PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2018

Question/Answer booklet

Yr 12 SPECIALIST
UNIT 3
Section Two:
Calculator-assumed

Your Name____

Your Teacher's Name________________________________

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time:	one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items:	pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorized material. If you have any unauthorized material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	21	21	100	95	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **(95 Marks)**

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(4 marks)**

Using vectors and the vector property $\underset{\sim}{c}.\underset{\sim}{c} = |\underset{\sim}{c}|^2$, prove the following inequality

$$|\underset{\sim}{a} + \underset{\sim}{b}| \leq |\underset{\sim}{a}| + |\underset{\sim}{b}|$$

Question 10 **(9 marks)**

Consider the following system of linear equations where $p \,\&\, m$ are constants.

$$x + 2y - 3z = 3$$
$$2x + 7y - 4z = p$$
$$-2x + 5y + mz = 7$$

Determine the values of $p \,\&\, m$:

(a) for which there is a unique solution (4 marks)

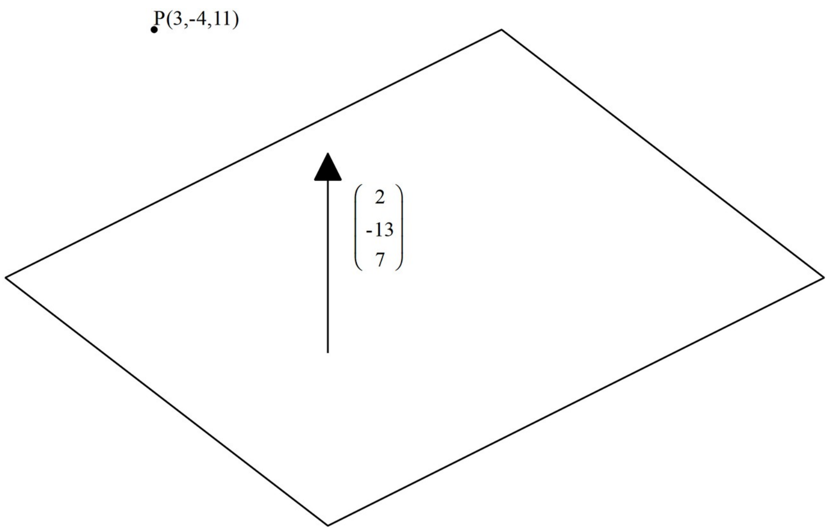
(b) for which there are infinite solutions. (3 marks)

(c) for which there are no solutions. (2 marks)

Question 11 **(4 marks)**

Consider the plane $\underset{\sim}{r} \cdot \begin{pmatrix} 2 \\ -13 \\ 7 \end{pmatrix} = 15$ as shown below.

Determine the distance of point P $(3, -4, 11)$ from the plane to two decimal places.



Question 12 **(4 marks)**

Given that $\left|A \times B\right| = \left|A\right|\left|B\right|\sin\theta$ use cross product to determine the distance of point P $(-5,1,19)$ from the line $\underset{\sim}{r} = \begin{pmatrix} 6 \\ -7 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 11 \end{pmatrix}$ to one decimal place.

Question 13 **(4 marks)**

The three vertices of a triangle have position vectors $\underset{\sim}{a}, \underset{\sim}{b}$ & $\underset{\sim}{c}$. Given that

$$\underset{\sim}{a} \times (\underset{\sim}{b} + \underset{\sim}{c}) = \underset{\sim}{a} \times \underset{\sim}{b} + \underset{\sim}{a} \times \underset{\sim}{c}$$

Show that the area of the triangle is given by $\frac{1}{2}\left|\underset{\sim}{a} \times \underset{\sim}{b} + \underset{\sim}{b} \times \underset{\sim}{c} + \underset{\sim}{c} \times \underset{\sim}{a}\right|$

Question 14 **(6 marks)**

Consider the polynomial $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ where $a, b, c, d \ \& \ e$ are real constants. Determine the values of $a, b, c, d \ \& \ e$ given the following information for $P(x)$

$(x + 1 - \sqrt{7}i)$ is a factor of $P(x)$.

