

STUDENT'S NAME _____

DATE: Friday 1st April

TIME: 33 minutes

MARKS: 33

INSTRUCTIONS:

Standard Items:

Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (7 marks)

Determine $\frac{dy}{dx}$ for each of the following. Do not simplify.

(a) $y = e^{\sqrt{x}}$

(b) $y = \sin\left(e^{2x}\right)$

(c) $y = (\cos x)e^{\cos x}$

[3]

[2]

[2]

9. (8 marks)

The amount A of a drug (in milligrams) in the bloodstream will decline at a rate proportional to

the current amount . That is $\frac{dA}{dt} = -\left(\frac{1}{k}\right)A$.

where k hours is a constant called the elimination time and time t is measured in hours.

(a) Write down the formula for $A(t)$, the amount of the drug in the bloodstream after t

hours , in terms of t , k and the initial amount A_0 .

[2]

(b) What proportion of the drug remains in the bloodstream after k hours ?

[3]

The drug sodium pentobarbital can be used to tranquilize animals. A dog is tranquilized if its bloodstream contains at least 45 milligrams of the drug for each kilogram of the dog's weight. The elimination time for the drug is 6 hours.

(c) What single dose of this drug should be given in order to tranquilize a 12 kilogram dog for 1 hour?

[3]

2. (10 marks)

(a) Evaluate exactly $\int_0^2 x e^{4-x^2} dx$ [4]

(b) Determine $\int \frac{4e^{2x} + 4x}{(e^{2x} + x^2)^3} dx$ [3]

(c) Determine $\int_{\pi}^{x^2} \left(\frac{d}{dt} e^{e^{-t}} \right) dt$ [3]

8. (5 marks)

A particular rock is dropped into a swimming pool and it sinks vertically to the bottom. Due to water resistance, the rock does not have a constant velocity on the way to the bottom. Its velocity, v centimetres per second, t seconds after it hits the surface of the water is given by $v = 8(2 - e^{-0.8t})$ for $0 \leq t \leq 7$

(a) What is the initial velocity of the rock in the water? [1]

(b) What is the acceleration of the rock after 4 seconds? [2]

(c) Terminal velocity is an expression used to describe the velocity that is approached but never exceeded. Determine the terminal velocity reached by the rock in the water. [2]

7.

(4 marks)

Sugar is being dissolved in a solution at a rate given by $\frac{dS}{dt} = -20e^{-0.1t}$ where S is the amount, in grams, of undissolved sugar after t seconds.

(a) how much sugar is initially in the solution

[2]

(b) how long does it take for half the sugar to dissolve.

[2]

3.

(4 marks)

Given $y = \frac{e^x}{3 + e^x}$

(a) determine $\frac{dy}{dx}$

[2]

4.

(5 marks)

(a) Determine $\frac{dy}{dx}$ given $y = xe^x$

[2]

(b) Hence determine $\int xe^x dx$

[3]

5. (4 marks)

Given $y = \int_{-3}^x \frac{t^2 - 2}{\sqrt{t}} dt$, use the incremental formula $\delta y \approx \frac{dy}{dx} \times \delta x$ to determine the change in y if x changes from 4 to 4.02.



Mathematics Methods Year 12
Test 2 2016

Section 2 Calculator Assumed
Exponential Function, Fundamental Theorem

STUDENT'S NAME _____

DATE: Friday 1st April

TIME: 20 minutes

MARKS: 21

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

6. (4 marks)

Determine the value of x for which $\int_x^{-1} (1 - t^2) dt$ has a relative minimum. Justify it is a minimum value.

[4]