CALCULATOR FREE

Question 1 **(6 marks)**

Differentiate the following with respect to x, without simplifying.

(a) $f(x)=\frac{1+e^x}{1-e^{-2x}}$ [2 marks]

$$f'(x) = \frac{e^x(1-e^{-2x}) - (1+e^x).2e^{-2x}}{(1-e^{-2x})^2}$$

✓ correct use of quotient rule
✓ all terms correct

(b) $g(x)=(3x+1)(1+x^2)^3$ [2 marks]

$$g'(x) = 3(1+x^2)^3 + (3x+1).3(1+x^2)^2.2x$$

✓ correct use of product rule
✓ all terms correct

(c) $h(x)=\int_1^{x^2}(1-4t)^3\,dt$ [2 marks]

$$h'(x) = (1-4x^2)^3.2x$$

✓ substitutes x^2
✓ multiplies by $2x$

6

See next page

Question 2 **(5 marks)**

(a) Evaluate $\int_{0.5}^{1}\left(\sqrt{2x-1}\right)^3 dx$ [3 marks]

$$= \int_{0.5}^{1}(2x-1)^{3/2}\,dx$$

$$= \left[\frac{(2x-1)^{5/2}}{2\times 5/2}\right]_{0.5}^{1}$$ ✓ $(2x-1)^{5/2}$

$$= \left[\frac{(2x-1)^{5/2}}{5}\right]_{0.5}^{1}$$ ✓ divides by 5

$$= \frac{1}{5} - 0$$ ✓ substitutes correctly

$$= \frac{1}{5}$$

(b) Determine $\int xe^{1+x^2}dx$ [2 marks]

$$\int x e^{1+x^2} dx = \frac{1}{2}e^{1+x^2} + c$$

✓ e^{1+x^2} and $+c$
✓ factor of $\frac{1}{2}$

5

See next page

Question 3 **(5 marks)**

Calculate the maximum and minimum values of $\frac{x^2+4}{x}$ in the interval $1 \le x \le 5$.

Let $S = \frac{x^2+4}{x} = x + \frac{4}{x}$

$\frac{dS}{dx} = 1 - \frac{4}{x^2}$ ✓ $\frac{dS}{dx}$

$\frac{dS}{dx} = 0 \Rightarrow x = \pm 2$ ✓ $x = 2$

$\frac{d^2S}{dx^2} = \frac{8}{x^3}$ At $x = 2$ $\frac{d^2S}{dx^2} = 1 > 0$ ∴ Minimum.

or

x	1	2	3
$\frac{dS}{dx}$	-3	0	$5/4$
	\	—	/

∴ Minimum ✓ shows MIN at $x=2$

$S(1) = 1+4 = 5$

$S(2) = \frac{4+4}{2} = 4$

$S(5) = \frac{25+4}{5} = 5.8$ ✓ evaluates end values

∴ Maximum value is 5.8 when $x=5$ ✓ conclusion

Minimum value is 4 when $x=2$

5

See next page

Question 4 **(5 marks)**

Solve the inequality $\frac{1-4x}{x+12} \le 3$

$$\frac{1-4x}{x+12} - \frac{3(x+12)}{x+12} \le 0$$

$$\Rightarrow \frac{-7x-35}{x+12} < 0$$

$$\Rightarrow \frac{-7(x+5)}{x+12} < 0$$ ✓ algebraic rearrangement

For $x < -12$ $\frac{-ve \times -ve}{-ve} < 0$ ✓ TRUE ✓ critical value $x=-12$

$-12 < x \le -5$ $\frac{-ve \times -ve}{+ve} > 0$ ✗ FALSE ✓ critical value $x=-5$

$x \geq -5$ $\frac{-ve \times +ve}{+ve} < 0$ ✓ TRUE ✓ testing

✓ statement

∴ $x < -12$ or $x \geq -5$

5

See next page

Question 5 (7 marks)

Let $f(x)=e^{-x}$ and $g(x)=\frac{1}{1-x}$.

