

Note: All part questions worth more than 2 marks require working to obtain full marks.

Materials required: Up to 3 calculators/classpads allowed

Number of questions: 6

Time allowed for this task: 40 mins

Task type: Response

Student name: _____ Teacher name: _____

Course Methods Year 12 test three 2022

PERTH MODERN SCHOOL
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Q1 (3, 3 & 2 = 8 marks)

Consider the discrete random variable X and the table of probabilities below.

X	0	1	2	3	4
$P(X = x)$	0.2	a	0.3	b	0.25

- a) Given that the expected value of X is 2.15, determine the values of a & b .

Solution

0.2 + a + 0.3 + b + 0.25 = 1
 $a + 2 \times 0.3 + 3b + 4 \times 0.25 = 2.15 | a, b$

{a=0.1, b=0.15}

Specific behaviours

- ✓ sets up one equation for a & b
- ✓ sets up two equations for a&b
- ✓ solves for a&b

- b) Determine the standard deviation of X to 3 dp, showing all reasoning.

Solution

$$V(x) = (0 - 2.15)^2 \cdot 0.2 + (1 - 2.15)^2 \cdot 0.1 + (2 - 2.15)^2 \cdot 0.3 + (3 - 2.15)^2 \cdot 0.2 + (4 - 2.15)^2 \cdot 0.25$$

$$= 2.028$$

$$std = 1.424$$

Specific behaviours

- ✓ shows a sequence for variance
- ✓ determines variance
- ✓ determines std to 3 dp

- c) Determine the $E(3X + 4)$ and $\text{Variance}(3X + 4)$.

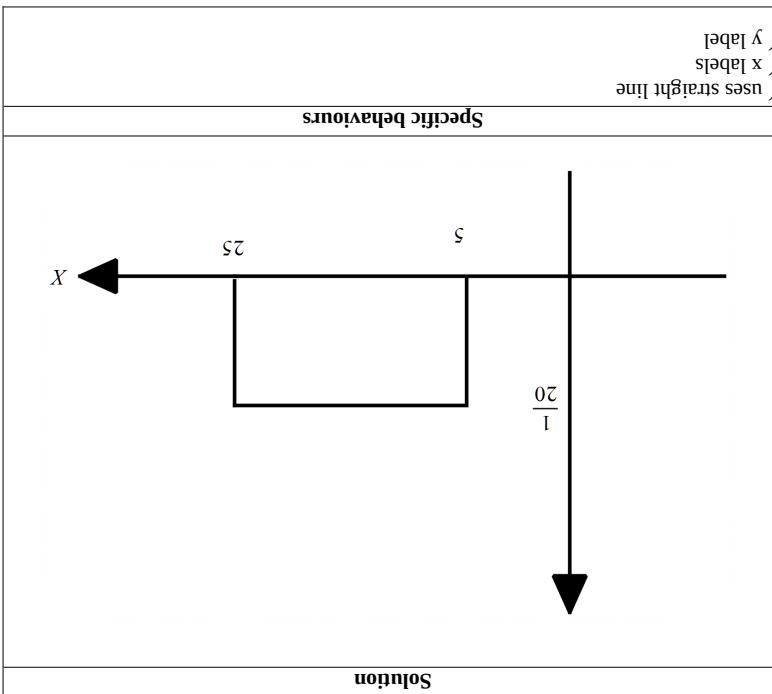
Solution

$$E(3X + 4) = 10.45$$

$$\text{Variance}(3X + 4) = 18.2975.$$

	Solution
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- Q2 continued
- b) Determine the probability that the train will be less than 12 mins late given that it is at least 7 mins late.



The number of X minutes late a train arrives at a particular station is a uniform probability distribution from 5 mins to 25 mins.

a) Sketch the probability density function for X showing all relevant features and labels.

