Additional working space

Question number: _____

Christ Church Grammar School

2020
TEST 5

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

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See next page

Question 9 **(2 marks)**

The 95% confidence interval for the proportion of Rotto ferry tickets that are cancelled on the intended departure day from Rous Head is calculated from a large sample to be (0.039, 0.121). Determine the sample proportion from which the interval was constructed.

Irrespective of the confidence level, the confidence interval is symmetric about the sample proportion, so:

$$\hat{p} = \frac{0.039 + 0.121}{2}$$ ✓

$$\hat{p} = 0.080$$ ✓

2

End of questions

Question 8 **(8 marks)**

Stratified sampling was used to find the proportion of teenagers that like mountain biking. There were 40 teenagers in the sample and the proportion ($\hat{p}$) that liked mountain biking was 0.68.

Given the standard error of the sample proportion ($se(\hat{p})$) is given by the equation

$se(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{40}}$,

(a) calculate the standard error of the sample proportion, correct to three decimal places. (2 marks)

$se(\hat{p}) = \sqrt{\frac{0.68 \times 0.32}{40}}$ ✓

$= 0.074$ ✓

(b) what is the 95% confidence interval within which one would expect the proportion, p, of teenagers that like mountain biking to lie. (2 marks)

95% ⇒ 1.96

$0.68 \pm 1.96(0.074) \Rightarrow 0.535 \leq p \leq 0.825$ ✓ ✓

(c) how many teenagers should be in the sample in order for the standard error for the sample proportion ($se(\hat{p})$) to be less than 0.05? (2 marks)

$0.05 = \sqrt{\frac{0.68 \times 0.32}{n}}$

$n = 87.04$ ✓

$\therefore n = 88$ teens ✓ (must round up)

(d) Show that if the actual proportion of teenagers who like mountain biking is 0.75, then for a sample of 40 teenagers, the standard deviation of the sample proportion ($sd(\hat{p})$) is less than the standard error of the sample proportion ($se(\hat{p})$). (2 marks)

$sd(\hat{p}) = \sqrt{\frac{0.75 \times 0.25}{40}}$

$= 0.068$ ✓

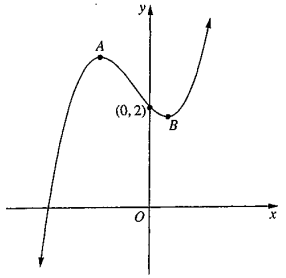
$se(\hat{p}) = 0.074$ (from (a))

$\therefore sd(\hat{p}) < se(\hat{p})$ as required ✓ (statement)

8

Question 1 **(6 marks)**

The graph of $y = x^3 + x^2 - x + 2$ is sketched below. Points A and B are the turning points.



(a) Determine the coordinates of A and B. (3 marks)

(b) For what values of x is the curve concave up? Give reason for your answer. (2 marks)

(c) For what values of k has the equation $x^3 + x^2 - x + 2 = k$ three real solutions? (1 mark)

Question 2 **(5 marks)**

If $f(x) = (1 - x^2)^{\frac{3}{2}}$,

(a) determine $f''(x)$. Do not simplify your answer. (3 marks)

(b) determine the domain of $f''(x)$. (2 marks)

Question 7 **(4 marks)**

Some of the world's oldest paintings can be found in the caves near the town of Lascaux in France. To determine the age of these paintings use is made of the Carbon-14 method. Over a long period of time, the radioactive Carbon-14 atoms (in the wood in the cave, and paint flakes on the ground close to the paintings) decay. It is estimated that the half-life of these atoms is 5568 years.
The level of radioactivity in the atoms is modelled by the equation $\frac{dR}{dt} = -kR$, where $k > 0$.

(a) Determine the value of k. (Give your answer to 6 decimal places) (2 marks)

$R = R_0 e^{-kt}$ half life $\frac{R}{R_0} = 0.5$ ✓

$\therefore 0.5 = e^{-k(5568)}$

$\therefore k = 0.000124$ (6 dp) ✓

(b) The level of radioactivity of the Carbon-14 atoms was found to have decreased by 97%. Determine the approximate age of the paintings. (2 marks)

$R = 0.03 R_0$

$\therefore 0.03 = e^{-0.000124(t)}$ ✓

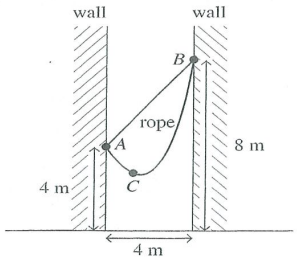
Paintings ≈ 28279 yrs (if 6 dp used) ✓

or ≈ 28168 yrs (if all dp used)

4

Question 5 continued

A straight rod joining the points **A** and **B** is now placed against the walls, as shown below.



