

Marking Key

Yr 12 SPECIALIST

UNIT 3

Semester One Examination, 2018

Question/Answer booklet

Section Two:
Calculator-assumed
Section One:
Non-calculator

PERTH MODERN SCHOOL

INDEPENDENT PUBLIC SCHOOL

Excellence, Integrity, Leadership, Responsibility

No other items may be taken into the examination room. It is **your responsibility** to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Formula sheet (relating from Section One)

This Question/Answer booklet

To be provided by the supervisor

Materials required/recommended for this section

Working time:
Reading time before commencing work: ten minutes
Working time: one hundred minutes

To be provided by the supervisor

Formula sheet (relating from Section One)

This Question/Answer booklet

To be provided by the candidate

No other items may be taken into the examination room. It is **your responsibility** to ensure that

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	21	21	100	95	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Additional working space

Question number: _____

Question 10

(9 marks)

Consider the following system of linear equations where p & m are constants.

$$x + 2y - 3z = 3$$

$$2x + 7y - 4z = p$$

$$-2x + 5y + mz = 7$$

Determine the values of p & m :

(a) for which there is a unique solution

(4 marks)

Solution
$\begin{bmatrix} 1 & 2 & -3 & 3 \\ 2 & 7 & -4 & p \\ -2 & 5 & m & 7 \end{bmatrix}$
$\begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & -3 & -2 & 6 - p \\ 0 & 9 & -6 + m & 13 \end{bmatrix}$
$\begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & -3 & -2 & 6 - p \\ 0 & 0 & -12 + m & 31 - 3p \end{bmatrix}$
Unique $m \neq 12$ $p \in R$
Specific behaviours
<ul style="list-style-type: none"> ✓ eliminates one variable from two equations ✓ eliminates two variables from one equation ✓ identifies all allowed values for m ✓ states that all real values of p allowed

(b) for which there are infinite solutions.

(3 marks)

Solution
$m = 12$ $p = \frac{31}{3}$
Specific behaviours
✓ shows that a line of zeros required or gives reasoning

See next page

Additional working space

Question number: _____

End of questions

Given that $-\pi < \operatorname{Arg}(z) \leq \pi$, determine the minimum value of $\operatorname{Arg}(z)$ in the region in (a). (Give to two decimal places)	
<ul style="list-style-type: none"> ✓ states value for m ✓ states value for p 	$m = 12$ $p \neq \frac{31}{3}$
(c)	for which there are no solutions. (2 marks)

Question 11 Consider the plane $r - 13 = 15$ as shown below.	
Determine the distance of point $P(3, -4, 11)$ from the plane to two decimal places.	
$\sqrt{3^2 + (-4)^2 + 11^2} = \sqrt{155} \approx 12.43$	$\sqrt{3^2 + (-4)^2 + 11^2} = \sqrt{155} \approx 12.43$
(4 marks)	

Solution

Solution
Specific behaviours $\operatorname{Arg}(z) = -\frac{\pi}{4} - \tan^{-1} \frac{3}{4} - \sin^{-1} \frac{5}{13} = -2.8578$ <ul style="list-style-type: none"> ✓ uses tangent at top of circle through origin ✓ determines both $\alpha \& \beta$ angles in diagram above ✓ determines minimum argument in allowed interval (Principal)
See next page

$$r = \begin{pmatrix} 3 \\ -4 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -13 \\ 7 \end{pmatrix} = \begin{pmatrix} 3+2\lambda \\ -4-13\lambda \\ 11+7\lambda \end{pmatrix}$$

Edit Action Interactive

0.5 1 $\frac{d}{dx}$ $\int dx$ Simp $\int dx$

```
dotP([3+2·λ], [2])
      [-4-13·λ], [-13]
      [11+7·λ], [7]

    7·(7·λ+11)+13·(13·λ+4)+2·(2·λ+3)
solve(7·(7·λ+11)+13·(13·λ+4)+2·(2·λ+3)=15, λ)
{λ=-0.5405405405}

norm([3+2·λ]
      [-4-13·λ])
      | λ=-0.5405405405
      8.057227744
```

