Semester 1 (Unit 3) Examination, 2016

Question/Answer Booklet

MATHEMATICS METHODS

Section Two: Calculator-assumed

Student Name/Number: ____

Teacher Name: ____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: hundred minutes

Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on ____ unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	9	9	50	50	35
Section Two: Calculator-assumed	13	13	100	98	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the ______________________________________*School/College assessment policy*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Acknowledgements

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Section Two: Calculator-assumed (98 Marks) Weighting 65%

This section has **(thirteen) 13** questions. Answer **all** questions. Write your answers in the spaces provided. Spare pages are included at the end of this booklet.

Suggested working time: **90 minutes**.

Question 10 **(4 marks)**

Use the incremental formula to determine the approximate percentage change in the radius of a hemisphere when its volume is increased by 1.5%.

Question 11 **(9 marks)**

A six-sided die shows the number 1 on two faces, 2 on three faces and 3 on the remaining face. This die is thrown once and the number on the uppermost face is noted.

Y is a discrete random variable representing “the number on the uppermost face”.

(a) Complete the probability distribution table for Y (1 mark)

y	1	2	3
$P(Y = y)$	$\frac{1}{3}$		

The die is thrown twice and the number on the uppermost face is noted after each throw.

(b) Determine the probability that

(i) the die shows a 1 after each throw. (1 mark)

(ii) the number on the die is a 2 after the first throw. (1 mark)

(iii) the number on the die is a 2 after the first throw and a 1 after the second throw (1 mark)

(iv) the die shows the same number after each throw. (2 marks)

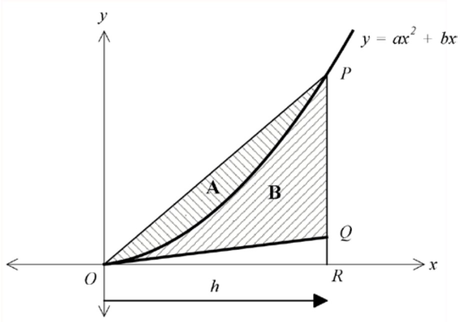
(v) the sum of the numbers on each die is 4. (3 marks)

Additional working space

Question number: ___________________

Question 22 **(6 marks)**

The diagram shows part of the parabola $y = ax^2 + bx$. PQ is the line $x = h$, P is on the curve, and OQ is a tangent to the curve at the origin.



Prove that the parabola divides the area of the triangle OPQ in the ratio 1:2.
i.e. A:B = 1:2

End of questions

Question 12 **(7 marks)**

(a) Use calculus to show that the function $y = a - 2ae^{x^2}$, where a is a positive constant, only has one stationary point which is located at $(0, -a)$. (3 marks)

(b) Use the second derivate to determine the nature of the stationary point. (4 marks)

Question 13 **(6 marks)**

Given the function $y = x\sin(x)$

(a) Determine $\frac{dy}{dx}$ (2 marks)

(b) Using part (a), show how you would determine $\int x\cos(x)\ dx$ (4 marks)

Question 21 **(8 marks)**

A colony of native Australian animals had a population of 1500 at the start of 2010. The population was growing continuously such that $\frac{dP}{dt} = 0.07P$ where P is the number of animals in the colony t years after the start of 2010.

(a) Determine the number of animals in the colony at the start of 2013. (2 marks)

(b) During which year will the colony reach 2000 animals? (2 marks)

(c) At the start of 2016 a disease caused the colony to begin to continually decrease at the rate of 5% per year. If this rate continues, when, to the nearest month, will the colony have a population equal to its size at the start of 2010? (4 marks)

Question 20 **(9 marks)**

A particle is undergoing rectilinear motion with a velocity $v(t) = \left(2e^{2t} - 10\right)$ m/s. Given that it has an initial displacement of 3 m, determine:

(a) when the particle is at rest. (1 mark)

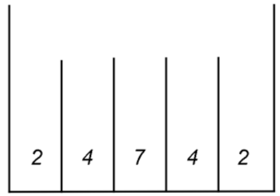
(b) the initial acceleration of the particle. (2 marks)

(c) the displacement of the particle when $t = 2$ s (3 marks)

(d) the total distance the particle travels in the first 4 seconds. (3 marks)

Question 14 **(10 marks)**

In a game, a player rolls two balls (a Red and a Blue) down an inclined plane so that each ball finally settles in one of five slots and scores the number of points allotted to that slot as shown in the diagram below:



It is possible for both balls to settle in one slot and it may be assumed that each slot is equally likely to accept either ball. The player's score is the sum of the points scored by each ball.

(a) If the discrete random variable X is the score obtained by the two balls, complete the following probability distribution table for X. (4 marks)

x	4	6	8	9	11	14
$P(X = x)$	0.16					0.04

A player pays 10 cents for each game and receives back a number of cents equal to their score.

