Christ Church Grammar School

2020
TEST 2

MATHEMATICS METHODS Year 11

Section One:
Calculator-free

Your name Solutions

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

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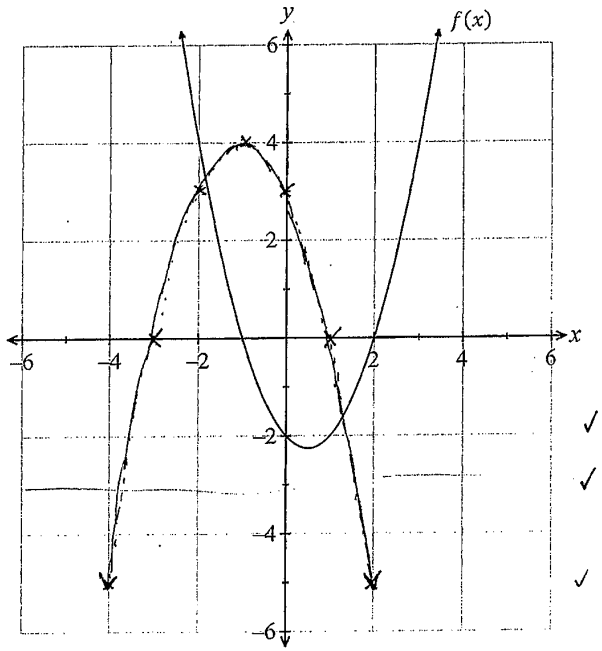
6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 9 3 (4 marks)

The graph of $f(x) = (x+1)(x-2)$ is shown on the set of axes below.



✓ x into
✓ correct shape and max TP.
✓ y intercept.

(a) Draw the graph of $g(x) = -(x+1)^2 + 4$ on the above set of axes. (3 marks)

Question 1 (4 marks)

For the graph of $y = 10x - 2x^2 + 4$ determine the

(a) coordinates of the y axis intercept. (1 mark)

let $x = 0$ $(0, 4)$ ✓ correct coordinate

(b) equation of the line of symmetry (2 marks)

L.O.S $x = -b/2a$

$= -10/2(-2)$

$x = 5/2$

✓ ✓

✓ states $x =$
✓ correct value

(c) location and nature of the turning point. (2 marks)

substitution:
let $x = 2.5$

$y = 10(2.5) - 2(2.5)^2 + 4$

$= 33/2$.

TP $(5/2, 33/2)$ ✓

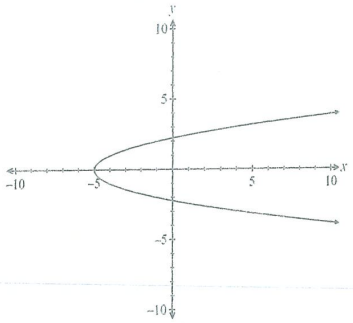
states co-ordinate

Nature maximum ✓

See next page

Question 2 **(4 marks)**

The graph below contains two functions, $f(x) = \sqrt{5+x}$ and $g(x)$.



(a) Write the equation of $g(x)$ and explain why combining $f(x)$ and $g(x)$ as shown in the graph does not represent one single function. **(2 marks)**

$g(x) = -\sqrt{x+5}$ ✓ Correct function.

- fails vertical line test ✓ valid reason
- x values have more than one y value ⇒ not a function

(b) Write the domain of $f(x)$. **(1 mark)**

$\{x : x \in \mathbb{R};\ x \geq -5\}$ ✓ states domain correctly.

accept $x \geq -5$

(c) Write the range of $g(x)$. **(1 mark)**

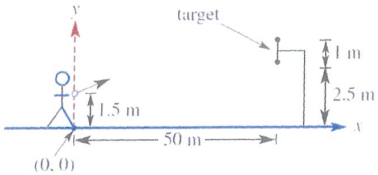
$\{y : y \in \mathbb{R};\ y \leq 0\}$ ✓ states range correctly.

Note if (b) $\{x : x \in \mathbb{R};\ x > -5\}$ then $\{y : y \in \mathbb{R};\ y \leq 0\}$.

