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Rossmoyne Senior High School

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNITS 1&2

SOLUTIONS

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

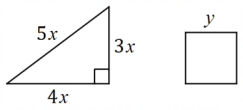
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Supplementary page

Question number: _____

Question 21 **(7 marks)**

A length of wire 72 cm long is cut into two pieces. One piece is bent into a right triangle with sides of length $3x, 4x$ and $5x$ cm and the other piece is bent into a square of side y cm.



(a) Show that the combined area of the triangle and square in terms of x is $A = 15x^2 - 108\,x + 324$ (3 marks)

Solution
$12x + 4y = 72 \Rightarrow y = 18 - 3x$ $A = \frac{1}{2}(4x)(3x) + y^2$ $= 6x^2 + (18 - 3x)^2$ $= 15x^2 - 108x + 324$
Specific behaviours
✓ equation relating x and y
✓ total area in terms of x and y
✓ total area in terms of x

(a) Use a calculus method to determine the value of x that minimises this combined area and find this minimum area. (4 marks)

Solution
$\frac{dA}{dx} = 30x - 108$ $30x - 108 = 0$ $x = \frac{18}{5} = 3.6$ $A(3.6) = 15(3.6)^2 - 108(3.6) + 324$ $= \frac{648}{5} = 129.6$ The minimum total area is $129.6\ \text{cm}^2$.
Specific behaviours
✓ derivative
✓ equates derivative to 0
✓ optimum value of x
✓ calculates and states minimum area

Section Two: Calculator-assumed **65% (97 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

**Allow answer only for 2 mark questions unless otherwise stated

Question 9 **(5 marks)**

Sector POQ subtends an angle of 80° in a circle with centre O and radius r.

(a) Express 80° as an exact and simplified radian measure. (1 mark)

Solution
$80° = \frac{4\pi}{9}$ radians
Specific behaviours
✓ value

The area of sector POQ is 50π cm^2.

(b) Determine the radius of the circle. (2 marks)

Solution
$\frac{1}{2}r^2 \times \frac{4\pi}{9} = 50\pi$ $r = 15$ cm
Specific behaviours
If use degrees 0 marks
✓ indicates equation
✓ calculates radius

(c) Determine the area of the minor segment bounded by arc PQ and chord PQ. (2 marks)

Solution
$A = \frac{1}{2}(15)^2\left(\frac{4\pi}{9} - \sin\frac{4\pi}{9}\right)$ $= 46.3\ \text{cm}^2$
Specific behaviours
If use degrees 0 marks
✓ indicates equation
✓ calculates area

See next page

Question 10 **(8 marks)**

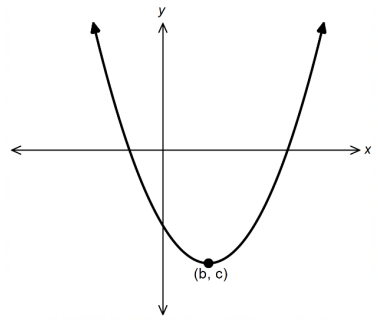
The graphs of the following equations are drawn below.

(i) $y = 2^x - 2$ (ii) $y = -x(1-x)^2$ (iii) $y = (1-x)^3 + 1$

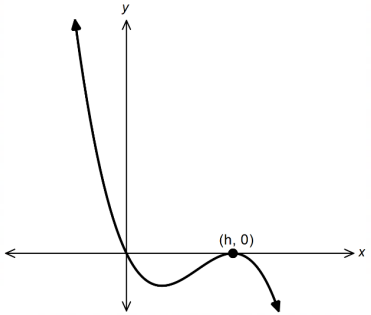
(iv) $y = 2 - 2^x$ (v) $y = (1-x)^2 - 3$ (vi) $y = (x+1)^2 - 4$

a, b, c, d, f, g and h are arbitrary constants, x and y are variables.

A



B



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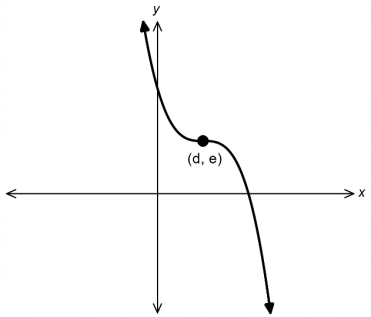
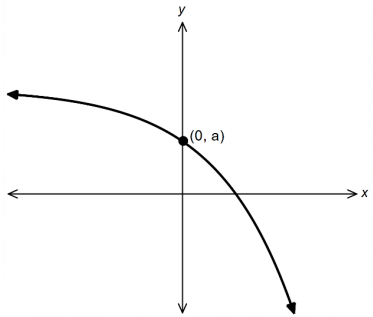


D



(d) Determine the length of time, correct to 3 decimal places, during the first 3 seconds for which $h_C > h_A > h_B$. (3 marks)

Solution

Use CAS to graph heights and identify required interval.

