## Calculator Free Anti-Differentiation Techniques

Time: 45 minutes Total Marks: 45 Your Score: / 45



## Question One: [2, 2, 2, 3, 3, 3, 3, 3 = 21 marks] CE

positive indices. Anti-differentiate each of the following, showing all working. Leave all answers with

 $1p\frac{z^{2}}{t}$ 

(a) 
$$\int_{\frac{1}{2}}^{\frac{1}{2}} dt$$

(p) np nz uis -∫

$$xp \left( S - xb \right)$$
 (5)

$$xb (\overline{x} - x\pi x + x^{-x}) dx$$

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Mathematics Methods Unit 3

$$\int \frac{4t^6 - 6t^2}{8t^2} \ dt$$

(e

$$\int (x^2 - 2)^3 dx$$

$$\int \cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} dx$$

(8

$$\int (e^{-2x} + 1)(e^{3x} - 2) dx$$

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Mathematics Methods Unit 3 Question Two: [3, 3, 3 = 9 marks]

CE

Calculate the following integrals, showing all working.

$$xb(1-^{z}x)_{1}^{z}$$
(a)

$$xb x \varepsilon \operatorname{nis}_{\frac{\varepsilon}{2}}^{\frac{\kappa}{2}} \Sigma - (d$$

$$\int_{1}^{\infty} \left(-e^{4x} + 2\right) dx$$

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 $= \frac{1}{\sqrt{2}} = \frac$ Mathematics Methods Unit 3

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Mathematics Methods Unit 3

Question Three: [3 marks] CF

The derivative of f(x) is given by  $f'(x) = 2e^{2x} + 3x^2$  . Given that  $f(1) = 4 + e^2$  , find an f(x)expression for .

Question Four: [6 marks] CF

The gradient function of f(x) is given by  $f'(x) = ax^2 + b$  . Determine the values of aand b if f(-2) = 28, f(0) = 1 f(1) = 7 and b.

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Mathematics Methods Unit 3

Question Five: [1, 2, 3 = 6 marks]

$$\int_{1}^{2} f(x) dx = 4 \qquad \int_{1}^{7} f(x) dx = 10$$
Given that and

, determine:

$$2\int_{-1}^{7}f(x)\ dx$$

(a)

$$=2\times10$$

$$\int_{0}^{2} f(x) dx$$

$$= \int_{-1}^{7} f(x)dx - \int_{-1}^{2} f(x)dx \quad \checkmark$$

$$\int_{0}^{2} (f(x) + x) dx$$

Mathematics Methods Unit 3 Question Five: [1, 2, 3 = 6 marks]

resulon five: [1, 2, 3 = 0 marks]

CE

 $0I = xb(x) \int_{1}^{x} b = xb(x) \int_{1}^{x}$  band that the state of the properties of the state o

 $xb(x) \int_{1}^{7} \zeta$  (a)

 $(q) xp(x) \iint_{z}^{L}$ 

 $xp(x+(x)\downarrow)\int_{1}^{z}$  (5)

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Mathematics Methods Unit 3 28 = 4a + b  $f(x) = \frac{ax^3}{3} + bx + c$  1 = c  $7 = \frac{a}{3} + b + 1$  28 = 4a + b  $26 = \frac{a}{3} + b$   $6 = \frac{a}{3} + b$   $7 = \frac{a}{3} + b$   $8 = \frac{11}{3}a$   $9 = \frac{11}{3}a$   $1 = \frac{11}{3}a$   $1 = \frac{11}{3}a$   $2 = \frac{11}{3}a$   $4 = \frac{11}{3}a$   $5 = \frac{11}{3}a$   $6 = \frac{11}{3}a$   $6 = \frac{11}{3}a$   $7 = \frac{11}{3}a$   $8 = \frac{11}{3}a$   $9 = \frac{11}$ 

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## SOLUTIONS Calculator Free Anti-Differentiation Techniques

Time: 45 minutes Total Marks: 45 Your Score: / 45

Question One: [2, 2, 2, 3, 3, 3, 3, 3 = 21 marks]

CF

Anti-differentiate each of the following, showing all working. Leave all answers with positive indices.

