

No other items may be taken into the examination room. It is **Your** responsibility to ensure that you do not have any unauthorised material with you, hand it to the supervisor **before** reading any room. If you have any unauthorised notes or other items of a non-personal nature in the examination room, if you have any unauthorised calculators approved for use in the WACE examinations, and up to three calculators approved for use in the WACE examinations.

Important note to candidates

Special items: drawing instruments, templates, ruler, highlighters and up to three calculators approved for use in the WACE examinations.

Standard items: pens(blue/black preferred), pencils(including coloured), sharpener, correction fluid, erasers, ruler, highlighters

To be provided by the supervisor
Formula Sheet (retained from Section One)
This Question/Answer booklet

To be provided by the supervisor
Working time for paper:
one hundred minutes

Reading time before commencing work:
ten minutes

Material required/recommended for this section

Working time for paper:
one hundred minutes

Teacher's Name:

Sowthon

Student Name:

Calculator-assumed
Section Two:

MATHEMATICS
METHODS UNIT 3

Question/Answer Booklet
Semester One Examination 2018



Structure of this paper

	Number of questions available	Number of questions to be attempted	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	9	9	50	52	35
Section Two Calculator-assumed	13	13	100	96	65
					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2018*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Section Two: Write answers in this Question/Answer Booklet. Answer **all** questions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

(1a)

$$\% \text{ Error} = 0.8153 \quad \checkmark$$

$$\% \text{ Error} = \frac{0.889 - 0.8153}{0.8153} \times 100$$

$$K\left(\frac{\pi}{4}\right) = 5.6569$$

- (b) Determine the exact increase in K and hence determine the percentage error in your approximation from (a). Give your answer to one decimal place. (2 marks)

$$= \frac{\sqrt{2}}{2} \text{ or } 0.889 \quad \checkmark$$

$$8K = 8 \cos\left(\frac{\pi}{4}\right)\left(\frac{\pi}{20}\right)$$

$$8K = \frac{dK}{d\theta} \times 8\theta \quad \leftarrow \quad 8K = 8 \cos\left(\frac{\pi}{20}\right)$$

$$\frac{dK}{d\theta} = 8 \cos\theta \text{ at } r=4, \quad 8\theta = \frac{\pi}{20} \text{ or } 0.05\pi$$

- (a) Use the incremental formula to approximate the increase in K , as θ changes from $\frac{\pi}{4}$ to 0.3π in a circle of radius 4 cm. $8\theta = 0.3\pi - \frac{\pi}{4}$ (3 marks)

and θ is the central angle.

A sector of a circle has area K , given by $K = \frac{1}{2}r^2 \sin \theta$, where r is the radius of the circle,

Question 10 (5 marks)

Working time: 100 minutes

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Counting an answer: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
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Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

This section has thirteen (13) questions. Attempt all questions. Write your answers in the spaces provided.

96 marks

Section Two: Calculator-Assumed

Question 11 (7 marks)

A curve has equation $y = ax^3 - bx^2 + cx - 9$.

There is a stationary point at $(-3, 0)$.

There is a point of inflection at $x = -\frac{5}{3}$.

Determine a , b and c . Show your working.

$$\text{at } (-3, 0) \rightarrow -27a - 9b - 3c - 9 = 0 \quad \checkmark$$

$$\frac{dy}{dx} = 3ax^2 - 2bx + c \quad \checkmark$$

$$\text{stationary point at } x = -3 \rightarrow \frac{dy}{dx} = 0$$

$$27a + 6b + c = 0 \quad \checkmark$$

$$\frac{d^2y}{dx^2} = 6ax - 2b \quad \checkmark$$

$$\text{p.o.i at } x = -\frac{5}{3} \rightarrow \frac{d^2y}{dx^2} = 0$$

$$-10a - 2b = 0 \quad \checkmark$$

$$\begin{array}{l} \textcircled{1} \sim 27a - 9b - 3c - 9 = 0 \\ \textcircled{2} \quad 27a + 6b + c = 0 \\ \textcircled{3} \quad -10a - 2b = 0 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Solve with classpad}$$

$$\therefore a = 1, b = -5, c = 3 \quad \checkmark$$

(7 marks)

