



PERTH MODERN SCHOOL
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Independent Public School

Course _____ **Specialist** _____ **Year** 12

Student name: _____ Teacher name: _____

Date: 17 June Weds p3 (Advo)

Task type: Response

Time allowed for this task: 45 mins

Number of questions: 7

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: 42 marks

Task weighting: 12%

Formula sheet provided: Yes

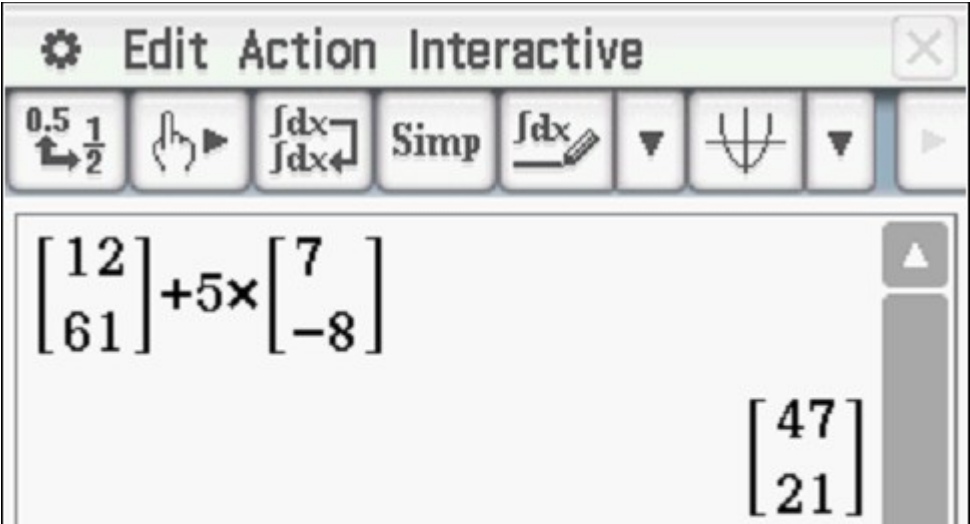
Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3.3.5- 3.3.6)

(2 & 3 = 5 marks)

Consider a car A that has an initial position vector $\begin{pmatrix} 12 \\ 61 \end{pmatrix}$ km and moving with a constant velocity of $\begin{pmatrix} 7 \\ -8 \end{pmatrix}$ km/h.

(a) Determine the position vector in 5 hours from now.

Solution

Specific behaviours
<ul style="list-style-type: none"> ✓ multiplies velocity by time ✓ states position vector

Consider a second car B that has an initial position $\begin{pmatrix} 57 \\ -29 \end{pmatrix}$ km and a constant velocity of $\begin{pmatrix} -2 \\ 10 \end{pmatrix}$ km/h.

(b) Determine if the two cars collide and if they do the position vector of this point of collision and the time it occurs.

Solution	
<p>The calculator screen displays the following work:</p> $\begin{bmatrix} 12 \\ 61 \end{bmatrix} + t \times \begin{bmatrix} 7 \\ -8 \end{bmatrix}$ $\begin{bmatrix} 7 \cdot t + 12 \\ -8 \cdot t + 61 \end{bmatrix}$ $\begin{bmatrix} 57 \\ -29 \end{bmatrix} + t \times \begin{bmatrix} -2 \\ 10 \end{bmatrix}$ $\begin{bmatrix} -2 \cdot t + 57 \\ 10 \cdot t - 29 \end{bmatrix}$ <p>solve(7·t+12=-2·t+57, t)</p> <p>{t=5}</p> <p>solve(-8·t+61=10·t-29, t)</p> <p>{t=5}</p>	
Collide at (47,21) Km	
Specific behaviours	
<ul style="list-style-type: none"> ✓ obtains expression for position vectors in terms of time ✓ solves for i components ✓ solves for j component and states pt of intersection 	

