Year 12 – Physics
Motion and Forces in a Gravitational Field
Part 2 – Circular Motion
Notes - 2013

Reviewed 14/02/2011

Outcomes / Objectives from Motion and Forces in a Gravitational Field Covered in this booklet

- *describe and apply the principle of conservation of energy*
- *resolve, add and subtract vectors in one plane*
- draw free body diagrams, showing the forces acting on objects, from descriptions of real life situations involving forces acting in one plane
- explain and apply the concept of centre of mass
- explain and apply the concepts of centripetal acceleration and centripetal force, as applied to uniform circular motion—this will include *applying the relationships*:

$$a_c = \frac{v^2}{r}, \quad \text{resultant F} = ma = \frac{\text{mv}^2}{r}$$

Context

Student unit learning contexts for **motion and forces in a gravitational field** may include**:**
- playground equipment
- physics in sport
- space travel
- planetary motion
- fairground physics
- bridges and buildings.

Texts
- Heinemann
- Stawa

Name ______________________________

Parts of Booklet
Newton's Laws Theory
Free Body Diagrams
Horizontal Circles
Vertical Circles
Ridiculous Answers
Word Answer Questions
Additional Resources / Questions
Appendix on Friction.
Appendix on Matrix Algebra and Simultaneous Equations
Free Body Diagram Solutions

What are Newton's 3 laws of motion (again)?

1st ____________________

____

____

2nd ____________________

____

____

3rd ____________________

____

____

What physical situation does each law describe?

1st ____________________

____

____

2nd ____________________

____

____

3rd ____________________

____

____

What behaviours does an object exhibit when it **is not** experiencing an unbalanced force? ($\Sigma F = 0$)

____

____

What behaviours does an object exhibit when it **is** experiencing a net / unbalanced / resultant force? ($\Sigma F = ma \neq 0$)

____

____

*__Activity –__ For each situation state if the net force is zero and indicate what behaviours the object exhibited that lead you to this answer.

	Situation	**ΣF = 0?**	**Behaviour of object that supports this.**
1.	Car travels in a straight line at a constant speed along a flat road.		
2.	Car turns around a corner at a constant speed.		
3.	Car drives in a straight line up a hill of 30.0^0 at a constant speed.		
4.	Car hits a tree and changes shape.		
5.	Ball dropped from the hand on the moon accelerates towards the ground.		
6.	Ball <u>collides</u> with the ground causing it to change direction (down to up).		
7.	A ball <u>in flight</u> having been thrown upwards on the moon by an astronaut.		
8.	Ball falling through the earth's atmosphere at terminal velocity.		

9. Are objects moving in a circle experiencing $\Sigma F = 0$ or $\Sigma F \neq 0$? Explain with supporting observations / diagrams.

What are the 3 circular motion formula?

Formula 1 - Circular Speed

To calculate circular speed we use a variation of…

$$v = \frac{s}{t}$$

that is …

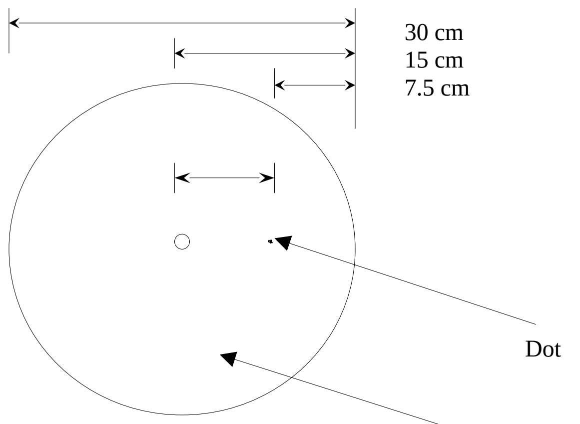
$$v = \frac{2\pi r}{T}$$

Where

Symbol	Definition	Vector / Scalar	Units
v =	Speed at which the object is moving in a circle	Scalar	$m\ s^{-1}$
$2\pi r$ =	Distance (circumference) around the circular path the object follows	Scalar	m
r =	Radius of the circle the object follows	Scalar	m
T =	Time (period) taken for the object to complete one lap of the circle	Scalar	s

Example - Circular speed calculation

A record of diameter 30 cm is placed on a 33 $^1/_3$ RPM turntable. A dot of liquid paper is placed on the record, half way from the centre. What is the speed of the dot?



$f = 33 \frac{1}{3}$ RPM
f = 33.333 cycles per minute.
f = 33.333 cycles per 60 seconds
f = 33.333 / 60
f = 0.5555 cycles per sec.
f = 0.555 Hz

T = 1 / 0.55555
T = 1.8 s

Diameter = 30.0 cm
r = 15.0 cm
Dist from centre to dot = 7.5 cm = **7.5×10^{-2} m**

$v = 2\pi r / T$
$v = 2\pi \times 7.5 \times 10^{-2} / 1.8$

$v = 2.62 \times 10^{-1}$ m/s

Activity

A car is spinning its wheels on ice and getting nowhere. A chalk mark is on one of the spinning tyres. A computer takes a picture of the wheel every 0.1 seconds. If the radius of the wheel is 35.0 cm, at what speed(s) can the wheel travel so that each photograph shows the chalk mark in exactly the same position.

(2.20×10^{1} m s^{-1} or whole number multiples there of.)

Formula 2 - Centripetal Acceleration

Centripetal acceleration is the acceleration of the object towards the centre of the circle. F_c and a_c both have the same direction (towards the centre to the circle). The centripetal acceleration is caused by the centripetal force (resultant force).

$$a_c = \frac{v^2}{r}$$

Symbols	Definition	Vector / Scalar	Units
a_c =	Centripetal acceleration caused by the component of the resultant force that is towards the centre of the circle.	Vector	$m\ s^{-2}$
v =	Speed of the object as it moves around the circle.	Scalar	$m\ s^{-1}$
r =	Radius of the circle.	Scalar	m

Derivation – still under development.

Imagine a car travelling around a round-about in a continuous circle.

When on one side of the circle the car is travelling to the left. When it has gon half way around it is travelling to the right.

Acceleration is given by the formula ...

$$a = \frac{v - u}{t}$$

If to the right is +ve and the left is –ve and the cars is travelleing at a constant speed the formula can be modified to ...

$$a = \frac{v - (-v)}{t}$$

If the car is travelling at a constant speed then $v = 2\pi r / t$

The time taken to travel half way round the circdle is so given by ...
$T = \pi r / v$

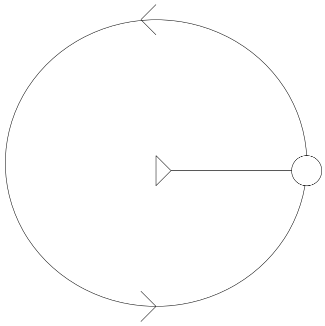
Subbing this in we get

$$a = \frac{2v}{\pi r / v}$$

a =

Example - Centripetal acceleration calculation

An Olympic hammer thrower will spin a hammer at 3.00 Hz just before release. The radius of the circle is 0.700 m. What is the centripetal acceleration?



r = 0.700 m
f = 3.00Hz
T = 1/f
T = 1/3
T = 0.3333s

a_c =

$a_c = v^2/ r$

v = ?

$v = 2\pi r / T$
$v = 2\pi \times 0.700 / 0.3333$

$v = 1.31 \times 10^1$ m/s

$a_c = v^2/ r$
$a_c = (1.31 \times 10^1)^2/ 0.7$

$a_c = 2.49 \times 10^2$ m/s^2 towards the centre of the circle.

Activity – (the school has a machine to demonstrate this)

A test tube is in a centrifuge. A centrifuge spins objects in a circle to separate different parts of colloids (e.g. blood). The centrifuge diameter is 20.0 cm. At what speed must the fluid in a test tube move in order to experience a centripetal acceleration, 3 times the strength of gravity towards the centre of the circle?

