PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS
UNIT 3 & 4
Section Two:
Calculator-assumed

Your Name: _____

Your Teacher's Name: _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
7		7	13		10
8		9	14		11
9		9	15		9
10		8	16		8
11		10	17		7
12		12			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	50	33
Section Two: Calculator-assumed	11	11	100	100	67
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-assumed **(100 Marks)**

This section has **eleven** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 7 **(7 marks)**

A projectile is fired upward from a $15.3\,m$ cliff at a speed of $19.6\,m/s$ and allowed to fall into a valley below. The acceleration g due to Earth's gravity is about $9.8\,m/s^2$ downward.

(a) Given that $a(t) = -9.8\,m/s^2$, determine an expression for $v(t)$ and use it to find the time at which the projectile reaches its maximum height. (2 marks)

(b) Determine this maximum height of the projectile. (2 marks)

(c) Determine the **total distance** traveled over the time interval $0 \le t \le 3$. (3 marks)

Question 8 **(9 marks)**

The table below summaries census information about the number of children in the households of an Australian town.

Number of children	0	1	2	3	4 or more
Percentage of households	23	32	35	7	3

A random sample of 20 households is selected from this town.

a. State the distribution and determine the probability that the sample will contain:

i. 3 households only with no children. (2 marks)

ii. more than half the households, with at least 2 children. (3 marks)

A new random sample of n households is selected from this town. The probability that this new sample contains a household with 4 or more children is more than 10%.

b. Determine the smallest value of n. (4 marks)

Question 9 **(9 marks)**

When refuelling the car, the rate of flow of petrol into the tank is given by

$$\frac{dV}{dt} = 9\,e^{-(t+2)}(8-t) \text{ for } 0 \le t \le 8,$$

where V is the litres of petrol in the tank at time t in minutes. Initially the tank has two litres of petrol.

(a)

i. Determine the **exact** initial rate of flow of petrol into the tank. (1 mark)

ii. Determine the value of t for which $\frac{dV}{dt} = 0$. (1 mark)

iii. Determine the time, to the nearest second, when the rate is 1 litre per minute. (2 marks)

(b) Determine the amount of petrol in the tank, to one decimal place, when $t = 8$.
(3 marks)

(c) Determine the time, **to the nearest second**, when there are 10 litres of petrol in the tank. (2 marks)

Question 10 **(8 marks)**

The volume, $V\,cm^3$, of a plastic bottle is given by $V=\pi r^2 h+\frac{2}{3}\pi r^3$, where h cm is the height and r cm is the radius of its cap.

(a) Given that $h=3\,cm$, what is the approximate increase in Volume when the radius expands by $p\,cm$ from $2\,cm$? Use a calculus method to find your answer. (4 marks)

(b) If $h = 6\,cm$ and $r = 4\,cm$, use calculus to find the approximate percentage change, to one decimal place, in the volume when the radius increases by $q\,\%$.

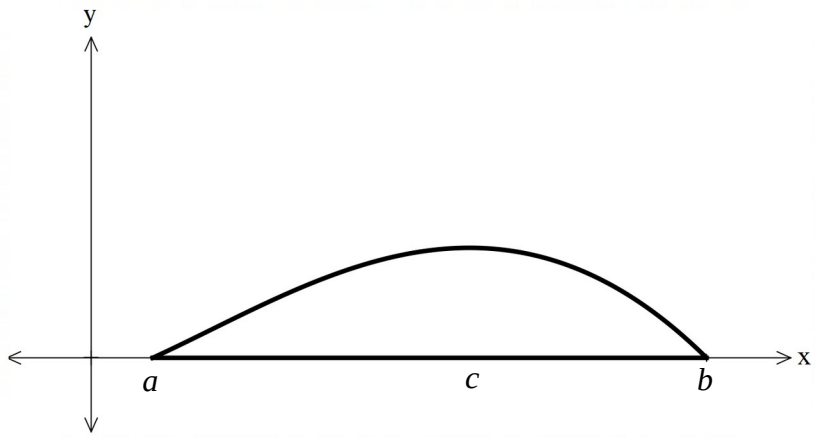
(4 marks)

Question 11 **(10 marks)**

An aircraft designer is trailing a new wing shape with a cross section at a certain point enclosed by the function

$$y=-k(x¿¿2-0.01)\ln(x)¿$$

where k is a positive constant, together with an interval of the x-axis from $x=a$ to $x=b$.



(a) Determine the values of a and b (given that $a<b$). (2 marks)

(b) Determine the value of c (to 3 decimal places) above which the highest point of the top edge of the cross section occurs. (3 marks)

(c) Find, to 3 decimal places, the value of k such that the maximum height of the cross section is 0.15 units. (3 marks)

(d) Determine, to 3 decimal places, the area of the cross section for the value of k determined in part (c). (2 marks)

Question 12 **(12 marks)**

Two electrical engineering companies produce light globes – one company produces type A and the other produces type B. Both types are normally distributed. A light globe is considered premium if it has a lifespan longer than 850 hours.

(a) The company producing type A globes claims that their product has a mean lifespan of 818 hours and a standard deviation of 112 hours.

i. What is the probability of a type A globe being premium? (2 marks)

ii. What is the probability of a type A globe being premium, given that its lifespan exceeds 800 hours? (2 marks)

iii. What lifespan is exceeded by 90% of all type A globes produced? (2 marks)

iv. If the company selects 50 type A batteries one at a time, find the probability that it takes a selection of eight batteries before six premium batteries are selected. (3 marks)

(b) The company producing type B light globes claims that 31% of their product has a lifespan over 860 hours, and 19% have a lifespan below 700 hours. Determine the mean and standard deviation for the type B light globes. (3 marks)

Question 13 **(10 marks)**

A discrete random variable Y is defined by $P(Y=y)=a\log(x-1)$ for $x=6,11$ and 21

(a) Determine the value of a. (3 marks)

(b) Determine $P(Y=21|Y>6¿$ in **exact** form. (3 marks)

(c) The expected value is

$$E(Y)=\frac{38}{3}+m\log n,$$

where the constants m and n are prime numbers. Determine the values of m and n.

