

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Important note to candidates

Special items: nil

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
To be provided by the candidate

Formula sheet
This Question/Answer booklet
To be provided by the supervisor
Materials required/recommended for this section

Working time: fifty minutes
Reading time before commencing work: five minutes
Time allowed for this section

Student Name:

Calculator Free
Section One:

UNIT 1

MATHEMATICS METHODS

SOLUTIONS

Question/Answer Booklet

Semester 1 Examination, 2019

Mathematics Department
Year 11 Mathematics Methods



Christ Church
Grammar School

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	14	14	100	98	65
Total					100

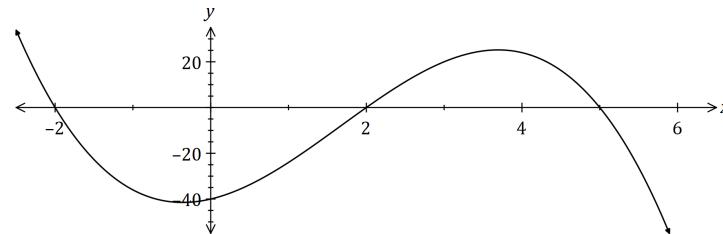
Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Question 22

(3 marks)

The graph of the cubic function $y = f(x)$ is shown below. Determine $f(10)$.

**Solution**

$$f(x) = a(x + 2)(x - 2)(x - 5)$$

$$-40 = a(2)(-2)(-5) \Rightarrow a = -2$$

$$f(10) = -2(12)(8)(5) = -960$$

Specific behaviours

- ✓ cubic in factored form
- ✓ correct value of a
- ✓ required value

End of questions

35% (52 Marks)

This section has eight (8) questions. Answer all questions. Write your answers in the spaces provided.

(S) (R) (W) (ZG) 8/66

This section has eight (8) questions. Answer all questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (5 marks)

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both correct solutions

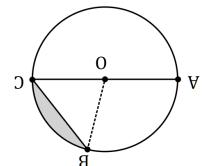
(a) The circle shown has centre O and diameter AC of length 50 cm. Determine the shaded area given that $2 \times \angle AOB = 3 \times \angle BOC$. (4 marks)

CALCULATOR-ASSUMED

(a) The circle shown has centre O and diameter AC of length 50 cm. Determine the shaded area given that $2 \times \angle AOB = 3 \times \angle BOC$. (4 marks)

Solution	$\angle AOB = \frac{3}{2} \angle BOC$ $= \frac{3}{2} \times 72^\circ = 108^\circ$
Specific behaviours	$A = \frac{1}{2}(25) \left(\frac{2\pi}{5} - \sin \frac{\pi}{5} \right)$ $= 95.5 \text{ cm}^2$
equation using angles	$\Delta ABC + \frac{2}{3}\angle BOC = \pi \Leftrightarrow \angle BOC = \frac{2\pi}{5} = 72^\circ$
correct angle for segment	$\angle BOC = \frac{2\pi}{5}$
substitutes correctly into formula	$A = \frac{1}{2}(25) \left(\frac{2\pi}{5} - \sin \frac{\pi}{5} \right)$ $= 95.5 \text{ cm}^2$

Solution	Solving simultaneously gives $2r + r\theta = 112$	$\frac{1}{2}r^2\theta = 735$	$r = 21, \theta = \frac{10}{3}$ or $r = 35, \theta = \frac{6}{5}$	Hence $r = 21$ or $r = 35$ cm	Specific behaviors equation for perimeter solution for area solution of equations states both values of r
	Solving simultaneously gives $2r + r\theta = 112$	$\frac{1}{2}r^2\theta = 735$	$r = 21, \theta = \frac{10}{3}$ or $r = 35, \theta = \frac{6}{5}$	Hence $r = 21$ or $r = 35$ cm	Specific behaviors equation for perimeter solution for area solution of equations states both values of r



Question 2

(5 marks)

- (a) A circle of radius 2 has its centre at the point $(1, -4)$. Determine the equation of the circle in the form $x^2 + y^2 = ax + by + c$. (3 marks)

