ALL SAINTS' COLLEGE

Mathematics Specialist

Test 3 2016

Vectors in 3D

NAME: _____

TEACHER: MLA

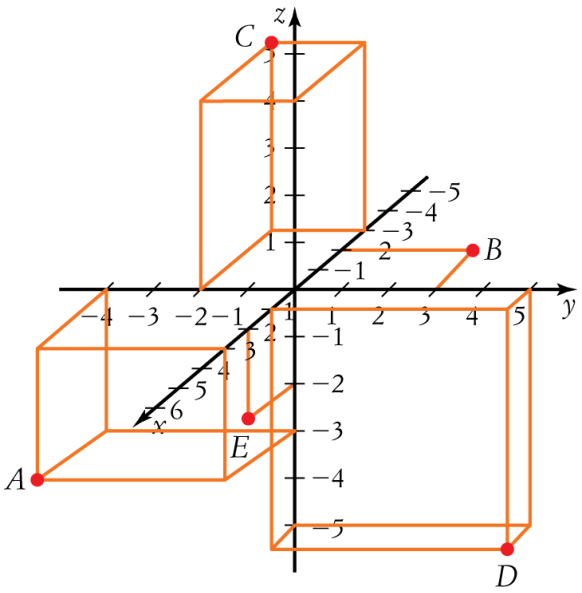
45 marks

45 minutes

One unfolded A4 page of notes, SCSA formulae booklet and ClassPad calculator permitted

Question 1 [2 marks]

Use the diagram below to determine the (exact) distance from C to D:



Question 2 [1, 1 & 1 = 3 marks]

Vector **u** has norm 8, and both azimuthal (in the x-y plane) and altitude angles of 60°.

Vector $v = 2i - j + 4k$

(a) State the (exact) rectangular form for **u**

(b) Find the acute angle between **u** and **v**, correct to two decimal places.

(c) Determine the (exact) vector orthogonal to both **u** and **v**

Question 3 [4 marks]

Find the projection of $\mathbf{a} = (-4, 2, -3)$ on $\mathbf{b} = (2, -5, 1)$

Question 4 [4 marks]

C is the midpoint of the line segment AB. D is a point **not** on the line AB such that $DC = CA$.

Use vector methods to prove that DA is perpendicular to DB.

Question 5 [4 marks]

Find the angle between the line $r = \begin{bmatrix} 1-2\lambda \\ \lambda \\ 2+3\lambda \end{bmatrix}$ and the plane $r \cdot \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = -4$

Question 6 [5 marks]

Find the equation of the plane containing the points E (4, −3, 1), F (5, 0, 2) and G (3, 2, 5)

Question 7 [4 & 1 = 5 marks]

(a) Determine the vector equation of the plane consisting of all points that are equidistant from the points P $(-1, 2, -3)$ and Q $(4, -2, 2)$.

(b) Hence, or otherwise, state the Cartesian form of the plane in (a)

Question 8 [5 & 2 = 7 marks]

Use elementary row operations to reduce the following system of equations to echelon form:

$$x+y+(k-3)z=1$$
$$2x+4y+4z=3$$
$$-x+y+(7-2k)z=m$$

(a) State the values of k and m if the system is to have:

(i) A unique solution

(ii) Infinite solutions

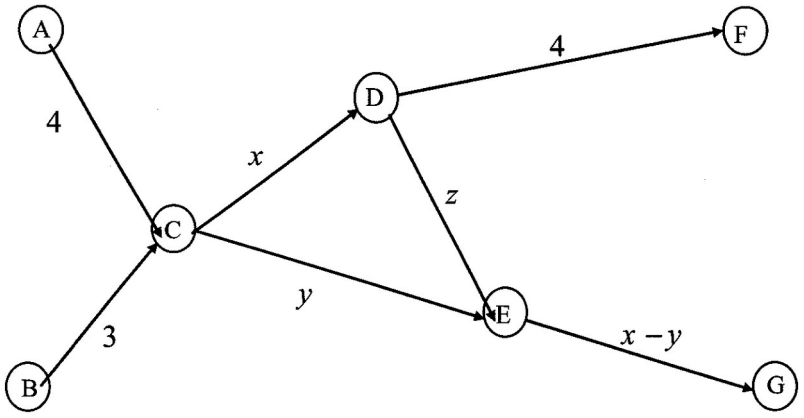
(iii) No solutions

(b) In the case of infinite solutions, find the particular solution for which $x=1$.

Question 9 [6 marks]

The schematic diagram below shows the volume of passengers (in tens of thousands) through airports A, B, C, D, E, F and G in the month of January.

Use Gaussian elimination to find the values of x, y and z if there are 10 000 less arrivals than departures at C, an equal number of arrivals and departures at D, and 20 000 more departures than arrivals at E.



Question 10 [5 marks]

$At\ time\ t=0$, the position ($\mathbf{r}$) and velocity ($\mathbf{v}$) vectors of a British warship (B) and German submarine (G) are as follows:

$$r_B=\begin{bmatrix}1150\\827\\0\end{bmatrix}m;\ v_B=\begin{bmatrix}10\\-2\\0\end{bmatrix}m/sec \qquad r_G=\begin{bmatrix}1345\\970\\0\end{bmatrix}m;\ v_G=\begin{bmatrix}-5\\-13\\-4\end{bmatrix}m/sec$$

If both vessels maintain their respective velocities, determine the time at which the submarine is directly beneath the warship, and the depth of the submarine at this instant.

Note. The x-y plane represents the surface of the ocean

End of Test 3