SCHOOL

Year 12 Trial WACE Examination, 2013

Question/Answer Booklet

MATHEMATICS 3A/3B
Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures ☐☐☐☐☐☐☐☐

In words ______________________________

Your name ______________________________

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	12	12	100	100	67
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

Question 19 **(8 marks)**

Triangle DEF has three sides of lengths 6 cm, 13 cm and 15cm.

(a) Determine the size of the largest angle in triangle DEF. (3 marks)

Largest angle opposite longest side:

$$\cos\theta = \frac{6^2 + 13^2 - 15^2}{2 \times 6 \times 13}$$
$$\theta = 97.4^\circ$$

Triangle PQR has three sides of lengths of lengths 12 cm, 19 cm and 21 cm.

(b) A student argued that because the three sides of triangle PQR were all 6 cm longer than the corresponding sides of triangle DEF, then the two triangles must be similar.

Explain whether this is a valid argument. (2 marks)

Not a valid argument - the triangles are not similar because the ratio of corresponding side lengths must be the same, not the difference.

Triangle ABC has an area of 350 cm^2 and is such that the length of AB is 5 cm more than the length of BC and the size of $\angle ABC = 30^\circ$.

(c) Determine the length of the side AB. (3 marks)

Let length of $AB = x$. Then

$$350 = \frac{1}{2}x(x - 5)\sin 30^\circ$$
$$x = -35, 40$$

Hence length is 40cm.

Question 18 **(6 marks)**

The pressure, P, in an air bubble varies inversely with the volume, V, of the bubble.

It is known that $P = 2.4$ kPa when $V = 5$ cm³.

(a) Find the value of the constant k in the equation $P = \frac{k}{V}$. (1 mark)

$$2.4 = \frac{k}{5}$$
$$2.4 \times 5 = k$$
$$k = 12$$

(b) Determine

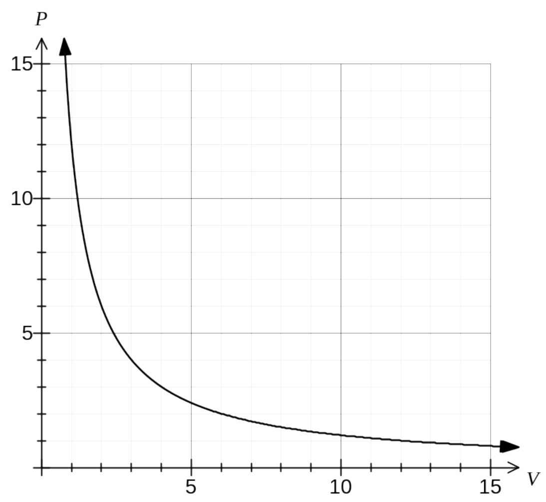
(i) the value of P when $V = 2.5$ cm³. (1 mark)

$$P = \frac{12}{2.5} = 4.8 \text{ kPa}$$

(ii) the value of V when $P = 10$ kPa. (1 mark)

$$10 = \frac{12}{V}$$
$$V = 1.2 \text{ cm}^3$$

(c) On the axes below, draw a graph to show how P varies with V. (3 marks)



Section Two: Calculator-assumed **(100 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(7 marks)**

The number of new accounts created on a recently launched social networking site increase by a factor of three every day. On Day 1, 140 new accounts were created, on Day 2, 420 new accounts were created and so on.

(a) How many new accounts were created on Day 4? (1 mark)

$420 \times 3 \times 3 = 3780$ new accounts

(b) Write a recursive rule for T_n, the number of new accounts created on Day n. (2 marks)

$$T_n = 3T_{n-1}$$
$$T_1 = 140$$

(c) On which day did the number of new accounts created first exceed one million and how many new accounts were created on that day, to the nearest thousand? (2 marks)

$$T_n > 1000000$$
$$n \geq 10$$
$$T_{10} = 2755620$$

First exceeded on day 10 when 2 756 000 new accounts created

(d) The site earns five cents in advertising revenue for each new account. How much did the site earn in the first seven days? (2 marks)

