SCHOOL

Year 11 Examination, 2013

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3A/3B

SOLUTIONS

Section One: Calculator-free

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	13	13	100	100	67
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

See next page

Section One: Calculator-free **(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(6 marks)**

Differentiate the following with respect to x, simplifying your answers as far as possible.

(a) $y = (1 - \log_e 2x)^3$ (2 marks)

$$\frac{dy}{dx} = 3 \cdot \left(-\frac{2}{2x} \right) (1 - \log_e 2x)^2$$
$$= -\frac{3}{x}(1 - \log_e 2x)^2$$

(b) $y = x^2 e^{2x}$ (2 marks)

$$\frac{dy}{dx} = 2x \cdot e^x + x^2 \cdot 2e^{2x}$$
$$= 2xe^{2x}(1 + x)$$

(c) $y = \frac{1 + 5x}{1 - 5x}$ (2 marks)

$$\frac{dy}{dx} = \frac{5(1 - 5x) - (-5)(1 + 5x)}{(1 - 5x)^2}$$
$$= \frac{5 - 25x + 5 + 25x}{(1 - 5x)^2}$$
$$= \frac{10}{(1 - 5x)^2}$$

Question 2 **(7 marks)**

(a) If $z = 3 - i$, express $\frac{z}{\overline{z}}$ in simplified form. (2 marks)

$$\frac{3-i}{3+i} \times \frac{3-i}{3-i} = \frac{8-6i}{10}$$
$$= \frac{4}{5} - \frac{3}{5}i$$

(b) Determine the complex number z, if $2\overline{z} - iz + 4 + i = 0$. (5 marks)

$$z = x + iy$$

$$2(x - iy) - i(x + iy) + 4 + i = 0$$
$$2x - 2iy - ix + y + 4 + i = 0$$

Real parts $2x + y + 4 = 0$

Imaginary parts $-2y - x + 1 = 0 \Rightarrow x = 1 - 2y$

$$2(1 - 2y) + y = -4$$
$$-3y = -6$$
$$y = 2$$
$$x = -3$$
$$z = -3 + 2i$$

See next page

Question 3 **(7 marks)**

(a) Express y in terms of x if $2\log_e x + 1 = \frac{\log_e 3y}{2}$. (3 marks)

$$\frac{\log_e 3y}{2} = \log_e x^2 + \log_e e$$
$$\log_e 3y = 2\log_e ex^2$$
$$3y = (ex^2)^2$$
$$y = \frac{e^2x^4}{3}$$

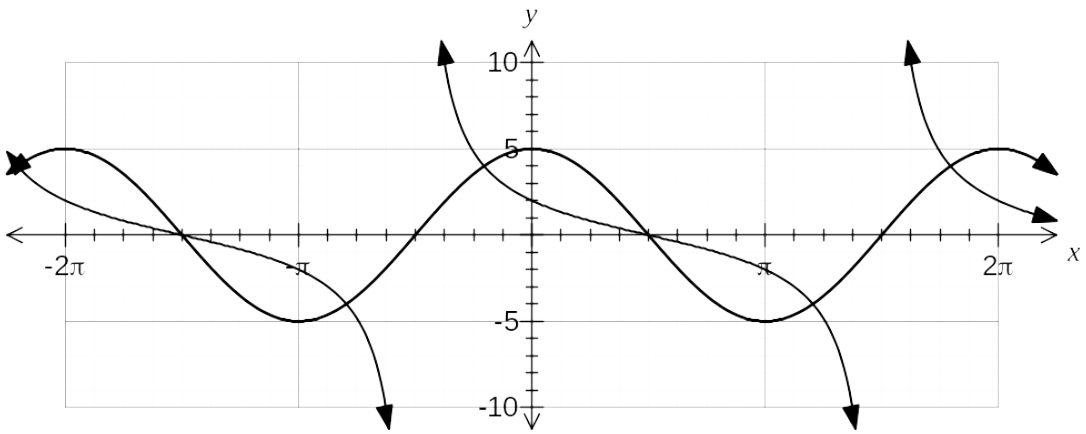
(b) Solve $2(4^x) - 3(2^x) + 1 = 0$. (4 marks)

$$2 \cdot (2^x)^2 - 3 \cdot 2^x + 1 = 0$$
$$(2 \cdot 2^x - 1)(2^x - 1) = 0$$
$$2^x = \tfrac{1}{2} \qquad 2^x = 1$$
$$x = -1 \qquad x = 0$$

See next page

Question 4 **(7 marks)**

The function $f(x) = a\tan(b(x + c))$ has been graphed below.



