Revision Examination Assessment Papers (REAP)

Semester 1 Examination 2012

Question/Answer Booklet

(This paper is not to be released to take home before 25/6/2012)

MATHEMATICS   3A

Section Two:
Calculator-assumed

Name of Student: _____Marking key_____

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for this section: 100 minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the student

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler,

Highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,

and up to three calculators satisfying the conditions set by the Curriculum

Council for this examination

Important note to students

No other items may be used in this section of the examination. It is **your** responsibility to ensure
that you do not have any unauthorised notes or other items in the examination room.
If you have any unauthorised material with you, hand it to the supervisor **before**
reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	13	13	100	100	

			Total	150	100

Instructions to students

1   Write your answers in the spaces provided in this Question/Answer Booklet.
        Spare pages are included at the end of this booklet. They can be used for
        planning your responses and/or as additional space if required to continue an
        answer. If you need to use the space to continue an answer, indicate in the
        original answer space where the answer is continued. i.e. give the page number.
        Fill in the number of the question(s) that you are continuing to answer at the top
        of the page.

2   **Show all your working clearly**. Your working should be in sufficient detail to
        allow your answers to be checked readily and for marks to be awarded for
        reasoning. Incorrect answers given without supporting reasoning cannot be
        allocated any marks. For any question or part question worth more than two
        marks, valid working or justification is required to receive full marks. If you

repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3   It is recommended that you **do not use pencil**, except in diagrams.

Section Two: Calculator-assumed
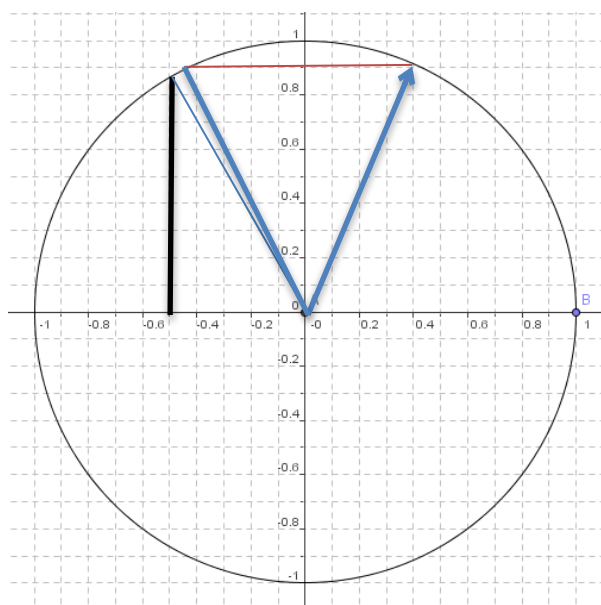   **(100 marks)**

This section has **thirteen (13)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

_____

Question 7   **(5 marks)**

Use the Unit circle below (there must be evidence of using the unit circle before full marks will be awarded)

(i)   to evaluate the value of $2\cos 120^0$ (2)

Solution
From unit circle, cos 120^0 = -0.5
Therefore $2\cos 120^0$ = 2 x (-0.5) = -1
Specific behaviours
✓ value for 120^0
✓ correct value for $2\cos 120^0$

(ii)   to find the value(s) of x , to the nearest whole number, such that

$$\sin 2x = 0.9$$ (3)

Solution
Read the two angles such that the y value is 0.9, giving 66^0 and 114^0 $(\pm 1^0)$

DIvide both angles by 2 to give x = 33^0 and 57^0
Specific behaviours
✓ two values for 2x ✓✓ two angles for x

Question 8   **(9 marks)**

Consider the following sequence of numbers 2, 5, 2, 9, 2, 13, 2, 17, …

    (i)   Write down the next two terms of the sequence, if the pattern continues. (1)

Solution
2,21
Specific behaviours
✓ or X

    (ii)   In this sequence, $T_1=T_3=T_5=$ etc, complete the sentence,

          " If n is an odd number, then
              $T_n=$ __ , $T_1 = $ "   (1)