$$S_7 = 153020$$

$153020 \times 0.05 = \$7651$

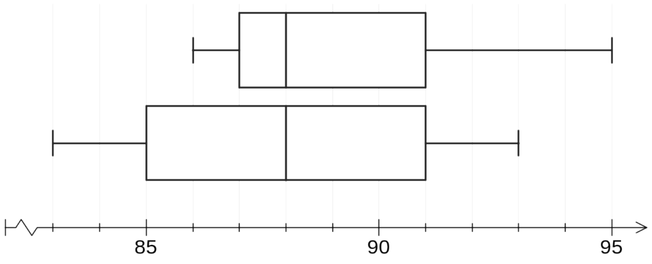
Question 9 **(10 marks)**

200 tagged catfish were released into a stretch of river. The next day at the same stretch of river, a sample of 140 catfish was caught of which 3 were tagged.

(a) Use this information to calculate an estimate for the number of catfish in the river at this location, to the nearest one hundred fish. (3 marks)

$$\frac{200}{P}=\frac{3}{140}$$
$$P=9333.\overline{3}$$
$$\approx 9300 \text{ to the nearest one hundred}$$

Over the next few days this method of estimating the population of catfish to the nearest one hundred was repeated another ten times at the same location. The population estimates are summarised in the box and whisker plot below.



Catfish population estimates (in hundreds of fish)

(b) State the median and interquartile range for this set of data. (2 marks)

Median is 88 hundred fish.
IQR is $91-87=4$ hundred fish.

The same sampling procedure was also carried out at a spot 500 m upriver from the first location, giving rise to a second set of population estimates, in hundreds of fish:

84, 93, 85, 93, 88, 91, 89, 83, 88, 89 and 88

(c) Construct a box and whisker plot for the second set of data on the diagram above. (3 marks)

Ordered data: **83**, 84, **85**, 88, 88, **88**, 89, 89, **91**, 93, **93**

Median: 88, Q1 & Q3: 85 & 91 Min & Max: 83 & 93

(d) Which location, the original spot or 500 m upriver, produced the most consistent population estimates? Justify your answer. (2 marks)

The original spot, because both the range and IQR were larger for the stretch 500 m upriver.

Question 17 **(10 marks)**

A function is given by $f(x)=200+32x^2-x^4$ for $-3\le x\le 5$.

(a) Use calculus techniques to determine the coordinates of both stationary points of the function for the given domain. (4 marks)

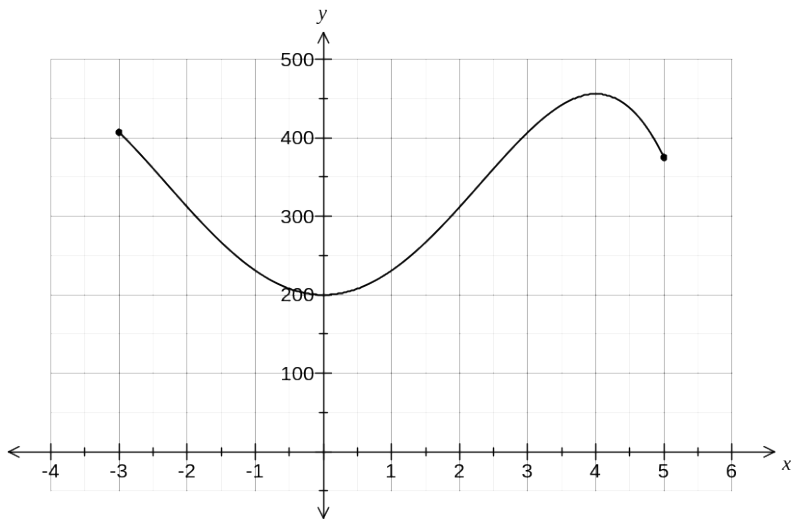
$f'(x)=64x-4x^3$

$64x-4x^3=0$ when $x=-4,0,4$

$f(0)=200$
$f(4)=456$

Over given domain, stationary points at (0, 200) and (4, 456)

(b) Sketch the graph of $f(x)$ for $-3\le x\le 5$ on the axes below. (4 marks)



(c) Does the graph above have any inflection points? If so, how many and where(approx.)? (2 marks)

Two Points of inflection at (-2.3, 342.2) and (2.3, 342.2).

