

Nuclear binding energy Solution

1. *Of the two hydrogen isotopes, deuterium, ${}^2_1\text{H}$, and tritium, ${}^3_1\text{H}$, which has the highest binding energy per nucleon?*

$$\begin{array}{lcl} \text{mass defect} & = & \text{mass of component neutrons and protons -} \\ \text{mass} & & \text{of nucleus} \end{array}$$

$$\text{mass defect for } {}^2_1\text{H} = \{m(1 \text{ proton} + 1 \text{ neutron})\} - m({}^2_1\text{H})$$

$$= 1.00728 + 1.00867 - 2.01350$$

$$= 0.00245 \text{ u}$$

$$= 0.00245 \times 1.6606 \times 10^{-27} \text{ kg}$$

$$= 4.068 \times 10^{-30} \text{ kg}$$

$$\text{Binding energy} = mc^2$$

$$= 4.068 \times 10^{-30} \times (3.00 \times 10^8)^2$$

$$= 3.662 \times 10^{-13} \text{ J}$$

$$\text{Binding energy per nucleon } {}^2_1\text{H} = 3.662 \times 10^{-13} / 2$$

$$= 1.831 \times 10^{-13} \text{ J}$$

$$\text{mass diff. for } {}^3_1\text{H} = \{m(1 \text{ proton} + 2 \text{ neutrons})\} - m({}^3_1\text{H})$$

$$= 1.00728 + 2 \times 1.00867 - 3.01550$$

$$= 0.00912 \text{ u}$$

$$= 0.00912 \times 1.6606 \times 10^{-27} \text{ kg}$$

$$= 1.514 \times 10^{-29} \text{ kg}$$

$$\text{Binding energy} = mc^2$$

$$= 1.514 \times 10^{-29} \times (3.00 \times 10^8)^2$$

$$= 1.363 \times 10^{-12} \text{ J}$$

$$\text{Binding energy per nucleon } {}^3_1\text{H} = 1.363 \times 10^{-12} / 3$$

$$= 4.54 \times 10^{-13} \text{ J}$$

Tritium has the higher binding energy per nucleon

2. *A small proportion of all the carbon in living organisms is the radioactive isotope carbon-14. Calculate the binding energy per nucleon of both the carbon-12 and carbon-14 nuclei and state which one is the most stable. The atomic number of carbon is 6.*

$$\begin{aligned} \text{mass defect. for } {}^{12}_6\text{C} &= \{m(6 \text{ proton} + 6 \text{ neutrons})\} - m({}^{12}_6\text{C}) \\ &= 6 \times 1.00728 + 6 \times 1.00867 - 11.99671 \\ &= 0.09899 \text{ u} \end{aligned}$$

$$= 0.09899 \times 1.6606 \times 10^{-27} \text{ kg}$$

$$= 1.644 \times 10^{-28} \text{ kg}$$

$$\text{Binding energy} = mc^2$$

$$= 1.644 \times 10^{-28} \times (3.00 \times 10^8)^2$$

$$= 1.4794 \times 10^{-12} \text{ J}$$

$$\text{Binding energy per nucleon } {}^{14}_6\text{C} = 1.4794 \times 10^{-12} / 12$$

$$= 1.233 \times 10^{-12} \text{ J}$$

$$\text{mass defect for } {}^{14}_6\text{C} = \{m(6 \text{ proton} + 8 \text{ neutrons})\} - m({}^{14}_6\text{C})$$

$$= 6 \times 1.00728 + 8 \times 1.00867 - 13.99995$$

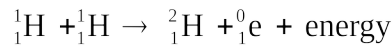
$$= 0.11309 \text{ u}$$

$$= 0.11309 \times 1.6606 \times 10^{-27} \text{ kg}$$

$$= 1.878 \times 10^{-28} \text{ kg}$$

$$\begin{aligned}
 \text{Binding energy} &= mc^2 \\
 &= 1.878 \times 10^{-28} \times (3.00 \times 10^8)^2 \\
 &= 1.690 \times 10^{-12} \text{ J} \\
 \text{Binding energy per nucleon } {}^{14}_6\text{C} &= 1.690 \times 10^{-12} / 14 \\
 &= 1.207 \times 10^{-12} \text{ J}
 \end{aligned}$$

3. *One of the simplest fusion reactions is*



a. *What mass does a single fusion reaction convert to energy?*

$$\begin{aligned}
 \text{mass diff. for reaction} &= m({}^2_1\text{H}) + m({}^0_{+1}\text{e}) - 2 m({}^1_1\text{H}) \\
 &= 2.01350 + 0.000549 - 2 \times 1.00728 \\
 &= -0.000511 \text{ u} \\
 &= -0.000511 \times 1.6606 \times 10^{-27} \text{ kg} \\
 &= -8.49 \times 10^{-31} \text{ kg}
 \end{aligned}$$

