

Western Australian Certificate of Education

Examination, 2014

Question/Answer Booklet

MATHEMATICS

3C/3D

Section Two:

Calculator-assumed

Place one of your candidate identification labels in this box.

↑

↓

Ensure the label is straight and within the lines of this box.

Student Number:

In figures

In words

Time allowed for this section

Reading time before commencing work:

Working time for section:

ten minutes

one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

- To be provided by the candidate

Standard items:

Special items:

pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Copyright © School Curriculum and Standards Authority 2014



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	9	9	50	50	33⅓
Section Two: Calculator-assumed	13	13	100	100	66⅔
Total					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2014*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Additional working space

Question number: \_\_\_\_\_

This section has 13 questions. Answer all questions. Write your answers in the spaces provided.

- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

**Question 10 (5 marks)**

The freshly hatched caterpillar weighs approximately 1.5 milligrams. It will grow to 1000 times its initial body weight in just three weeks. The rate of growth of the weight can be expressed by the differential equation  $\frac{dw}{dt} = kw$  where  $w$  = weight of the caterpillar at time  $t$ , in days, and  $k$  is a constant.

- (a) Determine the value of  $k$  to three decimal places. (3 marks)

- (b) When is the caterpillar double its initial body weight? (2 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question number: \_\_\_\_\_

Question 11

(11 marks)

A family is going camping for a week and will be taking their pet with them. They can only find room for five kilograms of food for their pet and will bring only two types of pet food, type X and type Y. Each kilogram of type X food has 10 grams of fat, 9 grams of carbohydrate and 12 grams of protein. Each kilogram of type Y has 15 grams of fat, 9 grams of carbohydrate and 6 grams of protein. The pet will need at least 30 grams of fat, 27 grams of carbohydrate and 24 grams of protein for the week.

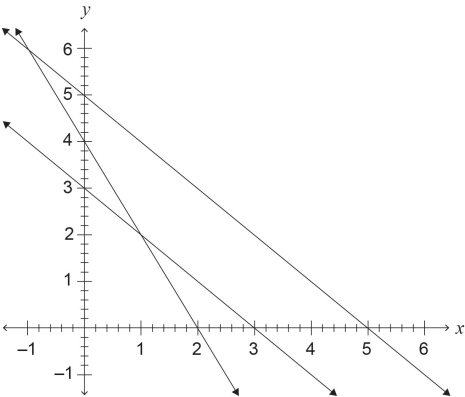
The cost of type X food is 20 cents per kilogram and type Y is 30 cents per kilogram.

Let  $x$  = number of kilograms of type X food and let  $y$  = number of kilograms of type Y food.

- (a) The constraints above can be written as inequalities. One constraint is missing. Write the missing constraint as an inequality in terms of  $x$  and  $y$  in simplified form. (2 marks)

$$9x + 9y \geq 27$$
$$12x + 6y \geq 24$$
$$x + y \leq 5$$
$$x \geq 0$$
$$y \geq 0$$

- (b) The inequalities are shown on the graph below. Add the missing inequality and shade the feasible region. (2 marks)



See next page

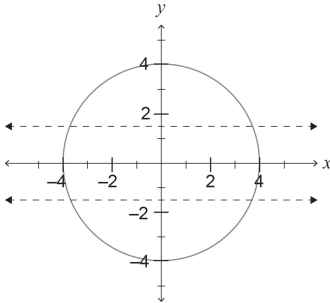
DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 22

(8 marks)

- (a) The volume of a spherical balloon is increasing at a rate of 1200 cubic metres per minute. Determine the rate of change of the radius, in metres per minute, when the radius is 12 metres. (3 marks)

- (b) A solid wooden sphere with a radius of four metres has a cylindrical hole drilled through the centre with a diameter of three metres, as shown in the diagram below. The cross-section of the sphere is defined by  $x^2 + y^2 = 16$ . Determine the volume, in cubic metres, of the remaining material in the sphere to three decimal places. (5 marks)



