Your Senior High School

Question/Answer Booklet

Name

_____

MATHEMATICS:SPECIALIST 3AB

Semester One Exam 2010

Section Two:
Calculator-assumed

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for this section: 100 minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters
Special items: drawing instruments, templates, and up to three calculators, (CAS, graphic or scientific), which satisfy the conditions set by the Curriculum Council for this course. Up to two (2) A4 pages of notes (handwritten, photocopied or typed on both sides) that may be either personally or commercially produced. *It must not be folded, have anything stuck to it or have correction fluid/tape on it.*

Important note to candidates
No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Suggested working time (minutes)	Marks available
Section One: Calculator-free	11	11	50	40
Section Two: Calculator-assumed	14	14	100	80
				120

Instructions to candidates

1. Answer the questions according to the following instructions.

   Section Two: Write answers in this Question/Answer Booklet. **All** questions should be answered.

   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

   For any question or part question worth more than two marks, valid working or justification is required to receive full marks.

   If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

   It is recommended that you **do not use pencil** except in diagrams.

2. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

MARK ALLOCATION AND RECORDS:

Section	Question	Marks	Awarded
TWO	12	3	
	13	3	
	14	6	
	15	8	
	16	10	
	17	7	
	18	4	
	19	4	
	20	5	
	21	5	
	22	6	
	23	5	
	24	8	
	25	6	
	Penalties	- 1/2/3	
	ONE	**40**	
	TWO	**80**	
	TOTAL	**120**	

Penalties	
Rounding (-1)	
Units (-1)	
Notation (-1)	

%

Question 12 [3 marks]

Using your graphics calculator or otherwise, solve $4 - |2x| < |2x + 3| + 1$.

Question 13 [1, 2 marks]

For $g(x) = \frac{4}{x} - 6$ determine each of the following.

(a) $g(-4)$

(b) the value of p for which $g(p) = 34$

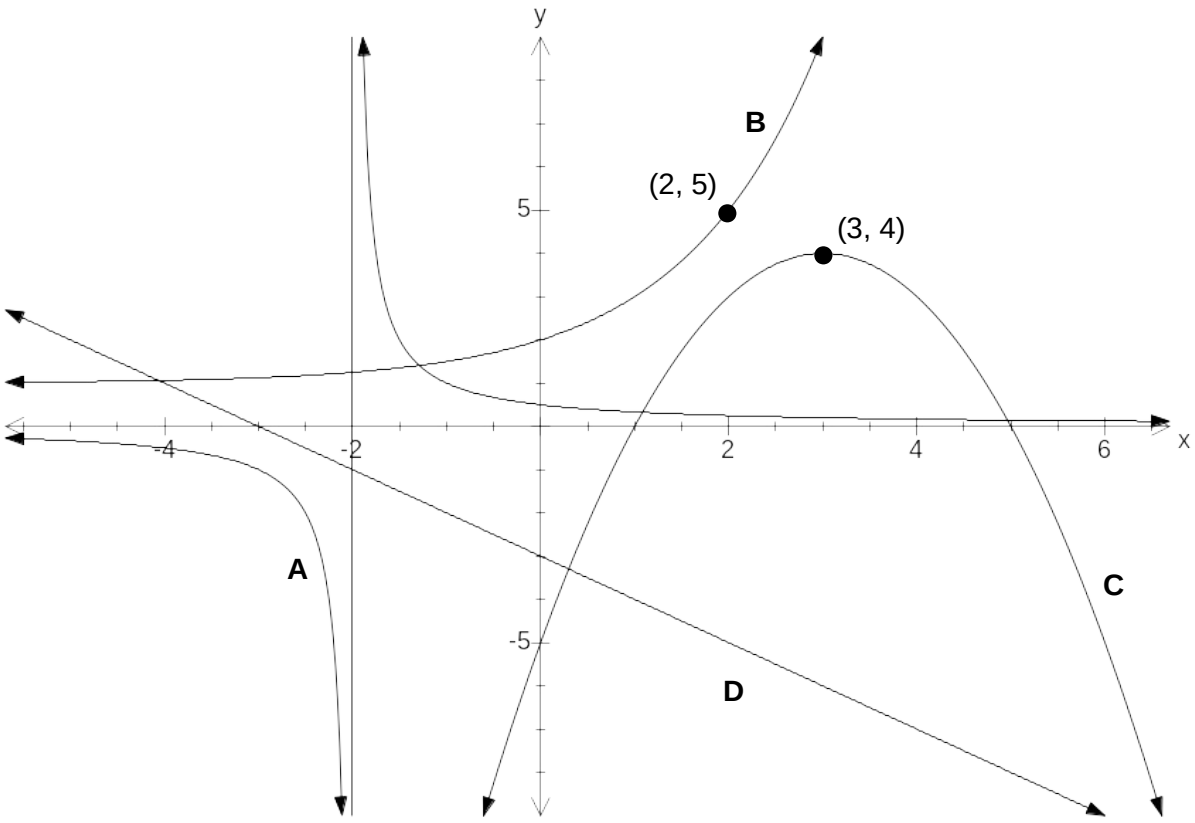
Question 14 [2, 4 marks]

