Semester One Examination, 2013

Question/Answer Booklet

3AB PHYSICS

Please place your student identification label in this box

Student Number: In figures

In words

Time allowed for this paper
Reading time before commencing work: Ten minutes
Working time for paper: Two and one half hours

Materials required/recommended for this paper
To be provided by the supervisor
This Question/Answer Booklet
Formulae and Constants Sheet

To be provided by the candidate
Standard items: pens, pencils, eraser, correction fluid, ruler, highlighters
Special items: non-programmable calculators satisfying the conditions set by the School Curriculum and Standards Authority for this course

Important note to candidates
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

	Section 1	Section 2	Section 3	Total
Score				
Out of	45	75	30	150
%				

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Suggested working time (minutes)	Marks available	Percentage of exam
Section One: Short response	10	10	45	45	30
Section Two: Problem-solving	7	7	75	75	50
Section Three: Comprehension	2	2	30	30	20
					100

Instructions to candidates

1. All numerical answers must be stated to 3 significant figures. Questions containing estimates should be stated to two significant figures.

2. The rules for the conduct of School Curriculum and Standards Authority examinations are detailed in the *Student Information Handbook.* Sitting this examination implies that you agree to abide by these rules.

3. Write answers in this Question/Answer Booklet.

4. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

5. Working or reasoning should be clearly shown when calculating or estimating answers.

6. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

Planning:
If you use the spare pages for planning, indicate this clearly at the top of the page.

Continuing an answer:
If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Section One: Short response - 30% (45 Marks)

This section has **10** questions. Answer **all** questions. Write your answers in the space provided.

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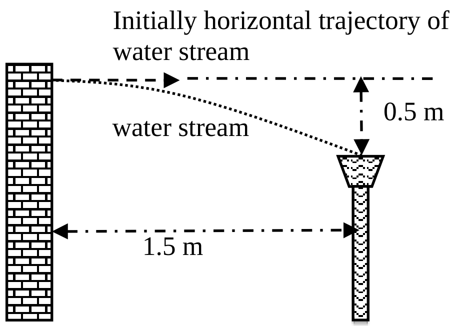
Suggested working time for this section is 45 minutes.

Question 1 (3 marks)

A fountain designer wanted a nozzle embedded in a wall to fire a stream of water horizontally to land in a small bowl, 1.5 m away and 0.5 m lower than the nozzle. At what speed should the designer have planned for the water to exit the nozzle? Air resistance should be ignored.



Question 2 **(5 marks)**

Two students threw a ball backwards and forwards between them. They stood 20 m apart and caught the ball from the same height at which they threw it. The ball was thrown at a constant speed of 15 m s^{-1}. Air resistance should be ignored. They found that two possible launch angles could be used.

a) Sketch a diagram showing the two possible trajectories of the ball. (1 mark)

b) What are the two possible launch angles to the horizontal?
You may need the formula: $\sin(2\theta) = 2 \times \sin\theta \times \cos\theta$ (4 marks)

Question 3 **(5 marks)**

To help cars corner safely, many highway corners are banked slightly so the road slopes downwards towards the inside of the bend. Over time, roads can settle and develop “reverse camber” with the banking sloping downwards towards the outside of the bend.

What frictional force must a 1500 kg car’s tyres provided on a “reverse camber” bend at an angle of 5° and having a radius of 100 m if the car is travelling at 72 km h^{-1}?

Question 4 **(5 marks)**

An electrical generation wind turbine as pictured is a large three bladed fan that spins at a constant speed under constant load in a steady breeze. The mass of the blades is not distributed uniformly but each blade can be approximated to a 15 tonne point mass 35 m from the axis of rotation. The blades typically spin at 22 rpm.

a) Calculate the tension in a blade at its point of attachment at the instant in time that it is pointing vertically up. (3 marks)

b) Calculate the tension in the blade at its point of attachment at the instant in time that it is pointing vertically down (2 marks)

Question 5 **(5 marks)**

Using a powerful optical telescope, astronomers have been able to observe planets orbiting nearby stars. One such planet was found to have a circular orbit of radius $9.27 \mathrm{X} 10^{11}$ m about its star. Its orbital period was $5.23 \mathrm{x} 10^{3}$ days.

a) What was the mass of the star it orbited? (3 marks)

b) A second planet, also in a circular orbit around that star, was discovered. It had an orbital period 2.2 times more than that of the first observed planet. What would its orbital radius have been? (2 marks)

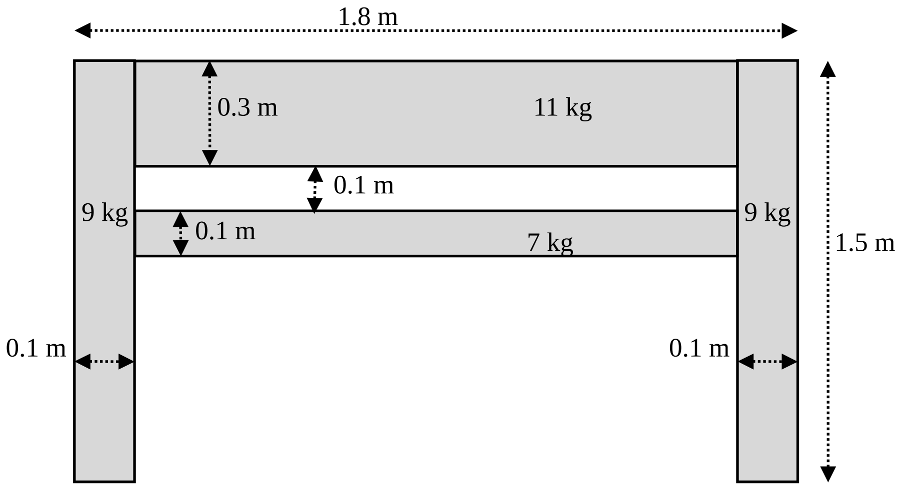
Question 6 **(5 marks)**

a) A satellite in a low circular orbit about the Earth began to spiral in towards the Earth. Explain in terms of forces and energy why this happened and suggest a cause. (3 marks)

b) If the satellite was equipped with directional rocket engines, what action could be taken to restore the satellite to its original orbit? (2 marks)

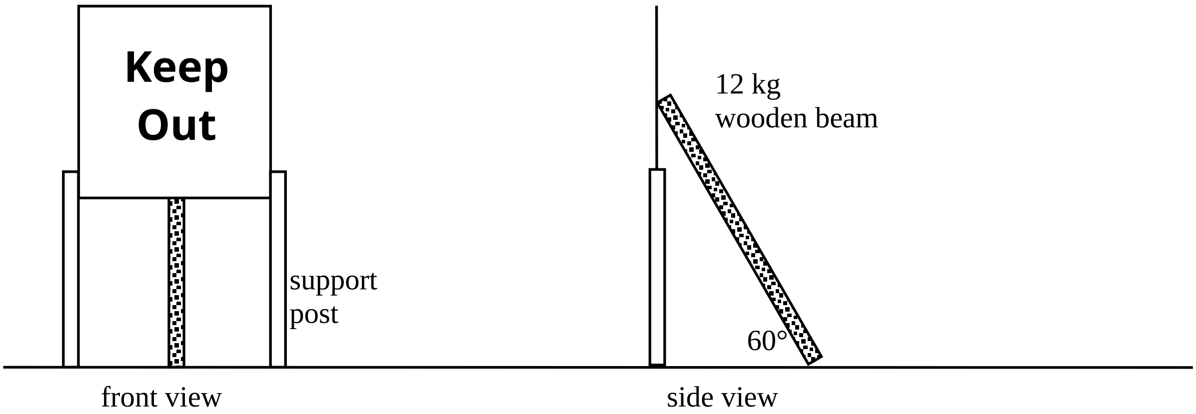
Question 7 **(4 marks)**

Calculate the position of the centre of mass of the heavy wooden bedhead pictured below and mark its position on the diagram. The bedhead is horizontally symmetrical and made of four uniform rectangular pieces of wood joined at right angles to each other. All masses and dimensions are as shown.



Question 8 **(6 marks)**

A “Keep Out” sign that was in danger of being blown over because its support posts were being eaten by termites was propped up with 12 kg beam of wood leant against it as shown. The beam was 2.8 m long. The beam was not attached to the ground or to the smooth back of the sign.

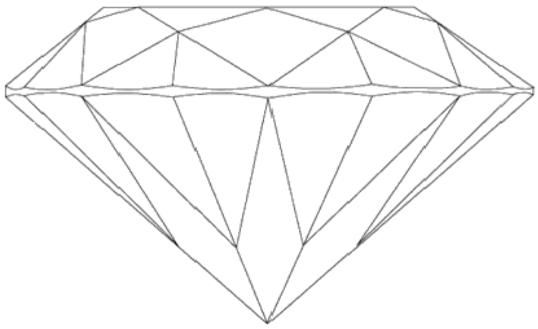


a) What was the force exerted on the back of the sign by the beam if no wind was blowing? (3 marks)

b) What was the force exerted on the beam by the ground if no wind was blowing? (3 marks)

Question 9 **(3 marks)**

Most gem quality diamonds are clear and transparent. They are cut and polished as irregular polyhedra, solid shapes with many flat sides (facets). In any diamond, those facets have a wide range of orientations. Using the picture of a diamond below, explain why someone looking at a diamond moving through direct sunlight sees many flashes of light coming from the diamond.



Question 10 **(4 marks)**

Annabelle lived next door to a house in which a party was being held. She complained to the party's host, Bruce, that she could not sleep with her window open a crack because of the "thump thump" of the music that was being played at the party. Bruce responded that he was surprised that there was a problem because:

- only the windows of his house facing the road were open and they were only open a crack so he thought the music would not even head toward Annabelle's house.
- the music was not "base heavy dance or trance" but music with a good balance of base, mid-range and treble.

Assuming that Bill and Annabelle were honest, explain why the base sounds are heard by Annabelle but the higher frequencies are not.

End of Section One

Section Two : Problem-Solving - 50% **(75 Marks)**

This section has **seven (7)** questions. You must answer **all** questions. Write your answers in the space provided.

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Suggested working time for this section is 75 minutes.

Question 11 **(13 marks)**

A tiny satellite with a mass of 1.00 kg is to be launched into orbit.

a) What is the kinetic energy of a 1.00 kg mass at sea level at the equator due to its rotation with the Earth's surface about the Earth's centre? (2 marks)

b) What is the kinetic energy of a 1.00 kg mass at sea level at the South Pole due to its rotation with the Earth's surface about the Earth's centre? (1 mark)

c) What is the kinetic energy of a 1.00 kg mass in a geostationary orbit above Earth? (4 marks)

The change in gravitational potential energy when an object of mass m is moved from a distance r_1 above Earth's centre to a distance r_2 above Earth's centre is given by:

$$\Delta E_{\text{gravitational-potential}} = -G \times m \times M \times \left(\frac{1}{r_2^{\,2}} - \frac{1}{r_1^{\,2}} \right)$$

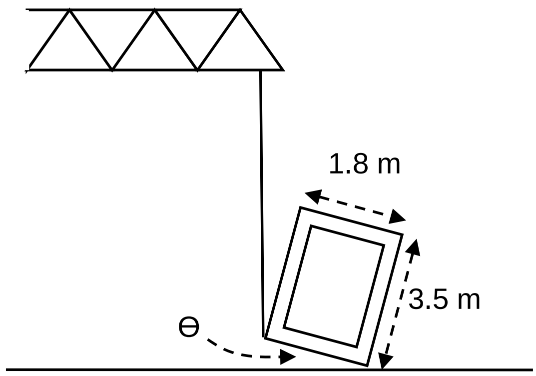
d) What is the increase in potential energy of the object if it is moved from Earth's surface to geostationary orbit? (2 marks)

e) What is the increase in total energy ($E_{\text{gravitational-potential}} + E_K$) of the object if it is moved from Earth's surface at the equator to geostationary orbit? (2 marks)

f) Making reference to the above answers if it assists you, explain why satellites are generally launched from nearer the equator than from nearer the poles. (2 marks)

Question 12 **(16 marks)**

A crane was to be used to lift a rectangular hollow concrete box which was standing on its end on level ground. The box was known to have uniform thickness walls (meaning that its centre of mass was at its geometric centre). Its mass was not known but its external dimensions were. The crane had a lifting capacity that could not be exceeded. In order to check the mass of the box without risking toppling the crane, the crane was used to tilt the box through a range of angles and the torque used to achieve that tilt was measured. From the resulting torque versus tilt graph, the mass of the box was calculated. The crane cable was attached to the lower left corner of the box. See the illustration to the right.



a) Assuming that the crane's lifting cable was hanging vertically and was itself weightless, write a formula for the torque applied to the box about its pivot point in terms of the tension in the cable, T, the width of the box , w, and the angle of tilt, θ. Ensure that units and direction are included. (2 marks)

For this box, the formula relating the tilt angle to the lifting torque in SI units is believed to be: $\tau = m \times g \times 1.968 \times \sin(27.22^{\circ} - \theta)$. The table below shows the data that was collected.

Variable	Dependent or Independent	Units	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6
Angle (θ)		degrees	0	4	8	12	16	20
τ		N m	221000	13000	159000	127000	94000	61000
	manipulated							

b) Identify the independent and dependent variable in the table. (1 mark)

c) Manipulate the appropriate data in the data table to prepare it for a creating a straight line graph. Label the row containing the manipulated data with the manipulated variable. State the units of the variable. (2 marks)

d) Graph the manipulated data below. Identify any outliers by drawing a circle around them on the graph and then draw the line of best fit. (4 marks)

e) Calculate the gradient of the line fit of best on the graph itself. (2 marks)

f) In this copy of formula that followed *part a*, circle the part of the formula that represents the gradient of the line of best fit. $\tau = m \times g \times 1.968 \times \sin(27.22^{\circ} - \theta)$ (1 mark)

g) Using the measured gradient and your answer to part f, estimate the mass of the box. (2 marks)

h) What is the significance of the intersection between the line of best fit and the angle related axis? (2 marks)

Question 13 **(7 marks)**

A salad spinner (illustrated at right) consists of a perforated bowl that can be rotated at high speed inside another bowl. The device is used to remove the water that was used to rinse salad ingredients. The wet ingredients are placed in the inner bowl which is spun rapidly. The water passes through the perforations into the outer bowl leaving the ingredients much drier.

a) The inner perforated bowl is spun at 400 rpm. Calculate its period of rotation in SI units. (1 mark)

b) The inner perforated bowl has a radius of 25 cm. What is the magnitude of the centripetal force on a 50 g lettuce leaf pressed against its side? (3 marks)

c. Explain how spinning the salad removes the water. (3 marks)

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Question 14 **(10 marks)**

The tallest mountain in the solar system is Olympus Mons on Mars at 21870 m high; so tall that its summit is almost into the vacuum of outer space. The radius of Mars is 3.40×10^6 m and its mass is 6.42×10^{23} kg. A cannon that could fire a 10 kg projectile horizontally towards the north pole of the planet was placed on Olympus Mons' summit. A not-to-scale diagram is shown below.

a) The cannon ball was fired with the velocity required for it to land at *point A*. On the diagram above, sketch the trajectory of the cannon ball, ignoring any air resistance. (1 mark)

