

it to the supervisor before reading any further.  
you do not have any unauthorized material if you have any unauthorized material with you, hand  
NO other items may be taken into the examination room. It is your responsibility to ensure that

#### Important note to candidates

Special items: drawing instruments, templates, notes on two unframed sheets of A4 paper,  
and up to three calculators approved for use in this examination

Standard items: correction fluid/tape, eraser, rule, highlighters,  
pens (blue/black preferred), pencils (including coloured), sharpener,

#### To be provided by the candidate

Formula sheet (retained from Section One)

This Question/Answer booklet

#### Materials required/recommended for this section

<input type="checkbox"/>	Number of additional answer books used	Reading time before commencing work: one minute(s) (if applicable):
		Working time: ten minutes

Your name \_\_\_\_\_

In words \_\_\_\_\_

WA student number: In figures: \_\_\_\_\_

**MATHEMATICS**  
**METHODS**  
**UNITS 1&2**  
**Section Two:**  
**Calculator-assumed**  
**SOLUTIONS**

Question/Answer booklet

Semester Two Examination, 2020



**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					<b>100</b>

**Instructions to candidates**

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

This section has thirteen questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes.

(5 marks)

Question 9

For the events A and B,  $P(A) = 0.52$  and  $P(B) = 0.25$ .

Determine  $P(A \cup B)$  when

(1 mark)

(a) A and B are mutually exclusive.

(b)  $P(A \cup B) = 0.19$ .

(1 mark)

(c)  $P(A \cup B) = 0.33$ .

(2 marks)

(d) A and B are independent.

Solution	
<input checked="" type="checkbox"/> Specific behaviours	<input checked="" type="checkbox"/> correct probability
$P(A \cup B) = 0.52 \times 0.25 = 0.13$	
$= 0.64$	
$P(A \cup B) = 0.52 + 0.25 - 0.13$	
	<input checked="" type="checkbox"/> correct probability
	<input checked="" type="checkbox"/> $P(A \cup B)$

Solution	
<input checked="" type="checkbox"/> Specific behaviours	<input checked="" type="checkbox"/> correct probability
$P(A \cup B) = 1 - 0.33$	
$= 0.67$	

Solution	
<input checked="" type="checkbox"/> Specific behaviours	<input checked="" type="checkbox"/> correct probability
$P(A \cup B) = 0.19 + 0.25$	
$= 0.44$	

Solution	
<input checked="" type="checkbox"/> Specific behaviours	<input checked="" type="checkbox"/> correct probability
$P(A \cup B) = 0.52 + 0.25$	
$= 0.77$	

Solution	
<input checked="" type="checkbox"/> Specific behaviours	<input checked="" type="checkbox"/> correct probability
$P(A \cup B) = 0.52 + 0.25 - 0.13$	
$= 0.64$	

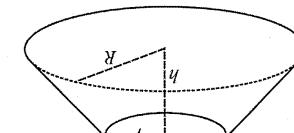
(b)

Use a calculus method to determine the value of  $x$  that maximises the volume of frustum  $F$  and state this maximum volume, rounding your answer to the nearest cm<sup>3</sup>. (4 marks)

Solution	
<input checked="" type="checkbox"/> Specific behaviours	<input checked="" type="checkbox"/> states root of derivative for maximum volume
$V(24) = 29952\pi \approx 94097$	<input checked="" type="checkbox"/> indicates derivative must equal zero
$x(312 - 13x) = 0$	<input checked="" type="checkbox"/> derivative is zero when
$x = 0, x = 24$	