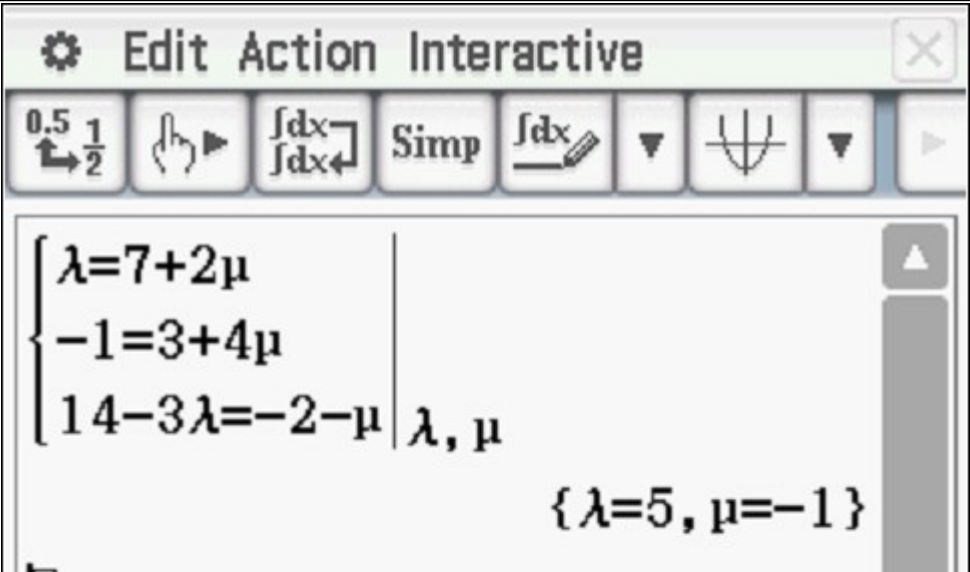
Q2 (3.3.1, 3.3.3)

(3 & 2 = 5 marks)

$$L_1 : r = \begin{pmatrix} 0 \\ -1 \\ 14 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} \quad L_2 : r = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$$

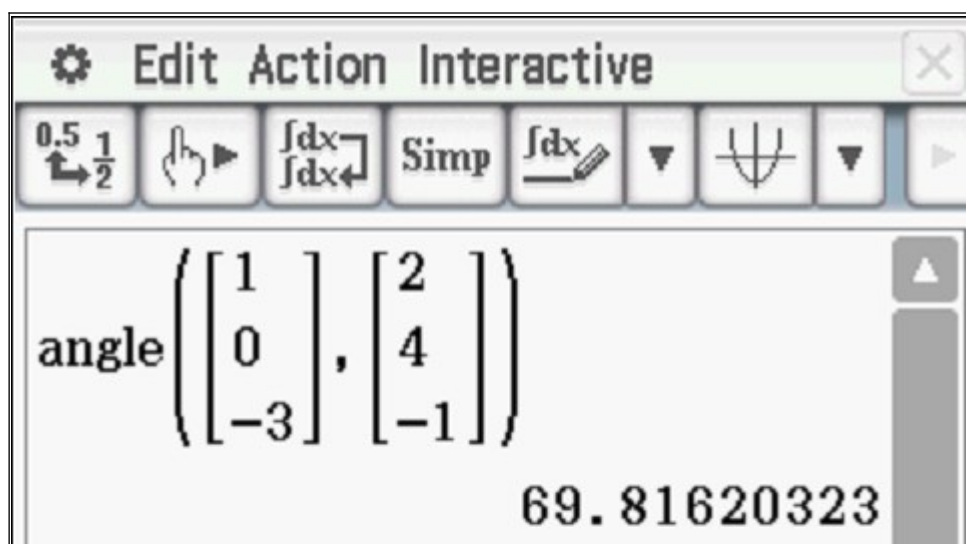
Consider the two lines

(a) Determine the point of intersection, if any.

