Mathematics
Specialist
Test 4 2016

Integration Techniques &
Applications of Integral Calculus

NAME: _____

TEACHER: MLA

Resource Free Section

30 marks
30 minutes

Question 1   [3, 3 & 3 = 9 marks]

Determine the following indefinite integrals:

(a) $\int 5\tan^2(5x)\,dx$

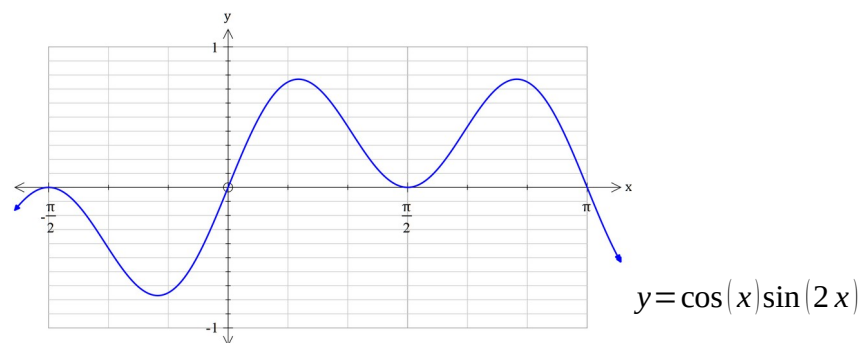(b) $\int 27\tan^2(3x)\sec^2(3x)\,dx$

(c) $\int 8\sin^2(2x)\,dx$

Question 2   [5 marks]

Use the substitution $u = 1 + \sin(x)$ to evaluate $\displaystyle\int_{0}^{\frac{\pi}{2}} \frac{4\cos(x)}{\sqrt{1+\sin(x)}}$
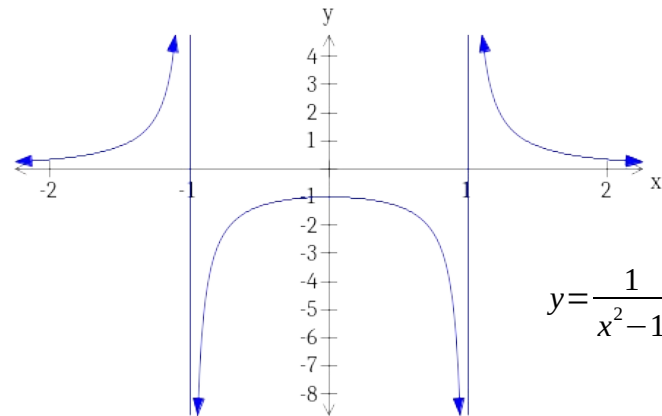
Question 3   [3 & 3 = 6 marks]

(a) $\textit{If } f'(x) = \cos(x)\sin(2x)$, determine f(x).

(b)   Hence, calculate the area between the curve $y = \cos(x)\sin(2x)$ and the x-axis from $x = \dfrac{-\pi}{2} \textit{ и } x = \pi$.

$y = \cos(x)\sin(2x)$

Question 4   [5 marks]

Calculate the exact volume generated by revolving the area trapped $between\ y = \dfrac{1}{x^2-1}$, the vertical axis and the lines $y=-e^2$ and $y=-1\ about\ the\ y\ axis$.

$$y = \frac{1}{x^2-1}$$

Question 5   [2 & 3 = 5 marks]

(a)   If $y = \ln\left(x^{x^2}\right)$, determine $\dfrac{dy}{dx}$

Hint 1: Apply a suitable log law to $y = \ln\left(x^{x^2}\right)$ before differentiating

Hint 2: Do not factorise your final answer

(b)   Hence, find $\int 2x\ln(x)\,dx$

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20 marks

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One unfolded A4 page of notes, SCSA formulae booklet and ClassPad calculator permitted

Question 6   [1 & 1 = 2 marks]

(a)   Express $\int \dfrac{x^2-x+1}{(x+3)(x^2+4)}dx$, in exact terms

(b)   Evaluate $\displaystyle\int_0^{4\pi} \dfrac{x^2-x+1}{(x+3)(x^2+4)}dx$, $correct \; i \; 2 \, decimal \, places$

Question 7   [6 marks]

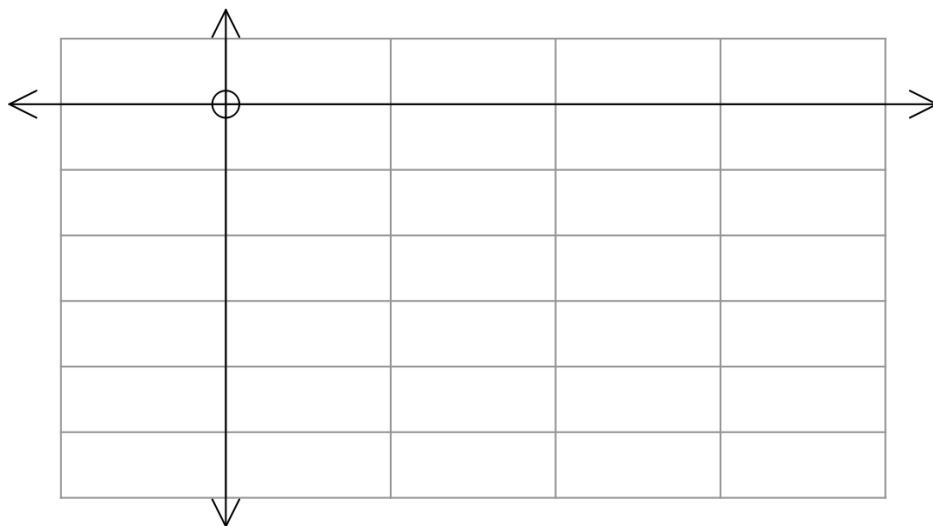Use your knowledge of partial fractions to determine $\int \dfrac{7x^2-2x+5}{(x-1)(x^2+1)}dx$

Show clear working.

Question 8   [3, 2, 2 & 1 = 8 marks]

Consider the functions $f(x)=\dfrac{\sqrt{x}(x^2-5x)}{2}\wedge g(x)=-3\sqrt{x}$

A, B and (0, 0) are the three points of intersection of the aforementioned functions.

(a)   Draw a neat sketch of f(x) and g(x) on the axes below. Label points A and B.

(b)   State the ordered pairs for points A and B, correct to 2 decimal places.

(c)   State the definite integral that defines the area enclosed by f(x) and g(x) between points A and B.

(d)   Use your Classpad to determine the area described in (c).

Question 9   [4 marks]

In geometry, a torus is a surface of revolution generated by revolving a circle in 3-dimensional space about an axis co-planar with the circle.
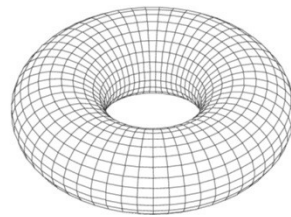
Use calculus to determine the volume of the torus formed by rotating the circle with equation $x^2+(y-2)^2=1$ about the x-axis.