

Rossmoyne Senior High School

Year 12 Trial WACE Examination, 2014
Question/Answer Booklet

MATHEMATICS 2C/2D
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures In words Your name

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MARKING KEY

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates
No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33%
Section Two: Calculator-assumed	12	12	100	100	66%
Total				150	100

Instructions to candidates

- The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.

See next page

Additional working space

Question number: _____

Question 8 (6 marks)

In order to estimate the toad population in a nature reserve, one evening researchers caught and tagged 54 toads and then released them back into the reserve. A few weeks later, the researchers returned to the reserve and noted that of 38 toads observed, four of them were tagged.

- (a) Use the capture-recapture technique to estimate the total number of toads in the reserve. (3 marks)

$$\frac{54}{4} = \frac{n}{38}$$
$$n = 513$$
$$n \approx 510 \text{ toads}$$

The following day, a different group of researchers observed another 63 toads. Using the capture-recapture technique, this group of researchers estimated there were close to 500 toads in the reserve.

- (b) Determine how many of the toads that this group observed were tagged. (3 marks)

$$\frac{54}{x} = \frac{500}{63}$$
$$x = 6,804$$
$$7 \text{ toads were tagged.}$$

Question 9 (8 marks)

The table below, based on figures from the Australian Taxation Office, shows the tax rates for the current financial year 2014 - 2015.

Annual taxable income	Annual tax on this income
\$1 - \$18 200	Nil
\$18 201 - \$37 000	19c for each \$1 over \$18 200
\$37 001 - \$80 000	\$3 572 plus 32.5c for each \$1 over \$37 000
\$80 001 - \$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 47c for each \$1 over \$180 000

A single person, who works away from his home on a remote mine site, expects to earn a taxable income of \$11 500 per month throughout this financial year.

- (a) Calculate the annual taxable income for this person. (1 mark)

11500 × 12 = \$138 000 ✓

- (b) Calculate the annual tax payable by this person. (3 marks)

138000 – 80000 = 58000 ✓
58000 × 0.37 = 21460 ✓
17547 + 21460 = \$39 007 ✓

The Medicare Levy Surcharge is added to a person's tax if they do not have private health insurance. The surcharge is calculated as a percentage of the annual taxable income using the rates shown in this table:

Taxable income (\$)	1-90 000	90 001-105 000	105 001-140 000	140 001 and over
Surcharge	0%	1%	1.25%	1.5%

- (c) This person did not have private health insurance.

- (i) Calculate their Medicare Levy Surcharge. (2 marks)

138000 × 1.25 ÷ 100 = \$1 725 ✓

- (ii) Private health insurance would cost this person \$28.57 per week. Comment on whether this person should consider such insurance from a financial point of view. (2 marks)

They should take out insurance, as the cost of health insurance is \$1485.64, which is less than the surcharge. ✓

Additional working space

Question number: _____

Question number: _____

Question 10 (9 marks)

The prices of second-hand bicycles advertised on a website are summarised in the table below.

Price (\$)	Frequency
0-49	5
50-99	12
100-149	14
150-199	23
200-249	21
250-299	22
300-349	18

(a) How many bicycles were advertised on the site? (1 mark)

115

(b) For these prices, determine

(i) the modal class (1 mark)

150-199

(ii) the median class (1 mark)

200-249

(iii) the mean (1 mark)

\$203.20

(iv) the standard deviation (1 mark)

\$85.99

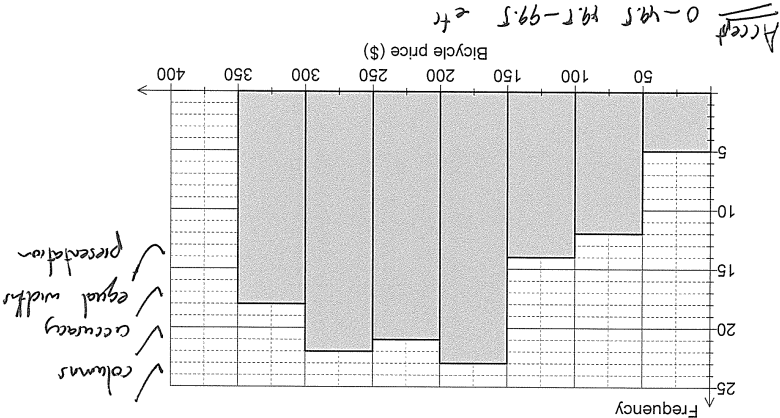
✓

✓

✓

✓

(c) Construct a frequency histogram for these prices on the axes below. (4 marks)



See next page

Question 11

(8 marks)

The table below shows the cost of building direct light rail links between seven different locations, in millions of dollars.

