PERTH MODERN SCHOOL
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Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS
UNIT 1 AND 2

Section Two:
Calculator-assumed

Name: ____

Teacher's Name:____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Question	Mark
7		12	
8		13	
9		14	
10		15	
11			
		TOTAL	

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	50	33
Section Two: Calculator-assumed	9	9	100	100	67
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **(100 Marks)**

This section has **8** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 7 **(10 marks)**

A circle is centered at $(5,6)$ and has radius 13.

(a) Find an equation for this circle. (2 marks)

Solution
$(x-5)^2+(y-6)^2=13^2$

The straight line l with equation $y=x-6$ intersects the circle at the points A and B and thus forms the chord AB.

(b) Determine the coordinates of A and the coordinates of B. (4 marks)

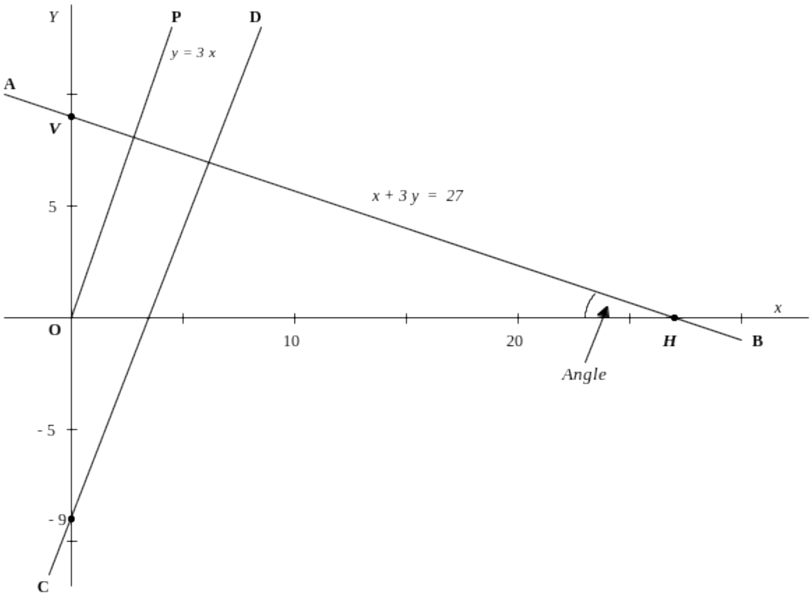
Solution
Solve : $(x-5)^2+(x-6-6)^2=13^2 \rightarrow x=0 \vee 17$ $x=0 \rightarrow y=-6, x=17 \rightarrow y=11$ Hence, the two points are $(0,-6) \wedge (17,11)$
✔ substituting y in terms of x into equation ✔ finding the solutions ✔✔ stating the two correct points

(c) Determine the equation of the line which passes through the centre and bisects the chord AB. (4 marks)

Solution
Midpoint of AB: $\left(\frac{17}{2}, \frac{5}{2}\right)$ Equation through the midpoint and the centre: $m=-1 \quad 6=-1(5)+c \rightarrow c=11$ $\therefore\ y=-x+11.$
✔ finding the midpoint ✔ finding the slope ✔✔ stating the correct equation

Question 8 **(12 marks)**

The line AB, $x + 3y = 27$ cuts both of the lines OP and CD. OP and CD are parallel to each other. OP is represented by the equation $y = 3x$.



(a) Write the equation of the line CD. (2 marks)

Solution
$y = 3x - 9$

(b) Find the vertical intercept at ***V*** and the horizontal intercept at ***H***. (2 marks)

Solution
$at\ x = 0 \rightarrow y = 9$ $at\ y = 0 \rightarrow x = 27$ $\therefore V = (0, 9) \wedge H = (27, 0)$

(c) Using (b), find the angle (in degrees), to 1 decimal place, the line AB makes with the horizontal axis as shown in the diagram. (2 marks)

Solution
$\theta = \tan^{-1}(\frac{9}{27})$ $\theta = 18.4^0$

(d) Find the coordinates of both points where the line AB intersects the lines OP and CD. (4 marks)