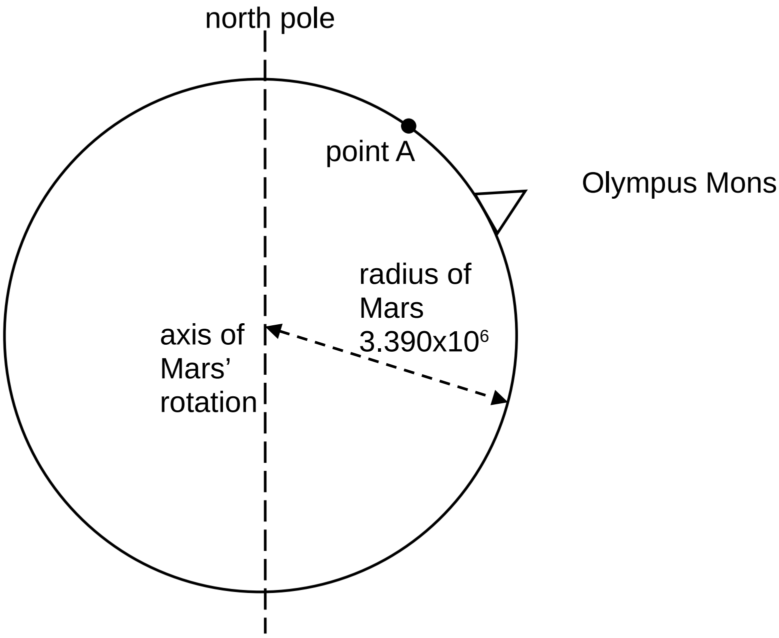
b) For this part of *Question 14* only, ignore the curvature of the surface and friction with the atmosphere, and assume that the gravitational field strength at the top of the mountain is equal to that at the bottom.

i) Calculate the time taken for the cannon ball to land at *point A*, 200 km horizontally from the summit and at 0 m height. (3 marks)

ii) Calculate the initial speed of the cannon ball. (1 mark)

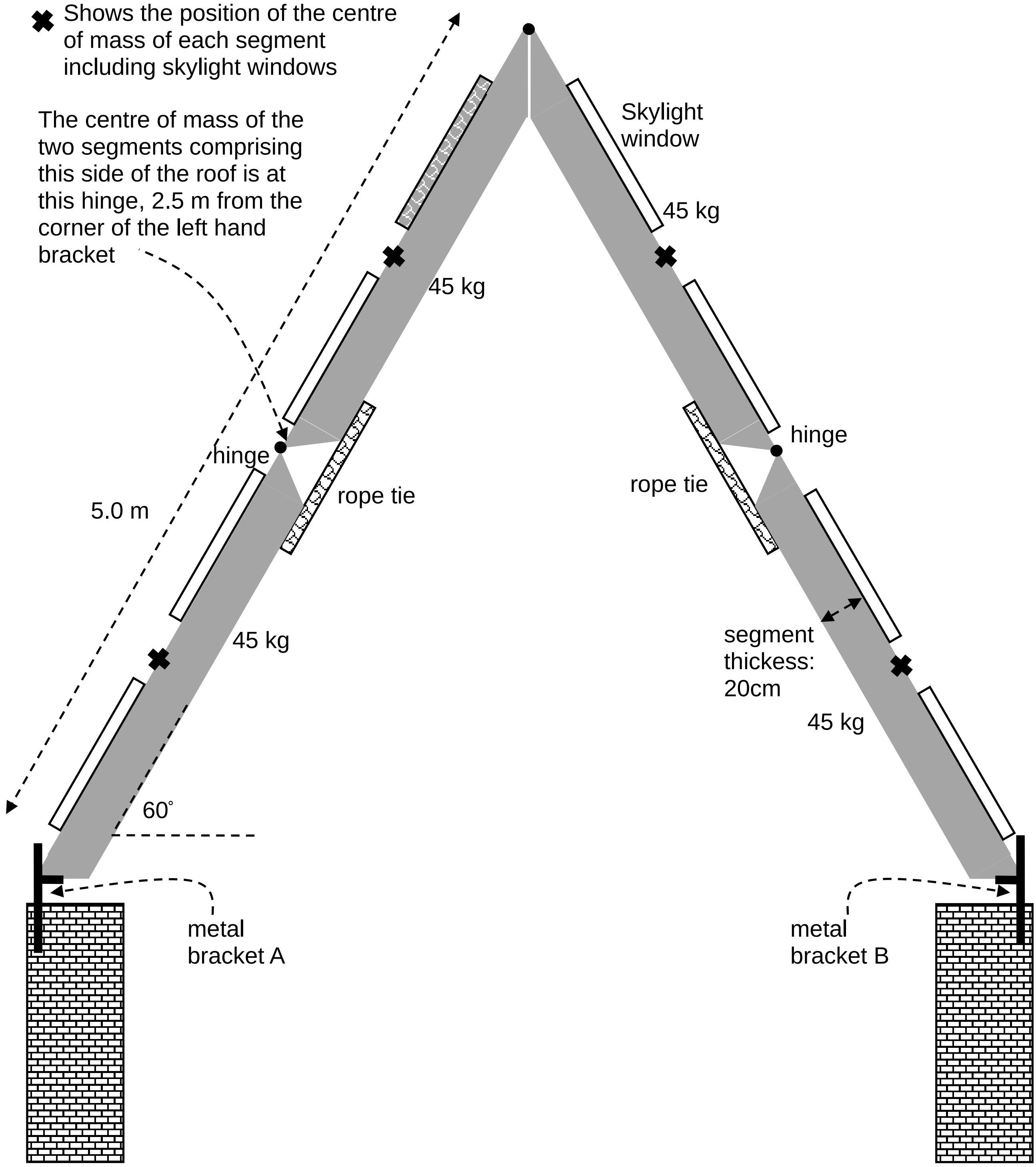
c) The cannon ball was fired with the velocity calculated for it to land at *point A* when zero air resistance was assumed. On the diagram above, sketch the trajectory of the cannon ball assuming that there was actually air resistance. (2 marks)

d) The cannon ball was fired again; this time with sufficient speed to enter a circular orbit around Mars. Calculate that speed, again ignoring any air resistance. The not-to-scale diagram below may assist you. (3 marks)



Question 15 **(10 marks)**

An alpine shelter was built with a steep symmetrical roof. The roof was made in four segments hinged together to allow for easy transport. There were two heavy skylight windows on each segment. The hinged segments each had a mass of 45 kg and a thickness (excluding the windows) of 20 cm. Rope ties were firmly attached in order to prevent the roof from bending at the side hinges when erected. The roof segments rested upon metal brackets set into low walls that supported the outside edges of the lower two segments. Dimensions are as shown below.



a) What was the vertical force on the roof exerted by the metal bracket A? (2 marks)

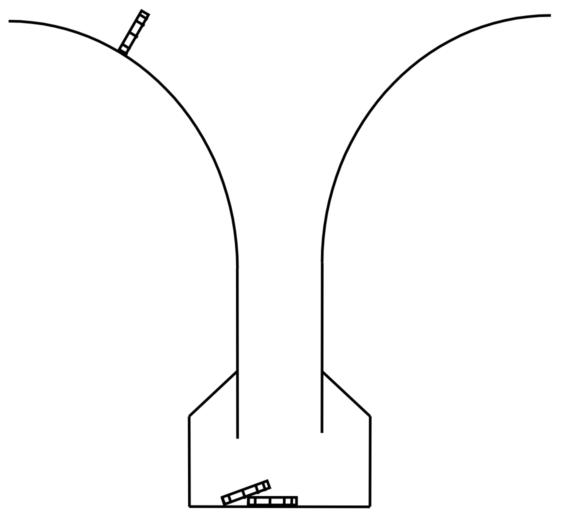
b) By treating each side of the roof as a single piece and by taking moments about metal bracket A, show that the force of the right hand side of the roof on the left hand side of the roof is 255 N to the left. (2 marks)

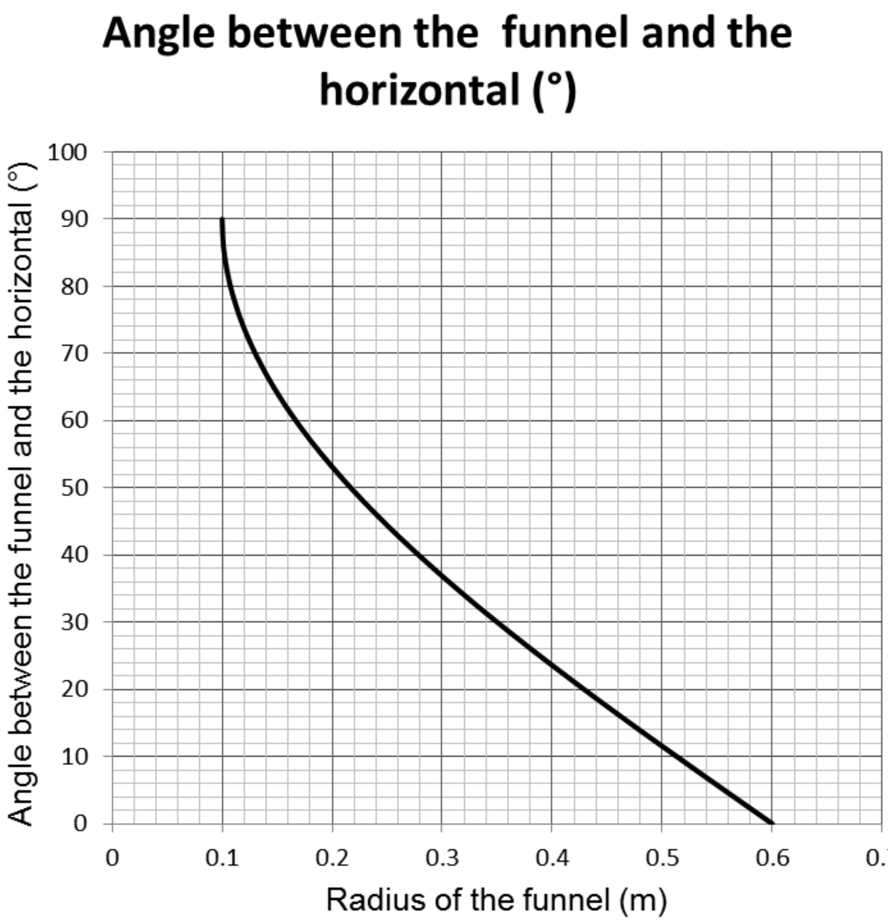
c) What is the horizontal force exerted on the roof by metal bracket A? (1 mark)

d) Calculate the tension in the left hand rope tie that will result in the net torque on the lower left hand roof segment being zero. (Hint, take moments about the hinge in the middle of the left hand side of the roof.) (5 marks)

Question 16 **(10 marks)**

A novelty device used to collect loose change for charities consists of a conical funnel around which a coin can be rolled before finally ending up in a collection bowl. Such a device is illustrated below in cross-section. The funnel has a polished hard surface on which a coin will roll with little friction. In the questions below, assume that the coin being rolled is a one dollar coin with a mass of 9.00 g. As the diagram is not-to-scale, a graph is provided to assist you.





a) A coin is to be rolled in a circular path of radius 0.42 m. Assuming no friction, with what speed must it be rolled? (3 marks)

b) In *part a*, in what direction should the coin have been launched? (1 mark)

c) The coin of *part a* was launched at the same height and speed as required by *part a* but at a slightly different direction to that described in *part b* so that it began heading slightly downwards. Sketch in top down view the trajectory that the coin would follow. For comparison, sketch the circular trajectory of *part a* as a dotted line on the same diagram. Mark the launch point on the diagram. Indicate the direction of travel. (3 marks)

d) Describe and explain the changes to the speed of the coin and the shape of its trajectory that would result from the coin launched again as it was in *part a* but with friction now being a significant factor. (3 marks)

Question 17 **(9 marks)**

The device illustrated at right is a type of siren. When the lower handle is turned, the perforated disk at the top spins rapidly. In order to make the siren sound, the disk is spun and a stream of air is blown through one of the rings of perforations. If we consider the stream of air after it passes through the disk, it rapidly oscillates from a stream to no flow. This oscillation in the flow results in vibrations in the air or sound. The rate at which this oscillation occurs is the frequency of the sound.

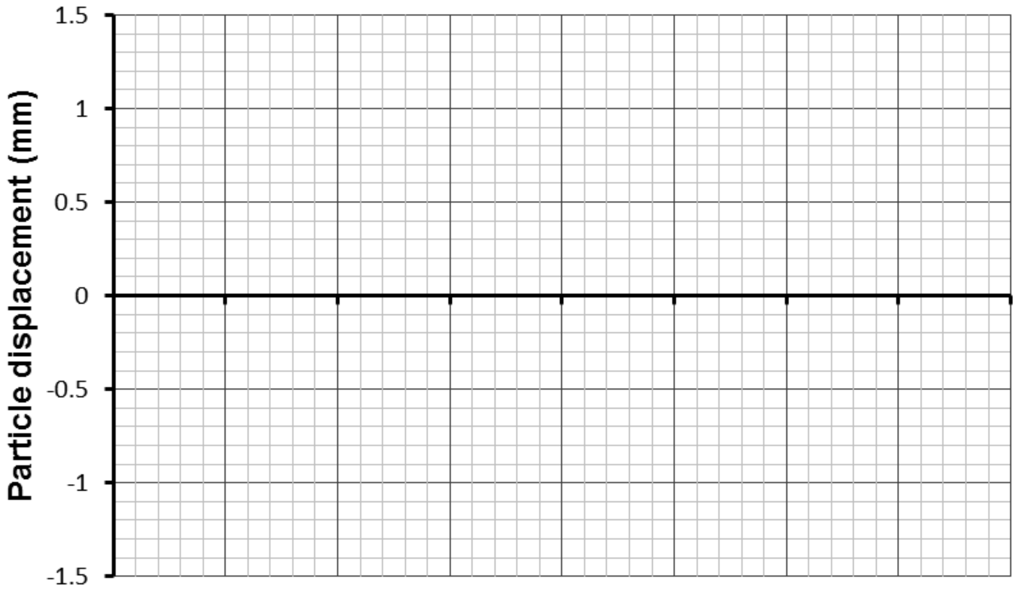
a) If the perforated disk was spun at 2400 rpm what is the frequency at which it spins? (1 mark)

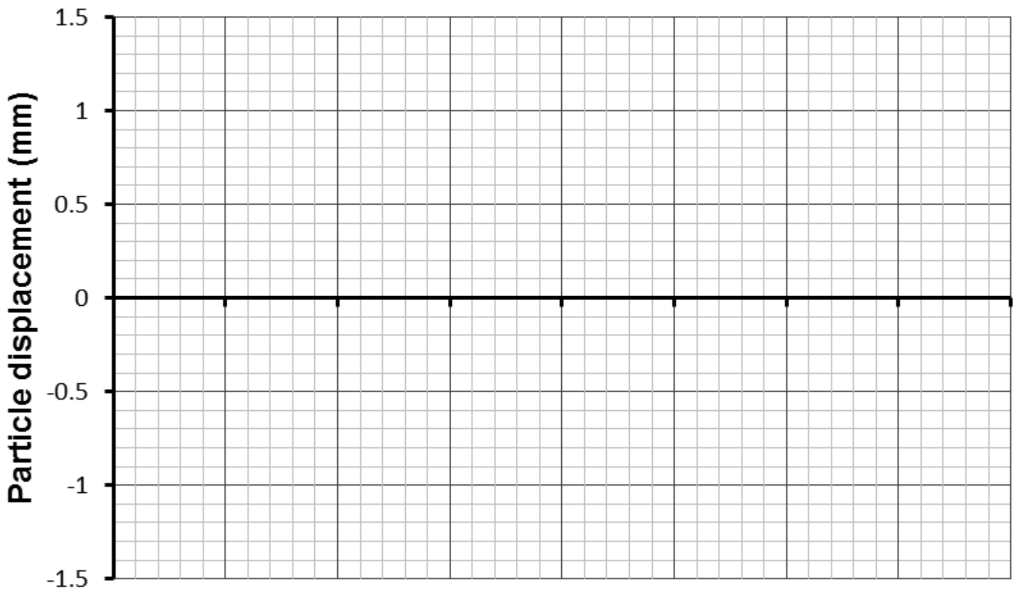
b) If there are ten holes in the ring of holes through which the air is blown, what is the frequency of the sound produced? (2 marks)

c) What is the period of the sound produced in *part b*? (1 mark)

The perforated disk is now rotated at a different speed and produces a sound frequency of 1000 Hz in air with a peak particle displacement of 1 mm.

d) Draw graphs on the axes below each showing 2 complete oscillations for this sound. Each graph will have a vertical axis of "Particle displacement" but they will have different horizontal axes. Annotate the graphs, to show the wavelength, period and amplitude. Accurately plot all the maxima and minima (4 marks)





End of Section Two

Section Three: Comprehension - 20% **(30 Marks)**

This section contains **two (2)** questions. You must answer both questions. Write your answers in the space provided.

