ALL SAINTS' COLLEGE

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 21 **(5 marks)**

(a) Determine the value of the constant a and the value of the constant b that make each of the following statements true, given that $f(x)$ is a polynomial:

(i) $\int_a^1 f(x)dx+\int_1^b f(x)dx=\int_{-3}^2 f(x)dx$. (1 mark)

Solution
$a=-3, b=2$
Specific behaviours
✓ correct values

(ii) $\int_0^2 f(x)dx-\int_1^2 f(x)dx+\int_{-1}^0 f(x)dx=\int_a^b f(x)dx$. (2 marks)

Solution
$a=-1, b=1$
Specific behaviours
✓ value of a
ü value of b

(b) Show that $\frac{d}{dx}\left(\int_{u(x)}^{v(x)} f(t)dt\right)=f(v(x))v'(x)-f(u(x))u'(x)$. (2 marks)

Solution
$\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)dt\right]=\frac{d}{dx}\left[\int_{u(x)}^{0} f(t)dt\right]+\frac{d}{dx}\left[\int_{0}^{v(x)} f(t)dt\right]$ ¿ $\frac{d}{dx}\left[\int_{0}^{v(x)} f(t)dt\right]-\frac{d}{dx}\left[\int_{0}^{u(x)} f(t)dt\right]$ ¿ $f(v(x))v'(x)-f(u(x))u'(x)$
Specific behaviours
✓ uses additivity to split integral   ü correctly uses fundamental theorem

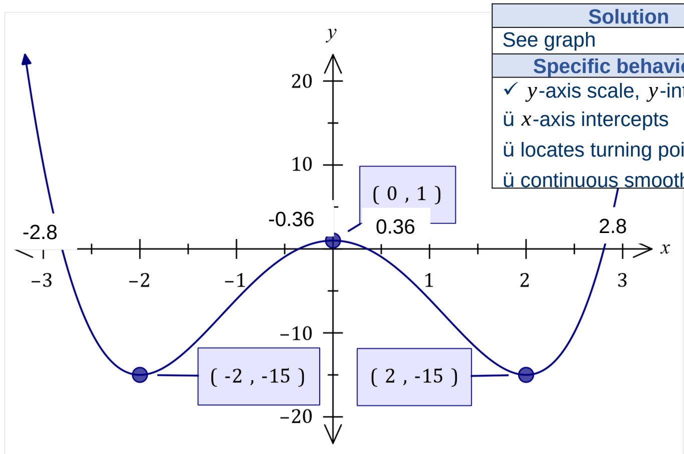
Alternative Solution
Let $F(t)$ be an antiderivative of $f(t)$ so that $F'(t)=f(t)$.   Then $\int_{u(x)}^{v(x)} f(t)dt=[F(t)]_{u(x)}^{v(x)}$ ¿ $F(v(x))-F(u(x))$ Hence $\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)dt\right]=\frac{d}{dx}[F(v(x))-F(u(x))]$ ¿ $F'(v(x))v'(x)-F'(u(x))u'(x)$ ¿ $f(v(x))v'(x)-f(u(x))u'(x)$
Specific behaviours
✓ defines antiderivative and obtains definite integral   ü correctly differentiates

- (a) was quite well done.
- (b) was not but only worth 2 marks so no biggie. The part in red was non negotiable and if you did not say it – everything after that was incorrect.

Question 9 **(8 marks)**

Let $f(x)=x^4+ax^2+1$.

(a) Sketch the graph of $y=f(x)$ when $a=-8$. (4 marks)



Solution
See graph
Specific behaviours
✓ y-axis scale, y-intercept   ü x-axis intercepts   ü locates turning points   ü continuous smooth curve

(b) Show that the graph of $y=f(x)$ will always have a maximum turning point at $x=0$ if $a<0$. (4 marks)

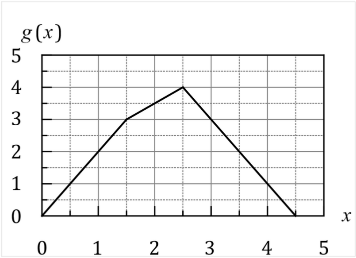
Solution
$f'(x)=4x^3+2axf'(0)=0$ Hence curve always stationary when $x=0$. $f''(x)=12x^2+2af''(0)=2a$ If $a<0$ then $f''(0)<0$ and so the curve will always be concave down.   Hence a maximum at $x=0$.
Specific behaviours
ü shows $f'(0)=0$ ü states always stationary when $x=0$ ü shows $f''(0)=2a$ ü justifies maximum using second derivative

