



PRESBYTERIAN LADIES' COLLEGE
A COLLEGE OF THE UNITING CHURCH IN AUSTRALIA

MATHEMATICS DEPARTMENT

Year 12 MATHEMATICS SPECIALIST

DATE: 4th December 2015

Name _____

Reading Time: 3 minutes

SECTION ONE: CALCULATOR FREE

TOTAL: 27 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet.

WORKING TIME: 25 minutes (maximum)

SECTION TWO: CALCULATOR ASSUMED

TOTAL: 25 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing instruments, templates, up to 3 Calculators,

1 A4 page of notes (one side only), SCSA formula sheet.

WORKING TIME: 20 minutes (minimum)

| SECTION 1 Question | Marks available | Marks awarded | SECTION 2 Question | Marks available | Marks awarded |
|-----------------------|--------------------|------------------|-----------------------|--------------------|------------------|
| 1 | 6 | | 5 | 7 | |
| 2 | 6 | | 6 | 4 | |
| 3 | 10 | | 7 | 7 | |
| 4 | 5 | | 8 | 7 | |
| | | | | | |
| Total | 27 | | | 25 | |

Section One: Calculator-free**[27 marks]**

This section has **Four (4)** questions. Answer **all** questions. Write your answers in the spaces provided

Question 1 [6 marks]

Simplify each of the following expressions, writing your answer in exact polar form.

(a) $(\sqrt{3} - i)^2 (\sqrt{3} - i)$

[2]

(b) $3\text{cis}\left(\frac{\pi}{4}\right) \times \left[2\text{cis}\left(\frac{-\pi}{3}\right)\right]^{-1}$

[2]

(c) $\frac{1}{\sqrt{2\text{cis}\left(\frac{\pi}{2}\right)}}$

[2]

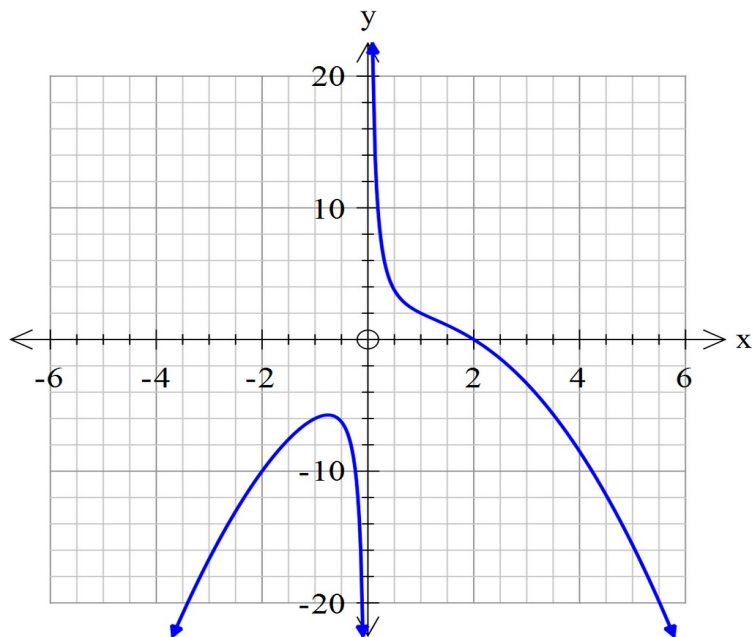
Question 2 [6 marks]

- (a) (i) Find the quotient and the remainder for $\frac{z^3 - 2z^2 + 4z - 1}{z^2 - z + 1}$, hence rewrite $z^3 - 2z^2 + 4z - 1$ in the form $H(z) \times (z^2 - z + 1) + R(z)$ [3]

- (ii) Hence, solve $z^3 - 2z^2 + 4z - 1 = 2z$ [3]

Question 3 [10 marks]

