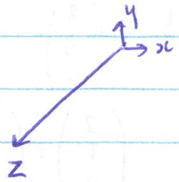
SADLER UNIT 3. CHAPTER 5

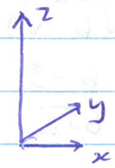
EXERCISE 5A

Q1

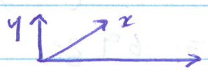
a)



b)



c)



Q2 $a = \begin{pmatrix}2\\6\\3\end{pmatrix}$ $b = \begin{pmatrix}3\\8\\-1\end{pmatrix}$

a) $a + b$

$= \begin{pmatrix}2\\6\\3\end{pmatrix} + \begin{pmatrix}3\\8\\-1\end{pmatrix}$

$= \begin{pmatrix}5\\14\\2\end{pmatrix} = 5i + 14j + 2k$

b) $a - b$

$= \begin{pmatrix}2\\6\\3\end{pmatrix} - \begin{pmatrix}3\\8\\-1\end{pmatrix}$

$= \begin{pmatrix}-1\\-2\\4\end{pmatrix} = -i - 2j + 4k$

c) $2a + b$

$= 2\begin{pmatrix}2\\6\\3\end{pmatrix} + \begin{pmatrix}3\\8\\-1\end{pmatrix}$

$= \begin{pmatrix}7\\20\\5\end{pmatrix} = 7i + 20j + 5k$

d) $2(a + b)$

$= 2\begin{pmatrix}5\\14\\2\end{pmatrix}$

$= \begin{pmatrix}10\\28\\4\end{pmatrix} = 10i + 28j + 4k$

e) $a \cdot b$

$= \begin{pmatrix}2\\6\\3\end{pmatrix} \cdot \begin{pmatrix}3\\8\\-1\end{pmatrix}$

$= 6 + 14 - 3 = \underline{17}$

f) $b \cdot a = a \cdot b$

$= \underline{17}$

g) $|a|$

$= \sqrt{2^2 + 6^2 + 3^2}$

$= \sqrt{49}$

$= \underline{7}$

h) $|a + b|$

$= \left|\begin{pmatrix}5\\14\\2\end{pmatrix}\right|$

$= \sqrt{25 + 196 + 4}$

$= \sqrt{225}$

$= \underline{15}$

Q3 $c = \begin{pmatrix}-1\\4\\3\end{pmatrix}$ $d = \begin{pmatrix}2\\0\\4\end{pmatrix}$

a) $\begin{pmatrix}-1\\4\\3\end{pmatrix} + \begin{pmatrix}2\\0\\4\end{pmatrix}$

$= \begin{pmatrix}1\\4\\7\end{pmatrix}$

b) $c - d$

$= \begin{pmatrix}-1\\4\\3\end{pmatrix} - \begin{pmatrix}2\\0\\4\end{pmatrix}$

$= \begin{pmatrix}-3\\4\\-1\end{pmatrix}$

c) $2c + d$

$= \begin{pmatrix}-2\\8\\6\end{pmatrix} + \begin{pmatrix}2\\0\\4\end{pmatrix}$

$= \begin{pmatrix}0\\8\\14\end{pmatrix}$

d) $2(c + d)$

$= 2\begin{pmatrix}1\\4\\7\end{pmatrix}$

$= \begin{pmatrix}2\\8\\14\end{pmatrix}$

e) $c \cdot d$

$= \begin{pmatrix}-1\\4\\3\end{pmatrix} \cdot \begin{pmatrix}2\\0\\4\end{pmatrix}$

$= -2 + 12$

$= \underline{10}$

f) $d \cdot c = c \cdot d$

$= \underline{10}$

g) $|c|$

$= \sqrt{1 + 16 + 9}$

$= \underline{\sqrt{26}}$

h) $|c + d|$

$= \sqrt{1 + 16 + 49}$

$= \underline{\sqrt{66}}$

Q4 $e = \begin{pmatrix}1\\4\\-3\end{pmatrix}$ $f = \begin{pmatrix}-1\\2\\0\end{pmatrix}$

a) $e - f$

$= \begin{pmatrix}1\\4\\-3\end{pmatrix} - \begin{pmatrix}-1\\2\\0\end{pmatrix}$

$= \begin{pmatrix}2\\2\\-3\end{pmatrix}$

b) $e - 2f$

$= \begin{pmatrix}1\\4\\-3\end{pmatrix} - 2\begin{pmatrix}-1\\2\\0\end{pmatrix}$

$= \begin{pmatrix}3\\0\\-3\end{pmatrix}$

c) $2e + f$

$= \begin{pmatrix}2\\8\\-6\end{pmatrix} + \begin{pmatrix}-1\\2\\0\end{pmatrix}$

$= \begin{pmatrix}1\\10\\-6\end{pmatrix}$

d) $e + f$

$= \begin{pmatrix}1\\4\\-3\end{pmatrix} + \begin{pmatrix}-1\\2\\0\end{pmatrix}$

$= \begin{pmatrix}0\\6\\-3\end{pmatrix}$

e) $e \cdot f$

$= \begin{pmatrix}1\\4\\-3\end{pmatrix} \cdot \begin{pmatrix}-1\\2\\0\end{pmatrix}$

$= -1 + 8$

$= \underline{7}$

f) $2e \cdot 3f$

$= 6\, e \cdot f$

$= \underline{42}$

g) $(e - f) \cdot (e - f)$

$= |e - f|^2$

$= \left|\begin{pmatrix}2\\2\\-3\end{pmatrix}\right|^2$

$= 4 + 4 + 9$

$= \underline{17}$

h) $|e - f|$

$= \underline{\sqrt{17}}$

Q5

$\vec{OA} = \begin{pmatrix}2\\3\\-4\end{pmatrix}$

$\vec{OB} = \begin{pmatrix}3\\2\\1\end{pmatrix}$

$\vec{OC} = \begin{pmatrix}-5\\-1\\1\end{pmatrix}$

a) $\vec{AB} = \vec{AO} + \vec{OB}$

$= \vec{OB} - \vec{OA}$

$= \begin{pmatrix}3\\2\\1\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix}$

$= \begin{pmatrix}1\\-1\\5\end{pmatrix}$

$= i - j + 5k$

b) $\vec{BC} = \vec{BO} + \vec{OC}$

$= \vec{OC} - \vec{OB}$

$= \begin{pmatrix}-5\\-1\\1\end{pmatrix} - \begin{pmatrix}3\\2\\1\end{pmatrix}$

$= \begin{pmatrix}-8\\-3\\0\end{pmatrix}$

$\therefore -8i - 3j + 0k$

c) $\vec{CA} = \vec{CO} + \vec{OA}$

$= \vec{OA} - \vec{OC}$

$= \begin{pmatrix}2\\3\\-4\end{pmatrix} - \begin{pmatrix}-5\\-1\\1\end{pmatrix}$

$= \begin{pmatrix}7\\4\\-5\end{pmatrix}$

$\therefore 7i + 4j - 5k$

d) $\vec{AC} = -\vec{CA}$

$= -\begin{pmatrix} 7 \\ 4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ -4 \\ 5 \end{pmatrix}$

$\therefore -7\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$

Q6

a) $\underset{\sim}{p} + \underset{\sim}{q}$

$= \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 1 \\ 4 \end{pmatrix}$

b) $\underset{\sim}{q} + \underset{\sim}{r}$

$= \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -1 \\ 4 \end{pmatrix}$

c) $(\underset{\sim}{p} + \underset{\sim}{q}) \cdot (\underset{\sim}{q} + \underset{\sim}{r})$

$= \begin{pmatrix} 7 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ 4 \end{pmatrix}$

$= 42 - 1 + 16$

$= \underline{\underline{57}}$

Q7. $\underset{\sim}{u} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$

$\underset{\sim}{v} = \begin{pmatrix} 2 \\ 14 \\ 5 \end{pmatrix}$

a) $|\underset{\sim}{u}| = \sqrt{9+4+36}$

$= \sqrt{49}$

$= \underline{7}$

b) $|\underset{\sim}{v}| = \sqrt{4+196+25}$

$= \sqrt{225}$

$= \underline{15}$

c) $\underset{\sim}{u} \cdot \underset{\sim}{v} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 14 \\ 5 \end{pmatrix}$

$= 6 - 28 + 36$

$= \underline{\underline{8}}$

d) $\cos\theta = \dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{u}||\underset{\sim}{v}|}$

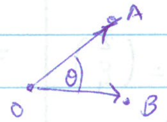
$= \dfrac{8}{7 \times 15}$

$\theta = \cos^{-1}\left(\dfrac{8}{105}\right)$

$= \underline{\underline{85.63^\circ}}$

Q8. $\vec{OA} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$\vec{OB} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$



$\cos\theta = \dfrac{\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}}{\sqrt{3}\ \sqrt{9}}$

$= \dfrac{2-1-2}{3\sqrt{3}}$

$\theta = \cos^{-1}\left(\dfrac{-1}{3\sqrt{3}}\right)$

$= 101.10$

$\approx \underline{\underline{101^\circ}}$

Q9. $\underset{\sim}{p} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$

$\underset{\sim}{q} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$

$\therefore \cos\theta = \dfrac{\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}}{\sqrt{6}\ \sqrt{6}}$

$= \dfrac{1+2-2}{6}$

$\theta = \cos^{-1}\left(\dfrac{1}{6}\right)$

$= 80.41$

$\approx \underline{\underline{80^\circ}}$

Q10. $\underset{\sim}{s} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

$\underset{\sim}{t} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$

$\cos\theta = \dfrac{\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}}{\sqrt{6}\ \sqrt{18}}$

