

Semester Two Examination 2012

Question/Answer Booklet

Section One:
Calculator-free

Student Name: _____

Solutions / Marking guide

Teacher Name: _____

Time allowed for this section

Reading time before commencing work: Five (5) minutes

Working time for this section: Fifty (50) minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section One: Calculator-free

(50 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the space provided. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

Question 1 (2, 3, 2, 3 = 10 marks)

- (a) Determine $\frac{dy}{dx}$ in each of the following (there is no need to simplify answers)

(i) $y = \frac{\sqrt{x^2-16}}{x}$ $x \cdot \frac{1}{2}(x^2-16)^{-\frac{1}{2}} \cdot 2x - (x^2-16)^{\frac{1}{2}} \cdot 1$
 x^2

(ii) $y = \int_3^{e^{1-x}} \left(\frac{t-2}{t^2-1} \right) dt$ $\left(\frac{e^{1-x}-2}{e^{2-2x}-1} \right) \cdot e^{1-x} \cdot (-1)$

- (b) (i) determine $\frac{d}{dx} (8x e^{4x})$ $8x \cdot 4e^{4x} + 8e^{4x}$

- (ii) hence or otherwise, determine $\int 32x e^{4x} dx$

$$\int \frac{d}{dx} (8x e^{4x}) dx = \int 32x e^{4x} + \int 8e^{4x} dx$$

$$\therefore \int 32x e^{4x} dx = 8x e^{4x} - 2e^{4x} + c$$

Question 8 (6 marks)

A cubic function $f(x)$ whose derivative $f'(x)$ is given by $f'(x) = 2x^2 - 5x + 2$ has a local or relative minimum value of 2. Determine the local or relative maximum value.

$$f''(x) = 4x - 5$$

$$f'(x) = 0 \Rightarrow 2x^2 - 5x + 2 = 0$$

$$(2x-1)(x-2) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = 2$$

$$f''\left(\frac{1}{2}\right) = -3 \quad \therefore \text{max}$$

$$f''(2) = 3 \quad \therefore \text{min}$$

$\therefore f(2)$ produces the min value of 2

$$f(x) = \frac{2x^3}{3} - \frac{5x^2}{2} + 2x + c$$

$$2 = 2 \cdot \frac{8}{3} - 5 \cdot \frac{4}{2} + 4 + c$$

$$c = 8 - \frac{16}{3} \Rightarrow c = \frac{8}{3}$$

$$\therefore f(x) = \frac{2x^3}{3} - \frac{5x^2}{2} + 2x + \frac{8}{3}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{12} - \frac{5}{8} + 1 + \frac{8}{3}$$

$$= \frac{2-15+24+64}{24}$$

$$= \frac{75}{24}$$

END OF SECTION 1

Solve for p , q and r $\left\{ \begin{array}{l} \text{Eqn } (1) - (3) \\ r = 0.2 \end{array} \right.$

$$2.0 \approx 1$$

$$2.0 = 1$$

Question 7 (1, 3, 3, 2 = 9 marks)

(a) Functions f, g, h are defined:

Determine: $f(x) = \sqrt{4-x}$ $g(x) = 2^x - 12$ $h(x) = e^{2x-1} - 1$

Determine:

$$\frac{x^2 - 91}{(x^2 - 4)(x^2 - 12)} =$$

(ii) the domain and range of $f \circ g(x)$

$$h \rightarrow x : x^*$$

$$h_4 > h_5 = 0 \quad : \quad R_h$$

$$x-1 = (x)f \circ f \circ \dots \circ f \circ (x)f \quad (III)$$

| | |
|------------------|--------------------------------|
| ${}_2 n - 1 = k$ | $\overline{{}_2 n - 1} = (k)!$ |
| $k - 1 = {}_2 n$ | $({}_2 n - 1) - 1 = (n)!$ |
| ${}_2 n - 1 = n$ | $n - 1 = (n - 1)!$ |

(b) The function $k(x)$ is a transformation of $h(x)$ consisting of a horizontal dilation of factor 2 followed by a vertical translation of 1 unit.

$$1 + 1 - \left(\frac{2-x}{1} \right)^2 = k(x)$$

$$\Rightarrow k(x) = x^{\frac{1}{2}}$$

Question 2 (1, 2, 2 = 5 marks)