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Suggested working time for this section is 30 minutes.

Question 18 **(15 marks)**

Roller Coasters

Paragraph 1
In 1827, a mining company in Summit Hill, Pennsylvania constructed the Mauch Chunk gravity railroad sketched in figure 1. A 14 km downhill track was used to deliver coal to the processing plant at Mauch Chunk from the mine in the hills above. The wagons were dragged up to the mine by mules and then loaded with coal. Their brakes were then released and the coal filled wagons rolled down the wooden rails to the processing plant. By the 1850s, the "Gravity Road" (as it became known) was providing rides to thrill-seekers for 50 cents a ride.

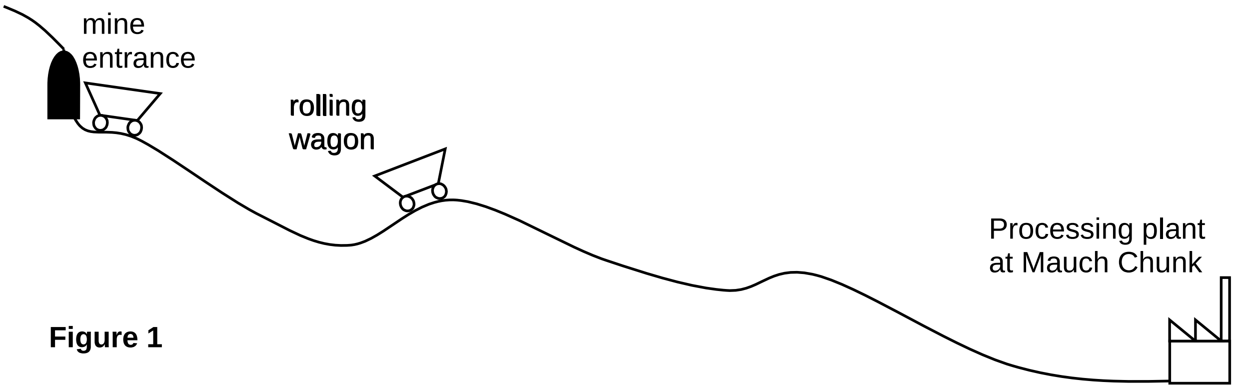


Figure 1

Paragraph 2
The earliest purpose built roller coasters used steam powered winches to drag wagons full of brave thrill seekers to the tops of man-made hills from which the wagons were allowed to roll down wooden rails via a series of smaller man made hills and valleys and around curves of varying radii back to the base of hill at the start. The limiting factor in the complexity and duration of the ride for these devices were the very high levels of friction caused by the use of wooden rails and roughly formed wheels. It has been estimated for the descent phase, such roller coasters typically operated at only 35% efficiency.

Paragraph 3
From their primitive beginnings, roller coasters have evolved into the technically sophisticated contraptions that are loosely called roller coasters today. Efficiencies have been vastly improved with the introduction of polished metal rails and low friction wheels. In some cases, the initial hills have become frighteningly tall, up to nearly 140 m. Other modern roller coasters have dispensed with an initial hill altogether, beginning the ride by catapulting the wagons along level track to start with. Some roller coasters provide an additional push to the wagons at various points along the track. Perhaps the biggest advance in roller coaster was the introduction of so called up-lift wheels.

Paragraph 4
Originally, the wheels at each corner of the wagon were flanged as per figure 2. They were replaced by up-lift wheels. Up-lift wheels are a block of six wheels as illustrated in the side view photograph of figure 3 below. These wheels are arranged so that they not only run along the top of the rails but also along the sides and the undersides of the rails. The wheels that run along the sides of the rails hold the wagons on the track during sharp left or right hand cornering. The flange of the wheels achieved a similar result but with considerably more friction. The wheels that run along the underside of the track hold the wagon on the track during rapid downward accelerations, a result not achieved by flanged wheels.

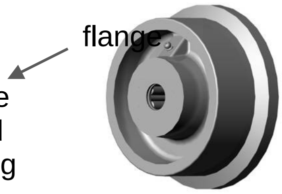


Figure 2: flanged wheel

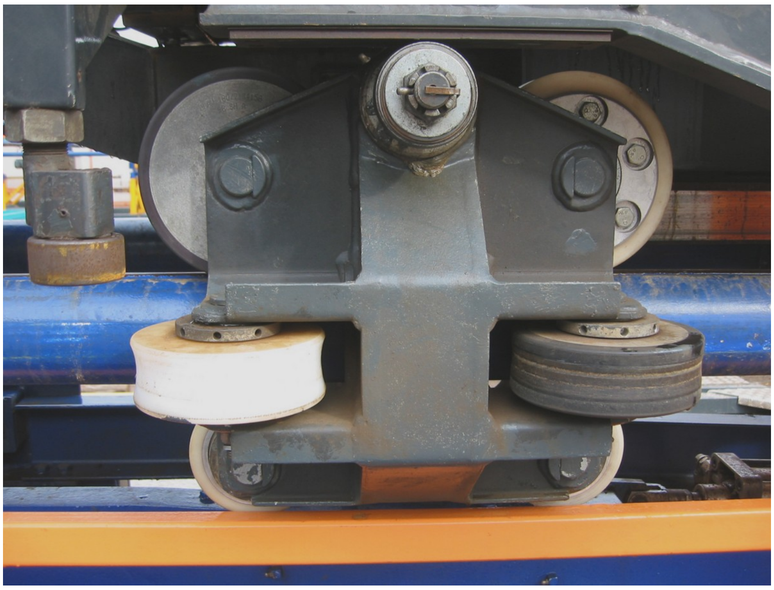


Figure 3: Up-lift wheels – side view

Paragraph 5
Up-lift wheels and the other advances mean that modern roller coasters are able to achieve very high accelerations as they pass over intermediate hills and as they negotiate sudden slope decreases on the way up hills and sudden slope increases on the way down hills. Accelerations of 4.7 g (4.7 times the acceleration due to gravity) are achieved by the most extreme roller coasters.

Paragraph 6
The advent of up-lift wheels and safe passenger harnesses has also meant the roller coaster wagons can now safely operate upside down. This has meant that loop-the-loops are now possible. Even the most modern roller coasters have their wagons “coast” around loop-the-loops, that is, the wagons are not powered as they go around the loop. The Law of Conservation of Energy means that, if roller coaster designers know the speed at which a wagon coasts into the bottom of a loop, they can work out how fast it will be going around the top of the loop (and, yes, they do ignore friction.) They usually try to design the loop height and radius so the passengers are apparently weightless at the top of the loop. You may notice that loop-the-loops are rarely circular. One typical loop-the-loop has a radius of curvature at its top of 7.5 m with a height of 18 m.

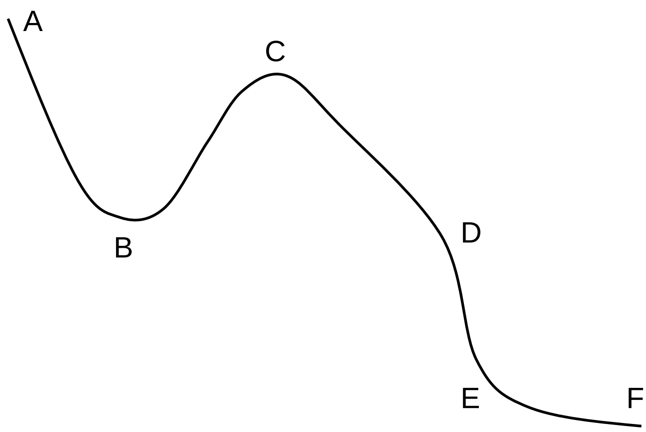
Paragraph 7
Of course, the greatest thrill of any roller coaster is the sensation of “your stomach in your mouth” that results from going over the crest of an intermediate hill at high speed. Our stomach is relatively free to move in the cavity of our abdomen so that if our bodies accelerate with a wagon because we are strapped to it, our stomach initially does not accelerate and gets left behind until the top of our abdominal activity pushes down hard enough on our stomach to give it the same acceleration as our body. That force on our stomach can be enough to result in unpleasant consequences for passengers foolish enough to travel on a roller coaster with a stomach full of soft drink and greasy food.

Questions.

a) For the earliest purpose built roller coasters, what devices were used to provide the initial gravitational potential energy to the wagons and their occupants? (1 mark)

b) If one of the earliest purpose built roller coasters with a typical efficiency lifts its wagons to a height of 60 m, calculate the speed would you expect the wagons to have after the descent phase? (2 marks)

c) The diagram below shows a roller coaster track on which wagons with up-lift wheels travel from left to right. Circle the two points at which the wheels on the underside of the track might exert a force on the track. Explain why. (3 marks)



d) On one modern roller coaster, the wagons round a 10.7 m radius horizontal bend at high speed. The force exerted by the wheels pushing against the side of the track as the wagon negotiates this bend results in the horizontal acceleration of 4.7 g mentioned in paragraph 5. At what speed must the wagon have rounded the bend? (2 marks)

e) Considering the loop-the-loop of paragraph 6 and making the same assumptions as the roller coaster designer, with what speed should a wagon enter the bottom of the loop-the-loop for the passengers to feel weightless at the top? (4 marks)

f) What force is exerted on a passenger's full stomach, with a mass of 2.00 kg, **by the top of their abdominal cavity** if the roller coaster travels over a hill of radius 12 m at a speed of 18 m s^{-1}? (3 marks)

Question 19 (15 marks)

The Pantheon

Figure 1 - The Pantheon from outside

Paragraph 1
The Pantheon is an Ancient Roman temple in Rome Italy. The building consists of two major pieces or rooms. They are the rectangular front and the circular back.

Paragraph 2
The front piece is the smaller of the two. Its roof is held up over the rectangular shape by stone columns. This front section is known as the portico. It acts as an entrance or preliminary room. There are eight columns in the front row and two groups of four behind.

Figure 2 – Inside the Pantheon looking up.

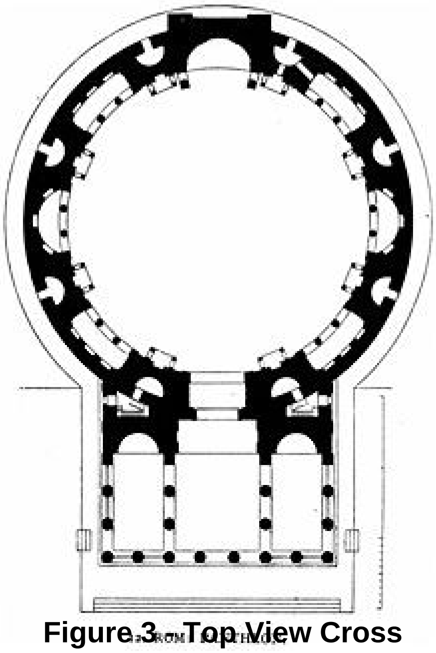


Figure 3 – Top View Cross Section Diagram

Paragraph 3
The second piece is much larger and represents the building proper. This section has a circular floor plan and a huge domed ceiling with a hole at the top of the dome called the oculus. The dome is made from ancient re-enforced concrete. Almost two thousand years after it was built, the Pantheon's dome is still the world's largest unreinforced concrete dome. The height of the oculus above the floor is 43.3 m. The ceiling forms a perfect spherical shape of diameter 43.3 m.

Figure 4 - Artist impression of the inside of the Pantheon

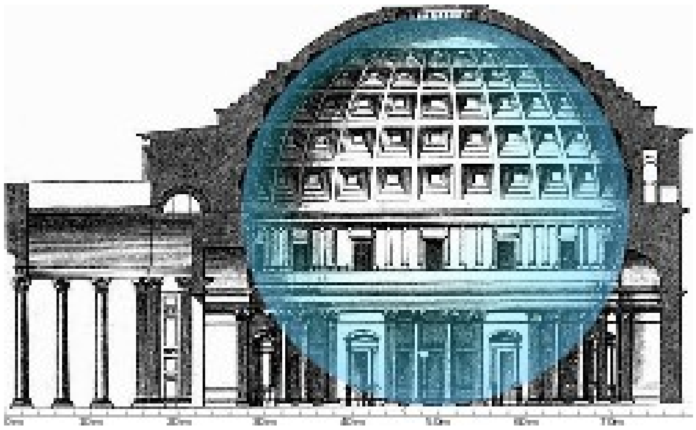


Figure 5 - Cross-section of the Pantheon showing how a 43.3 m diameter sphere fits under its dome.

Paragraph 4
The ceiling was made from molded sections of concrete that are approximately square in shape. They serve both a structural and a decorative purpose. The name given to this effect of square shapes molded from concrete is “coffered”. It is thought that the molded sections were mounted onto temporary scaffolding until a complete ring was in place. The scaffolding was then removed leaving each ring as a self supporting structure.

Paragraph 5
The shape of the roof has a great deal in common the simpler structure of the keystone arch. The domed ceiling however does not require a keystone due to its spherical rather than arched shape. The hole in the top (absence of the keystone) allows natural light to enter the room providing natural illumination to the space (an advantage in an ancient society without the benefit of electricity).

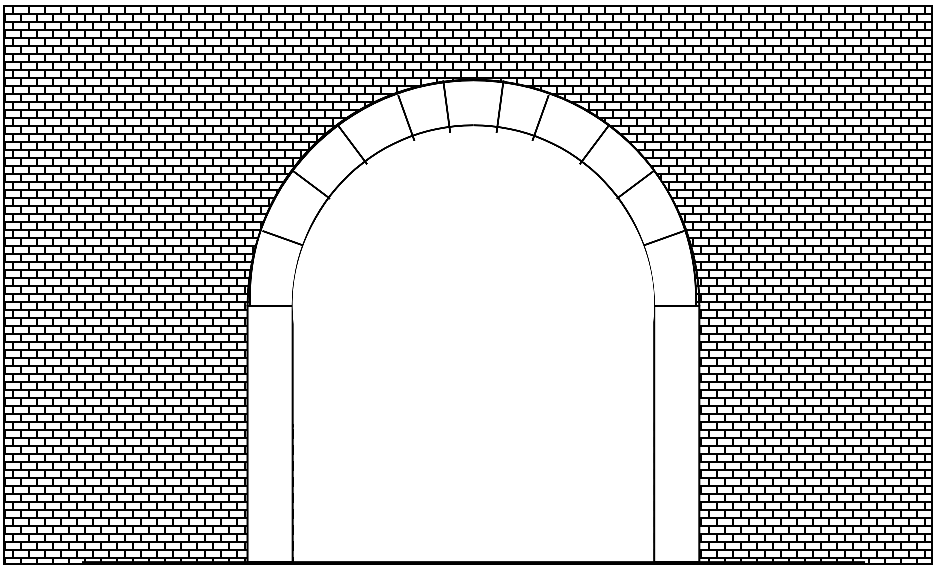
References
http://en.wikipedia.org/wiki/Pantheon,_Rome Extracted 1/4/2013

Questions

a) The Pantheon is largely made from stone and concrete. Explain why the spacing between the pillars in both the front and back rooms cannot be too large and why the stones that are placed across the tops of the pillars must be thick. (2 marks)

b) Why is it that the domed ceiling of the pantheon (second room) has a hole (oculus) but can still stand without collapsing? Use diagrams to assist your explanation. (3 marks)

c) Onto the diagram below indicate how the weight of the structure is supported and transferred around the arch. (3 marks)



d) Why do the outer walls of the pantheon have a wide base? (1 mark)

e) In what structural ways are the “coffered” (hollowed) repeated concrete pieces similar to and different from metal “I” beams. (2 marks)

	Concrete “Coffered” Pieces	Metal I Beams
One Similarity		
One Difference		

f) Would engineers be allowed to build an unreinforced concrete structure like this in modern times? Explain why or why not. (2 marks)

End of Exam

Additional working space

Additional working space

Additional working space

Additional working space

Semester One Examination, 2013

Question/Answer Booklet

3AB PHYSICS

SOLUTIONS

Please place your student identification label in this box

Student Number: In figures

In words

Time allowed for this paper
Reading time before commencing work: Ten minutes
Working time for paper: Two and one half hours

Materials required/recommended for this paper
To be provided by the supervisor
This Question/Answer Booklet
Formulae and Constants Sheet

To be provided by the candidate
Standard items: pens, pencils, eraser, correction fluid, ruler, highlighters
Special items: non-programmable calculators satisfying the conditions set by the School Curriculum and Standards Authority for this course

Important note to candidates
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

	Section 1	**Section 2**	**Section 3**	**Total**
Score				
Out of	45	75	30	150
%				

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Suggested working time (minutes)	Marks available	Percentage of exam
Section One: Short response	10	10	45	45	30
Section Two: Problem-solving	7	7	75	75	50
Section Three: Comprehension	2	2	30	30	20
					100

Instructions to candidates

1. All numerical answers must be stated to 3 significant figures. Questions containing estimates should be stated to two significant figures.

