

Semester Two Examination 2017 Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 & 4

Section One: Calculator-free

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula Sheet

To be provided by the candidate

Standard items: pens(blue/black preferred), pencils(including coloured), sharpener,
correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	8	8	50	50	35
Section Two Calculator—assumed	12	12	100	98	65
					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2017*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section One: Calculator-free**50 marks**

This section has **eight (8)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 50 minutes

Question 1 (4 marks)

- (a) Determine the following derivative.

$$\frac{d}{dx}(e^{\cos x} + 5)$$

(2 marks)

- (b) Hence determine the following integral.

$$\int (\sin x \cdot e^{\cos x}) dx$$

(2 marks)

Question 2 (5 marks)

Consider the function $f(x) = e^{2x} - \ln x$, where $x > 0$.

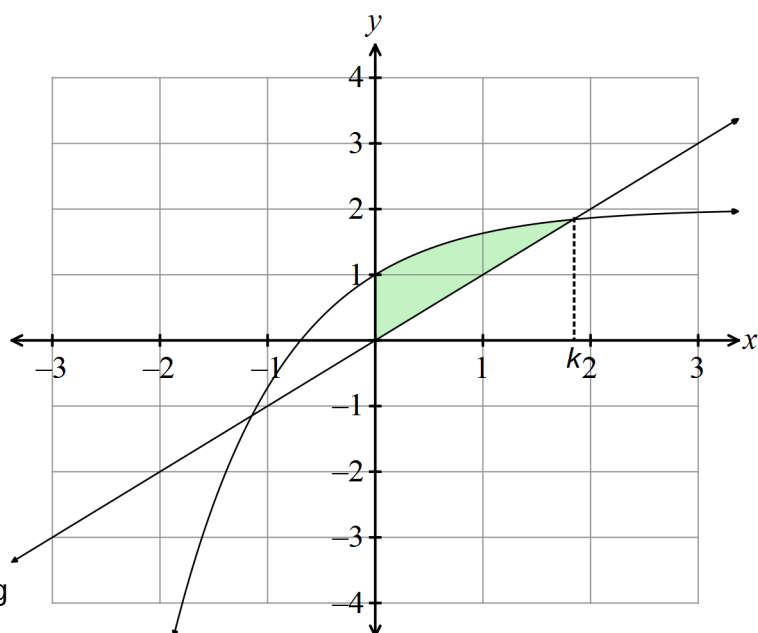
- (a) Show that there exists a stationary point where $x = 0.5 e^{-2x}$ (3 marks)

It can be shown that $f''(x) = 4e^{2x} + \frac{1}{x^2}$

- (b) Determine the nature of the stationary point determined in (a). (2 marks)

Question 3 (5 marks)

Consider the graphs of $y = x$ and $y = 2 - e^{-x}$



(a) Write an integ

(2 marks)

(b) Determine the area in terms of k .

(3 marks)

Question 4 (7 marks)

Determine the following, giving your answers in exact form.

(a) $\int (5x - \cos 5x) \, dx$ (2 marks)

(b) $\int_0^4 (e^{2x} - \sqrt{x}) \, dx$ (3 marks)

(c) $\frac{d}{dx} \left(\int_{\pi}^{2x} \sin t \, dt \right)$ (2 marks)

Question 5 (7 marks)

The velocity (v) of a body at time t seconds travelling along a linear path is given by the rule
 $v = 2\cos 2t - 2e^{-2t}$

- (a) Determine the displacement of the body, given that the body is initially 1 unit to the right of the origin. (3 marks)

- (b) Determine the acceleration of the body at any time t . (2 marks)

- (c) Show that the relationship between displacement and acceleration is NOT of the form $a = -k^2 x$. (2 marks)

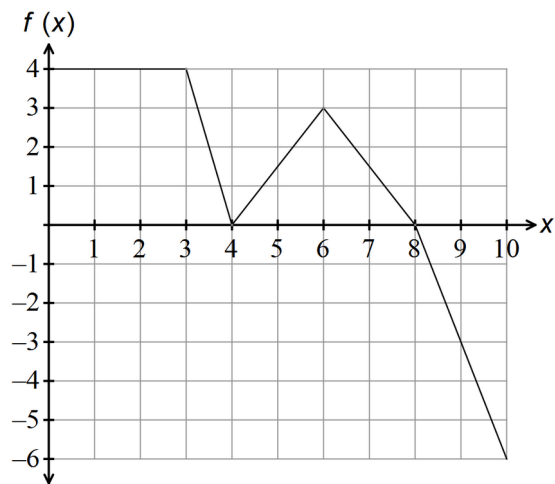
Question 6 (7 marks)

- (a) Evaluate $\log\left(\frac{a}{b}\right) + \log\left(\frac{b}{c}\right) + \log\left(\frac{c}{a}\right)$. (2 marks)

- (b) Solve these simultaneous equations and show that $x = \frac{\log 3}{\log 6}$.
 $2^x = 3^y$ and $x + y = 1$ (5 marks)

Question 7 (8 marks)

Consider the graph of $y = f(x)$ shown below.



(a) Determine the value of:

(i) $\int_0^4 f(x) \, dx$ (1 mark)

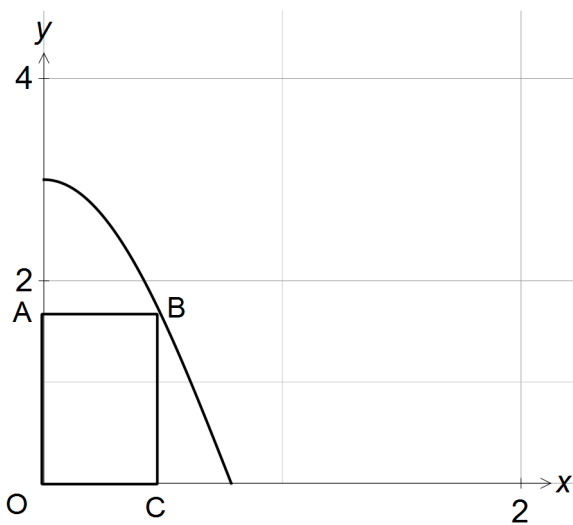
(ii) $\int_0^{10} f(x) \, dx$ (2 marks)

(iii) $\int_0^8 2 f(x) \, dx$ (2 marks)

(b) Find integers a , b and c , where $a \neq c$, if $\int_a^b f(x) \, dx = - \int_b^c f(x) \, dx$ (3 marks)

Question 8 (7 marks)

The first quadrant of $y = 3\cos 2x$ is shown.



- (a) Show that the area of rectangle OABC = $3x \cdot \cos 2x$. (1 mark)

- (b) Show that for the area of OABC to be a maximum, $2x \cdot \tan 2x - 1 = 0$. (3 marks)

- (c) Given the result from (b), use the second derivative to show that $\sin 2x + x \cos 2x > 0$.
(3 marks)

End of Section One

Additional working space

Question number(s):