Solution
$T_n = 2$, $T_1 = 2$
Specific behaviours
✓ or X

    (iii)   Write a recursive formula for the sequence 5, 9, 13, 17 ,…   (2)

Solution
$T_{n+1} = T_n + 4$, $T_1 = 5$
Specific behaviours
✓✓ or X

    (iv)   Write down the values of T_{99} and T_{100}.   (2)

Solution
$T_{99} = 2$, $T_{100} = 201$
Specific behaviours
✓✓ or X

    (v)   Which term is 61?   (1)

Solution
From CAS, T_{30} is 61
Specific behaviours

✓ or X

(vi)   Calculate the sum of the first 100 terms of the sequence.   (2)

Solution
$\displaystyle\sum_{n=1}^{50} 2 = 100$ 5 + 7 + 9 + … + 201 = 5150   Sum of the first 100 terms = 5150 + 100 = 5250
Specific behaviours
✓ sum of the fifty 2's and sum of 5,7,9,…201   ✓ sum of the first 100 terms

Question 9   **(6 Marks)**

Four digit numbers may be formed using the digits 0, 2, 4, 6, 8 such that no digit can be used more than once. How many of these 4- digit numbers

(i)   may be formed?   (1)

Solution
5! = 120
Specific behaviours
✓ or X

(ii)   begin with 8?   (1)

Solution
1x4x3x2 = 24
Specific behaviours
✓ or X

(iii)   end in a 2 or a 4?   (1)

Solution
1x4x3x2 x2 = 48
Specific behaviours
✓ or X

(iv)   are less than 486?   (2)

Solution
400's: 5 200's: 6 Therefore total is 11
Specific behaviours
✓✓

(v)   What is the probability that a randomly picked 4-digit number from this set is more than 8640?

(1)

Solution
Only 8642

$$\therefore P(>8640) = \frac{1}{120}$$

Specific behaviours
✓ or X

Question 10   **(6 marks)**

(a) The distribution of heights of a plant is normally distributed with a mean of 72 cm and a standard deviation of 6 cm. If there were 100 plants, estimate the number of plants which are less than 64cm in height.

(2)

Solution
P(X<64) = 0.0912
Number of plants = 0.0912 x 100 = 9
Specific behaviours
✓ probability
✓ Number

(b)   Achievement awards in a national Mathematics competition are allocated as

- High Distinctions to the top 15% of performances
- Distinctions to the next 25%
- Credits to the next 30%
- Participations to the remainder

If the scores were normally distributed with a mean of 75 and a standard deviation of 12, find the minimum score a competitor has to achieve to be awarded a
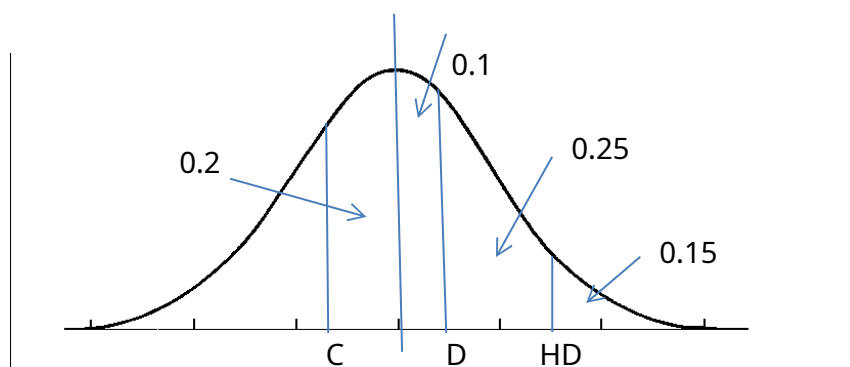
(i)   Credit   (2)

Solution
P(X<Credit) = 0.3
C = 68.7
Minimum score for a Credit is 68.7
Specific behaviours

