CATHOLIC EDUCATION WESTERN AUSTRALIA | ViSN Virtual School Network

2021 Year 12 ViSN Mathematics Specialist Unit 3

Test 2 – Functions

Section One – Calculator Free

Mr Daniel Comtesse

Mandurah Catholic College

daniel.comtesse@cewa.edu.au

Calculator Free:______/29

Calculator Assumed:_____/11

Result: _____/40 ______%

Student Name: Solutions

School: ______________________________

Time allowed: Section One - 30 minutes

Section Two – 15 minutes

Assessment Date: 22 March 2021

Material required/recommended

To be provided by the supervisor

This Question/Answer Paper

SCSA Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Submission Details

Timed Assessments are to be returned to the ViSN teacher by the ViSN mentor (scan completed assessment and email to teacher above) within 24 hours of assessment date (above).

Instructions to Students

1. **ALL** questions should be attempted.
2. Write your answers in the spaces provided in this Question/Answer Booklet.
3. **SHOW ALL YOUR WORKING CLEARLY**. Your working should be sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Correct answers given without supporting reasoning may not be allocated full marks. Incorrect answers given without supporting reasoning cannot be allocated any marks.
4. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.
5. It is recommended that you **do not use pencil**, except in diagrams.

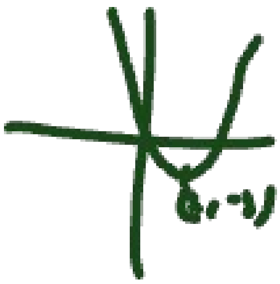
Question 1 **[1, 3 = 4 marks]**

Consider the function $f(x) = (x-2)^2 - 3$.

(a) Explain why it is necessary to restrict the natural domain of *f(x) so that* its inverse is also a function.

As $f(x)$ is not a one-to-one function, its inverse will not be a function. Hence, f needs to have its domain restricted to be one-to-one. ✓

(b) State a minimal restriction to the domain of *f(x)* that includes $x=1$, and then use this restriction to show that $f^{-1}(x) = 2 - \sqrt{x+3}$.



Restriction to domain: $\{x \in \mathbb{R}, x \le 2\}$ ✓ Domain

Let $y = (x-2)^2 - 3$

$x = (y-2)^2 - 3$

$\pm\sqrt{x+3} + 2 = y$

$2 \pm \sqrt{x+3} = y$ ✓ shows inverse

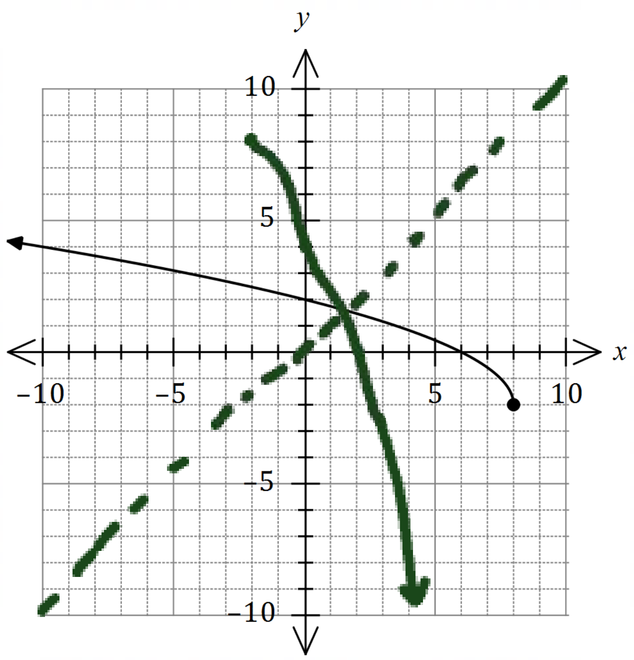
$f(1) = -2$ ✓ explain

Hence, the point $(1,-2)$ on $f(x)$ becomes $(-2,1)$ on $f(x)$ this only exists on $f^{-1}(x) = 2 - \sqrt{x+3}$.

4

Question 2 **[3, 3 = 6 marks]**

The graph of $y = f(x)$ is shown below.



