Kennedy Baptist College

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures ☐☐ ☐☐☐ ☐☐☐

In words ______________________

____

Your name ______________________

Number of additional answer booklets used (if applicable): ☐

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

Question number: _________

Question 19 **(7 marks)**

The values of the polynomial functions f, g and h and some of their derivatives are shown in the table below.

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$
2	-1	-6	6	2	8	0
3	2	4	5	4	12	-2
4	7	18	2	6	16	-4

(a) Given that $h''(2) = -2$, describe the graph of $y = h(x)$ near $x = 2$. Justify your answer. (2 marks)

Solution

There is a stationary point that is a maximum because $h'(2) = 0$ and $h''(2) < 0$.

Specific behaviours

✓ correct description
✓ justification

(b) Evaluate the derivative of $f(x) \cdot g(x)$ at $x = 2$. (2 marks)

Solution

$$\frac{d}{dx}\big(f(x)\cdot g(x)\big)_{x=2} = f'(2)\cdot g(2) + f(2)\cdot g'(2)$$
$$= 2 \times (-6) + (-1) \times 8 = -20$$

Specific behaviours

✓ correct use of product rule
✓ correct value

(c) Evaluate the derivative of $\dfrac{f(g(x))}{h(x)}$ at $x = 3$. (3 marks)

Solution

$$\frac{d}{dx}\left(\frac{f(g(x))}{h(x)}\right) = \frac{\frac{d}{dx}\big(f(g(x))\big)\cdot h(x) - f(g(x))\cdot h'(x)}{h(x)^2}$$
$$= \frac{g'(x)\cdot f'(g(x))\cdot h(x) - f(g(x))\cdot h'(x)}{h(x)^2}$$

$$\frac{d}{dx}\left(\frac{f(g(x))}{h(x)}\right)_{x=3} = \frac{g'(3)\cdot f'(g(3))\cdot h(3) - f(g(3))\cdot h'(3)}{h(3)^2}$$
$$= \frac{12 \times f'(4) \times 5 - f(4) \times (-2)}{5^2}$$
$$= \frac{12 \times 6 \times 5 - 7 \times (-2)}{25}$$
$$= \frac{374}{25} = 14.96$$

Specific behaviours

✓ correct use of quotient rule - first line
✓ correct derivative for $f(g(x))$ - second line (chain rule required)
✓ substitutes to obtain correct value

End of questions

Section Two: Calculator-assumed **65% (100 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(5 marks)**

A hire company have a fleet of n bicycles in a city. On any given day, the probability that one of their bicycles needs a repair is independent with a constant value of p.

The random variable X is the daily number of bicycles needing a repair and it has a mean of 53.76 and standard deviation 6.72.

(a) Determine the value of n and the value of p. (3 marks)

Solution

The distribution of $X \sim B(n, p)$.

$$np = 53.76, \qquad np(1-p) = 6.72^2 = 45.1584$$
$$1 - p = 45.1584 \div 53.76 = 0.84$$
$$p = 0.16, \qquad n = 53.76 \div 0.16 = 336$$

Specific behaviours

✓ forms equations using mean and variance of binomial distribution
✓ value of p
✓ value of n

(b) The daily cost to the hire company of these repairs C, in dollars, is also a random variable. It consists of a fixed amount of \$840 to cover workshop and labour costs plus an average of \$38.50 per bicycle repaired for parts and consumables.

Determine the mean and standard deviation of the daily repair cost. (2 marks)

Solution

$$C = 38.5X + 840$$
$$\text{mean} = 38.5 \times 53.76 + 840 = 2069.76 + 840 = \$2909.76$$
$$\text{sd} = 38.5 \times 6.72 = \$258.72$$

Specific behaviours

✓ correct mean
✓ correct standard deviation

See next page

Question 9 **(6 marks)**

A barrel is filled with 34 balls numbered with the integers $1, 2, 3, \ldots, 33, 34$, but otherwise identical.

Let the random variable X be the number on a ball drawn at random from the barrel.

(a) Explain why X has a uniform distribution. (1 mark)

Solution
Every outcome is equally likely.
Specific behaviours
✓ reasonable explanation indicating equally likely outcomes

(b) Determine the expected value of X. (1 mark)

Solution
Using the symmetry of a uniform distribution, $\mathrm{E}(X) = 17.5$
Specific behaviours
✓ correct value

Let the random variable Y take the value 1 when $X < 10$ and the value 0 otherwise.