✓ correct probability
✓ minimum score for Credit

(ii)   Distinction   (1)

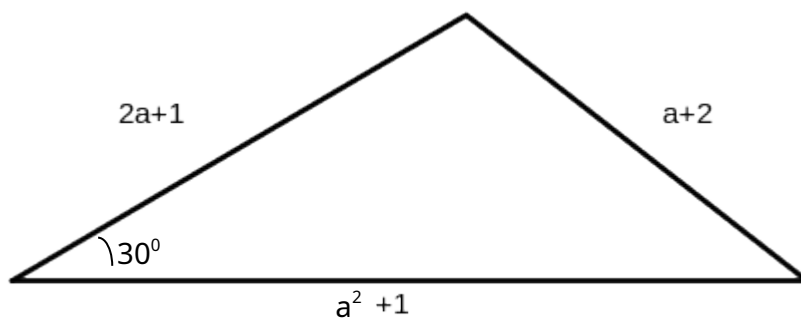| Solution |
| --- |
| P(X>D) = 0.40 |
| D = 78 |
| Minimum score for Distinction is 78 |
| **Specific behaviours** |
| ✓ minimum score for a Distinction |

(iii)   High Distinction   (1)

Solution
P(X>HD) = 0.15
HD = 87
Minimum score for a High Distinction is 87
Specific behaviours
✓ or X

Question 11   **(6 marks)**

Given the triangle below,

(i)   Find a simplified algebraic expression for the area of the triangle, given that

$$\sin 30^0 = \frac{1}{2}$$.

(2)

Solution
Area = $\frac{1}{2} \times (2a+1) \times (a^2+1) \times \sin 30^0$ = $\frac{(2a+1) \times (a^2+1)}{4}$
Specific behaviours
✓ area of non -right triangle formula with the correct sides   ✓ correct expression

(ii)   If the area is 17.5 units^2, determine the value of 'a'.   (2)

Solution
$\frac{(2a+1) \times (a^2+1)}{4} = \frac{35}{2}$ $a = 3$
Specific behaviours
✓ equates area expression to 17.5   ✓ solves correctly for "a"

(iii)   Hence, state the lengths of the triangle.   (2)

Solution
Lengths are 7, 5, 10
Specific behaviours
✓ substitutes a = 3 into each of the three sides   ✓ three correct lengths

Question 12   **(8 marks)**

(a)   A group of fifteen students travelling to Tasmania have their bags weighed at the airport
check-in counter. The weights of the bags in kilograms are listed in order as follows;

m, 9, n, 10, 15, 18, 21, p, 25, 29, 35, q, 38, r, 43.

The following statistics of this set of data are known.

$\bar{x}$ =24, Median = 21, Interquartile range = 27.

There are two modes of which 10 is one of them.

The minimum score is 2 less than the smaller mode.

Determine the values of m, n, p, q, r.

(5)

Solution
p = 21
q = 37
n = 10
m = 8
r = 41
Specific behaviours
✓✓✓✓✓ 1 mark each

Question 12 (continued)

(b)   In collecting data, biased samples and poor methods of collection can lead to invalid inferences. Consider the following method to determine the extent of unemployment in a local community.

"A team of data collectors phoned five randomly selected households on every twentieth page of the local telephone directory during regular office hours".

(i)   List two ways which might suggest this data collection method cause bias?   (2)

Solution
-   during office hours, some unemployed people will be contacted but so too will home-carers, retired individuals, people running a business from home, etc
-   not all people are listed on the telephone directory

- Thus those interviewed will not provide the right kind of data to make supportable inferences
Specific behaviours
✓✓ 2 valid reasons

(ii)   Suggest how this bias could be avoided.   (1)

Solution
- Door knock after regular working hours
- Ring after regular working hours to enable more people to be reached
- Survey form
Specific behaviours
✓ any 1 valid suggestion

Question 13 **(10 marks)**

(a)   The graph of *f(x)* is dilated parallel to the *x*-axis by a scale factor of 2, followed by a translation of 4 units in the positive direction of the *x*-axis. What is the equation of the final graph obtained?

