

**Papers written by
Australian Maths
Software**

SEMESTER ONE

MATHEMATICS SPECIALIST

REVISION 1

UNIT 3

2016

SOLUTIONS

Section One

1. (10 marks)

(a)

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 6 \\ 1 & 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & -4 \\ 0 & 1 & 0 & -2 \end{bmatrix}$$

$$R_1 - R_2 \quad R_1 - R_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 0 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2 - 2R_3$$

$$0 = 0$$

\therefore There are an infinite number of solutions.

None of the planes are parallel.

There are either two or three identical planes or the three planes intersect in a common line.

None of the planes have identical/equivalent equations so the three planes meet in a common line. (4)

(b) Two of the planes are parallel. Therefore there is no intersection.

$$x + y + z = 2 \text{ and } -x - y - z = 1 \Leftrightarrow x + y + z = -1 \text{ are parallel} \quad (2)$$

(c)

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 11 \\ 0 & 4 & 4 & 20 \end{bmatrix}$$

$$2R_1 - R_2 \quad 3R_1 - R_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 11 \\ 0 & 1 & 1 & 5 \end{bmatrix}$$

$$R_3 \div 4$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 11 \\ 0 & 0 & 2 & 6 \end{bmatrix}$$

$$R_2 - R_3$$

$$z = 3$$

$$y + 9 = 11 \rightarrow y = 2$$

$$x + 2 + 3 = 6 \rightarrow x = 1$$

The point of intersection is (1, 2, 3) (4)

2. (6 marks)

(a) $f(x) = \sin(|x|)$ ✓✓ (2)

(b) $g(x) = |x^3 + 1|$ ✓✓ (2)

(c) $p(x) = e^{x-1}$
 $q(x) = 1 + \ln(x)$ (2)

3. (13 marks)

(a) $z^3 - z^2 - 4 = 0$

Let $P(z) = z^3 - z^2 - 4$

$$P(2) = 8 - 4 - 4 = 0$$

$\therefore z = 2$ so $z - 2$ is a factor

$$\begin{array}{r} z^2 + z + 2 \\ z - 2 \overline{) z^3 - z^2 + 0z - 4} \end{array}$$

$$- (z^3 - 2z^2)$$

$$\hline z^2 + 0z$$

$$- (z^2 - 2z)$$

$$\hline 2z - 4$$

$$- (2z - 4)$$

$$\hline 0$$

$$z = 2 \text{ or } z^2 + z + 2 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1 - 8}}{2}$$

$$\Delta = -7 = 7i^2$$

$$z = \frac{-1 \pm i\sqrt{7}}{2}$$

$$\therefore z = \frac{-1 \pm i\sqrt{7}}{2} \text{ or } z = 2$$

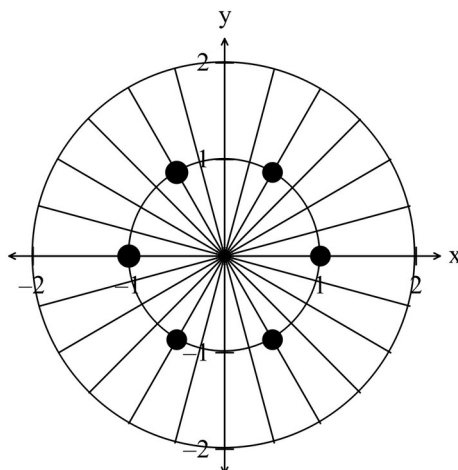
✓✓

(4)

(b) $z^3 = 8\text{cis}(\pi)$ ✓✓
 $z^3 = -8$

(3)

(c) (i)



✓✓ -1 per error

(2)

(ii) $z^6 = 1$

$$z^6 = \text{cis}(0 + 2n\pi) \quad n \in \mathbb{R}$$

$$z = (\text{cis}(2n\pi))^{\frac{1}{6}}$$

$$z = \text{cis}\left(\frac{2n\pi}{6}\right)$$

$$z = \text{cis}\left(\frac{n\pi}{3}\right)$$

$$n=0, \quad z = \text{cis}(0) = 1$$

$$n=1, \quad z = \text{cis}\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{i\sqrt{3}}{2}$$

-1 per error (4)

$$n=2, \quad z = \text{cis}\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$$