Question 22 (9 marks)

A drop of oil is spreading on a glass surface. The region covered is circular in shape, and the radius, r cm, of the circle is given as a function of time, t seconds.

$$r = -e^{-t} + 4$$

(a) Find the rate at which the radius is increasing when:

$$(i) \quad t = 4. \quad r' = e^{-t} \quad \checkmark$$

$$r'(4) = e^{-4} = 0.018 \text{ cm/s} \quad \checkmark$$

$$(ii) \quad t = 5.$$

$$r'(5) = e^{-5} = 0.0067 \text{ cm/s} \quad \checkmark$$

(b) Use your knowledge of exponential functions and its graph, or derivatives, to determine when the radius is increasing at its fastest rate.

(2 marks)

the graph of $r'(t)$ is decreasing \checkmark

or the graph of $r''(t)$ has the greatest slope at $t=0$.

\therefore the radius is increasing at its highest rate at $\underline{\underline{t=0}} \checkmark$

(c) At what rate is the area of the circle increasing when $t = 4$?

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r \quad (\frac{dr}{dt} = e^{-t} \text{ from (a)})$$

$$\begin{aligned} \frac{dA}{dt} &= \frac{dA}{dr} \times \frac{dr}{dt} \\ &= 2\pi r \times e^{-t} \end{aligned} \quad \checkmark$$

Other Method: (4 marks)

$$A = \pi r^2$$

$$A = \pi(-e^{-t} + 4)^2 \quad \checkmark$$

$$\frac{dA}{dt} = 2\pi(-e^{-t} + 4)(e^{-t}) \quad \checkmark$$

at $t=4$ \checkmark sub

$$\frac{dA}{dt} = 0.458 \text{ cm}^2/\text{s} \quad \checkmark$$

$$\text{at } t=4 \rightarrow \frac{dA}{dt} = 2\pi(-e^{-4} + 4)(e^{-4}) = 0.458 \text{ cm}^2/\text{s}$$

\checkmark

END OF QUESTIONS

$$\text{What time is } x = \frac{\pi}{2}$$

$$\downarrow$$

$$\checkmark \quad \frac{\pi}{2} \quad \therefore x = \frac{\pi}{2}, \quad \frac{\pi}{2}$$

$$\frac{\pi}{2} = \frac{\pi}{4}, \quad \frac{\pi}{4}$$

$$\cos \frac{\pi}{2} = \frac{\pi}{2}$$

$$-4 \cos \frac{\pi}{2} + 4 = 4 - 2\sqrt{2} \quad \checkmark$$

$$\checkmark \quad = 1000 L$$

$$\int_{200}^0 (10 - \frac{dt}{2}) dt$$

(c) How much gas flows in total after the pipe is closed?

(2 marks)

~~at 3 mins do sqa~~

$$\checkmark \quad t = 300 \text{ seconds}$$

$$10 - \frac{t}{20} = 0$$

(b) How many seconds does it take till the flow stops?

(1 mark)

(b) State the next value of x where the graph will have the same height (ie. $4 - 2\sqrt{2}$). Explain your reasoning.

$$\therefore f(x) = -4 \cos \frac{x}{2} + 4 \quad \checkmark$$

$$C = 4$$

$$-4(\frac{\pi}{2}) + C = 4 - 2\sqrt{2}$$

$$f(\frac{\pi}{2}) = -4 \cos \frac{\pi}{4} + C = 4 - 2\sqrt{2} \quad \checkmark$$

$$\checkmark \quad f(x) = \int 2 \sin \frac{x}{2} dx = -4 \cos \frac{x}{2} + C$$

$$\text{if } t=0 \leftarrow \frac{dF}{dt} = 10 \text{ L/sec} \quad \checkmark$$

(a) What is the initial rate of flow?

