

Rossmoyne Senior High School

Year 12 Trial WACE Examination, 2015

Question/Answer Booklet

MATHEMATICS
3CD
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33 $\frac{1}{3}$
Section Two: Calculator-assumed	13	13	100	100	66 $\frac{2}{3}$
Total				150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free

(50 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(6 marks)

Three unbiased coins are tossed together. Event A occurs when there are no heads, event B occurs when there is at least one head and event C occurs when there are at least two tails.

State whether the statements below are true or false, giving a reason for each answer.

(a) A and B are mutually exclusive.

(2 marks)

True.

If A occurs then B cannot, so $P(A \cap B) = 0$

(b) A and B are independent.

(2 marks)

False.

A and B are mutually exclusive, $P(A) \neq 0$ and $P(B) \neq 0$
so $P(A \cap B) \neq P(A)P(B)$.

(c) A and C are independent.

(2 marks)

False.

$P(A) = \frac{1}{8}$ but $P(A|C) = \frac{1}{4}$, so not independent.

Question 2

(7 marks)

- (a) Evaluate $\int_0^2 8x(x^2 - 1)^3 dx$.

(2 marks)

$$\begin{aligned}\int_0^2 8x(x^2 - 1)^3 dx &= \left[(x^2 - 1)^4 \right]_0^2 \\ &= 3^4 - 1 \\ &= 80\end{aligned}$$

- (b) Determine $\frac{d}{dx}(x^2 e^{3x})$.

(2 marks)

$$\frac{d}{dx}(x^2 e^{3x}) = 2xe^{3x} + 3x^2 e^{3x}$$

- (c) Determine $f'(1)$ if $f(x) = \frac{x^2 - 1}{2x + 1}$.

(3 marks)

$$\begin{aligned}f'(x) &= \frac{(2x)(2x + 1) - (x^2 - 1)(2)}{(2x + 1)^2} \\ f'(1) &= \frac{(2)(3) - (0)(2)}{(3)^2} = \frac{2}{3}\end{aligned}$$

Question 3

(6 marks)

A student spent \$60 at an online store, buying a total of 28 apps for their phone.

- (a) If x of the apps bought cost \$1 each, y of cost \$2 each and the remaining z apps cost \$5 each, use the above information to write down two equations. (2 marks)

$$\begin{aligned}x + y + z &= 28 \\x + 2y + 5z &= 60\end{aligned}$$

- (b) If the total number of \$1 and \$2 apps bought was three times the number of \$5 apps bought, write down another equation. (1 mark)

$$x + y = 3z$$

- (c) How many of each of the different priced apps did the student buy? (3 marks)

Since $x + y = 3z$,
then $x + y + z = 28$ becomes
 $3z + z = 28 \Rightarrow 4z = 28, z = 7$.

$x + 2y + 35 = 60 \Rightarrow x + 2y = 25$

$$\begin{aligned}x + 2y &= 25 \\x + y &= 21 \\y &= 4 \\x &= 17\end{aligned}$$

Student bought **17** @ \$1, **4** @ \$2 and **7** @ \$5.

Question 4

(4 marks)

If $y = x^3 + 6x^2 + 6x - 24$, show that $3 \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y - x^3 = 0$.

$$y = x^3 + 6x^2 + 6x - 24$$

$$\frac{dy}{dx} = 3x^2 + 12x + 6$$

$$\frac{d^2 y}{dx^2} = 6x + 12$$

$$3 \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y - x^3 = 3(6x + 12) - 2(3x^2 + 12x + 6) + x^3 + 6x^2 + 6x - 24 - x^3$$

$$= 18x + 36 - 6x^2 - 24x - 12 + 6x^2 + 6x - 24 + x^3 - x^3$$

$$= 18x - 24x + 6x - 6x^2 + 6x^2 + 36 - 12 - 24$$

$$= 0$$

Question 5

(6 marks)

Let $f(x) = \sqrt{x-3}$ and $g(x) = x-1$.

(a) State $f \circ g(x)$ with its domain and range.

(3 marks)

$$\begin{aligned} f \circ g(x) &= \sqrt{x-1-3} \\ &= \sqrt{x-4} \end{aligned}$$

Domain: $x \geq 4$.

Range: $y \geq 0$.

(b) If $h(x) = ax^2 + b$ and $h \circ g(x) = 2x^2 - 4x$, determine the values of a and b .

(3 marks)

$$\begin{aligned} h \circ g(x) &= h(x-1) \\ 2x^2 - 4x &= a(x-1)^2 + b \\ 2x^2 - 4x &= ax^2 - 2ax + a + b \end{aligned}$$

$$\begin{aligned} a &= 2 \\ b &= -2 \end{aligned}$$

Question 6

(7 marks)

- (a) Solve for a , where $\int_3^a 2x - 6 \, dx = 16$.

(3 marks)

$$\begin{aligned} \left[x^2 - 6x \right]_3^a &= 16 \\ a^2 - 6a - 9 + 18 &= 16 \\ a^2 - 6a - 7 &= 0 \\ (a - 7)(a + 1) &= 0 \\ a &= 7, a = -1 \end{aligned}$$

- (b) Solve the inequality $\frac{1}{x} \leq \frac{2}{3x - 2}$.

