PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

Course Specialist Year 12 Test Two 2022

Student name: _____ Teacher name: _____

Task type: **Response**

Time allowed for this task: **_____40_____ mins**

Number of questions: **_____6_____**

Materials required: Upto 3 Calculators with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **__41____ marks**

Task weighting: **_10___%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2, 3 & 3= 8 marks)

Consider the functions $f(x) = \sqrt{x - 2}$ and $g(x) = \frac{1}{x}$

a) Determine the natural domains of f & g.

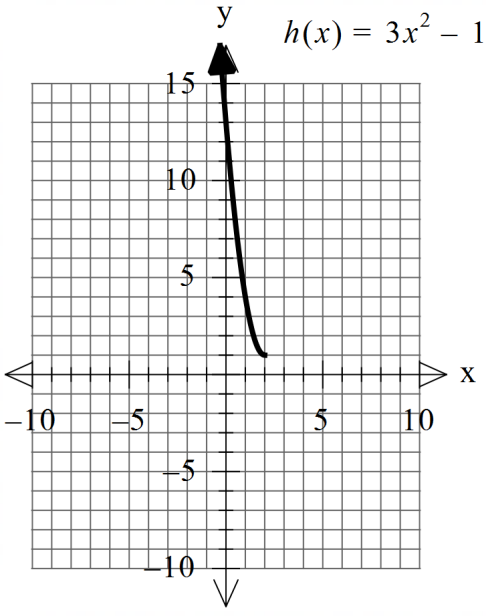
b) Does $f \circ g(x)$ exist over the natural domain of g ? Explain.

c) State the rule and largest possible domain for $g \circ f(x)$ and its corresponding range.

Q2 (2, 4, 1 & 3 = 10 marks)

The function $h(x)$ is defined below for $x \leq 2$.

$h(x) = 3x^2 - 12x + 13$



a) Sketch the inverse function $h^{-1}(x)$ on the axes above.

Q2 continued

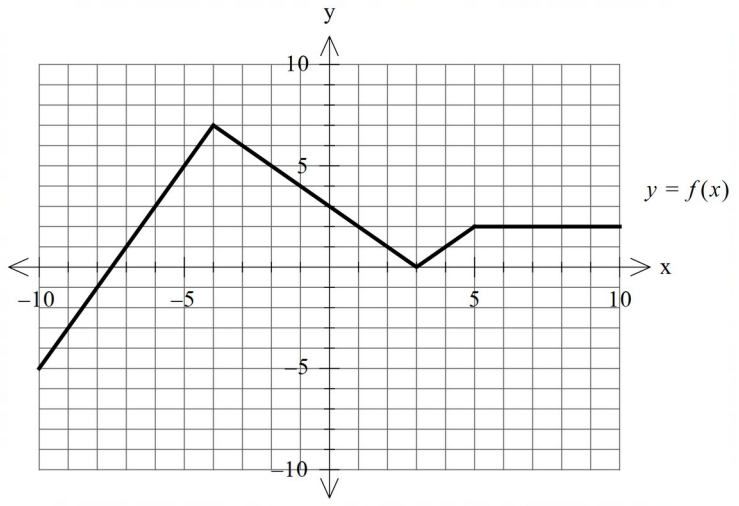
b) Determine the rule for $h^{-1}(x)$ and its domain showing **full working**.

c) Determine $h \circ h^{-1}(x)$.

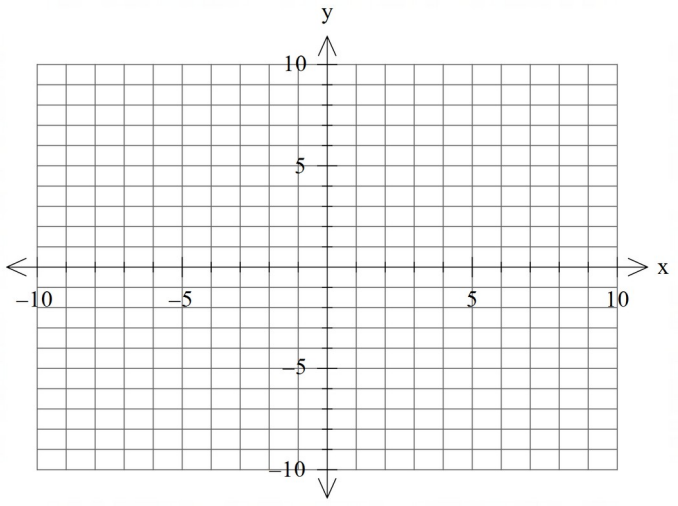
d) Determine the exact coordinates (if any) for where $h(x) = h^{-1}(x)$.

Q3 (2 & 3 = 5 marks)

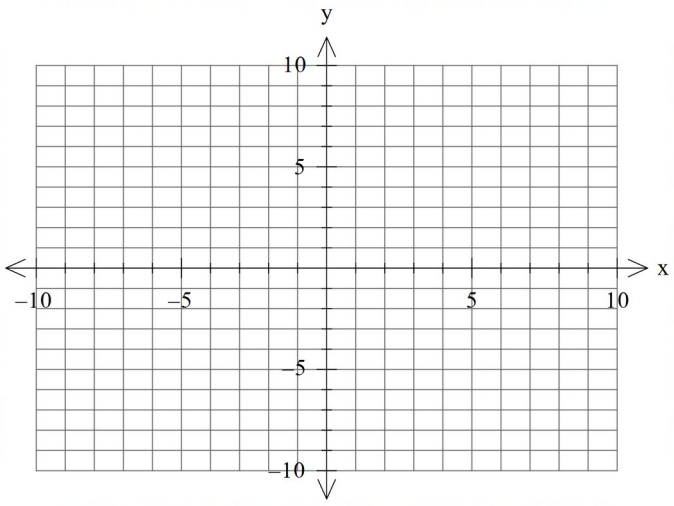
Consider the function $y = f(x)$ which is plotted below.



a) Sketch $y = f(-|x|)$



b) Sketch $y = \frac{1}{|f(x)|}$



Q4 (4 marks)

Consider two moving objects A & B such that at $t = 0$ seconds $r_A = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix}, r_B = \begin{pmatrix} 11 \\ 15 \\ -9 \end{pmatrix}$ metres and $v_A = \begin{pmatrix} 2 \\ 8 \\ -12 \end{pmatrix}, v_B = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix}$ metres per second. Determine the closet approach using **vector** methods.

Q5 (6 marks)

Consider a sphere $\left| r - \begin{pmatrix} 1 \\ -5 \\ \alpha \end{pmatrix} \right| = 7$ with α a constant and the line $r = \begin{pmatrix} 4 \\ -9 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}$.

Determine all possible real values of α such that:

(i) the line meets the sphere at two pints.

(ii) the line is a tangent to the sphere.

(iii) the line misses the sphere completely.

Q6 (2, 3 & 3 = 8 marks)

Consider the plane Ω given by $2x - 3y + 5z = 11$.

a) The point $A(5, -8, 3)$ is on a plane parallel to Ω. Determine the cartesian equation of this plane.

b) Determine the distance between these two planes. Show full reasoning.

c) Consider the lines $r_A = \begin{pmatrix} 2 \\ -9 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$ and $r_B = \begin{pmatrix} 3 \\ 11 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ -8 \\ 5 \end{pmatrix}$. Determine the distance between these lines.

Extra working space