PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 3&4

If required by your examination administrator, please place your student identification label in this box

Section One: Calculator-free

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	49	35
Section Two: Calculator-assumed	12	12	100	91	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	6	
2	7	
3	8	
4	6	
5	7	
6	8	
7	7	
S1 Total	49	
S1 Wt (×0.7143)	35%	
S2 Wt	65%	
Total	100%	

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Section One: Calculator-free **35% (49 Marks)**

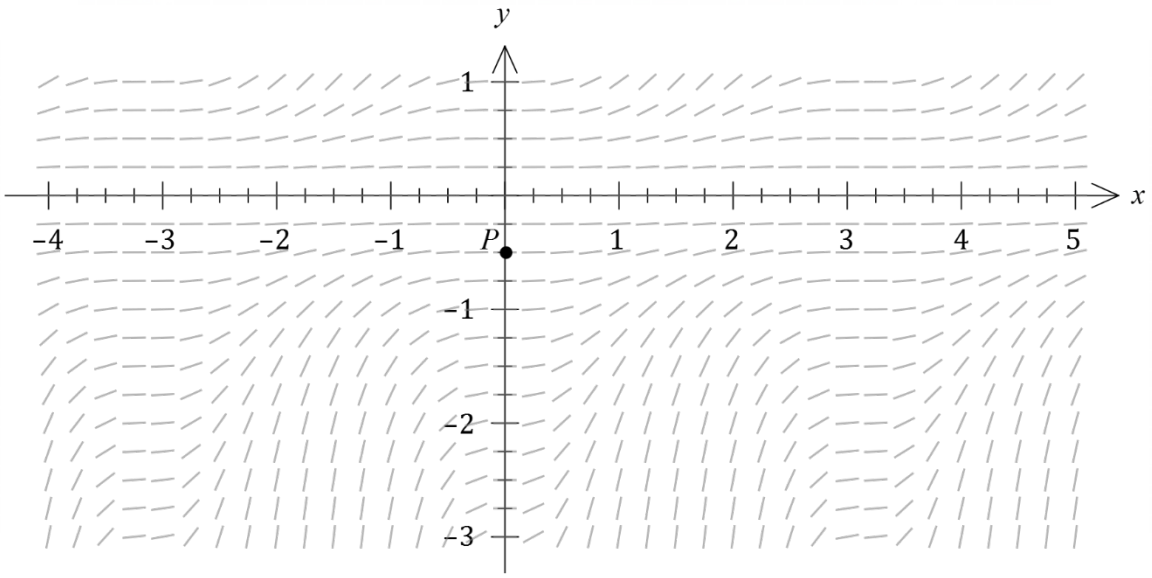
This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

See next page

Question 1 **(6 marks)**

The point $P(0,-0.5)$ and part of the slope field given by $\frac{dy}{dx}=y^2\sin^2 x$ is shown below.



(a) Determine the equation of the solution curve through P in the form $y=f(x)$. (4 marks)

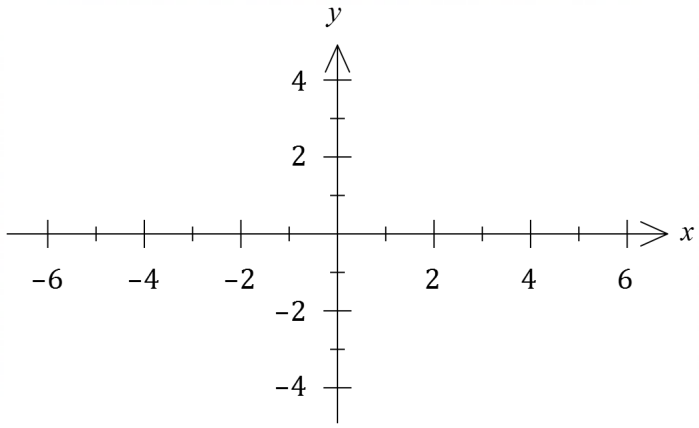
(b) Draw the solution curve through P on the slope field above. (2 marks)

See next page

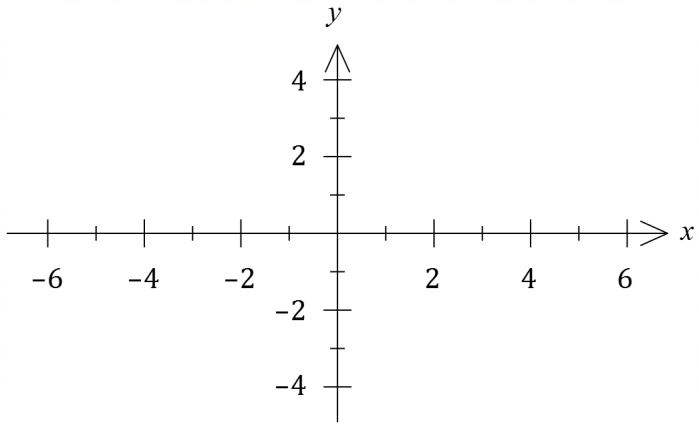
Question 2 **(7 marks)**

Let $f(x)=\dfrac{(x+1)(3-x)}{2}$.