$= \dfrac{6-3}{6\sqrt{2}}$

$= \dfrac{1}{2\sqrt{2}}$

$\theta = \cos^{-1}\left(\dfrac{1}{2\sqrt{2}}\right)$

$= 69.30$

$\approx \underline{\underline{69^\circ}}$

Q11. $\underset{\sim}{r} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ $\underset{\sim}{s} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$

a) $\hat{\underset{\sim}{r}} = \dfrac{1}{|\underset{\sim}{r}|}\underset{\sim}{r}$

$= \dfrac{1}{\sqrt{4+9+36}} \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$

$= \dfrac{1}{7}\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$

$= \dfrac{2}{7}\underset{\sim}{i} - \dfrac{3}{7}\underset{\sim}{j} + \dfrac{6}{7}\underset{\sim}{k}$

b) $|\underset{\sim}{s}| = \sqrt{9+16}$

$= 5$

$\therefore |\underset{\sim}{s}|\hat{\underset{\sim}{r}} = \dfrac{10}{7}\underset{\sim}{i} - \dfrac{15}{7}\underset{\sim}{j} + \dfrac{30}{7}\underset{\sim}{k}$

c) $\hat{\underset{\sim}{s}} = \dfrac{1}{|\underset{\sim}{s}|}\underset{\sim}{s}$

$= \dfrac{1}{5}\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$

$= \dfrac{3}{5}\underset{\sim}{i} + \dfrac{4}{5}\underset{\sim}{k}$

$\therefore |\underset{\sim}{r}|\hat{\underset{\sim}{s}} = \dfrac{21}{5}\underset{\sim}{i} + \dfrac{28}{5}\underset{\sim}{k}$

d) $\cos\theta = \dfrac{\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}}{7 \times 5}$

$= \dfrac{6+24}{35}$

$\theta = \cos^{-1}\left(\dfrac{30}{35}\right)$

$= 31.00$

$\approx \underline{\underline{31^\circ}}$

Q12 a) $\begin{pmatrix}2\\-3\\1\end{pmatrix}, \begin{pmatrix}4\\-6\\2\end{pmatrix}$

are parallel as one is a scalar multiple of the other.

b) $\begin{pmatrix}3\\2\\-1\end{pmatrix} \cdot \begin{pmatrix}1\\-1\\3\end{pmatrix}$

$= 3 - 2 - 3$

$= -2$

Neither perpendicular nor parallel.

c) $\begin{pmatrix}1\\3\\-2\end{pmatrix} \cdot \begin{pmatrix}-2\\3\\1\end{pmatrix}$

$= -2 + 9 - 2$

$= 5$

Neither perpendicular nor parallel.

d) $\begin{pmatrix}1\\2\\3\end{pmatrix} \cdot \begin{pmatrix}3\\3\\-3\end{pmatrix}$

$= 3 + 6 - 9$

$= 0$

$\therefore$ Perpendicular.

e) $\begin{pmatrix}3\\2\\-1\end{pmatrix} \cdot \begin{pmatrix}5\\-7\\1\end{pmatrix}$

$= 15 - 14 - 1$

$= 0$

$\therefore$ Perpendicular.

f) $\begin{pmatrix}-2\\6\\8\end{pmatrix} \cdot \begin{pmatrix}1\\-3\\4\end{pmatrix}$

$= -2 - 18 + 32$

$= 12$

Neither perpendicular nor parallel.

g) $\begin{pmatrix}3\\1\\5\end{pmatrix} \cdot \begin{pmatrix}6\\2\\-4\end{pmatrix}$

$= 18 + 2 - 20$

$= 0$

Perpendicular.

Q13

$\underset{\sim}{F}_1 = \begin{pmatrix}5\\10\\5\end{pmatrix}$

$\underset{\sim}{F}_2 = \begin{pmatrix}3\\5\\5\end{pmatrix}$

$\underset{\sim}{F}_3 = \begin{pmatrix}-2\\3\\-1\end{pmatrix}$

$\underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 = \underset{\sim}{R}$

$= \begin{pmatrix}5+3-2\\10+5+3\\5+5-1\end{pmatrix}$

$= \begin{pmatrix}6\\18\\9\end{pmatrix}$

$\therefore |\underset{\sim}{R}| = \left|\begin{pmatrix}6\\18\\9\end{pmatrix}\right|$

$= \sqrt{36+324+81}$

$= \sqrt{441}$

$= \underline{\underline{21}}$ N

Q14. $\vec{OA} = \begin{pmatrix}2\\3\\-4\end{pmatrix}$

$\vec{BA} = \begin{pmatrix}-1\\3\\4\end{pmatrix}$

$\vec{BA} = \vec{BO} + \vec{OA}$

$= \vec{OA} - \vec{OB}$

$\vec{OB} = \vec{OA} - \vec{BA}$

$= \begin{pmatrix}2\\3\\-4\end{pmatrix} - \begin{pmatrix}-1\\3\\4\end{pmatrix}$

$= \underline{\underline{\begin{pmatrix}3\\0\\-8\end{pmatrix}}}$

$\therefore 3\underset{\sim}{i} - 8\underset{\sim}{k}$

Q15 $\underset{\sim}{a} + \underset{\sim}{b} = \begin{pmatrix}7\\1\\2\end{pmatrix}$

$+ \quad \underset{\sim}{a} - \underset{\sim}{b} = \begin{pmatrix}3\\3\\-4\end{pmatrix}$

$2\underset{\sim}{a} = \begin{pmatrix}10\\4\\-2\end{pmatrix}$

$\underset{\sim}{a} = \begin{pmatrix}5\\2\\-1\end{pmatrix}$

$\therefore \underset{\sim}{b} = \begin{pmatrix}7\\1\\2\end{pmatrix} - \begin{pmatrix}5\\2\\-1\end{pmatrix} = \underline{\underline{\begin{pmatrix}2\\-1\\3\end{pmatrix}}}$

Q16 $\underset{\sim}{a} = k\underset{\sim}{b}$

$\begin{pmatrix}2\\3\\1\end{pmatrix} = k\begin{pmatrix}4\\6\\p\end{pmatrix}$

$k = \frac{1}{2}$

$\therefore \underline{\underline{p = \frac{1}{2}}}$

$\underset{\sim}{c} \cdot \underset{\sim}{a} = 0$

$\begin{pmatrix}7\\q\\-2\end{pmatrix} \cdot \begin{pmatrix}2\\3\\1\end{pmatrix} = 0$

$14 + 3q - 2 = 0$

$12 = -3q$

$\underline{\underline{q = -4}}$

$\underset{\sim}{d} \cdot \underset{\sim}{b} = 0$

$\begin{pmatrix}3\\-4\\r\end{pmatrix} \cdot \begin{pmatrix}4\\6\\p\end{pmatrix}$

$12 - 24 + \frac{1}{2}r = 0$

$-12 = -\frac{1}{2}r$

$\underline{\underline{r = 24}}$

Q17. $\underset{\sim}{r} = \begin{pmatrix}-4\\-4\\11\end{pmatrix} + t\begin{pmatrix}2\\4\\-2\end{pmatrix}$

a) $\underset{\sim}{r} = \begin{pmatrix}-4\\-4\\11\end{pmatrix} + \begin{pmatrix}2\\4\\-2\end{pmatrix}$

$= \underline{\underline{\begin{pmatrix}-2\\0\\9\end{pmatrix}}}$

b) $\underset{\sim}{r} = \begin{pmatrix}-4\\-4\\11\end{pmatrix} + \begin{pmatrix}4\\8\\-4\end{pmatrix}$

$= \underline{\underline{\begin{pmatrix}0\\4\\7\end{pmatrix}}}$

c) $\underset{\sim}{r} = \begin{pmatrix}-4\\-4\\11\end{pmatrix} + 3\begin{pmatrix}2\\4\\-2\end{pmatrix}$

$= \begin{pmatrix}2\\8\\5\end{pmatrix}$

$\left|\begin{pmatrix}2\\8\\5\end{pmatrix}\right| = \sqrt{4+64+25}$

$= \underline{\underline{\sqrt{93}}}$ m $= \underline{\underline{9.64}}$ m

d) $\left|\begin{pmatrix}-4\\-4\\11\end{pmatrix}+t\begin{pmatrix}2\\4\\-2\end{pmatrix}\right|=15$

$(-4+2t)^2+(-4+4t)^2+(11-2t)^2=225$

$16-16t+4t^2+16-32t+16t^2+121-44t+4t^2=225$

$24t^2-92t-72=0$

Using the CAS,

$t=-\frac{2}{3},\ 4.5$

$\therefore$ 4.5 seconds after leaving.

Q18 $r_{AB}=\begin{pmatrix}7\\5\\0\end{pmatrix}+\lambda\begin{pmatrix}3-7\\1-5\\-4-0\end{pmatrix}$

$=\begin{pmatrix}7\\5\\0\end{pmatrix}+\lambda\begin{pmatrix}-4\\-4\\-4\end{pmatrix}$

Check $\begin{pmatrix}2\\0\\-5\end{pmatrix}=\begin{pmatrix}7\\5\\0\end{pmatrix}+\lambda\begin{pmatrix}-4\\-4\\-4\end{pmatrix}$

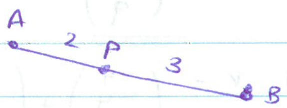
$\begin{pmatrix}-5\\-5\\-5\end{pmatrix}=\lambda\begin{pmatrix}-4\\-4\\-4\end{pmatrix}$

$\lambda=\frac{5}{4}$

$\therefore$ A, B and C are collinear.

