PERTH MODERN SCHOOL
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Independent Public School

Course Methods Year 12 test two 2022

Student name: ______________ Teacher name: ______________

Task type: **Response**

Time allowed for this task: **____40_____ mins**

Number of questions: **___7______**

Materials required: **Upto 3 calculators/classpads allowed**

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, **one page of A4 notes doublesided**

Marks available: **__40__ marks**

Task weighting: **_10__%**

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2 & 2 = 4 marks) (3.2.1)

Let $f'(x) = 6x^3 + 1$,

a) Determine an expression for the rate of change of $f'(x)$.

Solution
$f''(x) = 18x^2$
Specific behaviours
P recognises that derivative is needed P correct expression

b) Determine $f(x)$ given that $f(3) = 1$.

Solution
$f'(x) = 6x^3 + 1$ $f(x) = \frac{3}{2}x^4 + x + c$ $1 = \frac{3^5}{2} + 3 + c$ $c = -123.5$ $f(x) = \frac{3}{2}x^4 + x - 123.5$
Specific behaviours
P anti differentiates and uses a constant c P solves for constant

Q2 (3 marks) (3.2.3-3.2.9)

Determine x in terms of t given that $\frac{dx}{dt} = \frac{-5}{(3t+5)^3}$ and $x = 10$ when $t = 1$.

Solution

Extra working space

$\frac{dx}{dt} = \frac{-5}{(3t+5)^3} = -5(3t+5)^{-3}$ $x = \frac{5}{6(3t+5)^2} + c$ $10 = \frac{5}{6(64)} + c$ $c = \frac{3835}{384}$ or ~ 9.99 or 9.98
Specific behaviours
P anti- diffs and uses plus c P sets up equation to solve for c P solves for c (accept approx.)

Q3 (4 marks) (3.2.21-3.2.22)

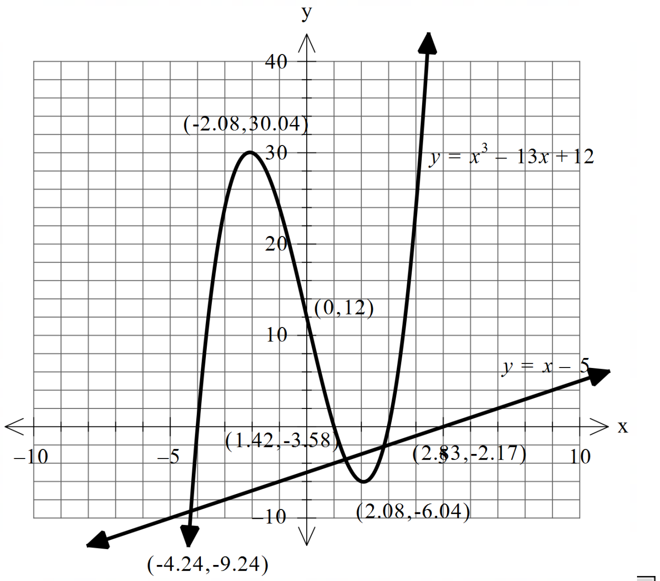
A particle travels along a straight line such that its acceleration at time t seconds is equal to $(3t^2+2t+1)\,m/s^2$. When $t=0$ the displacement is 10 metres and when $t=2$ the displacement is 20 metres. Determine the displacement when $t=3$.

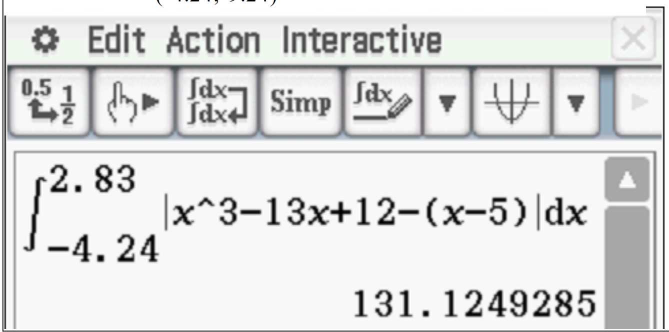
Solution
$a = (3t^2+2t+1)\,m/s^2$ $v = t^3+t^2+t+c$ $x = \frac{t^4}{4}+\frac{t^3}{3}+\frac{t^2}{2}+ct+p$ $t=0, x=10, p=10$ $20 = 4+\frac{8}{3}+2+2c+10$ $c = \frac{2}{3}$ $x = \frac{3^4}{4}+9+\frac{9}{2}+2+10 = \frac{183}{4}$
Specific behaviours
P anti diff to find v with constant stated P anti diff to find x with a new constant P solves for both constants P displacement for t=3

Q4 (6 marks) (3.2.19-3.2.20)

Make a sketch showing the graphs of $y = x^3 - 13x + 12$ and $y = x - 5$ indicating clearly on your sketch the coordinates (2 dp) of any stationary points, inflection (if any) and of any points where the functions intersect each other.
Determine the area between the graphs to 2 dp.