(a) Draw the graph of $y = f^{-1}(x)$ on the same axes.

✓ start point ✓ shape
✓ [illegible]

(b) Given that $f(x) = \sqrt{16 - 2x} - 2$, determine the defining rule for $f^{-1}(x)$.

Let $y = \sqrt{16 - 2x} - 2$

$x = \sqrt{16 - 2y} - 2$

$(x+2)^2 = 16 - 2y$

$\dfrac{(x+2)^2 - 16}{2} = -y$ ✓ Rule

$-\frac{1}{2}(x+2)^2 + 8 = f^{-1}(x)$, $\{x \in \mathbb{R}, x \geq -2\}$

✓ simplify ✓ states domain restriction.

6

Question 3 **[1, 1, 2, 2 = 6 marks]**

Functions f, g and h are defined as

$$f(x)=x+3,\ g(x)=\sqrt{x},\ h(x)=\frac{4}{2-x}.$$

(a) Determine

(i) $h \circ g \circ f(6)$.

$h \circ g \circ f(6) = -4$ ✓

(ii) the defining rule for $h \circ g \circ f(x)$.

$h \circ g \circ f(x) = \dfrac{4}{2-\sqrt{(x+3)}}$ ✓

(b) Determine the domain of $h \circ g \circ f(x)$.

$D: \{x \in \mathbb{R},\ x \geq -3,\ x \neq 1\}$ ✓✓ Both restrictions

$2 - \sqrt{x+3} \neq 0$

$x+3 \neq 4$

$x \neq 1$

(c) Determine the range of $h \circ g \circ f(x)$.

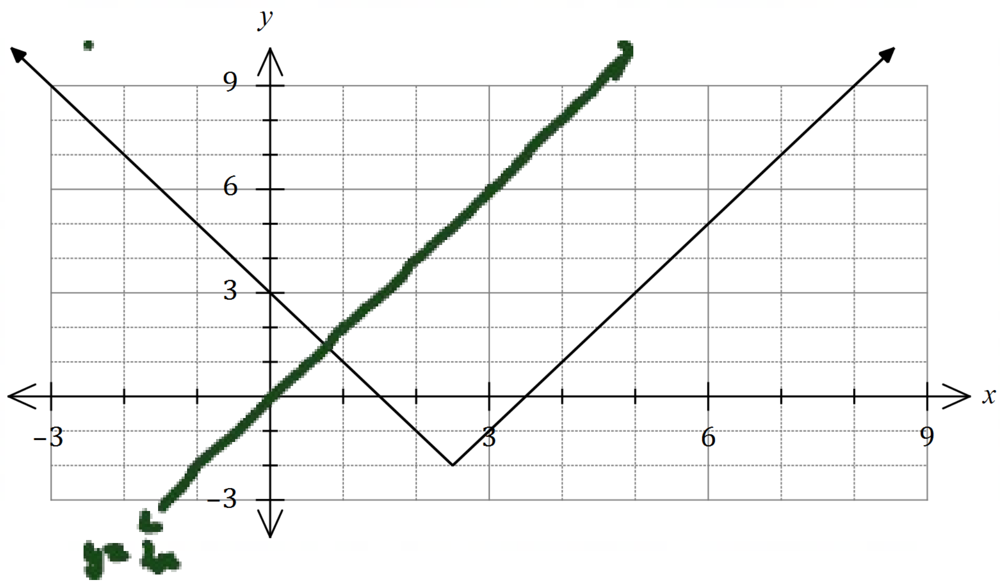
$R: \{y \in \mathbb{R},\ y \geq 2 \cup y < 0\}$ ✓✓ Both restrictions

6

Question 4 ~~[1, 1, 2~~ = 4 marks]

2, 4, 1, 3 = 8 marks

The graph of $f(x)=|ax+b|+c$ is shown below.



(a) Using the graph, or otherwise, solve

(i) $f(x)=5$.

$x = -1, 6$ ✓✓

(ii) $f(x)=x$.

$x = 1, 7$ ✓✓

(iii) $f(x)+2x=3$.

