

Important note to candidates

You do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

No other items may be taken into the examination room. It is **your responsibility** to ensure that

Special items: and up to three calculators approved for use in the WACE examinations

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

To be provided by the candidate

Formula Sheet (referred from Section One)

This Question/Answer Booklet

To be provided by the supervisor

MATERIALS REQUIRED/RECOMMENDED FOR THIS SECTION

Time allowed for this section

Reading time before commencing work: ten minutes

Working time for section: one hundred minutes

Your name _____
In words _____

Student Number: _____ In figures _____

Calculator-assumed
Section Two:

METHODS
UNITS 3 AND 4
MATHEMATICS

Semester Two Examination, 2016
Question/Answer Booklet

Perth Modern School

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total			150	150	100

Additional working space

Question number: _____

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

(2 marks)

- (c) The water supply pipe was seriously compromised when the mussel density reached 85 thousand shellfish per square metre. After how many days from the commencement of observations did this happen? (2 marks)

Solution	$85000 = 200 e^{0.0866 t}$
Specific behaviours	$t = 69.9 \approx 70 \text{ days}$
Substitutes values into equation	
Solves for number of days	

(2 marks)

- (b) Determine the value of k , rounded to four decimal places.

The mussel density was observed to double every eight days.

Solution	$t=0 \Leftrightarrow D=200$
Specific behaviours	$e^k = 2 \Rightarrow k = 0.0866$
Substitutes values into equation	
Solves to required degree of accuracy	

(1 mark)

- (a) What was the mussel density in the colony when observations began?

$$D = 200 e^{-kt}$$

is a positive constant.

Zebra mussels are an invasive species of shellfish recently discovered in some North American waterways. The mussel density, D , in shellfish per square metre, observed in a power station water supply pipe t days after a colony began, was modelled by the following equation, where k

(5 marks)

Question 8

Working time for this section is 100 minutes.

provided.

This section has **thirteen (13)** questions. Answer all questions. Write your answers in the spaces provided.

(6 marks)

Question 9

The speeds of 250 vehicles, on a section of freeway undergoing roadworks with a speed limit of 60 kmh^{-1} , had a mean and standard deviation of 56.9 kmh^{-1} and 3.6 kmh^{-1} respectively. A summary of the data is shown in the table below.

Speed ($x \text{ kmh}^{-1}$)	$45 \leq x < 50$	$50 \leq x < 55$	$55 \leq x < 60$	$60 \leq x < 65$	$65 \leq x < 70$
Relative frequency	0.024	0.272	0.504	0.188	0.012

- (a) Use the table of relative frequencies to estimate the probability that the next vehicle to pass the roadworks

- (i) was not exceeding the speed limit.

(1 mark)

Solution
$0.024 + 0.272 + 0.504 = 0.8$

Specific behaviours
✓ states probability

- (ii) had a speed of less than 65 kmh^{-1} , given they were exceeding the speed limit.

(1 mark)

Solution
$\frac{0.188}{1 - 0.8} = 0.94$

Specific behaviours
✓ calculates probability

- (b) Subsequent tests on the measuring equipment discovered that it had been wrongly calibrated. The correct speed of each vehicle, v , could be calculated from the measured speed, x , by increasing x by 6% and then adding 1.7.

- (i) Calculate the adjusted mean and standard deviation of the vehicle speeds.

(2 marks)

Solution
$\bar{v} = 56.9 \times 1.06 + 1.7 \approx 62.0 \text{ kmh}^{-1}$ $s_d v = 3.6 \times 1.06 \approx 3.82 \text{ kmh}^{-1}$

Specific behaviours
✓ calculates new mean
✓ calculates new sd

- (ii) Determine the correct proportion of vehicles that were speeding.