Solution	
<input checked="" type="checkbox"/> Specific behaviours	<input checked="" type="checkbox"/> clear steps to obtain final expression
$r = x, R = 3x, h = 36 - x$	<input checked="" type="checkbox"/> substitutes and simplifies $(r^2 + R^2 + RH)$ term
$\therefore V = \frac{\pi}{3}(36-x)(x^2 + (3x)^2 + x(3x))$	<input checked="" type="checkbox"/> expresses $r, R$ and $h$ in terms of $x$
$= (12\pi - \frac{\pi x}{3})(13x^2 + 3x^3)$	<input checked="" type="checkbox"/> substitutes and simplifies $(r^2 + R^2)$ term
$= 156\pi x^2 - \frac{3}{3} \pi x^3$	<input checked="" type="checkbox"/> correct steps to obtain final expression

(a) Show that the volume of frustum  $F$  is  $156\pi x^2 - \frac{3}{3} \pi x^3$  cm<sup>3</sup>. (3 marks)



Consider frustum  $F$  where  $r = x$  cm.  
 $R = 3r$  and  $h = 36 - x$  cm.

The volume of such a solid is  $V = \frac{\pi h}{3}(r^2 + R^2 + rR)$ ,  
 where  $r$  is the radius of the smaller circle and  $R$  is the radius of the larger circle and  $h$  is the perpendicular distance between the two parallel circles.

The frustum shown at right is a truncated right cone.  
 The radius of the smaller circle and  $R$  is the radius of the larger circle and  $h$  is the perpendicular distance between the two parallel circles.

(5 marks)

Question 21  
METHODS UNITS 1&2

This section has thirteen questions. Answer all questions. Write your answers in the spaces provided.

65% (98 Marks)

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65% (98 Marks)

## Question 10

(6 marks)

The cost,  $C$  dollars, for a gigabyte of computer memory between the end of year 2006 ( $t = 0$ ) and the end of year 2016 ( $t = 10$ ) can be modelled by the equation  $C = 14.5(0.84)^t$ .

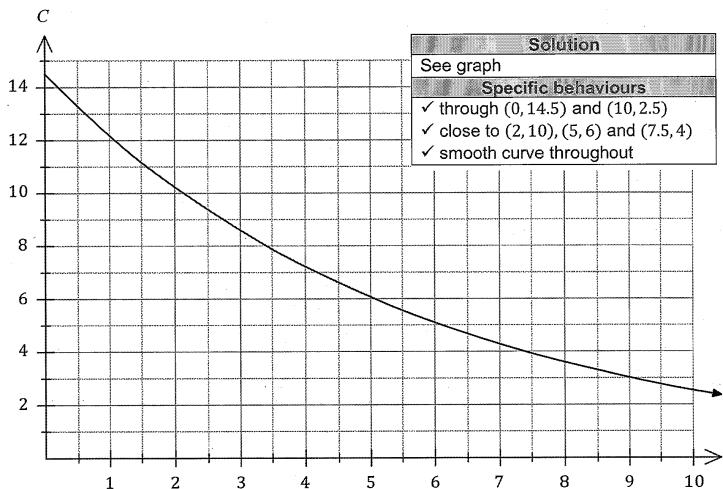
- (a) Calculate  $C$  at the end of year 2010.

Solution
$C(4) = \$7.22$
✓ correct cost, to nearest cent

(1 mark)

- (b) Draw the graph of  $C$  against  $t$  on the axes below.

(3 marks)



- (c) Assuming that the model continues to be valid, during which year will the cost of computer memory fall below \$1 per gigabyte?

(2 marks)

Solution
$C(t) = 1 \Rightarrow t = 15.3$
Hence during the year $2006 + 16 = 2022$
✓ correct value of $t$ ✓ correct year

## Question 20

(7 marks)

A reader bought 14 different novels, planning to read a selection of them when on holiday.

- (a) Determine the number of different combinations of novels the reader could choose from if they select

- (i) six novels.

(1 mark)

Solution
$\binom{14}{6} = 3\ 003$ combinations
✓ correct number

- (ii) five or six novels.

(2 marks)

Solution
$\binom{14}{5} + \binom{14}{6} = 2002 + 3003$ $= 5\ 005$ combinations
✓ ways to choose five ✓ correct number

Four of the 14 different novels are by the author Harper.

