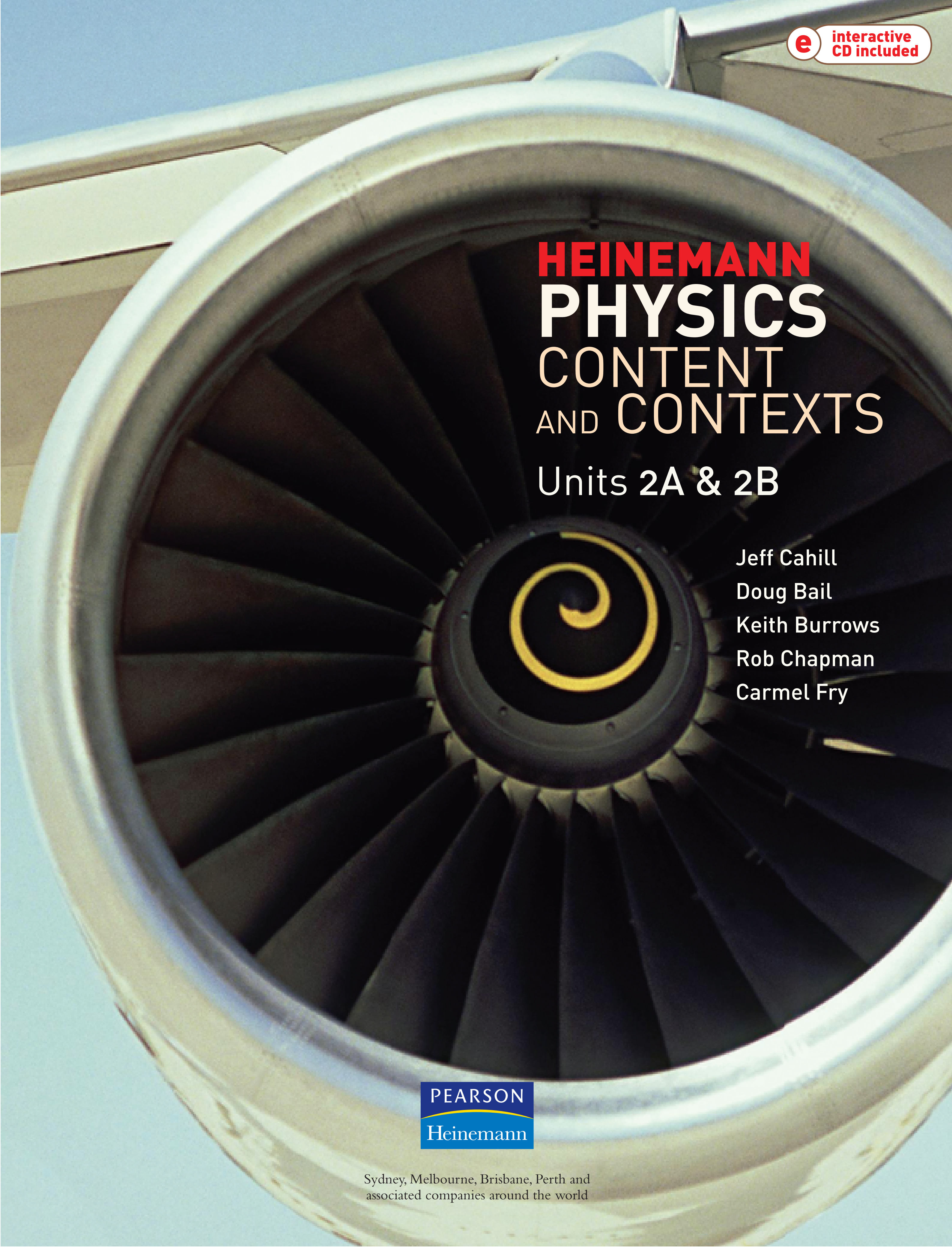

interactive CD included

HEINEMANN PHYSICS CONTENT AND CONTEXTS

Units 2A & 2B

Jeff Cahill
Doug Bail
Keith Burrows
Rob Chapman
Carmel Fry

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HEINEMANN PHYSICS CONTENT AND CONTEXTS

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The complete package for the Western Australian Physics Course of Study

Heinemann Physics—Content and Contexts Units 2A & 2B is the most up-to-date and complete package for the Physics Course of Study in Western Australia. Physics is presented in meaningful contexts, and the unique structure offers teachers the flexibility to address the content in the way that suits the needs and interests of their students.

Heinemann Physics—Content and Contexts Units 2A & 2B is written to cover Stage 2 of the course of study and *Heinemann Physics—Content and Contexts Units 3A & 3B* covers Stage 3.

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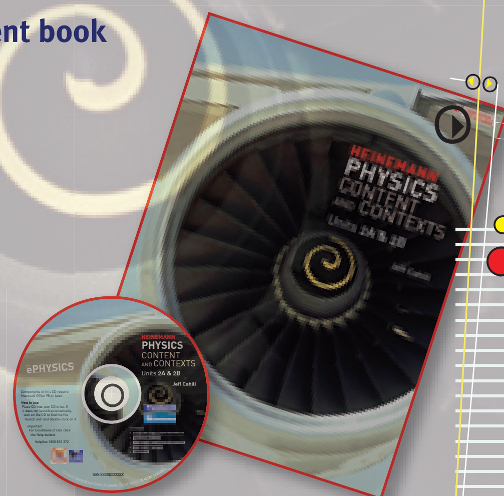

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- Specifically written for Stage 2 of the physics course in Western Australia
- Includes contexts and chapters
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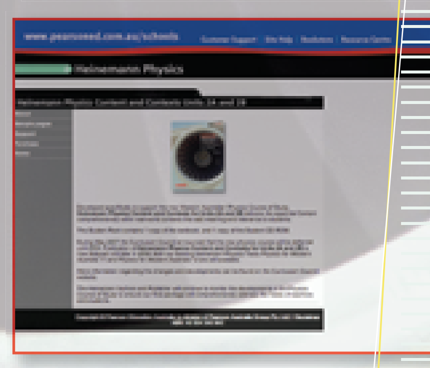


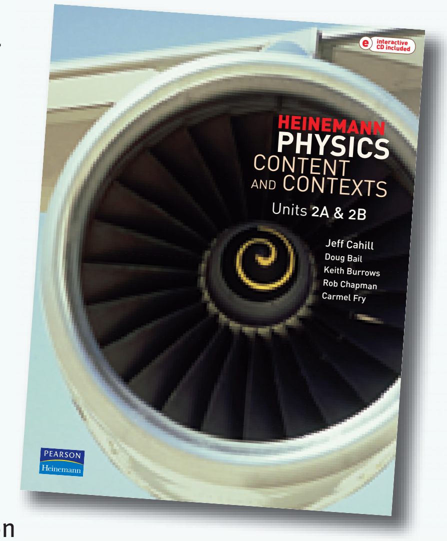
The *Companion Website* contains all the advice and support needed to readily and effectively implement the Western Australian Physics Course of Study. It includes:

- Teaching program
- Worked solutions to all exercises in the contexts and questions in the chapters
- Practical safety notes for all activities on ePhysics

For more information on **Heinemann Physics—Content and Contexts Units 2A & 2B Companion Website** visit www.hi.com.au/physics

Introduction

Heinemann Physics—Content and Contexts Units 2A & 2B is part of the Heinemann Physics—Content and Contexts series, which has been written to meet the needs of Western Australian students and teachers, and has been created specifically to match the Western Australian Physics Course of Study to be implemented from 2009.



The authors have written a text that will support students' learning in physics while making the subject interesting, enjoyable and meaningful. The book uses clear and concise language throughout. All concepts have been fully explored, first in general and then developed in context. Illustrative material is fresh, varied and appealing to a wide range of students.

Each of the book's chapters has been divided into a number of self-contained sections. At the end of each section is a set of homework-style questions that are designed to reinforce the main points. More demanding questions are included at the end of the chapter. The large number of questions is designed to access students' understanding of basic concepts as well as giving them practice at problem solving.

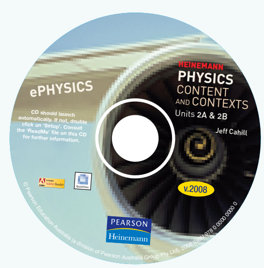
The chapter section questions are useful for both tutorial classes and homework assignments. A teacher might typically select some questions for immediate work in class and assign the rest as homework. There are over 600 questions in the text. Answers are supplied at the end of the text and fully worked solutions are available on the companion website.

CW hi.com.au/physics

The authors have retained many features of the previously highly successful textbooks for Year 11 and Year 12 physics. Within each section, the concept development and worked examples occupy the main column. The minor column has been set aside for some of the 500 photographs and diagrams, as well as small snippets of Physics file information. The longer pieces of high interest and context material are contained in the full-page width Physics in action sections. Both Physics in actions and Physics files are clearly distinguishable from remaining material, yet are well integrated into the general flow of information in the book. These features enhance students' understanding of concepts and context.

Newly featured in this series of texts are additional new areas that are entirely contextual. The contexts at the beginning of each unit cover a broad context that relate to the conceptual content of the course. The contexts include additional exercises and activities at appropriately challenging cognitive levels. Students and teachers choosing to cover a context first can be directed to the appropriate chapter sections when required. Alternatively, a more traditional approach including coverage of the content chapters prior to undertaking the context will enhance student learning due to the enrichment associated with additional exposure to applied and contextual material.

The textbook includes an interactive CD, ePhysics, which will enhance and extend the content of the texts. Included are:



- fully interactive tutorials that allow students to explore important concepts that may be too difficult, dangerous or expensive to do first hand in the classroom
- an innovative range of short and long practical investigations. These have been fully trialled and tested
- a complete electronic copy of the textbook
- an ICT toolkit with tutorials on spreadsheets, databases, Web use and more.

The *Heinemann Physics—Content and Contexts Units 2A & 2B* Companion Website supports the text and ePhysics and assists teachers implement, program and assess the course of study. The Companion Website includes: hi.com.au/physics

- a teaching program
- worked solutions to all exercises in the contexts and questions in the chapters
- detailed practical notes for all activities on ePhysics.

Acknowledgments

The publisher would like to acknowledge and thank the following writers who have contributed material to this text and interactive CD: Doug Bail, Rob Chapman, Carmel Fry and Keith Burrows.

We thank the following for their contributions to reproduce photographs in our student book.

The following abbreviations are used in this list: T= Top, B = Bottom, C = Centre, L = Left, R = Right.

AAP: pp. 132, 141L, 141R, 234, 314.

Alamy: pp. 213, 216.

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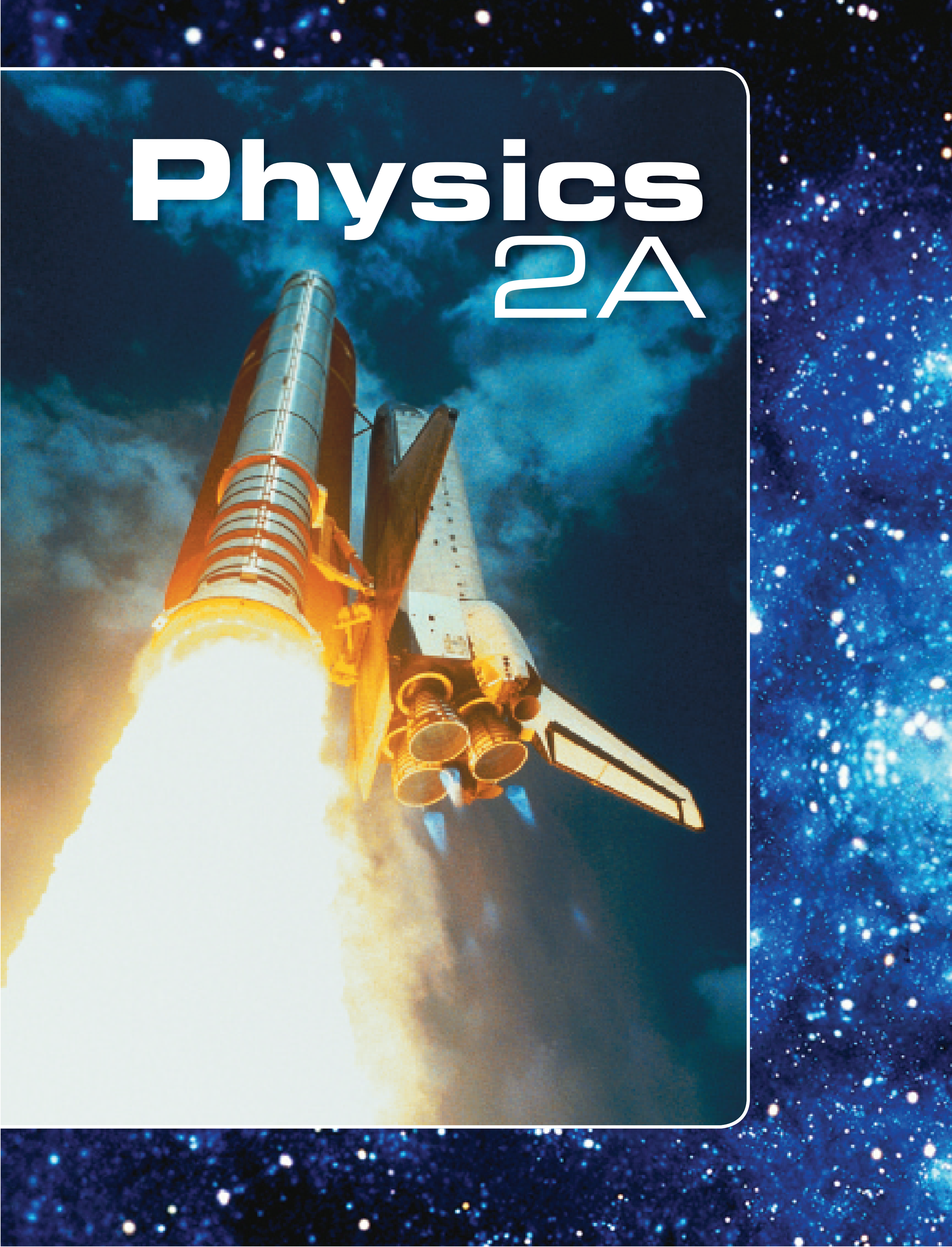
Heinemann Physics Content and Contexts for Units 2A & 2B

Physics 2A



The first two context sections and content Chapters 1–5 cover the content required for the Physics CoS Unit 2A.

Outcomes

The Physics Outcomes are as follows:

Outcome 1: Investigating and communicating in physics

Students investigate physical phenomena and systems, collect and evaluate data, and communicate their findings.

In achieving this outcome, students:

- develop questions and ideas about the physical world to prepare an investigation plan
- conduct experiments and investigations
- analyse data and draw conclusions based on evidence
- evaluate the accuracy and precision of experimental data and the effectiveness of their experimental design
- communicate and apply physics skills and understandings in a range of contexts.

Outcome 2: Energy

Students apply understanding of energy to explain and predict physical phenomena.

In achieving this outcome, students:

- apply understanding of conceptual models and laws relating to energy
- apply understanding of mathematical models and laws relating to energy.

Outcome 3: Forces and fields

Students apply understanding of forces and fields to explain physical phenomena.

In achieving this outcome, students:

- apply understanding of conceptual models and laws relating to forces and fields
- apply understanding of mathematical models and laws relating to forces and fields
- apply understanding of the vector nature of some physical quantities.

It is envisaged that students will fulfil the requirements of Outcome 1 through investigative work in the classroom. Investigative work will typically be related to the content covered in Chapters 2–5. Chapter 1 covers the basic requirements for understanding and processing measured quantities in the investigative work.

Chapters 2–5 cover the required content for Outcomes 2 and 3.

Unit description

Unit 2A focuses on two broad areas of physics: **Motion and Forces** (Chapters 2–4) and **Nuclear Physics** (Chapter 5).

In **Motion and Forces**, students learn about the macroscopic behaviour of matter under the influence of forces. They learn about Newton's laws of motion and acceleration due to applied forces. They add and resolve vectors to analyse quantitative physical concepts relating to forces.

In **Nuclear Physics**, students learn atomic structure and subatomic particles to understand and appreciate phenomena such as those that lead to the emission of nuclear radiation, and nuclear energy.

Contexts

The context material, preceding the content chapters, supports the content of Unit 2A:

- Aerospace Physics
- Medical Physics: Atoms in Action.

aerospace physics

By the end of this context

you will have covered material including:

- applying ideas on forces to the principles of flight
- applying concepts of forces, moments, centre of mass and equilibrium to balancing an aircraft
- an explanation of the forces inherent in flight
- an explanation of lift
- modelling, experimentally, aircraft performance
- an analysis of the performance of aircraft.

If you've ever flown on a large, modern jet then perhaps you too have been inspired to wonder at just how such an extraordinary machine stays in the air. Carrying over 500 passengers, and with a mass of almost 400 000 kg at take-off, it flies with an ease that would have been considered nothing short of miraculous a bare 100 years ago.

And since, as you'll soon find, the main emphasis of this study is investigation, let's start with a question. Who actually invented powered flight just over one lifetime ago? If you said Wilbur and Orville Wright with their 'Wright Flyer' then there are many who would agree with you. Others, with a more Eurocentric view of history, may refer to Alberto Santos Dumont's '14bis', recognised as the first to achieve powered, controlled flight in Europe. From a scientific standpoint perhaps we need to look at the question in a broader sense. Both the Wright brothers and Dumont paid tribute to the many others working in aviation who had pioneered aspects of their basic designs; people such as Gabriel Voisin, Otto Lilienthal and, notably, Australia's Lawrence Hargrave. More important to their whole work was an understanding of the basic principles upon which flight was based, developed by scientists including Galileo, Newton and Bernoulli. For the past 500 years, science has been developed by a community of scholars in which individual contributions became essential to the process. Powered flight is one very visible and practical achievement of that process.

This context is an investigation of forces in flight for aircraft rather than aerospace technology in general terms. It is an opportunity to develop an understanding not only of the basis of the physics behind flight but also of how seemingly disconnected ideas can be brought together to achieve new and exciting ideas through the continuous interaction between theoretical development and empirical results.

Forces in flight

The field of aerospace and its application is too broad to cover in the time available. From wind surfers and car design to jet aircraft and spacecraft, including launch vehicles, satellites, control systems and much more, aerospace is one of the fastest advancing areas of technology today; it is also one of the most heavily funded. Total annual investment worldwide exceeds Australia's entire gross national product many times over. It is an area of science that truly captures the imagination of the majority.

Underpinning these often amazing advances is some fundamental physics. The basic principles apply equally well to Hargrave's box kite as they do to the space shuttle. In this context we will be confining the study of forces in flight to aircraft, while touching briefly on other technologies. This context provides you with an opportunity to apply the same principles to other areas of personal interest.

Figure ae.1 The field of aerospace in general covers an amazing variety of some of the most advanced technology under development today.

The four forces of flight

The remainder of this context assumes that you have studied Newton's laws of motion including the material on forces and vectors covered in Chapter 3. Before going any further make sure you review this area of study.

Whatever the form of an aircraft, its motive power or purpose, there are four basic aerodynamic forces: weight, lift, thrust and drag. These can be considered as two pairs of opposing forces. Lift raises the aircraft upwards and weight pulls it down; thrust propels it forwards and drag slows it down. When an aeroplane flies level and straight at a steady speed, these forces are balanced and Newton's first law of motion applies.

NEWTON'S FIRST LAW: An object remains at rest or will travel at constant speed in a straight line unless an unbalanced force acts upon it.

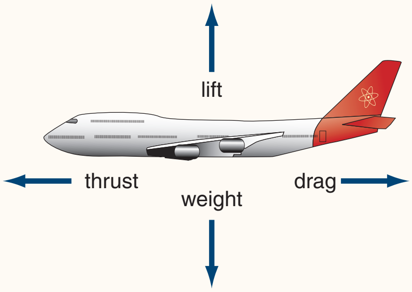


Figure ae.2 The four basic aerodynamic forces are thrust, drag, lift and weight. In this illustration, the size of each pair of forces is equal and opposite: thrust = drag and lift = weight. Hence, the aeroplane will fly straight and level at a constant speed.

For an aircraft travelling at constant velocity this means that:

the size of the thrust force = the size of the drag force

the size of the lift force = the size of the weight force

If for any reason the amount of thrust exceeds drag, the aircraft will speed up. Similarly, if the drag becomes larger than the thrust, the aircraft slows down.

In a vertical direction, if the amount of lift falls below the weight, the aircraft will begin to fall or descend. Increasing the lift allows the aircraft to climb or increase its altitude. In practice, the forces are difficult to consider in isolation because lift and drag increase when speed increases. However, considering the effective size of each of these forces at any point in time is the basis of understanding flight.

Lift

While the development of flying machines can be traced back to Leonardo Da Vinci's drawings and musings, there is no doubt that many before him gazed at birds in awe and wonder, and wished that they too could fly. The apparent freedom of birds to lift on air currents and leave the world behind them has entranced people through the eons. Lighter-than-air craft—principally hot air balloons—realised part of the dream by the 19th century. However, it wasn't until comparatively recently that two parallel lines of development produced the discoveries that were needed to achieve the dream of true aerodynamic, powered flight. One group developed sufficient understanding of the underlying forces involved in flight and another invented the comparatively light and efficient internal combustion engine.

Converging with the development by Karl Benz, and others, of the internal combustion engine was an apparently unrelated investigation into fluid flow; that is, of the flow of water around a propeller. It was the early 1800s and paddle steamers were taking the first tentative steps toward replacing sailing boats. In 1842, Isambard Kingdom Brunel was working on the plans for the largest ship yet to be built—the steam-powered *Great Britain*. He conceived the idea of replacing cumbersome side-mounted paddles with a single huge propeller. The fluid dynamic principles on which he based the successful development of the propeller for his ship are the same as apply to the lift of an aircraft wing or thrust of an aircraft propeller. A Swiss mathematician who knew nothing about flight developed those principles more than 150 years before the *Great Britain* was to sail the seas. His name was Daniel Bernoulli and he died in 1782, shortly before even the first hot air balloon took off.

Figure ae.3 The *Great Britain* set many records for steam-powered ships including for size, speed and luxury. One of the most lasting initiatives was the use of a single bronze propeller at the rear of the ship. It was an early practical application of Bernoulli's principle.

Bernoulli was investigating the flow of air through chimneys and tunnels—another example of fluid flow. In essence, Bernoulli's principle states that the faster any fluid—whether liquid or gas—moves, the less pressure it exerts on a surface over which it passes.

BERNOULLI'S PRINCIPLE: Where the velocity of a fluid is high, the pressure on a surface over which it flows is low, and where the velocity is low, this pressure is high.

This means that where an object has fluid flowing either side of it, and the flow on one side has a higher velocity than the flow on the other, the slower moving fluid will exert a larger force on the object. That might seem a little strange but the higher pressure, and hence force, must be associated with the slower moving fluid. If it were associated with the higher velocity it would slow the fluid down.

Bernoulli also developed an equation that expresses the principle quantitatively. Bernoulli's equation is an expression of the law of conservation of energy and can be derived by considering that the net work done on a system is equal to its change in kinetic energy.

BERNOULLI'S EQUATION: Assuming a fluid with constant density, ρ, then, within a system:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

where P_1 and P_2 are the fluid pressures in N m^{-2} either side of a relatively thin, horizontal surface where there is no appreciable change in height,
ρ is the fluid density in kg m^{-3}, and
v_1 and v_2 are the respective fluid velocities in m s^{-1}.

The equation ignores the effects of fluid friction (viscosity) and the compressibility of the fluid.

Bernoulli's equation tells us quantitatively that where the speed of gas or fluid moving over an aerofoil is high the pressure is low, and vice versa. The principle is important for aircraft wings and explains many other common phenomena. It also explains why so many early attempts to fly met with failure. Too often these early attempts revolved around

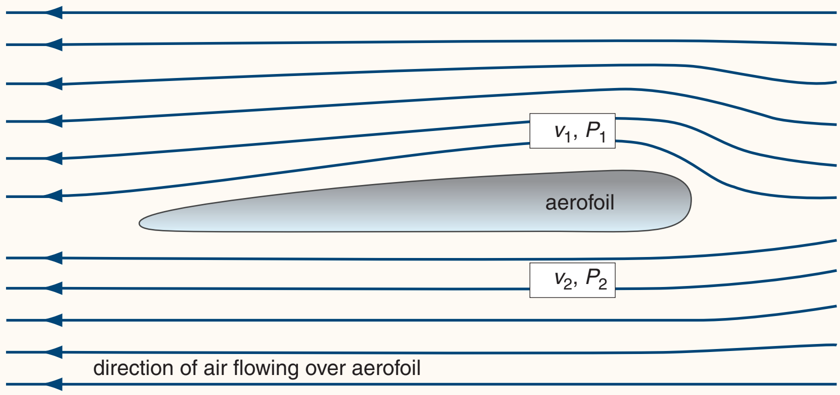


Figure ae.4 An aerofoil section generates lift for any wing, be it bird or aircraft. The pressure, P_1, of the fluid with a higher velocity, v_1, will be lower than pressure P_2 where the fluid has a lower velocity, v_2. The result is an upwards force or lift that allows an aircraft to take off and fly.

people trying to imitate birds by flapping wings. No one had separated thrust from lift. Birds flap to move forward, not simply to lift themselves into the air. The aerofoil wing shape of a bird's wing provides the lift once they are moving. The first person to demonstrate this important factor was English scientist, Sir George Cayley. In 1804 he built the world's first successful glider using wings with an aerofoil shape.

The aerofoil shape is successful because of the comparative distances that air must flow as it moves through the air. Air flowing over the top of the aerofoil shown in Figure ae.4 must travel further and faster than the air flowing under it. The slower moving air underneath the wing has a greater pressure and pushes the wing up. Similar aerofoils can be seen in the propellers of aircraft and boats, the sails of yachts and on boomerangs—surely the oldest human invention to successfully apply Bernoulli's principle to simple flight. Upside-down aerofoils can be found on the back of racing cars and sports cars. The air pressure in this application is pushing down and assists in keeping the car on the ground while also improving traction between rear wheels and the ground.

In practice, calculating the pressure distribution around a wing and the exact amount of lift it produces is more difficult than simply applying Bernoulli's equation. Wings are usually mounted with a slight upward tilt referred to as the *angle of attack*. Air striking the bottom surface is deflected downwards, the resulting change in momentum of the rebounding molecules providing an additional upward force on the wing. The lift coefficient of an aerofoil is a number that relates its lift-producing capability to air speed, air density, wing area and the angle of attack. Turbulence also plays an important role.

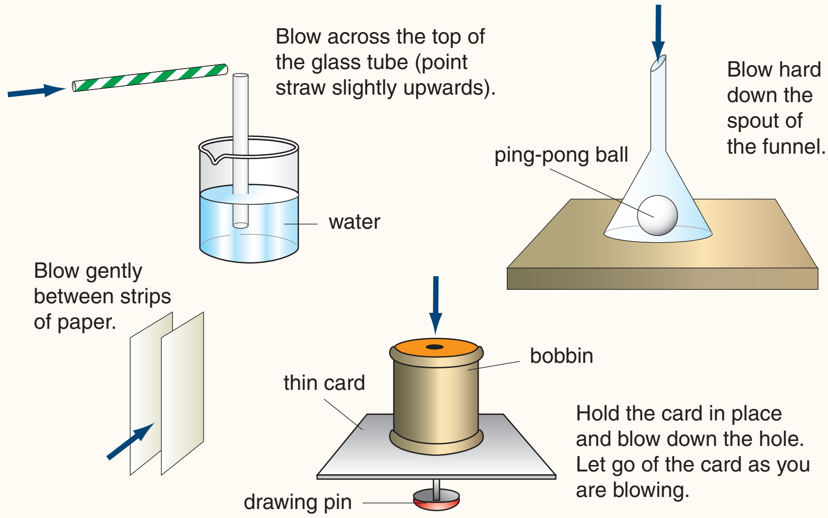


Figure ae.5 While more often associated with flight in modern times, Bernoulli's principle has much broader applications. Try these simple experiments to see Bernoulli's principle in action. Can you identify the positions of higher and lower pressure in each case?

The lift coefficient can be found from a derivation of Bernoulli's equation:

$$L = C_l \times \tfrac{1}{2} \times \rho v^2 A$$

where L is the lift in newtons, C_l is the coefficient of lift, ρ is the fluid density in kg m^{-3}, v is the velocity in m s^{-1} of the aircraft, and A is the wing area in m^2 at right angles to the direction of travel. The lift coefficient is dependent upon the angle of attack and must be recalculated for any change in the angle. Scientists conduct wind tunnel tests to measure the lift generated for a particular shape of aerofoil and use the results to publish tables of coefficients versus angle of attack. The lift of any wing can then be calculated. One thing immediately apparent from Bernoulli's equation and the equation for lift is the high dependence on velocity. A wing going twice as fast as another can generate four times as much lift.

Physics in action — Sailing upwind

A modern yacht cannot sail directly into the wind, but can 'tack' upwind by making use of Bernoulli's principle. The normal atmospheric pressure behind the larger mainsail is larger than the pressure in front of it due to the fast moving air in the slot between mainsail and jib (Figure ae.6). The net force on the sail—wind plus sail pressure—acts almost perpendicular to the sail. That would tend to move the boat sideways if it wasn't for a centreboard or deep keel extending far below the boat. The water exerts a force on the keel almost perpendicular to the keel. The resultant of the two forces is almost directly forward. The famed winged keel of *Australia II*, Australia's first successful challenger for the America's Cup, ensured that the area of the keel, and hence the force, stayed larger as the boat heeled over when travelling upwind, and allowed *Australia II* to sail faster and more directly upwind than its opponents.

Other yachts, such as catamarans, have single sails battened to form clean aerofoils. In the C-class catamaran of the Little America's Cup, the concept is taken still further with rigid aerofoil masts making up more than half the total sail area.

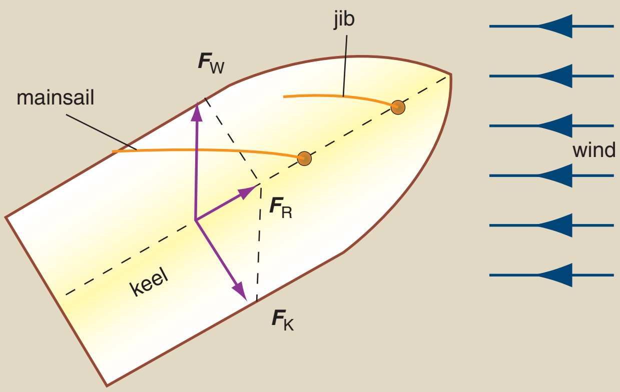


Figure ae.6 A yacht moving against the wind makes use of Bernoulli's principle as illustrated in the diagram. The force of wind and sail pressure, $\boldsymbol{F}_W$, together with the force of the water on the underwater keel, $\boldsymbol{F}_K$, combine to produce a resulting force, $\boldsymbol{F}_R$, which pushes the boat forward.

Testing aerofoils

The wing shapes of early aircraft were tested, like many other things, through a process of trial and error. The aircraft was built and flown and the results, sometimes humorous but too often tragic, were observed. Incremental improvements were made and the process repeated. It was a long drawn-out process and often costly in terms of materials and test pilots. In the case of large aircraft, it was usual to build a smaller-scale proof-of-concept prototype; this was less costly in terms of materials but still very dangerous for the test pilots. One advantage of the process was that it was the real aircraft that was being tested and it was being done under real conditions. If it worked as a prototype, designers could be pretty confident that the production model would also work.

A big step forward in the design process, and a huge relief to pilots, was the invention of the wind tunnel. Scale models of aircraft, and even some full-scale components, could be tested on the ground before the complete aircraft was built. Today wind tunnels are used for pre-testing just about all aerodynamic shapes, with considerable savings in time, cost and human life.

For details on how to build your own simple wind tunnel, see the instructions on page 25. Manufactured versions are also available.

On a simple level, a wind tunnel is just a large chamber with a massive fan at one end. A model is suspended in the chamber in as near to true flying position as possible. Smoke released into the airstream can give a visual indication of airflow while sensors attached to the model give real-time readings of pressure, temperature and strain. Vast amounts of data can be collected on just how the model performs in these simulated conditions. Similar 'tunnels' filled with water are used for testing fluid flow past the hull shapes of boats and their keels.

While a vast improvement over testing the real thing, testing in a wind tunnel is still expensive. The facilities are large and complex, the models are still relatively expensive to make, and testing still needs to happen in real time for meaningful results to be achieved. At the higher end of performance it is also difficult to replicate the high air speeds and differing air densities encountered by fighter aircraft and rockets as they soar through the upper reaches of the atmosphere at supersonic speeds. So it's becoming increasingly common for simulations to be carried out

Figure ae.7 Solar car testing in the Monash University wind tunnel.

by computer. Tests can be done by measuring the virtual flow of air over a virtual wing. The design can be rapidly changed on-screen to produce the best shape. The time for the design process is reduced from years with a real aircraft, to months with a wind tunnel, through to days with computer modelling. Of course, computer modelling is only as reliable as the original data that the models of air movement are built upon, so further testing in wind tunnels is performed both to verify and improve the computer model and to confirm the virtual results.

Weight

According to Newton's first law of motion, for an aircraft to be flying level when the aerofoil is applying lift there must be a balancing force. Weight, the force of gravity, provides that balancing force. The weight force is equivalent to the value predicted by Newton's second law.

WEIGHT IS A FORCE: $\boldsymbol{F} = m\boldsymbol{g}$
where $\boldsymbol{F}$ is the force in newtons (N) acting on the aircraft as a result of a gravitational field of strength $\boldsymbol{g}$ and m is the mass in kilograms (kg). In this case g has a value equivalent to the acceleration an object will experience in free fall and has a value of approximately 9.8 m s^{-2} at the Earth's surface.

Physics in action — The vomit comet

Astronauts preparing for space travel and the feeling of apparent weightlessness in orbit travel in a specially prepared Boeing aircraft. The aptly named 'vomit comet' performs a series of loops. At the top of each loop both aircraft and the people in it are in free fall along a parabolic path. Because everything is falling at the same rate, the astronauts experience a few seconds of apparent weightlessness. The effect of this on the stomach has led to the aircraft's nickname.

Figure ae.8 The aptly named 'vomit comet' allows astronauts to experience weightlessness as both aircraft and occupants free fall at the same rate through a parabolic path from the top of a loop.

Investigation

YOU TOO CAN DEMONSTRATE APPARENT WEIGHTLESSNESS:

- Attach a spring balance or force sensor to the top of a box by a hook or other secure fastener.
- Attach a 1 kilogram mass to the spring balance. The balance should be showing a value equivalent to a 1 kilogram weight force; that is, approximately 9.8 N.
- When you're ready, drop the box, mass and spring balance.

During the fall, the spring balance and mass are falling at the same rate. The recorded mass on the spring balance will be zero (a force sensor attached to a datalogger will show this clearly) as both mass and spring balance will be accelerating at the same rate. The resultant force between mass and force measurer is zero. Since the mass has obviously not lost any mass, this is a good example of apparent weightlessness.

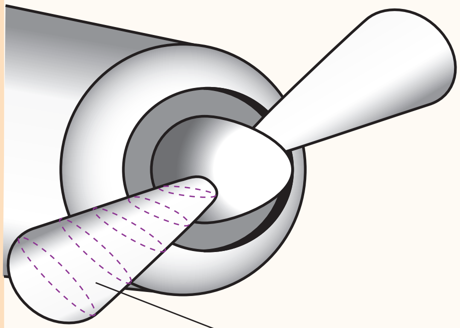


Figure ae.9 Take a series of slices through one blade of a propeller and its relation to an aerofoil is clearly visible.

Thrust

Thrust is the force that propels a bird, yacht or aircraft forward. The aerofoils of a wing won't generate lift when an aircraft is stationary. An aircraft needs to travel very quickly along a runway so that air rushes under and over the aerofoil in order to produce the lift predicted by Bernoulli. Once in the air, thrust is required both to keep the aircraft in the air and to carry it forward (you'll recall that we noted that no single force in aeronautics can be considered in isolation). In modern times, two basic types of engine are used to provide the required thrust: jet engines and propeller engines. Turbojets are hybrid, largely propeller-driven engines, which include some of the economy and thrust benefits of jets.

While seemingly quite different in concept, both jets and propellers create thrust according to Newton's third law.

NEWTON'S THIRD LAW: For every action there is an equal and opposite reaction.

Propellers are really just like a twisted wing. Brunel's massive brass propeller for the *Great Britain* was simply three aerofoils twisted at an angle to each other. Take a series of slices through the blade of a propeller and this is clearly visible. The propeller of most aircraft drives a stream of air backwards and, by doing so, pushes itself forwards, pulling (or pushing in some instances) the aircraft with it. The angle of attack along the propeller section varies along the length of the propeller. The angle is greatest at the centre because the speed of the propeller is slowest close to the hub. More elaborate three- and four-blade propellers have adjustable pitch mechanisms, allowing the pilot to adjust the propeller's angle of attack depending on air speed and altitude.

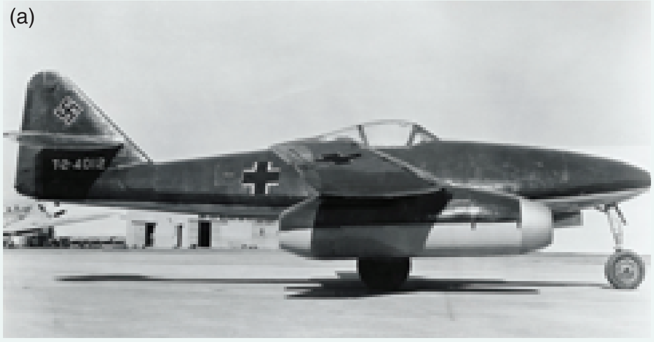
Investigation

The thrust of a propeller can be observed and measured through two simple models. One option is to attach an electric motor to a propeller and mount this unit on a low friction trolley. Clearly the propeller moves the trolley forward. A simpler arrangement is the rubber-band-propelled model aircraft. Simple measurements should permit the thrust of either arrangement to be investigated. The thrust of propellers with two, three, four or more blades can be compared. The effect of pitch or angle of the blades can also be readily investigated with this simple equipment.

Jet engines work on the same basic principle as propellers—air rushes out the back and the aircraft is pushed forward. A turbojet engine has three main sections as shown in Figure ae.10: compressor, combustion chamber and turbine. Air is drawn into the engine and compressed, making the air hot (you may have experienced the same basic effect when pumping air through a bicycle pump). In the combustion chamber the air is mixed with kerosene fuel. The air and kerosene mix burns explosively, and a high-pressure stream of hot gas rushes out the back. The gas turns the blades of the turbine, which pushes the aircraft forward.

Aircraft engines exert thrust to overcome the drag from air resistance, to climb against the weight force of gravity, and to accelerate. An aircraft's performance is limited by the rate at which it can do work. Even when an aircraft is travelling along in level flight at constant speed, it

(a)



(b)

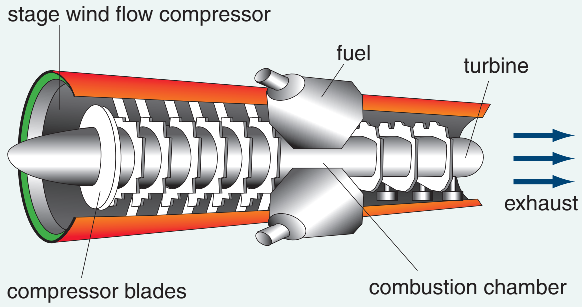


Figure ae.10 (a) The Messerschmitt Me 262 first flew with jet engines in November 1941. While ultimately highly successful, it was fortunate that this test flight still had a nose-mounted piston engine installed—both jets failed on the point of lift-off! (b) The basic principles of the turbojet engine haven't changed since the first production jet reached the skies during World War II.

needs power just to provide the lift needed to balance the weight force and the retarding force of drag. These forces depend on the size, mass and shape of the aircraft and flying conditions, and they can be enormous when compared with the forces acting on early aircraft. A Boeing 747 with a mass of over 390 000 kg is powered by four turbo-fan engines, each exerting over 250 kN of thrust. The power generated by the engine can be expressed in terms of the net force, $\boldsymbol{F}$, applied to an object and its speed, v. Since:

$$P = \frac{W}{t} \text{ and } W = Fx$$

where x is the effective distance moved, then:

$$P = \frac{W}{t} = \frac{Fx}{t} = Fv$$

where P is power in watts (W), F is the net force in newtons (N), and v is the average speed of the object in m s^{-1}.

Worked Example ae.1

If the average air resistance encountered by a particular aircraft is 5 kN when it is travelling at a average velocity of 720 km h^{-1} at an altitude of 5000 m, what power must the engines develop solely to overcome air resistance?

Solution

$$v = 720 \text{ km h}^{-1} = \frac{720 \times 1000 \text{ m}}{3600 \text{ s}} = 200 \text{ m s}^{-1}$$

and $P = Fv = 5000 \times 200 = 1\,000\,000$ W or 1 MW

(Of course, the engine must develop considerably more power than this in order to also keep the aircraft aloft!)

Drag

Drag—also known as air or fluid resistance—simply slows down any object moving in a fluid, whether it be a gas or a liquid. Riding your bike, you can feel the drag that your body creates. The amount of drag will depend on the shape, surface area, surface material (so-called skin friction), its speed and the fluid the object is moving through. The faster the object moves, the greater the drag experienced. Drag is produced by the work required to move the fluid out of the path of an object. If drag

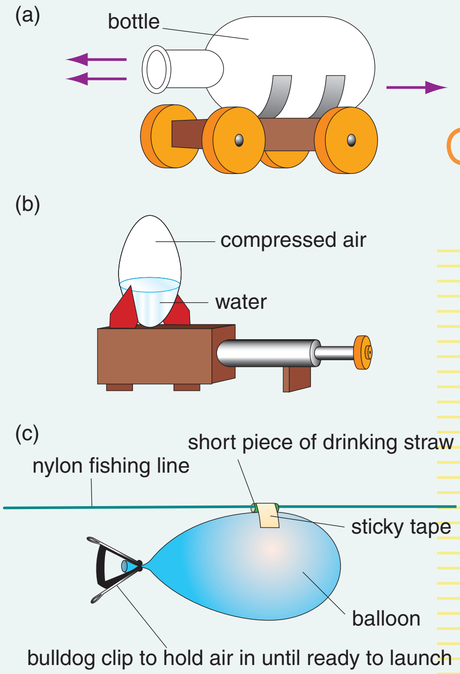


Figure ae.11 Realistic testing of jet engines is impractical outside a jet propulsion laboratory. However, the basic principle can easily be investigated using various sources of compressed air. Both commercial models and home-made alternatives allow measurement of force versus thrust.

is to be reduced, then an aircraft needs to have a shape that best allows it to move through the air smoothly. A great example of drag reduction is the position of speed skaters at the winter Olympics. Whenever they get the chance, they crouch down and reduce their frontal profile as much as possible. The drag the skaters create is decreased and that allows them to move faster and more efficiently around the track. Modern aircraft retract the undercarriage as soon as possible after take-off. Just like the skater, the pilot is trying to minimise drag by making the profile of the aircraft as small as possible. The amount of drag produced by the undercarriage of a large jet is huge.

The drag coefficient is a derivation of Bernoulli's equation:

$$D = C_D \times \tfrac{1}{2} \times \rho v^2 A$$

where D is the drag in newtons, C_D is the coefficient of drag, ρ is the fluid density in kg m^{-3}, v is the velocity in m s^{-1} of the aircraft, and A is the wing area in m^2 at right angles to the direction of travel. Like lift, the equation for drag is highly dependent on velocity. A wing going twice as fast as another can generate four times as much drag.

Drag is caused by the turbulence created as an aircraft moves through the air—the break-up of the smooth flow of the fluid into swirls around and behind the aircraft. The faster an aircraft travels, the greater the drag produced. Ultimately drag will equal thrust and the aircraft will no longer accelerate. You can see the effect by introducing a little smoke or the vapour of dry ice into a wind tunnel, or through a simple activity with a flame and a small beaker.

Investigation

Try dropping different shaped objects through a thick liquid, such as glycerol, inside a tall measuring cylinder. More streamlined shapes will continue to accelerate quite uniformly while others that are less streamlined will not reach the same speed by the time they reach the bottom of the cylinder. Video analysis of the acceleration can permit the investigation and calculation of terminal velocities for different shaped objects in the fluid.

Investigation

Place a candle behind a beaker as shown in Figure ae.12. Blow on the front, or opposite side, of the beaker and the flame should move in toward the beaker as the air curls into turbulence behind the smooth sides of the beaker. Try streamlining the back of the beaker and observe the changes in the movement of the flame.

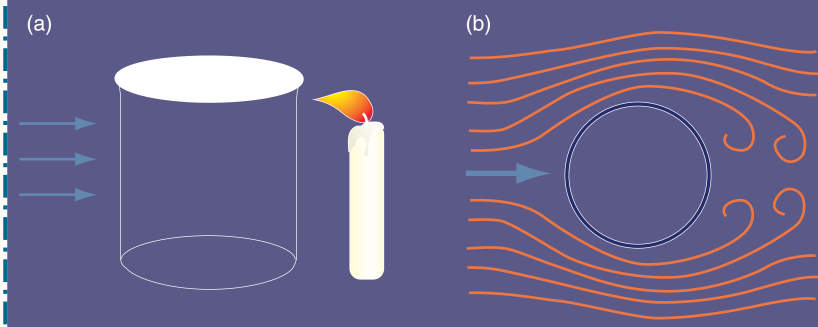


Figure ae.12 (a) Blowing on one side of the beaker causes the flame of the candle to move back toward the beaker. (b) The blown air curls into turbulence behind the smooth sides of the beaker.

A special example of the effect of drag is terminal velocity. Drop any object and as its velocity increases due to the acceleration, so too does drag, just as in level flight. Eventually the force due to drag will equal that of gravity and the object will no longer accelerate. It will continue to fall at a constant velocity referred to as *terminal velocity*. Skydivers stretch their arms and legs to increase drag during free fall, to control their rate of descent, and to perform aerobatics.

Physics in action — Thorpe takes to the air

The fundamentals of fluid flow developed within this study as airflow apply to any fluid—including plain old water. In developing Ian Thorpe's new swimsuit, the Audi wind tunnel was utilised. While usually employed for improving the aerodynamics of cars, its size and sophistication allow a broad range of other applications. Thorpe wore his new suit in the wind tunnel as smoke was blown into an air stream that was travelling at 100 km h^{-1} over him. A specially built platform allowed Thorpe to lie in the airstream in a similar position to that he would attain while swimming. Turbulence due to any imperfections in the suit design was immediately visible. Swedish and Norwegian ski teams have also improved the aerodynamics of their equipment using the Audi facility. The RMIT maintains similar facilities at its Department of Mechanical Engineering here in Victoria.

Figure ae.13 The study of fluid flow and aerodynamics is regularly used by sports equipment manufacturers and athletes in the search for that extra 0.01 second.

Exercises

E1 Explain, using a suitable vector diagram, the principles by which a jet engine provides an aircraft with forward thrust.

E2 A plane has a designed stall speed of 200 km h^{-1}. Flying into a sudden wind shear (a sudden change in wind direction and speed relative to the aircraft) of 70 km h^{-1} relative to the ground at an indicated air speed of 300 km h^{-1}, the aircraft may still stall. Explain how this could happen.

E3 Stunt planes often fly upside down. Since the aerofoil shape of the wing is what generates lift when the plane is flying upright, explain how the plane can continue to generate lift and fly when upside down.

E4 A plane doubles its indicated air speed. What is the proportional increase in the lift generated by the wing due to the change in speed?

E5 The performance of a glider can be best measured by its glide ratio. This is the ratio of horizontal distance travelled to fall in altitude. Modern gliders have glide ratios of more than 60:1 compared with a jet liner's 10:1. Drag reduction is the most important means of increasing glide ratio. Suggest three ways this can be achieved in glider design.

E6 A number of modern fighter jets have wings whose geometry can be altered during flight—swept back for high speed flight, brought forward to a straight position for landing. What advantages are there to this form of design?

E7 A frequent problem in older aircraft was the surface of the upper wing 'lifting off' at higher speeds. (Modern testing techniques usually highlight the problem *before* flight.) Explain this phenomenon using Bernoulli's principle. Is it 'lifted' off or 'pushed' off?

E8 Dangle two pieces of paper vertically, a few centimetres apart and blow between them. Try it and explain what you see.

E9 Aeroplanes normally take off into the wind. Why?

E10 A hummingbird drinks pollen from a flower while hovering in front of the flower—it doesn't land while doing so. It expends about 15–20 times as much energy in hovering as it does in normal flight. Explain why.

E11 Try this: Sit a 5-cent coin on a tabletop. Blow (very hard!) over the top of the coin. Explain what you see happen to the coin. Why does this occur?

E12 What is the lift, in newtons, due solely to Bernoulli's principle, of a wing of area 100 m^2 if the air passes over the top surface and bottom surface at speeds of 960 and 800 km h^{-1}, respectively? (Use density of air = 1.29.) Note: You will need to work out the relationship between pressure, and lift and area, from their units.

The following information applied to questions E13–E16. A jet plane is travelling at a speed of 200 m s^{-1} relative to the air around it. Each second the engines of the plane take in a total of 150 kg of air. The air is used to burn 2.0 kg of fuel each second. Finally, the air and burnt fuel mix is expelled from the rear of the engines at a speed of 450 m s^{-1} relative to the plane. Use the following relationships:

change in momentum (impulse), $\Delta p = m\Delta v = F\Delta t$ (N s)
power, $P = F v$ (W)

E13 What is the magnitude of the total force exerted on the plane by the intake of the air?

E14 What is the magnitude of the force exerted on the plane by the air and burnt fuel being expelled from the engines?

E15 What is the net thrust produced by the engines?

E16 What is the power developed as a result of the net thrust?

Modelling forces in flight

So far we have discussed and developed the four basic aerodynamic forces as if they were acting at a single point. The initial advantage of considering the basic forces as acting on a point source is that it allows us to isolate the effect of these forces from other complexities. This was one of the strengths of Newton's approach to understanding forces. He was able to isolate cause from effect. The analysis of many complicated systems can be simplified in a similar manner.

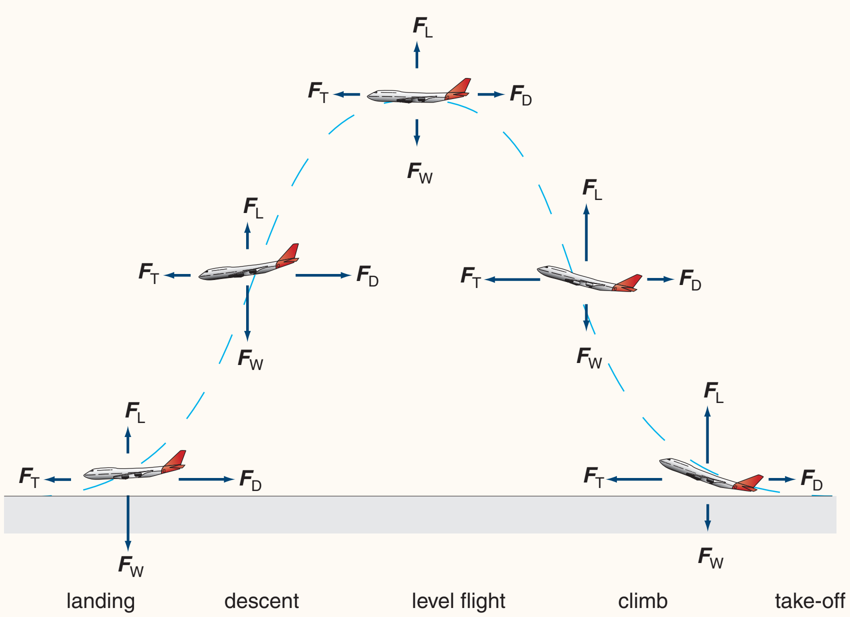


Figure ae.14 The relative size of the four basic aerodynamic forces during take-off, climb, cruise, descent and landing acting about the centre of gravity.

Centre of mass and gravity

The mass of the aircraft, no matter how complex the shape, can be considered to be concentrated into a single point whose path will follow exactly the same path as a single point would under the same net force. This single point is called the centre of mass.

The motion of any extended rigid body can be simplified to the motion of a particle of the same mass located at the centre of mass when it is subjected to the same forces.

If a body is no more than a uniform length of wire, its centre of mass will lie exactly at the centre. In two or three dimensions, the centre of mass will be central for each dimension. It is even possible for the centre of mass to lie outside the body, as with a doughnut (the centre of mass is in the hole!). For a simple non-uniform system, the position of the centre of mass can be calculated easily. For the more complex shape of an aircraft, more calculations are involved.

A closely related concept to centre of mass is the centre of gravity. Instead of being a point particle whose motion equates to the whole extended body or system, the centre of gravity is the position from which the entire weight of the body or system is considered to act. As a consequence, the centre of gravity is the position at which the body will balance. For just about all practical purposes, the centre of gravity is exactly the same as the centre of mass. It is only when a body is so massive that its own gravitational field becomes significant, that the centre of gravity no longer coincides with the centre of mass—hardly a concern for an aircraft.

Investigation

The centre of gravity of an aircraft can be found more easily by experiment than by calculation with the mathematical tools available to us at this level. Figure ae.15 shows a simple model of a thin aerofoil suspended vertically from a pivot point and free to move. When released, the model will swing until its centre of gravity lies directly beneath the pivot point. A plumb line is then drawn vertically down from the pivot. If the aerofoil is hung again from a second, different pivot point, the centre of gravity will again lie on a line beneath the pivot, so a second plumb line can be drawn. The aerofoil's centre of gravity will be at a position where the two plumb lines intersect. If the process is repeated, all such plumb lines will intersect at the same point. This method will work for any plane object. For a three-dimensional model of an aircraft, the centre of gravity will be found using a third line (although it may be hard to draw the plumb lines through the model).

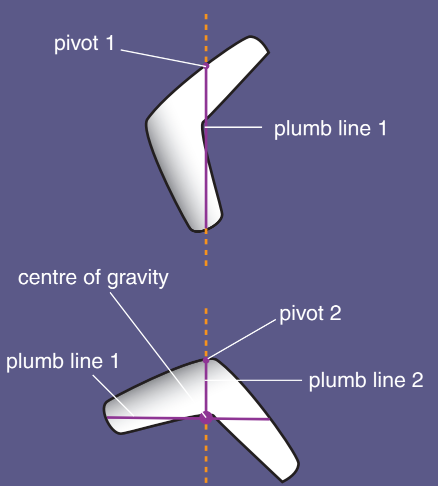


Figure ae.15 In order to determine its centre of gravity, a model aerofoil is suspended from a pivot, and a vertical plumb line is drawn through the pivot point.

Suspending an aerofoil from its centre of gravity in a wind tunnel will keep it in a stable, flying attitude while the forces created by airflow are investigated.

Equilibrium

We've already seen that when the two pairs of basic forces of flight are balanced, the aircraft will remain at rest or at constant speed. This was the situation described by Isaac Newton in his first law of motion. When all the forces acting on the aircraft add up to a zero net force, the aircraft is said to be in *translational equilibrium*.

A body is said to be in ***TRANSLATIONAL EQUILIBRIUM*** when the sum of the forces acting on the body is zero, i.e. $\Sigma \boldsymbol{F} = 0$.

So the motion of the centre of gravity of the aircraft can be described and predicted. Increase, or decrease, any one of the four basic forces and the aircraft as a whole will accelerate.

But an aircraft may be in translational equilibrium and still not be stable. The turning propeller, ailerons, wings, flaps, stabilisers and rudder, even the body of the aircraft itself, all exert additional turning, twisting forces called *torques* that will rotate the aircraft. They are an essential part of the flying and control of an aircraft and their individual effects on the rotation of the aircraft must be considered in any comprehensive investigation of forces in flight.

For an aircraft to be completely in equilibrium it must also be in *rotational equilibrium*. You can imagine an aircraft quickly falling into a spin if these forces were not controlled. In other instances, the application of a rotational force is what steers an aircraft and is highly desirable.

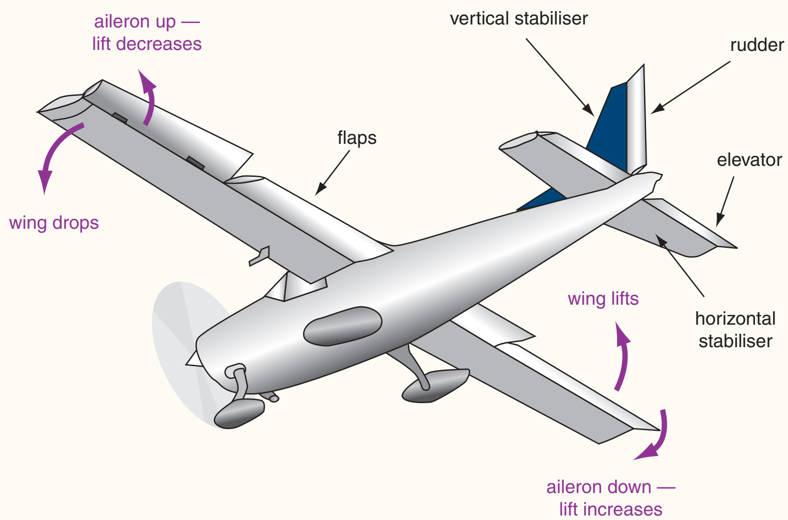


Figure ae.16 An aircraft in flight is affected not only by the translational forces of lift, weight, drag and thrust, but also by the rotational forces introduced by control surfaces and other key components of the aircraft.

For an aircraft to be in rotational equilibrium, all of the rotational forces, introduced by control and other surfaces, acting about a point must be zero. The net clockwise rotational force or torque must equal the net anticlockwise torque.

For an aircraft to be in ***ROTATIONAL EQUILIBRIUM***, the sum of all torques acting about a point must be zero, i.e. $\Sigma\tau = 0$ where τ is torque in newton metre (N m).

If the aircraft is not experiencing any translational acceleration or rotation then it is said to be in *static equilibrium*. That's not to say it is not moving. It could be stationary or it could be moving at high, but constant, speed in a straight line without any rotation.

For an aircraft to be in ***STATIC EQUILIBRIUM***, it must be in translational and rotational equilibrium, i.e. $\Sigma\boldsymbol{F} = 0$ and $\Sigma\tau = 0$.

Torque (Optional Concept for Unit 2A)

The size of a torque exerted by a control or other surface will depend on three factors. The first is the magnitude of the force. If all other things are equal, a larger force will result in a larger torque. The second is the point at which the torque acts. When analysing the rotation of a system, the position of the axis of rotation is an important reference point. A door, for example, moves in a circular path around its hinges. The line of the hinges is the axis of rotation. A force applied at the hinge itself will not create a turning effect; the maximum effect will be achieved by applying a force to the door as far from the hinge as possible. That's why an aircraft's vertical stabiliser and rudder are positioned firmly at the rear of the aircraft; this is where the stabiliser will have maximum effect in overcoming any horizontal rotation of the aircraft along the line of the fuselage.

An aircraft will experience turning effects in all three dimensions about its centre of gravity.

Finally, the turning effect of a force depends on the direction in which it is applied. In closing a door, for example, the maximum effect is achieved if the force you apply is at 90° to the door surface. If the angle is reduced, a smaller component of the force is perpendicular to the door and so a smaller torque is produced. If the force is applied along the line of the door (i.e. 0°, directly towards the hinges), the door will not rotate no matter how great a force you apply. These three factors combine to form the definition of torque.

The ***TORQUE***, τ, acting on a body is given by the product of the component of the applied force acting perpendicular to the lever arm, $\boldsymbol{F}_\perp$, and the distance from the axis of rotation to the applied force, r:

$$\tau = r\boldsymbol{F}_\perp \sin\theta$$

Where the force is perpendicular to r, as in the vertical stabiliser, $\sin\theta = 1$, so:

$$\tau = r\boldsymbol{F}_\perp$$

The symbol for torque is the Greek letter tau, and its unit is the newton metre (N m).

Torque is a vector quantity. A clockwise rotation is considered to be negative and an anticlockwise rotation to be positive. This convention can be useful when a number of torque forces are acting and the net torque has to be calculated.

Physics File

A propeller of an aircraft introduces considerable torque forces that must be controlled in some way. In single-engine aircraft the pilot must use the control surfaces to balance the tendency of the aircraft to roll in the direction that the propeller is spinning. In multi-engine aircraft the propellers are often 'handed'. That is, the propellers on one side will be spinning clockwise while the propellers on the other will be spinning anticlockwise. The torque effects of the propellers balance themselves.

Worked Example ae.2

A small plane of length 10.0 metres has a mass of 8000 kg. A large container of mass 200 kg is placed in the rear cargo bay, 4.2 metres behind the centre of gravity of the unladen aircraft. A reserve fuel tank is located 1.8 metres forward of the centre of gravity. (The mass of the fuel tank is included in the mass of the aircraft.) What mass of fuel must be pumped into this tank to offset the torque exerted by the cargo?

Solution

The torques from each object will act in different directions around the centre of gravity since one is forward and the other aft of the centre of gravity. (Let t stand for fuel tank and c, the container.) So:

$$\tau_t = \tau_c$$

Since both forces will be acting at right angles to the centre line of the aircraft:

$$m_t \times g \times d_t = m_c \times g \times d_c$$

Simplifying: $m_t \times d_t = m_c \times d_c$

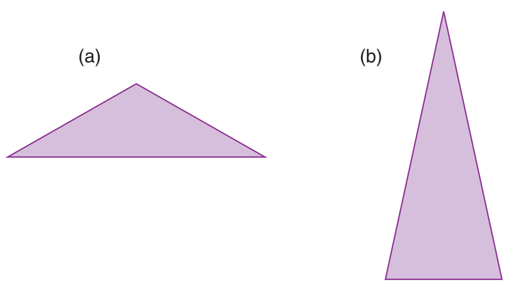
$$m_t \times 1.8 = 200 \times 4.2 \text{ and } m_t = 467 \text{ kg}$$

So, 467 kg of fuel must be pumped into the tank to offset the torque exerted by the cargo.

Exercises

E17 Before it ceased flying, the Concorde supersonic airliner set speed records for commercial airliners between Europe and North America. Its prodigious appetite for fuel and small payload led to its demise. The Concorde used fuel not only for keeping the engines running but also to restore aircraft stability. The pilots adjusted the fuel backwards and forwards in 'trim' tanks. Explain the need to do so.

E18 Determine, by geometry or otherwise, the centre of gravity of the two delta wing forms shown. Assume, for the purposes of this exercise, that the wings are of uniform density and composition.



E19 What implications do your solutions for question E18 have for aircraft incorporating the wings shown in the figure?

E20 An aircraft starts a flight with 3000 kg of fuel evenly distributed between two fuel tanks to balance the weight about the centre of gravity (cg) of the plane. One of the fuel tanks is 2.0 m forward of the plane's cg, the other is 5.0 m to the rear of the cg. The rear tank is used and emptied first. How much fuel must the pilot pump to the rear tank in order to restore the plane's trim?

E21 Explain, with a suitable force diagram or otherwise, the role of ailerons in turning an aircraft (see Figure ae.16).

E22 A Boeing 747 of length 70 m has a mass of 200 000 kg when unladen, which can be considered to be acting at a centre of gravity 30 m from the front of the aircraft. Two large cargo containers are to be loaded onto the aircraft. One of mass 15 000 kg is to be loaded into a cargo bay 20 m behind the aircraft's centre of gravity. Where must the second container, of mass 25 000 kg, be located in order for the combined torques to offset each other?

E23 During flight the aircraft in question E22 encounters severe turbulence, causing the rear load to shift a further 2.5 m towards the rear of the aircraft, causing the rear of the aircraft to fall slightly. The pilot can quickly restore the aircraft's flying attitude by altering the position of the elevators on the rear wing while the cargo is re-secured. What lift must the rear wings and elevators exert in order to restore the aircraft's original flying attitude? (Assume that the elevators are at the rear of the aircraft.)

E24 In practice a pilot can deliberately alter the aircraft's flying attitude or angle of

Experimental investigations

The technology involved in many areas of aerospace are well beyond the understanding and equipment available in a secondary physics course. However, the basic principles of flight—lift, thrust, drag and weight—invite investigation. A simple wind tunnel can be made in the course of a single lesson. This is a process we can model on a small scale in a meaningful investigation of our own design.

Investigating

The process of investigation is quite different from the traditional teacher-structured practical activity where often the outcome is already known. It is quite probable that your results will not have been duplicated nor be available in any published resource. A few simple guidelines will assist you in making those results meaningful. The following guide will assist you in planning your investigation, including deciding on your own topic and detailing your aims and results.

Getting started

The choice of topic is often the biggest hurdle to overcome. Be imaginative and investigate a topic of genuine interest to you, within the constraints of the study of aerospace and aircraft design. Your investigation should concentrate on the physics involved and make use of the established principles of scientific investigation. The topic should be simple enough for you to begin immediately. Here are some simple guidelines that can help you make a good choice.

- Straightforward topics are almost always better than more complicated ones when just getting started. Don't choose a topic where the background research occupies most of the available time. You should search for topics using a wide range of resources and search terms. Don't narrow your topic too early before you have discovered good-quality information from a number of sources.
- Develop some preliminary aims as part of your initial topic choice. They may change as your investigation develops but they will be useful for checking that your topic is not too complicated. Focus your aims on particular, measurable tasks. Broad aims are hard to achieve.

- Think about what you intend to do before you start investigating. Identify potential problems and plan ways of overcoming them. Thinking ahead will make it easier to identify and isolate other problems along the way.
- Check that you have sufficient resources available for both research and practical tasks.
- Reference material can be helpful but don't expect to find all the answers. You are running a practical investigation. Try and generate the answers from your own activities.
- Use simple or readily available equipment that can be easily modified or adapted as the investigation evolves. Don't spend the whole time making a piece of equipment that leaves no time or opportunity for investigation.
- Establish a time line for each task and stick to it. Your time is limited and can't be extended. Ask for advice quickly if you feel as though the investigation is getting bogged down with problems you hadn't anticipated.
- Make good use of a written record. Write everything down. Record ideas, revise your aims, develop a new hypothesis—think as you go. Record both success and failure. You don't know what will become valuable when it comes time to complete the report. A thorough and easily followed logbook of your investigation will make it easy to write the report.

Your investigation

*

You can find full details on standards for analysis and presentation of results in Chapter 1 of the text.

What follows is a suggested approach. You do not have to follow it exactly and should discuss your project regularly with your teacher. Make sure that throughout your investigation you use correct scientific standards for units and other conventions. Use the following suggested questions and activities to guide your research. Record all of your work, ideas and data in an exercise book or in an electronic file on your computer.

1. **What is your topic area of choice?** Is it aerofoil design, power versus thrust of a propeller, the angle of attack of a wing?
2. **Define your topic in terms of a measurable task.** For example, 'Investigating the lift of a aerofoil under varying wind speeds' or 'A determination of the optimum number of blades and the optimum blade angle for a propeller' or, more simply, 'Comparing the performance of three different aerofoils with the same cross-section but different chord length'. Make sure that your intent is clear.
3. **Write a brief introduction to clarify your intentions both to the reader and to yourself.** Consider this propeller example:

   'Early propellers were simple fixed-pitch two-bladed affairs. Later aircraft used three- and four-blade propellers with variable pitch, and there have been some aircraft using propellers with more blades. This investigation will compare three propellers using blades of the same length and section, but of varying blade-number, from 2 through to 4. The angle of attack of the two-blade propeller will be varied to investigate the effect of this variable.'
4. **Break your topic down into two or three distinct, achievable aims.** For our example they may be:
   a. To investigate the comparable effectiveness of 2, 3 and 4 blades of the same length and section for a constant wind speed.

b To investigate the effect on propeller effectiveness of the angle of the propeller blades to the wind direction.

c To consider the implications of the practical findings in the design and construction of an aircraft propeller.

5 **What variables are being considered in your investigation?** Group the quantities you're going to measure as:

a what you are going to try to measure or change (e.g. blade number, blade angle, wind speed)

b the constants of your investigation (blade length, blade profile, wind direction relative to the propeller are some that could be included in our example)

c outside influences that you will attempt to minimise but which are beyond your immediate control. You may also like to note how you're going to try and minimise or control the effect of these influences. In our example, air temperature can alter air density and affect the results a little with large temperature changes. Height of the blades above a surface and other factors affecting turbulence around the blades would have a marked effect. We'll attempt to test under similar conditions.

6 **Research relevant background material.** In our propeller example, the following points would be worth discussing in brief:

- the history of the development of propellers
- how a propeller works
- the application of Bernoulli's equation to propeller design.

7 **Write up your experimental method and results.** Get away from the standard recipe style here. You are not instructing someone on what to do but rather telling them what you did. A narrative style interspersed with your raw data, diagrams of equipment arrangement, perhaps photos of special equipment, would all help to tell the story. This is the stage where a well-kept logbook will help you.

8 **Analyse your results.** Make sure tables are constructed to show clearly the controlled and measured variables. Graphs are a very useful means of analysing results. Other background material on the analysis of practical results and the consideration of errors has been included in this book (see Chapter 1). Refer to this during the investigation. There's nothing worse than packing everything away and then finding out that your results don't make sense. Electronic measuring can have significant advantages. By producing graphs as you work, trends and errors are highlighted and you can make changes to your experimental design as you go.

9 **State your conclusion.** Your conclusion should simply state your results in a manner that answers your original aims. A valid comparison could be included.

10 **Evaluate your results.** Were your results what you expected? Are four blades really best for aircraft or are other variables influencing that choice? How does pitch need to be varied with wind speed? Your evaluation should discuss to what degree and with what degree of certainty your investigation achieved your aims. Avoid the 'It was really good and I learnt a lot' style. Explain the science and the implications your results have on the larger scale.

11 **List your references and/or bibliography.** Give credit where credit is due. Your references should be correctly and completely annotated together with appropriate recognition of partners. References are usually listed alphabetically based on the (first) author's last name, with the title of the published work in italics:
Adams, Phillip 1987, 'Black and white and read no more?', *Weekend Australian Magazine*, 7–8 Feb., p.2.
Angelucci, Enzo 1984, *World Encyclopedia of Civil Aircraft*, Willow Books, London.

Starting points

Throughout this study, a number of suitable starting points or ideas for investigation have been suggested. While by no means comprehensive, the following are some suggestions for investigation requiring little equipment beyond what could reasonably be expected in a senior secondary physics room. Further ideas, background and information are available from a variety of sources. The website hi.com.au/physics will publish regular updates and links to selected worthwhile resources.

Video analysis, or force sensors and loggers, provide a more reliable method of recording real-time maximum and minimum forces than simple spring balances or beam balances. A simple and cheap video analysis can be achieved using a video played back through a television and traced onto graph paper. Check on the availability of other electronic measuring alternatives or simulation software with your teacher.

- **Investigating airflow in a wind tunnel**—Investigate the effect of shape, speed and basic aerofoil support structures on the flow of air through a simple wind tunnel. Improve the design of the wind tunnel and observe the effect on airflow.
- **Power versus thrust of a propeller (I)**—Vary the voltage supplied to an electric motor driving a small propeller. Measure the resulting thrust produced by measuring the displacement of an object suspended in the airflow from the propeller. Graph and compare power versus thrust for two different propellers.
- **Power versus thrust of a propeller (II)**—Use a rubber-band-powered aircraft to investigate power versus thrust. Graph the number of twists of the rubber band versus flight range and comment on the results.
- **Efficiency of a propeller**—Compare length, pitch, number of blades, or any other variable associated with propellers. Graph speed of rotation against airspeed for each propeller and comment on comparative efficiencies.
- **Modelling and testing aerofoils**—Construct a simple wind tunnel using the plans provided (opposite). Test varying size, shape and thickness of aerofoils. Graph lift against airspeed for each aerofoil. Use your graphs to compare the flight characteristics of each aerofoil.
- **Angle of attack**—Construct a simple wind tunnel and a standard aerofoil shape. Vary the angle of attack and measure the lift generated for a constant airspeed. Graph angle of attack versus lift and comment on the results.
- **Control surfaces**—Suspend a simple model aircraft in a wind tunnel. Attach measuring lines (i.e. lines attached to force measurers) to the sides of the model. Vary the position of the vertical stabiliser and/or rudder. Graph rotational force measured versus wind speed and comment on the results. Vary the profile of the model or size of the stabiliser. Graph size versus force and comment on the results.
- **Simulating force**—Use a force simulation program to simulate the forces acting on an aircraft. Vary the size of the forces and investigate the effect on take-off, climb, cruise and descent. Comment on your results.

Investigation – MAKING A MODEL WIND TUNNEL

Introduction

Wind tunnels need not be expensive or large in order to achieve meaningful results. A simple wind tunnel that will allow testing of aerofoils and other simple aerodynamic models can be constructed in just one lesson.

The shape of an aerofoil is the most important factor in modern aircraft design. A well-designed wing should be able to do three things:

- lift the aircraft into the air
- keep the aircraft stable in flight
- ensure maximum fuel efficiency during flight.

Consider working as a team; while you're making the wind tunnel, other team members can be making aerofoils for testing.

Plan carefully before starting. Allocate tasks, set a time limit and stick to it. Your investigation is judged on results, not on the model!

Requirements

Substitute for similar items where necessary. (Refer to Figure ae.17 to ascertain the purpose of each item.)

- large, strong cardboard box
- small electric fan
- two or more pulleys as guides for the measuring line
- triple beam balance or spring balance and retort stand
- Substitute a force sensor and logger for real-time measurement and recording of instantaneous values as carried out in industry. Video cameras and video analysis through software tools like VideoPoint can also be useful.
- light fishing line
- drinking straws or other rigid tube
- balsa wood, perspex, card or other material for making aerofoils.

Procedure

1. Open a box at both ends to create an open 'tube' (the squareness doesn't matter).
2. Locate and set up a small electric fan to provide the airflow. A fan with variable speed settings is ideal as it increases the variables available for investigation. You may find it useful to cut some windows in the sides of your 'tube' and cover them with clear plastic to allow viewing or videotaping of the test in real time.
3. Find the centre of gravity of your aerofoil. Attach two short straws or other rigid tube to the sides as shown in Figure ae.17b.
4. Fix the aerofoil inside your wind tunnel by threading two pieces of fishing line through the drinking straws and fixing the line securely to the top and bottom of the cardboard box. It is essential that these lines are taut and as straight as possible.
5. Fix another piece of line to the bottom of the aerofoil, pass it through the pulleys and attach it to the force measurer you've chosen. The pulleys are used to ensure that the fishing line is free to move with the application of small forces. Check that the line is free to move.

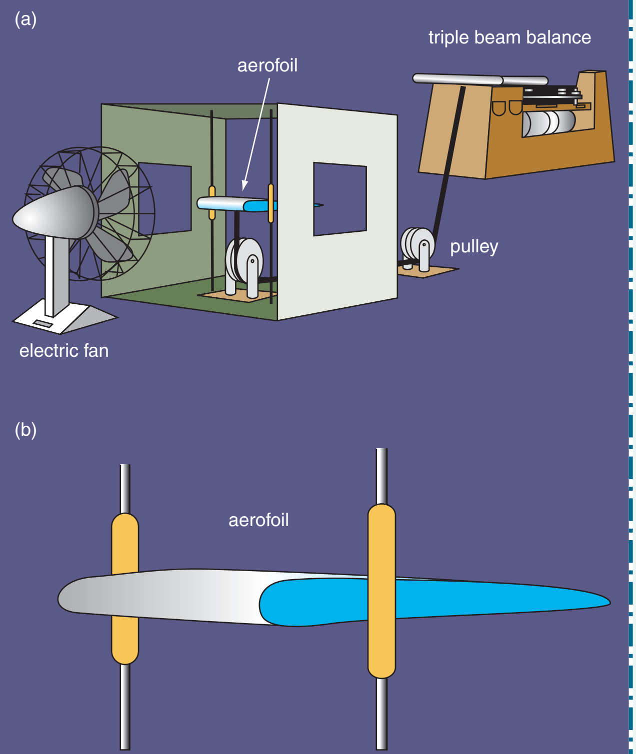


Figure ae.17 A model wind tunnel showing (a) the final model and (b) the standard mounting of an aerofoil in the tunnel.

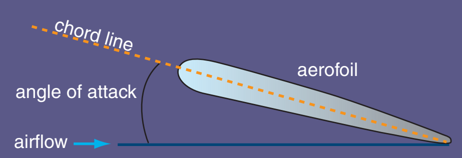


Figure ae.18 Angle of attack or pitch angle of an aerofoil.

6. Record the following measurements for each aerofoil:
   - the length of upper and lower surfaces of the aerofoil (a string laid along the surface may assist you with measurement along a curved surface)
   - the mass of each aerofoil
   - the lifting force recorded by the force measurer.
7. Introduce the smoke from a burning candle or mist from the vaporising of dry ice (solid carbon dioxide) to make the airflow over the aerofoil visible.

Analysis

Graph the relationship between the upper length of the aerofoil and the measured lift force.

Extension

Consider how you can modify your basic arrangement to test the effect of the angle of attack of your aerofoil.

medical physics: atoms in action

By the end of this context
you will have covered materials including:

- radiotherapy
- radioisotopes in medicine
- positron emission tomography
- radiation dosimetry.

Humans have always been exposed to radiation from natural sources. This is unavoidable. This background radiation originates from a variety of sources. The Earth is continually bombarded by cosmic radiation from the sun and from deep space, although our atmosphere blocks most of this. The ground we walk on is radioactive and every time we inhale, we take minute quantities of radioactive radon into our lungs. Even the food we eat and the water we drink contain trace amounts of radioactive elements from the natural environment. It is accepted that exposure to high levels of high-energy radiation leads to the development of cancerous tumours and leukaemia. This was vividly displayed in the aftermath of the nuclear explosions at Hiroshima and Nagasaki in 1945 and after the nuclear disaster at Chernobyl in 1986. However, radiation and radioactive elements can be used in a variety of applications that are of real benefit to people. The photograph on this page shows a cancer patient receiving radiation treatment for a tumour in his neck. A lead vest and mask protects the rest of his body from this radiation. There are many other useful applications of nuclear radiation.

Ionising radiation

Cancer is a general term that actually incorporates hundreds of different diseases. The term *tumour* describes the growth of abnormal cells. Tumours are not all cancerous. Benign tumours are abnormal, but not spreading, and often grow slowly. They can be single growths that remain in one place. Sometimes doctors don't even remove them unless they are affecting a nearby part of the body.

Cancer cells (malignant tumours) can grow in just about any part of the body. Malignant tumour cells can grow into and invade the surrounding healthy tissue. Some cells may break away and be carried by the bloodstream; they then settle in other parts of the body, spreading the cancer. Their growth can be aggressive and they can severely affect the ability of the invaded body parts to function.

It is well accepted that exposure to radiation can cause cancer, but it can often also provide a cure. A common method of cancer treatment is the use of *ionising radiation* to destroy the cancerous cells.

Particular types of electromagnetic radiation, namely X-rays and gamma rays, can kill living cells by ionising molecules in them. *Ionisation* means the forming of ions—positively and negatively charged particles. When cells are ionised, cell death can result. Therapeutic ionising radiation with X-rays or gamma rays targets the DNA in the cancerous cell. Radiation either breaks the DNA strand, resulting in cell death, or at least damages the DNA so that the cell cannot reproduce. Ionising radiation can produce ionised water molecules in the cells and highly reactive hydroxide ions that can damage parts of the cell, including DNA.

Cancerous cells are rapid growth cells. This means that they reproduce at a faster rate than most other cells in the body, but this also means that they are more susceptible to being destroyed by radiation. Generally, normal healthy cells recover from radiation treatment more quickly than cancer cells. Regardless, *radiation therapy* programs must be designed to provide the optimal dose of radiation to the cancerous cells while minimising the exposure of healthy cells. There are some relatively rapidly reproducing cells in the body that don't cope with radiation exposure, such as the reproductive organs, the kidneys, the liver, bone marrow and the eyes. Radiation exposure of healthy cells can cause ionisation and the production of malformed cells. So while the 'benefit outweighs the risk' for the patient with a malignant tumour, doses should always be kept to a minimum for the healthy cells of the patient.

Radioisotopes in medicine

Your studies of radioactivity (Chapter 5) will have introduced you to the three different types of radiation that can be emitted by unstable or radioactive atoms—alpha (α), beta (β) and gamma (γ) radiation. The branch of medicine that uses radioisotopes is called *nuclear medicine*. Just as was the case with both X-rays and ultrasound, radioisotopes can be used in two different ways in medicine—either to aid the diagnosis of disease or as therapy in order to treat disease.

The widespread use of nuclear medicine did not start until the early 1950s. Today, there are over 100 different nuclear medicine imaging and therapeutic procedures. They can provide information about every major organ system in the body. The Australian Nuclear Science and Technology Organisation (ANSTO) is responsible for the production of the majority of medical radioisotopes used in Australia. This organisation supplies a wide range of radioisotope products and services to nuclear medicine and industry throughout Australia, New Zealand and Asia. These isotopes are mainly produced in Australia's only nuclear reactor at Lucas Heights near Sydney and in the Cyclotron at Camperdown, New South Wales. Many large hospitals have their own nuclear medicine departments where products from ANSTO are processed within the hospital immediately before use.

Radioactive isotopes are produced in nuclear reactors and cyclotrons. There are some differences in the products of each, and so both types of facilities are needed. In reactors, medical radioisotopes are usually produced by bombarding a stable isotope of a substance with neutrons, which are plentiful in a reactor. This creates a radioactive isotope of the original substance. The most commonly used radioisotope, technetium-99m, is the daughter product of an atom produced in reactors and is used in the diagnosis of brain and heart disease and for studying lung, kidney and liver function. Iodine-131, which is used to detect and treat thyroid cancer, is also a reactor-produced radioisotope. In a thermal nuclear reactor a rod of the element tellurium-130 is bombarded with slow moving neutrons, forming the unstable tellurium-131 isotope according to the equation:

$$^{130}_{52}\text{Te} + {}^{1}_{0}\text{n} \rightarrow {}^{131}_{52}\text{Te} + \gamma$$

The tellurium-131 undergoes spontaneous decay that results in the production of iodine-131:

$$^{131}_{52}\text{Te} \rightarrow {}^{131}_{53}\text{I} + {}^{0}_{-1}\beta + \bar{\nu}$$

After a month or so in the reactor, the rod is removed and the tellurium and iodine are chemically separated from one another.

This context assumes you have studied nuclear energy in Chapter 5.

In cyclotrons, radioactive isotopes are produced by bombarding atoms with charged particles. Cyclotron-produced radioisotopes have only been produced in Australia since 1992. According to ANSTO, three of the most widely used cyclotron-produced radioisotopes in medicine are gallium-67, thallium-201 and iodine-123. Gallium-67 is used mainly in the detection of soft tissue tumours and infections. Thallium-201 is instrumental in detecting heart disease. Iodine-123 is used in imaging the thyroid gland and can be labelled onto a range of drugs to detect stroke, epilepsy, kidney and heart diseases, as well as some cancers.

Diagnostic imaging with radioisotopes

The radioisotopes used in diagnostic procedures must be able to provide useful information such as an image of an internal structure in the body, or an image of the various stages in the function of an organ. We have already looked at the use of barium and iodine as non-radioactive providers of contrast in X-ray images.

In diagnostic procedures the gamma radiation output by radioisotopes that have been put into the body is detected by gamma cameras (rather than the films used with X-ray radiation). The radioisotope is first attached to a compound (drug) that will be taken up by specific organs or disease sites within the body. This is called a radiopharmaceutical. The compound is therefore called the carrier and the gamma radiation from the radioisotope produces the image.

A ***RADIOPHARMACEUTICAL*** is a radioactive material combined with a chemically or biologically active compound that is taken up in the body.

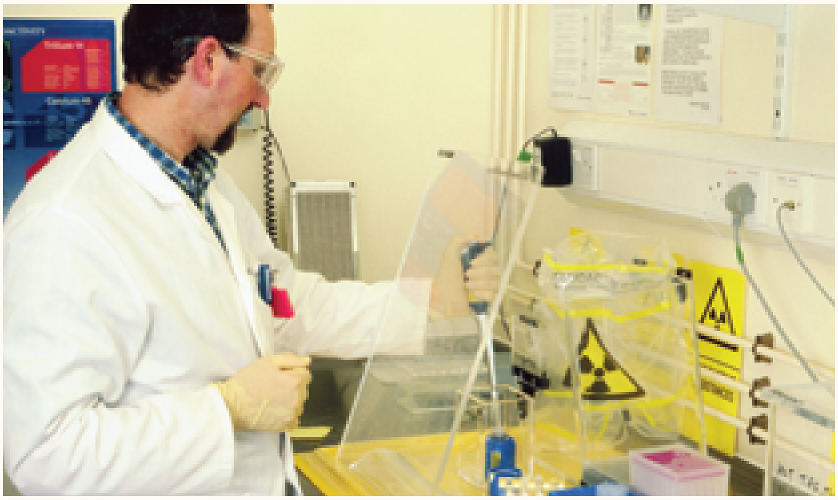


Figure mp.1 Production of a radiopharmaceutical.

One particular radioisotope may be attached to a number of different types of compounds, each one designed for a particular application. For example, technetium-99 is attached to one carrier compound when used to investigate the thyroid and another when it is used to investigate the brain. The brain consumes large quantities of glucose, whereas the thyroid takes up iodine.

Once the radiopharmaceutical has delivered the radioisotope to the designated part of the body, the gamma camera is simply placed in the vicinity of the patient for a set time so that the emitted radiation

Figure mp.2 For image clarity a gamma camera prevents the entry of scattered gamma rays.

produces a digitised image. Gamma cameras are designed for use with gamma radiation of a small range of energy values—a typical gamma camera will detect radiation between 126 and 154 keV. Scattered gamma rays are prevented from entering the camera, as its front plate is a thick layer of lead with narrow tubes drilled through it. These tubes ensure that only gamma rays travelling perpendicularly out from the body are imaged. Although this is necessary for the clarity of the image, it means that a large portion of the patient's radiation exposure does not contribute to the image.

Choice of radioisotope for diagnostic imaging

According to the supplier ANSTO, for a radioisotope to be used for *diagnostic imaging* it must:

- have a short half-life (hours), which is appropriate for the time taken for the diagnostic procedure
- not emit alpha or beta radiation, because these particles would be trapped in the patient's tissues and they would not be detected externally
- emit gamma radiation of an energy that can be detected by a gamma camera
- be available in the highest possible activity, but not be toxic to the patient or react with drugs used at the same time.

According to ANSTO, the reactor-produced technetium-99m is used in more than 80% of the estimated 100 000 patient studies that are performed worldwide each day. After technetium-99m, a series of cyclotron-produced radioisotopes, such as thallium-201, gallium-67, indium-111 and iodine-123, are the next most frequently used.

The production of technetium-99m

Technetium-99 is the semistable daughter nucleus of the atom molybdenum ($^{99}_{42}Mo$), which is produced in a nuclear reactor at Lucas Heights. Each week some molybdenum is brought to hospitals that have a technetium generator. Technetium generators consist of a central column of alumina to which the molybdenum is chemically bonded. As the molybdenum undergoes beta decay, technetium is formed on the outside of the molybdenum layer (see Figure mp.3). A saline solution is then washed over the technetium and this solution is collected in

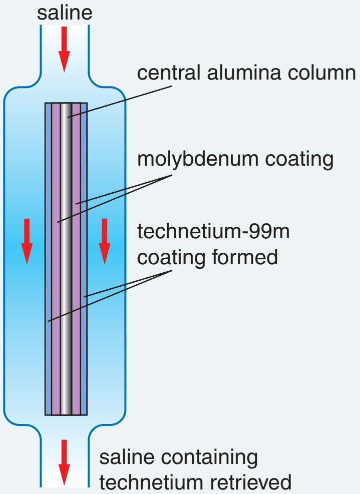


Figure mp.3 A technetium generator. Hospitals must order new samples of the mother isotope molybdenum each week, but this is frequently enough to ensure a continual supply of the widely used radioisotope technetium-99m, which decays via a gamma emission with a half-life of 6 hours.

table mp.1 Medical isotopes manufactured by ANSTO

Reactor-produced radioisotopes	
Chromium-51	Used to label red blood cells and quantify gastrointestinal protein loss
Copper-64	Used to study genetic disease affecting copper metabolism
Iodine-131	Used to diagnose and treat various diseases associated with the human thyroid. Also used in diagnosis of the adrenal medullary and for imaging suspected neural crest and other endocrine tumours
Iridium-192	Supplied in wire form for use as an internal radiotherapy source
Molybdenum-99	Used as the 'parent' in a generator to produce technetium-99m, the most widely used isotope in nuclear medicine
Phosphorus-32	Used in the treatment of excess red blood cells
Samarium-153	Used with ethylenediaminetetramethylene phosphonate (Quadramet) to reduce the pain associated with bony metastases of primary tumours
Technetium-99m	Used to image the brain, thyroid, lungs, liver, spleen, kidney, gall-bladder, skeleton, blood pool, bone marrow, salivary and lachrymal glands and heart blood pool and to detect infection
Yttrium-90	Used for liver cancer therapy
Cyclotron-produced radioisotopes	
Gallium-67	Used in imaging to detect tumours and infections
Iodine-123	Used in imaging to monitor thyroid function and detect adrenal dysfunction
Thallium-201	Used in imaging to detect the location of damaged heart muscle
Carbon-11, nitrogen-13, oxygen-15, fluorine-18	Used in positron emission tomography (PET) to study brain physiology and pathology, for detecting the location of epileptic foci and in dementia, and psychiatry and neuropharmacology studies. They are also used to detect heart problems and diagnose certain types of cancer

Source: ANSTO, *Radioisotopes: Their role in society today*, available on ANSTO website.

a syringe. This solution contains concentrated technetium and can be diluted for use. Of course all of this is housed within a lead-lined evacuated chamber so that the radioactivity is contained.

A recent development in the use of the technetium-99m isotope is the development of a new radiopharmaceutical that, when injected into the body, will accumulate at the site of any ulcerous infections in the body. It can be used to investigate local areas in the body or for whole body investigations. The pharmaceutical takes 5 minutes to prepare and the patient will be ready for scanning 1–2 h after the injection.

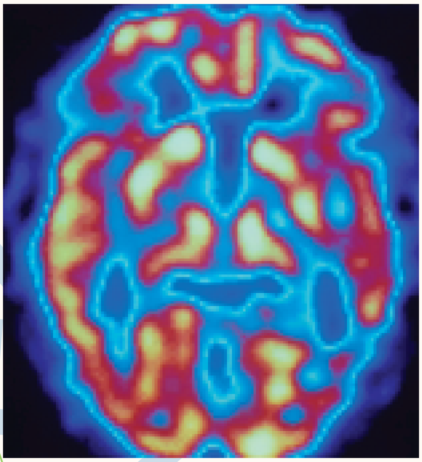


Figure mp.4 A PET scan.

Positron emission topography (PET)

Positron emission tomography (a *PET* scan) is used to study the function of the brain, heart and tissues throughout the body. PET scanning uses similar principles to diagnostic imaging with radioisotopes. A radioisotope is introduced to the body attached to a compound that will cause it to accumulate in a particular part of the body. Since PET scans are often used to scan the brain, the radiopharmaceutical often involves glucose (sugar) as the carrier substance.

The radiopharmaceuticals used in PET scans are designed so that electrons from the investigated tissue combine with positrons (small positively charged particles) from the radioactive tracer to produce gamma rays (similar to X-rays). These rays are recorded by a gamma camera and reconstructed into 3D pictures by the computer. The radioactive materials that are used in the PET scan only last a short time so a nuclear particle accelerator cyclotron is needed to make them in a location close to the patient. A cyclotron accelerates protons to a very high energy and these are then caused to collide into a stable gas or liquid isotope. This isotope is then temporarily radioactive. It will become a radiopharmaceutical by attachment to an appropriate carrier substance, for example sugar.

A PET scan can produce very high resolution 'slices' of the brain. It can show which tissues are living and using energy. Therefore, it would show up any dead tissue, in the muscles of the heart for example. PET scans can also show live or dead brain tissue after a stroke and where seizures may be occurring in the brain. It can even show the functioning of the brain during thought processes. PET can be used to find tumours and then monitor how well they respond to treatment. PET scans are very expensive and involve patient exposure to ionising radiation, but they can decrease the patient's overall medical costs by avoiding exploratory surgery or showing where surgery can do the most good.

Therapeutic treatments with radioisotopes

The medical approach of treating cancerous growths with *ionising radiation* has been discussed earlier. When using radioisotopes, the source of the ionising radiation is placed inside the body; it is usually localised in the affected organ. This can be achieved in a number of different ways. A radiopharmaceutical can be selected that when ingested or injected will accumulate in a particular part of the body. Iodine-131 or technetium-99m will accumulate in the thyroid and radiate a cancerous growth there. Phosphorus-32 will accumulate in the bone marrow. Technetium-99m can be used in a number of radiopharmaceuticals that will cause it to accumulate in the lungs or kidneys or brain, for example. Samarium-153 can accumulate in breast or prostate tumours.

Brachytherapy uses physical sources that are temporarily implanted in the body so that they radiate and kill the cells in the local area. For example, iridium-192 is produced in a thin wire that is then inserted via a catheter into the tumour. This is often used for tumours in the head and breast.

When selecting isotopes for use in therapy, different criteria apply. Unlike diagnostic isotopes, isotopes used in therapy should be sources of alpha and beta radiation. These are ionising radiations and need to be of sufficient energy to penetrate the diseased section of the body. It is convenient if the isotope is also a weak source of gamma radiation so that its presence in the body can be monitored by the gamma cameras discussed earlier. Radioisotopes commonly used in therapy include iodine-131, phosphorus-32 and yttrium-90, and several others are being investigated for possible application.

Exercises

E1 a What is the most common radioisotope used in nuclear medicine?
b How is it supplied to hospitals?

E2 Technetium-99m is the semistable daughter nucleus of the atom molybdenum ($^{99}_{42}Mo$). Write the decay equation for the production of technetium-99m.

E3 What characteristics should an isotope possess that will be used as a *radioactive tracer*?

E4 Briefly explain how ionising radiation kills cancer cells.

E5 Why do diagnostic radioisotope tracers need to be gamma emitters, but therapeutic radioisotopes need to be alpha, beta and gamma emitters?

E6 Which areas of the body need special protection from ionising radiation during nuclear medicine procedures? Why?

E7 Why are radiation treatments usually given in courses of lots of low doses spread over days or weeks?

E8 State two reasons why a cancer patient undergoing radiation treatment should be given ultrasounds, CAT scans or X-rays prior to their treatment.

E9 What is a radiopharmaceutical?

E10 Explain the apparent contradiction that ionising radiation can both cause and cure cancer.

Activities

1 Compare and contrast the imaging techniques of DSA (digital subtraction angiography) and PET. What are the advantages and disadvantages of each?
2 What is the greatest advantage of MRI (magnetic resonance imaging) as an imaging technique?
3 Describe the advantages and disadvantages of radiation therapy.

Extended response

Select a medical ailment and investigate:

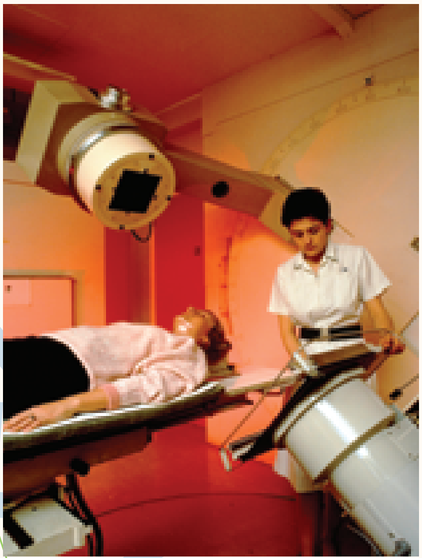
- who gets it and why
- physics principles underlying the treatment procedure
- dangers associated with the procedure
- success rate and future possibilities.

Ailments may include breast cancer and malignant melanomas.

Investigation

If your school has some radioactive sources, then investigate the shielding effects of various materials. You could use photographic materials or a Geiger counter to collect data. This activity must be done with due consideration for safety and under appropriate supervision.

Radiation dosimetry



Exposure to radiation is impossible to avoid. Radiation has been part of the natural environment since the Earth formed, and humans have evolved to live with a certain amount of exposure. Even though we live in an environment with ionising radiation, our senses are unable to detect its presence.

The background level of ionising radiation to which we are continually exposed is not a significant health problem. However, exposure to above-average levels of ionising radiation is dangerous. It may lead to long-term problems such as cancer and genetic deformities in future generations. Extremely high levels of exposure can cause death, and in extreme cases this can happen within just a few hours.

It is important that people who work with radiation in fields such as medicine, mining, nuclear power generation and industry are able to monitor closely the amount of radiation to which they are exposed.

Figure mp.5 Radiologists administer very precise doses of radiation that are designed to destroy cancer cells. This treatment is successful because rapidly dividing cancer cells are more susceptible to damage from ionising radiation than are normal body cells.

Furthermore, radiologists, who administer courses of radiation treatment to cancer patients, also need to be able to measure the amount of radiation that they are applying.

Absorbed dose

When a person is exposed to high-energy radiation, the energy of the radiation acts to break apart molecules and ionise atoms in the person's body cells. The severity of this exposure depends on the amount of the radiation energy that has been absorbed by the individual's body. This quantity is known as the *absorbed dose*.

The absorbed dose is the radiation energy that has been absorbed per kilogram of the target material.

$$\text{Absorbed dose} = \frac{\text{energy absorbed by tissue}}{\text{mass of tissue}}$$

Absorbed dose is measured in joule/kilogram ($J\ kg^{-1}$) or gray (Gy), i.e. $1\ Gy = 1\ J\ kg^{-1}$.

To illustrate this, if a 25 kg child absorbed 150 J of radiation energy, then the absorbed dose would be 6 Gy. This is a massive dose and would be enough to kill the child within a few weeks. However, an adult, being much larger, would be less severely affected by this radiation. If a 75 kg adult absorbed 150 J of radiation energy, the absorbed dose would be just 2 Gy. This dose would give the adult a severe case of radiation sickness but would probably not be fatal. You can think of dose in the same way as one administers medicine. The small mass of a child means that taking just half an aspirin might be equivalent to an adult taking two tablets.

Dose equivalent

Different forms of radiation have different abilities to ionise, and so cause different amounts of damage as they pass through human tissue. Alpha particles are the most ionising form of radiation, and so an absorbed dose of alpha radiation is far more damaging than an equal absorbed dose of beta or gamma radiation.

The different types of radiation are given weightings to reflect their biological impact. These are called the *quality factors*, shown in Table mp.2.

A measure of radiation dose that combines absorbed dose and the quality factor weightings will give a more accurate picture of the actual effect of the radiation on a person. This is the *dose equivalent*. Dose equivalent is defined as the product of the absorbed dose and the quality factor of the radiation. Dose equivalent is measured in sievert (Sv), although millisievert (mSv) and microsievert (μSv) are more commonly used.

Dose equivalent = absorbed dose × quality factor
Dose equivalent is measured in sievert (Sv).

table mp.2 Quality factors of different types of radiation

Radiation	Quality factor
Alpha particles	20
Neutrons* (>10 keV)	10
Beta particles	1
Gamma rays	1
X-rays	1

* Radiation from neutrons is only found around nuclear reactors and neutron bomb explosions.

For example, an absorbed dose of just 0.05 Gy of alpha radiation is biologically equally as damaging as an absorbed dose of 1.0 Gy of beta radiation. While the energy carried by the alpha particles is lower than the beta particles, each alpha particle does far more damage. In both cases, the dose equivalent is 1 Sv, and 1 Sv of any radiation causes the same amount of damage.

It is important to appreciate that 1 Sv is a massive dose of radiation and, while not being fatal, would certainly lead to a severe case of radiation illness.

In Australia the average annual background radiation dose is around 2.0 mSv, or 2000 μSv. A microsievert is a millionth of a sievert. Use Table mp.3 to estimate your annual dose.

Worked Example mp.1

A 10 g cancer tumour absorbs 0.002 J of energy from an applied radiation source.

a What is the absorbed dose for this tumour?

b Calculate the dose equivalent if the source is an alpha emitter.

c Calculate the dose equivalent if the source is a gamma emitter.

d Which radiation source is more damaging to the cells in the tumour?

Solution

a Absorbed dose $= \dfrac{\text{energy absorbed}}{\text{mass of tissue}} = \dfrac{0.002\ \text{J}}{0.01\ \text{kg}}$

$= 0.20$ Gy.

b Dose equivalent = absorbed dose × quality factor

$= 0.20 \times 20$

$= 4.0$ Sv for the alpha emitter

c Dose equivalent = absorbed dose × quality factor

$= 0.20 \times 1$

$= 0.20$ Sv for the gamma emitter

d The alpha particle source is more damaging. It causes more ionisation in the cells and so has a higher dose equivalent.

table mp.3 Annual radiation doses in Australia

Radiation source	Average annual dose (μSv)	Local variations
Cosmic radiation	300	Plus 200 mSv for each round-the-world flight. Plus 20 mSv for each 10° of latitude. Plus 150 mSv if you live 1000 m above sea level.
Rocks, air and water	1350	Plus 1350 mSv if you live underground. Plus 1350 mSv if your house is made of granite. Minus 140 mSv if you live in a weatherboard house.
Radioactive foods and drinks	350	Plus 1000 mSv if you have eaten food affected by the Chernobyl fallout.
Manufactured radiation	60	Plus 60 mSv if you live near a coal-burning power station. Plus 30 mSv from nuclear testing in the Pacific. Plus 20 mSv if you watch 20 hours of television each week.
Medical exposures	–	Plus 30 mSv for a chest X-ray. Plus 300 mSv for a pelvic X-ray. Plus 1000 mSv if you have had a ‘barium milkshake’ cancer diagnosis. Plus 40 000 000 mSv for a course of radiotherapy using cobalt-60.

Activities

1 In external beam radiotherapy, why are several low-dose beams aimed at the tumour from different angles rather than a single, fixed, high-dose beam of radiation?

2 Why are low half-life sources better for radioactive tracing and longer half-life sources better for the treatment of tumours?

Exercises

E11 Use Table mp.2 to answer this question. Calculate the dose equivalent from a radiation source if the absorbed dose is 0.50 mGy and the radiation is:
a alpha radiation b beta radiation c gamma radiation.

E12 An 80 kg tourist absorbs a gamma radiation dose of 200 mGy during a return flight to London. Calculate:
a the dose equivalent that has been received
b the amount of radiation energy that has been absorbed.

E13 a Which one of the following is the most damaging radiation dose?
A 200 mGy of gamma radiation
B 20 mGy of alpha radiation
C 50 mGy of beta radiation
b Which one of these is the most damaging radiation dose?
A 200 mSv of gamma radiation
B 20 mSv of alpha radiation
C 50 mSv of beta radiation

E14 Which one of the following is most appropriate for use as a radioactive tracer to detect the presence of a brain tumour?
A Radon-222; alpha emitter; half-life = 3.8 days
B Sulfur-35; beta emitter; half-life = 97 days
C Cobalt-60; gamma emitter; half-life = 5.3 years
D Technetium-99m; gamma emitter; half-life = 6 hours

E15 When in space, astronauts usually receive a radiation dose of approximately 1000 mSv per day. The maximum allowable annual dose for people working with radiation is 50 mSv.
a The normal annual background dose per year on Earth is 2 mSv. How many days does it take for astronauts to exceed this dose?
b How long would astronauts have to be in space before they exceeded the maximum annual dose for radiation workers?
c The record for time spent in space is held by a cosmonaut who was on the Mir space station for 439 days. How much radiation (in mSv) was the cosmonaut exposed to in this time?

E16 Discuss some strategies you could use to minimise your exposure to ionising radiation.

E17 To treat cancer of the uterus, a radioactive source is implanted directly into the affected region.
a If the uterus receives a dose of 0.40 Gy per hour from the source, how many hours should it be left there to deliver a dose of 36 Gy?
b Explain why caesium-137, a beta emitter with a half-life of 33 years, is well suited to perform this task.

1 Measurement and data

Motion, electricity and magnetism, and time travel are as diverse as physics topics might be, but all have something in common. They have all been studied and verified by practical investigations and experiments. Once physicists have described phenomena with mathematical relationships, it is possible to predict the behaviour of the physical objects being studied.

During the 17th century, Galileo explored the motion of bodies and quantified the relationships between displacement, velocity and acceleration. His work was based in experimentation and measurement.

In the 18th century, Isaac Newton proposed elegant relationships between forces and their effects on motion. The relationships between forces and movement are now well understood. Engineers and scientists can use mathematical relationships to predict and understand the movement of objects as diverse as a planet in an orbit, a car being tested for engine performance and an individual atom in a nuclear reaction. Nearly 300 years of experimentation by scientists and students have repeatedly confirmed that Newton's ideas are correct for macroscopic bodies—as long as they are not travelling too fast. Experimentation and measurement have also revealed the reality of the circumstances under which Newton's laws, as predicted by Albert Einstein, will fail.

Michael Faraday was one of the greatest experimental scientists, making many contributions to physics and chemistry. In the 19th century, he discovered and quantified relationships between electrical and magnetic phenomena. The lights and power points in your home are arranged according to the well-established behaviour of electric currents and electrical energy.

Early in the 20th century, Albert Einstein proposed that the experience of time is dependent on relative velocity. Experiments and measurements a few decades later confirmed that fast particles do, indeed, experience time at a slower rate. Time travel is yet to be incorporated into everyday technology.

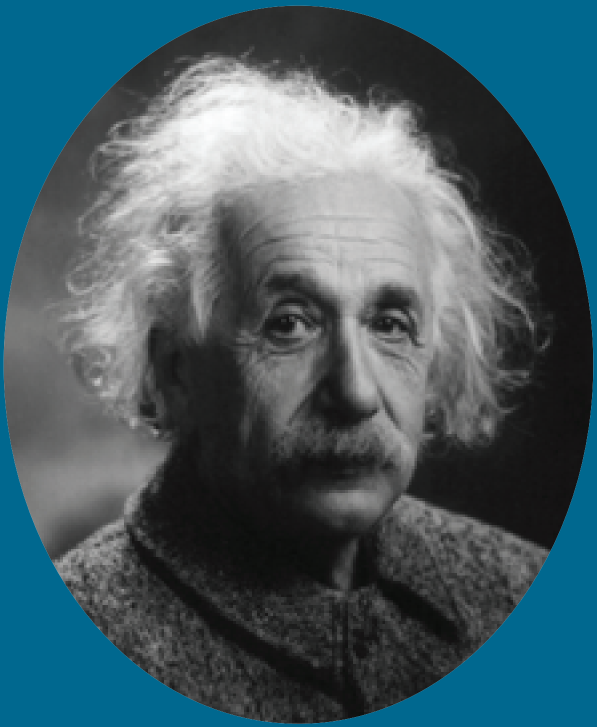
By the end of this chapter

you will have covered material from the study of the analysis of data including:

- the SI system of quantities for units
- the correct use of measurement units and notation
- accuracy and precision of data
- calculation of uncertainty in experimental results
- graphical analysis of data.

Theory and experiment go 'hand-in-hand' in the physical sciences. Although great theories might result from purely mathematical analysis and a creative imagination, they are not accepted until observations and measurements provide evidence for the theories.

Measurements have limitations to their precision and accuracy. These limitations need to be considered when interpreting data. In physics, you calculate the consequences of the imprecision in your measurements. In this chapter, we focus on the nature of measurement and the use of standard units for measuring quantities. Experimental measurements provide direct evidence of the relationship between variables. You will be introduced to techniques that will enable you to use your data to find the mathematical equations describing the relationships between your experimental variables.



1.1 Measurement and units

In every area of physics we have attempted to quantify the phenomena we study. In practical demonstrations and investigations we generally make measurements and process those measurements in order to come to some conclusions. Scientists have a number of conventional ways of interpreting and analysing data from their investigations. There are also conventional ways of writing numerical measurements and their units.

Units and symbols

For almost all of the work done in Years 11 and 12, the SI (Système International) system of units will be used. There are just seven fundamental quantities that form the basis of all measurements in science. They are metre (m), second (s), kilogram (kg), ampere (A), kelvin (K), mole (mol), and candela (cd). All other units can be derived from these. The fundamental units are summarised in Table 1.1.

Physics file

The metric system was originally developed in France and is known as the *Système International* (SI). It was adopted in France in 1840 as the official system of units, although it had been developing in that country since 1545. It has remained in use ever since and has gradually been adopted by most other countries. It has been modified a little over the years and now, in Australia, we use SI units that have been standardised by the International Standards Organisation (ISO) since the 1960s. Some countries such as France, Italy and Spain use an earlier form of the metric system that is slightly different. The USA still measures almost everything in the old imperial units such as pounds for mass and feet for distance but, even there, scientists use the SI system of units. There are two major advantages of using the metric system. It is easier to use than other systems in that derived units are straightforward and various sizes of units are created using multiples of ten. The other very big advantage is the international nature of the standards and units. All units are standardised, making comparisons straightforward.

table 1.1 The seven fundamental quantities of the SI system

Unit	Symbol	Physical quantity
metre	m	distance
second	s	time
kilogram	kg	mass
ampere	A	electrical current
kelvin	K	thermodynamic temperature
mole	mol	amount of substance
candela	cd	luminous intensity

The routine use of standard units in physics makes calculations and conversions easier. In Years 11 and 12, standard units or units derived from them are used for most topics.

A derived unit is a combination of fundamental units. An example of a derived unit is the unit of velocity: *metre* per *second*. It is written m s^{-1}. Notice that there is a space between the *m* and the *s*. Another example is the cubic metre. It is written m^3.

Correct use of unit symbols

The correct use of unit symbols removes ambiguity, as symbols are recognised internationally. The symbols for units are not abbreviations and should not be followed by a full stop unless they are at the end of a sentence.

Upper-case letters are not used for the names of any physical quantities of units. For example, we write newton for the unit of force, while we write Newton if referring to someone with that name. Upper-case letters are only used for the **symbols** of the units that are named after people. For example, the unit of energy is joule and the symbol is J. The joule was named after James Joule who was famous for studies into energy conversions. The exception to this rule is 'L' for litre. We do this because a lower-case 'l' looks like the numeral '1'. The unit of distance is metre and the symbol is m. The metre is not named after somebody.

The product of a number of units is shown by separating the symbol for each unit with a dot or a space. Most teachers prefer a space but a dot is perfectly correct. The division or ratio of two or more units can be shown in fraction form, using a slash, or using negative indices. Most teachers prefer negative indices. Prefixes should not be separated by a space.

table 1.2 Some examples of the use of symbols for derived units

Preferred	Correct also		WRONG!	
m s^{-2}	m.s^{-2}	m/s^2	ms^{-2}	
kW h	kW.h		kWh	k Wh
kg m^{-3}	kg/m^3	kg.m^{-3}	kgm^{-3}	
μm			μ m	
N m	N.m		Nm	

Units named after people can take the plural form by adding an 's' when used with numbers greater than one. Never do this with the unit symbols. It is acceptable to say 'two newtons' but wrong to write 2 Ns. It is also acceptable to say 'two newton'.

Numbers and symbols should not be mixed with words for units and numbers. For example, twenty metres and 20 m are correct while 20 metres and twenty m are incorrect.

Scientific notation

To overcome confusion or ambiguity, measurements are often written in scientific notation. Quantities are written as a number between one and ten and then multiplied by an appropriate power of ten. Note that 'scientific notation', 'standard notation' and 'standard form' all have the same meaning.

Examples of some measurements written in scientific notation are:

0.054 m = 5.4×10^{-2} m

245.7 J = 2.457×10^2 J

2080 N = 2.080×10^3 N or 2.08×10^3 N

You should be routinely using scientific notation to express numbers. This also involves learning to use your calculator intelligently. Scientific and graphics calculators can be put into a mode whereby all numbers are displayed in scientific notation. It is useful when doing calculations to use this mode rather than frequently attempting to convert to scientific notation by counting digits on the calculator display. It is quite acceptable to write all numbers in scientific notation although most people prefer not to use scientific notation when writing numbers between 0.1 and 1000.

An important reason for using scientific notation is that it removes ambiguity about the precision of some measurements. For example, a measurement recorded as 240 m could be a measurement to the nearest metre; that is, somewhere between 239.5 m and 240.5 m. It could also be a measurement to the nearest ten metres, that is, somewhere between 235 m and 245 m. Writing the measurement as 240 m does not indicate either case. If the measurement was taken to the nearest metre, then it would be written in scientific notation as 2.40×10^2 m. If it was taken to the nearest ten metres only, then it would be written as 2.4×10^2 m.

Figure 1.1

A scientific calculator.

Prefixes and conversion factors

Conversion factors should be used carefully. You should be familiar with the prefixes and conversion factors in Table 1.3. The most common mistake made with conversion factors is multiplying rather than dividing. Some simple strategies can save you this problem. Note that the table gives all conversions as a multiplying factor.

table 1.3 Prefixes and conversion factors

Multiplying factor		Prefix	Symbol
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	µ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p

Do not put spaces between prefixes and unit symbols. It is important to give the symbol the correct case (upper or lower case). There is a big difference between 1 mm and 1 Mm.

There is no space between prefixes and unit symbols. For example, one-thousandth of an ampere is given the symbol mA. Writing it as m A is incorrect. The space would mean that the symbol is for a derived unit — a metre ampere.

✓ Worked Example 1.1A

The diameter of a cylindrical piece of copper rod was measured at 24.8 mm with a vernier caliper. Its length was measured at 35 cm with a tape measure.

a Find the area of cross-section in m^2.
b Find the volume of the copper in m^3.

Solution

a The area of cross-section is πr^2. The radius is calculated by dividing the diameter by two. Hence the radius is 12.4 mm. To calculate the area in m^2, first halve the diameter and convert it to metres. The radius is $24.8/2 = 12.4$ mm $= 12.4 \times 10^{-3}$ m. The radius is not written in scientific notation. This is not necessary. All you need to do is multiply by the appropriate factor. The conversion factor for mm to m is 10^{-3}. Just multiply by the conversion factor and don't bother to rewrite the result in scientific notation. This is because it is only going to be used in a calculation and is not a final result.
The area of cross-section is $\pi r^2 = \pi(12.4 \times 10^{-3})^2 = 4.8 \times 10^{-4}$ m^2.

b The volume is $\pi r^2 h$, where h is the length of the cylinder. The length is 35 cm $= 35 \times 10^{-2}$ m.
Hence the volume is $\pi(12.4 \times 10^{-3})^2(35 \times 10^{-2}) = 1.69 \times 10^{-4}$ m^2.

Worked Example 1.1B

a A car is traveling at 110 km h^{-1}. How fast is this in m s^{-1}?

b Convert 35 miles per hour to metres per second. A mile is approximately 1600 m.

Solution

a 110 km h^{-1} is 110×10^3 metres per 3600 s.

$$\frac{110 \times 10^3}{3600} = 30.6,$$

Hence 110 km h^{-1} = 30.6 m s^{-1}.

b 35 miles per hour is 35×1600 metres per 3600 s.

$$\frac{35 \times 1600}{3600} = 15.6,$$

Hence 35 mph = 15.6 m s^{-1}.

1.1 SUMMARY Measurement and units

- The SI system of units has seven fundamental units. They are metre (m), second (s), kilogram (kg), ampere (A), kelvin (K), mole (mol) and candela (cd).
- A derived unit is a combination of fundamental units.
- Scientific notation can be used for measurements. Numbers are written in scientific notation as the product of a number between 1 and 10 and a power of ten.
- Symbols for derived units are written with single spaces between each of the individual unit symbols.
- There must be no space between a prefix multiplying factor and a unit symbol.

1.1 Questions

1 Classify the following quantities as either fundamental or derived:
  a area
  b electric charge
  c temperature
  d electric current
  e force
  f mass

The following information relates to questions 2–7.

The distance between Darwin and Alice Springs is about 1.4×10^6 m.

The mass of the Earth is about 5.98×10^{24} kg.

2 Convert the distance from Darwin to Alice Springs to km.

3 Convert the distance from Darwin to Alice Springs to cm.

4 Convert the distance from Darwin to Alice Springs to Mm.

5 Convert the mass of the Earth to g.

6 Convert the mass of the Earth to Gg.

7 Convert the mass of the Earth to μg.

The following information relates to questions 8–10.
The dimensions of a block of wood were measured to be 10 cm × 5 cm × 13 cm.

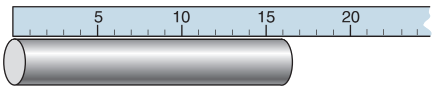
8 Calculate the volume of the block in mm^3.

9 Calculate the volume of the block in m^3.

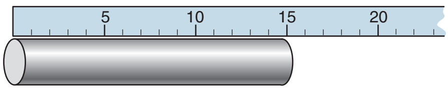
10 Calculate the surface area of the block in m^2.

11 Convert the speed 90 km h^{-1} to m s^{-1}.

12 Convert the speed 178 cm s^{-1} to km h^{-1}.



good tape measure



stretched tape measure

Figure 1.2
The diagram shows that a correctly manufactured tape measure correctly measures the cylinder to be 16 cm long while the stretched tape measure gives a wrong measurement of 15 cm. The stretched tape measure is inaccurate.

1.2 Data

Physicists and physics students collect, analyse and interpret experimental data. This requires a good understanding of the meaning and limitations of measurement.

You can expect to find questions in your examinations specifically designed to test your understanding of data and skill in analysing experimental data. You may well be studying this chapter before the end of the course and data analysis skills should be practical and refined by carefully analysing and reporting on your practical work. Don't forget that this material is about the analysis of data from real investigations and experiments rather than a purely mathematical exercise.

Accuracy and precision

Two very important aspects of any measurement are accuracy and precision. Accuracy and precision are not the same thing. The distinction between the two ideas is only hard to grasp because the two words are defined in a similar way in the dictionary. We often hear the words used together and in general conversation they tend to be used interchangeably.

Instruments are said to be *accurate* if they truly reflect the quantity being measured. For example, if a tape measure is correctly manufactured it can be used to measure lengths accurately to the nearest centimetre.

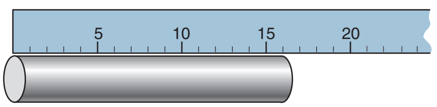
Imagine that the tape measure is accidentally stretched during the manufacturing process. It would still be used to measure length to the nearest centimetre but all measurements would be wrong. It would be inaccurate.

Suppose an accurate ruler had 3 cm snapped off the end. It would now give readings all too large by 3 cm if no allowance were made for the missing piece. This ruler measure would be inaccurate.

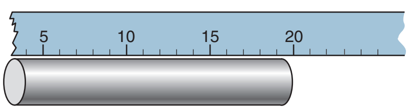
In these two examples, the tape measure or ruler is used to measure to the nearest centimetre but is inaccurate. Inaccurate means just plain wrong. Instruments are said to be *precise* if they can differentiate between slightly different quantities. Precision refers to the fineness of the scale being used.

Consider the metre rule, the tape measure and the measuring wheel used to mark out sports fields. All three measure distance. All three can be accurate. The metre rule is more *precise* because it measures to the nearest millimetre, the tape measure has less precision due to measuring only to the nearest centimetre, while the wheel measures only to the nearest metre.

The tape measure is a more precise instrument than the measuring wheel. Suppose two distances of 2673 and 2691 mm are being measured with these two instruments. Each distance would be measured as 3 metres, to the nearest metre, by the wheel. They would be measured differently as 2.67 and 2.69 metres, to the nearest centimetre, by the tape measure. The tape measure is more precise because it has a finer scale. We might also say that it has greater resolution. The measuring wheel has such low precision that it can't be used to measure which of the two distances is greater or smaller. Measuring instruments with less precision give measurements that are less certain. The uncertainty in the measurement is due to a coarser scale. The measuring wheel gives less certain measurements than the tape measure even though both instruments may be equally accurate.



good ruler



broken ruler

Figure 1.3
The diagram shows that an undamaged ruler correctly measures the cylinder to be 16 cm long while the broken ruler gives a wrong measurement of 19 cm. The broken ruler is inaccurate but equally as precise as the unbroken ruler.

All measurements have some amount of *uncertainty*. The uncertainty is generally one half of the finest scale division on the measuring instrument. The measuring wheel has an uncertainty of 0.5 m. The metre rule has an uncertainty of 0.5 mm. The tape measure has an uncertainty of 0.5 cm. The electronic balance set to measure grams to two decimal places has an uncertainty of 0.005 g.

Sometimes this uncertainty is referred to as error. It is not error, in that it is not a mistake or something wrong. All measuring instruments have limited precision and, in general, the uncertainty is half of the smallest scale division on the instrument.

The uncertainty is, indeed, the measure of the precision of an instrument. It is not related to accuracy. A micrometer screw gauge, which measures length to the nearest one-hundredth of a millimetre and hence is very precise, may not be accurate. Usually they are, but if one has been badly manufactured or bent by being over-tightened repeatedly, it most likely will be inaccurate. But its precision will still be ±0.000 005 m, or half of one-hundredth of a millimetre.

The uncertainty gives us the range in which a measurement falls. If we measured the length of a stick with a metre rule then we would get a measurement 'plus or minus' half a millimetre.

Any stick between 127.5 and 128.5 mm long would be measured as 128 mm to the nearest millimetre. We would record this as 128 ± 0.5 mm.

When using an analogue scale, you might think that you can 'judge by eye' fractions of a scale division and hence get greater precision than half a scale division. You should be able to judge to the nearest half a scale division. You might think you can judge to the nearest tenth of a division. You can't. Research shows that despite the fact that people try to judge the spaces between scale divisions to better than half a division, as soon as this is done, inconsistent measurements are obtained. That is, different people get different measurements of the same thing.

The best judgement you can definitely claim is one half of a scale division. The uncertainty we will still assume, however, is a full half scale division. Hence, you might measure another stick, one that has a length somewhere between 154 and 155 mm, as 154.5 ± 0.5 mm.

Of course, you don't have the option of adding an extra decimal place containing a 0 or a 5 if you are using a digital instrument.

The uncertainty can be recorded as the *absolute* uncertainty as we have done above. The absolute uncertainty is the actual uncertainty in the measurement. In this case it is 0.5 mm. Alternatively, it is often useful to write the uncertainty as a *percentage*. 0.5 mm is 0.32% of 154.5. Hence, the above length would be recorded as 154.5 mm ± 0.32%.

Percentage uncertainty is also called *relative* uncertainty. It is the size of the uncertainty relative to the size of the measured quantity.

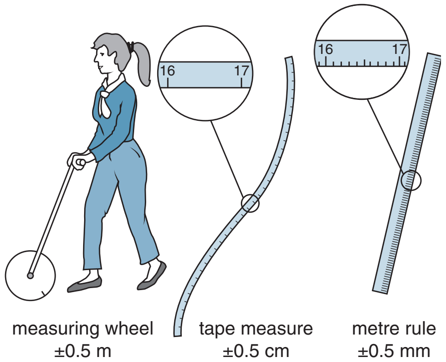


Figure 1.4
The measuring wheel has low precision and only measures to the nearest metre. It has an uncertainty of 0.5 m. The tape measure has more precision and has an uncertainty of 0.5 cm or 0.005 m. The metre rule has an uncertainty of 0.5 mm or 0.0005 m.

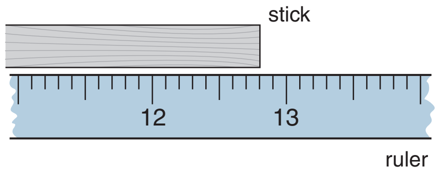


Figure 1.5
A stick anywhere between 127.5 mm and 128.5 mm would be recorded as having a length of 128 mm if measured by a metre rule with a scale division of 1 mm. Conversely, a measurement recorded as 128 mm could be of an object of length anywhere between 127.5 mm and 128.5 mm.

Estimating the uncertainty in a result

An experiment or a measurement exercise is not complete until the uncertainties have been analysed. The report should include an estimate of the total uncertainty. This gives the reader of the report some idea of your confidence in the result.

The following three processes are used for estimating uncertainty. They are demonstrated in Worked Example 1.2A.

- When adding or subtracting data, add the absolute uncertainties.
- When multiplying or dividing data, add the percentage uncertainties.
- When raising data to power *n*, multiply the percentage uncertainty by *n*.

Physics file

Many people use the term 'error' to refer to uncertainty and many other things. The problem with referring to uncertainty as error is that it is not actually error. Things that are a normal consequence of the limitations of measuring instruments must happen, and are not mistakes. If they are not mistakes or 'something gone wrong' then it makes no sense to call them errors.

Errors are the factors that limit the accuracy of your results. For example, if you perform a calorimetry experiment and do not use a good enough insulator, you will get inaccurate results due to heat losses to the environment. This will contribute to the error in your measurement. Suppose you measured the refraction of light in glass but did not place the protractor in the correct place when measuring angles. This would also cause error.

Many different things can contribute to experimental error. Some are unavoidable. Some are factors in the design of experiments. Good experimental design seeks to eliminate or at least minimise potential sources of error.

Never quote 'human error' as a source of error. Your data should be examined carefully and mistakes eliminated or at least ignored. So-called human errors, or lack of care, have no place in your experimental work. If you make mistakes then you should repeat the measurements.

In Worked Example 1.2A, the analysis of uncertainty reveals the *precision* of an experimental result.

✓ Worked Example 1.2A

Last year, you might have measured the specific heat of a metal. You could have calculated your result using:

$$c_{metal} = \frac{c_{water} m_{water} \Delta T_{water}}{m_{metal} \Delta T_{metal}}$$

Suppose you had the following data included in your tables.

Quantity		Absolute uncertainty	% uncertainty
c_{water}	4180 J kg^{-1} K^{-1}	5 J kg^{-1} K^{-1}	0.120
m_{water}	72.5×10^{-3} kg	0.05×10^{-3} kg	0.069
ΔT_{water}	5°C	1°C*	20
m_{metal}	87.3×10^{-3} kg	0.05×10^{-3} kg	0.057
ΔT_{metal}	72°C	1°C*	1.389

*Note that the ΔT values have an absolute uncertainty of 1°C because they are calculated by subtracting one temperature measurement from another.

You would calculate as follows:

$$c_{metal} = 241 \text{ J kg}^{-1}\text{ K}^{-1}$$
$$\text{Uncertainty (\%)} = 0.120 + 0.069 + 20 + 0.057 + 1.389$$
$$= 21.6\%$$

Hence, you would obtain the following result:

$$c_{metal} = 241 \text{ J kg}^{-1}\text{ K}^{-1} \pm 21.6\%$$
$$c_{metal} = 241 \pm 52 \text{ J kg}^{-1}\text{ K}^{-1}$$

Once you have done all of this you can consider the relative success of your measurement exercise.
Your result is:

$$189 \text{ J kg}^{-1}\text{ K}^{-1} \leqslant c_{metal} \leqslant 293 \text{ J kg}^{-1}\text{ K}^{-1}$$

If measurements by other people, such as the constants published in data books, fall within this range then you can conclude that your experiment is consistent with established values. That is, within the precision of your technique, there are probably no significant errors although the final measurement is rather imprecise in this case. We might say that it is accurate within the limitations of the equipment.
You are also now in a position to refine the experiment by reducing the larger uncertainties. In this case, the largest uncertainty was in the temperature change for the water. Hence, it would not be very helpful to measure the masses to greater precision because the limit to precision in this activity would be the temperature differences. Getting greater precision in the temperature changes would be a useful refinement. You could consider ways of getting larger temperature changes in the water and hence obtain a smaller percentage uncertainty in the temperature change. Alternatively, you might consider ways of measuring the temperatures to greater precision.
If your measurement range does not include the result you expect, you should think about the origin of the errors. In other words, if you are sure that c_{metal} is less than 189 J kg^{-1} K^{-1} or more than 293 J kg^{-1} K^{-1} then there must be some error in your experimental technique or more uncertainty than you realised.
When reviewing an experiment or a measurement exercise, it is a good idea to consider both errors *and* uncertainties.

Significant figures

The number of significant figures in a measurement is simply the number of digits used when the number is written in scientific notation. Once you have done a calculation, your calculator usually has eight or ten digits in the display but most of them are meaningless. You must round off your answer appropriately.

Consider the result of the experiment described in Worked Example 1.2A. It would make no sense to quote the result to two decimal places (or five significant figures) when clearly the precision of the experiment gives less than three significant figures.

Calculated results never have more significant figures than the original data and might have fewer than the original data. If you are not doing a full analysis of the uncertainties, it is customary to give your answers to the same number of significant figures as the least precise piece of data. For example, in Worked Example 1.2A, the least precise data is the change in temperature of the water with only a single digit. The value for the specific heat might then be quoted simply as 2×10^2 J kg^{-1} K^{-1}, but doing the full calculation of the uncertainty in the result is much more informative.

Physics file

In some classes, students are instructed to quote all results to two decimal places or to three significant figures. You should be able to see from Worked Example 1.2A that these rules are not absolutely correct when applied to real data. For ordinary calculations in assignments, tests and examinations, you might just give your answers to three figures. If a calculation is done in several stages then you should not round off any intermediate results. This will add rounding error to your calculations. Use the memories on your calculator so that there is no rounding until the end of your calculation.

1.2 SUMMARY Data

- Accuracy of a measurement relates to how close to reality the measurement is.
- Precision of a measurement relates to how fine the scale of the measuring instrument is.
- The precision of an instrument is assumed to be plus or minus half of the finest scale division.
- Absolute uncertainty in a measurement is half the scale division, whereas relative uncertainty is the % of the measurement represented by the absolute uncertainty.
- When adding or subtracting data, add the absolute uncertainties. When multiplying or dividing data, add the percentage uncertainties. When raising data to power n, multiply the percentage uncertainty by n.
- Final calculated results should be rounded off to a sensible number of figures based on the estimate of the uncertainty in the result.

1.2 Questions

The following information relates to questions 1–5. You measure a TV screen with a tape measure to find the area of the screen. You obtain the following data: height = 21 cm and width = 28 cm.

1. What is the absolute uncertainty in each of these measurements?
2. What is the relative uncertainty in each of these measurements?
3. What is the area of the screen in cm^2?
4. What is the percentage uncertainty in the area of the screen?
5. What is the absolute uncertainty in the area of the screen?

The following information relates to questions 6–9. The diameter of a cylindrical piece of copper rod was measured at 24.8 ± 0.05 mm with a vernier caliper. Its length was measured at 135 ± 0.5 mm with a metre rule.

6. Calculate the percentage uncertainty in the diameter.
7. Calculate the percentage uncertainty in the length.
8. Calculate the percentage uncertainty in the volume of the cylinder.
9. Calculate the absolute uncertainty in the volume of the cylinder in mm^3.
10. Select any measurement practical exercise that you have done this year and calculate the relative and absolute uncertainty in your result.

Physics file

When plotting graphs, put the correct variable on the horizontal axis. The *independent* variable goes on the horizontal axis. The *independent* variable is the quantity being changed while performing the experiment. The dependent variable is the one that may change as a consequence of the changes to the independent variable. For example, if you are manipulating the potential difference across an electrical circuit by changing the 'voltage' setting on the power pack, then you plot the measurements of potential difference on the horizontal axis.

When analysing data from a linear relationship, it is first necessary to obtain a graph of the data and an equation for the line that best fits the data. This line is often called the regression line. The entire process can be done on paper but most people will use a computer spreadsheet, the capabilities of a scientific or a graphics calculator, or some other computer-based process. In what follows, it is not assumed that you are using any particular technology.

Starting with a graph of your data has a number of uses including identifying any data that you may wish to ignore or else repeat the measurements. Often, some data is obviously suspect and it is common practice to acknowledge that the data was collected but eliminate it from the mathematical analysis. A graph of the data will also indicate when more data is needed. If there are gaps in the range of data then more data can be collected. The graph can also indicate that the relationship being studied is not linear and may need more complex analysis. Graphs should be created as the experiment proceeds.

If you are plotting your graph manually on paper then proceed as follows:

1 Plot each data point on clearly labelled, unbroken axes.
2 Identify and label but otherwise ignore any suspect data points.
3 Draw, by eye, the 'line of best fit' for the points. The points should be evenly scattered either side of the line.
4 Locate the vertical axis intercept and record its value as 'c'.
5 Choose two points on the line of best fit to calculate the gradient. Do not use two of the original data points as this will not give you the gradient of the line of best fit.
6 Write $y = mx + c$, replacing x and y with appropriate symbols, and use this equation for any further analysis.

1.3 Graphical analysis of data

A major problem with doing a calculation from just one set of measurements is that a single incorrect measurement can significantly affect the result. Scientists like to take a large amount of data and observe the trends in that data. This gives more precise measurements and allows scientists to recognise and eliminate problematic data.

Physicists commonly use graphical techniques to analyse a set of data. In this chapter section, the basic techniques that they use will be outlined and a general method for using a set of data that fits a known mathematical relationship will be developed.

Linear relationships

Some relationships studied in physics are linear, while others are not. It is possible to manipulate non-linear data so that a linear graph reveals a measurement. Linear relationships and their graphs are fully specified with just two numbers: gradient, m, and vertical axis intercept, c. In general, linear relationships are written:

$$y = mx + c$$

The gradient, m, can be calculated from the coordinates of two points on the line:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1, y_1) and (x_2, y_2) are any two points on the line. Don't forget that m and c have units. Omitting these is a common error.

Worked Example 1.3A

Some students used a computer with an ultrasonic detector to obtain the velocity–time data for a falling tennis ball. They wished to measure the acceleration of the ball as it fell. They assumed that the acceleration was nearly constant and that the relevant relationship was $v = u + at$, where v is the speed of the ball at any given time, u was the speed when the measurements began, a is the acceleration of the ball and t is the time since the measurement began.

Their computer returned the following data:

Time (s)	Speed (m s^{-1})
0.0	1.25
0.1	2.30
0.2	3.15
0.3	4.10
0.4	5.25
0.5	6.10
0.6	6.95

Find their experimental value for acceleration.

Solution

The data is assumed linear with the relationship $v = u + at$, which can be thought of as being $v = at + u$, which makes it clear that putting v on the vertical axis and t on the horizontal axis gives a linear graph with gradient a and vertical intercept u. A graph of the data is shown in Figure 1.6.

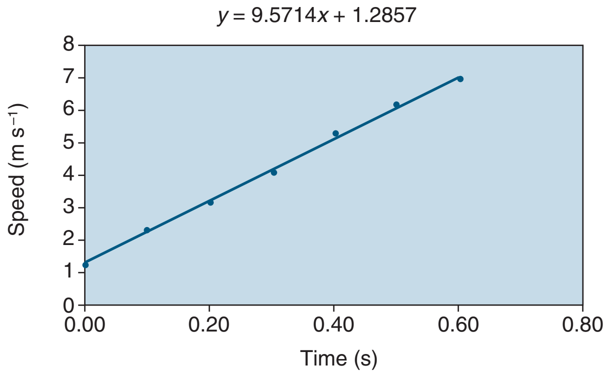


Figure 1.6
Velocity–time profile for a falling tennis ball.

This graph of the data was created on a computer spreadsheet. The line of best fit was created mathematically and plotted. The computer calculated the equation of the line. Graphics calculators can also do this. Scientific calculators can find the equation of the line but not draw the graph.

A scientific calculator or graphics calculator or spreadsheet gives the regression line as $y = 9.57x + 1.2857$. If this is rearranged and the constants are suitably rounded, the equation is $v = 1.3 + 9.6t$. This indicates that the ball was moving at 1.3 m s^{-1} at the commencement of data collection and the ball was accelerating at 9.6 m s^{-2}.

Manipulating non-linear data

Suppose we were examining the relationship between two quantities B and d and had good reason to believe that the relationship between them is

$$B = \frac{k}{d}$$

where k is some constant value. Clearly, this relationship is non-linear and a graph of B against d will not be a line. By thinking about the relationship it can be seen that in 'linear form':

$$\begin{array}{ccc} B & = & k\,\dfrac{1}{d} \\ \uparrow & & \uparrow\ \uparrow \\ y & = & m\ x + c \end{array}$$

A graph of B (on the vertical axis) against $\frac{1}{d}$ (on the horizontal axis) will be linear. The gradient of the line will be k and the vertical intercept, c, will be zero. The line of best fit would be expected to go through the origin because, in this case, there is no constant added and c is zero.

In the above example, a graph of the raw data would just show that B is larger as d is smaller. It would be impossible to determine the mathematical relationship just by looking at a graph of the raw data.

Physics file (cont)

If you are using a computer or a graphics calculator then proceed as follows:

1 Plot each data point on clearly labelled, unbroken axes.
2 Identify suspect data points and create another data table without the suspect data.
3 Plot a new graph without the suspect data. Keep both graphs as you don't actually discard the suspect data but do eliminate it from the analysis.
4 Plot the line of best fit—the regression line. The manner in which you do this depends on the model of calculator or the software being used.
5 Compute the equation of the line of best fit that will give you values for m and c.
6 Write $y = mx + c$, replacing x and y with appropriate symbols, and use this equation for any further analysis.

A graph of raw data will not give the mathematical relationship between the variables but can give some clues. The shape of the graph of raw data may suggest a possible relationship. Several relationships may be tried and then the best is chosen. Once this is done, it is not proof of the relationship but, possibly, strong evidence.

When an experiment involves a non-linear relationship, the following procedure is followed:

A Plot a graph of the original raw data.

B Choose a possible relationship based on the shape of the initial graph and your knowledge of various mathematical and graphical forms.

C Work out how the data must be manipulated to give a linear graph.

D Create a new data table.

Then follow steps 1–6 given in the Physics file on pages 46–47. It may be necessary to try several mathematical forms to find one that seems to fit the data.

Worked Example 1.3B

Some students were investigating the relationship between current and resistance for a new solid-state electronic device. They obtained the data shown in the table.

Current, i (A)	Resistance, R (Ω)
1.5	22
1.7	39
2.2	46
2.6	70
3.1	110
3.4	145
3.9	212
4.2	236

According to the theory they had researched on relevant Internet sites, the students believed that the relationship between i and R is $R = di^3 + g$, where d and g are constants.

By appropriate manipulation and graphical techniques, find their experimental values for d and g. The following steps would be used:

a Plot a graph of the raw data.

b Work out what you would have to graph to get a straight line.

c Make a new table of the manipulated data.

d Plot the graph of manipulated data.

e What is the equation relating i and R?

