



PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2023
Question/Answer booklet

MATHEMATICS METHODS

UNIT 3

Section Two:
Calculator-assumed

Your Name: _____
Your Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

The calculator/assumed

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items are to be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Question	Marks	Max	Question	Marks	Max
7	8	13			12
8	6	14			11
9	12	15			8
10	10				
11	10				
12	13				

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	51	36
Section Two: Calculator-assumed	10	10	100	90	64
Total					100

Additional working space

Question number: _____

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be allocated for correct answers given without supporting reasoning. You cannot be awarded any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

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Question 8

A barrel is filled with 34 balls numbered with the integers 1,2,3,...,33,34, but otherwise identical.

Let the random variable X be the number on a ball drawn at random from the barrel.

- (a) Explain why X has a uniform distribution. (1 mark)

- (b) Determine the expected value of X . (1 mark)

Let the random variable Y take the value 1 when $X < 10$ and the value 0 otherwise.

- (c) State the particular name given to two-outcome random variables such as Y . (1 mark)

- (d) Determine $P(Y=1)$. (1 mark)

- (e) Three balls are drawn at random from the barrel. Determine the probability that exactly two of the balls are marked with single digit numbers. (2 marks)

(6 marks)

Additional working space

Question number: _____

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The student rolls the two dice, and then writes down the following fractions. Add them up. The student who gets the highest total wins.

Mathematics teacher gives a student two six-sided dice. One die is coloured red and the other is coloured blue.

Initially a barrel took 300 litres. Water is pumped into the tank at the rate of $W(t) = 55 - 15\cos\left(\frac{\pi}{12}t\right)$ litres/hour.

At the same time 50 litres/hour is pumped from the tank.

Water is pumped into the tank at the rate of $R(t) = 12$ litres/hour.

(a) Show that the probability of getting a fraction less than 1 is $\frac{1}{2}$.

(b) Is the level of water rising or falling when $t = 67$ hours?

Question 11 **(10 marks)**

Fredeon F	$P(F < 1)$	$P(1 \leq F < 2)$	$P(2 \leq F < 3)$	$P(3 \leq F < 4)$	$P(4 \leq F < 5)$	$P(F \geq 5)$
Fredeon F	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$

Number of red die	Probability $(F = f)$	Completing the table, providing the missing probabilities
Number of red die	Probability $(F = f)$	Completing the table, providing the missing probabilities

Given that the probability is less than 3, determine the radius of a random ball is at least 2.	(2 marks)
Determine the radius of a student's ball is at least 2.	(2 marks)
Given that the probability is less than 3, a student draws a random ball is at least 2.	(2 marks)

(b) Calculate $P(X \geq X_4)$.

(c) Determine the expected value of X and interpret your answer in the context of this question.

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(c) Determine the expected value of X and interpret your answer in the context of this question. (2 marks)

(d) Calculate $P(X \geq X_3)$. (3 marks)

(e) Use your result from part (a) to show that $\int_{-\infty}^{\infty} x^2 dx = c^2 + \frac{2}{3}$, where c is a constant. (3 marks)

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(f) Use your result from part (g) to show that $\int_{-\infty}^{\infty} x^2 dx = c^2 + \frac{2}{3}$, where c is a constant. (3 marks)

(g) Determine the expected value of X and interpret your answer in the context of this question. (2 marks)

(h) three damaged tyres. (1 mark)

(i) more than two damaged tyres. (1 mark)

(j) Determine the probability that three passengers (1 mark)



(a) Use the quotient rule to show that $\int_{-\infty}^{\infty} x^2 dx = c^2 + \frac{2}{3}$. (3 marks)

(b) Use the quotient rule to show that $\int_{-\infty}^{\infty} x^2 dx = c^2 + \frac{2}{3}$. (3 marks)

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(c) Use calculus to determine and justify the maximum height of the inselberg above the surrounding plain. (7 marks)

(d) Calculate the Standard deviation of X . (2 marks)

(e) The factory owner decides to introduce a points scheme using the rule $Y = 20 - 5X$. Find the points Jake will expect to receive and state the variance of his points score. (Give your answers to the nearest unit) (2 marks)

(f) Use your result from part (g) to show that $\int_{-\infty}^{\infty} x^2 dx = c^2 + \frac{2}{3}$, where c is a constant. (3 marks)

(g) Determine the expected value of X and interpret your answer in the context of this question. (2 marks)