PERTH MODERN SCHOOL

UNIT 3C/3D MAS – 2010

TEST 4 – POLAR COORDINATES, COMPLEX NUMBERS & PROOFS

NAME: ____ DATE: ______________

[To achieve full marks and to allow assessment of particular outcomes, working and reasoning should be shown.]
[A maximum of 2 marks will be deducted for incorrect rounding, units, etc.]

This is *Resource Free, no calculator allowed – 50 minutes for 40 marks*:

Question 1 [3 marks]

Express $1 - i$ and $1 + \sqrt{3}\,i$ in polar form and ***hence*** simplify $(1 + \sqrt{3}\,i)^5 \div (1 - i)^4$.

Question 2 [2 marks]

Find the **exact** distance between the points A $[6, 25^0]$ and B $[10, 145^0]$

Question 3 [5marks]

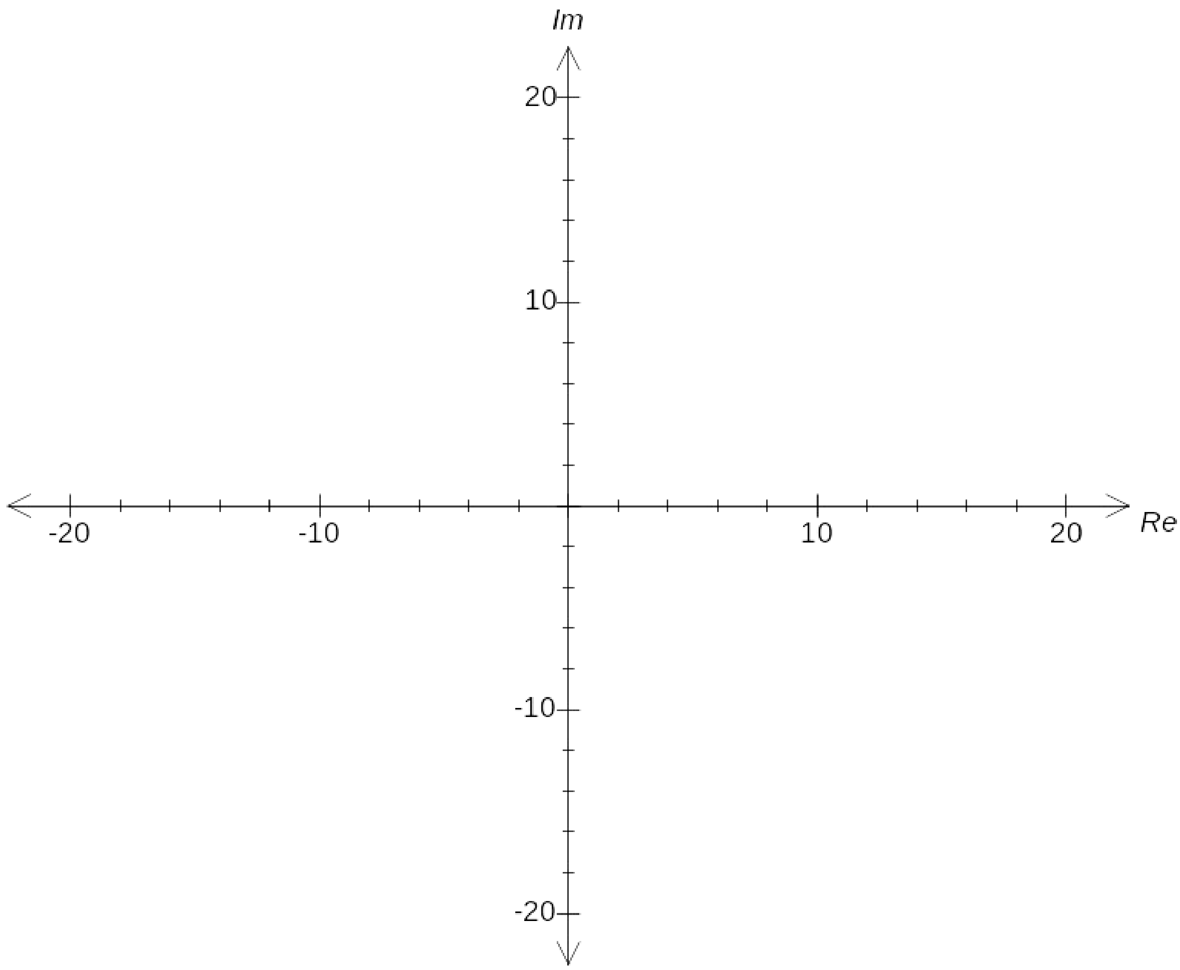
Given $\mathbf{z} = 3 - 3i$ indicate on a single Argand Diagram the points representing

