MATHEMATICS METHODS

MAWA Semester 2 (Units 3 and 4) Examination 2016

Calculator-free

Marking Key

The release date for this exam and marking scheme is

- **the end of week 1 of term 4, 2016**

Section One: Calculator-free **(54 Marks)**

Question 1(a)

Solution

$\ln m = \frac{3}{2} \Rightarrow m = e^{\frac{3}{2}}$

Marking key/mathematical behaviours	Marks
• identifies correct base	1
• determines correct power	1

Question 1(b)

Solution

$\log[(m+3)m] = 1$

$(m+3)m = 10^1$

$m^2 + 3m - 10 = 0$

$(m+5)(m-2) = 0$

$m = -5$ or 2 but since m has to be greater than zero, $m = 2$ is the only solution.

Marking key/mathematical behaviours	Marks
• applies logarithmic rule for a product correctly	1
• recognises base 10	1
• creates equation with correct trinomial	1
• solves equation correctly giving the correct value of m	1

Question 2(a)(i)

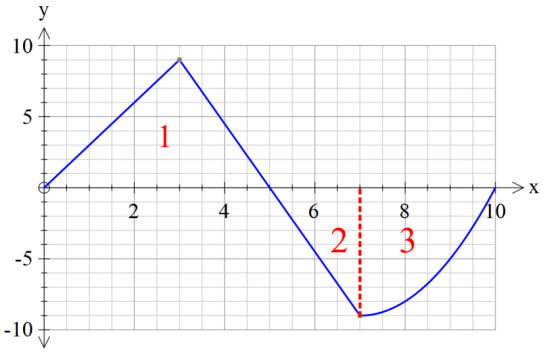
Solution

$\frac{dy}{dx} = \frac{(6x^4 - x^3 + e)(4e^x) - (4e^x)(24x^3 - 3x^2)}{\left(6x^4 - x^3 + e\right)^2}$

Marking key/mathematical behaviours	Marks
• differentiates the 1st term on numerator correctly	1
• differentiates the 2nd term on numerator correctly	1
• squares factor on denominator	1

Question 10(b)

Solution



$\text{Area } 2 = \frac{1}{2} \times 2 \times 9$

$= 9 \text{ square units}$

$\text{Area } 3 = 50 - 22\frac{1}{2} - 9$

$= 18\frac{1}{2} \text{ square units}$

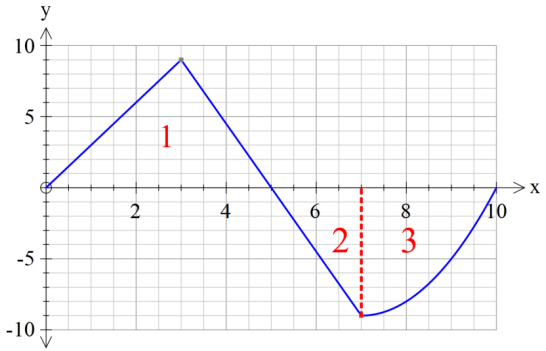
$\int_7^{10} f(x)dx = -\text{ Area } 3$

$= -18\frac{1}{2}$

Marking key/mathematical behaviours	Marks
• calculates area 2	1
• calculates area 3	1
• determines integral	1

Question 10(a)

Solution



$$\int_0^5 f(x)dx = \text{area of triangle (Area 1)}$$

$$= \frac{1}{2} \times 5 \times 9$$

$$= 22\frac{1}{2}$$

Marking key/mathematical behaviours	Marks
• identifies integral as area of correct triangle	1
• determines integral.	1

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Question 2(a)(ii)

Solution

$$\frac{dy}{dx} = \frac{d}{dx}[\ln(5x^3+3) - \ln(\sin(x)]$$

$$= \frac{d}{dx}[\ln(5x^3+3)] - \frac{d}{dx}[\ln(\sin(x)]$$

$$= \frac{15x^2}{(5x^3+3)} - \frac{\cos(x)}{\sin(x)}$$

Marking key/mathematical behaviours	Marks
• applies correctly logarithmic rule for quotients	1
• differentiates correctly 1st term	1
• differentiates correctly 2nd term	1

Question 2(b)

Solution

Let $u = x^2 - \cos(x) \Rightarrow \frac{du}{dx} = 2x + \sin(x)$ and $\frac{dy}{du} = \frac{e^u}{2}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{e^u}{2} \times (2x + \sin(x)) = \frac{e^{x^2-\cos(x)}}{2}(2x + \sin(x))$$

Marking key/mathematical behaviours	Marks
• differentiates correctly to determine1st factor in chain rule	1
• differentiates correctly to determine 2nd factor in chain rule	1
• expresses $\frac{dy}{dx}$ in terms of x	1

Question 3(a)

Solution Discrete random variable	
Marking key/mathematical behaviours	Marks
• determines correct category	1

Question 3(b)

Solution Non-random variable	
Marking key/mathematical behaviours	Marks
• determines correct category	1

Question 3(c)

Solution Continuous random variable	
Marking key/mathematical behaviours	Marks
• determines correct category	1

Question 9(c)(i)

Solution

n_2 is larger than n_1

To increase confidence a larger interval is required for a stable sample size. Increasing n reduces the standard error and thus the interval can remain the same.