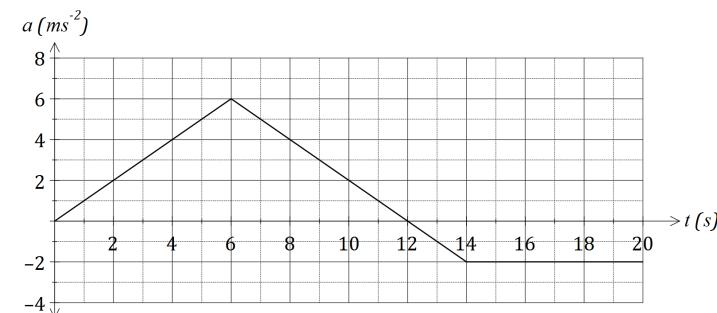
(2 marks)

Solution
$60 = x \times 1.06 + 1.7 \Rightarrow x = 55$ Hence $0.504 + 0.188 + 0.012 = 0.704$ is correct proportion.

Specific behaviours
✓ determines x
✓ states proportion

Question 20

A particle, initially stationary and at the origin, moves subject to an acceleration, $a \text{ ms}^{-2}$, as shown in the graph below for $0 \leq t \leq 20$ seconds.



- (a) Determine the velocity of the object when

- (i) $t = 6$.

Solution
$v = \frac{1}{2} \times 6 \times 6 = 18 \text{ m/s}$

Specific behaviours
✓ calculates area

- (ii) $t = 20$.

Solution
$v _{[20]} = 18 + 18 - 2 - 12 = 36 - 14 = 22 \text{ m/s}$

Specific behaviours
✓ calculates area above axes
✓ calculates area below axes and subtracts from area

- (b) At what time is the velocity of the body a maximum, and what is the maximum velocity?

Solution
When $t = 12$ seconds, $v_{MAX} = 36 \text{ m/s}$

Specific behaviours
✓ identifies time
✓ states maximum velocity

- (c) Determine the distance of the particle from the origin after 3 seconds.

(3 marks)

Solution
$a = t \Rightarrow v = \frac{t^2}{2} \Rightarrow x = \frac{t^3}{6}$ $x _{[3]} = \frac{27}{6} = 4.5 \text{ m}$

Specific behaviours
✓ expresses a in terms of t
✓ integrates twice to obtain displacement

(10 marks)

Question 11

- (a) The graph below shows the curve $y=f(x)$, where $f(x)=\frac{12}{2x-1}$.



Use the five centred rectangles shown to estimate the shaded area under the curve from $x=1.25$ to $x=3.75$. (3 marks)

Solution

$$f[1.5]=6, f[2]=4, f[2.5]=3, f[3]=2.4, f[3.5]=2$$

$$A \approx \frac{1}{2} \times (6+4+3+2.4+2)$$

$$A \approx 8.7 \text{ sq units}$$

Specific behaviours

- ✓ calculates five rectangle heights
- ✓ sums rectangles
- ✓ states area estimate

- (b) Given $\int_a^b h(x) dx = k$ and $h(x)$ is a polynomial, determine the following in terms of the constants a, b and k :

(i) $\int_a^b 3h(x) dx.$

Solution
$3k$

(1 mark)

Specific behaviours
✓ states answer

(ii) $\int_a^b 2-h(x) dx.$

(2 marks)

Solution
$\int_a^b 2-h(x) dx = \int_a^b 2 dx - \int_a^b h(x) dx$ $\therefore 2b - 2a - k$

Specific behaviours
✓ splits integral ✓ simplifies

Question 18

From a random sample of n people, it was found that 54 of them subscribe to a streaming music service. A symmetric confidence interval for the true population proportion who subscribe is $0.1842 < p < 0.2958$.

- (a) Determine the value of n , by first finding the mid-point of the interval. (3 marks)

Solution

$$\frac{0.1842+0.2958}{2} = 0.24 \quad p = 0.24 = \frac{54}{n}$$

$$n = 54 \div 0.24 = 225$$

Specific behaviours

- ✓ calculates mid-point
- ✓ writes equation using mid-point for p
- ✓ determines n

- (b) Determine the confidence level of the interval. (4 marks)

