$= \frac{1}{3}\left[\sqrt{3} - 1\right]$

[1 mark]

MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 3 - 2017
Applications and Integration
RESOURCE RICH - SOLUTIONS

Multiple-choice questions **[2 marks each]**

1 $y = 3x^3 + 4x^2 + 5$

$y' = 9x^2 + 8x$

When $x = 2$

$y' = 52$

$\delta y = 52 \times 0.03$

$= 1.56$

$\therefore$ **D**

2 $y = 2x^3 + 12x^2 - 18x - 5$

$y' = 6x^2 + 24x - 18$

$y'' = 12x + 24$

concave upwards when $y'' > 0$

$12x + 24 > 0$

$12x > -24$

$x > -2$

$\therefore$ **B**

3 Let the two numbers be x and y.

Then $xy = 72$ and the sum $S = 2x + 4y$

$y = \frac{72}{x}$

Substitute into $S = 2x + 4y$

$S = 2x + 4\left(\frac{72}{x}\right)$

$S = 2x + \frac{288}{x}$

$\frac{dS}{dx} = 2 - \frac{288}{x^2}$

Stationary point when $\frac{dS}{dx} = 0$,

$2 - \frac{288}{x^2} = 0$

$2x^2 = 288$

$x^2 = 144$, since x is positive

$x = 12$

$y = \frac{72}{12}$

$y = 6$

$\therefore$ **A**

4 The width of each rectangle is 0.25 units and the centres are at x = 0.125, 1.375, 1.625 and 1.875

Heights are $f(1.125) = 1.125^4$, $f(1.375) = 1.375^4$, $f(1.625) = 1.625^4$ and $f(1.875) = 1.875^4$

$A = 0.25 \times 1.125^4 + 0.25 \times 1.375^4 + 0.25 \times 1.625^4 + 0.25 \times 1.875^4$

$= 0.25 \times (1.125^4 + 1.375^4 + 1.625^4 + 1.875^4)$

$\therefore$ **D**

5 The algebraic area between $x = -4$ and $x = 1$ is negative, so $-\int_{-4}^{1} f(x)dx$ will give the physical area.

$\therefore$ **E**

6 $\int_0^4 (2x^3 - x^2 + 5x + 4)dx - \int_0^4 (x^3 + 2x^2 - 3x - 1)dx$

$= \int_0^4 (2x^3 - x^2 + 5x + 4 - x^3 - 2x^2 + 3x + 1)dx$

$= \int_0^4 (x^3 - 3x^2 + 8x + 5)dx$

$\therefore$ **A**

7 $\int_0^a (2x - 1)^2 dx = \int_0^a (4x^2 - 4x + 1)dx$

$= \left[\frac{4x^3}{3} - \frac{4x^2}{2} + x\right]_0^a$

$= \frac{4a^3}{3} - \frac{4a^2}{2} + a$

$\therefore$ **B**

$T = \frac{\sqrt{1 + x^2}}{2} + \frac{1 - x}{3}$ **[1 mark]**

$\frac{dT}{dx} = \frac{x}{2\sqrt{1 + x^2}} - \frac{1}{3}$ **[1 mark]**

$= 0$ when $\frac{x}{2\sqrt{1 + x^2}} - \frac{1}{3} = 0$

$\frac{x}{2\sqrt{1 + x^2}} = \frac{1}{3}$ **[1 mark]**

$3x = 2\sqrt{1 + x^2}$

$9x^2 = 4(1 + x^2)$

$9x^2 = 4 + 4x^2$

$5x^2 = 4$

$x^2 = \frac{4}{5}$

$x = \frac{2}{\sqrt{5}} = 0.89$ since $0 \le x \le 1$ **[1 mark]**

[1 mark]

Substitute into T to find $T \approx 0.706$ hours $\approx$ 42 minutes 22 seconds **[1 mark]**

17 a $\frac{dy}{dx} = 3 \times \frac{1}{\cos^2(3x)}$ **[1 mark]**

$= \frac{3}{\cos^2(3x)}$ **[1 mark]**

b $\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \frac{1}{\cos^2(3x)}dx = \int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \frac{1}{3} \times \frac{3}{\cos^2(3x)}dx$ **[1 mark]**

$= \frac{1}{3}\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \frac{3}{\cos^2(3x)}dx$ **[1 mark]**

$= \frac{1}{3}[\tan(3x)]_{\frac{\pi}{12}}^{\frac{\pi}{9}}$ **[1 mark]**

$= \frac{1}{3}\left[\tan\left(3 \times \frac{\pi}{9}\right) - \tan\left(3 \times \frac{\pi}{12}\right)\right]$

$= \frac{1}{3}\left[\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)\right]$

$$A = 2\int_0^3 2x^3\,dx$$ **[1 mark]**

$$= \int_0^3 4x^3\,dx$$

$$= \left[\frac{4x^4}{4}\right]_0^3$$

$$= \left[x^4\right]_0^3$$ **[1 mark]**

$= 3^4 - 0^4$

$= 81$

The area is 81 units^2. **[1 mark]**

15 $\frac{dy}{dx} = 8x - 7$

$y = 4x^2 - 7x + c$ **[1 mark]**

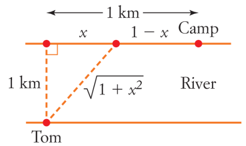
$y = 13$ when $x = -1$, so $13 = 4 \times (-1)^2 - 7 \times -1 + c$

$13 = 11 + c$

$c = 2$ **[1 mark]**

$y = 4x^2 - 7x + 2$ **[1 mark]**

16 Swim to a point approximately 0.89 km along the river towards his camp and then walk approximately 0.11 km to his camp. This will take approximately 42 minutes 22 seconds.



