Government of **Western Australia**
Curriculum Council

WACE
Western Australian Certificate of Education

MATHEMATICS

2C/2D

Calculator-free

WACE Examination 2011

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

When examiners design an examination, they develop provisional marking keys that can be reviewed at a marking key ratification meeting and modified as necessary in the light of candidate responses.

Question 1 **(10 marks)**

(a) How many significant figures are there in the number 0.02070? (1 mark)

Solution
4
Specific Behaviours
✓ determines number of significant figures

(b) Write 0.0038 in scientific notation. (1 mark)

Solution
3.8×10^{-3}
Specific Behaviours
✓ expresses answer in scientific notation correctly

(c) Factorise $4x^2 - 9$ (1 mark)

Solution
$(2x + 3)(2x - 3)$
Specific Behaviours
✓ factorises expression correctly

(d) Determine the gradient of the straight line given by $7x + 2y = 4$. (1 mark)

Solution
$-\frac{7}{2}$
Specific Behaviours
✓ determines gradient correctly

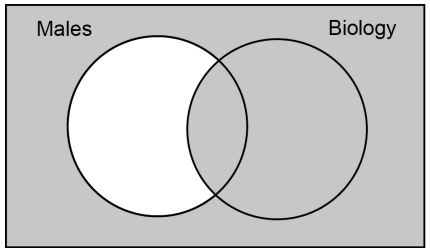
(e) Given $f(x) = 5 - x^2$, determine the value of $f(-2)$. (1 mark)

Solution
1
Specific Behaviours
✓ evaluates function when $x = -2$

(f) Write a recursive definition for the following sequence:
1, 1, 2, 3, 5, … . (1 mark)

Solution
$t_n = t_{n-1} + t_{n-2}$ With $t_0 = t_1 = 1$
Specific Behaviours
✓ states correct recursive definition with boundary conditions.

(c) Let M represent the set of Males and B represent the set of Biology students. Shade the region in the Venn diagram drawn below that is represented by the set $\bar{M} \cup B$. (1 mark)



Solution
shading on diagram above diagram
Specific Behaviours
✓ shades region correctly

(d) Determine $P(\bar{M} \cup B)$. (1 mark)

Solution
$\frac{17}{32}$
Specific Behaviours
✓ determines probability correctly (based on response in part (b) and/or (c))

(e) In the context of the question, describe the meaning of $n((M \cup B) \mid \bar{M})$. (2 marks)

Solution
$n((M \cup B) \mid \bar{M})$ The number of students that are female and take biology.
Specific Behaviours
✓ states that the 'n' represents the number of students ✓ states that the students are female and studying biology or other correct interpretation

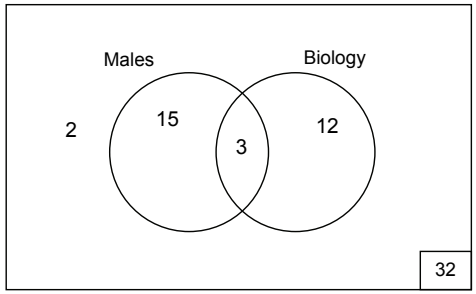
Question 6 **(9 marks)**

In a class of 32 students, of which 14 are females, it is found that three males are taking Biology. Fifteen students in the class are enrolled in Biology.

(a) Express the above data in a two-way table and complete the missing entries.(3 marks)

Solution			
	Biology	Not in Biology	Row Totals
Males	3	15	18
Females	12	2	14
Column Totals	15	17	32
Specific Behaviours			
✓ constructs table with appropriate headings ✓ calculates entries for cells of the table ✓ sums entries correctly to determine totals			

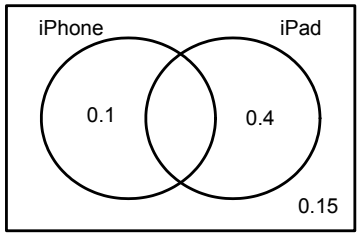
(b) The above data can also be expressed as a Venn diagram. Complete the Venn diagram below by writing the relevant number of students in every region. (2 marks)



Solution
On diagram
Specific Behaviours
✓✓ determines correct numbers for each of the four regions ✓ determines numbers for two or three of the regions

(g) A group of students was surveyed about ownership of iPads and iPhones. The Venn diagram shows some of the probabilities associated with the responses.