Alg Decimal Cplx Rad

Specific behaviours

- ✓ determines vector equation of line through P
- ✓ subs vector eqn of line into plane and uses dot product equaling 15
- ✓ solves for parameter
- ✓ determines distance (no need to round to 2 dp)

OR

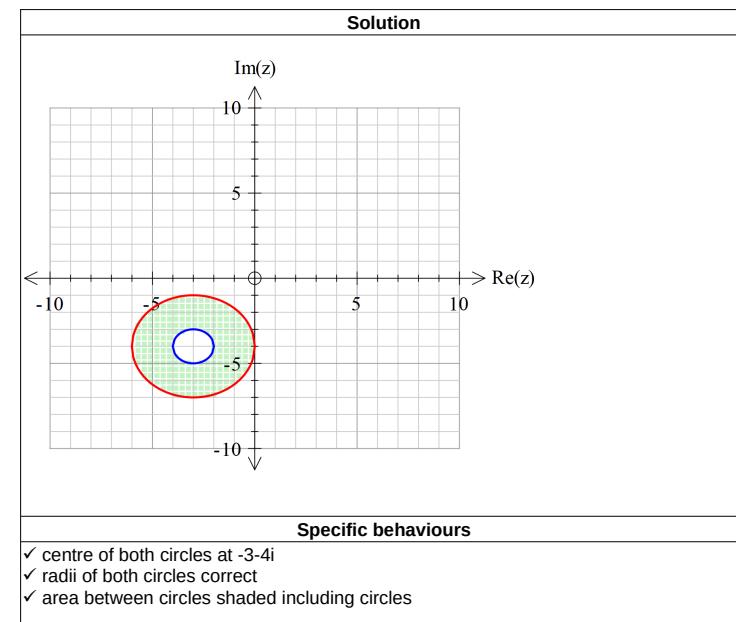
- ✓ determines any point on plane B
- ✓ determines vector PB
- ✓ dots this vector with unit normal
- ✓ determines distance

See next page

Consider the region defined by $1 \leq |z + 3 + 4i| \leq 3$ in the complex plane.

- (a) Sketch the region on the axes below.

(3 marks)



See next page

Specific behaviours

- ✓ determines vector AP
- ✓ uses unit vector parallel to line
- ✓ uses cross product of the above vectors
- ✓ determines magnitude of cross product (no need to round to one dp)

See next page

- (b) Express $|OG|^2, |DF|^2, |BH|^2 \text{ & } |CE|^2$ in terms of $b, d \text{ & } e$ (4 marks)

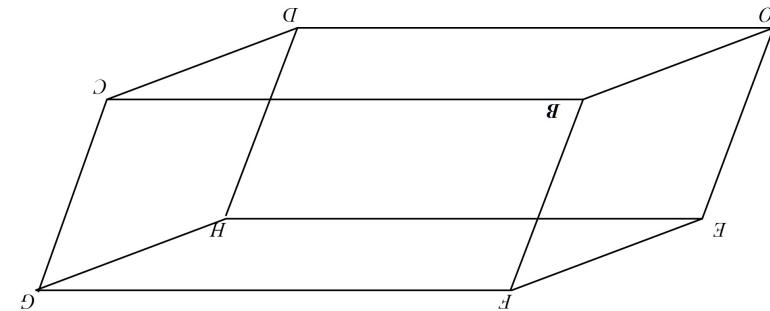
Solution
$OG \cdot OG = (d + b + e) \cdot (d + b + e) = d.d + d.e + b.b + b.e + e.e$ $= d ^2 + 2d.e + b ^2 + e ^2$ $\rightarrow \cancel{b.e} = 0 = b.d$ $DF \cdot DF = (-d + b + e) \cdot (-d + b + e) = d ^2 - d.e + b ^2 - e.d + e ^2$ $= d ^2 - 2d.e + b ^2 + e ^2$ $\rightarrow \cancel{b.e} = b ^2 + d ^2 + 2d.e + e ^2$ $CE \cdot CE = (-b - d + e) \cdot (-b - d + e) = d ^2 - 2d.e + b ^2 + e ^2$
Specific behaviours
<ul style="list-style-type: none"> ✓ dots vector with itself to obtain magnitude squared ✓ recognizes that $b.e = 0 = b.d$ ✓ obtains two correct expressions ✓ obtains all four correct expressions