$$\int_{\overline{t^2}}^4 dt$$

(a

$$\int 4t^{-2} dt$$

$$= \frac{4t^{-1}}{-1} + c \quad \checkmark$$

$$= \frac{-4}{t} + c \quad \checkmark$$

(b) 
$$\int -\sin 2u \ du$$

$$= \frac{\cos 2u}{2} + c$$

(c) 
$$\int (4x-5)^3 dx$$

$$= \frac{(4x-5)^4}{4\times 4} + c$$

$$= \frac{(4x-5)^4}{16} + c$$

Mathematics Methods Unit 3

Question Three: [3 marks]

CF

The derivative of f(x) is given by f'(x) = 
$$2e^{2x} + 3x^2$$
 is given by f(1) =  $4 + e^2$ , find an expression for .

$$f(x) = \int 2e^{2x} + 3x^2 dx$$

$$f(x) = e^{2x} + x^3 + c \checkmark$$

$$4 + e^2 = e^2 + 1 + c \checkmark$$

$$c = 3$$

$$f(x) = e^{2x} + x^3 + 3 \checkmark$$

Question Four: [6 marks]

The gradient function of is given by is given by is given by . Determine the values of 
$$a$$
 and  $b$  if  $f'(-2) = 28$ ,  $f(0) = 1$   $f(1) = 7$  and  $f'(-2) = 28$ .

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Mathematics Methods Unit 3  $\int_{\mathbb{R}^{5}} e^{-5\pi x} - x\pi x - \sqrt{x} \int_{\mathbb{R}^{5}} dx$ (b)

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1.

Mathematics Methods Unit 3  $= \left[ \sqrt{\frac{e^{4x}}{4}} + 2X \right]^{\frac{2}{4}} = \left[ \sqrt{\frac{e^{-4}}{4}} - 2 \right] - \left( \sqrt{\frac{e^{-4}}{4}} - 2 \right) = \left( \sqrt{\frac{e^{-4}}{4}} + 2 \right) = \left( \sqrt{\frac{e^{-4}}{4$ 

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OI

Mathematics Methods Unit 3

$$\int \frac{4t^6 - 6t^2}{8t^2} dt$$

(e)

$$= \int_{8t^2}^{4t^6} -\frac{6t^2}{8t^2} dt$$

$$= \int_{2}^{t^4} -\frac{3}{4} dt$$

$$= \frac{t^5}{10} - \frac{3t}{4} + c$$

$$\int (x^2 - 2)^3 dx$$

$$= \int (x^6 + 3(x^2)^2(-2) + 3(x^2)(-2)^2 + (-2)^3) dx$$

$$= \int (x^6 - 6x^4 + 12x^2 - 8) dx$$

$$= \frac{x^7}{7} - \frac{6x^5}{5} + \frac{12x^3}{3} - 8x + c$$

$$\int \cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} dx$$

$$\int \cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} dx$$

$$(g) \qquad \qquad \checkmark$$

$$= 3\sin\left(\frac{x}{3}\right) + \frac{3(6x)^{\frac{4}{3}}}{8} + c$$

$$\int (e^{-2x} + 1)(e^{3x} - 2)dx$$

$$= \int (e^x - 2e^{-2x} + e^{3x} - 2) dx$$

$$= e^x + \frac{1}{e^{2x}} + \frac{e^{3x}}{3} - 2x + c$$

Mathematics Methods Unit 3

Question Two:

[3, 3, 3 = 9 marks]

CF

Calculate the following integrals, showing all working.

$$\int_{1}^{2} (x^2 - 1) dx$$

(a)

$$= \left[\frac{x^3}{3} - x\right]_{-1}^2 \checkmark$$

$$= \left(\frac{8}{3} - 2\right) - \left(\frac{-1}{3} + 1\right) \checkmark$$

$$= \frac{9}{3} - 3$$

$$= 3 - 3$$

$$= 0 \checkmark$$

$$-2\int_{\frac{\pi}{3}}^{\frac{\pi}{3}}\sin 3x \ dx$$

(b)

$$=-2\left[\frac{-\cos 3x}{3}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$=-2\left[\frac{-\cos \pi}{3} - \frac{-\cos \frac{\pi}{2}}{3}\right]$$

$$=-2\left(\frac{1}{3} + 0\right)$$

$$=\frac{-2}{3}$$

$$\int_{1}^{3} (-e^{4x} + 2) dx$$
 (c)