



PERTH MODERN SCHOOL  
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## Semester 1 Examination 2012

### Question/Answer Booklet

#### MATHEMATICS: SPECIALIST 3CD

#### Section One: Calculator-free

Name of Student: \_\_\_\_\_ Marking Key \_\_\_\_\_

#### Time allowed for this section

Reading time before commencing work: 5 minutes  
Working time for this section: 50 minutes

#### Materials required/recommended for this section

##### *To be provided by the supervisor*

This Question/Answer Booklet  
Formula Sheet

##### *To be provided by the student*

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters  
Special items: nil

#### Important note to students

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

### Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator-free	6	6	50	50	
Section Two Calculator-assumed	11	11	100	100	
			Total	150	100

### Instructions to students

- 1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
- 2 **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- 3 It is recommended that you **do not use pencil**, except in diagrams.

**Section One: Calculator-free**

**(50 marks)**

This section has **six (6)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 50 minutes

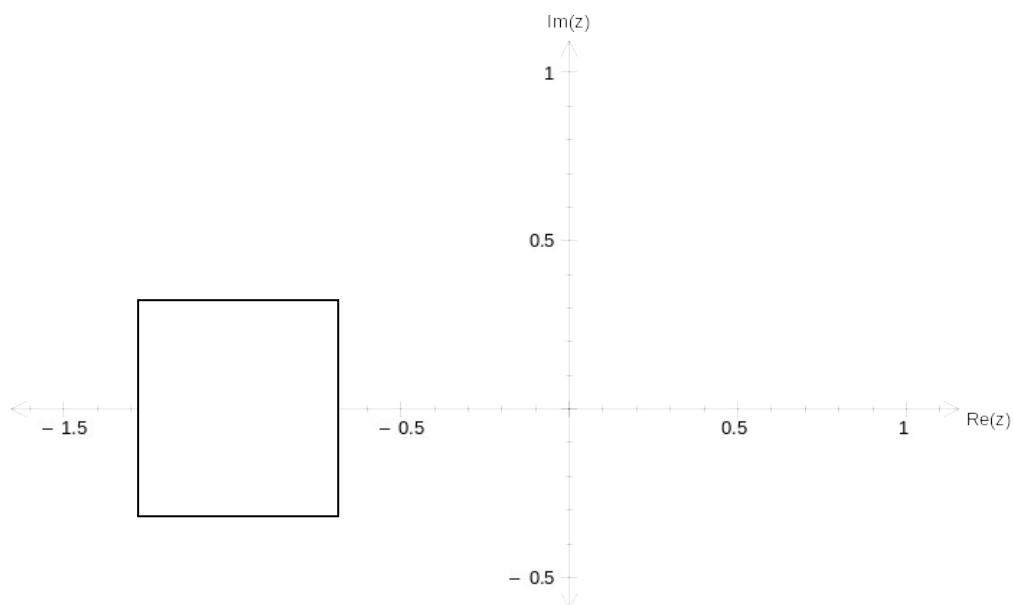
**Question 1**

**(9 marks)**

- (a) (i) Sketch on the complex plane below the region defined by

$$|z + 1| \leq \frac{1}{4}$$

**(3)**



Answer

- (ii) Hence, find the minimum value of  $|z|$  **(1)**

**Question 1 (continued)**

- (b) The curve C is defined parametrically by  $x = t^2$ ,  $y = t + 2$ ,  $t > 0$ . Find the area of the region bounded by the curve, C, the x-axis,  $x = 1$  and  $x = 4$ . (5)

**Question 2****(7 marks)**

Determine each of the following integrals.

(a)  $\int (3x+1)e^{3x^2+2x-1} dx$  (2)

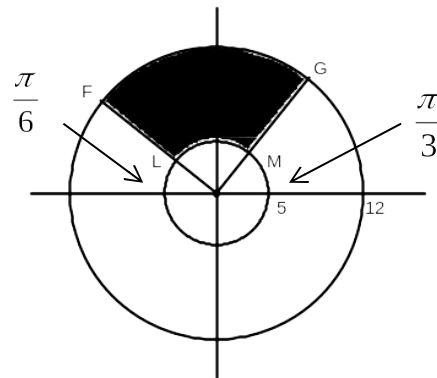
(b)  $\int (1 - e^{\cos x}) \sin x dx$  (2)

(c)  $\int \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{5 - 4 \cos x} dx$  (3)

**Question 3**

**(6 marks)**

- (a) Use **polar inequalities** to describe the region bounded by minor arcs LM and FG and the straight lines, FL and GM. (2)



- (b) If the graph of  $r = k\theta$ ,  $k > 0$  passes through M, find a possible value for  $k$ . (2)

(c) Find the distance between F and M.

(2)

**Question 4****(9 marks)**

The line L has equation  $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda (\mathbf{i} + 2\mathbf{j} + \mathbf{k})$  where  $\lambda$  is real parameter

- (i) Find the acute angle  $\theta$  between the x - y plane and the line L. (3)
- (ii) Show that the point A with coordinates (7, -4, 3) lies on the line which passes through (3,2,1) and is parallel to the vector  $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ . (3)
- (iii) Find the value of m such that  $2\mathbf{i} + \mathbf{j} + m\mathbf{k}$  is perpendicular to the vector  $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ . (3)



**Question 5****(10 marks)**

- (a) Express each of the following in polar form such that  $r > 0, -\pi \leq \theta \leq \pi$ .

(i)  $\left( \frac{1}{2} + \frac{i\sqrt{3}}{2} \right)$  (2)

(ii)  $\left( \frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2} \right)$  (2)

(iii)  $\left( \frac{1}{2} + \frac{i\sqrt{3}}{2} \right) \left( \frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2} \right)$  (2)

**Question 5 (continued)**

(b) Simplify and express in exact rectangular form. (2)

$$\left( \frac{1}{2} + \frac{i\sqrt{3}}{2} \right) \left( \frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2} \right)$$

© Hence, find the exact value of  $\sin \frac{\pi}{12}$  and  $\cos \frac{\pi}{12}$  (2)

**Question 6**

**(9 marks)**

- (a) If  $z$  is a complex number such that  $\arg(z + 4) = \frac{\pi}{4}$ , find the least value of  $|z|$ .  
(3)

- (b) A complex number  $z$  satisfies the equation  $z + 2\overline{z} = 2 + i$ , find the complex number.  
Hint: let  $z = x + yi$ .  
(6)