# 3CMAS MATHEMATICS: SPECIALIST

## Section One (calculator-free)

Name: _____

Teacher: _____

## Time allowed for this section

**Section One**
Reading time before commencing work:       5 minutes
Working time for paper:                     50 minutes

## Material required/recommended for this paper

**To be provided by the supervisor**
Question/answer booklet for Section One and a formula sheet (from Curriculum Council) which can be used for Section Two.

**To be provided by the candidate**
**Section One:**
*Standard items:*     pens, pencils, pencil sharpener, highlighter, eraser, ruler

## Important note to candidates

No other items may be taken into the examination room.  It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room.  If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Suggested working time (minutes) | Marks available |
|---|---|---|---|---|
| **Section One: Calculator-free** | **8** | **8** | **50** | **40** |
| Section Two: Calculator-assumed | 9 | 9 | 100 | 80 |
| | | | | 120 |

## Instructions to candidates

1. The rules for the conduct of Western Australian examinations are detailed in the *Year 12 Information Handbook 2010.* Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

   **Section One:**      Write answers in this Question/Answer Booklet. **All** questions should be answered.
   **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

   It is recommended that you **do not use pencil** except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
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## MARK ALLOCATION AND RECORDS:

| Section | Question | Marks | Awarded |
|---------|----------|-------|---------|
| ONE | 1 | 7 | |
| | 2 | 3 | |
| | 3 | 7 | |
| | 4 | 2 | |
| | 5 | 7 | |
| | 6 | 3 | |
| | 7 | 5 | |
| | 8 | 6 | |
| | Penalties | - 1/2/3/4 | |
| | ONE | **40** | |
| | TWO | **80** | |
| | **TOTAL** | **120** | |

| Penalties | |
|-----------|--|
| Rounding (-1) | |
| Units (-1) | |
| Notation (-1) | |
| Missing '+c' (-1) | |

| % |
|---|

## Section One (calculator-free) 40 marks

This section has **eight (8)** questions. Attempt **all** questions.

Suggested working time: **50 minutes**

The following exact value table may be useful to answer questions in this examination.

|        | $0^0$ | $30^0$ | $45^0$ | $60^0$ | $90^0$ |
|--------|-------|--------|--------|--------|--------|
| Sin    | 0     | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| Cos    | 1     | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| Tan    | 0     | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | undefined |

**Question 1: [7 marks – 1, 2, 1, 1, 1, 1]**
Differentiate each of the following, simplifying fully:

**a)**  $y = 3x^5 + 4x^3 - \dfrac{8}{x^2}$

**b)**  $y = \dfrac{(x-1)(x+1)}{x^4 + 4}$

**c)**  $y = e^x \ln x$

**d)**  $y = (\ln(x^4))^2$

**Question 1 cont…**

**e)** $y = \cos(x^2 - 4)$

**f)** $y = x \sin x$

**Question 2: [3 marks]**

Determine $\dfrac{dy}{dx}$ in terms of $t$ for: $x = 6t^4$, $y = \dfrac{1}{t+2}$.

[3]

**Question 3: [7 marks – 1, 1, 1, 1, 1, 2]**

Integrate the following:

**a)** $\displaystyle\int 6x^2 - 7 \; dx$

**b)** $\displaystyle\int (2x - 7)^4 \; dx$

**c)** $\displaystyle\int \sqrt{3 - x} \; dx$

**d)** $\displaystyle\int \sin 6x \; dx$

**e)** $\displaystyle\int \frac{8x}{x^2 + 3} \; dx$

**f)** $\displaystyle\int 2 \tan x \; dx$

**Question 4: [2 marks]**

Determine a general solution to the differential equation: $\frac{dy}{dx} = \frac{4x - 3}{y + 1}$. Simplify your solution fully.

[2]

**Question 5: [7 marks]**

Determine $z$ if:     $z\bar{z} \; + \; 2z = \dfrac{1 + 4i}{4}$.