2. The rules for the conduct of School Curriculum and Standards Authority examinations are detailed in the *Student Information Handbook.* Sitting this examination implies that you agree to abide by these rules.

3. Write answers in this Question/Answer Booklet.

4. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

5. Working or reasoning should be clearly shown when calculating or estimating answers.

6. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

Planning:
If you use the spare pages for planning, indicate this clearly at the top of the page.

Continuing an answer:
If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Section One: Short response - 30% **(45 Marks)**

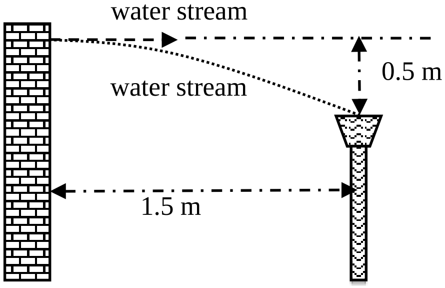
This section has **10** questions. Answer **all** questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Suggested working time for this section is 45 minutes.

Question 1 **(3 marks)**

A fountain designer wanted a nozzle embedded in a wall to fire a stream of water horizontally to land in a small bowl, 1.5 m away and 0.5 m lower than the nozzle. At what speed should the designer have planned for the water to exit the nozzle? Air resistance should be ignored.



Vertical
s = ut + ½at^2 and u = 0
=> t = sqrt(2 x s /a)
= sqrt(2 x 0.5 /9.8) = 0.319 s

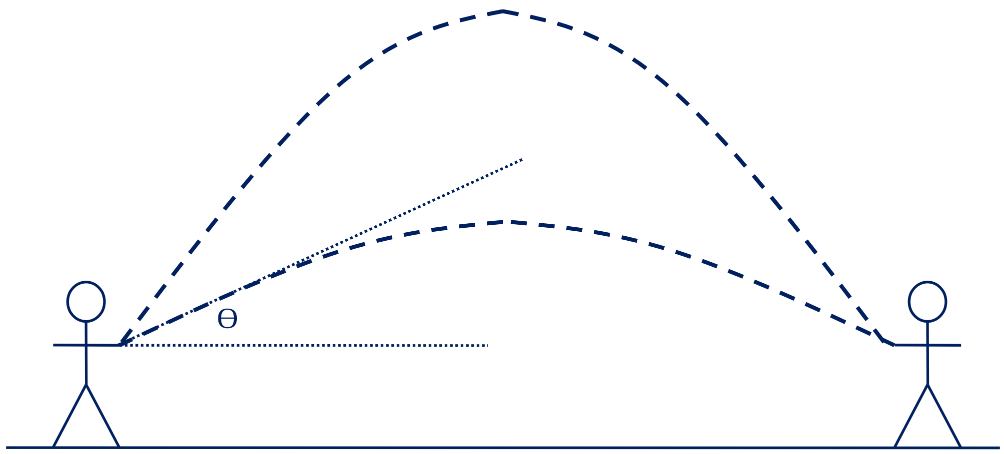
Horizontal
s = ut
=> u = s/t = 1.5/t

Combine
u = s/t = 1.5/0.391 = 4.70 m s^{-1} horizontal to the right

Question 2 **(5 marks)**

Two students threw a ball backwards and forwards between them. They stood 20 m apart and caught the ball from the same height at which they threw it. The ball was thrown at a constant speed of 15 m s^{-1}. Air resistance should be ignored. They found that two possible launch angles could be used.

a) Sketch a diagram showing the two possible trajectories of the ball. (1 mark)



b) What are the two possible launch angles to the horizontal?
You may need the formula: sin(2θ) = 2 x sinθ x cosθ (4 marks)

Vertical
v = u + at and v = -u
=> t = -2u/a = 2 x 15 x sinθ/9.8

Horizontal
s = ut
=> t = s/u = 20/(15 x cosθ)

Combine
2 x 15 x sinθ/9.8 = 20/(15 x cosθ)
=> 2(sinθ x cos θ) = 20 x 9.8 / 15^2) = 0.8711
=> sin(2θ) = 0.8711 => 2θ = 60.59° or 180 – 60.9 = 119.41
=> θ = 30.3° or 59.7° upwards from the horizontal

Question 3 **(5 marks)**

To help cars corner safely, many highway corners are banked slightly so the road slopes downwards towards the inside of the bend. Over time, roads can settle and develop “reverse camber” with the banking sloping downwards towards the outside of the bend.

What frictional force must a 1500 kg car’s tyres provided on a “reverse camber” bend at an angle of 5° and having a radius of 100 m if the car is travelling at 72 km h^{-1}?

$\Sigma F = 0$

Horz

$\Sigma F = mv^2/r$

$F \times \cos\theta - N \times \sin\theta = mv^2/r$

Vert

$\Sigma F = 0$

$F \times \sin\theta + N \times \cos\theta = mg$

$\Rightarrow N = (mg - F \times \sin\theta)/\cos\theta$

Combined

$F \times \cos\theta - ((mg - F \times \sin\theta)/\cos\theta) \times \sin\theta = mv^2/r$

$\Rightarrow F \times (\cos\theta + \sin^2\theta/\cos\theta) = mv^2/r + mg \times \sin\theta/\cos\theta$

$\Rightarrow F/\cos\theta = mv^2/r + mg \times \sin\theta/\cos\theta$

$\Rightarrow F = mv^2 \cos\theta /r + mg \times \sin\theta$

$= 1500 \times (72/3.6)^2 \times \cos5°/100 + 1500 \times 9.8 \times \sin5° = 7258.4 = 7.26\times10^3$ N up the slope

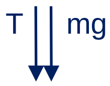
Question 4 **(5 marks)**

An electrical generation wind turbine as pictured is a large three bladed fan that spins at a constant speed under constant load in a steady breeze. The mass of the blades is not distributed uniformly but each blade can be approximated to a 15 tonne point mass 35 m from the axis of rotation. The blades typically spin at 22 rpm.

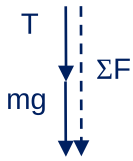
a) Calculate the tension in a blade at its point of attachment at the instant in time that it is pointing vertically up. (3 marks)

$v_{com} = 2 \times \pi \times r / T = 2 \times \pi \times r \times f = 2 \times \pi \times 35 \times 22/60 = 80.6 \text{ m s}^{-1}$
$\Sigma F = mv^2/r = 15\times10^3 \times 80.6^2/35 = 2.79\times10^6 \text{ N}$

Free body diagram



Sum of the forces diagram

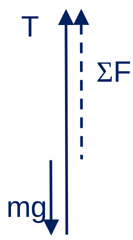


$T = \Sigma F - mg = 2.79\times10^6 - 15\times10^3 \times 9.8 = 2.64\times10^6 \text{ N}$

b) Calculate the tension in the blade at its point of attachment at the instant in time that it is pointing vertically down (2 marks)

Free body diagram

Sum of the forces diagram



$T = \Sigma F + mg = 2.79\times10^6 + 15\times10^3 \times 9.8 = 2.93\times10^6 \text{ N}$

Question 5 **(5 marks)**

Using a powerful optical telescope, astronomers have been able to observe planets orbiting nearby stars. One such planet was found to have a circular orbit of radius $9.27\text{X}10^{11}$ m about its star. Its orbital period was $5.23\text{x}10^{3}$ days.

a) What was the mass of the star it orbited? (3 marks)

$$GM/4\pi^2 = r^3/T^2$$
$$\Rightarrow M = 4\pi^2 r^3/(G \text{ x } T^2)$$
$$= 4\pi^2 \text{ x } (9.27\text{X}10^{11})^3/(6.67\text{x}10^{-11} \text{ x } (5.23\text{x}10^3 \text{ x } 24 \text{ x } 3600)^2)$$
$$= 2.31\text{x}10^{30} \text{ kg}$$

b) A second planet, also in a circular orbit around that star, was discovered. It had an orbital period 2.2 times more than that of the first observed planet. What would its orbital radius have been? (2 marks)

$$r_1^3/ T_1^2 = r_2^3 /T_2^2$$
$$\Rightarrow T_2^2/ T_1^2 = r_2^3/ r_1^3$$
$$\Rightarrow (T_2/ T_1)^2 = (r_2/ r_1)^3$$
$$\Rightarrow (T_2/ T_1)^{2/3} = r_2/ r_1$$
$$\Rightarrow r_2 = (T_2/ T_1)^{2/3} \text{ x } r_1 = 2.2^{2/3} \text{ x } 9.27\text{x}10^{11} = 1.57\text{x}10^{12} \text{ m}$$

Question 6 **(5 marks)**

a) A satellite in a low circular orbit about the Earth began to spiral in towards the Earth. Explain in terms of forces and energy why this happened and suggest a cause. (3 marks)

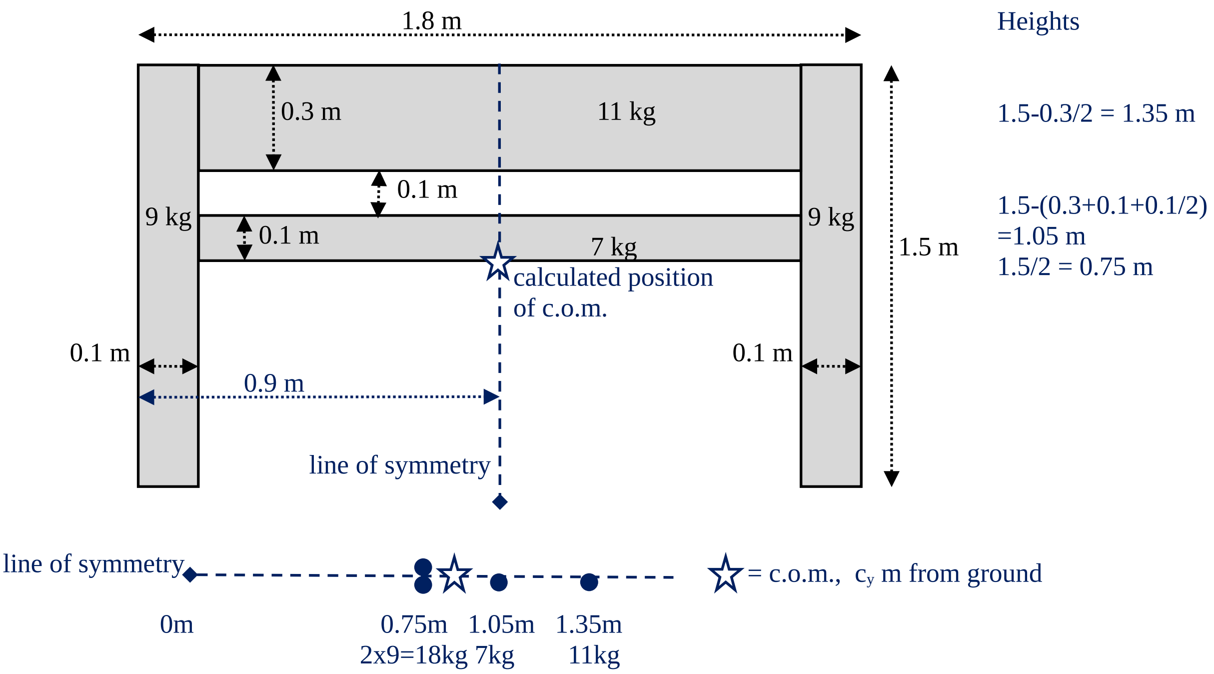
A force in the opposite direction to the instantaneous velocity of the satellite must be acting on the satellite. The force results in energy being transferred away from the satellite so the total energy of the satellite falls. Given that the satellite is in low Earth orbit, the most likely cause of the force is air resistance.

b) If the satellite was equipped with directional rocket engines, what action could be taken to restore the satellite to its original orbit? (2 marks)

The rocket engines could be fired on the opposite direction to the satellites instantaneous velocity.

Question 7 **(4 marks)**

Calculate the position of the centre of mass of the heavy wooden bedhead pictured below and mark its position on the diagram. The bedhead is horizontally symmetrical and made of four uniform rectangular pieces of wood joined at right angles to each other. All masses and dimensions are as shown.



Realise that c.o.m. will fall on line of symmetry. Draw this line. Its x coord is x coord of c.o.m..
Project the c.o.m. of each block of wood onto that line.
Draw the line horizontally.
Take moments about the c.o.m. (or use c.o.m. formula: $x_{com}=(m_1x_1+m_2x_2+ \dots +m_nx_n)/(m_1+m_2+ \dots +m_n)$

$\Sigma\tau = 0 \quad \Rightarrow \tau_c = \tau_{ac}$

$\Rightarrow g(c_y-0.75)18=g((1.05- c_y)7+(1.35- c_y))11$ (g cancels)

$\Rightarrow 18c_y -13.5= -7 c_y+7.35-11c_y +14.85$

$\Rightarrow c_y(18+7+11) = 13.5+7.35+14.85$

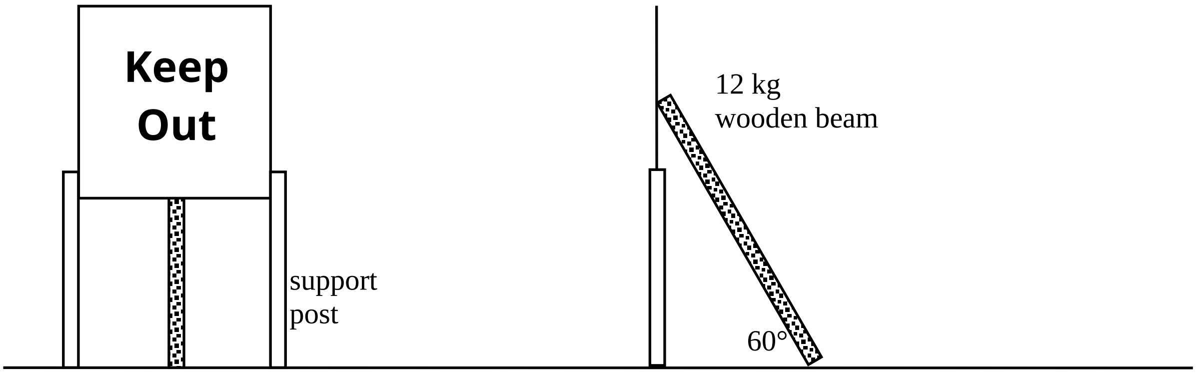
$\Rightarrow 36c_y = 35.7$

$\Rightarrow c_y=35.7/36 = 0.9917=0.992$ m

Therefore c.o.m. is 0.992 m above the ground and 0.9 m from either side of the bedhead