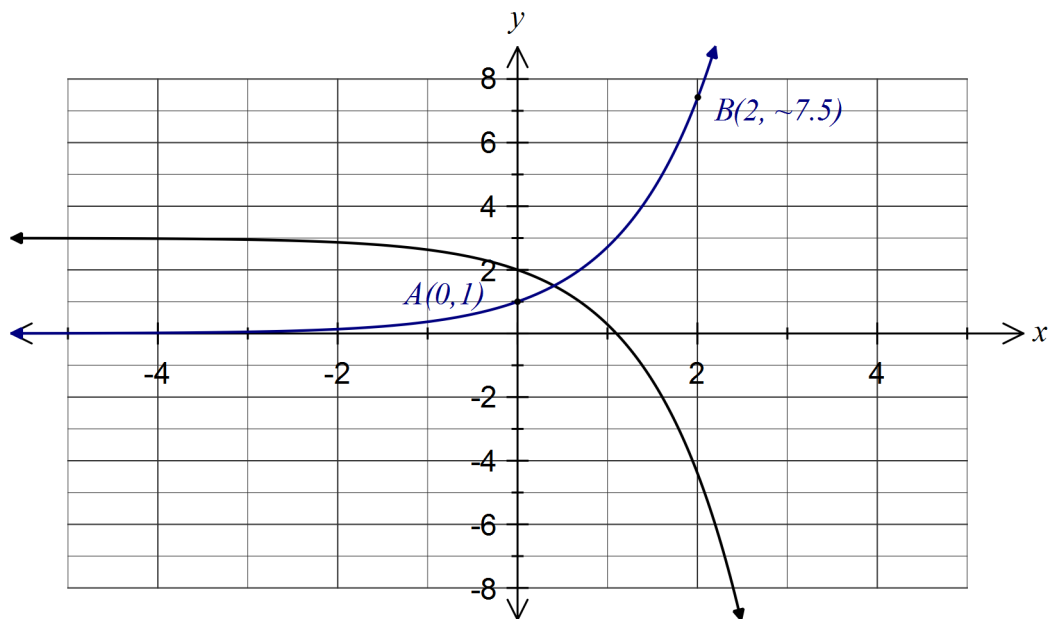
(4 marks)

$$\begin{aligned} \frac{1}{x} - \frac{2}{3x - 2} &\leq 0 \\ \frac{3x - 2 - 2x}{x(3x - 2)} &\leq 0 \\ \frac{x - 2}{x(3x - 2)} &\leq 0 \\ x < 0, \frac{2}{3} < x &\leq 2 \end{aligned}$$

Question 7

(8 marks)

The function $f(x) = 3 - e^x$ is graphed on the axes below.



- (a) If $\int_{-2}^{-1} f(x) dx = a$, $\int_{-1}^0 f(x) dx = b$ and $\int_0^1 f(x) dx = c$, evaluate each of the following definite integrals in terms of the constants a , b and c .

(i) $\int_0^1 f(-x) dx$.

$$\int_0^1 f(-x) dx = \int_{-1}^0 f(x) dx = b$$

(1 mark)

(ii) $\int_{-2}^0 -f(x) dx$.

$$\begin{aligned} \int_{-2}^0 -f(x) dx &= - \int_{-2}^0 f(x) dx \\ &= -(a + b) \end{aligned}$$

(2 marks)

(iii) $\int_{-1}^0 2f(x-1) dx$.

$$\begin{aligned} \int_{-1}^0 2f(x-1) dx &= 2 \int_{-1}^0 f(x-1) dx \\ &= 2 \int_{-2}^{-1} f(x) dx \\ &= 2a \end{aligned}$$

(2 marks)

- (b) On the axes above, sketch the graph of $y = 3 - f(x)$, showing all relevant features.

(3 marks)

Question 8

(6 marks)

Let $P(a, b) = \frac{a}{b} + \frac{b}{a}$, where a and b are positive integers.

- (a) Determine $P(4, 1)$.

(1 mark)

$$P(4, 1) = \frac{4}{1} + \frac{1}{4} = \frac{17}{4}$$

- (b) What can be said about $P(a, b)$ if $a = b$? Justify your answer.

(2 marks)

$$\begin{aligned} a &= b \\ P(b, b) &= \frac{b}{b} + \frac{b}{b} = 2 \end{aligned}$$

- (c) A conjecture was made that $P(a, b)$ will always be 2 or more. Prove this conjecture.

(3 marks)

$$\begin{aligned} &\text{If } P(a, b) \geq 2 \\ &\text{then } \frac{a}{b} + \frac{b}{a} - 2 \geq 0 \\ &\frac{a^2 + b^2 - 2ab}{ab} \geq 0 \\ &\frac{(a - b)^2}{ab} \geq 0, \text{ which is always true for +ve integers.} \\ &\text{Hence } P(a, b) \text{ will always be 2 or more.} \end{aligned}$$

Additional working space

Question number: _____

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