

Semester One Examination, 2020
Question/Answer Booklet



MATHEMATICS METHODS ATAR Year 12 Section Two: Calculator-assumed

Please circle your teacher's name:

Student Name: _____
Teacher: Miss Long Miss Rowden Ms Stone

Time allowed for this paper:
Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Materials required/recommended for this paper
To be provided by the supervisor
This Question/Answer Booklet
Number of additional answer books used (if applicable): _____

To be provided by the candidate
Standard items:
Pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters
Special items:
Drawing instruments, templates, notes on two unfolded sheets of A4
paper, and up to three calculators approved for use in this
examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Instructions to candidates

Section	Percentage of examination questionnaire	Number of questions suggested by examinee	Number of questions to be answered by examinee	Marks available	Percentage of examination questionnaire available	Number of questions to work through (minutes)	Number of questions to be answered (minutes)	Percentage of examination questionnaire available	Section One: Calculator-free	Section Two: Calculator-assumed	Total	100
	35	52	50	8	8	50	13	13	65	97	65	
	35	52	50	8	8	50	13	13	65	97	65	

Structure of this paper

MATHEMATICS METHODS

8

CALCULATOR-ASSUMED

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

6. It is recommended that you do not use pencil, except in diagrams.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be awarded for reasoning. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

4. Give the page number.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

2. Write your answers in this Question/Answer booklet. These rules:

1. Supplemental pages for the use of planning/containing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. countinue

The rules of the conduct of the AAT comprise examinations are detailed in the Year 12 Information Handbook 2020. Sitting this examination implies that you agree to abide by the rules of the conduct of the AAT.

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Formulae

It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

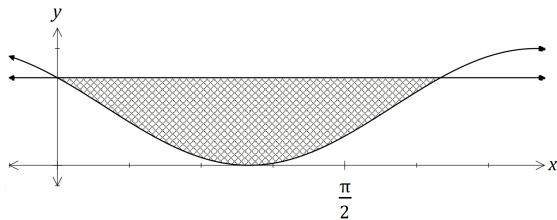
This section has thirteen (13) questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 9 (4 marks)

The graphs of $y = \cos^2\left(x + \frac{\pi}{6}\right)$ and $y = \frac{3}{4}$ are shown below. Determine the exact area of the shaded region they enclose.



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(8 marks)

Question 10
A small body moving in a straight line has displacement x cm from the origin at time t seconds given by

$$x = 8 \cos(0.5t - 2) + 1.5, 0 \leq t \leq 12.$$

(4 marks)

(a) Use derivatives to justify that the maximum displacement of the body occurs when

$$t = 4.$$

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(2 marks)

(b) Determine the time(s) when the velocity of the body is not changing.

(2 marks)

(c) Express the acceleration of the body in terms of its displacement x .

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(8 marks)

Question 11

The voltage, V volts, supplied by a battery t hours after timing began is given by

$$V = 8.95 e^{-0.265t}$$

(a) Determine

(i) the initial voltage.

(1 mark)

(ii) the voltage after 3 hours.

(1 mark)

(iii) the time taken for the voltage to reach 0.03 volts.

(1 mark)

(b) Show that $\frac{dV}{dt} = aV$ and state the value of the constant a .

(2 marks)

(c) Determine the rate of change of voltage 3 hours after timing began.

(1 mark)

(d) Determine the time at which the voltage is decreasing at 5% of its initial rate of decrease.

(2 marks)

See next page

Supplementary page

Question number: _____

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End of questions

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Question 12
(6 marks)

Given that $f(-2) = -2$, $f'(-2) = -1$, $g(-2) = 4$ and $g'(-2) = 3$, evaluate $h'(-2)$ in each of the following cases:

(a) $h(x) = |f(x)|^5$.

(b) $h(x) = \frac{f(x)}{g(x)}$.

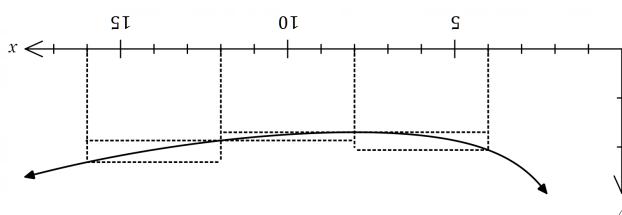
(c) $h(x) = g(f(x))$.

(2 marks)

(2 marks)

(2 marks)

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(a) Complete the missing values in the table below, rounding to 2 decimal places.

x	$f(x)$
4	
8	
12	
16	

(1 mark)

(b) Use the areas of the rectangles shown on the graph to determine an under- and over-estimate for $\int_4^{16} f(x) dx$.

(1 mark)

(c) Use your answers to part (b) to obtain an estimate for $\int_4^{16} f(x) dx$.

(2 marks)

(d) State whether your estimate in part (c) is too large or too small and suggest a modification to the numerical method employed to obtain a more accurate estimate.

Question 13

Functions f and g are such that

$$f(4)=2, f'(x)=18(3x-10)^{-2}$$

$$g(-4)=2, g'(x)=18(3x+10)^{-2}$$

- (a) Determine $f(6)$.

(3 marks)

- (b) Use the increments formula to determine an approximation for $g(-3.98)$.

(3 marks)

- (c) Briefly discuss whether using the information given about f and the increments formula would yield a reasonable approximation for $f(6)$.