(d) How far from the left wall is the separation between the rod and the rope greatest? (2 marks)

Line equ $r(x) = x+4$

dist apart $d(x) = r(x) - h(x)$ (separation equ)

$d(x) = -x^2 + 4x$ ✓

Max $d'(x) = 0 \Rightarrow -x^2+4x = 0$

$\therefore x = 2$ m ✓ (dist from left wall)

Greatest separation when 2m from left wall

Question 6 **(4 marks)**

Given $\int_2^5 f(x)dx = 15$, evaluate:

(a) $\int_5^2 3f(x)dx$ (1 mark)

$= -3\int_2^5 f(x)\,dx = -45$ ✓

(b) $\int_2^4 [f(x)+3]\,dx + \int_4^5 f(x)dx$ (3 marks)

$= \int_2^4 f(x)\,dx + \int_2^4 3\,dx + \int_4^5 f(x)\,dx$ ✓ (separates Integrals)

$= \int_2^5 f(x)\,dx + \int_2^4 3\,dx$ ✓ (Group f(x))

$= 15 + 6$

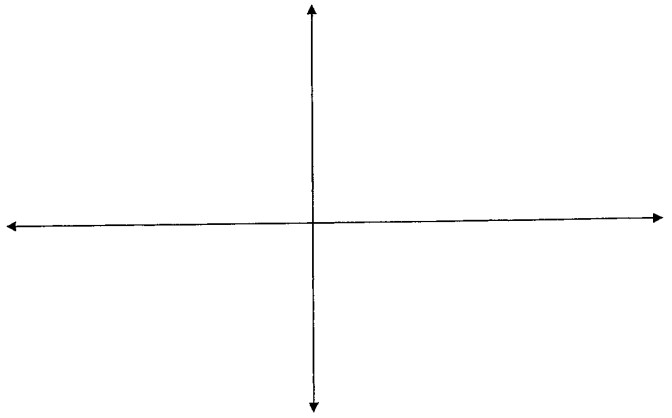
$= 21$ ✓ (Answ)

6

See next page

Question 3 **(4 marks)**

(a) Sketch the graphs of $y = e^{-x}$ *and* $y = e^x$ on the one set of axes and clearly indicate, by shading the region, the area given by $\int_0^2 e^{-x}\,dx + \int_{-2}^0 e^x\,dx$. (2 marks)



(b) Evaluate $\int_0^2 e^{-x}\,dx + \int_{-2}^0 e^x\,dx$. (2 marks)

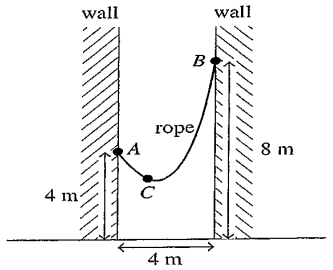
End of questions

Additional working space

Question number: _____

Question 5 **(5 marks)**

A rope is hanging freely between two walls 4 metres apart. One end of the rope is attached to a point 8 metres above ground level, whilst the other end is attached to the left wall 4 metres above ground level.



The height of the rope x m from the left wall is modelled by the equation $h(x) = ax^2 + bx + c$, for $0 \leq x \leq 4$. The rope hangs in such a way that it passes through the point C, a distance of one metre from the left wall, and 2 metres above the ground.

(a) State the coordinates of points **A**, **B** and **C** (1 mark)

A (0,4)
B (4,8)
C (1,2)

✓ (must have all correct)

(b) State the value of c. (1 mark)

$c = 4$ ✓

It is known that $b + 4a = 1$ and $b + a = -2$. (1 mark)

(c) Determine $h(x)$, for $0 \leq x \leq 4$.

