Revision Examination Assessment Papers (REAP)

Semester 1 Examination 2012

Question/Answer Booklet

(This paper is not to be released to take home before 25/6/2012)

MATHEMATICS 3C

Section Two:
Calculator-assumed

Name of Student: ____________________Marking key________________________

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for this section: 100 minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the student

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

Important note to students

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	12	12	100	100	
			Total	150	100

Instructions to students

1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2 **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you

repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

Section Two: Calculator-assumed
(100 marks)

This section has **twelve (12)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

Question 7 **(10 marks)**

(a) Emily is a very strong soccer player who has a probability of $\frac{3}{5}$ of scoring a goal with each attempt. She has 15 attempts. Find the probability that the number of goals she scores is less than 7. (2)

Solution
$P(X<7)=P(X\leq 6)=B\left(15,6,\frac{3}{5}\right)=0.0950$
Specific behaviours
✓ identifies Binomial distribution ✓ correct probability

(b) Suppose that Y is distributed normally with unknown mean μ and standard deviation σ.
Given that $P(\mu-2.5\leq Y\leq\mu+2.5)=0.9$, find the value of σ. (2)

Solution
$1.645=\frac{\mu+2.5-\mu}{\sigma}$ or $-1.645=\frac{\mu-2.5-\mu}{\sigma}$ $\sigma=15.20$
Specific behaviours
✓ z score of 1.645 ✓ correct value for σ

(c) Alice, Bronwyn and Cathy independently each think of an integer in the set {1, 2, 3, 4, 5, 6, 7}
Find the **probability** that, of the three integers selected,

(i) all three are greater than 4 (1)

Question 7 (continued)

(c) (ii) all three integers are greater than 5 (1)

(iii) the least integer is 5. (1)

(iv) the three integers are different given that the least integer selected is 5.
(2)

(v) the sum of the three integers is more than 15. (1)

Solution		
(i)	$\frac{27}{343}$	✓
(ii)	$\frac{8}{343}$	✓
(iii)	$\frac{27}{343} - \frac{8}{343} = \frac{19}{343}$	✓
(iv)	$\frac{6}{19}$	✓✓
(v)	$\frac{56}{343}$	✓
Specific behaviours		
As above		

Question 8 **(7 marks)**

(a) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth 'y' of fluid in the tank ,'t' hours after the valve is opened is given by

$y = 6\left(1 - \frac{t}{12}\right)^2$ metres.

(i) Find the rate $\frac{dy}{dt}$ m/hour at which the tank is draining at time, t. (2)

Solution
$\frac{dy}{dt} = -\left(1 - \frac{t}{12}\right)$
Specific behaviours
✓✓

(ii) When is the fluid in the tank draining fastest and slowest?

What are the values of $\frac{dy}{dt}$ at these times? (2)

Solution
Slowest when t = 12, $\frac{dy}{dt} = 0$ Fastest when t = 0, $\frac{dy}{dt} = -1$
Specific behaviours
✓ times ✓ values of $\frac{dy}{dt}$

Question 8 (continued)

(b) If the volume of a cylinder is given by $V = 2\pi r^3$, find the appropriate percentage change in V when r changes by $\frac{1}{2}$ %

(3)

Solution
$\frac{\delta r}{r} = \frac{5}{1000}$ $\frac{dV}{dr} = 6\pi r^2$ $\delta V = \frac{dV}{dr} \times \delta r$ $\therefore \frac{\delta V}{V} = \frac{6\pi r^2}{2\pi r^3} \times \delta r$ $\frac{\delta V}{V} = 3 \times \frac{\delta r}{r}$ $\therefore \frac{\delta V}{V} = 3 \times \frac{5}{1000} \times 100\% = 1.5\%$
Specific behaviours
✓ $\frac{dV}{dr} = 6\pi r^2$ ✓ $\frac{\delta V}{V} = \frac{6\pi r^2}{2\pi r^3} \times \delta r = 3 \times \frac{\delta r}{r}$ ✓ correct answer of 1.5 %

