

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Important note to candidates

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Formula Sheet (retained from Section One)

Materials required/recommended for this section
To be provided by the supervisor

Working time for section: one hundred minutes
Reading time before commencing work: ten minutes

Time allowed for this section

Your name

In words

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Student Number: In figures

Calculator-assumed
Section Two:

MATHEMATICS
UNIT 3
METHODS
QUESTION/ANSWER BOOKLET

QUESTION/ANSWER BOOKLET

Semester One Examination, 2016



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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	48	35
Section Two: Calculator-assumed	13	13	100	101	65
		Total		149	100

Additional working space

Question number: _____

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

- (a) Assuming that the rate of growth of the population can be modelled by the equation
- $$\frac{dp}{dt} = kp$$
- where P is the population of Australia at time t months, determine the value of the constant k .
 A recent news report said that it took 34 months for the population of Australia to increase from 23 to 24 million people.
 (5 marks)
- Question 9

Solution	$p = p_0 e^{kt}$
$24 = 23e^{kt}$	
$k = 0.001252$	
$t = 32.6 \text{ months}$	
Specific behaviours	✓ sets up correct values in equation, using k to at least 3sf ✓ solves for t

(b)

- Assuming the current rate of growth continues, how long will it take for the population to increase from 24 million to 25 million people?
 (2 marks)

Solution	$p = p_0 e^{kt}$
$24 = 23e^{kt}$	
$k = 0.001252$	
$t = 32.6 \text{ months}$	
Specific behaviours	✓ uses growth and decay equation ✓ substitutes correctly ✓ solves for k

(b)

- Assuming the current rate of growth continues, how long will it take for the population to increase from 24 million to 25 million people?
 (2 marks)

Working time for this section is 100 minutes.

- This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.
- Section Two: Calculator-assumed**
Section Three: Calculator-assumed
 65% (101 Marks)

Additional working space
 Question number: _____

Question 10

(7 marks)

A small object is moving in a straight line with acceleration $a = 6t + k \text{ ms}^{-2}$, where t is the time in seconds and k is a constant. When $t = 1$ the object was stationary and had a displacement of 4 metres relative to a fixed point O on the line. When $t = 2$ the object had a velocity of 1 ms^{-1} .

- (a) Determine the value of k and hence an equation for the velocity of the object at time t . (4 marks)

Solution

$$v = 3t^2 + kt + c$$

$$t = 1, 3 + k + c = 0$$

$$t = 2, 12 + 2k + c = 1$$

$$k = -8$$

$$c = 5$$

$$v = 3t^2 - 8t + 5$$

Specific behaviours

- ✓ antidifferentiates acceleration, adding constant
- ✓ derives simultaneous equations from information
- ✓ solves equations
- ✓ writes velocity equation

- (b) Determine the displacement of the object when $t = 2$. (3 marks)

Solution

$$s = t^3 - 4t^2 + 5t + c$$

$$t = 1, 4 = 1 - 4 + 5 + c$$

$$c = 2$$

$$s = t^3 - 4t^2 + 5t + 2$$

$$s(2) = 8 - 16 + 10 + 2$$

$$= 4 \text{ m}$$

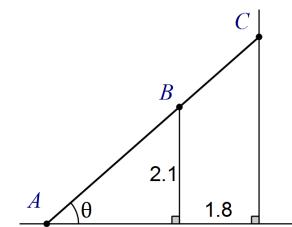
Specific behaviours

- ✓ antidifferentiates velocity
- ✓ determines constant
- ✓ evaluates displacement

Question 21

(7 marks)

A vertical wall, 2.1 metres tall, stands on level ground and 1.8 metres away from the wall of a house. A ladder, of negligible width, leans at an angle of θ to the ground and just touches the ground, wall and house, as shown in the diagram.