Determine $P(\overline{iPad})$.



(1 mark)

Solution
$P(\overline{iPad})$ = 0.25
Specific Behaviours
✓ determines probability correctly

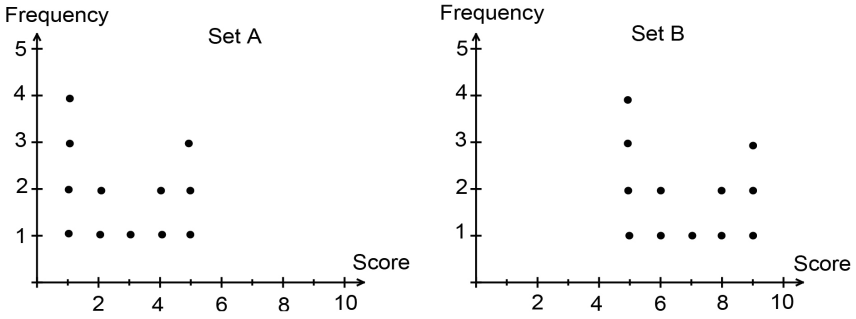
(h) For the cubic function $y = (x + 2)(x - 3)(x + 1)$, determine the coordinates of the y-axis intercept (sketch not required). (1 mark)

Solution
(0, – 6)
Specific Behaviours
✓ determines coordinates for y-axis intercept correctly

(i) The following is a set of scores in ascending order: 4, 4, 6, x, 9, 11. If the median is 7, determine the value of the x. (1 mark)

Solution
x = 8
Specific Behaviours
✓ calculates correct value of x

(j) Is the standard deviation of set A larger, smaller or the same as the standard deviation of set B? (1 mark)



Solution
Same
Specific Behaviours
✓ states that the standard deviations are the same

Question 5 **(4 marks)**

Jacqui was testing her conjecture that:

'for any three consecutive positive even numbers, the product of the first and third numbers is equal to the second number squared.'

(a) Show that Jacqui's conjecture does not work. (1 mark)

Solution
Answers may vary. For example, using 2, 4, and 6. $2 \times 6 = 12 \neq 4^2 = 16$
Specific Behaviours
✓ tests a set of three consecutive even numbers that disprove the conjecture

(b) Test the conjecture for **two (2)** more sets of three consecutive positive even numbers. (2 marks)

Solution
Answers may vary. For example, using 4, 6 and 8 $4 \times 8 = 32 \neq 6^2 = 36$ Similarly, using 6, 8, and 10 $6 \times 10 = 60 \neq 8^2 = 64$
Specific Behaviours
✓✓ tests conjecture correctly for two other sets of three consecutive even positive numbers

(c) Jacqui realised that she could revise her conjecture so that it is true. Her revised conjecture is:

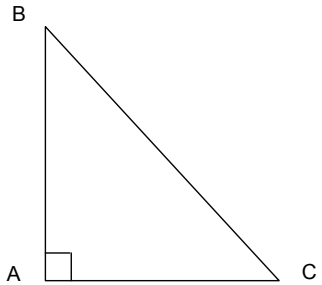
'for any three consecutive positive even numbers, the product of the first and third numbers is equal to the middle number squared take 4.'

Complete her revised conjecture. (1 mark)

Solution
The middle number squared take/minus/less four.
The square of the middle number take/minus/less four.
Specific Behaviours
✓ completes conjecture correctly

Question 4 **(5 marks)**

Consider the triangle ABC where tan $\angle$ABC = $\frac{3}{4}$. (Note the diagram is not drawn to scale.)



