ALL SAINTS' COLLEGE

WA Exams Practice Paper B, 2016

Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 AND 4

Section One: Calculator-free

SOLUTIONS

Student Number: In figures

In words ______________________________________

Your name ______________________________________

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Additional working space

Question number: _________

Question 8 **(5 marks)**

Given that $F(x)=\int_0^x f(t)\ dt$, $\dfrac{d^2F}{dx^2}=x^2$ and $F(2)=4$, determine the function $f(x)$.

$$F(x)=\int_0^x f(t)\ dt$$
$$\frac{dF}{dx}=f(x)$$

$$\frac{d^2F}{dx^2}=f'(x)=x^2$$
$$f(x)=\frac{x^3}{3}+c$$

$$F(2)=\int_0^2\left(\frac{t^3}{3}+c\right)\ dt$$
$$4=\left[\frac{t^4}{12}+ct\right]_0^2$$
$$4=\frac{4}{3}+2c$$
$$c=\frac{4}{3}$$

$$f(x)=\frac{x^3}{3}+\frac{4}{3}$$

Section One: Calculator-free **35% (53 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(7 marks)**

(a) Evaluate $\int_0^2 8(2x-1)^3\ dx$. (2 marks)

$$\int_0^2 8(2x-1)^3\ dx=\left[\frac{8(2x-1)^4}{2\times4}\right]_0^2$$
$$=3^4-(-1)^4$$
$$=80$$

(b) Determine $\dfrac{d}{dx}\left(\cos(4x)\cdot e^{3x}\right)$. (2 marks)

$$\frac{d}{dx}\left(\cos(4x)\cdot e^{3x}\right)=-4\sin(4x)\times e^{3x}+\cos(4x)\times 3e^{3x}$$

(c) Determine $f'(1)$ if $f(x)=\dfrac{\ln(3-2x)}{2x+1}$. (3 marks)

$$f'(x)=\frac{\left(\frac{-2}{3-2x}\right)(2x+1)-\left(\ln(3-2x)\right)(2)}{(2x+1)^2}$$

$$f'(1)=\frac{(-2)(3)-(0)(2)}{(3)^2}=-\frac{2}{3}$$

Question 2 **(6 marks)**

A small slider moves along a straight track so that its displacement, x cm, from a fixed point O is given by $x = 150 - 90\cos\left(\frac{\pi t}{3}\right)$.

Determine exact values for

(a) the initial displacement of the slider. (2 marks)

$$x(0) = 150 - 90\cos(0)$$
$$= 150 - 90$$
$$= 60 \text{ cm}$$

(b) the velocity of the slider when $t = \frac{1}{2}$ second. (2 marks)

$$v = 90 \times \frac{\pi}{3} \times \sin\left(\frac{\pi t}{3}\right)$$
$$= 30\pi \sin\left(\frac{\pi t}{3}\right)$$
$$v\left(\frac{1}{2}\right) = 30\pi \sin\left(\frac{\pi}{6}\right)$$
$$= 15\pi \text{ cm/s}$$

(c) the acceleration of the slider after one second. (2 marks)

$$a = 30\pi \times \frac{\pi}{3} \times \cos\left(\frac{\pi t}{3}\right)$$
$$= 10\pi^2 \cos\left(\frac{\pi t}{3}\right)$$
$$a(1) = 10\pi^2 \cos\left(\frac{\pi}{3}\right)$$
$$= 5\pi^2 \text{ cm/s}^2$$

Question 7 **(6 marks)**

A motor vehicle slows down from an initial velocity of 25 ms^{-1} until it is stationary. During this interval, its acceleration t seconds after the brakes were applied is given by $a(t) = \frac{t}{2} - 5 \text{ ms}^{-2}$.

(a) Determine the velocity of the vehicle after four seconds. (3 marks)

$$v = \int \frac{t}{2} - 5 \, dt$$
$$= \frac{t^2}{4} - 5t + c$$
$$v(0) = 25 \Rightarrow c = 25$$
$$v = \frac{t^2}{4} - 5t + 25$$
$$v(4) = 4 - 20 + 25$$
$$= 9 \text{ ms}^{-1}$$

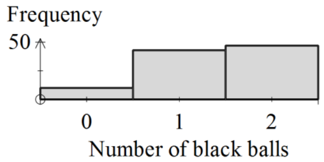
(b) Calculate the distance travelled by the vehicle in the first two seconds after the brakes were applied. (3 marks)

$$s = \int \frac{t^2}{4} - 5t + 25 \, dt$$
$$= \frac{t^3}{12} - \frac{5t^2}{2} + 25t + c$$
$$s(0) = 0 \Rightarrow c = 0$$
$$s = \frac{t^3}{12} - \frac{5t^2}{2} + 25t$$
$$s(2) = \frac{8}{12} - \frac{5 \times 4}{2} + 25 \times 2$$
$$= \frac{122}{3} \text{ m}$$

Question 6 **(8 marks)**

A barrel contains a large number of black and white balls, such that the ratio of black to white balls is 7:3.

The graph below shows the results of a simulation of an experiment in which two balls are randomly drawn from the barrel, the number of black balls noted and then the balls are replaced, for a total of 100 times.



