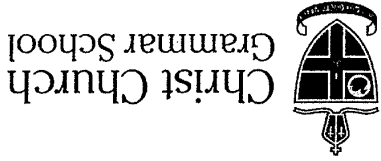


Solutions



Christ Church
Grammar School

2018
UNIT TEST 3

1 for work on
white
paper

MATHEMATICS METHODS Year 11

Section Two:
Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specific to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 5 (4 marks)

Consider the circle described by the relationship:

$$x^2 + y^2 = 14x - 8y + 16$$

(a) Determine the centre and radius of the circle. (3 marks)

$$\begin{aligned} x^2 - 14x + y^2 + 8y &= 16 \\ x^2 - 14x + 49 - 49 + y^2 + 8y + 16 - 16 &= 16 \\ (x-7)^2 + (y+4)^2 - 65 &= 16 \\ (x-7)^2 + (y+4)^2 &= 81 \end{aligned}$$

✓ (Completing the square for x and y)
centre is (7, -4) ✓ (for calculating centre)

radius is $\sqrt{81} = 9$ ✓ (for calculating radius)

(b) Determine the domain for the relationship. (1 mark)

$$7-9 \leq x \leq 7+9$$

domain is $\{x \in \mathbb{R} : -2 \leq x \leq 16\}$ ✓ (for calculating domain)

$$-2 \leq x \leq 16 \text{ or } 16 \leq x \leq -2 \text{ also OK}$$

See next page

Question 6

(4 marks)

- (a) Determine the equation of the image of the graph of $y = \sqrt{x}$ when the following sequence of transformations has been applied: a reflection in the y axis, followed by a translation of 3 units right. (2 marks)

reflection in y axis $y = \sqrt{-x}$

✓ (for correct reflection in y axis)

translation 3 units right

$$y = \sqrt{-(x-3)}$$

$$\text{or } y = \sqrt{-x+3}$$

✓ (for correct final answer)

- (b) Determine the equation of the image of the graph of:

$$y = 3x^3 + x^2 - 5x + 2$$

when the graph has firstly been reflected in the x axis and then translated 2 units up. (2 marks)

reflected in x axis $y = -(3x^3 + x^2 - 5x + 2)$ ✓ (for reflection in x axis)

translated 2 up $y = -(3x^3 + x^2 - 5x + 2) + 2$ ✓ (for correct final answer)
 $\text{or } y = -3x^3 - x^2 + 5x$

See next page

Additional working space

Question number: _____

A polynomial $P(x)$ has been factorised as follows:

$$P(x) = (x - 1)^2(3x + 4)$$

For the standard form of $P(x)$, that is with the brackets expanded, determine the following:

(a) The coefficient of the x term. (1 mark)

expand using classpad
 $P(x) = 3x^3 - 2x^2 - 5x + 4$

coefficient of x term is -5

✓ (for giving correct coefficient)

(b) The constant term. (1 mark)

constant term is 4

✓ (for giving correct constant term)

See next page

Question 8

(4 marks)

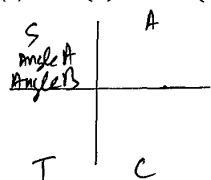
The angles A and B are both obtuse angles (that is, they are both in the range $90^\circ < \theta < 180^\circ$), such that:

$$\sin(A) = \frac{3}{5} \text{ and } \cos(B) = \frac{-12}{13}$$

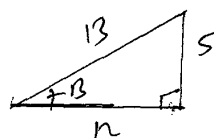
Determine the **exact** values of the following:

(a) $\cos(A)$ and $\sin(B)$.

(2 marks)



Angles A and B are in second quadrant so sine is positive and cos is negative



$$\cos A = -\frac{4}{5}$$

✓ (for $\cos A$)

$$\sin B = \frac{5}{13}$$

✓ (for $\sin B$)

(b) $\sin(A - B)$.

