5 a) Differentiate $y = x^3 \ln x$ with respect to x. (2)

b) Using your result from a), or otherwise, determine $\int x^2 \ln x \, dx$ (4)

SHENTON COLLEGE

ATMAM Mathematics Methods

Test 4 (2019) Calculator Free

Name: .....

Teacher: Friday Smith Ai

Time Allowed : 30 minutes

Marks /30

Materials allowed: Formula Sheet.

Attempt all questions. Questions 1 to 5 are in this section.
All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

For all questions, assume that the domain of x is restricted to ensure valid logarithms.

1 Differentiate with respect to x. [3, 2, 1, 2]

a) $y = \ln\left(\frac{3x^2}{\sin x}\right)$

b) $y = \ln\sqrt{(x^2-4)^3}$

c) $y = \ln\left(\frac{3}{5}\right)^2$

d) $y = \ln\left(\frac{1}{x}\right)$

2 Determine the following indefinite integrals.

a) $\int \frac{4}{3x} dx$ (2)

b) $\int \frac{\sin x + \cos x}{\cos x - \sin x} dx$ (2)

c) $\int \frac{x^2 + 2x + 1}{x^2 + 1} dx$ (3)

3 Evaluate the following definite integral, giving your answer as a **single logarithm**. (4)

$\int_2^3 \frac{6x}{x^2 - 3} dx$

4 If $f'(x) = \frac{x^2 - 3x + 2}{x}$ and $f(2) = 2 + \ln 4$, determine the equation of $f(x)$. (5)