(1 mark)

$$\text{is given by } \frac{dF}{dt} = 10 - \frac{t}{20}.$$

(a) Determine the equation of f given that it passes through the point $(\frac{\pi}{2}, 4 - 2\sqrt{2})$. (3 marks)

A long gas pipe is closed, but gas continues to flow out. The rate of flow $\frac{dF}{dt}$, in litres per second,

Question 13 (4 marks)

The exponential series can be expressed as:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

- (a) Use the first five terms of the series to evaluate e as a fraction. (Hint: $e = e^1$) (2 marks)

$$\begin{aligned} e^1 &= 1 + \frac{1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} \\ &= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} \\ &= \frac{65}{24} \text{ or } 2\frac{17}{24} \quad \checkmark \end{aligned}$$

- (b) Use the series to show that the derivative of e^x is e^x . (2 marks)

$$\begin{aligned} \frac{d}{dx} e^x &= \frac{d}{dx} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots \right) \\ &= 0 + 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \quad \checkmark \\ &= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \\ &= e^x \quad \checkmark \end{aligned}$$

See next page

Question 20 (10 marks)

Consider f where $f(x) = \cos x + \sin x$ where $0 \leq x \leq 2\pi$.

- (a) Determine:

- (i) $f'(x)$.

$$f'(x) = -\sin x + \cos x \quad \checkmark$$

(1 mark)

- (ii) $f''(x)$.

$$f''(x) = -\cos x - \sin x \quad \checkmark$$

(1 mark)

- (b) State the exact maximum value of f over the given domain. Prove it is the maximum using the second derivative test.

(4 marks)

Maximum when $f'(x) = 0$

$$-\sin x + \cos x = 0$$

$$x = \frac{\pi}{4} \rightarrow f(x) = \sqrt{2} \quad \checkmark$$

$$f''\left(\frac{\pi}{4}\right) < 0 \therefore \text{maximum} \quad \checkmark$$

- (c) Identify any points of inflection over the domain. Give the exact answer.

(4 marks)

P.o.i when $f''(x) = 0$

$$-\cos x - \sin x = 0 \quad \checkmark$$

$$x = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4} \quad \checkmark$$

$$\therefore \left(\frac{3\pi}{4}, 0\right) \text{ and } \left(\frac{7\pi}{4}, 0\right) \quad \checkmark$$

See next page

* accept 109

$$\text{y-intercept} = (0, 109)$$

$$c = 109$$

$$a + (6, 1) \leftarrow 1 = 216 - 324 + c$$

$$M(x) = x^3 - 9x^2 + c$$

$$M(x) = 3x^2 - 18x$$

(4 marks)

(b) Determine the value of the y -intercept on the graph of M .

(3 marks)

(b) When would the tree cover an area of 5 ha?

$$S = 0.6 e^{0.05t}$$

$$\therefore t = 42.4 \text{ hours}$$

$$t = 42 \text{ hours (nearest hour)}$$

(a) Calculate the value of k . (2 marks)

A stationary point occurs at (6, 1).

A function $M(x)$ has $M'(x) = 3x^2 - kx$, where k is a constant.

Question 14 (6 marks)

(a) What will be the area of the triangle after it was discovered?
Give your answer correct to two decimal places.
(3 marks)

A forest fire is estimated to be spreading at a rate of 5% per hour. The area, A , in ha, covered by the fire at any time, t , in hours since the fire was discovered, is defined by $\frac{dA}{dt} = 0.05A$. At the moment of discovery, the area covered was 0.6 ha.

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CALCULATOR-ASSUMED

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MATHEMATICS METHODS UNIT 3

MATHEMATICS METHODS UNIT 3

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Question 19 (6 marks)

Question 15 (15 marks)

75% of adults in a certain town graduated from high school.

- (a) Thirty adults are randomly selected using the council records of the town.

- (i) Calculate the probability that twenty five of these adults graduated from high school. State the probability distribution, and any parameters associated with that distribution.

$$X \sim \text{Bin}(30, 0.75) \quad \checkmark$$

(3 marks)

$$P(X=25) = 0.1047 \quad \checkmark$$

- (ii) Given that at least twenty five graduated, find the probability that more than twenty eight graduated.

