Kennedy
Baptist College

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNITS 1&2

Section One: Calculator-free

SOLUTIONS

WA student number: In figures

In words

Your name

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

The quadratic function $f(x) = ax^2 + bx - 6$ has roots at $x = 1$ and $x = -3$.

(a) Determine the value of the constant a and the value of the constant b. (3 marks)

Solution
Roots → factors: $f(x) = a(x-1)(x+3)$ $= a(x^2 + 2x - 3)$ Using last term: $-6 = -3a \Rightarrow a = 2$ $b = 2a = 4$
Specific behaviours
✓ uses factors to expand ✓ value of a ✓ value of b ✓ if $a=1$, $b=2$

OR
$f(1) = a+b-6 = 0$
$f(-3) = 9a-3b-6 = 0$ ✓
Solving simultaneously
$a=2$, $b=4$

(b) State the range of the function f. (2 marks)

Solution
Minimum turning point midway between roots: $x = \dfrac{1-3}{2} = -1$ $f(-1) = 2(-2)(2) = -8$ Hence range is $y = \{y \in \mathbb{R}, y \geq -8\}$
Specific behaviours
✓ locates turning point ✓ obtains range

SN245-182-3

Question 2 **(6 marks)**

(a) Evaluate $f'(3)$ when $f(x) = 10x^2 - 5x^4$. (2 marks)

Solution
$f'(x) = 20x - 20x^3$ $= 20x(1 - x^2)$ $f'(3) = 60(1 - 9)$ $= -480$
Specific behaviours
✓ obtains $f'(x)$ ✓ correct value

(b) Determine $\frac{d}{dx}\big((5x - 6)(5x + 6)\big)$. (2 marks)

Solution
$(5x - 6)(5x + 6) = 25x^2 - 36$ $\frac{d}{dx}(25x^2 - 36) = 50x$
Specific behaviours
✓ expands into polynomial ✓ obtains derivative

(c) The volume of water in a tank at time t seconds is given by $V(t) = t^3 - 3t + 1$ cm^3. Determine the instantaneous rate of change of volume when $t = 5$. (2 marks)

Solution
$V'(t) = 3t^2 - 3$ $V'(5) = 3(25) - 3$ $= 72 \text{ cm}^3/\text{s}$
Specific behaviours
✓ obtains $V'(t)$ ✓ correct rate of change

Must have correct units for second mark.

Question 3 **(6 marks)**

(a) Solve $(x-8)^2 - 16 = 0$. (2 marks)

Solution
$(x-8)^2 = 16$ $x - 8 = \pm 4$ ✓ $x = 12, \quad x = 4$ ✓ ✓
Specific behaviours
✓ indicates appropriate method ✓ obtains correct solutions

Let $g(x) = x^3 - 5x^2 + 2x + 8$.

(b) Evaluate $g(2)$. (1 mark)

Solution
$g(2) = 8 - 20 + 4 + 8 = 0$
Specific behaviours
✓ obtains zero

(c) Factorise $g(x)$. (3 marks)

Solution
$g(x) = (x-2)(x^2 + bx + c)$ By inspection: $g(x) = (x-2)(x^2 - 3x - 4)$ $= (x-2)(x-4)(x+1)$
Specific behaviours
✓ uses result from (b) to obtain one factor ✓ obtains quadratic factor ✓ completes factorisation

Question 4 **(7 marks)**

(a) Determine the function f given that $f(1) = 5$ and $f'(x) = 3 - 4x$. (3 marks)

Solution
$f(x) = 3x - 2x^2 + c$ $f(1) = 3(1) - 2(1)^2 + c = 5$ $c = 4$ $\therefore f(x) = 3x - 2x^2 + 4$
Specific behaviours
✓ obtains antiderivative ✓ evaluates constant ✓ clearly states function

(b) Determine the equation of the tangent to the curve $y = x^4 + 4x^3 - 10x - 2$ at the point where $x = -2$. (4 marks)

Solution
Gradient function: $\frac{dy}{dx} = 4x^3 + 12x^2 - 10$ ✓ Gradient of tangent: $m = 4(-2)^3 + 12(-2)^2 - 10$ $= 4(-8) + 12(4) - 10$ $= 6$ ✓ y-coordinate of point of tangency: $y = (-2)^4 + 4(-2)^3 - 10(-2) - 2$ $= 16 - 32 + 20 - 2$ $= 2$ ✓ Hence tangent: $y - 2 = 6(x - (-2))$ $y = 6x + 14$ ✓
Specific behaviours
✓ obtains gradient function ✓ calculates gradient of tangent ✓ obtains y-coordinate ✓ obtains equation of tangent

Question 5 **(7 marks)**

(a) The first term of an arithmetic sequence is 6 and the 13th term is three times the 4th term. Determine the sum of the first 12 terms of this sequence. (4 marks)