$x \le 1.5$ ✓

(c) Determine the values of a, b, c ~~and~~ d.

$c = -2$ (from graph) ✓

Sub $(6, 3)$

$3 = |b| - 2$

$\pm 5 = b$ ✓

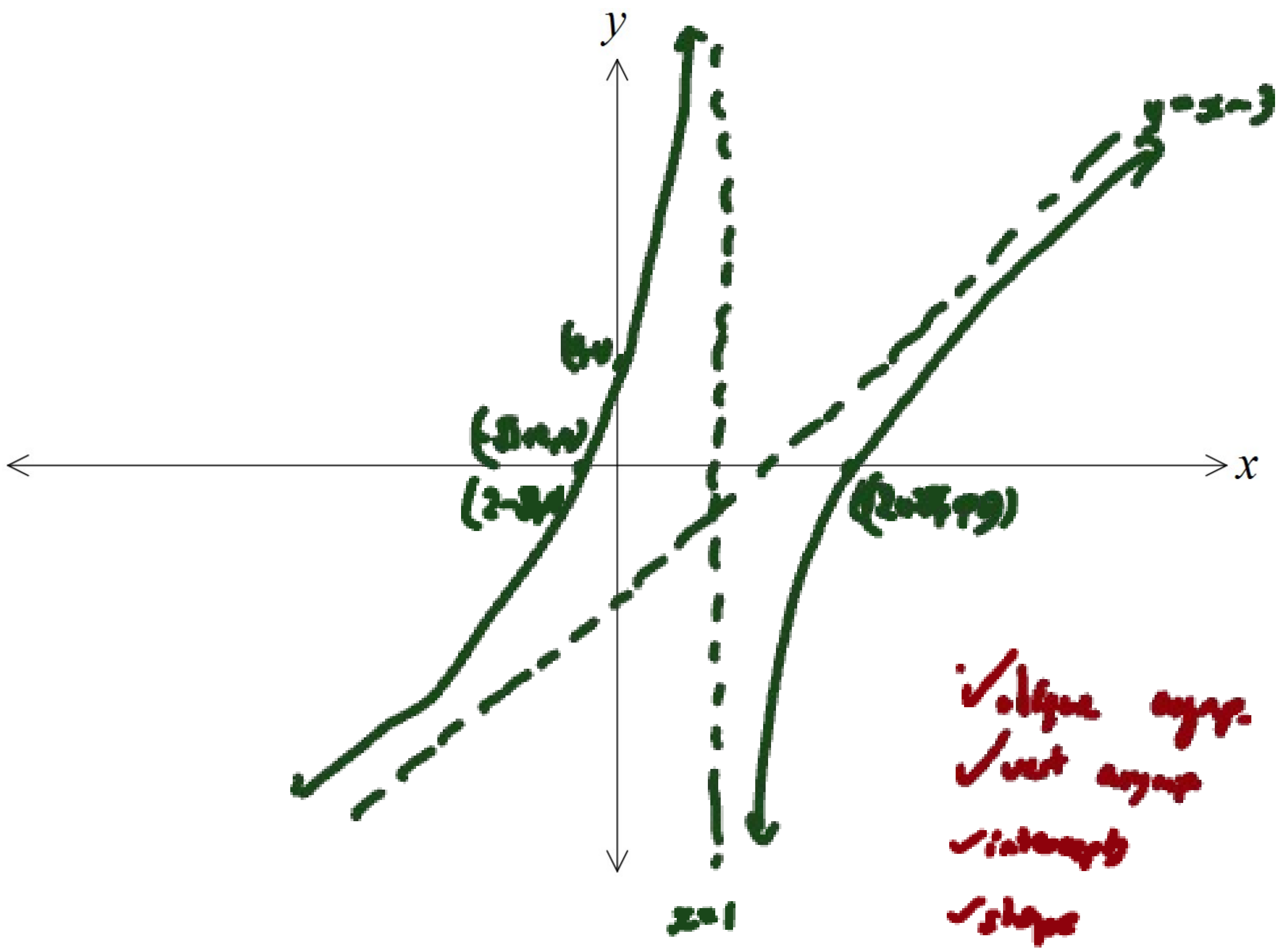
$a = \pm 2$ ✓

$a = 2, b = -5$ or $a = -2, b = 5$

8

Question 5 **[4 marks]**

Let $f(x)=\dfrac{x^2-4x-2}{x-1}$. Sketch $f(x)$ on the axes provided below.



