PERTH MODERN SCHOOL
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Independent Public School

Course Methods test 3 Year 12

Student name: ______________ Teacher name: ______________

Task type: **Response**

Time allowed for this task: ___40________ mins

Number of questions: ____8_______

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: ___46___ marks

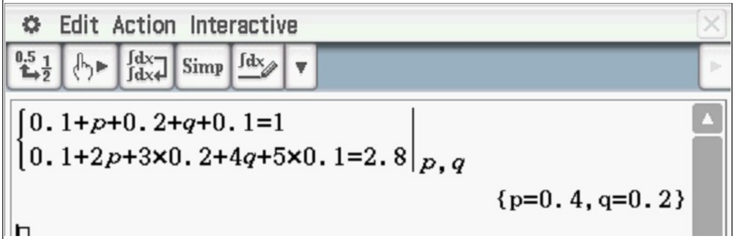
Task weighting: _10___%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3 marks)
The expected value of the discrete probability distribution given below is 2.8. Determine the values of $p \,\&\, q$ and hence determine Var(X), the variance of X .

x	1	2	3	4	5
P($X = x$)	0.1	p	0.2	q	0.1

Solution

Specific behaviours
✓ sets up one equation with p&q ✓ sets up two equations with p&q ✓ solves for both p&q (Note: max 2 marks if no working shown)

Q2 (9 marks)
A students wishes to play a gambling game on mufti day involving throwing two regular fair dice, each numbered 1 to 6. To play the game the student must pay $2 for each throw of two dice. If they score a double i.e two 1s, two 2s etc they win $6. If they throw a total of 7 they win $11 and anything else they receive nothing.
Let $ X equal the profit a player receives on a single play.

a) Describe the random variable X . (1 mark)

Solution
Discrete random variable
Specific behaviours
✓ states discrete

✓ solves for a & b to two decimal places **(must round)**

Q8 (5 marks)

Consider a continuous random variable, X, that has the following probability density function.

$f(x)=\begin{cases} ae^{-bx} & ,0 \le x \le 5 \\ 0 & ,elsewhere \end{cases}$ with $a\,\&\,b$ being constants.

a) Determine the cumulative distribution function, $P(X \le x)$, in terms of $a\,\&\,b$. (2 marks)

Solution
$P(X \le x)=\int_0^x ae^{-bu}\,du$ $=\left[\frac{-a}{b}e^{-bu}\right]_0^x=\frac{-a}{b}(e^{-bx}-1)$
Specific behaviours
✓ integrates with correct limits (no need to change variables)   ✓ states cumulative function

b) Given that $P(X \le 3)=0.7$ solve for approximate values of $a\,\&\,b$ to two decimal places. (3 marks)

Solution
Edit Action Interactive $\left\{\begin{array}{l} e^{-5b}=1-\frac{b}{a} \\ \frac{-a}{b}(e^{-3b}-1)=0.7 \end{array}\right.\Bigg\vert_{a,b}$ {a=0.2995948184, b=0.1742122177}   a=0.30 & b=0.17
Specific behaviours
✓ sets up equation using total prob of one with x=5   ✓ sets up second equation at x=3

b) Complete the following table for X. (3 marks)

Solution

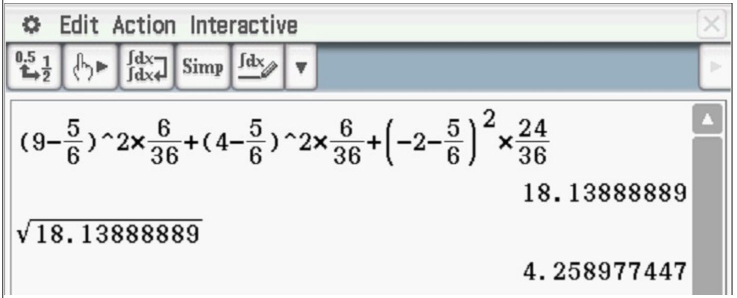
x	\$9	\$4	-\$2
P($X=x$)	$\frac{6}{36}$	$\frac{6}{36}$	$\frac{24}{36}$

Specific behaviours
✓ correct values for x   ✓ at least 1 probs correct   ✓ all probs correct

c) Determine the expected profit by a player on a single game. (3 marks)