Question 9 **(10 marks)**

(a) Give two reasons why the following cannot be a probability distribution. (2)

x	3	1	2	3	5	0
P(X=x)	0.05	0.1	0.4	0.1	0.2	0.3

Solution
- Different probabilities for same value of x i.e. P(X=3) = 0.05 and P(X=3) = 0.1 - Sum of all the probabilities is greater than 1 (1.15)
Specific behaviours
✓✓ 1 mark each

(b) The probability distribution of x where random variable, X is the sum of the uppermost numbers when two fair die are rolled is tabulated below.

x	2	3	4	5	6	7	8	9	10	11	12
P(X=x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Find

(i) $P(X > 3)$ (2)

Solution
$\frac{33}{36} = \frac{11}{12}$
Specific behaviours
✓✓

(ii) $P(X < 10 \mid X > 3)$ (2)

Solution

$\frac{P(3<X<10)}{P(X>3)} = \frac{27/36}{11/12} = \frac{9}{11}$
Specific behaviours
✓✓

(iii) If event A is $X>3$ and event B is $X<10$, are these two events independent? Justify your answer. (4)

Solution
$P(A\cap B)=\frac{27}{36}$, $P(A)=\frac{33}{36}, P(B)=\frac{30}{36}$ Now $\frac{11}{12}\times\frac{30}{36}=\frac{55}{72}\neq\frac{27}{36}$ i.e. $P(A)\times P(B)\neq P(A\cap B)$ ∴ A and B are not independent events
Specific behaviours
✓ $P(A\cap B)=\frac{27}{36}$ ✓ $P(A)=\frac{33}{36}, P(B)=\frac{30}{36}$ ✓ shows that $P(A)\times P(B)\neq P(A\cap B)$ ✓ states not independent

Question 10 **(7 marks)**

(a) The function *f(x)* is differentiable for all $x\varepsilon R$ and satisfies the conditions

$f'(x) < 0$ where $x < 2$

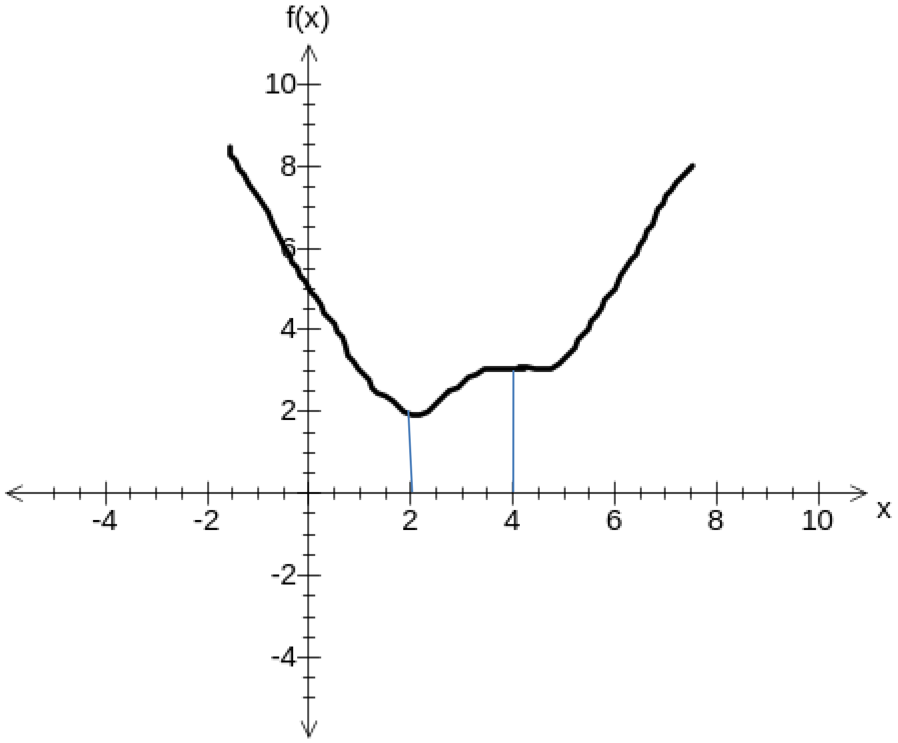
$f'(x) = 0$ where $x = 2$

$f'(x) = 0$ where $x = 4$

$f'(x) > 0$ where $2 < x < 4$

$f'(x) > 0$ where $x > 4$

(i) Draw a sketch of this function *f(x)*. (3)



