This section has **12** questions Attempt **all** questions.

Question 10 [1 + 2 = 3 marks]

Tessa guesses randomly the answers of each of the ten multiple choice questions on her test. In each question there are 3 alternative answers, one of which is correct

Determine the probability that:

(a)   Tessa answers exactly four of the ten questions correctly

(b)   The tenth question she attempts is only the second question she answers correctly

Question 11 [5 + 2 = 7 marks]

Given $y = e^x(x-3)$ use calculus methods to find *exactly*

(a)   the stationary point(s)

(b)   the points of inflection

Question 12 [2 + 2 + 2 + 3 = 9 marks]

The Weather Company said that the probability it rains today is 0.75. The company also said that the probability of it raining the day after it is wet is 0.95 while the probability of it raining the day after it is fine is 0.2.

Determine the probability that:

(a)   It does not rain today and it rains tomorrow

(b)   It rains for the next week.

(c)   It is fine for at least one day next week.

(d)   Given it rains today, it will rain at least once in the next two days

Question 13 [1+3 = 4 marks]
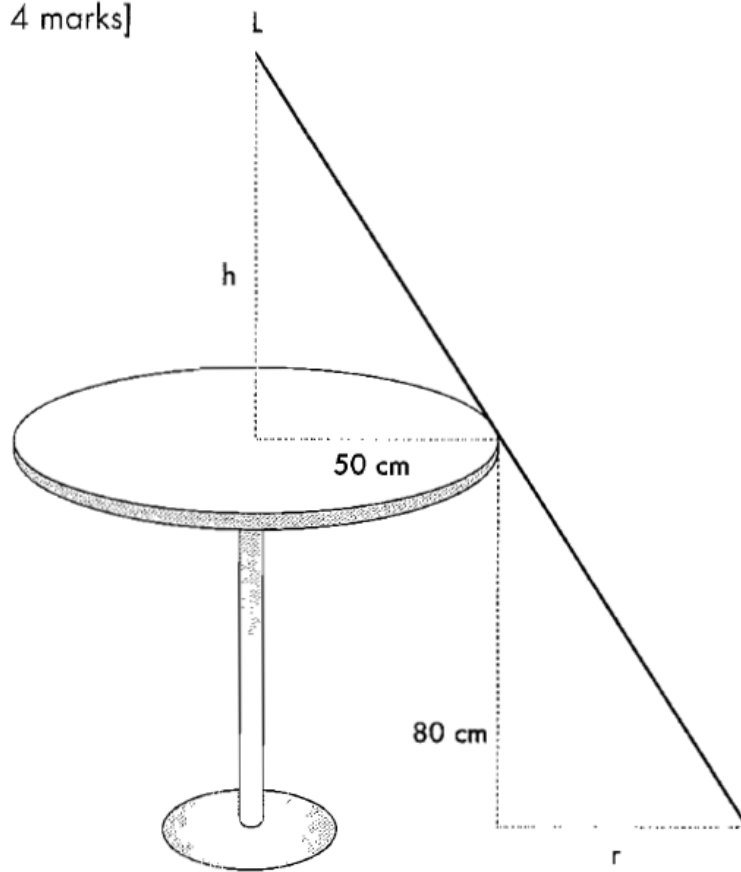
A table has a radius of 50 cm and a height of 80 cm A light (L) is lowered vertically downwards from a point above the centre of the table at a constant rate of 0 2 cm per second

When the light is h cm above the table it casts a shadow that extends r cm from the edge of the table

(a)   Show that $r = \dfrac{4000}{h}$

(b)   Find the rate at which r is changing when h = 60

Question 14 [2 + 2 + 2 + 2 = 8 marks]

A particle beginning at the origin moves in rectilinear motion such that its displacement (in metres) at any time (in seconds) is given by:

$$x(t) = \begin{cases} 2t^3 - 3t^2 & 0 \le t < 2 \\ -4(t^2 - 5t + 5) & 2 \le t \le 4 \end{cases}$$

Determine in this interval:

(a)   the velocity of the particle at any time t

(b)   when and where the particle is at rest for the first time

(c)   the distance travelled in the first three seconds

(d)   the minimum velocity

Question 15 [3 + 2 = 5 marks]

A new game involving chance involves several rounds. The winner receives one point for a win in each round The person who receives a total of **three** points is declared the winner of the game.

