Additional working space

Question number: __________

SHENTON COLLEGE

Examination Semester Two 2017
Question/Answer Booklet

MATHEMATICS
METHODS UNITS 3 and 4
Section One
(Calculator-free)

Teacher (Circle One) Mrs Friday Mr Smith

Your name Solution.

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section
To be provided by the supervisor
Question/answer booklet for Section One.
Formula sheet.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

	Number of questions	Working time (minutes)	Marks available
Section One Calculator Free	**8**	**50**	**52**
Section Two Calculator Assumed	13	100	96
		Total	148

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the Year 12 *Information Handbook 2017.* Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil, except in diagrams**.

STRUCTURE OF THIS PAPER

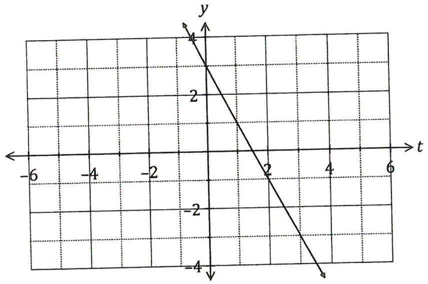
QUESTION	MARKS AVAILABLE	MARKS AWARDED
1	6	
2	6	
3	7	
4	7	
5	7	
6	8	
7	6	
8	5	
TOTAL	**52**	

Additional working space

Question number: _________

Question 8 **(5 marks)**

Part of the graph of the linear function $y = f(t)$ is shown below.



Another function $A(x)$ is given by

$$A(x) = \int_1^x f(t)\,dt\,.$$

Use the increments formula to estimate the change in A as x increases from 8.5 to 8.6.

$A'(x) = \frac{d}{dx}\int_1^x f(t)\,dt = f(x)$ ✓ Indicates $A'(x)$

$f(x) = -2x+3$

$\frac{\delta A}{\delta x} \approx \frac{dA}{dx}$ ✓ uses $x = 8.5$, $\delta x = 0.1$

$\delta A \approx \left.\frac{dA}{dx}\right|_{x=8.5} \cdot \delta x$ ✓ uses increments formula

$= (-2(8.5)+3)(0.1)$ ✓ determines change in A.

$= -1.4$

✓ states as decrease

End of questions

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

(a) Determine n, if $\log_3 n = 1 - 2\log_3 7 + \log_3 5$. (3 marks)

$1 - 2\log_3 7 + \log_3 5$

$= \log_3 3 - \log_3 7^2 + \log_3 5$ ✓ $2\log_3 7 = \log_3 7^2$

$= \log_3 \left(\frac{3 \times 5}{49}\right)$ ✓ $1 = \log_3 3$

✓ $n = \frac{15}{49}$.

$n = \frac{15}{49}$

(b) Determine the exact solution to $5(2)^{x-3} = 30$. (3 marks)

$2^{x-3} = 6$ ✓ $\div 5$

$(x-3)\log 2 = \log 6$ ✓ takes logs of any base both sides

$x = \frac{\log 6}{\log 2} + 3$ ✓ $x =$ solve.

OR $2^{x-3} = 6$ ✓ $\div 5$

$(x-3)\log_2 2 = \log_2 6$ ✓ $\log_2$

$x = \log_2 6 + 3$ ✓ $x =$ solve

See next page

Question 2 **(6 marks)**

The discrete random variable X is defined by

$$P(X=x)=\begin{cases}\dfrac{k}{x+1} & x=0,1\\ 0 & \text{elsewhere.}\end{cases}$$

(a) Determine the value of the constant k. (2 marks)

$\frac{K}{1}+\frac{K}{2}=1$ ✓ Σ to 1

$\frac{3K}{2}=1$ ✓ $K=$

$K=\frac{2}{3}$

(b) Determine

(i) $E(5-3X)$. (2 marks)

$E(X)=p=\frac{1}{3}$ ✓ Uses $E(X)=p$, $p(X=1)$

$E(5-3X)=5-3\left(\frac{1}{3}\right)$

$=4$ ✓ $E(X)$

(ii) $\mathrm{Var}(1+6X)$. (2 marks)

$\mathrm{Var}\ X=p(1-p)$ ✓ Uses $\mathrm{Var}(X)=p(1-p)$

$=\frac{2}{9}$

$\mathrm{Var}(1+6X)=6^2\times\frac{2}{9}$ ✓ $\mathrm{Var}(1+6X)$

$=8$

Question 7 **(6 marks)**

The functions f and g intersect at the point $(-1,7)$.

