WESLEY COLLEGE

By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER TWO 2017
TEST 3: Applications of Calculus

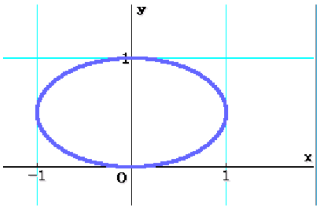
Name: ________________________

Monday 15th August		Time: 50 minutes		Mark		/45

Section 1 – Calculator free 25 marks

1. [5 marks – 4 and 1]

The curve defined by the equations $\begin{cases} x(t) = \sin 2t \\ y(t) = \cos^2 t \end{cases}$ for $0 \le t \le 2\pi$ generates the ellipse shown.



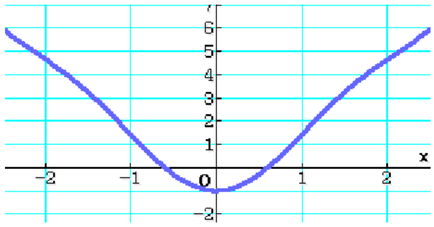
(a) Show that $\frac{dy}{dx} = -\frac{1}{2}\tan 2t$

(b) What is the slope of this curve at the point where $t = \frac{\pi}{8}$?

2. [4 marks]

If $y = \cos^3 x$ and $\frac{dy}{dt} = 2$, determine the rate of change of x when $x = \frac{\pi}{6}$

3. [4 marks – 2 and 2]



$f(x) = x^2 - \cos 2x$ is an even function, symmetric about the y-axis, as shown.

(a) Show clearly that $\int_0^{\frac{\pi}{2}} f(x)\, dx = \frac{\pi^3}{24}$

(b) Evaluate $A < 0$ and B so that $\int_A^{\frac{\pi}{2}} B\, f(x)\, dx = \pi^3$

4. [5 marks – 3 and 2]

(a) Calculate $\pi \int_0^{\frac{\pi}{4}} \cos^2\theta - \sin^2\theta \; d\theta$

(b) Describe, terms of the curves $y = \cos\theta$ and $y = \sin\theta$, the quantity represented by your calculation in part (a).

5. [7 marks – 3, 1 and 3]

Determine each of the integrals given:

(a) $\int \frac{x+4}{x^2-2x}\,dx$ where $\frac{x+4}{x^2-2x} = \frac{A}{x} + \frac{B}{x-2}$

(b) $\int \frac{\cos(\ln x)}{x}\,dx$

(c) Use the substitution $t = 2 + \cos x$ to evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x}{2+\cos x}\,dx$ in simplest exact form.

Section 2 – Calculator assumed 20 marks   Name:_________________________

6. [9 marks – 3, 2 and 4]

The equation of a curve in the plane is given by $x^2 + 3y^2 + 2xy = 12$

(a) Derive an expression, in terms of x and y, for $\frac{dy}{dx}$. Check on ClassPad.

(b) Find the equation of the tangent at the point $(0, 2)$

(c) At which points on the curve is the tangent parallel to the y axis?

7. [6 marks – 3, 2 and 1]

Compare the volumes generated when the ellipse with equation $\frac{x^2}{9}+\frac{y^2}{4}=1$ is revolved about the *x* axis and then the *y* axis.

Briefly explain why the larger value is the greater.

8. [5 marks]

David is flying a kite, which maintains a constant height of 26 m.

The string from David's hand, 1 m above the ground, to the kite is taut (i.e. forms a straight line) and he is releasing this string at a rate of 1.2 m per second.

Describe the motion of the kite when the length of the string is 65 m.