(2)

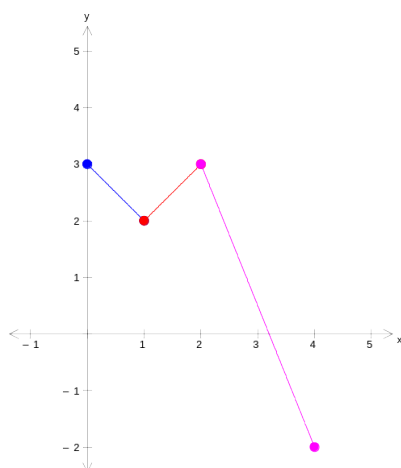| Solution |
|---|
| $f\left[\dfrac{1}{2}(x-4)\right]=f\left(\dfrac{1}{2}x-2\right)$ |
| **Specific behaviours** |
| ✓✓ $\dfrac{1}{2}$ and -4 OR ✓✓ $\dfrac{1}{2}$ and -2 |

(b)   The function $y=x^3$ is transformed to $y = -2(x+5)^3$. Describe the transformation in order. (3)

Solution
Translated 5 units to the left, followed by a dilation parallel to the y-axis by a scale factor of 2 and a reflection in the x-axis
Specific behaviours
✓ transformation in correct order ✓✓ translation 5 left, dilation 2 parallel to y axis and reflection in the x-axis

(c)   The graph of *y=f(x)* is given below
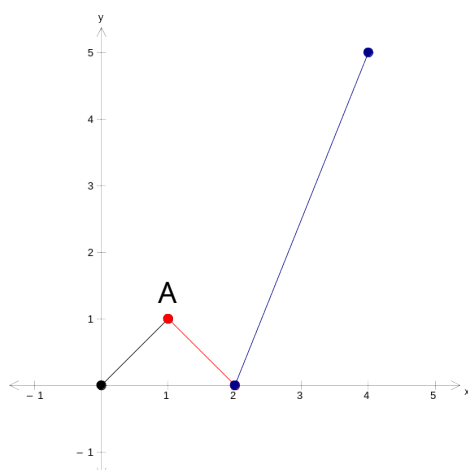
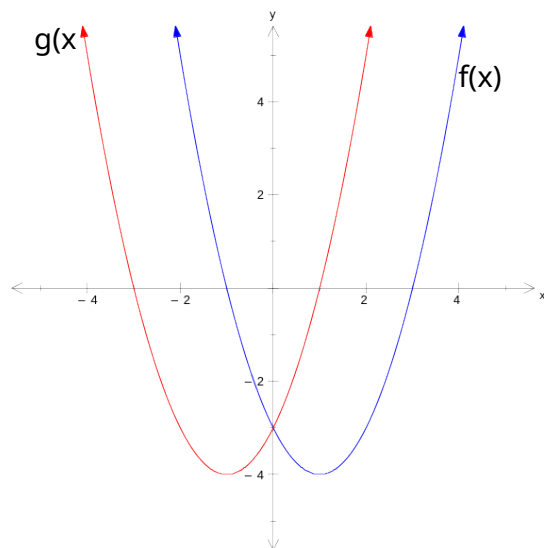(i)   Sketch the graph of *y=3 - f(x)* on the axes below.   (2)

Solution
As shown above
Specific behaviours
✓ reflected shape
✓ all correct points (0,3), (1,2), (4,-2)

Question 13 (continued)

(ii)   Write the coordinates of A if *y = - f(-x+3)*.   (2)

Solution
$(1,1) \rightarrow (-1,1) \rightarrow (2,1) \rightarrow (2,-1)$
Coordinates of A = (2, -1)
Specific behaviours
✓ working
✓ correct coordinates of A

