YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2017
TEST 1: Complex Numbers

WESLEY COLLEGE
By daring & by doing

Name: ________________________

Thursday 9th March

Time: 55 minutes **Mark** **/50 =** **%**

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet in **both** sections of the test.
- You are permitted one A4 page (one side) of notes in the calculator assumed section.

Calculator free section **Suggested time: 20 minutes** **/20**

1. [11 marks]

Determine each of the following in rectangular form $a+bi$

a) z if $2z-\bar{z}=3-6i$ [3]

b) $\dfrac{\overline{3+i}}{(2+i)^2}$ [3]

c) one solution to $z^3=8\,cis\left(\dfrac{3\pi}{4}\right)$ [2]

d) $\left(1-\sqrt{3}\,i\right)^5$ [3]

2. [6 marks]

$(z+2)$ is a factor of $P(z) = z^3 + pz^2 + 14z + 20$.

a) Evaluate *p*

[2]

b) Rewrite $P(z)$ in the form $P(z) = (z+2)Q(z) + R$

[2]

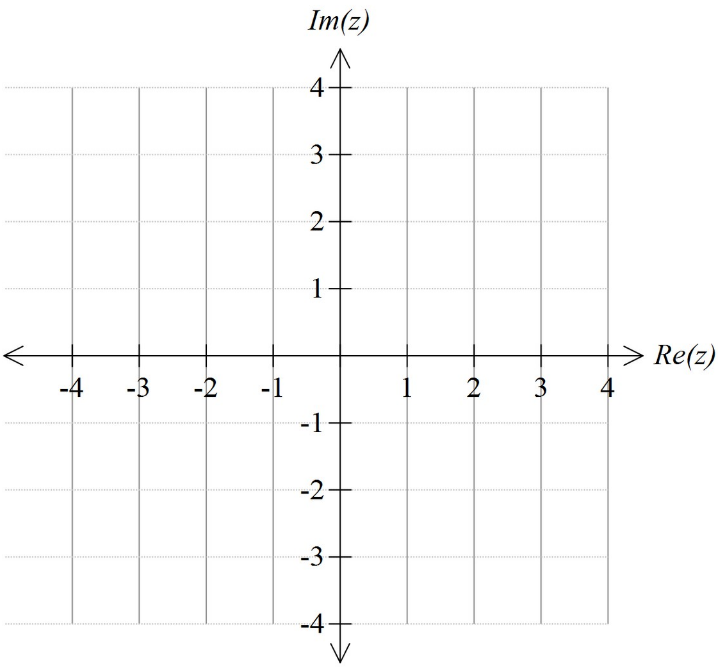
c) Determine all solutions to $P(z) = 0$

[2]

3. [3 marks]

When graphed on an Argand diagram, four of the solutions to $z^8 = k$ form a square with vertices $(1, i)$, $(-1, i)$, $(-1, -i)$ and $(1, -i)$.

Evaluate *k* and then write down the remaining solutions to $z^8 = k$



Calculator assumed section **Suggested time: 35 minutes** **/30**

Name: ________________________

4. [4 marks]

$z = 4\,cis\left(-\frac{\pi}{3}\right)$ and $\omega = 2\,cis\left(\frac{\pi}{6}\right)$

For which values of *n*, $-12 \le n \le 12$, will $\sqrt{z}.\omega^n$ be real?

5. [4 marks]

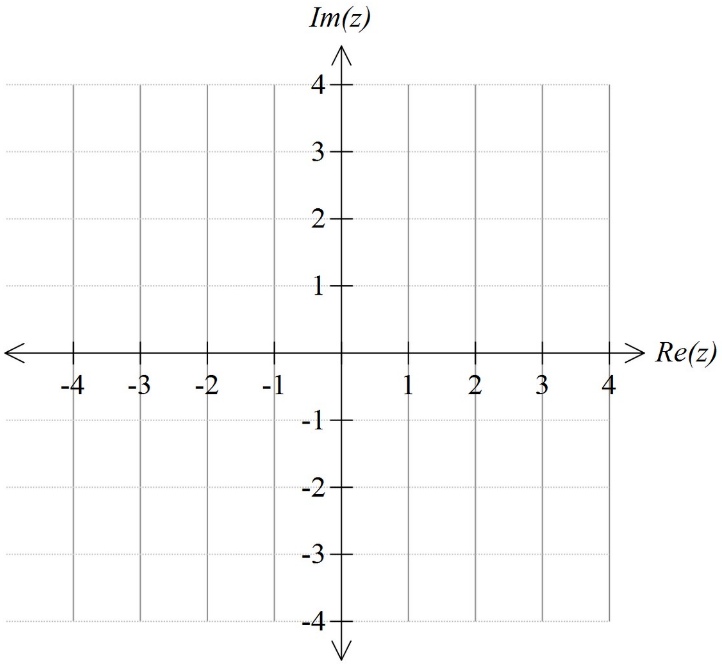
Determine, in Cartesian form $a + bi$, all solutions to the equation $z^4 = -16i$