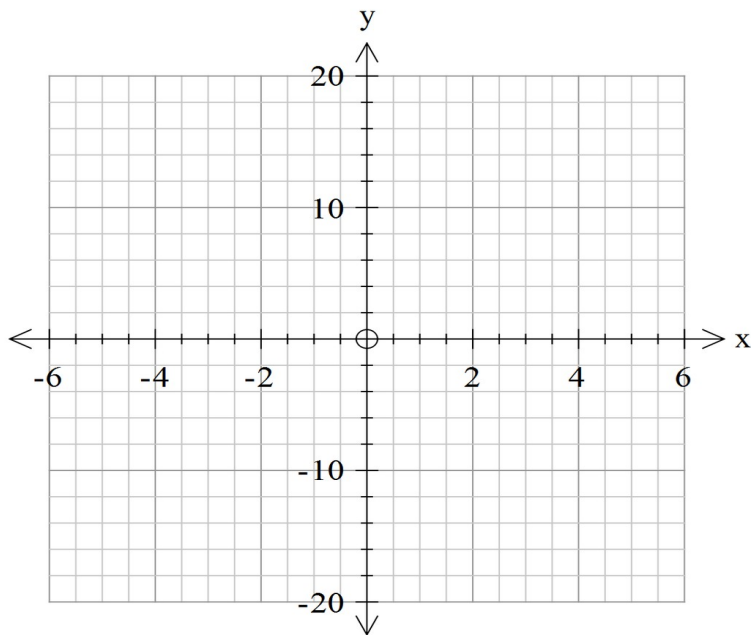
Given the graph of $y = f(x)$ is given as follows;



Sketch the graph of;

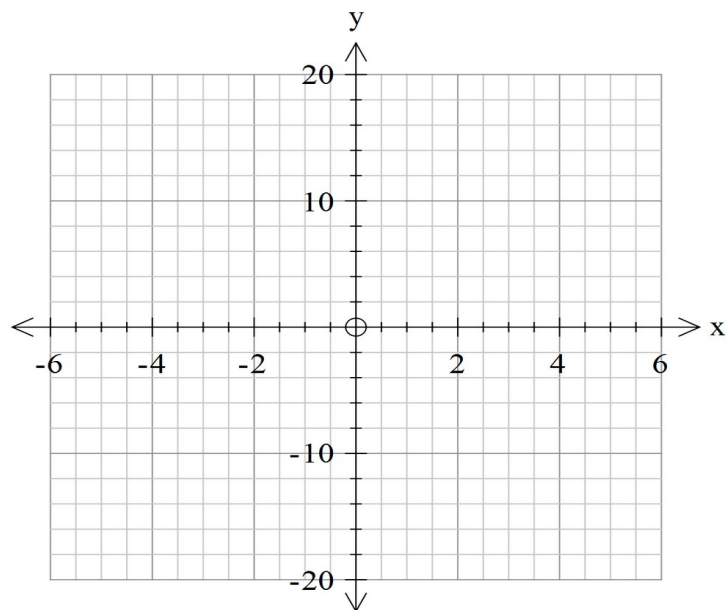
(a) (i) $y = f\left(\left|\frac{x}{2}\right|\right)$

[3]



(ii) $y = |f(x)| + 1$.

[3]