(a) State the natural domain and corresponding range for $f(x) = \sqrt{x+5}$.

(b) Find the inverse function for $f(x) = \frac{1}{x - 2}$ and state the domain and range for this inverse function.

Question 15 [8 marks]

Consider the four graphs shown on the axes below. Use information obtained from the graphs to complete the tables below. ***Note: Not all equations in the table are graphed and hence it is not possible to find a value for each variable.***



Equation	Letter label on graph e.g. **A**
$y = mx + c$	
$y = \log (x + b)$	
$y = a(x - d)^2 + e$	

Equation	Letter label on graph
$y = \frac{1}{(x + j)} + k$	
$y = n^x + p$	
$y = \lvert qx + r \rvert$	

Variable	**a**	**b**	**c**	**d**	**e**	**j**	**k**	**m**	**n**	**p**	**q**	**r**
Value												

Question 16 [1, 1, 2, 2, 4 marks]

(a) Write $\log_3 5 = x$ has an exponential statement.

(b) Express $\sqrt{1000} \div 10^{-2.5}$ as a power of 10.

(c) Evaluate $\log 20 + 2\log_2 17$ correct to two decimal places.

(d) Express $2\log x + \log 7 - \log y$ as a single logarithm.

(e) Solve $5^{x-3} = 2^{x+2}$ correct to two decimal places showing full working.

Question 17 [1, 1, 5 marks]

A $40 000 car depreciates at a rate of 18% per year for the first four years.

(a) Write down the equation for V, which represents the value of the car after n years.

(b) Using the formula, find the value of the car after 3 years.

(c) The car loses 10% in value during the fifth year.
How much has it **lost** in value after 5 years?

Question 18 [4 marks]

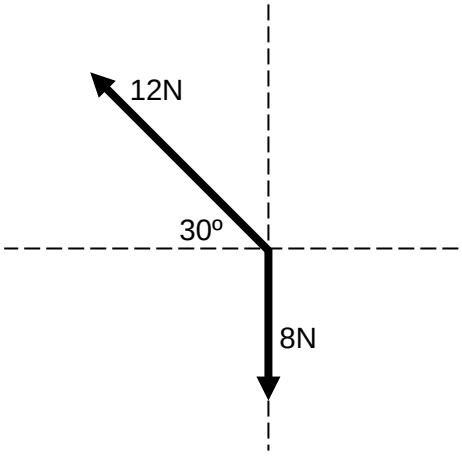
The concentration of a certain type of pollutant is measured using the formula $C = C_0 (0{\cdot}88)^n$

where C is the concentration after n filtering processes and C_0 is the original concentration.

How many filtering processes are required before the final concentration is less than 15% of the original concentration?

Question 19 [4 marks]

Determine the magnitude and direction of the resultant force in this system.