Note: No need to give y coordinate
and accept 2 to 2.6
one mark for each point

Question 16 **(8 marks)**

A variety of coffee blends are made by mixing together Brazilian beans that cost $31 per kg with Arabica beans that cost $45 per kg.

(a) Blend X is made by mixing together 8 kg of Brazilian beans with 12 kg of Arabica beans. What is the cost per kg of this blend? (2 marks)

$$\frac{8 \times 31 + 12 \times 45}{8 + 12} = \$39.40 \text{ per kg}$$

(b) Blend Y is made by mixing together b kg of Brazilian beans with 3 kg of Arabica beans. Write an expression for the cost per kg of blend Y and hence determine b, given that blend Y costs $32.40 per kg. (2 marks)

Cost per kg is $\frac{31b + 3 \times 45}{b + 3}$

$$\frac{31b + 3 \times 45}{b + 3} = 32.4$$
$$b = 27 \text{ kg}$$

(c) One kilogram of blend Z contains a kg of Arabica beans.

(i) Show that the cost of one kg of blend Z is $31 + 14a$. (2 marks)

Cost for 1 kg is $31(1 - a) + 45a$
$= 31 - 31a + 45a$
$= 31 + 14a$

(ii) Blend Z costs $38.70 per kg. Determine the weight of Brazilian beans in a 300 g bag of blend Z. (2 marks)

$$31 + 14a = 38.7$$
$$a = 0.55$$
$$(1 - 0.55) \times 300 = 135 \text{ g}$$

Question 10 **(7 marks)**

The lengths, X, of plastic pipes produced by a machine are normally distributed with a mean of 302 cm and a standard deviation of 1.5 cm.

(a) A pipe is chosen from the production line and has a length of 305 cm.

(i) How many standard deviations from the mean is the length of this pipe? (1 mark)

$$\frac{305 - 302}{1.5} = 2 \text{ sd's}$$

(ii) 95% of the pipes produced by the machine will lie within k cm of the mean. Use the 68%, 95%, 99.7% rule to determine the value of k. (1 mark)

95% lie within 2 sd's of mean: $k = 1.5 \times 2 = 3$

(b) A pipe is chosen at random from the production line. Determine $P(X > 301)$. (1 mark)

$P(X > 301) = 0.7475$

(c) 65 randomly chosen pipes are packed into a crate. Estimate how many of these pipes would be shorter than 300 cm. (2 marks)

$P(X < 300) = 0.0912$

$0.0912 \times 65 = 5.93$

Expect 6 of the pipes to be shorter than 300 cm.

(d) An average of one out of every eight pipes produced by the machine exceeds l cm. Determine the value of l, rounding your answer to four significant figures. (2 marks)

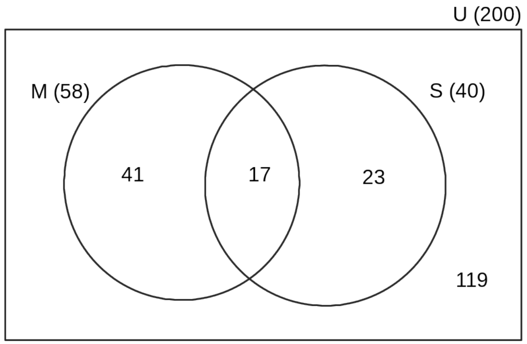
$$P(X > l) = \tfrac{1}{8}$$
$$l = 303.7255$$
$$\approx 303.7 \text{ to 3 sf}$$

Question 11 **(10 marks)**

Two subsets, M and S, belong to a universal set of 200 students. Students belonging to subset M have attended a math revision seminar and students belonging to subset S have attended a science revision seminar.

It is known that $n(M)=58$, $n(S)=40$ and $n(M\cup S)=81$.

