education equals

**Calculator Free**
**Logarithm Laws and Solving Equations**

Time: 45 minutes
Total Marks: 45
Your Score: / 45

**Question One: [2, 2, 2, 2 = 8 marks]** **CF**

Express each of the following as a single logarithm:

(a) $2\log 3 + \log 2 - \log 6$

(b) $2\log_x y - 3\log_x y + 6$

(c) $3\log_a m + 4\log_a n - 5\log_a t$

(d) $(\log x)^3 \div (\log x)^2 + \log x^2$

**Question Two: [3, 3, 3 = 9 marks]** **CF**

Evaluate each of the following showing full working:

(a) $3\log_2 6 - \log_2 27$

(b) $1.5\log_8 4$

(c) $\dfrac{\log 135 - \log 5}{\log 3^2}$

**Question Three: [1, 3 = 4 marks]** **CF**

If $\log x = y$, where $x$ is positive, express each of the following in terms of $y$.

(a) $\log x^2$

(b) $\log xm^3 - 3\log m$

**Question Four:** **[2, 3, 3, 3, 3, 3, 4, 3 = 24 marks]** **CF**

Solve each of the following equations, showing all working.

(a) $\log_y 64 = 2$

(b) $8x^{\frac{1}{3}} + 12x^{\frac{1}{3}} = 40$

(c) $\log_5 x + \log_2 8 = 0$

(d) $\frac{10^{x+2}}{100^{4x}} = 10000^{x-1}$

(e) $3^{x+1} = 12$

(f) $2^{x-3} = 5^{2x+1}$

(g) $4^{2x} - 4^{x} - 6 = 0$

(h) $5e^{2-x} = 100$

**SOLUTIONS**
**Calculator Free**
**Logarithm Laws and Solving Equations**

Time: 45 minutes
Total Marks: 45
Your Score: / 45

**Question One: [2, 2, 2, 2 = 8 marks]** **CF**

Express each of the following as a single logarithm:

(a) $2\log 3 + \log 2 - \log 6$

$= \log 9 + \log 2 - \log 6$ ✓

$= \log 18 - \log 6$

$= \log 3$ ✓

(b) $2\log_x y - 3\log_x y + 6$

$= \log_x y^2 - \log_x y^3 + 6\log_x x$ ✓

$= \log_x \frac{1}{y} + \log_x x^6$

$= \log_x \frac{x^6}{y}$ ✓

(c) $3\log_a m + 4\log_a n - 5\log_a t$

$= \log_a m^3 + \log_a n^4 - \log_a t^5$ ✓

$= \log_a \frac{m^3 n^4}{t^5}$ ✓

(d) $(\log x)^3 \div (\log x)^2 + \log x^2$

$= \log x + \log x^2$ ✓

$= \log x^3$ ✓

**Question Two: [3, 3, 3 = 9 marks]** **CF**

Evaluate each of the following showing full working:

(a) $3\log_2 6 - \log_2 27$

$=\log_2 216 - \log_2 27$ ✓

$=\log_2 8$

$=\log_2 2^3$ ✓

$=3\log_2 2$

$=3$ ✓

(b) $1.5\log_8 4$

$=\log_8 (\sqrt{4})^3$ ✓

$=\log_8 8$ ✓

$=1$ ✓

(c) $\dfrac{\log 135 - \log 5}{\log 3^2}$

✓

$=\dfrac{\log 27}{2\log 3}$

$=\dfrac{\log 3^3}{2\log 3}$ ✓

$=\dfrac{3\log 3}{2\log 3}$

$=\dfrac{3}{2}$ ✓

**Question Three:** **[1, 3 = 4 marks]** **CF**

If $\log x = y$, where $x$ is positive, express each of the following in terms of $y$.

(a) $\log x^2$

$=2\log x$

$=2y$ ✓

(b) $\log xm^3 - 3\log m$ ✓ ✓

$= \log x + 3\log m - 3\log m$

$= y$ ✓

**Question Four:** **[2, 3, 3, 3, 3, 3, 4, 3 = 24 marks]** **CF**

Solve each of the following equations, showing all working.

(a)

$\log_y 64 = 2$

$y^2 = 64$

$y = 8 \ (y > 0)$

(b)

$8x^{\frac{1}{3}} + 12x^{\frac{1}{3}} = 40$

$20x^{\frac{1}{3}} = 40$

$x^{\frac{1}{3}} = 2$

$x = 8$

(c)

$\log_5 x + \log_2 8 = 0$

$\log_5 x + 3\log_2 2 = 0$

$\log_5 x = -3$

$x = 5^{-3}$

$x = \frac{1}{125}$

(d)

$\frac{10^{x+2}}{100^{4x}} = 10000^{x-1}$

$\frac{10^{x+2}}{10^{8x}} = 10^{4x-4}$

$10^{2-7x} = 10^{4x-4}$

$2 - 7x = 4x - 4$

$6 = 11x$

$x = \frac{6}{11}$

(e)

$3^{x+1} = 12$

$$(x+1)\log 3 = \log 12$$

$$x+1 = \frac{\log 12}{\log 3}$$ ✓

$$x = \frac{\log 12}{\log 3} - 1$$ ✓

(f) $2^{x-3}=5^{2x+1}$

$(x-3)\log 2=(2x+1)\log 5$

$x\log 2-3\log 2=2x\log 5+\log 5$

$x\log 2-2x\log 5=\log 5+3\log 2$

$x(\log 2-2\log 5)=\log 5+3\log 2$

$x=\dfrac{\log 5+3\log 2}{\log 2-2\log 5}$

(g) $4^{2x}-4^{x}-6=0$

Let $y=4^{x}$

$y^{2}-y-6=0$

$(y-3)(y+2)=0$

$y=3 \quad y=-2$

$4^{x}\neq -2$

$4^{x}=3$

$x\log 4=\log 3$

$x=\dfrac{\log 3}{\log 4}$

(h) $5e^{2-x}=100$

$e^{2-x}=20$

$(2-x)\ln e=\ln 20$

$2-x=\ln 20$

$x=2-\ln 20$