Do NOT turn this page over until you are instructed to do so.

- 40 minutes are allocated for this task.

- CAS and/or scientific calculators are permitted.

- No notes of ANY nature are permitted.

- **Full marks may not be awarded to correct answers unless sufficient justification is given.**

- **Use the method specified (if any) in the question to show your working (otherwise, no marks awarded)**

Name : [   ]

Score : [   ]
(out of 42)

Test 4
Applications of Integration and
Discrete Random Variables
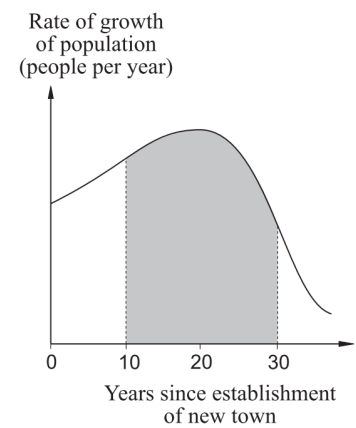
[This test contributes 6% towards the final year mark]

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1. [2 marks]
   Describe, in words, what quantity is represented by the shaded area in the graph below.

6. [3 + 2 + 2 + 3 = 10 marks]
   On a long train journey, a statistician is invited by a gambler to play a dice game. The game uses two ordinary dice which the statistician is to throw.

   If the total score is 12, the statistician is paid $6 by the gambler. If the total score is 8, the statistician is paid $3 by the gambler. However, if both or either dice show a 1, the statistician pays the gambler $2. Otherwise, no money changes hands.

   Let $\$X$ be the amount paid to the statistician by the gambler.

   (a) Complete the table below.

x		0	3	6
$P(X=x)$				

   (b) Explain why the table in part (a) describes a probability distribution for the discrete random variable X.

   (c) Show that, if the statistician played the game 100 times, his expected loss would be $2.78, to the nearest cent.

   (d) Find the amount, $\$a$, that the $6 would have to be changed to in order to make the game unbiased.

End of test

2. [5 marks]
The graph below shows the sketch of the curve $y = 2x(6-x)$ and the lines $y = x$ and $y = 3x$.

Determine the **exact** area of the shaded region.

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5. [3 + 4 + 3 = 10 marks]
The discrete random variable X can only take the values 0, 1, 2, 3, 4, 5. The probability distribution of X is given by the following:

$$P(X = 0) = P(X = 1) = P(X = 2) = a$$
$$P(X = 3) = P(X = 4) = P(X = 5) = b$$
$$P(X \geq 2) = 3P(X < 2)$$

where a and b are constants.
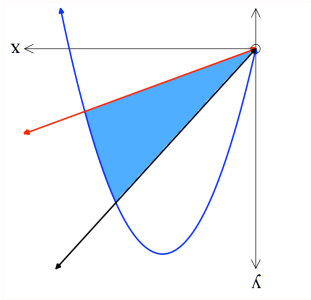
(a) Determine the values of a and b.

(b) Show that the expectation of X is $\frac{23}{8}$ and determine the exact variance of X.

(c) Determine the exact probability that the sum of two independent observations from this distribution exceeds 7.

3. [3 + 2 + 2 = 7 marks]
    A petrol tank, when full, contains 36 litres of petrol. It develops a small hole which widens
    as time goes by. The rate at which fuel leaks out (in litres per day) is given by the
    expression:

    $$0.009t^2 + 0.08t + 0.01$$

    where t is the time in days. When $t = 0$ the tank is full.

    (a) Determine a formula for the amount of fuel lost after t days.

    (b) How many litres of fuel does the tank lose on the tenth day?

    (c) How much fuel is left in the tank after 15 days?

4. [4 + 2 + 2 = 8 marks]
    An object is thrown vertically upward from a point O (at ground level) with velocity 49 ms^{-1}.
    The acceleration due to gravity is 9.8 ms^{-2} towards the centre of the Earth.

    Determine:

    (a) the height above O at any time t,

    (b) the time(s), correct to 3 decimal places, the object is 15 metres above the ground,

    (c) the maximum height reached.