PERTH MODERN SCHOOL
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Independent Public School

Course Methods Year 12 test one 2022

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: **_____40_____ mins**

Number of questions: **____8_____**

Materials required: **No calculators nor classpads allowed**

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper.

Marks available: **___40___ marks**

Task weighting: **__10__%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3, 4 & 3 = 10 marks)
Differentiate the following:

a) $(3x - 1)^5$

Solution
$5(3x - 1)^4 3$
Specific behaviours
P correct power
P uses factor of 5
P uses factor of 3 (no need to simplify)

b) $(5x^2 - 1)^7 3x^2$ and simplify

Solution
$(5x^2 - 1)^7 3x^2$
$(5x^2 - 1)^7 6x + 3x^2 7(5x^2 - 1)^6 10x$
$(5x^2 - 1)^6 2x\left[3(5x^2 - 1) + 105x^2\right]$
$(5x^2 - 1)^6 2x\left[120x^2 - 3\right]$
$(5x^2 - 1)^6 6x\left[40x^2 - 1\right]$
Specific behaviours
P uses product rule
P uses chain rule for bracket term
P obtains a correct expression
P shows a fully simplified expression

c) $\frac{3x+1}{\sqrt{7 - 2x}}$ (do not simplify)

Solution
$\frac{\sqrt{7 - 2x}(3) - (3x+1)\frac{1}{2}(7 - 2x)^{\frac{-1}{2}}(-2)}{7 - 2x}$

$T = A(800 - 20A) = 800A - 20A^2$
$\frac{dT}{dA} = 800 - 40A = 0$
$A = 20 \quad ha$
$\frac{dT^2}{dA^2} = -40$
$A = 20 \quad A'' = -40 \therefore local \max$
Specific behaviours
P determines expression for total amount of corn
P differentiates and equates to zero
P solves for A (no units required)
P shows using a derivative test that this is a local max

Q8 (5 marks)

Let the cost, $\$C$, to make x items in a factory be given by $C = 3x^3 - 12x^2 + 40x$ dollars. Using calculus show that the minimum **average cost** per item is equal to the marginal cost at this number of items.

Solution
$C = 3x^3 - 12x^2 + 40x$
$Av = \frac{C}{x} = 3x^2 - 12x + 40$
$(Av)' = 6x - 12 = 0 \quad , x = 2$
$(Av)'' = 6 \therefore local \min$
$Av(2) = 12 - 24 + 40 = 28$
$M\arg inal(x) = 9x^2 - 24x + 40$
$M\arg inal(2) = 36 - 48 + 40 = 28$
QED
Specific behaviours
P determines exp for average and differentiates
P equates derivative to zero and solves for x
P shows with derivative test that local min
P shows marginal cost formula
Pshows both equal at required x value

Note: No follow through if sketch is wrong as original function given & do not accept turning pt)

Q6 (2 & 3 = 5 marks)

Consider the function $y = g(x)$ where $g(2) = 10, \quad g'(2) = 5$.

a) Using the increments formula (small change) determine an approximate value for $g(2.1)$ and express this as an approximate percentage change again using the increments formula.

Solution
$\Delta y \approx \frac{dy}{dx}\Delta x = g'(2)0.1 = 0.5$ $g(2.1) \approx 10.5$
Specific behaviours
P uses increments formula P determines approx. g(2.1)

b) The volume of a sphere of radius r metres is given by $V = \frac{4}{3}\pi r^3$. Using the increments formula determine the approximate percentage change in volume for a 3% change in the radius.

Solution
$\frac{\Delta V}{V} \approx \frac{4\pi r^2 \Delta r}{\frac{4}{3}\pi r^3} = 3\frac{\Delta r}{r} = 9\%$
Specific behaviours
P sets up an expression for percentage change in volume P simplifies expression Psubs % change for r to give approx. % change in V

Q7 (4 marks)

Let A equal the number of hectares that a farmer will use to grow corn one season. The amount of corn to be harvested per hectare is given by $(800 - 20A)$ kg for $A \leq 40$. **Using calculus** determine the number of hectares that should be used to maximise the amount of corn produced.

