PERTH MODERN SCHOOL

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Independent Public School

Course ______12 Methods______ Year __12___

Student name: ______________ Teacher name: ______________

Task type: **Response/Investigation**

Time allowed for this task: **____40_______ mins**

Number of questions: **_____7______**

Materials required: No calculators nor classpads

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **___40___ marks**

Task weighting: **_10___%**

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2, 3 & 3 = 8 marks) (3.1.7-3.1.8)

Determine $\frac{dy}{dx}$ for each of the following.(No need to simplify)

a) $y = \frac{3}{x}$

Solution
$y = \frac{3}{x} = 3x^{-1}$ $y' = -3x^{-2} = \frac{-3}{x^2}$
Specific behaviours
✓ correct coefficient ✓ correct power (no need for positive power)

b) $y = (3x^2 + 4x)(5x - 1)$

Solution
$y = (3x^2 + 4x)(5x - 1)$ $y' = (3x^2 + 4x)5 + (5x - 1)(6x + 4) \rightarrow$ *full marks* $= 15x^2 + 20x + 30x^2 + 14x - 4$ $= 45x^2 + 34x - 4$
Specific behaviours
✓ uses product rule ✓ one correct product ✓ two correct products (no need to simplify)

c) $y = \frac{x+1}{5 - x^2}$

Solution

$y=\dfrac{x+1}{5-x^2}$ $y'=\dfrac{(5-x^2)-(x+1)(-2x)}{(5-x^2)^2} \rightarrow$ *full marks* $=\dfrac{5+x^2+2x}{(5-x^2)^2}$
Specific behaviours
✓ uses quotient rule ✓ correct denominator ✓ correct numerator(no need to simplify)

Q2 (2 & 3 = 5 marks) (3.1.8)

Consider $f(x)=(4x-2)^5$.

a) Determine $f'(0)$

Solution
$f(x)=(4x-2)^5$ $f'(x)=5(7x-2)^4\,4$ $f'(0)=20(16)=320$
Specific behaviours
✓ uses chain rule ✓ evaluates derivative

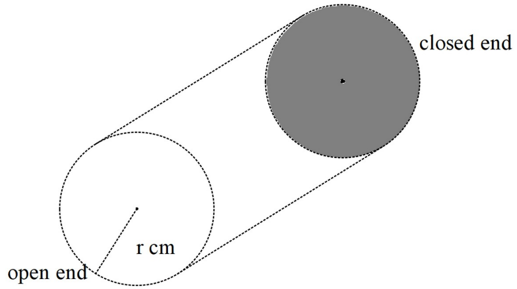
b) Determine the equation of the tangent at $x=0$

Solution
$f(0)=(-2)^5=-32$ $y=mx+c=320x+c$ $c=-32$ $y=320x-32$
Specific behaviours

✓ solves for y value at x=0 ✓ solves for constant ✓ states tangent equation

.Q7 (4 marks) (3.1.16)

Consider a cylindrical container that has an open end. The surface area of the container is $50cm^2$. Determine the exact value of the radius of the closed end that maximises the volume. (Justify)



Total surface area 50cm^2

Solution
$2\pi rh + \pi r^2 = 50$ $h = \dfrac{50 - \pi r^2}{2\pi r}$ $V = \pi r^2\left(\dfrac{50 - \pi r^2}{2\pi r}\right) = \dfrac{r}{2}\left(50 - \pi r^2\right) = \dfrac{50r - \pi r^3}{2}$ $\dfrac{dV}{dr} = \dfrac{50 - 3\pi r^2}{2}$ $50 - 3\pi r^2 = 0$ $r = \sqrt{\dfrac{50}{3\pi}}$ $\dfrac{d^2V}{dr^2} = -3\pi r < 0 \therefore local$ max
Specific behaviours
✓ obtains constraint equation containing h & r ✓ obtains expression for V in terms of one variable only ✓ obtains derivative and equates to zero ✓ obtains optimal value and confirms with second derivative

Solution

$x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 1$

$v = t^2 - 5t + 6$

$a = 2t - 5 = 0$

$t = \frac{5}{2}$

Specific behaviours

✓ differentiates velocity
✓ solves for t value

Q6 (3 marks) (3.1.10)

The period T of a swinging pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{10}}$.
Using the increments formula, determine the approximate percentage change in T if l changes by 3%