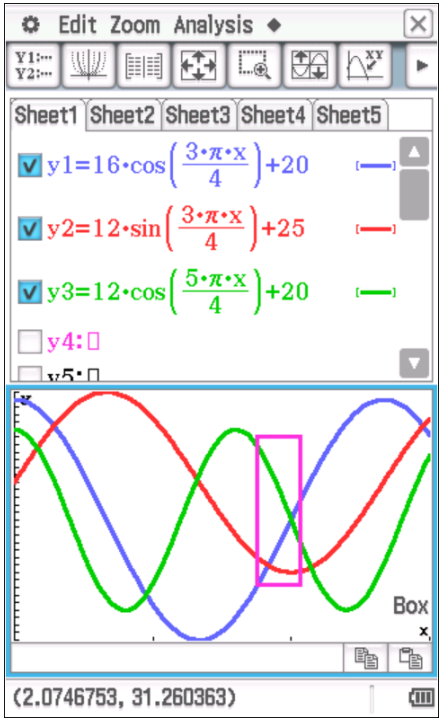
$$h_A = h_B \rightarrow t = 1.8341$$
$$h_A = h_C \rightarrow t = 2$$

Length of time:

$$\Delta t = 0.1659$$
$$\approx 0.166 \text{ s (3 dp)}$$

Specific behaviours

✓ indicates one endpoint
✓ indicates second endpoint
✓ calculates difference to **3 decimal places** correctly



Question 20 **(9 marks)**

Three small weights A, B and C, each attached to a spring, are oscillating vertically above level ground. The height, h cm, above the ground of each weight at time t seconds, $t \geq 0$, is given by

$$h_A = 16\cos\left(\frac{3\pi t}{4}\right) + 20, \qquad h_B = 12\sin\left(\frac{3\pi t}{4}\right) + 25, \qquad h_C = 12\cos\left(\frac{5\pi t}{4}\right) + 20.$$

(a) State which two weights are oscillating with the same amplitude, and state what this common amplitude is. (1 mark)

Solution
Weights B and C - their amplitude is 12 cm.
Specific behaviours
✓ correct weights and amplitude

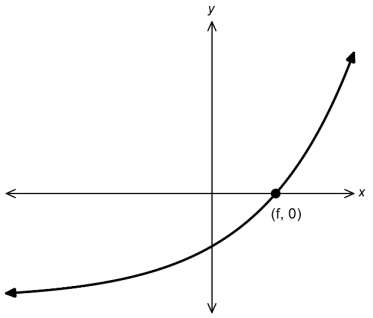
(b) State which two weights are oscillating with the same period, and state what this common period is. (2 marks)

Solution
Weights A and B - their period is $2\pi \div \frac{3\pi}{4} = \frac{8}{3}$ s.
Specific behaviours
✓ correct weights ✓ correct period

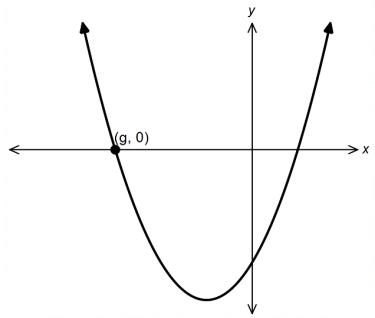
(c) State which of the weights reaches furthest above the ground, state this height and find the time at which it first reaches this position. (3 marks)

Solution
$h_A = 20 + 16 = 36, \quad h_B = 25 + 12 = 37, \quad h_C = 20 + 12 = 32$ Hence weight B reaches furthest above the ground with a height of 37 cm. When: $\sin\left(\frac{3\pi t}{4}\right) = 1 \Rightarrow \frac{3\pi t}{4} = \frac{\pi}{2} \Rightarrow t = \frac{2}{3}$ This first occurs when $t = \frac{2}{3}$ s.
Specific behaviours
✓ states correct weight ✓ states this height (**with units**) ✓ states correct time (**with units**)

E



F



(a) Match the equations with the sketches by writing **(i)** to **(vi)** corresponding to the equation in the table below. (3 marks)

Specific behaviours
✓ one or two correct ✓ three or four correct ✓ five or all correct

A	B	C	D	E	F
v	ii	iii	iv	i	vi

Hence,

(b) determine the values of a, b, c, d, e, f, g and h. (5 marks)

a	b	c	d	e	f	g	h
1	1	-3	1	1	1	-3	1

Specific behaviours
No follow through from a)
✓ one or two correct
✓ three or four correct
✓ five or six correct
✓ seven
✓ all correct

Question 11 **(9 marks)**

A function is defined by $f(x) = x^4 - 6x^2 + 8x + 13$.