When $P(x)$ is divided by $(x - 1)$ there is a remainder of 165

$P(0) = 32$ and $P(-2) = 0$

575=(x+2)5(23)

Question 15 **(9 marks)**

At noon a rocket is launched from position $(3,-4,11)$ km with a velocity of $\begin{pmatrix}-2\\6\\7\end{pmatrix}$ km/h.

Two hours later a second rocket is launched from position $(-10,11,5)$ km with a velocity of $\begin{pmatrix}-3\\21\\5\end{pmatrix}$ km/h.

Assume that both rockets move with constant velocity at all times and that the rockets do not collide.

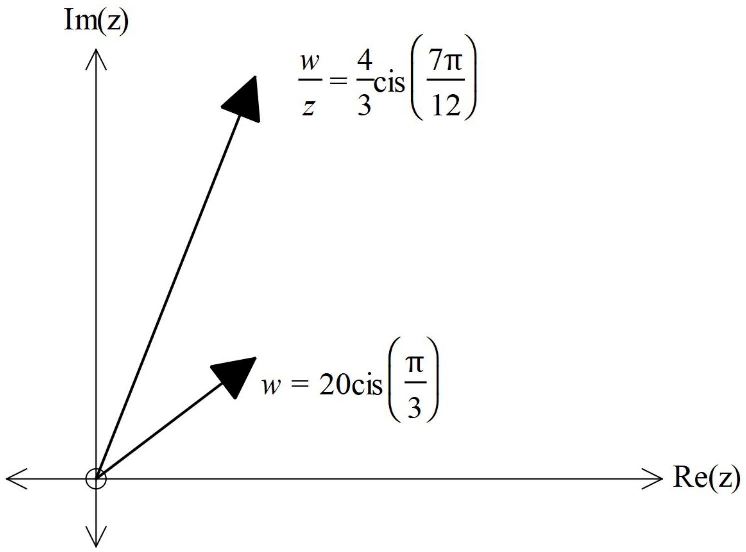
(a) Determine the distance between the rockets at 2:30pm that day to one decimal place (3 marks)

(b) Determine the times that the distance between the rockets is less than 50 km.
(4 marks)

(c) Determine the distance of closest approach and the time that this occurs. (2 marks)

Question 16 **(9 marks)**

Consider the complex numbers drawn in the complex plane below.



(a) Determine the exact value of z in the form of $a+bi$ (3 marks)

Consider the equation $7cis\,\dfrac{5\pi}{12} = \dfrac{5\sqrt{2}}{2} + \dfrac{5\sqrt{2}}{2}i + rcis\theta$
where $r > 0$ and $-\pi < \theta \leq \pi$

(b) Represent the above equation as a triangle in the complex plane below (3 marks)

Cont-

(c) Hence or otherwise solve for r & θ to one decimal place. (3 marks)

Question 17 **(9 marks)**

Consider a sphere with centre $(-3,4,7)$ and radius of 5 units.

(a) Write down the vector equation for this sphere (2 marks)

Consider a line parallel to vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$ and containing the point $(-4,a,11)$ where a is a constant.

(b) Write down the vector equation of the line in terms of a. (2 marks)

(c) Determine the possible values of a, to 2 decimal places, if the line is a tangent to the sphere..

(5 marks)

Question 18 **(11 marks)**

A particle moves with acceleration $\begin{pmatrix} -4\cos(2t) \\ \sin t \end{pmatrix} m/s^2$ at time t seconds. The initial velocity is $\begin{pmatrix} 1 \\ -7 \end{pmatrix} m/s$ and initial displacement of $\begin{pmatrix} 0 \\ 11 \end{pmatrix} m$.

