SEMESTER TWO

MATHEMATICS
METHODS
UNITS 1 and 2

2017

SOLUTIONS

Calculator-free Solutions

1. Origin is at (2, 8)

Radius $= \sqrt{(2-6)^2 + (8-8)^2}$

$= 4$ ✓

$(x-2)^2 + (y-8)^2 = 16$ ✓ [2]

2. (a) $\sin 2x = \dfrac{\sqrt{3}}{2}$ ✓

$\therefore 2x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, -\dfrac{4\pi}{3}, -\dfrac{5\pi}{3}$ ✓

$\therefore x = \dfrac{\pi}{6}, \dfrac{\pi}{3}, -\dfrac{2\pi}{3}, -\dfrac{5\pi}{6}$ ✓

(b) cos x cannot be more than 1. ✓ [4]

3. (a) $\dfrac{dy}{dx} = \dfrac{9}{5}kx^2$ ✓✓

(b) $y = 3 - \dfrac{3x^2}{2} + \dfrac{x^{-\frac{1}{2}}}{2}$ ✓

$\dfrac{dy}{dx} = -3x - \dfrac{1}{4x^{\frac{3}{2}}}$ ✓✓

(c) $y = 4x^3 - 9x$ ✓

$\dfrac{dy}{dx} = 12x^2 - 9$ ✓ [7]

4.

$y = -2^{-x} - 1$	$y = \left(\dfrac{1}{2}\right)^x$	$y = 2^x + 3$	$y = 2^x - 1$ ✓✓✓✓ [4]
D	A	B	C

5. (a) (i)

$(2^4 \times 3^4)^{\frac{1}{4}}$

$= 6$ ✓

(ii) $\dfrac{(ab^2c^{-3})^2\sqrt{a^4b^{-2}c^6}}{a^3b^2c}$

$= \dfrac{ab}{c^4}$ ✓✓

(iii) $\dfrac{24^{\frac{2}{3}}.\sqrt{18} \times 3^{\frac{1}{3}}}{\sqrt[6]{8}\,.(27)^{\frac{1}{3}}}$

$= \dfrac{(2^3.\,3)^{\frac{2}{3}}\,3.\,2^{\frac{1}{2}} \times 3^{\frac{1}{3}}}{(2^3)^{\frac{1}{6}}\,3}$ ✓✓

$= 2^2 \times 3$

$= 12$ ✓

(b) (i) $\dfrac{2^{4x-4}}{2^{-4}} = 2^{-3}$ ✓

$2^{4x} = 2^{-3}$ ✓

$4x = -3$

$x = -\dfrac{3}{4}$ ✓

(ii) $(2^x - 8)(2^x - 1) = 0$

$2^x = 8$ or $2^x = 1$ ✓

$x = 3$ or $x = 0$ ✓ [11]

6. (a) (i) 0.2 ✓

(ii) 0.4 ✓

(iii) 0.7 ✓

(b) $^5\mathbf{C}_2 \times 2^3 \times 3^2 = 720$ ✓✓ [5]

7. (a) (i) $\displaystyle\int x^2 + 1dx = \frac{x^3}{3} + x + c$ ✓✓

(ii) $\displaystyle\int t^{-2} - 2t + \pi t^{-3}\,\mathrm{d}t$ ✓

$= -\dfrac{1}{t} - t^2 - \dfrac{\pi}{2t^2} + c$ ✓✓

(b) $y = 3x + \dfrac{x^2}{2} - \dfrac{2x^5}{5} + c$ ✓

$2 = 3 + \dfrac{1}{2} - \dfrac{2}{5} + c$

$\therefore c = -\dfrac{11}{10}$ ✓

$y = -\dfrac{2x^5}{5} + \dfrac{x^2}{2} + 3x - \dfrac{11}{10}$ ✓ [8]

8. (a) $\dfrac{64}{2}(4 - b) = 0$

$b = 4$ ✓

(b) $p(x) = \dfrac{x^4}{2} - 2x^3$

$p'(x) = 2x^3 - 6x^2 = 0$ ✓

$2x^2(x - 3) = 0$

$x = 0$ or $x = 3$ ($x = 0$ is not a minimum) ✓

When $x = 3$ $y = \dfrac{3^4}{2} - 2(3^3)$

$\left(3, -\dfrac{27}{2}\right)$ ✓

(c) Stationary point at $x = 0$, $p'(0) = 0$

$p''(x) = 6x^2 - 12x = 0$ ✓

$6x(x - 2) = 0$

$x = 0, 2$

$\therefore x = 0$ $p'(x) = p''(x) = 0$ ✓ [6]