Solution

a Figure 1.7 shows the graph obtained using a spreadsheet.
It might be argued that the second piece of data is suspect. The rest of this solution supposes the students chose to ignore this data.

b We can see what to graph if we think of the equation like this:

$$\begin{array}{ccccccc} R & = & d & i^3 & + & g \\ \uparrow & & \uparrow & \uparrow & & \uparrow \\ y & = & m & x & + & c \end{array}$$

A graph of R on the vertical axis and i^3 on the horizontal axis would have a gradient equal to d and a vertical axis intercept equal to g.

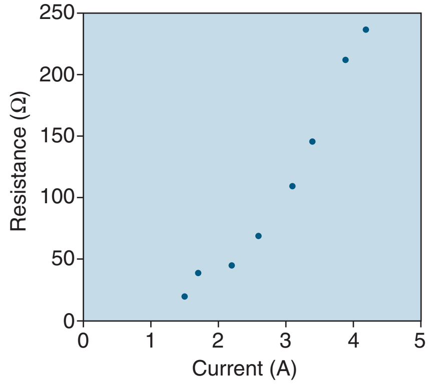


Figure 1.7
Current, resistance characteristic.

c The data is manipulated by finding the cube of each of the values for current.

Current cubed, i^3 (A^3)	Resistance, R (Ω)
3.38	22
10.65	46
17.58	70
29.79	110
39.30	145
59.32	212
74.09	236

d The graph in Figure 1.8 was obtained from the spreadsheet.

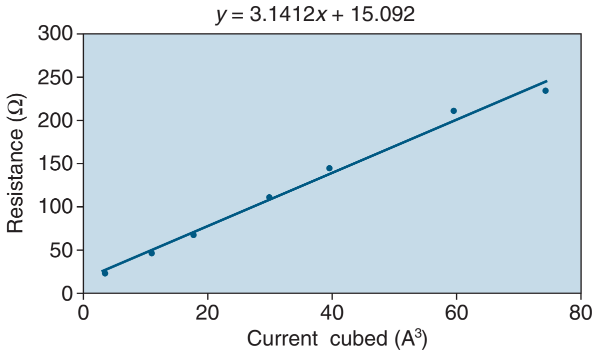


Figure 1.8
Current, resistance characteristic (manipulated data).

e The regression line has the equation $y = 3.1x + 15.1$, so the equation relating i and R is $R = 3.1i^3 + 15.1$. Hence, the value of d is 3.1 Ω A^{-3} and the value of g is 15.1 Ω.

1.3 SUMMARY Graphical analysis of data

- Graphical analysis of data decreases the effect of the inaccuracy or imprecision of individual data points.
- Graphs of data assist in identifying suspect data, which may be eliminated from further analysis.
- If data is linear or can be manipulated to behave in a linear manner, then constants associated with the relationships between variables can be obtained from the equation of the line of best fit through the data.
- The fact that, once manipulated, the data appears to fit a particular mathematical relationship does not prove the assumed relationship to be true, but is supporting evidence.

1.3 Questions

The following information relates to questions 1–5. Some chemistry students were investigating the behaviour of a sealed sample of gas and obtained the following data.

Pressure, P (kPa)	Temperature, T (°C)
128	5
136	19
140	36
145	49
163	52
165	82
171	98

They believed that the relevant relationship is $P = kT + P_o$ where P_o is the pressure at 0°C and k is the gradient of the graph for this particular sample of gas. They wanted to find the values of k and P_o for their sample of gas. They also wanted to find an experimental value for the absolute zero of temperature that would be the temperature at which the value of P is zero.

1 Plot a graph of the above data.

2 How should the graph be interpreted to find the values of k and P_o?

3 Write the equation obtained for the relationship between P and T.

4 What are the values of k and P_o? Make sure that you include their units.

5 What is their experimental value for the absolute zero of temperature?

The following information relates to questions 6–13. Some students were investigating a new type of light-dependent resistor (LDR). They measured the resistance in ohm when light of varying intensity was allowed to fall on the resistor. Consider the following data obtained from such an experiment.

Light intensity, I (lux)	Resistance, R (kΩ)
0.5	9.4
1.2	12.8
4.5	20.5
9.0	27.6
10.7	29.9
25.9	43.8
44.6	57.8

Their teacher told them that the relationship between the two measured variables is $R = a\sqrt{I} + b$. Their task was to decide whether or not this relationship seemed to be supported by their data and to find the values of a and b.

6 Plot the graph of this raw data.

7 What should they graph on each of the axes to get a linear relationship?

8 Construct a new table for manipulated data.

9 Plot a graph of the manipulated data.

10 What is the gradient of your graph? (Include units.)

11 What is the vertical intercept of your graph?

12 Write the equation expressing the relationship between resistance and incident light intensity for the LDR.

13 What are the experimental values for a and b?

Chapter 1 Review

The following information relates to questions 1–6.
The dimensions of a classroom were measured to be 9.5 m wide, 11.4 m long and 3.2 m ceiling height. Each measurement was made to the nearest dm, so each measurement is ±0.05 m.

1 Calculate the floor area of the classroom in m^2.

2 a Calculate the percentage uncertainty in the area of the classroom.
  b Calculate the absolute uncertainty in the area of the classroom.

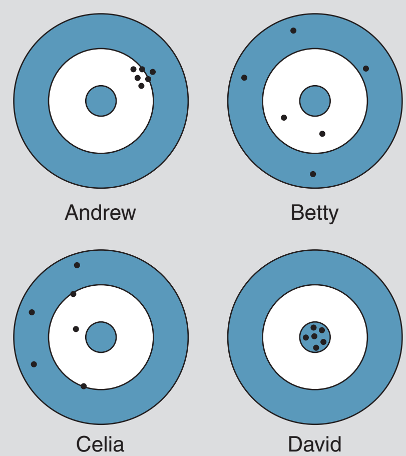
3 Calculate the volume of the classroom in m^3.

4 a Calculate the percentage uncertainty in the volume of the classroom.
  b Calculate the absolute uncertainty in the volume of the classroom.
  c State the range of possible values for the volume of the classroom.

5 Calculate the volume of the classroom in litres. One litre is 1000 cm^3.

6 Calculate the volume of the classroom in cubic feet. One foot is equal to 30.48 cm or 0.3048 m.

The following information relates to questions 7–10.
Four archers each shoot six arrows into targets.
Archer Andrew shoots all six arrows so that they penetrate the target within a few centimetres of each other but none of them penetrates the bull's-eye.
Archer Betty shoots all six arrows such that they are evenly distributed all over the target.
Archer Celia shoots the arrows so that they are widely spaced but all to the left of the bull's-eye.
None of them penetrated the bull's-eye.
Archer David shoots the arrows into the bull's-eye.



The diagrams show the distribution of the arrows on the target for each of the four archers.

7 Which archer is accurate but not precise?
Andrew
Betty
Celia
David

8 Which archer is precise but not accurate?
Andrew
Betty
Celia
David

9 Which archer is both precise and accurate?
Andrew
Betty
Celia
David

10 Which archer is neither accurate nor precise?
Andrew
Betty
Celia
David

The following information relates to questions 11–15.
Prior to the use of hand-held radar guns, the police sometimes checked the speed of motorists from an aircraft. They would paint two marks on the road a known distance apart. They would use a stopwatch to time the car travelling between the two marks on the road. A simple calculation of distance divided by time would give the speed of the motorist and radio contact with police officers on the ground would allow them to accost motorists as appropriate.

In one such case, the lines were painted across the road a distance of 150 m apart with an uncertainty of 5 m. A motorist was timed to take 4.6 s to travel this distance. Reaction time for use of a stopwatch is about 0.1 s, giving an uncertainty in the measurement of 0.2 s, as the button has to be pushed twice.

11 What speed do these measurements indicate for the car? Answer in m s^{-1} and km h^{-1}.

12 The speed limit on the highway was 110 km h^{-1}. Should the motorist have been fined for speeding?

13 Calculate the percentage uncertainty in the measured speed of the car.

14 Calculate the absolute uncertainty in the measured speed of the car and state the range of speeds at which the car may have been travelling according to the measurements.

15 Was the motorist speeding?

The following information relates to questions 16–21.
Some students were measuring the refractive index of a block of transparent plastic. They did this by shining a beam of light through the glass and measuring the incident and refracted angles at the surface of the glass. It is well established that the relationship between these two values is $\sin(i) = n\sin(r)$ where i is the incident angle, r is the refracted angle, and n is the refractive index. They obtained the following data.

i (degrees)	*r* (degrees)
12	10
25	19
33	26
52	37
70	48

16 Plot a graph of the raw data.

17 What would you have to graph to get a linear plot?

18 Make a new table of the manipulated data.

19 Plot the graph of manipulated data.

20 What is the equation relating i and r?

21 What is the experimental value of n?

The following information relates to questions 22–29.
An experiment was performed to find the acceleration of a car in third gear. The car cannot start off in third gear. Once in third gear and maintaining a particular speed, the driver accelerated when he passed a predetermined position. A series of radar guns were placed at 20 m intervals, the first being 40 m from the place where the car began to accelerate. The radar guns measure the speed of the car.

The experimenters, consequently, collected a set of data for the speed of the car at various distances from the place where the driver began to accelerate.

Their data was as follows:

Distance, *s* (m)	Speed, *v* (m s^{-1})
40	20.0
60	22.5
80	25.2
100	25.8
120	29.5

The established relationship between v and s is $v^2 = u^2 + 2as$ where u is the initial speed, v is the speeds at the various positions, a is the acceleration of the car, s is the various positions at which the speeds are measured.

22 Work out how you are going to manipulate the data so that you will get a straight-line graph.

a What will you graph on the vertical axis?

b What will you graph on the horizontal axis?

23 a What will the slope of the line represent?
b What will the vertical intercept represent?

24 Manipulate the relevant data and create a new data table.

25 Plot the graph and create the 'line of best fit'.

26 What is the equation of the line of best fit?

27 a What is the slope of the line of best fit? Write the number and the correct unit.
b What is the vertical intercept of the line of best fit? Write the number and the correct unit.

28 What is their experimental value of the acceleration of the car during the experiment? Write the number and the correct unit.

29 What was the initial speed of the car? Write the number and the correct unit.

30 This question is fairly challenging and is a purely fictional situation. If you can do this without assistance, then you should confidently approach any of this material during assessment. In an experiment to discover the properties of the newly invented Heinemann Electrostatic Flux Configurator, students subjected it to a variable potential difference, *P*, and measured the Field Inversion Factor, *F*.
Their data was as follows:

Field inversion *F* (N C^{-1})	Potential difference *P* (V)
1.52	3.58
2.45	2.84
4.07	2.40
4.53	1.94
5.73	1.64
7.05	1.40

The established relationship between F and P is:

$$F = \frac{\beta}{cP} - \frac{\phi}{\pi}$$

where β is the configurator constant, c is 3.00×10^8 m s^{-1} (the speed of light), and ϕ is the flux constant.
Use this data to find the experimental values of the flux constant and the configurator constant.

2 Describing motion

People have always been fascinated by speed and, throughout history, have sought to devise means of travelling faster. Horses have been used for many centuries both for individual riders and to pull carriages along. In western societies horses are now mostly used by recreational riders. In horse racing, the form of thoroughbreds is closely followed by punters who are keen to back a winner. In these races, the horses and their jockeys can travel at speeds of up to 70 km h^{-1}.

Greater speed has also been a major goal of athletes through the ages. In recent times, improved diet and a more scientific approach to training has meant that athletes have run faster than ever before. The winner of the women's 100 m sprint at the Melbourne Olympic Games in 1956 was the Australian Betty Cuthbert. Her time was 11.5 s. However, this would have left her around 10 m behind the 1988 world record holder, Florence Joyner-Griffith of the USA, whose time was 10.49 s.

Cars have been one of the dominant influences in the development of cities and the lifestyles of people in the 20th century. Cars today achieve performance levels and speeds far superior to those of earlier models. The first cars were so slow that people were able to outrun them. Modern cars, however, cruise comfortably at speeds in excess of 100 km h^{-1}. Unfortunately, this has contributed to the tragic death and injury toll due to road accidents. Safety features such as seatbelts, air bags, crumple zones, anti-lock braking systems and electronic sensing devices have helped to reduce the road toll to a certain extent. However, it is still tragically high, particularly in the 18–25 age group.

By the end of this chapter

you will have covered material from the study of movement including:

- graphical description of motion
- instantaneous and average velocities
- motion with constant acceleration; described by using graphs and equations of motion
- vertical motion under gravity.

2.1 Describing motion in a straight line

Motion, from the simple to the complex, is a fundamental part of everyday life. The motion of a gymnast performing a routine and that of a mosquito trying to avoid your desperate attempts to swat it would be considered complex forms of motion. Far simpler examples are a tram travelling in a straight line along a road, and a bowling ball rolling straight towards the ten-pins. In this chapter, the simplest form of motion—*straight line motion*—will be analysed.

In this section, terms that are useful in describing the motion of an object—*position*, *distance* and *displacement*—will be discussed.

Position and distance travelled

Consider a swimmer, Sophie, doing laps in a 50 m pool. To simplify this situation, we will treat Sophie as a simple point mass. The pool can be treated as a one-dimensional number line with the starting blocks chosen to be the *origin*. The right of the starting blocks is taken to be positive.

The *position* of Sophie is her *location with respect to the origin*. For example, her position as she is warming up behind the starting block in Figure 2.1a is –10 m. The negative sign indicates the direction from the origin, i.e. to the left. At the starting block, Sophie's position is 0 m; then after half a length she is +25 m or 25 m to the right of the origin.

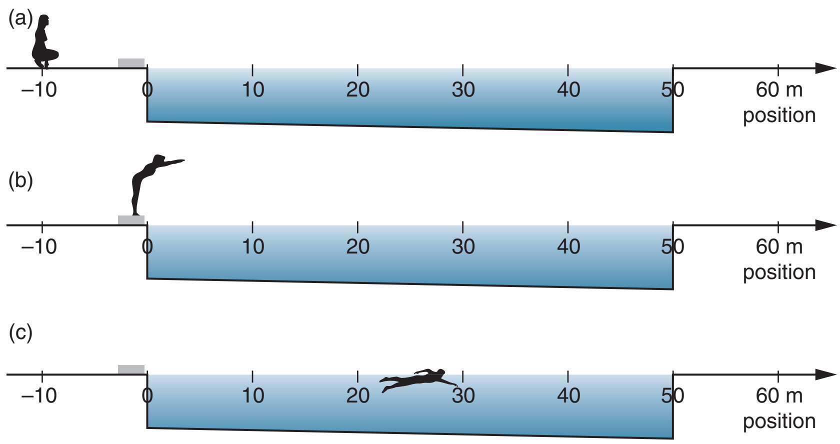


Figure 2.1
In this situation, the position of the swimmer is given with reference to the starting block. (a) While warming up, Sophie is at –10 m. (b) When she is on the starting block, her position is zero. (c) After swimming for a short time, she is at a position of +25 m.

Figure 2.2
Geoff Huegill won silver and bronze medals in swimming at the Sydney Olympic Games in 2000.

Distance travelled

Distance travelled is a measure of the *actual distance covered* during the motion. For example, if Sophie completes three lengths of the pool, the distance travelled during her swim will be 50 + 50 + 50 = 150 m.

DISTANCE TRAVELLED, *s*, is how far a body travels during a motion. Distance travelled is measured in metres (m).

The distance travelled does not distinguish between motion in a positive or negative direction. For example, if Sophie completes one length of the pool travelling from the starting block, i.e. in a positive direction, the distance travelled will be 50 m. If she swam one length from the far end back to the start, the distance travelled will also be 50 m.

Displacement

Displacement is a term related to position and distance travelled, but it has a different meaning. Displacement, s, is defined as the *change in position* of an object. Displacement takes into account only where the motion starts and finishes; whether the motion was directly between these points or took a complex route has no effect on its value. The sign of the displacement indicates the direction in which the position has changed.

> ***DISPLACEMENT*** is defined as the change in position of a body.
> Displacement s = final position − initial position
> Displacement is measured in metres (m) and is given a direction.

Consider the example of Sophie completing one length of the pool. During her swim, the distance travelled is 50 m, and the displacement is:

s = final position − initial position
= +50 − 0
= +50 m, i.e. 50 m in a positive direction

If Sophie swims two lengths, she will have travelled a distance of 100 m, i.e. 50 m out and 50 m back. However, her displacement during this swim will be:

s = final position − initial position
= 0 − 0
= 0

Even though she has swum 100 m, her displacement is zero because the initial and final positions are the same. Displacement only considers the starting and finishing positions of the motion; it does not indicate anything about the route taken by the person or object in getting from the initial to the final position.

Scalars and vectors

Many physical quantities are fully described by a magnitude (or size) only. For example, the distance travelled by the swimmer when completing three lengths of the pool was 150 m. There was no direction associated with this quantity—it would not matter if she had started from the other end of the pool. Distance travelled has been fully described by just one numerical piece of information—150 m.

A quantity that requires a *magnitude only* to describe it fully is known as a *scalar*. Other examples of scalar quantities are density, temperature and refractive index.

Other physical quantities, by definition, require two pieces of information to describe them completely—a magnitude and a direction. For example, when describing the displacement of the swimmer who had completed three lengths of the pool as +50 m, it was necessary to indicate how far she finished from her starting point (50 m) as well as the direction (positive) in which she finished relative to the starting point.

Quantities such as this are called *vectors*. A vector can be represented as a directed line segment where the length of the line represents the

Physics file

table 2.1 Recent world record distances

Activity	Record (m)
Men's pole vault	6.15
Women's javelin	72
Paper plane flight (indoors)	58.8
Longest golf drive	471
Cow pat toss	56
Ski jump	239
Spitting a watermelon pip	21

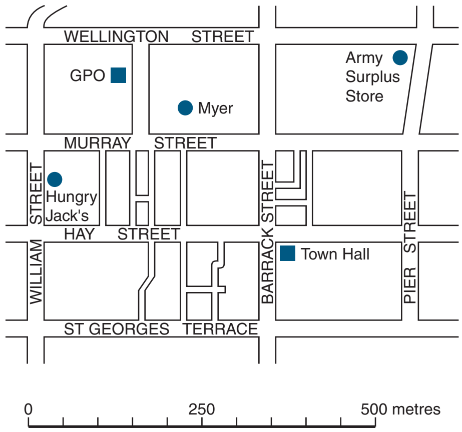


Figure 2.3
In the city one day, you walk from the Murray Street entrance of the Myer store to Hungry Jack's in William Street. After eating, you walk to the Army Surplus Store in Wellington Street near Pier Street. You will have walked a *distance* of approximately 1 km. Your *displacement* for this journey, however, is only approximately 300 m. You should confirm this with a ruler and the scale on the diagram.

Figure 2.4

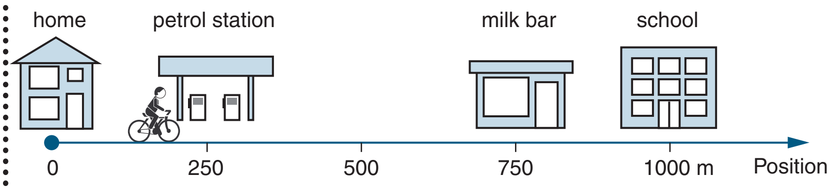
A displacement of +50 m can be represented by the displacement vector ***s***.

In situations where only the magnitude of a vector quantity needs to be considered, the quantity will be represented in *light italics*.

magnitude or size of the quantity, and the arrowhead indicates the direction. In the text, a vector is indicated by bold italic type. This is why the symbol for displacement is shown as ***s***.

✓ Worked Example 2.1A

As Benjamin rides to school, he passes a petrol station and a milk bar.



Upon reaching the milk bar, he stops for an ice-cream, then realises that he has spent his lunch money and so returns home before continuing on to school.

a What is the distance travelled in his journey?
b What is the displacement as he travels from home to school?
c Calculate Benjamin's displacement as he travels from the petrol station to the milk bar and then returns home.

Solution

a The distance that Benjamin travels must take account of his back-and-forth route.
Distance travelled $s = 750 + 750 + 1000 = 2500$ m

b Displacement $\boldsymbol{s}$ = final position – initial position = $1000 - 0 = +1000$ m

c Displacement $\boldsymbol{s} = 0 - 250 = -250$ m

Adding vector quantities

When analysing movement, it is often necessary to add vector quantities. Consider the earlier example of Sophie, the swimmer, again. We will now analyse her displacement as she travels from the warm-up position to the starting block, and then from the starting block to the far end of the pool. Her resultant, or total, displacement can be determined by *vector addition* of the two separate displacements.

The *addition of two vectors* is performed by simply *placing the tail of the second vector at the head of the first*. The sum of these vectors gives the total displacement ***s***, which runs from where the first vector starts to where the second vector ends, i.e. $\boldsymbol{s} = \boldsymbol{s}_1 + \boldsymbol{s}_2$ … Figure 2.5 shows how the resultant displacement of +60 m is obtained.

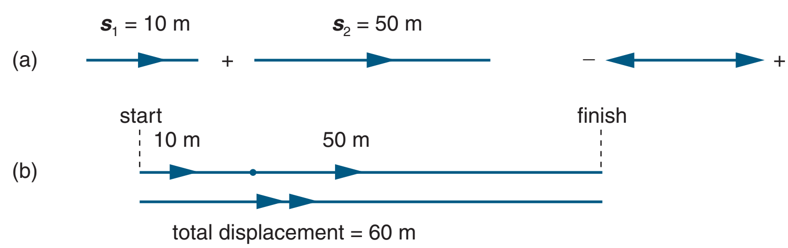


Figure 2.5

(a) Sophie walks 10 m from her warm-up position to the starting block, then swims 50 m to the other end of the pool. (b) Her total displacement ***s*** can be found by adding vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, and is a vector that runs from the start of $\boldsymbol{s}_1$ to the end of $\boldsymbol{s}_2$.

Multiplying a vector by a scalar

When a vector is multiplied by a scalar, the magnitude of the vector changes accordingly. For example, if ***s*** is a displacement of 5 m north, then $4\boldsymbol{s}$ is $\boldsymbol{s} + \boldsymbol{s} + \boldsymbol{s} + \boldsymbol{s}$ = 20 m north. If the vector is multiplied by a negative term, then the direction of the vector will reverse. So, using $\boldsymbol{s}$ as 5 m north once again, $-\boldsymbol{s}$ will be a displacement of 5 m south, and $-3\boldsymbol{s}$ will be 15 m south.

✓ Worked Example 2.1B

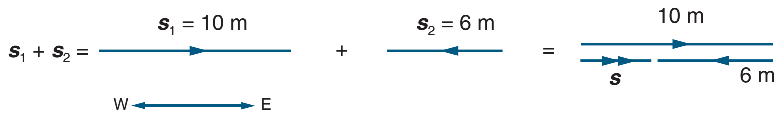
If $\boldsymbol{s}_1$ is a displacement of 10 m east and $\boldsymbol{s}_2$ is 6 m west, determine:

a $\boldsymbol{s}_1 + \boldsymbol{s}_2$

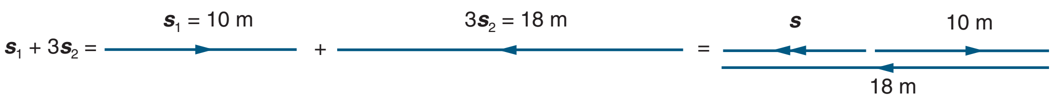
b $\boldsymbol{s}_1 + 3\boldsymbol{s}_2$.

Solution

a The total displacement vector is: $\boldsymbol{s} = \boldsymbol{s}_1 + \boldsymbol{s}_2$ = 10 m east + 6 m west. As shown below, this is 4 m east.



b The total displacement vector is: $\boldsymbol{s} = \boldsymbol{s}_1 + 3\boldsymbol{s}_2$ = 10 m east + 18 m west. As shown below, the total displacement vector $\boldsymbol{s}$ is 8 m west.



Physics in action — Motion in two dimensions

Vectors can be used to analyse the motion of a person or body moving in two dimensions. Consider Jessica, a bushwalker, who hikes 5 km north then 8 km east. The distance travelled is 5 km + 8 km = 13 km; however, her total displacement is:

$$\boldsymbol{s} = 5 \text{ km north} + 8 \text{ km east}$$

The magnitude of Jessica's total displacement can be determined by using Pythagoras's theorem:

$$\text{magnitude of displacement} = \sqrt{(5^2 + 8^2)}$$
$$= \sqrt{89} = 9.4 \text{ km}$$

The direction of the displacement can be determined by using trigonometry:

$$\tan\theta = 8/5 = 1.6$$
$$\theta = \tan^{-1} 1.6 = 58°$$

This direction can be stated as N58°E. The resultant displacement of the bushwalker is 9.4 km N58°E. The distance travelled is 13 km.

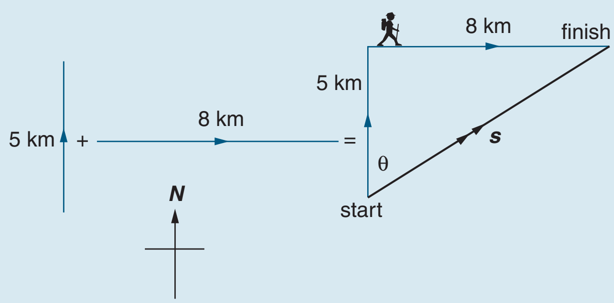


Figure 2.6

The displacement, $\boldsymbol{s}$, of the hiker is a vector that runs directly from the start to the finish of her journey. The magnitude of her displacement is different from the distance of 13 km that she has walked.

Physics in action — The standard units of measurement

The accurate and easy measurement of quantities is essential both in everyday life and for scientific investigation. Over the centuries, many different systems of measuring physical quantities have developed. For example, length can be measured in chains, fathoms, furlongs, yards, feet, rods and microns. Some units were based on parts of the body. The cubit was defined as the distance from the elbow to the fingertip, and so the amount of cloth that might be obtained from a tailor depended on the physical size of the person selling it to you.

The metric system was established by the French Academy of Science at the time of the French Revolution (1789–1815) and is now used in most countries. This system includes units such as metres, litres and kilograms. Countries of the British Empire adopted the British Imperial system of miles, gallons and pounds. These two systems developed independently and their dual existence created problems in areas such as trade and scientific research. In 1960, an international committee set standard units for fundamental physical quantities. This system was an adaptation of the metric system and is known as the 'Système International' (SI) system of units.

table 2.2 The SI units identify the seven fundamental quantities, whose basic values are defined to a high degree of accuracy.

Fundamental quantity	SI unit
Mass	kilogram (kg)
Length	metre (m)
Time	second (s)
Electric current	ampere (A)
Temperature	kelvin (K)
Luminous intensity	candela (cd)
Amount of substance	mole (mol)

The units for mass, length and time are the most commonly used fundamental SI units.

Mass

The kilogram was originally defined as the mass of one litre of water at 4°C. This is still approximately correct, but a far more precise definition is now used. Since 1897, the measurement standard for the kilogram has been a cylindrical block of platinum–iridium alloy kept at the International Bureau of Weights and Measures in France. Australia has a copy of this standard mass at the CSIRO Division of Applied Physics in Sydney. At times it is returned to France to ensure that the mass remains accurate.

table 2.3 Some derived SI quantities and their units

Quantity	SI unit	Equivalent unit
Velocity	metres per second ($m\ s^{-1}$)	
Acceleration	metres per second per second ($m\ s^{-2}$)	
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg\ m\ s^{-2}$
Energy/work	joule (J)	$kg\ m^2\ s^{-2}$

Length

The metre was originally defined in 1792 as one ten-millionth of the distance from the Equator to the North Pole, which is approximately 10 000 km. This definition has changed a number of times since.

Figure 2.7
One metre was originally defined as one ten-millionth of the distance from the Equator to the North Pole.

In 1983, to give a more accurate value, the metre was redefined as the distance that light in a vacuum travels in $\frac{1}{299\,792\,458}$ second. This standard can be reproduced all over the world, as light travels at a constant speed in a vacuum.

Time

Time has always been based on the apparent motion of the heavens. The second was once defined in terms of the motion of the sun. Until 1960, one second was defined as $\frac{1}{60}$ of $\frac{1}{60}$ of $\frac{1}{24}$ of an average day in 1900. This reflected the rate of the Earth's rotation on its axis; however, its rotation is not quite uniform. In 1967, a more accurate definition was adopted—one not based on the motion of the Earth. One second is now defined as the time required for a caesium-133 atom to undergo 9 162 631 770 vibrations. These vibrations are stimulated by an electric current and are extremely stable, allowing this standard to be reproduced all over the world.

Derived units

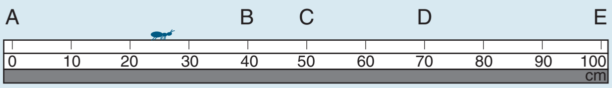
Apart from the seven fundamental quantities, a wide variety of other physical quantities can be measured. You may have encountered some of these, such as frequency, velocity, energy and density, already. A derived quantity is defined in terms of the fundamental quantities. For example, the SI unit for area is square metres (m^2).

2.1 SUMMARY Describing motion in a straight line

- Position defines the location of an object with respect to a defined origin.
- Distance travelled, *s*, tells how far an object has actually travelled. Distance travelled is a scalar.
- Displacement, ***s***, is a vector and is defined as the change in position of an object in a given direction. Displacement ***s*** = final position – initial position.
- Vector quantities require a magnitude and a direction, whereas scalar quantities can be fully described by a magnitude only.
- When adding vectors, place the tail of the second vector at the head of the first. The resultant vector is from where the first vector starts to where the second vector ends.

2.1 Questions

1 A somewhat confused ant is moving back and forth along a metre ruler.



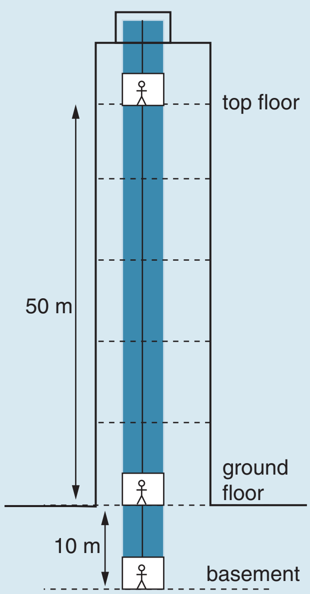
Determine both the displacement and distance travelled by the ant as it moves from:

a A to B
b C to B
c C to D
d C to E and then to D.

2 During a training ride, a cyclist rides 50 km north then 30 km south.

a What is the distance travelled by the cyclist during the ride?
b What is the displacement of the cyclist for this ride?

3 A lift in a city building carries a passenger from the ground floor down to the basement, then up to the top floor.



a Determine the displacement as the passenger travels from the ground floor to the basement.
b What is the displacement of the lift as it travels from the basement to the top floor?
c What is the distance travelled by the lift during this trip?
d What is the displacement of the lift during this trip?

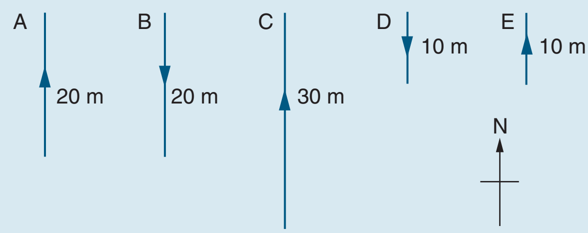
4 Which of these physical quantities are vectors: mass, displacement, density, distance, temperature?

5 Construct vector additions to find the resultant displacement vector for:

a 10 m east plus 15 m east
b 12 m west plus 8 m west.

6 If $\boldsymbol{s}_1$ is 20 m south and $\boldsymbol{s}_2$ is 10 m north, which of the vectors A–E represents:

a $\boldsymbol{s}_1 + \boldsymbol{s}_2$?
b $\boldsymbol{s}_2 + \boldsymbol{s}_1$?
c $3\boldsymbol{s}_2$?
d $-\boldsymbol{s}_1$?



7 Given that $\boldsymbol{s}_1$ = 10 m north and $\boldsymbol{s}_2$ = 20 m south, use graph paper and an appropriate scale to determine the magnitude and direction of these vector sums:

a $\boldsymbol{s}_1 + \boldsymbol{s}_2$
b $\boldsymbol{s}_2 + 2\boldsymbol{s}_1$.

8 Liam, aged 7, buried some 'treasure' in his backyard and wrote down these clues for you to find it: start at the clothes line, walk 10 steps south, then four steps east, 15 steps north, five steps west, and five steps south.

a What distance (in steps) is travelled while tracing the 'treasure'?
b Where is the 'treasure' buried?
c What is your displacement (in steps) as you follow the instructions?

2.2 Speed, velocity and acceleration

For thousands of years, humans have tried to travel at greater speeds. This desire has contributed to the development of all sorts of competitive activities, as well as to major advances in engineering and design. The records for some of these pursuits are given in Table 2.4.

table 2.4 Recent world speed records

Speed activity	Record speed ($m\ s^{-1}$)	Record speed ($km\ h^{-1}$)
Cycling	36	130
Train	161	581
Tennis serve	73	263
Water skiing	69	250
Downhill skiing	67	241
Horse racing	19	70

Speed and velocity are both quantities that give an indication of how fast an object moves, or more precisely of how quickly the position of an object is changing. Both terms are in common use and are often assumed to have the same meaning. In physics, however, these terms are defined differently.

- *Speed* is defined in terms of the *distance travelled* and so, like distance, speed is a *scalar*. Thus, a direction is not required when describing the speed of an object.
- *Velocity* is defined in terms of *displacement* and so is a *vector* quantity.

The SI unit for speed and velocity is metres per second ($m\ s^{-1}$). Kilometres per hour ($km\ h^{-1}$) is also commonly used.

Instantaneous speed and velocity

Instantaneous speed and *velocity* give a measure of how fast something is moving at a particular *moment* or *instant in time*. If the speedometer on a car shows 60 $km\ h^{-1}$, it is indicating the instantaneous speed of the car. If another car is detected on a police radar gun and registers 120 $km\ h^{-1}$, it indicates that this car's instantaneous speed is above the speed limit.

Average speed and velocity

Average speed and *velocity* both give an indication of how fast an object is moving over a *time interval*. For example, the average speed of a car that takes $2\frac{1}{2}$ hours to travel 200 km from Perth to Bunbury is 80 $km\ h^{-1}$. However, this does not mean that the car travelled the whole way at this speed. In fact, it is more likely that the car was moving at 60 $km\ h^{-1}$ for a significant amount of time, but some time was also spent not moving at all.

AVERAGE SPEED $v_{av} = \dfrac{\text{distance travelled}}{\text{time taken}} = \dfrac{s}{\Delta t}$

Speed is measured in metres per second ($m\ s^{-1}$).

Physics file

When converting a speed from one unit to another, it is important to think about the speeds to ensure that your answers make sense.

From $km\ h^{-1}$ to $m\ s^{-1}$: 100 $km\ h^{-1}$ is a speed that you should be familiar with as it is the speed limit for most freeways. Cars that maintain this speed would travel 100 km in 1 hour. Since there are 1000 m in 1 km and $60 \times 60 = 3600$ s in 1 hour, this is the same as travelling 100 000 m in 3600 s.

$$\begin{aligned} 100\ km\ h^{-1} &= 100 \times 1000\ m\ h^{-1} \\ &= 100\,000\ m\ h^{-1} \\ &= 100\,000 \div 3600\ m\ s^{-1} \\ &= 27.8\ m\ s^{-1} \end{aligned}$$

So $km\ h^{-1}$ can be converted to $m\ s^{-1}$ by multiplying by 1000/3600 (i.e. ÷ 3.6).

From $m\ s^{-1}$ to $km\ h^{-1}$: A champion Olympic sprinter can run at an average speed of close to 10 $m\ s^{-1}$, i.e. each second the athlete will travel approximately 10 metres. At this rate, in 1 hour the athlete would travel: $10 \times 3600 = 36\,000$ m, i.e. 36 km

$$\begin{aligned} 10\ m\ s^{-1} &= 10 \times 3600\ m\ h^{-1} \\ &= 36\,000\ m\ h^{-1} \\ &= 36\,000 \div 1000\ km\ h^{-1} \\ &= 36\ km\ h^{-1} \end{aligned}$$

So $m\ s^{-1}$ can be converted to $km\ h^{-1}$ by multiplying by 3600/1000 (i.e. × 3.6).

AVERAGE VELOCITY $\mathbf{v}_{av} = \dfrac{\text{displacement}}{\text{time taken}} = \dfrac{\mathbf{s}}{\Delta t}$

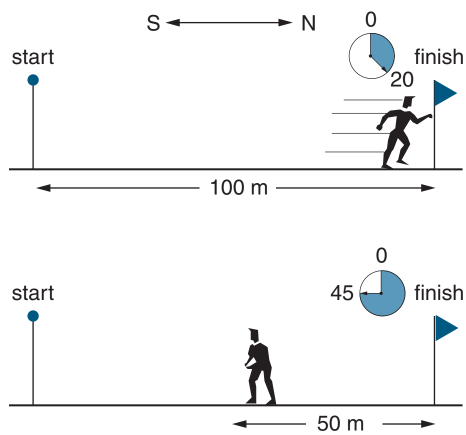
Velocity is measured in metres per second (m s^{-1}) and requires a direction.

A *direction* (such as north, south, up, down, left, right, positive, negative) must be given when describing a *velocity*. It will always be the same as that of the displacement.

Worked Example 2.2A

Consider Damian, an athlete performing a training routine by running back and forth along a straight stretch of running track. He jogs 100 m north in a time of 20 s, then turns and walks 50 m south in a further 25 s before stopping.

a Calculate Damian's average speed as he is jogging.
b What is his average velocity as he is jogging?
c What is the average speed for this 150 m exercise?
d Determine the average velocity for this activity.
e What is Damian's average velocity in km h^{-1}?



Solution

a His average speed when jogging is:

$$v_{av} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{s}{\Delta t} = \frac{100\text{ m}}{20\text{ s}} = 5.0\text{ m s}^{-1}$$

b His average velocity when jogging is:

$$\mathbf{v}_{av} = \frac{\text{displacement}}{\text{time taken}} = \frac{\mathbf{s}}{\Delta t} = \frac{100\text{ m north}}{20\text{ s}} = 5.0\text{ m s}^{-1}\text{ north}$$

Note that speed has been treated as a scalar and velocity as a vector.

c Damian has covered a distance of 150 m in 45 s. His average speed is:

$$v_{av} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{150\text{ m}}{45\text{ s}} = 3.3\text{ m s}^{-1}$$

d He has finished 50 m to the north of where he started, i.e. his displacement is 50 m north. His average velocity is:

$$\mathbf{v}_{av} = \frac{\mathbf{s}}{\Delta t} = \frac{50\text{ m north}}{45\text{ s}} = 1.1\text{ m s}^{-1}\text{ north}$$

i.e. Damian could have ended up at the same place in the same time by travelling with this average velocity.

e His average velocity is:

$$1.1\text{ m s}^{-1}\text{ north} = \frac{1.1 \times 3600}{1000} = 4.0\text{ km h}^{-1}\text{ north}$$

Measuring speed in the laboratory

A variety of methods can be employed to determine the speed of an object in a motion experiment. Common techniques include ticker timers, ultrasound transducers, photogates and multiflash photography.

A *ticker timer* has a hammer that vibrates with a frequency of 50 Hz and produces a series of dots on a piece of ticker tape that is being dragged along by a moving body. Since the hammer strikes the paper at regular intervals, the distance between the dots gives an indication of the speed of the body. Where the dots are widely spaced, the body is moving faster than when they are close together. Precise values of speed can be determined by measuring the distances between the dots. Consider the section of tape shown in Figure 2.8. The tape had been attached to a student to measure walking speed.

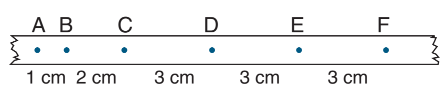


Figure 2.8
Ticker tape was commonly used to analyse the motion of objects. If the frequency of the timer is known and the distance between the dots has been measured, the average speed of the object can be determined.

The *average speed* of the student is calculated by measuring the distance travelled and taking account of the time elapsed. Since the hammer strikes the tape 50 times per second, each interval between the dots represents 1/50 s, i.e. 0.02 s. Thus the average speed between A and F, a distance consisting of five intervals, is:

$$v_{av} = \frac{\text{distance}}{\text{time}} = \frac{12\ \text{cm}}{5 \times 0.02\ \text{s}} = 120\ \text{cm s}^{-1} \text{ or } 1.2\ \text{m s}^{-1}$$

The *instantaneous speed* gives a measure of the speed at one particular time. This can be estimated with reasonable accuracy by calculating the average speed for the interval one dot either side of the point being analysed. For example, the instantaneous speed at point B can be estimated by calculating the average speed between points A and C:

$$v_{inst}(\text{B}) \approx v_{av}(\text{A to C}) = \frac{\text{distance travelled}}{\text{time taken}} = \frac{3\ \text{cm}}{2 \times 0.02\ \text{s}} = 75\ \text{cm s}^{-1} \text{ or } 0.75\ \text{m s}^{-1}$$

Multiflash photography was a useful method for analysing more complex motion. A photograph was taken by a camera with the shutter open and a strobe light that flashes at a known frequency. This was analysed in a similar manner to ticker tape. If the frequency of the flash is known, the time between each flash, i.e. the *period* of the flash, is easily found. For example, a flash with a frequency of 20 Hz has a period of 0.05 s. By measuring the appropriate distance from the photograph, average speed can be calculated and instantaneous speed estimated.

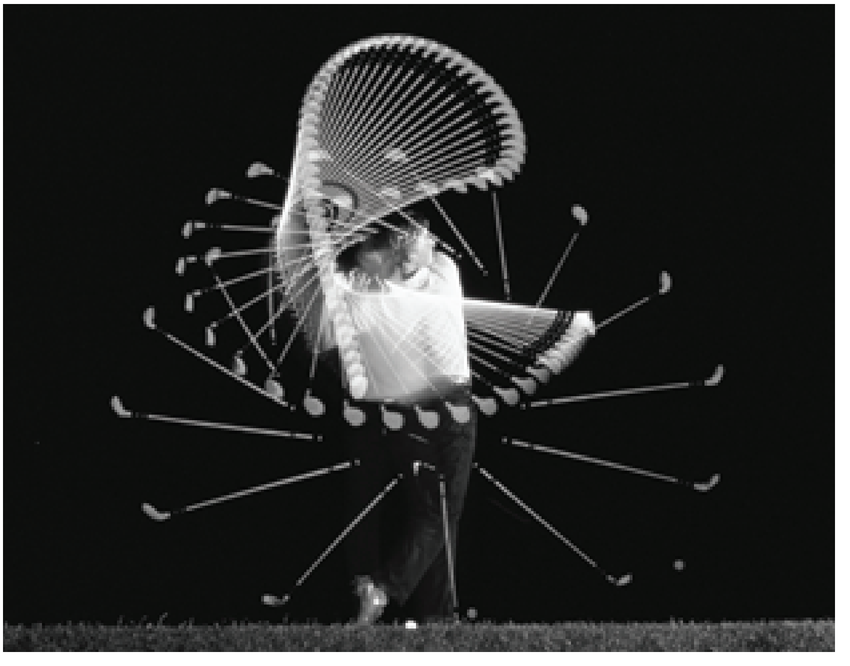


Figure 2.9

A multiflash photograph of this golf swing allows the motion of the club and ball to be analysed in detail. Given that the flash frequency is 120 Hz, and the scale of the photograph is 1:25, you should be able to show that the initial speed of the golf ball is approximately 70 m s^{-1}.

A *photogate* consists of a light source and sensor that triggers an electronic timing device when the light beam is broken. Photogates are designed to measure time to millisecond accuracy, and so give very accurate speed data. Some are calibrated to give a direct reading of speed. Others will simply give a measure of the time interval between two light beams being broken. The average speed of a falling mass that passes between two photogates can be calculated by considering the distance between the photogates and the time that the mass took to pass between them.

An *ultrasonic motion sensor* gives a direct and instantaneous measure of the speed of a body. These devices emit a series of high-frequency sound pulses that are reflected from the moving object, giving an indication of its position. The data are then processed to give a measure of the speed. Ultrasonic sensors allow complex motions such as a sprinter starting a race, or a ball bouncing several times, to be analysed in great detail.

Acceleration

If you have been on a train as it has pulled out of a station, you will have experienced an acceleration. Also, if you have been in a jumbo jet as it has taken off along a runway, you will have experienced a much greater acceleration. *Acceleration* is a measure of how quickly *velocity changes*.

Consider the following velocity information for a car that starts from rest at an intersection as shown in Figure 2.10.

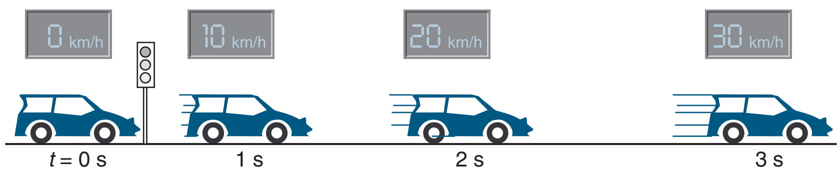


Figure 2.10
The velocity of the car increases by 10 km h^{-1} each second, and so its acceleration is said to be +10 kilometres per hour per second.

Each second, the velocity of the car increases by 10 km h^{-1}. In other words, its velocity changes by +10 km h^{-1} per second. This is stated as an *acceleration* of +10 kilometres per hour per second or +10 km h^{-1} s^{-1}.

More commonly in physics, velocity information is given in metres per second. The athlete in Figure 2.11 takes 3 s to come to a stop at the end of a race.

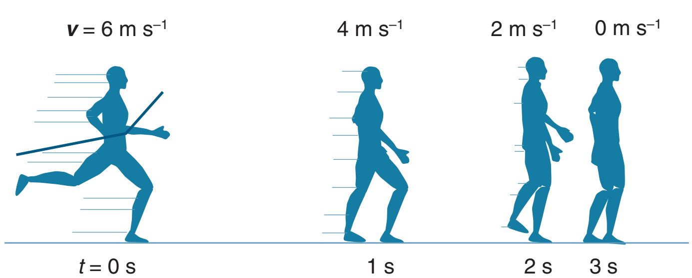


Figure 2.11
The velocity of the athlete changes by –2 m s^{-1} each second. The acceleration is –2 m s^{-2}.

Each second, the velocity of the athlete changes by –2 m s^{-1}, and so the acceleration is –2 metres per second per second. This is usually expressed as –2 metres per second squared or –2 m s^{-2}.

Acceleration is defined as the *rate of change of velocity*. Acceleration is a vector quantity whose direction is that of the velocity change. A negative acceleration can mean that the object is *slowing down* in the direction of travel or *speeding up* in the opposite direction.

AVERAGE ACCELERATION is the rate of change of velocity:

$$\boldsymbol{a}_{av} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{(\boldsymbol{v} - \boldsymbol{u})}{\Delta t}$$

where $\boldsymbol{v}$ is the final velocity (m s^{-1})
$\boldsymbol{u}$ is the initial velocity (m s^{-1})
Δt is the time interval (s).

Acceleration is measured in metres per second squared (m s^{-2}).

Worked Example 2.2B

A cheetah running at 30 m s^{-1} slows down as it approaches a stream. Within 3 s, its speed has reduced to 9 m s^{-1}. Calculate the average acceleration of the cheetah.

Solution

The average acceleration of the cheetah is:

$$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{(\boldsymbol{v} - \boldsymbol{u})}{\Delta t}$$

$$= \frac{(9 - 30)}{3}$$

$$= -\frac{21}{3}$$

$$= -7 \text{ m s}^{-2}$$

i.e. each second, the cheetah is slowing down by 7 m s^{-1}.

Finding velocity changes

When finding the *change* in any physical quantity, the initial value is taken away from the final value. Thus, the change in velocity is the final velocity minus the initial velocity:

$$\Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u}$$

In algebra, a subtraction is equivalent to the addition of a negative term, e.g. $x - y = x + (-y)$. The same rationale can be used when subtracting vectors. *Vector subtraction* is performed by *adding the opposite* of the subtracted vector:

$$\Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u} = \boldsymbol{v} + (-\boldsymbol{u})$$

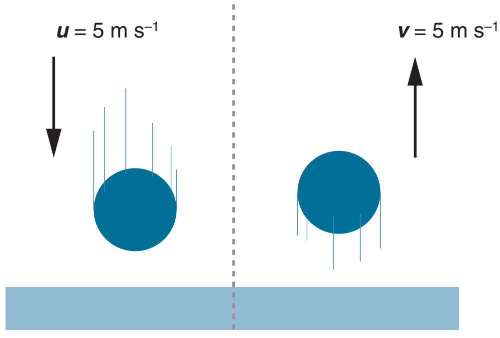
The *negative* of a vector simply points the *opposite* way, i.e. if $\boldsymbol{u}$ is 5 m s^{-1} north, then $-\boldsymbol{u}$ is 5 m s^{-1} south.

✓ Worked Example 2.2C

A golf ball is dropped onto a concrete floor and strikes the floor at 5.0 m s^{-1}. It then rebounds at 5.0 m s^{-1}.

a What is the change in speed for the ball?

b Calculate the change in velocity for the ball.



$\Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u}$

$= 5\uparrow - 5\downarrow$

$= 5\uparrow + 5\uparrow$

$= \uparrow 10$ m s^{-1}

$= 10$ m s^{-1} up

Solution

a Both the initial and final speed of the ball are 5.0 m s^{-1}, so the change in speed for the ball is:

$\Delta v = v - u = 5.0 - 5.0 = 0$

Since speed is a scalar quantity, the direction of motion of the ball is not a consideration.

b To determine the change in velocity of the ball:

$\Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u} = 5.0$ m s^{-1} up $- 5.0$ m s^{-1} down

Let up be the positive direction, so:

$\Delta \boldsymbol{v} = +5.0 - (-5.0) = +5.0 + 5.0 = 10$ m s^{-1} up

As can also be seen in the diagram, the change in velocity of the ball is 10 m s^{-1} up. Velocity is a vector quantity and the change in direction of the ball is responsible for its velocity change.

Physics in action — Breaking the speed limit!

table 2.5 Some recent world record times and speeds for men and women

Event	Distance	Time (h:min.s)	Average speed
Men			
Running	100 m	0:9.78	10.2 m s^{-1}
	200 m	0:19.32	10.4 m s^{-1}
	400 m	0:43.18	9.1 m s^{-1}
	800 m	1:41.11	7.9 m s^{-1}
	1500 m	3:26.00	7.2 m s^{-1}
	Marathon—42.2 km	2:04.55	5.5 m s^{-1}
Swimming	50 m freestyle	0:20.81	2.3 m s^{-1}
	1500 m freestyle	14:34.56	1.7 m s^{-1}
Cycling—the Hour	56.375 km	1:00.00	56.38 km h^{-1}
Downhill skiing			241 km h^{-1}*
Women			
Running	100 m	0:10.49	9.5 m s^{-1}
	200 m	0:21.34	9.4 m s^{-1}
	400 m	0:47.60	8.4 m s^{-1}
	800 m	1:53.28	7.1 m s^{-1}
	1500 m	3:50.46	6.5 m s^{-1}
	Marathon—42.2 km	2:15.25	2.9 m s^{-1}
Swimming	50 m freestyle	0:23.25	2.0 m s^{-1}
	1500 m freestyle	15:42.54	1.6 m s^{-1}
Cycling—the Hour	48.159 km	1:00.00	48.16 km h^{-1}
Downhill skiing			225 km h^{-1}*

* Instantaneous speed.

A common goal of sportsmen and women around the world is to run, swim, ski, skate or ride faster than their opponents. A combination of intensive training, improved diets, specialised equipment and a knowledge of biophysics has helped athletes to keep setting new world records. For example, in 1972 the Australian swimmer Shane Gould held every world freestyle record from 100 m to 1500 m. Her time in the 400 m at the Olympics was 4 minutes 19.04 seconds. The current world record for women for this event, set in 2006, is 4 minutes 2.13 seconds. At her winning time, Shane Gould would have finished around half a lap behind the winning swimmer in this race.

Figure 2.12

In 2002, Ian Thorpe was the world record holder for the 200, 400 and 800 metres freestyle, and a member of the world record relay team for the 4 × 100 metres and the 4 × 200 metres freestyle.

Over the past hundred years, advances in engineering and technology have led to the development of faster and faster machines. Today cars, planes and trains can move people at speeds that were thought to be unattainable and life-threatening a century ago.

The one-mile land speed record is 1019 km h^{-1} (283 m s^{-1}). This was set in 1983 in Nevada, USA, by Richard Noble driving his jet-powered Thrust 2. The fastest passenger-carrying aircraft is the Concorde. It cruises at up to 2300 km h^{-1} (640 m s^{-1}). The fastest speed reached by a passenger-carrying train is 515 km h^{-1} (143 m s^{-1}). This was achieved by the French TGV Atlantique in 1990.

A dragster only has to travel 400 m, but it has to do it rather quickly. A piston engine (as opposed to rocket-powered!) dragster can cover the 400 m in 4.9 s and reach a maximum speed of 475 km h^{-1}. It can achieve a peak acceleration of 56 m s^{-2} during its trip and a parachute has to be used to slow it down.

In the 1950s, the American Air Force used a rocket sled to determine the effect of extremely large accelerations on humans. It consisted of an 800 m long railway track and a sled with nine rocket motors. Volunteers were strapped into the sled and accelerated to speeds of over 1000 km h^{-1} in a very short time. Then water scoops were used to stop the sled in just 0.35 s. This equates to a deceleration of 810 m s^{-2}. Hopefully they hadn't just finished a big lunch!

Figure 2.13

These photos show the face of Lieutenant Colonel John Stapp while he was travelling in a rocket-powered sled. As the sled blasted off, it achieved an acceleration of 120 m s^{-2}. The effect of this acceleration of 120 m s^{-2} is evident on his face.

2.2 SUMMARY Speed, velocity and acceleration

- The average speed of a body, v_{av}, is defined as the rate of change of distance and is a scalar quantity:

$$v_{av} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{s}{\Delta t}$$

- The average velocity of a body, $\boldsymbol{v}_{av}$, is a vector and is the rate of change of displacement:

$$\boldsymbol{v}_{av} = \frac{\text{displacement}}{\text{time taken}} = \frac{\boldsymbol{s}}{\Delta t}$$

- The SI unit for both speed and velocity is metres per second (m s^{-1}).
- Instantaneous velocity is the velocity at a particular instant in time.
- The average acceleration of a body, $\boldsymbol{a}_{av}$, is defined as the rate of change of velocity. Acceleration is a vector:

$$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

2.2 Questions

1 Estimate the speed:
- **a** at which you walk
- **b** of a snail crawling
- **c** of a cricket ball bowled by Shane Warne
- **d** of a ten-pin bowling ball.

2 Toni rides her bicycle to school and travels the 2.5 km distance in a time of 10 min.
- **a** Calculate her average speed in kilometres per hour (km h^{-1}).
- **b** Calculate her average speed in metres per second (m s^{-1}).
- **c** Is Toni's average speed a realistic representation of her actual speed? Explain.

3 A sports car, accelerating from rest, was timed over 400 m and was found to reach a speed of 120 km h^{-1} in 18.0 s.
- **a** What was the average speed of the car in m s^{-1}?
- **b** Calculate the average acceleration of the car in km h^{-1} s^{-1}.
- **c** What was its average acceleration in m s^{-2}?
- **d** If the driver of the car had a reaction time of 0.60 s, how far would the car travel while the driver was reacting to apply the brakes at this speed of 120 km h^{-1}?

4 Draw these vector subtractions and find the resultant vector for:
- **a** 10 m s^{-1} east minus 15 m s^{-1} east
- **b** 12 m s^{-1} west minus 8 m s^{-1} west.

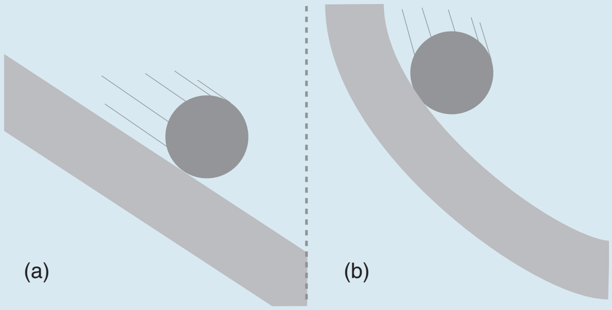
5 A squash ball travelling east at 25 m s^{-1} strikes the front wall of the court and rebounds at 15 m s^{-1} west. The contact time between the wall and the ball is 0.050 s.
- **a** Calculate the change in speed of the ball.
- **b** Calculate the change in velocity of the ball.
- **c** What is the acceleration of the ball during its contact with the wall?

6 A bus travelling north along a straight road at 60 km h^{-1} slows down uniformly and takes 5.0 s to stop.
- **a** Calculate its acceleration in km h^{-1} s^{-1}.
- **b** Calculate its acceleration in m s^{-2}.

7 During a world record 1500 m freestyle swim, Kieren Perkins completed 30 lengths of a 50 m pool in a time of 14 min 42 s.
- **a** What was his distance travelled during this race?
- **b** What was his average speed (in m s^{-1})?
- **c** What was his displacement during the race?
- **d** What was his average velocity during his record-breaking swim?

8 Christopher is travelling by car to a city 100 km away. For the trip, he wants to average 50 km h^{-1}. However, owing to mechanical problems, he finds that when he has travelled halfway his average speed is only 25 km h^{-1}. Which one of the following best describes the required speed for the second half of the trip for Christopher's average speed of 50 km h^{-1} to be achieved?
- **A** 75 km h^{-1}
- **B** 100 km h^{-1}
- **C** 150 km h^{-1}
- **D** It is not possible.

9 A ball rolls down an incline as shown in (a) below. Which one of the following best describes the speed and acceleration of the ball?
- **A** The speed and acceleration both increase.
- **B** The speed increases and the acceleration is constant.
- **C** The speed is constant and the acceleration is zero.
- **D** The speed and acceleration are both constant.


(a) (b)

10 A ball rolls down the slope shown in (b) above. Which one of the following best describes its speed and acceleration?
- **A** Its speed and acceleration both increase.
- **B** Its speed and acceleration both decrease.
- **C** Its speed increases and its acceleration decreases.
- **D** Its speed decreases and its acceleration increases.

Questions 11 and 12 require you to make reasonable estimates for any data that you need. You should perform these calculations without a calculator and then check them with your calculator. Assume that the speed of electromagnetic radiation is 3×10^8 m s^{-1}.

11 Estimate the number of metres in a light-year. A light-year is the distance that light travels in one year.

12 Suppose you are sitting on your chair in the loungeroom and you press the off button on the remote control. Estimate the amount of time it takes the infrared signal to reach the television set.

2.3 Graphing motion: position, velocity and acceleration

At times, the motion of an object travelling even in a straight line can be complicated. The object may travel forwards or backwards, speed up or slow down, or even stop. Where the motion remains in one dimension, the information can be presented in graphical form. The main advantage of a graph compared with a table is that it allows the scope of the motion to be seen clearly. Information that is contained in a table is not as readily accessible or as easy to interpret as that presented graphically.

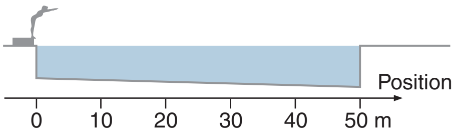


Figure 2.14
This swimmer will travel to the 50 m mark, then return to the 25 m mark. Her position is shown in Table 4.6.

Graphing position

A *position–time* graph indicates the position of an object at any time for motion that occurs over an extended time interval. However, the graph can also provide additional information.

Consider once again Sophie swimming laps of a 50 m pool. Her position–time data is shown in Table 2.6. The starting point is treated as the origin for this motion.

table 2.6 The positions and times of a swimmer completing one and a half lengths of a pool

Time	0	5	10	15	20	25	30	35	40	45	50	55	60
Position (m)	0	10	20	30	40	50	50	50	45	40	35	30	25

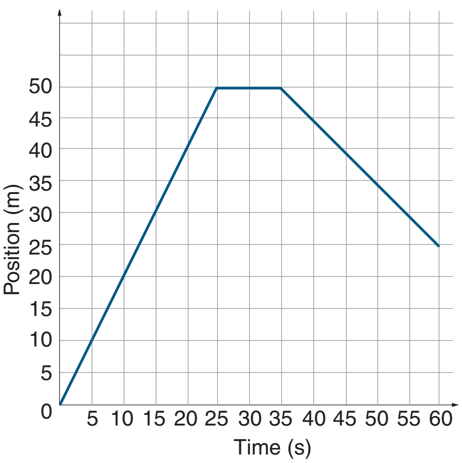


Figure 2.15
This graph represents the motion of a swimmer travelling 50 m along a pool, then resting and swimming back towards the starting position. The swimmer finishes halfway along the pool.

An analysis of the table reveals several features of the swim. For the first 25 s, Sophie swims at a constant rate. Every 5 s she travels 10 m in a positive direction, i.e. her velocity is +2 m s^{-1}. Then from 25 s to 35 s, her position does not change; she seems to be resting, i.e. stationary, for this 10 s interval. Finally from 35 s to 60 s, she swims back towards the starting point, i.e. in a negative direction. On this return lap, she maintains a more leisurely rate of 5 metres every 5 seconds, i.e. her velocity is –1 m s^{-1}. However, Sophie does not complete this lap but ends 25 m from the start. These data can be conveniently shown on a position–time graph.

The *displacement* of the swimmer can be determined by comparing the initial and final positions. Her displacement between 20 s and 60 s is, for example:

$$\boldsymbol{s} = \text{final position} - \text{initial position} = 25 - 40 = -15 \text{ m}$$

By further examining the graph in Figure 2.15, it can be seen that during the first 25 s, the swimmer has a displacement of +50 m. Thus, her *average velocity* is +2 m s^{-1}, i.e. 2 m s^{-1} to the right. This value can also be obtained by finding the *gradient* of this section of the graph.

Velocity is given by the gradient of a position–time graph. A positive velocity indicates that the object is moving in a positive direction, and a negative velocity indicates motion in a negative direction.

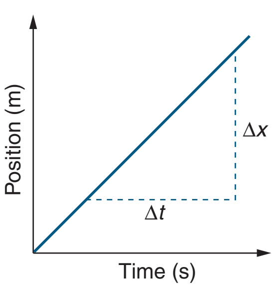


Figure 2.16
From the units of the rise and run, it can be seen that the units for the gradient are m s^{-1}, confirming that this is a measure of velocity.

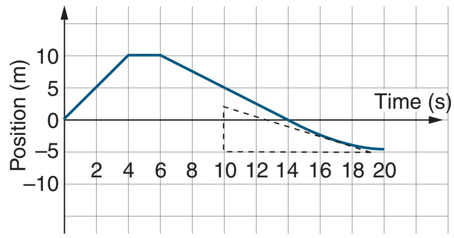
If the position–time graph is *curved*, the velocity will be the *gradient of the tangent* to the line at the point of interest. This will be an *instantaneous velocity*. Dimensional analysis can be used to confirm that the gradient of a position–time graph is a measure of velocity:

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$$

The units of gradient are metres per second (m s^{-1}), i.e. gradient is a measure of velocity.

Worked Example 2.3A

A car driven by a learner driver travels along a straight driveway and is initially heading north. The position of the car is shown in the graph.



a Describe the general motion of the car.
b What is the displacement of the car during the first 10 s of its motion?
c What distance has the car travelled during the first 10 s?
d Calculate the average velocity of the car during the first 4 s.
e Calculate the average velocity of the car between $t = 6$ s and $t = 20$ s.
f Calculate the average velocity of the car during its 20 s trip.
g Calculate the average speed of the car during its 20 s trip.
h Calculate the instantaneous velocity of the car at $t = 18$ s.

Solution

a The car initially travels 10 m north in 4 s. It then stops for 2 s. From $t = 6$ s to $t = 20$ s, the car travels towards the south, i.e. it reverses. It passes through its starting point after 14 s, and finally stops 2 m south of this point after 20 s.
b The displacement of the car is given by its change in position. From the graph, we can see that the car started from zero, and after 10 s its position is 5 m, so its displacement is +5 m or 5 m north.
c The distance travelled is an indication of the ground covered by the car. During the first 10 s the car travels 10 m north, then stops and travels 5 m south. Therefore, it travels a distance of 15 m.
d The average velocity is given by the gradient during the first 4 s:

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{10}{4} = +2.5 \text{ m s}^{-1} \text{ or } 2.5 \text{ m s}^{-1} \text{ north}$$

e Again, the average velocity is given by the gradient:

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = -\frac{15}{14} = -1.1 \text{ m s}^{-1} \text{ or } 1.1 \text{ m s}^{-1} \text{ south}$$

f To determine the average velocity for the 20 s:
$\boldsymbol{v}_{av} = \boldsymbol{s}/t = -5 \text{ m}/20 \text{ s} = -0.25 \text{ m s}^{-1}$ or 0.25 m s^{-1} south
This could also be found by calculating the gradient of the line from the start to the end of the motion.
g The car travels a distance of 10 m + 10 m + 5 m = 25 m in 20 s. Its average speed is:
$v_{av} = \text{distance}/\text{time} = 25/20 = 1.25 \text{ m s}^{-1}$
h The graph is curved at this time, so to find the instantaneous velocity it is necessary to draw a tangent to the line and calculate the gradient of the tangent:

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{-5}{10} = -0.50, \text{ i.e. } \boldsymbol{v}_{inst} = 0.50 \text{ m s}^{-1} \text{ south}$$

Graphing velocity

A graph of *velocity against time* shows how the velocity of an object changes with time. This type of graph is useful for analysing the motion of an object moving in a complex manner, for example a ball bouncing up and down. A velocity–time graph can also be used to obtain additional information about the object.

Consider the example of a small girl, Eleanor, running back and forth along an aisle in a supermarket. A study of the velocity–time graph in Figure 2.17 reveals that Eleanor is moving with a positive velocity, i.e. in a positive direction, for the first 6 s. Between the 6 s mark and the 7 s mark, she is stationary, then she runs in the reverse direction, i.e. has negative velocity, for the final 3 s.

This graph directly shows Eleanor's velocity at each instant in time. She moves in a *positive direction* with a constant speed of 3 m s^{-1} for the first 4 s. From 4 s to 6 s, she continues moving in a positive direction but slows down, until 6 s after the start she comes to a stop. Then during the final 3 s, when the line is below the time axis, her *velocity is negative*; she is now moving in a *negative direction.*

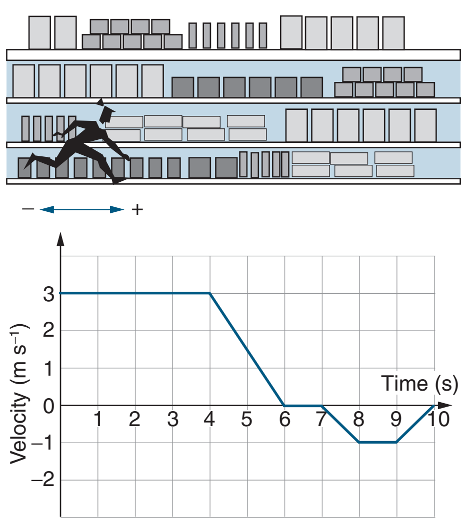


Figure 2.17
This graph shows the straight line motion of a girl running back and forth along a supermarket aisle.

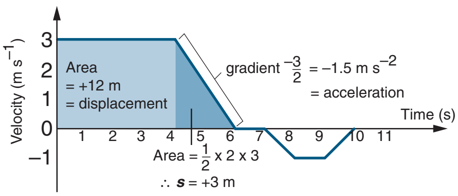


Figure 2.18

The displacement of the girl is given by the area under the graph. During the first 6 s, her displacement is +15 m.

Physics file

The *area* under a velocity–time graph is a measure of *displacement*. When the units on the axes are multiplied when finding the area, a displacement unit results. From Figure 2.19a:

area units = m s^{-1} × s = m

i.e. a displacement

The *gradient* of a velocity–time graph is the *acceleration* of the object. When finding the gradient, the units are divided. From Figure 2.19b:

gradient units = m s^{-1}/s = m s^{-2}

i.e. an acceleration

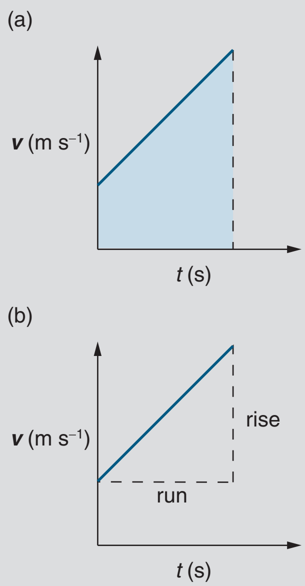


Figure 2.19

(a) The units on the axes of a v–t graph confirm that the area under the graph represents a displacement. (b) The gradient of the line is the acceleration.

A *velocity–time graph* can also be used to find the *displacement* of the body under consideration. In the first 6 s of Eleanor's motion she moves with a constant velocity of +3 m s^{-1} for 4 s, then slows from 3 m s^{-1} to zero in the next 2 s. Her displacement during this time can be determined from the v–t graph:

$\boldsymbol{v} = \boldsymbol{s}/\Delta t$, so

$\boldsymbol{s} = \boldsymbol{v} \times \Delta t$ = height × base = area under v–t graph.

From Figure 2.18, the *area* under the graph for the first 4 s gives the displacement of the girl during this time, i.e. +12 m. The displacement from 4 s to 6 s is represented by the area of the shaded triangle and is equal to +3 m. Thus the total displacement during the first 6 s is:

+12 m + 3 m = +15 m

Displacement is given by the *area under a velocity–time graph* (or the area between the line and the time axis). It is important to note that this applies for any graph, and that an area below the time axis indicates a negative displacement, i.e. motion in a negative direction.

The *acceleration* of an object can also be found from a velocity–time graph. Consider the motion of the girl in the 2 s interval between 4 s and 6 s. She is moving in a positive direction but slowing down from 3 m s^{-1} to rest. Her acceleration is:

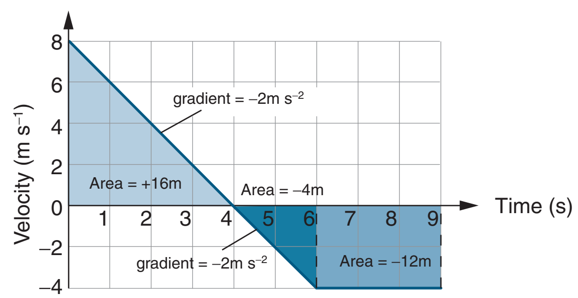
$\boldsymbol{a} = \Delta\boldsymbol{v}/\Delta t = (\boldsymbol{v} - \boldsymbol{u})/\Delta t = (0 - 3)/2 = -1.5$ m s^{-2}

Since acceleration is the velocity change divided by time taken, it is also given by the *gradient* of the v–t graph. As can be seen from Figure 2.18 once again, the gradient of the line between 4 s and 6 s is –1.5 m s^{-2}.

The *gradient of a velocity–time* graph is the average *acceleration* of the object over the time interval. If the acceleration is changing, the velocity–time graph will be curved, and so the gradient of the tangent will give an instantaneous acceleration.

Worked Example 2.3B

The motion of a marble rolling across a floor is represented by the following graph.



Use this graph to help you to:

a describe the motion of the marble
b calculate the displacement of the marble during the first 4 s
c determine the displacement for the 9 s shown
d find the acceleration during the first 4 s
e find the acceleration from 4 s to 6 s.

Solution

a The marble is initially moving in a positive direction at 8 m s^{-1}. It slows down and comes to a stop after 4 s, then reverses and travels in a negative direction. From 4 s to 6 s, the marble gains speed in the negative direction, then maintains a constant velocity of –2 m s^{-1} for the final 3 s.

b The displacement is given by the area under the graph; in this case the triangular area as shown. The marble's displacement during the first 4 s is +16 m.
c The displacement for the complete motion is given by the total area under the graph: +16 − 4 − 12 = 0, i.e. the marble finishes where it started.
d The acceleration is given by the gradient of the line. For the first 4 s, this is −2 m s^{-2}. This indicates that the marble is slowing down by 2 m s^{-1} each second while travelling in a positive direction.
e The gradient of the line from 4 s to 6 s is also −2 m s^{-2}. This now indicates that the marble is speeding up by 2 m s^{-1} each second while travelling in a negative direction.

Graphing acceleration

An *acceleration–time graph* simply indicates the acceleration of the object as a function of time. The area under an acceleration–time graph is found by multiplying an acceleration and a time value:

$$\text{area} = \boldsymbol{a} \times t = \Delta \boldsymbol{v}$$

The *area* will give the *change in velocity* ($\Delta \boldsymbol{v}$) of the object. In order to establish the actual velocity of the object, the initial velocity must be known.

Consider the marble from Worked Example 2.3B once again. The change in velocity during the first 6 s can be determined from the acceleration–time graph. As shown in Figure 2.20, the velocity changes by −12 m s^{-1}. This can be confirmed by looking at the velocity–time graph in Worked Example 2.3B. It shows that the marble slows down from +8 m s^{-1} to −4 m s^{-1}, a change of −12 m s^{-1}, during this time.

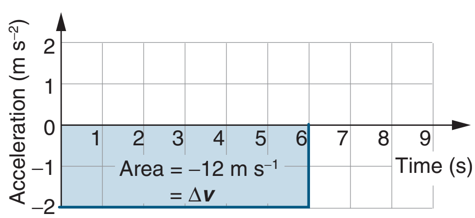


Figure 2.20
The acceleration–time graph for the marble rolling across a floor. It was drawn by taking account of the gradient values of the velocity–time graph. The marble's change in velocity is given by the area under the graph.

Physics file

A useful and time-saving technique of finding the area under a graph is that of 'counting squares'. To determine the area under a graph by counting squares:

- calculate the area of one grid square
- use a pencil to check off the number of complete squares under the graph
- if the graph is curved or contains part squares, estimate the combined total of these incomplete squares
- add these two amounts to determine the total number of squares
- multiply this value by the area of each square to determine the area under the graph.

For example, in the graph in Worked Example 2.3B, the area of each grid square is 2 × 1 = 2 m.

Up to 4 s, in the shaded triangular area, the complete and part squares combine to make 8 squares. The total displacement during this time is 8 squares × 2 m = +16 m.

Physics in action — Timing with precision

Until 1964, all timing of events at the Olympic Games was recorded by hand-held stopwatches. The reaction times of the judges meant an uncertainty of 0.2 s for any measurement. An electronic quartz timing system introduced in 1964 improved accuracy to 0.01 s, but in close finishes the judges still had to wait for a photograph of the finish before they could announce the placings.

Figure 2.21
At the 1968 Mexico Olympic Games, the judges used hand-held stopwatches to measure the times of the athletes.

Currently the timing system used is a vertical line-scanning video system (VLSV). Introduced in 1991, this is a completely automatic electronic timing system. The starting pistol triggers a computer to begin timing. At the finish line, a high-speed video camera records the image of each athlete and indicates the time at which the chest of each one crosses the line. This system enables the times of all the athletes in the race to be precisely measured to one-thousandth of a second.

Figure 2.23
This athlete has made a false start. A pressure pad in each starting block registers the starting time of each athlete. The cable leading from each starting block connects to a computer which instantly indicates the false start.

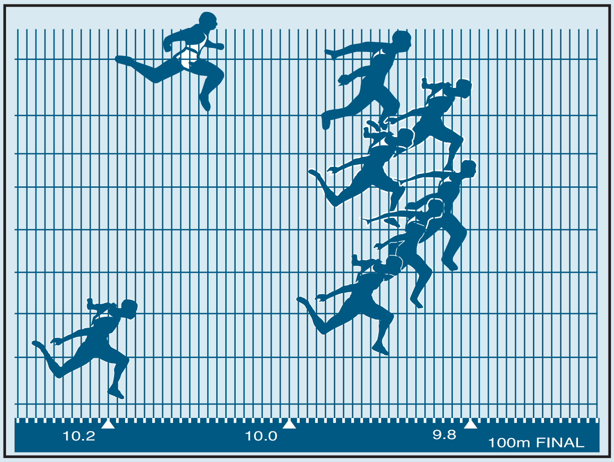


Figure 2.22
This is the finish of a world championship men's 100 m race. The time scale along the bottom of the film allows the time of each runner to be determined. The winning time here was 9.85 s and was a new world record set by Ben Johnson. He was later stripped of this record after a positive drug test.

Another feature of this system is that it indicates when a runner 'breaks' at the start of the race. Each starting block is connected by electronic cable to the timing computer and a pressure sensor indicates if a runner has left the blocks early. In fact, to ensure that a runner has not anticipated the pistol, a reaction time of 0.11 s is incorporated into the system. This means that a runner can still commit a false start even if their start was *after* the pistol. A start that is less than 0.11 s after the pistol has fired is registered as being false.

Physics in action — Overtake with care

Figure 2.24
A head-on collision is considered to be the most serious type of car accident.

Head-on collisions involving cars are a major cause of deaths and injuries on our roads. These accidents are often the result of a poorly judged overtaking manoeuvre. The construction of divided roads and freeways, and the increased use of right-turn arrows at intersections, has reduced the number of head-on collisions.

To gain an understanding of the risks associated with overtaking, consider this example. You are the driver of a car with a six-cylinder engine and you are driving behind a truck that is travelling at a constant speed of 90 km h^{-1} (25 m s^{-1}). You decide to overtake the truck but you are conscious of the speed limit and so do not travel faster than 100 km h^{-1} (28 m s^{-1}). At these speeds, your car is capable of accelerating at 3.0 m s^{-2}. This information can be represented in graphical form.

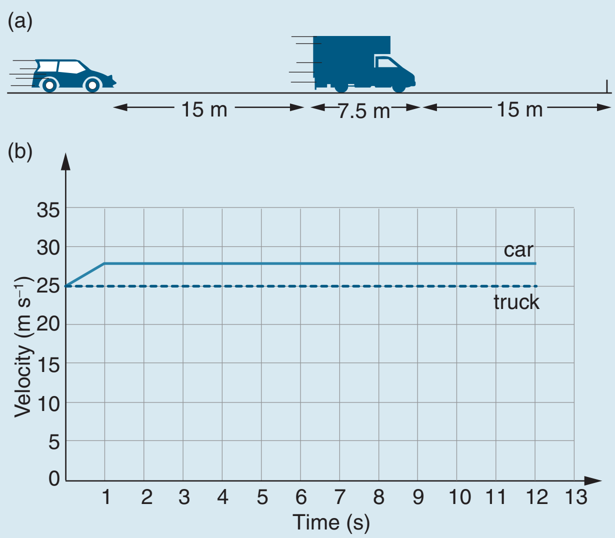


Figure 2.25
(a) The car commences its passing manoeuvre when it is 15 m behind the truck, and will overtake to a distance of 15 m ahead of the truck. (b) This velocity–time graph shows the motion of both the car and the truck. The area between the two lines represents the additional distance travelled by the car in passing the truck.

The area between the two lines is the extra distance that the car travels in passing the truck. If the truck is 7.5 m long and you start overtaking when 15 m behind and finish when 15 m ahead, then your car will have travelled 15 + 7.5 + 15 = 37.5 m further than the truck. So the area between the lines is 37.5 m, and the time needed to complete the overtaking manoeuvre can be found. From Figure 2.25, the area between the lines is equal to 37.5 m after 13 s. In other words, the car takes 13 s to overtake the truck.

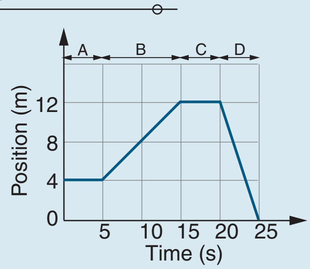
It is important to have an appreciation of the distances involved in overtaking. In the 13 s that the car takes to pass the truck, the car travels over 360 m. This is given by the area under the graph for the car. If there was an oncoming vehicle also moving at 100 km h^{-1}, then it too would travel around 360 m. Thus, including a safety margin of 100 m, the oncoming car must be at least 800 m away if you are to safely complete this overtaking manoeuvre. This is the reason that many drivers make poor decisions. They have not been aware of the large distances needed. The performance of the car and the relative speeds of the vehicles are the determining factors. How would the distances compare if a less powerful four-cylinder car was involved, or if the passing speed was only 5 km h^{-1} faster than the other vehicle?

2.3 SUMMARY Graphing motion: position, velocity and acceleration

- A position–time graph can be used to determine the location of a body directly. Additional information can also be derived in the following ways:
  - The displacement is given by the change in position.
  - The velocity of the body is given by the gradient of the position–time graph.
  - If the graph is curved, the gradient of the tangent at a point gives the instantaneous velocity.
- A velocity–time and acceleration–time graph can also be analysed to derive further information relating to the motion of a body.
- The gradient of a velocity–time graph is the acceleration of the object.
- The area under a velocity–time graph is the displacement of the object.
- The area under an acceleration–time graph is the change in velocity of the object.

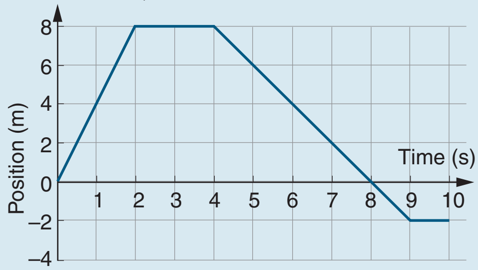
2.3 Questions

1 The graph shows the position of a dancer moving across a stage.



a What was the starting position of the dancer?
b In which of the sections (A–D) is the dancer at rest?
c In which of the sections is the dancer moving in a positive direction?
d In which of the sections is the dancer moving with a negative velocity?

The following information relates to questions 2–6. The graph represents the straight-line motion of a radio-controlled toy car.



2 Describe the motion of the car.

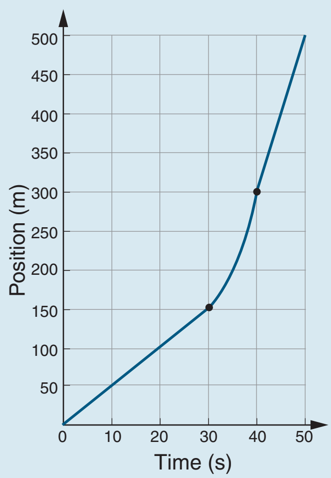
3 What was the position of the car after:
a 2 s? **b** 4 s? **c** 6 s? **d** 10 s?

4 When did the car return to its starting point?

5 What was the velocity of the car:
a during the first 2 s? **b** after 3 s?
c from 4 s to 8 s? **d** at 8 s?
e from 8 s to 9 s?

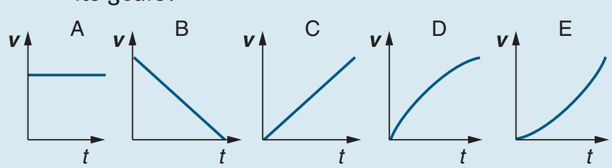
6 During its 10 s motion, what was the car's:
a distance travelled? **b** displacement?

7 The following position–time graph is for a cyclist travelling along a straight road.

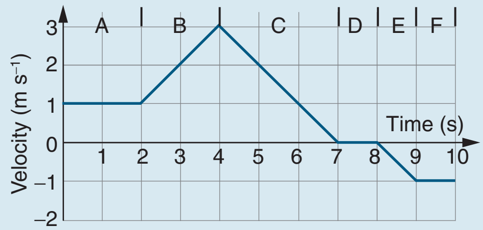


a Describe the motion of the cyclist.
b What was the velocity of the cyclist during the first 30 s?
c What was the cyclist's velocity during the final 10 s?
d Calculate the cyclist's instantaneous velocity at 35 s.
e What was the average velocity of the cyclist between 30 s and 40 s?

8 Which of the velocity–time graphs A–E best represents the motion of:
a a car coming to a stop at a traffic light?
b a swimmer moving with constant speed?
c a cyclist accelerating from rest with constant acceleration?
d a car accelerating from rest and changing through its gears?



The following information relates to questions 9–11. The following graph shows the motion of a dog running along a footpath. In this problem, north is considered to be positive.

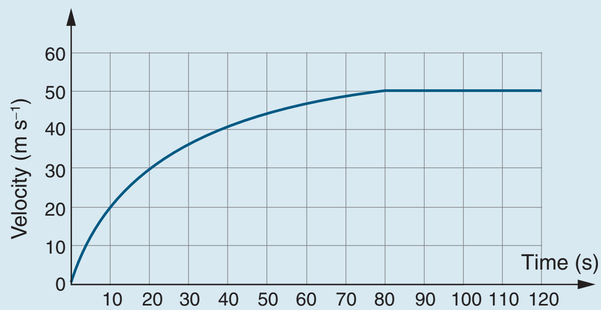


9 Describe the motion of the dog during these sections of the graph.
a A **b** B **c** C **d** D **e** E **f** F

10 Calculate the displacement of the dog after:
a 2 s **b** 7 s **c** 10 s.

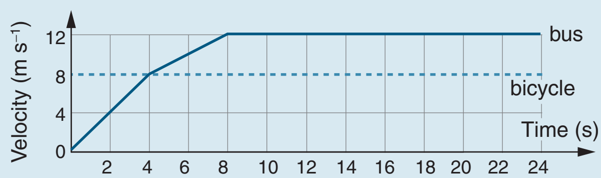
11 Plot a position–time graph of the dog's motion.

12 The straight-line motion of a high-speed intercity train is shown below.



a How long does it take the train to reach its cruising speed?
b What is the acceleration of the train 10 s after starting?
c What is the acceleration of the train 40 s after starting?
d What is the displacement of the train after 120 s?

13 The velocity–time graphs for a bus and a bicycle travelling along the same straight stretch of road are shown below. The bus is initially at rest and starts moving as the bicycle passes it.



a Calculate the initial acceleration of the bus.
b When does the bus first start gaining ground on the bicycle?
c At what time does the bus overtake the bicycle?
d How far has the bicycle travelled before the bus catches it?
e What is the average velocity of the bus during the first 8 s?

14 **a** Draw an acceleration–time graph for the bus discussed in question 13.
b Use your acceleration–time graph to determine the change in velocity of the bus over the first 8 s.

2.4 Equations of motion

Equations for uniform acceleration

A graph is an excellent way of representing motion because it provides a great deal of information that is easy to interpret. However, a graph is time-consuming to draw and at times values have to be estimated rather than precisely calculated. The previous section used the graph of a motion to determine the different quantities needed to describe the motion of a body. In this section, we will examine a more powerful and precise method of solving problems involving *uniform acceleration*. This method involves the use of a series of *equations* that can be derived from the basic definitions developed earlier.

Consider a body moving in a straight line with an *initial velocity* $\boldsymbol{u}$ and a *uniform acceleration* $\boldsymbol{a}$ for a time interval t. After time t, the body is travelling with a *final velocity* $\boldsymbol{v}$. Its acceleration will be given by:

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{(\boldsymbol{v} - \boldsymbol{u})}{t}$$

This can be rearranged as:

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t \qquad \text{......................... (i)}$$

The *average velocity* of this object is:

$$\boldsymbol{v}_{\text{av}} = \frac{\text{displacement}}{\text{time taken}} = \frac{\boldsymbol{s}}{t}$$

An *average velocity* $\boldsymbol{v}_{\text{av}}$ can also be found as the average of the initial and final velocities, i.e. $\boldsymbol{v}_{\text{av}} = \dfrac{(\boldsymbol{u} + \boldsymbol{v})}{2}$

So: $\dfrac{\boldsymbol{s}}{t} = \dfrac{(\boldsymbol{u} + \boldsymbol{v})}{2}$

This gives:

$$\boldsymbol{s} = \frac{(\boldsymbol{u} + \boldsymbol{v})t}{2} \qquad \text{...................... (ii)}$$

A graph describing this particular motion is shown in Figure 2.26. The displacement $\boldsymbol{s}$ of the body is given by the area under the velocity–time graph. The area under the velocity–time graph in Figure 2.26 is given by the combined area of the rectangle and the triangle:

Area = $\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2} \times t \times \Delta \boldsymbol{v}$

Since $\boldsymbol{a} = \Delta \boldsymbol{v}/t$ then $\Delta \boldsymbol{v} = \boldsymbol{a}t$ and this can be substituted for $\Delta \boldsymbol{v}$:

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2} \times t \times \boldsymbol{a}t$

$$\boldsymbol{s} = \boldsymbol{u}t + \tfrac{1}{2}\boldsymbol{a}t^2 \qquad \text{..................... (iii)}$$

Now making $\boldsymbol{u}$ the subject of equation (i) gives: $\boldsymbol{u} = \boldsymbol{v} - \boldsymbol{a}t$.

You might like to derive another equation yourself by substituting this into equation (ii). You should get:

$$\boldsymbol{s} = \boldsymbol{v}t - \tfrac{1}{2}\boldsymbol{a}t^2 \qquad \text{..................... (iv)}$$

Rewriting equation (i) with t as the subject gives: $t = (\boldsymbol{v} - \boldsymbol{u})/\boldsymbol{a}$.

Now if this is substituted into equation (ii):

$$\boldsymbol{s} = \left(\frac{\boldsymbol{u} + \boldsymbol{v}}{2}\right)t = \left(\frac{\boldsymbol{u} + \boldsymbol{v}}{2}\right) \times \frac{(\boldsymbol{v} - \boldsymbol{u})}{\boldsymbol{a}} = \frac{v^2 - u^2}{2\boldsymbol{a}}$$

Finally, transposing this gives: $v^2 = u^2 + 2\boldsymbol{a}\boldsymbol{s}$ .... (v)

Three of these equations are commonly used to solve problems where the acceleration is constant. They are summarised below.

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$\boldsymbol{s} = \boldsymbol{u}t + \tfrac{1}{2}\boldsymbol{a}t^2$$
$$v^2 = u^2 + 2\boldsymbol{a}\boldsymbol{s}$$

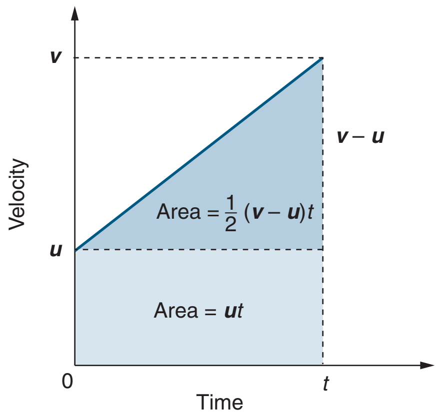


Figure 2.26
The area of the shaded rectangle and triangle represents the displacement $\boldsymbol{s}$ of the body for each time interval.

Physics file

The three equations of motion

$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$\boldsymbol{s} = \boldsymbol{u}t = \frac{1}{2}\boldsymbol{a}t^2$

$v^2 = u^2 + 2\boldsymbol{as}$

are equations that each describe the same thing—a body moving with uniform acceleration.

They all have initial velocity and acceleration since these two quantities determine what will happen to the body—they are the initial parameters. Each equation has just two of the other three quantities: $\boldsymbol{v}$, $\boldsymbol{a}$, t.

In any situation involving constant acceleration, choose the equation that has the quantities that you need to work with.

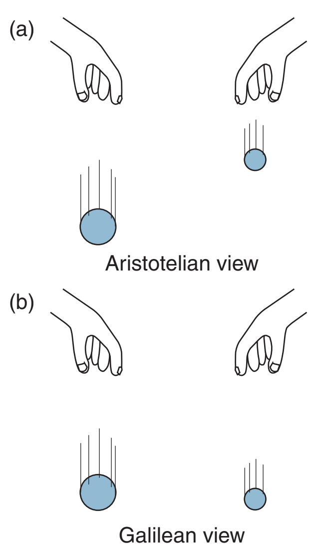


Figure 2.27
(a) Up until the 17th century, it was commonly thought that a heavy object would fall faster than a light object.
(b) After research by Galileo Galilei, it was shown that, if air resistance can be ignored, all bodies fall with an equal acceleration.

These equations can also be used with the scalar quantities speed and distance.

When solving problems by using these equations, it is important that you think about the problem and try to visualise what is happening. The following steps are advisable.

Step 1 Draw a simple diagram of the situation.

Step 2 Neatly write down the information that has been given in the question, using positive and negative values to indicate directions. Convert all units to SI form.

Step 3 Select the equation that matches your data.

Step 4 Use the appropriate number of significant figures in your answer.

Step 5 Include units with the answer and specify a direction if the quantity is a vector.

Worked Example 2.4A

A snowboarder in a race is travelling at 10 m s^{-1} as she crosses the finishing line. She then decelerates uniformly until coming to rest over a distance of 20 m.

a What is her acceleration as she pulls up?

b How long does she take to come to rest?

c Calculate the average speed of the snowboarder as she pulls up.

Solution

a When the snowboarder stops, her velocity is zero.

$\boldsymbol{u} = 10$ m s^{-1}, $\boldsymbol{v} = 0$, $\boldsymbol{s} = 20$ m, $\boldsymbol{a} = ?$

$v^2 = u^2 + 2\boldsymbol{as}$

$0 = 10^2 + 2 \times \boldsymbol{a} \times 20$

$\boldsymbol{a} = -2.5$ m s^{-2}

b $\boldsymbol{u} = 10$ m s^{-1}, $\boldsymbol{v} = 0$, $\boldsymbol{a} = -2.5$ m s^{-2}, $\boldsymbol{s} = 20$ m, $t = ?$

$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$0 = 10 - 2.5 \times t$

$t = 4.0$ s

c v_{av} = distance/time = 20/4.0 = 5.0 m s^{-1}

This could also have been determined by using:

$v_{av} = (u + v)/2 = (10 + 0)/2 = 5.0$ m s^{-1}

Vertical motion under gravity

Falling bodies are an interesting example of motion with a *constant acceleration*. Consider the motion of a golf ball that has been dropped. It is not greatly affected by air resistance and so *accelerates* as it falls. This is well known and explains why it would be no problem to catch a golf ball that had fallen just 1 m, but painful to catch one that had been dropped from a five-storey building. The longer the ball falls, the faster it travels. This property of falling bodies has been known since ancient times, but up until the 17th century it was thought that the acceleration depended on the mass of the body. In other words, people thought that a heavy mass would fall faster than a light mass.

Galileo Galilei changed this theory. He did a great deal of research on the motion of bodies on inclined planes—a sort of 'diluted acceleration due to gravity'. His work on these experiments laid the groundwork for Isaac Newton and his laws of motion.

Some falling objects are greatly affected by *air resistance*, for example a feather and a balloon. This is why these objects do not speed up as they fall. However, if air resistance is not significant, free-falling bodies near the Earth's surface will move with an equal downwards acceleration. In other words, the *mass of the object does not matter*. This is clearly shown in the multiflash photograph of Figure 2.28 where a tennis ball of mass 0.23 kg can be seen to fall at the same rate as a shotput of mass 5.4 kg. Given that the flash rate is 15 Hz and the markings are 10 cm apart, you should be able to calculate the acceleration of these objects and obtain a value close to 9.8 m s^{-2}. This value of 9.8 m s^{-2} is the *acceleration of bodies falling due to gravity* and is commonly represented as ***g***.

At the Earth's surface, the acceleration due to gravity is:
$\boldsymbol{g}$ = 9.8 m s^{-2} down

Free fall simply implies that the motion of the body is affected only by gravity, i.e. there is no air resistance and there are no rockets firing. It is also important to note that the acceleration of a free-falling body is always 9.8 m s^{-2} down, and does not depend on whether the body is falling up or down. For example, a coin that is dropped from rest will be moving at 9.8 m s^{-1} after 1 s, 19.6 m s^{-1} after 2 s, and so on. Each second, its speed increases by 9.8 m s^{-1}.

However, if the coin was launched straight up at 30 m s^{-1}, then after 1 s its speed would be 20.2 m s^{-1}, and after 2 s it would be moving at 10.4 m s^{-1}. In other words, each second it would slow down by 9.8 m s^{-1}.

Since the acceleration of a free-falling body is constant, it is appropriate to use the equations for uniform acceleration. It is often necessary to specify up or down as the positive or negative direction when doing these problems.

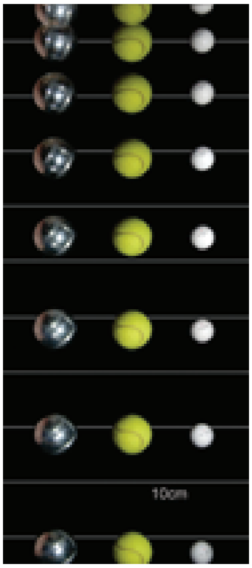


Figure 2.28
This multiflash photograph is taken with a frequency of 15 Hz and the scale markings are 10 cm apart. The photograph shows the relative motions of a tennis ball, shotput and a golf ball in free fall. Even though the shotput is over 20 times heavier than the tennis ball and the golf ball, all three objects fall with an acceleration of 9.8 m s^{-2} down.

Worked Example 2.4B

A construction worker accidently knocks a brick from a building so that it falls vertically a distance of 50 m to the ground. Using g = 9.8 m s^{-2}, calculate:

a the time the brick takes to fall the first 25 m
b the time the brick takes to reach the ground
c the speed of the brick as it hits the ground.

Solution

Down will be treated as the positive direction for this problem since this is the direction of the displacement.

a $u = 0$, $s = 25$ m, $a = 9.8$ m s^{-2}, $t = ?$

$s = ut + \frac{1}{2}at^2$

$25 = 0 + \frac{1}{2} \times 9.8 \times t^2$

$t^2 = 5.1$

$t = 2.3$ s

b $u = 0$, $a = 9.8$ m s^{-2}, $s = 50$ m, $t = ?$

$s = ut + \frac{1}{2}at^2$

$50 = 0 + \frac{1}{2} \times 9.8 \times t^2$

$t^2 = 10.2$

$t = 3.2$ s

Notice how the brick takes less time, only 0.9 s, to travel the final 25 m. This is because it is accelerating.

c $u = 0$, $a = 9.8$ m s^{-2}, $s = 50$ m, $t = 3.2$ s, $v = ?$

$v = u + at$

$= 0 + 9.8 \times 3.2 = 31$ m s^{-1}

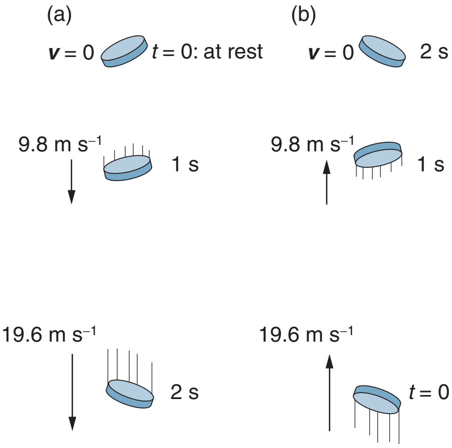


Figure 2.29
These coins are both moving with an acceleration of 9.8 m s^{-2} down.
(a) The speed of a coin falling vertically *increases* by 9.8 m s^{-1} each second, i.e. it has an acceleration of 9.8 m s^{-2} down. (b) The speed of a coin thrown upwards *decreases* by 9.8 m s^{-1} each second. It too has an acceleration of 9.8 m s^{-2} down.

Physics file

The acceleration of a falling object near the Earth's surface is 9.8 m s^{-2}. This quantity, g, can be used when describing large accelerations. For example, an acceleration of 19.6 m s^{-2} is $2g$. An astronaut will experience an acceleration of about $4g$ (39.2 m s^{-2}) at take-off. The forces involved give a crushing sensation similar to if the astronaut had three identical astronauts lying on top of him or her! Space missions are designed so that the acceleration does not exceed $6g$. Sustained accelerations greater than this can lead to the astronauts losing consciousness.

Physics file

The acceleration due to gravity on Earth varies according to the location. The strength of gravity is different on the surface of different bodies in the solar system depending on their mass and size.

table 2.7 The acceleration due to gravity at various locations around the solar system

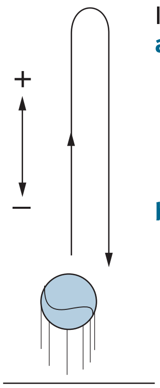
Location	Acceleration due to gravity (m s^{-2})
Perth	9.80
South Pole	9.83
Equator	9.78
Moon	1.60
Mars	3.60
Jupiter	24.600
Pluto	0.67

Worked Example 2.4C

On winning a tennis match the victorious player, Michael, smashed the ball vertically into the air at 30 m s^{-1}. In this example, air resistance will be ignored and the acceleration due to gravity will be taken as 10 m s^{-2}.

a Determine the maximum height reached by the ball.
b Calculate the time that the ball takes to return to its starting position.
c Calculate the velocity of the ball 5.0 s after being hit by Michael.
d Determine the acceleration of the ball at its maximum height.
e Draw an acceleration–time graph of the ball's motion.
f Draw a velocity–time graph of the ball's motion.

Solution



In this problem, up will be taken as positive.

a At the maximum height, the velocity of the ball is momentarily zero.
$\boldsymbol{u} = 30$ m s^{-1}, $\boldsymbol{v} = 0$, $\boldsymbol{a} = -10$ m s^{-2}, $\boldsymbol{s} = ?$
$v^2 = u^2 + 2\boldsymbol{as}$
$0 = (30)^2 + 2(-10)s$
$\therefore \boldsymbol{s} = +45$ m, i.e. the ball reaches a height of 45 m.

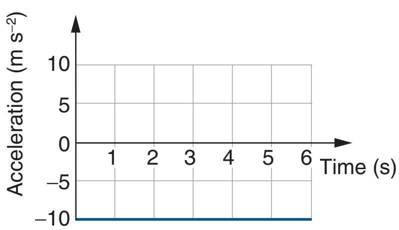
b To work out the time for which the ball is in the air, it is often necessary to first calculate the time that it takes to reach its maximum height.
$\boldsymbol{u} = 30$ m s^{-1}, $\boldsymbol{v} = 0$, $\boldsymbol{a} = -10$ m s^{-2}, $\boldsymbol{s} = 45$ m, $t = ?$
$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$
$0 = 30 + (-10 \times t)$
$\therefore t = 3.0$ s
The ball takes 3.0 s to reach its maximum height. It will therefore take 3.0 s to fall from this height back to its starting point and so the whole trip will last for 6.0 s.

c $\boldsymbol{u} = 30$ m s^{-1}, $\boldsymbol{a} = -10$ m s^{-2}, $t = 5.0$ s, $\boldsymbol{v} = ?$
$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$
$= 30 + (-10 \times 5.0) = -20$ m s^{-1}
The ball is travelling downwards at 20 m s^{-1}.

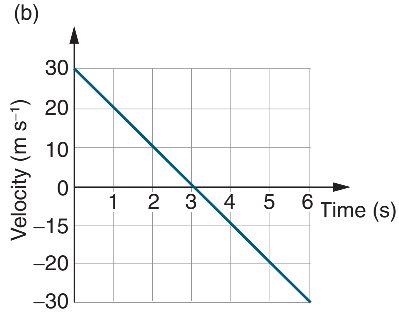
d The ball moves with an acceleration of 10 m s^{-2} down throughout its entire flight. Thus at its highest point, where its velocity is zero, its acceleration is still 10 m s^{-2} down.

e Since the acceleration of the ball is a constant –10 m s^{-2}, its acceleration–time graph will be as shown below in graph (a).

(a)



(b)



f The velocity–time graph above in graph (b) shows an initial velocity of 30 m s^{-1} reducing to zero after 3 s, then speeding up to –30 m s^{-1} after 6 s. The gradient of this graph is constant and is equal to –10, i.e. the acceleration of the ball.

Physics in action — How police measure the speeds of cars

Road accidents cause for the deaths of over 2000 people in Australia each year. Many times this number are seriously injured. Many steps have been taken to reduce the number of road fatalities. Some of these include random breath testing, speed cameras, mandatory wearing of bicycle helmets, and the zero alcohol level for probationary drivers.

One of the main causes of road trauma is speeding. In their efforts to combat speeding motorists, police employ a variety of speed-measuring devices. The two mostly used are radar and laser.

Speed camera radar

Camera radar units are usually placed in parked, unmarked police cars. These units take flash photographs of speeding vehicles, and also emit a radar signal of frequency 24.15 GHz (2.415×10^{10} Hz).

The radar antenna has a parabolic reflector that enables the unit to produce a directional radar beam 5° wide, thus allowing individual vehicles to be targeted. The radar signal allows speeds to be determined by using the Doppler principle, whereby the reflected radar signal from an approaching vehicle has a higher frequency than the original signal. Similarly, the reflected signal from a receding vehicle has a lower frequency. This change in frequency or 'Doppler shift' is processed by the unit and gives an instant measurement of the speed of the target vehicle.

Camera radar units are capable of targeting a single vehicle up to 1.2 km away. In traffic, the units can distinguish between individual cars and take two photographs per second. The photographs and infringement notices are then mailed to the offending motorists.

Laser speed guns

These devices are used by police to gain an instant measure of the speed of an approaching or receding vehicle. The unit is usually hand-held and is aimed directly at a vehicle by using a target sight. It emits a pulse of infrared radiation of frequency 331 THz (3.31×10^{14} Hz). As with camera radar units, speed is determined by the Doppler shift produced by the target vehicle. The infrared pulse is very narrow and directional—just one-sixth of 1° wide. This allows vehicles to be targeted with great precision. Hand-held units can be used at distances up to 800 m. If the speed registers as over the limit, the police are then likely to apprehend the offending driver.

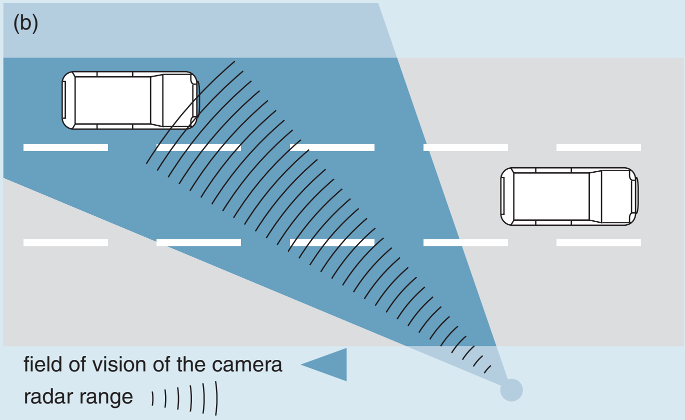


Figure 2.30
(a) A speed camera radar unit. (b) The unit emits a radar beam 5° wide which reflects from a target car and gives a measure of its speed.

Physics in action — Theories of motion: Aristotle and Galileo

Aristotle was a Greek philosopher who lived in the 4th century BC. He was such an influential individual that his ideas on motion were generally accepted for around 2000 years. Aristotle did not do experiments as we know them today, but simply thought about different bodies in order to arrive at a plausible explanation for their motion. He had spent a lot of time classifying animals, and so adopted a similar approach in his study of motion. His theory gave inanimate objects, such as rocks and rain, similar characteristics to living things. Aristotle organised objects into four terrestrial groups or elements: *earth*, *water*, *air* and *fire*. He said that any object was of a mixture of these elements in a certain proportion.

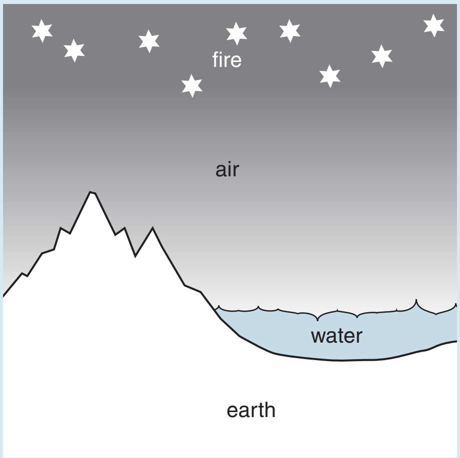


Figure 2.31
The Aristotelian terrestrial world consisted of earth, water, air and fire. According to this model, any type of matter has an inherent and natural tendency to return to its own state.

According to Aristotle, a body would move because of a tendency that could come from inside or outside the body. An internal tendency would cause 'natural' motion and result in a body returning to its proper place. For example, if a rock, which is an earth substance, is held in the air and released, its natural tendency would be to return to Earth. This explains why it falls down. Similarly, fire was thought to head upwards in an attempt to return to its proper place in the universe.

An external push that acts when something is thrown or hit was the cause of 'violent' motion in the Aristotelian model. An external push acted to take a body away from its proper place. For example, when an apple is thrown into the air, a 'violent' motion carries the apple away from the Earth, but then the 'natural' tendency of the apple takes over and it returns to its home. Aristotle's theory worked quite well and could be used to explain many observed types of motion. However, there were also many examples that it could not successfully explain, such as why some solids floated instead of sinking.

Aristotle explained the behaviour of a falling body by saying that its speed depended on how much earth element it contained. This suggested that a 2 kg cat would fall twice as fast and in half the time as a 1 kg cat dropped from the same height. Many centuries later, Galileo Galilei noticed that at the start of a hailstorm, small hailstones arrived at the same time as large hailstones. This caused Galileo to doubt Aristotle's theory and so he set about finding an explanation for the motion of freely falling bodies.

A famous story in science is that of Galileo dropping different weights from the Leaning Tower of Pisa. This story may or may not be true, but Galileo did perform a very detailed analysis of falling bodies. Galileo used inclined planes because freely falling bodies moved too fast to analyse. He completed extensive and thorough experiments that showed conclusively that Aristotle was incorrect. By using a waterclock to time balls as they rolled down different inclines, he was able to show that the balls were accelerating and that the distance they travelled was proportional to the square of the time, i.e. $x \propto t^2$. This relationship is evident when finding the distance travelled by a falling body released from rest:

$$s = ut + \frac{1}{2}at^2$$
$$s = 4.9t^2$$
$$\text{i.e. } s \propto t^2$$

Galileo found that this also held true when he inclined the plane at larger and larger angles, allowing him to conclude that freely falling bodies actually fall with a uniform acceleration.

2.4 SUMMARY Equations of motion

- Equations of motion can be used to analyse problems involving constant acceleration. These equations are:

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$\boldsymbol{s} = \boldsymbol{u}t + \tfrac{1}{2}\boldsymbol{a}t^2$$
$$v^2 = u^2 + 2\boldsymbol{as}$$

- If air resistance is not significant, bodies falling freely near the Earth will move with the same constant acceleration.
- The acceleration due to gravity is represented by $\boldsymbol{g}$ and is equal to 9.8 m s^{-2} in the direction of the centre of the Earth.
- The equations for uniform acceleration can be used to solve vertical motion problems. It is often necessary to specify a positive and negative direction.

2.4 Questions

1 A car starts from rest and accelerates uniformly for 8.0 s. It reaches a final speed of 16 m s^{-1}.
- **a** What is the acceleration of the car?
- **b** What is the average velocity of the car?
- **c** Calculate the distance travelled by the car.

2 A new model BMW can start from rest and travel 400 m in 16 s.
- **a** What is its average acceleration during this time?
- **b** Calculate the final speed of the car.
- **c** How fast is this final speed in km h^{-1}?

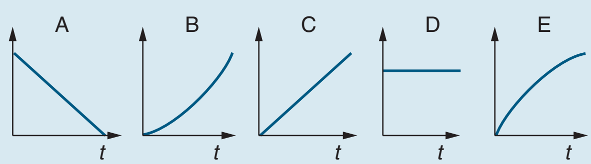
3 A car is travelling along a straight road at 75 km h^{-1}. In an attempt to avoid an accident, the motorist has to brake to a sudden stop.
- **a** What is the car's initial speed in m s^{-1}?
- **b** If the reaction time of the motorist is 0.25 s, what distance does the car travel while the driver is reacting to apply the brakes?
- **c** Once the brakes are applied, the car has an acceleration of –6.0 m s^{-2}. How far does the car travel whilst pulling up?
- **d** What total distance does the car travel from when the driver first notices the danger to when the car comes to a stop?

4 A billiard ball rolls from rest down a smooth ramp that is 8.0 m long. The acceleration of the ball is constant at 2.0 m s^{-2}.
- **a** What is the speed of the ball when it is halfway down the ramp?
- **b** What is the final speed of the ball?
- **c** How long does the ball take to roll the first 4.0 m?
- **d** How long does the ball take to travel the final 4.0 m?

5 A cyclist is travelling at a constant speed of 12 m s^{-1} when he passes a stationary bus. The bus starts moving just as the cyclist passes, and accelerates uniformly at 1.5 m s^{-2}.
- **a** When does the bus reach the same speed as the cyclist?
- **b** How long does the bus take to catch the cyclist?
- **c** What distance has the cyclist travelled before the bus catches up?

For questions 6–10, the acceleration due to gravity is 9.8 m s^{-2} down and air resistance is considered to be negligible.

6 A student drops a golf ball from a height of 5.0 m and uses a multiflash photograph to analyse its motion. The student designates down as the positive direction.



- **a** Which of the graphs A–E best represents the velocity of the ball?
- **b** Which of the graphs A–E best represents the acceleration of the ball?
- **c** How long does the ball take to reach the ground?
- **d** Calculate the speed of the ball as it hits the ground.

7 A book is knocked off a bench and falls vertically to the floor. If the book takes 1.0 s to fall to the floor, calculate:
- **a** its speed as it lands
- **b** the height from which it fell
- **c** the distance it falls during the first 0.5 s
- **d** the distance it falls during the final 0.5 s.

8 While celebrating her eighteenth birthday, a girl pops the cork off a bottle of champagne. The cork travels vertically into the air. Being a keen physics student, the girl notices that the cork takes 4.0 s to return to its starting position.
- **a** How long does the cork take to reach its maximum height?
- **b** What was the maximum height reached by the cork?
- **c** How fast was the cork travelling initially?
- **d** What was the speed of the cork as it returned to its starting point?
- **e** Describe the acceleration of the cork at each of these times after its launch:
  i 1.0 s **ii** 2.0 s **iii** 3.0 s.

9 At the start of a football match, the umpire bounces the ball so that it travels vertically and reaches a height of 15.0 m.
  a How long does the ball take to reach this maximum height?
  b One of the ruckmen is able to leap and reach to a height of 4.0 m with his hand. How long after the bounce should this ruckman endeavour to make contact with the ball?

10 A hot-air balloon is 80 m above the ground and travelling vertically downwards at 8.0 m s^{-1} when one of the passengers accidently drops a coin over the side. How long after the coin reaches the ground does the balloon touch down?

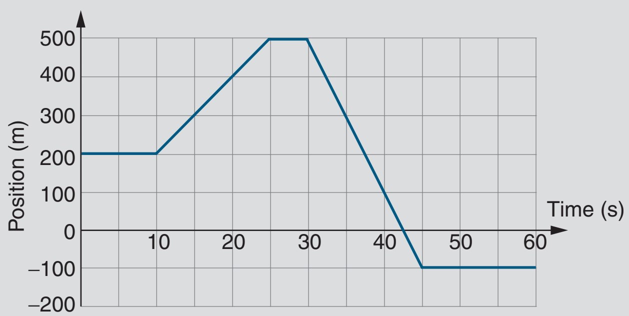
Chapter 2 Review

For the following questions, the acceleration due to gravity is 9.8 m s^{-2} down and air resistance is considered to be negligible.

The following information relates to questions 1–3.
During a game of mini-golf, a girl putts a ball so that it hits an obstacle and travels straight up into the air, reaching its highest point after 1.5 s.

1 Which one of the following statements best describes the acceleration of the ball while it is in the air?
  A The acceleration of the ball decreases as it travels upwards, becoming zero as it reaches its highest point.
  B The acceleration is constant as the ball travels upwards, then reverses direction as the ball falls down again.
  C The acceleration of the ball is greatest when the ball is at the highest point.
  D The acceleration of the ball is constant throughout its motion upwards and downwards.
2 What was the initial velocity of the ball as it launched into the air?
3 Calculate the maximum height reached by the ball.
4 Which one of the following is given by the area under a velocity–time graph:
  A the distance travelled during the time interval?
  B the acceleration of the object during the time interval?
  C the displacement of the object during the time interval?
  D the average velocity during the time interval?

The following information relates to questions 5–8.
The graph shows the position of a motorbike along a straight stretch of road as a function of time. The motorcyclist starts 200 m north of an intersection.



5 During what time interval is this motorcyclist:
  a travelling in a northerly direction?
  b travelling in a southerly direction?
  c stationary?
6 When does the motorcyclist pass back through the intersection?
7 Calculate the instantaneous velocity of the motorcyclist at each of the following times:
  a 15 s
  b 35 s.
8 For the 60 s motion, calculate the:
  a average velocity of the motorcyclist
  b average speed of the motorcyclist.

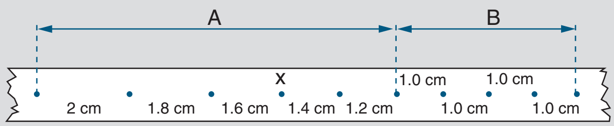
The following information relates to questions 9 and 10.
A skier is travelling along a horizontal ski-run at a speed of 10 m s^{-1}. After falling over, the skier takes 10 m to come to rest.

9 Which one of the following best describes the average acceleration of the skier?
  A –1 m s^{-2}
  B –10 m s^{-2}
  C –5 m s^{-2}
  D zero
10 Calculate the time that the skier takes to come to a stop.

The following information relates to questions 11 and 12.
An athlete in training for a marathon runs 15 km north along a straight road before realising that she has dropped her drink bottle. She turns around and runs back 5 km to find her bottle, then resumes running in the original direction. After running for 2.0 h, the athlete reaches 20 km from her starting position and stops.

11 Calculate the average speed of the athlete in km h^{-1}.
12 Calculate her average velocity in:
  a km h^{-1}
  b m s^{-1}.
13 A jet-ski starts from rest and accelerates uniformly. If it travels 2.0 m in its first second of motion, calculate:
  a its acceleration
  b its speed at the end of the first second
  c the distance that the jet-ski travels in its second second of motion.

The following information relates to questions 14 and 15.
A student performing an experiment with a dynamics cart obtains the ticker tape data as shown below. The ticker timer has a frequency of 50 Hz.



14 Calculate the average speed of the cart during:
  a section A
  b section B
  c its total journey.

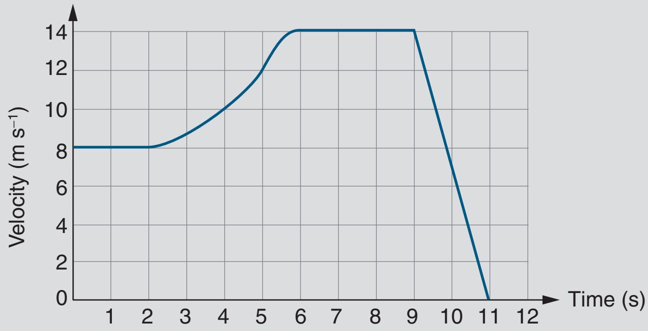
15 **a** What was the instantaneous speed of the cart when dot X was made?
  b Calculate the acceleration of the cart during section A.

The following information relates to questions 16 and 17.
Two physics students conduct the following experiment from a very high bridge. Thao drops a 1.5 kg shot-put from a vertical height of 60.0 m while at exactly the same time Benjamin throws a 100 g mass with an initial downwards velocity of 10.0 m s^{-1} from a point 10.0 m above Thao.

16 Calculate the time that:
  a the shot-put takes to reach the ground
  b the 100 g mass takes to reach the ground.

17 At what time will the 100 g mass overtake the shot-put?

The following graph relates to questions 18–20.
The velocity–time graph is for an Olympic road cyclist as he travels north along a straight section of track.



18 What is the average velocity of the cyclist during this 11 s interval?

19 Which one or more of the following statements correctly describes the motion of the cyclist?
  A He is always travelling north.
  B He travels south during the final 2 s.
  C He is stationary after 8 s.
  D He returns to the starting point after 11 s.

20 Calculate the acceleration of the cyclist at each of the following times:
  a 1 s
  b 5 s
  c 10 s.

21 Estimate the number of rotations of a roller-blade wheel while a skater is skating at a speed of 5 m s^{-1} for 5 minutes.

3 Forces and their effects

Although he did not know it at the time, Isaac Newton's work in the 17th century signalled an end to the transition in the way the world was conceived and understood. The transition was begun by Copernicus and Galileo, but Newton was able to use mathematics to develop laws and theories that could account for the motion of the heavens. These showed the universe to be a mechanism, regulated by simple natural laws that require no continuous external forces. The universe as taught by Aristotle and accepted until then was one that needed continual maintenance from a divine being.

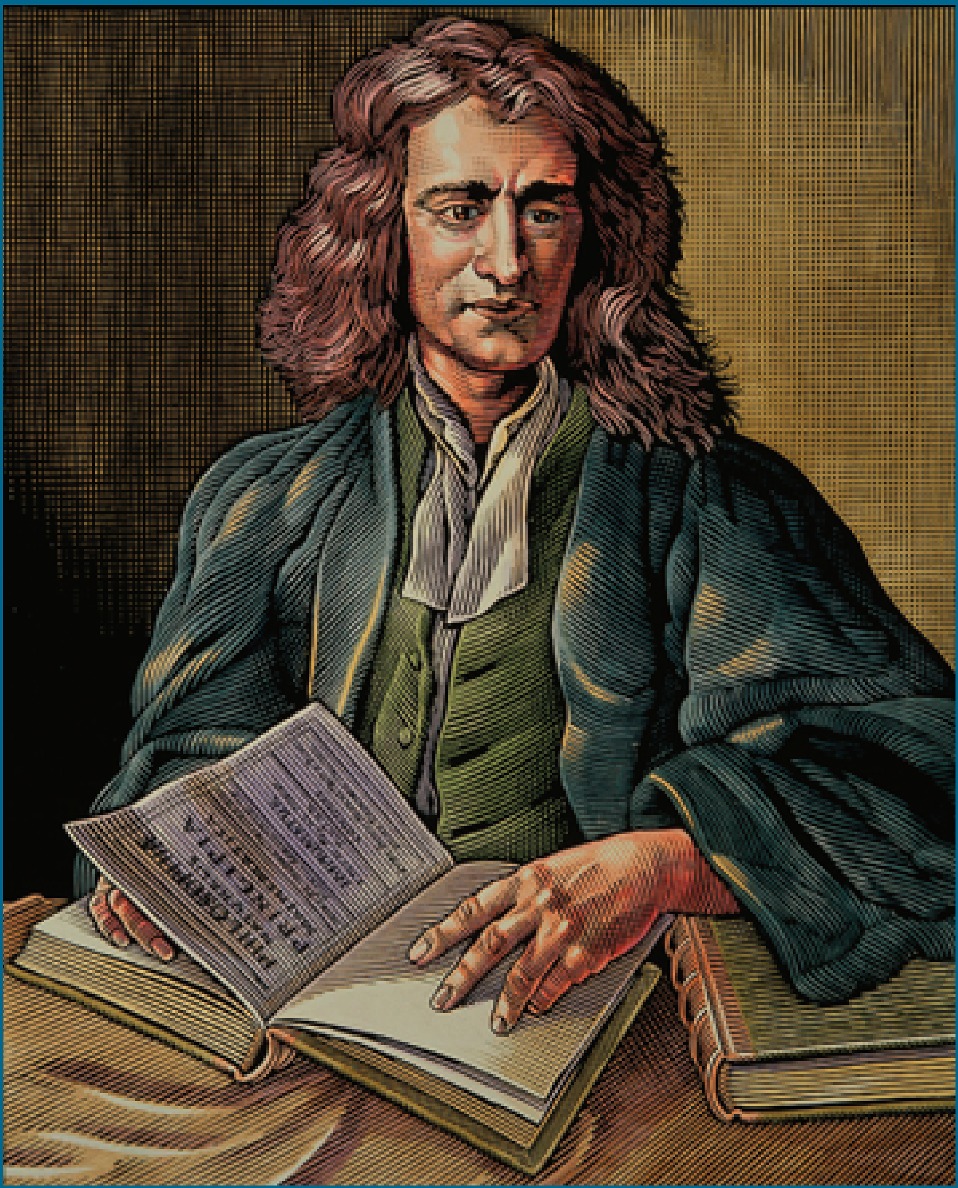
Isaac Newton was born in 1664 in rural England. He so impressed his mentors that he was made professor of mathematics at Cambridge University at the age of 26. His interests spanned light and optics, mathematics (he invented calculus), astronomy and the study of mechanics. His greatest achievement was the law of universal gravitation. This, along with a complete explanation of the laws that govern motion, is laid out in his book *Principia*, which was published in 1687 and is one of the most influential publications in natural science. Newton's framework for understanding the universe remained intact right up to the advent of Einstein's theory of relativity over 200 years later.

Newton died in 1727 a famous man. He remained self-critical and shy. That he could 'see so far' was only because he was 'able to stand on the shoulders of giants'. In saying this, he was referring to the work of Galileo.

By the end of this chapter

you will have covered material from the study of movement including:

- vector techniques in two dimensions
- forces in two dimensions
- Newton's laws of motion
- problems in mechanics including weight and friction.



3.1 Force as a vector

Chapter 2 developed the concepts and ideas needed to describe the motion of a moving body. This branch of mechanics is called *kinematics*.

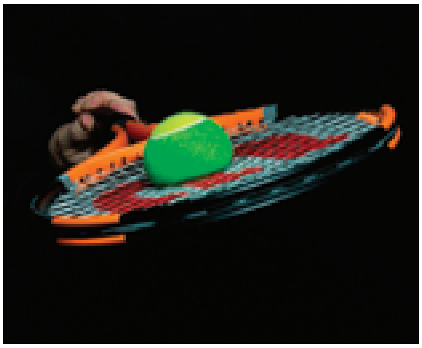
In this chapter, rather than simply describe the motion, we will consider the forces that cause the motion to occur. Treating motion in this way falls within the branch of mechanics called *dynamics*. In simple terms, a force can be thought of as simply a push or a pull, but forces exist in a wide variety of situations in our daily lives and are fundamental to the nature of matter and the structure of the universe. Consider each of the photographs in Figure 3.1 and identify each force—push or pull—that is acting.

In each of the situations depicted in Figure 3.1, forces are acting. Some are applied directly to an object and some act on a body without touching it. Forces that act directly on a body are called *contact forces*, because the body will only experience the force while contact is maintained. Forces that act on a body at a distance are *non-contact forces*.

Contact forces are the easiest to understand and include the simple push and pull that are experienced daily in people's lives. Examples of these include the forces between colliding billiard balls, the force on a bouncing ball and so on. Other contact forces include:

- adhesion—a force of attraction between different materials (e.g. Sellotape and paper, or the attraction between water and glass tubing that gives rise to a meniscus)
- cohesion—a force of attraction between particles of the same material (e.g. the attraction between water molecules that become rain droplets)
- surface tension—a force of attraction between molecules across the surface of a liquid, which can be strong enough in water to support the weight of some insects
- any force that occurs when two or more bodies actually touch.

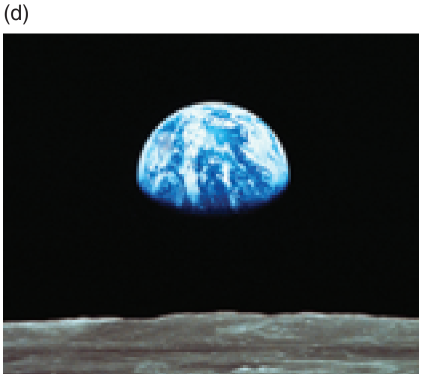
(a)



(b)

(c)

(d)



(e)

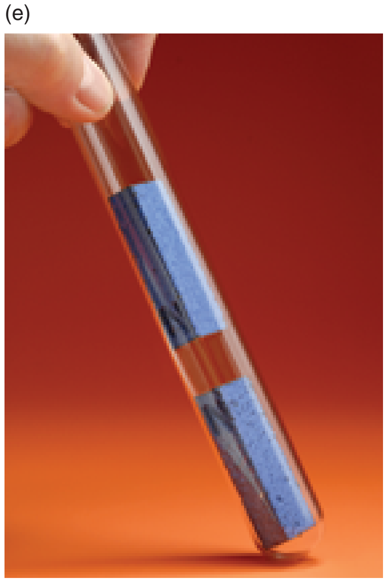


Figure 3.1
(a) At the moment of impact by a tennis racquet, a tennis ball is distorted to a significant extent. (b) The frictional force between the tyres and the road enables the Formula One car to complete the turn. (c) A continual force causes the clay to deform into the required shape. (d) The gravitational force between the Earth and the moon is responsible for two high tides each day. (e) The magnet is suspended in mid-air owing to a force of repulsion.

Non-contact forces occur when the object causing the push or pull is physically separated from the object that experiences the force. These forces are said to 'act at a distance'. Gravitation, magnetic and electric forces are non-contact forces.

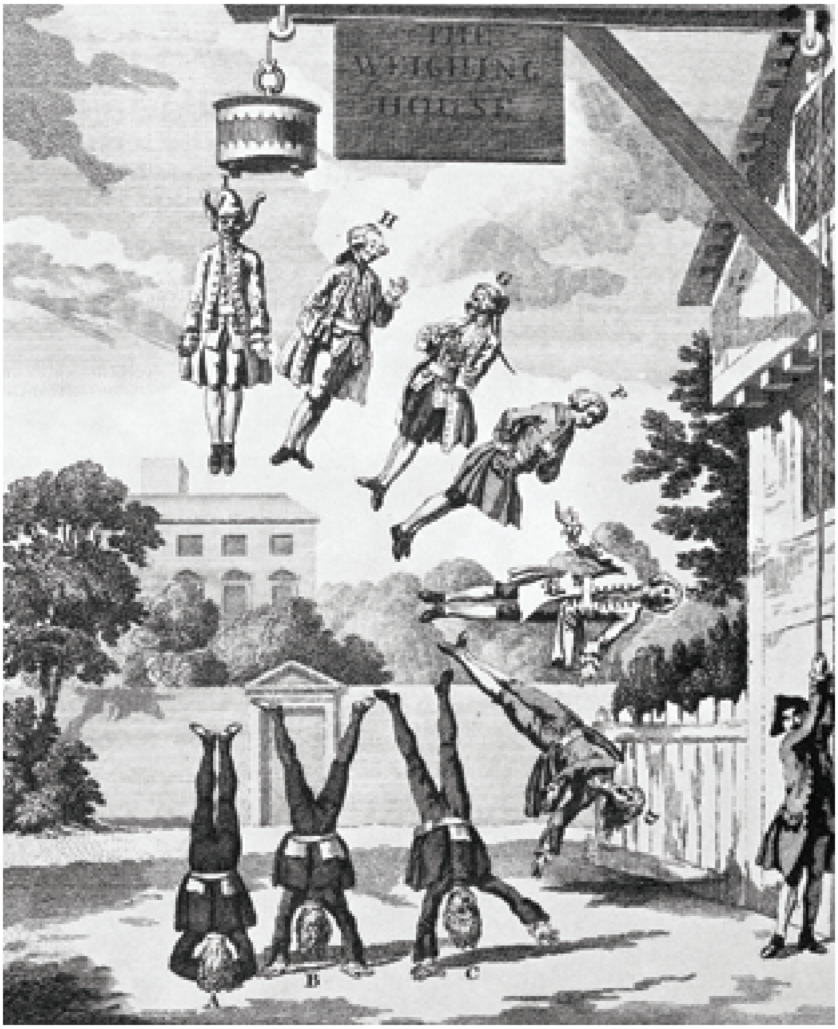


Figure 3.2
By the time of his death, Sir Isaac Newton had made outstanding contributions to the study of light and optics, astronomy, mathematics, mechanics and other areas of scientific inquiry. However, Newton's work was treated with scepticism by some. This is a cartoon from the times. Some of the captions are: A, absolute gravity; C, partial gravity; E, horizontal or good sense; H, partial levity or part fool; and I, absolute levity or stark fool.

The action of a force is usually recognised through its effect on an object or body. A force may do one or more of a number of things to the object. It may change its shape, change its speed or change only the direction of its motion. The tennis player in Figure 3.1a has applied a force to the tennis ball, and, as a consequence, the speed of the ball changes along with its direction. The ball also changes shape while the force acts!

The amount of force acting can be measured by using the SI unit called the *newton*, which is given the symbol N. The unit, which will be defined later in the chapter, honours Sir Isaac Newton (1642–1727), who even today is considered to be one of the most significant physicists to have lived. A force of one newton, 1 N, is approximately the force you have to exert when holding a 100 g mass against gravity. In everyday life this is about the same as holding a small apple. Table 3.1 provides a comparison of the magnitude of some forces.

Table 3.1 A comparison of the magnitude of various forces

Force	Magnitude (N)
Force on the electron in a hydrogen atom	10^{-7}
Holding a small apple against gravity	1
Opening a light door	10
Pedalling a bicycle	300
Thrust of a Boeing 747 at take-off	10^{6}
Gravitational force between the Earth and Sun	4×10^{22}

Force: a vector quantity

The magnitude of a force is not enough to completely define the force. The direction in which a force acts is also needed if the effect of the force is to be determined. When shooting to score in netball or basketball, not only must the force on the ball be of the correct size, but its direction must also be right. A force which is too large or small would cause the ball to either fly over the ring or fall short. The direction of the force is also crucial. If the force on the ball is delivered directly up in the air, there is no chance at all for the player to score. Again, if the ball is pushed horizontally, it will not gain any height and so cannot pass through the ring. In sport, as in most situations, a combination of the right magnitude of the force and its direction is needed.

Quantities associated with motion are either *vectors* or *scalars*. Scalars like time and mass cannot be given a direction. Only a magnitude can be provided. Quantities that require a direction in addition to their magnitude are vectors. Force is a vector quantity. Table 3.2 contains a summary of the scalar and vector quantities that have been covered in this book so far.

Figure 3.3
The netball will only go through the hoop if it is given a force of the right magnitude and direction. As force is a vector, it can only be completely specified if the direction is given as well as the magnitude.

A ***SCALAR*** quantity is completely described by a magnitude alone.
A ***VECTOR*** quantity requires both a magnitude and a direction.

Table 3.2 Common vector and scalar quantities and their symbols

Scalar quantity	Symbol	Vector quantity	Symbol
Time	t	Displacement	$\boldsymbol{s}$
Distance travelled	s	Velocity	$\boldsymbol{v}$
Speed	v	Acceleration	$\boldsymbol{a}$
Mass	m	Force	$\boldsymbol{F}$
Energy	E		
Temperature	T		

Some scalar and vector quantities are similar, but must not be confused. Speed and velocity have the same units, but are defined in different ways. An instantaneous speed of 5 m s^{-1} would become a velocity if it were known that the speed was in a certain direction, e.g. north. The velocity would therefore be 5 m s^{-1} north. But, if a car returns to the same place after completing a shopping trip, the average velocity would be zero (as the total displacement is zero) while the average speed would have a value, but no direction.

So a vector quantity, such as force or acceleration, must always include a direction in addition to the magnitude to be completely described. If a question only requires the magnitude of a vector, the direction can be ignored.

Representing force as a vector

Pictorially, a vector is represented as a 'directed line segment'. This is a line segment drawn to a specified scale whose length represents the magnitude of the vector, and whose direction is the same as that of the vector. A force $\boldsymbol{F} = 4$ N left, acting on a toy car, could be drawn as a 2 cm long line if a scale of 1 cm represented a 2 N force. The line is drawn with an arrow pointing toward the left as shown.

Figure 3.4
The force of 4 N to the left on the vehicle is represented by a 2 cm line segment pointing to the left, where the scale is 1 cm corresponding to 2 N.

If the force applied to the car is doubled, the force becomes $2\boldsymbol{F}$, i.e. 8 N left. Representing this force requires the length of the vector to be doubled to 4 cm with the same direction. Multiplying the force by a number (a scalar) affects only the magnitude of the force—the direction of the force remains unaffected. In the same way, half of the force, $\frac{1}{2}\boldsymbol{F}$, would be 2 N left (Figure 3.5).

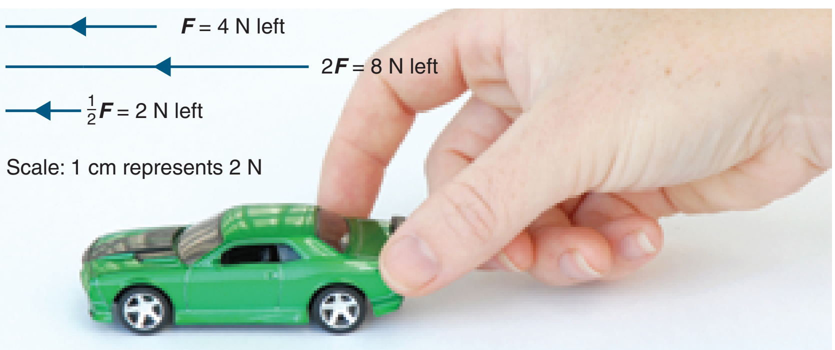


Figure 3.5
The force $\boldsymbol{F} = 4$ N left, which acts on the car, is doubled to $2\boldsymbol{F}$. This means that the magnitude of the force increases to 8 N with the same direction. If the force were halved, the force would be reduced to 2 N left.

When a vector is multiplied by a negative number, the negative sign is interpreted as reversing the direction of the vector. Hence a force of $-\boldsymbol{F}$ on the car constitutes a force of 4 N to the right (Figure 3.6).

Figure 3.6
When a vector is multiplied by a negative value, the direction of the vector is reversed. A force of ***F*** = 4 N left acts on the car. A force of –***F*** will be the same magnitude, 4 N, but directed to the right.

The product of a vector (e.g. force) and a scalar (e.g. a number) is an example of *scalar multiplication*. The effect is to change the magnitude of the vector but not its direction, unless the scalar is negative. Scalar multiplication can also occur when a vector quantity is multiplied by a scalar quantity in a mathematical relationship. In the previous chapter, the average velocity of a body was defined as the displacement of the body divided by the time taken. This can be thought of as the vector $\Delta \boldsymbol{s}$ being multiplied by the scalar $1/\Delta t$, i.e. $\boldsymbol{v}_{av} = \Delta \boldsymbol{s}/\Delta t$. The direction of the average velocity is identical to the direction of the displacement. The effect of the time taken in the relationship changes the magnitude of the vector only.

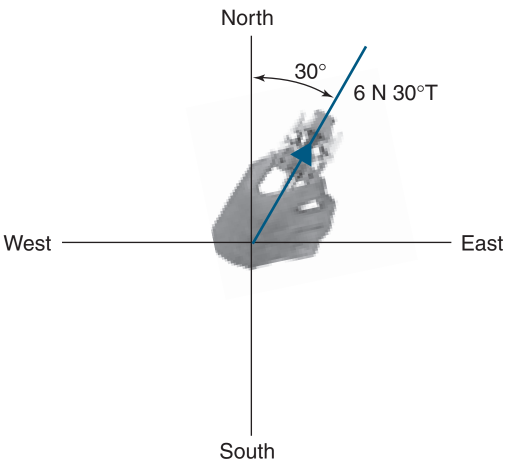


Figure 3.7
A force of 6 N 30°T is drawn as a vector 6 N long in a direction of 30° clockwise from north. The scale is: 1 cm represents 2 N.

