

Student Name: \_\_\_\_\_



Methodist Ladies' College Semester 2, 2010

## 3CD MATHEMATICS

Question/Answer Booklet – Section 1 – Calculators NOT allowed – Notes sheets NOT allowed

Teacher's Name: \_\_\_\_\_ **SOLUTIONS** \_\_\_\_\_

***Time allowed for this paper***

Section	Reading	Working
<b>Calculator-free</b>	5 minutes	50 minutes
<b>Calculator-assumed</b>	10 minutes	100 minutes

**Materials required/recommended for this paper**

**Section One (Calculator-free): 40 marks**

**To be provided by the supervisor**

Section One Question/Answer booklet

Formula sheet

**To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

**Section Two (Calculator-assumed): 80 marks**

**To be provided by the supervisor**

Section Two Question/Answer booklet

Formula sheet

**To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

***Important Note to candidates***

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



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**Instructions to candidates**

1. **All** questions should be attempted.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare answer pages may be found at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued (i.e. give the page number).
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.

**Structure of this paper**

Questions	Marks available	Your score
1	5	
2	5	
3	4	
4	4	
5	2	
6	4	
7	7	
8	4	
9	5	
<b>Total:</b>	<b>40</b>	
10	5	
11	3	
12	5	
13	8	
14	10	
15	7	
16	8	
17	6	
18	7	
19	5	
20	7	
21	6	
22	3	
<b>Total:</b>	<b>80</b>	
<b>Total marks = 120</b>		
		%

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Section One: Calculator-free

(40 Marks)

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the space provided.

Suggested working time for this section is 50 minutes.

Question 1

(5 marks)

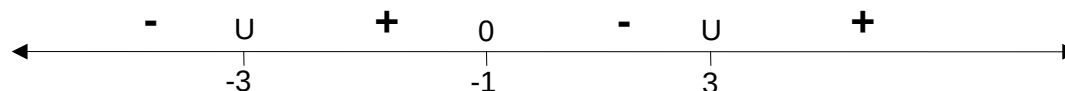
Solve  $\frac{2x^2+13x+15}{x^2-9} \geq \frac{2x+1}{x+3}$

Solution

$$\frac{2x^2+13x+15-(2x+1)(x-3)}{(x-3)(x+3)} \geq 0$$

$$\frac{18x+18}{(x-3)(x+3)} \geq 0$$

Critical values:  $x = -1, x = \pm 3$



Solution:  $\{x : -3 < x \leq -1 \text{ or } x > 3, x \in R\}$

Specific behaviours

- ✓ recognizes common denominator
- ✗ multiplies by common denominator correctly
- ✗ simplifies
- ✗ identifies critical values
- ✗ correct solution and notation (set notation not necessary)

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Question 2

(5 marks)

Find the following:

(a)  $\int x^2 \sqrt{x^3+5} dx$  [2]

Solution
$\frac{1}{3} \int (x^3+5)^{\frac{1}{2}} \cdot 3x^2 dx = \frac{1}{3} (x^3+5)^{\frac{3}{2}} \cdot \frac{2}{3} + c = \frac{2}{9} (x^3+5)^{\frac{3}{2}} + c$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ express integral in terms of <math>\int [f(x)]^n f'(x) dx</math></li> <li>■ integrates correctly and adds constant</li> </ul>

(b)  $\int_0^3 e^{-4x} dx$  [2]

Solution
$\int_0^3 e^{-4x} dx = \left[ \frac{-1}{4} e^{-4x} \right]_0^3 = -\frac{1}{4} (e^{-12} - e^0) = \frac{1}{4} (1 - e^{-12})$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ finds the integrand</li> <li>■ substitutes limits of integration and simplifies</li> </ul>