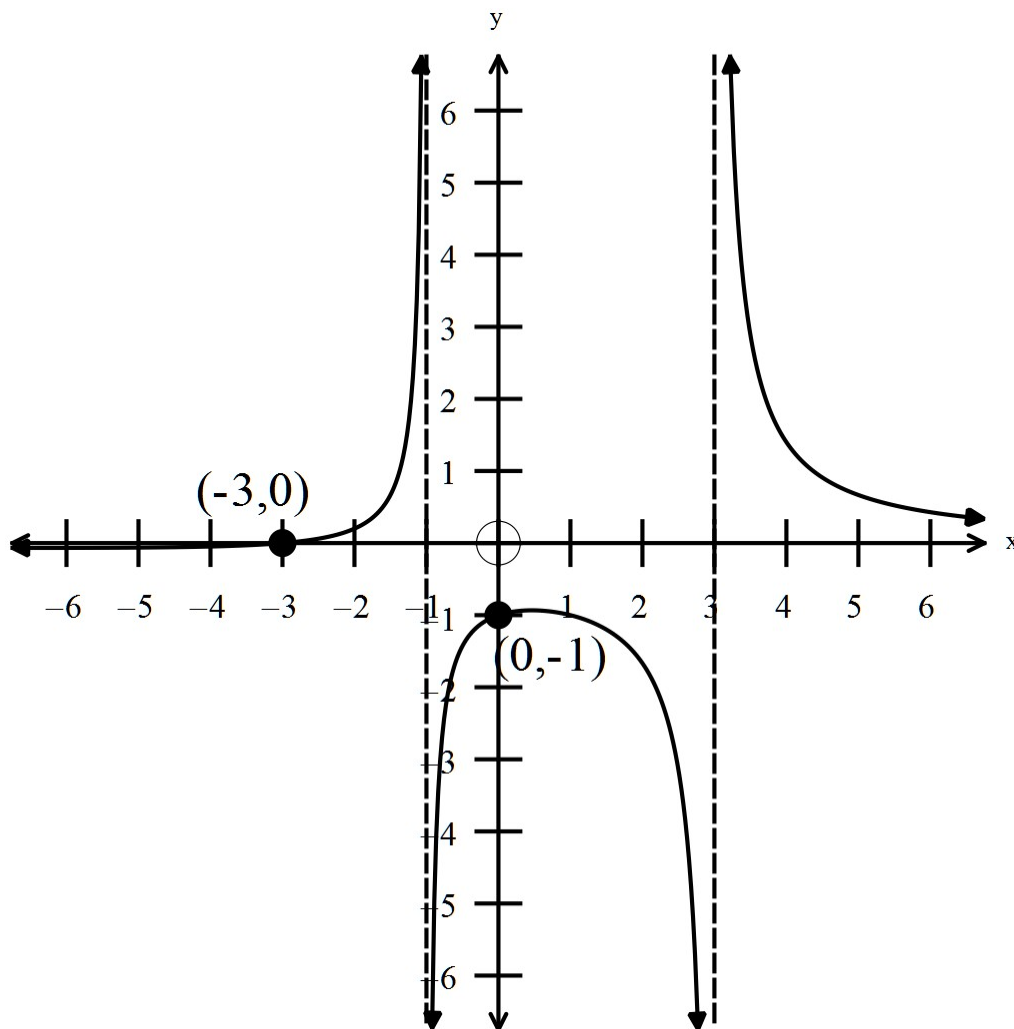
$$n=3, \quad z = \text{cis}(\pi) = -1$$

$$n=-1, \quad z = \text{cis}\left(-\frac{\pi}{3}\right) = \frac{1}{2} - \frac{i\sqrt{3}}{2}$$

$$n=-2, \quad z = \text{cis}\left(-\frac{2\pi}{3}\right) = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$$

4. (8 marks)