Let the random variable X be the number of rounds needed to **complete** the game and reach three points

If in a game Morgan has two points and Todd has one point, and the probability that Morgan wins any particular game is k:

(a)   Construct a probability distribution table for the above game

(b)   Determine the expected mean, i e E(x)

Question 16 [4 + 1 + 3 = 8 marks]

On the axes below linear inequalities are drawn with a feasible region.
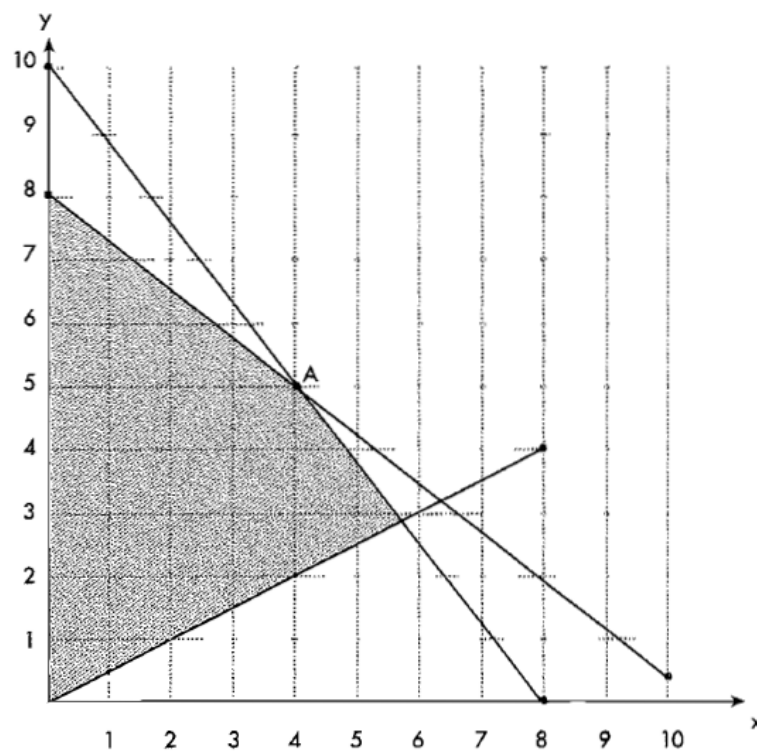
(a)   State the inequalities represented on the axes above

(b)   Determine the coordinates of A

(c)   The profit equation p = kx + 6y is maximised at point A What range of value(s) of k enable the profit equation to be maximised at A

Question 17 [3 + 1 + 2 = 6 marks]

A supplier of household wood logs is concerned that the 50 kg bags are on average underweight. He randomly selects a sample of 10 bags of wood logs and calculates the mean of 49.2 kg.

(a) The weights of the bags of wood logs are approximately normally distributed with a mean of μ and a standard deviation of 1.6 kg.

    (i) Determine a 95% confidence interval for μ.

    (ii) Is the concern of the supplier justified? Explain.

(b) The supplier decides that he requires the width of the 95% confidence interval of μ to be within 0.75 kg. Find the minimum sample size.

Question 18 [3 + 2 + 2 = 7 marks]

An animal drinking trough is constructed from stainless steel in the shape of a trapezoidal prism, with height 'h' metres and length of 1 metre

The cross section of the prism is an isosceles trapezium with acute angles of 45°, base 'b' metres and area of 60 m².
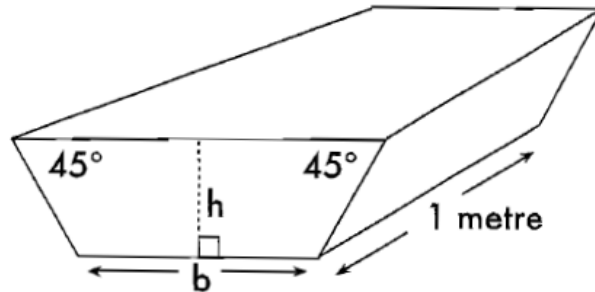
(a)　Show that: $b = \dfrac{60}{h} - h$