(a) Determine the values of the constants a, b and c. (4 marks)

Period of $\tan x$ is π, $f(x)$ is 2π, so $b = \frac{1}{2}$.

$y = \tan x$ has a root at $x = 0$. $f(x)$ has root at $\frac{\pi}{2}$, so $c = -\frac{\pi}{2}$.

$f(0) = a\tan(\frac{1}{2}(0 - \frac{\pi}{2}))$

$= a\tan(-\frac{\pi}{4})$ but $f(0) = 2$ so $a = -2$

$= -a$

$a = -2,\ b = \frac{1}{2},\ c = -\frac{\pi}{2}$

(b) On the same axes, sketch the graph of $y = 5\cos x$. (2 marks)

(c) State the number of solutions to the equation $5\cos x = f(x)$ over the domain $0 \le x \le 2\pi$. (1 mark)

3 solutions

Question 5 **(5 marks)**

The exact values of the sine and cosine of 36° are $\frac{\sqrt{10 - 2\sqrt{5}}}{4}$ and $\frac{1+\sqrt{5}}{4}$ respectively.

Using the identities $\sin A = \cos(90^\circ - A)$ and $\cos(2A) = \cos^2 A - \sin^2 A$, or otherwise, determine the exact value of the sine of 18° in the form $\frac{\sqrt{a} - b}{c}$, clearly stating the values of the positive integers a, b and c.

$$\sin 18 = \cos 72$$

$$= \cos(2 \times 36)$$

$$= \cos^2 36 - \sin^2 36$$

$$= \left(\frac{1+\sqrt{5}}{4}\right)^2 - \frac{10 - 2\sqrt{5}}{16}$$

$$= \frac{1 + 2\sqrt{5} + 5 - 10 + 2\sqrt{5}}{16}$$

$$= \frac{\sqrt{5} - 1}{4}$$

$$\therefore a = 5,\ b = 1,\ c = 4$$

See next page

Question 6 **(10 marks)**

(a) A line passes through the points with position vectors $10\mathbf{i} + \mathbf{j}$ and $-2\mathbf{i} + 6\mathbf{j}$. Determine a vector equation of the line through these two points in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{b}$ is a unit vector. (4 marks)

$$\mathbf{b} = \begin{bmatrix} 10 \\ 1 \end{bmatrix} - \begin{bmatrix} -2 \\ 6 \end{bmatrix} = \begin{bmatrix} 12 \\ -5 \end{bmatrix}$$

$$\mathbf{b}_{UNIT} = \frac{1}{13}\begin{bmatrix} 12 \\ -5 \end{bmatrix}$$

$$\mathbf{r} = \begin{bmatrix} 10 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} \frac{12}{13} \\ \frac{-5}{13} \end{bmatrix}$$

(b) The point A lies on the line with equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(2\mathbf{i} - \mathbf{j})$ and the point B has position vector $4\mathbf{i} - 5\mathbf{j}$. Use a method involving a dot product to determine the position vector of A so that the distance from A to B is a minimum. (6 marks)

$$\vec{BA} = \begin{bmatrix} 2 + 2\lambda \\ 1 - \lambda \end{bmatrix} - \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} 2\lambda - 2 \\ 6 - \lambda \end{bmatrix}$$

$$\begin{bmatrix} 2\lambda - 2 \\ 6 - \lambda \end{bmatrix} \bullet \begin{bmatrix} 2 \\ -1 \end{bmatrix} = 0$$

$$4\lambda - 4 - 6 + \lambda = 0$$

$$5\lambda = 10$$

$$\lambda = 2$$

$$\vec{OA} = \begin{bmatrix} 2 + 2(2) \\ 1 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ -1 \end{bmatrix}$$

See next page

Question 7 **(8 marks)**

Solve the following over the given domains.

(a) $2\cos A + \sqrt{3} = 0,\ -180^\circ \le A \le 180^\circ$ (2 marks)

$$\cos A = -\frac{\sqrt{3}}{2}$$
$$A = -150^\circ, 150^\circ$$

(b) $\tan 3A = \sqrt{3},\ 0 \le A \le \pi$. (2 marks)

$$0 \le 3A \le 3\pi$$
$$3A = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}$$
$$A = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$$

(c) $\sin A = \sin 2A,\ 0 \le A \le 360^\circ$. (4 marks)

$$\sin A - \sin 2A = 0$$
$$\sin A - 2\sin A\cos A = 0$$
$$\sin A(1 - 2\cos A) = 0$$

$\sin A = 0$: $A = 0, 180, 360$

$1 - 2\cos A = 0$: $\cos A = \frac{1}{2}$, $A = 60, 300$

$$A = 0, 60, 180, 300, 360$$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

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