Solution
$(x - 1)^2 + (y + 4)^2 = 2^2$
$x^2 - 2x + 1 + y^2 + 8y + 16 = 4$
$x^2 + y^2 = 2x - 8y - 13$
Specific behaviours
✓ writes equation of circle ✓ correctly expands ✓ writes in required form

- (b) The graph of $x = y^2$ passes through the point $(9, q)$. Determine the value(s) of q and hence explain why y is a relation but not a function of x . (2 marks)

Solution
$9 = q^2 \Rightarrow q = \pm 3$
A relation exists as we are told that $x = y^2$.
The relation is not a function because it is not one-to-one (for most values of x there is more than one value of y).
Specific behaviours
✓ both possible values ✓ explains why relation not a function

Question 20

(9 marks)

- (a) Show, using one or more identities from the formula sheet and without using the value of any trigonometric term, that (3 marks)

$$\cos 15^\circ \cos 65^\circ + \sin 15^\circ \sin 65^\circ = \sin 140^\circ$$

Solution
$\cos 15^\circ \cos 65^\circ + \sin 15^\circ \sin 65^\circ = \cos(65^\circ - 15^\circ)$ $= \cos 50^\circ$ $= \sin(90^\circ + 50^\circ)$ $= \sin 140^\circ$
Specific behaviours
✓ shows uses of difference identity ✓ reduces LHS to $\cos 50^\circ$ ✓ shows use of relationships between trigonometric ratios to obtain result

- (b) Simplify $\sin(A + B) \cos B - \cos(A + B) \sin(B)$. (2 marks)

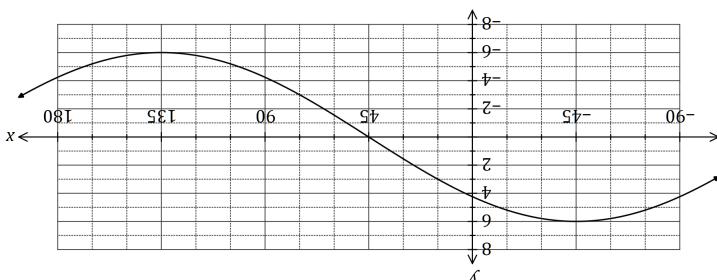
Solution
$\sin(A + B) \cos B - \cos(A + B) \sin(B) = \sin(A + B - B)$ $= \sin A$
Specific behaviours
✓ indicates use of difference identity ✓ correct result

- (c) (i) Show that $\sqrt{2} \sin(x + 45^\circ) = \sin x + \cos x$ (2 marks)

Solution
$\sqrt{2} \sin(x + 45^\circ) = \sqrt{2} \sin x \cos 45^\circ + \sqrt{2} \sin 45^\circ \cos x$ $= \sin x + \cos x$
Specific behaviours
✓ correctly expands LHS ✓ concludes correctly

- (ii) Hence, show that the exact value of $\sin 75^\circ = \frac{(1+\sqrt{3})}{2\sqrt{2}}$ (2 marks)

Solution
$\sqrt{2} \sin 75^\circ = \sqrt{2} \sin(45^\circ + 30^\circ) = \sin 30^\circ + \cos 30^\circ$ $= \frac{1}{2} + \frac{\sqrt{3}}{2}$ or $\frac{1}{2}(1 + \sqrt{3})$
Therefore $\sin 75^\circ = \frac{(1+\sqrt{3})}{2\sqrt{2}}$
Specific behaviours
✓ substitutes $x = 30$ into expression in (c) (i) ✓ uses exact values to obtain correct conclusion

Question 3
(6 marks)(a) Let $p = \cos 130^\circ$ and $q = \sin 35^\circ$.
Give your answers to the following in terms of p and/or q .(a) The graph of $y = a \cos(x + b)$ is shown below, where a and b are constants.

(2 marks)

Determine the value of a and the value of b , where $-90^\circ \leq b \leq 180^\circ$.

(1 mark)

(b) Given that $0^\circ \leq x \leq 360^\circ$, solve

(i) $\cos(x) = \frac{1}{2}$

(ii) $8 \cos(x + 30^\circ) + 4\sqrt{3} = 0$.