The first derivatives of the functions are $f'(x)=30(5x+7)^2$ and $g'(x)=10\pi\sin(\pi(1-2x))$.

Determine an expression for each function.

$f'(x)=30(5x+7)^2$

$f(x)=30\int(5x+7)^2\,dx$

$=\frac{30(5x+7)^3}{3(5)}+c$ ✓ antidifferentiates $f'(x)$

$f(x)=2(5x+7)^3+c$ ✓ evaluates c

at $(-1,7)$ ✓ states $f(x)$ in simplest form

$7=2(-5+7)^3+c$

$c=7-16$

$c=-9$

$f(x)=2(5x+7)^3-9$

$g'(x)=10\pi\sin(\pi(1-2x))$ ✓ antidifferentiates $g'(x)$

$g(x)=10\pi\int\sin(\pi-2\pi x)\,dx$

$=10\pi\left[\frac{-\cos(\pi-2\pi x)}{-2\pi}\right]+c$ ✓ evaluates c

$=5\cos(\pi-2\pi x)+c$ ✓ states $g(x)$ in simplest form

at $(-1,7)$

$7=5\cos(\pi+2\pi)+c$

$7=5(-1)+c$

$c=12$

$g(x)=5\cos(\pi-2\pi x)+12$

or $g(x)=5\cos(\pi(1-2x))+12$

Question 6 **(8 marks)**

A curve has first derivative $\frac{dy}{dx} = 3x(x-4)$ and passes through the point $P(1,-5)$.

(a) Determine the value(s) of x for which $\frac{d^2y}{dx^2} = 0$. (2 marks)

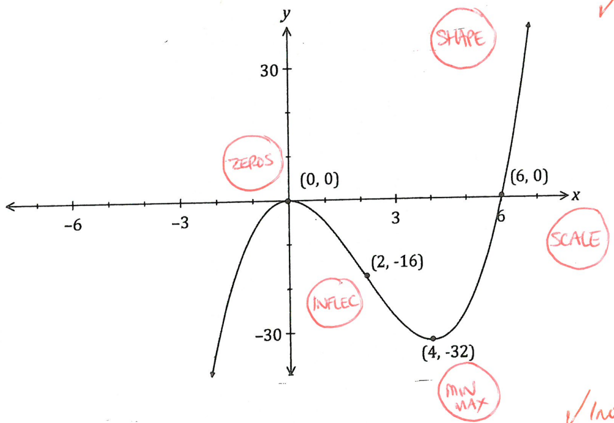
$\frac{d^2y}{dx^2} = 3(x-4) + (1)(3x)$

$= 3x - 12 + 3x$ ✓ differentiates

$\therefore 6x - 12 = 0$ ✓ states value

ie $x = 2$

(b) Sketch the curve on the axes below, clearly indicating the location of all axes intercepts, stationary points and points of inflection. (6 marks)



✓ Indicates scale

✓ Smooth continuous curve

✓ obtains zeroes of y.

✓ Indicates (ie lab location on graph of Min Max P of I Roots

✓ Indicates co-ordinates of P of I and Min.

$\frac{dy}{dx} = 3x(x-4)$

$0 = 3x(x-4)$

$x = 0$ or $x = 4$

$\frac{dy}{dx} = 3x^2 - 12x$

$y = x^3 - 6x^2 + c$

at $(1,-5)$ $-5 = 1 - 6 + c$

$c = 0$

$y = x^3 - 6x^2$ ✓ $y =$

$0 = x^3 - 6x^2$

$0 = x^2(x-6)$

$x = 0$ or $x = 6$

See next page

Question 3 **(7 marks)**

The rate of change of displacement of a particle moving in a straight line at any time t seconds is given by

$$\frac{dx}{dt} = 3 + 2e^{0.1t} \text{ cm/s}.$$

Initially, when $t = 0$, the particle is at A, a fixed point on the line.