a) $\mathbf{z}^{-1}$

b) $\mathbf{z}^{2}$

c) $i\mathbf{z}$

d) $\mathbf{z}\bar{\mathbf{z}}$



Question 4 [2, 3 marks]

a) Express the following in the form (a, b) where (a, b) represents the complex number a + b*i*:

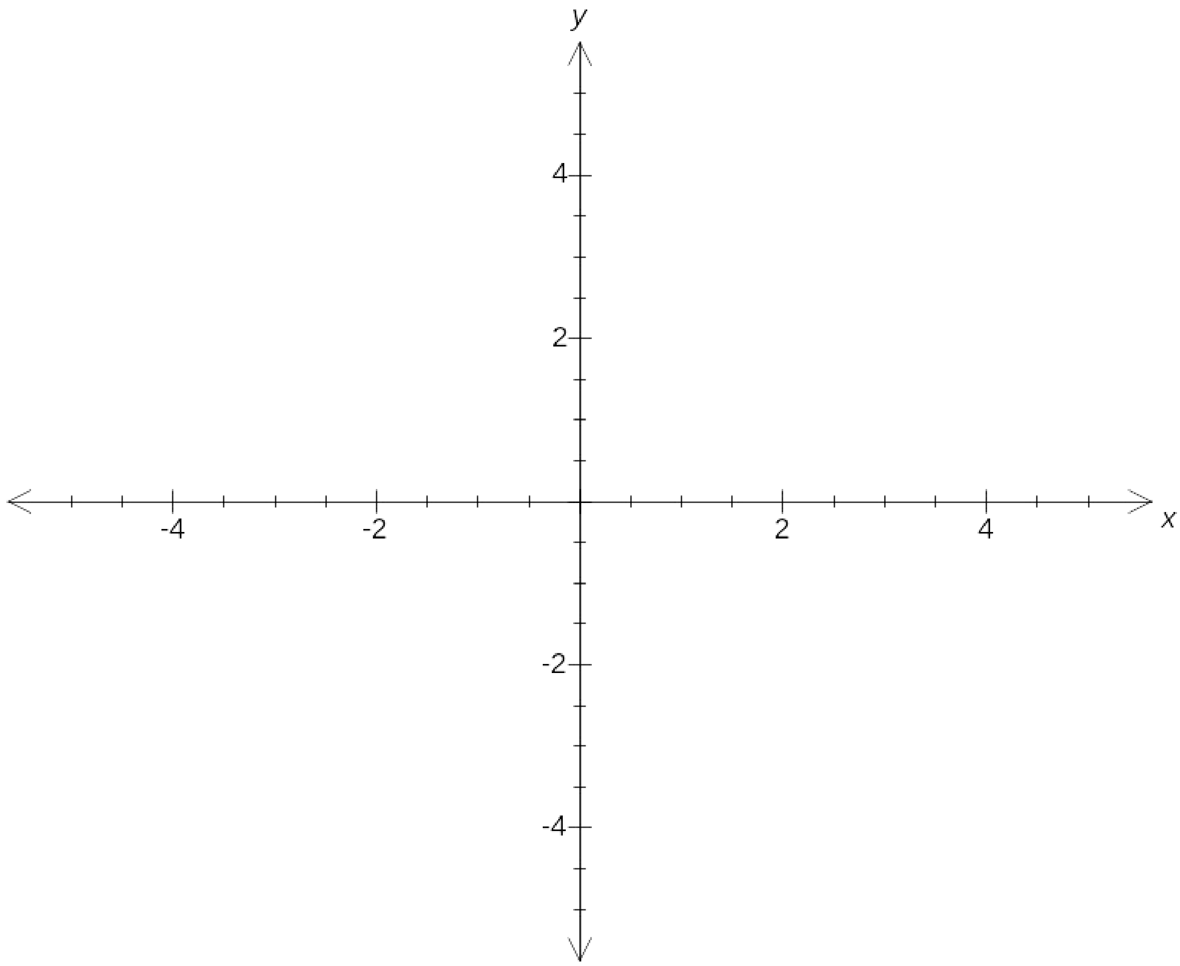
$e^{2 - 0.5\pi i}$

b) Using $\cos\theta + i\sin\theta = e^{i\theta}$ and $\cos\theta - i\sin\theta = e^{-i\theta}$ obtain an expression for both $\sin\theta$ and $\cos\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$.

Hence prove $\sin(2\theta) = 2\sin\theta\cos\theta$.

Question 5 [2, 2 marks]

a) Draw the graph of $y = |2x + 1| + |2x - 1|$



b) Hence, or otherwise, define y as a piecewise function

Question 6 [3 marks]