$$(a) \quad f(x) = \frac{(x+3)}{(x+1)(x-3)}$$



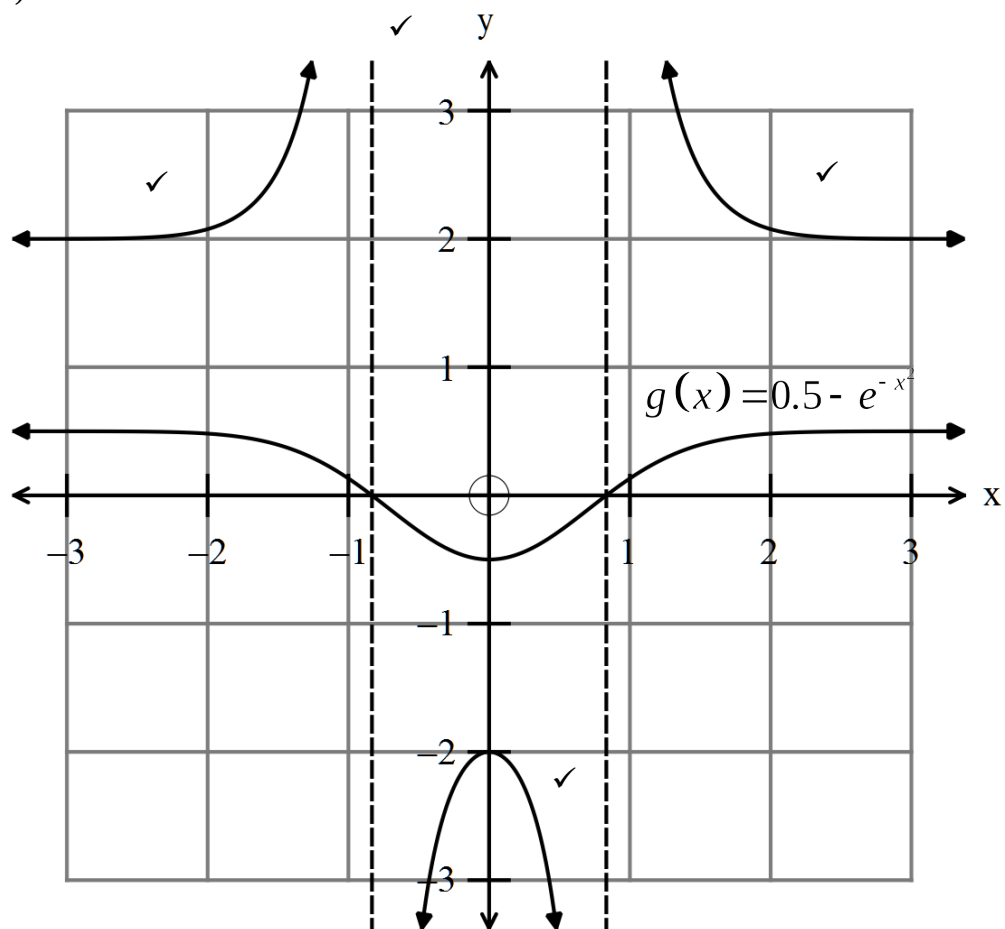
Vertical asymptotes at $x = -1$, $x = 3$ ✓ ✓

x intercept at $x = -3$ ✓

y intercept at $y = -1$ ✓

General shape – maximum turning point (w/o cutting x axis); limits as $x \rightarrow \pm \infty$;
limits about asymptotes ✓ -1/error (4)

(b)



(4)

5. (13 marks)

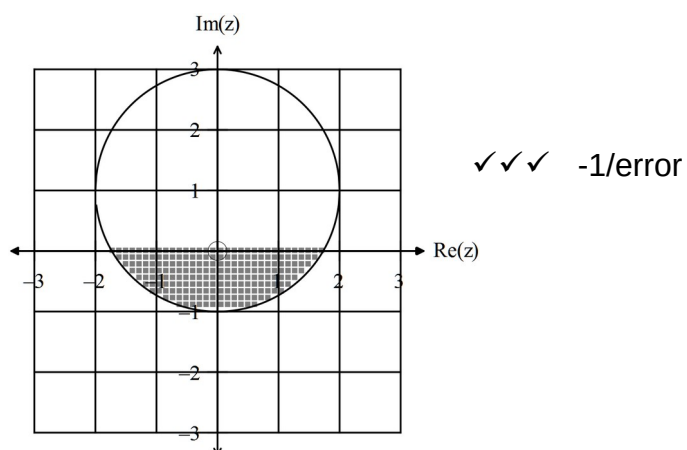
(a) $(3\sqrt{3} + \sqrt{2}i)(2\sqrt{3} - 2\sqrt{2}i)$.

$$= 18 + 2i\sqrt{6} - 6i\sqrt{6} - 4i^2 \quad \checkmark\checkmark$$

$$= 22 - 4i\sqrt{6} \quad \checkmark$$

(3)

(b)



(3)

(c)

$$\begin{aligned}
 (1-i)^{10} &= \left(\sqrt{2} \operatorname{cis} \left(-\frac{\pi}{4} \right) \right)^{10} \\
 &= (\sqrt{2})^{10} \operatorname{cis} \left(-\frac{10\pi}{4} \right) \\
 &= 2^5 \operatorname{cis} \left(-\frac{5\pi}{2} \right) \quad \checkmark \\
 &= 32 \operatorname{cis} \left(-\frac{\pi}{2} \right) \\
 &= 32(0-i) \\
 (1-i)^{10} &= -32i \quad \checkmark
 \end{aligned}$$

(2)

$$\begin{aligned}
 \text{(d)} \quad \operatorname{Re} \left(\frac{3-4i}{1+2i} \right) &= \operatorname{Re}(-1-2i) = -1 \\
 &\quad \checkmark \quad \checkmark
 \end{aligned}$$

(2)

$$\text{(e) (i)} \quad z = \operatorname{cis} \left(\frac{2\pi}{3} \right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \Rightarrow \bar{z} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i \quad \checkmark \quad (1)$$

$$\text{(ii)} \quad z^2 = \operatorname{cis} \left(\frac{2\pi}{3} \right)^2 = \operatorname{cis} \left(\frac{4\pi}{3} \right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i \quad \checkmark \quad (1)$$

$$\text{(ii)} \quad \frac{1}{z^2} = \frac{1}{\left(\operatorname{cis} \left(\frac{2\pi}{3} \right) \right)^2} = \frac{1}{\operatorname{cis} \left(\frac{4\pi}{3} \right)} = \operatorname{cis} \left(-\frac{4\pi}{3} \right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \quad \checkmark \quad (1)$$

