PERTH MODERN SCHOOL
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Independent Public School

Year 12 Methods
TEST 4 2019
Friday 23 August
TIME: 45 minutes working
One page of notes is allowed.
Calculator Assumed
46 Marks 6 Questions

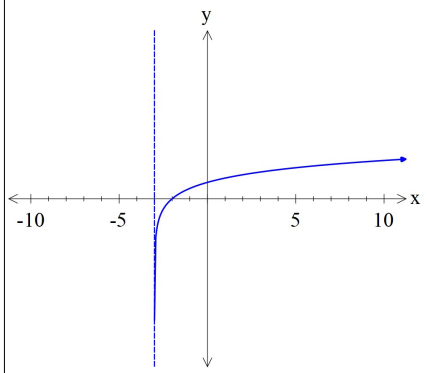
Name: _____ Teacher: _____

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 **(8 marks)**

Consider the function $f(x)=\log_a(x+3)$, $a>1$.

a) Sketch the function on the axes below showing all major features. (3 marks)

Solution

Specific behaviours
✓ asymptote at x=-3 ✓ shape ✓ positive y intercept

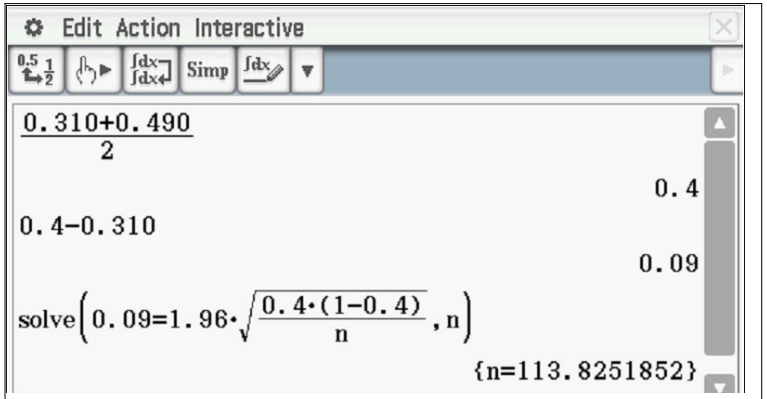
b) Determine the value of p given that $f(p)=3$. (2 marks)

$3=\log_a(p+3)$ $p+3=a^3$ $p=a^3-3$
Specific behaviours
✓ converts to a power statement ✓ expresses p in terms of a

c) Consider the new function $y=f(x-4a-3)+2$, determine the x coordinate where $y=3$ on this new function. (Note; a is the same constant as above.)

$3=\log_a(x-4a-3+3)+2$ $1=\log_a(x-4a-3+3)$ $a^1=x-4a$ $x=5a$
Specific behaviours
✓ obtains correct equation for x ✓ uses a power statement ✓ final expression for x in terms of a

(3 marks)



n=114

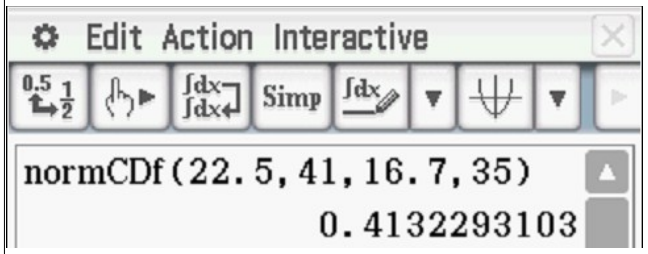
Specific behaviours
✓ uses correct p value
✓ sets up an equation for n with correct z score
✓ states a rounded up n value

Question 2 **(15 marks)**

A company makes circuit boards to be used to make computers. The length of the circuit boards is estimated to be Normally distributed with a mean of 35 cm and a standard deviation of 16.7 cm.

