MATHEMATICS METHODS

MAWA Semester 1 (Unit1) Examination 2015

Calculator-Assumed

Marking Key

Section Two: Calculator-assumed (90 Marks)

Question 8(a)

Solution

$m = \dfrac{-1-(-3)}{4-3} = 2 \Rightarrow y = 2x + c$

using $(3, -3)$; $-3 = 2(3) + c \Rightarrow c = -9$

$\therefore y = 2x - 9$

Marking key/mathematical behaviours	Marks
• calculates gradient	1
• uses a point to calculate c and states equation	1

Question 8(b)

Solution

$3x + 2y + 7 = 0$ has gradient $-\dfrac{3}{2}$

$\therefore y = -\dfrac{3}{2}x + c$

using $(2, 3)$; $3 = -3 + c \Rightarrow c = 6$

$\therefore y = -\dfrac{3}{2}x + 6$

Marking key/mathematical behaviours	Marks
• calculates gradient	1
• uses the given point to calculate c and states equation	1

Question 8(c)

Solution

$5x - 3y = 1$ has $m = \dfrac{5}{3} \therefore m_{\perp} = -\dfrac{3}{5}$

$\therefore y = -\dfrac{3}{5}x + c$

using $(5, -4)$; $-4 = -\dfrac{3}{5}(5) + c \Rightarrow c = -1$

$\therefore y = -\dfrac{3}{5}x - 1$

Marking key/mathematical behaviours	Marks
• calculates gradient	1
• determines perpendicular gradient	1
• uses the given point to calculate c and states equation	1

Question 8(d)

Solution

$2y - 3x - 4 = 0 \Rightarrow y = \frac{3}{2}x + 2$

∴ gradient of reflected line is $-\frac{3}{2}$ ⇒ line is $y = -\frac{3}{2}x + 2$

Marking key/mathematical behaviours	Marks
• determines gradient of original line	1
• determines gradient of reflected line and states equation	1

Question 9(a)

Solution

(i) $P(A) = 0.33$

$P(B) = 0.01 + 0.06 + 0.11 + 0.23$ read directly from the graph

(ii) $= 0.41$

(iii) $P(A \cup B) = 0.74$ (by the addition principle)

Marking key/mathematical behaviours	Marks
• reads $P(A)$ correctly from the graph	1
• provides the correct result for $P(B)$	1
• states correct result for $P(A \cap B)$	1
• correctly states $P(A \cup B)$	1

Question 9(b)

Solution

The events are mutually exclusive, since $P(A \cap B) = 0$

Marking key/mathematical behaviours	Marks
• states events are M.E.	1
• provides a valid reason	1

Question 9(c)

Solution

Since we know (given) that the selected person takes at least 5 attempts ($P(B) = 0$), this reduces the probability sample space to 0.59

Hence the probability that a selected person takes 5 attempts = $\frac{0.33}{0.59} = \frac{33}{59}$

Marking key/mathematical behaviours	Marks
• uses the idea of a reduced sample space	1
• states the correct response	1

Question 10(a)

Solution

$\angle ACB = 6°$

Using the sine rule:

$$\frac{BC}{\sin 21°} = \frac{19}{\sin 6°} \Rightarrow BC = 65.14 \text{ cm}$$

Marking key/mathematical behaviours	Marks
• calculates angle ACB	1
• uses the sine rule to calculate the length of BC	1

Question 10(b)

Solution

$\angle ABC = 153°$

Using the area rule:

$$\text{Area} = \frac{1}{2}(19)(65.14)\sin 153° \Rightarrow \text{Area} = 280.94 \text{ cm}^2$$

Marking key/mathematical behaviours	Marks
• calculates angle ABC	1
• uses the area formula to calculate the required area	1

Question 10(c)

Solution

$\angle ABC = 153°$

Let the mid-point of BC be D. BD = 32.57 cm

Using the cosine rule:

$$AD^2 = (32.57)^2 + (19)^2 - 2(19)(32.57)\cos 153° \Rightarrow AD = 50.25 \text{ cm}$$

