Government of **Western Australia**
Curriculum Council

WACE
Western Australian Certificate of Education

MATHEMATICS

2C/2D

Calculator-assumed

WACE Examination 2011

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

When examiners design an examination, they develop provisional marking keys that can be reviewed at a marking key ratification meeting and modified as necessary in the light of candidate responses.

Question 7 **(7 marks)**

Three friends move into a rented house on 1 March 2011. When their first water account was received from the Water Corporation, they could see that the water meter was read on 31 May.

(a) How many days had they been in the house when the water meter was read? (1 mark)

Solution
92 days
Specific Behaviours
✓ determines correct number of days

(b) The account shows that the water usage was 530 kL. The table below shows the charges associated with water usage. Calculate the amount they are required to pay for their water usage. (3 marks)

Rates for reading the water meter	
Usage (KL) per year	**Meters read January–June 2011**
First 150 kL	98.2 c/kL
next 200 kL	123.7 c/kL
next 150 kL	133.2 c/kL
next 50 kL	144.3 c/kL
next 400 kL	179.5 c/kL
over 950 kL	196.0 c/kL

Solution
$(150 × 0.982) + (200 × 1.237) + (150 × 1.332) + (30 × 1.443) = $637.79 147.30 + 247.40 + 199.80 + 43.29 = $637.79
Specific Behaviours
✓ identifies correct break up of water usage ✓ determines correct cost of each part ✓ calculates the correct total amount

(c) The Water Corporation has been encouraging all consumers to use 60 L of water less each day to help save our limited water supply. If these three friends had each used 60 L less each day of this billing period, how much would have been saved on their water account? (Note 1000 L = 1 kL) (3 marks)

Solution
Water consumption reduced by 16.56 kL So total consumption is now 513.44 kL Savings 16.56 x 144.3= 2389.608c = $23.90
Specific Behaviours
✓ calculates total water reduction ✓ identifies the water usage for the billing period or recognises the reduction is at the rate of 144.3 c/kL ✓ calculates the total savings in dollars and cents

ACKNOWLEDGEMENTS

Section Two:

Question 7 Data source: Water Corporation. (n.d.). *Water use charges* [Table]. Retrieved April 29, 2011, from www.watercorporation.com.au/A/accounts_rates_metro_res.cfm.

Question 11 Data source: Australian Bureau of Statistics. (n.d.). 6401.0-*Consumer price index, Australia.* Retrieved April 28, 2011, from www.abs.gov.au.

Question 15 Data source: Australian Bureau of Statistics. (2010, November 23). *Asthma: Medications and actions taken* [Table]. Retrieved April 28, 2011, from www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/4364.02007-2008%20(Reissue).

Published by the Curriculum Council of Western Australia
27 Walters Drive
OSBORNE PARK WA 6017

(c) Use the graph to determine the value/s of x that would produce a rectangle of area of 10. (2 marks)

Solution
x = 1.4, x = 6.1 (calculator gives 1.3915047: 6.1084953) Accept correct ordered pairs
Specific Behaviours
✓ determines x = 1.4 ✓ determines x = 6.1

(d) Use your calculator to determine the maximum possible area of this rectangle. Give the answer correct to **two (2)** decimal places. (2 marks)

Solution
Maximum area is 21.13 square units
Specific Behaviours
✓ states maximum area ✓ rounds to two decimal places correctly

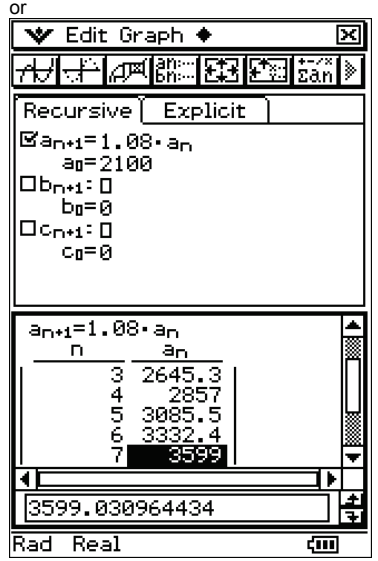
Question 8 **(6 marks)**

The recursive formula $T_{n+1} = 1.08T_n,\ T_0 = 2100$ can be used to calculate the value of an investment compounded annually for n years in the Farmers' Bank of Western Australia.