(3 marks)

Solution	$a = 6, b = 45$
Specific behaviours	
Specification	
SN18-132-3	

(b) Given that $0^\circ \leq x \leq 360^\circ$, solve

(i) $\cos(x) = \frac{1}{2}$

(ii) $8 \cos(x + 30^\circ) + 4\sqrt{3} = 0$.

(3 marks)

(1 mark)

(c) Determine an expression for $\tan 145^\circ$.

Solution	$\tan 145^\circ = \frac{\sin 145^\circ}{\cos 145^\circ} = -\sqrt{1-q^2}$
Specific behaviours	
Specification	
SN18-132-8	

(1 mark)

Solution	$\cos 145^\circ = \pm \sqrt{1-q^2}$
Specific behaviours	
Specification	
SN18-132-9	

(3 marks)

(b) Determine an expression for $\cos 145^\circ$.

Solution	$\cos 145^\circ = -\cos 30^\circ = -p$
Specific behaviours	
Specification	
SN18-132-9	

(1 mark)

(iii) $\cos 50^\circ$.

Solution	$\sin 145^\circ = \sin 35^\circ = q$
Specific behaviours	
Specification	
SN18-132-9	

(1 mark)

(i) $\sin 145^\circ$.

(1 mark)

(a) Write down an expression for

(1 mark)

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(1 mark)

Question 4

(a) Determine the coordinates of the

- (i)
- y
- intercept of the graph of
- $y = -2(x + 4)^2 + 12$
- .

Solution
$x = 0, y = -2(4)^2 + 12 = -32 + 12 = -20$

At $(0, -20)$

Specific behaviours
✓ correct coordinates

- (ii) turning point of the graph of
- $y = (x - 3)(x + 1)$
- .

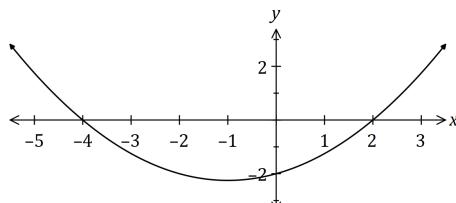
Solution
$x = (3 - 1) \div 2 = 1$

$y = (1 - 3)(1 + 1) = -4$

At $(1, -4)$

Specific behaviours
✓ correct x -coordinate ✓ correct y -coordinate

- (b) The graph of
- $y = ax^2 + bx + c$
- is shown below. Determine the value of the coefficients
- a
- ,
- b
- and
- c
- . (4 marks)



Solution
$y = a(x + 4)(x - 2)$

$-2 = a(4)(-2) \Rightarrow a = \frac{1}{4}$

$y = \frac{1}{4}(x^2 + 2x - 8)$

$a = \frac{1}{4}, b = \frac{1}{2}, c = -2$

Specific behaviours
✓ uses roots to write in factored form ✓ uses y -intercept to determine a ✓ expands quadratic ✓ states all coefficients

See next page

(7 marks)

(1 mark)

Question 18

(8 marks)

- (a) The equation of the axis of symmetry for the graph of
- $y = 2x^2 + 8x + 5$
- is
- $x = m$
- . Determine the value of
- m
- , using a method that does not refer to the graph of the parabola.

(2 marks)

Solution
$x = -\frac{8}{2 \times 2} = -2$

$\therefore m = -2$

Specific behaviours
✓ uses $x = -b \div (2a)$ or partially completes the square ✓ value of m

(2 marks)

- (b) A parabola with equation
- $y = ax^2 + bx + c$
- has a turning point at
- $(4, -5)$
- and passes through the point
- $(2, -17)$
- . Determine the value of
- a
- , the value of
- b
- and the value of
- c
- . (3 marks)

Solution
$y = a(x - 4)^2 - 5$

$-17 = a(2 - 4)^2 - 5 \Rightarrow a = -3$

$y = -3(x - 4)^2 - 5$

$= -3x^2 + 24x - 53$

$a = -3, b = 24, c = -53$

Specific behaviours
✓ correctly writes in turning point form ✓ solves for a using point ✓ expands and states all values ✓ correctly solves all 3 variables

- (c) Determine the value of the discriminant for the quadratic equation
- $4x^2 - 28x + 47 = 0$
- and use it to explain how many solutions the equation
- $(x + 3)(4x^2 - 28x + 47) = 0$
- will have. (3 marks)

Solution
$d = (-28)^2 - 4(4)(47) = 32$

When $d > 0$, quadratic will have two solutions.

