9. (8 marks)

The amount A of a drug (in milligrams) in the bloodstream will decline at a rate proportional to the current amount . That is $\frac{dA}{dt} = -\left(\frac{1}{k}\right)A$.
where k hours is a constant called the <u>elimination time</u> and time t is measured in hours.

(a) Write down the formula for $A(t)$, the amount of the drug in the bloodstream after t hours , in terms of t, k and the initial amount A_0 . [2]

$A = A_0 e^{-\frac{t}{k}}$

(b) What proportion of the drug remains in the bloodstream after k hours ? [3]

$A = A_0 e^{-\frac{k}{k}}$

$= A_0 e^{-1}$

$= \frac{A_0}{e}$

PROPORTION LEFT IS $\frac{1}{e}$ OR 0.368

The drug sodium pentobarbitol can be used to tranquilize animals. A dog is tranquilized if its bloodstream contains at least 45 milligrams of the drug for each kilogram of the dog's weight. The elimination time for the drug is 6 hours.

(c) What single dose of this drug should be given in order to tranquilize a 12 kilogram dog for 1 hour? [3]

$540 = A_0 e^{-\frac{1}{6}}$

$A_0 = 638$ milligrams

TRINITY COLLEGE

Mathematics Methods Year 12
Test 2 2016

Section 1 Calculator Free
Exponential Function, Fundamental Theorem

STUDENT'S NAME SOLUTIONS

DATE: Friday 1st April **TIME:** 33 minutes **MARKS:** 33

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

Determine $\frac{dy}{dx}$ for each of the following. Do not simplify.

(a) $y = e^{\sqrt{x}}$ [2]

$y = e^{x^{\frac{1}{2}}}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} e^{x^{\frac{1}{2}}}$

(b) $y = \sin\left(e^{2x}\right)$ [2]

$y' = \cos e^{2x} \cdot 2e^{2x}$

(c) $y = (\cos x)\, e^{\cos x}$ [3]

$y' = -\sin x\, e^{\cos x} + \cos x(-\sin x)\, e^{\cos x}$

2. (10 marks)

(a) Evaluate exactly $\int_0^2 x e^{4-x^2} dx$ [4]

$$= -\frac{1}{2}\int_0^2 -2x e^{4-x^2} dx$$

$$= -\frac{1}{2}\left[e^{4-x^2}\right]_0^2$$

$$= -\frac{1}{2}\left[e^0 - e^4\right]$$

$$= -\frac{1}{2}\left(1 - e^4\right)$$

$$\text{or} \quad \frac{e^4 - 1}{2}$$

(b) Determine $\int \frac{4e^{2x}+4x}{(e^{2x}+x^2)^3} dx$ [3]

$$= \int 2(2e^{2x}+2x)(e^{2x}+x^2)^{-3} dx$$

$$= \frac{2\left(e^{2x}-x^2\right)^{-2}}{-2} + c$$

$$= -\left(e^{-2x}-x^2\right)^{-2} + c$$

(c) Determine $\int_{\pi}^{x^2} \left(\frac{d}{dt} e^{e^{-t}}\right) dt$ [3]

$$= \int_{\pi}^{x^2} \left(-e^{-t} e^{e^{-t}}\right) dt$$

$$= \left[e^{e^{-t}}\right]_{\pi}^{x^2}$$

$$= e^{e^{-x^2}} - e^{e^{-\pi}}$$

8. (5 marks)

A particular rock is dropped into a swimming pool and it sinks vertically to the bottom. Due to water resistance, the rock does not have a constant velocity on the way to the bottom. Its velocity, v centimetres per second, t seconds after it hits the surface of the water is given by

$v = 8(2 - e^{-0.8t})$ for $0 \le t \le 7$

(a) What is the initial velocity of the rock in the water? [1]

$$v_0 = 8(2 - e^0)$$
$$= 8$$

(b) What is the acceleration of the rock after 4 seconds? [2]

$$v = 16 - 8e^{-0.8t}$$
$$a = 6.4e^{-0.8t}$$
$$(t=4) \quad = 0.26 \text{ m/s}$$

(c) Terminal velocity is an expression used to describe the velocity that is approached but never exceeded. Determine the terminal velocity reached by the rock in the water. [2]

16 m/sec.

7. (4 marks)

Sugar is being dissolved in a solution at a rate given by $\frac{dS}{dt} = -20e^{-0.1t}$ where S is the amount, in grams, of undissolved sugar after t seconds.

(a) how much sugar is initially in the solution [2]

$\frac{dS}{dt} = -0.1(200)e^{-0.1t}$

$\therefore S_0 = 200$

(b) how long does it take for half the sugar to dissolve. [2]

$100 = 200e^{-0.1t}$

$t = 6.9$ secs

3. (4 marks)

Given $y = \frac{e^x}{3+e^x}$

(a) determine $\frac{dy}{dx}$ [2]

$y' = \frac{e^x(3+e^x) - e^x \cdot e^x}{(3+e^x)^2}$

$= \frac{3e^x + \cancel{e^{2x}} - \cancel{e^{2x}}}{(3+e^x)^2}$

(b) explain why $\frac{dy}{dx} \neq 0$ [2]

$3e^x > 0$

$(3+e^x)^2 > 0$

$\therefore \frac{dy}{dx} \neq 0$

4. (5 marks)

(a) Determine $\frac{dy}{dx}$ given $y = xe^x$ [2]

$y = e^x + xe^x$

(b) Hence determine $\int xe^x dx$ [3]

$\int (e^x + xe^x)dx = xe^x$

$\int e^x dx + \int xe^x dx = xe^x$

$\int xe^x dx = xe^x - \int e^x dx$

$= xe^x - e^x + c$

5. (4 marks)

Given $y=\int_{-3}^{x}\frac{t^2-2}{\sqrt{t}}\,dt$, use the incremental formula $\delta y \approx \frac{dy}{dx}\times \delta x$ to determine the change in y if x changes from 4 to 4.02.

$$y' = \frac{d}{dx}\int_{-3}^{x}\frac{t^2-2}{\sqrt{t}}\,dt$$

$$= \frac{x^2-2}{\sqrt{x}}$$

$$\delta y \approx \frac{x^2-2}{\sqrt{x}} \times \delta x$$

$\delta x = 0.02$

$x = 4$

$$\approx \frac{16-2}{\sqrt{4}} \times 0.02$$

$$= 7 \times 0.02$$

$$= 0.14$$

TRINITY COLLEGE

Mathematics Methods Year 12
Test 2 2016

Section 2 Calculator Assumed

Exponential Function, Fundamental Theorem

STUDENT'S NAME ____________________

DATE: Friday 1st April **TIME:** 20 minutes **MARKS**: 21

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

6. (4 marks)

Determine the value of x for which $\int_{x}^{-1}(1-t^2)\,dt$ has a relative minimum. Justify it is a minimum value. [4]

$$\frac{d}{dx}\int_{x}^{-1}(1-t^2)\,dt$$

$$= -\frac{d}{dx}\int_{-1}^{x}(1-t^2)\,dt$$

$$= -(1-x^2)$$

$$-(1-x^2) = 0$$

$$1-x^2 = 0$$

$$x = \pm 1$$

$$\frac{d^2}{dx^2} = 2x$$

$x=1$ $\quad \frac{d^2}{dx^2} = 2 \quad \therefore$ MIN

$x=-1$ $\quad \frac{d^2}{dx^2} = -2 \quad \therefore$ MAX