(a) Sketch the graph of $y=f(x)$. (1 mark)



(b) On the axes below, sketch the graph of $y=¿f(|x|)\vee ¿$. (2 marks)



Consider $g(x)=\dfrac{2}{3+2x-x^2}$.

(c) Briefly describe how the graph of $y=f(x)$ can be used to sketch the graph of $y=g(x)$ and hence state the domain and range of $g(x)$. (4 marks)

See next page

Question 3 **(8 marks)**

The equations of planes Π_1, Π_2 and Π_3 are shown below.

$$\Pi_1 \qquad\qquad \Pi_2 \qquad\qquad \Pi_3$$
$$r = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \quad r \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 18 \quad x - y + z = 15$$

(a) Show that the equation of plane Π_1 in Cartesian form is $2x + y - z = 6$. (3 marks)

(b) The origin lies on the surface of sphere S. Determine the vector equation of S, given that its centre is the point of intersection of the three planes. (5 marks)

See next page

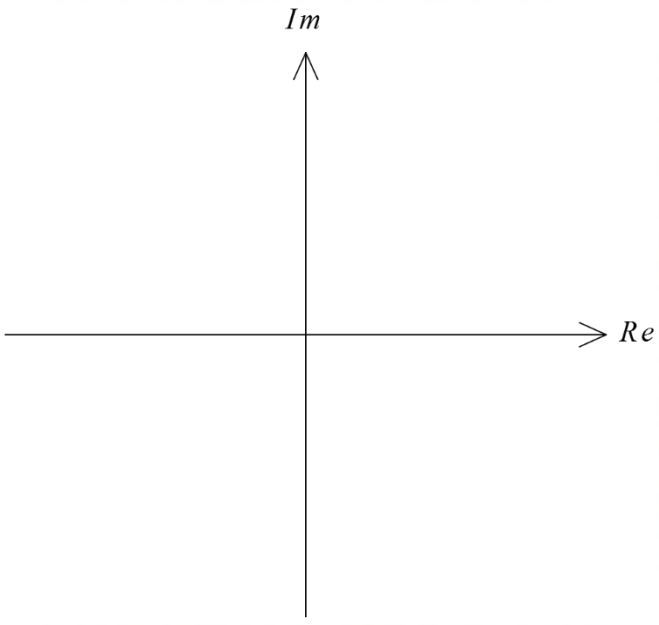
Question 4 **(6 marks)**

Let $f(z)=3z^3+pz^2+16z+q$, where p and q are real constants.

One of the roots of $f(z)$ is $z=1+\sqrt{3}i$.

(a) Determine the value of the constant p and the value of the constant q. (4 marks)

(b) Show all the roots of $f(z)$ in the complex plane below. (2 marks)



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Question 5 **(7 marks)**

An electronic circuit will remain stable when x and y, the resistances in ohms of two variable resistors in the circuit, satisfy

$$\frac{3}{x}+\frac{5}{y}=\frac{1}{10}.$$

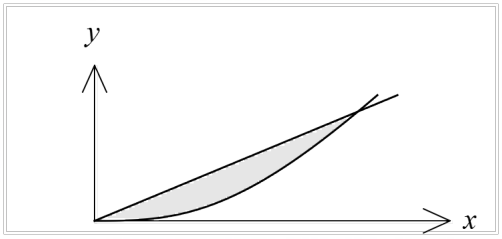
(a) When y is decreasing at a rate of 5 ohms per second, determine the rate that x must be changing for the circuit to remain stable when $y=200$ ohms. (4 marks)

(b) The circuit is stable, the resistance of x is increasing at 6 ohms per second and has just reached 60 ohms. Use the technique of increments to calculate the approximate change in the resistance of y in the next tenth of a second. (3 marks)

See next page

Question 6 **(8 marks)**

(a) The line $y=x$ and the curve $y=\dfrac{2x^3}{x^2+4}$ are shown in the diagram. They intersect at the origin and at $x=2$.



Determine the area between the curve and the line in the first quadrant.

(5 marks)

(b) Given that $\int_2^8 f\left(\dfrac{x}{x^2}\right)dx=6$, determine $\int_1^4 f\left(\dfrac{8}{x}\right)dx$. (3 marks)

See next page

Question 7 **(7 marks)**

(a) Determine the value of the constant A and the value of the constant B so that

$$\frac{5}{(3x-2)(x+1)}=\frac{A}{3x-2}+\frac{B}{x+1}.$$

(2 marks)

(b) Hence use the substitution $u=\cos x$ to determine

$$\int \frac{5\sin x}{2\cos 2x-\cos^2 x+\cos x}\,dx.$$

(5 marks)

End of questions

Supplementary page

Question number: _____