Swim: 2 km/h, Walk: 3 km/h

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time = Swim time + Walk time

8 The algebraic area between $x = 1$ and $x = 3$ is negative, so $-\int_1^3 (x^2 - 3x)\,dx$ will give the physical area.

∴ **B**

9 $\int_0^{\frac{\pi}{6}} \sin(x)\,dx = \left[-\cos(x)\right]_0^{\frac{\pi}{6}}$

$= -\cos\left(\frac{\pi}{6}\right) - [-\cos(0)]$

$= -\frac{\sqrt{3}}{2} + 1$

$= \frac{2}{2} - \frac{\sqrt{3}}{2}$

$= \frac{2 - \sqrt{3}}{2}$

∴ **B**

10 Total change $= \int_a^b R'(t)\,dt$

$= \int_0^5 10e^{0.2t}\,dt$

$= \left[\frac{10e^{0.2t}}{0.2}\right]_0^5$

$= \left[50e^{0.2t}\right]_0^5$

$= 50e^1 - 50e^0$

$= 85.914...$

∴ **B**

11 **a** The average score is decreasing.

[1 mark]

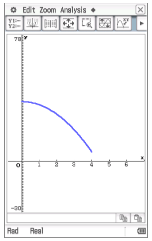
b The rate at which the average score is decreasing is increasing.

[1 mark]

c **[1 mark]** for concave downwards for x

[1 mark] for decreasing curve

[1 mark] for y-intercept of 38



12 Volume = $\pi r^2 h$

$500 = \pi r^2 h$ **[1 mark]**

$$h = \frac{500}{\pi r^2}$$ **[1 mark]**

Surface area = $2\pi r^2 + 2\pi rh$

$$A = 2\pi r^2 + 2\pi r \times \frac{500}{\pi r^2}$$ **[1 mark]**

$$= 2\pi r^2 + \frac{1000}{r}$$

$$\frac{dA}{dr} = 4\pi r - \frac{1000}{r^2}$$

$$= 0 \text{ when } 4\pi r - \frac{1000}{r^2} = 0$$ **[1 mark]**

$$4\pi r = \frac{1000}{r^2}$$

$$r^3 = \frac{1000}{4\pi}$$

$$r = \sqrt[3]{\frac{1000}{4\pi}}$$

$$r = 4.3 \text{ correct to 2 sig. fig.}$$

$$\frac{d^2A}{dr^2} = 4\pi + \frac{1000}{r^3}$$ **[1 mark]**

$$> 0 \text{ for all } r \geq 0 \therefore \text{minimum}$$ **[1 mark]**

13 a $$\int_1^3 (2x - 9)dx = \left[x^2 - 9x\right]_1^3$$ **[1 mark]**

$= (3^2 - 9 \times 3) - (1^2 - 9 \times 1)$

$= 9 - 27 - 1 + 9$

$= -10$ **[1 mark]**

b $$\int_2^6 e^x\,dx = \left[e^x\right]_2^6$$ **[1 mark]**

$= e^6 - e^2$ **[1 mark]**

$= e^2(e^4 - 1)$

c $$\int_0^\pi \cos(x)dx = \left[\sin(x)\right]_0^\pi$$ **[1 mark]**

$= \sin(\pi) - \sin(0)$

$= 0 - 0$

$= 0$ **[1 mark]**

d $$\int_{-2}^1 (x^2 - 3x + 5)dx = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 5x\right]_{-2}^1$$ **[1 mark]**

$$= \left(\frac{1^3}{3} - \frac{3 \times 1^2}{2} + 5 \times 1\right) - \left(\frac{(-2)^3}{3} - \frac{3(-2)^2}{2} + 5 \times -2\right)$$

$$= \left(\frac{1}{3} - \frac{3}{2} + 5\right) - \left(\frac{-8}{3} - 6 - 10\right)$$

$$= \frac{1}{3} - \frac{3}{2} + 5 + \frac{8}{3} + 6 + 10$$

$= 22\frac{1}{2}$ **[1 mark]**

14 a $$\int_{-3}^3 2x^3\,dx = \left[\frac{2x^4}{4}\right]_{-3}^3$$

$$= \left[\frac{x^4}{2}\right]_{-3}^3$$ **[1 mark]**

$$= \frac{3^4}{2} - \frac{(-3)^4}{2}$$

$$= \frac{81}{2} - \frac{81}{2}$$ **[1 mark]**

$= 0$ **[1 mark]**

b