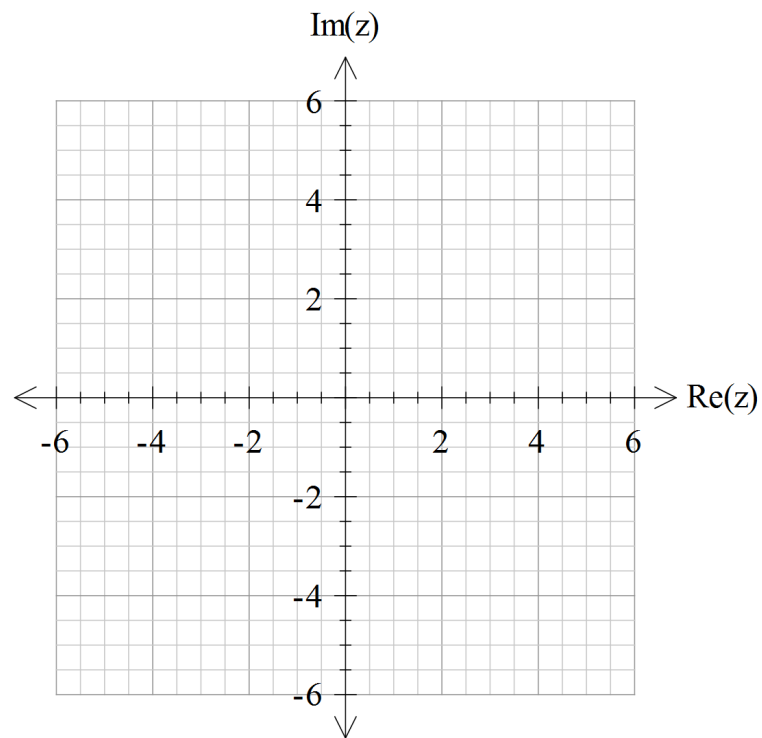
- (b) Given that $g(x) = \sqrt{3x-1}$ and $h(x) = \frac{x+2}{x+1}$, find the domain and range of the composite function $goh(x)$

[4]

Question 4 [5 marks]

- (a) On an Argand diagram sketch the loci of points and that satisfy the following condition;

$$|z - 2i| \leq |z - 3 + i|$$



- (b) Give the equation of the locus in Cartesian form. [3]

[2]

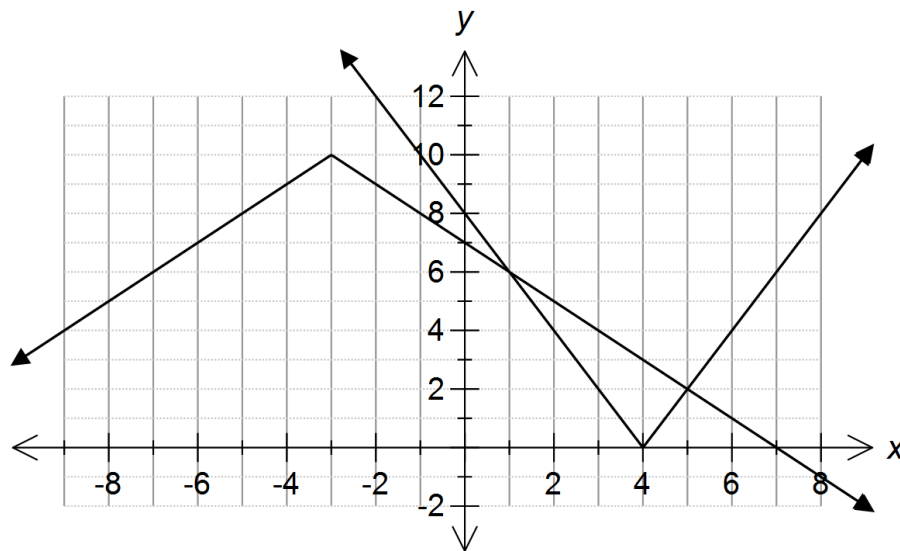
NAME: _____

Section Two: Calculator-assumed

[25 marks]

This section has **four (4)** questions. Answer **all** questions. Write your answers in the spaces provided

Question 5 [7 marks]



(a) Use the diagram above to solve for x in the following.

