PERTH MODERN SCHOOL

UNIT 3C/3D MAS – 2011

TEST 1 – POLAR COORDINATES & COMPLEX NUMBERS

NAME: ____ DATE: _______________

[To achieve full marks and to allow assessment of particular outcomes, working and reasoning should be shown.]
[A maximum of 2 marks will be deducted for incorrect rounding, units, etc.]

This is *Resource Rich – 50 minutes for 53 marks*:

1. [1, 1, 1, 1 = 4 marks]
   Convert:
   a) (4,-6) into polar coordinates with $-180^\circ < \theta \leq 180^\circ$.

   b) $(3,-\sqrt{3})$ into ***exact*** polar coordinates with $-\pi < \theta \leq \pi$.

   c) $[4,35^\circ]$ into Cartesian coordinates.

   d) $[8,-\frac{3}{4}\pi]$ into ***exact*** Cartesian coordinates.

2. [3, 3 = 6 marks]
Clearly show how you obtain your answers, find:
a) the distance between [20,-210°] and [$\sqrt{5}$,-50°].

b) the ***exact*** distance between $\left[\frac{\sqrt{5}}{3},\ -\frac{2\pi}{3}\right]$ and $\left[10,\ -\frac{7\pi}{6}\right]$.

3. [8 marks]

Find the ***exact*** distance between ($\sqrt{3k}$, $\sqrt{\boldsymbol{k}}$) and $\left[\sqrt{\mathbf{k}}\ ,\ \frac{5\pi}{6}\right]$. Draw a diagram.

4. [3, 2 = 5 marks]

a) Find, in ***exact*** form, the modulus and principal argument of $-\sqrt{3} + i$, and hence rewrite $-\sqrt{3} + i$ in ***exact*** polar (***cis***) form.

b) Convert $2\ cis\left(\frac{\pi}{4}\right)$ into ***exact*** algebraic Cartesian/rectangular form.

5. [3, 3 = 6 marks]

Evaluate, giving answers in ***exact*** form:

a) $4\ cis\,\frac{\pi}{3} \times 2\ cis\,\frac{3\pi}{4}$

b) $$\frac{4\ cis\left(-\frac{5\pi}{6}\right)}{2\ cis\left(\frac{5\pi}{6}\right)}$$

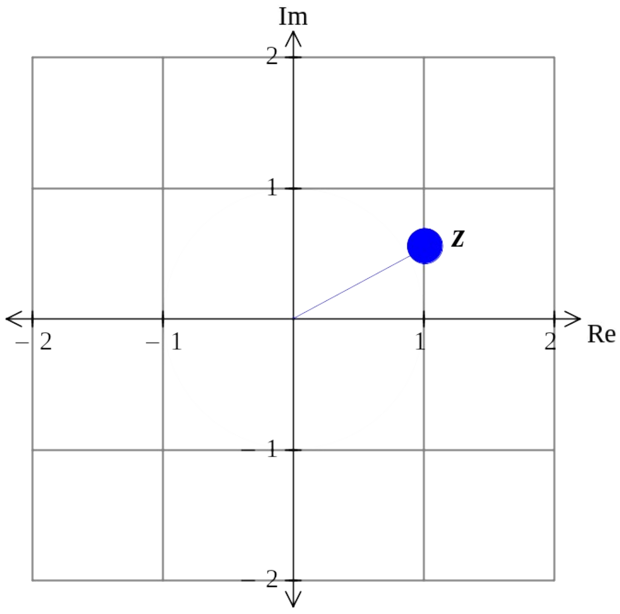
6. [4 marks]

Given $z = 2\ cis\ \frac{\pi}{4}$, express z^{-1} and $\bar{z}$ in ***exact*** polar and rectangular form.

7. [2, 2, 2, 2 = 8 marks]

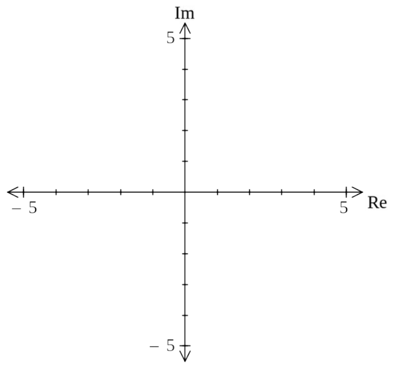
The Argand diagram below shows the point representing the complex number z where $|z| > 1$. Plot on the same diagram, the points representing the complex numbers:

a) $\bar{z}$ b) iz c) z^2 d) $\frac{1}{z}$

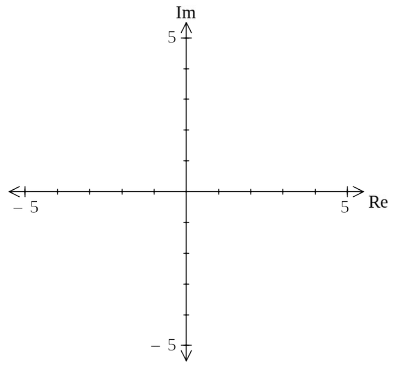


8. [3, 3, 3, 3 = 12 marks]
Sketch on an Argand diagram, the locus of the point $z = x + iy$, satisfying each of the following conditions. In each case, give the Cartesian equation or inequality of the locus.

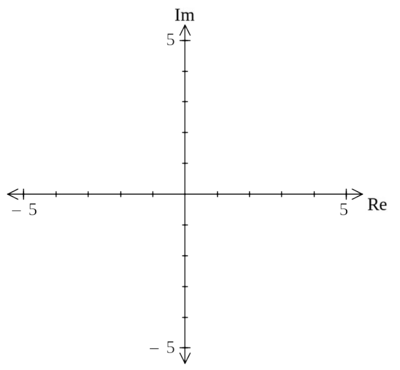
a) $\text{Re}(z) = -2$



b) $\text{Im}(z) = \text{Re}(z)$



c) $\text{Re}(z) + 2\text{Im}(z) > 3$



d) $\text{Re}(z).\text{Im}(z) = 1$