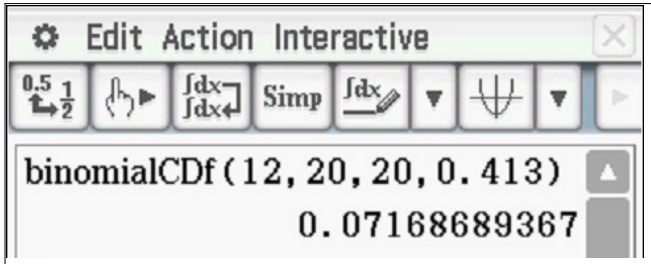
A customer will only buy circuit boards that are between 22.5 and 41 cm.

a) Determine the probability that a circuit board will meet the customer's requirements. (2 marks)

Solution

Specific behaviours
✓ uses correct parameters
✓ states probability (at least 2 dp)

b) If the company made 20 circuit boards, determine the probability that at least 12 boards would be suitable for the customer. (3 marks)

Solution
$X \sim Bin(20, 0.413)$ $P(X \geq 12)$



Specific behaviours

✓ states binomial distribution

✓ uses correct parameters

✓ states probability

Note: Answer only- 2 marks out of 3

The government will tax the circuit boards made by the company according to its length. Complete the table below by determining the probabilities to 4dp.

c) (4 marks)

Solution

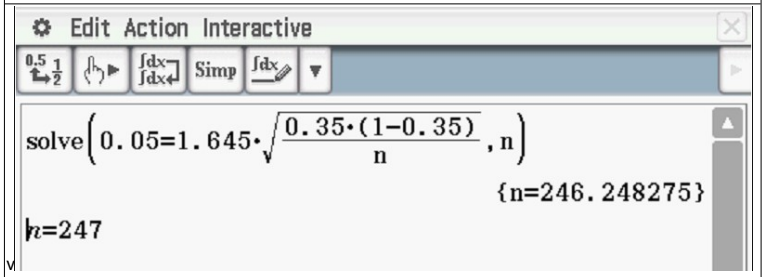
Length of circuit board	$length \leq 15cm$	$15 < length \leq 30cm$	$30 < length \leq 55cm$	$length > 55cm$
Tax $	$5	$7.50	$9	$11.50
Probability	0.0975 Or 0.1156	0.2668	0.5021	0.1155 Or 0.1336

Question 6 **(6 marks)**

It is believed that a toy company produces defective toys at a proportion of $\hat{p} = 0.35$.

a) A consultant wishes to determine the true proportion p of defective toys within 5% and with a confidence of 90%. Determine how many toys should be taken for sampling. (3 marks)

Solution



Specific behaviours

✓ uses appropriate z score

✓ sets up an equation for n

✓ states a rounded up value for n

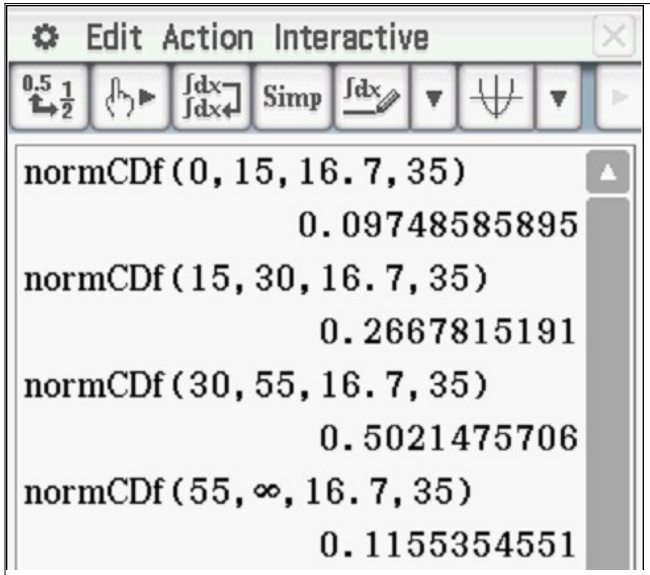
b) A year later another sample is taken and a 95% confidence interval for the proportion of defective toys is calculated as $(0.310, 0.490)$. Determine the sample size. (3 marks)

