Additional working space

Question number: _____

See next page

Christ Church Grammar School

2019
TEST 4

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

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See next page

Question 11 **(5 marks)**

The random variable X has a probability density function $f(x) = \frac{\sqrt{x}}{18}$ for $0 \leq x \leq 9$.

(a) Find the mean μ and the standard deviation σ for X. (2 marks)

$\mu = \int_0^9 x \times \frac{\sqrt{x}}{18}\, dx = 5.4$ ✓ (mean)

$\sigma^2 = \int_0^9 (x-5.4)^2 \times \frac{\sqrt{x}}{18}\, dx = 5.554 \left(\frac{972}{175}\right)$

$\therefore \sigma = 2.3568 \left(\frac{18\sqrt{21}}{35}\right)$ ✓ (σ)

(b) Find the mean μ for T if $T = 2X + 5$. (1 mark)

$\mu_T = 2(5.4) + 5$

$\mu_T = 15.8 \left(\frac{79}{5}\right)$ ✓

(c) Find the standard deviation σ for P if $P = \frac{X-\mu}{\sigma}$. (2 marks)

$\sigma_P = \frac{X}{\sigma} - \frac{\mu}{\sigma}$

$\sigma_P = \frac{1}{\sigma} X$ ✓ (USES CHANGE OF SCALE ONLY) $\therefore \frac{18\sqrt{21}}{35} \times \frac{35}{8\sqrt{21}}$

$\sigma_P = 1$ ✓ (ANSW)

5

Question 9 **(2 marks)**

In the case below, state whether or not the sampling method is fair, and if it isn't, state one kind of bias involved.

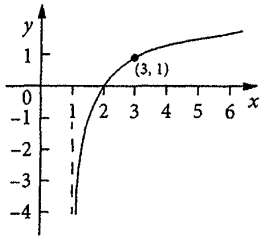
A reality TV show eliminates one contestant each week by having people SMS their choice of who gets eliminated to a particular number each week. They have the system set up so that only one vote is accepted for each mobile number.

Not fair - Bias ✓ (Bias)

Only attracts those interested in the show. ✓ (Reason)
They may influence their friends.

Question 10 **(3 marks)**

The function f has a domain $(1, \infty)$ and its graph is as shown below. Given that $y = \log_a(x + b)$, find the equation of this function.



$y = \log_a(x-1)$ ✓ (shift right) (b. value)

sub in (3,1)

$1 = \log_a(3-1)$

$1 = \log_a 2$

$\therefore a = 2$ ✓ (a value)

hence $\boxed{y = \log_2(x-1)}$ ✓ (equation)

5

See next page

Question 1 **(4 marks)**

Write the following equations without logarithms:

(a) $logA = logb + 2logC$ (2 marks)

(b) $lnM = 3\,ln(a) - 2$ (2 marks)

See next page

Question 2 **(7 marks)**

(a) Calculate each of the following definite integrals, simplifying your answers.

(i) $\int_0^1 x^2 e^{x^3} dx$ (2 marks)

(ii) $\int_1^2 \frac{2x^3+1}{x^4+2x}\ dx$ (3 marks)

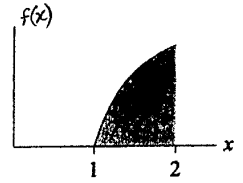
(b) Simplify 2019^{w}, where $w = \frac{1}{\ln 2019}$ (2 marks)

See next page

Question 7 **(3 marks)**

The equation and graph of a probability density functions is given. Find the value of k, clearly showing your method.

$f(x) = \begin{cases} k - \frac{k}{x^2}, & 1 \le x \le 2 \\ 0, & elsewhere \end{cases}$



(limits) ✓ (equates to 1) ✓

$\int_1^2 k - \frac{k}{x^2}\ dx = 1$ (solve on ClassPad)

$\therefore$ $k = 2$ ✓

Question 8 **(2 marks)**

A-Max finance has 49 office staff, 28 store workers and 21 delivery drivers. How many of each should be selected to make a stratified random sample of 10?

$49 + 28 + 21 = 98$ staff in total ✓ (total 98)

$\therefore$ Office Staff : $\frac{49}{98} \times 10$ = 5

Store workers : $\frac{28}{98} \times 10$ = 3

Delivery Drivers : $\frac{21}{98} \times 10$ = 2

10

✓ (All must be correct)

5

See next page

Question 6 **(6 marks)**

The number of customers entering Aldi's per day is modelled by a normal random variable with a mean of 350 and a standard deviation of 18.

(a) Determine the probability that tomorrow there will be:

(i) less than 340 customers. (1 mark)

$P(X < 340) = 0.2893$ ✓

(ii) less than 370 given that there are more than 340 customers. (2 marks)

$P(X < 370 \mid X > 340) = \dfrac{P(340 < X < 370)}{P(X > 340)}$

$= \dfrac{0.57748}{0.71074}$ ✓ correct denominator

$= 0.8125$ ✓

(b) Determine the probability that over the next 5 days there will be between 340 and 370 customers on exactly 3 of those days. Show all distributions and relevant parameters that you use. (3 marks)

Binomial Dist : $Bin(n, p)$ (✓ correct Dist.)

$Bin(5, 0.57748)$ ✓ Parameters

$= 0.3438$ ✓

6

See next page

Question 3 **(4 marks)**

The gradient function of a curve is given by $\frac{dy}{dx} = e^x + \frac{x}{1+x^2} - 1$. Find the equation of the curve given that it passes through the point $(0, -2)$.