Solution
 <p>Intersect at (5,-1,-1)</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ uses two parameters ✓ sets up three simultaneous equations ✓ states pt of intersection

(b) Determine to the nearest degree the acute angle between the two lines.
(Consider the plane that contains both lines)

Solution



Specific behaviours

- ✓ uses vectors parallel to lines
- ✓ states acute angle between lines

Q3 (3.3.8)

(2, 3 & 3 = 8 marks)

Consider a plane that contains the point $(5, -1, 3)$ and has a normal vector $\begin{pmatrix} 1 \\ 7 \\ -8 \end{pmatrix}$.

(a) Determine the vector equation of the plane.

Solution

$$r \cdot \begin{pmatrix} 1 \\ 7 \\ -8 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 7 \\ -8 \end{pmatrix} = -26$$

$$r \cdot \begin{pmatrix} 1 \\ 7 \\ -8 \end{pmatrix} = -26$$

Specific behaviours

- ✓ uses dot product with normal
- ✓ right hand side correct scalar

$$r = \begin{pmatrix} 3 \\ 12 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 6 \\ -7 \end{pmatrix}$$

(b) Determine the point of intersection of the line $\begin{pmatrix} -5 \\ -7 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ with the plane above.

Solution

The calculator screen displays the following steps:

- dotP**: The dot product of vectors $\begin{bmatrix} 3+2\lambda \\ 12+6\lambda \\ -5-7\lambda \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 7 \\ -8 \end{bmatrix}$ is calculated.
- solve**: The equation $7 \cdot (6\lambda + 12) + 8 \cdot (7\lambda + 5) + 2\lambda + 3 = -26$ is solved for λ , yielding $\{\lambda = -1.53\}$.
- Matrix Entry**: A matrix is entered with values corresponding to the vector components at $\lambda = -1.53$:

$$\begin{bmatrix} -\frac{3}{50} \\ \frac{141}{50} \\ \frac{571}{100} \end{bmatrix}$$
- Decimal Conversion**: The matrix values are converted to decimal form:

$$\begin{bmatrix} -0.06 \\ 2.82 \\ 5.71 \end{bmatrix}$$

The bottom of the screen shows the mode settings: **Alg**, **Decimal**, **Real**, and **Deg**.

- ✓ uses dot product and solves for parameter
- ✓ states pt of intersection, allow approx. decimal

(c) Determine the distance of point $(11, -3, 6)$ from the plane above.

Solution	
Choose any point on plane $(0,0,26/8)$	
<div style="border: 1px solid #ccc; padding: 5px; margin-bottom: 10px;"> <div style="display: flex; justify-content: space-between; align-items: center;"> ⚙ Edit Action Interactive ✕ </div> <div style="display: flex; justify-content: space-between; align-items: center; border-top: 1px solid #ccc; border-bottom: 1px solid #ccc;"> 0.5 $\frac{1}{2}$ $\int dx$ $\int dx \leftarrow$ Simp $\int dx$ $\int dx \leftarrow$ ▼ </div> </div> <div style="display: flex; align-items: center;"> <div style="flex: 1;"> $\text{dotP} \left(\begin{bmatrix} 0 \\ 0 \\ \frac{26}{8} \end{bmatrix} - \begin{bmatrix} 11 \\ -3 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ -8 \end{bmatrix} \cdot \frac{1}{\sqrt{1^2 + 7^2 + 8^2}} \right)$ </div> <div style="flex: 1; text-align: right; padding-right: 20px;"> <p>2.997074597</p> </div> </div> <div style="display: flex; align-items: center;"> <div style="flex: 1;"> <p>2.997074597</p> </div> <div style="flex: 1; text-align: right; padding-right: 20px;"> <p>$\frac{17368344}{5795099}$</p> </div> </div>	
OR	

Q4 (3.3.9-3.3.10) (3 & 3 = 6 marks)

(a) Solve the following system of linear equations. Working must be shown.

$$\begin{array}{r} 3x - 5y + 7z = 43 \\ x + 2y + 3z = 9 \\ 2x - 3y + 2z = 20 \end{array}$$

