Christ Church Grammar School

WA Exams Practice Paper A, 2015

Question/Answer Booklet

MATHEMATICS
METHODS
UNITS 1 AND 2
Section One:
Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free **(52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(6 marks)**

(a) The table shows the temperature of a liquid over a period of time.

Time (minutes)	0	5	10	15	20	25
Temperature (°C)	58	44	32	25	21	19

Determine the average rate of change of temperature of the liquid

(i) over the first ten minutes. (1 mark)

(ii) between 15 and 20 minutes. (1 mark)

(b) Determine the equation of the tangent to the curve $y = x^3 - 2x^2 + x + 2$ when $x = 2$. (4 marks)

See next page

Question 2 **(7 marks)**

(a) The vertices of three points are A(1, 1), B(-1, 2) and C(-2, -1).

(i) Use gradients to explain whether the lines AB and BC are perpendicular. (2 marks)

(ii) Determine the equation of the line through A that is parallel to the line BC. (1 mark)

(iii) If B is the mid-point of A and D, determine the coordinates of D. (2 marks)

(b) Solve $\frac{x-3}{3} - 3x = 4$. (2 marks)

(b) The displacement, x m, of a particle from a fixed point O is given by $x = 2t^3 - 3t^2 - 12t + 1$, $t \geq 0$, where t is the time, in seconds.

(i) Determine the initial velocity of the particle. (2 marks)

$v = x'(t) = 6t^2 - 6t - 12$
$v(0) = -12$ m/s

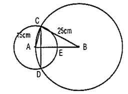
(ii) Determine the displacement of the particle at the instant it is stationary. (2 marks)

$v(t) = 0 \Rightarrow 6t^2 - 6t - 12 = 0$
$\cancel{t = -1}, t = 2$

$x(2) = -19$ m

Question 21 **(6 marks)**

The circumference of a circle of radius 25cm passes through the centre of a circle of radius 15cm. Find the area of intersection of the two circles.



$\cos\angle CAB = \frac{15^2 + 25^2 - 25^2}{2 \times 15 \times 25}$
$\angle CAB = 1.266^r$
$2\angle CAB = 2.532^r$

$\cos\angle CBA = \frac{25^2 + 25^2 - 15^2}{2 \times 25 \times 25}$
$\angle CBA = 0.6094^r$
$2\angle CBA = 1.219^r$

Segment CED:
$\frac{1}{2} \times 15^2 \times (2.532 - \sin 2.532) = 220.44$

Segment CAD:
$\frac{1}{2} \times 25^2 \times (1.219 - \sin 1.219) = 87.58$

$220.44 + 87.58 = 308.02$
Total area $\approx 308\text{cm}^2$

Question 19 **(6 marks)**

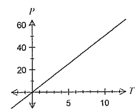
(a) The quantity P is directly proportional to the quantity T, and it is known that when $T = 12$, $P = 60$.

(i) Determine an equation for the relationship between P and T. (2 marks)

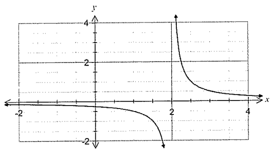
$P = kT$
$60 = k \times 12 \Rightarrow k = 5$

$P = 5T$

(ii) Sketch a graph of the relationship between P and T. (2 marks)



(b) The graph below shows $y = \dfrac{a}{x-b}$, where a and b are constants.



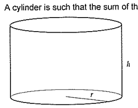
Determine the values of a and b. (2 marks)

From vertical asymptote, $b = 2$.

Using (2.5, 1), $1 = \dfrac{a}{2.5-2} \Rightarrow a = 0.5$

Question 20 **(10 marks)**

(a) A cylinder is such that the sum of the height and three times the radius is 50 cm.



(i) Write an equation for the height, h, in terms of the radius, r, for this cylinder. (1 mark)

$h + 3r = 50 \Rightarrow h = 50 - 3r$

(ii) Show that the total surface area of the cylinder is given by $A = 100\pi r - 4\pi r^2$. (2 marks)

$A = 2\pi rh + 2\pi r^2$
$= 2\pi r(50-3r) + 2\pi r^2$
$= 100\pi r - 4\pi r^2$

(iii) Using calculus techniques, determine the dimensions of this cylinder to obtain the maximum possible surface area, and state this area. (3 marks)

$\dfrac{dA}{dr} = 100\pi - 8\pi r$
$0 = 100\pi - 8\pi r \Rightarrow r = 12.5$ cm

$h = 50 - 3(12.5) = 12.5$ cm

$A = 625\pi$ cm^2
$(\approx 1960$ cm$^2)$

Question 3 **(5 marks)**

Solve the following equations.