- (b) The reader makes a random selection of six novels. Determine the probability that

- (i) none of the novels selected are by Harper.

(2 marks)

Solution
Must choose from 10 not by Harper: $\binom{10}{6} = 210$
$P = \frac{210}{3003} = \frac{10}{143} \approx 0.0699$
✓ ways to choose ✓ correct probability

- (ii) one of the novels selected is by Harper.

(2 marks)

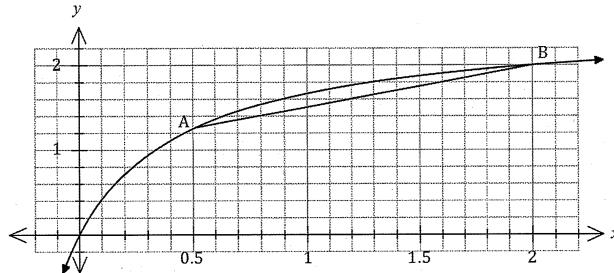
Solution
$P = \frac{\binom{10}{5}\binom{4}{1}}{3003} = \frac{252 \times 4}{3003} = \frac{48}{143} \approx 0.3357$
✓ indicates correct method ✓ correct probability



## Question 12

(8 marks)

Part of the graph of  $y = f(x)$  is shown below, where  $f(x) = \frac{5x}{2x+1}$ .



Points A and B lie on the curve and have  $x$ -coordinates of 0.5 and 2 respectively.

- (a) Draw the chord to the curve between A and B on the axes above and determine the gradient of this chord. (3 marks)

Solution	
$m = \frac{f(2) - f(0.5)}{2 - 0.5} = \frac{2 - 1.25}{1.5} = 0.5$	
Specific behaviours	
✓ draws chord on graph	
✓ correct $y$ -values	
✓ correct gradient	

Point C, with an  $x$ -coordinate of  $0.5 + h$ , lies on the curve between A and B. The gradient of the chord AC is  $m_{AC}$ .

- (b) Calculate  $m_{AC}$  for the values of  $h$  shown in the table below, recording the gradients in the table to 3 decimal places. (3 marks)

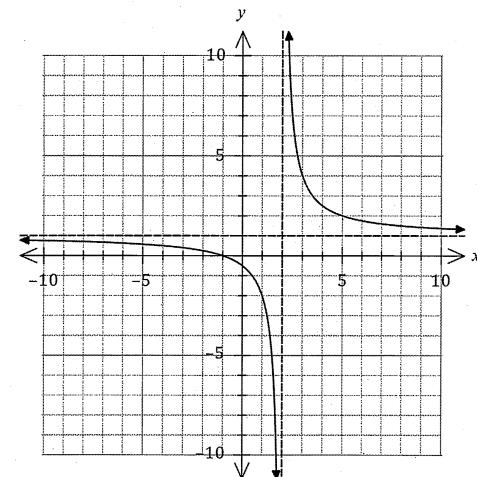
$h$	1	0.5	0.1	0.05	0.01
$m_{AC}$	0.625	<b>0.833</b>	<b>1.136</b>	<b>1.190</b>	<b>1.238</b>

Solution	
See table	
Specific behaviours	
✓ one correct gradient	
✓ at least three correct gradients	
✓ all correct gradients	

See next page

SN245-162-4

- (ii) Draw the graph of  $y = 1 + \frac{a}{x+b}$  on the axes below, clearly indicating any asymptotes. (3 marks)



Solution	
See graph	
Specific behaviours	
✓ both asymptotes	
✓ LHS, smooth curve, through (1, -2)	
✓ RHS, smooth curve, through (3, 4)	

See next page

SN245-162-4

- (a) Point A(11, -5) lies on the circumference of a circle with centre (-4, 3). Determine the equation of the circle. (3 marks)

Using given point:
$15^2 + (-8)^2 = k \Leftrightarrow k = 289 (= 17^2)$
Equation:
$(x + 4)^2 + (y - 3)^2 = 289$

