Chapter 3

Vectors- Basics Idea

Vector Quantities:

How to solve:

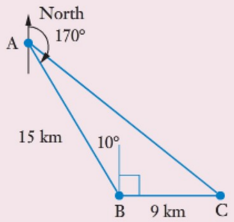
- Sketch the situation and use trigonometry to determine the final location relative to the initial position
- Make an accurate scale drawing to determine the final location relative to the initial position

e.g.

A boat sails 15 km on a bearing 170° followed by 9 km due East. Find the distance and bearing of the boat's final position from its initial position.

Solution

Method One: By calculation (i.e. sketch and use trigonometry)



If A is the initial position and C is the final position, then the distance from A to C is given by AC.

Now $AC^2 = 15^2 + 9^2 - 2 \times 15 \times 9 \times \cos 100°$ (cosine rule)

$\therefore$ $AC \approx 18.79$

The bearing of C from A is $(170° - \angle BAC)$

By the sine rule $\dfrac{9}{\sin \angle BAC} = \dfrac{AC}{\sin 100°}$

giving $\angle BAC \approx 28.2°$ Obtuse solution to equation not applicable as triangle already has an obtuse angle.

The final position of the boat is approximately 19 km from the initial position and on a bearing 142°.

Important note: Had there not already been an obtuse angle in $\triangle BAC$, finding $\angle BAC$, rather than $\angle ACB$, is a wise strategy because $\angle BAC$ is not opposite the longer side and so cannot be obtuse.

Vectors:

Are quantities that have magnitude and direction

e.g. Displacement, velocity, force, acceleration

Scalar:

Quantities that only have magnitude

e.g. distance, speed, magnitude of a force, magnitude of acceleration, energy

Adding Vectors:

We need to draw the lines/ vectors with nose to tail

And then using trigonometry we are able to find the resultant and the direction

e.g.

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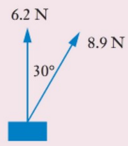
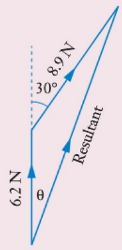
EXAMPLE 2



Forces of 6.2 Newtons vertically upwards and 8.9 Newtons acting at 30° to the vertical act on a body, see diagram. Determine the magnitude and direction of the single force that could replace these two forces (i.e. determine the **resultant** of the two forces).

Solution

First sketch the vector triangle remembering the 'nose to tail' idea for vector addition. By the cosine rule:



$$(\text{Magnitude of resultant})^2 = 6.2^2 + 8.9^2 - 2(6.2)(8.9)\cos 150°$$

$\therefore$ Magnitude of resultant $\approx$ 14.602 Newtons

By the sine rule: $\dfrac{\text{Magnitude of resultant}}{\sin 150°} = \dfrac{8.9}{\sin\theta}$

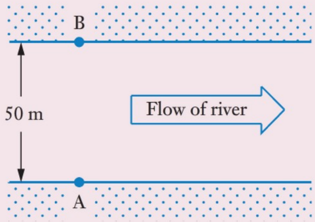
$\therefore$ $\theta \approx 17.7°$

(Obtuse solution not applicable, θ not opposite longest side, and triangle already has obtuse angle.)

The resultant of the two forces is a force of magnitude 14.6 Newtons (correct to one decimal place) acting at 18° to the vertical (to the nearest degree).

Things I Stuggled with , go over again(3C)

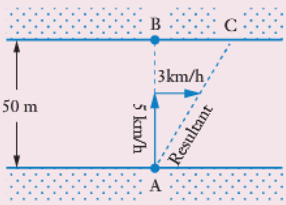
A canoeist wishes to paddle her canoe across a river from point A on one bank to the opposite bank. The canoeist can maintain a constant 5 km/h in still water. However the river is flowing at 3 km/h (see diagram).



a If the canoeist wishes to journey across the river as quickly as possible, how long will the journey take and how far down river will the canoeist travel?

b If instead she wishes to reach point B on the opposite bank, directly opposite A, in which direction should she paddle and how long will the journey take?

a If she wishes to journey across the river as quickly as possible, she must put all her efforts into getting across and let the current take her down river.



