Mathematics Department
Year 11 Mathematics Methods

Semester 1 Examination, 2020

Question/Answer Booklet

Christ Church
Grammar School

MATHEMATICS METHODS

UNIT 1

Section One:
Calculator Free

Solutions

Student Name: _____

Time allowed for this section

Reading time before commencing work:	five minutes
Working time:	fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	14	14	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the Christ Church Grammar School reporting and assessment policy. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

Question number: _____

Section One: Calculator-free   **35% (53 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1   **(5 marks)**

The point $M(-2,5)$ is the midpoint of point $A(-6,3)$ and point B.

(a)   Determine the coordinates of point B.   (2 marks)

Solution
$\dfrac{-6+x}{2} = -2 \Rightarrow x = 2$
$\dfrac{y+3}{2} = 5 \Rightarrow y = 7$
$B(2,7)$
Specific behaviours
✓ correct x-coordinate
✓ correct y-coordinate

(b)   Determine the equation of the straight line that passes through point $C(4,-1)$ and is perpendicular to the line through points A and M.   (3 marks)

Solution
$m_{AM} = \dfrac{5-3}{-2-(-6)} = \dfrac{1}{2}$
$m_\perp = -1 \div \dfrac{1}{2} = -2$
$y - (-1) = -2(x - 4)$
$y = -2x + 7$
Specific behaviours
✓ gradient of AM
✓ perpendicular gradient
✓ correct equation

Question 8   **(8 marks)**

Solve the following equations for x.

(a) $x^2 + 20x - 21 = 0$.   (2 marks)

Solution
$x^2 + 20x - 21 = (x-1)(x+21)$
Hence $x = 1$, $x = -21$.
Specific behaviours
✓ factorises
✓ states both solutions

(b) $(x - 1)^2 - 4 = 2x - 3$.   (3 marks)

Solution
$x^2 - 2x + 1 - 4 = 2x - 3$
$x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0$, $\quad x = 4$
Specific behaviours
✓ expands and equates to zero
✓ factorises
✓ states both solutions

(c) $x^3 - 2x^2 - 11x + 12 = 0$.   (3 marks)

Solution
$x^3 - 2x^2 - 11x + 12\big|_{x=1} = 13 - 13 = 0$
$x^3 - 2x^2 - 11x + 12 = (x-1)(x^2 - x - 12)$
$= (x-1)(x+3)(x-4)$
$x = -3, \quad x = 1, \quad x = 4$
Specific behaviours
✓ indicates linear factor
✓ factorises
✓ states all solutions

Question 2 (5 marks)

Functions f and g are defined by $f(x) = 4x^2 - 4x + 5$ and $g(x) = 2x^2 - 8x + 6$.

(a)   Determine the discriminant of f and the discriminant of g.   (2 marks)

Solution
$\Delta_f = (-4)^2 - 4(4)(5) = -64$
$\Delta_g = (-8)^2 - 4(2)(6) = 16$
Specific behaviours
✓ discriminant of f
✓ discriminant of g

(b)   State, with justification, which function has no zeros and determine all zeros of the other function.   (3 marks)

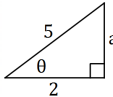| Solution |
|---|
| f has no zeroes as $\Delta_f < 0$. |
| |
| $g(x) = 0$ when |
| $\qquad x = \dfrac{-(-8) \pm \sqrt{16}}{2(2)}$ |
| $\qquad x = 2 \pm 1$ |
| |
| g has zeros when $x = 1, x = 3$. |
| **Specific behaviours** |
| ✓ states f or $f(x)$ has no zeroes |
| ✓ indicates appropriate method to find zeros |
| ✓ both zeros of g or $g(x)$ |

Question 7 (8 marks)

(a)   Determine an exact value for $\sin 80° \cos 110° - \cos 80° \sin 110°$.   (2 marks)

Solution
$\sin 80° \cos 110° - \cos 80° \sin 110° = \sin(80° - 110°)$
$= \sin(-30°)$
$= -\dfrac{1}{2}$
Specific behaviours
✓ uses double angle formula
✓ states exact value

