



PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2023

Question/Answer booklet

12 SPECIALIST MATHEMATICS

Section One: Calculator-free

Your Name _____

Your Teacher's Name _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Max	Question	Mark	Max
1		8	5		10
2		8	6		9
3		8			
4		7			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	50	34
Section Two: Calculator-assumed	12	12	100	97	66
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free**(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1**(8 marks)**

Consider the functions $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{2x+5}$.

a) Determine the natural domain and range of $g(x)$. (2 marks)

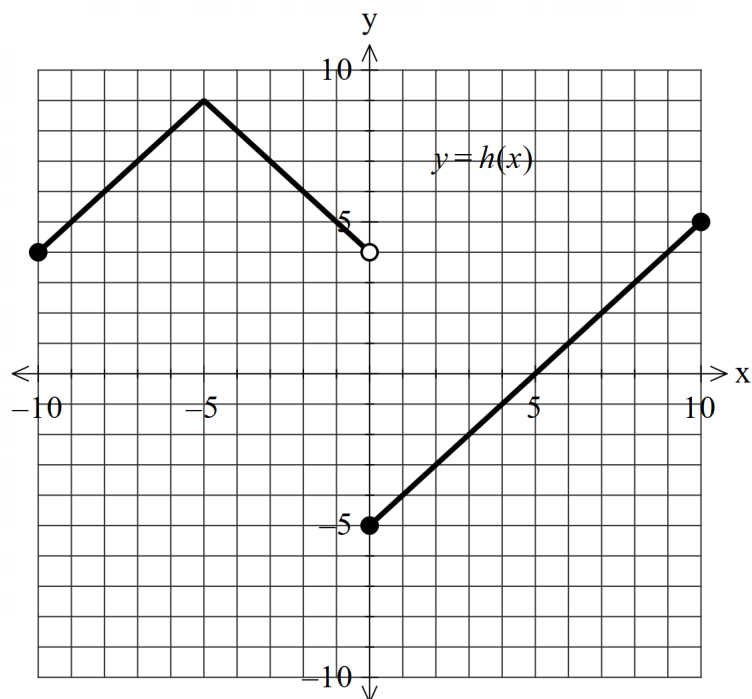
b) Does $f \circ g(x)$ exist over the natural domain of $g(x)$? Explain. (3 marks)

c) Determine $f \circ f(x)$ and its natural domain. (3 marks)

Question 2

(8 marks)

Consider the function $h(x)$ which is graphed below.

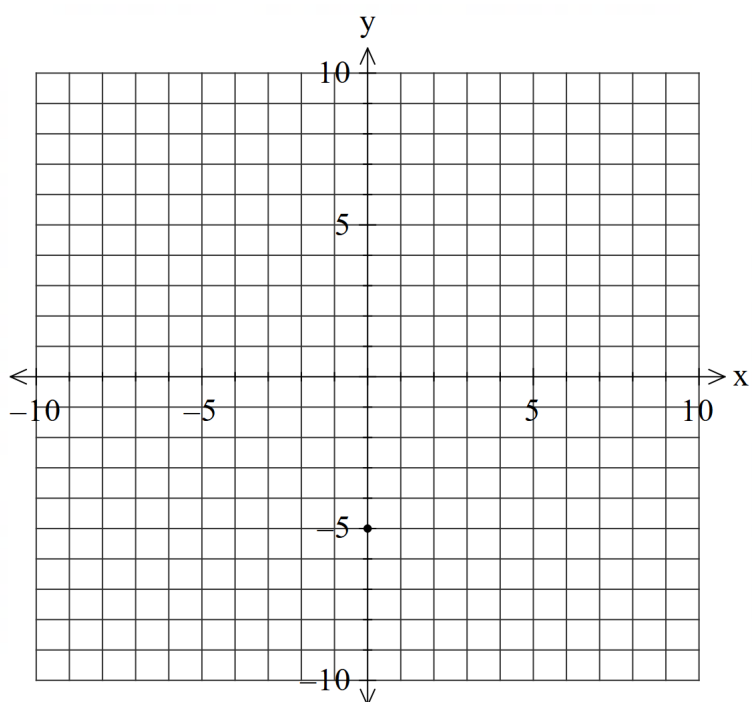


a) Solve for $|h(x)| = 5$.

