PERTH COLLEGE
Year 12

Semester One Examination 2012
Question/Answer booklet

MATHEMATICS SPECIALIST
3C/3D

Section One:
Calculator – free

Student Name: _____

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section
To be provided by the supervisor
Question/answer booklet for Section One
Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Working Time (minutes)	Marks available
Section One Calculator-free	**5**	**5**	**50 minutes**	**50**
Section Two Calculator-assumed	12	12	100 minutes	100
			Total marks	150

Instructions to candidates

1. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer
   a. Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   b. Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.

3. It is recommended that you **do not use pencil,** except in diagrams

Question 1   **[7 marks]**

Consider the functions $f(x)=|2x-5|$ and $g(x)=|x-4|$.
Let $h(x)=f(x)+3g(x)$.

a)   Write $h(x)$ as a piecewise function free of absolute values.

[5]

b)   Determine the range of $h(x)$.

[2]

Question 2   [14 marks]

a)   Determine $\dfrac{dy}{dx}$ for each of the following. Only minor simplification is required.

(i) $y = x^3 \sin^3(4x)$

[3]

(ii) $e^y = xy^3$

[3]

(iii) $y = \ln\left(\dfrac{\sqrt{\cos x}}{x^2 \sin x} \right)$

[4]

4

b)   Determine the exact value of $\displaystyle \lim_{h \to 0} \frac{\sin^3\left(\frac{\pi}{6} + h\right) - \frac{1}{8}}{h}$.

[4]

Question 3   **[15 marks]**

Consider matrix $\mathbf{T} = \begin{bmatrix} 4 & 2 \\ -5 & k \end{bmatrix}$

a)   In each case, state the value of k which satisfies the given condition.

   (i)   **T** is singular.
                                                                                                      [2]

   (ii)   **T** maps the vertices of a quadrilateral onto a line.
                                                                                                      [1]

   (iii)   **T** maps the triangle with vertices P (0, 0), Q (1, 0), R (0, –1) onto the triangle with
           vertices P' (0,0) , Q' (4, –5) and R' (–2, –3).
                                                                                                      [2]

   (iv) $\mathbf{T}^2 = \begin{bmatrix} 6 & 2 \\ -5 & -1 \end{bmatrix}$
                                                                                                      [2]

(v) $\mathbf{T} - 2\mathbf{I} = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$, where **I** is the 2 × 2 Identity Matrix.

[1]

(vi)   The point (3, 4) is transformed to the point (4, −7) after applying $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, and then applying **T**.

[3]

b)   If $k = 2$ and $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$, find **B**, given that $\mathbf{AB} + \mathbf{B} = \mathbf{T}$.

[4]

Question 4   **[7 marks]**

Determine each of the following indefinite integrals:

a) $\displaystyle\int \frac{3x}{5x^2 - 2}\, dx$

[2]

b) $\displaystyle\int 6\sin(\pi - 3t)\, dt$

[2]

c) $\displaystyle\int \frac{\cos x - \sin x}{(\sin x + \cos x)^2}\, dx$

[3]

Question 5   **[7 marks]**

A curve is defined parametrically by the equations $x = \dfrac{1}{e^t}$ and $y = (t+1)^3$.

Find each of the following in terms of t, fully simplifying your answers.

a) $\dfrac{dy}{dx}$

[3]

b) $\dfrac{d^2 y}{dx^2}$

[4]

EXTRA PAGES FOR WORKING
Clearly number any questions you do here.

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Clearly number any questions you do here.

Your Name: _____

	Question	Marks Available	Your Mark
	1	7	
	2	14	
	3	15	
	4	7	
	5	7	
	TOTAL SECTION 1	50	
	TOTAL SECTION 2	100	
	OVERALL	150	