(a) What is the annual interest rate? (1 mark)

Solution
8% per annum.
Specific Behaviours
✓ determines correct interest rate

(b) Calculate the value of the investment after seven years. (2 marks)

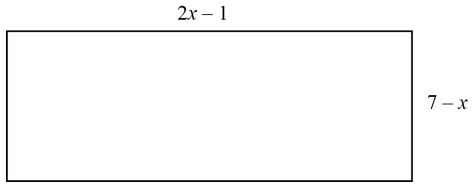
Solution	
$T_7 = (1.08)^7\ T_0$ $= 1.713824\ T_0$ $= \$3599.03$ Accept \$3599	or 
Specific Behaviours	
✓ expresses T_7 in terms of T_0 ✓ calculates of T_7 correctly	✓✓ uses calculator to express T_7

(c) Determine the simple interest rate that would produce the same value for the investment above after an equal time of seven years. (3 marks)

Solution
$SI = P \times R \times t = 2100 \times R \times 7$ $3599.03 = 2100 \times R \times 7 + 2100$ $R = \dfrac{1499.03}{14700}$ $= 0.101975 \approx 10.2\%$
Specific Behaviours
✓ equates result from (b) to sum of principle and simple interest ✓ expresses rate, R as $R = \dfrac{1499.03}{14\ 700}$ ✓ expresses rate as a percentage or decimal (follow through)

Question 19 **(8 marks)**

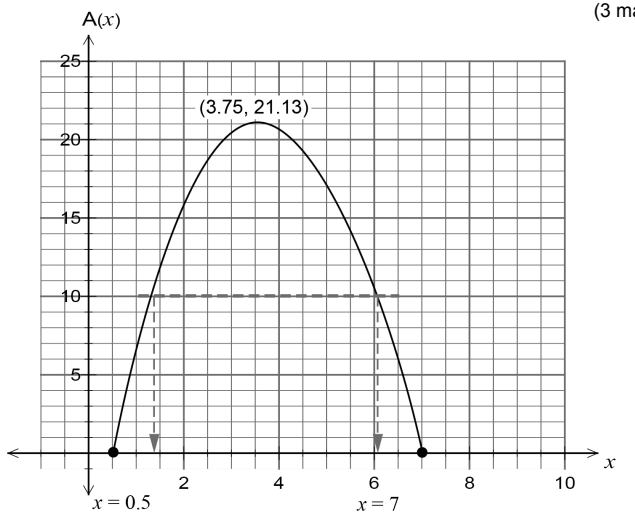
The rectangle shown in the diagram has sides of length $(2x - 1)$ and $(7 - x)$



(a) Give an expression for the area A(x) of this rectangle. (1 mark)

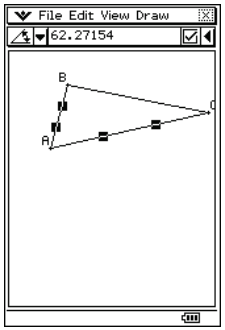
Solution
$(2x - 1)(7 - x)$ or $-2x^2 + 15x - 7$
Specific Behaviours
✓ states expression correctly

(b) On the axes below, sketch the area function from (a), labeling important features. (3 marks)



Solution
As shown on grid
Specific Behaviours
✓ draws shape correctly ✓ indicates intercepts on axis correctly ✓ labels turning point

(b) Given that BD = 2662.7 m, calculate the size of ∠BCD correct to **two (2)** decimal places. (3 marks)

Solution
$\cos\angle BCD = \frac{(1200)^2 + (3000)^2 - (2662.7)^2}{2 \times 1200 \times 3000}$ $= 0.465282$ $\angle BCD = 62.27°$ Or Accept $\angle BCD = \cos^{-1} 0.47$ $= 61.97°$ Or 
Specific Behaviours
✓ applies cosine rule ✓ calculates the of cos ∠ BCD ✓ calculates the size of ∠ BCD to two decimal places or ✓ notes the use of the draw function of the calculator by specifying included angle in a diagram ✓✓ states solution correct to two decimal places

(c) Using trigonometry, calculate the area of the quadrilateral ABCD. (3 marks)

