PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2019

Question/Answer booklet

Yr 12 SPECIALIST
UNIT 3 & 4

Section Two:
Calculator-assumed

Your Name

Your Teacher's Name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
8		4	15		9
9		8	16		8
10		9	17		8
11		7	18		9
12		7	19		7
13		7	20		7
14		10			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	34
Section Two: Calculator-assumed	13	13	100	100	66
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **(100 Marks)**

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

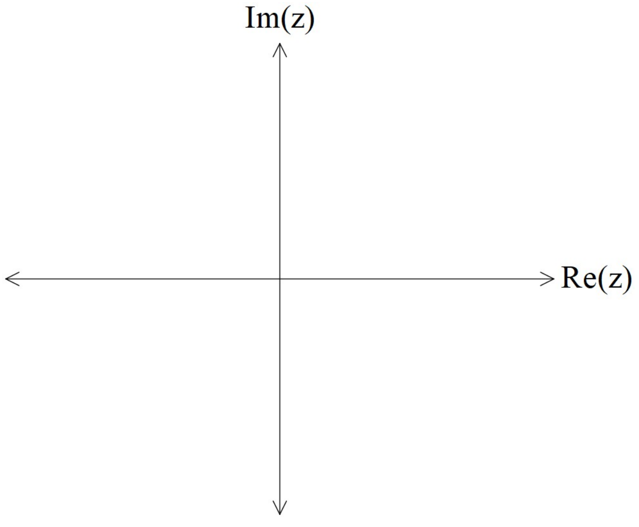
Question 8 **(4 marks)**

Consider the complex number $z = cis\theta$. By using De Moivre's theorem show that $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

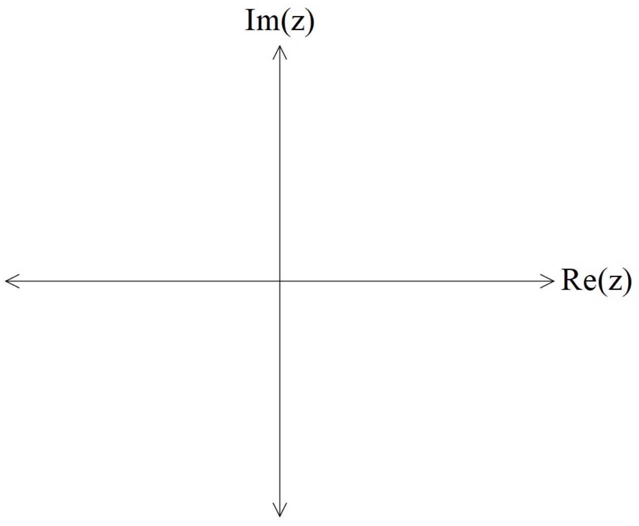
Question 9 **(8 marks)**

Sketch the following regions in the complex plane.

a) $\operatorname{Im}(z) \le \operatorname{Re}(z) + 4$ (2 marks)



b) $|z - 5 + 2i| > 3$ (2 marks)



Q9 Cont-

The solution to $|z-4+7i|=|z-a-bi|$, where $a \& b$ are real constants, is given by $\operatorname{Im}(z)=3\operatorname{Re}(z)-2$.

c) Determine the exact values of $a \& b$. (4 marks)

Question 10 **(9 marks)**

Consider an electronics company that manufactures transistors with weights that forms a Normal distribution of mean 95 milligrams and a standard deviation of 23 milligrams. A sample of 75 transistors is taken and the sample mean weight $\overline{X}$ of this sample of 75 is examined.

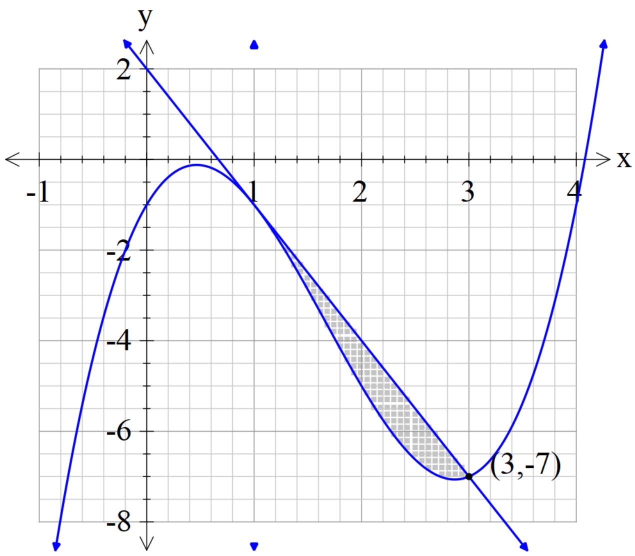
a) State the distribution $\overline{X}$ with its mean and standard deviation. (3 marks)