(i) $-|x + 3| + 10 = 7$

[1]

(ii) $-|x + 3| + 10 \geq |2x - 8|$

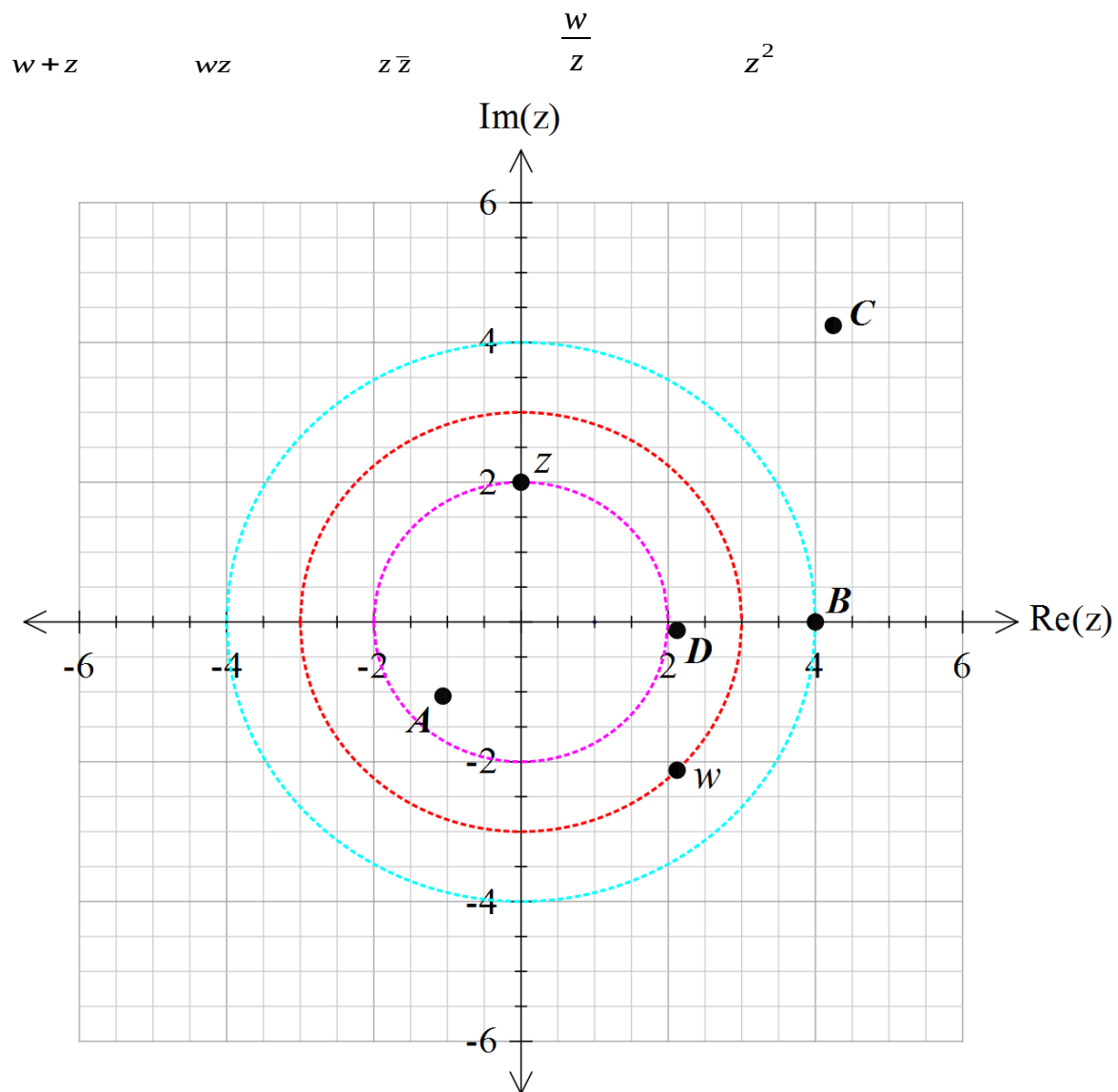
[2]

(b) Solve the following algebraically $4 + |3 + 2x| > |x - 5|$

[4]

Question 6 [4 marks]

Given the position of z and w on the Argand diagram below. Label the points A, B, C and D using the following options.



