



Year 12 Mathematics Methods Test 4  
Logarithmic functions and Calculus of Log functions

Name: \_\_\_\_\_

**Section 1: Calculator Free                      39 marks                      40 minutes**

**1.      [1, 1, 1, 2 marks]**

Suppose that two variables  $x$  and  $y$  are related by  $y = 6^x$ .

a)      Use the *definition of a logarithm* to express  $x$  in terms of  $y$ .

b)      Given that  $\log_6 2 = q$ , write the following in terms of  $q$ :

i)       $\log_6 24$

ii)       $\log_6 0.5$

iii)       $\log_6 3$

2. [2, 2, 2 marks]

Solve the following, giving your answers in exact form involving logarithms where necessary.

a)  $3^{x-4} = 14$

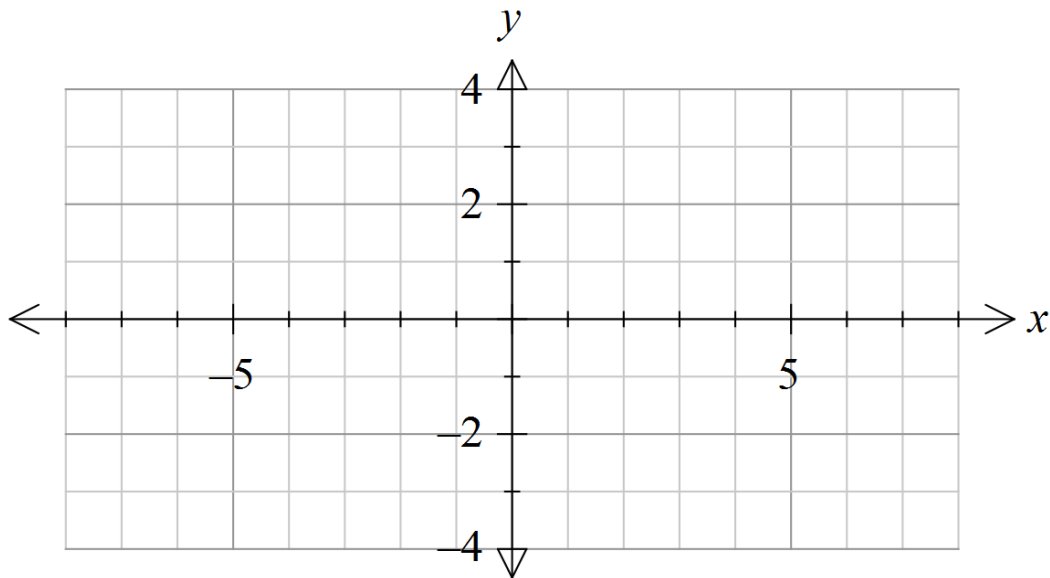
b)  $\log(x+4) - \log(x-5) = 1$

c)  $11(3^x) = 5 + 3^{x+2}$

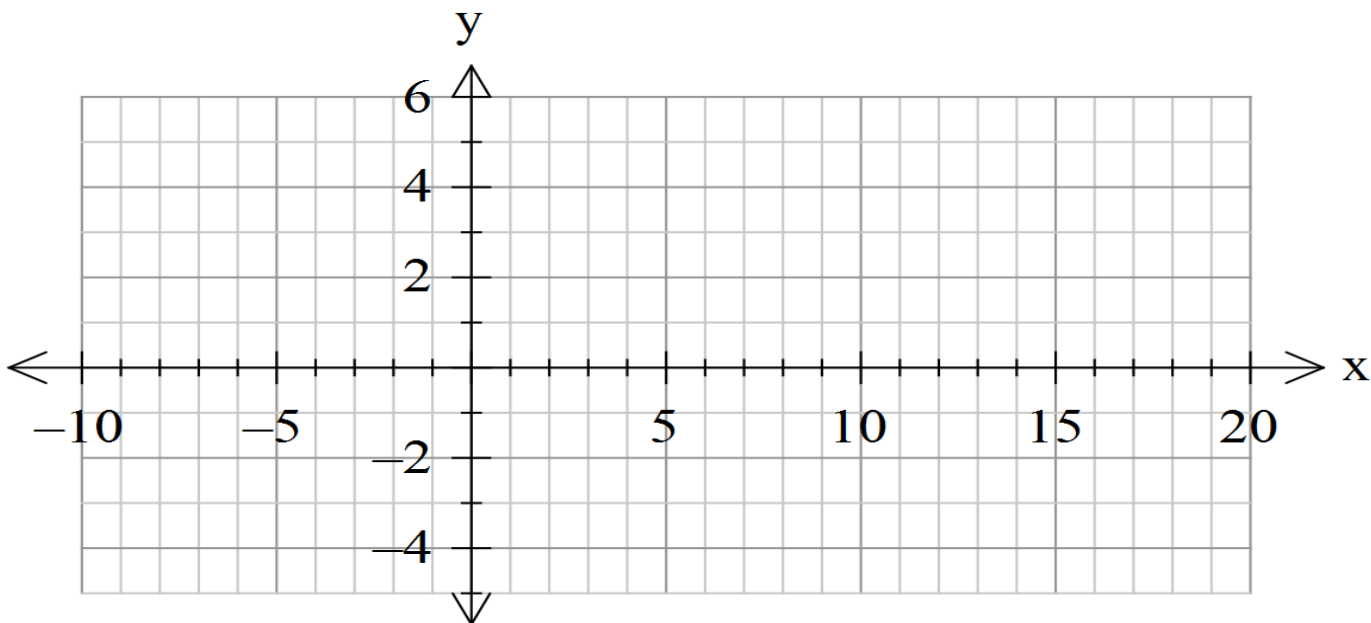
3. [3, 3, 2 marks]