	A	B	C	D	E	F	G
A	-	50	58	-	-	54	35
B	50	-	44	65	32	33	58
C	58	44	-	42	43	-	-
D	-	65	42	-	45	68	-
E	-	32	43	45	-	32	61
F	54	33	-	68	32	-	49
G	35	58	-	-	61	49	-

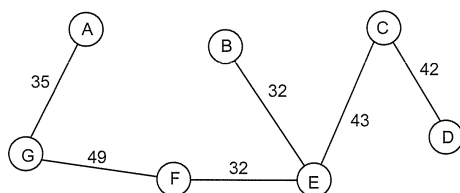
- (a) Use Prim's algorithm to determine the minimum spanning tree for this network and clearly state the cost of building the minimum spanning tree. (4 marks)

Minimum spanning tree connects circled locations in table above.

Total cost is \$233 million.

✓ circles above
✓ method used
✓ 233
✓ units

- (b) Show the minimal spanning tree on the network below. (2 marks)



✓ ✓ fit
-1 each error/division

- (c) An option exists to use an existing railway line between locations A and F, making it possible to halve the cost of this link. If this option is taken, describe the effect, if any, on the cost of building the minimal spanning tree. (2 marks)

Link AF now costs \$27 m and so replaces link GF, saving \$22 m.

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Question 19

(8 marks)

A lump sum of money is invested in a savings fund that compounds interest annually.

The amount of money in the savings fund, T_n , is given by the recursive rule $T_{n+1} = 1.05T_n$, $T_0 = 2250$, where n is the number of years after the initial deposit.

- (a) State the initial amount of the lump sum. (1 mark)

\$2 250

- (b) State the annual interest rate offered by the savings fund. (1 mark)

5% pa

- (c) Determine

- (i) the amount of money, to the nearest cent, in the savings fund after four years. (2 marks)

$2250(1.05)^4 = \$2\,734.89$

- (ii) the amount of interest accumulated during the first four years. (1 mark)

$3734.89 - 2250 = \$484.89$

- (d) Another savings fund was offering an interest rate of 4.9% pa compounded monthly. Would this fund be a better choice for savers investing the same amount for four years? Justify your answer. (3 marks)

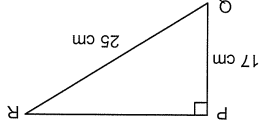
$2250 \left(1 + \frac{4.9}{12 \times 100} \right)^{12 \times 4} = \$2\,736.09$

Yes, this fund returns an extra \$1.20 in interest over four years.

End of questions

Question 18

(a) Using trigonometry, determine the size of angle PQR in this right triangle. (2 marks)



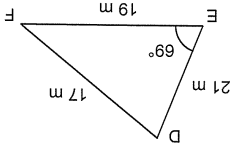
$$\angle PQR = \cos^{-1} \frac{17}{25}$$

$$= 47.16$$

$$\approx 47^\circ$$

no need to round, answer only

(b) Using trigonometry, determine the area of the triangular plot of land DEF to the nearest square metre.



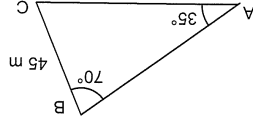
$$\text{Area} = \frac{1}{2}(21)(17) \sin 69$$

$$= 186.23$$

$$\approx 186 \text{ m}^2$$

(3 marks)

(c) Using trigonometry, determine the length of fencing required for side AB of the triangular plot of land ABC shown below.



$$\frac{45}{\sin 35} = \frac{AB}{\sin 75}$$

$$AB = 75.78$$

$$\approx 76 \text{ m}$$

May use opposite of CAS if they give every side/angle

See next page

Question 12

(a) The equation of a straight line is given by $y = 2x - 3$. (8 marks)

(i)

Determine the equation of a line parallel to this one that passes through the point with coordinates (15, 20).

$$y = 2x + c$$

$$20 = 2(15) + c \Rightarrow c = -10$$

$$y = 2x - 10$$

(ii)

