

**Question 20**

(7 marks)

Given only two events A and B are possible and  $P(A) = 0.2$ ,  $P(B) = x$  and  $P(A \cup B) = p$ :

- (a) Find in terms of  $x, p$  and/or any numeric value,  $P(A \cap B)$

(1)

- (b) If event A is a subset of event B determine a range of values for  $p$ .

(1)

- (c) If  $x=0.6$ , determine for what values of  $p$  are

- (i) events A and B mutually exclusive?

(2)

- (ii) events A and B are independent?

(3)

**END OF EXAM**

**Additional working space if needed**

Question number(s): \_\_\_\_\_



**MATHEMATICS**

**3CMAT**

**Section Two:**

**Calculator-assumed**

Student Name: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: 10 minutes  
Working time for paper: 100 minutes

**Material required/recommended for this section**

To be provided by the supervisor  
Question/answer booklet  
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler, correction fluid/tape.

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

**Important note to candidates**

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

### Structure of this paper

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One Calculator—free	7	7	50 minutes	40
Section Two Calculator—assumed	13	13	100 minutes	80
Total marks				120

### Instructions to candidates

1. Answer the questions in the spaces provided.
2. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.

### Question 19

(4 marks)

Talia has calculated the arrival time of her mother when picking her up after school on any one day can be modelled by a uniform probability function with a maximum arrival time of 30 minutes. If this probability function proves a good estimate of future events, determine the probability on the next date, Talia will wait:

- (a) 20 minutes (1)
- (b) at least 25 minutes (1)
- (c) at least 25 minutes if she has to wait at least 10 minutes. (2)

QUESTION	MARKS ALLOCATED
8	7
9	6
10	4
11	9
12	8
13	4
14	9
15	4
16	5
17	8
18	5
19	4
20	7
TOTAL	80

(5 marks)

Question 18

In the first five seconds of inflation, the relationship between the radius ( $r$  cm) and time ( $t$  sec) of a spherical party balloon are related by the formula

$$r = -t(t - 10)$$

(a) Show that the relationship between volume ( $V$  cm<sup>3</sup>) and time is given by

$$V = \frac{4\pi(10t - t^2)^3}{3}$$

(b) Determine the exact volume of the balloon 3 seconds after inflation commenced. (1)

(c) Determine the approximate change in volume as  $t$  increases from 3 to 3.01 sec. (3)

## Section Two: Calculator-assumed

80 marks

This section has ten (13) questions. Attempt all questions.

### Question 8

(7 marks)

- (a) The function  $y = f(x) = e^{x(x^2-1)}$  is transformed to  $y = 2e^{x(x^2-1)} + 1$ .  
Describe the transformation. (2)

- (b) Find the maximum and minimum values of the function  $f(x) = 104 + 8x + \frac{288}{x}$ ,  
over the interval  $1 \leq x \leq 7$ . Show calculus techniques to gain full marks. (5)

### Question 17

(8 marks)

- (a) In the following table,  $x$  is a score in a game and  $P(X)$  is the probability of getting that score. The expected mean of the discrete probability distribution is 3.2. Find the values of  $m$  and  $n$ . (3)

$x$	1	2	3	4	5
$P(X = x)$	0.2	$m$	0.2	$n$	0.2

- (b) The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that
- (i) exactly 5 survive? (1)
- (ii) at least 10 survive? (2)
- (iii) from 3 to 8 survive? (2)

**Question 15**

Given that  $\int_k^{25} e^{2x-5} dx = \frac{e-1}{2}$ , find the value of  $k$ .

**(4 marks)**

**Question 16**

**(5 marks)**

Consider the function  $f(x) = 2x^3 + ax^2 + 3x + b$  where  $a$  and  $b$  are constants

(a) Find an expression for the gradient of the curve (1)

(b) Given that the tangents at  $A(0, b)$  and  $B(3, 8)$  are parallel, find the values of  $a$  and  $b$ . (4)

**Question 9**

**(6 marks)**

(a) Greg tells the truth 3 out of 5 times and Ian tells a lie 4 out of 7 times. If they are asked about the same fact independently, what is the probability that they either both tell the truth or both lie? (3)

(b) A shipment of 10 television sets contains 3 defective sets. In how many ways can the Boulton Hotel purchase 4 of these sets and receive at least 2 of the defective sets? (3)

**Question 10**

**(4 marks)**

The variables  $y$  and  $t$  are related by the equation  $y = ke^{-0.0231t}$  where  $k$  is a constant.

