

## MATHEMATICS METHODS

**MAWA Semester 1 (Unit 1) Examination 2015**

**Calculator-free**

**Marking Key**

**Section One: Calculator-free**

**(60 Marks)**

**Question 1(a)**

Solution	
$\frac{\pi}{6} = 30^\circ \Rightarrow \frac{7\pi}{6} = 7 \times 30^\circ = 210^\circ$	
Marking key/mathematical behaviours	Marks

- $\frac{7\pi}{6} = 210^\circ$
- Determines

1

**Question 1(b)**

Solution	
$\cos \theta = -\sqrt{1 - \sin^2 \theta} = -\sqrt{1 - \left(-\frac{1}{2}\right)^2} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$ by using the right triangle identity or	
$\cos \theta = \cos \frac{7\pi}{6} = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$ by knowledge of exact values	
$\tan \theta = \tan \left(\frac{7\pi}{6}\right) = \tan \left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$	
Similarly	
Marking key/mathematical behaviours	Marks
• indicates use of $\cos \frac{7\pi}{6} = -\cos \frac{\pi}{6}$ or uses $\sin \theta = -\frac{1}{2}$ meaningfully	1
• states correct exact value of $\cos \frac{7\pi}{6}$ (accept $-\frac{\sqrt{3}}{2}$ or $-\frac{3}{2\sqrt{3}}$ )	1
• indicates use of $\tan \frac{7\pi}{6} = \tan \frac{\pi}{6}$	1

Marks	Marks
1	1
• solves for $x$ • factorises trinomial	Marking key/mathematical behaviours

**Question 2(b)**

Solution

$$\begin{aligned}x^2 + x - 72 &= 0 \\(x - 8)(x + 9) &= 0 \\x = -9 \text{ or } x &= 8\end{aligned}$$

Marks	Marks
1	1
• solves for $x$ • expands brackets and simplifies • multiplies the equation by the LCD	Marking key/mathematical behaviours

**Question 2(a)**

Solution

$$\begin{aligned}9x &= 27 \Leftrightarrow x = 3 \\5x + 15 - 4x + 12 &= 10x \\5(x + 3) - 4(x - 3) &= 10x \\x + 3 - x - 3 &= \frac{5}{2} \\&\theta = \frac{3\pi}{4} \text{ or } -\frac{3\pi}{4}\end{aligned}$$

Marks	Marks
1	1
• indicates both correct values of $\theta$ • indicates method of determination on diagram	Marking key/mathematical behaviours

**Question 1(c)**

Solution

From exact values and use of unit circle,  
 $\theta = \frac{3\pi}{4} \text{ or } -\frac{3\pi}{4}$

MATHEMATICS METHODS	CALCULATOR-FREE	SEMESTER 1 (UNIT 1) EXAMINATION	MARKING KEY
• states correct sample space	1	• determines that there are 6 pairs of numbers (listing or logic) that have an even product and determines the correct probability of an even product	(i)

(i)	1	• provides some form of exhaustive listing of the sum of two numbers
(ii)	1+1	• indicates that the only way that the product can be prime is if one of the cards has a one on it.
(iii)	1	• determines the correct probability of a prime product
(iv)	1	• determines that there are only 4 possibilities of prime sums

**Question 7 (a)**

Solution

$$\left(\alpha + \frac{\beta}{2}\right)^4 = \alpha^4 + 4\alpha^3 \cdot \frac{\beta}{2} + 6\alpha^2 \left(\frac{\beta}{2}\right)^2 + 4\alpha \left(\frac{\beta}{2}\right)^3 + \left(\frac{\beta}{2}\right)^4$$

Marking key/mathematical behaviours

- uses the correct binomial coefficients
- each term has the correct powers for  $\alpha$  and  $\beta$
- uses  $\frac{\beta}{2}$  correctly in each term

Marks

1

1

1

**Question 7(b)**

Solution

The mouse has six choices to enter and five choices to leave by another door.

$$\text{So, } 6 \times 5 = 30$$

Marking key/mathematical behaviours

- states the correct result

Marks

1

**Question 7(c)**

Solution

$$(i) \quad \binom{7}{2} = \frac{7 \times 6}{1 \times 2} = 21$$

- (ii) There is only one card with an even number on it.(2)  
There are six other numbers that the 2 can be combined with to give an even product.

$$\therefore P(\text{even product}) = \frac{6}{21}$$

- (iii) For the product to be prime, one of the cards must be ONE(1)  
Any of the other six cards can be combined with 1 to give a prime product

$$\therefore P(\text{prime product}) = \frac{6}{21}$$

- (iv) There are only 4 possible combinations of two of the numbers that sum to a prime number. 1+2, 2+3, 2+5, 2+11.

