WESLEY COLLEGE
SOUTH PERTH

Semester One Examination, 2011

Question/Answer Booklet

MATHEMATICS 3C/3D

Section Two: Calculator-assumed

Student Name: ____

Time allowed for this section

Reading time before commencing work: Ten (10) minutes
Working time for this section: One hundred (100) minutes

Material required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	40	33 1/3
Section Two: Calculator-assumed	11	11	100	80	66 2/3
				120	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

Section Two: Calculator-assumed **(80 Marks)**

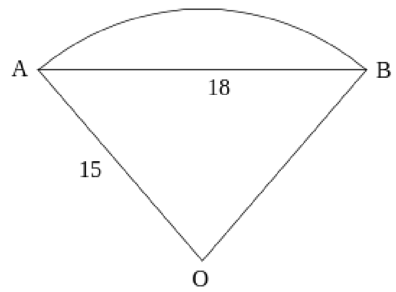
This section has **eleven (11)** questions. Answer **all** questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

The working time for this section is 100 minutes.

Question 9 **(3 marks)**



The sector of a circle, as shown, is divided by the line *AB* (which is a chord of the full circle) into a triangle *OAB* and a segment (above *AB*).

For $AB = 18$ and $OA = 15$, calculate:

(a) the altitude of ΔOAB (2 marks)

(b) the area of ΔOAB (1 mark)

Question 10 **(9 marks)**

A particle has displacement (position) x at time t defined by $x(t) = t^3 - 6t^2 + 9t + 4$

Determine expressions or values that describe:

(a) the rate of change of displacement, at time t (1 mark)

(b) when the particle is stationary (2 marks)

(c) the maximum displacement that occurs between $t = 0$ and $t = 4$ (2 marks)

(d) the distance travelled between $t = 0$ and $t = 4$ (1 mark)

(e) when, between $t = 0$ and $t = 4$, the particle is travelling fastest (3 marks)

Question 11 **(7 marks)**

(a) Calculate the equation of the tangent to the curve $y = e^{\frac{x}{2}} = \exp\left(\frac{x}{2}\right)$ drawn at the point $(2, e)$

(2 marks)

(b) Identify the *y*-value of the point on this tangent where $x = 2.02$ (1 mark)

(c) Starting at $(2, e)$, use the incremental method to estimate the value of $y = e^{\frac{x}{2}}$ where $x = 2.02$

(2 marks)

(d) With the aid of a sketch, explain the connection between your answers to parts (b) and (c)

(2 marks)

Question 12 **(5 marks)**

(a) Complete this table of values for the different integer values of *n* listed (2 marks)

n	2	3	4	5	6
$2n$	4	6			
$n^2 - 1$	3	8			
$n^2 + 1$	5				

(b) How do these values relate to the lengths of the sides of a right angled triangle? (1 mark)

(c) Prove that your conjecture is true for all integer values of $n \geq 2$ (2 marks)