Solution
$9\times\frac{6}{36}+4\times\frac{6}{36}-2\times\frac{24}{36}$ $\frac{5}{6}$ $\frac{5}{6}$ 0.8333333333
Specific behaviours
✓ multiplies x by prob   ✓ shows sum of products   ✓ states expected profit, approx. or exact (no need for units)   (Note two marks for answer only)

d) Determine the standard deviation of X. (3 marks)

Solution

Specific behaviours
✓ shows calculation ✓ determines variance ✓states standard deviation Note: Answer only two marks- third mark refers to shown working

Q3 (7 marks)

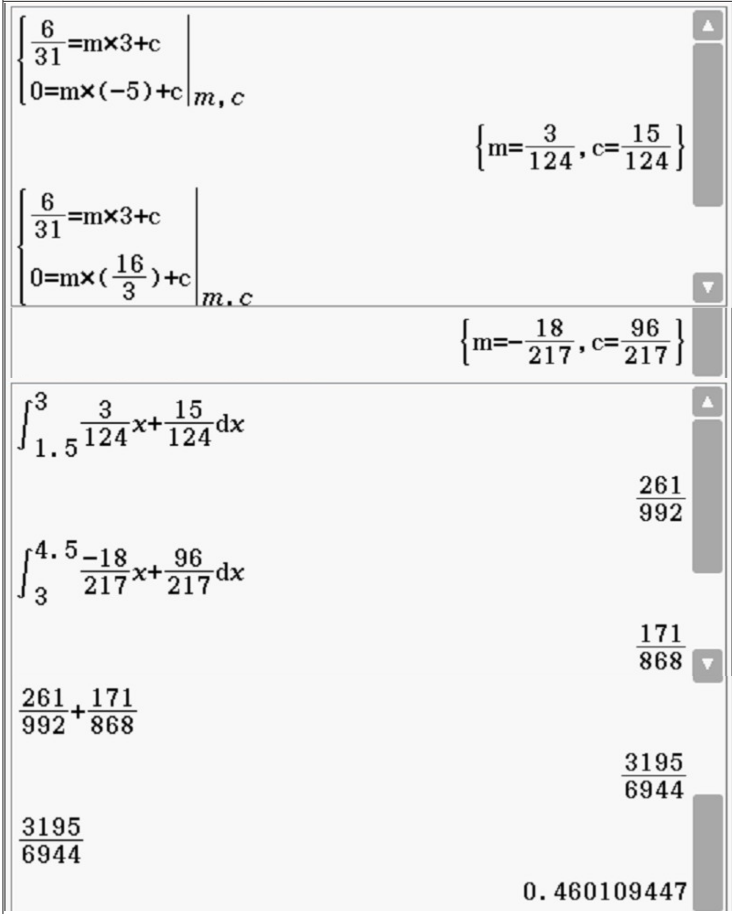
A factory produces toy cars. The probability that any toy car being defective is 0.15. If 20 toy cars are selected at random, let X equal the number of defective cars out of 20.

a) Describe the distribution X. (2 marks)

Solution
$X \sim Bin(20, 0.15)$
Specific behaviours
✓ states Binomial ✓ states n & p

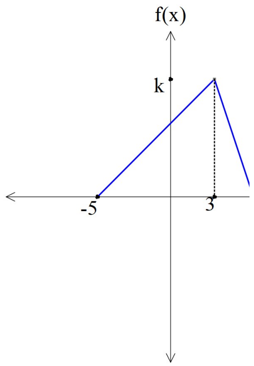
b) Determine that probability that exactly 4 cars will be defective. (2 marks)

Solution

Solution
 **Prob = 0.4601**
Specific behaviours
✓ determines equation of one side ✓ determines equations of both sides ✓ states integrals with correct limits for total area ✓ states approx. area to 4 decimal places (accept exact)

Q7 (6 marks)

Consider the continuous random variable X and its probability density function shown below.



a) Determine the exact value of k. (2 marks)

Solution
$\text{solve}\left(\frac{1}{2}\cdot\left(5+\frac{16}{3}\right)\cdot k=1, k\right)$ $\left\{k=\frac{6}{31}\right\}$
Specific behaviours
✓ uses total area of one   ✓ solves for exact value of k

b) Determine Prob $(1.5 < X < 4.5)$

(4 marks)

binomialPDf(4, 20, 0.15)   0.1821216721
Specific behaviours
✓ uses correct parameters   ✓ states prob

c) Determine the probability that at least 4 cars will be defective given that we know at least 2 cars are defective. (3 marks)

