

# MATHEMATICS METHODS

**MAWA Semester 1 (Unit 1) Examination 2015**

**Calculator-free**

**Marking Key**

Section One: Calculator-free

(60 Marks)

**Question 1(a)**

Solution

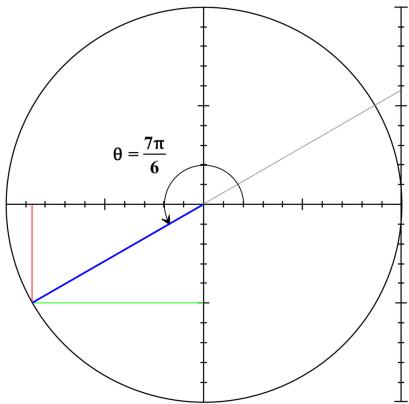
$$\frac{\pi}{6} = 30^\circ \Rightarrow \frac{7\pi}{6} = 7 \times 30^\circ = 210^\circ$$

Marking key/mathematical behaviours

	Marks
$\frac{7\pi}{6} = 210^\circ$ • Determines	1

**Question 1(b)**

Solution



$$\cos \theta = -\sqrt{1 - \sin^2 \theta} = -\sqrt{1 - \left(-\frac{1}{2}\right)^2} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2} \text{ by using the right triangle identity or}$$

$$\cos \theta = \cos \frac{7\pi}{6} = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2} \text{ by knowledge of exact values}$$

$$\tan \theta = \tan \left(\frac{7\pi}{6}\right) = \tan \left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$$

Similarly

Marking key/mathematical behaviours	Marks
$\cos \frac{7\pi}{6} = -\cos \frac{\pi}{6}$ or uses $\sin \theta = -\frac{1}{2}$ meaningfully	1
• indicates use of $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ or $-\frac{3}{2\sqrt{3}}$	1
• states correct exact value of $\cos \frac{7\pi}{6}$ (accept $-\frac{\sqrt{3}}{2}$ or $-\frac{3}{2\sqrt{3}}$ )	1
• indicates use of $\tan \frac{7\pi}{6} = \tan \frac{\pi}{6}$	

Marks	Marks
1 1 1	<ul style="list-style-type: none"> <li>solves for <math>x</math></li> <li>factors trinomial</li> <li>marking key/mathematical behaviours</li> </ul> <p>Solution  <math display="block">(x - 8)(x + 9) = 0</math> <math display="block">x^2 + x - 72 = 0</math> <math display="block">x = -9 \text{ or } x = 8</math> </p>

Question 2(b)

Marks	Marks
1 1 1	<ul style="list-style-type: none"> <li>solves for <math>x</math></li> <li>expands brackets and simplifies</li> <li>multiples the equation by the LCD</li> <li>marking key/mathematical behaviours</li> </ul> <p>Solution  <math display="block">5(x + 3) - 4(x - 3) = 10x</math> <math display="block">5x + 15 - 4x + 12 = 10x</math> <math display="block">9x = 27 \Leftrightarrow x = 3</math> <math display="block">\frac{4}{x+3} - \frac{5}{x-3} = \frac{x}{2}</math> </p>

Question 2(a)

Marks	Marks
1 1 1	<ul style="list-style-type: none"> <li>states both correct values of <math>\theta</math></li> <li>indicates method of determination on diagram</li> <li>marking key/mathematical behaviours</li> </ul> <p>From exact values and use of unit circle,  <math>\theta = \frac{3\pi}{4} \text{ or } -\frac{3\pi}{4}</math></p>

Question 2(c)

Marks	Marks
1	<ul style="list-style-type: none"> <li>states correct sample space</li> </ul>

MATHEMATICS METHODS  
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MARKING KEY

(i)	<ul style="list-style-type: none"> <li>determines that there are 6 pairs of numbers (listing or logic) that have an even product and determines the correct probability of an even product</li> </ul>
(ii)	<ul style="list-style-type: none"> <li>provides some form of exhaustive listing of the sum of two numbers</li> </ul>
(iii)	<ul style="list-style-type: none"> <li>indicates that the only way that the product can be prime is if one of the cards has a one on it.</li> </ul>
(iv)	<ul style="list-style-type: none"> <li>determines the correct probability of a prime product</li> <li>provides some form of exhaustive listing of the sum of two numbers</li> <li>determines that there are only 4 possibilities of prime sums</li> <li>determines the correct probability of the sum being prime</li> </ul>

MATHEMATICS METHODS  
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MARKING KEY

$$\frac{7}{10} \times \frac{3}{9} = \frac{7}{30} = 0.233$$

one mark denominator one mark numerator of answer

Question 7(b)

$$\frac{7}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{7}{9} = \frac{14}{30} = 0.466$$

one mark for two products, one mark denominator one mark numerator of answer

Question 7(c)

$$\frac{7}{10} \times \frac{6}{9} + \frac{3}{10} \times \frac{2}{9} = \frac{48}{90} = \frac{8}{15} = 0.533$$

one mark for two products, one mark denominator one mark numerator for answer

Question 7(d)

$$\frac{\frac{7}{10} \times \frac{6}{9}}{\frac{7}{10} \times \frac{6}{9} + \frac{3}{10} \times \frac{2}{9}} = \frac{7}{8} = 0.875$$

