SHENTON COLLEGE

ATMAM Mathematics Methods

Test 2 (2018) Calculator Free

Name: MARK L. O'KAESHN

Teacher: Friday Smith

Time Allowed : 30 minutes

Marks	/36

Materials allowed: Formula Sheet.

All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

1 Determine the following indefinite integrals.

a) $\int 12x^3 - 4x\,dx$ (1)

$3x^4 - 2x^2 + c$

b) $\int x(x+1)^2\,dx$ (2)

$= \int x(x^2+2x+1)\,dx$ ✓ expand

$= \int x^3 + 2x^2 + x\,dx$ ✓ integrate

$= \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + c$

c) $\int \frac{3x^4 - 2x^3 + 1}{x^2}\,dx$ (3)

$= \int 3x^2 - 2x + \frac{1}{x^2}\,dx$ ✓ split & cancel

$= x^3 - x^2 - \frac{1}{x} + c$

✓ integrate polynomial terms

✓ integrate $\frac{1}{x^2}$

d) $\int e^{3x-2}\,dx$ (2)

$= \frac{e^{3x-2}}{3} + c$

✓ integrate $e^{f(x)}$

✓ divide by $f'(x)$

e) $\int 3(4-2x)^5\,dx$ (2)

$= \frac{3(4-2x)^6}{6\times(-2)} + c$

$= \frac{(4-2x)^6}{-4} + c$

✓ correct treatment of bracket term

✓ divide by derivative of bracket.

2 Evaluate the following definite integrals

a) $\int_1^4 3x^2 + 1\,dx$ (2)

$= x^3 + x\Big]_1^4$

$= (4^3 + 4) - (1^3 + 1)$

$= 66$

✓ integrate

✓ substitute

b) $\int_{-1}^{2} \pi\,dx$ (2)

$= \pi x\Big]_{-1}^{2}$

$= 2\pi - (-\pi)$

$= 3\pi$

✓ integrate

✓ answer

c) $\int_0^{\frac{\pi}{4}} \sin 2x \ dx$ (5)

$= -\frac{\cos 2x}{2}\Big]_0^{\frac{\pi}{4}}$

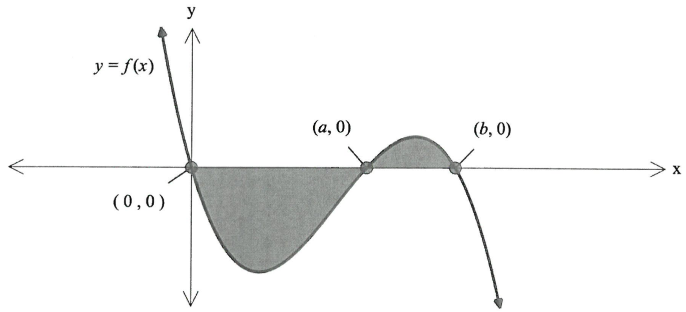
$= -\frac{\cos\frac{\pi}{2}}{2} - \left(-\frac{\cos 0}{2}\right)$

$= 0 + \frac{1}{2}$

$= \frac{1}{2}$

✓ $-\cos 2x$
✓ $\overline{\ \ 2\ \ }$
✓ substitute
✓ exact values
✓ answer

3 Circle all of the expressions that would give the area shaded below. (4)



$\int_0^b f(x)dx$	$\left\lvert\int_0^b f(x)dx\right\rvert$
$-\int_0^a f(x)dx + \int_a^b f(x)\,dx$	$\int_0^a f(x)dx - \int_a^b f(x)\,dx$
$\int_0^a f(x)dx + \int_a^b f(x)\,dx$	$\int_a^0 f(x)dx + \int_a^b f(x)\,dx$
$\left\lvert\int_0^a f(x)dx\right\rvert + \left\lvert\int_a^b f(x)\,dx\right\rvert$	$\int_0^b \lvert f(x)\rvert dx$

4 If $f''(x) = 6x - 2$ and given that $f(2) = 9$ and $f(-1) = -6$, determine $f(x)$. (7)

$\int 6x - 2 \, dx = 3x^2 - 2x + c$ ✓ integrate

$\int 3x^2 - 2x + c \, dx = x^3 - x^2 + cx + d$ ✓ integrate

✓ different constant

$9 = 2^3 - 2^2 + 2c + d$

$5 = 2c + d \quad \ldots ①$ ✓ eq 1

$-6 = (-1)^3 - (-1)^2 - c + d$ ✓ eq 2

$-4 = -c + d \quad \ldots ②$ ✓ solve c & d

② − ① ✓ f(x)

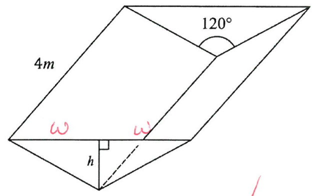
$9 = 3c$

$c = 3$

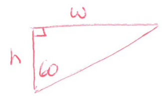
$d = -1$

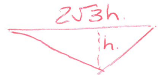
$f(x) = x^3 - x^2 + 3x - 1$

4 A steel trough in the shape of an isosceles prism is slowly being filled with water.



a) Show that the volume of water in the trough (in m^3) is given by the equation $V = 4\sqrt{3}h^2$, where h is the height of the water. (2)





$\tan 60° = \frac{w}{h}$

$w = \sqrt{3}h$

$\text{Area} = \frac{1}{2} \times 2\sqrt{3}h \times h$

$= \sqrt{3}h^2$

$\text{Volume} = 4\sqrt{3}h^2$

✓ use of exact value.