Specific behaviours
forms equation using centre and constant
$\text{Find } \sqrt{(x+4)^2 + (y-3)^2} \text{ see}$
correct equation (any form)

- (c) Determine a limiting value for  $m_{AC}$  as  $A$  becomes very close to 0 and state what feature of the graph of  $y = f(x)$  this value represents. (2 marks)

Solution
As $n \rightarrow 0$ then $m_{AC} \rightarrow 1.25$ . This is the gradient of $y = f(x)$ at the point A.
✓ limiting value
✓ states gradient at the point A ( $0.5, 1.25$ )

Equation:
$(x + 4)^2 + (y - 3)^2 = k$
Using given point:
$15^2 + (-8)^2 = k \Leftrightarrow k = 289 (= 17^2)$

- (b) The graph of  $y = 1 + \frac{x}{a} + \frac{b}{x}$  passes through the points (1, -2) and (3, 4). Determine the value of each of the integer constants  $a$  and  $b$ . (3 marks)

Solution
$-2 = 1 + \frac{1+b}{a}, \quad 4 = 1 + \frac{3+b}{a}$
Solve simultaneously for $a = 3, b = -2$ .
✓ uses points to form two equations ✓ solves simultaneously ✓ both values correct

## Question 13

(7 marks)

A set of 175 undergraduates were asked to choose their electives for the following year.  
85 chose calculus, 58 chose statistics and 67 chose neither calculus nor statistics.

- (a) Determine how many of the undergraduates chose both calculus and statistics. (2 marks)

Solution	
$n(C \cup S) = 175 - 67 = 108$	
$n(C \cap S) = 85 + 58 - 108 = 35$	
Hence 35 chose both electives.	
Specific behaviours	
✓ indicates union of sets	
✓ correct number	

- (b) Determine the probability that a randomly chosen undergraduate from the set chose

- (i) statistics.

Solution	
$P(S) = \frac{58}{175} \approx 0.3314$	(1 mark)
Specific behaviours ✓ correct probability	

- (ii) statistics but not calculus.

Solution	
$P(S \cap C) = \frac{58 - 35}{175} = \frac{23}{175} \approx 0.1314$	(1 mark)
Specific behaviours ✓ correct probability	

- (iii) statistics given that they chose calculus.

Solution	
$P(S C) = \frac{35}{85} = \frac{7}{17} \approx 0.4118$	(1 mark)
Specific behaviours ✓ correct probability	

- (c) Use your answers above to explain whether the choice of statistics and calculus electives is independent for these undergraduates. (2 marks)

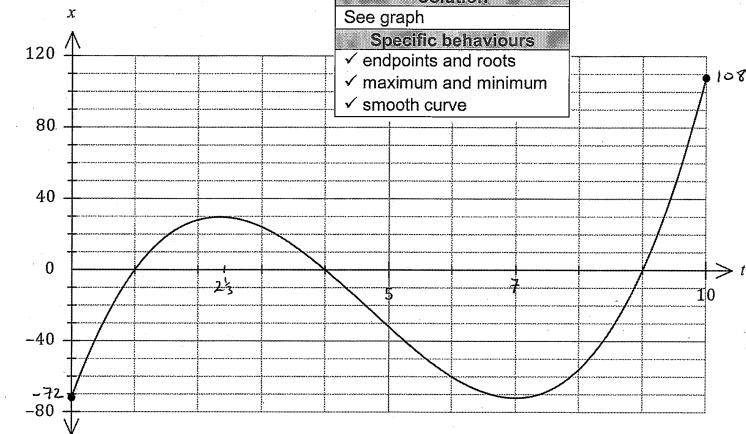
Solution	
Choice is not independent, as $P(S) \neq P(S C)$ .	
<i>(Undergraduates are more likely to choose statistics if they have chosen calculus.)</i>	
Specific behaviours ✓ states not independent ✓ explanation using existing probabilities	