Q2 (3, 2, 1 & 3 = 9 marks)

	Specific behaviours
	<ul style="list-style-type: none"> ✓ determines new mean ✓ determines new variance

The calculator screen shows the following steps:

$$\frac{(12-7)\frac{1}{20}}{(25-7)\frac{1}{20}}$$

$$\frac{5}{18}$$

$$\frac{5}{18}$$

$$0.277777778$$

At the top, the menu bar includes "Edit", "Action", "Interactive", and various function keys like $\frac{1}{2}$, $\int \! dx$, $\int \! dx \leftarrow$, $Simp$, $\frac{d}{dx}$, and $\frac{d}{dx} \leftarrow$.

Specific behaviours

- correct denominator
- correct numerator

- c) Determine the mean number of minutes late.

Solution
Mean = 15 mins
Specific behaviours
<input checked="" type="checkbox"/> states midpoint

- d) Determine the standard deviation of X showing all reasoning.

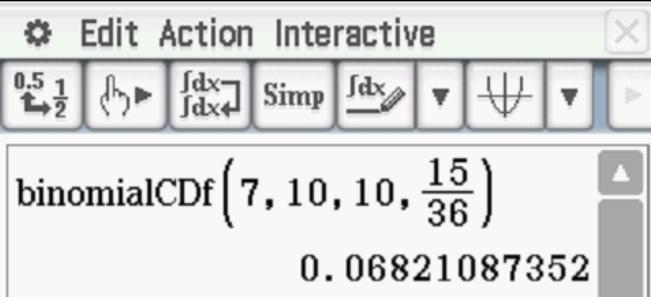
Solution

- a) Determine the probability of a win. Show reasoning.
- top numbers is added, if the sum is a prime number ($2, 3, 5, 7, \dots$) etc then this is considered a win.
- Consider a game where two ordinary dice are thrown into the air and then land and the sum of the two top numbers is added.
- Q3 (3, 3 & 2 = 8 marks)

✓ evaluates $x=0$ and $x=1$ showing both values for two terms after integration
✓ changes factor to give required definite integral (no need to simplify)

Solution																																																							
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Prime numbers 2,3,5,7,11																																																							
Pr(prime)=15/36																																																							
Specific behaviours																																																							
<ul style="list-style-type: none"> ✓ shows sample space ✓ number of favourable outcomes shown ✓ states probability 																																																							

- b) If this game was played 10 times, determine the probability that a win occurs at least 7 times.
Show all reasoning.

Solution						
$X \sim B(10, \frac{15}{36})$						
$P(X \geq 7)$						
						
Specific behaviours						
<ul style="list-style-type: none"> ✓ states Binomial with parameters ✓ uses cumulative expression ✓ states prob 						

- a) Let X = sum of the top numbers of both dice. Determine the mean and standard deviation for X .

a) Show without the use of a classpad how to $\frac{d}{dx} [5x \ln(3x+1)]$

Solution	
$\frac{d}{dx} [5x \ln(3x+1)] = 5x \frac{3}{3x+1} + 5 \ln(3x+1)$	
Specific behaviours	
<ul style="list-style-type: none"> ✓ uses product rule ✓ obtains correct expression 	

- b) Using (a) above and without the use of a classpad, show how to evaluate $\int 3 \ln(3x+1) dx$

$$\text{Hint-use } \frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$$

Solution	
$\frac{d}{dx} [5x \ln(3x+1)] = 5x \frac{3}{3x+1} + 5 \ln(3x+1)$	
$\int \frac{d}{dx} [5x \ln(3x+1)] dx = 5 \int \frac{3x}{3x+1} dx + 5 \int \ln(3x+1) dx$	
$\left[5x \ln(3x+1) \right]_0^1 = 5 \int \left(1 - \frac{1}{3x+1} \right) dx + 5 \int \ln(3x+1) dx$	
$5 \ln 4 = 5 \left[x - \frac{1}{3} \ln(3x+1) \right]_0^1 + 5 \int \ln(3x+1) dx$	
$5 \ln 4 = 5(1 - \frac{1}{3} \ln 4) + 5 \int \ln(3x+1) dx$	
$3 \ln 4 = 3(1 - \frac{1}{3} \ln 4) + 3 \int \ln(3x+1) dx$	
$3 \int \ln(3x+1) dx = 3 \ln 4 - 3 + \ln 4 = 4 \ln 4 - 3$	
Note- zero marks if answer given without any working!	
Specific behaviours	
<ul style="list-style-type: none"> ✓ integrates expression from part a ✓ uses FTC ✓ changes $\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$ and shows integration of these two terms (or uses $u=3x+1$) 	

