Test 5 : Friday 1st July

Logarithms

This assessment contributes 6% towards the final year mark.

45 minutes are allocated for this test.

No notes or calculators of ANY nature are permitted.

Full marks may not be awarded to correct answers unless sufficient justification is given.

Name : Solutions

Non-Calculator

45 minutes

Total $= \overline{48}$

Do NOT turn over this page until you are instructed to do so.

Question 1 **(4 marks)**

Evaluate:

(a) $\log_2 16$

4 ✓

(b) $\log_3 \frac{1}{9}$

-2 ✓

(c) $\log_e 1$

0 ✓

(d) $\log_9 27$

$\frac{3}{2}$ ✓

Question 2 **(4 marks)**

Solve $\log(x) + \log(x-3) = 1$.

$\log x(x-3) = \log 10$ ✓ converts 1 to log 10

$x^2 - 3x = 10$ ✓ equates arguments

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$ ✓ solves quadratic

$\underline{\underline{x = 5}}$ or $x = -2$ ✓ interprets solutions correctly

ignore $(x > 3)$

8

Question 3 **(6 marks)**

Given that $\log_a 3 = x$ and $\log_a 5 = y$,

(a) write expressions, in terms of *x* and *y*, for:

(i) $\log_a 0.6$ $= \log_a 3 - \log_a 5$ ✓ uses log laws correctly (2 marks)

$= x - y$ ✓ expresses in terms of x, y.

(ii) $\log_a 45$. (2 marks)

$= \log_a (3^2 \times 5)$ ✓ uses log laws

$= 2\log_a 3 + \log 5$

$= 2x + y$ ✓ express in terms of x, y

(b) Evaluate exactly a^{4x} . (2 marks)

$x = \log_a 3$

$a^x = 3$ ✓ uses log definition

$(a^x)^4 = 3^4 = \underline{\underline{81}}$ ✓ evaluates

6

Question 4 **(9 marks)**

Solve the following exactly using natural logarithms.

(a) $4^x = 28$ (2 marks)

$$\ln 4^x = \ln 28$$
$$x \ln 4 = \ln 28$$
$$x = \frac{\ln 28}{\ln 4}$$

✓applies log laws.

✓solves in base e. $\left(1+\frac{\ln 7}{\ln 4}\right)$

(b) $5^x = 7^{x+2}$ (3 marks)

$$x \ln 5 = (x+2)\ln 7$$

✓applies log laws

$$x\ln 5 - x\ln 7 = 2\ln 7$$

✓collect like terms

$$x = \frac{2\ln 7}{\ln 5 - \ln 7}$$

✓ solves $\left(\frac{-2\ln 7}{\ln 7 - \ln 5}\right)$

(c) $16^x - 2(4^x) - 3 = 0$ (4 marks)

