

$X$  is a continuous random variable, denoting the number of minutes in excess of two hours which a person takes to travel from one town to another. The probability density function is defined as follows.

$$f(x) = \begin{cases} k(10+x) & -10 \leq x < 0 \\ k(10-x) & 0 \leq x \leq 10 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Determine the value of  $k$ . [4]

- (b) Determine the probability that a person will take longer than 15 minutes to reach the next town. [2]



**Mathematics Methods Unit 3,4**  
**Test 4 2016**

Section 1 Calculator Free

Calculus Involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME \_\_\_\_\_

DATE: Friday 22 July

TIME: 25 minutes

MARKS: 25

**INSTRUCTIONS:**

Standard Items:

Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (4 marks)

Determine the equation of the tangent to the curve  $y = x \ln x$  at the point (e,e)

2. (4 marks)

(a)  $\int \frac{\sin x}{1 + \cos x} dx$  [2]

(b)  $\int \frac{8 - 6x^2}{x^3 - 4x + 1} dx$  [2]

(e) Determine  $\text{Var}(1 - 2T)$ , where  $\text{Var}$  is the variance. [2]

(f) (i) For the random variable  $T$ , give the cumulative distribution function  $F(t)$ . [3]

(ii) Determine  $P(T \geq 10)$  [2]

6.

(18 marks)

The time, in minutes, between telephone calls received at a pizza shop is a continuous random variable,  $T$ , with a density function given by

$$f(t) = \begin{cases} 0.25e^{-0.25t} & \text{for } t \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Calculate the probability that the next call occurs within 8 minutes. [2]

- (b) Calculate the probability that the next call occurs between 3 and 6 minutes given it occurs within 8 minutes. [2]

- (c) Determine the expected time to the next call. [3]

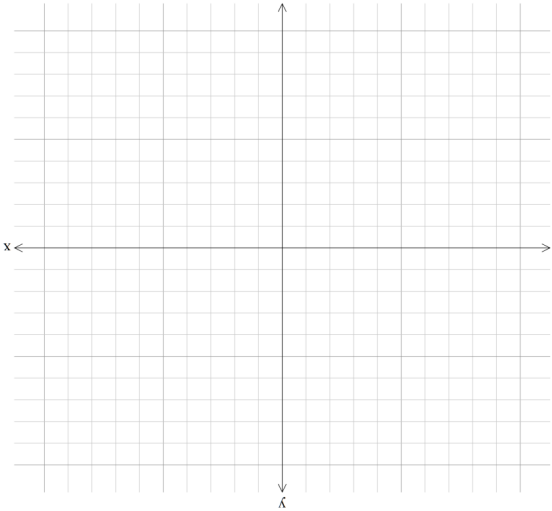
- (d) Determine the interval of time that is within one standard deviation of the expected completion time. [4]

3.

(5 marks)

Consider the functions  $y = 3 - x$  and  $xy = 2$ .

- (a) Draw a sketch of these two functions which clearly shows the enclosed area. [2]



- (b) Determine the exact value of the enclosed area. [3]

4. (12 marks)

Differentiate each of the following functions. Do NOT simplify.

(a)  $y = \ln \frac{2x}{x^2 - 1}$  [3]

(b)  $y = \ln \tan 2x$  [3]

(c)  $y = \ln \ln x^2$  [3]

(d)  $y = \ln(e^x(1 - e^{-x}))$  [3]



## Mathematics Methods Unit 3,4 Test 4 2016

Section 2 Calculator Assumed  
Calculus Involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME \_\_\_\_\_

DATE: Friday 22 July

TIME: 30 minutes

MARKS: 29

### INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

5. (5 marks)

The time,  $t$ , in hours that a fox spends hunting each night is a continuous random variable with probability density function

$$f(t) = \begin{cases} \frac{k}{32}t(4-t) & \text{for } 0 \leq t \leq 4 \\ 0 & \text{otherwise} \end{cases}$$



(a) Determine the value of  $k$ . [3]

(b) Calculate the probability the fox spends more than 3 hours hunting on one night. [2]