Solution

Standard error:

$$\sqrt{\frac{0.24 \times (1-0.24)}{225}} = 0.02847$$

$$0.24 + z \times 0.02847 = 0.2958$$

$$z = 1.96$$

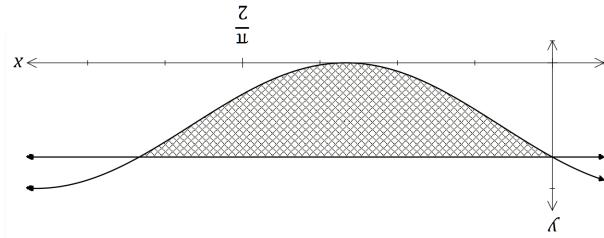
Hence a 95% confidence interval

Specific behaviours

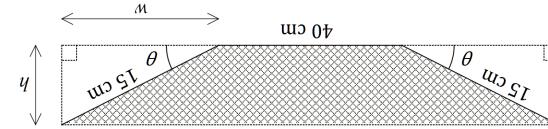
- ✓ calculates standard error
- ✓ uses interval formula

See next page

- Question 17
A trough for holding water is to be formed by taking a length of metal sheet 70 cm wide and folding 15 cm on either end, up through an angle of θ . The following diagram shows the cross-section of the trough with the cross-sectional area, A , shaded.
- (c) The graphs of $y = \cos^2 x + \frac{\pi}{6}$ and $y = \frac{4}{3}$ are shown below. Determine the exact area of the shaded region they enclose. (4 marks)



- Question 17
A trough for holding water is to be formed by taking a length of metal sheet 70 cm wide and folding 15 cm on either end, up through an angle of θ . The following diagram shows the cross-section of the trough with the cross-sectional area, A , shaded.
- (a) Determine A in terms of w and h . (1 mark)



- (b) Show that $A = 600 \sin \theta + 225 \sin \theta \cos \theta$. (2 marks)

Solution
$w = 15 \cos \theta$ and $h = 15 \sin \theta$
$A = 40 \times 15 \sin \theta + 15 \cos \theta \times 15 \sin \theta$
$A = 600 \sin \theta + 225 \sin \theta \cos \theta$
Specific behaviours
writes expressions for w and h in terms of θ
substitutes and simplifies into expression from

- (c) Use calculus to determine the maximum possible cross-sectional area. (4 marks)

Solution
$\frac{dA}{d\theta} = 600 \cos \theta + 225 (\cos^2 \theta - \sin^2 \theta)$
$\frac{dA}{d\theta} = 0$ when $\theta = 1.26$ (1.26) $= 636.77$ $A \approx 637$ sq cm
Specific behaviours
differentiates
solves derivative equal to zero
substitutes optimum value into area formula
evaluates rounded area

(8 marks)

Question 12

A box contains a large number of packets of buttons. The number of buttons in a packet may be modelled by the random variable X , with the probability distribution shown below. It is also known that $E(X) = 6.25$.

x	3 or fewer	4	5	6	7	8	9 or more
$P(X=x)$	0	0.05	a	b	0.25	0.15	0

- (a) Two packets are randomly chosen from the box. Determine the probability that there are at least 15 buttons altogether in the two packets. (2 marks)

Solution
$P=0.25 \times 0.15 + 0.15 \times 0.25 + 0.15 \times 0.15$
$P=0.0975$
Specific behaviours

- ✓ chooses (7,8), (8,7) and (8,8)
- ✓ calculates probability

- (b) Determine the values of a and b . (3 marks)

Solution
From sum of probabilities, $a+b=1-0.45=0.55$
From $E(X)$, $5a+6b=6.25-3.15=3.1$
Solve simultaneously to get $a=0.2$, $b=0.35$
Specific behaviours

- ✓ uses sum to 1
- ✓ uses $E(X)=6.25$
- ✓ solves for a and b

- (c) Calculate $\text{Var}(X)$. (1 mark)

Solution
Using technology, $\text{Var}(X)=1.1875$
Specific behaviours

- ✓ calculates variance

- (d) As part of a fundraiser, patrons pay 75 cents to select a packet at random and then win back 10 cents for each button in the packet. If the random variable W represents the net gain per game for a patron in cents, determine the mean and variance of W . (2 marks)

Solution
$E(W)=10 \times E(X)-75=10 \times 6.25-75=-12.5$
$\text{Var}(W)=10^2 \times \text{Var}(X)=118.75$
Specific behaviours

- ✓ calculates mean
- ✓ calculates variance

See next page

- (b) The stationery company that supplies pens to the conference centre claim that no more than 3 in 50 pens fail to write. Use your previous working to comment on the validity of this claim. (2 marks)

Solution
$3 \div 50=0.06$
The interval calculated in (a) contains 0.06 and so the claim is valid.