- Some of the graphs were really dodgy – I was quite generous but for 4 give away marks – take the time to do a good job.
- I chose to be tough on (b)…as it was realy poorly expressed. You must first SHOW it is a TP before you can show the TP is a max…otherwise the 2nd deriv only shows concavity.
- Common error: $f(x)=x^4+ax^2+1$ so the $f'(x)=4x^3+2ax+1$
- Common error – students carried on with a=-8. If they could do PERFECT work with this, then I gave2 marks – but anything less resulted in loss of marks – some students thought doing a 1 or 2 examples was enough – it wasn't.
- Sloppy working for statement in red.

Question 10 **(8 marks)**

The graph of function g, and a table of values for function f and its derivatives are shown below.

x	1	2	3
$f(x)$	2	1	3
$f'(x)$	3	2	2
$f''(x)$	-1	-2	1



(a) Evaluate $h'(k)$ when

(i) $h(x)=g(f(x))$ and $k=3$. (3 marks)

Solution

$$h'(3)=g'(f(3))\times f'(3) \text{¿} g'(3)\times 2$$
$$\text{¿} -2\times 2=-4$$

Specific behaviours

✓ correct application of chain rule

ü correct value for $g'(x)$

ü correct value

(ii) $h(x)=f(x)\div g(x)$ and $k=1$. (3 marks)

Solution

$$h'(1)=\frac{f'(1)g(1)-f(1)g'(1)}{g(1)^2}$$
$$\text{¿} \frac{(3)(2)-(2)(2)}{(2)^2}=\frac{1}{2}$$

Specific behaviours

✓ correct application of quotient rule

ü correct values for $g(x)$ and $g'(x)$

ü correct value

(b) Evaluate $h''(2)$ when $h'(x)=f'(x)\times g'(x)$. (2 marks)

Solution

$$h''(2)=f''(2)g'(2)+f'(2)g''(2) \text{¿} (-2)(1)+(2)(0) \text{¿} -2$$

Specific behaviours

✓ uses product rule with at least two correct values

ü correct result

This was either pretty well done – showing students reviewed both the test and last years paper for this concept was examined in both – OR really bad showing a lack of preparation

Question 20 **(8 marks)**

Small body P moves in a straight line with acceleration a cm/s² at time t s given by

$$a=At+B$$

Initially, P has a displacement of 8 cm relative to a fixed point O and is moving with a velocity of 4 cm/s. Three seconds later, P has a displacement of 3.8 cm and a velocity of -5.9 cm/s.

(a) Determine the value of the constant A and the value of the constant B. (6 marks)

Solution

Velocity:

$$v=\int At+B\,dt\quad v(t)=\frac{At^2}{2}+Bt+c\quad v(0)=4\Rightarrow c=4$$

Displacement:

$$s(t)=\int \frac{At^2}{2}+Bt+4\,dt\quad s(t)=\frac{At^3}{6}+\frac{Bt^2}{2}+4t+k$$
$$s(0)=8\Rightarrow k=8$$

$$v(3)=4.5A+3B+4=-5.9$$
$$s(3)=4.5A+4.5B+20=3.8$$

Solve:

$$A=0.6,\ B=-4.2$$

Specific behaviours

ü antiderivative for velocity, constant evaluated

ü integral for displacement

ü displacement, constant evaluated

ü expressions for $v(3)$ and $s(3)$

ü value of A

ü value of B

(b) Determine the minimum velocity of P. (2 marks)

Solution

$$v=0\Rightarrow 0.6t-4.2=0\Rightarrow t=7$$

$$v(7)=-10.7\text{ cm/s}$$

Specific behaviours

✓ indicates time for minimum

ü correct minimum velocity

- quite well done – showing good understanding BUT absolutely appalling setting out and too many things not included. I don't need to say it but I will – **6 marks will NOT be awarded for leaps of logic without showing the process you took to get there.**
- You CANNOT use the same letter for the constant of integration for both – use c and k. (or d, e, ……but NOT c again!!!!)
- SOOOO many people said m/s….I did not take off a mark – but could have. Be careful.

Question 19 **(8 marks)**

When an electronic device is run, it randomly generates one of the first four square numbers. The discrete random variable X is the number generated in one run of the device and the table below shows its probability distribution.

x	1	4	9	16
$P(X=x)$	a	0.2	b	0.15

The mean of X is 8.25.