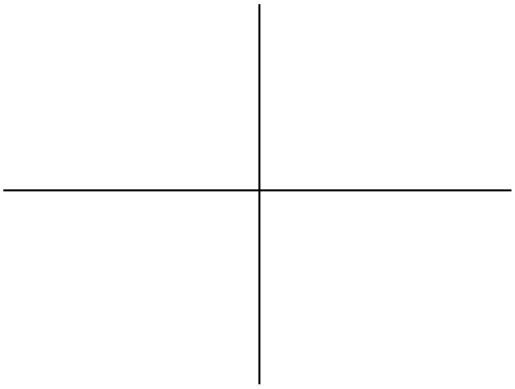


Question 20 [5 marks]

If A, B and C have position vectors <7, 2> , <2, -3> and <1, 1> respectively, find a vector in the same direction as $\overrightarrow{AB}$ but with the same magnitude as $\overrightarrow{BC}$.

Question 21 [5 marks]

A trapezium is a polygon with at least one pair of parallel sides. Use vectors to prove that the points **A** (-4, 6), **B** (1, 5), **C** (3, 2) and **D** (2, -3) form a trapezium.



Question 22 [2, 4 marks]

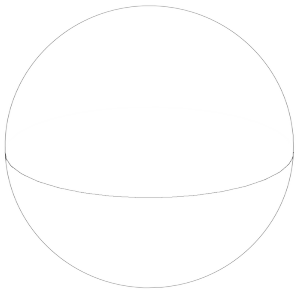
(a) If $\mathbf{V}_A = 3\mathbf{i} - 5\mathbf{j}$ and ${}_A\mathbf{V}_B = 8\mathbf{i} + 3\mathbf{j}$, find $\mathbf{V}_B$.

(b) To a person jogging due East at 8 km/h, the wind seems to come from the South with a speed of 5 km/h. Find the true magnitude and direction from which the wind is blowing.

Question 23 [5 marks]

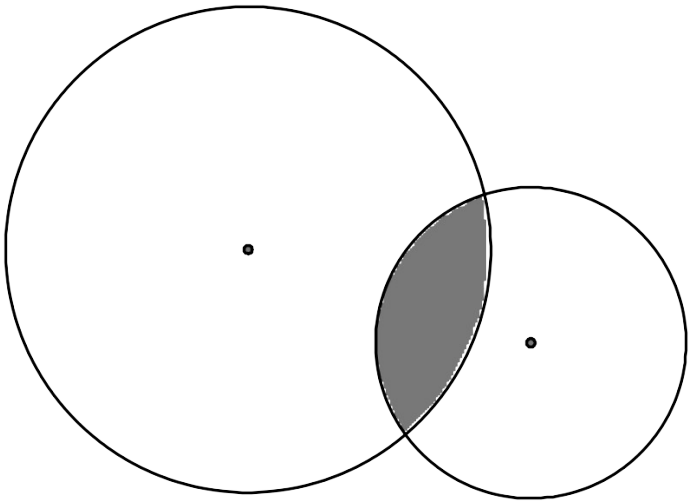
For this question assume that the radius of the Earth is 6350 km.

Jonah leaves his post at Brest, France(48°N,4.5°W) and travels for 5000 km due East. What is the latitude and longitude of his new position?



Question 24 [8 marks]

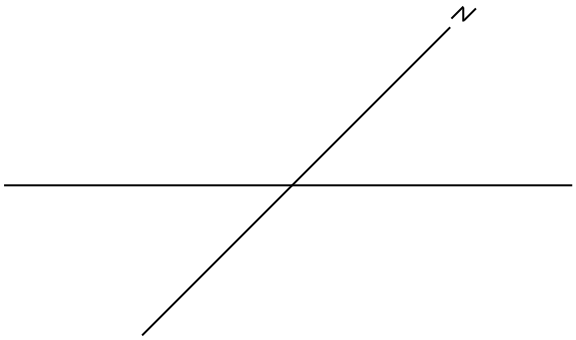
In the accompanying diagram, the two radii are 12cm and 16cm and the two centres are 20cm apart. Showing full working, find the perimeter and area of the shaded region. Round your answers to the nearest unit.