Solution
✓ shape ✓ turns at x = 2 ✓ point of inflection at x = 4
Specific behaviours
As above

(ii) State whether the following statement is true or false.
"The graph *f(x)* has a stationary point of inflection where *x=4*". (1)

Solution
True
Specific behaviours
✓ or X

(b) If $\int_0^a f(x)\,dx = a$, find $2\int_0^{5a}\left[f\left(\frac{x}{5}\right)+3\right]\,dx$ (3)

Solution

Now if $u=\frac{x}{5}$, $5u=x$ $\int_0^{5a} f\left(\frac{x}{5}\right)dx=5\int_0^{a} f(u)du=5a$ $2\int_0^{5a}\left[f\left(\frac{x}{5}\right)+3\right]dx$ $=2\int_0^{5a} f\left(\frac{x}{5}\right)dx+2\int_0^{5a}3\,dx$ $=2(5a)+2(15a)=40a$
Specific behaviours
✓ $\int_0^{5a} f\left(\frac{x}{5}\right)dx=5\int_0^{a} f(u)du=5a$ ✓ $2\int_0^{5a} f\left(\frac{x}{5}\right)dx+2\int_0^{5a}3\,dx$ ✓ integrates correctly

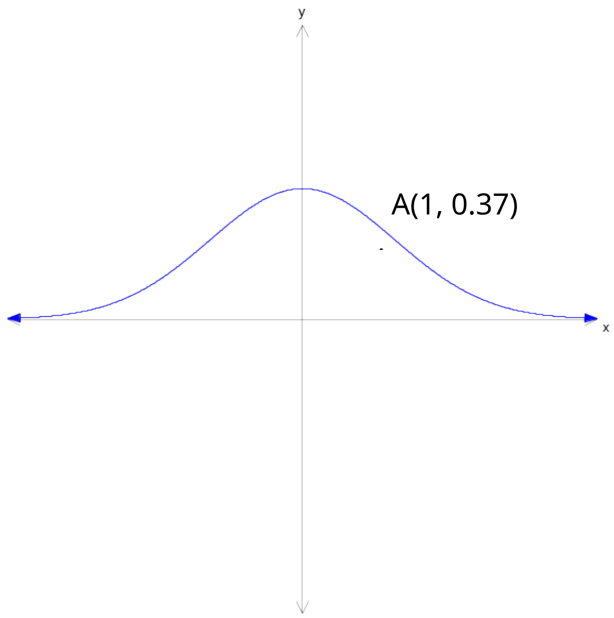
Question 11 **(7 marks)**

(a) The function $y = e^{x(x-1)(x+1)}$ is transformed to $y = -e^{x(1-x^2)}$.

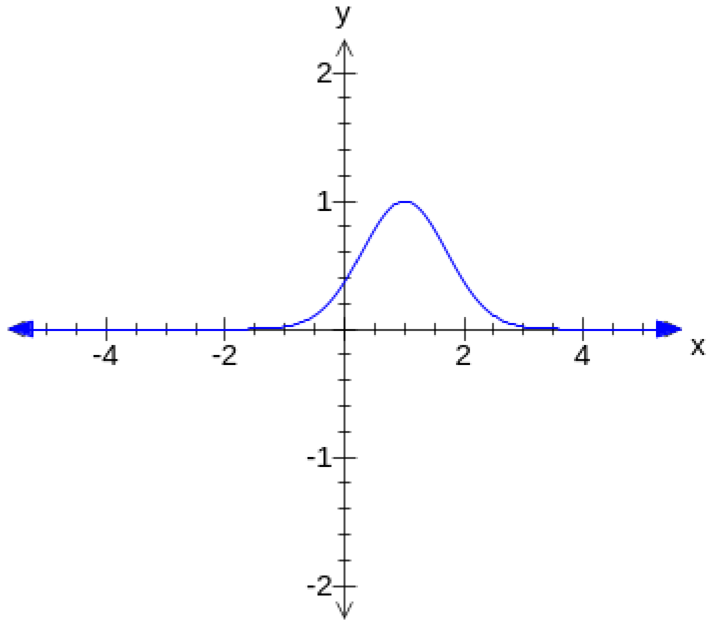
Describe the transformation in order. (3)