Solution
$Line\ AB: y=\frac{27-x}{3}\ Line\ OP: y=3x\ Line\ CD: y=3x-9$ $\frac{27-x}{3}=3x$ and $\frac{27-x}{3}=3x-9$ $x=2.7 \rightarrow y=8.1\ x=5.4 \rightarrow y=7.2$ $\therefore (2.7, 8.1) \wedge (5.4, 7.2)$
✔ equating line AB and OP ✔ equating line AB and CD ✔✔ two coordinates

(e) Find the length of the sloping side ***VH*** to 2 decimal places. (2 marks)

Solution
$9^2+27^2=VH^2$ $28.46\ units=VH$

Question 9 **(14 marks)**

Given $y=x^4+2x^3-3x^2-4x+4$, use a calculus method to find:

(a) the derivative of the function, (1 mark)

Solution
$\frac{dy}{dx}=4x^3+6x^2-6x-4$

(b) the location of the stationary points and their nature, (6 marks)

Solution
$\frac{dy}{dx}=0,0=4x^3+6x^2-6x-4\rightarrow x=-2,-\frac{1}{2},1$ at $x=-2, y=0\rightarrow(-2,0)\, is\, a\, minimum$ $x=1, y=0\rightarrow(1,0)\, is\, a\, minimum$ $x=\frac{-1}{2}, y=\frac{81}{16}\rightarrow\left(\frac{-1}{2},\frac{81}{16}\right) is\, a\, maximum$
✔ setting derivative to zero ✔ finding all three x values (-✔ for every incorrect answer) ✔ finding corresponding y coordinates ✔✔✔ for indicating nature

(c) the intervals on which y is increasing and decreasing. (4 marks)

Solution
Increasing for $-2<x\leftarrow\frac{1}{2}\wedge x>1$ Decreasing for $x\leftarrow\frac{2\wedge-1}{2}<x<1$
✔✔ increasing intervals ✔✔ decreasing intervals

(d) the intervals on which the rate of change is increasing and decreasing. (3 marks)

Solution
Increasing for $-1.37<x\wedge 0.37<x$ Decreasing for $-1.37<x<0.37$
✔✔ increasing intervals

✔ decreasing interval

Question 10 **(13 marks)**

The Adams family consist of two parents and two children, Wednesday and Pugsley. Usually the parents do not take their children shopping with them as it takes much longer (due to the extra shops the children want to visit) and often includes a stop for a drink. If Mr and Mrs Adams are shopping without their children they never stop for a drink.

The probability that Mr and Mrs Adams take only Wednesday with them when they go to shopping is 0.2 and is equal to the probability that they take only Pugsley. The probability that both children go shopping with their parents is 0.04.

(a) Are the events *taking Wednesday shopping* and *taking Pugsley shopping* independent? Justify your answer. (2 marks)

Solution
Yes $P(A \cap B) = P(\text{taking both children})$ $¿ 0.04$ $P(A) \times P(B) = P(\text{taking Wednesday}) \times P(\text{taking Pugsley})$ $¿ 0.2 \times 0.2$ $¿ 0.0576$ $P(A \cap B) = P(A) \times P(B)$ *which means the two events are not independent.*
✔ Yes   ✔ justification

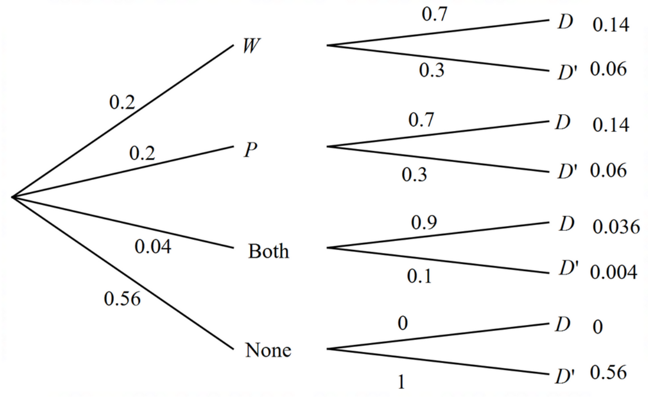
(b) Find the probability that, on the next visit to the shops, Mr and Mrs Adams take neither of the children with them. (2 marks)

Solution
$P ¿$ $¿ 0.56$

(c) If Mr and Mrs Adams take one of the children to the shops then the probability of stopping for a drink is 0.7. If they take both children shopping the probability for stopping for a drink is 0.9.