(a) Comment on the distribution shown above. (2 marks)

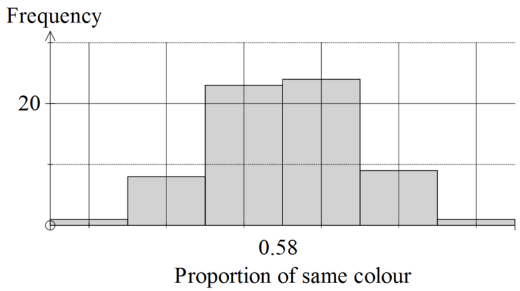
> Distribution is binomial and because p is large (0.7), the distribution is negatively skewed.

(b) Determine the probability that when two balls are randomly drawn from the barrel, both balls are the same colour. (3 marks)

> X is number of black balls:
> $P(X=0)=0.3^2=0.09$
> $P(X=2)=0.7^2=0.49$
> $P(Same\ colour)=0.09+0.49$
> $=0.58$

The same simulation is repeated 75 times, and the proportion of draws in which both balls are the same colour is noted for each simulation.

(c) Sketch a frequency histogram to illustrate the likely distribution of these proportions, noting any key features of your sketch. (3 marks)



> Histogram should show roughly normal distribution centred around 0.58. May note that standard deviation is about 0.05, so that most proportions will fall between 0.43 and 0.73.

Question 3 **(8 marks)**

(a) Solve, exactly, the following for x.

(i) $\log_x 64 = 3$. (2 marks)

> $x^3 = 64$
> $x = 4$

(ii) $e^{3x} = 30$. (2 marks)

> $3x = \ln 30$
> $x = \dfrac{\ln 30}{3}$

(b) If $a = \log_5 4$ and $b = \log_5 8$, express the following in terms of a and b:

(i) $\log_5 32$ (2 marks)

> $\log_5 32 = \log_5(4\times 8)$
> $= \log_5 4 + \log_5 8$
> $= a + b$

(ii) $\log_5 400$ (2 marks)

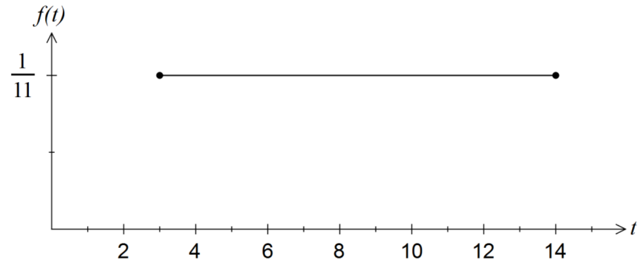
> $\log_5 400 = \log_5 (5\times 4)^2$
> $= 2(\log_5 5 + \log_5 4)$
> $= 2(1 + a)$

Question 4 **(7 marks)**

As part of a local arts festival, an artist plans to create an installation in which a concealed water cannon blasts a stream of water into the air for a few seconds at random intervals.

The lengths of the intervals between each firing of the cannon can be modelled by the uniformly distributed random variable T, where $3 \le t \le 14$ minutes.

(a) Sketch the probability density function $f(t)$ for the interval between each firing on the axes below. (2 marks)



(b) Determine the probability that a randomly chosen interval between firings is

(i) at least seven minutes. (1 mark)

$$P(T>7)=\frac{14-7}{14-3}=\frac{7}{11}$$

(ii) at least six minutes given that it is less than ten minutes. (2 marks)

$$\begin{aligned}&P(T>6\,|\,T<10)\\&=\frac{P(6<T<10)}{P(T<10)}\\&=\frac{4}{11}\div\frac{7}{11}\\&=\frac{4}{7}\end{aligned}$$

(c) Determine the value of t for which $P(T<t)=P(T>4t)$. (2 marks)

$$P(T<t)=\frac{t-3}{11}$$

$$P(T>4t)=\frac{14-4t}{11}$$

$$\frac{t-3}{11}=\frac{14-4t}{11}\ \Rightarrow\ t=\frac{17}{5}=3.4\text{ minutes}$$

Question 5 **(6 marks)**

The gradient function of a curve is given by $f'(x)=3\sin(2x+a)$, where a is a constant such that $0\le a\le\pi$.

Determine $f(x)$, given that the curve has a maximum at $\left(\frac{2\pi}{3},\ 4\right)$.

$$3\sin\left(2\left(\frac{2\pi}{3}\right)+a\right)=0$$

$$\frac{4\pi}{3}+a=\ldots,\ 0,\ \pi,\ 2\pi,\ldots$$

$$a=\frac{2\pi}{3}$$

$$f'(x)=3\sin\left(2x+\frac{2\pi}{3}\right)$$

$$f(x)=-\frac{3}{2}\cos\left(2x+\frac{2\pi}{3}\right)+c$$

$$4=-\frac{3}{2}\cos\left(2\left(\frac{2\pi}{3}\right)+\frac{2\pi}{3}\right)+c$$

$$4=-\frac{3}{2}+c$$

$$c=\frac{11}{2}$$

$$f(x)=-\frac{3}{2}\cos\left(2x+\frac{2\pi}{3}\right)+\frac{11}{2}$$