

(24 marks)

Section One: Calculator Free

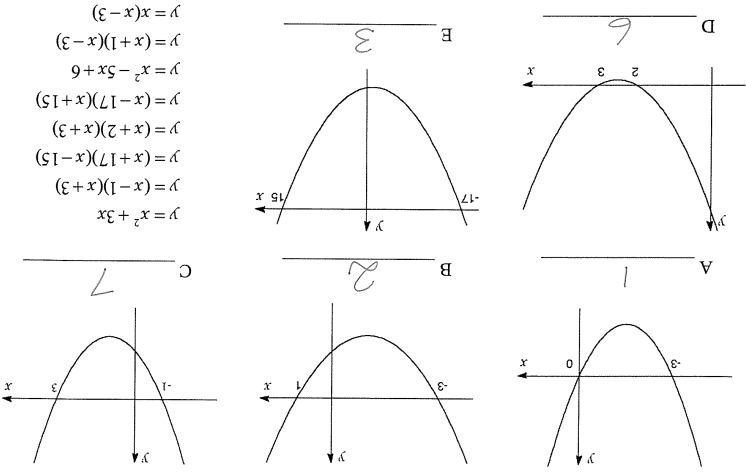
This section has five (5) questions. Answer all questions. Write your answers in the spaces provided.

Working time for this section is 25 minutes.

(3 marks)

Question 1

Shown are eight quadratic functions, numbered 1 to 8, and five graphs, lettered A to E. Each graph corresponds to one of the functions. Decide which function goes with which graph. You will have three functions left over.



Solution		Specific behaviours		Point
$A \ y = x^2 + 3x$ $B \ y = (x - 1)(x + 3)$ $C \ y = (x + 1)(x - 3)$ $D \ y = x^2 - 5x + 6$ $E \ y = (x + 17)(x - 15)$		✓ Identifies two quadratics.	✓ Identifies four quadratics	
		✓ Identifies all five.		?

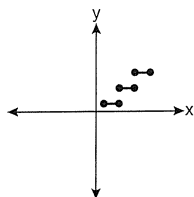
Question 2

(5 marks)

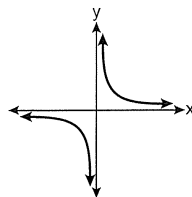
Circle the correct answer for the following questions:

- (a) Which represents a relation that is not a function? (1 mark)

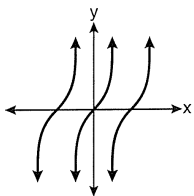
A



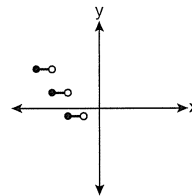
B



C



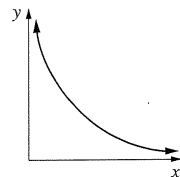
D



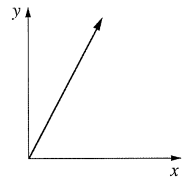
Solution	Specific behaviours
A	✓ Correct answer.

- (b) Which graph shows that y is directly proportional to x ? (1 mark)

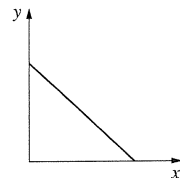
A



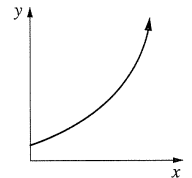
B



C



D

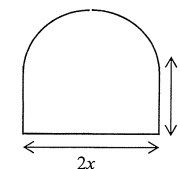


Solution	Specific behaviours
B	✓ Correct answer.

Question 9

(9 marks)

A window pane is to be made from 12 metres of steel.
The pane is to have a rectangular base and a semi-circular top as shown.



If the base of the pane is $2x$ metres and the side y metres then:

- (a) show that $y = 6 - x - \frac{\pi}{2}x$. (2 marks)

Solution	Specific behaviours	Point
$12 = 2x + 2y + \frac{1}{2} \times 2\pi x$ $12 = 2x + \pi x + 2y$ $2y = 12 - 2x - \pi x$ $y = 6 - x - \frac{\pi}{2}x$	✓ Works out equation for perimeter. ✓ Rearranges into required form.	?

- (b) show that the area enclosed by the pane is given by $A(x) = 12x - 2x^2 - \frac{\pi}{2}x^2$. (3 marks)

Solution	Specific behaviours	Point
$A(x) = 2xy + \frac{1}{2}\pi x^2$ $A(x) = 2x \left(6 - x - \frac{\pi}{2}x \right) + \frac{\pi}{2}x^2$ $A(x) = 12x - 2x^2 - \pi x^2 + \frac{\pi}{2}x^2$ $A(x) = 12x - 2x^2 - \frac{\pi}{2}x^2$	✓ Works out equation for area. ✓ Substitutes in answer from part $y = 6 - x - \frac{\pi}{2}x$. ✓ Expands and shows required formula.	?

- (c) calculate the maximum area enclosed by the pane, and the length of the base and side that gives the maximum area. State all answers correct to 1 decimal place. (4 marks)

Solution	Specific behaviours	Point
max TP at (1.68, 10.08) Maximum area is 10.1 m ² Base length is 3.4 m Side length is 1.7 m	✓ Determines maximum TP. ✓ States maximum area. ✓ States base length. ✓ States side length.	?