(a) $x(x+2)(2x-3) = 0$. (1 mark)

(b) $x^2 + 4x + 6 = 2x^2 + 5x - 6$. (2 marks)

(c) $2(x-2)^2 = 100$. (2 marks)

Question 4 **(7 marks)**

(a) Determine $\frac{dy}{dx}$ in simplified form if

(i) $y = 2x^3 - x + 3$. (1 mark)

(ii) $y = \frac{5x^3}{6} - \frac{x^4}{12}$. (1 mark)

(b) Determine the coordinates of the point on the curve $y = 3x^2 - 7x - 10$ where the gradient is 5. (2 marks)

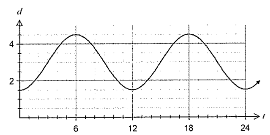
(c) Determine $f(x)$ given that $f'(x) = 5 + 2x - 6x^2$ and $f(1) = 0$. (3 marks)

Question 17 **(9 marks)**

The depth of water in a harbour, d measured in metres, t hours after midnight, can be modelled by the function $d(t) = a\cos(bt) + c$.

The minimum depth of 1.5 meters first occurred at midnight, followed by a maximum of 4.5 metres six hours later.

(a) Sketch how the depth varied over the first 24 hours on the axes below. (3 marks)



(b) Explain, with reasoning, why $a = -1.5$, $b = \frac{\pi}{6}$ and $c = 3$. (3 marks)

c is mean depth of water:
$c = \frac{1.5 + 4.5}{2} = 3$

b adjusts period to 12 hours:
$b = \frac{2\pi}{12} = \frac{\pi}{6}$

a is amplitude of function:
$1.5 = a\cos(0) + 3 \Rightarrow a = -1.5$

(c) For what percentage of a day is the depth of water at least 2.5 metres? (3 marks)

Solve $2.5 = -1.5\cos(\frac{\pi t}{6}) + 3$ to get first solution of $t = 2.351$.

$\frac{6 - 2.351}{6} \times 100 = 60.8\%$

Question 18 **(8 marks)**

A government organisation estimated that the world population was 6,768,167,712 on the first of July 2009 and was 6,774,705,647 one month later on the first of August.

(a) If the population is assumed to be growing exponentially, determine

(i) the monthly percentage growth rate in the population. (2 marks)

$6774705647 \div 6768167712 = 1.000965983$
$0.000965983 \times 100 = 0.0966\%$ per month

(ii) an expression for the population t months after the first of July 2009. (2 marks)

$6768167712(1.000965983)^t$

(b) If this rate of growth continues, determine

(i) the world population on the first of July 2010, to the nearest million. (2 marks)

$6768167712(1.000965983)^{12} = 6847041102$
$\approx 6\ 847$ million

(ii) in which year and month the population of the world was expected to reach 7,000 million. (2 marks)

$6768167712(1.000965983)^t = 7 \times 10^9$
$t = 34.88$ months

During May 2012.

Question 15 **(7 marks)**

A function is given by $f(x)=200+32x^2-x^4$ for $-3\le x\le 5$.

(a) Use calculus techniques to determine the coordinates of both stationary points of the function for the given domain. (4 marks)

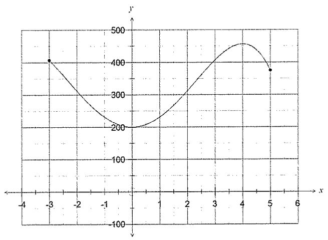
$f'(x)=64x-4x^3$

$64x-4x^3=0$ when $x=-4,0,4$

$f(0)=200$
$f(4)=456$

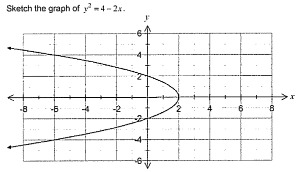
Over given domain, stationary points at (0, 200) and (4, 456)

(b) Sketch the graph of $y=f(x)$ for $-3\le x\le 5$ on the axes below. (3 marks)



Question 16 **(8 marks)**

(a) Sketch the graph of $y^2=4-2x$. (3 marks)



(b) State, with reasons, whether these relationships are also functions:

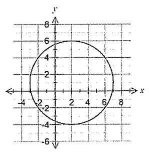
(i) $y^2=4-2x,\ x\ge 0$. (1 mark)

No - graph fails vertical line test.

(ii) $y^2=4-2x,\ y\ge 0$. (1 mark)

Yes - there is a one-to-one mapping.

