PERTH MODERN SCHOOL
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Independent Public School

Mathematics Specialist   Year 11

Student name: _____Teacher name: _____

Date: Friday 23rd July 2021

Task type:	**Response**
Time allowed:	**45 minutes**
Number of questions:	**6**
Materials required:	Calculator with CAS capability (to be provided by the student)
Standard items:	Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	Drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations
Marks available:	**40 marks**
Task weighting:	10%
Formula sheet provided:	Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

1. [7 marks]
   Use mathematical induction to prove that

   $$3 \times 5 + 6 \times 6 + 9 \times 7 + \ldots + 3n(n+4) = \frac{n(n+1)(2n+13)}{2}$$

   for all positive integers n.

2. [2 marks]

A question in a Specialist exam paper asked students to prove the following statement:

'$3n$ is odd if and only if n is odd (where n is an integer)'.

One student wrote the answer below. Explain clearly why they should **not** receive full marks for this answer.

Proof:
We prove the contrapositive. Assume that n is an even integer. Then $n = 2k$ for some integer k. Now

$$3n = 3(2k) ¿ 2(3k)$$

which is even since $3k$ is an integer. Hence if n is even then $3n$ is even, which implies that $3n$ is odd if and only if n is odd.

3. [9 = 3+3+3 marks]

Write whether each of the following statements is true or false, and prove or disprove it accordingly.

a) For all positive real numbers x
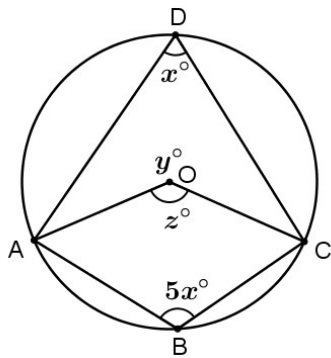
$$x^3 - x \geq x^2 - x$$

b) There exist distinct prime numbers p and q such that $p-q=2$.

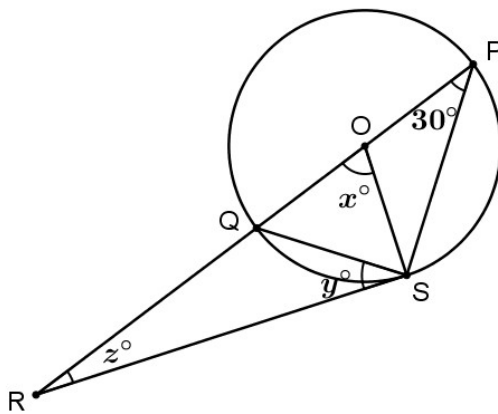c) There exist distinct prime numbers p and q such that $p^2-q^2=2$.

4. [6 marks]

Find the values of x, y and z in each of the following:

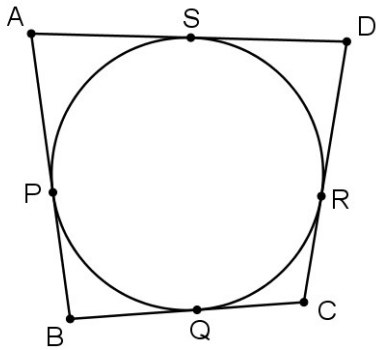a) A, B, C and D all lie on the circle with centre O:

b) $\overline{RS}$ is tangent to the circle with centre O.

5. [5 marks]

   $ABCD$ is a quadrilateral such that each of the four sides is tangent to the same circle, at the points P, Q, R and S, as illustrated below. If $AB=15$, $BC=10$ and $CD=12$, find the length AD.

6. [11 = 3+4+4 marks]
   Solve each of the following trigonometric equations for x in the stated domain.
   Show all working to support your answers.
   a) $2\cos(x)=\sqrt{3}$ for $0\le x\le 2\pi$

   b) $\sin\left(x+\dfrac{\pi}{4}\right)=\dfrac{-1}{\sqrt{2}}$ for $-\pi\le x\le\pi$

c) $\dfrac{1}{\sqrt{3}}\tan(5x)=1$ for $0 \le x \le \pi$