PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.

Test Two

Semester One 2016
Year 12 Mathematics Methods
Calculator Free

Teacher:

_____**Mr Staffe**

_____**Mrs. Carter**

_____**Mr Bertram**

_____**Mr Roohi**

_____**Ms Cheng**

Name:

- ***Complete all questions***
- ***Show all necessary working***
- ***Total Marks = 20***
- ***20 minutes***

1. [9 marks]

Find $\frac{dy}{dx}$ in each of the following, by using the appropriate rule. DO NOT SIMPLIFY.

(a) $y = x^2 e^x$ [2]

$$\frac{dy}{dx} = 2x\, e^x + x^2 e^x$$

(b) $y = \frac{e^{5x-2}}{e^{3x-2}} = e^{2x}$ [2]

$$\frac{dy}{dx} = 2\, e^{2x}$$

(c) $y = e^{2x}\left(5x - e^{\sqrt{x}}\right)$ [3]

$$\frac{dy}{dx} = 2\, e^{2x}\left(5x - e^{\sqrt{x}}\right) + e^{2x}\left(5x - \frac{1}{2}x^{-\frac{1}{2}}\, e^{\sqrt{x}}\right)$$

(d) Simplify $y = \frac{d}{dx}\int_3^{3x} 3t^2 - 6t + 4\, dt$ [2]

$$= \left(3(3x)^2 - 6(3x) + 4\right) \times 3$$
$$= 81x^2 - 54x + 12$$

2. [11 marks]

Simplify the following integrals

(a) $\int \frac{5x+2}{\sqrt{10x^2+8x-3}}\,dx$ [3]

$$= \int (5x+2)\left(10x^2+8x-3\right)^{-\frac{1}{2}}\,dx$$
$$= \frac{1}{4}\int (20x+8)\left(10x^2+8x-3\right)^{-\frac{1}{2}}\,dx$$
$$= \frac{1}{2}\sqrt{10x^2+8x-3}+c$$

(b) $\int e^{3x}\,dx$ [2]

$$= \frac{1}{3}e^{3x}+c$$

(c) $\int (40x-12)\,e^{5x^2-3x}\,dx$ [3]

$$= 4\int (10x-3)\,e^{5x^2-3x}\,dx$$
$$= 4\,e^{5x^2-3x}+c$$

(d) Evaluate $y = \int_0^1 4\left(\sqrt{e^x}+x^2\right)dx$ Leave as exact value [3]

$$= \int_0^1 4\sqrt{e^x}+4x^2\,dx$$
$$= 8\,e^{\frac{1}{2}} - \frac{20}{3}$$

5. [6 marks]

A Colour dye with initial concentration of 0.7 units is placed into a tub of water, and the rate of change of the dye is given by $\frac{dC}{dt} = -.75C$ units per minute where C = C(t) is the concentration of the dye at any time t minutes after being placed into the tub.

(a) Find C as a function of t. [1]

$C = 0.7\, e^{-0.75t}$

(b) Calculate the concentration of dye 2 minutes after the dye was placed into the water.

$C(2) = 0.7\, e^{-1.5}$
$= 0.156$ [2]
$\therefore$ The concentration is 0.156 after 2 minutes

(c) How long does it take for the concentration of dye to be 0.1 units ? [3]

$0.7\, e^{-0.75t} = 0.1$
$t = 2.595$
it takes about 2.6 minutes or 2 mins 36 secs to get to a concentration of 0.1 units

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Semester One 2016
Year 12 Mathematics Methods
Calculator Assumed

Teacher:

_____**Mr Staffe**
_____**Mrs. Carter**
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_____**Mr Roohi**
_____**Ms Cheng**

Name:

- ***Complete all questions***
- ***Show all necessary working***
- ***Total Marks = 30***
- ***30 minutes***

1. [8 marks]

(a) Find the area enclosed by the curve $y = x^2 - 3x - 4$ between the values x = 2 and x = 5, and the x – axis.

$A = \int_2^4 x^2 - 3x - 4\ dx + \int_4^5 x^2 - 3x - 4\ dx$
$= \frac{22}{3} + \frac{17}{6}$
$= 10\frac{1}{6}$ sq units

(b) Find the area between the curves curve f(x) = (x + 1)(x – 1)(x – 2) and $g(x) = 4x^2 - 28$

$(x+1)(x-1)(x-2) = 4x^2 - 28$
$\Rightarrow$ x = -2 , 3 , 5
$A = \int_{-2}^{3} (x+1)(x-1)(x-2) - (4x^2 - 28)\ dx + \int_3^5 (4x^2 - 28) - (x+1)(x-1)(x-2)\, dx$
$= \frac{375}{8} + 8$
= 101.75 sq units

2. [6 marks]

A sweeping "circular" driveway actually has two parabolas as its edges to allow parking near the house. The x – axis is the edge of the roadway and the driveway lies between the curves $y = \frac{x^2 - 22x + 21}{10}$ and $y = \frac{x^2 - 22x + 72}{10}$ (in metres).

(a) Draw a **sketch** showing the situation . Does not have to be to scale. [1]

(b) Find the area of the driveway and hence the cost of concreting to a depth of 15cm, with concreting costing \$350 / m^3 . [5]

$\frac{x^2 - 22x + 21}{10} = 0$ and $\frac{x^2 - 22x + 72}{10} = 0$ (in metres).

$x = 4 , 18$ and $x = 1 , 21$

$$\text{Area} = \int_1^{21} \frac{x^2 - 22x + 21}{10}\, dx - \int_4^{18} \frac{x^2 - 22x + 72}{10}\, dx$$

$$= \frac{400}{3} - \frac{686}{10}$$

$$= 87.6\ m^2$$

$V = 87.6 \times 0.15 = 13.14\ m^3$

cost = \$350 × 13.14 = \$4599

3. [6 marks]

Following the Second World War, there was a significant increase in the birth rates among the western countries. If it is assumed that the rate of births in millions of babies per year for the post war years is approximated by B'(t) = 2t + 5 for $0 \le t \le 15$, find

(a) How many babies were born in the first 15 years after the war? [3]

$$\int_0^{15} 2t + 5 \ \text{dt}$$

$= 300$

$\therefore$ 300 million babies were born in the first 15 years after the war.

(b) How long did it take for the number of babies born after the war to reach 104 million? [3]

$t^2 + 5t = 104$

$t = 8 , -13$

$\therefore$ 8 years after the war the numer of babies will reach 104 million

4. [4 marks]

The velocity of a particle moving in a straight line is given by $\frac{dx}{dt} = 20 - 8e^{-0.4t}$. Calculate the total distance travelled by the particle in the first 3 seconds.

$$\text{Displacement} = \int \text{Velocity}$$

$$= \int_0^3 20 - 8\, e^{-0.04t}\ \text{dt}$$

$$= 46.024 \text{ units}$$

The Particle travels 46.024 units in the first 3 seconds.