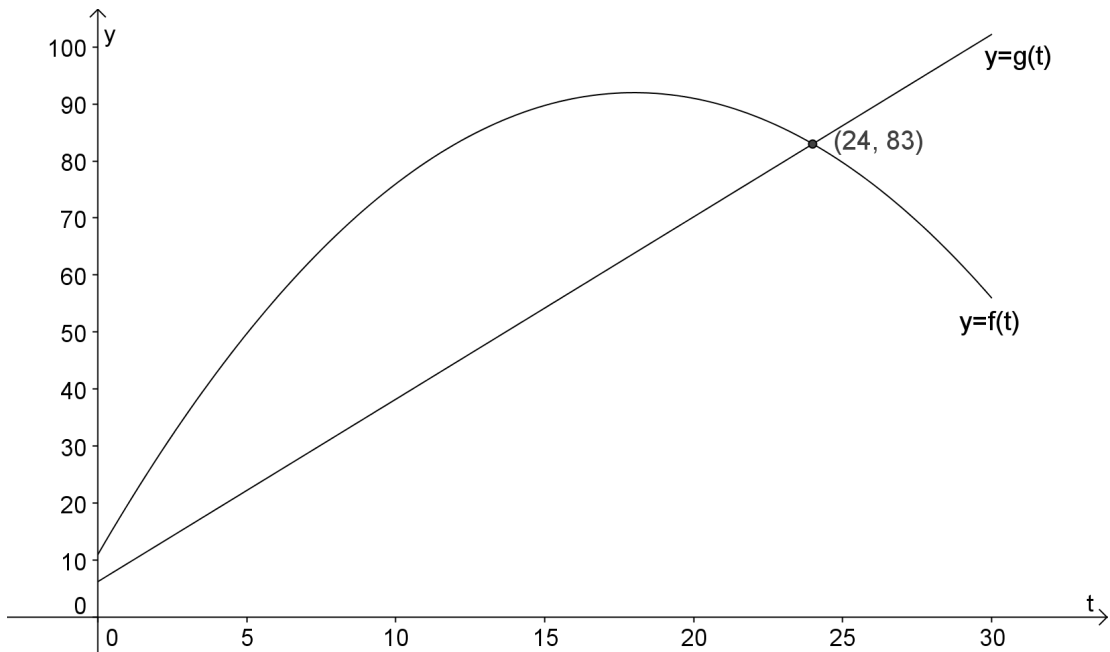
(c)  $\frac{d}{dx} \int_3^x \frac{8t}{\sqrt{t+9}} dt$  [1]

Solution
$\frac{8x}{\sqrt{x+9}}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ applies the Fundamental Theorem of Calculus correctly</li> </ul>

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Question 3 (4 marks)

The graphs of  $f(t) = -0.25t^2 + 9t + 11$  and  $g(t) = 3.2t + 6.2$  are shown on the axes below.



- (a) Write down an expression for the area enclosed by the two graphs and the vertical axis. [1]

Solution
$\text{Area} = \int_0^{24} f(t) - g(t) dt$
Specific behaviours
✓ correct expression

- (b) If  $f(t)$  represents the marginal revenue (in hundreds of dollars) for a product, where  $t$  is measured in months and  $g(t)$  represents the marginal cost (also in hundreds of dollars) for the product, what does this enclosed area represent? [2]

Solution
Profit for the 24 month period.
Specific behaviours
✓ recognizes that the area represents profit ✎ states correct time period

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- (c) Write down an expression for the volume of the solid generated when the part of the curve  $y=f(t)$  between  $t=5$  and  $t=25$  is rotated about the horizontal axis. [1]

Solution
$\text{Volume} = \pi \int_5^{25} [f(t)]^2 dt$
Specific behaviours
✓ correct expression

**Question 4**

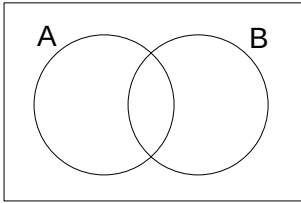
(4 marks)

In a probability experiment, events  $A$  and  $B$  are such that

$$P(A) = \frac{1}{4}, \quad P(B|A) = \frac{1}{3} \quad \text{and} \quad P(\bar{A} \cap \bar{B}) = \frac{1}{4}.$$

Find

- (a)  $P(A \cup B)$  [2]

Solution
$P(A \cup B) = 1 - P(\bar{A} \cap \bar{B}) = \frac{3}{4}$ 
Specific behaviours
✓ uses complement ■ correct answer

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(b)  $P(B)$

[2]

Solution	
$P(B A) = \frac{P(B \cap A)}{P(A)} P(B \cap A) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$	
Hence,	$P(B) = \frac{1}{12} + \frac{1}{2} = \frac{7}{12}$
Specific behaviours	
✓ uses conditional probability rule	
■ correct answer	

**Question 5**

(2 marks)

Which of the following statements is true for two events, each with probability greater than 0? Justify your answer.

- A: If the events are mutually exclusive, they must be independent.
- B: If the events are independent, they must be mutually exclusive.
- C: If the events are not mutually exclusive, they must be independent.
- D: If the events are not independent, they must be mutually exclusive.
- E: If the events are mutually exclusive, they cannot be independent.

Solution	
<p>Statement E is true.</p> <p>If events M and N are mutually exclusive, then <math>P(M \cap N) = 0</math></p> <p>If events M and N are independent, then <math>P(M \cap N) = P(M).P(N) &gt; 0</math> because both <math>P(M) &gt; 0</math> and <math>P(N) &gt; 0</math>.</p> <p>Hence, if events M and N are mutually exclusive, then they cannot be independent.</p>	
Specific behaviours	
✓ identifies E as the only true statement	
■ justifies choice of event E	

