



Question/Answer Booklet

Circle your teacher's initials			
BAH	GJ	JIB	VMU

MATHEMATICS 3CD
SEMESTER 2
Section Two
(Calculator Assumed)

Time allowed for this section

Your name _____

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section

Two Question/Answer Booklets
Formula Sheet (retained from Section One)

To be provided by the supervisor

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Structure of this examination

		Number of questions	Working time (minutes)	Marks available
Booklet 1 (Section 1)	Calculator Free	9	50	50
Booklet 2 This Booklet (Section 2)	Calculator Assumed	6	100	50
		7		50
Total marks		150		

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the booklet *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet.
3. Spare answer pages are provided at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning : If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued i.e. give the page number. Fill in the number(s) of the question(s) that you are continuing at the top of the page.
4. **Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you **do not use pencil**, except in diagrams.

Question 10 (8 marks)

(a) Calculate $P(A)$ [3 marks]

$$P(A \cup B) = 0.12, \quad P(A' \cup B) = 0.32 \quad \text{and} \quad P(B|A) = 0.4$$

For events A and B;

(b) Calculate $P(B)$ [1 mark]

(c) Calculate $P(A|B)$ [2 marks]

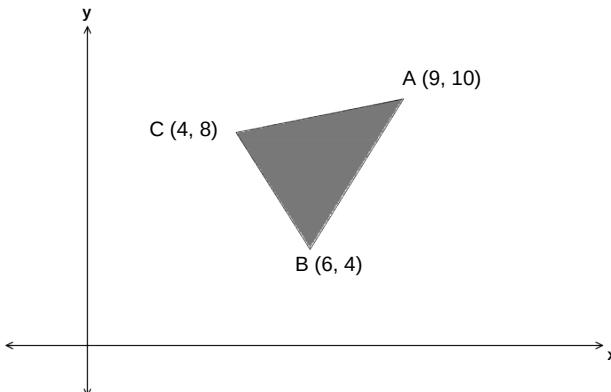
(d) Are events A and B independent? Justify your answer. [2 marks]

Question 11 (9 marks)**Spare page for extra working**

A local supermarket sells two brands of milk, X and Y. They sell x thousand cartons of brand X and y thousand cartons of brand Y.

Each carton of brand X makes a profit of \$0.80 but each carton of brand Y makes a loss of \$0.20.

The feasible region for the supermarket's weekly sales is shown in the diagram below.



- (a) Determine the inequality satisfied by x and y that corresponds to the edge CA of the feasible region. [3 marks]
- (b) Find the amount of each brand of milk that should be sold for maximum profit. Show your working. [3 marks]

- (c) If the profit on each brand X carton remains as \$0.80, to what value can the loss on a carton of brand Y rise before there is a change to the point in part b) that creates maximum profit? [3 marks]

Question 11 (Cont)

Question 12 (9 marks)

A continuous random variable, X, has a probability density function given

by

$$f(x) = \begin{cases} kx & 0 \leq x \leq 2 \\ 2k & 2 < x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

Spare page for extra working

- (a) Determine the value of k. [3 marks]

- (b) Find
i) $P(X \leq 4)$ [1 mark]

- ii) $P(X \geq 2 | X \leq 4)$ [2 marks]

- iii) M, the median of the distribution. [3 marks]

Question 22 (7 marks)

- (a) In a statistical experiment a coin is tossed repeatedly until a certain number of "Heads" have been obtained. On any particular toss of the coin there is a probability of 0.5 that it lands on "Heads". The score recorded is the number of tosses.

- (b) Find the probability that 3 "Heads" are obtained in exactly 3 tosses.

- (c) Find the probability that the third "Heads" is obtained on the 4th toss of the coin.

- (d) Write down a formula, in terms of r , for the probability that the a^r "Heads" is obtained on the r^{th} toss of the coin, $r \geq 3$.

State the sequence of transformations, in the correct order so that the graph of $y = 1 + 3e^{-2x}$ is transformed to $y = -2 + 9e^{-2x-1}$.