Solution
$T_n = 6 + (n-1)d$ $T_{13} = 3T_4$ $6 + (13-1)d = 3(6 + (4-1)d)$ $6 + 12d = 18 + 9d$ $3d = 12$ $d = 4$ $S_{12} = \frac{12}{2}\big(2(6) + (12-1)(4)\big)$ $= 6(12 + 44)$ $= 6 \times 56$ $= 336$
Specific behaviours
✓ formulates equation ✓ solves for d ✓ correct use of sum formula ✓ calculates sum

(b) Determine S_∞ for the following geometric sequence: (3 marks)

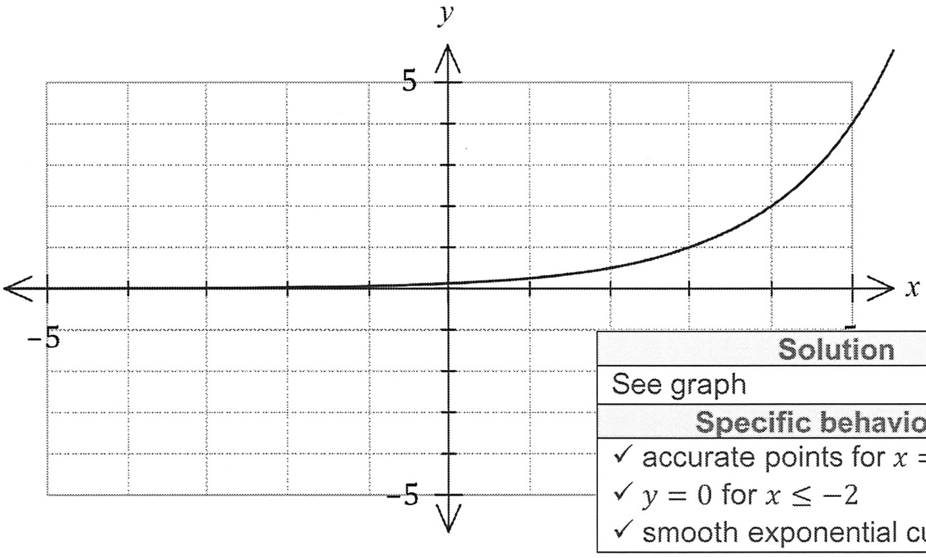
$$\frac{5}{3}, \quad \frac{5}{9}, \quad \frac{5}{27}, \quad \frac{5}{81}, \cdots$$

Solution
$a = \frac{5}{3}, \quad r = \frac{1}{3}$ $S_\infty = \frac{5}{3} \div \left(1 - \frac{1}{3}\right)$ $= \frac{5}{3} \div \frac{2}{3}$ $= \frac{5}{2}$
Specific behaviours
✓ indicates a and r ✓ correct use of formula ✓ correct sum to infinity

Question 6 **(7 marks)**

Let $f(x) = 2^{x-3}$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ accurate points for $x = \{3,4,5\}$
✓ $y = 0$ for $x \le -2$
✓ smooth exponential curve

(b) Solve $f(x) = \sqrt[3]{2}$ for x. (2 marks)

Solution
$2^{x-3} = 2^{\frac{1}{3}}$ $x - 3 = \frac{1}{3}$ $x = 3\frac{1}{3} = \frac{10}{3}$
Specific behaviours
✓ forms equation with fractional index on RHS
✓ correct solution

ft if incorrect fractional index

(c) Evaluate $f\left(\frac{1}{2}\right)$, giving your answer in simplest form without the use of indices. (2 marks)

Solution
$f\left(\frac{1}{2}\right) = 2^{\frac{1}{2}-3}$ $= 2^{-\frac{5}{2}}$ $= \frac{1}{\sqrt{2^5}}$ $= \frac{1}{4\sqrt{2}}$
Specific behaviours
✓ eliminates fractional or negative index
✓ correct value as required

Question 7 **(7 marks)**

(a) Solve the equation $\tan(2x - 10°) = \sqrt{3}$ when $0 \leq x \leq 180°$. (3 marks)

Solution
$2x - 10° = 60°, 240°$ $2x = 70°, 250°$ $x = 35°, 125°$
Specific behaviours
✓ indicates $\tan^{-1}\sqrt{3} = 60°$ ✓ one correct solution ✓ second correct solution

(b) In triangle ABC, the length of side AC is 9 cm, $\sin C = 0.4$ and $\sin B = 0.6$. Determine the length of side AB. (2 marks)

Solution
Using sin rule: $\frac{AB}{0.4} = \frac{9}{0.6}$ $AB = \frac{0.4 \times 9}{0.6}$ $= \frac{2}{3} \times 9 = 6\text{ cm}$
Specific behaviours
✓ indicates correct use of sin rule ✓ correct length

(c) Triangle PQR has sides of length 4, 5 and 6 cm. Given that PQ is the shortest side in the triangle, determine the value of $\cos R$. (2 marks)

Solution
Using cosine rule: $\cos R = \frac{5^2 + 6^2 - 4^2}{2(5)(6)}$ $= \frac{45}{60} = \frac{3}{4}$
Specific behaviours
✓ indicates correct use of cosine rule ✓ correct value

Question 8 **(7 marks)**

Determine the coordinates of the point(s) where the line $x - 2y = 5$ intersects the circle with centre $(2, 1)$ and radius 5.