Direction conventions

Until this point, directions in one dimension only—left and right, east and west etc.—have been described. More realistic situations require a description of the direction of a vector in a two-dimensional plane. Two common methods for describing the direction of a vector are available. In each case, the direction has to be referenced to a known direction.

A 'full circle bearing' describes north as 'zero degrees true'—written as 0°T. In this convention, all directions are given as a clockwise angle from north. 90°T is 90° clockwise from north, i.e. due east. Further examples have 180°T as 180° clockwise from north, i.e. due south, and 270°T as due west. A force of 6 N 30°T will be given as a vector 6 N long in a direction of 30° clockwise from north (Figure 3.7).

An alternative method rarely used in practical situations these days is to provide a 'quadrant bearing', where all angles are between 0° and 90° and so lie within one quadrant. The particular quadrant is identified by using two cardinal directions, the first being either north or south. In this method, 30°T becomes N30°E, literally '30° east of north'. North-west (NW) would be 315°T or N45°W (Figure 3.8).

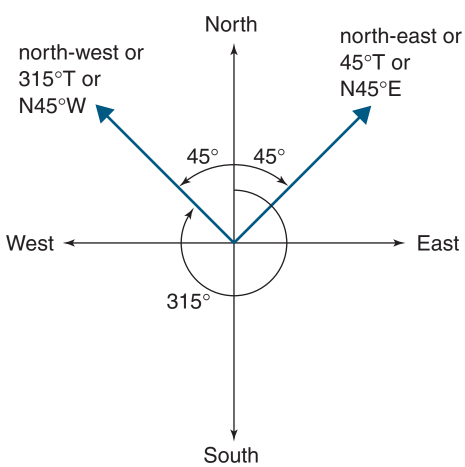


Figure 3.8
The directions north-east and north-west can be written as N45°E and N45°W respectively. North-east can also be shown as 45°T, while north-west is 315°T.

Worked Example 3.1A

The force applied by an athlete to a javelin is $\boldsymbol{F}$ = 200 N south 70° east.

a Determine values for $3\boldsymbol{F}$.

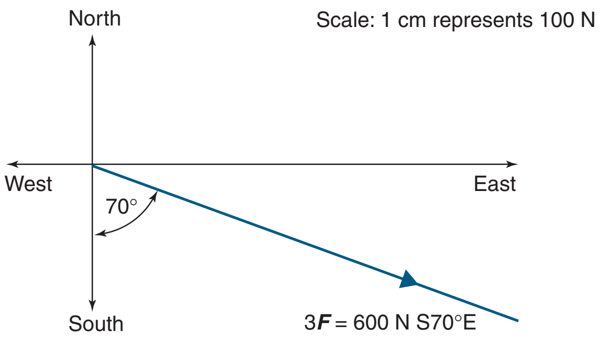
b Determine values for $-\boldsymbol{F}$.

c Determine the magnitude of $\frac{1}{4}\boldsymbol{F}$.

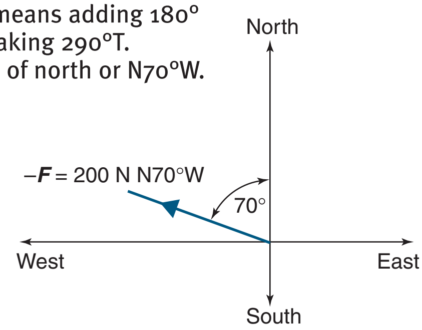
In parts a and b, draw accurate force vectors on a scale of 1 cm to represent 100 N.

Solution

a $3\boldsymbol{F} = 3 \times 200$ N south 70° east = 600 N south 70° east or 600 N S70°E



b $-\boldsymbol{F}$ = 200 N north 70° west as the direction has to be reversed. The direction S70°E is the same as 110°T. Reversing this means adding 180° to the angle, making 290°T. This is 70° west of north or N70°W.



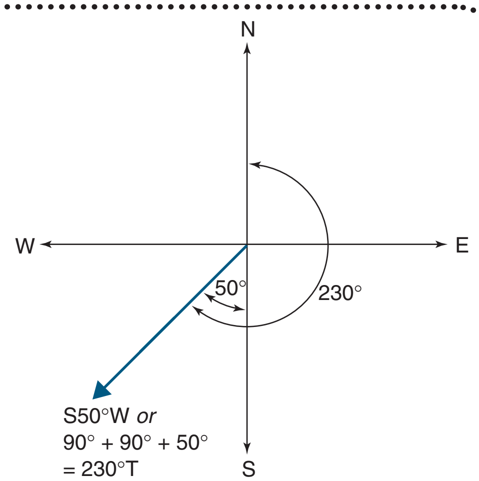
c The magnitude of $\frac{1}{4}\boldsymbol{F} = |\frac{1}{4}\boldsymbol{F}|$ = 50 N. There is no direction since only the magnitude is required.

Worked Example 3.1B

Convert the quadrant bearing S50°W to a full circle bearing in °T.

Solution

S50°W is 50° to the west of south. South is 180°T, so S50°W is 50° further west, i.e. 230°T. You can check this by using a compass or protractor.



3.1 SUMMARY Force as a vector

- A force is a push or a pull. Some forces act on contact while others can act at a distance.
- Force is a vector quantity whose SI unit is the newton (N).
- A vector can be represented by a directed line segment whose length represents the magnitude of the vector and whose direction is the direction of the vector.
- Directions can be defined by a full circle bearing (°T), or a quadrant bearing.
- When a vector is multiplied by a scalar, its magnitude is changed and its direction remains the same. A negative sign will reverse the direction of the vector.

3.1 Questions

1 a In your own words, explain the difference between a vector and a scalar quantity.
  b Under the headings 'scalar' and 'vector', list all the quantities about which you have learned during this year.
  c Use an everyday example of motion to explain the difference between the quantities 'average speed' and 'average velocity'.

2 If the force you have to exert when holding a small apple is about 1 N and that when holding a kilogram of sugar is 10 N, estimate the force required for:
  a using a stapler
  b kicking a beachball
  c lifting your school bag
  d opening this textbook.

3 Estimate the maximum force that you can exert when pulling on an anchored rope. What would be the approximate force that could be exerted by a ten-person tug-of-war team?

4 Which of the following directions are identical?
  A 40°T and S40°E
  B 140°T and S40°E
  C 200°T and S20°W
  D 280°T and N80°W

5 Convert the following into full circle bearings (i.e. °T).
  a N60°E
  b N40°W
  c S60°W
  d SE
  e NNE

6 If a force $\boldsymbol{F}$ is identified as 30 N east, find values for:
  a $2\frac{1}{2}\boldsymbol{F}$
  b $-6\boldsymbol{F}$
  c $-\frac{1}{2}\boldsymbol{F}$.

7 Use the vectors below to determine the forces represented in the following situations.

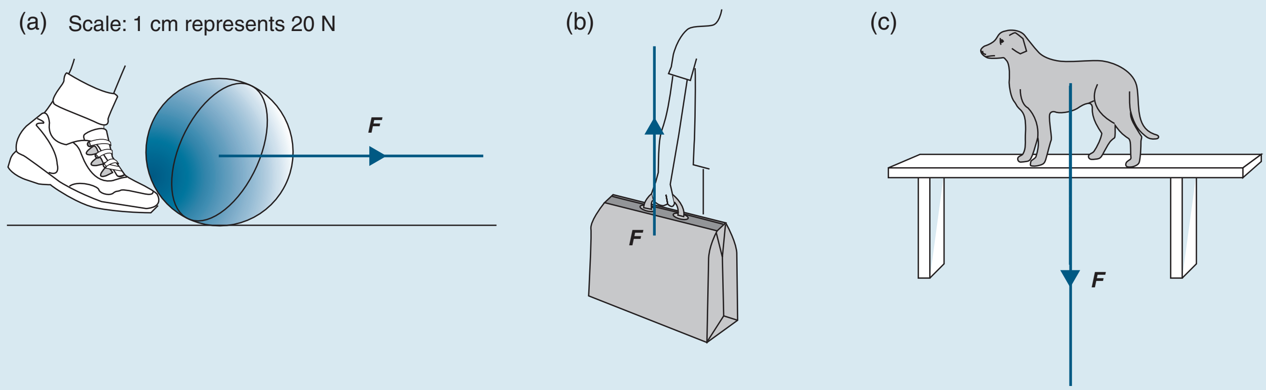


Figure 3.9
The golf ball moves in the direction of the applied force, which is in the direction of the line joining the centre of the club-head with the centre of the ball.

3.2 Vector techniques

If only one force acts on a body initially at rest, the body has no alternative but to move in the direction of that force. Striking a golf ball with a club illustrates this. When struck centrally, the ball moves off in the direction provided by the force from the club. The skill when playing golf is to ensure that the force is in the right direction!

If more than one force acts on a body at the same time, the body still behaves as if only one force is acting. This force is the *vector sum* of all the forces acting. Whenever more than one force acts, the sum of the forces is called the *resultant* or *net force*, $\Sigma \boldsymbol{F}$.

The **NET FORCE** acting on a body experiencing a number of forces acting simultaneously is given by the vector sum of all the individual forces acting:

$$\Sigma \boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \ldots + \boldsymbol{F}_n$$

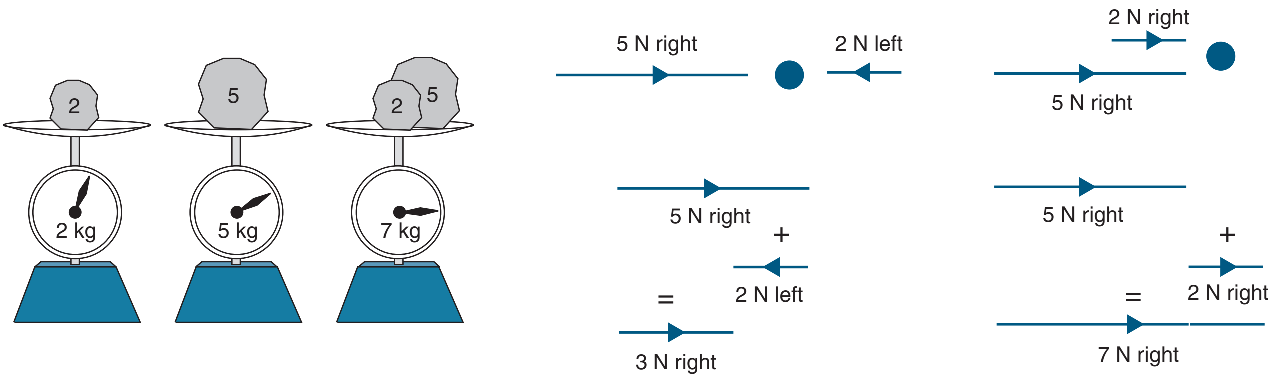


Figure 3.10

Mass is a scalar quantity, and so adding 2 kg and 5 kg of sugar can only produce a total of 7 kg. Force, on the other hand is a vector quantity, and the sum will depend on the direction of the individual forces. Adding 5 N and 2 N can give a magnitude of anywhere between 3 N and 7 N.

Because force is a vector quantity, the addition of a number of forces must be undertaken with the directions of the individual forces in mind. When scalars are added, the magnitudes add together just like numbers: 5 kg of sugar added to 2 kg of sugar will always be 7 kg.

When adding forces together, a vector sum is required. 5 N added to 2 N could be anything from 3 N to 7 N depending on the direction of the individual forces. If the forces were in opposite directions, e.g. 5 N to the right and 2 N to the left, the sum of the forces would be 3 N to the right. If the forces were in the same direction, say to the right, the sum will be 7 N to the right. Where the directions of the force are at an angle to each other, the resultant force will have a magnitude somewhere between the extremes of 3 N and 7 N.

Adding vectors graphically

When adding vectors graphically, a *vector diagram* is constructed, from which the required value is taken (Figure 3.11). The diagram can either be drawn to scale—in which case the resultant vector will be measured—or sketched, and a mathematical treatment used to get the solution. The following steps describe the process of adding vectors.

Step 1. Draw a small reference grid of the significant directions in two dimensions, e.g. north, south, east and west or up, down, left and right.

Step 2. Determine an appropriate scale for the diagram. Enough space should be made available to complete the diagram on the page. If the resultant vector is to be measured directly from the diagram, it has to be large enough for an accurate solution.

Step 3. Write out the vector equation to be represented, e.g. $\Sigma \boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2$.

Step 4. Draw the first vector from the equation to scale, $\boldsymbol{F}_1$, being careful to represent its length and direction accurately.

Step 5. From the head (arrow end) of the first vector, draw the vector to be added, $\boldsymbol{F}_2$, taking care with its length and direction. Repeat this step for any third or fourth vectors.

Step 6. The resultant vector (the net force, $\Sigma \boldsymbol{F}$) will be the directed line segment from the tail of the first vector to the head of the final vector. The magnitude of the resultant will be the length of the vector, and its direction is the direction of this vector.

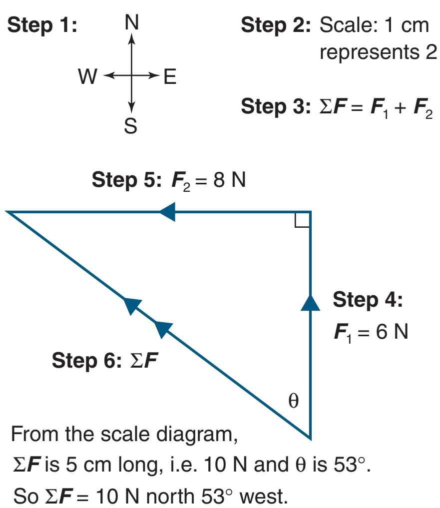
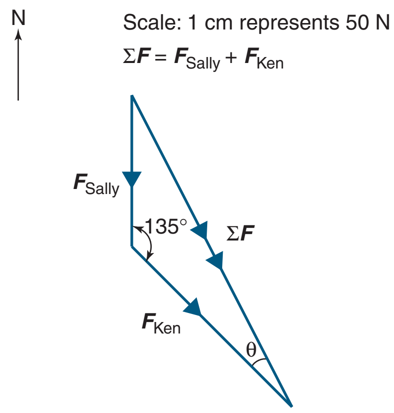


Figure 3.11

$\boldsymbol{F}_1$ = 6 N north and $\boldsymbol{F}_2$ = 8 N west are added using the steps in the text. The net force, $\Sigma \boldsymbol{F}$, is determined either mathematically or directly from the diagram. Placing the vector arrows in the centre of each vector ensures that arrows never 'collide'.

✓ Worked Example 3.2A

While playing at the beach, Sally and Ken kick a stationary beach ball simultaneously with forces of 100 N south and 150 N south-east, respectively. The ball moves as if it were only subjected to the net force. In what direction will it travel, and what is the magnitude of the force on the ball?



Solution

The net force is given by:

$\Sigma \boldsymbol{F} = \boldsymbol{F}_{Sally} + \boldsymbol{F}_{Ken}$

and a scale diagram is constructed by using the steps explained in the text. The magnitude of the force is determined by measuring the length of the resultant and converting it to newtons by using the scale,

i.e. 4.7 cm × 50 = 235 N.

The angle θ from the scale diagram is measured by a protractor at 27°, and so the direction in which the ball will travel is S63°E.

(Tip: The magnitude and direction could have been determined by mathematical techniques. In this case, the sine and cosine rules are needed. Pythagoras's theorem and simple trigonometry are used for situations involving a right-angled triangle.)

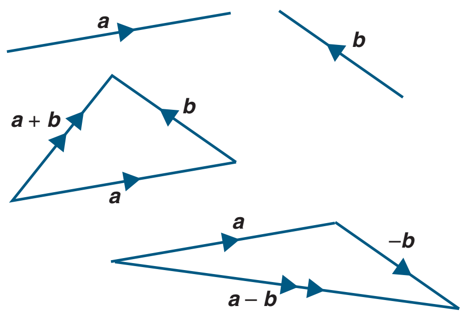


Figure 3.12

General vectors ***a*** and ***b*** are used to illustrate vector addition and subtraction. Vector subtraction is an addition where the direction of the second vector is reversed, e.g. ***a*** – ***b*** is the same as ***a*** + (–***b***).

On occasions, it is necessary to subtract one vector from another. Vector subtraction is straightforward if it is treated in the same way as vector addition (Figure 3.12). The difference is that the direction of the vector being subtracted has to be reversed. This means that in order to calculate say, $\boldsymbol{F}_1 - \boldsymbol{F}_2$, the equation is considered as $\boldsymbol{F}_1 + (-\boldsymbol{F}_2)$. The second vector—the one to be subtracted—is turned through 180°, i.e. reversed, and the problem reverts to vector addition.

Another way to approach vector subtraction is to use simple algebra and turn the equation into an addition. If $\boldsymbol{F}_3 = \boldsymbol{F}_2 - \boldsymbol{F}_1$, then rearranging, $\boldsymbol{F}_2 = \boldsymbol{F}_3 + \boldsymbol{F}_1$.

Vector components

It is often necessary to divide a force acting in a two-dimensional plane into two vectors. These two vectors are called the *components* of the force. This can be done because the force can be considered to act in each of the two directions at once. Consider, for example, wind directed to the south-west (S45°W). The force has some effect in a southerly direction and some in a westerly direction. The amount of force acting in each direction is called the component of the total force that acts in that direction. When the components are added together, the original force is the resultant.

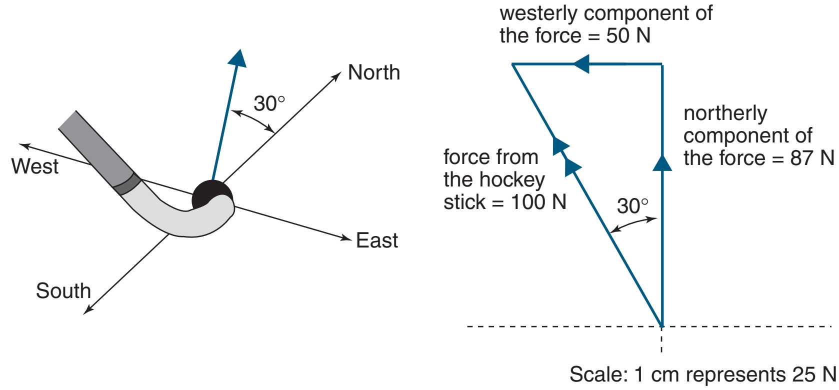
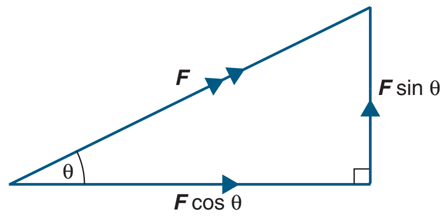


Figure 3.13

Striking a hockey ball with a force of 100 N north 30° west is equivalent to simultaneous forces of 87 N north and 50 N west. These two forces are the components of the force that acts on the ball.

To determine the components of a force, it is usual to construct a right-angled triangle around the force vector. The force vector is made the hypotenuse of the triangle, and the adjacent and opposite sides become the components of the force. Either trigonometry or a scale diagram can be used to calculate the components. To illustrate this, consider a stationary hockey ball struck with a force of 100 N north 30° west (Figure 3.13). The ball behaves as if it were simultaneously struck with forces of 50 N west and 87 N north. These are the components of the force.

In general terms, a force $\boldsymbol{F}$ acting at an angle θ to a given direction will have two components. $\boldsymbol{F}\cos\theta$ is the component along the reference direction, and $\boldsymbol{F}\sin\theta$ is perpendicular to that direction (Figure 3.14).



$\boldsymbol{F} = \boldsymbol{F}_{parallel} + \boldsymbol{F}_{perpendicular}$

Figure 3.14
The magnitude of the perpendicular components of a force $\boldsymbol{F}$ will be $\boldsymbol{F}\cos\theta$ and $\boldsymbol{F}\sin\theta$. A vector addition of the components will produce the initial vector.

Worked Example 3.2B

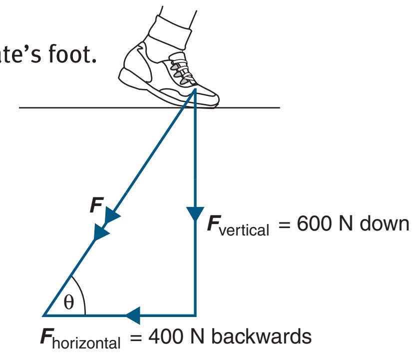
When walking, a person's foot pushes backwards and downwards at the same time. While playing basketball, Kate's foot pushes back along the court with a force of 400 N, and down with a force of 600 N. What is the actual force applied by Kate's foot?

Solution

400 N horizontal and 600 N down are the components of the force supplied by Kate's foot.
Therefore, the force she supplies will be
$\boldsymbol{F} = \boldsymbol{F}_{horizontal} + \boldsymbol{F}_{vertical}$ and a vector diagram is needed.
Using Pythagoras's theorem,

$$\boldsymbol{F} = \sqrt{\boldsymbol{F}_{horizontal}{}^2 + \boldsymbol{F}_{vertical}{}^2} = 400^2 + 600^2$$
$$= \sqrt{520\,000}$$
$$= 721\text{ N}$$
$$\theta = \tan^{-1}(600/400)$$
$$= \tan^{-1}1.5$$
$$= 56°$$

So Kate supplies a force of 720 N backwards at 56° down from the horizontal.

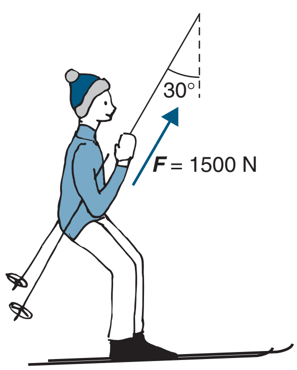


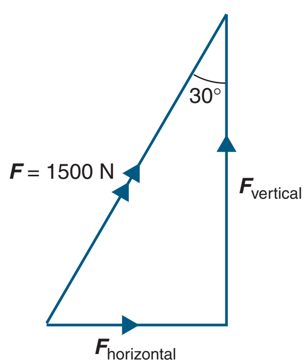
Worked Example 3.2C

A ski tow poma pulls a skier with a force of 1500 N at an angle of 30° to the vertical. Determine the horizontal and vertical components of the force.

Solution

A vector diagram is used to identify the horizontal and vertical components of the force.
Using trigonometry:
$\boldsymbol{F}_{vertical} = \boldsymbol{F}\cos\theta = 1500 \times \cos 30° = 1300$ N
and $\boldsymbol{F}_{horizontal} = \boldsymbol{F}\sin\theta = 1500 \times \sin 30° = 750$ N



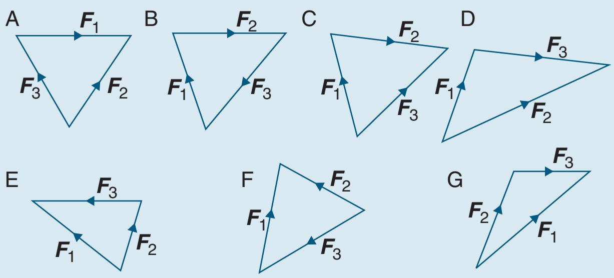


3.2 SUMMARY Vector techniques

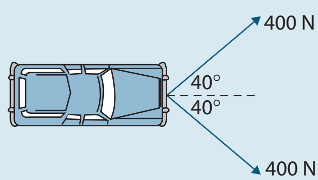
- The net force acting on a body that experiences a number of forces acting simultaneously is given by the vector sum of all the individual forces acting:
$$\Sigma F = F_1 + F_2 + \ldots + F_n$$
- Steps for adding vectors graphically:
Step 1. Draw a reference grid.
Step 2. Determine a scale for the diagram.
Step 3. Write the vector equation to be represented.
Step 4. Accurately draw the first vector from the equation to scale.
Step 5. From the head of the first vector, accurately draw the vector to be added. Repeat this step for any third or fourth vectors.
Step 6. The resultant vector is the directed line segment from the tail of the first vector to the head of the final vector.
- A vector addition may be also achieved by using a sketch vector diagram, which is solved using trigonometry.
- The perpendicular components of a force give the magnitude of the force acting in each of two perpendicular directions as a consequence of the force. A force $\boldsymbol{F}$ acting at an angle θ to a given direction will have components $F\cos\theta$ along the reference direction, and $F\sin\theta$ perpendicular to that direction.

3.2 Questions

1. Two forces are defined as F_1 = 25 N north and F_2 = 25 N east. Use scale diagrams to find the total force in the following vector additions.
   a $F_1 + F_2$
   b $2F_1$
   c $2F_1 + F_2$
   d $2F_1 + 2F_2$
2. If F_1 and F_2 are defined as in question 1, use a vector diagram to show that $F_1 + F_2 = F_2 + F_1$.
3. Consider the vector diagrams drawn below. Which one/s represent the following vector equations?
   a $F_1 + F_2 + F_3 = 0$
   b $F_1 + F_2 = F_3$
   c $F_1 - F_2 + F_3 = 0$
   d $F_1 - F_2 - F_3 = 0$
   e $F_2 + F_3 = F_1$



4. **a** A chair in a kindergarten is being pulled simultaneously by Elisa, Hugh and Rachel. Hugh pulls at 50 N north, Elisa applies a force of 50 N south 60° east and Rachel pulls at 50 N south 60° west. Use a scale diagram to find the resultant force on the chair.
   b Rachel now pulls a little harder at 60 N south 60° west. What is the new resultant force?
   c If Rachel lets go, in what direction will the chair move now?
5. Either use trigonometry or a scale diagram to add the following forces.
   a 3 N east and 4 N south
   b 60 N west and 80 N north
   c 5 N north 60° east and 5 N north 30° west
   d 1.5 N 20°T and 2.0 N 110°T
6. A small car is pulled by two people using ropes. Each person supplies a force of 400 N at an angle of 40° to the direction in which the car travels. What is the total force applied to the car?



7. Resolve the following forces into their perpendicular components around the north–south line. In the case of d, use the horizontal and vertical directions.
   a 100 N south 60° east
   b 60 N north
   c 300 N 160°T
   d 3.0×10^5 N 30° upward from the horizontal
8. What are the horizontal and vertical components of a 300 N force that is applied along a rope at 30° to the horizontal used to drag a Christmas tree from the boot of the car across the backyard?
9. A block of wood rests on an inclined plane which is tilted 30° to the horizontal. The weight of the wood can be considered as a force of 200 N acting downwards. Determine the component of the weight acting along the incline.
10. Two hockey players converge on a hockey ball and each player strikes the ball at the same time. If the two forces acting on the ball are 51 N south-west and 68 N north-west, what is the magnitude of the net force acting on the ball?

3.3 Newton's first law of motion

Aristotle and Galileo

The first attempt to explain why bodies move as they do was made over 2000 years ago by the Greek philosopher Aristotle. Aristotle and his followers felt that there was a 'natural state' for matter and that all matter would always tend towards its 'natural place' where it would be at rest. Aristotle's thesis was based on the everyday observation that a moving body will always slow down and come to rest unless a force is continually applied. Try giving this book a (gentle) push along a table top and see what happens!

Aristotle's ideas were an attempt to explain the motion of a body as it was seen, but they do not help to explain why a body moves as it does. It was not until the early 17th century that Galileo Galilei was able to explain things more fully. Galileo performed experiments that led him to conclude that the natural state of a moving body is not at rest. Significantly, Galileo introduced the idea that friction was a force which, like other forces, could be added together. A generation later, Newton developed Galileo's ideas further to produce what we now call the *first law of motion.*

To understand Newton's first law of motion, follow the logic in this thought experiment. Imagine you have to push an ice hockey puck across a roughened surface with a constant speed in a straight line. To do this, a relatively large force is needed to get the puck moving, but, once it is moving, a smaller, constant force only needs to be applied. The difference in the size of the forces for this situation is common in everyday experience.

If the puck is to be moved over a smoother surface than the first, the force that has to be supplied to keep it moving is reduced. Nonetheless, a constant force is still needed.

Imagine now, that the puck is struck over 'ideal, frictionless ice'. This surface no longer rubs against the puck. There is no force to slow the puck down, so it will simply keep moving with constant velocity. This is the breakthrough in understanding that Newton was able to make. Any body will continue with constant velocity if no net force ($\Sigma \boldsymbol{F} = 0$) acts upon it.

NEWTON'S FIRST LAW OF MOTION states that a body will either remain at rest or continue with constant speed in a straight line (i.e. constant velocity) unless it is acted on by a net force.

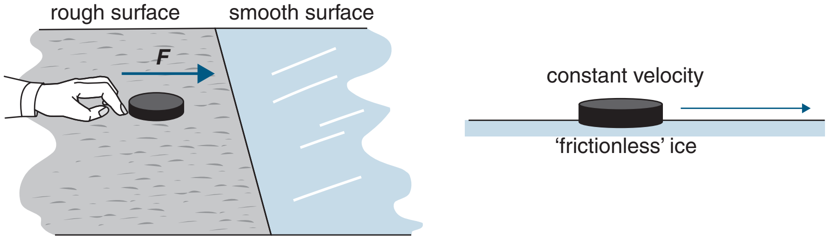


Figure 3.15

In order to keep the ice hockey puck moving with a constant velocity, a force is needed to overcome the friction that acts between it and the surface. When the puck moves over ice, where no friction acts, no force need be applied for the puck to maintain a constant velocity. This is the essence of Newton's first law of motion: that an object will continue with constant velocity unless acted upon by a net force.

Physics file

At the time of the Roman Empire some 2000 years ago, it cost as much money to have a bag of wheat moved 100 km across land as it did to transport it across the whole Mediterranean Sea. One of the reasons for this stemmed from the enormous friction that acted between the wheel and the axle in the cart of the day. Some animal fats were used as crude lubricants, but the effect was minimal. It was only during the last century or so that engineering provided a mechanical solution.

Today's wheels are connected to the axle by a wheel bearing. An outer ring is attached to the wheel, and an inner ring is attached to the axle. Separating the rings are a number of small ball bearings, which are able to roll freely between the rings. In this way, the area of contact and the friction between the wheel and axle are reduced dramatically.

Physics file

Whenever an object moves through a fluid (a liquid or a gas), fluid friction or drag acts. Air resistance is a very common example of a fluid friction. As an object moves through a fluid, the fluid will resist the motion so that the force is proportional to the speed: $\boldsymbol{F} \propto v$. Think about a spoon falling into honey!

To fly, an aircraft has to move the air in front of it out of its way. While air has only a small mass, the force required to do this is small, but as the speed increases, not only is there more mass to move, but the effect of the relative speed of the aircraft and the gas enhances the effect. Hence, drag forces increase with the square of the speed of the plane ($\boldsymbol{F} \propto v^2$). At high speed, drag forces become very large indeed. In fact, for a car travelling on the open road, air resistance will be the largest frictional force it experiences. For this reason, all transport designers must take drag forces very seriously, and these days, only the most aerodynamic of designs are used.

Figure 3.17
This car is typical of modern vehicles in that the body has been designed to offer the least possible air resistance. In turn, this increases the fuel efficiency of the car. Engineers and designers use computer modelling and experiments with models in wind tunnels in order to achieve the best result.

The motion of a spacecraft in deep space is a good example of a body moving with constant velocity as required by Newton's first law. Since there is no air in space to retard the craft, a satellite will continue with constant velocity so long as it is not retarded by a force in the opposite direction. For this reason, there is no need to make a satellite aerodynamic in shape.

Figure 3.16
Two Voyager spacecraft were launched from Cape Canaveral in 1977 with the mission to investigate the outer planets of the Solar System at close hand. Both craft completed the mission successfully, passing Saturn in 1981, Uranus in 1986 and Neptune in 1989. Voyagers 1 and 2 have now left the Solar System and since they have effectively no net force acting on them, they continue to travel away from the Earth with a constant velocity. Miraculously, both craft are still functioning—well beyond their expected life.

Forces in equilibrium

Newton's first law states that constant velocity will be achieved when no net force acts. This is easy to comprehend when considering the motion of, say, a spacecraft travelling at a great distance from the Earth. If gravitation is ignored, there is nothing to slow the craft down, and so it will continue with a constant velocity. This state of affairs can also occur where forces are involved, but their vector sum must be zero, i.e. $\Sigma \boldsymbol{F} = 0$. When the net force is zero, the forces are said to be in *equilibrium* or 'the forces are *balanced*'.

Worked Example 3.3A

During a car accident, a passenger travelling without a fastened seatbelt can fly through the windscreen and land on the road. Explain, using Newton's first law of motion, how this will occur.

Solution

During the accident, the car is brought to rest suddenly. The passenger without a fastened seatbelt will continue to travel with the original speed of the car until a force acts to slow him or her down. If the seatbelt were fastened, this would provide the necessary force to slow the passenger within the car. In the absence of an opposing force, the passenger continues to move—often crashing through the windscreen.

(The injuries received as a consequence of not wearing a seatbelt are usually far more serious than those received if the person were fixed in the car during an accident. This is the reason for the law requiring that seatbelts be worn.)

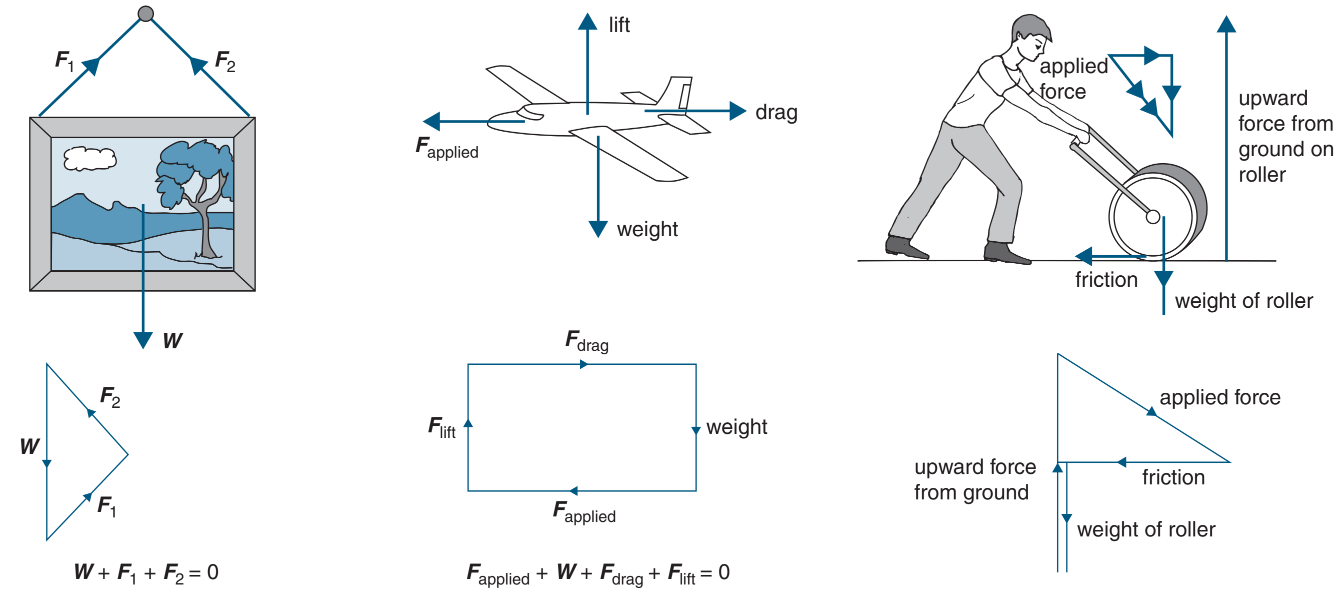


Figure 3.18

In these examples, the forces acting on the objects are in equilibrium—the net force is zero. The body will either remain at rest like the picture hanging on the wall, or continue with constant velocity like the aircraft. A more complex situation involves the groundsman pushing the heavy roller with constant velocity. The horizontal component of the force he applies along the handle exactly balances the frictional force that opposes the motion of the roller in the horizontal direction. The vertical component of the applied force acts downwards, and adds to the weight of the roller, but these two downward forces are balanced by an upward force provided by the ground.

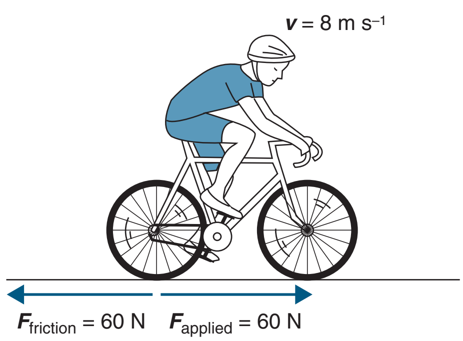
Worked Example 3.3B

A cyclist keeps her bicycle travelling with a constant velocity of 8.0 m s^{-1} west on a horizontal surface by continuing to pedal. A force (due to friction and air resistance) of 60 N acts against the motion. What force must be supplied by the rear wheel of the bicycle?

Solution

If the cyclist is to continue at a constant 8.0 m s^{-1} west, then the forces that act and the bicycle must be in equilibrium, i.e. $\Sigma \boldsymbol{F} = 0$. This means that just the forces due to air resistance and friction are exactly overcome by pedalling. A force of 60 N west must be produced at the rear wheel.

(The cyclist will actually have to produce more than 60 N as the gearing of the bike is designed to increase speed, not reduce the force that has to be applied.)



$$\Sigma \boldsymbol{F} = \boldsymbol{F}_{\text{applied}} + \boldsymbol{F}_{\text{friction}} = 0$$

Physics in action — Galileo Galilei—revolutionary

Galileo Galilei was born into an academic family in Pisa, Italy, in 1564. Galileo made significant contributions to physics, mathematics and the scientific method through intellectual rigor and the quality of his experimental design. But more than this, Galileo helped to change the way in which the universe was understood.

Galileo's most significant contributions were in astronomy. Through his development of the refracting telescope, he discovered sunspots, lunar mountains and valleys, the four largest moons of Jupiter (now called the Galilean moons) and the phases of Venus. In mechanics, he demonstrated that projectiles moved with a parabolic path and that different masses fall at the same rate (the law of falling bodies).

These developments were most important because they changed the framework within which mechanics was understood. This framework had been in place since Aristotle had constructed it in the 4th century BC. By the 16th century AD, the work of the Greek philosophers had become entrenched, and it was widely supported in the universities. It was also supported at a political level. In Italy at that time, government was controlled by the Catholic Church. Today one would think that Galileo would have been praised by his peers for making such progress, but so ingrained and supported was the Aristotelian view that Galileo actually lost his job as a professor of mathematics in Pisa in 1592.

Galileo was not without supporters, though, and he was able to move from Pisa to Padua where he continued teaching mathematics. At Padua Galileo began to use measurements from carefully constructed experiments to strengthen his ideas. He entered into vigorous debate in which his ideas (founded as they were on observation) were pitted against the 'philosophy' of the past and the politics of the day. The most divisive debate involved the motion of the planets. The ancient Greek view was that the Earth was at the centre of the Solar System and that all the planets, the Moon and the Sun were in orbit around it (known as the Ptolemaic view after the Greek astronomer Ptolemy). This view was taught by the Church and was also supported by 'common sense'. As such, it was accepted as the establishment view. In 1630 Galileo published a book in which he debated the Ptolemaic view and the new 'sun-centred' model proposed by Copernicus. Based on his own observations, Galileo supported the Copernican view of the universe. However, despite the book having passed the censors of the day, Galileo was summoned to Rome to face the Inquisition for heresy. The finding went against Galileo, and all copies of his book had to be burned and he was sentenced to permanent house arrest for the term of his life.

Galileo died in 1642 in a village near Florence. He had become an influential thinker across Europe and the scientific revolution he had helped start accelerated in the freer Protestant countries in northern Europe. For its part, the Catholic Church under Pope John Paul II began an investigation into Galileo's trial in 1979, and in 1992 a papal commission reversed the Church's condemnation of him.

Figure 3.19

Galileo Galilei (1564–1642) made significant contributions to our understanding of the forces that act on moving bodies. In his book *Principia*, Newton was quick to acknowledge his debt to Galileo's genius. This portrait was drawn 8 years before his trial.

3.3 SUMMARY Newton's first law of motion

- Aristotle understood that the natural state of matter was to be at rest in its natural place.
- Galileo concluded that the natural state of a moving body was not at rest, and that friction was a force.
- Newton developed Galileo's ideas further and devised the first law of motion, stated as 'A body will either remain at rest or continue with constant velocity unless it is acted on by a non-zero net force (or an unbalanced force).'
- Where the net force on a body is zero, i.e. $\Sigma \boldsymbol{F} = 0$, the forces are said to be balanced and are in equilibrium.

3.3 Questions

1 In a few sentences, distinguish between the understanding held by Aristotle and Newton about the 'natural state of matter'. Describe an experiment that might help support each of these views.

2 If a person is standing up in a moving bus that stops suddenly, the person can easily fall forwards. Has a force acted on the person? Use Newton's first law of motion to explain what is happening.

3 Why is it difficult to begin walking on smooth ice wearing shoes with hard leather soles? How is an ice skater able to gather speed on the ice?

4 What horizontal force has to be applied to a 'wheelie bin' if it is to be wheeled to the street on a horizontal path against a frictional force of 20 N at a constant 1.5 m s^{-1}?

5 When flying at constant speed at a constant altitude, a light aircraft has a weight of 50 kN down, and the thrust produced by the engines is 12 kN to the east. What is the lift force required by the wings of the plane, and what drag is acting?

6 A young boy is pulling his go-kart at a constant velocity with a horizontal rope. The kart opposes the motion due to a force of friction of 25 N.
  a What force must the boy apply to the cart?
  b The boy's father then attaches a longer rope to the kart because the short rope is uncomfortable to use. The rope now makes an angle of 30° to the horizontal. What is the horizontal component of the force that the boy needs to apply in order to move the kart with constant velocity?
  c What is the force acting along the rope that must be supplied by the boy?

7 Use Newton's first law of motion to help explain the reasons for wearing a seatbelt in a car or aircraft.

8 Consider the following situations, and name the force that causes each object to *not* move in a straight line.
  A The Earth moves in a circle around the Sun with constant speed.
  B An electron orbits the nucleus with constant speed.
  C A cyclist turns a corner at constant speed.
  D An athlete swings a hammer in a circle with constant speed.

3.4 Newton's second law of motion

Figure 3.20
This sprinter has just left the starting blocks because there is a net force acting on her. The starting blocks enable her to increase the horizontal component of the force produced by her legs, thus increasing her forward acceleration.

Newton's first law of motion explains that when all the forces on a body are balanced, the body can only remain at rest or continue with constant velocity—the real 'natural state' for matter. The second law of motion described by Newton deals with situations involving a body influenced by a non-zero net force, i.e. $\Sigma \boldsymbol{F} \neq 0$, in other words, when the forces are unbalanced.

When there is a non-zero net force acting on a body, the body will accelerate in the direction of the net force. Newton explained that the rate of the acceleration will depend on both the size of net force and the mass of the body. Experiment shows that the acceleration produced by a given accelerating force is directly proportional to the size of the net force acting, i.e. $\boldsymbol{a} \propto \Sigma \boldsymbol{F}$.

Also from experiment, it is possible to show how the acceleration produced by a given net force depends on the mass of the body. We know that a greater mass has a greater inertia, and so it will be more difficult to accelerate. Not surprisingly, experiments reveal that the acceleration produced is inversely proportional to the mass of the body, i.e. $a \propto \frac{1}{m}$. You only have to think of throwing a light and a heavy shot-put to comprehend this relationship.

If the two relationships are combined, we get:

$$\boldsymbol{a} \propto \Sigma \boldsymbol{F} \times \frac{1}{m}$$

or:

$$\boldsymbol{a} \propto \frac{\Sigma \boldsymbol{F}}{m}$$

The relationship can be converted into an equality by including a constant of proportionality, so:

$$\boldsymbol{a} = k\frac{\Sigma \boldsymbol{F}}{m}$$

By definition, 1 newton is the force needed to accelerate a mass of 1 kg at 1 m s^{-2}, so this makes the constant k equal to 1. The relationship is therefore simplified to $\Sigma \boldsymbol{F} = m\boldsymbol{a}$, a mathematical statement of Newton's second law of motion.

NEWTON'S SECOND LAW OF MOTION states that the acceleration of a body, $\boldsymbol{a}$, is directly proportional to the net force acting on it, $\Sigma \boldsymbol{F}$, and indirectly proportional to its mass, m:

$$\Sigma \boldsymbol{F} = m\boldsymbol{a}$$

The SI unit, the newton (N), will be required for force when the mass of the accelerating body is given in kilograms (kg) and its acceleration is provided in metres per second squared (m s^{-2}). 1 N = 1 kg m s^{-2}.

$\Sigma \boldsymbol{F} = m\boldsymbol{a}$ is a vector equation where the direction of the acceleration is the same as for the net force. This can be easily seen if the equation is seen as an acceleration vector $\boldsymbol{a}$ multiplied by the scalar m. If only one force acts, the acceleration will be in the direction of that force.

Worked Example 3.4A

Determine the size of the force required to accelerate an 80 kg athlete from rest to 12 m s^{-1} in 5.0 s.

Solution

$\Sigma \boldsymbol{F} = m\boldsymbol{a}$

$= m(\boldsymbol{v} - \boldsymbol{u})/t$

$= 80 \times \dfrac{12 - 0}{5}$

$= 192$ N

If more than one force acts on a body, the acceleration will be in the direction of the net force, i.e. the vector sum of all of the forces.

Worked Example 3.4B

A 150 g hockey ball is simultaneously struck by two hockey sticks. If the sticks supply a force of 15 N north and 20 N east respectively, determine the acceleration of the ball, and the direction in which it will travel.

Solution

To begin, it is necessary to calculate the net force on the ball:

$\Sigma \boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2$

$\Sigma F = \sqrt{F_1^{\,2} + F_2^{\,2}}$

$= \sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625} = 25$ N

$\boldsymbol{a} = \dfrac{\Sigma F}{m} = \dfrac{25}{0.15}$

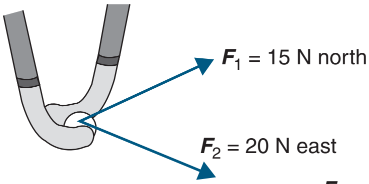
$= 170$ m s^{-2}

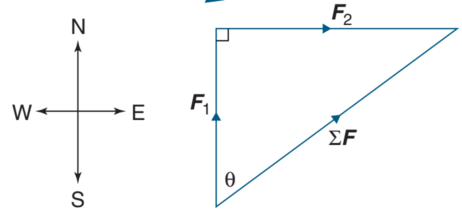
From the vector diagram showing the addition of the forces, we can see that θ will be given by:

$\tan\theta = \dfrac{F_2}{F_1} = \dfrac{20}{15}$

so $\theta = 53°$

So the ball will travel in the direction N53°E.





Worked Example 3.4C

A freestyle swimmer whose mass is 75 kg applies a force of 350 N as she begins a race. The water opposes her efforts to accelerate with a drag force of 200 N. What is her initial acceleration?

Solution

The net force on the swimmer in the horizontal direction will be:

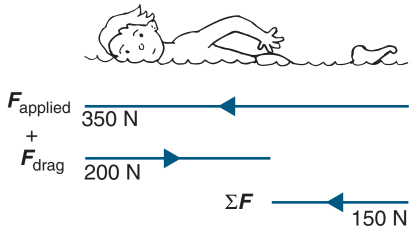
$$\Sigma \boldsymbol{F} = \boldsymbol{F}_{\text{applied}} + \boldsymbol{F}_{\text{drag}}$$

Since these forces are in opposite directions, $\Sigma F = 350 - 200 = 150$ N forwards:

So, $\boldsymbol{a} = \Sigma \boldsymbol{F}/m = 150/75$

$= 2.0$ m s^{-2} in the direction of the applied force

(It is worth noting that the drag applied by the water will increase with the swimmer's speed.)



Weight and mass

Mass of a body

To this point, the idea of the *mass* of an object has been taken for granted. However, the concept of a body's mass is rather subtle and, importantly in physics, the mass of a body is a fundamentally different quantity from its weight—even though people (even physics teachers) tend to use these expressions interchangeably in everyday life.

Physics file

Mass is usually considered to be an unchanging property of an object. This is true in Newtonian mechanics where the speed with which an object is considered to travel matches everyday experience. However, at very high speeds, Newton's laws of motion do not apply, and the theory of relativity must be used. In 1905, Albert Einstein showed that a body with a rest mass (i.e. mass when stationary) m_0 will experience an increase in mass as it gets faster. This increase is usually undetectable except when the object nears the speed of light. At these very high speeds, the mass will start to become greater and greater, tending to infinity as the speed approaches the speed of light.

The equation that Einstein found gives the mass, m, of a body whose rest mass is m_0, when travelling with speed v. The speed of light is given the symbol c.

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Table 3.3 gives the mass of a 1 kg block if it were to travel at speeds of $0.1c$ (10% of the speed of light or 3×10^7 m s^{-1}), $0.8c$ and $0.99c$.

Table 3.3 The mass of a 1 kg block at different speeds

Speed	Mass
$0.1c$	1.0050 kg (i.e. 5 g increase)
$0.8c$	1.6667 kg (667 g increase)
$0.99c$	7.1 kg (over 600% increase)

Relativistic mass increase provides the reason why no object can travel at the speed of light. To do so would require an infinite quantity of energy since the mass of the body would itself be infinite. Only objects with no mass (such as light 'particles') can travel at the speed of light.

In earlier science courses, mass may have been defined as 'the amount of matter' in an object. To understand what mass really is, this description says very little. The international standard for the kilogram is not very helpful either. Since the time of the French Revolution (late 1700s), the kilogram has been defined in terms of an amount of a standard material. At first, one litre of water at 4°C was used to define the kilogram. More recently an international mass standard has been introduced. This 1 kg cylinder of platinum–iridium alloy is kept in Paris, and copies are made from the standard and sent around the world.

Newton's second law can help to provide a better understanding of mass through the effect of a force on a massive body. Think about a mass resting on a frictionless surface. If a force is applied to the mass in the horizontal direction, an acceleration is produced given by $\boldsymbol{a} = \boldsymbol{F}/m$. The greater the mass, the smaller will be the acceleration. If the mass is reduced, the acceleration will increase. Here, the mass can be seen as the property of the body resisting the force. Mass is the closest quantity in physics to the concept of inertia.

If the above experiment is repeated, the same net force acting on the body will give the same acceleration regardless of where the experiment is performed. This is because—on Earth, on the Moon, in space—the mass of the body remains the same. Mass is a property of the body, and is not affected by its environment. In fact, for any situation at this level in physics, the mass of a body will be a constant value.

The (inertial) ***MASS*** of a body is its ability to resist acceleration when the body is acted on by a net force. Mass is a constant property of the body.

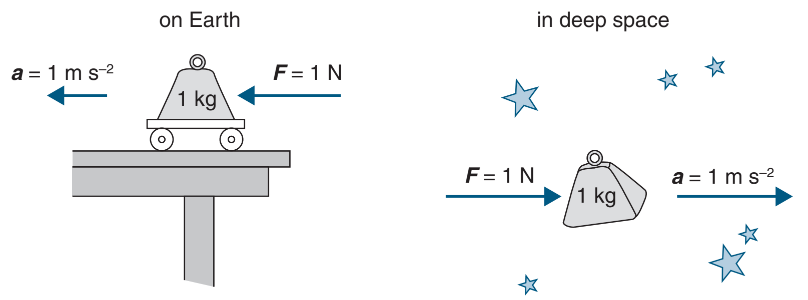


Figure 3.21
Regardless of the external conditions, the inertial qualities of a mass remain the same. A net force of 1 N will always produce an acceleration of 1 m s^{-2} for a 1 kg mass. In this way, mass can be understood as the resistance to a force. The greater the mass of the body, the smaller the acceleration caused by the force.

Weight of a body

In the late 1500s, Galileo was able to show that all objects dropped near the surface of the Earth accelerate at the same rate, $\boldsymbol{g}$, towards the centre of the Earth. The force that produces this acceleration is the force of gravity. In physics, the force on a body due to gravity is called the *weight* of a body, $\boldsymbol{W}$, and it can be determined using a revised Newton's second law, $\boldsymbol{W} = m\boldsymbol{g}$.

Consider a mass allowed to fall under the influence of gravity near the surface of the Earth. Here, the net force acting on the body will be its weight, $\Sigma\boldsymbol{F} = \boldsymbol{W}$. If the equation for weight is rearranged so that the acceleration of the body is made the subject of the formula, then $\boldsymbol{g} = \boldsymbol{W}/m$. From this we see that $\boldsymbol{g}$ could be given the unit N kg^{-1} (newtons per kilogram). This provides another insight into the nature of $\boldsymbol{g}$—that its magnitude, g, can be considered as a measure of the strength of the gravitational field.

Figure 3.22
The weight of this boulder is the force it experiences due to gravity given by $\boldsymbol{W} = m\boldsymbol{g}$. This is approximately 2.5×10^6 N or 2.5 MN directed to the centre of the Earth. The mass of the boulder is approximately 250 000 kg. If the boulder were taken to outer space where the gravitational field is nearly zero, then the boulder would have the same mass but very little weight.

In the Earth's gravitational field, any 1 kg mass will experience a force of 9.8 N (i.e. will have a weight of 9.8 N). If g had a different value, the weight force of the mass would be different.

This means that the acceleration of a mass due to gravity must be numerically identical to the gravitational field strength g. The two quantities have different names and different units but are numerically equal since $\boldsymbol{a} = \Sigma\boldsymbol{F}/m = \boldsymbol{W}/m = \boldsymbol{g}$ and 1 m s^{-2} can be shown to equal 1 N kg^{-1}.

The ***WEIGHT*** of a body, $\boldsymbol{W}$, is defined as the force of attraction on a body due to gravity:

$$\boldsymbol{W} = m\boldsymbol{g}$$

where m is the mass of the body
$\boldsymbol{g}$ is the acceleration due to gravity.

The ***ACCELERATION*** of a body falling under the influence of gravity alone will be numerically equal to the gravitational field strength, g, of the field, thus:

$$1 \text{ m s}^{-2} = 1 \text{ N kg}^{-1}$$

Figure 3.23
The weight of the school bag will change depending on the strength of the external gravitational field. If the bag is allowed to fall, its acceleration will equal the strength of the gravitational field.

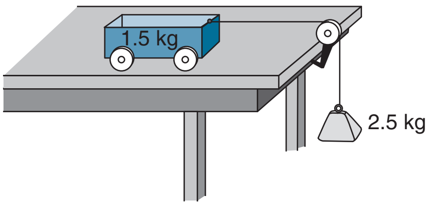
As a consequence of this, the weight of a body will change as it is placed in different gravitational fields. A 5.0 kg barbell will have a weight of $5.0 \times 9.8 = 49$ N downwards on the Earth. On the Moon, the gravitational field strength is lower at 1.6 N kg^{-1}, and so the barbell will be easier to lift since its weight is now only $5.0 \times 1.6 = 8$ N. In the depths of space where $g = 0$, the barbell would be truly weightless.

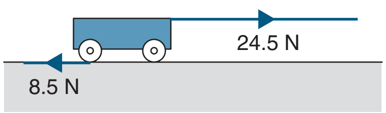
✓ Worked Example 3.4D

A 1.5 kg trolley cart is connected by a cord to a 2.5 kg mass as shown. The cord is placed over a pulley and allowed to fall under the influence of gravity.

a Assuming that the cart can move over the table unhindered by friction, determine the acceleration of the cart.

b If a frictional force of 8.5 N acts against the cart, what is the acceleration now?





Solution

a The cart and mass experience a net force equal to the weight of the falling mass. So $\Sigma F = W = mg = 2.5 \times 9.8 = 24.5$ N. This force has to accelerate not only the cart but the falling mass, and so the total mass to be accelerated is $1.5 + 2.5 = 4.0$ kg.

$a = \Sigma F/m$
$= 24.5/4.0 = 6.1 \text{ m s}^{-2}$

b In analysing the forces that now act on the cart, the net force:

$\Sigma F = ma = W - F_{\text{friction}}$
so $\Sigma F = 24.5 - 8.5 = 16$ N
and $a = \Sigma F/m = 16/4.0 = 4.0 \text{ m s}^{-2}$

3.4 SUMMARY Newton's second law of motion

- Newton's second law of motion states that the acceleration a body experiences is directly proportional to the net force acting on it, and inversely proportional to its mass:

$$\Sigma \boldsymbol{F} = m\boldsymbol{a}$$

where m is measured in kilograms (kg)
$\boldsymbol{a}$ is measured in metres per second squared (m s^{-2})
$\Sigma \boldsymbol{F}$ is in newtons (N).

- The mass of a body can be considered to be its ability to resist a force. Mass is a constant property of the body and is not affected by its environment.
- The weight of a body $\boldsymbol{W}$ is defined as the force of attraction on a body due to gravity. This will be given by $\boldsymbol{W} = m\boldsymbol{g}$ where m is the mass of the body and $\boldsymbol{g}$ is the strength of the gravitational field.

3.4 Questions

1 During a tennis serve, a 57 g tennis ball was accelerated from rest to 30 m s^{-1} in 7.0 ms. Determine the size of the average net force acting on the ball.

2 What is the average force required of the brakes in a 1200 kg car in order for it to come to rest from 60 km h^{-1} in a distance of 150 m?

3 Use Newton's laws to explain why a 1.0 kg shot-put can be thrown further than a 1.5 kg shot-put.

4 In a game of soccer, the ball is simultaneously kicked by two players who impart forces of 100 N east and 125 N south on the ball. Determine:
  a the net force acting on the ball
  b the direction in which the ball will travel
  c the acceleration of the ball if its mass is 750 g.

5 When travelling at 100 km h^{-1} a car has to overcome a drag force due to air resistance of 800 N. If the car has a mass of 900 kg, determine the average force that the motor needs to apply if it is to accelerate at 2 m s^{-2}.

6 In physics, weight and mass are different quantities measured in newtons and kilograms respectively. Why is it possible to refer to the weight of an object in kilograms in everyday language? What would be the reading from a set of bathroom scales that were designed for use on the Earth if they were taken to the Moon for a 72 kg man? ($g_{moon} = {}^{1}/_{6}\, g_{Earth}$)

7 On the surface of the Earth, a geological hammer has a mass of 1500 g. Determine its mass and weight on Mars where g = 3.6 m s^{-2}.

8 A 60 kg water-skier is accelerated from rest by a powerful boat through a tow rope that initially makes an angle of 10° down from the horizontal. It the rope provides a force of 1015 N along its length, what is the horizontal force acting on the skier? If the skier experiences a frictional drag due to the water of 400 N, what is her acceleration?

9 Consider a 70 kg parachutist leaping from an aircraft and taking the time to reach terminal velocity before activating the parachute. Draw a sketch graph of the net force against time for the parachutist in the period from the start of the jump until terminal velocity has been reached. Explain your reasoning.

3.5 Newton's third law of motion

Newton's first two laws of motion consider the motion of a body resulting from the forces that act on that body. The third law is easily stated and seems to be widely known by students. It is a very important law in physics which helps to understand the origin and nature of forces.

Newton realised that all forces exist in pairs, and that each force in the pair acts on a different body. To illustrate this, consider using a hammer to drive a tent peg into the ground. The hammer strikes the peg, providing a net force that causes the peg to accelerate in the direction of the hammer, i.e. into the ground. The hammer is said to apply an *action force* to the peg. As a consequence of this, however, the hammer slows down due to the force it experiences from the peg pushing upward. The force on the hammer is a *reaction force* which is in the opposite direction to the force on the peg (Figure 3.24).

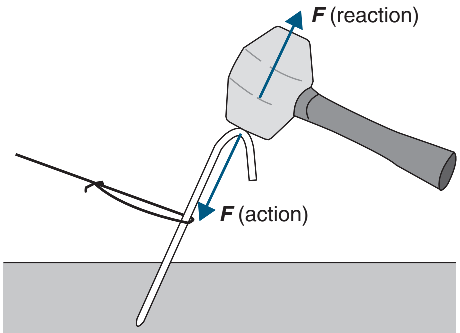


Figure 3.24
The hammer drives the peg into the ground by providing an action force. In turn, the peg causes the hammer to decelerate by providing a reaction force. Newton described this situation in his third law of motion.

From this example, it is easy to see that the action and reaction forces are in opposite directions, but Newton showed that the magnitude of each force is the same. The size of the force on the hammer is the same as the force on the peg. These points are combined to become Newton's third law.

> ***NEWTON'S THIRD LAW OF MOTION*** states that for every action force (object A on B), there is an equal and opposite reaction force (object B on A):
>
> $\boldsymbol{F}(\text{A on B}) = -\boldsymbol{F}(\text{B on A})$

It is important to recognise that the action force and the reaction force in Newton's third law act on different objects.

- When you begin to bounce a basketball, the force you apply to the ball with your hand is the action force. In return, the reaction force produced by the ball acts on your hand.
- The force exerted by a statue on a table is the action force. To remain in position, the table supplies an equal and opposite reaction force on the statue.
- A trailer is accelerated by a car. The action force supplied by the car acts on the trailer in a forward direction. In response, the trailer pulls back on the car with an equal (reaction) force.
- The action force on the skydiver is the skydiver's weight (i.e. Earth attracts the person). The reaction force to all weight forces acts on the Earth (i.e. person attracts the Earth).

Motion explained

Newton's third law also explains how we are able to move around. In fact, the third law is needed to explain all locomotion. Consider walking. Your leg pushes backwards with each step. This is an action force acting on the ground. A component of the force acts downwards and a component pushes backwards horizontally along the surface of the Earth. The force is transmitted through the friction between your shoe and the Earth's surface. In response, the ground then pushes forward on your leg. This forward component of the reaction force enables you to move forward.

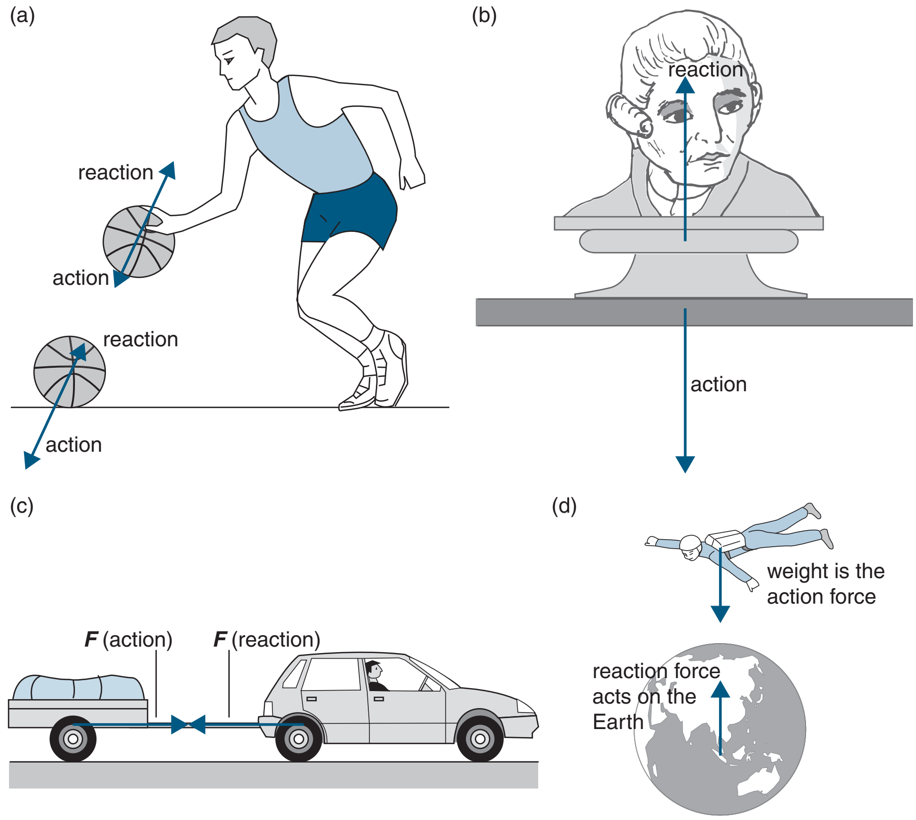


Figure 3.25

Some action/reaction force pairs. (a) The basketball player pushes down on the ball, which pushes back on the hand. When the ball hits the ground, the ball pushes on the ground (action) and in response, the ground pushes back on the ball. (b) The statue pushes down on the table, and in return, the table pushes upwards on the statue. (c) In this situation, the car must provide an action force to the trailer to cause it to accelerate. The reaction force acts on the car. (d) The weight of the skydiver is the action force and the reaction force will act on the Earth.

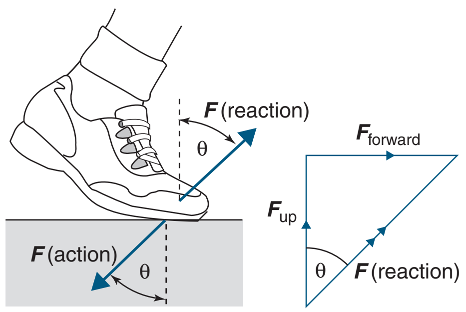


Figure 3.26

Walking relies on an action and reaction force pair in which the foot will push down and backwards with an action force. In response, the ground will push upwards and forwards. The forward component of the reaction force is responsible for the body moving forward as a whole, while the back foot remains at rest.

The act of walking relies on there being some friction between your shoe and the ground (Figure 3.26). Without it, there is no grip, and it is impossible to supply the action force to the ground. Consequently, the ground cannot supply the reaction force needed to enable forward motion. Walking on smooth ice is a good example of this, and so mountaineers will use crampons (basically a rack of nails) attached to the soles of their boots in order to gain purchase in icy conditions.

The situation outlined above is fundamental to all motion.

Table 3.4 All motion can be explained in terms of action and reaction force pairs

Motion	Action force	Reaction force
Swimming	Hand pushes back on water	Water pushes forward on hand
Jumping	Legs push down on Earth	Earth pushes up on legs
Bicycle/car	Tyre pushes back on ground	Ground pushes forward on tyre
Jet aircraft and rockets	Hot gas is forced backwards out of engine	Gases push craft forward
Skydiving	Force of gravitation on the skydiver from Earth	Force of gravitation on Earth from skydiver

Worked Example 3.5A

Larry is a 60 kg surfer who is able to stand on his 10 kg Malibu surfboard in calm water. He starts to walk to the front of the board with an acceleration of 0.5 m s^{-2}.

a Explain what happens to the surfboard.

b What is the acceleration of the board?

Solution

a In moving forward on the surfboard, Larry must experience a net force of $\Sigma \boldsymbol{F} = m\boldsymbol{a} = 60 \times 0.5 = 30$ N.
This means that he applies an action force of 30 N to the board which makes it move in the opposite direction to his motion.

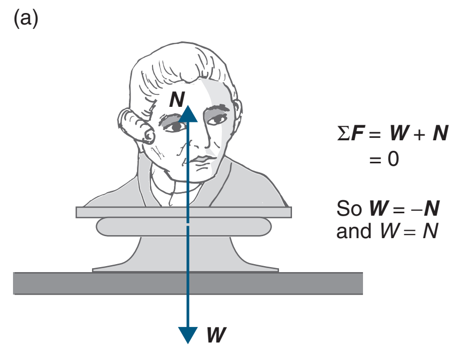
b The board receives a net force of 30 N, so its acceleration will be $\boldsymbol{a} = \Sigma \boldsymbol{F}/m = 30/10 = 3.0$ m s^{-2} in the opposite direction to Larry.

The normal force

When an object is allowed to fall under the influence of gravity, it is easy to see the effect of the force of gravity. The net force acting is the weight of the body, which therefore accelerates at $\boldsymbol{g}$.

When the body is at rest on a table, the force of gravity ($\boldsymbol{W} = m\boldsymbol{g}$) is still acting. Since the object fails to move, there must be another force (equal in magnitude and opposite in direction) acting to balance the weight. This force is provided by the table. Because of the weight of the object, the table is deformed a little, and being elastic, it will push upwards. The elastic force it provides is perpendicular to its surface and is called a *normal reaction force* (often abbreviated to the *normal force*). This means that there are two forces that act on the object which will completely balance each other so that the net force is zero.

The weight force and the normal force act on the same body, so they cannot be an action and reaction force pair. How can the action and reaction force pairs be identified? The reaction to the normal force will be an equal and opposite force exerted by the body on the table. The other action/reaction force pair involves the Earth. Since the weight of the body is the force due to gravity, the reaction force to this is the force of gravity on the Earth caused by the object in question. By symmetry, if the Earth causes a force on the object, the object causes a force on the Earth (see Figure 3.25d).



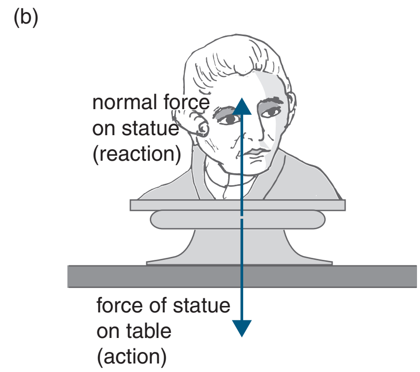


Figure 3.27
(a) The statue remains at rest, and so its weight is balanced by an upward force—a normal force—produced by the table:

$$\Sigma \boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N} = 0$$

The weight force on the statue and the normal force are not an action/reaction pair since they both act on the statue.
(b) The reaction force to the weight of the statue acts on the Earth, and the normal force and the force acting on the table are an action/reaction pair.

Worked Example 3.5B

An 8.0 kg computer rests on a table.

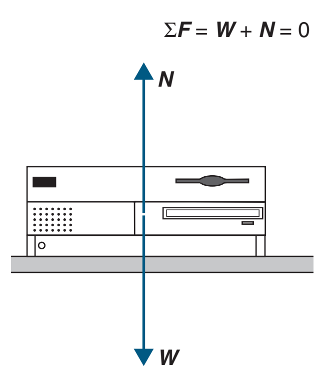
a Identify the forces that act on the computer and the table.

b If a 3.0 kg monitor is placed on the computer box, determine the new normal force acting on the computer.

Solution

a The weight of the computer will be $\boldsymbol{W} = m\boldsymbol{g} = 8.0 \times 9.8 = 784$ N down so that if the net force on the computer is zero, the normal force supplied by the table must be 784 N upwards: $\Sigma \boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N} = 0$.

b If a 3.0 kg monitor is placed on top of the computer, the table must supply a further $3.0 \times 9.8 = 294$ N, bringing the total normal force to 1078 N. (The computer will also have to provide a normal force of 294 N upwards to balance the weight of the monitor.)



The inclined plane

The Guinness Book of Records identifies the steepest road in the world as being at an angle of 51° to the horizontal. It is located in Dunedin, New Zealand. Living on such a road requires the residents to ensure that the handbrake in their car is always in good repair! To determine the force required by the handbrake for a car parked on this steep road, the physics of forces acting on a body on an inclined plane must be used.

Start by thinking of a body at rest on a horizontal surface. Two forces act on the body: the weight of the body $\boldsymbol{W}$ and the normal force $\boldsymbol{N}$ supplied by the surface. The weight force always acts downwards and is given by $\boldsymbol{W} = m\boldsymbol{g}$. The normal force is supplied by the surface and will vary depending on the situation, but it will always act upwards and perpendicular to the surface. This means that the net force on the body will be the sum of these two forces and in this case it has to be zero since the body does not move: $\Sigma\boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N} = 0$.

If the surface is tilted so that it makes an angle to the horizontal, the weight force remains the same: $\boldsymbol{W} = m\boldsymbol{g}$. However, the normal force continues to act at right angles to the surface and will change in magnitude, getting smaller as the angle increases. If there is no friction between the body and the surface, the two forces will not cancel and a non-zero net force will be directed down the incline as shown in Figure 3.28: $\Sigma\boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N} \neq 0$.

Physics file

Each time a person begins to walk, the Earth will immediately accelerate in the opposite direction as a consequence of Newton's third law. Walking applies an action force to the Earth. The effect of this is negligible owing to the enormous mass of the planet. The mass of the Earth is approximately 6×10^{24} kg, so if a 60 kg person were to accelerate at 10 m s^{-2}, the Earth would accelerate at $\frac{60 \times 10}{6 \times 10^{24}} = 10^{-22}$ m s^{-2}. This acceleration is minuscule. What if most people (say 5 billion) on the Earth were to accelerate in the same direction at once? What effect would this have? If each person were assumed to have a mass of 60 kg (which they do not) and they were to accelerate at 10 m s^{-2} in the same direction, the acceleration would still only be 2×10^{-12} m s^{-2}—again, an absolutely undetectable value.

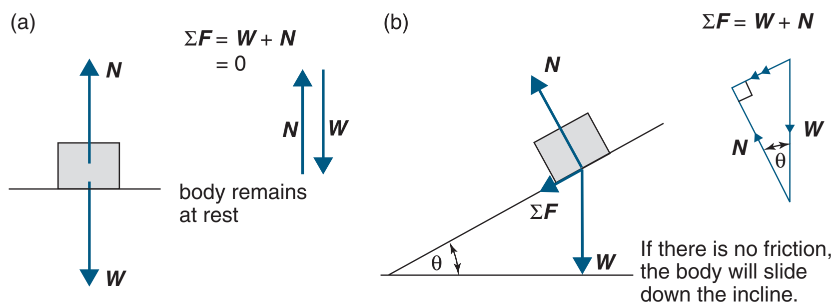


Figure 3.28
(a) Where the surface is perpendicular to the weight force, the normal force will act directly upwards and cancel the weight force. (b) On an inclined plane, $\boldsymbol{N}$ is at an angle to $\boldsymbol{W}$ and as long as no friction acts, there will be a net force down the incline. The body will accelerate.

From the vector diagram of the forces:

$$\Sigma\boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N} = \boldsymbol{W}\sin\theta = m\boldsymbol{g}\sin\theta$$

From Newton's second law, the net force is:

$$\Sigma\boldsymbol{F} = m\boldsymbol{a}$$

so:

$$m\boldsymbol{a} = m\boldsymbol{g}\sin\theta$$

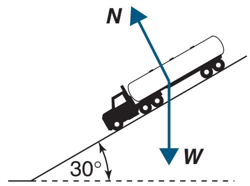
or:

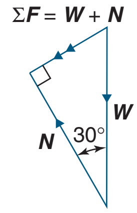
$$\boldsymbol{a} = \boldsymbol{g}\sin\theta$$

This means that the acceleration down an incline is a function of the angle of the incline alone, and not the mass of the body. Ignoring any friction, any car rolling down the steep street in Dunedin will accelerate at $\boldsymbol{a} = \boldsymbol{g}\sin\theta = 9.8 \times \sin 51° = 7.6$ m s^{-2}—quite a rate!

Worked Example 3.5C

A mistake allows a 5-tonne truck to roll down a steep road inclined at 30° to the horizontal. As it is a high-technology vehicle, there is no friction between the wheels of the truck and the wheel bearings. Find the acceleration of the truck if the acceleration due to gravity is taken as 9.8 m s^{-2}.





Solution

$\Sigma \boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N}$

From the vector diagram:

$\Sigma \boldsymbol{F} = \boldsymbol{W}\sin\theta$

So, $m\boldsymbol{a} = m\boldsymbol{g}\sin\theta$

$\boldsymbol{a} = \boldsymbol{g}\sin 30°$

$= 9.8\sin 30°$

$= 4.9$ m s^{-2} down the road

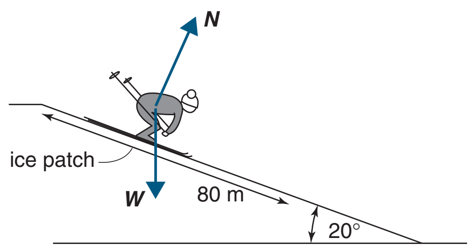
If friction exists between a body on an inclined plane and the surface, its direction will be along the incline but against the motion. If a frictional force is great enough to balance the sum of the normal force and the weight of the body, the net force is zero and the body will either travel with a constant velocity or remain in a stationary position. Worked Example 3.5D illustrates this point.

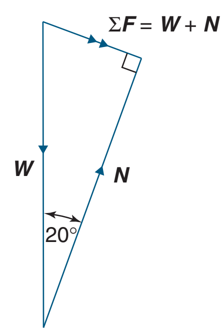
Worked Example 3.5D

Kristie is a 60 kg skier. At the start of a ski-slope that is at 20° to the horizontal, she crouches into a tuck. The surface is very icy, so there is no friction between her skis and the ice.

a If Kristie starts from rest, what is her speed after travelling a distance of 80 m on the ice?

b The snow conditions change at the end of the ice patch so that Kristie continues down the slope with a constant velocity. What is the force due to friction that must be acting between Kristie's skis and the snow?





Solution

a The net force on Kristie will be $\Sigma \boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N}$. This is a vector addition.

From the vector diagram:

$\Sigma \boldsymbol{F} = \boldsymbol{W}\sin\theta$

So, $\boldsymbol{a} = \boldsymbol{g}\sin 20°$

$= 9.8\sin 20°$

$= 3.35$ m s^{-2}

If this acceleration continues over 80 m, the final speed would be:

$v^2 = u^2 + 2as$

so $v^2 = 0 + 2 \times 3.335 \times 80 = 534$

$v = 23$ m s^{-1} (83 km h^{-1})

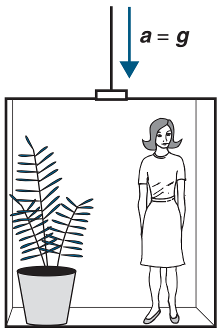
b Kristie is travelling with a constant velocity, so the force of friction would balance her weight and the normal force: $\Sigma \boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N} + \boldsymbol{F}_{\text{friction}} = 0$

So $\boldsymbol{F}_{\text{friction}} = m\boldsymbol{a} = 60 \times 3.35 = 200$ N up the incline.

Weight and apparent weight

Have you ever been in a lift and, as the lift accelerates, felt your weight change? This situation occurs because how heavy you feel does *not* depend on the force of gravity that acts on you; rather it depends on the size of the normal force pushing upwards on you from the floor. This is called your *apparent* weight. To investigate this, consider the following situation.

(a)



$\Sigma \boldsymbol{F} = m\boldsymbol{g} = \boldsymbol{W}$

$\therefore \boldsymbol{N} = 0$

(b)

(c)

Figure 3.30

(a) When the lift falls under the influence of gravity, no normal force is supplied by the floor and everything within the lift experiences apparent weightlessness. This situation is similar to that experienced by (b) thrillseekers on a funpark ride that will be in free fall momentarily, and (c) astronauts in orbit around the Earth. The astronauts are continually falling to Earth, but the initial velocity of the craft means that they will stay in a circular orbit.

Kate has a mass of 55 kg, and so the force of gravity acting on her—her weight—is 55 × 9.8 = 540 N downwards. When she is standing in a stationary lift, the net force acting on her must be zero. The two forces involved here are Kate's weight and the normal force pushing upwards from the floor, i.e. $\Sigma \boldsymbol{F} = 0 = \boldsymbol{W} + \boldsymbol{N}$. So, the normal force must exactly balance her weight, i.e. $\boldsymbol{N}$ = 540 N upward. She feels her usual self.

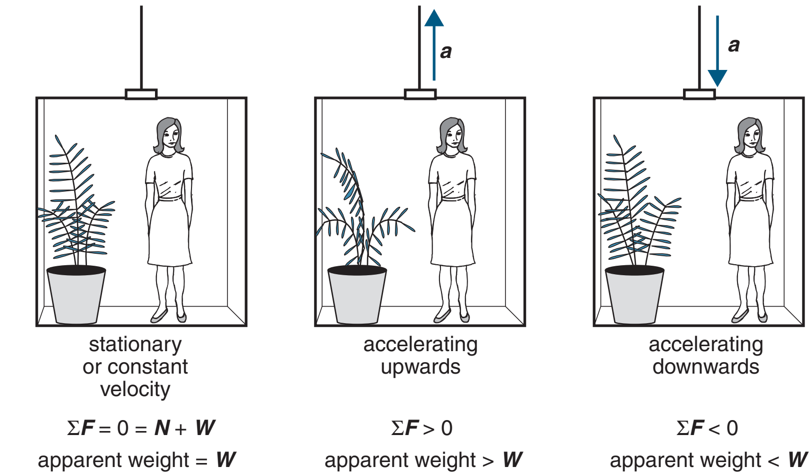


Figure 3.29

Kate is riding in a lift, and her apparent weight will vary with the acceleration of the lift. In (a) Kate's weight is normal as the lift is not accelerating. (b) The lift accelerates upwards and so the normal force supplied by the floor increases. The normal force has to overcome her weight and provide the necessary net force to accelerate her with the lift. (c) The lift accelerates downwards, allowing the normal force to be reduced. Some of Kate's weight provides the necessary net force, and so the normal force is reduced—she feels lighter.

When the lift is travelling with constant velocity (Figure 3.29a), Kate will also feel the same. This is because the lift is travelling with a constant velocity, and by Newton's first law, the net force, $\Sigma \boldsymbol{F}$, must be zero.

It is when the lift begins to accelerate (Figure 3.29b) that the situation becomes more complex. Let the lift now accelerate from rest at 2.0 m s^{-2} upwards, and think about what you would experience if you were in the lift. For such an acceleration, the overall net force on Kate must be $\Sigma \boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N}$ = 55 × 2.0 = 110 N upwards. Since $\boldsymbol{W}$ is fixed at 540 N downwards, $\boldsymbol{N}$ must not only balance Kate's weight but be big enough to accelerate her. $\boldsymbol{N} = \Sigma \boldsymbol{F} - \boldsymbol{W}$ = 110 N up – 540 N down = 110 N up – (–540 N up) = 650 N upwards. Kate will feel as if the lift is pushing harder on her, and so feel that she is heavier.

If Kate were to stand on a set of scales calibrated to read forces in newtons, then the scales would read 650 N—Kate's apparent weight.

APPARENT WEIGHT is the normal reaction force acting on an object.

What happens if the lift were to accelerate downwards at 2.0 m s^{-2} (Figure 3.29c)? In this case the net force acting on Kate must be $\Sigma \boldsymbol{F} = \boldsymbol{W} + \boldsymbol{N}$ = 55 × 2.0 = 110 N downwards. $\boldsymbol{W}$ is still 540 N downwards, so $\boldsymbol{N} = \Sigma \boldsymbol{F} - \boldsymbol{W}$ = 110 N down – 540 N down = –430 N down = 430 N upwards. As a result, Kate feels lighter, and if she were standing on scales, they would register this fact.

If the lift were to fall under the influence of gravity alone, the only force acting on Kate would be gravity, and so the net force $\Sigma\boldsymbol{F} = 55 \times 9.8 = 540$ N down. The normal force, $\boldsymbol{N} = \Sigma\boldsymbol{F} - \boldsymbol{W} = 540$ N down – 540 N down = 0. So Kate experiences no normal force, and is apparently weightless.

In fact, any body falling solely under the influence of gravity has zero apparent weight, and experiences the sensation of weightlessness. This will happen when the floor on which you are standing provides no normal force. Many people will have experienced this phenomenon on rides at a fun park, and filmmakers simulate the environment of space by shooting scenes in an aircraft freefalling from a great height.

Figure 3.31
The motion of the hammer places the wire under tension and acts at every point along the wire.

Tension

Another aspect of forces, common in everyday life, are the tension forces found in ropes and wires. When an athlete winds up to throw a hammer, you can imagine that the effect will be to stretch the wire connected to the ball. This stretching force is called *tension*. Because a stretching force will act in both directions along the length of the wire, tension has no direction. A magnitude only is given.

Calculations involving tension can be illustrated by the following example. A naughty monkey of mass 15 kg has escaped backstage in a circus (Figure 3.32). Nearby is a rope threaded through an ideal (frictionless and massless) pulley. Attached to one end of the rope is a 10 kg bag of sand. The monkey climbs a ladder and jumps onto the other end of the rope.

The rope is now subjected to a net force of:

$$\Sigma\boldsymbol{F} = 15 \times 9.8 - 10 \times 9.8 = 49 \text{ N down}$$

on the side of the monkey (Figure 3.32b). As a consequence, both masses and the rope will accelerate at $\boldsymbol{a} = \Sigma\boldsymbol{F}/m = 49/25 = 1.96$ m s^{-2}.

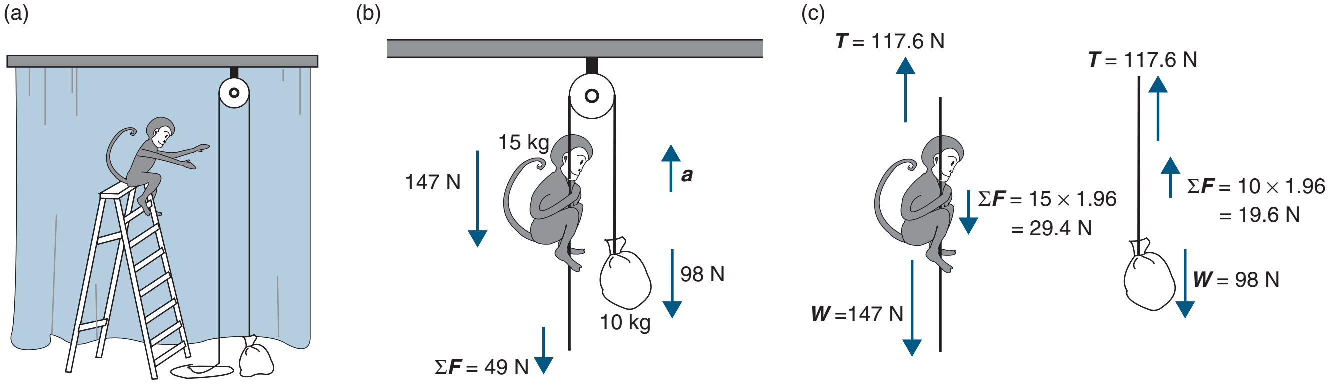


Figure 3.32
The monkey has a greater weight than the sand bag, and so the rope will accelerate in the direction of the monkey. The tension in the rope is found by considering the forces that act on each weight.

While all of this is occurring, the rope is under tension. To find the amount, we look at the forces acting on one of the masses (Figure 3.32c). Take the monkey: the net force on the monkey will be $\Sigma\boldsymbol{F} = \boldsymbol{W} + \boldsymbol{T}$.

So, $\boldsymbol{T} = \Sigma\boldsymbol{F} - \boldsymbol{W}$

$= 15 \times 1.96$ down $- 15 \times 9.8$ down

$= 29.4$ N down + 147 N up = 117.6 N

To check, find the tension acting on the sandbag.

Again, $\Sigma\boldsymbol{F} = \boldsymbol{W} + \boldsymbol{T}$.

So, $\boldsymbol{T} = \Sigma\boldsymbol{F} - \boldsymbol{W} = 10 \times 1.96$ up $- 10 \times 9.8$ down

$= 19.6$ N up + 98 N up = 117.6 N

Intuitively, you might have thought that the tension would have been $(15 + 10) \times 9.8 = 245$ N since the two weights are pulling in opposite directions. This is not the case because the system is allowed to accelerate, with a reduction in tension.

Physics in action — Frictional forces

Friction is a reaction force. This means that it does not exist until there is movement to create an action force. Suppose you want to push your textbook along the table. This simple experiment can reveal a significant amount of information about the nature of friction. As you start to push the book, you find that, at first, the book does not move. You then increase the force that you apply, and suddenly, at a certain critical value, the book starts to move relatively freely.

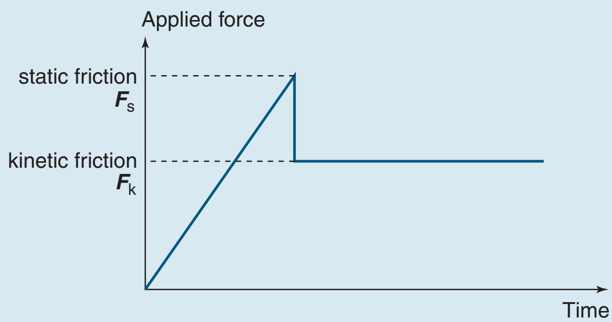
The maximum frictional force resists the onset of sliding. This force is called the *static friction* force, $\boldsymbol{F}_s$. Once the book has begun to slide, a much lower force than $\boldsymbol{F}_s$ is needed to keep the book moving. This force is called the *kinetic friction* force, and is represented by $\boldsymbol{F}_k$.

This phenomenon can be understood when it is realised that even the smoothest surfaces are quite jagged at the microscopic level. When the book is resting on the table, the jagged points of its bottom surface have settled into the valleys of the surface of the table, and this helps to resist attempts to try to slide the book. Once moving, the surfaces do not have any time to settle into each other, and so less force is required keep it moving.

Another fact that helps to explain friction arises from the forces of attraction between the atoms and molecules from the two different surfaces that are in contact. This produces weak bonding between the particles within each material; before one surface can move across the other, these bonds must be broken. This extra effort adds to the static friction force. Once there is relative motion between the surfaces, the bonds cannot re-form.

In everyday life, there are situations where friction is desirable (e.g. walking) and others where it is a definite problem. Consider the moving parts within the engine of a car. Friction can rob an engine of its fuel economy and cause it to wear out. Special oils and lubricants are introduced in order to prevent moving metal surfaces from touching. If the moving surfaces actually moved over each other they would quickly wear, producing metal filings that could damage the engine. Instead, both metal surfaces are separated by a thin layer of oil. The oils are chosen on the basis of their viscosity (thickness). For example, low viscosity oils can be used in the engine while heavier oils are needed in the gear box and differential of the car where greater forces are applied to the moving parts.

At other times, we want friction to act. When driving to the snow, if there is any ice on the road, drivers are required to fit chains to their cars. When driving over a patch of ice, the chain will break through the ice, and the car is again able to grip the road. Similarly, friction is definitely required within the car's brakes when the driver wants to slow. In fact, modern brake-pads are specially designed to maximise the friction between themselves and the brake drum or disk.



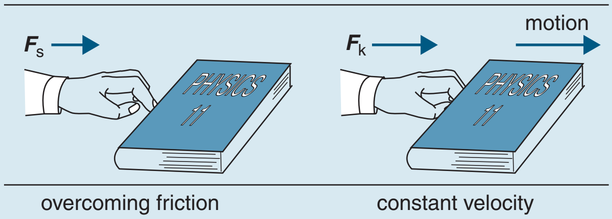


Figure 3.33
To get things moving, the static friction between an object and the surface must be overcome. This requires a larger force than that needed to maintain constant velocity. This can be illustrated by the good fuel economy that large cars can produce when travelling on the open road with constant speed. Where they have to stop and start in the city, the fuel economy is poorer.

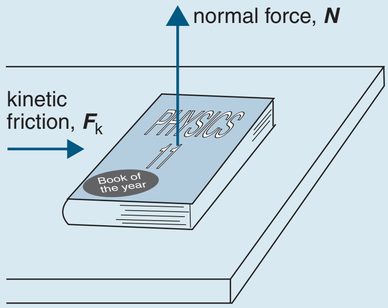


Figure 3.34 The kinetic friction needed to keep an object moving with a constant velocity is determined by the size of the normal force acting on the body and the roughness of the surfaces in contact. The rougher the surface, the higher the value for *m*, the coefficient of friction, and so a greater applied force is needed.

When a car is braking in a controlled fashion, the brake-pads grip a disk which is attached to the wheel of the car. The retarding force, applied through friction, slows the disk and hence the car will come to rest. If the brakes are applied too strongly they may grab the disk, locking up the wheels in the process. The car then slides over the road, with two undesirable consequences. First, the car usually takes about 20% longer to come to rest. This is because the car is relying on the kinetic frictional force between the tyres and the road to stop. As was seen when pushing the book over the desk, this force is less than the static friction force. The other consequence is that the car has lost its grip with the road, and so the driver can no longer steer the car. Many modern cars now employ anti-lock braking systems (ABS) to overcome the possibility of skidding. This is achieved by using feedback systems that automatically reduce the pressure applied by the brake-pads regardless of the pressure applied by the driver to the brake pedal.

When an object slides over another object, the kinetic friction force required to keep the object sliding can be determined from the nature of the two surfaces involved. For any two given surfaces, the coefficient of friction is a constant that describes the extent of the friction acting between the two surfaces. When an object is sliding on a horizontal surface, the kinetic friction, F_k, is proportional to the force between the two surfaces—which is the normal force $\boldsymbol{N}$. If the surface is level, $N = W$, but if the surface is inclined, the magnitude of N will be less than the weight of the body:

$$F_k \propto N$$

The coefficient of friction, μ, is included as the constant of proportionality. So:

$$F_k = \mu N$$

Both F_k and N will be measured in newtons and so μ will not have a unit. Some values for the coefficient of friction for various surfaces are displayed in Table 3.5. Where a body is moving along a surface, the value for μ relates only to the frictional characteristics of the surfaces and is independent of the speed of the body.

Table 3.5 Kinetic coefficient of friction for common surfaces

Surface 1	Surface 2	μ
Wood	Snow	~0.06
Metal	Ice	0.02–0.1
Metal	Metal (with oil)	0.07
Wood	Wood	0.25
Tyre	Icy road	0.1
Tyre	Traffic-smoothed road	0.4
Tyre	Dry gritty road	almost 1.0

When reconstructing the events that have led up to traffic accidents, the Police Accident Investigation Squad can use skid marks to help determine the speed of a car before the collision. On a level road, $F_k = \mu N$ is reduced to $a = \mu g$ (dividing by the mass of the car) as $N = W$, a is the acceleration of the car as it slows, and g is the acceleration due to gravity. Using an estimate of the coefficient of friction for the tyres and the road, the police can estimate the acceleration of the car as it skids. The acceleration and the length of the skid can be then used to determine a value for the initial speed of the car ($v^2 - u^2 = 2as$).

3.5 SUMMARY Newton's third law of motion

- Newton's third law of motion states that for every action force, there is an equal and opposite reaction force:

$$\boldsymbol{F}(\text{A on B}) = -\boldsymbol{F}(\text{B on A})$$

- Whenever a force acts against a fixed surface, the surface provides a normal force, $\boldsymbol{N}$, at right angles away from the surface. The size of the normal force depends on the orientation of the surface to the contact force.
- All locomotion is made possible through the existence of action and reaction force pairs.
- A person's apparent weight is determined by the normal force, $\boldsymbol{N}$, acting from the floor beneath them. In free fall with an acceleration equal to $\boldsymbol{g}$, a person will have an apparent weight of zero.

3.5 Questions

1 Determine the action and reaction forces involved in the following situations:
   a hitting a ping-pong ball with a bat
   b a pine cone falling from the top of a tree to the ground
   c letting go of an untied balloon filled with air.

2 A 70 kg fisherman is quietly fishing in a 40 kg dinghy at rest on a still lake when, suddenly, he is attacked by a swarm of wasps. To escape, he leaps from the boat into the water with a force of 140 N.
   a What is the force acting on the boat?
   b With what acceleration will the boat move?
   c If the force on the fisherman lasted for 0.5 s, determine the speed attained by both the man and boat.

3 A 100 kg astronaut (including the space suit) becomes untethered during a space-walk and drifts to a distance of 100 m from the mother ship. To get back to the ship, he throws his 2.5 kg tool kit away with an acceleration of 8.0 m s^{-2} which acts over 1.0 s.
   a How does throwing the tool kit away help the astronaut in this situation?
   b What force acts on the tool kit and the astronaut?
   c With what speed will the astronaut drift to the mother ship?
   d How long will it take him to reach the ship?

4 A 2.0 kg bowl strikes the stationary 'jack' whose mass is 1.0 kg during a game of bowls. It is a head-on collision, and the acceleration of the jack is found to be 25 m s^{-2} north. What is the acceleration of the bowl?

5 A 5.0 kg speaker is placed on a bookshelf. Determine the forces that act on the speaker. What is the reaction force to the weight of the speaker?

6 During the Winter Olympics, a 65 kg competitor in the women's luge has to accelerate down a course that is inclined at 50° to the horizontal.
   a Name the forces acting on the competitor.
   b Ignoring friction (because it's an icy slope), determine the forces that act.
   c Determine the magnitude of the net force on the competitor.
   d What acceleration will she experience?

7 A cyclist is coasting down a hill that is inclined at 15° to the horizontal. The mass of the cyclist and her bike is 110 kg, and for the purposes of the problem, no air resistance or other forces are acting. After accelerating to the speed limit, she applies the brakes a little. What braking force is needed for her to be able to travel with a constant velocity down the hill?

8 Why do you feel heavier in a lift that is accelerating upwards from rest? Describe another situation in which you also feel heavier in a lift.

9 Calculate the apparent weight of Paul, whose mass is 80 kg, in a lift that is:
   a at rest
   b accelerating upwards at 2.2 m s^{-2}
   c travelling upwards at a constant 4.0 m s^{-1}
   d slowing upwards at 2.2 m s^{-2}
   e accelerating downwards at 2.2 m s^{-2}.

10 A rope is allowed to move freely over a 'frictionless' pulley backstage of a theatre. At one end, a 30 kg sandbag is attached which is at rest on the ground. A 50 kg work-experience student, standing on a ladder, grabs onto the rope to lower himself.
   a What is the net force on the rope?
   b With what acceleration will the system move?
   c What is the tension in the rope?

Chapter 3 Review

1 Explain three examples of situations requiring the use of Newton's first law.

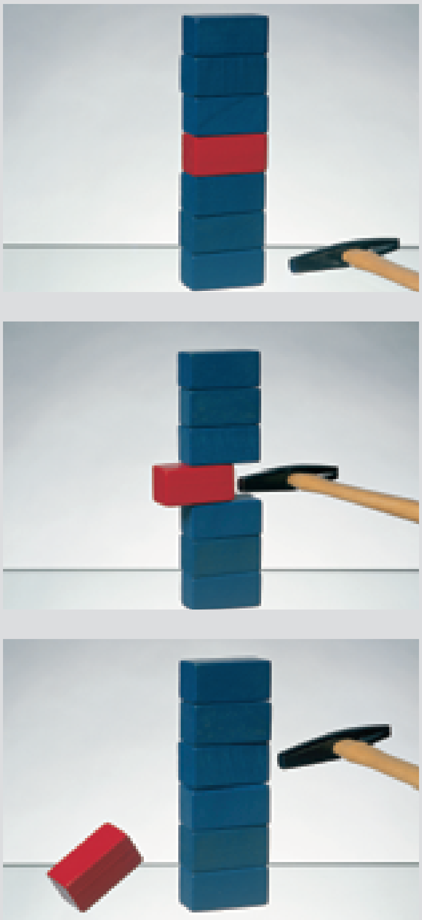
2 When pushing a shopping trolley along a horizontal path, James has to continue to provide a force of 30 N just to maintain his speed. If the trolley (and shopping) has a mass of 35 kg, what is the total horizontal force that he will have to provide to accelerate the cart at 0.5 m s^{-2}?

3 A force of 25 N is applied to a 15 kg ten-pin bowling ball for 4.0 s. If the ball was initially at rest, what is its final speed?

4 Identify the action and reaction pairs for three situations that involve Newton's third law.

5 Jane has a mass of 55 kg. She steps into a lift which goes up to the second floor. The lift accelerates upwards at 2.0 m s^{-2} for 2.5 s, then travels with constant speed until it accelerates at 2.0 m s^{-2} downwards for 2.5 s.
   a What is the maximum speed that the lift attains as it travels between floors?
   b What is Jane's apparent weight:
      i when the lift is stationary?
      ii when the lift is accelerating upwards?
      iii when the lift travels with constant speed?
      iv when the lift is slowing down?

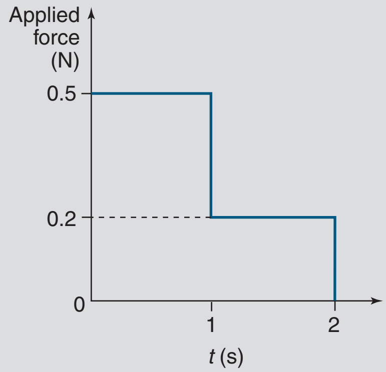
6 a What is the weight of an 85 kg astronaut on the surface of Earth, g = 9.8 m s^{-2}?
   b What is the weight of an 85 kg astronaut on the surface of the Moon, g = 1.6 m s^{-2}?
   c What is the weight of an 85 kg astronaut on the surface of Mars, g = 3.6 m s^{-2}?
   d What is the weight of an 85 kg astronaut during re-entry to Earth where the spacecraft is in free fall? Assume that g = 9.8 m s^{-2}.
   e What is the apparent weight of the astronaut in part d?
   f What is the apparent weight of the astronaut during lift-off from the Earth with $\boldsymbol{a}$ = 5.0 m s^{-2}?



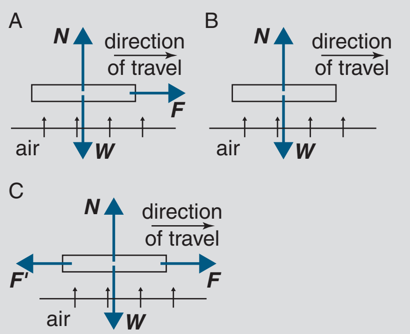
7 The series of photographs shows a stack of smooth blocks in a tall pile. One of the blocks in the pile is struck by a hammer and the blocks above it fall onto the block below, and the pile remains standing. Explain this in terms of Newton's laws of motion.

8 A force of 120 N is used to push a 20 kg shopping trolley along the line of its handle—at 20° down from the horizontal. This is enough to cause the trolley to travel with constant velocity to the north along a horizontal path.
  a Determine the perpendicular components of the force applied to the trolley.
  b What is the value of the frictional force acting against the trolley?
  c What is the normal force that must be supplied by the ground on which the trolley is pushed?
  d Why is it often easier to pull a trolley than to push it?

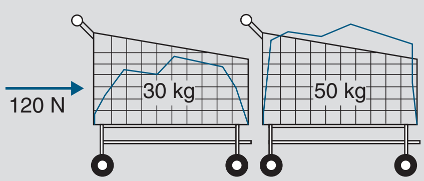
9 A 100 g glider is at rest on a horizontal air track, and a force is applied to it as shown in the graph below. What will be its speed at the end of the time interval?



10 The diagrams below show force vectors on a puck travelling across an air table in a games arcade. The puck experiences no friction as it moves across its 'cushion of air'. Which diagram A–C correctly shows the forces which act on the puck?



11 Two shopping trolleys with masses 30 kg and 50 kg stand together. A force of 120 N is applied to the trolleys.



  a With what acceleration will the trolleys move?
  b What is the contact force that the second trolley experiences from the first?

12 A young girl of mass 40 kg leaps off her stationary 10 kg skateboard. Assuming that no frictional forces are involved, determine the following ratios:
  a $\dfrac{\text{force on girl}}{\text{force on skateboard}}$
  b $\dfrac{\text{acceleration of the girl}}{\text{acceleration of the skateboard}}$
  c $\dfrac{\text{velocity of the girl}}{\text{velocity of the skateboard}}$

13 A rope has a breaking tension of 100 N. How can a full bucket of mass 12 kg be lowered using the rope?

14 The breaking strain of a fishing line is rated at 50 N. Determine the mass of the heaviest fish that can be caught if it is assumed that a fish may accelerate at 2.2 m s^{-2} downwards just as it breaks through the surface of the water.

15 A car begins to roll down a steep road that has a grade of 1 in 5 (i.e. a 1 m drop for every 5 m in length). If friction is ignored, determine the speed of the car in km h^{-1} after it has travelled a distance of 100 m if it begins its journey at rest.

16 A small boy's racing set includes an inclined track along which a car accelerates at ½*g* (i.e. 4.9 m s^{-2}). At what angle is the track to the horizontal?

17 When skiing down an incline, Eddie (the eagle!) found that there was a frictional force of 250 N acting up the incline of the mountainside due to slushy snow. The slope was at 35° to the horizontal, so if Eddie had a mass of 70 kg, what was his acceleration?

18 On a sketch, draw vectors to indicate the forces that act on a tennis ball:
  a at the instant it is struck
  b an instant after it has been struck.

19 Two masses, 5.0 kg and 10.0 kg, are suspended from the ends of a rope that passes over a frictionless pulley. The masses are released and allowed to accelerate under the influence of gravity. What is the acceleration of the system, and what is the tension in the rope?

20 Consider the single bounce on a concrete floor of a ball of weight ***W***. Sketch a graph of the normal force that the ball experiences during the bounce. Indicate the value for the weight in your graph.

4 Energy and momentum

A giant asteroid hurtles towards the Sun, its orbit bringing it into a collision course with the Earth. Unaware, the inhabitants go about their daily activities. It takes weeks but finally the asteroid is close enough to be visible as a bright star, but still it is not recognised for the danger it brings. Finally, it ploughs into the Earth. Huge quantities of dust are blown into the atmosphere, obscuring the Sun and bringing on a new ice age. Whole species become extinct as the weather patterns drastically change.

An asteroid of the size that may have caused the extinction of the dinosaurs is thought to be long overdue. The most recent body of notable size landed in Siberia in 1908 felling many square kilometres of forest. If it had hit just a few hours later, it could have totally destroyed a city the size of London or New York. Plans are being made to ensure that early detection and destruction of any such asteroid will protect us from the consequences of such an enormous collision. Energy provides a link between the many ideas of science used to explain the possible effects of such a collision, or the smaller everyday ones between cars, people, balls, right down to the subatomic level. The kinetic, or moving, energy of an asteroid is converted into heat, sound and other forms. The forces involved in the collision can be calculated by considering the energy involved and momentum.

The effect of external force on an object has been explained in terms of Newton's three laws of motion. Force played the central role as the cause of motion ($\boldsymbol{F} \propto \boldsymbol{a}$). An alternative, and very practical, derivation can be completed in terms of momentum and energy. In all circumstances these quantities are conserved. Hence, an alternative insight into practical problems involving motion can be gained by a study of these quantities. The laws of conservation of momentum and energy are particularly useful in the typical practical situation where consideration of all the forces involved would be difficult or time-consuming. They are particularly useful when investigating more complex motions or the changing forces during the collision or interaction between two or more objects, even that on the scale of a collision between Earth and an asteroid.

By the end of this chapter

you will have covered material including:

- force and change in momentum
- conservation of momentum
- work done as a change in energy
- kinetic and gravitational potential energy
- efficiency in mechanical systems
- elasticity of collisions
- power.

4.1 The relationship between momentum and force

Figure 4.1
When two footballers collide, they exert an equal and opposite force on each other. The effect this force will have on the velocity of each footballer can be investigated using the concept of momentum.

Figure 4.2
The enormous mass of a large ship endows it with very large momentum despite its relatively slow speed. After turning off its engines, it can continue against the resistance of the water for over 4 km if no other braking is applied.

Consider a collision between two footballers on the football field. From Newton's second law, each force can be expressed as:

$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$$

and using the relationship for acceleration:

$$\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{\Delta t} \qquad \boldsymbol{F} = \frac{m(\boldsymbol{v} - \boldsymbol{u})}{\Delta t}$$

where $\boldsymbol{a}$ is the acceleration during the collision (m s^{-2})
Δt is the time of contact (s)
$\boldsymbol{u}$ is the velocity of either one of the footballers before the collision (m s^{-1})
$\boldsymbol{v}$ is the velocity of the footballer after the collision (m s^{-1}).

Simplifying:

$$\boldsymbol{F}\Delta t = m(\boldsymbol{v} - \boldsymbol{u})$$

or:

$$\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v}$$

This relationship introduces two important ideas.

- The product of force and the contact time is referred to as impulse, $\boldsymbol{I}$. The idea of impulse applies to objects during collisions when the time of contact is small. More on this concept will be explained later.
- The product of the mass of an object and its velocity is referred to as momentum:

$$\boldsymbol{p} = m\boldsymbol{v}$$

where $\boldsymbol{p}$ is momentum (kg m s^{-1})
m is the mass of the object (kg)
$\boldsymbol{v}$ is the velocity of the object (m s^{-1}).

Momentum can be thought of as the tendency of an object to keep on moving with the same speed in the same direction. It is a property of any moving object and its size depends solely on that object, not on outside forces. As it is the product of a scalar quantity (mass) and a vector quantity (velocity), momentum is a vector quantity. The direction of the momentum of an object is the same as the direction of the velocity of that object. The unit for momentum is kg m s^{-1} which is readily determined from the product of the units for mass and velocity.

Momentum often indicates the difficulty a moving object has in stopping. A fast moving car has more momentum than a slower one of the same mass; equally so, an elephant will have more momentum than a person travelling at the same speed (just as a greater force is needed to cause the same acceleration). The more momentum an object gains as its velocity increases, the more it has to lose to stop and the greater the effect it will have if involved in a collision. A football player is more likely to be knocked over if tackled by a heavy follower than a light rover since the product $\boldsymbol{p} = m\boldsymbol{v}$ will be larger for the heavy follower.

Newton originally stated his second law in terms of momentum. That is:

$$\boldsymbol{F} = \frac{\Delta\boldsymbol{p}}{\Delta t}$$

where $\boldsymbol{F}$ is the average net force applied to the object during the collision in newtons (N)
$\Delta\boldsymbol{p}$ is the change in momentum during contact for a time Δt.

The change in momentum of a body is proportional to the net force applied to it:

$$\Delta \boldsymbol{p} \propto \boldsymbol{F}$$

An unbalanced force is required to change the momentum of an object, to increase it, decrease it or change its direction. This force might result from a collision or an interaction with another object. The change in momentum ($\Delta \boldsymbol{p}$) of the object will be given by:

Change in momentum = final momentum − initial momentum

$$\Delta \boldsymbol{p} = \boldsymbol{p}_f - \boldsymbol{p}_i$$

Worked Example 4.1A

A footballer collides with a goal post and comes to rest while trying to take a mark. The footballer has a mass of 80 kg and was travelling at 8.2 m s^{-1} at the time of the collision.

a What was the change in momentum during the collision for the footballer?

b Estimate the average force the footballer experienced in this collision.

Solution

a Prior to the collision the footballer's momentum was given by:

$\boldsymbol{p} = m\boldsymbol{v}$

$= 80 \text{ kg} \times 8.2 \text{ m s}^{-1}$ towards the pole

$= 656 \text{ kg m s}^{-1}$

After the collision the momentum was zero since the footballer stopped moving.

So:

$\Delta \boldsymbol{p} = 0 - 656$

$= -656 \text{ kg m s}^{-1}$ towards the pole

or:

$\Delta \boldsymbol{p} = 656 \text{ kg m s}^{-1}$ away from the pole

b The negative value for the change in momentum indicates that the direction of the momentum, and hence the force applied to the footballer, is opposite to the direction in which the footballer was travelling. The time that the footballer took to stop has not been given but a reasonable estimate of the force can be made by estimating the stopping time. Keeping to magnitudes of 10 for easy working, it would be reasonable to assume that the stopping time in this sort of collision would be less than 1 s (10^0) and greater than 0.01 s (10^{-2}). Something in the order of 0.1–0.5 s (10^{-1}) would make sense on the basis of observations of similar situations.

Then using:

$\boldsymbol{F} = \Delta \boldsymbol{p}/\Delta t$

$= \dfrac{656}{10^{-1}}$

$= 6560$

$\approx 7 \times 10^3$ N away from the pole, i.e. a retarding force

Physics file

The familiar form of Newton's second law, $\boldsymbol{F} = m\boldsymbol{a}$, is made more general in terms of momentum and easily applied since it allows for changes in mass while a force is being applied.

Impulse

Think about what it feels like to fall onto a concrete floor. Even from a small height it can hurt. A fall from the same height onto a tumbling mat is barely felt. Your speed is the same, your mass hasn't changed and gravity is still providing the same acceleration. So what is different about the fall onto the mat that reduces the force you experience?

Remember that, according to Newton's second law of motion, the velocity of an object only changes when a force is applied to that object. A larger force will be more effective in creating a change in the velocity of the object. The faster the change occurs (i.e. a smaller time interval Δt), the greater the force that is needed to create that change. Landing on a concrete floor changes the velocity very quickly as you are brought to an abrupt stop. When landing on a tumble mat the change occurs over a much greater time. The force needed to create the change is smaller.

Another illustration of this could be a tennis player striking a ball with a racquet. At the instant the ball comes in contact with the racquet the applied force will be small. As the strings distort and the ball compresses, the force will increase until the ball has been stopped. The force will then decrease as the ball accelerates away from the racquet. A graph of force against time will look like that in Figure 4.3b.

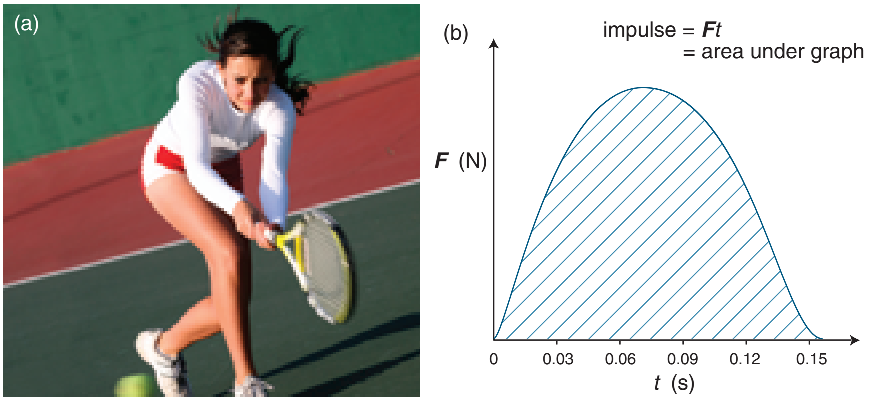


Figure 4.3

(a) When a tennis player hits a ball, an unbalanced force is applied to the ball, creating a change in its momentum; hence an impulse is applied to the ball. The magnitude of the force will change over time. (b) The impulse can be found from the area under the force–time graph since the area = x-axis × y-axis = $\boldsymbol{F} \times t$ = impulse.

The impulse affecting the ball at any time will be the product of applied force and time, i.e. $\boldsymbol{I} = \boldsymbol{F}\Delta t$. The total impulse during the time the ball is in contact will be $\boldsymbol{I} = \boldsymbol{F}_{av} \times t$, where $\boldsymbol{F}_{av}$ is the average force applied during the collision. This is equivalent to the total area under the force–time graph. The total impulse for any collision can be found in this way.

The ***IMPULSE*** affecting an object during a collision is the product of the net average applied force and the time of contact and is the area under a force–time graph.

The concept of impulse is appropriate when dealing with forces during any collision since it links force and contact time, e.g. a person hitting the ground, as described above, or a ball being hit by a bat or racquet. If applied to situations where contact is over an extended time, the average net force involved is used since the forces are generally changing (as the ball deforms, for example). The average net applied force can be found directly from the formula for impulse. The instantaneous applied force at any particular time during the collision must be determined from a graph of the force against time.

From the derivation at the start of the chapter, the impulse is also equal to the change of momentum for an object. Previously we had:

$$\boldsymbol{F} = \frac{m\Delta \boldsymbol{v}}{\Delta t}$$

Multiplying by the time interval Δt:

$$\boldsymbol{F}\Delta t = m\Delta \boldsymbol{v}$$

or:

$$\boldsymbol{I} = \boldsymbol{F}\Delta t = \Delta \boldsymbol{p}$$

The units for impulse and momentum in this context are the same, and are a combination of the unit for force and the unit for time—newton second (N s).

✓ Worked Example 4.1B

Figure 4.3 demonstrates how the force that a tennis racquet applies to a tennis ball changes during the contact of the ball with the racquet. The tennis ball has a mass of 58 g and was originally travelling towards the racquet at 55 m s^{-1}.

a Find the change in momentum as the ball is momentarily brought to a halt by the racquet.
b Find the magnitude of the impulse during this collision.
c Find the average force applied during the time it takes to stop the ball.

Solution

a Initial momentum:

$\boldsymbol{p}_i = m\boldsymbol{v}_i$
$= 0.058 \times 55$
$= 3.19$ kg m s^{-1}

Final momentum:

$\boldsymbol{p}_f = m\boldsymbol{v}_f$
$= 0.058 \times 0$
$= 0$ kg m s^{-1}

Change in momentum:

$\Delta\boldsymbol{p} = 0 - 3.19$
$= -3.19$ kg m s^{-1} in the direction of travel, i.e. ~3.2 kg m s^{-1} in the opposite direction.

b Impulse = change in momentum:

$\boldsymbol{I} = 3.19 \approx 3.2$ N s

c From the graph in Figure 4.3b, it took 0.15 s for the ball to come to a stop.

Using $\boldsymbol{I} = \boldsymbol{F}_{av}\Delta t$
then $\boldsymbol{F}_{av} = \boldsymbol{I}/\Delta t$
so $\boldsymbol{F}_{av} = 3.19/0.15$
$= 21.27$ N
≈ 21 N in the opposite direction to the ball's travel

Worked Example 4.1C

This example puts some of the earlier material on motion to practical use in an investigation of the forces on landing. A person weighing 60 kg lands on a concrete surface after jumping from a height of 6.0 m. The impact time when landing with straight legs, knees locked, is 0.002 s and with knees bent 0.050 s. Will the person's tibia bone fracture? Assume that the tibia has a cross-sectional area of 3.0 cm²
and cross-sectional bone strength of 170 N mm⁻². Use $\boldsymbol{g}$ = 9.8 m s⁻².

Solution

$\boldsymbol{a} = \boldsymbol{g} = 9.8$ m s⁻², $\boldsymbol{u} = 0$ m s⁻¹, $\boldsymbol{s} = 6.0$ m

From the equations of motion in Chapter 2:

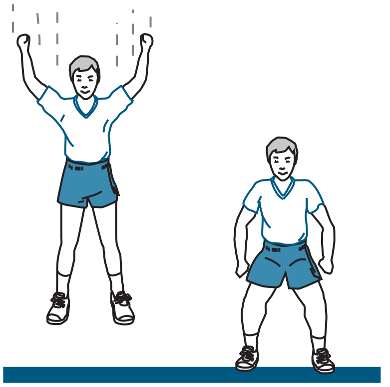
$$v^2 = u^2 + 2as$$
$$= 0 + 2 \times 9.8 \times 6.0 = 117.6$$

and so the landing speed is $v = 10.8$ m s⁻¹.

When the person strikes the ground, the momentum in the lower leg quickly becomes zero.

So:

$$F\Delta t = \Delta p$$
$$= 0 - 60 \times 10.8$$
$$= -651 \text{ N s}$$



The negative sign indicates that the impulse is opposite in direction to the original motion, i.e. upwards. The compressive breaking force for bone is 170 N mm⁻²; therefore for a tibia of cross-section 3.0 cm² or 300 mm², the force required to break it is:

$170 \times 300 = 5.1 \times 10^4$ N

Landing stiff legged:

$$F\Delta t = 651 \text{ N s}$$
$$\Delta t = 0.002 \text{ s}$$

So:

$$F = \frac{651}{0.002}$$
$$= 3.26 \times 10^5 \text{ N}$$

This is the net force on the tibia, being the sum of the upward force exerted by the ground and the downward force of gravity, as shown in Figure 4.3. Therefore:

$$F_{ground} = F + mg$$
$$= 3.26 \times 10^5 + 60 \times 9.8$$
$$= 3.26 \times 10^5 \text{ N}$$

This is significantly larger than the force required to break the tibia. Landing with a bent knee:

$F\Delta t = 651$ N s as before

$$\Delta t = 0.050 \text{ s}$$

So:

$$F = \frac{651}{0.050}$$
$$= 1.30 \times 10^4 \text{ N}$$

and as before:

$$F_{ground} = F + mg$$
$$= 1.30 \times 10^4 + 60 \times 9.8$$
$$= 1.36 \times 10^4 \text{ N}$$

As this is less than the compressive force required to fracture the tibia, the bone won't fracture. This calculation was based on applying the force to one leg. If the person landed on both legs, the force would be divided by two and so reduced. However, the tendons in the leg are much weaker than the bone so there still may be an injury.

Physics in action — Forces during collisions

Try this simple experiment. Grab a can of fruit or similar relatively soft non-corrugated steel can. Place your finger flat on a bench top and, carefully avoiding the can's seam, bring the side of the can crashing down on your finger. (We take no responsibility for you using the wrong part of the can!)

Were you actually game to try it? If you did, how much did it hurt? Not nearly as much as you expected, right? Why?

Bringing a rigid hammer down on your finger in similar circumstances would have caused considerable damage to the finger. Yet the can crumpled in around your finger and, even though it had a similar mass to the hammer and travelled at a reasonable velocity, it caused no damage to the finger and little pain. This observation can be nicely explained with the concept of impulse.

By assuming a mass of 500 g for both hammer and can and an impact speed of 20 m s^{-1}, the magnitude of the change in momentum, and hence impulse, can be estimated:

$$\begin{aligned} \boldsymbol{I} &= \Delta \boldsymbol{p} = m\Delta \boldsymbol{v} \\ &= 0.5 \times 20 \\ &= 10 \text{ N s} \end{aligned}$$

The hammer, being rigid, will quickly come to a stop. This time can be estimated at around 0.1 s, so:

$$\begin{aligned} \boldsymbol{F} &= \frac{\boldsymbol{I}}{\Delta t} \\ &= 10/0.1 \\ &= 100 \text{ N} \end{aligned}$$

This is a considerable force which could be expected to do some damage to the finger!

The can is able to compress and so the stopping time will be somewhat longer, say around 0.5 s. The average force will be:

$$\begin{aligned} \boldsymbol{F} &= \frac{\boldsymbol{I}}{\Delta t} \\ &= 10/0.5 \\ &= 20 \text{ N} \end{aligned}$$

Increasing the stopping time by five times has reduced the average force applied to the finger to one-fifth that applied by a rigid object to something which, while perhaps not totally pain-free, is quite tolerable and will do no real damage. The applied force is inversely proportional to the stopping time. Increase the stopping time and the applied force is decreased. Try it on a friend and see if you can prove this bit of physics to them!

This simple idea is the basis upon which the absorbency systems of sports shoes, crash helmets, the airbags and crumple zones of cars and other safety devices are designed.

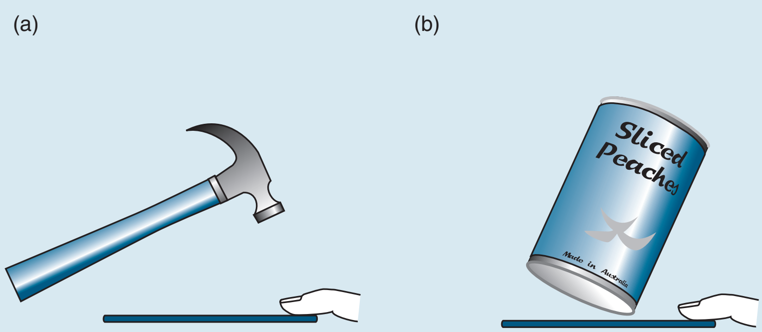


Figure 4.4

A simple example of the effect on the applied force of the stopping time during a collision can be achieved with nothing more complicated than a hammer, a can of fruit and your finger. (a) A rigid object, such as the hammer, will stop quickly. The applied force will be large. (b) A can will experience the same change in momentum, but, having a simple crumple zone, will stop more slowly, thus reducing the applied force to a tolerable amount. If you try this, then hit your finger with the side of the can and use a large (800 g) can of fruit.

Physics in action — Walking, running and sports shoe design

As athletes walk or run, they experience action–reaction forces due to gravity, the surface of the track and the air around them. These forces have been investigated in some detail in the previous chapter.

The force the ground exerts on a runner creates a change in momentum as the runner's feet strike the ground. This force can be quite large and cause considerable damage to the runner's ankles, shins, knees and hips as it is transmitted up the bones of the leg. Jogging in bare feet can increase the forces experienced to nearly three times that applied when simply standing still. Table 4.1 lists the relative size of some forces associated with common movements in sport.

table 4.1 Relative size of forces associated with some common movements in sport

Movement	Footwear	Ratio of normal to weight force
Standing still	Barefoot or shoes	1.0
Walking	Barefoot	0.6
Jogging	Barefoot	2.9
Jogging	Running shoes	2.2
Sprinting	Barefoot	3.8
Fast bowling	Cricket spikes	4.1
Long-jump take-off	Athletic spikes	7.8

An understanding of the forces generated and the elastic properties of materials are used by designers in the development of tracks, playing surfaces and sports shoes. Elastic materials can reduce the forces developed between foot and track by increasing the stopping time. Based on an understanding of impulse, sports shoes are designed with soles that include gels, air and cushioning grids, which extend the stopping time and thus reduce the force applied to the runner's body. Sophisticated modern sports shoes, properly fitted to suit the wearer, have substantially reduced the size and effect of forces on the runner with consequential benefits for the runner's knees and hips.

Figure 4.5
The forces developed between track and foot can do considerable damage to a runner's body unless they are reduced by increasing the stopping time through cushioning of the foot. Modern track shoes incorporate sophisticated design principles to increase stopping time and thus decrease the forces generated.

The running surface can also be designed to minimise the forces and produce fewer injuries. Cushioned surfaces reduce the impact considerably. Grass is actually quite effective but sometimes can be too springy. The extra time spent rebounding from the surface slows the runner down. The response must be quick if good running times are to be achieved. As a result artificial surfaces such as polyurethane, 'AstroTurf' and 'Rebound Ace' have become popular. They offer good cushioning but are more responsive, allowing faster take-offs, than grass.

Physics in action — Vehicle safety design

Designing a successful car is a complex task. A vehicle must be reliable, economical, powerful, visually appealing, secure and safe. Public perception of the relative importance of these issues varies. Magazines and newspapers concentrate on appearance, price and performance. The introduction of air-bag technology into most cars has altered the focus towards safety. Vehicle safety is primarily about crash avoidance. Research shows potential accidents are avoided 99% of the time. The success of accident avoidance is primarily attributable to accident avoidance systems like antilock brakes. When a collision does happen, passive safety features like the air bag come into operation. Understanding the theory behind accidents involves an understanding primarily of impulse and force.

Physics in action — The air bag

Seatbelts save lives. They also create injuries. Strangely, the number of people receiving serious injury has increased since the introduction of seatbelts. Previously many of today's survivors would have died instantly in the accident. A further safety device is required to minimise these injuries.

The air bag in a car is designed to inflate within a few milliseconds of a collision to reduce secondary injuries during the collision. It is designed to inflate only when the vehicle experiences an 18–20 km h^{-1} or greater impact with a solid object. The required deceleration must be high or accidental nudges with another car would cause the air bag to inflate. The car's computer control makes a decision in a few milliseconds to detonate the gas cylinders that inflate the air bag. The propellant detonates and inflates the air bag while the driver collapses towards the dashboard. As the body lunges forward into the air bag, the bag deflates, allowing the body to slow in a longer time as it moves towards the dashboard. Injury is thus minimised.

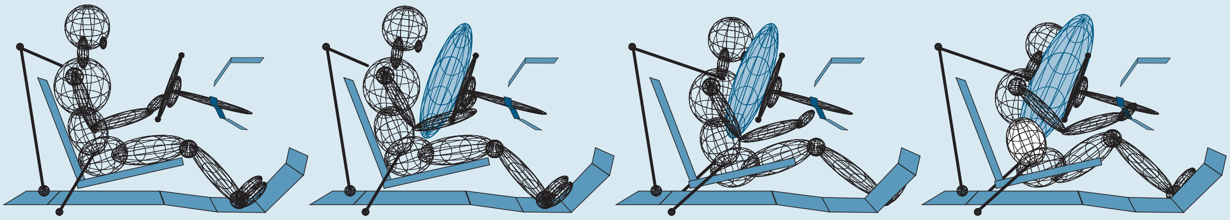
Calculating exactly when the air bag should inflate, and for how long, is a difficult task. Many cars have been crash tested and the results painstakingly analysed. High-speed film demonstrates precisely why the air bag is so effective. During a collision the arms, legs and head of the occupants are restrained only by the joints and muscles. Enormous forces are involved due to the large deceleration. The shoulders and hips can, in most cases, sustain the large forces for the short duration. However, the neck is the weak link. Victims of road accidents regularly receive neck and spinal injuries. An air bag reduces the enormous forces the neck must withstand by extending the duration of the collision, a direct application of the concept of impulse.

The extent of injuries during a collision is not only dependent on the size of the force but also on the duration and deflection resulting from the applied force. An increase in localised pressure will result in a greater compression or deflection of the skull. The air bag reduces the localised pressure by increasing the contact surface area and decreasing the force. The effect can be seen by the relationship:

$$P = \frac{F}{A}$$

where P is the pressure (N m^{-2})
F is the force (N)
A is the contact area (m^2).

An air bag has a contact surface area of around 0.2 m^2 compared with 0.05 m^2 for a seatbelt. This reduces injuries caused by seatbelts, such as bruising, broken ribs and collar bones since it increases the stopping time. It also supports the head and chest, preventing high neck loads caused by the seatbelt restraining the upper torso. Most importantly, it prevents the high forces caused by contact of the head with the steering wheel. The air bag ensures that the main thrust of the expansion is directed outwards instead of towards the driver. The deflation rate, governed by the size of the holes in the rear of the air bag, provides the optimum deceleration of the head for a large range of impact speeds.



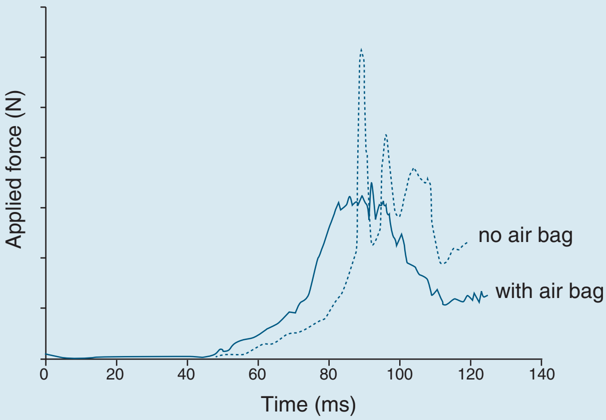


Figure 4.6
The air bag is one of a number of passive safety features incorporated into the design of modern cars. It extends the duration of the stopping time, significantly reducing the forces on the head and neck during a collision. It also distributes the force required to decelerate the mass of the driver or passenger over a larger area than a seatbelt. The deflation rate of the bag is governed by the size of holes in the rear of the air bag, and is designed to provide the optimum deceleration of the head for a large range of impacts.

The air bag is not the answer to all safety concerns associated with a collision but is one of many safety features that form a chain of defence in a collision.

4.1 SUMMARY The relationship between momentum and force

- The momentum of a moving object is the product of its mass and its velocity:

$$\boldsymbol{p} = m\boldsymbol{v}$$

 where $\boldsymbol{p}$ is in kg m s^{-1}
 m is in kg
 $\boldsymbol{v}$ is in m s^{-1}.

 Momentum is a vector quantity.
- The change in momentum ($\Delta\boldsymbol{p}$) = final momentum ($\boldsymbol{p}_f$) – initial momentum ($\boldsymbol{p}_i$).
- Impulse is the product of the net force during a collision and the time interval Δt during which the force acts: $\boldsymbol{I} = \boldsymbol{F}\Delta t$. It can also be found from the area under a force–time graph and is measured in newton seconds (N s). Impulse is equal to the change in momentum, $\Delta\boldsymbol{p}$, caused by the action of the net applied force.
- Extending the time over which a collision occurs will decrease the average net force applied since:

$$\boldsymbol{F} \propto \frac{1}{\Delta t}$$

 This is the principle behind many safety designs.

4.1 Questions

Use g = 9.8 m s^{-2} where required.

1 What is the momentum, in kg m s^{-1}, of a 20 kg cart travelling at:
   a 5 m s^{-1}?
   b 5 cm s^{-1}?
   c 5 km h^{-1}?

2 The velocity of an object of mass 8 kg increases from an initial 3 m s^{-1} to 8 m s^{-1} when a force acts on it for 5 s.
   a What is the initial momentum?
   b What is the momentum following the action of the force?
   c How much momentum is the object gaining each second when the force is acting?
   d What impulse does the object experience?
   e What is the magnitude of the force?

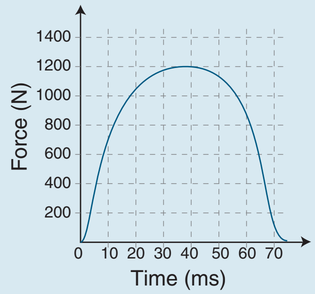
3 Which object has the greater momentum—a medicine ball of mass 4.5 kg travelling at 3.5 m s^{-1} or one of mass 2.5 kg travelling at 6.8 m s^{-1}?

4 Calculate the momentum of an object:
   a of mass 4.5 kg and velocity 9.1 m s^{-1}
   b of mass 250 g and velocity 3.5 km h^{-1}
   c once it has fallen freely from rest for 15 s if it has a mass of 3.4 kg
   d that experiences a force of magnitude 45 N if the force is applied for 3.5 s.

5 A tennis ball may leave the racquet of a top player with a speed of 61 m s^{-1} when served. If the mass of the ball is 65 g and it is actually in contact with the racquet for 0.032 s:
   a what momentum does the ball have on leaving the racquet?
   b what is the average force applied by the racquet on the ball?

6 A 200 g cricket ball is struck by a cricket bat. The ball and bat are in contact for 0.05 s, during which time the ball is accelerated to a speed of 45 m s^{-1}. Assume that the intitial velocity of the ball is zero.
   a What is the magnitude of the impulse the ball experiences?
   b What is the net average force acting on the ball during the contact time?
   c What is the net average force acting on the bat during the contact time?

7 The following graph shows the net vertical force generated as an athelete's foot strikes an asphalt running track.



   a Estimate the maximum force acting on the athlete's foot during the contact time.
   b Estimate the total impulse during the contact time.

8 A 25 g arrow buries its head 2 cm into a target on striking it. The arrow was travelling at 50 m s^{-1} just before impact.
   a What change in momentum does the arrow experience as it comes to rest?
   b What is the impulse experienced by the arrow?
   c What is the average force that acts on the arrow during the period of deceleration after it hits the target?

9 Crash helmets are designed to reduce the force of impact on the head during a collision.
   a Explain how their design reduces the net force on the head.
   b Would a rigid 'shell' be as successful? Explain.

10 Describe, with the aid of diagrams, a simple collision involving one moving object and one fixed in position. Estimate, by making reasonable estimates of the magnitudes of the mass and velocity of the moving object, the net force acting on the objects during the collision.

4.2 Conservation of momentum

Physics file

The principle of conservation of momentum was responsible for the interpretation of investigations that led to the discovery of the neutron. Neutral in charge, the neutron could not be investigated through the interactions of charged particles that had led to the discovery of the proton and electron. In 1932 Chadwick found that in collisions between alpha particles and the element beryllium, the principle of conservation of momentum only held true if it could be assumed that there was an additional particle within the atom which had close to the same mass as a proton but no electric charge. Subsequent investigations confirmed his experiments and led to the naming of this particle as the neutron.

The most significant feature of momentum is that it is conserved. This means that the total momentum in any complete system will be constant. For this reason momentum is very useful in investigating the forces experienced between two colliding objects—as long as they are unaffected by outside forces. The law of conservation of momentum, as it is known, is derived from Newton's third law.

From Newton's third law, the force applied by each object in a collision will be of the same magnitude but opposite in direction:

$$\boldsymbol{F}_1 = -\boldsymbol{F}_2$$

From Newton's second law, $\boldsymbol{F} = m\boldsymbol{a}$, the forces could be expressed as:

$$m_1\boldsymbol{a}_1 = -m_2\boldsymbol{a}_2$$

and using $\boldsymbol{a} = \dfrac{\boldsymbol{v} - \boldsymbol{u}}{\Delta t}$ we get:

$$m_1\frac{(\boldsymbol{v}_1 - \boldsymbol{u}_1)}{\Delta t} = -m_2\frac{(\boldsymbol{v}_2 - \boldsymbol{u}_2)}{\Delta t}$$

where $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ are the respective accelerations of the two objects during the collision (m s^{-2})
Δt is the time of contact (s)
$\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ are the velocities of the objects prior to collision (m s^{-1})
$\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ the velocities after collision (m s^{-1}).

Since the time that each object is in contact with the other will be the same, Δt will cancel out:

$$m_1(\boldsymbol{v}_1 - \boldsymbol{u}_1) = -m_2(\boldsymbol{v}_2 - \boldsymbol{u}_2)$$

or:

$$m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$$

In other words, the total momentum before colliding is the same as the total momentum after the collision.

THE LAW OF CONSERVATION OF MOMENTUM states that, in any collision or interaction between two or more objects in an isolated system, the total momentum of the system will remain constant; that is, the total initial momentum will equal the total final momentum:

$$\Sigma\boldsymbol{p}_\mathrm{i} = \Sigma\boldsymbol{p}_\mathrm{f}$$

or:

$$m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$$

It is most important to realise that momentum is only conserved in an isolated system. This is a system in which no external forces affect the objects involved. The only forces involved are the action/reaction forces on the objects in the collision. Consider two skaters coming together on a near-frictionless ice rink. In this near-ideal situation it is realistic to apply the law of conservation of momentum. The only significant horizontal forces between the two skaters are those of the action/reaction pair as the two skaters collide.

If the skaters were to skate through a puddle of water as they come together, then friction would become noticeable. This force is an external force since it is not acting between the two skaters. The interaction between a skater and the puddle would constitute a separate isolated system (as would that between puddle and ice, ice and ground etc.). Momentum would still be conserved within this other separate system. It is virtually impossible to find a perfectly isolated system here on Earth because of the presence of gravitational, frictional and air-resistance forces. Only where any external forces are insignificant in comparison to the collision forces is it reasonable to apply the law of conservation of momentum.

Also important is that in any collision involving the ground, Earth itself must be part of the system. Theoretically, any calculation based on conservation of momentum should include the Earth as one of the objects, momentum only being conserved when all the objects in the system are considered. In practice, the very large mass of the Earth in relation to the other objects involved means that there is a negligible change in the Earth's velocity and it can be ignored in most calculations. Of course, a collision with a fast moving asteroid, as described in the introduction to this chapter, would be another matter!

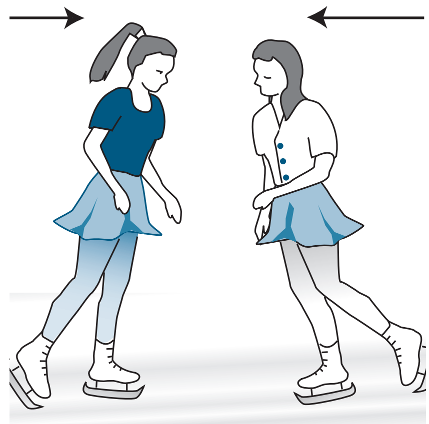


Figure 4.7
When two skaters collide on a near-frictionless skating rink, they exert an equal opposite force on each other. The total momentum of the two skaters before the collision will equal the total momentum of the two skaters after the collision. Because no large forces are involved other than those in the collision, this can be considered as an isolated system.

