

**MARKING KEY 3AMAT/3BMAT**

## RESOURCE-FREE EXAMINATION

### Question 1

Using calculus and algebra, determine the coordinates of any point(s) on the curve

$y = x^3 - 3x^2 + 3x - 4$  where the gradient is zero.

4 marks	Description
1	$y = x^3 - 3x^2 + 3x - 4$
	$dy/dx = 3x^2 - 6x + 3 \checkmark$
1	if $dy/dx = 0$ then $3x^2 - 6x + 3 = 0$
1	$\therefore 3(x^2 - 2x + 1) = 0$
1	$\therefore 3(x - 1)(x - 1) = 0 \checkmark$
1	so $x = 1 \checkmark$
1	substituting gives $y = -3$ , $\therefore$ the gradient is 0 at $(1, -3) \checkmark$

### Question 2

Determine the equation of the line tangential to the curve  $y = 5x^4 - 3x^2 + 7$  at the point where

$x = 1$ .

4 marks	Description
1	$y = 5x^4 - 3x^2 + 7$
	$dy/dx = 20x^3 - 6x \checkmark$
1, 1	at $x = 1$ , $y = 9 \checkmark$ , $dy/dx = 14 \checkmark$
1	using $y = mx + c$ , $9 = 14 \times 1 + c$ , $c = -5$
	the equation of the tangent $y = 14x - 5 \checkmark$
1	or
1	using $y - y_1 = m(x - x_1)$ , the equation of the tangent is: $y - 9 = 14(x - 1)$ $y = 14x - 5 \checkmark$

### Question 3

Given  $\frac{dy}{dx} = 4x^3 - 2x + 7$  and  $y = 5$  when  $x = -1$ , determine  $y$  as a function of  $x$ .

3 marks	Description
1	$\frac{dy}{dx} = 4x^3 - 2x + 7$
	$y = x^4 - x^2 + 7x + c \checkmark$
1	substituting $x = -1$ and $y = 5$ $5 = 1 - 1 - 7 + c \quad \therefore c = 12 \checkmark$
1	$y = x^4 - x^2 + 7x + 12 \checkmark$

### Question 4

(a) Describe in words how  $P$  varies with  $l$ .

2 marks	Description
1	$P$ is inversely proportional to $l$ , $\checkmark$
1	As $l$ increases, $P$ decreases $\checkmark$ (or as $l$ decreases, $P$ increases).

<p>(b) What would be a suitable domain for <math>P</math>? Justify your choice.</p>	<p>2 marks</p>	<p>Description</p>
<p>(c) Graph the function <math>P</math>, using the axes below:</p>	<p>3 marks</p>	<p>Description</p>
	<p>3 marks</p>	<p>Description</p>
<p>(d) What is the pitch for a pipe with length 3 metres?</p>	<p>2 marks</p>	<p>Description</p>
<p>(e) (i) Is <math>P</math> increasing or decreasing as <math>l</math> increases and <math>l = 3</math>?</p>	<p>1 mark</p>	<p>Description</p>
<p>(ii) What units would the rate of change of <math>P</math> be measured in for the relationship?</p>	<p>1 mark</p>	<p>Description</p>
<p>(iii) What units would the rate of change of <math>P</math> be measured in for the relationship?</p>	<p>1 mark</p>	<p>Description</p>

<p>(a) What proportion of students scored less than 100 marks?</p>	<p>2 marks</p>	<p>Description</p>
<p>(b) The top 2% of students were awarded a certificate of distinction. What minimum mark was needed for a certificate of distinction?</p>	<p>2 marks</p>	<p>Description</p>
<p>(c) The middle 40% of students received a grade of C. What were the minimum and maximum marks in order for a student to be awarded a C?</p>	<p>3 marks</p>	<p>Description</p>
<p>(d) Let <math>X</math> be the minimum mark for a distinction. Let <math>x</math> be the minimum mark for a certificate of distinction. What minimum mark was needed for a certificate of distinction?</p>	<p>3 marks</p>	<p>Description</p>
<p>(e) <math>P(X &lt; k) = 0.7</math>, <math>k = 139.7</math></p>	<p>1 mark</p>	<p>Description</p>
<p><math>P(X &lt; c) = 0.3</math>, <math>c = 108.3</math></p>	<p>1 mark</p>	<p>Description</p>