(a) Determine the time(s), $0 \le t \le \pi$, that the particle is travelling parallel to the y axis. (4 marks)

(b) Determine the first two times that the particle crosses the y axis. (4 marks)

(c) Determine the cartesian equation of the path of a new particle with the following position vector $\underset{\sim}{r} = \begin{pmatrix} \sin t - 1 \\ 3\cos(2t) + 5 \end{pmatrix} m$ (3 marks)

Question 19 **(9 marks)**

Consider the function f where $f(x) = ax^2 + bx + c$ and $a, b \,\&\, c$ are positive constants with $x \leq \frac{-b}{2a}$

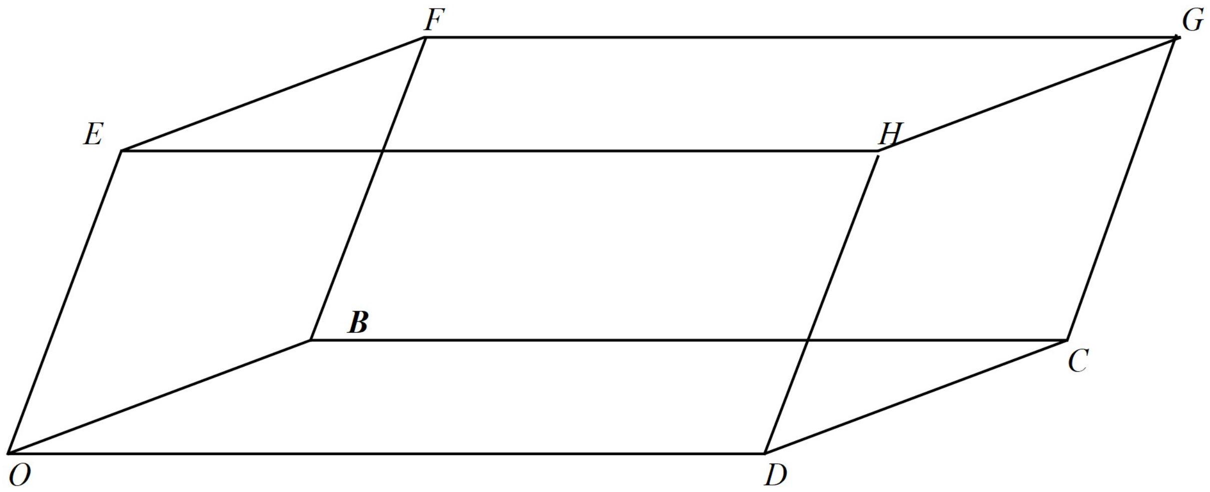
(a) Given that the inverse function does exist obtain an expression for $f^{-1}(x)$ in terms of $a, b \,\&\, c$ (3 marks)

(b) Given that there is only one point where $f(x) = f^{-1}(x)$ determine the x value in terms of $a, b \,\&\, c$ (3 marks)

(c) Given that $g \circ h(x) = ax^2 + bx + c$ and $h(x) = 3x - 1$, determine the function $g(x)$ in terms of $a, b \,\&\, c$ (3 marks)

Question 20 **(11 marks)**

Consider $OBCDEFGH$ drawn below, where each face is a parallelogram. Let $\underset{\sim}{b} = OB$, $\underset{\sim}{d} = OD$ and $\underset{\sim}{e} = OE$ with $\underset{\sim}{b}$ perpendicular to plane containing vectors $\underset{\sim}{d}$ & $\underset{\sim}{e}$.



