Don't forget Q.E.D when done

Geometric Proofs:

To prove something, we must present evidence to convince others of the truth of the statement

Axions:

Statements that are simply accepted as being true without the need of proof

Theorems:

Are statements that can be proved to be true using accepted deffinions, axions and other (proved_ theorems

Definition:

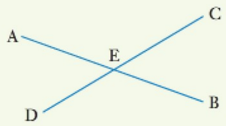
Due to how they are defined

e.g. of proofs

Proof of: When two straight lines intersect, the vertically opposite angles are equal.

Solution

Given: Two straight lines AB and CD intersecting at E.



To prove: Vertically opposite angles are equal.

I.e., in the diagram on the right,

$$\angle AED = \angle BEC$$

and $\angle AEC = \angle DEB$.

Proof:

$\angle AED + \angle AEC = 180°$ (Angle sum of straight line DC.)

$\therefore$ $\angle AED = 180° - \angle AEC$

$\angle BEC + \angle AEC = 180°$ (Angle sum of a straight line AB.)

$\therefore$ $\angle BEC = 180° - \angle AEC$

Hence $\angle AED = \angle BEC$ as required. (Each equal to $180° - \angle AEC$.)

Also $\angle AEC = 180° - \angle AED$ (Angle sum of straight line DC.)

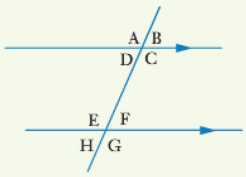
$\angle DEB = 180° - \angle AED$ (Angle sum of straight line AB.)

Hence $\angle AEC = \angle DEB$ as required. (Each equal to $180° - \angle AED$.)

Proof of: When a transversal cuts parallel lines, alternate angles are equal.

Solution

Given: A transversal cutting a pair of parallel lines with angles A, B, C, D, E, F, G and H as shown in the diagram on the right.



To prove: Alternate angles are equal.

I.e., in the diagram on the right, $C = E$

and $D = F$.

Proof:

$C = G$ (Corresponding angles are equal.)

But $E = G$ (Vertically opposite angles are equal, just proved.)

$\therefore$ $C = E$, as required.

Similarly $D = H$ (Corresponding angles are equal.)

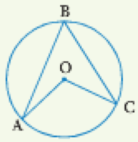
But $F = H$ (Vertically opposite angles are equal, just proved.)

$\therefore$ $D = F$, as required.

Chapter 5

EXAMPLE 3

Proof of the above statement.

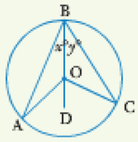


Solution

Given: Points A, B and C lying on the circumference of a circle centre O, as shown in the diagram.

To prove: In the given diagram $\angle AOC = 2 \times \angle ABC$.

Construction: Draw a line from B to pass through O to some point D. Let $\angle ABO = x°$ and $\angle CBO = y°$.



Proof: $OA = OB$ (Radii)

$\therefore$ $\triangle OAB$ is isosceles.

Thus $\angle OAB = \angle OBA = x°$ (Base angles of isosceles triangle)

$\therefore$ $\angle AOB = 180° - 2x°$ (Angle sum of a triangle)

and so $\angle AOD = 2x°$ (Angle of straight line BD)

Similar reasoning for $\triangle OCB$ gives

$\angle COD = 2y°$

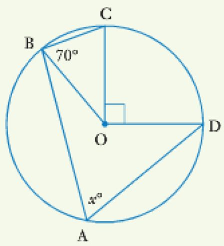
Now $\angle AOC = \angle AOD + \angle COD$

$= 2x° + 2y°$

$= 2(x° + y°)$

$= 2 \times \angle ABC$, as required.

In the diagram on the right, point O is the centre of the circle and points A, B, C and D lie on the circle.



$\angle CBO = 70°$,

$\angle COD = 90°$

and $\angle BAD = x°$.

Prove that $x = 65$.

Solution

To prove: That for the given diagram, $x = 65$.

Proof: $OB = OC$ (Radii)

$\therefore$ $\triangle OBC$ is isosceles.

Thus $\angle OCB = 70°$ (Base angles of isosceles triangle)

and $\angle BOC = 40°$ (Angle sum of a triangle)

Now $\angle BOD = \angle BOC + \angle COD$

$= 40° + 90°$

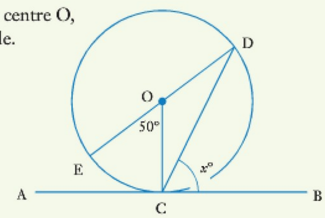
$= 130°$

Hence $\angle BAD = 65°$ (Angle at centre is twice angle at circumference)

Thus $x = 65$, as required.

EXAMPLE 5

In the diagram on the right, AB is a tangent to the circle centre O, with C the point of contact. ED is a diameter of the circle.



Given that $\angle EOC = 50°$

and $\angle DCB = x°$

prove that $x = 65$.

Solution

Given: Diagram as shown.

To prove: $x = 65$

Proof: $\angle ODC = 25°$ (Angle at centre $= 2 \times$ angle at circumference)

$\triangle OCD$ is isosceles. (OD and OC are radii)

$\therefore$ $\angle OCD = 25°$ ($\angle OCD = \angle ODC$, base angles of isosceles triangle)

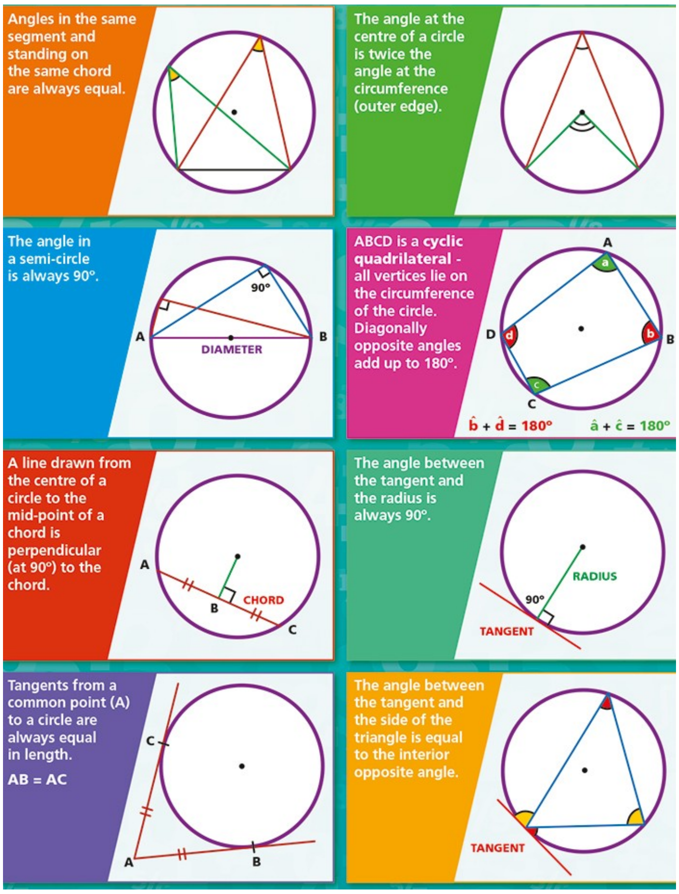
But $\angle OCD + \angle DCB = 90°$ (Angle between tangent and radius)

$\therefore$ $25 + x = 90$

and so $x = 65$, as required.

Chapter 5

In summary…



Similar Triangles:

AA

SAS

AAA

SSS

Hyp and S…

Chapter 5