$a = 1$, $b = -3$ => Classpad

$\therefore h(x) = x^2 - 3x + 4$ ✓

See next page

3

Question 4 **(7 marks)**

Twelve percent of the population is left-handed. What is the probability that in a randomly selected group of four people

(a) all are left-handed? (2 marks)

$X \sim Bin(4, 0.12)$ ✓

$P(x=4) = 0.000207$ ✓

(b) there are exactly three right-handed people? (1 mark)

$P(x=1) = 0.3271$ ✓

(c) there are more left-handed people than right-handed people? (2 marks)

$P(L > R) = P(x \geq 3)$ ✓

$= 0.0062899$ ✓

(d) they are all left-handed, given that there are more left-handed people than right-handed people in the group? (2 marks)

$P(x=4 / x \geq 3)$ ✓ $= \frac{0.0002}{0.0063}$

$= 0.0330$ ✓

See next page

7

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Christ Church Grammar School

2020
TEST 5

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name • Solutions •

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section
To be provided by the supervisor
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Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

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2020
TEST 5

MATHEMATICS METHODS Year 12
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Calculator-assumed

Your name _____

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Question 3 **(4 marks)**

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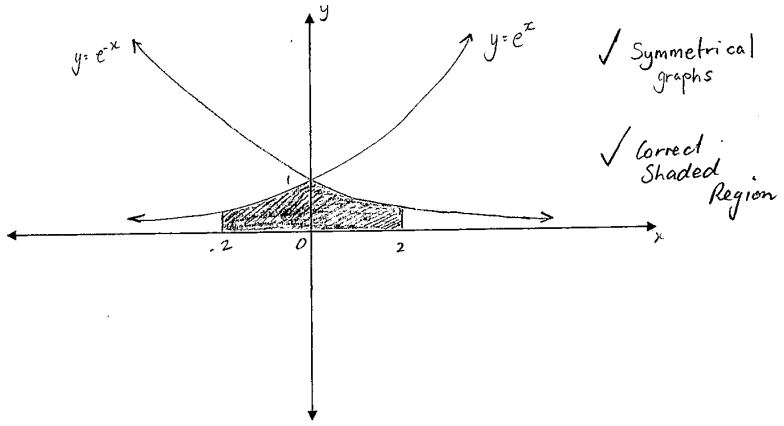


✓ Symmetrical graphs

✓ Correct Shaded Region

(b) Evaluate $\int_0^2 e^{-x}\,dx + \int_{-2}^0 e^{x}\,dx$. (2 marks)

$= \left[-e^{-x}\right]_0^2 + \left[e^{x}\right]_{-2}^0$ ✓ (Anti-differentiates correctly)

$= -e^{-2} - (-1) + 1 - e^{-2}$

$= -2e^{-2} + 2 \Rightarrow 2 - \frac{2}{e^2}$

$\Rightarrow 2\left(1 - \frac{1}{e^2}\right)$

Either ✓

4

Question 2 **(5 marks)**

If $f(x) = (1-x^2)^{\frac{3}{2}}$,

(a) determine $f''(x)$. Do not simplify your answer. (3 marks)

$f'(x) = \frac{3}{2}(1-x^2)^{1/2}(-2x)$ ✓ (Chain Rule)

Product Rule

$f'(x) = -3x(1-x^2)^{1/2}$ $u'v+uv'$

$f''(x) = -3(1-x^2)^{1/2} - 3x(\frac{1}{2})(1-x^2)^{-1/2}(-2x)$

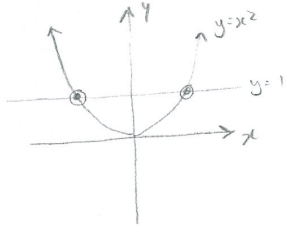
$f''(x) = -3(1-x^2)^{1/2} + 3x^2(1-x^2)^{-1/2}$ ✓✓ (product Rule)

(b) determine the domain of $f''(x)$. (2 marks)

for $f''(x)$ $1-x^2 > 0$ ✓

$1 > x^2$

$-1 < x < 1$ ✓ (inequality)



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Question 4 **(7 marks)**

12% of the population is left-handed. Determine the probability that in a randomly selected group of four people

(a) all are left-handed. (2 marks)

(b) there are exactly three right-handed people. (1 mark)

(c) there are more left-handed people than right-handed people. (2 marks)

(d) they are all left-handed, given that there are more left-handed people than right handed people in the group. (2 marks)

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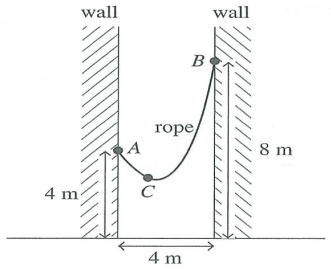

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(a) State the coordinates of points A, B and C. (1 mark)

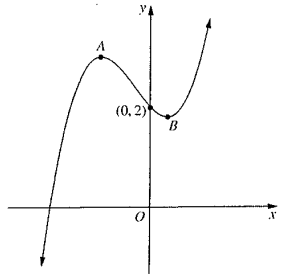
(b) State the value of c, as modelled by the equation $h(x) = ax^2 + bx + c$. (1 mark)

It is known that $b + 4a = 1$ and $b + a = -2$. (1 mark)

(c) Determine $h(x)$, for $0 \leq x \leq 4$.