END OF SECTION ONE

Section Two

6. (6 marks)

$$(a) \quad \mathbf{v}(t) = (2\cos(2t))\mathbf{i} + (-\sin(t))\mathbf{j}$$

$$\mathbf{r}(t) = \int ((2\cos(2t))\mathbf{i} + (-\sin(t))\mathbf{j}) dt$$

$$\mathbf{r}(t) = (\sin(2t))\mathbf{i} + (\cos(t))\mathbf{j} + \mathbf{c}$$

$$\mathbf{r}(\pi) = -\mathbf{j} \text{ so}$$

$$-\mathbf{j} = (\sin(2\pi))\mathbf{i} + (\cos(\pi))\mathbf{j} + \mathbf{c}$$

$$-\mathbf{j} = -\mathbf{j} + \mathbf{c} \Rightarrow \mathbf{c} = \mathbf{0}$$

$$\therefore \mathbf{r}(t) = (\sin(2t))\mathbf{i} + (\cos(t))\mathbf{j}$$

$$\mathbf{a}(t) = (-4\sin(2t))\mathbf{i} + (-\cos(t))\mathbf{j}$$

(3)

$$(b) \quad \mathbf{r}\left(\frac{3\pi}{2}\right) = (\sin(3\pi))\mathbf{i} + \left(\cos\left(\frac{3\pi}{2}\right)\right)\mathbf{j} = 0\mathbf{i} + 0\mathbf{j} = \mathbf{0} \quad \checkmark$$

$$\mathbf{v}\left(\frac{3\pi}{2}\right) = (2\cos(3\pi))\mathbf{i} + \left(-\sin\left(\frac{3\pi}{2}\right)\right)\mathbf{j} = -2\mathbf{i} + \mathbf{j} \quad \checkmark$$

(2)

(c) Show that $4\mathbf{r}(t) + \mathbf{a}(t) = 3\cos(t)\mathbf{j}$.

$$4\mathbf{r}(t) + \mathbf{a}(t) = 4 \begin{pmatrix} \sin(2t) \\ \cos(t) \end{pmatrix} + \begin{pmatrix} -4\sin(2t) \\ -\cos(t) \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 3\cos(t) \end{pmatrix} \quad \checkmark$$

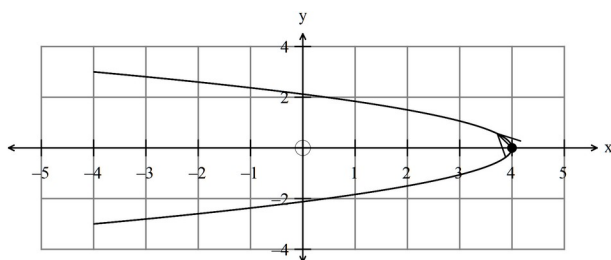
(1)

$$= (3\cos(t))\mathbf{j}$$

7. (20 marks)

$$(a) \quad \mathbf{r}(0) = (4\cos(0))\mathbf{i} + (3\sin(0))\mathbf{j} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad \checkmark$$

$$\mathbf{r}(0^+) = (4\cos(0^+))\mathbf{i} + (3\sin(0^+))\mathbf{j} = \begin{pmatrix} 4^- \\ 0^+ \end{pmatrix} \quad \checkmark$$

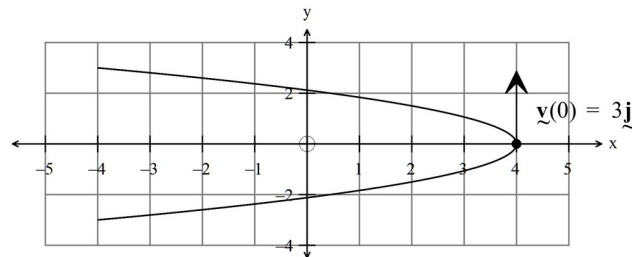


✓

(3)

$$(b) \quad \mathbf{v}(t) = (-8 \sin(2t))\mathbf{i} + (3 \cos(t))\mathbf{j} \quad \checkmark \checkmark$$

$$\mathbf{v}(0) = 3\mathbf{j} \quad \checkmark$$



✓

(4)

$$(c) \quad \mathbf{a}(t) = (-16 \cos(2t))\mathbf{i} + (-3 \sin(t))\mathbf{j} \quad \checkmark \checkmark$$

