

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

### **Important note to candidates**

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination.

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

*To be provided by the candidate*  
This Question/Answer booklet  
Formula sheet (retained from Section One)  
Working time:  
Reading time before commencing work: ten minutes  
one hundred minutes

*To be provided by the supervisor*  
Materials required/recommended for this section  
Working time:  
Reading time before commencing work: ten minutes  
one hundred minutes

Your name \_\_\_\_\_  
In words \_\_\_\_\_

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Student Number: In figures \_\_\_\_\_

Calculator-assumed

Section Two:

UNITS 1 AND 2

METHODS

MATHEMATICS

**SOLUTIONS**

Question/Answer booklet

Semester Two Examination, 2017

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					<b>100</b>

Additional working space

Question number: \_\_\_\_\_

**Instructions to candidates**

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

<b>Specific behaviours</b> $f(1) = 10$ $f(3) = 0 \Leftrightarrow c = \frac{4}{39}$ $f(x) = \frac{2}{3x^2} - \frac{4}{x^3} - x + c$	$\checkmark$ determines constant $\checkmark$ antiderivatives
<b>Solution</b>	

(3 marks)

(b) If  $f(x) = 3x - x^3 - 1$  and  $f(3) = 0$ , determine  $f(1)$ .

<b>Specific behaviours</b> $y = 0 \Leftrightarrow x = a = 4$ $y - 12 = -6 x - 2 $ or $y = -6x + 24$ $\frac{dy}{dx} = -6 - 4x = 2 - 4 2  = -6$	$\checkmark$ writes equation of tangent $\checkmark$ determines gradient of tangent
<b>Solution</b>	

(3 marks)

(a) Determine the value of  $a$ .  
The tangent to the curve  $y = 16 + 2x - 2x^2$  at  $(2, 12)$  intersects the  $x$ -axis at  $(a, 0)$ .

(6 marks)

**Question 9**

Additional working space \_\_\_\_\_

Working time: 100 minutes.

This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

**Section Two: Calculator-assumed**  
65% (98 Marks)

**Question 10**

A group of 160 students were asked whether they had bought a drink or a snack from the school canteen. 32 had bought neither, 32 had bought both and 112 had bought a drink.

- (a) Determine the number of students who only bought a snack. (2 marks)

<b>Solution</b>	
$n(S \cup D) = 160 - 32 = 128$	
$n(S \cap D) = 128 - 112 = 16$	
<b>Specific behaviours</b>	
<input checked="" type="checkbox"/> Venn diagram or other method <input checked="" type="checkbox"/> correct number	

- (b) Determine the probability that a randomly chosen student from the group had bought

- (i) a drink or a snack.

<b>Solution</b>	
$P = \frac{128}{160} = \frac{4}{5}$	
<b>Specific behaviours</b>	
<input checked="" type="checkbox"/> correct probability	

- (ii) only a drink.

<b>Solution</b>	
$P = \frac{112 - 32}{160} = \frac{80}{160} = \frac{1}{2}$	
<b>Specific behaviours</b>	
<input checked="" type="checkbox"/> correct probability	

- (iii) a drink given that they had bought a snack.

(2 marks)

<b>Solution</b>	
$P = \frac{32}{32 + 16} = \frac{32}{48} = \frac{2}{3}$	
<b>Specific behaviours</b>	
<input checked="" type="checkbox"/> uses conditional probability <input checked="" type="checkbox"/> correct probability	

- (c) For this group of students, are the events buying a snack and buying a drink independent? Justify your answer.

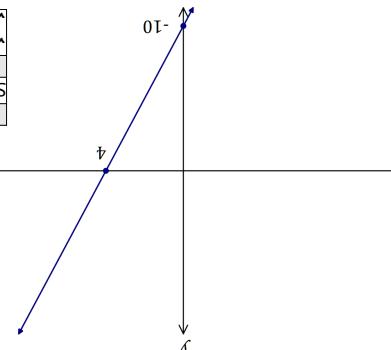
(2 marks)

<b>Solution</b>	
$P(D) = \frac{112}{160} = \frac{7}{10}$ and $P(D S) = \frac{2}{3}$ and so $P(S) \neq P(S \vee D)$ and so events are NOT independent.	
<b>Specific behaviours</b>	
<input checked="" type="checkbox"/> uses probabilities <input checked="" type="checkbox"/> correct conclusion	

(2 marks)

**Question 21****Question 11**

Line  $L_1$  has equation  $5x - 2y = 20$ . Sketch the graph of  $L_1$ .