Solution
$y = -e^{x(1-x)(1+x)} = -e^{x(1-x^2)} = -e^{-x(x^2-1)}$ -reflected in the x-axis followed by a reflection in the y-axis
Specific behaviours
✓ $y = -e^{x(1-x)(1+x)} = -e^{x(1-x^2)} = -e^{-x(x^2-1)}$ ✓✓ transformation in order

(b) The curve C has equation $y = e^{-x^2}$ and is drawn below



(i) Sketch the graph of $y = f(-x+1)$. (2)



Solution
Reflected across the y-axis and right 1 unit
Specific behaviours
✓ shape ✓ passes through points (1,1) and (0, 0.37)

Question 11 (continued)

(ii) State the coordinates of A if the curve is transformed to $y = -f\left(\frac{1}{2}x\right) + 2$

(2)

Solution
$(1, 0.37) \rightarrow (2, 0.37) \rightarrow (2, -0.37) \rightarrow (2, 1.63)$ Coordinates of A = (2, 1.63)
Specific behaviours
✓✓ 1 mark each for x-coordinate and y-coordinate

Question 12 **(9 marks)**

(a) A company produces fruit balls coated in either dark chocolate or milk chocolate. A large number of these fruit balls are placed in a box. Twenty per cent of the fruit balls in the box are coated with dark chocolate.

(i) Calculate $C_4^{10}(0.2)^4(0.8)^6$ (1)

Solution
0.08808
Specific behaviours
✓ or X

(ii) A random sample of ten fruit balls is taken from the box.

Explain the meaning of $C_4^{10}(0.2)^4(0.8)^6$ with respect to this sample. (2)

Solution
In a sample of 10 fruit balls, the probability of picking exactly 4 coated in dark chocolate is approximately 0.0881
Specific behaviours
✓✓

(b) (i) Find *n* given that $C_0^n(0.2)^0(0.8)^n$ =0.167 772 16 (1)

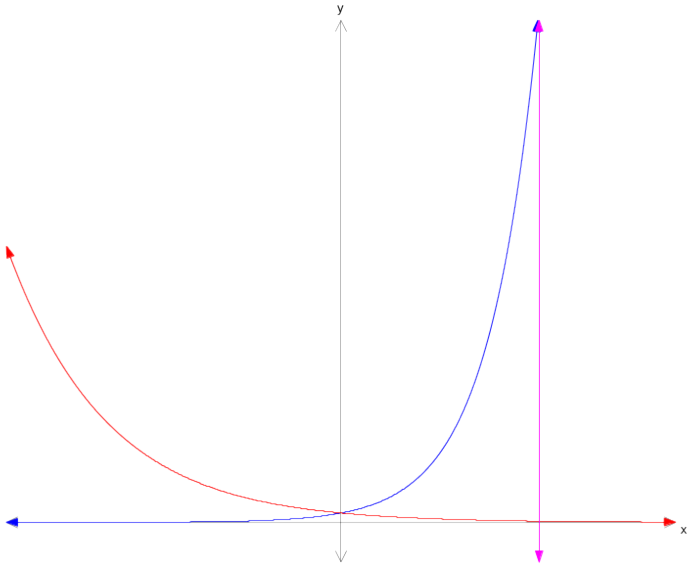
Solution
Using CAS, n = 8
Specific behaviours
✓

(ii) Explain the meaning of your answer to part (b)(i) with respect to the fruit balls. (2)

Solution
The probability of picking no dark chocolate fruit ball from 8 is 0.16777216
Specific behaviours
✓picking none ✓ from 8

Question 12 (continued)

(c) The curve $y = e^{2e}$ and $y = e^{-x}$ intersect at the point (0, 1) as shown in the diagram.



Find the area enclosed by the curves and the line *x=2.*
Leave your answer in terms of '*e*'.