See next page

The three vertices of a triangle have position vectors \vec{a} , \vec{b} & \vec{c} . Given that $\vec{d} = \vec{OD}$ and $\vec{e} = \vec{OE}$ with \vec{b} perpendicular to plane containing vectors \vec{d} & \vec{e} .

$$\text{Show that the area of the triangle is given by } \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$$

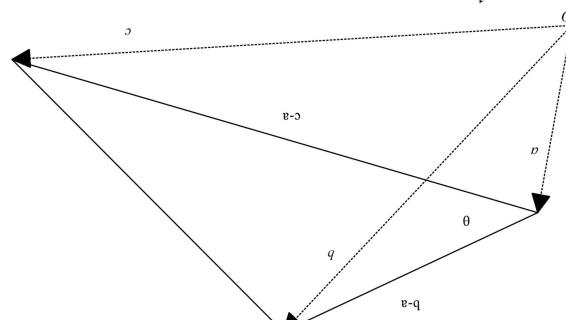
Consider $OBCDEFGH$ drawn below, where each face is a parallelogram. Let $\vec{b} = \vec{OB}$, $\vec{c} = \vec{OC}$ and $\vec{d} = \vec{OE}$ with \vec{b} perpendicular to plane containing vectors \vec{d} & \vec{e} .



(a) Express each of the vectors OG , DF , BH & CE in terms of \vec{b} , \vec{d} & \vec{e} (4 marks)

Solution	
Specific behaviours	
$OG = \vec{d} + \vec{b} + \vec{e}$	(one mark for each vector)
$DF = -\vec{d} + \vec{b} + \vec{e}$	
$BH = -\vec{b} + \vec{d} + \vec{e}$	
$CE = -\vec{b} - \vec{d} + \vec{e}$	

$\text{Area} = \frac{1}{2} \vec{b} - \vec{a} \vec{c} - \vec{a} \sin \theta$ $= \frac{1}{2} (\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})$ $= \frac{1}{2} (\vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{a} \times \vec{c} + \vec{a} \times \vec{a})$ $= \frac{1}{2} (\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}) \quad \text{as } \vec{a} \times \vec{a} = -\vec{b} \times \vec{a} \quad \text{and } \vec{a} \times \vec{a} = 0$	<ul style="list-style-type: none"> ✓ uses area formula for triangle using adjacent sides and included angle ✓ uses difference vectors and magnitude of cross product to determine this area ✓ uses that a vector crossed itself is zero and changing order negates sign ✓ summarises to show required result
Specific behaviours	



Solution

Question 14

(6 marks)

Consider the polynomial $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d & e are real constants. Determine the values of a, b, c, d & e given the following information for $P(x)$

$$(x + 1 - \sqrt{7}i) \text{ is a factor of } P(x)$$

When $P(x)$ is divided by $(x - 1)$ there is a remainder of 165

$$P(0) = 32 \text{ and } P(-2) = 0$$

Solution

Conjugate of $-1 + \sqrt{7}i$ is also a root

$$(x + 1 - \sqrt{7}i)(x + 1 + \sqrt{7}i) = (x^2 + 2x + 8)$$

$$P(x) = (ax + b)(x + 2)(x^2 + 2x + 8)$$

$$P(0) = 32$$

$$32 = b16$$

$$b = 2$$

$$165 = (a + 2)(3)(1 + 2 + 8)$$

$$a = 3$$

$$P(x) = (3x + 2)(x + 2)(x^2 + 2x + 8)$$

$$P(x) = 3x^4 + 14x^3 + 44x^2 + 72x + 32$$

Specific behaviours

- ✓ uses conjugate to determine new factor
- ✓ uses factor of $x-2$
- ✓ subs $x=0$ $y=32$
- ✓ subs $x=1$ $y=165$
- ✓ solves for all linear factors
- ✓ expands factors to determine coefficients