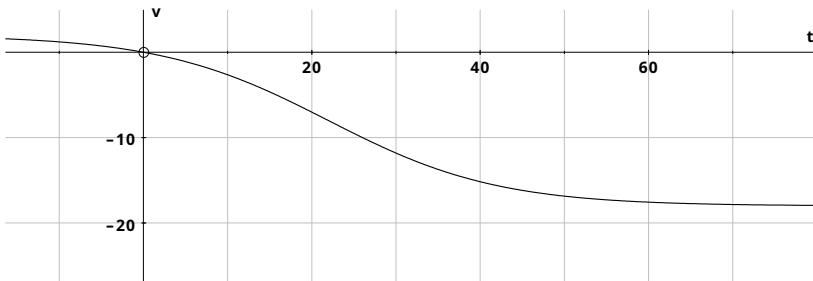
Question 13 (6 marks)

Question 14 (9 marks)

A skydiver drops out of a plane from a height of 1000 m. At time t seconds after she drops out of the plane her velocity in metres per second is given by the formula,

$$v = \frac{18(1 - e^{-0.1t})}{(9 + e^{-0.1t})}$$

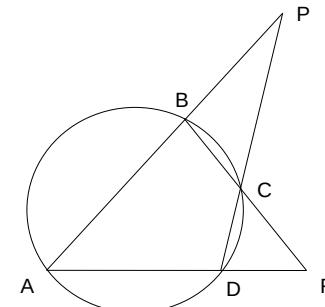
The graph below shows the velocity at time t seconds after she jumps.



- (a) Find the velocity of the skydiver after 20 seconds. [1 mark]
- (b) Find the acceleration of the skydiver after 20 seconds. [2 marks]
- (c) Find the time (to the nearest 0.1s) when the skydiver's speed is increasing at the fastest rate. [2 marks]

Question 21 (6 marks)

Consider the diagram below which shows a cyclic quadrilateral ABCD. The sides of the quadrilateral have been extended and these lines meet at the points P and R as shown.



Given that $\angle PBC = 2 \times \angle CRD$ prove that triangle ABR is isosceles.

- (c) After a number of weeks the company estimates that the standard deviation on each production run is 0.04 g. How large a sample do they need to take so that they can be 99% confident that the sample mean will be within 0.005 g of the true mean? [3]
- (d) Find the time (to the nearest 0.1s) taken for her to fall to the ground. [2 marks]
- (e) Find her speed (in metres per second to 1 decimal place) when she hits the ground. [2 marks]

Question 14 (Cont)

Question 20 (Cont)

Question 15 (9 marks)

An amateur golfer plays 18 holes with a professional player. The probability that the amateur player wins any particular hole is 0.4.

- a) Find the probability that the amateur player wins
- i) less than seven holes in the full round of 18 holes, [2]
 - ii) at least two of the first nine holes and at least two of the second nine holes. [3]
- b) If the players decide to play less than the full 18 holes, how many holes should they play so that the amateur has at least a 70% chance of winning at least 4 holes. [4]

Question 20 (9 marks)

A company manufactures ball bearings. The machine they use is set to a certain measurement for each week's production which amounts to 10 000 ball bearings.

- (a) In the first week of production a sample of 200 ball bearings is taken and found to have a mean mass of 0.824 g with a standard deviation of 0.042 g.

Given that the standard deviation observed in the sample matches the true standard deviation, find a 99% confidence interval for the true mean mass of the ball bearings produced during that week.

[3]

- (b) In the second week the machine is reset. From a sample of 100 ball bearings the company calculates that they are 95% confident that the true mean mass lies between 0.812 g and 0.842 g.

Find the mean and standard deviation of the sample that the company took for that week.

[3]

Question 19 (7 marks)

At the University of Western Australia the time taken to log on to the computer network is known to be distributed uniformly between 30 seconds and 150 seconds. For this distribution it is known that the mean is 90 seconds and the variance is 1200 seconds.² A group of students studying statistics record the time to log on to the network for samples of 40 randomly selected students.

