

No other items may be used in this section of the examination. It is **YOUR** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

### Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

### To be provided by the candidate

Formula Sheet (referred from Section One)

This Question/Answer Booklet

MATERIALS REQUIRED/RECOMMENDED FOR THIS SECTION

Working time for this section: one hundred minutes

Reading time before commencing work: ten minutes

### Time allowed for this section

Your name \_\_\_\_\_

In words \_\_\_\_\_



## SOLUTIONS

Question/Answer Booklet

Semester One Examination, 2013

ROSSMOYNE SENIOR HIGH SCHOOL

MATHEMATICS 3A  
Section Two:  
CALCULATOR-ASSUMED

Student Number: \_\_\_\_\_

In figures \_\_\_\_\_

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**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
<b>Total</b>			150	100	

**Instructions to candidates**

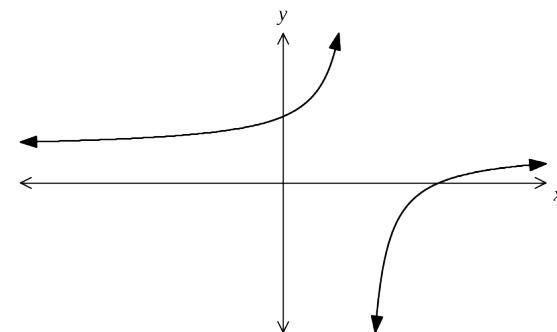
1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil**, except in diagrams.

**Question 20**

(8 marks)

The graph of the function  $y = f(x)$  is shown, with a root at  $(4, 0)$  and  $y$ -intercept at  $(0, 2)$ .

The hyperbola has asymptotes with equations  $x = 2$  and  $y = 1$ .



Determine the

- (a) coordinates of the root of the graph  $y = f(x - 2)$ .

(2 marks)

Was  $(4, 0)$   
Becomes  $(6, 0)$

- (b) equation of the horizontal asymptote for the graph of  $y = f(x - 2) + 3$ .

(2 marks)

Was  $y = 1$   
Becomes  $y = 1 + 3 \Rightarrow y = 4$

- (c) equation of the vertical asymptote for the graph of  $y = f(2(x + 3))$ .

(2 marks)

Was  $x = 2$   
Becomes  $x = \frac{1}{2} \times 2 - 3 \Rightarrow x = -2$

- (d) coordinates of the  $y$ -intercept of the graph  $y = 3 - f(x + 4)$ .

(2 marks)

Need to consider known root at  $(4, 0)$   
Becomes  $(4, 0) \rightarrow (0, 0) \rightarrow (0, 3)$

(1 mark)

- (iv) a Ragdoll, given that they are female?

28
87

(1 mark)

- (iii) a Persian or a male?

111
179

(1 mark)

- (ii) a male?

92
179

(1 mark)

- (b) What is the probability that a cat chosen at random from the sample is

22
179

(2 marks)

- (a) Complete the missing entries in the table above.

	Female	Persian	Ragdoll	Siamese	Other	Total	Total
Male	28	28	22	18	87	92	179
Total	19	47	45	48	39	179	179

(7 marks)

- Question 8 A sample of 179 cats brought to a veterinary clinic over the course of one month were categorised on the basis of breed and gender as follows:

Working time for this section is 100 minutes.  
 This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

(100 Marks)

Section Two: Calculator-assumed

MATHMATICS 3A

Question 19 CALCULATOR-ASSUMED

3 MATHMATICS 3A

- (e) If the repayment was repeatedly halved, would you expect the interest to roughly double each time? Explain your answer.

No. Once the repayment has been halved three times, it is less than the first month's interest, and so the loan would never be paid off, with interest accruing for ever.

(2 marks)

- (d) Show that the total interest over the life of the loan approximately doubles when the repayment is halved to \$110 per month.

Now takes 23 months, with final payment of \$106.99. Total interest is  $22 \times 110 + 106.99 - 2300 = \$226.99$ , which is just over double previous figure.

(3 marks)

- (c) How much interest is paid over the life of this loan?

Month 11. Final repayment is \$211.53

(2 marks)

- (b) At the end of which month is the loan repaid in full, and what is the amount of the final repayment?

$T_1 = 2300$   
 $T_{n+1} = T_n \times 1.008 - 220$

(2 marks)

Month 11. Final repayment is \$211.53

(2 marks)

- (a) Write a recursive rule to determine the amount of the loan  $T_n$  at the start of month  $n$ .