$$\therefore P(\text{prime sum}) = \frac{4}{21}$$

Question 6(a)	
Marking key/mathematical behaviours	
Solution	$y = \frac{3}{x} + 1$
Marks	1
Question 6(b)	
Marking key/mathematical behaviours	
Solution	$\therefore y = 3 \tan \frac{2}{x} + 1$
Marks	1
Question 3	
Marking key/mathematical behaviours	
Solution	$\begin{aligned} x^2 - 4x + 1 &= 0 \\ (x - 2)^2 - 4 + 1 &= 0 \\ (x - 2)^2 &= 3 \\ x - 2 &= \pm \sqrt{3} \\ x &= 2 \pm \sqrt{3} \end{aligned}$
Marks	1

Question 6(b)	
Marking key/mathematical behaviours	
Solution	
Marks	1
Question 3	
Marking key/mathematical behaviours	
Solution	$\begin{aligned} x^2 - 4x + 1 &= 0 \\ (x - 2)^2 - 4 + 1 &= 0 \\ (x - 2)^2 &= 3 \\ x - 2 &= \pm \sqrt{3} \\ x &= 2 \pm \sqrt{3} \end{aligned}$
Marks	1
Question 3	
Marking key/mathematical behaviours	
Solution	$\begin{aligned} x^2 - 4x + 1 &= 0 \\ (x - 2)^2 - 4 + 1 &= 0 \\ (x - 2)^2 &= 3 \\ x - 2 &= \pm \sqrt{3} \\ x &= 2 \pm \sqrt{3} \end{aligned}$
Marks	1
Question 3	
Marking key/mathematical behaviours	
Solution	$\begin{aligned} x^2 - 4x + 1 &= 0 \\ (x - 2)^2 - 4 + 1 &= 0 \\ (x - 2)^2 &= 3 \\ x - 2 &= \pm \sqrt{3} \\ x &= 2 \pm \sqrt{3} \end{aligned}$
Marks	1

**Question 4**

Solution

$$\text{Graph A: } y = -\sqrt{x+3}$$

$$\text{Graph B: } y = \frac{-3}{x+1}$$

Marking key/mathematical behaviours	Marks
• Graph A <ul style="list-style-type: none"> <li>◦ correct horizontal translation</li> <li>◦ recognition of reflection in <math>x</math>- axis</li> </ul>	1
	1
• Graph C <ul style="list-style-type: none"> <li>◦ correct horizontal translation</li> <li>◦ correct dilation factor</li> <li>◦ recognition of reflection in <math>x</math>- axis</li> </ul>	1
	1

**Question 5(a)**

Solution

$$(i) P(X \cup Y) = 0.9 \Rightarrow x = P(X \cap Y) = 0.3 \therefore P(X) = 0.7$$

$$(ii) \text{ From part (i), } P(X) = 0.7 \text{ and } P(Y) = 0.5$$

$$P(X) \times P(Y) = 0.35 \neq 0.3$$

$\therefore X$  and  $Y$  are not independent

Marking key/mathematical behaviours	Marks
(i) <ul style="list-style-type: none"> <li>• determines <math>x = 0.3</math></li> <li>• determines correct value for <math>P(X)</math></li> </ul>	1
	1
(ii) <ul style="list-style-type: none"> <li>• determines <math>P(X) \times P(Y)</math></li> <li>• shows that <math>P(X) \times P(Y) \neq P(X \cap Y)</math></li> <li>• concludes that the two events are not independent</li> </ul>	1
	1
	1

**Question 5(b)**

Solution

$$(i) P(X | Y) = \frac{P(X \cap Y)}{P(Y)} \Rightarrow \frac{2}{7} = \frac{x}{0.2+x}$$

$$\therefore 0.4 + 2x = 7x$$

$$\text{i.e. } 0.4 = 5x$$

$$\text{i.e. } 0.08 = x \quad \therefore P(X) = 0.48$$

$$(ii) \text{ From part (i) } P(X) = 0.48 \text{ and so } P(X \cup Y) = 0.48 + 0.2 = 0.68$$

$$P(\overline{X \cup Y}) = 1 - P(X \cup Y) = 1 - 0.68 = 0.32$$

Marking key/mathematical behaviours	Marks
(i) <ul style="list-style-type: none"> <li>• applies the conditional probability formula</li> <li>• substitutes correctly</li> <li>• multiplies correctly and simplifies</li> <li>• determines correct value for <math>P(X)</math></li> </ul>	1
(ii) <ul style="list-style-type: none"> <li>• determines <math>P(X \cup Y)</math></li> <li>• applies complementary property and arrives at the correct result</li> </ul>	1