Solution
Equation of circle: $$(x-2)^2 + (y-1)^2 = 25$$ Use line to substitute $x = 2y + 5$: $$(2y + 5 - 2)^2 + (y-1)^2 = 25$$ $$(2y+3)^2 + (y-1)^2 = 25$$ Expand: $$4y^2 + 12y + 9 + y^2 - 2y + 1 - 25 = 0$$ Simplify: $$5y^2 + 10y - 15 = 0$$ $$y^2 + 2y - 3 = 0$$ Solve quadratic: $$(y+3)(y-1) = 0$$ $$y = -3 \Rightarrow x = 2(-3) + 5 = -1$$ Or $$y = 1 \Rightarrow x = 2(1) + 5 = 7$$ Intersect at the points $(-1, -3)$ and $(7, 1)$.   ✓ ✓
Specific behaviours
✓ writes equation of circle   ✓ substitutes line to eliminate x or y ✓ expands   ✓ simplifies   ✓ solves quadratic   ✓ one correct point   ✓ second correct point

End of questions

Supplementary page

Question number: _________

Kennedy
Baptist College

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNITS 1&2

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

In words ______________________________

____

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

Sector POQ subtends an angle of $108°$ in a circle with centre O and radius r.

(a) Express $108°$ as an exact and simplified radian measure. (1 mark)

Solution
$108° = \dfrac{3\pi}{5}$ radians
Specific behaviours
✓ value

The area of sector POQ is 120π cm^2.

(b) Determine the radius of the circle. (2 marks)

Solution
$\dfrac{1}{2}r^2 \times \dfrac{3\pi}{5} = 120\pi$ $r = 20$ cm
Specific behaviours
✓ indicates equation ✓ calculates radius

(c) Determine the area of the minor segment bounded by arc PQ and chord PQ. (2 marks)

Solution
$A = \dfrac{1}{2}(20)^2\left(\dfrac{3\pi}{5} - \sin\dfrac{3\pi}{5}\right)$ $= 187$ cm^2
Specific behaviours
✓ indicates equation ✓ calculates area

Question 10 **(4 marks)**

The value V of a block of land, in thousands of dollars, t years after the start of the year 2010, can be modelled by the equation $V = 65r^t$, where r is a positive constant.

At the start of 2015, the land was valued at \$92 000.

(a) Show that the value of r is 1.072, when rounded to 3 decimal places. (2 marks)

Solution
$65r^5 = 92$ $r = 1.07195 \approx 1.072 \text{ to 3 dp}$
Specific behaviours
✓ writes equation ✓ solves to more than 3 dp (and then rounds)

(b) Assuming that the model remains valid into the future, determine the year in which the value of the block will reach \$500 000. (2 marks)

Solution
$65(1.072)^t = 500$ $t = 29.4 \text{ years}$ Hence during the year 2039.
Specific behaviours
✓ writes and solves equation ✓ states correct year

Accept during 30th year

Do not accept solutions that "use" 1.072 in part (a)

Question 11 **(9 marks)**

A function is defined by $f(x) = x^4 - 8x^3 + 18x^2 - 16x + 21$.

(a) Complete the following table. (2 marks)

Solution
See table
Specific behaviours
✓✓ −1 per error

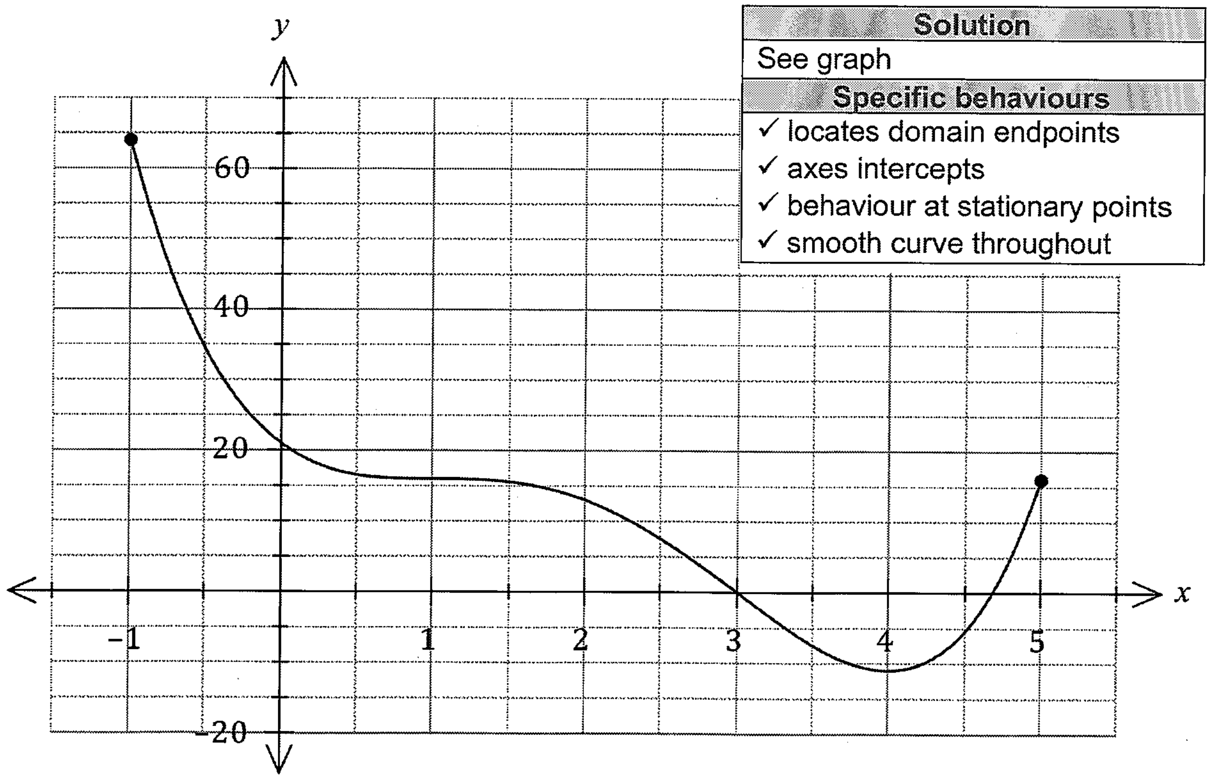
x	-1	0	1	2	3	4	5
$f(x)$	64	21	16	13	0	−11	16

