PERTH MODERN SCHOOL
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Independent Public School

Course Methods Test 2 Year 12

Student name: _______________ Teacher name: _______________

Task type:	**Response**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions:	_____**4**______
Materials required:	Upto three calculators/classpads
Standard items:	Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	Drawing instruments, templates, notes on one unfolded sheet of A4 paper,
Marks available:	**42 marks**
Task weighting:	**13%**

Formula sheet provided: no but formulae listed on next page.

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

Q1 (2, 3, 3 & 2 = 10 marks)
Consider the functions $f(x)\,\&\,g(x)$ and the table of values below.

Determine the following showing full working.

a) $\frac{d}{dx}\left(f(x)g(x)\right) \quad ,x=3$

c
$\frac{d}{dx}\left(f(x)g(x)\right) = fg' + f'g$ $= f(3)g'(3) + f'(3)g(3)$ $= 9(8) + 5(12)$ $= 132$
Specific behaviours
✓ uses product rule ✓ determines derivative at x=3

b) $\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right) \quad ,x=4$

c
$\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)$ $= \frac{fg' - gf'}{f^2} = \frac{f(4)g'(4) - g(4)f'(4)}{(f(4))^2}$ $= \frac{13(-9) - 18(-7)}{13^2}$ $= \frac{9}{169}$
Specific behaviours
✓ uses quotient rule ✓ subs correct values for numerator ✓ obtains derivative at x=4

Working out space

c) $\frac{d}{dx}\left[f(g(x))\right] \quad , x=5$

c
$\frac{d}{dx}\left[f(g(x))\right] = f'(g)g'$ $= f'\left[g(5)\right]g'(5)$ $= f'(3)(-2)$ $=5(-2)$ -10
Specific behaviours
✓ uses chain rule ✓ determines derivative of f at x=5 ✓ states derivative at x=5 NOTE: max of 1 out of 3 if no chain rule used

d) $\frac{d}{dx} f(3x) \quad , x=1$

c
$\frac{d}{dx} f(3x) = f'(3x)3$ $= f'(3)3$ $=5(3)$ $=15$
Specific behaviours
✓ uses chain rule ✓ states derivative NOTE: zero marks if no factor of 3 (chain rule) used!

Q2 (1, 2, 3, 2 & 3 = 11 marks)

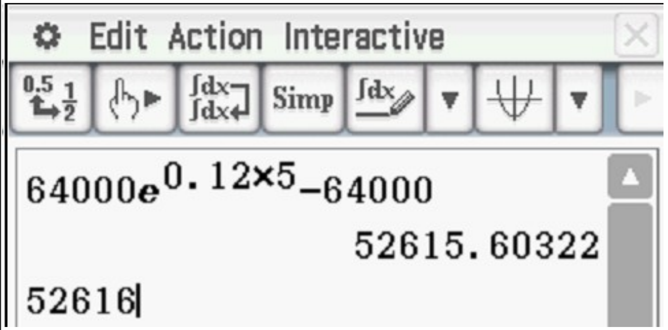
Consider a group of kangaroos living in an isolated habitat such that the number of kangaroos, N at time t years ($t=0$ at the start of 2012), is given by $N=64000e^{0.12t}$.

a) Determine the number of kangaroos at the start of 2012.

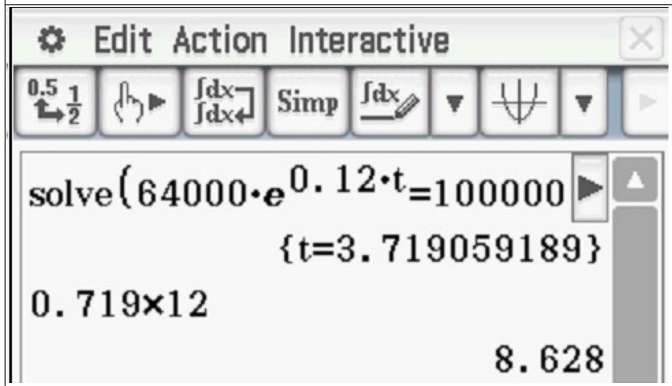
c

64000 kangaroos
Specific behaviours
✓ states number

b) Determine the increase in kangaroos over the first 5 years.

c

Specific behaviours
✓ suns t=5 ✓ rounds up to nearest integer

c) Determine to the nearest month when the population first exceeds 100000.

