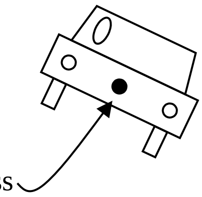
3 solutions to Question 6 of Circular Motion Booklet P13

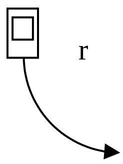
Front-on



centre of mass

Side-on

+ve



r

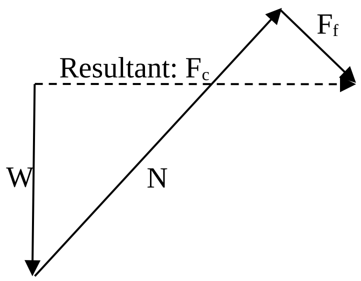
+ve

Sign

convention

θ

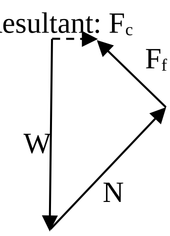
Solution 1: F_f is down the slope



Horz: $N_h + F_{f:h} = F_{c:h}$

Vert: $W_v + N_v + F_{f:v} = 0$

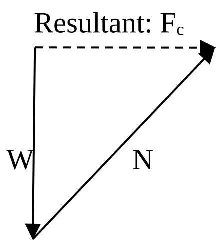
Solution 2: F_f is up the slope



Horz: $N_h + F_{f:h} = F_{c:h}$

Vert: $W_v + N_v + F_{f:v} = 0$

Solution 3: F_f is zero



Horz: $N_h + F_{f:h} = F_{c:h}\, F_{f:h} = F_{c:h}$

Vert: $W_v + N_v + F_{f:v} = 0$ $\qquad F_{f:v} = 0$

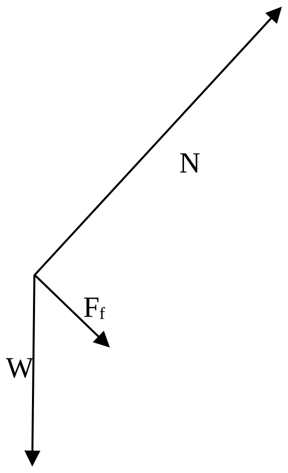
	There are three solutions to the combination of W (the weight force), N (the normal reaction force) and F_f (the frictional force) that will give F_c (the resultant force) that is horizontal and towards the centre of circular path around which the car is turning. In all cases: $\Sigma F=0$ in the vertical direction $\Sigma F=F_c$ in the horizontal direction

Note that the equations are the same in all cases!

Do we need to draw these sum of the forces diagrams?

It is not always necessary to draw a sum of the forces diagram, instead a free body diagram cam be the starting point. Consider the free body diagram where F_f is down the slope.

Free body diagram for Solution 1:
F_f is down the slope



We know that the sum of these forces must be the centripetal acceleration, i.e.
$\Sigma F = mv^2/r$
As this the centripetal acceleration is in the horizontal plane, we can use a approach to solve for the forces that involves resolution into horizontal and vertical components.

Horizontal

$\Sigma F = mv^2/r$

Vertical

$\Sigma F = 0$

Over the next few pages, I have set out approaches to solving problems represented by the scenarios on the first page of these notes, using resolution into horizontal and vertical components

Calculation of values of the forces

Vertical
$|W_v| = |m \times g|$
$|N_v| = |N \times \cos\theta|$
$|F_{f:v}| = |F_f \times \sin\theta|$
$F_{c:v} = 0$

Horizontal
$|W_h| = 0$
$|N_h| = |N \times \sin\theta|$
$|F_{f:h}| = |F_f \times \cos\theta|$
$|F_{c:h}| = |m \times v^2 / r|$

Irrespective of which of the above 3 situations is involved, the formulae relating these are:

Horizontal: $N_h + F_{f:h} = F_{c:h}$
Vertical: $W_v + N_v + F_{f:v} = 0$

A non-general solution with the P18 Tricky Activity as an example.

To solve problems where F_f is not known, e.g., the Tricky Activity on P18,

- Assume that F_f is down the slope (Solution 1, the first diagram in the table).
- Write down the formulae relating the actual forces N, F_f, and W and the resultant force F_c in the horizontal and vertical directions.
- Substitute **values** for those forces.
  - Ensure that a sign convention is assigned
  - Ensure correct signs are given to the forces to allow scalar addition of the forces.
- Simultaneously solve for N or F_f.
  If F_f is positive, negative or zero, the Solution is 1, 2 or 3 respectively.
- Use N to find F_f or vice versa

Horizontal: $N_h + F_{f:h} = F_{c:h} \Rightarrow |N| \times \sin\theta + |F_f| \times \cos\theta = mv^2/r$
$\Rightarrow |N| \times \sin 30° + |F_f| \times \cos 30° = 2000 \times (110/3.6)^2/50$
$\Rightarrow 0.5|N| + 0.86603|F_f| = 3.7356\text{x}10^4$ I

Vertical: $W_v + N_v + F_{f:v} = 0 \Rightarrow -|m \times g| + |N| \times \cos\theta - |F_f| \times \sin\theta = 0$
$\Rightarrow -9.8 \times 2000 + \cos 30°|N| - |F_f| \times \sin 30° = 0$
$\Rightarrow -1.96\text{x}10^4 + 0.86603|N| - 0.5|F_f| = 0$ II

From I $|F_f| = (3.7356\text{x}10^4 - 0.5|N|) / 0.86603$ III

Substitute III into II
$\Rightarrow -1.96\text{x}10^4 + 0.86603|N| - 0.5(3.7356\text{x}10^4 - 0.5|N|) / 0.86603 = 0$
$\Rightarrow |N| (0.86603 + 0.5 \times 0.5 / 0.86603) - 0.5 \times 3.7356\text{x}10^4 / 0.86603 - 1.96\text{x}10^4 = 0$
$\Rightarrow 1.1547|N| - 4.1667\text{x}10^4 = 0$
= $\mathbf{|N|} = 4.1667\text{x}10^4 / 1.1547 = 3.56\text{x}10^4$ N
substitute for |N| in equation III
$\mathbf{|F_f|} = (3.7356\text{x}10^4 - 0.5 \times 3.56\text{x}10^4) / 0.86603 = 2.25\text{x}10^4$ N

A general solution with the P18 Tricky Activity as an example.