✓ Worked Example 4.2A

One of the skaters in Figure 4.7, with mass 80 kg, was skating in a straight line with a velocity of 6.0 m s^{-1}, while the other, of mass 70 kg, was skating in the opposite direction, also with a speed of 6.0 m s^{-1}.

a The two skaters collide and the heavier skater comes to rest. Assuming that friction can be ignored, what will happen to the lighter skater after the collision?

b What would happen if the two skaters had hung on to each other and stayed together after the collision?

Solution

a From conservation of momentum:
$\Sigma \boldsymbol{p}_i = \Sigma \boldsymbol{p}_f$
or $m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$
and $m_1 = 80$ kg, $m_2 = 70$ kg
As both velocity and momentum are vector quantities, a positive direction should be established and taken into consideration.
Adopting the motion of the heavier skater's direction as the positive direction:
$\boldsymbol{u}_1 = 6.0$ m s^{-1}, $\boldsymbol{u}_2 = -6.0$ m s^{-1}, $\boldsymbol{v}_1 = 0$ m s^{-1}, $\boldsymbol{v}_2 = ?$
Substituting into equation:
$80 \times 6.0 + 70 \times (-6.0) = 80 \times 0 + 70 \times \boldsymbol{v}_2$
and $\boldsymbol{v}_2 = +0.86$ m s^{-1}.
The 70 kg skater bounces back in the opposite direction with a speed of 0.86 m s^{-1}.

b Treating the two skaters as one mass after the collision:
$80 \times 6.0 + 70 \times (-6.0) = (80 + 70) \times \boldsymbol{v}_2$
and now $\boldsymbol{v}_2 = +0.4$ m s^{-1}.
A different outcome after the collision results in a different velocity for each skater. There is no unique answer when applying the idea of conservation of momentum. The final velocity of any object depends on what happens to all the objects involved in the collision.

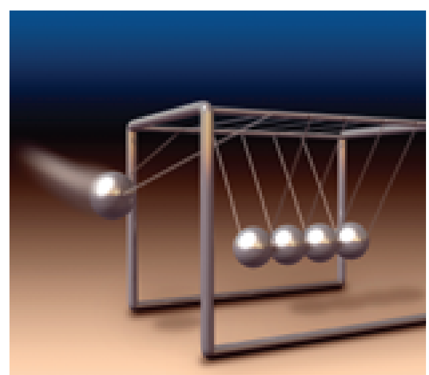
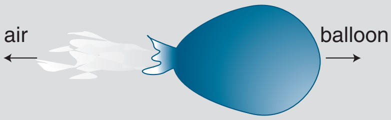


Figure 4.8
Newton's cradle, or Newton's balls to some, is an instructive 'executive toy' based on the principle of conservation of momentum extended over a number of objects.

The law of conservation of momentum can be extended to any number of colliding objects. The total initial momentum is found by calculating the vector sum of the initial momentum of every object involved. The total final momentum will then also be the vector sum of each separate momentum involved. Separation into two or more parts after the 'collision' (interaction is a better word since it does not have to be destructive), for example releasing a javelin, can also be dealt with in the same manner.

Physics file

If you release an inflated rubber balloon with its neck open, it will fly off around the room. In the diagram, the momentum of the air to the left is moving the balloon to the right. Momentum is conserved. This is the principle upon which rockets and jet engines are based.



Both rockets and jet engines employ a high-velocity stream of hot gases that are vented after the combustion of a fuel/air mixture. The hot exhaust gases have a very large momentum as a result of the high velocities involved and can accelerate rockets and jets to high velocities as they acquire an equal momentum in the opposite direction. Rockets destined for space carry their own oxygen supply, while jet engines use the surrounding air supply.

Worked Example 4.2B

An athlete of mass 63 kg is completing a run-up to throw a 500 g javelin. Just prior to the point of release, the athlete has reached a speed of 7.5 m s^{-1}. After releasing the javelin, the athlete continues with a speed of 7.0 m s^{-1}. Assuming that momentum is conserved (ignoring frictional forces acting on the athlete's feet), with what speed does the javelin leave the athlete's hand?

Solution

m_1(person) = 63 kg, m_2(javelin) = 500 g = 0.5 kg, $\boldsymbol{u}_1 = \boldsymbol{u}_2 = 7.5$ m s^{-1},
$\boldsymbol{v}_1 = 7.0$ m s^{-1}, $\boldsymbol{v}_2 = ?$
Using conservation of momentum:
$\boldsymbol{u}_1 \times (m_1 + m_2) = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$
so
$7.5 \times (63 + 0.5) = 63 \times 7.0 + 0.5 \times \boldsymbol{v}_2$
$7.5 \times 63.5 - 63 \times 7.0 = 0.5 \times \boldsymbol{v}_2$
and
$\boldsymbol{v}_2 = 70.5$ m s^{-1} in the direction of the athlete's original run-up.

Physics in action — Collisions and pedestrians

A car is designed to keep its occupants safe. Unfortunately, however, very little can be done to protect a pedestrian from the onslaught of a 1400 kg car travelling at 60 km h^{-1}. Bull bars on cars in residential areas are currently under review because of the enormous damage they inflict on a pedestrian. The effect on the pedestrian will depend on the person's height and mass, the height of the front of the oncoming vehicle, and the speed, mass and shape of the vehicle.

Consider the following possibilities:

- a pedestrian being struck by a truck moving at 30 km h^{-1}
- a pedestrian being struck by a car moving at 30 km h^{-1}
- a pedestrian being struck by a cyclist moving at 30 km h^{-1}.

The injuries to the pedestrian are due largely to the change in the pedestrian's momentum. Being hit by the cyclist will obviously result in the least injury to the pedestrian because of the lower momentum of the cycle and its rider. The mass of the other vehicles is such that they have a far larger momentum to impart to the pedestrian.

Consider the following situation. A 1400 kg car is travelling at 60 km h^{-1} when it strikes a stationary 70 kg pedestrian. The pedestrian lands on the bonnet of the car and travels with the car until it finally comes to a halt. Assuming that frictional forces are minimal:

total momentum before the collision = total momentum after the collision

Before the collision:

$\boldsymbol{p}_{car} = m\boldsymbol{v}$

$= 1400 \times (60 \times 1000/3600)$

$= 2.3 \times 10^4$ kg m s^{-1}

$\boldsymbol{p}_{pedestrian} = 0$ (pedestrian is stationary)

After the collision:

$\boldsymbol{p}_{total} = 2.3 \times 10^4 \text{ kg m s}^{-1} = (m_{car} + m_{pedestrian})\boldsymbol{v}$

so:

$$1470\boldsymbol{v} = 2.3 \times 10^4 \text{ kg m s}^{-1}$$

and:

$\boldsymbol{v}$ (both car and pedestrian) ≈ 16 m s^{-1} or 57 km h^{-1}

This means that the pedestrian accelerates from rest to a speed of 57 km h^{-1} in the short duration of the collision. A similar collision between the pedestrian and a cyclist travelling at 30 km h^{-1} would result in a final speed of 5.2 m s^{-1} (19 km h^{-1}). The speed of the car changes very little. The speed of the cyclist is almost halved.

Antilock brakes, excellent road handling and reduced speed limits in some areas reduce the likelihood of a vehicle striking a pedestrian. Unfortunately, accidents can still happen.

There are essentially two possibilities that can occur when a pedestrian is struck by a car.

(a) The pedestrian bounces off the front of the car and is projected through the air. This type of motion tends to happen when the vehicle is travelling relatively slowly. The pedestrian is rapidly accelerated forwards to near the velocity of the vehicle. Injuries occur to the pedestrian when the car strikes and again when they land on the ground. Leg and hip injuries are general in this form of pedestrian/vehicle collision.

Head injuries result from the pedestrian colliding with the road. Very little of a car's design will alter the severity of head injuries received.

Vehicles can be designed with a low, energy-absorbing bumper bar to reduce knee and hip damage. If the pedestrian's knee strikes the bumper bar, knee damage is very likely. Knees do not heal as well as broken legs. A lower, energy-absorbing bar is, for this reason, preferable.

(b) When a vehicle is moving very fast at the point of impact, the pedestrian's inertia acts against rapid acceleration. The pedestrian does not initially move forward with the same velocity as the vehicle. If the pedestrian remains in approximately the same place, he or she will go either over or under the car. The pedestrian will be either run over or run under (i.e. the car goes under the pedestrian).

Being run over usually results in serious injury or fatality. Massive head injuries occur as the pedestrian's head strikes the ground. It is therefore preferable for the pedestrian to be run under. The relative height of the vehicle's bumper bar and the height of the pedestrian determines whether they will be run over or under. Most bumper bars are below adult waist level. Small children, however, have much more chance of being run over as the height of the bumper bar is relatively much higher.

If the pedestrian is run under, 'passive' safety features of modern car design come into play. Removal of protruding hood ornaments is essential since they can easily penetrate the body of a person and cause enormous injuries. The bonnet of a car acts as a good impact absorber, particularly in comparison with the hard surface of the road. Bull bars, however, can block the path of the pedestrian, making it more likely that they be run over. Further, they have little impact-absorbing ability. The logic for a ban on bull bars in residential areas can be seen easily.

4.2 SUMMARY Conservation of momentum

- The law of conservation of momentum states that in any collision or interaction between two or more objects in an isolated system, the total momentum of the system will remain constant. The total initial momentum will equal the total final momentum:
$$\Sigma \boldsymbol{p}_i = \Sigma \boldsymbol{p}_f$$
- Conservation of momentum can be extended to any number of colliding objects within an isolated system.

4.2 Questions

Use $\boldsymbol{g}$ = 9.8 m s^{-2} when required.

1 A white billiard ball of mass 100 g travelling at 2 m s^{-1} across a low-friction billiard table has a head-on collision with a black ball of the same mass initially at rest. The white ball stops while the black ball moves off. What is the velocity of the black ball?

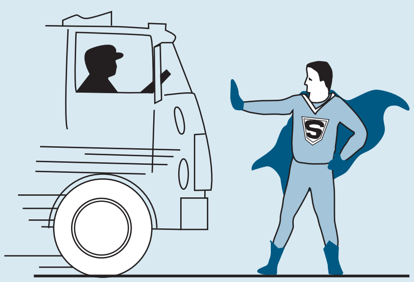
2 A girl with mass 50 kg running at 5 m s^{-1} jumps onto a 4 kg skateboard travelling in the same direction at 1.0 m s^{-1}. What is their new common velocity?

3 A man of mass 70 kg steps forward out of a boat and onto the nearby river bank with a velocity, when he leaves the boat, of 2.5 m s^{-1}. The boat has a mass of 400 kg and was initially at rest. With what velocity does the boat begin to move?

4 A railway car of mass 2 tonnes moving along a horizontal track at 2 m s^{-1} runs into a stationary train and is coupled to it. After the collision the train and car move off at a slow 0.3 m s^{-1}. What is the mass of the train alone?

5 A trolley of mass 4.0 kg and moving at 4.5 m s^{-1} collides with, and sticks to, a stationary trolley of mass 2.0 kg. Their combined speed in m s^{-1} after the collision is:
A 2.0
B 3.0
C 4.5
D 9.0

6 Superman stops a truck simply by blocking it with his outreached arm.



a Is this consistent with the law of conservation of momentum? Explain.
b Using reasonable estimates for the initial speed and mass of the truck and Superman demonstrate what will happen. Use appropriate physics concepts.

7 A car of mass 1100 kg has a head-on collision with a large four-wheel drive vehicle of mass 2200 kg, after which both vehicles stop. The four-wheel drive vehicle was travelling at 50 km h^{-1} prior to the collision in an area where the speed limit was 70 km h^{-1}. Was the car breaking the speed limit?

8 A 100 g apple is balanced on the head of young Master Tell. William, the boy's father, fires an arrow with a mass of 80 g at the apple. It reaches the apple with a velocity of 35 m s^{-1}. The arrow passes right through the apple and goes on with a velocity of 25 m s^{-1}. With what speed will the apple fly off the boy's head (assume no friction between apple and head)?

9 A space shuttle of mass 10 000 kg, initially at rest, burns 5.0 kg of fuel and oxygen in its rockets to produce exhaust gases ejected at a velocity of 6000 m s^{-1}. Calculate the velocity that this exchange will give to the space shuttle.

10 A small research rocket of mass 250 kg is launched vertically as part of a weather study. It sends out 50 kg of burnt fuel and exhaust gases with a velocity of 180 m s^{-1} in a 2 s initial acceleration period.
a What is the velocity of the rocket after this initial acceleration?
b What upward force does this apply to the rocket?
c What is the net upward acceleration acting on the rocket? (Use $\boldsymbol{g}$ = 10 m s^{-2} if required.)

4.3 Work

The notion of energy was not developed until a relatively short time ago, and was only fully understood in the early 1800s. Today, the concept has become one of the most fundamental in science. In physics an object is said to have energy if it can cause some change to occur. Energy is a conserved quantity and is useful not only in the study of motion but in all areas of the physical sciences. Before discussing energy, it is necessary to first examine the concept of work.

In common usage work has a variety of meanings. Most convey the idea of something being done. At the end of a long, tiring day we might say that we have done a lot of work. It could also be because the body's reserves of energy have been used up. Imagine lifting a heavy book up onto a high shelf. The heavier the book, the more force must be applied to overcome its weight. The higher the shelf, the greater the displacement over which the force must be applied. A very heavy book lifted to a high shelf will require a considerably greater effort than moving a few pieces of paper from floor to table. Work in physics describes what is accomplished on an object by the action of a force.

WORK is defined as the product of the applied net force and its displacement in the direction of the force. When work is done on a body, the energy of the body changes. When the force and displacement are in the same direction:

$$W = Fs$$

where W is the work in joules (J)
F is the magnitude of the force in newtons (N)
s is the magnitude of the displacement in metres (m).

Work is the area under a force–displacement graph.

One ***JOULE*** of work is done when the point of application of a net force of one newton moves an object through a distance of one metre in the direction of the force.

From the definition of work, it can be seen that if a person pushes a load for a distance of 5 m by exerting a horizontal force of 30 N on the load, then the person does 150 J of work on the load.

This seems fairly straightforward. Something is achieved so work is done. If it was done against a frictional force then the load would continue at a constant velocity and the work would be converted into heat and sound energy and transferred to the ground. On a frictionless surface, the load would accelerate, increasing its kinetic energy.

More difficult to comprehend is the idea that a force can be exerted on an object and yet do no work. For example, a person carries an armload of books. No work is done on the books since the direction of the applied force (i.e. up) is at right angles to the displacement (i.e. horizontal).

Walking horizontally involves the body doing work; for a start we rise and fall as we walk in order to travel horizontally. More particularly, our heart has to pump blood (do work) to supply the muscles with the chemical energy needed to do work (pushing legs and things around). The person carrying the books will tire.

Physics file

The unit for work, the joule (J), is used for all forms of energy in honour of James Prescott Joule, an English brewer and physicist, who pioneered work on energy in the 19th century.

Physics file

The symbol W is a little over-used in this area of physics. In the area of motion and mechanics it can stand for work, weight or the abbreviation of watt. Be careful to read the context when you come across the symbol!

Whenever the net force is perpendicular to the direction of motion, no work is done.

This may be apparent from the original definition of work. Situations also regularly occur where a constant force acts at an angle to the direction of motion. The work achieved by a force acting at an angle will be less than that when the force is acting in the direction of the displacement. The component of the force in the direction of motion, $F\cos\theta$, is used in calculating the work done in the required direction. So:

$$W = Fs\cos\theta$$

where θ is the angle between the applied force and the direction of motion.

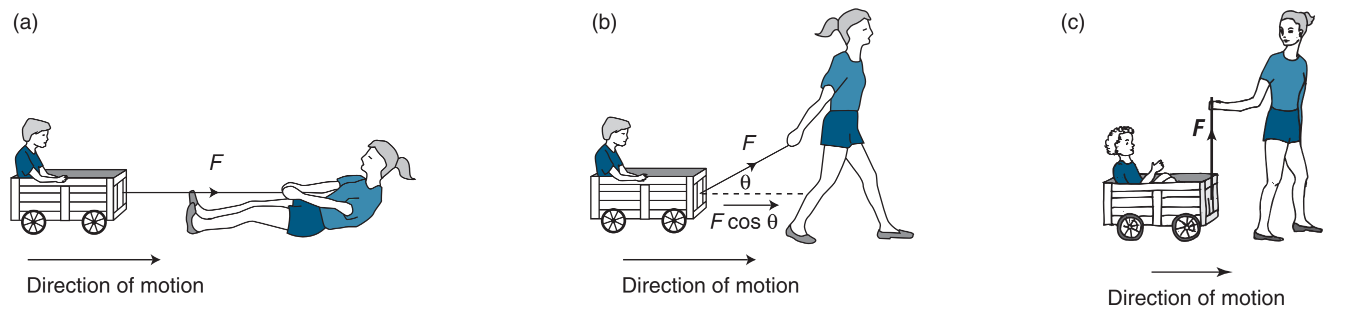


Figure 4.9

(a) If a force is applied in the direction of motion of the cart, then the force is at its most effective in moving the cart. (b) When the force is applied at an angle θ to the direction of motion of the cart, the force is less effective. The component of the force in the direction of the displacement, $F\cos\theta$, is used to calculate the work. (c) When the angle at which the force is acting is increased to a right angle ($\theta = 90°$), then the component of the force in the direction of the intended displacement is zero and it does no work on the cart—provided of course that it doesn't lift the cart, in which case work would also be done against gravity.

Figure 4.10

The annual Rialto Tower Run-up is a race to climb 1254 steps up the Rialto Tower building in Melbourne. Contestants climb a vertical distance of approximately 240 m and the winner takes around 7 minutes to complete the race.

Worked Example 4.3A

Calculate the work done against gravity by an athlete of mass 60 kg competing in the Rialto Tower Run-up illustrated in Figure 4.10.
Use $g = 9.8$ m s^{-2}.

Solution

Only the weight force need be considered in this example as the work in the vertical direction is all that is required.

$m = 60$ kg, $g = 9.8$ m s^{-2}, $s = \Delta h = 242$ m

Force required = weight = $mg = 60 \times 9.8 = 588$ N

$W = Fs$
$\quad = 588 \times 242$
$\quad = 142\,296$ N m $= 1.4 \times 10^5$ J

Worked Example 4.3B

The girl in Figure 4.9 pulls the cart by applying a force of 50 N at an angle of 30° to the horizontal. Assuming a force due to friction of 10 N is also acting on the wheels of the cart, calculate the work done if the cart is moved 10 m along the ground in a straight line.

Solution

$F_1 = 50$ N, $\theta = 30°$, $\boldsymbol{F}_f = 10$ N, $\boldsymbol{s} = 10$ m

$\Sigma F = F_1 \times \cos 30° - F_f$
$\quad = 50 \times 0.866 - 10 = 33.3$ N

Now $W = \Sigma Fs$,
and work $= 33.3 \times 10 = 330$ J

Force–displacement graphs

A graphical approach can also be used to understand the action of a force and the work achieved in the direction of motion. This is particularly useful in situations where the force is changing with displacement. The area under a graph of force against displacement always represents the work produced by the force, even in situations when the force is changing, such as during a collision. The area can be shown to be equivalent to work as follows. From Figure 4.11:

$$\text{the area of the square enclosed by the graph} = F_{av} \times s$$
$$\text{Work} = F_{av} \times s$$

When the force is changing, a good estimate of the area can be found by dividing the area into small squares and counting the number or by dividing it into thin segments. The segments can be considered to be rectangles with an area equal to the work for that small part of the displacement. The total work will be the sum of the areas of all the separate rectangles.

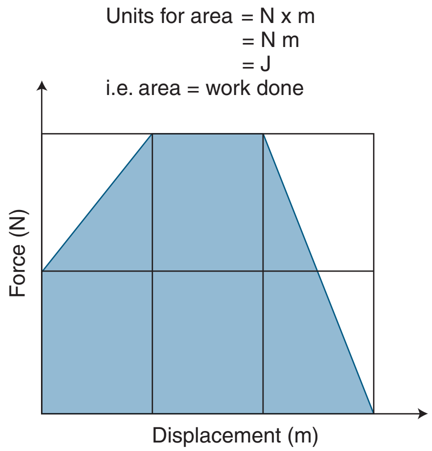
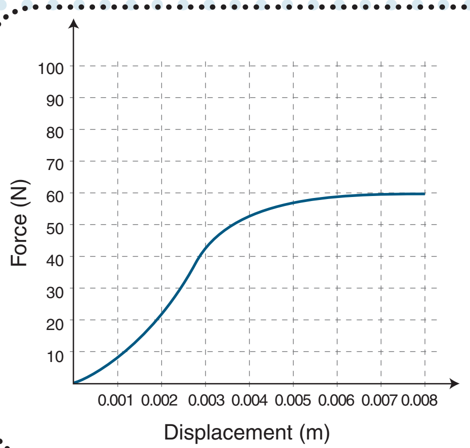


Figure 4.11
The area under a force–displacement graph is equivalent to the work done by a force acting in the direction of the displacement. Where the net applied force is changing, then the area can be found by counting squares or dividing the area into segments. The area of each segment then equals the work done by a constant force during that small displacement and the total area will represent the total work.

Worked Example 4.3C



The force–displacement graph represents the work done on the sole of a sports shoe as it compresses against the surface of a rigid track. The displacement shown represents the amount of compression the sole undergoes. Find the work done on the shoe by the compressive forces.

Solution

This is a simple case of working out the area represented by each square and then counting the total number of squares to find the total work done. Be careful to consider the scale of each axis in your working.

Area of one square = 10 N × 1 mm = 10 N × 0.001 m = 0.1 J

Total number of squares (part squares can be added to give whole squares) = 33.

Work = 33 × 0.1 = 3.3 J

Impulse and work

The concepts of impulse and work seem quite similar and, when solving problems, can easily be confused. Actually, problems focusing on forces in collisions may be solved using either concept but it should be understood that each is derived from a different idea. Impulse comes from an understanding of the action of a force on an object over time and is equal to the change in momentum the force creates. Work is related to the action of a force on an object as it moves the object, or part of it, through some displacement. As we shall see, in the next section, this equals the change in the object's energy, ΔE.

Physics file

The area under a force–displacement graph can also be found by using calculus if the equation of the graph is known. In most instances a good estimate by counting squares or segments is sufficient.

Summarising:

- *Impulse* is equal to $\boldsymbol{F} \times \Delta t$, is equivalent to Δp, has the units newton seconds (N s), and can be determined from the area under a force–time graph.
- Work is equal to $F \times s$, is equivalent to ΔE, has the units joules (J), and can be determined from the area under a force–displacement graph.

4.3 SUMMARY Work

- When a force does work on an object resulting in a change in displacement, there is a change in the energy of the object.
- Work, W, in joules (J) is the product of the net applied force and its displacement in the direction of the force:

$$W = Fs$$

- The work done by a force acting at an angle to the displacement is given by $Fs\cos\theta$, where θ is the angle between the force and the direction of the displacement. When the force is at right angles to the direction of the displacement, no work is done in that direction.
- The area under a force–displacement graph is equivalent to the work done. The area under the graph from a variable force can be found by counting squares or narrow segments.
- The concepts of impulse and work may both be used to find the size of the force acting on an object during a collision but are derived from a consideration of different ideas.

4.3 Questions

Where appropriate use $\boldsymbol{g} = 9.8\ \text{m s}^{-2}$.

1 How much work is done when a mass of 4.5 kg is lifted vertically at a constant speed through a displacement of 6.0 m?

2 A bushwalker climbs a hill 250 m high. If her mass is 50 kg and her pack has an additional mass of 10 kg, calculate the work she needs to do in climbing to the top of the hill.

3 Is the quantity you calculated in question 2 the only work that the bushwalker has done? Explain.

4 The work done by a force is:
- i calculated by multiplying the force by the distance moved
- ii measured in joules
- iii not affected by the angle at which the force acts.

Which statement/s is/are correct?

A i, ii, iii **B** i, ii **C** ii, iii
D ii **E** iii

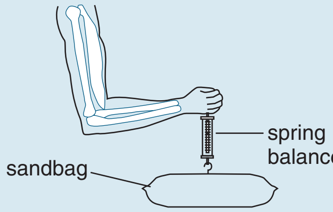
5 A removalist is loading five boxes onto a truck. Each has a mass of 10 kg and a height of 30 cm. The tray of the truck is 1.5 m above the ground and the removalist is placing each box on top of the previous one.
- **a** How much work does the removalist do in lifting the first box onto the truck tray?
- **b** How much energy has the removalist used in lifting this first box?
- **c** What is the total work done by the removalist in lifting all the boxes onto the truck as described?

6 If a lift of mass 500 kg is raised through a height of 15 m by an electric motor:
- i the weight of the lift is 4900 N
- ii the useful work done on the lift is 73 500 J
- iii the useful work done is the only energy used by the motor.

Which statement/s is/are correct?

A i, ii, iii **B** i, ii **C** ii, iii
D ii **E** iii

7 The diagram shows the position of a student's arm as the weight of a sandbag is measured using a spring balance. The balance is held still to take the reading. How much work is being done while the measurement is being made?

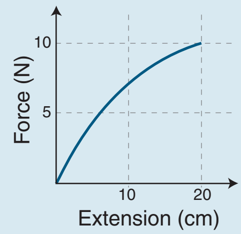


8 A weightlifter raises a 100 kg mass 2.4 m above the ground in a weightlifting competition. After holding it for 3 s he places it back on the ground.
- **a** How much work has been done by the weightlifter in raising the mass above his head?
- **b** How much additional work is done during the 3 s he holds it steady?

9 A rope that is at 35° to the horizontal is used to pull a 10.0 kg crate across a rough floor. The crate is initially at rest and is dragged for a distance of 4.00 m. The tension in the rope is 60.0 N and the frictional force opposing the motion is 10.0 N.

- **a** Draw a diagram illustrating the direction of all relevant forces.
- **b** Calculate the work done on the crate by the tension in the rope.
- **c** Find the total work done on the crate.
- **d** Determine the energy lost from the system as heat and sound due to the frictional force.

10 The graph represents the size of a variable force, ***F***, as a rubber band is stretched to a length of 20 cm. Estimate the total work done on the rubber band by the force.



4.4 Mechanical energy

In this section, we look at the forms of energy specifically related to motion.

Energy can take on many forms. In the study of heat, the energy of particles within a material can have both kinetic and potential energy. The particles in a material that undergoes a rise in temperature gain kinetic energy and, usually, potential energy. Kinetic energy and potential energy are two forms of mechanical energy. In this section, the ideas of kinetic and potential energy will be developed further within the context of movement in general.

ENERGY is the ability to do work.

A hockey puck gains energy when hit because work has been done by the stick on the puck. The amount of work done on the puck equals the puck's change in energy. However, the idea of work may be applied to many forms of energy. The common thread is that, regardless of the form of energy, whenever work is done there is a change in energy from one form to another. In order for any energy transformation to occur, say from motion to heat, work must be done.

Some comparative energy transformations are included in Table 4.2.

table 4.2 Comparison of various energy transformations

Energy use	Amount of energy
Household in 1 day	150 MJ
Fan heater in 1 hour	8.6 MJ
Adult food intake in 1 day	20 MJ
Making 1 Big Mac	2.1 MJ
Climbing a flight of stairs	5 MJ
Lifting 10 kg to a height of 2 m	200 J

Kinetic energy

An object in motion has the ability to do work and therefore is said to possess energy. This energy carried by a moving object is called kinetic energy (from the Greek word 'kinesis' literally meaning 'motion').

If a moving object of mass, m, and initial velocity, $\boldsymbol{u}$, experiences a constant force, $\boldsymbol{F}$, for time, t, then a uniform acceleration results. The velocity will increase to a final value, $\boldsymbol{v}$. Work will have been done during the time the force is applied. Since work is equivalent to the change in kinetic energy of the object, then there should be a relationship linking

Figure 4.12

The kinetic energy of any object depends on its mass and the square of its speed. Doubling the velocity will increase the kinetic energy by a factor of four.

Physics file

The derivation described for kinetic energy is actually that for translational kinetic energy, the movement of a body along a path. A body can also have rotational kinetic energy, as does the Earth, if it is spinning. A different relationship is required to calculate the kinetic energy of rotation.

the two quantities. This can be found from the definition for work when the net applied force is in the direction of the displacement:

$$W = Fs$$

now substituting Newton's second law $\boldsymbol{F} = m\boldsymbol{a}$ we get:

$W = m\boldsymbol{as}$ .... (i)

Using one of the earlier equations of motion: $v^2 = u^2 + 2\boldsymbol{as}$

and rearranging:

$$\boldsymbol{s} = \frac{v^2 - u^2}{2\boldsymbol{a}}$$

Substitute this for s in equation (i).

$$W = \left(\frac{v^2 - u^2}{2\boldsymbol{a}}\right)$$

Rearranging gives:

$$W = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

so

$$W = \Delta E$$

If it is accepted that the work done results in a change in kinetic energy then an object of mass m with a speed v has kinetic energy equal to $\tfrac{1}{2}mv^2$.

The ***KINETIC ENERGY***, E_k, of a body of mass m and speed v is:

$$E_k = \tfrac{1}{2}mv^2$$

Like all forms of energy, kinetic energy is a scalar quantity and is measured in joules (J). There is no direction associated with it. The kinetic energy of an object depends solely on its mass and velocity. The approximate kinetic energy of various moving objects is given in Table 4.3.

table 4.3 Kinetic energy of moving objects

Object	Mass (kg)	Average speed (m s^{-1})	E_k (J)
Earth in orbit	6×10^{24}	3×10^4	2.7×10^{33}
Orbiting satellite	100	8×10^3	3×10^9
Large car	1400	28	5.5×10^5
Netball player	60	8	1900
Footballer	90	8	2900
Electron in a TV tube	9×10^{-31}	7×10^7	2.2×10^{-15}

Worked Example 4.4A

Calculate the kinetic energy of an athlete of mass 60 kg running at a speed of 8 m s^{-1}.

Solution

$m = 60$ kg, $v = 8$ m s^{-1}

Using $E_k = \tfrac{1}{2}mv^2$:

$E_k = \tfrac{1}{2} \times 60 \times 8^2$

$= 1920$ J

Worked Example 4.4B

Blood is pumped by the heart into the aorta at an average speed of 0.15 m s^{-1}. If 100 g of blood is pumped by each beat of an adult human's heart find:

a the amount of work done by the heart during each contraction

b the energy used by the heart each day in pumping blood through the aorta (use an adult's average resting rate of 70 beats per minute). Assume that there are no other energy losses.

Solution

a The work done by the heart is equal to the kinetic energy the blood gains as it is pumped into the aorta.

$m = 0.10$ kg, $v = 0.15$ m s^{-1}, $u = 0$ m s^{-1}

Using $W = \Delta E_k = \Delta \frac{1}{2}mv^2$:

$W = \frac{1}{2} \times 0.10 \times (0.15 - 0)^2$

$W = 1.125 \times 10^{-3}$ J = 1.1 mJ

b If there are 70 beats each minute then the amount of energy transferred:

E_k per minute = $1.125 \times 10^{-3} \times 70 = 0.07875$ J per minute

E_k per day = 0.07875 J per minute × 60 min per hour × 24 hour per day

E_k = 113.4 J per day ≈ 110 J per day

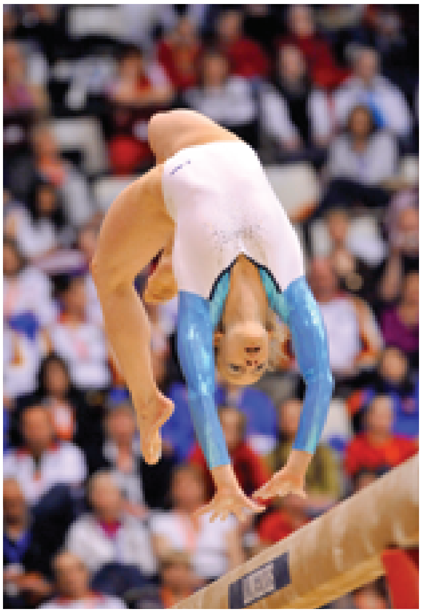


Figure 4.13
The energy gained or lost due to a change in height within a gravitational field is called gravitational potential energy. An increase in height will require the transformation of energy from other sources. A decrease will usually increase the kinetic energy of the body.

Gravitational potential energy

An object can have energy not only because of its motion, but also as a result of its shape or position. This is called potential energy. A gymnast, crouched ready to jump, has potential energy. As the jump takes place, work is being done by the force exerted by the gymnast, and potential energy is converted into kinetic energy from the stores of chemical energy in the muscles of the gymnast's body.

Figure 4.14
This photograph of a pole vaulter illustrates the conversion of stored chemical potential within the muscles of the athlete into strain potential energy stored in the bent pole and then to gravitational potential energy. A small amount of potential energy is also converted into kinetic energy, heat and sound.

Physics file

The relationship for gravitational potential energy used here is only appropriate when the weight force due to gravity is constant. This will only be the case when the change in height is relatively small. As the distance from the Earth's surface changes so will the strength of the gravitational field according to the relationship $g \propto \frac{1}{r^2}$, where r is the distance (in metres) from the centre of the Earth to the body's position. The area under a force–displacement graph can then be used to find the change in potential energy due to a change in position and the varying weight force.

For the purposes of this study, only relatively small changes in height close to the Earth's surface will be considered for which the weight force can be considered constant.

There are many different forms of potential energy—chemical, strain, elastic etc. Potential energy is a stored energy giving the body potential to do work or create a force creating motion. In this particular study we are mainly concerned with gravitational potential energy which, for the present, we will denote E_p. An athlete at the top of a high-jump has gravitational potential energy because of her position.

The work done by a body in changing position with or against a gravitational field can be quantified by considering the force acting on it. The force on the body will be the body's weight and is caused by the gravitational field of the Earth:

$$\boldsymbol{F} = \text{weight} = m\boldsymbol{g}$$
$$\text{and work} = Fs$$

Substituting for $\boldsymbol{F}$ and with $\boldsymbol{s}$ the change in height, Δh, since the displacement is in a vertical direction then:

$$W = m\boldsymbol{g}\Delta h$$

The work done in raising the object against a gravitational field is stored as gravitational potential energy; hence, the athlete will now have a change in energy, $\Delta E = m\boldsymbol{g}\Delta h$.

The change in gravitational potential energy is due to the work done against a gravitational field and is given by:

$$E_p = m\boldsymbol{g}\Delta h$$

where E_p is the gravitational potential energy measured in joules (J)
m is the mass of the body (kg)
Δh is the change in height (m).

The E_p of a body depends only on the vertical height of the object above some reference point, in this case the ground. It does not depend on the path taken since it is based on the direction of the gravitational field. It is the work done against or by the force of gravity that leads to changes in gravitational potential energy. Similarly, the work it can do when falling does not depend on whether the object falls vertically or by some other path, but only on the vertical change in height Δh.

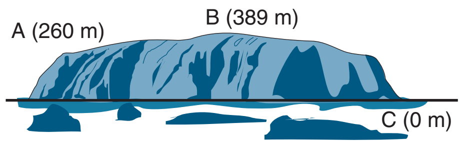
The reference level from which the height is measured does not matter as long as the same reference level is used throughout a given problem solution. It is only changes in potential energy that are important. For example, the height of the high-jumper is best referenced to the ground she jumped from. The height of a luggage locker in an aircraft makes a lot more sense when referenced to the floor of the aircraft than it would referenced to the ground.

The need for considering a change in height in comparison to a reference level is also made apparent when considering a person standing at ground level. If the person is standing beside a hole, she can have gravitational potential energy in reference to the bottom of the hole. It could also be quite justifiable to suggest that even in reference to the ground she has gravitational potential energy; she could fall over! Gravity would do work on her and her gravitational potential energy would change. The change in height would be in reference to the person's centre of mass.

Worked Example 4.4C

A ranger with a mass of 60 kg, checking the surface of Uluru for erosion, walks along a path that takes him past points A, B and C.

a What is his gravitational potential energy at points B and C relative to A?
b What is the change in potential energy as the ranger walks from B to C?
c If the ranger was to walk from B to C via A would it alter your answer to part b? Explain.



Solution

a In this question heights are being referenced to point A. The ranger would have had zero gravitational potential energy at A using this reference.
$m = 60$ kg, $\boldsymbol{g} = 9.8$ m s^{-2}, $h_A = 260$ m, $h_B = 389$ m, $h_C = 0$ m
Potential energy change from A to B: $\Delta E_p = mg(h_B - h_A)$
$= 60 \times 9.8 \times (389 - 260)$
$= 7.6 \times 10^4$ J
Potential energy change from A to C: $\Delta E_p = mg(h_C - h_A)$
$= 60 \times 9.8 \times (0 - 260)$
$= -1.5 \times 10^5$ J

b There is no need to calculate ΔE_p from B to C separately as the difference between the two previous results can be used.
Potential energy change from B to C: $\Delta E_p = \Delta E_p(\text{A to C}) - \Delta E_p(\text{A to B})$
$= -1.5 \times 10^5 - 7.6 \times 10^4$
$= -2.3 \times 10^5$ J

c It makes no difference what path is taken to achieve the change in height. Potential energy change throughout this example is being determined relative to an initial height. In general, if an object is originally at a height h_o, then the change in potential energy as it moves to a different height, h, is:
$\Delta E_p = mgh - mgh_o$
$= mg(h - h_o)$
In general terms, the change in potential energy of an object when it is moved between two heights is equal to the work needed to take it from one point to another.

4.4 SUMMARY Mechanical energy

- Energy is the ability to do work. Whenever work is done, energy is transformed from one form to another.
- Kinetic energy is the energy a body has because of its motion:
$$E_k = \tfrac{1}{2}mv^2$$
where E_k is the kinetic energy in joules (J), m is the mass in kilograms (kg) and v is the speed in metres per second (m s^{-1}).
- Potential energy is stored energy with the potential to allow work to be done. It may take many forms including chemical and spring
- Gravitational potential energy is the energy a body has because of its position within a gravitational height; $\Delta E_p = mg\Delta h$, where E_p is the gravitational potential energy in joules (J), m is the mass in kilograms (kg) and Δh is the change in height from a reference height in metres (m).

4.4 Questions

Where appropriate use $\boldsymbol{g} = 9.8$ m s^{-2}.

1 Calculate the kinetic energy of a:
  a 1 kg mechanics trolley with a velocity of 2.5 m s^{-1}
  b 5 g bullet travelling with a velocity of 400 m s^{-1}
  c 1200 kg car travelling at 75 km h^{-1}.

2 Calculate the gravitational potential energy relative to the ground when a:
  a mass of 1 kg is 5 m above the ground
  b bird of mass 105 g is 400 m above the ground
  c 1200 kg car has travelled a vertical height of 10 m up a slope.

3 A 100 g rubber ball falls from a height of 2.5 m onto the ground and rebounds to a height of 1.8 m. What is the gravitational potential energy relative to the ground of the ball at its:
  a original position?
  b final position?
  c final position relative to its original position?

4 Which object has the greatest amount of energy?
  A a cricket ball of mass 150 g thrown at 25 m s^{-1}
  B a cricket ball of mass 150 g stuck on the roof of a grandstand 14 m above the ground
  C a cricket ball of mass 150 g travelling at 10 m s^{-1} at a height of 10 m above the ground

5 What net braking force must be applied to stop a car with a mass of 900 kg initially travelling at a velocity of 100 km h^{-1} within a straight-line distance of 50 m.

6 A small steel ball with a mass of 80 g is released from a resting height of 1.25 m above a rigid metal plate. Calculate the:
   a velocity of the ball just before impact
   b kinetic energy of the ball just before impact
   c change in gravitational potential energy.

The following information relates to questions 7 and 8. A high-jumper with a mass of 65 kg just clears a height of 2.13 m and drops onto a 0.30 m thick landing mat.

7 What average net force does the landing mat exert on the high-jumper if it is compressed by 0.18 m during the landing?

8 What would be the effect on the force if the high-jumper were to land in sand that compresses only 5 cm?

The following information relates to questions 9 and 10. An arrow with a mass of 80 g is travelling at 80 m s^{-1} when it reaches its target. It penetrates the target board a distance of 24 cm before stopping. Half of its kinetic energy is converted into heat and sound. The other half permanently distorts the target and arrow.

9 Calculate the arrow's kinetic energy just before impact.

10 Calculate the average net force between arrow and target.

4.5 Energy transformation and power

Besides the mechanical energy discussed in section 4.4, other forms of energy exist, for example nuclear, heat, electrical, chemical and sound energy. Atomic theory has led to each of these other forms being understood as either kinetic energy or potential energy at the molecular level. This will be discussed in some depth in the study of heat in Chapter 6. Energy stored in food or fuel can be considered as potential energy stored as a result of the electrical forces in the molecules.

Despite the apparently different nature of the various forms of energy, any form of energy can be transformed from one form to another. The connecting factor is that all forms can do work on a body and therefore can be measured and compared in this way. A stone dropped from some height loses gravitational potential energy as its height decreases; at the same time its kinetic energy will increase as its speed increases.

Transformation of energy

Energy transfers or transformations enable people and machines to do work and processes and changes to occur. Elastic potential energy stored in a diving board must be transformed into the kinetic energy of the diver. Contracting a muscle converts chemical potential energy stored in the muscle to the kinetic energy of a person's motion. In each example, the transformation of energy means work is being done.

> Work is done whenever energy is transformed from one form to another.

In many cases a transformation of energy produces an often unwanted consequence—substantial amounts of the energy are lost as heat energy. Of a typical adult's daily food intake of around 15 MJ at least 80% is converted into heat energy during normal activity. Such transfers can be depicted by an energy conversion flow diagram.

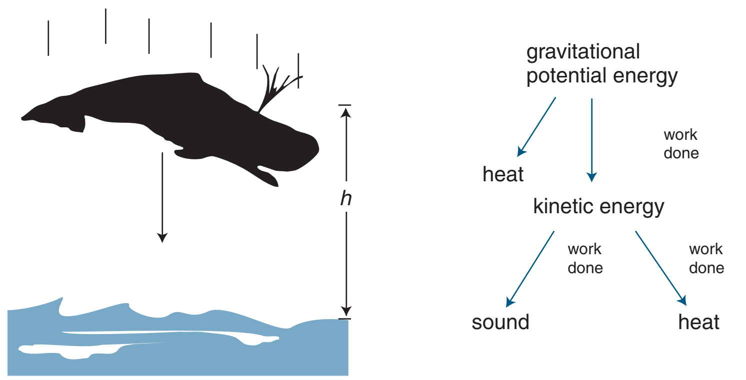


Figure 4.15

Whenever work is done, energy is transformed from one form to another. As a body falls, gravitational potential energy is transformed to kinetic energy and heat, from the friction with the air. Once the body lands, further energy transformations will take place. An energy conversion flow diagram can be useful in visualising the transformations that take place.

A simple, although infinitely unlikely, example is shown in Figure 4.15. As the body falls to the ground there will be a number of energy transformations. An energy flow diagram illustrates these changes. While the body falls, work will be done on the body by the gravitational field, and gravitational potential energy becomes kinetic energy, the energy of movement. There will also be some energy converted into heat by the action of air resistance. When the body hits the ground, the kinetic energy is converted into elastic potential energy by the compression of the body, and to other forms, particularly heat but also sound and kinetic energy. Each transformation requires a force to do work on the body.

The efficiency of energy transformations

The percentage of energy which is transformed to a useful form by a device is known as the efficiency of that device. All practical energy transformations 'lose' some energy as heat. The effectiveness of a transfer from one energy form to another is expressed as:

$$\text{efficiency (\%)} = \frac{\text{useful energy transferred} \times 100}{\text{total energy supplied}} = \frac{\text{useful output} \times 100}{\text{total input}}$$

table 4.4 Efficiencies of some common energy transfers

Device	Desired energy transfer	Efficiency (%)
Large electric motor	Electric to kinetic	90
Gas heater or boiler	Chemical to heat in water	75
Steam turbine	Heat to kinetic	45
High-efficiency solar cell	Radiation to electric	35
Coal-fired electric generator	Chemical to electric	30
Compact low-energy fluorescent light	Electric to light	25
Human body	Chemical to kinetic	25
Car engine	Chemical to kinetic	25
Open fireplace	Chemical to heat	15
Filament lamp	Electric to light	5

In all the energy transformations included in Table 4.4, the energy lost in the transfer process is mainly converted into heat. Most are caused by the inefficiencies involved in the process of converting heat into motion. In the real world, energy must be constantly provided for a device to continue operating. A device operating at 45% efficiency is converting 45% of the supplied energy into the new form required. The other 55% is lost to the surroundings, mainly as heat but also some sound.

Figure 4.16
In each of these situations an energy transformation is taking place. Can you identify the forms involved in each transformation?

Worked Example 4.5A

A team of huskies pulls a snow sled with a total force of 2800 N. After they have pulled on the traces for 25 m, the 200 kg sled is moving at a speed of 6 m s^{-1}. Calculate the efficiency with which the huskies are using their energy to pull the sled.

Solution

To calculate the efficiency of the huskies' efforts, it is first necessary to calculate the energy they have used, energy input, and the energy actually transferred to the motion of the sled, energy output.

Energy input = work = Fs

$W = 2800 \times 25$

$= 70\,000 \text{ J} = 70 \text{ kJ}$

Energy output = kinetic energy of the sled = $\frac{1}{2}mv^2$

$E_k = \frac{1}{2} \times 200 \times 6^2$

$= 3600 \text{ J} = 3.6 \text{ kJ}$

$$\text{Efficiency (\%)} = \frac{\text{energy output}}{\text{energy input}} \times 100$$

$$= \frac{3.6 \text{ kJ}}{70 \text{ kJ}} \times 100 = 5.1\%$$

This first part of the motion is when the dogs have the most friction to overcome between sled and snow (freezing to the surface, static friction).

Physics in action — Air resistance in sport

Air, like water, is a fluid and like water there is friction between the particles of the air and the surface of any moving object. This frictional force is air resistance or drag. At low speeds the effect of air resistance is slight. However, it has been found that air resistance is proportional to the square of the velocity (i.e. $F_A \propto v^2$). A doubling of speed will increase air resistance approximately four times. At the racing speeds that Olympic cyclists reach of 50 km h^{-1} or more, 90% of the cyclist's energy is required just to push the bicycle and rider through the surrounding air. The remaining 10% is needed to overcome frictional forces between the wheels and the ground.

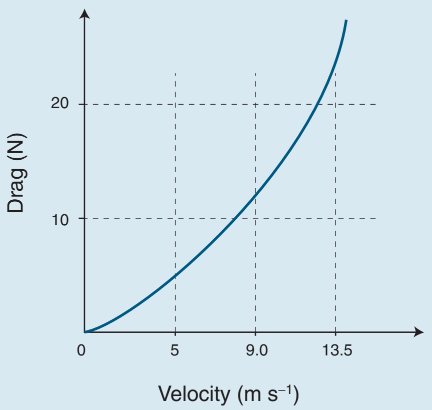


Figure 4.17
The efficiency of a cyclist is affected by the velocity of the bike. As the velocity increases, the drag or air resistance can use as much as 90% of the energy the cyclists input.

Air resistance is affected by the frontal area of cross-section and the shape of the bike. Designers have tried to reduce the frontal area of racing bikes and their riders by dropping the handle bars and raising the position of the pedals. This allows the rider to race bent forward, reducing the area presented to the air.

Streamlining the bike helps still further. Some shapes move through fluids more easily than others. Streamlined bicycles, such as those used by the Australian Cycling Team in international competition, have low-profile frames with relatively smaller front wheels. Brake and gear cables are run through the frame rather than left loose to create drag. Moulded three-spoke and solid-disk wheels help still further.

Figure 4.18
The technological advances in bike design, pioneered by The Royal Melbourne Institute of Technology, have led to efficient designs used by Australian riders in recent international competition.

The rider's clothing and helmets have also been streamlined. Cyclists wear pointed shoes, streamlined helmets and skin-tight one-piece lycra body suits. Together these reduce the air resistance on a rider by as much as 10% at higher speeds. At race time, even the men shave their legs to reduce energy losses just that little bit more.

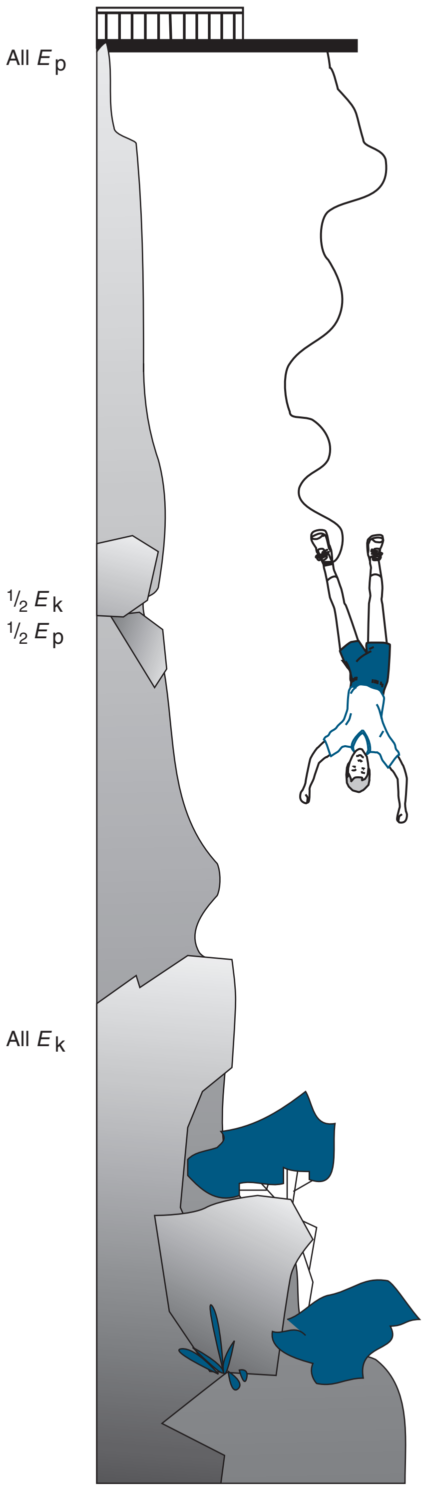


Figure 4.19

A bungee jumper loses gravitational energy but gains kinetic energy during the fall. The total energy remains the same.

Conservation of energy

No matter what energy transformation occurs overall, no energy is gained or lost in the process. It is a fundamental law of nature that energy is conserved.

Consider the example of a bungee jumper as depicted in Figure 4.19. When the jumper is at the top, her or his gravitational potential energy will be at a maximum. As the jumper free falls, the potential energy decreases but the kinetic energy will increase with the increased velocity. A small amount of energy will be converted into heat due to contact with the air. The moment before the jumper reaches the reference level, all the potential energy has been converted into other energy forms, mostly kinetic energy. The total at this point will be exactly equal to the potential energy at the top. The total energy in the system remains the same. Energy has been conserved.

> The total energy in a closed system is neither increased nor decreased by any transformation. Energy can be transformed from one kind to another, but the total amount stays the same.

This applies to any situation involving energy transfer or transformations in an isolated system. In this particular case the sum of the gravitational potential energy and the kinetic energy at any point is called the total mechanical energy. It is always constant in this situation.

> Kinetic energy + gravitational potential energy = total mechanical energy

Here, the total mechanical energy remains constant. As an object falls, potential energy decreases but kinetic energy increases to compensate so that the total remains constant. At any point during the object's fall:

$$\text{total mechanical energy} = \tfrac{1}{2}mv^2 + mgh$$

There are many examples of this conservation of energy. In athletics, the pole vaulter and high-jumper referred to earlier in this chapter base their techniques on this principle. Throwing a ball in the air is another example. When the ball leaves the hand, its kinetic energy is at a maximum. As it rises, its velocity falls, reducing the kinetic energy, and its potential energy increases by the same amount. At any point $E_k + E_p$ will equal the initial kinetic energy. At the top of the throw, the ball will have fallen to a vertical velocity of zero and, in a vertical direction, the energy will be totally gravitational potential energy (any horizontal motion will be represented by a remaining amount of E_k). The transformation will reverse as the ball falls. Gravitational potential energy will decrease as the ball returns towards its original height and, with its speed increasing, the kinetic energy will increase once more.

A more complex example is provided by the interactions as a gymnast repeatedly bounces on a trampoline. Figure 4.20 is a series of frames from a video of a gymnast carrying out a stunt on a trampoline. Kinetic energy and gravitational potential energy changes are shown in the graph below the frames. Despite the complexity of the motion, illustrated in the graph, the total energy of the gymnast remains the same during each airborne phase. On landing on the bed of the trampoline, the gymnast's energy is transferred to become elastic potential energy within the trampoline and both kinetic and potential energy fall. On take-off, some of this energy will be permanently transferred to the trampoline and its surrounds, thus lowering the total available to the gymnast. This is represented by the reduced total energy for each successive jump. Were the gymnast to flex his legs then additional energy would be added to the mechanical energy available and this total could be maintained or even increased until the gymnast finally ran out of available energy himself.

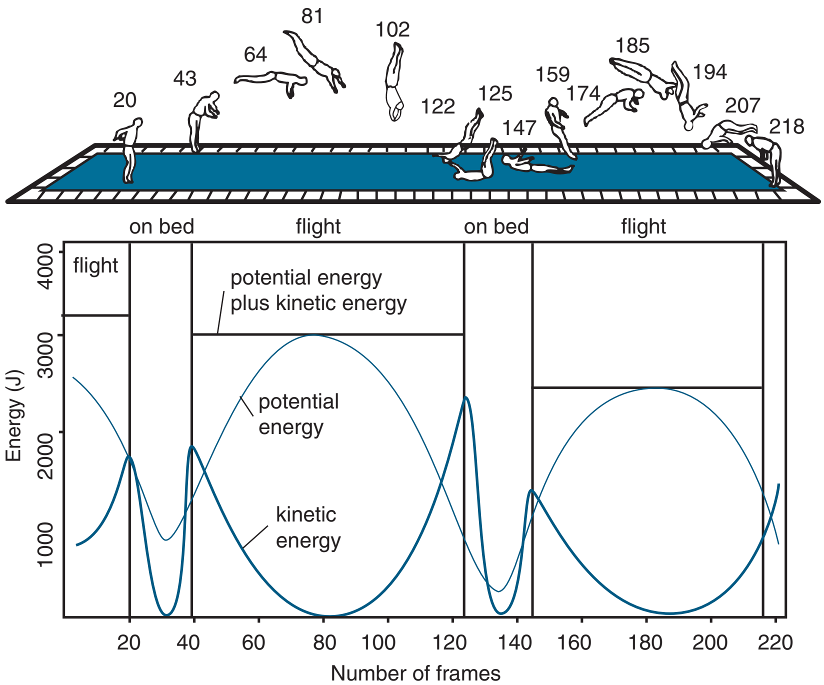


Figure 4.20
During each airborne stage of a gymnast's trampoline stunt, mechanical energy is conserved. The graph shows the relationship between total energy and gravitational potential energy and kinetic energy. Each time the gymnast lands, energy is transferred to the trampoline. The energy returning from the springs after each landing allows the stunt to continue.

✓ Worked Example 4.5B

Calculate the kinetic energy and, hence, the velocity required for a pole vaulter to pass over a 5.0 m high bar. Assume the vaulter reaches a maximum height equivalent to the level of the bar with a horizontal velocity of 1.2 m s^{-1}. Use g = 9.8 m s^{-2}.

Solution

To solve this problem the total mechanical energy of the pole vaulter must be considered. Small losses due to air resistance and friction will be ignored. As an equation:
E_k before the vault = $E_k + E_p + E_s$ (the pole's spring potential energy) during the vault:
$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\Delta h + 0$ (at top)
The mass cancels out, giving an expression independent of the mass of the athlete. The same speed at take-off will be required for a light person as a heavy one. (If this doesn't seem to make sense, remember that all objects fall at the same rate regardless of their mass.)
$\frac{1}{2}u^2 = \frac{1}{2}v^2 + g\Delta h$
Substituting the values from the question:
$\frac{1}{2}u^2 = \frac{1}{2} \times 1.2^2 + 9.8 \times 5.0$
$= 49.72$
and $u = 2 \times \sqrt{49.72} = 9.97$ m s^{-1}
In reality the take-off speed will need to be a little greater since there will be some losses to friction and air resistance and the athlete would try to allow some extra reserve.

Worked Example 4.5C

A climber abseiling down a cliff uses friction between gloved hands and the climbing rope to slow down. If a climber of mass 75 kg abseiling down a cliff of height 45 m reaches a velocity of 5.2 m s^{-1} by the time the ground is reached, calculate the average frictional force applied. Use $g = 9.8$ m s^{-2}.

Solution

$m = 75$ kg, $u = 0$ m s^{-1}, $v = 5.2$ m s^{-1}, $h = 45$ m

Gravitational potential energy at the top of the cliff:

$E_p = mgh$

$= 75 \times 9.8 \times 45 = 33\,075$ J

Kinetic energy at ground level:

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 75 \times 5.2^2 = 1014$ J

Total energy transformed to forms other than gravitational potential and kinetic energy:

$\Delta E = E_p - E_k$

$= 33\,075 - 1014 = 32\,061$ J

This change in energy will be equivalent to the work done by the frictional force, that is,

work $= \Delta E = F_{fr} \times s$

and:

$F_{fr} = \Delta E/s$

$= 32\,061 \div 45 = 712$ N ≈ 710 N

Physics file

The British Imperial unit for power is the horsepower, hp, dating from the time of the Industrial Revolution when the performance of steam engines was compared with the horses they were replacing. 1 hp = 746 W. The SI unit for power honours the inventor of the steam engine, James Watt.

Figure 4.21

It is not the amount of energy required that stops the rest of us from winning the 400 m hurdles but the rate at which we can effectively convert it to useful work.

Power

Why is it that running up a flight of stairs can leave you more tired than walking up if both require the same amount of energy to overcome the force of gravity?

The answer lies in the rate at which the energy is used. When horses were first replaced by steam engines, the engine was rated by how fast it could perform a given task compared with a horse. An engine that could complete a task in the same time as one horse was given a rating of one horsepower. More formally, power is defined as the rate at which energy is transformed or the rate at which work is done.

$$\textbf{POWER} = \frac{\text{work done}}{\text{time taken}} = \frac{\text{energy transformed}}{\text{time taken}}$$

or

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

where P is the power developed in watts (W) resulting from an energy transformation ΔE occuring in time Δt.

Determining the power developed is fairly straightforward when mechanical work is done; but consider a situation where a person pushes a lawnmower, say, at constant speed. Here, there is no increase in kinetic energy, but energy is being transformed to overcome the frictional forces acting against the lawnmower.

As $W = Fs$ then:

$$P = \frac{Fs}{\Delta t}$$

and as $v = s/\Delta t$ then:

$$P = F_{av}v_{av}$$

This is useful when finding the power required to produce a constant speed against a frictional or gravitational force.

The rate of energy use is as much a limiting factor of the work a person can do as the total energy required. A person may be able to walk or climb a long distance before having to stop because all available energy is used. The same person will fall over exhausted after a much shorter time if the same journey is attempted at a run. The limiting factor is power, the rate at which a person's body can transform chemical energy into mechanical energy. Few humans can maintain one horsepower, about 750 W, for any length of time. Table 4.5 includes comparative figures for the power developed in various activities and devices.

table 4.5 Average power ratings for various human activities and machines

Activity or machine	Power rating (W)
Sleeping adult	100
Walking adult	300
Cycling (not racing)	500
Incandescent light globe	60
Television	200
Fast-boil kettle	2400
Family car	150 000

Figure 4.22
Power is the rate at which energy is transformed. In high jumping kinetic energy is rapidly transformed into gravitational energy.

✓ Worked Example 4.5D

The fastest woman to scale the Rialto building stairs in the annual Rialto Tower Run-up, in a particular year climbed the 1254 steps, which are a total of 247 m high, in 7 min 58 s. Given that her mass is 60 kg, what rate was she using energy to overcome the gravitational force alone? Use $g = 9.8$ m s^{-2}.

Solution

The work is against gravity so:

$P = E_g/\Delta t = mg\Delta h/\Delta t$

$m = 60$ kg, $g = 9.8$ m s^{-2}, $\Delta h = 247$ m

$\Delta t = (7 \text{ min} \times 60) + 58$ s

$= 478$ s

$$P = \frac{60 \times 9.8 \times 247}{478}$$

$= 304$ W

4.5 SUMMARY Energy transformation and power

- Whenever work is done, energy is converted from one form into another.
- The efficiency of an energy transfer from one form to the required form is:

$$\text{efficiency (\%)} = \frac{\text{energy output}}{\text{energy input}} \times 100$$

- Whenever energy is transformed, the total amount of energy in the system remains constant. This conservation of energy is a fundamental natural principle.
- The total mechanical energy will remain constant in an isolated system. That is, $E_k + E_p = \text{constant}$.
- Power, P (in watts, W), is the rate at which work is done or energy transformed:

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} = Fv$$

4.5 Questions

Use $g = 9.8 \text{ m s}^{-2}$ where required.

1 Describe the energy transformations which take place when:
   a a car slows to rest
   b a high jumper leaps into the air
   c a swimmer dives off a diving board
   d an athlete's foot hits a track.

2 Draw an energy transformation flow chart for a swimmer diving off a diving board and into a pool of water.

3 A boy of mass 46 kg runs up a 12 m high flight of stairs in 12 s.
   a What is the gain in gravitational potential energy for the boy?
   b What is the average power he develops?

4 A coach is stacking shot-puts, from the shot-put event, onto a shelf 1.0 m high. Each shot-put has a mass of 500 g and all are being lifted from the ground. The coach stacks a total of 15 shot-puts, at the same level, in 2 minutes.
   a How much useful work has been done in lifting all the shot-puts?
   b What is the total gravitational potential energy of all the shot-puts once on the shelf?
   c What was the coach's power output in performing this task?
   d The actual power output would be considerably greater than the answer to part c. Suggest two possible reasons for this difference.

5 One of the shot-puts in question 4 rolls off the shelf just after the coach is finished.
   a What is the gravitational potential energy of the shot-put when it is halfway to the ground?
   b What is the kinetic energy of the shot-put when it is halfway to the ground?
   c What happens to the kinetic energy of the shot-put when it hits the ground?

6 Tarzan is running at his fastest speed (9.2 m s^{-1}) and grabs a vine hanging vertically from a tall tree in the jungle.
   a How high will he swing upwards while hanging on to the end of the vine?
   b What other factors that have not been considered may affect your answer?

7 In high-jumping, the kinetic energy of an athlete is transformed into gravitational potential energy. With what minimum speed must the athlete leave the ground in order to just clear a bar 1.80 m high with a remaining horizontal velocity of 0.50 m s^{-1}?

8 A 100 g apple falls from a branch 5 m above the ground.
   a With what speed would it hit the ground if air resistance could be ignored?
   b If the apple actually hits the ground with a speed of 3.0 m s^{-1}, what was the average force of air resistance exerted on it?

9 A load of 500 N is raised 0.50 m by a pulley and lever system. The engine powering the system used 485 J of electrical energy in doing so. Calculate the efficiency of this electrical engine.

10 An engine has a power rating of 5.0 kW. How long will it take to move a crate of 80 kg, 10 m horizontally against a frictional force between the crate and floor of 450 N, assuming that the engine is 100% efficient?

4.6 Elastic and inelastic collisions

In all collisions, momentum is conserved. Energy, in all its forms, is also conserved every time. These two principles—conservation of momentum and conservation of energy—hold true at all levels: at the grand scale of stars and galaxies, in everyday situations such as collisions between balls on a billiard table, through to the atomic interactions between the particles that make up matter.

Although energy is conserved in a collision, it is unusual for a particular form of energy to be conserved. However, there are situations in which no kinetic energy is lost—these are elastic collisions. Perfectly elastic collisions do not exist in everyday situations, but they do exist in the interactions between atoms and subatomic particles. A collision between two billiard balls is almost elastic because very little kinetic energy is lost as heat and sound.

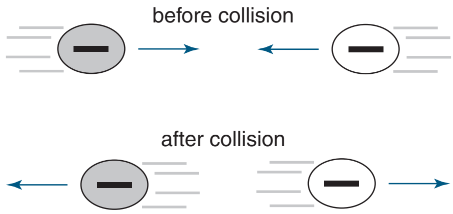


Figure 4.23
When two electrons collide, the force of repulsion due to their like charge prevents them from coming into contact. No kinetic energy is lost in this collision; it is perfectly elastic.

An ***ELASTIC COLLISION*** is one in which the kinetic energy and momentum of the system are both conserved.

Inelastic collisions can vary from almost elastic to perfectly inelastic. *Almost elastic* collisions include those where little friction acts, e.g. between billiard balls, between air track gliders with repelling magnets. Collisions such as a bouncing basketball, a gymnast on a trampoline and a tennis ball being hit are moderately elastic with about half the kinetic energy of the system being retained. *Perfectly inelastic* collisions are those in which the colliding bodies stick together after impact. Some car crashes, a collision between a meteorite and the Moon, and a collision involving two lumps of plasticine, would be perfectly inelastic. In these collisions, much, and sometimes all, of the initial kinetic energy of the system is lost.

An ***INELASTIC COLLISION*** is a collision in which momentum is conserved but kinetic energy is transformed into other forms of energy.

(a)

(b)

Figure 4.24
(a) Most car crashes are inelastic collisions. Most, if not all, of the initial kinetic energy of the car is transformed into heat and sound energy.
(b) Collisions between players in football games are also inelastic. A large proportion of the initial kinetic energy of the players is transformed into heat energy when they collide.

Energy changes during collisions

In an elastic collision, the total final kinetic energy of the system is equal to the total initial kinetic energy of the system. This does not mean, however, that the kinetic energy of the system has remained constant at all times throughout the collision. Consider an almost elastic collision between two air track gliders, A and B, fitted with spring bumpers (Figure 4.25).

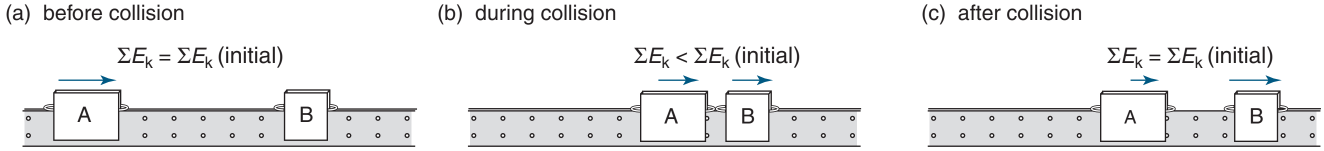


Figure 4.25
During this collision, the total kinetic energy of the system decreases when the gliders are in contact. At this time, some energy is stored in the spring bumpers, and then transformed back into kinetic energy. The total kinetic energy before the collision is approximately equal to the total kinetic energy after the collision.

Before the collision, glider A carries the total kinetic energy of the system. When the gliders are in contact, glider A slows down and glider B begins to move. The total kinetic energy of the gliders has actually decreased during this time, because some of the kinetic energy is momentarily stored in the springs. When the spring bumpers return to their original length, this elastic potential energy is transformed back into kinetic energy as the gliders move apart. After the contact, the total kinetic energy of the gliders is equal to the initial kinetic energy of the system. These energy changes are shown graphically in Figure 4.26a.

In an inelastic collision, not all of the kinetic energy that is transformed into elastic potential energy during the contact is returned to the system as kinetic energy. The missing energy is transformed into heat and sound, so the total kinetic energy of the system after contact is less than it was before (Figure 4.26b).

✓ Worked Example 4.6A

A car of mass 1000 kg travelling west at 20 m s^{-1} crashes into the rear of a stationary bus of mass 5000 kg. The vehicles lock together on impact.

a What is their joint velocity immediately after the collision?
b What is the total initial kinetic energy of the system?
c What is the total kinetic energy of the system after the collision?
d Is this an elastic or inelastic collision? Explain.

Solution

a For this problem, west will be taken as the positive direction. Conservation of momentum must be used to determine the final velocity $\boldsymbol{v}'$ of the vehicles.

$\Sigma \boldsymbol{p}_i = \Sigma \boldsymbol{p}_f$

$\therefore \boldsymbol{p}_i(\text{car}) + \boldsymbol{p}_i(\text{bus}) = \boldsymbol{p}_f(\text{bus and car})$

$\therefore (1000 \times 20) + 0 = (1000 + 5000)\boldsymbol{v}'$

$\therefore \boldsymbol{v}' = \dfrac{20\,000}{6000}$

$= 3.3$ m s^{-1}, i.e. 3.3 m s^{-1} west

b The initial kinetic energy of the system is the initial kinetic energy of the car. It has a mass of 1000 kg and is moving at 20 m s^{-1} prior to the collision.

$E_k = \frac{1}{2}mv^2$

$= 0.5 \times 1000 \times 20^2$

$= 2.0 \times 10^5$ J

c After the collision, the car and bus are locked together and moving at 3.3 m s^{-1}. Their total kinetic energy is:

$E_k = \frac{1}{2}mv^2$

$= 0.5 \times (1000 + 5000) \times 3.32$

$= 3.3 \times 10^4$ J

d This is an inelastic collision. A large amount of kinetic energy has been lost and so the collision is inelastic. The lost kinetic energy has been transformed into heat and sound, and the vehicles have been permanently deformed.

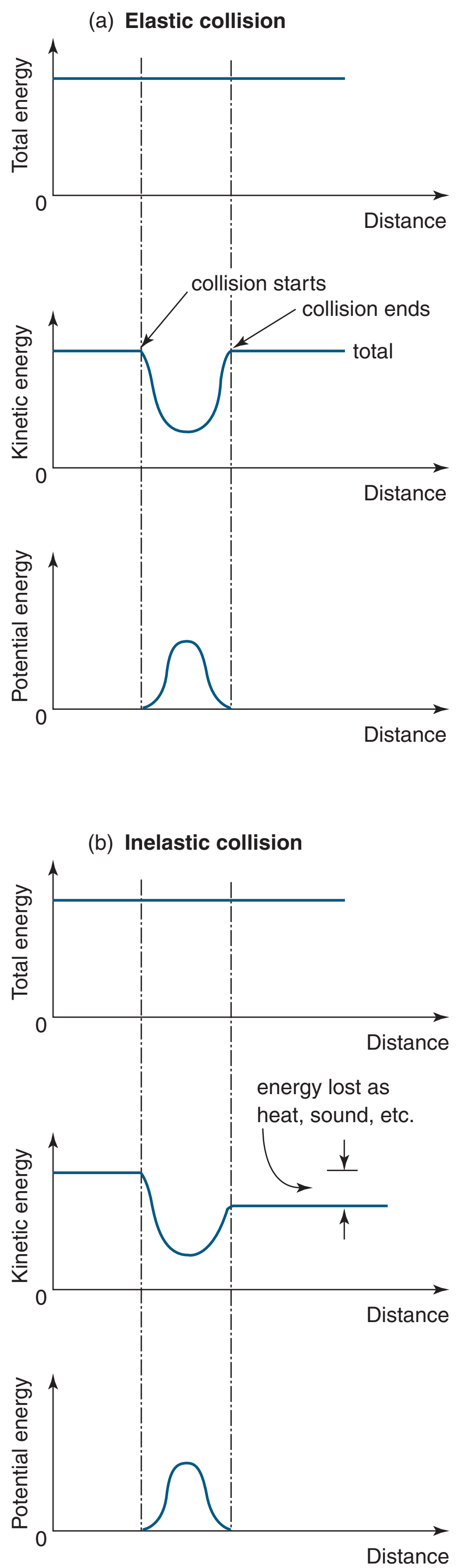


Figure 4.26

(a) In an elastic collision, the total mechanical energy (i.e. kinetic and potential energy) remains constant at all times. During the contact between the bodies, some energy is stored as potential energy and the total kinetic energy of the bodies is reduced. However, this potential energy is all returned as kinetic energy. The total kinetic energy before the collision is equal to the total kinetic energy after the collision.

(b) In an inelastic collision, some mechanical energy is lost as heat and sound. Not all of the energy that was stored as potential energy is returned as kinetic energy, so the total final kinetic energy is less than the total initial kinetic energy.

Physics in action — Efficiency of energy conversions

The efficiency with which kinetic energy is conserved during a collision or interaction can be expressed as a percentage. In a perfectly elastic collision the efficiency is 100%. The kinetic energy of the system before the collision is completely transformed into kinetic energy after the collision. The efficiency of energy conversion in an inelastic collision is less than 100%. The efficiency of an energy conversion is given by:

$$\text{efficiency (\%)} = \frac{\text{final kinetic energy}}{\text{initial kinetic energy}} \times 100$$

In an elastic collision between two protons (Figure 4.27), the forces that act as the particles approach are exactly the same size as those that act as they move away from each other. The work done on the protons (as given by the area under the *F*–*x* graph) is exactly the same when they approach as when they move apart, and no energy is transformed into other forms.

When the volleyball in Figure 4.28 is compressed 2.0 cm, it has 0.80 J of stored strain energy. As the ball expands back to its original shape, the expansion forces do not do as much work as the compression forces did. The amount of kinetic energy that the ball regains is approximately 0.50 J (the area under the expansion phase curve). The kinetic energy that has been lost as heat and sound is given by the area between the two lines—approximately 0.30 J. The efficiency for this bounce is:

$$\begin{aligned}\text{efficiency (\%)} &= \frac{\text{final kinetic energy}}{\text{initial kinetic energy}} \times 100\\ &= \frac{0.50}{0.80} \times 100\\ &= 63\%\end{aligned}$$

So the volleyball regains just 63% of its initial kinetic energy, and 37% of its original kinetic energy has been dissipated as heat and sound.

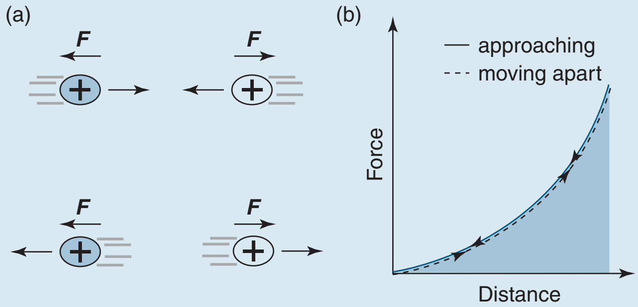


Figure 4.27
(a) As two protons interact, the forces that act as they are approaching are identical to the forces that act as they move away from each other. (b) The work done on the protons as they approach is equal to the work done as they move apart, so there is no loss of energy.

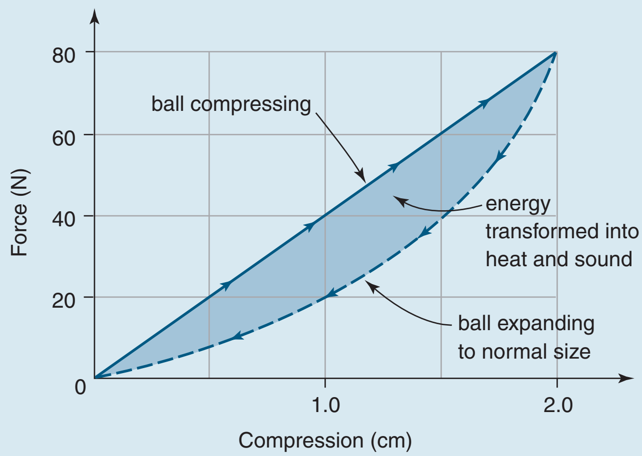


Figure 4.28
A graph of force vs compression for a bouncing volleyball. During this bounce, some kinetic energy has been transformed into heat and sound. The amount of energy lost to other forms is given by the area between the compression and expansion lines.

4.6 SUMMARY Elastic and inelastic collisions

- Momentum and energy are conserved in all collisions and interactions.
- In an elastic collision, the total kinetic energy of the system is also conserved. No kinetic energy is lost as heat and sound. Perfectly elastic collisions do not occur in everyday situations.
- In an inelastic collision, some kinetic energy is lost as heat and sound.
- The efficiency in a collision can be given by:

$$\text{efficiency (\%)} = \frac{\text{final kinetic energy}}{\text{initial kinetic energy}} \times 100$$

4.6 Questions

1 In an elastic collision between two objects, which of the following is true?
  A Kinetic energy is conserved but momentum is not.
  B Momentum is conserved but kinetic energy is not.
  C Both kinetic energy and momentum are conserved.

2 A 200 g snooker ball with initial velocity 9.0 m s^{-1} to the right collides with a stationary snooker ball of mass 100 g. After the collision, both balls are moving to the right. The 200 g ball has a speed of 3.0 m s^{-1}, while the 100 g ball has a speed of 12 m s^{-1}.
  a Calculate the total kinetic energy of the system before the collision.
  b Determine the total kinetic energy of the system after the collision.
  c Is this collision elastic or inelastic? Justify your answer.
  d Is this situation realistic? Justify your answer.

The following information applies to questions 3–5.

Two cars, each of mass 1500 kg and travelling in opposite directions at 72 km h^{-1}, have a head-on collision and become locked together.

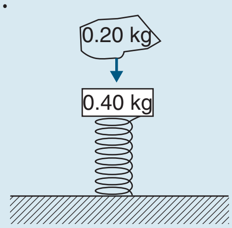
3 a What is the total momentum of the two cars before the collision?
  b What is the total momentum of the two cars after the collision?
  c Calculate the final velocity of the cars.

4 a What is the total kinetic energy of the two cars before the collision?
  b What is the total kinetic energy of the two cars after the collision?
  c Is this collision elastic or inelastic? Justify your answer.

5 a How much kinetic energy is lost during this collision?
  b Explain what has happened to this lost energy.
  c What is the energy efficiency of this collision?

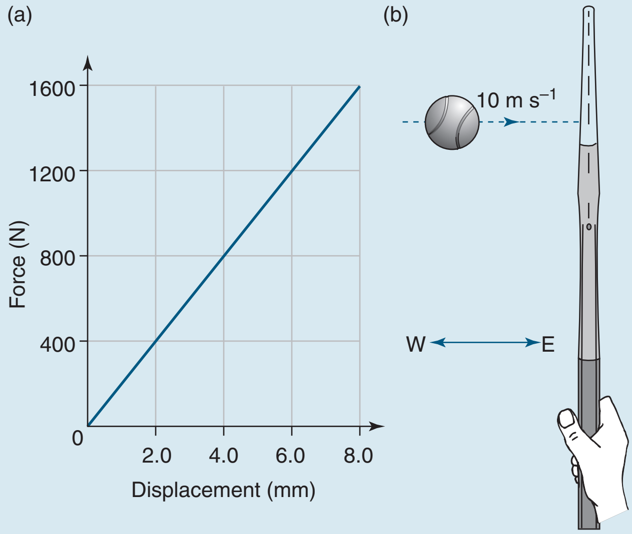
6 Two identical bowling balls, each of mass 4.0 kg, move towards each other across a frictionless horizontal surface with equal speeds of 3.0 m s^{-1}. During the collision, 20 J of the initial kinetic energy is transformed into heat and sound. After the collision the balls move apart from each other. Calculate the velocity of each ball after the collision.

7 A 200 g lump of plasticine falls 2.0 m vertically and sticks to a wooden block of mass 400 g attached to the top of a light, compressible spring. Assume that g is 9.8 m s^{-2}.



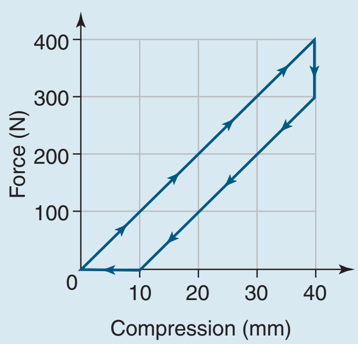
  a What is the kinetic energy of the plasticine just before impact?
  b What is the kinetic energy of the plasticine/block system immediately after impact?

8 The force–displacement graph for the strings of a new type of graphite-head tennis racquet is shown in diagram (a). The racquet is tested in a laboratory by being secured vertically and then having a special type of non-deforming tennis ball fired at it horizontally, as shown in diagram (b). The initial velocity of the ball as it strikes the racquet is 10 m s^{-1} east. After striking the racquet, the ball has a velocity of 9.5 m s^{-1} west. The mass of the ball is 100 g.



What is the maximum displacement of the strings of the racquet during this interaction?

9 The following diagram represents an idealised version of the forces acting on a bouncing basketball as it first compresses and then resumes its original shape during a typical rebound off the backboard.



  a Calculate the energy stored in the ball when it has undergone a compression of 40 mm.
  b How much work is done in restoring the ball to its original shape after such a compression?
  c How do you account for the difference in the energy changes that occur during the compression and the expansion phases?

Use $g = 9.8$ m s^{-2} where required.

1 Two players collide during a game of netball. Just before impact one player of mass 55 kg was running at 5.0 m s^{-1} while the other player, of mass 70 kg, was stationary. After the collision they fall over together. What is the velocity as they fall, assuming that momentum is conserved?

2 A 300 kg marshalling boat for a rowing event is floating at 2.0 m s^{-1} north. A starting cannon is fired from its bow, launching a 500 g ball, travelling at 100 m s^{-1} south as it leaves the gun, into the water across the start line. What is the final velocity of the marshalling boat?

3 A 150 g ice puck collides head-on with a smaller 100 g ice puck, initially stationary, on a smooth, frictionless surface. The initial speed of the 150 g puck is 3 m s^{-1}. After the collision the 100 g ice puck moves with a speed of 1.2 m s^{-1} in the same direction. What is the final velocity of the 150 g ice puck?

4 'When I jump, the Earth moves'. Is this true? Using reasonable estimates and appropriate physics relationships explain your answer.

The following information relates to questions 5–7.
In a horrific car crash, a car skids 85 m before striking a parked car in the rear with a velocity of 15 m s^{-1}. The cars become locked together and skid a further 5.2 m before finally coming to rest. The mass of the first car, including its occupants, is 1350 kg. The parked car has a mass of 1520 kg.

5 What is the velocity of the two cars just after impact?

6 What is the impulse on each car during the collision?

7 What is the average size of the frictional force between road and car that finally brings them to rest?

8 A stone of mass 3 kg is dropped from a height of 5 m. Neglecting air resistance, what will the kinetic energy of the stone be in joules just before hitting the ground?
A 3
B 5
C 15
D 147
E 150

The following information relates to questions 9 and 10.
An object of mass 2 kg is fired vertically upwards with an initial kinetic energy of 100 J. Assume no air resistance.

9 Which of A–E is the speed of the object in m s^{-1} when it first leaves the ground?
A 5
B 10
C 20
D 100
E 200

10 Which of A–E in question 9 is the maximum height in metres that it will reach?

The following information relates to questions 11–16.
After a particularly wet winter, a weir is overflowing at the rate of 800 litres of water every second (1 litre of water has a mass of 1 kg). The water takes 1.3 s to fall to the river below.

11 With what vertical velocity does the water hit the river below?

12 What height does the water fall through to the river below?

13 What weight of water falls over the weir every 10 s?

14 Calculate the work which has been done on this weight of water by gravitational forces by the time it reaches the river.

15 Calculate the power developed by the falling water at the instant before it hits the river.

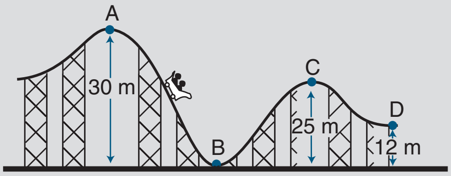
16 Use a diagram to illustrate the energy transformations which occur as the water falls from the weir to the river below.

The following information relates to questions 17–20.
A roller coaster is shown below. Assume no friction.

17 Calculate the speed at points B,C and D, assuming an initial speed of 4.0 m s^{-1} at point A.

18 Draw a graph of potential energy and kinetic energy against vertical displacement for this motion. Use separate lines for each form of energy and draw in a third line to represent the total mechanical energy, assuming no frictional losses

It is found that the roller coaster actually just reaches point C with no remaining speed.



19 What are the energy losses due to friction and air resistance between A and C?

20 With what efficiency is the roller coaster operating over this section of track?

The following information applies to questions 21–23.
Two air-track gliders, both travelling at 2.0 m s^{-1}, approach each other on an air track. The gliders, with masses of 300 g and 100 g, are fitted with magnets so that the opposite poles are facing each other. The heavier glider is initially moving towards the east. During the subsequent collision, the magnets stick together and the gliders move off with a common velocity.

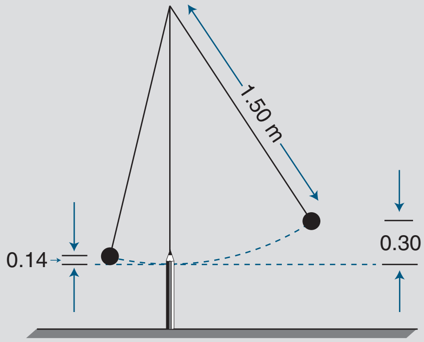
21 Calculate the value of their common velocity.

22 What is the energy efficiency of this collision?

23 Is the collision elastic or inelastic? Justify your answer.

24 A machine lifts a load of 30 kg vertically upwards at a constant speed of 1.2 m s^{-1}. If the machine is 75% efficient, what power input does the machine require?

The following information relates to questions 25 and 26.
An object of 0.40 kg hangs from a string 1.5 m long. While the string is kept taut, the object is drawn aside a vertical distance of 0.30 m, as shown below. A pencil is fixed in a clamp so that when the object is released it will swing down and break the pencil. The object swings on, but now only moves through a vertical distance of 0.14 m.



25 Calculate the velocity of the object at the instant before it strikes the pencil.

26 Calculate the work required to break the pencil.

27 An engine pulls a line of carriages along a flat track with a steady force but, instead of accelerating, the whole train travels at a constant velocity. How can this be consistent with Newton's first and second laws of motion?

28 When serving his first serve, a good tennis player can impart a velocity of 90 km h^{-1} to the ball. He follows through so that the ball and racquet are in contact for 5 ms. The mass of a tennis ball is 57 g.
a What is the average force exerted on the ball?
b What is the average force exerted by the ball on the racquet?

29 A cricket ball has a mass of 160 g. It is travelling at 200 km h^{-1} when it strikes a fielder on the head and is brought to rest in 5 ms.
a What is the total impulse the ball delivers to the fielder's head?
b What is the average force the ball exerts on the fielder's head?
c What average acceleration would the fielder's head—with a mass of 3.6 kg—experience if this was the only force acting on it?

30 Divers at Acapulco, Mexico, regularly dive head-first from a height of 36 m into water 3.6 m deep. (Assume that $g = 9.8$ m s^{-2} and that air resistance is negligible.)
a What would be the vertical momentum of a 50 kg diver when he reached the water?
b If he lost his vertical velocity by the time he reached a depth of 3.0 m, what average force must the water exert on him?
c Comment on the likely results if the initial force as he entered the water was ten times the average force.
d The rocks at the base of the cliff jut more than 6 m out into the water. If a diver hit these he would be killed. Why?

5 Nuclear energy

The nuclear age dawned in 1896. In that year, Henri Becquerel discovered radioactivity in salts of uranium. At the turn of the 20th century, Marie Curie showed that radioactivity was not a chemical effect, but a phenomenon associated with just a few known atoms. New Zealander Ernest Rutherford went on to show that there were three different types of radiation emitted from radioactive materials—alpha, beta and gamma radiation.

In the late 19th and early 20th centuries, a considerable amount of research was devoted to understanding the structure of the atom. The electron was discovered by J.J. Thomson in 1897, but it was mainly work done by Rutherford that led to our current understanding of the overall structure of the atom: a small dense nucleus surrounded by orbiting electrons. Considerable scientific effort then went into unlocking the secrets of the nucleus. In 1919, Rutherford identified the proton and, in 1932, the neutron was identified by James Chadwick.

The developing understanding of the nature of the atomic nucleus had far-reaching ramifications. Scientists began bombarding atoms with neutrons and producing artificial isotopes and radioisotopes. In 1934, nuclear fission was discovered by Enrico Fermi. The energy of the atom had been unlocked. This would lead to the development of nuclear weapons during World War II and, eventually, nuclear reactors. As a consequence, nuclear issues have directly shaped the political landscape for the latter half of the 20th century and continue to do so in the 21st century.

A multitude of subatomic particles—apart from the proton, electron and neutron—have been found. These particles are produced in high-energy collisions in particle accelerators. The photograph on the next page shows a scientist examining the tracks made by some of these particles. It is now generally accepted that all subatomic particles are made of two or three of the six known ‘quarks’.

After studying this chapter, you should understand the nature and effects of radiation from both natural and artificial sources. Natural sources expose us to radiation that can ultimately be dangerous, but background radiation is usually

By the end of this chapter

you will have covered material from the study of radioactivity and nuclear energy including:

- the origin, nature and properties of α, β and γ radiation
- stable, unstable, natural and artificial isotopes
- the half-life of a radioactive isotope
- nuclear fission in uranium and plutonium
- conditions for fission chain reactions
- nuclear fusion.

low enough to be considered safe. Large doses of radiation are well known to result in damage such as the development of cancer.

Artificial sources of radiation have a number of well-established benefits. For example, radioactive elements are used in the treatment and diagnosis of cancer, PET (positron emission tomography) scans use radiation to study and diagnose brain-related conditions, such as schizophrenia, epilepsy and Alzheimer's disease, and smoke detectors contain a small sample of a radioactive element that enables them to detect the presence of smoke particles.

In 1905, Albert Einstein theorised that mass and energy were interchangeable through the equation $E = mc^2$. This led to the realisation that vast amounts of energy lay, unharnessed, within the nuclei of atoms. However, it was not until 1932 that Einstein's theory could be verified. Chadwick's discovery of the neutron in 1932 enabled scientists to fire these particles at stationary atomic nuclei and, in some circumstances, to 'split the atom'. The ramifications of this work were first seen in 1938 when Enrico Fermi created the first nuclear pile in Chicago.

The use of nuclear energy is an issue that has been debated for many years.

5.1 Atoms, isotopes and radioisotopes

Physics file

To gain an idea of the emptiness of atoms and matter, consider this example. If the nucleus of an atom was the size of a pea and this was placed in the centre of the WACA ground, the electrons would orbit in three-dimensional space that would extend beyond the grandstands.

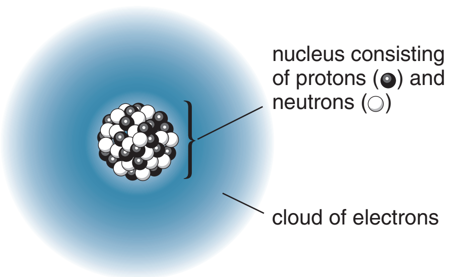


Figure 5.1
The nucleus of an atom occupies about 10^{-12} of the volume of the atom, yet it contains over 99% of its mass. Atoms are mostly empty space!

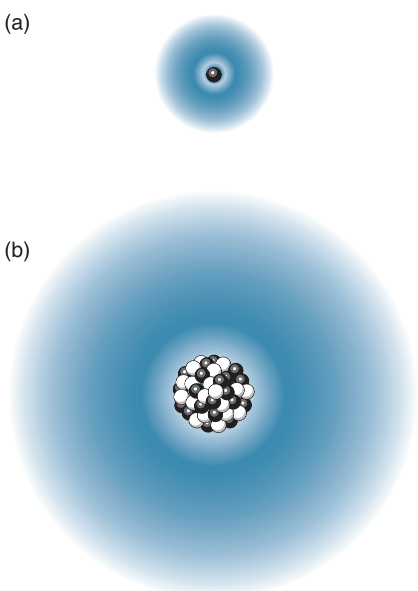


Figure 5.2
(a) Hydrogen is the simplest atom. It consists of just one proton and one electron. (b) Uranium-238 is the heaviest naturally occurring atom. Its nucleus contains 238 nucleons—92 protons and 146 neutrons.

Atoms

In order to understand radioactivity, it is necessary to be familiar with the structure of the atom. The central part of an atom, the *nucleus*, consists of particles known as *protons* and *neutrons*. Collectively, these particles are called *nucleons*, and are almost identical in mass and size. However, they have very different electrical properties. Protons have a positive charge, whereas neutrons, being electrically neutral, have no charge. The nucleus contains nearly all of the mass of the atom, but accounts for less than a million millionth (10^{-12}) of its volume. Most of an atom is empty space that is only occupied by negatively charged particles called *electrons*. These are much smaller and lighter than protons or neutrons and they orbit the nucleus of the atom at high speed.

The simplest atom is hydrogen. It consists of just a single proton with a single electron orbiting at a distance of about 5×10^{-11} m. Compare this with a uranium-238 atom. Its nucleus contains 92 protons and 146 neutrons. Its 92 electrons orbit the nucleus. Uranium-238 is the heaviest atom found in the Earth's crust.

> Two important terms that are used to describe the nucleus of an atom are its:
> - ***ATOMIC NUMBER*** (Z)—the number of protons in the nucleus of an atom.
> - ***MASS NUMBER*** (A)—the total number of protons and neutrons in the nucleus.

A particular atom can be identified by using the following format:

$$\text{mass number} \rightarrow {}^{A}_{Z}\mathrm{X} \leftarrow \text{element symbol}$$
$$\text{atomic number} \rightarrow Z$$

The atomic number defines the element. Atoms with the same number of protons will all belong to the same element. For example, if an atom has six protons in its nucleus (i.e. $Z = 6$) then it is the element carbon. Any atom containing six protons is the element carbon regardless of the number of neutrons.

In an electrically neutral atom, the number of protons is equal to the number of electrons. Any neutral atom of uranium ($Z = 92$) has 92 protons and 92 electrons.

The complete list of elements is shown in the periodic table in Figure 5.3.

The atomic number is also called the *charge number* as it is the charge on the nucleus. The notation described above is also used for particles that are not atoms.

For example, an electron is symbolised as ${}^{0}_{-1}\mathrm{e}$. The mass number is zero and the charge is –1. It would be incorrect to say the atomic number is –1. The mass number being zero means that there are no protons or neutrons in an electron. It does not mean that the electron has no mass.

Period	Group 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Period 1									1 H 1.01									2 He 4.00
2	3 Li 6.94	4 Be 9.01											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
3	11 Na 22.99	12 Mg 24.31	Transition elements										13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.06	17 Cl 35.45	18 Ar 39.95
4	19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.90	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.93	28 Ni 58.71	29 Cu 63.54	30 Zn 65.37	31 Ga 69.72	32 Ge 72.59	33 As 74.92	34 Se 78.96	35 Br 79.91	36 Kr 83.80
5	37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92.91	42 Mo 95.94	43 Tc (99)	44 Ru 101.07	45 Rh 102.91	46 Pd 106.4	47 Ag 107.87	48 Cd 112.40	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126.90	54 Xe 131.30
6	55 Cs 132.91	56 Ba 137.34	57 La 138.91	72 Hf 178.49	73 Ta 180.95	74 W 183.85	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.09	79 Au 196.97	80 Hg 200.59	81 Tl 204.37	82 Pb 207.19	83 Bi 208.98	84 Po (210)	85 At (210)	86 Rn (222)
7	87 Fr (223)	88 Ra (226)	89 Ac (227)	104 Rf (261)	105 Db (262)	106 Sg (263)	107 Bh (262)	108 Hs (265)	109 Mt (266)	110 Ds (269)	111 Rg (272)	112 Uub (277)	113	114 Uuq (289)	115	116 Uuh (289)	117	118 Uuo (293)

Lanthanides

58 Ce 140.12	59 Pr 140.91	60 Nd 144.24	61 Pm (145)	62 Sm 150.35	63 Eu 151.96	64 Gd 157.25	65 Tb 158.92	66 Dy 162.50	67 Ho 164.93	68 Er 167.26	69 Tm 168.93	70 Yb 173.04	71 Lu 174.97

Actinides

90 Th 232.04	91 Pa (231)	92 U 238.03	93 Np (237)	94 Pu (242)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (249)	99 Es (254)	100 Fm (253)	101 Md (256)	102 No (254)	103 Lr (257)

Every isotope of these elements is radioactive.

Figure 5.3
The periodic table of elements.

Isotopes

All atoms of a particular element will have the same number of protons, but may have a different number of neutrons. For example, lithium exists naturally in two different forms. All stable lithium atoms must have three protons, but they may have either three or four neutrons in the nucleus. These different forms of lithium are *isotopes* of lithium. Isotopes are chemically identical to each other. They react and bond with other atoms in precisely the same way. The number of neutrons in the nucleus does not influence the way in which an atom interacts with other atoms; it is the electrons that determine this. The difference between isotopes lies in their physical properties. More neutrons in the nucleus will mean that these atoms have a higher density.

> ***ISOTOPES*** are atoms that have the same number of protons but different numbers of neutrons. Isotopes have the same chemical properties but different physical properties.

When referring to a particular nucleus, we talk about a *nuclide*. In this case, we ignore the presence of the electrons. For example, the nuclide lithium-6 has three protons and three neutrons. Stable isotopes can be found for almost all nuclei and, in all, there are about 270 stable isotopes in nature. Tin ($Z = 50$) has ten stable isotopes, while aluminium ($Z = 13$) has just one.

(a)
(b)

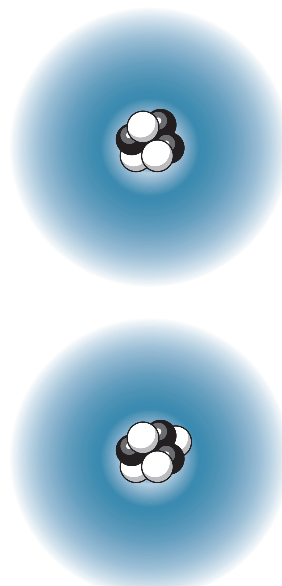


Figure 5.4
Isotopes of lithium. (a) The nucleus of a lithium-6 atom contains three protons and three neutrons. (b) The nucleus of a lithium-7 atom contains three protons and four neutrons.

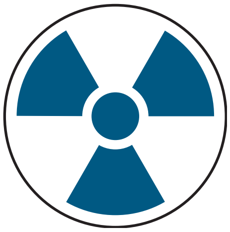


Figure 5.5
This symbol is used to label and identify a radioactive source.

Radioisotopes

Most of the atoms that make up the world around us are stable. Their nuclei have not altered in the billions of years since they were formed and, on their own, they will not change in the years to come.

However, there are also naturally occurring isotopes that are unstable. An unstable nucleus may spontaneously lose energy by emitting a particle and so change into a different element or isotope. Unstable atoms are *radioactive* and an individual radioactive isotope is known as a *radioisotope*. By way of illustration, carbon has two stable isotopes, carbon-12 and carbon-13, and one isotope in nature that is not stable. This is carbon-14. The nucleus of a radioactive carbon-14 atom may spontaneously decay, emitting high-energy particles that can be dangerous.

Nearly all elements found in the Earth's crust have naturally occurring radioactive isotopes. There are over 2000 known radioisotopes, most of which are artificially produced. This process of *artificial transmutation* is discussed in more detail in section 5.2.

Physics file

The heaviest stable isotope in the universe is $^{209}_{83}Bi$. Every isotope of every element with more than 83 protons, i.e. beyond bismuth in the periodic table, is radioactive. For example, every isotope of uranium (Z = 92) is radioactive. Technetium (Z = 43) and promethium (Z = 61) are the only elements with an atomic number below bismuth (Z = 83) that do not have any stable isotopes. Uranium is the heaviest element that occurs naturally on Earth. All of the elements with atomic numbers greater than 92 have been artificially manufactured.

Worked Example 5.1A

Use the periodic table in Figure 5.3 to determine:

a the name of element $^{95}_{42}X$

b the number of protons, neutrons and nucleons in this isotope.

Solution

a From the periodic table, the element with an atomic number of 42 is molybdenum.

b The lower number is the atomic number, so this isotope has 42 protons. The upper number is the mass number. This indicates the number of particles in the nucleus, i.e. the number of nucleons, so this atom has 95 nucleons. The number of neutrons can be found by subtracting 42 from the mass number. This isotope has 53 neutrons.

Physics in action — Quarks!

Our understanding of the atom has changed greatly in the past 100 years. It was once thought that atoms were like miniature billiard balls: solid and indivisible. The word atom comes from the Greek 'atomos' meaning indivisible. However, the first subatomic particles—the electron, the proton and then the neutron—were discovered in the period 1897–1932.

Since World War II, further research has uncovered some 200 other subatomic particles. Examples of these include pi-mesons, mu-mesons, kaons, tau leptons and neutrinos. For many years, physicists found it difficult to make sense of this array of subatomic particles. Then in 1964 Murray Gell-Mann put forward a simple theory. He suggested that most subatomic particles were themselves composed of a number of more fundamental particles called quarks. For example, the proton and the neutron would be made of three quarks while all the mesons would be made of two quarks.

A significant amount of effort has been directed to testing his theory—both theoretically and experimentally—and today the quark theory is accepted. Currently, it is accepted that there are six different quarks, each with different properties. As seen in Table 5.1, quarks have unusual names!

Table 5.1 Fundamental particles

Quarks	Leptons
Up	Electron
Down	Muon
Charm	Neutrino
Strange	Tau
Top	Electron-neutrino
Bottom	Tau-neutrino

Subatomic particles that consist of quarks are known as hadrons. Hadrons are the more massive subatomic particles. The proton consists of two 'up' quarks and one 'down' quark, while neutrons consist of one 'up' quark and two 'down' quarks.

Apart from the hadrons, there are the light particles—the leptons. These are indivisible, point-particles like the electron. Leptons are not composed of quarks.

The theory suggests that quarks and leptons are the ultimate fundamental particles. They cannot be further divided.

5.1 SUMMARY Atoms, isotopes and radioisotopes

- The nucleus of an atom consists of positively charged protons and neutral neutrons. Collectively, protons and neutrons are known as nucleons. Negatively charged electrons orbit the nucleus.
- The atomic number, *Z*, is the number of protons in the nucleus. The mass number, *A*, is the number of nucleons in the nucleus, i.e. the combined number of protons and neutrons.
- Isotopes of an element have the same number of protons but a different number of neutrons. Isotopes of an element are chemically identical to each other, but have different physical properties.
- An unstable isotope—a radioisotope—may spontaneously decay by emitting a particle from the nucleus.

5.1 Questions

1 How many protons, neutrons and nucleons are in the following nuclides? Use the periodic table to help you.
- **a** ^{45}Ca
- **b** ^{197}Au
- **c** ^{235}U
- **d** ^{230}Th

2 Use the periodic table to identify each element.
- **a** $^{66}_{30}X$
- **b** $^{14}_{6}X$
- **c** $^{31}_{14}X$
- **d** $^{69}_{31}X$

3 How many protons and neutrons are in these atoms? A periodic table will help.
- **a** cobalt-60
- **b** plutonium-239
- **c** carbon-14

4 What is the difference between a stable isotope and a radioisotope? Give three examples of stable isotopes.

5 Which of these atoms are definitely radioactive:

$^{24}_{12}Mg$, $^{69}_{27}Co$, $^{195}_{78}Pt$, $^{210}_{84}Po$, $^{238}_{92}U$?

Explain how you made your choice.

6 A proton has a radius of 1.07×10^{-15} m and a mass of 1.67×10^{-27} kg. Using the fact that the volume of a sphere is $V = \frac{4}{3}\pi r^3$:
- **a** calculate the volume of a proton
- **b** calculate the density of a proton.
- **c** If we assume that the density of an atomic nucleus is equal to that of a proton, determine the mass of 1 cm^3 of nuclear material.
- **d** How many 1 tonne cars would it take to balance 1 cm^3 of nuclear material?
- **e** What does this tell you about the density of normal matter compared to the density of atomic nuclei?

7 The nucleus of a gold atom has a radius of 6.2×10^{-15} m, while the atom itself has a radius of 1.3×10^{-10} m. Using the volume formula from question 6, determine the value of the fraction:

$$\frac{\text{volume of nucleus}}{\text{volume of atom}}$$

8 As part of a science project, a student wanted to make a scale model of a gold atom using a marble of radius 1.0 cm as the nucleus. Calculate the radius of the sphere to be occupied by the electrons in this model. Use the information in question 7 to assist your calculations.

9 Krypton-84 is stable while krypton-89 is radioactive.
- **a** Discuss any differences in how these atoms would interact chemically with other atoms.
- **b** Describe the difference in the composition of these two atoms.

10 A particular artificial radioisotope is manufactured by bombarding the stable isotope ^{27}Al with neutrons. The radioisotope is produced when each atom of ^{27}Al absorbs one neutron into the nucleus. Identify the radioisotope that is produced as a result of this process.

5.2 Alpha, beta and gamma radiation

Alpha, beta and gamma radiation is spontaneously emitted from unstable nuclei. The radiation does not emanate from the electron cloud surrounding the nucleus.

About 100 years ago, Ernest Rutherford and Paul Villard discovered that there were three different types of emission from radioactive substances. They named these *alpha*, *beta* and *gamma* radiation. Further experiments showed that the alpha and beta emissions were actually particles expelled from the nucleus. Gamma radiation was found to be high-energy electromagnetic radiation, also emanating from the nucleus.

Alpha decay ${}^{4}_{2}\alpha$

When a heavy nucleus undergoes radioactive decay, it may eject an alpha particle. An alpha particle is a positively charged chunk of matter. It consists of two protons and two neutrons that have been ejected from the nucleus of a radioactive atom. An alpha particle is identical to a helium nucleus and can also be written as ${}^{4}_{2}He^{2+}$, α^{2+}, ${}^{4}_{2}\alpha$ or simply α.

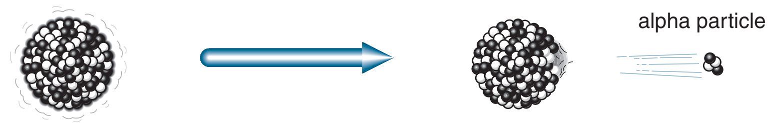


Figure 5.6
When the nucleus of uranium-238 decays, it will spontaneously eject an alpha particle that consists of two protons and two neutrons at high speed. The remaining nucleus is thorium-234.

Uranium-238 is radioactive and may decay by emitting an alpha particle from its nucleus. This can be represented in a nuclear equation where the changes occurring in the nuclei can be seen. Electrons are not considered in these equations—only nucleons. The equation for the alpha decay of uranium-238 is:

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}\alpha + \text{energy}$$

or

$${}^{238}_{92}U \xrightarrow{\alpha} {}^{234}_{90}Th$$

In the decay process, the *parent nucleus* ${}^{238}_{92}U$ has spontaneously emitted an alpha particle (α) and has changed into a completely different element, ${}^{234}_{90}Th$. Thorium-234 is called the *daughter nucleus*. The energy released is mostly kinetic energy carried by the fast moving alpha particle.

In any nuclear reaction, including radioactive decay, atomic and mass numbers are conserved.

When an atom changes into a different element, it is said to have undergone a *nuclear transmutation*. In nuclear transmutations, electric charge is conserved—seen as a conservation of atomic number. In the above example 92 = 90 + 2. The number of nucleons is also conserved: 238 = 234 + 4.

Beta decay ${}^{0}_{-1}\beta$

Beta particles are electrons, but they are electrons that have originated from the *nucleus* of a radioactive atom, not from the electron cloud. A beta particle can be written as ${}^{0}_{-1}e$, β, β^{-} or ${}^{0}_{-1}\beta$. The atomic number of –1 indicates that it has a single negative charge and the mass number of zero indicates that its mass is far less than that of a proton or a neutron.

Beta decay occurs in nuclei where there is an imbalance of neutrons to protons. Typically, if a light nucleus has too many neutrons to be stable, a neutron will spontaneously change into a proton, and an electron and an uncharged massless particle called an antineutrino $\bar{\nu}$ are ejected to restore the nucleus to a more stable state.

Consider the isotopes of carbon: ${}^{12}_{6}C$, ${}^{13}_{6}C$ and ${}^{14}_{6}C$. Carbon-12 and carbon-13 are both stable, but carbon-14 is unstable. It has more neutrons and so undergoes a beta decay to become stable. In the process, one of the neutrons changes into a proton. As a result, the proton number increases to seven, and so the product is not carbon. Nitrogen-14 is formed.

The nuclear equation for this decay is:

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}\beta + \bar{\nu} + \text{energy}$$

The transformation taking place inside the nucleus is:

$${}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + \bar{\nu}$$

Once again, notice how in all these equations, the atomic and mass numbers are conserved. (The antineutrino has no charge and has so little mass that both its atomic and mass numbers are zero.)

Gamma decay γ

Generally, after a radioisotope has emitted an alpha or beta particle, the daughter nucleus holds an excess of energy. The protons and neutrons in the daughter nucleus then rearrange slightly and off-load this excess energy by releasing gamma radiation (high-frequency electromagnetic radiation). Gamma rays—like all light—have no mass and are uncharged and so their symbol is ${}^{0}_{0}\gamma$. Being a form of light, gamma rays travel at the speed of light.

A common example of a gamma ray emitter is iodine-131. Iodine-131 decays by beta and gamma emission to form xenon-131.

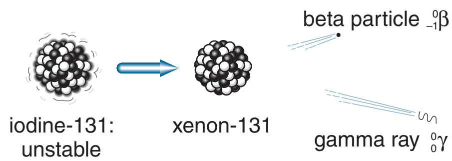


Figure 5.8

In the beta decay of iodine-131, a high-energy gamma ray photon is also emitted. This high-energy electromagnetic radiation carries no mass or charge—just energy.

The equation for this decay is:

$${}^{131}_{53}I \rightarrow {}^{131}_{54}Xe + {}^{0}_{-1}e + \gamma$$

or

$${}^{131}_{53}I \xrightarrow{\beta,\,\gamma} {}^{131}_{54}Xe$$

As gamma rays carry no charge and have no mass, they have no effect when balancing the atomic or mass numbers in a nuclear equation.

The graph in Figure 5.9 identifies the 272 stable nuclides, as well as some radionuclides and decay modes.

Artificial transmutation: how radioisotopes are manufactured

There are about 200 naturally occurring radioactive isotopes—usually in the Earth's crust, but some are also found in the atmosphere! These natural radioisotopes were used in the early days of research into radiation. Today, most of the radioisotopes that are used in industrial and medical applications are synthesised by *artificial transmutation*. There

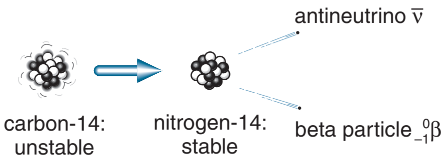


Figure 5.7

The nucleus of carbon-14 is unstable. In order to achieve stability, one neutron transforms into a proton, and an electron and antineutrino are emitted in the process. The emitted electron is a beta particle, and it travels near the speed of light.

Physics file

A different form of beta decay occurs in atoms that have too many protons. An example of this is the radioactive decay of unstable nitrogen-12. There are seven protons and five neutrons in the nucleus, and a proton may spontaneously change into a neutron and emit a neutrino and a positively charged beta particle. This is known as a β^+ (beta-positive) decay and the positively charged beta particle is called a *positron*.

The equation for this decay is:

$${}^{12}_{7}N \rightarrow {}^{12}_{6}C + {}^{0}_{+1}e + \nu + \text{energy}$$

Positrons, ${}^{0}_{+1}e$ or ${}^{0}_{+1}\beta$, have the same properties as electrons, but their electrical charge is positive rather than negative. Positrons are an example of *antimatter*.

Physics file

Neutrinos are particles with the lowest mass in nature, and they permeate the universe. Neutrinos have no charge and their mass has only recently been discovered to be about one-billionth that of a proton, i.e. about 10^{-36} kg. While you have been reading these sentences, billions of neutrinos have passed right through your body, and continued on to pass right through the Earth! Fortunately neutrinos interact with matter very rarely and so are completely harmless. It has been estimated that if neutrinos passed through a piece of lead 90 light-years thick, they would still have only a 50% chance of being absorbed!

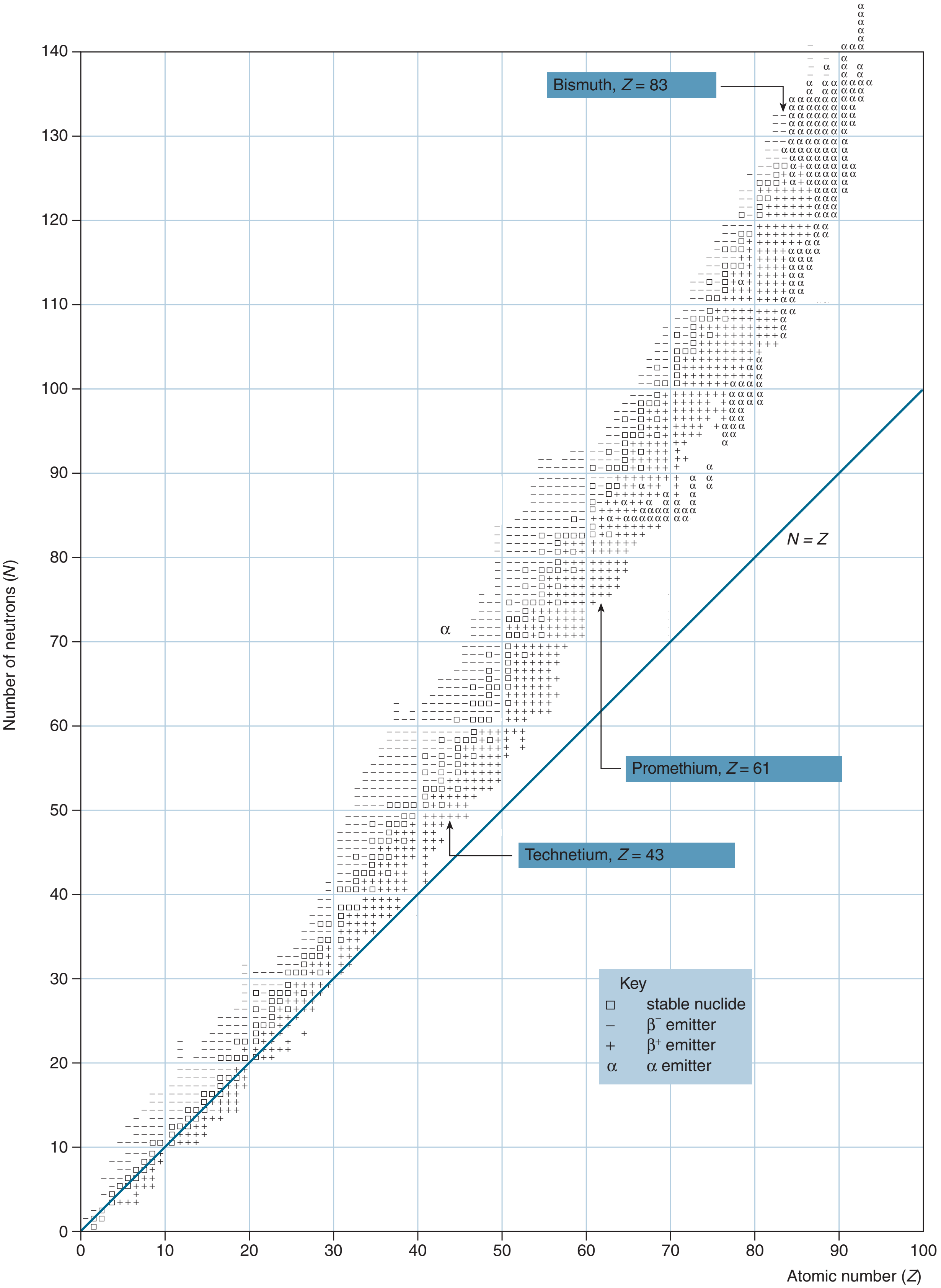


Figure 5.9

From this graph of stable isotopes and radioisotopes, it is evident that for larger nuclei there is a distinct imbalance between the number of protons and neutrons. The 'line of stability' of the stable nuclides can be seen as a line that curves away from the $N = Z$ line.

are now over 2000 such artificial radioisotopes. In the periodic table, every element with an atomic number greater than 92, i.e past uranium, is radioactive and is produced in this way.

One of the ways that artificial radioisotopes are manufactured is by *neutron absorption*. (In Australia, this is done at the Lucas Heights reactor near Sydney.) In this method, a sample of a stable isotope is placed inside a nuclear reactor and bombarded with neutrons. When one of the bombarding, or irradiating, neutrons collides with a nucleus of the stable isotope, the neutron is absorbed into the nucleus. This creates an unstable isotope of the sample element, which usually becomes a beta emitter.

This is how the radioisotope cobalt-60 (widely used for cancer treatment) is manufactured. A sample of the naturally occurring and stable isotope cobalt-59 is irradiated with neutrons. Some of the cobalt-59 nuclei absorb neutrons and this results in a quantity of cobalt-60 being produced. The nuclear transformation is: ${}^{1}_{0}\text{n} + {}^{59}_{27}\text{Co} \rightarrow {}^{60}_{27}\text{Co}$.

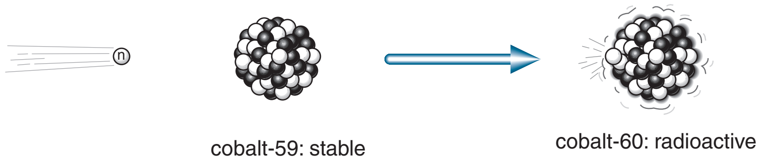


Figure 5.10

The artificial radioisotope cobalt-60 is used extensively in the treatment of cancer. It is produced by bombarding a sample of cobalt-59 with neutrons.

Worked Example 5.2A

Strontium-90 decays by radioactive emission to form yttrium-90. The equation is:

$${}^{90}_{38}\text{Sr} \rightarrow {}^{90}_{39}\text{Y} + \text{X} + \gamma$$

Determine the atomic and mass numbers for X and identify the type of radiation that is emitted during this decay.

Solution

The atomic and mass number for a gamma ray is zero. By balancing the equation, it is found that X has a mass number of zero and an atomic number of −1. X is an electron and so this must be beta decay. The full equation is: ${}^{90}_{38}\text{Sr} \rightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e} + \gamma$.

Why radioactive nuclei are unstable

Inside the nucleus, there are two completely different forces acting. The first is an electric force of repulsion between the protons. On its own, this would blow the nucleus apart, so clearly a second force must act to bind the nucleus together. This is the nuclear force, a powerful force of attraction between nucleons, which acts only over a very short range.

In a stable nucleus, there is a delicate balance between the repulsive electric force and the attractive strong force. For example, bismuth-209, the heaviest stable isotope, has 83 protons and 126 neutrons, and the forces between the nucleons balance to make the nucleus stable. Compare this with bismuth-211. It has two extra neutrons and this upsets the balance between forces. The nucleus of ${}^{211}\text{Bi}$ is unstable and it ejects an alpha particle in an attempt to attain nuclear stability.

Figure 5.9 shows all the stable nuclei with their proton and neutron numbers. It is evident that there is a 'line of stability' along which the nuclei tend to cluster. Nuclei away from this line are radioactive. For

Physics file

Gamma decay alone occurs when a nucleus is left in an energised or excited state following an alpha or beta decay. This excited state is known as a *metastable state* and it usually only lasts for a short time. An example of this is the radioactive decay of iodine-131, usually a two-stage process. First, a beta particle is emitted and the excited nuclide xenon-131m is formed. Then, the nucleus undergoes a second decay by emitting a gamma ray:

$${}^{131}_{53}\text{I} \rightarrow {}^{131\text{m}}_{54}\text{Xe} + {}^{0}_{-1}\text{e}$$

$${}^{131\text{m}}_{53}\text{Xe} \rightarrow {}^{131}_{53}\text{Xe} + \gamma$$

The 'm' denotes an unstable or metastable state. Cobalt-60 and technetium-99 also exist in metastable states.

Physics file

PET scans are used to diagnose and examine brain-related problems. PET—positron-emission topography—involves a positron-emitting radioisotope, ${}^{11}_{5}\text{C}$, being tagged to molecules that are injected into the patient's bloodstream. When the carbon decays, the emitted positrons (antimatter) meet with electrons (matter) and annihilate each other to release energy in the form of gamma rays. The gamma rays are detected externally and can be computer analysed to produce an image inside the brain. This technique is widely used in the identification and treatment of epilepsy, Parkinson's disease, Alzheimer's disease and schizophrenia.

Figure 5.11

Marie Curie performed pioneering work on radioactive materials. In fact, Marie Curie coined the term 'radioactivity' and, apart from Einstein, is the only scientist to have been awarded two Nobel prizes.

small nuclei with atomic numbers up to about 20, the ratio of neutrons to protons is close to one. However, as the nuclei become bigger, so too does the ratio of neutrons to protons. Zirconium ($Z = 40$) has a neutron to proton ratio of about 1.25, while for mercury ($Z = 80$) the ratio is close to 1.66. This indicates that for higher numbers of protons, the nuclei must have even more neutrons to remain stable. These neutrons dilute the repelling forces that act between the extra protons. Elements with more protons than bismuth ($Z = 83$) simply have too much repulsive charge and additional neutrons are unable to stabilise their nuclei. All of these atoms are radioactive.

5.2 SUMMARY Alpha, beta and gamma radiation

- Radioactive isotopes may decay, emitting alpha, beta and gamma radiation from their nuclei.
- An alpha particle, α, consists of two protons and two neutrons. It is identical to a helium nucleus and can be written as ${}^{4}_{2}\alpha$ or α^{2+}.
- A beta particle, β, is an electron that has been emitted from the nucleus of a radioactive atom as a result of a neutron transmuting into a proton.
- A gamma ray, γ, is high-energy electromagnetic radiation that is emitted from the nuclei of radioactive atoms. Gamma rays usually accompany an alpha or beta emission.
- In any nuclear reaction, both atomic and mass numbers are conserved.
- Artificial radioisotopes are manufactured by bombarding stable nuclei with neutrons. This process is known as artificial transmutation.

5.2 Questions

1 From which part of a radioisotope, the nucleus or the electron cloud, are the following emitted?
a alpha particles **b** beta particles **c** gamma rays

2 Discuss the physical differences between α particles, β particles and γ rays.

3 Identify each of these particles.
a ${}^{0}_{-1}A$ **b** ${}^{1}_{1}B$ **c** ${}^{4}_{2}C$ **d** ${}^{1}_{0}D$

4 Determine the atomic and mass numbers for the unknown elements in these decay equations, then use a periodic table to identify the elements.
a ${}^{218}_{84}Po \rightarrow X + \alpha + \gamma$ **b** ${}^{235}_{92}U \xrightarrow{\alpha,\gamma} X$
c ${}^{228}_{88}Ra \rightarrow X + \beta + \gamma$ **d** ${}^{198}_{79}Au \xrightarrow{\beta,\gamma} X$

5 Determine the mode of radioactive decay for each of the following transmutations.
a ${}^{218}_{86}Rn \rightarrow {}^{214}_{84}Po + X + \gamma$ **b** ${}^{234}_{91}Pa \xrightarrow{X,\gamma} {}^{234}_{92}U$
c ${}^{214}_{82}Pb \rightarrow {}^{214}_{83}Bi + X + \gamma$ **d** ${}^{239}_{94}Pu \xrightarrow{X,\gamma} {}^{235}_{92}U$
e ${}^{60m}_{27}Co \rightarrow {}^{60}_{27}Co + X$

6 When the stable isotope boron-10 is bombarded with neutrons, it transmutes by neutron capture into a new element X and emits alpha particles. The equation for this reaction is:
${}^{10}_{5}B + {}^{1}_{0}n \rightarrow X + {}^{4}_{2}He$. Identify the final element formed.

7 Identify the unknown particles in these nuclear transmutations.
a ${}^{14}_{7}N + \alpha \rightarrow {}^{17}_{8}O + X$ **b** ${}^{27}_{13}Al + X \rightarrow {}^{27}_{12}Mg + {}^{1}_{1}H$
c ${}^{14}_{7}N + X \rightarrow {}^{14}_{6}C + {}^{1}_{1}p$ **d** ${}^{23}_{11}Na + X \rightarrow {}^{26}_{12}Mg + {}^{1}_{1}H$

8 Carbon-14 decays by beta emission to form nitrogen-14. The equation for this is:
${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e$. It can be seen that the nucleus initially has six protons and eight neutrons.
a List the particles that comprise the decay side of this equation.
b Analyse the particles and determine which particle from the parent nucleus has decayed.
c Write an equation that describes the nature of this decay.

9 Use the graph in Figure 5.9 to answer these questions.
a List all the stable nuclides of calcium, $Z = 20$.
b How many stable nuclides does niobium, $Z = 41$, have?
c ${}^{48}_{19}K$ has a large imbalance of neutrons over protons and so is radioactive. Find potassium-48 on the graph and determine whether it is an alpha or beta emitter.
d Write the decay equation for potassium-48 and determine whether the daughter nucleus is itself stable or radioactive.
e Calculate the ratio of neutrons to protons for each of potassium-48 and its daughter nucleus.
f ${}^{217}_{87}Fr$ is a radioisotope. Is it an alpha or beta emitter?
g Determine the decay processes that francium-217 undergoes before it becomes a stable nuclide; identify this nuclide.

10 Gold has only one naturally occurring isotope, ${}^{197}Au$. If a piece of gold foil is irradiated with neutrons, neutron capture will occur and a radioactive isotope of gold will be produced. This radioisotope is a beta emitter. Write an equation that describes this:
a absorption process **b** decay process.

5.3 Properties of alpha, beta and gamma radiation

Alpha particles, beta particles and gamma rays all originate from the same place—the *nucleus* of a radioisotope. The properties of these emissions, however, are distinctly different from each other.

Alpha particles

Alpha particles, α, consist of two protons and two neutrons. Because an alpha particle contains four nucleons, it is relatively heavy and slow moving. It is emitted from the nucleus at about 20 000 km s^{-1} (2.0×10^7 m s^{-1}), just less than 10% of the speed of light.

Alpha particles have a double positive charge. This, combined with their relatively slow speed, makes them very easy to stop. They only travel a few centimetres in air before losing their energy, and will be completely absorbed by thin card. They have a poor *penetrating ability*.

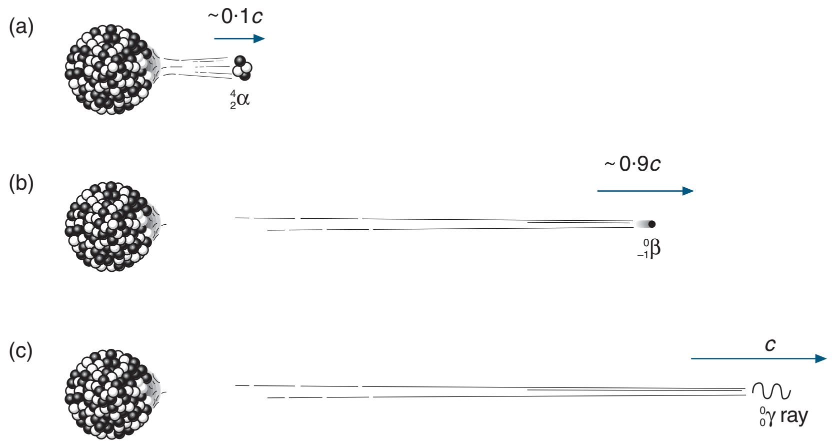


Figure 5.12

The relative speeds of alpha, beta and gamma radiation. (a) Alpha particles are the slowest of the radioactive emissions. Typically they are emitted from the nucleus at up to 10% of the speed of light. (b) Beta particles are emitted from the nucleus at speeds up to 90% of the speed of light. (c) Gamma radiation, being high-energy light, travels at the speed of light (3.0×10^8 m s^{-1}).

Beta particles

Beta particles, β, are fast moving electrons, created when a neutron decays into three parts—a proton, an electron (the beta particle) and an antineutrino. Beta particles are much lighter than alpha particles, and so they leave the nucleus with far higher speeds—up to 90% of the speed of light.

Beta particles are more penetrating than alpha particles, being faster and with a smaller charge. They will travel a few metres through air but, typically, a sheet of aluminium about 1 mm thick will stop them.

Gamma rays

Gamma rays, γ, being electromagnetic radiation with a very high frequency, have no mass and travel at the speed of light—3.0×10^8 m s^{-1} or

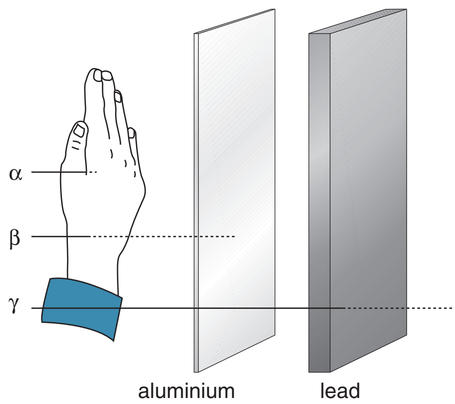


Figure 5.13

Gamma rays can pass through human tissue and sheets of aluminium quite readily. A 5 cm thick sheet of lead is needed to stop 97% of the gamma rays in a beam. By comparison, alpha particles are not capable of penetrating through a sheet of paper or beyond the skin of a person.

Physics file

X-rays and gamma rays are ionising radiations. They are both high-energy forms of electromagnetic radiation (released as high-energy photons), but gamma rays have higher energies. This means that gamma rays are more highly penetrating than X-rays. The defining distinction between X-rays and gamma rays is the method of production.

X-rays are created from electron transitions within the electron cloud, whereas gamma rays are emitted from the nuclei of radioactive atoms. Gamma rays and X-rays may have similar properties, but X-rays are not the result of radioactive decay.

300 000 km s^{-1}. They have no electric charge. Their high energy and uncharged nature makes them a very penetrating form of radiation. Gamma rays can travel an almost unlimited distance through air and even a few centimetres of lead or a metre of concrete would not completely absorb a beam of gamma rays.

The ionising abilities of alpha, beta and gamma radiation

When an *alpha particle* travels through air, its slow speed and double positive charge cause it to interact with just about every atom that it encounters. The alpha particle dislodges electrons from many thousands of these atoms, turning them into ions. Each interaction slows it down a little, and eventually it will be able to pick up some loose electrons to become a helium atom. This takes place within a centimetre or two in air. As a consequence, the air becomes quite ionised, and the alpha particles are said to have a *high ionising ability*. Since the alpha particles don't get very far in the air, they have a poor penetrating ability.

Beta particles, having a negative charge, are repelled by the electron cloud of atoms that they interact with. This means that when a beta particle travels through matter, it experiences a large number of glancing collisions and loses less energy per collision than an alpha particle. As a result, beta particles do not ionise as readily and will be more penetrating.

Gamma rays, having no charge and moving at the speed of light, are the most highly penetrating form of radiation. Gamma rays interact with matter infrequently when they collide directly with a nucleus or electron. The low density of an atom makes this a relatively unlikely occurrence. Gamma rays pass through matter very easily—they have a very *poor ionising ability* but have a high penetrating ability.

Measuring the energy of α, β and γ radiation

The energy of moving objects like cars and tennis balls is measured in joules. However, alpha, beta and gamma radiation have such small amounts of energy that the joule is an inappropriate unit. The energy of radioactive emissions is usually expressed in *electron-volts* (eV). An electron-volt is the energy that an electron would gain if it was accelerated by a voltage of 1 volt.

> One ***ELECTRON-VOLT*** is an extremely small quantity of energy equal to 1.6×10^{-19} J, i.e. 1 eV = 1.6×10^{-19} J.

Alpha and beta particles are ejected from unstable nuclei with a wide range of speeds. *Alpha particles* typically have energies of 5–10 million electron-volts (5–10 MeV). This corresponds to speeds of around 16 000 km s^{-1}, about 5% of the speed of light.

Beta particles are usually ejected with energies up to a few million electron-volts. For example, sodium-24 emits beta particles with a maximum energy of 1.4 MeV. This is equivalent to 2.24×10^{-13} J. These particles are travelling quite close to the speed of light.

Gamma rays normally have less than a million electron-volts of energy. They may even have energy as low as a hundred thousand

Physics file

Domestic smoke detectors contain a small radioactive source. The radioisotope most commonly used is americium-241, an artificial isotope which is an alpha particle emitter. The smoke detector contains a pair of oppositely charged metal electrodes. When the alpha particles pass between these electrodes, they ionise the air molecules present. These ions are then attracted to the electrodes. However, when smoke (or steam) is present, the ions attach themselves to these particles. The flow of charges to the electrodes reduces greatly because these charged smoke (or steam) particles are much bigger and so much less mobile than the ionised air molecules. It is this reduction in the flow of charges reaching the electrodes that triggers the alarm.

electron-volts. For example, the gamma rays emitted by the radioactive isotope gold-198 have a maximum energy of 412 000 eV (412 keV) or 6.6×10^{-14} J. Increasing the energy of a gamma ray does not increase its speed; it increases the frequency of the radiation.

Table 5.2 The properties of alpha, beta and gamma radiations

Property	α particle	β particle	γ ray
Mass	heavy	light	none
Charge	+2	−1	none
Typical energy	~5 MeV	~1 MeV	~0.1 MeV
Range in air	a few cm	1 or 2 m	many metres
Penetration in matter	$\sim 10^{-2}$ mm	a few mm	high
Ionising ability	high	reasonable	poor

Chemical and nuclear reactions compared

The energy released during any nuclear reaction (including radioactive decay) is many times greater than that released in a typical chemical reaction. For example, the chemical reaction of a sodium ion capturing an electron releases about 1 eV of energy.

$$Na^+ + e^- \rightarrow Na + 1\text{ eV}$$

Nuclear reactions such as alpha, beta and gamma decays typically release energies of the order of megaelectron-volts, MeV, i.e. nuclear reactions release about a million times more energy than chemical reactions.

Worked Example 5.3A

Uranium-238 emits alpha particles with a maximum energy of 4.2 MeV.

a Explain why a sample of this radioisotope encased in plastic is quite safe to handle yet, if inhaled as dust, would be considered very dangerous.

b Calculate the energy of an alpha particle in joules.

Solution

a The alpha particles have a poor penetrating ability and so would be unable to pass through the plastic casing. However, if the radioactive uranium was on a dust particle and was inhaled, the alpha-emitting nuclei would be in direct contact with lung tissue and the alpha particles would damage this tissue.

b $4.2\text{ MeV} = 4.2 \times 10^6\text{ eV}$
$= 4.2 \times 10^6 \times 1.6 \times 10^{-19}\text{ J}$
$= 6.7 \times 10^{-13}\text{ J}$

Radiation and radioactive materials

People often confuse the concepts of *radiation* and *sources of radiation*. In Worked Example 5.3A, the *radiation* is alpha particles. Alpha particles are dangerous because of their ionising ability, but rapidly gain electrons and become harmless helium atoms. The uranium-238 is the *source* of radiation and is dangerous as a continuing source of alpha particles.

When a spill or release of radioactive material occurs, such as at Chernobyl in 1986, dust containing radioactive atoms is released to the environment. This dust is a *source of radiation* and is carried by the wind. It is incorrect to say that the wind blows *radiation* from the accident site into the environment.

Physics file

Beta particles can be used to monitor the thickness of rolled sheets of metal and plastic during manufacture. A beta particle source is placed under the newly rolled sheet and a detector is placed on the other side. If the sheet is being made too thick, fewer beta particles will penetrate and the detector count will fall. This information is instantaneously fed back to the rollers and the pressure is increased until the correct reading is achieved, and hence the right thickness is attained.

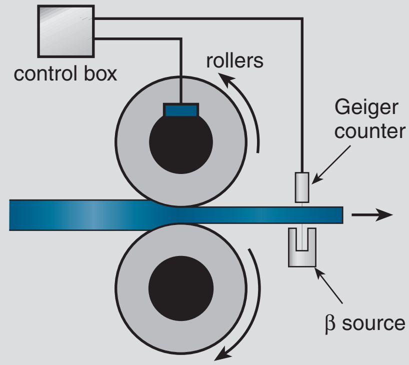


Figure 5.14
The thickness of a sheet of metal is monitored by using a strontium-90 isotope. A beam of beta particles is directed into the metal and those penetrating the metal sheet are counted by a detector on the other side. This count gives an indication of the thickness of the metal sheet. The thicker the sheet, the lower the count rate.

Physics in action — How radiation is detected

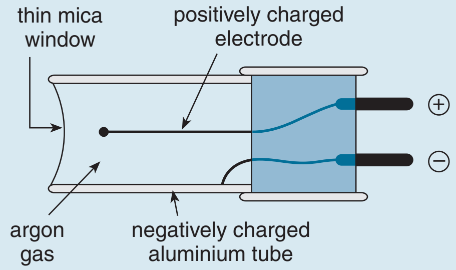


Figure 5.15

A radioactive emission that enters the tube in a Geiger counter will ionise the argon gas and cause a pulse of electrons to flow between the electrodes. This pulse registers as a count on a meter.

Figure 5.16

Film badges are used to monitor the exposure levels of doctors, radiologists, dentists and technicians who work with radiation.

Our bodies cannot detect alpha, beta or gamma radiation. A number of devices have therefore been developed to detect and measure radiation.

A common detector is the *Geiger counter*. These are used:

- by geologists searching for radioactive minerals such as uranium
- to monitor radiation levels in mines
- to measure the level of radiation after a nuclear accident such as the one at Chernobyl
- to check the safety of nuclear reactors
- to monitor radiation levels in hospitals and factories.

A Geiger counter consists of a Geiger–Müller tube filled with argon gas as shown in Figure 5.15. A voltage of about 400 V is maintained between the positively charged central electrode and the negatively charged aluminium tube. When radiation enters the tube through the thin mica window, the argon gas becomes ionised and releases electrons. These electrons are attracted towards the central electrode and ionise more argon atoms along the way. For an instant, the gas between the electrodes becomes ionised enough to conduct a pulse of current between the electrodes. This pulse is registered as a count. The counter is often connected to a small loudspeaker so that the count is heard as a ‘click’.

People who work where there is a risk of continuing low-level exposure to radiation usually pin a small radiation-monitoring device to their clothing. This could be either a film badge or a TLD (thermoluminescent dosimeter). These devices are used by personnel in nuclear power plants, hospitals, airports, dental laboratories and uranium mines to check their daily exposure to radiation. When astronauts go on space missions, they wear monitoring badges to check their exposure to damaging cosmic rays.

Film badges contain photographic film in a light-proof holder. The holder contains several filters of varying thickness and materials covering a piece of film. After being worn for a few weeks, the film is developed. Analysis of the film enables the type and amount of radiation to which the person has been exposed to be determined.

Thermoluminescent dosimeters are more commonly used than film badges. These contain a disk of lithium fluoride encased in plastic. Lithium fluoride can detect beta and gamma radiation as well as X-rays and neutrons. Thermoluminescent dosimeters are a cheap and reliable method for measuring radiation doses.

