PERTH MODERN SCHOOL
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MATHEMATICS METHODS
UNIT 1 AND 2
Section One:
Calculator-free

Name: _____SOLUTIONS_____

Teacher's Name:_____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
                  correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised material. If you have any unauthorised material with you,
hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
1		7	6		3
2		4	7		6
3		4	8		6
4		6	9		4
5		6			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	50	34
Section Two: Calculator-assumed	14	14	100	85	66
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free (44 Marks)

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

_____

Question 1 {2.2.1, 2.2.7} (7 marks)

(a)   The second term of a geometric sequence is 15 and the fourth term is 135.

Determine the sum of the first 4 terms of the sequence. (4 marks)

$$ar = 15$$
$$ar^3 = 135$$
$$r^2 = 9$$
$$r = \pm 3$$
$$a = \pm 5$$
$$5 + 15 + 45 + 135 = 200$$
$$-5 + 15 - 45 + 135 = 100$$

(1 mark) obtains an equation for one variable only
(1 mark) solves one value of r
(1 mark) solves for two values of a
(1 mark) solves for two sums

(b)   Determine the sum of the first 100 integers
i.e 1+2+3+ ……+99+100. (3 marks)

$$s = \frac{100}{2}(1+100) = 5050$$

(1 mark) recognizes arithmetic
(1 mark) uses sum formula
(1 mark) evaluates sum

Question 2 {2.2.5, 2.1.3}   **(4 marks)**

The fifth term and the ninth term of an arithmetic sequence are 26 and 42 respectively.
Determine the twenty-first term.

$a + 4d = 26$

$a + 8d = 42$

$4d = 16$

$d = 4$

$a = 10$

$t_{21} = 10 + 20(4) = 90$

(1 mark) Sets up two equations for parameters

(1 mark) solves for difference

(1 mark) solves for first term

(1 mark) solves for required term

Question 3 {1.1.24}   **(4 marks)**

Determine the natural domain and corresponding range for the following:

functions	Domain	Range
$f(x) = 5 - x$	R	R
$f(x) = -x^2 + 1$	R	$y \leq 1$
$f(x) = \sqrt{x - 1}$	$x \geq 1$	$y \geq 0$
$f(x) = \dfrac{1}{x + 2} + 3$	$x \neq -2$	$y \neq 3$

(1 mark) two correct domains
(1 mark) four correct domains

(1 mark) two correct ranges
(1 mark) four correct ranges

Question 4 {1.2.8, 1.2.10, 1.2.12, 1.2.13}   **(3, 3 = 6 marks)**

     a)   Solve the following trigonometric equation

$$\sin(3x) = \frac{\sqrt{3}}{2} \quad , 0 \leq x \leq 2\pi$$

$$\sin(3x) = \frac{\sqrt{3}}{2} \quad n = 0, \pm1, \pm2...$$

$$3x = \begin{array}{l} \dfrac{\pi}{3} + 2n\pi \\[2mm] \dfrac{2\pi}{3} + 2n\pi \end{array}$$

$$x = \begin{array}{l} \dfrac{\pi}{9} + \dfrac{6}{9}n\pi \\[2mm] \dfrac{2\pi}{9} + \dfrac{6}{9}n\pi \end{array}$$

$$x = \frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}, \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9}$$

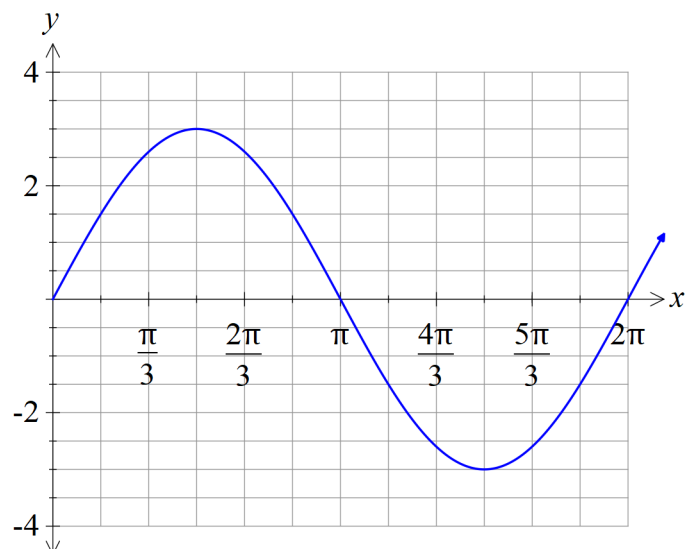(1 mark) solves for one angle for 3x
(1 mark) solves for two angles for x
(1 mark) solves for at least four angles for x