$n$	Balance (Start of month)	Interest	Repayment	Balance (End of month)
3	1895.19	15.16	220.00	1690.35
2	2098.40	16.79	220.00	1895.19
1	2300.00	18.40	220.00	2098.40
				$T_1 = 2300$

- A loan of \$2300 is to be repaid by monthly payments of \$220. Interest, of 9.6% pa of the balance, is added at the end of each month, just before the repayment is made. The table below shows the balance at the end of each month, just before the repayment with the monthly interest and repayment for the first few months.

The table below shows the balance at the start and end of each month, together with the monthly interest and repayment for the first few months.

**Question 9**

(6 marks)

The number of page views of a website per day, following a TV advertising campaign, is shown in the table below.

Day ( $n$ )	1	2	3	4
Views ( $T_n$ )	275 562	91 854	30 618	10 206

Assume that this pattern continues over the next few weeks.

- (a) How many page views are expected on day five?

(1 mark)

$$10206 \div 3 = 3402 \text{ views}$$

- (b) Write a recursive rule for the number of page views per day.

(2 marks)

$$T_{n+1} = T_n \div 3 \quad T_1 = 275562$$

- (c) What is the total number of page views of the website after one week?

(1 mark)

$$S_7 = 413154$$

- (d) Will the total number of page views reach half a million? Justify your answer.

(2 marks)

No. From the rule,  $T_{12} = 0.5$  and so after day 12 the rule predicts no-one is viewing the website. At this time  $S_{12} \approx 413343$  so 500000 will never be reached.

**Question 18**

(5 marks)

Using the digits 1, 2, 3, 4 and 5, with no digit used more than once, how many integers between 1000 and 9999 can be made if

- (a) there are no other restrictions?

(1 mark)

$$5 \times 4 \times 3 \times 2 = 120$$

- (b) the number must begin with the digit 5?

(1 mark)

$$1 \times 4 \times 3 \times 2 = 24$$

- (c) the number must be even?

(1 mark)

$$2 \times 4 \times 3 \times 2 = 48$$

- (d) the number must begin with the digit 5 **and** be even?

(1 mark)

$$1 \times 2 \times 3 \times 2 = 12$$

- (e) the number must begin with the digit 5 **or** be even?

(1 mark)

$$24 + 48 - 12 = 60$$

## Question 10

(7 marks)

Recent research into introducing a size limit for catches of longtail tuna in Australian coastal waters included the data below from a group of recreational fishermen.

$f(x)$  has a root at  $x = -2$  and  $g(x)$  passes through the point  $(2, -3)$ .  
 Two functions are given by  $f(x) = ax^2 + 2$  and  $g(x) = x^3 + bx^2 + x + 3$ , where  $a$  and  $b$  are constants.

## (12 marks)

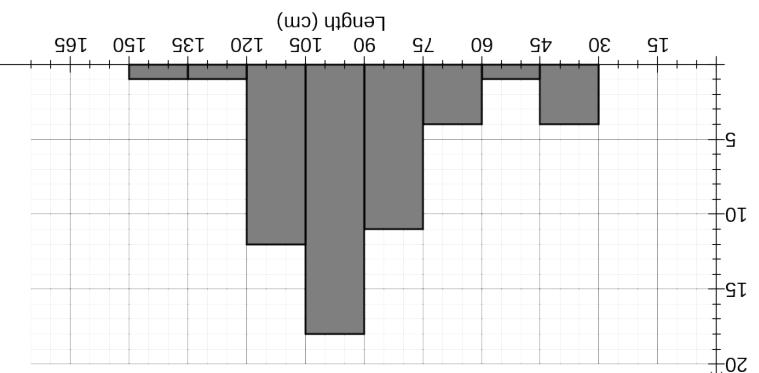
(a) Show that  $a = -\frac{1}{2}$ .

Question 17

(7 marks)

Length (cm)	Number of fish
30 < $L \leq 45$	4
45 < $L \leq 60$	1
60 < $L \leq 75$	1
75 < $L \leq 90$	11
90 < $L \leq 105$	18
105 < $L \leq 120$	12
120 < $L \leq 135$	1
135 < $L \leq 150$	1
Total	52

(a) Draw a histogram for the lengths of longtail tuna. (3 marks)



(a) Draw a histogram for the lengths of longtail tuna. (3 marks)

(b) Calculate an estimate for the mean and standard deviation of the lengths of longtail tuna. (2 marks)

$$\begin{aligned} x &= 91.44 \text{ cm} \\ s_d &= 22.45 \text{ cm} \end{aligned}$$

(c) If the original data was available for all 52 longtail tuna, so that the median length could be determined, would you expect it to be the same, smaller than, or greater than the mean length? Justify your answer with reference to the histogram. (2 marks)

Median would be **greater than** the mean, because of the large number of lengths in the LH column of the histogram (negative skew). This group compares to the otherwise fairly symmetric histogram. This group acts as an outlier, pulling the mean towards it and away from the median.