Solution
$P(X \geq 4 \mid X \geq 2) = \frac{P(X \geq 4)}{P(X \geq 2)}$ binomialCDf(2, 20, 20, 0.15)   0.8244421239   binomialCDf(4, 20, 20, 0.15)   0.3522748258 $\frac{0.3522748258}{0.8244421239}$ 0.4272887272
Specific behaviours
✓ uses conditional prob reasoning   ✓ shows numerator and denominator values   ✓ states prob

Q4 (4 marks)

Sound loudness, L dB, is measured by comparing the intensity of the sound, I, with the intensity of a sound that is just detectable by the human ear, I_O.

$L = 10\log_{10}\left(\frac{I}{I_O}\right)$.

a) If the noise level in a room was 65 dB, express the intensity of sound in this room in terms of . I_O. (1 mark)

Solution
$65 = 10\log_{10}\left(\frac{I}{I_O}\right)$ $\left(\frac{I}{I_O}\right) = 10^{6.5}$ $I = I_O 10^{6.5}$
Specific behaviours
✓ states expression

b) How many times is the intensity of a 105 dB noise level that of the intensity of a 35 dB noise level? (3 marks)

Solution
$L = 10\log_{10}\left(\frac{I}{I_O}\right)$ $\frac{I}{I_O} = 10^{\frac{L}{10}}$ $\frac{10^{10.5}}{10^{3.5}} = 10^{7}$
Specific behaviours
✓ uses index form ✓ shows the powers of 10 for both levels ✓ states simplified ratio

Q6 (6 marks)

a) Determine $\frac{d}{dx}\left(x^3 \ln x\right)$. (simplify) (3 marks)

Solution
$\frac{d}{dx}\left(x^3 \ln x\right) = x^3 \frac{1}{x} + 3x^2 \ln x$ $= x^2 + 3x^2 \ln x$
Specific behaviours
✓ shows use of product rule ✓ at least one product correct ✓ states simplified derivative

b) Using your result in a) above and **NOT using your classpad** determine $\int 10x^2 \ln x \, dx$. Show all working. (3 marks)

Solution
$\frac{d}{dx}\left(x^3 \ln x\right) = x^2 + 3x^2 \ln x$ $x^3 \ln x = \int x^2 dx + 3\int x^2 \ln x dx$ $\int x^2 \ln x dx = \frac{1}{3}\left(x^3 \ln x - \frac{x^3}{3} \right) + c$ $\int 10x^2 \ln x dx = \frac{10}{3}\left(x^3 \ln x - \frac{x^3}{3} \right) + c$
Specific behaviours
✓ uses linearity (integrates exp in (a) above) ✓ integrates squared term and adds a constant ✓ obtains exp for required integral (no need to factorise) (zero marks for answer only- from classpad)

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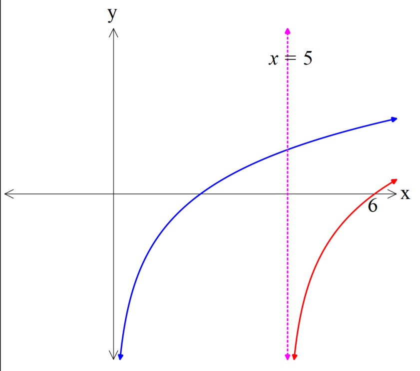
Q5 (5 marks)

Below is a graph of $y = \log_a x$ where a is a positive constant.

a) Sketch on the axes above $y = \log_a (x - 5)$ labelling major features. (2 marks)

Solution



(Not drawn to scale)

Specific behaviours

✓ labels new x intercept at x=6 AND appears to translate to the right
✓ shows a dotted line as asymptote and labels equation

b) Determine the values of $a, b \,\&\, c$ given that $y = \log_a (x + b) + c$ contains points $(-1, -1) \,\&\, (0, 5)$ and has a vertical asymptote at $x = -2$. (3 marks)

Solution

$asymptote\ b = 2$

$-1 = \log 1 + c$

$c = -1$

$5 = \log_a 2 - 1$

$6 = \log_a 2$

$a^6 = 2$

$a = 2^{\frac{1}{6}}$

Specific behaviours

✓ states value of b
✓ sets up equations containing $a \,\&\, c$

✓ states exact values of $a \,\&\, c$