Solution
Area of Δ ABD = ½ (2500) (2800) sin 60° = 3 031 088.914 m^2 Area of Δ BCD = ½ (1200)(3000) sin 62.27° = 1 593 270.2 m^2 (allow variation for rounding) Area of ABCD = 4 624 359.105 m^2
Specific Behaviours
✓ shows correct use of trigonometry formula for area of ΔABD ✓ shows correct use of trigonometry formula for area of ΔBCD ✓ calculates area of ABCD

Question 9 **(5 marks)**

A special six-sided die is rolled 100 times. The results are tabulated below:

Number on top of die	1	2	3	4	5	6
Frequency	14	42	27	11	1	5

(a) Determine the following:

(i) the mean of these data. (1 mark)

Solution
2.58
Specific Behaviours
✓ calculates mean correctly

(ii) the median of these data. (1 mark)

Solution
2
Specific Behaviours
✓ calculates median correctly

(b) Given that the mode of these data is 2, which of the measures mode, mean or median, most accurately indicates the central value of these data? (1 mark)

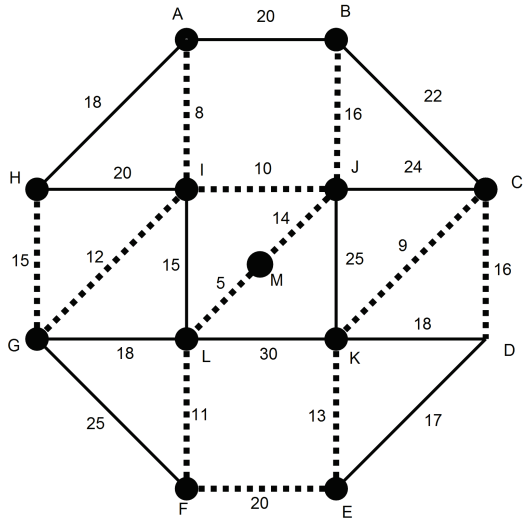
Solution
Mode or median
Specific Behaviours
✓ recognises that either mode or median is the correct answer

(c) Do you consider this die to be fair or biased? Justify your answer. (2 marks)

Solution
Biased. Expect centre to be 3.5 for a fair die (all numbers equally likely) or any reasonable statement identifying the uneven distribution of frequencies.
Specific Behaviours
✓ recognises bias ✓ justifies choice with reason

Question 10 **(5 marks)**

A delivery access network linking 13 depots A, B, C … L, M is to be constructed. The possible connections are given in the following network. The number on each arc represents the cost, in thousands of dollars, of establishing the connection.



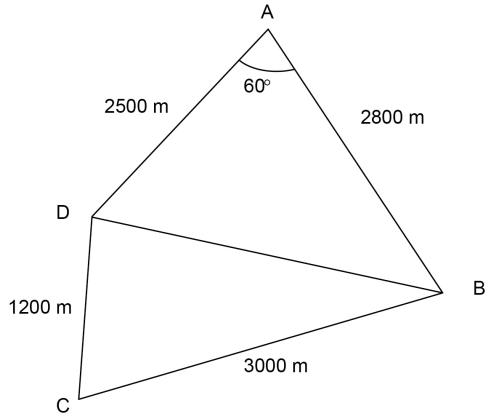
To minimise the cost, the network designers decide to use a minimal spanning tree solution.

(a) Determine the minimum cost and indicate clearly the minimal spanning tree solution on the network above. (3 marks)

Solution
Shown above. Minimum cost is $149 000
Specific Behaviours
Carries through to a degree of accuracy with no cycle evident ✓ shows at least 8 correct connections ✓ shows at least 10 connections correctly ✓ states correct minimum cost

Question 18 **(8 marks)**

The diagram below (not drawn to scale) is a survey plan of a parcel of land ABCD to be developed as an adventure park. The lengths, in metres, of the sides of ABCD are shown, as well as the size of the ∠BAD.



(a) Write an expression for the distance between B and D from the information provided in the diagram. (2 marks)

Solution
$BD^2 = AB^2 + AD^2 - 2AB \times AD\cos\angle DAB$ $BD = \sqrt{(2800)^2 + (2500)^2 - 2(2800)(2500)\cos 60^\circ}$
Specific Behaviours
✓ substitutes values into cosine rule correctly ✓ includes square root in expression

Question 17 **(6 marks)**

The table below produced by the Australian Bureau of Statistics shows the results of interviews with over 2 million asthma sufferers.