Specific behaviours
✓ uses correct integral
✓ limits correct
✓ states mean (Note: 2 marks for answer only)

d) the standard deviation of X. (3 marks)

Solution
$\frac{-6}{343}\int_{-2}^{5}(x-1.5)^2(x+2)(x-5)dx$ → 2.45
$\sqrt{2.45}$ → 1.565247584

Specific behaviours
✓ uses correct integral for variance
✓ uses correct limits
✓ states the standard deviation (i.e. square root of variance)



Specific behaviours
✓ at least two correct probs
✓ at least 3 correct probs
✓ all four correct
✓ all rounded to 4 dp

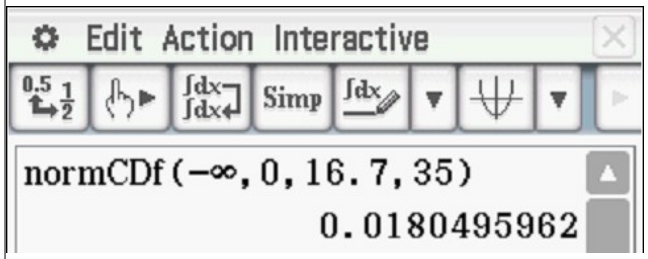
d) Determine the expected tax bill for a circuit board. (2 marks)

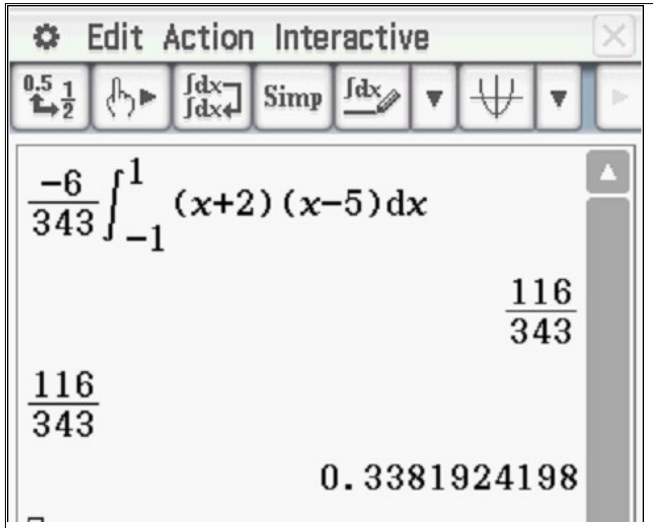
E(Tax)=$8.49 or $8.43 or $8.54
Specific behaviours
✓ states to at least 2 dp
✓ states units

e) Determine the standard deviation for the tax of a circuit board. (2 marks)

Standard deviation =$ 1.635 or 1.686 or 1.669
Specific behaviours
✓ states to at least 2 dp ✓ shows calculation Note: full marks for answer only, no need for units

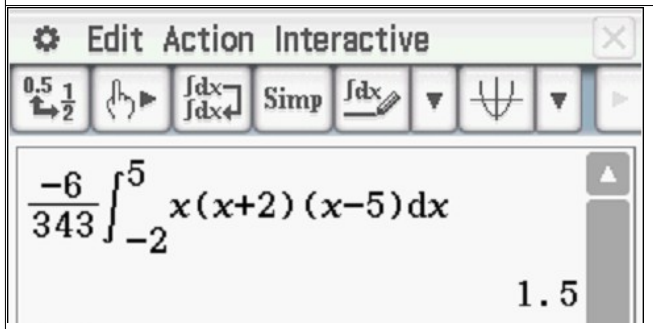
f) **Show** one reason why the Normal probability model is not appropriate for the lengths. (2 marks)

Solution
$P(X<0) =$ 0.018 which cannot be as length cannot be negative. 
Specific behaviours
✓ states that lengths cannot be negative ✓ states prob that length is less than zero