Q19. $\vec{OA}=\begin{pmatrix}3\\4\\4\end{pmatrix}$

$\vec{OB}=\begin{pmatrix}-2\\9\\-1\end{pmatrix}$



$\vec{AP}=\frac{2}{5}\vec{AB}$

$\vec{AO}+\vec{OP}=\frac{2}{5}(\vec{AO}+\vec{OB})$

$\vec{OP}=\frac{2}{5}\vec{AO}+\frac{2}{5}\vec{OB}-\vec{AO}$

$=\frac{2}{5}\vec{OB}+\frac{3}{5}\vec{OA}$

$=\frac{1}{5}\left(\begin{pmatrix}-4\\18\\-2\end{pmatrix}+\begin{pmatrix}9\\12\\12\end{pmatrix}\right)$

$=\frac{1}{5}\begin{pmatrix}5\\30\\10\end{pmatrix}$

$=\begin{pmatrix}1\\6\\2\end{pmatrix}$

Q20. $\vec{OA}=\begin{pmatrix}3\\2\\-1\end{pmatrix}$ $\vec{OB}=\begin{pmatrix}4\\-1\\1\end{pmatrix}$

$\vec{AB}=\vec{BP}$

$\vec{AO}+\vec{OB}=\vec{BO}+\vec{OP}$

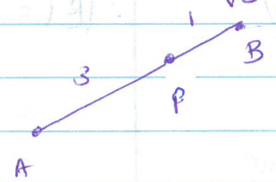
$\vec{OB}-\vec{OA}=-\vec{OB}+\vec{OP}$

$\vec{OP}=2\vec{OB}-\vec{OA}$

$=\begin{pmatrix}8\\-2\\2\end{pmatrix}-\begin{pmatrix}3\\2\\-1\end{pmatrix}$

$=\begin{pmatrix}5\\-4\\3\end{pmatrix}=5\underset{\sim}{i}-4\underset{\sim}{j}+3\underset{\sim}{k}$

Q21. $\vec{OA}=\begin{pmatrix}5\\-2\\3\end{pmatrix}$ $\vec{OB}=\begin{pmatrix}9\\6\\-9\end{pmatrix}$



$\vec{AP}=\frac{3}{4}\vec{AB}$

$\vec{AO}+\vec{OP}=\frac{3}{4}(\vec{AO}+\vec{OB})$

$=\frac{3}{4}(\vec{OB}-\vec{OA})$

$=\frac{3}{4}\begin{pmatrix}9-5\\6+2\\-9-3\end{pmatrix}$

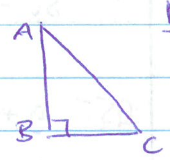
$\vec{OP}=\frac{3}{4}\begin{pmatrix}4\\8\\-12\end{pmatrix}-\vec{AO}$

$=\begin{pmatrix}3\\6\\-9\end{pmatrix}+\begin{pmatrix}5\\-2\\3\end{pmatrix}$

$=\begin{pmatrix}8\\4\\-6\end{pmatrix}=8\underset{\sim}{i}+4\underset{\sim}{j}-6\underset{\sim}{k}$

Q22. $\vec{OA}=\begin{pmatrix}2\\3\\2\end{pmatrix}$ $\vec{OB}=\begin{pmatrix}0\\0\\3\end{pmatrix}$ $\vec{OC}=\begin{pmatrix}4\\-3\\2\end{pmatrix}$

RTP: $\vec{AB}\cdot\vec{BC}=0$.



Proof: $\vec{AB}=\vec{AO}+\vec{OB}$

$=\vec{OB}-\vec{OA}$

$=\begin{pmatrix}0\\0\\3\end{pmatrix}-\begin{pmatrix}2\\3\\2\end{pmatrix}$

$=\begin{pmatrix}-2\\-3\\1\end{pmatrix}$

$\vec{BC}=\vec{BO}+\vec{OC}$

$=\vec{OC}-\vec{OB}=\begin{pmatrix}4\\-3\\2\end{pmatrix}-\begin{pmatrix}0\\0\\3\end{pmatrix}=\begin{pmatrix}4\\-3\\-1\end{pmatrix}$

$$\begin{pmatrix}-2\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}4\\-3\\-1\end{pmatrix}$$

$= -8+9-1$

$= 0$

$\therefore$ $\overrightarrow{AB}$ is perpendicular to BC and so ΔABC is a right-angled triangle.

Q23.

With the x axis, $\rightarrow$

$$\cos\theta = \frac{\begin{pmatrix}2\\3\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}}{\sqrt{14}\ \sqrt{1}}$$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$$

$= 57.69°$

$\approx \underline{\underline{58°}}$

With the y axis, $\nearrow$

$$\cos\theta = \frac{\begin{pmatrix}2\\3\\-1\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}}{\sqrt{14}\cdot\sqrt{1}}$$

$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right)$$

$= 36.70$

$\approx \underline{37°}$

With the z axis $\uparrow$

$$\cos\theta = \frac{\begin{pmatrix}2\\3\\-1\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{14}\ \sqrt{1}}$$

$$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{14}}\right)$$

$= 105.50$

$\therefore 180-105.50 = 74.50$

$\approx \underline{\underline{75°}}$

Q24. $\underset{\sim}{a} = \begin{pmatrix}1\\-2\\3\end{pmatrix}$ $\quad \underset{\sim}{b} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$ $\quad \underset{\sim}{c} = \begin{pmatrix}4\\-1\\3\end{pmatrix}$

$$\underset{\sim}{d} = \begin{pmatrix}7\\-5\\10\end{pmatrix} = \lambda\begin{pmatrix}1\\-2\\3\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix} + \eta\begin{pmatrix}4\\-1\\3\end{pmatrix}$$

$\lambda + 2\mu + 4\eta = 7$

$-2\lambda + \mu - \eta = -5$

$3\lambda - \mu + 3\eta = 10$

$$\left[\begin{array}{ccc|c}1 & 2 & 4 & 7\\-2 & 1 & -1 & -5\\3 & -1 & 3 & 10\end{array}\right]\begin{array}{l}R_1\\R_2 \Rightarrow R_2+2R_1\\R_3 \Rightarrow R_3-3R_1\end{array}$$

$$\left[\begin{array}{ccc|c}1 & 2 & 4 & 7\\0 & 5 & 7 & 9\\0 & -7 & -9 & -11\end{array}\right]\begin{array}{l}\\\\R_3 \Rightarrow 5R_3+7R_2\end{array}$$

$$\left[\begin{array}{ccc|c}1 & 2 & 4 & 7\\0 & 5 & 7 & 9\\0 & 0 & 4 & 8\end{array}\right]$$

$4\eta = 8$

$\underline{\underline{\eta = 2}}$

$5\mu + 14 = 9$

$5\mu = -5$

$\underline{\underline{\mu = -1}}$

$\lambda - 2 + 8 = 7$

$\lambda + 6 = 7$

$\underline{\underline{\lambda = 1}}$

$$\underset{\sim}{e} = \begin{pmatrix}1\\-5\\8\end{pmatrix} = \lambda\begin{pmatrix}1\\-2\\3\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix} + \eta\begin{pmatrix}4\\-1\\3\end{pmatrix}$$

$\lambda + 2\mu + 4\eta = 1$

$-2\lambda + \mu - \eta = -5$

$3\lambda - \mu + 3\eta = 8$

$$\left[\begin{array}{ccc|c}1 & 2 & 4 & 1\\-2 & 1 & -1 & -5\\3 & -1 & 3 & 8\end{array}\right]\begin{array}{l}R_1\\R_2 \Rightarrow R_2+2R_1\\R_3 \Rightarrow R_3-3R_1\end{array}$$

$$\left[\begin{array}{ccc|c}1 & 2 & 4 & 1\\0 & 5 & 7 & -3\\0 & -7 & -9 & 5\end{array}\right]\begin{array}{l}\\\\R_3 \Rightarrow 5R_3+7R_2\end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 0 & 5 & 7 & -3 \\ 0 & 0 & 4 & 4 \end{array}\right]$$

$4\eta = 4$

$\underline{\underline{\eta = 1}}$

$5\mu + 7 = -3$

$5\mu = -10$

$\underline{\mu = -2}$

$\lambda - 4 + 4 = 1$

$\underline{\lambda = 1}$

$$f = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \eta \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

$\lambda + 2\mu + 4\eta = 0$

$-2\lambda + \mu - \eta = 2$

$3\lambda - \mu + 3\eta = -2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -2 & 1 & -1 & 2 \\ 3 & -1 & 3 & -2 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 + 2R_1 \\ R_3 \Rightarrow R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 5 & 7 & 2 \\ 0 & -7 & -9 & -2 \end{array}\right] R_3 \Rightarrow 5R_3 + 7R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 5 & 7 & 2 \\ 0 & 0 & 4 & 4 \end{array}\right]$$

$4\eta = 4$

$\underline{\underline{\eta = 1}}$

$5\mu + 7 = 2$

$5\mu = -5$

$\underline{\underline{\mu = -1}}$

$\lambda + 2 + 4 = 0$

$\underline{\underline{\lambda = -2}}$

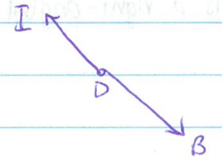
Q25

a) $\vec{DC} = 10\underset{\sim}{i}$

$\vec{DB} = 10\underset{\sim}{i} + 4\underset{\sim}{k}$

$\vec{DI} = 3\underset{\sim}{j} + \underset{\sim}{k}$

b) Let $\angle IDB = \theta$



$$\cos\theta = \frac{\vec{DI} \cdot \vec{DB}}{|\vec{DI}||\vec{DB}|}$$

$$= \frac{\begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 0 \\ 4 \end{pmatrix}}{\sqrt{10}\,\sqrt{116}}$$

$$= \frac{4}{\sqrt{1160}}$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{1160}}\right)$$

$= 83.26°$

$\approx \underline{\underline{83°}}$

Q26

a) Let $\angle OAE = \theta$.