Question 10

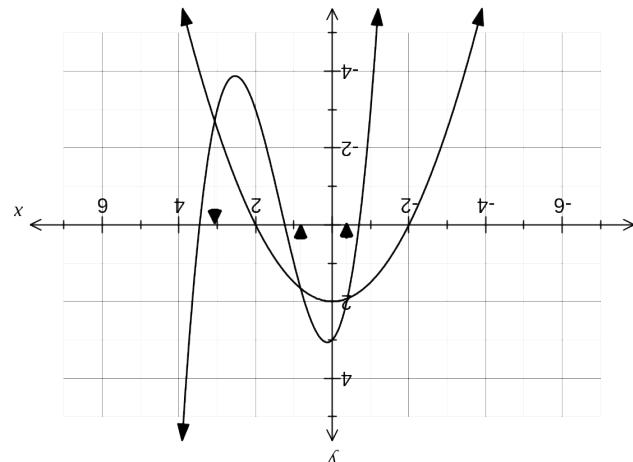
(7 marks)

(e) Show how your graph can be used to obtain the solutions to (d). (2 marks)

See graph - must clearly indicate  $x$ -coordinate only.

(d) Solve  $f(x) = g(x)$ , giving your solution(s) rounded to two decimal places. (2 marks)

$$x = -0.39, x = 0.83, x = 3.07$$

(c) Sketch the graphs of  $y = f(x)$  and  $y = g(x)$  on the axes below. (5 marks)

$$\begin{aligned} a &= -\frac{1}{2} \\ 4a &= -2 \\ 0 &= a(-2)^2 + 2 \\ -3 &= 8 + 4b + 5 \\ 4b &= -16 \\ b &= -4 \end{aligned}$$

(b) Show that  $b = -4$ . (2 marks)

(7 marks)

**Question 11**

Let the universal set  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ .

Two subsets, R and Q, are such that  $R = \{2, 3, 5, 7\}$  and  $Q = \{1, 3, 5, 7, 9\}$ .

(a) Determine

(i)  $R \cap Q$ 

{ 3, 5, 7 }
-------------

(1 mark)

(ii)  $n(R \cup Q)$ 

6
---

(1 mark)

(b) If a number is chosen at random from the universal set, determine

(i)  $P(R \cap \bar{Q})$ 

$\frac{1}{9}$
---------------

(1 mark)

(ii)  $P(Q | \bar{R})$ 

$\frac{2}{5}$
---------------

(2 marks)

(c) A third subset A is such that  $A \cap Q = \emptyset$ ,  $n(A \cap R) = 1$  and  $n(A) = 2$ . List all such possible subsets for A.

(2 marks)

$A = \{2, 4\} \text{ or } \{2, 6\} \text{ or } \{2, 8\}$
--

**Question 16**

(7 marks)

(a) In a science experiment, quantity  $P$  was observed to be related to the another quantity  $T$  by the equation  $P = \frac{T}{255}$ .

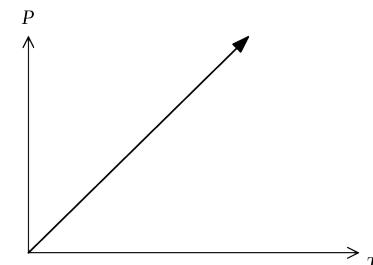
(i) Describe the relationship between  $P$  and  $T$ .

(1 mark)

P is directly proportional to T
---------------------------------

(ii) Sketch the relationship between  $P$  and  $T$ .

(2 marks)



(b) In another experiment, the quantity  $V$  was observed to be related to the another quantity  $A$  by the equation  $VA = 110$ .

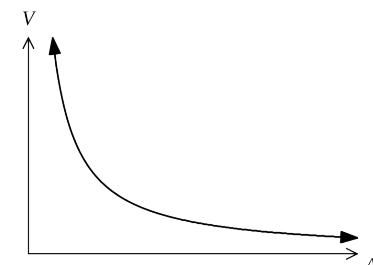
(i) Calculate  $A$  when  $V = 215$ .

(2 marks)

$$\begin{aligned} A &= \frac{110}{V} \\ &= \frac{110}{215} \\ &= 0.512 \end{aligned}$$

(ii) Sketch the relationship between  $V$  and  $A$ .