(3)

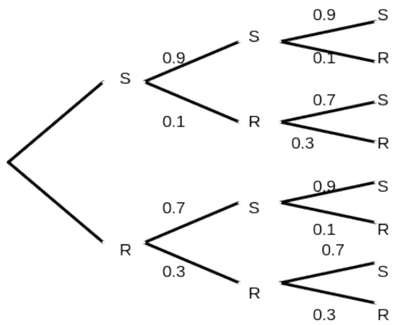
Solution
Required area = $\int_0^2 e^{2x}dx - \int_0^2 e^{-x}dx$ $= \left[\frac{e^{2x}}{2} - (-e^{-x})\right]_0^2$ $= \frac{e^4}{2} + e^{-2} - \frac{3}{2}$
Specific behaviours
✓ $\int_0^2 e^{2x}dx - \int_0^2 e^{-x}dx$ ✓ integrates each term correctly ✓ substitutes limits of integration to get exact value of $\frac{e^4}{2} + e^{-2} - \frac{3}{2}$

Question 13 **(8 marks)**

Adam paints garden gnomes to sell. He sends the garden gnomes to his father (a qualified quality controller) in the order of completion, who classifies them as either 'Superior' or 'Regular', depending on the quality of their finish.

If the garden gnome is Superior, then the probability that the next garden gnome is superior is 0.9. If the garden gnome is Regular, then the probability that the next garden gnome is superior is 0.7.

(a) If the first garden gnome inspected is Superior, find the probability that the third gnome is Regular. (2)

Solution
 P(SSR) + P(SRR) = (0.9)x(0.1) + (0.1x(0.3)=0.12
Specific behaviours
✓✓

(b) If the first garden gnome inspected is Superior, find the probability that the next three gnomes are Superior. (1)

Solution
0.9x0.9x0.9 = 0.729
Specific behaviours
✓ or x

(c) A group of 3 consecutive garden gnomes is inspected and the first is a Regular. It is also found that of these three gnomes,

P(no Superior) = 0.09
P(1 Superior) = 0.28
P(2 Superior) = 0.63

Find the expected number of these gnomes that will be Superior. (2)

Solution
Expected number = 0 x 0.09 + 1 x 0.28 + 2 x 0.63 = 1.54
Specific behaviours
✓ calculation ✓ correct answer of 1.54

Question 13 (continued)

(d) Adam's little brother, Brodie joins in this business venture. The probability that any one of
Brodie's painted garden gnomes is Regular is 0.8. He wants to ensure that the probability that he paints at least two Superior is at least 0.9. Calculate the minimum number of garden
gnomes that Brodie would need to paint to achieve this aim. (3)

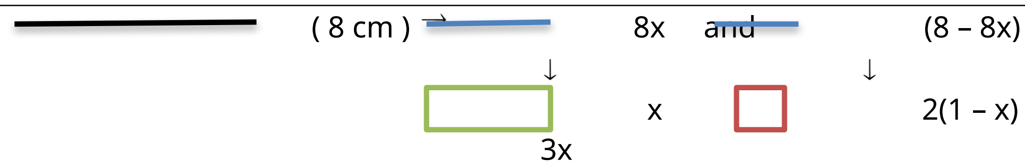
Solution
P(R) = 0.8, P(S) = 0.2 $P(X \geq 2) \geq 0.9$ $1 - P(X \leq 1) \geq 0.9$ $P(X \leq 1) \leq 0.1$ i.e. ${}^{n}C_{0}(0.2)^{0}(0.8)^{n} + {}^{n}C_{1}(0.2)^{1}(0.8)^{n-1} \leq 0.1$ Using CAS to solve, n= 17.95 ∴ minimum number is 18
Specific behaviours

Question 14 **(9 marks)**

A piece of wire 8cm long is cut into two unequal parts. One part is used to form a rectangle that has a length three times its width. The other part of the wire is used to form a square.

(i) If the width of the rectangle is *x* units, determine an equation that will give the sum of the areas of the rectangle and the square in terms of *x*.

