

EXTRA WORK SPACE



KINGSWAY CHRISTIAN COLLEGE
MATHS DEPARTMENT

Mathematics Methods Year 12

Test 1 – Exponential Functions & Differentiation

Sol Key

16th February 2017

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Course:

Assessment Task:

Student Name:

Date:

Assessment Score:

Year Score:

Comments:

Teacher signature: _____

Parent/ Guardian signature: _____

Comments: _____

Question 1: [3 Marks]

The population of a certain fish in the Ningaloo Reef grows continuously at a rate of 5% per year. The number of fish on 1st January, 2016 was estimated at 2500.

- a) Find an expression to model P , the number of fish, t years into the study.

$$P = P_0 e^{kt}$$

$$\therefore P = 2500 e^{0.05t} \quad \checkmark$$

- b) Find the population at 1st January, 2020. Give your answer in terms of e .

$$P = P_0 e^{kt} \quad \therefore t = 4 \text{ yrs}$$

$$= 2500 e^{0.05(4)}$$

$$\therefore P = 2500 e^{0.2} \quad \checkmark$$

- c) Give the calculator algorithm you would use to calculate the time, t , when the population will quadruple in size.

$$\text{Solve } (10\,000 = 2500 e^{0.05t}, t) \quad \checkmark$$

Question 7: [2, 4 = 6 Marks]

- a) Simplify $y = \frac{4x+12}{x^2-9}$, stating any exclusions from the domain.

$$y = \frac{4(x+3)}{(x+3)(x-3)} = \frac{4(x+3)}{(x-3)} \quad ; \quad x \neq \pm 3.$$

$$= \frac{4}{x-3} \quad \text{(OR)}$$

Hence, make use of the chain rule with Leibnitz notation, to determine:

- b) $\frac{dz}{dy}$, if $z = \frac{1}{3x}$ and $y = \frac{4x+12}{x^2-9}$

$$\therefore z = \frac{1}{3} x^{-1}$$

$$\therefore \frac{dz}{dx} = -\frac{1}{3x^2}$$

From (a)

$$\therefore y = \frac{4}{x-3} = 4(x-3)^{-1}$$

$$\therefore \frac{dy}{dx} = -4(x-3)^{-2} \cdot 1$$

$$\therefore \frac{dy}{dx} = \frac{-4}{(x-3)^2}$$

$$\text{then } \frac{dx}{dy} = \frac{(x-3)^2}{-4}$$

$$\therefore \frac{dz}{dy} = \frac{dz}{dx} \times \frac{dx}{dy}$$

$$= -\frac{1}{3x^2} \times \frac{(x-3)^2}{-4}$$

$$\therefore \frac{dz}{dy} = \frac{(x-3)^2}{12x^2}$$

Question 5: [3, 2 = 5 Marks]

Differentiate the following, without simplifying:

a) $y = \frac{x-1}{x^2+4}$

$$\frac{dy}{dx} = \frac{1(x^2+4) - (x-1)(2x)}{(x^2+4)^2}$$

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b) $y = e^{2x-x^2}$

$$\frac{dy}{dx} = e^{2x-x^2} \cdot (2-2x)$$

$$2 \cdot e^{2x-x^2} \cdot (1-x)$$

$$= 2 \cdot (1-x) \cdot e^{2x-x^2}$$

Question 6: [4 Marks]

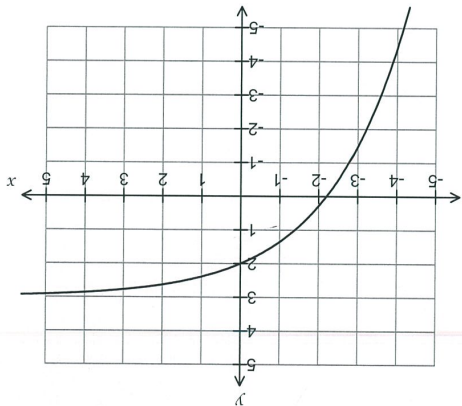
Show that $y = \frac{2e^{-x^2}}{1+e^{3x-1}}$ can be differentiated without using the product or quotient rule.

$$y = \frac{2e^{-x^2}}{1+e^{3x-1}} + \frac{2}{e^{3x-1}}$$

$$\frac{dy}{dx} = \frac{1}{2} e^{x^2} \cdot 2x + \frac{1}{2} e^{x^2+3x-1} \cdot (3x+3)$$

Question 2: [2, 2 = 4 Marks]

The graph of $y = ae^{bx} + c$ is shown below. The graph passes through the point (0, 2), and $y \rightarrow 3$ as $x \rightarrow \infty$.



a) Is b positive or negative? Justify your answer.

