Christ Church Grammar School

2020
TEST 5

MATHEMATICS METHODS Year 11

Section One:
Calculator-free

Your name Marking Key

Teacher's name _____

Time and marks available for this section

Working time:	30 minutes
Marks available:	27 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable or gel pens.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

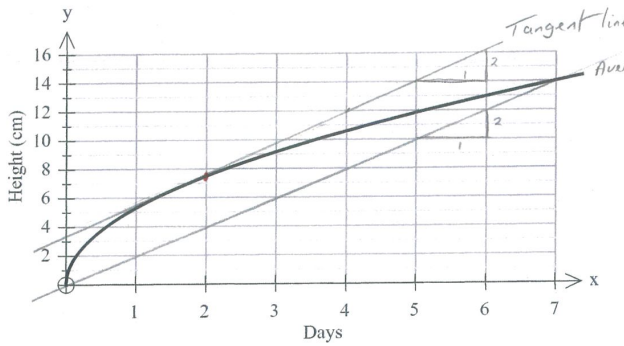
See next page

Additional working space

Question number: _____

Question 1 **(3 marks)**

The graph below shows the growth of bean plants over the seven days following the first appearance of the sprout.



(a) Calculate the average rate of growth over the seven days. (1 mark)

Average rate = $\frac{\text{Rise}}{\text{Run}}$

$= \frac{14}{7}$

$= 2$ cm/day. ✓

Calculates gradient from (0,0) & (7, 14) correctly.

(b) On what day does the instantaneous growth rate appear to be equal to the seven-day average growth rate. Draw a tangent on the graph at this point. (2 marks)

Day 2

✓ Draws tangent with gradient of 2

✓ Identifies day 2

3

See next page

Question 2 **(5 marks)**

Determine the derivative of the following functions.

(a) $f(x) = 7x^3 - 2x^2 - x - 6$ (1 mark)

$f'(x) = 21x^2 - 4x - 1$ ✓ Correctly determines derivative for all terms

(b) $f(x) = 2\pi x + \frac{x^3}{3} - \frac{x^4}{5}$ (2 marks)

$f'(x) = 2\pi + x^2 - \frac{4}{5}x^3$

✓ Correctly differentiates x term

✓ Correctly differentiates x^3 and x^4 terms

(c) $f(x) = (2x-1)^2$ (2 marks)

$(2x-1)^2 = (2x-1)(2x-1)$

$= 4x^2 - 4x + 1$ ✓ Expands f(x)

$f'(x) = 8x - 4$ ✓ Differentiates correctly

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See next page

Additional working space

Question number: _____

Question 8 **(7 marks)**

Consider the quartic function $y = 3x^4 - 4x^3 - 12x^2 + 12$,

(a) State the coordinates of any stationary points and give the nature of each. (3 marks)

From calculator

$(-1, 7)$ $(2, -20)$ Minimum ✓✓ Correct minimum points

$(0, 12)$ maximum ✓ correct maximum

If y coords not given $(\frac{1}{3})$ marks Points must be identified.

(b) For what values of x is the function decreasing. (2 marks).

$x < -1$ and $0 < x < 2$

✓ 1 mark for correct inequality ✓ 1 mark for correct inequality

(c) Determine the global maximum and the global minimum values for this function in the interval $-2 \le x \le 3$. (2 marks)

Global maximum 44 at $x = -2$ ✓ States correct global max

Global minimum -20 at $x = 2$ ✓ states correct global min.

7

End of questions

Question 3 **(5 marks)**

Consider the function $y = t^3 - 5t^2$

(a) Determine the instantaneous rate of change of the function at $t = 2$. (2 marks)

$y' = 3t^2 - 10t$

$y'(t=2) = 3(2)^2 - 10(2)$ ✓ Substitutes into derivative correctly

$= -8$ ✓ Evaluates correctly

(b) When is the instantaneous rate of change of y equal to 25? (3 marks)

$3t^2 - 10t = 25$ ✓ Formulates correct equation

$3t^2 - 10t - 25 = 0$

$(3t + 5)(t - 5) = 0$ ✓ Factorises correctly

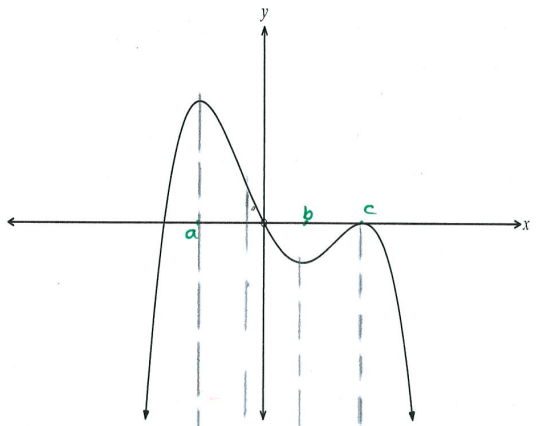
$t = -5/3$ or $t = 5$ ✓ States 2 values for t