A _____

[1]

B _____

[1]

C _____

[1]

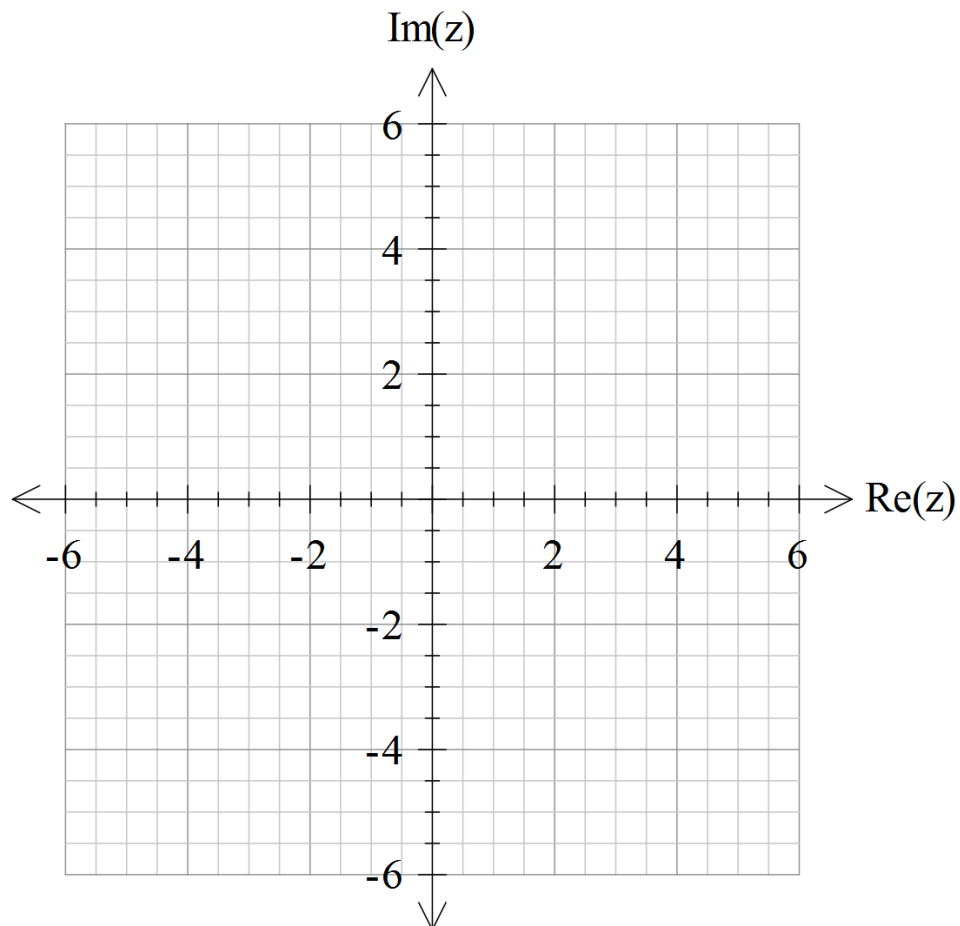
D _____

[1]

Question 7 [7 marks]

- (a) Represent on the Argand diagram provided below, the loci of points, that satisfy the following conditions;

$$|z + 2 - i| \leq 4, \quad \frac{-5\pi}{6} \leq \arg(z) < \frac{\pi}{3} \quad \text{and} \quad 4\operatorname{Im}(z) + 3\operatorname{Re}(z) + 8 \geq 0$$



[4]

- (b) Given that $|z + 2 - i| \leq 4$, state the minimum and maximum value of $|z|$.

[3]

Question 8 [7 marks]

- (a) Using your CAS calculator (or otherwise) find all the solutions to $z^5 = 512(\sqrt{3} - i)$ in exact polar form, where $z = r(\cos \theta + i \sin \theta)$, $-\pi < \theta \leq \pi$ and $r \geq 0$. [4]

- (b) Draw the solutions from (a) on the complex plane below. Show all major features. [3]