5.3 SUMMARY Properties of alpha, beta and gamma radiation

- Alpha particles, α, are ejected with a speed of about 10% of the speed of light. Alpha particles have a double positive electrical charge and are relatively heavy. They are a highly ionising form of radiation, but their penetrating ability is poor.
- Beta particles have a single negative electrical charge and are much lighter than alpha particles. They are a moderately ionising and penetrating form of radiation.
- Gamma rays are high-energy electromagnetic radiation and so have no electrical charge. They have a high penetrating ability, but a weak ionising ability.
- The energy of alpha, beta and gamma radiation is usually measured in electron-volts (eV):

$$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}$$

5.3 Questions

1 As part of an experiment, a scientist fires a beam of alpha, beta and gamma radiation at a brick. If the three radiation types are of equal energy, arrange them in order of:
  a increasing penetrating ability
  b increasing ionising ability.

2 Which one of the following correctly explains how penetrating ability relates to the ionising ability of a radioactive emission?
  A Emissions with more ionising ability have greater penetrating ability.
  B Emissions with less ionising ability have more penetrating ability.
  C There is no relationship between the ionising ability and penetrating ability of a radioactive emission.

3 An external source of radiation is used to treat a brain tumour. Which type of radioactive emission is best suited for this treatment?

4 A radiographer inserts a radioactive wire into a breast cancer with the intention of destroying the cancerous cells in close proximity to the wire. Should this wire be an alpha, beta or gamma emitter? Explain your reasoning.

5 Cancer patients being treated with an external source of radiation have to wear lead aprons to protect their other tissue from exposure. Which forms of radiation is the lead apron shielding them from?

6 Calculate the energy in joules of:
  a an alpha particle with 8.8 MeV of energy
  b a beta particle with 0.42 MeV of energy
  c a gamma ray with 500 keV of energy.

7 Alpha particles travelling through air ionise about 100 000 atoms each centimetre. Each time they ionise an atom, the alpha particles lose about 34 eV of energy.
  a How much energy will alpha particles lose as they pass through 1 cm of air?
  b Calculate the approximate distance that an alpha particle with 5.6 MeV will travel in air before it loses all of its energy.

8 Which one of the following has the greatest penetrating ability?
  A an alpha particle with 5.3 MeV of energy
  B a beta particle with 1.2 MeV of energy
  C a gamma ray with 700 keV of energy
  D a gamma ray with 0.81 MeV of energy

9 Which radiation identified in question 8 will be the most damaging to human tissue?

10 Explain why the penetrating ability of a radioactive emission is inversely related to its ionising ability.

5.4 Half-life and activity of radioisotopes

Different radioisotopes will emit radiation and decay at very different rates. For example, a Geiger counter held close to a small sample of polonium-218 will initially detect a significant amount of radiation, but the *activity* will not last for very long. After half an hour or so, there will hardly be any radiation detected at all.

Compare this with a similar sample of radium-226. A Geiger counter directed at the radium will show a sustained but low count rate—much lower than that of the polonium-218 sample. Furthermore, the activity will remain relatively steady for a very long time. In fact, no change in the count rate would be noticed for decades!

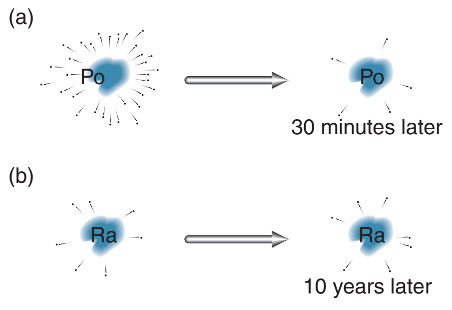


Figure 5.17

(a) The emissions from polonium-218 only last for a relatively short time. Its activity decreases very rapidly. (b) The emissions from a sample of radium-226 remain steady for a very long time. Its activity does not change significantly.

To explain this, you need to know that radionuclides are unstable but to different degrees. Consider again the sample of polonium-218. If the sample initially contains 100 million undecayed polonium-218 nuclei, as shown in Figure 5.18, after 3 minutes about half of these will have decayed, leaving just 50 million nuclei. A further 3 minutes later, half of these remaining polonium-218 nuclei will decay, leaving approximately 25 million of the original radioactive nuclei, and so on.

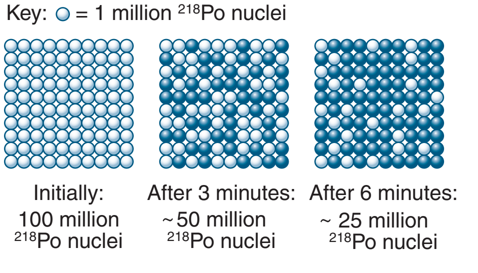


Figure 5.18

During one half-life, the number of nuclei of the radioisotope sample decreases by half (i.e. to 50%). After two half-lives, only one-quarter (25%) of the original radioisotope nuclei will remain.

This time that it takes for half of the nuclei of a radioisotope to decay is known as the *half-life* of that radioisotope. The half-life of polonium-218 is 3 minutes.

The decay rate of a radioisotope is measured in terms of its half-life ($t_{1/2}$). The ***HALF-LIFE*** of a radioisotope is the time that it takes for half of the nuclei of the sample radioisotope to spontaneously decay.

As time passes, a smaller and smaller proportion of the original radioisotope remains in the sample.

Time (t)	0	$t_{1/2}$	$2t_{1/2}$	$3t_{1/2}$	$4t_{1/2}$	$5t_{1/2}$	$nt_{1/2}$
Sample remaining	N_0	$\frac{N_0}{2}$	$\frac{N_0}{4}$	$\frac{N_0}{8}$	$\frac{N_0}{16}$	$\frac{N_0}{32}$	$\frac{N_0}{2^n}$

It is important to appreciate that, while the behaviour of a large sample of nuclei can be predicted, it is impossible to predict when any one particular nucleus will decay. The decay of the individual nuclei in a sample is random. It is rather like throwing dice. If 60 dice are thrown, then on average, 10 will roll up '6'. You just don't know which ones!

Furthermore, the half-life of a radioisotope is constant and is unaffected by any external conditions such as temperature, magnetic fields or the chemical environment. It is related only to the instability of the nucleus of the radioisotope.

Look at Figure 5.17 once again. It is evident that radium-226 has a very long half-life when compared with polonium-218. In fact, the half-life of radium-226 is about 1600 years. Clearly, a sample of radium-226 will emit particles and decay for centuries. The half-lives of some common radioisotopes are shown in Table 5.3. This table also illustrates that the half-life of a radioisotope is a factor in its application. For example, most medical applications using a radioisotope as a tracer require a short half-life. This is so that no radioactivity remains in the body any longer than necessary. On the other hand, the radioisotope used in a smoke detector is chosen because of its long half-life. The detector can continue to function for a very long time.

Table 5.3 Some common radioisotopes and their half-lives

Isotope	Emission	Half-life	Application
Natural			
Polonium-214	α	0.000 16 seconds	Nothing at this time
Carbon-14	β	5730 years	Carbon dating of fossils
Uranium-235	α	700 000 years	Nuclear fuel, rock dating
Uranium-238	α	4500 million years	Nuclear fuel, rock dating
Artificial			
Technetium-99m	β	6 hours	Medical tracer
Sodium-24	β	15 hours	Medical tracer
Iodine-131	β	8 days	Medical tracer
Phosphorus-32	β	14.3 days	Medical tracer
Cobalt-60	γ	5.3 years	Radiation therapy
Americium-241	α	460 years	Smoke detectors
Plutonium-239	α	24 000 years	Nuclear fuel, rock dating

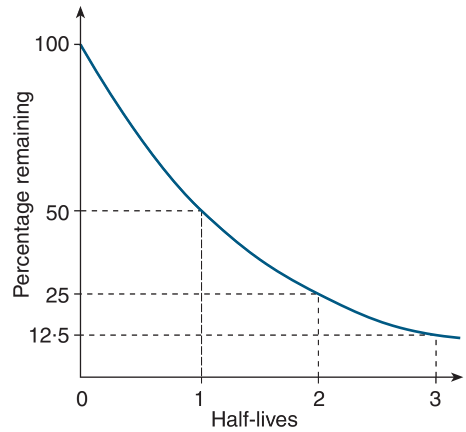


Figure 5.19
The amount of the original isotope halves as each half-life passes. This is an exponential relationship.

Physics file

In general, the decay of a radioactive isotope will follow a mathematical relationship as follows:

$$A = A_0(\tfrac{1}{2})^n$$

where n is the number of half-lives gone by.

A and A_0 can represent numbers of atoms of isotope in the sample, or mass of isotope in the sample, or activity of the sample. Each of these quantities follows the same pattern of decay:

$$n = \frac{t}{t_{1/2}}$$

where t is the time transpired and $t_{1/2}$ is the half-life.

A generalised formula is:

$$A = A_0(\tfrac{1}{2})^{\frac{t}{t_{1/2}}}$$

In mathematics classes you may also have encountered the generalised formula for growth and decay:

$$A = A_0e^{kt}$$

You may like to explore the relationship between k and half-life.

Physics file

Radioactive decay is a random and unpredictable process. It is impossible to predict when any particular unstable nucleus will decay. Half-life is the time during which any individual atom has a 0.5 probability of decay. Consequently, it is the time during which approximately half of a large number of atoms will decay.

Activity

A Geiger counter records the number of radioactive decays occurring in a sample each second. This is the *activity* of the sample. Activity is measured in becquerels, Bq, where 1 Bq = 1 disintegration per second.

Over time, the activity of any sample of a radioisotope will decrease. This is because more and more of the radioactive nuclei have decayed and will no longer emit radiation. So, *over one half-life, the activity of any sample will be reduced by half.* If the sample of polonium-218, discussed previously, has an initial activity of 2000 Bq, then after one half-life (i.e. 3 minutes) its activity will be 1000 Bq. After 6 minutes, the activity of the sample will have reduced to 500 Bq and so on.

Short-lived radioisotopes have an initially high activity. Their nuclei decay at a fast rate and so the sample lasts only for a short time. High-activity samples are extremely dangerous and must be handled with great caution.

✓ Worked Example 5.4A

A sample of the radioisotope thorium-234 contains 8.0×10^{12} nuclei. The half-life of ^{234}Th is 24 days. How many thorium-234 atoms will remain in the sample after:

a 24 days?
b 48 days?
c 96 days?

Solution

a Initially, there were 8.0×10^{12} ^{234}Th nuclei. 24 days is one half-life, so half of these will decay, leaving 4.0×10^{12} ^{234}Th nuclei.

b 48 days is two half-lives. This means that there will be $\frac{1}{2} \times \frac{1}{2} \times 8.0 \times 10^{12} = 2.0 \times 10^{12}$ nuclei.

c 96 days corresponds to four half-lives. In this time the number of atoms of the original radioisotope will have halved four times. This means that $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$ or one-sixteenth of the original ^{234}Th nuclei remain, i.e. 5.0×10^{11} atoms.

✓ Worked Example 5.4B

In 2 hours, the activity of a sample of a radioactive element falls from 240 to 30 Bq. What is the half-life of this element?

Solution

During each half-life, the activity of the radioisotope will fall by half. The activity of this element has decreased from 240 → 120 → 60 → 30 counts per second, so it has decayed through three half-lives in this 2-hour (120-minute) period. Thus the half-life must be 120/3 = 40 minutes.

Physics in action — Radiocarbon dating

Carbon dating is a technique used by archeologists to determine the age of fossils and ancient objects that were made from plant matter. In this method, the proportion of two isotopes of carbon, carbon-12 and carbon-14, in the specimen are measured and compared.

Carbon-12 is a stable isotope whereas carbon-14 is radioactive. Carbon-14 only exists in trace amounts in nature. In fact, carbon-12 atoms are about 1 000 000 000 000 (10^{12}) times more prevalent than carbon-14 atoms.

Carbon-14 has a half-life of 5730 years and decays by beta emission to nitrogen-14. Its decay equation is:

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}\beta$$

Both carbon-12 and carbon-14 can combine with other atoms in the environment, for example with oxygen to form carbon dioxide. While plants and animals are alive, they take in carbon-based molecules and so all living things will contain the same percentage of carbon-14. In the environment, the production of carbon-14 is matched by its decay and so the proportion of carbon-14 atoms to carbon-12 remains constant.

After a living thing has died, the amount of carbon-14 will decrease as these atoms decay to form nitrogen-14, and are not replaced. The number of atoms of carbon-12 does not change as this is a stable atom. So, over time, the proportion of carbon-14 to carbon-12 atoms falls. By comparing the proportion of carbon-14 to carbon-12 in a dead sample with that found in living things, and knowing the half-life of carbon-14 (5730 years), the approximate age of the specimen can be determined.

Consider this example. The count rate from a 1 g sample of carbon that has been extracted from an ancient wooden spear is 10 MBq. A 1 g sample of carbon from a living piece of wood gives a count rate of 40 MBq. For its count rate to have reduced from 40 to 10 MBq ($40 \rightarrow 20 \rightarrow 10$), the spear must be two half-lives of carbon-14 old, i.e. about 11 500 years old.

In 1988, scientists used carbon-dating techniques to show that the Shroud of Turin was probably a medieval forgery. Carbon-dating tests on samples of the cloth the size of a stamp established that there was a high probability that it was made between 1260 and 1390 AD, not around the time of Christ.

Figure 5.20
Carbon-dating techniques were used to show that the Shroud of Turin was most probably made around the 14th century.

More recent tests have cast doubt on the age of the shroud. Its age has again become a controversy.

Radiocarbon dating is an important aid to anthropologists who are interested in finding out about the migration patterns of early peoples—including the Australian Aboriginal people. This technique is very powerful since it can be applied to the remains of ancient campfires.

Physics in action — Decay series

Generally, when a radionuclide decays, its daughter nucleus is not completely stable, and is itself radioactive. This daughter will then decay to a granddaughter nucleus which may also be radioactive, and so on. Eventually a stable isotope is reached and the sequence ends. This is known as a decay series.

The Earth is 4.5 billion years old (4.5 giga-years)—enough to have only four naturally occurring decay series remain active. These are:

- the uranium series in which uranium-238 eventually becomes lead-206
- the actinium series in which uranium-235 eventually becomes lead-207
- the thorium series in which thorium-232 eventually becomes lead-208
- the neptunium series in which neptunium-237 eventually becomes bismuth-209. (Since neptunium-237 has a relatively short half-life, it is no longer present in the crust of the Earth, but the rest of its decay series is still continuing.)

Geologists analyse the proportions of the radioactive elements in a sample of rock to gain a reasonable estimate of the rock's age. This technique is known as rock dating.

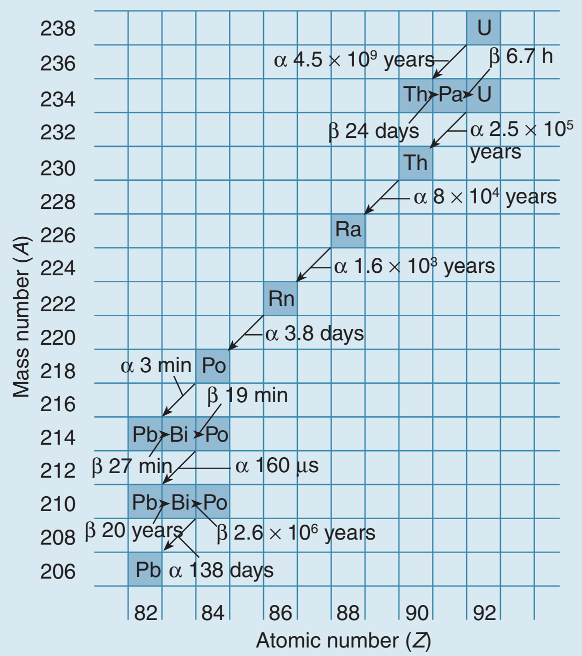


Figure 5.21
The uranium decay series. The half-life and emissions are indicated on each of the decays as radioactive uranium-238 is transformed into stable lead-206.

5.4 SUMMARY Half-life and activity of radioisotopes

- The rate of decay of a radioisotope is measured by its half-life. The half-life, $t_{½}$, of a radioisotope is the time that it takes for half of the nuclei in a sample of the radioisotope to decay.
- The activity of a sample indicates the number of radioactive decays that are occurring in the sample each second. Activity is measured in becquerels (Bq) where 1 Bq = 1 disintegration per second.
- The activity of any radioactive sample will decrease with time. Over a half-life, the activity of a sample will halve.
- $A = A_0(\frac{1}{2})^{\frac{t}{t_{½}}}$, where A is the decaying quantity remaining, A_0 is the initial quantity decaying, t is the time transpired, and $t_{½}$ is the half-life.

5.4 Questions

1 A radioactive isotope has a half-life of 1 hour. If a sample initially contains 100 mg of this isotope, which one of the following correctly gives the amount of the radioisotope remaining after 2 hours have elapsed?
  - **A** none
  - **B** 50 mg
  - **C** 25 mg
  - **D** 100 mg

2 A radioactive element has a half-life of 15 minutes. If you start with a 20 g sample of this element, how much of the original radioisotope will remain after:
  - **a** 15 minutes?
  - **b** 30 minutes?
  - **c** 45 minutes?
  - **d** 1.5 hours?

3 A Geiger counter measures the radioactive disintegrations from a sample of a certain

radioisotope. The count rate recorded is shown below.

Count rate (Bq)	400	280	200	140	100	70
Time (minutes)	0	10	20	30	40	50

a Plot a graph of count rate against time.
b Use your graph to estimate the activity of the sample after 15 minutes.
c What is the half-life of this element? Use both your graph and the table to determine your answer.
d Determine the activity of the sample after 60 minutes have elapsed.

4 The activity of a radioisotope changes from 6000 Bq to 375 Bq over a period of 1 h. What is the half-life of this element?

5 Gold-198 is a radioisotope with a half-life of 2.7 days. Consider one particular nucleus in a small sample of this substance. After 2.7 days this nucleus has not decayed. What is the probability that it will decay in the next 2.7-day period?

6 A hospital in Alice Springs needs 12 μg of the radioisotope technetium-99m, but the specimen must be ordered from a hospital in Sydney. If the half-life of ^{99m}Tc is 6 hours and the delivery time between hospitals is 24 hours, how much must be produced in Sydney to satisfy the Alice Springs order?

7 Radioactive materials are considered to be relatively safe when their activity has fallen to below 0.1% of their initial value.
a How many half-lives does this take?
b Plutonium-239 is a by-product of nuclear reactors. It has a half-life of about 24 000 years. For what period of time does a quantity of ^{239}Pu have to be stored until it is considered safe to handle?

8 Uranium-235 has a half-life of 700 000 years, while the half-life of uranium-238 is many times longer at 4.5×10^9 years.
a If you had 1 kg of each of these radioisotopes, which one would have the greater activity?
b The uranium that is mined in Australia and other parts of the world is 99.3% ^{238}U and only 0.7% ^{235}U. Explain why ^{235}U currently exists in trace amounts only.

9 Sodium-24 has a half-life of 15 hours. If a sample of this radioisotope has an activity of 10 million decays per second now, determine its activity in 5 days time.

10 A geologist analyses a sample of uranium ore that has been mined at Roxby Downs in South Australia. You may refer to Figure 5.21, the decay series graph for uranium, when answering this question.
a Explain why the sample would be expected to contain significant traces of lead.
b Explain why the geologist would be unlikely to find any ^{214}Po in the sample.

11 Sodium-24 has a half-life of 15 hours. How much of a 30 g sample will remain undecayed after one day?

12 A sample of iodine-131 was measured to have an activity of 137 Bq. The half-life of iodine-131 is 8 hours. How much time will it take for the activity to fall to 52 Bq?

13 Cobalt-60 has a half-life of 5.3 years. What percentage of a sample of cobalt-60 will remain after 14 years?

14 Some journalists quote the half-life as the time it takes for a radioactive substance to decay. For example, statements such as 'Stored strontium-90 will not decay for 28 years' or 'Strontium-90 will not be safe for 28 years' are common when referring to the problem of storing radioactive materials. Given that the half-life of strontium-90 is 28 years, what is it that these journalists do not understand?

5.5 Splitting the atom: nuclear fission

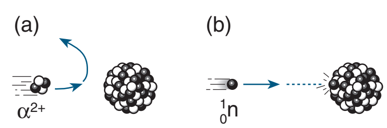


Figure 5.22
(a) Alpha particles are repelled so strongly by large atomic nuclei that collisions are not possible. (b) Neutrons, having no charge, are capable of colliding directly with the nucleus of an atom.

Until 1932, scientists used alpha particles from radioactive sources as probes to explore the nature of atomic nuclei. The alpha particles, acting as high speed 'bullets', bombarded target nuclei, and the resulting interactions were analysed. The problem with this method was that both the alpha particles and the target nuclei were positively charged and so repelled each other. It *was* possible for very energetic alpha particles to actually smash into the nuclei of small atoms such as nitrogen and aluminium. However, target nuclei with atomic numbers above 19, i.e. larger than potassium, repelled the alpha particles so strongly that collisions were not possible.

Physics file

Enrico Fermi was born in Italy in 1901. He completed his doctorate and postdoctorate work in physics at the University of Pisa and in Germany. Fermi had emigrated to the United States by the time the nuclear age dawned in the 1930s. The neutron had just been discovered in 1932 and this enabled scientists for the first time to fire neutral particles at atomic nuclei. Fermi was at the forefront of this research. He bombarded uranium atoms with neutrons and found that the uranium nuclei absorbed the neutrons and formed a radioactive isotope of uranium. This isotope then decayed to neptunium and then plutonium, two completely new elements. Fermi had successfully produced the world's first artificial and transuranic (i.e. after uranium) elements. The nuclear reactions for this process are:

$^{1}_{0}n + ^{238}_{92}U \rightarrow ^{239}_{92}U$ (neutron absorbed)

$^{239}_{92}U \rightarrow ^{239}_{93}Np + ^{0}_{-1}\beta$ (β decay)

$^{239}_{93}U \rightarrow ^{239}_{94}Np + ^{0}_{-1}\beta$ (β decay)

After the commencement of World War II, Fermi was commissioned by President Roosevelt to design and build a device that would sustain a nuclear chain reaction. In Chicago in 1942, Fermi succeeded in this task. Fermi died of cancer in 1954. One year after his death, the element with atomic number 100 was artificially produced, and named fermium, Fm.

The discovery of the neutron by James Chadwick in 1932 enabled scientists to explore the behaviour of larger atomic nuclei. Being neutral, a neutron is not repelled by the target nucleus, and can be absorbed into the nucleus of the target atom. This makes it very useful as a form of bombarding radiation, and it is used in many experiments to artificially transmute different isotopes (e.g. $^{1}_{0}n + ^{b}_{a}X \rightarrow ^{b+1}_{a}X$).

Enrico Fermi, an Italian-born scientist working in the United States, was conducting one such experiment in 1934. He bombarded uranium nuclei with neutrons and obtained some unexpected results. Some of the uranium nuclei absorbed the neutrons and split in two! Fermi was the first to observe *nuclear fission*.

NUCLEAR FISSION occurs when an atomic nucleus splits into two or more pieces. This is often triggered by the absorption of a neutron.

Nuclides that are capable of undergoing nuclear fission after absorbing a neutron are said to be *fissile*. Fissile nuclides are very uncommon. Uranium-235 and plutonium-239 are readily fissile. Uranium-238 and thorium-232 are only slightly fissile, requiring a very high-energy neutron to induce fission.

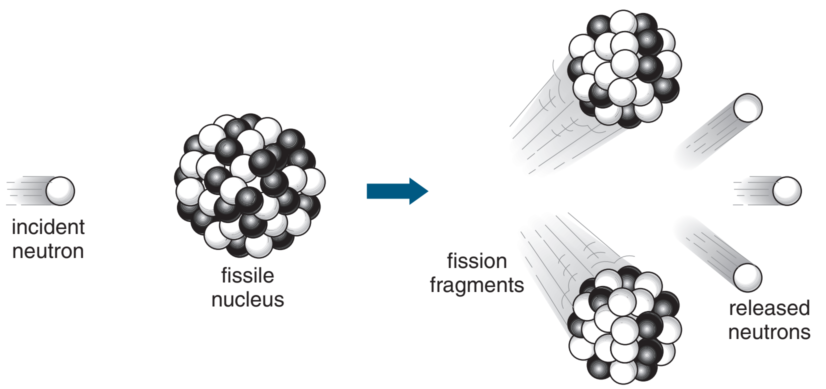


Figure 5.23
When a fissile nucleus absorbs a neutron, the nucleus splits in two, releasing a number of neutrons. This is an example of nuclear fission.

Figure 5.24
Enrico Fermi was awarded the Nobel Prize in physics for producing the first transuranic elements.

The release of neutrons during fission

Uranium-235 and plutonium-239 are the fissile nuclides most commonly used in nuclear reactors and nuclear weapons. This is because they release energy upon fission. When a uranium-235 or plutonium-239 nucleus absorbs either a slow or a fast moving neutron, it becomes unstable and spontaneously undergoes fission. However, fission is more likely to be induced by a slow moving neutron.

A typical fission reaction for uranium-235 is:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{91}_{36}Kr + ^{142}_{56}Ba + 3^{1}_{0}n + \text{energy}$$

Krypton-91 and barium-142 are known as the fission products or *fission fragments*. Three neutrons are freed from the uranium nucleus when it splits.

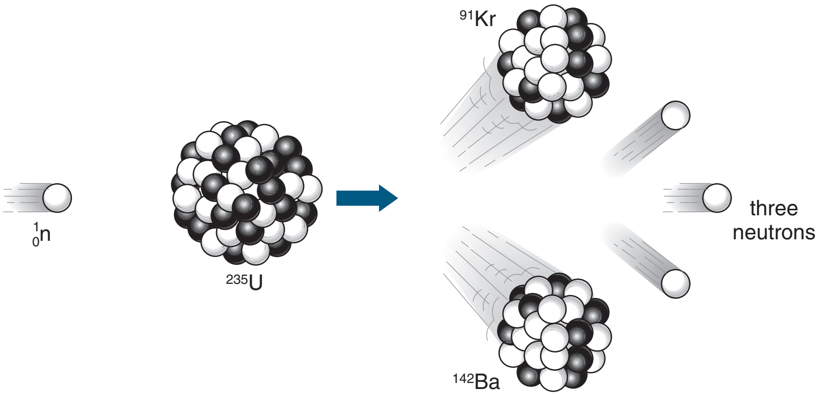


Figure 5.25

When a uranium-235 nucleus absorbs a neutron, nuclear fission occurs. The uranium nucleus splits in two, forming in this example krypton-91 and barium-142. Three neutrons are released.

A uranium-235 nucleus may split in many different ways, so when a sample of uranium-235 undergoes fission, a wide variety of fission products are produced. Usually either two or three neutrons are released. For uranium-235, an average of 2.47 neutrons per fission has been determined. Over 40 different pairs of fission fragments of uranium-235 have been found, and most of these are radioactive beta emitters. It is these *radioactive fission fragments* that comprise the bulk of the *high-level waste* produced by nuclear reactors.

Plutonium-239 will also undergo fission in a variety of ways. It releases an average of 2.89 neutrons per fission, slightly more than uranium-235.

The energy released during nuclear fission

The chemical reactions that you have probably performed at school typically release only a few electron-volts of energy. Compared to this, an enormous amount of energy, about 200 MeV, is released during each fission reaction. This energy is mainly in the form of the kinetic energy of the fission fragments, with the neutrons and emitted gamma radiation also having some energy. It was Albert Einstein who provided the explanation of the origins of this energy. He showed that *mass* and *energy*, instead of being completely independent quantities, were in fact *completely equivalent*: an amount of energy has an equivalent amount of mass.

In any fission reaction, the combined mass of the incident neutron and the target nucleus is always greater than the combined mass of the fission fragments and the released neutrons. For example, in Figure 5.25 the mass of incident neutron and the uranium-235 nucleus is greater than the combined masses of the fission products—barium-142, krypton-91 and three neutrons. The energy released as a result of this mass decrease is given by Einstein's famous equation.

Physics file

Energy is released in nuclear reactions because of variations in binding energy. Binding energy is the energy equivalence of the mass that is apparently missing when a nucleus is created from protons and neutrons. The total mass of the nucleons (protons and neutrons) that make up an atom is greater than the actual mass of the atom. The mass defect is a measure of the atom's binding energy. The greater the binding energy per nucleon in the atom, the greater the atom's stability. To calculate the binding energy of a nucleus, add the mass of the individual nucleons, and then subtract the mass of the atom itself. The mass left over is then converted into its energy equivalent.

$E_b = (Z \times m_H + N \times m_n - m_{isotope}) \times 931.5$ MeV/amu

where E_b = binding energy, in MeV
Z = number of protons
m_H = mass of a hydrogen atom (1.007 825 amu)
N = number of neutrons
m_n = mass of a neutron (1.008 664 904 amu)
$m_{isotope}$ = actual mass of the isotope

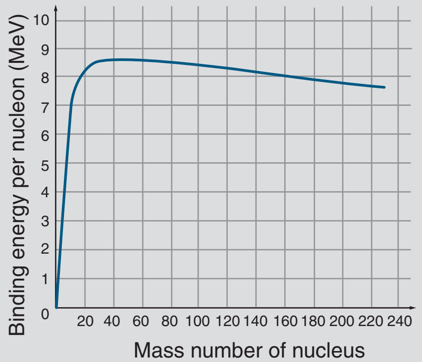


Figure 5.26

Graph of binding energy per nucleon against mass number.

Greater binding energy *per nucleon* of an atom gives greater stability. The binding energy per nucleon is the total binding energy divided by the mass number of the nucleus. The graph of the binding energy per nucleon against mass number shows that the nuclei of the light elements are generally less stable than those heavier nuclei with a mass number up to around 56. The nuclei of the heavier elements are less stable than the nuclei that have a mass number of around 56. This explains the two processes of converting mass into useful amounts of energy: fusion and fission. Fusion is discussed in section 5.7

$$E = \Delta mc^2$$

where E is the energy released from the reaction in joules (J)
Δm is the mass decrease for the reaction (kg)
c is the speed of light, i.e. 3.0×10^8 m s^{-1}.

It is important to note that only a very small proportion of the original mass of the nuclei is available as usable energy—typically around 0.1%. The energy released during the fission process is usually expressed in either joules (J) or electron-volts (eV).

Worked Example 5.5A

Plutonium-239 is a fissile material. When a plutonium-239 nucleus absorbs a neutron, it can split in many different ways. Consider the example of a nucleus that splits into barium-145 and krypton-93 and releases some neutrons. The nuclear equation for this is:

$$^{1}_{0}\text{n} + ^{239}_{94}\text{Pu} \rightarrow ^{145}_{58}\text{Ce} + ^{93}_{36}\text{Kr} + x\,^{1}_{0}\text{n} + \text{energy}$$

Use the following data: mass of neutron = $1.674\,95 \times 10^{-27}$ kg,
mass of plutonium-239 = $3.969\,60 \times 10^{-25}$ kg,
mass of cerium-145 = $2.406\,60 \times 10^{-25}$ kg,
mass of krypton-93 = $1.543\,18 \times 10^{-25}$ kg.

a How many neutrons are released during this fission process, i.e. what is the value of x?
b What is the decrease in mass (in kg) for this example?
c Calculate the amount of energy that was released during fission of a single plutonium-239 nucleus. Answer in both electron-volts and joules.

Solution

a Two neutrons are needed to make the mass numbers balance.

b The decrease in mass is the difference between the masses of the reactants ($^{1}_{0}$n and $^{239}_{94}$Pu) and the products ($^{145}_{58}$Ce, $^{93}_{36}$Kr and two $^{1}_{0}$n) of this fission reaction.

Mass of reactants:
$= (1.674\,95 \times 10^{-27}) + (3.969\,60 \times 10^{-25})$
$= 3.986\,35 \times 10^{-25}$ kg

Mass of fission products:
$= (2.406\,60 \times 10^{-25}) + (1.543\,18 \times 10^{-25}) + 2 \times (1.674\,95 \times 10^{-27})$
$= 3.983\,28 \times 10^{-25}$ kg

Decrease in mass Δm:
$= (3.986\,35 \times 10^{-25}) - (3.983\,28 \times 10^{-25})$
$= 3.07 \times 10^{-28}$ kg

c The energy released during the fission of this plutonium nucleus can be found by using:

$E = \Delta mc^2$
$= 3.07 \times 10^{-28} \times (3.0 \times 10^8)^2$
$= 2.76 \times 10^{-11}$ J

To determine the energy released in electron-volts, it is necessary to convert joules into electron-volts.

The energy released is: $\dfrac{2.76 \times 10^{-11}}{1.6 \times 10^{-19}} = 1.73 \times 10^8$ eV or 173 MeV.

Physics in action — The Manhattan Project

At the time World War II that was about to break out in Europe, Enrico Fermi was attempting to establish the first nuclear chain reaction. It was this development, and the concern that Germany would develop and use atomic weapons first, that prompted Albert Einstein to write his famous letter, dated 2 August 1939, to US President Roosevelt. Einstein suggested that an enormously powerful weapon could be made with uranium as the fuel, and proposed that a special team be established to explore this possibility.

Roosevelt responded quickly and set up a study committee which reported back to him in 1941 that Einstein's ideas were feasible. Roosevelt then established the top secret Manhattan Project headed by Robert Oppenheimer. The aim of the project was to produce the world's first nuclear-fission powered weapons.

Three teams of scientists worked to separate out the few kilograms of fissile material needed for a bomb. One team tried a technique on uranium known as electromagnetic separation. A second team also tried to separate uranium by a gas diffusion method. Enrico Fermi headed the third team. They produced plutonium-239 by bombarding uranium-238 in a nuclear reactor. After almost 4 years of research, two of the methods were successful—gas diffusion and the irradiation of uranium-238. These methods were used to produce fuel for three bombs.

On 16 July 1945, the first atomic bomb was exploded at the Trinity test site in New Mexico. Several weeks later, on 6 August 1945, an atomic bomb was dropped on Hiroshima in Japan. The third bomb was dropped on Nagasaki 3 days later. Around 200 000 people were killed by these explosions. Japan surrendered on 15 August 1945.

After the war, both Einstein and Oppenheimer became staunch opponents of the further development of nuclear weapons.

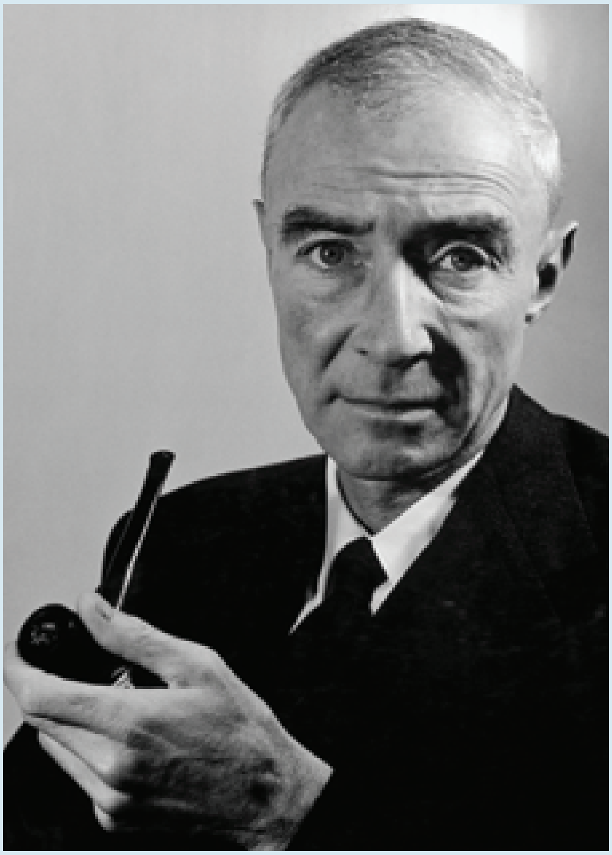


Figure 5.27
Robert Oppenheimer graduated from Harvard with a degree in chemistry after completing the 4-year degree program in 3 years. He pursued graduate studies in England and Germany and received his PhD in theoretical physics in 1927 from German Göttingen University. He moved back to the United States to convey his new discoveries in physics. In 1942 he organised a conference in California where top physicists discussed the possibility of nuclear weapons. In 1943 he became the director of the Manhattan Project and was involved with every step of the project.

5.5 SUMMARY Splitting the atom—nuclear fission

- Nuclear fission occurs when a nucleus is caused to split and release a number of neutrons. This can happen after a fissile nucleus has been struck by a neutron. A relatively large amount of energy is released during this fission process.
- Elements that are capable of undergoing nuclear fission are known as fissile materials. Only a handful of isotopes have this property, including uranium-235 and plutonium-239. These are the most common nuclear fuels.
- When a nucleus splits during nuclear fission, a number of neutrons are released.
- When a nucleus undergoes fission, the mass of the fission fragments is always less than the mass of the original particles. This decrease in mass is equivalent to the energy that is released during each fission and can be determined by using $E = \Delta mc^2$.

5.5 Questions

1 Determine the number of neutrons, x, released during each of these fission reactions:

a $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{148}_{57}La + ^{85}_{35}Br + x^{1}_{0}n$

b $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{142}_{54}Xe + ^{90}_{38}Sr + x^{1}_{0}n$

c $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{127}_{50}Sn + ^{104}_{42}Mo + x^{1}_{0}n$

2 Determine the value of the unknown mass number x and atomic number y in this fission reaction:

$^{1}_{0}n + ^{x}_{94}Pu \rightarrow ^{130}_{54}Xe + ^{106}_{y}Zr + 4^{1}_{0}n$

3 Which one of the following particles is best able to split an atomic nucleus?

A a proton
B a neutron
C an electron
D an alpha particle

4 Which one of the following statements is correct?

A Nuclear fission is more likely to occur in small nuclei.
B It is possible to produce fission in any nucleus provided the bombarding particles have enough energy.
C All radioactive nuclei are fissile.
D Only a few nuclei are fissile.

5 Einstein said that mass and energy are equivalent. In one particular nuclear fission reaction, a decrease in mass of 3.48×10^{-28} kg occurs.

a Express the energy equivalent of this in terms of joules and MeV.

b The highest amount of energy that is released during a single radioactive decay is around 10 MeV. Comment on the energy released during nuclear fission in comparison to this.

6 During the radioactive decay of cobalt-60, a gamma ray with 1.33 MeV of energy is released. Calculate the decrease in mass (in kg) of the cobalt-60 nucleus as a result of this emission.

7 When James Chadwick discovered the neutron in 1932, he was irradiating a sample of beryllium-9 with a radioactive emission. The equation for this is:

$^{9}_{4}Be + X \rightarrow ^{12}_{6}C + ^{1}_{0}n$

By balancing the equation, determine the nature of the bombarding particle.

8 Uranium-235 can undergo fission in the following manner:

$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3^{1}_{0}n$ + energy

(mass of neutron = $1.674\,95 \times 10^{-27}$ kg, mass of uranium-235 = $3.903\,05 \times 10^{-25}$ kg, mass of barium-144 = $2.389\,92 \times 10^{-25}$ kg, mass of krypton-89 = $1.476\,53 \times 10^{-25}$ kg)

a What is the decrease in the mass of the nuclear particles involved in this fission reaction?

b How many joules of energy are released during the fission of this uranium-235 nucleus?

c Express the decrease in mass as a percentage of the mass of the initial nuclear particles.

d If a 5 kg lump of pure uranium-235 completely underwent fission, how much energy (in joules) would be released?

9 When a uranium-235 atom absorbs a slow moving neutron, one of its possible fission reactions is:

$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{148}_{57}La + ^{85}_{35}Br + 3^{1}_{0}n$ + energy

(mass of neutron = $1.674\,95 \times 10^{-27}$ kg, mass of uranium-235 = $3.903\,05 \times 10^{-25}$ kg, mass of lanthanum-148 = $2.456\,98 \times 10^{-25}$ kg, mass of bromine-85 = $1.410\,45 \times 10^{-25}$ kg)

a What is the mass of the energy produced during this fission?

b Calculate the energy (in joules) that has been released by this fission process.

c When Enrico Fermi used uranium-235 to established the world's first nuclear reactor in 1942, he was able to generate just 0.6 W of power. How many fission reactions were occurring each second in his reactor?

10 In non-fissile nuclei, the nuclear forces holding the nucleons together are much stronger than the electrostatic forces trying to push them apart.

a Comment on the relative strengths of these forces in fissile atoms.

b Explain why a collision with a single neutron is enough to split a fissile nucleus.

5.6 Nuclear fission weapons

The nuclear arms race that developed between the United States and the then Soviet Union after World War II has shaped the world's political landscape over the past 60 years. At one time, it was estimated that the nuclear arsenal of these countries was capable of killing half of the population of Earth. In this section, the physics of how these weapons of destruction are able to release nuclear energy will be examined.

Nuclear fission weapons release an enormous amount of energy in a split second; they are capable of causing death and devastation on a massive scale. This was tragically evident in the bombing of Hiroshima and Nagasaki in 1945. The energy released in the one bomb dropped over Hiroshima was equivalent to 20 000 tonnes of TNT but involved a fission reaction in only 40 kg of uranium. The explosion is estimated to have killed around 150 000 people.

Figure 5.28
The aftermath of the bombing of Hiroshima.

The properties of uranium-235, uranium-238 and plutonium-239

Uranium-235 is most likely to undergo fission when struck by a slow moving, or *thermal*, neutron with energy as low as 0.01 eV. A slow moving neutron can be absorbed into a uranium-235 nucleus, forming the highly unstable uranium-236 isotope. This then undergoes *fission* and releases energy.

A fast, high-energy neutron is not easily captured by a uranium-235 nucleus and so is less likely to induce fission. It is difficult for a nucleus to capture a fast neutron because the neutron does not stay close to the nucleus for long enough for the strong, short-range nuclear force to drag it in.

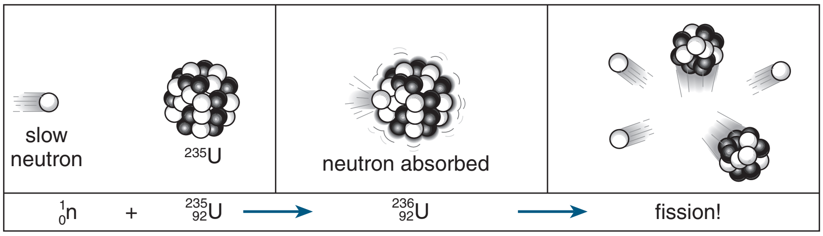


Figure 5.29
A slow neutron is absorbed by a uranium-235 nucleus, converting it into uranium-236 which is highly unstable. This nucleus then undergoes nuclear fission.

Uranium-238 is only slightly fissile. It requires a neutron with a large amount of energy (about 1 MeV) to cause fission in a uranium-238 nucleus. Because of its effectively non-fissile nature, uranium-238 is not suitable for use as a nuclear fuel, but it does have a role to play in nuclear energy production. A uranium-238 nucleus is far more likely to simply capture a neutron and become uranium-239. This then goes through a series of radioactive decays to become plutonium-239, itself a fissile substance. Uranium-238 is known as a *fertile* material because of this ability to capture a neutron and transform into a fissile substance.

Plutonium-239 is fissile in the same manner as uranium-235. However, plutonium nuclei require *fast neutrons* to bring about nuclear fission. Slow moving, or thermal, neutrons do not cause fission in plutonium-239 nuclei.

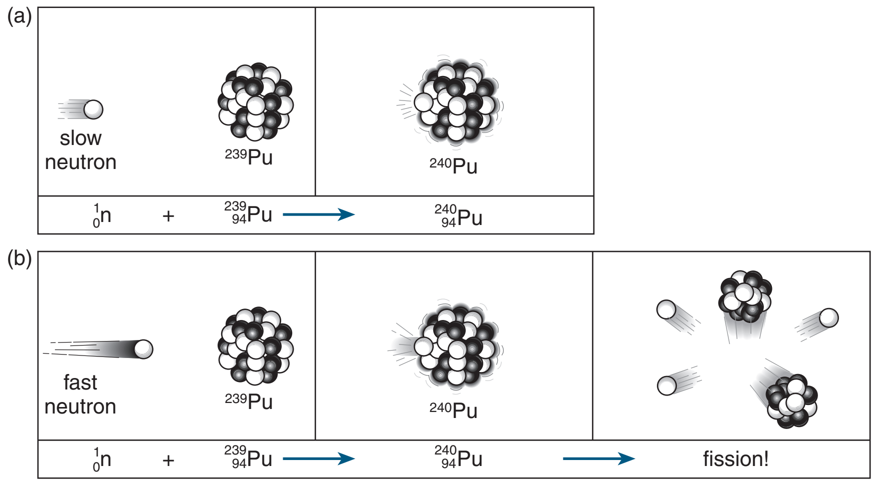


Figure 5.30

(a) When a plutonium-239 nucleus absorbs a slow neutron, the isotope plutonium-240 is formed. (b) The additional energy possessed by a fast moving neutron causes the plutonium-240 nucleus to distort and split into fission fragments.

Chain reaction

The scientists working on the Manhattan Project during World War II knew that nuclear energy could be released from a single fissile nucleus. The problem that they faced was how to obtain energy from a vast number of fissile nuclei. The nuclear fission bomb that was dropped with such devastating effect over Hiroshima in 1945 exploded as a result of an *uncontrolled chain reaction* in its uranium-235 fuel. When uranium-235 undergoes fission, it releases two or three neutrons each time. Each of these neutrons is then able to cause fission in another uranium-235 nucleus, which in turn will also release two or three neutrons. Within a very short time, the number of released neutrons and fission reactions has escalated in a process known as a *chain reaction*.

In Figure 5.31, two neutrons are released during the fission reaction. The number of nuclei undergoing fission doubles each generation and within a small fraction of a second an enormous number of nuclei have undergone fission. Only a minuscule amount of energy (of the order of 10^{-13} J) is released by each fission reaction, but in this uncontrolled chain reaction there are so many reactions occurring in such a short time that an explosion results. In 1 kg of uranium-235, so many reactions occur that about 8×10^{13} J of energy is released in just over one-millionth of a second!

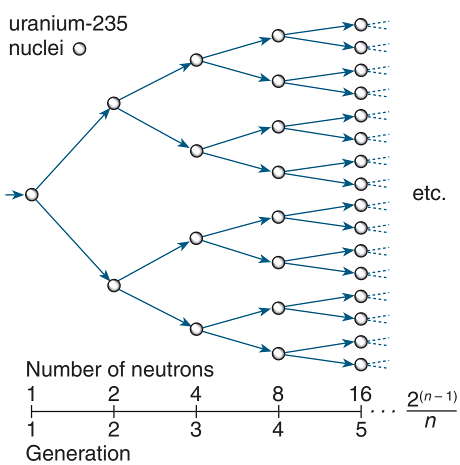


Figure 5.31

A single slow moving neutron causes fission, and two neutrons which are both capable of splitting another nucleus are released. After five nuclear generations, 16 neutrons are capable of triggering fission.

Nuclear fuel

In the 4.5 billion years since the Earth was formed, the radioactive isotopes have been decaying to form more stable isotopes. By far the two most common isotopes of uranium are uranium-238 and uranium-235. These have half-lives of 4.5 billion years and 710 million years respectively, and so uranium-235 has been decaying at a faster rate than uranium-238. This means that far less uranium-235 remains in the Earth's crust so that today the uranium that is mined from the ground consists of:

- 99.3% uranium-238—the non-fissile isotope
- 0.7% uranium-235—the readily fissile isotope.

This means that a chain reaction cannot occur in a sample of uranium taken from the ground because the proportion of fissile uranium-235 is too low. To be useful as a nuclear fuel, the ore has to be *enriched*. This involves increasing the proportion of uranium-235 relative to uranium-238, a very difficult and expensive process. The slightly different masses of the isotopes enables separation to be achieved. The three common enrichment methods are the ultracentrifuge, electromagnetic and gaseous diffusion separation techniques.

Nuclear weapons require fissile material that has been enriched to over 90% purity. The bomb that was dropped over Hiroshima contained 40 kg of 95% pure uranium-235. Nuclear reactors require fissile material enriched to about 3% uranium-235.

Critical mass

In their efforts to produce an explosion, the scientists working on the Manhattan Project had to establish a nuclear chain reaction in a sample of nuclear fuel. They found that the explosive ability of a sample of fissile material depended on its *purity*, *shape* and *size*.

In a sample of nuclear material where the *concentration* of uranium-235 or plutonium-239 is too low, a chain reaction cannot be established. This is because the neutrons have only a small chance of being absorbed by a fissile nuclei and causing a further fission reaction. The chain reaction will die out. The fuel used in nuclear fission weapons is enriched to a high degree of purity so that a chain reaction can be sustained.

The *shape* of the nuclear fuel is an important factor in its explosive ability. A 20 kg sample of enriched uranium-235 in the shape of a sphere will spontaneously explode, whereas 20 kg of uranium-235 flattened into a sheet will not. The flat piece has a very large surface area and so an enormous number of neutrons are able to escape from the uranium into the air. These neutrons do not cause further fission reactions and so the chain reaction will die out. In the spherical piece of uranium, the surface area is much smaller and a greater proportion of neutrons remain in the uranium to sustain the chain reaction.

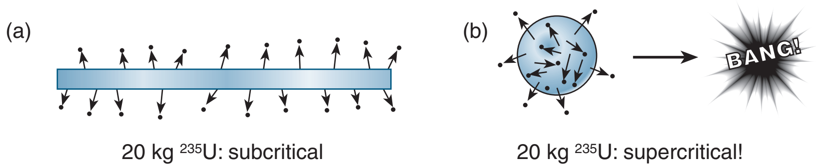


Figure 5.32
(a) The large surface area of the flat piece of uranium-235 enables a large proportion of neutrons to escape into the air, causing the chain reaction to die out. (b) In the spherical piece of uranium-235, a sufficient proportion of neutrons remain inside the material to maintain the chain reaction, leading to an explosion.

The explosive ability of a fissile material also depends on its *physical size*. For example, a piece of uranium-235 the size of a marble will not explode but a piece the size of a grapefruit most definitely will. The small piece has more surface area compared with its volume than the large piece. In the marble-sized lump, a greater proportion of neutrons escape into the air and so the chain reaction dies out. This is a subcritical mass. In the sample the size of a grapefruit, a higher proportion of neutrons is available to continue the chain reaction within the material. This is a *supercritical* piece, capable of causing a nuclear explosion.

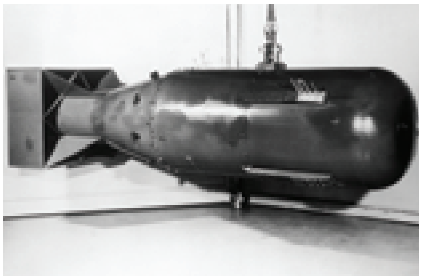
The minimum amount of enriched fissile material in the shape of a sphere that leads to a sustained chain reaction is known as the ***CRITICAL MASS***.

The design of nuclear fission weapons

The design of a fission bomb is really quite simple. It has to contain separate subcritical masses of a fissile material that are then combined at the desired time to make one supercritical mass when the explosion is required.

The nuclear bomb that was dropped on Hiroshima contained two hemispherical subcritical pieces of 95% pure uranium-235. This bomb, known as Little Boy, was dropped by a B29 bomber called the *Enola Gay* on 6 August 1945. When it reached an altitude of 580 m, an explosive charge fired one piece into the other, creating one supercritical mass of uranium-235, which exploded within one-millionth of a second after this.

(a)



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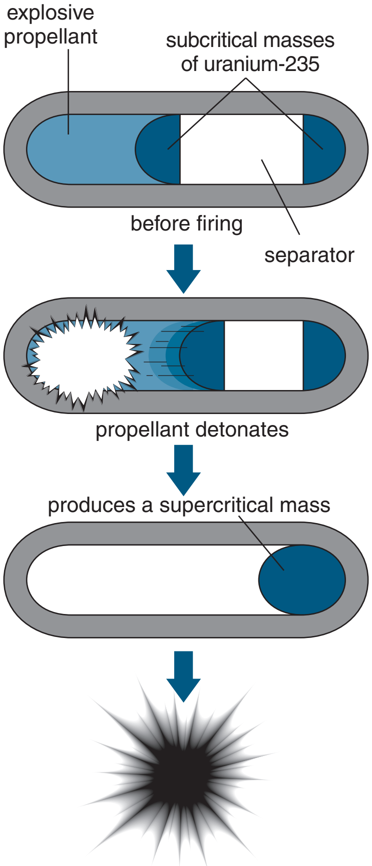
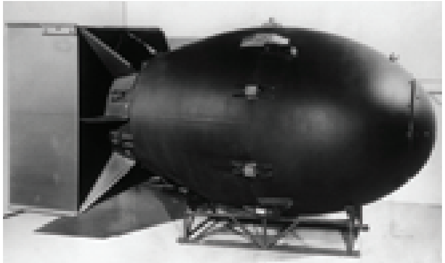


Figure 5.33
(a) A replica of Little Boy, the uranium-235 bomb that was dropped on Hiroshima on 6 August 1945.
(b) A simple fission bomb contains two subcritical pieces of uranium-235. The two pieces are kept separate until they are forced together, creating a supercritical mass and leading to an explosion within one-millionth of a second.

Three days after the bombing of Hiroshima, a second bomb was dropped over Japan. The target city was originally Kokura, but low cloud and fog resulted in the attack being changed to Nagasaki. This bomb, nicknamed Fat Man, contained many small subcritical pieces of plutonium-239. It used a spherical implosion to force these pieces together and make one supercritical mass. As with Little Boy, this led to an uncontrolled fission reaction and consequent nuclear explosion. This one killed or maimed almost 100 000 people. Twelve hours after the explosion, pilots could see Nagasaki burning from over 300 km away.

(a)



(b)

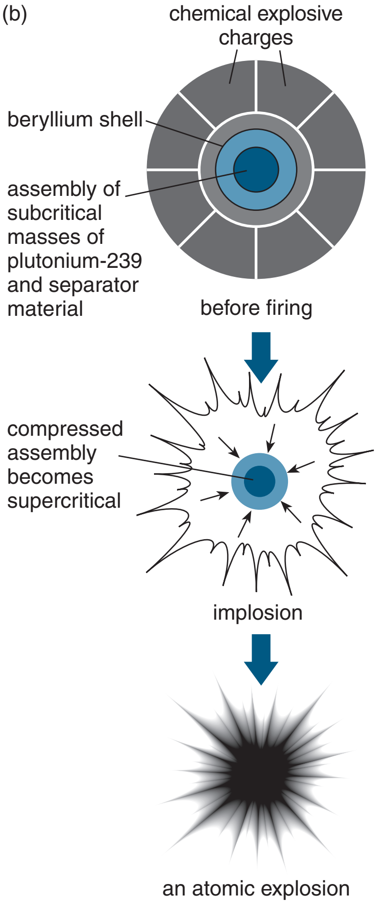


Figure 5.34

(a) A replica of Fat Man, the plutonium bomb that was dropped on Nagasaki on 9 August 1945. (b) Fat Man was a spherical fission bomb. In this type of bomb, an implosion forces a large number of subcritical pieces of plutonium-239 into one supercritical mass, producing a nuclear explosion.

Physics file

By the 1980s, the United States and the then Soviet Union had accumulated around 65 000 nuclear warheads. This was enough for each country to totally destroy the other's cities and nuclear targets some 17 times over. In 1991, President Bush of the USA and President Yeltsin of Russia signed the historic Strategic Arms Limitation Treaty, known as SALT II. This committed both countries to reducing their arsenal by over 90% to just 3000 nuclear warheads each by 2005.

Physics file

The power of nuclear weapons is generally expressed in kilotonnes (kt) or megatonnes (Mt). This gives an indication of their explosive ability in comparison to a conventional chemical bomb containing TNT.

The chain reactions that occur in nuclear weapons are actually very inefficient. The percentage of the available nuclei that undergo fission is very low. For example, in a typical uranium bomb with a yield of 15 kt, a critical mass of 30 kg is needed, but only about 1% of the fissile nuclei actually undergo fission. The proportion in a plutonium bomb is higher, but is still only about 10% of the available fissile nuclei. This means that a large amount of uranium and plutonium is scattered by a nuclear explosion, and forms part of the radioactive fallout. One problem associated with this is that plutonium is extremely toxic. At Maralinga, South Australia, in the early 1960s, the British government exploded hundreds of detonating devices, scattering about 20 kg of plutonium in the immediate area. The clean-up of this plutonium is currently taking place.

Physics in action — The effect of a nuclear weapon dropped over Perth

A number of nations including the USA, Russia, France, the UK and China maintain an arsenal of nuclear weapons. These typically have a power of around 1 Mt. In the past, nuclear weapons far more powerful than this have been assembled. The most powerful bomb ever exploded was a 57 Mt hydrogen bomb tested by the USSR in 1963. Table 5.4 and the map in Figure 5.34 will give you some idea of the devastating effect of nuclear weapons.

Table 5.4 The explosive effects of a 1 Mt nuclear blast over Perth CBD

Distance from explosion	Effects of small (1 Mt) bomb
Up to 5 km from central city	There would be no survivors. Radiation levels would be very high. Overpressure due to the shock wave would be approximately double normal atmospheric pressure. Winds due to the explosion would be approximately 500–1000 km h^{-1}. Multistorey reinforced concrete buildings would be destroyed.
5–7 km from central city	Approximately 50% of the population would be killed. Winds would be up to 200 km h^{-1}. Unreinforced buildings such as wooden and brick houses would be destroyed or severely damaged.
7–13 km from central city	Approximately 10% of the population would be killed. Winds would be approximately 100 km h^{-1}. Roofs of buildings would be severely damaged, walls would be cracked. A large proportion of the population would be injured by flying debris.

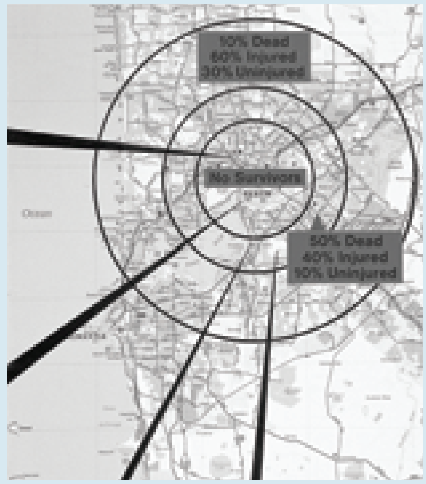


Figure 5.35
The map shows predictions of the immediate effect of a 1 Mt bomb dropped over the centre of the city of Perth.

5.6 SUMMARY Nuclear fission weapons

- Uranium-235 is readily fissile when it is struck by a slow neutron. Plutonium-239 is fissile when struck by a fast moving neutron.
- Uranium-238 is effectively non-fissile. It is more likely to absorb stray neutrons and become plutonium-239, itself a fissile material by radioactive decay. For this reason, uranium-238 is known as a fertile material.
- If a fission reaction is taking place that releases at least one neutron per fission, a chain reaction may be established.
- The critical mass is the least amount of material that will sustain a chain reaction. The critical mass for a material depends on its concentration, shape and size.

5.6 Questions

1 Which one of the following isotopes is most suitable as a fuel for a nuclear weapon?
A ^{238}U **B** ^{235}U **C** ^{234}U **D** ^{236}U

2 The uranium ore that is dug from the ground contains two different isotopes: ^{235}U and ^{238}U. Explain why uranium in this form is not immediately suitable as a fuel for a nuclear weapon.

3 The uranium that is used as the fuel for a nuclear weapon has been enriched so that its uranium-235 content is around:
A 0.7% **B** 3% **C** 10% **D** 95%

4 During an atomic bomb explosion, approximately 10^{24} uranium-235 nuclei undergo fission in just over 1 μs. During each fission reaction, about 200 MeV of energy is released.
a Calculate the energy that is released (in joules) as a result of each fission.
b How much energy (in joules) is released during the explosion?
c 1 tonne of TNT releases approximately 4×10^9 J of energy. What is the TNT equivalent (in tonnes) of this explosion?

5 Discuss how the chain reaction that releases the energy in a nuclear fission explosion is established. Explain why the chain reaction grows rather than dies out.

6 The critical mass of uranium-235 is about 1 kg. Explain then why it is that a 5 kg piece of ^{235}U that is flattened like a sheet is not capable of exploding.

7 Nuclear fission bombs require a supercritical mass of highly fissile material to explode.
a How is the bomb transported to the target site without exploding?
b Describe the process by which the bomb is detonated once the target site is reached.

8 During a 1 Mt nuclear fission explosion, approximately 4×10^{15} J of energy was released. By how much did the mass of the nuclear particles decrease during this explosion?

5.7 Nuclear reactors

Since the 1950s, it has been possible to control nuclear fission within a nuclear reactor for the purpose of producing electrical power for domestic use. Australia does not produce any electricity in this way, but in over 30 countries around the world there are several hundred nuclear power plants in operation. Many more reactors have been constructed for medical, military and research purposes.

A nuclear power plant will produce electricity in much the same way as a coal-burning power plant. The primary difference is how the heat is produced. The power stations in the La Trobe Valley generate electricity by burning coal to produce heat that creates the steam which is used to turn the generator turbines. A nuclear power station simply has a different way of producing heat—by nuclear fission.

Table 5.5 Approximate percentage of electricity produced by nuclear power plants

Country	Percentage of electricity produced by nuclear power plants
USA	20
France	80
Sweden	50
Switzerland	40
South Korea	40
Japan	30
Britain	20
Belgium	55

Physics file

The first nuclear reactor was designed by Enrico Fermi as part of the Manhattan Project during World War II. The reactor contained layers of graphite, and over 40 t of uranium and uranium oxide. It was constructed in a squash court under a grandstand at the University of Chicago. A self-sustained chain reaction was first established at 3:45 p.m. on 2 December 1942. This historic event marked the beginning of the atomic age.

The reactor initially generated just 0.6 W of power—not even enough to light a torch bulb! It was later modified to produce about 200 W.

Thermal nuclear reactor

In their endeavours to harness the energy from the nuclear fission reactions, nuclear reactor designers had to overcome three major difficulties.

First, the neutrons released from uranium-235 when it undergoes fission are travelling at very high speeds—around 20 000 km s^{-1}. Uranium-235 is most fissile when irradiated by slow moving neutrons. Thus, these emitted neutrons needed to be slowed down. Second, the fission of each uranium-235 nucleus releases an average of 2.47 neutrons. This can lead to a chain reaction that results in an explosion. A way had to be found to absorb some of these emitted neutrons and maintain a steady chain reaction. Third, the heat generated in the reactor by the fission process had to be somehow collected and used to create steam to drive the turbine and generate electricity.

A thermal nuclear reactor generates energy through the fission of uranium-235, an isotope that is most likely to undergo fission when it is hit by *slow moving*, or thermal, neutrons. There are many different varieties of thermal nuclear reactors, but they all include the following design elements:

- fuel rods—long, thin rods containing pellets of enriched uranium
- a moderator—a material that slows the neutrons
- control rods—a material that absorbs neutrons
- a coolant—a liquid to absorb heat energy that has been produced by nuclear fission
- radiation shield—a thick concrete wall that prevents neutrons escaping from the reactor.

Nuclear fuel rods

Uranium-235 is used as nuclear fuel since it is readily fissile with slow moving neutrons. However, this isotope comprises only 0.7% of naturally occurring uranium, not enough to sustain a chain reaction. The predominant (99.3%) isotope, uranium-238, is effectively non-fissile and has the property of capturing slow moving neutrons. Therefore the proportion of uranium-235 in the ore has to be increased. That is, the uranium ore has to be *enriched* before it can be used as reactor-grade fuel. Uranium enriched for use in a thermal nuclear reactor contains 97.7% uranium-238 and 2.3% uranium-235.

Figure 5.36
The nuclear research reactor at Lucas Heights, New South Wales.

The proportion of uranium-235 is increased from 0.7% to 2.3%, i.e. the uranium is *enriched* by a factor of about three. By comparison, the fuel for a nuclear weapon is enriched to at least 90% uranium-235—a far more expensive and technically difficult procedure. The fuel used in a nuclear reactor is thus not suitable for use in nuclear weapons.

The enriched uranium, in pellet form, is then packed into a thin aluminium tube, known as a *fuel rod*. This is usually 3–5 m long. A large nuclear reactor has over 1000 fuel rods in its core. A fuel rod will eventually become depleted in uranium-235. This means that, over time, the concentration of uranium-235 falls to a level where it cannot sustain the fission chain reaction. Each fuel rod needs to be replaced every 4 years or so, and a typical 1000 MW reactor produces around 30 tonnes of spent fuel each year.

The moderator

In a nuclear reactor, the problem of fast moving neutrons is overcome by including a material that slows down, or *moderates*, the speed of the free neutrons. It has been found that substances whose nuclei are small will slow the neutrons down to speeds at which neutron capture will occur. When the emitted neutrons collide with these small nuclei, they lose most of their kinetic energy and so slow down. After many collisions, the neutrons have been slowed down to about 2 km s^{-1} and have less than 1 eV of energy.

Some materials that are commonly used as *moderators* are:

- graphite—consisting of carbon atoms
- normal water—H_2O
- heavy water—containing deuterium (^2_1H), an isotope of hydrogen
- carbon dioxide—CO_2.

Each of these materials works well as a moderator because not only does it slow the neutrons, but it does so without absorbing a significant number of them. Heavy water is the most effective moderator, but is also the most expensive. Water is the cheapest material, but absorbs more neutrons than the others and so reduces the extent of the chain reaction. Graphite is less effective than water because carbon nuclei are heavier than hydrogen nuclei. In losing their energy, the emitted neutrons have to collide with about 120 carbon nuclei, but only about 25 water molecules.

Control rods

A nuclear fission bomb can produce vast quantities of energy. The number of neutrons released during the uncontrolled fission chain reaction grows exponentially, releasing enormous amounts of energy in a split second.

A nuclear reactor can also produce great amounts of energy. However, a steady controlled energy release is required. This is achieved by controlling the number of neutrons that are involved in the fission chain reaction of uranium-235. This task is performed by the *control rods*.

A control rod contains material that has the ability to *absorb neutrons*. Cadmium and boron steel are commonly used in control rods. When a neutron strikes the nucleus of either of these atoms, it is absorbed into the nucleus and so takes no further part in the chain reaction.

Putting it all together

The *core* of a thermal nuclear reactor consists of the moderating material with fuel rods and control rods placed in it. These rods could be inserted into holes drilled into a pile of graphite several metres thick, or immersed into a volume of water or heavy water. The reactor that blew up at Chernobyl had about 1600 fuel rods and over 200 control rods in its graphite core.

When a neutron is released during the chain reaction, it is slowed by the moderating material. This enables it to be absorbed by a further uranium-235 nucleus, induce fission, and so continue the chain reaction.

The *rate* of the chain reaction is controlled by raising or lowering the *control rods*. If the operators wish to reduce the energy output of the reactor, or even shut it down completely, they will lower the control rods further into the core. This has the effect of absorbing more neutrons and reducing or stopping the chain reaction.

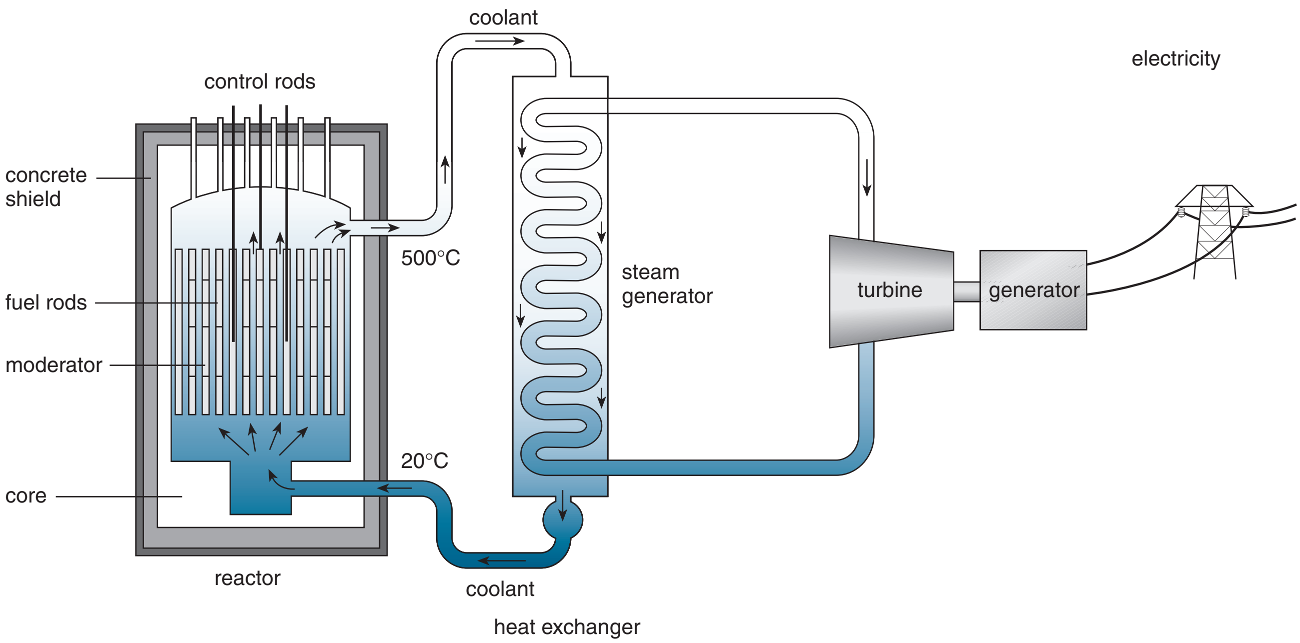


Figure 5.37
A schematic diagram of a nuclear power plant. The heat is removed from the core of the reactor by the coolant. The coolant then heats steam that turns the turbine that drives the generator. In this way electricity is generated. The primary difference between this and a coal-fired generator is in the way the heat is produced—a nuclear reactor uses the fission process and a coal generator burns coal. A typical 1000 MW power plant consumes about six million tonnes of black coal each year, or about 25 tonnes of enriched uranium that has been obtained from around 75 000 tonnes of ore.

The fission reaction in the reactor core produces an enormous amount of heat energy. The core of a reactor is typically at temperatures of 500–1500°C. Heat energy is removed from the core by pipes that contain a *coolant*. Liquid sodium, water, carbon dioxide gas, and heavy water are commonly used as coolants since they have high specific heat capacities. A *heat exchanger* then transfers this energy into pipes containing water. The water is converted into steam which is then used to rotate the turbines that drive the generator.

The core of the reactor is encased in a protective *radiation shield* about 2 m thick. This consists of layers of concrete, steel, graphite and lead. The function of this shield is to prevent neutrons and gamma rays from escaping from the reactor core, and so protect the workers at the plant from damaging radiation. The layers of graphite in the shield act to reflect escaping neutrons back into the core to take part in the chain reaction. The workers at a nuclear power plant are continually monitored to ensure that they are not exposed to unacceptably high levels of radiation. However, their allowed dose would still be much higher than that of the general population.

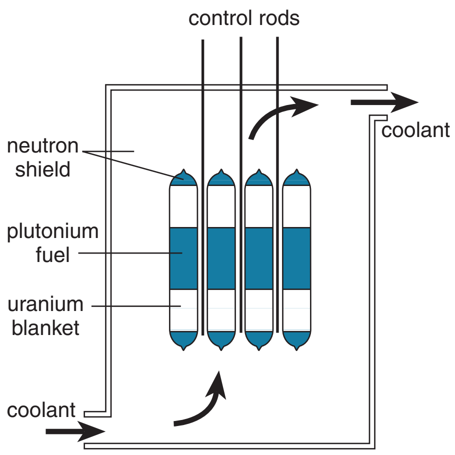


Figure 5.38

The core of a fast breeder reactor does not include a moderator. The plutonium in the fuel rods is surrounded by uranium. It is here that new plutonium is bred.

Fast breeder reactors

The Earth's known supply of the relatively scarce uranium-235 may run out within a few decades if its use is increased. In this event, many current nuclear reactors will become obsolete. A different type of reactor, known as a *fast breeder reactor*, may therefore come into greater prominence in the future. Fast breeder reactors make use of the most abundant uranium isotope, uranium-238.

Uranium-238 is only slightly fissile and comprises over 99% of naturally occurring uranium. It is not suitable for use as a fuel in a thermal nuclear reactor because it is only likely to split when it captures a high-energy neutron. There is only a low probability of this happening. However, despite its non-fissile properties, uranium-238 can play an important role in the production of nuclear energy. This is due to its ability to *absorb neutrons*. In a thermal nuclear reactor, there are many slow moving neutrons flying around. About 40% of these are captured by uranium-235 nuclei and induce fission, but 35% are captured by the non-fissile uranium-238. The remainder of the thermal neutrons are absorbed by other materials or escape from the core.

The neutrons that are captured by uranium-238 nuclei may cause the uranium to change into a highly unstable isotope, uranium-239. This decays almost immediately by beta emission to form neptunium-239, which has a half-life of 2.35 days and decays by beta emission to form the artificial element plutonium-239. The decay series for this process is:

$${}^{1}_{0}n + {}^{238}_{92}U \rightarrow {}^{239}_{92}U \qquad t_{½}\,({}^{239}U) = 23.5 \text{ minutes}$$

$${}^{239}_{92}U \rightarrow {}^{239}_{93}Np + {}^{0}_{-1}e \qquad t_{½}\,({}^{239}Np) = 2.35 \text{ days}$$

$${}^{239}_{93}Np \rightarrow {}^{239}_{94}Pu + {}^{0}_{-1}e \qquad t_{½}\,({}^{239}Pu) = 24\,400 \text{ years}$$

In other words, plutonium is a by-product of nuclear reactors; but it is a by-product that has great significance. Plutonium is a fissile material but only when it captures fast moving neutrons. The slow moving neutrons in most nuclear reactors do not induce fission of the plutonium nuclei that have formed there. Plutonium is extracted from the spent fuel rods of these nuclear reactors and used as the fissile material in a different type of reactor—a fast breeder reactor—which uses fast neutrons and actually breeds more plutonium than it uses!

A fast breeder reactor has many similarities to a standard nuclear reactor. It has *control rods* to control the rate of the fission reaction. A *coolant* flows through the core of the reactor, removing the heat energy. This energy is then used to produce the steam that drives the turbines to generate electricity. However, a fast breeder reactor *does not require a moderator* since fast neutrons are needed to trigger fission in the plutonium.

The fuel rods in a fast breeder reactor contain a core of plutonium and it is in this core that the fission process takes place. Fast neutrons released during fission are captured by plutonium nuclei to produce further fission. A great deal of heat energy is generated by this chain reaction.

The *breeding* of new plutonium occurs in the following way. The plutonium core of the fuel rods is surrounded by a 'blanket' of uranium-238. Many neutrons from the chain reaction escape into this uranium-238 and are absorbed by these nuclei, leading to the formation of plutonium-239. Thus, as the plutonium in the core of the fuel rod is used up as it undergoes fission, more plutonium is bred in the surrounding uranium-238. This plutonium is eventually extracted for use in the next generation of fuel rods.

Physics file

Plutonium is not a naturally occurring element in the Earth's crust. Any plutonium that exists has been created in a nuclear reactor. Since there are only a few fast breeder reactors in operation, most of the plutonium that is produced by thermal nuclear reactors is not re-used as nuclear fuel, but is stored as nuclear waste. Plutonium is an alpha emitter and so it is not difficult to protect against its emissions. However, it has a half-life of over 24 000 years. Furthermore, plutonium is an extremely toxic substance. This means that it must be safely stored for many thousands of years. At present, the plutonium is kept underground in bunkers and salt mines. However, many people are concerned that this is not an entirely satisfactory solution.

Technically, it is relatively easy to use plutonium as fuel in a nuclear weapon. There is great concern that a terrorist group could steal a quantity of plutonium from a reactor, or while it is being transported for disposal, and use it to manufacture an atomic bomb. Countries that use plutonium are required by the Atomic Energy Commission to account for every gram of the material, yet despite every safeguard, quantities of plutonium have gone missing.

Fast breeder reactors are very expensive to build and they suffer from many technological difficulties. At present, there are only a handful of fast breeder reactors in operation in France, the United States, Britain, Russia and Japan.

Physics in action — Chernobyl

In the early morning of 25 April 1986, electrical engineers at the Reactor Number 4, Chernobyl, in the then Soviet Union, began conducting a series of tests. They were trying to determine whether the generator would, as it was running down, continue to power the emergency systems in the plant. Over the next 24 hours, in performing the tests, they violated six different safety procedures. At one stage during the tests, the control rods were almost completely withdrawn from the core of the reactor. At 1:23 a.m. on 26 April, the power output began to rise rapidly, so the engineers decided to fully insert the control rods and shut down the reactor. However, each control rod was designed with 5 m of graphite on its end. This acted as an additional moderator and sped up the reaction even more. Within 4 s, the power had surged to 100 times the reactor's capacity, and an enormous steam explosion occurred. This lifted the concrete and steel dome many metres into the air and started a fire that burned out of control for 5 days. More than 50 tonnes of radioactive fallout was carried by the northerly winds across Ukraine, Belarus, Scandinavia and other parts of Europe. This was almost 200 times that of the waste from the bombs dropped at Hiroshima and Nagasaki in 1945.

The Chernobyl reactor stands in what is now Ukraine, close to the border of Belarus. Vast regions within these countries now stand empty. Approximately one-quarter of Belarus is considered to be uninhabitable. Thousands of villages lie abandoned, livestock has been slaughtered and schools have been closed. The immediate toll from the explosion was 31 deaths. This included firefighters and workers at the power plant. Far more serious has been the toll from the radioactive fallout. The incidence of cancer in the region has risen dramatically. For example, in the 1980s there were just seven reported cases of thyroid cancer among children in Belarus. From 1990 to 1996, this had increased to over 300. Not many of these have been fatal, but it does indicate that the problems will be long term, and projections vary from 4000 to 75 000 deaths.

Following the accident, a concrete and steel sarcophagus was constructed around the damaged reactor. The Chernobyl plant remained in service until December 2000, when it was finally shut down under pressure from the rest of the world. A 30 km exclusion zone around the site is still maintained, although this is being reconsidered.

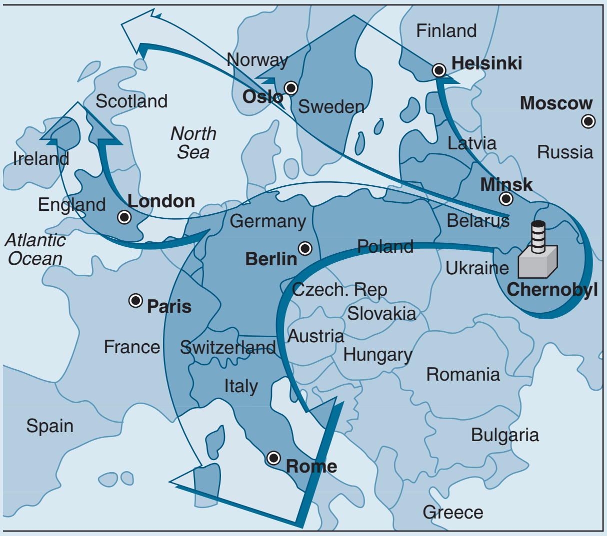


Figure 5.39
This map shows how the radioactive fallout from the nuclear accident at Chernobyl spread across Europe.

Nuclear fusion

An important type of nuclear reaction that occurs in nature is *fusion*. Fusion occurs in the Sun and the stars. It is how heavier elements are formed from light nuclei such as hydrogen and helium.

Fusion involves the joining of two small nuclei to form a larger one. When this occurs, energy is released and the mass of the products is less than the mass of the reactants. The mathematics of the mass defect is exactly the same as for fission.

Physics file

The three isotopes of hydrogen are: hydrogen $^{1}_{1}H$, deuterium $^{2}_{1}H$ and tritium $^{3}_{1}H$.

They are involved in many known fusion reactions. One of the people who discovered tritium was Mark Oliphant, who, as well as being a distinguished scientist, was Governor of South Australia from 1972 to 1977.

Physics file

The following definitions and conversions are useful:

Mass

The atomic mass unit, u, is defined as one-twelfth of the mass of a carbon-12 atom.

$1 \text{ u} = \frac{1}{12}$ mass of ${}^{12}_{1}\text{C}$

$= 1.6606 \times 10^{-27}$ kg

Energy

$1 \text{ eV} = 1.60 \times 10^{-19}$ J

$1 \text{ MeV} = 1.60 \times 10^{-13}$ J

Mass energy equivalence

$1 \text{ u} \equiv 931$ MeV

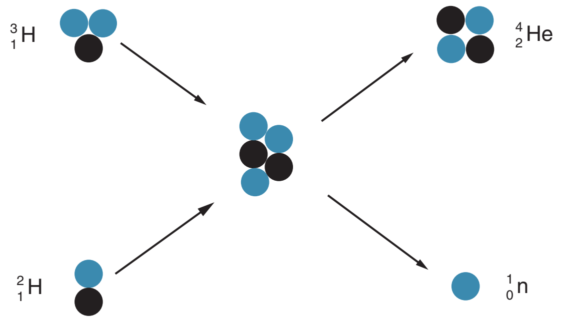


Figure 5.40

One example of fusion involves the combination of the two heavier isotopes of hydrogen. Tritium and deuterium can fuse to form an alpha particle and a neutron: ${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n}$

Physics in action — Nuclear fusion reactors—the way of the future

Since the 1950s, a great deal of research has been devoted to recreating the conditions by which the Sun and other stars produce energy—*nuclear fusion*. In nuclear fusion, two light nuclei combine to form a heavier nucleus. The combined mass is less than the mass of the initial nuclei, and so energy is also released. The attractiveness of nuclear fusion is that no radioactive by-products are created. On the Sun, the principle reaction is the fusion of hydrogen nuclei to form helium. An example of a fusion process that takes place on the Sun is:

$${}^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + {}^0_1\text{e} + \text{energy}$$

$${}^2_1\text{H} + {}^1_1\text{H} \rightarrow {}^3_2\text{He} + \gamma + \text{energy}$$

$${}^3_2\text{He} + {}^1_1\text{H} \rightarrow {}^4_2\text{He} + {}^0_1\text{e} + \text{energy}$$

The reactions that take place on the Sun are difficult to sustain in fusion reactors. This is because extremely high densities, temperatures and pressures are required. Fusion researchers are instead using two isotopes of hydrogen, deuterium (${}^2_1\text{H}$) and tritium (${}^3_1\text{H}$), as fuel. Deuterium can be extracted in vast quantities from lakes and oceans, but tritium is radioactive with a half-life of 12.3 years and must be artificially produced. The nuclear reactions used in current fusion reactors are as follows:

$${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^1_0\text{n}$$

$${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_1\text{H} + {}^1_1\text{H}$$

$${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n}$$

The main obstacle to the fusion of two nuclei is that the nuclei each have positive charges and so are naturally repelled from each other. In order to give the nuclei sufficient energy to overcome this force of repulsion, extremely high temperatures must be achieved. This has been the major difficulty facing researchers. Temperatures of the order of hundreds of millions of degrees are needed to trigger a self-sustaining fusion reaction. Current fusion reactors have achieved temperatures of about 100 million degrees, but only for very short periods.

A commercial nuclear fusion reactor is perhaps several decades into the future. At present, research is being carried out into *tokamaks*—doughnut-shaped reactors that use magnetic fields to contain the reaction.

Nuclear fusion may perhaps be the ultimate energy solution. The fuel for nuclear fusion, deuterium, can be readily obtained from sea water. Vast quantities of energy are released during the fusion process, yet a relatively small amount of radioactive waste—consisting of the reactor parts that have suffered neutron irradiation—is created.

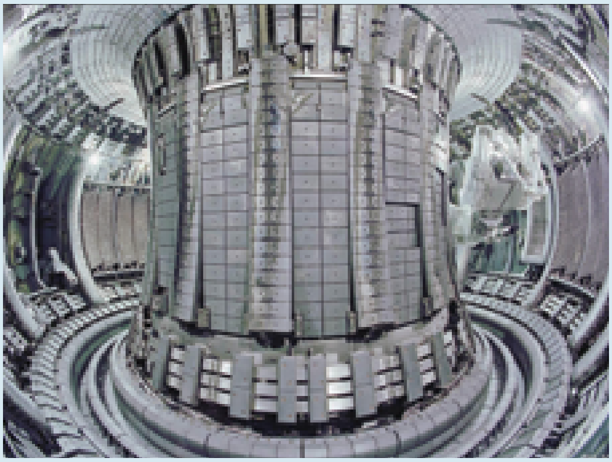


Figure 5.41

The doughnut shape or torus of the JET fusion reactor can be clearly seen in this photograph. Temperatures of hundreds of millions of degrees have been achieved in the torus.

5.7 SUMMARY Nuclear reactors

- A nuclear reactor uses enriched uranium as its fuel. The fuel rods in a nuclear reactor contain uranium that has been enriched to 3% or 4% uranium-235.
- The core of the reactor consists of a material (e.g. graphite, water) that acts to slow the neutrons that are emitted during fission. This is called a moderator. These slowed neutrons are then able to induce fission in the uranium-235 nuclei.
- The rate of the nuclear reaction in the reactor core is determined by the control rods. These consist of a material (e.g. cadmium, boron steel) that absorbs neutrons. The control rods are raised and lowered to control the chain reaction and so produce a steady release of energy.
- The coolant is a liquid that flows through the reactor core. It extracts heat energy from the core. This energy is then used to produce steam that drives turbines to produce electricity.
- A fast breeder reactor uses plutonium as its fuel. Plutonium-239 is fissile when struck by fast moving neutrons. A fast breeder reactor does not need a moderator.
- Uranium-238 is also placed in the core of a fast breeder reactor. It absorbs neutrons and transmutes into plutonium-239, which can then be used as fuel for a fast breeder reactor.
- Nuclear fusion involves joining two small nuclei under conditions of high energy to produce one larger nucleus. The principles of conservation of mass number and charge apply as in nuclear fission reactions.
- Fusion reactions release energy and demonstrate a mass defect. The energy from the Sun and stars is a result of fusion reactions.
- Research into controlled fusion is ongoing but scientists do not expect to have controlled fusion reactors for some decades.

5.7 Questions

1 The moderator in a nuclear reactor has the ability to:
A absorb neutrons **B** slow neutrons
C release energy **D** remove heat energy

2 The function of the control rods in a nuclear reactor is to:
A reduce the energy of the neutrons
B make the neutrons more likely to cause fission
C prevent the reactor core from overheating
D absorb neutrons and maintain a controlled chain reaction

3 **a** Outline the process by which a nuclear power plant produces electricity.
b Discuss the primary difference between how electricity is produced in a nuclear power station and in a coal-burning power station.
c What aspects of the production of electricity do coal-fired and nuclear power stations have in common?

4 Explain why lead ($Z = 82$) would be unsuitable for use as a moderator.

5 Describe the effect on the operation of a nuclear reactor if the number of neutrons per fission that is able to continue the chain reaction is:
a equal to one
b less than one
c greater than one.

6 The fissile material that is used in a nuclear reactor is uranium-235. What is the effect on the nuclei of this isotope when it is bombarded with:
a fast neutrons?
b slow moving neutrons?

7 Approximately 97% of the uranium in the fuel rods of nuclear reactors is uranium-238. When struck by a neutron, a uranium-238 nucleus is most likely to absorb the neutron.
a In what way does this change the uranium nuclei?
b Why does this lead to problems in disposing of the nuclear waste from the reactor?

8 A fast breeder reactor is a simpler design than a thermal nuclear reactor.
a What fuel is used in fast breeder reactors?
b Why is it called a 'fast' reactor?
c Why is it called a 'breeder' reactor?
d In what way is the design of a fast breeder reactor different from that of a thermal nuclear reactor?

9 During the fission of plutonium-239, the average number of neutrons released is 2.91 per fission. This is higher than the average of 2.47 released during the fission of uranium-235. How is this of benefit in a fast breeder reactor?

10 During the lifetime of a reactor, the control rods need to be gradually removed over a period of months in order to maintain the energy production at a constant rate. Explain why this procedure is necessary.

11 Hydrogen-2 (deuterium) has a mass of 3.34354×10^{-27} kg. Hydrogen-3 (tritium) has a mass of 5.00742×10^{-27} kg. Which of these two isotopes has the greatest binding energy per nucleon?

12 It is possible for two deuterium particles to fuse to produce tritium. Write the nuclear equation for this reaction and name the other particles that the reaction produces.

Chapter 5 Review

The following information relates to questions 1–5.

One particular fission reaction of uranium-235 is:

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n$$

Data: $m(\text{neutron}) = 1.674\,95 \times 10^{-27}$ kg,
$m(^{235}U) = 3.903\,05 \times 10^{-25}$ kg,
$m(^{141}Ba) = 2.339\,79 \times 10^{-25}$ kg,
$m(^{92}Kr) = 1.524\,51 \times 10^{-25}$ kg.

1 Calculate the decrease in the mass of the nuclear particles during this fission reaction.

2 How much energy is released during the fission of one uranium-235 nucleus?

3 Calculate the percentage of the initial mass that is lost.

4 If a 20 kg lump of pure uranium-235 completely underwent fission, by how much would the mass of the nuclear particles decrease?

5 Calculate the amount of energy released as a result of this mass loss.

The following information relates to questions 6 and 7.

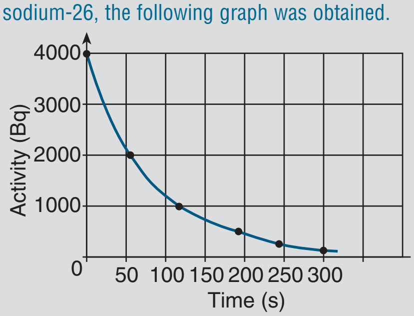
The artificial radioisotope plutonium-239 is formed when a uranium-238 nucleus absorbs a neutron and undergoes two beta decays. During each decay, a gamma ray is also emitted.

6 Write an equation for the absorption process.

7 Write equations to describe the decay process that results in plutonium-239.

The following information relates to questions 8–11.

In an experiment to determine the half-life of sodium-26, the following graph was obtained.



8 Use the graph to work out the half-life of sodium-26.

9 If the initial sample contained 150 g of sodium-26, how much of this radioisotope will remain after 5 minutes?

10 Sodium-26 is a beta emitter. Write the nuclear equation for its decay.

11 If a particular atom in the sample has not decayed during the first half-life, which one of the following statements best describes its fate?

A It will definitely decay during the second half-life.
B It has a 50% chance of decaying during the second half-life.
C There is no way of determining the probability that it will decay.
D If it does not decay during the first half-life, it will not decay at all.

12 A large piece of pure plutonium-239 will spontaneously explode, whereas a smaller piece will not. Explain why this is so.

13 When James Chadwick discovered the neutron in 1932, he was performing an experiment in which alpha particles were fired at beryllium-9 nuclei, and carbon-12 nuclei were formed. Write the nuclear equation for this process.

14 Uranium ore is 99.3% uranium-238 and 0.7% uranium-235. Given that the half-lives of these radioisotopes are 4.5×10^{9} and 7.1×10^{8} years respectively, determine whether the proportion of uranium-235 is increasing or decreasing.

The following information relates to questions 15 and 16.

Some of the survivors of the bombing of Hiroshima received a radiation dose of 4 Sv.

15 What somatic effects would this dose have on these people?

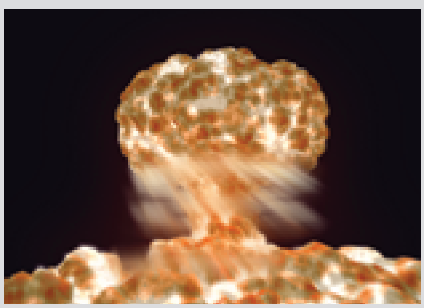


Figure 5.42
Survivors of an atomic explosion will be exposed to potentially harmful radiation.

16 Would they suffer any genetic damage? If so, what could be the consequences of this damage?

The following information relates to questions 17 and 18.

The Hartlepool nuclear power plant in England has two reactors; each produces 1500 MW of heat energy. Carbon dioxide is used as the coolant. The plant generates 1250 MW of electrical power.

17 Explain the primary function of the coolant.

18 Calculate the efficiency with which thermal energy is converted into electrical energy.

The following information relates to questions 19 and 20.

A nuclear scientist has prepared equal quantities of two radioisotopes of bismuth, ^{211}Bi and ^{215}Bi. These isotopes have half-lives of 2 and 8 min respectively. Assume that each sample has the same number of atoms.

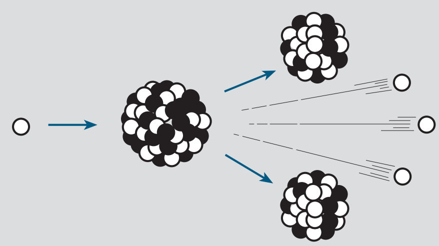
19 Which one of the following statements best describes the activities of these samples?

A The samples start with an equal activity, then bismuth-211 has the greater activity.
B Bismuth-211 initially has four times the activity of bismuth-215.
C Bismuth-215 initially has four times the activity of bismuth-211.
D Bismuth-211 initially has twice the activity of bismuth-215.

20 How will the activity of these samples compare after 8 minutes?

The following information relates to questions 21–26.

When neutrons are released from the fission of a uranium-235 nucleus, they have around 1 MeV of energy. The mass of a neutron is 1.675×10^{-27} kg. A typical fission of uranium-235 is shown below.



21 How many neutrons are released in the fission reaction shown?

22 What is the speed with which these neutrons are released?

23 Why does a thermal nuclear reactor use a moderator to slow the neutrons?

24 Discuss the characteristics of the moderating material that enable it to perform its task.

25 Fast breeder reactors do not have moderators. Explain why this is so.

26 Plutonium does not naturally exist on the Earth. Explain how the plutonium that is used in fast breeder reactors is formed.

The following information relates to questions 27–30.
The isotope $^{16}_{7}N$ is radioactive. It emits beta particles and has a half-life of 7.3 s.

27 Explain what beta particles are and where they come from.

28 How many protons, neutrons and nucleons does this nuclide have?

29 Write the decay equation for nitrogen-16.

30 Beta emitters are often used as surgical implants to treat tumours. Would this radionuclide be suitable for this application? Explain.

31 In the thorium decay series, the successive nuclides formed are: $^{232}_{90}Th$, $^{228}_{88}Ra$, $^{228}_{89}Ac$, $^{228}_{90}Th$, $^{224}_{88}Ra$, $^{220}_{86}Rn$ and $^{216}_{84}Po$. Determine the order of the alpha and beta decays that take place in this series.

32 It is possible for two deuterium atoms to fuse to form a tritium atom and a proton. 4.0 MeV is released when this happens. Write the nuclear equation for this reaction. How much energy would be released by such fusion of 100 g of deuterium?

The following information relates to questions 33 and 34.
A series of fusion reactions that occur in the Sun begins with two protons fusing to form a deuterium and a positron. A positron is just like an electron but is positively charged. A deuterium and another proton can fuse to form a helium-3. Two helium-3 nuclei can fuse to form an alpha particle and two protons. This last reaction releases 12.98 MeV for each alpha particle produced.

33 Write the reaction equations for the three fusion reactions described above.

34 What might be the power of a reactor that could fuse 100 g of helium-3 into alpha particles and protons in one day? The mass of one atom of tritium is 3.014 93 u. Give your answer in W.

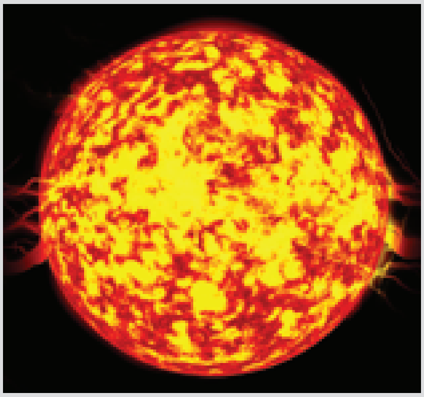
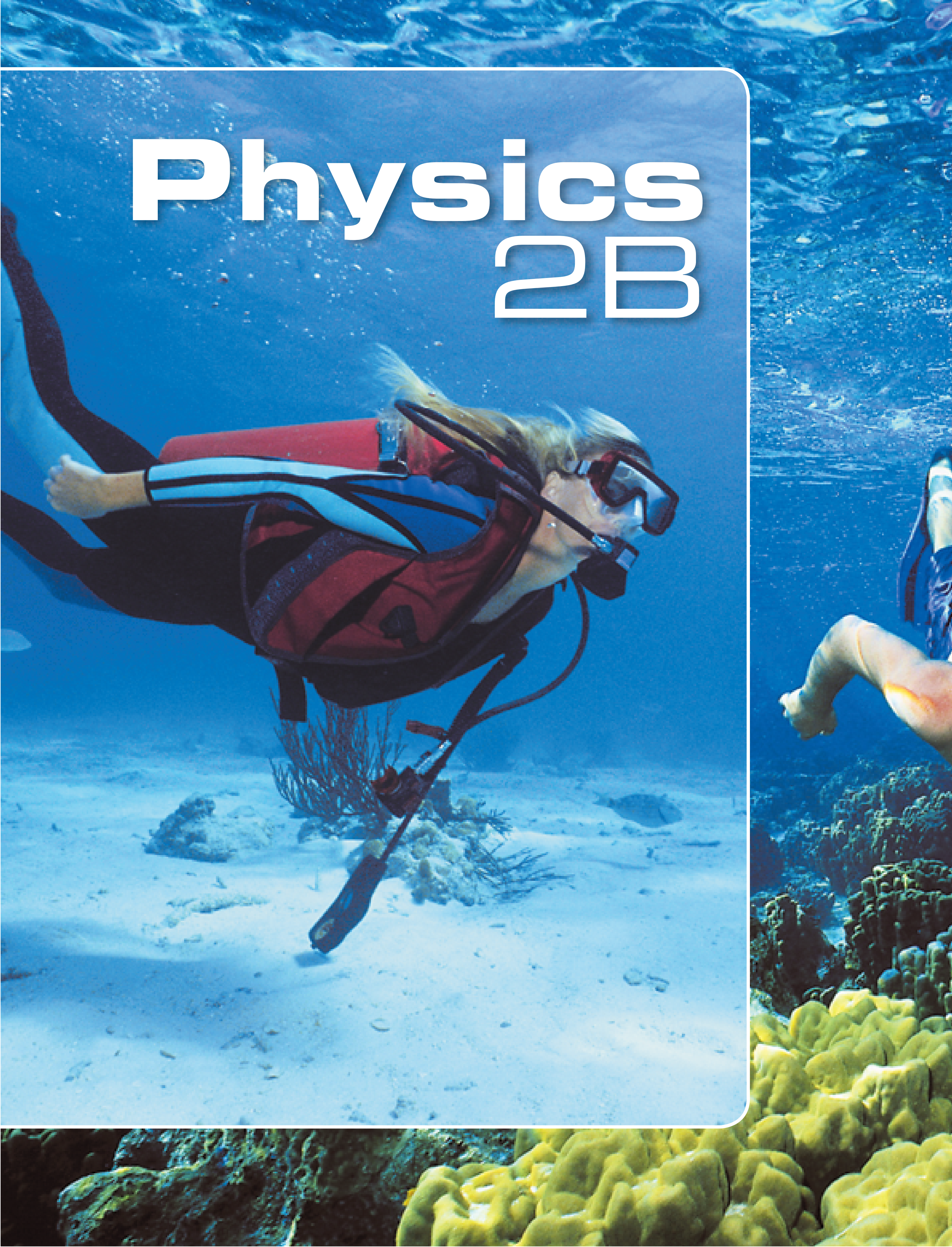


Figure 5.43
Fusion occurs within stars including our own sun.


Physics
2B

The following two context sections and content Chapters 1, 6 and 7 cover the content required and some extension content for the Physics CoS Unit 2B.

Outcomes

The Physics Outcomes are as follows:

Outcome 1: Investigating and communicating in physics

Students investigate physical phenomena and systems, collect and evaluate data, and communicate their findings.

In achieving this outcome, students:

- develop questions and ideas about the physical world to prepare an investigation plan
- conduct experiments and investigations
- analyse data and draw conclusions based on evidence
- evaluate the accuracy and precision of experimental data and the effectiveness of their experimental design
- communicate and apply physics skills and understandings in a range of contexts.

Outcome 2: Energy

Students apply understanding of energy to explain and predict physical phenomena.

In achieving this outcome, students:

- apply understanding of conceptual models and laws relating to energy
- apply understanding of mathematical models and laws relating to energy.

Outcome 3: Forces and fields

Students apply understanding of forces and fields to explain physical phenomena.

In achieving this outcome, students:

- apply understanding of conceptual models and laws relating to forces and fields
- apply understanding of mathematical models and laws relating to forces and fields
- apply understanding of the vector nature of some physical quantities.

It is envisaged that students will fulfil the requirements of Outcome 1 through investigative work in the classroom. Investigative work will typically be related to the content covered in Chapters 6 and 7. Chapter 1 covers the basic requirements for understanding and processing measured quantities in the investigative work.

Chapters 6 and 7 cover the content for Outcomes 2 and 3.

Unit description

Unit 2B focuses on two broad areas of physics: **Heating and Cooling** (Chapter 6) and **Electricity** (Chapter 7).

In **Heating and Cooling** students gain insight into temperature measurement, internal energy, conduction, convection and radiation to develop understandings about how energy is transferred by heat through different types of materials. They also examine the thermal properties of substances, including thermal expansion, specific heat capacity and latent heat.

In **Electricity**, students will use the concepts of charge and energy transfer to situations involving both electrostatics and current electricity. They study characteristics of electric circuits and learn about electrical safety.

Contexts

The context material preceding the content chapters supports the content of Unit 2B:

- Controlling our Environment
- Electricity at Home.

controlling our environment

By the end of this context
you will have covered material including:

- heat transfer
- refrigeration
- breathing underwater.

Humans differ from other animals in our ability to manipulate and control the thermodynamic nature of our environment. From the relative simplicity of good housing design to maximise the heating effect of the Sun in a cold climate to the sophisticated technology that scuba divers use to explore a reef, we control our environment to ensure our comfort and survival. We need to live in a narrow range of temperatures and pressures, and have the right mixture of gases to breathe. Without these conditions, our survival would be very brief.

Heat transfer in the home

If you have ever put a metal poker in a fire you will be aware of the rate at which heat travels through it! This is due to metals having a high *thermal conductivity*, *k*. The rate at which heat energy passes though a material is called the thermal current, *I*, and has the units of joules per second or watts (W). The thermal current through a material is calculated using equation ce.1.

$$\textbf{equation ce.1} \quad I = kA\frac{\Delta T}{\Delta x}$$

where I is the thermal current (W), k is the thermal conductivity of the material (W m^{-1} K^{-1}), A is the cross-sectional area (m^2), ΔT is the temperature difference (K or °C), and Δx is the distance through which the heat moves (m).

Equation ce.1 can be rearranged to give $\Delta T = IR$, where R is called the *thermal resistance* of a material and is equal to:

$$R = \frac{\Delta x}{kA}$$

It can be shown that if there is a series of materials in contact with one another, then the effective thermal resistance of all of the materials is given by:

$$R_e = R_1 + R_2 + \ldots$$

Note the similarity to the effective resistance of electrical resistors in series.

Worked Example ce.1

The surface temperature of a human body is about 34°C. You are wearing a leather jacket which is 3 mm thick. It is a cold morning of 4°C. At what rate is heat conducted through the jacket, assuming its total surface area is 0.5 m^2?

Solution

Assume that $k = 0.17$ W m^{-1} K^{-1}.
We are given $A = 0.5$ m^2, $\Delta T = 34°\text{C} - 4°\text{C} = 30°\text{C}$ and $\Delta x = 0.003$ m. Therefore:

$$I = kA\frac{\Delta T}{\Delta x} = 0.17 \times 0.5 \times \frac{30}{0.003} = 850 \text{ W} = 850 \text{ J s}^{-1}$$

*

The context assumes that you have studied heating and cooling in Chapter 6.

The answer to Worked Example ce.1 would suggest that your body would be losing heat by conduction at a rate equivalent to eight-and-a-half 100 W light bulbs. This, however, would not be the case as long as there was a layer of air between the surface of your skin and the inside of the leather jacket. This is because air is a poor conductor of heat, and therefore the transfer of heat through air occurs at a minimal rate. As a consequence, air is potentially a good insulator but the problem is that it is a fluid and fluids flow.

Air that is heated expands (due to the increase in motion of its particles) and consequently becomes less dense. Cold air, which is denser, would fall under the hot air and push it up. This is the means by which a *convection current* is established. So if we wish to make use of air's low thermal conductivity we need to trap it. Glass wool, which is the main constituent of 'ceiling batts', is a typical porous material that effectively traps air. Between the inner and outer surfaces of ovens and refrigerators, there are sheets of urethane or polystyrene that trap air within, creating insulation.

In other situations, where we want heat to be spread around, convection currents are clearly very convenient. Fan-forced ovens have a fan in them that moves the hot air around so the oven has a uniform temperature. Room heaters are more effective if they have a fan that encourages the circulation of air and hence heat flow.

Radiation has a bad name, but that is because the word is generally associated with radioactivity. Radiation, or more fully 'electromagnetic (EM) radiation', is literally all around us. We radiate infrared (IR) radiation (i.e. heat) at a rate of about 80 J s^{-1}, even when we are not doing any physical work; this is why a room full of people gets hot.

Experimental investigations

Investigate the cooking times of potatoes with and without metal skewers inserted. Consider how you will decide when a potato is cooked and how your experiment will be controlled. Discuss your findings.

Extended response

1 When a ball is dropped, does it increase in thermal energy?

2 What would be the most appropriate material to use for the handles of pots and pans?
(See Table 6.5 page 258.)

Figure ce.1 Glass wool and other materials used for ceiling batts trap air which enables the low conductivity of air to be used to advantage and create an insulating layer between the ceiling and the roof of a house.

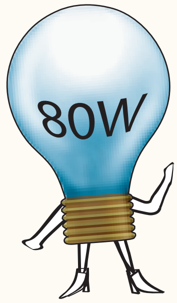


Figure ce.2 You are an 80 W light bulb!

Late in the nineteenth century, scientists were very interested in the concept of heat and established the discipline of *thermodynamics*. The relationship between the temperature of a substance and the rate at which it emits EM radiation is known as Stefan–Boltzmann's law:

$$I = \frac{\Delta Q}{\Delta t} = e\sigma AT^4$$

We usually say that 'hot air rises'; remember that it does so because it is pushed up by cold air—it doesn't just move against gravity of its own accord!

where I is the emissive power of a body (W), e is the *emissivity* of the body, σ is 5.67×10^{-8} W m^{-2} K^{-4} (called the Stefan constant), A is the radiating area (m^2), and T is the absolute temperature of the radiating body (K).

The emissivity of a material is defined as the ratio of the emissive power of a body at a particular temperature to the emissive power of a 'black body' at the same temperature. A 'perfectly black' material is one that absorbs *all* incident radiation, not just visible light, and that is able to emit all wavelengths of EM radiation. Perfectly black materials have an emissivity of 1. Most materials are generally black for some wavelengths of light but not others, so their emissivity is less than 1.

As we shall see later in this context, a body emits a range of wavelengths of EM radiation. These wavelengths are not all emitted with the same intensity, however. There is a relationship between the absolute temperature of a body and the wavelength of its most intense emission, λ_{max}. Equation ce.2 is known as Wien's displacement law, after Wilhelm Wien (1864–1928).

equation ce.2 $$\lambda_{max} = \frac{2.898 \times 10^{-3}}{T}$$

where λ_{max} is the wavelength with the greatest intensity emitted by a substance at an absolute temperature of T (K).

So how do we deal with radiation in the home? Because of the fourth power in Stefan–Boltzmann's law, we know that hotter objects emit thermal radiation at a far greater rate. We notice this when we sit in front of a fire or a bar heater. The emissivity of materials varies for different ranges of wavelengths of radiation. An object that is white or shiny to visible

Experimental investigations

Investigate the efficiency of energy transfer to the air from an electric heater in a room. First, determine the rate at which it uses energy (what is its power rating?). Then time how long it takes the room to be warmed to some predetermined temperature. You could extend this further and compare the efficiency of different heating methods, but you may need to heat a smaller body of air.

Extended response

1 How will a thin nylon jacket keep you warm on a cool windy day?

2 You get over-enthusiastic with your ceiling batts and squash an extra layer in the ceiling. Why does this actually decrease the effectiveness of the batts?

Worked Example ce.2

Determine the rate of thermal radiation emission of a human body, if the emissivity of the body is about 0.5, the surface area of the average adult is about 2 m^2 and the surface temperature is 34°C.

Solution

We know that $e = 0.5$, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}, $A = 2$ m^2, $T = 34°C = 307$ K. Substituting into Stefan–Boltzmann's law:

$$I = e\sigma AT^4 = 0.5 \times 5.67 \times 10^{-8} \times 2 \times (307)^4 = 504 \text{ W}$$

Note that this is significantly higher than the 80 W quoted earlier. This figure would be *all* the EM radiation you would emit (not just IR) if you were placed somewhere there was no energy incident on yourself (i.e. in deep space!).

light may be almost 'black' to infrared light. Bright clothing made of such materials will keep you cooler in the day as it reflects the visible sunlight, and warmer in the night as it hinders the radiation of your body heat due to its 'blackness' to infrared radiation.

The most efficient insulator is a vacuum since convection and conduction cannot occur within one. A Thermos bottle (a modern 'Dewar flask' as invented by Sir James Dewar in the late 1800s) is by far the most effective way of maintaining temperature. A Thermos has one wall that encloses the drink and another wall around the first with a small gap between them. This gap is evacuated of air to form a near vacuum. To reduce the flow of heat between the walls by radiation, the surfaces of the walls are mirrored as this reduces their emissivity.

Worked Example ce.3

The solar spectrum has a maximum intensity at about 480 nm. What does this suggest is the surface temperature of the Sun?

Solution

From Wien's displacement law:

$$\lambda_{max} = \frac{2.898 \times 10^{-3}}{T}$$

$$\Rightarrow T = \frac{2.898 \times 10^{-3}}{\lambda_{max}}$$

$$= \frac{2.898 \times 10^{-3}}{480 \times 10^{-9}} = 6040 \text{ K} = 5765°C$$

This illustrates one way in which astronomers are able to measure celestial phenomena without actually going there!

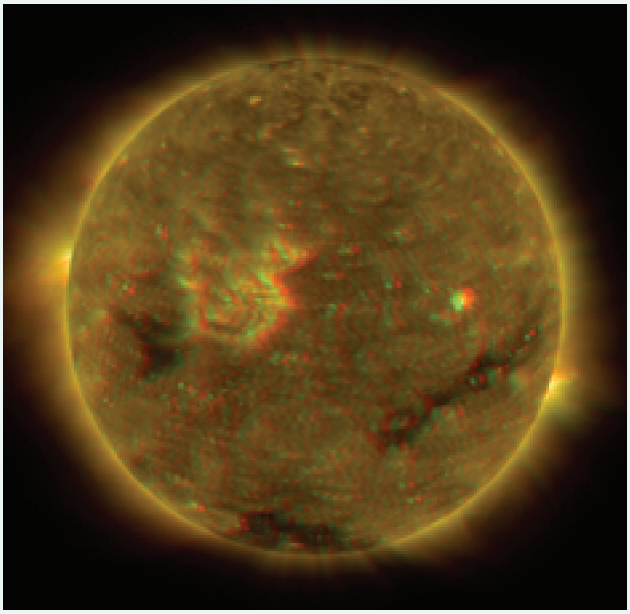


Figure ce.3 Our Sun emits vast amounts of EM radiation, but not all of the wavelengths that are emitted are of the same intensity.

Heat appliances

The natural way for many hot (animal) bodies to lose heat is by sweating. The evaporation of the sweat, or liquid, off the surface of our skin requires energy and this energy comes from our body. On hot days when we want this to occur at a greater rate, we use electric fans to circulate the air so the moisture-laden air on our skin moves on to allow less moist air to continue the process. A problem, however, arises when it is

Figure ce.4 The vacuum between the double-layered walls of the container within a Thermos flask provides the perfect insulator.

Experimental investigations

Measure the internal temperature changes in different-coloured cars left in the sun.

Extended response

Explain why aluminium foil is more effective at keeping food warm than clear plastic wrap.

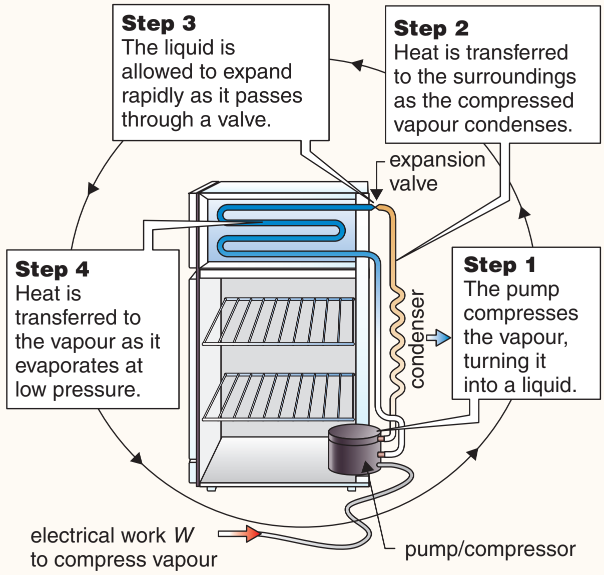


Figure ce.5 A refrigerator has the following components: evaporator, compressor, condenser and a constriction.

a humid day. At a given temperature, air can only hold so much water vapour. We feel uncomfortable on humid days because the air can't carry any more water vapour so the process of evaporation is slowed, causing us to cool down less effectively.

This is when we turn to the air-conditioner. The air-conditioner not only provides us with cold air, but also air of relatively low humidity so that the cooling process can occur effectively. The operation of air-conditioners is quite complicated because they have to circulate air and regulate its humidity. In essence though, air-conditioners and refrigerators are the same thing: they are both *heat pumps* that work to move heat from one place to somewhere else that is generally hotter.

The refrigerant is the substance that flows around the pipes in the refrigerator, and has the special ability of being a gas at low pressure and a liquid at high pressure over the temperature range at which the refrigerator operates. Chlorofluorocarbons (CFCs) were originally used as refrigerants, but hydrocarbons are used now as they are less harmful to the environment. Table ce.1 summarises the refrigeration process.

Clearly it is not natural to take heat to a place that is hotter. To make this possible, energy must be an input to this heat pump process. When we plug in our refrigerator, the electricity is used to operate the compressor which is essentially a pressure pump.

In colder climates, wood stoves are used as a means of heating. Wood stoves are designed so that the air that is used for combustion does not mix with the air (in the room) that is to be heated. The amount of heat a wood stove will generate is dependent on the *heat of combustion*, *H*, of the fuel (presumably wood). Table ce.2 lists the heats of combustion of a few fuels.

table ce.1 The refrigeration process

Component	Function	Nature of refrigerant
Evaporator	long metal tube that carries the refrigerant through the region to be cooled	• The low-pressure liquid refrigerant absorbs heat easily, causing it to evaporate and hence take heat away from the region to be cooled. • The refrigerant leaves the evaporator as a cool low-pressure gas.
Compressor	to compress the gaseous refrigerant	• The gaseous refrigerant increases in its temperature and pressure.
Condenser	long metal tube that carries the refrigerant to where the heat is to be released	• The refrigerant cools down by releasing its heat via conduction and convection in the surrounding air. • The refrigerant condenses to a high-pressure cool liquid.
Constriction	to reduce the pressure of the refrigerant	• The liquid refrigerant enters the narrowing, speeds up and hence reduces in pressure (see Bernoulli's equation). • Then back to the evaporator ...

table ce.2 Heat of combustion of some common fuels

Material	Heat of combustion ($\times 10^7$)
Wood	1.40 J kg^{-1}
Coal	3.26 J kg^{-1}
Natural gas	4.35 J m^{-3}
Diesel	4.50 J kg^{-1}
Domestic fuel oil	4.60 J kg^{-1}
Kerosene	4.60 J kg^{-1}
Petrol	4.70 J kg^{-1}
Propane	8.80 J m^{-3}

Activity

Research how wood stoves are designed to enable them to distribute heat by the three heat-transfer mechanisms.

Worked Example ce.4

How many cubic metres of natural gas are required to heat 12 L of water from 20°C to 90°C if 40% of the heat is lost to the surroundings?

Solution

We know 1 L of water has a mass of 1 kg; therefore $m = 12$ kg, $c = 4816$ J kg^{-1} K^{-1} and $\Delta T = 90 - 20 = 70$°C. The heat required to increase the temperature of the water is:

$$\Delta Q = mc\Delta T$$
$$= 12 \times 4186 \times 70 = 3\,516\,240 \text{ J}$$

We also know the heat of combustion of natural gas from table ce.2, $H = 4.35 \times 10^7$ J m^{-3}. The units of heat of combustion (J m^{-3}) imply that

$H = \dfrac{\Delta Q}{V}$; therefore:

$$V = \frac{\Delta Q}{H} = \frac{3\,516\,240}{4.35 \times 10^7} = 0.0808 \text{ m}^3$$

Since 40% of the energy from a given volume of gas is lost to the environment, this volume of 0.0808 m^3 will be 60% of the total volume of gas needed to provide the required energy:

$$V = V_{total} \times \frac{60}{100}$$

$$\Rightarrow V_{total} = V \times \frac{100}{60} = 0.0808 \times \frac{100}{60} = 0.135$$

Hence, 0.135 m^3 of natural gas is required to heat the water.

Exercises

E1 A 15 cm thick outer wall of a building has a temperature gradient of 86.7°C m^{-1} across it. If it is 35°C outside, what is the temperature inside?

E2 Two cubes of lead and silver each have an edge length of 2 cm. They are placed together between two walls. One wall is held at a temperature of 0°C and the other at 100°C. Determine the net thermal current through the two cubes and the temperature at the interface between the two cubes using equation ce.1.

E3 Betelgeuse is one of the stars in the constellation Orion. It is a red giant in astronomical terms. Its spectrum has a peak wavelength of approximately 600 nm. What would be the surface temperature of Betelgeuse?

E4 A 6 cm diameter ball is taken from a furnace with its temperature at 120°C. It is found to emit radiant energy at a rate of 12 W. What is the emissivity of the sphere's material?

E5 A copper-bottomed saucepan containing 0.8 L of boiling water boils dry in 10 min. Assuming that all the heat flows through the flat copper bottom, which has a diameter of 15 cm and a thickness of 3.0 mm, calculate the temperature of the outside of the copper bottom while some water is still in the pan.

Breathing under water

Breathing involves a movement of air in and out of the lungs, which occurs automatically under normal circumstances. The physics of breathing is quite simple while we're on land and so we tend not to even consider what's happening, but under water, getting new air to the lungs can be quite a challenge.

Lung capacity varies greatly between individuals, although the lungs always retain a residual volume of approximately 1.5 L so that they don't collapse. While they are partially inflated the lungs are more readily inflated further.

There is a low-pressure area between the outside of the lungs and the walls of the thoracic cavity, which means that the air within the lungs, which is at atmospheric pressure, provides a constant outwards force. This force ensures that the alveoli don't collapse.

The alveoli are coated in a surfactant that reduces their surface tension. There is therefore less force pulling the walls of the alveoli together, again helping to reduce the likelihood of collapse.

How our lungs work

Our respiratory system includes two input channels (nose and mouth) that combine into one (the trachea) before again splitting into two (the bronchi). Each bronchus then leads to a lung; we have two separate lungs. Our lungs are a little like a sponge, in that they are made up of a network of air sacs called *alveoli*. This structure is important for maximising the surface area within our lungs, and allows as many blood vessels as possible to come into contact with the air we breathe, so that we absorb as much oxygen as we can with each breath.

Approximately 18 times per minute, the muscles in an adult's chest pull the ribs upwards and the diaphragm that separates our lungs from our abdominal cavity contracts, straightens and pulls down. The result is that our lungs expand to a greater volume, and so does the air within them. When these muscles relax, the ribs move down again and the diaphragm returns to an upwardly curved shape, reducing the volume of the lungs and the amount of air they contain.

One of the properties of matter is that gases expand to fill all of the available space, and Boyle's law states that increasing the volume of a gas decreases its pressure. This means that if the lungs were closed to the outside air, attempting to inhale would expand the air inside the lungs until the particles were evenly spread throughout the lungs, giving a lower pressure inside the lungs. However, our lungs are not a closed space; they are always open to the atmosphere! When we inhale and increase the volume of our lungs, the pressure of the air inside our lungs begins to reduce. Some of the air from the outside atmosphere immediately moves into our lungs in order to balance the pressure inside and the pressure outside. In the same way, when we exhale and decrease the volume of our lungs, the pressure of the air inside our lungs

Although we can easily close our mouths, it takes a conscious effort to close off a nose. It's also worth noting that the trachea includes rings of cartilage that keep it open; a 'crushed windpipe' is when a force has collapsed some of these rings and thereby closed off the trachea. If you run your fingers down the front of your neck, you can feel how strong these rings of cartilage are. It's certainly reassuring to know that we are designed to be able to breathe even when we're asleep (or unconscious)!

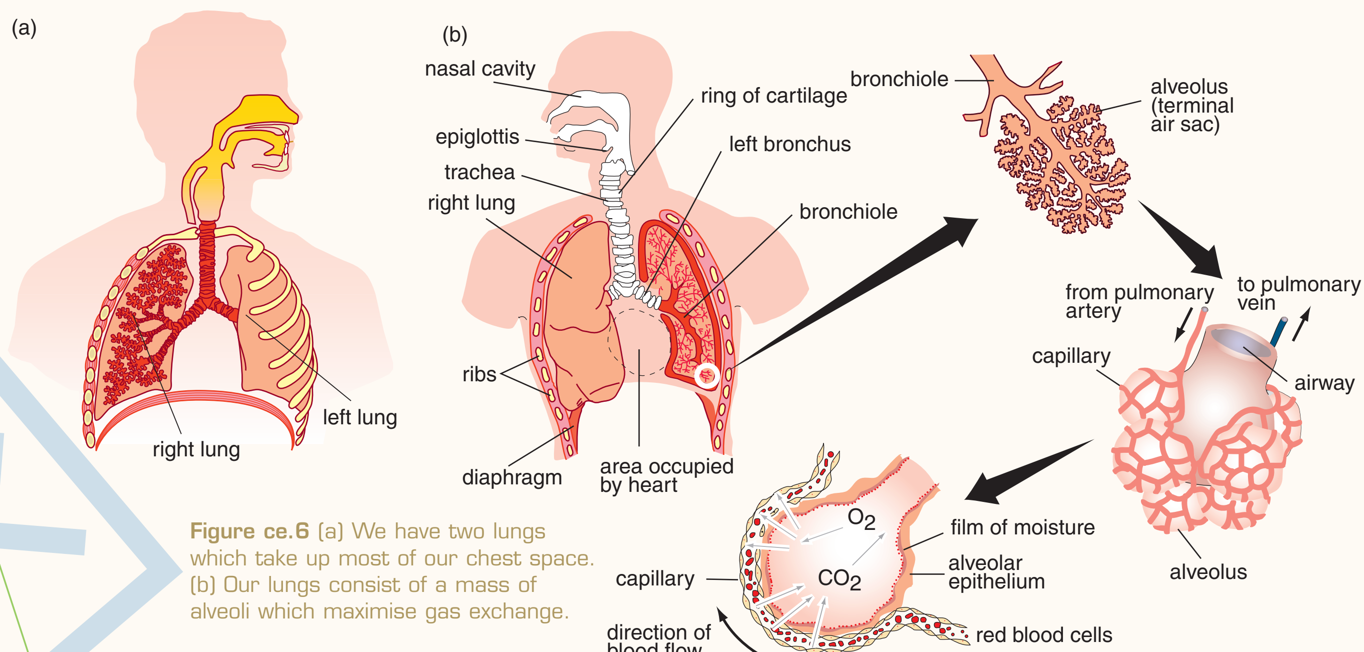


Figure ce.6 (a) We have two lungs which take up most of our chest space. (b) Our lungs consist of a mass of alveoli which maximise gas exchange.

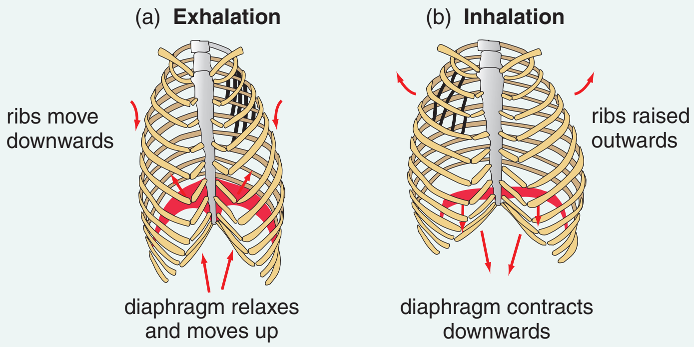


Figure ce.7 Air moves in and out of the lungs to maintain the balance between the pressure inside the lungs and the atmospheric pressure. (a) When we exhale, we make our chest cavities smaller so air must move out of the lungs to keep the pressures balanced. (b) When inhaling, we make our chest cavities larger, so air must move into the lungs to keep the pressures balanced.

Activity

Create a model of your lungs

Materials

- large glass jar with bottom removed; e.g. bell jar, 2 L or 4 L wine flagon
- rubber glove
- large rubber band
- wide sticky tape
- glass (or Perspex) tubing: straight piece, T-piece, 2 elbows
- 2 rubber balloons
- large rubber stopper with a hole in the centre

Method

1. Connect the glass tubing to form a model of the trachea dividing into two bronchi, as shown in Figure ce.8.
2. Attach a balloon to the bottom of each elbow, and place the tubing and balloons inside the glass jar.
3. With the rubber stopper outside of the glass jar, push the top of the glass tubing through the stopper.
4. Cut the rubber glove to form a flat rubber bottom for the glass jar (preferably leaving a thumb for a handle for the bottom), and secure firmly with the rubber band and tape.
5. With the rubber stopper loose, push the rubber bottom approximately 2 cm into the glass jar and then tighten the seal of the stopper and the jar. (Two straight pieces of glass tubing will work instead of the elbows and T-piece as long as there is an extra hole in the rubber stopper.)
6. Alternate between pulling down the rubber bottom, which simulates a diaphragm, and pushing it up.

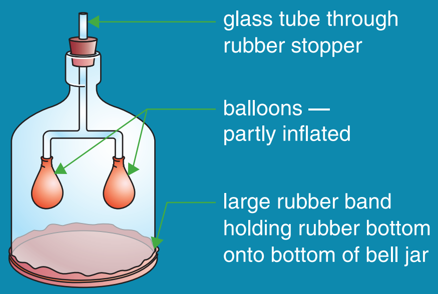


Figure ce.8 A glass jar can be set up to model the way that the changing volume of your chest cavity moves air into and out of the lungs.

Discussion

The balloons, which simulate the lungs, will inflate and deflate, in much the same way as our lungs inflate when our chest cavity expands and then deflate when we relax. Pay close attention to the tubing sticking out of the top of the rubber stopper. Can you tell that air is being sucked into the balloons and then pushed out of them?

begins to increase, and some of the air from our lungs moves out to the lower pressure air in the atmosphere. Sucking and blowing are due to the way that gases move to try to balance areas of different pressure.

Using a snorkel

When swimming around looking at the reef from the surface of the water, a snorkel allows us to breathe without losing our magnificent view. Even scuba divers should have a snorkel: to enable them to rest and swim at the surface without unnecessarily using the air in their tanks, and to make it easier to swim to the boat or shore if they run out of air.

Figure ce.9 A snorkel makes it possible to breathe while swimming face down in the water.

Quite simply, a snorkel is a tube that reaches from the mouth to the air at the back of the head. But not any tube will do. Try breathing through a drinking straw for a while, and you'll find that the width of the tube is important. It's easier to move a hundred people across a sports oval than through a narrow hallway, and the same applies to the particles in the air that we're trying to breathe. A snorkel's diameter (or bore) needs to be large enough to allow air to move through it relatively easily, and should be matched to our individual lung capacities: generally around 2 cm.

It might seem like a good idea to make the snorkel as long as possible, so that it's possible to dive well below the surface and still breathe, but there is a limit to the useful length of the barrel (the upper portion). The minimum length is such that the snorkel reaches the back of the head, but if the water is choppy then the snorkel needs to be long enough to reach above the tops of the waves.

The reason for the maximum length isn't quite as obvious: if the barrel is too long, then it not only can be difficult to clear the snorkel of any water that gets into it, but it also means that exhaled air gets rebreathed. Only enough air moves out of the snorkel to allow the gas pressures of the lungs and the atmosphere to balance; we not only can't push more air out of our lungs/snorkel system than our lungs contain, but we usually do not empty our lungs, only reduce the volume of air in them. If the volume contained within the combined length of the snorkel and trachea is larger than the difference between our lung volume when exhaling and inhaling (i.e. the amount of air that we breathe in or out), then our exhaled air can't escape and simply moves up and down the snorkel.

Activity

A drinking straw is like a snorkel

Try using a drinking straw to suck up only a portion of a drink. You can control the suction so that the liquid is always within the straw but moves up and down, trapping the air above it. As you move the level of the liquid up and down, the same air is sucked back up every time. This is the same principle as the air in a snorkel that is too long.

With the straw of course you can simply blow more air, and have the old air leave the bottom as bubbles. With a snorkel, we don't have any more air left to blow. The air that remains trapped in the snorkel is the only air that we are able to breathe.

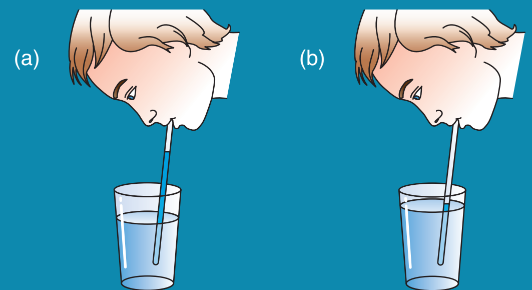


Figure ce.10 The air cannot escape from the straw and is rebreathed each time.

Think: How could you use the same air movement in the straw and still blow bubbles? What could you do to the straw?

The ideal length of a snorkel is therefore a balance between reaching up as far as possible, and allowing the exhaled air to escape into the wider atmosphere so that the snorkeller gets fresh air each breath. This ideal length is about 40 cm.

The bend in a snorkel should be smooth rather than sharp. This is important for easy breathing, as a bend that is too sharp will interfere with the fluid flow of the air and make it harder to breathe. The snorkel also fits closely to the contours of the head, thereby minimising drag due to water resistance.

Experimental investigations

Investigate the efficiency of different-shaped snorkels.

Scuba

Physics in action — Jacques Cousteau

As Jacques Yves was born on 11 June 1910 in the French town of Saint-Andre-de-Cubzac, Daniel and Elizabeth Cousteau could never have dreamed how influential their son would become. There may have been a few clues in his childhood, including frequent travel because of his father's work, time spent swimming and sailing, the construction of models and mechanical toys and a keen interest in films, but it was a change to a stricter high-school environment that allowed him to thrive academically.

Jacques entered the French Naval Academy in 1930. He graduated as a gunnery officer and then began training to be a pilot. But his flying career was cut short when the headlights on his car failed and his car careered off a mountain road, leaving him seriously injured. Swimming was part of his recovery and in 1936 Jacques took to the underwater world and his fascination with the ocean was truly born. His attempts to have more freedom when diving by carrying an air supply under water eventually led to the development of the aqualung in 1942. Scuba diving had been created, and the oceans were suddenly open for exploration!

Figure ce.11 Jacques Cousteau.

Jacques Cousteau did not stop there, though. He also pioneered underwater photography, produced more than 115 films and 50 books, served as the director of the Oceanographic Museum of Monaco for 30 years, was showered with awards and prizes, and created The Cousteau Society with the slogan 'To know, to love, to protect'. He and the society have been responsible for the creation of a number of submersible vehicles, as well as floating and underwater laboratories. Built in 1963, Conshelf II (later followed by Conshelf III) was basically a small village constructed 10 m down on the floor of the Red Sea, where five oceanauts could live for a month. It was the precursor for many of the experiments later used to train astronauts.

Jacques was an avid protector of the environment, drawing attention to the effects of pollution, speaking out against the damage done by spearfishing, launching a worldwide campaign to successfully protect Antarctica from mineral exploration, and denouncing nuclear energy as a threat to all life on the planet (including public opposition to France's tests on Mururoa Atoll). He died on 25 June 1997.

To truly explore the depths of the reef we need to be able to take an air supply with us. Unless some kind of underwater vehicle like a submarine is handy, this would most likely take the form of a tank of pressurised air strapped to the back as part of the now common **s**elf-**c**ontained **u**nderwater **b**reathing **a**pparatus (scuba).

Figure ce.12 Scuba gear allows a diver to breathe under water.

Note that 1 bar = 10^5 Pa.

Scuba tanks commonly contain 8 L, 10 L, 12 L or 15 L of air. A 12 L tank is the same size as a bottle that contains 12 L of water.

The pressure within a scuba tank is typically between 170 and 200 bars, although it can be more than 320 bars. Although there are slight differences, 1 bar is the approximate pressure of the air around us. Remember the way that the higher pressure air of the atmosphere is forced inside your lungs as the pressure begins to decrease when you inhale, even though the pressure difference is relatively small. Imagine what would happen if 15 L of 200-bar air were able to equalise with the approximately 1-bar air contained in your nowhere-near-as-strong-as-a-scuba-tank, somewhere-near-5 L lungs!

Figure ce.13 A regulator reduces the pressure of the air in the scuba tank so that it becomes breathable.

Clearly the pressurised air of a scuba tank cannot be allowed an open connection to the diver's respiratory system, so a device called a *regulator* (or *demand valve*) is used. The first stage of a regulator attaches to the scuba tank valve and reduces the pressure of the air to less than 10 bars above the surrounding water pressure. The second stage of a regulator has a mouthpiece, and reduces the first stage's intermediate pressure to that of the surrounding water, which is equivalent to the pressure within a diver's lungs at that depth.

As shown in Figure ce.14, the second stage of the regulator makes use of a lever-operated inlet valve to control the flow of air from the first stage. The lever rests against the diaphragm, and blocks the inlet valve between breaths. When we inhale, the lower air pressure pulls the diaphragm forward, causing the lever to pivot and open the inlet valve and thereby allowing the air to move into the mouthpiece. When we exhale, the higher pressure air pushes the diaphragm back and closes the inlet valve, and forces the used air out of a one-way exhaust valve.

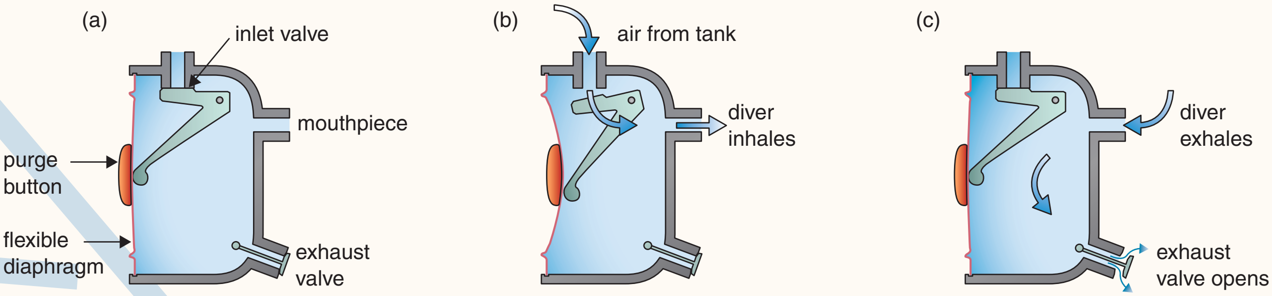


Figure ce.14 The functioning of the second stage of a regulator: (a) at rest, (b) during inhalation and (c) during exhalation.

Clearing masks and snorkels

Unlike a pair of goggles, a diving mask is a single air space that also covers the nose. If water gets into the mask, an increase in air pressure is required to clear it. While holding the top firmly against the forehead, the diver looks up slightly and exhales slowly through the nose; increasing the amount of air in the mask increases the pressure, thereby forcing the water out through the open bottom. Some masks even have a purge valve at the bottom to make this job a little easier. In this case, the mask would then be held against the face while the diver looks down and exhales slowly through the nose as before, and the water is forced out the valve.

To clear water from a snorkel may initially appear to be as simple as shooting water through a straw, nothing more complicated than a sudden blast of air that takes the water out with it. But what if there is still water left behind? Just as you can breathe out and slowly push bubbles through water, thanks to the J-shape of a snorkel, you can breathe in carefully and fresh air will bubble past the small amount of water in the bend of the snorkel. Usually a second blast will clear the snorkel of water.

Figure ce.15 Exhale slowly through the nose to clear a diving mask.

If you become nauseated while scuba diving, you can't make a sudden dash for the surface. Although it certainly isn't appealing, it is possible to vomit under water and keep diving. The key is to not remove the regulator. If you take the regulator out of your mouth in order to vomit directly into the water, when you automatically breathe in after vomiting, you'll get a lungful of sea water! It is preferable to hold the regulator firmly in your mouth and force the vomit into it and out of the exhaust valve.

If the chunks get caught in the exhaust valve, the result will be a steady flow of air from the first stage of the regulator, through the second stage, and directly out of either the exhaust valve or the mouthpiece. You will still be able to breathe, although you will use up the air in your tank more quickly.

Exercises

E6 If the lung capacity of a small person is 4.0 L with a residual volume of 1.2 L, calculate the maximum length for a snorkel of diameter 2.5 cm. (Assume the snorkel is a cylinder, although they tend to be more of an oval than a circle.)

E7 A young man has a lung capacity of 7.8 L. The air in his lungs between breaths is balanced with the air around him at 1 bar. Determine the pressure of the air in his lungs if the air was unable to escape as he decreases the volume of his lungs to 2.4 L in an attempt to exhale from a deep breath.

E8 A 12 L scuba tank contains air at a pressure of 180 bars. To what volume would this air expand if it was released:
a at sea level?
b at a depth of 10 m under water?

E9 A scuba diver is breathing under water, where the pressure around her is 2.10 bars. As she inhales 3.20 L of air from her regulator, this air leaves her 8.0 L scuba tank, thereby reducing the amount of air in the tank. As the required volume of air leaves the tank, the air that remains spreads evenly to fill in the space, and so has a reduced pressure. If the pressure of the air in the tank was 172 bars before this breath, what will be the pressure within the tank afterwards?

