PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

Independent Public School

Course ________Specialist______ Year ____12______

Student name: _________________ Teacher name: _________________

Task type: **Response**

Time allowed for this task: ____40_______ mins

Number of questions: _____7______

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: ___38___ marks

Task weighting: _10___%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2, 2 & 3 = 7 marks) (3.1.1 to 3.1.3)

If $z=3-4i$ & $w=-1+2i$ determine the following.

a) $w\bar{z}$

Solution
$(-1+2i)(3+4i)=-11+2i$
Specific behaviours
✓ shows conjugate of z ✓ obtains result

b) $\dfrac{z}{w}$

Solution
$\dfrac{3-4i}{-1+2i}\times\dfrac{-1-2i}{-1-2i}=\dfrac{-11-2i}{5}$
Specific behaviours
✓ uses conjugate ✓ obtains simplified result

c) $\dfrac{1}{z}-\dfrac{1}{w}$

Solution
$\dfrac{1}{3-4i}\times\dfrac{3+4i}{3+4i}=\dfrac{3+4i}{25}$ $\dfrac{1}{-1+2i}\times\dfrac{-1-2i}{-1-2i}=\dfrac{-1-2i}{5}=\dfrac{-5-10i}{25}$ $\dfrac{3+4i}{25}+\dfrac{5+10i}{25}=\dfrac{8+14i}{25}$
Specific behaviours
✓ expresses one fraction with real denominator showing use of conjugates ✓ expresses both fractions with real denominators showing use of conjugates ✓ simplified result (answer only one mark)

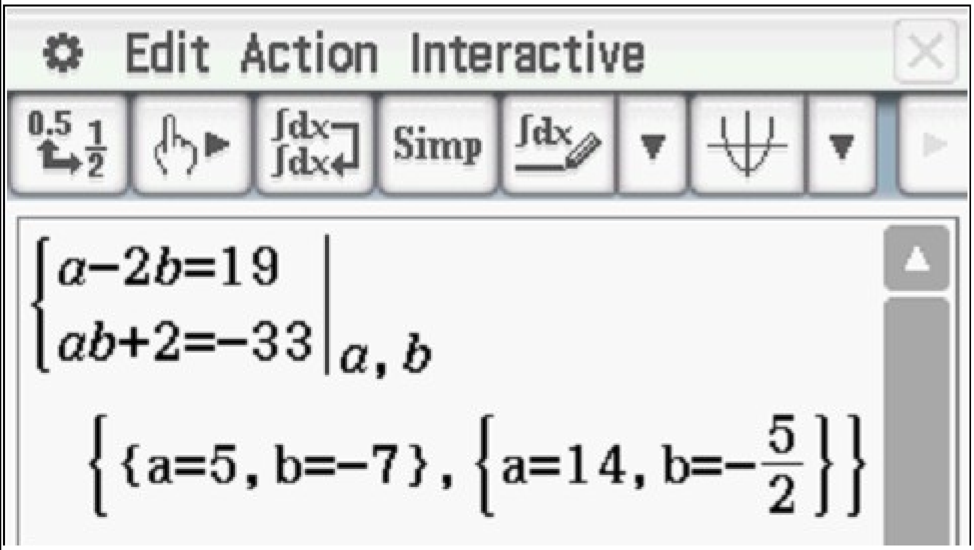
Q2 (3 marks) (3.1.2)

Determine all possible pairs of real numbers $a \& b$ such that $\dfrac{19-33i}{a+2i}=1+bi$