Solution
$\begin{bmatrix} 1 & 2 & 3 & 9 \\ 2 & -3 & 2 & 20 \\ 3 & -5 & 7 & 43 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 7 & 4 & -2 \\ 0 & 11 & 2 & -16 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 7 & 4 & -4 \\ 0 & -15 & 0 & 30 \end{bmatrix}$ $y = -2$ $-14 + 4z = -2$ $z = 3$ $x - 4 + 9 = 9$ $x = 4$ $(4, -2, 3)$
Specific behaviours
<ul style="list-style-type: none"> ✓ eliminates one variable from two equations ✓ eliminates two variables from one equation ✓ solves for all three variables

Consider the constants p & q in the system below.

$$3x - 5y + 7z = p$$

$$x + 2y + qz = 9$$

$$2x - 3y + 2z = 20$$

(b) Determine all the value(s) of p & q such that:

- (i) There will be an unique solution
- (ii) There will be infinite solutions
- (iii) There will be no solutions

Solution
$\begin{bmatrix} 1 & 2 & q & 9 \\ 2 & -3 & 2 & 20 \\ 3 & -5 & 7 & p \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & q & 9 \\ 0 & 7 & 2q-2 & -2 \\ 0 & 11 & 3q-7 & 18-p \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & q & 9 \\ 0 & 7 & 2q-2 & -2 \\ 0 & 0 & 27+q & 7p-211 \end{bmatrix}$ <p>i) $q \neq -27$</p> <p>ii) $q = -27$ & $p = \frac{211}{7}$</p> <p>iii) $q = -27$ & $p \neq \frac{211}{7}$</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ obtains row with two variables eliminated ✓ determines values for infinite solns ✓ determines values for unique and no solution

Q5 (3.3.11 – 3.3.15)

(3 & 3 = 6 marks)

Consider an object moving with acceleration $\ddot{r} = \begin{pmatrix} 5\cos(2t) \\ -3\sin t \end{pmatrix} \text{ m/s}^2$ at time t seconds. The initial velocity is $\begin{pmatrix} 5 \\ -2 \end{pmatrix} \text{ m/s}$ and initial displacement $\begin{pmatrix} -7 \\ 5 \end{pmatrix} \text{ m}$.

(a) Determine the position vector at time t seconds.

Solution

$\ddot{r} = \begin{pmatrix} 5\cos(2t) \\ -3\sin t \end{pmatrix}$ $\dot{r} = \begin{pmatrix} \frac{5}{2}\sin(2t) \\ 3\cos t \end{pmatrix} + \underline{c}$ $\begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \underline{c} \quad \underline{c} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$ $\dot{r} = \begin{pmatrix} \frac{5}{2}\sin(2t) + 5 \\ 3\cos t - 5 \end{pmatrix}$ $r = \begin{pmatrix} \frac{-5}{4}\cos(2t) + 5t \\ 3\sin t - 5t \end{pmatrix} + \underline{w}$ $\begin{pmatrix} -7 \\ 5 \end{pmatrix} = \begin{pmatrix} \frac{-5}{4} \\ 0 \end{pmatrix} + \underline{w} \quad \underline{w} = \begin{pmatrix} -\frac{23}{4} \\ 5 \end{pmatrix}$ $r = \begin{pmatrix} \frac{-5}{4}\cos(2t) + 5t - \frac{23}{4} \\ 3\sin t - 5t + 5 \end{pmatrix}$
Specific behaviours
<ul style="list-style-type: none"> ✓ integrates to find velocity with a vector constant ✓ integrates to find position with a vector constant ✓ solves correctly for both vector constants

(b) Determine the distance travelled in the first 10 seconds.(One decimal place)

Solution

$$\int_0^{10} \text{norm} \left(\begin{bmatrix} \frac{5}{2} \sin(2t) + 5 \\ 3 \cos(t) - 5 \end{bmatrix} \right) dt$$

75.60358851

Distance = 75.6 metres

Specific behaviours

- ✓ uses magnitude of velocity(shown)
- ✓ states integral
- ✓ states distance to one decimal place

Q6 (3.3.15)

(3 & 2 = 5 marks)

$$r = \begin{pmatrix} 3t^2 \\ 3+t \\ t^3 - 2t \end{pmatrix} \text{ km}$$

Consider an aircraft with position vector at time t hours. At the top of a building