See next page

SN245-162-4

- (c) Use the axes below to sketch the displacement of the body over the given domain. (3 marks)

Solution	
See graph	
Specific behaviours	
✓ endpoints and roots	
✓ maximum and minimum	
✓ smooth curve	



- (d) State the number of times the body passed through 0 and determine the minimum speed and maximum speed of the body as it passed through this point. (3 marks)

Solution	
Passes through 0 when $t = 1, 4, 9$ s - on 3 occasions.	
$v(1) = 48$ $v(4) = -30$ $v(9) = 80$	
Hence minimum speed is 30 cm/s and maximum speed is 80 cm/s.	
Specific behaviours	
✓ correct number of times	
✓ minimum speed (must be +ve)	
✓ maximum speed	

See next page

SN245-162-4

(a) Obtain an expression for the velocity of the body in the form  $v(t) = (at + b)(ct + d)$ , where  $a, b, c$  and  $d$  are integer constants. (3 marks)

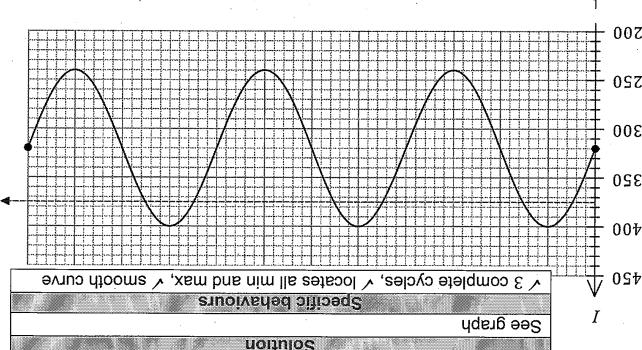
$$I = 320 + 80 \sin\left(\frac{\pi t}{10}\right)$$

$$x(t) = 2t^3 - 28t^2 + 98t - 72, \quad 0 \leq t \leq 10.$$

Question 17  
A small body is moving in a straight line. Relative to a fixed point  $O$ , it has a displacement of  $x$  cm at time  $t$  seconds given by

When an alternating current is used to power a light globe, the intensity of light emitted from the globe,  $I$  lumens, varies with time  $t$  milliseconds and can be modelled by the formula

(a) Draw the graph of  $I$  against  $t$  on the axes below for  $0 \leq t \leq 60$ . (3 marks)



(a) Draw the graph of  $I$  against  $t$  on the axes below for  $0 \leq t \leq 60$ . (3 marks)

(b) State the period of  $I$ . (1 mark)

5 10 15 20 25 30 35 40 45 50 55 60

(c) Determine the percentage of each cycle that the intensity of light exceeds 375 lumens. (2 marks)

Solution	
Period is 20 milliseconds	
Correct Period	
$I = 375 \Rightarrow t = 7.587, 2.413$	
$7.587 - 2.413 = 5.174$	
$\frac{5.174}{20} \times 100 \approx 26\%$	
Indicates interval in ms	
Correct percentage to nearest whole number	

(c) Determine the percentage of each cycle that the intensity of light exceeds 375 lumens. (2 marks)

Solution	
Period is 20 milliseconds	
Correct Period	
$I = 375 \Rightarrow t = 7.587, 2.413$	
$7.587 - 2.413 = 5.174$	
$\frac{5.174}{20} \times 100 \approx 26\%$	
Indicates interval in ms	
Correct percentage to nearest whole number	

(c) Determine the percentage of each cycle that the intensity of light exceeds 375 lumens. (2 marks)

(i) the displacement of the body at the instant(s) that it is stationary. (3 marks)

Solution	
$v(0) = 98 \text{ cm/s}$	
Correct Velocity	
$v(t) = 98$	

(i) the initial velocity of the body. (1 mark)

(i) mark)

Solution	
$v(0) = 98 \text{ cm/s}$	
Correct Velocity	
$v(t) = 98$	

(ii) the displacement of the body at the instant(s) that it is stationary. (3 marks)

