Papers written by
Australian Maths
Software

SEMESTER ONE

REVISION 3

MATHEMATICS METHODS

UNIT 3

2016

SOLUTIONS

SECTION ONE

1. (5 marks)

(a) $f'(x) = x^2 - 3$ and $f(1) = 2$

$f(x) = \int (x^2 - 3)\,dx$

$f(x) = \dfrac{x^3}{3} - 3x + c$

$f(1) = 2 \Rightarrow \ 2 = \dfrac{1^3}{3} - 3(1) + c$

$c = \dfrac{14}{3}$

$\therefore f(x) = \dfrac{x^3}{3} - 3x + \dfrac{14}{3}$

$\therefore f(1) = \dfrac{1}{3} - 3 + \dfrac{14}{3} = 2$

(3)

(b) $f''(x) = 2x$

$f''(3) = 6$

(1)

(c) $f''(x) = 0$ at $x = 0$

$f(0) = \dfrac{14}{3}$

(1)

2. (9 marks)

(a) (i) $x = (t-2)(t-3) = t^2 - 5t + 6$

$v = \frac{dx}{dt} = 2t - 5$

When $v = 0,\ t = 2.5\ m\,s$

(3)

(ii) $a = \frac{dv}{dt} = 2\,m\,s^{-2}$

The acceleration is always constant as the velocity is linear and the acceleration is the gradient function of the velocity. The gradient of the $x - t$ graph is always increasing so the acceleration is always positive. (3)

(b) Where $v = 0$ there is a turning point in the displacement graph. ✓
As the acceleration graph is positive ($a = 2$) at that point, ✓
the turning point in the displacement graph is a minimum. ✓ (3)

3. (8 marks)

(a) (i) $f(x) = e^x(\sin(\pi x))$

$f'(x) = e^x(\sin(\pi x)) + \pi(\cos(\pi x))e^x$ ✓ ✓

$f'(x) = e^x(\sin(\pi x) + \pi(\cos(\pi x)))$ (2)

(ii) $g(x) = \frac{x^2}{\tan(x)}$

$g'(x) = \frac{2x(\tan(x)) - x^2\left(\frac{1}{\cos^2(x)}\right)}{\tan^2(x)}$ ✓✓ -1/error

$g'(x) = \frac{x}{\tan^2(x)} \times \left(2\tan(x) - \frac{x}{\cos^2(x)}\right)$

(2)

(b) ((i) $f'(1) = e^1(\sin(\pi) + \pi(\cos(\pi)))$

$f'(1) = -e\pi$ (2)

(ii) $g'\left(\frac{\pi}{4}\right) = \frac{2 \times \frac{\pi}{4} \times 1 - \left(\frac{\pi}{4}\right)^2 \times (2)}{1}$ (2)

$g'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{\pi^2}{8}$

4. (6 marks)

(a) $\int(3x^2+4x^3-2)dx = x^3+x^4-2x+c$ ✓ (1)

(b) (i) $\frac{dy}{dx} = 1\times sin(x)+x(cos(x)) = sin(x)+x(cos(x))$ ✓✓ (1)

(ii) $\int x\,cos(x)dx = x(sin(x)) - \int sin(x)dx$ ✓ ✓

$\int x\,cos(x)dx = x(sin(x))+cos(x)+c$ ✓ (3)

5. (7 marks)

(a) $\int_0^1 \sqrt{x}\,dx = \frac{2}{3}\left[x^{3/2}\right]_0^1 = \frac{2}{3}(1-0) = \frac{2}{3}$

✓ ✓ (2)

(b) $\int_1^2\left(\frac{1}{x^2}+2x^3-4\right)dx = \int_1^2\left(x^{-2}+2x^3-4\right)dx$

$= \left[-\frac{1}{x}+\frac{x^4}{2}-4x\right]_1^2$

$= \left(-\frac{1}{2}+8-8\right)-\left(-1+\frac{1}{2}-4\right)$

$= 4$

(3)