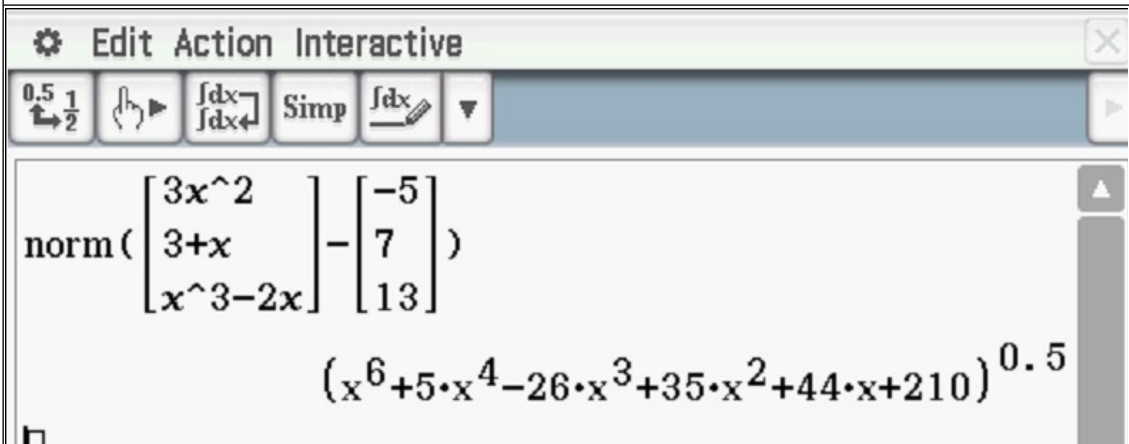
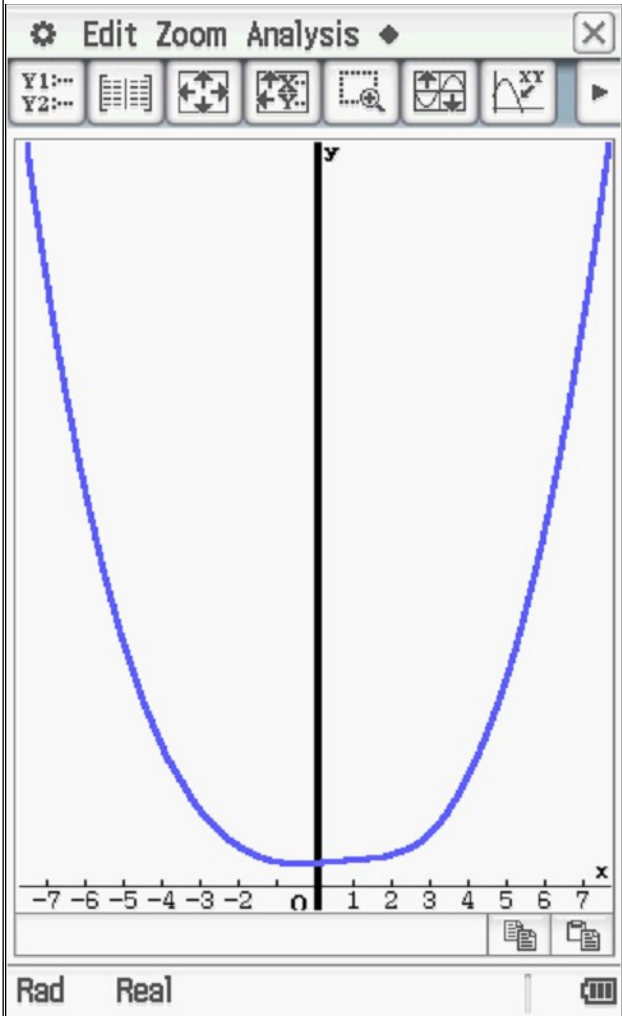
$$r = \begin{pmatrix} -5 \\ 7 \\ 13 \end{pmatrix} \text{ km}$$

stands an antenna with the position vector of the highest point being

(a) Determine the times the aircraft is less than 100 km from the top of the antenna.

