WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER TWO 2017
QUESTIONS OF REVIEW 7: Differential Equations

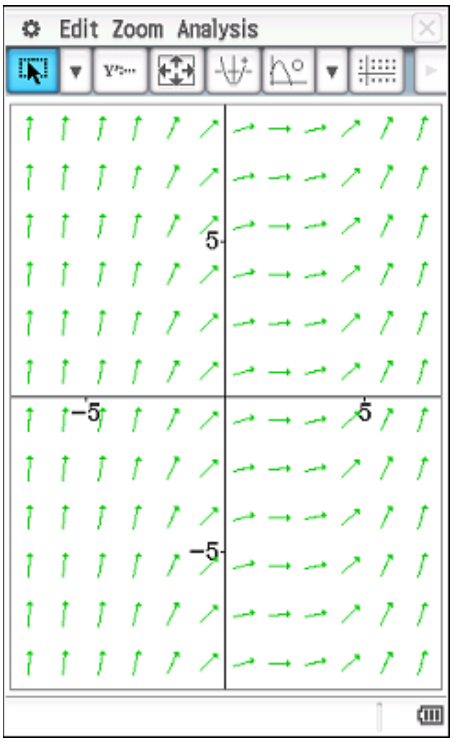
Name: _____

Thursday 31st August Time: 35 minutes Mark /28

Calculators assumed.

1. [10 marks – 3, 3, 2 and 2] – based on 2016 WACE paper

This slope field represents the family of polynomials $y = f(x) + c$ with $f'(2) = 0$ and $f'(0) = \frac{1}{2}$



a) Explain why $f'(x)$ takes the form

$f'(x) = a(x+b)^2$ and evaluate *a* and *b*

b) Use Euler's method, with an increment of $\partial x = 0.2$ to estimate $f(0.6)$, starting from $f(0) = 3$

x	$y = f(x)$	∂x	∂y
0	3		
0.2			
0.4			
0.6		-	-

c) Determine $f(x)$ given $f(0) = 3$

d) Compare the actual value of $f(0.6)$ with your estimate in (b) and explain the difference.

2. [18 marks – 2, 4, 2, 1, 1, 1, 1, 3, 1, 1 and 1] – based on 2016 SA Specialist paper

A country's population growth depends on the internal growth rate *r* and the number of immigrants *I*, so that $\frac{dP}{dt} = rP + I$

(a) Use separation of variables to show that $\int \frac{1}{P + \frac{I}{r}} dP = \int r \, dt$

(b) Hence solve the differential equation $\frac{dP}{dt} = rP + I$ and show that $P(t) = P_0 e^{rt} + \frac{I}{r}(e^{rt} - 1)$ where $P(0) = P_0$

A country in 2017 has $P_0 = 24.5$ million, $I = 0.21$ million and $r = 0.0062$

(c) Estimate the population in 5 years time (the year 2022)

(d) Estimate the population in 50 years time

(e) Identify one assumption in these estimates

(f) How reliable is the model in the longer term?

A logistic equation for this country's population is derived from $\frac{dP}{dt} = 0.016P\left(\frac{45 - P}{45}\right)$ with a 2017 population of $P_0 = 24.5$ million and an increased *r* value to include immigration.

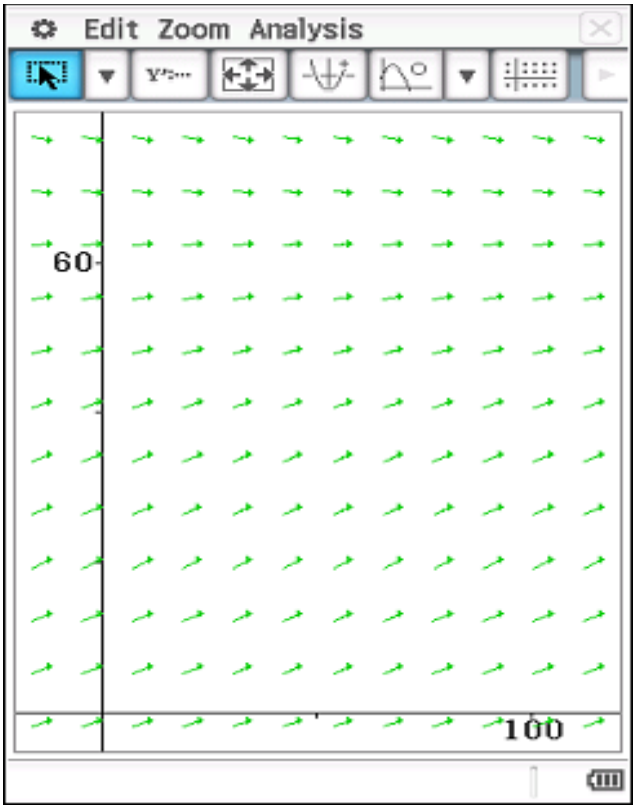
(g) What is the maximum population?

(h) Estimate the population in 5 years time (2022) and after 50 years

(i) What is the major difference between these logistic results and the earlier model?

A combined equation has $\frac{dP}{dt} = 0.0062P\left(\frac{45 - P}{45}\right) + 0.21$

(j) Draw the solution curve with $P_0 = 24.5$ million on this slope field.



(k) How do the results from this model compare with the logistic model?