MOUNT LAWLEY SENIOR HIGH SCHOOL

Semester 2 Examination, 2011

Question/Answer Booklet

MATHEMATICS SPECIALIST
MAS 3C/3D

Section Two
Calculator- assumed

NAME________________________

Time allowed for this section
Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	6	6	50	40
Section Two: Calculator-assumed	13	13	100	80
			Total	120

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2011*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

This section has **thirteen (13)** questions.
Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 7 **(5 marks)**

The temperature, I°C, of a liquid in an insulated flask at any time t seconds can be described by the differential equation $\frac{dI}{dt} = -0.003I$.

(a) How long will it take for the liquid in the flask to fall by 10%? [2]

The temperature of a liquid in another, uninsulated, flask is decreasing exponentially at a continuous percentage decay rate of 0.75%. Given the initial temperatures of the liquids in the insulated and uninsulated flasks are 65˚C and 95˚C respectively and knowing that in the cooling process there will be multiple times when the liquids will be at the same temperature,

(b) determine the first two times when the difference in temperature between the two liquids is 10˚C. [3]

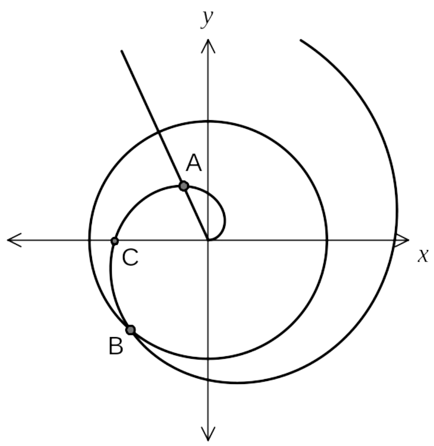
Question 8 **(6 marks)**

(a) Find the distance between the points with polar coordinates $\left(5, \frac{2\pi}{3}\right)$ and $\left(12, -\frac{3\pi}{4}\right)$, where distances are in centimetres and angles in radians. [3]

(b) The graphs of

$\theta = \alpha$, $r = \beta$ and $r = n\theta$

are shown together with the points A and B which have polar coordinates of $(1, 2)$ and $(\beta, 4)$ respectively.



Find the values of β, n and the polar coordinates of point C.

[3]

Question 9 **(8 marks)**

The point A has position vector $3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and O is at the origin.

(a) Find the value of a if the vectors $\overrightarrow{OA}$ and $a\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ are perpendicular. [1]

(b) Find the size of the angle between $\overrightarrow{OA}$ and the z-axis, to the nearest degree. [2]

(c) Find the value of b if the points $(7, b, 2)$ and $(-1, 2, 5)$ are in the plane perpendicular to the vector $\overrightarrow{OA}$. [2]

(d) Find the value of c if the point $(15, -14, c)$ lies on the straight line through A and the point $(-1, 2, 5)$. [3]

Question 10 **(5 marks)**

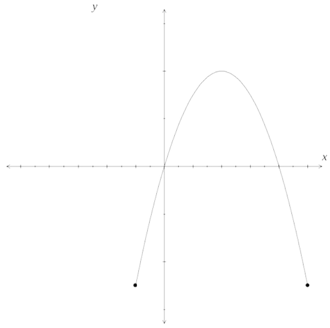
When an object is at a distance u cm from a lens of focal length 20 cm, an image is created at a distance of v cm from the lens.

The variables are related by the formula $\frac{1}{20} = \frac{1}{u} + \frac{1}{v}$.
An object is moving with a constant speed of 2 cm/s towards the lens.

At the instant when the image is 30 cm from the lens, in what direction and with what speed is the image moving?

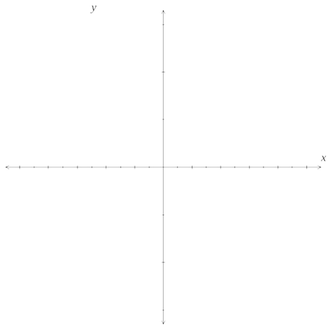
Question 11 **(4 marks)**

(a) The graph of $y = f(x)$ is shown.

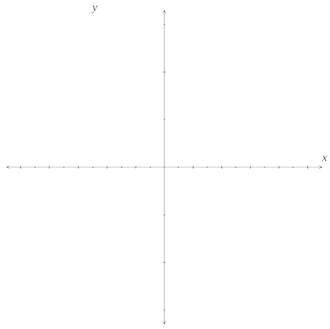


Sketch the graphs of [2]

(i) $y = |f(x)|$



(ii) $y = f(|x|)$



(b) The equation $|ax + b| = |x - 4|$ has solutions $x = -0.2$ and $x = -3$.

Find all the simultaneous values values of a and b. [2]

Question 12

(5 marks)

Prove that $\dfrac{1+sin(2\theta)-\ cos(2\theta)}{1+sin(2\theta)+cos(2\theta)}=tan\,\theta$.

Question 13 **(6 marks)**

Two complex numbers are given by $u = 3i$ and $v = \dfrac{3\sqrt{3} - 3i}{2}$.

(a) Express $u^3 v$ in the form $r\,cis\theta$ where $-\pi \le \theta \le \pi$ and $r \ge 0$. [2]

(b) List all solutions for z in exponential form, given that $z^4 = u^3 v$. [2]

(c) Show that the sum of all the solutions from part (b) is 0. [2]

Question 14 **(5 marks)**

At a school with 108 boarders, boarders can either eat breakfast or not eat breakfast.