(a) Determine expressions for $f(g(x))$ and $g(f(x))$. [2 marks]

$f(g(x)) = e^{-\frac{1}{1-x}}$ ✓ answer

$g(f(x)) = \frac{1}{1-e^{-x}}$ ✓ answer

(b) Evaluate $f(g(0))$ and $g(f(0))$. [2 marks]

$f(g(0)) = e^{-1}$ ✓ $g(f(0))$ undefined ✓

(c) Determine the domain of $f(g(x))$. [1 mark]

$D_{fg} = \{x : x \neq 1, x \in \mathbb{R}\}$ ✓ $x \neq 1$

(d) Determine the range of $g(f(x))$. [2 marks]

$R_{gf} = \{y : y < 0 \text{ or } y > 1\}$

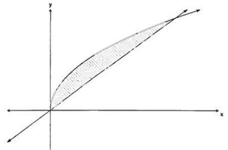
✓ $y > 1$

✓ $y < 0$

7

Question 6 (6 marks)

The diagram below shows graphs of $y=\sqrt{x}$ and $y=0.5x$. Find the shaded area.



Intersection: $\sqrt{x} = 0.5x$

$2 = \sqrt{x}$

$x = 4$ ✓

Area $= \int_0^4 \sqrt{x} - 0.5x \, dx$ ✓ limits ✓ integrand

$= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{4}\right]_0^4$ ✓✓ antideriv

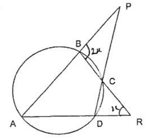
$= \frac{2}{3} \times 8 - 4$

$= \frac{4}{3}$ units^2 ✓ answer

6

Question 21 (6 marks)

Consider the diagram below which shows a cyclic quadrilateral ABCD. The sides of the quadrilateral have been extended and these lines meet at the points P and R as shown.



Let $\angle CRD = x$

$\therefore \angle PBC = 2x$

Given that $\angle PBC = 2 \times \angle CRD$ prove that triangle ABR is isosceles.

Statement	Reason
$\angle ABR = 180 - 2x$	angles on a line add to 180°
$\angle ADC = 2x$	opp. ∠s in cyclic quad add to 180°
$\angle RDC = 180 - 2x$	angles on a line add to 180°
$\angle DCR = 180 - (180 - 2x) - x$ $= x$	angle sum of Δ DCR
$\angle BCD = 180 - x$	angles on a line add to 180°
$\angle BAD = x$	opp ∠'s in a cyclic quad
$\angle BAR = \angle BRA = x$	
$\therefore \Delta BAR$ isosceles with $AB = RB$	

✓ clear
✓ sequential
✓ complete

✓ partial reasoning
✓ nearly complete reasoning
✓ complete reasoning

6

Question 22 (7 marks)

In a statistical experiment a coin is tossed repeatedly until a certain number of "Heads" have been obtained. On any particular toss of the coin there is a probability of 0.5 that it lands on "Heads". The score recorded is the number of tosses.

(a) Find the probability that 3 "Heads" are obtained in exactly 3 tosses. [1]

$P(H,H,H) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ ✓

(b) Find the probability that the third "Heads" is obtained on the 4th toss of the coin. [2]

THH H
HTH H
HHT H

$P(4) = 3 \times \left(\frac{1}{2}\right)^4 = \frac{3}{16}$ ✓ ✓

(c) Write down a formula, in terms of r, that the third "Heads" is obtained on the rth toss of the coin, $r \geq 3$ [2]

Need 2 of first $r-1$ tosses "Heads" – followed by Head

$P(r) = {}^{r-1}C_2 \times \left(\frac{1}{2}\right)^r$ ✓ ✓

(d) Write down a formula, in terms of a and r for the probability that the ath "Heads" is obtained on the rth toss of the coin, $r \geq a$. [2]

$P(r) = {}^{r-1}C_{a-1} \times \left(\frac{1}{2}\right)^r$ ✓ ✓

Need $a-1$ out of $r-1$ and then a final "Head".

7

End of Booklet 3

Question 7 (6 marks)

Let A denote the set $\{1, 2, 3, \ldots, 999, 1000\}$, the set of positive integers up to 1000.

(a) How many numbers in set A are not multiples of either 4 or 5 or both? [3 marks]

Multiples of 4: 4, 8, 12, ..., 1000 — 250
Multiples of 5: 5, 10, 15, ..., 1000 — 200 ✓ 250, 200
Multiples of 20: 20, 40, 60, ..., 1000 — 50 ✓ 50

$\therefore$ There are $1000 - 250 - 200 + 50$

$= 600$ numbers ✓ answer

(b) How many numbers in set A that have at least 2 digits start and finish with the same digit? [3 marks]

2 digits: 11, 22, ... 99 — 9 numbers

3 digits: 1 . 1, 2 . 2, 3 . 3, ... 9 . 9

$9 \times 10 = 90$ numbers

$\therefore$ Total $= 9 + 90 = 99$ numbers

✓ breaks into 2/3 digits
✓ shows calculations
✓ answer

6

Question 8 (6 marks)