(b) Use calculus to determine the coordinates of all stationary points of the graph $y = f(x)$. (3 marks)

Solution
$f'(x) = 4x^3 - 24x^2 + 36x - 16$ $f'(x) = 0 \Rightarrow x = 1, 4$ $f(x)$ is stationary at $(1, 16)$ and $(4, -11)$.
Specific behaviours
✓ shows $f'(x)$ ✓ solves $f'(x) = 0$ ✓ states coordinates of both points

(c) Sketch the graph of $y = f(x)$ on the axes below for $-1 \le x \le 5$. (4 marks)

Solution
See graph
Specific behaviours
✓ locates domain endpoints ✓ axes intercepts ✓ behaviour at stationary points ✓ smooth curve throughout



Question 12 **(8 marks)**

Data from repairs to 495 smartphones showed that 340 were Android and the remainder iOS. The type of repair was classified as screen or other, and of the 346 smartphones that required screen repairs, 265 were Android.

(a) Determine, to 3 decimal places, the probability that a randomly selected smartphone from those repaired

(i) was an iOS smartphone. (2 marks)

Solution
$495 - 340 = 155$ $P(I) = \dfrac{155}{495} \approx 0.313$
Specific behaviours
✓ calculates numerator   ✓ correct probability

(ii) required a screen repair or was an Android smartphone. (2 marks)

Solution
$346 + 340 - 265 = 421$ $P(S \cup A) = \dfrac{421}{495} \approx 0.851$
Specific behaviours
✓ calculates numerator   ✓ correct probability

(iii) was an iOS smartphone given that it required a screen repair. (2 marks)

Solution
$346 - 265 = 81$ $P(I\|S) = \dfrac{81}{346} \approx 0.234$
Specific behaviours
✓ indicates use of conditional probability   ✓ calculates probability

(b) Use two of the above probabilities to explain whether the repair data indicates possible independence of type of smartphone and type of repair. (2 marks)

Solution
Independence appears unlikely since $P(I) = 0.313$ is not close to $P(I\|S) = 0.234$.
Specific behaviours
✓ states independence unlikely   ✓ justifies by comparing relevant probabilities

Question 13 (7 marks)

An aeroplane takes off from an airport situated at an altitude of 150 metres above sea level and climbs 450 metres during the first minute of flight. In each subsequent minute, its rate of climb reduces by 4%.

(a) Determine the **increase in altitude** of the aeroplane during the fourth minute. (2 marks)

Solution
$\Delta A = 450(0.96)^{4-1}$ $= 398 \text{ m}$
Specific behaviours
✓ indicates use of appropriate method   ✓ correct increase

(b) Deduce a rule in simplified form for the **altitude** A_n of the aeroplane at the end of the n^{th} minute. (~~2~~ marks) 3 *

Solution
A_n will be sum of terms plus initial altitude: $A_n = \dfrac{450(1-0.96^n)}{1-0.96} + 150$ $= 11\,250(1-0.96^n) + 150$ $= 11\,400 - 11\,250(0.96)^n$
Specific behaviours
✓ correct use of sum formula   ✓ includes initial altitude   ✓ simplifies (to last or second last line)

(c) Determine the altitude of the aeroplane after 12 minutes. (1 mark)

Solution
$A_{12} = 4357 + 150 = 4507 \text{ m}$
Specific behaviours
✓ calculates correct term

(d) Determine the maximum altitude the aeroplane can reach. (~~2~~ marks) 1 *

Solution
$A_\infty = 11\,250(1-0.96^\infty) + 150$ $= 11\,400\text{m}$
Specific behaviours
✓ correct altitude

Question 14 **(8 marks)**

Two events A and B are such that $P(A) = 0.35$ and $P(B) = 0.48$.