Question 13 **(9 marks)**

Consider the function defined by the equation $y = \dfrac{x^2}{x - 4}$

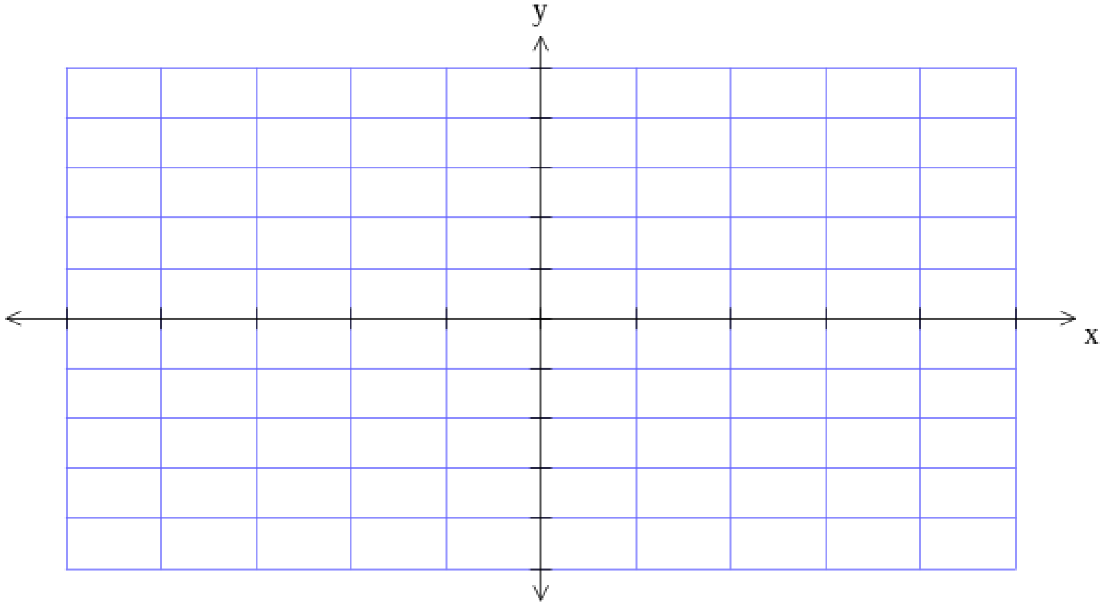
(a) For which *x* value(s) is the function not defined? (1 mark)

(b) Determine a simplified expression for $\dfrac{dy}{dx}$ (2 marks)

(c) Show how $\dfrac{dy}{dx}$ can be used to locate the stationary points of $y = \dfrac{x^2}{x-4}$ (2 marks)

(d) Use the second derivative test to classify these stationary points (2 marks)

(e) Complete a sketch of $y = \dfrac{x^2}{x-4}$ on these axes, clearly showing the asymptote(s) and stationary points. (2 marks)



Question 14 **(6 marks)**

A rectangle is drawn in the first quadrant with two sides lying on the (positive) *x* and *y* axes and the remaining vertex on the curve $y = \sqrt{9 - x}$

(a) Draw a sketch of this situation (2 marks)

(b) Write the area of the enclosed rectangle in terms of *x*, the ordinate of the remaining vertex (1 mark)

(c) Show clearly how a calculus technique can be used to determine the maximum possible area of such a rectangle. (3 marks)

Question 15 **(7 marks)**

The lifetime (L hours) of a fully charged battery in a notebook computer is believed to decrease at a rate proportional to the current lifetime, which is defined as a function of the computer's age t years.

(a) Show, by differentiation that $L = L_0 e^{kt}$ satisfies this condition. Specify any necessary restrictions on the value of k (3 marks)

My notebook computer had an 8 hour battery life when it was brand new and, after 6 months, this lifetime has declined to 7.2 hours.

Evaluate:

(b) k (2 marks)

(c) the battery lifetime expected for a three year old computer (1 mark)

(d) the age when the battery needs to be replaced because it is lasting less than three hours. (1 mark)

Question 16 **(4 marks)**

In a learning/memory experiment, a student could remember M words after t minutes, where $M(t) = 0.015t^3 - 0.45t^2 + 4t$

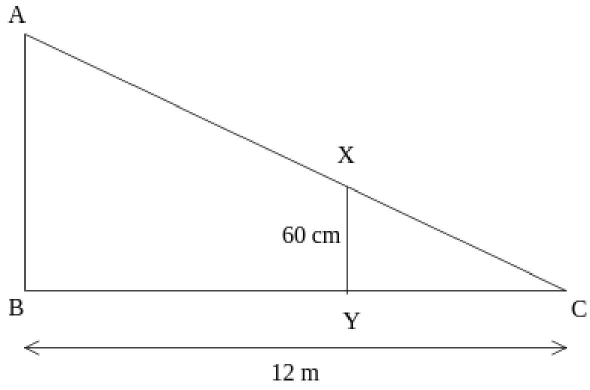
With the help of your ClassPad, determine:

(a) his initial rate of learning (2 marks)

(b) the general pattern of the number of words learned at different t values between $t = 0$ and $t = 14$ (2 marks)

Question 17 **(10 marks)**

A spotlight, mounted on horizontal ground at point C, is directed towards a 60 cm ruler XY so that a shadow is cast on the wall AB. Both XY and AB are vertical and $BC = 12$ m.



