PERTH MODERN SCHOOL
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Independent Public School

Year 12 Methods
TEST 7 June 2019

TIME: 45 minutes working

Calculator Assumed
44 Marks 6 Questions

Name: _____ Teacher: _____

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 **(5 marks)**

(a) Differentiate $x \sin x$ (2 marks)

(b) Hence find $\int_0^{\frac{\pi}{2}} x \cos x \, dx$ **using** the result in(a) above. (3 marks)

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Question 2 **(3 marks)**

Determine the *x*-coordinates of all points on the graph of $f(x)=2\cos(x)+x$ for $-\pi \le x \le \pi$ where the tangent line is horizontal. (Justify your answers)

Question 3 **(7 marks)**

A survey conducted by a local bank shows that 75% of its customers use an ATM at least once a month.

(a) Find the probability that in a random sample of 8 customers, **at least 75%** of them use an ATM machine at least once a month. (2 marks)

(b) If the random variable X follows a binomial distribution with n=12 and p=0.75, what is the mean of this distribution and what is P¿X≥mean¿? (3 marks)

(c) If the sample size became very large what would you expect P($X \geq$ mean) to approach? Briefly explain your answer. (2 marks)

Question 4 **(10 marks)**

The discrete random variable X can only take the values 2, 3 or 4. For these values the cumulative distribution function is defined by

$$P(X \leq x) = \frac{(x+k)^2}{25}$$

for $x = 2, 3 \wedge 4$, where k is a positive constant integer.

(a) Find the value for k. (3 marks)

(b) Complete the following table for X. (3 marks)

X	2	3	4
$P(X \leq x)$			
$P(X = x)$			

(c) Hence find $E(X)$ and $SD(X)$. (2 marks)

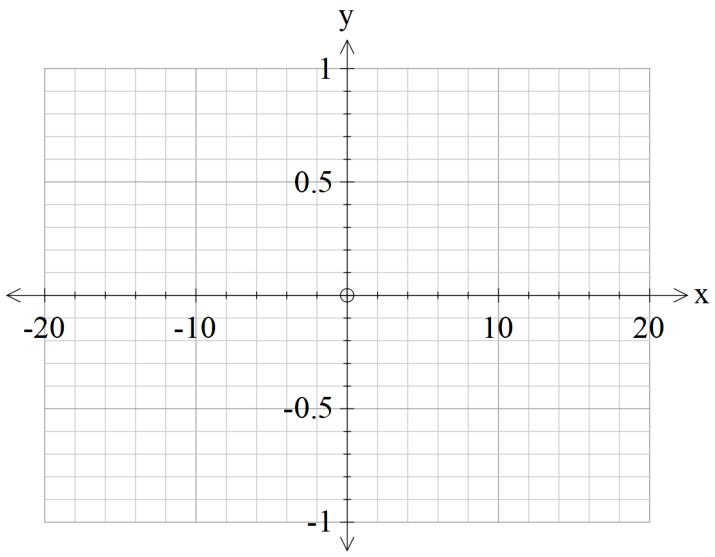
(d) Calculate $Var(3-2X)$ giving your answer to 2 decimal places. (2 marks)

Question 5 **(8 marks)**

Consider the function $f(x)=\frac{1-\cos x}{x}$ where x is in radians.

a) Sketch $f(x)$ on the axes below for $-20 \le x \le 20$ on the axes below. Clearly label undefined points (if any).

(3 marks)



b) As x approaches zero from the positive side, state the value that $f(x)$ approaches.

(1 mark)

c) As x approaches zero from the negative side, state the value that $f(x)$ approaches.

(1 mark)

d) Use the above to define a value for $f(x)$ as x approaches zero, that is the following limit $\lim_{x\to 0}\frac{1-\cos x}{x}$.

(1 mark)

It can be shown that $\frac{d}{dx}(\cos x)=-\cos x\lim_{h\to 0}\frac{1-\cosh}{h}-\sin x\lim_{h\to 0}\frac{\sinh}{h}$.

e) Using the fact that $\lim_{h\to 0}\frac{\sinh}{h}=1$ and the above results, show that $\frac{d}{dx}(\cos x)=-\sin x$.

(2 marks)

Question 6 **(11 marks)**

A game is played by throwing two standard six-sided dice into the air once. The sum of the uppermost numbers are added together and if the sum is greater than 8 the player wins $5.

Determine:

a) the probability of winning $5 in one game. (2 marks)

b) the probability of winning exactly $15 in 5 games. (3 marks)

c) the probability of winning at least $15 in at most 5 games. (3 marks)

d) the minimum number of games to be played so that the probability of winning at least $15 is greater than 0.47. (Justify) (3 marks)

Working out space

Working out space