National Health Survey: Summary of Results, 2007–2008
Asthma: Medications and actions taken, Persons

Persons with asthma (in '000)	Age group (years)					Males	Females	Persons
	0–14	15–24	25–44	45–64	65 years and over			
Has a written asthma action plan	198.3	40.1	67.1	77.5	46.7	189.3	240.5	429.8
Does not have a written asthma action plan	216.8	278.7	517.4	392.6	214.3	720.6	899.3	1 620.0
Total persons with asthma	415.2	318.8	584.5	470.1	261.0	909.9	1 139.8	2 049.7

Note: All data are rounded.

(a) Determine the probability that an asthma sufferer is in the 65 years and over age group. (2 marks)

Solution
$\frac{261}{2049.7} = 0.127336$
Specific Behaviours
✓ provides correct numerator ✓ provides correct denominator

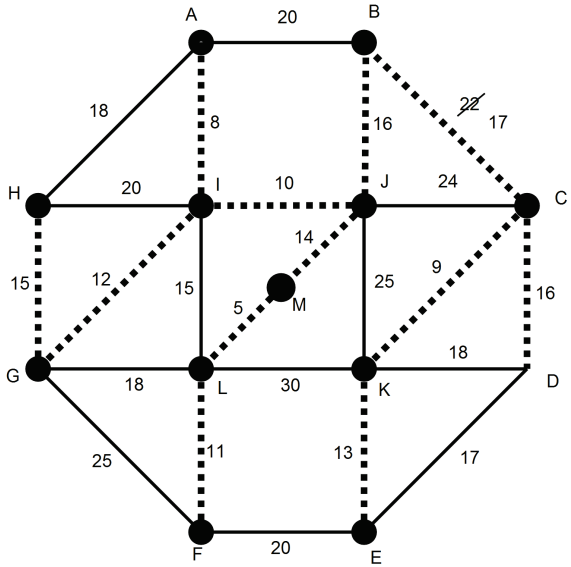
(b) Determine the probability that an asthma sufferer is a male without a written action plan. (2 marks)

Solution
$\frac{720.6}{2049.7} = 0.351564$
Specific Behaviours
✓ provides correct numerator ✓ provides correct denominator

(c) Given that an asthma sufferer is under the age of 25, determine the probability that they have a written action plan. (2 marks)

Solution
$\frac{198.3+40.1}{415.2+318.8} = \frac{238.40}{734} = 0.324796$
Specific Behaviours
✓ provides correct numerator ✓ provides correct denominator

(b) The cost of constructing BC has been overestimated by $5000. By how much does this information change the minimum cost of constructing the network? Justify your solution. (2 marks)



Solution
The link BC replaces the link FE resulting in a saving of $3000.
Specific Behaviours
✓ identifies connections affected by changed conditions ✓ describes the effect of changed conditions: a savings of $3000.

Question 11 **(4 marks)**

The owners of a marron farm in the South West need to estimate the number of marron in a large pond. They released 100 tagged marron into the pond and after one month netted three samples of marron from different parts of the pond. The results were as follows.

	Number of marron netted	Number of tagged marron netted
Sample 1	36	10
Sample 2	26	7
Sample 3	30	8

Use the capture/recapture method to estimate the total number of marron in the pond.

Solution

Sample 1: $\frac{10}{36}=\frac{100}{P}$ $P=\frac{3600}{10}=360$

Sample 2: $\frac{7}{26}=\frac{100}{P}$ $P=\frac{2600}{7}=371.43$ (Accept 371)

Sample 3: $\frac{8}{30}=\frac{100}{P}$ $P=\frac{3000}{8}=375$

Therefore the estimated population = $\frac{360+371.43+375}{3}=368.81$

≈ 368

Number of tagged marron = 25

Or Total number in sample = 92

So $\frac{25}{92}=\frac{100}{P}$ $P=\frac{9200}{25}=368$

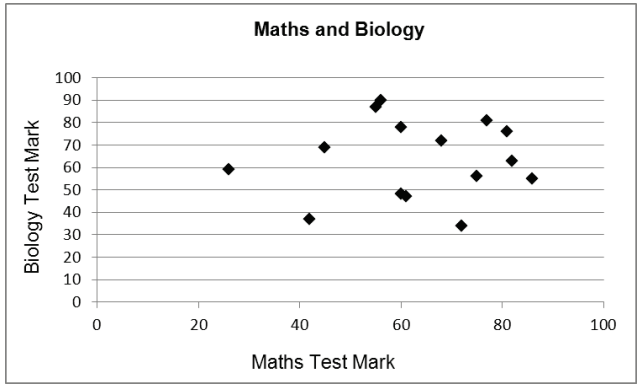
Therefore there are an estimated 368 marron in the pond. (Accept 369)