End of questions

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 21

For any two unequal positive numbers  $a$  and  $b$ , the arithmetic mean is defined by  $\frac{a+b}{2}$  while the geometric mean is defined by  $\sqrt{ab}$ .  
It is conjectured that the arithmetic mean of two unequal positive numbers is always greater than the geometric mean.  
(a) Provide **two** pairs of numbers to demonstrate that the conjecture is true. (2 marks)

(b) Prove algebraically that the conjecture is true for all unequal positive numbers  $a$  and  $b$ . (4 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

(c) Determine the optimal amounts of each type of pet food to minimise the cost. State this minimum cost. (4 marks)

(d) To ensure a larger amount of type Y food, by how much should the price per kilogram of type Y food fall to maintain the minimum cost from part (c)? (3 marks)

See next page

Question 12

(5 marks)

A bicycle is travelling at a constant speed of 20 kilometres per hour.

- (a)
- Determine the distance, in metres, that the bicycle travels in one second.
- (1 mark)

When the brakes of the bicycle are applied, this results in a deceleration (negative acceleration) of 10 metres per second squared. Let  $t$  represent the time, in seconds, from when the brakes are initially applied.

- (b)
- State the velocity function of the bicycle, in metres per second, in terms of  $t$  after the brakes are applied.
- (2 marks)

- (c)
- How far will the bicycle travel while braking before it stops?
- (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 20

(8 marks)

Five members of a complex of flats form a strata committee. One of these members is an elected president. Let event  $A$  = the number of subcommittees formed with a **prime** number of people from the strata committee. Let event  $B$  = the number of subcommittees that contain the president from the strata committee. Assume that all possible subcommittees are equally likely. A subcommittee consists of one or more people from the strata committee.  
(Note: the number **one** is not a prime.)

Determine

- (a)
- the total number of subcommittees possible.
- (2 marks)

- (b)
- $P(A \cap B)$ .
- (3 marks)

- (c)
- $P(A \cup B)$ .
- (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 19 (10 marks)

A pay TV service sends a signal through the telephone lines to each of its customers every hour. The length of the signal is between two and nine seconds and follows a uniform distribution.

(a) Define the probability density function for the length of the signal. (2 marks)

(b) Determine the probability that the signal is longer than four seconds. (2 marks)

(c) Determine the probability that, on any given day, at least half of the signals are greater than four seconds. (3 marks)

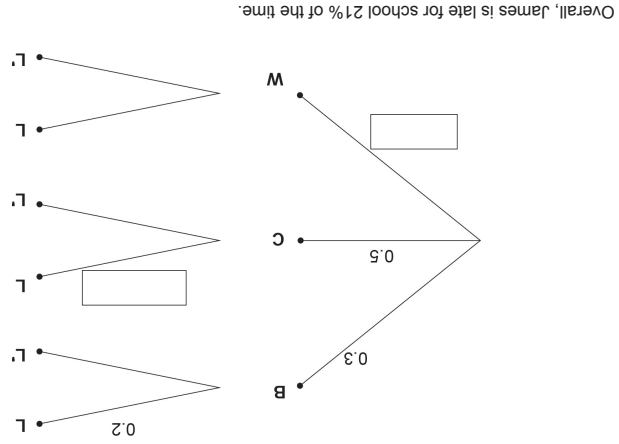
(d) One particular day, fewer than 21 signals were longer than four seconds. Determine the probability that at least 15 were longer than four seconds. (3 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 13 (6 marks)

James travels to school in one of three ways. Thirty per cent of the time he rides his bicycle (B), 50% of the time his mother drives him (C) and the rest of the time he walks (W). When he rides his bicycle, there is a 20% chance of his having a puncture that will make him late for school (L). On the days he walks, he is never late for school.