- (a) Show that the length of the ladder, L , is given by $L = \frac{2.1}{\sin \theta} + \frac{1.8}{\cos \theta}$. (3 marks)

Solution

$$\sin \theta = \frac{2.1}{AB} \Rightarrow AB = \frac{2.1}{\sin \theta}, \cos \theta = \frac{1.8}{BC} \Rightarrow BC = \frac{1.8}{\cos \theta}$$

$$L = AB + BC$$

$$= \frac{2.1}{\sin \theta} + \frac{1.8}{\cos \theta}$$

Specific behaviours

- ✓ shows on diagram and writes length of AB
- ✓ shows on diagram and writes length of BC

- (b) Use a calculus method to determine the length of the shortest ladder that can touch the ground, wall and house at the same time. (4 marks)

Solution

$$L = 2.1(\sin \theta)^{-1} + 1.8(\cos \theta)^{-1}$$

$$\frac{dL}{d\theta} = -2.1\cos \theta (\sin \theta)^{-2} - 1.8(-\sin \theta)(\cos \theta)^{-2}$$

$$= \frac{1.8\sin^3 \theta - 2.1\cos^3 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$\frac{dL}{d\theta} = 0 \Rightarrow 1.8\sin^3 \theta - 2.1\cos^3 \theta = 0$$

$$\tan^3 \theta = \frac{2.1}{1.8} \Rightarrow \theta = \tan^{-1} \sqrt[3]{\frac{2.1}{1.8}} \approx 0.8111$$

$$L(0.8111) \approx 5.51 \text{ metres}$$

Specific behaviours

- ✓ shows first derivative of L (may use CAS, but must show key results)
- ✓ solves derivative equal to 0
- ✓ obtains acute angle solution
- ✓ substitutes into equation to obtain minimum length

METHODS UNIT 3		CALCULATOR-ASSUMED		Question 20							
(a)	Complete the table below.	x	0	1	2						
(b)	Consider the function $f(t) = \frac{t - 4}{2}$ and the function $A(x) = \int f(t) dt$. Calculate all values correctly.	A(x)	0	-1.75	-3	-3.75	-4	-3.75	-3	0	1.75
(c)	On the axes below, sketch the graph of $y = A(x)$ for $0 \leq x \leq 6$.	x	2	4	6	y	4	2	-2	-4	
(d)	Determine one of these samples has no students who study advanced mathematics.	Solution		See graph							
(e)	If $n = 22$, determine the probability that three or more than three of the students in the sample study advanced mathematics.	Solution		P(X = 3) = 0.2370							
(f)	If $n = 22$, determine the probability that none of the students in the sample study advanced mathematics.	Solution		P(X > 4) = 0.4248							
(g)	None of the students in the sample study advanced mathematics.	Solution		P(X = 0) = 0.0280							
(h)	One of these samples has no students who study advanced mathematics.	Solution		P(Y > 1) = 0.247							
(i)	If ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study advanced mathematics.	Solution		Y ~ B(10, 0.028)							
(j)	When $A(x) = 0$	Solution		x = 4							
(k)	Determine if ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study advanced mathematics.	Solution		P(Y > 1) = 0.247							
(l)	The function $A(x)$ in terms of x .	Solution		A(x) = $\int \frac{t - 4}{2} dt = \frac{x^2}{4} - 2x$							
(m)	States function in terms of x .	Solution		A(x) = $\frac{x^2}{4} - 2x$							
(n)	States specific behaviours	Solution		evaluates probability							
(o)	States binomial distribution with parameters	Solution		specific behaviours							
(p)	States binomial distribution	Solution		evaluates probability							
(q)	States parameters of binomial distribution	Solution		evaluates probability							
(r)	Describes the distribution of X .	Solution		X ~ B(n, 0.15) - binomial distribution with n trials and $p = 0.15$.							
(s)	Variable X is the number of those in the sample who study advanced mathematics.	Solution		A random sample of n students is selected from all Year 12s in this country, and the random variable X is known that 15% of Year 12 students in a large country study advanced mathematics.							
(t)	It is known that 15% of Year 12 students in a large country study advanced mathematics.	Solution		it is known that 15% of Year 12 students in a large country study advanced mathematics.							
(u)	Question 11	Question 20		METHODS UNIT 3							
(v)	Calculator	CALCULATOR-ASSUMED		METHODS UNIT 3							

Question 12

(8 marks)

The height of grain in a silo, initially 0.4 m, is increasing at a rate given by $h'(t) = 0.55t - 0.05t^2$ for $0 \leq t \leq 11$, where h is the height of grain in metres and t is in hours.