Circle the equation(s) of any of the following lines that are perpendicular to the line $y = 2x - 3$. (2 marks)

$y = -2x + 3$ $y = \frac{1}{2}x - \frac{1}{3}$ $y = 3 - \frac{1}{2}x$ $y = -0.5x - 0.3$

(b)

Two lines, AB and AC, are drawn through the points A(1, -2), B(4, 2) and C(14, 26). Which line has the largest gradient? Justify your answer. (2 marks)

$$m_{AB} = \frac{2 - (-2)}{4 - 1} = \frac{4}{3} \approx 1.33$$

$$m_{AC} = \frac{26 - (-2)}{14 - 1} = \frac{28}{13} \approx 2.15$$

AC has the largest gradient.

(ii)

Determine the distance from B to C, if one unit on each axis is one centimetre. (2 marks)

$$\sqrt{(14 - 4)^2 + (26 - (-2))^2} = \sqrt{10^2 + 28^2}$$

$$= 26 \text{ cm}$$

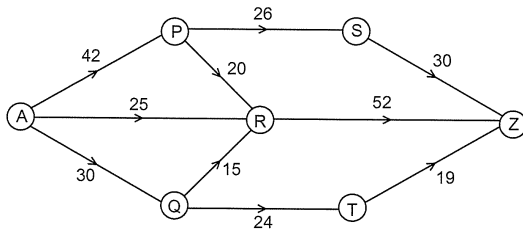
No need for units

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Question 13

(6 marks)

At a mine site, coal is moved through a system of conveyor belts from A to Z, as shown on the network below. The number on each arc represents the maximum weight of coal, in tonnes per hour, which can be moved along that conveyor belt.



- (a) What is the maximum weight of coal that can be transported from A to Z each hour? Show systematic working to allow your solution to be checked. (4 marks)

$APSZ = 26$
 $APRZ = 16$
 $ARZ = 25$
 $AQRZ = 11$
 $AQTZ = 19$
 Total = 97 tonnes per hour

- (b) What effect, if any, would there be on the maximum weight of coal that can be transported from A to Z each hour if mechanical problems slowed the belt from Q to R from 15 to just 8 tonnes per hour? Justify your answer. (2 marks)

From (a) there is spare capacity of 4 t/h on this belt, so would lose $7 - 4 = 3$ tonnes per hour from A to Z.

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Question 17

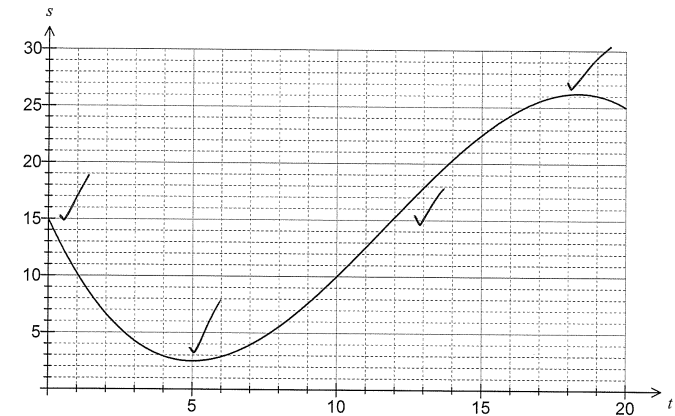
(10 marks)

The average wind speed, s metres per second, recorded at a weather station during a 20 hour period was modelled by the function $s = 15 - 5.5t + 0.7t^2 - 0.02t^3$, for $0 \leq t \leq 20$ hours.

- (a) Calculate the average wind speed when $t = 1.3$. (1 mark)

22.5 m/s

- (b) Sketch the graph of average wind speed against time on the axes below. (4 marks)



- (c) At what time was the average wind speed

- (i) least? (1 mark)

$t = 5$

- (ii) greatest? (1 mark)

$t = 18.3$

- (d) For what percentage of the 20 hours was the average wind speed no more than 9 metres per second? (3 marks)

From $t = 1.3$ until $t = 9.6$
 $9.6 - 1.3 = 8.3$ hours
 $= \frac{8.3}{20} \times 100\% = 41.5\%$

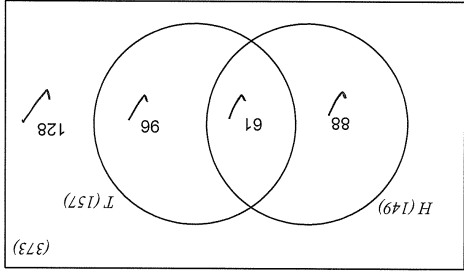
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Question 16 (11 marks)

In a random sample of 373 investors who own shares in the Australian stock market, it was found that 149 owned shares in the health sector and of these 149, 61 also owned shares in the technology sector. A total of 157 investors owned shares in the technology sector.

