Mathematics: Specialist
Formula sheet Units 3A and 3B

Vectors

$\left|(a_1, a_2)\right| = \sqrt{a_1^{\,2} + a_2^{\,2}}$

$\left|\mathbf{a} + \mathbf{b}\right| \le \left|\mathbf{a}\right| + \left|\mathbf{b}\right|$

$\mathbf{a} \cdot \mathbf{b} = \left|\mathbf{a}\right|\left|\mathbf{b}\right|\cos\theta = a_1 b_1 + a_2 b_2$

Vector equation of a circle in the plane:

$\left|\mathbf{r} - \mathbf{d}\right| = \rho$

Trigonometry

In any triangle ABC:

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\text{Area} = \dfrac{1}{2}\, a\, b \sin C$

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

In a circle of radius r, for an arc subtending angle θ (radians) at the centre:

$\text{Length of arc} = r\theta$

$\text{Area of sector} = \dfrac{1}{2} r^2 \theta$

$\text{Area of segment} = \dfrac{1}{2} r^2 (\theta - \sin\theta)$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\cos^2\theta + \sin^2\theta = 1$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\tan(\theta \pm \phi) = \dfrac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Exponentials and logarithms

For $a, b > 0$ and m, n real,

$a^m a^n = a^{m+n}$

$a^{m-n} = \dfrac{a^m}{a^n}$

$a^0 = 1$

$a^{-n} = \dfrac{1}{a^n}$

$(a^m)^n = a^{mn}$

$a^m b^m = (ab)^m$

For m an integer and n a positive integer:

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

For $a, y > 0$ then

$1 = a^0 \Leftrightarrow \log_a 1 = 0$

$\log_a (mn) = \log_a (m) + \log_a (n)$

$y = a^x \Leftrightarrow \log_a y = x$

$a = a^1 \Leftrightarrow \log_a a = 1$

$\log_a (m^n) = n \log_a (m)$

Functions

If $f(x) = y$, then $f'(x) = \frac{dy}{dx}$

If $f(x) = x^n$, then $f'(x) = n x^{n-1}$

If $f(x) = e^x$, then $f'(x) = e^x$

If $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$

Product rule:

If $y = f(x)\, g(x)$ or If $y = uv$

then $y' = f'(x)\, g(x) + f(x)\, g'(x)$ then $\frac{dy}{dx} = \frac{du}{dx} v + u \frac{dv}{dx}$

Quotient rule:

If $y = \frac{f(x)}{g(x)}$ or If $y = \frac{u}{v}$

then $y' = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$ then $\frac{dy}{dx} = \frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2}$

Chain rule:

If $y = f(g(x))$ or If $y = f(u)$ and $u = g(x)$

then $y' = f'(g(x))\, g'(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Powers: $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$

Exponentials: $\int e^x dx = e^x + c$

Trigonometric:

$\int \sin x \, dx = -\cos x + c$

$\int \cos x \, dx = \sin x + c$

$\int \frac{1}{\cos^2 x} dx = \tan x + c$

Fundamental Theorem of Calculus:

$\frac{d}{dx} \int_a^x f(t) dt = f(x)$ and $\int_a^b f'(x) dx = f(b) - f(a)$

Absolute value function : $|x| = \begin{cases} x, \text{ for } x \geq 0 \\ -x, \text{ for } x < 0 \end{cases}$

Sign function : $\operatorname{sgn}(x) = \begin{cases} 1, \text{ for } x > 0 \\ 0, \text{ for } x = 0 \\ -1, \text{ for } x < 0 \end{cases}$

Greatest integer function :

$\operatorname{int}(x)$ = greatest integer $\leq x$ for all x

Measurement

Circle : $C = 2\pi r = \pi D$, where C is the circumference, r is the radius and D is the diameter

$A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2} b h$, where b is the base and h is the perpendicular height

Parallelogram: $A = b h$

Trapezium : $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular height

Prism: $V = Ah$, where V is the volume, A is the area of the base and h is the perpendicular height

Pyramid: $V = \frac{1}{3} \mathrm{A} h$

Cylinder : $S = 2\pi r h + 2\pi r^2$, where S is the total surface area

$V = \pi r^2 h$

Cone : $S = \pi r s + \pi r^2$ where s is the slant height

$V = \frac{1}{3} \pi r^2 h$

Sphere : $S = 4\pi r^2$

$V = \frac{4}{3} \pi r^3$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.