(c) State the particular name given to two-outcome random variables such as Y. (1 mark)

Solution
Bernoulli random variable.
Specific behaviours
✓ correct name

(d) Determine $\mathrm{P}(Y = 1)$. (1 mark)

Solution
$\mathrm{P}(Y = 1) = \dfrac{9}{34}$
Specific behaviours
✓ correct probability

(e) Three balls are drawn at random from the barrel with each being replaced before the next is taken. Determine the probability that exactly two of the balls are marked with single digit numbers. (2 marks)

Solution
$W \sim B\left(3, \dfrac{9}{34}\right), \qquad \mathrm{P}(W = 2) = 0.1546$ Alternative: $p = \left(\dfrac{9}{34}\right)^2 \times \dfrac{25}{34} \times 3 = \dfrac{6075}{39304} = 0.1546$
Specific behaviours
✓ indicates correct method ✓ correct probability

(b) Determine the maximum area of R. (5 marks)

Solution
Maximum area when g is tangential to f, at point $x = k$. Then using $(0, 0)$ and $(k, e^{0.125k})$ we get $m = \dfrac{e^{0.125k}}{k}$. Also, $m = f'(k) \to 0.125e^{0.125k}$. Hence $k = 1 \div 0.125 = 8$ and $m = 0.125e^{0.125\times 8} = 0.125e$. $A_{MAX} = \int_0^8 (e^{0.125x} - 0.125ex)\,dx$ $= 4e - 8 \approx 2.873\ \mathrm{u}^2$
Specific behaviours
✓ indicates area maximised when g is tangential to f ✓ one equation relating m and k ✓ second equation relating m and k ✓ solves for m and k ✓ correct maximum area

Question 18 **(10 marks)**

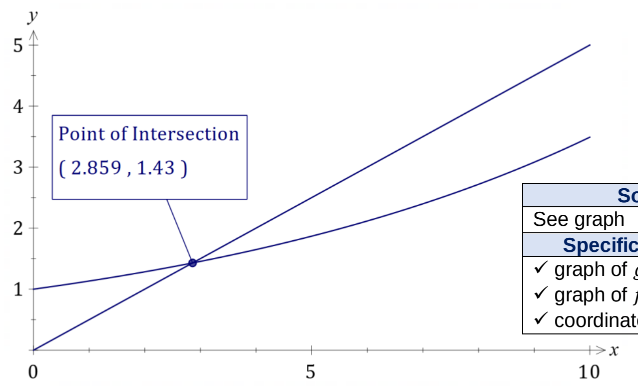
Consider the functions $f(x) = e^{0.125x}$ and $g(x) = mx$ for $x \geq 0$.

The positive constant m is such that the graphs of f and g always intersect.

Let R be the region enclosed by the y-axis and the graphs of f and g.

(a) Let $m = 0.5$.

(i) Sketch the graphs of f and g for $0 \leq x \leq 10$, showing the coordinates of the point where they intersect on the boundary of R. (3 marks)



Solution
See graph
Specific behaviours
✓ graph of g ✓ graph of f and vert scale ✓ coordinates

(ii) Determine the area of R. (2 marks)

Solution
$A = \int_0^{2.859} (e^{0.125x} - 0.5x)\,dx$ $= 1.393\ \text{u}^2$
Specific behaviours
✓ correct integral ✓ correct area

Question 10 **(8 marks)**

45 mg of a radioisotope with a half-life of 77 hours was injected into a patient before a CT scan. The mass M of the radioisotope decays continuously so that t hours after administration, the mass remaining is given by $M = M_0 e^{-kt}$, where M_0 and k are constants.

(a) Determine the value of the constants M_0 and k. (3 marks)

Solution
$t = 0 \Rightarrow M = M_0 = 45$ $\frac{M}{M_0} = 0.5 = e^{-77k} \Rightarrow k = 0.009$
Specific behaviours
✓ states M_0 ✓ equation for k ✓ value of k

(b) Determine the mass of the radioisotope that remains in the patient exactly one week after their injection. (1 mark)

Solution
$t = 7 \times 24 = 168\ \text{h}, \quad M = 45e^{-0.009 \times 168} = 9.92\ \text{mg}$
Specific behaviours
✓ calculates mass M

(c) Eventually, the mass of the remaining radioisotope falls to 5 mg.

