

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Important note to candidates

Special items: **nil**

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
To be provided by the candidate

Materials required/recommended for this section
To be provided by the supervisor

Working time for this section: **forty minutes**
Reading time before commencing work: **five minutes**
Time allowed for this section

Your name _____
In words _____



MATHEMATICS 3C
Section One:
Calculator-free
Student Number: _____
in figures
in words _____

Question/Answer Booklet

Semester One Examination, 2015

Rossmanyne Senior High School

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Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|------------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|--------------------|
| Section One: Calculator-free | 7 | 7 | 50 | 50 | 33½ |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 100 | 66⅔ |
| Total | | | | 150 | 100 |

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Question 7

(3 marks)

1, $\binom{n}{1}(-2x)^1$, $\binom{n}{2}(-2x)^2$, $\binom{n}{3}(-2x)^3$, ..., $\binom{n}{n}(-2x)^n$
 The derivatives of the sequence
 are $0, \binom{n}{1}(-2), \binom{n}{2}(-4)(-2x)^1, \binom{n}{3}(-6)(-2x)^2, \dots, \binom{n}{n}(-2n)(-2x)^{n-1}$.

When n is a positive even integer, the sum of the series

$$1 + \binom{n}{1}(-2x)^1 + \binom{n}{2}(-2x)^2 + \binom{n}{3}(-2x)^3 + \dots + \binom{n}{n}(-2x)^n = (2x - 1)^n$$

Show that when n is a positive even integer, the sum of the series of derivatives

$$0 + \binom{n}{1}(-2) + \binom{n}{2}(-4)(-2x)^1 + \binom{n}{3}(-6)(-2x)^2 + \dots + \binom{n}{n}(-2n)(-2x)^{n-1} = \frac{2n}{2x - 1}(2x - 1)^n$$

If sum of series is $(2x - 1)^n$, then sum of series of derivatives will be $\frac{d}{dx}(2x - 1)^n$.

$$\begin{aligned} \frac{d}{dx}(2x - 1)^n &= 2n(2x - 1)^{n-1} \\ &= 2n(2x - 1)^n(2x - 1)^{-1} \\ &= \frac{2n}{2x - 1}(2x - 1)^n \end{aligned}$$

(8 marks)

Question 2

Determine the following, simplifying where possible.

$$(a) \frac{d}{dx} \left(\frac{2x^2 - 1}{1 - 3x} \right)$$

(2 marks)

$$\boxed{\frac{4x(1 - 3x) - (2x^2 - 1)(-3)}{(1 - 3x)^2} = \frac{4x - 6x^2 - 3}{(1 - 3x)^2}}$$

$$(b) \int_{2\sqrt{x}} \frac{1}{5} dx$$

(2 marks)

$$\boxed{\sqrt{x} - \frac{x^4}{20} + c}$$

$$(c) \frac{d}{dx} (x^2 \sqrt{x+1})$$

(2 marks)

$$\boxed{2x\sqrt{x+1} + x^2 \times \frac{1}{2}(x+1)^{-\frac{1}{2}} = 2x\sqrt{x+1} + \frac{x^2}{2\sqrt{x}}}.$$

$$(d) \int 3xe^{x^2+1} dx$$

(2 marks)

$$\boxed{\frac{3}{2}e^{x^2+1} + c}$$

- (c) The equation of the tangent to the curve $y = \frac{(x+1)(x-3)}{(x-1)(x+3)}$ at the point (3, 0) is $y = ax + b$. Determine the values of a and b .

$$y = \frac{(x+1)(x-3)}{(x-1)(x+3)}$$

(4 marks)

$$y = 1 - (x-1)^{-1} - 3(x+3)^{-1}$$

$$\begin{aligned} \frac{dy}{dx} &= 0 + (x-1)^{-2} + 3(x+3)^{-2} \\ &= \frac{1}{(x-1)^2} + \frac{3}{(x+3)^2} \end{aligned}$$

$$x = 3, \frac{dy}{dx} = \frac{1}{4} + \frac{3}{36} = \frac{1}{3}$$

$$y - 0 = \frac{1}{3}(x - 3)$$

$$y = \frac{1}{3}x - 1 \Rightarrow a = \frac{1}{3}, b = -1$$

| Question 5 | | Question 6 | |
|--|--|--|---|
| MATHEMATICS 3C | CALCULATOR-FREE | 8 | MATHEMATICS 3C |
| 8 marks | 8 marks | 8 marks | 8 marks |
| Two functions are defined as $f(x) = 2x + 5$ and $g(x) = 2 - \sqrt{2x + 5}$ | (a) State $g \circ f(x)$ with its domain and range. Two functions are defined as $f(x) = 2x + 5$ and $g(x) = 2 - \sqrt{2x + 5}$ | (a) Show that $\frac{1}{x-1} - \frac{x+3}{3} = \frac{(x-1)(x+3)}{(x+1)(x-3)}$. | (b) Determine the domain and range of $f\left(\frac{x}{2}\right)$. |
| (3 marks) | (3 marks) | (3 marks) | (2 marks) |
| $\begin{aligned} R: & \sqrt{2x+5} \geq 0 \Leftrightarrow y \geq 2 \\ D: & 2x+5 \geq 0 \Leftrightarrow x \geq -2.5 \\ g \circ f(x) = & 2 - \sqrt{2x+5} \end{aligned}$ | $\begin{aligned} R: & y \neq 5 \\ D: & x \neq 0 \\ f\left(\frac{x}{2}\right) = & \left(\frac{x}{2}\right)^2 + 5 \end{aligned}$ | $\begin{aligned} & \frac{(x-1)(x+3)}{(x+1)(x-3)} = \\ & \frac{(x-1)(x+3)}{x^2 - 2x - 3} = \\ & \frac{(x-1)(x+3)}{x^2 + 2x - 3 - 3x} = \\ & \frac{(x-1)(x+3)}{x^2 - x - 3} = \\ & \frac{1}{x-1} - \frac{x+3}{3} = \frac{(x-1)(x+3) - (x+1)(x-3)}{(x+1)(x-3)} \end{aligned}$ | $\begin{aligned} & y = 1 - \frac{1}{x+3} \text{ has a root at } (3, 0). \\ & y = 1 - \frac{1}{x+3} \Leftrightarrow x+3 = \frac{1}{y-1} \Leftrightarrow x = \frac{1-y}{y-1} \Leftrightarrow x = \frac{y-2}{y-1} \end{aligned}$ |
| (3 marks) | (3 marks) | (1 mark) | (3 marks) |

(10 marks)

Question 4

A function is given by $f(x) = (7 - x)(x - 1)^2$.

- (a) Determine the coordinates of the axes intercepts of the graph of $y = f(x)$. (2 marks)

(0, 7), (1, 0) and (7, 0)

- (b) Determine the coordinates of the stationary points of the graph of $y = f(x)$. (4 marks)

$$\begin{aligned}f'(x) &= 0 \text{ where} \\(-1)(x-1)^2 + (7-x)(2)(x-1) &= 0 \\-(x^2 - 2x + 1) + 2(8x - 7 - x^2) &= 0 \\-3x^2 + 18x - 15 &= 0 \\-3(x^2 - 6x + 5) &= 0 \\-3(x-1)(x-5) &= 0 \Rightarrow x\end{aligned}$$

(1, 0) and

- (c) Determine the location of the point of inflection of the graph of $y = f(x)$. (2 marks)

$$\begin{aligned}f''(x) &= -6x + 18 \\-6x + 18 &= 0 \Rightarrow x = 3\end{aligned}$$

(3, 16)

- (d) Sketch the graph of $y = f(x)$ on the axes below. (2 marks)

