

Note: All part questions worth more than 2 marks require working to obtain full marks.

Formula sheet provided: Yes

Task weighting: 10 %

Marks available: 44 marks

Special items: Drawing instruments, templates, one page of A4 notes
double-sided

Standard items: Pens (blue/black preferred), pencils (including coloured),
sharpener, correction fluid/tape, eraser, ruler, highlighters

Materials required: Up to 3 calculators/classpads allowed

Number of questions: 6

Time allowed for this task: 40 mins

Task type: Response

Student name: _____ Teacher name: _____

COURSE METHODS YEAR 12 TEST FOUR 2022



Q1 (4 marks) 4.2.5

The exam results, out of a 100, for a Methods exam at a particular school was found to be Normally Distributed. It was found that 21% of the students scored a result greater than 85 and 17% scored a mark less than 55. Determine the mean and standard deviation.

Solution

```

Edit Action Interactive
0.5 1
invNormCDF("R", 0.21, 1, 0)
0.806421247
invNormCDF("L", 0.17, 1, 0)
-0.9541652531

```

$$\begin{cases} 0.8064 = \frac{85-\mu}{\sigma} \\ -0.9542 = \frac{55-\mu}{\sigma} \end{cases} \quad \mu, \sigma$$

$$\{\mu=71.25922981, \sigma=17.03964558\}$$

Specific behaviours

- ✓ determines both z scores
- ✓ sets up one equation for mean and stdev
- ✓ sets up two equations for mean and stdev
- ✓ solves for mean and stdev

Note- answer only max of 2 marks

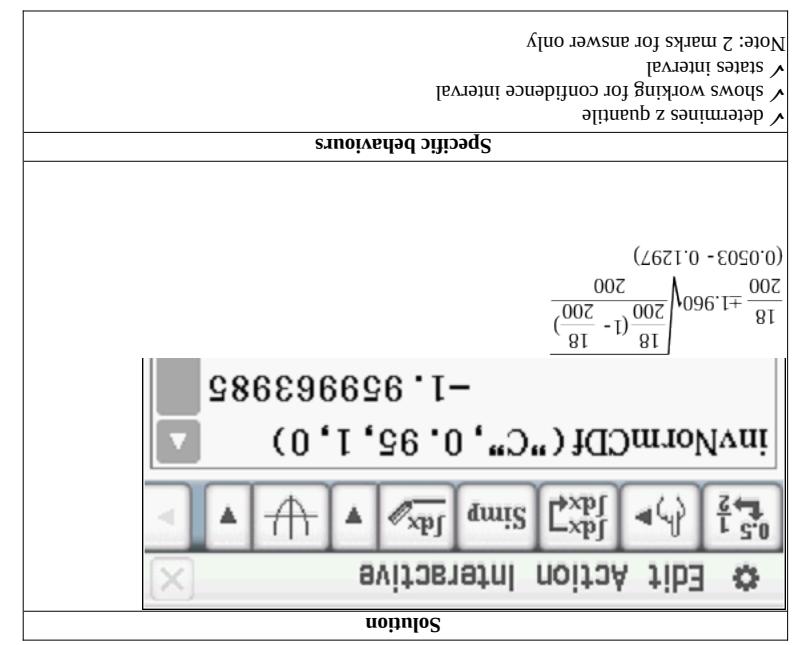
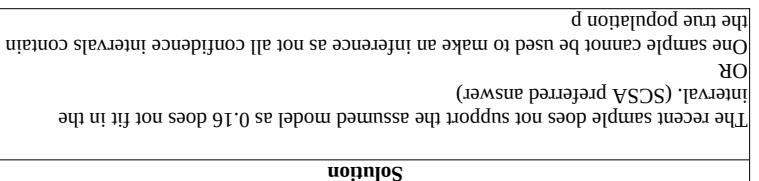
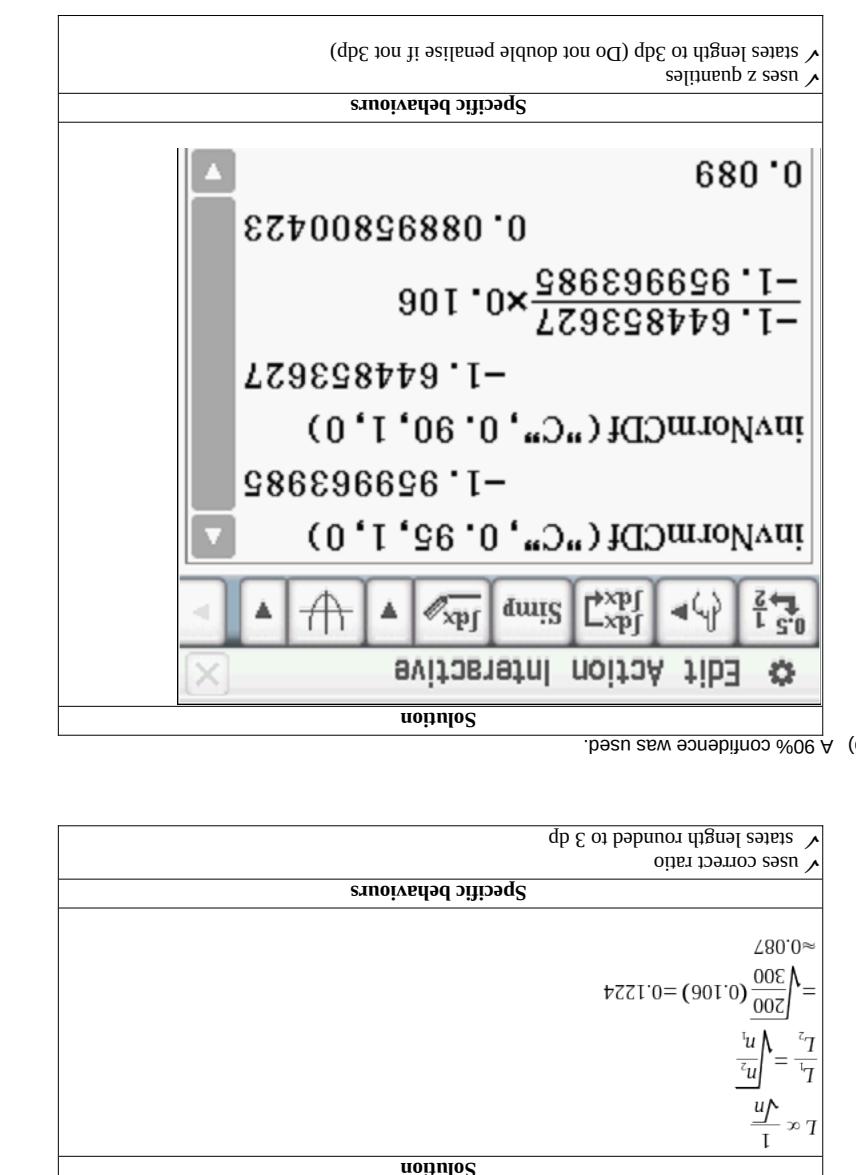
Q2 (2, 2, 3 & 2 = 9 marks) 4.3.8