The function $r(x)=ax+bx^2+\frac{c}{x}$ has the following properties:

- $r(2) = 20$
- $r(x)$ has a stationary point when $x = 1$
- $r(x)$ has a point of inflection at $x = -2$

(a) Show that the constants a, b and c satisfy the simultaneous equations:

$4a + 8b + c = 40, \quad a + 2b - c = 0, \quad 8b - c = 0.$ [3 marks]

$r(2) = 20 \Rightarrow 2a + 4b + \frac{c}{2} = 20$

$\Rightarrow 4a + 8b + c = 40$ ✓ justifies

$r'(1) = 0 \Rightarrow a + 2bx - \frac{c}{x^2}\Big|_{x=1} = 0$

$\Rightarrow a + 2b - c = 0$ ✓ justifies

$r''(-2) = 0 \Rightarrow 2b + \frac{2c}{x^3}\Big|_{x=-2} = 0$

$\Rightarrow 2b + \frac{2c}{-8} = 0$

$\Rightarrow 8b - c = 0$ ✓ justifies

3

Question 18 (8 marks)

The lifetime of a particular type of bicycle tyre is known to follow a normal distribution with mean 800 hours and standard deviation 60 hours.

(a) Find the probability that a bicycle tyre lasts at least 900 hours. [1 mark]

$N(800, 60^2)$ $P(X > 900) = 0.0478$ (4dp)

✓ answer

(b) The company wants to advertise their tyres using the words, "99% of our tyres last longer than x hours". What should the value of x be? [1 mark]

$P(X > a) = 0.99$ $a = 660$ hours ✓ answer

(c) If a sample of 200 tyres is taken, state estimates for the mean and standard deviation of the sample. [2 marks]

Mean = 800 hrs s.d. = 60 hours ✓ 800 ✓ 60

(d) If a sample of 200 tyres is taken, find the probability that the mean of the sample is greater than 810 hours. [2 marks]

$\bar{x} \sim N\left(800, \left(\frac{60}{\sqrt{200}}\right)^2\right)$ ✓ distribution

$P(\bar{x} > 810) = 0.0092$ (4dp) ✓ answer

(e) The tyre company has a new manufacturing process which increases the mean lifetime to 900 hours. They find that 90% of their tyres have lifetimes greater than 850 hours. What is the standard deviation of the lifetimes of these new tyres? [2 marks]

$x = 850$ corresponds to $z = -1.28155$ ✓ z value

$\therefore$ $-1.28155\,\sigma = -50$

$\therefore$ $\sigma = 39.0$ hours ✓ answer

8

See next page

Question 19 (7 marks)

At the University of Western Australia the time taken to log on to the computer network is known to be distributed uniformly between 30 seconds and 150 seconds. For this distribution it is known that the mean is 90 seconds and the variance is 1200 seconds^2.

A group of students studying statistics record the time to log on to the network for samples of 40 randomly selected students.

(a) Find the probability that the time taken to log on for one student is greater than 2 minutes. [1]

$P(X > 120) = \frac{30}{120} = 0.25$ ✓ answer

(b) Find the probability that the mean time to log on for one sample is greater than 95 seconds. [2]

$\bar{X} \sim N\left(90, \frac{1200}{40}\right)$ ✓ distribution

$P(\bar{X} > 95) = 0.1807$ (4dp) ✓ answer

(c) If 20 samples of size 40 are taken, how many would be expected to differ from the true mean by more than 5 seconds? [2]

$P(\bar{x} > 95) + P(\bar{x} < 85) = 2 \times 0.1807$ ✓ 0.36

Expect $20 \times 2 \times 0.1807 = 7.23$

Expect 7 or 8 ✓ answer

(d) Find the probability that at least 8 of the 20 samples collected differ from the true mean by more than 5 seconds. [2]

$Y \sim Bin(20, 0.36131)$ ✓ distribution

$P(Y \geq 8) = 0.4410$ (4dp) ✓ answer

3

See next page

Question 20 (9 marks)

A company manufactures ball bearings. The machine they use is set to a certain measurement for each week's production which amounts to 10 000 ball bearings.

(a) In the first week of production a sample of 200 ball bearings is taken and found to have a mean mass of 0.824 g with a standard deviation of 0.042 g.

