SOLUTIONS

Christ Church Grammar School

2018
UNIT TEST 5

MATHEMATICS METHODS Year 11

Section Two:
Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	31 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specific to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that **you do not use pencil**, except in diagrams.

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Question 5 **(2 marks)**

The profit in dollars of a Perth company t years after the 1st of January 2007 is given by $P(t)$. Give an expression for the average rate of change of the profit of the company between 1st of January 2009 and 1st of January 2018.

2009 is $t=2$
2018 is $t=11$
} ✓ (for correct t values)

expression is $\frac{P(11)-P(2)}{11-2} = \frac{P(11)-P(2)}{9}$

✓ (for final answer)

See next page

Question 6 **(4 marks)**

A group of students conducted a science experiment on cooling rates. They measured the temperature $T\,°C$ of some liquid in a container over a period of time t minutes. The results are shown in the table below:

time (t minutes)	3	6	9	12	15	18	21
Temperature ($T\,°C$)	71.5	59	49	40.7	34	28	23.5

(a) Give an exponential model equation for this data in the form $T = a \times b^t$. (2 marks)

using 'ab exponential' regression in Classpad.

$T = 85.7 \times 0.94^t$

✓ (for 'a' value) ✓ (for 'b' value)

(Note: allow 85.7 ± 0.5 for 'a')

(b) If the room temperature for the experiment was $15°C$, calculate how long it took for the cooling of the liquid to cease. (2 marks)

Using Classpad Solve $(15 = 85.7 \times 0.94^x)$

✓ (for equation to solve)

$t = 28.2$ minutes ✓ (for final answer)

(Note: allow other answers corresponding to allowable values of 'a' in part (a))

See next page

Additional working space

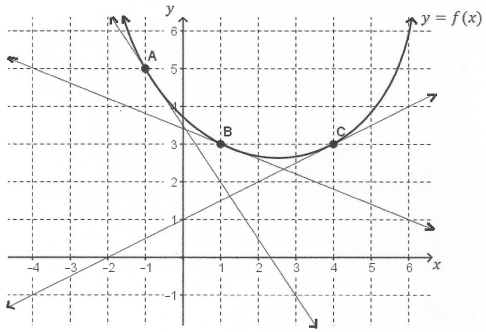
Question number:_____

Additional working space

Question number:_____

Question 7 **(4 marks)**

The graph below shows the curve $y = f(x)$. The points A, B and C all lie on the curve. The tangent lines at the points A, B and C are also shown on the graph.



Calculate the following:

(a) The average rate of change of $f(x)$ from A to C. (1 mark)

A(-1,5) to C(4,3)

$\frac{3-5}{4-(-1)} = -\frac{2}{5}$ ✓(for final answer)

(b) The instantaneous rate of change of $f(x)$ at B. (1 mark)

use (6,1) and (1,3) on tangent line

$\frac{3-1}{1-6} = \frac{2}{-5} = -\frac{2}{5}$ ✓(for final answer)

(c) The equation of the tangent line of $f(x)$ at B. (2 marks)

line is $y = -\frac{2}{5}x + c$ ✓(for giving equation of line in terms of constant c)

use (1,3) $3 = -\frac{2}{5} \times 1 + c$ so $y = -\frac{2}{5}x + 3\frac{2}{5}$

$c = 3\frac{2}{5}$ (or $y = -\frac{2}{5}x + \frac{17}{5}$)

(or $2x + 5y = 17$)

✓(for final answer)

See next page

Question 8 **(4 marks)**

If $y = f(x)$ then finding $\frac{dy}{dx}$ from first principles involves calculating:

$$\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

Calculate $\frac{dy}{dx}$ from first principles if $f(x) = \frac{1}{1+x^2}$

$\frac{dy}{dx} = \lim_{h\to 0}\left(\frac{\frac{1}{1+(x+h)^2} - \frac{1}{1+x^2}}{h}\right)$ ✓(substitutes into first principles formula correctly)

$= \lim_{h\to 0}\left[\frac{1}{h}\left(\frac{1+x^2-(1+(x+h)^2)}{(1+(x+h)^2)(1+x^2)}\right)\right]$ ✓(uses correct common denominator)

$= \lim_{h\to 0}\left[\frac{1}{h}\left(\frac{1+x^2-(1+x^2+2xh+h^2)}{(1+(x+h)^2)(1+x^2)}\right)\right]$

$= \lim_{h\to 0}\left[\frac{1}{h}\left(\frac{-2xh-h^2}{(1+(x+h)^2)(1+x^2)}\right)\right]$

$= \lim_{h\to 0}\left[\frac{-2x-h}{(1+(x+h)^2)(1+x^2)}\right]$ ✓(gives simplified expression in terms of h)

$= \frac{-2x}{(1+x^2)(1+x^2)}$

$= \frac{-2x}{(1+x^2)^2}$ ✓(gives final answer)

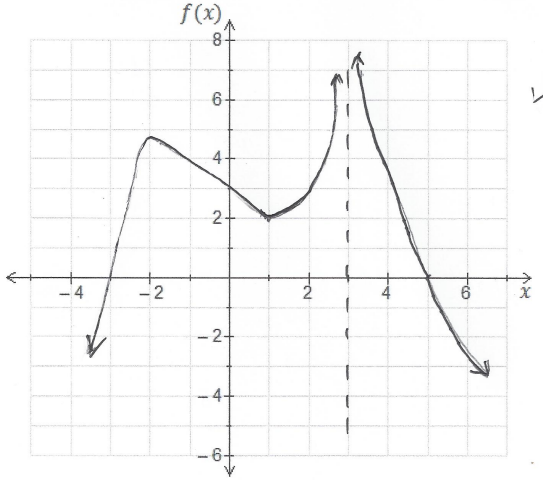
See next page

Question 12 **(5 marks)**

A function $f(x)$ satisfies **all** of the following properties:

- $x+3$ and $x-5$ are both factors of $f(x)$
- $f(0) = 3$
- $f'(-2) = 0$ and $f'(1) = 0$
- $f'(x) > 0$ for $x < -2$, $1 < x < 3$
- $f(3)$ is undefined

On the axes below, draw the graph of $f(x)$.