Solution

$$\frac{19-33i}{a+2i}=1+bi$$

$$19-33i=(1+bi)(a+2i)=a-2b+i(ab+2)$$



Specific behaviours

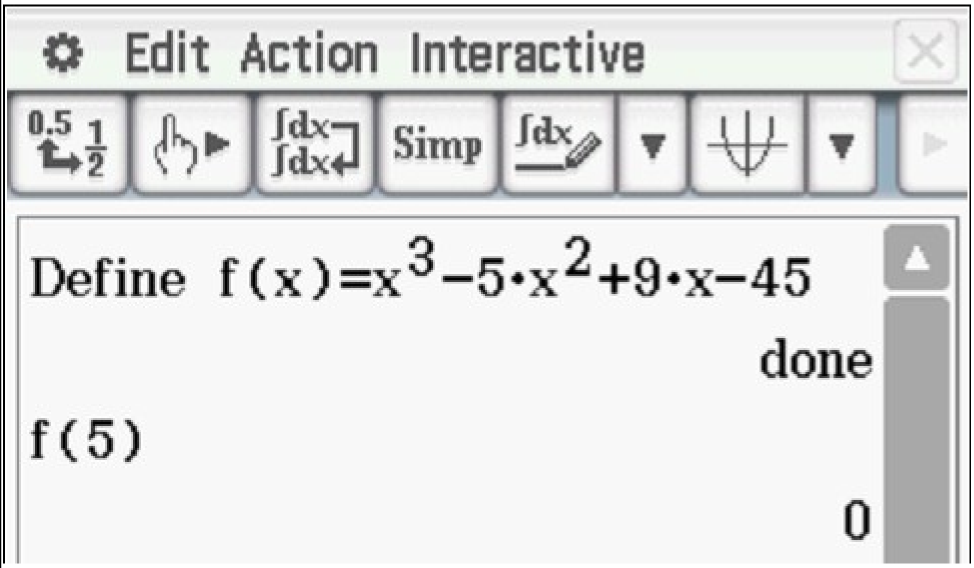
✓ obtains one equation for a & b
✓ states two simultaneous equations and solves for at least one pair
✓ states two pairs of values

Q3 (2 & 3 = 5 marks) (3.1.13- 3.1.15)

Consider the function $f(x)=x^3-5x^2+9x-45$.

a) Determine the remainder of $f(x)$ when divided by $x-5$.

b) Solution



Specific behaviours

✓ subs x=5

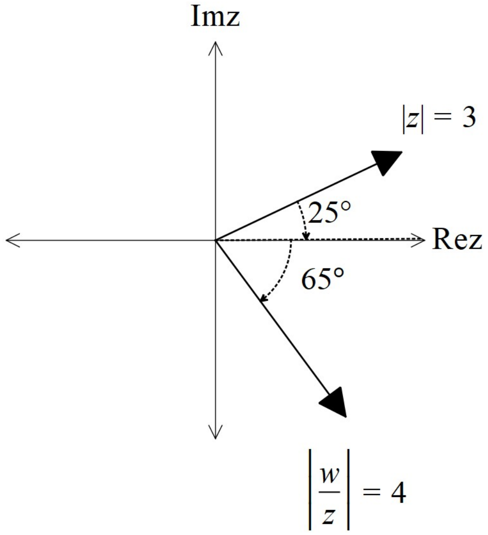
✓ states zero remainder

c) Show that $x - 3i$ is a factor of $f(x)$ and hence determine all linear factors.

Solution
$(3i)^3 - 5(3i)^2 + 9(3i) - 45 = -27i + 45 + 27i - 45 = 0$ $(x - 3i)(x + 3i)(x - 5)$
Specific behaviours
✓ subs x=3i and shows the result of **each** term with the sum being zero ✓ uses conjugate root stating two complex linear factors ✓ states all 3 linear factors

Q4 (3 marks) (3.1.9)

Determine the complex number w in the form $rcis\theta$ with $r \geq 0$ & $-180 < \theta \leq 180$.



