

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

IMPORTANT NOTE TO CANDIDATES

Special items: drawing instruments, templates, notes on up to two unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations.

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

To be provided by the candidate

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

Working time for section: 100 minutes

Reading time before commencing work: 10 minutes

TIME ALLOWED FOR THIS SECTION

Teacher: _____

Name: _____

(Calculator-assumed)

Section Two

2016

REVISION 3

Units 3-4

MATHEMATICS METHODS

SEMINSTER TWO
YEAR 12
Papers written by
Australian Maths
Software

To be provided by the supervisor

Question/answer booklet for Section Two.
Formula sheet retained from Section One.

1. The rules for the conduct of this examination are detailed in the Information Handbook.
2. Write your answers in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

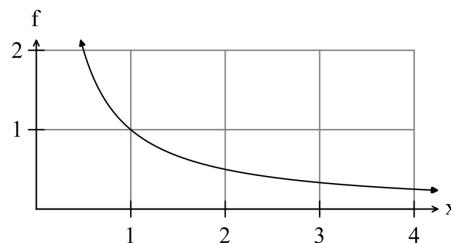
Instructions to Candidates

Percentage of exam	Marks	Available time (minutes)	Number of questions to be answered	Working time (minutes)	Number of questions to be answered	Calculator-free	Section One	Calculator-assumed	Section Two	Calculator—assumed	Total marks	100

Structure of this examination

6. (4 marks)

- (a) The equation of the graphed function is $f(x) = \frac{1}{x}$.



Determine the area between the function, the x axis for $1 \leq x \leq 3$ correct to three decimal places.

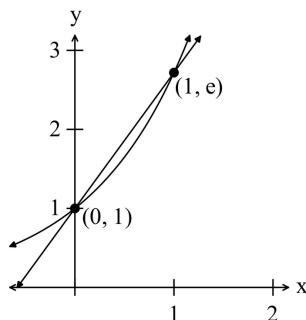
(2)

- (b) The function $f(x) = e^x$ graphed below.

Find the expression for the area between the line and the function $f(x) = e^x$.

(2)

NB You do not have to calculate the area.



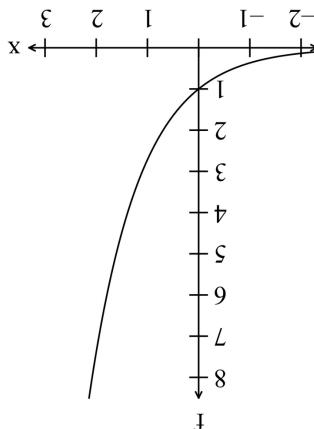
- (c) Residents living in East Perth are thought to want no more high rise developments. It was planned to conduct a survey to determine the proportion of residents that object to more high rise development.

How large should the sample be, allowing for an error margin of 10% with a 95% confidence level? (5)

(i) Determine an estimate for the probability that the people in the Council area wanted garden rubbish collected more than once a year. (4)

- (ii) Eighty-five out of 100 people surveyed by Stirring Council wanted garden rubbish collected more than once a year.
- (iii) Determine the 90% confidence level that the probability that the people in the Council area wanted garden rubbish collected more than once a year. (4)

(a) The function $f(x) = e^x$ is sketched on the set of axes below.



7. (10 marks)

(ii) In your discussion you should mention turning points, points of inflection, concavity, gradient, and how they relate to the function f and its graph. (3)

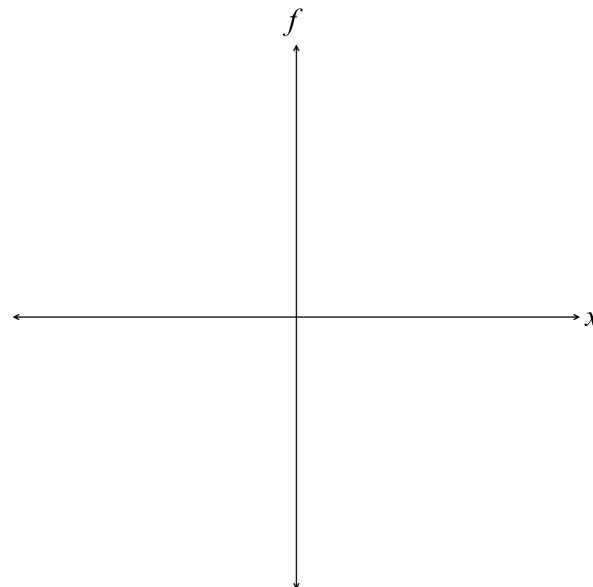
(iii) Discuss fully the relationship between the functions f , f' , and f'' .