(a) Calculate the initial velocity of the particle. (1 mark)

$\frac{dx}{dt} = 3 + 2e^0$

$= 5$ cm/s ✓ $\frac{dx}{dt}$.

(b) Determine the distance of the particle from A after 20 s. (3 marks)

$x = 3t + 20e^{0.1t} + c$

$t = 0$
$x = 0$

$20e^0 + c = 0$ ✓ integrates

$c = -20$ ✓ evaluates c

$x = 3(20) + 20e^2 - 20$

$x = 40 + 20e^2$ cm ✓ substitutes to obtain distance

(c) Determine when the acceleration of the particle is 7 cm/s^2. (3 marks)

$\frac{d^2x}{dt^2} = 0.2e^{0.1t}$ ✓ differentiation

$\therefore 0.2e^{0.1t} = 7$ ✓ eliminates e

$0.1t = \ln 35$

$t = 10\ln 35$ s ✓ t = solve.

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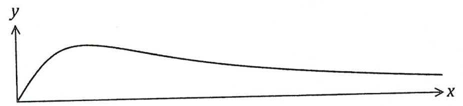
Question 4 **(7 marks)**

The graph of $y = f(x),\ x \geq 0$, is shown below, where $f(x) = \dfrac{4x}{x^2+3}$.



(a) Determine the gradient of the curve when $x = 2$. (3 marks)

$$f'(x) = \frac{(x^2+3)(4) - 4x(2x)}{(x^2+3)^2}$$ ✓ uses Quotient Rule

$$f'(2) = \frac{4(7) - 8(4)}{(7)^2}$$ ✓ correct $f'(x)$

$$= \frac{-4}{49}$$ ✓ correct gradient.

(b) Determine the exact area bounded by the curve $y = f(x)$ and the lines $y = 0$ and $x = 2$, simplifying your answer. (4 marks)

$$A = \int_0^2 f(x)\,dx$$ ✓ writes integral

$$= 2\int_0^2 \frac{2x}{x^2+3}\,dx.$$ ✓ antidifferentiatio

$$= 2\left[\ln|x^2+3|\right]_0^2$$ ✓ substitution

$$= 2\left[\ln 7 - \ln 3\right]$$ ✓ simplifies

$$= 2\ln\frac{7}{3}$$

Question 5 **(7 marks)**

A function is defined by $f(x) = \dfrac{2 + 2\ln x}{3x}$.

(a) State the natural domain of f. (1 mark)

$x > 0$ ✓ correct domain

(b) Show that $f'(1) = 0$. (3 marks)

$$f'(x) = \frac{3x\left(\frac{2}{x}\right) - (2 + 2\ln x)(3)}{(3x)^2}$$ ✓ uses Quotient Rule

$$f'(1) = \frac{6 - (2 + 2\ln 1)(3)}{9}$$ ✓ correct $u'v$ and uv'

$$= \frac{6-6}{9}$$ ✓ shows $f'(1) = 0$

$$= 0$$

(c) Use the second derivative test to determine the nature of the stationary point of the function at $x = 1$. (3 marks)

$$f'(x) = \frac{6 - 6 - 6\ln x}{9x^2}$$ ✓ simplifies $f'(x)$ to be able to use Q rule.

$$= \frac{-2\ln x}{3x^2}$$

$$f''(x) = \frac{(3x^2)\left(-\frac{2}{x}\right) - \left[(-2\ln x)(6x)\right]}{(3x^2)^2}$$ ✓ Differentiate correctly

$$f''(1) = \frac{3(-2) - \left[-2\ln 1\,(6)\right]}{9}$$

$$= \frac{-6 - 0}{9}$$ ✓ interprets $f''(1)$ correctly.

$$= -\frac{6}{9}$$

$f''(1) < 0 \ \therefore$ Maximum T.P.