ALL SAINTS' COLLEGE

WA Exams Practice Paper E, 2016

Question/Answer Booklet

MATHEMATICS METHODS UNIT 3

Section One: Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Additional working space

Question number: _____

Additional working space

Question number: _____

Section One: Calculator-free **35% (52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(6 marks)**

The random variable X has two outcomes and can be modelled by a Bernoulli distribution with parameter $p = \frac{1}{3}$.

(a) Calculate $E(X)$ and $Var(X)$. (2 marks)

$$E(X) = p = \frac{1}{3}$$

$$Var(X) = p(1-p) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

(b) When n independent Bernoulli trials are carried out with $p = \frac{1}{3}$, the number of successes, Y, can be modelled by another distribution.

(i) State the distribution of Y. (1 mark)

$$Y \sim B(n, \tfrac{1}{3})$$

(ii) Given that the standard deviation of Y is 4, determine $E(Y)$. (3 marks)

$$\sqrt{np(1-p)} = 4$$

$$n \times \frac{1}{3} \times \frac{2}{3} = 16$$

$$n = 72$$

$$E(X) = np = \frac{1}{3} \times 72 = 24$$

Alternative

$$\sqrt{np(1-p)} = 4$$

$$np \times \frac{2}{3} = 16$$

$$E(X) = np = 16 \times \frac{3}{2} = 24$$

See next page

Question 2 **(9 marks)**

Differentiate the following with respect to x.

(a) $\int_0^x \left(\frac{2}{3\sqrt{t}}\right) dt$. (1 mark)

$$\frac{2}{3\sqrt{x}}$$

(b) $y = e^{\cos(5x-2)}$. (2 marks)

$$\frac{dy}{dx} = -5\sin(5x-2)e^{\cos(5x-2)}$$

(c) $y = e^{3x}\sqrt{x}$. (3 marks)

$$\frac{dy}{dx} = \left(3e^{3x}\right)\left(\sqrt{x}\right) + \left(e^{3x}\right)\left(\frac{1}{2\sqrt{x}}\right)$$

$$= 3e^{3x}\sqrt{x} + \frac{e^{3x}}{2\sqrt{x}}$$

(d) $y = \frac{\cos(2x)}{\sin(3x)}$. (3 marks)

$$\frac{dy}{dx} = \frac{\left(-2\sin(2x)\right)\left(\sin(3x)\right) - \left(\cos(2x)\right)\left(3\cos(3x)\right)}{\sin^2(3x)}$$

$$= \frac{-2\sin(2x)\sin(3x) - 3\cos(2x)\cos(3x)}{\sin^2(3x)}$$

See next page

Additional working space

Question number: _____

Additional working space

Question number: _____

Question 3 **(6 marks)**

The first derivative of the function f is $f'(x) = 3x^2 - 2ax - 8x + 8a$, where a is a constant.

Determine $f(x)$, given that f has a point of inflection at (3, 5).

$f''(x) = 6x - 2a - 8$

$f''(3) = 0$

$6(3) - 2a - 8 = 0$

$a = 5$

$f'(x) = 3x^2 - 18x + 40$

$f(x) = x^3 - 9x^2 + 40x + c$

$f(3) = 5$

$27 - 81 + 120 + c = 5 \Rightarrow c = -61$

$f(x) = x^3 - 9x^2 + 40x - 61$

See next page

Question 4 **(10 marks)**

(a) Evaluate $\int_0^1 \left(\frac{e^{1-2x}}{3}\right) dx$. (3 marks)

$$\int_0^1 \left(\frac{e^{1-2x}}{3}\right) dx = \left[\frac{e^{1-2x}}{-6}\right]_0^1$$
$$= \frac{e^{-1}}{-6} - \frac{e}{-6}$$
$$= \frac{e}{6} - \frac{1}{6e}$$

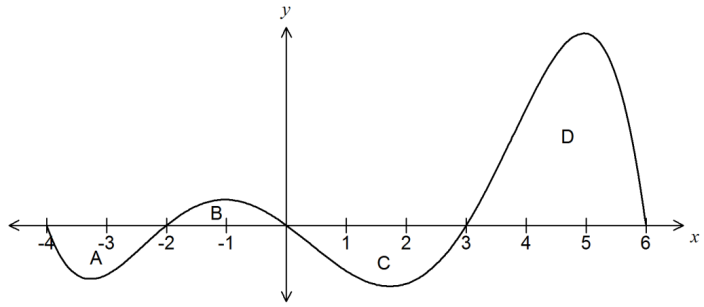
(b) Determine $\int \left(\frac{1}{x^2} - \sin\left(\frac{x}{3}\right)\right) dx$. (3 marks)

$$\int \left(\frac{1}{x^2} - \sin\left(\frac{x}{3}\right)\right) dx = \int \left(x^{-2} - \sin\left(\frac{x}{3}\right)\right) dx$$
$$= \frac{x^{-1}}{-1} + 3\cos\left(\frac{x}{3}\right) + c$$
$$= 3\cos\left(\frac{x}{3}\right) - \frac{1}{x} + c$$

See next page

Question 7 **(7 marks)**

The graph of the function $y = f(x)$ is shown below for $-4 \le x \le 6$.



