



Question/Answer Booklet

Semester Two, 2010

St Mary's Anglican Girls' School



MATHEMATICS
Year 12 3C/3D
Section Two:
Calculator-assumed

Reading time before commencing work: 10 minutes

Time allowed for this section

Working time for this section: 100 minutes

Material required/recommended for this section

To be provided by the candidate

Formula Sheet (retained from Section One)

This Question/Answer Booklet

To be provided by the supervisor

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	8	8	50	40
Section Two: Calculator-assumed	13	13	100	80
				120

Instructions to candidates

1. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
2. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or

year 12 3CD Mathematics	Semester II 2010	Calculator-Assumed
Justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.	3.	It is recommended that you do not use pencil except in diagrams.
Calculator-Assumed	Semester II 2010	year 12 3CD Mathematics
Calculator-Assumed	Semester II 2010	year 12 3CD Mathematics

9. [3 marks]

When air is released from an inflated balloon it is found that the rate of decrease of the volume of the balloon is proportional to the volume of the balloon. This can be represented by

the differential equation $\frac{dv}{dt} = -kv$, where v is the volume (in cm^3), t is the time (in seconds) and k is the constant of proportionality.

- (a) If the initial volume of the balloon is v_0 , find an expression, in terms of k , for the volume of the balloon at time t . (1)

$$V = V_0 e^{-kt}$$

- (b) If it takes 20 seconds for the volume to halve, find k . (2)

$$\frac{V_0}{2} = V_0 e^{-k(20)}$$

$$\frac{1}{2} = e^{-20k}$$

$$0.5 = e^{-20k}$$

Solve:

$$k = 0.0347 \text{ (4 dp)}$$

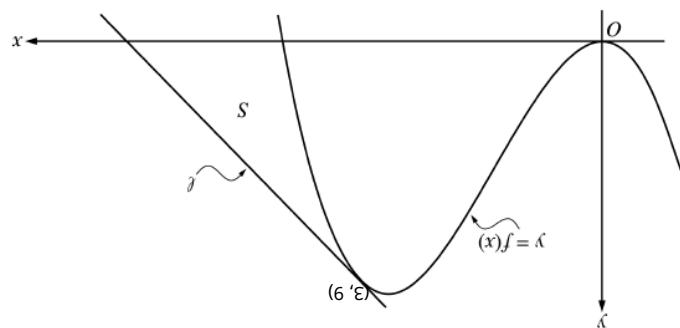
Additional Working Space

(3)

(a) Show that f has equation $y = 18 - 3x$

$$\begin{aligned}
 & y = -3x + 18 \\
 & 18 = c \\
 & y = -3(3) + c \\
 & y = -3x + c \\
 & -3 = c \\
 & f(3) = 8(3) - 3(3^2) \\
 & f(x) = 8x - 3x^2
 \end{aligned}$$

Let f be the function given by $f(x) = 4x^2 - x^3$, and let S be the tangent to the graph of f .



10. [7 marks]

Additional Working Space

Let S be the region bounded by the graph of f , the line l , and the x -axis, as shown on the previous page.

(b) Find the area of S .

(4)

$$x\text{-int of } f = (4, 0) \quad x\text{-int of } l = (6, 0)$$

$$\begin{aligned} \text{Area of } S &= \int_3^4 (18 - 3x - (4x^2 - x^3)) dx \\ &\quad + \int_4^6 (18 - 3x) dx \\ &= 7.916 \end{aligned}$$

Additional Working Space

$$\frac{13}{24} = \frac{13/4}{3 \times 8}$$

(2)

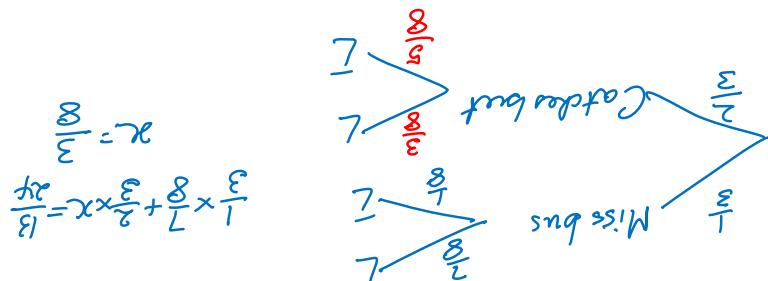
(ii) Josie misses her bus, given that she is late for school.

$$\frac{2}{5} = \frac{4}{10} = \frac{3}{5} \times \frac{2}{3}$$

(1)

(i) Josie catches the bus and is not late for school.

