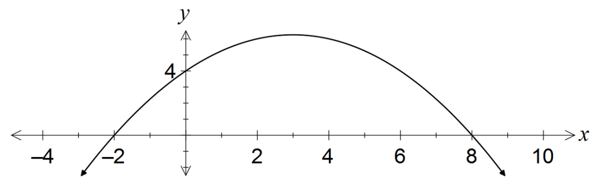
Question 1 **(7 marks)**

(a) Part of the graph of $y = ax^2 + bx + 4$ is shown below.



Determine the values of the coefficients a and b. (3 marks)

Solution
$y = a(x+2)(x-8)$ $(0,\ 4) \Rightarrow 4 = a(2)(-8) \Rightarrow a = -\frac{1}{4}$ $y = -\frac{1}{4}\left(x^2 - 6x - 16\right)$ $= -\frac{1}{4}x^2 + \frac{3}{2}x + 4 \Rightarrow a = -\frac{1}{4},\ b = \frac{3}{2}$
Specific behaviours
✓ uses roots to express in factored form ✓ uses y-intercept to find a ✓ expands and states b

(b) A quadratic has equation $y = x^2 - 6x + 2$. Determine

(i) the coordinates of its turning point. (2 marks)

Solution
$x^2 - 6x + 2 = (x-3)^2 - 3^2 + 2$ $= (x-3)^2 - 7$ At $(3, -7)$
Specific behaviours
✓ completes square, or uses $x = -b/2a$ ✓ states coordinates

(ii) the exact values of the zeros of the quadratic. (2 marks)

Solution
$(x-3)^2 - 7 = 0$ $x - 3 = \pm\sqrt{7}$ $x = 3 \pm \sqrt{7}$
Specific behaviours
✓ uses quadratic formula or completes square ✓ states both roots in exact form

(c) Is it possible to bend a 12 cm length of wire to form the perpendicular sides of a right angled triangle with area 20cm?
(4 marks)

Solution

Height = *x* , Base = *12-x*

Area: 20 ¿ $\frac{1}{2}x(12-x)$

$40=\frac{1}{2}x(12-x)$

$12x-x^2-40=0$
$x^2-12x+40=0$

Discriminant = $(-12)^2-4(1)(40)$
¿ $-16\, which\, is<0$
There are no real solutions, indicating this situation is impossible.

Specific behaviours

- ✓ Use of x and $12-x$ correctly.
- ✓ Substituting into area of a triangle formula
- ✓ Correct general formula
- ✓ Use of discriminant to indicate no real solutions.

Note: 1 mark if they indicate that they would need two numbers which add to 12 and multiply to 40, 1 mark if they try some values to show that it is not possible, 1 mark if they set out a table in an orderly manner and reach the maximum of 6x6 giving 36 (ie max area is 18sq m) and 1 mark for demonstrating that this is the maximum by extending the table etc. Basically use your professional judgement and (generously) allocate a mark out of 4 accordingly.

Question 5 (1.1.24) **(1, 1, 2, 2 = 6 marks)**

Suppose $G(x)=\frac{2x-3}{x-4}$.

a) Evaluate $G(2)$

Solution
✓ $\frac{-1}{2}$

b) Find a value of x such that $G(x)$ does not exist.

Solution
✓ $x=4$

c) Find $G(x+2)$ in simplest form.

Solution
$g(x+2)=(2(x+2)-3)/¿$ $g(x+2)=\frac{2x+1}{x-2}$
Specific behaviours
✓Substitute correctly   ✓ Answer

d) Find x such that $G(x)=-3$.

Solution
$-3=\frac{2x-3}{x-4}$ $x=3$
Specific behaviours
✓Sets equation up correctly   ✓ Answer

Question 2 **(8 marks)**

(a) A circle of radius 5 has its centre at (6, -4).

(i) Determine the equation of this circle. (2 marks)

Solution
$(x-6)^2+(y+4)^2=25$
Specific behaviours
✓ uses standard circle form with correct radius   ✓ correct equation

(ii) State, with justification, whether the point (9, -8) lies on the circle. (1 mark)

Solution
$(9-6)^2+(-8+4)^2=9+16=25 \Rightarrow$ Does lie on circle
Specific behaviours
✓ substitutes point into equation from (a) and interprets

(b) Determine the centre and radius of the circle with equation $x^2+y^2-4x+6y+9=0$
.
(3 marks)

Solution
$(x-2)^2-4+(y+3)^2-9+9=0$ $(x-2)^2+(y+3)^2=4=2^2$ Hence centre at (2, -3) and radius 2
Specific behaviours
✓ factors x terms   ✓ factors y terms   ✓ states centre and radius

(c) Find the equation of the curve drawn below. (3 marks)

Solution
$y=k\sqrt{x+b}+c$ $y=2\sqrt{x+3}-2$
Specific behaviours
✓ $a=3$ ✓ $k=2$ ✓ $c=-2$

Question 3 (1.1.14) **(2, 2, 2 = 6 marks)**

A rectangular hyperbola has asymptotes with equation $x=-2$ and $y=4$.

a) Write two possible equations for this function

Solution
$y=\frac{a}{x+2}+4$ so a could be any number eg $y=\frac{1}{x+2}+4$ and $y=\frac{-1}{x+2}+4$
Specific behaviours
✓✓ two possible equations

b) Write the equation of this function if it has a *y*-intercept at (0,5)

Solution
$5=\frac{a}{0+2}+4$ so a=2
Specific behaviours
✓ substitutes correctly into equation ✓a=2

c) Write the equation of this function if it passes through the point (3,5)

Solution
$5=\frac{a}{3+2}+4$ so a=5 therefore y¿ $\frac{5}{x+2}+4$
Specific behaviours
✓ substitutes correctly into equation ✓ states equation

Question 4 (1.1.24) **(1, 2, 1, 2 = 6 marks)**

a) Given $f(x)=x^2-2x$

i) What type of correspondence does f show? Circle one of the following.

Many-to-one One-to-many One-to-one

Specific behaviours
✓ Many to one

ii) If the domain of f is $f(x) \in R, -4 \le x \le 5$, find the range of f.

Specific behaviours
✓✓ $-1 \le y \le 24$

b) Given $y=2+\sqrt{4-x^2}$

i) What is the largest possible value of y.

Specific behaviours
✓ $y=¿4$

ii) Determine the domain and range.

Specific behaviours
✓ $-2 \le x \le 2$ ✓ $2 \le y \le 4$