b) Determine the probability that the sample mean is greater than 102 milligrams. (2 marks)

c) A new sample size is chosen such that the probability that the sample mean is no more than 12 milligrams from 95 milligrams is 92%. Determine the new sample size. (4 marks)

Question 11 **(7 marks)**

Consider the graph of $f(x) = x^3 - 5x^2 + 4x - 1$ and the tangent line drawn at $x = 1$.
The area between the graph and the tangent is shaded as seen below.



a) Determine the shaded area. (Exact) (3 marks)

The shaded area is then revolved around the x axis.

b) Determine the exact volume of the resulting solid. (4 marks)

Question 12 **(7 marks)**

A super-heated metal rod cools according to the differential equation $\frac{dT}{dt} = k(T - T_o)$ where T_o is a constant representing the room temperature and k is a constant. $T(t)$ represents the temperature of the rod in degrees at time t seconds that the rod has been left in the room,

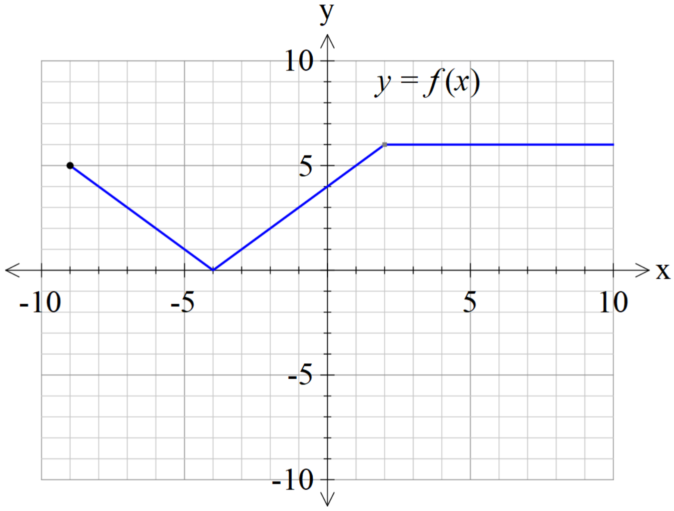
a) Determine an expression for the temperature $T(t)$ at **any time** in terms of t and the constants k & T_o. (4 marks)

It is known that the room temperature is 18 degrees and that the initial temperature is 65 degrees and $k = -0.5$.

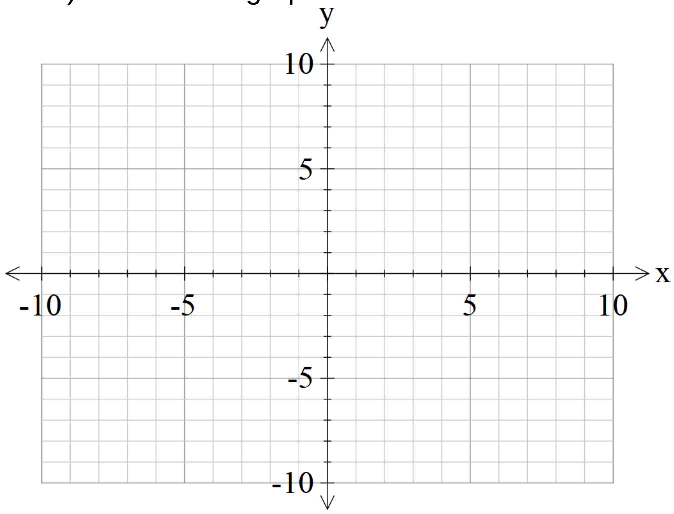
b) Determine the time taken for the temperature of the rod to cool to 32 degrees. (3 marks)

