

(2 marks)

$$(a) h(x) = \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{4}x^{\frac{1}{4}}$$

$$h(x) = x^{\frac{3}{4}} - x^{\frac{1}{4}}$$

$$(b) h(x) = \sqrt[4]{x^3} - \sqrt[4]{x}$$

(2 marks)

$$\boxed{(3-2x)^3 + \frac{12x^2}{(3-2x)^4}}$$

OR

$$\frac{4x^6}{(3-2x)^6}$$

$$= \frac{(3-2x)^3(4x) - 2x^2(3)(3-2x)^2(-2)}{(3-2x)^6} = \frac{(3-2x)^3(4x) + 12x^2(3-2x)^2}{(3-2x)^6}$$

(2 marks)

$$(a) f(x) = 3x^{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{2}$$

(2 marks)

Determine the derivative for each of the following, ensuring all answers are expressed with positive indices.

(6 marks)

Working time for this section is 50 minutes.

This section has **seven** (7) questions. Answer all questions. Write your answers in the spaces provided.

35% (50 Marks)

MATHEMATICS METHODS

CALCULATOR-FREE

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Solution	$P(t)=22(\ln(t+3)) = 100$ million
Behaviour	92nd day or 1st December
Solves equation	Sets up equation

(iii) On what day will the population reach 100 million? (3 marks)

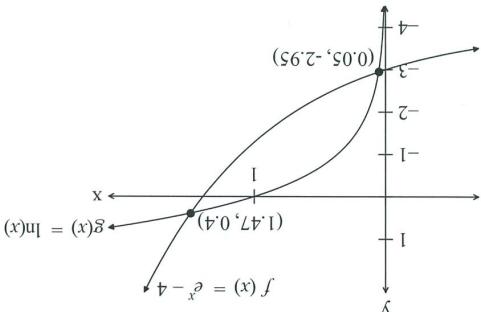
Solution	$P(0)=22(\ln(0+3)) = 24.17$ million
Behaviour	Sets up equation

(i) What was the population on the 1st of September? (2 marks)

Behaviour	Where $t$ is days starting on 1st September 2018.
Solution	The total population (in millions) can be modelled by the equation $P(t)=22(\ln(t+3))$ month. The population of this colony was studied in over a she is testing for growth characteristics. She has a colony of an experimental bacteria that

Solution	$\int_{1.47}^{0.05} f(x) dx - \int_{1.47}^{0.05} g(x) dx$
Behaviour	Calculates value.

(2 marks)



The points of intersection are shown.

Question 20	$f(x) = e^x - 4$ and $g(x) = \ln(x)$ as shown in the diagram below.
Behaviour	Use your calculator to find the area enclosed between the two functions

(7 marks)

Solution	$P(t)=22(\ln(t+3)) = 100$ million
Behaviour	Solves equation
Solves equation	Sets the correct day

**Question 2**

(8 marks)

Millie got 90% on the Law Admission Test (LAT). The average LAT applicant got 80% and the standard deviation was 5. Malcolm got 80% on the Undergraduate Medicine and health Science Admission Test (UMAT). The UMAT applicants result was 70% and standard deviation was 4.

- (a) Who did relatively better their respective test, Millie or Malcolm?

(3 marks)

$$\begin{aligned} Z_{M_i} &= \frac{90-80}{5} \\ &= 2 \quad \checkmark \\ Z_{M_e} &= \frac{80-70}{4} \\ &= 2.5 \quad \checkmark \\ \therefore \text{Malcolm did better as } 2.5 \text{ std dev. above Mean} \end{aligned}$$

- (b) Molly walks to work five days a week day, going through two sets of traffic lights.

Let  $X$  be the probability density function that gives the probability of the number of red lights Molly encounters in one day.

$x$	0	1	2
$P(X=x)$	0.5	0.3	0.2

- (i) Determine the expected number of red lights per trip.

(2 marks)

$$\begin{aligned} E(X) &= 0 \times 0.5 + 1 \times 0.3 + 2 \times 0.2 \quad \checkmark \\ &= 0 + 0.3 + 0.4 \\ &= 0.7 \quad \checkmark \end{aligned}$$

- (ii) What is the probability that Molly had 2 red lights two days in a row? (1 marks)

$$\begin{aligned} P(R_1, R_2) &= 0.2 \times 0.2 \\ &= 0.04 \quad \checkmark \end{aligned}$$

OR

- (iii) Determine the standard deviation of the distribution of  $X$ . Leave your answer un-simplified. (2 marks)

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= 0.3 \times 1^2 + 0.2 \times 2^2 \\ &= 0.3 + 0.8 \\ &= 1.1 \quad \checkmark \\ \text{Var}(X) &= 1.1 - 0.7^2 \\ &= 1.1 - 0.49 \\ &= 0.61 \end{aligned} \quad \left| \begin{aligned} \sigma &= \sqrt{\sum (x_i - \bar{x})^2 p(x)} \\ &= \sqrt{(0 - 0.7)^2 \times 0.5 + (1 - 0.7)^2 \times 0.3 + (2 - 0.7)^2 \times 0.2} \\ &= \sqrt{-0.7^2 \times 0.5 + 0.3^2 \times 0.3 + 1.3^2 \times 0.2} \\ &= \sqrt{0.245 + 0.027 + 0.338} \\ &\text{See next page} \\ \therefore \text{Std dev} &= \sqrt{0.61} \end{aligned} \right.$$

The set of samples obtained from the second site has a mean sample grade of  $4.7 \text{ mLm}^{-3}$ . The data was given to Sophie as a box-plot with the median of  $4.72 \text{ mLm}^{-3}$ , the lower quartile of  $4.2 \text{ mLm}^{-3}$  and the upper quartile of  $5.2 \text{ mLm}^{-3}$ .