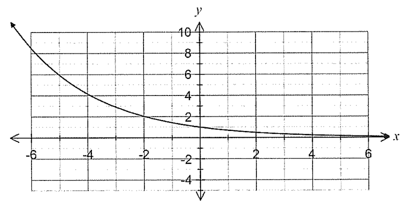
(c) The equation of the following graph is $x^2+y^2+ax+by=c$. Determine the values of a, b and c. (3 marks)



$(x-2)^2+(y-1)^2=5^2$
$x^2+y^2-4x-2y=20$
$a=-4,\ b=-2,\ c=20$

Question 5 **(7 marks)**

(a) The graph of $y=a^x$ is shown below.



On the same axes, sketch the graphs of

(i) $y=a^{x+2}$. (1 mark)

(ii) $y=a^x-3$. (1 mark)

(b) Evaluate $\left(3.6\times10^{-3}\right)\div\left(1.2\times10^{-4}\right)$. (1 mark)

(c) Solve for x:

(i) $27^{2x-1}=81$. (2 marks)

(ii) $x^{-2}=6\frac{1}{4}$. (2 marks)

Question 6 **(5 marks)**

(a) Determine the exact value of $\sin 210°$. (1 mark)

(b) Solve the equation $\cos\left(\frac{1}{2}x\right) = \frac{\sqrt{3}}{2}$ for $-\pi \le x \le \pi$. (2 marks)

(c) Expand $(n-1)^4$. (2 marks)

Question 13 **(9 marks)**

For two events, A and B, $P(A \cap \overline{B}) = 0.3$, $P(\overline{A} \cap \overline{B}) = 0.1$ and $P(B \cap \overline{A}) = x$.

(a) Determine an expression for $P(A \cap B)$ in terms of x. (2 marks)

$$\boxed{\begin{aligned} P(A \cap B) &= 1 - 0.3 - 0.1 - x \\ &= 0.6 - x \end{aligned}}$$

(b) State the maximum possible value of $P(A)$. (1 mark)

$$\boxed{x = 0 \Rightarrow P(A) = 0.9}$$

(c) Determine the value of x under each of the following conditions.

(i) A and B are mutually exclusive. (1 mark)

$$\boxed{P(A \cap B) = 0 \Rightarrow x = 0.6}$$

(ii) $P(A \mid B) = \frac{1}{5}$. (2 marks)

$$\boxed{\begin{aligned} \frac{0.6 - x}{0.6} &= \frac{1}{5} \\ 0.6 - x &= 0.12 \\ x &= 0.48 \end{aligned}}$$

(iii) A is independent of B. (3 marks)

$$\boxed{\begin{aligned} (0.3 + 0.6 - x)(0.6) &= 0.6 - x \\ 0.54 - 0.6x &= 0.6 - x \\ 0.4x &= 0.06 \\ x &= \tfrac{3}{20} = 0.15 \end{aligned}}$$

Question 14 **(9 marks)**

Sequence A is geometric and has n^{th} term ($n \ge 1$) given by $T_n = 5(0.8)^n$.

(a) What is the first term of Sequence A? (1 mark)

$$\boxed{T_1 = 4}$$

(b) How many terms of Sequence A are greater than 1? (1 mark)

$\boxed{\text{7 terms}}$

(c) A student added together the first m terms of Sequence A and obtained a total between 21.9 and 22. Explain why the student must have made a mistake, even though the number m is not known. (2 marks)

> The sum to infinity for this sequence is 20, so impossible for any number of terms to exceed this number.

Sequence B is also geometric with a common ratio of 1.2. The sum of its first two terms is 0.22.

(d) Determine the first term of Sequence B. (2 marks)

$$\boxed{\begin{aligned} a + 1.2a &= 0.22 \\ a &= 0.1 \end{aligned}}$$

(e) How many terms of Sequence B are less than 1? (1 mark)

$\boxed{\text{13 terms.}}$

(f) What is the fewest number of terms of Sequence B that must be summed to obtain a total of at least 100? Justify your answer. (2 marks)

> 30 terms.
>
> $S_{29} \approx 98.4$
>
> $S_{30} \approx 118.2$

Question 11 **(9 marks)**

After the failure of a computer containing the details of 412 clients, Chris was given the job of re-entering all the client information into a new computer. On the first day he managed to re-enter the details of 15 clients. On each subsequent day, he was given more and more time for this job and managed to add two more clients each day than on the previous day.

(a) How many clients did Chris re-enter on the fifth day? (1 mark)

$\boxed{T_5 = 23}$

(b) How many clients had Chris re-entered altogether after 8 days? (1 mark)

$\boxed{S_8 = 176}$

(c) During which day did Chris finish the job and how many clients did Chris add on this day? (2 marks)

$S_{14} = 392$
$T_{15} = 43$
Chris finished on day 15, when 20 new clients were added.