Hence equation will have three solutions - one from the linear factor and two from the quadratic factor.

Specific behaviours
✓ value of discriminant ✓ uses discriminant to say quadratic will have two solutions ✓ explains why equation has three solutions

Question 5

(a) Expand $x(x+5)^2$.
(2 marks)

Solution	
\checkmark expands quadratic correctly	
$x(x^2 + 10x + 25) = x^3 + 10x^2 + 25x$	
Solution	

(b) Let $f(x) = x^3 + 2x^2 - 11x - 12$.
(1 mark)

Solution	
$f(-1) = (-1)^3 + 2(-1)^2 - 11(-1) - 12$	
$= -1 + 2 + 11 - 12$	
$= 0$	
Solution	

(i) Determine $f(-1)$.
(1 mark)

(ii) Solve $f(x) = 0$.
(4 marks)

Solution	
$x^3 + 2x^2 - 11x - 12 = (x+1)(x^2 + bx - 12)$	
$-11x = bx - 12x \Rightarrow b = 1$	
$x^2 + x - 12 = (x+4)(x-3)$	
$x + 1)(x + 4)(x - 3) = 0 \Leftrightarrow x = -4, -1, 3$	
Solution	

$$\begin{aligned} & \text{(i) the maximum distance of the mass from the top of the spring.} \\ & x^2 + x - 12 = 0 \\ & -11x = bx - 12x \Rightarrow b = 1 \\ & x^3 + 2x^2 - 11x - 12 = (x+1)(x^2 + bx - 12) \end{aligned}$$

(b) Mark on your graph point M , where the mass is 40 cm from the top of the spring and moving downwards.
Moving downwards.

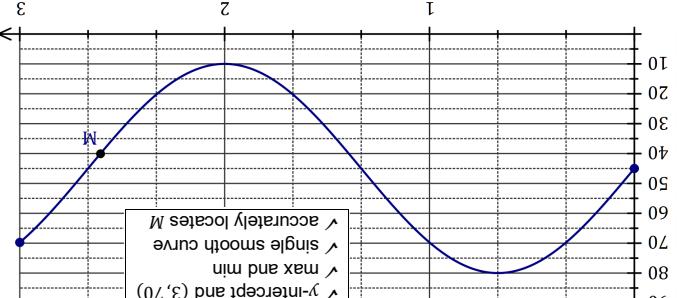
(c) Determine
(1 mark)

- (i) the time taken for the mass to return to its initial position.
(1 mark)
- (ii) the distance moved by the mass between $t = 1$ and $t = 2$.
(1 mark)

Solution	
$t = \frac{3}{4} = 1.3 \text{ s}$	
80 cm	
Solution	

$$\begin{aligned} & \text{(iii) the distance moved by the mass between } t = 1 \text{ and } t = 2. \\ & d(1) - d(2) = 69.75 - 10 \\ & = 59.75 \text{ cm} \end{aligned}$$

(d) Sketch the graph on the axes below for $0 \leq t \leq 3$.
(3 marks)



(a) Sketch the graph on the axes below for $0 \leq t \leq 3$.
(3 marks)

The mass from the top of the spring after t seconds can be modelled by
A small mass, attached to the bottom of a spring, oscillated up and down. The distance, d cm, of
 $d = 45 + 35 \sin\left(\frac{3\pi t}{4}\right)$

Question 6

(7 marks)

- (a) Describe the behaviour of the y values for each of the following graphs, given the behaviour of the x values:

(i) $y = x^4$, as $x \rightarrow \infty$.

Solution
$y \rightarrow \infty$
Specific behaviours ✓ describes correct behaviour

(1 mark)

(ii) $y = (2 - x)^3$, as $x \rightarrow \infty$.