Specific behaviours

- ✓ uses equal proportions to estimate the population for *one* sample
- ✓ uses equal proportions to estimate the population for *all* samples
- ✓ calculates the mean of the population estimates

Or

- ✓✓ combines samples as one
- ✓ states the estimated population as a whole number value



(a) A new student, Ben, has arrived and was able to complete the English and Maths tests, but missed the Biology test. It has been decided to use one of the other tests to help estimate a Biology test result for Ben. Which test should be used and why? (2 marks)

Solution

English as there is a better fit (from graphs)

Specific Behaviours

- ✓ states that English should be used
- ✓ justifies choice with valid reason noting the better fit of the graph

(b) Ben received test marks of 55% for Maths and 71% for English. Ben's Biology mark is to be estimated using the test selected in (a) above. The equations of the trend lines are:

English (E) and Biology (B): B = 0.98 E + 1.89

Maths (M) and Biology (B): B = 0.06 M + 59.64.

Use your selection from (a) and the appropriate trend line equation to determine Ben's predicted Biology test mark. (2 marks)

Solution

71.47 ≈ 71%

Specific Behaviours

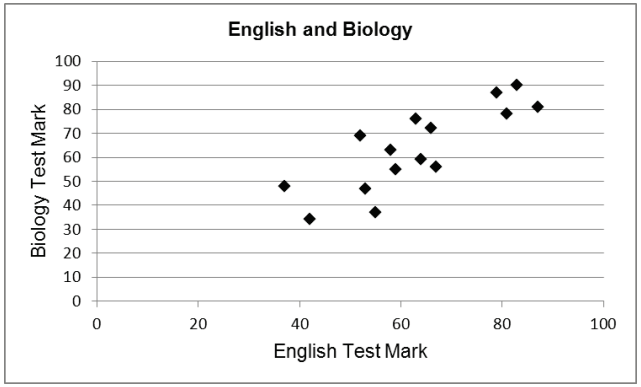
- ✓ uses choice from (a) to calculate predicted mark
- ✓ calculates Biology mark as a percentage

Question 16 **(4 marks)**

A group of 15 students have had their Biology test marks graphed against their English and Maths test marks respectively, as displayed below.

Test Marks (%)

Student	English	Maths	Biology
1	59	86	55
2	42	72	34
3	81	60	78
4	67	75	56
5	63	81	76
6	55	42	37
7	64	26	59
8	52	45	69
9	37	60	48
10	83	56	90
11	87	77	81
12	58	82	63
13	79	55	87
14	66	68	72
15	53	61	47

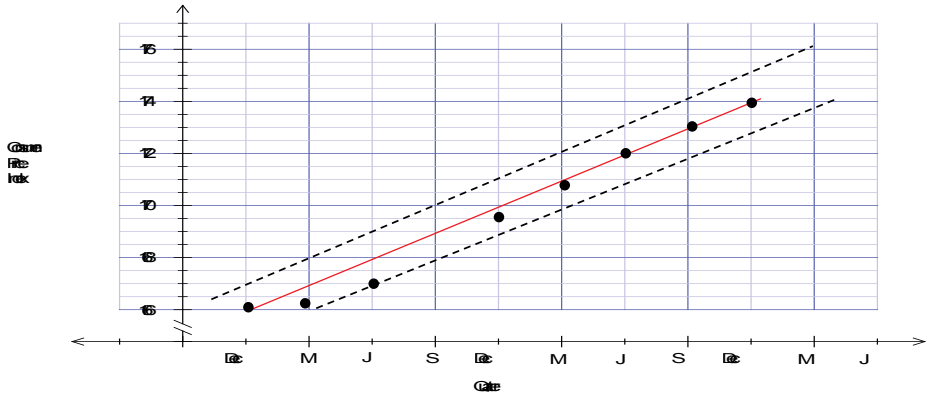


Question 12 **(8 marks)**

The Consumer Price Index (CPI) is measured every three months and is used to help determine inflation rates. The following information has been provided by the Australian Bureau of Statistics.