Determine the following probabilities.

(a) $P(\overline{A \cup B})$ when A and B are mutually exclusive. (2 marks)

Solution
$P(A \cup B) = 0.35 + 0.48 = 0.83$ $P(\overline{A \cup B}) = 1 - 0.83 = 0.17$
Specific behaviours
✓ indicates $P(A \cup B)$ ✓ correct probability

(b) $P(A \cup B)$ when $P(\bar{A} \cap B) = 0.25$. (2 marks)

Solution
$P(A \cup B) = P(A) + P(\bar{A} \cap B)$ $= 0.35 + 0.25 = 0.6$
Specific behaviours
✓ indicates suitable method ✓ correct probability

(c) $P(A \cap \bar{B})$ when A and B are independent. (2 marks)

Solution
$P(A \cap B) = 0.35 \times 0.48 = 0.168$ $P(A \cap \bar{B}) = 0.35 - 0.168 = 0.182$
Specific behaviours
✓ indicates $P(A \cap B)$ ✓ correct probability

(d) $P(A|B)$ when $P(B|A) = 0.2$. (2 marks)

Solution
$P(A \cap B) = 0.35 \times 0.2 = 0.07$ $P(A\|B) = 0.07 \div 0.48 = \frac{7}{48} \approx 0.146$
Specific behaviours
✓ indicates $P(A \cap B)$ ✓ correct probability

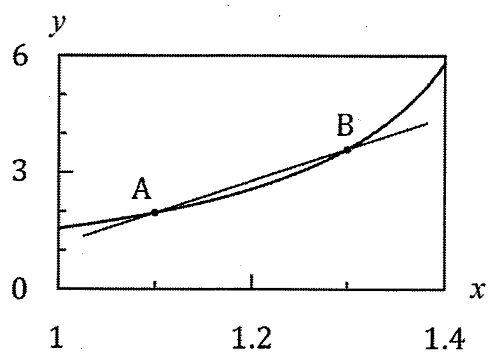
Question 15 **(7 marks)**

Let $f(x) = \tan x$, where x is measured in radians.

The graph of $y = f(x)$ is shown.

Two points, A and B, lie on the curve with x-coordinates 1.1 and $1.1 + h$ respectively, where $h > 0$.

The secant through AB is also shown.



(a) Use the difference quotient $\dfrac{\delta y}{\delta x} = \dfrac{f(x+h) - f(x)}{h}$ to calculate, to 3 decimal places, the slope of secant AB when

(i) $h = 0.2$. (2 marks)

Solution
$\dfrac{\delta y}{\delta x} = \dfrac{\tan(1.3) - \tan(1.1)}{0.2} \approx 8.187$
Specific behaviours
✓ uses correct values in quotient ✓ correct value

(ii) $h = 0.05$. (1 mark)

Solution
$\dfrac{\delta y}{\delta x} = \dfrac{\tan(1.15) - \tan(1.1)}{0.05} \approx 5.395$
Specific behaviours
✓ correct value

(b) Show use of the difference quotient to determine ~~an~~ a better estimate, correct to 3 decimal places, for the slope of secant AB as the value of h tends to 0. (3 marks)

Solution
$h = 0.01 \Rightarrow \dfrac{\delta y}{\delta x} \approx 4.958$ $0 < h \leq 0.000\,02 \Rightarrow \dfrac{\delta y}{\delta x} \approx 4.860$ To 3 dp, best estimate for gradient as $h \to 0$ is 4.860.
Specific behaviours
✓ calculates quotient with $0 < h < 0.05$ ✓ calculates another quotient with smaller h ✓ correct estimate, to 3 dp

(c) Briefly explain how your answer to part (b) relates to a feature of the graph of $y = f(x)$ at the point A. (1 mark)

Solution
It is the slope of the graph at the point A.
Specific behaviours
✓ states slope at point

Question 16 **(6 marks)**

The sum of the first n terms of a sequence is given by $S_n = 3n^2 + 2n$.

(a) Determine S_5. (1 mark)

Solution
$S_5 = 3(5)^2 + 2(5)$ $= 85$
Specific behaviours
✓ correct value

(b) Determine T_5, where T_n is the n^{th} term of the sequence. (2 marks)

Solution
$S_4 = 3(4)^2 + 2(4) = 56$ $T_5 = S_5 - S_4$ $= 85 - 56$ $= 29$
Specific behaviours
✓ calculates S_4 ✓ calculates T_5