b) Sketch the graph of the following equation

$$y = 3\cos\left(x - \frac{\pi}{2}\right) \quad , 0 \leq x \leq 2\pi$$

(1 mark) amplitude
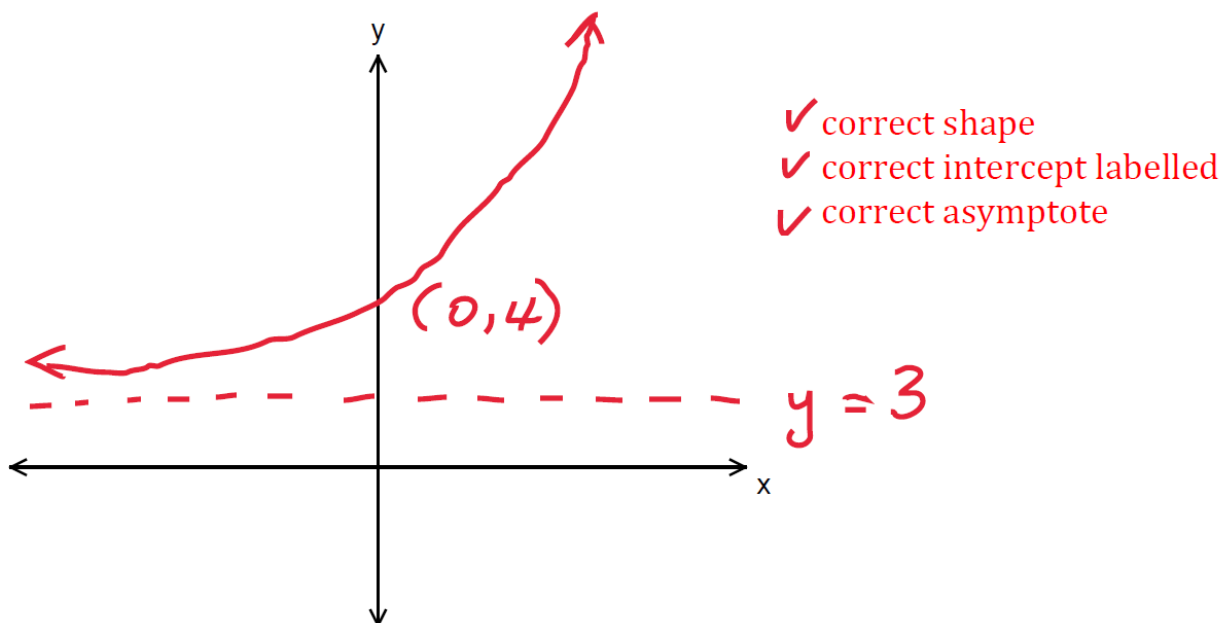(1 mark) all x intercepts
(1 mark) shape AND y intercept
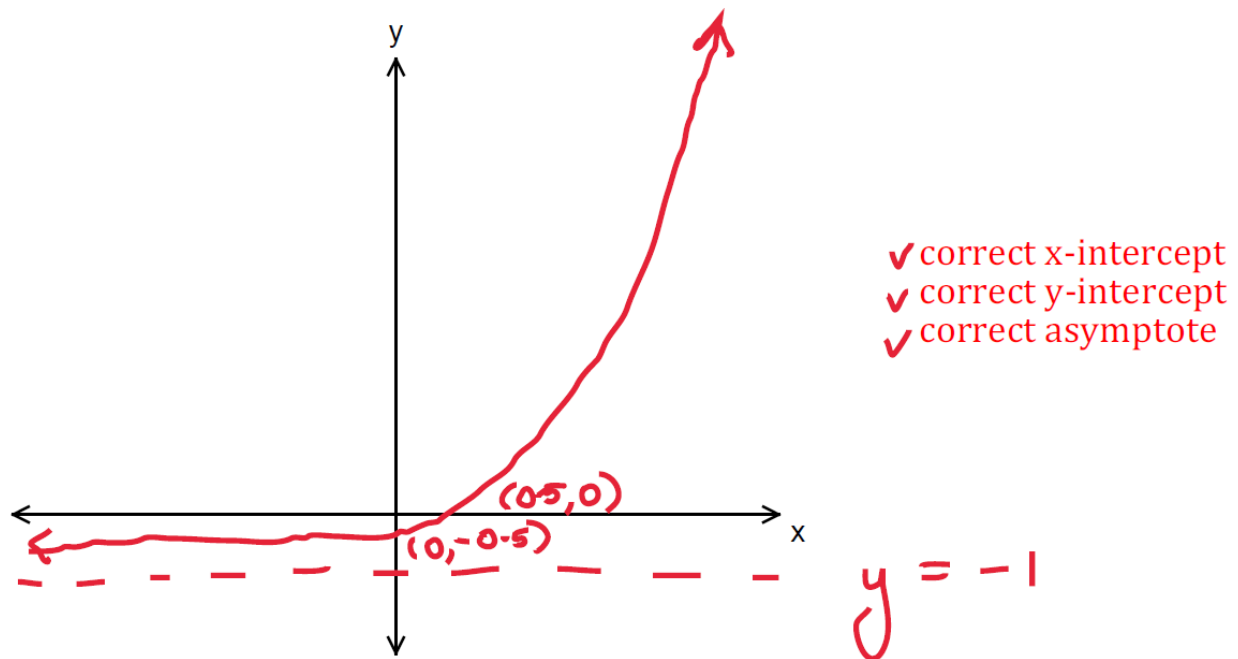
Question 5 {2.1.2, 2.1.7}   **(3, 3 = 6 marks)**