(4)

Solution

Area, A = $3x^2 + (2 - 2x)^2$ $= 7x^2 - 8x + 4$
Specific behaviours
✓✓ expressions for 8x and (8 – 8x) ✓✓ areas of rectangle and square

(ii) Using Calculus find the length of each part of the wire when the sum of the areas is a minimum. (5)

Solution
$A = 7x^2 - 8x + 4$ $\frac{dA}{dx} = 14x - 8$ $\frac{d^2A}{dx^2} = 14 \Rightarrow Min$ $14x - 8 = 0$ $x = \frac{4}{7}$ Lengths of each part of wire are $x = 4\frac{4}{7}\left(\frac{32}{7}\right)$ and $3\frac{3}{7}\left(\frac{24}{7}\right)$
Specific behaviours
✓ first derivative ✓ second derivative to confirm x value gives a minimum area ✓ x value

✓✓ the two lengths of $\left(\dfrac{32}{7}\right)$ and $\left(\dfrac{24}{7}\right)$

Question 15 (11 marks)

Nuts and Bolts Company manufactures 120mm bolts which are normally distributed with a mean length of 120mm and a standard deviation of 1mm. Only bolts which are between 118.6mm and 121.4mm pass inspection and are packaged as 120mm bolts.

(a) Find the probability of a randomly selected bolt being an acceptable length. (2)

Solution
$P(118.6 \leq X \leq 121.4) = NCDF(118.6, 121.4, 1, 120) = 0.838487$
Specific behaviours
✓✓

(b) Find the expected number of acceptable bolts in a batch of 100 000 (1)

Solution
$0.838487 \times 100000 = 83849$
Specific behaviours
✓ or X

(c) Is this a reasonable outcome for the company? Justify your answer. (2)

Solution
% of acceptable bolts = $\frac{83849}{100000} \times 100\% = 83.85\%$ % of unacceptable bolts = 16.15% which is too high – too much waste ∴ outcome is not really reasonable.
Specific behaviours
✓ % of unacceptable bolts ✓ outcome not reasonable

(d) A new quality controller suggests adjusting the settings on the machines so that the standard deviation becomes 0.85mm and that only the shortest 5% and the longest 5% of the bolts are rejected.

(i) Find the new minimum and maximum acceptable lengths correct to the nearest 0.1mm. (3)

Solution

$\bar{x} = 120, \quad \sigma = 0.85$ $P(a < X < b) = 0.9$ INV NormCDF('c',0.9,0.85,120) results in a = 118.6 and b = 121.4
Specific behaviours
✓ uses inverse NormCDF ✓✓ "a" and "b" values

(ii) Do the packages now contain bolts that are more consistent in length?

(1)

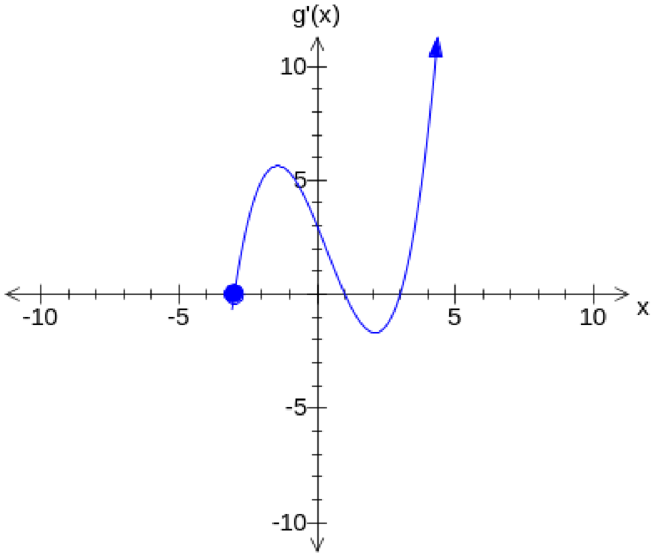
Solution
Range of size of bolts is the same at 118.6 mm to 121.4 mm ∴ in terms of consistency the bolts have the same range.
Specific behaviours
✓ valid reason

(iii) Is the manufacturer better off? Justify. (2)

Solution
Yes, as wastage is reduced from 16.15% to 10%. i.e. 6150 more bolts will be accepted
Specific behaviours
✓ yes ✓ justification

Question 16 **(7 marks)**

The graph of g'(x) is given below.