The area of each region enclosed by the curve and the x-axis is shown in the following table.

Region	A	B	C	D
Area of region	5	3	11	25

(a) Determine the area enclosed between the graph of $y = f(x)$ and the x-axis, from $x = -4$ to $x = 6$. (1 mark)

$$5 + 3 + 11 + 25 = 44$$

(b) Determine the value of

(i) $\int_{-2}^{6} f(x)\, dx$. (2 marks)

$$\int_{-2}^{6} f(x)\, dx = 3 - 11 + 25$$
$$= 28 - 11 = 17$$

(ii) $\int_{-2}^{3} 3f(x)\, dx$. (2 marks)

$$\int_{-2}^{3} 3f(x)\, dx = 3\int_{-2}^{3} f(x)\, dx$$
$$= 3(3 - 11) = -24$$

(iii) $\int_{0}^{6} 4 - f(x)\, dx$. (2 marks)

$$\int_{0}^{6} 4 - f(x)\, dx = \int_{0}^{6} 4\, dx - \int_{0}^{6} f(x)\, dx$$
$$= 24 - \big((-11) + 25\big) = 10$$

End of questions

Question 6 **(5 marks)**

Given that $y = \sqrt{x}$, use $x = 100$ and the increments formula $\delta y = \frac{dy}{dx} \times \delta x$ to determine an approximate value for $\sqrt{104}$.

$x = 100,\ \delta x = 4$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\delta y = \frac{1}{2\sqrt{x}} \times \delta x$

$= \frac{1}{2\sqrt{100}} \times 4 = \frac{1}{5} = 0.2$

$\sqrt{104} = \sqrt{100} + 0.2$

$= 10.2$

(c) Evaluate $\int_0^5 \sqrt{3x+1}\ dx - \int_1^5 \sqrt{3x+1}\ dx$. (4 marks)

$\int_0^5 \sqrt{3x+1}\ dx - \int_1^5 \sqrt{3x+1}\ dx = \int_0^1 \sqrt{3x+1}\ dx$

$= \int_0^1 (3x+1)^{0.5}\ dx$

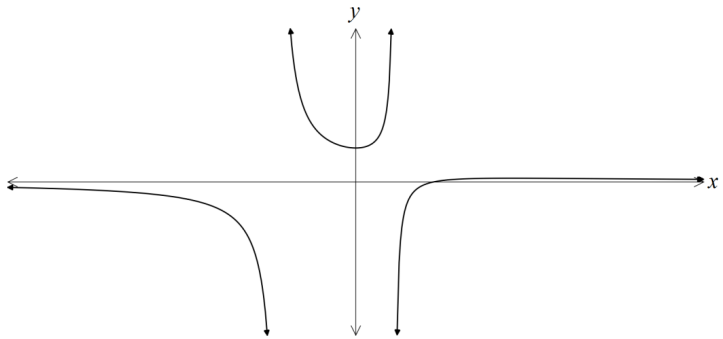
$= \left[\frac{(3x+1)^{1.5}}{1.5 \times 3} \right]_0^1$

$= \frac{8}{4.5} - \frac{1}{4.5}$

$= \frac{14}{9}$

Question 5 **(9 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = \dfrac{x-2}{x^2+x-2}$.



(a) Show that $f'(x) = \dfrac{x(4-x)}{\left(x^2+x-2\right)^2}$. (4 marks)

$$f'(x) = \frac{(1)\left(x^2+x-2\right)-(x-2)(2x+1)}{\left(x^2+x-2\right)^2}$$

$$= \frac{x^2+x-2-2x^2+3x+2}{\left(x^2+x-2\right)^2}$$

$$= \frac{4x-x^2}{\left(x^2+x-2\right)^2} = \frac{x(4-x)}{\left(x^2+x-2\right)^2}$$

(b) The graph has a stationary point at (0, 1). Determine the coordinates of the other stationary point. (3 marks)

$x(4-x) = 0 \Rightarrow x = \cancel{0},\ 4$

$$f(4) = \frac{4-2}{4^2+4-2} = \frac{2}{18} = \frac{1}{9}$$

$\left(4,\ \dfrac{1}{9}\right)$

(c) Given that $f''(x) = \dfrac{2(x^3-6x^2-4)}{\left(x^2+x-2\right)^3}$, use the second derivative test to determine the nature of stationary point found in (b). (2 marks)

$$f''(4) = \frac{2(64-6\times16-4)}{16+4-2}$$

$$= \frac{-ve}{+ve} = -ve$$

Since second derivative is negative, then curve is concave down and so point is a maximum.