SEMESTER TWO

YEAR 11

MATHEMATICS SPECIALIST
Units 1 & 2
2016

REVISION 3

Section Two
(Calculator–assumed)

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 10 minutes

Working time for section: 100 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items*:* pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: drawing instruments, templates, notes on up to two unfolded sheets of A4 paper, and up to three calculators approved for use in examinations.

IMPORTANT NOTE TO CANDIDATES
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

To be provided by the supervisor
Question/answer booklet for Section Two.
Formula sheet retained from Section One.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	7	7	50	52	35
Section Two Calculator—assumed	**11**	**11**	**100**	**98**	**65**
Total marks				150	

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answer in the Question/Answer booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula Sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator–assumed   **98 marks**

This section has **eleven (11)** questions. Attempt **all** questions.
Working time: 100 minutes

Question 8   **(7 marks)**

(a)   Three little boys are pulling the dog's tail. The dog seems to understand they are little, but if they pull with a combined force of 10N against the dog, he will snap and possibly bite them.

The forces exerted by the young lads are shown in the diagram below.

Boy 1; 4N at 045°

Dog

Boy 2; 5N at 090°

Boy 2; 6N at 120°
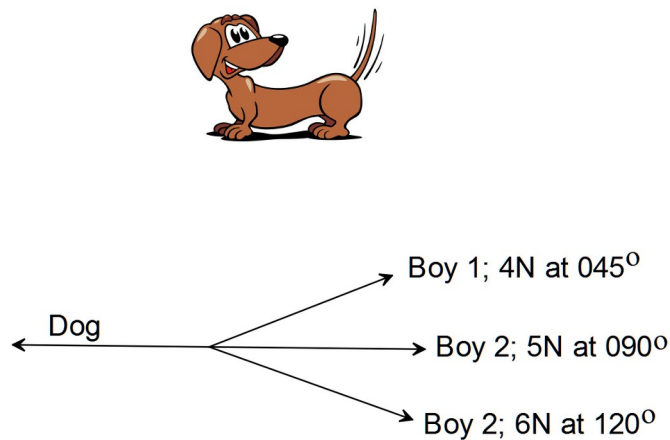
(i)   What is the resultant force in the direction against the dog?   (2)

(ii)   Are the boys in danger from the dog?   (1)

(b)   Use a vector proof to show that

       "*The midpoints of the sides of a quadrilateral join to from a parallelogram*."   (4)

Question 9 (8 marks)

(a)   Simplify $\dfrac{(1+2i)}{(1+i)(1-2i)}$.   (3)

(b)   Given $z_1 = a + bi$ and $z_1 = c + di$

     (i)   Find $\mathrm{Re}(z_1 + z_2)$ and $\mathrm{Im}(z_1 + z_2)$.   (3)

     (ii)   Show that $\mathrm{Im}\big((z_1 - z_2)^2\big) = 2(a-c)(b-d)$ (2)

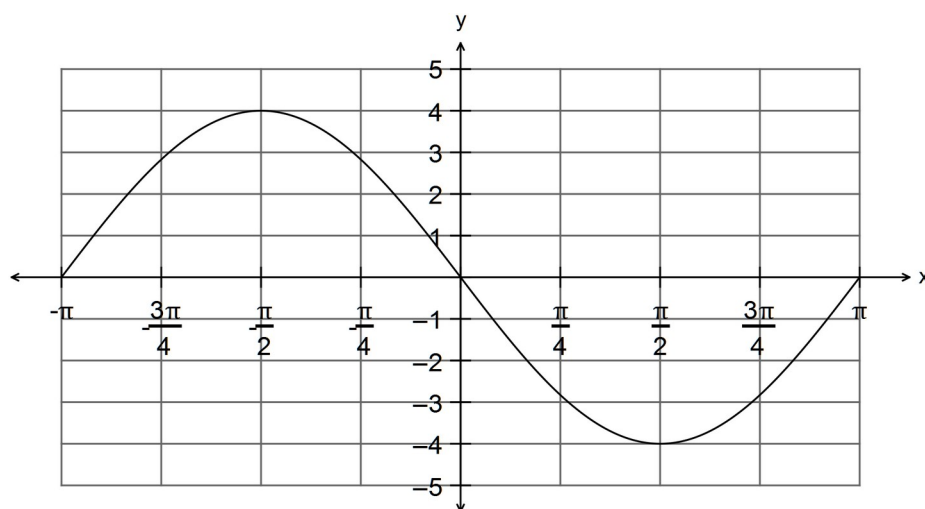**Question 10**   **(14 marks)**

(a)   (i)   Express $\cos(\theta) - \sqrt{3}\sin(\theta)$ in the form of $R\cos(\theta + \alpha)$ for $R > 0$ and