Question 25 [2, 4 marks]

Kate is on top of a 28m high cliff. She observes her friend Tim who is due East of the base of the cliff on level ground at an angle of depression of 8^o. Kate also observes Jack who is on the same level ground as Tim, on a bearing of 145^o from the base of the cliff and at an angle of depression of 6^o.

(a) Draw a diagram to represent the information provided above.



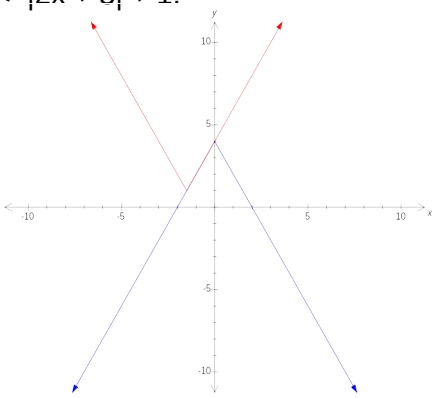
(b) How far apart are Jack and Tim?

MAS 3AB Section Two (Calculator assumed) SOLUTIONS

Question 12 [3 marks]

Using your graphics calculator or otherwise, solve $4 - |2x| < |2x + 3| + 1$.

Solution = {x | x < -1·5 or x > 0 } ✓✓✓



Question 13 [1, 2 marks]

For $g(x) = \frac{4}{x} - 6$ determine each of the following.

(a) g(-4)

$= -7$ ✓

(b) the value of p for which g(p) = 34

$\frac{p}{4} - 6 = 34$ ✓

$\frac{p}{4} = 40$

$p = \frac{1}{10}$ ✓

Question 14 [2, 4 marks]

(a) State the natural domain and corresponding range for $f(x) = \sqrt{x+5}$.

$D_x = \{ x \mid x \geq -5 \}$ ✓ $R_y = \{ y \mid y \geq 0 \}$ ✓

(b) Find the inverse function for $f(x) = \frac{1}{x - 2}$ and state the domain and range for this inverse function.

Let $y = \frac{1}{x - 2}$

$x - 2 = \frac{1}{y}$

$x = \frac{1}{y} + 2$

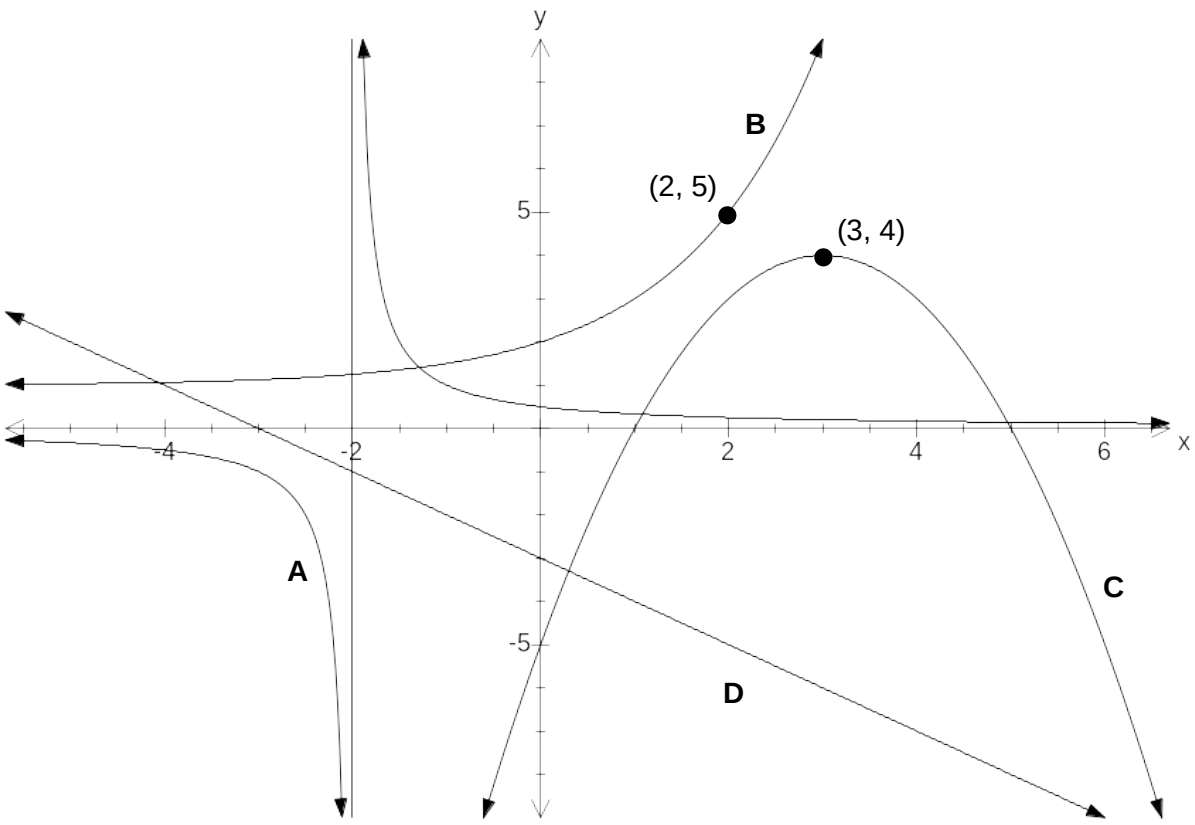
$\Rightarrow f^{-1}(x) = \frac{1}{x} + 2$ ✓✓