(a) Express each of the vectors OG, DF, BH & CE in terms of $\underset{\sim}{b}, \underset{\sim}{d}$ & $\underset{\sim}{e}$

(4 marks)

(b) Express $|OG|^2, |DF|^2, |BH|^2$ & $|CE|^2$ in terms of $\underset{\sim}{b}, \underset{\sim}{d}$ & $\underset{\sim}{e}$ (4 marks)

Cont-

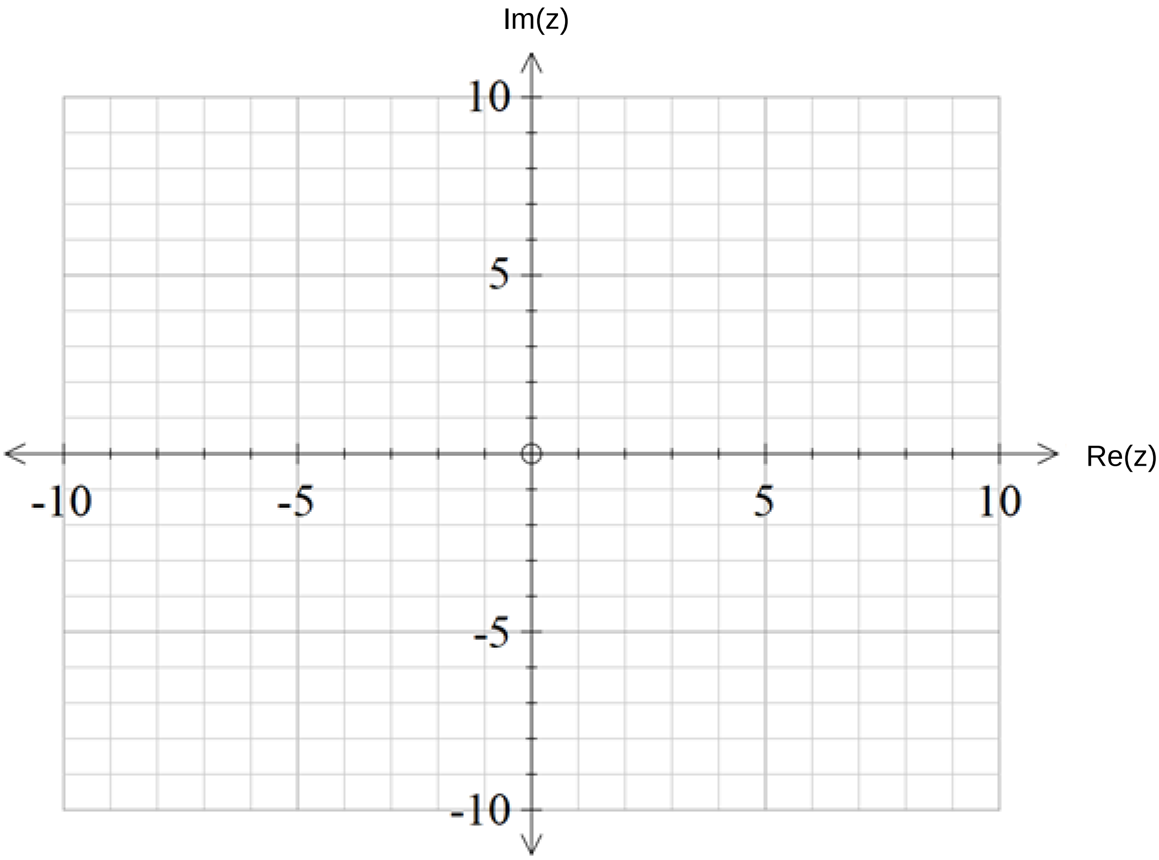
(c) Hence show that $\left|OG\right|^2 + \left|DF\right|^2 + \left|BH\right|^2 + \left|CE\right|^2 = 4\left(\left|\underset{\sim}{b}\right|^2 + \left|\underset{\sim}{d}\right|^2 + \left|\underset{\sim}{e}\right|^2\right)$

(3 marks)

Question 21 **(6 marks)**

Consider the region defined by $1 \leq |z+3+4i| \leq 3$ in the complex plane.

(a) Sketch the region on the axes below. (3 marks)



(b) Given that $-\pi < Arg(z) \leq \pi$, determine the minimum value of $Arg(z)$
In the region in (a).(Give to two decimal places) (3 marks)

Additional working space

Question number: ________

End of questions