(i) Draw a tree diagram to represent all information and results. (4 marks)

Solution



✔ for every column in the tree diagram

(ii) What is the probability that, on the next visit to the shops, Mr and Mrs Adams do not stop for a drink? (2 marks)

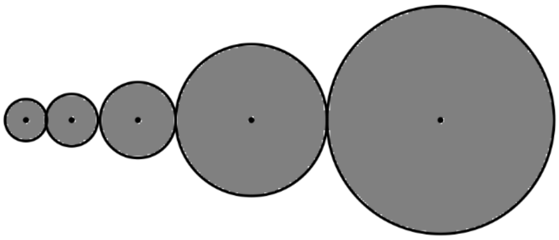
Solution
$P(D')=0.06+0.06+0.004+0.56$ ¿0.68

(iii) If Mr and Mrs Adams went to the shops and did not stop for a drink, what is the probability that neither child was with them? (3 marks)

Solution
P ¿ ¿0.82
✔✔ setting up conditional probability ✔ final answer in 2dp

Question 11 **(10 marks)**

The figure below shows a pattern of 5 circles, touching externally, whose centres lie on a straight line of length *L* units.



The radii of these circles form a geometric progression where the radius of the smaller circle is 3 units and that of the fifth (larger) circle is 48 units.

(a) Find the common ratio of the geometric progression. (2 marks)

Solution
$t_1=3$ $\quad\quad 48=3r^4$ $t_5=48$ $\quad\quad r=2$

The pattern is extended by 5 more circles to a total of 10 circles.

(b) Determine the length added to the original value of *L*. (4 marks)

Solution
$L_{10}=2\times S_{10}$ $\quad\quad L_5=2\times S_5$ ¿ $2\times\frac{3(2^{10}-1)}{2-1}$ $\quad\quad$ ¿ $2\times\frac{3(2^5-1)}{2-1}$ ¿ $6\,138\,units$ $\quad\quad$ ¿ $186\,units$ $\rightarrow 6138-186=5952\,units$
✔✔ finding Lengths of 10 and 5 circles ✔✔ finding additional length

(b) Calculate, in terms of π, the total area of the 10 circles of the new pattern. (4 marks)

Solution
$A=\pi\times(3\times2^0)^2+\pi\times(3\times2^1)^2+\pi\times(3\times2^2)^2+\pi\times(3\times2^5)^2+\ldots+\pi\times(3\times2^9)^2$

$¿9\pi+36\pi+144\pi+\ldots+2359296\pi$ $¿9\pi¿$ $¿9\pi\times\frac{1(4^{10}-1)}{4-1}$ $¿3\,145\,725\pi\, units^2$
✔✔ expressing area as a sum of the geometric sequence ✔✔ determining the area with correct unit ($units^2¿$

Question 12 **(12 marks)**

On Phillip Island the number of hours of daylight in each day of the year varies from 10 hours to 14 hours over the 365 days in the calendar year.

The longest day of the year occurs 10 days before the beginning of the calendar year.

(a) Write a trigonometric equation that will model the number of hours of daylight in each day of the year where h is the hours of daylight and d is the number of days after midnight from the start of the year. (4 marks)

Solution
$h=12+2\cos\left(\frac{2\pi}{365}(d+10)\right)\rightarrow rad$ or $h=12+2\cos\left(\frac{360}{365}(d+10)\right)\rightarrow deg$
✔ vertical translation of 12 units ✔ amplitude of 2 ✔ phase shift of 10 ✔ period of 365

(b) How many hours of daylight are there on the first day of the year? (2 marks)

Solution
Let $d=1$ $h=13.96\, hours\ of\ daylight$

(c) The prices of tourist accommodation in the island are highest when the hours of daylight per day are 13 hours or more. How many days after the beginning of the year does the high season end? (3 marks)