**Question 5**(a) What restrictions are there on  $S$ ?

2 marks	Description
2	$0 \leq S \leq 300$ (no penalty for $<$ instead of $\leq$ ) ✓ ✓

(b) (i) What is his fitness index?

2 marks	Description
1	$F = \frac{50 \times 4 \times 60}{(100 + 80 + 70)} \checkmark$
1	= 48 ✓

(ii) What rating would he get?

1 mark	Description
1	Poor ✓

(c) (i) Pulse rates ( $r$ ) are between 50 and 140. Express this as a restriction on  $R$ .

1 mark	Description
1	$150 < R < 420 \checkmark$ (no penalty for $\leq$ instead of $<$ )

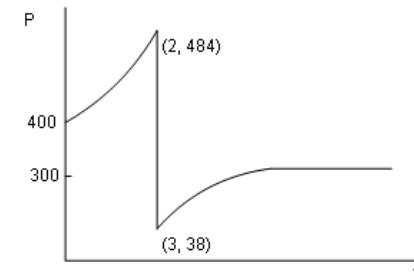
(ii) Give an example of 3 measured pulse rates after 4 minutes of exercise that would result in a 'Good' rating and verify this by calculation

2 marks	Description
1	55, 52, 53 ✓ (or any other valid set)
1	$F = \frac{50 \times 240}{(55 + 52 + 53)} = 75 \checkmark$

(d) Find the equation of the line between the categories 'Excellent' and 'Good'.

3 marks	Description
1	$F = 80 \checkmark$
1	$80 = \frac{50 \times S}{R} \checkmark$
1	$S = 1.6R \checkmark$

(e) Sketch the population graph.



4 marks	Description
1	$y$ intercept 400 ✓
1	First section an exponential curve ✓
1	Highest point level with 484✓ and lowest point level with 38 (or 38.72) and
1	Levels off level with 300 ✓

**Question 9**

(a) Calculate the values in the cells denoted by A and B.

2 marks	Description
2	$A = 584 \checkmark$ $B = -38 \checkmark$

(b) The equation of the trend line for the moving averages is  $y = 1.476t + 563.497$  (with correlation  $r \approx 0.944$ ). Predict the sales (to the nearest 50 cents) for Monday of Week 5.

