PERTH COLLEGE

YEAR 12

Semester One Examination, 2010

Question/Answer Booklet

MATHEMATICS 3CMAT/3DMAT

Section Two: Calculator-assumed

Student Name: _____

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for this section: 100 minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section Two: Calculator-assumed (80 Marks)
Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	7	7	50	40
Section Two: Calculator-assumed	13	13	100	80
				120

Instructions to candidates

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the space provided.

1. The rules for the conduct of Western Australian external examinations are detailed in the Year 12 Information Handbook 2010. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

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Question 8 [7 marks]

Ashleigh wanted to choose to study two languages from the table below. Due to timetable clashes she could only choose from List 1 and List 2 in the following way:

List 1	French(F), Italian(I), German(G)
List 2	Russian(R), Japanese(J), Chinese(C), Arabic(A)

She must first choose one language from List 1 and another language from List 2.
She cannot study Japanese unless she also studies French.
She cannot study Russian unless she also studies German.

(a) Display Ashleigh's possible language combinations in a tree diagram. [2]

(b) How many possible combinations are there for Ashleigh? [1]

(c) If each combination is equally likely, what is the probability that Ashleigh

(i) will choose French and Japanese? [1]

(ii) will not choose German? [1]

(iii) will choose Chinese given that she has chosen French? [2]

Question 9 [7 marks]

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At the beginning of a mice plague there were 15 mice in a farm shed and the number of mice, N, increased continuously at a daily rate equal to 8% of the existing number of mice.

(a) Write an equation for the rate of change of the number of mice. [1]

(b) Find the number of mice in the farm shed after one week. [1]

(c) When will the population of mice double? [2]

(d) The number of mice peaked at 368. From then on the number of mice decreased continuously at a daily rate equal to 20% of the number present. On which day of the plague did the number of mice return to 15? [3]

Question 10 [3 marks]

See next page

(a) Evaluate $\int_{-1}^{3} (2x+1)\,dx$ [1]

(b) Comment on whether the following statement is true or false.
The integral in (a) gives the area between the line y = 2x + 1 and the x-axis, from x = -1 to x = 3. [2]

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Question 11 [10 marks]

Four boys and three girls, Adam, Bob, Charles, David, Eva, Felicity and Grace, stand in a row for a photo.

(a) How many different arrangements are possible? [1]

(b) How many different arrangements have all the girls together? [1]

(c) What is the probability of the arrangement if

(i) boys and girls must alternate? [2]

(ii) Charles and Bob refuse to stand together? [2]

(d) A group of four people is to be selected from seven students above.
What is the probability that

(i) the four selected will contain all three girls? [2]

(ii) the four selected will contain at most two boys given that at least one girl will be selected? [2]

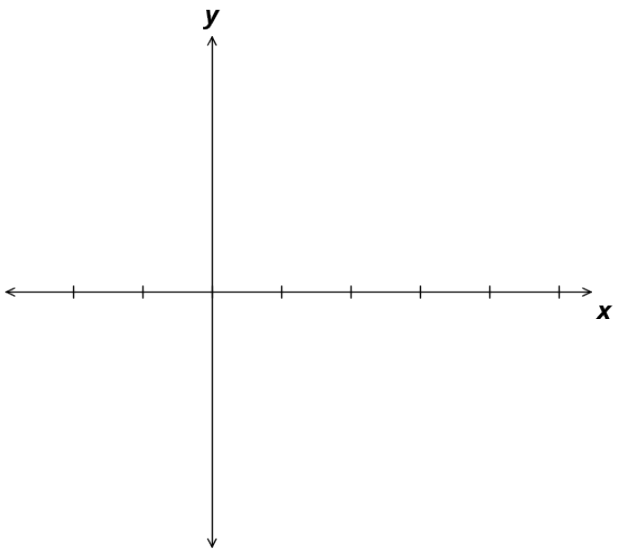
See next page

Question 12 [7 marks]

(a) Use calculus methods, showing all working clearly, to determine the **exact** coordinates of any stationary points and point of inflection on the curve

$$y = (3 - x)\, e^{x}$$ [5]

(b) Sketch the curve $y = (3 - x)\, e^{x}$, for $-2 \le x \le 4$, showing clearly all the main features where necessary, round all values to two decimal places. [2]



Question 13 [7 marks]