- (a) At what time is the height of grain rising the fastest? (2 marks)

Solution

$$h''(t) = 0.55 - 0.1t$$

$$0.55 - 0.1t = 0 \Rightarrow t = 5.5 \text{ hours}$$

Specific behaviours

- ✓ differentiates rate of change
- ✓ solves derivative equal to zero to obtain time.

- (b) Determine the height of grain in the silo after 11 hours. (3 marks)

Solution

$$\Delta h = \int_0^{11} 0.55t - 0.05t^2 dt$$

$$\approx 11.09$$

$$h = 11.09 + 0.4$$

$$= 11.49 \text{ m}$$

Specific behaviours

- ✓ shows integral of rate of change
- ✓ evaluates integral to obtain change in height
- ✓ adds initial height

- (c) Calculate the time taken for the grain to reach a height of 4.45 m. (3 marks)

Solution

$$\Delta h = 4.45 - 0.4 = 4.05$$

$$\Delta h = \int_0^k 0.55t - 0.05t^2 dt$$

$$= \frac{11k}{40} - \frac{k^3}{60}$$

$$\frac{11k}{40} - \frac{k^3}{60} = 4.05 \Rightarrow k = 4.5 \text{ hours}$$

Specific behaviours

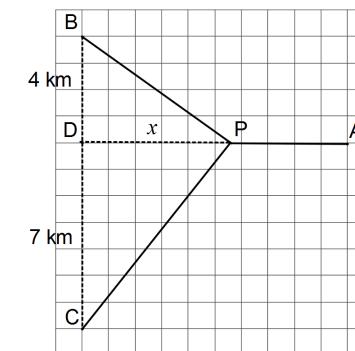
- ✓ determines change in height
- ✓ evaluates integral using constant
- ✓ solves equation, ignoring solutions outside domain

Question 19

(7 marks)

Three telecommunication towers, A, B and C, each need underground power cable connections directly to a new power station, P, that is to be built x km from depot D on a 10 km road running east-west between D and A.

Tower B lies 4 km due north of depot D and tower C lies 7 km south of the depot, as shown in the diagram.



- (a) Determine an expression for the total length of underground cable required to connect A, B and C directly to P. (2 marks)

Solution

$$L = \sqrt{16 + x^2} + \sqrt{49 + x^2} + 10 - x$$

Specific behaviours

- ✓ uses Pythagoras' theorem for BP and CP
- ✓ determines correct expression

- (b) Show that the minimum length of cable occurs when $\frac{x}{\sqrt{16+x^2}} + \frac{x}{\sqrt{49+x^2}} = 1$. (3 marks)

Solution

$$\frac{dL}{dx} = \frac{1}{2}(2x)(16+x^2)^{-0.5} + \frac{1}{2}(2x)(49+x^2)^{-0.5} - 1$$

$$\frac{dL}{dx} = 0 \Rightarrow \frac{x}{\sqrt{16+x^2}} + \frac{x}{\sqrt{49+x^2}} = 1$$

Specific behaviours

- ✓ uses chain rule to determine derivative
- ✓ shows that derivative must equal zero
- ✓ simplifies equation to required result

- (c) Determine the minimum length of cable required, rounding your answer to the nearest hundred metres. (2 marks)

Solution

$$x \approx 3.025536$$

$$L = \sqrt{16 + x^2} + \sqrt{49 + x^2} + 10 - x \Big|_{x \approx 3.025536} \approx 19.6 \text{ km}$$

Specific behaviours

- ✓ solves equation from (b)
- ✓ substitutes to find length

Question 14

(14 marks)

- (a) Determine the mean of a Bernoulli distribution with variance of 0.24.

