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Calculator Assumed
General Discrete Random Variables

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 3 = 7 marks] **CA**

Determine, with reasoning, whether each of the following represent a discrete random variable.

(a)

x	0	1	2	3	4
$P(X=x)$	0.3	0.1	0.4	0.05	0.15

(b)

x	-2	-1	1	3	5
$P(X=x)$	0.2	0	-0.3	0.6	0.5

(c) $P(X=x)=\left(\frac{1}{2}\right)^x ; x=1,2,3,4....$

Question Two: [1, 2, 2, 2, 1, 2 = 10 marks] CA

A regular 8-sided dice is rolled.

(a) Explain why this experiment yields a uniform discrete random variable.

(b) Define the cumulative probability function of this random variable in a table below.

(c) In the long run, what is the value we expect to obtain on one roll of the dice?

(d) What is the standard deviation of these outcomes?

(e) If instead of an 8-sided dice, we roll a 16-sided dice, by what scale do each of the probability values change?

(f) Would the rule $E(aX+b)=aE(X)+b$ hold in this situation. Explain your answer.

Question Six: [3, 2, 2 = 7 marks] CA

The probability distribution for X is given by:

$$P(X=x)=\begin{cases}\dfrac{x+1}{k} & x=0,1,2\\ \dfrac{200-kx}{825} & x=3,4\end{cases}$$

(a) Determine the value of k.

$$\frac{1}{k}+\frac{2}{k}+\frac{3}{k}+\frac{200-3k}{825}+\frac{200-4k}{825}=1$$ ✓✓

$$k=10$$ ✓

(b) Calculate $E(X)$.

$$E(X)=0\times\frac{1}{10}+1\times\frac{2}{10}+2\times\frac{3}{10}+3\times\frac{170}{825}+4\times\frac{160}{825}$$ ✓

$$E(X)=2.19$$ ✓

(c) Calculate $P(X<3\,|\,X\geq 2)$

$$=\frac{\frac{3}{10}}{\frac{3}{10}+\frac{330}{825}}$$ ✓

$$=0.4286$$ ✓

Question Three: [1, 2, 3, 2 = 8 marks] CA

Each of the following represent discrete probability functions. Determine the value of k for each.

(a) $P(x)=\frac{1}{k};\ x=1,2,3,\ldots 12$

(b)

x	0	1	2	3	4
$P(X=x)$	0.2	k	0.3	0.1	0.15

(c) $P(x)=\frac{x}{2k-1};\ x=1,2,3,4,5,6$

(d)

x	1	2	3	5	7
$P(X=x)$	$2k$	k	k	$5k$	$6k$

Question Four: [4 marks] CA

Consider the discrete probability function represented in the table below.

x	1	3	4	5	7
$P(X=x)$	a	0.2	b	0.15	0.3

Determine the values of a and b such that $E(X)=4.7$

Question Five: [1, 2, 2, 2, 2 = 9 marks] CA

A probability distribution for Y is:

y	1	2	3	4	5
$P(Y\leq y)$	0.1	0.4	0.65	0.8	1

Determine:

(a) $P(Y=3)$

(b) $P(2\leq Y<4)$

(c) $P(Y<2\cup Y>4)$

(d) $P(\mathrm{Y}\leq 2\,|\,\mathrm{Y}\leq 3)$

(e) $P(Y>2\,|\,Y\leq 4)$

Question Four: [4 marks] CA

Consider the discrete probability function represented in the table below.

x	1	3	4	5	7
$P(X=x)$	a	0.2	b	0.15	0.3

Determine the values of a and b such that $E(X)=4.7$

$a+b=0.35$ ✓
$a+4b=1.25$ ✓
$a=0.05$ ✓
$b=0.3$ ✓

Question Five: [1, 2, 2, 2, 2 = 9 marks] CA

A probability distribution for Y is:

y	1	2	3	4	5
$P(Y\leq y)$	0.1	0.4	0.65	0.8	1

Determine:

(a) $P(Y=3)$

$=0.25$ ✓

(b) $P(2\leq Y<4)$

$=0.3+0.25=0.55$ ✓✓

(c) $P(Y<2\cup Y>4)$

$=0.1+0.2=0.3$ ✓✓

(d) $P(\mathrm{Y}\leq 2\,|\,\mathrm{Y}\leq 3)$

$=\frac{0.4}{0.65}=0.6154$ ✓✓

(e) $P(Y>2\,|\,Y\leq 4)$

$=\frac{0.4}{0.8}=0.5$ ✓✓

Question Three: [1, 2, 3, 2 = 8 marks] **CA**

Each of the following represent discrete probability functions. Determine the value of k for each.