(2 marks)

(2 marks)

Determine the equation of line  $L_2$  that is parallel to  $L_1$  and passes through the point with coordinates  $(-3, -11)$ .

Solution	
States equation	
Uses correct gradient	
$5x - 2y = 5$ (or $y = \frac{5}{2}x - \frac{5}{2}$ )	
$5(-3) - 2(-11) = -15 + 22 = 7$	
Solution	

(2 marks)

Determine the equation of line  $L_3$  that is perpendicular to  $L_1$  and has the same y-intercept as  $L_1$ .

Solution	
Uses correct gradient	
$2x + 5y = -50$ (or $y = -\frac{2}{5}x - 10$ )	
$2(0) + 5(-10) = -50$	
Solution	

Determine the equation of line  $L_3$  that is perpendicular to  $L_1$  and has the same y-intercept as  $L_1$ .

(b) Use differentiation to determine the values of  $r$  and  $h$  that will maximise the volume of the solid, and state this maximum volume.

Solution	
Determines root of derivative	
States optimum value of $r$	
Calculates height $h$	
States optimum value of $V$	
Solution	

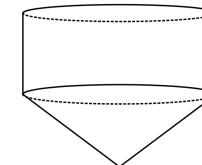
(b) Use differentiation to determine the values of  $r$  and  $h$  that will maximise the volume of the solid, and state this maximum volume.

(c) Use differentiation to determine the values of  $r$  and  $h$  that will maximise the volume of the solid, and state this maximum volume.

Solution	
Writes $h$ in terms of $r$	
Substitutes into sum of cone and cylinder volumes	
Writed as $48\pi r^2 - 4\pi r^3$	
$V = \frac{1}{3}\pi r^2 h + \pi r^2 h = \frac{3}{4}\pi r^2 h = \frac{3}{4}\pi r^2 (36 - 3r)$	
Solution	

(a) Show that the volume of the solid is given by  $V = 48\pi r^2 - 4\pi r^3$ .

The dimensions are such that the sum of  $h$  and  $3r$  is 36 cm.

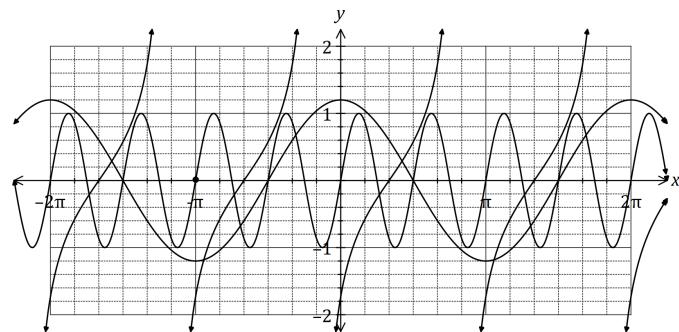


A composite solid is made from a cone and a cylinder, both of height  $h$  cm and radius  $r$  cm, as shown below.

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## Question 12

- (a) The graphs of  $y = a \cos x$ ,  $y = \sin(bx)$  and  $y = \tan(x+c)$  are shown below.



Determine the values of the constants  $a$  and  $b$  and the smallest positive value of the constant  $c$ . (3 marks)

**Solution**

$$a=1.2, b=4, c=\frac{2\pi}{3}$$

**Specific behaviours**

- value of  $a$
- value of  $b$
- value of  $c$

- (b) One day, the depth of water in a tidal basin was modelled by  $d = 5.5 + 3.7 \sin(0.5t - 0.6)$ , where  $d$  was the depth in metres and  $t$  was the time, in hours, after midnight. For this day, determine

- (i) the depth of water at 9.30 am. (2 marks)

**Solution**

$$9.30 \text{ am} \Rightarrow t=9.5$$

$$d|_{9.5}=2.37 \text{ m}$$

**Specific behaviours**

- converts time
- determines depth

- (ii) the last time in the afternoon that the depth of water was 8.3 m. (2 marks)

**Solution**

$$5.5 + 3.7 \sin(0.5t - 0.6) = 8.3$$

Using CAS:  $t = 18.33 \text{ h}$ , or 6.20 pm

**Specific behaviours**

- writes equation
- solves equation over stated domain

## Question 20

Five different letters are selected from the eleven in the word COMRADESHIP. The order in which the letters are selected is not important, so that the selection COMRA is the same as the selection RAMOC, and so on.