Solution
$y \rightarrow -\infty$
Specific behaviours ✓ describes correct behaviour

(1 mark)

(iii) $y = \frac{1}{x}$, as $x \rightarrow -\infty$.

Solution
$y \rightarrow 0$
Specific behaviours ✓ describes correct behaviour

(1 mark)

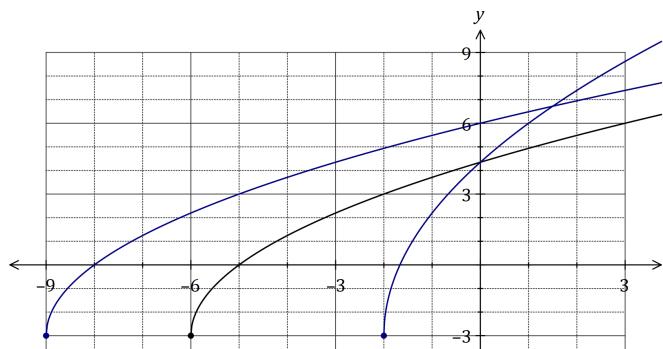
- (b) The graph of $y = f(x)$ is shown below. On the same axes sketch the graph of

(i) $y = f(x + 3)$.

(2 marks)

(ii) $y = f(3x)$.

(2 marks)



Solution (i)
See graph
Specific behaviours ✓ smooth curve starting at $(-9, -3)$ ✓ intercepts at $(-8, 0)$ and $(0, 6)$

Solution (ii)
See graph
Specific behaviours ✓ smooth curve starting at $(-2, -3)$ ✓ same y -intercept as $f(x)$

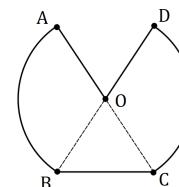
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SN018-132-3

Question 16

(7 marks)

- In shape $OABCD$ below, $\angle AOB = 117^\circ$ and AC, BD are diameters of the circle with centre O and radius 42 cm.



- (a) Calculate the perimeter of $OABCD$.

(4 marks)

Solution
 $117^\circ = \frac{13\pi}{20}, \quad \pi - \frac{13\pi}{20} = \frac{7\pi}{20} = 63^\circ$

$$AB, DC: 42 \times \frac{13\pi}{20} = \frac{273\pi}{10} \approx 85.765$$

$$BC^2 = 42^2 + 42^2 - 2(42)(42) \cos 63^\circ$$

$$BC \approx 43.9$$

$$P_{TOTAL} = 2(42) + 2(85.765) + 43.9 \approx 299 \text{ cm}$$

Specific behaviours
✓ indicates length of arc AB
✓ indicates use of cosine rule for BC
✓ correct length BC
✓ correct total perimeter

- (b) Calculate the area of $OABCD$.

(3 marks)

Solution
 $AOB + DOC: 2 \times \frac{1}{2} \times 42^2 \times \frac{13\pi}{20} = \frac{5733\pi}{5} \approx 3602$

$$BOC: \frac{1}{2}(42)(42) \sin 63^\circ \approx 786$$

$$A_{TOTAL} = 3602 + 786 = 4388 \text{ cm}^2$$

Specific behaviours
✓ sector area AOB
✓ triangle area BOC
✓ correct total area

See next page

SN018-132-8

Question 8

- (a) Evaluate
- $\sin\left(\frac{39\pi}{4}\right)$
- .

(7 marks)

(2 marks)

Solution
$\sin\frac{39\pi}{4} = \sin\frac{(39-32)\pi}{4} = \sin\frac{7\pi}{4}$
$\sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$
Specific behaviours
✓ reduces angle ✓ exact value

- (b)
- A
- is an acute angle and
- B
- is an obtuse angle such that
- $\cos A = \frac{1}{3}$
- and
- $\sin B = \frac{2}{3}$
- .