(i) Determine how long after their injection that this occurs. (2 marks)

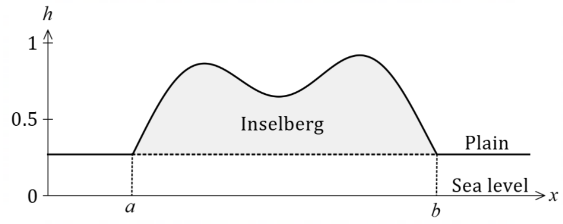
Solution
$5 = 45e^{-0.009t} \Rightarrow t = 244\ \text{h}$
Specific behaviours
✓ substitutes to form equation ✓ uses CAS to solve for t

(ii) Determine the rate at which the radioisotope is decaying at this time. (2 marks)

Solution
$\frac{dM}{dt} = -kM$ $= -0.009 \times 5 = -0.045\ \text{mg/h}$
Specific behaviours
✓ uses rate of change equation ✓ correct rate

Question 11 **(11 marks)**

A vertical cross section through the highest point of an inselberg, a mountain range that rises above a surrounding level plain, is shown in the figure below.



The height of the plain and the inselberg above sea level h, in kilometres, is given by

$$h(x)=\begin{cases} x-\dfrac{1}{5}\left(x^2+2+\sin\left(\dfrac{13x}{4}\right)\right) & a\le x\le b \\ 0.27 & \text{otherwise}\end{cases}$$

where x is the horizontal displacement in kilometres from an arbitrary origin.

(a) Determine the value of a and the value of b, the x displacements where the inselberg meets the surrounding plain. (2 marks)

Solution
$x-\dfrac{1}{5}\left(x^2+2+\sin\left(\dfrac{13x}{4}\right)\right)=0.27$ Using CAS to solve results in $a=0.88$ and $b=4.04$.
Specific behaviours
✓ writes equation   ✓ states both values

(b) Use calculus to determine the cross-sectional area of the inselberg shaded in the figure above. (3 marks)

Solution
$A=\displaystyle\int_{0.88}^{4.04}\left(x-\frac{1}{5}\left(x^2+2+\sin\left(\frac{13x}{4}\right)\right)-0.27\right)dx$ $=1.417\ \text{km}^2$
Specific behaviours
✓ correct integrand   ✓ correct bounds of integration   ✓ correct area, with units

Question 17 **(9 marks)**

Spinners A and B are used in a game of chance, with equally likely outcomes of $2,3,4,5,6$ for spinner A and $2,3,4,5$ for spinner B after each has been spun.

A player pays \$2 for one play of the game and will win \$5 if the outcomes of spinner A and spinner B are the same, \$2 if their outcomes differ by one, and nothing otherwise.

Let X be the profit (winnings minus payment) in dollars made by a player in one play of the game.

(a) Explain why X is a random variable and list all possible values it can take. (2 marks)

Solution
X is a random variable because its value is the result of a random event and cannot be predicted. The values X can take are $3, 0$ and -2.
7 **Specific behaviours**
✓ correct values   ✓ reasonable explanation

(b) Determine the expected value of X. (4 marks)

Solution
Total number of outcomes is $n_A\times n_B=5\times 4=20$. Of these, $(2,2),(3,3),(4,4),(5,5)$ are the same and $(2,3),(3,4),(4,5),(3,2),(4,3),(5,4),(6,5)$ differ by one. Hence $\text{P}(X=3)=\dfrac{4}{20},\quad \text{P}(X=0)=\dfrac{7}{20},\quad \text{P}(X=-2)=\dfrac{9}{20}$ $\text{E}(X)=\dfrac{3\times4-0\times7-2\times9}{20}=-\dfrac{3}{10}$
Specific behaviours
✓ correct number of all possible outcomes   ✓ one correct probability   ✓ all correct probabilities   ✓ correct expected value

(c) Calculate the variance of X. (2 marks)