Negative. The graph is reflected in the y-axes (and also in the x-axes).

b) Evaluate a and c.

$$C = 3$$

$$y = a e^{bx} + 3$$

$$2 = a \cdot e^{b \cdot 0} + 3$$

$$2 = a \cdot 1 + 3$$

$$-1 = a$$

Question 3: [3, 2 = 5 Marks]

Find $\frac{dy}{dx}$ if:

a) $y = \frac{\sqrt[3]{x^2 - 6x^2}}{2x}$

$$\therefore y = \frac{x^{\frac{2}{3}}}{2x} - \frac{6x^0}{2x}$$

$$\therefore y = \frac{x^{-\frac{1}{3}}}{2} - 3x$$

$$\therefore \frac{dy}{dx} = -\frac{1}{3} \left(\frac{1}{2} x^{-\frac{4}{3}} \right) - 3$$

$$\therefore \frac{dy}{dx} = -\frac{1}{6\sqrt[3]{x^4}} - 3$$

b) $y = 2ax^a - 4a^2$, where a is constant and $a > 0$

$$\therefore \frac{dy}{dx} = 2a^2 x^{a-1}$$

(2) * (- if -8a as well.)

Question 4: [2, 2, 3, 3, 3 = 13 Marks]

Find the derivative of each of the following. Simplify all answers.

a) $y = (2x - 5)(x^2 - 3x + 4)$

$$\frac{dy}{dx} = 2(x^2 - 3x + 4) + (2x - 5)(2x - 3)$$

$$= 2x^2 - 6x + 8 + 4x^2 - 16x + 15$$

$$\therefore \frac{dy}{dx} = \underline{6x^2 - 22x + 23}$$

b) $y = \frac{3x-2}{3x^2+1}$

$$\therefore \frac{dy}{dx} = \frac{3(3x^2+1) - (3x-2)(6x)}{(3x^2+1)^2}$$

$$= \frac{9x^2 + 3 - 18x^2 + 12x}{(3x^2+1)^2} = \underline{\underline{\frac{-9x^2 + 12x + 3}{(3x^2+1)^2}}}$$

c) $(\sqrt[4]{x^2+4})^3$

$$\therefore y = (x^2+4)^{\frac{3}{4}}$$

$$\therefore \frac{dy}{dx} = \frac{3}{4} (x^2+4)^{-\frac{1}{4}} \cdot 2x$$

$$= \frac{6x}{4 \cdot (x^2+4)^{\frac{1}{4}}}$$

$$= \frac{3x}{2\sqrt[4]{(x^2+4)}}$$

d) $y = \frac{3x^5}{e^{2x}}$

$$\therefore \frac{dy}{dx} = \frac{15x^4 \cdot e^{2x} - 3x^5 \cdot e^{2x} \cdot 2}{(e^{2x})^2}$$

$$= \frac{e^{2x} (15x^4 - 6x^5)}{(e^{2x})^2}$$

$$\frac{dy}{dx} = \frac{15x^4 - 6x^5}{e^{2x}}$$

e) $y = \frac{3}{\sqrt{1+e^{5x}}}$

$$y = 3 \cdot (1+e^{5x})^{-\frac{1}{2}}$$

$$\therefore \frac{dy}{dx} = -\frac{3}{2} (1+e^{5x})^{-\frac{3}{2}} \cdot e^{5x} \cdot 5$$

$$= \underline{\underline{\frac{-15 e^{5x}}{2 \sqrt[3]{(1+e^{5x})^3}}}}}$$