Marking key/mathematical behaviours	Marks
• calculates the length of BD	1
• uses the cosine rule to calculate the length of AD	1

Question 11 (a)

$\frac{4}{8} = \frac{1}{2}$ **one mark (no need to simplify)**

Question 11 (b)

$\frac{2}{8}$ **one mark for denominator, one mark for numerator**

Question 11 c

$\frac{4}{8}$ **one mark for denominator, one mark for numerator**

Question 12(a)

Solution

By substitution of $t = 0$ into $x = 5\cos\left(\pi t - \frac{\pi}{2}\right)$ we get $x = 5\cos\left(-\frac{\pi}{2}\right) = 0$. That is, the weight is at the rest (or 0 position).

Marking key/mathematical behaviours	Marks
• Determines that the weight is at the rest position	1

Question 12(b)

Solution

The period of the weights oscillation is 2 seconds. Hence it goes through the rest position twice every 2 seconds. That is once every second. Because it starts at the rest position and finishes at the rest position, we need to add one. Hence the answer is 6 times.

Alternatively, we note that $5\cos\left(\pi t - \frac{\pi}{2}\right) = 0$ for every whole number value of t.

i.e. when $t = 0, 1, 2, 3, 4 \text{ and } 5$. So 6 times.

Marking key/mathematical behaviours	Marks
• Provides a reasonable explanation as to how arrived at the number of times the weight is at the rest position	1
• Determines the correct number of times (i.e. 6 times)	1

Question 12(c)

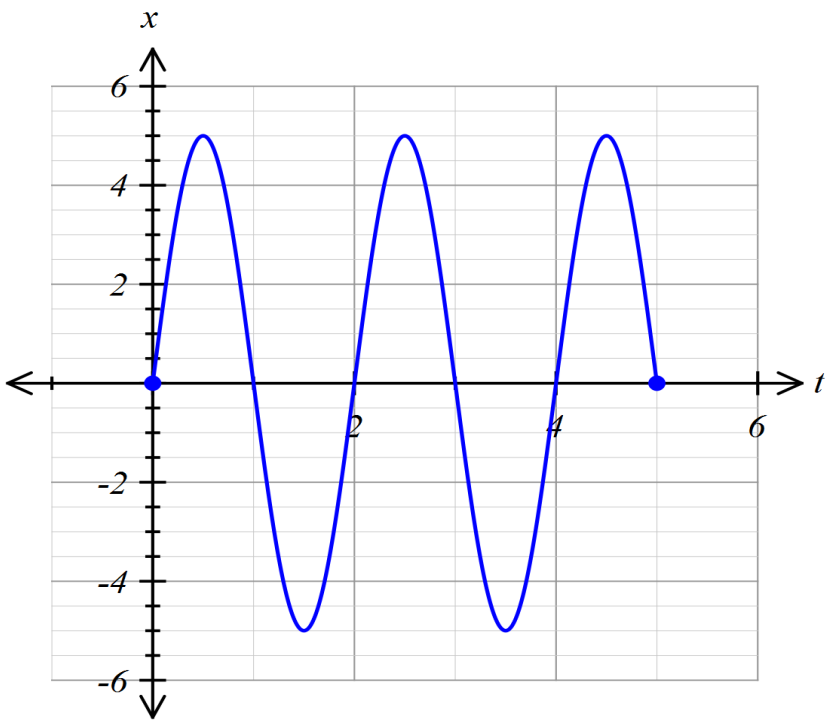
Solution

Negative values of x represent the distance that the weight is below the rest position.
The negative represents 'below the rest position' the magnitude of the number represents the distance.