(2 marks)

b) Sketch $y = h(-|x|)$ on the axes below.

(3 marks)

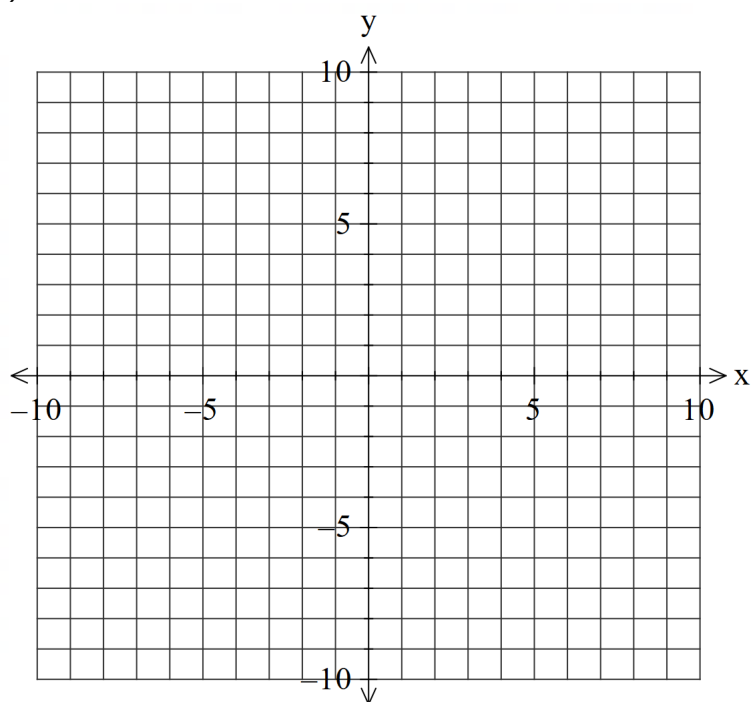


See next page

Q2 continued-

c) Sketch $y = \frac{1}{h(x)}$ on the axes below.

(3 marks)



Question 3

(8 marks)

Consider the following planes:

$$x + 2y - 3z = 10$$

$$2x - 3y + 5z = -26$$

$$3x - 5y + 2z = -36$$

a) Show that none of these planes are parallel.

(2 marks)

Q3 continued-

b) Solve the system of simultaneous equations.

(3 marks)

c) Consider the system of equations below with p & q being constants.

(3 marks)

$$x + 2y - 3z = p$$

$$2x - 3y + qz = -26$$

$$3x - 5y + 2z = -36$$

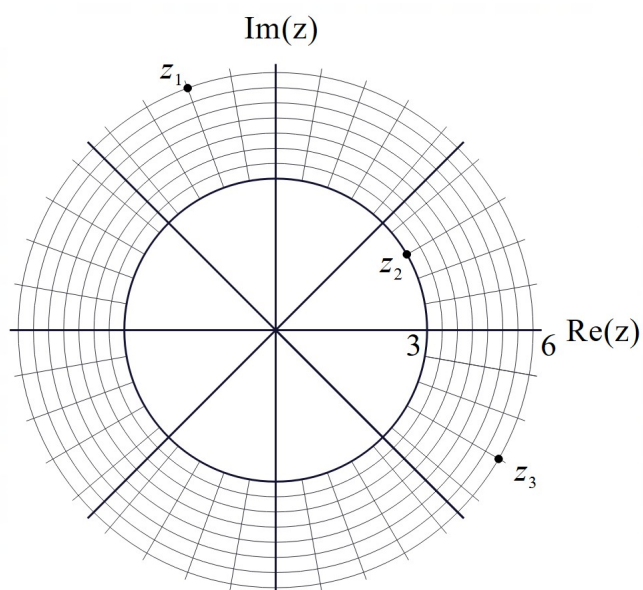
Determine all possible values of p & q such that there are:

- i) No solutions.
- ii) Infinite solutions.
- iii) A unique solution.