(a) Use calculus to determine the coordinates of all stationary points of the graph $y = f(x)$ and then use the sign test to determine their nature. (5 marks)

Solution

$$f'(x) = 4x^3 - 12x + 8$$

$$f'(x) = 0 \Rightarrow x = -2, 1$$

Sign test	x	-2^-	-2	-2^+
	$f'(x)$	$-$	0	$+$
Sign test	x	1^-	1	1^+
	$f'(x)$	$+$	0	$+$

$(-2, -11)$ is a minimum turning point and
$(1, 16)$ is a horizontal point of inflection

Specific behaviours

✓ shows $f'(x)$
✓ solves $f'(x) = 0$
✓ correct use of sign test for both
✓ states correct nature of both stationary points
✓ states coordinates of both points

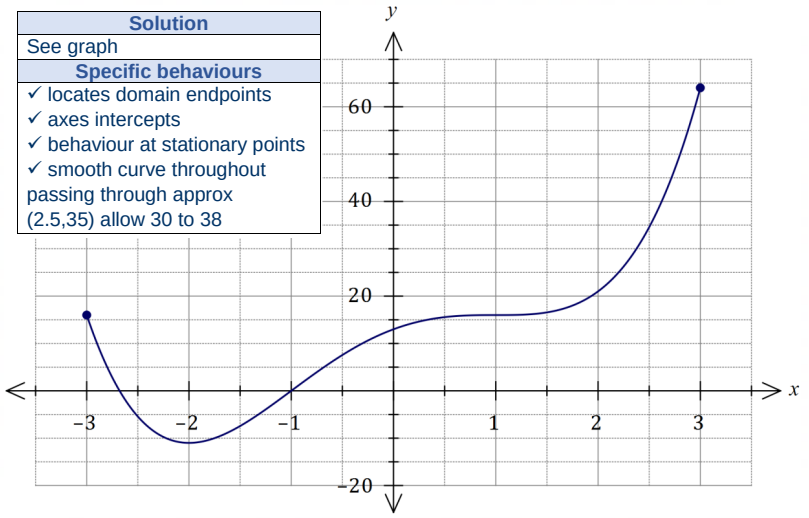
(b) Sketch the graph of $y = f(x)$ on the axes below for $-3 \leq x \leq 3$. (4 marks)

Solution

See graph

Specific behaviours

✓ locates domain endpoints
✓ axes intercepts
✓ behaviour at stationary points
✓ smooth curve throughout passing through approx (2.5,35) allow 30 to 38



(c) Add the line $y = 2x + 3$ to the graph of the hyperbola and state the number of points of intersection it will have with the hyperbola. (2 marks)

Solution

See graph for line.
It will have 2 points of intersection with the hyperbola.

Specific behaviours

✓ correct line
✓ correct number of intersections

(d) The line $y = mx + 3$ is tangential to the hyperbola, where m is a constant. Use an algebraic method to determine all possible values of m. (4 marks)

Solution

Require one solution to intersection of lines:

$$\frac{3}{x-3} = mx + 3$$
$$3 = (x-3)(mx+3)$$
$$mx^2 + (3-3m)x - 12 = 0$$

For one solution, quadratic discriminant $\Delta = b^2 - 4ac = 0$:

$$\Delta = (3-3m)^2 - 4(m)(-12) = 0$$

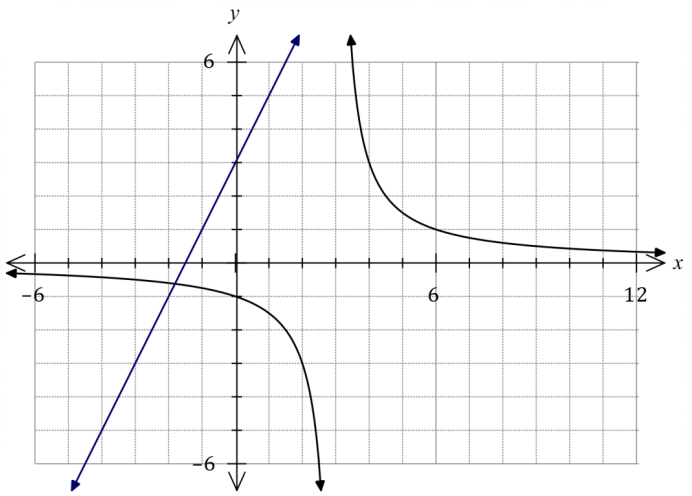
Using CAS: $m = -3,\ m = -\frac{1}{3}$.

Specific behaviours

✓ forms equation by equating both functions
✓ obtains quadratic from equating both lines
✓ uses discriminant to form equation in m
✓ both correct values

Question 19 **(10 marks)**

The graph of the hyperbola $y = \frac{a}{x+b}$ is shown below, where a and b are constants.



(a) State the equations of all asymptotes of the hyperbola. (2 marks)

Solution
Horizontal: $y = 0$ Vertical: $x = 3$.
Specific behaviours
✓ equation for horizontal asymptote ✓ equation for vertical asymptote

(b) Determine the value of a and the value of b. (2 marks)

Solution
From asymptote, $b = -3$. Using $(0, -1)$: $-1 = \frac{a}{0-3} \Rightarrow a = 3$
Specific behaviours
✓ value of a ✓ value of b

Question 12 **(8 marks)**

Data from repairs to 405 smartphones showed that 274 of them were Android. The type of repair was classified as battery or other, and of the 136 smartphones that required battery repairs, 98 were Android.

(a) Complete the missing entries in the table below. (3 marks)

Specific behaviours
✓records 98 correctly ✓ 274 and 131 correct ✓ all correct in table

	Battery	Other	Total
Android	98	176	274
Not Android	38	93	131
Total	136	269	405

(b) Determine the probability that a randomly selected smartphone from those repaired

(i) did not require a battery repair. (1 marks)

* Part (b) allow follow through from (a),
No need to simplify fraction for probability. If rounded to 2dp -1 mark once.

Solution
$P(\bar{B}) = \frac{269}{405} \approx 0.664$
Specific behaviours
✓ correct probability

(ii) was an Android smartphone or required battery repairs. (2 marks)

Solution
$274 + 136 - 98 = 312$ $P(A \cup B) = \frac{312}{405} = \frac{104}{135}$ ≈ 0.770
Specific behaviours
✓ calculates numerator ✓ correct probability

(iii) did not require a battery repair given that it was an Android smartphone. (2 marks)

Solution
$274 - 98 = 176$ $P(\bar{B}\|A) = \frac{176}{274} = \frac{88}{137} \approx 0.642$
Specific behaviours
✓ uses denominator 274 ✓ calculates probability

Question 13 **(8 marks)**

An aeroplane takes off from an airport situated at an altitude of 150 metres above sea level and climbs 450 metres during the first minute of flight. In each subsequent minute, its rate of climb reduces by 4%.

(a) Determine the **increase in altitude** of the aeroplane during the second minute. (1 mark)

Solution

$\Delta A = 450(0.96)$
$= 432 \text{ m}$

Specific behaviours

✓ correct increase

(b) Determine the actual **altitude** of the aeroplane at the end of 2 minutes. (1 mark)

Solution

$A_2 = 432 + 450 + 150$
$= 1032 \text{ m}$

Specific behaviours

✓ correct altitude

(c) Deduce a rule in simplified form for the **altitude** A_n of the aeroplane at the end of the n^{th} minute. (3 marks)

Solution

A_n will be sum of terms plus initial altitude:

$$A_n = \frac{450(1-0.96^n)}{1-0.96} + 150$$
$$= 11\,250(1-0.96^n) + 150$$
$$= 11\,400 - 11\,250(0.96)^n$$

Specific behaviours

✓ correct use of sum formula
✓ includes initial altitude
✓ simplifies (to last or second last line)

(d) Determine the altitude of the aeroplane at the end of 12 minutes. (1 mark)

Solution

$A_{12} = 4357 + 150 = 4507 \text{ m}$

Specific behaviours

✓ calculates correct term

(e) Determine the maximum altitude the aeroplane can reach. (2 marks)

Solution

$A_\infty = 11\,250(1-0.96^\infty) + 150$
$= 11\,400\text{m}$

Specific behaviours

✓ recognises S_∞ needed
✓ correct altitude

Alternative Solution

$$A_\infty = \frac{450}{1-0.96} + 150$$
$= 11\,400\text{m}$

Specific behaviours

✓ recognises S_∞ needed
✓ correct altitude

See next page

Question 18 **(9 marks)**

A random selection of 4 paint brushes is made from a collection of 16 different brushes, 9 of which are flat and the remainder round.

