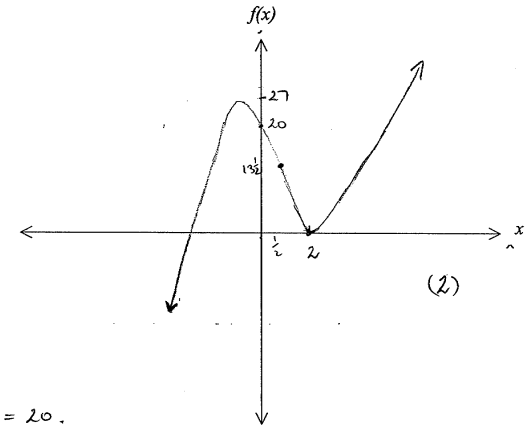
Question 8 (7 marks)

Determine all turning points and points of inflection of the function $f(x) = 2x^3 - 3x^2 - 12x + 20$, and use these to sketch its graph.



(2)

y int = 20.

$f'(x) = 6x^2 - 6x - 12$ (1)

$f'(x) = 0$ for max/min

$0 = x^2 - x - 2$

$= (x-2)(x+1)$

$x = 2 \quad x = -1$ (2,0) (-1,27) } (1)

$y = 0 \quad y = 27$

$f''(x) = 12x - 6$

$f''(2) > 0 \therefore$ min

$f''(-1) < 0 \therefore$ max } (1)

$f''(x) = 0$ for pts of inflection

$12x - 6 = 0$ (1)

$x = 1/2 \quad y = 13\,1/2$ $(1/2, 13\,1/2)$ (1)

Mathematics 3C/3D

Semester 1 Exam, 2010

Name: _____

Teacher: Solutions

Section One: Calculator-free
TIME ALLOWED FOR THIS PAPER:

Reading time before commencing work: Five minutes
Working time for paper: Fifty minutes

Material required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks
Section One: Calculator-free	8	8	50	40
Section Two: Calculator-assumed	12 15	12 15	100	80
				120

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you do not use pencil except in diagrams.

Section One: Calculator-free (40 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the space provided.

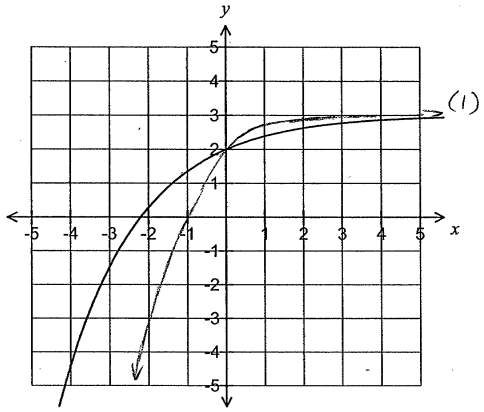
Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.

Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Question 7 (4 marks)

The graph of $y = ae^{bx} + c$ is shown below. The graph passes through the point $(0, 2)$, and $y \to 3$ as $x \to \infty$.



(a) Is b positive or negative? Justify your answer. (1 mark)

As $y \to 3$, $x \to -\infty$
$\therefore e^{bx} \to 0$ as $x \to -\infty$
$\therefore$ b must be negative (similar logic statements)

(b) Evaluate a and c. (2 marks)

as $y \to 3$ $c = 3$ (1)
$y(0) = ae^0 + 3$
$= a + c$
$= 2$ $\therefore a = -1$ (1)

(c) Sketch on the same axes the graph of $y = ae^{2bx} + c$. (1 mark)

Question 5 (4 marks)

Determine the following integrals:

(a) $\int \frac{x^2-1}{(x^3-3x)^2}\,dx$ (2 marks)

$= \frac{(x^3-3x)^{-1}(x^2-1)}{-1\cdot(3x^2-3)} + C$

$= \frac{-1}{3(x^3-3x)} + C$ -1 each error

Alternative method.

$\int (x^2-1)(x^3-3x)^{-2}\,dx$

$= \frac{1}{3}\int (3x^2-3)(x^3-3x)^{-2}\,dx$

$= \frac{1}{3}\frac{(x^3-3x)^{-1}}{-1} + C$

$= \frac{-1}{3(x^3-3x)} + C.$

(b) $\int_0^5 e^{-2x}\,dx$ (2 marks)

$= \left[\frac{e^{-2x}}{-2}\right]_0^5$ (1)

$= \frac{e^{-10}}{-2} + \frac{1}{2}$ or $\frac{-1}{2e^{10}} + \frac{1}{2}$ (1)

Question 6 (4 marks)

a) Simplify each of the following.