[7]

Determine $z$ if:     $z\bar{z} \; + \; 2z = \dfrac{1 + 4i}{4}$.

**Question 6: [3 marks]**

If $z_1 = 2 \operatorname{cis}\left(\dfrac{\pi}{12}\right)$ and $z_2 = 5 \operatorname{cis}\left(\dfrac{\pi}{6}\right)$, prove that: $\quad z_1 z_2 = 5\sqrt{2}\,(1 + i)$

[3]

**Question 7: [5 marks]**

Determine the gradient of the curve defined by the parametric equations: $x = 2 \sin t$ and $y = 7 \cos 3t$,

at the point where $t = \dfrac{\pi}{6}$.

[5]

**Question 8: [6 marks]**

Determine the coordinates of the points on the graph of: $5x^2 + y^2 - 20x + 3y = 8$, where the tangent to the curve is horizontal.
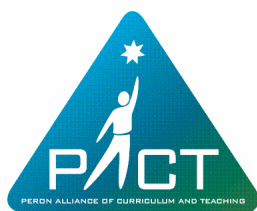
[6]

**END OF SECTION ONE**

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# 3CMAS MATHEMATICS: SPECIALIST

## Section Two (calculator-assumed)

Name: _____

Teacher: _____

## Time allowed for this section

**Section Two**

Reading time before commencing work:     10 minutes

Working time for paper:     100 minutes

## Material required/recommended for this paper

**To be provided by the supervisor**

Question/answer booklet for Section Two and a formula sheet (from Curriculum Council) which was provided with Section One.

**To be provided by the candidate**

**Section Two:**

*Standard items:*     *pens, pencils, pencil sharpener, highlighter, eraser, ruler*

*Special items:*     drawing instruments, templates, and up to three calculators, (CAS, graphic or scientific), which satisfy the conditions set by the Curriculum Council for this course. Up to two (2) A4 pages of notes (handwritten, photocopied or typed on both sides) that may be either personally or commercially produced. *It must not be folded, have anything stuck to it or have correction fluid/tape on it.*

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## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Suggested working time (minutes) | Marks available |
|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 40 |
| **Section Two: Calculator-assumed** | **9** | **9** | **100** | **80** |
| | | | | 120 |

## Instructions to candidates

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2.   Answer the questions according to the following instructions.

   **Section Two:**        Write answers in this Question/Answer Booklet.  **All** questions should be answered.

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## MARK ALLOCATION AND RECORDS:

| Section | Question | Marks | Awarded |
|---------|----------|-------|---------|
| TWO | 9 | 4 | |
| | 10 | 6 | |
| | 11 | 6 | |
| | 12 | 9 | |
| | 13 | 4 | |
| | 14 | 4 | |
| | 15 | 11 | |
| | 16 | 15 | |
| | 17 | 7 | |
| | 18 | 14 | |
| | Penalties | - 1/2/3/4 | |
| | **TWO** | **80** | |

| Penalties |
|-----------|

| Rounding (-1) | |
|---------------|--|

| Units (-1) | |
|------------|--|

| Notation (-1) | |
|---------------|--|

| Missing '+c' (-1) | |
|-------------------|--|

## Section Two: Calculator – 80 marks

This section has **eleven (11)** questions. Attempt **all** questions.

Suggested working time: **100 minutes**

**Question 9:          [4 marks]**

Vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are as follows: $\underset{\sim}{a} = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 3 \\ t \\ -6 \end{pmatrix}$. Determine the value of $t$ if $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are:

**a)**     Parallel to each other.

[1]

**b)**     Perpendicular to each other.

[3]

**Question 10:** **[6 marks]**

Prove that the square of an integer NOT divisible by 5 leaves a remainder of 1 or 4 when divided by 5.