OABC is a rhombus with $\overrightarrow{\mathbf{OA}} = \mathbf{a}$ and $\overrightarrow{\mathbf{OC}} = \mathbf{c}$.

Use vector methods to prove the diagonals of a rhombus are perpendicular to each other

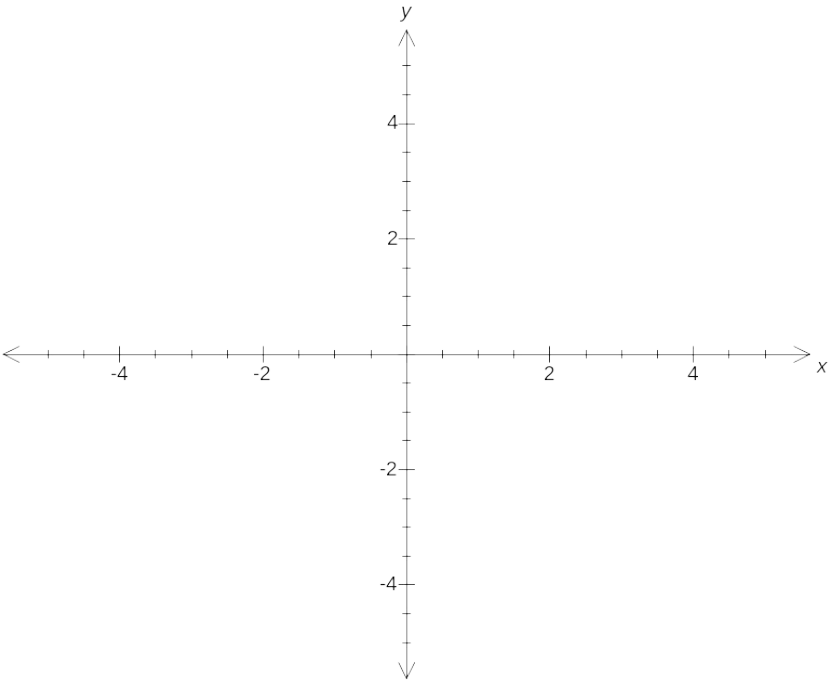
Question 7 [4 marks]

Prove by exhaustion that for a positive integer n then n(n + 1) (n + 2) is a multiple of 3.

Question 8 [2, 2, 2 marks]

a) Find the set of values for x for which $1 - x > 2|x + 1|$.

b) Graph the function $y = 1 - x - 2|x + 1|$.



c) ***Explain*** how your graph can be used to solve part a).

Question 9 [3 marks]

Determine exact values for all the roots of $\mathbf{z^3 = -8 + 8\sqrt{3}\,\boldsymbol{i}}$

Question 10 [5 marks]

Find all complex numbers Z satisfying $\dfrac{1}{z} + \dfrac{2}{\bar{z}} = 1 + i$