- (d) Examine the above statistics to determine if the data for the second potential site could be represented by a normal distribution. **Justify your conclusion.** (3 marks)

Solution	4.72 (Median) $\sim$ 4.7 (Mean), not equal but close (1 mark) 4.72 – 4.2 $\sim$ 5.2 – 4.72, not symmetrical, but close (1 mark)  Conclusion can be Yes/No, as long as justified. (1 mark)
Behaviour	States conclusion. Justifies position with understanding of characteristics of normal distribution.

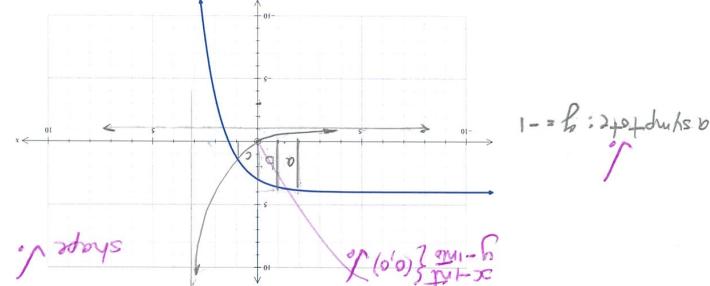
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(8 marks)

## CALCULATOR-FREE

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Question 3

The function  $f(x) = -e^x + 4$  is graphed on the axes below.

<p><b>Question 19</b></p> <p>Sophie is a petroleum engineer working for Vechiron Limited in charge of choosing between two sites for the construction of an offshore drilling rig. The government will only allow drilling of one site. Sophie is examining recent samples for both sites to help with his decision. A sample taken from the first site has a mean deviation of <math>0.56 \text{ mL}^{-3}</math>. Sophie found that the data for the samples are normally distributed.</p> <p>(a) Determine the probability that a randomly chosen sample contains a grade that is greater than <math>3.5 \text{ mL}^{-3}</math>.</p> <p><b>Solution</b> <math>P(X &gt; 3.5) = 0.975</math></p> <p><b>Behaviour</b> Calculate probability distribution.</p> <p>(1 mark)</p>
<p>(i) exactly <math>4.6 \text{ mL}^{-3}</math>.</p> <p><b>Solution</b> <math>P(X = 4.6) = 0</math></p> <p><b>Behaviour</b> Continuous data has zero probability at point</p> <p>(1 mark)</p>
<p>(ii) exactly <math>4.6 \text{ mL}^{-3}</math>.</p> <p><b>Solution</b> <math>\mu = 4.6 \text{ mL}^{-3}</math></p> <p><b>Behaviour</b> Notes the mean is median in normal distribution.</p> <p>(1 mark)</p>
<p>(iii) <math>\int_0^2 f(x) dx</math>.</p> <p><b>Solution</b> <math>\int_0^2 f(x) dx = 4.22 \text{ mL}^{-3}</math></p> <p><b>Behaviour</b> Calculate maximum grade sample from probability distribution.</p> <p>(1 mark)</p>
<p>(c) The probability that another sample contained less than the particular grade was 0.25 or 25%. Determine the maximum grade for the sample.</p> <p><b>Solution</b> <math>\text{invNormCDF}("l", 0.25, 0.56, 4.6) = 4.22 \text{ mL}^{-3}</math></p> <p><b>Behaviour</b> Calculate maximum grade sample from probability distribution.</p> <p>(1 mark)</p>

<p>(2 marks)</p> <p><math>y = -f(x) + 3</math></p>
<p>(i) <math>\int_0^2 -f(x) dx</math>.</p> <p><b>Solution</b> <math>\int_0^2 -f(x) dx = 2 \int_1^2 f(x) dx = 2 \int_1^2 \text{Note: } 2 \text{ cte} \text{ } \Rightarrow \text{area}</math></p>
<p>(ii) <math>\int_0^2 -f(x) dx</math>.</p> <p><b>Solution</b> <math>\int_0^2 -f(x) dx = \int_0^2 2f(x-1) dx</math></p>
<p>(iii) <math>\int_0^2 f(x-1) dx</math>.</p> <p><b>Behaviour</b> Calculate maximum grade sample from probability distribution.</p> <p>(1 mark)</p>

