Calculator Assumed
Applications of Logarithms

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [3, 3, 3 = 9 marks] CA

The magnitude of an earthquake, known as the Richter scale, is given by:

$$M = \log_{10} I - \log_{10} S$$

where M is the magnitude of the earthquake, I is the intensity measurement of the earthquake and S is the standard intensity of an earthquake.

(a) The magnitude of the 2011 Christchurch earthquake measured 6.3 on the Richter scale. Find an expression for the intensity of this earthquake in terms of S.

(b) If in the same year there was another earthquake that was three times stronger in intensity, what was the magnitude of this second earthquake?

(c) In 2013 an earthquake in the Solomon Islands measured 8.0 on the Richter scale. How much more intense was this earthquake compared with the one in Christchurch?

Question Two: [1, 2, 2, 2 = 7 marks] **CA**

The mass *M,* in grams, of a radioactive substance after *t* years is given by :

$$M = 13.8 - \ln(t + 43.1)$$

(a) State the initial mass of the substance.

(b) Determine an expression for the instantaneous rate of change of the mass with respect to time.

(c) Calculate the decrease in mass in the 100th year.

(d) Calculate the average decrease in the mass over 100 years.

Question Three: [2, 2, 3, 2, 3 =12 marks] CA

Kepler's Third Law states that the square of the orbit of the period of a planet, T, is proportional to the cube of its average distance from the sun, a.

(a) State Kepler's Third Law is terms of T and a.

(b) The table below gives values of a and T, using Earth units.

Planet	Earth	Mercury	Mars	Jupiter	Saturn
a	1	0.3870	1.523	5.203	9.539
T	1	0.2410	1.881	11.86	29.46
$\ln a$					
$\ln T$					

Calculate the rows for $\ln a$ and $\ln T$.

(c) Using your CAS calculator, fit a linear regression model using the data from the table. Input the values for $\ln a$ in List 1 and the values for $\ln T$ in List 2. State the linear regression model.

(d) By stating the value of the correlation coefficient, r, explain why a linear model is appropriate.

Since the vertical intercept of the answer to (c) is almost 0, we can say that $\ln T \approx 1.5 \ln a$

(e) Hence show using algebraic manipulation that Kepler's Third Law (as stated in part (a)) holds true from our data.

Question Four: **[2, 2, 2, 3 = 9 marks]** **CA**

Laura is starting a new fitness routine and she completes 2 sets of 5 repetitions of squats each day. Her aim is to get stronger and lift heavier each day.

Laura models her progress over t days by the function: $f(t) = 5 + k\ln(t+10)$, where $f(t)$ is the weight in kilograms of her squat each day.

(a) Calculate the value of k if initially Laura lifts 30 kg.

(b) After 2 weeks of training, by how many kilograms has her strength increased?

(c) Calculate the rate of change of Laura's strength with respect to time.

(d) Determine when Laura's increase in strength is half of what it was initially.

Question Five: [3, 2, 2 =7 marks] **CA**

The instantaneous rate of change of the number of fish over t weeks, being farmed in a fish farm can be modelled by $P'(t) = \frac{-4970}{t+e}$ where $P(t)$ is the population after t weeks.

(a) If after 5 weeks there are 12 000 fish left, determine an expression for $P(t)$.

(b) Calculate the initial number of fish when the study began.

(c) When the decline in fish each week falls below 500, the farmer is no longer as concerned for his fish stock. During which week does this occur?

SOLUTIONS
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Question One: [3, 3, 3 = 9 marks] **CA**

The magnitude of an earthquake, known as the Richter scale, is given by:

$$M = \log_{10} I - \log_{10} S$$

where M is the magnitude of the earthquake, I is the intensity measurement of the earthquake and S is the standard intensity of an earthquake.

(a) The magnitude of the 2011 Christchurch earthquake measured 6.3 on the Richter scale. Find an expression for the intensity of this earthquake in terms of S.

$$6.3 = \log_{10} I - \log_{10} S \checkmark$$
$$6.3 = \log_{10} \frac{I}{S} \checkmark$$
$$10^{6.3} = \frac{I}{S}$$
$$I = 10^{6.3} \times S \checkmark$$

(b) If in the same year there was another earthquake that was three times stronger in intensity, what was the magnitude of this second earthquake?

$$M = \log_{10} \frac{3I}{S} \checkmark$$
$$M = \log_{10} 3I - \log_{10} S$$
$$M = \log_{10} 3 + \log_{10} I - \log_{10} S \checkmark$$
$$M = \log_{10} 3 + \log_{10} \frac{I}{S}$$
$$M = \log_{10} 3 + 6.3$$
$$M = 6.78 \checkmark$$

(c) In 2013 an earthquake in the Solomon Islands measured 8.0 on the Richter scale. How much more intense was this earthquake compared with the one in Christchurch?

$$8.0 = \log_{10} I - \log_{10} S$$
$$8.0 = \log_{10} \frac{I}{S}$$
$$10^8 = \frac{I}{S}$$
$$I = 10^8 \times S$$
$$\frac{I_{Solomon}}{I_{Christchurch}} = \frac{10^8 \times S}{10^{6.3} \times S} = 10^{1.7} \approx 50$$

Approximately 50 times more intense.