- (b) Given that there is only one point where $f(x) = f^{-1}(x)$ determine the x value in terms of a, b & c (3 marks)

i

Solution

$$f(x) = x = ax^2 + bx + c$$

$$x = ax^2 + bx + c$$

$$0 = ax^2 + bx + c - x$$

$$0 = ax^2 + (b - 1)x + c$$

$$x = \frac{-(b - 1) \pm \sqrt{(b - 1)^2 - 4ac}}{2a}$$

$$x = \frac{-(b - 1) - \sqrt{(b - 1)^2 - 4ac}}{2a} \text{ as } x \leq \frac{-b}{2a}$$

Specific behaviours

- ✓ equates $f(x)$ to x
- ✓ solves using quadratic formula or completing the square to solve for x
- ✓ uses negative sign to give one answer for x

- (c) Given that $g \circ h(x) = ax^2 + bx + c$ and $h(x) = 3x - 1$,

determine the function $g(x)$ in terms of a, b & c (3 marks)

Solution

$$g(y) = ax^2 + bx + c \text{ where } y = 3x - 1$$

$$x = \frac{y+1}{3}$$

$$g(y) = a\left(\frac{y+1}{3}\right)^2 + b\left(\frac{y+1}{3}\right) + c$$

$$g(x) = a\left(\frac{x+1}{3}\right)^2 + b\left(\frac{x+1}{3}\right) + c$$

Specific behaviours

- ✓ expresses x in terms of y for $h(x)$
- ✓ subs into composite to give expression in terms of y only
- ✓ states $g(x)$ in terms of x only

Question 15 (9 marks)

At noon a rocket is launched from position $(3, -4, 11)$ km with a velocity of 7 km/h . Two hours later a second rocket is launched from position $(-10, 11, 5)$ km with a velocity of 5 km/h . Assume that both rockets move with constant velocity at all times and that the rockets do not collide.

(a) Determine the distance between the rockets at 2:30pm that day to one decimal place (3 marks)

Specific behaviours

Uses vector equations for paths of both rockets
Uses appropriate time values
Determines distance apart (no need to round to 1 dp)

Solution

(a) Determine the distance between the rockets at 2:30pm that day to one decimal place (3 marks)

Question 19 (9 marks)

Given that the inverse function does exist obtain an expression for $f^{-1}(x)$ in terms of $a, b \neq c$

Solution

$$x = ay^2 + by + c$$

$$y = \frac{-b \pm \sqrt{b^2 - 4a(c - x)}}{2a}$$

$$y \leq 0$$

$$f^{-1}(x) = \frac{-b - \sqrt{b^2 - 4a(c - x)}}{2a}$$

swaps x and y
uses quadratic formula or completing the square to solve for inverse
uses negative as $\frac{-b}{2a}$

Specific behaviours

Swaps x and y
uses quadratic formula or completing the square to solve for inverse
uses negative as $\frac{-b}{2a}$

Solution

$$0 = ay^2 + by + c$$

$$y = \frac{-b \pm \sqrt{b^2 - 4a(c - x)}}{2a}$$

$$y \leq 0$$

$$f^{-1}(x) = \frac{-b - \sqrt{b^2 - 4a(c - x)}}{2a}$$

- (b) Determine the times that the distance between the rockets is less than 50 km.
(4 marks)

Solution

Specific behaviours

- ✓ determines vector equation for rocket in terms of t
- ✓ determines vector equation of second rocket for $t-2$
- ✓ determines expression for magnitude of separation in terms of t and equates to 50k
- ✓ identifies $0 \leq t \leq 4.575$ approx (no need to round)

See next page

- (c) Determine the cartesian equation of the path of a new particle with the following position

$$\text{vector } \vec{r} = \begin{pmatrix} \sin t - 1 \\ 3\cos(2t) + 5 \end{pmatrix} m$$

(3 marks)

Solution

$x = \sin t - 1, \sin t = x + 1$
 $y = 3\cos(2t) + 5 = 3(1 - 2\sin^2 t) + 5$
 $y = 3(1 - 2(x+1)^2) + 5$
 $y = 8 - 6(x+1)^2$

Specific behaviours

- ✓ obtains $\sin t$ in terms of x
- ✓ uses double angle formula to rearrange y expression
- ✓ obtains a cartesian equation (unimplified)