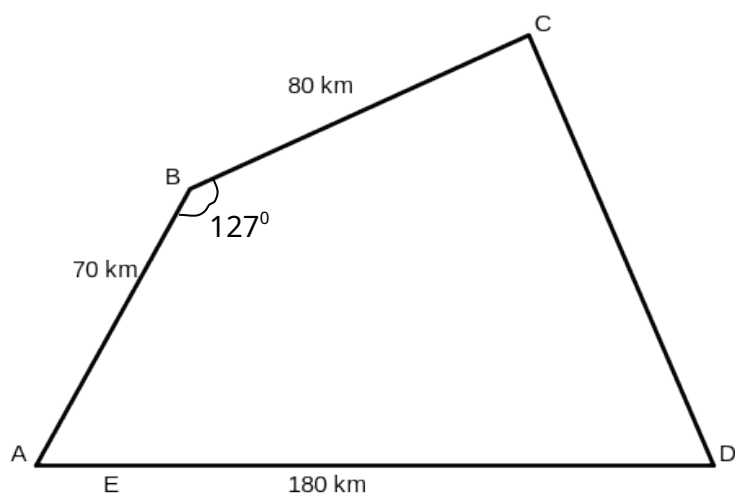(d)   Describe how the graph of *f(x)* can be used to obtain the graph of *g(x).*
       (1)

Solution
Reflection about the y-axis
Or f(x) is translated 2 units left
Specific behaviours
✓ or X

Question 14   **(7 marks)**

The diagram shows four towns, A, B, C and D on level ground.

D is 180km due east of A and E is a rest station along AD such that $AE = \dfrac{1}{9}AD$.

The bearing of B from C is 261^0.

$\angle$ABC=127^0, AB=70km and BC=80km.

(a)   Calculate

(i)   The distance of A from C   (2)

Solution
$AC^2 = 70^2 + 80^2 - 2(70)(80)\cos 127^0$ AC = 134 km
Specific behaviours
✓ uses cosine rule   ✓ correct answer of 134 km

(ii) $\angle$ACB using the SINE rule   (2)

Solution
$\dfrac{70}{\sin \angle ACB} = \dfrac{134}{\sin 127^0}$ $\angle ACB = 24.7^0$
Specific behaviours
✓ uses Sine rule   ✓ correct answer

 (iii)   The bearing of A from C.   (1)

Solution
$180^0 + (81^0 - 24.7^0) = 236.3^0$
Specific behaviours
✓ or X

(b)   In town A, the highest peak has a height of 7500m. Find the angle of elevation of the peak from the rest station at E.

    (2)

Solution
$\tan \theta = \dfrac{7.5}{20}$ $\theta = 20.6^0$ Nagle of elevation is 20.6^0
Specific behaviours
✓ tangent ratio ✓ correct angle of elevation

Question 15   **(8 marks)**

(a)   The following table shows statistics of the results of the five mathematics tests for John, Ryan, Julie and Gary.

	Mean	Standard deviation	Range
John	13.6	3.8	7
Ryan	13.9	4.2	7
Julie	13.8	2.7	7
Gary	14.0	3.4	7

Which student has the most consistent results? State a reason why you chose this student.   (2)

Solution
Julie because she has the smallest spread of marks (standard deviation)
Specific behaviours
✓ Julie ✓ lowest standard deviation

(b)   Mrs Kendall decided to survey a sample of 10% of the students at her school. The school enrolment is shown in the table below.

Year	7	8	9	10	11	12	Total
Number of students	225	232	233	230	150	130	1200

She surveyed the same number of students in each year group. Explain how would the numbers of students surveyed in Year 10 and Year 11 have changed if she had chosen to use a stratified sample based on year groups?

