



Semester Two Examination, 2023

Question/Answer booklet

12 SPECIALIST MATHEMATICS

Section One: Calculator-free

Your Name _____

Your Teacher's Name _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

| Question | Mark | Max | Question | Mark | Max |
|----------|------|-----|----------|------|-----|
| 1 | | 9 | 5 | | 7 |
| 2 | | 7 | 6 | | 8 |
| 3 | | 8 | | | |
| 4 | | 9 | | | |

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|---------------------------|
| Section One: Calculator-free | 6 | 6 | 50 | 48 | 33 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 97 | 67 |
| Total | | | | | 100 |

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free**(48 Marks)**

This section has **six (6)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1**(9 marks)**

Consider the functions $g(x) = \frac{1}{x^2 - 1}$ and $h(x) = \sqrt{x - 4}$.

a) Determine the natural domain and range of $g(x)$. (3 marks)

b) Does $g \circ h(x)$ exist over the natural domain of $h(x)$? Explain. (3 marks)

c) State $h \circ g(x)$ and its natural domain. (3 marks)

Question 2**(7 marks)**

Determine the following integrals.

a) $\int \frac{2x}{\sqrt{1-3x}} dx$.

(3 marks)

b) $\int_0^{\pi} 5 \sin^3(2x) dx$.

(4 marks)

Question 3**(8 marks)**

$$f(x) = \frac{2x^2 + 11x + 7}{(x+2)(x^2 + 6x + 1)}$$

- a) The function can be expressed in the form

$$\frac{a}{x+2} + \frac{bx+c}{x^2+6x+1}$$

where a, b & c are constants.

a, b & c

Determine the values of .

(4 marks)

$$\int \frac{4x^2 + 22x + 14}{(x+2)(x^2 + 6x + 1)} dx$$

- b) Hence determine .

(4 marks)

Question 4**(9 marks)**

Consider a herd of 25 horses, N in an isolated habitat such that the growth rate after t years is

$$\frac{dN}{dt} = \frac{N}{4} - \frac{N^2}{1000}$$

given by

$$N(t)$$

- a) By using separation of variables and partial fractions, derive $N(t)$ showing all working. (5 marks)

Q4 continued

b) Determine the limiting value of the number of horses. (2 marks)

c) Set up an equation, but do not solve, that will allow the time to be calculated where the growth rate is a maximum. (2 marks)

Question 5

(7 marks)

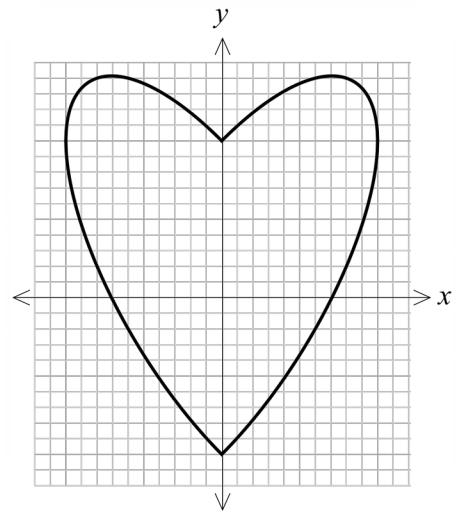
A designer creates a heart-shaped pendant for Valentine's Day shown on the right, using the function

$$2|x|^2 - 2|x|y + y^2 - 1 = 0$$

For $x \geq 0$ this equation becomes

$$2x^2 - 2xy + y^2 - 1 = 0$$

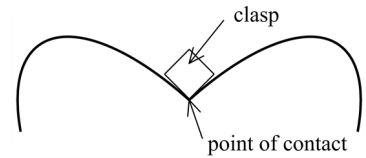
(a) Show that $\frac{dy}{dx} = \frac{2x-y}{x-y}$ for $x \geq 0$ (3 marks)



(3 marks)

Q5 continued-

The jewellery plans to attached a small square shaped clasp to the pendant. One corner of the square will sit in the cusp on the curve at the point of contact. The situation is illustrated on the right below.



- (b) (i) Determine the coordinates of the point of contact.
(1 mark)

- (ii) At the point of contact, will the gradient of the heart match that of the clasp?
Justify your answer. (3 marks)

Question 6**(8 marks)**

Let $v = 1 + \sqrt{3}i$.

(a) Determine the three cube roots of v .

(3 marks)

(b) Consider the polynomial $P(z) = z^4 - 8z^3 + kz^2 - 46z + 44$, where k is a real constant.

Given that $P(v) = 0$, solve the equation $P(z) = 0$.

(5 marks)

Working out space

Working out space

Working out space.