(b) Find the probability that



(3)

(a) Represent this information on a tree diagram.
(b) Show all probabilities on the branches of the tree.

$$P(L) = \frac{2}{4}$$

Let M be the event "she misses the bus" and L the event "she is late for school".Josie travels to school on a bus. On any day, the probability that Josie will miss the bus is $\frac{3}{7}$. If she misses the bus, the probability that she will be late to school is $\frac{8}{7}$.Josie travels to school on a bus. On any day, the probability that Josie will miss the bus is $\frac{3}{7}$. If she misses the bus, the probability that she will be late to school is $\frac{8}{7}$.

11. [9 marks]

Additional Working Space
End of Paper $\approx 96 \text{ or } 97$

$$n = 96.06$$

$$10.204269 = \frac{\sqrt{n}}{100}$$

The cost for each bus journey is \$1. Josie goes to school on Monday and Tuesday. If she misses the bus she must walk to school. She always gets a lift home from school.

- (c) Complete the probability distribution table. (2)

1 miss

X (Cost in dollars)	0	1	2
$P(X)$	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$

- (d) What is the expected cost for Josie's bus trips on Monday and Tuesday? (1)

$$\begin{aligned} E(X) &= 0 \times \frac{1}{9} + 1 \times \frac{4}{9} + 2 \times \frac{4}{9} \\ &= \$1.33 \end{aligned}$$

- (d) When is the object speeding up? (2)

$$1 < t < 3, \quad t > 5$$

21. [4 marks]

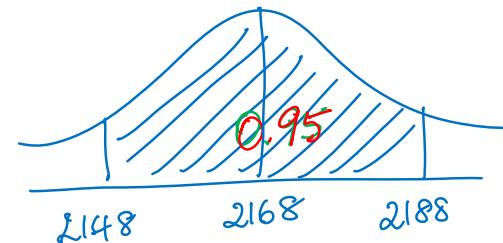
A chicken farmer wishes to find a confidence interval for the mean weight of his chickens. He therefore randomly selects n chickens and weighs them. Based on his results, he obtains the following 95% confidence interval.

[2148 grams, 2188 grams]

The weights of the chickens are known to be normally distributed with a standard deviation of 100 grams.

Find the value of n .

$$\begin{aligned} \mu &= \frac{2148 + 2188}{2} \\ &= 2168 \\ S &= \frac{100}{\sqrt{n}} \end{aligned}$$



$$P(2148 < X < 2188) = 0.95$$

$$\text{Solve: } S = 10.204269$$

The cell requires 14 batteries.

$$\therefore x = 14$$

$$x \geq 13.425 \dots$$

$$1 - 0.8^x \leq 0.95$$

(2)

(3)

the probability of the satellite still operating at the end of one year is at least 0.95.

(b) For a satellite with n solar cells, find the smallest number of solar cells required so that

$$P(x \geq 7) = P(x \leq 10) = 0.8$$

(2)

(a)

For a satellite with ten solar cells, find the probability that more than 6 cells have

failed within one year.

A satellite relies on solar cells for its power and will operate provided that at least one of the cells is working. Cells fail independently of each other, and the probability that an individual cell fails within one year is 0.8.

12. [9 marks]

$t < 2.2$ seconds

(1)

$$t = 1 \text{ sec}, t = 5 \text{ sec}$$

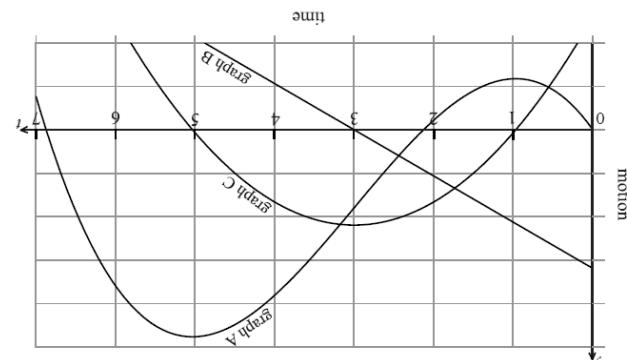
(1)

(b) When is the object turning around?

Function	Graph
displacement	A
acceleration	B

(1)

(a) Complete the following table by noting which graph A, B or C corresponds to each function.



The following diagram shows the graphs of the horizontal displacement, velocity and acceleration of a moving object as functions of time, t .

20. [5 marks]

$$\frac{45}{1} = \underline{\underline{8145060}}$$

(c) What is the probability that your chosen 6 numbers win the Lotto draw?

(1)

(1)

19. [5 marks]

Suppose that the random number generator on your calculator was programmed to print whole numbers from 1 to 45 inclusive.

