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# Test 3

Calculus of Trigonometric Functions  
Discrete Random Variables  
Binomial Distributions  
**Semester One 2018**  
**Year 12 Mathematics Methods**  
**Calculator Assumed**

Name:

Date: Wed 2<sup>nd</sup> May

You may have a formula sheet for this section of the test.  
Classpad Calculators  
1 page of Notes

Total\_\_\_\_\_/41

45 minutes +5 minutes READING

Teacher:

\_\_\_\_\_ Mr McClelland

\_\_\_\_\_ Mrs. Carter

\_\_\_\_\_ Mr Gannon

\_\_\_\_\_ Ms Cheng

\_\_\_\_\_ Mr Staffe

\_\_\_\_\_ Mr Strain

## Question 1

(5 marks)

The discrete random variable  $X$  has the probability distribution shown in the table below.

$x$	0	1	2	3
$P(X = x)$	$\frac{2a^2}{3}$	$\frac{1 - 3a}{3}$	$\frac{1 + 2a}{3}$	$\frac{4a^2}{3}$

Determine the value of the constant  $a$ .

**Question 2****(8 marks)**

- (a) Differentiate  $e^{-3x} \sin(2x)$  with respect to  $x$ , showing full working. (2 marks)

- (b) Hence find the following indefinite integral. (3 marks)

$$-3 \int e^{-3x} \sin(2x) dx + 2 \int e^{-3x} \cos(2x) dx.$$

And using a similar process as part (a), find the indefinite integral for

$$-3 \int e^{-3x} \cos(2x) dx - 2 \int e^{-3x} \sin(2x) dx.$$

- (c) Use the two equations from (b) to determine  $\int e^{-3x} \sin(2x) dx$ . (3 marks)

**Question 3****(6 marks)**Differentiate with respect to  $x$ , (show full working)

(a)  $y = \sin^3(2x+1)$

(3 marks)

Evaluate the following, showing full working.

(b)  $\int_{\pi/6}^{\pi/2} \cos(2x) dx$

(3 marks)

**Question 4****(9 marks)**

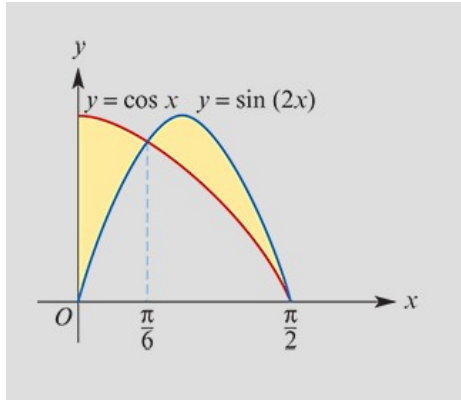
75% of the avocados produced by a farm are known to be first grade, the rest being second grade. Trays of 24 avocados are filled at random in a packing shed and sent to market.

Let the random variable  $X$  be the number of first grade avocados in a single tray.

- (a) Explain why  $X$  is a discrete random variable, and identify its probability distribution. (2 marks)
- (b) Calculate the mean and standard deviation of  $X$ . (2 marks)
- (c) Determine the probability that a randomly chosen tray contains
- (i) 18 first grade avocados. (1 mark)
- (ii) more than 15 but less than 20 first grade avocados. (2 marks)
- (d) In a random sample of 1000 trays, how many trays are likely to have fewer first grade than second grade avocados. (2 marks)

**Question 5****(4 marks)**

Find the area between the two curves from  $0 \leq x \leq \frac{\pi}{2}$ , showing full algebraic reasoning.



**Question 6****(9 marks)**

(a) A sample of six objects is to be drawn from a large population in which 20% of the objects are defective. Find the probability that the sample contains:

(i) three defectives. (2 marks)

(ii) fewer than three defectives. (2 marks)

(b) Another large population contains a proportion  $p$  of defective items.

(i) Write down an expression in terms of  $p$  for  $P$ , the probability that a sample of six items contains exactly two defectives. (2 marks)

(ii) By differentiating to find  $\frac{dP}{dp}$ , show that  $P$  is greatest when  $p = \frac{1}{3}$ . (3 marks)