$\vec{OA} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ $\vec{OE} = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$

$\vec{AE} = \vec{AO} + \vec{OE}$

$= \vec{OE} - \vec{OA}$

$= \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} -4 \\ -2 \\ 8 \end{pmatrix}}}$

$$\cos\theta = \frac{\vec{AE} \cdot \vec{OA}}{|\vec{AE}||\vec{OA}|}$$

$$= \frac{\begin{pmatrix} -4 \\ -2 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}}{\sqrt{84}\,\sqrt{20}}$$

$$\theta = \cos^{-1}\left(\frac{-20}{\sqrt{1680}}\right) = 119.21°$$

$\therefore \underline{\underline{60.79°}}$

b) $\vec{AE} = \begin{pmatrix} -4 \\ -2 \\ 8 \end{pmatrix}$

$\vec{DB} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ 4 \\ 0 \end{pmatrix}$

$\therefore \cos\theta = \dfrac{\begin{pmatrix} -4 \\ -2 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ 4 \\ 0 \end{pmatrix}}{\sqrt{84} \times \sqrt{80}}$

$\theta = \cos^{-1}\left(\dfrac{24}{\sqrt{6720}}\right)$

$= 72.98°$

$\approx \underline{73°}$

Q27. $\vec{OA} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$

$\vec{OB} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$

a) $\vec{AB} = \vec{OB} - \vec{OA}$

$= \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$

$= \underline{\begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}}$

$\vec{BC} = \vec{OC} - \vec{OB}$

$= \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$

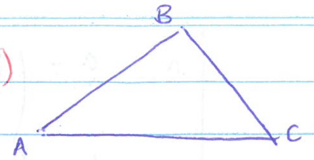
$= \underline{\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}}$

$\vec{AC} = \vec{OC} - \vec{OA}$

$= \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$

$= \underline{\begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix}}$

b)



RTP: Two sides equal in length.

$|\vec{AB}| = \sqrt{36+4+1}$

$= \sqrt{41}$

$|\vec{BC}| = \sqrt{1+4+9}$

$= \sqrt{14}$

$|\vec{AC}| = \sqrt{25+16}$

$= \sqrt{41}$

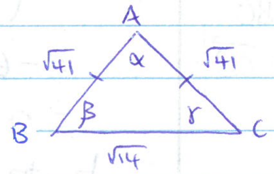
$\therefore \triangle ABC$ is isosceles with $\underline{AB = AC}$

c) $\vec{AC} \cdot \vec{AC}$

$= |\vec{AC}|^2$

$= \underline{41}$

d)



$\cos\alpha = \dfrac{\vec{AB} \cdot \vec{AC}}{41}$

$\alpha = \cos^{-1}\left(\dfrac{\begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix}}{41}\right)$

$= \cos^{-1}\left(\dfrac{34}{41}\right)$

$= 33.98°$

$\approx \underline{34°}$

$\cos\beta = \dfrac{\vec{AB} \cdot \vec{BC}}{\sqrt{41}\sqrt{14}} = \cos\gamma$

$\beta = \cos^{-1}\left(\dfrac{\begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}}{\sqrt{574}}\right)$

$= \cos^{-1}\left(\dfrac{-7}{\sqrt{574}}\right)$

$= 106.99°$

$\therefore \underline{73.01°} = \beta = \gamma$

Q28

RTP: $|\vec{OG}| = |\vec{OE}| = |\vec{OB}| = |\vec{BE}|$
$= |\vec{BG}| = |\vec{GE}|$.

Proof:

$|\vec{OG}| = \sqrt{1+1} = \sqrt{2}$

$|\vec{OE}| = \sqrt{1+1} = \sqrt{2}$

$|\vec{GE}| = \sqrt{1+1} = \sqrt{2}$

$\therefore \triangle OGE$ is an equilateral triangle with all side lengths $\sqrt{2}$,

⋮

(Similar method for other three triangles).

⋮

$\therefore$ 4 congruent equilateral triangle faces.

$\Rightarrow$ OBGE is a regular tetrahedron.

Q29

RTP: $\vec{OF}$, $\vec{AG}$, $\vec{BD}$ and $\vec{CE}$ intersect at some point P such that P divides all diagonals in the ratio 1 : 1.

Proof:

$\vec{OF} = \mathbf{c} + \mathbf{a} + \mathbf{d}$

$\vec{AG} = \mathbf{c} - \mathbf{a} + \mathbf{d}$

$\vec{BD} = -\mathbf{c} - \mathbf{a} + \mathbf{d}$

$\vec{CE} = \mathbf{a} - \mathbf{c} + \mathbf{d}$

$\therefore$ Midpoints of all above have position vectors.

$\frac{1}{2}\vec{OF} = \frac{1}{2}(\mathbf{c} + \mathbf{a} + \mathbf{d})$

$\frac{1}{2}\vec{AG} = \frac{1}{2}(\mathbf{c} + \mathbf{d} - \mathbf{a})$

$\frac{1}{2}\vec{BD} = \frac{1}{2}(\mathbf{d} - \mathbf{a} - \mathbf{c})$

$\frac{1}{2}\vec{CE} = \frac{1}{2}(\mathbf{a} + \mathbf{d} - \mathbf{c})$

But $|\frac{1}{2}\mathbf{a}| = |-\frac{1}{2}\mathbf{a}|$

$|\frac{1}{2}\mathbf{c}| = |-\frac{1}{2}\mathbf{c}|$

$|\frac{1}{2}\mathbf{d}| = |-\frac{1}{2}\mathbf{d}|$

$\therefore$ All intersect at P, midpoint

Q30.

RTP: If $\vec{OC} \cdot \vec{BA} = 0$ and $\vec{OB} \cdot \vec{CA} = 0$, then $\vec{OA} \cdot \vec{CB} = 0$.

Proof: $\underset{\sim}{c} \cdot (\underset{\sim}{a} - \underset{\sim}{b}) = 0$

$\underset{\sim}{c} \cdot \underset{\sim}{a} - \underset{\sim}{c} \cdot \underset{\sim}{b} = 0$

$\underset{\sim}{c} \cdot \underset{\sim}{a} = \underset{\sim}{c} \cdot \underset{\sim}{b}$

$\underset{\sim}{b} \cdot (\underset{\sim}{a} - \underset{\sim}{c}) = 0$

$\underset{\sim}{b} \cdot \underset{\sim}{a} - \underset{\sim}{b} \cdot \underset{\sim}{c} = 0$

$\underset{\sim}{b} \cdot \underset{\sim}{a} = \underset{\sim}{b} \cdot \underset{\sim}{c}$

But $\underset{\sim}{b} \cdot \underset{\sim}{c} = \underset{\sim}{c} \cdot \underset{\sim}{b}$

$\therefore \underset{\sim}{c} \cdot \underset{\sim}{a} = \underset{\sim}{b} \cdot \underset{\sim}{a}$

$\underset{\sim}{c} \cdot \underset{\sim}{a} - \underset{\sim}{b} \cdot \underset{\sim}{a} = 0$

$\underset{\sim}{a} \cdot (\underset{\sim}{c} - \underset{\sim}{b}) = 0$

$\vec{OA} \cdot \vec{BC} = 0$

$\Rightarrow \vec{OA}$ is perpendicular to $\vec{BC}$

$\Rightarrow \vec{OA}$ is perpendicular to $\vec{CB}$.

EXERCISE 9B

Q1. Let $\underset{\sim}{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$,

then $\underset{\sim}{b} = \lambda \underset{\sim}{a}$ is parallel to $\underset{\sim}{a}$ such that

$$\underset{\sim}{b} = \begin{pmatrix} \lambda x \\ \lambda y \\ \lambda z \end{pmatrix}.$$

$$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} \lambda x \\ \lambda y \\ \lambda z \end{pmatrix}$$

$$= \begin{pmatrix} \lambda yz - \lambda yz \\ \lambda xz - \lambda xz \\ \lambda xy - \lambda xy \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\therefore$ two parallel vectors produce the 0 vector when cross product is applied.

Q2 $\underset{\sim}{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

$\underset{\sim}{b} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$

$$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3+3 \\ 1-2 \\ 6+3 \end{pmatrix}$$

$$\underset{\sim}{c} = \begin{pmatrix} 6 \\ -1 \\ 9 \end{pmatrix}$$

$$\underset{\sim}{a} \cdot \underset{\sim}{c} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \begin{pmatrix} 6 \\ -1 \\ 9 \end{pmatrix}$$

$= 12 - 3 - 9$

$= 0$

$\therefore \perp$ to $\underset{\sim}{a}$

$$\underset{\sim}{b} \cdot \underset{\sim}{c} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ 9 \end{pmatrix}$$

$= -6 - 3 + 9$

$= 0$

$\therefore \perp$ to $\underset{\sim}{b}$.

Q3. $\underset{\sim}{c} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$

$\underset{\sim}{d} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$\underset{\sim}{c} \times \underset{\sim}{d} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 1-5 \\ 5 \end{pmatrix}$$

$$\underset{\sim}{e} = \begin{pmatrix} -1 \\ -4 \\ 5 \end{pmatrix}$$

$$\underset{\sim}{c} \cdot \underset{\sim}{e} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \\ 5 \end{pmatrix}$$

$= -5 + 5$

$= 0$

$\therefore \perp$ to $\underset{\sim}{c}$

$$\underset{\sim}{d} \cdot \underset{\sim}{e} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \\ 5 \end{pmatrix}$$

$= -1 - 4 + 5$

$= 0$

$\therefore \perp$ to $\underset{\sim}{d}$.