Given that the standard deviation observed in the sample matches the true standard deviation, find a 99% confidence interval for the true mean mass of the ball bearings produced during that week. [3]

$\bar{x} - 2.5758\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + 2.5758\frac{\sigma}{\sqrt{n}}$

$0.824 - \frac{2.5758 \times 0.042}{\sqrt{200}} \leq \mu \leq 0.824 + \frac{2.5758 \times 0.042}{\sqrt{200}}$

$0.816 \leq \mu \leq 0.832$ (3dp)

✓ formula
✓ substitution
✓ answer

(b) In the second week the machine is reset. From a sample of 100 ball bearings the company calculates that they are 95% confident that the true mean mass lies between 0.812 g and 0.842 g.

Find the mean and standard deviation of the sample that the company took for that week. [3]

$\bar{x} = \frac{0.812 + 0.842}{2} = 0.827$ g ✓ mean 0.827

$0.842 - 0.827 = \frac{1.96 \times \sigma}{\sqrt{100}}$ ✓ eq^n

$\sigma = \frac{0.015 \times 10}{1.96} = 0.077$ ✓ s.d.

6

See next page

Question 20 (Cont)

(c) After a number of weeks the company estimates that the standard deviation on each production run is 0.04 g. How large a sample do they need to take so that they can be 99% confident that the sample mean will be within 0.005 g of the true mean? [3]

$2.5758\frac{\sigma}{\sqrt{n}} < 0.005$ ✓ inequality

$\frac{2.5758 \times 0.04}{0.005} < \sqrt{n}$ ✓ substitute

$n > 424.63$

Need at least 425 in the sample. ✓ answer

3

See next page

Question 8 (cont)

(b) Evaluate the constants a, b and c by solving the equations in part (a). [3 marks]

$4a + 8b + c = 40$ (1)
$a + 2b - c = 0$ (2)
$8b - c = 0$ (3)

(1) + (2) $5a + 10b = 40$ (4)
(1) + (3) $4a + 16b = 40$
$\Rightarrow$ $a + 4b = 10$ (5)
(5) − (4) $10b = 10$

$b = 1$
$c = 8$
$a = 6$

✓ rearrange and eliminates 2 variables
✓ finds b
✓ finds a, c

$\therefore$ $\boxed{a = 6, \quad b = 1, \quad c = 8}$

3

See next page

Question 9 (4 marks)

Consider the conjecture:
"every prime number greater than 3 is one more or less than a multiple of 6".

(a) Show that the conjecture is true for three values. [1 mark]

$11 = 12 - 1$
$17 = 18 - 1$
$37 = 36 + 1$

✓ Shows true for 3 values > 3

(b) Prove the conjecture. [3 marks]

Any number can be written as

$6n$: it is a multiple of 6 and not prime

$6n+1$:

$6n+2 = 2(3n+1)$ $\therefore$ not prime when $n \neq 0$
$6n+3 = 3(2n+1)$ $\therefore$ not prime when $n \neq 0$
$6n+4 = 2(3n+2)$ $\therefore$ not prime when $n \neq 0$
$6n+5$

$\therefore$ All primes are of form $6n+1$ or $6n+5$

ie 1 more or less than multiple of 6

(provided > 3)

✓ Considers all cases 4
✓ shows not prime if $6n, 6n+2, 6n+3, 6n+4$
✓ concludes correctly

End of paper

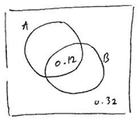
CALCULATOR ASSUMED

Question 10 (8 marks)

For events A and B;

$P(A \cap B) = 0.12$, $P(A' \cap B') = 0.32$ and $P(B|A) = 0.4$

(a) Calculate $P(A)$ [3 marks]



$P(B|A) = \frac{P(B \cap A)}{P(A)}$ ✓ equation

$0.4 = \frac{0.12}{P(A)}$ ✓ substitutes correctly

$P(A) = 0.3$ ✓ answer

(b) Calculate $P(B)$ [1 mark]

$P(B) = 1 - 0.32 - 0.18$
$= 0.5$ ✓ answer

(c) Calculate $P(A|B')$ [2 marks]

$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{0.18}{0.5}$ ✓ numerator ✓ denominator

$= 0.36$

(d) Are events A and B independent? Justify your answer. [2 marks]

$P(B|A) = 0.4$ $P(B) = 0.5$ ✓ evidence

These are different $\therefore$ not independent. ✓ answer

8

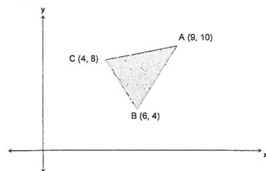
See next page

Question 11 (9 marks)

A local supermarket sells two brands of milk, X and Y. They sell x thousand cartons of brand X and y thousand cartons of brand Y.