Question 4

(7 marks)

Consider the three complex numbers plotted below in the Argand diagram.



a) Determine the complex number z_2 in exact cartesian form. (2 marks)

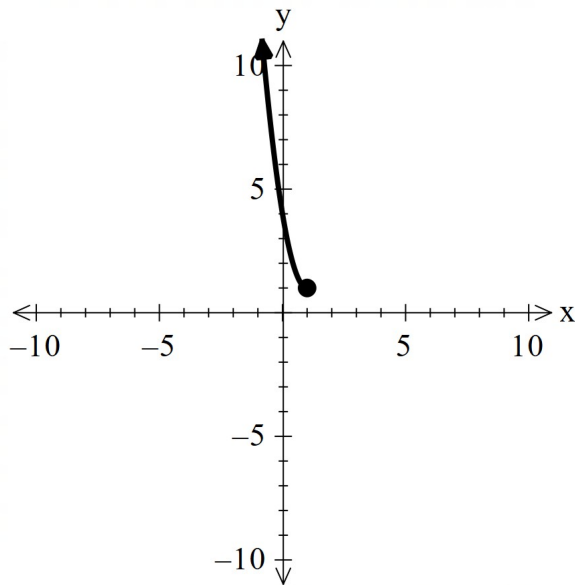
b) Plot the complex number $z_1 \times (z_3)^{-1}$ on the axes above. (2 marks)

c) State the modulus and argument of $\frac{(1+i)z_2}{\sqrt{3}\bar{z}_1z_3}$. (3 marks)

Question 5

(10 marks)

Consider the function $f(x) = 3x^2 - 6x + 4$, $x \leq 1$ which is graphed below.



- a) On the axes above, plot $f^{-1}(x)$. (2 marks)
- b) Determine the rule for $f^{-1}(x)$ and state its domain. (3 marks)

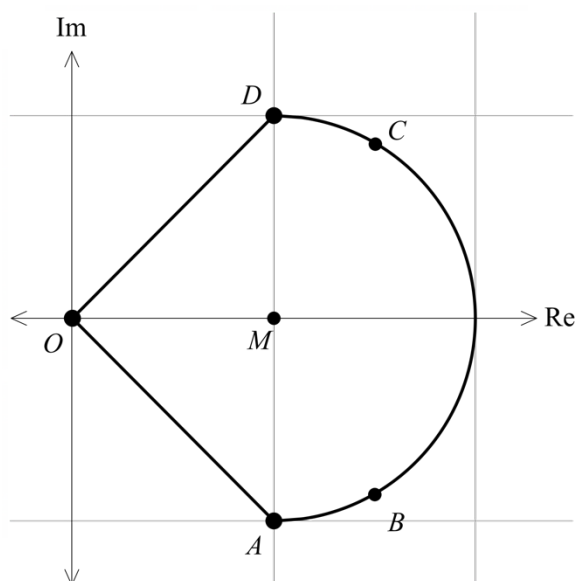
Consider the function $g(x) = ax^3$ where a is a positive constant.

- c) Does $g(x)$ have an inverse function? Explain. (2 marks)
- d) Determine the x values in terms of a for where $g(x) = g^{-1}(x)$. (3 marks)

Question 6

(9 marks)

The Argand diagram below shows a right-angled triangle AOD , with semicircle $ABCD$ centred at M .



- (a) Given A represents the complex number $1 - i$, determine the complex number representing D . (1 mark)
- (b) State the locus of points that define semicircle $ABCD$. (3 marks)

The rays MC and MB form angles of $\frac{\pi}{3}$ with the positive direction of the real axis.

Let C be the complex number z_C , and B be the complex number z_B .

(c) Determine, in polar form, z_C . (3 marks)

(d) Explain why $\overline{z_C} = z_C \operatorname{cis}\left(\frac{-\pi}{3}\right)$. (2 marks)

End of section one

Working out space.

Working out space.

End of questions