<p>(2 marks)</p> <p><math>\therefore \int_0^2 -f(x) dx = -(a+b) \text{ oe } -a-b</math></p>
<p><math>\therefore \int_0^2 -f(x) dx = \int_0^2 f(c-x) dx</math>.</p>
<p><b>Behaviour</b> Probability that another sample contained less than the particular grade was 0.25 or 25%. Determine the maximum grade for the sample.</p> <p>(1 mark)</p>

<p>(1 mark)</p> <p><math>\therefore \int_0^2 f(-x) dx</math>.</p>
<p><b>Solution</b> <math>\int_0^2 f(-x) dx = \int_{-1}^2 f(x) dx = a</math>, <math>\int_0^1 f(x) dx = b</math> and <math>\int_1^2 f(x) dx = c</math>, evaluate each of the following definite integrals in terms of the constants <math>a</math>, <math>b</math> and <math>c</math>.</p>
<p><b>Behaviour</b> Notes the mean is median in normal distribution.</p> <p>(1 mark)</p>

<p>(1 mark)</p> <p><math>\int_0^2 f(x) dx</math>.</p>
<p><b>Solution</b> <math>\int_0^2 f(x) dx = 4.6 \text{ mL}^{-3}</math></p>
<p><b>Behaviour</b> Calculate probability distribution.</p>
<p>(1 mark)</p>

<p>(1 mark)</p> <p><math>\int_0^2 -f(x) dx</math>.</p>
<p><b>Solution</b> <math>\int_0^2 -f(x) dx = 4.6 \text{ mL}^{-3}</math></p>
<p><b>Behaviour</b> Calculate probability distribution.</p>
<p>(1 mark)</p>

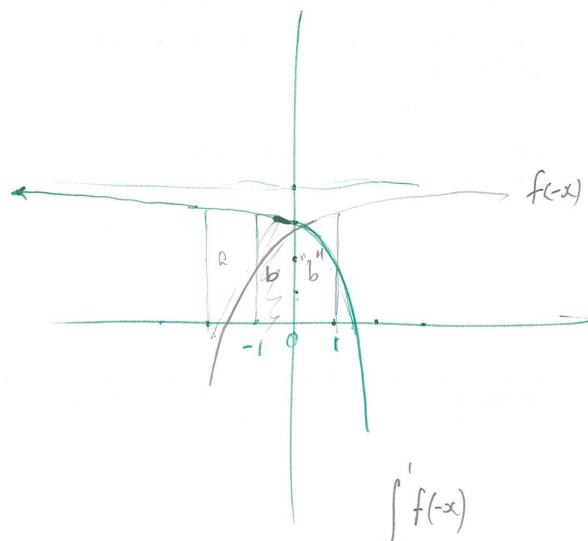
<p>(7 marks)</p> <p><b>Question 3</b></p> <p>The function <math>f(x) = -e^x + 4</math> is graphed on the axes below.</p>
<p><b>Sophie</b> is a petroleum engineer working for Vechiron Limited in charge of choosing between two sites for the construction of an offshore drilling rig. The government will only allow drilling of one site. Sophie is examining recent samples for both sites to help with his decision. A sample taken from the first site has a mean deviation of <math>0.56 \text{ mL}^{-3}</math>. Sophie found that the data for the samples are normally distributed.</p> <p>(a) Determine the probability that a randomly chosen sample contains a grade that is greater than <math>3.5 \text{ mL}^{-3}</math>.</p> <p><b>Solution</b> <math>P(X &gt; 3.5) = 0.975</math></p> <p><b>Behaviour</b> Calculate probability distribution.</p> <p>(1 mark)</p>
<p>(b) Determine the median score.</p> <p><b>Solution</b> <math>\mu = 4.6 \text{ mL}^{-3}</math></p> <p><b>Behaviour</b> Notes the mean is median in normal distribution.</p> <p>(1 mark)</p>
<p>(c) The probability that another sample contained less than the particular grade was 0.25 or 25%. Determine the maximum grade for the sample.</p> <p><b>Solution</b> <math>\text{invNormCDF}("l", 0.25, 0.56, 4.6) = 4.22 \text{ mL}^{-3}</math></p> <p><b>Behaviour</b> Calculate maximum grade sample from probability distribution.</p> <p>(1 mark)</p>

**Question 18**

(11 marks)

In a certain library, there is one computer with internet access that is open for public use. The internet usage (in minutes) is a continuous random variable  $X$  with pdf given by

$$f(x) = \begin{cases} 0.047e^{-0.047x}, & x \geq 0, \\ 0, & \text{elsewhere.} \end{cases}$$



Q3.

(a) (i)

b

(This is correct.)

$$\int_0^b -e^{-x+4} dx = e^{-1} + 3 = \boxed{b}$$

$$\int_0^1 -e^{-x+4} dx = e^{-1} + 3 = \boxed{b}$$

- (a) Find the probability that the visitor will use library internet access for:

(i) less than 15 minutes

$$P(X < 15) = \int_0^{15} 0.047 e^{-0.047x} dx$$

$$= 0.5059$$

- (ii) longer than 30 minutes.

$$P(X > 30) = 1 - P(X \leq 30) = 1 - \int_0^{30} 0.047 e^{-0.047x} dx$$

$$= 0.2441$$

- (b) Calculate the rate of internet usage after 30 minutes.