Let H represent the set of investors who owned shares in the health sector and T represent the set of investors who owned shares in technology sector.

(a) Show this information on the Venn diagram below, completing all regions and showing set totals. (4 marks)



(b) Determine

(i) $n(H \cap T)$.

88

fit

(iii) $n(H \cap T)$.

312

✓

(c) If one of the investors is selected at random, determine

(i) $P(H \cap T)$.

$\frac{245}{373}$

✓

(iii) $P(T | H)$.

$\frac{96}{224}$

✓

(d) Determine the percentage of investors in the random sample who owned shares in the technology sector but not in the health sector. (2 marks)

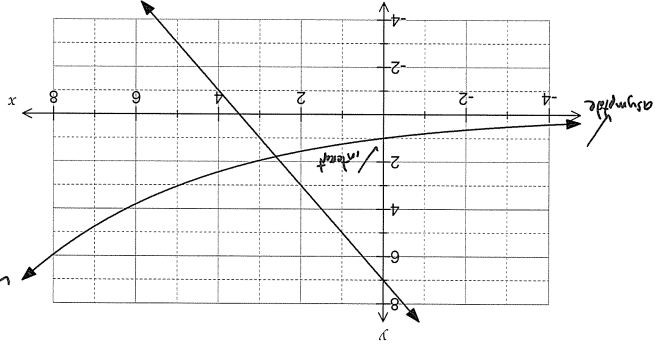
$\frac{96}{373} \times 100 = 25.7\%$

✓

See next page

Question 14

The graph of $y = 7 - 2x$ is shown below.



(8 marks)

(c)

Using the graph, or otherwise, solve the equations

(i) $7 - 2x = -3$.

$x = 5$

✓

(1 mark)

(ii) $1.25^x = 7 - 2x$.

$x = 2.6$

✓

(1 mark)

(iii) $1.25^x = 5$.

$x = 7.2$

✓

(1 mark)

See next page

Question 15

(10 marks)

The table below summarises the use of various health services on at least one occasion by people from different household structures over a 12 month period.

Household structure	All persons	No use	GP	Specialist	Dentist	Hospital	Other
Person living alone	2 330	151	2 074	994	1 056	379	583
Couple only	4 834	226	4 372	2 155	2 475	724	1 284
Couple with children	10 198	915	8 259	2 716	4 887	998	1 908
All other households	4 744	512	3 824	1 242	1 925	498	769
Total	22 106	1 804	18 529	7 107	10 343	2 599	4 544

For example, out of the 22 106 people surveyed, 10 198 lived in a household structured as a couple with children, and of these 10 198 people, 2 716 had used the services of a health care specialist on at least one occasion over the past 12 months.

- (a) Of those people who lived alone, how many had used the services of a dentist? (1 mark)

1 056

- (b) How many people in the survey did not live alone? (1 mark)

22106 - 2330 = 19 776

- (c) Of those people who lived in a household structure with either a couple only or a couple with children, what percentage had used the services of a GP? (2 marks)

$\frac{4372 + 8259}{4834 + 10198} \times 100 = \frac{12631}{15032} \times 100$
= 84.0%

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- (d) If one person is selected at random from all those surveyed, determine the probability that

- (i) they had made use of 'other' health services? (1 mark)

$\frac{4544}{22106}$
(≈ 0.206)

- (ii) they had used the services of a specialist and their household structure was 'all other'.

$\frac{1242}{22106}$
(≈ 0.056)

- (iii) they had made no use of health services, given that they lived alone. (2 marks)

$\frac{151}{2330}$
(≈ 0.065)

- (e) Which type of household structure had the greatest proportion of people who had used the services of a hospital? Justify your answer. (2 marks)

Person living alone (16.3%)
 $\frac{379}{2330} \approx 16.3\%$
 $\frac{724}{4834} \approx 15.0\%$
 $\frac{998}{10198} \approx 9.8\%$
 $\frac{498}{4744} \approx 10.5\%$

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