ALL SAINTS' COLLEGE

MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 5 - 2016
Normal Distribution and Sample Proportions
Resource Rich

Name: _____ **Teacher:** _____

Marks: 34
Time Allowed: 45 minutes

Instructions: You are allowed to use Calculators and 1 page (two sides) of notes.
NO formula sheet will be provided.

---

1 Which of the following could be probability density functions on the intervals given?

I $f(x) = 0.2$ on the interval [5, 10]

II $f(x) = \frac{x^2}{3}$ on the interval [0, 3]

III $f(x) = \begin{cases} 0.125(x-2) \text{ for } 2 \le x \le 6 \\ -0.125(x-10) \text{ for } 6 \le x \le 10 \end{cases}$

IV $f(x) = 2x - 6$ on the interval [3, 4]

A All of I–IV

B I and II only

C I, II and IV

D Only I

E I and IV only **[1 mark]**

2 The expected value of a uniform probability density function on the interval [12, 38] is:

A 24

B 19

C 25

D $\frac{1}{19}$

E $\frac{1}{26}$ **[1 mark]**

3 The standard deviation of a uniform probability density function on the interval [10, 30] is:

A 33.3

B 5.8

C 4.5

D 10

E 6.1 **[1 mark]**

4 What is the area under the standard normal distribution for −1.2 ≤ *Z* ≤ 1.5?

A 0.0035

B 0.0483

C 0.1151

D 0.8181

E 0.8849 **[1 mark]**

5 *X* is a random normal variable with a mean of 35.2 and a standard deviation of 3.7. Find the probability that a random value of *X* lies between 33 and 36.

A 0.3095

B 0.2057

C 0.0673

D 0.1383

E 0.0323 **[1 mark]**

6 What is the most accurate statement about a survey where an interviewer stands on a corner in central Perth at lunchtime and asks passers-by, 'Do you support the death penalty?'

A It has completion bias.

B It has non-response.

C It has reporting bias.

D It has interviewer bias.

E It is fair. **[1 mark]**

7 In order to make the sample in question **1** fairer, the interviewer also asks the people interviewed whether they are aged under 20, 20–29, 30–39, 40–49, 50–59 or over 59.

To reflect the proportions of the Australian population in different age groups, the answers used were for
those of:
the first 3 men and 3 women under 20, the first 9 men and 9 women aged 20–29, the first 10 men and first 12 women over 59, and the first 8 men and 8 women in each of the other age groups.
What kind of sampling has been used?

A Stratified random sampling

B Convenience sampling

C Systematic Sampling

D Quota sampling

E Purposive sampling **[1 mark]**

8 There are 340 male players, 230 female players, 480 male supporters and 250 female supporters who are members of a sports club. For a survey of 60 members, how many females should be chosen?

A 12

B 15

C 20

D 22

E 23 **[1 mark]**

9 From 30 Year 12 students who have obtained their driver licences, 12 passed on their first test and 15 on their second test. What was the sample proportion of those taking more than 2 tests to get their licence?

A 0.1

B 0.9

C 3

D 0.05

E 0.333… **[1 mark]**

10 A card is randomly cut from a standard deck of playing cards and then replaced. What is the standard deviation of the sample proportion of hearts from doing this 20 times?

A 0.25

B 0.097

C 0.43

D 0.0094

E 0.11 **[1 mark]**

11 It is known that 35% of Australians aged 20–24 are attending university or TAFE. What is the probability that at least half of a random group of 20 Australians aged 20–24 are attending university or TAFE?

A 0.35

B 0.01

C 0.11

D 0.13

E 0.08 **[1 mark]**

12 A continuous random variable, *X,* is transformed to the random variable, *Y,* according to the equation
Y ■ 3*X* ◆ 4. The mean and standard deviation of *X* are 13.4 and 3.5 respectively. What are the mean and standard deviation of *Y*?

[2 marks]

13 Francis got 53 on a Maths test for which the class average was 39 and the standard deviation was 12.
For another test, he got 59, but on this test the class average was 46 and the standard deviation was 9.
In which test did he do better compared to the rest of the class?

[3 marks]

14 In an art class, 30% of the students making a sketch complete it within 17 minutes, while 50% complete it within 20 minutes. Assuming a normal distribution, what percentage of the students complete it within 28 minutes?

[2 marks]

15 From 28 English students, 2 got an A, 5 got a B, 14 got a C, 6 got a D and 1 got an E. What is the sample proportion of students getting at least a passing grade (C)?

[2 marks]

16 In a fresh water fishing competition with 40 competitors on the Swan river, 32 caught at least one fish of legal size, with 105 legal-sized fish being caught altogether.

a Estimate the probability of catching at least one legal-sized fish on the Swan.

b State any problems with this estimate.

[2 marks]

17 The probability of loaded dice landing with a 6 uppermost is 0.25. Two loaded dice are tossed 40 times. What is the expected proportion of double sixes and the standard deviation of this proportion?

[2 marks]

18 Find the proportion of sample proportions you would expect to lie between 0.3 and 0.5 for samples with $n = 35$ and $p = 0.3$.