- (a) Determine the number of different selections

- (i) of five letters. (2 marks)

Solution
$n = \binom{11}{5} = 462$
Specific behaviours
✓ uses combination ✓ correct number

- (ii) of five letters that contain one vowel and four consonants. (2 marks)

Solution
$n = \binom{4}{1} \times \binom{7}{4} = 4 \times 35 = 140$
Specific behaviours
✓ splits selections ✓ multiplies selections

- (b) Determine the probability that a random selection of five different letters

- (i) includes the letters M and R. (2 marks)

Solution
$P = \frac{\binom{2}{2} \times \binom{9}{3}}{\binom{11}{5}} = \frac{84}{462} = \frac{2}{11}$
Specific behaviours
✓ selects (i) M & R (ii) other four ✓ states probability

- (ii) includes at least one vowel. (2 marks)

Solution
$P = 1 - \frac{\binom{4}{0} \times \binom{7}{5}}{\binom{11}{5}} = 1 - \frac{21}{462} = \frac{441}{462} = \frac{21}{22}$
Specific behaviours
✓ selects no vowels ✓ states probability

at  $x = -4$  and

(a) Determine the values of the constants  $a$ ,  $b$  and  $c$ .

(d) Are multi-axle vehicles excluded from the market? (1 mark)

(1) A and B are unusually exclusive.

(a) Determine  $P(B)$  when

(卷三十一)

Writies in factored form	Specific behaviours
$a = \frac{2}{-1} = -0.5, b = \frac{2}{3}, c = 1.5, d = 14$	
$? = -\frac{2}{1}x^2 + \frac{2}{3}x + 14$	
$f(x) = -\frac{2}{1}(x+4)(x-7)$	
$a = \frac{2}{-1}$	
$f(5) = 9 = a(9)(-2)$	
$f(x) = a(x+4)(x-7)$	Solutions

Solution	$P(B = P(A \cup B) - P(A))$	0.68 - 0.15 = 0.53
Specific behaviors uses probability laws states probability	$\rightarrow$ dependent.	

(Continued)

$y = -3(14) = -42$ <b>Solution</b>	$y = -3f(x)$ <b>Specific behaviours</b>
$y = -3x$ <b>Correcl y-value</b>	

Solution	<p>Let <math>P(A \cup B) = x</math> and <math>P(A) \times P(B) = P(A \cap B)</math></p> $0.2 \times (0.48 + x) = x$ $x = 0.12$ $P(B) = 0.48 + 0.12 = 0.6$
Determining $P(A \vee B)$ if $P(A \cap B) = 0.36$ . (3 marks)	<p>Uses probability law for independence</p> <p>Uses probability law for independence</p> <p>Determines <math>P(A \cup B)</math></p>

(c) State the location of the roots of the graph  $y = f(4x)$ . (2 marks)

Specific behaviours	
$P(A B) = \frac{0.48 + 0.02}{0.02} = \frac{0.5}{0.02} = 0.04$	determines $P(A \cup B)$
$P(A \cap B) = 0.2 - 0.18 = 0.02$	
$x = 0.18$	
$\frac{x+0.32}{x} = 0.36$	
Let $P(A \cap B) = x$ and $P(A \cup B) = 0.32$	
Solution	

**Question 14**

(8 marks)

- (a) Evaluate the following and leave your solution in standard form (scientific notation).

$$\frac{\sqrt{m}}{n^4} \text{ if } m = 1.44 \times 10^6 \text{ and } n = 2 \times 10^{-2}$$

(3 marks)

$$\begin{aligned} & \frac{\sqrt{1.44 \times 10^6}}{(2 \times 10^{-2})^4} \\ &= \frac{1.2 \times 10^3}{1.6 \times 10^{-7}} \\ &= 0.75 \times 10^{10} \\ &= 7.5 \times 10^9 \end{aligned}$$

✓

✓

✓

- (b) The number of bacteria,
- $P$
- , in a culture increases with time, in hours, according to the equation
- $P = P_0 r^t$
- . The initial number of bacteria in the culture was 1200 and six hours later there were
- $1.5 \times 10^4$
- bacteria.

- (i) Determine the values of
- $P_0$
- and
- $r$
- .

(2 marks)

$$\begin{aligned} 1.5 \times 10^4 &= 1200(r)^6 \\ P_0 &= 1200 \\ r &= 1.5234 \end{aligned}$$

✓

✓

- (ii) How many bacteria would there be after 12 hours?

