(f) $2^{x-3} = 5^{2x+1}$

$(x-3)\log 2 = (2x+1)\log 5$ ✓
$x\log 2 - 3\log 2 = 2x\log 5 + \log 5$
$x\log 2 - 2x\log 5 = \log 5 + 3\log 2$
$x(\log 2 - 2\log 5) = \log 5 + 3\log 2$ ✓
$x = \dfrac{\log 5 + 3\log 2}{\log 2 - 2\log 5}$ ✓

(g) $4^{2x} - 4^{x} - 6 = 0$

Let $y = 4^x$ ✓
$y^2 - y - 6 = 0$ ✓
$(y-3)(y+2) = 0$
$y = 3 \quad y = -2$ ✓
$4^x \neq -2$
$4^x = 3$ ✓
$x\log 4 = \log 3$
$x = \dfrac{\log 3}{\log 4}$

(h) $5e^{2-x} = 100$

$e^{2-x} = 20$ ✓
$(2-x)\ln e = \ln 20$ ✓
$2 - x = \ln 20$
$x = 2 - \ln 20$ ✓

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Logarithm Laws and Solving Equations

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 = 8 marks] **CF**

Express each of the following as a single logarithm:

(a) $2\log 3 + \log 2 - \log 6$

(b) $2\log_x y - 3\log_x y + 6$

(c) $3\log_a m + 4\log_a n - 5\log_a t$

(d) $(\log x)^3 \div (\log x)^2 + \log x^2$

Question Two: **[3, 3, 3 = 9 marks]** **CF**

Evaluate each of the following showing full working:

(a) $3\log_2 6 - \log_2 27$

(b) $1.5\log_8 4$

(c) $\dfrac{\log 135 - \log 5}{\log 3^2}$

Question Three: **[1, 3 = 4 marks]** **CF**

If $\log x = y$, where x is positive, express each of the following in terms of y.

(a) $\log x^2$

(b) $\log xm^3 - 3\log m$

Question Four: **[2, 3, 3, 3, 3, 3, 4, 3 = 24 marks]** **CF**

Solve each of the following equations, showing all working.

(a) $\log_y 64 = 2$

$y^2 = 64$ ✓

$y = 8 \ (y > 0)$ ✓

(b) $8x^{\frac{1}{3}} + 12x^{\frac{1}{3}} = 40$

$20x^{\frac{1}{3}} = 40$ ✓

$x^{\frac{1}{3}} = 2$ ✓

$x = 8$ ✓

(c) $\log_5 x + \log_2 8 = 0$

$\log_5 x + 3\log_2 2 = 0$ ✓

$\log_5 x = -3$

$x = 5^{-3}$ ✓

$x = \dfrac{1}{125}$ ✓

(d) $\dfrac{10^{x+2}}{100^{4x}} = 10000^{x-1}$

$\dfrac{10^{x+2}}{10^{8x}} = 10^{4x-4}$ ✓

$10^{2-7x} = 10^{4x-4}$

$2 - 7x = 4x - 4$ ✓

$6 = 11x$

$x = \dfrac{6}{11}$ ✓

(e) $3^{x+1} = 12$

$(x+1)\log 3 = \log 12$ ✓

$x + 1 = \dfrac{\log 12}{\log 3}$ ✓

$x = \dfrac{\log 12}{\log 3} - 1$ ✓

Question Two: **[3, 3, 3 = 9 marks]** **CF**

Evaluate each of the following showing full working:

(a) $3\log_2 6 - \log_2 27$

$= \log_2 216 - \log_2 27$ ✓
$= \log_2 8$
$= \log_2 2^3$ ✓
$= 3\log_2 2$
$= 3$ ✓

(b) $1.5\log_8 4$

$= \log_8 \left(\sqrt{4}\right)^3$ ✓
$= \log_8 8$ ✓
$= 1$ ✓

(c) $\dfrac{\log 135 - \log 5}{\log 3^2}$

$= \dfrac{\log 27}{2\log 3}$ ✓
$= \dfrac{\log 3^3}{2\log 3}$ ✓
$= \dfrac{3\log 3}{2\log 3}$
$= \dfrac{3}{2}$ ✓

Question Three: **[1, 3 = 4 marks]** **CF**

If $\log x = y$, where x is positive, express each of the following in terms of y.

(a) $\log x^2$

$= 2\log x$
$= 2y$ ✓

(b) $\log xm^3 - 3\log m$

✓ ✓
$= \log x + 3\log m - 3\log m$
$= y$ ✓

Question Four: **[2, 3, 3, 3, 3, 3, 4, 3 = 24 marks]** **CF**

Solve each of the following equations, showing all working.

(a) $\log_y 64 = 2$

(b) $8x^{\frac{1}{3}} + 12x^{\frac{1}{3}} = 40$

(c) $\log_5 x + \log_2 8 = 0$

(d) $\dfrac{10^{x+2}}{100^{4x}} = 10000^{x-1}$

(e) $3^{x+1} = 12$

(f) $2^{x-3} = 5^{2x+1}$

(g) $4^{2x} - 4^{x} - 6 = 0$

(h) $5e^{2-x} = 100$

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Calculator Free
Logarithm Laws and Solving Equations

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 = 8 marks] **CF**

Express each of the following as a single logarithm:

(a) $2\log 3 + \log 2 - \log 6$

$= \log 9 + \log 2 - \log 6$ ✓
$= \log 18 - \log 6$
$= \log 3$ ✓

(b) $2\log_x y - 3\log_x y + 6$

$= \log_x y^2 - \log_x y^3 + 6\log_x x$ ✓
$= \log_x \frac{1}{y} + \log_x x^6$
$= \log_x \frac{x^6}{y}$ ✓

(c) $3\log_a m + 4\log_a n - 5\log_a t$

$= \log_a m^3 + \log_a n^4 - \log_a t^5$ ✓
$= \log_a \frac{m^3 n^4}{t^5}$ ✓

(d) $(\log x)^3 \div (\log x)^2 + \log x^2$

$= \log x + \log x^2$ ✓
$= \log x^3$ ✓