*paper said at 30 min*

$$y = 0.047 e^{-0.047(30)}$$

$$\frac{dy}{dx} = -0.047 e^{-0.047(30)} \downarrow$$

$$= -5.39 \times 10^{-4}$$

Should this be zero?

- (c) Alex has been waiting for his turn to use the internet for at least 10 minutes. What is the probability that he will have to wait for at least 10 more minutes?

$$\frac{P(X > 20)}{P(X > 10)} = \frac{1 - \int_0^{20} 0.047 e^{-0.047x} dx}{1 - \int_0^{10} 0.047 e^{-0.047x} dx}$$

$$= \frac{0.3906}{0.6250}$$

$$= 0.62496. \checkmark$$

(d)  $0.8 = \int_0^a 0.047 e^{-0.047x} dx \Rightarrow a = 34.24 \therefore \text{return in } 35 \text{ min.}$

**Question 4**

(a) Determine  $\int \sqrt{x} - \frac{1}{x^2} dx$  (5 marks)

$$\int x^{1/2} - x^{-2} dx = \frac{2}{3} x^{3/2} + \frac{1}{x} + C$$

**Question 4**

(b) If  $f(x) = 6(x^2 - 7)^2$  and  $f(2) = -20$ , determine  $f(3)$ . (3 marks)

$$f(x) = 6(x^2 - 7)^2$$

$$f(2) = 6(2^2 - 7)^2 = -20$$

$$-20 = (2^2 - 7)^2 + C$$

$$C = 7$$

$$f(3) = 6(3^2 - 7)^2 + C$$

$$f(3) = (3^2 - 7)^2 + 7$$

$$f(3) = 15$$

**Question 4**

(c) If  $f(x) = 6x(x^2 - 7)^2$  and  $f(2) = -20$ , determine  $f(3)$ . (3 marks)

$$f(x) = 6x(x^2 - 7)^2$$

$$f(2) = 6(2)(2^2 - 7)^2 = -20$$

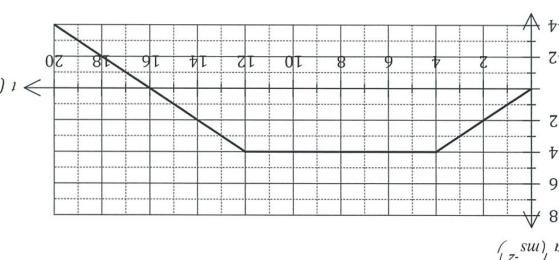
$$-20 = 6(2)(2^2 - 7)^2 + C$$

$$C = 7$$

$$f(3) = 6(3)(3^2 - 7)^2 + 7$$

$$f(3) = 1269.33m$$

(a)	Determine the velocity of the object when $t = 4$ (1 mark)	Solution $v(4) = 8 \text{ ms}^{-1}$	Behaviour $v(4) = 8 \text{ ms}^{-1}$
(b)	At what time is the velocity of the body a maximum, and what is the maximum velocity? (2 marks)	Solution $v(12) = 40 \text{ ms}^{-1}$	Behaviour $v(12) = 40 \text{ ms}^{-1}$
(c)	Determine the distance of the particle from the origin after 20 seconds. (3 marks)	Solution $s(t) = \int \frac{4}{t} dt + \int_{t=4}^{t=20} 4 dt \int_{t=12}^{t=20} (-t+16) dt$	Behaviour $s(t) = \int \frac{4}{t} dt + \int_{t=4}^{t=20} 4 dt \int_{t=12}^{t=20} (-t+16) dt$
(d)	Calculate the total displacement over the intervals to obtain total displacement. (3 marks)	Solution $s(16) = 1269.33m$	Behaviour $s(16) = 1269.33m$



A particle, initially stationary and at the origin, moves subject to an acceleration,  $a \text{ ms}^{-2}$ , as shown in the graph below for  $0 \leq t \leq 20$  seconds.

In the graph below, calculate the total displacement of the particle over the time interval  $0 \leq t \leq 20$  seconds.

Calculator-free

## Question 5

(7 marks)

Two hundred and fifty randomly selected students were surveyed to determine if an overseas trip for students should be planned for students in Year 10, or in Year 11 or in Year 12.

The results are in the table below.

Year	10	11	12
Preference	90	90	70

- (a) (i) Convert the data to form a probability density function.