Additional working space

Question number: _____

Question 5 **(4 marks)**

It is known that 42% of students studying medicine at UWA have a parent who is a doctor of medicine. A sample of 50 students studying medicine at UWA was taken. Describe the distribution of the sample proportion and estimate the probability that the sample proportion of students with parents who are doctors of medicine is no more than 0.5.

By CLT (sample > 30) ✓ (states Normal)

Distribution ~ Normal with

mean = 0.42 & $\sigma = \sqrt{\frac{0.42(1-0.42)}{50}}$

$\sigma = 0.06980$ ✓ Mean ✓ SD

$\therefore P(\hat{p} \leq 0.50) \approx 0.87413$ ✓ (Prob)

4

Question 4 **(5 marks)**

Suppose that a random variable X has the probability density function with rule:

$$f(x) = \begin{cases} cx, & if\ 0 \le x \le 2 \\ 0, & if\ x > 2\ or\ x < 0 \end{cases}$$

(a) Find the value of c that makes f a probability density function. (2 marks)

Since f is a pdf $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

$\therefore \int_0^2 cx\,dx = 1$ ✓ Integral statement

$\therefore 2c = 1$

$c = \frac{1}{2}$ ✓ Answer.

(b) Find $\Pr(X > 1.5)$. (2 marks)

$\int_{1.5}^{2} 0.5x\,dx$ ✓ (correct limits of integration)

$= 0.4375$ ✓ answer. $\frac{7}{16}$

(c) Find $\Pr(1 \le X \le 1.5)$. (1 mark)

$\int_1^{1.5} 0.5x\,dx = 0.3125$ ✓ $\frac{5}{16}$

5

Christ Church Grammar School

2019
TEST 4

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section
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This Question/Answer Booklet
Formula Sheet (retained from Section One)

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CALCULATOR-ASSUMED 2 MATHEMATICS METHODS Year 12

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Christ Church Grammar School

2019
UNIT TEST 4

MATHEMATICS METHODS Year 12

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Calculator-assumed

Student name _____

Teacher name _____

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(b) Find $\Pr(X > 1.5)$. (2 marks)

(c) Find $\Pr(1 \le X \le 1.5)$. (1 mark)

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The gradient function of a curve is given by $\frac{dy}{dx} = e^x + \frac{x}{1+x^2} - 1$. Find the equation of the curve given that it passes through the point $(0, -2)$.

If $\frac{dy}{dx} = e^x + \frac{x}{1+x^2} - 1$

then $y = e^x + \frac{1}{2}\ln|1+x^2| - x + c$ ✓✓ (Anti-diff)

But when $x = 0$, $y = -2$.

$\therefore -2 = e^0 + \frac{1}{2}\ln 1 - 0 + c$

$-2 = 1 + 0 - 0 + c$

$\therefore c = -3$ ✓ (c-value)

$\therefore$ Equation is $y = e^x + \frac{1}{2}\ln|1+x^2| - x - 3$ ✓ (Equ)

4

Question 2 **(7 marks)**

(a) Calculate each of the following definite integrals, simplifying your answers.

(i) $\int_0^1 x^2 e^{x^3} dx$ (2 marks)

$\frac{1}{3}\int_0^1 3x^2 e^{x^3} dx = \left[\frac{1}{3}e^{x^3}\right]_0^1$ ✓ (Integrates)

$= \frac{1}{3}e - \frac{1}{3}$

$= \frac{1}{3}(e-1)$ ✓ (Answ. must be factorised)

(ii) $\int_1^2 \frac{2x^3+1}{x^4+2x} dx$ (3 marks)

$\frac{1}{2}\int_1^2 \frac{4x^3+2}{x^4+2x} dx$

$= \frac{1}{2}\left[\ln|x^4+2x|\right]_1^2$ ✓ (Integrates)

$= \frac{1}{2}\left[\ln 20 - \ln 3\right]$ ✓ (substitutes)

$= \frac{1}{2}\ln\left(\frac{20}{3}\right)$ ✓ (simplifies)

(b) Simplify $2019^{\frac{1}{\ln 2019}}$ (2 marks)

$y = 2019^{\frac{1}{\ln 2019}}$

$\ln y = \frac{1}{\ln 2019} \cdot \ln 2019$ ✓ (uses log laws)

$\ln y = 1$

$\therefore y = e$ ✓ (simplifies)

7

See next page

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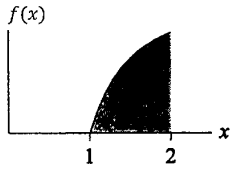

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Question 1 **(4 marks)**

Write the following equations without logarithms:

(a) $\log A = \log b + 2\log c$ (2 marks)

$\log A = \log b + \log c^2$ ✓ Uses log law

$\log A = \log bc^2$

$\therefore A = bc^2$ ✓ Simplifies

(b) $\ln M = 3\ln a - 2$ (2 marks)

$\ln M = \ln a^3 - \ln e^2$ ✓ Uses log law

$\ln M = \ln \frac{a^3}{e^2}$

$M = \frac{a^3}{e^2}$ ✓ Simplifies

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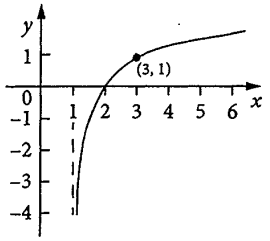

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(b) Find the mean μ for T if $T = 2X + 5$. (1 mark)

(c) Find the standard deviation σ for P if $P = \frac{X-\mu}{\sigma}$. (2 marks)

End of questions

Christ Church Grammar School

2019
UNIT TEST 4

MATHEMATICS METHODS Year 12

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Teacher name - SOLUTIONS -

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