Solution

Specific behaviours
P uses quotient rule P correct denominator P correct numerator

Q2 (4 marks)

Determine the equation of the tangent to $y = (5x - 1)(2x^3)$ at $(1, 8)$

Solution
$y' = (5x - 1)6x^2 + 10x^3$ $x = 1, \quad y' = 34$ $y = 34x + c$ $8 = 34 + c$ $c = -26$ $y = 34x - 26$
Specific behaviours
P uses product rule P determines gradient P sets up a constant and equation to solve P states tangent line

Q3 (5 marks)

Determine the coordinates of the stationary points and their nature for $y = x^3 - 2x^2 - x + 2$. Justify.

Solution
$y = x^3 + 2x^2 + x + 2$ $y' = 3x^2 + 4x + 1 = (3x + 1)(x + 1)$ $y'' = 6x + 4$ $y' = 0, \quad x = \frac{-1}{3}, x = -1$ $x = \frac{-1}{3} \Rightarrow y'' = 2 \therefore local\ \min \quad y = \frac{-1}{27} + \frac{2}{9} - \frac{1}{3} + 2 = \frac{-1}{27} + \frac{6}{27} - \frac{9}{27} + \frac{54}{27} = \frac{50}{27}$ $x = -1 \Rightarrow y'' = -2 \therefore local\ \max \quad y = -1 + 2 - 1 + 2 = 2$ $\left(\frac{-1}{3}, \frac{50}{27}\right) \& (-1, 2)$

Specific behaviours
P determines first derivative
P equates derivative to zero
P solves for x values of both stationary pts
P uses a derivative test and shows values to determine nature
P determines y values of stationary pts

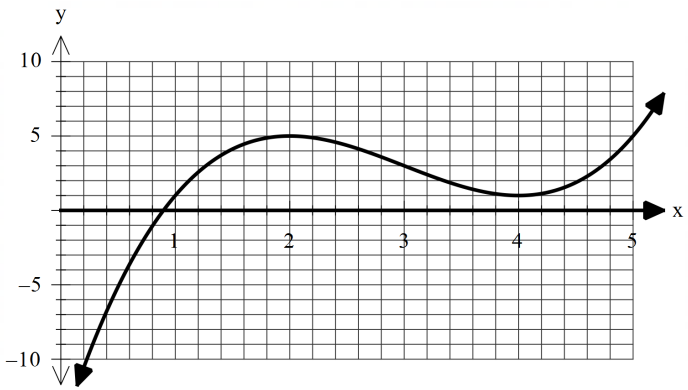
Q4 (3 marks)
The displacement of a body from an origin O, at time t seconds, is x metres where

$$x = t^3 - 3t^2 + 5t + 1, \quad t \geq 0$$

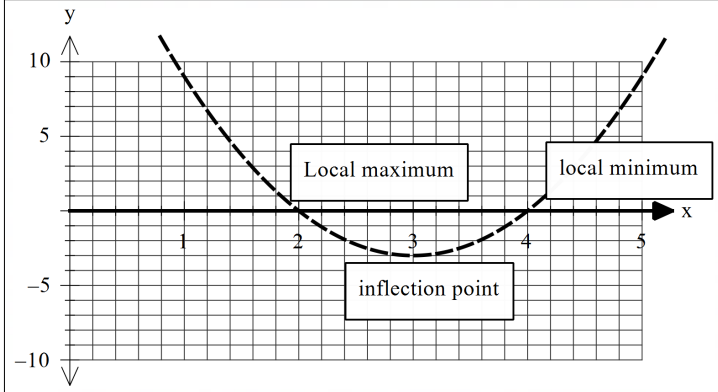
Determine the velocity and the displacement of the body when the acceleration is zero.

Solution
$x = t^3 - 3t^2 + 5t + 1, \quad t \geq 0$ $v = 3t^2 - 6t + 5$ $a = 6t - 6 = 0$ $t = 1$ $x = 1 - 3 + 5 + 1 = 4$ $v = 3 - 6 + 5 = 2$
Specific behaviours
P differentiates to determine velocity and acceleration
P equates acceleration to zero and solves for t
P states velocity and displacement for this time

Q5 (4 marks)
Consider the function $f(x)$ which is graphed below.



On the **axes below**, sketch the gradient function $f'(x)$ indicating on your sketch the location of any stationary points and any inflection points. (labelled)

Solution

Specific behaviours
P correct shape being concave up with a min turning pt (location may differ)
P labels local minimum (accept min)
P labels inflection pt
P labels local max (accept max)