(a) Determine the value of the constant a and the value of the constant b. (3 marks)

Solution
Sum of probabilities:
$a+b+0.35=1$
Mean:
$a+9b+3.2=8.25$
Solving simultaneously:
$a=0.1, b=0.55$
Specific behaviours
✓ equation using sum ü equation using mean
ü both correct values

(b) The electronic device is run 3 times. Determine the probability that

(i) the number 16 is generated exactly once. (2 marks)

Solution
$Y \quad B(3, 0.15)$
$P(Y=1)=0.3251$
Specific behaviours
✓ indicates correct method
ü probability

(ii) the sum of the numbers generated is at least 35. (3 marks)

Solution
Require $16,16,16$ or $16,16,9$ or $16,16,4$ in any order.
$P=0.15^3+3(0.15)^2(0.55)+3(0.15)^2(0.2)\text{¿}\frac{27}{500}=0.054$
Specific behaviours
✓ indicates required events
ü indicates correct probabilities for at least two events
ü correct probability

- (a) and (b)(i) were quite well done –
- (b)(ii) was not – in fact less than 5 people got it right. Most people missed all the ways to get over 35 – and missed the number of ways to arrange them.
- I was quite generous with the marks and gave 2/3 if you got most of them.

Question 11 **(7 marks)**

(a) List A contains the digits in the first 250 decimal places of π. The relative frequencies of the digits are:

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	0.088	0.104	0.116	0.096	0.112	0.096	0.088	0.068	0.124	0.108

Determine the probability that a randomly selected digit from list A

(i) is odd. (1 mark)

Solution
$P=0.104+0.096+0.096+0.068+0.108=0.472$
Specific behaviours
✓ correct probability

(ii) is a factor of 14, given that it is not odd. (2 marks)

Solution
$P=\frac{0.116}{1-0.472}=\frac{29}{132}\approx 0.2197$
Specific behaviours
✓ numerator ü denominator and simplifies

(b) The discrete random variable X is defined by

$$P(X=x)=\begin{cases}\frac{1}{8} & x=0,1,2,3,4,5,6,7 \\ 0 & \text{otherwise}\end{cases}$$

(i) State the name of this type of distribution. (1 mark)

Solution
Discrete **uniform** distribution
Specific behaviours
✓ states uniform distribution

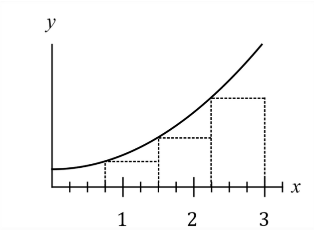
(ii) Calculate the expected value and variance of X. (3 marks)

Solution	
$E(X)=\frac{28}{8}=\frac{7}{2}=3.5$ $Var(X)=\frac{2(3.5^2+2.5^2+1.5^2+0.5^2)}{8}\text{¿}\frac{21}{4}=5.25$	Or $E(X)=\frac{28}{8}=\frac{7}{2}=3.5$ $E(X^2)=\frac{1+4+9+16+25+36+49}{8}=\frac{140}{8}$ $VAR=\frac{140}{8}-\left(\frac{28}{8}\right)^2=\frac{21}{4}$
Specific behaviours	
✓ $E(X)$ ü indicates calculation or use of CAS	ü variance

- This is a good example of: If the question is worth only 2 marks you only need the answer HOWEVER if you make a mistake and show no working you CANNOT receive ANY marks – even if I know what you did. This came up in (a) (ii) here but in many other questions.
- (b) (ii) SD = 2.29 and VAR = 5.25 ....if you used calc you need to square all of the SD and show it: 3 marks!!!!

Question 12 **(8 marks)**

The graph of $y=f(x)$ is shown at right with 3 equal width inscribed rectangles. An estimate for the area under the curve between $x=0.75$ and $x=3$ is required.



The function f is defined as $f(x)=4x^2+5$ and let the area sum of the 3 rectangles be S_3.