(2 marks)

$$\begin{aligned} \sin(A - B) &= \sin A \cos B - \cos A \sin B \\ &= \frac{3}{5} \times \left(\frac{-12}{13}\right) - \left(-\frac{4}{5}\right) \times \left(\frac{5}{13}\right) \quad \checkmark \text{ (correct expansion)} \\ &= -\frac{36}{65} + \frac{20}{65} \\ &= -\frac{16}{65} \quad \checkmark \text{ (final answer)} \end{aligned}$$

See next page

Question 13

(6 marks)

The depth in water, in metres, in a harbour at a certain point at time t hours is given by $D(t)$, where:

$$D(t) = 8 + 2\sin\left(\frac{\pi t}{6}\right), \quad 0 \leq t \leq 24$$

(a) Determine the period of the function $D(t)$.

(2 marks)

$$\begin{aligned} \text{period of sine function} &= \frac{2\pi}{b}, \quad b = \frac{\pi}{6} \\ \text{period} &= \frac{2\pi}{\pi/6} \quad \checkmark \text{ (gives basic expression for period)} \\ &= 2/1/6 \\ &= 12 \text{ hours} \quad \checkmark \text{ (gives final answer for period)} \end{aligned}$$

(b) Give the value of t when the depth of the water is first 9 metres.

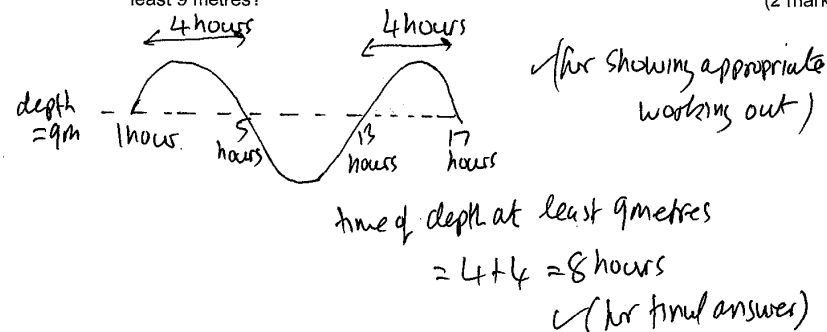
(2 marks)

$$9 = 8 + 2\sin\left(\frac{\pi t}{6}\right) \quad \checkmark \text{ (setup basic equation)}$$

Solve on classpad $t = 1 \text{ hour} \quad \checkmark \text{ (give final answer)}$

(c) For how many hours in the 24 hour period under consideration, is the depth at least 9 metres?

(2 marks)



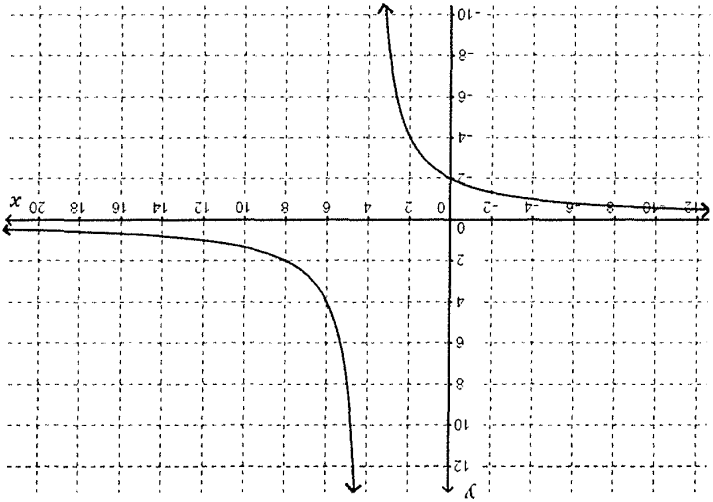
End of questions

(3 marks)

Question 9

The equation of the curve in the graph below is in the form:

$$y = \frac{x-b}{a}$$



- (a) Determine the values of a and b .
(2 marks)

vertical asymptote at $x=4$ so $b=4$ (correct value of b)
 $y = \frac{a}{x-4}$ use point $(0, -2)$ so $a=8$ (correct value of a)
 $-2 = \frac{a}{0-4}$

