

	<b>MATHEMATICS SPECIALIST 3CD</b>	Semester 1 2011 EXAMINATION
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NAME:  <hr/>
TEACHER:

## Section One: Calculator-free

### Time allowed for this section

Reading time before commencing work: 5 minutes

Working time for this section: 50 minutes

### Material required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer Booklet

Formula Sheet

#### *To be provided by the candidate*

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

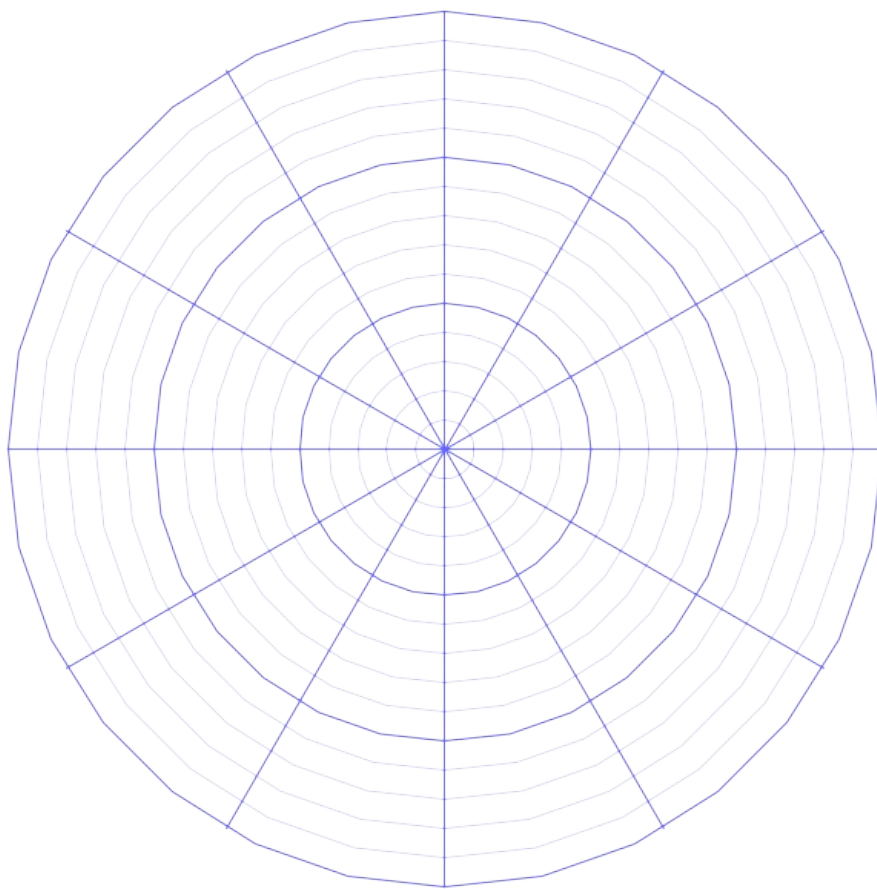
Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	9	9	50	40
Section Two: Calculator-assumed	13	13	100	80
				120

## Instructions to candidates

- Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil** except in diagrams.