$D_x = \{ x \mid x \in IR, x \neq 0, x \neq 2 \}$ ✓

$R_y = \{ y \mid y \in IR\ y \neq 0, y \neq 2 \}$ ✓

Question 15 [8 marks]

Consider the four graphs shown on the axes below. Use information obtained from the graphs to complete the tables below. ***Note: Not all equations in the table are graphed and hence it is not possible to find a value for each variable.***



Equation	Letter label on graph e.g. **A**
$y = mx + c$	**D**
$y = \log (x + b)$	
$y = a(x - d)^2 + e$	**C**

Equation	Letter label on graph
$y = \frac{1}{(x + j)} + k$	**A**
$y = n^x + p$	**B**
$y = \lvert qx + r \rvert$	

Variable	**a**	**b**	**c**	**d**	**e**	**j**	**k**	**m**	**n**	**p**	**q**	**r**
Value	**-1**	x	**-3**	**3**	**4**	**2**	**0**	**-1**	**2**	**1**	x	x

8 marks ½ mark off for each error

Question 16 [1, 1, 2, 2, 4 marks]

(a) Write $\log_3 5 = x$ has an exponential statement.

$5 = 3^x$ ✓

(b) Express $\sqrt{1000} \div 10^{-2\cdot5}$ as a power of 10.

$= 10^{1\cdot5} \cdot 10^{2\cdot5}$

$= 10^4$ ✓

(c) Evaluate $\log 20 + 2\log_2 17$ correct to two decimal places.

$= 9\cdot48$ ✓✓

(d) Express $2\log x + \log 7 - \log y$ as a single logarithm.

$= \log 7x^2 - \log y$ ✓

$= \log \frac{7x^2}{y}$ ✓

(e) Solve $5^{x-3} = 2^{x+2}$ correct to two decimal places showing full working.

$(x - 3) \cdot \log 5 = (x + 2) \cdot \log 2$ ✓

$x \log 5 - x \log 2 = 2 \log 2 + 3 \log 5$ ✓

$x(\log 2\cdot5) = \log (4 \times 125)$

$x = \frac{\log 500}{\log 2.5}$ ✓

$x = 6\cdot78$ ✓

Question 17 [1, 1, 5 marks]

A $40 000 car depreciates at a rate of 18% per year for the first four years.

(a) Write down the equation for V, which represents the value of the car after n years.

$V = \$40000 \times (0\cdot82)^n$ ✓

(b) Using the formula, find the value of the car after 3 years.