(i) Write down the derivatives $y = f'(x)$ and $y = f''(x)$. (4)

- (b) (i) Sketch a function that has the following properties

x	-2	0	2
f	16	0	-16
f'	0	-12	0
f''	-	0	+

(3)



- (ii) Write down the equation of the function.

(3)

18. (14 marks)

- (a) The probability of an Australian lady over 40 years of age not having had children is 0.1.

In a sample of 60 ladies over 40, what are the chances that between 5 and 9 of them have had no children? (4)

17. (8 marks) MATHEMATICS METHODS, Semester Two
Calculator-assumed
- The probability that the baker runs out of sultana buns by the time Bill leaves work is about $\frac{1}{3}$. Bill calls in every Monday to buy a sultana bun on his way to night class. What is the probability that on the next 6 Mondays Bill manages to buy a sultana bun each time?
- (a) on the next 6 Mondays Bill manages to buy a sultana bun each time? (2)

- The volume of a sphere is $V = \frac{4}{3}\pi r^3$. Use a calculus method to determine the increase in the volume if the radius increases from 1 to 1.05 metres.
- (b) on the next 6 Mondays Bill only manages to buy a sultana bun 4 times out of 6? (2)

- (c) Bill can buy a sultana bun if on the last three Mondays the shop had run out of buns. (2)
- (d) on the next six Mondays Bill is able to buy a sultana bun only on alternate Mondays? (2)
- (e) Use your calculator to evaluate $\frac{4}{\pi} - \frac{1}{h}$ as $h \rightarrow 0$. (2)

9. (5 marks)

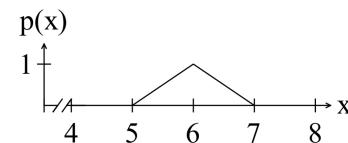
(a) Find $\frac{d}{dx} \int^{\sqrt{x}} \frac{2}{(1-t^4)^2} dt$

(3)

16. (6 marks)

Consider the probability density function $p(x) = \begin{cases} x - 5 & \text{for } 5 \leq x \leq 6 \\ -x + 7 & \text{for } 6 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$

graphed below.



(b) (i) If $f(x) = \sqrt{x}$ find $y = f'(x)$.

(1)

(a) Find $P(5.5 \leq x \leq 6.5)$

(2)

(ii) Hence find $\int^4 \left(\frac{1}{2\sqrt{x}} \right) dx$

(1)

(b) Find the mean and the variance of the distribution.

(4)

NB Professor Markus Schlaich of Perth conducted trials with a new procedure to combat high blood pressure in March 2016.

- (b) What percentage of people had high blood pressure at age 70? (2)

The 2014-15 National Health Survey found that 23% of Australian aged 18 and over had hypertension (high blood pressure). For those aged 75 and over, 46.9% had hypertension.
 Assume the percentage, P , of those with high blood pressure rises exponentially i.e. according to the law $P = P_0 e^{kx}$, where x represents age.

10. (6 marks)



(3)

One week Fred took three of his friends and they all had steak
 than 300 grams?
 (c) What is the probability that at least three of the men had steaks greater

(b) Given that Fred's steak was not below the advertised weight, what are the chances it was at least 300 grams?

(2)

(a) What is the probability that Fred is served a steak under the advertised weight?
 Fred is fussy about his steaks.

The weight of a piece of steak in Fred's favourite restaurant is advertised as at least 280 grams.
 The weight of steaks actually served are normally distributed with a mean of 300 grams with a standard deviation of 10 grams.