Q4. $\underset{\sim}{p} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$

$\underset{\sim}{q} = \begin{pmatrix} -1 \\ 6 \\ -4 \end{pmatrix}$

$$\underset{\sim}{p} \times \underset{\sim}{q} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \times \begin{pmatrix} -1 \\ 6 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} -12+12 \\ 2+4 \\ 6+3 \end{pmatrix}$$

$$\underset{\sim}{r} = \begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix}$$

$$\underset{\sim}{p} \cdot \underset{\sim}{r} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix}$$

$= 18 - 18$

$= 0$

$\therefore \perp$ to $\underset{\sim}{p}$.

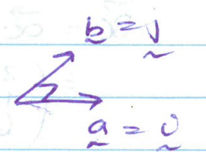
$$\underset{\sim}{q} \cdot \underset{\sim}{r} = \begin{pmatrix} -1 \\ 6 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix}$$

$= 36 - 36$

$= 0$

$\therefore \perp$ to $\underset{\sim}{q}$.

Q5



then $\underset{\sim}{a} \times \underset{\sim}{b} = \underset{\sim}{k}$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}}$$

Q6

a) $\underset{\sim}{a} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$

$\underset{\sim}{b} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$

$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} -2+2 \\ 3-1 \\ -2+6 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$

$|\underset{\sim}{a} \times \underset{\sim}{b}| = \left| \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} \right|$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= \underline{2\sqrt{5}}$

b) $\sin\theta = \dfrac{|a \times b|}{|a||b|}$

$= \dfrac{2\sqrt{5}}{\sqrt{6} \times \sqrt{14}}$

$\theta = \sin^{-1}\left(\dfrac{2\sqrt{5}}{\sqrt{84}}\right)$

$= 29.21°$

$\approx \underline{\underline{29°}}$

Q7. $\underset{\sim}{p} \times \underset{\sim}{q} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 9-2 \\ 1+6 \\ 4+3 \end{pmatrix}$

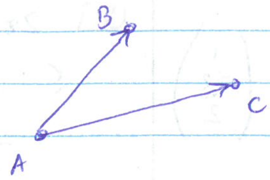
$\underset{\sim}{r} = \begin{pmatrix} 7 \\ 7 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\therefore \hat{\underset{\sim}{r}} = \dfrac{1}{7\sqrt{3}} \begin{pmatrix} 7 \\ 7 \\ 7 \end{pmatrix}$

$= \dfrac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= \dfrac{1}{\sqrt{3}}\underset{\sim}{i} + \dfrac{1}{\sqrt{3}}\underset{\sim}{j} + \dfrac{1}{\sqrt{3}}\underset{\sim}{k}$

Q8



$\vec{AB} = \vec{AO} + \vec{OB}$

$= \vec{OB} - \vec{OA}$

$= \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$

$\vec{AC} = \vec{AO} + \vec{OC}$

$= \vec{OC} - \vec{OA}$

$= \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$\therefore \vec{AB} \times \vec{AC}$

$= \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -2 \\ -3 \end{pmatrix}$

$= -\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$

$\therefore \hat{\underset{\sim}{u}} = \dfrac{-1}{\sqrt{17}} \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$

EXERCISE 5C

Q1a) $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$

$= (3+2\lambda)\underset{\sim}{i} + (2-\lambda)\underset{\sim}{j} + (2\lambda-1)\underset{\sim}{k}$

b) $x = 3+2\lambda$

$y = 2-\lambda$

$z = 2\lambda - 1$

Q2a) $\underset{\sim}{a} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$ $\underset{\sim}{b} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

$\underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$

$\therefore \underset{\sim}{r} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 4-\lambda \\ 2-\lambda \\ 3-2\lambda \end{pmatrix}$

b) $x = 4-\lambda$

$y = 2-\lambda$

$z = 3-2\lambda$

Q3. $\underset{\sim}{r} \cdot \underset{\sim}{n} = \underset{\sim}{a} \cdot \underset{\sim}{n}$

$\underset{\sim}{r} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$

$= 6+3+10$

$\underline{\underset{\sim}{r} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = 19}$

Q4 $\underset{\sim}{r} \cdot \underset{\sim}{n} = \underset{\sim}{a} \cdot \underset{\sim}{n}$

$\underset{\sim}{r} \cdot \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$

$= 10+1-9$

$\underline{\underset{\sim}{r} \cdot \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = 2}$

Q5. $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$

or

$\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -12 \\ -11 \end{pmatrix}$

$\underset{\sim}{r} \cdot \begin{pmatrix} 6 \\ -12 \\ -11 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -12 \\ -11 \end{pmatrix}$

$\underset{\sim}{r} \cdot \begin{pmatrix} 6 \\ -12 \\ -11 \end{pmatrix} = \underline{\underline{-22}}$

Q6.

$$\underset{\sim}{r} = \begin{pmatrix}-3\\2\\-1\end{pmatrix} + \lambda\begin{pmatrix}2\\0\\-3\end{pmatrix} + \mu\begin{pmatrix}1\\-3\\2\end{pmatrix}$$

or

$$\begin{pmatrix}2\\0\\-3\end{pmatrix} \times \begin{pmatrix}1\\-3\\2\end{pmatrix} = \begin{pmatrix}-9\\-7\\-6\end{pmatrix}$$

$$\underset{\sim}{r} \cdot \begin{pmatrix}-9\\-7\\-6\end{pmatrix} = \begin{pmatrix}-3\\2\\-1\end{pmatrix} \cdot \begin{pmatrix}-9\\-7\\-6\end{pmatrix}$$

$$= 27 - 14 + 6$$

$$\underset{\sim}{r} \cdot \begin{pmatrix}-9\\-7\\-6\end{pmatrix} = 19$$

Q7.

$$\underset{\sim}{r} = \begin{pmatrix}2\\6\\-1\end{pmatrix} + \lambda\begin{pmatrix}-3\\1\\2\end{pmatrix}$$

$$\begin{pmatrix}a\\7\\5\end{pmatrix} = \begin{pmatrix}2\\6\\-1\end{pmatrix} + \lambda\begin{pmatrix}-3\\1\\2\end{pmatrix}$$

$5 = -1 + 2\lambda$

$6 = 2\lambda$

$\lambda = 3$

$\therefore a = 2 + 3(-3)$

$a = -7$

$7 = b + 3(1)$

$b = 4$

Q8 let $\underset{\sim}{r} = \begin{pmatrix}x\\y\\z\end{pmatrix}$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} \cdot \begin{pmatrix}3\\2\\-1\end{pmatrix} = 21$$

$3x + 2y - z = 21$ //

Q9. $2x - 3y + 7z = 5$

$$\underset{\sim}{r} \cdot \begin{pmatrix}2\\-3\\7\end{pmatrix} = 5$$

Q10.

$$L: \underset{\sim}{r} = \begin{pmatrix}2\\8\\-3\end{pmatrix} + \lambda\begin{pmatrix}-6\\-4\\2\end{pmatrix}$$

$$\pi: \underset{\sim}{r} \cdot \begin{pmatrix}3\\2\\-1\end{pmatrix} = c$$

If a line is perpendicular to a plane, then the directional vector must be parallel to the normal.

$$\begin{pmatrix}-6\\-4\\2\end{pmatrix} = k\begin{pmatrix}3\\2\\-1\end{pmatrix}$$

$-6 = 3k$

$k = -2$

$-4 = 2k$

$k = -2$

$2 = -k$

$k = -2$

$\therefore L$ is $\perp$ to π.

Q11.

$$\underset{\sim}{r}_1 = \begin{pmatrix}10\\5\\-2\end{pmatrix} + \lambda\begin{pmatrix}4\\1\\-2\end{pmatrix}$$

$$\underset{\sim}{r}_2 = \begin{pmatrix}0\\8\\-6\end{pmatrix} + \mu\begin{pmatrix}-1\\-3\\5\end{pmatrix}$$

$$\begin{pmatrix}10+4\lambda\\5+\lambda\\-2-2\lambda\end{pmatrix} = \begin{pmatrix}-\mu\\8-3\mu\\-6+5\mu\end{pmatrix}$$

$10 + 4\lambda = -\mu$

$4\lambda + \mu = -10$ (1)

$5 + \lambda = 8 - 3\mu$

$\lambda + 3\mu = 3$. (2)

$-\ \ 12\lambda + 3\mu = -30$

$-11\lambda = 33$

$\lambda = -3$

$\mu = -10 - 4\lambda$

$= -10 + 12$

$= 2$

$$\underset{\sim}{r}_1 = \begin{pmatrix}10-12\\5-3\\-2+6\end{pmatrix} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$$

$$\underset{\sim}{r}_2 = \begin{pmatrix}-2\\8-6\\-6+10\end{pmatrix} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$$

$\therefore$ Lines intersect at $\begin{pmatrix}-2\\2\\4\end{pmatrix}$

Q12

$$\underset{\sim}{r}_1 = \begin{pmatrix}1\\-2\\3\end{pmatrix} + \lambda\begin{pmatrix}-1\\3\\2\end{pmatrix}$$

$$\underset{\sim}{r}_2 = \begin{pmatrix}3\\13\\-15\end{pmatrix} + \mu\begin{pmatrix}-1\\0\\4\end{pmatrix}$$

$$\begin{pmatrix}1-\lambda\\-2+3\lambda\\3+2\lambda\end{pmatrix} = \begin{pmatrix}3-\mu\\13\\-15+4\mu\end{pmatrix}$$