(2 marks)

$x$	10	11	12
$P(X=x)$	0.36	0.36	0.28
	$\frac{90}{250}$	$\frac{90}{250}$	$\frac{70}{250}$

- (ii) If two students from the school were selected at random, what is the probability that they both thought Year 12 students should not go on an overseas trip.  
(Do not simplify)

(2 marks)

$$\text{P(Yr12, Yr12)} = 0.28 \times 0.28$$

OR  ${}^2C_2 \left( \frac{140}{500} \right)^2 \left( \frac{360}{500} \right)^0 = 0.28^2 = \left( \frac{7}{25} \right)^2 = \frac{49}{625}$

Which of the following represents a probability density function? Give your reasons. (3 marks)

(b) (i)

$x$	0	1	2	3
$P(X=x)$	0.1	0.2	0.3	0.4

$\sum P(x=x) = 1$

∴ Yes

If an No  
reasons  
(ii)  
for (i) & (ii)  
1 mark  
only

$x$	10	11	12	13
$P(X=x)$	0.2	0.3	0.1	0.5

$$\sum P(x=x) > 1$$

∴ No

$$P(x=8) < 0$$

∴ No

(iii)

$x$	6	7	8	9
$P(X=x)$	0.4	0.5	-0.3	0.4

See next page

## Question 16

(8 marks)

The moment magnitude scale  $M_w$  is used by seismologists to measure the size of earthquakes in terms of the energy released. It was developed to succeed the 1930's-era Richter magnitude scale.

The moment magnitude has no units and is defined as  $M_w = \frac{2}{3} \log_{10}(M_0) - 10.7$ , where  $M_0$  is the total amount of energy that is transformed during an earthquake, measured in dyn·cm.

- (a) On 21 December 1937, an estimated  $6.31 \times 10^{13}$  dyn·cm of energy was transformed during an earthquake in the Simpson Desert in NT. Calculate the moment magnitude for this earthquake.

$$\frac{2}{3} \log_{10} 6.31 \times 10^{13} - 10.7 \quad \text{(2 marks)}$$

$$M_w = 8.43 \quad \text{---} \quad -10.7$$

- (b) A few years later, on 27 July 1941, there was another earthquake with moment magnitude 6.5 in the Simpson Desert. Calculate how much energy was transformed during this earthquake. (2 marks)

$$6.5 = \frac{2}{3} \log_{10}(M_0) - 10.7 \quad \text{---}$$

$$17.2 = \frac{2}{3} \log_{10}(M_0) \quad \therefore M_0 = 6.31 \times 10^{25} \text{ dyn.cm}$$

$$25.8 = \log_{10}(M_0)$$

- (c) Show that an increase of 2 on the moment magnitude scale corresponds to the transformation of 1000 times more energy during an earthquake. (4 marks)

$$M_w = 6.5 \Rightarrow M_{6.5} = 6.31 \times 10^{25} \quad \text{---}$$

$$M_w = 8.5 \Rightarrow M_{8.5} = 10^{(8.5+10.7) \times 1.5} = 6.31 \times 10^{28} \quad \text{---}$$

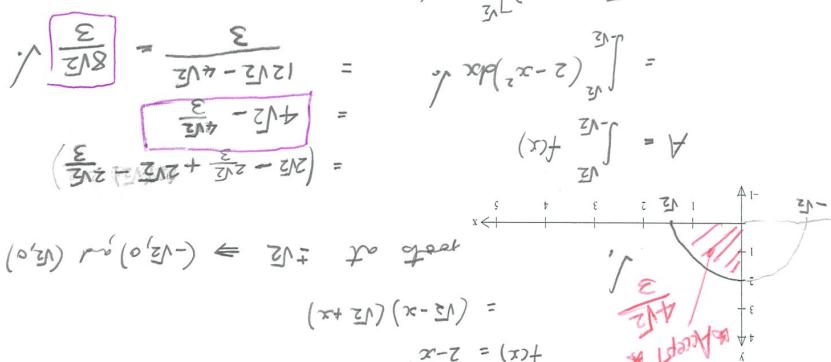
$$\therefore \frac{M_{8.5}}{M_{6.5}} = 1000 \text{ times greater energy} \quad \text{---}$$

$$\begin{aligned} M_w &= 2x \\ M_{w2} &= M_{w1} + 2 \\ &= 2 \end{aligned}$$

See next page

(6 marks)

(a) Find the area between the function  $f(x) = 2 - x^2$  and the  $x$ -axis.



(4 marks)

$$f(x) = 2 - x^2$$

$$= (\sqrt{3} - x)(\sqrt{3} + x)$$

$$\text{area } \Delta x \leftarrow (-\sqrt{2}, 0) \cup (0, \sqrt{2})$$

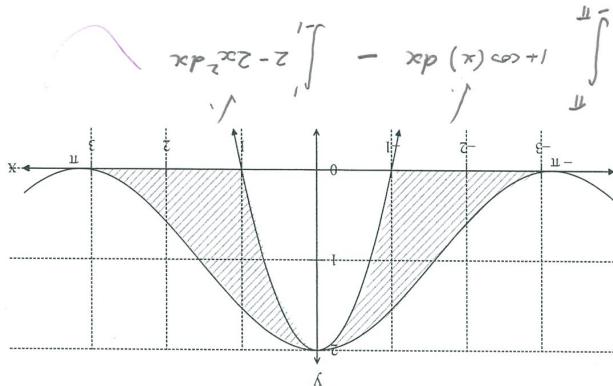
(4 marks)

(a)

(b)

(2 marks)

- (b) Write down the expression for the area between the function  $y = 2 - 2x^2$  and  $y = 1 + \cos(x)$  between  $-\pi \leq x \leq \pi$  that is illustrated in the diagram below.