$V(3) = \$40000 \times (0\cdot82)^3$

$= \$22054\cdot72$ ✓

(c) The car loses 10% in value during the fifth year.
How much has it **lost** in value after 5 years?

$V(5) = V(4) \times 0\cdot9$ ✓

$V(5) = \$40000 \times (0\cdot82)^4 \times 0\cdot9$

$= \$16276\cdot38$ ✓✓

$\therefore$ Lost value = Original cost - V(5)

$= \$40000 - \$16276\cdot38$

$= \$23723\cdot62$ ✓✓

Question 18 [4 marks]

The concentration of a certain type of pollutant is measured using the formula $C = C_0 (0{\cdot}88)^n$ where C is the concentration after n filtering processes and C_0 is the original concentration. How many filtering processes are required before the final concentration is less than 15% of the original concentration?

Let $C = 0{\cdot}15\, C_0$ ✓

$\Rightarrow \quad 0{\cdot}15\, C_0 = C_0 (0{\cdot}88)^n$

$\Rightarrow \log(0{\cdot}15) = n \log(0{\cdot}88)$ ✓

$$n = \frac{\log(0 \bullet 15)}{\log(0 \bullet 88)}$$

$= 14{\cdot}84$ ✓

∴ 14 processes not enough, 15 processes required. ✓

Question 19 [4 marks]

Determine the magnitude and direction of the resultant force in this system.

$R^2 = 8^2 + 12^2 - 2.8.12.\cos 60^o$

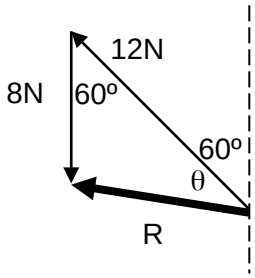
$R = \sqrt{112}$ N ✓✓

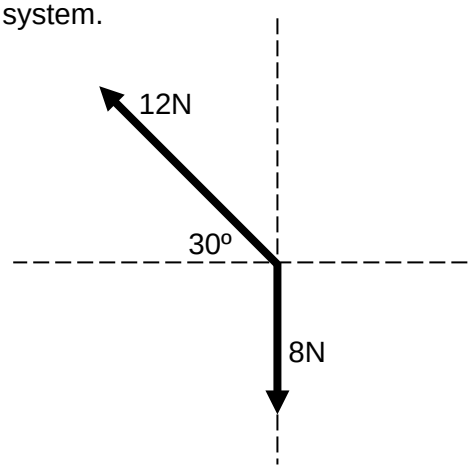
$$\frac{\sin\theta}{8} = \frac{\sin 60^\circ}{\sqrt{112}}$$

$\theta = 41^o$ (nearest degree) ✓

∴ Magnitude of resultant force is $\sqrt{112}$ N

with a bearing of 259º ✓





Question 20 [5 marks]

If A, B and C have position vectors <7, 2> , <2, -3> and <1, 1> respectively, find a vector in the same direction as $\vec{AB}$ but with the same magnitude as $\vec{BC}$.

$\vec{AB} = \langle 2, -3\rangle - \langle 7, 2\rangle$ ✓

$= \langle -5, -5\rangle$

$|\vec{AB}| = \sqrt{50}$ ✓

$\vec{BC} = \langle 1, 1\rangle - \langle 2, -3\rangle$ ✓

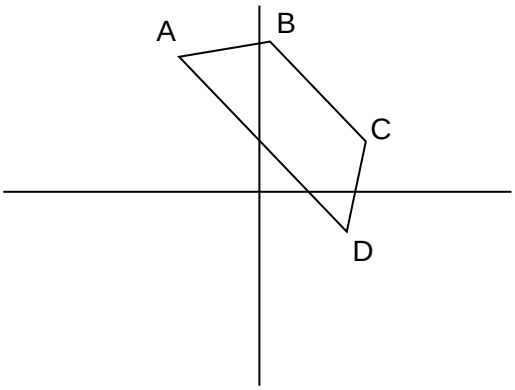
$= \langle -1, 4\rangle$

$|\vec{BC}| = \sqrt{17}$ ✓

$\therefore V = \frac{\sqrt{17}}{\sqrt{50}} \langle -5, -5 \rangle$ ✓

Question 21 [5 marks]

A trapezium is a polygon with at least one pair of parallel sides. Use vectors to prove that the points **A** (-4, 6), **B** (1, 5), **C** (3, 2) and **D** (2, -3) form a trapezium.