Question Two: [1, 2, 2, 2 = 7 marks] **CA**

The mass *M,* in grams, of a radioactive substance after *t* years is given by:

$$M = 13.8 - \ln(t + 43.1)$$

(a) State the initial mass of the substance.

$$M = 13.8 - \ln 43.1 = 10.04g$$

(b) Determine an expression for the instantaneous rate of change of the mass with respect to time.

$$\frac{dM}{dt} = \frac{-1}{t + 43.1}$$

(c) Calculate the decrease in mass in the 100th year.

$$M = [13.8 - \ln(100 + 43.1)] - [13.8 - \ln(99 + 43.1)]$$
$$M = -0.00701g$$

A decrease of 0.00701 g

(d) Calculate the average decrease in the mass over 100 years.

$$\frac{M(100) - M(0)}{100} = -0.012g / year$$

Question Three: [2, 2, 3, 2, 3 =12 marks] CA

Kepler's Third Law states that the square of the orbit of the period of a planet, T, is proportional to the cube of its average distance from the sun, a.

(a) State Kepler's Third Law is terms of T and a.

$T^2 = a^3$

(b) The table below gives values of a and T, using Earth units.

Planet	Earth	Mercury	Mars	Jupiter	Saturn
a	1	0.3870	1.523	5.203	9.539
T	1	0.2410	1.881	11.86	29.46
$\ln a$	0	-0.949	0.421	1.649	2.255
$\ln T$	0	-1.423	0.632	2.473	3.383

Calculate the rows for $\ln a$ and $\ln T$.

(c) Using your CAS calculator, fit a linear regression model using the data from the table. Input the values for $\ln a$ in List 1 and the values for $\ln T$ in List 2. State the linear regression model.

$\ln T = 1.5 \times \ln a + 0.00027$

(d) By stating the value of the correlation coefficient, r, explain why a linear model is appropriate.

The correlation coefficient is 1, giving a perfect linear relationship.

Since the vertical intercept of the answer to (c) is almost 0, we can say that $\ln T \approx 1.5 \ln a$

(e) Hence show using algebraic manipulation that Kepler's Third Law (as stated in part (a)) holds true from our data.

$$\ln T = \frac{3}{2}\ln a \checkmark$$
$$2\ln T = 3\ln a \checkmark$$
$$\ln T^2 = \ln a^3 \checkmark$$
$$T^2 = a^3$$

Question Four: [2, 2, 2, 3 = 9 marks] CA

Laura is starting a new fitness routine and she completes 2 sets of 5 repetitions of squats each day. Her aim is to get stronger and lift heavier each day.

Laura models her progress over t days by the function: $f(t) = 5 + k\ln(t+10)$, where $f(t)$ is the weight in kilograms of her squat each day.

(a) Calculate the value of k if initially Laura lifts 30 kg.

$$30 = 5 + k\ln 10 \checkmark$$
$$k = 10.86 \checkmark$$

(b) After 2 weeks of training, by how many kilograms has her strength increased?

$$f(14) = 5 + 10.86\ln(24) = 39.51 \checkmark$$
$$\therefore 9.51\ kg \checkmark$$

(c) Calculate the rate of change of Laura's strength with respect to time.

$$f'(t) = \frac{10.86}{t+10} \checkmark\checkmark$$

(d) Determine when Laura's increase in strength is half of what it was initially.

$$f'(0) = \frac{10.86}{0+10} = 1.086 kg/day \checkmark$$
$$0.543 = \frac{10.86}{t+10} \checkmark$$
$$t = 10 \checkmark$$

Question Five: [3, 2, 2 =7 marks] **CA**

The instantaneous rate of change of the number of fish over t weeks, being farmed in a fish farm can be modelled by $P'(t) = \dfrac{-4970}{t+e}$ where $P(t)$ is the population after t weeks.

(a) If after 5 weeks there are 12 000 fish left, determine an expression for $P(t)$.

$P(t) = -4970\ln(t+e)+c$

$12000 = -4970\ln(5+e)+c$

$c = 22156.65$

$P(t) = -4970\ln(t+e)+22156.65$

(b) Calculate the initial number of fish when the study began.

$P(0) = 17186.65$

(c) When the decline in fish each week falls below 500, the farmer is no longer as concerned for his fish stock. During which week does this occur?

$-500 > \dfrac{-4970}{t+e}$

$t = 7.22$

During the 8th week.