(a) What can be said about the gradient of the function g(x) between x =-3 to x= 1? (1)

Solution
Gradient is positive
Specific behaviours
✓ or X

(b) When does the function, g(x) have a negative gradient? (2)

Solution
1 < x < 3
Specific behaviours
✓✓ correct interval

(c) State an equation for the tangent to the graph of g(x) at x = 3. (1)

Solution
Horizontal line y = k where k is a constant
Specific behaviours
✓ or X

(d) Find the value of *x* at which $g(x)$ has a relative maximum for $-3 \le x \le 4$ (1)

Solution
x = 1
Specific behaviours

✓ or X

(e) Find the *x*-coordinate of each point of inflection of the graph of $g(x)$ for $-3 \le x \le 4$

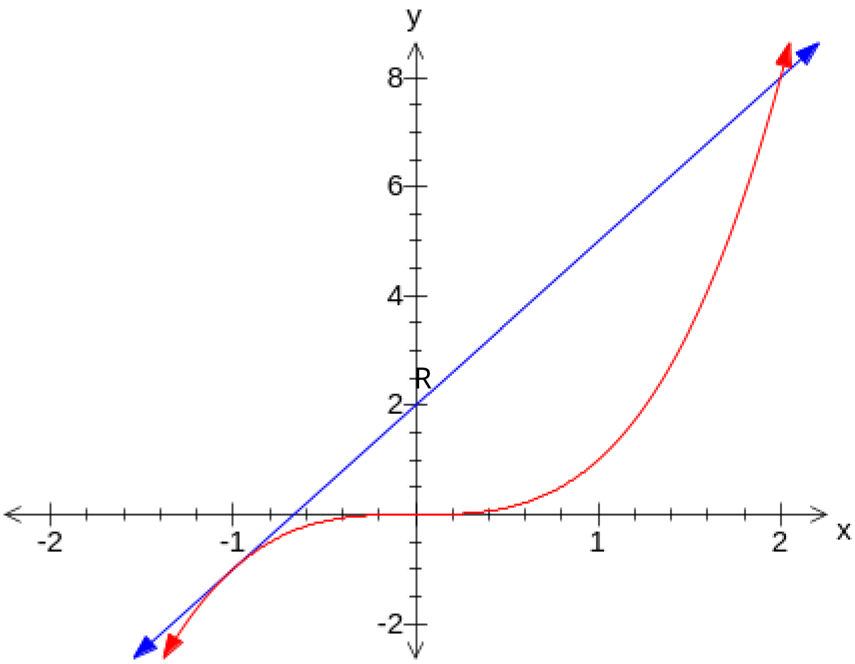
(2)

Solution
x = -1.5 and x = 2
Specific behaviours
✓✓ 1 mark each

Question 17 **(9 marks)**

(a) Shade the region, R bounded by the curves $y = x^3$, $y = 3x + 2$, and $x = 0$ in the diagram.

Find the area of the region R, showing all working steps. (4)



Solution
Area of region R = $\int_{-1}^{0} 3x + 2 - x^3 \, dx$ $= \left[\frac{3x^2}{2} + 2x - \frac{x^4}{4}\right]_{-1}^{0}$ $= \frac{3}{4}$ units²
Specific behaviours
✓ correct shading of R   ✓ $\int_{-1}^{0} 3x + 2 - x^3 \, dx$ ✓ integrates each term correctly   ✓ correct answer

Question 17 (continued)

(b) A group of anthropologists found that human tooth size is continuing to decrease, such that

$$\frac{dS}{dt} = kS$$

In Northern Europeans, for example, tooth size reduction now has a rate of 1% per 1000 years.

(i) If ***t*** represents time in years and ***S*** represents tooth size, find the value of **k**.

(2)

Solution
$S = S_0 e^{kt}$ $0.99S_0 = S_0 e^{1000k}$ i.e. $0.99 = e^{1000k}$ k = -0.000 01
Specific behaviours
✓ $0.99S_0 = S_0 e^{1000k}$ ✓ solves correctly for k

(ii) In how many years will human tooth size be 90% of their present size?