[2 marks]

19 [BAD QUESTION – DO NOT INCLUDE IN ANY FUTURE ASSESSMENT]

A good tennis player will serve into the correct area of the court 85% of the time when practising. However, some players improve and others do worse under match pressure. Players who improve may do so by about 5%, while those who do worse might drop their service record by as much as 15%. A typical 3-set match will involve about 100 serves. A double fault occurs when a player misses the correct area twice in a row.

Find the probability of more than 3 double faults in a match for:

a A good player whose service is unaltered under match conditions

b A good player whose service improves by 5% under match conditions

c A good player whose service record drops by 10% under match conditions.

[3 marks]

20 It is known that 11% of Australian males are colour blind but only about 1.2% of females. For a group of 180 Australians, what is the probability that there is at least one woman and 11 men who are colour blind, given that there are equal numbers of men and women in the group?

[5 marks]

******** END OF TEST ********

ALL SAINTS' COLLEGE

MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 5 - 2016
Normal Distribution and Sample Proportions
Resource Rich SOLUTIONS

Part A – Multiple-choice questions **[1 mark each]**

1 Only I and IV have integrals of 1 ∴ **E**

2 $\frac{12 + 38}{2} = 25$

∴ **C**

3 $\frac{30 - 10}{2\sqrt{3}}$ □ 5.8

∴ **B**

4 Using a calculator, $P(\blacksquare 1.2 \blacksquare Z \blacksquare 1.5)$ □ 0.8181

∴ **D**

5 Using a calculator, $P(33 \blacksquare X \blacksquare 36)$ □ 0.3095

∴ **A**

6 Some people will not answer.

∴ **B**

7 The interviewer has just filled quotas for each category.

∴ **D**

8 There are 1300 altogether, so $\frac{230\ 1\ 250}{1300} \times 60 = 22.153...$

∴ **D**

9 $\frac{3}{30} = 0.1$

∴ **A**

10 $\sqrt{\frac{0.25 \times 0.75}{20}} = 0.0968...$

∴ **B**

11 s = $\sqrt{\frac{0.35 \times 0.65}{20}} \approx 0.1067$, using cdf(0.5, `, 0.35, 0.1067), $P = 0.0798...$

∴ **E**

12 $E(Y) = 3 \times 13.4 + 4 = 44.2$, $SD(Y) = 3 \times 3.5 = 10.5$ **[2 marks]**

13 $Z_{1st\ test} = \frac{53 - 39}{12} = 1\frac{1}{6}$

$Z_{2nd\ test} = \frac{59 - 46}{9} = 1\frac{4}{9} > 1\frac{1}{6}$

Francis did better on the second test. **[3 marks]**

14 Since it is a normal distribution, m = 20 by symmetry.

For a standard normal distribution, the 0.3 quantile is at $Z = -0.5244...$ from the inverse normal distribution function.

Thus $20.5244... = \frac{17 - 20}{\sigma}$, so s = 5.7208...

Using the normal cdf function, $P(-\infty < 28) = 0.9190...$

About 92% will have completed the sketch within 28 minutes. **[2 marks]**

15 Sample proportion = $\frac{21}{28} = 0.75$ **[2 marks]**

16 **a** $P(\text{Catching a fish}) = \frac{32}{40} = 0.8$

b The people who enter a fishing competition are likely to be good fishers, and the competition is also likely to be held in a place on the river that you are likely to catch fish.

[2 marks]

17 Probability of a double 6 = $(0.25)^2 = 0.0625$, so proportion expected is 0.0625.

SD = $\sqrt{\frac{0.0625 \times 0.9375}{40}} \approx 0.038$ **[2 marks]**

18 Sampling distribution $\sigma = \sqrt{\frac{0.3 \times 0.7}{35}} \approx 0.07746$

Using a calculator, $P(0.3 \leq X \leq 0.5) \approx 0.4951$ **[2 marks]**

19

a The probability of a double fault is $(0.15)^2 = 0.0225$.

With n = 100, np<5

Using Binomial Distribution : n= 100, p = 0.0225 and for

df >3 out of 100

$P(p > 0.03) \approx 0.189$ **[1 mark]**

b The probability of a double fault is $(0.1)^2 = 0.01$

With n = 100, np<5

Using Binomial Distribution : n=100, p = 0.01 and for

df > 3 out of 100

$P(p > 0.03) \approx 0.0184$ **[1 mark]**

c The probability of a double fault is $(0.25)^2 = 0.0625$

With n = 100, np<10

Using Binomial Distribution : n = 100, p=0.0625

$P(p > 0.03) \approx 0.8779$ **[1 mark]**

20 For the males, sampling mean = 0.11, n = 90

Probability of 11 or more men who are colour blind from 90

From the Binomial distribution ≈0.403558

For the females, population proportion = 0.012, n=90

Probability of 1 or more women who are colour blind from 90

From the binomial distribution ≈ 0.6626156

Probability of at least 11 men and one woman who are colour blind = 0.403558 X 0.6626156

≈ 0.2674 **[5 marks]**