$$(4^x)^2 - 2(4^x) - 3 = 0$$

$$(4^x - 3)(4^x + 1) = 0$$

✓Factorises

$4^x = 3$ OR $4^x = -1$ ✓NFL

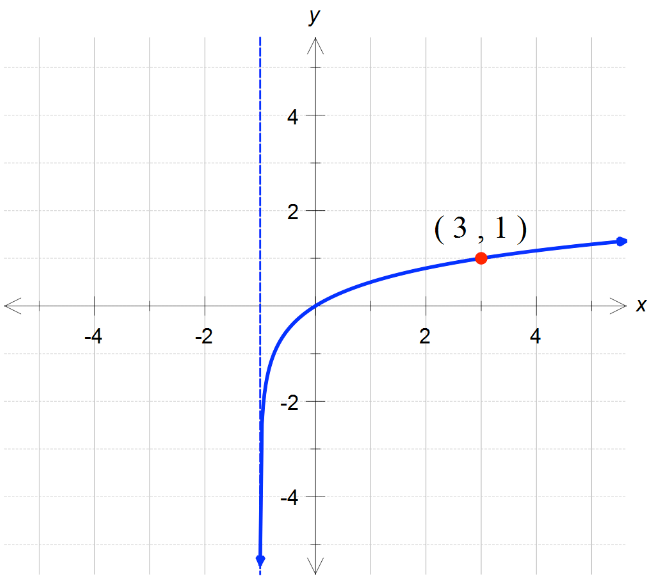
$x = \frac{\ln 3}{\ln 4}$ ✓solves

No solution ✓solves

9

Question 5 **(5 marks)**

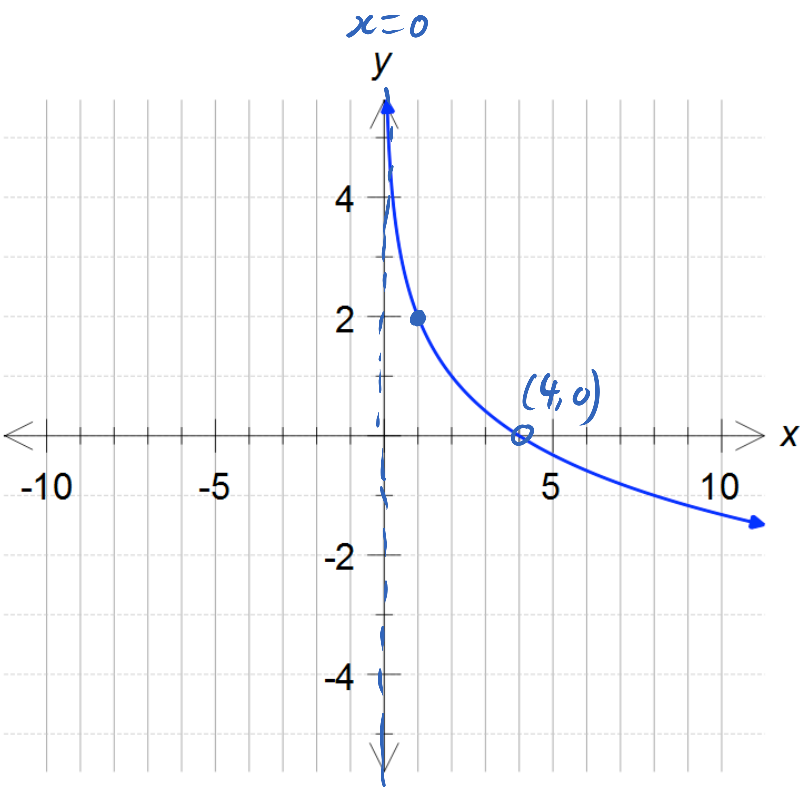
(a) State the equation of the graph below. (2 marks)



$y = \log_4(x+1)$ ✓ ✓shift

Base 4

(b) Sketch the graph of $y = -\log_2(x) + 2$ on the axes below. (3 marks)
Clearly label any key features.



✓Asymptote clearly indicated/labelled

✓y-intercept clearly indicated/labelled

✓accurate, smooth curve.

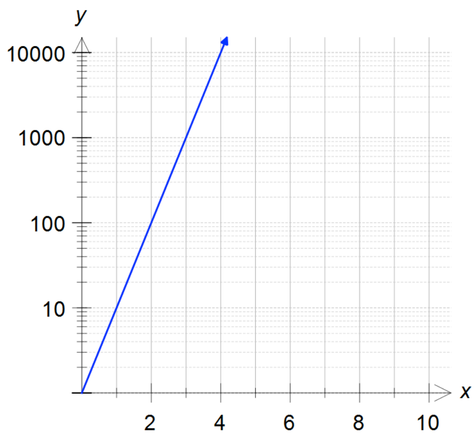
5

Question 6 **(4 marks)**

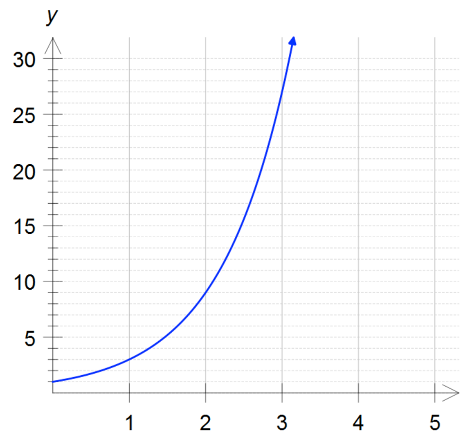
Match each graph below to one of the following equations:

A. $y = 3^x$	B. $y = 10x$
C. $y = \ln(x-1)$	D. $y = 10^x$
E. $y = \log_4(x-1)$	F. $y = \log_3(x)$
G. $y = \log_3(x-1)$	H. $y = \ln(x)$

