A water level of 6.0 metres is the critical level for boat safety.

(b) Write the equation (with an appropriate domain) that needs to be solved to determine when, on **28th March 2019**, a water level of 6.0 metres occurs. (2 marks)

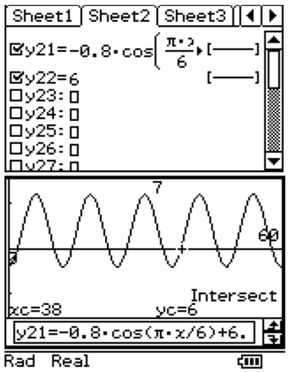
Solution
28th March 2019 corresponds to the period of time where $34 \le t \le 58$ hrs Need to solve $W(t)=6$ where $34 \le t \le 58$ i.e. $-0.8\cos\left(\frac{\pi t}{6}\right)+6.4 = 6$
Specific behaviours
✓ forms the equation correctly ✓ states the correct domain for 28th March 2019

1.2.16

The water level must be greater than 6.0 metres for boat safety. A mathematician wished to make a prediction for when the river will be safe for boats on **28th March 2019**.

(c) Determine when the river will be safe for boats on **28th March 2019**. (2 marks)

Solution

Solving $-0.8\cos\left(\frac{\pi t}{6}\right)+6.4 = 6$ where $34 \le t \le 58$
Using CAS: $t=34,\ t=38,\ t=46,\ t=50,\ t=58$.
 	
Part of CAS screenshot (note the domain restriction with the solve command)
`solve(-0.8×cos(πx/6)+6.4=6,x)|34≤x≤58`
`{x=34,x=38,x=46,x=50,x=58}`
 	
But for safety we require the graph of $W(t)$ above 6.0 metres.

$W(t)>6$ when
$38<t<46,\quad 50<t<58$
$\therefore$ Safe on 28th March:
4:00 am – 12 noon AND
4:00 pm – 12 midnight

Specific behaviours
✓ determines ONE period of values of t correctly ✓ determines the corresponding times during 28th March correctly

1.2.16

Christ Church Grammar School

2019
TEST 3

MATHEMATICS METHODS Year 11

MARKING KEY

Time and marks available:

Calculator-Free
Reading time for this section: 3 minutes
Working time for this section: 30 minutes
Marks available: **30 marks**

Calculator-Assumed
Working time for this section: 10 minutes
Marks available: **8 marks**

Materials required/recommended:
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

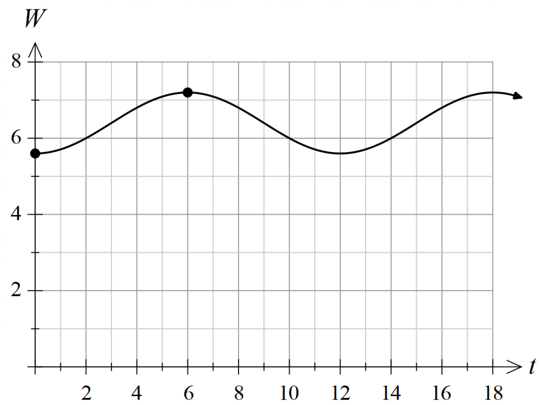
1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. Answer all questions.
4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
7. It is recommended that **you do not use pencil**, except in diagrams.

Calculator-Assumed Section (8 marks)

Question 8 (8 marks)

The water level at a fixed point in a river varies with the rise and fall of the tide, as shown in the diagram below. At 2:00 pm one afternoon on 26th March 2019, it is noticed that the water level is 5.6 metres at low tide. At 8:00 pm, the next time for high tide, the water level is 7.2 metres.

Let $t =$ the time in hours elapsed after 2:00 pm on 26th March 2019.
$W(t) =$ the water level measured in metres.