See next page

Question 8 **(4 marks)**

An archer's target is located 50 m away from her feet which are standing at (0,0) on a Cartesian plane. A circular target, of diameter 1 m, is 2.5 m off the ground, as shown below. She fires arrows at the target from a height of 1.5 m.



Assume that the distance travelled by the arrow can be represented using a linear function.

(a) Determine the gradient of the arrow's path to the bottom of the target. **(1 mark)**

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{50}$ or 0.02 ✓ Determines value.

(b) Determine the equation of the line for the arrow's path to the centre of the target. **(2 marks)**

$c = 1.5 \quad m = 1.5/50 = \frac{3}{100}$

✓ Gradient

$y = 0.03x + 1.5$ ✓ y intercept.

(c) Write an expression for the possible values of the gradient of the arrow's path for it to successfully hit the target. **(1 mark)**

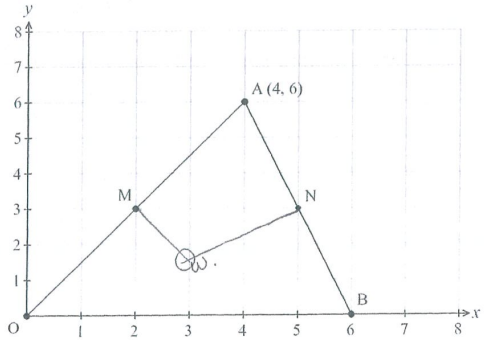
Archer needs to fire between

$0.02 \leq m \leq 0.04$ ✓ correct inequality

See next page

Question 7 **(5 marks)**

Consider the diagram below, where M is the midpoint of OA and N is the midpoint of AB.



(a) Determine the gradient of the perpendicular bisector of OA. (1 mark)

$m_{OA} = 3/2$

$\therefore \perp$ bisector $m_{MW} = -2/3$

✓ correct value for m

Let point W be the point where the perpendicular bisectors of OA and AB intersect.

(b) Determine the equation of the line MW. (2 marks)

$m_{MW} = -2/3$

midpoint $= (2,3)$

$3 = -2/3(2) + c$

$\therefore c = 4\,1/3$

$y = -2/3x + 4\,1/3$

✓ solution

✓ method (valid)

(c) Determine the coordinates of point W. (2 marks)

line $x = 3$ ⊥ bisects OA

$\Rightarrow W(3,)$

$y = -2/3(3) + 4\frac{1}{3}$

$y = -2 + 4\frac{1}{3}$

$= 2\frac{1}{3}$

$(3, 2\,1/3)$ with

✓ solution

✓ valid method

	Point	Gradient	Equation
i)	(0,0) (4,6)	$OA = 3/2$	$y = 3/2x$
ii)	(4,6)(6,0)	$AB = -3$	$y = -3x + 18$
iii)	(2,3)	$MW = -2/3$	$y = -2/3x + 4\frac{1}{3}$
iv)	(5,3)	$NW = 1/3$	$y = 1/3x + 1\frac{1}{3}$

Variety of methods that may include →

See next page

Question 3 **(3 marks)**

If $f(x) = x^3 + 4x^2 - 4x - 16 = (x-2)(ax^2 + bx + c)$, then determine the values of a, b and c.

Linear factors $(x-2)(x+2)(x+4)$ valid method

Hence $(x-2)(x^2 + 6x + 8)$ ✓ Demonstrates understanding of expansion

so $a = 1$

$b = 6$

$c = 8$

✓✓ solves for a, b, c

NOTE:

Some candidates may use cubic factorisation then factorise the quadratic.

$$(x-2)\overline{)\,x^3 + 4x^2 - 4x - 16}$$

$x^2 + 6x + 8$

$-(x^3 - 2x^2)$

$6x^2 - 4x$

$-(6x^2 - 12x)$

$8x - 16$

$-(8x - 16)$

0

$(x-2)(x+2)(x+4)$

See next page

Question 4 (4 marks)

The three points $A(p+11, 17)$, $B(-2, p)$ and $C(1, 7)$ are collinear. Determine the value(s) of p.