(2)

$$(d) \quad \mathbf{a}(t) = \begin{pmatrix} 16 \\ -3 \end{pmatrix} \Rightarrow t = \frac{\pi}{2} \quad \checkmark$$

$$\mathbf{r}\left(\frac{\pi}{2}\right) = (4 \cos(\pi))\mathbf{i} + \left(3 \sin\left(\frac{\pi}{2}\right)\right)\mathbf{j}$$

$$\mathbf{r}\left(\frac{\pi}{2}\right) = -4\mathbf{i} + 3\mathbf{j} \quad \checkmark$$

(2)

$$(e) \quad \mathbf{a}(t) = (-16 \cos(2t))\mathbf{i} + (-3 \sin(t))\mathbf{j}$$

$$\mathbf{r}(t) = (4 \cos(2t))\mathbf{i} + (3 \sin(t))\mathbf{j}.$$

$$\mathbf{a}(t) \neq k\mathbf{r}(t) \quad \checkmark$$

$$\text{as } -1 \times 3 = -3 \text{ but } -1 \times 4 \neq -16 \quad \checkmark$$

(2)

$$(f) \quad \text{If } \mathbf{a}(t) = \mathbf{0} \text{ then } \mathbf{a}(t) = (-16 \cos(2t))\mathbf{i} + (-3 \sin(t))\mathbf{j} = \mathbf{0}$$

$$x = -16 \cos(2t) = 0 \text{ and } y = (-3 \sin(t)) = 0$$

$$2t = \frac{\pi}{2} + k\pi \quad t = 0, \pi, 2\pi$$

$$t = \frac{\pi}{4} + \frac{k\pi}{2}$$

$$t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

The x and y coordinates are never zero at the same time so $\mathbf{a}(t) \neq \mathbf{0}$

(4)

$$(g) \quad \mathbf{r}(t) = (4 \cos(2t))\mathbf{i} + (3 \sin(t))\mathbf{j}$$

$$x = (4 \cos(2t)) \quad y = (3 \sin(t))$$

$$x = 4(1 - 2 \sin^2(t)) \quad \sin(t) = \frac{y}{3}$$

$$x = 4 \left(1 - \frac{2y^2}{9} \right)$$

(3)

8. (10 marks)

$$(a) \quad A(3, 4, 0), B(4, -3, 0) \text{ and } C(0, 0, 5).$$

$$\mathbf{AB} = \begin{pmatrix} 1 \\ -7 \\ 0 \end{pmatrix}, \quad \mathbf{AC} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$$

$$\mathbf{r}(t) = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ -7 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$$

There are alternative answers as they can also use **BC** as a direction vector and then any of the points A, B or C for the equation.

(2)

$$(b) \quad x^2 + y^2 + z^2 = 25 \quad \checkmark \checkmark$$

(2)

$$(c) \quad (i) \quad \mathbf{r}_{ball}(t) = \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$$

Hits the ground at $2 - t = 0$ i.e. $t = 2$ seconds \checkmark

(2)

$$(ii) \quad \mathbf{r}_{Paul}(t) = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3.5 \\ -1 \end{pmatrix}$$

$$\mathbf{r}_{Paul}(2) = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 3.5 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \\ 0 \end{pmatrix}$$

$$\mathbf{r}_{ball}(2) = \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \\ 0 \end{pmatrix}$$

Both Paul and the ball are at the same point $Q(5, 11, 0)$ at ground level when $t = 2$, so Paul catches the ball.

(3)

$$(ii) \quad 2 \times \left| \begin{pmatrix} 1 \\ 3.5 \\ -1 \end{pmatrix} \right| = 2 \times \sqrt{1 + 12.25 + 1} = 7.55 \text{ m} \quad (1)$$

9. (6 marks)

$$(a) \quad (i) \quad \mathbf{MN} = \begin{pmatrix} -4 \\ -6 \\ -8 \end{pmatrix}$$

$$\mathbf{r}(t) = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} -4 \\ -6 \\ -8 \end{pmatrix} \quad (2)$$

$$(ii) \quad x = 2 - 4t \quad y = 1 - 6t \quad z = 3 - 8t$$

$$t = \frac{2-x}{4} \quad t = \frac{1-y}{6} \quad t = \frac{3-z}{8}$$

$$\text{so} \quad \frac{2-x}{4} = \frac{1-y}{6} = \frac{3-z}{8}$$

(2)

$$(b) \quad L_1: \mathbf{r}_1(t) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \Rightarrow \begin{array}{l} x = 1 - t \\ y = 2 \\ z = 3 - t \end{array}$$

$$L_2: \mathbf{r}_2(s) = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + s \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \Rightarrow \begin{array}{l} x = 0 \\ y = 2 \\ z = 2 + 2s \end{array}$$

It can be seen that $y = 2$. If $x = 0 \Rightarrow t = 1$ i.e. $z = 2 \Rightarrow (0, 2, 2)$

If $z = 2$ then $s = 0$ which does not contradict the x and y values and gives $(0, 2, 2)$. ✓ logic

Yes, the lines intersect, and do so at $(0, 2, 2)$.