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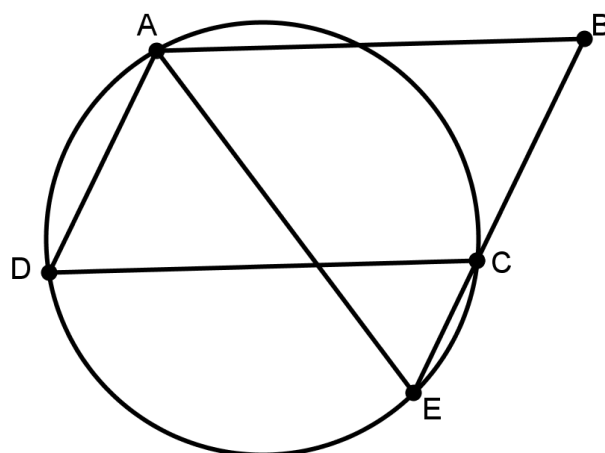
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Question 6

(4 marks)

In the diagram below, ABCD is a parallelogram.



Prove that  $\triangle ABE$  is isosceles.

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Solution		
Given: Parallelogram ABCD		
To Prove: $\triangle ABE$ is isosceles		
Proof:		
$\angle AEC = \angle ADC$		$\angle$ 's subtended by arc AC
In parallelogram ABCD, $\angle ADC = \angle ABC$		opposite $\angle$ 's of a parallelogram
In $\triangle ABE$ ,		
$\angle AEB = \angle AEC$		same angle
$\therefore \angle AEB = \angle ABC$		
Hence, $\triangle ABE$ is isosceles		two angles are congruent
Specific behaviours		
<input checked="" type="checkbox"/> Correctly reasons $\angle AEC = \angle ADC$ <input checked="" type="checkbox"/> Correctly reasons $\angle ADC = \angle ABC$ <input checked="" type="checkbox"/> Correctly reasons $\angle AEB = \angle ABC$ <input checked="" type="checkbox"/> Concludes $\triangle ABE$ is isosceles		

**Question 7**

(7 marks)

Consider the following system of equations:

$$\begin{aligned} 3x - 2y + z &= -7 \\ -x + 2y + 2z &= 11 \\ ax + bz &= 4 \end{aligned}$$

- (a) Determine the value of  $a$  and of  $b$  such that the system of equations has an infinite number of solutions. [2]

Solution		
	$3x - 2y + z = -7$	
	$-x + 2y + 2z = 11$	
	$ax + bz = 4$	
①+②	$2x + 3z = 4$	
	$a = 2$ and $b = 3$	
Specific behaviours		
<input checked="" type="checkbox"/> solves for $a$ <input checked="" type="checkbox"/> solves for $b$		

- (b) Solve the system when  $a = -2$  and  $b = 1$ . [5]

Solution	
$-2x + z = 4$	1234567-8

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③+④	$2x+3z=4$ $4z=8 \quad z=2$	④
Substitution gives	$y=3$ and $x=-1$	
<b>Specific behaviours</b>		
<input checked="" type="checkbox"/> ✓ Eliminates one variable from two pairs of equations <input checked="" type="checkbox"/> ✓ ✓ Evaluates each of the variables correctly		

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Question 8

(4 marks)

Given  $f(x) = \sqrt{x} + 2$ ,  $g(x) = \frac{1}{x+5}$ , and  $k(x) = \frac{1}{x} - 5$ , determine:

(a)  $g \circ f(1)$

[2]

Solution
$g \circ f(1) = g(3) = \frac{1}{8}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly evaluates <math>f(1)</math></li> <li>✓ correctly evaluates <math>g \circ f(1)</math></li> </ul>

(b) the domain and range of  $k \circ f$

[2]

Solution		
$x$	$f(x)$	$k(f(x))$
$D_{k \circ f} = D_f = \{x : x \geq 0, x \in R\}$ $R_{k \circ f} = \{y : -5 < y \leq -4.5, y \in R\}$		
Specific behaviours		
<div>✓ Correctly states domain</div> <div>✗ Correctly states range</div>		

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Question 9

(5 marks)

Determine the equation of the line tangential to the curve  $f(x) = \frac{9-x^2}{x}$  at the point where  $x=1$ .

Solution	
$f(x) = \frac{9}{x} - x$ $f'(x) = \frac{-9}{x^2} - 1$ $f(1) = 8 \text{ and } f'(1) = -10$ <p>Equation of tangent to curve at (1,8) with gradient -10,</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;">(1,8)</div> <div> <math display="block">y = -10x + c</math> <math display="block">8 = -10 + c</math> <math display="block">c = 18</math> <math display="block">y = -10x + 18</math> </div> </div> <p>Hence,</p>	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ correctly differentiates <math>f(x)</math></li> <li>■ determines <math>f(1)</math> and <math>f'(1)</math></li> <li>✓ recognizes that <math>f'(1)</math> is the gradient of the tangent line</li> <li>✓ uses the point (1,8) to determine the vertical intercept</li> <li>✓ states equation of the tangent</li> </ul>	

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