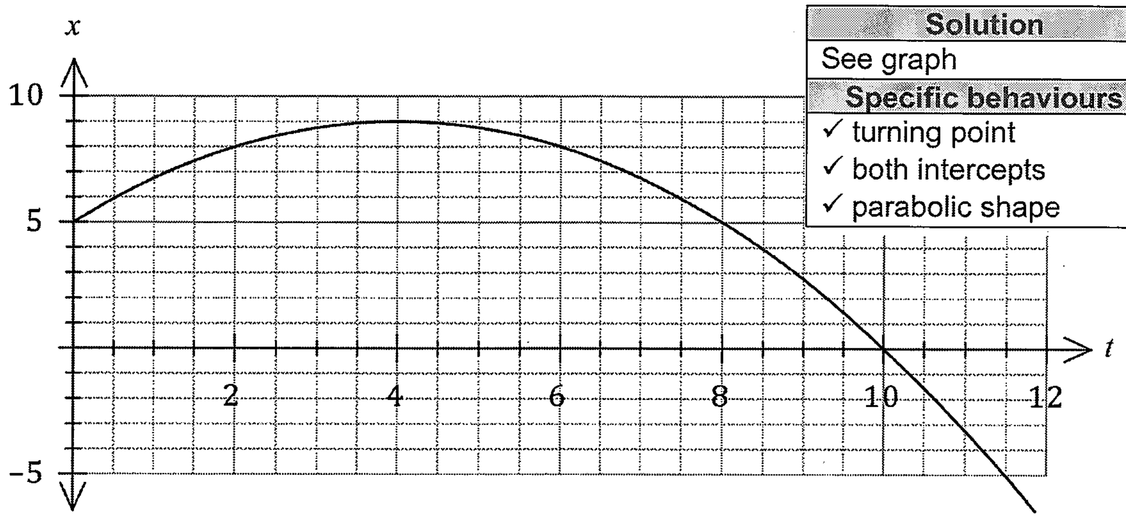
(c) Explain why the sequence must be arithmetic and hence deduce a rule for the n^{th} term of the sequence. (3 marks)

Solution
The rule for S_n is quadratic and so the second difference of the sums will be constant and equal to the common difference of the sequence. $T_1 = S_1 = 5$ $T_2 = S_2 - T_1 = 16 - 5 = 11$ $d = T_2 - T_1 = 11 - 5 = 6$ $T_n = 5 + (n-1)(6) = 6n - 1$
Specific behaviours
✓ reasonable explanation ✓ calculates common difference ✓ correct rule

Question 17 **(10 marks)**

Particle A is moving along the x-axis so that its displacement, in cm, at time t seconds, $t \geq 0$, is given by $x = 5 + 2t - 0.25t^2$.

(a) Sketch the displacement-time graph of particle A on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ turning point
✓ both intercepts
✓ parabolic shape

(b) Determine the velocity of particle A at the instant it reaches the origin. (3 marks)

Solution
Reaches origin when $x = 0 \Rightarrow t = 10$. $v = \dfrac{dx}{dt} = 2 - 0.5t$ $v(10) = 2 - 0.5(10) = -3 \text{ cm/s}$
Specific behaviours
✓ indicates correct time   ✓ obtains velocity function   ✓ correct velocity

(c) Particle B is also moving along the x-axis, but with a constant velocity. When $t = 5$, it has the same displacement and velocity as particle A. Determine when particle B reaches the origin. (4 marks)

Solution
$x(5) = 8.75, \quad v(5) = -0.5$ Displacement equation (tangent to curve at $t = 5$): $x - 8.75 = -0.5(t - 5)$ $x = 11.25 - 0.5t$ Reaches origin: $11.25 - 0.5t = 0 \Rightarrow t = 22.5$ Hence B reaches origin when $t = 22.5$ seconds.
Specific behaviours
✓ initial displacement and velocity   ✓ displacement equation   ✓ equates displacement to 0   ✓ solves for correct time

Question 18 **(8 marks)**

A random selection of 4 spanners is made from a collection of 15 different spanners, of which 6 are metric and the remainder imperial.

(a) Show that the probability the selection contains all imperial spanners is $\frac{6}{65}$. (3 marks)

Solution
Total possible selections is $\binom{15}{4} = 1365$.   Number of imperial spanners is $15 - 6 = 9$.   Ways to select all imperial is $\binom{9}{4} = 126$. $P(\text{All Imperial}) = \frac{126}{1365} = \frac{6}{65}$
Specific behaviours
✓ calculates number of all possible selections   ✓ calculates number of ways to select all imperial   ✓ uses counts to form probability

(b) Determine the probability that the selection contains

(i) all metric spanners. (2 marks)

Solution
Ways to select all metric is $\binom{6}{4} = 15$. $P(\text{All Metric}) = \frac{15}{1365} = \frac{1}{91}\ (\approx 0.01099)$
Specific behaviours
✓ calculates number of ways to select all metric   ✓ correct probability

(ii) at least one imperial spanner. (1 mark)

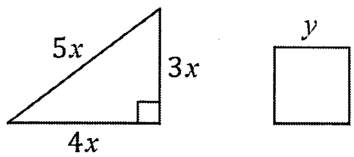
Solution
$P = 1 - \frac{1}{91} = \frac{90}{91} (\approx 0.98901)$
Specific behaviours
✓ correct probability

(iii) at least one metric spanner and at least one imperial spanner. (2 marks)

Solution
$P(\text{All of same type}) = \frac{6}{65} + \frac{1}{91} = \frac{47}{455}$ $P = 1 - \frac{47}{455} = \frac{408}{455}\ (\approx 0.8967)$
Specific behaviours
✓ probability all of same type   ✓ correct probability

Question 19 **(7 marks)**

A length of wire 72 cm long is cut into two pieces. One piece is bent into a right triangle with sides of length $3x, 4x$ and $5x$ cm and the other piece is bent into a square of side y cm.