(b) If the discrete random variable Y is the number of cents won per game, complete the following probability distribution table for Y. (3 marks)

y	–6					4
$P(Y = y)$	0.16					0.04

(c) Calculate the players expected gain or loss per 50 games. (3 marks)

Question 15 **(5 marks)**

Given $\frac{d}{dx}\int_a^x (\ f(t)+e^t\ )\ dt - 2\int_0^x \frac{d}{dt}\ (f(t)+e^{2t}\ )\ dt = 2$ and $f(0)=1$.

Show that $f(x)=2+e^x-2e^{2x}$.

Question 16 **(4 marks)**

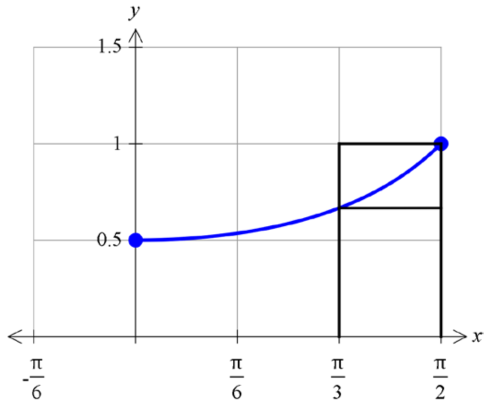
Show that $\int_0^k \left(\sqrt{k}-\sqrt{x}\right)^2 dx = \frac{k^2}{6}$

(b) Show that the derivative of $\frac{\sin x}{1+\cos x}$ is $\frac{1}{1+\cos x}$, given that $\cos^2 x+\sin^2 x=1$. (3 marks)

(c) Using the result of part (c), determine the exact area of the region enclosed by the curve $y=f(x)$, the x-axis, the y-axis and the line $x=\frac{\pi}{3}$. (3 marks)

Question 19 **(11 marks)**

The graph of $f(x)=\dfrac{1}{1+\cos x}$, for $0 \le x \le \dfrac{\pi}{2}$ is drawn below.



(a) Complete the tables below to estimate the area bounded by $f(x)$ and the x and y axes respectively by calculating the mean of the areas of the upper and lower rectangles.
(The rectangles for the interval $\dfrac{\pi}{3}$ to $\dfrac{\pi}{2}$ are shown on the graph above). (5 marks)

x	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$
y	0.5			1

Rectangle	$0-\dfrac{\pi}{6}$	$\dfrac{\pi}{6}-\dfrac{\pi}{3}$	$\dfrac{\pi}{3}-\dfrac{\pi}{2}$	Total
Lower rectangle area	0.26			
Upper rectangle area	0.28			
			Mean	

Question 17 **(8 marks)**

A store manager knows that 6% of the torches in a box are defective. Fifteen torches are selected randomly for testing.

(a) Determine the probability that two of the torches selected are defective. (1 mark)

(b) Determine the probability that fewer than two defective torches are selected. (1 mark)

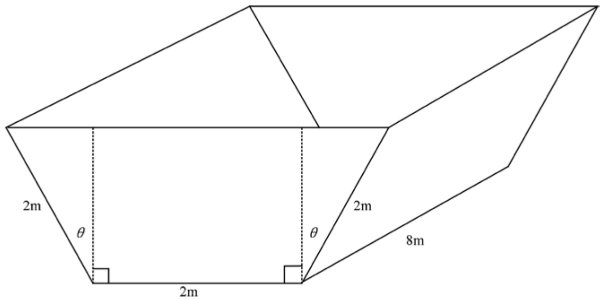
(c) Determine the probability that four or more of the torches selected are defective. (1 mark)

(d) What is the probability that at least one defective torch is selected? (2 marks)

(e) What is the greatest number of torches that can be selected from the box so that the probability that there is at least one defective torch selected is less than 0.55?
Explain how you determined your answer. (3 marks)

Question 18 **(11 marks)**

A company manufactures troughs designed to hold water for use on a farm. The diagram below shows their basic design with the base and sides made of rectangles measuring 8m by 2m and end pieces which are symmetrical trapeziums.



By changing the angle θ ($0 < \theta < \frac{\pi}{2}$) they can manufacture designs with different capacities.

(a) Show that the volume of the trough, V m^3, is given by $V = 32\cos\theta(1+\sin\theta)$ (3 marks)

(b) Determine the value of θ if the trough holds 10 000 Litres when full.
Remember that $1\ kL = 1\ m^3$ (2 marks)

(c) Using calculus methods, determine the value of θ if the trough is to have maximum capacity and state this maximum capacity. Give your answer in kL. (6 marks)