Yr 11/12

RSHS SCHOOL

Yr 11/12 sems 2 Examination, 2012

Question/Answer Booklet

MATHEMATICS 3A/3B
Section One:
Calculator-free

SOLUTIONS

MARKING KEY

Student Number: In figures

In words

Your name

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before reading any further.**

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	12	12	100	100	67
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

Additional working space

Question number: _____

Section One: Calculator-free (50 Marks)

This section has seven (7) questions. Answer all questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 (3 marks)

Find the values of x and y if $3x + 2y = -4$ and $5x - 3y = 25$.

Eqn 1 times 3 $9x+6y=-12$

Eqn 2 times 2 $10x-6y=50$

Add $19x=38$

$x = 2$

$y = -5$

Additional working space

Question number: _____

Question 2 (9 marks)

(a) If $a = 5 \times 10^2$ and $b = 8 \times 10^6$ evaluate $a^2 \div b^{1/3}$. (3 marks)

$$\frac{5^2 \times 10^{2 \times 2}}{8^{1/3} \times \left(10^6\right)^{1/3}} = \frac{25 \times 10^4}{2 \times 10^2} \quad \checkmark$$
$$= 12.5 \times 10^2 \quad \checkmark \quad \text{Either form}$$
$$= 1250$$

(b) Solve the following for x.

(i) $25^x = 125\sqrt{5}$ (3 marks)

$$5^{2x} = 5^3 \times 5^{0.5} \quad \checkmark$$
$$2x = 3.5 \quad \checkmark$$
$$x = \frac{7}{4} \quad (1.75) \quad \checkmark$$

(ii) $\dfrac{(2x-3)^3}{4} = 16$ (3 marks)

$$(2x-3)^3 = 64 \quad \checkmark$$
$$(2x-3)^3 = 4^3$$
$$2x - 3 = 4 \quad \checkmark$$
$$x = \frac{7}{2} \quad \checkmark$$

Question 7 (7 marks)

(a) Prove that the product of any two odd numbers will always be odd. (4 marks)

Let the odd numbers be $2n+1$ and $2m+1$. $\checkmark$

Then their product is

$$(2n+1)(2m+1) = 4nm + 2n + 2m + 1$$
$$\checkmark \qquad = 2(2nm + n + m) + 1 \quad \checkmark$$

Hence the product is odd as it is of the form $2p+1$ $\checkmark$

(b) A simple polygon is a closed two-dimensional shape, made of straight lines and with only one boundary that doesn't cross over itself.
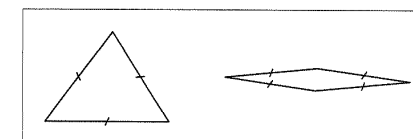
Consider the conjecture: "If all the sides are of length d, then as the number of sides of a polygon increases, so does the area of the polygon."

State whether the conjecture is true or false and justify your answer. (3 marks)

Conjecture is false. $\checkmark$

A polygon must have 3 or more sides. A polygon with 3 equal length sides of d must be an equilateral triangle, as shown. The polygon shown with 4 equal length sides of d will obviously have a smaller area, showing that as the number of sides increases, the area does not.

$\checkmark\checkmark$ Reasoning and/or diagram

Question 3   (8 marks)

The universal set $\{1, 2, 3, 4, 5, 6, 7\}$ has subsets $A = \{1, 3, 5, 6, 7\}$ and $B = \{4, 5, 6\}$.

(a)   Determine $n\ (\overline{A \cup B})$.

$$\boxed{6}$$

(1 mark)

(b)   List the elements of $A \cap \overline{B}$.

$$\boxed{\{1, 2, 3, 5, 7\}}$$

(2 marks)

(c)   A four-digit PIN number is made by randomly choosing the first two digits from subset A and the last two digits from subset B. An example of such a PIN number is 3346.