Question 8 **(6 marks)**

A “Keep Out” sign that was in danger of being blown over because its support posts were being eaten by termites was propped up with 12 kg beam of wood leant against it as shown. The beam was 2.8 m long. The beam was not attached to the ground or to the smooth back of the sign.



a) What was the force exerted on the back of the sign by the beam if no wind was blowing? (3 marks)

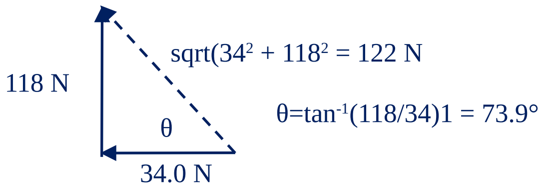
Find N_{sign}, the force of the back of the sign on the beam.
Take moments about point of contact of the beam with the ground:
$\Sigma\tau = 0$
$\tau_c = N_{sign} \times 2.8 \times \sin60° = \tau_{ac} = m \times g \times 2.8 \times ½ \times \cos60°$
$\Rightarrow N_{sign} = m \times g \times 2.8 \times ½ \times \cos60°/(2.8 \times \sin60°) = 12 \times 9.8 \times \cancel{2.8} / (\cancel{2.8} \times 2 \times \tan60°) = 34.0$ N
therefore the force of the back of the sign on the beam is 34.0 N to the right

b) What was the force exerted on the beam by the ground if no wind was blowing? (3 marks)

$\Sigma F = 0$

Vertical
$mg + N_{ground} = 0$
$\Rightarrow N_{ground} = -mg = -12 \times 9.8 = -117.6$ N i.e.118 N up

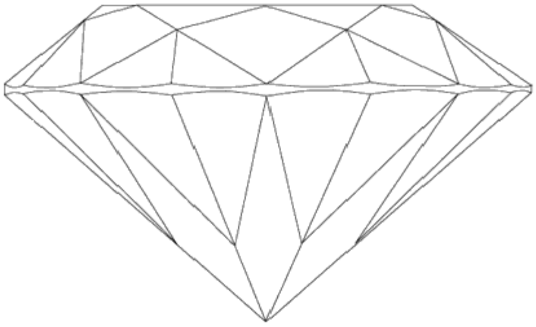
Horizontal
$F_{friction} + N_{sign} = 0$
$\Rightarrow F_{friction} = - N_{sign} = -34.0$ i.e. 34 N left



Force of ground on beam is 122 N left 73.9° up

Question 9 **(3 marks)**

Most gem quality diamonds are clear and transparent. They are cut and polished as irregular polyhedra, solid shapes with many flat sides (facets). In any diamond, those facets have a wide range of orientations. Using the picture of a diamond below, explain why someone looking at a diamond moving through direct sunlight sees many flashes of light coming from the diamond.



Diamonds have a very high refractive index.

For a diamond in air θc is therefore small.

As a result, light hitting a facet at other than a small angle of incidence will totally internally reflect with the angle of incidence equalling the angle of reflection.

Because there are a large number of facets, sunlight will reflect from many facets. Because these facets are all in different orientations, these reflections will be in a wide variety of directions.

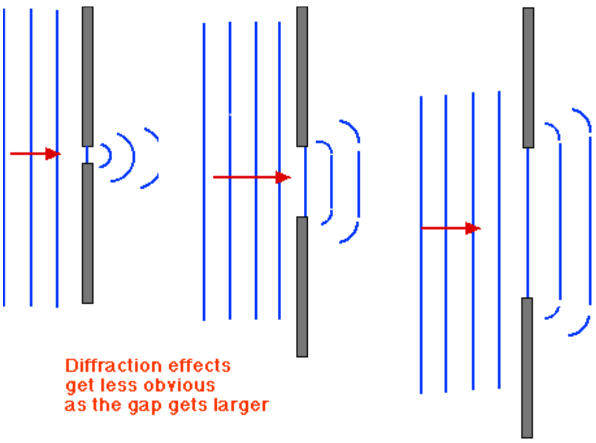
As the ring is moved about, many facets in turn will reflect light into and observer's eye which will be detected as flashes of light.

Question 10 **(4 marks)**

Annabelle lived next door to a house in which a party was being held. She complained to the party's host, Bruce, that she could not sleep with her window open a crack because of the "thump thump" of the music that was being played at the party. Bruce responded that he was surprised that there was a problem because:

- only the windows of his house facing the road were open and they were only open a crack so he thought the music would not even head toward Annabelle's house.
- the music was not "base heavy dance or trance" but music with a good balance of base, mid-range and treble.

Assuming that Bill and Annabelle were honest, explain why the base sounds are heard by Annabelle but the higher frequencies are not.



Include a diagram like the one beside. The smaller the gap compared to the wavelength, the more sound "bends" around the edges of the gap. Narrow window openings are small gaps for the long wavelengths of low frequencies but are relatively large gaps for the short wavelengths of the high frequencies. Annabelle will hear the low frequencies of the music that diffract but not hear the high frequencies that diffract very little as so travel in straight paths away from Bruce's house and not towards Annabelle's house.

End of Section One

Section Two : Problem-Solving - 50% **(75 Marks)**

This section has **seven (7)** questions. You must answer **all** questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Suggested working time for this section is 75 minutes.

Question 11 **(13 marks)**

A tiny satellite with a mass of 1.00 kg is to be launched into orbit.

a) What is the kinetic energy of a 1.00 kg mass at sea level at the equator due to its rotation with the Earth's surface about the Earth's centre? (2 marks)

$v = 2 \times \pi \times r / T = 2 \times \pi \times 6.38 \times 10^6 / (24 \times 3600) = 464 \text{ m s}^{-1}$
$E_K = ½ \times m \times v^2 = ½ \times 1 \times 464^2 = 1.08 \times 10^5 \text{ J}$

b) What is the kinetic energy of a 1.00 kg mass at sea level at the South Pole due to its rotation with the Earth's surface about the Earth's centre? (1 mark)

0 J

c) What is the kinetic energy of a 1.00 kg mass in a geostationary orbit above Earth? (4 marks)

$$\frac{GmM}{r^2} = \frac{m \times v^2}{r} \text{ and } v = \frac{2\pi r}{T} \Rightarrow \frac{GmM}{r^2} = \frac{m \times \left(\frac{2\pi r}{T}\right)^2}{r} \Rightarrow \frac{GM}{4\pi^2} = \frac{r^3}{T^2}$$

$$\Rightarrow \quad r_{geostationary} = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times (24 \times 3600)^2}{4\pi^2}} = 4.22 \times 10^7 \text{ m}$$

$$v_{geostationary} = \frac{2\pi r}{T} = \frac{2\pi\, 4.22 \times 10^7}{24 \times 3600} = 3.07 \times 10^3 \text{ m s}^{-1}$$

$$E_{K:geostationary} = \frac{1}{2} mv^2 = \frac{1}{2} \times 1 \times (3.07 \times 10^3)^2 = 4.71 \times 10^6 \text{ J}$$

The change in gravitational potential energy when an object of mass m is moved from a distance r_1 above Earth's centre to a distance r_2 above Earth's centre is given by:

$$\Delta E_{gravitational\text{-}potential} = -G \times m \times M \times \left(\frac{1}{r_2^{\,2}} - \frac{1}{r_1^{\,2}}\right) \quad \Delta E_{gravitational\text{-}potential} = -G \times m \times M \times \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

d) What is the increase in potential energy of the object if it is moved from Earth's surface to geostationary orbit? (2 marks)

$\Delta E_{gravitational\text{-}potential}$ = -GmM x $((1/r_2) – (1/r_1))$

$$= -\ 6.67 \times 10^{-11} \times 1 \times 5.97 \times 10^{24} \left(\frac{1}{(4.22 \times 10^7)^2} - \frac{1}{(6.38 \times 10^6)^2}\right)$$

= 9.56 J

$\Delta E_{gravitational\text{-}potential}$ = -GmM x $((1/r_2) – (1/r_1))$

$$= -\ 6.67 \times 10^{-11} \times 1 \times 5.97 \times 10^{24} \times \left(\frac{1}{4.22 \times 10^7} - \frac{1}{6.38 \times 10^6}\right)$$

= 5.30×10^7 J

e) What is the increase in total energy ($E_{gravitational\text{-}potential} + E_K$) of the object if it is moved from Earth's surface at the equator to geostationary orbit? (2 marks)

$$\Delta E_T = E_{T:geostationary} - E_{T:earths\text{-}surface\text{-}at\text{-}equator}$$
$$= E_{P:geostationary} + E_{K:geostationary} - (E_{P:earths\text{-}surface\text{-}at\text{-}equator} + E_{K:earths\text{-}surface\text{-}at\text{-}equator})$$
$$= (E_{P:geostationary} - E_{P:earths\text{-}surface\text{-}at\text{-}equator}) + (E_{K:geostationary} - E_{K:earths\text{-}surface\text{-}at\text{-}equator})$$
$$= 9.56 + (4.71\text{x}10^6 - 1.08\text{x}10^5)$$
$$= 4.61\text{x}10^6 \text{ J}$$

$$\Delta E_T = E_{T:geostationary} - E_{T:earths\text{-}surface\text{-}at\text{-}equator}$$
$$= E_{P:geostationary} + E_{K:geostationary} - (E_{P:earths\text{-}surface\text{-}at\text{-}equator} + E_{K:earths\text{-}surface\text{-}at\text{-}equator})$$
$$= (E_{P:geostationary} - E_{P:earths\text{-}surface\text{-}at\text{-}equator}) + (E_{K:geostationary} - E_{K:earths\text{-}surface\text{-}at\text{-}equator})$$
$$= 5.30\text{x}10^7 + (4.71\text{x}10^6 - 1.08\text{x}10^5)$$
$$= 5.76\text{x}10^7 \text{ J}$$

f) Making reference to the above answers if it assists you, explain why satellites are generally launched from nearer the equator than from nearer the poles. (2 marks)

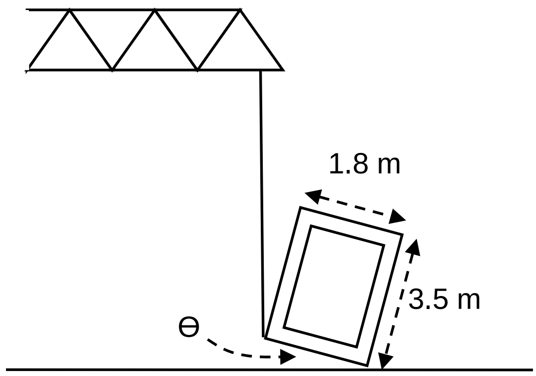
Because the less energy that is required to launch a satellite, the more economic is the launch, it is usual to launch from locations where required energy is minimised. The minimum energy required for launching a satellite into a geostationary orbit is given by the formula:

$(E_{P:geostationary} - E_{P:earths\text{-}surface}) + (E_{K:geostationary} - E_{K:earths\text{-}surface})$

$E_{P:geostationary}$, $E_{P:earths\text{-}surface}$ and $E_{K:geostationary}$ do not depend on the location on Earth's surface of the launch. As calculated in *parts a and b* above, $E_{K:earths\text{-}surface}$ does depend on launch location. As $E_{K:earths\text{-}surface}$ is subtracted when computing the total energy required,, it can be seen that the greater is $E_{K:earths\text{-}surface}$, the less the energy required to launch the satellite. That is, if the satellite already has some E_K at launch (due to the Earth's rotation), less additional energy will be needed to launch the satellite, provided that it is launched in the direction of Earth's rotation.

Question 12 **(16 marks)**

A crane was to be used to lift a rectangular hollow concrete box which was standing on its end on level ground. The box was known to have uniform thickness walls (meaning that its centre of mass was at its geometric centre). Its mass was not known but its external dimensions were. The crane had a lifting capacity that could not be exceeded. In order to check the mass of the box without risking toppling the crane, the crane was used to tilt the box through a range of angles and the torque used to achieve that tilt was measured. From the resulting torque versus tilt graph, the mass of the box was calculated. The crane cable was attached to the lower left corner of the box. See the illustration to the right.



a) Assuming that the crane's lifting cable was hanging vertically and was itself weightless, write a formula for the torque applied to the box about its pivot point in terms of the tension in the cable, T, the width of the box , w, and the angle of tilt, θ. Ensure that units and direction are included. (2 marks)

τ = T x w x cosθ N m clockwise

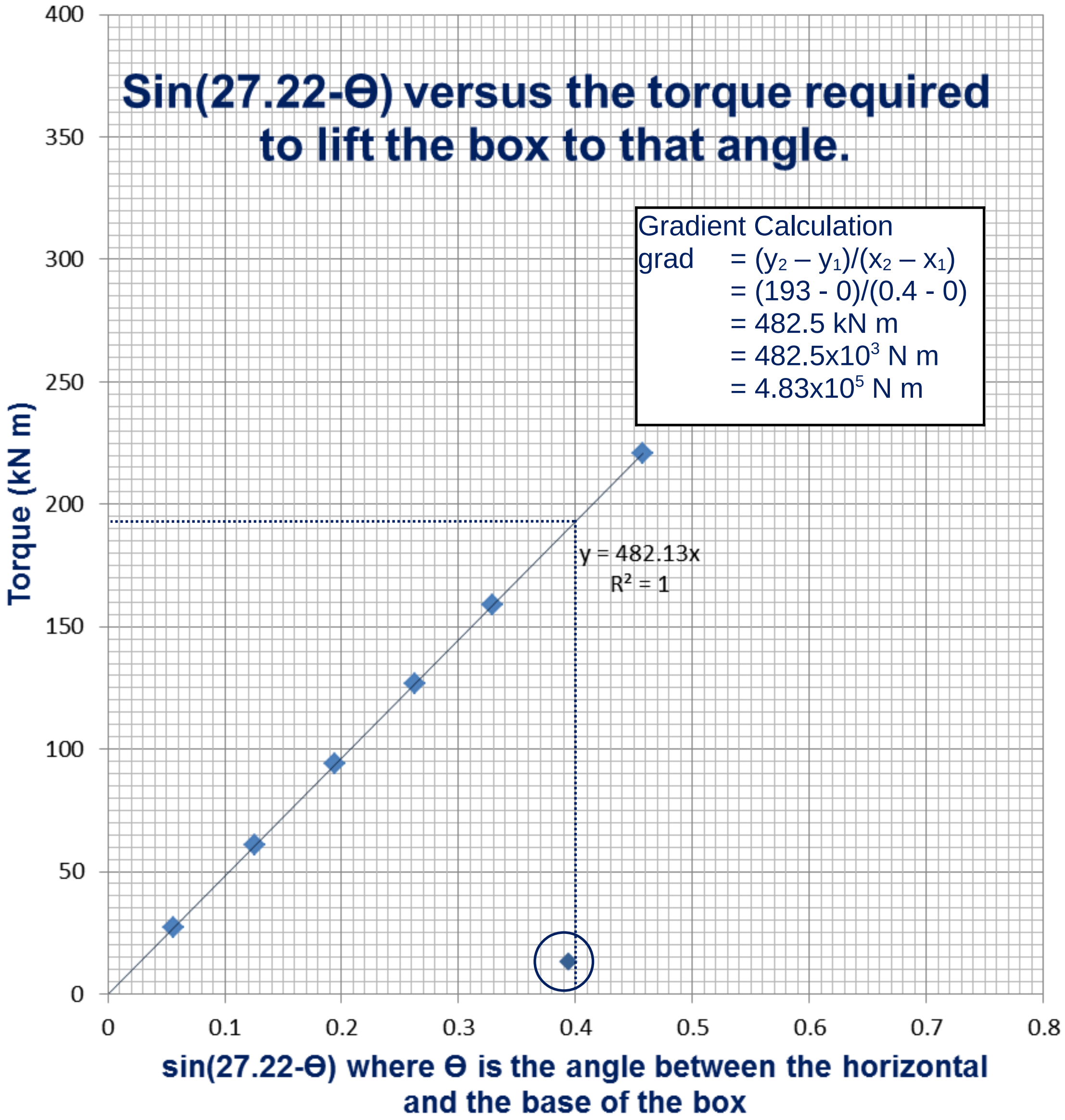
For this box, the formula relating the tilt angle to the lifting torque in SI units is believed to be: $\tau = m \times g \times 1.968 \times \sin(27.22^\circ - \theta)$. The table below shows the data that was collected.