See next page

<p>MATHEMATICS SPECIALIST UNIT 3</p> <p>CALCULATOR-ASSUMED</p> <p>13 MATHEMATICS SPECIALIST UNIT 3</p>	<p>20</p> <p>CALCULATOR-ASSUMED</p>
<p>(c) Determine the distance of closest approach and the time that this occurs. (2 marks)</p>	<p>(b) Determine the first two times that the particle crosses the y axis. (4 marks)</p>

Solution

$$f = \begin{pmatrix} -2\sin(2t) + 1 \\ -2\cos t - 6 \end{pmatrix}$$

$$\dot{f} = \begin{pmatrix} -4\cos(2t) \\ 2\sin t \end{pmatrix}$$

$$\ddot{f} = \begin{pmatrix} 8\sin(2t) + 4 \\ 2\cos t - 6 \end{pmatrix}$$

$$\text{norm} \left(\begin{pmatrix} -21 \cdot (t-2) + 6 \cdot t - 15 \\ 3 \cdot (t-2) - 2 \cdot t + 13 \end{pmatrix} \right)$$

$$\text{Min} \left(\begin{pmatrix} (t+7)^2 + (15-t-27)^2 + (2+t+16)^2 \\ 0.5 \cdot t^2 - 1 \end{pmatrix} \right)$$

$$\text{MinValue} = 22.13594362, t=2$$

Closest distance of 22.14 km at $t=2$ hours at time 2 pm

Specific behaviours

- ✓ determines minimum value of separation magnitude and states distance
- ✓ gives time that is within one minute of correct answer

Edit Action interactive

Solution

$$e = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} \cos(2t) + t \\ \sin t - 6t \end{pmatrix}$$

$$r = \begin{pmatrix} \cos(2t) + t - 1 \\ \sin t - 6t + 11 \end{pmatrix}$$

$$\{t=0, t=0.55720908, t=1.849076836\}$$

Specific behaviours

- ✓ anti-differentiates to determine position vector
- ✓ determines vector constant
- ✓ evaluates component to zero
- ✓ solves for first two positive values only (approx.)

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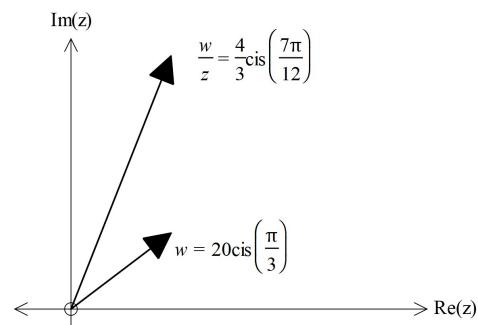
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Question 16

(9 marks)

Consider the complex numbers drawn in the complex plane below.



- (a) Determine the exact value of Z in the form of $a + bi$

(3 marks)

Solution

$$\begin{aligned} z &= \frac{20\text{cis}\frac{\pi}{3}}{\frac{4}{3}\text{cis}\frac{7\pi}{12}} = 15\text{cis}\left(\frac{4\pi}{12} - \frac{7\pi}{12}\right) = 15\text{cis}\left(-\frac{\pi}{4}\right) \\ &= 15\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) = \frac{15}{2}(\sqrt{2} - i\sqrt{2}) \end{aligned}$$

Specific behaviours

- ✓ determines modulus for z
- ✓ determines Arg for z
- ✓ express z in cartesian form (No need to rationalize denominator)

See next page

Question 18

(11 marks)

A particle moves with acceleration $\begin{pmatrix} -4\cos(2t) \\ \sin t \end{pmatrix} \text{ m/s}^2$ at time t seconds. The initial velocity is $\begin{pmatrix} 1 \\ -7 \end{pmatrix} \text{ m/s}$ and initial displacement of $\begin{pmatrix} 0 \\ 11 \end{pmatrix} \text{ m}$.

