



PERTH MODERN SCHOOL
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Independent Public School

Year 12 Methods
TEST 1
Friday 22 February 2019
TIME: 45 minutes working
One-page notes allowed
Calculator Assumed
39 marks 7 Questions

Name: _____ Teacher: _____

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 (6 marks)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	1	-2	-1
2	2	2	-1	1
3	1	-2	2	0

(a) Define $h(x) = \frac{f(x)}{g(x)}$, use the table to find the value for $h'(2)$. (3 marks)

(b) Define $I(x) = [g(x)]^5$, use the table to find the value for $I'(1)$. (3 marks)

Question 2**(3 marks)**

Find the equation of the line tangent to the function $y = (3x^2 - 2)^3$ at the point $(2, 2)$. Give your answer in the gradient-intercept form.

Question 3**(3 marks)**

If $\frac{dy}{dx} = (5x + 3)^3$, and $y = 50$ when $x = 1$, determine the expression of y in terms of x .

Supplementary Page

Question Number: _____

Question 4

(7 marks)

A company is purchasing a type of thin sheet metal required to make a closed cylindrical container with a capacity of 4000π cm³. Let the radius of the cylindrical base be r and the height be h .

(a) Show that the surface area of the cylinder can be expressed as
$$2\pi r^2 + \frac{8000\pi}{r}.$$

(3 marks)

(b) Using calculus, determine the least area of metal required to make a closed cylindrical container from thin sheet metal in order that it will have a capacity of 4000π cm³.
(Work to one decimal place)

(4 marks)

Question 7

(6 marks)

The position of a train on a straight mono rail, x metres at time t seconds, is modelled by the following formula for the velocity, v in metres/second, $v = pt^x - 12t + q$ where p & q are constants. The deceleration of the train is $8ms^{-2}$ when $t = 1$. The train has a position $x = \frac{4}{3}$ when $t = 2$ and is initially at the origin ($x = 0$).

a) Determine the values of the constants p & q .

(4 marks)

b) Determine the position of the train when the acceleration is $12ms^{-2}$.

(2 marks)

Question 5

(6 marks)

A share portfolio, initially worth \$26 000, has a value of f dollars after t months, and begins with a negative rate of growth. The rate of growth remains negative until after 20 months ($t=20$) when the value of the portfolio is momentarily stationary and then continues with negative growth for the life of the investment. The value of the portfolio, $f(t)$ after t months can be modelled by the following model, $f(t) = -2t^3 + bt^2 + ct + d$, $0 \leq t \leq 37$ months where b, c & d are constants.

Determine the values of the constants b, c & d .

Question 6

(8 marks)

The volume, V in cubic metres and radius R metres, of a spherical balloon are changing with time, t seconds. $V = \frac{4\pi R^3}{3}$. The radius of the balloon at any time is given by $R = 2t(t+3)^3$.

Determine the following:

- a) The value of $\frac{dR}{dt}$ when $t=1$. (3 marks)

- b) The value of $\frac{dV}{dt}$ when $t=1$. (3 marks)

Consider the volume of the balloon at $t=1$.

- c) Use the incremental formula to estimate the change in volume 0.1 seconds later (i.e. $t=1.1$) (2 marks)