See next page

1. only

$$\int_{-\pi}^{\pi} 1 + \cos(x) - 2 + 2x^2 dx$$

where

See next page

5a

(8 marks)

- (a) Determine the value of  $n$ , by first finding the mid-point of the interval  $0.1842 < p < 0.2958$ . A symmetric confidence interval for the true population proportion who watched the grand final is  $0.1842 < p < 0.2958$ .

$$\text{midpoint} \rightarrow 0.1842 + 0.2958 = 0.24$$

(4 marks)

(a)

- (b) Determine the confidence level of the interval.

$$\text{Margin of error (MSE)} = 0.2958 - 0.1842$$

(4 marks)

(b)

$$\begin{aligned} \text{MSE} &= \frac{0.2958 - 0.1842}{2n} \\ &= 0.0558 / 2 \\ &= 0.0558 \end{aligned}$$

$$\begin{aligned} z &= 1.9598 / (0.0558) \\ &= 34.58 \end{aligned}$$

$\therefore q_{5\%}$  confidence level  
~1.960

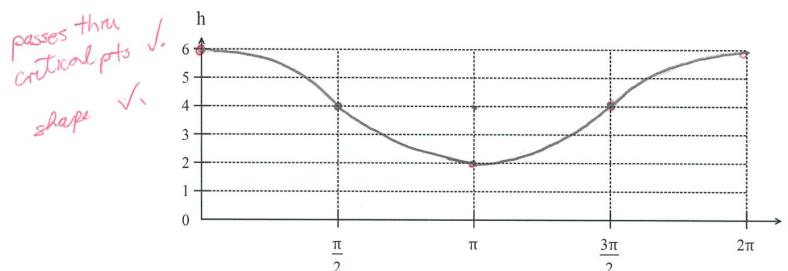
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## Question 7

(10 marks)

The height of the acrobat on a circus swing is given by  $h(t) = 2 \cos(t) + 4$  where  $h$  is measured in metres and  $t$  in seconds.

- (a) Sketch the height of the acrobat on the set of axes below for  $0 \leq t \leq 2\pi$ . (2 marks)



- (b) At what rate is the acrobat rising at  $t = \frac{3\pi}{2}$ ? (2 marks)

$$h(t) = 2 \cos(t) + 4$$

$$h'(t) = -2 \sin(t) \quad \checkmark$$

$$\text{when } t = \frac{3\pi}{2} \quad h'(t) = -2 \sin\left(\frac{3\pi}{2}\right) = 2 \text{ m/s.} \quad \checkmark$$

- (c) Explain using the graph in (a) why your answer to (b) gives the maximum rate of rise of the acrobat. (3 marks)

$\therefore$  P.O.I { Concave Up  $\Rightarrow \frac{\pi}{2} \leq t < \frac{3\pi}{2} \Rightarrow$  Velocity increasing. }  
 $\therefore$  P.O.I { Concave Down  $\frac{3\pi}{2} \leq t < 2\pi \Rightarrow$  Velocity decrease. }

$\therefore$  when  $t = \frac{3\pi}{2}$  Velocity is Max. ✓.

OR  $h''(t) = -2 \cos(t)$  and  $h''\left(\frac{3\pi}{2}\right) = 0$  which is a Max (P.O.I)

- (d) At what rate is the vertical velocity of the acrobat changing at  $t = \frac{3\pi}{2}$ ? (3 marks)

$$h''(t) = -2 \cos(t) \quad \checkmark$$

$$\therefore h''\left(\frac{3\pi}{2}\right) = 0 \text{ m/s}^2 \rightarrow \text{If Only or only } \begin{matrix} \text{No statement} \\ \hline \pm 1 \end{matrix}$$

$\therefore$  Velocity is Not changing ✓

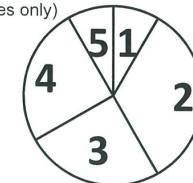
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## Question 14

(9 marks)

The simulation of a loaded (unfair) spinner is spun 60 times and the results recorded with the following results. (Below diagram is for illustrative purposes only)

Result	Count
1	5
2	20
3	15
4	15
5	5



- (a) Calculate the proportion of even numbers recorded in this simulation. (2 mark)

$$\frac{35}{60} = \frac{7}{12} \approx 0.583$$

- (b) Determine the mean and standard deviation for the sample proportion of even numbers in 60 tosses, using the results above. (2 marks)

$$\text{MEAN: } \hat{p} = \frac{7}{12} \approx 0.583 \quad \checkmark$$

$$\text{STD.DEV.: } \sigma_{\hat{p}} \approx \sqrt{\frac{0.583(1-0.583)}{60}} \approx 0.0636 \text{ or } \frac{\sqrt{21}}{72} \quad \checkmark$$

- (c) It has been decided to create a confidence interval for the proportion of even numbers using the simulation results. The level of confidence will be chosen from 90% or 95%. Explain which level of confidence will give the smallest margin error. State the margin of error. (3 marks)

90% confidence level has the smallest margin of error/as it has the smallest range. ✓

$$\begin{array}{|l} 90\% \text{ Margin of Error} = 1.645 \times \sigma \\ \hline \begin{array}{|l} 95\% \text{ Margin of Error} = 1.96 \times \sigma \\ \hline \approx 0.1047 \end{array} \end{array} \quad \approx 0.1247 \quad \checkmark$$

This simulation of 60 spins of the spinner is performed another 200 times, with the proportion of even numbers recorded each time and graphed.

