3CD MATHEMATICS SPECIALIST – INVESTIGATION 1 MARKING KEY

1. For *n* an integer, the product $(n-1).n.(n+1)$ is the product of three consecutive ✓
integers. In any 3 consecutive integers there will be at least one multiple of 2 and ✓
a multiple of 3. Any product which is a multiple of both 2 and 3 is a multiple of 6, ✓
and is therefore divisible by 6.

2. Prove: $1+4+7+\ldots+(3n-2)=\dfrac{n(3n-1)}{2}$ for all positive integers *n*.

Step 1: Verify the statement is true when *n* = 1.

L.H.S. = 1

R.H.S. = $\dfrac{1(3\times 1-1)}{2}$

$=\dfrac{1\times 2}{2}$

$=1$ ✓

⇒ Statement is true for *n* = 1.

Step 2: Assume the statement is true for *n* = *k*.

That is, $1+4+7+\ldots+(3k-2)=\dfrac{k(3k-1)}{2}$ ✓

Step 3: Prove the statement is true for *n* = *k* + 1

That is, prove $1+4+7+\ldots+(3k-2)+(3[k+1]-2)=\dfrac{(k+1)(3[k+1]-1)}{2}$

$=\dfrac{(k+1)(3k+2)}{2}$ ✓

L.H.S.$=1+4+7+\ldots+(3k-2)+(3[k+1]-2)$

$=\dfrac{k(3k-1)}{2}+(3k+1)$ (from step 2) ✓

$=\dfrac{k(3k-1)+2(3k+1)}{2}$ ✓

$=\dfrac{3k^2-k+6k+2}{2}$

$=\dfrac{3k^2+5k+2}{2}$ ✓

$=\dfrac{(3k+2)(k+1)}{2}$ ✓

= R.H.S.

⇒ The statement is true for *n* = *k* + 1 if it is true for *n* = *k*.

Step 4: As the statement is true for *n* = 1, it must be true for *n* = 2.

As the statement is true for *n* = 2, it must be true for *n* = 3 and so on.

Hence, $1+4+7+...+n=\dfrac{n(3n-1)}{2}$ is true for all positive integers *n*. ✓

3. Prove $(|z|\text{cis}\theta)^n = |z|^n \text{cis}(n\theta)$ for all positive integers n.

Step 1 Verify the statement is true for n = 1

LHS = $(|z|\text{cis}(\theta))^1$

= $|z|\text{cis}(\theta)$

RHS = $|z|\text{cis}(1\theta)$

= $|z|\text{cis}(\theta)$

⇒ Statement true for n=1 ✓

Step 2 Assume it is true for n = k

That is, $(|z|\text{cis}\theta)^k = |z|^k \text{cis}(k\theta)$ ✓

Step 3 Prove the statement is true for n = k + 1

That is, prove $(|z|\text{cis}\theta)^{k+1} = |z|^{k+1}\text{cis}((k+1)\theta)$ ✓

LHS = $(|z|\text{cis}\theta)^k (|z|\text{cis}\theta)^1$

= $|z|^k(\text{cis}(k\theta))|z|(\text{cis}\theta)$ Step 1 and 2 ✓

= $|z|^{k+1}\ \text{cis}(k\theta)\ \text{cis}(\theta)$

= $|z|^{k+1}(\cos(k\theta)+i\sin(k\theta))(\cos\theta+i\sin\theta)$ ✓

= $|z|^{k+1}(\cos(k\theta)\cos\theta + i\cos(k\theta)\sin\theta + i\sin(k\theta)\cos\theta + i^2\sin(k\theta)\sin\theta)$ ✓

= $|z|^{k+1}(\cos(k\theta)\cos\theta - \sin(k\theta)\sin\theta + i(\sin(k\theta)\cos\theta + \cos(k\theta)\sin\theta$ ✓

= $|z|^{k+1}(\cos(k\theta+\theta)+i\sin(k\theta+\theta))$ ✓

= $|z|^{k+1}\text{cis}(k\theta+\theta)$

= $|z|^{k+1}\text{cis}((k+1)\theta)$

= RHS

Step 4: As the statement is true for *n* = 1, it must be true for *n* = 2.
As the statement is true for *n* = 2, it must be true for *n* = 3 and so on.

Hence, $(|z|\text{cis}\theta)^n = |z|^n\text{cis}(n\theta)$ is true for all positive integers *n*. ✓