient is positive
✓ or X

- (b) When does the function, $g(x)$ have a negative gradient? (2)

Solution
$1 < x < 3$
✓✓ correct interval

- (c) State an equation for the tangent to the graph of $g(x)$ at $x = 3$. (1)

Solution
Horizontal line $y = k$ where k is a constant
✓ or X

- (d) Find the value of x at which $g(x)$ has a relative maximum for $-3 \leq x \leq 4$ (1)

Solution
$x = 1$
✓✓ correct answer

		Solution
(2)		$P(X < 10 X > 3)$

		Solution
		$\frac{33}{36} = \frac{11}{12}$

		Solution
(2)		$P(X > 3)$

		Solution
(b)		The probability distribution of x where random variable, x is the sum of the uppermost numbers when two fair die are rolled is tabulated below.

		Solution
(b)		The manufacturer better off justify.

		Solution
(b)		<ul style="list-style-type: none"> - Different probabilities for same value of x i.e. $P(X=3) = 0.05$ and $P(X=3) = 0.1$ - Sum of all the probabilities is greater than 1 (1.15)

		Solution
(2)		$P(X=x) = 0.05$ 0.1 0.4 0.1 0.2 0.3

		Question 9
(10)		(a) Give two reasons why the following cannot be a probability distribution. (b) Do the packages now contain bolts that are more consistent in length?

		CALCULATOR-ASSUMED
		MATHEMATICS 3C

		CALCULATOR-ASSUMED
		MATHEMATICS 3C

Question 16

The graph of $g(x)$ is given below.

		Find
(i)		$P(X > 3)$

		Yes
		<ul style="list-style-type: none"> ✓ justification

		(iii)
		Is the manufacturer better off justify.

		(ii)
		<ul style="list-style-type: none"> ✓ valid reason

		(i)
		Do the packages now contain bolts that are more consistent in length?

		INVERSE NORMCDF
		<ul style="list-style-type: none"> ✓ "a" and "b" values

$$\frac{P(3 < X < 10)}{P(X > 3)} = \frac{\cancel{27}/36}{\cancel{11}/12} = \frac{9}{11}$$

Specific behaviours

✓✓

- (iii) If event A is $X > 3$ and event B is $X < 10$, are these two events independent? Justify your answer.

(4)

Solution

$$P(A \cap B) = \frac{27}{36}, P(A) = \frac{33}{36}, P(B) = \frac{30}{36}$$

$$\text{Now } \frac{11}{12} \times \frac{30}{36} = \frac{55}{72} \neq \frac{27}{36}$$

$$\text{i.e. } P(A) \times P(B) \neq P(A \cap B)$$

\therefore A and B are not independent events

Specific behaviours

$$\checkmark P(A \cap B) = \frac{27}{36}$$

$$\checkmark P(A) = \frac{33}{36}, P(B) = \frac{30}{36}$$

$$\checkmark \text{ shows that } P(A) \times P(B) \neq P(A \cap B)$$

$$\checkmark \text{ states not independent}$$

Question 10

(7 marks)

- (a) The function $f(x)$ is differentiable for all $x \in R$ and satisfies the conditions

$$f'(x) < 0 \text{ where } x < 2$$

$$f'(x) = 0 \text{ where } x = 2$$

$$f'(x) = 0 \text{ where } x = 4$$

$$f'(x) > 0 \text{ where } 2 < x < 4$$

$$f'(x) > 0 \text{ where } x > 4$$

- (i) Draw a sketch of this function $f(x)$.

(3)

Question 15
marks)

(11

Nuts and Bolts Company manufactures 120mm bolts which are normally distributed with a mean length of 120mm and a standard deviation of 1mm. Only bolts which are between 118.6mm and 121.4mm pass inspection and are packaged as 120mm bolts.

- (a) Find the probability of a randomly selected bolt being an acceptable length. (2)

Solution

$$P(118.6 \leq X \leq 121.4) = NCDF(118.6, 121.4, 1, 120) = 0.838487$$

Specific behaviours

✓✓

- (b) Find the expected number of acceptable bolts in a batch of 100 000 (1)

Solution

$$0.838487 \times 100000 = 83849$$

Specific behaviours

✓ or X

- (c) Is this a reasonable outcome for the company? Justify your answer. (2)

Solution

$$\frac{83849}{100000} \times 100\% = 83.85\%$$

% of acceptable bolts = 83.85%
 % of unacceptable bolts = 16.15% which is too high – too much waste
 \therefore outcome is not really reasonable.