Marking key/mathematical behaviours	Marks
• Indicates that the negative represents the distance 'below'	1

Question 12(d)

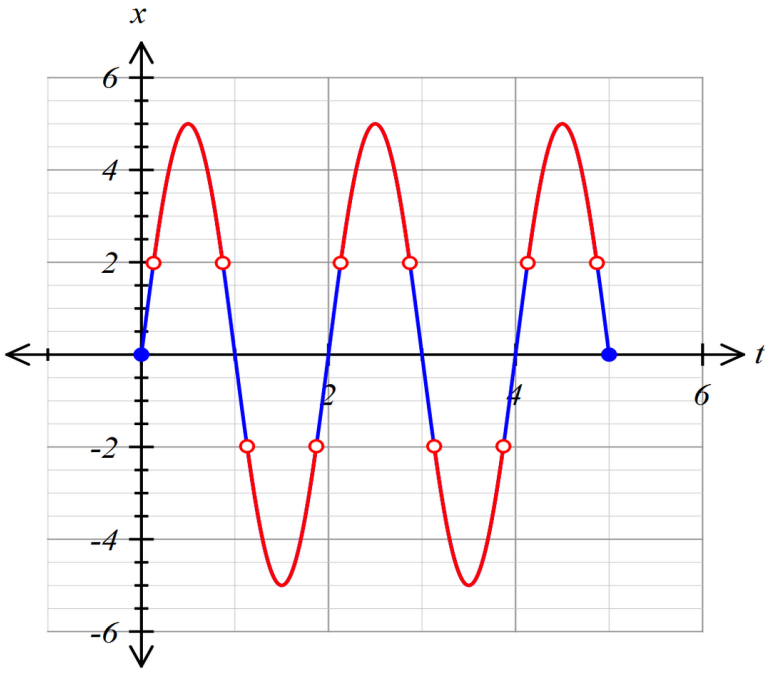
Solution



Marking key/mathematical behaviours	Marks
• graph is sinusoidal with correct amplitude and number of cycles	1
• graph passes through the t-axis at each of the whole number of seconds in the domain	1
• graph has smooth TP's and is acceptably accurate	1

Question 12(e)

Solution



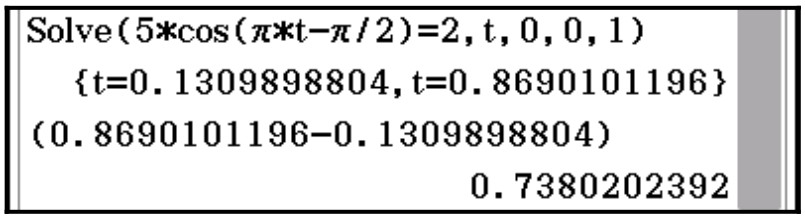
Marking key/mathematical behaviours	Marks
• Indicates (by use of colour or otherwise), the points of the curve where the magnitude of x is greater than 2	1
• Excludes when $x = 2$ or -2.	1

Question 12(f)

Solution

From the graph, the values of t for which $x > 2$ is approximately $0.13 < t < 0.87$ for the first second of motion. This is approx. 75% of the time. This is repeated during every second of the motion. Hence the fraction requested is approximately $\frac{3}{4}$.

For a more accurate answer, use a CAS calculator as follows:



This indicates that the weight is further than 2 cm from the rest position for approximately 73.8% of the time.

Marking key/mathematical behaviours	Marks
• Attempts to estimate the correct fraction of any of the cycles from the graph	1
• Provides an reasonably accurate estimate (70-80%)	1
• Use a calculator to refine the result to 73.8%	1

Question 13(a)

Solution

(i) $Q = \{(0,0),(1,0),(2,1),(3,2),(4,2),(5,3),(6,3),(7,4),(8,4),(9,4),(10,4)\}$

Hence domain of $Q = K$ ={0,1,2, …,10}

(ii) Range of Q ={0,1,2,3,4}

Marking key/mathematical behaviours	Marks
(i)	
• states the correct domain of Q	1
(ii)	
• provides a full listing of the elements of Q	1
• states the correct range of Q	1

Question 13(b)

Solution

(i) $Q' = \{(0,0),(0,1),(1,2),(2,3),(2,4),(3,5),(3,6),(4,7),(4,8),(4,9),(4,10)\}$

Hence domain of Q' ={0,1,2,3,4}

(ii) Range of $Q' = K$ ={0,1,2, …,10}

Marking key/mathematical behaviours	Marks
(i)	
• provides a listing of the elements of Q'	1
• states the correct domain of Q'	1
(ii)	
• states the correct range of Q'	1

Question 13(c)

Solution

Q is a function, Q' is not as it does not satisfy the vertical line test when graphed (or it has multiple y- vales for some x- values i.e. ((2,3) and (2,4) etc.