S_n, the area estimate using n inscribed rectangles can be calculated using

$$S_n=\sum_{i=1}^{i=n} f(x_i)\delta x$$

(a) State the values of x_1, x_2, x_3 and δx that should be used to determine S_3. (1 mark)

Solution
$x_1=0.75, x_2=1.5, x_3=2.25, \delta x=0.75$
Specific behaviours
✓ correctly states all values

(b) Calculate the value of S_3. (3 marks)

Solution
$S_3=0.75\left((4(0.75)^2+5)+(4(1.5)^2+5)+(4(2.25)^2+5)\right)$ ¿ $0.75(7.25+14+25.25)$ ¿ $0.75(46.5)$ ¿ $\frac{279}{8}=34.875\,\text{u}^2$
Specific behaviours
✓ indicates correct calculation for one rectangle   ü correct heights of all rectangles   ü correct value

(c) Explain, with reasons, how the value of δx and the area estimate S_n will change as the number of inscribed rectangles increase. (2 marks)

Solution
δx is the width of each rectangle and so must decrease. S_n will increase, approaching true area under curve, as area 'lost' between curve and rectangles will decrease.
Specific behaviours
✓ indicates δx will decrease as it's the rectangle width   ü indicates S_n will increase

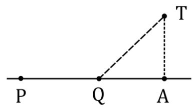
(d) Determine the limiting value of S_n as $n \to \infty$. (2 marks)

Solution	
$S_\infty=\int_{0.75}^{3} f(x)\,dx=\frac{747}{16}=46.6875\,\text{u}^2$	
Specific behaviours	
✓ correct integral	ü correct limiting value

- I did not penalise if you did not put units² – but it will be in future so check for this!!!!
- you MUST have the bit in red….the S does not merely increase (it can't do that forever!!) – it limits to the real value.

Question 18 **(8 marks)**

An offshore wind turbine T lies 9 km away from the nearest point A on a straight coast. It must be connected to a power storage facility P that lies on the coast 40 km away from A.



Engineers will lay the cable in two straight sections, from T to Q, where Q is a point on the coast x km from A, and then from Q to P.

The cost of installing cable along the coastline is $\$4000$ per km and offshore is $\$5000$ per km.

(a) Determine, to the nearest hundred dollars, the cost of installing the cable when Q lies midway from A to P. (2 marks)

Solution
$C=4000\times 20+5000\times\sqrt{20^2+9^2}¿\$189\,700$
Specific behaviours
✓ correct expression ü calculates cost

(b) Show that C, the cost in thousands of dollars, to run the cable from T to Q to P, is given by
$C=5\sqrt{x^2+81}-4x+160$. (2 marks)

Solution
$C_{TQ}=5\times QT=5\times\sqrt{x^2+9^2}$ $C_{QP}=4(40-x)=160-4x$ Hence $C=C_{TQ}+C_{QP}=5\sqrt{x^2+81}-4x+160$
Specific behaviours
✓ expression for cable from T to Q ✓ expression for cable from Q to P and shows sum

(c) Use calculus techniques to determine, with justification, the minimum cost of laying the cable from T to P. (4 marks)

Solution
$C'(x)=\frac{5x}{\sqrt{x^2+81}}-4$ or $\frac{5x-4\sqrt{x^2+81}}{\sqrt{x^2+81}}$ $C'(x)=0\Rightarrow x=12$ $C(12)=187$ $C''(12)\approx 0.12>0\Rightarrow$ minimum. Hence minimum cost is $\$187\,000$.
Specific behaviours
✓ correct derivative ü solves for optimum value of x ü justifies minimum with 2nd deriv or sign test. ü states minimum cost

- This was quite well done –
- mistakes were made with rounding – or the lack of it.
- (b) again – if you miss the "in thousands" you can't just drop it off…again when you use an = sign the LHS has to be equivalent to RHS.
- You MUST state the things in red.

Question 13 **(7 marks)**

A hot potato was removed from an oven and placed on a cooling rack. Its temperature T, in degrees Celsius, t minutes after being removed from the oven was modelled by

$$T=23+187e^{kt}.$$

The temperature of the potato halved between $t=0$ and $t=8$.

(a) Determine the value of the constant k. (3 marks)

Solution
$T_0=23+187=210$ $105=23+187e^{8k}\Rightarrow k=-0.103$
Specific behaviours
✓ indicates initial temperature ü equation for temperature halving ü solves for k

(b) The temperature of the potato eventually reached a steady state. Determine the time taken for its temperature to first fall to within $1\,°C$ of this steady state. (2 marks)

Solution
$T_\infty=23$ $24=23+187e^{-0.103t}\Rightarrow t=50.8$ minutes
Specific behaviours
✓ indicates steady state temperature ü correct time, to at least 1 dp

(c) Determine the time at which the potato was cooling at a rate of $1\,°C$ per minute. (2 marks)