Formulate an expression for the combined area of the triangle and square in terms of x and hence use calculus to determine the minimum value of this total area.

Solution
$12x + 4y = 72 \Rightarrow y = 18 - 3x$ ✓ $A = \frac{1}{2}(4x)(3x) + y^2$ ✓ $= 6x^2 + (18 - 3x)^2$ $= 15x^2 - 108x + 324$ ✓ $\frac{dA}{dx} = 30x - 108$ ✓ $30x - 108 = 0$ ✓ $x = \frac{18}{5} = 3.6$ ✓ $A(3.6) = 15(3.6)^2 - 108(3.6) + 324$ $= \frac{648}{5} = 129.6$ ✓   The minimum total area is 129.6 cm^2.

Specific behaviours
✓ equation relating x and y ✓ total area in terms of x and y ✓ total area in terms of x ✓ derivative   ✓ equates derivative to 0   ✓ optimum value of x ✓ calculates and states minimum area

Question 20 **(10 marks)**

Three small weights A, B and C, each attached to a spring, are oscillating vertically above level ground. The height, h cm, above the ground of each weight at time t seconds, $t \geq 0$, is given by

$$h_A = 15\sin\left(\frac{\pi t}{3}\right) + 35, \qquad h_B = 14\cos\left(\frac{2\pi t}{3}\right) + 30, \qquad h_C = 15\sin\left(\frac{2\pi t}{3}\right) + 30.$$

(a) State which two weights are oscillating with the same amplitude, and state what this common amplitude is. (2 marks)

Solution
Weights A and C - their amplitude is 15 cm.
Specific behaviours
✓ correct weights ✓ correct amplitude

(b) State which two weights are oscillating with the same period, and state what this common period is. (2 marks)

Solution
Weights B and C - their period is $2\pi \div \frac{2\pi}{3} = 3$ s.
Specific behaviours
✓ correct weights ✓ correct period

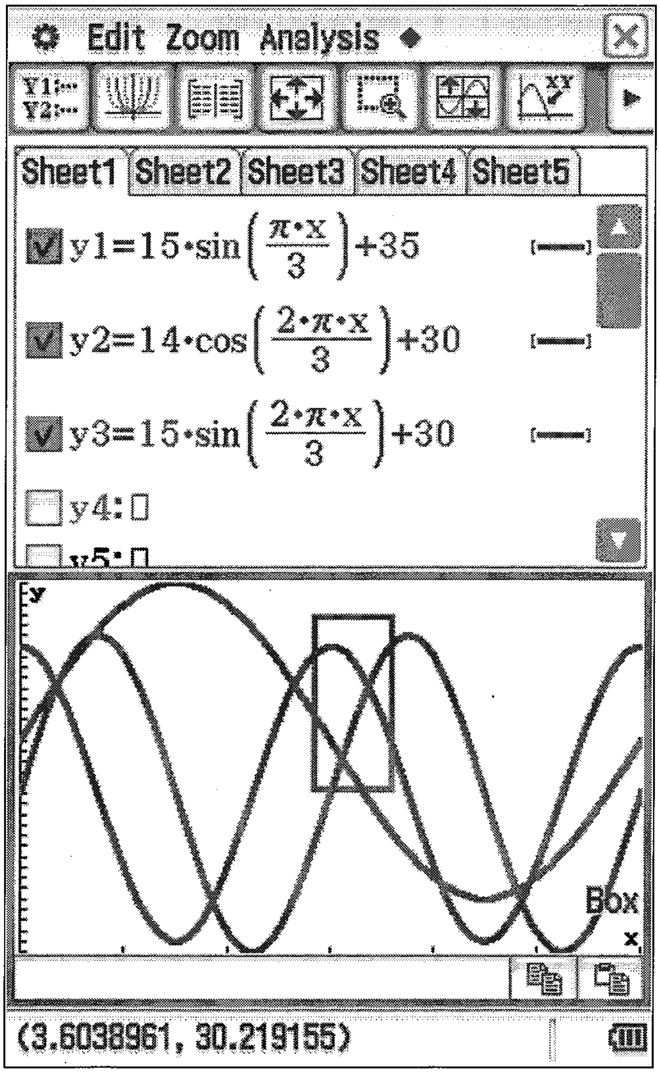
(c) State which of the weights reaches closest to the ground and state the time at which it first reaches this position. (3 marks)

