Maths Specialist

<u>Test 1</u>

Composite functions

A function is one that:

- passes the vertical line test
- is one-to-one or many-to one (each x value has only one corresponding y value)

A non-function is one that:

- fails the vertical line test
- is one-to-many (each x value has more than one corresponding y value)

Composite Functions:

Consider the functions f(x) and g(x)

x -> f(x) -> g(x)

Domain(f(x)) -> Range(f(x))=Domain(g(x)) -> Range(g(x))

Eg. If Domain(f(x)) = {2,4,6} and f(x) = 2x, g(x) = x^2 + 1

Range(f(x)) = {4,8,12} = Domain(g(x))

Range(g(x)) = {17,65,145}

Also: f(g(x))=fg(x)=f∘g(x)=2[g(x)]

=2(x^2 + 1)

Inverse functions

An inverse function is the function reflected in the line y = x

(x, y) -> (y, x), i.e. y = 3x + 7 -> x = 3y + 7

*Exchange x and y values y = $\frac{x-7}{3}$

$f^{-1}(x) = \frac{x-7}{3}$

*Take care for many-to-one functions!

y = x^2 reflected in the line y = x gives a reflection of y = $\pm\sqrt{x}$ (i.e. not a function), so a suitable restriction x>0 will give an inverse for y = x^2

Absolute value: equations $|x|=3 \Longrightarrow x=\pm 3$

$|x|<3 \Longrightarrow -3<x<3$

$|x+5|=7 \Longrightarrow x+5=7 \vee x+5=-7, x=-12, 2$

$|x+3|=|x-1| \Longrightarrow x+3=x-1\,(no\,solution) \vee x+3=-(x-1)$

$|2x+3|=|x+1| \Longrightarrow 2x+3=x+1 \vee 2x+3=-(x+1), x=-2, -\frac{4}{3}$

Solving inequalities:

$|2x-3|+|x+1|\geq 8$

$(2x-3)+(x+1)\geq 8 \vee (2x-3)-(x+1)\geq 8 \vee -(2x-3)-(x+1)\geq 8$

$3x\geq 10 \vee x\geq 12 \vee -3x\geq 12$

$x\geq 3.33, x\geq 12, x\leq -4$

$\{x\leq -4\} \cup \{x\geq \frac{10}{3}\}$

Graphs of rational functions

$y=\frac{1}{f(x)}$

Consider the graph of $y=x^2-4x+3=(x-3)(x-1)=(x-2)^2+1$

Vertical asymptotes where y = 0 on original graph

Reciprocal function -> hyperbolic graph

Sketching polynomials:

Consider some or all of:

- intercepts with the x and y axes
- vertical/horizontal/oblique asymptotes
- behaviour as x -> ±∞ and as y -> ±∞
- stationary/turning points (use differentiation)

$y=|f(x)|$, where y is the graph of f(x) but where f(x) is negative, it is reflected around the x axis

$y=f(|x|)$, where in quadrants 1 and 4 the graph will be the same as that of f(x), however quadrants 2 and 3 will be the parts of the graph of f(x) in quadrants 1 and 4 reflected in the y axis.

If $g(f)=(x-2)^2 \wedge f(x)=1-\sqrt{x}$

Domain of g(x): **R**, range of g(x): y≥0, domain of f(x): x≥0, range of f(x): y≤ 1

Why is fg(x) defined for all real x? g(x) is defined for all real x and the output from g(x) consists of numbers that are all within the domain of f(x), thus fg(x) is defined for all real x.