15. (8 marks)

11. (6 marks)

The displacement of a particle is given by $x = 3t^2 - 6t$ m, where t is measured in seconds, determine

(a) the displacement when the particle changes direction. (2)

(b) the acceleration at $t = 2$ s. (1)

(c) the distance travelled during the first ten seconds. (3)

(b) Find the mean and variance of the probability density function in (a). (4)

(c) Which of the following are not probability density functions?
State clearly why.

(i)

x	1	2	3
$P(X = x)$	0.3	0.4	0.5

(1)

(ii)

x	3	4	5
$P(X = x)$	0.6	-0.1	0.3

(1)

(iv) Find $P(\text{a score of } 6 \text{ or } 7)$

(iii) Find $P(\text{there is at least one odd number})$

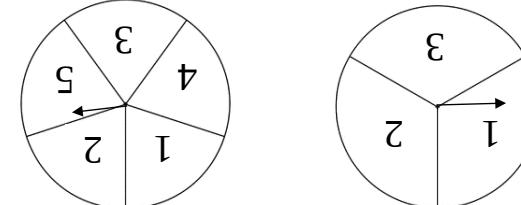
(ii) Find $P(\text{the score is odd})$

(i) Complete the following probability density table.

$P(\text{score})$							
Score when added	2	3	4	5	6	7	8

(3) The numbers the spinners land on are added. The possible combinations are shown in the table below.

Spinner of slide 5	1	2	3	4	5
Spinner of slide 3	1	2	3	4	5



(a) Two spinners as shown below are spun simultaneously.

(a) A toy teddy bear waves its arms up and down.

The motion of the arms is modelled on the equation

$$d = 2 \sin \left(\frac{3}{2\pi} t \right) \quad \text{where } t \text{ is measured in seconds and } d \text{ in cm.}$$

(i) How far up and down do the teddy bear's arms move?



(ii) How long does a complete wave of the teddy bear arms take?

(1)

(iii) If the teddy bear's arms started to move twice as fast, write down the equation that models the faster motion.

(2)

12. (8 marks)

(a) A toy teddy bear waves its arms up and down.

The motion of the arms is modelled on the equation

$$d = 2 \sin \left(\frac{3}{2\pi} t \right) \quad \text{where } t \text{ is measured in seconds and } d \text{ in cm.}$$

(i) How far up and down do the teddy bear's arms move?

(1)

(iii) If the teddy bear's arms started to move twice as fast, write down the equation that models the faster motion.

(2)

- (b) A vat is slowly being filled with whisky. The volume of the whisky in the vat follows the rule $V = \ln(10 + 3t)m^3$ from 9 a.m. when $t = 0$.

(i) Find the volume of whisky in the vat at 11 a.m.

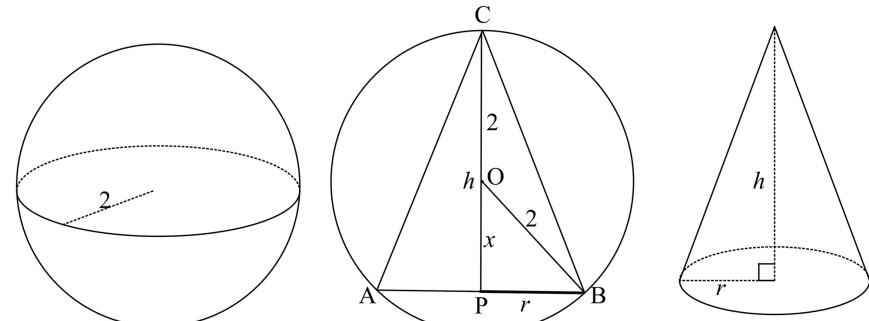
(2)

(ii) When will the vat be filled to $3 m^3$?

(2)

13. (7 marks)

A cone is inscribed in a sphere of radius 2.



(a) Find an expression for the volume of the cone in terms of r .

(2)

- (b) Explain precisely what you would need to do to determine the dimensions of the cone of maximum volume which can be inscribed in the sphere. (5)
NB You do not have to solve the problem.