- (i) Show that
- $\sin A = \frac{2\sqrt{2}}{3}$
- and determine the value of
- $\cos B$
- . (3 marks)

Solution
$\sin^2 A = 1 - \left(\frac{1}{3}\right)^2 = \frac{8}{9} \Rightarrow \sin A = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$
$\cos^2 B = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$
$\cos B = -\frac{\sqrt{5}}{3}$
Specific behaviours
✓ indicates how to obtain $\sin^2 A$ ✓ uses Pythagoras' to show value of $\sin A$ ✓ obtains the absolute value of $\cos B$ ✓ correct value of $\cos B$

- (ii) Determine the value of
- $\sin(A + B)$
- as a single fraction. (2 marks)

Solution
$\sin(A + B) = \frac{2\sqrt{2}}{3} \times \left(-\frac{\sqrt{5}}{3}\right) + \frac{1}{3} \times \frac{2}{3}$
$= \frac{2 - 2\sqrt{10}}{9}$
Specific behaviours
✓ substitutes correctly ✓ correct value as single fraction

Question 14

- (a) Convert, giving an exact answer

- (i)
- 40°
- to radians.

Solution
$\frac{2\pi}{9}$
Specific behaviours
✓ exact value

- (ii) 0.2 radians to degrees.

Solution
$\frac{36}{\pi}$
Specific behaviours
✓ exact value

- (b) Calculate, to the nearest degree, the acute angle between the line
- $y = 4.5x + 2$
- and the line
- $y = 1.5x - 3$
- . (3 marks)

Solution
$\theta_1 = \tan^{-1} 4.5 = 77.5$
$\theta_2 = \tan^{-1} 1.5 = 56.3$
$\theta_2 - \theta_1 = 21^\circ$
Specific behaviours
✓ indicates use of $\tan^{-1} m$ ✓ one correct angle with x -axis ✓ correct angle between lines

- (c) The sides adjacent to the right-angle in a right triangle have lengths 36 cm and 77 cm.

If the smallest angle in the triangle is α , then determine an exact value for

- (i)
- $\tan \alpha$
- .

Solution
$\tan \alpha = \frac{36}{77}$
Specific behaviours
✓ correct ratio

- (ii)
- $\cos(90^\circ - \alpha)$
- .

Solution
$\sqrt{36^2 + 77^2} = 85$
$\cos(90^\circ - \alpha) = \frac{36}{85}$
Specific behaviours
✓ calculates length of hypotenuse ✓ correct ratio

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	14	14	100	98	65
Total					100

Instructions to candidates

- The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Question 12

(8 marks)

- (a) The variables Q and v are directly proportional and when $v = 40$, $Q = 10$.

- (i) Determine an equation for the relationship between Q and v . (2 marks)

Solution
$Q = mv, m = \frac{10}{40} = 0.25 \Rightarrow Q = 0.25v$

Specific behaviours

- ✓ uses correct linear relationship
- ✓ calculates the proportion constant

$v = 4Q$ is also correct and is to be awarded full marks

- (ii) State the value of Q when $v = 80$. (1 mark)

Solution
$Q = 0.25(80) = 20$

Specific behaviours

- ✓ correct value

- (b) The time, t minutes, that a car takes to travel 250 m at a constant speed of s kmh⁻¹ is given by the formula $t = \frac{k}{s}$.

- (i) Determine the value of the constant k , given that when $s = 15$, $t = 60$. (1 mark)

Solution
$60 = \frac{k}{15} \Rightarrow k = 900$

Specific behaviours

- ✓ correct value

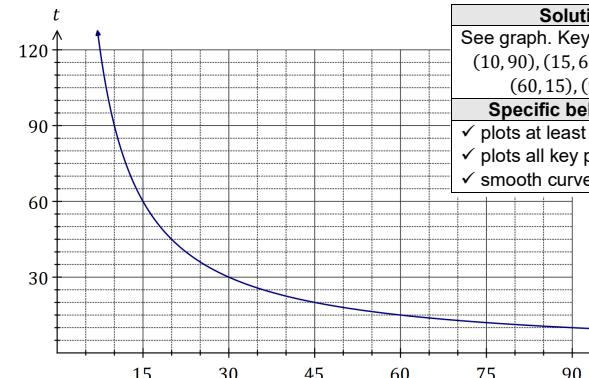
- (ii) Determine the value of t when $s = 10$. (1 mark)

Solution
$t = 900 \div 10 = 90$

Specific behaviours

- ✓ correct value

- (iii) On the axes below, draw a graph to show how t varies with s . (3 marks)