(d) Because of this new job, Chris began to spend less and less time on his usual job of answering client emails. His time spent answering emails followed an arithmetic progression such that on the 6th day of his new job he spent 2 hours 20 minutes answering emails and on the 10th day he spent just 1 hour 36 minutes.

(i) How long did Chris spend answering emails on the first day of his new job? (3 marks)

$T_6 = 140$
$T_{10} = 96$
$4d = -44 \Rightarrow d = -11$
$T_1 = 140 + 55 = 195$ minutes

(ii) After how many days in his new job will Chris have spent a total of at least 30 hours answering emails? (2 marks)

$60 \times 30 = 1800$
$S_{30} = 1800$
After 30 days.

See next page

Question 12 **(6 marks)**

Software has been developed to classify an email message as either good or spam. The software is not perfect: only 88% of spam is classified as such, and 4% of emails that are good are classified as spam.

A large number of emails, 15% of which were spam, were checked by the software.

(a) What is the probability that the software will classify a randomly chosen email as spam? (3 marks)

Let S=Spam email and C=Classified as spam by software
$P(C \cap S) + P(C \cap \bar{S})$
$= 0.15 \times 0.88 + 0.85 \times 0.04$
$= 0.132 + 0.034$
$= 0.166$

(b) Given that the software classifies an email as good, what is the probability that it is actually spam (3 marks)

$P(\bar{C}) = 1 - 0.166$
$= 0.834$

$P(S|\bar{C}) = \dfrac{P(S \cap \bar{C})}{P(\bar{C})}$
$= \dfrac{0.15 \times 0.12}{0.834}$
$= \dfrac{0.018}{0.834}$
$= \dfrac{3}{139} \approx 0.0216$

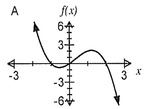
See next page

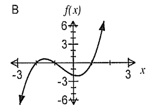
Question 7 **(8 marks)**

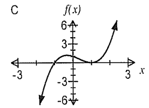
(a) If $(x-2)(x+2)(x+3) = ax^3 + bx^2 + cx + d$, determine the value of c. (2 marks)

(b) Match each function in the table below with its graph. (2 marks)

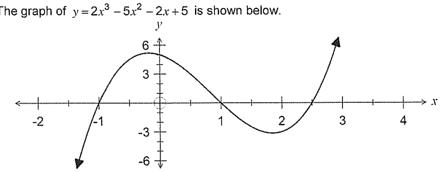
Function	Graph (A, B or C)
$f(x) = (x+1)(x-1)^2$	
$f(x) = x(1+x)(2-x)$	
$f(x) = (x-1)(x+1)(x+2)$	

A


B


C


(c) The graph of $y = 2x^3 - 5x^2 - 2x + 5$ is shown below.



(i) Solve $2x^3 - 2x = 5x^2 - 5$. (2 marks)

(ii) Factorise $2x^3 - 5x^2 - 2x + 5 = 0$. (2 marks)

See next page

Question 8 **(7 marks)**

(a) Calculate the gradient of $y = x^2 - 3x - 10$ at the points where $y = 8$. (3 marks)

(b) The function $f(x) = \frac{x^2}{2}(x-6)$ has a local minimum at (p, q), where $p > 0$.
Determine the values of p and q. (4 marks)

End of questions

SOLUTIONS

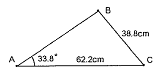
Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(6 marks)**

(a) Calculate the smallest possible length of AB in the triangle shown below. (3 marks)
(The triangle is not drawn to scale).

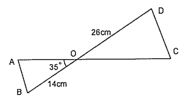


$\frac{\sin(B)}{62.2} = \frac{\sin(33.8)}{38.8} \Rightarrow B = 63.1° \text{ or } 116.9°$

For smallest AB, $C = 180 - 33.8 - 116.9 = 29.3°$

$\frac{AB}{\sin(29.3)} = \frac{38.8}{\sin(33.8)} \Rightarrow AB = 34.1 \text{ cm}$

(b) In the diagram below (not to scale), the line AC intersects the line BD at O. The angle AOB = 35°, and the lengths OB = 14cm, OD = 26cm and AC = 30cm. If the area of triangle ODC is twice that of triangle OAB, determine the length OA. (3 marks)



Let OA = x then OC = $30 - x$

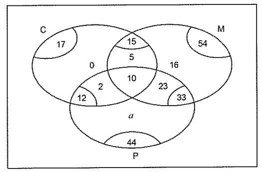
$2\left(\frac{1}{2} \times x \times 14 \times \sin(35)\right) = \frac{1}{2} \times (30 - x) \times 26 \times \sin(35)$

$28x = 26(30 - x)$

$x = 14.\overline{4}$ cm

Question 10 **(5 marks)**

The following Venn diagram shows the numbers of students electing to study at least one of Chemistry (C), Math (M) or Physics (P) in upper school.