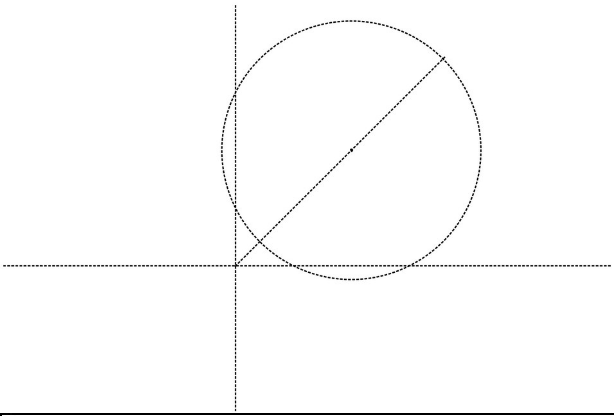
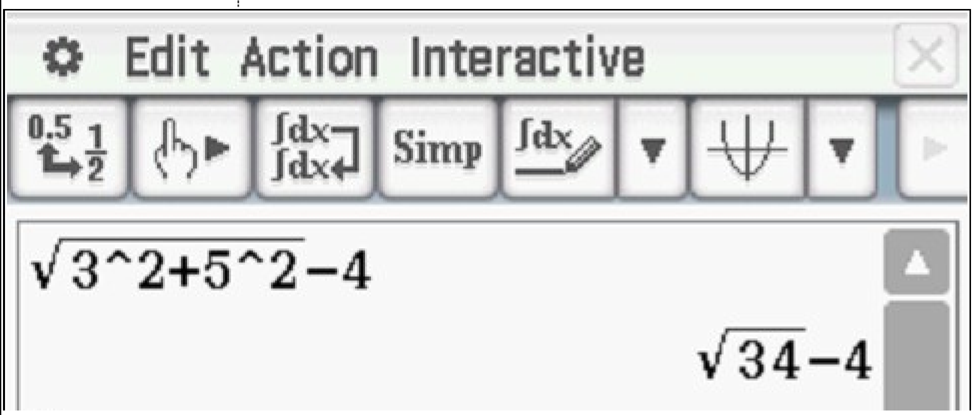
Solution
$Arg w - 25 = -65$ $Arg w = -40$ $\frac{\|w\|}{3} = 4$ $\|w\| = 12$ $w = 12cis(-40)$

Specific behaviours
✓ determines argument with working ✓ states modulus ✓ states in polar form with principal argument

Q5 (2, 2, 3 & 3 = 10 marks) (3.1.10)

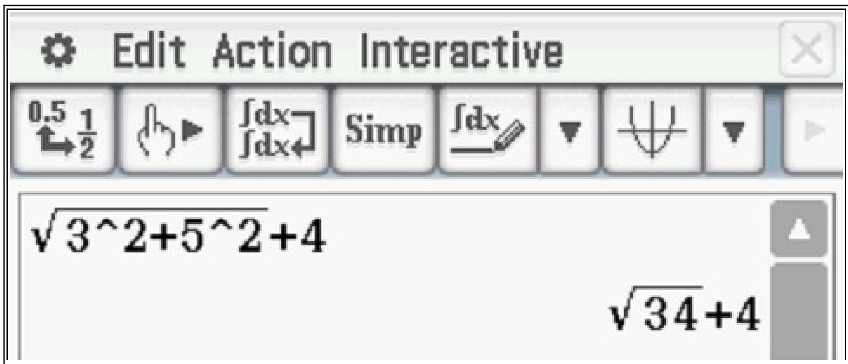
Consider the following set of complex numbers z such that $|z-5-3i|=4$.
Determine the following.

a) Minimum value of $|z|$.(exact)

Solution
 
Specific behaviours
✓ uses modulus of centre ✓ states exact value

b) Maximum value of $|\overline{z}|$.