Solution

$T = 2\pi\sqrt{\frac{l}{10}} = \frac{2\pi}{\sqrt{10}} l^{\frac{1}{2}}$

$\Delta T \approx \frac{\pi}{\sqrt{10}} l^{-\frac{1}{2}} \Delta l$

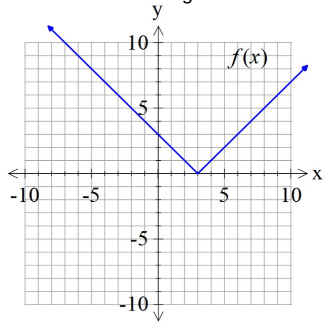
$\frac{\Delta T}{T} \approx \frac{\frac{\pi}{\sqrt{10}} l^{-\frac{1}{2}} \Delta l}{2\pi\sqrt{\frac{l}{10}}} = \frac{\Delta l}{2l} = \frac{3}{2}\%$

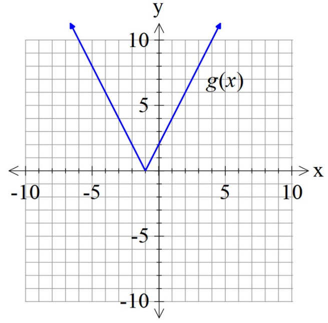
Specific behaviours

✓ uses increments formula
✓ obtains expression for approx. change in T
✓ obtains % change

Q3 (1, 1, 3 & 3 = 8 marks) (3.1.7-3.1.8, 3.1.15)

Consider the following functions f & g.





a) Determine the derivative of $f(x)$ when $x = -2$

Solution

Gradient = -1

Specific behaviours

✓ states gradient

b) Determine the derivative of $3g(x)$ when $x = 0$

Solution

Gradient = 6

Specific behaviours

✓ states gradient

c) Determine the derivative of $f(x)g(x)$ when $x = 0$.

Solution

$\frac{d}{dx} f(x)g(x) = f(x)g'(x) + f'(x)g(x)$

$= 3(2) + -1(2) = 4$

Specific behaviours

✓ uses product rule
✓ uses correct values for all variables

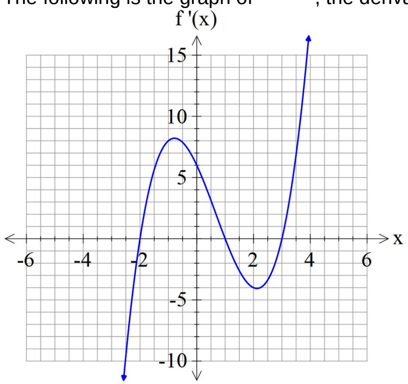
✓ states final value

d) Determine the derivative of $f(g(x))$ when $x=0$.

Solution
$\frac{d}{dx}f(g(x))=f'(g(x))g'(x)=f'(2)2=-2$
Specific behaviours
✓ uses chain rule and is demonstrated ✓ uses correct value for derivative of f ✓ states final value

Q4 (2, 3 & 2 = 7marks) (3.1.13 – 3.1.17)

The following is the graph of $f'(x)$, the derivative of $f(x)$.



a) State the x values of all stationary points of $f(x)$.

Solution
-2, 1 & 3
Specific behaviours
✓ states one correct x value ✓ states all three values

b) State the nature of each stationary point above and justify.

Solution
-2, local min as $f''>0$ 1, local max as $f''<0$ 3, local min as $f''>0$
Specific behaviours
✓ states nature of at least two stationary points ✓ states reason using first or second derivatives for at least two pts ✓ states nature and reason for all three stationary points

c) State approximate x value for an infection point(s) and explain why.

Solution
Near -1 & 2 as $f''=0$
Specific behaviours
✓ states near x values ✓ states reason using second derivative

Q5 (3 & 2 = 5 marks) (3.1.12)

The displacement of a body from the origin O, at time t seconds, is x metres where

$$x=\frac{t^3}{3}-\frac{5t^2}{2}+6t+1$$

a) Determine the time(s) that the velocity is zero metres/second.

Solution
$x=\frac{t^3}{3}-\frac{5t^2}{2}+6t+1$ $v=t^2-5t+6=(t-2)(t-3)$ $t=2,3$
Specific behaviours
✓ differentiates ✓ equates velocity to zero and factorises/quadratic formula ✓ states both t values

b) Determine when the acceleration is zero.