(a) Determine the value of a. (1 mark)

$a = 44 - 23 - 10 - 2$
$= 9$

(b) Determine $n(C \cup M \cup P)$. (1 mark)

$n(C \cup M \cup P) = 54 + 0 + 2 + 9$
$= 65$

(c) If one student is selected at random from the group, determine the probability

(i) they elected to study math but not physics. (1 mark)

$P(M \cap \overline{P}) = \frac{16+5}{65} = \frac{21}{65}$

(ii) they elected to study math and physics, given that they did not study chemistry. (1 mark)

$P(M \cap P | \overline{C}) = \frac{23}{65-17} = \frac{23}{48}$

(iii) they elected to study two subjects, given that they did not elect to study all three subjects. (1 mark)

$P = \frac{5+23+2}{65-10} = \frac{30}{55}$

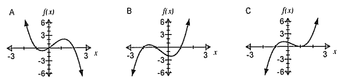
Question 7 **(8 marks)**

(a) If $(x-2)(x+2)(x+3)=ax^3+bx^2+cx+d$, determine the value of c. (2 marks)

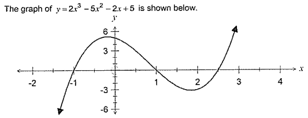
$\boxed{(x^2-4)(x+3)=\ldots-4x-\ldots \Rightarrow c=-4}$

(b) Match each function in the table below with its graph. (2 marks)

Function	Graph (A, B or C)
$f(x)=(x+1)(x-1)^2$	C
$f(x)=x(1+x)(2-x)$	A
$f(x)=(x-1)(x+1)(x+2)$	B



(c) The graph of $y=2x^3-5x^2-2x+5$ is shown below.



(i) Solve $2x^3-2x=5x^2-5$. (2 marks)

$2x^3-5x^2-2x+5=0$
$x=-1,\ x=1,\ x=2.5$

(ii) Factorise $2x^3-5x^2-2x+5=0$. (2 marks)

$\boxed{2(x+1)(x-1)(x-2.5)=(x+1)(x-1)(2x-5)}$

Question 8 **(7 marks)**

(a) Calculate the gradient of $y=x^2-3x-10$ at the points where $y=8$. (3 marks)

$x^2-3x-10=8$
$x^2-3x-18=0$
$(x+3)(x-6)=0$
$x=-3 \Rightarrow \frac{dy}{dx}=-9$
$x=6 \Rightarrow \frac{dy}{dx}=9$

(b) The function $f(x)=\frac{x^2}{2}(x-6)$ has a local minimum at $(p,\ q)$, where $p>0$.
Determine the values of p and q. (4 marks)

$f(x)=\frac{x^3}{2}-3x^2$
$f'(x)=\frac{3x^2}{2}-6x$
$0=x\left(\frac{3x}{2}-6\right) \Rightarrow \cancel{x=0},\ x=4$
$p=4$
$q=\frac{4^2}{2}(4-6)=-16$

End of questions

Christ Church Grammar School

WA Exams Practice Paper A, 2015

Question/Answer Booklet

MATHEMATICS METHODS UNITS 1 AND 2
Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures ☐☐☐☐☐☐☐☐

In words ______________

Your name ______________

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

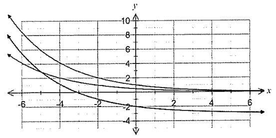
Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Question 5 **(7 marks)**

(a) The graph of $y = a^x$ is shown below.



On the same axes, sketch the graphs of

(i) $y = a^{x+2}$. (1 mark)

(ii) $y = a^x - 3$. (1 mark)

(b) Evaluate $(3.6 \times 10^{-3}) \div (1.2 \times 10^{-4})$. (1 mark)

$$\boxed{\frac{3.6}{1.2} \times \frac{10^{-3}}{10^{-4}} = 3 \times 10 = 30}$$

(c) Solve for x:

(i) $27^{2x-1} = 81$. (2 marks)

$$3^{3(2x-1)} = 3^4$$
$$6x - 3 = 4$$
$$x = \tfrac{7}{6}$$

(ii) $x^{-2} = 6\tfrac{1}{4}$. (2 marks)

$$\boxed{\frac{1}{x^2} = \frac{25}{4} \Rightarrow x^2 = \frac{4}{25} \Rightarrow x = \pm\frac{2}{5}}$$

Question 6 **(5 marks)**

(a) Determine the exact value of $\sin 210°$. (1 mark)

$$\boxed{-\frac{1}{2}}$$

(b) Solve the equation $\cos\left(\tfrac{1}{2}x\right) = \tfrac{\sqrt{3}}{2}$ for $-\pi \le x \le \pi$. (2 marks)

$$\frac{x}{2} = \pm\frac{\pi}{6}$$
$$x = \pm\frac{\pi}{3}$$

(c) Expand $(n-1)^4$. (2 marks)

$$(n-1)^4 = n^4 + 4n^3(-1) + 6n^2(-1)^2 + 4n(-1)^3 + (-1)^4$$
$$= n^4 - 4n^3 + 6n^2 - 4n + 1$$

Question 3 **(5 marks)**

Solve the following equations.