Solution

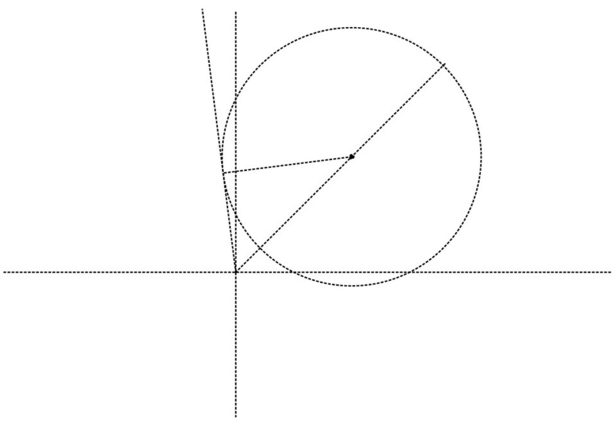


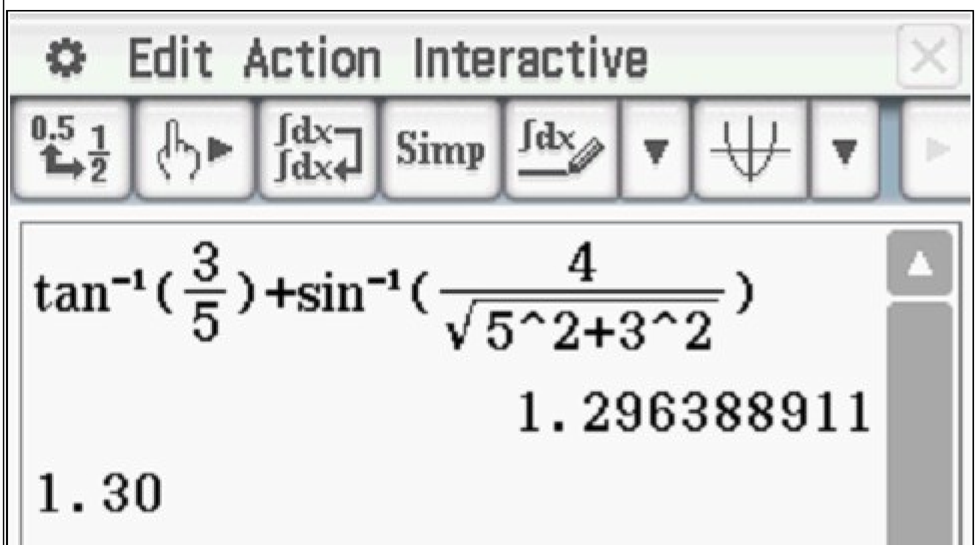
Specific behaviours

✓ uses modulus of centre and ignores conjugate
✓ states exact value

c) Maximum value of Arg(z) in radians to two decimal places.

Solution



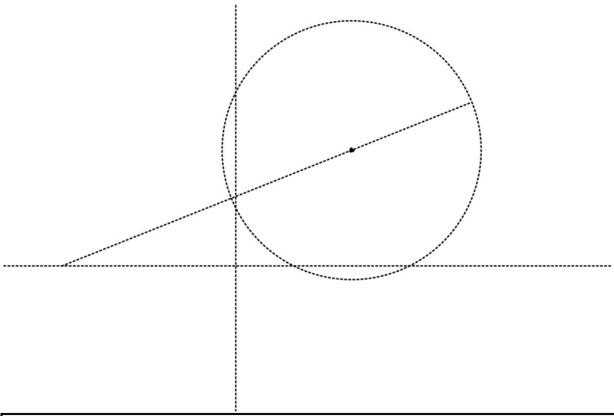


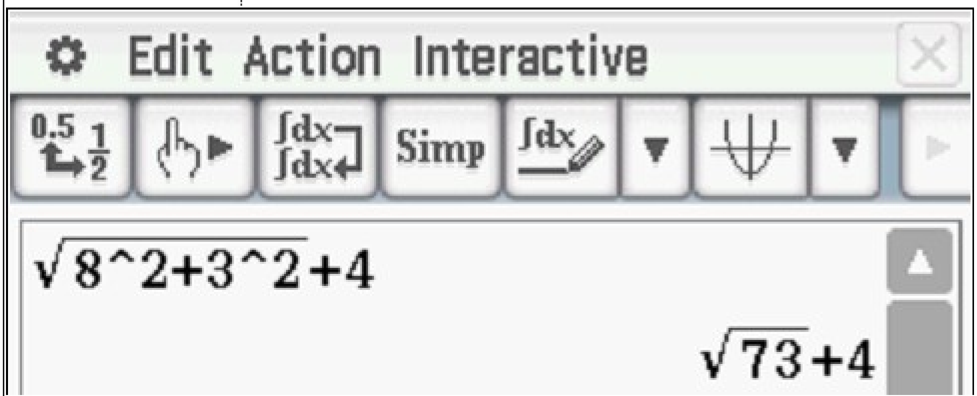
Specific behaviours

✓ uses tangent line and finds argument of centre ✓ uses inverse sine to find added argument to tangent ✓ states argument rounded to 2 dp radians

d) Maximum value of $|z+3|$ (exact)

