

 <p><b>PERTH MODERN SCHOOL</b> Exceptional schooling. Exceptional students. Independent Public School</p>	<p>Year 12 Specialist TEST 3 7 May 2018 TIME: 50 minutes working Classpads <b>allowed!</b> 39 Marks 7 Questions</p>
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Name: \_\_\_\_\_

Teacher: \_\_\_\_\_

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

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Q1 (2 & 2 = 4 marks)

$$x = 3 - 5\lambda$$

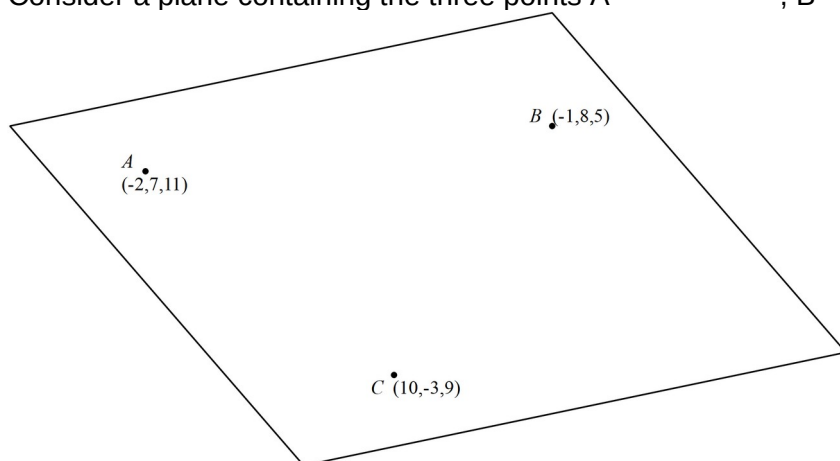
Consider a line with parametric equations  $y = -7 + 2\lambda$

i) Determine a vector equation

ii) Determine a cartesian equation.

Q2 (3 & 2 = 5 marks)

Consider a plane containing the three points A  $(-2, 7, 11)$ , B  $(-1, 8, 5)$  & C  $(10, -3, 9)$ .



i) Determine the vector equation of the plane.

Continued-

- ii) Determine the cartesian equation of the plane(simplified) .

Q3 (4 marks)

Determine the distance of point P  $(-5, 1, 3)$  from the line 
$$r = \begin{pmatrix} -1 \\ 7 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -8 \\ 1 \end{pmatrix}$$

Q4 (4 marks)

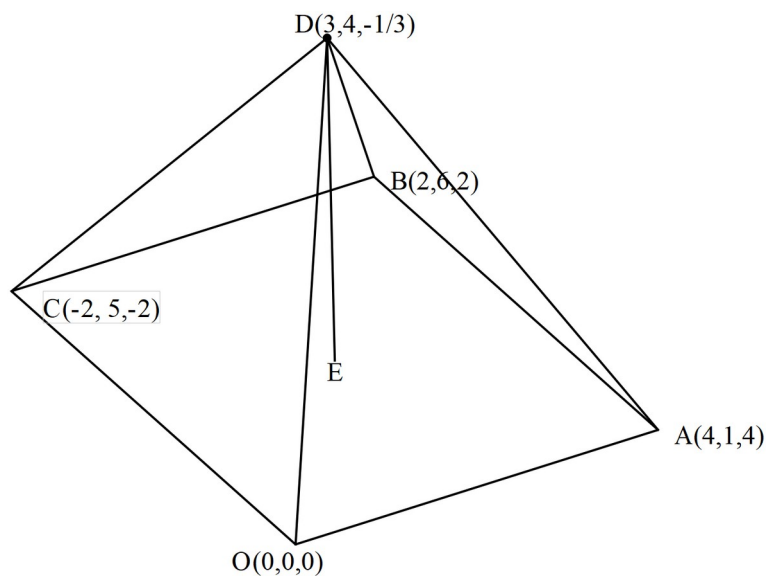
Consider two particles A and B whose position at  $t = 0$  is recorded as below moving with constant velocities  $v_A$  &  $v_B$ . Determine the distance of closest approach and the time that this occurs.

$$r_A = \begin{pmatrix} 2 \\ -5 \\ 9 \end{pmatrix} km \quad v_A = \begin{pmatrix} 11 \\ -5 \\ 7 \end{pmatrix} km/h$$

$$r_B = \begin{pmatrix} 1 \\ -1 \\ 9 \end{pmatrix} km \quad v_B = \begin{pmatrix} 12 \\ -10 \\ 2 \end{pmatrix} km/h$$

Q5 (2, 4 & 3 = 9 marks)

OABCD is a pyramid. The height of the pyramid is the length of DE, where E is the point on the base OABC such that DE is perpendicular to the base.



- i) Show that the base OABC is a rhombus.

The unit vector  $\frac{p\mathbf{i} + q\mathbf{j} + r\mathbf{k}}{\sqrt{p^2 + q^2 + r^2}}$  is perpendicular to both  $\overrightarrow{OA}$  and  $\overrightarrow{OC}$ .

- ii) Show that  $q = 0$  and determine the exact values of  $p$  &  $r$ .

- iii) Hence determine the exact height of the pyramid.

Q6 (5 marks)

Consider a sphere of centre  $(-3, 2, 7)$  and radius of  $a$  units, where  $a$  is a constant.

$$\vec{r} = \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix}$$

The line is a tangent to the above sphere.

Determine the possible value(s) of  $a$

Q7 (2, 3 & 3 = 8 marks)

Consider the function  $f(x) = ax^4 + bx^3 + cx^2 + dx$  where  $a, b, c$  &  $d$  are constants.

The graph has a stationary point ( $f' = 0$ ) at  $(1, 1)$  and passes through the point  $(-1, 4)$ .

i) Write down three linear equations satisfied by  $a, b, c$  &  $d$ .

ii) Express  $a, b$  &  $c$  in terms of  $d$  **without** the use of a classpad.

iii) Determine the value of  $d$  for which the graph has a stationary point where  $x = 4$   
(You may use a classpad here and show reasoning).