(2)

10. (9 marks)

$$(a) \quad \mathbf{AB} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}, \mathbf{AC} = \begin{pmatrix} 2 \\ -4 \\ -6 \end{pmatrix}$$

$$Area_{\Delta} = \frac{1}{2}a \times b \times \sin(C) \quad \text{and} \quad |\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin(\theta)$$

$$Area_{\Delta} = \frac{1}{2} |\mathbf{AB} \times \mathbf{AC}|$$

$$= \frac{1}{2} \left| \begin{pmatrix} -28 \\ -2 \\ -8 \end{pmatrix} \right|$$

$$= \frac{1}{2} \sqrt{784 + 4 + 64}$$

$$Area_{\Delta} = 14.6 \text{ units}^2$$

(4)

$$(b) \quad AB : AC = |\mathbf{AB}| : |\mathbf{AC}| = \sqrt{21} : \sqrt{56} = \frac{\sqrt{7 \times 3}}{\sqrt{7 \times 8}} = \sqrt{3} : 2\sqrt{2}$$

(2)

$$(c) \quad P(1.5, 3, 1) \quad Q(2, 0, 0)$$

$$\mathbf{PQ} = \begin{pmatrix} 0.5 \\ -3 \\ -1 \end{pmatrix} \quad \mathbf{BC} = \begin{pmatrix} 1 \\ -6 \\ -2 \end{pmatrix}$$

$$\mathbf{BC} = 2 \begin{pmatrix} 0.5 \\ -3 \\ -1 \end{pmatrix}$$

$$\therefore 2\mathbf{PQ} = \mathbf{BC}$$

Therefore PQ is parallel to BC .

(3)

11. (13 marks)

$$\begin{aligned}
 \text{(a)} \quad \frac{(xy)^3}{\sqrt{z}} &= \frac{\left(\text{cis}\left(\frac{\pi}{4}\right) \right)^3 (1-i)^3}{(1+\sqrt{3}i)^{1/2}} \\
 &= \frac{\left(\text{cis}\left(\frac{3\pi}{4}\right) \right) \left((\sqrt{2})^3 \text{cis}\left(-\frac{3\pi}{4}\right) \right)}{\left(2\text{cis}\left(\frac{\pi}{3}\right) \right)^{1/2}} \\
 &= \frac{2\sqrt{2} \left(\text{cis}\left(\frac{3\pi}{4}\right) \right) \left(\text{cis}\left(-\frac{3\pi}{4}\right) \right)}{\left(\sqrt{2} \text{cis}\left(\frac{\pi}{6}\right) \right)} \\
 &= 2 \left(\text{cis}\left(\frac{3\pi}{4} - \frac{3\pi}{4} - \frac{\pi}{6}\right) \right) \\
 &= 2 \left(\text{cis}\left(-\frac{\pi}{6}\right) \right) \\
 &= 2 \left(\frac{\sqrt{3}}{2} - \frac{i}{2} \right) \\
 &= \sqrt{3} - i
 \end{aligned}$$

(4)

$$\text{(b)} \quad \left\{ z: 1 \leq |z| \leq 2 \cap -\frac{3\pi}{4} \leq \arg(z) \leq \frac{\pi}{4} \right\} \quad \checkmark \text{ correct inequalities} \quad (3)$$

(c) (i) Given $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ show that $\overline{z_1 z_2} = \overline{z_1} \times \overline{z_2}$.

$$\begin{aligned}
 \overline{z_1} \times \overline{z_2} &= \overline{(x_1 + iy_1)} \times \overline{(x_2 + iy_2)} \\
 &= (x_1 - iy_1) \times (x_2 - iy_2) \\
 &= x_1 x_2 + i^2 y_1 y_2 + i(-x_1 y_2 - x_2 y_1) \\
 &= x_1 x_2 - y_1 y_2 - i(x_1 y_2 + x_2 y_1) \\
 \overline{z_1 z_2} &= \overline{(x_1 + iy_1)(x_2 + iy_2)} \\
 &= \overline{x_1 x_2 + i^2 y_1 y_2 + ix_1 y_2 + ix_2 y_1} \\
 &= \overline{x_1 x_2 - y_1 y_2 + i(x_1 y_2 + x_2 y_1)} \\
 &= x_1 x_2 - y_1 y_2 - i(x_1 y_2 + x_2 y_1) \\
 &= \overline{z_1} \times \overline{z_2}
 \end{aligned}$$