Marking key/mathematical behaviours	Marks
• indicates that Q is a function and that Q' is not	1
• states a valid reason	1

Question 14(a)

Solution

$x^2 + (y - 1)^2 = 16$

Marking key/mathematical behaviours	Marks
• uses the correct centre	1
• uses the correct radius	1

Question 14(b)

Solution

readable points from the graph are (0,1), (1,3) and (2,3)

$\Rightarrow$ axis of symmetry is at $x = 1.5$

$\therefore y = a(x - 1.5)^2 + c$

at $x = 0, y = 1 \Rightarrow 1 = \frac{9a}{4} + c$

at $x = 2, y = 3 \Rightarrow 3 = \frac{a}{4} + c$

Solving simultaneously we get that $a = -1$ and $c = \frac{13}{4}$

Hence the equation is $y = -(x - 1.5)^2 + \frac{13}{4} = -x^2 + 3x + 1$

Marking key/mathematical behaviours	Marks
• uses known points off graph	1
• determines axis of symmetry	1
• substitutes two points into a general, appropriate equation of a parabola	1
• solves for a and c	1
• states the equation in the required form	1

Question 14(c)

Solution

$\Delta = b^2 - 4ac = 3^2 - 4(-1)(1)$

$= 13$

> 0, but not a perfect square $\Rightarrow$ irrational roots

Marking key/mathematical behaviours	Marks
• determines the discriminant (of the parabola function equated to 0)	1
• interprets $\Delta > 0$	1

Question 15 (a)

Solution

substitute $x = 9$ into upper curve and get

$y = \frac{-2(9)^2}{27} + 8\frac{(9)}{3} - 9 = 9$

therefore coordinates are (9, 9)

Marking key/mathematical behaviours	Marks
• substitutes x = 9 into upper curve	1
• states coordinates	1

Question 15 (b)

Solution

using the coordinate (9, 9)

$9 = a(9)^2 \Rightarrow a = \frac{1}{9}$

∴ equation is $y = \frac{1}{9}x^2$

Marking key/mathematical behaviours	Marks
• substitutes x = 9 into lower curve to obtain value of a	1
• states equation of lower curve	1

Question 15 (c)

Solution

new upper curve: $y = \frac{-2x^2}{27} + \frac{8x}{3} - 9 + 3 \therefore y = \frac{-2x^2}{27} + \frac{8x}{3} - 6$

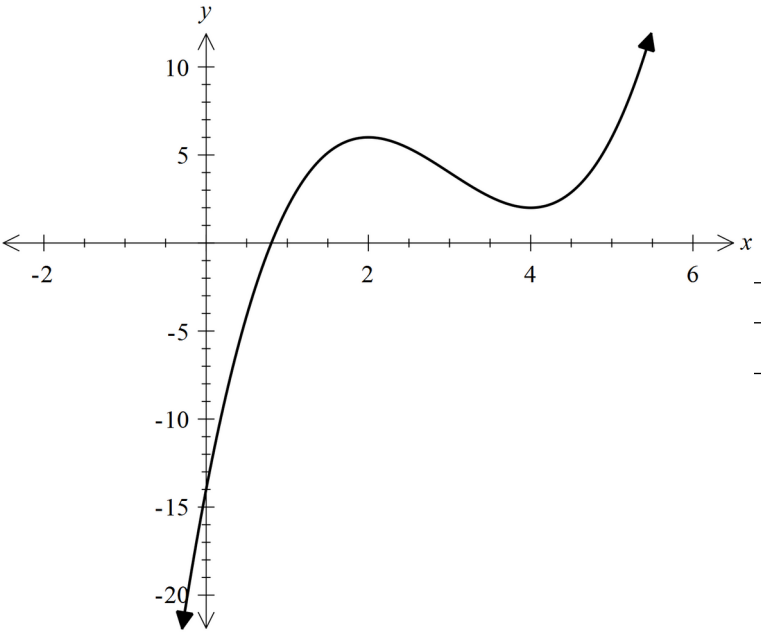
new lower curve: $y = \frac{1}{9}x^2 + 3$