(c) $\int_{\pi/4}^{3\pi/4} 2\cos 2z\,dz = \frac{2}{2}\times\left[\sin 2z\right]_{\pi/4}^{3\pi/4} = 1\left(\sin\left(\frac{3\pi}{2}\right)-\sin\left(\frac{\pi}{2}\right)\right) = -2$ (3)

✓ ✓

6. (7 marks)

(a) (i) $y = f\left(h(x)\right) = f\left(sin(x)\right) = sin^2(x)$ ✓ (1)

(ii) $\dfrac{dy}{dx} = 2\left(sin(x)cos(x)\right)$ ✓

At $x = \dfrac{\pi}{4}$ $\dfrac{dy}{dx} = 2\left(sin\left(\dfrac{\pi}{4}\right)cos\left(\dfrac{\pi}{4}\right)\right) = 2 \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = 1$ ✓ (2)

(b) (i) $y = g\left(f(x)\right) = g\left(x^2\right) = e^{x^2}$ (2)

✓ ✓

(ii) $\dfrac{dy}{dx} = 2x \times e^{x^2}$ ✓

At $x = \dfrac{1}{\sqrt{2}}$, $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{2}} \times e^{1/2} = \sqrt{2e}$ ✓ (2)

7. (7 marks)

(a) $\int_1^x cos(3t)dt = \left[\dfrac{sin(3t)}{3}\right]_1^x$ ✓

$= \dfrac{sin(3x) - sin(3)}{3}$ ✓ (2)

(b) $\dfrac{d}{dx}\left(\int_1^x cos(3t)dt\right) = cos(3x)$ ✓✓ (2)

(c) $\dfrac{d}{dx}\left(\int_1^x cos(3t)dt\right) = -1$

$cos(3x) = -1$ for $0 \le x \le \pi$

$3x = \pi,\ 3\pi$

$x = \dfrac{\pi}{3},\ \pi$

(3)

END OF SECTION ONE

SECTION TWO

8. (6 marks)

(a) $x = t^3 - 12t$

$0 = t\left(t^2 - 12\right)$

$t = 0 \;\; or \;\; t = \pm\sqrt{12}$

$But \; t > 0, \;\; t = 2\sqrt{3}$ ✓ ✓

(2)

(b) $v = \dfrac{dv}{dt} = 3t^2 - 12$ ✓

Changed direction when

$v = 0$ i.e. at $t = \pm 2$

but $t > 0, \; t = 2$

(2)

(c) $a = \dfrac{d^2v}{dt^2} = 6t$

$If \;\; 12 = 6t \Rightarrow t = 2$ ✓

$x = 8 - 24 \Rightarrow x = -16m$ ✓ (2)

9. (9 marks)

(a) (i) $a + 0.1 + a + 0.5 = 1$

$2a = 0.4$

$a = 0.2$ (1)

(ii) $P\left(x \le 30\right) = 0.5$ ✓ (1)

(iii) $E(X) = \Sigma xp(x)$

$E(X) = 10 \times 0.2 + 20 \times 0.1 + 30 \times 0.2 + 40 \times 0.5$

$E(X) = 30$ ✓

$\text{Var}(X) = E(X^2) - \mu^2$

$\text{Var}(X) = 10^2 \times 0.2 + 20^2 \times 0.1 + 30^2 \times 0.2 + 40^2 \times 0.5 - 30^2$ ✓

$\text{Var}(X) = 140$ ✓

$\text{Sd}(X) = \sqrt{140} \approx 11.83$ ✓ (4)

(b) (i) Variance $= 3.5^2 = 12.25$ ✓ (1)

(ii) $E(X) = 5$ ✓

$\text{Sd}(X) = 1.75$ ✓ (2)

10. (4 marks)

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\delta A \approx \frac{dA}{dr} \times \delta r$

$\delta A \approx 2\pi r \times \delta r$

$At\, \delta r = 0.5m, \quad r = 100m$

$\delta A \approx 2\pi \times 100 \times 0.5 = 100\pi$

$\delta A \approx 314.2\, m^2$

11. (8 marks)