$$
\begin{array}{r}
x-3 \\
x-1\,\overline{)\,x^2-4x-2} \\
-(x^2-x) \\
\hline
-3x-2 \\
-(3x-3)
\end{array}
$$

let $x=0$

$y = \frac{-2}{-1}$

y int @ $(0,2)$

let $y=0$

$0 = x^2-4x-2$

$0 = (x-2)^2-4-2$

$0 = (x-2)^2-6$

$2 \pm \sqrt{6} = x$

4

Question 6 **[1, 1, ~~3~~ 2 = ~~5~~ 4 marks]**

Let $g(x)=\frac{(x-2)(x+3)}{x^2+1}$.

(a) State the equation of the horizontal asymptote of the graph of $y=g(x)$.

$y=1$ ✓

(b) State the values of $g(6), g(7)$ and $g(8)$.

$g(6)=\frac{(4)(9)}{37}=\frac{36}{37}$

$g(7)=\frac{(5)(10)}{50}=1$

$g(8)=\frac{(6)(11)}{65}=\frac{66}{65}$

✓

(c) Use your previous two answers to explain why the graph of $y=g(x)$ must have a local maximum to the right of $x=7$.

As there is an asymptote at $y=1$, the graph must start approaching $y=1$ after $g(8)$. Hence, it should start decreasing again after $x=7$ and its maximum.

✓ explanation

✓

End of Section One

t

Extra Working Space

Question number: __________

2021 Year 12 ViSN
Mathematics Specialist Unit 3

Test 2 – Functions
Section Two – Calculator Assumed

Mr Daniel Comtesse
Mandurah Catholic College

Calculator Assumed:_____/11

daniel.comtesse@cewa.edu.au

Student Name: S. Cheng ________________________________

School: ________________________________

Time allowed: Section One - minutes

Section Two – 15 minutes

Assessment Date: 22 March 2021

Material required/recommended

To be provided by the supervisor

This Question/Answer Paper

SCSA Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

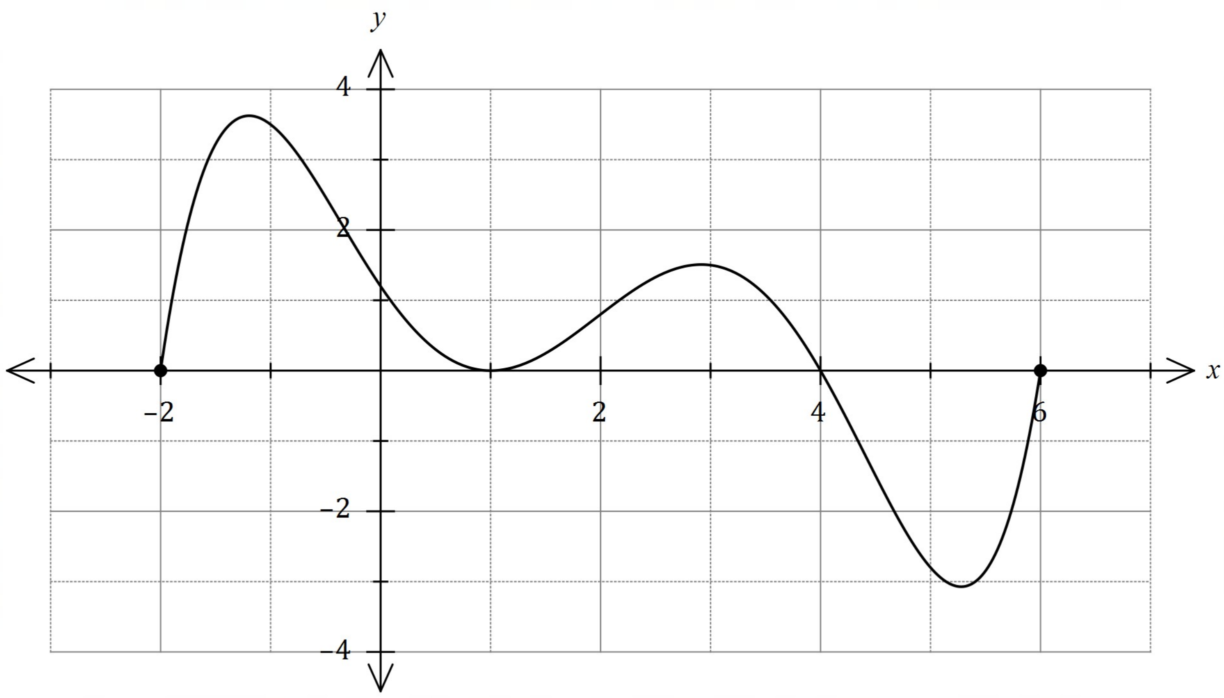
Special items: scientific and/or CAS calculator, 1 A4 (one sided) page of notes.

