Papers written by Australian Maths Software

SEMESTER TWO

YEAR 12

MATHEMATICS METHODS

Units 3-4

2017

Section One
(Calculator–free)

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 5 minutes
Working time for section: 50 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section One.
A formula sheet which may also be used for Section Two.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	**8**	**8**	**50**	**52**	**35**
Section Two Calculator—assumed	12	12	100	98	65
			Total marks	150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the Question/Answer booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

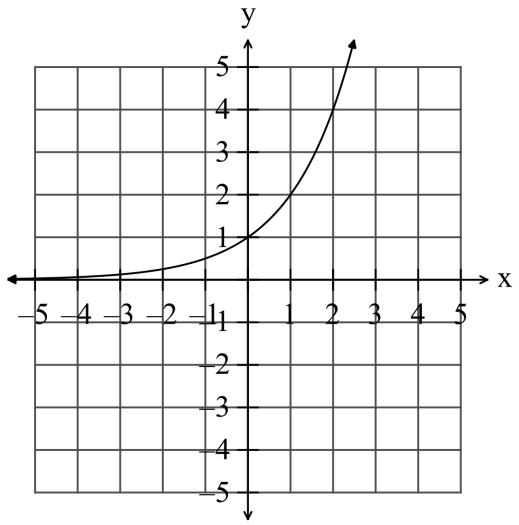
6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

Extra page for working if necessary

1. (6 marks)

(a) The graph of $y = 2^x$ is graphed on the set of axes below.



(i) Sketch $y = log_2 x$ on the set of axes above. (2)

(ii) Describe the relationship between $y = 2^x$ and $y = log_2 x$. (1)

(b) Prove that $ln(ab) = ln(a) + ln(b)$. (3)

2. (9 marks)

(a) Find the derivative of

(i) $y = e^{sin(x)}$ (1)

(ii) $y = x\,ln(x)$ (2)

(iii) $y = \frac{sin(2x)}{cos(3x)}$ (3)

(d) The sample proportion of university students who have taken a part time job in the past year is $p = 0.7$.

Ten random samples of students are selected and the students in each sample are asked if they have taken a part time job in the past year.

The proportion means of each sample is recorded.

(i) Explain why the distribution of sample means form a normal distribution. (2)

(ii) Write down the expression that if evaluated gives the standard deviation of the distribution. (1)

END OF SECTION ONE

8. (10 marks)

(a) Explain exactly how to obtain a random sample of 10 integers between 1 and 20 using your calculator. (3)

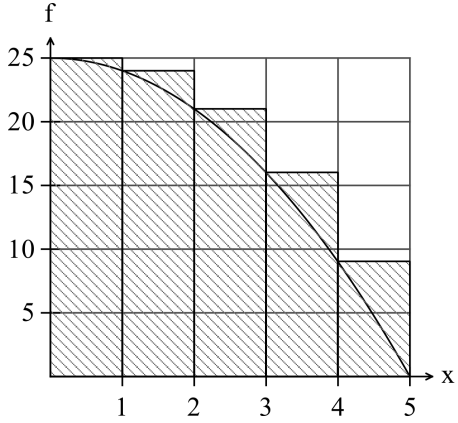
(b) A set of football attendees were asked their opinion on the increase in parking fees in the CBD.
Comment on any possible bias that may affect their response. (2)

(c) Explain why two random samples of the same size from the same distribution may have different means and standard deviations. (2)

(b) Let $f(x) = 4x + 3$ and $g(x) = ln(x)$, find $y = g(f(x))$ and hence find gradient at $x = 1$ of the function $y = g(f(x))$. (3)

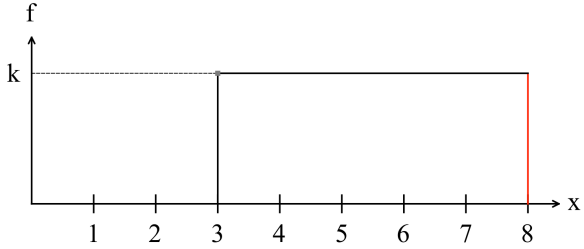
3. (5 marks)

(a) Estimate the area between the function $f(x) = 25 - x^2$ and the x axis for $0 \le x \le 5$ using rectangles from above. (3)



(b) Suggest whether using rectangles from above or below is the most accurate in this case and explain why. (2)

(b) The continuous random variable X has the probability density function graphed below.



Determine

(i) the value of k. (1)

(ii) $P(4 < x \le 8)$. (2)

7. (9 marks)

(a) The probability density function for the discrete random variable X is shown on the chart below.

x	0	1	2	3
$P(X = x)$	0.2	0.4	0.3	0.1

(i) Find the expected value nce of the distribution. (2)

(ii) Write down the expression that if evaluated gives the variance of the distribution. (2)

The variable X in (i) is transformed by the equation $Y = 4X - 3$.

(iii) Determine the expected value of Y. (1)

(iv) What number would you have to multiply the variance in (ii) by to obtain the variance of Y? (1)

4. (8 marks)

(a) Find

(i) $\int\left(x^6 - \frac{4}{x^2} + 2\sqrt{x} + \frac{1}{x}\right)dx$ (3)

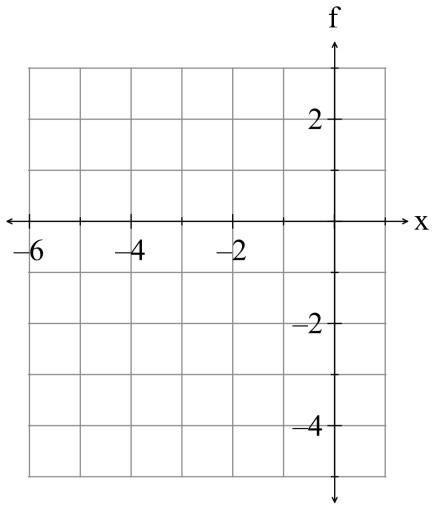
(ii) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\left(cos\left(2x\right) - sin\left(2x\right)\right)dx$ (3)

(iii) $\int\frac{2}{\left(2x-1\right)}dx$ (1)

(b) Simplify the expression $\frac{d}{dx}\left(\int_a^x \sqrt{1-t^2}\,dt\right)$. (1)

5. (3 marks)
Sketch the function on the set of axes below that has the following features. (3)

x	-5	-4	-3	-2	-1	0
f	-	$-\frac{8}{3}$	-	-4	-	+
f'	+	0	-	0	+	+
f''	-	-	0	+	+	+



6. (2 marks)

Given the function $y = f'(x)$ sketch the function $y = f(x)$ on the set of axes below. (2)