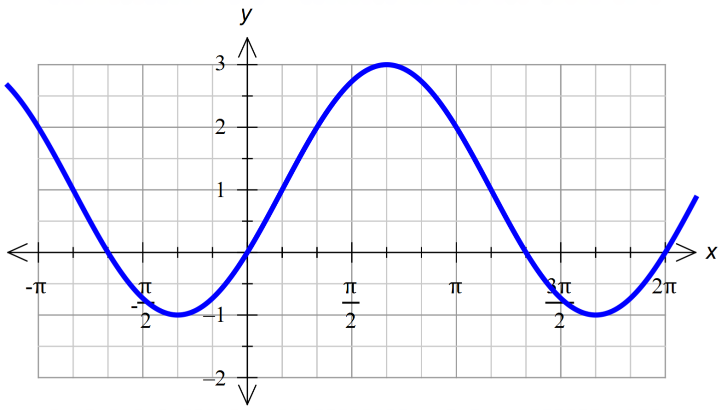
(a) If the water level is modelled by the function $W(t) = a\cos(bt)+c$ detemine the values of the constants a, b and c. (4 marks)

Solution
Equilibrium value $c = \dfrac{5.6+7.2}{2} = 6.4$ Period of function $T = 12\ hrs \quad \therefore\ 12 = \dfrac{2\pi}{b} \quad \therefore\ b = \dfrac{\pi}{6}$ Amplitude $\lvert a\rvert = \dfrac{7.2-5.6}{2} = 0.8 \quad \therefore\ a = -0.8$ (starts at low tide)   Hence $W(t) = -0.8\cos\left(\dfrac{\pi t}{6}\right)+6.4$
Specific behaviours
✓ determines the value of c correctly   ✓ determines the value of b correctly   ✓ states the amplitude is 0.8 metres   ✓ determines the value of a correctly i.e. $a = -0.8$

1.2.15

Question 7 **(4 marks)**

On the axes below, sketch the graph of $g(x) = 2\sin\left(x - \frac{\pi}{6}\right) + 1$ for $-\pi \le x \le 2\pi$

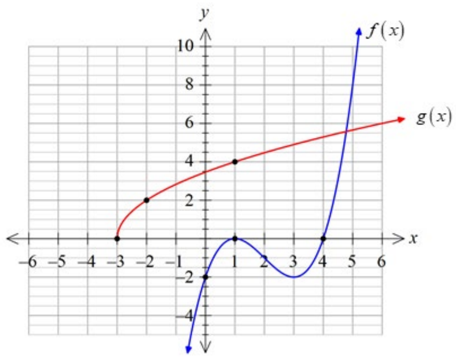


Solution
Shown above.
Specific behaviours
✓ indicates an amplitude of 2 units 1.2.10 ✓ indicates a period of 2π units 1.2.11 ✓ indicates a phase shift of $\frac{\pi}{6}$ to the right 1.2.12 ✓ indicates an equilibrium value at y = 1 (or *y*-intercept at (0,0)) 1.2.8

Calculator-Free Section (34 marks)

Question 1 **(7 marks)**

The graphs of functions $f(x)$ and $g(x)$ are shown below.



Determine the defining rule for function:

(a) $f(x)$ in the form $ax^3 + bx^2 + cx + d$. (4 marks)

Solution
$f(x) = k(x-1)^2(x-4)$ Using $f(0) = -2$ $-2 = k(0-1)^2(0-4)$ i.e. $-2 = -4k$ $\therefore\ k = 0.5$ i.e. $f(x) = 0.5(x-1)^2(x-4)$ $= 0.5x^3 - 3x^2 + 4.5x - 2$
Specific behaviours
✓ writes $f(x)$ with factor $(x-1)^2$ 1.1.18 ✓ writes $f(x)$ with factor $(x-4)$ 1.1.18 ✓ determines dilation factor $k = 0.5$ 1.1.18 ✓ expands correctly to determine standard form 1.1.17

(b) $g(x)$ (3 marks)

Solution
$g(x) = k\sqrt{x+3}$ Using $g(1) = 4$ $4 = k\sqrt{1+3}$ $4 = 2k$ $\therefore\ k = 2$ i.e. $g(x) = 2\sqrt{x+3}$
Specific behaviours
✓ identifies $g(x)$ as a square root function ✓ writes $g(x)$ with factor $\sqrt{x+3}$ ✓ determines the dilation factor $k = 2$

1.1.15

Question 2 **(8 marks)**

Consider the polynomial $P(x) = x^3 - 5x^2 + 2x + 8$.