(a) $x(x+2)(2x-3)=0$. (1 mark)

$$\boxed{x=0,\ x=-2,\ x=\tfrac{3}{2}}$$

(b) $x^2+4x+6=2x^2+5x-6$. (2 marks)

$$\boxed{\begin{aligned} 0 &= x^2+x-12 \\ (x+4)(x-3) &= 0 \\ x &= -4,\ x=3 \end{aligned}}$$

(c) $2(x-2)^2=100$. (2 marks)

$$\boxed{\begin{aligned} (x-2)^2 &= 50 \\ x-2 &= \pm 5\sqrt{2} \\ x &= 2+5\sqrt{2},\ x=2-5\sqrt{2} \end{aligned}}$$

Question 4 **(7 marks)**

(a) Determine $\frac{dy}{dx}$ in simplified form if

(i) $y=2x^3-x+3$. (1 mark)

$$\boxed{\frac{dy}{dx}=6x^2-1}$$

(ii) $y=\frac{5x^3}{6}-\frac{x^4}{12}$. (1 mark)

$$\boxed{\frac{dy}{dx}=\frac{5x^2}{2}-\frac{x^3}{3}}$$

(b) Determine the coordinates of the point on the curve $y=3x^2-7x-10$ where the gradient is 5. (2 marks)

$$\boxed{\begin{aligned} &\frac{dy}{dx}=6x-7 \\ &6x-7=5 \Rightarrow x=2 \\ &y=3(4)-7(2)-10=-12 \\ &\text{At } (2,-12) \end{aligned}}$$

(c) Determine $f(x)$ given that $f'(x)=5+2x-6x^2$ and $f(1)=0$. (3 marks)

$$\boxed{\begin{aligned} &f(x)=5x+x^2-2x^3+c \\ &c=0-5-1^2+2=-4 \\ &f(x)=5x+x^2-2x^3-4 \end{aligned}}$$

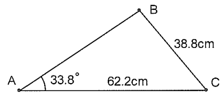
Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

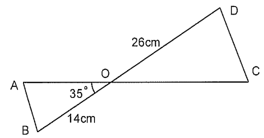
Working time for this section is 100 minutes.

Question 9 **(6 marks)**

(a) Calculate the smallest possible length of AB in the triangle shown below. (3 marks)
(The triangle is not drawn to scale).

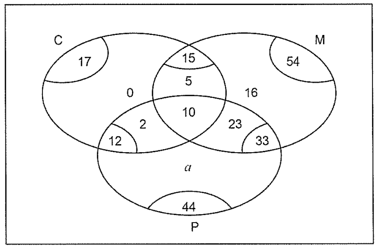


(b) In the diagram below (not to scale), the line AC intersects the line BD at O. The angle AOB = 35°, and the lengths OB = 14cm, OD = 26cm and AC = 30cm. If the area of triangle ODC is twice that of triangle OAB, determine the length OA. (3 marks)



Question 10 **(5 marks)**

The following Venn diagram shows the numbers of students electing to study at least one of Chemistry (C), Math (M) or Physics (P) in upper school.



(a) Determine the value of a. (1 mark)

(b) Determine $n(C \cup M \cup P)$. (1 mark)

(c) If one student is selected at random from the group, determine the probability

(i) they elected to study math but not physics. (1 mark)

(ii) they elected to study math and physics, given that they did not study chemistry. (1 mark)

(iii) they elected to study two subjects, given that they did not elect to study all three subjects. (1 mark)

SOLUTIONS

Section One: Calculator-free **(52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(6 marks)**

(a) The table shows the temperature of a liquid over a period of time.

Time (minutes)	0	5	10	15	20	25
Temperature (°C)	58	44	32	25	21	19

Determine the average rate of change of temperature of the liquid

(i) over the first ten minutes. (1 mark)

$$\boxed{\frac{32-58}{10-0}=-\frac{26}{10}=-2.6 \text{ °C/min}}$$

(ii) between 15 and 20 minutes. (1 mark)

$$\boxed{\frac{21-25}{20-15}=-\frac{4}{5}=-0.8 \text{ °C/min}}$$

(b) Determine the equation of the tangent to the curve $y=x^3-2x^2+x+2$ when $x=2$. (4 marks)

$$x^3-2x^2+x+2\Big|_{x=2}=4$$

$$\frac{dy}{dx}=3x^2-4x+1\Big|_{x=2}=5$$

$$y-4=5(x-2)$$
$$y=5x-6$$

Question 2 **(7 marks)**

(a) The vertices of three points are A(1, 1), B(-1, 2) and C(-2, -1).