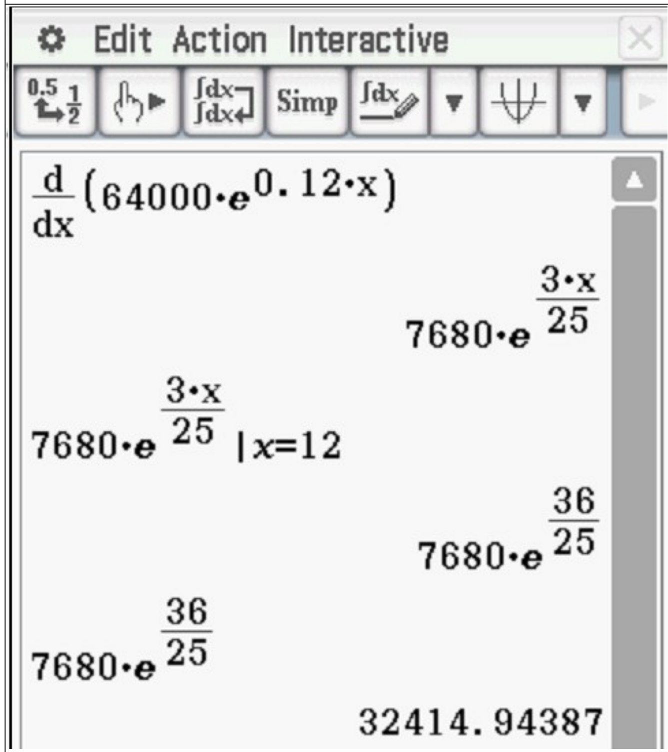
c


Working out space

Working out space

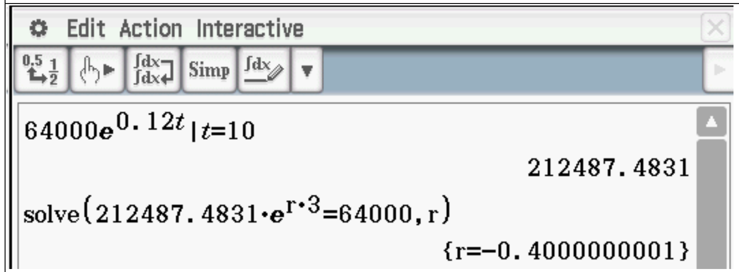
Three years and nine months.
Specific behaviours
✓ sets up equation ✓ solves for time in years ✓ rounds to nearest month (do not accept 8 months nor days)

d) Determine the rate of growth at the start of 2024.

c

Specific behaviours
✓ recognises that derivative is needed at time t=12 ✓ states rate, approx. or exact

After 10 years the number of kangaroos starts to decline according the formula $N = Ae^{rt}$ where $A \& r$ are constants.

e) Determine $A \& r$ if after 3 years after the decline of the kangaroos, the population is back to 64000.

c

Specific behaviours
✓ determines A constant, accept decimal ✓ sets up equation for r constant ✓ solves for r

Q3 (2, 2, 2, 2 & 4 = 12 marks)

An oscillating mass has a velocity, v given by $v = 3t^2 \sin\left(t - \dfrac{\pi}{4}\right)$, $t \geq 0$.
The velocity is measured in metres/second with the time, t in seconds.
Find below a graph of the velocity.

a) Determine the first two exact times that the mass changes direction, $t > 0$.

c
$\dfrac{\pi}{4}, \dfrac{5\pi}{4}$
Specific behaviours
✓ first time, t>0 ✓ second time

Working out space

✓ states anti-derivative ✓ states area (no need to round nor units)

c) Using calculus, determine the maximum distance of the trench below sea level.

c
Edit Action Interactive $\frac{d}{dx}\left((x-7)^2-\sin(3\cdot x-5)-6\right)$ $2\cdot x-3\cdot\cos(3\cdot x-5)-14$ solve(2·x−3·cos(3·x−5)−14=0 | 5.53<x<8.85, x) {x=6.493797662, x=7.551832135, x=8.173855977} $\frac{d^2}{dx^2}\left((x-7)^2-\sin(3\cdot x-5)-6\right)$ $9\cdot\sin(3\cdot x-5)+2$ 9·sin(3·x−5)+2 | x=6.493797662 10.47203228 9·sin(3·x−5)+2 | x=7.551832 −6.368831997 9·sin(3·x−5)+2 | x=8.174 7.606098668 Local min when x=6.494 & x=8.174 km (x−7)²−sin(3·x−5)−6 | x=6.494 −6.685095898 (x−7)²−sin(3·x−5)−6 | x=8.174 −5.244623852 Max distance below sea level 6.69 km
Specific behaviours
✓ states derivative ✓ equates derivative to zero and solves for three x values within domain in part a ✓ states second derivative for all three stationary points ✓ identifies correct stationary point and states depth as a positive number WITH units (no need to calculate both y values) Note: max of -1 for no units in this entire question.