Marking key/mathematical behaviours	Marks
• states equation of new upper curve	1
• states equation of new lower curve	1

Question 16 (a)

Solution

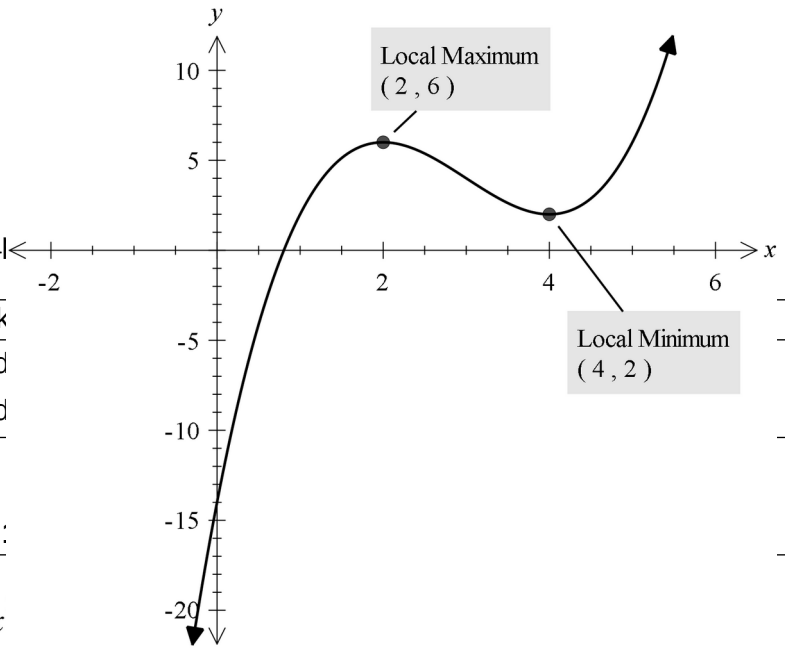
(i)



From graph

Marking ke	Mark
• ide	1

(ii)



From gra

Marking k	Mark
• id	1
• id	1

Question

Solution

$2(ax + b)(x$

$$(ax + b)(x^2 - x + c) = 2x^3 - x^2 - 13x + d$$

$$ax^3 - ax^2 + acx + bx^2 - bx + bc = 2x^3 - x^2 - 13x + d$$

$$ax^3 + (b - a)x^2 + (ac - b)x + bc = 2x^3 - x^2 - 13x + d \text{ equating coefficients}$$

$$a = 2$$

$$b - 2 = -1 \Rightarrow b = 1$$

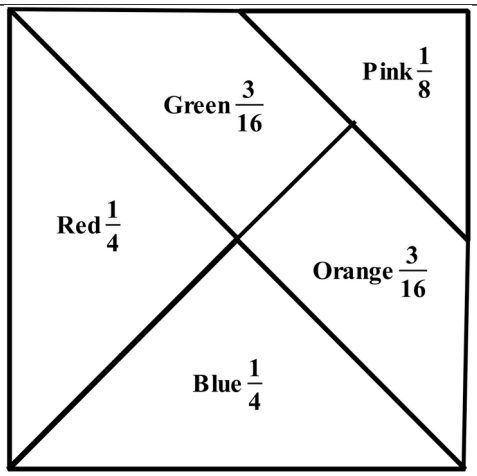
$$2c - 1 = -13 \Rightarrow c = -6 \text{ and } d = -6$$