- (a) Determine the time(s), $0 \leq t \leq \pi$, that the particle is travelling parallel to the y axis. (4 marks)

Solution
$\ddot{r} = \begin{pmatrix} -4\cos(2t) \\ \sin t \end{pmatrix}$ $\dot{r} = \begin{pmatrix} -2\sin(2t) \\ -\cos t \end{pmatrix} + \varsigma$ $\begin{pmatrix} 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} + \varsigma$ $\varsigma = \begin{pmatrix} 1 \\ -6 \end{pmatrix}$ $\dot{r} = \begin{pmatrix} -2\sin(2t) + 1 \\ -\cos t - 6 \end{pmatrix}$
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<p>(c) Determine the possible values of A, to 2 decimal places, if the line is a tangent to the sphere.</p> <p>Consider the equation $T \operatorname{cis} \theta = \frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i + r \operatorname{cis} \theta$ where $r > 0$ and $-\pi < \theta \leq \pi$</p>
<p>(b) Represent the above equation as a triangle in the complex plane below. (3 marks)</p> <p>The diagram shows a triangle in the complex plane with vertices labeled as follows:</p> <ul style="list-style-type: none"> Top vertex: $7 \operatorname{cis} \left(\frac{5\pi}{12}\right)$ Bottom-left vertex: $7 \operatorname{cis} \left(\frac{12\pi}{12}\right)$ Bottom-right vertex: $7 \operatorname{cis} \left(\frac{11\pi}{12}\right)$ <p>The horizontal axis is labeled $\operatorname{Re}(z)$ and the vertical axis is labeled $\operatorname{Im}(z)$.</p>

<p>CALCULATOR-ASSUMED</p> <p>15</p> <p>MATHEMATICS SPECIALIST UNIT 3</p>
<p>(b) Represent the above equation as a triangle in the complex plane below. (3 marks)</p>
<p>Consider the equation $T \operatorname{cis} \theta = \frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i + r \operatorname{cis} \theta$ where $r > 0$ and $-\pi < \theta \leq \pi$</p>

<p>Solution</p> <p>$\begin{aligned} -4 - A &= -3 \\ A + 3A &= 4 \\ -1 - A^2 &+ (A + 3A - 4)^2 + (4 + 8A)^2 &= 25 \\ 1 + 2A + A^2 + A^2 + 9A^2 - 8A - 24A + 16 + 64A^2 + 64A^2 - 25 &= 0 \\ 74A^2 + (42 + 6A)A^2 + A^2 - 8A + 8 &= 0 \\ A = (42 + 6A)^2 - 4(74)(A^2 - 8A + 8) &= 0 \\ \text{solve } ((42+6\cdot a)^2-4\cdot 74\cdot (a^2-8\cdot a+8)=0, a) &\\ a=0, 2144705358, a=10, 83168331 \end{aligned}$</p>
<p>Specific behaviours</p> <p>✓ subs r from line into vector eqn of sphere</p> <p>✓ expands an equation in terms of A and parameter</p> <p>✓ expands and adds like terms to give a quadratic equation where the quadratic formula may be used.</p> <p>✓ obtains two values for A (no need to give to 2 dp)</p>

Cont-

- (c) Hence or otherwise solve for r & θ to one decimal place. (3 marks)

Solution
<p>Length: 7.00 ∠: 106.88 Length: 5.00 ∠: 30.00 Length: 3.66</p>
$r = 3.66$ $\theta = 360 - 106.88 - 135 = 118.12 \text{ deg} = 2.06 \text{ radians}$
Specific behaviours
<ul style="list-style-type: none"> ✓ solves for r using geometry or cosine rule ✓ solves another angle in triangle ✓ determines Principal Argument in radians (No need to round to 1 dp)

See next page

Question 17

(9 marks)

Consider a sphere with centre $(-3, 4, 7)$ and radius of 5 units.

- (a) Write down the vector equation for this sphere (2 marks)

Solution
$\left \begin{pmatrix} x + 3 \\ y - 4 \\ z - 7 \end{pmatrix} \right = 5$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses difference of r from centre ✓ equates difference to radius

Consider a line parallel to vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$ and containing the point where a is a constant.

- (b) Write down the vector equation of the line in terms of a . (2 marks)

Solution
$\begin{pmatrix} -4 \\ a \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses parameter ✓ states correct vector equation

See next page