Saigon International College

Deduction for cheating (30%.)

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Saigon International College
Mathematics and Science Department
Year 11 Mathematics Methods (ATAR)
Investigation 2
Semester 2, 2022

Time Allowed: 60 minutes

Name: Chu Minh Đăng....

Question 1 **(8 marks)**

A light year (ly) is defined as the distance that light travels in one year (365.25 days).

$1 \text{ ly} = 9.4607 \times 10^{15} \text{ m}$

$1 m = 100 cm = 1000$ 1cm=10

(a) Use scientific notation to express 1 ly in mm, correct to two significant figures. (2)

9.46×10^{18} mm ✓ (1)

(b) Use the fact that 1 ly is estimated at 9.4607×10^{15} m to determine the number of km travelled in 1 second. Present your answer in standard form correct to the nearest m. (4)

second minute hour day

1 year how many seconds: 1 day: 86400 seconds

1 year: 31 557 600

$\frac{9.4607 \times 10^{12}}{31557600} = 299\,791.492 \text{ km}$ (1)

(c) Given that 1 sec = 10^6 µsec (microseconds), determine the number of microseconds that it would take light to travel 1 km. Express your answer to one significant figure. (2)

1 sec = 1000 000 µs

1s = 300 000 km

$1 km = \frac{1}{299791^6}$

$= 3.335657174 \times 10^{-6} \times 1000000$

$= 3.3 \text{ µs}$ (1)(1)

Question 2 **(9 marks)**

(a) For what values of m (a real number) does $m^{\frac{1}{5}}$ lie from 1 to 10 inclusive,

i.e., $1 \leq m^{\frac{1}{5}} \leq 10$? (2)

1 to 100 000 can take any number from 1 to 100 000

R: 1 – 100000

write as an interval

(b) Given m^5 lies between 1000 and 10 000, i.e. $1000 < m^5 < 10\ 000$, what values can m take if

(i) m is an integer

4, 5, 6

(ii) m is any real number (4)

can take any real number from 3.981 to 6.3096

R: 3.981 – 6.3096

(c) Given $m^{\frac{5}{6}}$ lies between a and b, i.e., $a < m^{\frac{5}{6}} < b$ state the range of values that m can take

(i) expressing your answer in fractional index form $a = m^{\frac{5}{6}}$

(ii) expressing your answer in radical form (3)

Question 3 **(9 marks)**

(a) Given $a^{\frac{p}{q}} \times a^{\frac{m}{n}} = a^{\frac{np+mq}{qn}}$, simplify $a^{\frac{3}{4}} \times a^{\frac{2}{5}}$ (2)

$a^{\frac{3}{4}} \times a^{\frac{2}{5}} = a^{\frac{15+8}{20}} = a^{\frac{23}{20}}$ ✓ ✓ (2)

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

(b) Express $a^{\frac{p}{q}} \times a^{\frac{m}{n}} = a^{\frac{np+mq}{qn}}$ in radical form. (2)

$\sqrt[qn]{a^{np+mq}}$ ✓ ✓ (2)

$\sqrt[qn]{a^{np} \times a^{mq}}$

(c) Determine a simplified expression for $a^{\frac{p}{q}} \div a^{\frac{m}{n}}$ (1)

$a^{\frac{p}{q}} \times \frac{1}{a^{\frac{m}{n}}} = a^{\frac{p}{q}} \times a^{-\frac{m}{n}} = a^{\frac{p}{q}} \times a^{\frac{-m}{n}} = a^{\frac{np-mq}{qn}}$ ✓ (2)

(d) Use your expression from (c) to simplify $a^{\frac{3}{4}} \div a^{\frac{2}{5}}$ (2)

$a^{\frac{np-mq}{qn}} = a^{\frac{15-8}{20}} = a^{\frac{7}{20}}$ ✓ ✓ (2)

(e) Given $a^{\frac{3}{k}} \times a^{\frac{1}{w}} = a^{\frac{23}{30}}$, determine k and w. (2)

$a^{\frac{3}{k}} \times a^{\frac{1}{w}} = a^{\frac{3w+k}{kw}} = a^{\frac{23}{30}}$

$3w+k = \frac{23kw}{30}$

$30(3w+k) = \frac{23kw}{30} \times 30$

$90w+30k = 23kw$

$\frac{90w+30k}{w} = 23k$

Question 4 **(10 marks)**

Exponential functions are to be used to predict the population growth of three different countries.

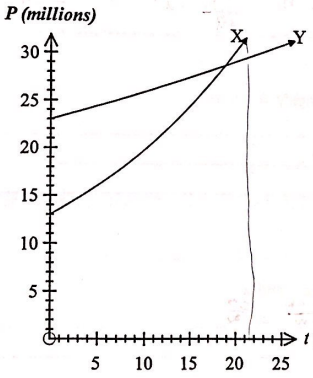
(a) For the first country the formula is $P = 15 \times 1.03^t$ where the current population is 15 million and P represents the population (in millions) after t years.

Determine t when $P = 30$ million. Describe what this value represents. (2)

It represents the population of 30 million by 23.4 years or 24 years

For the other two countries the graphs provided represent the population growth.

Country A: Current population is 23 million and the growth rate is 1.2%
Country B: Current population is 13 million and the growth rate is 4.3%



(b) Which graph represents Country A, X or Y?
Give two reasons for your choice. (3)

Y to represent population

Because we can see from the equation is an exponential but 1.2% is very not steep so it looks like a linear but it's actually a curve.
Y starts out with initial value of 23 million.

(c) The graphs intersect at the point (18.9, 28.8). Describe the values represented by this point. (2)

It represents the time after 18.9 years the population of country B surpasses the population of country A at the time when two country reach 28.8 million people.

(d) Write an equation with one variable t for which the solution is $t = 18.9$ (3)

$13 \times 1.043^t = 23 \times 1.012^t$

End of Investigation questions