Variable	Dependent or Independent	Units	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6
Angle (θ)	independent	degrees	0	4	8	12	16	20
τ	dependent	N m	221000	13000	159000	127000	94000	61000
sin(27.22 - θ))	manipulated	none	0.457	0.394	0.329	0.262	0.195	0.126

b) Identify the independent and dependent variable in the table. (1 mark)

c) Manipulate the appropriate data in the data table to prepare it for a creating a straight line graph. Label the row containing the manipulated data with the manipulated variable. State the units of the variable. (2 marks)

d) Graph the manipulated data below. Identify any outliers by drawing a circle around them on the graph and then draw the line of best fit. (4 marks)



e) Calculate the gradient of the line fit of best on the graph itself. (2 marks)

f) In this copy of formula that followed *part a*, circle the part of the formula that represents the gradient of the line of best fit. $\tau = m \times g \times 1.968 \times \sin(27.22^\circ - \theta)$ (1 mark)

g) Using the measured gradient and your answer to part f, estimate the mass of the box. (2 marks)

m x g x 1.968 = 4.825×10^5 => m = 4.825×10^5 / (9.8 x 1.968) = 2.50×10^4 kg

h) What is the significance of the intersection between the line of best fit and the angle related axis? (2 marks)

Sin(27.22 – θ) = 0 and torque = 0 where the line of best fit meets the angle related axis. If sin(27.22 – θ) = 0 then θ = 27.22. As 0 torque is required to further tilt the box at this angle, this is the tippling angle of the block.

Question 13 **(7 marks)**

A salad spinner (illustrated at right) consists of a perforated bowl that can be rotated at high speed inside another bowl. The device is used to remove the water that was used to rinse salad ingredients. The wet ingredients are placed in the inner bowl which is spun rapidly. The water passes through the perforations into the outer bowl leaving the ingredients much drier.

a) The inner perforated bowl is spun at 400 rpm. Calculate its period of rotation in SI units. (1 mark)

$f = 400/60\ s^{-1}$ $T = 1/f = 60/400 = 0.150\ s$

b) The inner perforated bowl has a radius of 25 cm. What is the magnitude of the centripetal force on a 50 g lettuce leaf pressed against its side? (3 marks)

$F = m \times v^2 / r$ and $v = 2 \times \pi \times r / T$

$\Rightarrow F = m \times (2 \times \pi \times r / T)^2 / r$

$= m \times 4 \times \pi^2 \times r/T^2$

$= 50\times10^{-3} \times 4 \times \pi^2 \times 25\times10^{-2}/0.150^2$

$= 21.9\ N$

c. Explain how spinning the salad removes the water. (3 marks)

Perforated bowl provides an unbalanced force on the leaf and causes it to accelerate towards the centre of the spinner, (centripetal acceleration). Therefore, the leaf changes direction.

The perforations mean that perforated bowl cannot push the water, so the water does not accelerate and continues in its original direction.

That is, the leaf accelerates away from the water thus drying the salad.

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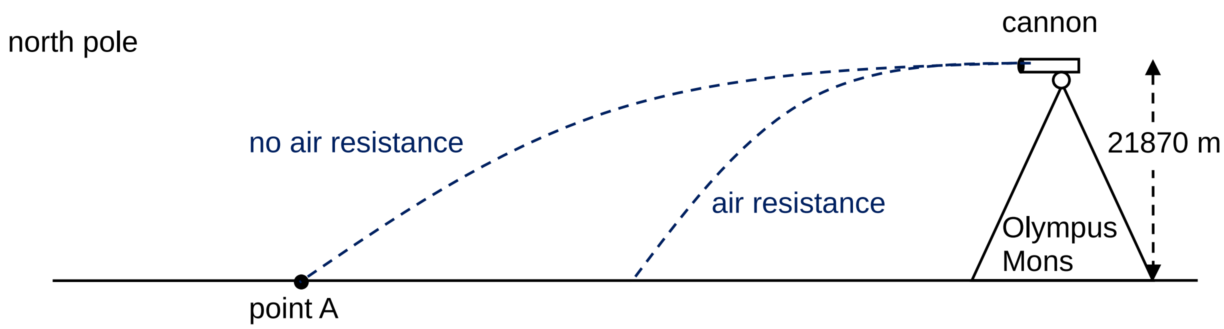
Question 14 **(10 marks)**

The tallest mountain in the solar system is Olympus Mons on Mars at 21870 m high; so tall that its summit is almost into the vacuum of outer space. The radius of Mars is 3.40×10^6 m and its mass is 6.42×10^{23} kg. A cannon that could fire a 10 kg projectile horizontally towards the north pole of the planet was placed on Olympus Mons' summit. A not-to-scale diagram is shown below.



a) The cannon ball was fired with the velocity required for it to land at *point A*. On the diagram above, sketch the trajectory of the cannon ball, ignoring any air resistance. (1 mark)

b) For this part of *Question 14* only, ignore the curvature of the surface and friction with the atmosphere, and assume that the gravitational field strength at the top of the mountain is equal to that at the bottom.

i) Calculate the time taken for the cannon ball to land at *point A*, 200 km horizontally from the summit and at 0 m height. (3 marks)

vert

a $= GM/r^2$
$= 6.67 \times 10^{-11} \times 6.42 \times 10^{23}/(3.40 \times 10^6)^2$
$= 3.70$

s $= ut + ½at^2$ $\quad u = 0$

=> s $= ½at^2$

=> t $= \text{sqrt}(2s/a)$
$= \text{sqrt}(2 \times 21870/3.70)$
$= 109$ s

ii) Calculate the initial speed of the cannon ball. (1 mark)

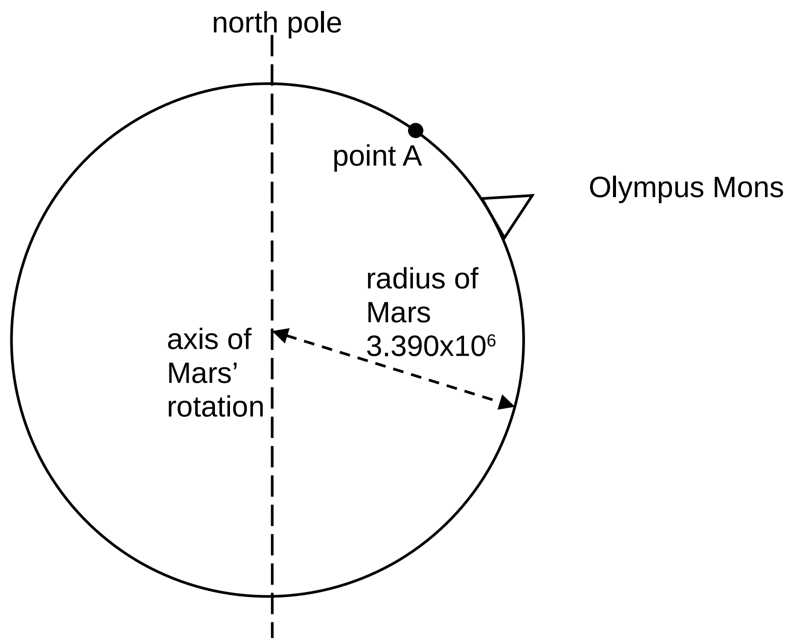
horz

s $= ut$

=> u $= s/t$
$= 200 \times 10^3/109$
$= 1.84 \times 10^3$ m s^{-1}

c) The cannon ball was fired with the velocity calculated for it to land at *point A* when zero air resistance was assumed. On the diagram above, sketch the trajectory of the cannon ball assuming that there was actually air resistance. (2 marks)

d) The cannon ball was fired again; this time with sufficient speed to enter a circular orbit around Mars. Calculate that speed, again ignoring any air resistance. The not-to-scale diagram below may assist you. (3 marks)



$GM/r^2 = v^2/r$

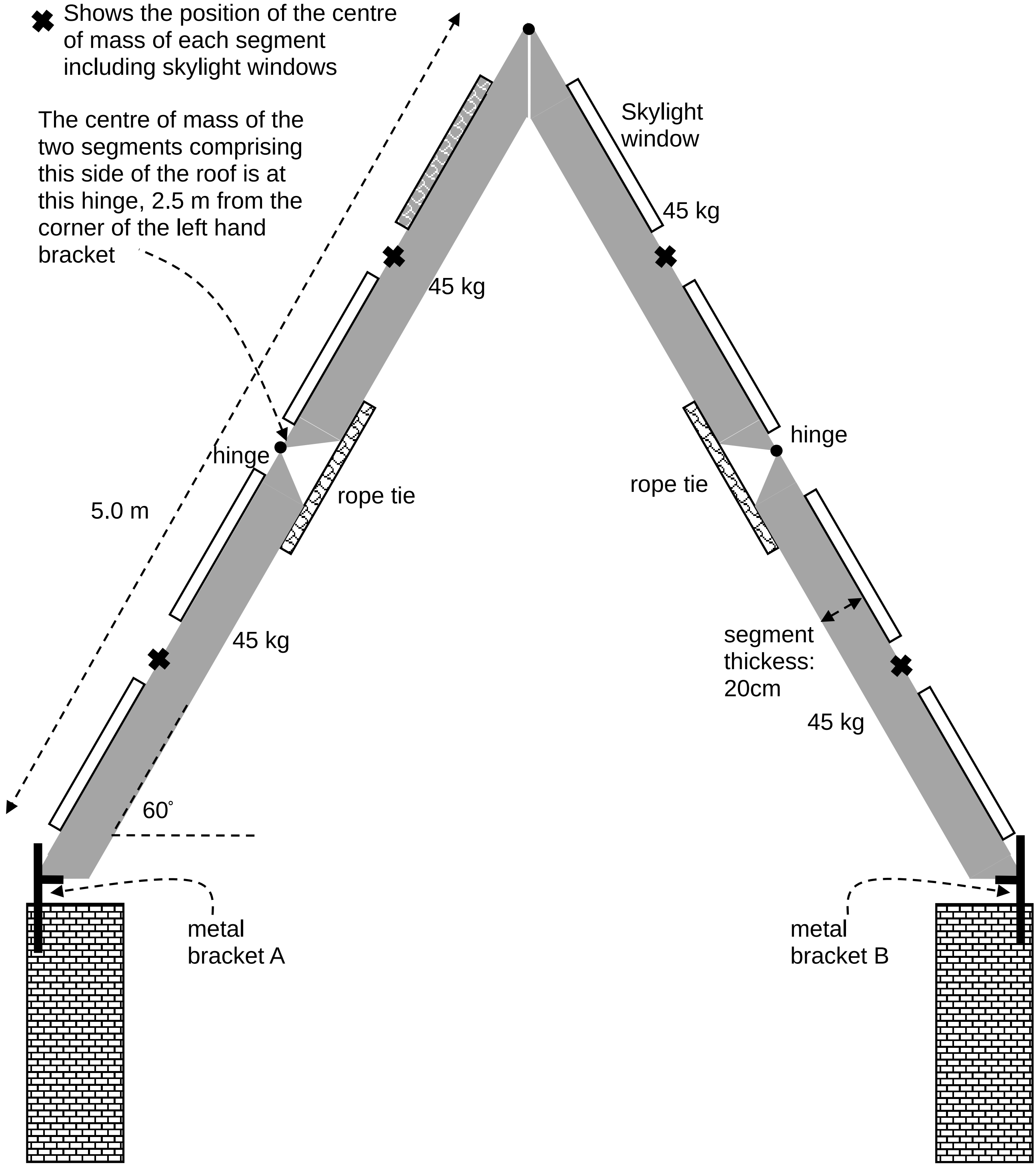
=> $v = \text{sqrt}(GM/r)$

$= \text{sqrt}(6.67\text{x}10^{-11} \text{ x } 6.42\text{x}10^{23}/(3.40\text{x}10^{6}+21870)$

$= 3.54\text{x}10^{3} \text{ m s}^{-1}$

Question 15 **(10 marks)**

An alpine shelter was built with a steep symmetrical roof. The roof was made in four segments hinged together to allow for easy transport. There were two heavy skylight windows on each segment. The hinged segments each had a mass of 45 kg and a thickness (excluding the windows) of 20 cm. Rope ties were firmly attached in order to prevent the roof from bending at the side hinges when erected. The roof segments rested upon metal brackets set into low walls that supported the outside edges of the lower two segments. Dimensions are as shown below.



a) What was the vertical force on the roof exerted by the metal bracket A? (2 marks)

$\Sigma F_{vert}=0 \Rightarrow |F_{bracket:A}|=(4 \times 45) \times g/2 = 882$ N upwards *(½ the total weight of the roof)*

b) By treating each side of the roof as a single piece and by taking moments about metal bracket A, show that the force of the right hand side of the roof on the left hand side of the roof is 255 N to the left. (2 marks)

Analyse left hand side of roof
Taking moments about LH bracket $\Sigma\tau = 0 \Rightarrow \tau_{clockwise} = \tau_{anticlockwise}$

$\tau_{clockwise}$ = weight x r = 2 x 45 x g x 5 x cos60°/2 *(5 x cos60°/2 is ¼ width of roof)*
$\tau_{anticlockwise}$ = $thrust_{RHS\_on\_LHS}$ x 5 x sin60° *(5 x sin60° is height of roof)*
$\tau_{clockwise} = \tau_{anticlockwise}$
=> 2 x 45 x g x 5 x cos 60°/2= $thrust_{RHS\_on\_LHS}$ x 5 x sin60°
=> $thrust_{RHS\_on\_LHS}$ = 2 x 45 x g x 5 x cos 60°/(2 x 5 x sin60°)
= 45 x g / tan60°
= 255 N to the left

c) What is the horizontal force exerted on the roof by metal bracket A? (1 mark)

255 N to the right

d) Calculate the tension in the left hand rope tie that will result in the net torque on the lower left hand roof segment being zero. (Hint, take moments about the hinge in the middle of the left hand side of the roof.) (5 marks)

Analyse the lower left hand roof segment
Taking moments about hinge in the middle of the left hand side of the roof.
$\Sigma\tau = 0 \Rightarrow \tau_{clockwise} = \tau_{anticlockwise}$

$\tau_{anticlockwise}$ = *(τ due to weight + τ due to $bracket_{horz}$ + τ due to Tension)*
= 45 x g x 5 x cos60°/4 + 255 x 5 x sin60°/2 +Tension x 0.2
(method 2:
when weight of LH segment of roof is appropriately "slid, its perpendicular distance from the hinge is or1/8 width of roof or 5 x cos60°/4
when the horizontal force of the LH bracket is "slid" its perpendicular distance from the hinge is ½ height of roof or 5 x sin60°/2
when the tension is appropriately "slid", its perpendicular distance from the hinge is 20 cm)

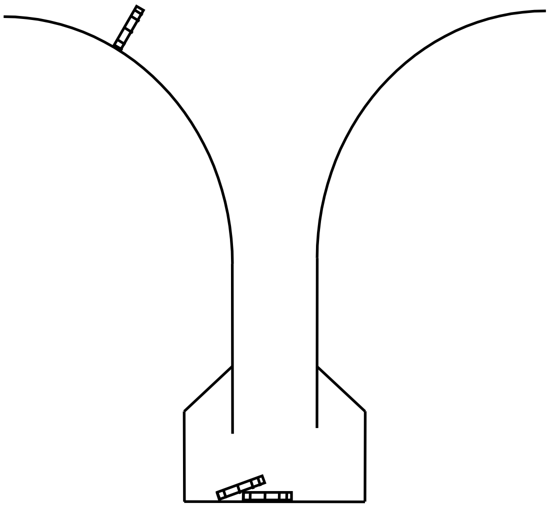
$\tau_{clockwise}$ = *τ due to $bracket_{vert}$*
= 882 x 5 x cos60°/2