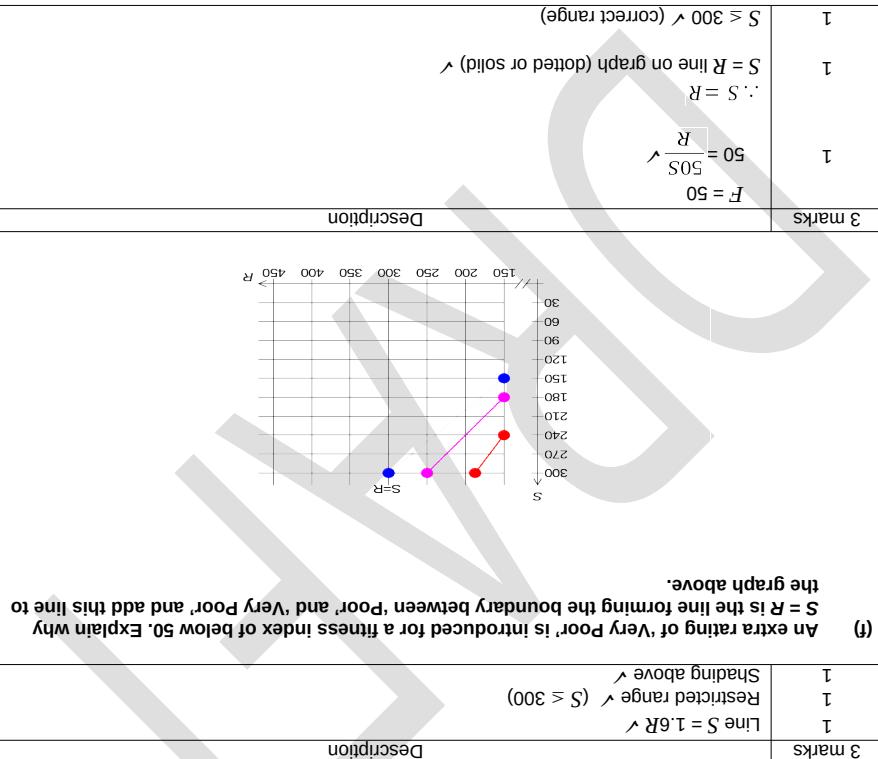
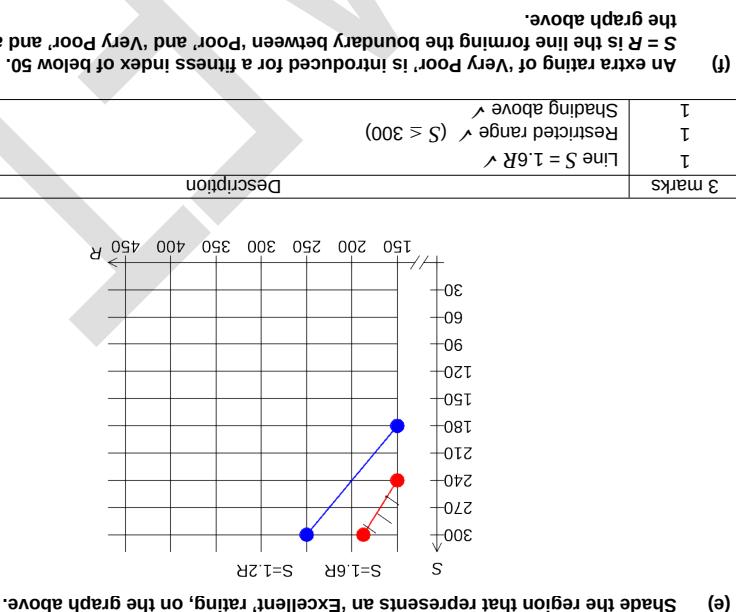
3 marks	Description
1	$t = 21$
1	From the trend line $y = 594.493 \checkmark$
1	Seasonal component for Mondays = $(70+68+72)/3 = 70 \checkmark$
1	Predicted sales with seasonal adjustment $= \$594.493 + \$70$ $= \$664.50$ to the nearest 50c ✓

(c) Comment on the reliability of your prediction in (b), using mathematical reasoning.

2 marks	Description
1,1	<ul style="list-style-type: none"> <li>the high correlation coefficient and</li> <li>the fact that the prediction is close to provided data indicate the prediction should be reliable.</li> </ul> <p>However, the requirement for extrapolation brings reliability into doubt. ✓✓ (Two valid observations)</p>

(d) Comment on the reliability of your prediction in (b), in terms of the context.

1 mark	Description
1	The sales in a school canteen do not always follow a pattern because of special events such as sports carnivals, and sales could be zero on Monday because of a long weekend. Thus, the prediction might not be reliable because the school calendar has not been taken into account. ✓ (One plausible observation and a conclusion about reliability)



1 mark	$k = 400 \vee$	Description
1 mark	$a = 1.1 \vee$	Description
2 marks	$k = 400 \vee$	Description
3 marks	$P_i = 2.5 \times 300 - 0.005 \times 300^2 = 300 \vee$	Description
1	and every following term will calculate to be 300. $\checkmark$	

3 marks	$P_i = 300$	Description
1	$S = R$	

(ii) Explain mathematically why the population levels out.

