Kennedy Baptist College

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS METHODS UNITS 1&2

Section One: Calculator-free

SOLUTIONS

WA student number: In figures | | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Supplementary page

Question number: _____

Question 8 **(7 marks)**

The line $y=3x+c$ is a tangent to the curve $y=x^3-3x^2-6x+7$. Determine the value(s) of the constant c.

Solution

Gradient of cubic:

$$\frac{dy}{dx}=3x^2-6x-6$$

Gradient of line is 3 so:

$$3x^2-6x-6=3 \quad x^2-6x-9=0 \quad x^2-2x-3=0 \quad (x-3)(x+1)=0$$
$$x=3, x=-1$$

At $x=3$:

$$y=27-27-18+7=-11$$

$$y+11=3(x-3) \quad y=3x-20 \Rightarrow c=-20$$

At $x=-1$:

$$y=-1-3+6+7=9$$

$$y-9=3(x+1) \quad y=3x+12 \Rightarrow c=12$$

Hence $c=12, c=-20$.

Specific behaviours

✓ gradient function for cubic

ü equates $\frac{dy}{dx}=3$

ü simplifies and factors quadratic

ü both solutions to quadratic

ü y-coordinate of point of tangency

ü one value of c

ü repeats for second value of c

End of questions

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

Solve the following equations.

(a) $18x=25x-28$. (1 mark)

Solution

$$7x=28 \quad x=4$$

Specific behaviours

✓ correct solution

(b) $9x^2=18x$. (2 marks)

Solution

$$9x^2-18x=0 \quad 9x(x-2)=0$$
$$x=0, x=2$$

Specific behaviours

✓ equates to 0 and factorises

ü both correct solutions

(c) $x^3-9x^2-25x+33=0$. (3 marks)

Solution

When $x=1$: LHS $=1-9-25+33=0$

$$x^3-9x^2-25x+33=(x-1)(x^2+bx-33)$$
$$b-1=-9 \Rightarrow b=-8$$

$$(x-1)(x^2-8x-33)=0 \quad (x-1)(x+3)(x-11)=0$$

$$x=1, x=-3, x=11$$

Specific behaviours

✓ indicates that $x-1$ is a factor

ü determines quadratic factor

ü all three solutions

See next page

Question 2 **(7 marks)**

(a) Simplify $\sqrt{4^{-5}}$. (2 marks)

Solution
$\sqrt{4^{-5}}=\left(\sqrt{4}\right)^{-5}¿2^{-5}¿\frac{1}{32}$
Specific behaviours
✓ eliminates square root
ü correct fraction

(b) Write the value of xy in scientific notation when $x=2.5\times10^{3}$ and $y=5\times10^{-7}$. (2 marks)

Solution
$2.5\times5\times10^{3}\times10^{-7}=12.5\times10^{-4}¿1.25\times10^{-3}$
Specific behaviours
✓ obtains equivalent expression of form $a\times10^{b}$
ü correct value using scientific notation

(c) Determine the value of n given that $9^{n+1}=\sqrt{27}$. (3 marks)

Solution
$9^{n+1}=\sqrt{27}\left(3^{2}\right)^{n+1}=\sqrt{3^{3}}3^{2n+2}=3^{\frac{3}{2}}$
$2n+2=\frac{3}{2}n=\frac{-1}{4}$
Specific behaviours
✓ expresses LHS in form 3^{a}
ü expresses RHS in form 3^{b}
ü correct value of n

Question 7 **(6 marks)**

Consider the function defined by $f(x)=2x^{2}+5$.

(a) Determine $f'(-3)$. (1 mark)

Solution
$f'(x)=4x$
$f'(-3)=4(-3)=-12$
Specific behaviours
ü correct value

(b) Show that when $x=3$, the expression $f(x+h)-f(x)$ simplifies to $12h+2h^{2}$. (3 marks)

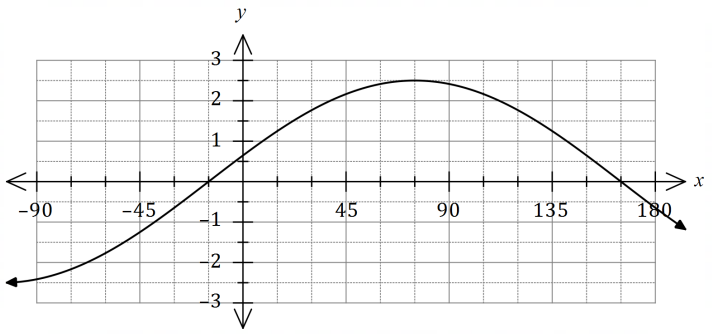
Solution
$f(3+h)-f(3)=2(3+h)^{2}+5-\left(2(3)^{2}+5\right)$
$¿2\left(9+6h+h^{2}\right)+5-18+5¿18+12h+2h^{2}-18¿12h+2h^{2}$
Specific behaviours
ü substitutes $x+h$ and x into function
ü clearly and correctly expands quadratic term
ü substitutes for x and simplifies

(c) Show use of the result in (b) and the formula $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ to determine the value of $f'(3)$. (2 marks)

Solution
$f'(3)=\lim_{h\to0}\frac{12h+2h^{2}}{h}¿\lim_{h\to0}(12+2h)¿12$
Specific behaviours
✓ uses result as numerator in limit
ü correctly eliminates h from denominator and simplifies limit

Question 6 **(7 marks)**

(a) Part of the graph of $y=a\cos(x-\theta)$ is shown below.