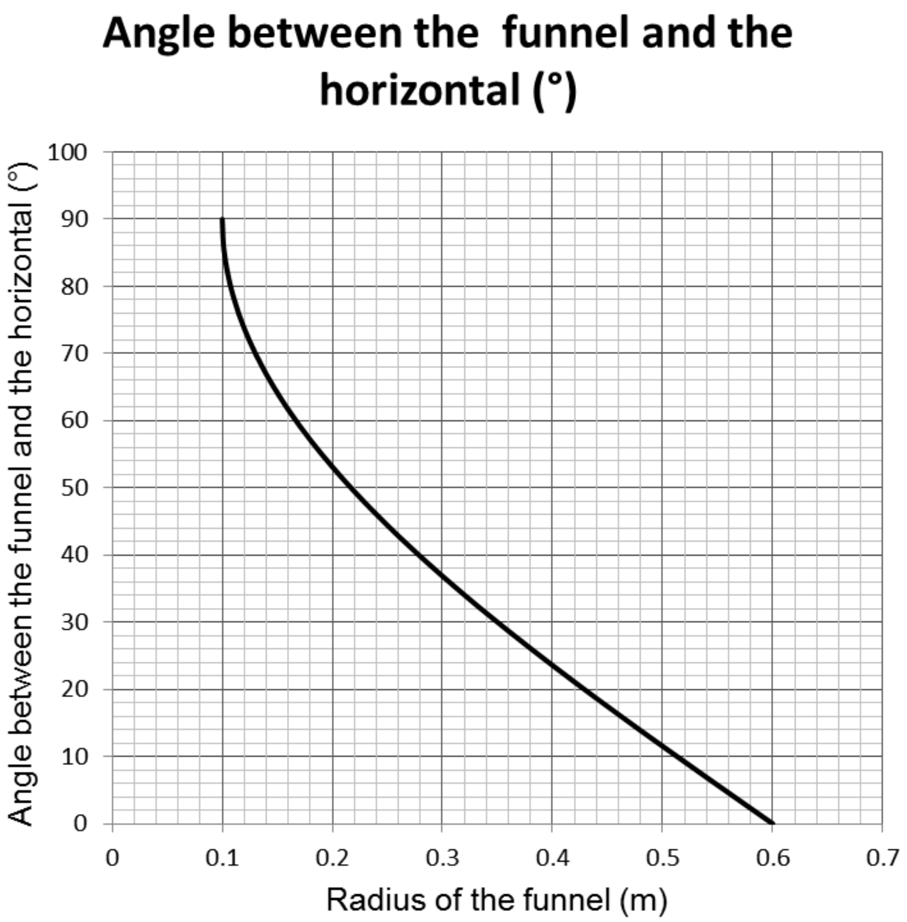
(method 2:
when the vertical force of the LH bracket is "slid" its perpendicular distance from the hinge is ¼ width of roof or 5 x cos60°/2)

$\tau_{clockwise} = \tau_{anticlockwise}$
=> 45 x g x 5 x cos60°/4 + 255 x 5 x sin60°/2 +Tension x 0.2= 882 x 5 x cos60°/2
=> Tension = (882 x 5 x cos60°/2- 45 x g x 5 x cos60°/4 - 255 x 5 x sin60°/2)/0.2
= 1.37×10^3 N (full precision gives 1.38×10^3 N

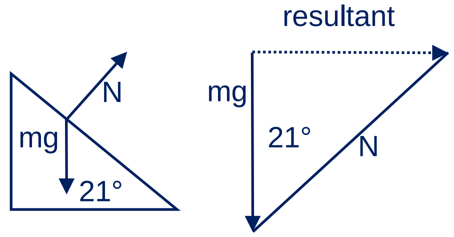
Question 16 **(10 marks)**

A novelty device used to collect loose change for charities consists of a conical funnel around which a coin can be rolled before finally ending up in a collection bowl. Such a device is illustrated below in cross-section. The funnel has a polished hard surface on which a coin will roll with little friction. In the questions below, assume that the coin being rolled is a one dollar coin with a mass of 9.00 g. As the diagram is not-to-scale, a graph is provided to assist you.





a) A coin is to be rolled in a circular path of radius 0.42 m. Assuming no friction, with what speed must it be rolled? (3 marks)



=>

$mg\tan 21 = mv^2/r$

$v = \text{sqrt}(r \times g \times \tan 21)$

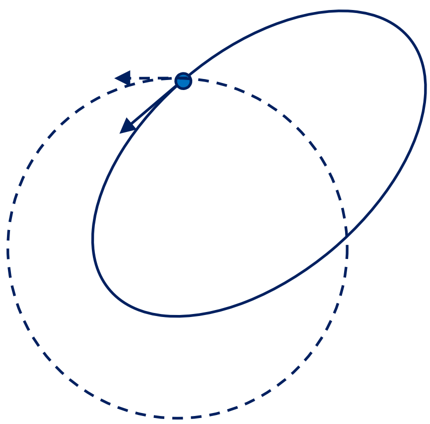
$= \text{sqrt}(0.42 \times 9.8 \times \tan 21)$

$= 1.26 \text{ m s}^{-1}$

b) In *part a*, in what direction should the coin have been launched? (1 mark)

tangential to the horizontal circle of radius 0.42 m

c) The coin of *part a* was launched at the same height and speed as required by *part a* but at a slightly different direction to that described in *part b* so that it began heading slightly downwards. Sketch in top down view the trajectory that the coin would follow. For comparison, sketch the circular trajectory of *part a* as a dotted line on the same diagram. Mark the launch point on the diagram. Indicate the direction of travel. (3 marks)



d) Describe and explain the changes to the speed of the coin and the shape of its trajectory that would result from the coin launched again as it was in *part a* but with friction now being a significant factor. (3 marks)

Coin would spiral down the funnel with its radius getting smaller and smaller as its energy is transformed to heat because of friction. However, as it falls, its speed would increase as required for circular motion with increased centripetal force due to the increase slope of the cone. (That increase would be offset by the decrease in in speed required due to the reduced radius – a smaller effect than the increased force.) The increased speed implies increased kinetic energy but the total energy of the coin falls because its potential energy falls so much as it gets lower in the funnel.

Question 17 **(9 marks)**

The device illustrated at right is a type of siren. When the lower handle is turned, the perforated disk at the top spins rapidly. In order to make the siren sound, the disk is spun and a stream of air is blown through one of the rings of perforations. If we consider the stream of air after it passes through the disk, it rapidly oscillates from a stream to no flow. This oscillation in the flow results in vibrations in the air or sound. The rate at which this oscillation occurs is the frequency of the sound.

a) If the perforated disk was spun at 2400 rpm what is the frequency at which it spins? (1 mark)

240/60 = 40 Hz

b) If there are ten holes in the ring of holes through which the air is blown, what is the frequency of the sound produced? (2 marks)

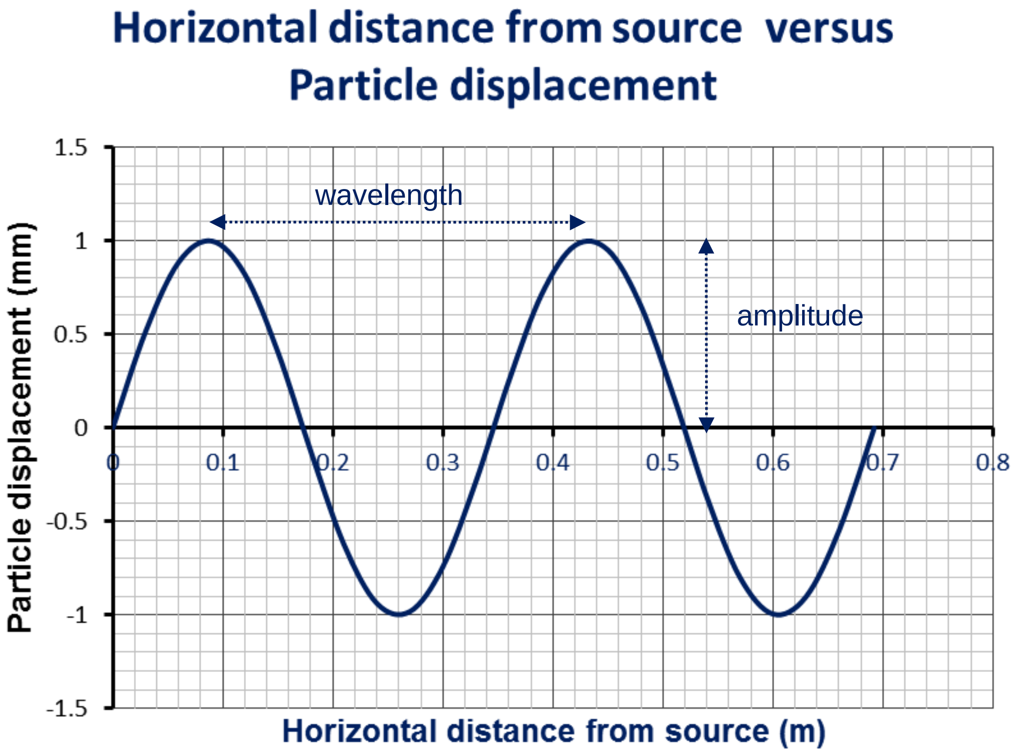
240 x 10 = 2400 per minute therefore 2400/60=400Hz

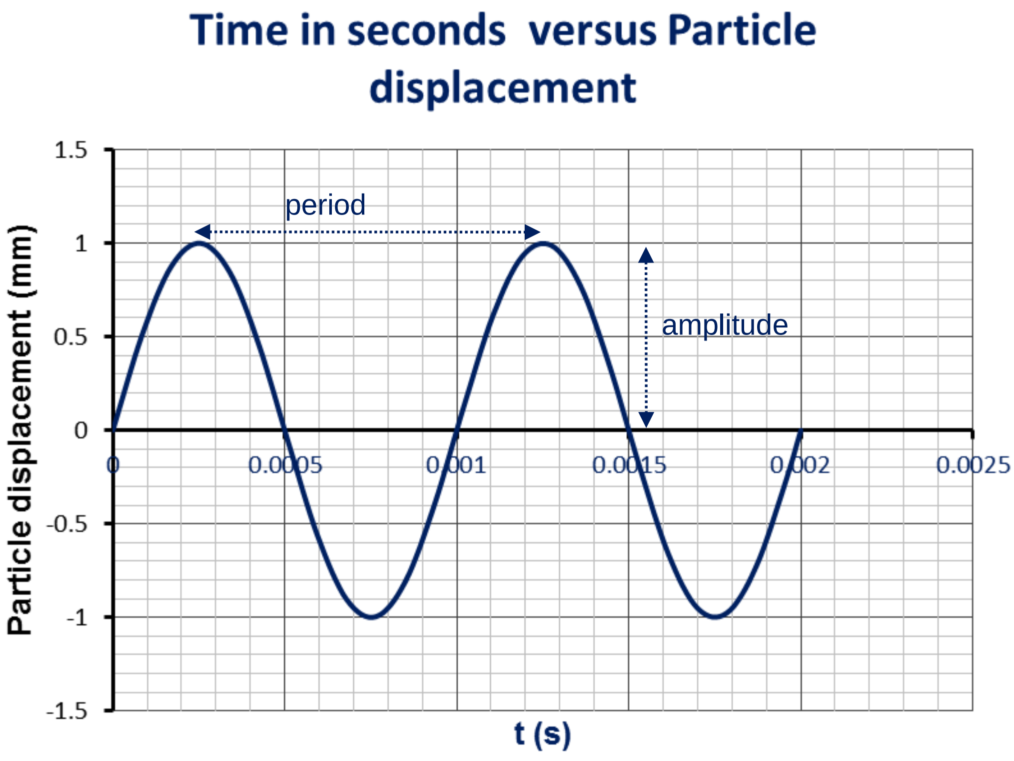
c) What is the period of the sound produced in *part b*? (2 mark)

1/400=0.0025 s

The perforated disk is now rotated at a different speed and produces a sound frequency of 1000 Hz in air with a peak particle displacement of 1 mm.

d) Draw graphs on the axes below each showing 2 complete oscillations for this sound. Each graph will have a vertical axis of "Particle displacement" but they will have different horizontal axes. Annotate the graphs, to show the wavelength, period and amplitude. Accurately plot all the maxima and minima (4 marks)





End of Section Two

Section Three: Comprehension - 20% **(30 Marks)**

This section contains **two (2)** questions. You must answer both questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Suggested working time for this section is 30 minutes.

Question 18 **(15 marks)**

Roller Coasters

Paragraph 1
In 1827, a mining company in Summit Hill, Pennsylvania constructed the Mauch Chunk gravity railroad sketched in figure 1. A 14 km downhill track was used to deliver coal to the processing plant at Mauch Chunk from the mine in the hills above. The wagons were dragged up to the mine by mules and then loaded with coal. Their brakes were then released and the coal filled wagons rolled down the wooden rails to the processing plant. By the 1850s, the "Gravity Road" (as it became known) was providing rides to thrill-seekers for 50 cents a ride.

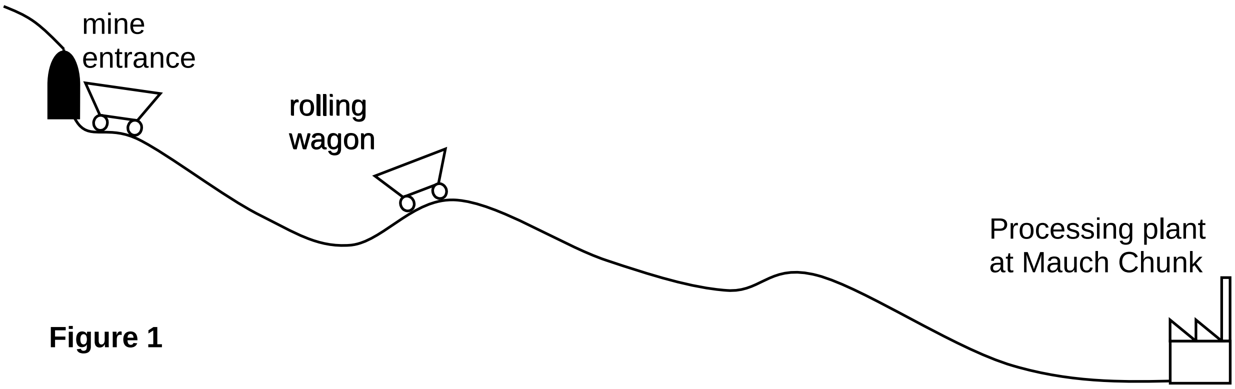


Figure 1

Paragraph 2
The earliest purpose built roller coasters used steam powered winches to drag wagons full of brave thrill seekers to the tops of man-made hills from which the wagons were allowed to roll down wooden rails via a series of smaller man made hills and valleys and around curves of varying radii back to the base of hill at the start. The limiting factor in the complexity and duration of the ride for these devices were the very high levels of friction caused by the use of wooden rails and roughly formed wheels. It has been estimated for the descent phase, such roller coasters typically operated at only 35% efficiency.

Paragraph 3
From their primitive beginnings, roller coasters have evolved into the technically sophisticated contraptions that are loosely called roller coasters today. Efficiencies have been vastly improved with the introduction of polished metal rails and low friction wheels. In some cases, the initial hills have become frighteningly tall, up to nearly 140 m. Other modern roller coasters have dispensed with an initial hill altogether, beginning the ride by catapulting the wagons along level track to start with. Some roller coasters provide an additional push to the wagons at various points along the track. Perhaps the biggest advance in roller coaster was the introduction of so called up-lift wheels.