(3)

$$\begin{aligned}
 \text{(ii)} \quad z &= x + iy \quad x = ? \quad y = ? \\
 z(1+i) + \bar{z}(1-i) + 2z &= 10 - 2i \\
 (x+iy)(1+i) + (x-iy)(1-i) + 2x + 2iy &= 10 - 2i \\
 (x-y) + i(x+y) + (x-y) + i(-x-y) + 2x + 2iy &= 10 - 2i \\
 (x-y+x-y+2x) + i(x+y-y-x+2y) &= 10 - 2i \\
 (4x-2y) + i(2y) &= 10 - 2i \\
 \text{Im} : 2y &= -2 \\
 y &= -1 \\
 \text{Re} : 4x - 2y &= 10 \\
 4x + 2 &= 10 \\
 x &= 2, \quad y = -1
 \end{aligned}$$

(3)

12. (6 marks)

$$\begin{aligned}
 \text{(a)} \quad \mathbf{a}(t) &= -9.8 \mathbf{j} \\
 \mathbf{v}(t) &= \int -9.8 \mathbf{j} dt = -9.8t \mathbf{j} + \mathbf{c}_1 \\
 \mathbf{v}(0) &= 20 \cos(60^\circ) \mathbf{i} + 20 \sin(60^\circ) \mathbf{j} = 10 \mathbf{i} + 10\sqrt{3} \mathbf{j} \Rightarrow \mathbf{c}_1 = 10 \mathbf{i} + 10\sqrt{3} \mathbf{j} \\
 \mathbf{v}(t) &= 10 \mathbf{i} + (10\sqrt{3} - 9.8t) \mathbf{j} \\
 \mathbf{r}(t) &= \int 10 \mathbf{i} + (10\sqrt{3} - 9.8t) \mathbf{j} dt = 10t \mathbf{i} + (10\sqrt{3}t - 4.9t^2) \mathbf{j} + \mathbf{c}_2 \\
 \mathbf{r}(0) &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{c}_2 = \mathbf{j} \\
 \mathbf{r}(t) &= 10t \mathbf{i} + (10\sqrt{3}t - 4.9t^2 + 1) \mathbf{j} \\
 \text{If } 10t &= 50, \quad t = 5 \quad \text{and} \quad h = 10\sqrt{3}t - 4.9t^2 + 1 \\
 \text{At } t &= 5, \quad h = -34.9 \text{ m} \\
 \text{This means the ball is not in flight for five seconds so Tom could not have} \\
 &\text{kicked the ball through the window.}
 \end{aligned}$$

(4)

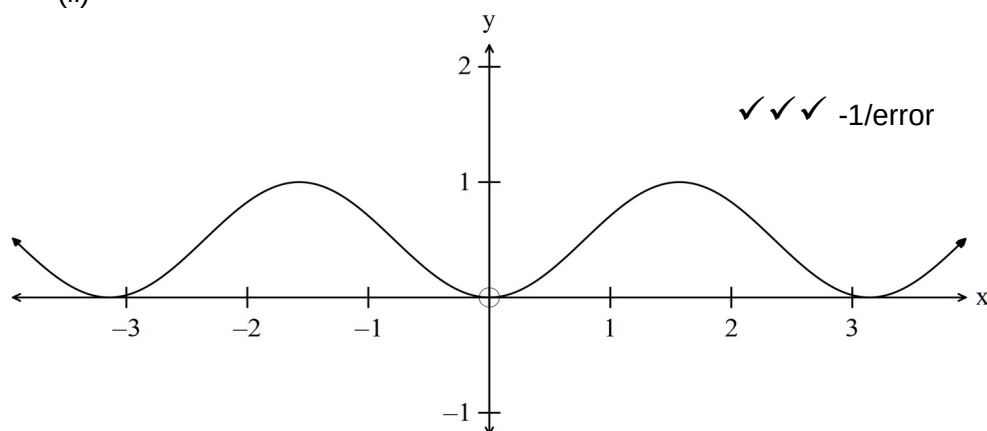
$$\begin{aligned}
 \text{(b)} \quad h &= 10\sqrt{3}t - 4.9t^2 + 1 \\
 \text{At } t &= 3, \quad h = 8.9 \text{ m} \\
 \text{The ball was still in flight so the deputy may have seen it.}
 \end{aligned}$$

(1)

13. (14 marks)

(a) (i) $y = g(f(x)) = g(\sin(x)) = (\sin(x))^2$ (1)