State the value of the constant a and the value of the constant θ, $0°\le\theta\le180°$. (2 marks)

Solution
$a=2.5, \theta=75°$
Specific behaviours
✓ correct amplitude
ü correct phase angle

(b) Show that $\cos(x+y)+\cos(x-y)=k\cos x\cos y$ and state the value of the constant k. (2 marks)

Solution
$\cos(x+y)+\cos(x-y)=\cos x\cos y+\sin x\sin y+\cos x\cos y-\sin x\sin y$
$\quad\quad 2\cos x\cos y$
$k=2$
Specific behaviours
✓ uses sum and difference identities correctly
ü states correct value of k

(c) Determine an exact value for $\cos75°+\cos15°$. (3 marks)

Solution
If $x=45°$ and $y=30°$ then $x+y=75°$ and $x-y=15°$. Hence
$\cos75°+\cos15°=2\cos45°\cos30°=2\times\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{6}}{2}$
Specific behaviours
✓ indicates suitable values for x and y
ü uses result from (b) and correct exact values
ü correct, simplified surd

Question 3 **(6 marks)**

(a) The turning point of a quadratic is at $(-3,-10)$ and the curve passes through $(0,8)$.
Determine the equation of the quadratic in the form $y=ax^2+bx+c$. (3 marks)

Solution
$y=a(x+3)^2-10$
$(0,8)\Rightarrow 8=9a-10$
$a=2$
$y=2(x+3)^2-10=2(x^2+6x+9)-10=2x^2+12x+8$
Specific behaviours
✓ writes in completed square form using constant
ü uses y-intercept to evaluate constant
ü correct equation in required form

(b) Functions f, g and h are defined by $f(x)=3+\sqrt{x-5}$, $g(x)=2f(x)$ and $h(x)=f(x+7)$.

State the

(i) domain of $f(x)$. (1 mark)

Solution
Require $x-5\ge0$:
$D_f\{x\in R: x\ge5\}$
Specific behaviours
✓ states restriction on x

(ii) range of $g(x)$. (1 mark)

Solution
Range of g is 2 × range of f:
$R_g\{y\in R, y\ge6\}$
Specific behaviours
✓ states restriction on y

(iii) domain of $h(x)$. (1 mark)

Solution
h is f translated 7 units left.
$D_f\{x\in R: x\ge-2\}$
Specific behaviours
✓ states restriction on x

Question 4 **(6 marks)**

(a) The point $A(1,3)$ lies on the curve with equation $y=x^3-4x^2+7x-1$. Determine the equation of the tangent to the curve at A. (3 marks)

Solution

$$\frac{dy}{dx}=3x^2-8x+7$$

When $x=1$

$$\frac{dy}{dx}=3-8+7=2$$

Equation of tangent:

$$y-3=2(x-1)$$

Or

$$y=2x+1$$

Specific behaviours

✓ derivative
ü gradient of tangent
ü equation of tangent

(b) Determine $g(1)$ given that $g(-1)=5$ and $g'(x)=12x^3+4x-3$. (3 marks)

Solution

$$g(x)=3x^4+2x^2-3x+c$$

Using $g(-1)=5$:

$$3+2+3+c=5c=-3$$

$$g(1)=3+2-3-3¿-1$$

Specific behaviours

✓ antiderivative
ü determines constant
ü correct value

Question 5 **(7 marks)**

(a) A sequence is defined by $T_{n+1}=T_n+0.3, T_1=5$. Determine

(i) T_{101}. (2 marks)

Solution

$$T_{101}=5+(100)(0.3)¿35$$

Specific behaviours

✓ indicates use of general term formula
ü correct term

(ii) the sum of the first 101 terms of the sequence. (2 marks)

Solution

$$S_{101}=\frac{101}{2}(2(5)+(100)(0.3))$$

$$¿\frac{101\times 40}{2}=101\times 20=2020$$

Specific behaviours

✓ indicates correct use of sum formula
ü correct sum

(b) The sum to infinity of the series $4+4k+4k^2+4k^3+\ldots$ is 10. Determine the sum of the first three terms of the series. (3 marks)

Solution

Series is geometric with $a=4$ and $r=k$.

$$\frac{4}{1-k}=10\,4=10-10k\,10k=6k=\frac{3}{5}=0.6$$

$$S=4+4(0.6)+4(0.6)^2¿4+2.4+1.44¿7.84$$

NB

$$S=4+4\left(\frac{3}{5}\right)+4\left(\frac{3}{5}\right)^2=4+\frac{12}{5}+\frac{36}{25}=\frac{100+60+36}{25}=\frac{196}{25}$$

Specific behaviours

✓ equation using sum to infinity
ü value of k
ü correct sum