(3 marks)

$$\frac{P(X \geq 29)}{P(X \geq 25)} = \frac{0.00196}{0.20260} \quad \checkmark$$

$$= 0.0097 \quad \checkmark$$

- (b) How many adults need to be randomly selected so that the probability that at least ten graduated is at least 99%?

(3 marks)

$$Y \sim \text{Bin}(n, 0.75) \quad \checkmark$$

$$P(Y \geq 10) \geq 0.99 \quad \checkmark$$

$$n = 19 \quad \checkmark$$

(trial and error on classpad)

The particle has a displacement, x metres, from the origin O on the line. It is initially at the origin.

- (d) Determine the displacement when $t = 3$.

(3 marks)

$$x = \int v dt$$

$$x = \frac{2}{3}t^3 - \frac{5}{2}t^2 + 3t + C$$

$$\text{at } t=0 \rightarrow x(0) = 0 \rightarrow C = 0 \quad \checkmark$$

$$x = \frac{2}{3}t^3 - \frac{5}{2}t^2 + 3t \quad \checkmark$$

$$x(3) = 4.5 \text{ m} \quad \checkmark$$

- (e) Determine the distance travelled in the first 3 seconds to the nearest cm.

(2 marks)

$$\text{Distance} = \int_0^3 |v(t)| dt$$

$$= \int_0^3 |2t^2 - 5t + 3| dt \quad \checkmark$$

$$= 4.58 \text{ m} \quad \checkmark$$

- (f) Explain why the answers to (d) and (e) are different.

(1 mark)

Distance travelled \neq Displacement \checkmark

OR because the particle stops and changes direction at $t=1$ and $t=1.5$.

Question 16 (6 marks)

- (a) Solve
- $\sin 2x + \cos x = 0$
- for
- $0 \leq x \leq 2\pi$
- .

(2 marks)

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

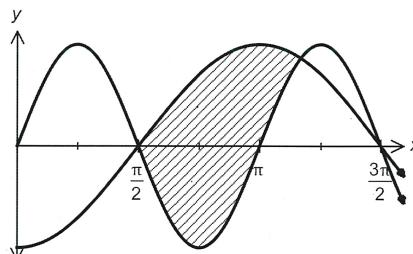
*solve on classpad

Hence, or otherwise,

- (b) determine the points of intersection of
- $y = \sin 2x$
- and
- $y = -\cos x$
- for
- $0 \leq x \leq 2\pi$
- . (2 marks)

$$\left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right), \left(\frac{7\pi}{6}, \frac{\sqrt{3}}{2}\right), \left(\frac{11\pi}{6}, -\frac{\sqrt{3}}{2}\right)$$

- (c) The graphs of
- $y = \sin(2x)$
- and
- $y = -\cos(x)$
- are drawn below. Determine the area of shaded region. (2 marks)



$$A = \int_{\frac{\pi}{2}}^{\frac{7\pi}{6}} (-\cos x - \sin 2x) dx$$

$$= 2.25 \text{ units}^2$$

$$\text{OR } \frac{9}{4} \text{ units}^2$$

See next page

Question 17 (5 marks)

Consider the function $y = \frac{x+1}{x-1}$

- (a) Determine the x-intercept(s) of the function.

(1 mark)

$$x = -1 \quad \text{or} \quad (-1, 0)$$

- (b) (i) Determine
- $\frac{dy}{dx}$
- .

(1 mark)

$$\frac{dy}{dx} = -\frac{2}{(x-1)^2}$$

- (ii) Prove the conjecture; "This function has no turning points."

(1 mark)

$$\frac{dy}{dx} = -\frac{2}{(x-1)^2} \neq 0 \text{ for } x \in R$$

$\therefore \text{True}$

- (c) (i) Determine
- $\frac{d^2y}{dx^2}$
- .

(1 mark)

$$\frac{d^2y}{dx^2} = \frac{4}{(x-1)^3}$$

- (ii) Prove the conjecture; "This function has no points of inflection."

(1 mark)

$$\frac{d^2y}{dx^2} = \frac{4}{(x-1)^3} \neq 0 \text{ for } x \in R$$

$\therefore \text{True}$

See next page