PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course   Specialist Test 4   Year 12

Student name: _____   Teacher name: _____

Task type: **Response**

Time allowed for this task: _____40_____ mins

Number of questions: _____7_____

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: _44_____ marks

Task weighting: _10___%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3 & 3 = 6 marks)
Solve the following.

a) $\dfrac{dy}{dx} = \dfrac{3x-2}{y(5-y^2)}$ given that when $x=1, y=1$.

b) $3x^4 \cos(2y)\dfrac{dy}{dx} = 10$ given that when $x=5, y=\pi$.

Q2 (4 marks)

An iron has a temperature of $54^\circ C$ is left in a room, of temperature $18^\circ C$, to cool such that the temperature $T^\circ C$ at time t minutes is given by $\dfrac{dT}{dt} = k(T-18)$. After 15 mins the temperature of the iron is $37^\circ C$. Determine the time taken for the iron's temperature to drop to $22^\circ C$.

Q3 (1, 5 & 2 = 8 marks)

The number N thousands, of bacteria cells living in a petri dish at time t hours is given by

$$\frac{dN}{dt} = 0.30N - 0.05N^2 .$$

The initial number of cells was 2 thousand.

    a) What is the limiting value of the number of cells as $t \rightarrow \infty$?

    b) Using calculus and partial fractions, show every step to express N in terms of t.
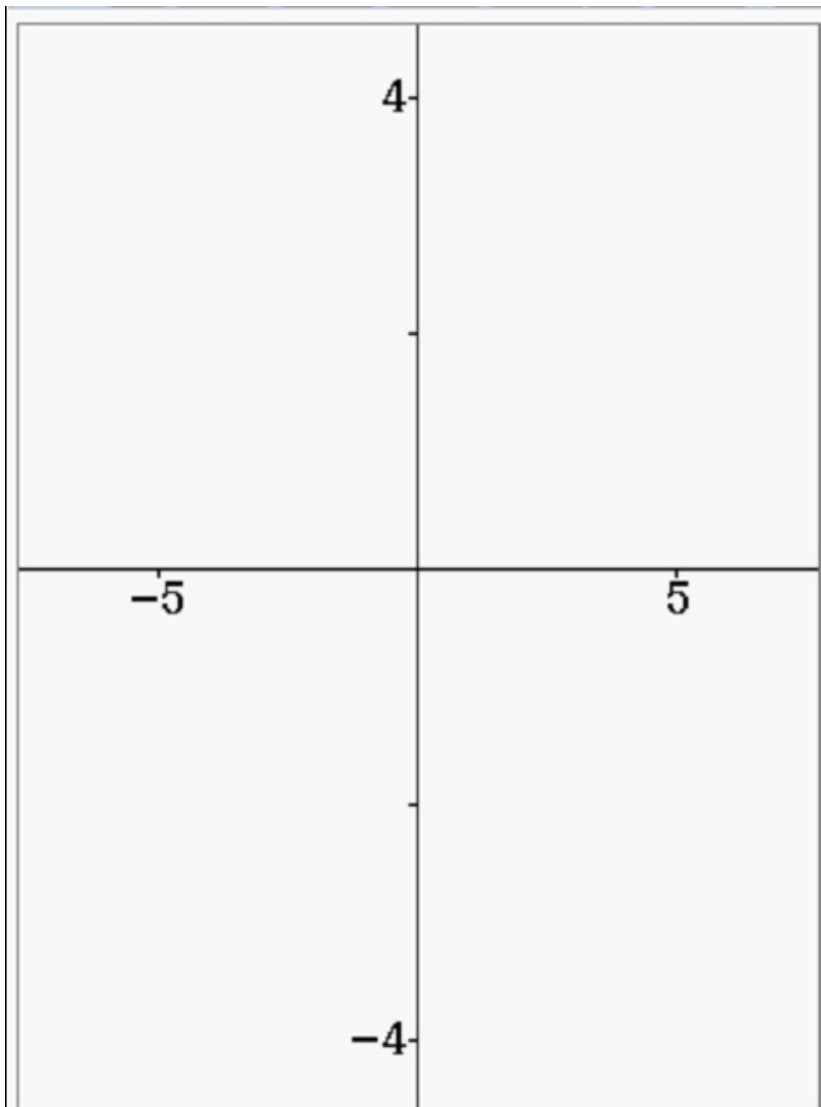
Q3-cont

c) Determine the number of cells after 15 hours.

Q4 (3, 2 & 2 = 7 marks)

Consider the slope field $\dfrac{dy}{dx} = (x-3)(x+2)$

   a) Sketch this field on the axes below.

   b) Draw the solution curve, axes above, that contains the point (1,1).

   c) Determine the equation of the solution curve that contains (1,1).

Q5 (2, 2 & 3 = 7 marks)

Consider an object that is moving with Simple Harmonic Motion such that $\ddot{x} = -9x$ with x, t in metres and seconds respectively. At $t = 0$, $x = 7$ metres and is a rest.

    a) Determine a rule for x in terms of t.

    b) Determine the exact speed when $x = 3$ metres.

    c) Determine the percentage of the time, to one decimal place, that the object is less than 3 metres from the mean position, $x = 0$.

Q6 (4 marks)

Consider an object that is initially at the origin and at rest such that its acceleration is given by

$\dfrac{dv}{dt} = \dfrac{1 + v^3}{v} \, m/s^2$ where v equals the speed in m/s at t seconds. Determine the exact speed when its displacement from the origin is $\ln(3)$ metres.

Q7 (2, 3 & 3 = 8 marks)
A lolly company makes jelly beans where the mass of one jelly bean is normally distributed with a mean of 23.4 mg and a standard deviation of 3.2 mg. (Note: 1g=1000mg)

a) Determine the probability to two decimal places that the total mass of 85 jelly beans is more than two grams.

b) Given that the probability that the mean mass of a jelly bean differs from the population mean by more than 0.35 mg is 5%, determine n, the number of jelly beans that need to be sampled.

c) On a particular day the operator of a machine that makes jelly beans is suspected of being faulty. A sample of 200 jelly beans had a sample standard deviation of 3.8 mg with a total mass of 5.4 grams. Present a mathematical argument to either support or to dismiss such a claim.