(i)   In how many ways can the first two digits be chosen?

$$\boxed{4 \times 4 = 16 \text{ ways}}$$

(1 mark)

(ii)   What is the probability that the last two digits chosen are both sixes?

$$\boxed{\dfrac{1 \times 1}{3 \times 3} = \dfrac{1}{9}}$$

(2 marks)

(iii)   What is the probability that the PIN number starts with a five but does not end with a five?

$$\boxed{\dfrac{1 \times 4 \times 3 \times 2}{4 \times 4 \times 3 \times 3} = \dfrac{1}{6}}$$

(2 marks)

Question 6   (9 marks)

(a) $P = (t^2 - 2)(1 - 3t + 2t^2)$.

Use the product rule to find $\dfrac{dP}{dt}$, simplifying your answer.

(4 marks)

$$\boxed{\begin{aligned}
\frac{dP}{dt} &= 2t(1 - 3t + 2t^2) + (t^2 - 2)(-3 + 4t) \\
&= 2t - 6t^2 + 4t^3 - 3t^2 + 6 - 8t \\
&= 8t^3 - 9t^2 - 6t + 6
\end{aligned}}$$

(b)   A polynomial function $f(x)$ passes through the point A(1, -2) and is such that
$f'(x) = 5 - 2x$.

(i)   Find the equation of the tangent to $f(x)$ at the point A.

(2 marks)

$$\boxed{\begin{aligned}
f'(1) &= 5 - 2(1) \\
&= 3 \\
y - (-2) &= 3(x - 1) \\
y &= 3x - 5
\end{aligned}}$$

(ii)   Find $f(x)$.

(3 marks)

$$\boxed{\begin{aligned}
f(x) &= 5x - x^2 + c \\
-2 &= 5(1) - (1)^2 + c \\
c &= -6 \\
f(x) &= 5x - x^2 - 6
\end{aligned}}$$

Question 4   **(6 marks)**

A set of test scores were 12, 9, 4, 16, 13, 2, 10, 11, 10, 15.

(a)   State the mode of the scores.   (1 mark)

> 10   ✓

(b)   The minimum and maximum scores are 2 and 16 respectively. Calculate the other
       statistics that would be required to construct a boxplot for these scores.   (3 marks)

> 2, 4, **9**, 10, 10   (M)   11, 12, **13**, 15, 16
>
> Median is 10.5 ✓
>
> Lower Quartile is 9 ✓
>
> Upper Quartile is 13 ✓

(c)   Use a calculation to decide whether or not the set of scores contains an outlier.   (2 marks)

> IQR = 13 - 9 = 4
>
> LQ - 1.5×IQR = 9 - 1.5×4 = 3
>
> Any score below 3 is an outlier, so the
> score of 2 is an outlier - the set of
> scores **does** contain an outlier.

✓ Calculation

✓

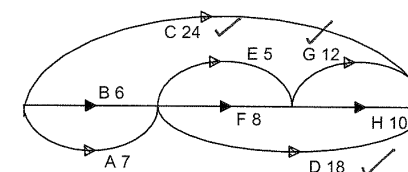**Question 5**   **(8 marks)**

The eight activities involved in a construction project, together with their completion times and
immediate predecessors are shown in this table:

Activity	Time (days)	Immediate Predecessor
A	7	-
B	6	-
C	24	-
D	18	A, B
E	5	A, B
F	8	A, B
G	12	E, F
H	10	E, F

(a)   Display this information as a project network.   (3 marks)

Marks awarded
for correct placement
of tasks C, G & D
as shown.

(b)   List the activities on the critical path and state the minimum completion time for this
       project.   (2 marks)

> CP is A, F, G. ✓
>
> MCT is 7 + 8 + 12 = 27 days ✓

follow through

(c)   Consider each of the questions below in isolation.

   (i)   How many days can activity E be delayed, without an increase in the minimum
          completion time?   (1 mark)

   > 3 days ✓

   (ii)   If the time taken by activity F is halved, what effect does this have on the minimum
          completion time and critical path?   (2 marks)

   > MCT decreases by 2 days to 25 days. ✓
   >
   > New CP: A, D. ✓