5

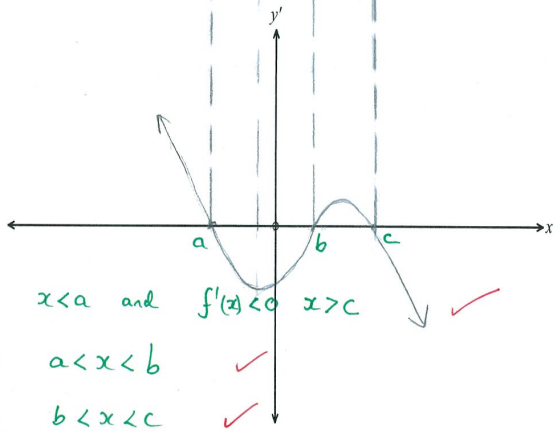
See next page

Question 4 **(3 marks)**

A function has been drawn on the axes below



Draw the gradient function for this function on the axes below.



$f'(x) > 0 \quad x < a$ and $f'(x) < 0 \quad x > c$ ✓

$f'(x) < 0 \quad a < x < b$ ✓

$f'(x) > 0 \quad b < x < c$ ✓

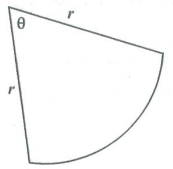
Graph of gradient function must line up correctly with original function. ✓

3

See next page

Question 7 **(5 marks)**

Part of a local children's playground, in the shape of a circular sector, is to have a new rubber surface installed.



OR

Perimeter $= 2\pi R \times \frac{\theta}{2\pi} + 2R$

$20 = R\theta + 2R$

etc.

(a) If the perimeter of this sector is 20 metres, then show that θ can be expressed as

$$\theta = \frac{20}{r} - 2$$ (2 marks)

Perimeter $= R + R + R\theta$

$= 2R + R\theta$

$2R + R\theta = 20$

$R\theta = 20 - 2R$

$\theta = \frac{20 - 2R}{R}$

$\theta = \frac{20}{R} - 2$

✓ states correct equation for perimeter.

✓ Re arranges to express in correct form.

(b) Hence, or otherwise, determine the angle and the radius required to maximise the area of the rubber surface.

(3 marks)

$A = \frac{1}{2} R^2 \theta$

$= \frac{R^2}{2}\left(\frac{20}{R} - 2\right)$ ✓

$= 10R - R^2$ ✓ States correct Area formula

$\frac{dA}{dR} = 10 - 2R$

Max $\Rightarrow$ $10 - 2R = 0$

$R = 5$ ✓ Differentiates and solves for R

$\theta = \frac{20}{5} - 2$

$= 2$ radians. ✓ States value of θ in radians.

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See next page

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Question 5 **(4 marks)**

The tangent to the curve $y = 8x - 2 - 2x^2$ at the point $(3,4)$ intersects the x-axis at $(a,0)$. Determine the value of a.

$y' = 8 - 4x$ ✓states correct derivative

$y'_{(x=3)} = 8 - 12$

$= -4$ gradient of tangent ✓calculates correct gradient

Equation of tangent : $y = -4x + c$

Using $(3,4)$ $4 = -4(3) + c$

$16 = c$ ✓calculates c correctly

Equation of tangent : $y = -4x + 16$

x intercept : $0 = -4x + 16$

$x = 4$

$a = 4$ ✓states correct value for a

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See next page

Question 6 **(7 marks)**

Consider the function $f(x) = x^2 - 5x$

(a) Use first principles to differentiate the function $f(x)$. (3 marks)

$f(x) = x^2 - 5x$

$f(x+h) = (x+h)^2 - 5(x+h)$

$= x^2 + 2xh + h^2 - 5x - 5h$

$f(x+h) - f(x) = x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x$

$= 2xh + h^2 - 5h$ Correctly evaluates $f(x+h) - f(x)$ ✓

$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 - 5h}{h}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2xh + h^2 - 5h}{h}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{h(2x + h - 5)}{h}$ States correct expression for limit ✓

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (2x + h - 5)$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 2x - 5$ Factorises to remove h and evaluates correctly ✓

$\frac{df}{dx} = 2x - 5$

if "$\lim_{h \to 0}$" not included on R.H.S. −1 mark.

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Christ Church Grammar School

2020
TEST 5

MATHEMATICS METHODS Year 11

Section Two:
Calculator-assumed

Your name Marking Key.

Teacher's name _____

Time and marks available for this section

Working time:	15 minutes
Marks available:	12 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates and up to three calculators approved for use in this assessment

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Question 6 continued

(b) Determine the gradient of $f(x)$ when $x = 2$ (1 mark)

$f'(2) = 2(2) - 5$

$= -1$

Evaluates $f'(2)$ correctly ✓

(c) Write the equation of the tangent to $f(x)$ at $x = 2$ (3 marks)

$y = -1x + c$

$-6 = -1(2) + c$

$-4 = c$

$y = -1x - 4$

Equation : $y = -x - 4$

$f(x) = x^2 - 5x$

$f(2) = -6$

$(2, -6)$

✓ Equation with correct gradient (i.e. $m = -1$)

✓ Calculates c correctly.

✓ States equation

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End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____