



MATHEMATICS DEPARTMENT  
MATHEMATICAL METHODS YEAR 12 – TEST 3

DATE: 27<sup>th</sup> June 2016

Name:

Mark Allwright

CALCULATOR FREE

Reading Time:

3 minutes

Working Time:

50 minutes

EQUIPMENT:

pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet  
(provided)

Question	Marks available	Marks awarded
1	4	
2	6	
3	6	
4	3	
5	6	
6	5	
7	6	
8	7	
9	9	
Total	52	

**Question 1****(4 marks)**

Evaluate the following:

(a)  $\log_3 27$

(1 mark)

$$= 3 \quad \checkmark$$

(b)  $\log_{15} 1$

(1 mark)

$$= 0 \quad \checkmark$$

(c)  $\log_{25} 0.2$

(2 marks)

$$25^x = \frac{1}{5} \quad \checkmark$$

$$5^{2x} = 5^{-1}$$

$$x = -\frac{1}{2} \quad \checkmark$$

**Question 9 continued****(9 marks)**(c) Determine the cumulative distribution function  $F(x)$ 

(3 marks)

$$\int_0^k \frac{2x}{9} dx \quad \checkmark$$
$$= \left[ \frac{x^2}{9} \right]_0^k$$
$$= \frac{k^2}{9} \quad \checkmark$$

$$p(X \leq x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{x^2}{9} & \text{for } 0 \leq x \leq 3 \\ 1 & \text{for } x > 3 \end{cases} \quad \checkmark$$

(d) Calculate  $P(1 < x < 2)$ 

(2 marks)

$$p(1 < x < 2) = \frac{2^2}{9} - \frac{1^2}{9} \quad \checkmark$$
$$= \frac{3}{9}$$
$$= \frac{1}{3} \quad \checkmark$$

End of Test

### Question 9

(9 marks)

The continuous random variable  $X$  is defined by the probability density function

$$f(x) = \begin{cases} \frac{2x}{9} & 0 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine  $E(X)$ .

$$E(x) = \int_0^3 \frac{2x^2}{9} dx = 2$$

$$= \left[ \frac{2x^3}{9} \right]_0^3 = 2$$

(b) The variance of  $X$ ,  $\text{Var}(X)$ , is  $\frac{1}{2}$ .

(i) Determine  $E(4X + 3)$

$$= 4 \times 2 + 3 = 11$$

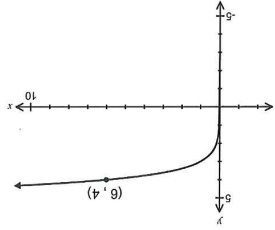
(ii) Determine  $\text{Var}(4X + 3)$

$$\frac{1}{2} \times 4^2 = 8$$

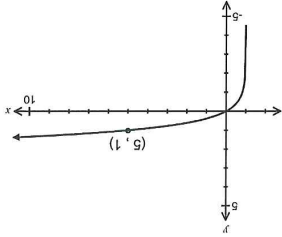
### Question 2

(6 marks)

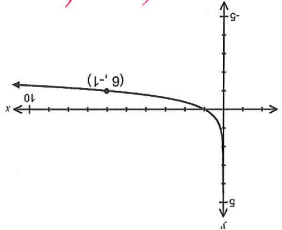
Match each of the following graphs with their equations from the given list. A graph may have more than one matching equation. Not all equations have a matching graph.



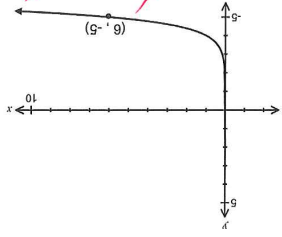
A ✓



B ✓



C ✓  
D ✓



D ✓  
F ✓  
I ✓

A  $y = \log_6(x+1)$

D  $y = -\log_6(x)$

G  $y = \log_5(x)$

B  $y = \log_6(x-1)$

E  $y = \log_6(x) + 3$

H  $y = \log_5(x+1)$

C  $y = \log_6(x)$

F  $y = -\log_6(x) - 4$

I  $y = \log_6\left(\frac{x}{1}\right) - 4$

### Question 3

(6 marks)

Differentiate with respect to  $x$ .

(a)  $f(x) = \ln(2x+1)$

(1 mark)

$$f'(x) = \frac{2}{2x+1} \quad \checkmark$$

(b)  $f(x) = \ln\left(\frac{x^2+2x}{x-5}\right)$

(2 marks)

$$= \ln(x^2+2x) - \ln(x-5)$$

$$f'(x) = \frac{2x+2}{x^2+2x} - \frac{1}{x-5} \quad \checkmark \quad \text{or} \quad \frac{x^2-10x-10}{(x^2+2x)(x-5)} \quad \checkmark$$

(c)  $f(x) = \frac{2\sqrt{x}}{\ln x} = \frac{2x^{1/2}}{\ln x}$

(3 marks)

$$f'(x) = \frac{\ln x \times x^{-1/2} - \frac{1}{x} \times 2x^{1/2}}{(\ln x)^2} \quad \checkmark \checkmark$$

$$= \frac{\ln x \cdot x^{-1/2} - x^{-1/2} \times 2}{(\ln x)^2}$$

$$= \frac{\frac{1}{\sqrt{x}}(\ln x - 2)}{(\ln x)^2} \quad \checkmark = \frac{\ln x - 2}{\sqrt{x}(\ln x)^2}$$