- (b) If the curve is subject to a dilation of scale factor 4 parallel to the y axis, give the equation of the horizontal asymptote of the new curve.
(1 mark)

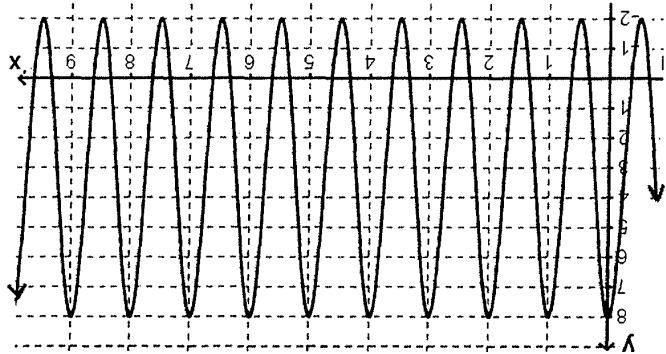
$y=0$ ✓ (for correct answer)

See next page

(3 marks)

Question 12

Determine the equation of the following cosine function:



$$\text{Period} = 1 \Rightarrow \frac{2\pi}{b} = 1 \Rightarrow b = 2\pi$$

translated 3 units up
amplitude = 5

$$y = 5 \cos(2\pi x/9 + 3)$$

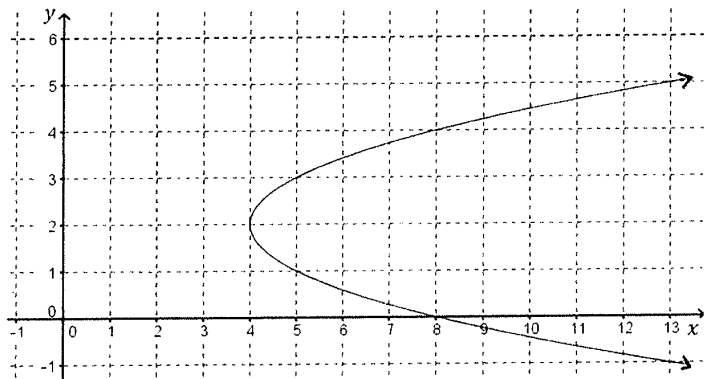
✓ for amplitude = 5
✓ for vertical shift = +3
✓ for x coefficient is $2\pi/9$

See next page

Question 10

(2 marks)

The following graph shows $y^2 = x$ after it has been translated either up, down, left or right, or a combination of these. Give the equation of the curve shown in the graph.



$$(y-2)^2 = x-4$$

✓ (for correct transformation of y)

✓ (for correct transformation of x)

or $x = (y-2)^2 + 4$
is also ok

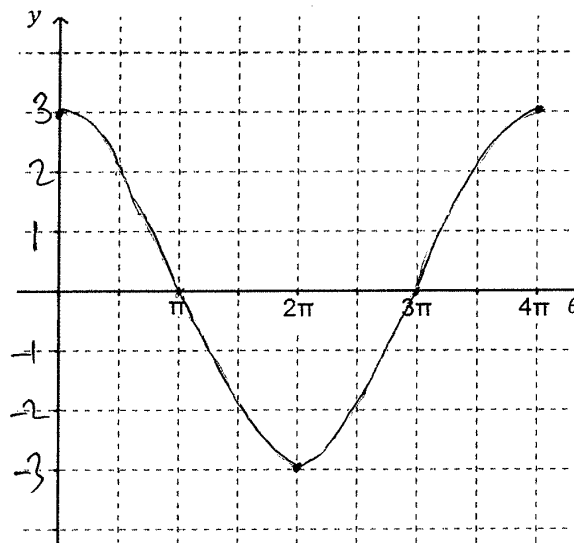
See next page

Question 11

(2 marks)

Use the axes below to sketch the following graph for $0 \leq \theta \leq 4\pi$:

$$y = 3\cos\left(\frac{\theta}{2}\right)$$



✓ (correct x axis intercepts)

✓ (correct max and min values)

See next page