Mercedes College

WA Exams Practice Paper D, 2016

Question/Answer Booklet

MATHEMATICS METHODS UNIT 3

Section One: Calculator-free

SOLUTIONS

Student Number: In figures

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	12	12	100	98	65
			Total	151	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

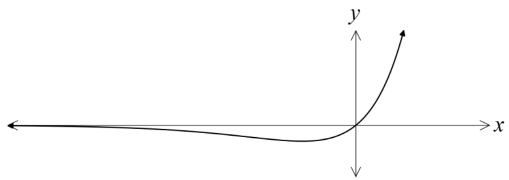
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Additional working space

Question number: _________

Question 8 **(9 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = xe^x$.



(a) Determine the exact location of the stationary point on the graph of $y = f(x)$. (3 marks)

$$f'(x) = 1\times e^x + xe^x$$
$$= e^x(1+x)$$

$$e^x(1+x) = 0 \Rightarrow x = -1$$

$$f(-1) = -1e^{-1} = -\frac{1}{e} \Rightarrow \left(-1,\ -\frac{1}{e}\right)$$

(b) Apply the second derivative test to show that the stationary point in (a) is a minimum. (3 marks)

$$f''(x) = e^x + 1\times e^x + xe^x$$

$$f''(-1) = e^{-1} + e^{-1} - e^{-1} = \frac{1}{e}$$

$$f''(-1) > 0 \Rightarrow \text{minimum}$$

(c) The graph of $y = f(x)$ has just one point of inflection. Determine the exact coordinates of this point. (3 marks)

$$f''(x) = e^x(2+x)$$

$$e^x(2+x) = 0 \Rightarrow x = -2$$

$$f(-2) = -\frac{2}{e^2} \Rightarrow \left(-2,\ -\frac{2}{e^2}\right)$$

End of questions

Section One: Calculator-free **35% (53 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

The gradient function of a curve is given by $\frac{dQ}{dt} = at - 2t^2$, where a is a constant. Determine the equation of the curve if it has a maximum when $t = 3$ and a zero when $t = 1$.

$$t = 3,\ \frac{dQ}{dt} = 0 \Rightarrow t(a-2t) = 0$$

$$3(a-6) = 0 \Rightarrow a = 6$$

$$Q = \int 6t - 2t^2\,dt$$
$$= 3t^2 - \tfrac{2}{3}t^3 + c$$

$$t = 1,\ Q = 0 \Rightarrow 3 - \tfrac{2}{3} + c = 0$$
$$c = -\tfrac{7}{3}$$

$$Q = 3t^2 - \tfrac{2}{3}t^3 - \tfrac{7}{3}$$

See next page

Question 2 **(8 marks)**

Differentiate the following with respect to x, simplifying your answers.

(a) $y = \cos^3(1-2x)$. (3 marks)

$$y = (\cos(1-2x))^3$$

$$\frac{dy}{dx} = (3)(-2)(-\sin(1-2x))(\cos(1-2x))^2$$

$$= 6\sin(1-2x)\cos^2(1-2x)$$

(b) $y = \dfrac{\sin(2x)}{e^{3x}}$. (3 marks)

$$\frac{dy}{dx} = \frac{2\cos(2x)\times e^{3x} - \sin(2x)\times 3e^{3x}}{\left(e^{3x}\right)^2}$$

$$= \frac{2\cos(2x) - 3\sin(2x)}{e^{3x}}$$

(c) $\int_x^2 \left(3-t^2\right) dt$. (2 marks)

$$\frac{d}{dx}\int_x^2 \left(3-t^2\right) dt = -\frac{d}{dx}\int_2^x \left(3-t^2\right) dt$$

$$= x^2 - 3$$

Question 7 **(6 marks)**

Calculate the area bounded by the functions $f(x) = (x-3)^2 - 2$ and $g(x) = 4 - 2x$.

Solve $f(x) = g(x)$

$$x^2 - 6x + 9 - 2 + 2x - 4 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$x = 1,\ x = 3$$

Integrate to find area

$$\int_1^3 g(x) - f(x)\ dx = -\int_1^3 f(x) - g(x)\ dx$$

$$= -\int_1^3 \left(x^2 - 4x + 3\right)\ dx$$

$$= -\left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^3$$

$$= -(9 - 18 + 9) + \left(\frac{1}{3} - 2 + 3\right)$$

$$= \frac{4}{3} \text{ square units}$$

Question 6 **(5 marks)**

A random sample of n people are selected from a large population of which the proportion p are known to have had a dental check-up in the last year.

It is known that the mean and standard deviation of the random variable X are 12 and 3 respectively, where X is the number of people in the sample who have had a check-up in the last year.

(a) Name the distribution of X. (1 mark)

Binomial with parameters n and p.