- (d) Comment briefly on the key features of this graph. (2 marks)

Binomial distribution approaches Normal when  $n$  is large.  
Distribution centred on  $\hat{p} \approx 0.583$  ✓  
etc.

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This section has thirteen (13) questions. Answer all questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Section Two: Calculator-assumed

65% (100 marks)

Calculator

(a) Working time for this section is 100 minutes. (3 marks)

The cash balance C (in thousands of dollars) for the first few months of a start-up company varies according to the function  $C = -6e^{0.1t} \sin(t)$ , where t represents months. Hint: Use radians.

Question 8

(b) State whether the cash balance function equals zero or is first derivative equal zero. (3 marks)

(d) State when in the first year when either the cash balance function equals zero or its first

Solution	$\text{Cash Balance is zero at } 3.14, 6.28, 9.42$ $\text{Stationary Points at } 1.67, 4.81, 7.95, 11.09$ <b>Specific behaviours</b> $\checkmark$ identifies when cash balance is zero $\checkmark$ identifies stationary point $\checkmark$ identifies all points.
----------	--

Solution	$\text{derivative equal zero.}$ $\text{derivative second derivative}$
----------	--

Solution	$\frac{d^2C}{dt^2} = -\frac{1}{100} e^{\frac{t}{10}} (20 \cos(t) - 99 \sin(t))$ $\frac{dC}{dt} = -\frac{1}{10} e^{\frac{t}{10}} (\sin(t) + \cos(t))$ $C = -e^{\frac{t}{10}} \sin(t)$ <b>Specific behaviours</b> $\checkmark$ states first derivative $\checkmark$ states second derivative
----------	---

(a) State the first and second derivatives of the cash balance function. (3 marks)

The cash balance C (in thousands of dollars) for the first few months of a start-up company varies according to the function  $C = -6e^{0.1t} \sin(t)$ , where t represents months. Hint: Use radians.

Question 8 (11 marks)

A lottery sells 1000 tickets and claims that there will be 10 winners. How many tickets should you buy so that you have a 20% chance of winning at least 1 prize.

Working time for this section is 100 minutes. (3 marks)

Section Three: Purchase 23 tickets

$$P(X \geq 1) = 0.20$$

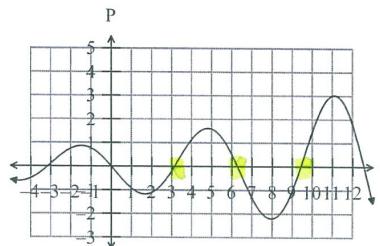
$$1 - \text{Bin}(1000, 0.01) = 0.2064$$

$$\text{Bin}(1, 23, 0.01) = 0.2064$$

Working time for this section is 100 minutes. (3 marks)

Section Four: Purchase 23 tickets

- (c) Sketch the cash balance equation on the set of axes.



## CALCULATOR-ASSUMED

(3 marks)

*x-int* ✓  
*T.P.* ✓  
*shape/inaccuracy* ✓

After the first five months, the owner employed more staff and it took a little while for sales to start to increase again.

- (d) Determine when the cash balance started to increase again. (1 mark)

Solution
Cash starts to increase after 7.95 months
Specific behaviours
✓ identifies when cash balance begins to increase

- (e) Determine when the cash balance become positive again. (1 mark)

Solution
Cash Balance become positive after 9.42 months
Specific behaviours
✓ identifies when cash balance returns to positive

See next page

## CALCULATOR-ASSUMED

## Question 12

12 A section of freeway had a speed limit of  $65 \text{ kmh}^{-1}$ , had a mean and standard deviation of  $56.9 \text{ kmh}^{-1}$  and  $3.6 \text{ kmh}^{-1}$  respectively. A summary of the data is shown in the table below.

Speed ( $x \text{ kmh}^{-1}$ )	9	10	10	11	11	12	12	13	13	14
Relative frequency	0.024		0.272		0.504		0.188		0.012	

- (a) Use the table of relative frequencies to estimate the probability that the next vehicle to pass the roadworks

- (i) was not exceeding the speed limit. (1 mark)

$$0.024 + 0.272 + 0.504 = 0.8 \checkmark$$

13

- (ii) had a speed of less than  $65 \text{ kmh}^{-1}$ , given they were exceeding the speed limit. (1 mark)

$$\frac{0.188}{0.188+0.012} = \frac{0.188}{0.2} = 0.94 \checkmark$$

- (b) Subsequent tests on the measuring equipment discovered that it had been wrongly calibrated. The correct speed of each vehicle,  $v$ , could be calculated from the measured speed,  $x$ , by increasing  $x$  by 6% and then adding 1.7.