A 95% confidence interval was determined for the proportion of faulty factory parts made at a company. The interval length is 0.106 and the sample size of 200.

Determine the **expected length** of the interval for each change in isolation to 3 decimal places.

Specific behaviours

- ✓ states that support cannot be made with any reason
- ✓ one of the reasons stated above



- c) A 88% confidence AND a sample size of 150 was used.

Solution
<code>invNormCDf("C", 0.88, 1, 0)</code> -1.554773595
$L \propto z \frac{1}{\sqrt{n}}$ $L_1 = z_1 \sqrt{n_1}$ $L_2 = z_2 \sqrt{n_2}$ $= \frac{1.5548}{1.960} \sqrt{\frac{200}{150}} (0.106) = 0.1224$ ≈ 0.097
Specific behaviours
<input checked="" type="checkbox"/> states quantile for 0.88 <input checked="" type="checkbox"/> shows ratio involving sample sizes <input checked="" type="checkbox"/> states length

- d) The true proportion of faulty parts does not lie in the stated interval. Does this suggest a sampling error was made? Justify.

Solution
Cannot tell if there is a sampling error as not all confidence intervals contain the true value of population proportion p.
Specific behaviours
<input checked="" type="checkbox"/> states no with any reason <input checked="" type="checkbox"/> states reason as above (Note – zero marks if no without any reason)

Edit Action Interactive
$\text{normCDF}\left(0.21, \infty, \sqrt{\frac{0.16 \cdot (1-0.16)}{300}}, 0.16\right)$ 9.081509174E-3
Specific behaviours
<input checked="" type="checkbox"/> uses correct parameters <input checked="" type="checkbox"/> states prob

Q4 cont-

In a recent survey it was found that 18 people out of a sample of 200 were left handed.
 c) For a 99% confidence interval, what is the margin of error based on this recent sample of 200?