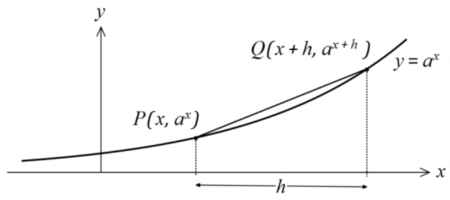
Solution
$\text{Var}(X)=\left(3+\dfrac{3}{10}\right)^2\times\dfrac{4}{20}+\left(\dfrac{3}{10}\right)^2\times\dfrac{7}{20}+\left(-2+\dfrac{3}{10}\right)^2\times\dfrac{9}{20}=3.51$
Specific behaviours
✓ indicates appropriate method   ✓ correct variance

(d) Determine what the cost of one play of the game should be so that in the long run, a player will break even. (1 mark)

Solution
Require $\text{E}(X)=0$ and so the profit per game must increase by 0.3 and hence the cost must be $2.00-0.30=\$1.70$ per play.
Specific behaviours
✓ correct cost per play

Question 16 **(6 marks)**

The graph of $y = a^x$ is shown in the diagram below, where a is a positive constant.



A secant is drawn between points P and Q that lie on the curve with x-coordinates x and $x + h$ respectively.

(a) Describe the property of the secant that $\dfrac{a^{x+h} - a^x}{h}$ represents. (1 mark)

Solution
Slope of the secant.
Specific behaviours
✓ correct description

(b) Describe the property of the curve that $\lim\limits_{h\to 0}\left(\dfrac{a^{x+h} - a^x}{h}\right)$ represents. (1 mark)

Solution
Slope of the curve at P.
Specific behaviours
✓ correct description

It can be shown that $\lim\limits_{h\to 0}\left(\dfrac{a^{x+h} - a^x}{h}\right) = a^x \lim\limits_{h\to 0}\left(\dfrac{a^h - 1}{h}\right)$.

(c) Complete the following table when $a = 3$, rounding values to 4 decimal places, and explain how the values can be used to obtain an approximation for the first derivative of 3^x with respect to x. (3 marks)

h	0.01	0.001	0.0001	0.00001
$\dfrac{a^h - 1}{h}$	**1.1047**	**1.0992**	**1.0987**	**1.0986**

Solution
The table shows that $\lim\limits_{h\to 0}\left(\dfrac{a^h - 1}{h}\right) \to 1.0986$ and so $\dfrac{d}{dx}(3^x) = 1.0986(3)^x$.
Specific behaviours
✓ one correct value ✓ all correct values ✓ correct explanation

(d) For what value of a does $\lim\limits_{h\to 0}\left(\dfrac{a^h - 1}{h}\right) = 1$? (1 mark)

Solution
$a = e$
(Euler's number)
Specific behaviours
✓ correct value

(c) Use calculus to

(i) determine the maximum height of the inselberg above the surrounding plain. (4 marks)

Solution
$h'(x) = 1 - \dfrac{8x + 13\cos\left(\frac{13x}{4}\right)}{20}$ In order to use cl   Using CAS to solve $h'(x) = 0$ gives $x = 1.625,\ x = 2.397,\ x = 3.238$.   From figure, middle value is a minimum, so check other values: $h(1.625) = 0.865, \quad h(3.238) = 0.919$ Hence maximum height is 919 m above sea level, which is $919 - 270 = 649$ m above plain.
Specific behaviours
✓ obtains first derivative of h ✓ shows all solutions to $h'(x) = 0$ ✓ shows reasoning for selecting root of $h'(x)$ that gives required maximum   ✓ correct height above plain, with units

(ii) verify that the stationary point on the curve that represents the highest part of the inselberg is a maximum. (2 marks)

Solution
$h''(x) = \dfrac{1}{80}\left(169\sin\left(\dfrac{13x}{4}\right) - 32\right)$ $h''(3.238) = -2.28$ As the sign of the second derivative at this stationary point is negative then the curve is concave down and thus a maximum.
Specific behaviours
✓ obtains second derivative   ✓ uses sign of second derivative for justification

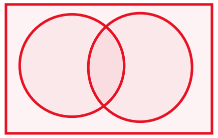
Question 12 **(11 marks)**

A random sample of 150 households within a large town revealed that 48 households owned a cat, 60 owned a dog and 27 owned both a cat and a dog. You may assume that point estimates of probabilities derived from this sample are reliable and representative of the whole town.