✓ volume shown.

b) Use the method of small change to find the change in the height of the water if the volume is increased from 0.8 m^3 to 0.81 m^3. Give your answer in millimetres, to two decimal places. (5)

$\delta h \simeq \frac{dh}{dV} \delta V$

$\delta V = 0.01$

$\frac{dV}{dh} = 8\sqrt{3}h$

$\frac{dh}{dV} = \frac{1}{8\sqrt{3}h}$

when $V = 0.8$, $h \approx 0.3398$

$\frac{dh}{dV}\Big|_{h=0.3398} = 0.2124$

$\delta h = 0.2124 \times 0.01$

$= 0.00212 \text{ m}$

$= 2.12 \text{ mm}$

✓ small change formula with correct variables

✓ $\frac{dV}{dh}$ ✓ $h = f(V)$

or

✓ $h =$ ✓ $\frac{dh}{dV}$

✓ evaluate derivative

✓ answer

3 The Neinz Quality Food Company are redesigning their can for their iconic pickled eel and ox tongue soup. The can is to be cylindrical and have a volume of 300mL.

a) Rearrange the volume formula to determine an equation for the height of the can in terms of the radius. (1)

$$300 = \pi r^2 h$$

$$h = \frac{300}{\pi r^2}$$

b) Write an expression for the surface area of the can in terms of the radius, simplifying where appropriate. (2)

$$A = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r\left(\frac{300}{\pi r^2}\right)$$

$$= 2\pi r^2 + \frac{600}{r}$$

c) The material for the sides of the can is relatively thin and costs 0.04c/cm^2. The material for the top and bottom of the can is much thicker and costs 0.06c/cm^2. Write an expression for the total material cost for the can, in terms of the radius only. (1)

$$C = 0.06 \times 2\pi r^2 + \frac{0.04 \times 600}{r}$$

$$= 0.12\pi r^2 + \frac{24}{r}$$

d) Use your answer for part c) to determine the dimensions of the can which minimise the material cost. Determine this minimum cost. (5)

$$\frac{dC}{dr} = 0.24\pi r - \frac{24}{r^2}$$ ✓ $\frac{dC}{dr}$ shown.

$$0 = 0.24\pi r - \frac{24}{r^2}$$ ✓ $\frac{dC}{dr} = 0$

$r \simeq 3.17$ cm. ✓ r

$h = 9.51$ cm. ✓ h

Minimum cost = 11.36 ¢ ✓ min cost.

5 Determine the area trapped between the curve $y = x^3 - 3x + 3$ and the line $y = x + 3$. (6)

f(x) g(x)

Intersection

$x^3 - 3x + 3 = x + 3$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x(x-2)(x+2) = 0$

$x = 0, \; x = 2, \; x = -2$

✓ intersections
✓ split integrals
✓ ensure positive results.
✓ integrate
✓ substitute
✓ answer

$$\left| \int_{-2}^{0} x^3 - 4x \, dx \right| + \left| \int_{0}^{2} x^3 - 4x \, dx \right|$$

$$= \left| \left[\frac{x^4}{4} - 2x^2 \right]_{-2}^{0} \right| + \left| \left[\frac{x^4}{4} - 2x^2 \right]_{0}^{2} \right|$$

$$= \left| (0) - (4-8) \right| + \left| (4-8) - (0) \right|$$

$$= 4 + |-4|$$

$$= 8$$

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ATMAM Mathematics Methods

Test 2 (2018) Calculator Assumed

Name: Mark L. O'Kaeshn

Teacher: Friday Smith

Time Allowed : 20 minutes

Marks	/24

Materials allowed: Classpad, Formula Sheet.

All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given to two decimal places.
Marks may not be awarded for untidy or poorly arranged work.

1 Given $\int_{-4}^{3} f(x)dx = 7$ and $\int_{1}^{3} f(x)\,dx = -4$, determine

a) $\int_{-4}^{1} f(x)dx$ (1)

$= 11$

b) $\int_{1}^{1} f(x)dx$ (1)

$= 0$

c) $\int_{1}^{3} 2f(x) + 1\,dx$ (2)

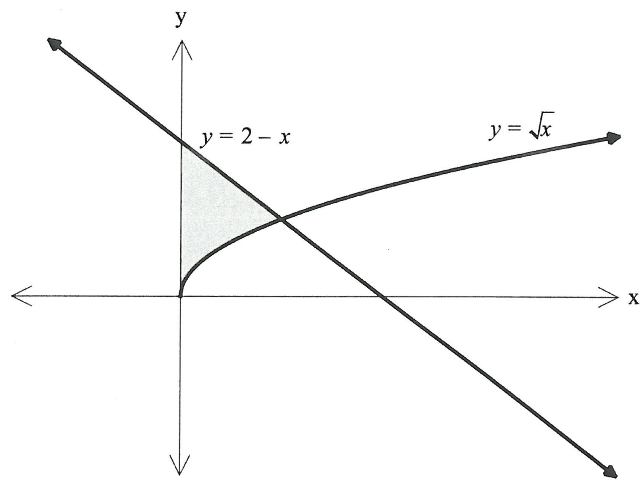
$= 2\int_1^3 f(x)\,dx + \int_1^3 1\,dx$ ✓ correct use of transformation

$= 2(-4) + [x]_1^3$ ✓ answer

$= -8 + 2$

$= -6$

2 Determine the area of the shaded region, clearly showing how you obtained your answer. (4)



Intersection

$2 - x = \sqrt{x}$

$x = 1$

$\int_0^1 (2-x) - \sqrt{x}\,dx$

$= \frac{5}{6}$

✓ shows intersection

✓ $\int_0^1 dx$

✓ top − bottom.

✓ answer