

Note: All part questions worth more than 2 marks require working to obtain full marks.

Materials required: Up to 3 calculators/classpads allowed

Number of questions: 7

Time allowed for this task: 40 mins

Task type: Response

Student name: _____ Teacher name: _____

COURSE METHODS YEAR 12 TEST TWO 2022

PERTH MODERN SCHOOL



Exceptional school living. Exceptional students.

Q1 (2 & 2 = 4 marks) (3.2.1)

Let $f'(x) = 6x^3 + 1$,

- a) Determine an expression for the rate of change of
- $f'(x)$
- .

| Solution |
|----------------------------|
| $f''(x) = 18x^2$ |
| Specific behaviours |

P recognises that derivative is needed

P correct expression

- b) Determine
- $f(x)$
- given that
- $f(3) = 1$
- .

| Solution |
|---|
| $f'(x) = 6x^3 + 1$ |
| $f(x) = \frac{3}{2}x^4 + x + c$ |
| $1 = \frac{3}{2}(3)^4 + 3 + c$ |
| $c = -123.5$ |
| $f(x) = \frac{3}{2}x^4 + x - 123.5$ |
| Specific behaviours |
| P anti differentiates and uses a constant c |
| P solves for constant |

Q2 (3 marks) (3.2.3-3.2.9)

Determine x in terms of t given that $\frac{dx}{dt} = \frac{-5}{(3t+5)^3}$ and $x = 10$ when $t = 1$.

| Solution |
|-----------------|
|-----------------|

| | |
|-----------------|--|
| | P solves for both constants |
| | P anti-diff to find x with a new constant |
| | P anti-diff to find v with constant stated |
| | P displacement for $t=3$ |
| Solution | |
| | |

$$\begin{aligned}x &= \frac{4}{3}t^3 + 9 + \frac{9}{2}t^2 + 2t + 10 = \frac{183}{4} \\c &= \frac{3}{2} \\20 &= 4 + \frac{3}{8}t^3 + 2 + 2c + 10 \\t=0, x=10, p=10 &= t^3 + t^2 + t + c \\x &= t^3 + \frac{3}{4}t^2 + \frac{1}{2}t + ct + p \\v &= t^2 + t + 1 + c \\a = (3t^2 + 2t + 1) m/s^2 &\end{aligned}$$

metres. Determine the displacement when $t = 3$.
When $t = 0$ the displacement is 10 metres and when $t = 2$ the displacement is 20
 $(3t^2 + 2t + 1) m/s^2$
A particle travels along a straight line such that its acceleration at time t seconds is equal to
Q3 (4 marks) (32.21-3.22)

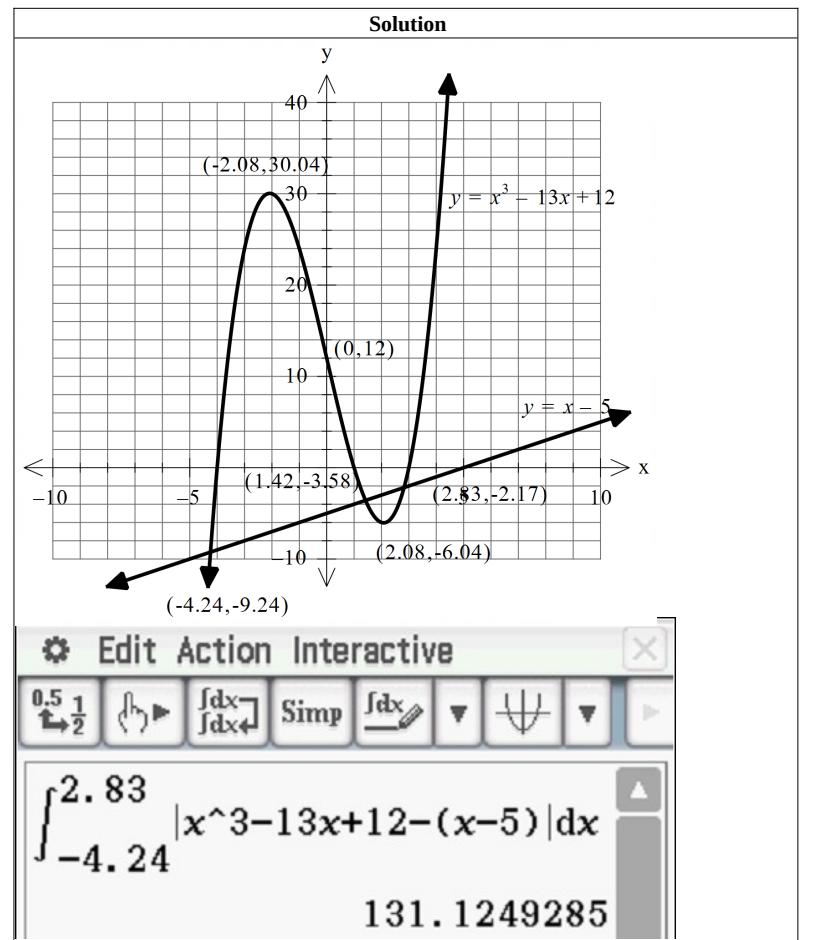
| | |
|-----------------|-------------------------------------|
| | P solves for c (accept approx.) |
| | P sets up equation to solve for c |
| | P anti-diffs and uses plus c |
| Solution | |
| | |