The lifetime of a particular component of a solar cell is Y years, where Y is a continuous random variable with probability density function

$$f(y) = \begin{cases} 0 & \text{when } y < 0 \\ 0.5e^{-y/2} & \text{when } y \geq 0. \end{cases}$$

- (c) Find the probability that a given component fails within six months.

$$\begin{aligned} P(Y < \frac{1}{2}) &= \int_0^{\frac{1}{2}} 0.5e^{-\frac{y}{2}} dy \\ &= 0.2212 \quad (4dp) \end{aligned} \quad (2)$$

Each solar cell has three components which work independently and the cell will continue to run if at least two of the components continue to work.

- (d) Find the probability that a solar cell fails within six months.

$$B \sim (3, 0.2212) \quad (3)$$

$$\begin{aligned} P(\text{cell fails}) &= P(X=2) + P(X=3) \\ &= 0.1143 + 0.0108 \\ &= 0.1251 \end{aligned}$$

- (a) If this was done many times, and probabilities calculated, what distribution would you expect? Explain your reasoning. *uniform*

(1)

Each no. has same chance of occurrence

The mean of this distribution is 23 and the standard deviation is 12.99 (2 d.p.).

Suppose now that the calculator is programmed to simulate the selection of 6 numbers from 1 – 45 (without repetition) as required to win in Weekend Lotto.



It is done 100 times and the mean of the 6 numbers is calculated each time.

- (b) To what distribution would the means approximate? State the statistics that define this distribution. *normal*

(3)

$$\bar{x} = 23$$

$$\sigma = \frac{12.99}{\sqrt{6}}$$

5.30314...

of the container. Now there will be

3 consecutive nos.

$$\cancel{(n)(n+1)(n+2)} =$$

$$\cancel{(1-n)(n+1)(n-1)} =$$

$$2(n^3 - n) = 2n(n^2 - 1)$$

(= 12k)

(b) Propose a number divisible by 12 using algebraic notation and then prove it.

(1)

12

be divisible by?

(a) When you follow the instructions, what is the largest number that the answer must

- Double your answer.

- Subtract the original number.

- Cube it.

- Choose any counting number greater than 1.

Follow these instructions:

13. [6 marks]

$$\text{Ans} = -5.89 \text{ cm}^3/\text{s}$$

$$= \frac{4}{\pi r^2} \times -0.3$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$= \frac{4}{\pi r^2} \frac{dV}{dh}$$

$$= \frac{\pi r^3}{12}$$

$$= \frac{3}{4} \pi r^3$$

$$= \frac{3}{4} \pi \left(\frac{r}{2}\right)^3$$

$$V = \frac{3}{4} \pi r^2 h$$

when $h = 5 \text{ cm}$. Indicate units of measure.

(4)

(b) Find the rate of change of the volume of water in the container, with respect to time,

(4)

Find the rate of change of the volume of water in the container, with respect to time,

- be an even no. and one divisible by 3.
 \therefore divisible by $2 \times 3 = 6$
 and with the 2 as a common factor
 it must be divisible by 12.
14. [4 marks]

The marginal cost (\$) for a refrigerator company producing x refrigerators per day is:

$$C'(x) = \frac{100}{\sqrt{x}} + 150$$

(a) Find $\int_{100}^{125} C'(x) dx$. (1)

$\$ 3986.07$

- (b) Interpret your answer to (a) in the context of this problem. (1)

This is the extra cost associated with making 125 rather than 100 fridges

- (c) Show how to use the incremental formula to approximate the value you found in (a). (2)

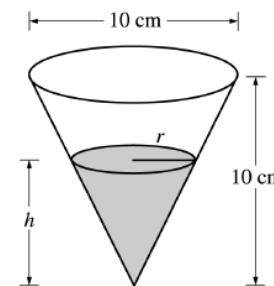
$$\frac{dc}{dx} \approx \frac{\Delta c}{\Delta x}$$

$$\frac{100}{\sqrt{x}} + 150 \approx \frac{\Delta c}{25}$$

$$x=100: (\frac{100}{10} + 150) \times 25 = \Delta c$$

OTHER

18. [6 marks]



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so the depth h is changing at the constant rate of $\frac{-3}{10}$ cm/h.

