

43 Marks 9 Questions

Determine how long it will take for the temperature of the item to cool to 100°C to the nearest second.

Q3 (2, 4 & 3 = 9 marks)

The logistical growth model is given by the following differential equation.

$$\frac{dy}{dx} = ay - by^2 \quad \text{where } a \text{ \& } b \text{ are positive constants and } y > 0$$

a) State the y value where the gradient will be zero and hence give the limiting value of y .

b) Using separation of variables and partial fractions, derive the logistical formula

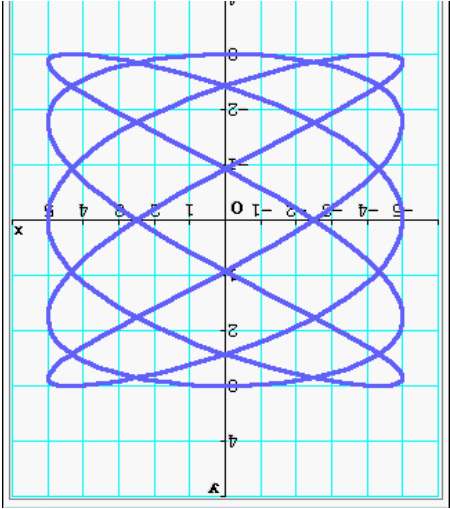
$$y = \frac{a}{b + Ce^{-ax}} \quad \text{where } C \text{ is a constant. Show all steps without the use of a classpad.}$$

Q9 (3 marks)

The Iron Man completes a race following a unique race track so that his position vector in metres

$$r = \begin{pmatrix} 5 \cos \frac{2\pi}{3}t \\ 3 \sin \frac{2\pi}{5}t \end{pmatrix} \text{ metres}$$

The motion is graphed as follows.

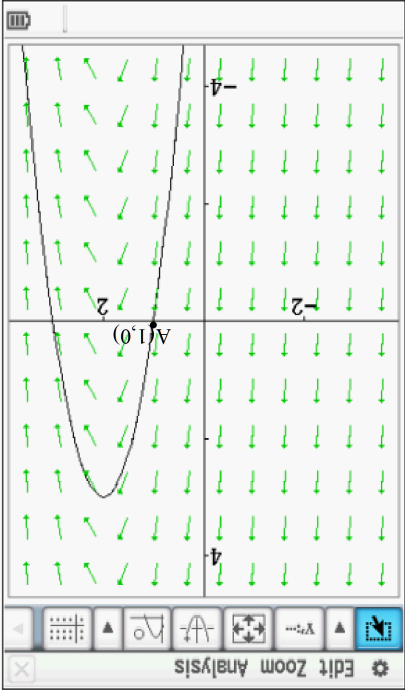


Determine the time taken to complete one circuit of the race track and the length of this circuit.

Q3 continued

c) Given that the Population P of a group of Kangaroos at t years (initially 285 kangaroos) can be modelled by the logistical growth model $\frac{dP}{dt} = \frac{1}{1}P - \frac{1}{13780}P^2$, determine the time taken for the population to reach 2000 kangaroos. Use your result from (b)

Q4 (4 marks)
A slope field is plotted below showing a particular line of force through point A(1,0). At point A the slope field is 6.



Given that the slopes are horizontal at $x=2$ and that the lines of force are parabolic:
Determine the equation of the line of force through point A and the slope field in terms of x .

Q5 (4 marks)

The Ant-Man is moving in a straight line so that his speed, v metres per second, at displacement x metres from the origin at time t seconds can be described by the following acceleration. The Ant-Man's speed is zero when $x=1$ metre from the origin.

$$\frac{dv}{dt} = x(5 + 3x^2)^5$$

Determine the approximate Ant-Man's speed when $x=5$ metres.

Q6 (4 marks)

A particle is undergoing Simple Harmonic Motion and can be described by $\ddot{x} = -36x$. Determine what percentage of the time that the particle is **less than three quarters** of the maximum distance from the origin.

Q7 (3 & 2 = 5 marks)

An object is undergoing SHM $\ddot{x} = -4x$ and is initially at rest with $x=15$ units but with a positive initial acceleration. Determine.

a) An expression for x in terms of time, t .

b) The distance travelled in the first 10 seconds.

Q8 (3 & 3 = 6 marks)

An object's displacement, x metres at time, t seconds is described by $x = 7 \cos(3t) - 5 \sin(3t)$

a) Show that the motion is Simple Harmonic.

b) Determine the Amplitude and the **exact** speed when $x=4$ metres.