

Note: All part questions worth more than 2 marks require working to obtain full marks.

Formula sheet provided: no but formulae stated on page 2

Task weighting: 13%

Marks available: 40 marks

WACE examinations

A4 paper SINGLE SIDE, and up to three calculators approved for use in the

Drawing instruments, templates, notes on one unruled sheet of

Special items:

Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Standard items:

Up to 3 classspads/calculators

Number of questions: 7

Working time allowed for this task: 40 mins

Reading time for this test: 5 mins

Task type: Response/investigation

Student name: _____ Teacher name: _____

Course Specialist Test 2 Year 12

PERTH MODERN SCHOOL



Independent Public School

Exceptional Schooling. Exceptional Students.

Working out space

Useful formulae**Complex numbers**

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = z = \sqrt{a^2 + b^2} = r$	$\text{Arg}(z) = \theta, \tan \theta = \frac{b}{a}, -\pi < \theta \leq \pi$
$ z_1 z_2 = z_1 z_2 $	$\left \frac{z_1}{z_2} \right = \frac{ z_1 }{ z_2 }$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z \bar{z} = z ^2$	$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{ z ^2}$
$\overline{z_1 + z_2} = \overline{z}_1 + \overline{z}_2$	$\overline{z_1 z_2} = \overline{z}_1 \overline{z}_2$
Polar form	
$z = a + bi = r(\cos \theta + i \sin \theta) = r \text{cis } \theta$	$\bar{z} = r \text{cis } (-\theta)$
$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$
$\text{cis}(\theta_1 + \theta_2) = \text{cis} \theta_1 \text{cis} \theta_2$	$\text{cis}(-\theta) = \frac{1}{\text{cis} \theta}$
De Moivre's theorem	
$z^n = z ^n \text{cis}(n\theta)$	$(\text{cis } \theta)^n = \cos n\theta + i \sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}} \left(\cos \frac{\theta + 2\pi k}{q} + i \sin \frac{\theta + 2\pi k}{q} \right), \quad \text{for } k \text{ an integer}$	

Working out space

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Length of arc = $r\theta$
$a^2 = b^2 + c^2 - 2bc \cos A$	Area of segment = $\frac{1}{2} r^2 (\theta - \sin \theta)$
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	Area of sector = $\frac{1}{2} r^2 \theta$
Identities	
$\cos^2 x + \sin^2 x = 1$	$1 + \tan^2 x = \sec^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\cos 2x = \cos^2 x - \sin^2 x$ $= 2 \cos^2 x - 1$ $= 1 - 2 \sin^2 x$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\sin 2x = 2 \sin x \cos x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
$\cos A \cos B = \frac{1}{2} (\cos(A-B) + \cos(A+B))$	$\sin A \cos B = \frac{1}{2} (\sin(A+B) + \sin(A-B))$
$\sin A \sin B = \frac{1}{2} (\cos(A-B) - \cos(A+B))$	$\cos A \sin B = \frac{1}{2} (\sin(A+B) - \sin(A-B))$

Consider complex numbers $z \propto w$. It is known that:

$$|z|=10, \operatorname{Arg}(z)=\theta \quad \text{where } \frac{\pi}{3} < \theta < \frac{4\pi}{3}$$

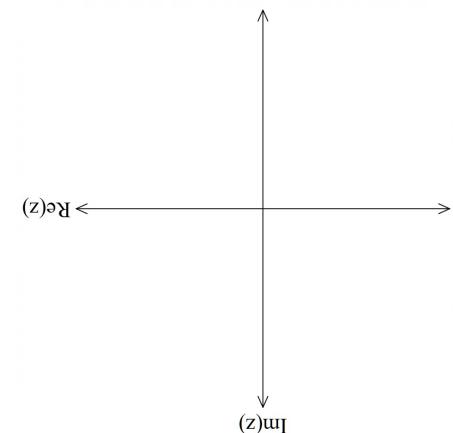
Consider the inequality $|3x - 7| \leq a$ which is only true for $b \leq x \leq 9$ where $a \neq b$ are constants.