(4 marks)

Question 14

A company that manufactures professional photography drones uses a particular model of accelerometer that fails when it becomes too hot. The temperatures T ℃ at which a randomly chosen accelerometer fails are normally distributed.

a) In preliminary laboratory tests, 97% of a random sample of accelerometers continued working at a temperature of at least 85 ℃, but only 5% continued working at least 105 ℃.

(i) Calculate the mean and standard deviation of the distribution.

(ii) What proportion of accelerometers will operate at least at $100\,℃$. (2 marks)

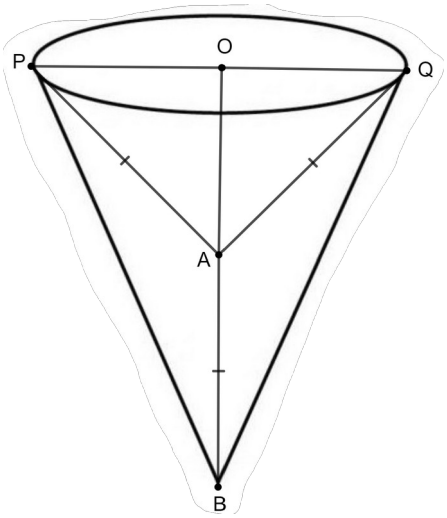
b) During the process of thorough testing, the manufacturer took repeated samples and one sample of 300 accelerometers revealed that 51 continued to function at $100\,℃$.

(i) State the approximate distribution of the sample proportions and justify your choice. (3 marks)

(ii) Provide a 99% confidence interval for the proportion of accelerometers that can function at a temperature of at least $100\,℃$. (2 marks)

Question 15 **(9 marks)**

An engineering project requires an original part PBQ that is conical in shape. At a later stage of the project, a smaller cone PAQ will be extracted from the original such that $AP = AB = AQ$. We will use the letter a to represent the three equal lengths AQ, AB and AP. We will use the letter r to represent the radii OP and OQ of the circular section of the cone. We will use the letter h to represent the altitude OB of the original cone. Both angles $\angle OAP$ and $\angle OAQ$ have a magnitude θ. It is advised that you label the diagram accordingly.



a) Find h, the altitude of the original cone, in terms of a and θ. (1 mark)

b) Find r, the radius of the circular section of the cone, in terms of a and θ. (1 mark)

The volume of the cone is given by $V = \frac{1}{3}\pi r^2 h$.

c) Use the results from (a) and (b) to show that (2 marks)

$$V = \frac{1}{3}\pi a^3 \sin^2\theta(1+\cos\theta)$$

d) Given that a is a constant, find $\frac{dV}{d\theta}$ and hence find the value(s) of θ for which the volume is maximised.

Question 16 **(8 marks)**

5000 boxes of a certain kind of cereal are stored in a warehouse. The cereal manufacturer wishes to estimate the proportion of boxes in the warehouse that weigh less than the 300g stated on the box. The manufacturer asks a warehouse employee to sample and weigh 200 cereal boxes. The boxes are stored in the warehouse on 50 numbered shelves (with 100 boxes arranged in a line on each shelf).

a) Briefly describe a suitable method for selecting the sample of 200 boxes. (2 marks)

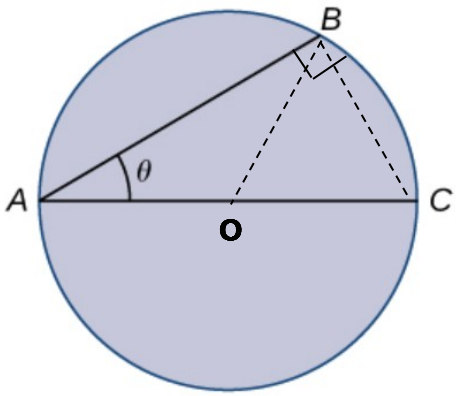
The warehouse employee finds that 38 of the 200 selected boxes weigh less than 300g.

b) Determine the sample proportion of underweight cereal boxes, and thus **show** that a 90% confidence interval for the proportion of underweight boxes (to 3 decimal places) is $(0.144, 0.236)$. (4 marks)

c) The manufacturer later consults factory records to conduct a census of the weights of all 5000 boxes in the warehouse, and finds that the proportion of underweight boxes is 0.2528 (assume that this is the correct proportion). The manufacturer accuses the employee of being careless in their measurements or calculations. Is this accusation justified? Briefly explain your answer. (2 marks)

Question 17 **(7 marks)**

Consider a lifeguard at a circular pool, at position A, with diameter of $60\,m$. He must reach someone who is drowning on the exact opposite side of the pool, at position C. The lifeguard is considering take a two-stage route, first swimming at angle θ with a speed $1\,m/s$ to position B and then leaving the pool and running around the pool at speed $3\,m/s$.



(a) Determine the function $T(\theta)$ as the total amount of time it takes to reach the drowning person, in terms of the swim angle θ (radians). Hint: $Angle\,BOC = 2\theta$.

(3 marks)

(b) Using Calculus, justify whether this two-stage route will **minimise** the time the lifeguard takes to reach the drowning person. If not, determine the route that will.

(4 marks)

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____