50 Minutes

ALL SAINTS' COLLEGE

Mathematics Specialist

Test 5 2016

Applications of Differentiation

NAME: _____

TEACHER: MLA

Resource Free Section

20 marks
20 minutes

Question 1 **[4 marks]**

Using the identity $\sin^2\theta+\cos^2\theta=1$, show that if $x=A\sin(\omega t+\alpha)$, then $v^2=\omega^2(A^2-x^2)$

Question 2 [5 marks]

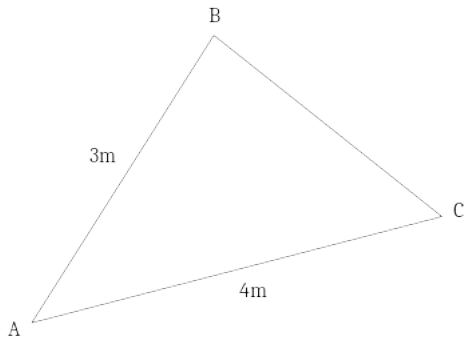
Solve the $\frac{dy}{dx} - 2y = 12 \text{ for } y \geq 0, \text{ if } y = 4 \text{ when } x = 0$

Question 3 **[5 marks]**

Show that the equation of the tangent (with gradient m) to the curve with equation $y^2 = 4ax$ can be expressed as $y = mx + \frac{a}{m}$

Question 4 **[6 marks]**

In ΔABC, $AB=3m, AC=4m \wedge \angle BAC=\theta$. If θ is increasing at a rate of π radians per minute, find the rate (in metres per minute), at which the length of the side BC is changing at the instant $\theta=\frac{\pi}{2}$.



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Resource Rich Section

30 marks
30 minutes

Question 5 **[5 marks]**

Particle P travels along a straight line. At time t = 0 seconds, it passes a fixed point O with velocity of 16 ms^{-1} and undergoes constant acceleration of$4\ ms^{-2}$.

Use calculus to find the velocity of P when it is 10 metres from O.

Question 6 **[2 & 4 = 6 marks]**

An object travels in a straight line such that its velocity at time t is given by $v = e^{\sin(t)}\, mm/s$

(a) Show that the object travels to the right for $0 \le t \le 10$

(b) If the object was 10 mm from a fixed observation point O at t = 0, use calculus to establish its displacement at t = 10 seconds.

Question 7 **[5, 1, 1 & 2 = 9 marks]**

According to Newton's Law of Cooling, the temperature $T(Celcius)$ of a brass plate left to cool down satisfies the equation $\frac{dT}{dt} = -k(T-20), where\ k$ is a positive constant and t is measured in minutes.

(a) After 20 minutes the temperature of the brass plate is 60°C, and 10 minutes later it has dropped to 30°C. Express T as a function of time.

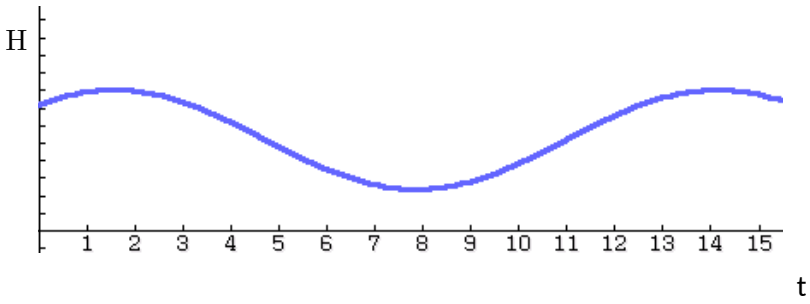
(b) What is the initial temperature of the brass plate?

(c) What is the final temperature of the brass plate?

(d) How long will it take for the temperature of the brass plate to drop to within $5^{\circ}C$ of its final temperature?

Question 8 [1, 1, 1, 1, 3 & 3= 10 marks]

At a point in a small bay near Cable Beach in Broome, the depth H (metres) of the water at time t (hours) is given by:



$$H = 5.2 + 2\sqrt{2}\sin\left(\frac{\pi}{4} + \frac{t}{2}\right)$$

$t = 0$ *corresponds ¿* 9.00 *am*.

The sinusoidal motion of the tide (shown above) can be replicated on your ClassPad.

If the depth of water in the bay is influenced only by the tide,

(a) State the water depth when $t = 0$

(b) About which depth does the tide oscillate?

(c) What is the exact range of depth in the bay?

(d) How much time lapses between successive high tides?

(e) How fast is the water depth changing at 1:00pm?

(f) Show that the **change** in water depth is an example of SHM.