$1 - \lambda = 3 - \mu$

$-2 = \lambda - \mu$ (1)

$-2 + 3\lambda = 13$

$3\lambda = 15$

$\lambda = 5$

$\therefore -2 = 5 - \mu$

$\mu = 7$

$\therefore$ Lines intersect at

$$\begin{pmatrix}1-5\\-2+15\\3+10\end{pmatrix} = \begin{pmatrix}-4\\13\\13\end{pmatrix}$$

Q13.

a) $\underset{\sim}{r}_1 = \begin{pmatrix}13\\1\\8\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\3\end{pmatrix}$

$\underset{\sim}{r}_2 = \begin{pmatrix}12\\2\\6\end{pmatrix} + \mu\begin{pmatrix}5\\3\\-8\end{pmatrix}$

$\begin{pmatrix}13+2\lambda\\1-\lambda\\8+3\lambda\end{pmatrix} = \begin{pmatrix}12+5\mu\\2+3\mu\\6-8\mu\end{pmatrix}$

$13+2\lambda = 12+5\mu$

$2\lambda-5\mu = -1$ (1)

$1-\lambda = 2+3\mu$

$\lambda+3\mu = -1$ (2)

$2\lambda+6\mu = -2$

$-\quad 2\lambda-5\mu = -1$

$11\mu = -1$

$\underline{\underline{\mu = -\frac{1}{11}}}$

$\lambda = -1-3\left(\frac{-1}{11}\right)$

$= -1+\frac{3}{11}$

$= \underline{\underline{-\frac{8}{11}}}$

But checking in the last component:

$8+3\left(\frac{-8}{11}\right) \neq 6-8\left(\frac{-1}{11}\right)$

$8-\frac{24}{11} \neq 6+\frac{8}{11}$

$88-24 \neq 66+8$

$64 \neq 74$

$\therefore$ Lines 1 and 2 do not intersect.

b) $\underset{\sim}{r}_1 = \begin{pmatrix}13\\1\\8\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\3\end{pmatrix}$

$\underset{\sim}{r}_3 = \begin{pmatrix}-5\\2\\-3\end{pmatrix} + \beta\begin{pmatrix}2\\1\\-1\end{pmatrix}$

$\begin{pmatrix}13+2\lambda\\1-\lambda\\8+3\lambda\end{pmatrix} = \begin{pmatrix}-5+2\beta\\2+\beta\\-3-\beta\end{pmatrix}$

$13+2\lambda = -5+2\beta$

$2\lambda-2\beta = -18$

$\lambda-\beta = -9$ (1)

$\lambda+\beta = -1$ (2)

$2\lambda = -10$

$\underline{\underline{\lambda = -5}}$

$\beta = -1+5$

$\underline{\underline{\beta = 4}}$

Check

$8+3(-5) = -3-4$

$\underline{\underline{-7 = -7}}$

$\therefore \begin{pmatrix}13-10\\1+5\\8-15\end{pmatrix}$

$= \underline{\underline{\begin{pmatrix}3\\6\\-7\end{pmatrix}}}$

Angle between the lines is the angle between the two directional vectors.

$\cos\theta = \dfrac{\begin{pmatrix}2\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}2\\1\\-1\end{pmatrix}}{\sqrt{14}\,\sqrt{6}}$

$\theta = \cos^{-1}\left(\frac{0}{\sqrt{84}}\right)$

$= 90°$

ie perpendicular.

Q14.

$L: \begin{pmatrix}1\\-2\\5\end{pmatrix} + \lambda\begin{pmatrix}5\\3\\-2\end{pmatrix}$

$\pi: \underset{\sim}{r}\cdot\begin{pmatrix}-1\\3\\2\end{pmatrix} = 3$

a) $\begin{pmatrix}-4\\-5\\7\end{pmatrix} = \begin{pmatrix}1\\-2\\5\end{pmatrix} + \lambda\begin{pmatrix}5\\3\\-2\end{pmatrix}$

$-4 = 1+5\lambda$

$-5 = 5\lambda$

$\underline{\underline{\lambda = -1}}$

$-5 = -2+3\lambda$

$-3 = 3\lambda$

$\underline{\underline{\lambda = -1}}$

$7 = 5-2\lambda$

$2 = -2\lambda$

$\underline{\underline{\lambda = -1}}$

$\therefore$ Point is on the line.

b) $\begin{pmatrix}10\\3\\2\end{pmatrix} = \begin{pmatrix}1\\-2\\5\end{pmatrix} + \lambda\begin{pmatrix}5\\3\\-2\end{pmatrix}$

$10 = 1+5\lambda$

$9 = 5\lambda$

$\underline{\underline{\lambda = \frac{9}{5}}}$

$3 = -2+3\lambda$

$5 = 3\lambda$

$\underline{\underline{\lambda = \frac{5}{3}}}$

$\therefore$ Not on line as $\frac{9}{5} \neq \frac{5}{3}$

c) $\begin{pmatrix}-4\\-5\\7\end{pmatrix}\cdot\begin{pmatrix}-1\\3\\2\end{pmatrix}$

$= 4-15+14$

$= \underline{\underline{3}}$ $\therefore$ on the plane

$\begin{pmatrix}10\\3\\2\end{pmatrix}\cdot\begin{pmatrix}-1\\3\\2\end{pmatrix}$

$= -10+9+4$

$= \underline{\underline{3}}$ $\therefore$ on the plane

d) If L is on Π then direction vector must be perpendicular to the normal.

i.e. $\begin{pmatrix}5\\3\\-2\end{pmatrix}\cdot\begin{pmatrix}-1\\3\\2\end{pmatrix}$

$= -5+9-4$

$= 0$

$\therefore L$ is on Π.

Q15

$\underset{\sim}{r}_A = \begin{pmatrix}-10\\20\\-12\end{pmatrix} + t\begin{pmatrix}5\\-10\\6\end{pmatrix}$

$\underset{\sim}{r}_B = \begin{pmatrix}-3\\-8\\2\end{pmatrix} + t\begin{pmatrix}4\\-6\\4\end{pmatrix}$

$\begin{pmatrix}-10+5t\\20-10t\\-12+6t\end{pmatrix} = \begin{pmatrix}-3+4t\\-8-6t\\2+4t\end{pmatrix}$

$-10+5t = -3+4t$

$t = 7$

$20-10t = -8-6t$

$-4t = -28$

$t = 7$

$-12+6t = 2+4t$

$2t = 14$

$t = 7$

$\therefore$ Collide after 7 seconds at the point

$\underset{\sim}{r}_A = \underset{\sim}{r}_B = \begin{pmatrix}-10+35\\20-70\\-12+42\end{pmatrix}$

$= \begin{pmatrix}25\\-50\\30\end{pmatrix}$

Q16. If the plane contains the line, then must have a directional component of $\begin{pmatrix}1\\-3\\-5\end{pmatrix}$.

Second directional component of

$\begin{pmatrix}2\\2\\1\end{pmatrix} - \begin{pmatrix}1\\2\\0\end{pmatrix}$

$= \begin{pmatrix}1\\0\\1\end{pmatrix}$

$\therefore \underset{\sim}{r} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-3\\-5\end{pmatrix} + \mu\begin{pmatrix}1\\0\\1\end{pmatrix}$

$\therefore \begin{pmatrix}1\\-3\\-5\end{pmatrix} \times \begin{pmatrix}1\\0\\1\end{pmatrix}$

$= \begin{pmatrix}-3\\-5-1\\3\end{pmatrix}$

$= \begin{pmatrix}-3\\-6\\3\end{pmatrix}$

$\therefore \underset{\sim}{r}\cdot\begin{pmatrix}-3\\-6\\3\end{pmatrix} = \begin{pmatrix}1\\2\\0\end{pmatrix}\cdot\begin{pmatrix}-3\\-6\\3\end{pmatrix}$

$\underset{\sim}{r}\cdot\begin{pmatrix}-3\\-6\\3\end{pmatrix} = -3-12$

$\underset{\sim}{r}\cdot\begin{pmatrix}-3\\-6\\3\end{pmatrix} = -15$

$\therefore -3x-6y+3z = -15$

or

$x+2y-z = 5$

Q17.

