Question 7 (a) **(4 marks)**

Solution	
$f'(x)=3x^2e^{-x}-x^3e^{-x}=x^2(3-x)e^{-x}$ $f'(x)=0 \Rightarrow x=0$ or $x=3$. $f(0)=0,\ f(3)=3^3e^{-3}$, so f has stationary points at $(0,0)$ and at $(3,3^3e^{-3})$ Since $f'(x)\geq 0$ if $x<3$ and $f'(x)<0$ if $x>3$, f has a point of inflection at $(0,0)$ and f has a local maximum at $(3,3^3e^{-3})$	
Mathematical behaviours	**Marks**
• differentiates correctly • equates $f'(x)=0$ and determines co-ordinates of stationary points • justifies nature of first stationary point • justifies nature of 2nd stationary point	1 1 1 1

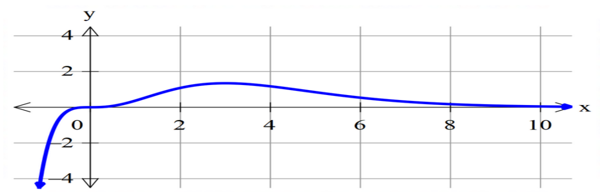
x	3^-	3	3^+
$f'(x)$	+ve	0	-ve

Question 7 (b) **(2 marks)**

Solution	
Yes. Reason: $f(3)=3^3e^{-3}=\left(\frac{3}{e}\right)^3>1$ since $0<e<3$	
Mathematical behaviours	**Marks**
• gives correct answer • gives a valid reason	1 1

Question 7 (c) **(3 marks)**

Solution



Mathematical behaviours	Marks
• shows inflection point at origin • shows maximum at $x=3$ • shows correct limits as $x\to\infty$ and $x\to-\infty$	1 1 1

MATHEMATICS METHODS

MAWA Semester 2 (Unit 3&4) Examination 2018
Calculator-free

Marking Key

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The release date for this exam and marking scheme is

- **the end of week 1 of term 4, Fri October 12th 2018**

Section One: Calculator-free **(54 Marks)**

Question 1 (a) **(3 marks)**

Solution

$$\int_1^4 \left(6x^2 + \frac{1}{2\sqrt{x}}\right) dx$$
$$= \left[2x^3 + \sqrt{x}\right]_1^4$$
$$= (2(64) + 2) - (2 + 1) = 127$$

Mathematical behaviours	Marks
• integrates square root function correctly	1
• substitutes limits into correct anti-derivative	1
• evaluates result	1

Question 1 (b) **(2 marks)**

Solution

$$g'(x) = e^{\frac{x+1}{2}}$$
$$g(x) = 2e^{\frac{x+1}{2}} + c$$
$$(3, e^2) \Rightarrow e^2 = 2e^2 + c \Rightarrow c = -e^2$$
$$\therefore g(x) = 2e^{\frac{x+1}{2}} - e^2$$

Mathematical behaviours	Marks
• anti-differentiates correctly	1
• substitutes in $(3, e^2)$ to determine c	1

Question 1 (c) **(2 marks)**

Solution

$$\int_0^{\frac{\pi}{2}} \frac{d}{du} \sin u \; du = [\sin u]_o^{\frac{\pi}{2}} = 1$$

Mathematical behaviours	Marks
• applies the fundamental theorem	1
• evaluates result	1

Mathematical behaviours	Marks
• states correct derivative	1
• integrates both sides	1
• applies Fundamental Theorem	1
• rearranges to arrive at correct result	1

Question 6 (a) **(2 marks)**

Solution

$\hat{p} = 1 \Rightarrow$ 5 heads in 5 tosses

$\therefore$ probability = $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

Mathematical behaviours	Marks
• identifies that each toss must result in a head	1
• determines probability	1

Question 6(b) **(4 marks)**

Solution

$\hat{p}$ is normally distributed with

$\mu = 0.5$ and $\sigma = \sqrt{\frac{0.5 \times 0.5}{100}} = .05$

$$z_{0.55} = \frac{0.55 - 0.5}{0.05} = 1$$

Hence, $P(\hat{p} > 0.55) = P(z > 1) \approx 0.16$

Mathematical behaviours	Marks
• identifies that $\hat{p}$ will be normally distributed	1
• determines mean and standard deviation for distribution of $\hat{p}$	1
• determines Z score associated with $\hat{p} = 0.55$	1
• determines probability	1

Question 6 (c) **(3 marks)**

Solution

$P(\hat{p}_1 = \hat{p}_2) = P(\hat{p}_1 = \hat{p}_2 = 0) + P\left(\hat{p}_1 = \hat{p}_2 = \frac{1}{3}\right) + P\left(\hat{p}_1 = \hat{p}_2 = \frac{2}{3}\right) + P(\hat{p}_1 = \hat{p}_2 = 1)$ (*)

¿¿¿ (**)

¿ $\left(\frac{1}{8}\right)^2 + \left(\frac{3}{8}\right)^2 + \left(\frac{3}{8}\right)^2 + \left(\frac{1}{8}\right)^2 = \frac{20}{64} = \frac{5}{16}$

Mathematical behaviours	Marks
• determines $\hat{p}$ values $0, \frac{1}{3}, \frac{2}{3}, 1$	1
• states calculation required to determine probability	1
• evaluates required sum	1