- ✓ compares proportion to confidence interval.
- ✓ states claim is valid

- (c) Comment on how the margin of error would change in (a) (ii) if (1 mark)

- (i) the quality of the pens had been better.

Solution
Decrease, as p is further from 0.5.
Specific behaviours

- ✓ states change

- (ii) the required level of confidence decreased. (1 mark)

Solution
Decrease, as z-score lower.
Specific behaviours

- ✓ states change

(1 mark)

See next page

(7 marks)

- A hardware store sells stakes, of nominal length 1.8 metres, to be used for supporting newly planted trees. The length, X metres, of these stakes can be modelled by a normal distribution with mean 1.85 and standard deviation σ .

- (a) If $\sigma = 0.035$, determine

- (i) the probability that a randomly chosen stake is shorter than 1.8 metres. (1 mark)
- | |
|-------------------------------------|
| Solution |
| $P(X < 1.8) = 0.0766$ |
| Specific behaviours |
| \checkmark calculates probability |
- (ii) the probability that a randomly chosen stake is longer than 1.79 m given that it is shorter than 1.8 metres. (2 marks)
- | |
|--|
| Solution |
| $P(1.79 < X < 1.8) = P(X < 1.8) - P(X < 1.79)$ |
| $= 0.033 - 0.0766 \approx 0.435$ |
| Specific behaviours |
| \checkmark calculates numerical probability |
- (iii) the value of k , if the longest 15% of stakes exceed k metres in length. (1 mark)
- | |
|---|
| Solution |
| $P(X > k) = 0.15 \Leftrightarrow k = 1.886$ |
| Specific behaviours |
| \checkmark determines k |

- (b) A large number of stakes were measured and it was found that 97% of them were longer than their nominal length. Show how to use this information to deduce that the value of σ is 0.027 when rounded to three decimal places. (3 marks)
- | |
|---|
| Solution |
| $P(Z > z) = 0.97 \Rightarrow z = -1.881$ |
| $-1.881 = -\frac{0.05}{\sigma}$ |
| $\sigma = \frac{0.05}{-1.881} = 0.02658$ |
| $0.02658 \approx 0.027$ (3 dp) |
| Specific behaviours |
| \checkmark shows use of standardising formula |
| \checkmark shows Z-score for 97% |
| \checkmark solves equation more than 3 dp |

- (c) A large number of stakes were measured and it was found that 97% of them were longer than their nominal length. Show how to use this information to deduce that the value of σ is 0.027 when rounded to three decimal places. (3 marks)

- (d) A large number of stakes were measured and it was found that 97% of them were longer than their nominal length. Show how to use this information to deduce that the value of σ is 0.027 when rounded to three decimal places. (3 marks)
- | |
|---|
| Solution |
| $P(Z > z) = 0.97 \Rightarrow z = -1.881$ |
| $-1.881 = -\frac{0.05}{\sigma}$ |
| $\sigma = \frac{0.05}{-1.881} = 0.02658$ |
| $0.02658 \approx 0.027$ (3 dp) |
| Specific behaviours |
| \checkmark shows use of standardising formula |
| \checkmark shows Z-score for 97% |
| \checkmark solves equation more than 3 dp |

(9 marks)

- (a) If p is the true proportion of pens that fail to write and \hat{p} is the corresponding sample proportion, use the above sample to determine

- (b) The management at a conference centre was concerned about the quality of the free pens that it provided in its meeting rooms. A staff member tested a random sample of 150 pens and found that 18 of them fail to write.
- The management at a conference centre was concerned about the quality of the free pens that it provided in its meeting rooms. A staff member tested a random sample of 150 pens and found that 18 of them fail to write.