(a) $A = xy \Rightarrow A = x\sqrt{100 - x^2}$ (2)

✓ ✓

(b) Maximum area when $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = -\frac{2x^2 - 100}{\left(-x^2 + 100\right)^{0.5}}$

$At\ \frac{dA}{dx} = 0,$

$2x^2 = 100$

$x = \pm\sqrt{50}$ but $x > 0$

$x = \sqrt{50}$

max or min?

x	0	$\sqrt{50}$	10
$\frac{dA}{dx}$	+	0	-

Therefore max at $x = \sqrt{50}$

$At\ \ x = \sqrt{50}$

$y^2 = 100 - 50$

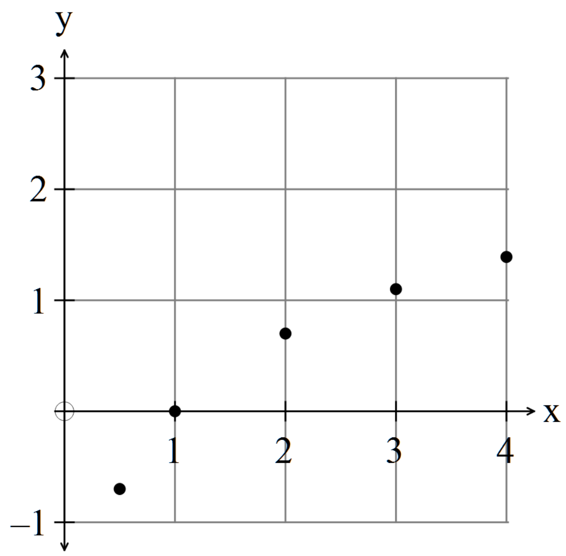
$y = \sqrt{50}\ \ as\ y > 0$

Therefore the maximum area occurs when $x = y$. (6)

12. (7 marks)

(a) $\left(\frac{1}{2}, -0.69\right), (1, 0), (2, 0.69), (3, 1.10), (4, 1.39)$ ✓✓✓✓-1/error (4)

(b)



✓✓

(2)

(c) $a = e \ (=2.7)$ ✓ (1)

13. (5 marks)

(a) $\int_2^5 \sqrt{1+2x^3}\, dx = 28.61 \ (2\,dp)$ ✓✓ (2)

(b) (i) $\int \frac{\cos(3x)}{2} dx = \frac{\sin(3x)}{6} + c$ ✓

$$\frac{5}{6} = \frac{\sin(3\pi)}{6} + c \ \Rightarrow \ c = \frac{5}{6}$$

$$\therefore f(x) = \frac{\sin(3x)}{6} + \frac{5}{6}$$

(2)

(iii) $f\left(\frac{\pi}{2}\right) = \frac{2}{3}$ ✓ (1)

14. (14 marks

(a) (i) $(-1.31,4)$ and $(3.14,4)$ ✓ ✓ (2)

(ii) Area = $\int_{-1.31}^{3.14}(4-x(x-1)(x-3)(x+1))dx$ ✓✓ (2)

(iii) Area = 25.53 units^2 ✓✓ (2)

(b) (i) From below

Area $\approx 1\times 0.5+1.875\times 0.5+2\times 0.5$

$=2.4375$

(2)

(ii) From above

Area $\approx 1.875\times 0.5+2\times 0.5+2.125\times 0.5$

$=3$

(2)

(iii) The area calculated from below is an underestimate.
The area calculated from above is overestimate.
The average combines both the underestimate and the overestimate and should be more accurate. ✓
Average is 2.71875
Area ≈ 2.72 units^2 ✓ (2)

(iv) $\int_0^{1.5}\left((x-1)^3+2\right)dx=2.765625$ ✓

Difference from estimate is $0.046875\approx 0.05\ (2dp)$ ✓ (2)

15. (9 marks)

