# PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

**Semester Two Examination, 2022**

**Question/Answer booklet**

# MATHEMATICS SPECIALIST UNITS 3&4

### Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number:    In figures  ☐☐ ☐☐☐ ☐☐☐

In words    _____

_____

Your name    _____

## Time allowed for this section
Reading time before commencing work:    ten minutes
Working time:    one hundred minutes

Number of additional answer booklets used (if applicable): ☐

## Materials required/recommended for this section
**To be provided by the supervisor**
This Question/Answer booklet
Formula sheet (retained from Section One)

**To be provided by the candidate**
Standard items:    pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items:    drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 49 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 91 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1.  The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.  Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.  You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4.  Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.  It is recommended that you do not use pencil, except in diagrams.

6.  Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.  The Formula sheet is not to be handed in with your Question/Answer booklet.

| Markers use only | | |
|---|---|---|
| Question | Maximum | Mark |
| 8 | 8 | |
| 9 | 6 | |
| 10 | 9 | |
| 11 | 6 | |
| 12 | 9 | |
| 13 | 8 | |
| 14 | 8 | |
| 15 | 7 | |
| 16 | 8 | |
| 17 | 7 | |
| 18 | 7 | |
| 19 | 8 | |
| S2 Total | 91 | |
| S2 Wt (×0.7143) | 65% | |

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**Section Two: Calculator-assumed** **65% (91 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

**Question 8** **(8 marks)**

The distribution of the weights $W$ of loaves of rye bread produced by a bakery has a mean and standard deviation of 735 g and 15 g respectively. Quality control frequently take random samples of 60 white loaves from the bakery and calculate the mean weight of each sample.

(a) Describe the expected distribution of these sample means. (3 marks)

Further production checks are made if the mean weight of a sample is less than a prescribed minimum value of 731.3 g.

(b) Over the course of the next 250 random samples, how many times would you expect that further production checks need to be made? (2 marks)

Quality control has to reduce the sample size from 60 to 49 and change the prescribed minimum value so that the frequency of further production checks remains the same.

(c) Determine the prescribed minimum value for the mean weight of a sample required for this change. (3 marks)

**Question 9**                                                                    **(6 marks)**

The position vectors of points $A$ and $B$ are $\begin{pmatrix} 2 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ respectively.

(a)     Determine the vector equation of line $L$ that passes through $A$ and $B$.          (1 mark)

The vector equation of curve $S$ is $r = \begin{pmatrix} 2\mu+3 \\ 4-4\mu^2 \end{pmatrix}$.

(b)     Determine the Cartesian equation of curve $S$.                                    (2 marks)

(c)     Determine the position vector(s) of the point(s) where curve $S$ meets line $L$.          (3 marks)

**Question 10** **(9 marks)**

Let $f(x)=\ln\left(5-\sqrt{x+4}\right)$ and $g(x)=x^2-4x$.

(a) State the domain and range of $f$. (3 marks)

(b) Determine $f^{-1}(x)$ and state its range. (3 marks)

(c) Determine an expression for $f \circ g(x)$ and state the domain for which the composite function is defined. (3 marks)

**Question 11** **(6 marks)**

The position vector of a particle moving in the Cartesian plane at time $t$ seconds is given by

$$r(t) = -7\cos(2t)i + 7\sin(2t)j.$$

(a) Show that the particle is moving at a constant speed. (2 marks)

(b) Calculate the scalar product of the position vector and the velocity vector of the particle and interpret the result. (2 marks)

(c) Determine the acceleration vector of the particle when its position vector is $-7j$. (2 marks)

**Question 12** **(9 marks)**

The mean and standard deviation of a random sample of $52$ physics teachers working in a region was $45.2$ and $6.4$ years respectively. The sample was taken to construct a confidence interval for the mean age of such teachers.

(a) State two reasons why it is appropriate to assume the approximate normality of the distribution of the sample mean for this data. (2 marks)

(b) State another assumption required to construct a valid confidence interval. (1 mark)

(c) Construct a $90\%$ confidence interval for the mean age of physics teachers working in the region. (3 marks)

(d) Based on another random sample, the $99\%$ confidence interval for the mean age of art teachers employed in the same region was calculated to be $(39.09, 44.71)$. Given that the standard deviation of the sample was $6.9$ years, determine the size of the sample. (3 marks)
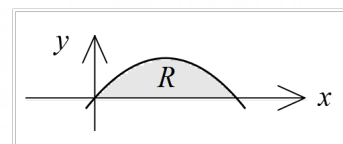
**Question 13**                                                                     **(8 marks)**

Let $f(x)=x(a-x)$, where $a$ is a constant, and region $R$
be the area between the $x$-axis and the curve $y=f(x)$.



All dimensions are in centimetres.