Question 13 **(7 marks)**

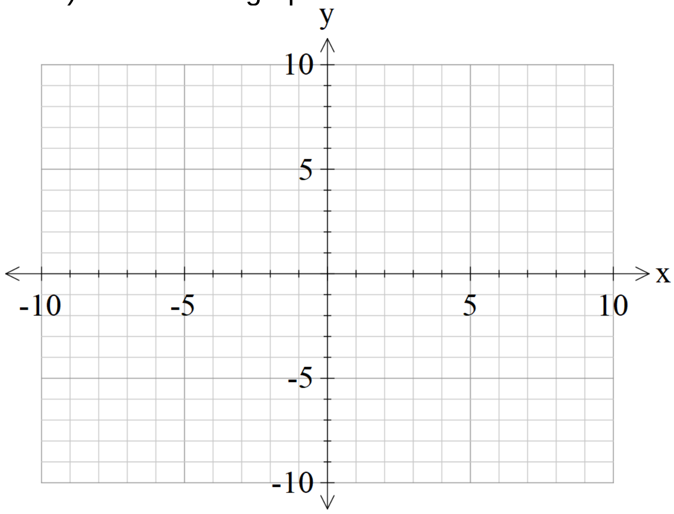
Consider the graph of the function $y = f(x)$ as shown below.



a) Sketch the graph $y = f(|x|)$ on the axes below. (3 marks)



b) Sketch the graph $y = \dfrac{1}{f(-|x|)}$ on the axes below. (4 marks)



Question 14 **(10 marks)**

An object with speed v and displacement x from the origin at time t is moving with the following accelerations.

a) $a = (v+3)^2$ with $v = 1$ at $t = 2$. Determine the speed at $t = 10$. (3 marks)

b) $a = e^{-(v^2+1)}$ with $v = 5$ at $x = 3$. Determine the speed at $x = 11$. (3 marks)

An object is known to be moving with **speed** given by the equation $v = 3\sqrt{(25 - x^2)}$
.

a) If initially at the origin, determine the displacement from the origin, x, at any time t.
(Hint- use the substitution $x = 5\sin u$) (4 marks)

Question 15 **(9 marks)**

A particle moves according to the following parametric equations.

$x = 3\cos(2t)$

$y = 4 - \sin t$ at time t seconds, $x \& y$ in metres.

a) Determine the cartesian equation. (3 marks)

b) Determine the equation of the tangent when $t = \frac{\pi}{6}$. (3 marks)

c) Determine $\frac{d^2y}{dx^2}$ when $t = \frac{\pi}{6}$. (Simplify) (3 marks)

Question 16 **(8 marks)**

A sample of 25 tyres are used to determine the population mean weight of the type of tyre.

The following 95% confidence interval was calculated $(6.651, 7.749)$ kg.

a) Determine the sample mean. (1 mark)

b) Determine the sample standard deviation. (3 marks)

State whether the following changes would increase or decrease the width of the confidence interval and give a reason.

i) Have a sample size greater than 25 tyres. (1 mark)

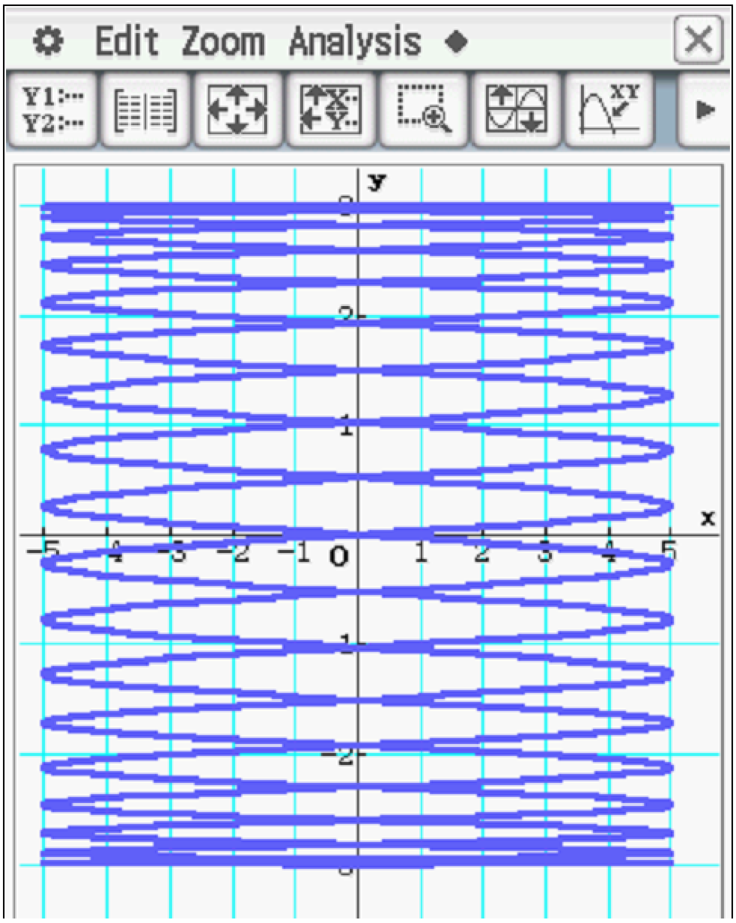
ii) Calculate a 90% confidence interval. (1 mark)