Instructions to Students

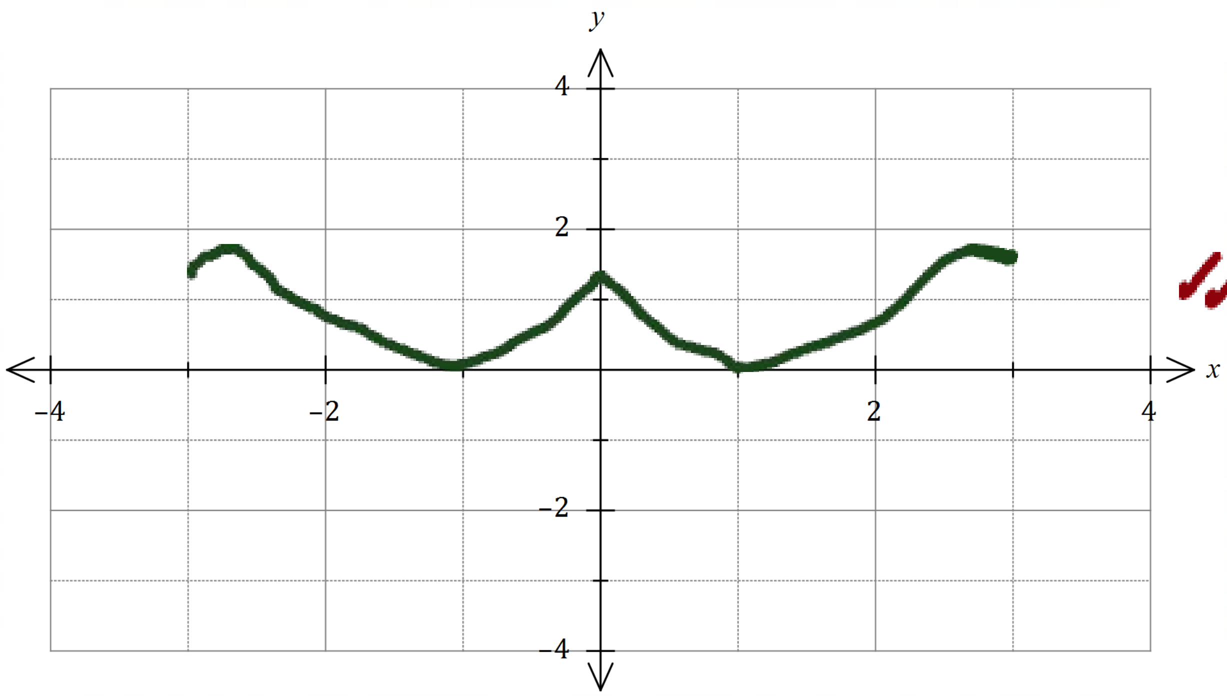
1. **ALL** questions should be attempted.
2. Write your answers in the spaces provided in this Question/Answer Booklet.
3. **SHOW ALL YOUR WORKING CLEARLY**. Your working should be sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Correct answers given without supporting reasoning may not be allocated full marks. Incorrect answers given without supporting reasoning cannot be allocated any marks.
4. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.
5. It is recommended that you **do not use pencil**, except in diagrams.

