PERTH MODERN SCHOOL
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Independent Public School

Course Methods Year 12 test one 2022

Student name: _______________ Teacher name: _______________

Task type: **Response**

Time allowed for this task: **____40_____ mins**

Number of questions: **___8_____**

Materials required: **No calculators nor classpads allowed**

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of
A4 paper.

Marks available: **__40__ marks**

Task weighting: **__10_%**

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3, 4 & 3 = 10 marks)
Differentiate the following:

a) $(3x-1)^5$

Solution
$5(3x-1)^4 3$
Specific behaviours
P correct power P uses factor of 5 P uses factor of 3 (no need to simplify)

b) $(5x^2-1)^7 3x^2$ and simplify

Solution
$(5x^2-1)^7 3x^2$ $(5x^2-1)^7 6x + 3x^2 7(5x^2-1)^6 10x$ $(5x^2-1)^6 2x\left[3(5x^2-1)+105x^2\right]$ $(5x^2-1)^6 6x\left[40x^2-1\right]$
Specific behaviours
P uses product rule P uses chain rule for bracket term P obtains a correct expression P shows a fully simplified expression

c) $\dfrac{3x+1}{\sqrt{7-2x}}$ (do not simplify)

Solution
$\dfrac{\sqrt{7-2x}(3)-(3x+1)\frac{1}{2}(7-2x)^{\frac{-1}{2}}(-2)}{7-2x}$
Specific behaviours

P uses quotient rule P correct denominator P correct numerator

Q2 (4 marks)

Determine the equation of the tangent to $y=(5x-1)(2x^3)$ at $(1,8)$

Solution
$y'=(5x-1)6x^2+10x^3$ $x=1, \quad y'=34$ $y=34x+c$ $8=34+c$ $c=-26$ $y=34x-26$
Specific behaviours
P uses product rule P determines gradient P sets up a constant and equation to solve P states tangent line

Q3 (5 marks)

Determine the coordinates of the stationary points and their nature for $y=x^3-2x^2-x+2$. Justify.

Solution
$y=x^3+2x^2+x+2$ $y'=3x^2+4x+1=(3x+1)(x+1)$ $y''=6x+4$ $y'=0, \quad x=\frac{-1}{3}, x=-1$ $x=\frac{-1}{3} \Rightarrow y''=2 \therefore local \min \quad y=\frac{-1}{27}+\frac{2}{9}-\frac{1}{3}+2=\frac{-1}{27}+\frac{6}{27}-\frac{9}{27}+\frac{54}{27}=\frac{50}{27}$ $x=-1 \Rightarrow y''=-2 \therefore local \max \quad y=-1+2-1+2=2$ $\left(\frac{-1}{3}, \frac{50}{27}\right) \& (-1,2)$
Specific behaviours

P determines first derivative P equates derivative to zero P solves for x values of both stationary pts P uses a derivative test and shows values to determine nature P determines y values of stationary pts

Q4 (3 marks)
The displacement of a body from an origin O, at time t seconds, is x metres where

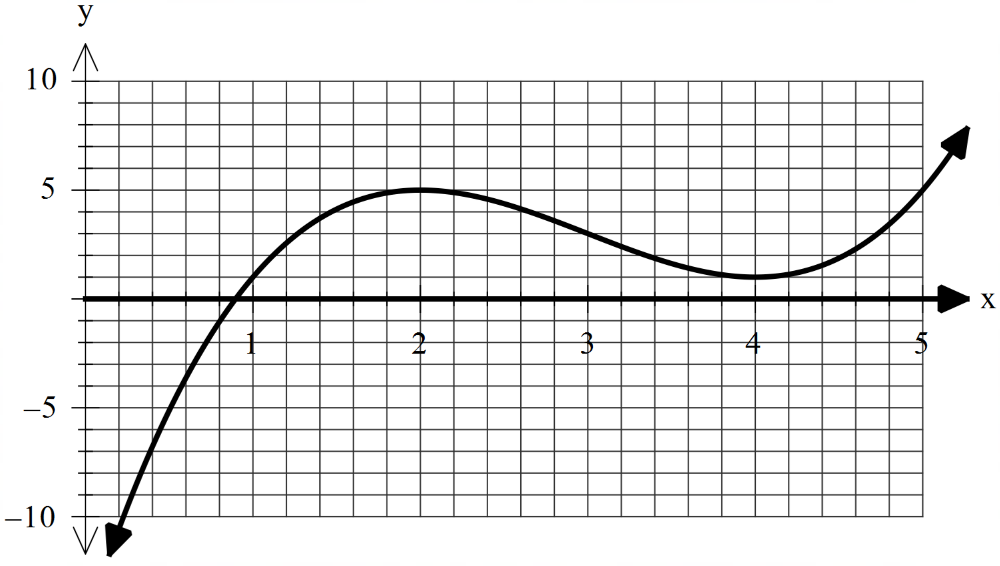
$x = t^3 - 3t^2 + 5t + 1, \quad t \geq 0$

Determine the velocity and the displacement of the body when the acceleration is zero.