On the axes provided, sketch each of the following exponential functions. Remember to label any intercepts or asymptotes.

(a) $y=2^x+3$

✔ correct shape
✔ correct intercept labelled
✔ correct asymptote

(0,4)

$y=3$

(b) $y=0.5\times 4^x-1$

✔ correct x-intercept
✔ correct y-intercept
✔ correct asymptote

$(0.5, 0)$

$(0, -0.5)$

$y = -1$

Question 6 {1.2.16}   **(3 marks)**

Solve for x:

$$4^x - 6(2^x) - 16 = 0$$

$$2^{2x} - 6(2^x) - 16 = 0$$

let $y = 2^x$

$$y^2 - 6y - 16 = 0$$

$$(y - 8)(y + 2) = 0$$

✔ correctly substitutes $y = 2^x$
✔ recognises no solutions
✔ correct answer

$$\therefore y = 8 \text{ or } -2$$

$$2^x = 8 \quad \text{or} \quad 2^x = -2$$

$$\therefore x = 3 \qquad \text{No solution}$$

Question 7 {1.1.17}   **(6 marks)**

Show that $x+1$ is a factor $x^3 - x^2 + 3x + 5$ of and hence find the other factor

$P(-1) = (-1)^3 - (-1)^2 + 3(-1) + 5$ ✓

$= -1 - 1 - 3 + 5$ ✓

$= 0$

$\therefore (x+1)$ is a factor

$x^3 - x^2 + 3x + 5 = (x+1)(ax^2 + bx + c) + r$

$a = 1$ ✓

As $\quad -x^2 = dx^2 + bx^2$

$-1 = 1 + b$

$b = -2$ ✓

As $\quad 3x = bx + cx$

$3 = -2 + c$

$c = 5$ ✓

As $\quad 5 = c + r \qquad \therefore (x+1)(x^2 - 2x + 5)$ ✓

$r = 0$

> ✓Substitutes -1
> ✓Equates to zero
>
> ✓Solves for a
> ✓Solves for b
> ✓Solves for c
> ✓States the factor.

Question 8 {1.3.1, 1.3.5}   **(2, 2 = 4 marks)**

The first four rows of Pascal's triangle are shown below.

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1
\end{array}
$$

Simplify all answers in this question.

    a.   Expand the expression $(x-2)^5$.

$1x^5(-2)^0 + 5x^4(-2)^1 + 10x^3(-2)^2 + 10x^2(-2)^3 + 5x^1(-2)^4 + 1x^0(-2)^5$ ✓

$x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$ ✓

b. Find the third term of the expansion of $(2x-3y)^4$

$$6(2x)^2(-3y)^2 \checkmark$$

$$= 6(4x^2)(9y^2)$$

$$= 216x^2y^2 \checkmark$$

✓ substitutes correct terms
✓ correct answer

Question 9 {2.3.15, 2.3.9}   **(2, 2 = 4 marks)**

a) Find the derivative of the function $f(x) = \dfrac{4}{x^2} + 2x$. Express your answer with a positive indices.

$$f(x) = 4x^{-2} + 2x$$

$$f'(x) = -8x^{-3} + 2 \quad \checkmark$$

$$= \frac{-8}{x^3} + 2 \quad \checkmark$$

✓ correct derivative
✓ expresses as a positive indice

b) Find the equation of the curve if the gradient at any point of the curve is given by $\dfrac{dy}{dx} = 1 - 2x$ and the curve passes through the point (1,2).

$$y = x - x^2 + c \quad \checkmark$$

At (1,2)

$$2 = 1 - 1^2 + c$$

$$\therefore c = 2$$

$$\therefore y = x - x^2 + 2 \quad \checkmark$$

✓ correct anti-derivative with c.
✓ Correct equation.

END OF SECTION 1

Additional working space

Question number: _____

Additional working space

Question number: _____