



# MOUNT LAWLEY SENIOR HIGH SCHOOL

Semester 2 Examination, 2011

Question/Answer Booklet

## MATHEMATICS SPECIALIST MAS 3C/3D

### Section One Calculator-free

NAME \_\_\_\_\_

#### Time allowed for this section

Reading time before commencing work: 5 minutes

Working time for paper: 50 minutes

#### Material required/recommended for this section

##### *To be provided by the supervisor*

This Question/Answer booklet

Formula sheet

##### *To be provided by the candidate*

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: nil

#### Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	6	6	50	40
Section Two: Calculator-assumed	13	13	100	80
<b>Total</b>				120

**Instructions to candidates**

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2011*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil**, except in diagrams.

This section has **six (6)** questions.

Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

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**Question 1**

**(6 marks)**

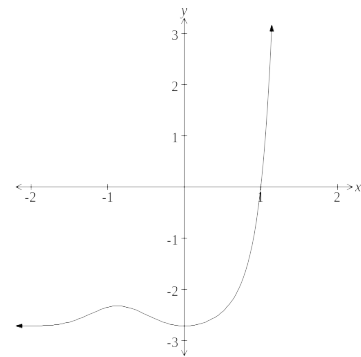
Consider the function  $f(x) = x^2 e^{x^3} - e$ .

a) Determine  $\int f(x) dx$ .

[1]

The graph of  $y = f(x)$  is shown.

b) Carefully shade the area contained between  $y = f(x)$ , the lines  $x = -1$ ,  $x = 1$ , and  $y = 2$ . [1]



c) Write down an integral whose value represents the shaded area.

[1]

d) Prove the shaded area is exactly equal to  $4 + \frac{1+5e^2}{3e}$ .

[3]

Question 2

(5 marks)

- (a) Find the equation of the tangent to the curve  $x^3 - y^3 = 2$  at the point on the curve where  $x = 1$ .

[2]

- (b) Evaluate  $\int_1^{e^2} \frac{(\ln x)^2}{x} dx$ .

[3]

**Question 3**

**(6 marks)**

The transformation matrix  $M = \begin{bmatrix} -2 & 1 \\ a & b \end{bmatrix}$ .

$M$  represents a shear of factor  $k$  parallel to the  $y$ -axis followed by a rotation of  $90^\circ$  clockwise.

(a) Use properties of the two transformations to explain why  $|M| = 1$  (ie  $\text{Det } M = 1$ ). [1]

(b) Determine the values of  $a$ ,  $b$  and  $k$ . [3]

The point  $P$  is transformed by  $M$  to the point  $(8,3)$ .

(c) Determine the coordinates of  $P$ . [2]

Question 4

(8 marks)

Consider the identity  $2i \sin(n\theta) = z^n - \frac{1}{z^n}$  where  $z = \text{cis } \theta$ .

(a) By initially letting  $n = 1$ , show how to use the identity to prove that:

$$\sin^3 \theta = \frac{3 \sin \theta - \sin(3\theta)}{4}. \quad [5]$$

Question 4 (continued)

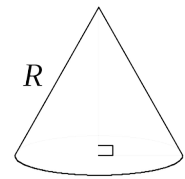
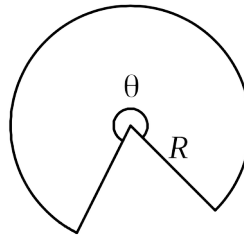
Hence,

(b) evaluate  $\int_0^{\pi} (9 \sin x - 12 \sin^3 x) dx$ . [3]

**Question 5**

**(7 marks)**

A minor sector of angle  $2\pi - \theta$  is removed from a circular piece of paper of radius  $R$ . The two straight edges of the remaining major sector are pulled together to form a right circular cone, with a slant height of  $R$ .



- (a) Show that the volume of the cone is given by  $V = \frac{R^3 \theta^2 \sqrt{4\pi^2 - \theta^2}}{24\pi^2}$ .

**[3]**



**Question 5 (continued)**

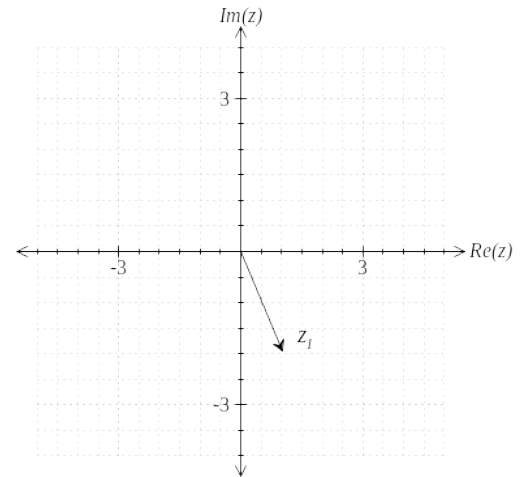
Assuming the radius,  $R$ , of the circular piece of paper to be fixed,

- (b) show the exact value of  $\theta$  which maximises the volume of cone is  $\frac{2\sqrt{2}\pi}{\sqrt{3}}$ . [4]

**Question 6**

**(8 marks)**

The Argand diagram below shows the complex number  $z_1 = a + ib$  as a position vector with  $a$  and  $b$  having integer values.



- (a) On the same diagram plot and label the complex numbers  $z_2$ ,  $z_3$  and  $z_4$  given by:

$$z_2 = z_1 \times \bar{z}_1 \quad z_3 = i^3 \times z_1 \quad z_4 = 10(z_1)^1$$

[4]

- (b) On the same diagram sketch the region given by  $|z + z_1| \leq 1$ .

[2]

- (c) Determine the Cartesian equation described by  $|z + 1| < |z - i|$ .

[2]