Solution
Solve for d when $h=13$. $13=12+2\cos\left(\frac{2\pi}{365}(d+10)\right)\rightarrow d=50.83\, days$ $\therefore The\ high\ season\ ends\ after\ 51\ days\ ¿ the\ beginnning\ of\ the\ year.$
✔ equating model to 13 ✔ determining d ✔ stating 51 $days$

(d) Philip Island Tourism tries to encourage visitors to the island during the winter months. They offer special deals for those visiting the island during the time of year when there are less than 11 hours of daylight. For how many days of the year are the special deals available to visitors? (3 marks)

Solution
Solve for d when $h=11$. Two points $(111.667, 11) \wedge (233.333, 11) \rightarrow 233.333-111.667=121.67$ $\therefore$ *There are* 122 *days with* 11 *hours of daylight.*
✔✔ stating two points where $h=11$ ✔ stating 122 days

Question 13 **(9 marks)**

A relay team of four runners is to be chosen from a training squad of eight runners. Assuming that all eight runners are of equal ability and have the same chance of being selected, find the following:

(a) the number of different teams that can be selected. (1 marks)

Solution
$\binom{8}{4}=70$

(b) the probability that you will be chosen, supposing that you are one of the eight runners. (2 marks)

Solution
$\frac{\binom{1}{1}\binom{7}{3}}{70}=\frac{1}{2}$

(c) Suppose that your friend is also one of the eight. What is the probability that you and your friend will be both selected? (2 marks)

Solution
$\frac{\binom{2}{2}\binom{6}{2}}{70}=\frac{3}{14}$

(d) What is the probability of you or your friend being in the team? (2 marks)

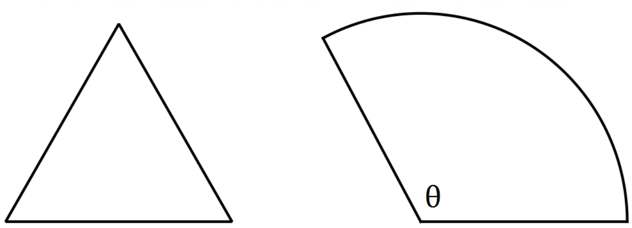
Solution
$\frac{1}{2}+\frac{1}{2}-\frac{3}{14}=\frac{11}{14}$

(e) Given that the team chosen is one that your friend is in, what is the probability that you will also be included? (2 marks)

Solution
$\dfrac{\frac{3}{14}}{\frac{1}{2}} = \dfrac{3}{7}$

Question 14 **(11 marks)**

A wire of total length $120\,cm$ is to be cut into two pieces for an art installation. The first piece is bent to form an equilateral triangle of side length $x\,cm$ and the second piece is bent to form a circular sector of radius $x\,cm$. The circular sector subtends an angle θ radians at the centre.



(a) Show that $x\theta = 120 - 5x$.(2 marks)

Solution
$3x + 2x + x\theta = 120$ $x\theta = 120 - 5x$
✔✔ setting up the sum of the perimeter of the shapes equal to 120.

(b) The total area of the two shapes is $A\,cm^2$.Show clearly that $A = \frac{x^2}{4}\left(\sqrt{3} - 10\right) + 60x$.

(4 marks)

Solution
$A_{triangle} = \frac{1}{2}x^2 \sin 60^0 \wedge A_{sector} = \frac{1}{2}x^2\theta$ ¿ $\frac{\sqrt{3}x^2}{4} = \frac{1}{2}x(120 - 5x)$

$$A=\frac{\sqrt{3}x^2}{4}+\frac{2(120x-5x^2)}{4}$$ $$A=\frac{\sqrt{3}x^2+240x-10x^2}{4}$$ $$A=\frac{x^2}{4}(\sqrt{3}-10)+60x$$
✔ setting up area for triangle ✔ setting up area for sector ✔✔ correct working out resulting to required equation for A (-✔ for every error)

(c) Use a calculus method to find correct to three significant figures, the maximum value of A, justifying the fact that it is indeed the maximum value of A.