(3)

Solution
Same number for each group i.e. $10\% \times \dfrac{1200}{6} = 20$ ✓ Year 10: 10% of 230 = 23 ∴ increase of 3 ✓ Year 11: 10% of 150 = 15 ∴ decrease of 5 ✓

Specific behaviours
As indicated above

Question 15 (continued)

(c)   Solve the indicial equation showing all working steps   (3)

$$2 \times 3^{2x+3} = 6 \times 9^{4x}$$

Solution
$2 \times 3^{2x+3} = 2 \times 3^{1+8x}$ Equating indices: $2x + 3 = 1 + 8x$ $$x = \frac{1}{3}$$
Specific behaviours
✓ expresses as powers of 3 ✓ equates indices ✓ solves correctly for x

Question 16   **(9 marks)**

Josh bought a second hand Hyundai for $4500. He borrowed the full sum of $4500 from NO Frills Finance Company and agreed to make a minimum payment of $300 a month. For a year he pays the minimum monthly payment. At the end of the year, he checks his statement from the Finance company and is surprised that he still owes $1683.50. He created a spreadsheet to check whether the amount owing is correct.

Month	Previous Balance	Interest	Balance Due	Payment	Balance after payment
1	4500	90	4590	300	4290
2	4290	85.80	4375.80	300	4075.80
3	4075.80	81.50	4157.30	300	3857.30
4	3857.30	77.10	3934.50	300	3634.50
5	3634.50	72.70	3707.20	300	3407.20
6	3407.20	68.10	3475.30	300	3175.30
7	3175.30	63.50	3238.80	300	2938.80
8	2938.80	58.80	2997.60	300	2697.60
9	2697.60	53.90	2751.50	300	2451.50
10	2451.50	49.10	2500.60	300	2200.60
11	2200.60	44.00	2244.60	300	1944.60
12	1944.60	38.90	1983.50	300	1683.50

Note: The values in this table have been rounded to the nearest ten cents.

(i)   Calculate the annual rate of interest Josh is paying.   (1)

Solution
$\dfrac{90}{4500} \times 12 \times 100 = 24\%$
Specific behaviours
✓ or X

(ii)   If Josh continues to pay the monthly payment of $300, it will take him a total of 19 months to pay off his car. What would be the total amount of his payments? (2)

Solution
Total amount of his payments = $18 \times 300 + (3.40 + 0.02 \times 3.40) = \$5403.50$
Specific behaviours
✓✓ calculation and correct answer

(iii)   Josh decides to reduce his social life activities in order to pay an extra $200 a month from the 13th payment onwards. Values have been rounded to the nearest ten cents.

Complete the table of calculation below.   (2)

Month	Previous Balance	Interest	Balance Due	Payment	Balance after payment
13	1683.50	**33.70**	1717.20	500	1217.20
14	1217.20	24.30	**1241.50**	500	741.50
15	741.50	14.80	756.30	500	**256.30**

Solution
As shown in table
Specific behaviours
✓✓ for the three correct values

Question 16 (continued)

(iv)   How long does it take Josh to repay the loan?   (1)

Solution
16 months
Specific behaviours
✓ or X

(v)   What is the total amount that Josh will pay over the period with the increased repayment ?   (2)

Solution
$12 \times 300 + (3 \times 500) + 256.30 + (0.02 \times 256.30) = \$5361.40$
Specific behaviours
✓✓ calculation and correct answer

(vi)   How much money does Josh save by paying in this way?   (1)

Solution
$ 5 403.50 - $ 5 361.40 = $ 42.10

Specific behaviours
✓ or X

Question 17   **(8 marks)**

The final result of the students in Mrs Bond's classes of 3CDMAT and 3CDMAS are summarised in the frequency table below.

Mark in percent	3CDMAT	Cumulative frequency	3CDMAS	Cumulative frequency
0 – 19	2	2	1	1
20 – 39	2	4	3	4
40 – 59	5	C	A	7
60 – 79	D	20	9	B
80 - 99	7	27	5	21

(a)   Find the values of A, B C and D   (2)

Solution
A = 3, B = 16, C = 9, D = 11
Specific behaviours
✓✓ for all four correct answers

(b)   Calculate the mean and standard deviation of the 3CDMAS marks   (2)

Solution
$\bar{x} = 62.83, \quad \sigma = 22.54$
Specific behaviours
✓✓ 1 mark each