(a) $v=\sqrt{1+t}$

$a=\frac{1}{2}(1+t)^{-1/2}=\frac{1}{2\sqrt{1+t}}$ ✓

As $t\ge 0, \sqrt{1+t}\ge 1$ so $a>0$ i.e. a is always positive. ✓ (2)

(b) $x=\int\sqrt{1+t}\ dt=\frac{2\sqrt{(1+t)^3}}{3}+c$ ✓

$At\ \ t=0,\ x=\frac{1}{3}$

$\frac{1}{3}=\frac{2\sqrt{(1)^3}}{3}+c\Rightarrow c=-\frac{1}{3}$

$\therefore x=\frac{2\sqrt{(1+t)^3}}{3}-\frac{1}{3}$

(2)

(c) At $v = 4\ ms^{-1}$, $4 = \sqrt{1+t} \Rightarrow 16 = 1 + t \Rightarrow t = 15$

$At\ \ t = 15,$

$a = \dfrac{1}{2\sqrt{1+15}}$

$a = \dfrac{1}{8}\ ms^{-2}$

(2)

(d) $v > 0\ \ \forall t > 0$ so toy does not change direction.

At $t = 0,\ x = \dfrac{1}{3}m$ At $t = 3,\ x = \dfrac{2\sqrt{(1+3)^3}}{3} - \dfrac{1}{3} = \dfrac{16}{3} - \dfrac{1}{3} = 5$

Therefore distance travelled is $4\dfrac{2}{3}m$ ✓ (3)

16. (8 marks)

(a)

1972	100 grams	
1987	50 grams	using the half-life
2002	25 grams	

In 2002 there will be 25 grams of DDT left so it will take 30 years. ✓ (1)

(b) $A = 100(a)^t$

At $t = 15$, $50 = 100(a)^{15}$ ✓

$a = 0.9548416039$ ✓ (2)

(c) $t = ?$ $25 = 100(0.9548416039)^t \Rightarrow t = 30$ years ✓ (1)

(d) $t = ?$ $1 = 100(0.9548416039)^t \Rightarrow t = 99.6578$ years ✓ ✓ (2)

$t \approx 100$ years ✓

(e) $2016 - 1972 = 44$ years ✓

$A = ?$ $A = 100(0.9548416039)^{44} \Rightarrow A = 13.09$ grams ✓ (2)

17. (8 marks)

(a)

x	0	1	2	3	4
$P(X=x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

✓✓

(2)

(b) $\frac{4}{16}\times\frac{4}{16}=\frac{1}{16}$ ✓✓ (2)

(c) (i) $n=10,\ P(X=0)=\frac{1}{16}$ ✓

$E(x)=np=10\times\frac{1}{16}=\frac{5}{8}$ ✓

i.e. expect one family to have no boys. ✓ (3)

(ii) You cannot have a fraction of a family, so no, you would never get exactly that number. ✓ (1)

18. (9 marks)

(a) 0.25 ✓✓ (2)

(b) $P(x=2)=0.263671875\approx0.26$ ✓✓ (2)

(c) $P(x=0)=0.2373046875\approx0.24$ ✓✓ (2)

(d) $P(x\geq2)=0.3671875\approx0.37$ ✓✓ (2)

(e) $E(x)=np=5\times0.25=1.25\approx1$ ✓ (1)

19. (8 marks)

(a)

x	1	2	3	4
$P(X=x)$	0.1	0.2	0.4	0.3

✓✓

(2)

(b) $P(X=2\text{ or }X>3)=0.2+0.3=0.5$ (2)

✓ ✓

(c) Yes, as the relative proportions could be estimated. ✓ (2)

(d) By inspection, about 3. ✓✓ (2)

20. (6 marks)

(a) $0.6 \times 20 = 12$

Peter will have to guess 8 questions. ✓ (1)

(b) 80% of $20 = 16$ Needs to get 16 or more correct to get 80% ✓

Peter knows 12, so needs to get at least 4 more correct out of the 8 he has to guess. ✓

$n = 8 \quad p = 0.2$

$P(x \geq 4) = 0.0562816$ ✓✓ (4)

END OF SECTION TWO