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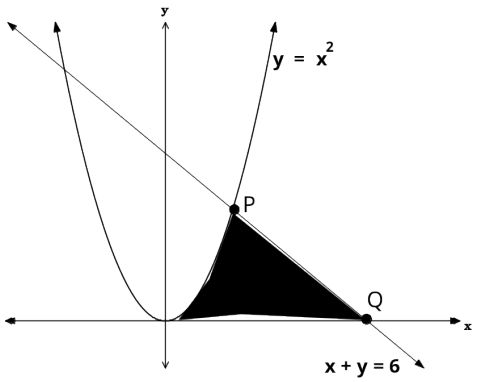
The diagram below shows a sketch of the curve $y = x^2$ and the line $x + y = 6$. Find

(a) the coordinates of points P and Q. [2]



(b) the area bounded by the curve $y = x^2$, the line $x + y = 6$ and the x-axis (i.e the area of the shaded region). For full marks to be awarded for this part of the question, you will have to state the integrals involved in working out the area. [3]

(c) the area enclosed by the curve $y = x^2$ and the line $x + y = 6$. [2]

See next page

Question 14 [6 marks]

(a) If $y = kx^3$ for some constant k, use the incremental formula to estimate the percentage increase in y required to yield a 1.5% increase in x. [3]

(b) A company sells goods such that its revenue, in dollars, from selling x items is given by the equation,

$$R(x) = 5x\,(20x - x^2)$$

(i) Determine the marginal revenue, when x =10. [2]

(ii) What does this represent? [1]

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Question 15 [6 marks]

The weather in Melbourne is difficult to predict in Autumn. On average, 3 mornings out of 5 are fine. If it is fine in the morning, there is a 60% chance that it will be fine in the afternoon. However if it is raining in the morning, there is only 20% chance that it will be fine in the afternoon.

(a) Display the above information in a tree diagram, indicating clearly the probabilities for all possible outcomes. [2]

(b) Find the probability that on a randomly selected Autumn day next year, the weather in Melbourne will be

(i) fine in the morning and raining in the afternoon. [1]

(ii) fine in the afternoon. [1]

Sanne travelled by plane from Perth to Melbourne, arriving on an Autumn afternoon and it was raining.

(c) Given this, what is the probability that it was fine in Melbourne that morning. [2]

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Question 16 [4 marks]

A box has 6 chocolates inside of it. There are two turkish delights and four coconut creams. Three chocolates are selected from the bag, one after the other, and eaten after each selection.

(a) Draw a probability distribution table, for the random variable X, the number of coconut creams selected in this process. [3]

(b) Find the mean value of X. [1]

See next page

Question 17 [5 marks]

The probability distribution function for a discrete random variable X is given by

$$\mathrm{P}\left(X=x\right)=\begin{cases}\mathrm{k}\left(5-x\right); & x=1,\,2,\,3,\,4 \text{ or } 5\\ 0 \quad ; & \text{else where}\end{cases}$$

(a) Calculate the value of k. [2]

(b) Find

(i) P(2 ≤ X < 4) [1]

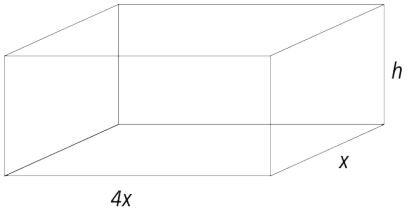
(ii) P(2 ≤ X < 4 | X ≥ 3) [2]

See next page

Question 18 [8 marks]

The diagram below shows a framework of a rectangular prism made using a 60 cm piece of wire. The length of the base is four times the width, *x* cm, of the prism.

(a) Show that the height, h of the prism in terms of *x,* is (15 – 5x) cm. [2]



(b) Find an expression for the volume enclosed by the framework in terms of *x*. [1]

(c) Using calculus methods and showing full reasoning,

(i) find the dimensions of the framework for maximum volume. [4]

(ii) determine the maximum volume. [1]

See next page

Question 19 [5 marks]

(a) If $y = (x - 3)\, e^{-2x}$, show using calculus techniques, that the exact value of the gradient of the tangent line to this curve at $x = 1$ is $\frac{5}{e^2}$. [3]

(b) Find the equation of the tangent to the curve $y = \frac{4}{(x-5)^3}$ at the point where x = 4. [2]

See next page

Question 20 [5 marks]

The gradient function of a curve is given by $\frac{dy}{dx} = \frac{k}{x^2} + 1$ where k is a constant

(a) Find the equation of this curve given that when x =1, $\frac{dy}{dx}$ = 3 and y = 3. [4]

(b) Find the value of y when x = 2. [1]

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END OF SECTION TWO

EXTRA PAGE FOR WORKING

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