(b)   Determine all possible values of $\tan \theta$ when $\cos \theta = \dfrac{2}{5}$.   (3 marks)

Solution
$a = \sqrt{25 - 4} = \sqrt{21}$
Note that θ can be in quadrant 1 or 4.
In Q1: $\tan \theta = \dfrac{\sqrt{21}}{2}$, in Q4 $\tan \theta = -\dfrac{\sqrt{21}}{2}$
Specific behaviours
✓ relevant use of right triangle
✓ one correct value
✓ both correct values

(c)   Determine an exact value for $\cos 105°$.   (3 marks)

Solution
$\cos 105° = \cos(45° + 60°)$
$\qquad = \cos 45° \cos 60° - \sin 45° \sin 60°$
$\qquad = \dfrac{\sqrt{2}}{2} \times \dfrac{1}{2} - \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{3}}{2}$
$\qquad = \dfrac{\sqrt{2} - \sqrt{6}}{4}$ or $\dfrac{\sqrt{2}}{4} - \dfrac{\sqrt{6}}{4}$
Specific behaviours
✓ uses double angle formula
✓ uses correct exact values
✓ simplifies to obtain final value

Question 3   (4 marks)

The expansion of $(x+1)^9$ is

$x^9 + 9x^8 + 36x^7 + 84x^6 + 126x^5 + 126x^4 + 84x^3 + 36x^2 + 9x + 1$.

(a) Determine the number of combinations of 6 objects taken from a set of 9 distinct objects.   (1 mark)

Solution
x^3 (or x^6) coefficient:
$\binom{9}{6} = 84$
Specific behaviours
✓ correct number

(b) Consider the simplified expansion of $(x+1)^{10}$. The first four terms in descending powers of x are

$x^{10} + ax^9 + bx^8 + cx^7$.

(i) State the number of terms in the complete simplified expansion.   (1 mark)

Solution
11 terms.
Specific behaviours
✓ correct number

(ii) Determine the value of each of the coefficients a, b and c.   (2 marks)

Solution
Using coefficients from expansion of $(x+1)^{10}$ and properties of Pascal's triangle:
$a = 1 + 9 = 10$
$b = 9 + 36 = 45$
$c = 36 + 84 = 120$
Specific behaviours
✓ at least 2 correct
✓ all three correct

Question 6   (8 marks)

(a) The variable P is inversely proportional to the variable t, so that when $t = 2.4$, $P = 20$.

(i) Explain how P will change as t decreases.   (1 mark)

Solution
As t decreases P will proportionally increase.
Specific behaviours

(ii) Determine t when $P = 6$.   (2 marks)

Solution
$P \times t = k \Rightarrow k = 2.4 \times 20 = 48$
$6t = 48 \Rightarrow t = 8$
Specific behaviours
✓ indicates appropriate method
✓ correct value

(b) Part of the graph of $y = \dfrac{a}{x-3}$ is drawn below.

(i) Determine the value of a.   (2 marks)

Solution
$(2, -3) \Rightarrow -3 = \dfrac{a}{2-3} \Rightarrow a = 3$
Specific behaviours
✓ substitutes point to calculate value
✓ correct value

(ii) Draw the remainder of the graph.   (3 marks)

Solution (b)(ii)
See graph
Specific behaviours
✓ asymptotes
✓ thru' $(0,-1),(4,3)$
✓ two smooth curves

Question 4   **(8 marks)**

(a)   A periodic function is defined by $f(x) = 2 - 2\sin(3x)$.