2 marks for denominator, one mark for numerator, one mark for final answer

Question 2(c)		Marking key/mathematical behaviours	
Solution		Marks	
$x^2 - 4x + 1 = 0$			
$(x - 2)^2 = 3$			
$x - 2 = \pm\sqrt{3}$			
$x = 2 \pm \sqrt{3}$			
Has the form $y = \text{atan}bx + c$			
Vertical translation 1 unit up, hence $c = 1$ .			
Period = $2\pi$ hence $b = \frac{1}{2}$			
Dilation parallel to $y$ axis, scale factor = 3. Hence, $a = 3$			
$\therefore y = 3\tan\frac{x}{2} + 1$			
Marking key/mathematical behaviours			
<ul style="list-style-type: none"> <li>solves for <math>x</math></li> <li>takes square root</li> <li>completes the square</li> <li>equates <math>(x - 2)^2 = 3</math></li> <li>determines the period and hence <math>b</math></li> <li>identifies vertical translation and determines <math>c = 1</math></li> <li>determines the correct equation</li> </ul>	1 1 1 1 1 1 1		
Solution			

Question 6(a)		Marking key/mathematical behaviours	
Solution		Marks	
$\therefore$			
Dilation parallel to $y$ axis, scale factor = 3. Hence, $a = 3$			
Vertical translation 1 unit up, hence $c = 1$ .			
Period = $2\pi$ hence $b = \frac{1}{2}$			
Has the form $y = \text{atan}bx + c$			
$\therefore$			
Marking key/mathematical behaviours			
<ul style="list-style-type: none"> <li>states the correct equation</li> <li>determines <math>a</math></li> <li>identifies vertical translation and determines <math>c = 1</math></li> <li>determines the period and hence <math>b</math></li> <li>determines the correct equation</li> </ul>	1 1 1 1 1		
Solution			

Question 6(b)		Marking key/mathematical behaviours	
Solution		Marks	
$\therefore$			
graph is drawn over the correct domain			
graph has the correct amplitude			
phase shift is correct			
graph is accurate passing through (0, -3) and has smooth turning points			
sketches $(x - 2)^2 + (y - 4)^2 = 4$ with correct radius and centre			
sketches $y = -x^2 - 4x - 3$ reflecting the correct turning point, orientation and intercepts			
sketches $2x + 3y = 6$ accurately, showing $x$ and $y$ intercepts			
sketches $2x + 3y = 6$ accurately, showing $x$ and $y$ intercepts	1+1		
graph is accurate passing through (0, -3) and has smooth turning points	2		
sketches $(x - 2)^2 + (y - 4)^2 = 4$ with correct radius and centre	2+1+1		
graph is drawn over the correct domain	1		
graph has the correct amplitude	1		
phase shift is correct	1		
graph is accurate passing through (0, -3) and has smooth turning points	1		
sketches $(x - 2)^2 + (y - 4)^2 = 4$ with correct radius and centre	2		

**Question 4**

Solution

$$\text{Graph A: } y = -\sqrt{x+3}$$

$$\text{Graph B: } y = \frac{-3}{x+1}$$

Marking key/mathematical behaviours

	Marks
• Graph A	
o correct horizontal translation	1
o recognition of reflection in $x$ - axis	1
• Graph B	
o correct horizontal translation	1
o correct dilation factor	1
o recognition of reflection in $x$ - axis	1

**Question 5(a)**

Solution

$$(i) P(X \cup Y) = 0.9 \Rightarrow x = P(X \cap Y) = 0.3 \therefore P(X) = 0.7$$

$$(ii) \text{ From part (i), } P(X) = 0.7 \text{ and } P(Y) = 0.5 \\ P(X) \times P(Y) = 0.35 \neq 0.3$$

Therefore not equal

	Marks
(i)	
• determines $x = 0.3$	1
• determines correct value for $P(X)$	1
(ii)	
• determines $P(X) \times P(Y)$	1
• shows that $P(X) \times P(Y) \neq P(X \cap Y)$	2

**Question 5(b)**

Solution

$$(i) P(X | Y) = \frac{P(X \cap Y)}{P(Y)} \Rightarrow \frac{2}{7} = \frac{x}{0.2+x}$$

$$\therefore 0.4 + 2x = 7x$$

$$\text{i.e. } 0.4 = 5x$$

$$\text{i.e. } 0.08 = x \quad \therefore P(X) = 0.48$$

$$(ii) \text{ From part (i) } P(X) = 0.48 \text{ and so } P(X \cup Y) = 0.48 + 0.2 = 0.68$$

$$P(\overline{X \cup Y}) = 1 - P(X \cup Y) = 1 - 0.68 = 0.32$$

	Marks
(i)	
• applies the conditional probability formula	1
• substitutes correctly	1
• multiplies correctly and simplifies	1
• determines correct value for $P(X)$	1
(ii)	
• determines $P(X \cup Y)$	1
• applies complimentary property and arrives at the correct result	1