(a) Part of the tree diagram is shown above. Write the two unknown probabilities in the boxes above. (3 marks)

(b) On a day when he arrives late for school, what is the probability that he has ridden his bicycle? (3 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 14

(16 marks)

A study found that 80 per cent of people exhibiting common influenza symptoms recovered without taking any medication. A random sample of 30 people who had developed influenza symptoms was taken.

Let  $X$  denote the number of people in this sample who recovered without taking any medication.

- (a)

Is  $X$  discrete or continuous?

(1 mark)
- (b)

State the probability distribution of  $X$  and the mean and standard deviation of this distribution.

(3 marks)
- (c)

What is the probability, correct to three decimal places, that

(i)

exactly 25 people recovered without any medication?

(1 mark)

(ii)

at least 24 but no more than 28 people recover without any medication?

(2 marks)
- (d)

Trial groups of 30 people from each of 15 different suburbs were then surveyed. Let  $\bar{Y}$  denote the mean number of people per trial group who recover without any medication.

(i)

State the probability distribution of  $\bar{Y}$  and the mean and standard deviation of this distribution.

(3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 18

(4 marks)

Let  $I(x) = \int_{-3}^x g(t)dt$  with  $I(5) = 208$  and  $\frac{d^2I}{dx^2} = 6x$ .

Determine the function  $g(x)$ .

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF



Question 17 (7 marks)

The label on a bottle states that it contains 330 millilitres of orange juice. The capacity of the orange juice in these bottles is normally distributed, with a mean of 365 millilitres and a standard deviation of 20 millilitres.

(a) In a batch of 100 bottles, how many bottles are expected to have less than the labelled amount? (2 marks)

(b) Samples of 10 bottles are tested and the mean capacity for each sample is recorded.

(i) State the distribution for the sample means and their mean and standard deviation. (2 marks)

(ii) Calculate the probability that the sample mean is less than 360 millilitres. (1 mark)

(c) Determine the sample size so that there is a 99% chance that the sample mean is no more than 5 millilitres from the population mean. (2 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

(ii) Determine  $P(\bar{Y} \geq 25)$ . (1 mark)

(iii) Determine a 95% confidence interval, to three decimal places, for the population mean number of people per trial who recover without medication. (3 marks)

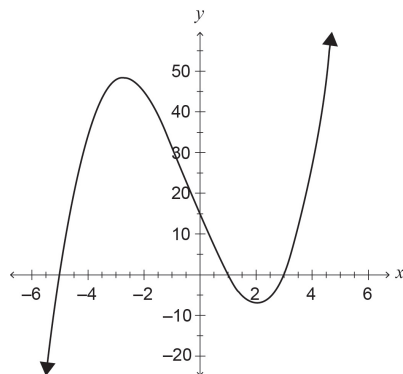
(iv) The researcher who conducted the trials in the 15 suburbs calculated a mean of 25 people who recovered without medication per trial. The researcher concluded that a smaller percentage of influenza sufferers take medication than had been assumed. Does this mean support her conclusion? Explain. (2 marks)

See next page

Question 15

(7 marks)

Consider the curve defined by the rule  $y = x^3 + x^2 - 17x + 15$  shown below.



- (a) Show that the equation of the tangent at  $x = -2$  is  $y = -9x + 27$ . (3 marks)

- (b) Determine the area enclosed between the curve and the tangent at  $x = -2$ . (4 marks)

See next page

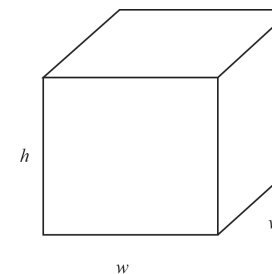
DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 16

(7 marks)

A closed box is constructed with a square base. Exactly 10 square metres of material is to be used in the construction of the box, without wastage.

Let  $h$  = height of the box,  $w$  = width of box = length of box.



- (a) Show that  $5 = w^2 + 2wh$ . (2 marks)

- (b) By using calculus, determine the maximum volume of the box and state the dimensions required to achieve this maximum. (5 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page