(2)

Solution
$0.9 = e^{-0.00001t}$ t = 10 536 years
Specific behaviours
✓ $0.9 = e^{-0.00001t}$ ✓ solves correctly for t

(iii) What will be our descendant’s tooth size 20 000 years from now? (1)

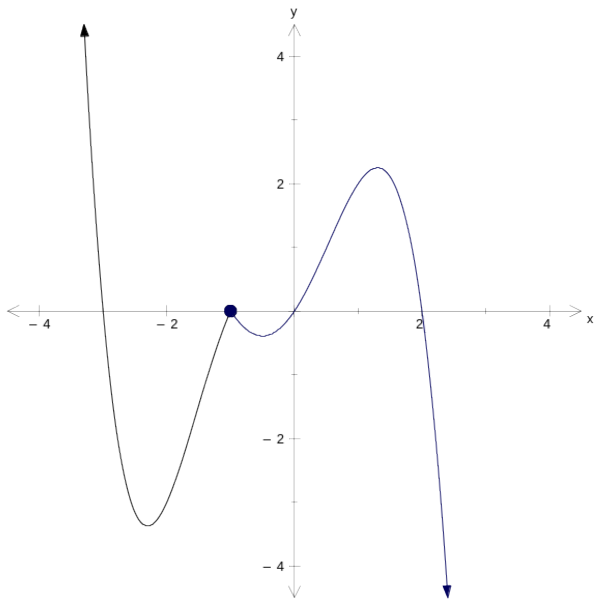
(as a percentage of our present tooth size)

Solution
$S = S_0 e^{-0.00001x20000}$ $S = 0.8187S_0$ $\therefore \ \sim 82\%$

Specific behaviours
✓ or X

Question 18 **(6 marks)**

(a) For the function $y = f(x)$ below



It is known that

$\int_{-3}^{-1} f(x)\,dx = 75$,

$\int_{-1}^{2} f(x)\,dx = 20$

The area under the curve from $x = -1$ to $x = 2$ is 80 square units.

Use the information above and mathematical reasoning to determine the value of each of the following.

(i) $\int_{-1}^{0} f(x)\,dx$ (2)

Solution
Let $\int_{-1}^{0} f(x)dx = p,\quad \int_{0}^{2} f(x)dx = q$ $-p + q = 20,\ \ p + q = 80$ Solving the two equations simultaneously, p = 30, q = 50 $\therefore \int_{-1}^{0} f(x)dx = -30$
Specific behaviours
✓✓

(ii) the area between the curve and the x-axis from $x = -3$ to $x = 0$ (1)

Solution
Area = 75 + 30 = 105
Specific behaviours
✓ or X

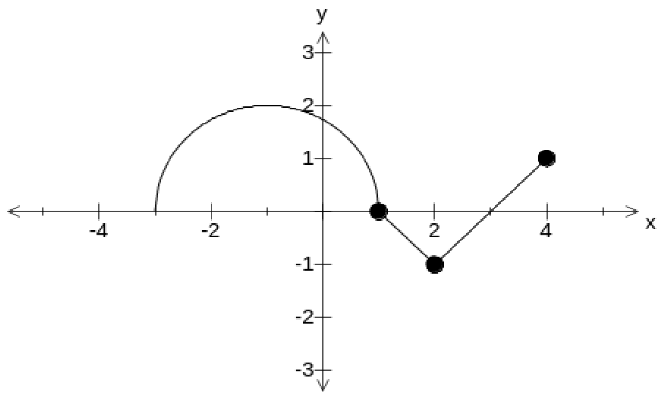
Question 18 (continued)

(iii) $\int_{-3}^{2} f(x)\, dx$ (1)

Solution
-74 – 30 + 50 = - 55
Specific behaviours
✓ or X

(b) The graph of a function $f(x)$ consists of a semi- circle and two line segments as shown.

Find the exact value of $\int_{-3}^{4} f(x)\, dx$ (2)



Solution
$\int_{-3}^{4} f(x)\, dx$ Exact value of $= \left(\frac{1}{2}\times\pi\times 2^2\right) + \left(\frac{-1}{2}\times 2\times 1\right) + \left(\frac{1}{2}\times 1\times 1\right)$ $= 2\pi - \frac{1}{2}$
Specific behaviours
✓✓