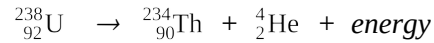
b. *What energy does a single fusion reaction release?*

$$\begin{aligned}
 \text{energy change} &= mc^2 \\
 &= -8.49 \times 10^{-31} \times (3.00 \times 10^8)^2 \\
 &= -7.64 \times 10^{-14} \text{ J}
 \end{aligned}$$

$$\text{energy released} = 7.64 \times 10^{-14} \text{ J}$$

(note that if the masses in kg are used rounding errors give an answer of $7.1 \times 10^{-14} \text{ J}$)

4. *Uranium-238 undergoes a series of radiocative decays, the first of which is:*



How much energy does each decay release?

$$\begin{aligned} \text{mass diff. for reaction} &= m({}_{90}^{234}\text{Th}) + m({}_2^4\text{He}) - m({}_{92}^{238}\text{U}) \\ &= 233.98418 + 4.00150 - 238.00028 \\ &= -0.0146 \text{ u} \\ &= -0.0146 \times 1.6606 \times 10^{-27} \text{ kg} \\ &= -2.42 \times 10^{-29} \text{ kg} \\ \text{energy change} &= mc^2 \\ &= -2.42 \times 10^{-29} \times (3.00 \times 10^8)^2 \\ &= -2.182 \times 10^{-12} \text{ J} \\ \text{energy released} &= 2.182 \times 10^{-12} \text{ J} \end{aligned}$$

5. *Hydrogen and deuterium fuse to give the isotope ${}_2^3\text{He}$.*

a. How much energy does a single fusion release?



$$\begin{aligned} \text{mass diff. for reaction} &= m({}_2^3\text{He}) - m({}_1^2\text{H}) - m({}_1^1\text{H}) \\ &= 3.01493 + 2.01350 - 1.00728 \\ &= -0.00585 \text{ u} \\ &= -0.00585 \times 1.6606 \times 10^{-27} \text{ kg} \\ &= -9.71 \times 10^{-30} \text{ kg} \\ \text{energy change} &= mc^2 \\ &= -9.71 \times 10^{-30} \times (3.00 \times 10^8)^2 \\ &= -8.74 \times 10^{-13} \text{ J} \end{aligned}$$

$$\text{energy released} = 8.74 \times 10^{-13} \text{ J}$$

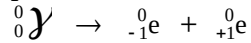
b. *How much energy does a kilogram of reactant release?*

$$(2.01350 + 1.00728) \times 1.6606 \times 10^{-27} \text{ kg produces } 8.74 \times 10^{-13} \text{ J}$$

$$1.000 \text{ kg produces } 8.74 \times 10^{-13} / ((2.01350 + 1.00728) \times 1.6606 \times 10^{-27}) \text{ J}$$

$$= 1.74 \times 10^{14} \text{ J}$$

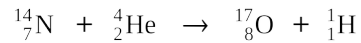
7. *Under certain circumstances, a gamma ray photon may suddenly change into an electron and a positron.*



Calculate the minimum energy of the photon.

$$\begin{aligned} \text{mass diff. for reaction} &= m({}^0_{+1}e) + m({}^0_{-1}e) - m({}^0_0\gamma) \\ &= 0.000549 + 0.000549 - 0.000000 \\ &= 0.001098 \text{ u} \\ &= 0.001098 \times 1.6606 \times 10^{-27} \text{ kg} \\ &= 1.823 \times 10^{-30} \text{ kg} \\ \text{energy change} &= mc^2 \\ &= 1.823 \times 10^{-30} \times (3.00 \times 10^8)^2 \\ &= 1.641 \times 10^{-13} \text{ J} \\ \text{minimum energy of photon} &= 1.641 \times 10^{-13} \text{ J} \end{aligned}$$

8. When 3.0000 MeV alpha particles bombard nitrogen 14, oxygen-17 forms and the reaction releases a proton. Calculate the energy this reaction releases.



$$\begin{aligned}
 \text{mass diff. for reaction} &= m({}^1_1\text{H}) + m({}^{17}_8\text{O}) - m({}^4_2\text{He}) - m({}^{14}_7\text{N}) \\
 &= 1.00728 + 16.99474 - 4.00150 - 13.99923 \\
 &= 0.00129 \text{ u} \\
 &= 0.00129 \times 1.6606 \times 10^{-27} \text{ kg} \\
 &= 2.142 \times 10^{-30} \text{ kg} \\
 \text{energy change} &= mc^2 \\
 &= 2.142 \times 10^{-30} \times (3.00 \times 10^8)^2 \\
 &= 1.928 \times 10^{-13} \text{ J} \\
 \text{input energy of } {}^4_2\text{He} &= 3.0000 \times 10^6 \times 1.60 \times 10^{-19} \text{ J} = 4.80 \times 10^{-13} \text{ J} \\
 \text{energy released} &= 4.80 \times 10^{-13} - 1.928 \times 10^{-13} \text{ J} \\
 &= 2.872 \times 10^{-13} \text{ J}
 \end{aligned}$$