($v = 1.71 \times 10^{0}\ m\ s^{-1}$)

Formula 3 - Centripetal Force

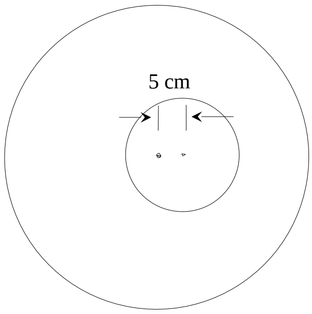
$$F_c = \frac{mv^2}{r}$$

Symbols	Definition	Vector / Scalar	Units
ΣF =	Sum total of all the forces acting on the object	Vector	N
m =	Mass of the object moving in a circle	Scalar	kg
a =	Acceleration of the object due to the resultant force	Vector	$m\ s^{-2}$
R =	Resultant force	Vector	N
F_c =	Centripetal Force	Vector	N
v =	Speed of the object moving in a circle	Scalar	$m\ s^{-1}$
r =	Radius of the circle	Scalar	m

Example – Centripetal force calculation

A person is learning pottery. They place a 3.00 kg lump of clay on the potter's wheel and set it spinning at 4.00 Hz. The clay is not quite in the middle of the wheel and begins to wobble. If the centre of the lump of clay is 5.00 cm from the centre of the wheel, what frictional force will be required to stop the clay slipping off the wheel?

$F_f = F_c$



$T = 1 / f$
$T = 1 / 4$
$T = 0.25$ s

$v = 2\pi r / T$
$v = 2\pi \times 5.00 \times 10^{-2} / 0.25$
$v = 1.26$ m/s

$F_c = m v^2 / r$
$F_c = 3 \times 1.26^2 / 0.05$
F_c = 95.2 N towards the centre of the potter's wheel.

Activity

What is the force of friction required to allow a 1.50 tonne car to successfully navigate a curve of radius 30.0 m if the car is travelling at 60.0 km h^{-1}?

($F_C = 1.38 \times 10^4$ N Towards the centre of the circle)

How do you calculate the centripetal force acting on an object moving in a horizontal circle when there is more than one force acting on the object?

You have to draw a clear free body diagram and then create vertical and horizontal mathematical equations from the free body diagram.

How do you draw free body diagrams for horizontal circular motion?

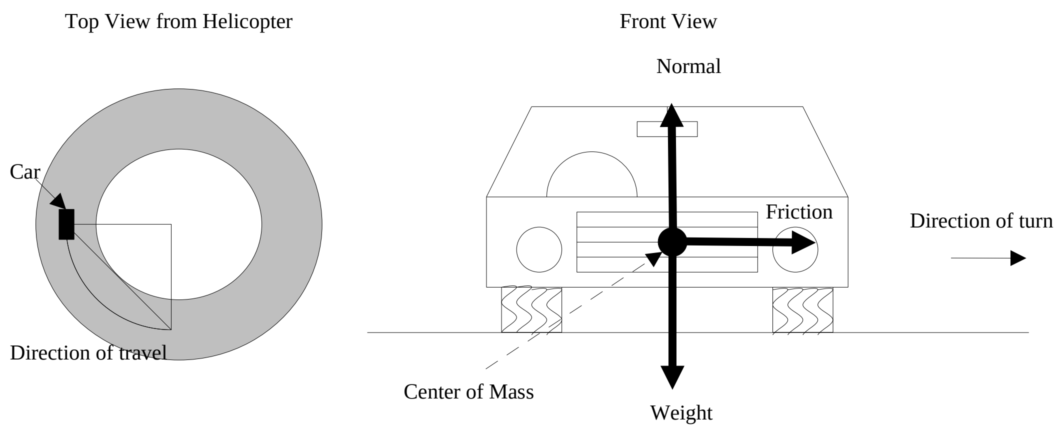
Free Body Diagrams

A “free body diagram” shows all the forces acting on a body. In circular motion situations, we are talking about all the forces acting on the body / object that is moving in a circle. All the forces are drawn where they attach to the object. In situations where we are ignoring torque (toppling effects of forces) we draw all of the forces acting through the centre of mass of the object. Torque will be discussed in the “Structures” section of the course and centre of mass will be discussed in the next booklet (universal gravitation and satellite motion).

Example

Draw a free body diagram of a car turning around a curve while travelling at a constant speed. The car is turning to the drivers left.

Free Body Diagram



Some important things to notice…

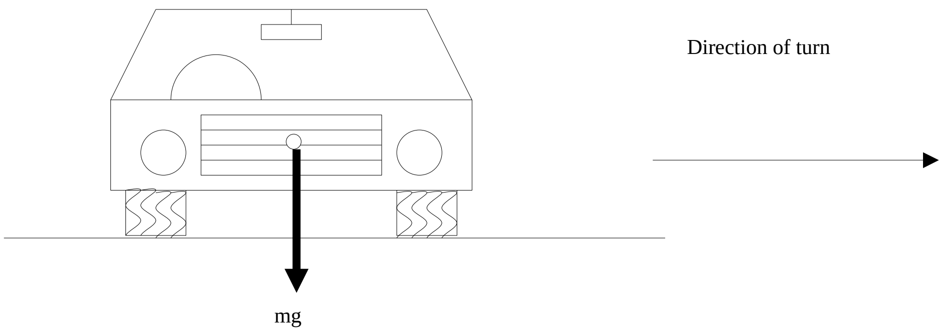
The ground is *not always* shown because the car is drawn as a free body that is not in contact with any of the bodies (objects). That is why it is called a free body diagram.

The force of the ground on the car is the Normal force. The Normal force is the force of the ground pushing on the car.

Only forces acting **on the car** are shown. I.e. Forces acting on other objects are not shown e.g. forces acting **on the road** etc.

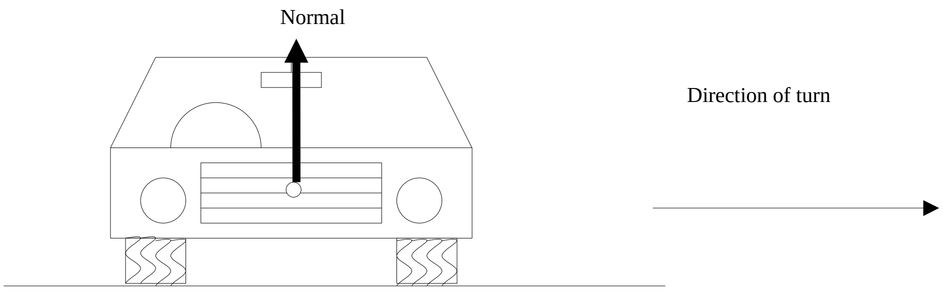
Some Forces that are commonly found on free body diagrams and how to draw them

1. **Weight Force**: - The weight force is the force of attraction between the object and the earth. It is equal to Wt = mg. *The direction of the weight is always <u>straight down</u> towards the centre of the earth under <u>all</u> circumstances*.
E.g. - in the car example

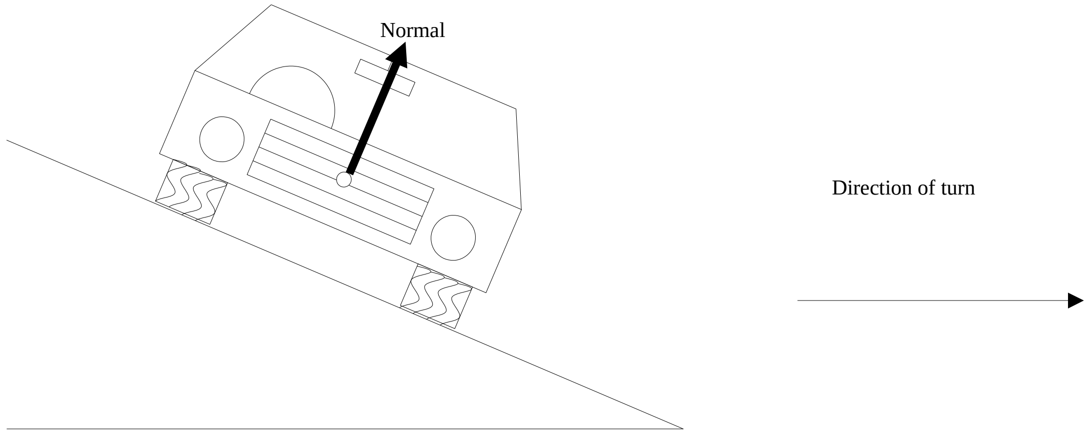


2. **Normal Force**: - The Normal force is the force of the ground on the object. The normal force is calculated differently depending on the situation. *The direction of the normal force is always at right angles to the ground. If the ground is sloped / banked, the normal is at right angles to the slope. The Normal force is **only** equal to the weight force when the car is on flat ground.*