Staying warm

Although the temperature of sea water varies throughout the world, ranging from about −2°C in polar regions to nearly 40°C in the tropics, the seasonal variation in a specific region is usually no greater than around 10°C. This means that while the requirements for keeping warm vary depending on where you choose to dive, once you're familiar with a given location there tends to be very little change.

Temperature and depth

In both salt and fresh water the temperature decreases as the depth increases, and distinct layers form. *Thermoclines* are regions of rapid temperature decline with depth and form abrupt boundaries, when the weather is calm, with the temperature difference between the layers able to reach around 10°C. Thermoclines can occur at quite shallow depths and tend to rise and fall with the seasons; it is even possible to see a distortion of the light travelling from one temperature layer to another.

The ocean can be divided into three zones based on temperature layers. Most scuba divers stay in the surface (mixed) layer, which is the most easily influenced by weather conditions and so is the most variable. Only the surface layer has any real seasonal variation. The next layer is the main thermocline, and is where there is the greatest change in temperature with depth. In the final deep water layer, the temperature change isn't as great from the top to the bottom, but the temperature is always less than 10°C, falling to just above the freezing point of sea water. Note that these layers are related to recognised density zones, but do not necessarily match because salinity also affects density.

The changing temperature with depth combines with the changing pressure to determine the volume of the gases in the lungs, masks and buoyancy compensation devices (BCDs). During a deep dive that is subject to temperature changes, buoyancy is determined by a combination of pressure and temperature, as well as the effects of adding and releasing quantities of air to the BCD.

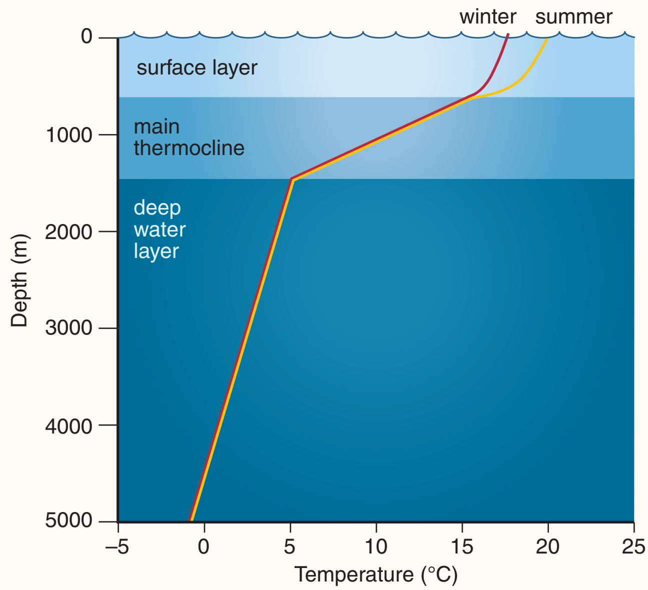


Figure ce.16 Temperature decreases with depth, but not at a constant rate. Water forms distinct layers based on temperature differences.

Extended response

1. By referring to thermoclines and ocean zones, investigate the influence of the El Niño current on weather patterns.
2. Explain what happens to the sea water and ice at the Amery Ice Shelf, and why this knowledge is important to our understanding of global weather patterns.
3. Investigate and/or build an example of Galileo's thermometer.

Surface effects

While considering the potential cold of the water, it is easy for us to forget about the Sun and the wind. Sunburn is caused by the effects of high-energy *electromagnetic radiation* on the skin. Ultraviolet light is a higher energy wave than visible light and so, while clouds block the visible light and cast the world in shadow, the ultraviolet light is generally able to pass through the clouds unseen. And the same applies to shallow water! Because we're shaded and cool when in the water, we won't necessarily feel the burn from ultraviolet radiation. Sunlight can also keep us warm or lead to overheating due to a form of heat-energy transfer known as *radiation*.

Being on the surface means that our wet bodies can be cooled by the wind. Wind chill occurs when the air moves around the body, and this movement allows for effective *evaporation* of the moisture from the skin. As the water molecules on the skin absorb energy from our bodies and evaporate, we lose heat and are cooled. The wind ensures that these molecules are continuously swept away, so that the next molecule can also evaporate quickly.

Insulating suits

For water temperatures between 10°C and 30°C, it's recommended we use a *wetsuit* (with long or short sleeves and legs). The closed-cell neoprene foam used to make wetsuits contains bubbles that don't connect like those in a sponge, and so don't soak up water or allow water to flow through them. The air trapped in these bubbles provides buoyancy as well as effective *insulation*, so that our bodies don't transfer heat to the water around us. It should be noted, however, that both their buoyancy and their effectiveness as an insulator decrease as the depth of the dive increases and the gas in these bubbles is compressed, reducing their volume.

Figure ce.17 A wetsuit is important for keeping warm while under water.

Figure ce.18 A drysuit is a better option than a wetsuit when the water temperature drops below 18°C.

Wetsuits have the added benefit that water enters through the openings at the wrists, ankles and neck, and this water then gets trapped between our skin and the suits. This water quickly heats via conduction of body heat, and because the neoprene suit is a good insulator, the water provides a persistent layer of warmth. The wetsuit must be a snug fit though, because if the water between the skin and the suit is able to circulate, then, either through the effects of motion or through *convection*, heat will be lost unnecessarily.

Rather than being a snug fit like a wetsuit, a *drysuit* is relatively loose and sealed at all openings. This traps a layer of air between the skin and the suit. Air is a better insulator than the insulating material of the suit alone and it has a conductivity that's one twenty-fourth that of water! To counter the problem of a reduced volume due to the greater pressure as a diver descends, air can be added to a drysuit from the air tank. Similarly, air can be released through an exhaust valve during the ascent.

Exercises

E10 Explain why thermal radiation causes us to warm up while sitting in the sun, but cool down while sitting in the shade.

E11 Assuming no other effects, how much heat would a person lose if a 1 mm layer of water covering the exposed 1.6 m^2 surface area of body were to evaporate?

E12 Explain why it's important for a wetsuit to be a snug fit.

E13 The thermal conductivity of water is 0.58 J s^{-1} m^{-1} °C^{-1} and that of air is 0.024 J s^{-1} m^{-1} °C^{-1}.

- a At what rate will your body lose heat through 0.8 mm of water, if the temperature of the surrounding environment is 15° lower than your body temperature?
- b At what rate will your body lose heat through 2.0 mm of air, if the temperature of the surrounding environment is 15° lower than your body temperature?

electricity at home

By the end of this context
you will have covered material including:

- distribution of household electricity from the power station
- 240 V AC mains electricity
- household wiring for mains electricity
- safety devices such as RCDs and fuses
- parameters for injury due to electrical shock.

A modern house without electricity is virtually unthinkable. Electrical wiring is always installed as the house is built and electrical appliances are often built in as well. Building codes specify the type of wiring, the number of fuses and much more to ensure our safety. However, although electricity is a very safe form of energy when used wisely, it does have the potential to kill.

Electricity is supplied to our homes as an alternating current (AC) that varies between +340 and −340 V. The figure of 240 V is the voltage of a DC (direct current) supply that would supply the same average power to a load such as a light globe or heater element. However, there are many things that a 240 V DC supply cannot do. Transformers, for example, will not work on DC, so any device that relies on one could not be used on DC. Many electric motors are designed to be used on AC and would not work at all on a DC supply; in fact, they would be damaged. Any circuit containing coils, such as those in transformers and motors, will behave very differently on AC and DC voltages.

Electrical distribution

This context assumes that you have studied electricity in Chapter 7. Before going any further, make sure you review this area of study.

Electrical energy is efficiently distributed over long distances from the generator. Most of the electrical energy used in Western Australia is generated at the Collie and Muja power stations, which are both located near Collie, approximately 200 km south of Perth. The electrical power is distributed at high voltages along the wires from Collie to Perth, where it is transformed down to lower voltages. Transformers are devices that change AC voltages to higher or lower voltages. Figure el.1 illustrates a typical power grid.

Transformers found on street power poles reduce the voltage to the 240 V that we are familiar with. What would be the peak voltage delivered to our homes, and would the frequency be different from that at the generator?

Note that Figure el.1 demonstrates that the secondary coil of one transformer, the primary coil of the next transformer and the transmission lines between them form a loop of wires. In fact, this loop is not complete. One end of the coil in each transformer is earthed (literally stuck in the ground) and the free electrons in the ground complete the loop. The electrons within this loop are forced to oscillate backwards and forwards. When the final transformer has dropped the voltage to 240 V, the active end of the secondary loop of this transformer enters the home along with a neutral wire that is earthed at the pole where the last transformer is located (see Figure el.2).

Once in the home there are usually a few circuits created for specific needs. The most common are lights, stove, hot-water system and usually at least two power circuits that supply energy to the power points. Figure el.3 illustrates a typical wiring system for a home. The most important point to note is that every circuit is in parallel so that every appliance is provided with a potential difference of 240 V across it. This, however, highlights the need for fuses or trip-switches which prevent wires from overheating.

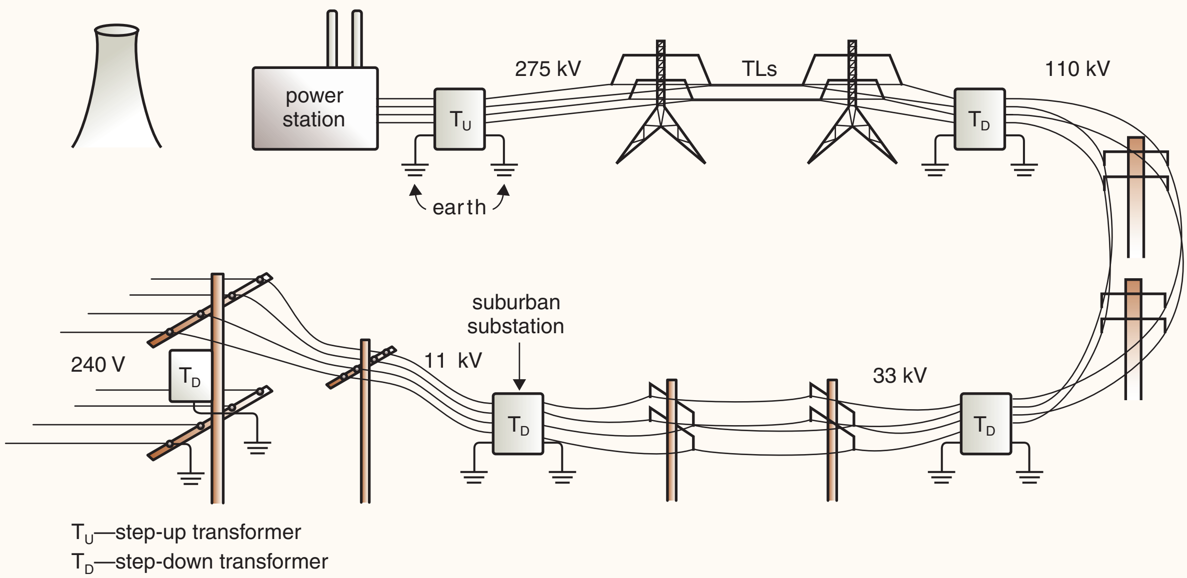


Figure el.1 A power grid. Electricity passes along transmission lines that run from the power stations, and through step-up and step-down transformers, before reaching our homes.

Extended response

1. Research the significance of AC voltages to the operation of transformers and, hence, to our power distribution system.
2. Figure el.2 shows there are four wires carried by the poles. One of these is the neutral wire. The others are active and are related to the fact that generators produce three phase power. Investigate what ‘three phase’ refers to with respect to power supply.
3. What is the purpose of the neutral wire that leaves your home?

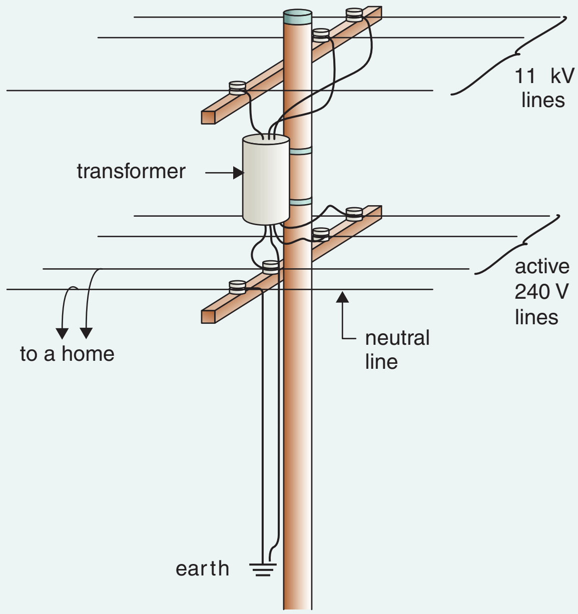


Figure el.2 Typical street power pole. Note that not all power poles will have the 11 kV lines at the top, but all will have the three active and one neutral wires. Only one of the active wires will go to a particular house along with a branch of the neutral wire.

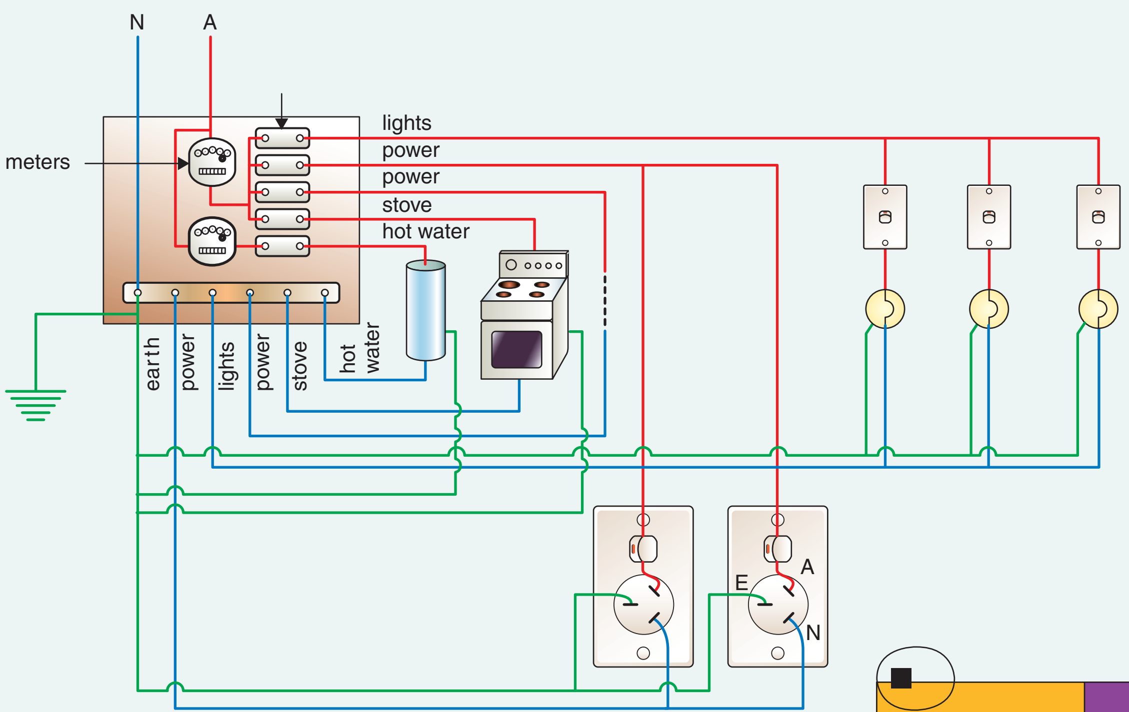


Figure el.3 A standard household wiring diagram. The active and neutral wires enter the house at the roof (or underground in some places) and go to the fuse box. The active wire first passes through the meters and then to the various appliances. All of the earth wires are eventually connected to a steel post that is stuck deep into the ground next to the house.

Figure el.3 is only for illustrative purposes. Only a qualified electrician should deal with electrical wiring.

House wiring

There are normally two cables carrying the electricity from the power lines in the street to our houses. One of these is the so-called 'active', the other is the 'neutral'. The potential of the active varies between +340 V and −340 V at a frequency of 50 times per second (50 Hz). The neutral will be very close to zero potential. It is electrically connected to the ground at every house and at every street transformer.

At the meter box the active wire goes to the meter and the main switch and then to the circuit breakers (or fuses). The neutral is connected to a brass strip, called the neutral bar, which you can sometimes see in the meter box. The cables running through the walls carry an active wire, a neutral wire and an earth wire. The active comes from the fuse, while both neutral and earth come from the neutral bar. Each fuse will supply a group of either lights or power points (but not both).

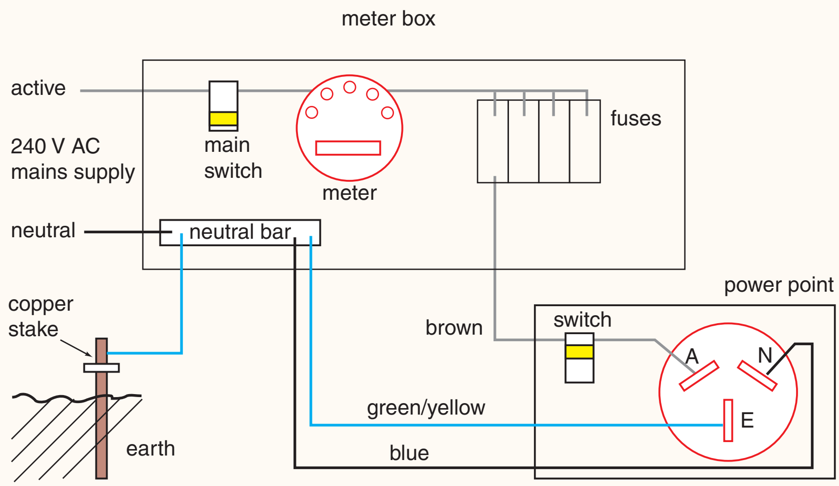


Figure el.4 The mains power from the street poles comes into the house via the meter box and then is distributed to power points and lights through cables carrying three conductors.

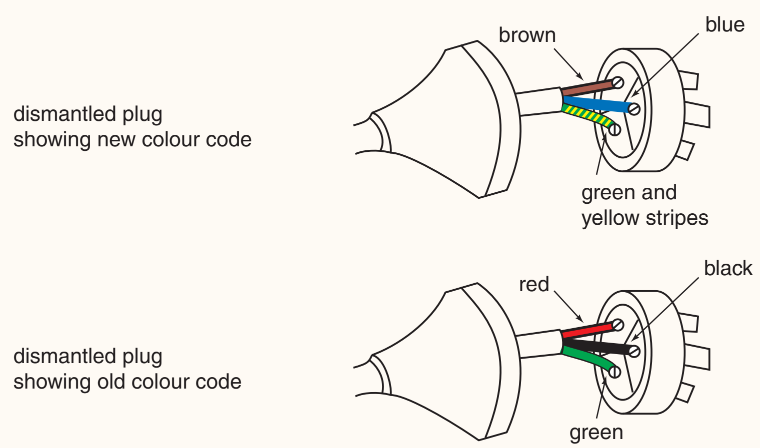


Figure el.5 This diagram shows correctly wired plugs using both the new and old colour codes. The new code is an international standard which should be used all around the world.

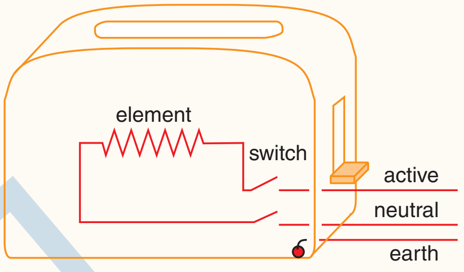


Figure el.6 The element of this toaster is connected between the active and neutral conductors. For safety the switch breaks both active and neutral conductors. The metal case is permanently connected to the earth wire.

At the power point the active wire first goes to the switch and then to the upper left terminal. This is so that when the switch is off there is no active voltage in the terminal sockets. The neutral and earth are connected to the upper right and lower terminals respectively. The plug on an appliance cord must be wired so that the correct conductors are connected to their appropriate terminal sockets. Figure el.5 shows the new and old colour codes used to identify the three conductors. The new code does not apply to the fixed house wiring.

The wiring of a typical appliance, a toaster, is shown in Figure el.6. The element is connected between the active and neutral conductors. The earth wire is connected to the metal case. Ideally the switch should break both active and neutral conductors as near as possible to where they enter the appliance. Often, however, the switch will only break the active wire, or worse, only the neutral wire. If the switch only breaks the neutral conductor it means that the element of the appliance remains 'live' even if it is not switched on. This is one reason why an appliance should always be completely unplugged before any work (including cleaning) is done on it. Even if the appliance was designed to have the switch in the active, the cable could have been wired incorrectly and the active and neutral conductors interchanged.

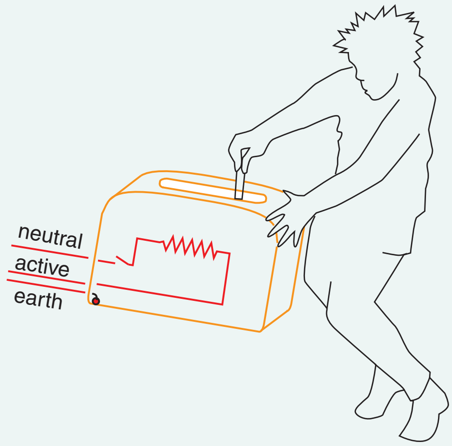


Figure el.7 A very dangerous condition! The toaster has been wired with the switch in the neutral instead of the active. It will work perfectly, but the element is still live when the switch is off. The person will receive a very bad shock as he is contacting the active with one hand and the earthed case with the other.

Activity

Have you noticed that some lights in buildings are controlled by two switches? The light can be turned on or off from either switch. The switches are 'two-way' switches.

Using two two-way switches, one power pack, one globe and electrical leads, design a circuit that will allow the light to be operated in the same way as a light with two switches would work at home.

Extended response

Use some simple switches, light globes, an ammeter, electrical leads, a power source etc. to develop a demonstration to explain the purpose of a fuse to a group of Year 8 students.

Electrical safety

There are a number of basic safety features built into our mains electric supply system.

1. The fuse In the event of a short circuit, for example as the result of a broken active conductor coming into contact with a neutral or earth, a heavy current will flow through all conductors, switches and plugs. This can easily cause them to become hot and even burn. To prevent this the fuse is designed to be the thinnest piece of conductor in the system. If the current becomes too heavy the fuse will melt and disconnect the rest of the circuit from the active. It is important to realise that a fuse will not save a person from electrocution if they touch a live wire. Household fuses will blow only when more than 8 A (light circuits) or 16 A (power-point circuits) flows. A current of even 0.1 A flowing through a person can be fatal.

2. Switches While power-point switches and those on appliances should always cut the active conductor, as we have seen this is not always the case. Although a switch in the neutral will turn the element off, it will leave it connected to the active wire. This can be dangerous if the element is touched by someone believing the appliance to be safe because it is off. For this reason, never touch any inner part of an electrical appliance unless it is unplugged completely. Don't try to remove burnt toast until the toaster plug is out of the power point!

3. The earth wire The separate earth wire, which parallels the neutral wire, is purely to provide protection against a fault occurring in the appliance. If, for example, the active wire breaks and contacts the case, the whole case would become active—with dire consequences for anyone who touched it. Provided the earth wire is connected to the case, however, this condition will cause a short circuit which will blow the fuse and disconnect the circuit.

As well as these basic safety features which are an inherent part of the household wiring system there are a number of others which have been introduced over the last decades.

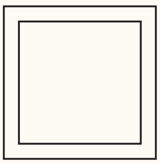


Figure el.8 This symbol is used to identify a 'double-insulated' appliance.

4. Double insulation Many appliances are now designed so that there are two effective barriers between the active wire and a person using the appliance. If, as well as the active wire being insulated and protected inside the appliance, the case is made of plastic, there is very little chance of the user touching an active part. This is actually safer than using an earthed case as it is still possible for the case to become live if damage to the cable happens to break the earth wire and it then comes into contact with the active wire. Double-insulated appliances have no earth wire and are characterised by their two-pin plugs and special symbol.

5. Earth leakage system Also known by the less enlightening term 'residual current device' (or RCD). In a properly operating system the current in the active and the neutral conductors should be exactly equal (but in opposite directions). An RCD is designed to detect any current lost from the active–neutral circuit. The most likely reason for such a loss is that it is going to earth through a fault or a person! The RCD uses the magnetic effects of an electric current to detect any difference between the active and neutral currents. Basically, the magnetic effects of two equal currents flowing in opposite directions will cancel each other. If, however, the currents are not equal a magnetic effect will occur. The RCD uses this effect to switch off the supply within around 20 milliseconds. Undoubtedly, the installation of these devices in households, schools and factories has saved many lives. In modern houses, regulations require that these be used instead of fuses.

Extended response

1. Compare and contrast the safety purposes behind fuses and RCDs. Consider both electrocution and fire hazard in your answer.
2. Some countries do not use an earth wire and their electrical sockets only have two holes. What do they do instead of having an earth wire?

Electric shock

Because our bodies are controlled by electrical impulses through the nerves, any current from an external source that flows in the body may interfere with our vital functions. In particular, any current flowing from one arm to the other may cause the chest muscles to contract and stop breathing. Current through the heart region can cause *ventricular fibrillation*, which means that the muscles become uncoordinated and the heart function stops. Depending on the actual path of the current through the body, even a brief current over about 80 mA may cause fibrillation. This is the main cause of electrical fatalities.

Despite all the safety features of modern electrical systems and appliances, every year around 50 Australians are killed in electrical accidents. Roughly half of these are the result of industrial accidents and the others are domestic or commercial. Many could have been prevented either by the use of residual current devices or by other simple precautions such as keeping equipment well maintained.

The unavoidable fact is that the amount of current that will flow through the body if it is in good contact with 240 V is well above the level that will cause death. That some people do survive an electric shock is because either it was very brief, the contact was not good or the current flowed through 'non-essential' parts of the body. The amount of current that will flow depends on the total resistance between the active wire and the neutral or earth. A person who touches a live wire while standing on carpet has considerably more resistance than someone standing barefoot on wet concrete or grass. Hot sweaty hands or skin will conduct better than cold dry skin.

One square centimetre of skin will have a resistance that can vary from around 100 kW if it is dry, to less than 1 kW if it is wet. As a rough guide, 1.5 kW can be taken as the resistance from one hand to the other of a normal perspiring worker. At 240 V this means that a current of 240 V/1.5 kW = 160 mA will flow across the chest and heart region of the body. As you can see from Table el.2 this would have very serious consequences. A multimeter with a resistance scale can be used to get an idea of the resistance of skin and the body. Indeed, so-called lie detectors are often no more than a resistance meter connected to terminals held by the subject. The theory is that a person who is lying will sweat more and conduct a (harmless) current better.

The time for which the current flows is crucial. The shock from a Van de Graaff machine charged to 100 000 V is harmless because it only lasts for around a microsecond. On the other hand the shock from a voltage source of only 100 V can be fatal if the contact is good and it lasts for a few seconds. Table el.3 shows the effect of a 50 mA current for various times.

Another potentially fatal result of the interference with nerve function of an electric current is that involuntary contraction of muscles may make it impossible to let go of the object, causing the shock. This can occur at currents as small as 10 mA, which means that situations which otherwise might be harmless may become very dangerous. If there is any suspicion at all about an electrical appliance, never touch it with an open hand; use the back of the hand so that any contraction will pull the hand away. Also keep one hand well away from any possible earth, so as to avoid providing a good path to earth through your arms and chest. Normally shoes will provide some resistance and so a shock from one hand to earth through the feet may not be quite as dangerous. It would be particularly dangerous, for example, to hold the earthed case of a toaster with one hand while trying to extract burnt toast with a metal knife held in the other hand! Always unplug the toaster before doing anything like that.

table el.2 The likely effect of a half-second electric shock. The actual current that flows will depend on the voltage and skin resistance

Current (mA)	Effect on the body
1	Able to be felt
3	Easily felt
10	Painful
20	Muscles paralysed—cannot let go
50	Severe shock
90	Breathing upset
150	Breathing very difficult
200	Death likely
500	Serious burning, breathing stops, death inevitable

table el.3 The likely effect on the human body of a 50 mA shock for various times

Time of 50 mA current	Likely effect
Less than 0.2 s	Noticeable but usually not dangerous
0.2–4 s	Significant shock, possibly dangerous
Over 4 s	Severe shock, possible death

Never try to repair any active part of an electrical appliance or install household wiring. Not only is it illegal, it could be potentially fatal. Many deaths from accidental electrocution have occurred as the result of faulty wiring. Any suspect electrical device should be unplugged and taken to a qualified electrician.

Preventing electric shocks

With wise use, electrical equipment can be perfectly safe. The reasons for the following precautions will be obvious when one understands the nature of electricity and the body's response to it.

- Never use electrical appliances when barefoot, particularly when outdoors.
- Be extremely careful with any electrical appliance anywhere near water. Never use a hair dryer while wet or near a bath.
- If there is any reason to suspect an appliance, touch it only with the back of the hand and keep the other hand well away from it.
- At the first sign of any shock, no matter how small, have the appliance checked by a qualified electrician.
- Never tamper with electrical equipment. Keep it in good order and have any damaged cords or elements repaired by a qualified person.

If the worst happens

In the event that you find a person who has suffered an electric shock:

- Look for the reason for the shock and pull out any plugs or turn off any switches.
- If that can't be done try to push the person away from the source by using an insulating object: a wooden pole or thick wad of dry clothing for example.
- Do not touch the person until you are sure that the source of voltage is removed. Check with the back of one hand before making good contact.
- If power lines have fallen, keep well away from them. If the lines have fallen on a car, do not touch the car and tell the occupants to stay inside it until you can be sure it is not live. The occupants will be safe as long as they stay right inside. They will be at a high potential, but there will be no potential difference across them unless they touch something at earth potential.
- Check to see if the victim is conscious by talking loudly or gently shaking them. If so, reassure them and treat any burns with cold water. If not, place them in the coma position (see Figure el.9) and check for breathing and pulse.
- If either is missing, send for urgent medical aid and begin mouth-to-mouth resuscitation and/or cardiac massage. Ideally everyone should learn these basic first-aid techniques. St John Ambulance and the Red Cross run regular courses.

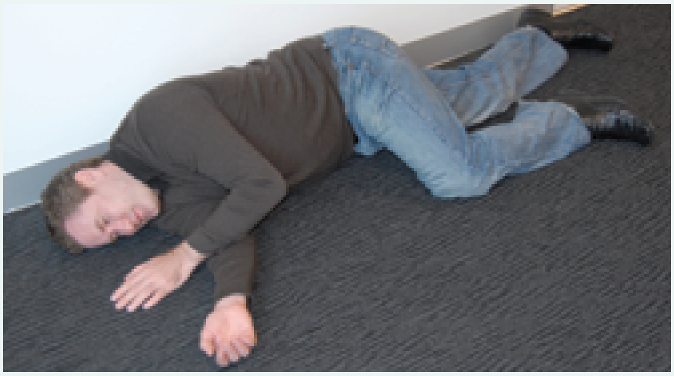


Figure el.9 The coma position. Check that the air passages are clear by opening the mouth and tilting the head back.

Exercises

E1 Australian houses are supplied with 240 V AC. The 240 V is the:
A maximum value of the alternating voltage
B average value of the alternating voltage
C value of a DC voltage that would supply the same power
D none of these.

E2 Why is it that there are only two cables coming into the house from the street and yet power points always have three connections?

E3 The function of a fuse is to burn out and, thus, turn off the current if the circuit is overloaded. Why is it always placed in the active wire at the meter box rather than the neutral one when this function could be fulfilled if it was in either?

E4 What is the function of the 'earth stake' that is normally found near a meter box?

E5 A toaster cable with conductors coloured red, black and green is to be joined to another cable with brown, blue and green/yellow conductors. Peter has joined the red and blue, black and brown, and green and green/yellow. Will the toaster work normally when it is plugged in and turned on? Why is the way he has connected the cables dangerous?

E6 An appliance was mistakenly wired between the active and earth instead of between the active and neutral. Explain why, although it will appear to work normally, this a very dangerous thing to do.

E7 What is the main advantage of a 'double-insulated' electrical appliance over a normal earthed one?

E8 Why is the shock received when a finger touches a live wire likely to be less severe than the shock received by a person who touches a live wire with a pair of uninsulated pliers?

E9 How much current would flow through a person with dry hands and a total contact resistance of 100 kW when they touch a 240 V live wire?

E10 Why is it normally more dangerous to use an electrical device outdoors? What precautions are particularly necessary outdoors?

6 Heating and cooling

Thermal energy is part of our everyday experience. Our bodies operate at a specific temperature and so we feel uncomfortable when the temperature of the surroundings is significantly different from this. We put effort into keeping warm during winter and cool during summer. We feel the radiant heat when sitting near a fire or when the sun shines. A hot water bottle helps us sleep more comfortably on a cold night. The effects of heat have been observed, recorded and studied for centuries, yet it has not been until recently that the true nature of heat has been understood. This chapter explores the way in which our understanding of heat has developed over the centuries. Heat involves the transfer of thermal energy. Our understanding of the particle nature of matter has enhanced our understanding of heat.

The human body is complex. It has systems and subsystems that can cope with an incredible variety of external conditions. Through homeostasis, the body will maintain a constant temperature—generating its own thermal energy to raise the temperature when cold and using control mechanisms to lose heat to the environment when overheated. Furthermore, human ingenuity has created technology that has further extended our control of thermal energy and temperature. Humans can thrive in the climatic extremes of the Earth, from the Saharan deserts to Arctic winters. The control of heat energy has been regulated to the point that humans can survive in space.

We consider the measurement of heat and the measurement of the change in temperature when matter gains or loses thermal energy. We also study what happens when matter changes state from a solid to a liquid, and from a liquid to a gas. Systems that allow us to control the temperature of our environment can be understood with the aid of the kinetic molecular theory. With knowledge of what happens at the particle level, we can understand, and therefore use and control, temperature and heat on the large scale. In this chapter, we study the mechanisms by which thermal energy can be moved around.

By the end of this chapter

you will have covered material about the particle nature of matter:

- early theories of heat
- kinetic molecular theory
- thermal behaviour of gases
- temperature scales
- specific heat capacity
- latent heat
- conduction
- convection
- radiation.

Physics file

Sir Francis Bacon (1561–1626) attended Cambridge University for two years, spent three years in France, and then, after the death of his father left him without financial support, went on to study law. His concern with the refusal of philosophers to consider logical development of theories, so common in his time, led him to write with an emphasis on the need for new methods of scientific investigation. His insistence that investigation should begin with observable facts, rather than theory, makes him an important contributor in the history of scientific thought. Ironically, Bacon died from the results of a chill contracted while stuffing a chicken carcass with snow, while attempting to investigate whether cold could slow decay. This was actually the only scientific experiment he is credited with doing.

Figure 6.1
Sir Francis Bacon.

Physics file

A common unit for heat energy was named from the caloric theory. The 'calorie' is the amount of energy required to raise the temperature of 1 gram of water by one degree Celsius. A more commonly used unit is the kilocalorie, or thousand calories, often expressed as 'Calorie' (with a capital C).

This unit is still used in the common, but no longer official, description of the amount of energy supplied by a quantity of food.

6.1 Heat: a historical perspective

It is easier to observe the effects of heat, or the absence of it, than it is to describe the nature of heat itself. The ancient Greek philosophers wrote of hot and cold as the cause of the evolution of the universe. They understood heat as causing expansion and vaporisation. For them, cold congealed and hardened the gases and liquids into planets. Like many other theories first proposed by ancient Greek philosophers, this view persisted for many centuries.

The history of an idea

It wasn't until the 16th century that a different view of the nature of heat was suggested. Sir Francis Bacon, an English statesman, essayist and philosopher, drew the conclusion from his reading of earlier philosophers that 'heat is motion'. He went on to write that heat was the rapid vibration of minute particles within every substance. At the time, molecules still had to be discovered, so Bacon's hypothesis did not meet with much support.

An opposing view to Bacon's was to persist well into the 19th century. The philosophers of the Middle Ages believed that heat was a 'fluid' that filled the spaces between the particles of a substance and 'flowed' from one substance to another. The model was persuasive—even today we talk of heat 'flowing' from one object to the next. This explanation came to be known as the 'caloric' theory.

Caloric theory

The caloric theory saw heat as a fluid-like substance called 'caloric', which filled all the gaps within a substance. Each object was assumed to contain a fixed amount of caloric. When an object was broken apart, the caloric was released and the object got hotter. If caloric flowed from one object to another, it caused the second object to heat up. Reducing the amount of caloric in a body resulted in its cooling down. Burning was believed to be the release of enormous amounts of caloric.

Although the theory fitted the observations of the day, caloric could not be detected in any experiment. Instead of changing the theory, scientists simply gave caloric more properties. They assumed that it had no mass, odour, taste or colour. The theory explained observations involving the 'flow' of heat from a hot body to a cold one. Other phenomena, such as the heat generated by friction, were much more difficult to explain.

Although the caloric theory has long been abandoned, we still have some reminders of it. Expressions such as the 'flow' of heat, as if heat were a fluid, are still commonly used.

Heat from friction

Benjamin Thompson (1753–1814; later known as Count Rumford) was born in America but sided with the British during the American War of Independence. After the war he became a supervisor in a Munich munitions factory where he became aware of a particular problem that occurred during the boring of cannon barrels. A considerable amount of heat was generated when the cutting tools bored into cannon barrels.

This continued even when the boring tools were dull and no metal was being cut (i.e. no caloric was being released)!

Thompson was not prepared to accept that caloric was responsible. He could observe no limit to the amount of heat that could be obtained from friction. If the caloric theory were correct, there would be a fixed amount of heat released from a piece of metal. To make more heat, Thompson only had to keep the motion between cutting tool and barrel going. He proposed that heat was produced by the motion 'taking place among the particles of the body', particles that were later to be identified as atoms and molecules.

Thompson's idea was pursued by others in the early 1800s, in particular an English brewer, James Prescott Joule (1818–1889). Joule found that when a falling weight caused a paddle wheel to turn, the resulting friction between the paddle wheel and the water would cause the temperature of the water to rise.

Physics file

James Prescott Joule was born in Lancashire, England, on 24 December 1818. He studied with the English chemist John Dalton at the University of Manchester in 1835 on the way to becoming a brewer. Always interested in physics, Joule published a number of papers describing his work on the heating effect of an electric current in a wire and on the mechanical equivalent of heat. In 1843 he published his value for the amount of work required to produce a unit of heat. In 1852, Joule and William Thompson (later known as Lord Kelvin) discovered that when a gas is allowed to expand, the temperature of the gas will fall. At a time when England was the largest importer of goods and foods in the world, this 'Joule–Thompson effect' became the basis upon which a large refrigeration industry was developed in the 19th century. The standard unit for all forms of energy, the joule, honours the significance of his work.

Heat as energy

By the mid 19th century, several scientists had begun to write of the heating process as an energy change from work (mechanical energy) to heat. It was eventually realised that all forms of energy were equivalent and that when a particular form of energy seemed to disappear, the process was always associated with the appearance of the same amount of energy in other forms. This led to the development of the principle of the conservation of energy. At this same time, Joule conducted a series of experiments fundamental to our present understanding of heat.

Figure 6.2
James Prescott Joule.

Joule noticed that stirring water could cause a rise in temperature. He designed a way of measuring the relationship between the energy used in stirring the water and the change in temperature. A metal paddle wheel was rotated by falling masses and this churned water around in an insulated can. The amount of work done was calculated by multiplying the weight of the falling masses by the distance they fell. The heat generated was calculated from the mass of the water and the temperature rise. Joule found that exactly the same quantity of heat was always produced by exactly the same amount of work. Heat was simply another form of energy, and 4.18 joules of work was equivalent to 1 calorie of heat.

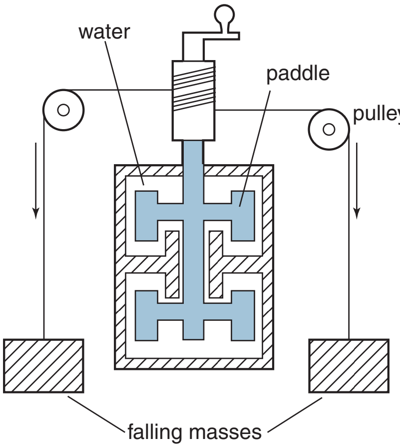


Figure 6.3
Joule's original apparatus for investigating the mechanical work equivalent of heat energy. The falling weights caused the paddle to turn. The friction between the wheel and the water created heat energy in the water. For the first time, heat energy could be measured and related to other forms of energy.

Joule's work led to some unusual conclusions for his day. He stated that as a container of cold water is stirred, the mechanical energy is being transformed into thermal energy, heating the water. Theoretically, this means that a cup of water stirred long enough and fast enough will boil—a novel, if laborious, way of making a cup of coffee. Of course, the rate at which we can normally add energy by stirring is less than the transfer of energy to the surrounding environment. For the cup of water to boil it would need to be very well insulated.

As a result of Joule's investigations and other experiments of the time, we now interpret the process of heating or cooling as a transfer of energy. When heat 'flows' from a hot object to a cold one, energy is being transferred from the hot to cold body. This idea is central to the kinetic theory of heat, which relies on the kinetic molecular theory.

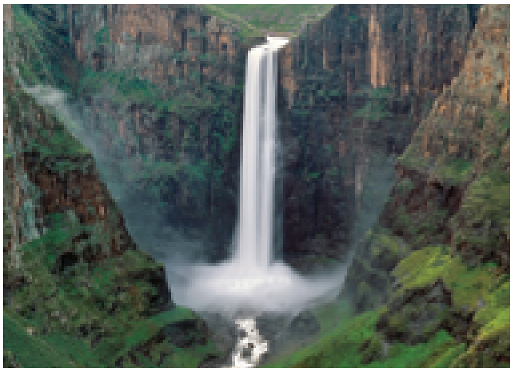


Figure 6.4

As a result of the considerable amount of mechanical work being done on the water of a waterfall, the water at the bottom of the falls is usually 1°C or 2°C higher in temperature than at the top.

Physics file

The convention for naming units in physics is to use small letters when writing the unit in full (e.g. joule, newton, metre) and a capital letter for the symbol only when the unit is named in recognition of a scientist's contributions (e.g. J for joule, N for newton, but m for metre).

Physics in action — Energy and power

Energy is a very important concept in the study of the physical world, and is a focus in all areas of scientific study.

Energy is a measure of an object's ability to do work. Raising an object's temperature, as Joule found, or lifting another object is referred to as doing work. Work is measured in joules (symbol J).

Kinetic energy is the energy of movement. It is equal to the amount of work needed to bring an object from rest to its present speed or return it to rest. Potential energy is stored energy. It is the amount of energy an object possesses which would, if used, allow it to reach its present speed. There are many forms of potential energy—gravitational, chemical and nuclear to name three.

Work can be done whenever energy is transformed from one form to another. Work is usually defined as the product of force and displacement, i.e. $W = F \times s$. The work done is a measure of the energy transformed and so is measured in joules.

The total amount of energy remains the same.

The rate at which energy is transformed from one form to another is the power that has been developed, measured in watts (W).

$$\text{Power} = \frac{\text{energy transformed}}{\text{time taken}}$$

$$\text{or} \quad P = E/t$$

This means that 1 watt of power is required to transform 1 joule of energy each second to another form, say mechanical energy to heat energy.

A 100 W light globe will use 100 J of energy every second. The meter on an electrical switchboard reads the total consumption in kilowatt hours (kW h), rather than joules, and is the unit used by your electricity supplier in calculating the cost of electricity consumed. One kilowatt hour is the energy used in one hour by an electrical appliance using energy at the rate of 1000 J per second (1 kW).

$$\begin{aligned} 1 \text{ kW h} &= 60 \times 60 \times 1000 \text{ J} \\ &= 3\,600\,000 \text{ J} \\ &= 3.6 \times 10^6 \text{ J} \\ &= 3.6 \text{ MJ} \end{aligned}$$

In other words, to calculate the total energy used in megajoules from the number of kilowatt hours shown on the electricity bill, simply multiply by 3.6.

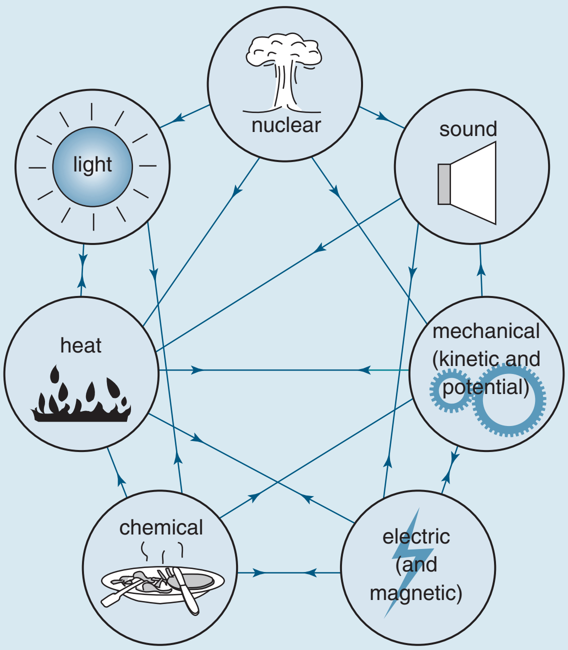


Figure 6.5

When work is done, energy is transformed from one form to another. The connecting lines indicate the many different ways that energy can be converted. Regardless of the number or type of conversions, the total energy remains the same.

Other forms of energy supplied to households, such as gas, are charged in terms of equivalent heating values. Gas is charged per megajoule of heating. The heating values of some typical domestic fuels are included in Table 6.1. These values represent the quantity of chemical potential energy released when the fuel is burnt.

Western Power electricity accounts show a complete usage calculation on the reverse of the account. The following is a typical household usage calculation for a Perth home for two months in 2002.

Usage Calculations

Tariff	*Reading Type*	*Meter Number*	*Current Meter Reading*	*Equals Total Units Used*
A1 Domestic Supply	Normal	15W2725	80993	2187

Current Account Details

A1 Domestic Supply Tariff

2187 units @ 12.67 cents per unit	\$277.09	
Supply Charge	\$13.33	
Domestic Supply TARIFF GST @ 10%	\$29.08	
Total	\$319.88	\$319.88
	Total Payable	**\$319.85**

Table 6.1 Approximate heating value of some domestic fuels

Fuel		Heating value
Gas	39 MJ m^{-3}	
Wood	17 MJ kg^{-1}	depending on type and moisture content
Briquettes	22 MJ kg^{-1}	depending on moisture content
Coal	25 MJ kg^{-1}	depending on moisture content
Heating oil	45 MJ kg^{-1}	

Australia is one of the largest per capita users of energy in the world. A typical household may use 7 500 000 000 joules (7500 MJ or 7.5 GJ) each month. This is more than 30 times that used by the average household in a developing nation like India. It has been estimated that if we conserved our energy more carefully, the whole world population of 8.5 billion (estimated for 2010) could enjoy a different, but perhaps more satisfying, lifestyle, and only consume the same energy as at present.

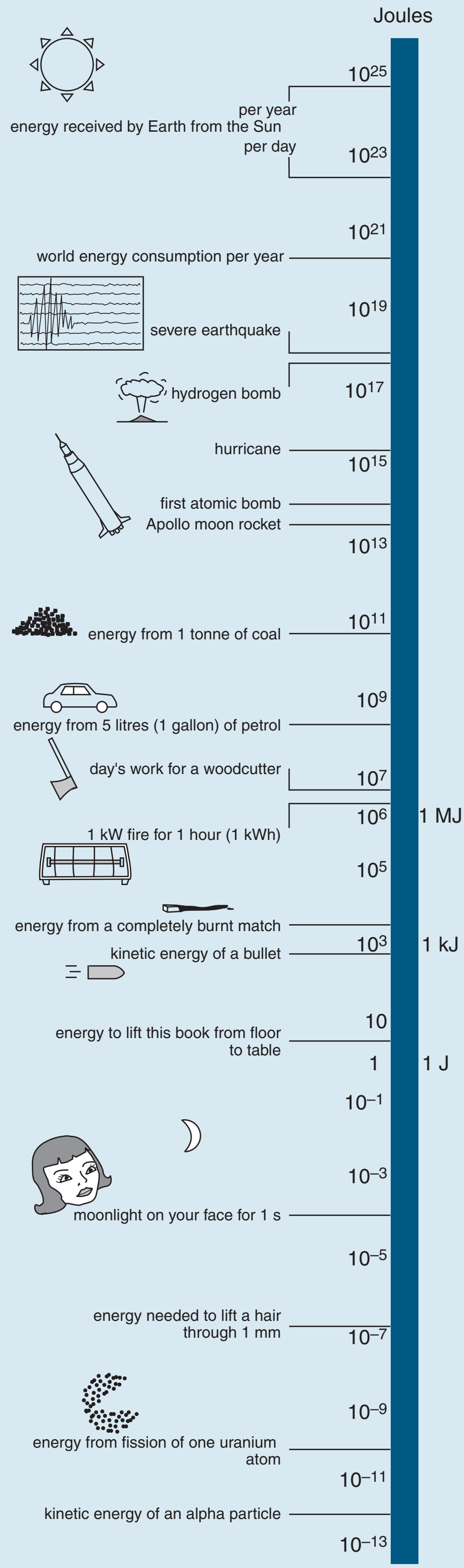


Figure 6.6

One joule is a very small amount of energy. It takes 4200 J to raise the temperature of 1 kg of water by 1°C. This diagram illustrates the comparative amounts of energy available from several common energy sources. Particularly notable is the energy received from the Sun. Imagine the environmental advantages that could be achieved from harnessing this energy!

✓ Worked Example 6.1A

The energy transformed from chemical potential energy to heat and light by completely burning one match is approximately 2000 J.

a If this takes 5 seconds, how much power does the match develop?

b If the match were held upright so that it took twice as long to burn, what effect would this have on the power produced?

Solution

a $E = 2000$ J, $t = 5$ s

$P = E/t$

so $P = 2000/5$

and $P = 400$ W

b Now $t = 10$ s, and doubling the time will mean halving the power developed.

$P = 2000/10$

$P = 200$ W

The flame will not be as large.

6.1 SUMMARY Heat: a historical perspective

- Early theories of heat were based upon fitting observations to theories. The caloric theory proposed that heat was a colourless, tasteless fluid.
- Joule suggested that heat was a form of energy. The transfer of mechanical energy to a material causes an increase in temperature, speed, height or other physical state. The unit for all forms of energy is the joule (J).
- Work is done whenever energy is transformed from one form to another. It is defined as the product of force and displacement ($W = F \times s$). During any energy transformation, the total amount of energy remains the same.
- The rate at which energy is transformed from one form to another is called power (P) and is measured in watts (W), i.e. $P = E/t$.

6.1 Questions

1. Explain the feeling of your hand cooling as you touch a cold surface in terms of:
   a the caloric theory
   b Joule's theories.
2. Describe the nature of heat as most scientists believed it to be prior to the work of Thompson. What did Thompson believe were the major flaws in these early ideas?
3. On what observations did Thompson base his understanding of heat? What conclusions did he draw? Why?
4. Place your hand in a basin of hot water. Take your hand out and hold another person's hand. Does their hand feel hot or cold? Explain your observations in terms of a transfer of energy.
5. If you rub your hands together, what happens to the energy you use to do the work of overcoming the friction between them? Explain.
6. In 1799 Humphrey Davy tried rubbing two blocks of ice together in a sub-zero temperature. The faces of the two blocks in contact with each other began to melt. How can this happen? Explain.
7. Using the energy forms shown in Figure 6.5, draw a flowchart to demonstrate the energy changes that take place when:
   a heating water over a wood fire
   b heating water in an electric kettle.
8. Running up a flight of stairs requires the same amount of energy as walking. Explain why it is then that you feel so much hotter and more tired after running up.
9. A typical incandescent light bulb is rated at 60 W. How many joules of electrical energy is being transformed to heat and light energy each second?
10. A billy of water needs about 350 kJ of energy to reach boiling point. If it takes 10 minutes to boil the billy over an open fire, how much total power is the fire developing? Assume in this case that half the total energy from the fire reaches the billy.

6.2 Kinetic theory

Much of our understanding of the behaviour of matter now depends on what we now call the 'kinetic molecular theory'. One of the first people to suggest that the behaviour of matter was due to randomly moving particles was the British botanist, Robert Brown (1773–1858). He noticed that pollen particles suspended in water could be observed, under a microscope, to be moving in a random fashion. He suggested that this might be due to collisions between the pollen grains and the molecules of the water.

The kinetic theory applies to solids, liquids and gases regardless of the phase, or state, of the matter. The assumptions of the kinetic theory are:

- All matter is made up of very small particles (atoms or molecules).
- There are many particles.
- The particles are in constant motion.
- Collisions between the particles are perfectly elastic in that no kinetic energy is lost or gained during a collision.
- There are forces of attraction and repulsion between the particles in a material.
- The forces between particles in a gas are insignificant.
- The distance between particles in a gas are large compared with the size of the particles.

Within a solid, the particles must clearly exert attractive forces or bonds on each other for the matter to hold together in a fixed shape. There are also repulsive forces without which the attractive forces would cause the solid to collapse. In a solid, the attractive and repulsive forces hold these particles in more or less fixed positions, usually in some sort of regular arrangement or lattice. The particles in a solid are in motion within the lattice—they vibrate around fixed average positions, the forces on individual particles being sometimes predominantly attractive and sometimes repulsive.

In a liquid, the forces between particles are weaker than in a solid. The particles have more freedom to move around and generally the liquid takes up a greater volume. Particles collide but remain attracted to each other; hence, the liquid remains within a fixed volume but with no fixed shape.

In a gas, particles are in constant random motion, colliding with each other and the container walls. There is almost no force of attraction between the particles; hence, there is no fixed shape or volume. Instead, the particles move rapidly in every direction, quickly filling any container and occasionally colliding with each other. The speeds are usually high enough that, on collision, the attraction is not strong enough to keep them close together and repulsion causes the particles to move off in new directions. In air, oxygen molecules are travelling at an average speed of 500 m s^{-1} and experience approximately 10 million collisions each second.

When a solid substance is 'heated', the particles within the material gain both kinetic energy (and move faster) and potential energy, as they move away from their equilibrium positions. However, they will continue to be held in place, due to relatively strong interparticle forces. In order for the substance to change state, from solid to liquid, it must receive enough energy to separate the particles from each other. While this is happening, work is done to overcome the interparticle forces, but the speed of the particles does not change. The energy increases the potential energy of the particles.

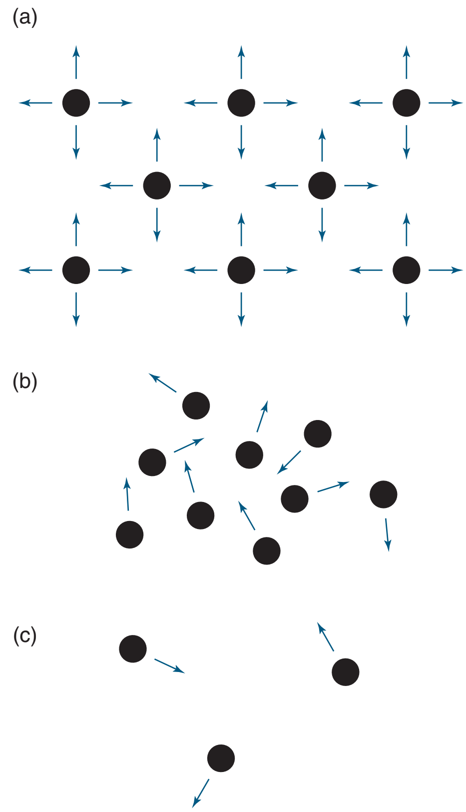


Figure 6.7
(a) The particles within a solid are in constant motion about fixed positions; hence, a solid remains in a relatively fixed shape. (b) In a liquid, particles have greater amounts of kinetic energy and move more freely past one another. Bonds between the particles still keep a liquid to a fixed volume but the shape can change. (c) In a gas, the particles are moving still quicker and are more widely separated again. Forces are no longer sufficient to keep a gas to a fixed volume or shape. The molecules in a gas are free to move independently of each other.

Physics file

A hundred years ago, we were not so sure that atoms existed. Albert Einstein's ideas about Brownian motion helped convince the world of the existence of atoms. In two papers published in 1905, Einstein showed how the movement of huge numbers of tiny molecules could add up to significant effects. This understanding is useful in areas as diverse as the construction industry, the dairy industry and environmental science. William Sutherland, an Australian, had the same ideas and published them shortly before Einstein. The botanist Brown was the botanist on Matthew Flinders' circumnavigation of Australia in 1801.

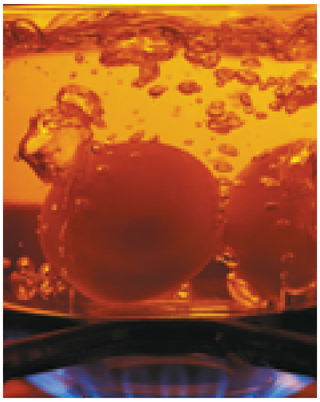


Figure 6.8

When water is heated over a stove, the burning gas particles transfer energy to the pot as they collide with it. The water gains energy too by the same process. The temperature of the water increases as the average kinetic energy of the particles in the water increases.

The kinetic theory is consistent with the idea of heat as a transfer of energy. We can illustrate the theory by examining the heating of a pot of water on a stove. The particles in the burner of the stove have a considerably higher kinetic energy, on average, than those in the cold water or pot. As the temperature of the flame increases, so does the speed of the gas particles in the flame. The particles from the burning gas collide with those in the metal of the pot, transferring some of their energy to the metal. Thus, the particles in the pot gain kinetic and potential energy. The particles in the pot transfer some of their energy, by collision, to the particles around them and thus to the water. Hence, the temperature of the pot, and consequently the water, increases.

The word 'heat' is properly used to refer to energy only as it is being transferred from one object to another during a temperature change or during a change of state. The term 'internal energy' is used to refer to the kinetic and potential energy of the molecules within a substance. An increase in internal energy will result in a gain in temperature only if there is a net gain in energy.

6.2 SUMMARY Kinetic theory

- Kinetic theory proposes that all matter is made of atoms or molecules (particles) which are in constant motion.
- In solids, the particles vibrate about fixed points. Bonds in liquids and gases are progressively weaker, as the particles within them gain energy from surrounding particles at a higher temperature.
- Heat energy is considered to be the energy transferred to a material that results in an increase in the kinetic and potential energy of the particles within the material.

6.2 Questions

1 The forces between molecules within a material are primarily:
A electrical
B gravitational
C chemical
D mechanical.

2 According to the kinetic molecular theory, which of the following statements is/are true?
A A substance heats up as it gains caloric.
B All solids, liquids and gases are composed of molecules in constant motion.
C Most gases cannot be compressed due to the small distances between molecules.
D The molecules of a solid all have the same kinetic energy.

3 The term 'internal energy' is sometimes used to describe the total energy of particles within a solid, liquid or gas (kinetic and potential). When a substance is heated the internal energy will:
A stay the same
B increase
C decrease
D all of the above.

4 A solid keeps its shape at room temperature while the particles within it vibrate about fixed points. Draw a diagram of a model which illustrates how the particles are arranged within a solid. Extend your model to illustrate a rise in temperature.

5 Very large amounts of energy are needed to change a liquid into a gas. What happens to the energy during this process?

6 The temperature of a container of water is found to be the same at all places. Are all the molecules moving at the same speed? Explain.

For each of the following questions, state whether the statements are true or false. Briefly explain your choice.

7 A hot drink does not stay hot on a cold day because heat transfers to the environment.

8 A refrigerator cools things by adding more cold to them.

9 Adding hot water to a bath of cold water will increase the temperature of the water by raising the average energy of the molecules within the cold water.

10 There is no such thing as cooling, only heating.

6.3 Heat and temperature

'Temperature is measured with thermometers.' Ask anybody to respond to that statement and they will wonder why you have stated the obvious. The use of a thermometer is well known and everyone knows what temperature is—or do they?

If heat is added to an object, that object may change temperature, it may expand or it may undergo a change of state. It is also possible that all these events will occur (although not at the same time). As energy is transferred to the material, the average kinetic energy of the particles increases and we record this as a rise in temperature. Imagine just a thousand particles in a cubic metre of space. (This is almost a perfect vacuum!) If those particles were moving at around 300 km s^{-1}, the total energy of the space would only be about 10^{-12} joule. However, because of the speed of the particles, the temperature of this very 'thin' gas would be over a hundred million degrees!

Temperature and heat—there is a difference

The kinetic molecular theory establishes a clear distinction between the measurement of temperature, heat and the internal energy of an object.

The measurement of temperature is related to the measurement of the average kinetic energy of the particles. Heating refers to the actual transfer of heat energy from an object at a higher temperature to one initially at a lower temperature. The transfer is to the object of lower temperature and, hence, lower average kinetic energy. It does not depend on the total energy and is never from cold to hot unless energy is supplied from other external sources.

Arbitrary scales

Only four centuries ago there were no thermometers and everyone described heating effects by terms such as hot, cold and lukewarm. Temperature was determined by touch or by observing effects such as boiling water or melting lead. It had been noted that many things expand when they get warmer but the principle had never been used to measure temperature and there were no numerical scales.

Galileo was one of the first to make a thermometer. His 'thermoscope' was not particularly accurate as it did not take into account changes in air pressure, but it did suggest some basic principles for determining a suitable scale of measurement.

Besides the tube of air, a scale was required so that a number could be used to denote the level of the water in the tube and hence indicate the temperature. Initially there was a multitude of scales. Each person wanting to measure temperature would put two marks on their thermometer and divide the space between into a number of parts. Galileo suggested that there be two fixed points—the hottest day of summer and the coldest day of winter. The space in between was to be divided into 360 parts in a similar way to a circle. Newton used a scale with only twelve divisions.

Such scales are referred to as arbitrary scales. The fixed points were arbitrarily based on limited observations and could not be duplicated or observed by others. Imagine trying to compare temperatures in Australia with a scale based on the hottest and coldest days in Europe, in some long

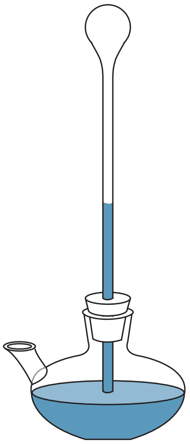


Figure 6.9
The glass bulb of Galileo's thermometer is warmed in a person's hands and is then turned upside down with the tube being dipped in water. As the bulb returns to air temperature, water rises in the tube. The amount it rises is related to the temperature of the surrounding air.

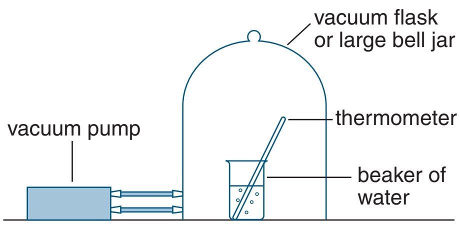


Figure 6.10

A simple demonstration can show the effect of air pressure on the boiling point of water. A beaker of cool water placed in a vacuum flask will boil as the air is removed. A thermometer, left standing in the beaker, will confirm that the temperature hasn't changed.

gone year! For any scale to have universal value, there must be agreement on the number of divisions and the fixed points which can be reproduced.

Fahrenheit and Celsius scales

A better known scale of past years, and one still used in the USA, was developed by the German physicist Gabriel Fahrenheit. He selected two reference points for the scale. His zero was the lowest temperature that he could obtain by mixing salt and ice. His upper reference point was the temperature of a healthy human body; he called this 96°. The Fahrenheit scale followed by dividing the region between these two temperatures equally. From these two points we get the boiling point of water, at 212°F, and the melting point of ice, at 32°F. The Fahrenheit scale is still arbitrary. It was initially based upon the difficult-to-repeat observations of one person.

The final scale was based upon boiling and melting points, which can vary with changes in pressure. On the slopes of the Himalayas, water will boil at temperatures as low as 150°F (approximately 60°C). Later, a new scale was proposed that took into account the effect of changes in pressure on the boiling and freezing of water. The Swiss Anders Celsius (1701–1774) was the first person to publicly propose a scale of 100 divisions. The Celsius scale uses the melting point of pure water at 0°C and the boiling point of pure water at 100°C at standard atmospheric pressure.

A later refinement of the Celsius scale took account of the effects of pressure. A temperature exists where the combination of temperature and air pressure allows all three states of water to coexist. This is the *triple point*. For water, the triple point is only slightly above the normal freezing point (0.01°C) and provides a unique and repeatable temperature with which to adjust the Celsius scale.

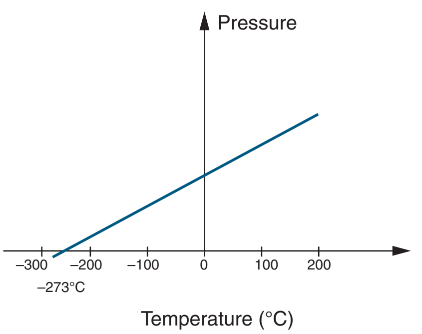


Figure 6.11

Graph showing the change in pressure for a constant volume of gas as temperature changes. At absolute zero the pressure exerted by the gas, and hence the volume, is zero. While absolute zero can not be achieved in practice, theory suggests that the pressure of the gas, and hence the motion of the molecules within it, would become zero at –273.15°C.

The absolute temperature scale

For a scale to be regarded as truly 'absolute', it should not have negative values. Its fixed points must be reproducible and it should have zero as its lowest value. The triple point of water provides one reliable temperature. The zero temperature must then be known to set a bottom limit to the scale. This scale is known as the absolute or kelvin temperature scale.

Experiments indicate that there is a limit to how cold things can get. The kinetic theory suggests that if a given volume of gas is heated or cooled, the resulting change in pressure can be plotted against temperature. This results in a straight line graph.

Assuming that the gas does not change to a liquid or a solid, extrapolation of the graph gives a point where the gas will exert zero pressure and hence have zero volume. The molecules will actually stop moving. This intersection is the theoretical lowest temperature attainable. It is called absolute zero and has a value usually accepted as –273°C (actually –273.15°C). This is the zero point for the absolute temperature scale. It is called the kelvin scale after Lord Kelvin, an English physicist who worked with Joule and proposed a scale based on absolute zero in the middle of the 19th century.

The kelvin scale is related to the Celsius scale. Absolute zero becomes zero for the kelvin scale and the triple point of water is 273.16 kelvin. The word 'degree' and the degree symbol are not used with the kelvin scale. A kelvin (symbol K) is defined as the temperature interval corresponding to $\frac{1}{273.16}$ of the interval between the triple point of water and absolute zero, thus making the interval the same size as one degree on the Celsius scale (i.e. 1 kelvin = 1°C). Kelvin is the standard SI unit for temperature.

Absolute temperature in kelvin = temperature in degrees Celsius + 273.

Conversions between the kelvin and the Celsius scales are made throughout this text by assuming that absolute zero is –273°C and the triple point lies at 0°C.

✓ Worked Example 6.3A

Convert 20°C to its equivalent temperature in kelvin.

Solution

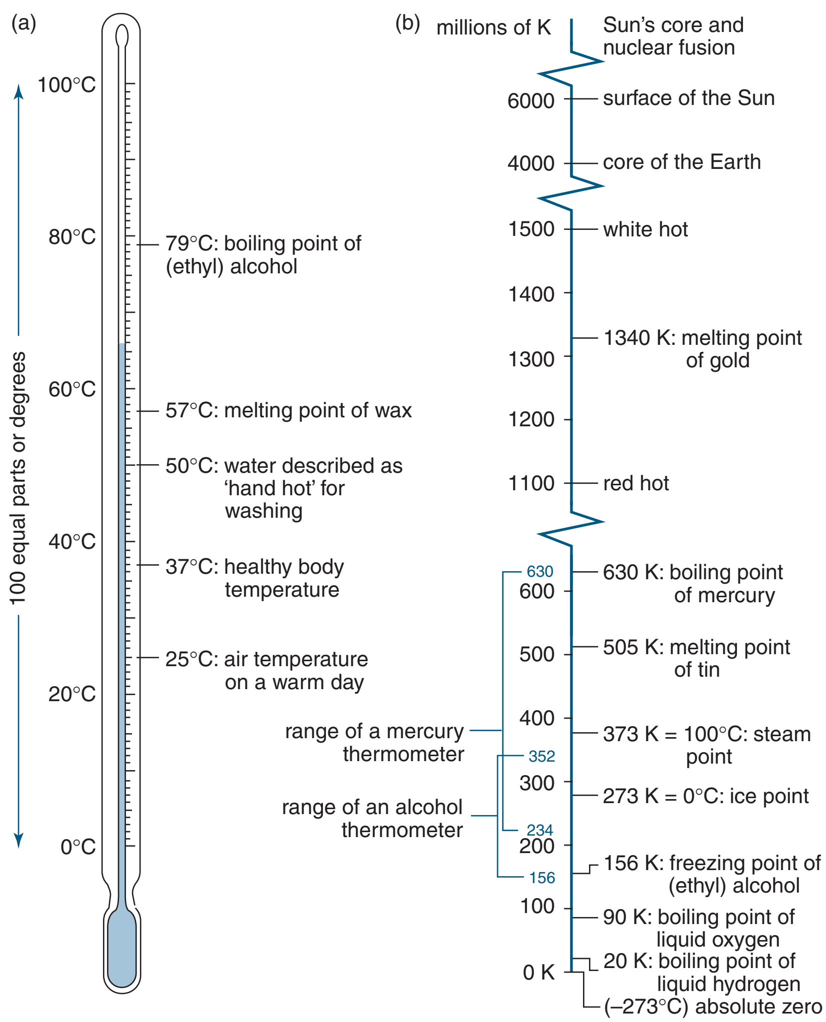
Absolute temperature = temperature in degrees Celsius + 273.
So, temperature in kelvin = 20 + 273 = 293 K.

✓ Worked Example 6.3B

Convert 323 K to its Celsius equivalent.

Solution

Temperature in degrees Celsius = temperature in kelvin – 273.
So, temperature in degrees Celsius = 323 – 273 = 50°C.



Physics file

At absolute zero, even some atoms start to behave in weird ways! Since the French physicist Guillaume Amontons first proposed the idea of an absolute lowest temperature in 1699, physicists have theorised about the effects of such a temperature and how it could be achieved. The laws of physics dictate that absolute zero itself can be approached but not reached. To do so would take an infinite amount of energy. In July 1995, researchers at the Joint Institute of Laboratory Physics in Colorado, USA, succeeded in cooling rubidium atoms to within a few billionths of a degree of absolute zero. At this temperature all elementary particles merge into a single state, losing their separate properties and behaving as a single 'super atom', a state first proposed by Einstein 70 years before. The Colorado researchers succeeded in producing these temperatures for up to 15 minutes at a time inside a magnetic 'bottle' and, in so doing, not only created the coldest, stillest place in the universe but also created a new form of matter!

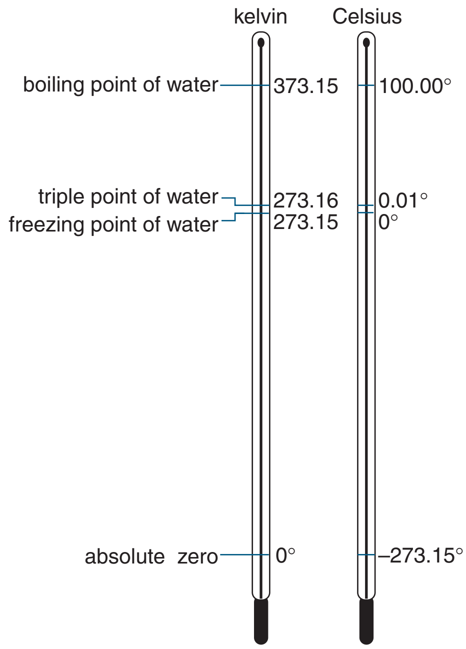


Figure 6.12
Comparison of the kelvin and Celsius scales. The size of each unit, 1°C or 1 K, is the same.

Figure 6.13
Comparative temperatures for some common physical features or phenomena. (a) Everyday temperatures on the Celsius scale. (b) Temperatures in nature.

Measuring temperature—thermal expansion

Galileo's original thermometer was based on the expansion of air as it was heated. An increase in the volume of gases, liquids and solids as they get hotter is still the principle used in most modern thermometers whether they be a simple alcohol thermometer or a more sophisticated digital model.

When a solid is heated, the particles within gain kinetic energy, vibrate more rapidly and occupy more space. Neighbouring particles are pushed further away and so increase in potential energy. A similar situation occurs when a liquid or uncontained gas is heated. The increase in volume of a liquid is much greater than that of solids and the increase for an uncontained gas is greater again.

Common thermometers make use of the expansion of a liquid, such as mercury or coloured ethyl alcohol, in a glass tube. The air is removed and the tube sealed so that changes in air pressure have no effect on the air in the tube. Mercury solidifies at the comparatively high temperature of –39°C but doesn't boil until 357°C. It makes a good laboratory thermometer as it will allow measurement of temperature well beyond the boiling point of water. Ethyl alcohol boils at 78°C at atmospheric pressures but doesn't become solid until –117°C, and is relatively harmless, making it ideal for clinical thermometers and temperatures below 0°C.

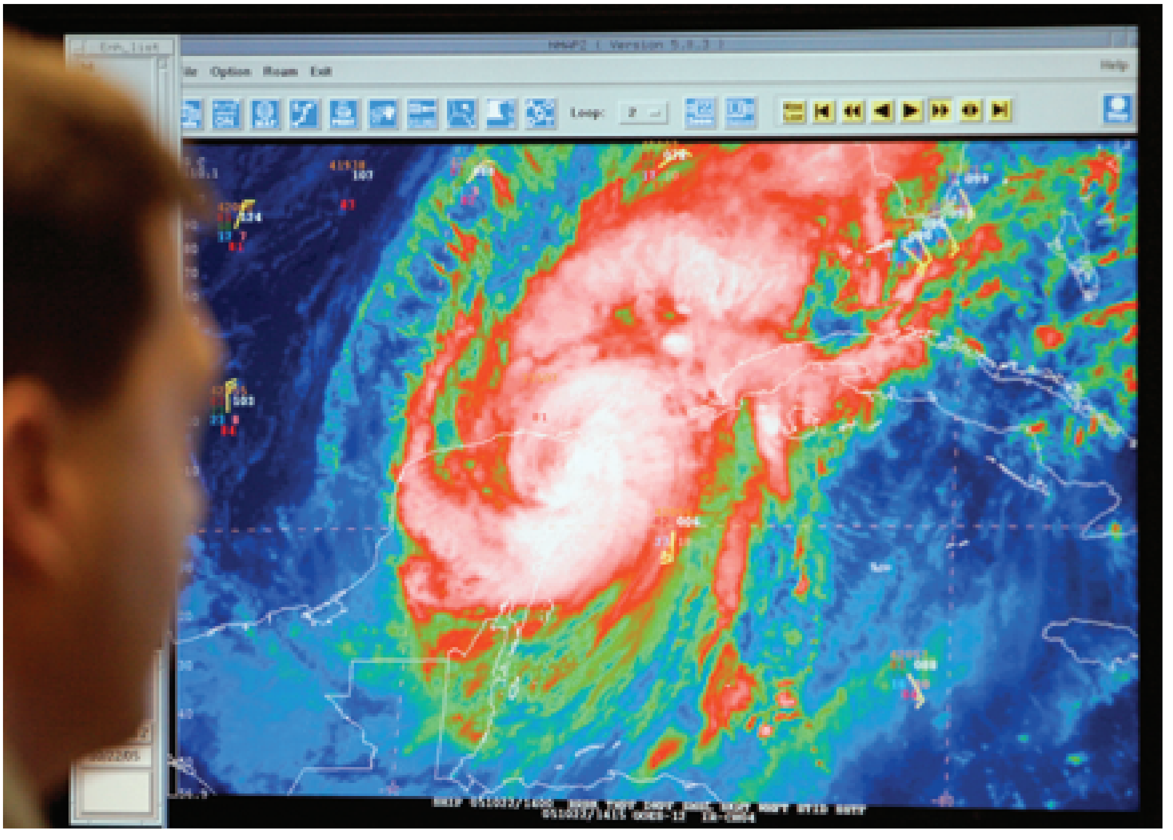


Figure 6.14

Remote-sensing techniques can be used to measure and compare temperatures. This satellite image is of a tropical cyclone over northern Australia. Infrared energy emitted by the Earth has been recorded and colour-enhanced in order to highlight cooler regions within the cyclone corresponding to thicker clouds and more severe cyclonic activity.

Older styles of thermostat, used to keep rooms and other open spaces at constant temperature, also work on the principle of expansion. Most thermostats employ a flexible strip made of two different metals. The metals, usually brass or copper and iron, are selected so that they expand or contract at different rates when heated or cooled. The metals are riveted or welded. When the temperature changes, this bimetallic strip changes curvature as the brass on the outside expands more than the iron on the inside. As it bends, it may open an electric circuit, turning off the heating system. When the required temperature is achieved, the bimetallic strip will have straightened enough to close the contacts and reactivate the heating system. While cheap, such systems are not reliable over time as the metal strip is subjected to repeated stress with each temperature change. Thermostats based on bimetallic strips are being replaced by electronic circuits that rely instead on semiconductors such as thermistors. These are based on a change of resistance with changing temperature.

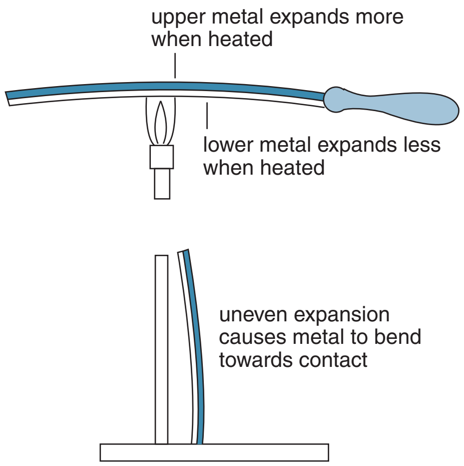


Figure 6.15
Thermostats and bimetallic strips. Prior to the introduction of semiconducting thermistors, thermostats worked on the expansion material. Two metals expanding at different rates caused a bimetallic strip to bend, making contact with an electrical or mechanical switch.

Thermal expansion of water—a special case

There is an exception to the general rule that materials shrink as they cool. Water, one of the most common compounds on Earth, has its smallest volume, and its maximum density, at about 4°C. When cooled below this temperature it expands. When it freezes, it expands still further, as you would know if you've ever put a bottle of water in the freezer. This turns out to be very important.

Ponds and lakes are warmed by solar radiation. As the top layer increases in temperature it expands. A given volume of warmed water will thus have less mass than cooler water and, as a result, the warmer water will remain at the top 'floating' on the colder water underneath. On a cool night, the warm layers at the top lose heat first and sink through the layers underneath. This causes a gradual mixing and results in a fairly even temperature throughout the entire body of water as winter approaches. This continues until the water reaches 4°C. As shown in Figure 6.16, from that point the colder water is expanding rather than contracting. Hence, water at 2°C will be less dense than water at 3°C and will stay on top. In areas where ponds and lakes freeze over, this colder surface water eventually freezes. However, the densest water, at about 4°C, will have sunk to the bottom where it will provide a relatively warm, ice-free environment for aquatic life. The ice on top provides a layer of thermal insulation. If water was not a special case of thermal expansion, ponds, lakes and even oceans would freeze to the very bottom.

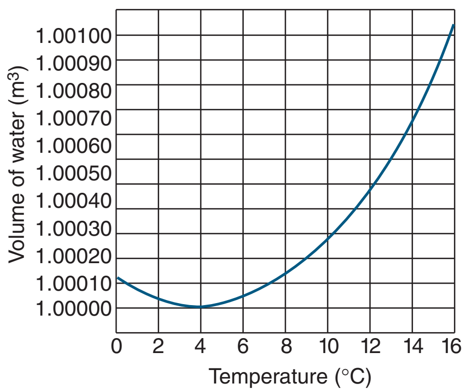


Figure 6.16
As water cools, it shrinks until the temperature reaches 4°C. Below that temperature it expands.

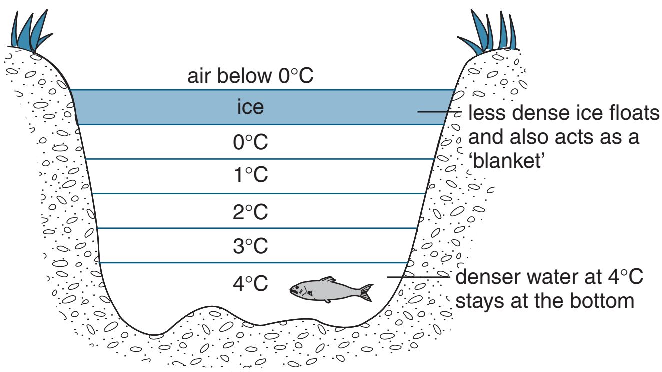


Figure 6.17
Water temperatures in a frozen pond. The unusual expansion of water ensures that large bodies of water rarely freeze to the very bottom.

Physics file

Silicon, germanium, water, sterling silver alloys, and lead–tin–antimony alloys all expand on freezing. Water is the only one that expands for the few degrees above its freezing temperature.

The behaviour of gases

An ideal gas is a model of the behaviour of gas based on the assumption that there are no forces between the molecules of the gas. Real gases behave slightly differently to an ideal gas, but the mathematical relationships for the behaviour of an ideal gas work well enough as long as the gases are not too compressed or too cold.

Boyle's law describes the behaviour of a sample of gas if its temperature is kept constant while its volume is changed. There is an inverse relationship between the pressure of a gas sample and its volume. You can test this qualitatively if you take a medical syringe, without the needle, and block the end. You can push on the plunger of the syringe and compress the gas into a smaller volume. When you release the plunger, it will move back towards its initial position owing to the increased pressure of the gas inside. Mathematically, Boyle's law is $PV = k$ and the useful form of this law is:

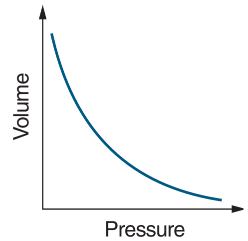


Figure 6.18

For a specific quantity of a gas at a given temperature, the pressure is inversely proportional to the volume.

$$P_1V_1 = P_2V_2$$

where P_1 is the initial pressure of the gas, V_1 is the initial volume of the gas, P_2 is the final pressure of the gas and V_2 is the final volume of the gas. Any pressure or volume units can be used as long as they are the same throughout the calculation. The usual units are pascal (Pa) for pressure and m^3 for volume.

Worked Example 6.3C

Suppose the teacher shows the class a syringe containing 25 cm^3 of air, open to the air on a day when the atmospheric pressure is 102 kPa. The syringe is then sealed and the plunger is pushed until the air is squeezed down to a volume of 14 cm^3. Calculate the new pressure inside the syringe.

Solution

$P_1 = 102$ kPa, $V_1 = 25$ cm^3, $V_2 = 14$ cm^3

Substitute these values into the equation for Boyles' law:

$P_1V_1 = P_2V_2$

$\Rightarrow (102)(25) = P_2(14)$

$\Rightarrow P_2 = 182$ kPa

Charles' law describes the behaviour of a sample of gas when the pressure is kept constant while the temperature is changed. In order to keep the pressure constant when a gas is heated, the volume must be allowed to increase. Mathematically, Charles' law is $V = kT$ and the useful form of this law is:

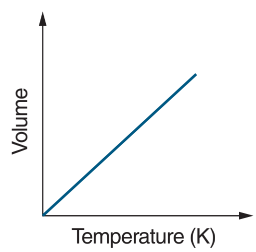


Figure 6.19

For a specific quantity of a gas at a given pressure, volume is directly proportional to temperature.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where T_1 is the initial temperature of the gas, V_1 is the initial volume of the gas, T_2 is the final temperature of the gas and V_2 is the final volume of the gas. Any volume units can be used as long as they are the same throughout the calculation. Temperature must be expressed in the absolute temperature scale, kelvin (K).

Worked Example 6.3D

Some air in a syringe is heated and expands from a volume of 12 cm^3 and a temperature of 20°C to a volume of 14 cm^3. If the pressure is kept constant, what is the final temperature of the gas?

Solution

T_1 = 20°C = 293 K, V_1 = 12 cm^3, and V_2 = 14 cm^3.
Substitute these values into the equation for Charles' law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \frac{12}{293} = \frac{14}{T_2}$$

$$\Rightarrow T_2 = 342 \text{ K} = 69°\text{C}$$

Gay-Lussac's law decribes the behaviour of a sample of gas when the volume is kept constant while the temperature is changed. If the gas is contained in a rigid vessel, then the volume will not change while the pressure increases as the temperature increases. Mathematically, Gay-Lussac's law is $P = kT$ and the useful form of this law is:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

where T_1 is the initial temperature of the gas, P_1 is the initial pressure of the gas, T_2 is the final temperature of the gas and P_2 is the final pressure of the gas. Any pressure units can be used as long as they are the same throughout the calculation. Temperature must be expressed in the absolute temperature scale, kelvin (K).

Worked Example 6.3E

Some air in a rigid 5.0 L container, initially at a pressure of 200 kPa and a temperature of 40°C, is cooled to a temperature of 10°C. What is the final pressure of the gas?

Solution

T_1 = 40°C = 313 K, P_1 = 200 kPa, and T_2 = 283 K.
Substitute these values into the equation for Gay-Lussac's law:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow \frac{200}{313} = \frac{P_2}{283}$$

$$\Rightarrow P_2 = 181 \text{ kPa}$$

Physics file

Sometimes more than one of pressure, volume and temperature are changed for a sample of gas, in which case it is useful to use the combined gas law:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

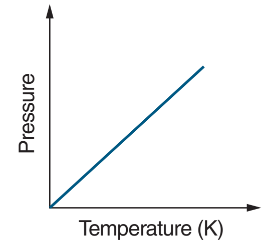


Figure 6.20
For a specific quantity of a gas contained within a given volume, the pressure is directly proportional to the temperature.

Physics file

The ideal gas law, $PV = nRT$, is a universal law that holds true for all gases and mixtures of gases. It relates pressure, volume and temperature to the number of particles in the gas sample. P is the pressure of the gas, V is the volume of the gas, T is the absolute temperature of the gas, n is the number of moles of the gas and R is a constant of proportionality called the universal gas constant. The value of R depends on the units used for P, V, and T. A mole of a gas is 6.02×10^{23} particles of the gas.

6.3 SUMMARY Heat and temperature

- The temperature of a material or object is a measure of the average kinetic energy of all the particles within it. Heat refers to the energy transferred due to a difference in temperature. Internal energy refers to the total kinetic and potential energy of the particles within an object.
- The temperature at which particles theoretically have no motion is referred to as zero kelvin (0 K) or absolute zero. At this temperature, a gas would exert no pressure and occupy zero volume.
- Any scale requires at least two fixed points, one of which is zero, which are then divided into a set number of points.
- For general use, temperature is measured on the Celsius scale. On this arbitrary scale, the melting and boiling temperatures for water at standard atmospheric pressure (average sea level pressure) form the fixed points.
- The absolute, or kelvin scale, is based upon the triple point of water and absolute zero. No degree symbols are used and there are no negative temperatures. K = °C + 273
- Most substances expand when heated and contract when cooled. Thermal expansion is the basis of design for many thermometers.
- Water expands when heated and contracts when cooled, except in the range of 0–4°C where it expands as it cools. The behaviour of water is unusual as it, along with a very few other materials, expands as it changes from liquid to solid.
- The ideal model of gases assumes that the particles of a gas do not attract or repel each other.
- The thermal behaviour of gas is described with the three gas laws, Boyle's law, Charles' law and Gay-Lussac's law, which relate the pressure, volume and temperature of a sample of gas. The three gas laws can be combined into the combined gas law:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

6.3 Questions

1. Complete the following conversions:
   - **a** to kelvin from Celsius
     - **i** 27°C
     - **ii** −27°C
     - **iii** 500°C.
   - **b** from the absolute scale to Celsius
     - **i** 0 K
     - **ii** 500 K
     - **iii** 1000 K.
2. A body is cooled from 100°C to 0°C. What is this temperature interval in kelvin?
3. A half-full jar of jam was sealed on a day when the air pressure was 103 kPa and the temperature was 25°C. The jar was then placed in the refrigerator and cooled to 4°C. Determine the pressure of the air trapped inside the jar.
4. **a** Explain the significance of lower and upper fixed points in setting up a temperature scale.
   b Describe how you would find one of these temperatures on an uncalibrated thermometer.
5. In a deep freshwater dam high up in the Victorian Alps, when the air temperature is −5°C, which of the following is true?
   - **A** The water at the bottom of the dam will be at a temperature of about 4°C.
   - **B** The surface will begin to ice over and will be at a temperature of about 2°C.
   - **C** The temperature of the water will be the same at all levels because the molecules are in constant motion and mix the water thoroughly.
   - **D** The coldest water will always be at the bottom regardless of the air temperature.
6. State two ways in which a clinical thermometer would differ from a thermometer for general use. In doing so, consider the particular requirements of such a thermometer.
7. In a physics experiment to explore Charles' law, a syringe and tube are used to maintain a constant pressure in a system. If the initial temperature was 0°C before raising the temperature to 25°C with a gas sample initially at 18 cm^3, how much additional space must be created by manipulating the syringe?
8. Does a negative temperature have meaning on the kelvin scale? Why?
9. Suppose the air and fuel vapour mixture, initially at 110 kPa, in the cylinder of a moped engine was compressed from a volume of 90 cm^2 to a volume of 15 cm^2 while, at the same time, the temperature was increased from 100°C to 250°C. By how much would the pressure in the cylinder increase?
10. The average temperatures in Antarctica lie well below zero throughout most of the year. Surface snow and ice rarely melt yet a vast lake of fresh water many thousands of years old has been found beneath the ice almost at the South geographic pole. Explain how this could occur.

6.4 Specific heat capacity

A small amount of water in a kettle will experience a greater change in temperature when heated than will a larger volume. Metal objects left in the sun get hotter to touch faster than the same mass of water. Larger heaters warm rooms more quickly than small ones. These simple observations suggest that the mass, material and amount of energy influence any change of temperature when objects are heated. In this section quantitive relationships will be established so the size of the energy transfers involved can be calculated.

Heat capacity

Try heating a pot of water for, say, 10 minutes. Double the mass of water and it takes twice as long to reach the same temperature. This simple activity suggests that twice the energy is required for twice the mass to be heated through the same temperature change. The amount of energy transferred is proportional to the mass of the substance:

$$\Delta Q \propto m$$

where ΔQ is the heat energy transferred to the material in joules (J)
m is the mass of the material in kilograms (kg).

Similar logic suggests that more energy will be required for a greater temperature change (ΔT) to occur:

energy transferred ∝ final temperature – starting temperature

or $\Delta Q \propto T_f - T_o$

and $\Delta Q \propto \Delta T$

where ΔT is the temperature change in kelvin or °C.

Simple heating experiments using different materials will confirm that these relationships hold true regardless of the material being heated.

Heating identical masses of different liquids through the same change in temperature will reveal a further relationship—it takes more energy for some substances than for others. A particular volume of water takes more energy to heat through the same temperature change than the same volume of alcohol or methylated spirits. The amount of energy required is dependent on the nature of the material being heated.

When these observations are combined, it can be said that the capacity of a given material to absorb energy as it changes temperature will depend on its mass, the kind of material and the size of the temperature change. This is referred to as a material's *heat capacity*.

In order to compare the heat capacity of different materials and masses for a fixed temperature change, a specific mass is adopted and therefore *specific heat capacity* is defined.

The ***SPECIFIC HEAT CAPACITY*** of a material is defined as the amount of energy that must be transferred to change the temperature of one kilogram of a material by 1°C or 1 K.

The symbol for heat capacity is c and it can be found from the following relationship:

$$c = \frac{\Delta Q}{m\Delta T}$$

where ΔQ is the heat energy transferred to the material in joules (J)
m is the mass of the material in kilograms (kg)
ΔT is the temperature change in °C or kelvin.

Thus the SI unit for specific heat capacity is J kg^{-1} K^{-1}.

Physics file

Δ, or capital delta from the Greek alphabet, is used in physics to represent a change in a quantity. Mathematics makes use of similar notation to represent a change, using both Δ and lower case delta, δ, or a 'd', to represent the very small changes inherent in calculus.

Physics file

The symbol for both specific heat capacity and the speed of light in a vacuum is c. As the two areas are unrelated, the chances of confusion are remote.

Table 6.2 Approximate specific heat capacities of some common materials

Material	Specific heat capacity ($J\ kg^{-1}\ K^{-1}$)
Human body	3500
Methylated spirits	2500
Air	1000
Aluminium	900
Glass	840
Iron	440
Copper	390
Brass	370
Lead	130
Mercury	140
Water	
ice	2100
liquid	4200
steam	2000

For example, 440 J of energy will raise the temperature of 1 kg of iron, which has a specific heat capacity of 440 $J\ kg^{-1}\ K^{-1}$, by 1°C or 0.5 kg by 2°C and so on.

Table 6.2 shows the average specific heat capacities for several common materials (the value for the human body is based on the various materials found in the body and their percentage of the body's total mass). In many cases, the specific heat capacity of a material changes as its temperature increases and its molecular structure changes. So, with the exception of ice, these values are quoted for normal room temperature. For small temperature changes the value can be treated as constant. To calculate the transfer of energy required for a particular temperature change the following general relation is used:

$$\Delta Q = mc\,\Delta T$$

where ΔQ is the heat energy transferred in joules (J)
m is the mass in kilograms (kg)
ΔT is the change in temperature in °C or kelvin
c is the specific heat capacity of the material or object ($J\ kg^{-1}\ K^{-1}$).

✓ Worked Example 6.4A

A hot water tank contains 135 litres of water. The water is initially at 20°C.

a Calculate the amount of energy that must be transferred to the water to raise the temperature to 70°C.

b Calculate the time this will take when a 5 kW electric water heater is used.

Solution

a Volume = 135 litres; hence, mass = 135 kg
$\Delta T = 70° - 20° = 50°C$
and from Table 6.2, $c = 4200\ J\ kg^{-1}\ K^{-1}$ for water.
$\Delta Q = cm\,\Delta T$
$\Delta Q = 4200 \times 135 \times 50$
$\Delta Q = 28\,350\,000$ joule = 28 MJ

b Recall that power (P) = energy/time.
Rearranging: time = E/P
$$= \frac{28\,350\,000}{5000}$$
= 5670 seconds = 94.5 minutes

Physics file

Since water is a familiar material, many of the examples in this section use it as the liquid being heated. 1 kg of pure water has a volume of 1 litre at 4°C.

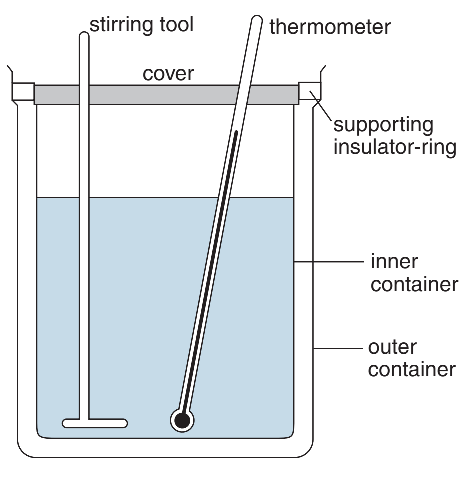


Figure 6.21 A simple calorimeter is used to determine the specific heat capacity of liquids by ensuring that energy does not escape or get into the liquid being tested. The inner container is insulated from the outer container in much the same way as a vacuum flask separates internal and external containers. Heat energy is introduced with an immersion heater. A voltmeter and ammeter, connected within the circuit supplying power to the immersion heater, are used to measure the amount of energy required for a measured temperature change.

Physics in action — The specific heat capacity of water

One of the notable values in Table 6.2 is the high value for water. The specific heat capacity of water is in fact higher than for all but a few of the more uncommon materials. As a result, water makes a very useful cooling agent, for example, in car radiators or heating systems. A liquid of lower specific heat capacity may not be able to hold sufficient energy, as it moves through the cooling system to the radiator, to prevent an engine from overheating.

Life on Earth has long depended on the large volumes of its surface water. Some 70% of the Earth's surface is covered by water. Water can absorb large quantities of thermal energy without great changes in temperature. Oceans both heat up and cool down more slowly than the adjacent land areas. The stability of ocean temperatures is also reflected in the relatively mild weather found on islands and in coastal regions where water acts to moderate extreme temperatures.

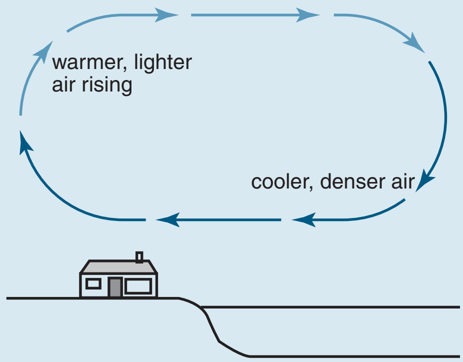


Figure 6.22
A sea breeze results when air over the land warms more quickly and hence is less dense than the air over the sea. Water has a higher specific heat capacity than land and will 'store' much of the Sun's energy, so air over the sea will become displaced upwards and form a convection current. At night the land will cool more quickly and the circulation may reverse to form a land breeze.

Mixtures

The hot water from your hot water system at home is probably maintained at a temperature somewhere between 60°C and 80°C. You have to mix the hot water with cold water in order to avoid burning yourself when washing. What happens to the energy from the hot water? How much cold water do you need to add?

As the hot water mixes in the bath, the higher energy molecules of the hot water collide with those of the cold water, transferring energy until all molecules have the same average kinetic energy. The hot water transfers energy and so cools and the cold water receives the energy and becomes warmer. Both will end up at the same average temperature. The mixture is said to reach 'thermal equilibrium'. The final temperature will lie somewhere between the two original temperatures. Exactly where depends on the relative masses and, where different materials are involved, their specific heat capacities. It will only lie halfway between if both masses and specific heat capacities are equal.

Whenever two materials are at different temperatures, the hotter material will lose energy by transfer to the colder one until both come to the same temperature.

For ***THERMAL EQUILIBRIUM***, the heat energy gained by the colder material equals the heat energy lost by the hotter material.

This does not necessarily mean that both materials will have the same internal energy. That would only occur if the two materials have the same mass and specific heat capacity.

Assuming no losses to the surrounding environment, the total energy remains the same (if additional energy were added, for example, both materials could end up at a higher temperature than the original temperature of either). This is referred to as 'conservation of energy' and is essential when solving problems involving a transfer of energy.

Physics file

An application of the calculations involving mixtures is the bomb calorimeter. The bomb calorimeter is a device that uses heat energy to measure the energy available from the complete metabolism of food. The food is burnt in a container immersed in water. This rapidly converts the molecules in the food to carbon dioxide and water and releases energy. The temperature rise of the water allows the energy available from the food to be calculated. This overestimates the energy available from some foods since foods containing fibre are not completely converted to water and carbon dioxide when digested and metabolised.

Worked Example 6.4B

The hot water tap of a bath delivers water at 80°C. Ten litres of hot water is added to a bath containing 30 litres of water at 20°C. Ignoring energy losses to the surrounding environment, what will be the final temperature of the bath water?

Solution

Heat energy gained by the cold water = heat energy lost by the hot water. In terms of the final temperature of the mixture:

$$cm(80 - T) = cm(T - 20)$$

Note that the specific heat of water will cancel out as it appears on both sides of the equation.

So, $10(80 - T) = 30(T - 20)$

Expanding: $800 - 10T = 30T - 600$

Rearranging: $800 + 600 = 30T + 10T$

and $1400 = 40T$

Hence $T = 35°\text{C}$

Worked Example 6.4C

50 grams of iron is heated over a flame for several minutes. It is then plunged into a closed container containing 1.0 litre of cool water, originally at 15°C. After the temperatures equalise, the water is now found to be at 17°C. If no water changed state to become steam and there were no losses to the surrounding environment, what was the temperature of the iron just prior to being immersed in the water?

Solution

Mass of iron = 50 g = 0.050 kg, mass of water = 1.0 kg

From Table 6.2, c for iron = 440 J kg^{-1} K^{-1}, c for water = 4200 J kg^{-1} K^{-1}.

Energy lost by iron = energy gained by water

$$cm\,\Delta T = cm\,\Delta T$$

$$440 \times 0.05 \times (T - 17) = 4200 \times 1 \times (17 - 15)$$

$$22T - 374 = 8400$$

$$22T = 8774$$

and $T = \frac{8774}{22} = 399°\text{C}\ (\sim 400°\text{C})$.

It may seem strange in this example that a temperature change for the water of only 2°C equates to such a large change for the iron but the specific heat capacity of water is almost ten times that of iron and there was twenty times as much water as iron.

6.4 SUMMARY Specific heat capacity

- During a transfer of heat energy the materials involved come to thermal equilibrium as the heat gained by materials receiving the energy is equal to the heat lost by the materials supplying energy.
- When heat energy is added to or taken from an object, the temperature change depends upon the amount of energy, the mass of material and the specific heat capacity of the material:

$$\Delta Q = cm\,\Delta T$$

where ΔQ is the change in energy (J)
m is the mass of the material involved (kg)
ΔT is the change in temperature (°C or K)
c is specific heat capacity (J kg^{-1} K^{-1}).

- The very high specific heat capacity of water makes it a useful coolant and accounts for the moderating effect of oceans, rivers and lakes on the world's climate.

6.4 Questions

For the following questions refer to Table 6.2 for specific heat capacities.

1 100 g of water is heated to change its temperature from 15°C to 20°C. How much energy has been transferred to the water?

2 150 g of water is heated from 10.0°C to 50.0°C. What amount of energy is required?

3 a Equal masses of water and aluminium are heated through the same temperature range. Which one requires more energy?
  b What is the ratio of the specific heat capacity of aluminium to that of water?
  c Equal amounts of energy are absorbed by equal masses of aluminium and water. What is the ratio of the temperature rise of the aluminium to that of the water?

4 A 2.0 kg block of copper at 100°C is put into a large pot containing 5.0 litres of water at 20°C. Assuming no energy is lost to the surrounding environment, what is the final temperature of the 'mixture'?

5 Which of the following statements about specific heat capacity are/is true?
  A All materials have the same specific heat capacity.
  B The specific heat capacity of liquid water is different from that of ice and steam.
  C Good conductors of heat generally have high specific heat capacities.
  D Specific heat capacity is independent of temperature.

6 A 2000 watt kettle holds 3.0 kg of water at 20°C. It is turned on for 5 minutes. If all the energy supplied is used to heat the water, will it boil?

The following information relates to questions 7 and 8. A relatively efficient electric motor has an input of 200 W and an output of 180 W.

7 If all the wasted energy is converted into thermal energy, how much thermal energy will be produced in 10 minutes?

8 The motor is made of iron. How many kilograms of iron could be heated from 20°C to 100°C by this energy (neglecting losses to the surrounding environment)?

9 Mercury, like many metals, has a low specific heat capacity. Is this an advantage or a disadvantage when designing a mercury thermometer? Explain.

10 Alice Springs and Brisbane are at similar latitudes yet experience very different temperature variations. Explain this in terms of the specific heat capacity of water.

Use your general knowledge to make reasonable estimates for any data that you need before doing calculations to answer questions 11 and 12.

11 Estimate, in joules, how much heat energy would be used to heat enough water to make a cup of coffee.

12 Estimate, in joules, how much heat energy would be used to heat enough water for you to take a shower.

Physics file

Once water is boiling, it will not get any hotter until it is all turned to steam. This is why it makes no difference to cooking time when preparing soup or stew if you boil it vigorously or simmer it gently.

Boiling temperature is elevated by pressure. This is the principle of the pressure cooker. The temperature inside a pressure cooker is well above 100°C so the food cooks faster and with the use of less fuel.

6.5 Latent heat

Try the following simple experiment.

- Put some paraffin wax and a thermometer in a test tube.
- Heat it gently with a heat pad or Bunsen burner until the paraffin melts.
- Remove the source of heat when the temperature reaches about 100°C.
- Record the temperature of the liquid paraffin at half-minute intervals until the paraffin solidifies once again, stirring regularly in the process.
- Plot a curve of temperature against time. This is referred to as the cooling curve for liquid paraffin.

Now consider the shape of this curve.

What causes the liquid paraffin to cool?

Do you think that cooling stops when the temperature of the paraffin remains constant as seen in the horizontal part of the cooling curve? What is happening? The cooling and heating curves of all materials undergoing a change of state are similar to that of paraffin. There are sections of increasing temperature and sections where the temperature remains constant while the material changes state.

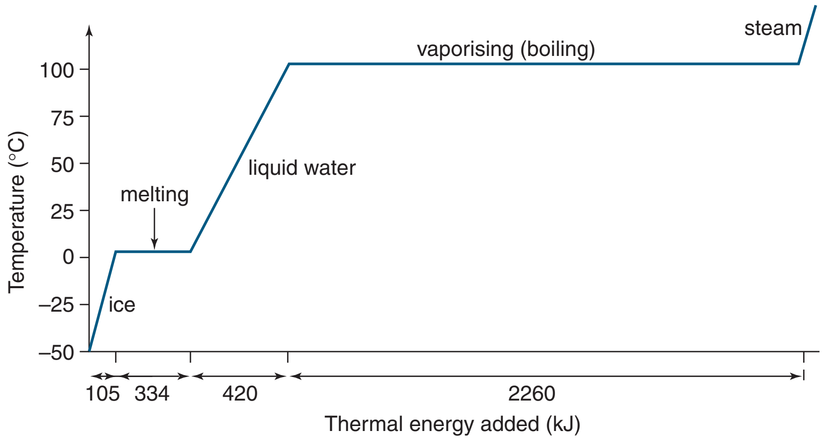


Figure 6.23

Heating curve for 1 kg of water showing how temperature changes as thermal energy is added at a constant rate. The temperature remains constant during the change in state from ice to liquid water and again from liquid water to steam.

Melting and the heat of fusion

As a solid receives extra heat energy, it heats up, and the particles within it gain kinetic and potential energy and increase their speed of vibration. At the point where the solid begins to melt, the particles begin to separate, reducing the strength of the bonds holding them in place. At this temperature, instead of the extra energy creating a rise in temperature, the energy increases the potential energy of the particles, overcoming the forces holding the particles in fixed positions. No change in temperature occurs until the whole of the material has been melted. All of the extra energy supplied is used in overcoming the forces between particles.

The amount of energy involved in melting a solid at constant temperature is exactly equal to the potential energy released as the liquid refreezes. This is called the ***LATENT HEAT OF FUSION***.

In the process of melting, the solid requires additional latent energy. When a liquid freezes, the reverse of this process occurs and the latent heat energy is released from the material at a constant temperature.

Latent means hidden or unseen. The heat energy which changes ice into water is hidden in the sense that when the ice melts it is no hotter than before it received the heat. The latent heat of fusion turns ice at 0°C into water at 0°C. There is no change in temperature within the material. The latent heat energy needed during melting comes from the surrounding environment. The energy released when water is frozen is transmitted to the same environment. The potential energy of the water molecules decreases and the energy of the surrounding environment increases.

The latent heat of fusion for water is 3.34×10^5 J kg^{-1}. This means that it takes 3.34×10^5 J of energy to completely melt 1 kg of water at 0°C. Table 6.3 identifies the latent heat of fusion of some other common materials.

If the latent heat of fusion is known, the amount of energy involved in freezing or melting a material can be calculated. In general, when a material freezes or melts:

heat energy transferred = mass × latent heat of fusion

$$\Delta Q = mL_f$$

where ΔQ is the energy transferred in joules (J)

m is the mass in kilograms (kg)

L_f is the latent heat of fusion in joules per kilogram (J kg^{-1}).

Table 6.3 Latent heat of fusion of some common materials

Material	Melting point (°C)	H_{fus} (J kg^{-1})
Water	0	3.34×10^5
Oxygen	−218.8	0.14×10^5
Lead	327	0.25×10^5
Ethyl alcohol	−114	1.05×10^5
Silver	961	0.88×10^5

Worked Example 6.5A

How much energy has to be removed from 2.5 kg of water at 10°C to produce a block of ice at 0°C? Express your answer in kilojoules.

Solution

$m = 2.5$ kg and, from Table 6.3, $L_f = 3.34 \times 10^5$ J kg^{-1}, from Table 6.2, $c = 4200$ J kg^{-1} K^{-1}.

Cooling the water to 0°C:

$\Delta Q = cm\,\Delta T$

$\Delta Q = 4200 \times 2.5 \times 10 = 105\,000$ J = 105 kJ

Freezing the water at 0°C to create ice at 0°C:

$\Delta Q = m \times L_f$

$\Delta Q = 2.5 \times 3.34 \times 10^5 = 835\,000$ J = 835 kJ

Total energy required = 105 kJ + 835 kJ = 940 kJ

Boiling and the heat of vaporisation

Try heating ice over a flame. Melting the ice completely will take a while. As Worked Example 6.5A demonstrated, a large amount of energy is required for this process. By comparison, the amount of heat energy

Physics file

Anyone who has been to the Victorian snowfields will know that temperatures need to drop to below zero before snow will begin to fall. Yet in spring, when air temperatures have risen well above zero, sheltered regions in the higher alps will still contain small snowdrifts. In New Zealand, glaciers remain all year round on the South Island despite temperatures being regularly above zero, and in the Irian Jayan highlands a glacier persists in tropical latitudes. This is due to the very large amounts of energy needed to melt ice and the highly reflective nature of the snow. Much of the incident energy is reflected, keeping the energy absorbed below that required to melt it.

Figure 6.24
Monte Rosa and the Gorner Glacier in Switzerland.

Physics file

It takes almost 80 times more energy to melt ice with no temperature change than it does to raise the temperature of water by 1°C. In terms of molecular theory this makes sense. Latent heat is a measure of the energy required to overcome the large intermolecular forces rather than simply adding kinetic and potential energy in moving the particles apart.

required to raise the temperature of the water from 0°C to 100°C is much less. Boiling the water may take half an hour or more and, during this time, the temperature doesn't change at all! Since it takes so long, the heat required to boil all the water must be relatively very large. One kilogram of water at 100°C requires 2250 kJ of heat energy to turn it into steam at 100°C. That is, the latent heat of vaporisation of water is 22.5×10^5 J kg^{-1}. This is almost seven times as much energy as to melt the same quantity of ice. The same amount of energy is released in converting steam back into water in the process of condensation.

Similarly, all liquids will require the additional energy of the latent heat of vaporisation in order to change to the gaseous state. The amount will depend on the particular liquid:

$$\Delta Q = mL_v$$

where L_v is the latent heat of vaporisation in joules per kilogram (J kg^{-1}).

Table 6.4 Latent heat of vaporisation of some common materials

Material	Boiling point (°C)	L_v (J kg^{-1})
Water	100	22.5×10^5
Oxygen	−183	2.2×10^5
Lead	1750	9.0×10^5
Ethyl alcohol	78	8.7×10^5
Silver	2193	23.0×10^5

The amount of energy required to convert a liquid into gas at constant temperature is exactly equal to the potential energy released as the gas condenses to a liquid. This is called the ***LATENT HEAT OF VAPORISATION***.

Some typical values of latent heat of vaporisation are included in Table 6.4.

A problem-solving approach

Problems involving heat energy often consider both latent heat and specific heat. They require a number of steps. A diagram will often help in the understanding of what steps are involved, and what form of heating is required for each step. Figure 6.25 illustrates the different processes involved in turning ice into steam. The processes are completely reversible.

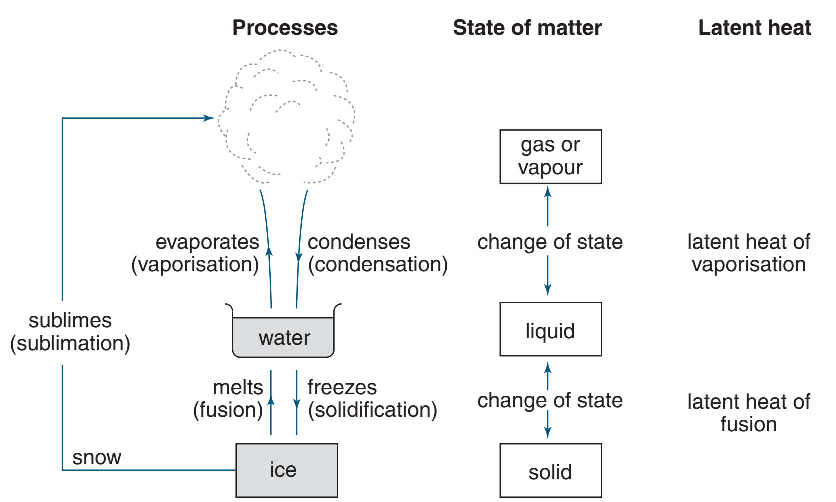


Figure 6.25
Converting ice into steam involves a number of changes of state. Each must be considered when calculating the total energy required. The energy released to the environment in reversing the process requires similar steps.

Worked Example 6.5B illustrates an approach as water is heated through changes of state.

Worked Example 6.5B

Calculate the heat required to convert 5 kg of ice at −20°C into steam at 100°C.

Solution

Four steps are involved in this process as a flow diagram will indicate.

a Warming of ice from −20°C to 0°C: specific heat

$\Delta T = 20°C$, $c = 2100$ J kg^{-1} K^{-1} for ice

$\Delta Q = cm\,\Delta T$

$\Delta Q = 2100 \times 5 \times 20$

$= 2.1 \times 10^5$ J = 0.21 MJ

b Melting the ice: latent heat of fusion

$\Delta Q = mL_f$, $L_f = 3.34 \times 10^5$ J kg^{-1} for ice

$\Delta Q = 3.34 \times 10^5 \times 5$

$= 1.67 \times 10^6$ J = 1.67 MJ

c Warming the water from 0°C to 100°C: specific heat

$\Delta T = 100°C$, $c = 4200$ J kg^{-1} K^{-1} for water

$\Delta Q = cm\,\Delta T$

$\Delta Q = 4200 \times 5 \times 100$

$= 2.1 \times 10^6$ J = 2.1 MJ

d Evaporating the water: latent heat of vaporisation

$\Delta Q = mL_v$, $L_v = 22.5 \times 10^5$ J kg^{-1} for water

$\Delta Q = 22.5 \times 105 \times 5$

$= 11.25 \times 10^6$ J = 11.25 MJ

Total energy required = a + b + c + d = 15.2×10^6 J or 15.2 MJ.

Notice that the vast majority of the energy required is used in converting water into steam.

6.5 SUMMARY Latent heat

- When a solid material changes state, energy is needed to separate the particles by overcoming the attractive forces between them.
- Latent heat is the energy required to change the state of 1 kg of material at constant temperature. L_f is the latent heat of fusion and L_v the latent heat of vaporisation.
- In general, when a material freezes or melts:

$$\Delta Q = mL$$

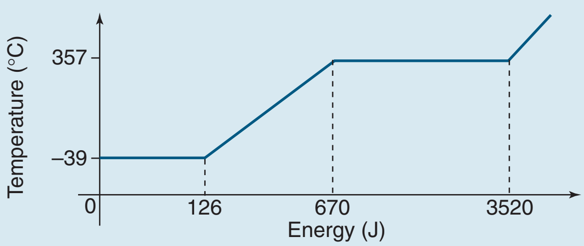
where ΔQ is the energy required or released (J)
m is the mass (kg)
L is the latent heat of fusion or vaporisation (J kg^{-1}).

6.5 Questions

Refer to Tables 6.3 and 6.4 for latent heats of fusion and vaporisation when required.

This information relates to questions 1–5.

This graph represents the heating curve for mercury. Thermal energy is added to 10 g of solid mercury, initially at a temperature of −39°C, until all the mercury is evaporated.



1. Explain why the temperature remains constant during the first part of the graph.
2. What is the melting point of mercury?
3. What is the boiling point of mercury?
4. What is the latent heat of fusion for mercury?
5. What is the latent heat of vaporisation for mercury?
6. The latent heat of fusion of sulfur is 0.38×10^5 J kg^{-1}. What does this mean? Explain in terms of kinetic molecular theory.
7. Why is energy needed to change a liquid into a gas? Explain using a kinetic molecular model and diagrams where appropriate.
8. Is there any difference between the average speed of a molecule of a particular liquid at boiling point and the average speed of molecules of a gas formed from this liquid and at the same temperature? Why?
9. How much heat energy is required for 100 g of water, initially at 100°C, to evaporate completely?
10. 50.0 g of water is heated from a room temperature of 20.0°C to a boiling point of 100°C. It is boiled at this temperature until it is completely evaporated. How much energy is required?

Figure 6.26

One of Australia's great inventions of past years was the Coolgardie meat safe. Prior to the availability of refrigerators, it acted as a container for food, which would otherwise spoil in hot weather. Hessian, often in the form of an old sack, was draped over the sides and kept constantly wet. Water evaporating from the hessian transferred energy away from the food, keeping it cool.

Physics file

Evaporative air-conditioning is used in many Perth homes. Small portable units can be placed near a window. Larger units are placed on roofs and the cool air is ducted through to various rooms in the house. In dry weather, such air-conditioning works well. Water does not evaporate as rapidly in humid air so evaporative air-cooling does not work so well in humid weather. Perth experiences a relatively dry climate so this type of air-conditioning is popular—and inexpensive to operate. In other hot cities such as tropical Brisbane, the humidity is usually too high for evaporative air-conditioning to be used.

6.6 Evaporation: heat energy in context

This section covers a broad range of ideas which draw on the theory and ideas established earlier in this chapter, and puts them into the context of the world around us.

Cooling effects of evaporation

Most people are familiar with the cooling felt when jumping out of a pool and into a breeze. This is due to evaporation and makes even a sunny day feel cold. A similar and greater effect is felt by putting a small amount of methylated spirits on the back of your hand. Methylated spirits evaporates at a faster rate than water and the cooling effect is very noticeable. Blowing over the methylated spirits increases the effect still further.

It is possible to demonstrate the cooling effect of evaporation by a simple laboratory experiment. A fume hood should be used.

- Set up a small beaker containing some ether as shown in Figure 6.27.
- Draw a stream of air through the ether, using a bike pump or vacuum pump. Don't attempt to draw the air through with your mouth. The vapour is dangerous if inhaled.
- Watch the sides of the beaker and feel them from time to time. Try lifting the beaker.

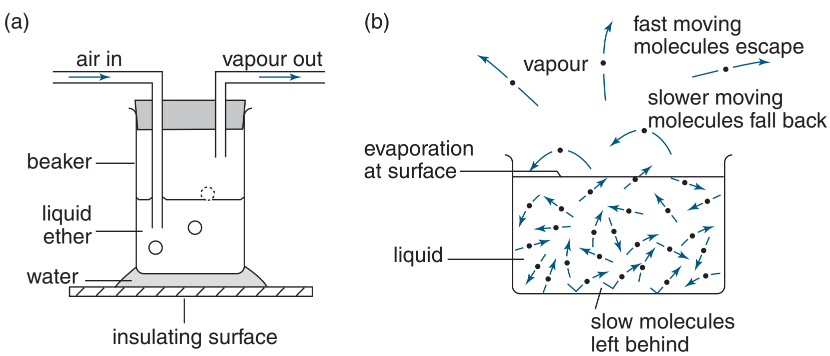


Figure 6.27

Demonstrating evaporation and cooling using ether. The high rate of evaporation of ether makes it possible to freeze the water condensing around the outside of the beaker.

Ether is a liquid that evaporates extremely easily. It is said to be 'volatile'. Like methylated spirits on skin, the evaporation of ether causes cooling. Water vapour condenses from the air on the outside of the beaker and may even turn to ice, causing the beaker to freeze to the table. This gives some idea of the significant cooling effects that are possible during evaporation.

Figure 6.27b illustrates why this happens. Particles at the surface of the liquid may either escape, if they have sufficient energy, or fall back into the liquid. As only the faster or higher energy particles escape, over time the average kinetic energy of the particles in the liquid is lowered. If the particles have less average kinetic energy then the liquid will cool.

This is one example of the process of evaporation and cooling. Evaporative cooling is experienced by a swimmer leaving the water. The faster water molecules soon evaporate, leaving slower, and hence cooler, water molecules on the skin. The initially warmer skin transfers energy to these slow molecules, speeding them up but reducing the temperature of the skin. The process will continue while there is water on the surface of the skin.

The ingenious Coolgardie meat safe and the simple water bag hung on the front of a travelling car made use of this principle.

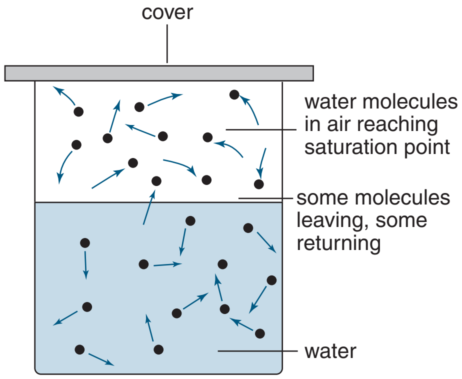


Figure 6.28
Evaporation is continuous in all liquids. In a closed container or room, vapour will build up above the liquid until equilibrium between the two states of matter is reached.

Rate of evaporation

The rate of evaporation is affected by a number of factors including the temperature, humidity and area exposed.

Temperature

Water will evaporate at any temperature while it is a liquid. Some will even evaporate from solid forms such as snow and ice. Even frozen clothes left on a line will dry if there is a strong enough breeze blowing. It is only the very high latent heat of vaporisation of water that prevents rivers and lakes evaporating completely.

The main factor affecting the rate of evaporation is temperature. In general, the warmer the water, the greater the number of molecules with sufficient energy to escape. The same is true of all liquids.

Humidity

Water molecules constantly escape from the water of a bath or shower. The rate at which they escape is influenced by the temperature of the water. Once molecules have escaped, they will travel freely within the confines of the bathroom, colliding with each other and with air particles and the walls of the room. Some will lose sufficient energy to once again become liquid water (this is most noticeable on the cold surface of a mirror). If this process continues, then eventually the number of particles evaporating from the water each second will equal the number of molecules condensing each second. The air is said to have reached 'saturation' point. There will be a constant movement of water particles in and out of the gas state, but the total number will remain the same. Hence, the rate of evaporation depends on the degree of saturation of the atmosphere. This is referred to as 'humidity'.

Humans are particularly sensitive to humidity. While a relative humidity of 40–50% is generally ideal, high humidity, particularly on hotter days, reduces the rate of evaporation from the surface of the skin. As this is one of the body's vital mechanisms for regulating body temperature, heavy work or exercise in these conditions can soon lead to overheating. Similarly, very low humidity can increase evaporation, drying the skin and also causing damage.

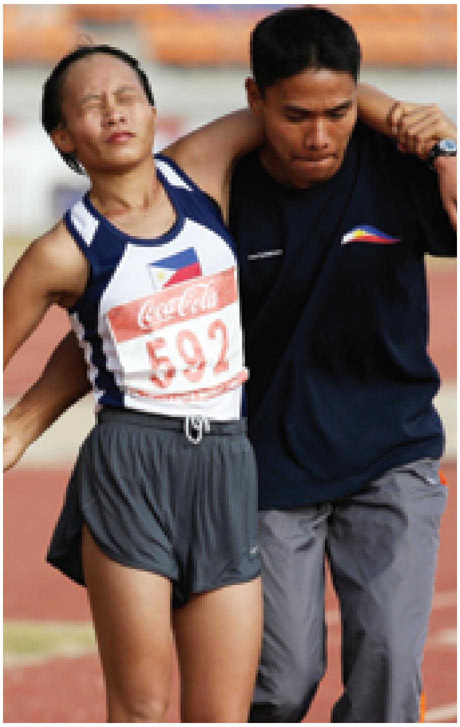


Figure 6.29
Uncontrolled heating during an athletics event such as the marathon can lead to a condition called hyperthermia if the runner's body is unable to lose heat to the surrounding air fast enough. High humidity will increase the chance of overheating as the rate of evaporative cooling from perspiring will decrease.

Area exposed

Large shallow expanses of water evaporate more quickly than equal volumes in deeper lakes with a smaller surface area. Greater surface areas allow a larger number of particles to escape and a larger volume of air is available to contain them. If a small proportion of the surrounding air becomes saturated, the humidity of the surrounding air may still be less than 100% and is able to accept more moisture. For this reason dams built in deep gorges are preferable to large shallow expanses.

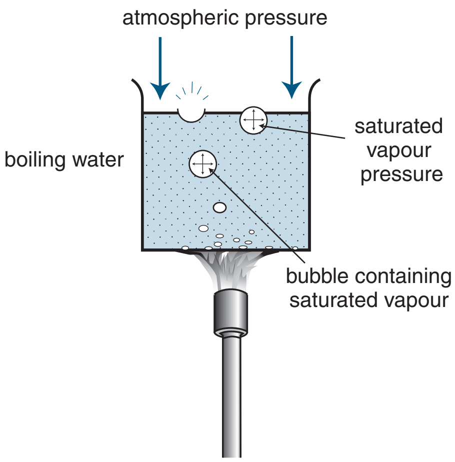


Figure 6.30

When water reaches boiling point, bubbles of steam form throughout the liquid. The pressure of the water vapour within the bubbles is greater than the external pressure within the liquid.

Air movement

The rate at which saturated air is replaced over the surface of an evaporating liquid is a major factor affecting evaporative cooling. Evaporation will only continue until the surrounding air is fully saturated. Unless the temperature changes, evaporation will cease. Additional particles from the liquid will leave the liquid, but only to replace particles which have returned to the liquid by losing kinetic energy. Faster air movement will carry moisture away and allow quicker replacement from the evaporating liquid. This will increase the chilling effect of wind on the body, a major factor in the aptly named 'wind chill factor'.

Vapour pressure and boiling

At higher temperatures more evaporation takes place, so the vapour pressure of a liquid will increase with temperature and warmer air will consequently hold more water vapour. Boiling occurs when the temperature of a liquid is raised to the point where the vapour pressure within the liquid is equivalent to the pressure in the surrounding environment.

As the boiling temperature approaches, tiny bubbles within the liquid indicate a change of some molecules into the gaseous state. If the vapour pressure inside the bubbles is less than the external pressure, then the bubbles will be crushed. As the temperature increases, the vapour pressure inside the bubbles rises to eventually exceed the external pressure. The bubbles increase in size and rise to the surface. Boiling has commenced.

The boiling point depends not only on the liquid but also on external air pressure. At elevation, the boiling point is less than that at sea level (recall the earlier discussion of temperature scales). At the summit of Mt Everest, where air pressure is only one-third of that at sea level, water will boil at around 70°C. Cooking food by boiling alone takes longer. Pressure cookers, on the other hand, increase pressure up to about twice that of standard air pressure; the boiling point of water is increased to 120°C and cooking time is reduced to about one-quarter of that under normal pressures.

Physics file

A common misconception confuses the terms humidity and relative humidity. Relative humidity is a ratio. It is not a direct measurement of the amount of water vapour in the air as a percentage of the total volume. 90% humidity on a hot day does not mean that 90% of the air is water vapour.

The water molecules in the atmosphere will be exerting an extra pressure on top of that normally exerted by the air alone. This is referred to as 'vapour pressure'. The 'relative humidity' is defined as the vapour pressure of the water in the air compared with the maximum possible at that temperature before saturation occurs. When humidity is close to 100%, the air is holding nearly all the water vapour it can for that particular temperature. If the temperature were to suddenly decrease, the ability of the air to hold the water vapour would decrease. The excess water would condense as dew, clouds, fog or rain, in much the same way as the water condensing on the mirror of a bathroom.

Thermal effects of changing gas pressure

Anyone who has pumped up a tyre or released gas from an air cylinder will be aware of the change in temperature as a gas changes pressure. As air is compressed, say by a bike pump, more work is done on the molecules in the air, giving them more kinetic energy and thus increasing their temperature. Whenever any gas is compressed, its temperature will increase.

The expansion of a gas will cause the reverse to occur. The gas molecules do work on the surroundings, losing kinetic energy themselves. The temperature decreases. This is most dramatically demonstrated when gas is released from an LPG bottle. Water vapour will condense on the cooling surface of the bottle and, if the gas continues to be released, will change to ice. Releasing the carbon dioxide from a soda bulb produces a similar readily observable effect.

The thermal effects of changes in gas pressure are the basis of refrigeration and air-conditioning.

Figure 6.31
When the stationary air surrounding a body is blown away by wind, its insulating effect is lost. This creates a cooling effect called wind chill.

Physics file

We feel cooler when standing in a breeze or near a fan. Evaporative cooling is accelerated. Wet clothing can rapidly chill the body but even the perspiration of heavy exercise can cause significant cooling. The moving air also carries away the relatively still air normally surrounding a body. Heated by the body, this still air acts as an effective insulator. When the still air is carried away by the wind, it feels as if the temperature has decreased. The effective air temperature decreases by 1°C for every 1 km h^{-1} of wind speed. This is called the 'wind chill factor'.

Physics in action — Refrigerators and air-conditioning

Refrigerators and air-conditioners operate on the same principle. They make things cold by removing energy from them. Energy is pumped from the space to be cooled to the outside air or, if heating is required, energy is pumped in. As a result, modern air-conditioning systems can be called 'heat pumps'. This is a particularly effective and energy-efficient method of heating and cooling.

Inside a heat pump, a volatile liquid, known as a refrigerant, is used to remove heat in the same way as ether did in the earlier demonstration. The refrigerant is circulated inside a closed circuit of pipes by a pump. Evaporation occurs inside the evaporator pipes as pressure is reduced through an expansion valve. The latent heat of vaporisation, required to evaporate to the liquid, is removed from the air, making it cooler.

In the condenser pipes, the gas is compressed. You will recall from the previous section that the boiling point increases as pressure increases. The higher pressure causes the gas to return to a liquid state, releasing the stored energy collected from

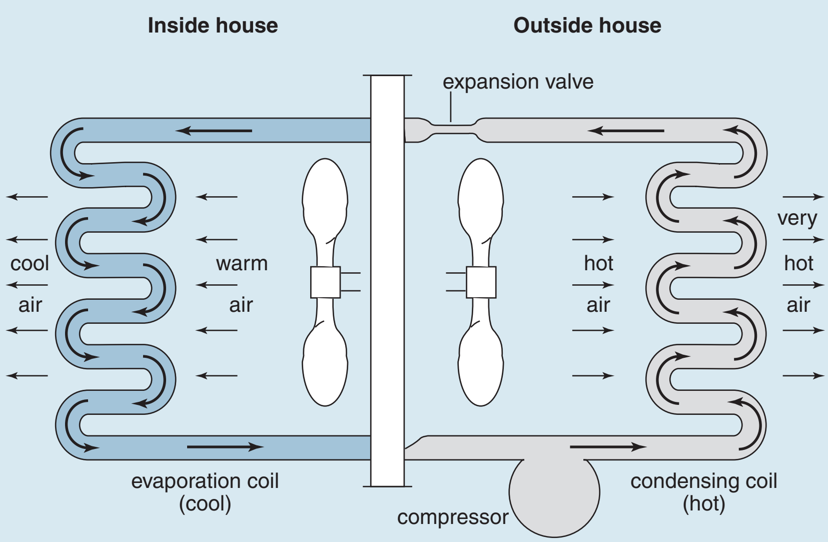


Figure 6.32
A refrigerative air-conditioner acts as a heat pump. Evaporating refrigerant transfers thermal energy from the surrounding environment. As the refrigerant condenses back to a liquid state outside the house, energy is released. Reversing the process allows a 'reverse-cycle' system to also act as a highly efficient heat pump.

inside the building. The refrigerant will transfer the heat energy to the surrounding air.

Modern reverse-cycle air-conditioners reverse this process, picking up heat energy from the outside of the house and releasing it inside once the vapour has condensed. It is an energy-efficient means of heating as it relies on only small energy inputs in addition to that provided by the local environment (approximately 20% of the heat being pumped). The environment itself supplies the heat. Only a small pump is required to circulate the refrigerant and change the pressure.

Gas refrigerators, working with the same principle, use an external source of heat, a gas flame, to increase the temperature (and therefore the pressure) inside the compressor tubes. This saves the need for mechanical energy. With an understanding of heat, it is relatively simple to see how heating is just the reverse of cooling—a transfer of energy!

6.6 SUMMARY Evaporation: heat energy in context

- The rate of evaporation of a liquid is increased by increasing air temperature, decreasing the vapour pressure and humidity of the surrounding air, increasing the circulation of the air, and a decreased latent heat of vaporisation of the material, i.e. a 'volatile' liquid.
- The temperature of a gas will decrease with decreasing pressure as the gas particles lose kinetic energy. The reverse occurs when gas pressure is increased.
- The boiling point of a liquid increases with increased pressure. This process is useful in the design and operation of energy-efficient refrigerators and cooling/heating systems.

6.6 Questions

1 If a container of water is partly covered by a lid then:
- **A** none of the water can evaporate
- **B** the space between the water and the cover will become saturated with water vapour
- **C** whether evaporation occurs will depend upon the average kinetic energy of the molecules in the water
- **D** the rate of evaporation will be faster than if it were uncovered.

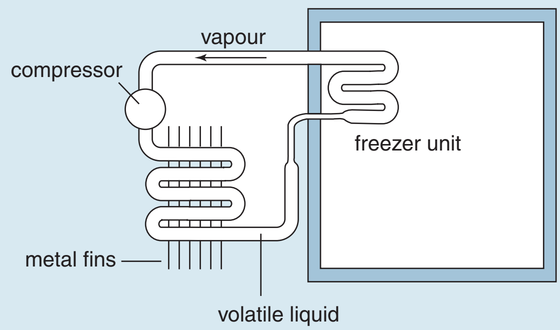
2 A popular method of keeping water cool on a long trip in summer is to hang a canvas bag on the front of the moving vehicle.
- **a** How does this method keep the water cool?
- **b** What conditions are important for this cooling method to work?

3
- **a** Why do humans perspire more on hot days? Explain why cooling occurs in terms of evaporation, kinetic theory and latent heat.
- **b** Would you expect perspiration to be as effective in humid weather? Explain.

4 Cats, kangaroos and other animals frequently lick themselves or pant on hotter days. Explain the process by which this helps to keep them cool.

5 Using simple kinetic theory, explain why a dish of petrol or methylated spirits placed in a draught of air will be at a lower temperature than its surroundings.

The following information applies to questions 6–9. The diagram illustrates the basic features of a simple refrigeration system.



6 Explain what happens in the pipes of the freezer unit, stating clearly how this section of the system manages to reduce temperature.

7 Why is this part of the unit situated toward the top of the refrigerator?

8 What is the purpose of the metal fins?

9 If the door was left open on a hot day, would such a refrigerator be able to cool the room? Why?

10 A marathon runner may lose several litres of water through perspiration during the course of a race. Typically, runners are able to convert stored chemical energy into useful energy with only 20% efficiency. The remainder of the energy is converted into heat which must be lost through evaporation if the body is not to overheat. If a particular runner uses 11 000 kJ during the course of a race, how many litres of water must be evaporated in order for the body to maintain a constant temperature?

6.7 Conduction and convection

Experience tells us that if two objects at different temperatures are in contact, then heat energy will transfer from the hot object to the colder object. There are only three possible means by which heat can be transferred: conduction, convection and radiation.

Conduction

Conduction is the process by which thermal energy is transferred from one place to another within a material. For example, if one end of a steel rod is placed in a fire, heat will travel along the rod so that the far end of the rod will also heat up despite the fact that nothing obviously moves along the rod. This process is most significant in solids; it is important in liquids but plays a lesser role in the case of gases. All materials will conduct heat to some extent. Materials that conduct readily are referred to as good conductors while materials conducting heat poorly are referred to as insulators.

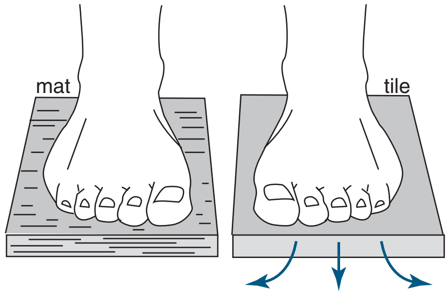


Figure 6.33
Materials can be classified as conductors or insulators of heat. Ceramic tiles conduct heat away from the foot readily so the foot feels cold. The mat is an insulator so heat is not removed from the foot so quickly. The part of the mat in contact with the foot warms up, reducing even further the rate at which heat is removed from the foot. The foot does not feel cold.

Conductors and insulators of heat

Why are some substances good conductors and others good insulators? Heat conduction happens in all materials at any temperature. Hence it is probably more appropriate to talk about good conductors and poor conductors rather than conductors and insulators. A material's ability to conduct heat depends on how conduction occurs within that material. It can happen in two ways: energy transfer through molecular or atomic collisions and energy transfer by free electrons.

Heat transfer by molecular collisions

The kinetic molecular theory of matter explains that particles in a solid substance are constantly vibrating within the crystal structure and so interact with neighbouring particles. If one part of the material is heated, then the particles in that region will vibrate more rapidly than their neighbours. Interactions with neighbouring particles will pass on the vibrational energy through the system through the bonds between particles.

This process can be quite slow since the mass of each particle is relatively large and their vibrational velocities are fairly low. Since the particles are vibrating relatively slowly, they will also be slow to pass on their vibrational energy. Substances where this method of conduction is the sole means of heat transfer are likely to be poor conductors, or insulators, of heat. Materials such as plastic, glass, wood, paper, ceramic and concrete are of this type.

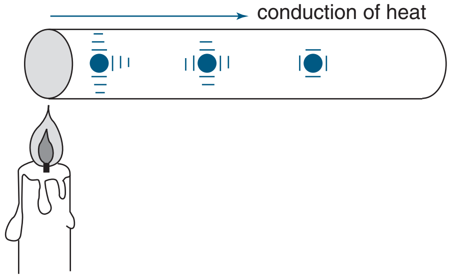


Figure 6.34
When a solid is heated, particles closest to the heat source gain energy and vibrate more rapidly. This heat energy is passed on during collisions between adjacent particles. Conduction in metals is enhanced by the extra collisions between free electrons within the metallic lattice.

Heat transfer by free electrons

Some substances, particularly metals, have electrons that are not involved in any one particular chemical bond; instead they are free to move throughout the entire solid. These free electrons also are referred to as 'mobile' or 'delocalised'.