To solve problems where F_f is not known, e.g., the Tricky Activity on P18,

- Assume that F_f is down the slope (Solution 1, the first diagram in the table).
- Write down the formulae relating the actual forces N, F_f, and W and the resultant force F_c in the horizontal and vertical directions.
- Substitute **equations** for those forces.
    - Ensure that a sign convention is assigned
    - Ensure correct signs are given to the forces to allow scalar addition of the forces.
- Simultaneously solve for N or F_f.
  If F_f is positive, negative or zero, the Solution is 1, 2 or 3 respectively.
- Use N to find F_f or vice versa

Horizontal:	$N_h + F_{f:h} = F_{c:h}$ =>	$\lvert N\rvert \times \sin\theta + \lvert F_f\rvert \times \cos\theta = mv^2/r$	I
Vertical:	$W_v + N_v + F_{f:v} = 0$	=> $-\lvert m \times g\rvert + \lvert N\rvert \times \cos\theta - \lvert F_f\rvert \times \sin\theta = 0$	II

From I $\quad |F_f| = (mv^2/r - |N| \times \sin\theta) / \cos\theta \quad$ III

Substitute III into II

$$
\begin{aligned}
0 &= -|m \times g| + |N| \times \cos\theta - \sin\theta \times (mv^2/r - |N| \times \sin\theta) / \cos\theta \\
&= |N| (\cos\theta + \sin^2\theta / \cos\theta) - |m \times g| - (\sin\theta / \cos\theta) \times mv^2/r \\
\Leftrightarrow \quad \mathbf{|N|} &= (|m \times g| + (\sin\theta / \cos\theta) \times mv^2/r) / (\cos\theta + \sin^2\theta / \cos\theta) \\
&= (|m \times g| + (\sin\theta / \cos\theta) \times mv^2/r) / ((\cos^2\theta + \sin^2\theta) / \cos\theta) \\
&= (|m \times g| + (\sin\theta / \cos\theta) \times mv^2/r) / (1 / \cos\theta) \\
&= (|m \times g| + (\sin\theta / \cos\theta) \times mv^2/r) \times \cos\theta) \\
&= \mathbf{\sin\theta \times mv^2/r + |m.g.\cos\theta|} \\
&= \sin 30° \times 2000 \times (110/3.6)^2 / 50) + 2000 \times 9.8 + \cos 30° \\
&= 3.56 \times 10^4 \text{ N}
\end{aligned}
$$

Now substitute the value calculated for N into equation III to find F_f.

or find F_f first

From I	$\lvert N\rvert$	$= (mv^2/r - \lvert F_f\rvert \times \cos\theta) / \sin\theta$	IIIa
Substitute III into II	0	$= -\lvert m \times g\rvert + \cos\theta \times (mv^2/r - \lvert F_f\rvert \times \cos\theta) / \sin\theta - \lvert F_f\rvert \times \sin\theta$	
		$= \lvert F_f\rvert \times -(\sin\theta + \cos^2\theta / \sin\theta) - \lvert m \times g\rvert + (\cos\theta / \sin\theta) \times mv^2/r$	
<=>	$\mathbf{\lvert F_f\rvert}$	$= (\lvert m \times g\rvert - (\cos\theta / \sin\theta) \times mv^2/r) / -(\sin\theta + \cos^2\theta / \sin\theta)$	
		$= (\lvert m \times g\rvert - (\cos\theta / \sin\theta) \times mv^2/r) / -((\sin^2\theta + \cos^2\theta) / \sin\theta)$	
		$= (\lvert m \times g\rvert - (\cos\theta / \sin\theta) \times mv^2/r) / -(1 / \sin\theta)$	
		$= (\lvert m \times g\rvert - (\cos\theta / \sin\theta) \times mv^2/r) \times -\sin\theta$	
		$= \mathbf{\cos\theta \times mv^2/r - \lvert m.g.\sin\theta\rvert}$	
		$= \cos 30° \times 2000 \times (110/3.6)^2 / 50) - 2000 \times 9.8 \times \sin 30°$	
		$= 2.25 \times 10^4$ N	

Now substitute the value calculated for F_f into equation IIIa to find N.

A Revaluation of the general solution

We derived two simple equations after much algebra:

$$\mathbf{\lvert N\rvert = \sin\theta \times mv^2/r + \lvert m.g.\cos\theta\rvert}$$
$$\mathbf{\lvert F_f\rvert = \cos\theta \times mv^2/r - \lvert m.g.\sin\theta\rvert}$$

This is simply resolving into components parallel to and perpendicular to the slope.

Question: Would not that have been simpler in the first place?
Answer: Yes, but who would have thought to do it?

So, as 3AB Physics students, how should we proceed?

The approach to solving such problems that we are least likely to forget is to always resolve into horizontal and vertical components and substitute in numerical values early in the manipulation to simplify the algebra.

That approach works in a very wide set of situations and, although it can result in significant complexity, careful setting out can address that difficulty.