(c)   If a student from one of these classes is picked at random, what is the probability that the
        student

   (i)   received a mark greater than or equal to 60 ?   (1)

   (ii)   was in the 3CDMAT class?   (1)

   (iii)   was in the 3CD MAS class and received a mark ≥ 80 ?   (1)

   (iv)   was in the 3CDMAS class if the student received a mark
           between 60 and 79 inclusive ?   (1)

Solution
(i) $\dfrac{2}{3}$ (ii) $\dfrac{27}{48} = \dfrac{9}{16}$ (iii) $\dfrac{5}{48}$ (iv) $\dfrac{9}{20}$

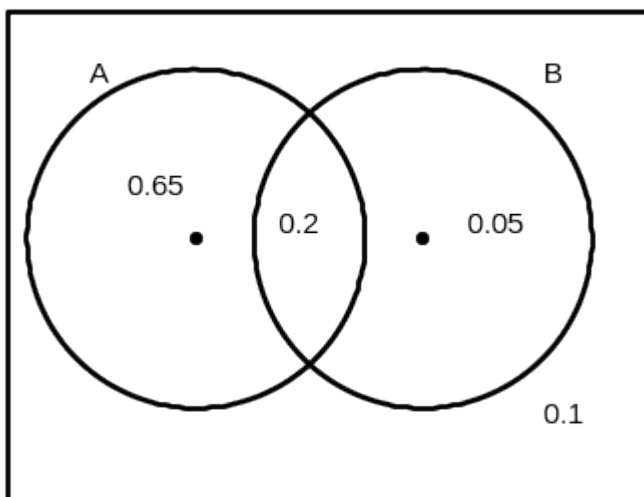| Specific behaviours |
|---|
| ✓✓✓✓ 1 mark each |

Question 18   **(9 marks)**

(a) A and B are events such that $P(A \cup B)=0.9, P(A \cap B)=0.2, P(A|B)=0.8$

   (i)   Represent the diagram in the Venn diagram below.   (3)

| Solution |
|---|
| As shown in diagram |
| **Specific behaviours** |
| ✓ 0.65 |
| ✓ 0.05 |
| ✓ 0.1 |

   (ii)   Hence, find P(A)   (1)

Solution
0.85
Specific behaviours
✓ or X

   (iii)   Hence find $P(\overline{B})$ (1)

Solution
0.75
Specific behaviours
✓ or X

Question 18 (continued)

(b) The test results for a group of 45 students involved in a reading program are displayed below in the form of a pair of box plots. The data collected shows the reading speed of the students both before and after the administration of the program.
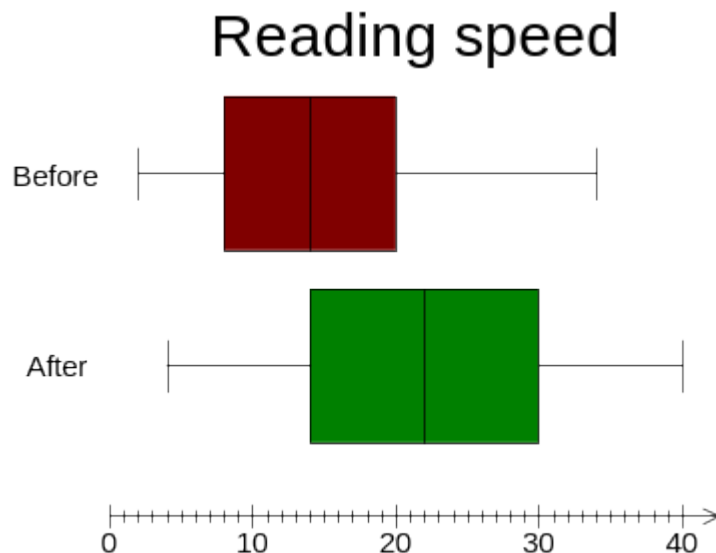
For each of the following, answer True (T) or False (F):-   (4)

(i)   For the "after " data, the median has increased by 8 and the interquartile range has increased by 4.