Hence $4x^3 - 2x^2 - 26x - 12 = 2(2x + 1)(x - 3)(x + 2)$

Marking key/mathematical behaviours	Marks
• divides by 2	1
• expands brackets	1
• collects like terms	1
• equates coefficients to solve for a, b, c and d	1
• factorises expression	1

Question 17(a)

Solution



Firstly we determine the proportional areas for each colour (gives the probability sample space). Hence the Probability that the coin lands on Blue is $\frac{1}{4}$

Marking key/mathematical behaviours	Marks
• represents sample space	1
• determines the correct probability	1

Question 17(b)

Solution

$$\frac{1}{4}+\frac{3}{16}=\frac{7}{16}$$

Marking key/mathematical behaviours	Marks
• identifies that needs to add the proportional areas	1
• adds the appropriate proportional areas correctly	1

Question 17(c)

Solution

$$\frac{1}{2}\times\frac{3}{16}=\frac{3}{32}$$

Marking key/mathematical behaviours	Marks
• identifies that two events need to occur simultaneously	1
• multiplies the appropriate probabilities to get the correct result	1

Question 17(d)

Solution

The events are independent i.e. probability of a tail is not affected by the colour it lands on

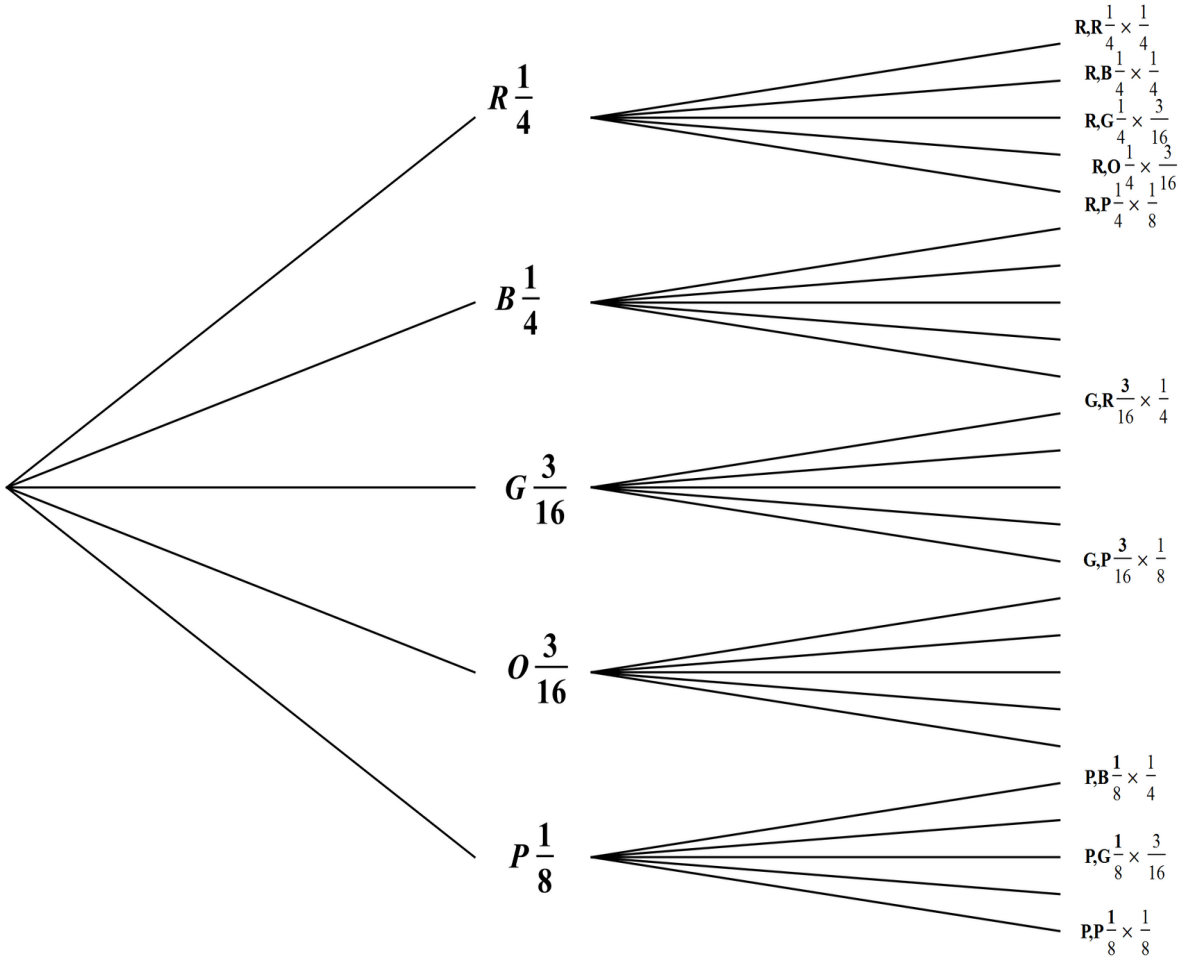
Hence, answer is $\frac{1}{2}$

Marking key/mathematical behaviours	Marks