(a) $P(x)=\frac{1}{k};\ x=1,2,3,...12$

$k=12$ ✓

(b)

x	0	1	2	3	4
$P(X=x)$	0.2	k	0.3	0.1	0.15

$k=1-0.2-0.3-0.1-0.15$

$k=0.25$ ✓ ✓

(c) $P(x)=\frac{x}{2k-1};\ x=1,2,3,4,5,6$

$1=\frac{1+2+3+4+5+6}{2k-1}$ ✓

$1=\frac{21}{2k-1}$ ✓

$k=11$ ✓

(d)

x	1	2	3	5	7
$P(X=x)$	$2k$	k	k	$5k$	$6k$

$2k+k+k+5k+6k=1$ ✓

$k=\frac{1}{15}$ ✓

Question Six: [3, 2, 2 = 7 marks] **CA**

The probability distribution for X is given by:

$$P(X=x)=\begin{cases}\frac{x+1}{k} & x=0,1,2\\ \frac{200-kx}{825} & x=3,4\end{cases}$$

(a) Determine the value of k.

(b) Calculate $E(X)$.

(c) Calculate $P(X<3|X\geq 2)$

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SOLUTIONS
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General Discrete Random Variables

Time: 45 minutes
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Question One: [2, 2, 3 = 7 marks] **CA**

Determine, with reasoning, whether each of the following represent a discrete random variable.

(a)

x	0	1	2	3	4
$P(X=x)$	0.3	0.1	0.4	0.05	0.15

Yes this table does represent a DRV, all probabilities add to 1 and there are no negative values. ✓✓

(b)

x	-2	-1	1	3	5
$P(X=x)$	0.2	0	-0.3	0.6	0.5

No, this table does not represent a DRV. Despite all the probability values adding to 1, as one is negative, this cannot represent a DRV. ✓✓

(c) $P(X=x)=\left(\frac{1}{2}\right)^x$; $x=1,2,3,4....$

The sequence of probabilities is: $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}...$ ✓

$S_\infty=\frac{0.5}{1-0.5}=1$ ✓

Therefore all probabilities will add to 1, and none are negative. ✓

Question Two: [1, 2, 2, 2, 1, 2 = 10 marks] **CA**

A regular 8-sided dice is rolled.

(a) Explain why this experiment yields a uniform discrete random variable.

The chance of each outcome is the same (1/8) therefore making it uniform, and they all add to 1, making it a DRV. ✓

(b) Define the cumulative probability function of this random variable in a table below.

x	1	2	3	4	5	6	7	8
$P(X\leq x)$	0.125	0.250	0.375	0.5	0.625	0.750	0.875	1

✓✓

(c) In the long run, what is the value we expect to obtain on one roll of the dice?

$E(X)=0.125(1+2+3+4+5+6+7+8)$ ✓
$E(X)=4.5$
$\therefore 4 \text{ or } 5$ ✓

(d) What is the standard deviation of these outcomes?

$\sigma_x=\sqrt{0.125((-3.5)^2+(-2.5)^2+(-1.5)^2+(-0.5)^2+(0.5)^2+(1.5)^2+(2.5)^2+(3.5)^2)}$ ✓
$\sigma_x=2.29$ ✓

(e) If instead of an 8-sided dice, we roll a 16-sided dice, by what scale do each of the probability values change?

Multiplied by a half ✓

(f) Would the rule $E(aX+b)=aE(X)+b$ hold in this situation. Explain your answer.

No. There has been no change of scale or origin in this situation, for example, no change from one unit to another. Instead, the number of outcomes have doubled. ✓✓