iii) Using a smaller sample standard deviation. (1 mark)

c) If 60 lots of 95% confidence interval were calculated, what number would you expect to contain the true population mean? (1 mark)

Question 17 **(8 marks)**

The position vector of a particle at time, t seconds, is given by $\underset{\sim}{r} = \begin{pmatrix} 5\sin 3t \\ -3\cos\frac{t}{6} \end{pmatrix}$ metres.
The path of the particle is shown as follows.



a) State the initial position and label on the path above. (1 mark)

b) Determine the acceleration when $t = \pi$ seconds. (3 marks)

c) Explain why the time of one complete circuit is 12π seconds. (2 marks)

d) Determine the distance travelled in one circuit. (2 marks)

Question 18 **(9 marks)**

a) Determine all positive values of the constant m for the function $f(x) = e^{mx}$ so that $f(x)$ will satisfy the differential equation $15\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 4y = 0$. (3 marks)

b) The section of the curve of the function $f(x) = e^{mx}$ in the interval $0 \le x \le a$ is rotated about the x axis. Show that for the value of m found in part a above, the volume of the solid produced after one rotation is $V = \frac{3\pi}{2}\left(e^{\frac{2a}{3}} - 1\right)$. (3 marks)

c) Show that if A is the area under the curve $f(x)$ in the interval $0 \le x \le a$, then $V = \frac{3\pi}{2}\left[\left(\frac{A}{3} + 1\right)^2 - 1\right]$. (3 marks)

Question 19 **(7 marks)**

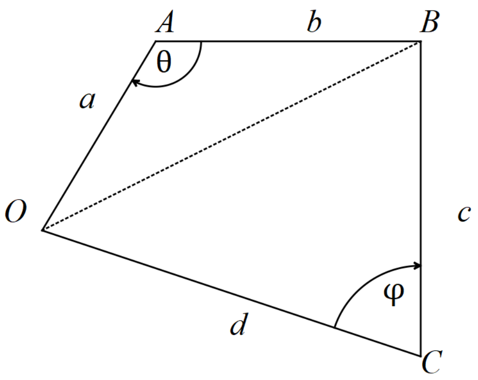
Two rockets A & B have initial positions $r_A = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} \& \, r_B = \begin{pmatrix} 0 \\ -1 \\ 14 \end{pmatrix}$ km at noon. They both move with constant velocities $v_A = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} \& \, v_B = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$ km/h.

a) The two rockets leave a smoke trail that stays in the air for a long period of time. Determine the point (if any) where the smoke trails cross. (3 marks)

b) Determine the shortest distance between the two rockets and the time that this occurs. (4 marks)

Question 20 **(7 marks)**

Consider the quadrilateral $OABC$ with fixed side lengths $a, b, c \,\&\, d$. Let $\theta \,\&\, \varphi$ be opposite angles.



a) Show that the area of the quadrilateral is $A = \frac{1}{2}ab\sin\theta + \frac{1}{2}cd\sin\varphi$. (1 mark)

b) By considering the common side $\overline{OB}$ to both triangles above, show that $\frac{d\varphi}{d\theta} = \frac{ab\sin\theta}{cd\sin\varphi}$.

(3 marks)

c) Hence show **using calculus** that the area of the quadrilateral is optimal, $\frac{dA}{d\theta} = 0$, when opposite angles are supplementary, $\theta + \varphi = \pi$. (3 marks)

Additional working space

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