Christ Church Grammar School

2016
UNIT TEST 4

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Student name Solution

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

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Additional working space

Question number: _____

Question 1 **(5 marks)**

Let $\log 2 = x$ and $\log 3 = y$. Express each of the following in terms of x and y.

(a) 6 (1 mark)

$6 = 10^x \times 10^y$ ✓

$= 10^{x+y}$

(b) $\log 45$ (2 marks)

$= \log(5 \times 9)$

$= \log 5 + \log 3^2$

$= \log \frac{10}{2} + 2y$ ✓

$= 1 - x + 2y$ ✓

(c) $\log 0.\overline{8}$ (2 marks)

$= \log \frac{8}{9}$ ✓

$= \log 2^3 - \log 3^2$

$= 3x - 2y$ ✓

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Question 2 **(6 marks)**

(a) Solve the equation $3^{(x-1)} = 15$, leaving your answer in exact values. (2 marks)

$x-1 = \log_3 15$ ✓

$x = \log_3 15 + 1$ ✓

$= \frac{\log 15}{\log 3} + 1$

(b) Solve the equation $2\log_x\left(\frac{1}{32}\right) = -5$. (2 marks)

$\frac{1}{32} = x^{-\frac{5}{2}}$

$x^{\frac{5}{2}} = 32$ ✓

$x = 32^{\frac{2}{5}}$

$= 2^2 = 4$ ✓

(c) Differentiate $\log_5(x^2)$ with respect to x. (2 marks)

$\frac{d}{dx}\left(\log_5 x^2\right) = \frac{d}{dx}\left(\frac{\ln x^2}{\ln 5}\right)$ ✓

$= \frac{2x}{x^2 \ln 5}$

$= \frac{2}{x \ln 5}$ ✓

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Question 8 **(5 marks)**

(a) A health care provider was considering a study of children in Australia who had suffered from influenza. After defining their target population, 6800 children who met the criteria were identified through available medical records. Clearly explain the steps involved in using systematic sampling to select a random sample of 400 of these children. (2 marks)

1. Number all children from 1 to 6800.
2. $\frac{6800}{400} = 17$

   Choose a random number between 1 to 17 and pick this child ✓
3. Pick every 17th child after the first child. ✓

(b) 50 tagged trout were released into a lake. Over weekly intervals, small samples of trout were caught from various locations around the lake, with the following results

	Number of trout in sample	Percentage of sample tagged
Week 1	40	5%
Week 2	20	10%
Week 3	50	4%

Use the capture-recapture method to estimate the number of trout in the lake. (2 marks)

Let n = no. of trout in lake

$\frac{50}{n} = \frac{0.05 \times 40 + 0.1 \times 20 + 0.04 \times 50}{40 + 20 + 50}$ ✓

$n = 916.67$

≈ 917 ✓ (accept 916 – 920)

(c) X is a continuous random variable which follows a uniform distribution for $2 \leq x \leq 9$. That is, $X \sim U[2, 9]$.

Write a ClassPad function to simulate a value for X. (1 mark)

7rand() + 2 ✓

Question 7 **(7 marks)**

Joe spends X minutes in the shower, with X having the probability density function $f(x)$ defined as follows,

$$f(x) = \begin{cases} \frac{a-x}{40} & for \quad 2 \le x \le 10 \\ 0 & for \quad all\ other\ values\ of\ x \end{cases}$$

(a) Determine the value of the constant a. (2 marks)

$$1 = \int_2^{10} \frac{a-x}{40}\,dx \quad \checkmark$$

$$= \left[\frac{ax - \frac{1}{2}x^2}{40}\right]_2^{10}$$

$$a = 11 \quad \checkmark$$

(b) Find $E(X)$, the expected value of X. (2 marks)

$$E(X) = \int_2^{10} x f(x)\,dx$$

$$= \int_2^{10} x\left(\frac{11-x}{40}\right)dx \quad \checkmark$$

$$= \frac{74}{15} \quad \checkmark$$

(c) Find the standard deviation of X. (2 marks)

$$Var(X) = E(X^2) - E(X)^2$$

$$= \int_2^{10} x^2\left(\frac{11-x}{40}\right)dx - \left(\frac{74}{15}\right)^2 \quad \checkmark$$

$$= \frac{944}{225} \quad \therefore s = 2.048 \quad \checkmark$$

(d) Determine $P(3 < X < 5)$. (1 mark)

$$P(3 < X < 5) = \int_3^5 \left(\frac{11-x}{40}\right)dx$$

$$= \frac{7}{20} \quad \checkmark$$

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Question 3 **(4 marks)**

Find the following:

(a) $\frac{d}{dx}\left(\frac{x}{\ln x}\right)$ (2 marks)

$$= \frac{(1)\ln x - x\left(\frac{1}{x}\right)}{(\ln x)^2}$$

$$= \frac{\ln x - 1}{(\ln x)^2} \quad \checkmark \quad \checkmark$$

(b) $\int \frac{12-20x}{5x^2-6x+1}\,dx$ (2 marks)

$$= -2\int \frac{10x-6}{5x^2-6x+1}\,dx$$

$$= -2\ln|5x^2 - 6x + 1| + C \quad \checkmark \quad \checkmark$$

Additional working space

Question number: _____

Question 6 **(7 marks)**

Clay Fishe, owner of a yabbie farm, has found that the distribution of weights of his yabbies is symmetrical, with approximately 68% of his yabbies weighing between 40 g and 66 g, and a further 13.5% of his yabbies weighing between 27 g and 40 g.

(a) State the appropriate distribution (with its parameters) for the weight of yabbies at Clay's farm. Justify your choice of distribution. (3 marks)

$W \sim N(53, 13^2)$ ✓

As it is a symmetrical distribution. with probability of 68% between 40g - 66g (1 σ) and 13.5% between $\mu - 2\sigma$ and $\mu - \sigma$.

(b) What proportion of Clay's yabbies are above the market weight of 30 g? (2 marks)

$P(W > 30)$

$= 0.96157$ ✓✓

(c) Yabbies increase in size by an average of 3 g per month. How many months will Clay have to wait so that at least 99% of his current stock will be above the market weight of 30 g? (2 marks)

Z - score for 99% is -2.326347874

$30 = \mu - 2.32634787\,\sigma$

$= \mu - (2.32634787)(13)$ ✓

$\mu = 60.24252231$

so an extra 7.24252231 g

ie $\frac{7.24252213}{3} = 2.4$ months

≈ 3 months ✓

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Question 5 **(7 marks)**

An intersection is controlled by a set of traffic lights. The traffic lights are timed so that the lights will be green for 20 seconds, and then red for 100 seconds. Motorist Di Hatsoo passes through these traffic lights each day on her way to and from work. The time that she arrives at these traffic lights on any given day is random and independent of travel on any other day.

Assuming that the lights are red when Di arrives at the intersection, and ignoring any delays caused by traffic or road conditions, the amount of time (T seconds) that Di has to wait at these lights can be modelled by a uniform distribution.

(a) State the probability density function for T. (2 marks)

$$P(T) = \begin{cases} \frac{1}{100} & 0 \le T \le 100 \\ 0 & \text{otherwise} \end{cases}$$

(b) Determine the probability that, on any given day, Di has to wait at this intersection for less than 90 seconds, given that she has to wait for at least one minute. (2 marks)

$$P(T < 90 \mid T > 60) = \frac{P(60 < T < 90)}{P(T > 60)} = \frac{30}{40} = \frac{3}{4}$$

(c) What is the mean waiting time for Di on the days that she arrives at the intersection when the lights are red? (2 marks)

$$E(T) = \frac{1}{2}(0 + 100) = 50 \text{ s}$$

(d) If the assumption that Di arrived at the intersection when the lights are red was removed, would it still be appropriate to model Di's waiting time using a uniform distribution? Justify your answer. (1 mark)

No. If it was green, there would be no waiting time, hence uniform dist. would not apply.

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Christ Church Grammar School

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UNIT TEST 4

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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Question 4 **(4 marks)**

Chemists define the acidity or alkalinity of a substance according to the formula: $pH = -\log[H^+]$ where $[H^+]$ is the hydrogen ion concentration, measured in moles/litre. Solutions with a pH value of less than 7 are acidic, solutions with a pH value greater than 7 are basic, and solutions with a pH of 7 (such as pure water) are neutral.

(a) The hydrogen ion concentration of apple juice is $[H^+] = 0.000\,28$. Find the pH value, rounded correct to 2 decimal places, and hence determine whether the juice is basic or acidic. (2 marks)

$pH = -\log(0.00028)$

$= 3.55$ so acidic ✓ ✓

(b) An unknown mixture is tested and found to have a pH value of 9.7. What is the hydrogen ion concentration of the unknown mixture? Write your answer in scientific notation, rounded correct to 3 significant figures. (2 marks)

$pH = -\log[H^+]$

$9.7 = -\log[H^+]$

$[H^+] = 10^{-9.7}$ ✓

$= 2.00 \times 10^{-10}$ ✓

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