(1 mark)

$$P = 187500 \checkmark$$

- (iii) Determine, to the nearest minute, when the number of bacteria will reach 1 000 000.

$$t = 15.987 \checkmark$$

(2 marks)

15 hours and 59 minutes.(plus or minus one minute)

✓

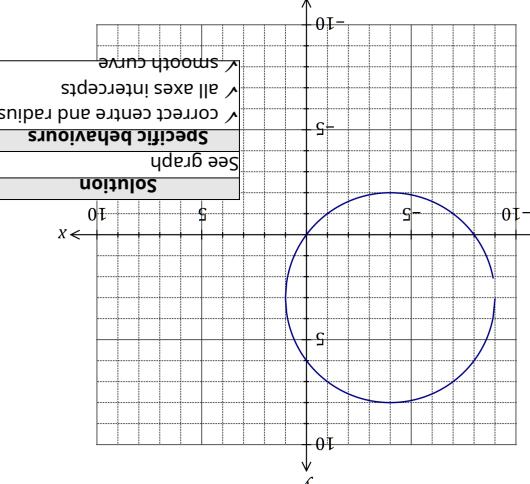
- (d) Determine the area common to both circles.

(5 marks)

Solution
Segment, centre $B$ :
Angle: $2 \times 49.58 \times \frac{\pi}{180} = 1.731$
$A = \frac{1}{2}(13)^2[1.731 - \sin(1.731)] = 62.83$
Segment, centre $A$ :
$\angle PAB = 98.21^\circ$
Angle: $2 \times 98.21 \times \frac{\pi}{180} = 3.428$
$A = \frac{1}{2}(10)^2[3.428 - \sin(3.428)] = 185.55$
Total:
$A = 62.83 + 185.55 = 248.4 \text{ cm}^2$
Specific behaviours
✓ uses segment formula with angles in radians
✓ states area of segment, centre $B$
✓ shows

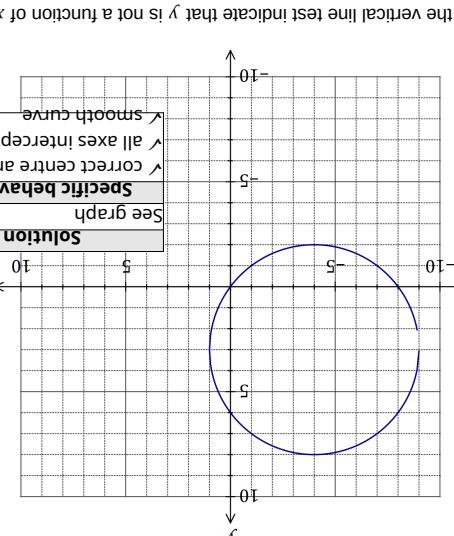
(a) Sketch the graph of this relationship.  
(3 marks)

- (a) The variables
- $x$
- and
- $y$
- are related by
- $(x+4)^2 + (y-3)^2 = 25$
- .



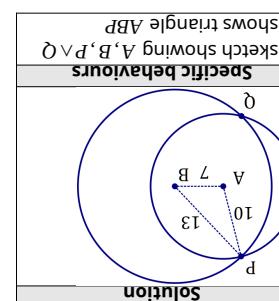
(i)

- Sketch the graph of this relationship.
- 
- (2 marks)



(ii)

- Sketch the graph of this relationship.
- 
- (2 marks)

(a) Sketch a diagram of the two circles and clearly show triangle  $ABP$ .  
(3 marks)(a) Sketch a diagram of the two circles and clearly show triangle  $ABP$ .  
(3 marks)

<b>Solution</b>
$\cos B = \frac{2(7)(13)}{7^2 + 13^2 - 10^2} \angle PBA = 49.583$
$\angle PBA = 49.6^\circ$ (1dp)
$\angle PBA = 49.6^\circ$ (1dp)
<b>Specific behaviours</b>
substitutes correctly into cosine rule
states angle to 2 or more dp then rounds

(c) Determine the length of the chord  $PQ$  to the nearest millimetre.  
(2 marks)

<b>Solution</b>
$PQ = 13 \sin 49.6^\circ$
$PQ = 19.8 \text{ cm}$
<b>Specific behaviours</b>
uses right-angle trig
states length, rounded to nearest mm

**See next page**

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NS085-102-A

**See next page****See next page**

(d)

- State the domain and range of the function
- $f(x) = 4 - \sqrt{x+3}$
- .
- 
- (2 marks)

<b>Solution</b>
$D_f = \{x : x \in \mathbb{R}, x \geq -3\}$
$R_f = \{y : y \in \mathbb{R}, y \leq 4\}$
<b>Specific behaviours</b>
indicates restriction on $x$ values
indicates restriction on $y$ values