On the sets of axes below, sketch the functions:

a)  $y = \log_3(x-2)$



b)  $y = \log_{0.5}x + 4$



c) Use the graph to solve  $\log_{0.5}x = 0.5$

**4. [3, 2 marks]**

Let  $g(x) = \frac{\ln x}{x^2}$ , for  $x > 0$ .

(a) Use the quotient rule to show that  $g'(x) = \frac{1 - 2 \ln x}{x^3}$ .

(b) The graph of  $g$  has a maximum point at A. Find the  $x$ -coordinate of A.

**5. [2 marks]**

Find the derivative with respect to  $x$  of  $y = \ln(x^3 + x^2)$

6. [2, 1, 2, 2, 2 marks]

a) Given the function  $g(x) = x \ln x - x + 1$ , determine  $g'(x)$

b) Hence determine an expression for  $\int \ln(x) dx$

c) Evaluate  $\int_1^2 \ln(x) dx$

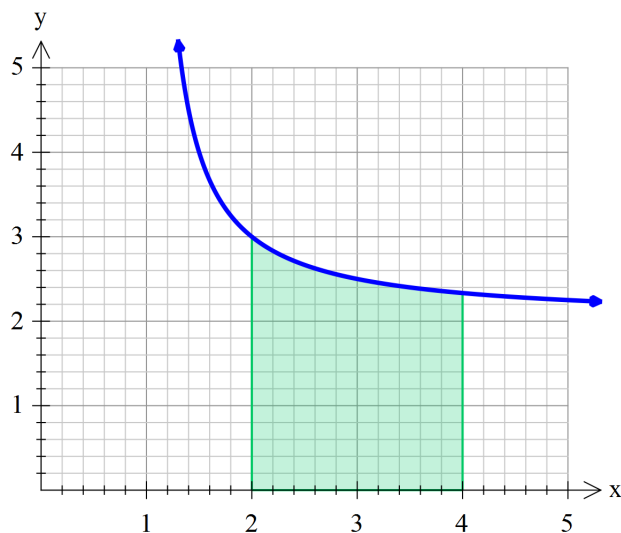
d) Evaluate  $\int_1^2 \ln \sqrt{x} dx$

e) Determine an expression for  $\int_a^b \ln(x) dx$  in terms of  $a$  and  $b$ , such that  $b > a > 0$

7. [2, 2 marks]

Consider the function  $f(x) = 2 + \frac{1}{x-1}; x > 1$

The region enclosed by the graph of  $f(x)$ , the  $x$ -axis and the lines  $x = 2$  and  $x = 4$ , is shaded below.



(a) Find  $\int f(x) dx$ .

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(b) Find a simplified expression for the exact area of A.

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**Section 2: Calculator & Notes Allowed      13 marks      15 minutes**

**8. [1, 1, 3 marks]**

The faintest sound that can be heard by the human ear has intensity

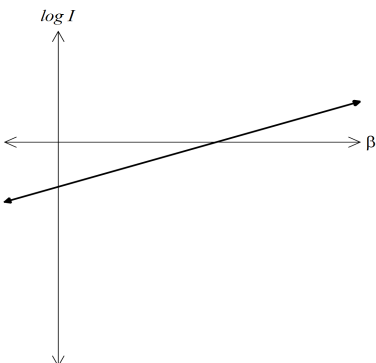
$$I_0 = 10^{-16} \text{ watts per square centimetre.}$$

Noise levels,  $\beta$ , are measured in decibels and are related to intensity:

$$\beta = 10 \log \frac{I}{I_0} \text{ decibels}$$

Where  $I$  is the intensity of sound in watts per square centimetre.