(a) Show that the probability the selection contains all round brushes is $\frac{1}{52}$. (3 marks)

Solution

Total possible selections is $\binom{16}{4} = 1820$.
Number of round brushes is $16 - 9 = 7$.
Ways to select all round is $\binom{7}{4} = 35$.

$$P(\text{All Round}) = \frac{35}{1820} = \frac{1}{52}$$

Specific behaviours

✓ calculates number of all possible selections 1820
✓ calculates number of ways to select all round 35
✓ uses counts to form probability no f/t

Alternative Solution

$$\frac{7}{16}\text{x}\frac{6}{15}\text{x}\frac{5}{14}\text{x}\frac{4}{13} = \frac{1}{52}$$

Specific behaviours

✓$\frac{7}{16}$
✓ uses no replacement
✓ uses counts to form prob no f/t

(b) Determine the probability that the selection contains

(i) all flat brushes. (2 marks)

Solution

Ways to select all flat is $\binom{9}{4} = 126$.

$$P(\text{All Flat}) = \frac{126}{1820} = \frac{9}{130} \ (\approx 0.06923)$$

Specific behaviours

✓ calculates number of ways to select all flat
✓ correct probability

Alternative Solution

$$\frac{9}{16}\text{x}\frac{8}{15}\text{x}\frac{7}{14}\text{x}\frac{6}{13} = \frac{9}{130}$$

Specific behaviours

✓$\frac{9}{16}$
✓ correct probability

(ii) at least one round brush. (2 mark)

Solution

$$P = 1 - \frac{9}{130} = \frac{121}{130} (\approx 0.93077)$$

Specific behaviours

✓Calculates no round brushes 9 (alternative method total 121 ok)
✓ correct probability

(iii) at least one round brush and at least one flat brush. (2 marks)

Solution

$$P(\text{All of same type}) = \frac{1}{52} + \frac{9}{130} = \frac{23}{260}$$
$$P = 1 - \frac{23}{260} = \frac{237}{260} (\approx 0.9115)$$

Specific behaviours

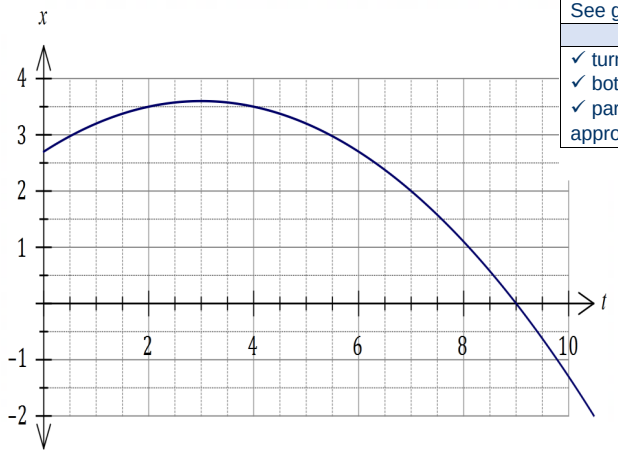
✓ probability all of same type (or similar appropriate method)
✓ correct probability

See next page

Question 17 **(6 marks)**

Particle P is moving along the x-axis so that its displacement, in cm, at time t seconds, $t \geq 0$, is given by $x = 2.7 + 0.6t - 0.1t^2$.

(a) Sketch the displacement-time graph of particle P on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ turning point
✓ both intercepts
✓ parabolic shape passes approx through (7,2) & (9.5,-0.6)

(b) Determine the velocity of particle P at the instant it reaches the origin. (3 marks)

Solution
Reaches origin when $x = 0 \Rightarrow t = 9$.
$v = \frac{dx}{dt} = 0.6 - 0.2t$
$v(9) = 0.6 - 0.2(9) = -1.2$ cm/s
Specific behaviours
✓ indicates correct time
✓ obtains velocity function
✓ correct velocity

Question 14 **(8 marks)**

Two events S and T are such that $P(S) = 0.46$ and $P(T) = 0.35$.