• identifies that events are independent	1
• states the correct result	1

Question 17(e)

Solution

Here, it is best to draw a tree diagram to represent what happens



What we want is the probability of:
Blue and Blue or Red and Red or Green and Green or Orange and Orange or Pink and Pink

$$\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}+\frac{3}{16}\times\frac{3}{16}+\frac{3}{16}\times\frac{3}{16}+\frac{1}{8}\times\frac{1}{8}$$

$$=\frac{1}{8}+\frac{18}{256}+\frac{1}{64}$$

$$=\frac{27}{128}$$

Marking key/mathematical behaviours	Marks
• applies the multiplication principle for simultaneously occurring independent events	1
• adds the mutually exclusive events	1
• identifies all the possibilities (by listing or other sample space representation)	1
• calculates correctly to give the correct result	1

Question 18(a)

Solution

If $C = -B$ then $\sin(A+C) = \sin(A+(-B))$

$\therefore \sin(A-B) = \sin A\cos(-B) + \cos A\sin(-B)$

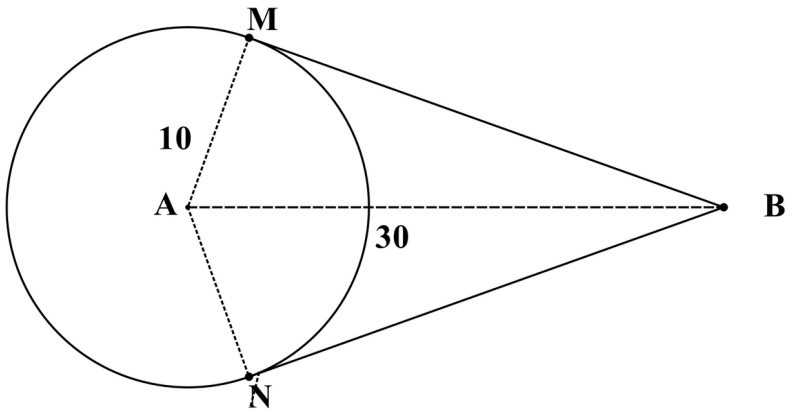
$= \sin A\cos B - \cos A\sin B$

$(\because \cos(-B) = \cos B$ and $\sin(-B) = -\sin B)$

Marking key/mathematical behaviours	Marks
• Substituting $-B$ for C into given identity	1
• Indicating $\cos(-B) = \cos B$ and $\sin(-B) = -\sin B$ and simplifying	1

Question 18(b)