Solution
$\frac{dT}{dt}=-19.261e^{-0.103t}$ $-19.261e^{-0.103t}=-1\Rightarrow t=28.7$ minutes
Specific behaviours
✓ indicates derivative ü correct time, to at least 1 dp

- Generally well done

Question 14 **(8 marks)**

(a) It is known that 12% of a large number of fire alarms in a complex of buildings are faulty. If an electrician randomly selects 6 alarms for inspection, determine

(i) the probability that none of the alarms will be faulty. (2 marks)

Solution
Let X be the number of faulty alarms. Then $X \; B(6, 0.12)$.
$P(X=0)=0.4644$ (to 4dp)
Specific behaviours
✓ defines distribution ü states probability

(ii) the probability that more than two alarms are faulty, given that at least one is faulty. (2 marks)

Solution
$P(X \geq 3)=0.0261$
$P¿$
Specific behaviours
✓ indicates $P(X \geq 3)$
ü calculates conditional probability

(iii) the standard deviation of the distribution of the number of faulty alarms. (1 mark)

Solution
$sd=\sqrt{6 \times 0.12 \times 0.88}=0.7960$
Specific behaviours
✓ correct value

(b) In a newer complex that also has a large number of fire alarms, only 4% are faulty. Determine, with reasoning, the minimum number of alarms that should be inspected so that the probability that at least one of them will be faulty is more than 98%. (3 marks)

Solution	
$Y \; B(n, 0.04)$ $P(Y \geq 1) \geq 0.98 \Rightarrow P(Y=0)<0.02$ $P(Y=0)=(0.96)^n$ $0.96^n<0.02$ $n=96$	OR $Y \; B(n, 0.04)$ $P(Y \geq 1) \geq 0.98$ *When n = 96 .....P=0.9801* *When n = 95 .... P= 0.979* $\therefore n=96$
Specific behaviours	
✓ identifies distribution and required probability ü expression for no faulty alarms, in terms of n OR show table of probabilities ü correct number	

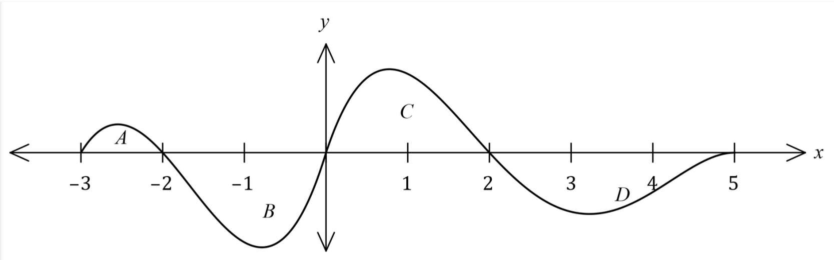
- Probability to 4 dp please – if you do less you must state degree of accuracy.
- In (c) it was worth 3 marks= working needed for 3 marks (2 options given).- you have been told that over and over again but some of you are STILL not listening.

- (b) You should ALWAYS state when you use the FTC (in red) – this will incur a loss of marks.

- OK this was a BIG one. $\Delta V = \int_0^1 V' dt = -21.7$ NOT 21.7… that is NOT correct – you ~~cannot just drop a negative because it doesn't suit you. When you use = that means~~ both sides are equivalent. Be mathematically accurate with your setting out.

Question 17 **(7 marks)**

Regions A, B, C and D bounded by the curve $y = f(x)$ and the x-axis are shown on this graph:



The areas of A, B, C and D are $9, 30, 23$ and 21 square units respectively.

(a) Determine the value of

(i) $\int_0^5 f(x)\,dx$. (1 mark)

Solution
$I = 23 - 21 = 2$
Specific behaviours
✓ correct value

(ii) $\int_{-3}^{2} 5f(x)\,dx$. (2 marks)

Solution
$I = 5(9 - 30 + 23) = 5(2) = 10$
Specific behaviours
✓ shows sum of signed areas
ü uses linearity to obtain correct value

(iii) $\int_{-2}^{5} (f(x) - 4)\,dx$. (2 marks)

Solution
$I = [-30 + 23 - 21] - ¿$
$I = -28 - (20 + 8) = -56$
Specific behaviours
ü uses linearity to obtain two integrals
ü correct value

(b) Explain why $\int_{-2}^{2} f'(x)\,dx = 0$. (2 marks)

Solution
Using fundamental theorem, result is $f(2) - f(-2)$.
Since $f(-2) = f(2) = 0$, then the difference is 0.
Specific behaviours
✓ uses fundamental theorem to obtain result
ü explains value of 0 using the two roots

- This was well done – and so it should have been – you have done these over and over again – if you got this wrong -you did not make sure you understood this concept.
- (a). (iii) mistakes were made with negatives.

