PERTH MODERN SCHOOL
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Independent Public School

Course Methods Year 12

Student name: _______________ Teacher name: _______________

Task type: **Response**

Time allowed for this task: **_____45______ mins**

Number of questions: **_____8______**

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **__49____ marks**

Task weighting: **__10__%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3.1.7) **(9 marks)**

Use the product rule and/or quotient rule to differentiate the following.(Simplify)

i) $y = (x - 11)(x^3 + 2)$ (3 marks)

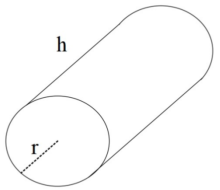
Solution
$\frac{dy}{dx} = (x-11)3x^2 + (x^3+2)(1)$ $= 3x^3 - 33x^2 + x^3 + 2$ $= 4x^3 - 33x^2 + 2$
Specific behaviours
✓ demonstrates use of product rule ✓ differentiates correctly ✓ simplifies NOTE: Zero for answer only as done by classpad

ii) $y = \frac{2x+1}{(3-x)}$ (3 marks)

Solution
$\frac{dy}{dx} = \frac{(3-x)2 - (2x+1)(-1)}{(3-x)^2}$ $= \frac{6 - 2x + 2x + 1}{(3-x)^2}$ $= \frac{7}{(3-x)^2}$ (May leave denominator in expanded form)
Specific behaviours
✓ demonstrates use of quotient rule ✓ differentiates correctly ✓ simplifies NOTE: Zero for answer only as done by classpad

Q8 **(3.1.16)** **(4 marks)**

Consider a closed hollow cylinder with end radius r and length h.



If the outside of the cylinder has a surface area of $300\,m^2$ determine the dimensions of the radius and length, nearest cm, to maximise the capacity of the cylinder **using calculus techniques**.

Solution
$2\pi r^2 + 2\pi rh = 300$ $h = \dfrac{150 - \pi r^2}{\pi r}$ $V = \pi r^2 h = \pi r^2 \dfrac{150 - \pi r^2}{\pi r} = 150r - \pi r^3$ $\dfrac{dV}{dr} = 150 - 3\pi r^2 = 0$ $r = \sqrt{\dfrac{50}{\pi}} \approx 3.98m \quad h \approx 7.99m$ $\dfrac{d^2V}{dr^2} = -6\pi r = -6\pi\sqrt{\dfrac{50}{\pi}} < 0 \quad \therefore local\ \max$
Specific behaviours
✓ states constraint equation in terms of r and h ✓ differentiates V and equates to zero ✓ solves for r and h, **must be in decimal** form but do not penalise if not rounded to nearest cm ✓uses second derivative test to show local max

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iii) $y = (5-2x)(x^2+1)^3$ (3 marks)

Solution
$(5-2x)3(x^2+1)^2\,2x + (x^2+1)^3(-2)$ $2(x^2+1)^2\left[3x(5-2x) - (x^2+1)\right]$ $2(x^2+1)^2\left[15x - 6x^2 - x^2 - 1\right]$ $2(x^2+1)^2\left[15x - 7x^2 - 1\right]$
Specific behaviours
✓ demonstrates use of product **and** chain rules correctly ✓ differentiates correctly for entire function ✓Simplifies correctly NOTE: Zero for answer only as done by classpad

Q2 **(3 marks)**

Determine the equation of the tangent to $y = (3x+1)^3$ at the point $(1, 64)$.

Solution
$\dfrac{dy}{dx} = 3(3x+1)^2\,3$ $\left.\dfrac{dy}{dx}\right\|_{x=1} = 144$ $y = 144x + c$ $64 = 144 + c$ $c = -80$ $y = 144x - 80$
Specific behaviours
✓ uses chain rule to differentiate ✓ solves for constant ✓states equation

Q7 (3.1.11) (6 marks)

A colony of bacteria is represented as a circle on a petri dish and is increasing in such a way that the number of bacteria present is given by N where $N=\sqrt{3x+2}$, x being the radius of the circle of bacteria.

a) Determine $N'(2)$ and explain its meaning. (3 marks)

Solution
$N'=\frac{3}{2}(3x+2)^{\frac{-1}{2}}$ $N'(2)=\frac{3}{2\sqrt{8}}=\frac{3}{4\sqrt{2}}\approx 0.53$ Rate of change of N at x=2 (SCSA preferred answer)
Specific behaviours
✓ states derivative in terms of x   ✓ states value at x=2(accept approx.)   ✓describes as rate of change **at x=2** (accept gradient of tangent at x=2)

b) Determine $N''(2)$ and explain its meaning. (3 marks)