$$\begin{aligned}c &= \frac{384}{385} \text{ or } \sim 0.99 \text{ or } 9.98 \\10 &= \frac{6(64)}{5} + c \\x &= \frac{6(3t^2 + 5)}{5} + c \\dx/dt &= -\frac{5}{5} = -5(3t^2 + 5)\end{aligned}$$

Q4 (6 marks) (3.2.19-3.2.20)

Make a sketch showing the graphs of $y = x^3 - 13x + 12$ and $y = x - 5$ indicating clearly on your sketch the coordinates (2 dp) of any stationary points, inflection (if any) and of any points where the functions intersect each other.

Determine the area between the graphs to 2 dp.



Solution

- b) Determine the values of any inflection points.

| | |
|----------------------------|---|
| Specific behaviours | $f(x) = x^3 - 27x^2$ $f'(x) = 3x^2 - 54x$ $f''(x) = 6x - 54$ $f'''(x) = 6$ |
| Solution | |

- a) Using calculus determine all stationary points and their nature.

Q5 (4 & 3 = 7 marks) (3.1.2-3.1.3)

$$\text{Let } f(x) = x^3 - 27x^2$$

| | |
|----------------------------|---|
| Specific behaviours | $f(x) = x^3 - 27x^2$ $f'(x) = 3x^2 - 54x$ $f''(x) = 6x - 54$ $f'''(x) = 6$ |
| Solution | $\text{Area} = 131.12 \text{ sq units (accept 131.13)}$ |

$(x^3 + 6x^2 + 6x)e^x = 0$

$x(x^2 + 6x + 6)e^x = 0$

Edit Action Interactive

$\frac{d}{dx} \left(-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3 \right)$

$-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2$

$\text{solve}\left(-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2 = 0 | 0 \leq x \leq 10, x\right)$

$\{x=1.944677644, x=5.90930399, x=8.227862951\}$

$-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3 | x=1.944677644$

-3.693220407

$-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3 | x=5.90930399$

13.40118367

$-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3 | x=8.227862951$

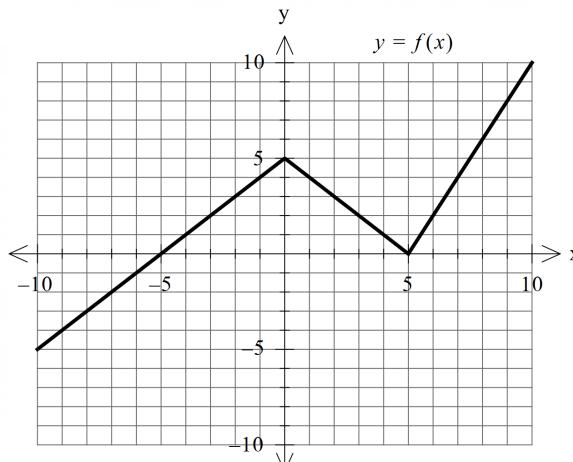
8.873150208

Specific behaviours

P shows second derivative equated to zero
P states at least two x values, accept approx
P states three x values, approx.

Q6 (2, 2, 2 & 2 = 8 marks) (3.2.15-3.2.17)

Consider the function $y = f(x)$ which is graphed below.