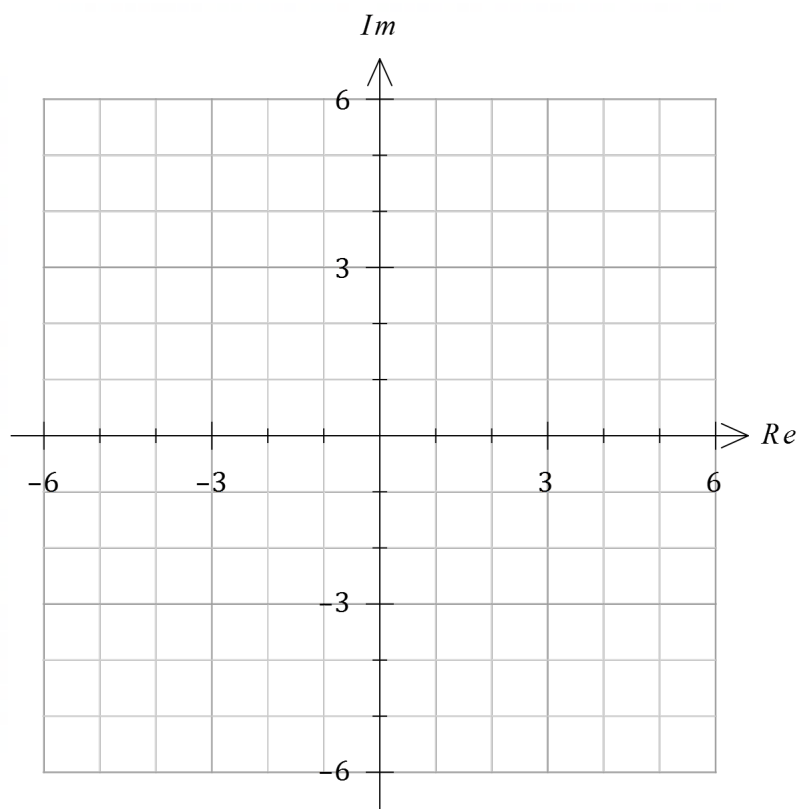(a)    When $a=18$, determine the volume of revolution when $R$ is rotated about the $y$-axis.

(5 marks)

(b)    When $R$ is rotated about the $x$-axis, the volume of revolution is $\dfrac{81\pi}{10}$ cm³.

Determine the value of $a$.                                                          (3 marks)
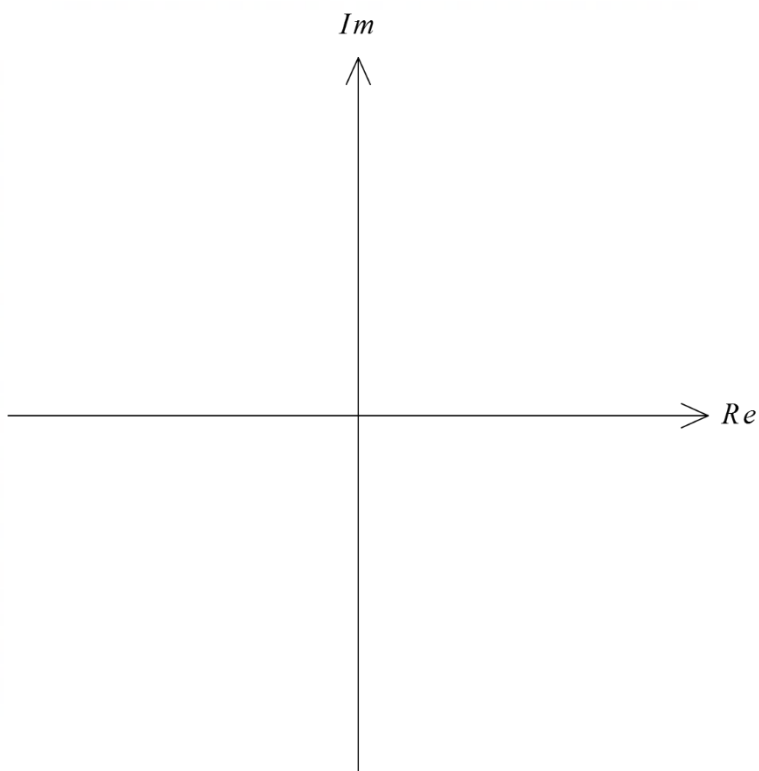
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**Question 14** (8 marks)

Consider the complex number $z$.

(a) Let $R$ be the subset of the complex plane that satisfies $|z+1| \geq |z-1+4i|$.

(i) Sketch the subset $R$. (3 marks)



(ii) Determine the exact minimum value of $|z-3i|$ in $R$. (2 marks)

(b)  Sketch the subset of the complex plane that satisfies the equation $arg\left(\dfrac{z+i}{z+1}\right)=\pm\,\pi$.

(3 marks)

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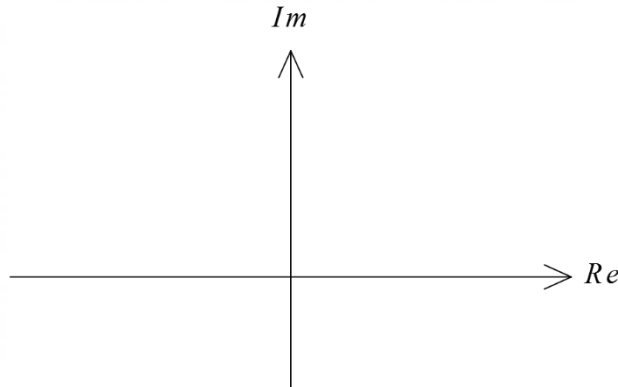SPECIALIST UNITS 3&4                    **12**                    CALCULATOR-ASSUMED

**Question 15**                                                          **(7 marks)**

Let $OABC$ be a rectangle in the complex plane, where $O$ is the origin. The points $A$ and $C$ represent the complex numbers $z$ and $-\sqrt{3}\,iz$ respectively, where $\Re(z)<0$ and $\Im(z)>0$.