(b)　Show that the surface area 'A' in m² is: $A = \dfrac{60}{h} - h + 2h\sqrt{2} + 120$

(c)　Find the depth of the drinking trough to the nearest mm, if the amount of stainless steel is to be kept to a minimum

Question 19 [6 marks]

A souvenir snow dome located on John's study desk contains liquid and can be modelled by rotating the parabola $y = 4 - x^2$ from $x = 0$ to $x = 2$, $360°$ about the y axis.
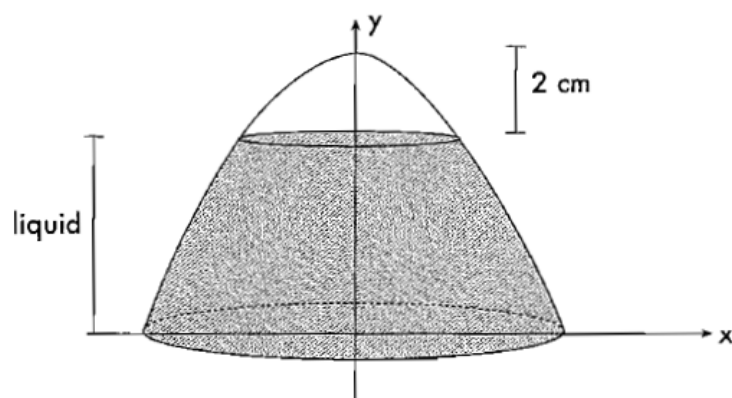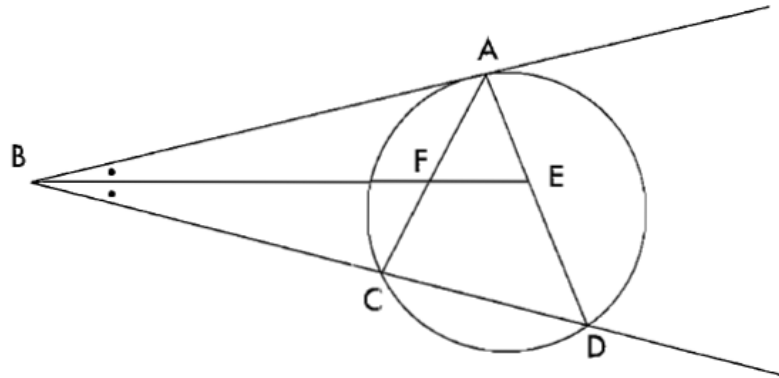

When the snow dome is placed on the desk the liquid is 2 cm below the top. If the snow dome is inverted determine the depth of the liquid

Question 20 [3 + 3 = 6 marks]

The diagram below shows that:

- EB bisects ∠ABC

- AB is a tangent to the circle at point A

(a) Show that ∠FCB = ∠EAB

(b) Prove that ∆FAE is isosceles

Question 21 [3 + 2 + 3 + 3 = 11 marks]

Granny Smith apples grown on Farmer Jack's farm are normally distributed with a mean weight of 121 g and a standard deviation of 2.7 g

(a)   Determine the probability that a randomly chosen apple:

   (i)   weighs more that 122 5 g

   (ii)   weighs less than 120 g given the apple weighs more than 117 g.

(b)   Farmer Jack sends the apples to market in cartons containing 10 bags of apples The probability that a bag contains a rotten apple is found to be 0.05. If more than two bags contain rotten apples all 10 bags are rejected

   Assume that rotten apples occur independently Calculate the probability that a randomly selected carton is rejected

(c)   On Farmer Jill's farm smaller Fuji apples are grown These apples are in bags of two for the lunchbox market The weight of these apples are normally distributed with a mean weight of 85 g and a standard deviation of 1 7 g Assuming the weights of the apples are independent, determine the probability that the combined weight of the two apples is less than 172 g, if the combined standard deviation is 2 4042 g

(d)   Farmer Joe grows Pink Lady apples The weights of these apples are normally distributed with a standard deviation of 2 4 g If 80% of the time the weight of a Pink Lady apple is more than 70 g, calculate the mean weight of these apples

END OF PAPER