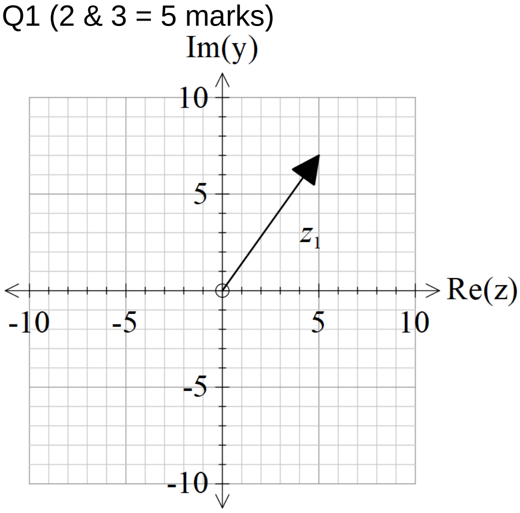
PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Specialist TEST 2 Monday 1 April 2019 TIME: 45 minutes working Classpads allowed One page of notes 45 marks 7 Questions

Name:________________________________

Teacher: _____________________________

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2 & 3 = 5 marks)



From the diagram, z_1 is a solution to $z^4 = k$ for complex k.

i) Determine k.

ii) Determine the other three roots and express in the form $a + bi$.

Q2 (2, 3 & 1 = 6 marks)

Let $f(x)=\sqrt{2x-1}$ and $g(x)=\dfrac{1}{x+5}$.

a) State the natural domain and range of $g(x)$.

b) Does $f\circ g(x)$ exist over the natural domain of g? If it does not, determine the largest possible domain for the composite to exist.

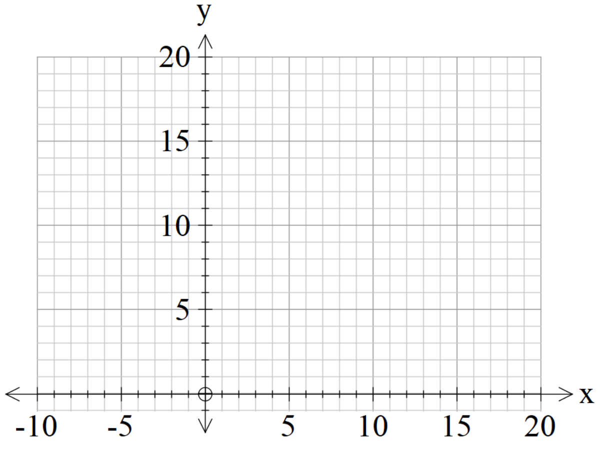
c) Determine $f\circ f^{-1}(x)$

Q3 (2, 3 & 2 = 7 marks)

Given that $f(x)=2x^2-12x+19$, $x\le 3$, determine the following.

a) $f^{-1}(x)$ and its domain.

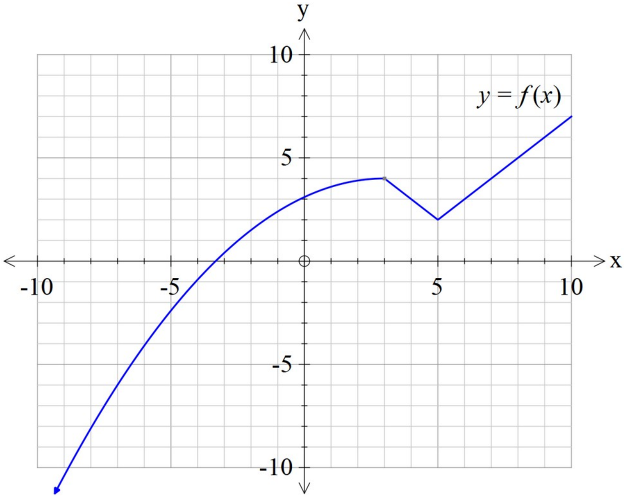
b) Sketch on the axes below, $f(x)\ \&\ f^{-1}(x)$



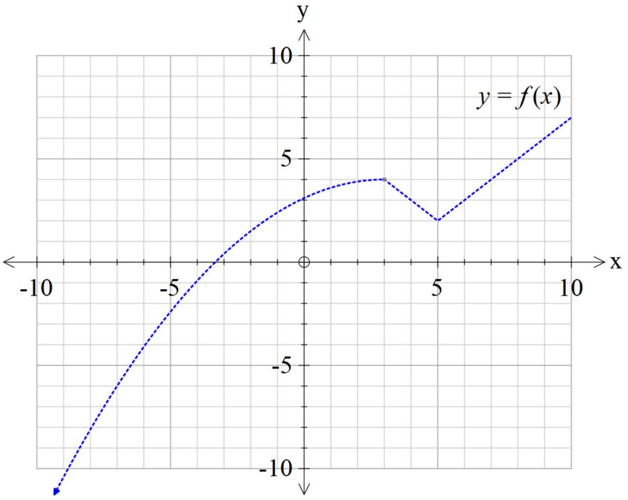
c) On the sketch above show the precise points where $f(x)=f^{-1}(x)$