Quarter	Dec 2008	Mar 2009	June 2009	Sept 2009	Dec 2009	Mar 2010	June 2010	Sept 2010	Dec 2010	Mar 2011	June 2011
Consumer Price Index	166.0	166.2	167.0	Not given	169.5	171.0	172.1	173.3	174.0	Not given	Not given

(a) Plot the above data as a scatterplot on the axes below. (2 marks)



Solution
As shown on grid.
Specific Behaviours
✓ plots at least five points correctly ✓ plots at least seven points correctly

(b) Draw a trend line by eye on the graph above. (1 mark)

Solution
As shown on grid
Specific Behaviours
✓ draws trend line within bounds shown on grid

(c) Use your trend line to predict the CPI for:

(i) September 2009. (1 mark)

Solution
169 ± 1
Specific Behaviours
✓ determines reasonable choice from graph

(ii) June 2011. (1 mark)

Solution
176 ± 1
Specific Behaviours
✓ determines reasonable choice from graph

(d) Comment on the reliability of each of your predictions in (c). (3 marks)

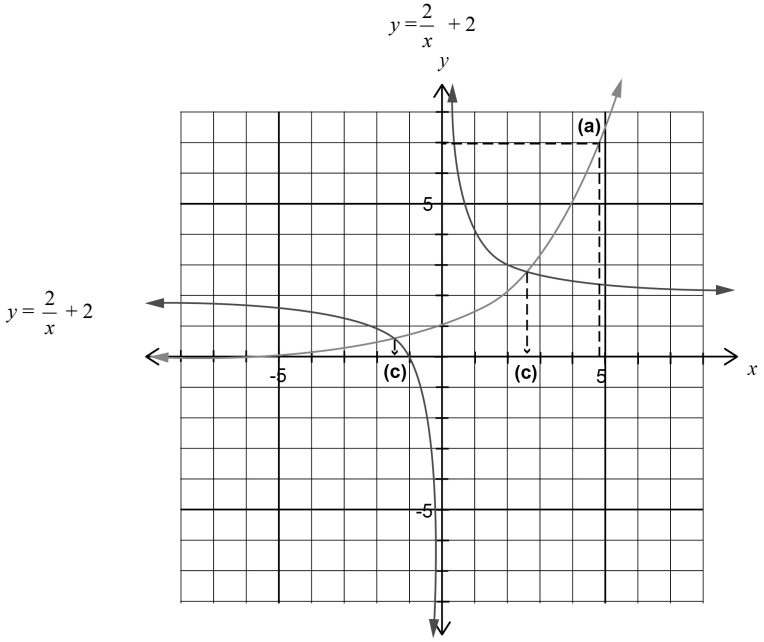
Solution
(c)(i) reliable as this is an interpolation (c)(ii) not reliable as this is an extrapolation
Specific Behaviours
✓ states (i) is reliable ✓ states part (ii) is not reliable ✓ uses terms 'interpolation' and 'extrapolation' or words to that effect in comments of reliability

(c) Solve the equation $1.5^x = \frac{2}{x} + 2$ (2 marks)

Solution
x = –1.39635 ≈ –1.40 x = 2.530771 ≈ 2.53 (Accept 2.5)
Specific Behaviours
✓ solves for x = –1.40 ✓ solves for x = 2.53

Question 15 **(7 marks)**

The graph of $y = 1.5^x$ is drawn on the grid below.