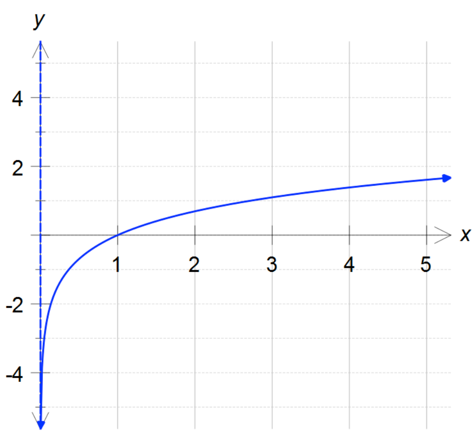
Graph i **equation:** D ✓



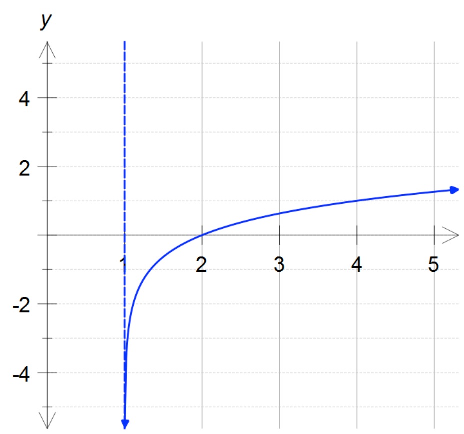
Graph ii **equation:** A ✓



Graph iii **equation:** H ✓



Graph iv **equation:** G ✓



4

Question 7 **(8 marks)**

(a) Differentiate the following. Do ***not*** simplify.

(i) $y = \ln(x^2 - 3x)\sin(x)$ (3 marks)

$$y = \frac{2x-3}{(x^2-3x)} \cdot \sin x + \ln(x^2-3x)\cos x$$

✓ $\frac{f'(x)}{f(x)}$

✓ $\frac{d}{dx}(\sin x)$

✓ uses product rule correctly.

(ii) $y = 4\log_7 x$ (2 marks)

$$= \frac{4\ln x}{\ln 7}$$

$$\frac{dy}{dx} = \frac{4}{x\ln 7}$$ ✓

✓ correct change of Base

(b) Determine $\int \frac{e^{2x}}{e^{2x}+3}\,dx$ (3 marks)

$$= \frac{1}{2}\ln\left(e^{2x}+3\right) + C$$

✓ coefficient

✓ $\ln(e^{2x}+3)$

✓ constant

8

Question 8 **(4 marks)**

The approximate apparent magnitudes of two heavenly bodies are listed in the table below:

Heavenly body	Apparent magnitude m
Sirius	–1.5
Antares	1

The ratio of brightness (or intensity) $\frac{I_A}{I_B}$ of two objects A and B, of apparent magnitudes m_A and m_B respectively, satisfies the equation

$$\log_e\left(\frac{I_A}{I_B}\right) = m_B - m_A$$

(a) Determine the ratio of brightness of Sirius to Antares, stating your answer exactly.

(2 marks)

$\ln\left(\frac{I_S}{I_A}\right) = 1 - (-1.5)$ ✓substitutes

$= 2.5$

$\frac{I_S}{I_A} = \underline{\underline{e^{2.5}}}$ ✓solves

(b) If the ratio $\frac{I_{Jupiter}}{I_{Sirius}}$ is $\sqrt{e}$, determine the apparent magnitude of Jupiter. (2 marks)

$\ln(\sqrt{e}) = -1.5 - m_J$ ✓substitutes

$\frac{1}{2} = -1.5 - m_J$

$\underline{\underline{m_J = -2}}$ ✓solves

4

Question 9 **(4 marks)**

The position, x, of a particle at time t is given by the equation:

$$x(t) = t + \ln(t - 3).$$

(a) Determine the velocity function for the particle. (1 mark)

$$x'(t) = 1 + \frac{1}{t-3} \quad \checkmark$$

(b) Does the particle ever stop moving? Justify your answer. (3 marks)

$$0 = 1 + \frac{1}{t-3} \qquad \checkmark\ x'(t) = 0$$

$$0 = t - 3 + 1$$

$$t = 2 \qquad \checkmark \text{solves for } t$$

But $t > 3$ $\therefore$ No does not stop. ✓ interprets.

4