Paragraph 4
Originally, the wheels at each corner of the wagon were flanged as per figure 2. They were replaced by up-lift wheels. Up-lift wheels are a block of six wheels as illustrated in the side view photograph of figure 3 below. These wheels are arranged so that they not only run along the top of the rails but also along the sides and the undersides of the rails. The wheels that run along the sides of the rails hold the wagons on the track during sharp left or right hand cornering. The flange o wheels achieved a similar result but with considerably more friction. The wheels that run along the underside of the track hold the wagon on the track during rapid downward accelerations, a result not achieved by flanged wheels.

Figure 2: flanged wheel

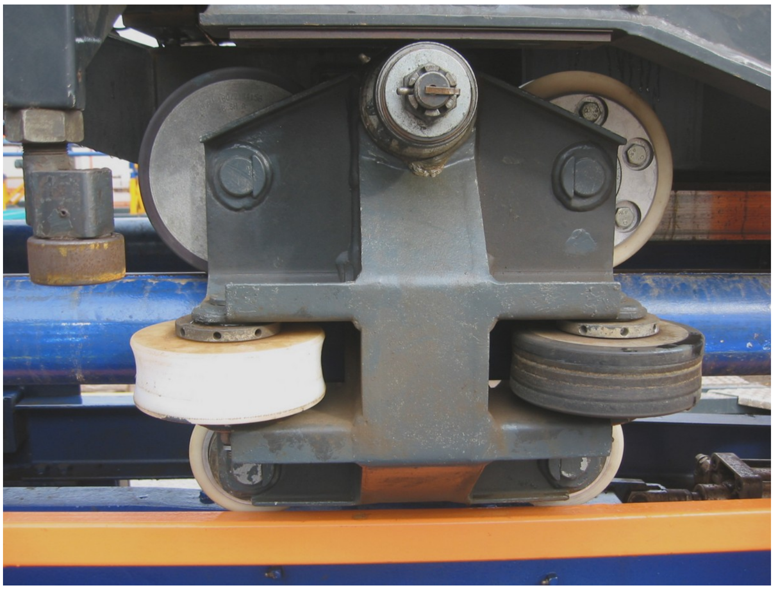


Figure 3: Up-lift wheels – side view

Paragraph 5
Up-lift wheels and the other advances mean that modern roller coasters are able to achieve very high accelerations as they pass over intermediate hills and as they negotiate sudden slope decreases on the way up hills and sudden slope increases on the way down hills. Accelerations of 4.7 g (4.7 times the acceleration due to gravity) are achieved by the most extreme roller coasters.

Paragraph 6
The advent of up-lift wheels and safe passenger harnesses has also meant the roller coaster wagons can now safely operate upside down. This has meant that loop-the-loops are now possible. Even the most modern roller coasters have their wagons “coast” around loop-the-loops, that is, the wagons are not powered as they go around the loop. The Law of Conservation of Energy means that, if roller coaster designers know the speed at which a wagon coasts into the bottom of a loop, they can work out how fast it will be going around the top of the loop (and, yes, they do ignore friction.) They usually try to design the loop height and radius so the passengers are apparently weightless at the top of the loop. You may notice that loop-the-loops are rarely circular. One typical loop-the-loop has a radius of curvature at its top of 7.5 m with a height of 18 m.

Paragraph 7

Of course, the greatest thrill of any roller coaster is the sensation of “your stomach in your mouth” that results from going over the crest of an intermediate hill at high speed. Our stomach is relatively free to move in the cavity of our abdomen so that if our bodies accelerate with a wagon because we are strapped to it, our stomach initially does not accelerate and gets left behind until the top of our abdominal activity pushes down hard enough on our stomach to give it the same acceleration as our body. That force on our stomach can be enough to result in unpleasant consequences for passengers foolish enough to travel on a roller coaster with a stomach full of soft drink and greasy food.

Questions.

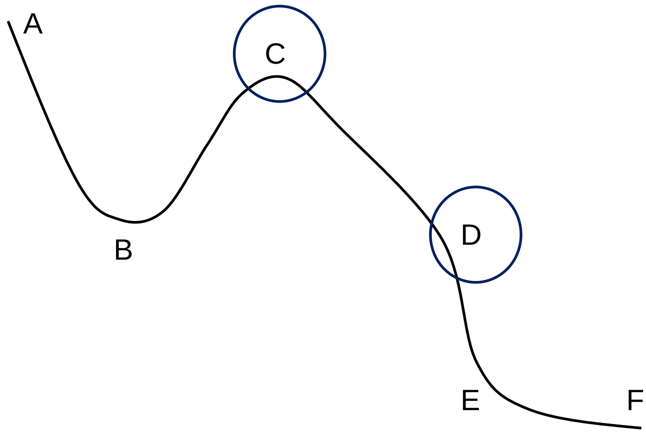
a) For the earliest purpose built roller coasters, what devices were used to provide the initial gravitational potential energy to the wagons and their occupants? (1 mark)

Steam powered winches

b) If one of the earliest purpose built roller coasters with a typical efficiency lifts its wagons to a height of 60 m, calculate the speed would you expect the wagons to have after the descent phase? (2 marks)

$0.35 \times m \times g \times h = \frac{1}{2} \times m \times v^2 \Rightarrow v = \text{sqrt}(0.35 \times 2 \times g \times h) = \text{sqrt}(0.35 \times 2 \times 9.8 \times 60) = 20.3 \text{ m s}^{-1}$

c) The diagram below shows a roller coaster track on which wagons with up-lift wheels travel from left to right. Circle the two points at which the wheels on the underside of the track might exert a force on the track. Explain why. (3 marks)



At both these points, the centre of curvature of the track is below the wagon and the wagon would have acceleration towards that centre. If the component of the weight force parallel to the radius vector at those points is less than the force required for centripetal acceleration at the wagon’s speed, the underside wheels would have to provide an additional force by pushing up on the track.

d) On one modern roller coaster, the wagons round a 10.7 m radius horizontal bend at high speed. The force exerted by the wheels pushing against the side of the track as the wagon negotiates this bend results in the horizontal acceleration of 4.7 g mentioned in paragraph 5. At what speed must the wagon have rounded the bend? (2 marks)

$a = v^2/r$

$\Rightarrow v = \text{sqrt}(ra)$

$= \text{sqrt}(10.7 \times 4.7 \times 9.8)$

$= 22.2 \text{ m s}^{-1}$

e) Considering the loop-the-loop of paragraph 6 and making the same assumptions as the roller coaster designer, with what speed should a wagon enter the bottom of the loop-the-loop for the passengers to feel weightless at the top? (4 marks)

$E_{K:bottom} = ½ \, m \, v_{bottom}^2$
$E_{K:top} = ½ \, m \, v_{top}^2 = ½ \, m \, v_{bottom}^2 – m \, g \, h$
=> $v_{top}^2 = v_{bottom}^2 – 2 \, g \, h$
$a_{c:top} = v_{top}^2/r$
=> $g = (v_{bottom}^2 – 2 \, g \, h)/r$ (if acceleration down is g, passenger is apparently weightless)
=> $v_{bottom} = \text{sqrt}(rg + 2 \, g \, h)$
$= \text{sqrt}(9.8(7.5 + 2 \times 18))$
$= 20.6 \text{ m s}^{-1}$

f) What force is exerted on a passenger’s full stomach, with a mass of 2.00 kg, **by the top of their abdominal cavity** if the roller coaster travels over a hill of radius 12 m at a speed of 18 m s^{-1}? (3 marks)

$\Sigma F_{stomach} = m \, v^2/r = 2 \times 18^2/12 = 54$ N
$\text{weight}_{stomach} = m \, g = 2 \times 9.8 = 19.6$ N
Force of top of abdominal cavity on stomach is 54.0 – 19.6 = 34.40 N down

Question 19 (15 marks)

The Pantheon

Figure 1 - The Pantheon from outside

Paragraph 1
The Pantheon is an Ancient Roman temple in Rome Italy. The building consists of two major pieces or rooms. They are the rectangular front and the circular back.

Paragraph 2
The front piece is the smaller of the two. Its roof is held up over the rectangular shape by stone columns. This front section is known as the portico. It acts as an entrance or preliminary room. There are eight columns in the front row and two groups of four behind.

Figure 2 – Inside the Pantheon looking up.

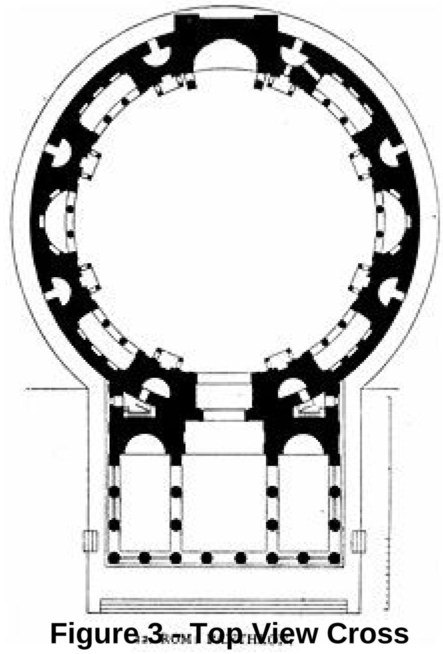


Figure 3 – Top View Cross Section Diagram

Paragraph 3
The second piece is much larger and represents the building proper. This section has a circular floor plan and a huge domed ceiling with a hole at the top of the dome called the oculus. The dome is made from ancient re-enforced concrete. Almost two thousand years after it was built, the Pantheon's dome is still the world's largest unreinforced concrete dome. The height of the oculus above the floor is 43.3 m. The ceiling forms a perfect spherical shape of diameter 43.3 m.

Figure 4 - Artist impression of the inside of the Pantheon

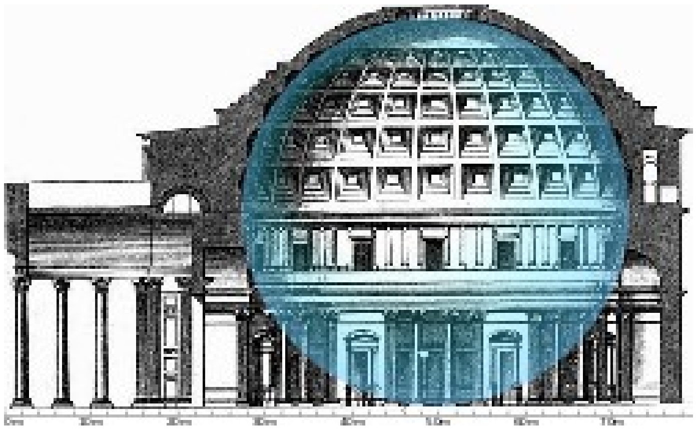


Figure 5 - Cross-section of the Pantheon showing how a 43.3 m diameter sphere fits under its dome.

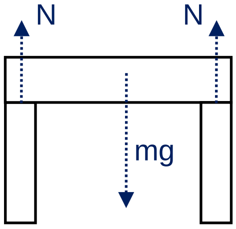
Paragraph 4
The ceiling was made from molded sections of concrete that are approximately square in shape. They serve both a structural and a decorative purpose. The name given to this effect of square shapes molded from concrete is "coffered". It is thought that the molded sections were mounted onto temporary scaffolding until a complete ring was in place. The scaffolding was then removed leaving each ring as a self supporting structure.

Paragraph 5
The shape of the roof has a great deal in common the simpler structure of the keystone arch. The domed ceiling however does not require a keystone due to its spherical rather than arched shape. The hole in the top (absence of the keystone) allows natural light to enter the room providing natural illumination to the space (an advantage in an ancient society without the benefit of electricity).

References
http://en.wikipedia.org/wiki/Pantheon,_Rome Extracted 1/4/2013

Questions

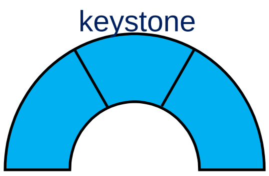
a) The Pantheon is largely made from stone and concrete. Explain why the spacing between the pillars in both the front and back rooms cannot be too large and why the stones that are placed across the tops of the pillars must be thick. (4 marks)

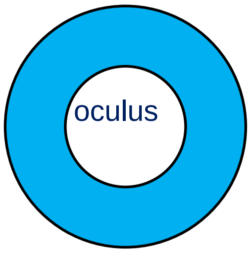


The stone beam will sag in the middle as its weight pulls it down while the supports keep it up. The greater the distance between the supports, the heavier the beam and the more it will sag. The top of the beam is in compression, the bottom in tension, the middle is neutral. If the beam's weight is too great, the tension is too great and the beam will fail. The thicker the beam, the more tension and compression in the beam for the same amount of sag as the top and bottom of the beam are further from the neutral line. Therefore, for the same amount of tension, the less sag that will result. This is an architecturally desirable result.

|

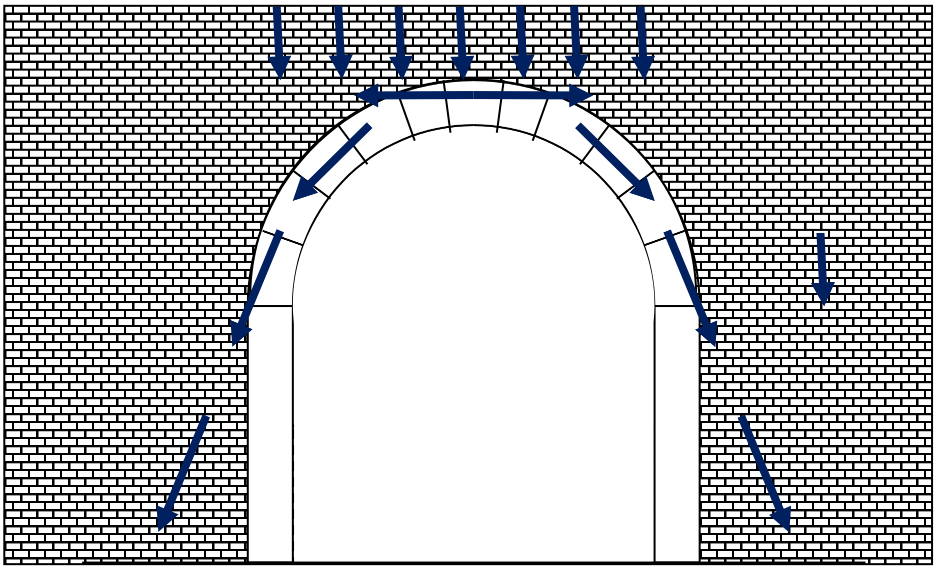
b) Why is it that the domed ceiling of the pantheon (second room) has a hole (oculus) but can still stand without collapsing? Use diagrams to assist your explanation. (3 marks)





All the stones of a cylindrical arch are in compression which means that the stones adjacent to the keystone push against it. A dome can be considered to be collection of cylindrical arches sharing a single keystone. If that keystone is removed, it will leave a circular hole with the stones adjacent to its circumference being all towards the centre of the hole. The stones surrounding the hole form a circular arch which is itself able to support the compressions from all sides so if the limits of the strength of that circular arch are not exceeded, the dome will stand.

c) Onto the diagram below indicate how the weight of the structure is supported and transferred around the arch. (3 marks)



d) Why do the outer walls of the pantheon have a wide base? (1 mark)

The wide base of the outer walls prevents them from tipping in response to the outward forces of the dome.

e) In what structural ways are the “coffered” (hollowed) repeated concrete pieces similar to and different from metal “I” beams. (2 marks)

	Concrete “Coffered” Pieces	Metal I Beams
One Similarity	Most of the material of I beams and “coffered” pieces is around the edges of the beam or piece where stresses are at their maxima.	
One Difference	Made of concrete therefore strong in compression and weak in tension.	Made of steel therefore strong in both compression and tension.

f) Would engineers be allowed to build an unreinforced concrete structure like this in modern times? Explain why or why not. (2 marks)

No. Earthquakes, subsidence and other earth movements can occur in almost all locations. This can result in unplanned tensions in concrete components Without reinforcing, concrete is weak in tension. These days it would be considered unacceptable to build a structure prone to collapse under even mild earth movement.

End of Exam

Additional working space

Additional working space

Additional working space

Additional working space