Specific behaviours

✓ % of unacceptable bolts

✓ outcome not reasonable

- (d) A new quality controller suggests adjusting the settings on the machines so that the standard deviation becomes 0.85mm and that only the shortest 5% and the longest 5% of the bolts are rejected.

- (i) Find the new minimum and maximum acceptable lengths correct to the nearest 0.1mm. (3)

Solution

Solution

(3)

$$\int_a^b f(x) dx = a \left[\frac{5}{x} + 3 \right]_a^b, \text{ find } 2 \int_a^b f(x) dx$$

(b)

$$\int_a^b f(x) dx = a$$

✓ or X

specific behaviours

True

Solution

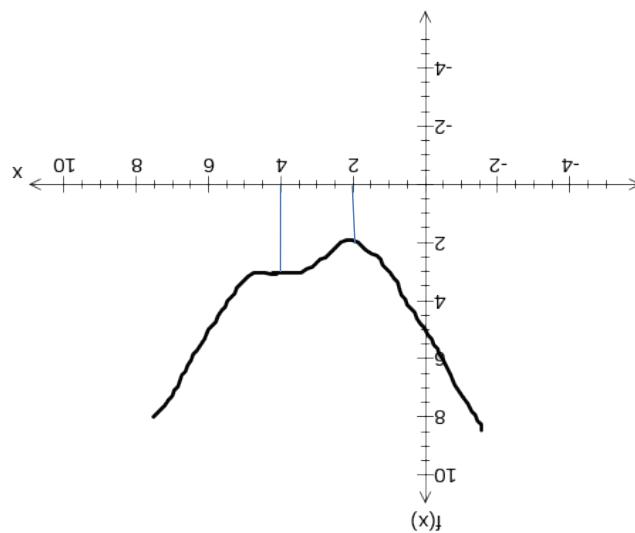
(1)

- "The graph $f(x)$ has a stationary point of inflection where $x=4$ ".
 (ii) State whether the following statement is true or false.

As above

specific behaviours✓ point of inflection at $x = 4$ ✓ turns at $x = 2$

✓ shape

Solution

✓ the two lengths of $\left(\frac{32}{7}\right)$ and $\left(\frac{24}{7}\right)$

Now if $u = \frac{x}{5}$, $5u = x$

$$\int_0^{5a} f\left(\frac{x}{5}\right) dx = \int_0^a f(u) du = 5a$$

$$2 \int_0^{5a} \left[f\left(\frac{x}{5}\right) + 3 \right] dx$$

$$= 2 \int_0^{5a} f\left(\frac{x}{5}\right) dx + 2 \int_0^{5a} 3 dx$$

$$= 2(5a) + 2(15a) = 40a$$

Specific behaviours

$\checkmark \int_0^{5a} f\left(\frac{x}{5}\right) dx = \int_0^a f(u) du = 5a$

$\checkmark 2 \int_0^{5a} \left[f\left(\frac{x}{5}\right) + 3 \right] dx$

\checkmark integrates correctly

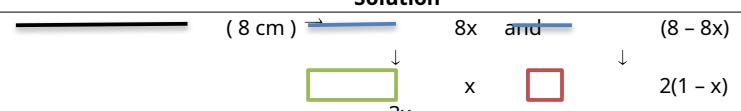
Question 14

(9 marks)

A piece of wire 8cm long is cut into two unequal parts. One part is used to form a rectangle that has a length three times its width. The other part of the wire is used to form a square.

- (i) If the width of the rectangle is x units, determine an equation that will give the sum of the areas of the rectangle and the square in terms of x .

(4)

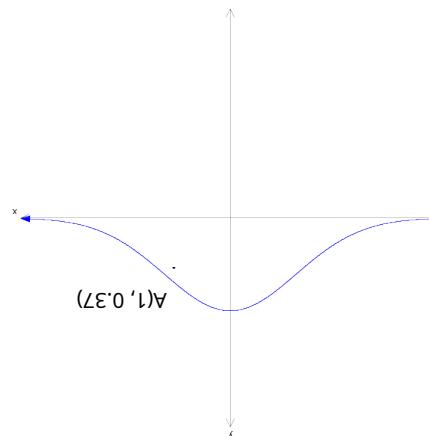
Solution
 <p>Area, $A = 3x^2 + (2 - 2x)^2$ $= 7x^2 - 8x + 4$</p>
Specific behaviours
<p>$\checkmark \checkmark$ expressions for $8x$ and $(8 - 8x)$</p> <p>$\checkmark \checkmark$ areas of rectangle and square</p>

- (ii) Using Calculus find the length of each part of the wire when the sum of the areas is a minimum.