Each carton of brand X makes a profit of $0.80 but each carton of brand Y makes a loss of $0.20.

The feasible region for the supermarket's weekly sales is shown in the diagram below.



(a) Determine the inequality satisfied by x and y that corresponds to the edge CA of the feasible region. [3 marks]

$m = \frac{10-8}{9-4}$ Eq^n : $y = 0.4x + 6.4$ ✓ equation

$= \frac{2}{5} = 0.4$ ✓ gradient $y \leq 0.4x + 6.4$ ✓ inequality

(b) Find the amount of each brand of milk that should be sold for maximum profit. Show your working. [3 marks]

Vertex	$P = 0.8x - 0.2y$
(4, 8)	$3.2 - 1.6 = 1.6$
(9, 10)	$7.2 - 2 = 5.2$
(6, 4)	$4.8 - 0.8 = 4.0$

✓ P formula

Should sell 9000 x and 10 000 y ✓ answer

✓ calculations

6

See next page

Question 11 (Cont)

(c) If the profit on each brand X carton remains as \$0.80, to what value can the loss on a carton of brand Y rise before there is a change to the point in part b) that creates maximum profit? [3 marks]

Vertex	$P = 0.8x + ay$
(4, 8)	$3.2 + 8a$
(9, 10)	$7.2 + 10a$
(6, 4)	$4.8 + 4a$

✓ sets up formula and evaluates

Comparing (4, 8) and (9, 10) $\quad 3.2 + 8a = 7.2 + 10a$

$a = -2$

Comparing (9, 10) and (6, 4) $\quad 7.2 + 10a = 4.8 + 4a$

$6a = -2.4$

$a = -0.4$

✓ solves equation

If the loss rises above \$0.40 per carton there would need to be a change ✓ answer

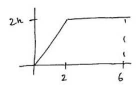
3

Question 12 (9 marks)

A continuous random variable, X, has a probability density function given

by $f(x) = \begin{cases} kx & 0 \le x \le 2 \\ 2k & 2 < x \le 6 \\ 0 & otherwise \end{cases}$

(a) Determine the value of k. [3 marks]



Area $= 1$ ✓ concept

$\Rightarrow \frac{1}{2} \times 2 \times 2k + 4 \times 2k = 1$ ✓ equat

$10k = 1$

$k = 0.1$ ✓ k = 0.

(b) Find

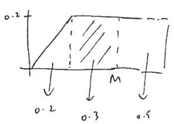
i) $P(X \le 4) = 2k + 4k$ [1 mark]

$= 0.6$ ✓ answer

ii) $P(X \ge 2 | X \le 4) = \frac{0.4}{0.6} = \frac{2}{3}$ [2 marks]

$= 0.\bar{6}$ ✓ numerat ✓ answer

iii) M, the median of the distribution. [3 marks]



$P(x \le M) = 0.5$ ✓ concept

$\therefore (6 - M) \times 0.2 = 0.5$ ✓ equati

$6 - M = 2.5$

$\underline{M = 3.5}$

✓ answer

9

Question 13 (6 marks)

State the sequence of transformations, in the correct order so that the graph of $y=1+3e^{2-x}$ is transformed to $y=-2+9e^{-2x-1}$.

$y = 1 + 3e^{2-x}$

$\to y = 1+3e^{-x-1}$ Translation of 3 units in the negative x direct

$\to y = 1+3e^{-2x-1}$ Dilation of factor 0.5 in the x direction

$\to y = 3+9e^{-2x-1}$ Dilation of factor 3 in the y direction

$\to y = -2+9e^{-2x-1}$ Translation 5 units in the negative y direction.

✓✓✓✓ each transformation

✓ order

✓ clarity of descriptions

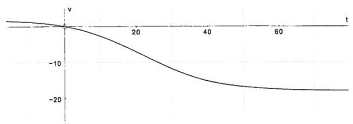
6

Question 14 (9 marks)

A skydiver drops out of a plane from a height of 1000 m. At time t seconds after she drops out of the plane her velocity in metres per second is given by the formula,

$$v = \frac{18\left(1-e^{0.1t}\right)}{\left(9+e^{0.1t}\right)}$$

The graph below shows the velocity at time t seconds after she jumps.



