8. [7 marks: 2, 3, 2]

The position vectors of the points P and Q are -2 **i** + 2 **j** -3 **k** and 6 **i** + 4 **j** + 5 **k** respectively.

(a) Find the position vector of K, the mid-point of the line joining P and Q.

The plane Π is the perpendicular bisector of the line joining the points P and Q.

(b) Find the vector equation of the plane Π.

(c) Find the acute angle the plane Π makes with the *x*-*y* plane.

9. [8 marks: 2, 4, 2]

A curve has equation $e^{y+x} + e^{y-x} - x^2 - 4e^y + 1 = 0$

(a) Find the exact value of the vertical intercept (y-intercept) of this curve.

(b) Use an analytical method to find $\frac{dy}{dx}$.

(c) Verify that the curve has a stationary point at its vertical intercept.

10. [8 marks: 3, 5]

A cool room for storing food is refrigerated so that the temperature in the room, *F*, in degrees Celsius, at *t* hours after midnight, is given by the formula

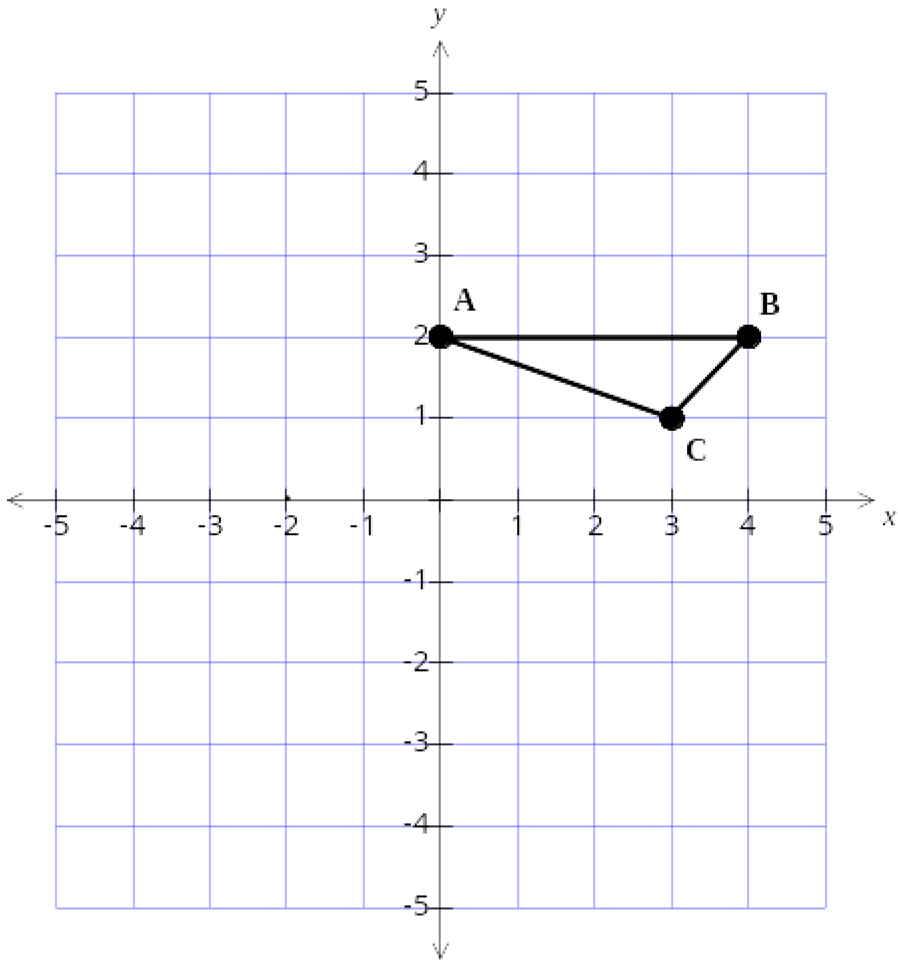
$$F = -4\cos\frac{\pi(t-3)}{12} \qquad \text{for } 0 \le t \le 24$$

(a) Show that *F* experiences fluctuations that are similar to a particle undergoing simple harmonic motion.

(b) The refrigeration system automatically switches on when the rate of change of temperature, with respect to time, is greater than or equal to 0.5°C. When the rate of change of temperature, with respect to time, is less than 0.5°C per hour it automatically switches off again. Find the actual times (e.g. 2.17 a.m.), to the nearest minute, at which the system switches on and then switches off, during a 24 hour period.