Solution





Specific behaviours

✓ use distance from -3 on real axis
✓ determine distance to centre from -3
✓ adds radius to give maximum distance

Q6 (3 & 3 = 6 marks) (3.1.6)

Let $p, q \& s$ be complex numbers such that

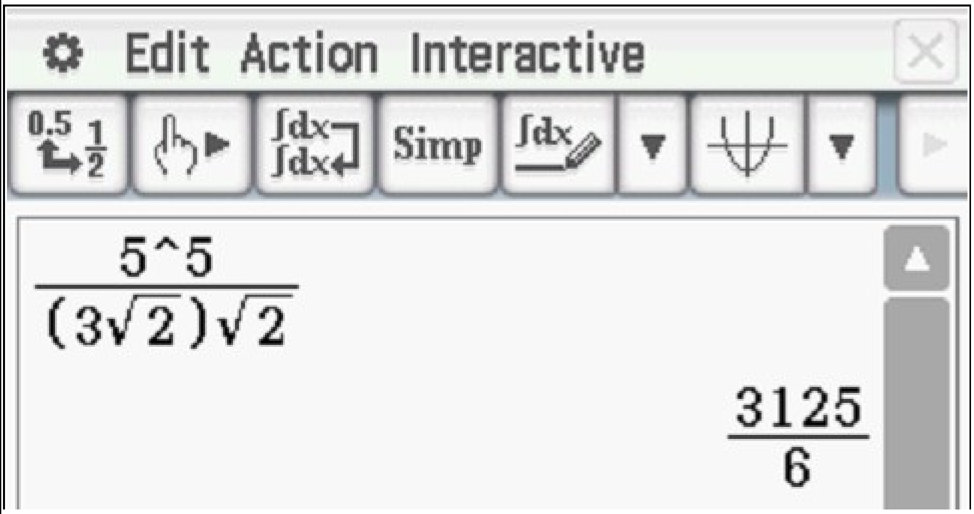
$|p| = 5 \quad Arg(p) = \frac{\pi}{6} \quad \bar{q} = 1 - i$

$s = \frac{p^5}{(3+3i)q}$

a) Determine the exact value of Arg(s) in principal form (i.e $-\pi < Arg(s) \leq \pi$)

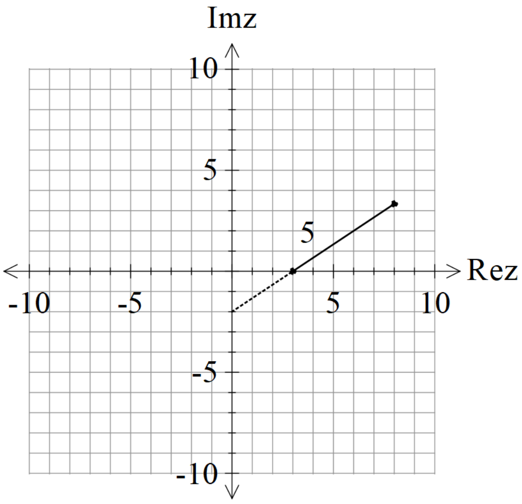
Solution
$\frac{5\pi}{6} - \frac{\pi}{4} - \frac{\pi}{4} = \frac{\pi}{3}$
Specific behaviours
✓ multiplies Arg(p) by 5 ✓ determines argument of q ✓ determines final principal argument of s

b) Determine the exact value of $|s|$

Solution

Specific behaviours
✓ raises mod of p to power of 5 ✓ determines mod of both terms in denominator ✓ determines exact result

Q7 (4 marks) (3.1.10)

Sketch the locus of complex numbers that satisfy **both** of the following $|z+2i| = |z-3| + \sqrt{13}$ **AND** $|z+2i| \leq \sqrt{13} + 5$ in the Argand diagram below.

Solution

Specific behaviours
✓ uses line that when extended passes through -2 on imaginary axis (dotted) ✓ has line passing through 3 on real axis ✓ only allows part of line above real axis ✓ shows that line only has a length of 5 units