Q4 (2 & 3 = 5 marks)

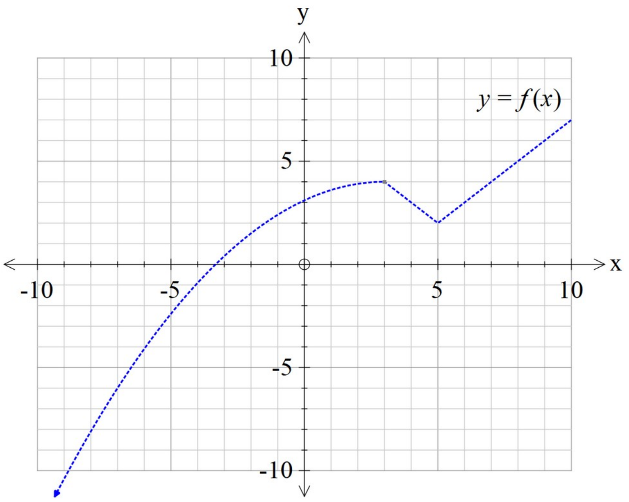
Consider the function $y = f(x)$ for the questions below.



a) Sketch the function $y = |f(x)|$ on the axes below.



b) Sketch the function $y = |f(-|x|)|$ on the axes below.



Q5 (3 & 4 = 7 marks)

a) Two moving objects have the following position vectors and constant velocities at time, $t = 0$:

$$r_a = \begin{pmatrix} 9 \\ -8 \end{pmatrix} m \quad v_a = \begin{pmatrix} -2 \\ 7 \end{pmatrix} m/s$$

$$r_b = \begin{pmatrix} 11 \\ -3 \end{pmatrix} m \quad v_b = \begin{pmatrix} 5 \\ -3 \end{pmatrix} m/s$$

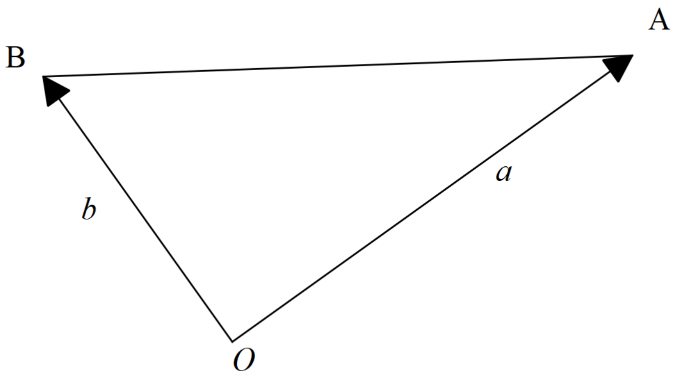
Determine the closest approach and the time that this will occur.

b) Let the circle S have a radius 3 units and centre $(1, \beta)$, where β is a constant, and the line $r = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ is tangential to this circle. Determine the value(s) of β .

c)

Q6 (1, 1, 1, 3, 1 & 3 =10 marks)

The diagram below shows a triangle with vertices with $O, A \& B$. Let O be the origin, with vectors $OA = a$ and $OB = b$.



a) Determine the following vectors in terms of $a \& b$.

i) MA, where M is the midpoint of the line segment OA.

ii) BA

iii) AQ, where Q is the midpoint of the line segment AB.

Let N be the midpoint of the line segment OB.

b) Use a vector method tom prove that the quadrilateral $MNQA$ is a parallelogram.

Q6 continued

Now consider the particular triangle OAB with $\overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \\ \sqrt{3} \end{pmatrix}$ and $OB = \begin{pmatrix} \alpha \\ 0 \\ 0 \end{pmatrix}$ where α is a positive constant, chosen so that triangle OAB is isosceles, with $|OB| = |OA|$.

c) Show that $\alpha = 4$.

d) Use a vector method to show that OQ is perpendicular to AB.

Q7 (5 marks)

Let $w = 1 + qi$ where q is a real constant. Let $p(z) = z^3 + bz^2 + cz + d$, where $b, c \,\&\, d$ are real constants. If $p(z) = 0$ for $z = w$ and all roots of $p(z) = 0$ satisfy $\left|z^3\right| = 8$, determine all possible values of $q, b, c \,\&\, d$.