Solution



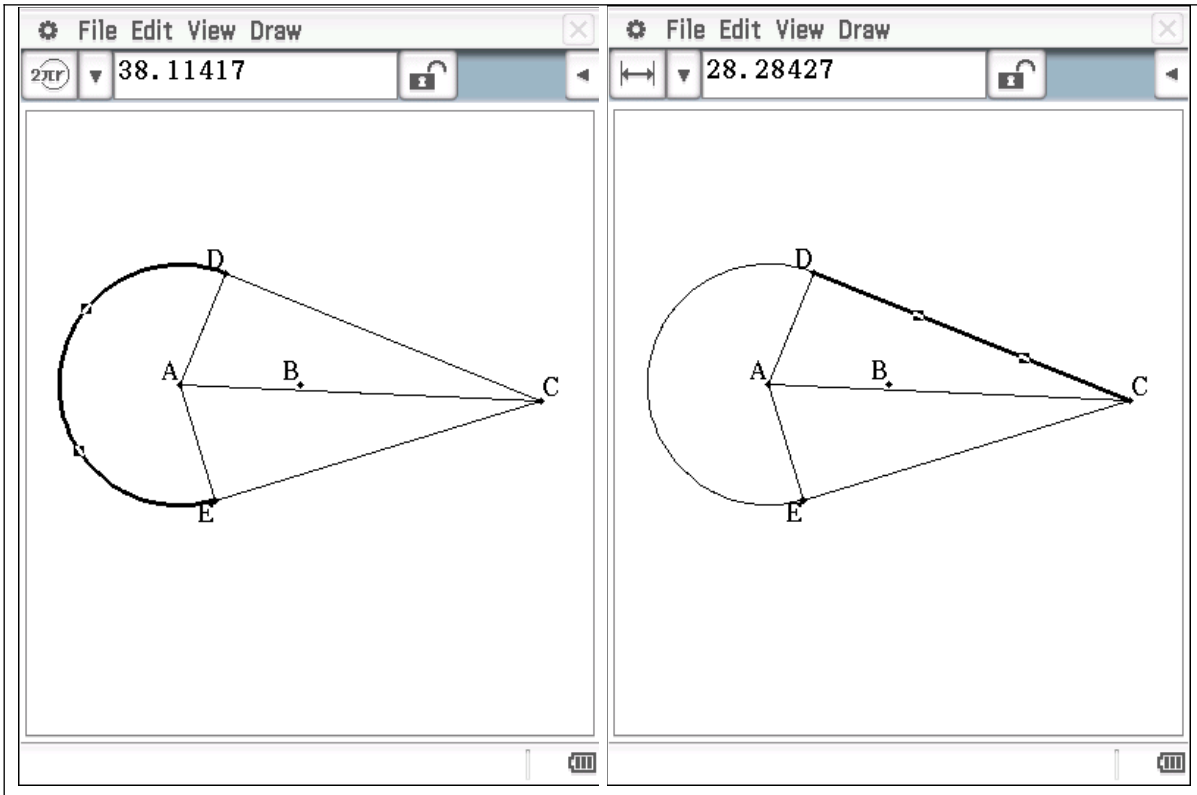
ΔAMB has a rt angle at $M \Rightarrow BM = \sqrt{30^2 - 10^2} = 20\sqrt{2} \approx 28.284$

$\angle MAN = 2\left(inv\left(cos\left(\frac{1}{3}\right)\right)\right) = 2.462$

$\therefore$ length of major arc $\overset{\frown}{MN} = (2\pi - 2.462) \times 10 = 38.213$

Hence the length of the rope $\approx 2 \times 28.284 + 38.2133 \approx 94.78$ cm

Alternatively, using the geometry app on a CAS



Length of rope = 38.11417+2(28.28427) = 94.68 cm (which is within 1 mm of the above answer, due to rounding)

Marking key/mathematical behaviours	Marks
• Indicates length of AB and shows appropriate central angle on the diagram	1
• Calculates length of tangents	1
• Calculates size of central angle	1
• Calculates appropriate arc length	1
• Determines correct length of rope (including units)	1
Or, calculates the length of belt using CAS	or
• Indicates length of the tangents (implies knowledge of length of AB)	2
• Indicates length of major arc ($\overset{\frown}{MN}$)	2
• Provides the correct length of the rope (including units)	1

Acknowledgements

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