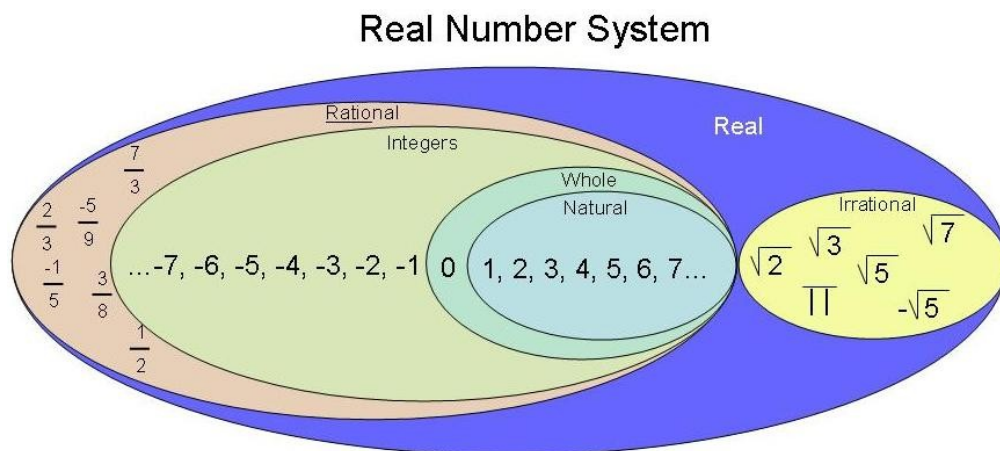


Conjectures: \Rightarrow (implies)e.g. $x = 4 \Rightarrow x^2 = 16$ \Leftrightarrow (equivalent)e.g. $x = \pm 4 \Leftrightarrow x^2 = 16$ Quantifiers: \forall : (For all) \mathbb{R} : (real set) \mathbb{Z} : (integer set) \exists : (there exists) \in : (element)e.g. $\exists x \in \mathbb{Z}$ Implications: \Rightarrow one wayEquivalence: \Leftrightarrow two-way

Converse:

** Works for most definitions

$$P \Rightarrow Q$$

Converse is $Q \Rightarrow P$

Is when the hypothesis and conclusion of a statement is **switched**. However, the converse of a true statement need **not be true**

e.g. if $x=2$ then $x^2=4$ **is true**

if $x^2 = 4$ **is false (because x could be -2)**

Although if the statement are true they are equivalent statements can be written 'P if and only Q'

e.g. A triangle has two sides of the same length if and only if it has two angles in size.

Contrapositive:

$$P \Rightarrow Q$$

The contrapositive is "If not Q then not P"

Is when the hypothesis and the conclusion of a conditional statement is **switches** and then **negating both**.

e.g. if $x=2$ then $x^2=4$

Contrapositive statement: if $x^2 \neq 4$ then $x \neq 2$

The contrapositive of a true statement is also true

e.g. if a polygon has exactly 4 sides then the polygon is a quadrilateral (**True statement**)

If a polygon is not a quadrilateral then it does not have exactly four sides (**The contrapositive is also true**)

Inverse:

$$P \Rightarrow Q$$

The inverse statement is: if not P then not Q

Negating both the hypothesis and the conclusion of a conditional statement.

Negation: (not)

If P is the statement

It is raining

Then the negation of P is the statement:

It is not raining

Assume the opposite and prove the opposite wrong

e.g. the statement:

You cannot have a right-angle triangle with one side of length $3x$ cm, another side length $(4x+5)$ cm and the longest side of length $(5x+4)$

Assume the opposite

Assume that we can indeed have a right-angle triangle with the given side lengths and then prove that this assumption leads to something that cannot be true.

Pigeon-Hole Principles:

If there are n pigeon holes, $n - 1$, and $n + 1$ pigeons go in them, then at least one pigeon hole must get two or more pigeons.

e.g. a letterman has 7 letters, but there's only 6 letter boxes. Therefore, one of the letter boxes will have at least 2 letters.

Concluding

Thus for this statement

if P then Q

The *converse* statement is

if Q then P

The *contrapositive* is

if not Q then not P

The *inverse* statement is

if not P then not Q

- The *contrapositive* statement involves both the effect of the *converse*, in its switch of P and Q, and the *inverse*, with its negations of both P and Q
- If the original statement is true then the *contrapositive* is also true but the *converse* and the *inverse* may not be