(a) Use the graph to solve $1.5^x = 7$. Show clearly on the graph where you found the solution. (2 marks)

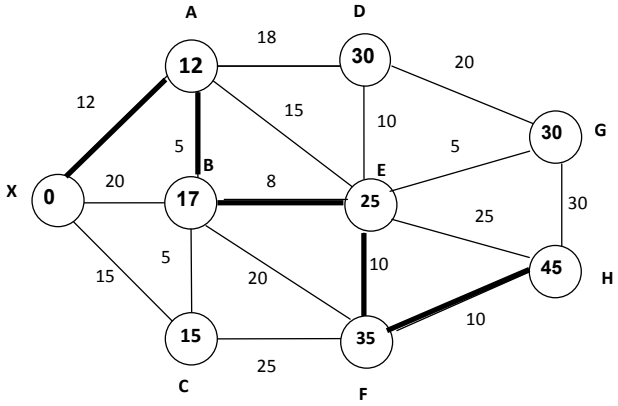
Solution
x = 4.8 (Accept $x = 4.7 - 4.9$)
Specific Behaviours
✓ identifies correct point on graph
✓ states correct solution

(b) On the axes above sketch $y = \frac{2}{x} + 2$. (3 marks)

Solution
On graph. (x intercept at (-1,0))
Specific Behaviours
✓ identifies x-intercept correctly
✓ identifies behavior for x tends to $\pm\infty$ i.e.asymptote and arrows
✓ identifies behavior for y tends to $\pm\infty$ i.e.asymptote and arrows

Question 13 **(7 marks)**

The network below represents the road transport network for a product distributor. The network consists of one distribution centre X and eight retail outlets A, B, C, …, H. The number on each arc represents the distance, in km, between nodes the arc joins.



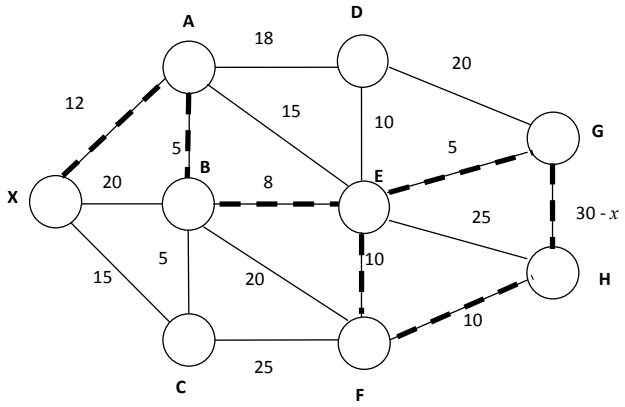
(a) Determine and state the shortest route from the distribution centre X to H. To obtain full marks, numbers must be added to the above diagram showing that an appropriate method has been used. (3 marks)

Solution
X, A, B, E, F, H.
Specific Behaviours
✓ shows at least three correct components
✓ applies method correctly
✓ states route

(b) State the distance of the route found in (a) above. (1 mark)

Solution
45 km
Specific Behaviours
✓ states correct distance

(c) The arc GH currently goes around a creek. A bridge is being proposed that can reduce the distance between G and H by x km. For what value(s) of x will the shortest route from X to H definitely use this bridge? Justify your answer. (3 marks)



Solution
Using the new bridge would result in path X, A, B, E, G, H having a length of (60 – x) km To ensure the use of the bridge we need to have 60 – x < 45 $x > 15$ Also $30 - x \geq 0$ so $x \leq 30$ $15 < x \leq 30$
Specific Behaviours
✓ Identifies the path that includes the bridge ✓ For determining that $x > 15$ ✓ For determining that $x \leq 30$

Question 14 **(5 marks)**

Kate wants to buy a new car and has narrowed her choices down to three models, A, B and C. Kate has used three categories – price, fuel economy and colour – to help her make her choice, and has given a score from 0 to 10 for each car. The higher the score, the more desirable the car. The scores are as follows:

Car model	Price	Fuel economy	Colour
A	7	5	6
B	2	9	8
C	8	3	2

(a) Determine an overall average score out of 10 for each car. (2 marks)

Solution
A: 6 B: 6.3 C: 4.3
Specific Behaviours
✓ calculates two averages correctly ✓ calculate all three averages correctly

(b) Kate has decided that the price factor is five times more important than colour. She has also decided that fuel economy is half as important as price. Determine the weighted score for each car and then state which car she should select. (3 marks)

Solution

Model \ Weights	Price	Fuel	Color	Weighted score
	5	2.5	1	
A	7	5	6	53.50
B	2	9	8	40.50
C	8	3	2	49.50

Weighted means:
A: 6.29
B: 4.76
C: 5.82

Specific Behaviours
✓ calculates two weighted scores/means correctly ✓ calculates all three weighted scores/means correctly ✓ identifies car A is the one she should select