(a) Find the velocity of the skydiver after 20 seconds. [1 mark]

$v(20) = -7.017 \text{ ms}^{-1}$ (3dp) ✓ answer

(b) Find the acceleration of the skydiver after 20 seconds. [2 marks]

$a(20) = \frac{dv}{dt}(20) = -0.495 \text{ ms}^{-1}$ (3dp) ✓ concept ✓ answer

(c) Find the time (to the nearest 0.1s) when the skydiver's speed is increasing at the fastest rate. [2 marks]

$\frac{d^2v}{dt^2} = 0 \Rightarrow t = 21.97$

✓ concept

$t \simeq \underline{\underline{22.0 \text{ s}}}$ ✓ answer

5

Question 14 (Cont)

(d) Find the time (to the nearest 0.1s) taken for her to fall to the ground. [2 marks]

$\int_0^a v(t)\,dt = -1000$ ✓ equation

$a = -510.54,\ 81.110$

$\therefore$ Takes $\underline{81.1 \text{ s}}$ ✓ answer

(e) Find her speed (in metres per second to 1 decimal place) when she hits the ground. [2 marks]

Hits at speed $|v(81.1)| = \underline{17.9 \text{ ms}^{-1}}$

✓ idea ✓ answer (1dp)

4

Question 15 (9 marks)

An amateur golfer plays 18 holes with a professional player. The probability that the amateur player wins any particular hole is 0.4.

a) Find the probability that the amateur player wins

i) less than seven holes in the full round of 18 holes, [2]

Bin (18, 0.4) ✓ $\quad P(x < 7) = 0.3743$ ✓ (4dp)

ii) at least two of the first nine holes and at least two of the second nine holes. [3]

Bin (9, 0.4) ✓ $\quad P(x \ge 2) = 0.9295$ ✓

$\therefore P(x \ge 2)^2 = 0.8639$ ✓ (4dp)

b) If the players decide to play less than the full 18 holes, how many holes should they play so that the amateur has at least a 70% chance of winning at least 4 holes. [4]

Bin (n, 0.4) $\quad$ Need $P(X \ge 4) > 0.7$ ✓ concept

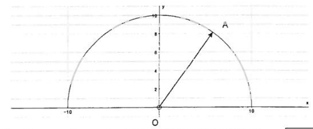
If $n = 10 \quad P(x \ge 4) = 0.6177$ ✓ evidence +

$n = 11 \quad P(x \ge 4) = 0.7037$ ✓ calculation

They must play at least 11 holes ✓ answer

9

Question 16 (6 marks)



The diagram above shows a semicircle with equation $y=\sqrt{100-x^2}$. The line OA is moving so that A moves around the circumference of the circle.

When the y coordinate is 8, the x coordinate of point A is increasing at a rate of 2 units per second. Find the rate at which the y coordinate of point A is changing at that same instant.

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ ✓ chain rule

$= -2x \times \frac{1}{2}(100-x^2)^{-\frac{1}{2}} \times 2$ ✓ differentiati

$= -\frac{2x}{\sqrt{100-x^2}}$

✓ $x = 6$

If $y = 8 \quad x^2 = 36 \Rightarrow x = \pm 6$ ✓ $x = -6$

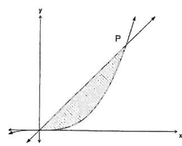
$x = 6$ y coordinate decreasing at $\frac{12}{8} = 1.5$ units/ ✓ answe

$x = -6$ y coordinate increasing at 1.5 units/s ✓ answer

(−2 marks if $x = -6$ not considered) 6

Question 17 (7 marks)

The diagram shows the graph of the curve $y=x^3$ and the line $y=4x$.



The area trapped between the curve and the line as shaded in the diagram is rotated 360° about the y – axis.

(a) Find the coordinates of the point, P, where the line and the curve intersect. [1 mark]

$x^3 = 4x \Rightarrow x = \pm 2$

P is (2, 8) ✓ answer

(b) Write down an expression to find the volume generated. [3 marks]

$\int_0^8 \pi x_1^2\,dy - \int_0^8 \pi x_2^2\,dy$ ✓ limits

$= \pi\int_0^8 y^{2/3}\,dy - \pi\int_0^8 \left(\frac{y}{4}\right)^2 dy$ ✓ 1st int ✓ 2nd integr

(c) If the shape generated represents a reservoir which could contain water, find the depth to which it needs to be filled so that it is half full. [3 marks]

$\pi\int_0^h y^{2/3} - \frac{y^2}{16}\,dy = \frac{\pi}{2}\int_0^8 y^{2/3} - \frac{y^2}{16}\,dy$ ✓✓ equation

$\therefore h = -13.31,\ 3.71$ or 11.24

$\therefore$ Should fill to depth $\underline{y = 3.71}$ ✓ answ

7