Start Calculation

One-Variable

Solution

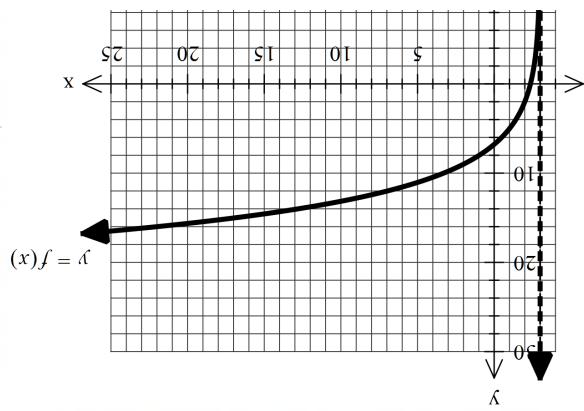
Specific behaviours

states mean of 7
states stdv of 2.415

Solve

$$\int_m^1 -\frac{5}{776} \cdot x^3 \cdot (x-5) dx = 0.5, m \in [10, m < 5]$$

Edit Action Interactive



Q4) (5 marks)
Consider $f(x) = r \log_s(x+p) + q$ where r, p, q are constants.

Solution

Specific behaviours

sets up equation for median only
sets one median only

Edit Action Interactive

c) Determine the median.

Specific behaviours

denominator
numerator

Solve

$$\frac{\int_2^4 x^3 (x-5) dx}{\int_2^2 x^3 (x-5) dx} = 0.3843503937$$

Edit Action Interactive

Using the graph above and given that the following points $(22, 16)$ & $(2, 9)$ lie on the curve $y = f(x)$, determine the values of r, p & q .

Solution
$f(x) = r \log_5(x + p) + q$
Asymptote $x = -3$, $p = 3$
$(2, 9)$
$9 = r \log_5(5) + q = r + q$
$(22, 16)$
$16 = r \log_5(25) + q = 2r + q$
$16 - 9 = r$
$r = 7$
$16 = 14 + q$
$q = 2$
Specific behaviours
<ul style="list-style-type: none"> ✓ identifies asymptote at $x = -3$ ✓ determines p ✓ sets up one equation with two unknowns ✓ sets up two equations with two unknowns ✓ states r & q

Q5 (2, 2 & 2 = 6 marks)

Consider the probability density function $f(x) = ax^3(x - 5)$, $1 \leq x \leq 5$ and zero for all other values of x .

a) Show that $a = \frac{-5}{776}$.

Solution

Edit Action Interactive
0.5 $\frac{1}{2}$ $\frac{\downarrow}{\uparrow}$ $\frac{\leftarrow}{\rightarrow}$ $\frac{\partial}{\partial}$ \int_{dx} \int_{dx} Simp \int_{dx} $\frac{\partial}{\partial}$ $\frac{\partial}{\partial}$ $\frac{\partial}{\partial}$ $\frac{\partial}{\partial}$
solve($\int_1^5 a \cdot x^3 \cdot (x-5) dx = 1, a$)
$\left\{ a = -\frac{5}{776} \right\}$
Specific behaviours
<ul style="list-style-type: none"> ✓ sets up integral for total area = 1 ✓ shows equation to solve for a

b) Determine the probability $\Pr(X \leq 3 | 2 \leq X \leq 4)$ for the above function.

Solution
$\Pr(X \leq 3 2 \leq X \leq 4) = \frac{\Pr(2 \leq X \leq 3)}{\Pr(2 \leq X \leq 4)}$