(a) If the length of the side AB is 400 metres, calculate the length of the side AC. (3 marks)

Solution
$\tan \angle ABC = \frac{3}{4} = \frac{AC}{AB}$ $\therefore AC = \frac{3}{4} AB$ $= \frac{3}{4}(400)$ $= 300 \text{ m}$
Specific Behaviours
✓ states the tangent ratio $\left(\tan \angle ABC = \frac{AC}{AB} = \frac{3}{4}\right)$ ✓ substitutes $AB = 400$ ✓ states correct value for AC

(b) Determine the value of sin$\angle$BCA. (2 marks)

Solution
$\sin \angle BCA = \frac{AB}{BC}$ $= \frac{400}{500}$ $= 0.8$ (Accept $\frac{4}{5}$)
Specific Behaviours
✓ identifies correct sides ✓ determines correct value

Question 2 **(7 marks)**

(a) Simplify the expression $7x - 2(3 - 4x) - 5x$. (2 marks)

Solution
$7x - 6 + 8x - 5x = 10x - 6.$
Specific Behaviours
✓ expands brackets correctly ✓ simplifies expression

(b) Solve the equation $18 - x^2 = 7x$. (3 marks)

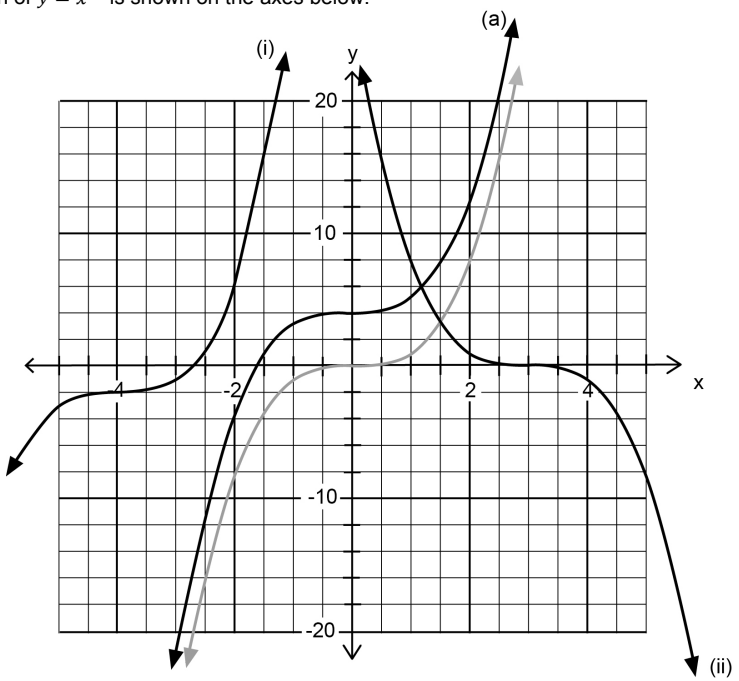
Solution
$x^2 + 7x - 18 = 0$ $(x - 2)(x + 9) = 0$ $x = 2$ or $x = -9$
Specific Behaviours
✓ rearranges equation in the form $f(x) = 0$ ✓ factorises quadratic correctly ✓ solves equation to find both values of x Or ✓✓ uses guess and check for each solution ✓ shows relevant working

(c) Determine the value of n given $\frac{5^3 \times 5^n}{5^4} = 5^6$. (2 marks)

Solution
$5^{n-1} = 5^6$ or $\frac{5^n}{5} = 5^6$ $n - 1 = 6$ $5^n = 5^6 \therefore n = 7$ $n = 7$
Specific Behaviours
✓ simplifies the expression using index laws ✓ determines correct value of n

Question 3 **(5 marks)**

The graph of $y = x^3$ is shown on the axes below.



(a) Describe how the graph of $y = x^3$ can be used to draw the graph $y = x^3 + 4$. (1 mark)

Solution
Vertical translation – move up 4 units (or 2 spaces) on scale (Graph shown but not needed) .
Specific Behaviours
✓ describes transformation as either a vertical translation or a shift upward by 4 units

(b) On the set of axes above sketch each of the following:

(i) $y = (x+4)^3 - 2$ (2 marks)

Solution
On diagram above
Specific Behaviours
✓ sketches shape of graph correctly ✓ sketches graph accurately (location of point of inflection and reasonably congruent shape)

(ii) $y = -(x-3)^3$ (2 marks)

Solution
On diagram Note: sketches should be labelled (i) and (ii).
Specific Behaviours
✓ sketches with correct orientation ✓ locates point of inflection correctly