Question 1 **(6 marks)**

The graph of $y = x^3 + x^2 - x + 2$ is sketched below. The points A and B are the turning points.



(a) Find the coordinates of A and B. (3 marks)

$\frac{dy}{dx} = 3x^2 + 2x - 1$ ⇒ Since A, B are stationary pts

$3x^2 + 2x - 1 = 0$

$(3x - 1)(x + 1) = 0$

$x = \frac{1}{3}$, $x = -1$ ✓

If $x = \frac{1}{3}$

$y = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right) + \frac{6}{3}$

$y = \frac{1}{27} + \frac{3}{27} - \frac{9}{27} + \frac{54}{27}$

∴ pt $\left(\frac{1}{3}, \frac{49}{27}\right)$ ✓

If $x = -1$

$y = (-1)^3 + (-1)^2 - (-1) + 2$

$y = -1 + 1 + 1 + 2$

$y = 3$ pt $(-1, 3)$ ✓

(b) For what values of x is the curve concave up? Give reason for your answer. (2 marks)

When $\frac{d^2y}{dx^2} > 0$

ie $6x + 2 > 0$ ✓ (2nd der)

$6x > -2$

$x > -\frac{1}{3}$ ✓ (inequality)

(c) For what values of k has the equation $x^3 + x^2 - x + 2 = k$ three real solutions? (1 mark)

$\frac{49}{27} < k < 3$ ✓

6

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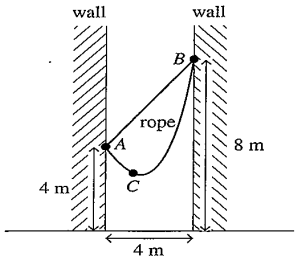
A straight rod joining the points A and B is now placed against the walls, as shown below.



(d) How far from the left wall is the separation between the rod and the rope greatest? (2 marks)

Question 6 **(4 marks)**

Given $\int_2^5 f(x)dx = 15$, evaluate:

(a) $\int_5^2 3f(x)dx$ (1 mark)

(b) $\int_2^4 [f(x)+3]\,dx + \int_4^5 f(x)dx$ (3 marks)

See next page

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Some of the world's oldest paintings can be found in the caves near the town of Lascaux in France. To determine the age of these paintings, use is made of the Carbon-14 method. Over a long period of time, the radioactive Carbon-14 atoms (in the wood in the cave, and paint flakes on the ground close to the paintings) decay. It is estimated that the half-life of these atoms is 5568 years.
The level of radioactivity in the atoms is modelled by the equation $\frac{dR}{dt} = -kR$, where $k > 0$.

(a) Determine the value of k. (Give your answer to 6 decimal places) (2 marks)

(b) The level of radioactivity of the Carbon-14 atoms was found to have decreased by 97%. Determine the approximate age of the paintings.

(2 marks)

See next page

Christ Church Grammar School

2020
TEST 5

MATHEMATICS METHODS Year 12

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Calculator-free

Your name • SOLUTIONS •

Teacher's name _____

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Stratified sampling was used to find the proportion of teenagers that like mountain biking. There were 40 teenagers in the sample and the proportion ($\hat{p}$) that liked mountain biking was 0.68.

Given the standard error of the sample proportion ($se(\hat{p})$) is given by the equation $se(\hat{p})=\sqrt{\frac{\hat{p}(1-\hat{p})}{40}}$,

(a) calculate the standard error of the sample proportion, correct to three decimal places. (2 marks)

(b) determine the 95% confidence interval within which one would expect the proportion, p, of teenagers that like mountain biking to lie. (2 marks)

(c) determine the number of teenagers that should be in the sample in order for the standard error for the sample proportion to be less than 0.05. (2 marks)

(d) Show that if the actual proportion of teenagers who like mountain biking is 0.75, then for a sample of 40 teenagers, the standard deviation of the sample proportion ($sd(\hat{p})$) is less than the standard error of the sample proportion ($se(\hat{p})$). (2 marks)

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The 95% confidence interval for the proportion of Rotto ferry tickets that are cancelled on the intended departure day from Rous Head is calculated from a large sample to be (0.039, 0.121). Determine the sample proportion from which the interval was constructed.

End of questions

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