11. [7 marks: 1, 1, 2, 1, 2]

A triangle ABC is shown on the grid below with A(0, 2), B(4, 2) and C(3, 1).



(a) On the same grid, sketch the image ΔA'B'C', after a transformation that rotates each point of the original triangle through 90° anti-clockwise about the origin.

(b) Also, sketch the image ΔA"B"C", when ΔA'B'C' is subjected to a shear transformation, of factor 2, in the *y*-direction.

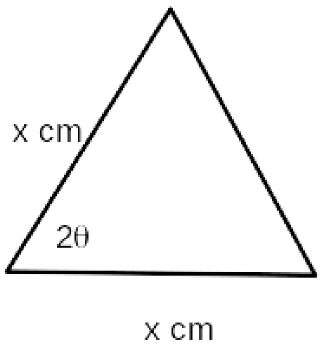
11. (c) Determine the single 2 × 2 matrix that will map ΔABC directly onto ΔA"B"C".

(d) Find the area of each triangle drawn on the grid.

(e) The matrix $\begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$, when used as a transformation matrix, will map all of the points in ΔABC onto a straight line. Give the Cartesian equation of that line.

12. [11 marks: 2, 4; 5]

The diagram below shows an isosceles triangle with two sides both *x* cm and the included angle 2θ radians.



(a) If the perimeter of the triangle is fixed at 100 cm.

(i) Prove that $\sin\theta = \dfrac{50 - x}{x}$.

(ii) Find the exact value(s) of *x* and θ when the area of the triangle is a maximum.

12. (b) The perimeter of the triangle is no longer fixed at 100 cm.

The sides with length x cm are increasing at a constant rate of 1cm per minute.
The included angle is increasing at a constant rate of 0.1 radians per minute.
Find the exact rate at which the area of the triangle is increasing

when x = 10 cm and $\theta = \frac{\pi}{6}$ radians.

13. [7 marks: 2, 2, 3]

Three people, Andrew, Benjamin, and Charles, kick a soccer ball to each other. There is a probability of $\frac{1}{4}$ that Andrew will kick the ball to Benjamin, there is a probability of $\frac{3}{5}$ that Benjamin will kick the ball to Charles and there is probability of $\frac{1}{3}$ that Charles will kick the ball to Andrew. Assume that each person does not kick the ball to himself. This information is summarized in a transition matrix

$$\mathbf{T} = \text{To}\; \begin{array}{c c} & \begin{array}{ccc} & \text{From} & \\ \text{A} & \text{B} & \text{C} \end{array} \\ \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \end{array} & \begin{pmatrix} 0 & \frac{2}{5} & \frac{1}{3} \\ \frac{1}{4} & 0 & \frac{2}{3} \\ \frac{3}{4} & \frac{3}{5} & 0 \end{pmatrix} \end{array}.$$

(a) Given that Andrew had the first kick, find the probability that Andrew will have the ball back after the ball has been kicked twice (this includes Andrew's first kick).

(b) Given that Benjamin had the first kick, find the probability that Charles will have the ball after the ball has been kicked five times.

13. (c) In the long term, who is most likely to end up with the ball? Justify your answer.

14. [7 marks: 3, 2, 2]

In a chemical process, the quantity of an enzyme (Q mg) is modelled by the equation

$\frac{dQ}{dt} = (200 - Q) \times t$ where t is time in hours.

(a) Use integration to find an expression for Q in terms of t.

(b) If the initial amount of the enzyme is 1000 mg, how much remains after 3 hours?

(c) Show clearly why the long term quantity of the enzyme is not dependent on its initial amount.

15. [6 marks]

Use the substitution $x = \frac{5}{2}\sin\theta$, to evaluate exactly $\displaystyle\int_0^{\frac{5}{4}} \frac{1}{\sqrt{25-4x^2}}\,dx$.
Show clearly each step of your working.

16. [6 marks]

A particle P moves in the x-y plane. Its equation of motion is given by:

$\frac{dy}{dt} = 2\sin(2t)$ and $\frac{dx}{dt} = \cos(t)$, where t is time in seconds. Given that the particle P starts from the point (0, 0), find the Cartesian equation of the path traced by this particle.

17. [6 marks]

Prove that $(1 + \cos 2\theta + i \sin 2\theta)^n = 2^n \cos^n \theta \, (cis\, n\theta)$.

18. [7 marks]

Using mathematical induction, prove that, for all counting numbers, n,
$2n\,(2n + 1)(2n - 1)$ is divisible by 6.

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