(a) Prove that ΔABC is similar to ΔXYC (2 marks)

(b) XY is positioned so that the shadow $AB = 1.8$ m. Determine the lengths CY and BY.
(2 marks)

(c) Let $AB = h$ and $CY = x$. Show clearly that $h = \frac{7.2}{x}$ m (1 mark)

(d) Calculate $\frac{dh}{dx}$ and then, for time t, use this result to express $\frac{dh}{dt}$ in terms of $\frac{dx}{dt}$
(2 marks)

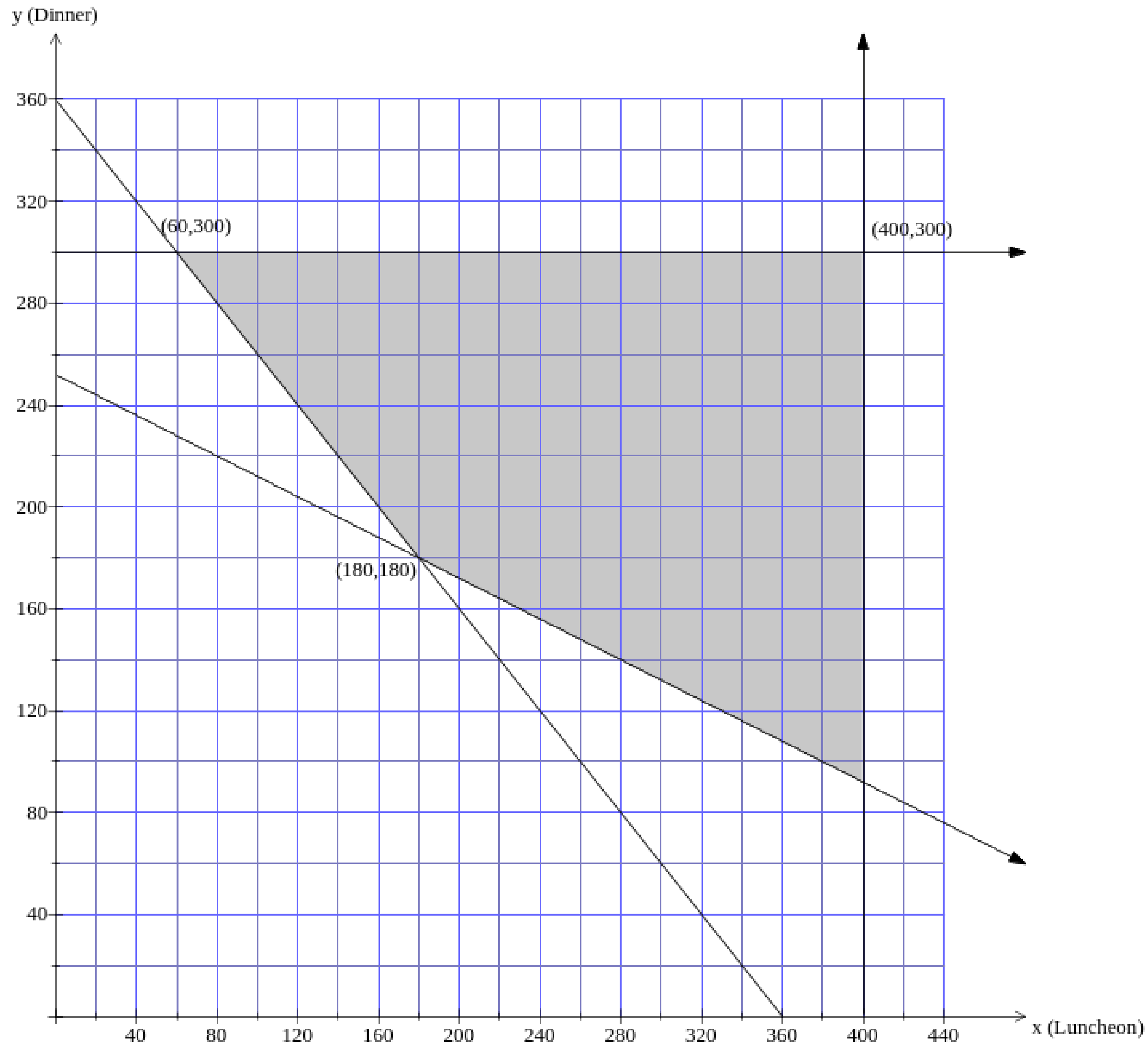
(e) The ruler *XY* is moving towards the wall at a rate of 0.1 m sec^{-1}. How is the length of the shadow changing when it is 1.8 m high? (3 marks)