E.g. car on flat surface



Car on a curved bank or berm of about 30^0

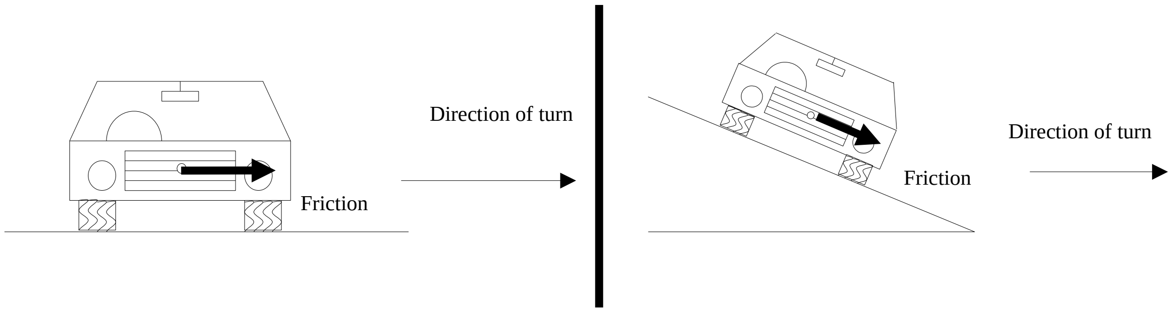


3. **Friction Force**: - Before we cover friction in free body diagrams it is wise to quickly revise the concept of friction from Year 11. These revision notes are located at the back of the booklet in “Appendix 1 - Supplementary Notes on Friction”

Friction…

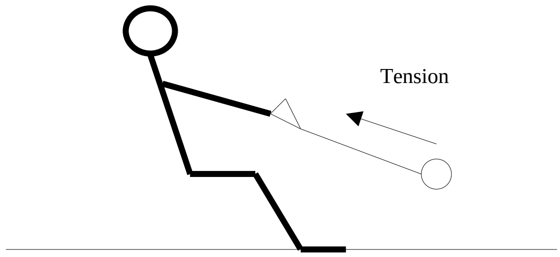
- Is always parallel with the ground.
- If the object uses kinetic friction (sliding) it is opposite in direction to the velocity
- If the object uses static friction (rolling), it can be in any number of directions depending on the situation. The direction of the static friction will become clear based on how the object is moving and the other forces applied. In circular motion problems, the friction on the object is…
    - Tangential to the circle for sliding (skidding) friction
    - Parallel to the radius of the circle for static (which includes rolling) friction.

The friction on this car making a turn to the drivers left is as follows…



4. **Tension**: - The tension force is the force of a string, wire of chain on the object. The tension force is calculated from the free body diagram of the situation. *Tension always acts in the direction **<u>from</u> the object <u>along</u> the string.***

Eg a hammer thrower swinging a hammer in a circle.



How do you draw horizontal circular motion free body diagrams?

By hopping in and giving it a go with teacher support. Try these with the help of your teacher.

<u>Horizontal Circular Motion Situations</u>

1. A car stationary on a flat road (front view)
2. A car moving at a constant velocity on a flat road in a straight line (front view)
3. A car moving at constant speed on a flat road and turning to the <u>drivers</u> left (front view).
4. A car parked sideways on a bank / hill (front view).
5. A car moving at constant velocity along a bank in a straight line (front view).
6. A car turning along a berm at constant speed that is banked to the drivers left (front view).

----

7. A person dragging an athletics hammer across some grass at a constant velocity in a straight line (side view).
8. A person dragging a hammer across some grass so that it accelerates forward in a straight line (side view).
9. A person moving a hammer in circular motion just before throwing it (side view).

----

10. A person riding a bike down a flat road at constant velocity in a straight line (front view).
11. *A person turning a corner on a bike on a flat road at constant speed (front view).
12. A person turning along a berm at constant speed (front view).

----

13. A person standing (front view).
14. A person walking forward in a straight line (front view).
15. A person running around a corner at constant speed on flat ground (front view).
16. A person running around a berm at constant speed (front view).

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16		

Solutions at end of booklet

Steps to Analysing and Solving a Circular Motion Question in the Horizontal Plane

1. **Draw a free body diagram** of the situation usually so that the objects velocity is pointing at you out of the page (side view or front view, not top view)

2. **State the law -** because the object is moving in a circle the forces on the object will not be zero. There will be a resultant force acting on the object towards the centre of the circle.

I.e. $\Sigma F = \text{Resultant} \approx F_c \neq 0$

3. **Write component equations -** because the motion is uniform circular motion in the horizontal plane, the resultant force will be horizontal towards the centre of the circle. All vertical forces will cancel each other out.

$$\Sigma F_v = 0$$

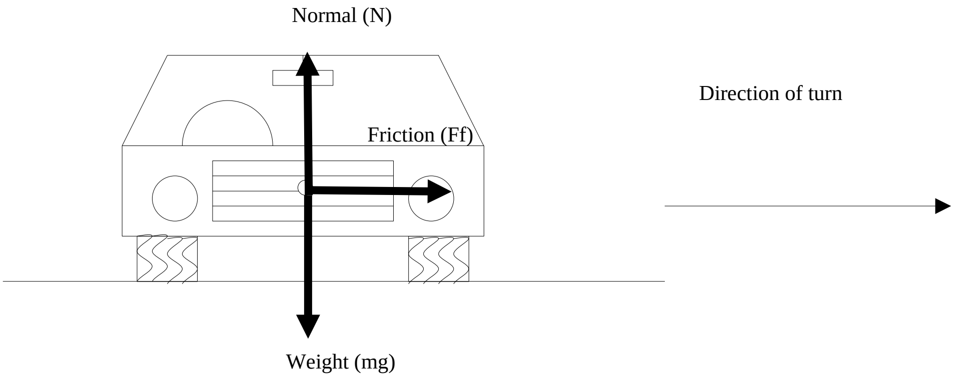
$$\Sigma F_h = F_c \quad \text{(Force Centripetal)} \quad = \frac{mv^2}{r}$$

4. **Allocate** the vertical and horizontal components of the vectors in the free body diagrams to the 2 equations **and solve** for the unknown quantity.

Example 1 A 1500 kg car is cornering around a bend of radius 20.0 m at 50.0 km h^{-1} on a flat road. What is …

a) the frictional force required
b) the normal force of the road on the car.

a)
Step 1 – Draw the free body diagram.



Step 2 – State the law
$\Sigma F = F_c$
Step 3 – Form vertical and horizontal equations

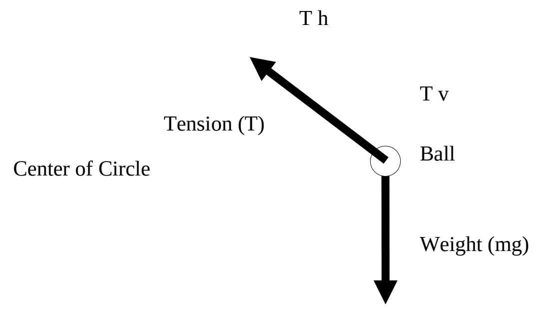
$\Sigma F_V = 0$	$\Sigma F_H \neq 0$
$(-mg) + (+N) = 0$ $(-1500 \times 9.8) + (N) = 0$ $\mathbf{N = 1.47 \times 10^4\ N}$	$+F_f = m v^2 / r$ $F_f = 1500 \times v^2 / 20.0$ 50 km/h = 13.8888 m/s $F_f = 1500 \times 13.888^2 / 20.0$ $\mathbf{F_f = 1.45 \times 10^4\ N}$

Note the sign convention!

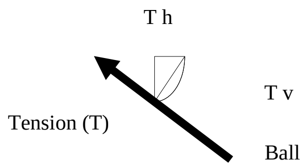
Example 2 A 200 g ball attached to a string is swung around your head at 10.0 m s^{-1} in a horizontal circle. The radial distance from the ball to the person is 40.0 cm.

a) What is the tension in the string?
b) What angle does the string form with the horizontal?

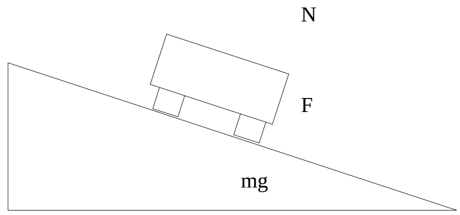
a)
Step 1 – Draw the free body diagram.



Step 2 – State the law
$\Sigma F = F_c$
*Step 3 – Form vertical and horizontal equations



Activity (Tricky) A car travels around a curve in the road. The diameter of the curve is 100 m. The surface of the curve is banked at 30.0^0 to the horizontal to assist the car in negotiating the curve. Calculate the normal force on the car if the car is travelling at 110 km h^{-1} and has a mass of 2.00 tonnes.