$L: \underset{\sim}{r} = \begin{pmatrix}2\\13\\1\end{pmatrix} + \lambda\begin{pmatrix}-1\\3\\-2\end{pmatrix}$

$= \begin{pmatrix}2-\lambda\\13+3\lambda\\1-2\lambda\end{pmatrix}$

$\Pi: \underset{\sim}{r}\cdot\begin{pmatrix}2\\-1\\-1\end{pmatrix} = 11$

$\begin{pmatrix}2-\lambda\\13+3\lambda\\1-2\lambda\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\-1\end{pmatrix} = 11$

$2(2-\lambda)-(13+3\lambda)-(1-2\lambda) = 11$

$4-2\lambda-13-3\lambda-1+2\lambda = 11$

$-3\lambda = 21$

$\lambda = -7$

$\therefore \underset{\sim}{r} = \begin{pmatrix}2-(-7)\\13+3(-7)\\1-2(-7)\end{pmatrix}$

$= \begin{pmatrix}9\\-8\\15\end{pmatrix}$

Q18

$\underset{\sim}{r}_D = \begin{pmatrix}1200\\3000\\900\end{pmatrix} + t\begin{pmatrix}2000\\-3600\\1000\end{pmatrix}$

$\underset{\sim}{r}_S = \begin{pmatrix}5750\\-13250\\3370\end{pmatrix} + t\begin{pmatrix}600\\1400\\240\end{pmatrix}$

$\underset{\sim}{r}_D = \underset{\sim}{r}_S$

$1200+2000t = 5750+600t$

$1400t = 4550$

$t = 3.25$

$3000-3600t = -13250+1400t$

$-5000t = -16250$

$t = 3.25$

$900+1000t = 3370+240t$

$760t = 2470$

$t = 3.25$

$\therefore$ After 3.25 hrs, bodies will collide

Q19.

$$\underset{\sim}{r}_A = \begin{pmatrix} 80 \\ 400 \\ 3 \end{pmatrix} + t\,\underset{\sim}{v}$$

$$\underset{\sim}{r}_B = \begin{pmatrix} 150 \\ 470 \\ 2 \end{pmatrix} + t \begin{pmatrix} 300 \\ 180 \\ 0 \end{pmatrix}$$

Let $t = \frac{1}{6}$ (ie 10 mins)

$$\underset{\sim}{r}_B = \begin{pmatrix} 150 \\ 470 \\ 2 \end{pmatrix} + \frac{1}{6}\begin{pmatrix} 300 \\ 180 \\ 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 200 \\ 500 \\ 2 \end{pmatrix}}}$$

$$\therefore \begin{pmatrix} 200 \\ 500 \\ 2 \end{pmatrix} = \begin{pmatrix} 80 \\ 400 \\ 3 \end{pmatrix} + \frac{1}{6}\underset{\sim}{v}$$

$$\frac{1}{6}\underset{\sim}{v} = \begin{pmatrix} 120 \\ 100 \\ -1 \end{pmatrix}$$

$$\underset{\sim}{v} = \underline{\underline{\begin{pmatrix} 720 \\ 600 \\ -6 \end{pmatrix}}} \text{ km/hr.}$$

Q20.

$$\underset{\sim}{r}_1 \cdot \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = 12$$

$$\underset{\sim}{r}_2 \cdot \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix} = 15$$

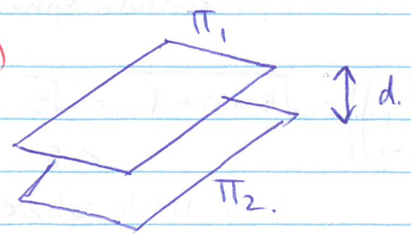
a) If Π_1 and Π_2 are parallel, then

$$\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = k \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$$

$\underline{\underline{k = -1}}$

$\therefore$ Planes are parallel.

b)



Consider a point on Π_1 such as $A: \begin{pmatrix} 1 \\ 1 \\ 12 \end{pmatrix}$.

Now finding the distance between the point $\begin{pmatrix} 1 \\ 1 \\ 12 \end{pmatrix}$ and the plane $\underset{\sim}{r} \cdot \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix} = 15$

$$D = \frac{|ax + by + cz - d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|-2(1) + 2(1) - 1(12) - 15|}{\sqrt{4 + 4 + 1}}$$

$$= \frac{|-2 + 2 - 12 - 15|}{\sqrt{9}}$$

$$= \frac{27}{3}$$

$$= \underline{\underline{9 \text{ units}}}$$

Q21.

$$\underset{\sim}{r}_A = \begin{pmatrix} 30 \\ -37 \\ -30 \end{pmatrix} + t \begin{pmatrix} 5 \\ 8 \\ 3 \end{pmatrix}$$

$$\underset{\sim}{r}_B = \begin{pmatrix} 2 \\ 40 \\ 26 \end{pmatrix} + t \begin{pmatrix} 8 \\ 0 \\ -2 \end{pmatrix}$$

$$\underset{\sim}{r}_A - \underset{\sim}{r}_B = \begin{pmatrix} 30 + 5t \\ -37 + 8t \\ -30 + 3t \end{pmatrix} - \begin{pmatrix} 2 + 8t \\ 40 \\ 26 - 2t \end{pmatrix} = \begin{pmatrix} 28 - 3t \\ -77 + 8t \\ -56 + 5t \end{pmatrix}$$

$$d = \sqrt{(28 - 3t)^2 + (-77 + 8t)^2 + (-56 + 5t)^2}$$

Using CASIO,

$$\frac{dd}{dt} = 0.$$

$$\underline{\underline{t = 10}} \text{ seconds.}$$

$$\therefore d = \underline{\underline{7\,m}}$$

EXERCISE 5D

Q1 $|\underset{\sim}{r}| = 16$

centre @ $(0,0,0)$

and

radius, $r = 16$

Q2 $x^2+y^2+z^2=100$

centre @ $(0,0,0)$

and

radius, $r=10$.

Q3. $\left|\underset{\sim}{r} - \begin{pmatrix}1\\1\\1\end{pmatrix}\right| = 25$

centre @ $(1,1,1)$

and

radius, $r = 25$

Q4. $\left|\underset{\sim}{r} - \begin{pmatrix}2\\-3\\4\end{pmatrix}\right| = 18$

centre @ $(2,-3,4)$

and

radius, $r = 18$.

Q5 $(x-3)^2+(y+1)^2+(z-2)^2=10$

centre @ $(3,-1,2)$

and

radius, $r = \sqrt{10}$.

Q6. $(x+4)^2+(y-1)^2+z^2=25$

centre @ $(-4,1,0)$

and

radius, $r = 5$.

Q7 $x^2+y^2-8y+16+z^2=50$

$x^2+(y-4)^2-16+16+z^2=50$

$x^2+(y-4)^2+z^2=50$

centre @ $(0,4,0)$

and

radius, $r = \sqrt{50} = \underline{\underline{5\sqrt{2}}}$.

Q8 $x^2+y^2+z^2-2x+6y=15$

$(x-1)^2-1+(y+3)^2-9+z^2=15$

$(x-1)^2+(y+3)^2+z^2=25$

$\therefore$ centre @ $(1,-3,0)$

and

radius, $r = 5$.

Q9. $x^2+y^2+z^2-6y+2z=111$

$x^2+(y-3)^2-9+(z+1)^2-1=111$

$x^2+(y-3)^2+(z+1)^2=121$

$\therefore$ centre @ $(0,3,-1)$

and

radius, $r = 11$.

Q10. $x^2+y^2+z^2+8x-2y+2z=7$

$(x+4)^2-16+(y-1)^2-1+(z+1)^2-1=7$

$(x+4)^2+(y-1)^2+(z+1)^2=25$

$\therefore$ centre @ $(-4,1,-1)$

and

radius, $r = 5$.

Q11. $\left|\begin{pmatrix}2\\-3\\4\end{pmatrix}\right| = \sqrt{4+9+16} = \sqrt{29}$

> 5

$\therefore$ outside sphere

Q12. $\left|\begin{pmatrix}-2\\3\\6\end{pmatrix}\right| = \sqrt{4+9+36} = \sqrt{49}$

$= 7$

$\therefore$ on sphere.

Q13. $\left|\begin{pmatrix}7\\12\\9\end{pmatrix}\right| = \sqrt{49+144+81} = \sqrt{284}$

> 16

$\therefore$ outside sphere

Q14. $\left|\begin{pmatrix}3\\1\\0\end{pmatrix} - \begin{pmatrix}1\\1\\-1\end{pmatrix}\right| = \sqrt{4+0+1} = \sqrt{5}$

< 8

$\therefore$ inside sphere

Q15 $\left|\begin{pmatrix}3\\5\\2\end{pmatrix}-\begin{pmatrix}2\\-1\\3\end{pmatrix}\right| = \sqrt{1+36+1}$

$= \sqrt{38}$

> 5

$\therefore$ outside sphere

Q16. $\left|\begin{pmatrix}2\\-2\\2\end{pmatrix}-\begin{pmatrix}7\\10\\2\end{pmatrix}\right| = \sqrt{25+144+0}$
$= 13$

$\therefore$ on sphere

Q17. $(x-1)^2+(y+3)^2+(z-2)^2 = 36$

$= (5-1)^2+(-6+3)^2+(-1-2)^2$

$= 16+9+9$

$= 34$

< 36

$\therefore$ Inside sphere

Q18 $x^2+y^2+z^2-4x-3y-z=61$

$= 1+0+64+4-0-8$

$= 61$

$\therefore$ on sphere.

Q19. $\left|\underset{\sim}{r}-\begin{pmatrix}1\\1\\-3\end{pmatrix}\right| = 5\sqrt{2}$

$\left|\begin{pmatrix}a\\5\\2\end{pmatrix}-\begin{pmatrix}1\\1\\-3\end{pmatrix}\right| = 5\sqrt{2}$

$\left|\begin{pmatrix}a-1\\4\\5\end{pmatrix}\right| = 5\sqrt{2}$

$(a-1)^2+16+25 = 50$

$(a-1)^2 = 9$

$a-1 = \pm 3$

$\underline{\underline{a=4}}$ or $a=-2$ (reject)

$\left|\begin{pmatrix}-4\\b\\-3\end{pmatrix}-\begin{pmatrix}1\\1\\-3\end{pmatrix}\right| = 5\sqrt{2}$

$\left|\begin{pmatrix}-5\\(b-1)\\0\end{pmatrix}\right| = 5\sqrt{2}$

$25+(b-1)^2 = 50$

$(b-1)^2 = 25$

$b-1 = \pm 5$

$\underline{\underline{b=6}}$ or $b=-4$ (reject)

$\left|\begin{pmatrix}2\\1\\c\end{pmatrix}-\begin{pmatrix}1\\1\\-3\end{pmatrix}\right| = 5\sqrt{2}$

$\left|\begin{pmatrix}1\\0\\c+3\end{pmatrix}\right| = 5\sqrt{2}$

$1+(c+3)^2 = 50$

$(c+3)^2 = 49$

$(c+3) = \pm 7$

$\underline{\underline{c=4}}$ or $c=-10$ (reject)

Q20. $\underset{\sim}{r} = \begin{pmatrix}-2-2\lambda\\16+5\lambda\\-1-2\lambda\end{pmatrix}$

$\left|\begin{pmatrix}-2-2\lambda\\16+5\lambda\\-1-2\lambda\end{pmatrix}-\begin{pmatrix}1\\-1\\3\end{pmatrix}\right| = 5\sqrt{2}$

$\left|\begin{pmatrix}-3-2\lambda\\17+5\lambda\\-4-2\lambda\end{pmatrix}\right| = 5\sqrt{2}$

$(-3-2\lambda)^2+(17+5\lambda)^2+(-4-2\lambda)^2 = 50$

Using CASIO,

$\lambda = -4$ or $\lambda = -2$

when $\lambda = -4$,

$\underset{\sim}{r} = \begin{pmatrix}-2+8\\16-20\\-1+8\end{pmatrix}$

$= \underline{\underline{\begin{pmatrix}6\\-4\\7\end{pmatrix}}}$

When $\lambda = -2$,

$\underset{\sim}{r} = \begin{pmatrix}-2+4\\16-10\\-1+4\end{pmatrix}$

$= \underline{\underline{\begin{pmatrix}2\\6\\3\end{pmatrix}}}$

Q21.