- The maximum intensity which a human ear can tolerate is  $10^{-4}$  watts per square centimetre. Determine the corresponding value of  $\beta$ .
- Busy motor traffic has a noise level of 70 decibels. Determine the corresponding intensity.
- The graph (without scales) of  $\log I$  against  $\beta$  is sketched below; it is linear. By expressing  $\log I$  in terms of  $\beta$ , determine the gradient and the intercept on the vertical axis.



9. [3, 2 marks]

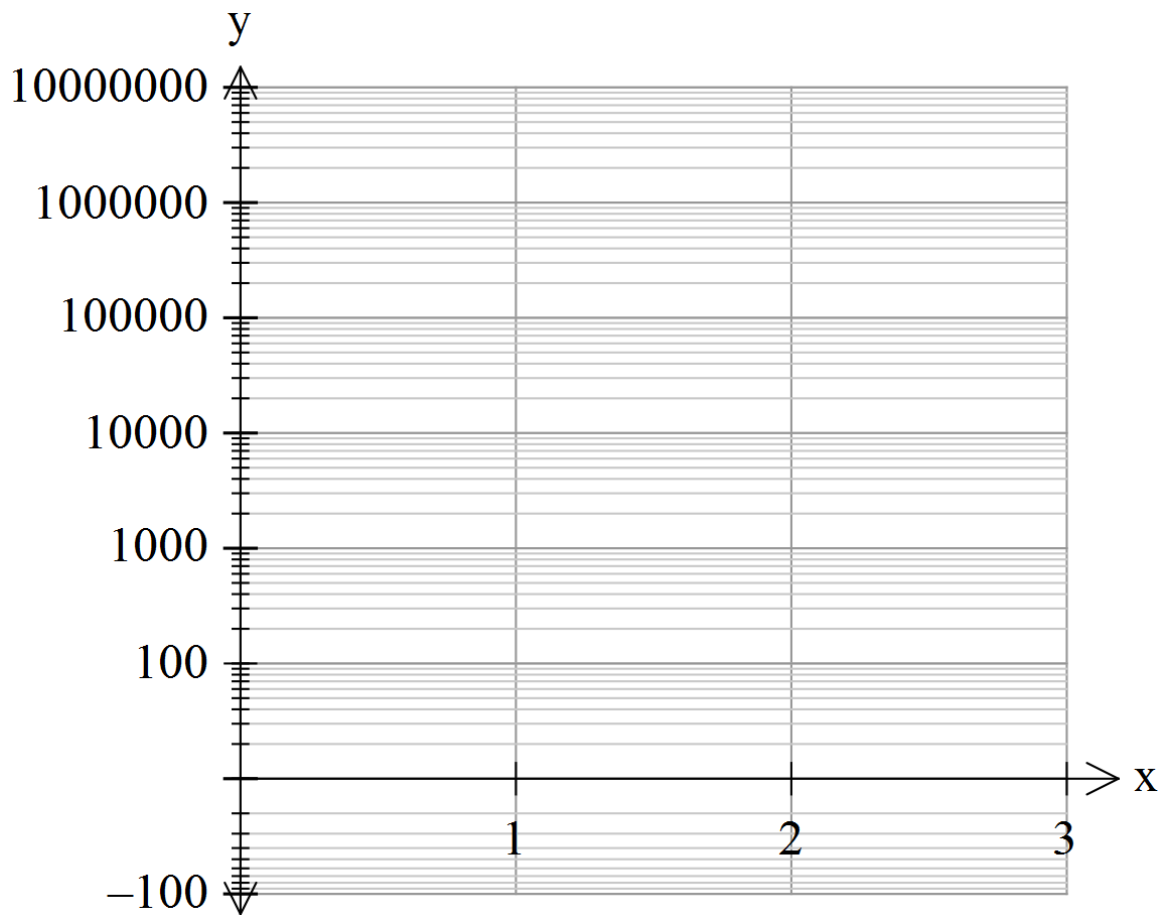
A particle P moves along a straight line. Its velocity,  $v \text{ ms}^{-1}$  at time  $t$  seconds, is given by  
$$v = 10\ln(t + 3) + 2 \text{ for } t \geq 0$$

(a) Find the initial velocity and acceleration

(b) Find the acceleration of P when its velocity is  $20 \text{ ms}^{-1}$



10. [3 marks]



Draw the graph of  $y = 2 \times 10^{2x}$ , for  $x \geq 0$ ; using semi-log grid.