✓(x intercepts at −3 and 5)
✓(y intercept at 3)
✓(max turning point at x=−2 and min turning point at x=1)
✓($f'(x)>0$ for $x<-2$ and $1<x<3$)
✓(vertical asymptote at $x=3$)

End of questions

Question 11 **(2 marks)**

At the beginning of an experiment there are 30 bacteria in a culture. After 90 minutes there are 1920 bacteria in the culture. What is the doubling period of the bacteria?

$30 \to 60 \to 120 \to 240 \to 480 \to 960 \to 1920$

bacteria doubles 6 times ✓(calculates number of doubling events)

$\therefore$ doubling period $= \frac{90}{6} = 15$ minutes

✓(gives final answer)

Question 9 **(4 marks)**

Radioactive materials decay so that the amount of radioactive material A present at time t (where t is in years) is given by the function:

$$A = A_0 2^{-kt}$$

where A_0 is the initial amount and k is a positive constant that depends on the type of material. The half-life of the material is the time required for half the material to decay.

Carbon-14 is a radioactive substance with a half-life of 5730 years. A Babylonian cloth fragment now has 40% of the carbon-14 that it contained originally. How old is the cloth fragment? Give your answer rounded to the nearest 100 years.

half-life is 5730

So $\frac{1}{2}A_0 = A_0 2^{-k \times 5730}$

$\frac{1}{2} = 2^{-k \times 5730}$ ✓(for equation for k)

Solve on Classpad $k = 1.745 \times 10^{-4}$ ✓(for correct k value)

[note: roundings allowed for written value of k, but final age of cloth answer must still be correct]

So we need to solve $0.4A_0 = A_0 \times 2^{-1.745 \times 10^{-4} t}$

$0.4 = 2^{-1.745 \times 10^{-4} t}$

✓(equation for t)

Solve on Classpad

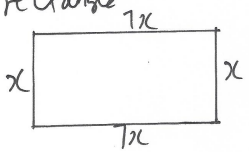
$t = 7600$ years, to nearest 100 years

✓(for final answer)

Question 10 **(6 marks)**

A piece of wire 60 *cm* long is cut into two pieces. One piece is bent into a square shape and the other into a rectangle shape whose length is seven times its width. **Clearly show the use of calculus techniques at each stage** to determine the width of the rectangle that will minimise the sum of the areas of the two shapes.

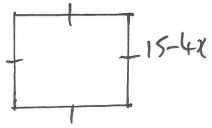
rectangle



✓(for mathematically representing dimensions of rectangle)

remaining length of wire = $60-16x$

$\therefore$ length of side of square = $15-4x$



✓(for side length of square)

Let A = sum of areas of shapes

$A = 7x^2 + (15-4x)^2 = 23x^2 - 120x + 225$

✓(correct simplified expression for sum of areas)

$\frac{dA}{dx} = 46x - 120$ ✓(for $\frac{dA}{dx}$)

for $\frac{dA}{dx} = 0$ $46x - 120 = 0$ $x = 2.61$ to 2d.p.

Sign table:

x	2	2.61	3
$\frac{dA}{dx}$	−	0	+
	\	—	/

$\therefore$ $x = 2.61$ is minimum point

✓(uses sign table of $\frac{dA}{dx}$ to show that point is a minimum
Note: cannot use sign of x^2 coefficient of expression for A as this is not a calculus technique)

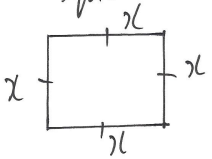
So width of rectangle is 2.61 cm to 2d.p ✓(final answer)

P.T.O. for alternative solution

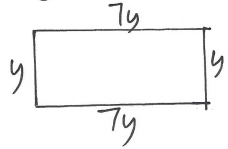
See next page

Q10 - Alternative solution

Square



remaining length of wire = $60-4x$



$\therefore 16y = 60-4x$

$y = \frac{60-4x}{16}$

$= \frac{15-x}{4}$

✓(for mathematically representing dimensions of square and rectangle)

✓(for expression for width of rectangle)

Let A = sum of areas of shapes

$= x^2 + 7\left(\frac{15-x}{4}\right)^2 = x^2 + \frac{7}{16}(x^2 - 30x + 225)$

$A = \frac{23}{16}x^2 - \frac{210}{16}x + \frac{1575}{16}$ ✓(correct simplified expression for sum of areas)

$\frac{dA}{dx} = \frac{46}{16}x - \frac{210}{16}$ ✓(for $\frac{dA}{dx}$)

for $\frac{dA}{dx} = 0$ $\frac{46}{16}x - \frac{210}{16} = 0 \Rightarrow x = 4.565$

Sign table:

x	4	4.565	5
$\frac{dA}{dx}$	−	0	+
	\	—	/

$\therefore$ $x = 4.565$ is a minimum point

✓(uses sign table of $\frac{dA}{dx}$ to show that point is a minimum
Note: cannot use sign of x^2 coefficient of expression for A as this is not a calculus technique)

So $y = \frac{15-4.565}{4} = 2.61$ cm to 2d.p

So width of rectangle is 2.61 cm to 2d.p.

✓(for final answer)