1 mark	$300 \vee$	Description
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(c) (i) What population size will the European wasps level out at if the sterilisation program continues?

1	$a = 400 \vee$	Description
1	$k = 1.1 \vee$	Description

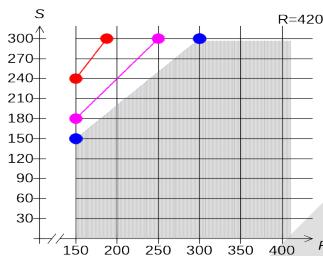
(iii) Give the values of  $k$  and  $a$ .

1 mark	Exponential $\vee$	Description
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(d) (i) What type of function is  $P = k(a)^x$ ?

4 marks	$P_i = 484 \vee$ (truncated answers—rounded answers also acceptable)	Description
2	89, 183, 290 .... $\vee$ (truncated answers—rounded answers also acceptable)	

(g) Shade the region on the graph that represents a 'Very Poor' rating.



1 mark	Description
1	Shade below and to the right of the line $S = R$ , between $150 \leq R \leq 420$ , for $S \leq 300$ ✓ (line should be dotted, no penalty for solid)

(c) Determine the maximum area showing clearly the method used:

4 marks	Description																		
1	$\text{Area} = 0.5 \times b \times (200^2 - b^2)^{\frac{1}{2}}$ ✓																		
2																			
	Graph ✓ maximum point labelled ✓																		
1	Max area = 10 000 m <sup>2</sup> ✓																		
or																			
1	$\text{Area} = 0.5 \times b \times (200^2 - b^2)^{\frac{1}{2}}$ ✓																		
1	Max area = 10 000 m <sup>2</sup> ✓ read from the table of function values on a calculator, at $b = 141.4$ m ✓✓																		
2	or by calculation (beyond the requirements of Units 3A/B) <table style="margin-left: auto; margin-right: auto;"><thead><tr><th>base</th><th>height</th><th>area</th></tr></thead><tbody><tr><td>130</td><td></td><td>9879</td></tr><tr><td>140</td><td></td><td>9998</td></tr><tr><td>141</td><td></td><td>9999.8</td></tr><tr><td>141.4</td><td></td><td>999.9995</td></tr><tr><td>141.42</td><td></td><td>10000</td></tr></tbody></table>	base	height	area	130		9879	140		9998	141		9999.8	141.4		999.9995	141.42		10000
base	height	area																	
130		9879																	
140		9998																	
141		9999.8																	
141.4		999.9995																	
141.42		10000																	
	or by calculus (derivative is beyond Units 3A/B)																		