Horizontal point of inflection

Or sign table:

	$x < 0$	$x = 0$	$0 < x < 3$	$x = 3$	$x > 3$

$p(x)$	↓	↔	↓	↔	↑
$p'(x)$	-	0	-	0	+

9. (a) $a_1 = 3$
$a_2 = 6$
$a_3 = 10$ ✓✓
(b) $T_{n+2} = T_n + T_{n+1}$ $T_1 = 1$ $T_2 = 1$ ✓ [3]

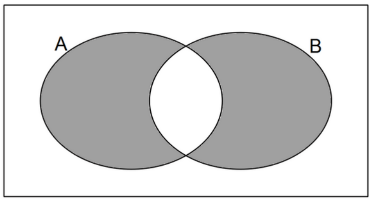
<u>Calculator-assumed Solutions</u>

10. (a) $x^2 = 4x^2 + 9 - 2(2x)(3)\cos Q$
$\dfrac{-3(x^2+3)}{-12x} = \cos Q$ ✓
$\cos Q = \dfrac{x^2+3}{4x}$ ✓
(b) $\cos Q = 0.9125$ ✓
$Q = 24.1468°$ ✓
Area $= \dfrac{1}{2}(3)(4.8)\sin 24.1468$
$= 2.945$ units^2 ✓ [5]

11. (a) $-\dfrac{b}{2a} = -\dfrac{2}{2a} = -\dfrac{1}{a}$
$y\left(-\dfrac{1}{a}\right) = a - \dfrac{1}{a}$ or $\dfrac{a^2-1}{a}$
Turning point $\left(-\dfrac{1}{a}, \dfrac{a^2-1}{a}\right)$ ✓✓

(b) $b^2 - 4ac > 0$
$4 - 4a^2 > 0$ ✓
$a^2 < 1$
$-1 < a < 1$ ✓ [4]

12. (a)



✓

(b) (i) 0.1 ✓
(ii) 0.2 ✓
(iii) 0.3 ✓
(iv) 0.9 ✓ [5]

13. (a) $C(t) = \dfrac{t^3}{3} - \dfrac{t^2}{2} - 12t + 105$ ✓
$C(2) = 81.7$ cents ✓
(b) $(t-4)(t+3) = 0$
$t = 4$ weeks ✓
$C(4) = 70.3$ cents ✓
(c) Week 6.8 therefore during week 7 ✓ [5]

14. (a) $d = k\sqrt{h}$

$4665 = k\sqrt{1.7}$

$\therefore k = 3577.89 \approx 3578$ ✓

$d = 3577.89\sqrt{(1.76 + 85)}$

$d = 33\,326.3\ m$ ✓

$d \approx 33\ km$ ✓

(b) $d = 3577.89\sqrt{0.8h}$ ✓

$d = 3577.89\ (0.8944)\sqrt{h}$

d decreases by 10.6%. ✓ [5]

15. (a) (i) $T_{n+1} = T_n + \sqrt{2}$ $T_1 = 3 - \sqrt{2}$ ✓✓

(ii) $T_{n+1} = 3\,T_n$ $T_1 = -2$ ✓✓

(b) $T_n = 17 + (n - 1)(3)$ ✓

$T_{25} = 17 + (24)(3)$

$= 89$ ✓ [6]

16. (a)

$h \to 0$	Limit =
0.1	3.05
0.01	3.005
0.0001	3.00005
0.000001	3.0000005

✓✓

$\therefore \lim\limits_{h \to 0} \dfrac{f(3 + h) - f(3)}{h} = 3$ ✓

(b) $f(x) = 3x^2$ ✓

$\therefore f'(x) = 6x$ ✓

(c) $4x + k = 0$ ✓

$4(3) + k = 0$

$\therefore k = -12$ ✓

(d) $y = \int 4x + 1\ dx$

$\therefore y = 2x^2 + x + c$ ✓

$2(1)^2 + 1 + c = -2$

$c = -5$

$y = 2x^2 + x - 5$ ✓ [9]