- (a) Find the volume V of the water in the container when $h = 5$ cm. (2)

Indicate units of measure: $\frac{1}{3}\pi \times 2.5^2 \times 5$

$$\begin{aligned} r &= 2.5 & V &= \\ h &= 5 & &= 32.72 \text{ cm}^3 \end{aligned}$$

17. [6 marks]

$\therefore \Delta TAC \text{ isosceles}$

$T_A = T_C \quad (\text{equal base angles})$

$\therefore TA = TC = 90 - x$

$\therefore TAC = 90 - x$

$\therefore \overline{BAC} = x \quad (\text{exterior angle of triangle})$

$\therefore \overline{BAC} = 90 - x$

$\therefore \overline{BAC} = 90 - (90 - x) - (90 - x)$

$\therefore \overline{BAC} = 90 - 90 + x - 90 + x$

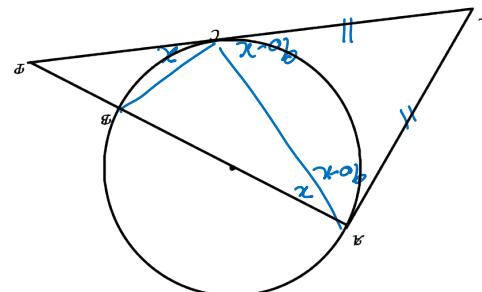
$\therefore \overline{BAC} = 2x$

Proof: Let $\angle BCP = x$

Extension to diagram: Join AC, BC

$\therefore ATC = 90^\circ$

AB is a diameter of the circle. The tangents at A and C meet at T . The lines TC and AB are produced to meet at P .

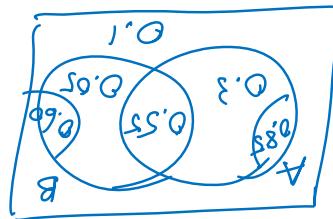


(3)

There was a team of ten people working on the building, including three electricians and two plumbers. The architect called a meeting with five of the team, and randomly selected people to attend. Calculate the probability that exactly two electricians and one plumber were called to the meeting.

(b)

$$\frac{3^2 C_2}{5^2 C_2} = \frac{60}{252}$$



(3)

(iii) All the materials arrive on time. Find the probability that the building will be completed on time.

compleeted on time.

$$P(A \cap B) \neq P(A) \times P(B) \therefore \text{not independent.}$$

$$P(A) \times P(B) = 0.85 \times 0.6 = 0.51$$

$$P(A \cap B) = 0.55$$

(2)

(i) Show that events A and B are not independent.

(a) At a building site the probability, $P(A)$, that all materials arrive on time is 0.85. The probability, $P(B)$, that the building will be completed on time is 0.60. The probability that the materials arrive on time and that the building is completed on time is 0.55.

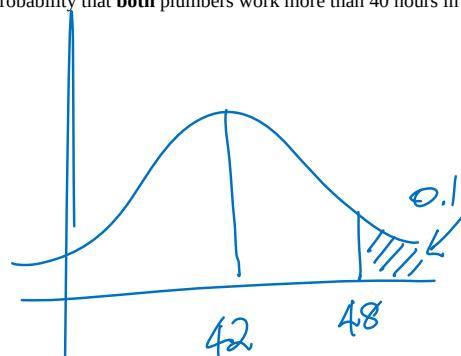
15. [12 marks]

Year 12 3CD Mathematics Semester II 2010 Semester II 2010 Calculator-Assumed Calculator-Assumed

4 < 4

- (c) The number of hours per week the people in the team work is normally distributed with a mean of 42 hours. 10% of the team work 48 hours or more a week. Find the probability that **both** plumbers work more than 40 hours in a given week.

(4)



$$P(X > 48) = 0.1$$

$$\sigma = 4.68$$

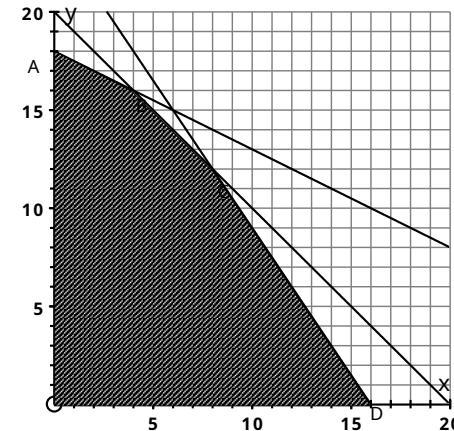
$$P(X > 40) = 0.6653772$$

$$P(\text{both} > 40) = (0.6653\ldots)^2 \\ = 0.4427 \quad (\text{4dp})$$

16. [4 marks]

The graph below shows the region satisfying the following inequalities:

$$\begin{aligned}x &\geq 0 \\y &\geq 0 \\x+y &\leq 20 \\x+2y &\leq 36 \\3x+2y &\leq 48\end{aligned}$$



The vertices of this region are A(0,18), B(4, 16), C(8,12) and D(16,0).

The profit (P) on each unit of x is \$6 and on each unit of y is \$5.50.

Point C maximises this profit.

By how much can the profit on each unit of y decrease before C is no longer the optimal solution?
Let $P = 6x + 5.50y$

$$\begin{aligned}P(16,0) &> P(8,12) \\96 &> 48 + 12b \\48 &> 12b\end{aligned}$$

It can decrease by \$1.50