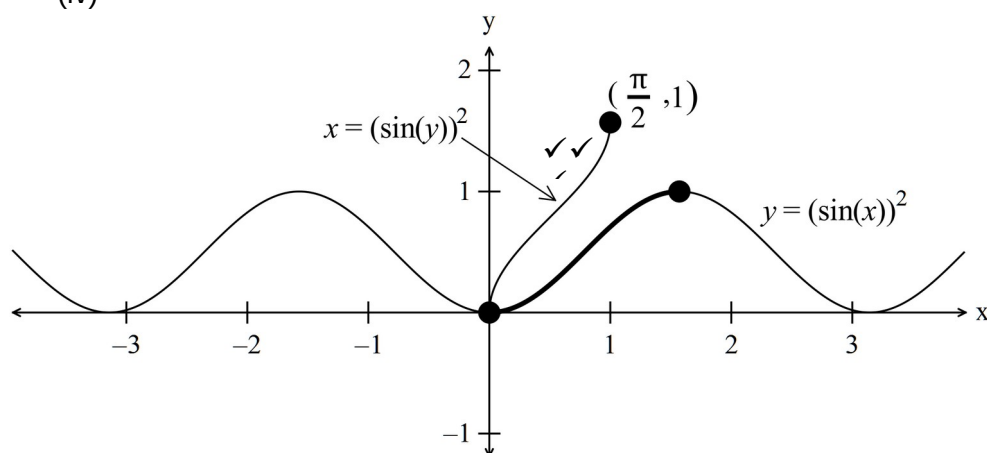
(ii)



(3)

(iii) $a = \frac{\pi}{2}$ ✓✓ (2)

(iv)



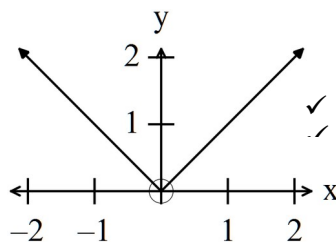
(2)

(b) (i) $h(x) = \sqrt{x}$
 $h^{-1}(x) = x^2$ for $x \geq 0$ ✓ *must have restricted domain* (1)

(ii) $h(x) \geq 0$ ✓✓ (1)

(c) $g(h(x)) = g(\sqrt{x}) = x$ ✓
 Defined on $[0, 2\pi]$ ✓ (2)

(d) (i)



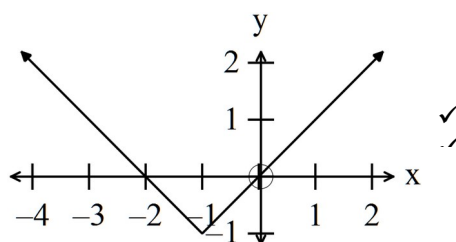
(1)

(ii) $y = |x|$ has no inverse because it is not a one to one function
i.e. for every x there is not a unique y . ✓

(1)

14. (7 marks)

(a)



$$y = |x+1| - 1$$

so $x < -2$ or $x > 0$ ✓

(2)

(b) Solve $|1+x| - 1 = |x-1|$

$$y = |1+x| - 1 = \begin{cases} x & \text{for } x \geq -1 \\ -x - 2 & \text{for } x < -1 \end{cases} \quad \checkmark$$

$$y = |x-1| = \begin{cases} x-1 & \text{for } x \geq 1 \\ 1-x & \text{for } x < 1 \end{cases} \quad \checkmark$$

For $x \geq 1$

$$x = x - 1 \quad \text{No solution} \quad \checkmark$$

For $-1 < x < 1$

$$x = 1 - x$$

$$2x = 1$$

$$x = \frac{1}{2} \quad \checkmark$$

For $x < -1$

$$-x - 2 = -x + 1$$

$$-2 = 1 \quad \text{No solution} \quad \checkmark$$

$$\therefore x = \frac{1}{2} \text{ only}$$

(5)

15. (4 marks)

$$y = \frac{x^2}{(x-1)(x+3)}$$

16. (5 marks)

$$\begin{aligned}
 \text{(i)} \quad (1-i) &= 1-i & = 1-i \\
 (1-i)^2 &= -2i & = -2i \\
 (1-i)^3 &= -2-2i & = -2(1+i) \\
 (1-i)^4 &= -4 & = -4 \\
 (1-i)^5 &= -4+4i & = -4(1-i) \\
 (1-i)^6 &= 8i & = 4(2i) \\
 (1-i)^7 &= 8+8i & = 8(1+i) \\
 (1-i)^8 &= 16 & = 16 \\
 (1-i)^9 &= 16-16i & = 16(1-i) \\
 (1-i)^{10} &= -32i & = -32i \checkmark \checkmark
 \end{aligned}$$

(2)

(ii) Every fourth result seems to be connected.

Starting with $n=1$, then $n=5$, the pattern seems to be a multiple of $(1-i)$.Starting with $n=2$, then $n=6$, the pattern seems to be a multiple of i .Starting with $n=3$, then $n=7$, the pattern seems to be a multiple of i .Starting with $n=4$, then $n=8$, the pattern seems to be a real multiple of 4.*✓ for any one of these up to 3 marks*

The coefficients are a pattern of powers of 2.

A lot more analysis is needed, but this is sufficient for 3 marks.

(3)

END OF SECTION TWO