- (i) Calculate the adjusted mean and standard deviation of the vehicle speeds. (2 marks)

$$Y = 1.06x + 1.7$$

$$\mu(Y) = 1.06 \times 11.4 + 1.7 = 13.78 \checkmark$$

$$\sigma(Y) = 1.06 \times 0.8 = 0.848 \checkmark$$

- (ii) Determine the correct proportion of vehicles that were speeding. (2 marks)

Let the incorrect speed limit be  $z$ .

$$1.06z + 1.7 = 12$$

$$z = 9.72 \text{ kmh}^{-1} \checkmark$$

$$0.272 + 0.504 + 0.188 + 0.012 =$$

$$= \underline{\underline{0.976}} \checkmark$$

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$$\frac{1}{10} = 0.064$$

$$= 1 - 0.6 - 0.4 \times 0.6 = 0.4 \times 0.6$$

$$1 - [P(DC1_{st}) + P(DC2_{st}) + P(DC3_{st})]$$

(b) What is the probability that a DC card is not drawn? (2 marks)

Up to three cards can be drawn. The draw stops once a DC card is selected.

$$\left(\frac{3}{5}\right)^2 \times \left(\frac{2}{5}\right) = \frac{12}{25} \text{ or } 0.48$$

(a) What is the probability that the first DC card drawn is the third card? (2 marks)

One card is selected, and its brand colour noted then it is replaced several times.

Michele collects comic book trading cards from two brands, Marvel and DC. She has eight (8) Marvel cards and twelve (12) DC cards. She shuffles the cards in a deck.

She thoroughly洗 the other cards again. This process is repeated several times.

It costs Fiona \$1 per roll and the payout is \$2 for a 2 or a 3 and nothing otherwise.

What is Fiona's average payout? (3 marks)

$$\$ -0.2 \quad \begin{matrix} \$0.8 \\ \cancel{\text{not}} \end{matrix}$$

$$= -0.3 + 0.2 + 0.2 - 0.3$$

$P(X=x)$	0.3	0.2	0.2	0.3
x	1	1	-1	-1

(b) Fiona bets on the outcome of an unfair 4-sided tetrahedral die with probabilities as in the table below.

(b) Fiona bet on the outcome of an unfair 4-sided tetrahedral die with probabilities as in the table below.

Question 9 (4 marks)

CALCULATOR-ASSUMED

MATHEMATICS METHODS

## Question 10

Australian population in 1880 was 2,231,489.  
The population in 1930 had grown to 6,501,012.

- (a) Taking  $t = 0$  in 1880, set up an equation in the form  $P = P_0 e^{kt}$  that can be used to estimate the population growth during the 50-year period. (2 marks)

$$6501012 = 2231489 e^{50k}$$

$$\text{Solve: } \frac{6501012}{2231489} = e^{50k}$$

$$k \approx 0.021386 \quad \checkmark$$

$$\therefore P = 2231489 e^{0.021386 t} \quad \checkmark$$

- (b) Write down the average annual population growth over that period. (1 mark)

$$\text{Ave Growth} = \frac{6501012 - 2231489}{50} \\ \approx 85390.46$$

Over the next 60 years to 1990, the population grew to 17,169,181.

- (c) Determine if the rate of growth during the 60 years from 1930 to 1990 is the same as the rate of growth from 1880 to 1930. (2 marks)

$$\text{Solve } \frac{17169181}{6501012} = e^{60k}$$

$$k \approx 0.016186 \quad (\text{Not the Same})$$

- (d) Use the data from 1930 to 1990 to predict the taxable income in 2016. (2 marks)

$$P = 17169181 e^{0.016186 \times 26}$$

$$\approx 26152666.6$$

$$\approx 26152700$$

NB. Australia's population in 2016 was 24,129,300.

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## Question 11

- (a) (i) Find the expected value and variance of the probability density function in the table below. (10 marks)

(2 marks)

$x$	1	2	3	4
$P(X=x)$	0.3	0.2	0.2	0.3

$$\begin{aligned} E(X) &= 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.2 + 4 \times 0.3 \\ &= 0.3 + 0.4 + 0.6 + 1.2 \\ &= 2.5 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= 0.3(1-2.5)^2 + 0.2(2-2.5)^2 + 0.2(3-2.5)^2 + 0.3(4-2.5)^2 \\ &= 0.3(-1.5)^2 + 0.2(-0.5)^2 + 0.2(0.5)^2 + 0.3(1.5)^2 \\ &= 1.45 \end{aligned}$$

$$\boxed{\text{Class pad}}: \bar{x} = 1.204 \quad \therefore \text{Var}(X) = 1.450 \quad \checkmark$$

- (ii) The values of set  $X$  are transformed so that  $Y = 2X + 1$ . Write down the expected value and variance of set  $Y$ . (2 marks)

$$\begin{aligned} E(2X+1) &= 2 \times 2.5 + 1 \\ &= 6 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{Var}(2X+1) &= 2^2 \times 1.45 \\ &= 5.8 \quad \checkmark \end{aligned}$$

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