(a) For households within the town, determine the probability that

(i) a randomly selected household owns neither a cat nor a dog. (2 marks)

Solution
Households owning at least one cat or dog is $60 + 48 - 27 = 81$.
$$\mathrm{P(Neither)} = \frac{150 - 81}{150} = \frac{69}{150} = 0.46$$
Specific behaviours
✓ number who own at least one cat or dog ✓ correct probability



(ii) in a random sample of 5 households, exactly 3 will not own a dog. (3 marks)

Solution
$\mathrm{P(Household\ does\ not\ own\ dog)} = (150 - 60) \div 150 = 0.6$
If X is number not owning dog in sample, then $X \sim \mathrm{B}(5, 0.6)$.
$\mathrm{P}(X = 3) = 0.3456$
Specific behaviours
✓ calculates probability of event ✓ states distribution is binomial, with parameters ✓ calculates probability

(iii) in a random sample of 9 households that own a dog, at least 2 will own a cat. (3 marks)

Solution
$\mathrm{P(Household\ owns\ cat} \mid \mathrm{owns\ dog)} = 27 \div 60 = 0.45$
If X is number owning cat in sample, then $X \sim \mathrm{B}(9, 0.45)$.
$\mathrm{P}(X \geq 2) = 0.9615$
Specific behaviours
✓ calculates conditional probability ✓ states distribution is binomial, with parameters ✓ calculates probability

(c) The height h of a plant, initially 9 cm, is changing at a rate given by $\frac{dh}{dt} = \frac{2t}{e^{0.5t}}$ cm/day, for $t \geq 0$.

(i) Determine an equation to model the height of the plant as a function of time and hence determine its height after 7 days. (3 marks)

Solution
$$h = \frac{-4t - 8}{e^{0.5t}} + c$$ $$c = 9 - \frac{-8}{e^{0}} = 17$$
$$h(t) = \frac{-4t - 8}{e^{0.5t}} + 17$$
$$h(7) = 15.9 \text{ cm}$$
Specific behaviours
✓ uses result from (b), changing variables ✓ evaluates constant c ✓ correct height

(ii) According to the model, what height will the plant never exceed? (1 mark)

Solution
As $t \to \infty$, $h \to 17$ cm.
Height will not exceed 17 cm.
Specific behaviours
✓ correct height

Question 15 **(10 marks)**

(a) Use the quotient rule to show that $\frac{d}{dx}\left(\frac{4x+2}{e^{0.5x}}\right) = \frac{3}{e^{0.5x}} - \frac{2x}{e^{0.5x}}$. (3 marks)

Solution
$u = 4x + 2, u' = 4, \qquad v = e^{0.5x}, v' = 0.5e^{0.5x}$ Using the quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$ $= \frac{4e^{0.5x} - (4x+2)0.5e^{0.5x}}{(e^{0.5x})^2}$ $= \frac{4 - 0.5(4x+2)}{e^{0.5x}}$ $= \frac{4 - 2x - 1}{e^{0.5x}} = \frac{3}{e^{0.5x}} - \frac{2x}{e^{0.5x}}$
Specific behaviours
✓ correct derivatives for u, v ✓ clearly shows use of quotient rule   ✓ clear simplification steps to obtain required result

(b) Use your result from part (a) to show that $\int \frac{2x}{e^{0.5x}}\,dx = \frac{-4x}{e^{0.5x}} - \frac{8}{e^{0.5x}} + c$, where c is a constant. (3 marks)

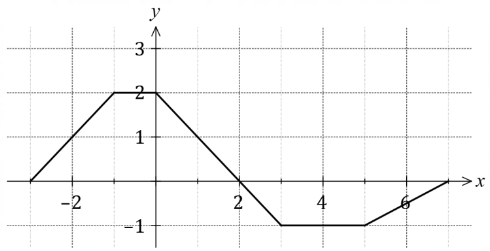
Solution
$\frac{d}{dx}\left(\frac{4x+2}{e^{0.5x}}\right) = \frac{3}{e^{0.5x}} - \frac{2x}{e^{0.5x}}$ Hence $\int \frac{d}{dx}\left(\frac{4x+2}{e^{0.5x}}\right)dx = \int \frac{3}{e^{0.5x}}\,dx - \int \frac{2x}{e^{0.5x}}\,dx$ $\frac{4x+2}{e^{0.5x}} = \frac{-6}{e^{0.5x}} - \int \frac{2x}{e^{0.5x}}\,dx + c$ $\int \frac{2x}{e^{0.5x}}\,dx = \frac{-6}{e^{0.5x}} - \frac{4x+2}{e^{0.5x}} + c$ $= \frac{-4x}{e^{0.5x}} - \frac{8}{e^{0.5x}} + c$
Specific behaviours
✓ uses result from (a), wrapping integrals around terms   ✓ simplifies two integrals, including constant   ✓ rearranges for required integral and simplifies