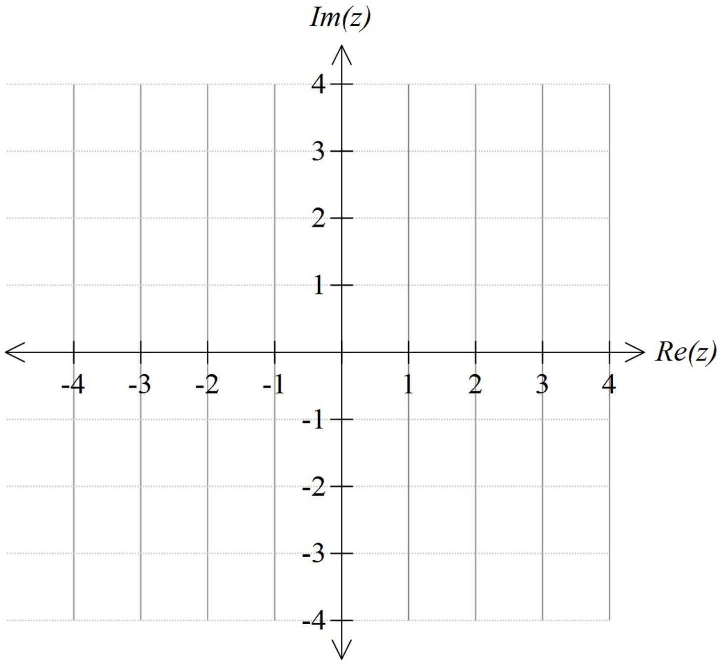
6. [12 marks]

a) On the Argand diagrams given, sketch

(i) $|z-(2+i)|=2$ [2]

(ii) $|z+3|<|z-1-i|$ [4]





b) For the points defined in (i), determine the:

(iii) maximum value of arg(z) [1]

(iv) minimum value of arg(z) [3]

(v) maximum value of $|z+i|$ [2]

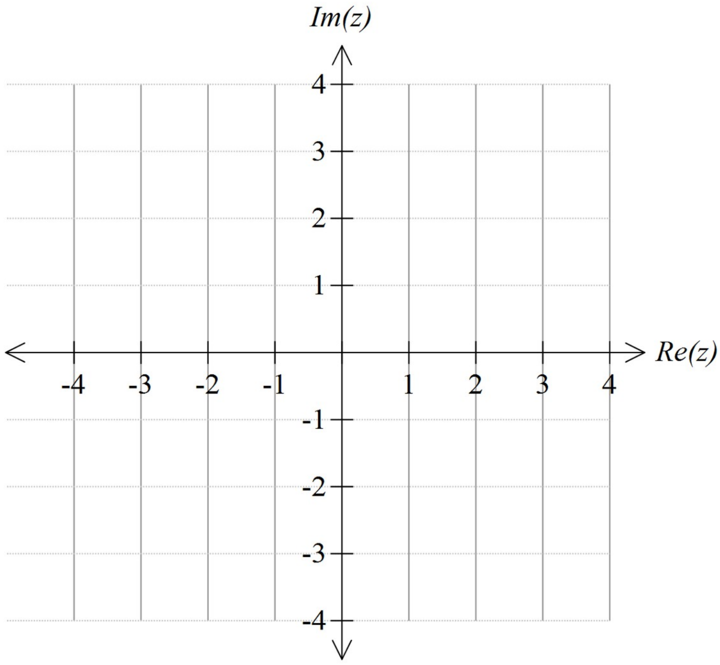
7. [10 marks]

The line segments joining the points $A\left(-\sqrt{3},0\right)$, $B(0,3i)$ and $C\left(\sqrt{3},-2i\right)$ form a triangle whose interior satisfies two inequalities:

$$\theta_1 \le \arg\left(z+\sqrt{3}\right) \le \theta_2$$

and

$$5\,\mathrm{Re}(z) + a\,\mathrm{Im}(z) \le b$$



Determine:

a) the values of:

a [2]

b [2]

θ_1 [2]

θ_2 [2]

b) the area of triangle *ABC*

[2]