End of Calculator Assumed Section

Question 8

(9 marks)

- (a) Show that the lines $y + 2x = 3$ and $2y - x = 1$ are perpendicular. At what point do they intersect? (3 marks)

Solution	$y = -2x + 3$ $y = \frac{1}{2}x + \frac{2}{2}$ $\frac{1}{2} \times (-2) = -1 \Rightarrow$ perpendicular $y + 2x = 3$ $2y - x = 1$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
Specific behaviours	<input checked="" type="checkbox"/> Rearranges. <input checked="" type="checkbox"/> Shows they are perpendicular. <input checked="" type="checkbox"/> Determines point of intersection.
Point	?

- (b) Determine the equation of the line, having an x-intercept of -4 , and which is parallel to the line connecting the turning point of $y = (x + 1)^2 + 3$ with $(3, 7)$. (3 marks)

Solution	$TP(-1, 3)$ $m = \frac{7 - 3}{3 - (-1)} = 1$ $y = x + c$ $0 = -4 + c$ $c = 4$ $y = x + 4$
Specific behaviours	<input checked="" type="checkbox"/> Determines turning point of parabola. <input checked="" type="checkbox"/> Determines gradient of line through turning point and $(3, 7)$. <input checked="" type="checkbox"/> Determines equation of line.
Point	?

- (c) The line with an angle of inclination to the positive x axis of 135° and y intercept of $-\frac{1}{2}$. State your answer in the form $ax + by = c$, where a , b and c are integers. (3 marks)

Solution	$m = \tan 135^\circ = -1$ $y = -x - \frac{1}{2}$ $x + y = -\frac{1}{2}$ $2x + 2y = -1$
Specific behaviours	<input checked="" type="checkbox"/> Determines gradient. <input checked="" type="checkbox"/> Determines equation of the line. <input checked="" type="checkbox"/> Rearranges into required form.
Point	?

- (c) George drew a correct diagram that gave the solution to the simultaneous equations $y = 2x - 5$ and $y = x + 6$. (1 mark)

Which diagram did he draw?

A

B

C

D

- (d) Which equation represents the relationship between x and y in this table? (1 mark)

x	0	2	4	6	8
y	1	2	3	4	5

- A** $y = 2x + 1$
B $y = 2x - 2$
C $y = \frac{2}{x} - 2$
D $y = \frac{2}{x} + 1$

Solution	<input checked="" type="checkbox"/> Correct answer.
Specific behaviours	

- (e) For the graph $y = ax^2 + bx + c$, if a and c are both positive, which of the following statements is true. (1 mark)

- ☒ A The graph will have a minimum turning point, and a positive y -intercept.
☐ B The graph will have a maximum turning point, and a positive y -intercept.
☐ C The graph will have a maximum turning point, and has two positive x intercepts.
☐ D The graph will have a minimum turning point, and a negative y -intercept.

Solution	Specific behaviours
A	✓ Correct answer.

Question 3

(2 marks)

The linear function $f(x) = 4 - x$ has range $-2 \leq f(x) < 6$.

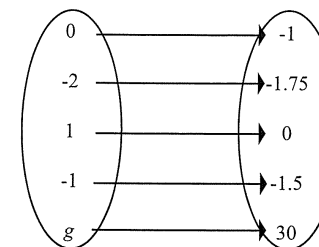
Determine the domain of the function.

Solution	Specific behaviours
$-2 = 4 - x$ $x = 6$ $6 = 4 - x$ $x = -2$ $D_x : \{x : -2 < x \leq 6\}$	✓ Works backwards from range to determine values for domain. ✓ States domain.

Question 7

(7 marks)

The mapping below is of the form $f: x \rightarrow a \times 2^x + b$ and maps the elements of x to elements of y .



- (a) List the elements in the domain of $f(x)$. (1 mark)

Solution	Specific behaviours
$D_x = \{-2, -1, 0, 1, g\}$	✓ Determines the domain.

- (b) List the elements in the range of $f(x)$. (1 mark)

Solution	Specific behaviours
$R_y = \{-1.75, -1.5, -1, 0, 30\}$	✓ Determines the range.

- (c) Find a and b . (3 marks)

Solution	Specific behaviours
$(0, -1) \Rightarrow -1 = a + b$ (1) $(1, 0) \Rightarrow 0 = 2a + b$ (2) $(2) - (1) \Rightarrow 1 = a$ $\Rightarrow b = -2$	✓ Sets up simultaneous equations. ✓ Correct value for a . ✓ Correct value for b .

- (d) Find the value of g . (2 marks)

Solution	Specific behaviours
$f(x) = 2^x - 2$ $30 = 2^g - 2$ $32 = 2^g$ $g = 5$	✓ Sets up equation. ✓ Correct value for g .

This section has four (4) questions. Answer all questions. Write your answers in the spaces provided.

Working time for this section is 30 minutes.