(3 marks)

Solution
$p(1-p) = 0.24$
$p = 0.4, 0.6 \Rightarrow$ mean is either 0.4 or 0.6
Specific behaviours
✓ writes variance equation
✓ solves equation
✓ states both values of p are possible means

- (b) A Bernoulli trial, with probability of success
- p
- , is repeated
- n
- times. The resulting distribution of the number of successes has an expected value of 5.76 and a standard deviation of 1.92. Determine
- n
- and
- p
- . (4 marks)

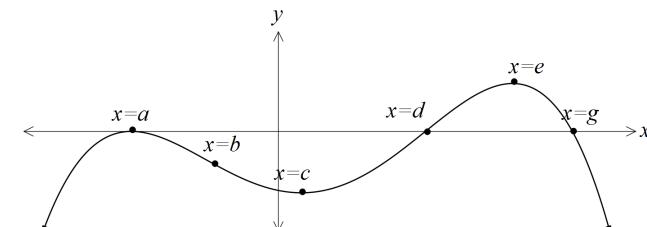
Solution
$X \sim B(n, p)$
$np = 5.76$
$np(1-p) = 1.92^2$
$1-p = 1.92^2 \div 5.76 = 0.64$
$p = 0.36$
$n = 16$
Specific behaviours
✓ identifies distribution of successes as binomial
✓ states equation for mean
✓ states equation for variance (or standard deviation)
✓ solves equations for n and p

See next page

Question 17

(8 marks)

The graph of $y = f'(x)$, the derivative of a polynomial function f , is shown below. The graph of $y = f(x)$ has stationary points when $x = a, x = c$ and $x = e$, points of inflection when $x = b$ and $x = d$, and roots when $x = a, x = d$ and $x = g$, where $a < b < c < d < e < g$.



- (a) For what value(s) of
- x
- does the graph of
- $y = f(x)$
- have a point of inflection? (1 mark)

Solution
$x = a, c, e$
Specific behaviours
✓ states all three values

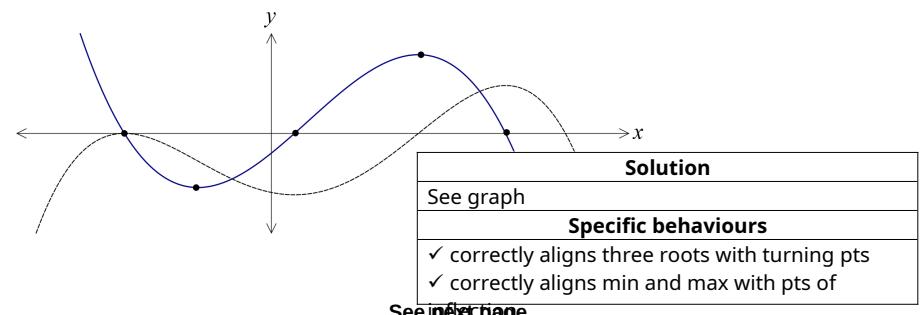
- (b) Does the graph of
- $y = f(x)$
- have a local maximum? Justify your answer. (2 marks)

Solution
Yes, as x increases through $x = g$, the gradient of f changes from positive to zero to negative, indicating a local maximum.
Specific behaviours
✓ responds yes, indicating when $x = g$
✓ explains reason

- (c) Does the graph of
- $y = f(x)$
- have a horizontal point of inflection? Justify your answer. (2 marks)

Solution
Yes, as x increases through $x = a$, the gradient of f changes from negative to zero to negative, indicating a horizontal pt of inflection.
Specific behaviours
✓ responds yes, indicating when $x = a$

- (d) On the axis below, sketch a possible graph of
- $y = f''(x)$
- . The graph of
- $y = f'(x)$
- is shown with a broken line for your reference. (3 marks)



See next page

- (c) The probability that a student misses their bus to school is 0.2, and the probability that they miss the bus on any day is independent of whether they missed it on the previous day.
- Over five consecutive weekdays, what is the probability that the student only misses the bus on Tuesday?

