PRESBYTERIAN LADIES' COLLEGE
A COLLEGE OF THE UNITING CHURCH IN AUSTRALIA

MATHEMATICS DEPARTMENT
MATHEMATICAL METHODS YEAR 12 – TEST 3

DATE: 27th June 2016 Name: _____

CALCULATOR FREE

Reading Time: 3 minutes

Working Time: 50 minutes

EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet (provided)

Question	Marks available	Marks awarded
1	4	
2	6	
3	6	
4	3	
5	6	
6	5	
7	6	
8	7	
9	9	
Total	**52**	

Question 1 **(4 marks)**

Evaluate the following:

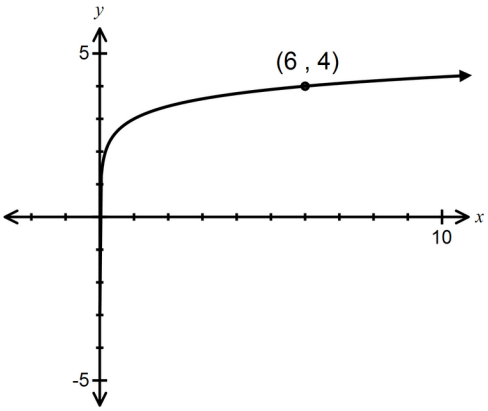
(a) $\log_3 27$ (1 mark)

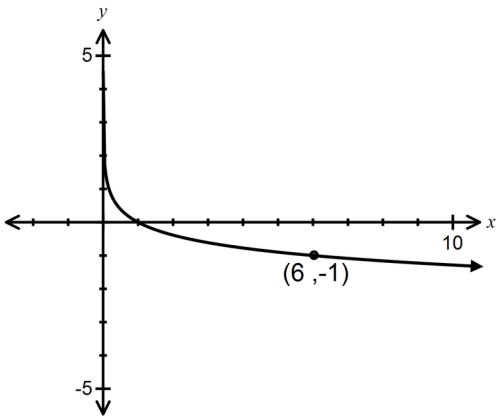
(b) $\log_{15} 1$ (1 marks)

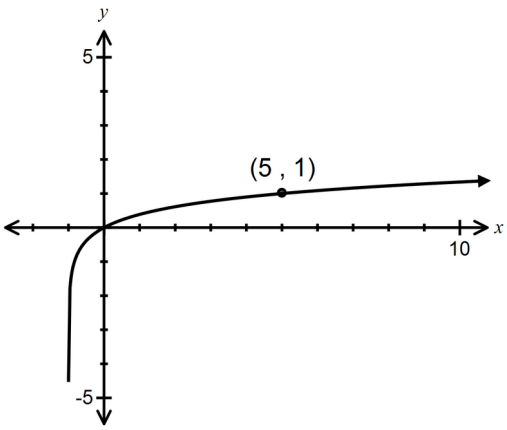
(c) $\log_{25} 0.2$ (2 marks)

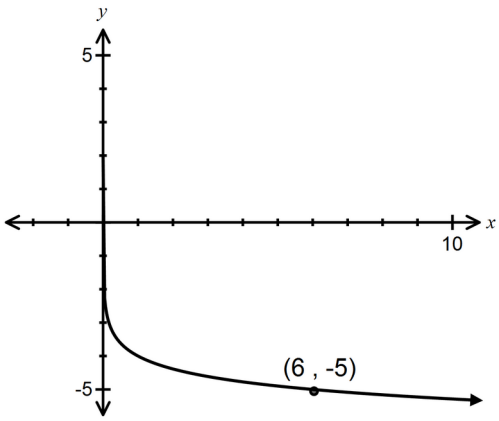
Question 2 **(6 marks)**

Match each of the following graphs with their equations from the given list.
A graph may have more than one matching equation. Not all equations have a matching graph.









A $y = \log_6(x+1)$

B $y = \log_6(x-1)$

C $y = \log_{\frac{1}{6}}(x)$

D $y = -\log_6(x)$

E $y = \log_6(x)+3$

F $y = -\log_6(x)-4$

G $y = \log_5(x)$

H $y = \log_5(x+1)$

I $y = \log_6(\frac{1}{x})-4$

Question 3 **(6 marks)**

Differentiate with respect to x.