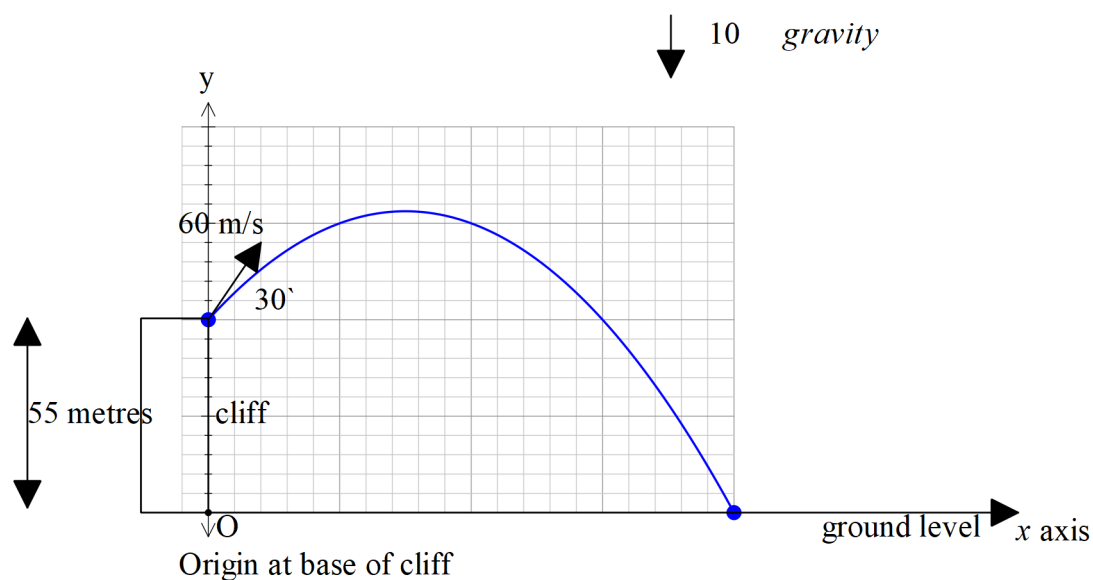
Solution	
<p>The calculator interface shows the following steps:</p> <ul style="list-style-type: none"> Input: $\text{norm}\left(\begin{bmatrix} 3t^2 \\ 3+t \\ t^3-2t \end{bmatrix} - \begin{bmatrix} -5 \\ 7 \\ 13 \end{bmatrix}\right)$ Result: $(t^6 + 5t^4 - 26t^3 + 35t^2 + 44t + 210)^{0.5}$ Input: $\text{solve}\left((t^6 + 5t^4 - 26t^3 + 35t^2 + 44t + 210)^{0.5} \leq 100, t\right)$ Result: $\{-4.234659064 \leq t \leq 4.573882795\}$ 	
Time between zero and 4.57 seconds	
Specific behaviours	
<ul style="list-style-type: none"> ✓ uses vector subtraction ✓ determines expression for distance apart ✓ solves for less than 10 km and states non negative values of time 	

(b) Determine the closest approach of the aircraft and the time it occurs.

Solution	
 <p>TI-Nspire CX CAS interface showing the norm function applied to a vector and a scalar expression.</p> <p>norm($\begin{bmatrix} 3x^2 \\ 3+x \\ x^3-2x \end{bmatrix} - \begin{bmatrix} -5 \\ 7 \\ 13 \end{bmatrix}$)</p> <p>$(x^6 + 5x^4 - 26x^3 + 35x^2 + 44x + 210)^{0.5}$</p>	
 <p>TI-Nspire CX CAS interface showing a graph of the function $(x^6 + 5x^4 - 26x^3 + 35x^2 + 44x + 210)^{0.5}$. The graph is a blue curve on a coordinate plane with x and y axes. The x-axis ranges from -7 to 7, and the y-axis ranges from 0 to 10. The curve is symmetric about the y-axis and has a minimum at the origin (0,0).</p>	
Min distance at t=zero seconds	
Specific behaviours	
<ul style="list-style-type: none"> ✓ graphs or uses fmin of expression for distance apart from part a ✓ states t=0 & 14.5 km 	

Q7 (3.3.15)

(4 & 3 = 7 marks)



Consider a football that is kicked off the top of a cliff of height 55 metres with an initial speed of 60 m/s at an angle of 30° with the horizontal. The acceleration due to gravity is -10 m/s^2 .