(ii) misses the bus at least twice? (2 marks)

Solution	
$0.2 \times 0.8^4 = 0.08192$	✓ uses 0.8 for not catching bus ✓ determines probability
$P(Y \geq 0 Y \leq 1) = \frac{0.2}{1} = 0.2$	✓ correct denominator ✓ correct numerator and simplification
$P(Y \geq 0 Y \leq 1) = 0.2$	(a) Calculate $P(Y \geq 0 Y \leq 1)$ (2 marks)

(iii) misses the bus on Tuesday and on two other days? (3 marks)

Solution	
$X \sim B(5, 0.2)$	$= -0.8 - 0.2 + 0.1 + 0.4$
$E(Y) = (-2)(0.4) + (-1)(0.2) + (0)(0.1) + (1)(0.1) + (2)(0.2)$	✓ forms products to obtain expected value
$E(Y) = 0.5$	(ii) Calculate $E(Y)$ (1 mark)

Solution	
$P(Y = 2) = 0.1536$	$P = 0.2 \times 0.1536$
$P = 0.2 \times P(Y = 2)$ where $Y \sim B(4, 0.2)$	$P(Y = 2) = 0.03072$
$P(Y = 2)$	\checkmark evaluates binomial situation for other two days
$Specific behaviours$	\checkmark determines probability

Solution	
$P(Y = 2)$	$P = 0.2 \times P(Y = 2)$ where $Y \sim B(4, 0.2)$
$P(Y = 2)$	\checkmark evaluates binomial situation for other two days
$Specific behaviours$	\checkmark determines probability

Solution	
$P(Y = 2)$	$P = 0.2 \times 0.1536$
$P = 0.03072$	\checkmark evaluates binomial situation for other two days
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Solution	
$P(Y = 2)$	$P = 0.2 \times 0.1536$
$P = 0.03072$	\checkmark evaluates binomial situation for other two days
$Specific behaviours$	\checkmark determines probability

Solution	
$P(Y = 2)$	$P = 0.2 \times 0.1536$
$P = 0.03072$	$\$

Question 15

(9 marks)

$$x(t) = \frac{t^2 + 3}{t + 1}, t \geq 0$$

A particle moves in a straight line according to the function $x(t) = \frac{t^2 + 3}{t + 1}$, $t \geq 0$, where t is in seconds and x is the displacement of the particle from a fixed point O , in metres.

- (a) Determine the velocity function, $v(t)$, for the particle. (2 marks)

Solution

$$\begin{aligned} v(t) &= \frac{d}{dt} x(t) \\ &= \frac{t^2 + 2t - 3}{(t + 1)^2} \end{aligned}$$

Specific behaviours

- ✓ relates velocity to first derivative of displacement wrt t
- ✓ determines the first derivative

- (b) Determine the displacement of the particle at the instant it is stationary. (2 marks)

Solution

$$v(t) = 0 \Rightarrow t^2 + 2t - 3 = 0 \Rightarrow t = -3, 1$$

$$x(1) = 2 \text{ m}$$

Specific behaviours

- ✓ solves $v=0$ over domain
- ✓ determines displacement

- (c) Show that the acceleration of the particle is always positive. (2 marks)

Solution

$$\begin{aligned} a(t) &= \frac{d}{dt} v(t) \\ &= \frac{8}{(t+1)^3} \\ t \geq 0 &\Rightarrow a(t) \geq 0 \end{aligned}$$

Specific behaviours

- ✓ determines acceleration function
- ✓ shows that acceleration always positive for $t \geq 0$

- (d) After five seconds, the particle has moved a distance of k metres.

- (i) Explain why $k \neq \int_0^5 v(t) dt$. (1 mark)

Solution

Integral will calculate change in displacement, but since particle turned around after one second, this will not be the same as distance travelled.

Specific behaviours

✓ explains change in displacement not distance travelled in this instance

- (ii) Calculate k . (2 marks)

Solution

$$\begin{aligned} k &= \left| \int_0^1 v(t) dt \right| + \int_1^5 v(t) dt \\ &= 1 + \frac{8}{3} \\ k &= \frac{11}{3} \end{aligned}$$

Specific behaviours

- ✓ separates into two integrals
- ✓ determines k