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Test 4

Logarithmic Functions

Semester One 2018
Year 12 Mathematics Methods
Calculator Assumed

Name:

Date: 29/06/2018 7.45am

You may have a calculator, a single-sided page of notes and a formula sheet for the test.

40 Minutes

Total_____/35 marks

Teacher:

_____ **Mr McClelland**

_____ **Mrs. Carter**

_____ **Mr Gannon**

_____ **Ms Cheng**

_____ **Mr Staffie**

_____ **Mr Strain**

Questions 1**(7 marks)**

Find the derivatives of the following. Do not simplify your answer.

(a) $\ln(2x^3 - 3x^2 + 4x - 1)^3$ (2 marks)

(b) $e^x \ln(x)$ (2 marks)

(c) $\ln(x) \cos(x) + \frac{\sin(x)}{x}$ (3 marks)

Question 2**(5 marks)**

(a) Use Polynomial Long division to simplify $\frac{x^2 - 2x + 5}{x - 3}$. (3 marks)

(b) Hence find $\int \frac{x^2 - 2x + 5}{x - 3} dx$. (2 marks)

Question 3**(5 marks)**

(a) Find the constants a and b given that for $\{x \in \mathbb{R} : x \neq 2, x \neq -3\}$.

(3 marks)

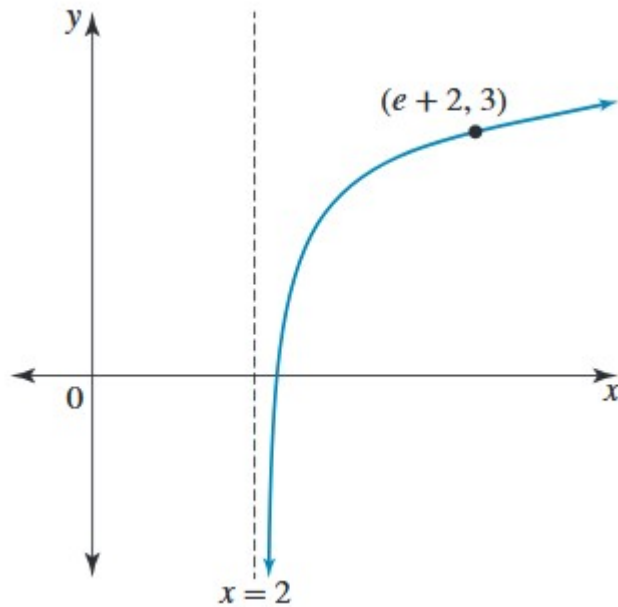
$$\frac{a}{x-2} + \frac{b}{x+3} = \frac{x+8}{x^2+x-6}$$

(b) Hence find $\int \frac{x+8}{x^2+x-6} dx$.

(2 marks)

Question 4**(2 marks)**

The rule for the function shown is $y = \ln(x - m) + n$. Find the values of m and n .

**Question 5****(3 marks)**

Solve the following equations for x . Show full algebraic reasoning.

$$3e^{2x} - 5e^x - 2 = 0$$

Question 6**(5 marks)**

The graph of the function with the rule $y = 3 \log(x+1) + 2$ intersects the axes at the point $(a, 0)$ and $(0, b)$. Find the exact values of a and b . Show full algebraic reasoning.

Question 7**(8 marks)**

There are two species of insects living in a suburb: the *Asla bibla* and the *Cutus pius*. The number of *Asla bibla* alive at time t days after 1 January 2000 is given by

$$N_A(t) = 10\,000 + 1000t, 0 \leq t \leq 15$$

The number of *Cutus pius* alive at time t days after 1 January 2000 is given by

$$N_C(t) = 8000 + 3 \times 2^t, 0 \leq t \leq 15$$

- (a) (i) Show full reasoning that $N_A(t) = N_C(t)$ if and only if $t = 3\log_2 10 + \log_2 \left(\frac{2+t}{3} \right)$.
(4 marks)

(ii) Solve the value for t if $t = 3 \log_2 10 + \log_2 \left(\frac{2+t}{3} \right)$. (2 marks)

- (b) It is found by observation that the model for *Cutus pius* does not quite work. It is known that the model for the population of *Asla bible* is satisfactory. The form of the model for *Cutus pius* is $N_C(t) = 8000 + c \times 2^t$. Find the value of c , correct to two decimal places, if it is known that $N_A(15) = N_C(15)$. (2 marks)

End of Test