$\Sigma F = Fc$
*

H	V
$F_h + N_h = F_c$ $F \cos 30 + N \sin 30 = mv^2 / r$	$-F_v + - mg + N_v = 0$ $(- F \sin 30) + (-mg) + (+N \cos 30) = 0$

(N = 3.56 x 10^4 N)
(F_F = 2.25 x 10^4 N

What is the “reaction” force?

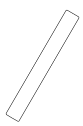
The reaction force is the vector sum of the normal force and the friction force. Both of these forces are as a result of the object being in contact with ground. These forces only occur when the weight force of the object pulls the object into contact with the ground.

i.e.
Action = object (through its) weight acts on the ground.
Reaction = ground through reaction force (Friction and Normal combined) act on object.

When bikes and runners (objects with a point base) move around flat curves the angle formed between the bike / runner and ground, matches the angle of the reaction force.

Activity – Onto the below diagrams draw the Centre of mass, Weight, Normal, Friction. In a different colour list the reaction force. Both the runner and bike are turning to the right hand side of the page.

Bike



Runner

IMPORTANT NOTE - Strictly speaking you should **only** list the Friction and Normal forces **or** the Reaction force. If you list both you are doubling up (showing that there are 2 sets of forces when there is only one).

<u>What Do We Do Now? (W^2D^2N)</u>

Heinemann 3AB – Book – Chapter 1.2 – Circular Motion in Horizontal Plane p33 – 41

Heinemann 3AB – Book – Chapter 1.2 – Questions p41 - 42

Stawa Set 3

How do draw vertical circular motion free body diagrams?
Try the below with the assistance of your

Vertical Circular Motion Situations

1. A glider moving at constant velocity (side view)
2. A glider doing a loop the loop...
   a) At the top of the loop (side view)
   b) At the bottom of the loop (side view)
3. A carriage on a ferris-wheel moving in a circle…
   a) At the top of the loop (side view)
   b) At the bottom of the loop (side view)
4. A gymnast swinging around a bar…
   a) At the top of the loop (side view)
   b) At the bottom of the loop (side view)
5. A racing car with aerofoils travelling straight along a pipe upside down (front view)
6. A tennis racket at the top of the curve during a tennis serve. Ignore the force of the ball on the racket (side view).
7. A golf club at the bottom of its swing. Ignore the force of the ball on the club (side view).
8. A roller coaster doing a loop the loop…
   a) At the top of the loop (side view)
   b) At the bottom of the loop (side view)

1	2a	2b
3a	3b	4a
4b	5	6
7	8a	8b

Solutions at end of booklet

Steps to Analysing and Solving a Circular Motion Question in the Vertical Plane

1. **Draw a free body diagram** of the situation usually so that you can see the full circle of the movement (side view not top or front view). The question will indicate if you should draw the object at he top or the bottom of the circle.

2. **State the law -** because the object is moving in a circle the forces on the object will not be zero. There will be a resultant.

Ie $\Sigma F = R \approx F_c \neq 0$

3. **Write component equations -** because the motion is circular motion in the vertical plane, the resultant force will have a component towards the centre of the circle. *If the object is at the top or the bottom of the circle the total resultant force will be towards the centre of the circle, not just a component. When the object is at the top or the bottom of the circle, all horizontal forces will cancel each other out.*

Equations for objects at the top or the bottom of the circle

$\Sigma F_h = 0$ (There are ususlly not horizontal forces on the object)

$\Sigma F_v = F_c$ (Force Centripetal) $= \frac{mv^2}{r}$

Your second equation is a conservation of energy equation.

$mgh_i + \frac{1}{2} mu^2 = mgh_f + \frac{1}{2} mv^2$

Remember the velocity will not be the same at the top of the circle as it is at the bottom. It is faster at the bottom (loose Ep and gain Ek).

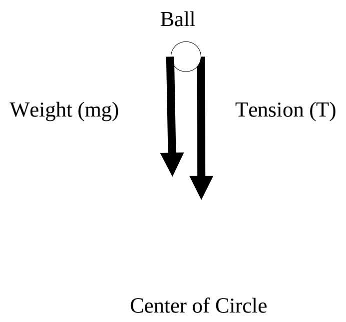
4. **Allocate** the vertical and horizontal components of the vectors in the free body diagrams to the 2 equations **and solve** for the unknown quantity.

Example 1 A ball is swung in a vertical circle or radius 30.0 cm on a string. The ball has a mass of 50.0 g and a velocity at the top of the string of 4.00 m s^{-1}. What is…

a) the tension in the string at the top of the loop
b) the tension in the string at the bottom of the loop
c) the speed at which the string will just start to go slack at the top of the loop?

a) the tension in the string at the top of the loop

Step 1 – Draw the free body diagram.



Step 2 – State the law

$\Sigma F = F_c$

Step 3 – Form vertical and horizontal equations

$\Sigma F_V = m v^2 / r$	$\Sigma F_H = 0$
$(- mg) + (- T) = -(mv^2 / r)$ $(mg) + (T) = mv^2 / r$ $(50 \times 10^{-3} \times 9.8) + T = 50 \times 10^{-3}\ 4^2 / 0.30$ $0.49 + T = 2.66666$ **T = 2.18 N**	No horizontal equation

b) the tension in the string at the bottom of the loop

Step 1 – Draw the free body diagram.

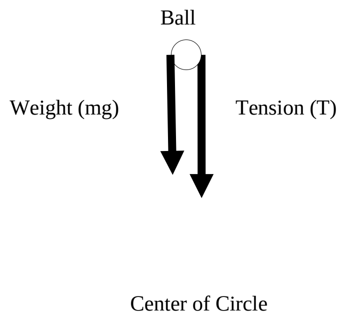
Step 2 – State the law
$\Sigma F = F_c$

Step 3 – Form vertical and horizontal equations

$\Sigma F_V = m v^2 / r$	**$\Sigma F_H = 0$ (Cons of energy)**
$(- mg) + (+ T) = = +mv^2 / r$ $(-50 \times 10^{-3} \times 9.8) + T = 50 \times 10^{-3} v^2/ 0.30$ Get v form the equation at the right … $-0.49 + T = 50 \times 10^{-3}\ 5.27^2/ 0.30$ $-0.49 + T = 4.629$ **$T = 5.12$ N**	No horizontal equation but by cons.of energy … $E_{top} = E_{bottom}$ $\frac{1}{2} m u^2 + mgh = \frac{1}{2} mv^2$ $\frac{1}{2} u^2 + gh = \frac{1}{2} v^2$ $0.5 \times 4^2 + 9.8 \times 0.6 = 0.5 \times v^2$ $v^2 = 27.76$ **$v = 5.27$ m s^{-1}**

c) the speed at which the string will just start to go slack at the top of the loop?

Step 1 – Draw the free body diagram.



Step 2 – State the law
$\Sigma F = F_c$

Step 3 – Form vertical and horizontal equations

$\Sigma F_V = 0$	$\Sigma F_H = 0$
Ball begins to fall out of loop when the tension in the string = 0 $(-mg) + (-T) = -mv^2 / r$ $(mg) + (T) = mv^2 / r$ Let $T = 0$ $(mg) += mv^2 / r$ $(50 \times 10^{-3} \times 9.8) = 50 \times 10^{-3} v^2 / 0.30$ **$v = 1.71$ m s^{-1}**	No vertical equation

Example 2 A 750 kg car travels over the top of a hill. The hill is circular near the top with a radius of 20.0 m.

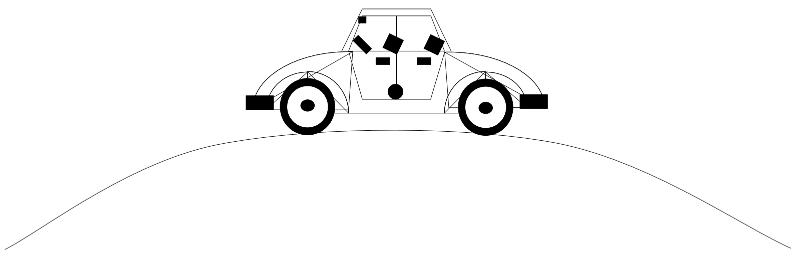
a) What speed is the car travelling if the normal force on the car is exactly half the weight force as it passes over the top of the hill?

b) What speed must the car travel at to just become airborne (“weightless”) as the car passes over the top of the hill?

c) If the car travels at 17.0 m/s over the hill, what is the direction of the normal force on the car if the car is required to stick to the road surface? Can the road provide a force in this direction? What consequences does this have for the car?

a) What speed is the car travelling if the normal force on the car is exactly half the weight force as it passes over the top of the hill?

Step 1 – Draw the free body diagram.