QUESTION	MARKS AVAILABLE	STUDENT MARK
1	4	
2	4	
3	4	
4	3	
5	4	
6	6	
7	6	
8	3	
9	6	
TOTAL	40	

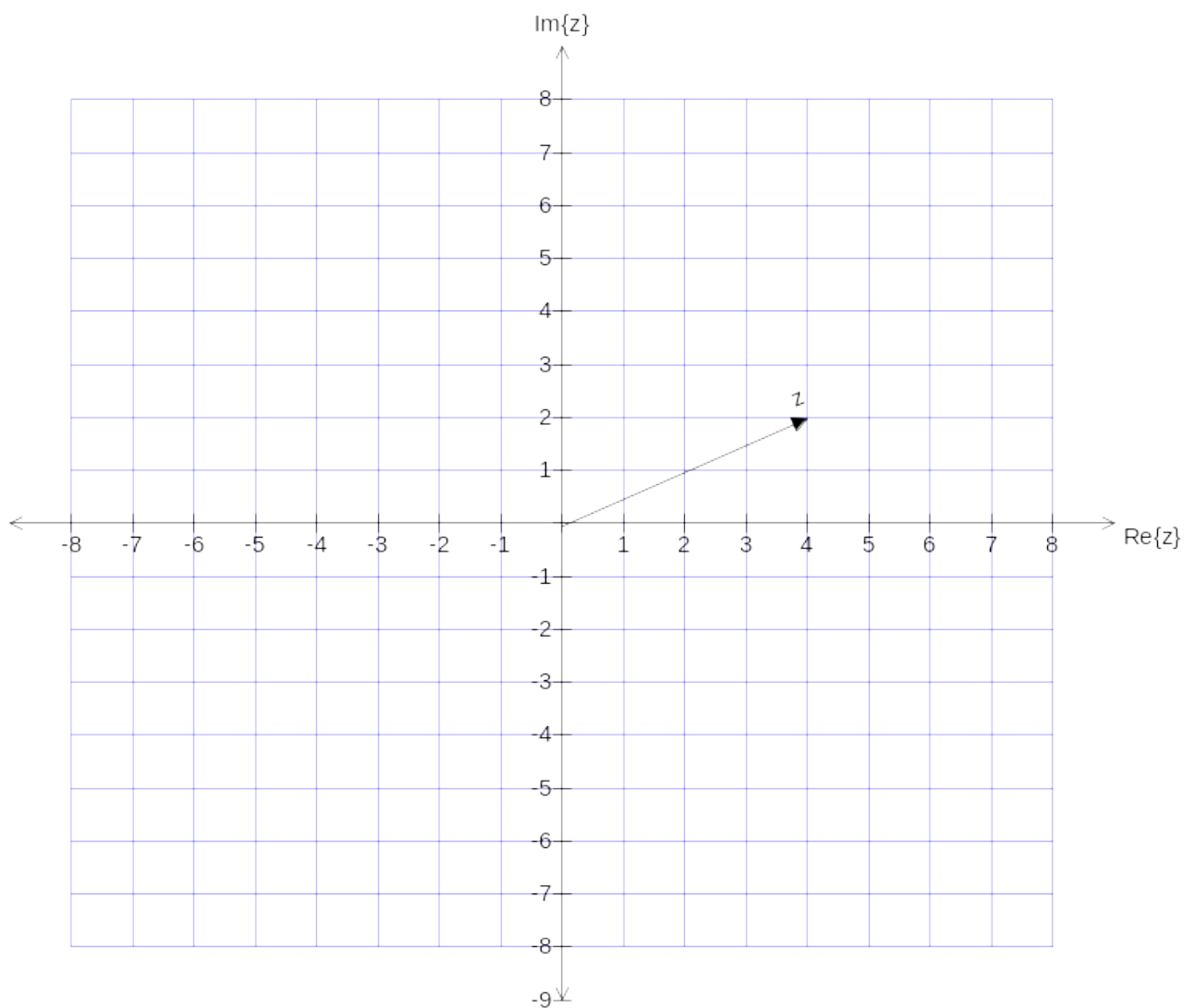
1. (3, 1 = 4 marks)  
(a) On the axes below, draw the graphs of  $r = 3\theta$  and  $r = 4\pi$ , for  $0 \leq \theta \leq 2\pi$ .



- (b). Determine the exact point of intersection of these two graphs in polar form.

2. (1, 1, 1, 1 = 4 marks)

The complex number  $z$  is shown on the Argand diagram below.