(i) Use gradients to explain whether the lines AB and BC are perpendicular. (2 marks)

$m_{AB}=-\frac{1}{2}$ $\quad m_{BC}=3$ $\quad -\frac{1}{2}\times 3=-\frac{3}{2}$

No, since perpendicular gradients have a product of -1.

(ii) Determine the equation of the line through A that is parallel to the line BC. (1 mark)

$$y=3x+c$$
$$1=3(1)+c$$
$$c=-2 \qquad \therefore y=3x-2$$

(iii) If B is the mid-point of A and D, determine the coordinates of D. (2 marks)

$$D(-1-2,\ 2+1)$$

$$D(-3,\ 3)$$

(b) Solve $\frac{x-3}{3}-3x=4$. (2 marks)

$$x-3-9x=12$$
$$-8x=15$$
$$x=-\frac{15}{8}$$

Question 21 **(6 marks)**

The circumference of a circle of radius 25cm passes through the centre of a circle of radius 15cm. Find the area of intersection of the two circles.

End of questions

Question 11 **(9 marks)**

After the failure of a computer containing the details of 412 clients, Chris was given the job of re-entering all the client information into a new computer. On the first day he managed to re-enter the details of 15 clients. On each subsequent day, he was given more and more time for this job and managed to add two more clients each day than on the previous day.

(a) How many clients did Chris re-enter on the fifth day? (1 mark)

(b) How many clients had Chris re-entered altogether after 8 days? (1 mark)

(c) During which day did Chris finish the job and how many clients did Chris add on this day? (2 marks)

(d) Because of this new job, Chris began to spend less and less time on his usual job of answering client emails. His time spent answering emails followed an arithmetic progression such that on the 6^{th} day of his new job he spent 2 hours 20 minutes answering emails and on the 10^{th} day he spent just 1 hour 36 minutes.

(i) How long did Chris spend answering emails on the first day of his new job? (3 marks)

(ii) After how many days in his new job will Chris have spent a total of at least 30 hours answering emails? (2 marks)

Question 12 **(6 marks)**

Software has been developed to classify an email message as either good or spam. The software is not perfect: only 88% of spam is classified as such, and 4% of emails that are good are classified as spam.

A large number of emails, 15% of which were spam, were checked by the software.

(a) What is the probability that the software will classify a randomly chosen email as spam? (3 marks)

(b) Given that the software classifies an email as good, what is the probability that it is actually spam. (3 marks)

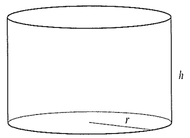
(b) The displacement, x m, of a particle from a fixed point O is given by $x = 2t^3 - 3t^2 - 12t + 1$, $t \geq 0$, where t is the time, in seconds.

(i) Determine the initial velocity of the particle. (2 marks)

(ii) Determine the displacement of the particle at the instant it is stationary. (2 marks)

Question 20 **(10 marks)**

(a) A cylinder is such that the sum of the height and three times the radius is 50 cm.



(i) Write an equation for the height, h, in terms of the radius, r, for this cylinder. (1 mark)

(ii) Show that the total surface area of the cylinder is given by $A = 100\pi r - 4\pi r^2$. (2 marks)

(iii) Using calculus techniques, determine the dimensions of this cylinder to obtain the maximum possible surface area, and state this area. (3 marks)

Question 13 **(9 marks)**

For two events, A and B, $P(A \cap \overline{B}) = 0.3$, $P(\overline{A} \cap \overline{B}) = 0.1$ and $P(B \cap \overline{A}) = x$.

(a) Determine an expression for $P(A \cap B)$ in terms of x. (2 marks)

(b) State the maximum possible value of $P(A)$. (1 mark)

(c) Determine the value of x under each of the following conditions.

(i) A and B are mutually exclusive. (1 mark)

(ii) $P(A \mid B) = \frac{1}{5}$. (2 marks)

(iii) A is independent of B. (3 marks)

Question 14 **(9 marks)**

Sequence A is geometric and has n^{th} term ($n \geq 1$) given by $T_n = 5(0.8)^n$.