Question 7 **[2, 4, 1 = 7 marks]**

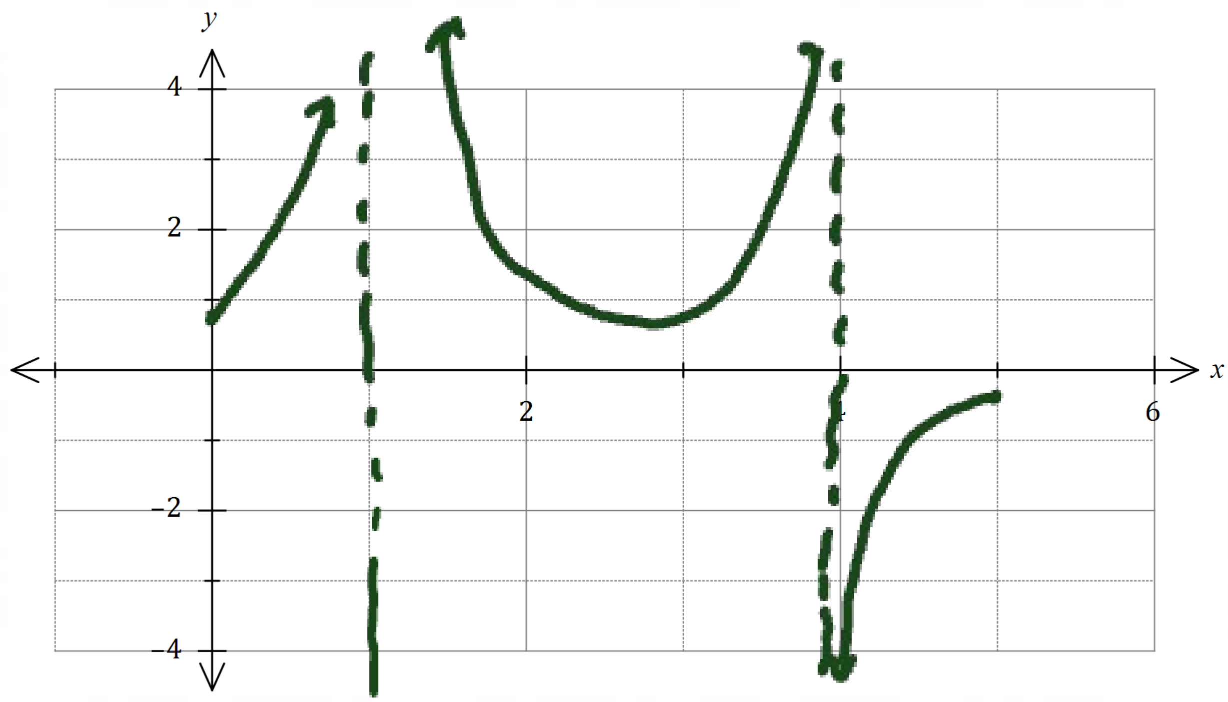
The graph of $y=f(x)$ is shown below over the domain $-2 \le x \le 6$.



(a) Sketch the graph of $y=¿f(|x|)¿$ over the domain $-3 \le x \le 3$ on the axes below.



(b) Sketch the graph of $y=\frac{1}{f(x)}$ on the axes below over the domain $0 \le x \le 5$.