Use this information to determine:

(a) $P(-1.15 < z < 1.15)$

$$\frac{3}{4} =$$

$$\frac{(51.1 - 2) \text{ d}}{(51.1 - 2) \text{ d}} =$$

$$\frac{\frac{8}{2}}{\frac{8}{4}} =$$

$$\frac{L}{1} =$$

(c) a 75% confidence interval (correct to one decimal place) for the mean of any sample of size 25 taken from a population of mean 50 and standard deviation 10.

$$50 - 1.15 \times \frac{5}{10} < \mu \leq 50 + 1.15 \times \frac{5}{10}$$

$$50 - 2.3 = 47.7$$

47.7 5 2.3

Question 3 (2, 3 = 5 marks)

A particle moves in a straight line so that its distance from a fixed point O is given by $x = \frac{4}{t+1}$ $t > 0$ \rightarrow (Vel is never zero)
where x is the distance in metres at time t seconds.

- (a) What is the average speed for the first two seconds?

$$\begin{aligned} x(0) &= 4 \\ x(2) &= \frac{4}{3} \end{aligned} \quad \left. \vphantom{\begin{aligned} x(0) &= 4 \\ x(2) &= \frac{4}{3} \end{aligned}} \right\} 1\frac{2}{3} \text{ m} \quad \therefore \text{Av Speed} = \frac{2\frac{2}{3}}{2} = 1\frac{1}{3} \text{ m/sec}$$

- (b) When does the particle reach a speed of $\frac{1}{4}$ m/sec

$$\dot{x} = \frac{-4}{(t+1)^2} \quad \therefore \text{Speed} = \frac{4}{(t+1)^2}$$

$$|\dot{x}| = \frac{1}{4} \Rightarrow \frac{4}{(t+1)^2} = \frac{1}{4} \Rightarrow t = 3 \text{ sec}$$

Question 4 (2, 2 = 4 marks)

The first stage of the 2013 AFL Training Camp for elite 18 year old footballers consists of three sets of skills testing A, B and C. From previous experience, the probabilities of success in skills A is 0.4, skills B is 0.6 and skills C is 0.75.

- (a) For the top 50 18 year olds who attend the Training Camp, how many would be expected to succeed in all three skills tests?

$$P(A \cap B \cap C) = \frac{2}{5} \times \frac{3}{5} \times \frac{3}{4} = \frac{9}{50}$$

\therefore Expect 9 to succeed in all 3 skills

To be accepted into the second stage of the Training Program, a trainee must pass Test A and at least one of the other two tests.

- (b) What is the probability that a person is accepted into the second stage?

$$\begin{aligned} &= 0.4 \times (1 - 0.4 \times 0.25) \\ &= 0.4 \times 0.9 \\ &= 0.36 \end{aligned}$$

Question 5 (5 marks)

Solve the inequality $\frac{x}{x-2} \leq \frac{8}{x}$

$$\begin{aligned} x^2 &= 8x - 16 \\ x^2 - 8x + 16 &= 0 \\ (x-4)^2 &= 0 \Rightarrow x = 4 \end{aligned}$$

check:

\therefore Soln $0 < x < 2 \cup x = 4$

Question 6 (3, 3 = 6 marks)

The probability distribution and cumulative probability distribution for a discrete random variable X are shown in the tables below:

| x | 0 | 1 | 2 | 3 | 4 |
|----------|------|-----|-----|-----|------|
| $P(X=x)$ | 0.05 | p | q | r | 0.25 |

| x | 0 | 1 | 2 | 3 | 4 |
|---------------|------|---|------|---|---|
| $P(X \leq x)$ | 0.05 | | 0.55 | | 1 |

and it is known that the expected value of the random variable is 2.5.

- (a) Form three equations from the given information

$$\begin{aligned} p + q + r &= 0.7 \quad \dots (1) \\ p + 2q + 3r + 1 &= 2.5 \Rightarrow p + 2q + 3r = 1.5 \quad \dots (2) \\ p + q + 0.05 &= 0.55 \Rightarrow p + q = 0.5 \quad \dots (3) \end{aligned}$$