

Question number:

Additional working space

MATHEMATICS METHODS

12

CALCULATOR-FREE

Perth Modern School



PERTH MODERN SCHOOL  
Semester Two Examination, 2018

Question/Answer Booklet

Excellence. Integrity. Responsibility.

## MATHEMATICS METHODS

Section One:  
Calculator-free

Student's name \_\_\_\_\_  
Your Teacher's name \_\_\_\_\_

Time allowed for this section  
Reading time before commencing work: five minutes  
Working time for section: fifty minutes

Materials required/recommended for this section  
To be provided by the supervisor

Formula Sheet  
This Question/Answer Booklet

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighter

No other items may be taken into the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

You do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

No other items may be taken into the examination room. It is your responsibility to ensure that

### Important note to candidates

Special items: nil

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	13	13	100	100	65
<b>Total</b>			150	100	

**Additional working space**

Question number: \_\_\_\_\_

**Instructions to candidates**

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

(2 marks)

$$(c) \quad h(x) = \frac{x^{\frac{1}{3}} - x^{\frac{1}{2}}}{x^{\frac{1}{3}}} \quad (2 \text{ marks})$$

(2 marks)

$$(b) \quad g(x) = \frac{3-2x}{2^{x^2}} \quad (\text{Do not simplify}) \quad (2 \text{ marks})$$

(2 marks)

$$(a) \quad f(x) = 3x^{\frac{5}{3}} - \frac{x^{\frac{3}{2}}}{2} \quad (2 \text{ marks})$$

(6 marks)

**Question 1**

Determine the derivative for each of the following, ensuring all answers are expressed with positive indices.

Working time for this section is 50 minutes.

Question number: \_\_\_\_\_

**Section One: Calculator-free**  
35% (50 Marks)

Additional working space

This section has seven (7) questions. Answer all questions. Write your answers in the spaces provided.

**Question 2****(8 marks)**

Millie got 90% on the Law Admission Test (LAT). The average LAT applicant got 80% and the standard deviation was 5. Malcolm got 80% on the Undergraduate Medicine and health Science Admission Test (UMAT). The UMAT applicants result was 70% and standard deviation was 4.

- (a) Who did relatively better their respective test, Millie or Malcolm? (3 marks)

- (b) Molly walks to work five days a week day, going through two sets of traffic lights. Let  $X$  be the probability density function that gives the probability of the number of red lights Molly encounters in one day.

$x$	0	1	2
$P(X = x)$	0.5	0.3	0.2

- (i) Determine the expected number of red lights per trip. (2 marks)

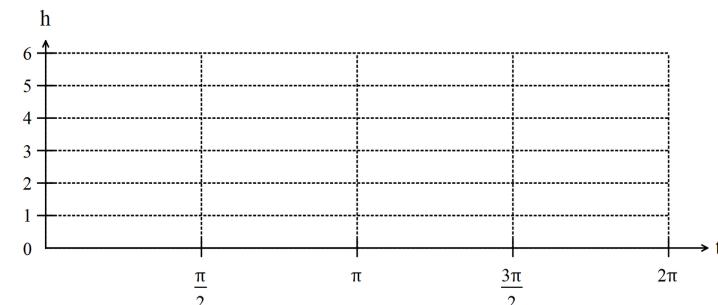
- (ii) What is the probability that Molly had 2 red lights two days in a row? (1 marks)

- (iii) Determine the standard deviation of the distribution of  $X$ . Leave your answer un-simplified. (2 marks)

**Question 7****(10 marks)**

The height of the acrobat on a circus swing is given by  $h(t) = 2 \cos(t) + 4$  where  $h$  is measured in metres and  $t$  in seconds.

- (a) Sketch the height of the acrobat on the set of axes below for  $0 \leq t \leq 2\pi$ . (2 marks)



- (b) At what rate is the acrobat rising at  $t = \frac{3\pi}{2}$ ? (2 marks)

- (c) Explain using the graph in (a) why your answer to (b) gives the maximum rate of rise of the acrobat. (3 marks)

- (d) At what rate is the vertical velocity of the acrobat changing at  $t = \frac{3\pi}{2}$ ? (3 marks)

(3 marks)

- (b) On the axes above, sketch the graph of  $y = 3 - f(x)$ , showing all relevant features.