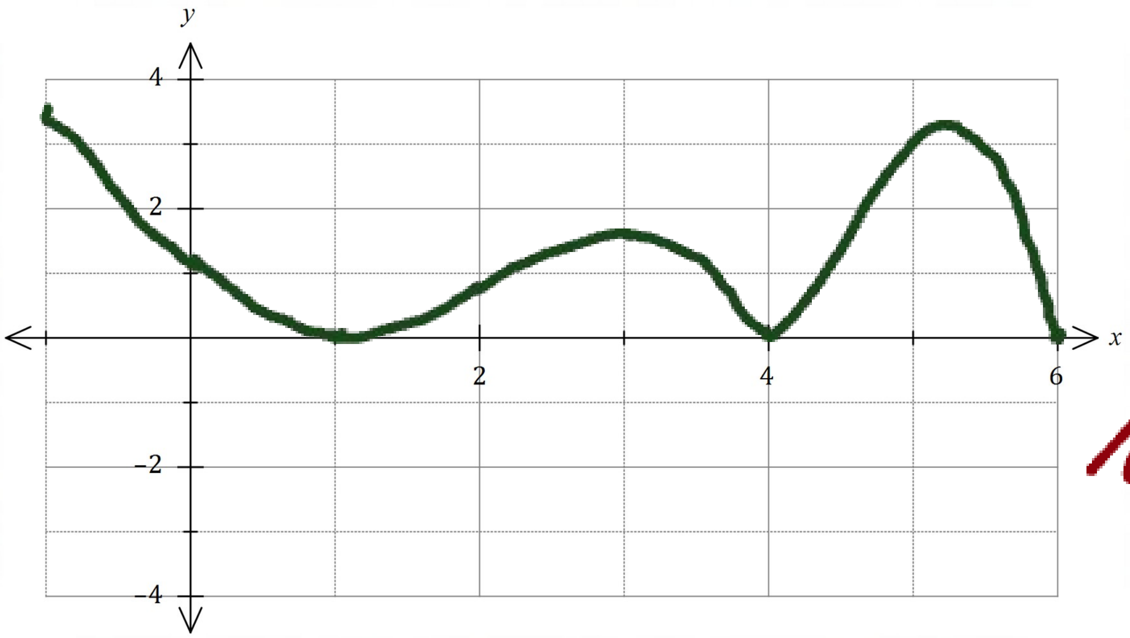


✓ $0 \le x \le 1$ ✓ $1 < x < 4$

✓ $4 \le x \le 5$

✓ vertical asymptotes

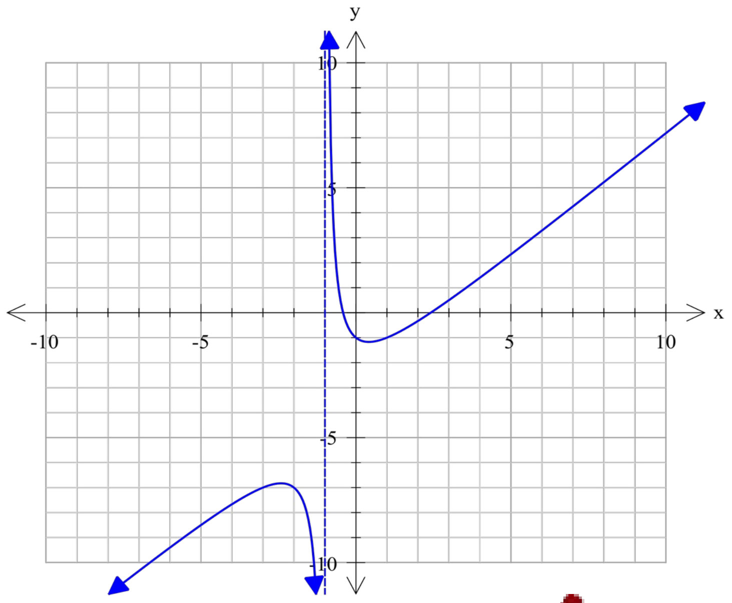
(c) Sketch the graph of $y=¿f(x)\vee¿$



correct

Question 8 **[3 marks]**

The function $f(x)$ is graphed on the axes below with oblique asymptote $y = x - 3$. Determine the equation for $f(x)$.



$f(x) = \dfrac{a}{x+1} + x - 3$ ✓ correct form

← Vert. asymptote at $x = -1$

$\therefore$ denominator $= x+1$

✓ Sets denominator.

$f(0) = -1 \Rightarrow -1 = a - 3$

$\therefore a = 2$

$\therefore f(x) = \dfrac{2}{x+1} + x - 3$ ✓a