Or $X \sim B(n, p)$

(b) Determine n and p. (4 marks)

$$np = 12$$

$$np(1-p) = 3^2$$
$$12(1-p) = 9$$

$$1-p = \frac{3}{4} \Rightarrow p = \frac{1}{4}$$

$$n = 12 \div \frac{1}{4} = 48$$

Question 3 **(5 marks)**

(a) State the following limits.

(i) $\lim_{h\to 0}\left(\frac{e^h - 1}{h}\right)$. (1 mark)

$\boxed{e}$

(ii) $\lim_{h\to 0}\left(\frac{\cos(x+h)-\cos(x)}{h}\right)$ (1 mark)

$\boxed{-\sin x}$

(b) If $x = 3\cos\left(\frac{t}{2}+1\right)$, show that $\frac{d^2x}{dt^2} = ax$ and state the value of a. (3 marks)

$$x = 3\cos\left(\frac{t}{2}+1\right)$$
$$\frac{dx}{dt} = \left(-\frac{1}{2}\right)\sin\left(\frac{t}{2}+1\right)$$

$$\frac{d^2x}{dt^2} = \left(-\frac{1}{4}\right)3\cos\left(\frac{t}{2}+1\right)$$

$$\frac{d^2x}{dt^2} = a\times x \Rightarrow a = -\frac{1}{4}$$

Question 4 **(8 marks)**

(a) Determine $\int\left(6\sqrt{x}+e^{6x}\right)\,dx$, simplifying your answer. (3 marks)

$$\int\left(6\sqrt{x}+e^{6x}\right)\,dx=\int\left(6x^{\frac{1}{2}}+e^{6x}\right)\,dx$$
$$=6\times\frac{2}{3}\times x^{\frac{3}{2}}+\frac{1}{6}e^{6x}+c$$
$$=4\sqrt{x^3}+\frac{e^{6x}}{6}+c$$

(b) Evaluate $\int_0^2 2\cos\left(\pi-\frac{\pi x}{4}\right)\,dx$. (3 marks)

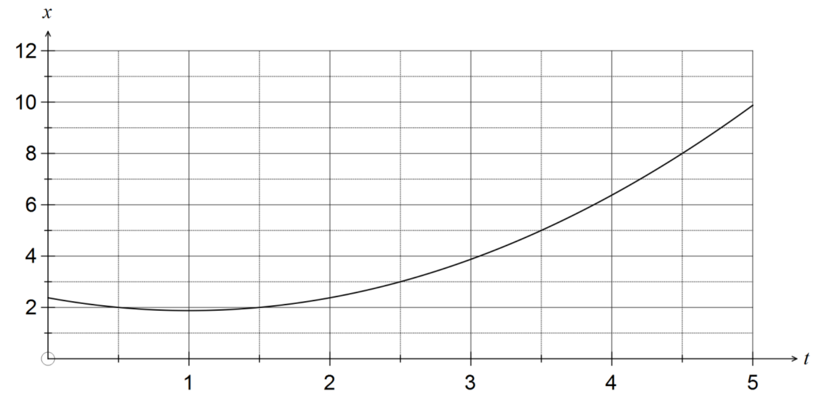
$$\int_0^2 2\cos\left(\pi-\frac{\pi x}{4}\right)\,dx=\left[-\frac{4}{\pi}\times 2\sin\left(\pi-\frac{\pi x}{4}\right)\right]_0^2$$
$$=-\frac{8}{\pi}\sin\frac{\pi}{2}--\frac{8}{\pi}\sin\pi$$
$$=-\frac{8}{\pi}$$

(c) Write, if possible, the three integrals below as a single integral. If not possible, explain why.
$\int_1^4 f(x)\,dx+\int_1^2 f(x)\,dx+\int_2^4 f(x)\,dx$. (2 marks)

$$I=\int_1^4 f(x)\,dx+\int_1^2 f(x)\,dx+\int_2^4 f(x)\,dx$$
$$=\int_1^4 f(x)\,dx+\int_1^4 f(x)\,dx$$
$$=2\int_1^4 f(x)\,dx$$

Question 5 **(7 marks)**

A toy car travels along a straight path on level ground so that its displacement, x metres, relative to a fixed point O, is shown on the graph below for the interval $0\le t\le 5$ seconds.



(a) Use the graph to complete the table below. (2 marks)

t	0.5	1.5	2.5	3.5	4.5
Displacement, x (m)	2	**2**	**3**	**5**	**8**

The area under the graph can be interpreted as the total distance travelled by the car during the first five seconds.

(b) Estimate the area under the graph for the interval $0\le t\le 5$ seconds using five centred rectangles of equal widths, and hence state the distance travelled by the toy car in this time. (2 marks)

$A=1\times(2+2+3+5+8)$
$=20$

Car has travelled 20 m.

(c) State, with reasons, whether your estimate in (b) is larger or smaller than the actual distance travelled by the car. (2 marks)

Smaller - each rectangle is slightly under-estimating the area as the curve is concave up.

(d) Suggest a way to improve the accuracy of this estimation method. (1 mark)

Use more than 5 rectangles by decreasing their widths.