$0 < \alpha < \dfrac{\pi}{2}$.   (3)

(ii)   Hence solve $\cos(\theta) - \sqrt{3}\sin(\theta) = -1$ for θ, correct to two decimal places,
on the domain $0 \leq \theta \leq \pi$.   (4)

(b)   Write down the equation of the function graphed below.   (2)

(c)   Given $p = 2i + 4j$ and $q = 3i - 5j$

find

(i) $3p - 4q$.   (2)

(ii)   the vector in the same direction as p but that has a magnitude of $\sqrt{5}$.   (3)

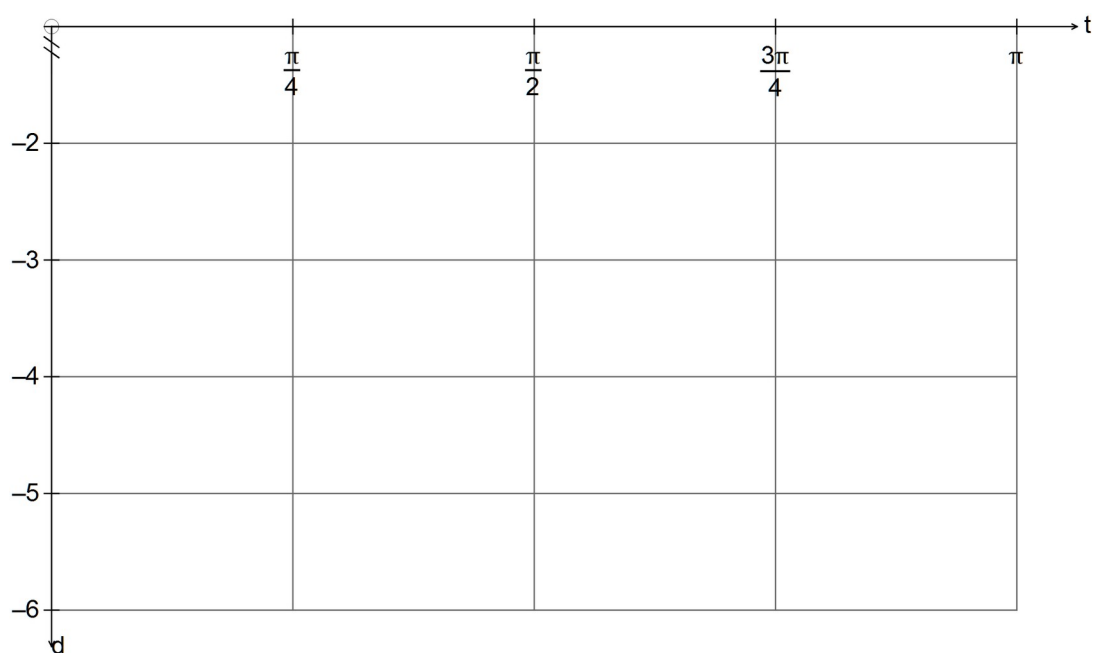**Question 11**   **(6 marks)**

Consider the two functions $y = 2\cos\left(3\left(x - \dfrac{\pi}{2}\right)\right) - 4$ and $y = 2\sin(3x) - 4$.

(a)   Are the functions equivalent functions? Explain carefully.   (3)

(b)   Sketch the function $y = 2\sin(3x) - 4$ on the set of axes below:

        (3)

Question 12   **(3 marks)**

(a)   Simplify $\cos(x+y)\cos(x-y)+\sin(x+y)\sin(x-y)$ (2)

(b)   Hence show that $\cos(x+y)\cos(x-y)+\sin(x+y)\sin(x-y)=1-2\sin^2(y)$ (1)

Question 13   **(11 marks)**

(a)   Solve the following equation $z^2 + 4z + 5 = 0$. Show all working.   (3)

(b)   Find the complex number u such that $\left(\bar{u}\right)^{-1} = 1 - 2i$.   (3)