(5 marks)

Solution
$$\frac{dA}{dx}=\frac{1}{2}(\sqrt{3}-10)x+30$$ $for\ max/min,\ \frac{dA}{dx}=0 \rightarrow 0=¿\frac{1}{2}(\sqrt{3}-10)x+30¿$ $x=7.26\,cm$ $therefore\ A=109\,cm^2$ $for\ x<7.26,\ \frac{dA}{dx}>0$ $for\ x>7.26,\ \frac{dA}{dx}<0$ $\rightarrow (7.26,109)\ is\ a\ maximum.$
✔ finding derivative ✔ setting derivative to 0 ✔ finding value for A ✔✔ valid justification for A as the maximum area

Question 15 **(9 marks)**

(a) Integrate $\frac{(x+1)(2x-1)}{2x^5}$ with respect to x. Express your answer with exactly 4 terms and without any negative exponents. (3 marks)

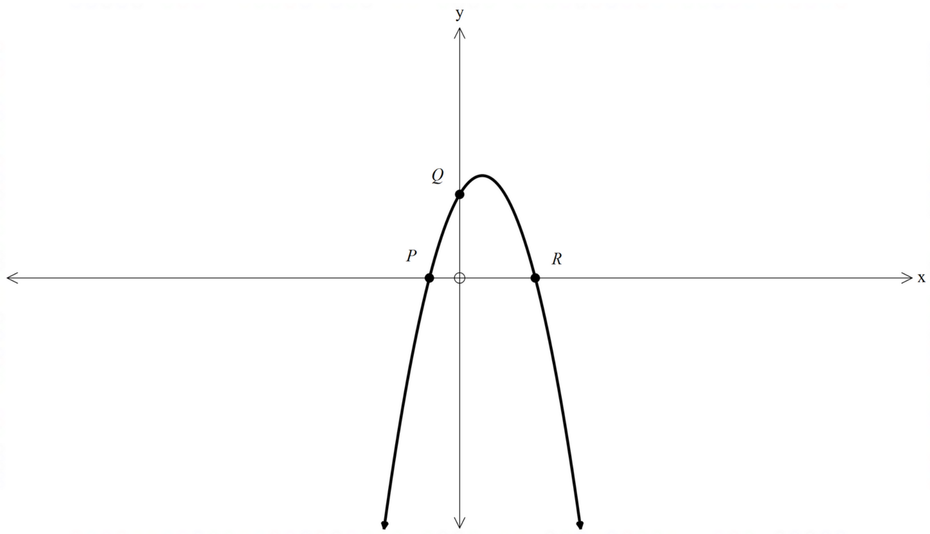
Solution
$\int \frac{(x+1)(2x-1)}{2x^5}dx = \int \frac{2x^2+x-1}{2x^5}dx$ $¿\int x^{-3}+\frac{x^{-4}}{2}-\frac{x^{-5}}{2}dx$ $¿\frac{-x^{-2}}{2}-\frac{x^{-3}}{6}+\frac{x^{-4}}{8}$ +C $¿-\frac{1}{2x^2}-\frac{1}{6x^3}+\frac{1}{8x^4}+c$
✔ correct antiderivative   ✔ positive exponents   ✔ C

(b) Justify the relevance of adding an arbitrary constant C in finding the antiderivatives of functions. (1 mark)

Solution
Adding a constant C, accounts for the fact that there are families of curves which will

have the same derivative. Unless we are given a point in the function, we can only write a general form for the antiderivative.

(c) The figure below shows the curve C which meets the coordinate axes at the points P, Q and R.



The gradient function of C is given by $f'(x)=3-4x$ and that $f(1)=2f(2)$, determine the coordinates of the points P, Q and R. (5 marks)

Solution
$f'(x)=3-4x \rightarrow f(x)=3x-2x^2+c$ $3-2+c=2(6-8+c)$ $c=5$ $f(x)=-2x^2+3x+5$

$\rightarrow f(0)=5$ $\rightarrow 0=-2x^2+3x+5$ $0=(x+1)(2x-5)$ $\rightarrow x=-1,\frac{5}{2}$ $\therefore Q\, is\,(0,5), P\, is\,(-1,0) \wedge R\, is\left(\frac{5}{2},0\right).$
✔ finding the function ✔ correct value for c ✔✔✔ coordinates for $P, Q \wedge R$

END OF SECTION 2

Additional working space

Question number: _________

Additional working space

Question number: ________

Additional working space

Question number: ________