(5)

Solution
$A = 7x^2 - 8x + 4$ $\frac{dA}{dx} = 14x - 8$ $\frac{d^2A}{dx^2} = 14 \Rightarrow \text{Min}$ $14x - 8 = 0$ $x = \frac{4}{7}$ $x = 4 \frac{4}{7} \left(\frac{32}{7} \right) \text{ and } 3 \frac{3}{7} \left(\frac{24}{7} \right)$ Lengths of each part of wire are
Specific behaviours
<p>\checkmark first derivative</p> <p>\checkmark second derivative to confirm x value gives a minimum area</p> <p>\checkmark x value</p>

(2)

(i) Sketch the graph of $y = f(-x+1)$ (b) The curve C has equation $y = e^{-x^2}$ and is drawn below

Solution
$y = -e^{-(x-1)^2} = -e^{-(x-1)(x+1)} = -e^{-(x^2-x-1)}$
\forall transformation in order
The curve C has equation $y = e^{-x^2}$ and is drawn below

(3)

(a) Describe the transformation in order.

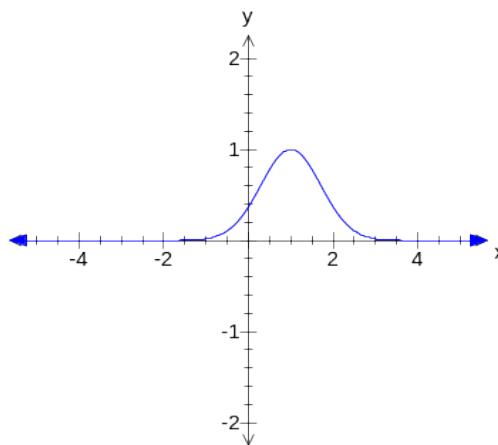
(a) The function $y = e^{-(x-1)(x+1)}$ is transformed to $y = -e^{-(x-z)}$ Question 11
(7 marks)

(d) Adam's little brother, Brodie joins in this business venture. The probability that Brodie's painted gnomes is Regular is 0.8. He wants to ensure that the probability that he paints at least two Superior is at least 0.9. Calculate the minimum number of gnomes that Brodie would need to paint to achieve this aim.

(3) any one of

Using CAS to solve, $n = 17.95$
i.e. $C_0(0.2)^n + C_1(0.2)(0.8)^{n-1} \leq 0.1$
$P(X \leq 1) \leq 0.1$
$P(X \geq 2) \geq 0.9$
$P(R) = 0.8, P(S) = 0.2$
Solution
Using CAS to solve, $n = 17.95$
i.e. $C_0(0.2)^n + C_1(0.2)(0.8)^{n-1} \leq 0.1$
Specific behaviours
∴ minimum number is 18

Specific behaviours
$P(X \leq 1) \leq 0.1$
$1 - P(X \leq 1) \geq 0.9$
$P(R) = 0.8, P(S) = 0.2$
Solution
Using CAS to solve, $n = 17.95$
i.e. $C_0(0.2)^n + C_1(0.2)(0.8)^{n-1} \leq 0.1$
Specific behaviours
∴ minimum number is 18



(c) A group of 3 consecutive garden gnomes is inspected and the first is a Regular.
It is also
found that of these three gnomes,

$$\begin{aligned}P(\text{no Superior}) &= 0.09 \\P(\text{1 Superior}) &= 0.28 \\P(\text{2 Superior}) &= 0.63\end{aligned}$$

Find the expected number of these gnomes that will be Superior. (2)

Solution
Expected number = $0 \times 0.09 + 1 \times 0.28 + 2 \times 0.63 = 1.54$
Specific behaviours
✓ calculation
✓ correct answer of 1.54

Solution
Reflected across the y-axis and right 1 unit
Specific behaviours
✓ shape
✓ passes through points (1,1) and (0, 0.37)

Question 11 (continued)

$$(ii) \quad \text{State the coordinates of A if the curve is transformed to} \quad y = -f\left(\frac{1}{2}x\right) + 2$$