Solution





Continue with Q7

Area = 131.12 sq units (accept 131.13)
Specific behaviours
P states coordinates on sketch for both turning points P states coordinates on sketch of inflection point P shows line meeting cubic at 3 points P states coordinates on sketch of all pts of intersection P states integral(s) to calculate area between functions Pstates area to 2 dp (Note: max -1 if not 2 dp) (Note: -2 if coordinates not given on sketch but stated elsewhere)

Q5 (4 & 3 = 7 marks) (3.1.2-3.1.3)

Let $f(x) = x^3 e^x$

a) Using **calculus** determine all stationary points and their nature.

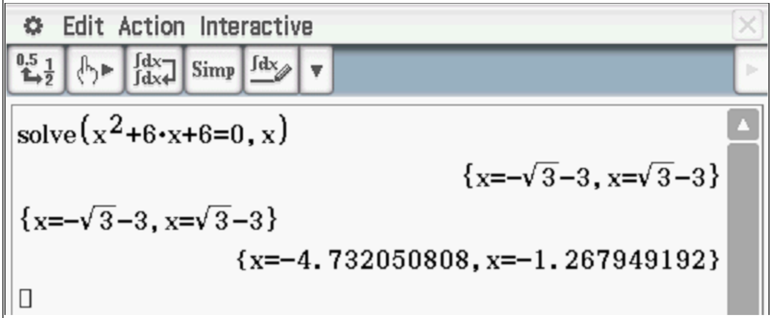
Solution
$f(x) = x^3 e^x$ $f'(x) = x^3 e^x + 3x^2 e^x = (x^3 + 3x^2)e^x = x^2(x+3)e^x$ $f''(x) = (3x^2 + 6x)e^x + (x^3 + 3x^2)e^x = (x^3 + 6x^2 + 6x)e^x$ $f'(x) = 0 \Rightarrow x^2(x+3) = 0, x = 0, -3$ $(0,0) \& (-3, -27e^{-3})$ *stationary* $(0,0), f''(0) = 0,$ *horizontal* inf *lection* $(-3, -27e^{-3}), f''(-3) = 9e^{-3},$ *local* min
Specific behaviours
P uses product rule to obtain first derivative P equates derivative to zero and solves for two values P uses first or second derivative test to determine nature with actual, values stated(accept approx) P gives coordinates and nature of each stationary point(accept approx)

b) Determine the values of any inflection points.

Solution

$(x^3+6x^2+6x)e^x=0$

$x(x^2+6x+6)e^x=0$

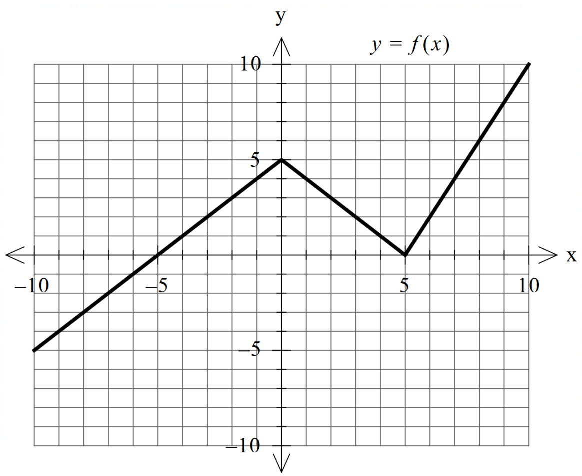


And x=0

Specific behaviours
P shows second derivative equated to zero P states at least two x values, accept approx P states three x values, approx.