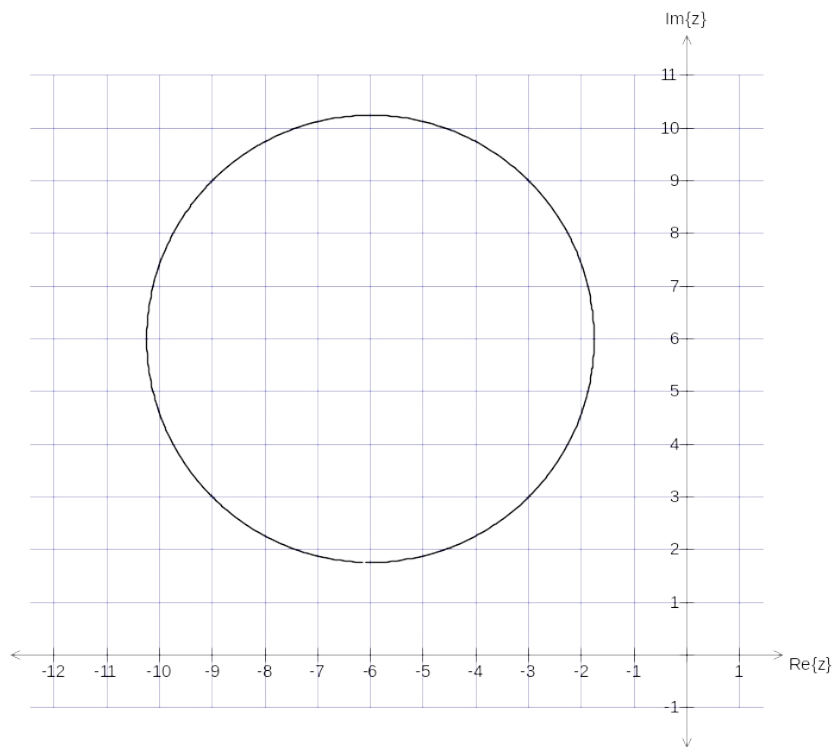
Show the following as vectors on the Argand diagram above.

(a)  $\bar{z}$   
(c)  $2z$

(b)  $-zi$   
(d)  $z(1+i)$

3. (1, 3 = 4 marks)

The locus of a complex number  $z$  is a circle of radius  $3\sqrt{2}$  units as shown below.

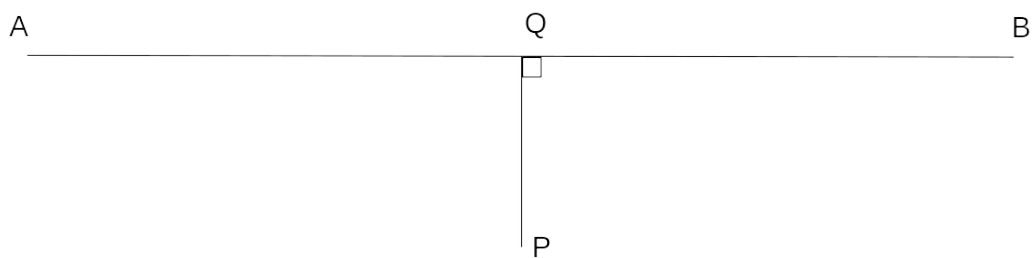


(a) Express the locus mathematically in terms of  $z$ .

(b) Determine the maximum value of  $\text{Arg}(z)$ .

4. (3 marks)  
Give the vector equation of the line that is perpendicular to the plane with equation  $\mathbf{r} \cdot (6\mathbf{i} - \mathbf{j} - \mathbf{k}) = 12$  and containing the  $z$  intercept of the plane.

5. (4 marks)  
Point  $P$  is a point in the plane not on  $\overline{AB}$ . Let  $Q$  be the point such that  $\overline{PQ}$  is perpendicular to  $\overline{AB}$ .



Prove, using the method of contradiction, that the shortest distance from point  $P$  to  $\overline{AB}$  is the distance  $PQ$ .

6. (3, 3 = 6 marks)

Differentiate the following with respect to  $x$ . Do not simplify.

(a)  $y = \sqrt{2x^3} \sin \frac{x}{3}$

(b)  $y = 5 \cos^4(x^2 - 3x)$



7. (5 marks)  
Determine the derivative of  $f(x) = \cos(5x)$  by first principles.

i.e. using  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

8. ( 3 marks)

Prove the following trigonometric identity.

$$\tan^2 x \equiv \frac{1 - \cos 2x}{1 + \cos 2x}$$

9. (2, 4 = 6 marks)

Function  $f$  is defined by  $f(x) = a^{\log_b x}$  for  $x > 0$  where  $a, b$  are positive real constants.

(a) Show clearly that  $f(x)$  can be written in the form  $e^{\frac{(\ln a)(\ln x)}{\ln b}}$ .

(b) Hence, using the expression from part (a), determine  $\int_e^{e^2} \frac{a^{\log_b x}}{x} dx$ .