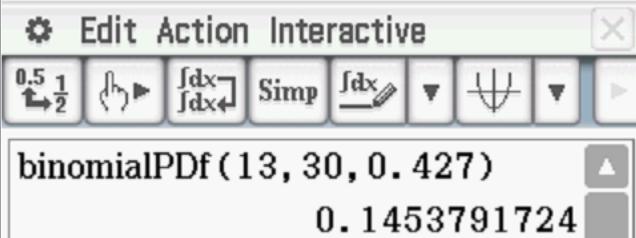
Solution
<code>invNormCDf("C", 0.99, 1, 0)</code> -2.575829304 $2.575829304 \sqrt{\frac{\frac{18}{200} \left(1 - \frac{18}{200}\right)}{200}}$ 0.05212475572
Specific behaviours
<input checked="" type="checkbox"/> determines z quantile

Mathematics Department		Perth Modern																					
Q3 (2, 2, 3, 3 & 3 = 18 marks) 4.2.3, 3.3.1, 3.3.6, 3.3.7		A parcel making factory makes boxes of the same width and heights but the lengths vary and are found to be Normally Distributed with a mean of 135 mm and a standard deviation of 27 mm.																					
a) Determine the percentage of boxes that are longer than 166mm.		s = 0.71 $(3.21 - 5.57)^2 \times 0.00043 + (4.12 - 5.57)^2 \times 0.097 + (5.20 - 5.57)^2 \times 0.476 + (6.30 - 5.57)^2 \times 0.427 = s^2$																					
Q4 (3, 2, 3 & 2 = 13 marks) 4.3.4, 4.3.5, 4.3.6, 4.3.9, 4.3.10		In Australia it has been found that 16% of people are left-handed. Samples of people are surveyed to ascertain the proportion that are left-handed. Let p denote the proportion of people in the sample who are left handed.																					
		a) State the approximate distribution of p for sample sizes of 100. $p \sim N\left(0.16, \sqrt{\frac{0.16(1-0.16)}{100}}\right)$																					
The boxes can be classified as the following.		a) Complete the missing probabilities in the above table.																					
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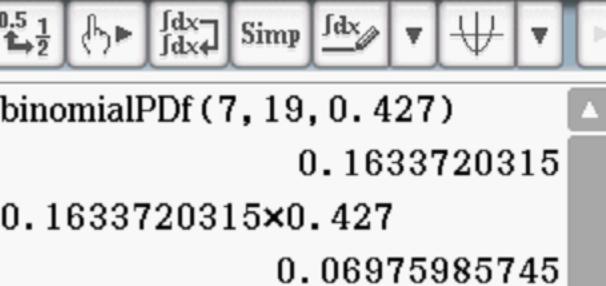
- ✓ discusses negative values
- ✓ discusses unlimited positive values

- d) If 30 boxes were taken off the assembly line, determine the probability that exactly 13 were gigantic lengths.

Solution
$X \sim \text{Bin}(30, 0.427)$ $P(X = 13)$

Specific behaviours
<ul style="list-style-type: none"> ✓ states Binomial with parameters ✓ uses $x=13$ ✓ states prob (no need to round)

- e) Determine the probability that it would take 20 boxes in a row off the assembly line before 8 gigantic boxes were found.

Solution
$X \sim \text{Bin}(19, 0.427)$ $P(X = 7) \times 0.427$

Edit Action Interactive


- Specific behaviours
- ✓ states Binomial with $n=19$
 - ✓ determines prob for $x=7$
 - ✓ states total prob (no need to round)

Continued on next page

Q3 cont

If the costs of each box were determined as follows.

Box	short	long	Very long	gigantic
Cost \$	\$3.21	\$4.12	\$5.20	\$6.30

- i) Determine the mean cost to two decimal places. Show all working.

Solution
$3.21 \times 0.00043 + 4.12 \times 0.097 + 5.20 \times 0.476 + 6.30 \times 0.427 = 5.57$
Specific behaviours
<ul style="list-style-type: none"> ✓ multiplies each x score by prob ✓ shows total series ✓ states mean to 2 dp

- ii) Determine the standard deviation to two decimal places. Show all working.

Solution