(1 mark)

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Question 20

(6 marks)

The moment magnitude scale M_w is used by seismologists to measure the size of earthquakes in terms of the energy released. It was developed to succeed the 1930's-era Richter magnitude scale.

The moment magnitude has no units and is defined as $M_w = \frac{2}{3} \log_{10}(M_0) - 10.7$, where M_0 is the total amount of energy that is transformed during an earthquake, measured in $\text{dyn}\cdot\text{cm}$.

- (a) On 28 June 2016, an estimated 2.82×10^{21} $\text{dyn}\cdot\text{cm}$ of energy was transformed during an earthquake near Norseman, WA. Calculate the moment magnitude for this earthquake.

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(1 mark)

- (b) A few days later, on 8 July 2016, there was another earthquake with moment magnitude 5.2 just north of Norseman. Calculate how much energy was transformed during this earthquake.

(2 marks)

- (c) Show that an increase of 2 on the moment magnitude scale corresponds to the transformation of 1000 times more energy during an earthquake.

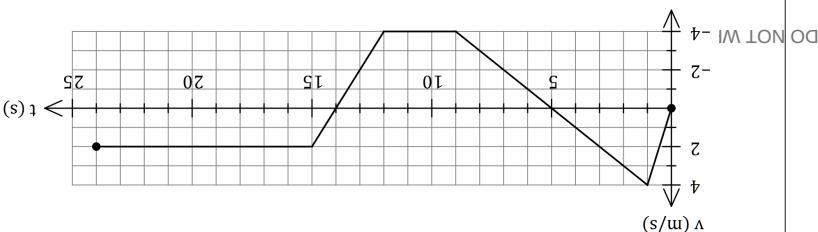
(3 marks)

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(1 mark)

- (c) Given that the water in the pool has a uniform depth of 145 cm, determine the capacity of the pool in kilolitres (1 kilolitre of water occupies a volume of 1 m^3).

- Q. The velocity v m/s of the body is shown in the graph below for $0 \leq t \leq 24$ seconds.
- A small body leaves point P and travels in a straight line for 24 seconds until it reaches point Q .



(3 marks)

- (b) Determine an expression, in terms of t , for the displacement of the body relative to P during the interval $1 \leq t \leq 9$.

(3 marks)

- (a) Use the graph to evaluate $\int_0^5 v \, dt$ and interpret your answer with reference to the motion of the small body.

- (c) Determine the time(s) at which the body was at point P for $0 < t \leq 24$.

(3 marks)

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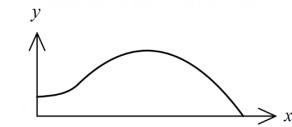
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(8 marks)

Question 19

The edges of a swimming pool design, when viewed from above, are the x -axis, the y -axis and the curves

$$y = -0.1x^2 + 1.6x - 1.5 \text{ and } y = 1.4 + e^{x-3}$$



where x and y are measured in metres.

- (a) Determine the gradient of the curve at the point where the two curves meet.

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(3 marks)

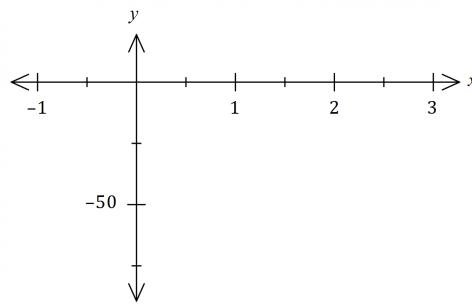
- (b) Determine the surface area of the swimming pool.

(4 marks)

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(2 marks)

- (c) Sketch the curve on the axes below.



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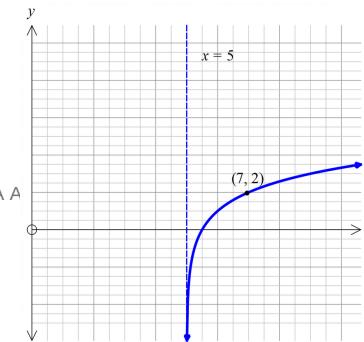
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(9 marks)

Question 18

- (a) The rule of the graph below is of the form $y = \log_2(x-b)+c$.

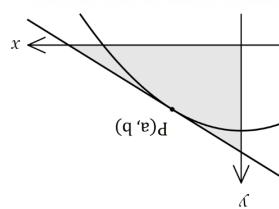
(2 marks)

Find the values of b and c .

See next page

Question 16 (8 marks)

Let $P(a, b)$ be a point in the first quadrant that lies on the curve $y = 5 - x^2$ and A be the area of the triangle formed by the tangent at P and the coordinate axes.



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(4 marks)

$$(a) \text{ Show that } A = \frac{4a}{a^2 + 5}.$$

(b) Use the increments formula to determine the percentage change in the radius of a cone if the height remains constant and V increases by 3%. (4 marks)

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- (b) Use calculus to determine the coordinates of P that minimise A .

(4 marks)

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Question 17

(8 marks)

- (a) The cost of producing x items of a product is given by $\$[5x + x\ln(x+2)]$. Each item is sold for \$24.90.

- (i) Determine the profit equation.

(1 mark)

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- (ii) Use differentiation to determine the profit associated with the sale of the 1001st item.

(3 marks)

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