(i) $\frac{x^2-1}{x} \div \frac{x^2+2x+1}{3x^2-6x} = \frac{(x-1)(x+1)}{x} \div \frac{(x+1)(x+1)}{3x(x-2)}$ (2 marks)

$= \frac{(x-1)(x+1)}{x} \times \frac{3x(x-2)}{(x+1)(x+1)}$ (1)

$= \frac{3(x-1)(x-2)}{x+1}$ or $\frac{3x^2-9x+6}{x+1}$ (1)

(ii) $\frac{5}{y^2-9} + \frac{6}{y-3} - \frac{7}{y+3} = \frac{5+6(y+3)-7(y-3)}{(y-3)(y+3)}$ (1) (2 marks)

$= \frac{5+6y+18-7y+21}{(y-3)(y+3)}$

$= \frac{44-y}{(y-3)(y+3)}$ (1)

STRUCTURE OF THE PAPER

QUESTION	MARKS	TOTAL
1	3	
2	10	
3	3	
4	5	
5	4	
6	4	
7	4	
8	7	
TOTAL	40	

Question 1 (3 marks)

Determine the domain and range of $f(g(x))$, given that $f(x)=\sqrt{x}$ and $g(x)=4-2^x$.

$f(4-2^x) = \sqrt{4-2^x}$ (1)

$D_x \Rightarrow \{ x \in \mathbb{R},\ x \leq 2 \}$ (1)

$R_y \Rightarrow \{ y \in \mathbb{R},\ 0 \leq y < 2 \}$ (1)

$4-2^x \geq 0$

$4 \geq 2^x$

$\therefore 2 \geq x$

(1) mark for some calculation of Domain

Question 2 (10 marks)

Differentiate the following, without simplifying:

(a) $y = e^{2x-x^2}$ (2 marks)

$\frac{dy}{dx} = (2-2x)e^{2x-x^2}$ (1) (1)

N.B. Brackets needed around $(2-2x)$

(b) $y = \frac{5x}{x^2+4}$ (2 marks)

$\frac{dy}{dx} = \frac{5(x^2+4) - 2x(5x)}{(x^2+4)^2}$ -1 each error.

(c) $y = e^{3x}(e^x + \sqrt{x} + 3\pi^3)$ (3 marks)

$\frac{dy}{dx} = 3e^{3x}(e^x + \sqrt{x} + 3\pi^3) + (e^x + \frac{1}{2}x^{-1/2})e^{3x}$

-1 each error

(d) $y = \sqrt{u^2-3}$ using chain rule notation $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ where $u = 2e^{2x}+3$

$\frac{dy}{du} = \frac{1}{2}(u^2-3)^{-1/2} \cdot 2u$ (1) $\qquad \frac{du}{dx} = 4e^{2x}$

(3 marks)

$\frac{dy}{dx} = 4e^{2x}\left((2e^{2x}+3)^2-3\right)^{-1/2}(2e^{2x}+3)$ (2)

Question 3 (3 marks)

The probabilities of two events A and B are given by: $P(A) = 0.6$ and $P(B) = 0.3$
Calculate $P(A \cup B)$, given that A and B are independent.

$P(A \cup B) = P(A) \times P(B)$ for independent events.

$= 0.6 \times 0.3$

$= 0.18$ (1)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.3 - 0.18$ (1)

$= 0.72$ (1)

Question 4 (5 marks)

Find the maximum and minimum values over the interval $1 \leq x \leq 4$ of the function

$f(x) = x + \frac{4}{x^2}$

$f'(x) = 1 - \frac{8}{x^3}$ (1)

$f'(x) = 0$ for max. or min. values

$1 - \frac{8}{x^3} = 0$ (1)

$\frac{8}{x^3} = 1$

$x = 2$ (1)

Needed to show some sort of calculate.

$f(2) = 3$

$f(1) = 5$

$f(4) = 4.25$

max. occurs at $x = 1$ Value 5 (1)

min. occurs at $x = 2$ Value 3 (1)