(a) Use this information to complete all regions of the Venn diagram below. (3 marks)



(b) How many students from the group attended just one of the revision seminars? (1 mark)

$$\boxed{41+23=64}$$

(c) What is the probability that a student selected at random from the group

(i) attended both revision seminars? (1 mark)

$$\boxed{\frac{17}{200}}$$

(ii) only attended a math revision seminar? (1 mark)

$$\boxed{\frac{41}{200}}$$

(d) If a student is selected at random from the group, determine

(i) $P(\bar{M}\cup S)$ (2 marks)

$$\boxed{\frac{17+23+119}{200}=\frac{159}{200}}$$

(ii) $P(\bar{M}\mid\bar{S})$ (2 marks)

$$\boxed{\frac{119}{41+119}=\frac{119}{160}}$$

(c) Determine the equation of the linear regression line that can be used to predict the moving average, M, from time, T. (2 marks)

$$\boxed{M=0.6327T+36.76}$$

(d) Draw the line of regression calculated in (c) on the graph. (2 marks)

(e) Calculate the seasonal component for the number of calls on Monday. (2 marks)

$$\boxed{\frac{10.4+11.2}{2}=10.8}$$

(f) Predict the number of calls to the enquiry centre on Monday of Week 4. (4 marks)

$$T=16$$

$$\begin{aligned}M&=0.6327(16)+36.76\\&=46.88\end{aligned}$$

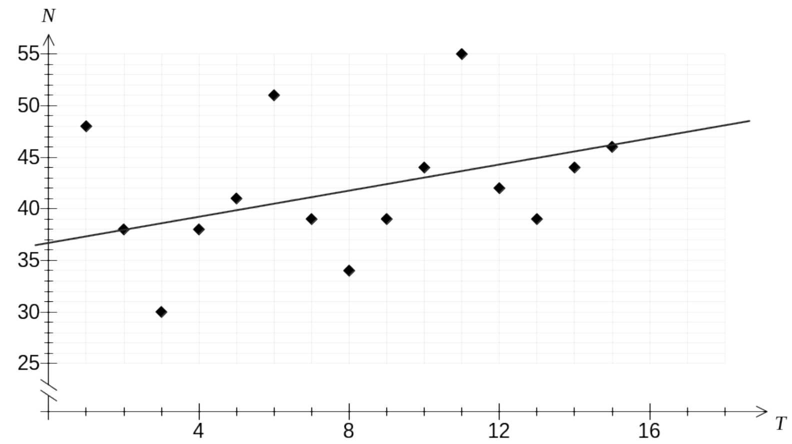
$$\begin{aligned}C&=46.88+10.8\\&=57.68\end{aligned}$$

$$\approx 58 \text{ calls}$$

Question 15 **(12 marks)**

The table and graph below show N, the number of calls per weekday to a new enquiry line, over a three week period, together with five-point moving averages, M, and associated residuals, R.

	Week 1					Week 2					Week 3				
Day	M	T	W	T	F	M	T	W	T	F	M	T	W	T	F
T	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
N	48	38	30	38	41	51	39	34	39	44	55	42	39	44	46
M			39.0	39.6	39.8	40.6	40.8	41.4	42.2	42.8	43.8	44.8	45.2		
R			-9.0	-1.6	1.2	10.4	-1.8	-7.4	-3.2	1.2	11.2	-2.8	-6.2		



(a) What is noticeable about the number of calls to the enquiry line on Wednesdays? (1 mark)

The number of calls on Wednesdays is always the lowest for each week.

(b) Write down the calculation that was used to determine the five-point moving average for Tuesday of Week 2. (1 mark)

$$\frac{41+51+39+34+39}{5}$$

Question 12 **(7 marks)**

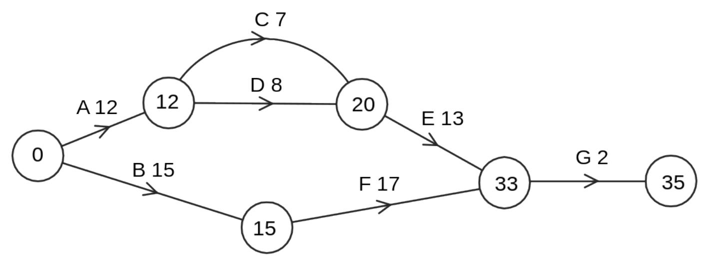
A manager of a building company has split a project into seven jobs that need to be completed. The time, in weeks, that each job requires, is shown in the table below.

Job	A	B	C	D	E	F	G
Time (Weeks)	12	15	7	8	13	17	2

Some jobs cannot start until other jobs have finished, as instructed below:

- C and D cannot start until A has finished
- F must wait until B has finished
- E must wait until C and D are finished
- G must wait until E and F are finished.