Solution
See graph. Key points: (10, 90), (15, 60), (30, 30), (60, 15), (90, 10)

Specific behaviours

- ✓ plots at least 2 key points
- ✓ plots all key points
- ✓ smooth curve

(a) The points A and B have coordinates $(7, -2)$ and $(-3, 6)$ respectively. If A is the midpoint of B and C, determine the coordinates of C.
(4 marks)

Question 9	
$\frac{2}{2} = 7 \Rightarrow x = 17$ and $\frac{6+2}{2} = -2 \Rightarrow y = -10$ Solution	<ul style="list-style-type: none"> ✓ obtains the correct coordinates of C ✓ writes equalities for midpoints of uses horizontal and vertical distance between points ✓ indicates perpendicular gradient ✓ states not parallel, comparing gradients

(a)

The points A and B have coordinates $(7, -2)$ and $(-3, 6)$ respectively. If A is the midpoint of B and C, determine the coordinates of C.

Working time: 100 minutes.

Question 9	
<p>State, with justification, if L is parallel to the line with equation $y = 0.6x + 4$. (2 marks)</p> $y = 3 - \frac{3}{5}x$ <p>Solution</p>	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(b) State, with justification, if L is parallel to the line with equation $y = 0.6x + 4$.
(2 marks)

Determine the equation of line P that is perpendicular to L and passes through the point with coordinates $(30, 19)$.
(2 marks)

Solution	
$y - 19 = \frac{3}{5}(x - 30)$ Solution	<ul style="list-style-type: none"> ✓ indicates perpendicular gradient ✓ indicates perpendicular gradient

(d) Determine the coordinates of the point of intersection of L and P .
(2 marks)

Solution	
$P: y = \frac{5}{3}x - 31$ $\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(b) The points D and E have coordinates $(-2p, q)$ and $(3q, -2p)$ respectively, where p and q are constants. Determine the value of p and the value of q if the midpoint of D and E is at $(11, -7)$.
(2 marks)

Solution	
$\frac{-2p + 3q}{2} = 11 \text{ and } \frac{q - 2p}{2} = -7$ Solution	<ul style="list-style-type: none"> ✓ value of p ✓ value of q

Solve simultaneously CAS to get $p = 16, q = 18$

(d) Determine the coordinates of the point of intersection of L and P .
(2 marks)

Solution	
$y = 0 \Leftrightarrow x = 5$ $At (5, 0)$	<ul style="list-style-type: none"> ✓ correct coordinates (no marks if not listed as coordinates)

(a) State the coordinates of the point where L intersects the x -axis.
(1 mark)

Solution	
L is not parallel to this line as gradients are different: $-\frac{3}{5} \neq 0.6$.	<ul style="list-style-type: none"> ✓ states not parallel, comparing gradients

(c) Determine the equation of line P that is perpendicular to L and passes through the point with coordinates $(30, 19)$.
(2 marks)

Solution	
$y - 19 = \frac{3}{5}(x - 30)$ Solution	<ul style="list-style-type: none"> ✓ indicates perpendicular gradient ✓ indicates perpendicular gradient

(b) State, with justification, if L is parallel to the line with equation $y = 0.6x + 4$.
(2 marks)

Solution	
$y = 3 - \frac{3}{5}x$ Solution	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(d) Determine the coordinates of the point of intersection of L and P .
(2 marks)

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(e) This section has fourteen (14) questions. Answer all questions. Write your answers in the spaces provided.