- (a) When  $t = 40$ ,  $y = 28$ , calculate the value of  $k$ . Express your answer to 3 significant figures. (1)

- (b) When  $t = 50$ , calculate the value of

- (i)  $y$  (1)

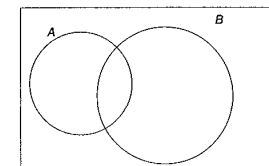
- (ii)  $\frac{dy}{dt}$  (2)

**Question 14**

**(9 marks)**

A Personal Identification Number (PIN) consists of 4 digits in order, each of which is one of the digits 0, 1, 2, ..., 9. Aimee has difficulty remembering her PIN. She tries to remember her PIN and writes down what she thinks it is. The probability that the first digit is correct is 0.8 and the probability that the second digit is correct is 0.86. The probability that the first two digits are both correct is 0.72.

By letting  $A$  = event that first digit is correct and letting  $B$  = event that second digit is correct complete the Venn diagram and answer the following questions



(1)

- (a) Find the probability that the

- (i) Second digit is correct given that the first digit is correct. (1)

- (ii) First digit is correct and the second digit is incorrect. (1)

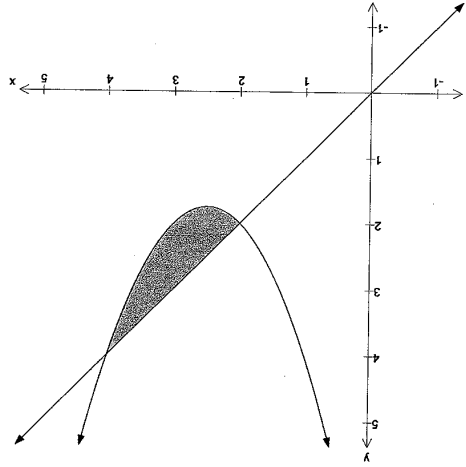
- (iii) First digit is incorrect and the second digit is correct. (1)

- (iv) Second digit is incorrect given that the first digit is incorrect. (2)

- (b) Assuming the probability that all 4 digits are correct is 0.7. On 12 separate occasions Aimee writes down independently what she thinks is her PIN. Find the probability that the number of occasions on which all four digits are correct is less than 10. (3)

Question 13

The line  $y = x$  intersects the curve  $y = x^3 - 5x + 8$  at  $A(2, 2)$  and  $B(4, 4)$ . The diagram shows the shaded region bounded by the line and the curve. Find the area of the shaded region. Show full working.



(4 marks)

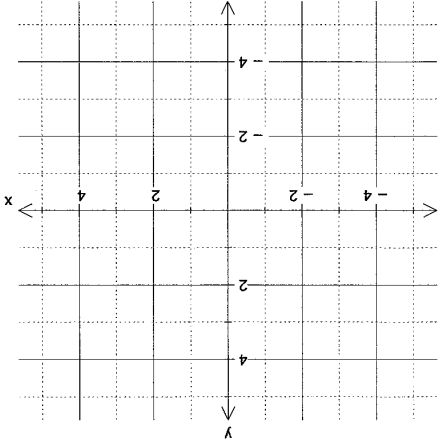
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Question 11

(9 marks)

- (a) Sketch a continuous curve for which
- $f(0) = 1$
  - $f'(x) < 0$  and  $f''(x) > 0$  for  $0 < x < 2$
  - $f'(2) = 0$  and  $f(2) = -2$
  - $f'(x) < 0$  and  $f''(x) < 0$  for  $x > 2$

(3)



- (b) Determine the domain and range of  $f(g(x))$  given that  $f(x) = \frac{12}{x+1}$  and  $g(x) = \sqrt{x+1}$ .

(3)

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(c) Given that  $f(x) = 2x + 3$  and  $g(f(x)) = 4x^2 + 12x + 11$ , find  $g(x)$ . (3)

(b) A factory produces tins of tomatoes. The mass of tomatoes in each tin is normally distributed with a mean mass of 460 g and a standard deviation of 5g.

(i) Find the percentage of tins in which the mass of tomatoes is less than 454 g. (2)

It is required that only 2% of the cans are to have less than 454 g.

(ii) If the standard deviation remains at 5g, find the mean mass required. (3)

**Question 12**

**(8 marks)**

(a) In a binomial probability distribution, there are  $n$  trials and the probability of success for each trial is  $p$ . If the mean is 8 and the standard deviation is  $\sqrt{4.8}$ , find the values of  $n$  and  $p$ . (3)