(a) Draw a project network to represent the above information. (3 marks)



(b) List, in order, the jobs that lie on the critical path and state the minimum completion time for this project. (2 marks)

Critical path is A - D - E - G.

MCT is 35 weeks.

(c) If job B was delayed by two weeks and job C delayed by three weeks, what effect does this have on the critical path and minimum completion time for the project? (2 marks)

Critical path becomes A - C - E - G.

MCT increases to 37 weeks.

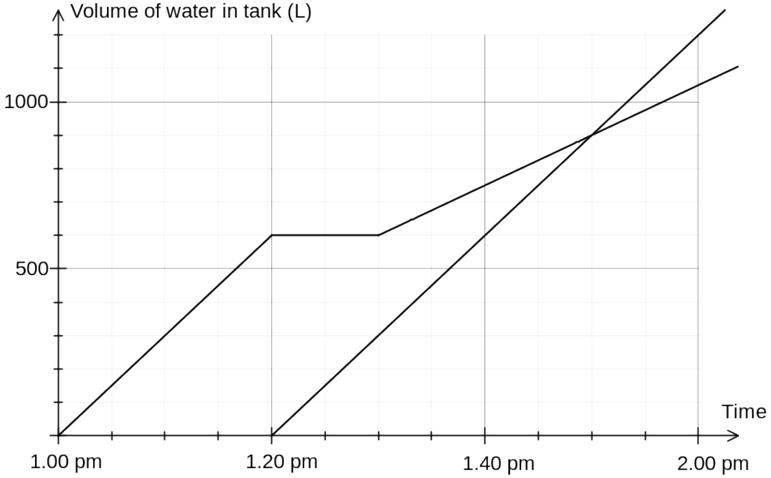
Question 13 **(7 marks)**

Two empty water tanks are to be filled with water.

At 1 pm, a hose delivering a steady 30 litres per minute is placed into tank A and left to run for 20 minutes, at which time it is switched from tank A into tank B.

Ten minutes after the hose is switched, a second hose is placed into tank A, this one delivering a steady 15 litres per minute.

The graph below shows the volume of water in tank B for the first hour.



(a) What volume of water was in tank A at 1.20 pm? (1 mark)

$20 \times 30 = 600$ L

(b) On the axes above, draw the graph for the volume of water in tank A during the first hour. (2 marks)

(c) Determine the time at which both tanks contained the same volume of water and state what this volume is. (2 marks)

At 1.50 pm both tanks held 900 L of water.

(d) Calculate the rate at which the second hose delivered water, in kilolitres per hour, given that there are 1000 litres in one kilolitre. (2 marks)

$15 \times 60 \div 1000 = 0.9$ kL/h

Question 14 **(8 marks)**

A young person began saving up for a deposit on a home by paying $275 at the start of each month into a First Home Saver account. Account interest and government bonuses combined to grow the balance of the account at a rate of 21% per annum, compounded monthly.

The table below shows the amount in the account at the start of each month, the interest added at the end of the month, the next payment and the balance carried forward to the start of the next month.

Month	Amount at the start of month	Interest for month	Payment	Balance carried forward
n	T_n	I_n	P_{n+1}	T_{n+1}
1	275.00	4.81	275.00	554.81
2	554.81	9.71	275.00	839.52
3	839.52	A	275.00	B
4				

(a) Calculate the values of entries A and B in the table above. (2 marks)

$$A = 839.52 \times \frac{21}{12 \times 100}$$
$$= \$14.69$$

$$B = 839.52 + 14.69 + 275$$
$$= \$1129.21$$

(b) Write a recursive rule to determine the amount in the account at the start of each month. (3 marks)

$$1 + \frac{21}{12 \times 100} = 1.0175$$

$$T_{n+1} = T_n \times 1.0175 + 275$$

$$T_1 = 275$$

(c) The young person has just made their ninth payment.

(i) How much money is now in their account? (1 mark)

$2655.51

(ii) How much interest has been earned to date? (2 marks)

Deposited $9 \times 275.00 = 2475.00$

$$I = 2655.51 - 2475.00$$
$$= \$180.51$$