Question 6 (6 marks)

An air balloon leaves its base at 12 noon and is moving such that its height, h metres, above sea-level at any time t hours, after 12 noon, is given as

$$h(t) = -\frac{3}{4}(t+1)(t-16) \quad \text{for } 0 \leq t \leq 20$$

Determine:

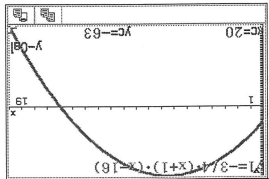
- (a) the initial height above sea-level. (1 mark)

Solution	$f = 0 \text{ kg} = 12 \text{ m}$
Specific behaviours	✓ Correct answer.

- (b) the maximum height, correct to one decimal place, to which the balloon rises. (1 mark)

Solution	$h(7.5) = 54.2 \text{ m}$
Specific behaviours	✓ Correct answer.

- (c) the minimum height to which the balloon sinks over the time interval. (2 marks)

Solution	
Specific behaviours	✓ Determines minimum occurs at (20, -63). ✓ Correctly interprets in the context of the question.

The balloon is to manoeuvre over a building of height 30 metres above sea level.

- (d) During what times, correct to two decimal places, will it be able to do this? (2 marks)

Solution	$30 = -\frac{3}{4}(t+1)(t-16)$ $1.82 \leq t \leq 13.18$
Specific behaviours	✓ Writes equation, or indicates on graph solution method. ✓ States times.

is between 1:49pm and 1:11am the next day.

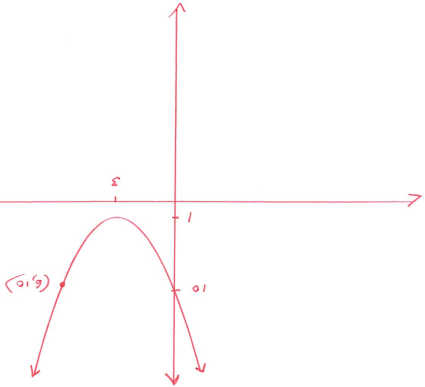
Question 4 (6 marks)

- (a) Complete the square for $x^2 - 6x + 10$. (2 marks)

Solution	$x^2 - 6x + 10 = (x-3)^2 - 9 + 10$ $= (x-3)^2 + 1$
Specific behaviours	✓ Uses completing the square. ✓ Correct answer.
Point	?

- (b) Using your result from part (a), sketch the graph $y = x^2 - 6x + 10$, showing all significant features. (3 marks)

Solution	✓ Turning point at (3, 1). ✓ y-intercept at (0, 10). ✓ Correct graph.
Specific behaviours	
Point	?



- (c) Explain how the graph can be used to show the following statement is always true: (1 mark)

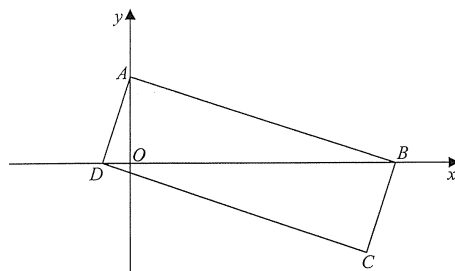
"When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive."

Solution	Since the curve is always above the x-axis, then $y = x^2 - 6x + 10$ would always be positive.
Specific behaviours	✓ Recognises that curve is always above x-axis, and hence answer is always positive.
Point	?

Question 5

(8 marks)

The figure below shows a rectangle $ABCD$.



The point A lies on the y -axis and the points B and D lie on the x -axis as shown.

Given that the straight line through the points A and B has equation $5y + 2x = 10$

- (a) show that the straight line through the points A and D has equation $2y - 5x = 4$. (3 marks)

Solution	Specific behaviours	Point
$AB: y = -\frac{2}{5}x + 2 \Rightarrow m_1 = \frac{5}{2}$	✓ Determines perpendicular gradient.	1.1.4
$AD: y = \frac{5}{2}x + 2$	✓ Determines equation of AD .	1.1.5
$2y = 5x + 4$		
$2y - 5x = 4$	✓ Rearranges.	

- (b) determine the coordinates of the points B and D . (2 marks)

Solution	Specific behaviours	Point
$2x = 10$		1.1.5
$x = 5 \Rightarrow B(5, 0)$	✓ Determines coordinates of B .	10A
$-5x = 4$		
$x = -\frac{4}{5} \Rightarrow D(-\frac{4}{5}, 0)$	✓ Determines coordinates of D .	

- (c) determine the coordinates of the midpoint of the diagonal BD . (1 mark)

Solution	Specific behaviours	Point
$\frac{-\frac{4}{5} + 5}{2} = \frac{21}{10} \text{ i.e. } (\frac{21}{10}, 0)$	✓ Determines coordinates of midpoint.	?

- (d) The diagonals of a rectangle bisect. *Use this fact*, along with your results from part (a) and (c), to determine the coordinates of the point C . (2 marks)

Solution	Specific behaviours	Point
$(\frac{0+x}{2}, \frac{2+y}{2}) = (\frac{21}{10}, 0) = (2.1, 0)$	✓ Determines midpoint formula, or 'steps it out'.	?
$C(\frac{21}{5}, -2) = (4.2, -2)$	✓ Determines coordinates.	

End of Calculator Free Section