Step 2 – State the law

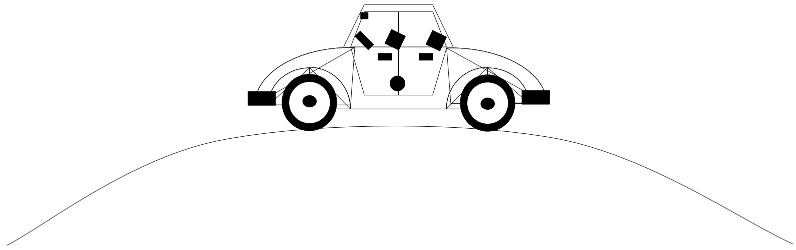
$\Sigma F = F_c$

*Step 3 – Form vertical and horizontal equations

$\Sigma F_V = m v^2 / r$	$\Sigma F_H = 0$
$(-mg) + (+N) = -mv^2 / r$ $(-mg) + (+\tfrac{1}{2}mg) = -mv^2 / r$ $(-g) + (+\tfrac{1}{2}g) = -v^2 / r$ $(-9.8) + (+4.9) = -v^2 / 20.0$ $(-4.9) = -v^2 / 20.0$ $\mathbf{v = 9.90\ m\ s^{-1}}$	No horizontal equation

b) What speed must the car travel at to just become airborne ("weightless") as the car passes over the top of the hill?

Step 1 – Draw the free body diagram.



Step 2 – State the law

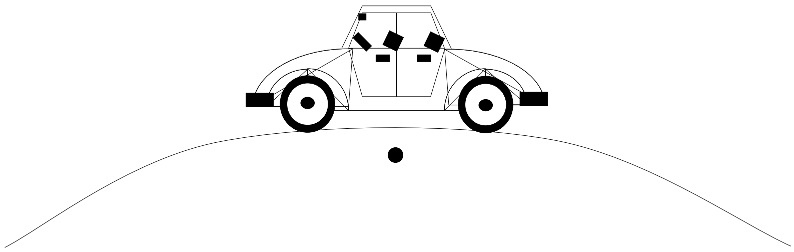
$\Sigma F = F_c$

*Step 3 – Form vertical and horizontal equations.

$\Sigma F_V = m v^2 / r$	$\Sigma F_H = 0$
The car becomes "weightless" when the normal force = 0 $(- mg) + (+ N) = -mv^2 / r$ Let N = 0 $(-mg) = -mv^2 / r$ $g = v^2 / r$ $9.8 = v^2 / 20.0$ **v = 14.0 m s^{-1}**	No horizontal equation

c) If the car travels at 17.0 m/s over the hill, what is the direction of the normal force on the car if the car is required to stick to the road surface? Can the road provide a force in this direction? What consequences does this have for the car?

Step 1 – Draw the free body diagram.



Step 2 – State the law
$\Sigma F = F_c$

*Step 3 – Form vertical and horizontal equations

$\Sigma F_V = m v^2 / r$	$\Sigma F_H = 0$
It is unclear in what direction the normal force will be acting. We will guess a direction (up) and then let the mathematics tell us if the guess was correct. $(- mg) + (+ N) = -mv^2 / r$ $(-750 \times 9.8) + (+ N) = -750 \times 17.0^2 / 20$ $(-7350) + (+ N) = -10837.5$ $N = -10837.5 + 7350$ **$N = - 3.49 \times 10^3$ N**	No horizontal equation

The minus sign means that the direction we guessed for the Normal force was incorrect. It is actually down. Surfaces (normal forces) cannot pull downwards, they can only push upwards. Consequently, this answer is physically impossible. If the car attempts to drive over this hill at 17.0 m/s it will become airborne and follow a pathway with a larger radius than 20.0 m i.e.($-mg = -mv^2/r$ such that N = 0).

Remember
Tension forces only pull.
Normal forces only push.

Activity A pilot is flying a glider in a vertical loop. The loop's radius is 250 m. The glider has a mass of 500 kg (including pilot). The speed of the glider is 300 km h^{-1} at the top of the loop.

a) Will the glider be flying faster or slower at the top of the loop? Explain why.

(Slower. Conservation of energy)

b) Calculate the "lift" force (L) force on the glider when it is at the top of the loop.

(N = 8.99×10^3 N down)

c) Calculate the lift force on the glider at the bottom of the loop.

(v = 129.4 m/s, L = 3.84×10^4 N)

Caution - Nonsensical Answers.

Mathematics can do anything but physics cannot. The mathematics of a situation will sometimes provide an answer that is physically impossible. Physics can be trusted but sometimes maths cannot. It just so happens that these nonsensical answers tend to happen at the top of vertical circles so this is the situation to be on you guard.

Situation 1 – The Normal Force

On your free body diagram always choose the direction of the Normal force to be pointing away from the surface on which the object sits. When you solve for the Normal force in the calculation, if the answer comes out negative, the situation is physically impossible and the object will change its radius and leave the surface (become airborne) and temporarily become a projectile.

i.e. – if Normal force is negative, the situation is physically impossible

Some Examples

Car over top of hill too quickly	**Roller coasters inside loop too slowly.**

Situation 2 – Tension

Always draw the direction of the Tension force from the object towards the string or wire attached. When you solve for the Tension force in the calculation, if the answer comes out negative, the situation is physically impossible. In reality the object will shorten its radius or string or wire will go slack and object will start moving as a projectile instead..

i.e. – If the Tension force is negative the situation is physically impossible.

Some Examples

.**Ball on end of string travels too slowly**	

In both the situations above, it relies on you drawing the Normal and or Tension forces according to the rules of free body diagrams…

Normal away from surface and tension towards the string.

The boundary between when an answer is sensible and is not sensible occurs when the tension or normal force = 0. To calculate the threshold value at which the behaviour changes set …

Normal or Tension = 0

Re inspect the previous examples you have tried again now that you know this.

Activity
Draw some free body diagrams and correctly draw and label the normal or tension force in each.

a) vehicle going over crest of hill	b) roller coaster upside down inner loop	c) ball on string at top of loop

What Do We Do Now? (W^2D^2N)

Heinemann 3AB – Book – Chapter 1.3 – Circular Motion in Vertical Plane
p42 – 49.

Heinemann 3AB – Book – Chapter 1.3 – Questions p49

Heinemann 3AB – Book – Chapter Review Questions
- (Projectile motion and circular motion)
p50 - 51

Stawa Set 3 (New) – Circular Motion – This only contain horizontal circular motion
Old Stawa Set14 – Motion in a circle – This contains vertical and horizontal circles.

List of possible practical activities.

- Rubber stopper on string prac.
- Coin in lamp shade.
- Marble in a filter very large filter funnel prac.
- Go cart practical.

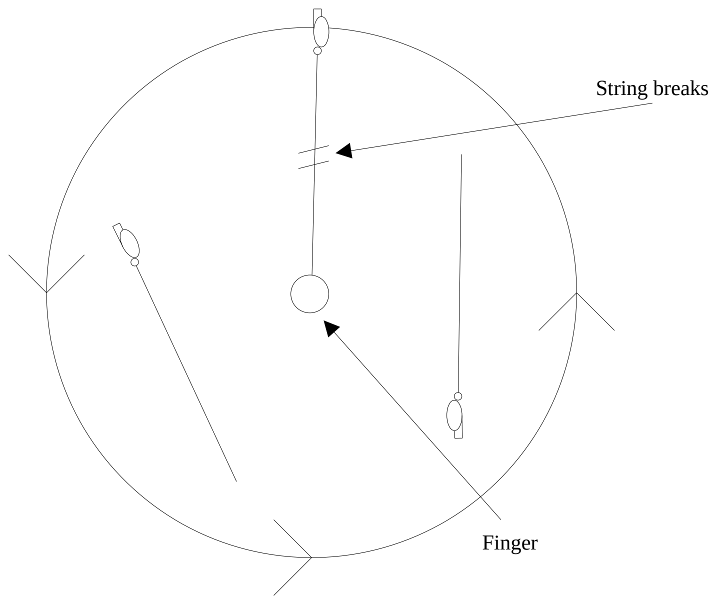
Word Answer Questions – General approach

- Draw a diagram.
- Use / build a mathematical equation.
- Talk about what happens to the answer when changes are made to the variables / equation.

Activity

A sports whistle on the end of a piece of string is swung in a circle on the end of a person's finger in empty outer space.

<u>Top view diagram of situation</u>



a) Why does the whistle move in a circle?

b) If the string breaks what will happen to the whistle? Why?

c) Estimate the centripetal force on the whistle if it moves with a frequency of 4.00 Hz on the end of a 15.0 cm piece of string.

