Papers written by Australian Maths Software

SEMESTER TWO

YEAR 12

MATHEMATICS SPECIALIST

UNIT 3-4

REVISION ONE

2016
Section One
(Calculator–free)

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 5 minutes
Working time for section: 50 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items*:* pens, pencils, pencil sharpener, highlighter, eraser, ruler.
Special items: nil.

IMPORTANT NOTE TO CANDIDATES
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor
Question/answer booklet for Section One.
A formula sheet which may also be used for Section Two.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	**8**	**8**	**50**	**52**	**35**
Section Two Calculator—assumed	12	12	100	98	65
			Total marks	150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answer in the Question/Answer booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

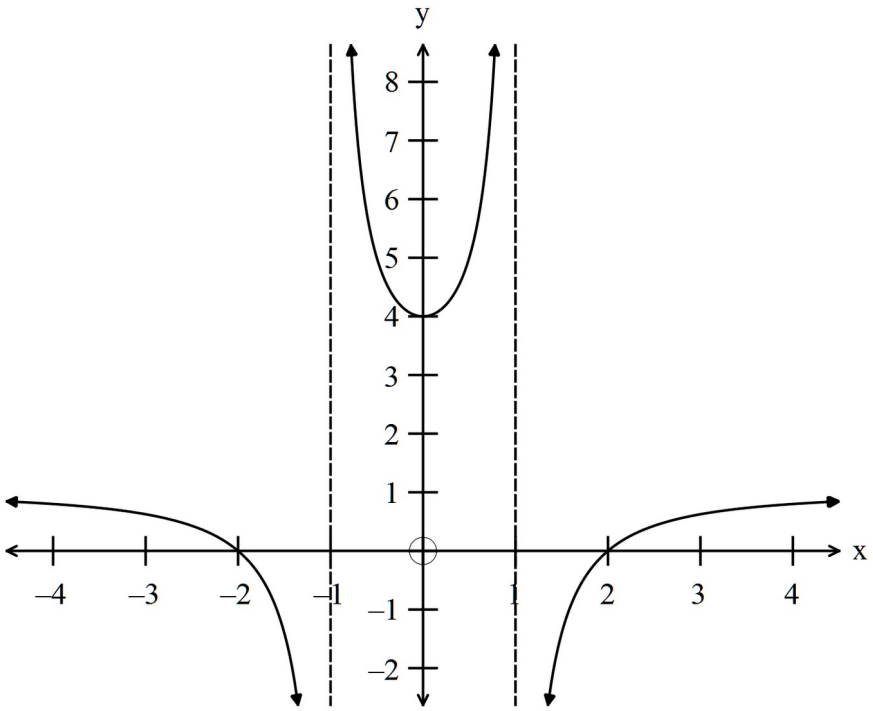
6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

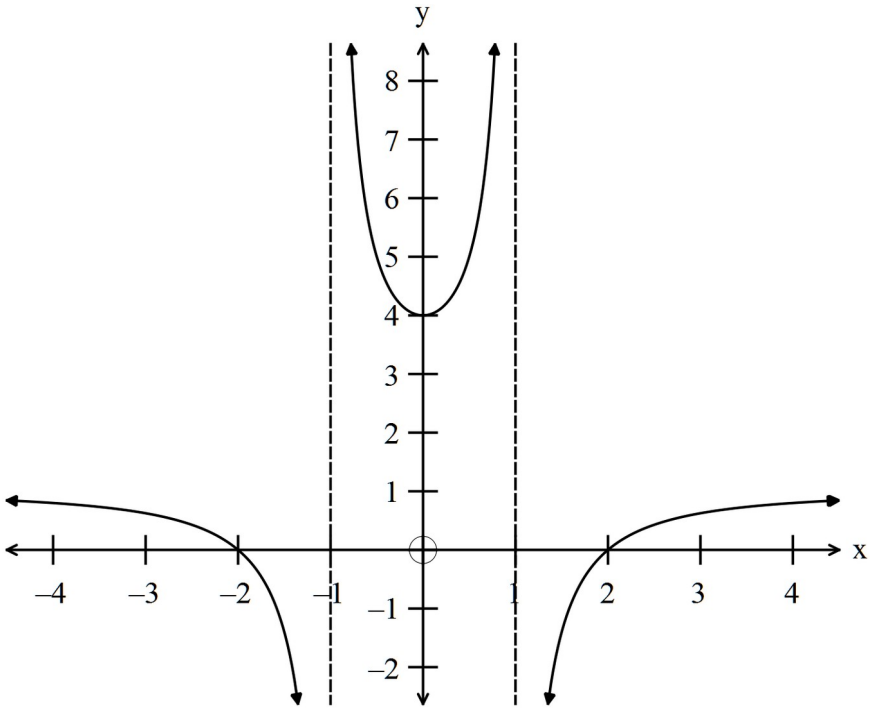
1. (5 marks)