[6]

**Question 11:        [6 marks]**

Determine the equations of A, B and C from the graph below:



A = _____

B = _____

C = _____

[6]
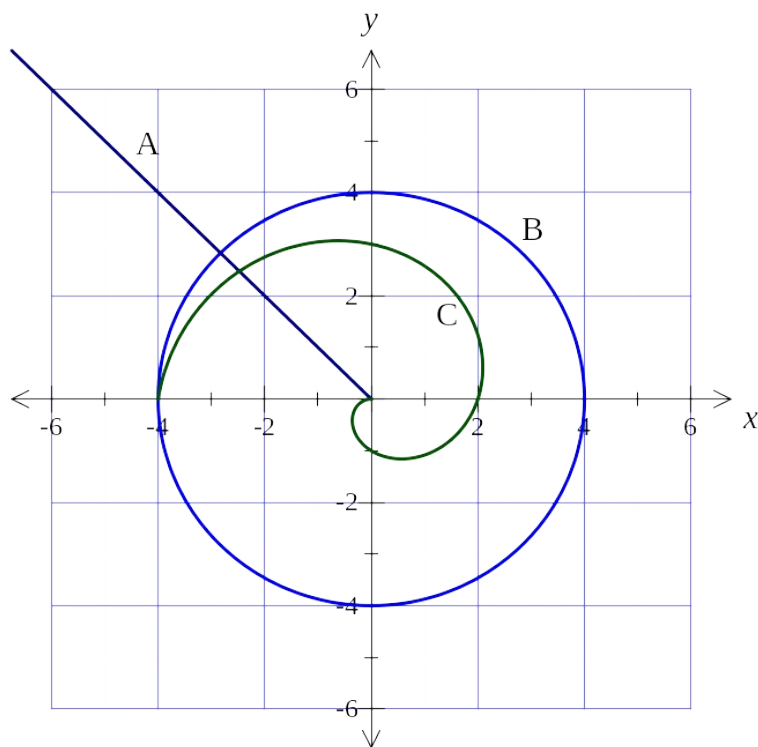
**Question 12:**    **[9 marks]**

The diagram on the right shows a rectangular prism.

Determine:

**a)**    $\overrightarrow{BH}$
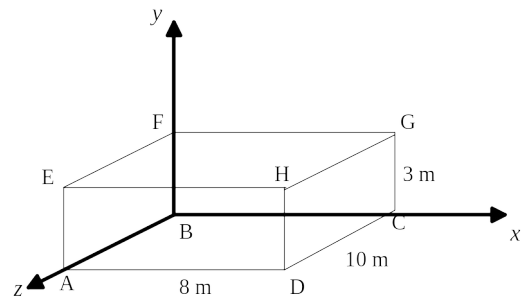
[1]

**b)**    $\overrightarrow{BD}$

[1]

**c)**    $\angle\,HBD$ (in degrees, to 1 decimal place).

[3]

**d)**    The acute angle between the skew lines BD and FE.

[4]

**Question 13:**          **[4 marks]**

Prove the following:

$$\frac{\sin A}{\cos B} + \frac{\cos A}{\sin B} = \frac{2 \cos (A - B)}{\sin 2B}.$$
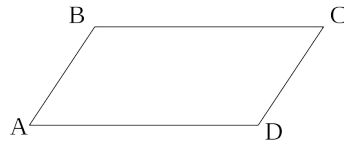
[4]

**Question 14:**          **[4 marks]**

Use calculus techniques to determine: $\displaystyle\int 5^{x} \, dx$.

[4]

**Question 15:**      **[11 marks]**

ABCD is a parallelogram with points E and F such that $\overrightarrow{AE} : \overrightarrow{EB} = 1 : 4$ and $\overrightarrow{BF} : \overrightarrow{FC} = 3 : 1$. $\overrightarrow{ED}$ and $\overrightarrow{AF}$ intersect each other at G. Let: $\overrightarrow{AB} = \underset{\sim}{a}$ and $\overrightarrow{AD} = \underset{\sim}{d}$.

**a)**      Complete the diagram below with the information given above.



[2]

**b)**      Determine the ratios in which $\overrightarrow{AF}$ and $\overrightarrow{ED}$ intersect each other, if the intersection point is at G.