(a) $f(x) = \ln(2x + 1)$ (1 mark)

(b) $f(x) = \ln\left(\frac{x^2 + 2x}{x - 5}\right)$ (2 marks)

(c) $f(x) = \frac{2\sqrt{x}}{\ln x}$ (3 marks)

Question 4 **(3 marks)**

Find the following indefinite integrals.
Assume denominators are greater than zero.

(a) $\int \frac{12}{x} dx$ (1 mark)

(b) $\int \frac{2e^x + 2\sin x}{e^x - \cos x} dx$ (2 marks)

Question 5 **(6 marks)**

Find the exact value of x satisfying the equation

$$(3^x)(4^{2x+1}) = 6^{x+2}.$$

Give your answer in the form $\frac{\ln a}{\ln b}$.

Question 6 **(5 marks)**

(a) The function f is defined for $x > 2$ by $f(x) = \ln x + \ln(x - 2) - \ln(x^2 - 4)$.

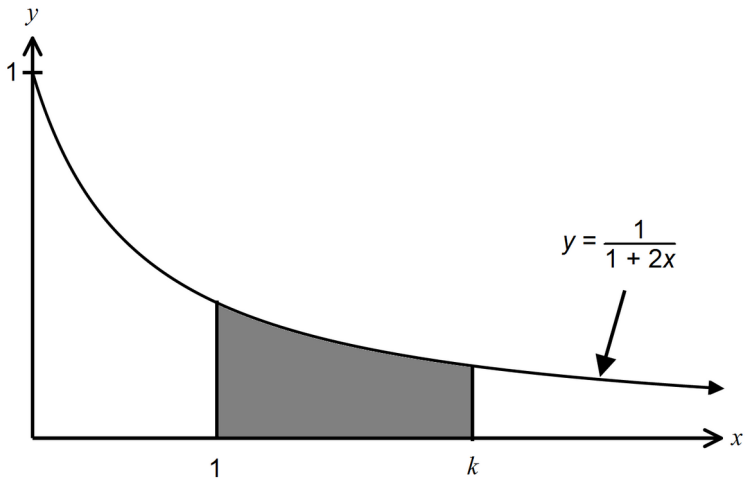
Express $f(x)$ in the form $\ln\left(\frac{x}{x + a}\right)$. (2 marks)

(b) Evaluate $\log\frac{1}{2} + \log\frac{2}{3} + \log\frac{3}{4} + \log\frac{4}{5} + \ldots + \log\frac{8}{9} + \log\frac{9}{10}$. (3 marks)

Question 7 **(6 marks)**

The shaded area is 0.2 units^2.

Find k exactly.



Question 8 **(7 marks)**

As part of a local arts festival, an artist plans to create an installation in which a concealed water cannon blasts a stream of water into the air for a few seconds at random intervals.

The lengths of the intervals between each firing of the cannon can be modelled by the uniformly distributed random variable T, where $3 \leq t \leq 14$ minutes.

(a) Sketch the probability density function $f(t)$ for the interval between each firing on the axes below. (2 marks)

(b) Determine the probability that a randomly chosen interval between firings is

(i) at least seven minutes. (1 mark)

(ii) at least six minutes given that it is less than ten minutes. (2 marks)

(c) Determine the value of t for which $P(T < t) = P(T > 4t)$ (2 marks)

Question 9 **(9 marks)**

The continuous random variable X is define by the probability density function

$$f(x) = \begin{cases} \frac{2x}{9} & 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine $E(X)$. (2 marks)

(b) The Variance of X, $\text{Var}(X)$, is $\frac{1}{2}$.

(i) Determine $E(4X+3)$ (1 mark)

(ii) Determine $\text{Var}(4X+3)$ (1 mark)

Question 9 continued **(9 marks)**

(c) Determine the cumulative distribution function $F(x)$ (3 marks)

(d) Calculate $P(1 < x < 2)$ (2 marks)

End of Test

Additional page for working.