(a) Given the sketch of the function $f(x) = \dfrac{(x^2 - 4)}{(x^2 - 1)}$ sketch

(i) $y = |f(x)|$ (2)



(ii) $y = \dfrac{1}{f(x)}$ (3)



2. (4 marks)

Solve the complex equation $z^4 = -16$. (4)

$\sqrt{\ }$ -1

"Why can't we be together?" "It is complex!"

3. (6 marks)

(a) Find the expression for $\frac{dy}{dx}$ given the relationship $e^{\cos(x)} + e^{\sin(y)} = e + 1.$

(4)

(b) Hence find $\frac{dy}{dx}$ at the point $(0, 0).$ (2)

4. (7 marks)

(a) Prove that "If $(x - a)$ is a factor of a polynomial, then $P(a) = 0$."

(4)

(b) Given $P(x) = x^3 + x^2 + x - 3$ find x such that $P(x) = 0$ and hence solve the equation $x^3 + x^2 + x - 3 = 0$. (3)

5. (10 marks)

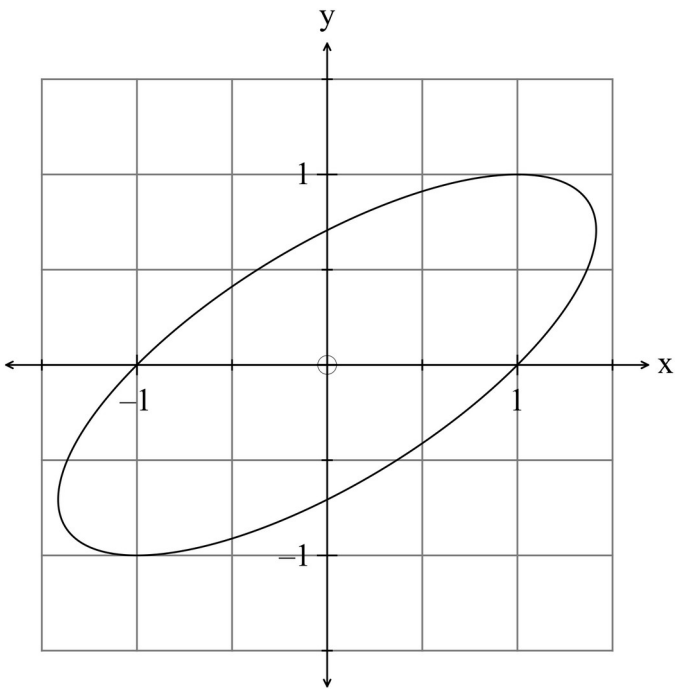
(a) Evaluate $\int_{\pi/6}^{\pi/3} (sin^2(x) + 2x)dx.$ (3)

(b) Evaluate $\int_0^1 \sqrt{1 - x^2}\ dx$ put $x = sin(\theta)$ (5)

(c) Determine $\int \frac{ln(x)}{x} dx$ put $u = ln(x)$ (2)

6. (12 marks)

(a) The position vector of a particle travelling on an elliptical path, as shown on the graph below, is given by $\boldsymbol{r}(t) = (\sin(t) + \cos(t))\boldsymbol{i} + (\cos(t))\boldsymbol{j}$ for any time t.



(i) Find when the particle is at (-1, -1). (2)

(ii) Find the initial position of the particle. (1)

(iii) Find the velocity and acceleration of the particle at $t = 0$. (3)