[9]

**Question 16:** **[15 marks]**

If: **a** = <8, -6, 0> and **b** = <-2, 4, -1>, determine:

**a)** 2**b** – **a**

[1]

**b)** A vector in the same direction as **a** but equal in magnitude to **b**.

[3]

**c)** The acute angle that vector **a** makes with the *y*-axis.

[4]

**Question 16 cont…**

**d)**     Point P lies on the line AB (from **a** to **b** ) and it is known that: $\overrightarrow{AP} : \overrightarrow{AB} = 2 : 3$. Determine the position vector of the point P.
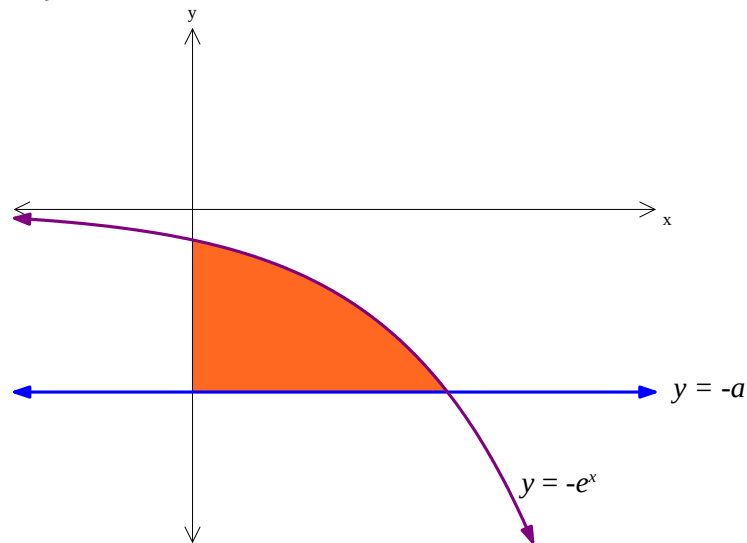
[4]

**e)**     If: **c** = <6, 5, -2> and **d** = <-10, -38, 11>, express **d** in the form: $\lambda$**a** + $\mu$**b** + $\eta$**c** , and hence determine $\lambda, \mu,$ and $\eta$ . (Hint: Use the vectors **a** and **b** from the start of the question).

[3]

**Question 17:**      **[7 marks]**

The graphs of $y = -a$ and $y = -e^x$ are shown below:



**a)**     Write an expression for the shaded area (above), in the form: $\displaystyle\int_0^b c\ dx$.

[3]

**b)**     Determine the values of $a$ and $b$, if it is known that: $-e^x = -a$, when $x = 1\cdot 7918$.

[3]

**c)**     Determine the area of the shaded region (to two decimal places).

[1]

**Question 18:          [14 marks]**

The two countries Zedlandia and Xenutia are at war. Zedlandia ($z$) fired a ground to air missile in order to intercept a missile coming in from Xenutia ($x$). When $z$ was launched, the position vectors (in metres), relative to the army base were:

$$r_z = \begin{pmatrix} 720 \\ 0 \\ 0 \end{pmatrix} \text{ and } r_x = \begin{pmatrix} 3000 \\ 4800 \\ 650 \end{pmatrix}.$$

Both $z$ and $x$ have constant velocities (m/s):

$$v_z = \begin{pmatrix} -200 \\ 208 \\ 40 \end{pmatrix} \text{ and } v_x = \begin{pmatrix} -255 \\ 127 \\ 3 \end{pmatrix}.$$

**a)**     Prove that the two missiles *did not* intercept.

[4]

**Question 18 cont…**

**b)**    Determine by how much (distance and time) the two missiles missed each other (Note: Enough working must be shown in order to gain full marks).

[6]

**Question 18 cont…**

c) In order for the two missiles to collide, the *z* missile changed its flight plan after 15 seconds. The *x* continued on its path and then interception occurred 25 seconds later. Determine the constant velocity that *z* maintained in this second stage for interception to occur.

[5]

**END OF EXAM**

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