PERTH MODERN SCHOOL
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Independent Public School

Course ___Methods_Test 4_ Year __12_____

Student name: _____ Teacher name: _____

Date: **Weds 26 August**

Task type: **Response**

Time allowed for this task: **_____45_____ mins**

Number of questions: **_____6_____**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

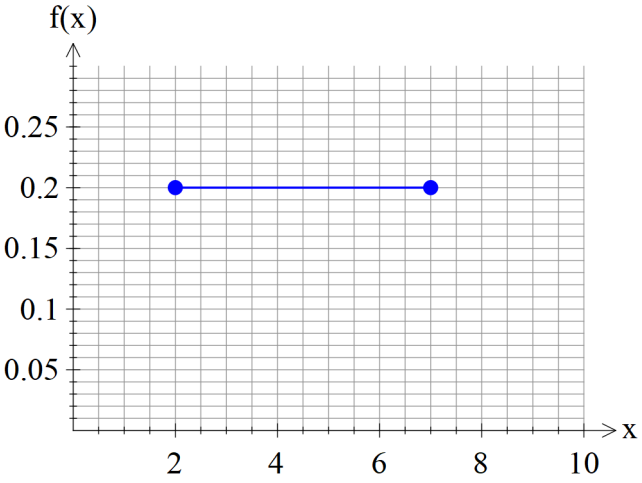
Marks available: **__46____ marks**

Task weighting: **__10__%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (1, 1, 1 & 3 = 6 marks)
Consider a continuous random variable X that is uniformly distributed as follows.



Determine the following:

a) $P(X > 3)$

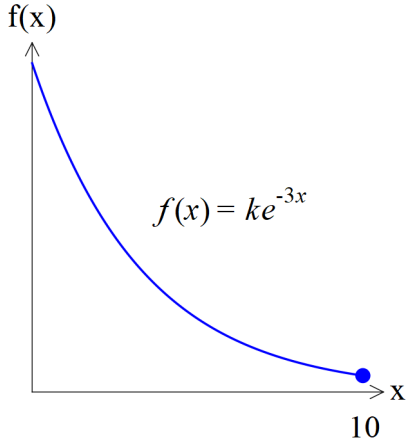
b) $P(X \geq 3)$

c) $P(1 < X \leq 7)$

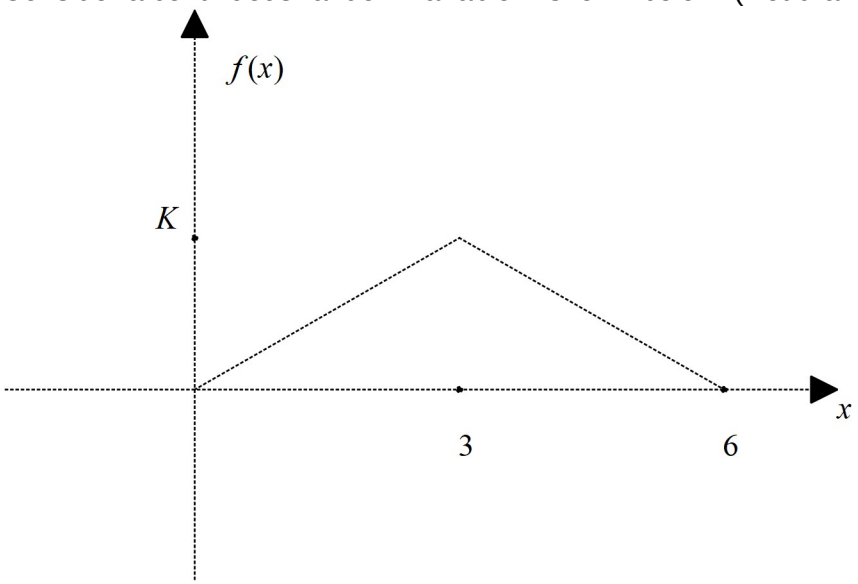
d) $P(X > 3 \mid x < 6)$

Q2 (3 marks)
Consider a continuous random variable X shown below.
Solve for the constant k **exactly.** (Show all working)

$$f(x) = \begin{cases} ke^{-3x} & 0 \leq x \leq 10 \\ 0 & elsewhere \end{cases}$$



Q3 (1 , 4, 1 & 2 = 8 marks)
Consider a continuous random variable X shown below. (Not drawn to scale)



a) Determine the value of the constant K.

b) Determine $P(1 < x < 4)$

c) Determine $E(X)$

d) Determine Standard deviation of X

Q4 (2, 2 ,2 & 1 = 7 marks)

A continuous random variable, X has a pdf $f(x) = \begin{cases} \frac{3}{16}(x-3)^2 & 1 \le x \le 5 \\ 0 & elsewhere \end{cases}$

Determine:

a) $E(x)$

b) $Var(X)$

c) Standard deviation

d) $Var(3x-1)$

Q5 (2, 2, 2 & 3 =9 marks)

The results for a class test, X can be modelled by a Normal Distribution given by $X \sim N(60, 15^2)$.
Determine:

a) The 78th percentile.

b) $P(55 \leq X \leq 72)$

c) The cut-off for an A grade given that this grade is only given to the top 20%.

d) A second test is a Normal Distribution with a mean of 55. Given that the 58th percentile is 62, determine the standard deviation.

Q6 (3, 3, 3, 2 & 2 =13 marks)

The time it takes to be served at a supermarket checkout, X seconds, can be modelled by a normal distribution as follows $X \sim N(103, 30^2)$ seconds. The assistant at the check out is paid according to the following scheme.

Time served In seconds	$0 \le X < 35$	$35 \le X < 60$	$60 \le X < 150$	$150 \le X < 200$	$X \ge 200$
Payment \$P	\$5	\$7	\$12	\$15	\$18
Probability To 4 decimal places					

a) Fill in the probability line of the above table rounded to three decimal places.

b) Determine the expected payment $E(P)$ showing full working.

c) Determine the variance of the payment $Var(P)$ showing full working.

d) If the payments were all increased by 30% and a bonus of \$2 added to each category, determine the new mean and standard deviation.

e) Explain a limitation of the Normal distribution model and show a calculation to support this.