TEST 1 (Complex Numbers & Vectors)

Worth 5% of the Year Mark 50 minutes permitted.

Name :

Score :
(out of 60)

1. [10 marks]

Given complex numbers **z** and **w** where $\mathbf{z} = 3 + 5\mathbf{i}$ and $\mathbf{w} = 4 - 7\mathbf{i}$

(a) Determine, exactly

(i) $|\mathbf{z} - \mathbf{w}|$

[2]

(ii) $\text{Re}(\mathbf{z}) - \text{Im}(\mathbf{w})$

[1]

(iii) $\dfrac{1}{\bar{z} - \bar{w}}$

[3]

(b) Find the value of a such that $a\mathbf{z} + 3\mathbf{w} = 6 - 31\mathbf{i}$

[4]

2. **[8 marks]**

Consider the complex numbers $\mathbf{u} = 2\sqrt{3} - 2\mathbf{i}$ and $\mathbf{v} = \mathbf{i} - 1$

(a) Write $\mathbf{u}$ and $\mathbf{v}$ in exact polar form.

[3]

(b) Simplify $\dfrac{u^2}{v^6}$, leaving your answer exactly in polar form.

[3]

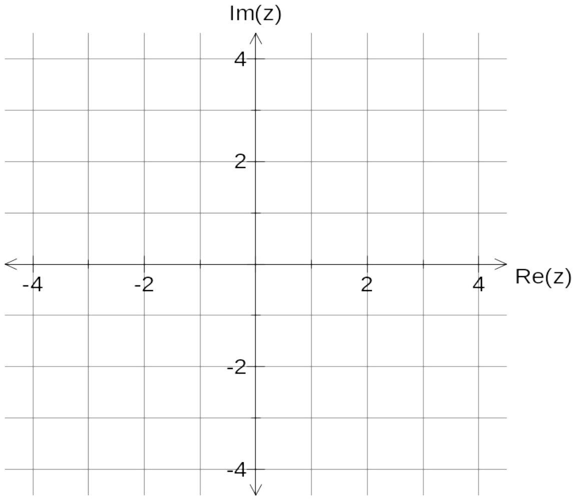
(c) Find exactly $|\mathbf{u} + 2\mathbf{v}|$

[2]

3. **[9 marks]**

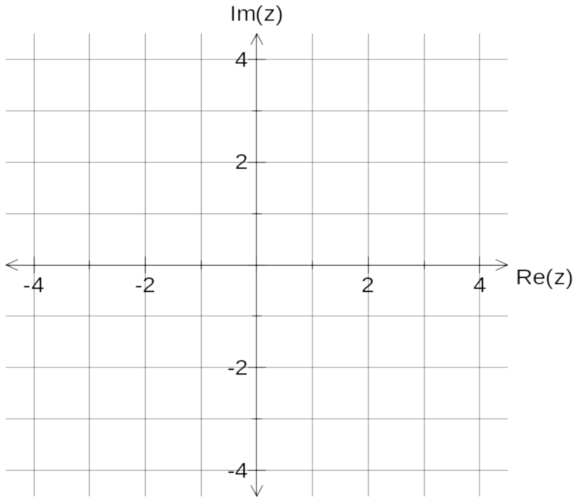
(a) Sketch the graphs in the Argand Plane to indicate the set of numbers z that satisfy :

(i) $\frac{z}{\bar{z}} = i$



[3]

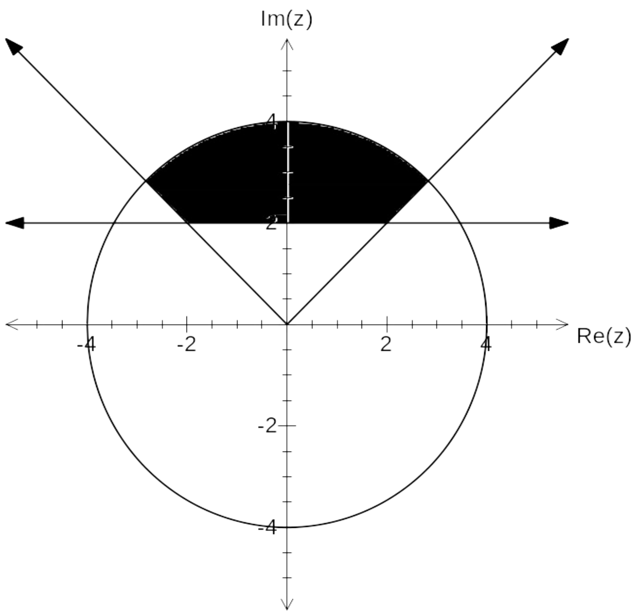
(ii) $-\frac{\pi}{6} \le Arg\left[(1+\sqrt{3}\,i)\,z\right] \le \frac{\pi}{3}$



[3]

(b) Describe the shaded region in the Argand plane below.

[3]



4. [5 marks]

For the region in the Argand plane defined by the inequality $|z - 4 - 2i| \leq 2$,

determine the maximum and minimum value for the argument of z.

5. [5 marks]

(a) State the geometrical relationship between the complex numbers $\mathbf{w}$ and $\mathbf{z}$ if it is known that $\mathbf{w} = i\mathbf{z}$

[2]

(b) The three points A, B and C in the Argand plane correspond to complex numbers $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ respectively. The triangle ABC is isosceles and has a right angle at A.

Write down algebraically the relationship between $\mathbf{z}_3 - \mathbf{z}_1$ and $\mathbf{z}_2 - \mathbf{z}_1$.
Explain how you arrived at your answer.

[3]

6. **[23 marks]**

Consider the following vectors in space :

a = $\begin{pmatrix} -2 \\ 2 \\ -3 \end{pmatrix}$, **b** = $\begin{pmatrix} x \\ 3 \\ -2 \end{pmatrix}$, **c** = $\begin{pmatrix} 5 \\ 2 \\ -z \end{pmatrix}$,

and **d** = $\begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$

Determine :

(a) vector **e** such that **e** is parallel to **d** and double its length.

[1]

(b) the acute angle between vectors **a** and **d** (to nearest degree).

[3]

(c) the relationship between x and z if **c** is perpendicular to **b**.

[2]

(d) the value of x such that **a** is parallel to **b**.

[3]

(e) vector **f** such that **f** is in the direction of **a** with a magnitude of 17 units.

[3]

(f) a vector which is perpendicular to both **a** and **d**.

[4]

Suppose that vectors **a** and **d** represent position vectors of points A and D respectively.

(f) Determine the position vector **p** for the point P which divides $\overline{AD}$ internally in the ratio 3:1.

[4]

(g) Determine the vector equation for the line in space that connects points A and D.

[3]

End of Test