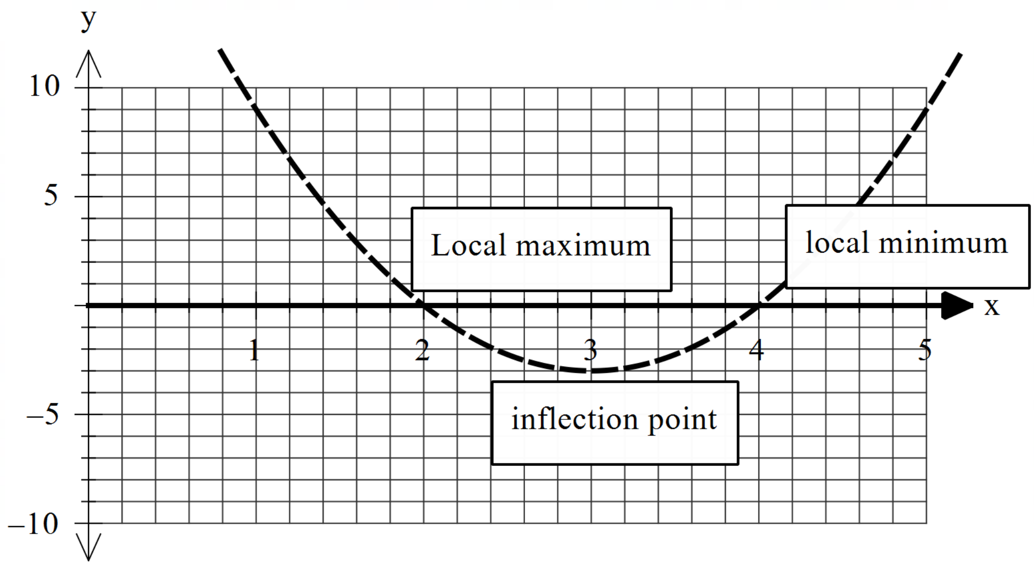
Solution
$x = t^3 - 3t^2 + 5t + 1, \quad t \geq 0$ $v = 3t^2 - 6t + 5$ $a = 6t - 6 = 0$ $t = 1$ $x = 1 - 3 + 5 + 1 = 4$ $v = 3 - 6 + 5 = 2$
Specific behaviours
P differentiates to determine velocity and acceleration P equates acceleration to zero and solves for t P states velocity and displacement for this time

Q5 (4 marks)

Consider the function $f(x)$ which is graphed below.



On the **axes below**, sketch the gradient function $f'(x)$ indicating on your sketch the location of any stationary points and any inflection points. (labelled)

Solution

Specific behaviours
P correct shape being concave up with a min turning pt (location may differ) P labels local minimum (accept min) P labels inflection pt P labels local max (accept max)

Note: No follow through if sketch is wrong as original function given & do not accept turning pt)

Q6 (2 & 3 = 5 marks)

Consider the function $y = g(x)$ where $g(2) = 10, \quad g'(2) = 5$.

a) Using the increments formula (small change) determine an approximate value for $g(2.1)$ and express this as an approximate percentage change again using the increments formula.

Solution
$\Delta y \approx \frac{dy}{dx}\Delta x = g'(2)0.1 = 0.5$ $g(2.1) \approx 10.5$
Specific behaviours
P uses increments formula P determines approx. g(2.1)

b) The volume of a sphere of radius r metres is given by $V = \frac{4}{3}\pi r^3$. Using the increments formula determine the approximate percentage change in volume for a 3% change in the radius.

Solution
$\frac{\Delta V}{V} \approx \frac{4\pi r^2 \Delta r}{\frac{4}{3}\pi r^3} = 3\frac{\Delta r}{r} = 9\%$
Specific behaviours
P sets up an expression for percentage change in volume P simplifies expression Psubs % change for r to give approx. % change in V

Q7 (4 marks)

Let A equal the number of hectares that a farmer will use to grow corn one season. The amount of corn to be harvested per hectare is given by $(800 - 20A)$ kg for $A \le 40$. **Using calculus** determine the number of hectares that should be used to maximise the amount of corn produced.

Solution

$T = A(800 - 20A) = 800A - 20A^2$ $\frac{dT}{dA} = 800 - 40A = 0$ $A = 20 \quad ha$ $\frac{dT^2}{dA^2} = -40$ $A = 20 \quad A'' = -40 \therefore local \max$
Specific behaviours
P determines expression for total amount of corn P differentiates and equates to zero P solves for A (no units required) P shows using a derivative test that this is a local max

Q8 (5 marks)

Let the cost, \$$C$, to make x items in a factory be given by $C = 3x^3 - 12x^2 + 40x$ dollars. Using calculus show that the minimum **average cost** per item is equal to the marginal cost at this number of items.

Solution
$C = 3x^3 - 12x^2 + 40x$ $Av = \frac{C}{x} = 3x^2 - 12x + 40$ $(Av)' = 6x - 12 = 0 \quad , x = 2$ $(Av)'' = 6 \therefore local \min$ $Av(2) = 12 - 24 + 40 = 28$ $M\arg inal(x) = 9x^2 - 24x + 40$ $M\arg inal(2) = 36 - 48 + 40 = 28$ QED
Specific behaviours
P determines exp for average and differentiates P equates derivative to zero and solves for x P shows with derivative test that local min P shows marginal cost formula Pshows both equal at required x value