17. (a) $\dfrac{\sqrt{1.44 \times 10^6}}{(2 \times 10^{-2})^4}$

$= \dfrac{1.2 \times 10^3}{1.6 \times 10^{-7}}$ ✓

$= 0.75 \times 10^{10}$ ✓

$= 7.5 \times 10^9$ ✓

(b) (i) $1.5 \times 10^4 = 1200(r)^6$

$P_0 = 1200$ ✓

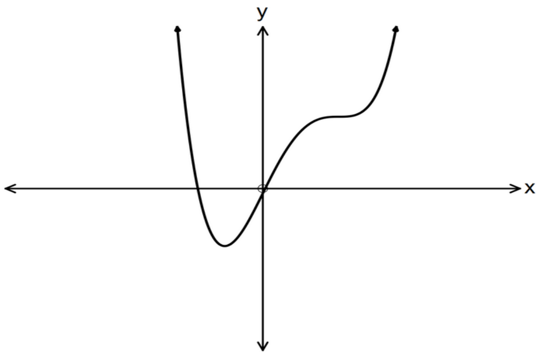
$r = 1.5234$ ✓

(ii) $P = 187500$ ✓

(iii) $t = 15.987$ ✓

15 hours and 59 minutes. ✓ [8]

18. (a)



✓✓✓

(b)

A	B	C
(iii)	(ii)	(i)

✓✓✓ [6]

19. (a) (i) $n = 13$
$T_{13} = 26 + (12)(2)$ ✓
$= 50$ seats ✓
(ii) $n = 23$ Row W ✓
(iii) $n = 24$
Sum of seats = 1176 ✓

(b) $205 = 5(2a + 9d)$
$710 = 10(2a + 19d)$ ✓
$a = 7 \quad d = 3$ ✓
$S_{30} = 15(14 + 29(3))$
$= 1515$ ✓

(c) $a = 254 \quad d = -3 \quad T_n = 176$
$176 = 254 + (n - 1)(-3)$ ✓
$n = 27$ ✓
$S_n = \dfrac{27}{2}(254 + 176)$
$= 5805$ ✓ [10]

20. (a) $V(x) = 3x(90 - 3x)\left(\dfrac{x}{3}\right)$ ✓
$= x^2(90 - 3x)$ ✓
$= 90x^2 - 3x^3$

(b) $V'(x) = 180x - 9x^2$
$9x(20 - x) = 0$ ✓
$x = 0$ or $x = 20$ ✓
$V''(x) = 180 - 18x \quad V''(20) < 0 \therefore$ Maximum ✓
Maximum volume is 12000 m^3
When length = 60 m, width = 30 m and depth= 6.67 m ✓ [6]

21. (a) (i) $A_1 = \frac{3}{2}$ $A_2 = \frac{9}{2}$ $A_3 = \frac{27}{2}$ ✓

(ii) $A_{n+1} = 3\,A_n$ $A_1 = \frac{3}{2}$ ✓✓

(iii) $S_{25} = 6.355 \times 10^{11}$ ✓✓

(b) (i) $T_n = 2\,T_{n-1}$ $T_0 = 6$

$T_{12} = 24\,576$ users ✓✓

(ii) After 18 months ✓✓

(Or during the 18th month)

(iii) $\frac{6000000}{1.15} = 5\,217\,391$ users ✓

During the 20th month ✓

(c) $S_\infty = \frac{a}{1-r}$

$= \frac{45}{100} \div \left(1 - \frac{1}{100}\right)$ ✓

$= \frac{45}{99}$

$= \frac{5}{11}$ ✓ [13]

22. (a) $s = -\frac{t^3}{2} + t^2 + 20t$. $s(3) = 55.5\ m$ ✓

(b) $\frac{55.5}{3} = 18.5\ m/s$ ✓

(c) $v = -\frac{3t^2}{2} + 2t + 20$ ✓

$v(2) = 18\ m/s$ ✓

(d) $-\frac{3t^2}{2} + 2t + 20 = 0$ ✓

$t = 4.38\ s$ ✓

Max displacement: $s(4.38) = 64.8\ m$ ✓

(e) $s(8) = -32$ ✓

Total distance = 64.8 + 64.8 + 32 = 161.6 m ✓ [9]

23. (a) $g'(x) = 2x - 6$

$m = \frac{1}{3} \therefore 2x - 6 = \frac{1}{3}$ ✓

$x = \frac{19}{6} = 3\frac{1}{6}$ ✓

$y = \frac{1}{36}$ $\left(\frac{19}{6}, \frac{1}{36}\right)$ ✓

(b) (i) $f'(x) = x^2 + 2ax + b$

$36 - 12a + b = 0$ ✓

$-72 + 36a - 6b = 0$ ✓

Solving simultaneously $a = 4$ $b = 12$ ✓

(ii) $x^2 + 8x + 12 = 0$

$x = -6$ or $x = -2$ ✓

Turning point $\left(-2, -\frac{32}{3}\right)$ ✓

$f''(x) = 2x + 8$ $f''(-2) > 0$ Therefore Local Minimum ✓ [9]

End of Questions