**See next page**

- (d) State the domain and range of the function
- $f(x) = 4 - \sqrt{x+3}$
- .
- 
- (2 marks)

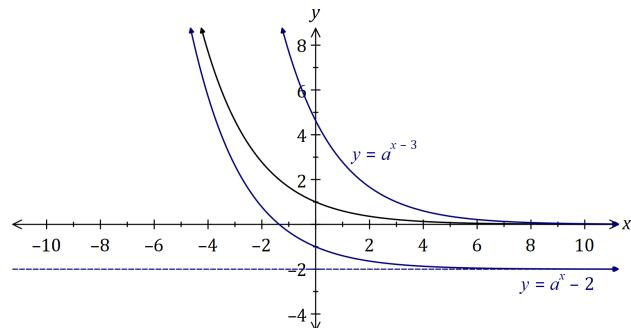
<b>Solution</b>
$PQ = 19.8 \text{ cm}$
$\frac{1}{2} PQ = 13 \sin 49.6^\circ$
<b>Specific behaviours</b>
uses right-angle trig
states length, rounded to nearest mm

**See next page**

- (d) State the domain and range of the function
- $f(x) = 4 - \sqrt{x+3}$
- .
- 
- (2 marks)

## Question 16

The graph of  $y=a^x$  is shown below, where  $a$  is a positive constant.



- (a) On the same axes, sketch and label the graphs of

(i)  $y=a^{x-3}$ .

Solution
See graph
<b>Specific behaviours</b>

(2 marks)

(ii)  $y=a^x-2$ .

Solution
(i) ✓ y-int close to (0, 5); ✓ touches x-axis close to (9, 0)

(2 marks)

- (b) The graph of  $y=a^{x-3}$  intersects the graph of  $y=1.2^x$  when  $x=2.1$ .

Determine, giving your answers to 3 significant figures,

- (i) the  $y$ -coordinate of the point of intersection.

(1 mark)

Solution
$y=1.2^{2.1}$
✓ 1.466495...

≈ 1.47 (3sf)

Specific behaviours
✓ value that rounds to 1.47

- (ii) the value of the constant  $a$ .

(3 marks)

Solution
$a^{2.1-3} = 1.2^{2.1}$
$a = 0.653497\dots$

$a \approx 0.653$  (3sf)

Specific behaviours
✓ writes equation
✓ writes solution to equation
✓ rounds answers to (b)(i) & (ii) correctly

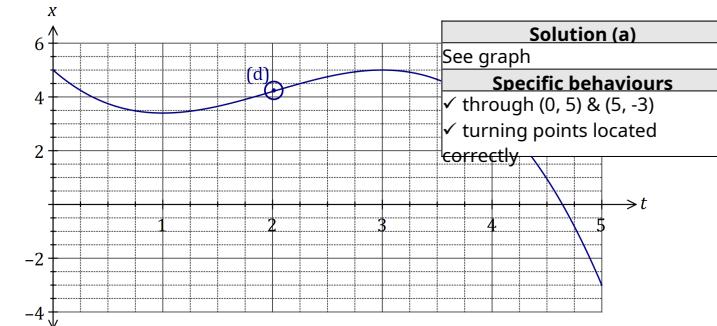
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## Question 17

A particle is moving along a straight line so that its displacement,  $x$  metres, from a fixed point  $O$  after  $t$  seconds is given by

$$x = 5 - \frac{18t}{5} + \frac{12t^2}{5} - \frac{2t^3}{5}.$$

- (a) Sketch the displacement of the particle on the axes below for  $0 \leq t \leq 5$ . (3 marks)



Solution (a)
See graph
<b>Specific behaviours</b>

✓ through (0, 5) & (5, -3)

✓ turning points located correctly

- (b) Determine the velocity of the particle when  $t=0.5$ . (2 marks)

Solution
$v = \frac{-(6t^2 - 24t + 18)}{5}$

$$v(0.5) = \frac{-3}{2} = -1.5 \text{ m/s}$$

Specific behaviours
✓ differentiates displacement

✓ states velocity

- (c) For how long during the first five seconds is the particle moving towards  $O$ ? (1 mark)

Solution
3 seconds (for $0 < t < 1$ and $3 < t < 5$ ).

Specific behaviours
✓ correct time

- (d) Circle the point on the graph where the particle is moving with the maximum velocity and explain what feature of the graph you used to choose this point. (2 marks)

Solution
See graph. Velocity is maximum at point where curve has greatest positive slope.

Specific behaviours
✓ indicates point of inflection

✓ explanation

See next page