Question 5 (a) **(3 marks)**

Solution	
$y = \ln\sqrt{3x - x^2}$ $= \frac{1}{2}\ln(3x - x^2)$ $\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{(3x-x^2)}\cdot\frac{(3-2x)}{1} = \frac{(3-2x)}{2(3x-x^2)}$	
Mathematical behaviours	Marks
• expresses $y = \ln\sqrt{3x-x^2}$ as $y = \frac{1}{2}\ln(3x-x^2)$	1
• uses $\frac{d}{dx}\ln x = \frac{1}{x}$	1
• applies chain rule correctly and simplifies	1

Question 5 (b) **(3 marks)**

Solution	
$\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{1+\sin^2 x}\,dx = \int_0^{\frac{\pi}{4}} \frac{2\sin x\cos x}{1+\sin^2 x}\,dx$ $= \ln\left\|1+\sin^2 x\right\|_0^{\frac{\pi}{4}}$ $= \ln\left\|1+\frac{1}{2}\right\| - \ln\|1\|$ $= \ln\frac{3}{2}$ or $\ln 3 - \ln 2$	
Mathematical behaviours	Marks
• states anti-derivative of function with bounds	1
• substitutes in limits of integration correctly using $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$	1
• evaluates result	1

Question 5 (c) **(4 marks)**

Solution

Let $y = x\cos x$

$\frac{dy}{dx} = -x\sin x + \cos x$

Hence $\int \frac{dy}{dx}\,dx = \int(-x\sin x + \cos x)\,dx$

ie $x\cos x = -\int x\sin x\,dx + \int \cos x\,dx + c$

ie $x\cos x = -\int x\sin x\,dx + \sin x$

ie $\int x\sin x\,dx = \sin x - x\cos x + c$

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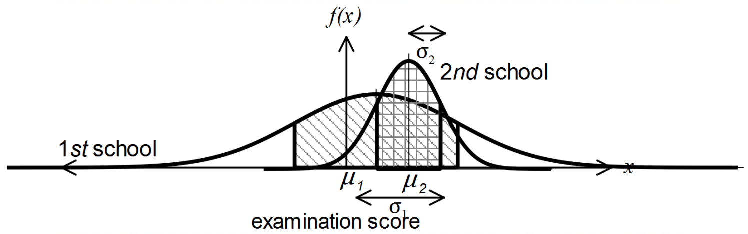
Question 2 (a) **(2 marks)**

Solution	
$X \sim N(45, 9^2)$ $z_{63} = \frac{63-45}{9} = 2$ ie 63 represents 2 std deviations above the mean 2.5% of the population is above 63 $\therefore 0.025 \times 150 = 3.75$ ie approximately 4 students scored above Joanne.	
Mathematical behaviours	Marks
• states that 63% represents 2 std deviations above the mean	1
• determines number of students above Joanne	1

Question 2 (b) **(2 marks)**

Solution



Mathematical behaviours	Marks
• diagram demonstrates that both distributions are normally distributed and $\mu_1 < \mu_2$	1
• diagram clearly depicts $\sigma_1 > \sigma_2$	1

Question 2 (c) **(3 marks)**

Solution	
$\mu_X = 45,\quad \sigma_X = 9$ $\mu_Y = 55,\quad \sigma_Y = 6$ $Y = aX + b$ $a = \frac{6}{9} = \frac{2}{3}$ $55 = \frac{2}{3}\times 45 + b$ $25 = b$ $\therefore a = \frac{2}{3}, b = 25$	
Mathematical behaviours	Marks
• uses standard deviations to determine a	1
• states equation needed to solve for b	1
• determines b value	1

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Question 3 (a) **(2 marks)**

Solution

X	5	(-3)
P($X=x$)	$\frac{1}{4}$	$\frac{3}{4}$

Mathematical behaviours	Marks
• correct entries for X values	1
• determines probabilities correctly	1

Question 3 (b) **(2 marks)**

Solution

$E(X) = 5\times\frac{1}{4}+(-3)\times\frac{3}{4}$

$= \frac{5}{4}-\frac{9}{4}$

= (-1)

On average, Michael will lose $1 per toss

Mathematical behaviours	Marks
• determines expected gain correctly	1
• explains meaning of the negative value	1

Question 3 (c) **(2 marks)**

Solution

With a loss of $1 per toss, this is not a "fair" game.

A game is considered "fair" if Michael will, on the average, come out even.
That is, an expected gain of zero will define a "fair" game.

Mathematical behaviours	Marks
• states game is "not fair"	1
• valid explanation	1

Question 4 (a) **(3 marks)**

Solution

$16^x - 5\times 8^x = 0$

ie $2^{4x} = 5\times 2^{3x}$

ie $4x\log 2 = \log 5 + 3x\log 2$

ie $x\log 2 = \log 5$

ie $x = \frac{\log 5}{\log 2}$

Mathematical behaviours	Marks
• rearranges equation and writes in exponential form	1
• applies log laws to each term of equation	1
• rearranges equation to arrive at result	1

Question 4 (b) **(3 marks)**

Solution

$5^{(2+\log_5 3)} + \log_{\frac{1}{5}} 125$

$=5^2.5^{\log_5 3} + \log_{\frac{1}{5}}(\frac{1}{5})^{-3}$

$=25\times 3 - 3$

$=75 - 3$

$=72$

Mathematical behaviours	Marks
• uses $a^m.a^n=a^{m+n}$ and $a^{\log_a b}=b$	1
• expresses $\log_{\frac{1}{5}} 125\, as \log_{\frac{1}{5}}(\frac{1}{5})^{-3}$, hence value of (-3)	1
• evaluates expression	1