(c)   A quadratic equation with real coefficients has one of its roots equal to $2 - 4i$.
      Determine the quadratic equation.   (3)

(d)   Write the converse of the statement "*If you study hard, you will pass your exams*."
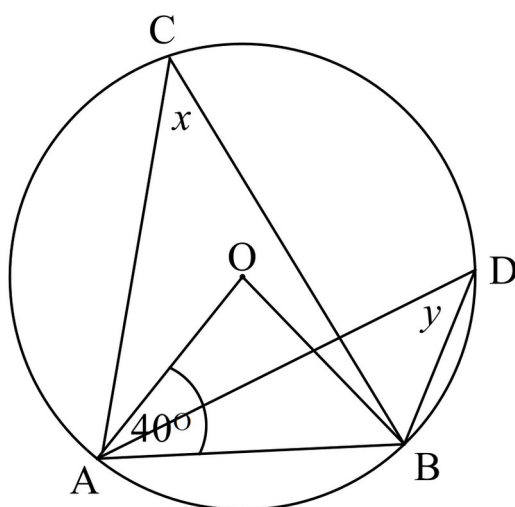
          (1)

(e)   Write the contrapositive of the statement
      "*If you have a TV, then you are can watch the news on Channel Two.*"   (1)
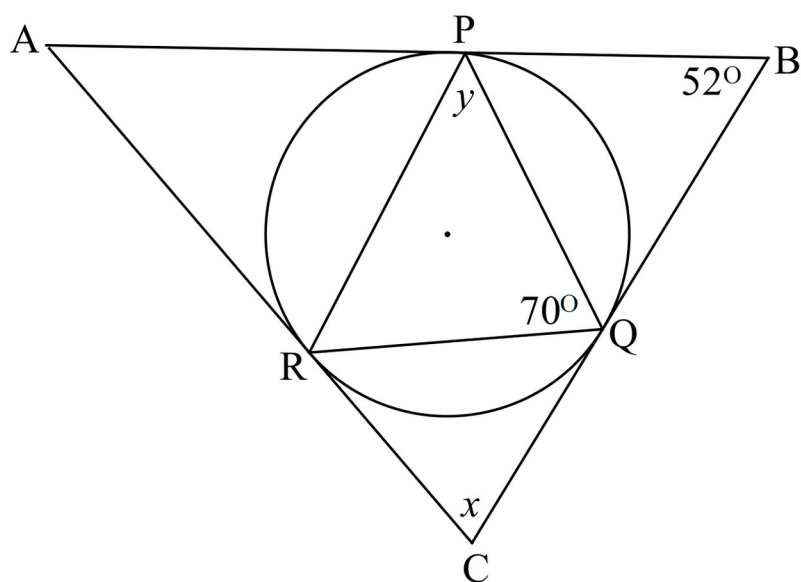
Question 14 **(9 marks)**

(a) Find real x and y given $(x+iy)^2 = 5 - 12i$. (4)

(b)   (i)   Solve for x and y, giving reasons.   (2)

(ii)   Solve for x and y, giving reasons.   (3)

Question 15 **(10 marks)**

(a)   There are 9 people that need to take a seat in a row of 12 chairs.

Show that there are at least three people seated in adjacent chairs.   (2)

(b)   (i)   In how many ways can students gain first, second and third positions if 8 students entered the competition?   (1)

(ii)   If the three prize winners are given identical tickets to the cinema. How many different groups are there that could go to the cinema?   (1)

(c)   Among the 18 students in Mr Bean's Maths class, 7 had black hair, 10 had blue eyes, and 10 were considered tall. Also, 3 had black hair and had blue eyes, 4 had black hair and were considered tall, and 5 had blue eyes and were considered tall. One student had black hair, blue eyes and was considered tall.