- (c) If p is the true proportion of pens that fail to write and \hat{p} is the corresponding sample proportion of pens that fail to write.

- (d) An approximate 98% confidence interval for p . (1 mark)
- | |
|---|
| Solution |
| $0.12 \pm 0.0617 \approx 0.0583 < \hat{p} < 0.1817$ |
| Specific behaviours |
| \checkmark evaluates interval |
- (e) An approximate 98% confidence interval for p . (3 marks)
- | |
|--|
| Solution |
| $98\% \Rightarrow z = 2.326$ |
| $se = \sqrt{\frac{0.12(1-0.12)}{150}} \approx 0.02653$ |
| $E = 2.326 \times 0.02653 \approx 0.0617$ |
| Specific behaviours |
| \checkmark calculates Z-score |
| \checkmark calculates margin of error |

(8 marks)

Question 14

The random variable X denotes the number of hours that a business telephone line is in use per nine hour working day.

The probability density function of X is given by $f(x) = \begin{cases} \frac{(x-a)^2+b}{k} & 0 \leq x \leq 9 \\ 0 & \text{otherwise} \end{cases}$,

where a , b and k are constants.

- (a) If $a=15$ and $b=3$, determine the value of k .

(2 marks)

Solution
$\int_0^9 \frac{(x-15)^2+3}{k} dx = 1 \Rightarrow \frac{1080}{k} = 1$
Hence $k=1080$.
Specific behaviours
✓ writes correct integral ✓ evaluates integral and states value of

- (b) Let $a=16$, $b=1$ and $k=1260$.

- (i) The business is open for work for 308 days per year. On how many of these days can the business expect the phone line to be in use for more than eight hours?

(2 marks)

Solution
$\int_8^9 \frac{(x-16)^2+1}{1260} dx = 0.0455$
$308 \times 0.0455 = 14$ days
Specific behaviours
✓ evaluates integral ✓ calculates number of days

- (ii) Determine, correct to two decimal places, the mean and variance of X . (4 marks)

Solution
$E(X) = \int_0^9 x \times \frac{(x-16)^2+1}{1260} dx = 3.39$
$Var(X) = \int_0^9 (x-3.39)^2 \times \frac{(x-16)^2+1}{1260} dx = 5.78$
Specific behaviours
✓ writes integral for mean ✓ determines mean ✓ writes integral for variance ✓ determines variance

(8 marks)

An analysis of the number of dogs registered by each household within a suburb resulted in the following information:

- (a) Number of dogs registered
- | | | | |
|---|---|---|-----------|
| 0 | 1 | 2 | 3 or more |
|---|---|---|-----------|
- Percentage of households
- | | | | |
|----|----|----|---|
| 21 | 44 | 27 | 8 |
|----|----|----|---|
- A council worker selects households at random to visit. What is the probability that the first five households visited all have at least one dog registered? (2 marks)

Solution
$p = 1 - 0.21 = 0.79$
$0.79^5 = 0.3077$

Specific behaviours

- ✓ calculates probability one household has at least one dog
- ✓ calculates probability

- (b) A random sample of 40 households within the suburb is selected.

Use a binomial distribution with $n=40$, together with relevant information from the table in each case, to determine the probability that the sample contains:

- (i) exactly 6 households with no dogs registered. (2 marks)

Solution
$X \sim B(40, 0.21)$
$P(X=6) = 0.1088$

Specific behaviours

- ✓ uses correct p
- ✓ calculates probability

- (ii) no more than 15 households with at least two dogs registered. (2 marks)

Solution
$0.27 + 0.08 = 0.35$
$X \sim B(40, 0.35)$
$P(X \leq 15) = 0.6946$

Specific behaviours

- ✓ uses correct p
- ✓ calculates probability

- (c) A random sample of 25 households within the city is to be selected. If X is the number of households in the sample that have exactly one dog registered, determine the mean and variance of X . (2 marks)

Solution
$n=25, p=0.44, \bar{x}=25 \times 0.44=11$
$\sigma^2=11 \times (1-0.44)=6.16$

Specific behaviours

- ✓ calculates mean
- ✓ calculates variance