Solution	
$x(7) = -72 \text{ cm}$	
Correct Displacement	
$x(7) = -72$	

(ii) the displacement of the body at the instant(s) that it is stationary. (3 marks)

Solution	
$x\left(\frac{3}{7}\right) = 29.63 \text{ cm}$	
Times When Stationary	
$x\left(\frac{3}{7}\right) = 29.63$	

(ii) the displacement of the body at the instant(s) that it is stationary. (3 marks)

Solution	
$x(t) = 0 \Leftrightarrow t = \frac{7}{3}, t = 7$	
Specific Behaviours	
$x(t) = 0$	

(ii) the displacement of the body at the instant(s) that it is stationary. (3 marks)

Solution	
$x(7) = -72 \text{ cm}$	
Both Correct	
$x(7) = -72$	

(ii) the displacement of the body at the instant(s) that it is stationary. (3 marks)

## Question 15

(7 marks)

A farmer was treating a large area of land for an invasive weed. The area treated on the first day was  $275 \text{ m}^2$ . Over the following months more resources were utilised so that the area treated each day was 7.5% more than the previous day.

- (a) Determine the area treated on the 28
- <sup>th</sup>
- day.

(2 marks)

Solution
$T_{28} = 275(1.075)^{(28-1)}$ = $1938 \text{ m}^2$
Specific behaviours
✓ indicates use of general term formula ✓ correct area

The cost of the treatment was 35.8 cents per square metre.

- (b) On which day did the cost of the days treatment first exceed \$10 000?

(3 marks)

Solution
$C_n = 0.358 \times 275(1.075)^{(n-1)}$ = $98.45(1.075)^{(n-1)}$
$98.45(1.075)^{(n-1)} \geq 10000$ $n \geq 65$ On day 65.
Specific behaviours
✓ adjusts sequence ✓ indicates equation/inequality to solve ✓ correct day

- (c) Determine, to the nearest ten dollars, the total cost of the first 15 days of treatment.

(2 marks)

Solution
$S_{15} = \frac{98.45(1 - 1.075^{15})}{1 - 1.075}$ ≈ \$2 570
Specific behaviours

✓ indicates use of sum formula  
✓ total cost, rounded as required  $\$2571.35$   
✓ rounded to nearest ten dollars

## Question 16

(8 marks)

A farm grows two varieties of apples - Fuji and Gala. 42% of all apples are grown in orchard A, 36% in orchard B and the remainder in orchard C. The proportion of Fiji apples that are grown in orchards A, B and C are 25%, 30% and 35% respectively. After harvesting, the farm stores all the apples together in a large silo before using them to make apple juice.

- (a) Determine the probability that an apple chosen at random from the silo is

- (i) a Fuji grown in orchard C.

(2 marks)

Solution
$P(C) = 1 - 0.42 - 0.36 = 0.22$ $P(C \cap F) = 0.22 \times 0.35 = 0.077$

Specific behaviours
✓ proportion grown in C ✓ correct probability

- (ii) a Gala.

(3 marks)

Solution
$P(A \cap G) = 0.42 \times 0.75 = 0.315$ $P(B \cap G) = 0.36 \times 0.7 = 0.252$ $P(C \cap G) = 0.22 \times 0.65 = 0.143$
$P(G) = 0.315 + 0.252 + 0.143 = 0.71$
Specific behaviours

Specific behaviours
✓ at least one correct proportion ✓ all correct proportions ✓ correct probability

- (b) Given that an apple selected at random is a Fuji, determine the probability that it was grown in orchard A.

(3 marks)

Solution
$P(F) = 1 - 0.71 = 0.29$
$P(A \cap F) = 0.42 \times 0.25 = 0.105$
$P(A F) = \frac{0.105}{0.29} = \frac{21}{58} \approx 0.362$
Specific behaviours

Specific behaviours
✓ $P(F)$ ✓ $P(A \cap F)$ ✓ correct probability