(a) Find the probability that the time taken to log on for one student is greater than 2 minutes.

(b) Find the probability that the mean time to log on for one sample is greater than 95 seconds.

(c) If 20 samples of size 40 are taken, how many would be expected to differ from the true mean by more than 5 seconds?

(d) Find the probability that at least 8 of the 20 samples collected differ from the true mean by more than 5 seconds.

Question 18 (8 marks)

The lifetime of a particular type of bicycle tyre is known to follow a normal distribution with mean 800 hours and standard deviation 60 hours.

- (a) Find the probability that a bicycle tyre lasts at least 900 hours.
[1 mark]

- (b) The company wants to advertise their tyres using the words, "99% of our tyres last longer than x hours". What should the value of x be?
[1 mark]

- (c) If a sample of 200 tyres is taken, state estimates for the mean and standard deviation of the sample.
[2 marks]

- (d) If a sample of 200 tyres is taken, find the probability that the mean of the sample is greater than 810 hours.
[2 marks]

- (e) The tyre company has a new manufacturing process which increases the mean lifetime to 900 hours. They find that 90% of their tyres have lifetimes greater than 850 hours. What is the standard deviation of the lifetimes of these new tyres?
[2 marks]

Marks available	Working time (minutes)	Number of questions	Calculator	Section 1)	Free	Booklet 1	Section 1)	Booklet 2	Section 2)	Calculator	Booklet 3	Assumed	Section 2)	This Booklet	Section 2)	Booklet 3	Assumed	Total marks
50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	150	

Structure of this examination

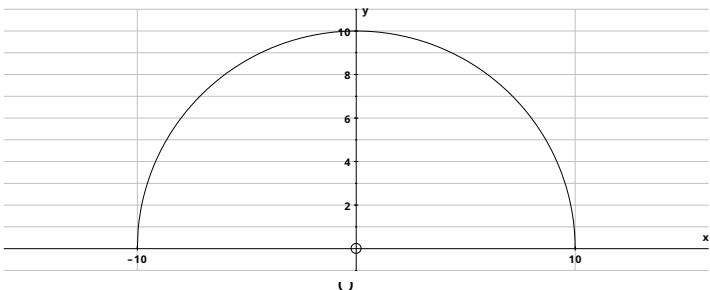
Your name _____

Circle your teacher's initials	BAH	GJ	JIB	VMU
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Question/Answer Booklet

MATHEMATICS 3CD
Section Two
(Calculator Assumed)
Booklet 3 of 3



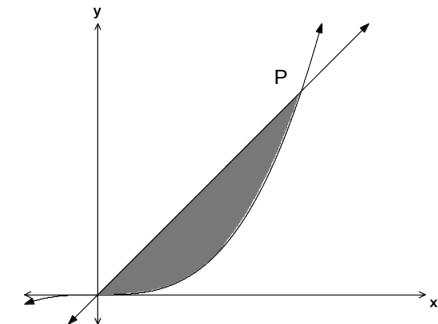
Question 16 (6 marks)

The diagram above shows a semicircle with equation $y = \sqrt{100 - x^2}$. The line OA is moving so that A moves around the circumference of the circle.

When the y coordinate is 8, the x coordinate of point A is increasing at a rate of 2 units per second. Find the rate at which the y coordinate of point A is changing at that same instant.

Question 17 (7 marks)

The diagram shows the graph of the curve $y = x^3$ and the line $y = 4x$.



The area trapped between the curve and the line as shaded in the diagram is rotated 360° about the y – axis.

- (a) Find the coordinates of the point, P, where the line and the curve intersect. [1 mark]

- (b) Write down an expression to find the volume generated. [3 marks]

- (c) If the shape generated represents a reservoir which could contain water, find the depth to which it needs to be filled so that it is half full. [3 marks]