(Fc = 1.89 N towards the center of the circle)

d) What is the force on the finger (assuming no friction)

(Fc = 1.89 N towards the whistle)

<u>Additional Resources</u>

Thought Experiment 1 - What is the center of mass of an object?

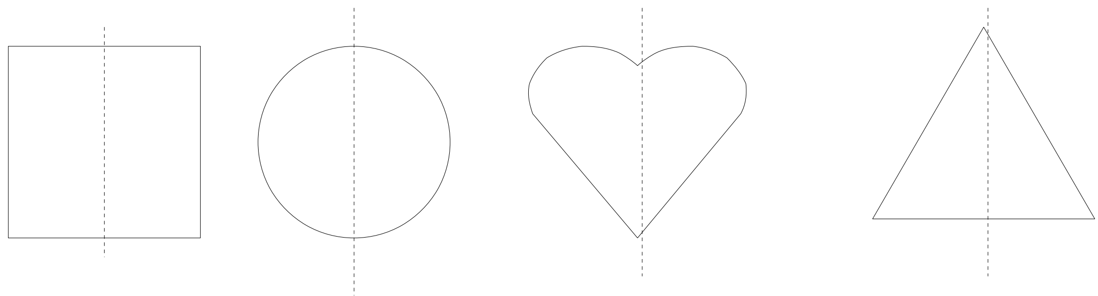
The center of mass of a body is the point (usually) inside the body through which the total mass (weight) of the body will act. This is commonly called the point of balance of the object.

Example

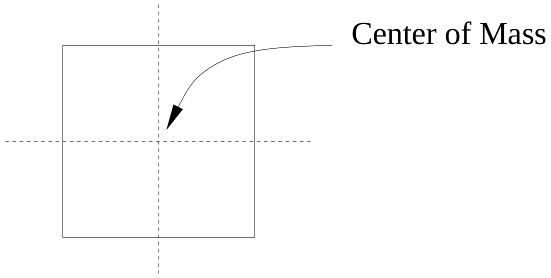
A see saw plank will balance at a point halfway along its length. This is because it has equal amounts of mass to either side of the middle of the plank. The center of mass of the plank will therefore be in the middle of the plank. The total weight of the plank will act straight down from the centre of mass. The pivot point of the seesaw is also placed in the middle (centre of mass) of the plank because this is where it can support the full weight of the plank. The total of the mass of the plank will act through the point called the centre of mass.

The centre of mass of a body will be found on any lines of symmetry (LoS) that an object (body) has. A line of symmetry is a line that can be drawn on an object such that everything to the left of the line of symmetry is an exact reflection of everything to the right of the line of symmetry.

Example One of the possible lines of symmetry is drawn on each of these shapes.



Where two lines of symmetry cross, marks the location of the centre of mass of the object.



A square has two lines of symmetry so the centre of mass of the square, is where the two lines cross (i.e. the centre of the square).

Activity - Can you draw any other lines of symmetry on the above shapes?

Summary

The centre of mass of an object is a point (usually inside the object) through which the full weight of the object is said to act. The object can be replaced with a theoretical "point object" which has no volume. The theoretical "point object" will have the same mass as the original object. The "point object" will be located in the exact position of the center of mass of the original object.

Exercise

a) Draw the following shapes in the table and indicate where their center of mass is by drawing on the shapes, lines of symmetry as dotted lines and putting a red dot where the lines cross. Label the center of mass with a dot.

Shapes and Prisms	**Diagram**
Rectangle	
Equilateral Triangle	
Circle	
Oval (Ellipse)	
Rhombus	
Diamond (sideways square)	
Cube	
Rectangular prism	
Sphere	
Boomerang	

b) Is the center of mass of an object always inside the object? Explain using an example.

____

Practical Experiment 2 – Rotation or revolution?

READ ALL THE BELOW INSTRUCTIONS FIRST

You will need safety glasses and a mathematical template.

a) Go outside.

b) Put on safety glasses.

c) Swing a math aid or math-o-mat around on the end of your finger.

d) WITHOUT SWINGING TOO HARD let the math aid or math-o-mat slip off.

1. In which situation was the math template revolving / rotating?

Revolution

____

Rotation

____

2. What is the difference between revolution and rotation?

____

____

____

3. In which of these situations (rotation / revolution) is a centripetal force acting?

Explain why or why not.

____

____

____

4. Why did the math template fly away when the finger was removed?

____

____

____

5. What types of energy does the flying (after release) template possess?

____

____

____

6. In this experiment list one …

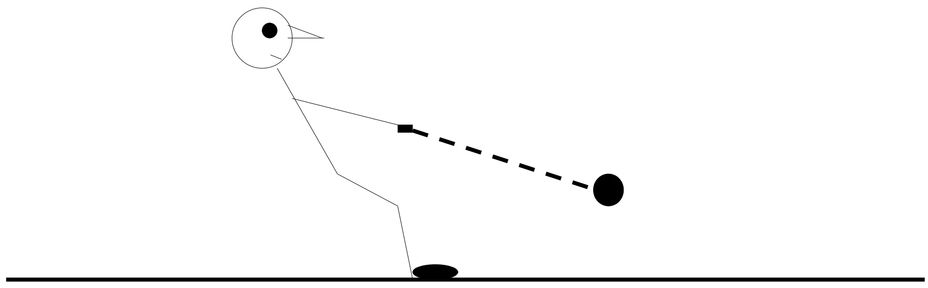
a) Energy transfer ____

b) Energy transform ____

7. When the template is spinning on the finger is work being done? Explain.

____

Past Exam Question - The Hammer Thrower



a) Calculate the centripetal force on a hammer that is being whirled in a horizontal circle of <u>horizontal</u> radius 1.80 m at a steady pace of 10.0 m s^{-1}. The mass of the hammer is 7.30 kg

(2 marks)

(4.06×10^2 N)

b) Draw on the above diagram the forces acting on the hammer.

(2 marks)

(weight, tension – in two components)

c) What is the magnitude of the tension in the chain of the hammer?

(2 marks)

(T_v = 71.5 N, T_h = 4.06×10^2 N, T = 4.12×10^2 N)

d) What is the angle formed between the chain and the horizontal?

(2 marks)

(10.0^0 relatine to th horizontal)

e) The hammer thrower is in fact also a mass revolving in a circle about a central point. If the hammer thrower has a mass of 130 kg, what is the horizontal radial distance from the hammer thrower to the central point about which he is rotating?

(4 marks)

(r = 1.01 x 10^{-1} m)

f) What variables are identical between the hammer and the hammer thrower and what variables are different?

	Hammer	Hammer Thrower
Identical		
Different		

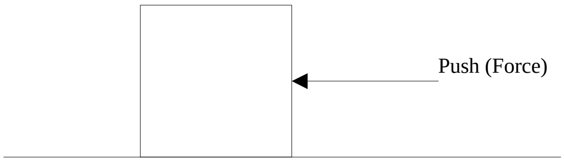
g) When the hammer thrower and the hammer are considered together as a combined system, the system is (rotating / revolving) (delete one) around the central point.

Appendix 1 - Supplementary Notes on Friction

Year 11 Friction Revision - (Solid Rubbing Against Solid)

Example Friction Situation

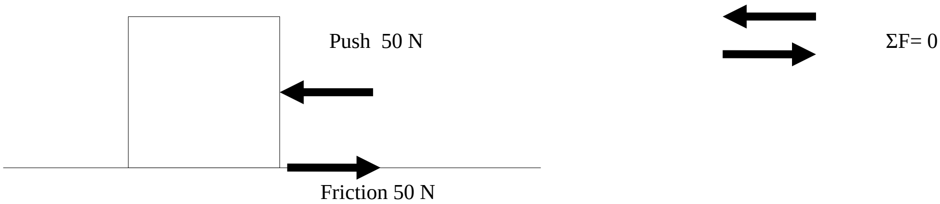
A person is trying to push a box across a level floor.



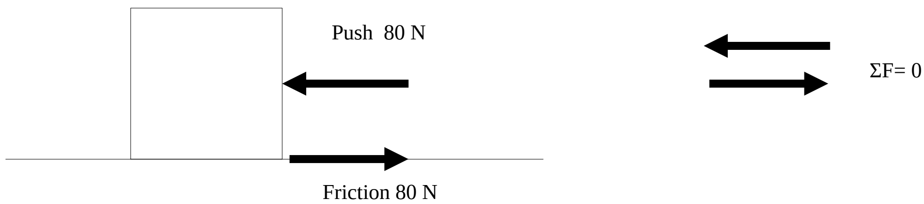
The person applies the pushing force gently at first, gradually pushing harder and harder until, finally, the box starts to move.