(a) State the degree of function $P(x)$. (1 mark)

Solution
Degree = 3 (cubic)
Specific behaviours
✓ states the correct value of the degree

1.1.16

(b) Show that $(x+1)$ is a factor of $P(x)$. (2 marks)

Solution
$P(-1) = (-1)^3 - 5(-1)^2 + 2(-1) + 8$ $= (-1) - 5 - 2 + 8 \quad ...(1)$ $= 0$
Specific behaviours
✓ substitutes $x=-1$ correctly into $P(x)$ ✓ demonstrates that $P(-1)=0$ by obtaining the expression (1)

1.1.19

OR

Alternative Solution	
$$\begin{array}{r\|l} & x^2 - 6x + 8 \\ \hline x+1 & x^3 - 5x^2 + 2x + 8 \\ & -(x^3 + x^2) \\ & -6x^2 + 2x \\ & -(-6x^2 - 6x) \\ & 8x + 8 \\ & -(8x+8) \\ & 0 \end{array}$$	Since there is ZERO remainder upon division by $x+1$ then it is a FACTOR of $P(x)$.
Specific behaviours	
✓ carries out the long division process correctly ✓ states that there is ZERO remainder when dividing by $x+1$	

1.1.19

Question 6 **(3 marks)**

A and ***B*** are acute angles with $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$.

Determine the exact value of $\cos(A-B)$.

$$\cos A = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= \frac{4}{5}$$

$$\sin B = \sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$= \frac{5}{13}$$

$$\therefore \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$$

$$= \frac{48+15}{65}$$

$$= \frac{63}{65}$$

✓ determining the exact values of cosA and sinB

✓ applying the correct expansion formula

✓ obtaining a correctly simplified answer as an exact value

(c) Hence fully factorise $P(x)$. (2 marks)

Solution
$P(x) = (x+1)(x^2-6x+8)$ $= (x+1)(x-2)(x-4)$
Specific behaviours
✓ determines the quadratic factor correctly (by long division or otherwise) ✓ determines all 3 factors correctly

1.1.19

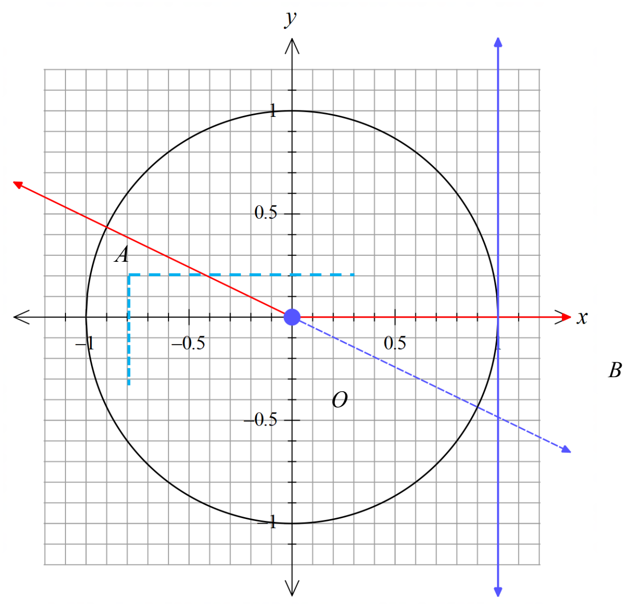
(d) Solve the equation $P(x) = -x^2 - x + 8$. (3 marks)

Solution
Solve $x^3 - 5x^2 + 2x + 8 = -x^2 - x + 8$ i.e. $x^3 - 4x^2 + 3x = 0$ i.e. $x(x^2 - 4x + 3) = 0$ i.e. $x(x-1)(x-3) = 0$ $\therefore$ $x = 0,\ x = 1,\ x = 3$
Specific behaviours
✓ forms the standard cubic equation correctly ✓ factorises fully the left hand expression correctly ✓ states correctly ALL solutions

1.1.20

Question 5 **(3 marks)**

The diagram below shows the unit circle with $s\angle AOB = \theta$ radians.