Solution
$h_A = 35 - 15 = 20, \quad h_B = 30 - 14 = 16, \quad h_C = 30 - 15 = 15$ Hence weight C reaches closest to ground. When: $\sin\left(\frac{2\pi t}{3}\right) = -1 \Rightarrow \frac{2\pi t}{3} = \frac{3\pi}{2} \Rightarrow t = \frac{9}{4}$ This first occurs when $t = 2.25$ s.
Specific behaviours
✓ states correct weight ✓ equates trig function to -1 ✓ solves for correct time Needs justification for full marks

(d) Determine the length of time to 3s.f, during the first 6 seconds for which $h_B > h_C > h_A$. (3 marks)

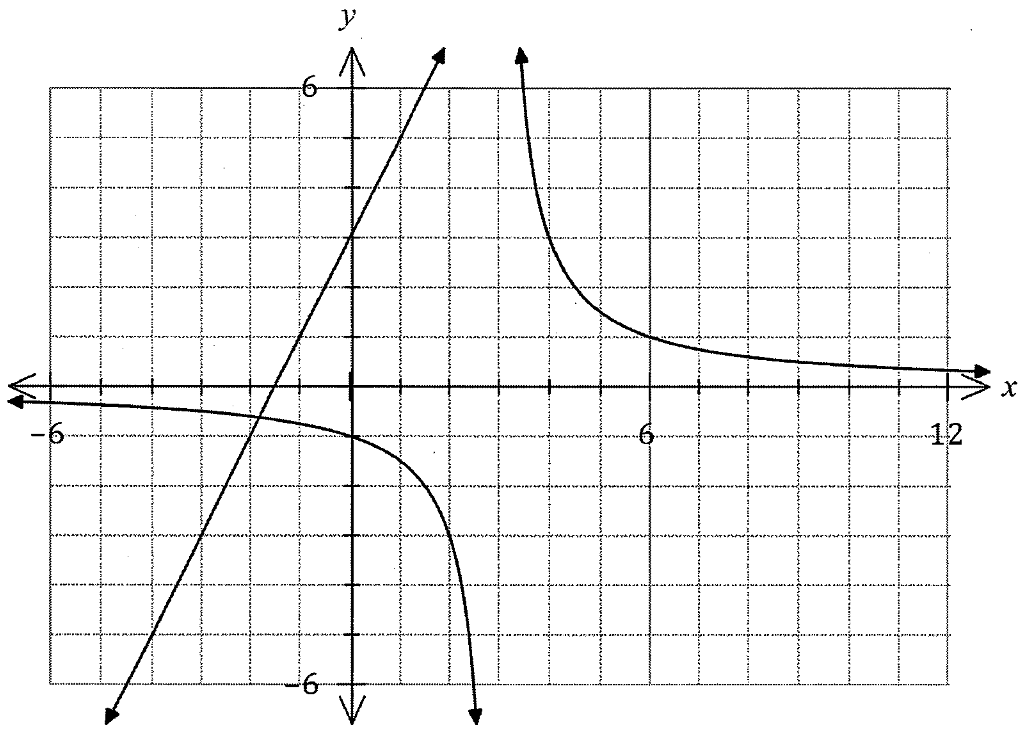
Solution
Use CAS to graph heights and identify required interval. $h_A = h_C \to t = 3.1068$ $h_B = h_C \to t = 3.3585$ Length of time: $\Delta t = 0.2517$ ≈ 0.252 s (3 sf)

Specific behaviours
✓ indicates one endpoint ✓ indicates second endpoint ✓ calculates difference



Question 21 **(9 marks)**

The graph of the hyperbola $y = \dfrac{a}{x+b}$ is shown below, where a and b are constants.



(a) State the equations of all asymptotes of the hyperbola. (2 marks)

Solution
Horizontal: $y = 0$ Vertical: $x = 3$.
Specific behaviours
✓ equation for horizontal asymptote ✓ equation for vertical asymptote

(b) Determine the value of a and the value of b. (2 marks)

Solution
From asymptote, $b = -3$. Using $(0, -1)$: $$-1 = \frac{a}{0-3} \Rightarrow a = 3$$
Specific behaviours
✓ value of a ✓ value of b

(c) Add the line $y = 2x + 3$ to the graph of the hyperbola and state the number of points of intersection it will have with the hyperbola. (2 marks)

Solution
See graph for line. It will have 2 points of intersection with the hyperbola.
Specific behaviours
✓ correct line ✓ correct number of intersections

(d) The line $y = mx + 3$ is tangential to the hyperbola, where m is a constant. Use an algebraic method to determine all possible values of m. (3 marks)

Solution
Require one solution to intersection of lines: $\frac{3}{x-3} = mx + 3$ $3 = (x-3)(mx+3)$ $mx^2 + (3-3m)x - 12 = 0$ For one solution, quadratic discriminant $\Delta = b^2 - 4ac = 0$: $\Delta = (3-3m)^2 - 4(m)(-12) = 0$ Using CAS: $m = -3,\ m = -\frac{1}{3}$.
Specific behaviours
✓ obtains quadratic from equating both lines ✓ uses discriminant to form equation in m ✓ both correct values

End of questions