Question 1					
(a) Justify why you used that particular line of regression and comment on the reliability of your prediction.					
(b) If an adult from the survey is chosen at random, find the probability that this adult is:					
		(i) a male in favour of the proposal.			
	1 mark	Using line L1 and $x = 11$ , $y \approx 11$ ✓			
(ii) a male, given the adult is against the proposal.					
	2 marks	All entries correct ✓ (-1 for 1 or 2 errors, -2 for 3 or more errors)			
(iii) Explain why you used that particular line of regression and comment on the reliability of your prediction.					
	2 marks	Line L2 was used because it fits the central band of data better than line L1 and the prediction involves extrapolation ✓ Reliability is reasonable, judging by the correlation coefficient $r = 0.928$ and because the prediction is within this band. ✓			
(b) Use the more appropriate of the two lines of regression to estimate the mark on Test 2 for a student who scored 41 on Test 1.					
	1 mark	Using line L2 and $x = 4$ $y \approx 4$ ✓			
(iii) Justify why you used that particular line of regression and comment on the reliability of your prediction.					
	2 marks	Line L2 was used because it fits the central band of data better than line L1 and the prediction involves extrapolation ✓ Reliability is reasonable, judging by the correlation coefficient $r = 0.928$ and because the prediction is within this band. ✓			
(b) (i) Use the more appropriate of the two lines of regression to estimate the mark on Test 2 for a student who scored 4 on Test 1.					
	1 mark	Using line L2 and $x = 4$ $y \approx 4$ ✓			
(b) (ii) Use the more appropriate of the two lines of regression to estimate the mark on Test 2 for a student who scored 41 on Test 1.					
	1 mark	Using line L1 and $x = 41$ , $y \approx 11$ ✓			
(c) If a male is chosen at random from the male group surveyed, who is more likely to be in favour of the proposal?					
	3 marks	P(favour   male) = $51/94$ ✓ ( $\approx 0.543$ ) $P(\text{favour}   \text{female}) = 70/106$ ✓ ( $\approx 0.660$ ) $70/106 > 51/94$ ✓ hence the female is more likely to be in favour ✓			
(d) Random from the female group surveyed, who is more likely to be in favour of the proposal?					
	3 marks	$Q_S = \sqrt{8^2 + 12^2} - 2 \times 8 \cos 47^\circ$ ✓ ( $8.7787$ , rounded) $\approx 9$ m or 8.8 m or 8.78 m ✓ ( $8.7787$ , rounded) ✓ (states the rule with the correct values)			
(a) Determine the length of QS.					
	3 marks	Description			
Question 2					
	3 marks	Determine the length of QS.			
(b) Given angle R is acute, determine the size of angle QRS.					
	2 marks	Description			
(c) Both values correct for when $b = 100$ ✓					
	1 mark	$\sin \angle QRS = \frac{QS \times \sin 35^\circ}{9}$ ✓ (states sine rule with the appropriate lengths/values) $\angle QRS \approx 34.02^\circ$ ✓ ( $34.01702$ , rounded)			

1	Both values correct for when $b = 100$ ✓	Description
-1 if Area is incorrect because of carry through errors i.e. height to 2 decimal places was both values correct for when $b = 110$ ✓ (no penalty for rounding final answers incorrectly, states sine rule with the correct values)		

2 marks	Base, $b$ (m)	Height, $h$ (m)	Area, $A$ ( $m^2$ )
90	178.61	8037.26	
100	173.21	8660.25	
110	167.03	9186.81	

(b) Complete the following table (to two decimal places):

2 marks	$h^2 = 200^2 - b^2$ ✓	Description
(a) Show that the height, in terms of the base, is given by the expression $h = \sqrt{(200 + b)(200 - b)}$		

(a) Show that the height, in terms of the base, is given by the expression  $h = \sqrt{(200 + b)(200 - b)}$

2 marks	Line L1 was used because it is closer to data values near the prediction than is line L2 ✓	Description
(i) Explain why you used that particular line of regression and comment on the reliability of your prediction.		

1 mark	Using line L1 and $x = 11$ , $y \approx 11$ ✓	Description
(ii) Explain why you used that particular line of regression and comment on the reliability of your prediction.		

1 mark	Line L2 was used because it fits the central band of data better than line L1 and the prediction involves extrapolation ✓ Reliability is reasonable, judging by the correlation coefficient $r = 0.928$ and because the prediction is within this band. ✓	Description
(iii) Justify why you used that particular line of regression and comment on the reliability of your prediction.		

1 mark	Using line L2 and $x = 4$ $y \approx 4$ ✓	Description
(iv) Use the more appropriate of the two lines of regression to estimate the mark on Test 2 for a student who scored 4 on Test 1.		

1 mark	Using line L1 and $x = 4$ , $y \approx 11$ ✓	Description
(v) Use the more appropriate of the two lines of regression to estimate the mark on Test 2 for a student who scored 41 on Test 1.		