If a metal is heated, not only will the larger atoms within the material gain extra energy, but so will these free electrons. As the electrons have only tiny masses compared to atoms, even a small energy gain will result in a very large gain in velocity. Consequently these fast moving electrons transfer heat energy throughout the whole material very quickly. It is

Table 6.5 Approximate thermal conductivities of some common substances

Substance	Conductivity (k) ($W\ m^{-1}\ K^{-1}$)
Silver	420
Copper	380
Aluminium	240
Steel	60
Ice	2.2
Brick, glass	~1
Concrete	~1 depending on composition
Water	0.6
Plaster	0.6
Human tissue	0.2
Wood	0.15
Polystyrene	0.08
Paper	0.06
Fibreglass	0.04
Air	0.025

therefore no surprise that metals, which are good electrical conductors because they have mobile electrons, are also good heat conductors for the same reason. Table 6.5 summarises the approximate thermal conductivities of some common materials. The relatively high values for metals are evident.

Factors affecting thermal conduction

The rate at which heat energy is transferred through any object will depend on the:

- temperature difference across the material. A greater temperature difference will result in a faster rate of heat energy transfer.
- thickness of the material. A thicker material requires a greater number of molecular collisions to transfer energy from one side of the material to the other, slowing the rate of conduction.
- surface area. Increasing the surface area increases the number of particles involved in the transfer process, thus increasing the rate of conduction.
- nature of the material. The larger a material's thermal conductivity (k), the more rapidly it will conduct heat energy.

Thus the rate of heat transfer (energy per time) through a material is given by:

$$\frac{Q}{t} = \frac{kA\,\Delta T}{L}$$

where Q is the heat energy transferred, in joules (J), per unit time (t)
k is the material's thermal conductivity ($W\ m^{-1}\ K^{-1}$)
A is the surface area (m^2)
ΔT is the temperature difference across the material (°C or K)
L is the thickness of the material (m).

Worked Example 6.7A

Calculate the amount of heat energy gained each second by a polystyrene drink box which is in the shape of a cube of sides 30 cm. The walls are 1.0 cm thick, the contents of the box are at 5°C and the outside air temperature is 40°C. Ignore any odd effects at the corners.

Solution

From Table 6.5, $k = 0.08\ W\ m^{-1}\ K^{-1}$.
Total surface area of the six surfaces of the container
$= 0.30 \times 0.30 \times 6 = 0.54\ m^2$
$\Delta T = 40 - 5 = 35$°C or 35 K
$L = 1.0$ cm or 0.01 m

Using $\frac{Q}{t} = \frac{kA\Delta T}{L}$

$$\frac{Q}{t} = \frac{0.08 \times 0.54 \times 35}{0.01}$$

and $Q/t = 150\ J\ s^{-1}$.
This means that the contents of the container are receiving heat at the rate of 150 joules per second.

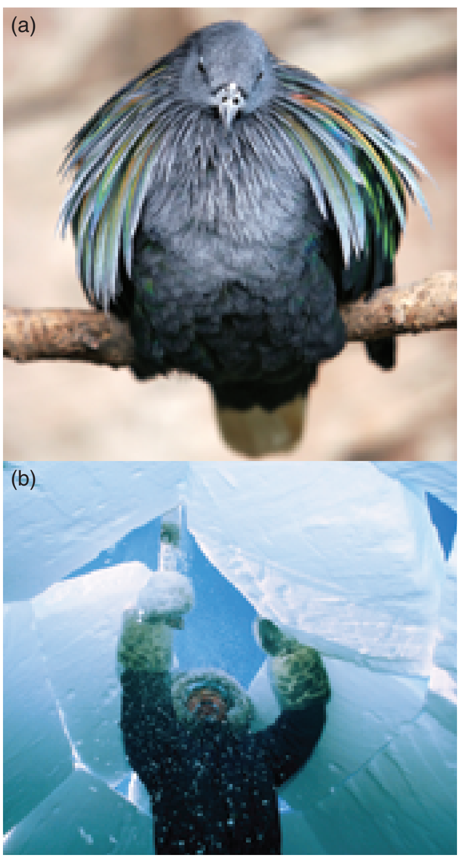


Figure 6.35
(a) Birds and other animals use air trapped within feathers and fur as a very effective insulator. (b) Humans employ similar ideas in housing and clothing. The ice of igloos provides quite effective thermal insulation as does thick fur clothing, trapping insulating air around the body.

Convection

Convection is the transfer of heat energy within a fluid (liquid or gas) where a hot parcel of fluid will move from one place to another. If one part of a fluid is heated, the material there will expand and so become less dense. The hotter material, being less dense, will rise and the colder, denser material will tend to sink. This gives rise to a 'convection current'.

Although liquids and gases are generally not very good conductors of heat, they can transfer heat quite rapidly through convection. Unlike other forms of heat energy transfer, convection involves the mass movement of particles over a distance.

Figure 6.36 shows convection of water in a beaker being heated from below. The entire beaker will be heated rather than just the water at the bottom. The same process occurs when air is heated in a room.

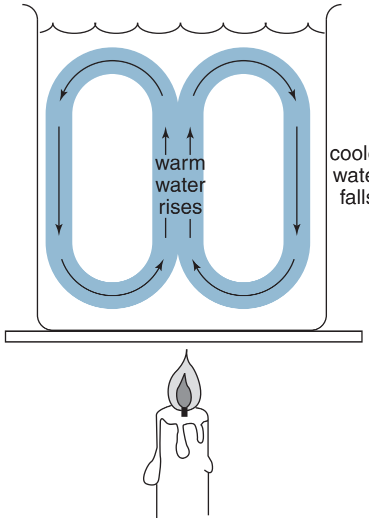


Figure 6.36
When a fluid (liquid or gas) is heated it becomes less dense and will rise, while the colder, denser fluid falls to replace it. As this fluid heats up it in turn will rise, creating a convection current. Coloured dye in a container or smoke in air will clearly show this movement.

Physics in action — Natural convection

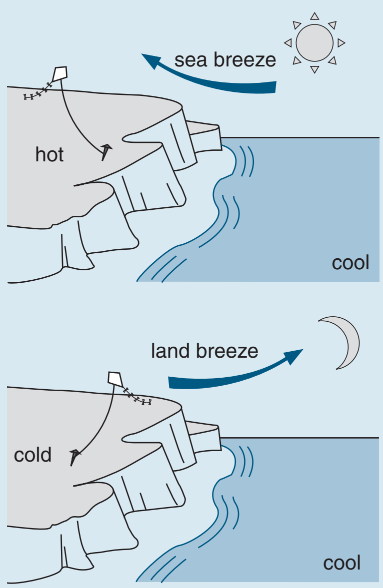


Figure 6.37
Natural convection in air.

At the coast there is often a temperature difference between the land and the sea. The water in the sea hardly changes temperature between night and day, but the land becomes much hotter through the day when heated by the Sun. As the air over the land is heated, it rises and is replaced by cooler, denser air from over the sea. This moving air creates a sea breeze experienced in most coastal areas in Australia during summer. These sea breezes generally make the summer weather near the coast more pleasant on hot days than that in inland regions.

During the night the process is reversed. The land cools more quickly, cooling the air above it. This denser air moves out over the ocean, displacing the now relatively lighter and warmer air, and a land breeze is created.

When relatively large diameter pipes are used, convection currents can be relied upon to carry hot water up from a boiler to a storage tank without the use of a pump. This is particularly useful in remote areas where a solar hot water system uses this principle. Systems using pipes of a regular household diameter may need a pump to assist the circulation.

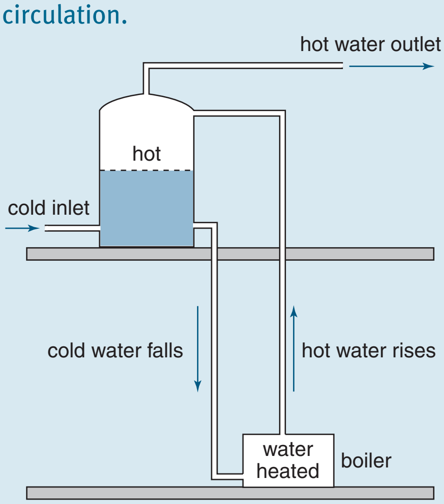


Figure 6.38
Natural convection in pipes.

As air is heated, the air particles gain kinetic energy and push apart, lowering the density of the air and rising. Colder air, with slower moving molecules, is more dense and heavier and so falls. A convection current forms. Warm or cold ocean currents, wind and weather are a result of convection on a large scale.

'Forced convection' refers to systems that use fans to blow the hot air around. Fan-forced ovens and fan heaters move hot air around quickly with a fan. This is not really convection as it does not result from buoyancy due to density changes.

The human body generates a great deal of heat itself; over 80% of the energy from the food we eat is transformed into heat. During light activity, if this heat energy were not lost to the surrounding environment, body temperature would rise by about 3°C per hour. Conduction is responsible for very little of the transfer of heat energy from inside the body's core that must occur if a stable body temperature is to be maintained. Instead heat is carried to the surface by blood. It acts as a convective fluid, transferring heat energy to the subsurface of the skin. Heat energy is then conducted the short distance to the surface of the skin where it is lost to the surrounding environment by a mix of convection, conduction and radiation (more on radiation in the next section).

It is difficult to derive an appropriate formula to deal with convection quantitatively as the rate of transfer depends upon a number of complex and interrelated factors. Some estimates can be made. The rate at which convection occurs (Q/t) is approximately proportional to:

- the temperature difference between the object and the convective fluid
- the surface area exposed to the convective fluid.

Thus:

$$\frac{\text{rise}}{\text{run}} = hA\,\Delta T$$

where h is a coefficient depending on speed of flow, conduction, shape of object and other factors
A is the surface area of the material (m^2)
ΔT is the change in temperature (°C or K).

In still air the human body loses energy by convection at about 6 J m^{-2} s^{-1} K^{-1}, doubling in an air speed of 1 m s^{-1} and increasing five times in an air speed of 5 m s^{-1}.

Physics in action — Wind chill

Convective effects are the main means of heat transfer that lead to the 'wind chill' factor. The wind blows away the thin layer of relatively still air near the skin which would normally act as a partial insulator in still air. Cooler air comes in closer contact with the skin and heat loss increases. It feels as if the 'effective' temperature of the surrounding air has decreased. In cold climates the wind chill factor can become significant. As a general approximation, or 'rule of thumb', each 1 km h^{-1} of wind speed creates an effective decrease in temperature of 1°C. An air temperature of 0°C has an effective temperature of −30°C in a 30 km h^{-1} wind. Skiers can experience similar effects simply from the wind created by their own motion.

The chilling effect is even more dramatic when the body or clothing is wet, increasing evaporative cooling. Bushwalkers look for clothing that dries rapidly after rain or river crossing and which carries moisture from the perspiration of heavy exertion away from the skin.

6.7 SUMMARY Conduction and convection

- Conduction is the process of heat transfer through a material without the overall transfer of the substance itself.
- All materials will conduct heat energy. Materials conducting heat readily are good conductors; materials conducting poorly are insulators.
- Whether a material is a good conductor depends on the method of conduction: heat transfer by molecular collisions alone—poor conductor; heat transfer by particles and free electrons—good.
- The rate of thermal conduction depends on the temperature difference, the thickness of the material, the surface area and the nature of the material.
- Convection is the transfer of heat energy by the mass movement of a hot fluid (i.e. a liquid or gas) from one place to another within the fluid.
- The circulation of a material due to convective movement of heat is called a convection current. Convection currents may be 'forced' or 'natural'.
- Sea and land breezes and ocean currents are due to large scale, natural convection currents.
- The rate of convection is affected by temperature difference and the exposed surface area.
- Wind chill is due to the convective effects of air moving over the skin and increasing heat loss. The 'effective' temperature falls by approximately 1°C for every 1 km h^{-1} of wind speed.

6.7 Questions

1 Which of the materials listed in Table 6.5 would be a suitable choice for:
  a insulation material inside the door of a refrigerator?
  b the heat exchanger of a ducted gas home heating system?
  c the handle of a saucepan?

2 a Calculate the rate at which thermal energy could be lost from a 3 mm thick copper pipe which is carrying water at 100°C, if the surrounding air is at 20°C.
  b In reality the rate of heat loss would be much less than this calculated value. Explain how the surrounding air reduces the heat loss.
  c Explain what measures are usually used to reduce heat loss even further from hot water pipes.

3 a Calculate the heat energy that would be lost through a glass window in one hour when the window measures 2.0 m by 1.4 m. The inside temperature is 20°C, the outside is 5°C and the glass is 7 mm thick.
  b Suggest two ways in which the rate of heat loss from the window could be reduced

4 Explain in terms of the kinetic molecular theory why you can 'burn' yourself by touching an extremely cold object such as dry ice.

5 On a cold day the plastic or rubber handles of a bicycle feel much warmer than the metal surfaces. Explain this in terms of the thermal conductivity of each material.

6 Explain what is meant by the term 'fan-forced convection oven' and explain its advantage over a conventional oven.

7 Ceiling fans fitted in houses or in schools are designed to rotate forward and in reverse so that they can be used in both summer and winter. Explain the reason for this.

8 Convection is referred to as a method of heat transfer in fluids, either liquids or gases. Could solids pass on their heat energy by convection? Explain.

9 On a reasonably mild winter's day a wind of 30 km h^{-1} is blowing. The temperature reaches 15°C yet people are complaining of the cold.
  a What is the effective temperature?
  b Why wouldn't a conventional thermometer record this effective temperature?
  c How could a thermometer be modified to allow it to measure the effective temperature?

10 The pilots of glider aircraft and hang-gliders, some birds such as eagles and albatross, and some insects rely on 'thermals' to give them extra lift. Explain how these rising columns of air are established.

6.8 Radiation

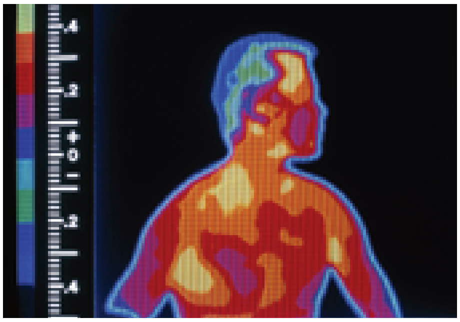


Figure 6.39

Radiant energy is emitted by all objects above absolute zero and the wavelengths of the radiation emitted will depend on the temperature of the object. Most objects at everyday temperatures emit some radiation in the infrared wavelengths. A thermograph detects infrared radiation and can be used to find a lost person, indicate where energy is being lost from a building, predict weather patterns or assist in medical diagnosis.

Both convection and conduction involve heat transfer through matter. Yet life on Earth depends upon the transfer of energy from the Sun through near-empty space. If heat energy can be transferred by the action of particles, as is the case for conduction and convection, then how can the Sun's energy come to the Earth? Radiation represents a case where heat is transferred without the movement of particles.

The phenomenon uses a form of energy transfer by *radiation*. Radiation is another expression for electromagnetic waves (which include such things as visible, ultraviolet and infrared light). These waves travel at the speed of light and can be focused, refracted and reflected. When electromagnetic radiation falls on an object, it is partly reflected, partly transmitted and partly absorbed like any other wave-like phenomenon. In this case, the absorbed part transfers heat energy to the absorbing object and causes a rise in temperature.

RADIATION, or transfer of heat energy from one place to another, is through electromagnetic waves.

Electromagnetic radiation is emitted by all objects whose temperature is above absolute zero (–273°C). A human body emits infrared radiation. Your hand is warmed if it is in close proximity to, say, your forehead. Your temperature is also revealed in infrared photography or thermography. The wavelength of the emitted radiation will decrease as the temperature of the emitting object increases. Visible light and ultraviolet wavelengths are also present if the object is very hot, for example light filaments, fire, a red hot poker and the Sun.

Emission and absorption of radiant energy

All objects emit some radiation but they will not all emit or absorb at the same rate as each other. Matt black surfaces emit radiant energy at a greater rate than shiny, light-coloured surfaces. Not only does this mean that a roughened, dark-coloured surface will heat up faster than a shiny one, but it will cool down faster too since it will also radiate energy more efficiently. An application of this is the car radiator which is painted black, not to increase the absorption of radiant energy but to increase the emission of energy collected from the car engine by conduction and convection.

There are a number of other factors that will affect the rate of both emission and absorption of radiation.

Surface area

With a greater exposed surface area, more energy can be radiated. The greater the surface area, the greater the rate.

Temperature

Both the temperature of the absorbing or emitting surface and the temperature of the surrounding objects can affect the rate of radiation. All objects simultaneously transfer energy to and from their surroundings. The temperature of the object will fall while the temperature of the surroundings will rise. It is estimated that 50% of heat energy loss from a person at normal room temperature occurs because of radiation.

Eventually bodies in contact will come to the same temperature as each other. They are then said to be in 'thermal equilibrium' since the rate at which they are losing energy will be exactly the same as the rate at which they are receiving it. Their respective temperatures will remain steady.

Wavelength of the incident radiation

It has already been noted that a matt black surface emits and absorbs radiant energy well. In fact it is an almost perfect absorber of radiation at all wavelengths. Polished, highly reflective surfaces are good reflectors of radiation at all wavelengths. On the other hand matt, light coloured surfaces behave differently toward radiation of different wavelengths. White surfaces absorb visible wavelengths poorly but may absorb infrared radiation just as well as black surfaces.

Surface colour and texture

The characteristics of the surface itself also determine how readily that particular surface will emit or absorb radiation. This is the *emissivity*, e, of the surface. Matt black surfaces have a value of e close to 1, whereas shiny surfaces have e values close to zero; they emit and absorb substantially less radiation. The emissivity of skin is surprisingly high—about 0.6 for light skin and 0.8 for dark skin. A perfect emitter and absorber has an emissivity of 1 and is referred to as a 'black body'.

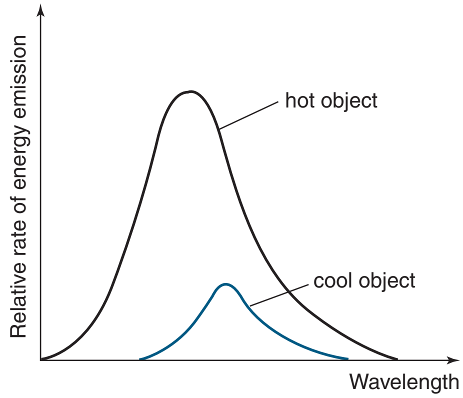


Figure 6.40
An object at a low temperature will emit very little radiation, and only of long wavelengths. As the temperature of the object increases, the largest proportion of the radiation emitted shifts to shorter wavelengths and the total radiant energy emitted also increases.

Figure 6.41
The silvered surface of an emergency blanket reflects, and hence retains, the radiant energy lost by the normal body at low temperatures. This simple method for thermal insulation can reduce rapid heat loss that can otherwise lead to hypothermia or, in the case of injury, deep shock.

Physics file

The transfer of radiant heat energy is given by the Stefan–Boltzmann equation:

$$Q = e\,\sigma\,A\,T^4\,t$$

and for small temperature differences (<20°C):

$$Q = e\,\sigma\,A\,(T_o^4 - T_s^4)\,t$$

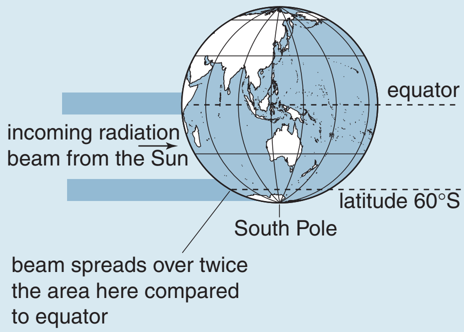
where e is the emissivity of the surface with a value between 0 and 1
σ is the Stefan–Boltzmann constant with a value of 5.67×10^{-8} J s^{-1} m^{-2} K^{-4}
A is the object's total surface area (m^2)
T_o is the temperature of the object (K)
T_s is the temperature of the surroundings (K)
t is the time (s).

Physics in action — Solar radiation and the greenhouse effect

In an age when the Earth's resources are being seriously depleted it seems incredible that the Earth receives 1.8×10^{17} J of radiant energy from the Sun every second. This means that the total energy output from the Sun in all directions is 3.9×10^{26} J s^{-1}. Radiant energy from the Sun is distributed across all types of electromagnetic radiation with the predominant amounts lying within or close to the visible spectrum.

Figure 6.42

The amount of incoming radiant energy received by each square metre of the Earth's surface is affected by latitude. At the equinox, when the Sun lies directly over the equator, the amount of radiation received at the equator is a maximum. The same amount of energy is spread over a large area in northern and southern latitudes. In December the tilt of the Earth on its axis brings the Sun over the Tropic of Capricorn, significantly increasing the amount of radiant energy received in southern regions leading to the higher temperatures of the southern summer. Seasons are not due to the small decrease this tilting causes in the distance to the Sun.



When the Sun is directly overhead, each square metre of the Earth's upper atmosphere receives an average of 1365 J of energy. Closer to either pole this amount of energy will spread over a larger area as Figure 6.42 suggests. This is why the poles are considerably cooler than the Earth's equatorial regions.

At the autumn and spring equinox, when the Sun is directly above the equator, on a relatively clear day every square metre at Perth's latitude will receive about 1000 J of energy each second at midday. Further south at about 60° latitude this falls to 700 J. As spring progresses into summer, the orientation of the Earth's tilt on its axis to the Sun brings the Sun more directly over our heads and increases the amount of radiant energy received over each square metre. The average temperature increases. In winter the reverse occurs.

About 80% of the radiant energy arriving at the outer edge of the Earth's atmosphere will pass through to the surface. Twenty per cent or so of the incident radiant energy is absorbed by the atmosphere, predominantly by the ozone layer but also by water vapour and greenhouse gases such as carbon dioxide. Of the remaining 80%, clouds reflect about 22% and the Earth's surface reflects about 3%. This leaves about 50% to be eventually absorbed by the Earth's surface.

The ozone layer, a concentrated layer of O_3 molecules in the outer atmosphere 20–50 km above the surface, plays an important role in reflecting and absorbing radiation in the ultraviolet region. This results in significant heating of this region, warming the Earth, but also reducing the amount of ultraviolet radiation reaching the Earth's surface. High-energy ultraviolet radiation can cause skin cancer in humans and the death or mutation of small cellular life, so any depletion of the ozone layer is of considerable concern. Ozone is generated naturally through lightning strikes and other processes but is broken down by chlorofluorocarbons, or CFCs, which were used until recently in aerosols and refrigerants. It will take some 80 years to recover fully.

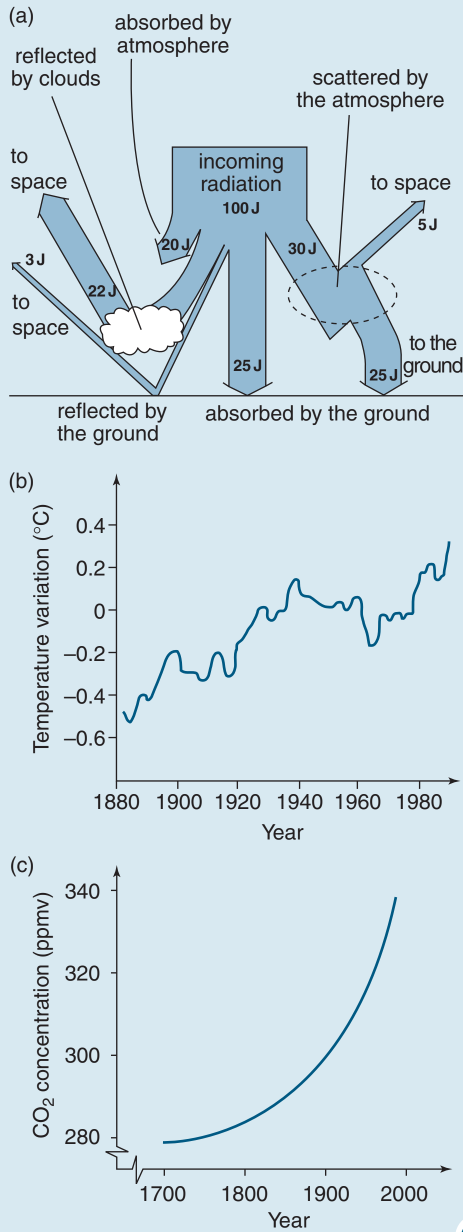


Figure 6.43

Like all electromagnetic radiation, radiant heat energy is reflected, transmitted and absorbed in proportions dependent on its wavelength and the material through which it passes. (a) Of every 100 J incident on the Earth's surface, only 50 J is eventually absorbed. The remainder is reflected back into space or absorbed by the atmosphere. This has led over time to the establishment of a long-term energy balance allowing life to evolve. Additional energy is re-radiated as infrared radiation.
(b) In recent years the average temperature of the Earth's atmosphere has increased, as the concentration of greenhouse gases, particularly carbon dioxide, has increased (graph c). The long-term implications of this trend have led to reviews of our current production of carbon dioxide from the burning of fossil fuels and clearing of forests.

If the Earth and its atmosphere were to stop receiving energy from the Sun, the atmosphere would gradually cool. Under normal conditions the average temperature remains relatively constant, with fluctuations of approximately 15°C. The atmosphere acts as a heat trap, absorbing both energy from the Sun and reflected and emitted radiant energy from the Earth's surface. Radiant energy emitted by Earth is of longer wavelengths, due to the cooler temperature of the Earth than the Sun. This is readily absorbed by the atmosphere, creating a desirable sort of greenhouse effect. It results in the consistent temperatures over day and night required by the life forms that have evolved on Earth.

In recent years, evidence has suggested that an enhanced greenhouse effect is leading to the atmosphere absorbing and retaining more of the emitted energy from the Earth's surface. Specific gases present in the lower atmosphere in very small amounts are responsible for most of the absorption of these longer wavelengths. The most significant of these gases have increased in concentration due to human activities over the past 100 years. These include carbon dioxide (CO_2), water vapour (H_2O), methane (CH_4) and nitrous oxide (N_2O). Chlorofluorocarbons and ozone also make significant contributions. Accumulating evidence supporting the greenhouse theory have recently led to the proposal of global policies aimed at limiting the increase in greenhouse gases. A predicted increase of 1.5°C to 4.5°C would lead to considerable changes in climate throughout the world with many low-lying countries likely to be largely inundated. Unfortunately, greenhouse gas reduction policies are not as successfully supported as those on ozone depletion.

6.8 SUMMARY Radiation

- Radiation transfer of heat energy from one place to another is by means of electromagnetic waves.
- Any object whose temperature is greater than absolute zero emits heat energy by radiation. The rate of emission or absorption will depend on the temperature of the object and of the surrounding environment, surface area of the object, wavelength of the radiation and surface characteristics or emissivity of the object.
- Emissivity varies between 0 and 1. Matt black surfaces have a value close to 1 and both emit and absorb radiant energy better than white shiny surfaces.

6.8 Questions

1. A person is sitting, unclothed, in a room which is at a temperature of 15°C. Calculate the rate at which heat energy is lost by radiation if the person's skin temperature is 34°C and $e = 0.70$. Assume also that the exposed surface area of the person is 1.5 m^2.
2. Electric hot-water urns, such as those used in canteens, are silvery on the outside. Why?
3. Some human populations have dark skins yet live in hot climates. Dark skin has an emissivity greater than that for fair skin. As a consequence dark skin will absorb more heat energy than will fair skin. Why do you think this evolutionary adaption may have occurred?
4. Using the same logic as in question 3 explain why:
   - **a** many people have fair skin and light hair in the northern regions of Europe
   - **b** Inuits (Eskimos) living in extreme northern latitudes have a darker skin.
5. Three beakers are of identical size and shape. One beaker is painted matt black, one is dull white and one is gloss white. The beakers are filled with near-boiling water. In which beaker will the water cool most quickly? Give reasons. Which one will be slowest?
6. In recent years, surfers' wetsuits have changed from being predominantly black to including large areas of bright, light colours, largely to follow trends in fashion. Considering the effect on transfer of radiant heat energy, will these new suits be more or less effective insulators?
7. Two cars are painted black and white respectively. Comment on the effect this will have on the internal temperature of each car.
8. Reflective foil insulation (i.e. sisalation) is used as a means of insulating the walls and roofs of houses. Explain why this will be better at reducing energy gain in summer than reducing energy loss in winter.
9. It has been suggested that radiant heaters, such as open fires, would be more efficient if they were used to warm the surfaces of a room rather than just the room's occupants. Is this true? Explain.
10. Infrared photography can be used to detect marijuana crops even in relatively thick bush. The crops show up as brighter regions on a photograph taken during daylight. Does this mean that marijuana leaves have a high or low emissivity?

Chapter 6 Review

1 Burning a match creates a transfer of energy from one form to another. The main change of energy is from:
A chemical energy to thermal energy
B thermal energy to chemical energy
C sound energy to kinetic energy
D light energy to heat energy
E wood energy to light energy.

2 The bulb of a simple alcohol thermometer is made of *thin* glass. The main reason for this is so that:
A it can magnify the liquid and make it easier to read
B alcohol can be used instead of mercury
C the liquid will expand further up the tube with less space in the bulb
D the liquid will begin to expand more quickly.

3 If the particles within two objects have the same average kinetic energy, are the two objects at the same temperature? Explain.

4 A thermometer is supplied with no markings. Describe the process you would go through to produce a calibrated scale. Would your scale be arbitrary or absolute? Explain.

5 A bucket is filled with equal amounts of hot and cold water. The hot water is originally at 80°C and the cold water at 10°C. The temperature of the final mixture will be approximately:
A 10°C
B 25°C
C 45°C
D 70°C
E 90°C.

6 How much energy, in joules, is needed to raise the temperature of 100 kg of water from a room temperature of 20.0°C to a comfortable bath temperature of 35.0°C? (Assume no losses to the surrounding environment.)

7 An electrical water heater is used to heat the bath water in question 6. If the heater is rated at 5.6 kW, how long will it take to heat the water?

8 4000 joules of energy is required to raise the temperature of 1 kg of paraffin by 2°C. How much energy is required to raise the temperature of 5 kg of paraffin by 1°C?
A 20 kJ
B 2 kJ
C 1 kJ
D 10 kJ
E 10 MJ

9 Hypothermia is the cooling of the body to levels considerably lower than normal. The body's functions slow, and death can result. A person may survive 12 hours in air at or near 0°C before suffering severe hypothermia yet only survive a few minutes in water of the same temperature. Why?

10 A student attempts to identify a metal by measuring its specific heat capacity. 100 g of the metal is heated to 75°C and then transferred to a 70 g copper calorimeter containing 200 g of water at 20°C. The temperature of the final mixture is 25°C. Using Table 6.2 as a comparison, what metal is the student probably testing?

11 Which of the following examples supports the statement that it takes a larger amount of thermal energy to melt ice than to warm air?
A Moisture forms on the outside of a glass of ice water.
B Ice cubes in a freezer can be colder than 0°C.
C Glaciers and ice flows last in some areas of New Zealand throughout summer.
D Snow storms can occur at low altitudes in winter during extreme weather conditions.

12 Energy must be supplied to ice in order for it to melt. The temperature of the resulting water is no higher than that of the original ice. Explain.

13 How many joules of energy would be required to melt exactly 100 g of ice, initially at –4°C? (Assume no losses to the surrounding environment.)

14 Vegetables have been brought to the boil in a saucepan on the stove. Will they now cook faster with the gas flame on high or left to simmer with the gas flame on low?

15 0.50 kg of ice at 0°C is mixed with 0.10 kg of steam at 100°C. What will be the final temperature?

16 Steam produces much worse burns than does boiling water. Compare the amount of energy the body, at a temperature of 37°C, receives from the cooling of 0.10 g of water from 100°C, with that received from the cooling of 0.10 g of steam originally at 100°C.

17 During mild exercise, a person transfers away from the body 5.0×10^5 J of energy by evaporation of water from the skin. What volume of water has been lost?

18 How much thermal energy must a refrigerator remove from 0.50 L of water at 20°C to make ice at –10°C?

19 Which of the following best describes the effect of an increase in pressure on the boiling point of water?
A lowered
B raised
C unchanged
D unrelated
E either A, B or C depending on temperature.

20 A 2.5 kW kettle is left switched on for one minute after the water in it has started to boil. The kettle initially contains 100 g of water. Would the kettle boil dry? Justify your answer by showing your working.

21 Refrigerators have the freezing unit at the top yet cold air is denser and falls. Explain, with the use of diagrams, why this design feature works.

22 Suppose some students did an experiment to examine Boyle's law. Their initial sample volume was 10.5 cm^3 and they increase the pressure of the sample from 105 kPa to 160 kPa. To what new volume would they reduce their sample?

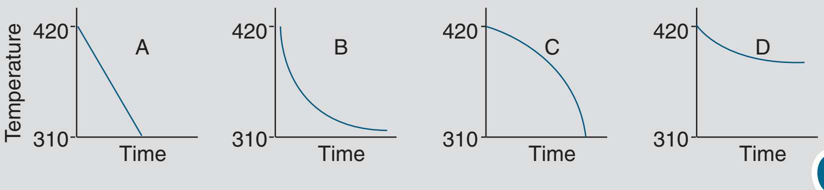
23 Design a system for a hot water storage tank which would permit hot water to be taken from the tank and replacement cold water to enter without the two mixing. Include in your design appropriate connections to allow the water to be circulated without the need for pumps.

24 If you are lost in the snow, you are advised to build yourself a snow cave. In terms of thermal conductivity, explain how it is possible to stay warm inside a cave made of snow or ice.

25 Explain why it is that you can stay comfortably warm in still air at 20°C but that you would become cold fairly quickly in still water at the same temperature.

26 An aluminium can with a paper label is left in a deep freeze for some time to cool the contents. On taking the can out of the freezer your hand sticks to the cold aluminium but not to the label.
a Which is at the lower temperature?
b The label is peeled off the can. Which will return to normal room temperature first? Why?

27 The graphs below correspond to the cooling curves of several different materials. Which material would make the best thermal insulator? Justify your answer.



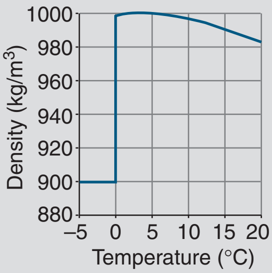
28 When equal masses of water and paraffin are supplied with heat at the same rate, the temperature of the paraffin rises faster because paraffin has a:
A lower density
B lower boiling point
C lower specific heat capacity
D lower thermal conductivity
E lower melting point.

29 Supermarkets use large, uncovered chest freezers to store and display frozen food.
a Explain, with the aid of diagrams, why this format of frozen food storage is effective.
b These chest freezers have maximum load lines (food must not be packed to lie above this line). Why is this necessary? Modify your diagram, if necessary, to illustrate your explanation.

30 Which of the following statements is/are true?
i In cold weather the wooden handle of a saucepan feels warmer than the metal pan because wood is a better conductor of heat.
ii Convection occurs when there is a change of density in the parts of a fluid.
iii Conduction and convection cannot occur in a vacuum.
A i, ii, iii
B i, ii
C ii, iii
D i
E iii

31 Which of the following statements is not true?
A Energy from the Sun reaches the Earth by radiation only.
B A dull black surface is a good absorber of radiation.
C A shiny white surface is a good emitter of radiation.
D The best heat insulation is provided by a vacuum.
E A vacuum flask is designed to reduce heat loss or gain by conduction, convection and radiation.

32 The graph below shows the variation of the density of water with temperature. It may be deduced from the graph that:
i a given mass of water has a minimum volume at 4°C
ii between 0°C and 4°C convection will not take place in a vessel of water heated at the bottom
iii expansion occurs when water freezes.
Which statement/s is/are correct?
A i, ii, iii
B i, ii
C ii, iii
D i
E iii



The following information relates to questions 33–38.
A power-generation system is set up in an area remote from mains power. It includes the following devices:
- a water-powered turbine operating at an efficiency of 45%
- a hot water service operating at an efficiency of 75%
- a storage battery operating at an efficiency of 90%
- a low-voltage fluorescent light operating at an efficiency of 50%.

Note that, in general, percentage efficiency is calculated as (output/input) × 100.

33 State the intended energy transfer occurring in the operation of each device.

34 State which of the energy forms identified is high-grade energy.

The turbine is connected directly to the hot water service.

35 What is the combined efficiency of the two devices operating in tandem?

36 If the water wheel is turned by moving water producing 32 MJ of energy, what volume of water could be heated by the hot water service from 20°C to 70°C (the specific heat capacity of water is 4200 J kg^{-1} K^{-1})?

37 What is the combined efficiency of the turbine, storage battery and light operating in tandem?

38 What power, in watts, must be developed by the turbine in order to keep a 30 W fluorescent globe glowing at its normal operating level? Note that one watt is one joule per second.

39 A student reduced the temperature of a gas bottle from 50°C to –10°C. What would be the percentage reduction of the pressure in the bottle?

40 Suppose the air and fuel vapour mixture, initially at 130 kPa, in the cylinder of a diesel engine was compressed from a volume of 450 cm^3 to a volume of 60 cm^3 while at the same time the temperature was increased from 100°C to 400°C. By how much would the pressure in the cylinder increase?

7 Electricity

Lightning is electricity in its raw, most magnificent and awe-inspiring form. To be close to an electrical storm is an impressive, if somewhat frightening, experience. It hints at the enormous forces locked up in ordinary matter. Lightning reminds us that electricity is a fundamental phenomenon in nature. The electric forces within and between atoms bring the enormous diversity and richness to the structure of matter and life that we see about us. Within our bodies, at any moment, there are millions of tiny electric currents in nerve fibres, not only obeying our conscious instructions, but also controlling and monitoring all the unconscious processes that keep us alive and well, and without any direct awareness on our part.

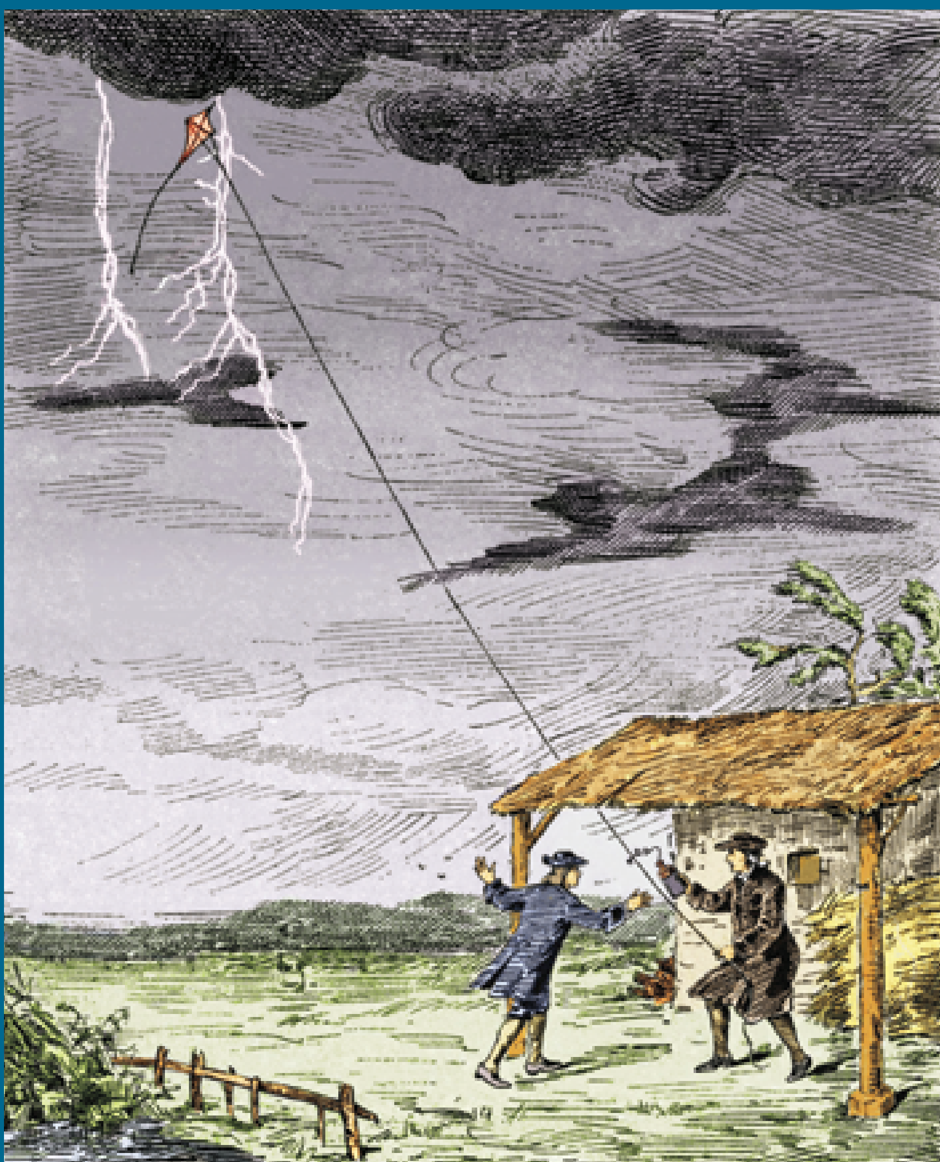
In between the hundreds of millions of volts of a lightning bolt and the millivolts driving the nerve currents are the more moderate voltages of batteries and household electricity supplies. A century ago, electric light totally changed the way we used time at night. Electric motors are used in an enormous variety of modern devices, such as trains, refrigerators, printing presses, lifts, hair driers, DVD players, toys and computers. Electronic devices, from a simple MP3 player to sophisticated robotic factories, have become an essential part of the modern world.

Many questions can be asked about the wise use of modern technology. For example, the use and generation of electricity involve fundamental questions for both developed and developing societies. There is a lot of talk of 'alternative' forms of energy, but much of it is ill-informed. This discussion needs to be taken seriously by a well-informed public. An understanding of the basics of electrical technology is a very good place to start for anyone who wishes to help find sensible answers to these questions. An understanding of electricity is vital, not only in order to come to grips with our modern technological society, but also to enable us to discover the fundamental nature of matter and the universe—and after all, that is really what physics is about.

By the end of this chapter

you will have covered material including:

- the concepts of electric charge and electric forces
- the concepts of current, EMF and electric potential
- resistance in ohmic and non-ohmic conductors
- electrical energy and power
- series and parallel circuits
- characteristics of cells, batteries and power supplies.



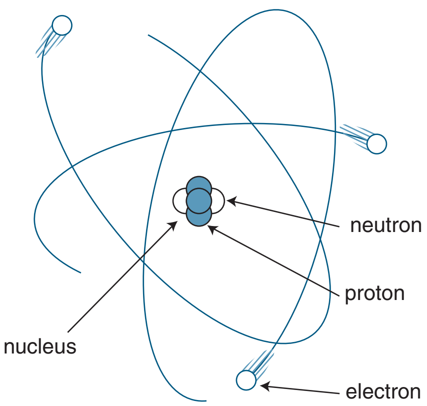


Figure 7.1
The way we draw an atom is not meant to be a representation of what it would 'really look like'. For a start, if it was drawn to scale both the nucleus and electrons would be so small they would be invisible!

7.1 Electrical charge

When a plastic pen, rubbed with a dry cloth, is brought near some small pieces of paper, the paper may 'dance'. Some of the bits of paper may even jump on and off the pen, seemingly at random. The pen has gained an *electrostatic charge*, which creates an *electric field* around it. This field will cause the paper to experience a force. Why?

As you will know, present theory suggests that all matter in the universe is constructed from around 100 different atoms. Further, all atoms are made up of just three fundamental particles: the proton, the electron and the neutron. You will have investigated the properties of these particles in Chapter 5, Nuclear energy.

Atomic charge

Inside the atom, the heavier protons and neutrons reside together in an extremely dense, positively charged sphere called the nucleus. The nucleus contains almost all the mass of the atom, but occupies only a tiny fraction of its volume. Orbiting at relatively large distances are the negatively charged electrons. These particles are very light indeed. Typically, the electrons only contribute about $^1/_{4000}$ of the mass of the atom. However, their orbits define the size of the atom, and, importantly, they balance the positive charge of the nucleus.

Ever since the time of Benjamin Franklin's experiments with electricity, it has been realised that electric charge appears to be indestructible. Franklin was the first to suggest that all matter is made up of equal amounts of positive and negative charge, normally in balance. Electrification, he suggested, is the transfer of some of this charge from one object to the other, resulting in an imbalance between the charges. To demonstrate this he had two people stand on insulated stools. One used a cloth to rub a glass rod held by the other. Afterwards, when they each took the charge from their cloth and glass respectively, and then brought their fingers close, a spark jumped between them and they both lost their charge.

This type of experiment led to the idea of 'conservation of charge'. That is, charge cannot be created or destroyed, only transferred from one object to another. If, for example, a glass rod is rubbed by a cloth and the rod acquires a positive charge, then the cloth will have acquired an equal amount of negative charge. Overall the charge is still zero. The principle of conservation of charge is now regarded, like conservation of energy, as one of the central principles of modern physics.

Physics file

The word 'atom' is derived from the Greek word 'atomos' meaning indivisible. However, during the past century, physicists have not only shown that there is an internal structure to the atom, but that there is even an internal structure to the protons and neutrons.

As well as their mass, the charge (or lack of it) carried by these atomic particles is an intrinsic feature of each of them. It is impossible to remove the charge from, say, an electron. One cannot have an electron without negative charge or a proton without positive charge! It is not possible to somehow 'charge' a neutron. When an object becomes charged it is because there has been a transfer of charged subatomic particles, normally electrons, either to or from the object.

Electric charge is conserved. This means that it cannot be created or destroyed, only transferred from one object to another.

Electrostatic charge

In all chemical reactions it is the outer electrons orbiting the atom that are either swapped or shared between atoms. Remember that the orbiting negative electrons are held to the nucleus by the attraction of the positive protons. The atoms of different elements 'hang on' to their electrons to varying degrees and these differences are responsible for the huge variety of chemical reactions that occur around us.

When two different materials are rubbed together, this tendency for electrons to move between atoms normally results in one of the materials gaining electrons at the expense of the other. The one which gains electrons will thus attain a negative charge and the other, now with fewer electrons than protons, will become positive. It is worth noting that it is the close proximity, rather than the rubbing itself, which is needed to produce an electrostatic charge. However, rubbing a plastic rod with wool brings many more atoms of the two materials into close contact than would occur simply by pressing them against each other.

> An excess of electrons causes an object to be negatively charged, and a deficit in electrons will mean the object is positively charged.

Perspex and polythene are two common materials that are easily charged by rubbing with cloth. In the process of rubbing, polythene tends to gain electrons and so becomes negatively charged. On the other hand Perspex tends to lose electrons and become positive.

If a charged polythene strip is brought near another charged polythene strip, the two strips will repel. However, attraction is seen when a charged polythene strip is brought near a charged Perspex strip. In both cases, the effect is greater the closer the strips are to each other.

> Like charges repel and unlike charges attract. The closer the charges are to each other, the stronger the force.

The Van de Graaff generator is often used as a source of electrostatic charge in the laboratory. In effect, it deposits the charge produced by the contact between a plastic roller and a rubber belt onto a metal dome. While the belt is running, the charges on the dome become more and more concentrated. However, anyone who has watched a Van de Graaff generator in action knows that the charge does not keep building up for ever. Eventually the concentration of charge becomes so great that charges start to jump off the dome—either as a bright spark across to an earthed object or as tiny crackling sparks into the air.

A unit for charge

In order to measure the actual amount of charge on a charged object, a 'natural' unit would be the charge on one electron or proton. This fundamental charge is often referred to as the 'elementary charge' and is given the symbol e. The proton therefore has a charge of $+e$ and the electron $-e$. Despite many experiments designed to look for smaller charges, no charge smaller than e has ever been found in nature. All larger charges are understood to be whole number multiples of e.

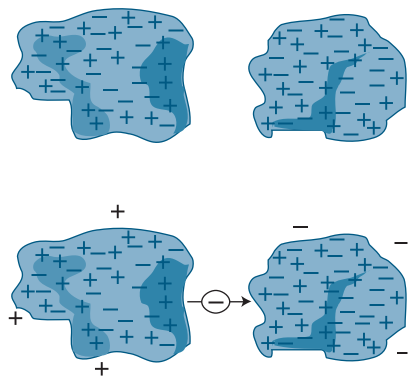


Figure 7.2
If as a result of being rubbed together some electrons are transferred from one object to another, the first will become positively charged and the second negatively.

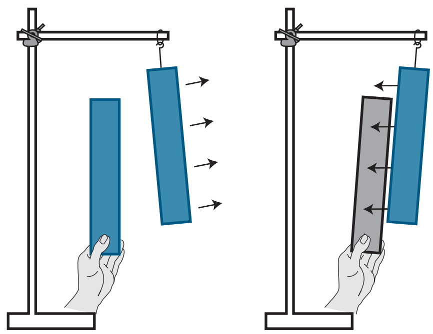


Figure 7.3
Like charges repel. Unlike charges attract.

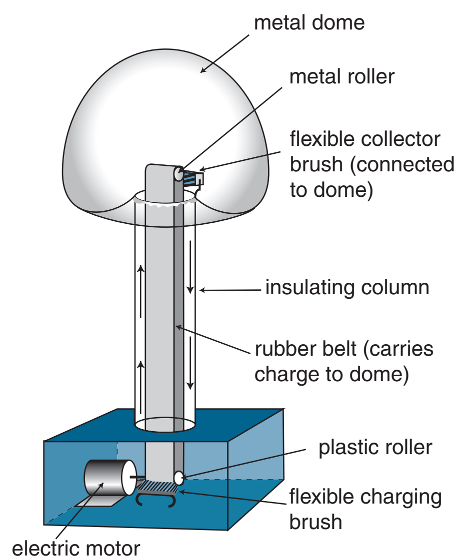


Figure 7.4
The Van de Graaff generator. In the base the rubber belt passes around a plastic roller. The close contact between rubber and plastic results in electrons being transferred to the rubber. The electrons are replaced by a metal foil or comb which is electrically connected to 'earth'. The electrons on the belt are carried to the top where they are picked up by another foil or comb and allowed to flow onto the dome. There, because they repel each other, they spread out over the conductive dome. A very large concentration of charge can build up.

Physics file

The nuclear particles, protons and neutrons, are thought to be made up of fundamental particles called quarks. The theoretical charges on quarks are positive and negative $\frac{1}{3}e$ and $\frac{2}{3}e$. While experimental evidence for these particles is strong, theory predicts that they can exist only in combinations which produce particles with exactly $1e$ and not by themselves. So we are unlikely to ever see particles with charges that are not whole multiples of e.

Physics file

That it is 6.242×10^{18} elementary charges which make up one coulomb of charge is a result of the original definition of electric current. As we shall see, charge and current are closely related, but in the early days of electrical experimentation the unit for current (the ampere) was defined before the coulomb. When dealing with electrostatics, the coulomb is a huge unit of charge. For this reason smaller units are often used:

$1\ \text{mC} = 10^{-3}\ \text{C}$
$1\ \mu\text{C} = 10^{-6}\ \text{C}$
$1\ \text{nC} = 10^{-9}\ \text{C}$
$1\ \text{pC} = 10^{-12}\ \text{C}$

Before going further, it is interesting to reflect on the nature of the proton and the electron for a moment. They have significantly different mass and size. The proton can be imagined to have a radius of around 10^{-15} m, while the electron is so small it is meaningless to give it a size. The mass of the proton is almost two thousand times that of the electron. Incredibly though, while being opposite in sign, their charges are absolutely identical in magnitude! Atoms containing equal numbers of protons and electrons are always exactly neutral.

The ***ELEMENTARY CHARGE***, e, is the magnitude of the charge on a proton or electron. It is the smallest charge found in nature.

The elementary charge is clearly a very small unit of charge. Even the small charge rubbed on the pen for the 'dancing paper' experiment would involve many billions of electrons—either lost or gained. For practical purposes a much larger unit of charge is used. The SI unit is the 'coulomb' (symbol C). It is equivalent to 6.242×10^{18} elementary charges. The reciprocal of this number is therefore the charge, in coulomb, on a proton or electron.

The elementary charge, e, the charge on a proton, is equal to 1.602×10^{-19} C. The charge on an electron is $-e$.

Worked Example 7.1A

a It has been stated that the charge on a rubbed pen would involve many billions of electrons. What is the charge in coulomb carried by 10 billion electrons?

b The charge on a school Van de Graaff generator might be around $-3.0\ \mu\text{C}$ ($1\ \mu\text{C} = 10^{-6}$ C). How many extra electrons are on the dome?

Solution

a In coulomb, the charge on 10 billion electrons is
$10 \times 10^9 \times 1.602 \times 10^{-19} = 1.6 \times 10^{-9}$ C.
This can be referred to as $0.0016\ \mu\text{C}$ or as 1.6 nC (nanocoulombs).

b The number of electrons in a charge is the magnitude of the charge divided by the charge on one electron, i.e. $n_e = q/e$.
The negative $3.0\ \mu\text{C}$ charge on the Van de Graaff dome consists of

$$n_e = q/e$$
$$= \frac{3.0 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.9 \times 10^{13} \text{ electrons.}$$

Any normal electrostatic charge involves huge numbers of electrons!

Physics file

Chemistry students will know that, as the atomic mass of aluminium is 27, there will be 6×10^{23} (Avogadro's number) atoms in 27 g of the metal. As there are 13 protons and electrons in each atom, the total number in the 500 g dome can be found from a little simple arithmetic.

It is interesting to compare the number of extra charges on a Van de Graaff dome with the number of charges in the metal itself. In Worked Example 7.1A it was found that a charged Van de Graaff dome might have an excess of nearly 20 million million electrons (2×10^{13}). This is a huge number of electrons! But, assuming the aluminium dome has a mass of around 500 g, there would be a total of about 10^{26} electrons and the same number of protons in the aluminium atoms of the dome. So the number of extra electrons on a fully charged dome is actually an extremely tiny fraction (2×10^{-13}) of the total number of electrons in the metal of the dome! You might like to confirm that this would be the equivalent of adding just a few litres of water to Fremantle Harbour.

Electrostatic induction

Recalling the dancing pieces of paper, how then did the charged plastic pen cause them to dance? We now know that the pen was negatively charged. It had created around it an *electric field* in which negative charges will move away from the pen and positive ones towards it. Any electrons in the paper that are free to move will therefore move to the opposite edge of the paper, leaving the edge closest to the pen with an excess of protons and thus a positive charge. While the paper is still neutral overall, this positive charge is closer to the pen than is the negative charge on the other edge and will therefore be attracted more strongly than the negative charge is repelled. If the pieces of paper are small enough, and the charge on the pen is great enough, the paper will be lifted from the table and may even jump onto the pen.

This process is called *electrostatic induction*. The charges in the paper are 'induced' to move by the presence of the charged object, thus creating 'induced charges' of opposite sign on opposite sides of the paper. Induction will occur regardless of the sign of the charge on the pen. If the pen were to be made positive, electrons in the paper would move towards it, causing the closer side to become negative and the further side to become positive.

The 'lightning rod', invented by Benjamin Franklin, is a good example of an application of the principle of electrostatic induction. A tall pointed metal rod on the highest part of a building is well connected to the ground by a heavy wire. When a charged thunder cloud moves overhead, charges of the opposite sign will be induced in the ground below, particularly in taller objects. The charge concentration on the end of the lightning rod can become so intense that the air molecules nearby become ionised and form a conducting path towards the cloud. This normally will have the effect of discharging the cloud sufficiently so that a lightning strike will not occur. If a strike does occur it will be conducted to ground through the lightning rod rather than the building, thus protecting the building and its inhabitants.

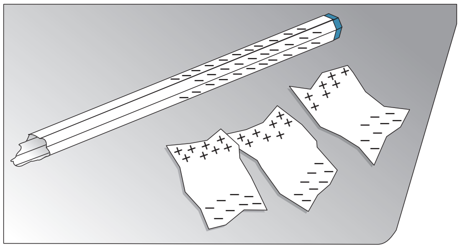


Figure 7.5
A negatively charged pen induces a positive charge on the nearer side of the paper and a negative charge on the opposite side. Because the positive side is closer the paper is attracted.

Physics file

The effect of a lightning rod can easily be demonstrated by placing a nail (vertically) on top of a Van de Graaff generator. Because of the intense electric field at the point of the nail, the charge dissipates into the air and the spark from the dome will not be nearly as impressive as that normally obtained.

Conductors and insulators

Any attempt to produce an electrostatic charge by rubbing a metal rod instead of a plastic or glass rod is normally unsuccessful. Charge transferred to the metal rod will flow away through the rod and your hand. Unlike plastic and glass, metals are *conductors*: they allow the movement of charge through their structure. The structure of metals is such that the outermost electrons are free to move around in the fixed crystal lattice made up of the atoms. Any excess of electrons in one place will soon be dispersed as the electrons flow away from each other. Materials such as plastic and glass do not allow the flow of electrons. They are called *insulators*.

There is not always a clear distinction between insulators and conductors. Wood, for example, will conduct electrostatic effects reasonably well, but certainly cannot be used as a conductor for household appliances! Wood can be classed as a poor conductor or a poor insulator depending on the situation. Another important group of materials are called *semiconductors*. Most notably these include silicon and germanium, the basis of the modern electronics industry.

Figure 7.6
This type of lightning rod did not become popular!

Table 7.1 Some common conductors and insulators

Conductors	Insulators
Good	
All metals, especially gold, silver, copper and aluminium	Plastics
Any ionic solution	Polystyrene
	Dry air
	Glass
	Porcelain
	Cloth (dry)
Moderate	
Water	Wood
Earth	Paper
Semiconductors, e.g. silicon, germanium	Damp air
Skin	Ice, snow

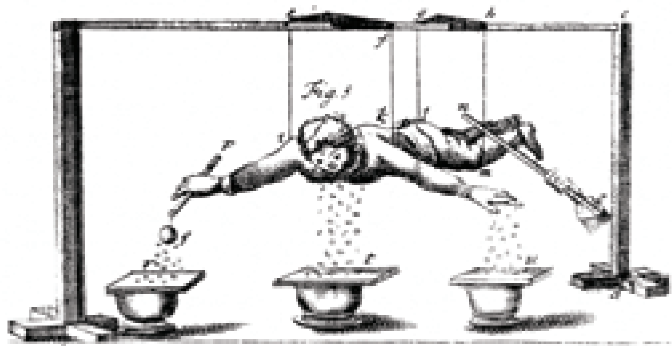


Figure 7.7
The distinction between insulators and conductors was discovered by Stephen Gray in 1729. He showed that the human body is a conductor of electric effects. He suspended a boy from silk cords and brought a charged glass rod near his legs. The electric effect was transmitted through the boy's body to his hands and face—as shown by the dancing paper. While the boy acted as a conductor, the silk acted as an insulator, preventing the charge from escaping to ground.

Pure semiconductors are not nearly as conductive as metals, but can be modified by 'doping' them with small amounts of certain elements so that they will conduct quite well. They are the materials from which transistors and integrated circuits are made.

Returning once more to the dancing paper experiment, you will probably now realise why the paper might jump off the pen after a little while. Because paper is not a very good insulator, it will slowly allow charge to move through it. As it picks up some of the charge from the pen it gradually becomes charged with the same charge as the pen. Once the charge builds up sufficiently, the paper will be repelled by the pen and fly off.

Physics in action — Robert Millikan and the elementary charge

By the late 19th century it was well established that electrons and protons had an identical, but opposite, charge. The big question was how this charge related to the coulomb, which was defined in terms of large-scale electrical phenomena such as electrolysis. An experiment was needed which linked the tiny forces on atomic charges to the more macroscopic electrical measurements.

Robert Millikan, at the University of Chicago, took up the challenge in 1907 and published his results two years later. His experiment was remarkably accurate and is now famous, both for the importance of the results and for the sheer elegance of its design.

In essence, Millikan measured the electrostatic force on tiny drops of oil sprayed from an atomiser. The drops acquired an electrostatic charge simply as a result of their motion through the air. His apparatus basically consisted of two horizontal plates held 1.6 cm apart which could be given opposite charges by an 8000 V battery. The oil drops were allowed to fall through the air in the space between these plates (they took over 20 seconds to fall 1 cm).

The rate at which such drops fall depends on the balance between the upward air-resistance force and the downward weight force. As the air resistance depends on the size, Millikan was able to calculate their weight from measurements of their speed. (The same theory applies to the rate at which a balloon falls through still air.)

When the plates were charged by the battery, the speed changed as a result of the added electric force on the drops. Some drops fell faster, others almost stopped or even rose. As the speed at which the drops fell was directly related to the total force on them, he was able to calculate the strength of the electric force. From this force he could calculate the electric charge on the drop. He found that this charge only came in whole number multiples of a certain smallest amount. This charge he assumed to be the charge on a single electron. He was able to show that one elementary charge was equal to 1.64×10^{-19} coulomb, which, considering the difficulties of the experiment, was very close to the currently accepted value of 1.60219×10^{-19} C.

In 1923 Millikan was awarded the Nobel prize for his work on the elementary charge, along with his later experimental work on Einstein's interpretation of the photoelectric effect.

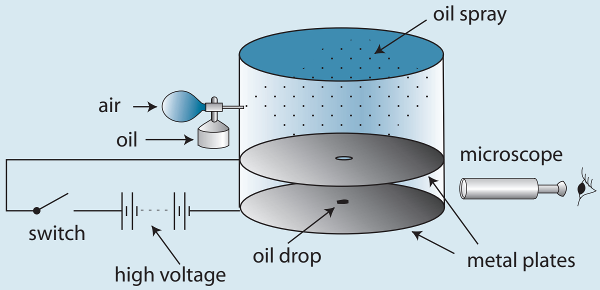


Figure 7.8

Millikan's apparatus consisted of an atomiser which sprayed a fine mist of oil into the chamber. Some of the drops fell into the space between the plates which could be charged via the switch from the battery.

Physics in action — Lightning

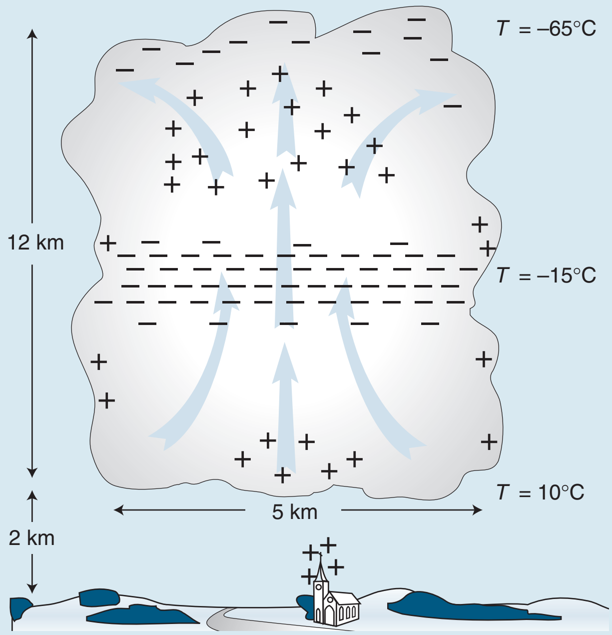


Figure 7.9

A thundercloud can be several kilometres wide and well over 10 km high. Strong updrafts drive the electrical processes that lead to the separation of charge. The strong negative charge will induce positive charges on tall objects on the ground. This may lead to a discharge which can form a conductive path for lightning.

Lightning is undoubtedly one of nature's greatest spectacles. No wonder it was for so long thought of as the voice of the gods. When Benjamin Franklin showed that it was basically the same sort of electrical phenomenon as could be achieved by rubbing a glass rod with wool, he didn't so much demystify it as move the mystery into another realm. How indeed can such enormous voltages be created in a cloud, something normally associated with the moisture that makes electrostatic experiments hard to perform!

A thundercloud normally has three charged regions. In the lower centre there is a strong negatively charged region, often less than a kilometre in thickness, but possibly several kilometres in width. The top of the cloud is mostly positively charged. There is normally also a smaller positively charged region at the bottom.

There will be strong electric fields between these regions of opposite charge. If they become sufficiently strong, electrons can be stripped from the air molecules (they become ionised). Because of the electric field, the free electrons and ions will gain kinetic energy and collide with more molecules, thus precipitating an 'avalanche of charges'. This is the lightning flash seen either within the cloud or between the Earth and the cloud. Most flashes are within the cloud; only a relatively small number actually strike the ground.

A typical lightning bolt to the ground bridges a potential difference of hundreds of millions of volts and transfers ten or more coulombs of negative charge to the ground in a brief current pulse of up to 10 000 A. A moderate thundercloud with a few flashes per minute generates several hundred megawatts of electrical power, the equivalent of a small power station.

The exact mechanisms that are responsible for the charge build-up in a cloud are still not entirely clear, but it is thought that charge is transferred in collisions between the tiny ice crystals that form as a result of the cooling of the upward flowing moist air and the larger, falling, hailstones. The actual charge transfer depends on the temperature difference between the particles and this is thought to be the explanation for the three different charge regions in the cloud. In the lower, warmer, regions the hailstones acquire a positive charge, whereas in the cooler upper regions they acquire a negative charge. Because they are falling, they carry this charge with them, thus causing the charge separation which results in huge voltages.

Once the electric field in the cloud has built up, electrostatic induction will further enhance the process. In the upper region of the cloud where the field points up, a positive charge will be induced on the bottom of the hailstone (and a negative charge on the top). In the collision, the lower ice crystal will therefore share some of this positive charge, leaving the hailstone with a negative charge which it carries downwards, further reinforcing the field.

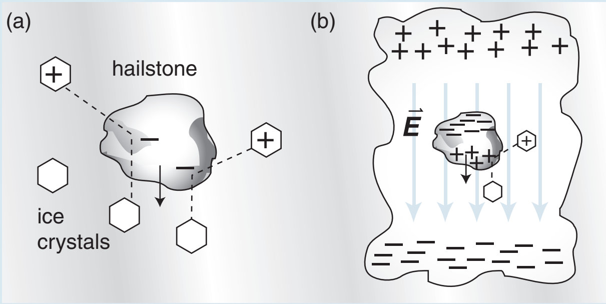


Figure 7.10
(a) When falling hailstones hit ice crystals of a different temperature, charge is transferred. (b) As the field in the cloud builds up, electrostatic induction further enhances the process.

7.1 SUMMARY Electrical charge

- Matter is made up of vast numbers of positive and negative charges (protons and electrons). Normally there is an equal number of each.
- Like charges repel and unlike charges attract.
- Charge cannot be created or destroyed, but it can be transferred from one object to another.
- An electrostatic charge involves an imbalance of positive and negative charges.
- The charges on a proton and an electron are equal in magnitude but opposite in sign. The magnitude of this charge is referred to as one elementary charge.
- One coulomb of charge is equal to 6.242×10^{18} elementary charges, or one elementary charge is equal to 1.602×10^{-19} C.
- If a charged object is placed near a conductor, an opposite charge will be induced on the side of the conductor nearer the charge and a like charge on the side further away from the charge.

7.1 Questions

1 In 1752, an American inventor, Benjamin Franklin, became famous for using a kite during thunderstorms to conduct lightning down the kite string in order to do experiments with electrostatic charge. Why was this so dangerous?

2 What are some of the ways in which our lives would be different if we did not have electricity?

3 List some of the electric motors in your household. How many do you think would be found in the average household?

4 Both simple electrostatic and magnetic experiments show forces that act through space between two different types of object. What similarities and differences can you think of in the results obtained from simple electrostatic experiments as compared to those performed with magnets?

5 Why do you think that early experimenters such as Franklin concluded that there were only two types of electric charge and not more?

6 Plastic strip A, when rubbed, is found to attract strip B. Strip C is found to repel strip B. What will happen when strip A and strip C are brought close together?

7 Some early theoreticians thought that electricity was a sort of fluid which could be transferred from one material to another. How could this model account for the fact that there were two types of charge?

8 Why does a Van de Graaff generator have a smooth round aluminium dome on the top rather than, say, an aluminium cube which would be easier to make?

9 Why is it not safe to stand under an isolated tree in a thunderstorm? What should you do if caught out in a thunderstorm?

10 We could use the terms ‘red’ and ‘black’ to describe the two different types of charge. What advantages do the terms ‘positive’ and ‘negative’ have over the use of colours as labels for charge?

7.2 Electric forces and fields

Coulomb's law

Electricity is clearly one of nature's fundamental forces. It was Charles Coulomb, in 1785, who first published the quantitative details of the force that acts between two electric charges (see Physics in action, page 280). The force between any combination of electrical charges can be understood in terms of the force between the simplest possible arrangement of charges: two so-called 'point charges' separated by a certain distance. The expression 'point charges' simply means that the two charges are regarded as being very much smaller than the distance between them. Remember that between like charges there will be repulsion and between unlike charges attraction.

Coulomb found that the force, whether repulsive or attractive, between two charges q_1 and q_2 a distance r apart was proportional to the product of the two charges, and inversely proportional to the square of the distance between them. This can be expressed by the simple equation below (where k is the proportionality constant).

Coulomb's law for the force between two charges q_1 and q_2 at a distance of r:

$$\boldsymbol{F} = \frac{kq_1q_2}{r^2}$$

It is not surprising that the force between two charges depends on the product of the two charges. Imagine that we found a force of 10 N between two particular charges A and B. If charge A was then doubled, for example by adding another identical charge, we would be surprised if the force between A and B did not increase to 20 N. If charge B was then also doubled, would we not expect the force between A and B now to increase to 40 N?

The force being inversely proportional to the square of the distance means that, for example, if the distance between A and B is doubled, the force will decrease to one-quarter of the previous value. There are a number of important 'inverse square' laws in physics. The reason for this is suggested in the adjacent Physics file.

The constant k has a value (in SI units) close to 9.0×10^9 N m^2 C^{-2}. This means that the force between two charges of one coulomb each, placed one metre apart, would be almost 10^{10} newtons—equivalent to the weight of about ten large battleships! This suggests that a one coulomb charge is a huge amount of charge. Imagine, for example, the repulsive force between two halves of any one coulomb charge on an ordinary-sized object. It would blow itself to pieces with enormous energy! In practice, the amount of charge that can be placed on ordinary objects is a tiny fraction of a coulomb. Even a highly charged Van de Graaff dome will have only a few microcoulombs (1 μC = 10^{-6} C) of excess charge.

Another way to get a feel for the magnitude of electrical forces is to realise that Mount Everest (or any other mountain) is supported by the electrostatic repulsion between the atoms at its base and those in the rock below! Of course there is no particular 'base'; the point is that the atoms in the rock beneath the mountain withstand its weight. Also, the strength of the hardest steel is due to the electrical forces between its atoms. In fact, only in the last stages of collapse of a giant dying star can

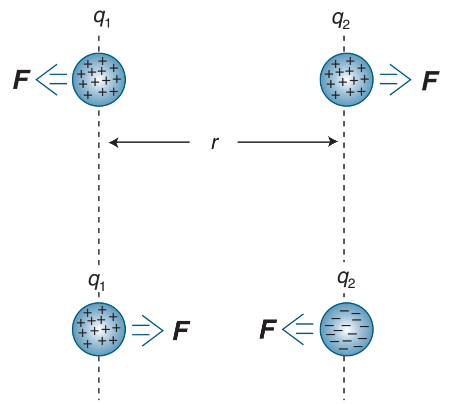


Figure 7.11
The nature of the force, $\boldsymbol{F}$, between two point charges q_1 and q_2 a distance r apart was discovered by Charles Coulomb in 1785.

Physics file

Both Coulomb's law for the force between electric charges and Newton's universal law of gravitation, for the gravitational force between any two masses, are examples of 'inverse square' laws. Another is the law for the intensity of light around a 'point source' of light. In all these cases something, whether it is physical (light), or simply an 'influence' (a force), can be imagined to be spreading out evenly from a point. The intensity of this 'something' at a certain distance will therefore be inversely proportional to the area of the sphere over which it is spread. As the area of this sphere increases with the square of the distance ($A = 4\pi r^2$), the intensity will therefore decrease with the square of the distance. This, of course, does not constitute a 'proof' that these laws should be inverse square laws. It simply suggests that it is reasonable that they would be.

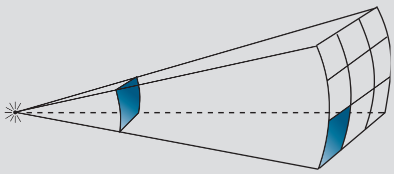


Figure 7.12
The inverse square law. Three metres from a point source of light, the light will be spread over an area nine times as large as that at one metre. The light will therefore appear only one-ninth as bright.

the gravitational forces overwhelm the electrical forces between atoms and cause them to collapse into the superdense state of matter that exists in what is called a neutron star.

Worked Example 7.2A

Two Van de Graaff machines are placed 50 cm apart and switched on. If they both attain a charge of 3 μC, of the same sign, what will be the force between them? (Ignore the size of the machines for the moment.) How would this force change if:

a one of the machines sparks and loses half its charge?
b the machines are moved to a distance of 1 m apart?
c the charges were of opposite sign instead of the same?

Solution

The force between them is given by $F = kq_1q_2/r^2$ where $k = 9 \times 10^9$ N m^2 C^{-2}, $q_1 = q_2 = 3 \times 10^{-6}$ C and $r = 0.5$ m.

Thus the force $F = \dfrac{9 \times 10^9 \times (3 \times 10^{-6})^2}{0.5^2} = 0.3$ N, not a large force,

but possibly noticeable. As both machines have charge of the same sign the force will be a repulsive one.

a If one machine loses half its charge we do not need to repeat the calculation, we know that the force will halve to 0.15 N.
b If the distance in an inverse square law doubles, the force will reduce to one quarter. Given the original charge of 3 μC the force will decrease to 0.3/4 = 0.08 N.
c If the charges were opposite, the force would be attractive rather than repulsive. (It is worth noting that if two Van de Graaff machines with the same sign were actually placed this distance apart, the force would probably be less than that calculated because the charges would repel and move to the opposite sides of the domes. On the other hand, if the charges were opposite, they would attract and move to the closer sides of the dome, thus increasing the force.)

Electric fields

The actual force between any pair of real charged objects is simply the vector sum of all the 'Coulomb forces' on it from all the pairs of charges on the objects. However, in many situations, to actually calculate the forces this way would be totally impractical. To avoid this sort of difficulty physicists use the concept of an electric field. Around any charged object there is a region of space in which a 'test charge' would experience a force. This test charge can be imagined as a tiny, positive, unit charge. The force this unit charge experiences is said to be the electric field. Electric field has both direction and strength, and so is a vector quantity.

The electric field $\mathbf{E}$ in any region of space is defined as the electric force per unit charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

The electric field is often shown by lines which represent the direction of the field. The closeness of the lines can normally be taken to represent the relative strength of the field. The shape of the field around some charged objects is shown in Figure 7.13. The shape of the field around a small charge is radial, pointing outward in the case of a positive charge and inward in the case of a negative charge.

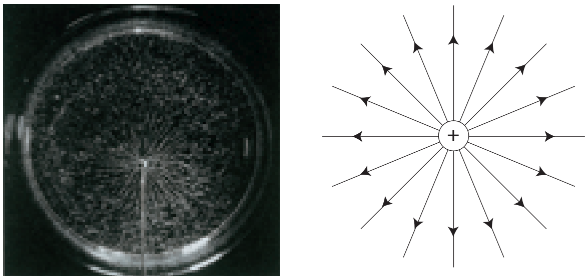
Physics file

The electric field $\mathbf{E}$ may be compared with the gravitational field $\mathbf{g}$. Both represent the strength of a 'force field' in a region without the need to specify on what the force is acting. While $\mathbf{g}$ is the gravitational force per kilogram of mass (for example, 9.8 N kg^{-1} on Earth), $\mathbf{E}$ is the electric force per coulomb of charge. This means that the magnitude of $\mathbf{E}$ is usually very large; remember that one coulomb is a huge charge. It is important to think of $\mathbf{E}$ as the ratio of the force to the charge. It is not necessary to have a one coulomb test charge in order to measure it!

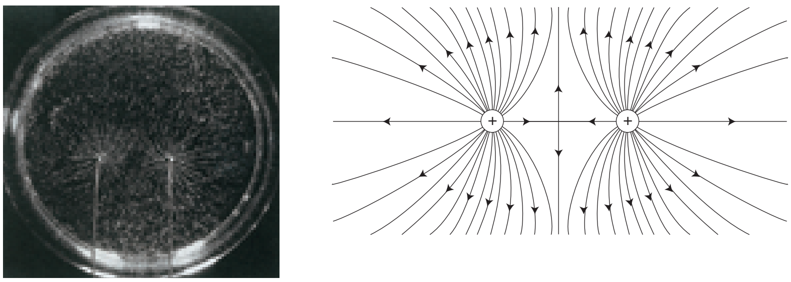
The force on a charge q in an electric field $\mathbf{E}$ is given by $\mathbf{F} = q\mathbf{E}$.

As shown in Figure 7.13, the electric field lines between two charged parallel plates are mostly parallel and uniform. This means the strength of the field has the same value everywhere. We will see later that there is a very simple way to calculate the strength of the field in such a region. In fact, a pair of parallel plates is often used as a convenient way of obtaining a uniform electric field of known strength. Millikan used such plates in his experiment described on page 274.

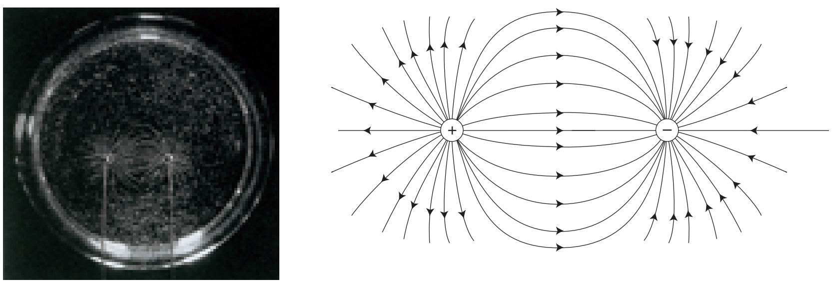
A single positive charge



Two positive charges



A positive and a negative charge



Parallel oppositely charged plates

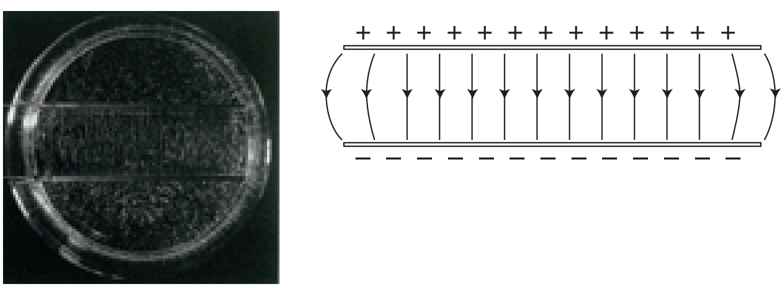


Figure 7.13
In these photographs, grass seeds suspended in oil make the electric field around charged objects visible. Electric fields can be represented by lines showing the direction of the field. The closeness of the lines suggests the strength of the field.

✓ Worked Example 7.2B

Robert Millikan measured the charge on an electron by finding the electric force on tiny oil drops in a known electric field. If the force on an oil drop was due to the charge of one single extra electron on the drop, and was found to be 8.0×10^{-14} N upwards, what was the strength and direction of the electric field he was using?

Solution

One elementary charge, the charge on an electron, is equal to 1.6×10^{-19} C. The strength of the field is therefore given by:

$$E = \frac{F}{q} = \frac{8.0 \times 10^{-14}}{1.6 \times 10^{-19}} = 5.0 \times 10^{5}\ \mathrm{N\ C^{-1}}$$

The direction of the field is downwards. Remember that the direction of the field is the direction of the force on a positive charge. A negative charge experiences a force in the opposite direction to the field. It is interesting to note that Millikan achieved this field by connecting a battery providing 8000 V across two parallel plates 1.6 cm apart.

Physics in action — Charles Coulomb and the force between two electric charges

Charles Coulomb (1736–1806) was an engineer who investigated variations in the Earth's magnetic field. In order to do this he invented a 'torsion balance', a device which could measure very small forces (Figure 7.14). The principle of a torsion balance is simple. A carefully balanced horizontal rod is suspended by a thin fibre. First the force required to twist the rod through a given angle is found. Then two charges are placed, one on a small fixed sphere as shown and the other on a similar sphere on one end of the rod (which is an insulator). By finding the amount of twist caused by the electric force between the charges, the magnitude of the force can be calculated. By varying the amount of charge on the spheres as well as the distance between them, he was able to show that the force between them was proportional to the amount of charge on each, and decreased with the square of the distance between them.

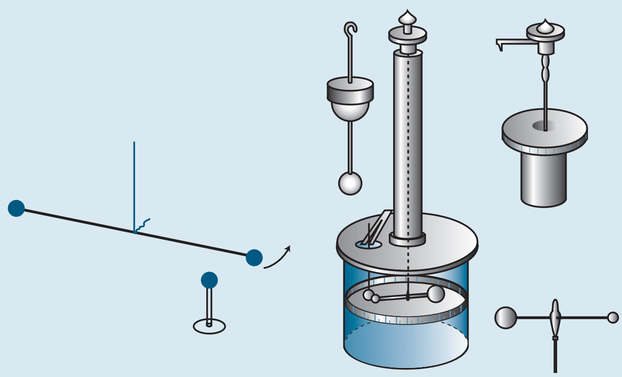


Figure 7.14
A sketch of Coulomb's original torsion balance. The torsion force in the fibre is measured by the degree of rotation of the knob at the top needed to compensate for the electrical force between the fixed and movable spheres.

Physics in action — Photocopiers

In 1938, American inventor Chester Carlson used an electrostatic effect to transfer an image from one piece of paper to another. He called it 'xerography' from the Greek words for 'dry' and 'writing'. But it wasn't until 1959 that the Xerox Corporation produced the first truly successful office copier, which became the basis of the current multibillion dollar industry.

When light is shone on some materials which have been given an electrostatic charge, they become conductive ('photoconductivity') and lose the charge. If the light is in the form of the image of some writing, an 'electrostatic image' can be formed. If fine carbon particles with the opposite charge are then sprinkled on the image they will adhere to the places that are charged. If they can then be 'fixed' in place we have a 'photocopy' of the original.

A modern photocopier is a very complex piece of machinery, but the basis of its operation is still this same photoelectric effect that Carlson used. At its centre is an aluminium drum which has two very thin (around 0.05 mm) coatings. The first is a layer of photoconductive material such as cadmium sulfide. This is coated with an even thinner transparent insulating film.

As the drum rotates, the various parts undergo different processes. First, it is given an electrostatic charge from a ‘corona wire’. This is a fine wire charged to around 5000 V. Because it is very thin, the electric field near it is intense and it ionises the air, creating ions (charged atoms) and free electrons. (This is the same effect that Benjamin Franklin used for his lightning rods.) If the wire is negative, the electrons will charge the surface of the drum and the underside of the thin insulating layer will become positively charged by induction as charges move through the photoconductive layer.

Next, the image of the work to be copied is projected onto the drum. Where it is bright, it increases the conductivity of the photoconductive layer and allows the charge to escape. The dark parts remain charged. Next the image is ‘developed’ by allowing positively charged toner particles to be attracted to the still negatively charged dark parts of the image on the drum. In order to transfer the toner powder, and hence the image, to the paper, the paper is given a strong negative charge from another corona wire and brought into contact with the drum. The paper is then taken through two hot rollers where adhesive in the toner melts and ‘fixes’ the image on the paper. At the same time the drum is cleaned and made ready for a fresh charge from the first corona wire.

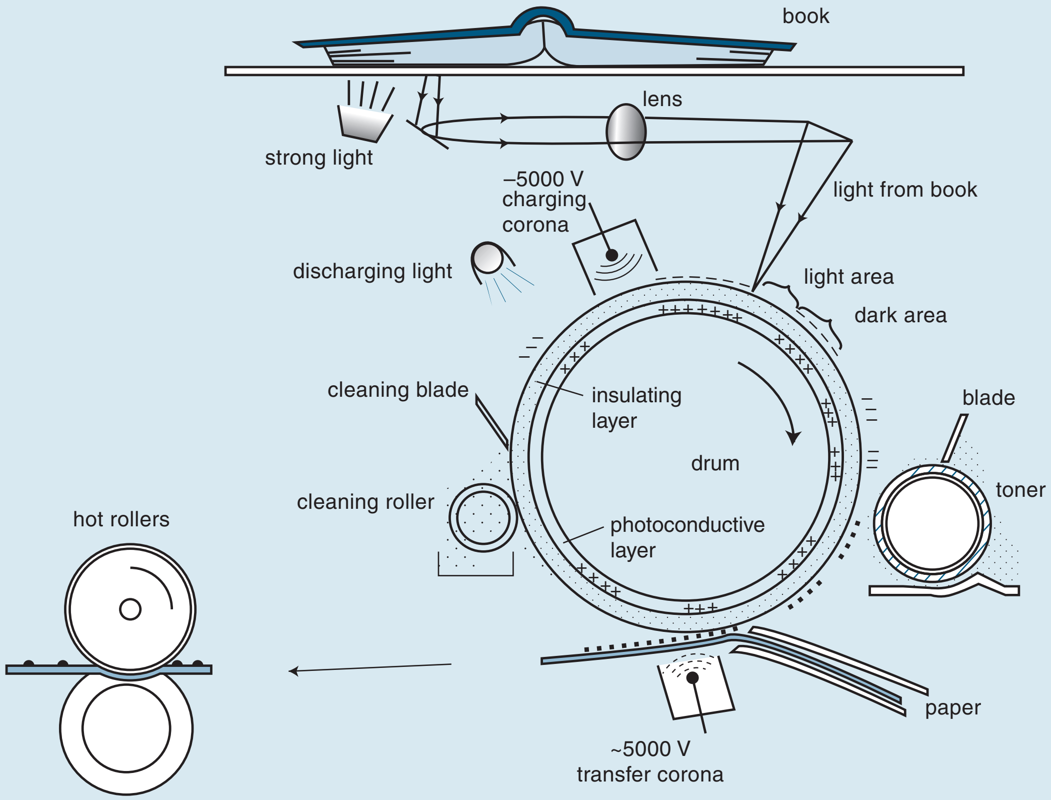


Figure 7.15
There are several types of photocopy machine, but the basic principles are illustrated here. The drum is given an electrostatic charge before being exposed to light from the image. The light enables the charge to dissipate. Black toner powder is attracted to the still charged dark parts of the image. The toner is then transferred to the paper and fixed by hot rollers.

7.2 SUMMARY Electric forces and fields

- Coulomb's law for the force between two charges q_1 and q_2 at a distance of r is:

$$\boldsymbol{F} = \frac{kq_1q_2}{r^2}$$

- The constant, k, in Coulomb's law has a value of 9×10^9 N m^2 C^{-2}, indicating that an electrostatic charge of one coulomb would be enormous.
- The electric field $\boldsymbol{E}$ in any region of space is the electric force per unit of charge in that space:

$$\boldsymbol{E} = \boldsymbol{F}/q$$

Conversely, the force on a charge q in an electric field $\boldsymbol{E}$ is given by $\boldsymbol{F} = q\boldsymbol{E}$.

7.2 Questions

1 A charge of $+q$ is placed a distance R from another charge also of $+q$. A repulsive force of magnitude $\boldsymbol{F}$ is found to exist between them. Describe the changes, if any, that will occur in the force when this situation is changed to each of the following:
   a one of the charges is doubled to $+2q$
   b both of the charges are doubled to $+2q$
   c one of the charges is changed to $-q$
   d the distance between the charges is changed to $\frac{1}{2}R$.

2 What force would exist between them if we could place two 1 C charges 100 m apart?

3 How practical would it be to set up the situation described in question 2?

4 Danielle and Daniel set up two Van de Graaff machines exactly 80 cm apart (centre to centre) on frictionless trolleys that allow them to measure the force between them. They read that the manufacturer states that it is possible to obtain a charge of 5 μC on the domes of the machines and then proceed to use Coulomb's law to calculate the force they expect to find between them.
   a What force do they expect to find between the machines?
   b Assuming that the charge on each machine is 5 μC, why do they find that the measured force is less than they expect?

5 Some small charged spheres are to be placed in an electric field which points downwards and has a strength of 5000 N C^{-1}.
   a What force would be experienced by charges of +2 μC and −5 μC?
   b A sphere with an unknown charge is found to experience an upwards force of 1×10^{-3} N in this field. What was the charge on the sphere?

7.3 Electric current, EMF and electrical potential

When a battery is connected to a conductor (such as a torch bulb, for example) one end of the conductor becomes positively charged and the other end becomes negative. Along the length of the conductor there will be a gradual change from an excess of positive charge at one end to an excess of negative charge at the other end. The effect of this is to set up an electric field along the wire directed away from the positive end and toward the negative end. Any charges in the wire will experience a force as a result of this field, and so the free electrons in the wire will tend to move (in the direction opposite to the field). This movement of charge constitutes an *electric current*.

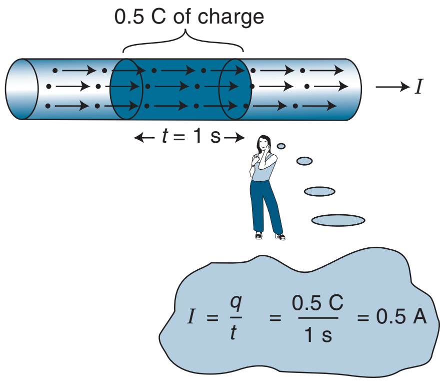


Figure 7.16
If a charge of 0.5 C passes a point in a conductor in 1 second, a current of 0.5 A is flowing.

Electric current

Just as a current in a river involves the flow of water, electric current is the flow of electric charge. Any moving charge constitutes an electric current. Whether it consists of electrons moving through the atomic structure of a metal, or protons flying through space from the Sun, moving charges make up a current.

The magnitude of the current is defined simply as the rate of transfer of charge. We can think of it as the amount of charge that flows past any point in a conductor in one second. A current of one ampere flows when one coulomb of charge flows past a point in one second. So 1 A = 1 C s^{-1}. Electric current is given the symbol I.

Electric current is the rate of transfer of charge:

$$I = \frac{q}{t}$$

The symbol q is for the charge transferred and t is the time taken. If the charge is measured in coulombs and the time in seconds, the current is measured in *amperes*. Conversely, the charge, in coulomb, carried by a current of I ampere in t seconds is given by $q = It$.

1 ampere (A) = 1 coulomb per second (C s^{-1}),
so 1 coulomb (C) = 1 ampere second (A s).

Worked Example 7.3A

Determine the charge that has flowed through a torch battery producing a current of 300 mA if it has been left on for 20 minutes.

Solution

$I = q/t$ so, $q = It$ where $I = 300 \times 10^{-3} = 0.300$ A and $t = 20 \times 60 = 1200$ s.
Thus $q = 0.300 \times 1200 = 360$ C.

Physics file

The ampere is named in honour of André Marie Ampère (1775–1836). You may know that a magnetic force exists between two wires carrying an electric current. It was Ampère who first worked out the mathematics of this force. The ampere is actually defined as the current which, when flowing in two long straight parallel wires one metre apart, produces a force of exactly 2×10^{-7} N between them. This force was typical of that produced by the current which could be obtained from the batteries of that time. The coulomb was later defined as the amount of charge carried by a current of one ampere in one second.

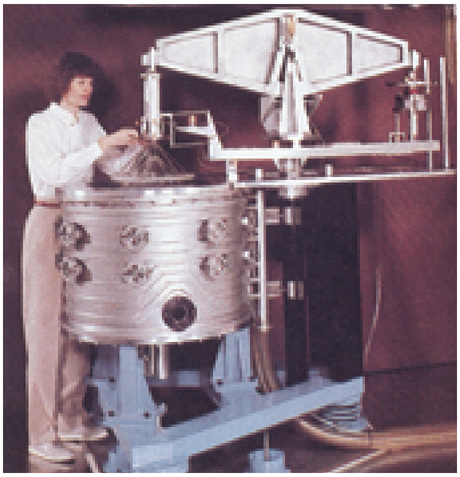


Figure 7.17
It is not practicable to measure the force between two very long wires 1 m apart, but, based on the definition, the force between two current-carrying coils can be calculated and used to set up a primary standard of current upon which other instruments can be calibrated. In a standard current balance the magnetic force is balanced against a known weight.

Table 7.2 Typical values for electric current

Situation	Current
Lightning	10 000 A
Starter motor in car	200 A
Fan heater	10 A
Toaster	3 A
Light bulb	400 mA
Pocket calculator	5 mA
Nerve fibres in body	1 μA

Figure 7.18
In an electrical circuit, an ammeter is used to measure the current. The ammeter is connected in series with the device. That is, in such a way that the current flows through it as well as the device. Typically, a meter will require the user to select a current range. If the current is too large it is possible to damage the meter. In this circuit a voltmeter is also connected across the batteries.

Physics file

Whenever the term 'current' is used in this book the direction is assumed to be that of the transfer of positive charge as defined by the equation $I = q/t$. This is sometimes called *conventional current*. The term *electron current*, which you may also come across, is equivalent to defining current by the equation $I = -q/t$ and will not be used in this book as it may lead to confusion. Simply remember that positive charge transfer occurs either as a result of positive charges moving in the direction of the current, by negative charges moving in the opposite direction (electrons in metals), or by both at once.

The direction of current

There is sometimes confusion about the direction of an electric current. In the case of a river, the direction of the current is clear; it is the direction of the water flow. However, unlike water, electric charge can be either positive or negative. The direction of an electric current is the direction of the flow of positive charge. It is important to realise that positive charge can be transferred either by positive charges moving in the direction of the current or by negative charges moving in the opposite direction. In a metal wire, for example, the current is carried by electrons moving in the direction opposite to that of the current. On the other hand, in a fluorescent tube the current is carried both by positive ions moving in the direction of the current and electrons moving in the opposite direction.

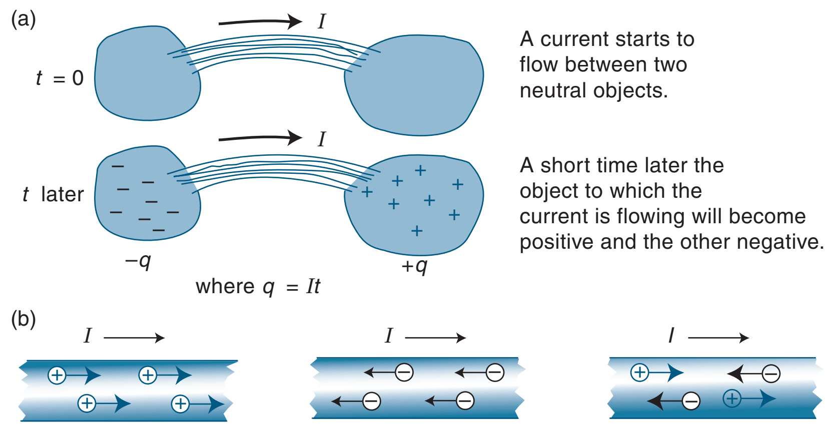


Figure 7.19
(a) If two objects are electrically connected and one is becoming more positive while the other becomes more negative, then we say there is a positive current flowing towards the one which is becoming more positive. (b) This might result either from positive charges moving in the direction of the current, from negative charges moving in the opposite direction, or both at once.

Because the electrons in a wire actually move in the direction opposite to the current, the terms *electron current* and *conventional current* are sometimes used to distinguish between them. It is important to remember, however, that *current* is the rate of transfer of *positive* charge.

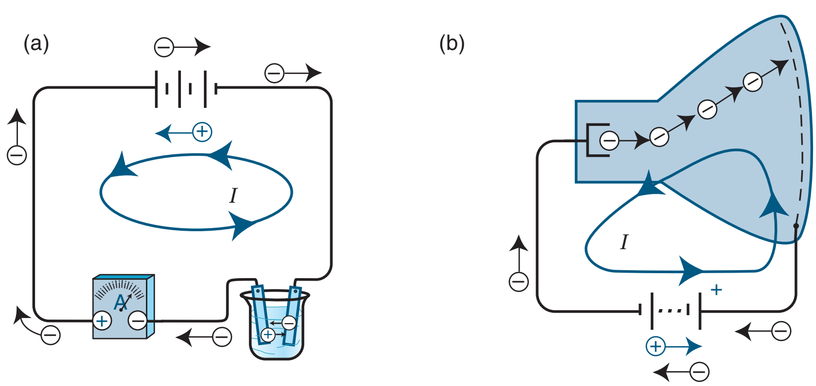


Figure 7.20
(a) Current flows around a circuit from the positive terminal of the battery to the negative. In the connecting wires the current is carried by electrons travelling in the opposite direction. In the battery itself, and in the salt solution, the current will involve the movement of both positive ions in the direction of the current and negative ions moving in the opposite direction. (b) The beam of electrons travelling down a cathode ray tube in a television set produces a positive current in the opposite direction.

EMF and electric potential

In order to drive a current around an electric circuit, the charges must be given energy. A battery or generator is the usual source of this energy. Another increasingly common source of electrical energy is the photovoltaic cell, or solar cell. These devices are all referred to as sources of *EMF*. The letters stand for 'Electromotive Force', but that is a rather unfortunate term because, as we shall see, the EMF involves the energy given to the charges rather than the force on them.

In order to understand the meaning of an EMF it is helpful to consider another source of EMF, the Van de Graaff machine (see Figures 7.4 and 7.21). The source of energy in this case is very obvious. The motor is pushing the charges on the rubber belt up against the electrostatic repulsion of the charges already on the dome. The more charge already on the dome, the greater the force, and hence the greater the work that has to be done to bring more charge to the dome. In fact, you may hear the motor slow down as the concentration of charge on the dome builds up.

In many ways the EMF can be visualised as this 'concentration of charge'. The more charges put on the dome, the more concentrated they become and the greater the force of repulsion between them. The work done pushing the charges together (by the motor in this case) is stored as electrical potential energy.

Just as the compressed spring in a jack-in-the-box contains potential energy, so do all the 'concentrated' charges. And just as the spring energy can be recovered when it is allowed to expand, so the electrical energy can be recovered when the charges are allowed to fly apart again. When a spark flies from the Van de Graaff generator we see the result. The potential energy is rapidly converted into kinetic energy, and as the charges collide with the air molecules it is turned into heat, light and sound energy.

EMF is defined as the amount of work done for each unit of charge in this process of charge concentration. Because it is actually the 'electric potential energy per unit charge', this quantity is most often abbreviated simply to *electric potential* or just *potential*. The EMF is then the electric potential given to charges by the device. A battery uses chemical energy to give the charges on its terminals this potential energy. A generator uses the kinetic energy of rotation and solar cells use the energy in sunlight.

The SI unit for potential is *joules per coulomb*. One joule per coulomb is given the name *volt*, in honour of Alessandro Volta, the inventor of the first chemical battery. It is easy to see why the terms 'potential' and 'voltage' are often used interchangeably. A normal dry cell has an EMF of $1\frac{1}{2}$ V. This means that every coulomb of charge on the positive terminal has $1\frac{1}{2}$ J of potential energy more than those at the negative terminal. The symbol used for EMF is usually $\mathscr{E}$ (a script E). So, for a dry cell, $\mathscr{E} = 1.5$ joules per coulomb, that is, 1.5 V.

A source of EMF gives charges electrical potential energy.
The EMF, $\mathscr{E}$, is the energy per unit of charge.
1 volt = 1 joule per coulomb (1 V = 1 J C^{-1}).

Worked Example 7.3B

The alternator of a car being driven at night with the headlights on is producing a 50 A current at an EMF of 12 V.

a How many coulombs of charge flow from the alternator each second?
b How many joules of energy does each coulomb of charge obtain?

Figure 7.21
A Van de Graaff machine does work pushing the charges on the belt up against the repulsive force from those already on the dome. If a person touches a Van de Graaff machine while standing on an insulated chair, the charge will spread from the dome and over them. The charged hairs repel each other.

Physics file

The EMF of a charged Van de Graaff generator, or any source of high voltage, can be estimated from the length of the spark it will produce. Between smooth spheres a spark will jump about 1 cm for every 25 000 V. Between pointed conductors it will jump 1 cm for every 10 000 V. Under good conditions a Van de Graaff generator might produce a spark of up to about 15 cm, which corresponds to nearly 400 000 V.

c How many joules of energy does the alternator produce each second?
d Where does this energy go?

Solution

a The 50 A current means that 50 C of charge flows each second ($q = It$).
b The 12 V EMF means that each 1 C of charge is given 12 J of energy.
c Each second, 50 C of charge each with 12 J of energy flows from the alternator, so the energy produced is 50 × 12 = 600 J.
d This energy will go to the headlights, the ignition system and any other electrical devices in operation. Some may also be used to recharge the battery.

Electric circuits and potential difference

Any electric circuit consists of at least one source of EMF, conductors which carry current (hopefully with very little loss of energy) and the various 'circuit elements'. The circuit elements are the working parts of the circuit: light bulbs, motors, loudspeakers, heating elements and so on. In these, the electric potential energy is converted into heat, light, motion or whatever other form of energy is required.

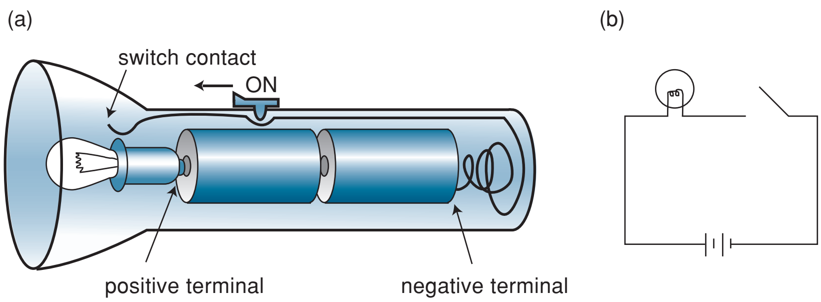


Figure 7.22
(a) A battery and light bulb connected by conductors in a torch constitute an electric circuit. (b) This circuit can be represented by a simple circuit diagram.

A torch consists of a battery, a switch and a bulb all connected by conductors to form an electrical circuit. The battery may consist of one or more *cells*. The current from the battery goes from the positive terminal to the bulb and then back to the base of the battery via the switch.

When a battery is connected in a circuit, it creates a *potential difference*; that is, a difference in the potential energy of the charges in the conductors connected to its terminals. The potential difference created by the battery can be imagined as a difference in 'charge concentration' on the conductors connected to either side of the battery. While the switch is off, all the conductors connected to the positive terminal, including the bulb and one side of the switch, will have a positive charge concentration and all those connected to the negative terminal will have a negative concentration. This initial distribution of charge, which ensures that all electrically connected conductors are at the same potential, will take place in the first fraction of a microsecond after the battery is put in place. Remember that there is no current flowing in the conductors at this stage.

When the switch is turned on, charges move through the circuit using up their potential energy as they move through any circuit element, such as the torch bulb. Some of the potential energy they carry will be converted into other forms of energy, light and heat energy in the light bulb, for example. Just as the EMF is the potential energy *given* to each

Physics file

Strictly speaking, the 'dry cells' we buy should not be called batteries. The term 'battery' really refers to a group of cells connected together, as in the torch shown. When a battery is connected in this way (in series) the total EMF is equal to the sum of the EMF values of the individual cells.

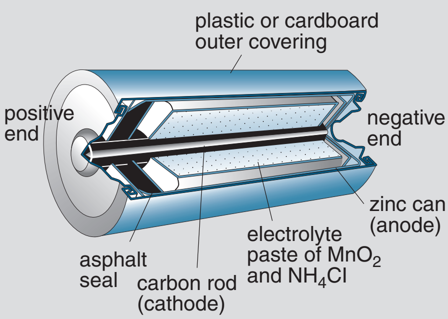


Figure 7.23
A modern dry cell is a complex mixture of chemicals designed to drive electrons from the top terminal to the bottom one.

unit of charge, the potential difference across a circuit element is the potential energy lost by each unit of charge in that element.

The potential difference, sometimes called potential drop or just p.d., across a circuit element is written ΔV, the Δ representing the fact that there is a change of potential. In practice, the Δ is often omitted as there is rarely a need to refer to anything other than a *change* of potential. As ΔV is the energy lost by one unit of charge, a charge of q coulomb will lose $q\Delta V$ joules of energy as it goes through a potential difference of ΔV.

A charge q moving through a potential difference ΔV will lose energy given by $E = q\Delta V$, or simply, $E = qV$.

Worked Example 7.3C

The potential difference across a torch bulb is found to be 2.7 V. The current flowing through it is 0.2 A.

a How much charge flows through the torch in 1 minute?
b How much energy is lost by this charge?

Solution

a $q = It = 0.2 \text{ A} \times 60 \text{ s} = 12 \text{ C}$.
b Each coulomb lost 2.7 J of energy. $E = qV = 12 \times 2.7 = 32.4 \text{ J}$

A useful analogy

It is helpful to compare an electric circuit to water in a dam. The water in the dam has potential energy relative to the water in the river at the base of the dam. The Sun is the source of the potential energy. It evaporated the water which eventually rained on the hills. The energy it gave to every kilogram of water could be regarded as the 'gravitational EMF' of the system. The initial equalising of charge concentration in the electrical circuit is rather like the process of pouring water into the dam. Any water which is higher than the average will run to a lower place until the whole dam is at the same level. The switch is rather like the dam wall. On one side the potential (or water level) is higher than on the other side, in the case of the switch on our torch, by 3 V.

What happens when the switch is closed? This is somewhat like opening the sluice gate which lets water over the dam spillway. As the water flows down the spillway its potential energy is transformed into kinetic energy, most of which ends up as heat and sound. The bulb in the electrical circuit is the equivalent of the spillway. Because the filament is very thin compared to the other connecting wires, the charges have more trouble moving through it and give up most of their energy to the atoms they collide with—which then appears as heat and light. The current in the other conductors is rather like the water above and below the dam wall. Just as water will slowly move through the dam itself as a result of the barely perceptible height difference created by the loss of water over the spillway, so charge will move around the conductors as a result of the very small potential difference created along the conductors by the movement of charge through the filament.

We can see and hear the potential energy being turned into kinetic energy as the water falls. The charges also gain kinetic energy as they 'fall' through the 3 V potential difference that the battery will maintain across the bulb. This kinetic energy is transferred to the atoms in the filament as the electrons collide with them. A typical incandescent light

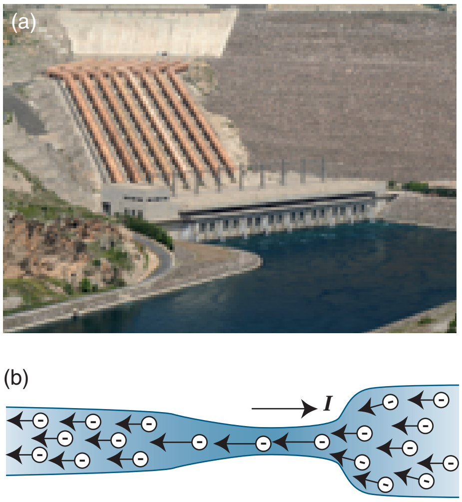


Figure 7.24
(a) It can be useful to compare charge going around a circuit with water in a dam. The current in the conductors in the torch is like the water in the dam or river, while the bulb is like the spillway of the dam. (b) Just as the water must go faster, and will lose more height going over the spillway, so the electrons in a wire will lose more energy, when they come to a thin conductor.

bulb is designed so that the heat energy produced raises the temperature of the filament to around 2200°C. At this temperature around 5% of the energy is actually radiated as visible light, the rest being lost as infrared radiation and heat conducted away through the filament supports and connections.

The water that has come over the spillway will now slowly flow along a broad river to the sea. The charges, having done their work in the bulb, will make a leisurely return to the battery through the (relatively) broad conductors in the torch. On reaching the sea, some of the water will gain enough energy from the Sun to rise in the atmosphere and become a cloud. On reaching the battery, some of the charges will eventually be given potential energy, by the chemical reactions, and find themselves on the positive terminal again.

There are many differences between electric currents and flowing water! Any analogy is only useful to a limited extent. For a start there is no such thing as 'negative water' while there is both positive and negative charge. The analogy is useful, however, because the 'electric potential' of charges can be related to the 'height' of water. Just as the actual kinetic energy released over the spillway of the dam depends on both the height and the amount of water, so the energy produced in the filament of the bulb depends on the potential difference and the amount of charge that flows. We shall return to the question of the energy released in electric circuits in section 7.5.

Physics in action — What is an electric current?

It is interesting to consider the nature of current flowing in the different states of matter. For any current to flow, mobile charge carriers are required. In the solid state, metals are the best conductors owing to the presence of many 'free' electrons. Metals, being ductile, can be drawn into wires, and so metal wires are the most familiar 'pipe' for transferring electricity.

Liquids in which there are free ions will also conduct an electric current. Although water molecules themselves are neutral, there are always a very small number of charged ions present (both positive and negative) and these allow even pure water to conduct electricity, although poorly. The addition of impurities, such as dissolved salts, raises the number of charged ions considerably and increases the conductivity of water. The electrolyte in a car battery (a solution of sulfuric acid) is an example of a good liquid conductor. In any liquid, the current is made up both of positive ions flowing in the direction of the current and negative ions flowing in the opposite direction.

Gases too can carry electric current as long as enough of the gas atoms become ionised. For example, lightning will occur when the electric field within a charged cloud is strong enough to strip electrons from the gas molecules in the air. Fluorescent lamps incorporate mercury vapour which ionises relatively easily. A plasma is a gas heated to the point where it ionises and becomes conductive. Again, conduction in gases involves both positive and negative charge carriers moving in opposite directions.

Figure 7.25
An electric current in air! This is a photograph of lightning, which occurs when the electric field within a charged cloud is strong enough to strip electrons from the gas molecules in air.

7.3 SUMMARY Electric current, EMF and electric potential

- A battery establishes an electric field in a conductor connected to its terminals. This electric field results in the movement of charge; that is, an electric current.
- Electric current is the time rate of transfer of charge. $I = q/t$ (1 A = 1 C s^{-1})
- The direction of an electric current is the direction of the transfer of positive charge. This can occur as a result of positive charge movement in that direction, negative charge movement in the opposite direction, or both.
- A source of EMF gives charges electrical potential energy. The EMF, $\mathscr{E}$, is the electric potential energy per unit of charge. 1 volt = 1 joule per coulomb.
- A potential difference, V, is the loss of potential energy per unit charge as charge flows through a circuit element. The energy lost is given by $E = qV$.

7.3 Questions

1 Using the values given in Table 7.2 find the amount of charge that would flow through a:
  a pocket calculator in 10 min
  b car starter motor in 5 s
  c light bulb in 1 h.

2 Do the values for the charge that you obtained in question 1 indicate the amount of energy required to operate the devices for those times? Explain.

3 The dome on a fully charged Van de Graaff machine may carry something of the order of 50 million million extra electrons. When running well it may take about three seconds to charge up. If we assume no loss of charge in this time, what is the current flowing up the belt to the dome?

4 When water runs through a hose at the rate of 0.5 litre per second, it can be calculated that 1.7×10^{26} electrons pass any point in the hose each second.
  a What electric current (in amps) does this represent?
  b Is this the actual electric current in the hose? Explain.

5 Although you will not normally get a shock if you put your hands on the terminals of a car battery, you will if you touch the spark plugs while the engine is running. Why is this?

6 The negative terminal of a 12 V car battery is connected to the car frame which can be regarded as ‘ground’, at a potential of 0 V. What is the potential of the other terminal?
  A 0 V
  B −12 V
  C +6 V
  D +12 V

7 A charge of 5 C flows from a battery through an electric water heater and delivers 100 J of heat to the water. What was the potential of the battery?

8 How much energy will each coulomb of charge flowing from a 9 V transistor radio battery possess?

9 How much charge must have flowed through a 12 V car battery if 2 kJ of energy was delivered to the starter motor?

10 In comparing the electrical energy obtained from a battery to the energy of water stored in a hydroelectric system dam in the mountains, the EMF of the battery could be likened to which *two* of these?
  A The amount of water in the dam.
  B The height of the dam above the power station.
  C The potential energy of each kilogram of water in the dam.
  D The total potential energy of the water stored in the dam.

7.4 Resistance, ohmic and non-ohmic conductors

When a potential difference is applied across an electrical device in a circuit, a current will flow. Generally speaking, the greater the potential applied, the greater the current that will flow. The actual relationship between the current and the potential difference applied is the subject of this section.

In order to measure the current through and the potential difference across a circuit element we must introduce two invaluable tools for any exploration of electric circuits: the voltmeter and the ammeter.

Ammeters and voltmeters

Figure 7.26

Voltmeters and ammeters come in a wide variety of shapes and forms. Some are digital and some are analogue.

As its name implies, a *voltmeter* is used to measure the potential difference across any circuit element or source of EMF. A voltmeter could be compared to a pressure gauge used to measure tyre pressure. Just as the pressure gauge measures the difference between the tyre pressure and atmospheric pressure, the voltmeter measures the difference in electrical potential between two points in a circuit. This is why the voltmeter connections are always placed *across* the circuit element.

The *ammeter* could be compared to the paddle-wheel water meter you might see on an irrigation channel. It is used to measure the current flowing in a circuit and is therefore placed so that the current flows through the ammeter as well as the circuit element. Never place an ammeter across a circuit element (as you would a voltmeter) because it would effectively create a short circuit which could damage the meter, the circuit or both.

Physics file

Ideally a voltmeter should draw no significant current from the circuit it is being used to measure. In practice, just as a pressure gauge may take a little air out of the tyre and reduce the pressure, a voltmeter will draw some current and change the circuit a little. It is important to ensure that any current drawn by the voltmeter is much less than that flowing in the circuit.

A water meter should not significantly slow down the rate of flow of the water that it is measuring. Likewise, an ammeter should offer no significant resistance to the current flowing through it and so it is important to ensure that the resistance of the ammeter is much less than that of the other components in the circuit.

Current–voltage graphs

Whether the device is a simple light bulb or a complex electronic component, a knowledge of the relationship between the current and the voltage, the so-called *I–V* characteristic, is necessary in order to predict the behaviour of the device or the power it will consume. This is often given in the form of a graph. The voltage is plotted on the horizontal axis because it is usually the 'independent variable', the quantity we set by using a certain battery or power supply. Some examples of I–V graphs for some common electrical devices are shown in Figure 7.28.

We can divide these devices into two groups: those which have a straight I–V graph and those which do not. The resistor is in the first group, the light bulb and the diode are in the second. Those with a straight I–V characteristic are called *ohmic conductors* and those which don't are (rather logically) called *non-ohmic conductors*.

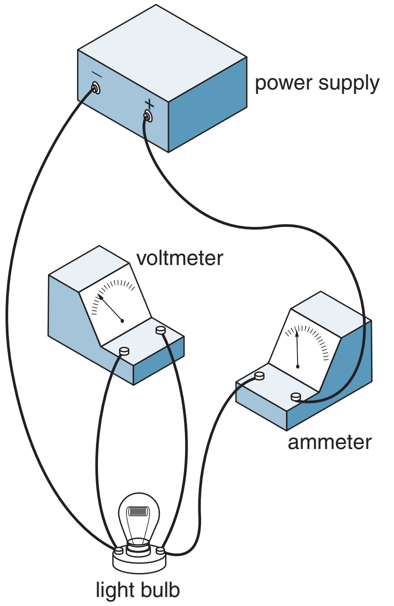


Figure 7.27
A simple circuit of one battery and one circuit element, together with an ammeter and voltmeter to measure the current and potential difference.

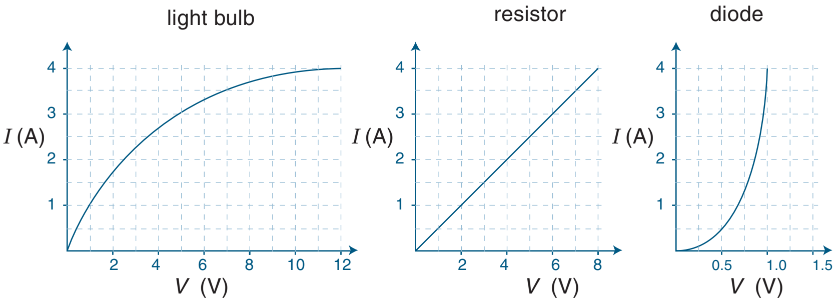


Figure 7.28
Examples of the relationship between current and applied potential difference for three common electrical devices.

Resistance

Georg Ohm (1789–1854) found that if the temperature of a metal wire was kept constant, the current flowing through it was directly proportional to the potential placed across it: $I \propto V$. This is known as Ohm's law.

OHM'S LAW states that $I \propto V$.

Rather than writing this as $I = kV$ (where k is the slope of the graph) this relationship is normally written the other way around as $V = IR$, where R is called the *resistance* (it is the inverse gradient of the graph). The resistance is defined then as the ratio of the potential difference across a conductor to the current flowing in it.

RESISTANCE is the ratio of potential difference to current:

$$R = \frac{V}{I} \text{ or } V = IR$$

The expression $V = IR$ is often referred to as Ohm's law, but in fact it is simply the definition of resistance. Ohm's law states that the resistance of certain conductors is constant. We can see from Figure 7.28 that while the resistance of the resistor is constant, that of the light bulb increases with increasing potential, whereas the resistance of the diode decreases with increasing potential. The resistor obeys Ohm's law, but the others do not.

Physics file

All circuits must have three things: a source of electrical energy, a load and a conductive pathway. Examples of energy sources are batteries, generators and solar cells. Loads can be things like light bulbs, fans and kettles. The conducting pathway is often wire made from copper.

Physics file

It is interesting to consider the reason for the increase in resistance of the light bulb. At 1 V the filament is barely glowing dull red, if at all. At 12 V it is shining very brightly and at a temperature of around 2200°C. At this temperature the atoms of the filament are vibrating much more rapidly. It is not surprising then that the electrons have a more difficult time getting through. In effect, their average drift speed (the speed due to the current as distinct from their thermal speed) is slowed. As a result, more voltage is needed to achieve the same current; that is, the resistance is greater.

Ohmic conductors

Many conductors do obey Ohm's law quite closely and so their *I*–*V* characteristic is completely specified by a single number, the resistance *R*. The unit for resistance is *volts per ampere* and is given the name *ohm* (symbol Ω, omega). It helps to think of the resistance as the number of volts needed to make a current of one ampere flow through the conductor. Ohmic conductors are often simply referred to as 'resistors':

$$1 \text{ ohm} = 1 \text{ volt per ampere } (1\ \Omega = 1\ \text{V A}^{-1})$$

✓ Worked Example 7.4A

A resistor of 5 Ω is supplied with a potential which can vary from 1 V to 100 V.

a What will be the range of current that will flow in it?

b How much energy will be dissipated in the resistor each second?

Solution

a 5 V are required to make 1 A flow in this resistor, therefore at 1 V the current will be $^1/_5$ A or 0.2 A. More formally:
At 1 V, $I = V/R = {}^1/_5 = 0.2$ A
At 100 V, $I = {}^{100}/_5 = 20$ A (or simply 100 times the previous answer).

b At 1 V, 0.2 C flow through the resistor each second. The energy is given by $E = qV = 0.2 \times 1 = 0.2$ J.
At 100 V, $E = 20 \times 100 = 2000$ J
Notice the very large increase in energy as the voltage is increased. As the voltage increased by 100 times, the energy released each second increased by 10 000 times because both voltage and current increased by 100 times.

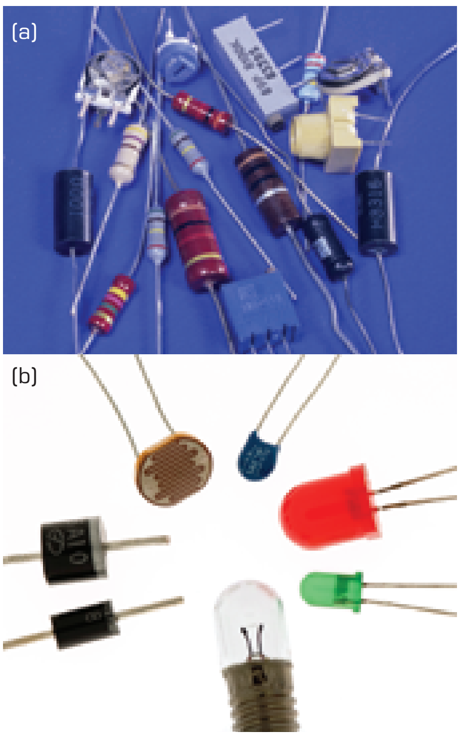


Figure 7.29
(a) An assortment of ohmic resistors, including a variable one. (b) Some non-ohmic devices including diodes, a light-dependent resistor (LDR), a thermistor, a torch bulb and light-emitting diodes (LEDs).

Non-ohmic conductors

A light bulb is a common example of a non-ohmic conductor. Typically, a car headlamp bulb may draw around 1 A at 1 V, but as the voltage increases, the current will not increase in proportion, as you can see in Figure 7.28. At 12 V the current might be 4 A. So while the resistance at 1 V is 1 Ω, at 12 V the resistance has increased to 3 Ω. While it may sometimes be useful to know the resistance of the bulb at its operating voltage of 12 V, it cannot be used to calculate the current flowing at other voltages. The bulb does not obey Ohm's law.

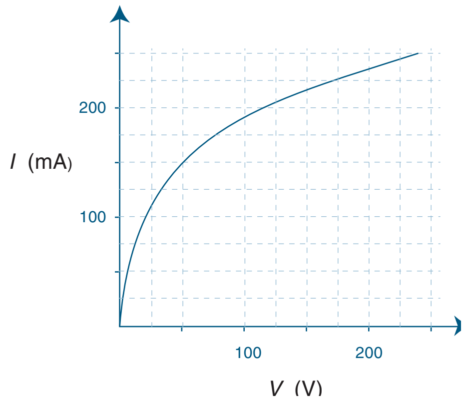
To quote the resistance of the diode in Figure 7.28 would be almost meaningless: it decreases very rapidly once the voltage reaches about 0.5 V. The important thing to know about the diode is that once the voltage exceeds a certain level the current increases, apparently without limit. In practice there will be a limit to the current because the power dissipated will reach a level where the diode will become too hot and burn out.

Other non-ohmic conductors include devices whose resistance changes with light or temperature. These are particularly useful as detectors in sensors which need to respond to changes in light or temperature levels.

Worked Example 7.4B

The graph represents the I–V characteristic of a 240 V, 60 W light bulb. What is the resistance at:

a 24 V?
b 120 V?
c 240 V?



Solution

Resistance is given by $R = V/I$ at any point on the graph. Note that the current is given in mA (100 mA = 0.1 A).

a At 24 V, $R = 24/0.10 = 240\ \Omega$
b At 120 V, $R = 120/0.20 = 600\ \Omega$
c At 240 V, $R = 240/0.25 = 960\ \Omega$

Resistance increases as the filament becomes hotter.

Resistance and resistivity

What are the factors that determine the resistance of a conductor? Given that the resistance of a piece of metal wire is a measure of the ability of the wire to somehow impede the flow of electrons along its length, it is reasonable to expect that:

1. If the wire is made longer there will be a greater resistance as there is more to impede the flow of the electrons.
2. If the wire is made thicker there will be less resistance as there is more pathway for the charge flow and so less impedance.
3. If a different metal is used it will most likely have a different structure and so a different impedance to the current.

This would suggest a simple relationship:

$$R = \frac{\rho L}{A}$$

where L is the length of the wire
A is the cross-sectional area
ρ (rho) is the constant of proportion.

The value of ρ will depend on the particular material used. When actual experiments are done it is found that this relationship does indeed hold, at least as long as the temperature is held constant. The constant of proportion, ρ, is called the *resistivity*, and it is seen as a measure of the inherent resistance of the material without regard to its shape or size.

The value of ρ varies over a huge range, about 23 orders of magnitude, from around $10^{-8}\ \Omega$ m for good conductors like copper right up to over $10^{15}\ \Omega$ m for good insulators. Table 7.3 gives some typical values.

Physics file

The value of R normally increases with the temperature of the material. For example, at 300°C the resistivity of copper is about twice its value at room temperature. This is because at higher temperatures the atoms of the metal are vibrating more vigorously and will therefore impede the progress of the electrons to a greater extent.

Table 7.3 Resistivity of some common materials at 20°C

Material	Resistivity, ρ (Ω m)
Silver	1.6×10^{-8}
Copper	1.7×10^{-8}
Aluminium	2.8×10^{-8}
Tungsten	5.5×10^{-8}
Tungsten (2000°C)	70×10^{-8}
Nichrome	100×10^{-8}
Doped silicon*	1×10^{-3}
Pure silicon	3×10^{3}
Pure water	5×10^{3}
Soils and rock	$10^{3} \rightarrow 10^{7}$
Wood	$10^{8} \rightarrow 10^{11}$
Glass	$10^{10} \rightarrow 10^{14}$
Fused quartz	10^{16}

* Doped silicon is used for transistors and integrated circuits.

Worked Example 7.4C

Normal household wiring uses 1.8 mm diameter copper wire. What is the resistance of a 10 m long piece of this copper wire? What voltage drop will there be along it if a current of 10 A is flowing through it?

Solution

The resistivity of copper is found from Table 7.3: $\rho = 1.7 \times 10^{-8}\ \Omega$ m.
The cross-sectional area of the wire is given by:
$A = \pi r^2 = 3.14 \times (0.9 \times 10^{-3})^2 = 2.5 \times 10^{-6}\ \text{m}^2$
The resistance is therefore found from:
$R = \rho L/A = 1.7 \times 10^{-8} \times 10/(2.5 \times 10^{-6}) = 0.068\ \Omega$
If the wire is carrying a 10 A current there will be a voltage drop of
$V = IR = 10 \times 0.068 = 0.68$ V along the length of the wire.

Physics in action — What really travels along a wire?

It is worth reflecting for a moment on the nature of an electric current in a metal. If it were actually possible to see all the atoms and electrons in a metal wire, we would see a constant blur of activity. The atoms would be vibrating madly and the free electrons would be rushing around at random with enormous speeds—rather like a game of air hockey gone wild! Now imagine that a current suddenly starts to flow in this wire. What difference would it make? The answer is none at all! At least not to any perceptible extent. However, if we could watch a single electron for a few seconds we would notice that, on top of its wild random dance, it had moved a few millimetres in one direction. If you think you might have noticed that, consider that in those few seconds, in its random dance, that same electron would have collided with millions of billions of atoms and covered thousands of kilometres!

If the electrons travel so slowly around the wires, why does the light come on almost instantaneously? In fact, electricity travels along a wire at close to the speed of light. Less than a millisecond after the switch is closed at the power station near Collie, 200 km from Perth, the electrical potential appears at the power line in Perth.

What travels down the power line is not really charge, but a wave of electric field. This wave of electric field pushes the charges a little closer (or a little further apart). It is this change in the concentration of the charges that gives rise to the potential, the voltage, which travels down the wire so quickly.

A simple analogy can be drawn with a garden hose. Provided it is full of water to begin with, water will come out the other end virtually as soon as the tap is turned on. The actual water that goes into the hose when the tap is turned on may take quite a few seconds to emerge from the other end. Again, what travels so quickly down the hose is a pressure pulse, not the actual water. The water then flows because the pressure at the tap end of the hose is higher than that at the other end. Likewise, if the potential at one end of a wire is greater than that at the other, a steady current will flow along the wire.

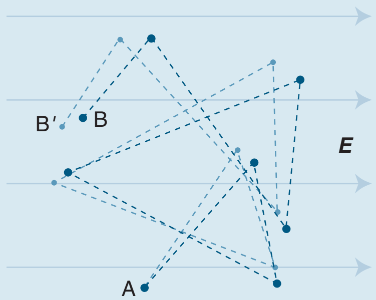


Figure 7.30

This diagram represents the path of an individual electron in a wire as it might appear without (AB) and with (AB′) an electric field (***E***). The motion due to the field, however, has been greatly exaggerated. Drawn to scale it would be almost impossible to notice any difference.

Physics in action — Resistance and the electric field in a wire

We can gain some insight into the nature of resistance from our simple model of the mechanism of charge movement in wires. For a current to flow in a wire, an electric field must be established in order to produce a force on the electrons in the wire. This electric field is set up by the source of EMF which produces a different 'charge concentration' (potential) at one end of the wire relative to that at the other end. This is shown diagrammatically in Figure 7.31.

The concentration of charge along the length will gradually change from being strongly positive at one end to strongly negative at the other end. It is this *changing* concentration which establishes the uniform electric field. At any point an electron 'sees' more positive charge to the left than to the right and will therefore experience a force to the left. (Remember that an electron experiences a force in the direction opposite to the field.) If in fact the field was stronger at one place than another, electrons would move more quickly there and this would weaken the field until it became uniform.

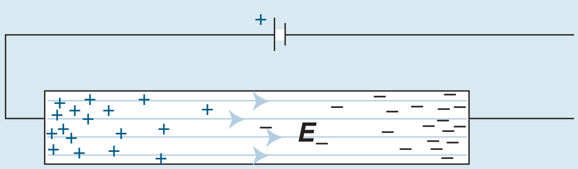


Figure 7.31

A potential difference applied to the ends of the metal conductor creates an electric field along the length of the conductor.

It is this electric field which 'drives' the current. It creates a steady force on the free electrons which results in their slow movement along the conductor. In an average conductor the electrons might take around a minute to move a millimetre. Don't forget, however, that there are huge numbers of electrons moving and so the number passing any point in one second will also be huge. In a current of 1 A there will be 6.2×10^{18} electrons flowing past any point each second.

The reason the electrons move so slowly is simple: they keep colliding with the atoms in the wire and giving up their energy (which is why the temperature of the wire rises). The stronger the field, the faster they move, however, and therefore the stronger the current. Not surprisingly, it turns out that the speed is directly proportional to the strength of the field and that the field is directly proportional to the potential difference applied. Thus the current is proportional to the potential difference, as Georg Ohm discovered.

Why is the electric field equal to the potential gradient along a wire? If we imagine a small positive test charge, $+q$, moving along the wire in a steady field $\boldsymbol{E}$, it will lose potential energy given by $E_p = q\boldsymbol{E}\Delta x$ (work = force × distance) where Δx is the distance it has travelled. If it travels the whole length, l, the energy lost will be $E_p = q\boldsymbol{E}l$. Note that the kinetic energy gained by the electron is given up almost immediately to the atoms the electron collides with.

Now the energy transferred *per unit of charge* is just what is meant by the potential difference, V. That is, $V = E_p/q$. From the previous expression we see therefore that $V = \boldsymbol{E}l$. This is the electrical equivalent of the expression 'work = force × distance'. We simply need to remember that V represents the 'work *per unit charge*' and $\boldsymbol{E}$ is the 'force *per unit charge*'. In this context remember that the work done is equal to the change of potential energy.

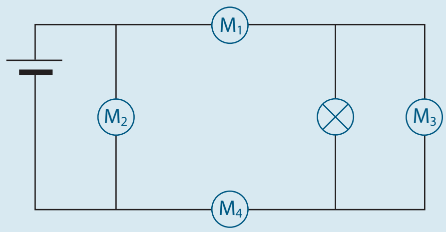
Rearranging, we can write $\boldsymbol{E} = V/l$. The electric field can thus be expressed as the 'potential gradient' along a wire. Indeed it is usually given units of 'volts per metre'.

It is now easy to see why the simple relationship between resistance, length and cross-sectional area of a conductor holds. The longer the wire (with a constant potential difference), the weaker the electric field and hence the lower the current that will flow. The field created in the wire is dependent on the potential difference and the length, not on its width, and so if one piece of wire has twice the cross-sectional area of another it will carry twice the current. The resistance will therefore decrease with increasing cross-sectional area.

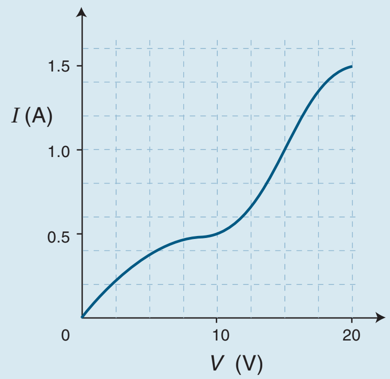
7.4 SUMMARY Resistance, ohmic and non-ohmic conductors

- The current–potential difference relationship for an electrical device describes the electrical behaviour of the device.
- A device with a linear I–V graph is known as an ohmic conductor. It obeys Ohm's law: the current is directly proportional to the voltage.
- While the current in a non-ohmic conductor will be dependent on the voltage, it is not directly proportional to it.
- The resistance of a conductor, whether ohmic or not, is defined as the ratio of the potential difference to the current ($1\,\Omega = 1\,\mathrm{V\,A^{-1}}$). Thus $V = IR$. The resistance of an ohmic conductor is constant.
- A current flows in a metal because of an electric field established by a potential difference along it. The amount of current depends on the field, the cross-sectional area and the length as well as the nature and temperature of the material.
- The resistance of a conductor is given by $R = \rho L/A$ where ρ is the resistivity, a characteristic of the material. The resistivity normally depends on the temperature.

7.4 Questions



1 Andy wishes to measure the *I*–*V* characteristic of a light bulb. He has set up a circuit as shown. In which of the positions M_1–M_4 can he place the:
   a voltmeter?
   b ammeter?



2 A strange electrical device has the *I*–*V* characteristic shown.
   a Is it an ohmic or non-ohmic device? Explain.
   b What current is drawn when a voltage of 10 V is applied to it?
   c What voltage would be required to double the current drawn at 10 V?
   d What is the resistance of the device at 10 V; at 20 V?

3 A student finds that the current through a resistor is 3.5 A while a voltage of 2.5 V is applied to it.
   a What is the resistance?
   b The voltage is then doubled and the current is found to increase to 7.0 A. Is the resistor ohmic or not?

4 Rose and Rachel are trying to find the resistance of an electrical device. They find that at 5 V it draws a current of 200 mA and at 10 V it draws a current of 500 mA. Rose says that the resistance is 25 Ω, but Rachel maintains that it is 20 Ω. Who is right and why?

5 Nick has an ohmic resistor to which he has applied 5 V. He measures the current at 45 mA. He then increases the voltage to 8 V. What current will he find now?

6 Lisa finds that when she increases the voltage across an ohmic resistor from 6 V to 10 V the current *increases* by 2 A.
   a What is the resistance of this resistor?
   b What current does it draw at 10 V?

7 The resistance of a certain piece of wire is found to be 0.8 Ω. What would be the resistance of:
   a a piece of the same wire twice as long?
   b two pieces of the same wire side by side?

8 If the resistance of a copper wire 20 m long and 1 mm in diameter is 0.44 Ω, what will be the resistance of the same length of wire 2 mm in diameter?

9 In Worked Example 7.4C the resistance of a piece of copper wire 1.8 mm in diameter and 10 m long was found to be 0.068 Ω.
   a What would be the resistance of a piece of aluminium wire of the same dimensions? (The resistivity of copper is 1.7×10^{-8} Ω m, aluminium is 2.8×10^{-8} Ω m.)
   b What voltage drop would occur along the aluminium wire if 10 A were flowing in it?

10 Two students have measured the *I*–*V* characteristics of two electrical resistors and have found them to be straight lines with different slopes. Elsa says that the one with the steeper slope has a greater resistance, but Cathryn says the one with the lower slope has the greater resistance. Who is right and why?

7.5 Electrical energy and power

Anyone who has been anywhere near a lightning strike knows the enormous electrical energy that nature can unleash in a fraction of a second. You are quite likely using a tamed version of that energy by which to read this book. One only needs to look around at all the devices which rely on it, in one form or another, to be reminded that electrical power is central to our modern way of life.

Electrical energy

Electrical potential energy is stored whenever charges are pushed close together (or separated). This potential energy can be transmitted long distances, by high-voltage power lines, from power stations or simply produced on demand from the chemical energy stored in batteries.

The EMF, or voltage, of a power source is a measure of the number of joules of energy stored for each coulomb of charge. As the charges move through the circuit they lose the energy given to them by the source. The energy lost by a charge q moving through a potential difference V is given by:

$$E = qV$$

As the current is the rate at which charge is moving, the total charge q can be expressed as $q = It$. The total energy produced is therefore given by:

Electrical energy (joules) = potential drop (volts) × current (amps) × time (seconds).

$$E = VIt$$

Physics file

Electrical energy is correctly called work. Work is done by the electric field in the circuit. The electric field puts a force on the charge and causes it to move. $W = Fs$.

Worked Example 7.5A

How much energy is used in one hour by a 240 V heater drawing 5 A?

Solution

Each coulomb of charge gives 240 J of heat energy, and in one hour the number of coulombs used is given by $q = 5\text{ A} \times 3600\text{ s}$. The total energy used is thus $E = VIt = 240 \times 5 \times 3600 = 4.3 \times 10^6\text{ J} = 4.3\text{ MJ}$.

Electric power

Power is the rate of energy use: $P = E/t$. (Remember that the SI unit for power is the watt, where 1 watt = 1 joule per second.) Dividing the previous expression by t gives:

$$E/t = VIt/t \text{ or } P = VI$$

This is an important expression. A simple example might help you see what it really means—as distinct from simply knowing the formula! Think of the energy delivered to a 12 V headlight bulb drawing 5 A from a car battery. Each second, 5 C of charge pass through the lamp filament. Each of these coulombs carry 12 J of energy from the battery. This means that every second 12 × 5 = 60 joules of energy are delivered to the lamp. The power is the energy delivered per second and so this lamp is operating at a power of 60 joules per second, that is, 60 watts. This can be summarised:

Total energy supplied to bulb each second	=	Energy provided by each unit of charge	×	Number of charges supplied each second

Power (watts) = voltage (volts) × current (amps):
$P = VI$

Table 7.4 Some examples of electric power consumption

Electric train	2 MW
Electric home heater	2.4 kW
Car starter motor	2 kW
TV set	200 W
Electric light	15–150 W
Walkman (tape)	300 mW
Calculator	0.4 mW

Physics file

It is easy to confuse the energy and power units for electricity because of the presence of the 'kW' in both.
Remember:
Units of energy 1 MJ = 10^3 kJ = 10^6 J
1 kW h = 10^3 W h
= 3.6 MJ
Units of power 1 MW = 10^3 kW
= 10^6 W

For the car bulb:
60 J/s = 12 J/C × 5 C/s, which is the same as: 60 W = 12 V × 5 A.

This relationship applies in all electrical situations. Fundamentally it is another expression of the principle of conservation of energy: the energy we can obtain from an electric current (each second) is equal to that put into it by the source of the voltage.

Worked Example 7.5B

Two different torch bulbs are rated as 2.8 V, 0.27 A, and 4.2 V, 0.18 A.
a Which will be the brightest?
b Could they be interchanged?

