



Name: \_\_\_\_\_

**Section 1: Resource Free**      **30 marks**      **30 minutes**  
**QUESTION 1**    [3, 1, 2, 7 marks]

a) If  $f(x) = \frac{1}{2x^3}$ , evaluate  $f''(-1)$

b) Find  $g'(x)$ , if  $g(x) = (1 + 2x - 2x^3)(x^2 - 1)$ ; do not simplify your answer

c) Use the chain rule to differentiate  $\frac{(x^3 + 2)^4}{2}$ ; apply basic simplification

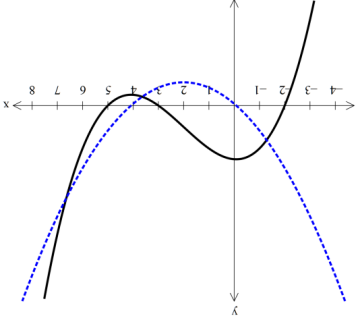
d) The 1<sup>st</sup> and 2<sup>nd</sup> derivative function of a function is shown. The x-coordinates of points where various features of the original function occur are shown below. State the nature of each of these points:

i.  $x = -2$

ii.  $x = 3$

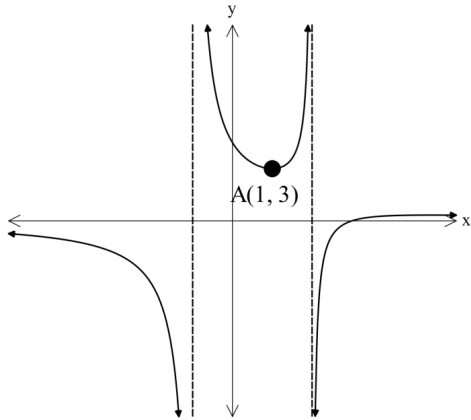
iii.  $x = 4$

iv.  $x = 5$



QUESTION 2 [3, 3 marks]

Consider the graph of  $f(x) = \frac{3x-9}{x^2-x-2}$  shown below with a local minimum at A(1, 3)

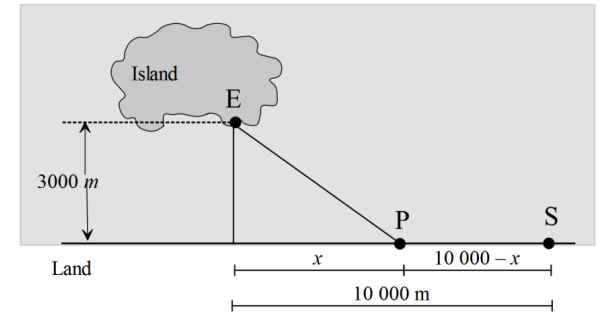


a) Show that  $f'(x) = \frac{-3(x-1)(x-5)}{(x^2-x-2)^2}$

b) Hence, or otherwise, determine the coordinates of the local maximum value of  $f(x)$ .

Question 10 [1, 2, 3 marks]

In the accompanying diagram, S represents the position of a power relay station located on a straight coast and E shows the location of a marine biology experimental station on an island. A cable is to be laid connecting the relay station with the experimental station. The cost of running cable on land is \$1.50 per metre and the cost of running the cable under water is \$2.50 per metre. Locate the point P that will result in a minimum cost.



- State the distance from E to P in terms of  $x$
- State the cost of the cabling in terms of  $x$
- Find the value of  $x$  that will minimise the cost

QUESTION 9 [2, 1, 4 marks]

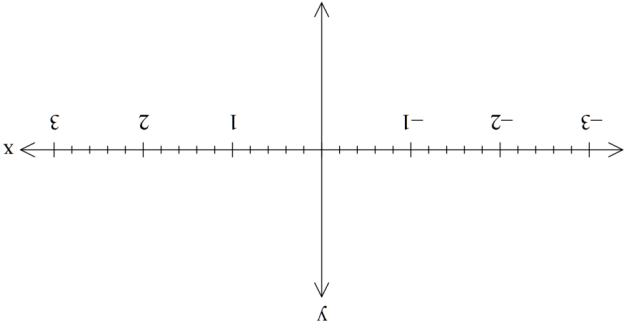
A POLYNOMIAL FUNCTION  $f(x) = ax^4 + bx^2 + c$ , where a, b and c are real constants, has the following features:

- $f(x) = 0$  only for  $x = -2$  and  $x = 2$
- $f'(x) = 0$  only for  $x = -1$ ,  $x = 0$  and  $x = 1$
- $f'(x) > 0$  only for  $-1 < x < 0$  and  $x > 1$
- $f''(0) > 0$

a) At the point where the curve intersects the y-axis, is it concave up or concave down? Explain your answer

b) Is c positive or negative? Explain your answer

c) Sketch a possible graph of the function on the axes below



QUESTION 3 [3 marks]

The volume of a solid sphere is given by  $\frac{4}{3}\pi r^3$  where r is the radius. If the radius is increased from 2 cm to 2.1 cm, use the incremental formula to find the approximate increase in volume. Give your answer simplified in terms of  $\pi$ .

QUESTION 4 [1, 2, 3, 2 marks]

The motion of a body is determined by  $x = t^3 - 3t^2 - 9t + 4$ , where x is measured in cm and t is in seconds. Find

a) The velocity-time equation

b) The acceleration-time equation

c) The time when the body is at rest

d) The acceleration when the body is at rest



Year 12 Maths Methods Test 1, 2017  
Differentiation Techniques and Applications of Differentiation

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**Section 2: Resource Rich 25 marks 25 minutes**

QUESTION 5 [4 marks]

The graph of the function with rule  $y = \frac{k}{2(x^3 + 1)}$  has gradient 1 when  $x = 1$ . Find the value of  $k$ .

QUESTION 6 [1, 1, 1, 2 marks]

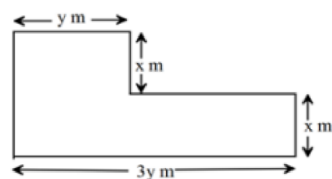
A flower bed is to be L-shaped, as shown in the diagram. Its perimeter is 48 m

a) Write down an expression for the area,  $A \text{ m}^2$ , in terms of  $y$  and  $x$

b) Find  $y$  in terms of  $x$

c) Write down an expression for  $A$  in terms of  $x$ .

d) Find the values of  $x$  and  $y$  that give the maximum area



QUESTION 7 [3 marks]

A coat of paint of thickness 0.05 cm is to be applied uniformly to the faces of a cube of edge 30 cm. Use calculus methods to find the amount of paint required for the job.

QUESTION 8 [4 marks]

The length of time, in seconds, a certain individual takes to learn a list of  $n$  items is approximated by  $f(x) = 4n\sqrt{n} - 4$ . Use calculus to find the percentage increase in time taken when the number of items in the list is increased by 1%