Q7 (3 marks)

Determine the values of $a \neq b$.

$$w=z+k \text{ such that } \operatorname{Arg}(w)=\pi - \theta \quad \text{where } \operatorname{Im}(k)=0, k < 0$$

a) Represent this information on the Argand Diagram below.



b) Determine a simplified expression for k in terms of θ . Justify your answer.

Determine if the ships will collide and if they do, determine the time and position of this collision.

$$\begin{aligned} t_B &= 6 \left[\frac{\text{km}}{\text{km/h}} \right] = 6 \left[\frac{\text{km}}{4} \right] \text{ km/h at } 11:30 \text{ am} \\ t_A &= 2 \left[\frac{\text{km}}{\text{km/h}} \right] = 2 \left[\frac{7}{2} \right] \text{ km/h at } 11:00 \text{ am} \end{aligned}$$

Q2 (4 marks)

Consider two ships A & B moving with constant velocities and the following noted position vectors.

$$|z|=10, \operatorname{Arg}(z)=\theta \quad \text{where } \frac{\pi}{3} < \theta < \frac{4\pi}{3}$$

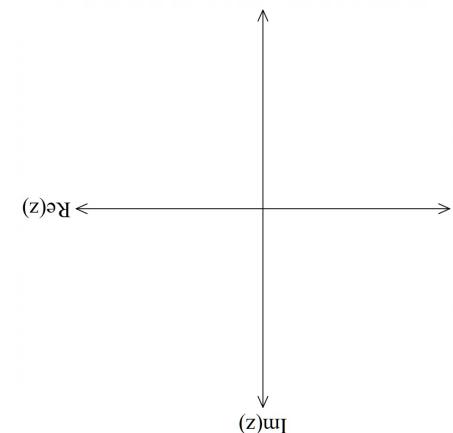
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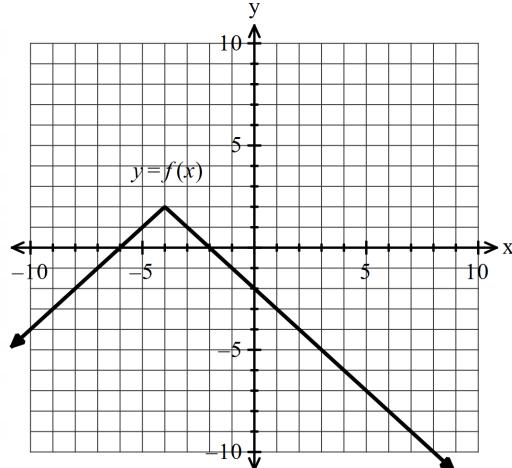
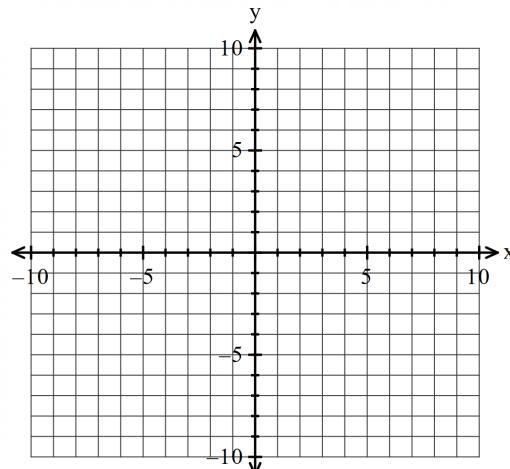
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Q2 (4 marks)

Consider two ships A & B moving with constant velocities and the following noted position vectors.

Q3 (2, 3 & 3 = 8 marks)

Consider the function $y = f(x)$ below.a) Sketch $|f(x)|$ on the axes below.