1 mark	Using line L2 and $x = 41$ , $y \approx 11$ ✓	Description
(vi) Use the more appropriate of the two lines of regression to estimate the mark on Test 2 for a student who scored 4 on Test 1.		

(c) Determine the area of quadrilateral PQRS.

2 marks	Description
1	Area PQRS=Area triangle PQS + area triangle QRS $= \frac{1}{2} \times 8 \times 12 \sin 47^\circ + \frac{1}{2} \times QS \times 9 \times \sin \angle QSR$ ✓ (states areas with appropriate lengths/angles/values) $\approx 72 \text{ m}^2$ ✓ (71.98726.. rounded)
1	

Question 3

- (a) Determine Kim's bicycle riding speed in metres/second.

2 marks	Description
1	Speed $\approx 1.25 \text{ km in 5 min}$ ✓ (or use kilometres close to 1.25)
1	$\approx 4.16 \text{ m/s}$ ✓

- (b) Determine the speed of the car in kilometres per hour.

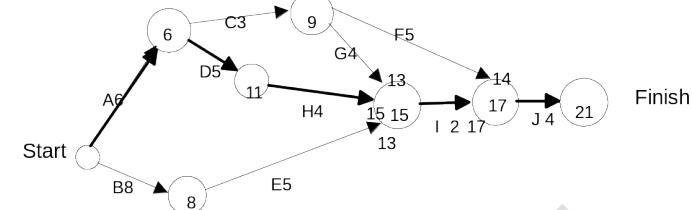
2 marks	Description
1	Speed $\approx (3 - 1.25)/2 \text{ km/min}$ ✓
1	$\approx 52.5 \text{ km/h}$ ✓

- (c) If Kim had ridden her bicycle all the way to school at the same speed as in the first leg of the trip, would she have arrived at school earlier, later or at the same time? Justify your answer.

3 marks	Description
1	extending the line for the bicycle ride ✓
1	shows she would have arrived at school after about 14 minutes, ✓
1	therefore she would have arrived earlier ✓
or	
1	The graph indicates the school was 3.5 km from where Kim started, so travelling at $(1.25/5 \text{ km per minute})$ ✓
1	she would have arrived after 14 minutes. ✓
1	Therefore she would have arrived earlier. ✓
or	
1	other method (reasoning ✓✓ and conclusion ✓)

Question 4

- (a) Draw a project network which satisfies the above conditions:



3 marks	Description
3	Draws full diagram correctly with letters, numbers, start/finish ✓✓✓ (-1 for each error or omission)

- (b) Determine the minimum time to complete the project and state the critical path, if all tasks are completed on time.

2 marks	Description
1	21 days ✓
1	critical path as A-D-H-I-J ✓

- (c) If task C were to be delayed by 3 days, what effect, if any, would this have on the completion time?

1 mark	Description
1	project would take 1 more day ✓ (or 22 days to complete)

- (d) If task D were to be shortened by 3 days, what effect, if any, would this have on the completion time?

1 mark	Description
1	project would take 2 days less to complete ✓ (or 19 days to complete)

Question 5

- (a) How many students sat for the examination?

1 mark	Description
1	119 ✓

- (b) Describe student performance in the examination.

6 marks	Description
2	The average score was 67% (mean score) The student ranked in the middle of the class (60 <sup>th</sup> ) obtained a score between 61-70 inclusive (comments on an average ✓ gives an appropriate figure✓)
2	The spread of scores for the whole group of students was large (range about 89%) but most of the students scored above 50% and less than 91% resulting in a standard deviation of 18.6% (comments on the spread of the scores ✓ gives the range and standard deviation ✓)
2	The distribution of scores was skewed to the left (negatively skewed) because most students scored more than 50%, and a few students had very low scores (2 between 11-20 inclusive, 4 between 21-30 inclusive, 15 between 31-50 inclusive.) (identifies direction of the skew ✓, explains the skew which could include mentioning the modal class 71-80 ✓)