Solution	
L has equation $\frac{x}{5} + \frac{y}{3} = 1$.	<ul style="list-style-type: none"> ✓ correct coordinates

(f) Section Two: Calculator-assumed
65% (98 Marks)

Solution	
$y = 0 \Leftrightarrow x = 5$ $At (5, 0)$	<ul style="list-style-type: none"> ✓ correct coordinates

(g) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{2}{2} = 7 \Rightarrow x = 17$ and $\frac{6+2}{2} = -2 \Rightarrow y = -10$ Solution	<ul style="list-style-type: none"> ✓ obtains the correct coordinates of C ✓ writes equalities for midpoints of uses horizontal and vertical distance between points ✓ indicates perpendicular gradient ✓ states not parallel, comparing gradients

(h) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(i) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$y = 3 - \frac{3}{5}x$ Solution	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(j) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(k) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$y = 3 - \frac{3}{5}x$ Solution	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(l) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(m) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$y = 3 - \frac{3}{5}x$ Solution	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(n) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(o) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$y = 3 - \frac{3}{5}x$ Solution	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(p) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(q) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$y = 3 - \frac{3}{5}x$ Solution	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(r) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(s) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$y = 3 - \frac{3}{5}x$ Solution	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(t) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(u) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$y = 3 - \frac{3}{5}x$ Solution	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(v) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates

(w) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$y = 3 - \frac{3}{5}x$ Solution	<ul style="list-style-type: none"> ✓ indicates gradient of L ✓ indicates gradients are different: $-\frac{3}{5} \neq 0.6$.

(x) METHODS UNIT 1
CALCULATOR-ASSUMED
3
METHODS UNIT 1

Solution	
$\frac{5}{3}x - 31 = 3 - \frac{3}{5}x \Rightarrow \frac{15}{5}x = 34 \Rightarrow x = 15$ $y = \frac{3}{5}(15) - 31 = -6$ $\text{Intersect at } (15, -6)$	<ul style="list-style-type: none"> ✓ equates equations and solves for x ✓ correct coordinates</li

Question 10

(8 marks)

The average wind speed, $S(t)$ in km/h, over an 18 hour period from midnight to 6pm during a stormy day was observed to follow $S(t) = \frac{t^3}{10} - \frac{5t^2}{2} + 16t + 28$ where t was the number of hours since midnight.

- (a) No data was available after 6pm as the measuring instrument broke at that time.
What was the average wind speed at 6pm? (1 mark)

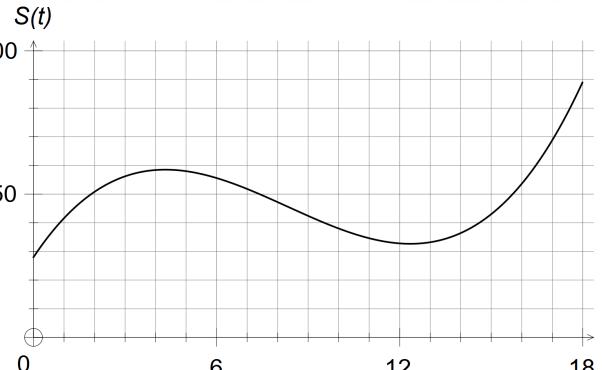
Solution

$$S(18) = 89.2 \text{ km/h}$$

Specific behaviours

✓ states correct value

- (b) On the grid below, sketch a graph to show how the average wind speed varied during the 18 hour period. (2 marks)

**Solution**

On grid above

Specific behaviours

- ✓ graph passes through (0, 28) approximately and (18, 89.2) approximately
✓ graph has correct shape

- (c) At the height of the storm in the morning, some properties suffered structural and other damage. At what time, to the nearest quarter of an hour, did this occur? (2 marks)

Solution

$$t = 4.319$$

= 4 hours and 19 mins

So height of storm occurred at 4.15 am

Specific behaviours

- ✓ determines t value for maximum $S(t)$
✓ states correct time

Question 10 continued

- (d) What was the lowest average wind speed recorded after 6am? (1 mark)

Solution

$$S(t) = 32.659 \text{ km/h}$$

Specific behaviours

✓ states correct value

- (e) For what percentage of the 18 hours did the average wind speed exceed 50 km/h? (2 marks)

Solution

$$\begin{aligned} &\text{From } t = 1.89 \text{ to } t = 7.40 \text{ and } t = 15.70 \text{ to } t = 18.00 \\ &= 5.51 \text{ hours} + 2.30 \text{ hours} \\ &= 7.81 \text{ hours} \end{aligned}$$

$$7.81/18 \times 100 = 43.4\%$$

Specific behaviours

- ✓ calculates the correct time periods
✓ calculates the correct percentage