(i)   How many of these students did not have black hair, did not have blue eyes and were not considered tall?   (2)

(ii)   How many of the students had blue eyes but not black hair?   (1)

(d)   Prove that $^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$ (3)

Question 16   **(10 marks)**
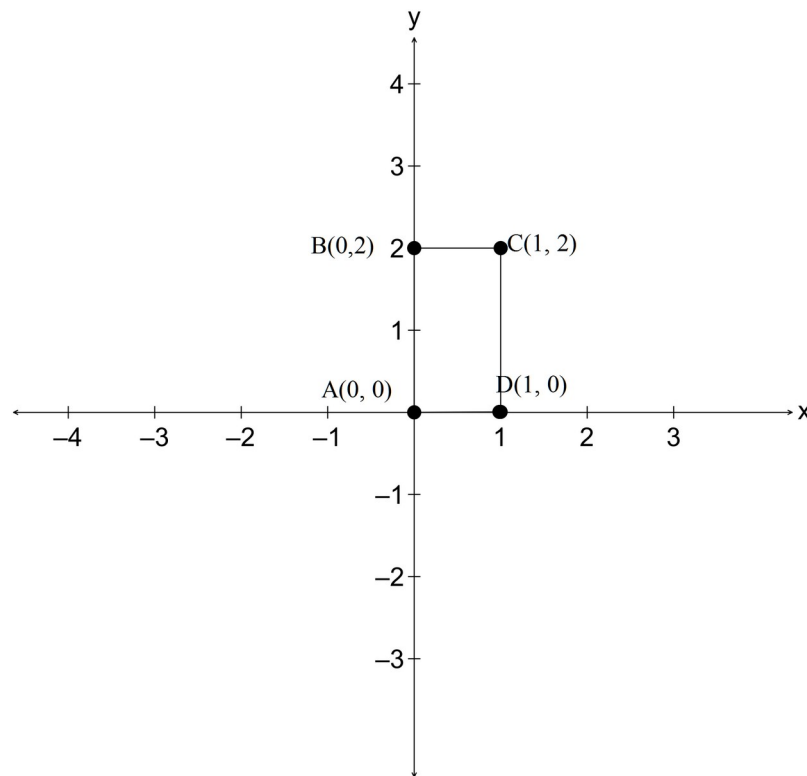
(a)   Prove, using a proof by contradiction, that $\sqrt{2}$ is an irrational number.   (6)

(b)   Prove using mathematical induction, that $1+3+5+....+(2n-1)=n^2$

for any integer $n \geq 1$. .   (4)

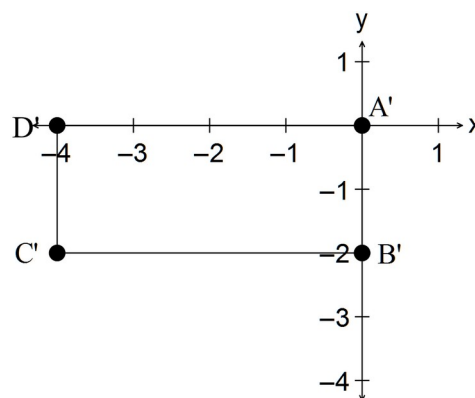**Question 17**   **(12 marks)**

(a)   Consider the rectangle ABCD as shown in the diagram below.

(i)   Write down the matrix that transforms the coordinates A, B, C and D into the
        coordinates A', B', C' and D' as shown in the diagram below   (1)

(ii)   The rectangle A' B' C' D' is rotated 90° anti-clockwise about the origin to form
        A''B''C''D''. Write down the rotational matrix.   (1)

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(iii)　Determine the coordinates of the rectangle A''B''C''D''.　　　　(2)

(iv)　Determine the single matrix that could have been used to change ABCD into A''B''C''D''.　　　　(2)

(v)　Determine the matrix that could be used to transform A''B''C''D'' back into the rectangle ABCD.　　　　(2)

(b)　(i)　Explain how the area of the rectangle A''B''C''D'' can be calculated from the area of the rectangle ABCD using the single transformational matrix found in (a) (iv).　　(2)

(ii)　Verify that the method works.　　　　(2)

Question 18   **(8 marks)**

It is known that the matrix M performs a reflection about the line $y = mx$

where $M = \begin{bmatrix} p & q \\ q & -p \end{bmatrix}$, $p = \dfrac{1-m^2}{1+m^2}$, $q = \dfrac{2m}{1+m^2}$ and m is the gradient of the line.

The point $P(5, 5)$ is reflected about the line $y = 2x$ and then <u>that point</u> is reflected about the line

$y = -\dfrac{x}{2}$.

(a)   Determine the first reflected point, P'(x, y).   (3)

(b)   Find the second reflected point, P''(x, y).   (3)

(c)   Show that the second reflected point is a rotation of P(5, 5) about the origin.   (2)

END OF SECTION TWO