Explanation of the above Friction Situation

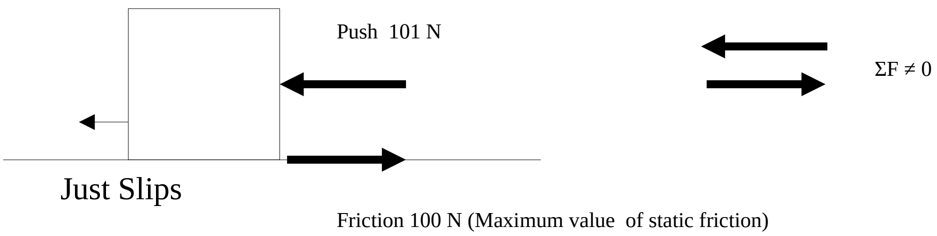
When we are trying to get the box moving, the force of friction is equal and opposite to the force we apply.



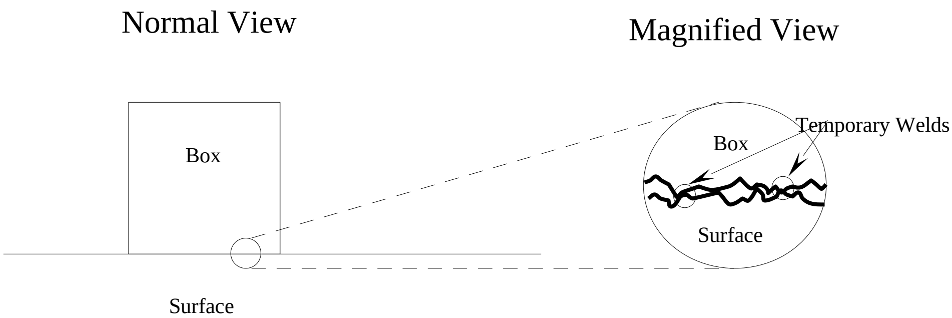
The friction involved in this situation is called static friction because the box is static (like the word statue which means not moving). As we increase the push on the box, the value (size) of static friction increases to perfectly match and counter balance the push.



We apply more and more force until the box finally begins to move. The force at which the box first slips is called the “maximum value of static friction”.

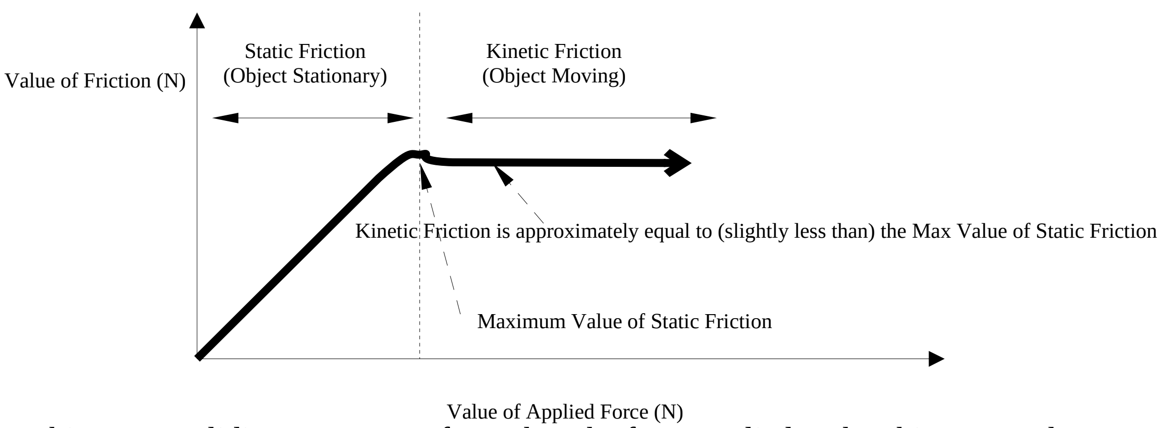


There is still friction when the box <u>is</u> moving. This is called kinetic friction. The value of kinetic friction is slightly less than the maximum value of static friction. This is because when the object is stationary, temporary welds are present between the object and the surface. These temporary welds are not present when the object is sliding.



In all the situations we will be encountering, the amount of <u>kinetic friction</u> stays constant between 2 <u>solid</u> surfaces regardless of how quickly one surface rubs against the other, and this kinetic friction will be approximately equal to the max value of static friction.

Applied Force verses Frictional Force Present



An object starts sliding across a surface when the force applied to the object exceeds the maximum value of static friction for that particular object on that particular surface.

<u>**Important Note :-**</u>

Static friction is in the opposite direction to *the* ***applied force***.

Kinetic friction is in the opposite direction to the ***objects velocity*** not the applied force.

Friction Involving Fluids

Friction occurs when 2 substances or surfaces attempt to move past each other. The substances involved in the friction action can be solids or fluids. Fluids are liquids and gases. Some general friction situations you can encounter are…

Solid rubbing against Solid
Solid rubbing against Fluid
Fluid rubbing against Fluid

Solid rubbing against solid friction has already been discussed above. We will now deal with the other two.

Solid on Fluid and Fluid on Fluid Situations

All situations involving fluids are outside the scope of the year 12 physics course (as far as calculations are concerned). What you do need to know is written below…

- Air resistance is a solid against fluid situation and only needs to be understood qualitatively (theoretically) not quantitatively (mathematically)
- The kinetic friction in (Solid v's Fluid) and (Fluid v's Fluid) situations increases as the relative velocity of the substances involved increases. E.g. a fast moving car experiences more air resistance that a slow moving car. [$F_{friction}$ = f (velocity)]

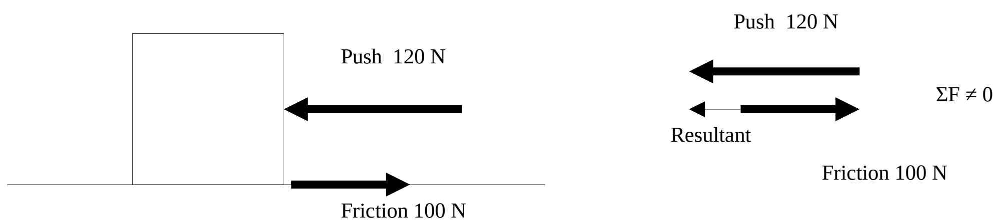
Here ends the fluid friction discussion.

Some Solid v's Solid Situations of Special Interest

*Dragging / Sliding Friction (An example of **kinetic** friction)*

When a solid object is dragged across the ground, the kinetic force of friction between the object and the ground remains constant.

Eg1 a box is pushed with a force of 120 N against a frictional force of 100 N.



The box accelerates forward because there is an unbalanced force of 20 N on it.

*The sliding friction is acting in the opposite direction to **the movement** of the object*

Eg2 A car's brakes lock on and the car skids to a halt. The sliding friction is in the opposite direction to the velocity of the car.

*Rolling Friction (An example of **static** friction)*

When a wheel is rolled across the ground it experiences static friction, not kinetic friction. The surfaces do not slide of drag past each other. Instead they are placed in contact and are lifted apart by the rolling action of the wheel. *The static friction present* ***acts in the opposite direction to the force on the wheel***. The force on the wheel is not always in the same direction as the movement of the object (especially when turning corners).

Solid on Solid Friction Summary

	Static Friction	**Kinetic Friction**
Direction of Acton	• Opposite to applied force • Not opposite to velocity	• Opposite to velocity • Not opposite to applied force
Size of frictional force	Varies form zero to some maximum value	Only one constant value
Max size	Max values of static	Slightly less than the maximum value of static friction.
Amount of friction	Depends on the features of the surface in contact. It depends on… • **Roughness** (increases friction) • **Weight** (increases friction) • **Lubrication** (decreases friction) • **Direction of Applied Force** (forces that push surfaces together increase friction)	

Activity

A car is rounding a bend on a wet road. As the car rounds the bend a kangaroo is spotted in the headlights. The car breaks heavily. What will happen to the pathway taken by the car? Explain why using a diagram and a written explanation.

Diagram

Explanation

____

____

____

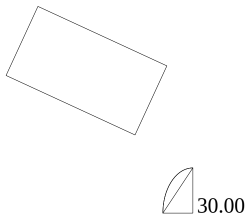
____

____

Appendix on Matrix Algebra.