She travels at 5 km/h across the river and the current takes her down river at 3 km/h. Her resultant velocity is as shown on the right.

Now speed $= \dfrac{\text{distance}}{\text{time}}$.

Thus to travel the 50 m *across* the river when her speed *across* is 5 km/h will take t_1 seconds where

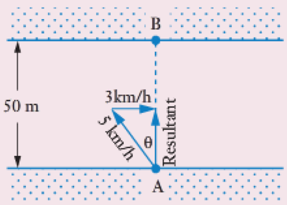
$$\frac{5 \times 1000}{60 \times 60} = \frac{50}{t_1} \qquad \therefore\ t_1 = 36.$$

The distance travelled downstream will be given by BC, see diagram.

By similar triangles $\dfrac{3}{5} = \dfrac{BC}{50}$ $\quad\therefore$ BC = 30.

If she wishes to journey across the river as quickly as possible it will take her 36 seconds and she will travel 30 metres down river.

b To reach point B on the far bank she needs to set a course such that the combined effect of her paddling and the flow of the river takes her directly across the river (see diagram).



By trigonometry: $\theta \approx 37°$

By Pythagoras: Resultant speed = 4 km/h.

To travel 50 metres at 4 km/h will take 45 seconds.

To reach point B on the opposite bank, directly opposite A, she should paddle upstream at 53° to the bank. The combined effect of this paddling and the current will cause her to travel to B in 45 seconds.

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Representing Vectors:

-If we are talking a line from one point (A) to another (B), we represent it by placing a=n arrow on top of both letters.

- If the sides are already lettered, we bold them by underlining them, e.g. vectors a = a

- and for magnitude we write it with | |, e.g. |a|

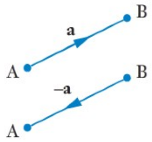
Equal Vectors:

Two vectors that have the **same magnitude** and the **same direction**

The negative of a vector:

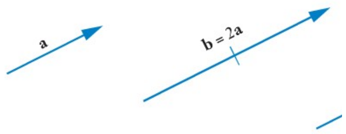
Same magnitude but opposite direction

- If a = ->AB, then -a = - ->AB = ->BA



Multiplication of a vector by a scalar:

If b=2a then b is the same direction as a but twice the magnitude



Parallel Vectors:

Two vectors that are parallel if one scalar multiple of the other

- If the scalar multiple is positive, the vectors are said to be *like parallel vectors*
- If the scalar multiple is negative, the vectors are said to be *unlike parallel vectors*

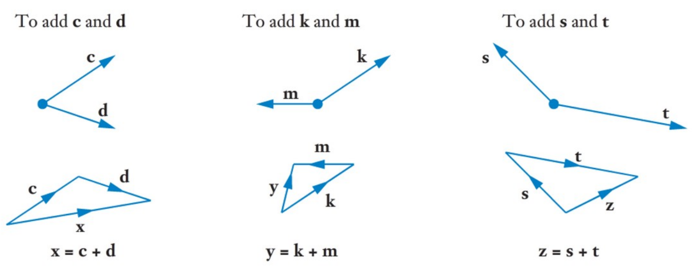
Addition of vectors:

- Is to add two vectors
- And also find the single/ resultant vectors (that replaces the two)

We do this by using a vector triangle

- Which the vectors are added from "nose to tail", which forms two sides of the triangle.

e.g.