(ii)   Both distributions have the same median.

(iii)   The 'after' set of scores is more evenly distributed than the 'before' set of scores.

(iv)   The range of both sets of scores is the same.

Solution
(i)   T
(ii)   F
(iii)   T

(iv)	F
Specific behaviours	
✓✓✓ 1 mark each	

Question 19   **(9 marks)**

The point P(1,k) lies on the curve with equation $f(x) = (x^2 - 6x)(x - 2) + 3x$

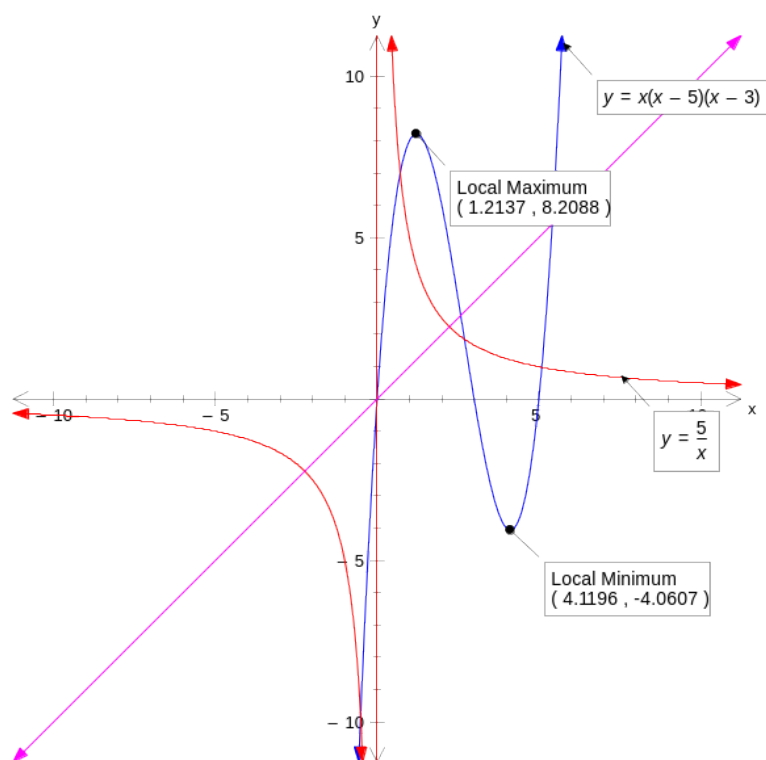(i)   Find the value of k.   (1)

Solution
k= 8
Specific behaviours
✓ or X

(ii)   Express $f(x)$ in a completely factorised form.   (2)

Solution
$f(x) = x(x^2 - 8x + 15) = x(x - 5)(x - 3)$
Specific behaviours
✓ expand ✓ factorised form

(iii)   On the axes below sketch the curves with the following equations

$f(x) = (x^2 - 6x)(x - 2) + 3x$ and $g(x) = \dfrac{5}{x}$, showing clearly where they meet the

axes.   (3)

Solution
As displayed in diagram
Specific behaviours
✓✓ 1 mark each for f(x) and g(x)
✓ shows clearly x and y intercepts, turning points

Question 19 (continued)

(iv)   Read from the graph the **number** of solutions to the equation.   (1)

$$(x-5)(x-3) = \frac{5}{x^2}$$

Solution
4 solutions
Specific behaviours
✓ or X

(v)   By adding **one** suitable function on the axes, show how you solve $x^2 = 5$ and
        estimate the solution(s) from your graph, correct to 1 decimal place.
                    (2)

Solution

$x^2 = 5$
$x = \dfrac{5}{x}$
$x = \pm 2.2$

Specific behaviours
✓ draws the line y=x as shown on the graph
✓ reads the x values where y=x intersects with $y = \dfrac{5}{x}$