(a) What is the first term of Sequence A? (1 mark)

(b) How many terms of Sequence A are greater than 1? (1 mark)

(c) A student added together the first m terms of Sequence A and obtained a total between 21.9 and 22. Explain why the student must have made a mistake, even though the number m is not known. (2 marks)

Sequence B is also geometric with a common ratio of 1.2. The sum of its first two terms is 0.22.

(d) Determine the first term of Sequence B. (2 marks)

(e) How many terms of Sequence B are less than 1? (1 mark)

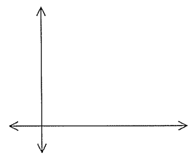
(f) What is the fewest number of terms of Sequence B that must be summed to obtain a total of at least 100? Justify your answer. (2 marks)

Question 19 **(6 marks)**

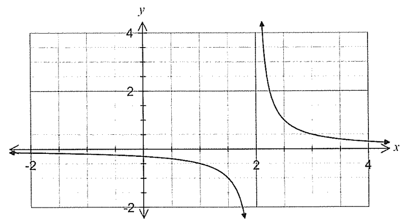
(a) The quantity P is directly proportional to the quantity T, and it is known that when $T = 12$, $P = 60$.

(i) Determine an equation for the relationship between P and T. (2 marks)

(ii) Sketch a graph of the relationship between P and T. (2 marks)



(b) The graph below shows $y = \dfrac{a}{x-b}$, where a and b are constants.



Determine the values of a and b. (2 marks)

Question 18 **(8 marks)**

A government organisation estimated that the world population was 6,768,167,712 on the first of July 2009 and was 6,774,705,647 one month later on the first of August.

(a) If the population is assumed to be growing exponentially, determine

(i) the monthly percentage growth rate in the population. (2 marks)

(ii) an expression for the population t months after the first of July 2009. (2 marks)

(b) If this rate of growth continues, determine

(i) the world population on the first of July 2010, to the nearest million. (2 marks)

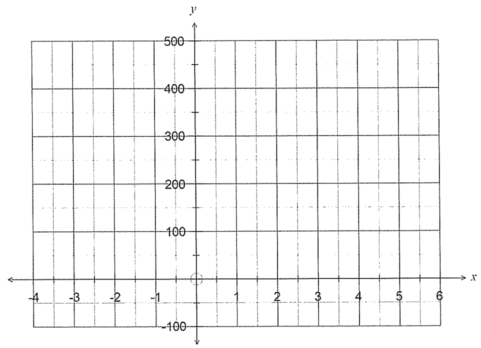
(ii) in which year and month the population of the world was expected to reach 7,000 million. (2 marks)

Question 15 **(7 marks)**

A function is given by $f(x) = 200 + 32x^2 - x^4$ for $-3 \le x \le 5$.

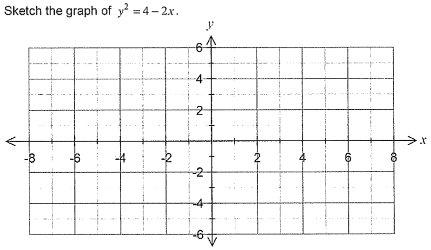
(a) Use calculus techniques to determine the coordinates of both stationary points of the function for the given domain. (4 marks)

(b) Sketch the graph of $y = f(x)$ for $-3 \le x \le 5$ on the axes below. (3 marks)



Question 16 **(8 marks)**

(a) Sketch the graph of $y^2 = 4 - 2x$. (3 marks)

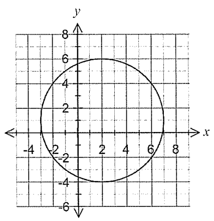


(b) State, with reasons, whether these relationships are also functions:

(i) $y^2 = 4 - 2x,\ x \geq 0$. (1 mark)

(ii) $y^2 = 4 - 2x,\ y \geq 0$. (1 mark)

(c) The equation of the following graph is $x^2 + y^2 + ax + by = c$. Determine the values of a, b and c. (3 marks)

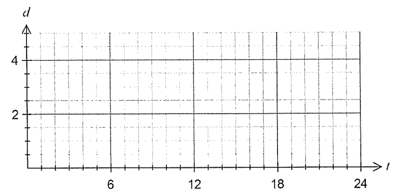


Question 17 **(9 marks)**

The depth of water in a harbour, d measured in metres, t hours after midnight, can be modelled by the function $d(t) = a\cos(bt) + c$.

The minimum depth of 1.5 meters first occurred at midnight, followed by a maximum of 4.5 metres six hours later.

(a) Sketch how the depth varied over the first 24 hours on the axes below. (3 marks)



(b) Explain, with reasoning, why $a = -1.5$, $b = \frac{\pi}{6}$ and $c = 3$. (3 marks)

(c) For what percentage of a day is the depth of water at least 2.5 metres? (3 marks)