Working out space

PERTH MODERN SCHOOL
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Independent Public School

Course _____Specialist_____ Year __12_____

Student name: _____ Teacher name: _____

Date: 24 Feb

Task type: **Response**

Time allowed for this task: **_____45_____ mins**

Number of questions: **_____8_____**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **__50____ marks**

Task weighting: **_10___%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3.1.1, 3.1.2, 3.1.3) (2, 2, 3 & 3 = 10 marks)

If $z = 2 + 3i$ and $w = -1 + 2i$ determine exactly the following. (Simplify)

a) $z\overline{w}$

b) $w\overline{w}$

c) $w \div \overline{w}$

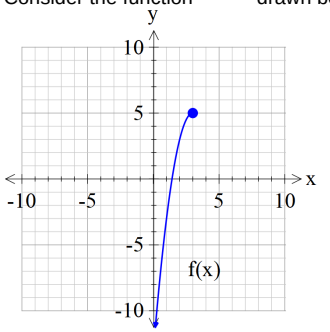
d) $\frac{1}{z} + \frac{1}{w}$

Q2 (3.1.3) (3 marks)

Determine all possible real values of $a \& b$ such that $\frac{43 - i}{a + 4i} = 5 + bi$

Q8 (3.2.3, 3.2.4) (2 , 3 & 3 = 8 marks)

Consider the function $f(x)$ drawn below.



a) Sketch $y = f^{-1}(x)$ on the axes above.

b) Given that $f(x) = -2x^2 + 12x - 13, \quad x \leq 3$, determine the defining rule for $y = f^{-1}(x)$. Show working for full marks.

c) Consider the function $h(x) = ax^3$ where a is a positive constant. Solve in terms of a, the solution(s) to $h(x) = h^{-1}(x)$.

Q7 (3.2.1, 3.2.2) (1, 2, 2 & 2 = 7 marks)

Consider the functions $f(x)=\sqrt{x-8}$ & $g(x)=x^3$.

a) Give the defining rule for $f\circ g(x)$.

b) Does $f\circ g(x)$ exist over the natural domain of $g(x)$? Explain

c) State the natural domain and range for $f\circ g(x)$.

Consider the function $h(x)=x-8$.

d) Does the function $[f(x)]^2=h(x)$? Justify your answer.

Q3 (3.1.14, 3.1.15) (3& 3 = 6 marks)

Consider the quadratic equation $x^2+bx+c=0$ where $b\&c$ are real.

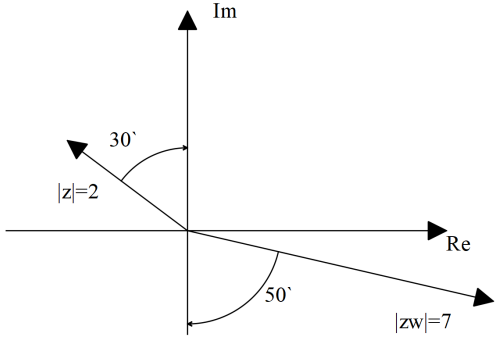
a) If one root of the above equation is $x=4-2i$, determine $b\&c$.

Consider the equation $x^3+px^2+qx+w=0$ where $p,q\&w$ are real.

b) If the cubic equation above has roots $x=2\&x=\sqrt{3}i$, determine $p,q\&w$.

Q4 (3.1.3, 3.1.3, 3.1.3) (2 marks)

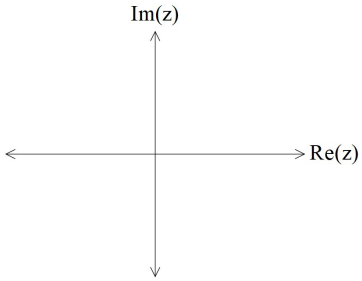
Determine $z\&w$ in the form $rcis\theta$ with $-\pi<\theta\leq\pi$. (Note: diagram not drawn to scale)



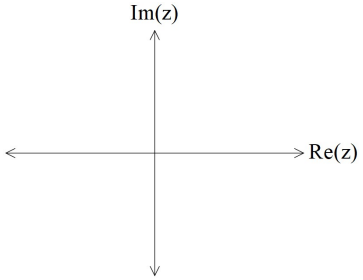
Q5 (3.1.10) (2, 2 & 3 = 7 marks)

Sketch the following regions in the complex plane showing major features.

a) $Arg(z) = \frac{3\pi}{4}$



b) $|z + 3 + 4i| \geq |z - 5 + i|$



c) Consider all the complex numbers z that satisfy $|z - (2 + 5i)| = 3$, determine the maximum possible value of $Arg(z)$, giving your answer in radians correct to two decimal places.

Q6 (3.1.7, 3.1.12) (4 & 3=7 marks)

a) Determine all the roots of $z^5 = \sqrt{3} + i$ expressing in the form $rcis\theta$ with $-\pi < \theta \leq \pi$.

b) Plot all of these roots on the diagram below.