

DATE: 27th June 2016

Name: _____

CALCULATOR FREE

Reading Time: 3 minutes

Working Time: 50 minutes

EQUIPMENT:

pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet (provided)

Question	Marks available	Marks awarded
1	4	
2	6	
3	6	
4	3	
5	6	
6	5	
7	6	
8	7	
9	9	
Total	52	

Question 1

(4 marks)

Evaluate the following:

(a) $\log_3 27$ (1 mark)

(b) $\log_{15} 1$ (1 marks)

(c) $\log_{25} 0.2$ (2 marks)

Question 9 continued

(9 marks)

(c) Determine the cumulative distribution function $F(x)$ (3 marks)

(d) Calculate $P(1 < x < 2)$ (2 marks)

Question 2

(6 marks)

Question 9

The continuous random variable X is define by the probability density function

$$f(x)=\begin{cases} \frac{9}{2x} & 0\leq x\leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine $E(X)$.

(2 marks)

(b) The Variance of X , $\text{Var}(X)$, is $\frac{1}{2}$.
Determine $E(4X + 3)$

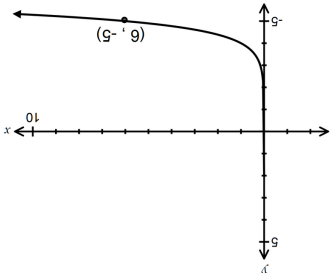
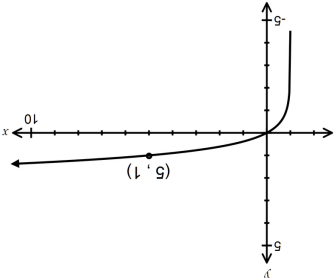
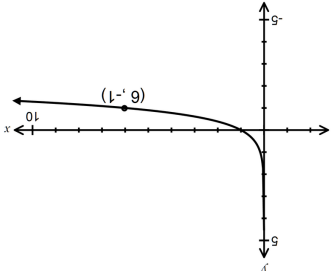
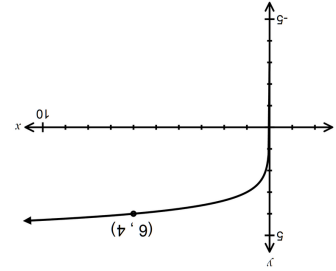
(1 mark)

(iii) Determine $\text{Var}(4X + 3)$

(1 mark)

(9 marks)

Match each of the following graphs with their equations from the given list.
A graph may have more than one matching equation. Not all equations have a matching graph.



A $y=\log_6(x+1)$

B

$y=\log_6(x-1)$

C

$y=-\log_6(x)-4$

F

$y=-\log_6(x)-4$

I

$y=\log_6(\frac{x}{1})-4$

G $y=\log_5(x)$

H

$y=\log_5(x+1)$

Question 3

(6 marks)

Differentiate with respect to x .

(a) $f(x) = \ln(2x+1)$

(1 mark)

(b) $f(x) = \ln\left(\frac{x^2 + 2x}{x - 5}\right)$

(2 marks)

(c) $f(x) = \frac{2\sqrt{x}}{\ln x}$

(3 marks)

Question 4

(3 marks)

Question 8

(7 marks)

As part of a local arts festival, an artist plans to create an installation in which a concealed water cannon blasts a stream of water into the air for a few seconds at random intervals.

The lengths of the intervals between each firing of the cannon can be modelled by the uniformly distributed random variable T , where $3 \leq t \leq 14$ minutes.

- (a) Sketch the probability density function $f(t)$ for the interval between each firing on the axes below. (2 marks)



- (b) Determine the probability that a randomly chosen interval between firings is

(i) at least seven minutes.

(1 mark)

(ii) at least six minutes given that it is less than ten minutes.

(2 marks)

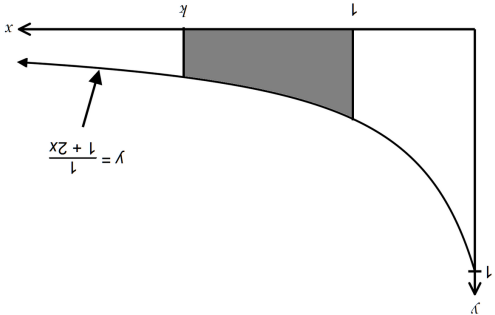
- (c) Determine the value of t for which $P(T < t) = P(T > 4t)$

(2 marks)

Question 7

The shaded area is 0.2 units².

Find k exactly.



(6 marks)

(a)

$$\int_{1/2}^x \frac{1}{x} dx$$

Find the following indefinite integrals. Assume denominators are greater than zero.

(1 mark)

(b)

$$\int \frac{e^x - \cos x}{2e^x + 2 \sin x} dx$$

(2 marks)

Question 5

(6 marks)

Find the exact value of x satisfying the equation

$$(3^x)(4^{2x+1})=6^{x+2}.$$

$$\frac{\ln a}{\ln b}.$$

Give your answer in the form

Question 6

(5 marks)

(a) The function f is defined for $x > 2$ by $f(x) = \ln x + \ln(x-2) - \ln(x^2-4)$.

Express $f(x)$ in the form $\ln\left(\frac{x}{x+a}\right)$.

(2 marks)

(b) Evaluate $\log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} + \log \frac{4}{5} + \dots + \log \frac{8}{9} + \log \frac{9}{10}$.

(3 marks)