(b) If another random sample of 276 households was drawn from within the town, determine the mean and standard deviation of the probability distribution that models the number of households that own either a cat or a dog in the sample. (3 marks)

Solution
$\mathrm{P}(\text{Household owns cat or dog}) = 81 \div 150 = 0.54$ If X is number owning cat or dog in sample, then $X \sim \mathrm{B}(276, 0.54)$. $\mathrm{E}(X) = 276 \times 0.54 = 149.04$ $\mathrm{sd} = \sqrt{276 \times 0.54(1 - 0.54)} = 8.28$
Specific behaviours
✓ states distribution is binomial, with parameters   ✓ calculates mean   ✓ calculates standard deviation

Question 13 **(8 marks)**

The graph of $y = f(x)$ is shown below.



Evaluate each of the following.

(a) $\int_{-2}^{4} f(x)\,dx$. (2 marks)

Solution

$$\int_{-2}^{4} f(x)\,dx = 5.5 - 1.5 = 4$$

Specific behaviours

✓ indicates use of signed areas
✓ correct value

(b) $\int_{0}^{7} (f(x) + 2)\,dx$. (2 marks)

Solution

$$\int_{0}^{7} (f(x) + 2)\,dx = \int_{0}^{7} f(x)\,dx + \int_{0}^{7} 2\,dx$$
$$= 2 - 3.5 + 14 = 12.5$$

Specific behaviours

✓ indicates use of linearity
✓ correct value

(c) $\int_{-1}^{-3} 3f(x)\,dx$. (2 marks)

Solution

$$\int_{-1}^{-3} 3f(x)\,dx = -3\int_{-3}^{-1} f(x)\,dx$$
$$= -3(2) = -6$$

Specific behaviours

✓ adjusts integral so that LH bound < RH bound
✓ correct value

(d) $\int_{0}^{5} f'(x)\,dx$. (2 marks)

Solution

$$\int_{0}^{5} f'(x)\,dx = f(5) - f(0)$$
$$= -1 - 2 = -3$$

Specific behaviours

✓ indicates use of fundamental theorem
✓ correct value

Question 14 **(9 marks)**

A particle is moving in a straight line with acceleration $a = 3e^{-0.25t}$ cm/s² after t seconds. When $t = 0$ it has a displacement of 1.5 m and a velocity of -7 cm/s.

(a) Determine the acceleration of the particle at the instant at which it comes to rest. (4 marks)

Solution

$$v = \int 3e^{-0.25t}\,dt$$
$$= -12e^{-0.25t} + c$$
$$v(0) = -12 + c = -7 \to c = 5$$
$$v = -12e^{-0.25t} + 5$$

$$v = 0$$
$$-12e^{-0.25t_1} + 5 = 0$$
$$t_1 = 3.502$$

$$a(t_1) = 1.25\ \mathrm{cm/s^2}$$

Specific behaviours

✓ integrates acceleration
✓ expression for velocity, including constant
✓ solves for root of velocity
✓ substitutes to obtain acceleration

(b) Determine an expression for the displacement of the particle in terms of t. (2 marks)

Solution

$$x = \int -12e^{-0.25t} + 5\,dt$$
$$= 48e^{-0.25t} + 5t + c$$
$$x(0) = 48 + c = 150 \to c = 102$$
$$x = 48e^{-0.25t} + 5t + 102$$

Specific behaviours

✓ integrates velocity
✓ expression for displacement, including constant

(c) Determine the velocity of the particle when it again has a displacement of 1.5 m. (3 marks)

Solution

$$x = 150$$
$$48e^{-0.25t_2} + 5t_2 + 102 = 150$$
$$t_2 = 8.43452$$

$$v(t_2) = 3.54\ \mathrm{cm/s}$$

Specific behaviours

✓ forms correct equation
✓ solves for correct time
✓ substitutes to obtain velocity