Working out space

PERTH MODERN SCHOOL
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Independent Public School

Course Methods Test 1 Year 12

Student name: ______________ Teacher name: ______________

Task type: **Response**

Reading time for this test : 5 mins

Working time allowed for this task: 40 mins

Number of questions: _____6______

Materials required: **No Cals allowed at all!**

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper single sided,

Marks available: **34 marks**

Task weighting: **13%**

Formula sheet provided: no, but formulae listed on next page.

Note: All part questions worth more than 2 marks require working to obtain full marks.

Useful formulae

$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
$\frac{d}{dx}e^{ax-b} = ae^{ax-b}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}dx = \ln x + c, \quad x > 0$
$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)}dx = \ln f(x) + c, \quad f(x) > 0$
$\frac{d}{dx}\sin(ax-b) = a\cos(ax-b)$	$\int \sin(ax-b)\,dx = -\frac{1}{a}\cos(ax-b) + c$
$\frac{d}{dx}\cos(ax-b) = -a\sin(ax-b)$	$\int \cos(ax-b)\,dx = \frac{1}{a}\sin(ax-b) + c$

Product rule	If $y = uv$ then $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$	or	If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$
Quotient rule	If $y = \frac{u}{v}$ then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	or	If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$
Chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or	If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$
Fundamental theorem	$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$	and	$\int_a^b f'(x)\,dx = f(b) - f(a)$
Increments formula	$\delta y \approx \frac{dy}{dx} \times \delta x$		
Exponential growth and decay	$\frac{dP}{dt} = kP \Leftrightarrow P = P_0e^{kt}$		

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Q5 (4 marks)

The cost $\$C$ for the production of x thousands units of a certain product is given by

$C = (3x+5)^4$, $x > 0$.

Determine the number of units for which the **average cost per unit** is a minimum and find this minimum average cost. Justify. (No need to simplify)

Q6 (4 marks)

Consider a train moving in a straight line. The displacement, x km, from its starting position at time t minutes is given by $x = \frac{t^3}{3} - \frac{3t^2}{2} + 2t$, $t \geq 0$. The train changes direction twice. Determine the distance in km between these two positions on the track. (Simplify)

No calculators allowed!!!

Q1 (2, 2 & 2 = 6 marks)

Determine the gradient function $\frac{dy}{dx}$ for each of the following.

i) $y = x^3 + \frac{1}{x^2}$

ii) $y = \frac{8x^4 - 5x}{x}$

iii) $y = (x^3 - 1)(5 + \sqrt{x})$

Q2 (4 marks)

Determine the equation of the tangent to the curve $y = \frac{5x - 7}{3x + 2}$ at the point $\left(1, \frac{-2}{5}\right)$.

Q3 (2, 2, 2 & 4= 10 marks)

The table below contains the values of the polynomial function $f(x)$ and its first and second derivatives for $x = 0, 1, 2, 3, 4, 5, 6$.
There are no stationary points for non-integer values of x.

x	0	1	2	3	4	5	6
$f(x)$	12	5	-2	-13	-20	-35	-5
$f'(x)$	-4	-12	-5	0	-11	0	15
$f''(x)$	-8	0	2	0	-5	7	10

a) Evaluate $\frac{d}{dx}\left[f(x)\right]^2$ when $x = 1$

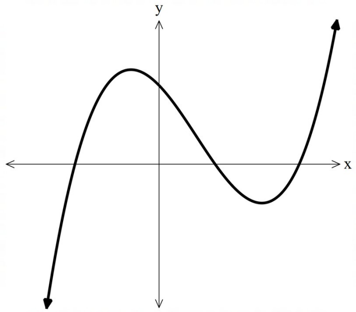
b) Evaluate $\frac{d}{dx}\left[f(2x)\right]$ when $x = 3$

c) Evaluate $\frac{d}{dx}\left[\frac{1}{f(x)}\right]$ when $x = 2$

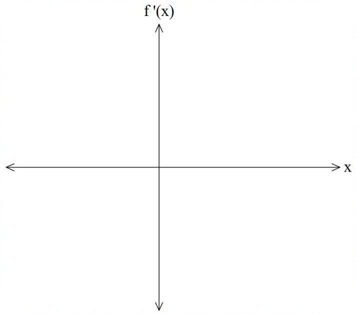
d) Determine the x-coordinate of any **stationary** points and their nature. Justify your answer.

Q4 (3 & 3 = 6 marks)

Consider the curve of $y = f(x)$ which is graphed below.



a) Sketch below a graph of the first derivative of $y = f(x)$. Label on this new graph stationary points.



b) Sketch below a graph of the second derivative of $y = f(x)$. Label on this new graph any inflection points(if any).