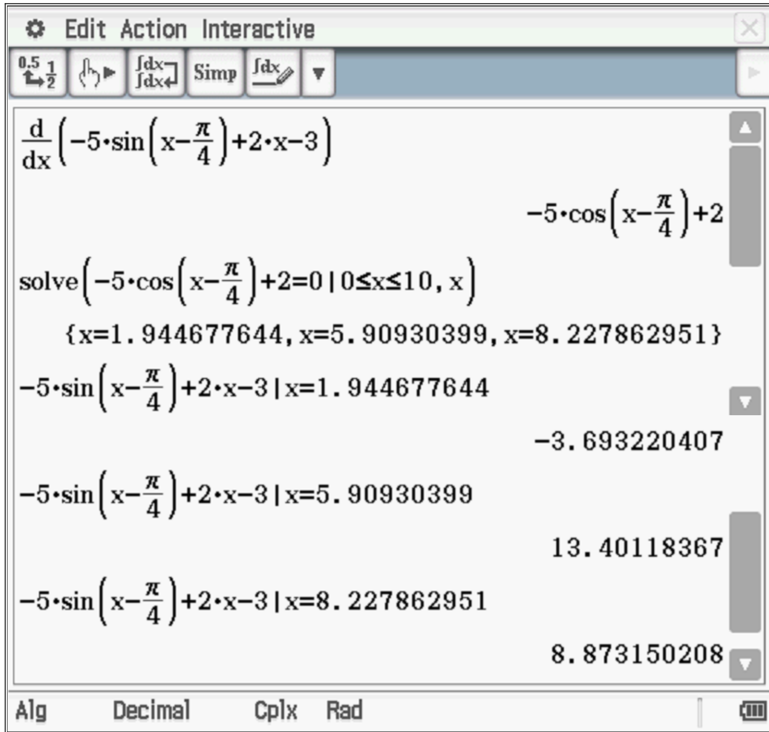
Q6 (2, 2, 2 & 2 = 8 marks) (3.2.15-3.2.17)

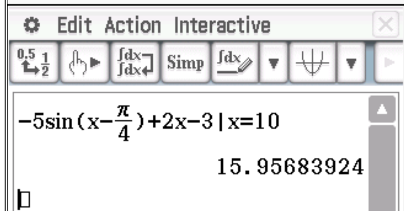
Consider the function $y=f(x)$ which is graphed below.



Determine the following.

a) $\int_{10}^{10} f(x)dx$.

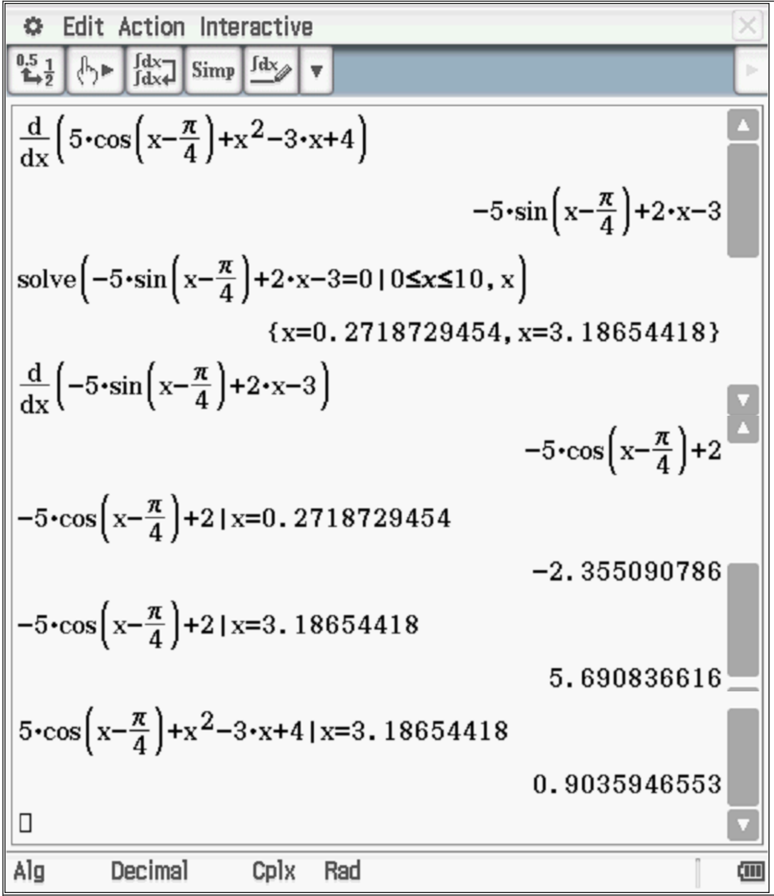




Steepest point is at x=5.91 metres & y = 23.20 metres or 23.19(do not accept 23.2)

Note: **accept** the endpoint if derivative is given although the endpoints were not supposed to be examined. endpoint of x = 10 metres & y = 69.11 metres
Behaviours unchanged