From the unit circle, state, correct to 0.01, the value for:

(a) $\cos\theta$ (1 mark)

Solution
$\cos\theta = -0.90$ (the x coordinate on the unit circle)
Specific behaviours
✓ states the correct value (allow values of $-0.89,\ -0.9,\ -0.91$) 1.2.7

(b) $\sin\theta$ (1 mark)

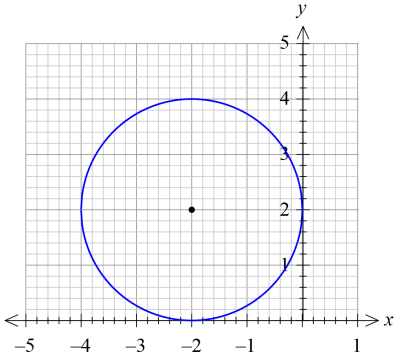
Solution
$\sin\theta = 0.44$ (the y coordinate on the unit circle)
Specific behaviours
✓ states the correct value (allow values of $0.43,\ 0.44,\ 0.45$) 1.2.7

(c) $\cos(\theta + \pi)$ (1 mark)

Solution
$\cos(\theta+\pi) = -\cos\theta = 0.90$
Specific behaviours
✓ states the correct value (allow values of $0.89,\ 0.9,\ 0.91$) OR states a value that is the OPPOSITE of the value given at part (a) 1.2.7

Question 3 **(2 marks)**

The graph of a circle is shown below. Determine the equation for this circle.

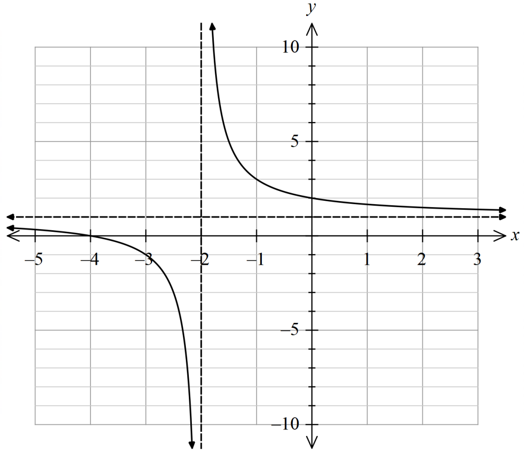


Solution
Centre is $(-2, 2)$ and radius $= 2$ units $\therefore\ (x+2)^2 + (y-2)^2 = 2^2$ OR $x^2 + 4x + y^2 - 4y + 4 = 0$
Specific behaviours
✓ observes the correct centre and radius ✓ forms the equation of the circle correctly

1.1.21

Question 4 **(3 marks)**

The diagram shows the graph of function $f(x) = \dfrac{2}{x+2} + 1$



In each question, write the specific defining rule if the following transformations are applied to $f(x) = \dfrac{2}{x+2} + 1$.

(a) Reflect about the y axis. (1 mark)

Solution
$y = \dfrac{2}{x+2} + 1 \xrightarrow{} y = \dfrac{2}{(-x)+2} + 1$ i.e. $y = -\dfrac{2}{x-2} + 1$ F
Specific behaviours
✓ writes the correct defining rule

1.1.27

(b) Translate 1 unit down, then dilate vertically about the *y*-axis with factor 2. (2 marks)

Solution
$y = \dfrac{2}{x+2} + 1 \xrightarrow[D]{} y = \dfrac{2}{x+2} \xrightarrow[H]{} y = \dfrac{4}{x+2}$
Specific behaviours
✓ subtracts 1 from constant (transformation D) 1.1.26 ✓ writes factor 4 in numerator (transformation H) 1.1.27

1.1.26