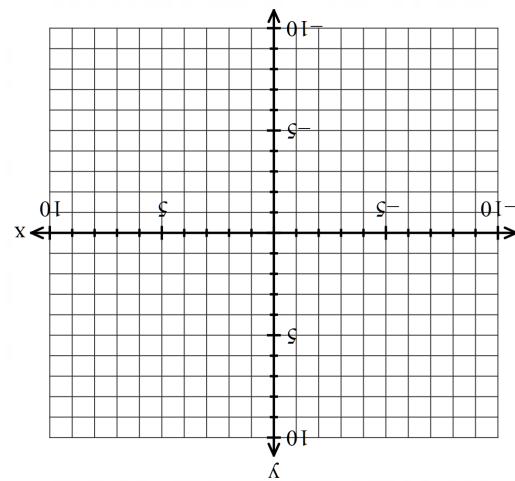
Q6 (5 marks)

$$r = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

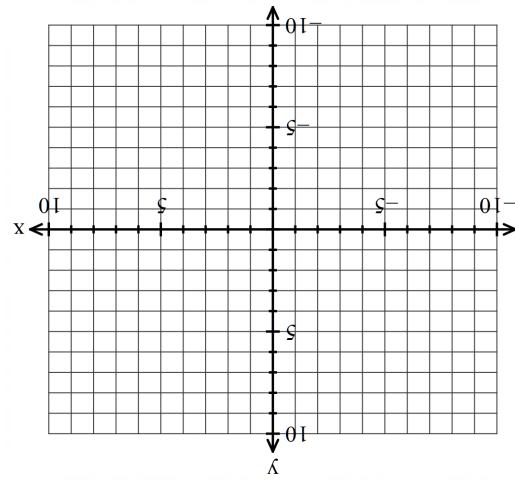
Consider the line and the circle $\left| r - \begin{pmatrix} 3 \\ \alpha \end{pmatrix} \right| = 9$ where α is a constant.

Determine all possible values of α such that:

- i) The line will be a tangent to the circle.
- ii) The line crosses the circle at two points.
- iii) The line will never meet the circle.



c) Sketch $f(x)$ on the axes below.



b) Sketch $f(-|x|)$ on the axes below.

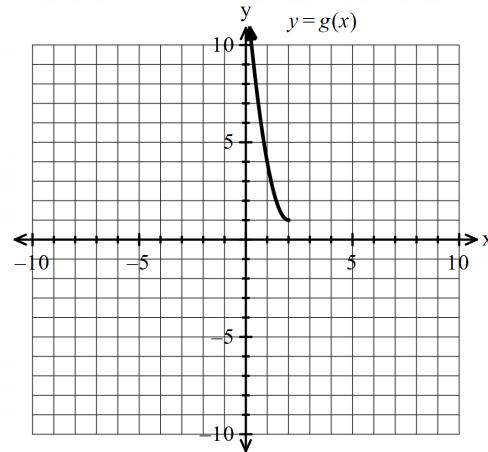
Consider two moving objects P & Q which at time $t = 0$ seconds have the following positions and constant velocities.

P : $\begin{pmatrix} 7 \\ -8 \end{pmatrix}$ m $V_p = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ m/s

Q : $\begin{pmatrix} 11 \\ -6 \end{pmatrix}$ m $V_q = \begin{pmatrix} 10 \\ -3 \end{pmatrix}$ m/s

Determine the minimum distance between them using vectors and the time that this occurs.

Q4 (2, 3, 1 & 3 = 9 marks)

Consider $g(x) = 3x^2 - 12x + 13$ for $x \leq 2$ which is plotted below.a) Sketch $g^{-1}(x)$ on the axes above.b) Determine the rule for $g^{-1}(x)$ showing full working and the domain.

Q4 cont-

c) Determine $g^{-1} \circ g(x)$.d) Determine all value(s) of x such that $g(x) = g^{-1}(x)$. Show reasoning for full marks.