How to solve simultaneous equations in physics if you know how to do matrices.

Activity (Tricky) A car travels around a curve in the road. The diameter of the curve is 100 m. The surface of the curve is banked at 30.0^0 to the horizontal to assist the car in negotiating the curve. Calculate the normal force on the car if the car is travelling at 110 km h^{-1} and has a mass of 2.00 tonnes.



Vertical	Horizontal
(-mg) + (+Nv) + (-Fv) = 0 (Nv) + (-Fv) = (-mg) N Cos (30) -F Sin (30) = -2000x9.8 N Cos (30) -F Sin (30) = -19600	(+Nh) + (Fh) = mv^2 / r N Sin (30) + F Cos (30) = 2000 x 30.6^2 / 50 N Sin (30) + F Cos (30) = 37500

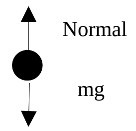
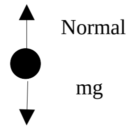
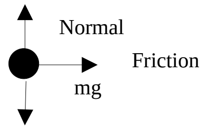
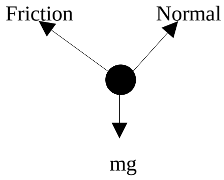
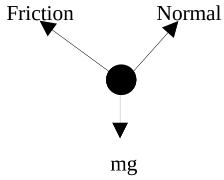
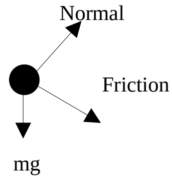
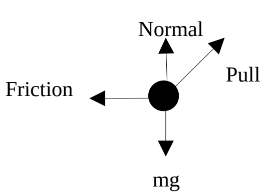
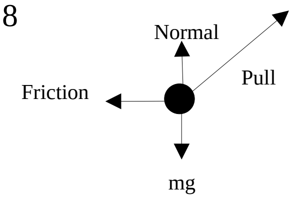
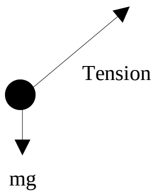
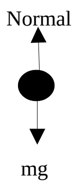
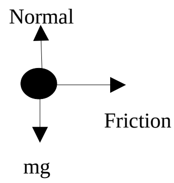
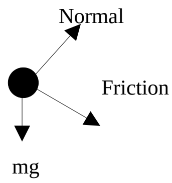
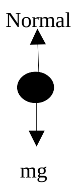
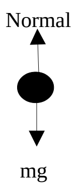
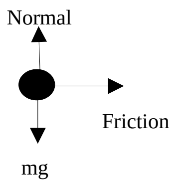
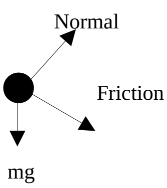
¿¿ $\begin{bmatrix} N \\ F \end{bmatrix} = \begin{bmatrix} -19600 \\ 37500 \end{bmatrix}$

¿¿ $\begin{bmatrix} N \\ F \end{bmatrix} = \begin{bmatrix} -19600 \\ 37500 \end{bmatrix}$

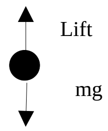
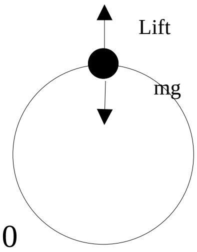
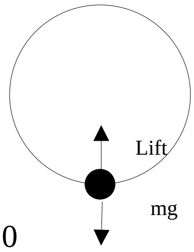
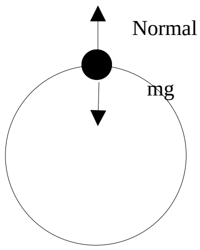
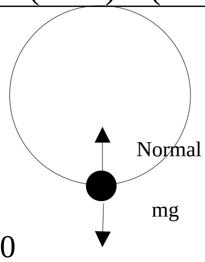
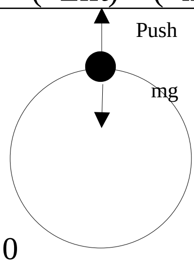
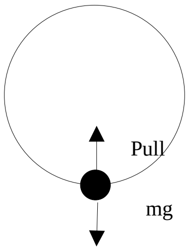
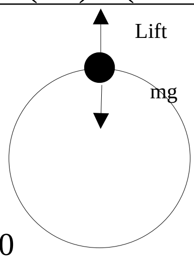
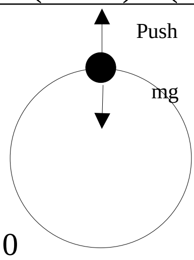
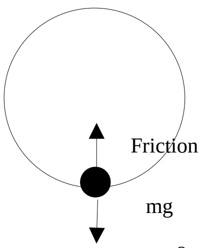
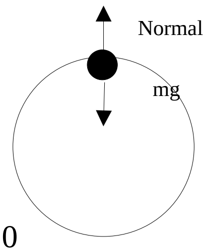
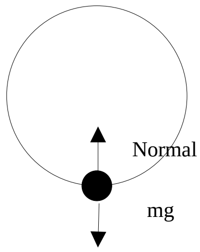
Apply matrix algebra …

$\begin{bmatrix} N \\ F \end{bmatrix} = \begin{bmatrix} 1776.5 \\ 42277 \end{bmatrix}$

Horizontal Solutions

1  $\Sigma F = 0$ $(-mg) + (+N) = 0$	2  $\Sigma F = 0$ $(-mg) + (+N) = 0$	3  $\Sigma F = F_{centripetal}$ $(-mg) + (+N) = 0$ $(Friction) = F_{centripetal}$
4  $\Sigma F = 0$ $(-mg)+(+N_v)+(+F_{Fri\ v}) = 0$ $(+N_h)+(-F_{Fri\ h}) = 0$	5  $\Sigma F = 0$ $(-mg)+(+N_v)+(+F_{Fri\ v}) = 0$ $(+N_h)+(-F_{Fri\ h}) = 0$	6  $\Sigma F = 0$ $(-mg)+(+N_v)+(-F_{Fri\ v}) = 0$ $(+N_h)+(+F_{Fri\ h}) = F_{centripetal}$
7  $\Sigma F = 0$ $(-mg)+(+N)+(+Pull_v) = 0$ $(+Pull_h)+(-F_{Fri\ h}) = 0$	8  $\Sigma F = ma$ $(-mg)+(+N)+(+Pull_v) = 0$ $(+Pull_h)+(-F_{Fri\ h}) = ma$	9  $\Sigma F = F_{centripetal}$ $(-mg)+(+T_v) = 0$ $(+T_h) = F_{centripetal}$
10  $\Sigma F = 0$ $(-mg)+(+N) = 0$	11  $\Sigma F = 0$ $(-mg)+(+N) = 0$ $(+F_{Fri}) = F_{centripetal}$	12  $\Sigma F = 0$ $(-mg)+(+N_v)+(-F_{Fri\ v}) = 0$ $(+N_h)+(+F_{Fri\ h}) = F_{centripetal}$
13  $\Sigma F = 0$ $(-mg)+(+N) = 0$	14  $\Sigma F = 0$ $(-mg)+(+N) = 0$	15  $\Sigma F = 0$ $(-mg)+(+N) = 0$ $(+F_{Fri}) = F_{centripetal}$
16 	16 cont. $\Sigma F = 0$ $(-mg)+(+N_v)+(-F_{Fri\ v}) = 0$ $(+N_h)+(+F_{Fri\ h}) = F_{centripetal}$	

Vertical Solutions

1  $\Sigma F = 0$ $(-mg) + (+Lift) = 0$	2a  $\Sigma F \neq 0$ $(-mg) + (+Lift) = (-mv^2/r)$	2b  $\Sigma F \neq 0$ $(-mg) + (+Lift) = (+mv^2/r)$
3a  $\Sigma F \neq 0$ $(-mg) + (+N) = (-mv^2/r)$	3b  $\Sigma F \neq 0$ $(-mg) + (+N) = (+mv^2/r)$	4a  $\Sigma F \neq 0$ $(-mg) + (+Push) = (-mv^2/r)$
4b  $\Sigma F \neq 0$ $(-mg) + (+Pull) = (+mv^2/r)$	5  $\Sigma F = 0$ $(-mg) + (+Lift) = 0$	6  $\Sigma F \neq 0$ $(-mg) + (+Push) = (-mv^2/r)$
7  $\Sigma F \neq 0$ $(-mg) + (+Fr) = (+mv^2/r)$	8a  $\Sigma F \neq 0$ $(-mg) + (+N) = (-mv^2/r)$	8b  $\Sigma F \neq 0$ $(-mg) + (+N) = (+mv^2/r)$