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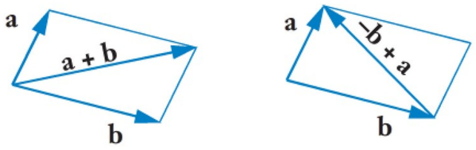
Subtraction of one Vector from Another:

Instead of a – b, instead be a + (-b)

With the parallelogram approach one diagonal is a+b and the other is a-b

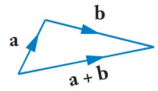


Also, the *triangle inequality* is…

With $|\mathbf{a}|$ as the magnitude of **a** it also follows that

$$|\mathbf{a}+\mathbf{b}| \le |\mathbf{a}| + |\mathbf{b}|$$

the triangle inequality.



The zero Vector:

When we add a vector to the negative of its self we obtain the zero vector:

P+(-P) = 0

-The zero vector has zero magnitude and an undefined direction, we can write the zero vector as 0

ha=kb

1. a and b are parallel – because one is a scalar multiple of the other)
2. or h=k=0

i.e. $(p-r)\mathbf{a} = (s-q)\mathbf{b}$,

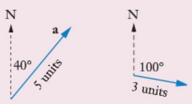
it follows that $p-r=0$ and $s-q=0$.

If **a** is a vector of magnitude 5 units in direction 040° and **b** is a vector of magnitude 3 units in direction 100°, find the magnitude and direction of:

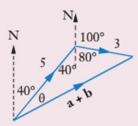
a $\mathbf{a}+\mathbf{b}$ **b** $\mathbf{a}-\mathbf{b}$.

Solution

First sketch **a** and **b**:



a



By the cosine rule $|\mathbf{a}+\mathbf{b}|^2 = 3^2+5^2-2\times3\times5\times\cos120°$

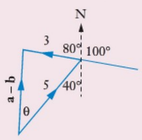
Thus $|\mathbf{a}+\mathbf{b}| = 7$

By the sine rule $\dfrac{|\mathbf{a}+\mathbf{b}|}{\sin120°} = \dfrac{3}{\sin\theta}$

Thus $\theta \approx 21.8°$

$\mathbf{a}+\mathbf{b}$ has a magnitude of 7 units and direction 062° (to the nearest degree).

b



By the cosine rule $|\mathbf{a}-\mathbf{b}|^2 = 3^2+5^2-2\times3\times5\times\cos60°$

Thus $|\mathbf{a}-\mathbf{b}| \approx 4.36$

By the sine rule $\dfrac{|\mathbf{a}-\mathbf{b}|}{\sin60°} = \dfrac{3}{\sin\theta}$

Thus $\theta \approx 36.6°$

$\mathbf{a}-\mathbf{b}$ has magnitude 4.4 units (correct to 1 decimal place) and direction 003° (to nearest degree).

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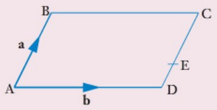
e.g. of using sides to describe other sides

In parallelogram ABCD, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$.

E is a point on DC such that DE : EC = 1 : 2.

Express each of the following vectors in terms of **a** and/or **b**.



a $\overrightarrow{DC}$ **b** $\overrightarrow{CB}$ **c** $\overrightarrow{DE}$ **d** $\overrightarrow{BE}$

Solution

a $\overrightarrow{DC}$ is the same magnitude and direction as $\overrightarrow{AB}$. Thus $\overrightarrow{DC} = \mathbf{a}$.

b $\overrightarrow{CB}$ is the same magnitude but opposite direction to $\overrightarrow{AD}$. Thus $\overrightarrow{CB} = -\mathbf{b}$.

c DE : EC = 1 : 2. Thus $\overrightarrow{DE} = \frac{1}{3}\overrightarrow{DC}$

$\therefore \quad \overrightarrow{DE} = \frac{1}{3}\mathbf{a}$.

d $\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CE}$ or $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DE}$

$= \mathbf{b} + \left(-\frac{2}{3}\mathbf{a}\right)$ $\qquad$ $= -\mathbf{a} + \mathbf{b} + \frac{1}{3}\mathbf{a}$

$= \mathbf{b} - \frac{2}{3}\mathbf{a}$ $\qquad$ $= -\frac{2}{3}\mathbf{a} + \mathbf{b}$

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