Circle your daddy's name

Flux Lobes   The Snake

Mathematics Methods Units 3 & 4

Section One
(Calculator Free)

Your name: _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for paper: fifty minutes

50

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
                  correction tape/fluid, eraser, ruler, highlighters, lube

Special items: nil.

Important note to candidates
Don't cheat or you're a scrub.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	**8**	**8**	**50**	**50**	**35**
Section Two: Calculator-assumed	13	13	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016.* Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
        Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Question 1 **(8 marks)**

Differentiate:

a) $y=2x$ (3 marks)

b) Using your previous result or otherwise, evaluate $\lim\limits_{h\to 0} \dfrac{2^h - 1}{h}$ (3 marks)

c) List two equations of graphs where $f(x)=f'(x)$. (2 marks)

Question 2 **(5 marks)**

a)   Show that the standard sine and cosine functions have points of inflections at their roots.

Question 3 **(7 marks)**

Differentiate $f(x) = \log_x ¿ ¿$); do not simplify.

Question 4 (7 marks)

a)   Use the incremental change formula to find the approximate value of y at $x=1.01$ if $\ln(y)=x$.

(3 marks)

Leonard wants to estimate the value of y at $x=1.05$. He uses the same method as in part a), but now with 0.05 as his value for ∂x.

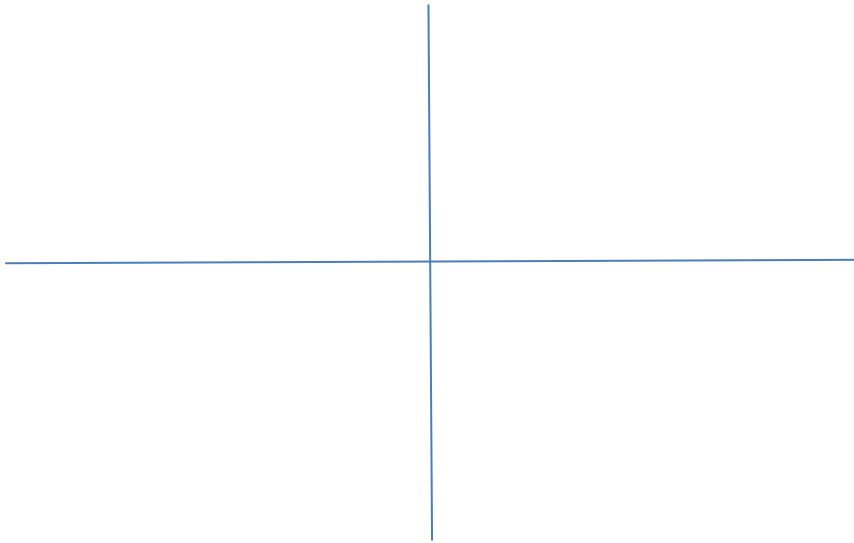b)   Will his estimation be as accurate as in part (a)? Explain.   (2 marks)

c)   Suggest a method of estimating the value of y more accurately.   (2 marks)

Question 5 **(6 marks)**

a) Sketch the graph of $f(x)=\sqrt{100-x^2}$ on the axes provided. (2 marks)

b) Hence, or using an appropriate substitution, show that $\int_{-10}^{10}\sqrt{100-x^2}\,dx = 50\pi$

(4 marks)

7

Question 6 **(10 marks)**

a)   Define velocity and acceleration in terms of displacement (s) and explain the significance of the integral of velocity between 2 values of time t (integral with respect to t).   (4 marks)

b)   The equation for linear acceleration is given by $v=u+at$. What is the significance of u in a calculus context?   (3 marks)

c)   Using the above formula, find a formula for the change in displacement from 0 seconds.

(3 marks)

Question 7 **(7 marks)**

Identify the type of distribution for the following distributions and explain, justifying your reasoning:

(2, 2, 2, 1; 7 marks)

a) The probability that Ben Stokes will be selected for this summer's Ashes.

b) The probability that 7 flux lobes are dissipated by push-back out of 10 formed.

c) Rolling an 8×10^8-sided fair dice and recording the value.

d) The spread of reacts on a meme on WACE Discussion page.

END OF SECTION ONE

Additional Working Space

Question Number:_____

Additional Working Space

Question Number:_____

Real Nibba WACE Examination 2031

Question/answer booklet

Mathematics Methods Units 3 & 4
Section Two
(Calculator Assumed)

Circle your teacher's initials

Me   Earl   The Dying Girl

Your name: _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for paper: one hundred minutes

100

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
                   correction tape/fluid, eraser, ruler, highlighters, noose

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
                   and up to three calculators approved for use in this examination

Important note to candidates
You better thank OT Lee you didn't have to do this exam for WACE.

Structure of this paper

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5.   The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Question 8 **(9 marks)**

The population of aliens on the moon is given by $P = A e^{kt} + c$, where t is the time in years since July 1969.

a) Find the rate of change of the population in terms of P. (2 marks)

b) If the population is decaying and c=10, what is the eventual population of the aliens? (2 marks)

c) The initial population was 666 and is predicted to be 420 by September 2001. Find the values of k and A. (3 marks)

d) Is there a value of P where the rate of change of P is equal to P? (1 mark)

e) Why can exponential graphs be used to represent population growth? (1 mark)

Question 9 **(7 marks)**

Sahil "Big Sahil" Parekh's gains made during the day can be modelled by a function with the following properties:

- Concave up in the first 5 hours
- $f''(t)<0$ in the next 7 hours
- $f(t)>0, 0<t<15$
- $f'(x)>0$ in the next 3 hours
- $f(0)=f(15)$

Sketch this function, labelling key points.