$\vec{AD} = \langle 2, -3 \rangle - \langle -4, 6 \rangle$ ✓

$= \langle 6, -9 \rangle$

$\vec{BC} = \langle 3, 2 \rangle - \langle 1, 5 \rangle$ ✓

$= \langle 2, -3 \rangle$

Now $\vec{AD} = \langle 6, -9 \rangle$

$= 3 \langle 2, -3 \rangle$ ✓

which is a scalar multiple of $\vec{BC}$. ✓

✓

$\therefore \vec{AD}$ is parallel to $\vec{BC}$ and so **ABCD** forms a trapezium.

Question 22 [2, 4 marks]

(a) If $\mathbf{V}_A = 3\mathbf{i} - 5\mathbf{j}$ and ${}_A\mathbf{V}_B = 8\mathbf{i} + 3\mathbf{j}$, find $\mathbf{V}_B$.

$\langle 8, 3 \rangle = \langle 3, -5 \rangle - \langle x, y \rangle$ ✓

$\langle x, y \rangle = \langle 3, -5 \rangle - \langle 8, 3 \rangle$

$= \langle -5, -8 \rangle$

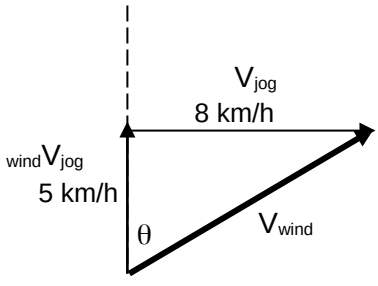
i.e. $\mathbf{V}_B = -5\mathbf{i} + -8\mathbf{j}$ ✓

(b) To a person jogging due East at 8 km/h, the wind seems to come from the South with a speed of 5 km/h. Find the true magnitude and direction from which the wind is blowing.

${}_{wind}V_{jog} = V_{wind} - V_{jog}$

$\therefore V_{wind} = V_{jog} + {}_{wind}V_{jog}$ ✓

$|V_{wind}| = \sqrt{89}$ ✓



$\text{Tan}\,\theta = \frac{8}{5}$

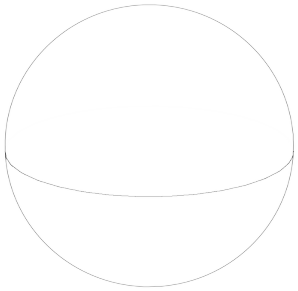
$\theta = 58^\circ$ (blowing to) ✓ ✓

$\therefore$ Magnitude is $\sqrt{89}$ and the wind is blowing from 238°.

Question 23 [5 marks]

For this question assume that the radius of the Earth is 6350 km.

Jonah leaves his post at Brest, France(48^o N,4.5^o W) and travels for 5000 km due East. What is the latitude and longitude of his new position?