Determine the following probabilities.

(a) $P(\overline{S \cup T})$ when S and T are mutually exclusive. (2 marks)

Solution
$P(S \cup T) = 0.46 + 0.35 = 0.81$
$P(\overline{S \cup T}) = 1 - 0.81 = 0.19$
Specific behaviours
✓ indicates $P(S \cup T)$
✓ correct probability

(b) $P(S \cup T)$ when $P(\bar{S} \cap T) = 0.22$. (2 marks)

Solution
$P(S \cup T) = P(S) + P(\bar{S} \cap T)$
$= 0.46 + 0.22 = 0.68$
Specific behaviours
✓ indicates suitable method
✓ correct probability

**Parts c) & d) no deduction if rounded to 2 decimal places, just comment (-1 mark once if 1 dp)

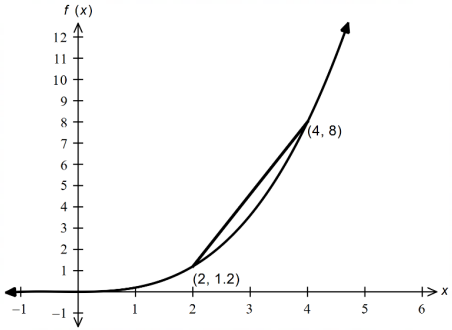
(c) $P(S \cap \bar{T})$ when S and T are independent. (2 marks)

Solution
$P(S \cap T) = 0.46 \times 0.35 = 0.161$
$P(S \cap \bar{T}) = 0.46 - 0.161 = 0.299$
Specific behaviours
✓ indicates $P(S \cap T)$
✓ correct probability

(d) $P(T|S)$ when $P(S|T) = 0.6$. (2 marks)

Solution
$P(S \cap T) = 0.35 \times 0.6 = 0.21$
$P(T\|S) = 0.21 \div 0.46 = \frac{21}{46} \approx 0.457$
Specific behaviours
✓ indicates $P(S \cap T)$
✓ correct probability

Question 15 **(5 marks)**



(a) Determine the average rate of change of the function $y = f(x)$ between $x = 2$ and $x = 4$. (2 marks)

Solution
$ARC = \dfrac{8 - 1.2}{4 - 2}$ $= 3.4$
Specific behaviours
✓ uses correct idea
✓ correct rate

The following table shows points on the curve.

x	3	2.1	2.01
y	3.6	1.37	1.216

(b) Use all the information in the table above to demonstrate how to use the difference quotient $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to show that the instantaneous rate of change of the function $y = f(x)$ at $x = 2$ is 1.6. (3 marks)

Solution
$\dfrac{f(3) - f(2)}{1} = \dfrac{3.6 - 1.2}{1} = 2.4$ ✓ $\dfrac{f(2.1) - f(2)}{0.1} = \dfrac{1.37 - 1.2}{0.1} = 1.7$✓ $\dfrac{f(2.01) - f(2)}{0.01} = \dfrac{1.216 - 1.2}{0.01} = 1.6$✓
Specific behaviours
See above

Question 16 **(5 marks)**

The sum of the first n terms of a sequence is given by $S_n = 4n^2 + 7n$.

(a) Determine S_4. (1 mark)

Solution
$S_4 = 4(4)^2 + 7(4)$ $= 92$
Specific behaviours
✓ correct value

(b) Determine T_4, where T_n is the n^{th} term of the sequence. (1 mark)

Solution
$S_3 = 4(3)^2 + 7(3) = 57$ $T_4 = S_4 - S_3$ $= 92 - 57$ $= 35$
Specific behaviours
✓ calculates T_4

(c) Determine a simplified rule for the n^{th} term of the sequence. (3 marks)

Solution
$T_1 = S_1 = 11$ $T_2 = S_2 - T_1 = 30 - 11 = 19$ $d = T_2 - T_1 = 19 - 11 = 8$ $T_n = 11 + (n-1)(8) = 8n + 3$
Specific behaviours
✓ $T_1 = 11$ ✓ calculates common difference   ✓ correct rule simplified

Alternative Solution
$T_n = S_n - S_{n-1}$ $= 4n^2 + 7n - (4(n-1)^2 + 7(n-1))$ $= 4n^2 + 7n - 4n^2 + 8n - 4 - 7n + 7$ $T_n = 8n + 3$
Specific behaviours
✓ forms equation   ✓ expands and simplifies   ✓ correct rule simplified