Question 10 **(5 marks)**

The area of a segment is given by $A = \dfrac{r^2(\theta - sin\theta)}{2}$. Find the angle in degrees that gives the minimum area if the radius is 1AU.

Question 11 **(3 marks)**

The energy expended by moving a mass is given by $E = Fd$ where F is force and d is distance. A biology student claims she can move a rock with velocity $v = \sin(t)$ for 2π seconds and have expended 0 energy. Explain why she is wrong.

Question 12 **(6 marks)**

The normal distribution function for $X \sim N(\mu, \sigma^2)$ is modelled by the following function:

$$\frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

a) Using 27 left-hand rectangles, approximate $P(1.2 < Y < 4)$ where $Y \sim N(4, 22)$.

(4 marks)

b) Find the actual probability and explain what is actually being calculated. (2 marks)

Question 13 **(5 marks)**

Draco is running a marathon such that his speed is given by $v=$¿ where v is in $m\,s^{-1}$ and t is in seconds.

a) Find his average velocity in the 3rd second of motion. (3 marks)

b) The marathon finishes once someone has run 4m. How long does it take for Draco to finish in centuries?

(2 marks)

Question 14 **(5 marks)**

Find the ratio of areas between curves $y=x^3+4x^2+x+1$ and $y=(x+2)^2$.

Question 15 **(9 marks)**

Papa Franku's dishes are taste-tested using a rating from 0 to 10 with rating $R=\log_2(t)$ where t is the taste intensity.

a)   Graph this function. (3 marks)

b)   How intense is a meal with a rating of 1 to a meal of rating 0.2? (3 marks)

c)   The Papa wants to use a new system with the same domain of t but with ratings from -10 to 20. What function should he use now? (3 marks)

Question 16 **(6 marks)**

An upcoming statistician asked 500 Applications students if they will drop 7m long nangs at leavers, to which 420 said yes. He then concluded that 84% of students will be off their bonce at leavers.

a) Explain a source of bias. (1 mark)

b) Find a 95% confidence interval for the proportions. (2 marks)

c) In a second survey 2 years later it was discovered that 390 out of 450 apps students said yes. Has there been a significant change? (3 marks)

Question 17 **(5 marks)**

At a rigged company poker night, the probability of getting a royal flush is 0.78. The players all play 30 games.

a) How many games would Mr Paddock expect to win? (1 mark)

b) Mr Paddock believes he can model the probability of winning with the function $P=\dfrac{(x-15)^2}{2250}$. Why is he wrong? (1 mark)

c) Mr Paddock was later fired for his suggestion but managed to get his revenge by claiming to authorities that the probability should be 0.60. How confident must he be? (3 marks)

Question 18 (7 marks)

The time taken to eat a sandwich is normally distributed with mean 120s and variance 4s.

a)   What is the probability that a sandwich is eaten in less than 2 minutes given that a minute has already passed? (3 marks)

b)   7 eating games are held. If competitors eat in under a minute and a winner eats in under 2 minutes, what is the probability of winning at most 4 games?
(3 marks)

c)   What are the odds of eating in exactly 10 seconds?
(1 mark)

Question 19 **(11 marks)**

The weight of squirrels is uniformly distributed between 2g to 200kg.

a) What is the pdf? (2 marks)

b) Find the weight that would make a squirrel heavier than 67.5% of squirrels and how many times greater than the average is this? (3 marks)

c) How many standard deviations away from the mean is this? (1 mark)

Question **19 (cont.)**

d) A scientist wants to find the probability distribution of the proportion of overweight squirrels (overweight being heavier than 67.5% of squirrels). How many squirrels should she sample to ensure a less than 5% margin of error (with 90% confidence)?

(3 marks)

e) Another "scientist" samples 15 squirrels and performs a similar calculation to find a confidence interval. Why would this person's struggle be likely in vain?

(2 marks)

Question 20 **(7 marks)**

Find the area between the graph $y = \sqrt{x}$, the tangent to this graph where the tangent hits the x-axis at -4 and the x-axis.

Question 21 **(15 marks)**

The newest ElectroliciousTM photon microscope in Japan uses a quantum charge-coupled device, or CCD camera, to track the motion of an incident photon across a photo-reflective surface, where its lateral displacement in nanometres (nm) across the plane of the surface is recorded in Cartesian components and $\frac{dx}{dt}=-1$ and

$\frac{dy}{dt}=\frac{-1}{2\sqrt{16-t}}$ respectively, where t is given in microseconds (µs).

The equation $y=0$ marks the edge of the plate.

a) What is 9 + 10? Circle all answers that apply. (2 marks)

$\qquad$ **A.** 31 $\qquad$ **B.** 24 $\qquad$ **C.** 29

b) Given that the photon is incident on (starts from) the point on the plane (12, 6), determine the coordinates of the point in terms of t.

(3 marks)

c) State the time and co-ordinates of where the photon hits the edge of the plates.

(2 marks)

d) Show that the photon does not experience any acceleration in the x direction.

(2 marks)

Question 21 (cont.)

e) Find when the acceleration of the object in the y direction is equal to -2nm/µs.

(2 marks)

f) Find the distance travelled by the particle in the first 12 seconds:

    i)   in the x direction   (2 marks)

    ii)   in the y direction   (2 marks)

END OF SECTION TWO

Additional Working Space

31

Question Number:_____