Latitude radius $= r_e \cos 48^\circ$

$= 4249$ km (nearest km) ✓

$A = r\theta$

$\theta = \frac{5000}{4249}$ ✓

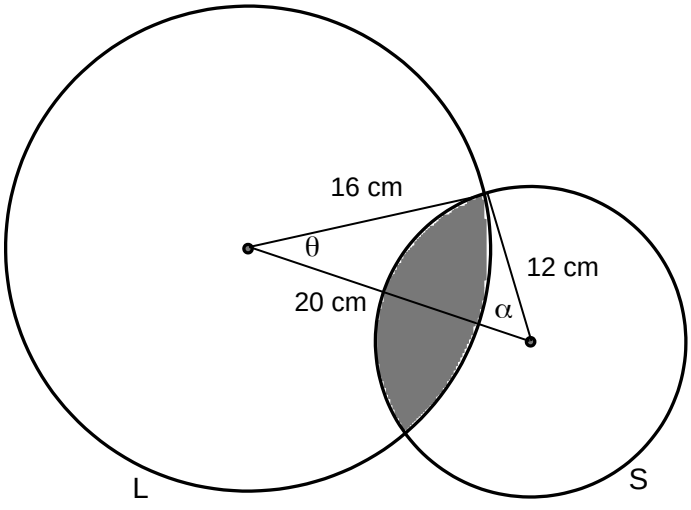
$= 1{\cdot}177$ radians

$\approx 67{\cdot}4^\circ$ ✓

$\therefore$ Latitude $= 48^\circ$ N ✓ and Longitude $= 62{\cdot}9^\circ$ E ✓

Question 24 [8 marks]

In the accompanying diagram, the two radii are 12cm and 16cm and the two centres are 20cm apart. Showing full working, find the perimeter and area of the shaded region. Round your answers to the nearest unit.



$12^2 + 16^2 = 400$

$= 20^2$ ✓

$\therefore$ Right-angled triangle.

$\text{Tan}\,\theta = \frac{12}{16}$

$\theta = 36{\cdot}9^\circ$

$2\theta = 1{\cdot}288$ radians ✓

$\alpha = 90^\circ - 36{\cdot}9^\circ$

$\alpha = 53{\cdot}1^\circ$

$2\alpha = 1{\cdot}854$ radians ✓

<u>Area</u>

$A_T = \text{Segment}_L + \text{Segment}_S$

$= \frac{1}{2}(16^2)(1{\cdot}288 - \sin(1{\cdot}288)) + \frac{1}{2}(12^2)(1{\cdot}854 - \sin(1{\cdot}854))$ ✓ ✓

$= 41{\cdot}948 + 64{\cdot}356$

$= 106\ \text{cm}^2$ (nearest unit) ✓

<u>Perimeter</u>

$P_T = \text{Arc}_L + \text{Arc}_S$

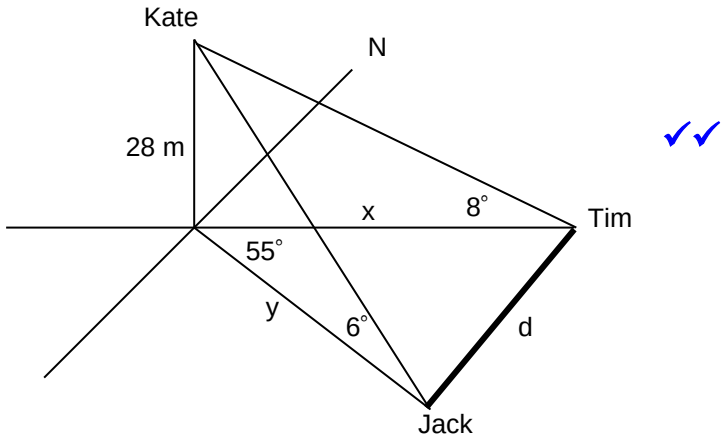
= 16 x 1·288 + 12 x 1·854 ✓

= 43 cm (nearest unit) ✓

Question 25 [2, 4 marks]

Kate is on top of a 28m high cliff. She observes her friend Tim who is due East of the base of the cliff on level ground at an angle of depression of 8°. Kate also observes Jack who is on the same level ground as Tim, on a bearing of 145° from the base of the cliff and at an angle of depression of 6°.

(a) Draw a diagram to represent the information provided above.



✓✓

(b) How far apart are Jack and Tim?

$\frac{28}{x}$ = Tan 8°

x = 199·23 m ✓

$\frac{28}{y}$ = Tan 6°

y = 266·4 m ✓

$d^2 = 199{\cdot}23^2 + 266{\cdot}4^2 - 2 \cdot 199{\cdot}23 \cdot 266{\cdot}4 \cdot \cos 55°$

$\therefore d = 223{\cdot}1$ m ✓✓