(i)   State the amplitude of the function.   (1 mark)

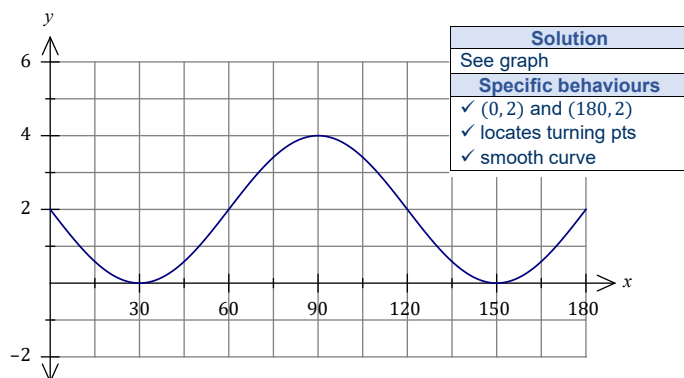| Solution |
|---|
| Amplitude is 2. |
| **Specific behaviours** |
| ✓ correct amplitude |

(ii)   State the period of the function in degrees.   (1 mark)

Solution
Period is $360 \div 3 = 120°$.
Specific behaviours
✓ correct period

(iii)   Sketch the graph of $y = f(x)$ on the axes below.   (3 marks)

Solution
See graph
Specific behaviours
✓ $(0, 2)$ and $(180, 2)$
✓ locates turning pts
✓ smooth curve

(b)   Solve the equation $2\cos(x - 15°) = \sqrt{3}$ where $0 \le x \le 360°$.   (3 marks)

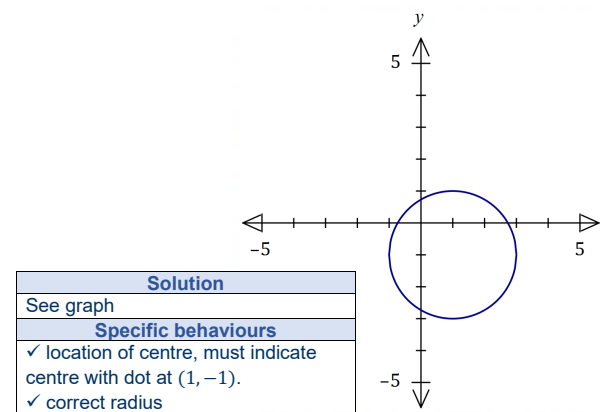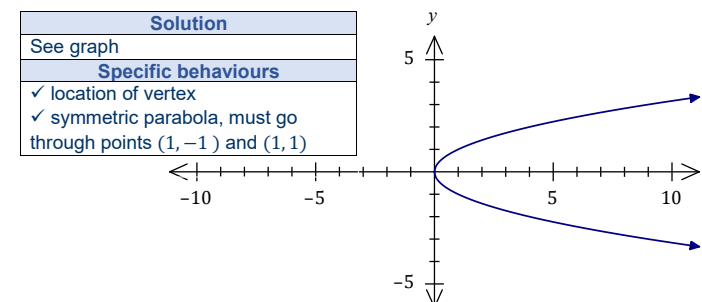| Solution |
|---|
| $\cos(x - 15°) = \dfrac{\sqrt{3}}{2}$ |
| $x - 15° = 30°, 330°$ |
| $x = 45°, 345°$ |
| **Specific behaviours** |
| ✓ determines an angle for $\frac{\sqrt{3}}{2}$ |
| ✓ determines one solution |
| ✓ both correct solutions |

Question 5   **(7 marks)**

(a)   Sketch the graph of $(x - 1)^2 + (y + 1)^2 = 4$ on the axes below.   (3 marks)

Solution
See graph
Specific behaviours
✓ location of centre, must indicate centre with dot at $(1, -1)$.
✓ correct radius

(b)   Sketch the graph of $y^2 = x$ on the axes below.   (2 marks)

Solution
See graph
Specific behaviours
✓ location of vertex
✓ symmetric parabola, must go through points $(1, -1)$ and $(1, 1)$

(c)   Explain whether y is a function of x in the relationship graphed in (b).   (2 marks)

Solution
y is NOT a function of x. This is because the graph of the relationship fails the vertical line test.
Specific behaviours
✓ states not a function
✓ explanation (VLT, one-to-many, etc)