Determine the following.

a) $\int_{-10}^0 f(x) dx$

Edit Action Interactive

$\frac{d}{dx} \left(-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3 \right)$

$-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2$

$\text{solve}\left(-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2 = 0 | 0 \leq x \leq 10, x\right)$

$\{x=1.944677644, x=5.90930399, x=8.227862951\}$

$-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3 | x=1.944677644$

-3.693220407

$-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3 | x=5.90930399$

13.40118367

$-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3 | x=8.227862951$

8.873150208

Alg Decimal Cplx Rad

Edit Action Interactive

$-5 \sin\left(x - \frac{\pi}{4}\right) + 2x - 3 | x=10$

15.95683924

Steepest point is at $x=5.91$ metres & $y = 23.20$ metres or 23.19(do not accept 23.2)

Note: accept the endpoint if derivative is given although the endpoints were not supposed to be examined. endpoint of $x = 10$ metres & $y = 69.11$ metres

Behaviours unchanged

Specific behaviours

P equates second derivative to zero.
P solves for 3 values of x
P states value of first derivative for at least 2 x values above.
P selects steepest point with x & y values rounded to 2 dp (2 possible answers)
Note : max -1 if 2 dp not used in entire question (no need for units)

Solution

Specific behaviours

P identifies negative result
P starts total value

$$\int_{-5}^5 f(x) dx = \int_{-5}^0 f(x) dx + \int_0^5 f(x) dx$$

$$= -\frac{1}{2}(5)^2 + \frac{1}{2}(10)(5) + \frac{1}{2}(5)(10)$$

$$= -\frac{25}{2} + 25 + 25$$

$$= \frac{75}{2}$$

Solution

Specific behaviours

P uses FTC
P states result

$$\int_a^b f(t) dt = f(x) \Big|_a^b = f(b) - f(a)$$

Solution

Specific behaviours

P solves for two x values in domain
P differentiates and equates to zero.
P solves for two x values in domain
P uses first or second derivative test to select local min with y value stated to 2 dp
P need for units)

$$5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2x - 3 = 0$$

$$x = 0, 2.718729454, x = 3.18654418$$

$$\frac{d}{dx} \left(-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2x - 3 \right)$$

$$-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2$$

$$-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3$$

$$\text{solve}\left(-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2x - 3 = 0 | 0 \leq x \leq 10, x\right)$$

$$0.9035946553$$

$$5.690836616$$

$$-2.355090786$$

$$-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2 \mid x = 3.18654418$$

$$-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2 \mid x = 0.2718729454$$

$$-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2 \cdot 3.18654418 = 0$$

$$-5 \cdot \cos\left(x - \frac{\pi}{4}\right) + 2 \cdot 0.2718729454 = 0$$

- d) The area enclosed between $y = f(x)$ and the line $y = 2$.

Solution

Edit Action Interactive

$\frac{1}{2}(6)(3) + \frac{1}{2}(3)(2)$

12

Specific behaviours

P uses two triangles
P states result
(If 4 triangles used giving 52.5 then 1 mark out of 2)

- a) Determine $\frac{dy}{dx}$.

Solution

Edit Action Interactive

$\frac{d}{dx} \left(5 \cdot \cos\left(x - \frac{\pi}{4}\right) + x^2 - 3x + 4 \right)$

$-5 \cdot \sin\left(x - \frac{\pi}{4}\right) + 2 \cdot x - 3$

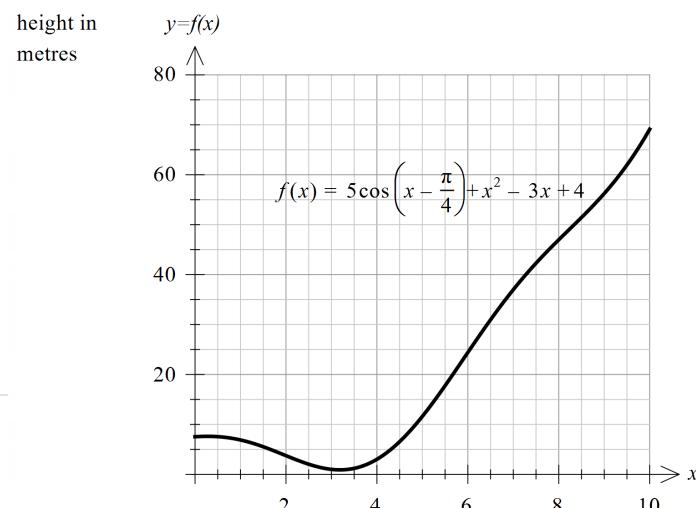
Specific behaviours

P states derivative

Q7 (1, 3x= & 4 = 8 marks) (3.2.5-3.1.6)

The cross section of a mountain can be given by $f(x) = 5 \cos\left(x - \frac{\pi}{4}\right) + x^2 - 3x + 4$ for $0 \leq x \leq 10$
metres where $f(x)$ = height in metres

cross-section of a mountain



- b) Determine the minimum height of the mountain to 2 decimal places. Justify.

Solution