(2 marks)

$$\text{(iii)} \quad \int_0^2 f(x-1) dx$$

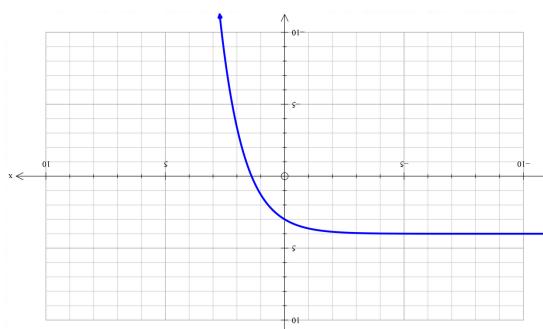
(2 marks)

$$\text{(ii)} \quad \int_0^{-2} xp(x) dx$$

(1 mark)

$$\text{(i)} \quad \int_1^0 f(-x) dx$$

- (a) If  $\int_{-2}^2 f(x) dp = a$ ,  $\int_0^1 f(x) dx = b$  and  $\int_1^2 f(x) dx = c$ , evaluate each of the following definite integrals in terms of the constants  $a$ ,  $b$  and  $c$ .



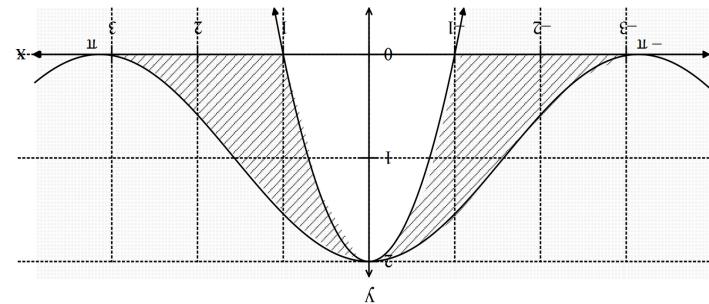
The function  $f(x) = -e^x + 4$  is graphed on the axes below.

(8 marks)

### Question 3

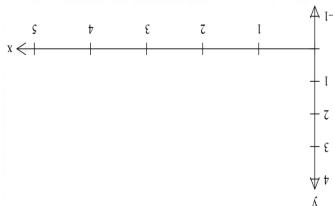
(2 marks)

- (b) Write down the expression for the area between the function  $y = 2 - x^2$  and  $y = 1 + \cos(x)$  between  $-\pi \leq x \leq \pi$  that is illustrated in the diagram below.



- (a) Find the area between the function  $f(x) = 2 - x^2$  and the  $x$ -axis.

(4 marks)



**Question 4****(5 marks)**

(a) Determine  $\int \sqrt{x} - \frac{1}{x^2} dx$

(2 marks)

(b) If  $f'(x) = 6x(x^2 - 7)^2$  and  $f(2) = -20$ , determine  $f(3)$ .

(3 marks)

**Question 5**

(7 marks)

Five hundred randomly selected students were surveyed to a date for an upcoming social.

The results are in the table below.

Day	Friday	Saturday	Sunday
Preference	180	180	140

(a) (i) Convert the data to form a probability density function. (2 marks)

$x$	Friday	Saturday	Sunday
$P(X = x)$			

(ii) If two students from the school were selected at random, what is the probability that they both thought the social should be on Sunday.  
(Do not simplify) (2 marks)

Which of the following represents a probability density function? Give your reasons. (3 marks)

(b) (i)

$x$	0	1	2	3
$P(X = x)$	0.1	0.2	0.3	0.4

(ii)

$x$	10	11	12	13
$P(X = x)$	0.2	0.3	0.1	0.5

(iii)

$x$	6	7	8	9
$P(X = x)$	0.4	0.5	-0.3	0.4

**Question 6**

(6marks)

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