Question 15 **(8 marks)**

The area A of a regular polygon with n sides of length x is given by

$$A = \frac{n x^2 \cos\left(\frac{\pi}{n}\right)}{4 \sin\left(\frac{\pi}{n}\right)}$$

(a) Simplify the above formula when $n = 12$ to obtain a function for the area of a regular dodecagon. (2 marks)

Solution
$A(x) = \dfrac{12x^2 \cos\left(\frac{\pi}{12}\right)}{4\sin\left(\frac{\pi}{12}\right)} = 3x^2(\sqrt{3}+2)$ or $\dfrac{3x^2(\sqrt{3}+1)}{(\sqrt{3}-1)}$ or 11.196 (to 3dp)
$A(x) = \dfrac{3x^2}{\left(\frac{\pi}{\ }\right)}$ for 1 mark as not simplified.

(b) Use the increments formula to estimate the change in area of a regular dodecagon when its side length increases from 10 cm to 10.3 cm. (3 marks)

Solution
$\dfrac{dA}{dx} = 12x(\sqrt{3}+2)¿$
$\delta A \approx \dfrac{dA}{dx}\delta x \approx 12(10)(\sqrt{3}+2)(0.3) \approx 18(\sqrt{3}+2) \approx 223.93 \times 0.3 \approx 67.2\,\text{cm}^2$
Specific behaviours
✓ derivative of A with respect to x
ü correct statement and use of increments formula
ü calculates change

(c) Use the increments formula to estimate the change in area of a regular polygon with sides of length 6 cm when its number of sides increases from 32 to 35. (3 marks)

Solution
$x = 6 \Longrightarrow \dfrac{dA}{dn} = \dfrac{-36\left(n\sin\left(\frac{2\pi}{n}\right) + 2\pi\right)}{4n\cos\left(\frac{2\pi}{n}\right) - 4n}, n = 32, \delta n = 3$
$\dfrac{dA}{dn}¿_{n=32} = 183.3$
$\delta A \approx \dfrac{dA}{dn}\delta n \approx 183.3 \times 3 \approx 550\,\text{cm}^2$
Specific behaviours
✓ ~~derivative of ^ with respect to (CAS)~~

- Because in this question it specifies using a particular method you MUST make the effort to clearly show that method (even if you don't need it) – that means the lot – the derivative, the substitution and the increments formula – the setting out here was pretty bad. If you did not show increments formula – no marks.
- This is an approximation so you have to use ≈ (-1)

Question 16 **(8 marks)**

The volume, V litres, of fuel in a tank is reduced between $t=0$ and $t=42$ minutes so that

$$\frac{dV}{dt}=-185\pi\sin\left(\frac{\pi t}{42}\right)$$

(a) Determine, to the nearest litre, the amount of fuel emptied from the tank

(i) in the first minute. (3 marks)

Solution
$\Delta V=\int_0^1 V'dt\text{¿}-21.7$
Hence 22 litres were emptied.
Specific behaviours
✓ writes integral for change
ü evaluates integral
ü answers as positive number of litres

(ii) in the last 5 minutes. (1 mark)

Solution
$\Delta V=\int_{37}^{42} V'dt=-537.1$
Hence 537 litres were emptied.
Specific behaviours
ü correct number of litres

The tank initially held 18 400 litres of fuel.

(b) Determine the volume of fuel in the tank 10 minutes after the volume in the tank reached 16 000 litres. (4 marks)

Solution
$\int_0^T V' dt=-2400T=10.8$
$\Delta V=\int_{10.8}^{20.8} V'dt\text{¿}-5253$
$V(20.8)=16\,000-5253\text{¿}10\,747\text{ L}$
Specific behaviours
✓ equation for $\Delta V=-2400$
ü determines T
ü determines ΔV
ü correct volume

Alternative Solution
$V(t)=\int V' dt=7770\cos\left(\frac{\pi t}{42}\right)+c$
$V(0)=18\,400\Rightarrow c=10\,630$
$V(T)=16\,000\Rightarrow T=10.8$
$V(20.8)=10\,747\text{ L}$
Specific behaviours
✓ antiderivative for $V(t)$
ü determines c
ü determines T
ü correct volume

- This was either really well understood or not at all.
- Some silly marks lost for not rounding as required