$m_{AB} = \dfrac{p-17}{-2-(p+11)}$

$= \dfrac{p-17}{-13-p}$

$m_{BC} = \dfrac{7-p}{1+2}$

$= \dfrac{7-p}{3}$ ✓ Finds gradient of two sets of points

$m_{AC} = \dfrac{10}{p+10}$

Collinear : when $m_{AB} = m_{BC}$

$\dfrac{p-17}{-13-p} = \dfrac{7-p}{3}$

$\therefore \quad 3(p-17) = (7-p)(-13-p)$ ✓ Equates gradients

$3p - 51 = -91 - 7p + 13p + p^2$

$0 = -40 + 3p + p^2$ ✓ Develops correct quadratic

$0 = (p+8)(p-5)$

$\therefore \quad p = -8 \text{ or } 5$ ✓ solves for both values. using Null Factor Law.

End of questions

Question 6 continued

(b) Determine the values of a, b, c and d. (4 marks)

$f(x) = a(x-3)(x+1)^2$

$6 = a(0-3)(0+1)^2$

$6 = a(-3)(1)$

Hence 'a' $= -2$.

✓ Generates equation with 'a' in linear factor form.

✓ solves for 'a'

$f(x) = -2(x-3)(x+1)^2$

$= -2x^3 + 2x^2 + 10x + 6.$

$\begin{cases} a = -2 \\ b = 2 \\ c = 10 \\ d = 6 \end{cases}$

✓✓ correct values for all three b, c, d.

(c) Comment on the behaviour of the function as $x \to \infty$. (1 mark)

as $x \to \infty$, $y \to -\infty$. ✓

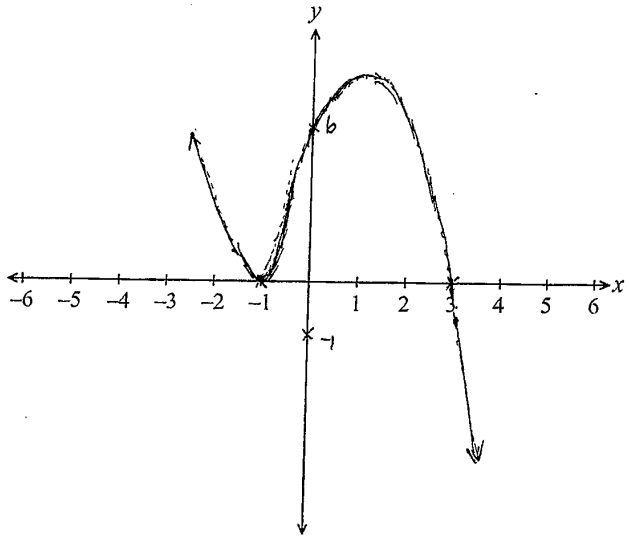
Correct range.

See next page

Question 6 **(8 marks)**

A function f is defined by $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants. The graph of f has intercepts located at $(3, 0)$, $(-1, 0)$ and $(0, 6)$ and a local minimum at $(-1, 0)$.

(a) Sketch the function $y = f(x)$ on the axes below. (3 marks)



✓ Locates x intercepts @ $(-1, 0)$ and $(3, 0)$

✓ Correct shape - inverted cubic
- repeated root @ $(-1, 0)$

✓ - y intercept at $(0, 6)$ but not T.P.

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Question 5 (4 marks)

(a) The coordinates of the midpoint of $(3k-1, 4-5k)$ and (x, y) are $(4k-1, 3.5-5k)$. Write an expression for x and y. (2 marks)

midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

[x] $4k-1 = \frac{3k-1+x}{2}$

$8k-2 = 3k-1+x$

$x = 5k-1$. Final solution.

[y] $3.5-5k = \frac{4-5k+y}{2}$

$7-10k = 4-5k+y$

$y = 3-5k$.

✓ Demonstrates awareness of midpoint and applies
✓ Isolates x and y correctly.

(b) Show that the midpoint given in part (a) lies on the line with the equation $5x+4y=9$ (2 marks)

let $x = (4k-1)$ $y = (3.5-5k)$

$5(4k-1) + 4(3.5-5k) = 9.$

$20k - 5 + 14 - 20k = 9.$

$9 = 9.$

Hence yes it does lie on the line.

✓ substitutes x and y co-ordinates correctly
✓ completes algebraic statement correctly ($9=9$.)

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