Question 18 **(13 marks)**

William and Catherine are planning a small wedding and will invite their guests to either a luncheon to be hosted by William's grandmother, Elizabeth, or to a buffet dinner at his father Charles' house.

No guest will be invited to both functions.

In the graph below, *x* represents the number of luncheon guests, *y* the number of dinner guests and the other constraints represent the maximum capacity of each venue, the minimum total number to be invited and a publicity/television need for $2x + 5y \geq 1260$.



(a) What is the maximum capacity of Charles' house? (1 mark)

(b) What is the minimum total number of guests that must be invited? (1 mark)

(c) After discussions in the family, it is agreed that the number of guests for dinner must be at least (i.e. greater than or equal to) half the number of luncheon guests. Add this constraint to the graph and clearly mark the resulting feasible region. (2 marks)

Catherine's parents are paying £250 for each luncheon guest and for each dinner guest and, naturally, wish to keep their contribution as low as possible. (£ = British pound.)

(d) How many guests should be invited to each function, in order that this cost be minimised? (3 marks)

Elizabeth's advisors suggest that she save some expenses for Catherine's parents by contributing more toward the luncheon costs. How should Elizabeth adjust her contribution so that, maintaining all the existing constraints, a single optimal solution has:

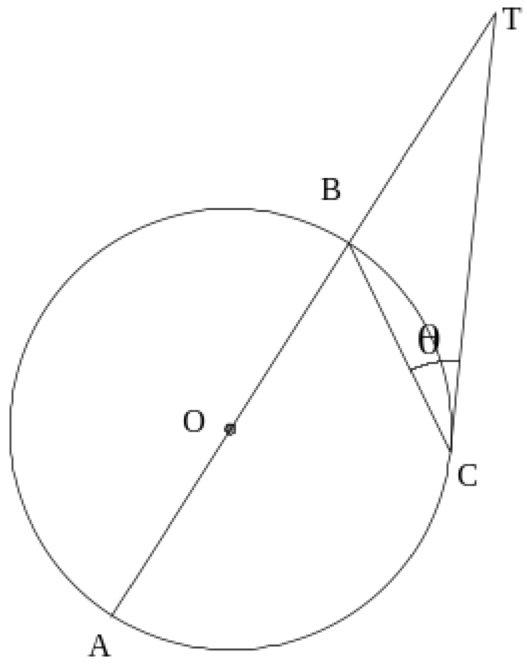
(e) as many luncheon guests as possible (3 marks)

(f) as few luncheon guests as possible (2 marks)

(g) equal numbers at the luncheon and the dinner (1 mark)

Question 19 **(7 marks)**

A circle with centre *O* and diameter *AB* has a tangent drawn at *C* that intersects the extension of *AB* at point *T*, as shown.



(a) $\angle BCT = \theta$. Derive, in terms of θ, an expression for $\angle ATC$ (3 marks)

(b) If $CT = t$ and $BT = x$, show that the radius of the circle is $\dfrac{t^2 - x^2}{2x}$ (2 marks)

(c) If $AC = CT = t$, $BC = BT = x$ and radius $AO = r$, set up a second equation involving *x*, *r* and *t*.

Solve these equations simultaneously on your ClassPad to find values of *r* and *t*.

Interpret these solutions in terms of the geometry of this problem. (2 marks)

Additional working space

Question number(s):_______________

Additional working space

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