Specific behaviours
✓ integrates over correct domain ✓ states prob to at least 2 dp or exact

c) the mean of X. (3 marks)



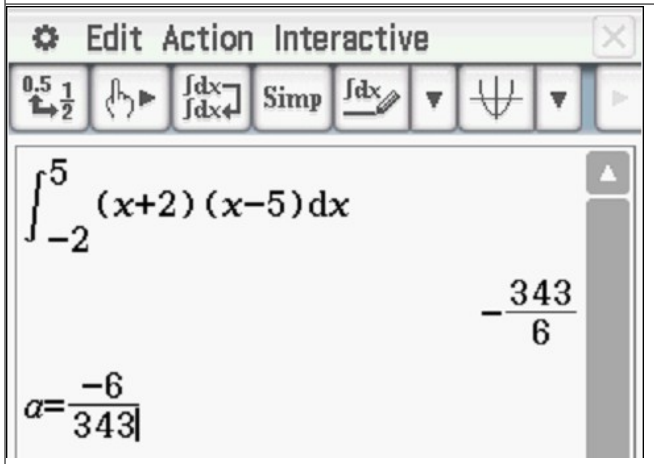
Question 5 **(10 marks)**

A probability density function is defined as the following.

$$f(x)=\begin{cases} a(x+2)(x-5) & ,-2\le x\le 5 \\ 0 & all\ other\ x\ values \end{cases}$$ where a is a constant.

Determine the following.

a) the exact value of a. (2 marks)



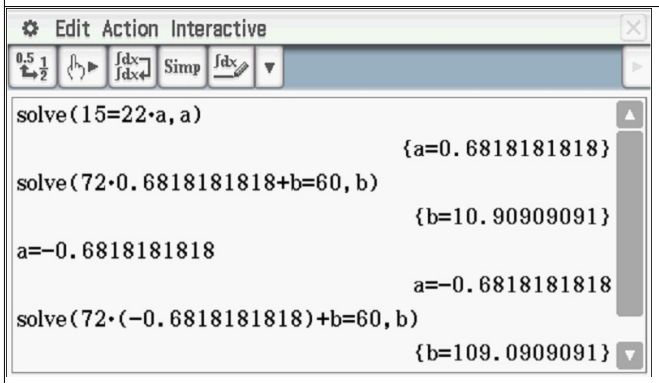
Specific behaviours
✓ integrates function above
✓ states exact value of a

b) $\Pr(-1\le x\le 1)$ (2 marks)

Question 3 **(4 marks)**

The exam data for a cohort of Year 12 Methods students at a school has a mean of 72% and a standard deviation of 22%. The Head of Department needs to scale the results so that the mean is 60% and a standard deviation of 15%. This will be done by multiplying the original scores by a constant a and adding a constant b (any order). Determine two possible pairs values of a & b and the order they should be applied.

Solution



OR

$a(72+b)=60$

$a22=15$

$a=0.6818$

$b=16$

OR

$a(72+b)=60$

$a22=-15$

$a=-0.6818$

$b=-160$

Specific behaviours
✓ shows an equation for value of a
✓ shows an equation for value of b
✓ solves for one pair of a&b values stating order
✓ states two possible pairs of a&b values and stating order

Question 4 **(3 marks)**

A pharmaceutical company wishes to gather information on a new form of headache tablets.

Comment on whether there is any bias in the following sampling methods, give reasons.

a) People were surveyed outside a dental clinic. (1 marks)

Bias as dental patients more likely to have headaches due to dental pain than average.
Specific behaviours
✓ States bias with a reason

b) People waiting at a central bus station in the city. (1 marks)

No bias as not connection between headaches and mode of travel in general
Specific behaviours
✓ states no bias with a reason Accept a reasonable argument of bias with reason for this part ONLY

c) People were contacted using random mobile numbers. (1 marks)

Bias as only people with mobiles contacted-those with landlines only are ignored
Specific behaviours
✓ Bias stated with reason