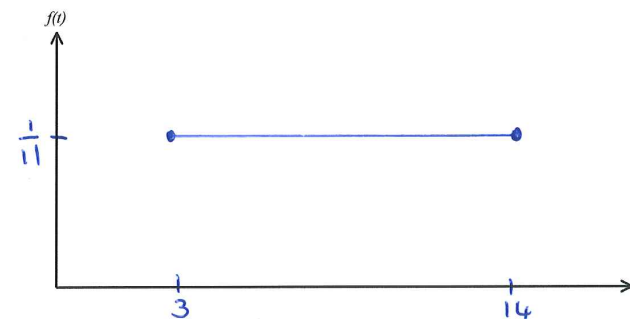
### Question 8

(7 marks)

As part of a local arts festival, an artist plans to create an installation in which a concealed water cannon blasts a stream of water into the air for a few seconds at random intervals.

The lengths of the intervals between each firing of the cannon can be modelled by the uniformly distributed random variable  $T$ , where  $3 \leq t \leq 14$  minutes.

- (a) Sketch the probability density function  $f(t)$  for the interval between each firing on the axes below. (2 marks)



- (b) Determine the probability that a randomly chosen interval between firings is

- (i) at least seven minutes. (1 mark)

$$P(7 \leq x \leq 14) = \frac{7}{11} \quad \checkmark$$

- (ii) at least six minutes given that it is less than ten minutes. (2 marks)

$$P(x \geq 6 | x \leq 10) = \frac{4/11}{7/11} \quad \checkmark = \frac{4}{7} \quad \checkmark$$

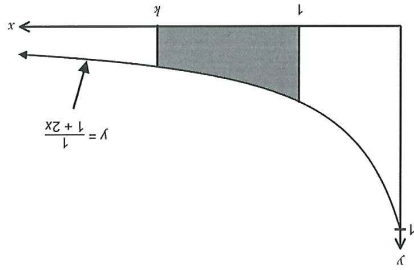
- (c) Determine the value of  $t$  for which  $P(T < t) = P(T > 4t)$  (2 marks)

$$\frac{t-3}{11} = \frac{14-4t}{11} \quad \checkmark$$

$$5t = 17 \\ t = \frac{17}{5} = 3\frac{2}{5} \quad \checkmark$$

# Question 7

The shaded area is 0.2 units<sup>2</sup>.  
Find  $k$  exactly.



(6 marks)

$$\int_k^1 \frac{1}{1+2x} dx = 0.2$$

$$\left[ \frac{1}{2} \ln(1+2x) \right]_k^1 = 0.2$$

$$\frac{1}{2} [\ln(1+2) - \ln(1+2k)] = 0.2$$

$$\ln\left(\frac{1+2k}{3}\right) = 0.4$$

$$e^{0.4} = \frac{3}{1+2k}$$

$$1+2k = 3e^{0.4}$$

$$2k = 3e^{0.4} - 1$$

$$k = \frac{3e^{0.4} - 1}{2}$$

# Question 4

Find the following indefinite integrals.  
Assume denominators are greater than zero.

(a)  $\int \frac{12}{x} dx$

$$= 12 \ln x + C$$

(1 mark)

(b)  $\int \frac{2e^x + 2 \sin x}{e^x - \cos x} dx$

$$= 2 \int \frac{e^x + \sin x}{e^x - \cos x} dx$$

$$= 2 \ln(e^x - \cos x) + C$$

(2 marks)

Question 5

(6 marks)

Find the exact value of  $x$  satisfying the equation

$$(3^x)(4^{2x+1}) = 6^{x+2}$$

Give your answer in the form  $\frac{\ln a}{\ln b}$ .

$$\ln(3^x \cdot 4^{2x+1}) = \ln(6^{x+2}) \quad \checkmark$$

$$x \ln 3 + (2x+1) \ln 4 = (x+2) \ln 6 \quad \checkmark$$

$$x \ln 3 + x \ln 4^2 + \ln 4 = x \ln 6 + \ln 6^2$$

$$x \ln 3 + x \ln 16 - x \ln 6 = \ln 36 - \ln 4 \quad \checkmark$$

$$x (\ln(\frac{3 \times 16}{6})) = \ln(\frac{36}{4}) \quad \checkmark$$

$$x \ln 8 = \ln 9$$

$$x = \frac{\ln 9}{\ln 8} \quad \checkmark$$

Question 6

(5 marks)

(a) The function  $f$  is defined for  $x > 2$  by  $f(x) = \ln x + \ln(x-2) - \ln(x^2-4)$ .

Express  $f(x)$  in the form  $\ln\left(\frac{x}{x+a}\right)$ .

(2 marks)

$$f(x) = \ln\left(\frac{x(x-2)}{x^2-4}\right) \quad \checkmark$$

$$= \ln\left(\frac{x(x-2)}{(x-2)(x+2)}\right)$$

$$= \ln\left(\frac{x}{x+2}\right) \quad \checkmark$$

(b) Evaluate  $\log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} + \log \frac{4}{5} + \dots + \log \frac{8}{9} + \log \frac{9}{10}$ .

(3 marks)

$$\log\left(\frac{1 \times 2 \times 3 \times 4 \times \dots \times 8 \times 9}{1 \times 2 \times 3 \times 4 \times \dots \times 9 \times 10}\right) \quad \checkmark$$

$$= \log\left(\frac{1}{10}\right) \quad \checkmark$$

$$= -\log 10$$

$$= -1 \quad \checkmark$$