PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Year 12 Test One 2022

Student name: _____   Teacher name: _____

Task type:	Response
Time allowed for this task:	40 ___ mins
Number of questions:	8 ___
Materials required:	Calculator with CAS capability (to be provided by the student)
Standard items:	Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations
Marks available:	42 ___ marks
Task weighting:	10 ___ %
Formula sheet provided:	Yes/No

Note: All part questions worth more than 2 marks require working to obtain full marks.

Working out space

Q1 (2, 3 & 3 = 8 marks) (3.1.1- 3.1.6)

Let $z = 5 - 3i$ and $w = 7 - i$.

Simplify the following.

a) $z^2 w$

b) $\dfrac{1}{w}$

c) $\dfrac{z}{w}$ (simplify)

Q2 (3 marks) (3.1.1-3.1.3)

Determine all possible real number pairs a, b such that $\dfrac{101 + 47i}{a - 5i} = 6 + bi$

Working out space

Q3 (3 marks) (3.1.13-3.1.15)

Consider the polynomial $f(z) = z^3 + bz^2 + cz + d$ where $b, c \,\&\, d$ are real numbers.

Given that $f(3) = 0$ and $f(2 - 5i) = 0$ determine the values of $b, c \,\&\, d$.

Q4 (3 marks) (3.1.8-3.1.10)

Using the diagram below determine the complex number w in exact cartesian form.
(Note: Not drawn to scale)

Im(y)

Re(z)

$|z| = 5$

$\left|\dfrac{w}{z}\right| = 3$

$40°$

$130°$

z

$\dfrac{w}{z}$

Q8 (4, 2 & 3 = 9 marks) (3.1.11-3.1.12)

a) Solve for all the roots $z^6 = 1 - i$ in polar form $z = rcis\theta$ with $-\pi < \theta \leq \pi$.
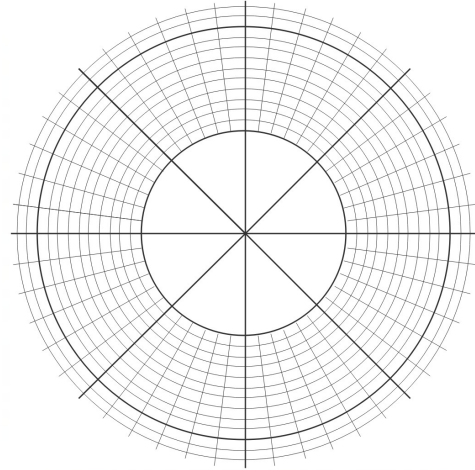
b) Plot these roots on the complex plane below.

c) Adjacent points can be joined by lines to form a polygon. Determine the exact area of this polygon.
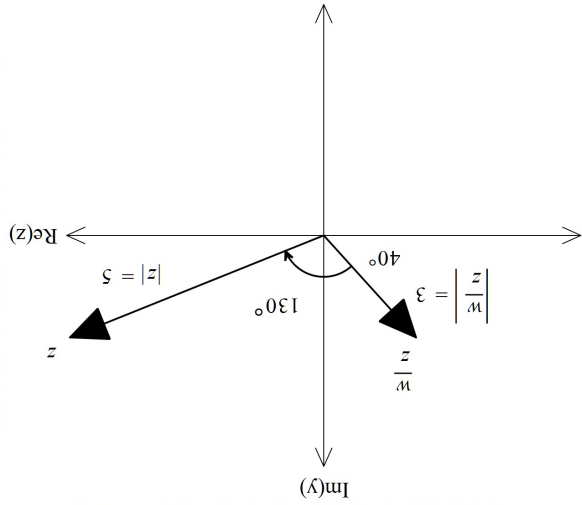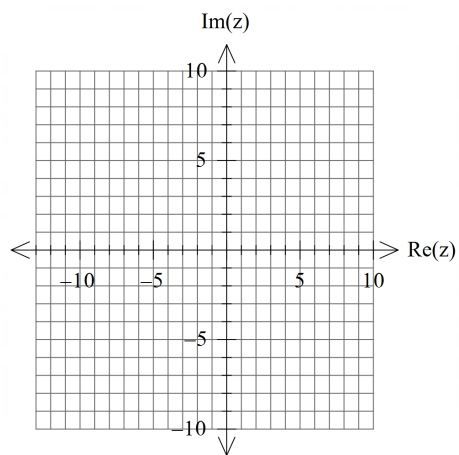
Q5 (3 & 3= 6 marks) (3.1.10)

Sketch the locus for the following labelling important features and points.

a) $|z - 3 + 7i| \geq |z - 5|$

Im(z)

b) $|z + 3 + 7i| = |z - 5| + \sqrt{113}$

Im(z)
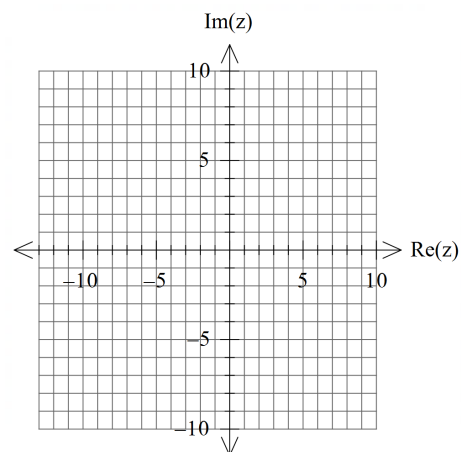
Q6 (2 & 4 = 6 marks) (3.1.10)

Consider the set of points z in the complex plane such that $|z - 2 - 3i| = 5$.

a) Determine the maximum value of $|z|$.

b) Determine the maximum value of $Arg(z + 12)$.

Q7 (4 marks) (3.1.7)

Using De Moivre's Theorem, derive an expression for $\sin(4\theta)$ in terms of $\cos(\theta) \& \sin(\theta)$.