(2)

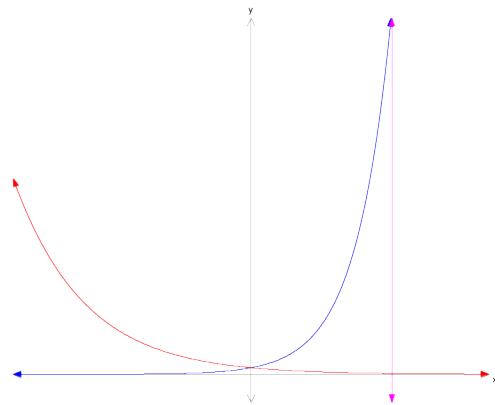
Solution
$(1, 0.37) \rightarrow (2, 0.37) \rightarrow (2, -0.37) \rightarrow (2, 1.63)$
Coordinates of A = (2, 1.63)
Specific behaviours
✓✓ 1 mark each for x-coordinate and y-coordinate

Question 12		9 marks)
Calculator		
(a) A company produces fruit balls coated in either dark chocolate or milk chocolate. A large number of these fruit balls are placed in a box. Twenty percent of the fruit balls in the box are coated with dark chocolate.	(i) Calculate $C_{10}^4 (0.2)^4 (0.8)^6$	(1)
Adam paints garden gnomes to sell. He sends the garden gnomes to his father (a superlative, or 'regular', depending on the quality of their finish).	If the garden gnome is Superior, then the probability that the next garden gnome is Superior is 0.9. If the garden gnome is Regular, then the probability that the next garden gnome is Superior is 0.7.	(2)
(ii) A random sample of ten fruit balls is taken from the box.	If the first garden gnome inspected is Superior, find the probability that the next third garden gnome is Superior.	(a)
Explain the meaning of $C_{10}^4 (0.2)^4 (0.8)^6$ with respect to this sample.	If the first garden gnome is Superior, then the probability that the next garden gnome is Regular.	(b)
In a sample of 10 fruit balls, the probability of picking exactly 4 coated in dark chocolate is approximately 0.0881	P(SRR) + P(SR) = (0.9)x(0.1) + (0.1)x(0.3) = 0.12	(1)
Using CAS, $n = 8$		(1)
Solution	Specific behaviours	Using CAS, $n = 8$
Solution	Specific behaviours	(i)
balls.	from 8	
(iii) Explain the meaning of your answer to part (b)(i) with respect to the fruit balls.	✓ picking none	
The probability of picking no dark chocolate fruit ball from 8 is 0.1677216	✓ or X	
Solution	Specific behaviours	

Question 13		8 marks)
Calculator-Assumed		
MATHEMATICS 3C	MATHEMATICS 3C	
CALCULATOR-ASSUMED	CALCULATOR-ASSUMED	
(a) Adam paints garden gnomes to sell. He sends the garden gnomes to his father (a superlative, or 'regular', in the order of completion), who classifies them as either 'superior' or 'regular', depending on the quality of their finish.	If the garden gnome is Superior, then the probability that the next garden gnome is Superior is 0.9. If the garden gnome is Regular, then the probability that the next garden gnome is Superior is 0.7.	(1)
(b) If the first garden gnome inspected is Superior, find the probability that the next three garden gnomes are Superior.	If the first garden gnome is Superior, find the probability that the next three garden gnomes are Superior.	(b)
Solution	Specific behaviours	Solution
Solution	Specific behaviours	

Question 12 (continued)

- (c) The curve $y = e^{2x}$ and $y = e^{-x}$ intersect at the point (0, 1) as shown in the diagram.



Find the area enclosed by the curves and the line $x=2$.

Leave your answer in terms of ' e '.

(3)

Solution
$\text{Required area} = \int_0^2 e^{2x} dx - \int_0^2 e^{-x} dx$ $= \left[\frac{e^{2x}}{2} - (-e^{-x}) \right]_0^2$ $= \frac{e^4}{2} + e^{-2} - \frac{3}{2}$
Specific behaviours
$\int_0^2 e^{2x} dx - \int_0^2 e^{-x} dx$ <ul style="list-style-type: none"> ✓ integrates each term correctly ✓ substitutes limits of integration to get exact value of $\frac{e^4}{2} + e^{-2} - \frac{3}{2}$