(2 marks)



(2 marks)

- (c) The sugar is packed onto pallets which can hold 440 bags. How many bags on a full pallet would be expected to weigh less than 500 g?

$$P(X < 500) = 0.00905$$

$$0.00905 \times 440 = 3.98 \approx 4 \text{ bags}$$

(2 marks)

- (b) 5% of the bags weigh less than  $w$  grams. Find the value of  $w$ , correct to three significant figures.

$$P(X < w) = 0.05$$

$$w = 507.906 \approx 508 \text{ g to 3sf}$$

(1 mark)

- (iii) between 520 and 530 g.

$$P(520 < X < 530) = 0.349$$

(1 mark)

- (a) Determine the probability that a randomly selected bag weighs

(i) more than 545 g

$$P(X > 545) = 0.042$$

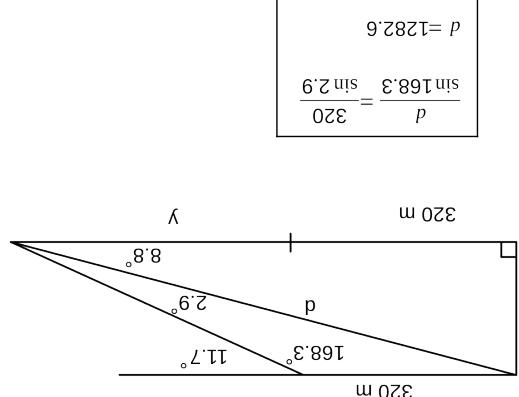
(6 marks)

- A machine fills bags of sugar so that the weights of the bags are normally distributed with a mean of 526 g and a standard deviation of 11 g.

**Question 12**

At one point during the flight, the operator of the drone noted that the angle of depression to the base of the tower was  $8.8^\circ$ . Eight seconds later, this angle had increased to  $11.7^\circ$ .

Calculate, to the nearest metre, the height the drone was flying at.



A remotely controlled drone flies at a constant speed of  $40 \text{ m s}^{-1}$  above level ground and at a steady height, directly towards a tower.

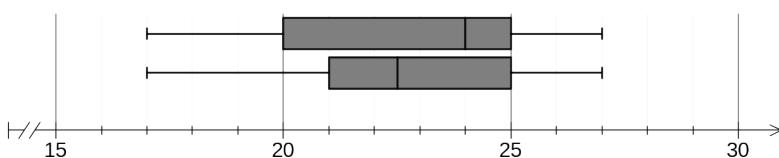
At one point during the flight the height the drone was flying at.

(10 marks)

**Question 13**

37 students in a school sat a Chemistry test that had a maximum possible score of 40.

The scores of the 19 students in Class M are summarised in this boxplot.



- (a) State the interquartile range for the scores of Class M. (1 mark)

5 marks

- (b) One student in Class M scored 27 out of 40. If they had scored an extra five marks, would their score be considered an outlier for the class? Justify your answer. (2 marks)

$1.5 \times 5 = 7.5$

$25 + 7.5 = 32.5$

Score of  $27 + 5 = 32$  is below  $1.5 \times \text{IQR}$  above  $Q_3$ , so not outlier.

The scores of the remaining students, in Class P, are listed below in ascending order.

17	18	19	20	21	21
22	22	22	23	24	25
25	25	26	26	26	27

- (c) Construct a boxplot for these 18 scores on the above diagram, next to that for Class M. (3 marks)

- (d) Explain which class

- (i) performed better in the test (2 marks)

Class M - had a higher median of 24 compared to 22.5 for Class P.

- (ii) had the least skewed marks (2 marks)

Class P - Its quartiles are fairly equal in width, but Class M has positive skew - width of upper half much less than width of lower half of boxplot.

(8 marks)

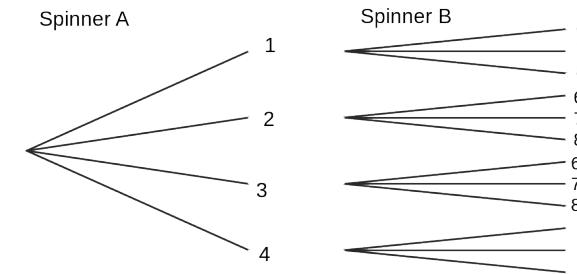
**Question 14**

Two spinners, A and B, are marked with both odd and even numbers.

Spinner A has sides marked 1, 2, 3 and 4 and each score has an equal chance of occurring.

Spinner B has sides marked 6, 7 and 8 and again, each score has an equal chance of occurring.

- (a) Draw a tree diagram to show all the ways spinner A and spinner B could stop. (3 marks)



- (b) Determine the probability that

- (i) both spinners stop on even numbers. (1 mark)

$\frac{4}{12}$

- (ii) one spinner stops on an even and the other stops on an odd number. (1 mark)

$\frac{6}{12}$

- (iii) spinner A or spinner B stops on an even number. (1 mark)

$\frac{10}{12}$

- (iv) both spinners stop on an odd number, given that at least one spinner has stopped on an odd number. (2 marks)

$\frac{2}{8}$