Solution
a The brightness is indicated by the power used—although only about 4% becomes light energy. $P = VI$ and so their powers are 2.8 × 0.27 = 0.76 W (or 760 mW) and 4.2 × 0.18 = 0.76 W. The bulbs will be the same brightness.
b Although the power of each bulb is similar, using them in the wrong torch will either result in the bulb burning out or running dim. The bulbs are designed to work at a certain voltage. If a greater voltage is used, too much current will flow in the bulb which will result in it burning out.

Another unit for electrical energy

The total amount of energy used by an appliance depends on the time for which it is switched on. The total energy is given by the product of the power and the time: $E = Pt$ (1 joule = 1 watt × 1 second).

When discussing domestic appliances, time is more likely to be measured in hours than seconds. Just as one 'watt second' is one joule, a 'watt hour' is also a unit of energy and will be equal to 3600 joules (as there are 3600 seconds in an hour). Similarly one 'kilowatt hour' will be 3 600 000 joules. The watt hour (W h) and kilowatt hour (kW h) are the amount of energy used in one hour by a device using a power of 1 watt or 1 kilowatt respectively.

Worked Example 7.5C

How much energy does a 100 W light bulb use in half an hour?

Solution
Here P = 100 W and t = 0.5 h.
So E = 100 W × 0.5 h = 50 W h.
This could also be given as 100 W × 1800 s = 180 000 J = 180 kJ.

Electricity supply companies install a meter in every home which measures the power consumed in kW h. We are charged around 12 cents for each kW h used on the normal tariff. In homes where off-peak electricity is used, for example a storage hot water heater which is only on in the early hours of the morning, a second meter will be installed as off-peak electricity is charged at a lower rate.

Worked Example 7.5D

At a rate of 12 cents per kW h, how much will it cost each week to run a 200 W television set for 4 hours per day? Compare this to the cost of running an electronic clock rated at 5 W.

Solution

The energy required for the television set each day is $E = Pt = 200 \times 4 = 800$ W h $= 0.8$ kW h. In one week the total will be seven times this, or 5.6 kW h. That will cost 67 cents. The clock will use $5 \times 24 \times 7 = 840$ W h, or 0.84 kW h in one week. This will cost 10 cents.

Electric power production and transmission

When electric power is generated on a large scale it is almost always 'AC', or alternating current power. What does this mean?

A battery, a solar cell and a Van de Graaff generator all produce what is known as a DC EMF. The letters stand for direct current but could just as easily mean direct voltage in many situations. In fact, although it is something of a contradiction in terms, the expression 'DC voltage' is commonly used. A DC source of EMF always pushes charges in one direction. The top (red or +) terminal of a dry cell is always positive, as the chemicals inside push electrons from the top terminal to the bottom. In any device connected to the dry cell, current will flow from the positive terminal of the cell to the negative terminal.

As the name suggests, an alternating current continually changes direction. The AC mains voltage we use in our homes reverses direction 50 times every second. The active terminal (often coloured red or brown) might have a positive potential at one moment, but it will have a negative potential $^{1}/_{50}$ of a second later. A detailed study of alternating current remains for next year, but for most purposes we can assume that the mains 240 V AC power will have the same effect as a 240 V DC source.

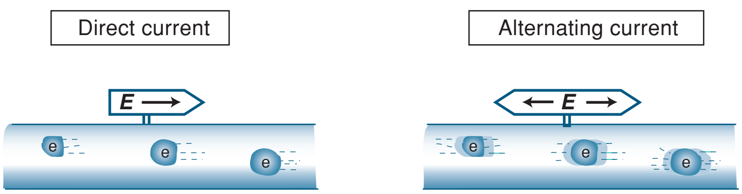


Figure 7.33

In a wire carrying a DC current the electrons are slowly moving in one direction. In an AC current the electrons hardly move at all, they just vibrate back and forward 50 times a second.

Most of the power used in cities is generated by large power stations a long way from where it is eventually used. The power is generated at around 20 kV (20 000 volts) but then 'transformed' to a much higher voltage, typically 400 kV, for transmission to the city. There it is transformed down to 22 kV for distribution. Transformers on the poles along our streets reduce it further to 240 volts for domestic use. We won't be studying the internal operation of transformers this year, but the law of conservation of energy tells us that all the power that goes into them must come out. In fact, a little will be converted into heat, but in a good transformer over 99% continues on as electrical energy but at a different voltage.

The power relation $P = VI$ tells us that if a transformer changes the voltage but does not alter the power, the current must also change. If the voltage was doubled, for example, the current would halve. This is exactly why the voltage is changed. At 400 000 V the current needed to transmit power is clearly far less than the current needed at lower voltages. As the size of the cables needed to transmit power depends on the current, considerable cost savings are achieved by using high voltages.

Figure 7.32

A typical household electricity energy meter. The dial on the right reads the number of unit kW h while each one to the left reads one power of ten higher. When reading a meter, be careful to note that each alternate dial turns in the opposite direction and to read the number which the pointer has most recently passed. This meter reads 92371 kW h. The small dial reads tenths of a kilowatt hour and the large horizontal disk, which drives the meter, spins at a rate which you will find stated on the meter ($266\frac{2}{3}$ revolutions per kW h in this case).

Physics file

In fact, the potential of the active terminal of an AC supply varies between +340 V and −340 V during one cycle. The neutral terminal will be at zero potential. The 240 V quoted as 'mains voltage' is actually an average potential. It is the DC potential that would be required to provide the same amount of power.

✓ Worked Example 7.5E

If the power from a generator operating at 20 kV and producing a current of 10 000 A is transformed up to 500 kV, what current will flow at the higher voltage? How much power is being produced? (Assume the transformer is 100% efficient.)

Solution

500 kV is 25 times 20 kV. This means that the current flowing at the higher voltage will be only $^1/_{25}$ of the 10 000 A flowing at 20 kV. The high voltage current will therefore be 400 A. The power can be calculated at either voltage:

$P = VI = 20\text{ kV} \times 10\,000\text{ A}$

$= 200\text{ MW}$ (megawatts) or $500\text{ kV} \times 400\text{ A} = 200\text{ MW}$

Table 7.5 Typical values for daily domestic electrical energy consumption

Appliance	Typical power (W)		Average use ($h\ day^{-1}$)		Energy use (average $W\ h\ day^{-1}$)	
	Min.	Max.	Min.	Max.	Min.	Max.
Kitchen						
Lights	11	100	1.00	3.00	11	300
Refrigerator	100	260	6.00	12.00	600	3120
Microwave oven	650	1200	0.17	0.25	111	300
Toaster	600	600	0.03	0.08	18	48
Laundry						
Lights	11	100	0.25	1.00	3	100
Iron	500	1000	0.17	0.42	85	420
Washing machine	500	900	0.22	0.33	110	300
Dryer	1800	2400	0.20	0.54	360	1300
Sewing machine	15	75	0.07	0.07	1	5
Water pumps	300	500	0.25	1.00	75	500
Lounge						
Lights	15	100	1.00	4.00	15	400
Television	25	200	0.50	5.00	13	1000
Video recorder	100	100	0.50	5.00	50	500
Stereo	60	80	0.50	3.00	30	240
Radio	10	40	0.33	3.00	3	120
Vacuum cleaner	100	1000	0.13	0.25	13	250
Bedrooms						
Lights	11	100	0.50	2.00	6	200
Bathroom						
Lights	11	100	0.17	1.00	2	100
Garage/external						
Lights	11	100	0.17	2.00	2	200
Power tools	200	800	0.17	0.17	34	136
Hot water						
New Dimension	1750	2500	4.00	6.00	7000	15 000
electric storage						24 000
Traditional storage	2000	3000	5.00	8.00	10 000	
Electric heating*						
Heat-banks	3000	6000	10.00	16.00	30 000	96 000
Fan heaters	2000	7000	6.00	12.00	12 000	84 000
Strip heaters	500	1500	0.50	1.00	250	1500
Oil-filled radiators	1000	3000	8.00	14.00	8000	42 000

* During winter months

Physics in action — Electricity and energy

The average Australian uses about 640 MJ of energy each day. This includes all energy used in mines, agriculture, industry and commerce as well as at home. This is about the same as Western Europe, about half that of North America and around three times the world average. Our modern way of life has seen a huge increase in the amount of energy used by the average person each day (Figure 7.34). The 8 MJ for the Stone Age person is simply that required for food; 966 MJ is an estimate of the consumption of a 'modern technological' person. Many suburban Australian households with two or three cars would fall into that category!

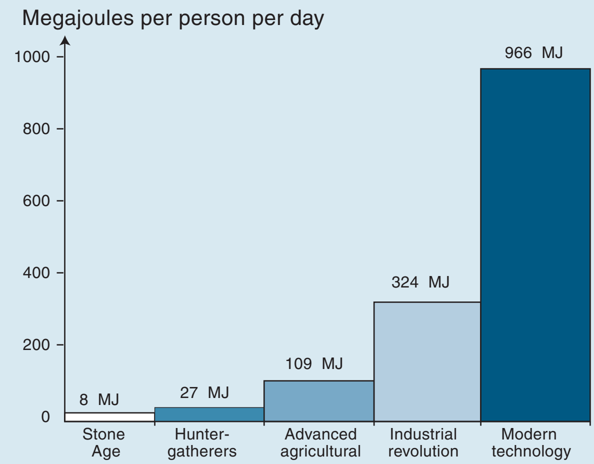


Figure 7.34
The average daily energy consumption in megajoules per person at various stages in history. The value of 966 MJ is not the current world average but an estimate for the average modern technological consumer. The current world average is of the order of 200 MJ.

By far the largest proportion of the energy consumed is derived from fossil fuels (coal, gas and petroleum). In Australia virtually all of our energy comes from this source, while about 17% of the world's electrical energy is generated from nuclear power. Most fossil fuel energy is used directly as fuel in industry, transportation or heating, but around one-third of it is used to generate electrical energy.

Electricity is a 'high quality' form of energy. While it is very hard to convert more than around 40% of the heat energy from coal or oil into electricity, it is possible to convert 100% of electrical energy into heat. A car engine is regarded as very efficient if 25% of the fuel energy can be converted into the kinetic energy of the car, whereas a large electric motor can be 95% efficient. Thus, electricity is a very important form of energy in the modern world. Furthermore, it is very difficult to run a radio directly on coal! Figure 7.36 indicates the main uses for electricity.

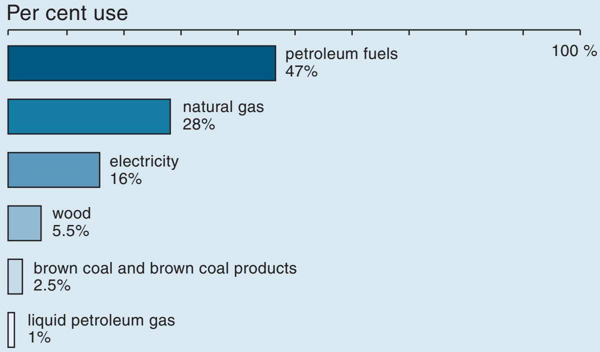


Figure 7.35
Most of the energy used by Australians is either fuel for their cars or air-conditioning (heating and cooling). Electricity makes up about one-sixth of the energy needs.

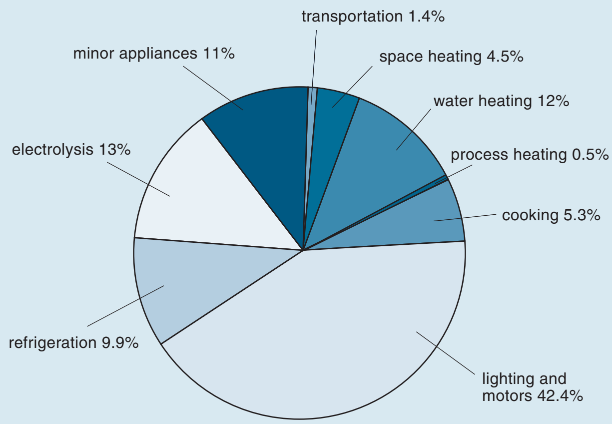


Figure 7.36
Electricity is a very adaptable form of energy. Its major uses include lighting, motors of many types and the production of aluminium (electrolysis).

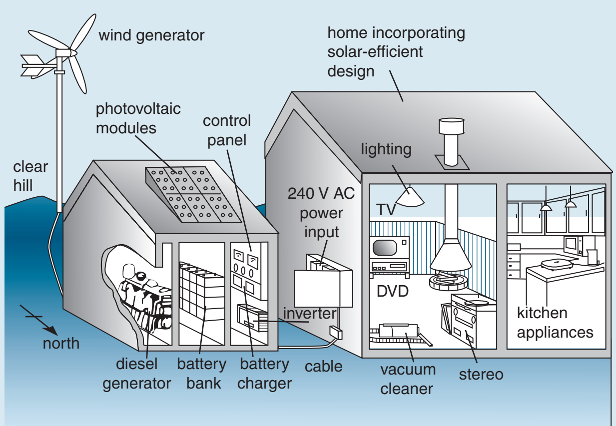
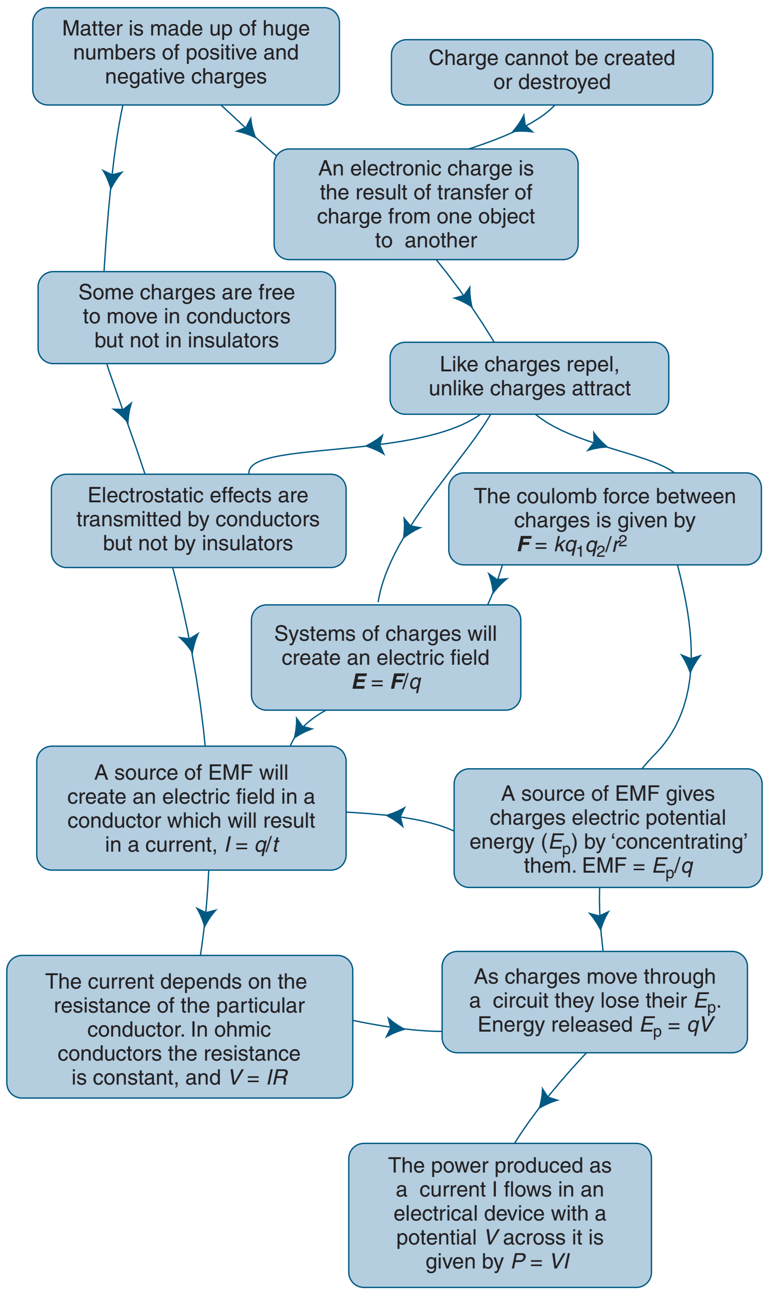


Figure 7.37
The importance of finding alternative sources of energy has never been greater. This diagram shows a simple 'self-sufficient' energy system.

7.5 SUMMARY Electrical energy and power

- The potential energy stored when charge is concentrated can be converted into other forms of energy in nature or in man-made devices.
- The rate at which this energy is released (the power) is given by the product of the potential (voltage) and the current, $P = VI$. In SI units 1 watt is equal to 1 volt × 1 ampere (1 W = 1 A V).
- Total energy produced is given by $E = Pt$ (1 J = 1 W s). The kilowatt hour (kW h) is an alternative energy unit and is equal to 3.6 MJ.
- The direction of an EMF may be either constant or alternating. A constant EMF gives rise to a direct current (DC) and an alternating EMF gives rise to alternating current (AC).
- Power is transmitted to cities at high voltages in order to keep the current needed relatively low.



7.5 Questions

1 A 4.5 V battery is used to power the motor of a toy car.
  - a How much energy (in joules) is given to each coulomb of charge that flows through the battery?
  - b How much electrical potential energy is released in the motor by each coulomb of charge that flows through it?
  - c Will all of the electrical potential energy released in the motor appear as kinetic energy of the car? Explain.

2 How much energy does one electron receive when it travels through a 1.5 V cell?

3 What is the power used by a:
  - a 3 V torch bulb drawing 0.2 A?
  - b starter motor which takes 200 A from a 12 V battery?
  - c mains-powered (240 V) toaster rated at 3 A?

4 How much current is used by a:
  - a 60 W, 240 V light globe?
  - b 1200 W, mains-powered heater?
  - c 90 W car windscreen wiper motor?

5 What is the voltage of a:
  - a 100 W spotlight which draws 4 A?
  - b 200 mW radio operating with a current of 23 mA?
  - c 7500 W (10 HP) industrial motor using 18 A?

6 A large power station generator might be rated as 500 MW with a 24 kV output. What current would it be generating?

7 How much energy is used by a 5 W digital clock in one week? Answer in kW h as well as in joules. If electric energy costs 12 cents per kW h, how much will it cost to run the clock for a year?

8 A step-down transformer is used to run a 12 V model railway from the 240 V mains. If the model engine operates at a power of 18 W, and the transformer can be assumed to be 100% efficient, what is:
  - a the current used by the engine?
  - b the input current to the transformer from the mains?

9 Briefly describe the difference between AC and DC electric power. Give examples of some sources of each.

10 Why is electric power transmitted to cities at very high voltages?

7.6 Simple electric circuits

Any electrical circuit consists of one or more sources of EMF connected by good conductors to various combinations of circuit elements. By 'good conductors' we mean that any potential drop (p.d.) in them will be very small compared with the potential drops across the circuit elements. Only in a *superconductor* is there no p.d. at all. By way of example, the p.d. along an extension cord used to run a 240 V heater might amount to 2 or 3 V.

In order to draw clear diagrams of electrical circuits, a range of more or less standard symbols and conventions is used. Some of these are given in Figures 7.38 and 7.39.

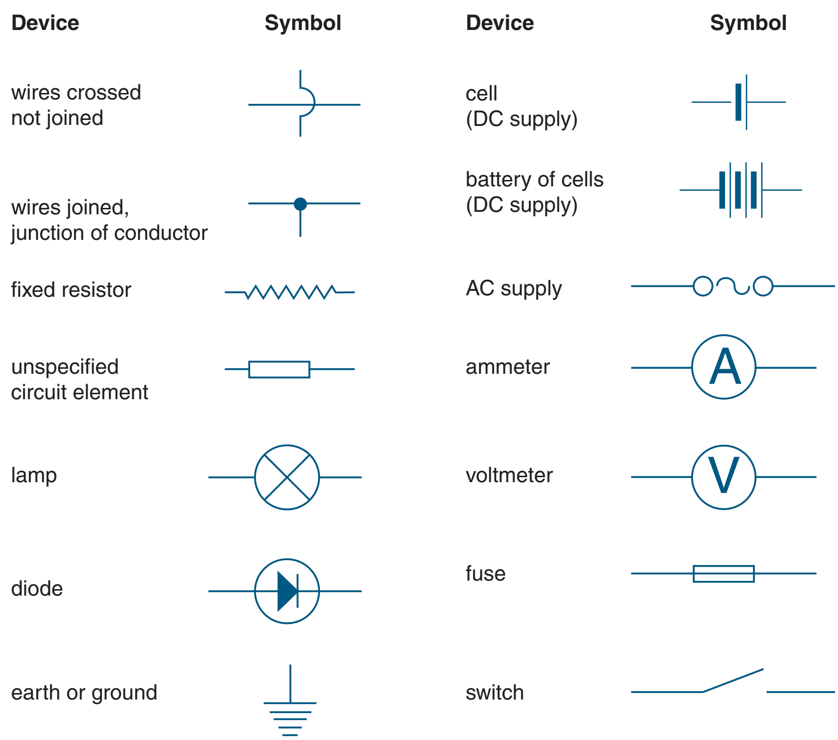


Figure 7.38

Some commonly used electrical devices and their symbols.

Physics file

Superconductors are materials which, at very low temperatures, have zero resistance. Mercury and lead become superconducting at temperatures of 4.2 K and 7.2 K respectively. Curiously, most metals that are good conductors at normal temperatures, gold and copper for example, do not become superconducting at all.

In 1986 a new class of so called 'warm superconductors' was discovered. It was found that various obscure compounds of metal oxides with rare earths became superconducting around the relatively high (but still rather cold!) temperature of liquid nitrogen (77 K). The significance of this was that liquid nitrogen is about as cheap as milk and so it made practical applications look much more feasible.

So far, the applications are mainly in the production of very strong magnets. This is because once an electric current is set up in a superconductor it simply keeps going as there is no resistance to absorb its energy. As you will probably know, a strong electric current creates a strong magnetic field.

Two circuit rules

The principle of conservation of charge and the principle of conservation of energy can be used to establish two very important rules that apply to all electric circuits.

> In any electrical circuit the sum of all currents flowing into any point is equal to the sum of the currents flowing out of it.

For example, if at a junction of three wires there is 2 A flowing in on one wire and 3 A flowing in on another, then there must be a current of 5 A flowing out on the third. As current is simply flowing charge, this 'rule' is basically a consequence of the principle of conservation of charge.

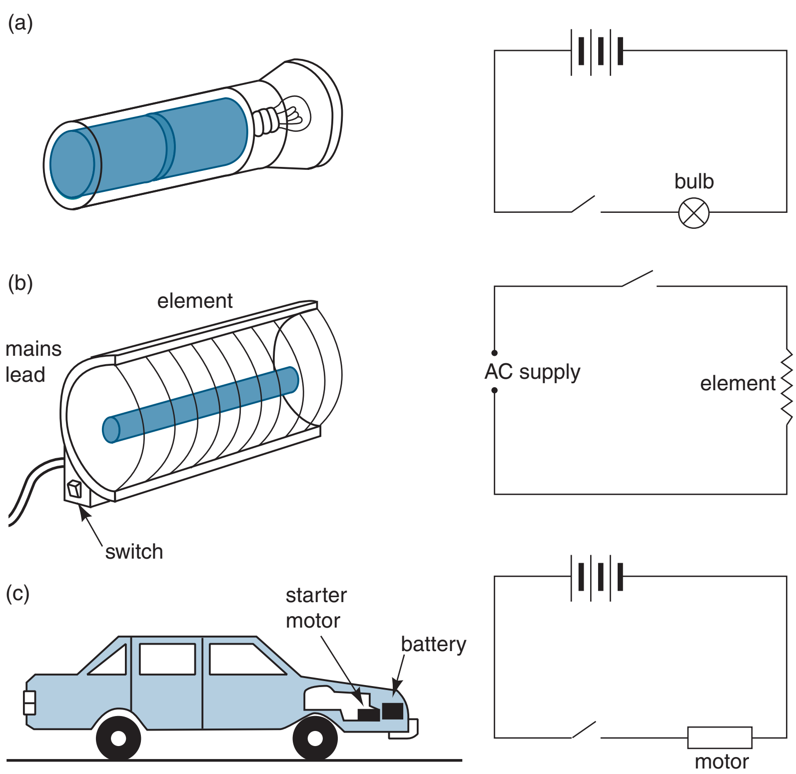


Figure 7.39

Some electrical devices and their circuit diagrams. (a) A torch. (b) A single bar radiator. (c) A car starter motor.

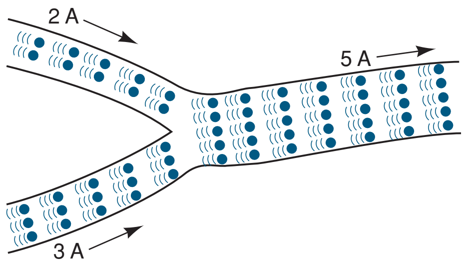


Figure 7.40

The current flowing into any junction must be equal to the current flowing out of it. Each little symbol could be imagined as the movement of 1 C of positive charge in 1 s.

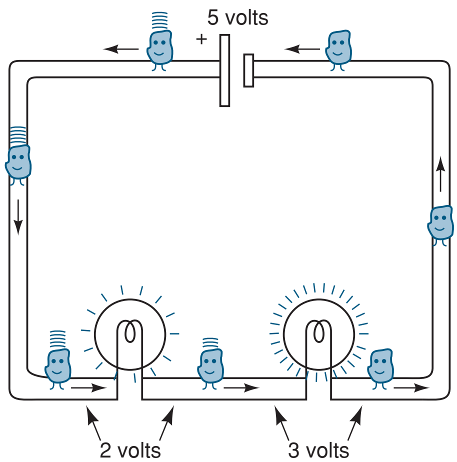


Figure 7.41

The little creatures represent coulombs of charge loaded with various amounts of energy from the battery, which they give up as they travel around the circuit. They will use up a little in the conductors, but that would be very small compared with the amount given to the light bulbs.

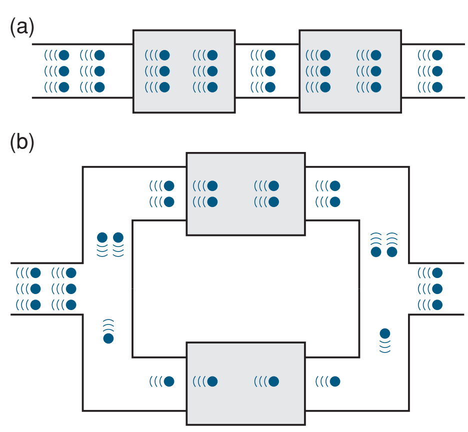


Figure 7.42

Two circuit elements (a) in series and (b) in parallel.

Sometimes this rule is abbreviated simply to 'the sum of all currents at a point is zero'. Remember that currents flowing into the point will be positive and those flowing away from the point negative. The second rule is an expression of the principle of conservation of energy.

> The total potential drop around a closed circuit must be equal to the total EMF in the circuit.

In the torch, for example, if the battery supplies an EMF of 3.0 V and we measure a 2.8 V p.d. across the bulb, there must be a 0.2 V drop somewhere else in the circuit—possibly across the switch contacts if they are a little dirty. As the EMF is the energy given to each unit of charge and the total potential drop is the energy obtained from each unit of charge, they must be equal.

These two rules are known as *Kirchoff's laws* after Gustav Kirchoff who first stated them in 1845. In terms of our water cycle analogy (section 7.3) the first is equivalent to saying that the total water flowing into a river junction is the same as that flowing out from it. The second is equivalent to saying that the height gained by water evaporated by the Sun is equal to the distance fallen as it returns to the sea. While the first might be a little questionable for water, as a result of some evaporation or seepage into the ground, the second obviously has to be exactly true. As electric charge does not 'evaporate' or seep into the insulation, both laws are very accurate for electric circuits. These two simple rules, or 'laws', are the basis for the analysis of any electrical circuit. In the rest of this chapter we will use them to look at some practical circuits.

Figure 7.43

Mains-powered Christmas tree lights are usually wired in series so that lower voltage globes can be used. If 20 globes are used in series the voltage across each is just 12 V. The problem is that if one globe burns out they all go out.

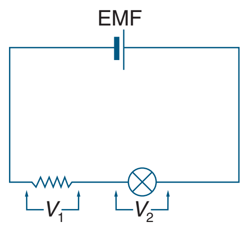


Figure 7.44

In a series circuit the sum of the two potential drops V_1 and V_2 will be equal to the total supplied potential difference.

Two ways of connecting circuits

No matter how complex a circuit, it can always be broken up into sections in which circuit elements are combined either in series or in parallel; that is, one after another, or beside each other.

Charge flowing in a series circuit flows through one element *and* then through the other. Charge flowing through a parallel circuit flows through one element *or* the other. Christmas tree lights are often wired in series. Normal house lights and power points are in parallel. We will consider series circuits in this section and parallel circuits in the next.

Series circuits

Consider the case of a battery in a circuit with a torch bulb and a resistor in series, as in Figure 7.44. The current flowing in this circuit will pass through both bulb and resistor. This will result in a potential drop across each and, as Kirchoff's second law tells us, the sum of the two potential drops will be equal to the total supplied potential difference.

It is the *I–V* characteristics (see section 8.4) that define the electrical behaviour of a circuit. For an ohmic resistor the *I–V* characteristic can be given simply by its resistance. For any other circuit device, such as the bulb in this circuit, the *I–V* graph is usually needed. If the characteristic is known then it is possible to determine the current and power at any particular voltage. The question, then, is if the *I–V* characteristics of the two individual devices are known, can the *I–V* characteristic of the series combination be found?

When circuit elements are placed in series, because the charge has to be 'pushed' through one element and then the next, common sense would lead us to expect that a greater total voltage would be needed to achieve a certain current flow. This is indeed the basis for finding the combined characteristic. If the two individual *I–V* characteristics are drawn on a graph (see Figure 7.45) the characteristic of the combination can be found by adding the two graphs 'sideways' (see Worked Example 7.6A). That is, at any particular current, add the two individual voltages to get that of the series combination. This procedure simply reflects the fact that at a particular current, the total voltage across the combination is equal to the sum of the individual voltages.

Worked Example 7.6A

The resistor and the light bulb whose *I–V* characteristics are given in the graphs in Figure 7.45 are to be combined in series.

a What is the resistance of the resistor?

b Draw the *I–V* characteristics of the series combination.

c The light bulb is designed to operate at a current of 4 A. It is necessary to run the light bulb from a power supply which can only produce voltages between 15 V and 25 V. Suggest a way in which this could be done using the resistor in conjunction with the bulb.

Solution

a The resistance is found from $V = IR$, that is, $R = V/I = 10/5 = 2\ \Omega$.

b In order to find the characteristics of the series combination the graphs are added sideways; that is, the voltages are added at constant currents. For example, to obtain a current of 1.0 A in the circuit requires 2.0 V across the resistor, but 0.5 V across the bulb; a total of 2.5 V is required. A table of values obtained this way follows:

1.0 A	2.0 A	3.0 A	4.0 A
2.5 V	5.5 V	10 V	18 V

These values are graphed in Figure 7.46.

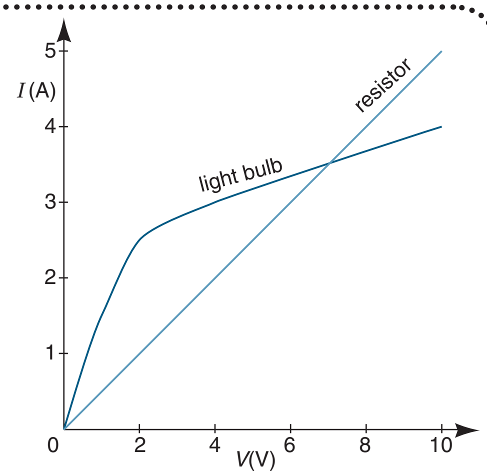


Figure 7.45

The *I–V* characteristics of a fixed resistor and a light bulb.

c The bulb requires 10 V to operate at 4 A. In order to operate it from a power supply over 10 V a resistor will be required in series to lower the potential across the bulb to 10 V. As can be seen from the graph of the series combination above, to obtain a current of 4 A through both, the voltage needs to be set at 18 V. At this point there is a p.d. of 8 V across the resistor and 10 V across the bulb.

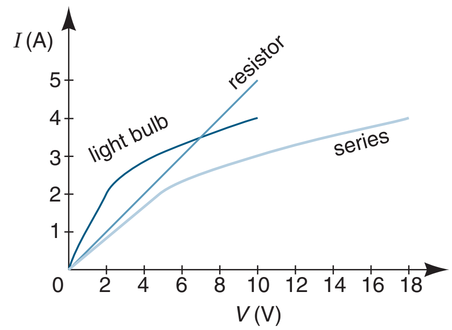


Figure 7.46
The resistor and light bulb graphs have been added at constant current, that is, sideways. The voltage of the series graph at any current is the sum of the two individual voltages.

Resistors in series

When ohmic resistors are placed in series with each other, the situation is somewhat simpler. Because the I–V characteristic of a resistor is a straight line they will add together to give a combined characteristic with another straight line. The use of Kirchoff's laws enables us to find the *effective resistance* of a combination of resistors in series. The effective resistance is the value of the single resistor which would have the same I–V characteristic as the two in series.

If two resistors are placed in series, we expect the effective resistance to be higher than that of either resistor separately. (Remember that the resistance can be thought of as the number of volts required to achieve a current of 1 A.)

Figure 7.47 shows two resistors of values R_1 and R_2 in series and another resistor of value R_e, the value of the single resistor which could replace the series combination of the first two. As the two resistors are in series, the sum of the voltages across them must add to the total voltage, that is the voltage across R_e. (V_1 is the voltage across R_1 and V_2 that across R_2.) The voltage across R_e is V, the sum of V_1 and V_2. That is:

$$V = V_1 + V_2$$

For two resistors in series the current, I, through each is the same. As the voltage across a resistor is always given by $V = IR$ we can write:

$$IR_e = IR_1 + IR_2$$

Dividing both sides by I we have:

$$R_e = R_1 + R_2$$

The effective resistance is simply the sum of the two resistances. If there are more than two resistors in series it is easy to show that the effective resistance is simply the sum of all of them:

$$R_e = R_1 + R_2 + R_3 + \ldots$$

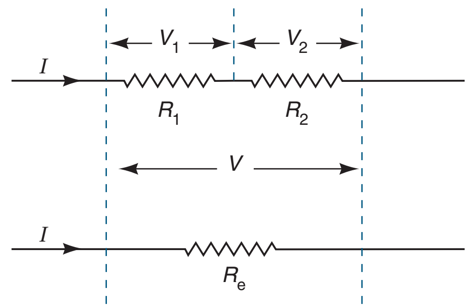


Figure 7.47
R_e is the value of the single resistor that could replace the series combination of R_1 and R_2.

Worked Example 7.6B

Two pieces of nichrome wire (as used in heater elements) have resistances of 10 Ω and 20 Ω.

a What current would flow through them, and what power will be produced in them, if they are separately connected to a 12 V battery?
b If they are connected in series what is their total resistance?
c When placed in series across the 12 V battery, what current will flow through them and what power will be produced?

Solution

a The current will be given by $I = V/R$, so for the two wires separately the currents will be $12/10 = 1.2$ A and $12/20 = 0.6$ A. The power is found from $P = VI$ and so will be $12 \times 1.2 = 14.4$ W and $12 \times 0.6 = 7.2$ W.
b When connected in series the total resistance will be $10 + 20 = 30$ Ω.
c The current that flows from the 12 V battery will be $I = V/R = 12/30 = 0.4$ A. The total power will be $12 \times 0.4 = 4.8$ W.

Physics file

While the term 'potential drop' is more strictly correct, the simpler term 'voltage' is very commonly used. In either case, remember that the term always refers to the *difference* in potential across a circuit element or source of EMF.

Physics file

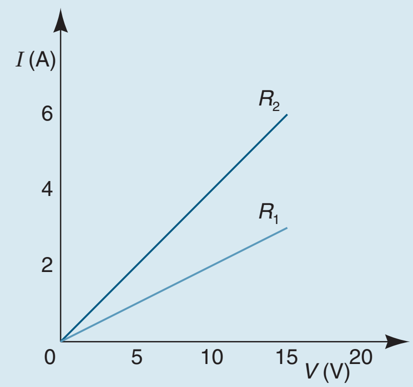
In summary, resistors in series have:
- the same current as each other
- their own potential drop.

7.6 SUMMARY Simple electric circuits

- An electric circuit normally consists of a source of EMF, conductors to carry current with little loss of energy, and circuit elements which turn the potential energy of the charges into some form of useful energy.
- Circuit diagrams are drawn by using a set of conventions (see Figure 7.38).
- Kirchoff's two laws enable us to analyse any circuit:
  1. In any electrical circuit the sum of all currents flowing into any point is equal to the sum of the currents flowing out of it.
  2. The sum of all EMF values around a circuit is equal to the sum of all potential drops around the circuit.
- All circuits are combinations of circuit elements in series (one after another) or in parallel (beside each other).
- In a simple series circuit the same current goes through each element in turn. The total voltage drop is the sum of the individual drops.
- The combined I–V characteristics of a series combination of circuit elements can be found by adding the individual characteristics 'sideways', that is, adding the voltages at constant currents.
- For ohmic resistors in series the effective resistance is given by $R_e = R_1 + R_2 + \ldots$

7.6 Questions

1. State the two circuit rules (Kirchoff's laws) which apply to all circuits. What are the two basic principles that they are based upon?
2. An auto-electrician finds that while the EMF of the car battery is 12 V there is only 10 V across one of the tail-light bulbs in a car. Can you suggest a reason for this?
3. Emily has found a point in her car where four wires are attached together. She finds currents of +2.5 A and +1.0 A in two of the wires, and −4.2 A in a third (+ means current into the point and − means current out of the point). What is the current in the fourth wire?
4. Two torch bulbs are placed in series with each other and a 4.5 V battery. The current through one is found to be 0.25 A and the voltage across it 2.1 V.
   a. What is the current through the other bulb?
   b. What is the voltage across the other bulb?
5. Bill has bought two 12 V headlamps for his truck but finds that it has a 24 V battery. He decides that the simplest way to overcome the problem is to wire them in series.
   a. Will the headlamps work correctly?
   b. Do you see any problem with his scheme?
6. Two equal resistors are placed in series and found to have a combined resistance of 34 Ω. What is the resistance of each one?
7. A 10 V power supply is used across two separate resistors. The current through one is found to be 0.4 A and through the other 0.5 A. When they are combined in series, what current will flow through them and what is their effective resistance?
8. A 400 Ω resistor and a 100 Ω resistor are placed in series across a battery with an EMF of 5 V.
   a. How much current will flow from the battery?
   b. What will be the voltage across each resistor?
9. Two ohmic resistors, whose I–V graphs are shown, are to be combined in series.



   a. Draw a sketch graph of the I–V characteristic of the two resistors combined in series.
   b. What is the effective resistance of the combination?
10. It is found that there is 60 V across one resistor of a pair in series and 20 V across the other. If the smaller resistance of the two is 5 Ω, what is the value of the other resistor?

7.7 Circuit elements in parallel

Putting two circuit elements in parallel increases the paths available to the current and so allows more current to flow for a given voltage. We would therefore expect the *I–V* graph of a parallel combination of elements to be rather higher (more current at a given voltage) than either of the two separate graphs. This is indeed the basis for finding the combined characteristic.

The *I–V* graph of parallel circuit elements

When a voltage is applied to a parallel combination it is applied to both elements. At a given voltage, the total current that flows will simply be the sum of the two separate currents. This time, the *I–V* characteristic for the combination is obtained by adding the individual characteristics 'upwards' (see Worked Example 7.7A). That is, at any particular voltage, add the two currents together to obtain the total current that will flow in the circuit.

Worked Example 7.7A

The resistor and the light bulb whose *I–V* characteristics were given in Figure 7.45 (and repeated in Figure 7.48) are to be combined in parallel.

a Draw the *I–V* characteristics of the two elements in parallel.

b The light bulb is designed to operate at a current of 4 A. If the two are placed in parallel across a power supply, what voltage setting will be required and what total current will flow from the power supply?

Solutions

a To find the parallel characteristic requires that the graphs are added upwards; that is, the currents at a particular voltage are added. At 2.0 V the total current is given by 1.0 + 2.5 = 3.5 A. The other values are given below:

2.0 V	4.0 V	6.0 V	8.0 V	10.0 V
3.5 A	5.0 A	6.4 A	7.7 A	9.0 A

The combined characteristics are shown in Figure 7.48.

b The bulb requires 10 V to operate at 4 A. If the resistor and bulb are in parallel this means that there will also be 10 V across the resistor and hence a current of 5 A flowing in it. The voltage setting required is 10 V and the total current drawn from the power supply will be 9 A.

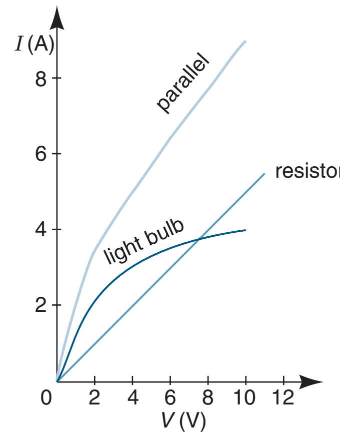


Figure 7.48
The resistor and light bulb graphs have been added at constant voltage. The current of the parallel graph at any voltage is the sum of the two individual currents.

Resistors in parallel

When ohmic resistors are placed in parallel with each other we can expect a greater current to flow at any particular voltage. In fact the current will be the sum of the two separate currents. We thus expect the effective resistance of two resistors in parallel to be somewhat less than either of them alone.

The combined characteristic will again be a straight line. As was the case with the series combination of resistors, the effective resistance of two resistors in parallel can be found by using Kirchoff's laws and a little algebra. Remember that the effective resistance is the value of the single resistor which would have the same characteristics as the parallel combination of the two.

If the two resistors are placed in parallel it is the sum of the two currents that must add to get the total current:

$$I = I_1 + I_2$$

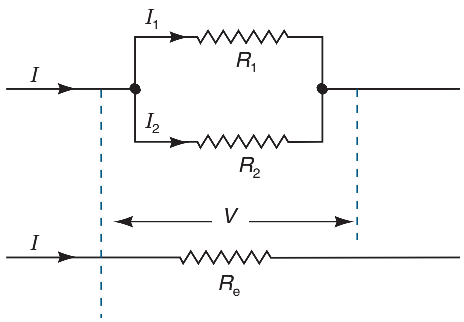


Figure 7.49
R_e is the value of the single resistor that could replace the parallel combination of R_1 and R_2.

Again, by re-expressing this relationship in terms of the voltages and resistances we can find what we are looking for. This time, replace I by V/R. $I = I_1 + I_2$ becomes:

$$\frac{V}{R_e} = \frac{V}{R_1} + \frac{V}{R_2}$$

Dividing both sides by V we have:

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2}$$

This time it is the reciprocals of the resistances that are to be added. Again, this is what might be expected. R_e will always be somewhat smaller than either of the other two. If, for example, both resistances are the same, say R, the above expression yields $\frac{1}{R_e} = \frac{2}{R}$, or $R_e = \frac{1}{2}R$. As might be expected, putting two similar resistances in parallel halves the total resistance.

If there are more than two resistors in parallel the effective resistance is found by adding all the reciprocals:

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Physics file

The reciprocal of the resistance is sometimes called the 'conductance' for the simple reason that a large value would mean a good conductor. It is given the symbol G, i.e. $G = 1/R$. The unit is then Ω^{-1} sometimes called a 'mho' (ohm backwards). The expression for two resistors in parallel then becomes $G = G_1 + G_2$. It is not surprising that the total conductance of two conductors in parallel is simply the sum of the individual conductances. (Combining two conductances in series yields an effective conductance given by $1/G = 1/G_1 + 1/G_2$.) In most circuits we deal with, the resistances have values in the hundreds or thousands of ohms and so it is normal to use resistance rather than the corresponding very small values of conductance. On the other hand, electric power engineers sometimes deal with conductance because their cables need to have very low resistances.

Worked Example 7.7B

As in Worked Example 7.6B, two pieces of nichrome wire (as used in heater elements) are found to have resistances of 10 Ω and 20 Ω.

a If they are connected in parallel what is their effective resistance?

b What total current will flow through them and what power will be produced if the combination is placed across a 12 V battery?

Solution

a The effective resistance is found from $\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20}$.
Thus $R_e = \frac{20}{3} = 6.7\ \Omega$.

b The total current is given by $I = \frac{V}{R} = 12/6.7 = 1.8$ A. The power is therefore $P = VI = 12 \times 1.8 = 21.6$ W.
(You will find that these answers can also be obtained by adding those in part a of Worked Example 7.6B. Can you see why?)

Power in ohmic resistors

Quite often we need to calculate the power dissipated in an ohmic resistor. In some cases the resistor may be a heating element specifically designed to produce heat. Because the temperature may affect the resistance it is important to measure the resistance at the appropriate temperature. In other situations the power (and therefore heat) may be a nuisance, in the confined space of a compact electronic device for example. The power lines that carry electricity from the power stations to the city are ohmic resistors (with very low values of resistance). The power lost in them represents not only wasted generating capacity but extra fuel used and extra Greenhouse gases produced.

We have already seen that the power dissipated in any electrical device is given by $P = VI$. This is true whatever the characteristics of the device. The power may appear as any form of energy: kinetic energy from a motor, sound energy from a loudspeaker or heat from a simple resistance wire.

In the case of an ohmic resistor the power will produce heat. Because of the simple relationship between the current, voltage and resistance it is possible to find two simple relationships between the power produced and these quantities. By substituting either $V = IR$ or $I = V/R$ into the power equation, $P = VI$, we can find the following expressions.

Physics file

In summary, resistors in parallel have:
- the same potential drop as each other
- their own current.

The power produced in a resistor is given by:

$$P = I^2R \text{ or } P = \frac{V^2}{R}$$

Worked Example 7.7C

In Worked Examples 7.6B and 7.7B the two wires with resistance 10 Ω and 20 Ω were placed in series and in parallel across a 12 V battery. Use these two new expressions for power (above) to find the power produced in each wire when they are:

a in series

b in parallel.

Solution

a When the wires were in series a current of 12/30 = 0.4 A flowed. The power in each can be found directly from the expression $P = I^2R$. For the first wire $P = 0.4^2 \times 10 = 1.6$ W, and for the second $P = 0.4^2 \times 20 = 3.2$ W. The total power is thus 4.8 W as found in Worked Example 7.6B, part c.

b When in parallel the expression $P = V^2/R$ can be used to find the power without the need to find the current first. For the first wire $P = 12^2/10 = 14.4$ W, and for the second $P = 12^2/20 = 7.2$ W, as was found in Worked Example 7.7B, part b.

Physics in action — Three-heat switches

Simple heaters of various sorts often have a 'three-heat' switch. An electric blanket will usually have 'low', 'medium' and 'high' settings, for example. Rather than making three different heating elements the manufacturer can use two elements in different series and parallel combinations to obtain the three heat settings. If the two elements are placed in series the total resistance is relatively high and therefore the power will be a minimum (as $P = \frac{V^2}{R}$). For the medium setting one of the elements will be used by itself. The high setting is then achieved by placing both elements in parallel.

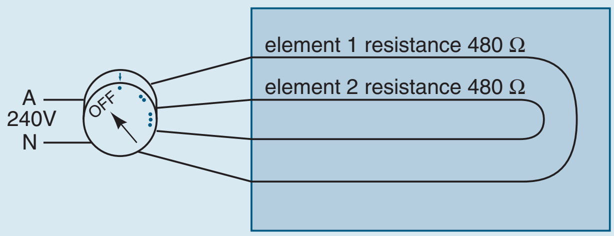
It is a simple matter to work out the relative power being used for the three settings. If it is assumed that the resistance of both of the elements is the same (R) and does not change appreciably with temperature, the effective resistance in the three cases will be given by:

Low heat (two elements in series): $R_e = R + R = 2R$

Medium heat (one element only): $R_e = R$

High heat (both in parallel): $R_e = \frac{1}{2}R$

As the power is inversely proportional to the resistance ($P = \frac{V^2}{R}$), if we call the high setting 100%, then the others will be 50% and 25%.



To obtain the various settings the following circuits are used:

OFF Not connected to power

• Resistors connected in series, $R = 960$ Ω, $I = 0.25$ A, $P = 60$ W

• • Only one resistive element is connected, $R = 480$ Ω, $I = 0.5$ A, $P = 120$ W

• • • Resistors connected in parallel, $R = 240$ Ω, $I = 1.0$ A, $P = 240$ W

Figure 7.50
An example of the use of series and parallel combinations of resistors to achieve three heat settings for an electric blanket.

7.7 SUMMARY Circuit elements in parallel

- In a simple parallel circuit the same voltage drop occurs across both elements. The total current that flows is the sum of the two individual currents.
- The combined *I–V* characteristics of a parallel combination of circuit elements can be found by adding the individual characteristics 'upwards'; that is, adding the currents at constant voltages.
- For ohmic resistors in parallel, the effective resistance is given by:

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

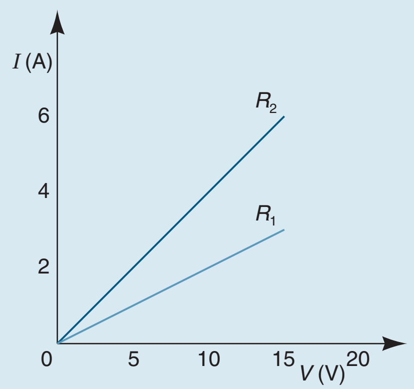
- The power produced in an ohmic resistance can be expressed as $P = I^2R$ or $P = \frac{V^2}{R}$.

7.7 Questions

1 Two torch bulbs are placed in parallel with each other across a 3.0 V battery. The current through the battery is 0.55 A. The current through one of the bulbs is 0.25 A.
  a What is the current through the other bulb?
  b What is the voltage across the other bulb?

2 What is the effective resistance of two 10 Ω resistors in:
  a series?
  b parallel?

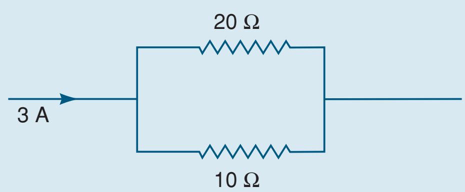
3 Two ohmic resistors, whose *I–V* graphs are shown, are to be combined in parallel.



  a Draw a sketch graph of the *I–V* characteristic of the two resistors combined in parallel.
  b What is the effective resistance of the combination?

4 Two equal resistors are placed in parallel and found to have a combined resistance of 34 Ω. What is the resistance of each one?

5 A 10 V power supply is used across two separate resistors. The current through one is found to be 0.4 A and through the other 0.5 A. When they are combined in parallel, what current will flow through them and what is their effective resistance?



6 A current of 3 A is found to be flowing through two resistors of 20 Ω and 10 Ω in parallel as shown.
  a What is the effective resistance of the combination?
  b What is the voltage across the pair of resistors?
  c How much current will be flowing in each resistor?

7 How much power will be dissipated in each of the two resistors in the previous question?

8 Three resistors of 900 Ω, 1.5 kΩ and 2.0 kΩ are to be used in a circuit. What is their effective resistance if they are all placed:
  a in series?
  b in parallel?

9 5 W of power is being produced in a resistor across which there is a voltage of 10 V.
  a What is the resistance of this resistor?
  b How much power will be produced if the voltage doubles to 20 V?

10 Ann has bought a hair dryer overseas labelled 220 V, 800 W, and she is going to use it on the Australian 240 V mains. She says to her flatmate Betty that as the voltage difference is less that 10% it will only produce about 10% more heat and that should not matter. Betty, however, claims that it is more likely to produce about 20% more heat which may cause it to burn out. Who is correct and why?

7.8 Cells, batteries and other sources of EMF

The source of the energy that the charges on a Van de Graaff machine dome obtain is obvious. The motor that drives the belt is literally doing work on the charges by pushing them up closer to other like charges. Although it is a source of EMF, in practice the power it can deliver is limited by the extremely small current it can maintain. Any source of EMF uses some form of energy to create electrical potential energy.

Chemical cells

In a battery, or cell as we should say, the source of EMF is the chemical energy stored in the materials used. As we saw at the beginning of the last chapter, atoms have varying abilities to attract electrons—the chemists call it electronegativity. A cell basically consists of two materials with different electronegativities, between which there is what we shall call a 'go-between' material. It is represented in Figure 7.51.

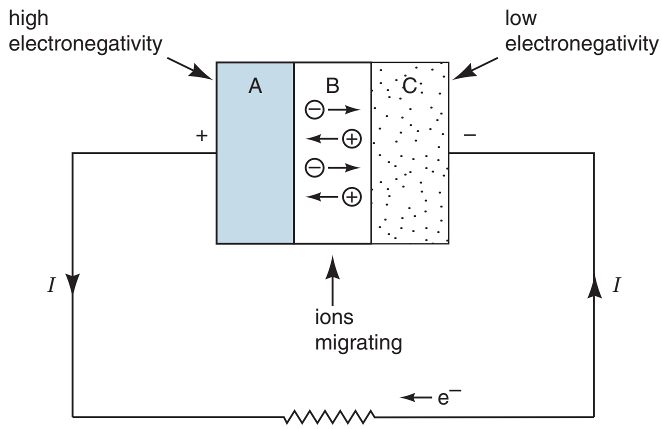


Figure 7.51

A cell consists of two materials that attract electrons to different degrees, along with a 'go-between' which acts as a conductor. In this diagram material A attracts electrons most (high electronegativity) and material C least. Material B acts as the go-between.

The chemistry of electrical cells can be very complex, but for our purpose it is sufficient to realise that electrons will flow from the material of lower electronegativity (for example zinc) to the one of higher electronegativity (for example copper) through the external circuit connected to the terminals.

In Figure 7.51 material A has the highest tendency to attract electrons and material C the lowest. Material B acts as the 'go-between'. It is effectively a conducting material which allows the other two to 'trade' electrons by undergoing a chemical reaction with each of them which either replaces the electrons lost (material C) or takes up the electrons gained (material A). In the course of this process the positive and negative ions in material B migrate towards A and C respectively. As a result of these reactions, material B is eventually used up and the product of the reactions replaces it. At this stage the cell stops working and we say it has gone 'flat'.

The properties of the materials chosen for A, B and C are very important. A and C must have as different electronegativities as possible

Physics file

The potential that a good Van de Graaff generator can develop is limited mainly by the size of the dome. The smaller it is, the more intense the electric field in the air around it and the greater the possibility of ionisation that allows the charges to escape. For the same reason, any sharp points or projections on the dome will reduce the potential.

Physics file

What is a motor? It is a common misconception that electric motors are a source of energy. Don't confuse motors with generators. Far from being a source of electrical energy, motors consume electrical energy and convert it to mechanical work. Motors are used to make things turn. Common examples are electric fans, microwave oven turntables, CD players, computer hard disks, food processors and washing machines.

but also undergo suitable reactions with material B. B must allow the migration of the ions (charged atoms) formed in the reaction and so is normally a liquid or a moist paste. In a dry cell, C is the outer zinc casing, B is a paste of ammonium chloride and other special substances (see Figure 7.23). Material A is not actually a metal but a manganese dioxide powder which is mixed with the ammonium chloride paste. The carbon rod in the centre of the cell is there to collect the current. In a charged car battery, A is lead dioxide coated on a lead plate, B is a solution of sulfuric acid and C is a lead plate. As current from the car battery is used, lead from the lead plate is converted into lead sulfate which remains as a coating on the plate. The lead dioxide is also converted into lead sulfate which remains on the other plate. Fortunately this process can be reversed by forcing an electric current through the battery in the opposite direction. This is the main purpose of the car's alternator.

There are many other types of cells in use. Some are single use and some (so-called 'Ni-Cads' for example) can be recharged. There is now considerable incentive for manufacturers to develop smaller, more efficient rechargeable batteries for use in electric cars and portable electronic devices.

Batteries in series and parallel

In many situations cells are combined in series, parallel or both in order to provide a battery with the required characteristics. The EMF of a set of cells joined in series, 'head-to-tail', will be equal to the sum of the individual EMF values. In this case each cell takes the potential of the previous cell and adds to it its own potential.

Cells combined in parallel must have the same EMF and be joined + to + and – to –. In this case, charge moving around the circuit will go through either one or the other of the cells but not both. As a result, the EMF will not be increased, but each battery will only need to provide half of the total current and so they will last twice as long.

Notice that if, in an attempt to combine the cells in parallel, they were actually joined head-to-tail, instead of head-head and tail-tail, it would create a closed circuit with two cells in series and no significant resistance. This is what is referred to as a *short circuit*. The batteries would run flat very quickly and could even become hot enough to cause a fire. When jumper leads are used to connect two car batteries in parallel it is particularly important to ensure that they are not inadvertently connected in series. If they were, an extremely large current could flow and burn the leads or even cause a battery to explode.

Figure 7.52
Two car batteries are connected in parallel when jumper leads are used to start a car with a flat battery. It is very important to ensure that the batteries are in parallel (+ to + and – to –) and not series in this situation!

Internal resistance of cells

Any chemical cell will have a certain amount of internal resistance due to the fact that the current must flow through the various chemicals and the electrodes. Although the chemical reactions always give the charges a certain amount of energy (the EMF of the cell), some of this energy will be lost as the charges move through the cell. As the cell gets 'flat' this loss of energy shows up as an increasing internal resistance. If the battery is fresh and being used in a situation where the current drawn does not exceed its design limits, the internal resistance can be assumed to be negligible relative to the other resistances in the circuit. Generally, to make a battery, with a very low internal resistance it has to be large; a 12 V car battery for example, might have an internal resistance of around 0.01 Ω. On the other hand, the small 9 V batteries often used in electronic devices have an internal resistance of about 10 Ω.

When a current is flowing through the battery there will be a voltage drop across the internal resistance. This results in the actual voltage appearing at the terminals of the battery being somewhat less than the EMF. A battery can be represented as in Figure 7.53; that is, a source of EMF in series with a resistor. The EMF can be measured by a voltmeter which draws no significant current, as then there will be no voltage drop across the internal resistance R_i. However, the actual voltage at the terminals when the battery is in use and a current is flowing will be lower than the EMF by the drop occurring across R_i. This drop will depend on both the current drawn and the value of R_i. The terminal voltage will therefore be given by the expression $V = \mathscr{E} - IR_i$. As the battery becomes flat the value of R_i rises and so the terminal voltage decreases.

Another advantage of using cells in parallel to make up a battery will now become clear. Because of the lower current in each cell, less energy is wasted in warming the battery (remember that $P = I^2R$) and the voltage at the terminals is closer to the EMF of the cells.

Virtually any source of EMF will involve some internal resistance. In designing a generator, for example, there is always a compromise between making the wire in the coils thin enough to keep the overall size small and the added internal resistance that this will involve.

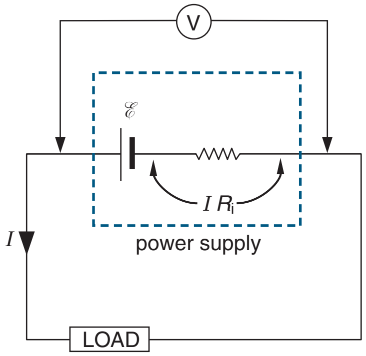


Figure 7.53
Any real power supply such as a cell or battery can be represented in a circuit diagram as an ideal source of EMF in series with a resistor called the internal resistance, R_i. The terminal voltage, V, is the potential drop across the load. The terminal voltage is given by $V = \mathscr{E} - IR_i$ where I is the current in the circuit. If there is no current in the circuit then $V = \mathscr{E}$. Relatively large internal resistance causes the potential drop across the load to be smaller.

Worked Example 7.8A

Michael and Mary-Ann have made small model electric motors. Mary-Ann is using two D cells in series to drive her motor. Michael, not to be outdone, decides to use a 9 V battery from a transistor radio, thinking that it should get better results than the 3 V that Mary-Ann is using. Unfortunately Michael finds that his motor hardly turns with the 9 V battery and yet when he uses Mary-Ann's battery it goes well. He tests the two batteries with a voltmeter and finds that they do indeed read 3 V and 9 V as expected. Mary-Ann then suggests reading the voltage while the motor is operating and they find that the voltage of her battery reads 2.5 V while Michael's reads only 2.0 V. Both motors were found to be drawing 0.5 A while this reading was taken.

a Why did Michael's battery not drive his motor properly despite the higher voltage?

b What was the value of the internal resistance of each battery?

c When he replaced the 9 V battery in the radio it worked perfectly, the battery showing a voltage of 8.0 V. How much current was the radio using?

Solution

a Michael's battery had a relatively high internal resistance and the motor was drawing too much current. As a result, the voltage drop across the internal resistance was very large.

b A current of 0.5 A caused a voltage drop of 0.5 V in Mary-Ann's battery (3.0 V – 2.5 V) and 7 V (9 V – 2.0 V) in Michael's battery. The internal resistance is given by $R_i = \Delta V/I = 0.5/0.5 = 1\ \Omega$ (Mary-Ann's) or $R_i = 7.0/0.5 = 14\ \Omega$ (Michael's).

c When used in the radio the voltage drop across the 14 Ω resistance was 1.0 V. Thus the current drawn was $I = \Delta V/R = 1.0/14 = 0.071$ A = 71 mA.

Other sources of EMF

By far the most common source of EMF is a generator or alternator (they are much the same for our present purposes). Very large generators at power stations can produce hundreds of megawatts of power at around 20 000 V. The alternator in a modern car is capable of producing around 500 W of power at 12 V. They both convert kinetic energy into electrical energy. You will learn about the physics of the means by which these devices convert kinetic energy into electrical energy if you undertake study of Physics Unit 3B.

There are a number of other interesting sources of EMF. Perhaps the most curious are animals such as the electric ray (*Torpedo nobilinia*) and the South American electric eel (*Electrophorus*). They can stun or even kill their prey with pulses of EMF reaching several hundred volts producing a current of up to 1 A. Electric eels use large numbers of cells called electroplaques, each producing 0.15 V at 1 mA. They have about four million of these cells arranged as 1000 parallel sets of around 4000 in series along the length of their bodies, giving a total of 1 A at 600 V.

Figure 7.54
An electric ray can deliver a brief 600 W electric pulse to stun or kill its prey. It is also a very effective defence against predators!

A thermocouple simply consists of two different metal wires joined at both ends. Iron and constantan (a copper–nickel alloy) are commonly used. When two different metals are in contact, electrons will have a tendency to move from one to the other. Normally this effect cannot be used as a source of EMF because any circuit will require two such joins and the two electromotive forces generated will oppose each other. However, this effect is temperature dependent and so if the two joins are held at different temperatures, one EMF will be greater than the other. This will result in a current flowing around the circuit.

By arranging many such joins in a series, quite a reasonable EMF can be produced. Such a device is called a *thermopile* and is used in situations where a reliable source of electric power is required and where a source of heat is available. The spacecraft which travel a long way from the Sun use the heat generated by a strong radioactive source to drive a thermopile to power their electrical systems. Thermopiles were once used widely to power remote weather stations and the like, but solar cells have almost completely taken over that role.

Solar cells

Solar cells, or more correctly photovoltaic cells, have been used widely to produce electricity from sunlight in areas where it is impractical or too expensive to use mains power. Producing electrical energy from solar cells on the roofs of our houses would be an ideal solution to many of the world's energy problems.

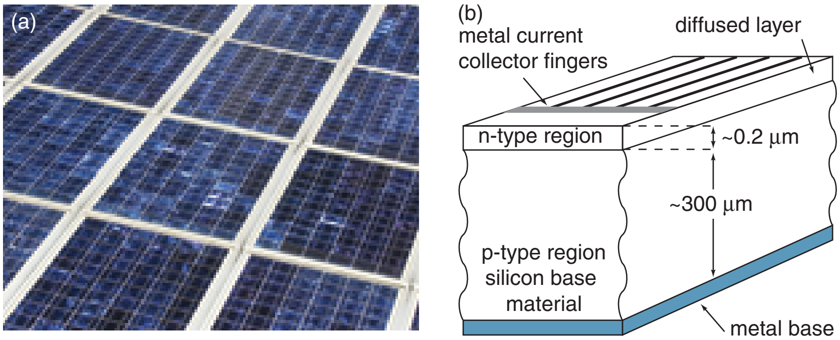


Figure 7.55
(a) An array of silicon photovoltaic cells. (b) Most of the cell is p-type silicon, but the very thin upper layer has n-type doping material diffused into it. Metal fingers on the top and a metal base collect the current.

Electrically, a photovoltaic cell is somewhat like a semiconductor diode. Both comprise two layers of silicon which are 'doped' with very small amounts of elements that either tend to add extra 'free' electrons to the structure or to leave a gap, thus creating 'holes' which can move as electrons move into them (in much the same way that the bubbles of air from a fish tank aerator are moving 'holes' in the water). The layer with the extra electrons is called an n-type semiconductor and that with the holes a p-type semiconductor. At the junction of the two layers, electrons from the n-type tend to fall into the holes in the p-type. But this means that electrons have been lost from the n-type layer and gained by the p-type layer, making the p-type layer in the region a little negative relative to the n-type layer.

When a photon of light falls on the cell it may knock an electron out of place, thus forming a new free electron and hole pair. If this occurs near the junction, because of their relative charges, the electron will tend to go towards the now positive n-type layer and the hole towards the more negative p-type layer. This charge movement results in the p-type region gaining an overall positive potential, while the n-type region gains a negative potential. If an external resistance (a so called 'load resistance') is now connected, current will flow through it from the p-type region to the n-type. As long as the light keeps creating the electron-hole pairs the process continues.

The overall effect is that the charges are given potential energy by the sun and the cell acts as a source of EMF. The characteristics of a typical cell are shown in Figure 7.56. The voltage and current obtained depend on the load being used. The power obtained is the product of the voltage and current. If the cell is being used as a source of energy it is important to try to operate the cell at the point where this product is a maximum. If the load resistance is too low, a high current will be obtained but with little voltage. On the other hand, if the load resistance is too high the full voltage will be obtained but with little current.

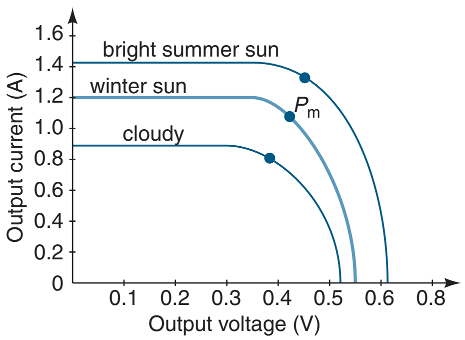


Figure 7.56
The characteristics of a typical 8 cm diameter solar cell. The points shown dotted (P_m) represent the operating conditions for maximum power in three different degrees of sunlight.

Physics in action — Solar energy: the big picture

The Sun's energy falls on the Earth's surface at the rate of about 1 kW on each square metre. This means that an average roof of around 200 m^2 receives 1000 kW h of energy in 5 hours of Sun. Given that an average household might use around 20 kW h of electrical energy in a day you can see the possibilities! Commercially available solar cells can convert about 15–25% of the energy falling on them into electricity and so a whole roof of solar cells could feasibly produce well over 100 kW h of energy per day.

The bad news

There are some problems, however. One is the cost of the cells. Most cells are made from single crystals of very high purity silicon. The cost of these cells is approximately \$6 per watt. To collect 20 kW h of energy over 5 hours would require solar panels capable of producing 4 kW. They would therefore cost about \$40 000. On top of that, there is the cost of storing the energy and converting it into a useful voltage. While this is high, for people in areas away from mains power it is feasible.

Another problem is the energy cost. In order to produce the very pure silicon crystals from which the cells are made, very high temperatures are needed and so considerable energy is used in the manufacture. At present it is estimated that it takes around 2–4 years for the cells to 'pay back' this energy. Given that the cells should last at least 20 years, however, this cost is reasonable.

Most households use their electricity at night and often use very little during the day when the Sun is shining. At present most solar electric systems use lead–acid batteries to store the energy. It is then converted from 12 V DC power into 240 V AC power for use in normal mains-powered appliances. Apart from the dollar cost of the batteries, the environmental cost of producing all the lead for the batteries is very considerable.

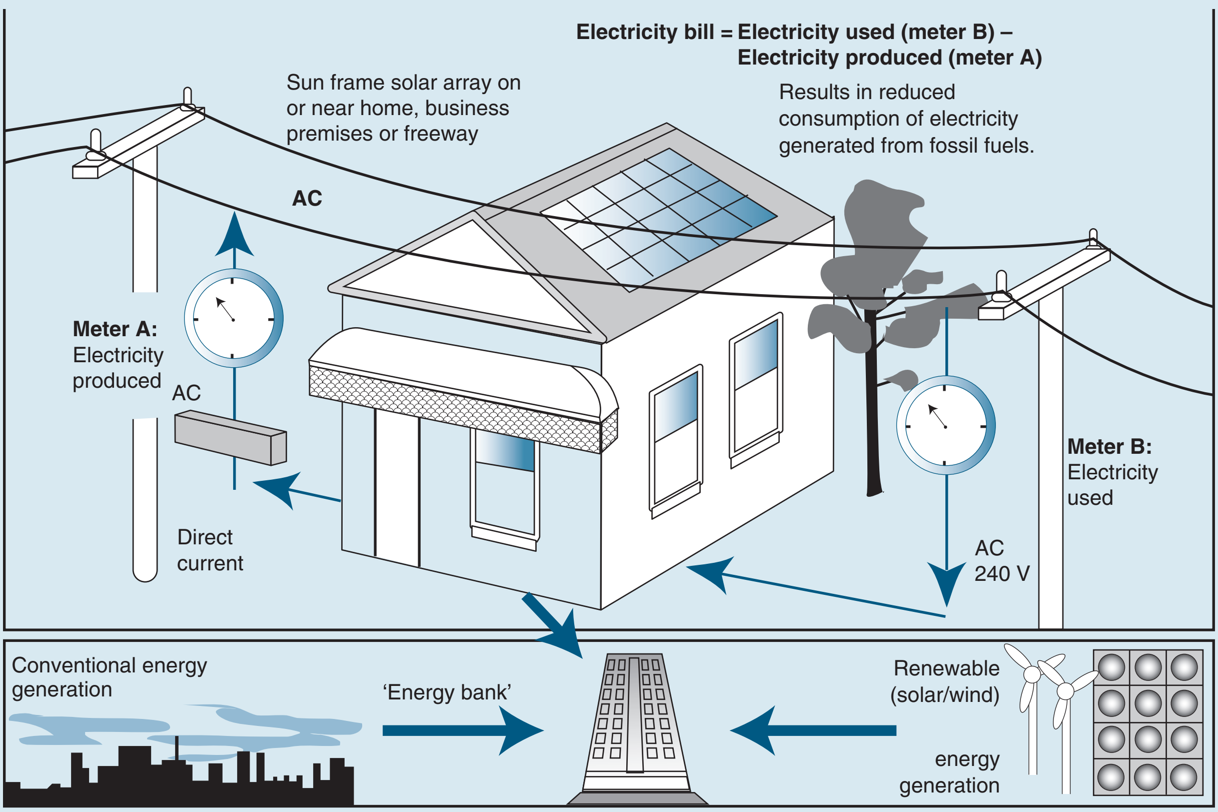


Figure 7.57
In the not-too-distant future it may become feasible for the roofs of our houses to collect all the energy we need.

The good news

There is good news on all these fronts, however. A new development, pioneered at the University of New South Wales, is to use 'multilayered amorphous silicon cells'. This avoids the need for high temperatures to make the pure crystals. The amorphous layers are very thin and can virtually be sprayed onto a convenient material. Although this technique is less efficient, the lower costs should in time dramatically reduce the cost of solar electricity, perhaps to around $1 per watt. At that price it begins to become competitive with electricity produced by conventional means.

It is very difficult to see how a 'self-sufficient' house can store enough energy for its needs without using lead–acid batteries or some other fairly costly method, both in dollars and environmental terms.

On the other hand, most of the electricity demand across our society is during the day—for transport, commerce and industry. If households were to feed their solar-generated power into the grid during the day it could reduce the demand for coal-fired power enormously. At night, households would use power from the grid generated by either coal-fired or hydro stations. In fact, if solar power eventually becomes cheap enough, excess power produced during the day could be stored by pumping water up into the hydro power station dams for use at night. This system would provide almost pollution-free energy and involve the production of virtually no Greenhouse gases. If suitable batteries for electric-powered cars could also be developed we might yet see the end of the 'fossil fuel era', at least in sunny Australia.

7.8 SUMMARY Cells, batteries and other sources of EMF

- A source of EMF uses some other form of energy to create electrical potential energy. Batteries use chemical energy, generators and Van de Graaff machines use mechanical energy, solar cells use the energy in sunlight and thermopiles convert heat directly into electricity.
- If several sources of EMF are combined in series, the total EMF will be given by the sum of the individual sources.
- Only sources of EMF that provide the same voltage should be combined in parallel. Cells in such a battery only need to provide half the current they would otherwise.
- The actual voltage on the terminals of a cell is often less than the ideal EMF because the cell will have some internal resistance. The terminal voltage is given by $V = \mathscr{E} - IR_i$.

7.8 Questions

1. Why could we not use zinc for both terminals of a dry cell?
2. In order to start a car with a flat battery sometimes another car battery is connected to the flat battery. How must this connection be made and what could happen if it was done the wrong way around?
3. What is meant by a 'short circuit'? What are the consequences of connecting batteries in such a way that they short circuit?
4. If a set of four $1\frac{1}{2}$ V dry cells was placed in series with a small 9 V battery what would be the total EMF of the combination?
5. Some torches use a battery of eight D cells ($1\frac{1}{2}$ V each) arranged as two parallel sets of four cells in series.
   a. What is the EMF of the battery?
   b. What is the advantage of this arrangement over one with four cells in series?
6. Can you explain why the headlights of a car dim when the starter motor is in operation?
7. When being used to start a car, a 12 V battery with an internal resistance of 0.02 Ω is supplying a current of 100 A. What will be the actual voltage across the terminals of the battery?
8. A 1.5 V torch battery is found to provide only 1.3 V when connected to a bulb which is drawing 0.2 A. What is the internal resistance of the cell?
9. What is the maximum possible current that a battery of EMF 6.0 V and internal resistance 0.5 Ω could provide, even if short circuited?
10. While using a solar cell with characteristics such as those shown in Figure 7.56, a student attempts to get more power from it by lowering the load resistance. Unfortunately she finds that this only lowers the power available. Can you explain why she obtains this result?

Chapter 7 Review

For some of these questions you may need to refer to data given in Chapter 7.

1 When a voltmeter is used to measure the potential difference across a circuit element it should be placed:
- **A** in series with the circuit element
- **B** in parallel with the circuit element
- **C** either in series or in parallel with the circuit element
- **D** neither in series nor in parallel with the circuit element.

2 When an ammeter is used to measure the current through a circuit element it should be placed:
- **A** in series with the circuit element
- **B** in parallel with the circuit element
- **C** either in series or in parallel with the circuit element
- **D** neither in series nor in parallel with the circuit element.

3 In discussing what is meant by the EMF of a battery, Alf claims that unless a battery is connected into a circuit it has no EMF as the charges in it are not moving and therefore have no energy. Bert, on the other hand, claims that the EMF is greater when the battery is not connected as the charges are not losing energy as they move through it. They are both wrong. Can you explain why?

4 Describe what would happen if, in an attempt to make a battery that would last longer, cells of different EMF were joined in parallel.

The following information relates to questions 5–7.
Belinda needs to operate a portable CD player requiring 12 V at 100 mA. She has some 1.5 V dry cells which each have an internal resistance of 0.3 Ω, and a small 9 V battery with an internal resistance of 5 Ω. She decides to connect two of the dry cells in series with the 9 V battery to provide an EMF of 12 V for her CD player. Assume the batteries are all fresh.

5 She measures the voltage of her new battery with a voltmeter to check that it will be satisfactory. What does she find?

6 She then connects the battery to the CD player and again measures the voltage at the terminals and finds that while it operates the CD player satisfactorily it is less than she had hoped. What voltage does she find?

7 What problems do you see with Belinda's arrangement?

The following information relates to questions 8–13.
Assume that when the dome of a Van de Graaff generator is fully charged it is at a potential of +400 000 V and that the current from the belt onto the dome is a continuous 2 μA.

8 Explain why the potential of the dome remains at 400 000 V even though the belt continues to supply charge at the rate of 2 μA.

9 How many electrons is the belt moving each second? Are these electrons being carried onto or off the dome?

10 At what power must the motor be working to keep supplying the current to the fully charged dome?

11 In what way is a charged Van de Graaff machine like a jack-in-the-box? Explain in terms of potential energy.

12 Each elementary charge on the dome of a fully charged Van de Graaff machine has, relatively speaking, a considerable amount of potential energy. Given that these charges have considerable energy, why is the electric shock one gets from a Van de Graaff machine relatively harmless?

13 If the dome is at a potential of 400 kV, what is the potential energy (in J) of each elementary charge on it?

14 Why is it that a plastic rod that has been rubbed with a piece of cloth can attract an uncharged piece of paper or other conducting material?

15 What are some of the problems and possibilities of using the solar energy that falls on the roofs of our houses as a source of energy for domestic purposes?

For questions 16–18 refer to the graph in Figure 7.56 showing the *I–V* characteristics of a solar cell.

16 If the cell was operating in bright summer sunshine and providing a voltage of 0.5 V to a resistor connected to it, what power would be dissipated in the resistor?

17 Explain why although reducing the resistance would increase the current flowing, the power produced in the resistor would decrease.

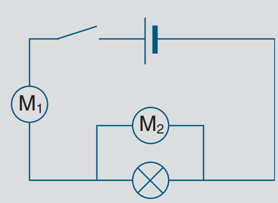
18 If the resistance was increased so that the voltage supplied became 0.6 V, how would the power dissipated change?

The following information relates to questions 19 and 20.
Two identical plastic balls A and B with a metallic coating are given identical but opposite charges and placed a fixed distance apart. There is found to be an attractive force of 0.04 N between them. A third identical, but initially uncharged ball, C, is now brought into contact with ball A and then removed.

19 What will be the force between A and B now?

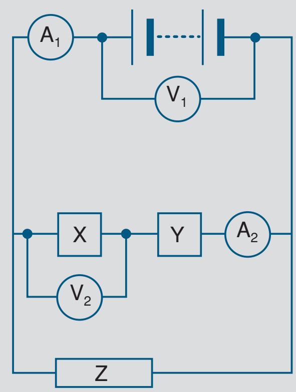
20 If ball C, still with the charge it obtained from ball A, is now brought into contact with ball B, and then removed, what will be the force between A and B now?

21 Josh wires up a circuit as shown in the diagram. He places a voltmeter in position M_1 and an ammeter at position M_2. When he presses the switch he finds that the voltmeter reads 1.5 V but the ammeter reads 0 and the lamp doesn't light.



- **a** Explain why he gets this result.
- **b** He then removes the voltmeter (M_1) from the circuit and replaces it with a direct connection. What is likely to happen when he presses the switch this time?
- **c** How should he connect the two meters correctly so that they each perform their appropriate function.

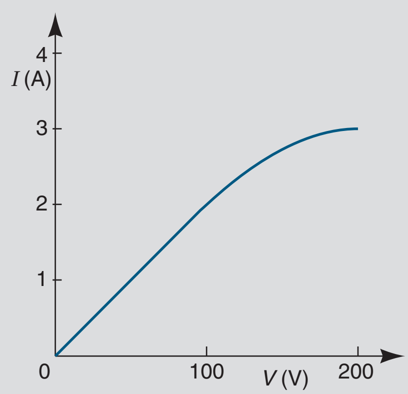
The following information relates to questions 22–23.
A student sets up the circuit shown. A_1 and A_2 are ammeters and V_1 and V_2 are voltmeters. X, Y and Z are various circuit elements. The reading on A_1 is initially found to be 4.5 A and that on V_1 is 6.0 V. A_2 gives a reading of 1.3 A while V_2 reads 4.9 V. (Assume 'ideal' meters.)



22 What is the current through:
- **a** X?
- **b** Y?
- **c** Z?

23 What is the potential difference across the elements:
- **a** X?
- **b** Y?
- **c** Z?

The following information relates to questions 24–27.
A heater element is found to have the *I–V* characteristic shown in the graph.



24 What current flows if a p.d. of 100 V is applied to the element?

25 What is the resistance of the element at 50 V, 100 V, 150 V and 200 V?

26 What power will be produced at 150 V?

27 Why do you think that the resistance at 200 V would be greater than at 100 V?

The following information relates to questions 28 and 29.

A student performs an experiment in which an electric motor is used to lift a 200 g weight through 2 m, thus increasing its potential energy by 4 J. From measurements of the rate at which the weight is lifted the efficiency of the motor is to be determined. Two different voltages were used and the current was measured.

28 In the first experiment at 6 V a current of 0.25 A was measured and the weight took 5 s to rise the 2 m. What was the efficiency of the motor?

29 In the next experiment the voltage was increased to 8 V. The current was found to be 0.30 A and the efficiency worked out to be 60%. How long did the motor take to lift the weight the 2 m this time?

30 If an electric motor converts 10 J of electrical energy from a 6 V battery into kinetic energy and some heat, how much charge has passed through the motor?

The following information relates to questions 31–33.

The 240 V supply cables to a certain house have a total resistance of 0.5 Ω. The maximum likely power use at any one time is estimated to be 10 kW, while the normal minimum load is estimated at 120 W. Note that the supply cables are in series with the loads in the house and there will be a potential drop along the supply cables due to their resistance.

31 What is the maximum current likely to be used by the household?

32 What will the voltage drop along the supply cables be under minimum load?

33 What will the voltage at the house switchboard be under maximum load?

The following information relates to questions 34–37.

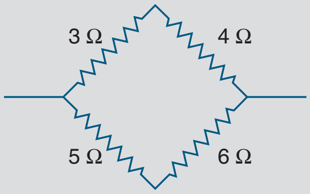
Bill and Mary are discussing the lighting for their living room. At present they have four 60 W, 240 V light bulbs in parallel. Bill suggests that it might be cheaper to replace these with four bulbs wired in series.

34 If this was to be done, what would be the voltage and power rating of each of the new bulbs they would need in order to produce the same amount of power?

35 What would be the total current flowing in the circuit and how would this compare to the total current flowing when the original parallel bulbs were used?

36 Bill says that they would save on electricity bills because the current is going through all four bulbs and therefore being used more effectively. Mary says this is not right and that the power bill would be exactly the same. Who is correct and why?

37 What other advantages or disadvantages of this arrangement could you suggest to Bill and Mary?



38 What is the effective resistance of the four resistors shown in this diagram?

39 Harry has three 4.7 kΩ resistors. What is the least and what is the greatest resistance he can achieve by combining all three in different ways?

40 Two resistors are connected in parallel across a battery. It is found that there is a total current through the battery of 9 A. One of the resistors is 10 Ω and the voltage across the other is 40 V.

a What is the current in the 10 Ω resistor?

b What is the resistance of the second resistor?

Appendix

Some useful quantities

Time: 60 s = 1 minute
60 min = 1 hour
24 hours = 1 day
365.25 days = 1 year

Length: 1 angstrom unit = 10^{-10} m = 0.1 nm

Volume: 1 litre = 1 dm^3 = 10^{-3} m^3 = 10^3 cm^3

Mass: 1 tonne = 1 t = 10^3 kg
1 kilogram of water at 4°C occupies a volume of 1 litre

SI units and symbols

Fundamental SI units

Quantity	Symbol	SI unit	SI symbol
length	l	metre	m
mass	m	kilogram	kg
time	t	second	s
electric current	I	ampere	A
thermodynamic temperature	T	kelvin	K
luminous intensity	I	candela	cd
amount of substance	n	mole	mol

Some derived SI units, names and symbols

Quantity	Symbol	SI unit	SI symbol
velocity	$\boldsymbol{v}$	metres per second	$m\ s^{-1}$ or m/s
acceleration	$\boldsymbol{a}$	metres per (second)^2	$m\ s^{-2}$ or m/s^2
momentum	$\boldsymbol{p}$	newton second	N s
force	$\boldsymbol{F}$	newton	N
energy, work, heat	E, W, Q	joule	J
power	P	watt	W
electrical charge	q, Q	coulomb	C
electric potential, potential difference	V (ΔV)	volt	V
electric resistance	R	ohm	Ω
capacitance	C	farad	F
frequency	f	hertz	Hz
electric field strength	E	newtons per coulomb	$N\ C^{-1}$ (= $V\ m^{-1}$)
magnetic flux	ϕ	weber	Wb
magnetic field intensity	B	tesla	T (= $Wb\ m^{2}$ = $N\ A^{-1}\ m^{-1}$)
gravitational field strength	g	newtons per kilogram	$N\ kg^{-1}$ or N/kg
pressure	P	newtons per (metre)^2	Pa (= $N\ m^{-2}$)
Young's modulus	E	newtons per (metre)^2	Pa (= $N\ m^{-2}$)

Metric prefixes

Multiplying factor	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Example: 1 GJ = 1 x 10^9 J

The periodic table of elements

Group

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Period 1									1 H 1.01									2 He 4.00
2	3 Li 6.94	4 Be 9.01											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
3	11 Na 22.99	12 Mg 24.31											13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.06	17 Cl 35.45	18 Ar 39.95
4	19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.90	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.93	28 Ni 58.71	29 Cu 63.54	30 Zn 65.37	31 Ga 69.72	32 Ge 72.59	33 As 74.92	34 Se 78.96	35 Br 79.91	36 Kr 83.80
5	37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92.91	42 Mo 95.94	43 Tc (99)	44 Ru 101.07	45 Rh 102.91	46 Pd 106.4	47 Ag 107.87	48 Cd 112.40	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126.90	54 Xe 131.30
6	55 Cs 132.91	56 Ba 137.34	57 La 138.91	72 Hf 178.49	73 Ta 180.95	74 W 183.85	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.09	79 Au 196.97	80 Hg 200.59	81 Tl 204.37	82 Pb 207.19	83 Bi 208.98	84 Po (210)	85 At (210)	86 Rn (222)
7	87 Fr (223)	88 Ra (226)	89 Ac (227)	104 Rf (261)	105 Db (262)	106 Sg (263)	107 Bh (262)	108 Hs (265)	109 Mt (266)	110 Ds (269)	111 Rg (272)	112 Uub (277)	113	114 Uuq (289)	115	116 Uuh (289)	117	118 Uuo (293)

Transition elements

Lanthanides

58 Ce 140.12	59 Pr 140.91	60 Nd 144.24	61 Pm (145)	62 Sm 150.35	63 Eu 151.96	64 Gd 157.25	65 Tb 158.92	66 Dy 162.50	67 Ho 164.93	68 Er 167.26	69 Tm 168.93	70 Yb 173.04	71 Lu 174.97

Actinides

90 Th 232.04	91 Pa (231)	92 U 238.03	93 Np (237)	94 Pu (242)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (249)	99 Es (254)	100 Fm (253)	101 Md (256)	102 No (254)	103 Lr (257)

Every isotope of these elements is radioactive

Answers

Chapter 1 Measurement and data

1.1 Measurement and units

1 a derived **b** derived **c** fundamental **d** fundamental
e derived **f** fundamental
2 1.4×10^3 km **3** 1.4×10^8 cm **4** 1.4 Mm **5** 5.98×10^{27} g
6 5.98×10^{18} Gg **7** 5.98×10^{33} μg **8** 6.5×10^5 mm^3
9 6.5×10^{-4} m^3 **10** 4.9×10^{-2} m^2 **11** 25 m s^{-1} **12** 6.4 km h^{-1}

1.2 Data

1 ±0.5 cm **2** 2.4% for height, 1.8% for width **3** 588 cm^2
4 2% **5** 24.5 cm^2 **6** 0.20% **7** 0.37% **8** 0.77% **9** 502 mm^3

1.3 Graphical analysis of data

2 P_o is the vertical intercept and k is the slope of the line of best fit **3** $P = 0.47T + 127$ **4** $k = 0.47$ kPa °C^{-1}, $P_o = 127$ kPa
5 −268°C **7** R on the vertical axis and $\sqrt{l}$ on the horizontal axis
10 8 kΩ lux$^{-0.5}$ **11** 3.7 kΩ **12** $R = 8\sqrt{l} + 3.7$
13 $a = 8$ kΩ lux$^{-0.5}$, $b = 3.7$ kΩ

CHAPTER 1 REVIEW

1 108.3 m^2 **2 a** 0.96% **b** 1.04 m^2 **3** 346 m^3
4 a 2.53% **b** 8.7 m^3 **c** 338.9 < volume < 356.3 **5** $3.47 \cdot 10^5$ L
6 12.2×10^4 ft^3 **7** Betty **8** Andrew **9** David **10** Celia
11 32.6 m s^{-1} or 117 km h^{-1} **12** Probably **13** 7.68%
14 9 km h^{-1}, 108 km h^{-1} < speed < 126 km h^{-1} **15** Possibly
17 sin(i) on the vertical axis and sin(r) on the horizontal axis
20 sin(i) = 1.29 sin(r) – 0.01 or sin(i) = 1.27 sin(r) if forced through the origin. **21** 1.3
22 a square of velocity **b** displacement
23 a $2a$ **b** u^2 **24** Fourth data point was eliminated
26 $v^2 = 5.93s + 158$ **27 a** 5.93 m s^{-2} **b** 158 m^2 s^{-2}
28 2.96 m s^{-2} **29** 12.6 m s^{-1}
30 5.4 N C^{-1}, 3.8×10^9 N C^{-1} V m s^{-1}

Chapter 2 Describing motion

2.1 Describing motion in a straight line

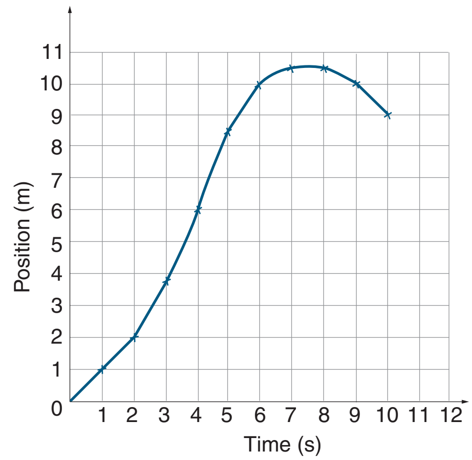
1 a +40 cm, 40 cm **b** –10 cm, 10 cm **c** +20 cm, 20 cm
d +20 cm, 80 cm **2 a** 80 km **b** 20 km north **3 a** 10 m down
b 60 m up **c** 70 m **d** 50 m up **4** displacement **5 a** 25 m east
b 20 m west **6 a** D **b** D **c** C **d** A **7 a** 10 m south **b** zero
8 a 39 steps **b** 1 step west of the clothes line
c 1 step west of the clothes line

2.2 Speed, velocity and acceleration

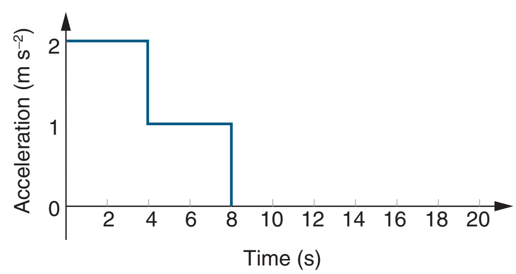
1 a ~2 m s^{-1} **b** ~1 mm s^{-1} **c** ~10 m s^{-1} **d** ~5 m s^{-1}
2 a 15 km h^{-1} **b** 4.2 m s^{-1}
c No, she would probably be travelling faster or slower than this average speed. It depends on the traffic and the terrain.
3 a 22.2 m s^{-1} **b** 6.7 km h^{-1} s^{-1} **c** 1.85 m s^{-2} **d** 20 m
4 a 5 m s^{-1} west **b** 4 m s^{-1} west **5 a** –10 m s^{-1} **b** 40 m s^{-1} west
c 800 m s^{-2} west **6 a** 12 km h^{-1} s^{-1} south **b** –3.3 m s^{-2} south
7 a 1500 m **b** 1.7 m s^{-1} **c** 0 **d** 0 **8** D **9** B **10** C
11, 12 Answers to these questions will vary according to estimations used in calculations.

2.3 Graphing motion: position, velocity and acceleration

1 a +4 m **b** A, C **c** B **d** D
2 The car initially moves in a positive direction and travels 8 m in 2 s. It then stops for 2 s. The car then reverses direction for 5 s, passing back through its starting point after 8 s. It travels a further 2 m in a negative direction before stopping after 9 s.
3 a +8 m **b** +8 m **c** +4 m **d** –2 m **4** 8 s **5 a** +4 m s^{-1} **b** 0
c –2 m s^{-1} **d** –2 m s^{-1} **e** –2 m s^{-1} **6 a** 18 m **b** –2 m
7 a The cyclist travels with a constant velocity in a positive direction for the first 30 s, travelling 150 m during this time. Then the cyclist speeds up for 10 s, travelling a further 150 m. The cyclist maintains this increased speed for the final 10 s, covering another 200 m in this time. **b** +5 m s^{-1}
c +20 m s^{-1} **d** ~13 m s^{-1} **e** +15 m s^{-1} **8 a** B **b** A **c** C **d** D
9 a Running north at 1 m s^{-1}
b Increasing speed from 1 m s^{-1} to 3 m s^{-1} while running north
c Running north but slowing to a stop **d** Stationary
e Accelerating from rest to 1 m s^{-1} while running south
f Running south at 1 m s^{-1} **10 a** 2 m north **b** 10.5 m north
c 9 m north
11



12 a 80 s **b** ~1.3 m s^{-2} **c** ~0.5 m s^{-2} **d** ~4900 m
13 a +2 m s^{-2} **b** 4 s **c** 10 s **d** 80 m **e** +7 m s^{-1}
14 a



b +12 m s^{-1}

2.4 Equations of motion

1 a +2.0 m s^{-2} **b** +8.0 m s^{-1} **c** 64 m **2 a** +3.1 m s^{-2} **b** 50 m s^{-1}
c 180 km h^{-1} **3 a** 21 m s^{-1} **b** 5.2 m **c** 37 m **d** 42 m
4 a 4.0 m s^{-1} **b** 5.7 m s^{-1} **c** 2.0 s **d** 0.83 s **5 a** 8.0 s **b** 16 s
c 192 m **6 a** C **b** D **c** 1.0 s **d** 9.9 m s^{-1} **7 a** 9.8 m s^{-1} **b** 4.9 m
c 1.2 m **d** 3.7 m **8 a** 2.0 s **b** 19.6 m **c** 19.6 m s^{-1} **d** 19.6 m s^{-1}
e i 9.8 m s^{-2} down **ii** 9.8 m s^{-2} down **iii** 9.8 m s^{-2} down
9 a 1.7 s **b** 3.2 s **10** 6.7 s

CHAPTER 2 REVIEW

1 D **2** 14.7 m s^{-1} up **3** 11.0 m **4** C
5 a from 10 s to 25 s **b** from 30 s to 45 s

c from 0 s to 10 s, 25 s to 30 s, and 45 s to 60 s
6 a After 42.5 s **7 a** 20 m s^{-1} north **b** 40 m s^{-1} south
8 a 5 m s^{-1} south **b** 15 m s^{-1} **9** C **10** 2.0 s **11** 15 km h^{-1}
12 a 10 km h^{-1} north **b** 2.8 m s^{-1} north **13 a** 4.0 m s^{-2}
b 4.0 m s^{-1} **c** 6.0 m **14 a** 0.80 m s^{-1} **b** 0.50 m s^{-1} **c** 0.67 m s^{-1}
15 a 0.75 m s^{-1} **b** –5.0 m s^{-2} **16 a** 3.50 s **b** 2.89 s **17** 1.00 s
18 ~10 m s^{-1} north **19** A **20 a** 0 **b** ~2.0 m s^{-2} north
c 7.0 m s^{-2} south
21 Answer will depend on estimations used in calculations.

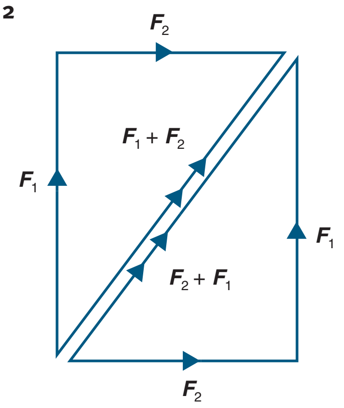
Chapter 3
Forces and their effects

3.1 Force as a vector

1 a A scalar is a quantity which is completely defined by its magnitude, whereas a magnitude and a direction are essential for a vector quantity to be fully defined. **b** Scalar: mass, distance travelled, average speed, time. Vector: displacement, velocity, acceleration, force. Your answer to this may vary depending on the sequence your class has followed.
c Average speed is a scalar quantity—defined as the rate of change of distance (a scalar). Average velocity is a vector quantity—defined as the rate of change of displacement (a vector). For the same motion, these quantities can be different, e.g. running around an oval and returning to the same point. The average speed will be a certain value, and the average velocity will be zero, i.e. $\boldsymbol{v}_{av} = 0$.
2 a 10 N **b** 100 N **c** 50–100 N **d** 1 N **3** 1000 N, 10 000 N
These estimates may vary widely.
4 B, C, D **5 a** 60°T **b** 320°T **c** 240°T **d** 135°T **e** 22.5°T
6 a 75 N east **b** 180 N west **c** 15 N west **7 a** 70 N right
b 50 N up **c** 80 N down

3.2 Vector techniques

1 a $\boldsymbol{F}_1 + \boldsymbol{F}_2$ = 35 N north 45° east **b** $2\boldsymbol{F}_1$ = 50 N north
c $2\boldsymbol{F}_1 + \boldsymbol{F}_2$ = 56 N north 27° east
d $2\boldsymbol{F}_1 + 2\boldsymbol{F}_2$ = 71 N north 45° east
2



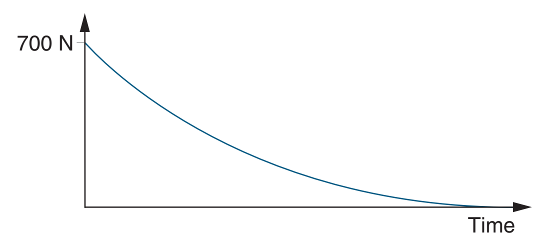
3 a B **b** C **c** A, D, F **d** E, G **e** E, G
4 a 0 N **b** 10 N south 60° west
c The chair will move in the direction of the sum of the forces provided by Hugh and Elisa, i.e. N60°E
5 a 5 N 143°T **b** 100 N 323°T **c** 7.1 N north 15° east **d** 2.5 N 73°T
6 613 N in a direction that bisects the two ropes
7 a 50 N south, 87 N east **b** 60 N north
c 282 N south, 103 N east
d 1.5 × 10^5 N up, 2.6 × 10^5 N horizontal
8 150 N upwards, 260 N horizontal
9 100 N acting down the incline **10** 85 N

3.3 Newton's first law of motion

1 Aristotle felt that the natural state for any object was at rest in its natural place. This meant that any moving body would come to rest of its own accord. Galileo introduced the idea that friction was a force that could be added to other forces that act on a moving body, but it was Newton who explained that the moving object should continue to travel with constant velocity unless a net force is acting.
2 No force acts on the person. In accordance with Newton's first law of motion, the bus slows, and the standing passenger will continue to move with constant velocity unless acted on by a force—usually the passenger will lose his or her footing and fall forward.
3 Hard leather soles provide little or no grip on the ice—there is no friction. As a consequence, no force can be applied to begin walking, and the net force on the person will be zero. To be propelled with ice skates, the blade of the skate has to be dug into the surface of the ice, allowing the skater a fixed point from which to 'push off'.
4 20 N in a forward direction, so that the net force will be zero
5 To exactly balance the other forces, lift = 50 kN up, and drag = 12 kN west **6 a** 25 N **b** 25 N horizontally
c 29 N at 30° to the horizontal
7 When the car or aircraft slows suddenly, a passenger will continue to travel with the same velocity as before, until being acted on by an unbalanced force. The purpose of the seatbelt is to supply that force, but across the body where the effects of the force are reduced.
8 A gravitation **B** electric force **C** friction between the tyres and the road **D** tension in the wire

3.4 Newton's second law of motion

1 244 N **2** 1110 N opposing the motion
3 The 1.5 kg shot-put is the larger of the two masses, and so for the same applied force, its acceleration will be lower. This means a lower speed on leaving the athlete's arm, and so it cannot be thrown as far as the lighter 1.0 kg ball. **4 a** 160 N
b 141°T **c** 213 m s^{-2} 141°T (but this is for a very short time)
5 2600 N in the direction of travel
6 On Earth's surface, the gravitational field strength is almost the same at every point, so the weight of a stationary object is proportional to its mass ($\boldsymbol{W} \propto m$, $\boldsymbol{g}$ constant). Scales make use of this fact and are calibrated in kilograms. Scales would be inappropriate for use on the Moon, reading 12 kg. Scales measure weight, not mass. **7** m = 1500 g, $\boldsymbol{W}$ = 5.4 N
8 10 m s^{-2} horizontally
9



As the parachutist leaves the aircraft, his or her weight will be the net force acting, accelerating at 9.8 m s^{-2} but as the speed increases, the drag force (air resistance) opposing the motion also increases until it equals the weight. At this point in time, the net force will be zero and the parachutist will travel with a constant speed.

3.5 Newton's third law of motion

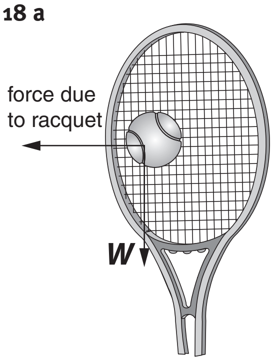
1

Situation	Action force	Reaction force
a	Force from the bat on to the ball	Ball exerts a force on the bat in the opposite direction
b	Weight of pine cone, i.e. gravitation pulling it to the Earth	Earth pulled toward the pine cone with the same force
c	Air under pressure in the balloon pushes backwards on the air escaping from the balloon	Air escaping from the balloon pushes forward on the air inside the balloon

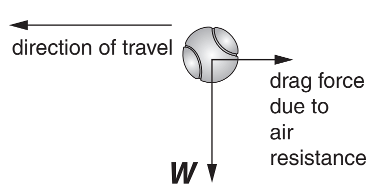
2 a 140 N in the opposite direction to the leaping fisherman **b** 3.5 m s^{-2} in the opposite direction to the fisherman **c** Fisherman: 1.0 m s^{-1}. Boat: 1.75 m s^{-1}
3 a If no other forces act, there will be an action/reaction force pair in which the action force is the force on the tool kit, and the reaction force will act on the astronaut in the opposite direction. If the kit is thrown directly away from the ship, hopefully the reaction force will propel him back to the craft. **b** 20 N act on each mass, but in opposite directions **c** 0.2 m s^{-1} **d** 500 s or 8 min 20 s **4** 12.5 m s^{-2} south
5 The force of gravitation on the speaker will be 49 N downwards and the normal force exerted by the bookshelf on the speaker will be 49 N upwards. The reaction force to the weight of the speaker will be a force on the Earth of 49 N acting towards the speaker.
6 a Weight vertically down, normal force up and perpendicular to the incline, friction up the incline **b** $\boldsymbol{W}$ = 637 N downwards, $\boldsymbol{N}$ = 409 N up and perpendicular to the track **c** $\Sigma \boldsymbol{F}$ = 488 N along the incline **d** 7.5 m s^{-2} along the track **7** 280 N
8 As the lift accelerates, the normal force that you experience from the floor increases—this is your apparent weight. The normal force has to balance your weight and provide extra force to accelerate you upwards. $\boldsymbol{N} = \Sigma \boldsymbol{F} - \boldsymbol{W}$
The same situation occurs when the lift comes to rest while 'going down' as the net force must still be upwards.
9 a 784 N down **b** 960 N down **c** 784 N down **d** 608 N down **e** 608 N down **10 a** 196 N down on the side of the student **b** 2.45 m s^{-2} down to the side of the student **c** 368 N

CHAPTER 3 REVIEW

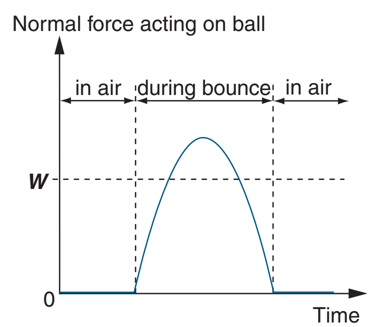
1 A person tripping—because their foot stops momentarily, and the rest of the body continues with constant velocity. The constant velocity of a satellite in deep space far from the influence of any gravitational fields. A racing car will leave the race track at a bend and continue in a straight line if it hits an oil patch owing to a lack of friction. **2** 47.5 N **3** 6.7 m s^{-1}
4 Kicking the tyre of a car hurts because you apply a force to the tyre (action) and the tyre will apply a reaction force to your foot (reaction). Hot gases are forced out of a jet engine (action) and the gases push the engine forward (reaction).
The weight of any object can be considered an action force—the Earth pulls on the body. The reaction force acts on the Earth—the object pulls on the Earth. **5 a** 5.0 m s^{-1} **b i** 539 N down **ii** 649 N down **iii** 539 N down **iv** 429 N down
6 a 833 N down **b** 136 N down **c** 306 N down **d** 833 N down (note the difference between weight and apparent weight) **e** 0 N (zero normal force) **f** 1260 N down
7 A block is struck with a sharp blow so that it overcomes the grip of the blocks with which it is in contact, and so it is ejected from the pile. The other blocks experience no horizontal net force, and so stay in the vertical stack.
8 a 113 N horizontally, 41 N down **b** 113 N to the south **c** $\boldsymbol{N}$ = 196 + 41 = 237 N upwards **d** When the trolley is pulled, the vertical component of the applied force is upwards rather than downwards, and so a smaller (upward) normal force is needed—helping the wheels to rotate more freely.
9 7.0 m s^{-1} **10** B **11 a** 1.5 m s^{-2} in the direction of the force **b** 75 N **12 a** 1:1 **b** 1:4 **c** 1:4
13 To maintain a tension of less than 100 N in the rope, the bucket must accelerate at greater than 1.47 m s^{-2} downwards. If the acceleration falls below this, the rope will snap.
14 4.2 kg **15** 71.3 km h^{-1} **16** 30°
17 2.05 m s^{-2} down the incline
18 a



b



19 3.27 m s^{-2}, 65.3 N
20



Chapter 4
Energy and momentum

4.1 The relationship between momentum and force

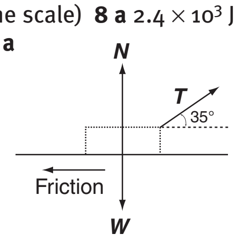
1 a 100 kg m s^{-1} **b** 1 kg m s^{-1} **c** 28 kg m s^{-1} **2 a** 24 kg m s^{-1} **b** 64 kg m s^{-1} **c** 8 kg m s^{-1} **d** 40 N s **e** 8 N **3** object 2
4 a 41 kg m s^{-1} **b** 0.24 kg m s^{-1} **c** 500 kg m s^{-1}
d 157(.5) kg m s^{-1}
5 a 4.0 kg m s^{-1} **b** 124 N **6 a** 9.0 N s
b 180 N in the direction of the ball's travel
c 180 N opposite the direction of the ball's travel **7 a** 1200 N
b 66 N s **8 a** 1.25 kg m s^{-1} opposite direction of flight
b 1.25 N s opposite direction of flight
c 1.6×10^{-3} opposite direction of flight
9 a Based on idea of impulse. Stopping time increased by collapsing shell reduces impact force. **b** Stopping time would be reduced and force increased therefore not as successful.
10 Need to include mass and initial velocity then $\Delta \boldsymbol{p} = m\Delta \boldsymbol{v}$. Impulse = $\Delta \boldsymbol{p} = \boldsymbol{F}\Delta t$ (units: N s). Estimate stopping time then $\boldsymbol{F} = \Delta \boldsymbol{p}/\Delta t$ (units: N)

4.2 Conservation of momentum

1 2 m s^{-1} in the direction of the white ball's original motion
2 4.7 m s^{-1} in the same direction
3 0.44 m s^{-1} in the opposite direction **4** 11(.33) tonnes **5** B
6 a Only if Superman is fixed to the ground so that the momentum of the truck is transferred directly to the ground.
b Final velocity = $\boldsymbol{p}_{truck}$/total mass of Superman + truck
7 v_{car} = 100 km h^{-1} so yes! **8** 8 m s^{-1}
9 3 m s^{-1} opposite the direction of the exhaust gases
10 a Rocket is losing 50 kg over this 2 s period so use average mass of rocket is 225 kg and v = 40 m s^{-1} **b** 4.5×10^3 N
c 10(.2) m s^{-2}

4.3 Work

1 $2.6(5) \times 10^2$ J **2** 1.5×10^5 J
3 The walker also does work travelling against the frictional forces as well as vertically against gravity.
4 D since statement i mentions distance rather than displacement
5 a 1.5×10^2 J **b** 1.5×10^2 J **c** $1.0(29) \times 10^3$ J **6** B since the motor won't convert all supplied energy to useful work
7 Nil since there is no change in position (don't need to read the scale) **8 a** 2.4×10^3 J **b** Nil
9 a



b 2.0×10^2 J **c** 1.6×10^2 J **d** 40 J **10** ~1 J

4.4 Mechanical energy

1 a 3(.125) J **b** 4×10^2 J **c** 2.6×10^5 J **2 a** 49 J **b** 412 J
c 1.2×10^5 J
3 a 2.5 J **b** 1.8 J **c** 0.69 J **4** A **5** 6.9×10^3 N **6 a** 4.9(5) m s^{-1}
b 0.98 J **c** 0.98 J **7** 7.1×10^3 N **8** 2.7×10^4 J **9** $2.5(6) \times 10^2$ J
10 1.07×10^3 N

4.5 Energy transformation and power

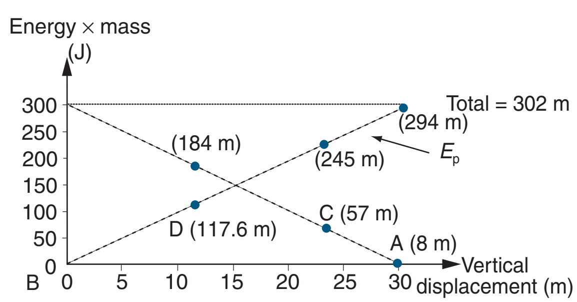
1 a kinetic energy → heat **b** kinetic → gravitational potential
c elastic → kinetic → gravitational potential → kinetic
d kinetic → heat/sound
2 elastic potential → kinetic → gravitational potential → kinetic (+ heat/sound) → heat (+ sound, kinetic of water rebounding)
3 a 5.4×10^3 J **b** 4.5×10^2 W **4 a** 73(.5) J **b** 73 J **c** 0.61 W
d Coach lifting own body, loses heat energy to the environment
5 a 2.4(5) J **b** 2.4(5) J
c Transfer to the ground, transformed into other energy forms
6 a 4.3(2) m **b** Length of rope insufficient to allow height change, air resistance and other factors leading to transfer/loss to the surrounding environment.
7 6.0 m s^{-1} **8 a** 9.9 m s^{-1} **b** 8.9×10^{-1} N **9** 52% **10** 0.90 s

4.6 Elastic and inelastic collisions

1 C **2 a** 8.1 J **b** 8.1 J **c** This collision is elastic since there is no loss of kinetic energy during the collision.
d No. Truly elastic collisions only occur at the atomic level.
3 a 0 **b** 0 **c** 0 **4 a** 6.0×10^5 J **b** 0
c Inelastic since kinetic energy is lost during the collision.
5 a 6.0×10^5 J **b** It is converted into other forms of energy during the collision such as heat and sound. **c** 0%
6 2.0 m s^{-1} left, 2.0 m s^{-1} right **7 a** 3.92 J **b** 1.31 J **8** 7 mm
9 a 8.0 J **b** 4.5 J
c Energy is converted into heat during this process.

CHAPTER 4 REVIEW

1 2.2 m s^{-1} in direction of moving player **2** 2.2 m s^{-1} north
3 2.2 m s^{-1} in same direction **4** Negligible movement but there is a change ($v = 2.7 \times 10^{-21}$ m s^{-1})
5 7.1 m s^{-1} **6** 1.1×10^4 N s **7** 1.4×10^4 N **8** D **9** B **10** A
11 13 m s^{-1} **12** 8.3 m **13** 7.8×10^4 N **14** 6.5×10^5 J
15 6.5×10^4 W **16** Gravitational potential + kinetic → kinetic (+heat/sound) → kinetic of ground + heat + sound + gravitational potential of rebounding water
17 B: 25 m s^{-1} C: 11 m s^{-1} D: 19 m s^{-1}
18



19 57 × mass J **20** 81% **21** 1.0 m s^{-1} east **22** 25%
23 The collision is inelastic because kinetic energy is lost.
24 470 W **25** 2.42 m s^{-1} **26** 0.63 J
27 Magnitude of thrust equals magnitude of drag and net force is zero. **28 a** 285 N **b** 285 N **29 a** 8.89 N s
b 1.78×10^3 N **c** 494 m s^{-2} **30** 1.33×10^3 kg m s^{-1} **b** 5880 N
c Possible bruising, sprains, or broken bones.
d Possibly as the answer to **b** is equivalent to 600 kg weight. Multiply this by ten, say, and the forces are dangerously large.

Chapter 5 Nuclear energy

5.1 Atoms, isotopes and radioisotopes

1 a 20 protons, 25 neutrons, 45 nucleons
b 79 protons, 118 neutrons, 197 nucleons
c 92 protons, 143 neutrons, 235 nucleons
d 90 protons, 140 neutrons, 230 nucleons **2 a** zinc
b carbon **c** silicon **d** gallium **3 a** 27 protons, 33 neutrons
b 94 protons, 145 neutrons **c** 6 protons, 8 neutrons
4 A radioisotope is an unstable isotope. At some time, it will spontaneously eject radiation in the form of alpha particles, beta particles or gamma rays from the nucleus. Three isotopes that are not radioisotopes could be any three stable isotopes, e.g. carbon-12, lead-206 and bismuth-209. **5** Polonium-210 and uranium-238. These have atomic numbers of 84 and 92 respectively; and every isotope beyond bismuth ($Z = 83$) in the periodic table is radioactive. **6 a** 5.13×10^{-45} m^3
b 3.25×10^{17} kg m^{-3} **c** 3.25×10^{11} kg (325 million tonnes)
d 325 million cars **e** The density of normal matter is far lower than the density of an atomic nucleus. **7** 1.1×10^{-13} **8** 210 m
9 a There are no differences. **b** ^{89}Kr has 5 more neutrons in its nucleus. **10** $^{28}_{13}$Al

5.2 Alpha, beta and gamma radiation

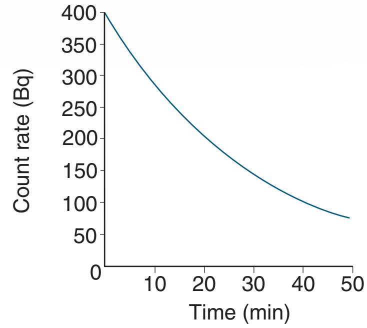
1 a the nucleus **b** the nucleus **c** the nucleus **2** For example, α is a helium nucleus, β is an electron, γ is electromagnetic radiation. **3 a** beta particle **b** proton **c** alpha particle
d neutron **4 a** $Z = 82$, $A = 214$, lead **b** $Z = 90$, $A = 231$, thorium
c $Z = 89$, $A = 228$, actinium **d** $Z = 80$, $A = 198$, mercury
5 a α **b** β^- **c** β^- **d** α **e** γ **6** lithium-7 **7 a** proton **b** neutron
c neutron **d** alpha particle
8 a 7 protons, 7 neutrons, 1 electron **b** A neutron has decayed.
c $^1_0\text{n} \rightarrow {}^1_1\text{p} + {}^{0}_{-1}\text{e} + \bar{\nu}$ **9 a** ^{40}Ca, ^{42}Ca, ^{43}Ca, ^{44}Ca, ^{46}Ca, ^{48}Ca **b** one
c beta **d** $^{48}_{19}\text{K} \rightarrow {}^{48}_{20}\text{Ca} + {}^{0}_{-1}\text{e}$ **e** K: 1.53, Ca: 1.40 **f** alpha
g $^{217}_{87}\text{Fr} \rightarrow {}^{213}_{85}\text{At} + {}^4_2\alpha$; $^{213}_{85}\text{At} \rightarrow {}^{209}_{83}\text{Bi} + {}^4_2\alpha$; bismuth-209
10 a $^{197}_{79}\text{Au} + {}^1_0\text{n} \rightarrow {}^{198}_{79}\text{Au}$ **b** $^{198}_{79}\text{Au} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e}$

5.3 Properties of alpha, beta and gamma radiation

1 a α, β, γ **b** γ, β, α **2** B **3** Gamma radiation is most suitable since its penetrating ability will enable it to reach the tumour.
4 A beta emitter would be best suited because its penetrating ability would enable it to irradiate a small volume of tissue around the source. Alpha radiation would not penetrate the tumour at all, and gamma radiation would pass out of the body, irradiating some healthy cells along the way.
5 Beta and gamma radiation. Alpha particles are soon absorbed by air, but beta and gamma radiation can travel through air for metres. **6 a** 1.4×10^{-12} J **b** 6.7×10^{-14} J
c 8.0×10^{-14} J **7 a** 3.4×10^6 eV **b** 1.6 cm **8** D **9** A
10 If a form of radiation interacts with and ionises matter easily, it will lose its energy very quickly and so penetrate only a small distance.

5.4 Half-life and activity of radioisotopes

1 C **2 a** 10 g **b** 5 g **c** 2.5 g **d** 0.31 g
3 a



b ~235 Bq **c** 20 min **d** 50 Bq **4** 15 minutes **5** 1 chance in 2
6 192 µg **7 a** 10 half-lives **b** ~240 000 years **8 a** uranium-235
b It has a much shorter half-life than uranium-238 and so has decayed much more rapidly since the formation of the Earth.
9 3.9×10^4 Bq **10 a** Over time, the radioisotopes transmute by a series of decays to form lead-206, which is stable. The percentage of lead in the sample will increase over time.
b ^{214}Po has such a short half-life (160 µs) that when ^{214}Bi nuclei decay to ^{214}Po, they almost instantaneously transmute to ^{210}Pb. **11** 9.9 g **12** 11.2 h **13** 16% **14** The decay begins immediately but half of the material will still be present after 28 years.

5.5 Splitting the atom: nuclear fission

1 a 3 **b** 4 **c** 5 **2** $x = 239$, $y = 40$ **3** B **4** D
5 a 3.1×10^{-11} J, 196 MeV **b** About 20 times more energy is released in the fission reaction. **6** 2.36×10^{-30} kg
7 alpha particles **8 a** 3.10×10^{-28} kg **b** 2.79×10^{-11} J
c 0.079% **d** 3.56×10^{14} J **9 a** 2.12×10^{-28} kg **b** 1.91×10^{-11} J
c 3.14×10^{10}
10 a In fissile nuclei, the nuclear forces of attraction are just stronger than the electrostatic forces of repulsion.
b The additional energy causes the nucleus to break apart.

5.6 Nuclear fission weapons

1 B **2** It doesn't have a high enough concentration of the fissile isotope, uranium-235. **3** D **4 a** 3.2×10^{-11} J **b** 3.2×10^{13} J
c 8000 tonnes **5** First, a neutron causes fission to occur in a uranium-235 nucleus, thus releasing 2 or 3 more neutrons. These then go on and induce fission in more uranium-235 nuclei, each resulting in the release of 2 or 3 neutrons and so on. The chain reaction grows very rapidly and energy is released in each fission reaction. **6** As a result of its shape, a very high proportion of neutrons are able to escape from the material, and so the chain reaction dies out.
7 a It is carried as two or more subcritical masses.
b The subcritical masses are forced together to form a combined supercritical mass. **8** 0.044 kg

5.7 Nuclear reactors

1 B **2** D **3 a** The fission process in the reactor core produces heat. This heat energy is conducted into the coolant which is flowing through the core. The energy is used to produce steam, which drives a turbine to generate electricity. **b** The difference is that the heat energy that makes the steam is produced by burning coal instead of a nuclear fission reaction.
c They both use steam to turn a turbine to generate electricity.

4 The nucleus is too heavy. When a neutron collides with a lead nucleus, the neutron will keep almost all of its energy, and so not slow down sufficiently to be captured by a fissile nucleus. **5 a** The chain reaction will be self-sustaining, i.e. critical, and a steady release of energy will result. **b** The chain reaction will die out because it is subcritical. This will lead to a decrease in the amount of energy produced. **c** The chain reaction will grow, causing an increasing amount of energy to be produced. This may be dangerous and could result in an explosion. **6 a** Fast neutrons are most unlikely to be captured by the nuclei. **b** Slow neutrons are likely to be absorbed by the nuclei and cause fission. **7 a** It results in the uranium-238 transmuting into plutonium-239. **b** Plutonium is highly radioactive and has a half-life of about 24 000 years.
8 a plutonium-239 **b** They rely on fast, high energy neutrons to induce fission in plutonium nuclei. **c** They produce, or breed, more of their own fuel, plutonium-239, when neutrons are absorbed by uranium-238 nuclei. **d** Fast breeder reactors do not have moderators. **9** Since only one neutron is required to sustain the chain reaction, the remaining neutrons are able to breed more plutonium. **10** Over a period of months, the fissile nuclei in the fuel rods become depleted, the number of fissions decreases, and so fewer neutrons are flying around in the core. In order to maintain the chain reaction, the control rods must be gradually withdrawn. **11** tritium
12 ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H$ proton

CHAPTER 5 REVIEW

1 5.25×10^{-28} kg **2** 4.73×10^{-11} J **3** 0.13% **4** 27 g
5 2.41×10^{15} J **6** ${}^{238}_{92}U + {}^{1}_{0}n \rightarrow {}^{239}_{92}U$
7 ${}^{239}_{92}U \rightarrow {}^{239}_{93}Np + {}^{0}_{-1}\beta + \gamma$, and finally ${}^{239}_{93}Np \rightarrow {}^{239}_{94}Pu + {}^{0}_{-1}\beta + \gamma$
8 60 s **9** 4.7 g **10** ${}^{26}_{11}Na \rightarrow {}^{26}_{12}Mg + {}^{0}_{-1}\beta$ **11** B
12 The smaller piece has more surface area per volume, loses a higher proportion of fission neutrons, resulting in the chain reaction dying out. In the larger mass, a smaller proportion of neutrons is lost and so it is capable of spontaneously exploding. **13** ${}^{9}_{4}Be + {}^{4}_{2}\alpha \rightarrow {}^{12}_{6}C + {}^{1}_{0}n$
14 ${}^{235}U$ has a shorter half-life and so is decaying at a slightly faster rate than ${}^{238}U$. Therefore the proportion of uranium-235 will be decreasing. **15** Leukaemia, tumours, radiation sickness, probable death within months.
16 Yes, genetic problems could arise in future generations.
17 To extract heat from the reactor core. This energy is used to produce steam which is used to drive turbines and generate electricity. **18** 42% **19** B **20** The bismuth-215 sample will have twice the activity of bismuth-211. **21** 3 **22** 1.4×10^{7} m s^{-1}
23 The fission neutrons are moving too fast to induce further fission in uranium-235 nuclei, and so a moderator is used to slow them down. **24** The moderating nuclei must be relatively light so that the incident neutrons lose some of their energy upon colliding. **25** Their fuel, plutonium-239, is highly fissile when struck by fast moving neutrons. **26** In the core of nuclear reactors when uranium-238 nuclei absorb neutrons and decay to form plutonium-239. **27** Electrons that are emitted from the nucleus of an unstable atom.
28 7 protons, 9 neutrons, 16 nucleons **29** ${}^{16}_{7}N \rightarrow {}^{16}_{8}O + {}^{0}_{-1}\beta$
30 No, its half-life is too short. **31** $\alpha, \beta, \beta, \alpha, \alpha, \alpha$
32 5.98×10^{25} MeV
33 ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{-1}e$, ${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He$, ${}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + 2{}^{1}_{1}p$
34 240 MW

Chapter 6 Heating and cooling

6.1 Heat: a historical perspective

1 a Heat is being drawn away from the hand and is flowing into the cold material. **b** The particles within the hand have more energy. The energy is being transferred to the cold surface, evening up the temperature of each. The energy transfer will cease when the temperature of both is the same.
2 Heat was present as caloric. Thompson could not confirm the existence of caloric. Heat was still produced without any breaking of material to release caloric.
3 Heat was being produced as long as work was being done. Thompson suggested that a transfer of energy was required.
4 The other person's hand will feel cold. It is at a lower temperature and heat energy is transferred from the warmer hand.
5 The energy is mainly converted to heat. Some will be converted to sound energy.
6 Mechanical work is being done on each face, increasing the internal energy of the ice and converting it into water.
7 a Chemical potential energy → heat and light energy → heat energy transferred to pot → heat energy transferred to water
b Chemical potential energy → electrical energy → heat energy transferred to water
8 Energy has been transformed at a faster rate than the body is able to release to its surroundings.
9 60 J **10** 1.2 kW (1167 W)

6.2 Kinetic molecular theory

1 A **2** B **3** B **4** Diagram should show greater movement about fixed points as temperature increases. **5** Potential energy
6 No. There is a distribution of speeds. The average of the distribution of kinetic energies indicates the temperature of the substance.
7 True
8 False. Refrigerators transfer energy away from the interior. reducing the average kinetic energy and, hence, the temperature. **9** True. Particles from the hot water will collide with those in the cold water transferring kinetic energy as they do so. The overall average energy of each particle increases.
10 True. Energy is always transferred from the higher temperature to the lower (heating). Less energy (cooling) cannot be transferred.

6.3 Heat and temperature

1 a i 300 K **ii** 246 K **iii** 773 K **b i** −273°C **ii** 227°C **iii** 727°C
2 100 K **3** 95.7 kPa **4 a** Before the size of one graduation can be established there must be a fixed amount to be divided. **b** Duplicate conditions under which the fixed point was established, e.g. for the Celsius scale it could be the freezing and boiling points of water. **5** A **6** Thin glass bulb, constriction, small section of total Celsius scale **7** 1.65 cm^{3}
8 Negative has no meaning on the kelvin scale. It is an absolute scale beginning at zero. **9** 815 kPa
10 Water becomes more dense when it approaches freezing. This results in water just above freezing point sinking and leaving liquid water below a frozen surface. (In this particular case, the pressure exerted by the ice above will also have

contributed to keeping the water from freezing—effects of pressure are detailed later in this chapter.)

6.4 Specific heat capacity

1 2.1 kJ **2** 25.2 kJ **3 a** Water—greater specific heat capacity **b** Al : water = 1 : 4.67 or 10 : 47
c Al : water = 4.67 : 1 or 47°C : 10°C **4** 23°C (22.9°)
5 B is the best answer. D could apply over a small temperature range. **6** T = 68°C so won't boil. A comparison of the energy available with the energy required to boil the water would also confirm this response. **7** 12 kJ **8** 0.34 kg
9 Mercury's low specific heat capacity is an advantage as it will not take much energy to increase its temperature and hence expand. A higher specific heat capacity would lead to a slow response time, assuming that the rate of heating is the same.
10 The ocean near Brisbane has a higher specific heat capacity than the deserts around Alice Springs. This moderates the extremes of hot and cold evident between day and night in Central Australia. **11, 12** Answers to these questions will vary according to estimations used in calculations.

6.5 Latent heat

1 Changing state—melting **2** –39°C **3** 357°C
4 1.26×10^4 J kg^{-1} **5** 2.85×10^5 J kg^{-1}
6 It is the potential energy required to overcome the intermolecular forces binding the sulfur in solid form to a point where the particles are free to move randomly within a fixed volume. **7** Energy is needed to overcome the weak intermolecular forces in a liquid, allowing particles to move freely with no fixed volume. **8** As the temperature has not changed, the kinetic energy and hence the speed remains the same. There is a difference in the total energy of each molecule. The potential energy has increased.
9 2.25×10^5 J **10** 1.29×10^5 J

6.6 Evaporation: heat energy in context

1 B **2 a** Water molecules with higher kinetic energy evaporate, reducing the average kinetic energy of the water remaining and hence reducing the temperature. **b** Temperature, air speed, surface area and humidity. **3 a** Perspiration allows evaporative cooling. Evaporating water molecules with high kinetic energy remove large amounts of energy, reducing the average kinetic energy of particles within the skin and cooling the blood flowing through. **b** High humidity slows evaporation, reducing the rate at which energy will be transferred away from the body. **4** Evaporation of the moisture transfers energy away from the body. **5** Molecules with high kinetic energy evaporate and reduce the average kinetic energy of the remaining liquid, lowering the temperature. A breeze will increase the rate of evaporation. **6** Refrigerant fluid in the pipes evaporates, removing energy from inside the refrigerator. **7** To aid natural convection inside the refrigerator. The cooler air is denser and will fall to the bottom of the refrigerator, cooling the whole interior. **8** Metal fins increase the surface area exposed to the outside air, increasing the transfer of heat away from the refrigerator. **9** No, as heat from inside the refrigerator is being returned to the room via the pipes and fins on the rear of the unit. **10** 3.9 litres

6.7 Conduction and convection

1 a fibreglass, paper, polystyrene
b copper (silver too expensive) **c** fibreglass, wood
2 a 10.1 MW m^{-2} **b** Surrounding air will act as an insulating layer if still so doesn't carry heat away by convection.
c Put pipes inside the walls rather than outside the house. Put insulating material around the pipes.
3 a 21.6 MJ **b** Double glazing, pelmets, thick curtains—trap layer of air around or within window. **4** Rapid rate of heat transfer via conduction due to large temperature difference.
5 Plastic, rubber = low conductivity = insulator
Metal = high conductivity = conductor hence transfers heat energy away from hand faster, cooling hand.
6 Fan to assist convection (forced convection) gives more even heat energy distribution than conventional natural convection.
7 Summer: want heat upwards and cool air down. Winter: want cool up and hot air down.
8 No free molecules to establish convection in solids.
9 a –15°C **b** Conventional thermometers shielded against convective losses. **c** Cover thermometer bulb in wet cloth. Cooling of cloth by convective transfer will also cool bulb and be recorded. **10** Hot air rising; hence name 'thermals'.

6.8 Radiation

1 120 W
2 Light, reflective colours have low radiant heat energy losses.
3 Radiate heat energy faster, assisting in transferring heat energy away from the body and reducing overheating.
4 a Reduce radiant heat energy losses. **b** Increase radiant gains when Sun visible—important to maximise for wellbeing. As exposure time is limited this becomes the most important evolutionary consideration of heat energy loss.
5 Light, shiny surfaces radiate slowest = gloss white beaker. Matt black are fastest. **6** More efficient since lighter colours both absorb and emit radiant heat energy slower.
7 Black car will heat up and cool down fastest.
8 In summer, foil will reduce radiant energy gains. In winter will not reduce conductive losses. **9** Radiant energy does not require heating of the environment; hence potentially more efficient. Convective losses need to be considered if it is to be more efficient. **10** High emissivity = high radiant heat transfer hence marijuana leaves have high emissivity.

CHAPTER 6 REVIEW

1 A **2** D **3** Yes. Temperature is a measure of the average kinetic energy of an object. **4** Two fixed points required then graduated scale. Only absolute if repeatable and based on one point being an absolute zero. **5** C **6** 6.3 MJ **7** 19 min **8** D
9 Greater specific heat capacity of water means more heat is needed from body to heat water directly in contact with the body. **10** 870 J kg^{-1} K^{-1} therefore aluminium. **11** C
12 Energy supplies latent heat of fusion **13** 34.2 kJ
14 Doesn't matter—temperature won't increase until all the water has boiled. **15** 39.7°C
16 Steam provides 225 J more energy (or 9.5 times more)
17 0.22 kg or 220 mL **18** 2.2×10^5 J **19** B
20 Less energy supplied than required to evaporate everything. 33 g would remain. **21** Top cooled directly. Middle cooled by convection as cold air falls. Crisper section chilled by cold air at lowest region in vegetable storage. **22** 6.9 cm^3
23 Tank needs to allow hot water to be drawn off at top, cold water enters at the bottom. Convective currents move water to top as it heats. Heating tank should lie below storage tank to assist this circulation. **24** Snow has low thermal conductivity

and acts as an effective insulator. **25** Conductive heat losses increased in water. Air has a lower thermal conductivity. **26 a** Both the same temperature. **b** Can has a higher conductivity but size may mean that it has considerably more energy to transfer. **27** D—Smallest gradient suggests that it both cools and therefore heats the slowest. **28** C
29 a Cold air is denser so largely remains inside the chest. **b** Conduction and convection currents will carry energy into the top section of the freezer, warming the air and the food.
30 C **31** C **32** A **33** Turbine: kinetic → electrical. Hot water: electrical → heat. Battery: chemical potential → electrical. Light: electrical → light **34** Kinetic, electrical, chemical potential (light if wavelength is towards the visible or higher). **35** 33(.75)%
36 51 L **37** 20(.25)% **38** 148 W **39** 18.6% **40** 1629 kPa pressure increase

Chapter 7 Electricity

7.1 Electric charge

1 He was providing lightning with an easy path to him.
2 Almost none of our modern technology would exist!
3 Maybe 20–30. **4** Both show repulsion and attraction. Magnetism relatively permanent, electrostatic effects temporary. Magnetic poles cannot be isolated, opposite charges can. **5** Charges either repel or attract, there are no other alternatives. **6** They will attract.
7 An excess of the fluid creates one type and a deficit the other. **8** Charge leaks from sharp points.
9 Lightning seeks the easiest route to ground and will take a path through the high point (tree). You should seek shelter inside a building or stay as low as possible and try to insulate yourself from the ground.
10 Positive and negative can cancel each other, as do the effects of opposite charges; colours do not.

7.2 Electric forces and fields

1 a 2***F*** **b** 4***F*** **c** –***F*** **d** 4***F*** **2** 900 kN **3** It is not!
4 a 0.35 N **b** The like charges will repel and therefore move to the furthest part of the dome, thereby increasing the effective distance.
5 a 0.01 N down, 0.025 N up **b** −0.2 μC

7.3 Electric current, EMF and electric potential

1 a 3 C **b** 1000 C **c** 1440 C **2** No, it depends on the energy carried by the charges in the current. The 'current' is down the belt as the electrons are moving up. **3** 3 μA
4 a 2.7×10^{7} A **b** No, because there are the same numbers of protons moving so the net current is zero. **5** Spark plugs 15 000 V **6** D **7** 20 V **8** 9 J **9** 167 C **10** B and C

7.4 Resistance, ohmic and non-ohmic conductors

1 a M_2 or M_3 **b** M_1 or M_4 **2 a** non-ohmic **b** 0.5 A **c** 15 V
d 20 Ω, 13.3 Ω **3 a** 0.71 Ω **b** ohmic
4 Both right at different voltages **5** 72 mA **6 a** 2 Ω **b** 5 A
7 a 1.6 Ω **b** 0.4 Ω **8** 0.11 Ω **9 a** 0.11 Ω **b** 1.1 V **10** Lower slope

7.5 Electrical energy and power

1 a 4.5 J **b** 4.5 J **c** No, some as heat **2** 2.4×10^{-19} J
3 a 0.6 W **b** 2400 W **c** 720 W **4 a** 250 mA **b** 5 A **c** 7.5 A
5 a 25 V **b** 8.7 V **c** 417 V **6** 21 kA **7** 0.84 kW h, 3.0 MJ, $5
8 a 1.5 A **b** 0.075 A **9** AC oscillates, DC steady
10 Less current needed, which means there is less power lost in the transmission lines.

7.6 Simple electric circuits

1 $I_{tot} = 0$ at a point. $E_{tot} = \Delta V_{tot}$ in a circuit.
2 2 V drop in circuit **3** +0.7 A **4 a** 0.25 A **b** 2.4 V
5 a Yes **b** Yes, if one light fails, then they will both go out. **6** 17 Ω **7** 0.22 A, 45 Ω **8** 0.01 A, 4 V and 1 V
9 a through (15 V, 2 A) **b** 7.5 Ω **10** 15 Ω

7.7 Circuit elements in parallel

1 a 0.30 A **b** 3 V **2 a** 20 Ω **b** 5 Ω **3 a** through (10 V, 6 A)
b 1.7 Ω **4** 68 Ω **5** 0.9 A, 11 Ω **6 a** 6.7 Ω **b** 20 V
c 1 A (in 20 Ω) and 2 A **7** 20 W (in 20 Ω), 40 W **8 a** 4.4 k Ω
b 0.44 kΩ **9 a** 20 Ω **b** 20 W **10** Betty is correct

7.8 Cells, batteries and other sources of EMF

1 Need different electronegativities.
2 + to + and – to –. If the connection is the wrong way round, the voltage could damage the electronics of the car. **3** No resistance across terminals.
4 15 V **5 a** 6 V **b** Batteries last longer
6 The heavy current causes an increase in the voltage drop across the battery's internal resistor.
7 10 V **8** 1 Ω **9** 12 A **10** Increased *I* leads to lower *V*.

CHAPTER 7 REVIEW

1 B **2** A **3** Alf is wrong because EMF is a potential energy, which exists even without current. Bert is wrong because EMF is not the same as the output voltage of the cell, which will vary depending on current. **4** They would go flat.
5 12 V **6** 11.4 V **7** The 9 V battery will go flat before the others.
8 Charges escape at the same rate. **9** 1.2×10^{13} off.
10 0.8 W **11** Both have potential energy due to 'compression'.
12 Not enough charge to sustain a current. **13** 6.4×10^{-14} J
14 Charges separate on the paper due to the charged rod. Opposite charges on the near side experience a stronger force than similar charges on the far side so attractive forces dominate. **15** Cloud cover, seasonal changes to angle of sun, no electricity at night.
16 0.6 W **17** V would drop more. **18** 0.24 W **19** 0.02 N
20 0.005 N **21 a** Voltmeter has very high resistance.
b Burn out ammeter. **c** M_1 = A, M_2 = V
22 a 1.3 A **b** 1.3 A **c** 3.2 A **23 a** 4.9 V **b** 1.1 V **c** 6.0 V
24 2 A **25** 50 Ω, 50 Ω, 55 Ω, 67 Ω **26** 409 W
27 Element is hotter. **28** 53% **29** 2.8 s **30** 1.7 C
31 42 A **32** 0.25 V **33** 219 V **34** 60 W, 60 V **35** 1 A, same
36 Mary **37** One out, all out! **38** 4.3 Ω **39** 1.6 kΩ, 14.1 kΩ
40 a 4.0 A **b** 8 Ω

Index

A

B

C

D

E

F

G

H

I

J

K

L

M

N

O

P

Q

R

S

T

U

V

W

X