Solution
$N''=\frac{-3}{4}(3x+2)^{\frac{-3}{2}}(3)$ $=\frac{-9}{4(8)^{\frac{3}{2}}}\approx -0.09943$ Rate of change of $N'(x)$ at x=2 (SCSA preferred answer)
Specific behaviours
✓ states second derivative in terms of x   ✓ states value at x=2(accept approx.)   ✓describes as rate of change of $N'(x)$ **at x=2** (accept gradient of dy/dx at x=2)

Note must mention at x=2 otherwise max 4 out of 6 marks

c) The distance travelled in the first 12 seconds. (2 marks)

Solution
t=0 x=18 t=5.5 x=-12.25 turns around t=12 x=30 Distance equals 18 +12.25 +12.25 +30=72.5 metres
Specific behaviours
✓ determines distance from start to turning pt ✓ determines total distance, no need for units.

d) $x''(1)$ and explain its meaning. (2 marks)

Solution
Acceleration of 2 at t=1 second
Specific behaviours
✓ states acceleration at time t=1 (accept rate of change of v at t=1) ✓ states 2 for second derivative

Q6 (3.1.10) (3 marks)

If $y=3x^5$ use the small increments formula $\delta y \approx \frac{dy}{dx}\delta x$ to determine the approximate percentage change in y when x decreases by 2%.

Solution
$\frac{\Delta y}{y} \approx \frac{\frac{dy}{dx}\Delta x}{y}$ $=\frac{15x^4\Delta x}{3x^5}=5\frac{\Delta x}{x}=10\%$
Specific behaviours
✓ uses increments formula ✓ obtains expression for approx. percentage change for y in terms of x ✓obtains approx. percentage change for y

Q3 (3.1.8) (8 marks)

Consider the functions $P(x) \& Q(x)$ and their derivatives $P'(x) \& Q'(x)$ with values given for the following x values.

X value	-1	3	7
$P(x)$	5	2	-4
$P'(x)$	0	1	-2
$Q(x)$	2	5	-3
$Q'(x)$	-1	-2	6

Determine the following **derivatives** at the given x values.'

a) $P(x)Q(x)$ at $x=3$ (2 marks)

Solution
$P(x)Q(x)$ $P(x)Q'(x)+Q(x)P'(x)$ $(2)(-2)+(5)(1)$ 1
Specific behaviours
✓ uses product rule ✓states result

b) $[Q(x)]^3$ at $x=-1$ (3 marks)

Solution
$3[Q(x)]^2 Q'(x)$ $3[2]^2(-1)$ -12
Specific behaviours
✓ demonstrates chain rule ✓ subs values correctly ✓states final result

c) $\frac{[P(x)]^2}{Q(x)}$ at $x = 7$ (3 marks)

Solution
$\frac{Q(x)2P(x)P'(x) - P^2(x)Q'(x)}{Q^2(x)}$ $\frac{(-3)2(-4)(-2) - (-4)^2(6)}{9}$ -16
Specific behaviours
✓ demonstrates quotient **and** chain rule   ✓ subs values correctly   ✓states final result

Q4 (3.1.14, 3.1.15) (7 marks)

Use calculus techniques to determine the **exact** coordinates of any stationary points on the following curves and use the second derivative test to determine the nature of the stationary point.

a) $y = (x-4)^3 - 1$ (3 marks)

Solution
$y' = 3(x-4)^2 = 0$ $x = 4$ $y'' = 6(x-4) \quad x = 4 \Rightarrow y'' = 0$ (4,-1) inflection
Specific behaviours
✓ determines first derivative   ✓ equates to zero and solves for stationary pt and states y value   ✓determines value of second derivative and states horizontal inflection

b) $y = 2x^3 + 9x^2 - 60x + 12$ (4 marks)

Solution
$y' = 6x^2 + 18x - 60 = 0$ $x^2 + 3x - 10 = (x+5)(x-2) = 0$ $x = -5, 2$ $y'' = 12x + 18$ $x = -5 \; y'' = -42 \quad \therefore local \max(-5, 287)$ $x = 2 \; y'' = 42 \quad \therefore local \min(2, -56)$
Specific behaviours
✓ determines first derivative and equates to zero   ✓ solves for stationary pts including y value   ✓ determines second derivative for stationary pts   ✓identifies nature for each stationary point

Q5 (3.1.12) (7 marks)

The displacement of a body from an origin O, at time t seconds, is x metres where $x = t^2 - 11t + 18$, $t \geq 0$.

Determine the following.

a) The velocity function. (2 marks)

Solution
$v = 2t - 11$
Specific behaviours
✓ differentiates   ✓ expresses in terms of t

b) The times and displacements when the body is at rest. (3 marks)

Solution
$2t - 11 = 0$ $t = 5.5$ $x = -12.25$
Specific behaviours
✓ equate velocity to zero   ✓ solves for time   ✓determines displacement