$\underset{\sim}{r} = \begin{pmatrix} 14 \\ 0 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ -9 \end{pmatrix}$

$\left| \underset{\sim}{r} - \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \right| = 7$

$\left| \begin{pmatrix} 14 \\ 0 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ -9 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \right| = 7$

$\left| \begin{pmatrix} 10 \\ -1 \\ -12 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ -9 \end{pmatrix} \right| = 7$

$(10+4\lambda)^2 + (\lambda-1)^2 + (-12-9\lambda)^2 = 49$

Using CASIO,

$\lambda = -2$ or $\lambda = -1$.

When $\lambda = -2$,

$\underset{\sim}{r} = \begin{pmatrix} 14 \\ 0 \\ -9 \end{pmatrix} - 2 \begin{pmatrix} 4 \\ 1 \\ -9 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 6 \\ -2 \\ 9 \end{pmatrix}}}$

When $\lambda = -1$,

$\underset{\sim}{r} = \begin{pmatrix} 14 \\ 0 \\ -9 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ -9 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 10 \\ -1 \\ 0 \end{pmatrix}}}$

Q22. $\underset{\sim}{r} = \begin{pmatrix} -2 \\ -1 \\ -11 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$

$\left| \underset{\sim}{r} - \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} \right| = 5$

$\left| \begin{pmatrix} -2+3\lambda-3 \\ -1+1 \\ -11+4\lambda-4 \end{pmatrix} \right| = 5$

$\left| \begin{pmatrix} -5+3\lambda \\ 0 \\ -15+4\lambda \end{pmatrix} \right| = 5$

$(-5+3\lambda)^2 + (-15+4\lambda)^2 = 25$

$\lambda = 3.$

$\therefore$ Given only one value of λ, line only intersects the sphere once and $\therefore$ is a tangent.

$\underset{\sim}{r} = \begin{pmatrix} -2 \\ -1 \\ -11 \end{pmatrix} + 3 \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 7 \\ -1 \\ 1 \end{pmatrix}}}$

Q23. $\underset{\sim}{r} = \begin{pmatrix} 9 \\ 18 \\ 20 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -4 \\ -3 \end{pmatrix}$

$\left| \underset{\sim}{r} - \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \right| = 7$

$\left| \begin{pmatrix} 9 \\ 18 \\ 20 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -4 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \right| = 7$

$\left| \begin{pmatrix} 11 \\ 17 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -4 \\ -3 \end{pmatrix} \right| = 7$

$(11-\lambda)^2 + (17-4\lambda)^2 + (17-3\lambda)^2 = 49$

Using CASIO,

$\underline{\underline{\lambda = 5}}$

Given one value of λ, line only intersects the sphere at $\underline{\text{one}}$ point

$\therefore$ tangential.

$\underset{\sim}{r} = \begin{pmatrix} 9 \\ 18 \\ 20 \end{pmatrix} + 5 \begin{pmatrix} -1 \\ -4 \\ -3 \end{pmatrix}$

$= \underline{\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}}$

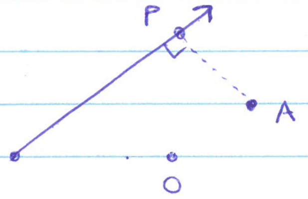
EXERCISE 5E

Q1 $\vec{OA} = \begin{pmatrix}1\\1\\1\end{pmatrix}$

$$\underset{\sim}{r} = \begin{pmatrix}2\\-1\\2\end{pmatrix} + \lambda\begin{pmatrix}3-2\\1+1\\-1-2\end{pmatrix}$$

$$\underset{\sim}{r} = \begin{pmatrix}2\\-1\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-3\end{pmatrix}$$

Let P be the pt on the line.



$$\vec{AP} = \vec{AO} + \vec{OP}$$
$$= \vec{OP} - \vec{OA}$$
$$= \begin{pmatrix}2\\-1\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-3\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix}$$
$$= \begin{pmatrix}1\\-2\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-3\end{pmatrix}$$

$$\therefore \begin{pmatrix}1\\2\\-3\end{pmatrix} \cdot \begin{pmatrix}1+\lambda\\-2+2\lambda\\1-3\lambda\end{pmatrix} = 0$$

$$1+\lambda-4+4\lambda-3+9\lambda=0$$
$$-6+14\lambda=0$$
$$14\lambda=6$$
$$\lambda = \frac{3}{7}$$

$$\therefore \vec{AP} = \begin{pmatrix}1\\-2\\1\end{pmatrix} + \frac{3}{7}\begin{pmatrix}1\\2\\-3\end{pmatrix}$$
$$= \begin{pmatrix}\frac{10}{7}\\-\frac{8}{7}\\-\frac{2}{7}\end{pmatrix}$$

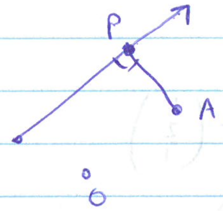
$$\therefore |\vec{AP}| = \sqrt{\frac{100}{49}+\frac{64}{49}+\frac{4}{49}}$$
$$= \frac{\sqrt{168}}{7}$$
$$= 1.85 \text{ (2dp)}$$

Q2.

$$\vec{OA} = \begin{pmatrix}4\\-2\\3\end{pmatrix}$$

$$\underset{\sim}{r} = \begin{pmatrix}2\\1\\-3\end{pmatrix} + \lambda\begin{pmatrix}3-2\\-1-1\\2+3\end{pmatrix}$$
$$= \begin{pmatrix}2\\1\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\5\end{pmatrix}$$

Let P be the closest point on the line



$$\vec{AP} = \vec{AO} + \vec{OP}$$
$$= \vec{OP} - \vec{OA}$$
$$= \begin{pmatrix}2\\1\\-3\end{pmatrix} - \begin{pmatrix}4\\-2\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\5\end{pmatrix}$$
$$= \begin{pmatrix}-2\\3\\-6\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\5\end{pmatrix}$$

$$\therefore \begin{pmatrix}1\\-2\\5\end{pmatrix} \cdot \begin{pmatrix}-2+\lambda\\3-2\lambda\\-6+5\lambda\end{pmatrix} = 0$$

$$-2+\lambda-6+4\lambda-30+25\lambda=0$$
$$30\lambda = 38$$
$$\lambda = \frac{19}{15}$$

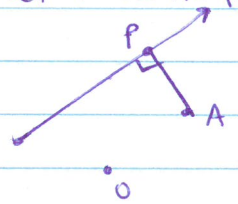
$$\therefore \vec{AP} = \begin{pmatrix}-2\\3\\-6\end{pmatrix} + \frac{19}{15}\begin{pmatrix}1\\-2\\5\end{pmatrix}$$
$$= \begin{pmatrix}-30+19\\45-38\\-90+95\end{pmatrix}\frac{1}{15}$$
$$= \frac{1}{15}\begin{pmatrix}-11\\7\\5\end{pmatrix}$$

$$|\vec{AP}| = \frac{1}{15}\sqrt{121+49+25}$$
$$= \frac{\sqrt{195}}{15} = 0.93 \text{ (2dp)}$$

Q3. $\vec{OA} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$$\underset{\sim}{r} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$$

Let P be the closest point on the line.



$$\vec{AP} = \vec{AO} + \vec{OP}$$
$$= \vec{OP} - \vec{OA}$$
$$= \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 + \lambda \\ -2 - 3\lambda \\ 2 + 4\lambda \end{pmatrix} = 0$$

$$1 + \lambda + 6 + 9\lambda + 8 + 16\lambda = 0$$
$$26\lambda = -15$$
$$\lambda = \frac{-15}{26}$$

$$\vec{AP} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} - \frac{15}{26}\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$$
$$= \begin{pmatrix} 26 - 15 \\ -52 + 45 \\ 52 - 60 \end{pmatrix} \frac{1}{26}$$
$$= \frac{1}{26}\begin{pmatrix} 11 \\ -7 \\ -8 \end{pmatrix}$$

$$\therefore |\vec{AP}| = \frac{1}{26}\sqrt{121 + 49 + 64}$$
$$= \frac{1}{26}\sqrt{234}$$
$$= \frac{3\sqrt{26}}{26}$$
$$= 0.59 \text{ (2dp)}$$