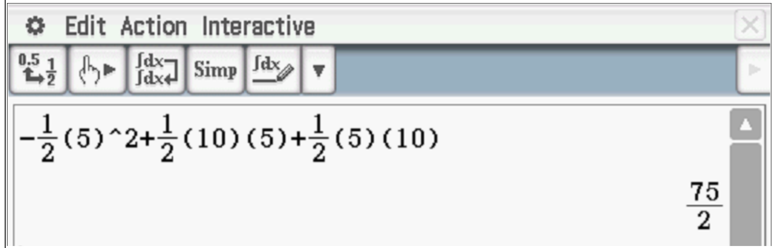
Specific behaviours
P equates second derivative to zero. P solves for 3 values of x P states value of first derivative for at least 2 x values above. P selects steepest point with x & y values rounded to 2 dp (2 possible answers) Note : max -1 if 2 dp not used in entire question (no need for units)



Specific behaviours
P differentiates and equates to zero. P solves for two x values in domain P uses first or second derivative test to select local min with y value stated to 2 dp (no need for units)

c) A water collection tank will be placed at the **steepest** part of the mountain. Determine the coordinates of this point to 2 decimal places. Justify.

Solution

Solution


Specific behaviours
P identifies negative result P states total value

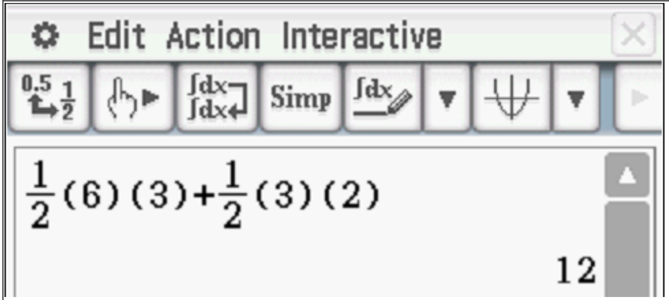
b) $\int_5^{10} f'(x)dx$.

Solution
$\int_5^{10} f'(x)dx = f(10) - f(-5) = 10 - 0 = 10$
Specific behaviours
P uses FTC P states result

c) $\frac{d}{dx}\int_5^x f(t)dt$ when $x=7$.

Solution
$\frac{d}{dx}\int_5^x f(t)dt = f(x) = f(7) = 4$
Specific behaviours
P uses FTC P states result

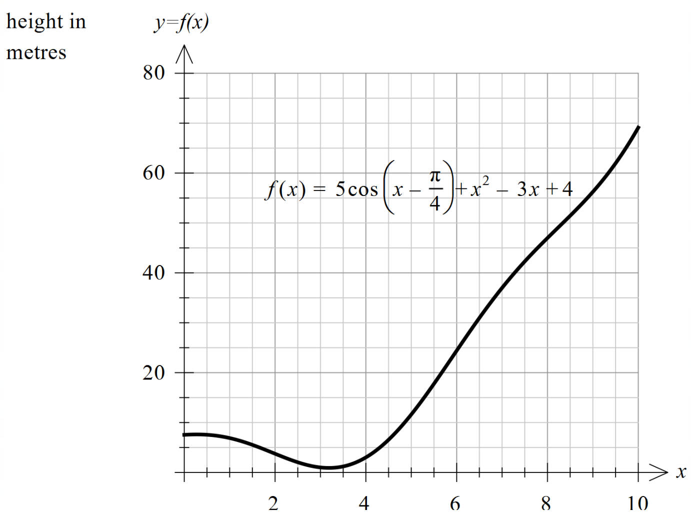
d) The area enclosed between $y = f(x)$ and the line $y = 2$.

Solution

Specific behaviours
P uses two triangles P states result (If 4 triangles used giving 52.5 then 1 mark out of 2)

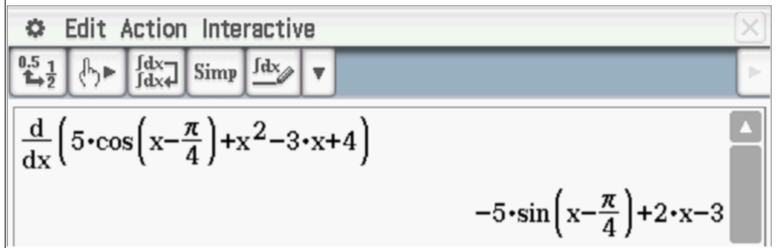
Q7 (1, 3x= & 4 = 8 marks) (3.2.5-3.1.6)

The cross section of a mountain can be given by $f(x) = 5\cos(x - \frac{\pi}{4}) + x^2 - 3x + 4$ for $0 \le x \le 10$ metres where $f(x)$ = height at x metres

cross-section of a mountain



a) Determine $\frac{dy}{dx}$.

Solution

Specific behaviours
P states derivative

b) Determine the minimum height of the mountain to 2 decimal places. Justify.

Solution