The canteen manager estimates that of those boarders who eat breakfast one morning, 5% of them will not eat breakfast the next morning and of those boarders who do not eat breakfast one morning, 55% of them eat breakfast the following morning.

(a) If 55 boarders eat breakfast on Monday, how many boarders should the canteen manager expect to eat breakfast on Wednesday? [3]

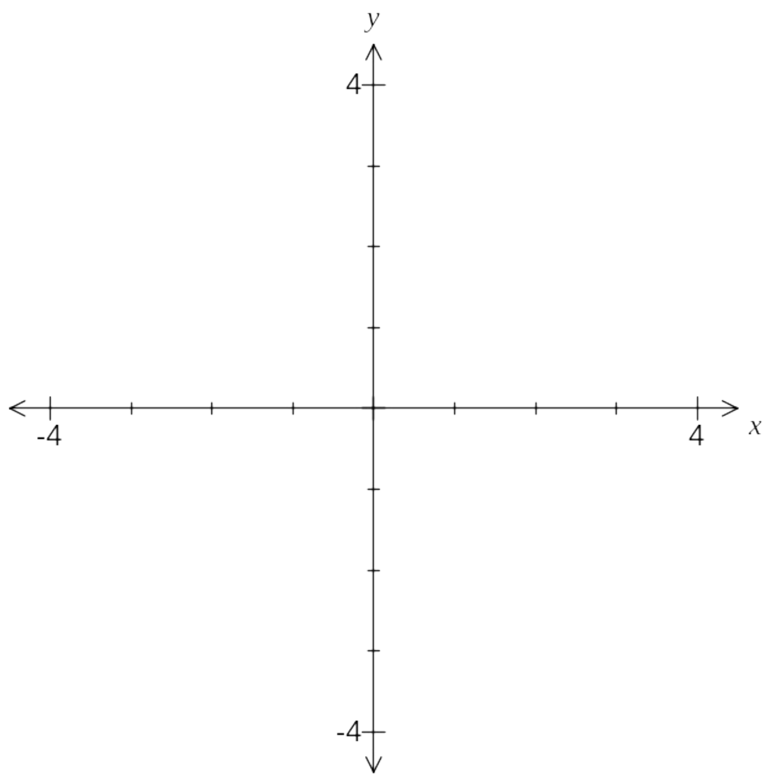
(b) In the long term, what proportion of boarders can be expected to eat breakfast? [2]

Question 15 **(7 marks)**

The graphs of the function $f(x)=2\ln(x+k)$, where k is a constant, and its inverse $y=f^{-1}(x)$, intersect where $x=-2$ and at one other point.

(a) Given the graphs of $y=f(x)$ and the inverse $y=f^{-1}(x)$ are symmetrical about the line $y=x$, show that the exact value of k is $\frac{1}{e}+2$. [2]

(b) Sketch the graphs of $y=f(x)$ and its inverse $y=f^{-1}(x)$ on the axes below, showing any asymptotes and showing the coordinates of all points of intersection and axes-intercepts. Where appropriate give values correct to 2 decimal places. [5]



Question 16 **(5 marks)**

(a) The matrix equation $AX = B$ could be used to solve the following system of equations.

$$2a + 3b = c - 5$$
$$b - 2c - 4a = 1$$
$$5 = c + b$$

Write down suitable matrices for A, X and B then illustrate how to solve the system of equations using a matrix method. **(Do not solve the equations)** [2]

(b) If $PQ = 3P + I$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 7 & -9 \\ 5 & -8 \end{bmatrix}$, find matrix P. [3]

Question 17 **(7 marks)**

The displacement $x(t)$ metres, of a small particle undergoing simple harmonic motion is given by $x(t) = A\cos(\omega t) + B\sin(\omega t)$, where A, B and ω are positive constants.

(a) Show that $x''(t) + \omega^2 x(t) = 0$. [2]

The particle completes 2.5 cycles every second and initially has a displacement of 1.5 m and a velocity of 7.5 ms^{-1}.

(b) Determine the exact values of the constants A, B and ω. [3]

(c) What is the amplitude of motion, correct to the nearest millimetre? [2]

Question 18 **(9 marks)**

Relative to itself, an anti-ballistic missile (ABM) launch site detects a ballistic missile at $11\mathbf{i} - 18\mathbf{j} + 10\mathbf{k}$ km headed at constant velocity for a target at $35\mathbf{i} + 14\mathbf{j} + \mathbf{k}$ km.

The ballistic missile is expected to hit the target in 50 seconds.

(a) How close does the ballistic missile come to the ABM launch site? [5]

Question 18 (continued)

The launch site plans to fire an ABM to hit the ballistic missile. The hit is timed to take place at the instant the ballistic missile comes within 8km of the target.

(b) Assuming the ABM instantly achieves a constant velocity of 1150 ms^{-1} as it is launched, how long from the time of detection should the defence site fire it?
Give your answer to the nearest tenth of a second. [4]

Question 19 **(8 marks)**

Every odd integer I can be written as ten times some integer, n, plus some constant c.

That is, $I = 10n + c$, where $c = 1, 3, 5, 7, 9$.

(a) Show how the odd integers 237 and -35 can be written this way. [1]

(b) By exhausting the five different cases for c prove that the square of every odd integer ends in 1, 5 or 9. [7]

End of questions