b) Shade on the diagram above the signed area that is represented by the integral

$$\int_{\frac{7\pi}{6}}^{\frac{4\pi}{3}} 3t^2\sin\left(t-\frac{\pi}{4}\right)dt.$$

c
Shades area between graph from $\frac{4\pi}{3}$ to $\frac{7\pi}{6}$
Specific behaviours
✓ shades positive signed area to intercept ✓ shades negative signed area from intercept to $\frac{7\pi}{6}$.

c) What does the integral $\int_{\frac{7\pi}{6}}^{\frac{4\pi}{3}} 3t^2\sin\left(t-\frac{\pi}{4}\right)dt$ represent for the mass?

c
Change in displacement from $\frac{4\pi}{3}$ to $\frac{7\pi}{6}$
Specific behaviours
✓ discusses change in displacement (Do not accept distance) ✓ states start and finish times

d) Determine the first time after $t=\pi$ that the acceleration is zero m/s^2. **(2 marks)**

c

Edit Action Interactive

$$\frac{d}{dx}\left(3\cdot x^2\cdot\sin\left(x-\frac{\pi}{4}\right)\right)$$

$$3\cdot x^2\cdot\cos\left(x-\frac{\pi}{4}\right)+6\cdot x\cdot\sin\left(x-\frac{\pi}{4}\right)$$

$$\text{solve}\left(3\cdot x^2\cdot\cos\left(x-\frac{\pi}{4}\right)+6\cdot x\cdot\sin\left(x-\frac{\pi}{4}\right)=0\mid\pi<x<2\cdot\pi,x\right)$$

$$\{x=5.828346949\}$$

Time= 5.83 seconds

Specific behaviours
✓ shows derivative of velocity ✓ solves for first time after pi for zero acceleration WITH units Note- full marks for answer only being a 2 mark question.

e) The displacement of the mass is given by $x = At^2\cos(t-\frac{\pi}{4}) + Bt\sin(t-\frac{\pi}{4}) + C\cos(t-\frac{\pi}{4})$ metres, where $A, B \& C$ are constants. Determine the values of $A, B \& C$.

c
$x = At^2\cos(t-\frac{\pi}{4}) + Bt\sin(t-\frac{\pi}{4}) + C\cos(t-\frac{\pi}{4})$ $v = 2At\cos(t-\frac{\pi}{4}) - At^2\sin(t-\frac{\pi}{4}) + Bt\cos(t-\frac{\pi}{4}) + B\sin(t-\frac{\pi}{4}) - C\sin(t-\frac{\pi}{4})$ $v = (2A+B)t\cos(t-\frac{\pi}{4}) - At^2\sin(t-\frac{\pi}{4}) + (B-C)\sin(t-\frac{\pi}{4})$ $= 3t^2\sin(t-\frac{\pi}{4})$ $A=-3$ $-6+B=0, B=6$ $B=C=6$

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Specific behaviours
✓ diffs x to obtain expression of v in terms of A,B&C ✓ sets up equations for constants ✓ Solves for B & C ✓ Solves for A

Q4 (2, 3 & 4 = 9 marks)

a) Determine the values of $x_1 \& x_2$ to two decimal places.

c
Edit Action Interactive $\text{solve}((x-7)^2-\sin(3\cdot x-5)-6=-3\mid 0\le x\le 3\cdot\pi, x)$ $\{x=5.527696098, x=8.850698127\}$ 5.53 & 8.85 Km

Specific behaviours
✓ equates D=-3 ✓ solves for x rounded to 2dp and gives units.

b) Using calculus, determine the cross-sectional area of the trench to one decimal place.

c
$\int_{5.53}^{8.85} -3-(x-7)^2+\sin(3x-5)+6\,dx$ $\left[-3x-\frac{(x-7)^3}{3}-\frac{1}{3}\cos(3x-5)+6x\right]_{5.53}^{8.85}$ $\left(-3(8.85)-\frac{(8.85-7)^3}{3}-\frac{1}{3}\cos(3(8.85)-5)+6(8.85)\right)$ $-\left(-3(5.53)-\frac{(5.53-7)^3}{3}-\frac{1}{3}\cos(3(5.53)-5)+6(5.53)\right)$ $=7.2787$ Area = 7.3 square Km

Specific behaviours
✓ sets up definite integral with correct limits