Marking key/mathematical behaviours	Marks
• states n_2 is larger with reason	1
• states correct reason	1

Question 9(c)(ii)

Solution

$$E_1 = 1 \times \sqrt{\frac{m(1-m)}{n_1}}$$

$$E_2 = 1.5 \times \sqrt{\frac{m(1-m)}{n_2}}$$

Same interval so $E_1 = E_2$

$$\sqrt{\frac{m(1-m)}{n_1}} = 1.5 \times \sqrt{\frac{m(1-m)}{n_2}}$$

$$\frac{m(1-m)}{n_1} = (1.5)^2 \frac{m(1-m)}{n_2}$$

$$\frac{n_2}{n_1} = 2.25$$

$$n_2 = 2.25 n_1$$

Marking key/mathematical behaviours	Marks
• equates E_1 and E_2	1
• squares both sides	1
• states relationship	1

Question 9(a)(ii)

Solution

$$E = 2 \times \sqrt{\frac{\frac{1}{5}(1-\frac{1}{5})}{100}}$$
$$= 2 \times \sqrt{\frac{4}{2500}}$$
$$= 2 \times \frac{2}{50}$$
$$= 0.08$$

95% CI is $(0.12, 0.28)$

Marking key/mathematical behaviours	Marks
• substitutes values for z, n and p	1
• simplifies square root	1
• simplifies E	1
• states interval	1

Question 9(b)

Solution

$$E = 1 \times \sqrt{\frac{m(1-m)}{n_1}}$$

68% CI is $(m - \sqrt{\frac{m(1-m)}{n_1}}, m + \sqrt{\frac{m(1-m)}{n_1}})$

Marking key/mathematical behaviours	Marks
• determines E	1
• states confidence interval.	1

Question 4

Solution

$$k\int_0^1 x - \frac{x^3}{3}\, dx = 1$$
$$k\left[\frac{x^2}{2} - \frac{x^4}{12}\right]_0^1 = 1$$
$$k\left[\frac{1}{2} - \frac{1}{12}\right] = 1 \quad \Rightarrow \quad k = \frac{12}{5}$$

Marking key/mathematical behaviours	Marks
• sets up integral and equates to one	1
• integrates correctly	1
• evaluates integral correctly	1
• calculates the value of k	1

Question 5

Solution

$$p(1-p) = \left(\frac{\sqrt{3}}{4}\right)^2 = \frac{3}{16}$$
$$16p^2 - 16p + 3 = 0$$
$$(4p-1)(4p-3) = 0 \quad \Rightarrow \quad p = \frac{1}{4} \text{ or } p = \frac{3}{4}$$

Marking key/mathematical behaviours	Marks
• sets up equation using variance of a Bernoulli distribution	1
• derives quadratic equation	1
• factorises trinomial	1
• solves correctly for p	1

Question 6(a)

Solution

Function is valid for $x > -3$

Marking key/mathematical behaviours	Marks
• correctly states the values of x for which the function is valid	1

Question 6(b)

Solution

$$\frac{dy}{dx} = \frac{2}{2x+6} = 4 \Rightarrow \frac{2x+6}{2} = \frac{1}{4} \Rightarrow x+3 = \frac{1}{4} \Rightarrow x = -2.75$$

Marking key/mathematical behaviours	Marks
• differentiates correctly	1
• solves equation correctly	1

Question 7(a)

Solution

y	0	1	2	3	4
$P(Y = y)$	**0**	$\boldsymbol{k}$	$4k$	$\boldsymbol{9k}$	$\boldsymbol{16k}$

Marking key/mathematical behaviours	Marks
• correctly completes two values	1
• correctly completes 4 values	1

Question 7(b)

Solution

$$k + 4k + 9k + 16k = 1$$

$$30k = 1 \quad \Rightarrow \quad k = \frac{1}{30}$$

Marking key/mathematical behaviours	Marks
• sums probabilities equal to one	1
• correctly solves equation for k	1

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Question 8

Solution

$$f(x) = \int f'(x)\,dx$$

$$= \int 2xe^{3x^2-1}\,dx$$

$$= \frac{1}{3}e^{3x^2-1} + c$$

since $f(0) = 0$:

$$0 = \frac{1}{3}e^{-1} + c$$

$$c = -\frac{1}{3e}$$

$$f(x) = \frac{1}{3}e^{3x^2-1} - \frac{1}{3e}$$

Marking key/mathematical behaviours	Marks
• determines indefinite integral	1
• substitutes initial conditions to calculate the constant c	1
• states f(x)	1

Question 9 (a)(i)

Solution

$$\hat{p} = \frac{20}{100} = \frac{1}{5}$$

Marking key/mathematical behaviours	Marks
• determines the proportion	1