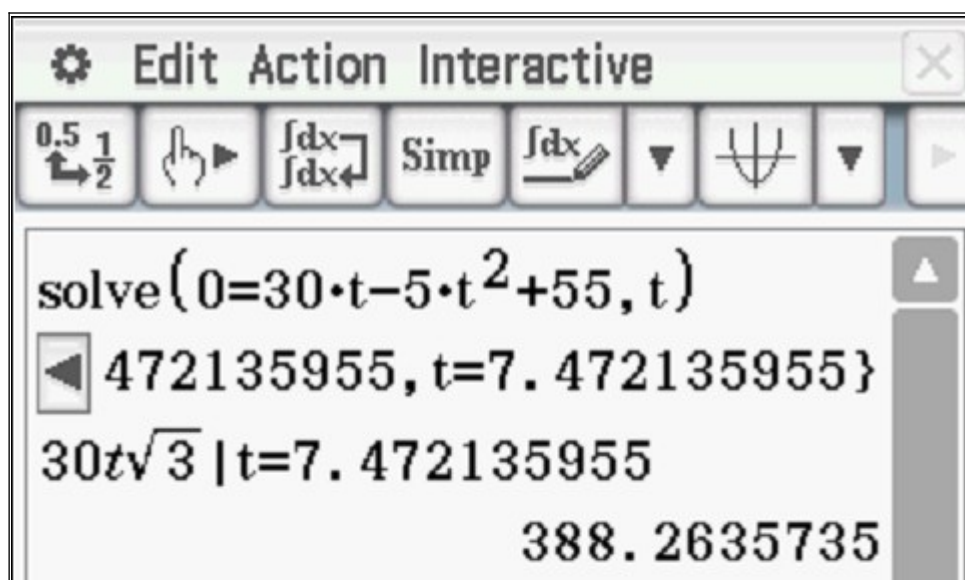
- (a) Show using **vector integration** how to determine the exact cartesian equation of the path using the base of the cliff as the origin.

Solution

$\ddot{r} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$ $\dot{r} = \begin{pmatrix} 0 \\ -10t \end{pmatrix} + \zeta$ $\begin{pmatrix} 60 \cos 30 \\ 60 \sin 30 \end{pmatrix} = \zeta = \begin{pmatrix} 30\sqrt{3} \\ 30 \end{pmatrix}$ $\dot{r} = \begin{pmatrix} 30\sqrt{3} \\ 30 - 10t \end{pmatrix}$ $r = \begin{pmatrix} 30t\sqrt{3} \\ 30t - 5t^2 \end{pmatrix} + w$ $w = \begin{pmatrix} 0 \\ 55 \end{pmatrix}$ $r = \begin{pmatrix} 30t\sqrt{3} \\ 30t - 5t^2 + 55 \end{pmatrix}$ $x = 30t\sqrt{3} \quad t = \frac{x}{30\sqrt{3}}$ $y = 30t - 5t^2 + 55 = \frac{x}{\sqrt{3}} - \frac{5x^2}{2700} + 55$
Specific behaviours
<ul style="list-style-type: none"> ✓ integrates acceleration and solves for vector constant ✓ integrates velocity and solves for vector constant ✓ obtains expression for t in terms of x ✓ obtains exact cartesian equation

- (b) Determine the time, one decimal place, taken to hit the ground and the horizontal distance of this point from the base of the cliff.

Solution



Specific behaviours

- ✓ equates y parametric equation to zero
- ✓ solves for time to one decimal place
- ✓ states approx. horizontal distance

End of test