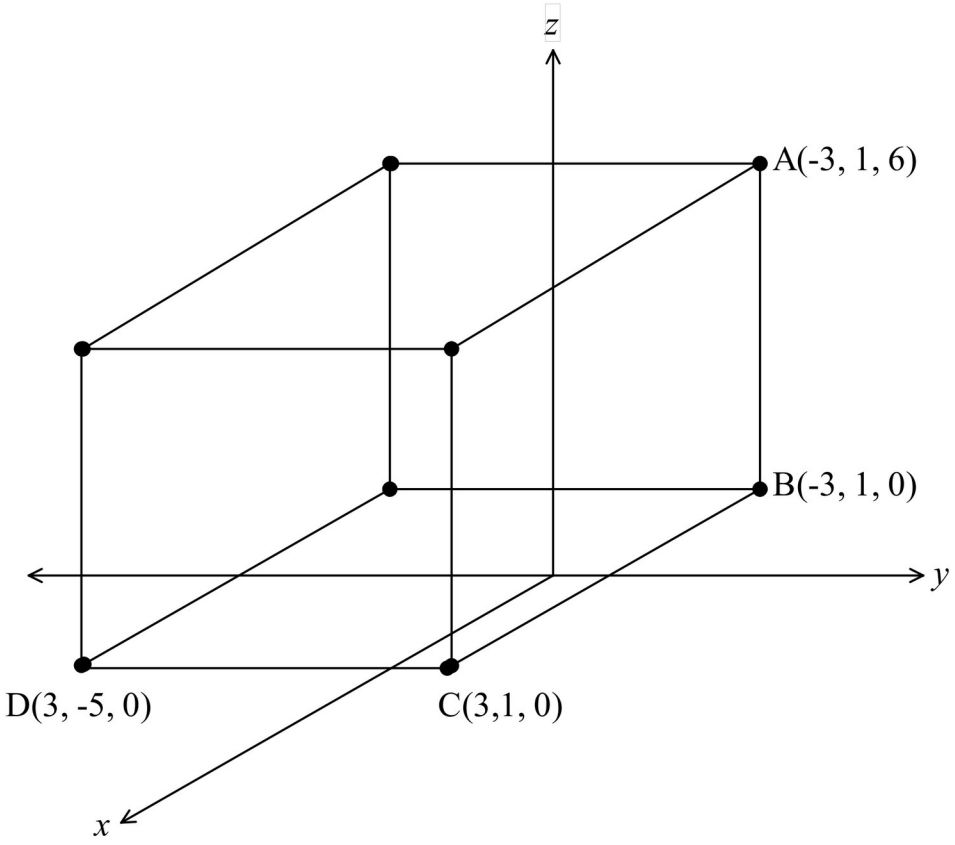
(iv) Plot the acceleration vector on the graph at $t = 0$. (2)

(v) Determine the values of t such that $\boldsymbol{a}(t) = -\boldsymbol{r}(t)$. (1)

(b) Fidn the vector equation of a plane that contains the line $\mathbf{r}(t) = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$ and the point $P(-1, 2, -4)$. (3)

7. (3 marks)

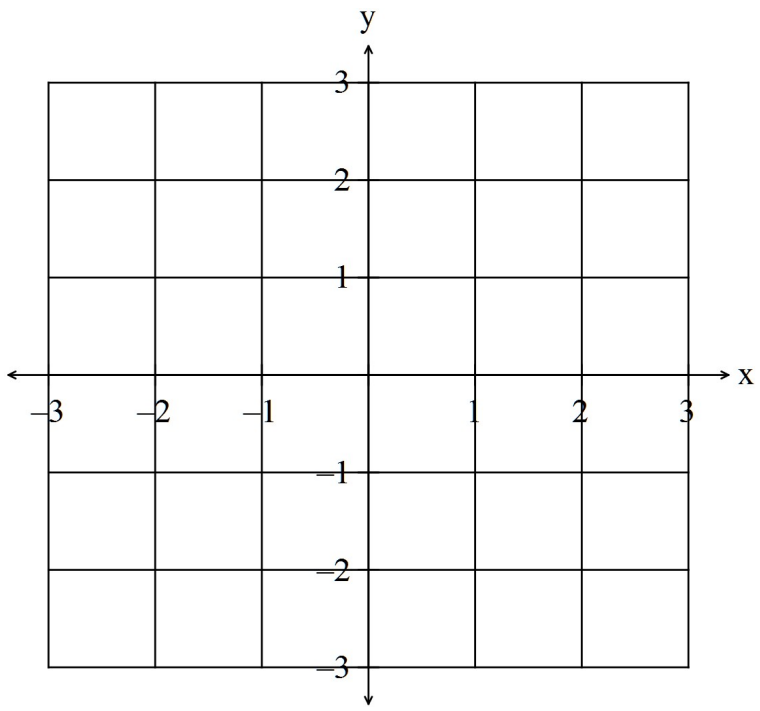
Find the equation of a sphere that fits exactly inside the cube on the diagram below.



(3)

8. (5 marks)

(a) Sketch $\{z : |x - 1 + iy| = 2|x + i(y - 1)|\}$ on the set of axes below. (3)



(b) If $z = \frac{1+i}{1-i} \times (3+3i)$ find the expression for $\bar{z}$. (2)

END OF SECTION ONE