(a)     Draw a labelled sketch of the rectangle in the complex plane.                 (2 marks)



(b)     Determine the complex number represented by $B$.                             (1 mark)

Rectangle $OABC$ is rotated $120°$ about $O$ in an anticlockwise direction to $OA'B'C'$.

(c)     Determine in exact Cartesian form the complex numbers represented by the points $A', B'$ and $C'$.                                                              (4 marks)
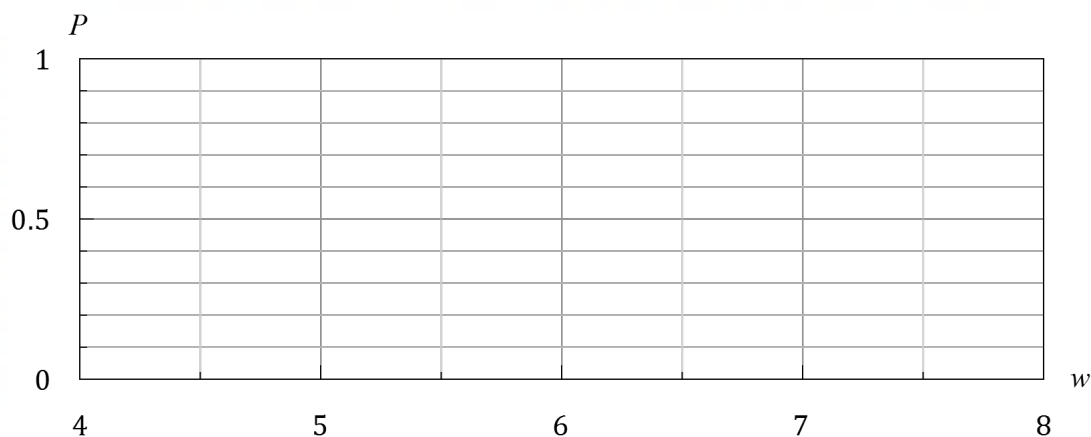
**Question 16** **(8 marks)**

The probability $P$ that an adult King Charles spaniel of weight $w$ kg is a female can be modelled by the logistic equation

$$P = \frac{1}{1 + 0.000\,006\, e^{2.1w}}.$$

(a)   Calculate the probability that a spaniel of weight 5.8 kg is a female. (1 mark)

(b)   Sketch the graph of this model on the axes below. (2 marks)



(c)   The logistic equation can be written in the form $\frac{dP}{dw} = rP(k-P)$. State the value of $r$ and the value of $k$. (2 marks)

(d)   The sensitivity $S$ of the model is defined as the absolute value of the change in $P$ for a one-gram increase in the weight of a spaniel. Determine the maximum value of $S$. (3 marks)

**Question 17** **(7 marks)**

(a) Let the complex numbers $z = r\,cis\,\theta$ and $w = \dfrac{\sqrt{3}+i}{3z}$. Determine the modulus and argument of $w$ in terms of the real constants $r$ and $\theta$.

(2 marks)

(b) Let the complex numbers $z = 3 + ai$ and $w = \dfrac{z-5}{z+1}$. Determine the value(s) of the real constant $a$ given that $w$ is purely imaginary. (2 marks)

(c) The complex number $w = 6\,cis\left(\dfrac{8\pi}{17}\right)$ is a root of the equation $z^n - (x + yi) = 0$, where $n$, $x$ and $y$ are real non-zero constants.

Determine two roots of the complex equation $z^{2n} - (x + yi) = 0$. (3 marks)

**Question 18** **(7 marks)**

A small body is moving in a straight line so that $t$ seconds after leaving fixed point $O$ its velocity is $v$ cm/s and its acceleration $a = bv + c$ cm/s$^2$, where $b$ and $c$ are constants.

Initially the body is at rest at $O$ and its acceleration is $3.6$ cm/s$^2$.

$T$ seconds later, its velocity is 4 m/s and its acceleration is 0.4 m/s$^2$.

(a)     Show that $5\dfrac{dv}{dt} + 4v - 18 = 0$. (3 marks)

(b)     Determine the exact value of $T$. (4 marks)

**Question 19** **(2 marks)**

Plane $\Pi$ contains triangle $OAB$.

Relative to $O$, the points $A$ and $B$ have position vectors $a = \begin{pmatrix} 6 \\ -9 \\ -2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ respectively.

(a)    State the unit vectors $\hat{a}$ and $\hat{b}$.  (1 mark)

(b)    Calculate $\angle AOB$.  (1 mark)

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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