

PRACTICE EXAM QUESTIONS (II)

[6, 5, 12 = 23 marks]

1.1 For the vectors $\mathbf{a} = \mathbf{OA} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = \mathbf{OB} = 5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, find:

- (a) the angle between \mathbf{a} and \mathbf{b}
- (b) a vector equation of the line passing through C (1, 2, -3) and D, the midpoint of AB
- (c) the points where the above line intersects the sphere $|\mathbf{r} - \langle 3, -4, 1 \rangle| = 10$.

PRACTICE EXAM QUESTIONS (II) – SOLUTIONS

$$\begin{aligned}
 \text{(a)} \quad |\mathbf{a}| &= \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29} \quad \checkmark \\
 |\mathbf{b}| &= \sqrt{5^2 + 1^2 + 3^2} = \sqrt{35} \quad \checkmark \\
 \mathbf{a} \cdot \mathbf{b} &= \langle 2, -3, 4 \rangle \cdot \langle 5, 1, -3 \rangle \\
 &= 10 - 3 - 12 \\
 &= -5 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \cos \theta &= \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} \\
 &= \frac{-5}{\sqrt{29} \sqrt{35}} \quad \checkmark \\
 \theta &= 99^\circ \quad \checkmark
 \end{aligned}$$

The angle between the vectors is 99° \checkmark

$$\begin{aligned}
 \text{(b)} \quad \mathbf{OD} &= \frac{1}{2} (\mathbf{OA} + \mathbf{OB}) \quad \leftarrow \text{-----} \\
 &= \frac{1}{2} (\langle 2, -3, 4 \rangle + \langle 5, 1, -3 \rangle) \\
 &= \frac{1}{2} \langle 7, -2, 1 \rangle \quad \checkmark \\
 &= \langle 3.5, -1, 0.5 \rangle \quad \checkmark \\
 \mathbf{CD} &= \mathbf{OD} - \mathbf{OC} \\
 &= \langle 3.5, -1, 0.5 \rangle - \langle 1, 2, -3 \rangle \quad \checkmark \\
 &= \langle 2.5, -3, 3.5 \rangle \quad \checkmark
 \end{aligned}$$

If A and B are the end points of a line and D is the midpoint of AB, then the position vector of D is given by

$$\mathbf{OD} = \frac{1}{2} (\mathbf{OA} + \mathbf{OB})$$

Using point C (1, 2, -3) on the line

$$\mathbf{r} = \langle 1, 2, -3 \rangle + t \langle 2.5, -3, 3.5 \rangle \quad \checkmark \text{-----}$$

$$\begin{aligned}
 \text{(c)} \quad |\mathbf{r} - \langle 3, -4, 1 \rangle| &= 10 \\
 |\langle x, y, z \rangle - \langle 3, -4, 1 \rangle| &= 10 \quad \checkmark \\
 |\langle x - 3, y + 4, z - 1 \rangle| &= 10 \quad \checkmark \\
 (x - 3)^2 + (y + 4)^2 + (z - 1)^2 &= 100 \quad \checkmark
 \end{aligned}$$

$$\mathbf{r} = \langle 1 + 2.5t, 2 - 3t, -3 + 3.5t \rangle \quad \checkmark$$

$$x = 1 + 2.5t, \quad y = 2 - 3t, \quad z = -3 + 3.5t \quad \checkmark$$

component form of the line
parametric equations

sub into sphere equation

$$(1 + 2.5t - 3)^2 + (2 - 3t + 4)^2 + (-3 + 3.5t - 1)^2 = 100 \quad \checkmark$$

$$(2.5t - 2)^2 + (6 - 3t)^2 + (-4 + 3.5t)^2 = 100 \quad \checkmark$$

$$6.25t^2 - 10t + 4 + 36 - 36t + 9t^2 + 16 - 28t + 12.25t^2 = 100 \quad \checkmark$$

$$27.5t^2 - 74t - 44 = 0 \quad \checkmark$$

$t = 3.192$ or -0.501 by using the quadratic formula or graphics calculator \checkmark

if $t = 3.192$, intersection point is (9.0, -7.6, 8.2) \checkmark

if $t = -0.501$, intersection point is (-0.3, 3.5, -4.8) \checkmark