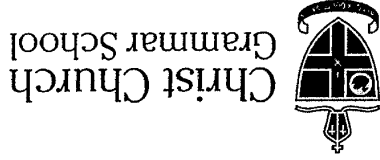


Question number: _____

See next page

MATHEMATICS METHODS Year 12
Section One:
Calculator-free



2017
UNIT TEST 3

Student name _____
Teacher name _____

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

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4. It is recommended that **you do not use pencil**, except in diagrams.

See next page

(c) Given that $SD(X) = 1.6$, determine:

- i) $SD(3X)$ (1 mark)

$$3 \times 1.6 = \underline{4.8} \checkmark$$

- ii) $Var(3X)$ (1 mark)

$$= 4.8^2 \\ = \underline{23.04} \checkmark$$

Question 8**(6 marks)**

It is known that 3% of cars manufactured in a particular assembly line will have some kind of defect.

(a) If a random sample of 7 cars is selected for testing, find that probability that:

- i) no cars have a defect. (1 mark)

$$X \sim B(7, 0.03) \quad \underline{0.8080} \checkmark$$

- ii) exactly 3 cars have a defect. (1 mark)

$$\underline{0.0008366} \checkmark$$

- iii) no cars have a defect, given that less than 5 cars have a defect. (2 marks)

$$\frac{0.8080}{1} \checkmark = \underline{0.8080} \checkmark$$

(b) What is the largest number of cars that can be selected in a random sample such that the probability of there being at least 1 defective car is less than 20%.

(2 marks)

$$\therefore \text{None Defective} > 0.8$$

$$0.97^n > 0.8 \checkmark$$

$$n < 7.33$$

$$\therefore \underline{n = 7} \checkmark$$

End of questions

8

Question 7

(8 marks)

Consider the discrete probability distribution shown below

x	P(X=x)	0.25	0.3	0.1	p	q
3		2				

(a) Determine:

i) $P(X > -1)$

0.75 ✓

(1 mark)

iii) $P(X = -1 | X \leq 1)$

$\frac{0.25}{0.65} = 0.3846$ ✓ $(\frac{5}{13})$

(1 mark)

(b) Given that $E(X) = 0.85$ determine:

i) the values of p and q.

$\sum P(X=x) = 1$ $\therefore p+q = 0.35$ ① ✓

$-0.25 + 0 + 0.1 + 2p + 3q = 0.85$
 $2p + 3q = 1 - ②$ ✓

Solve ① & ② in CP

$\therefore p = 0.05, q = 0.3$ ✓

iii) $E(2X - 1)$

$= 2(0.85) - 1 = 0.7$ ✓

(1 mark)

6

See next page

Question 1

(9 marks)

Evaluate the following.

(a) $\int_{-1}^2 \frac{d}{dx} x^2 e^{x^3} dx$

(2 marks)

(b) $\frac{d}{dx} (\sin(2x) + e^{x^3})$

(2 marks)

(c) $\int -3xe^{2x^2} dx$

(2 marks)

See next page

Question 1 continued

- (d) $\int_0^{\frac{\pi}{2}} 2 + \cos \frac{x}{2} dx$, giving your answer as an exact value. (3 marks)

See next page

Question 6

5
(4 marks)

A particle Q , with an initial displacement of 3m moves along a straight line. Its velocity $v \text{ ms}^{-1}$ after t seconds is given by $v(t) = 2 - 4e^{-0.5t}$.

(a) Determine:

- (i) The displacement of the particle in terms of t . (1 mark)

$$x(t) = 2t + 8e^{-0.5t} + c \quad \checkmark \quad \text{but } x(0) = 3$$

$$3 = 8e^0 + c$$

$$c = -5$$

$$\therefore x(t) = 2t + 8e^{-0.5t} - 5 \quad \checkmark$$

- (ii) The acceleration of the particle when it is at rest. (2 marks)

$$\text{At rest } v(t) = 0$$

$$2 - 4e^{-0.5t} = 0$$

$$t = 1.39 \quad \checkmark$$

$$a(t) = 2e^{-0.5t}$$

$$a(1.39) = 0.9981 \text{ ms}^{-2} \quad \checkmark$$

- (b) Describe the speed of Q for large values of t . (1 mark)

tends to 2 ms^{-1} \checkmark

See next page



Christ Church
Grammar School

2017
UNIT TEST 3

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Student name _____
Teacher name _____
- Solutions -

Time and marks available for this section
Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

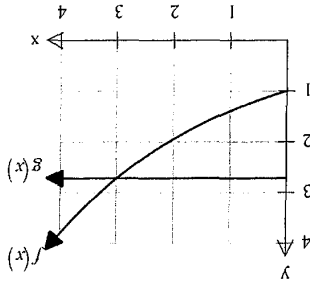
Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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Question 2

(3 marks)

The functions $f(x) = e^{\frac{x}{3}}$ and $g(x) = e$ are graphed below, intersecting at $(3, e)$.



Determine the area bound by the two curves and the y-axis.

See next page

Question 3

(3 marks)

Determine the x coordinates of all stationary points on the function

$$f(x) = \int_0^{x^2} e^{t^2} - e \, dt$$

End of questions

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See next page

Question 4

(6 marks)

$X \sim B[n, p]$, with $\text{Var}(X) = 1.5$ and $E(X) = 2$.

(a) Determine the value of n and p .

(4 marks)

$$\begin{aligned} \text{Var}(X) &: np(1-p) = 1.5, \\ E(X) &= np = 2 \\ \therefore 2(1-p) &= 1.5 \\ 1-p &= 0.75 \\ p &= 0.25 \\ \therefore n &= 8 \end{aligned}$$

(b) Determine an expression for $P(X \geq 1)$. Do not simplify. (2 marks)

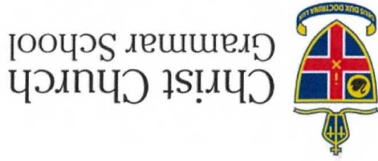
$$1 - 0.75^8 \quad \text{or} \quad \sum_{x=1}^8 C_x (0.25)^x (0.75)^{8-x}$$

See next page

6

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed



2017
UNIT TEST 3

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

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See next page

Question 5

(5 marks)

A population grows continuously such that $\frac{dP}{dt} = 0.09P$, where P is the size of the population t years after observation commenced. When observation commenced, the size of the population was 350 000.

- (a) Determine an expression for P in terms of t . (1 mark)

$$P_t = 350\,000 e^{0.09t} \quad \checkmark$$

- (b) How long will it take for the population to reach 1 000 000? (2 marks)

$$1\,000\,000 = 350\,000 e^{0.09t}$$

$$t = 11.66 \overset{469}{\text{yrs}} \quad \checkmark$$

$$t > 11.665$$

- (c) A second population is increasing at the same rate as that from part (a), however the initial population is unknown. How long will it take for this population to double in size? (2 marks)

$$2P_0 = P_0 e^{0.09t} \quad \checkmark$$

$$2 = e^{0.09t}$$

$$t = 7.70 \overset{163}{\text{yrs}} \quad \checkmark$$

$$\therefore t > 7.70163534$$

See next page

Question 3

(3 marks)

Determine the x coordinates of all stationary points on the function

$$f(x) = \int_{x^2}^0 e^{t^2} - e \, dt$$

$$f'(x) = [e^{(x^2)^2} - e][2x]$$

$$\therefore f'(x) = 2x(e^{x^4} - e) \quad \checkmark$$

$$\text{Stat pt } f'(x) = 0$$

$$\therefore 2x(e^{x^4} - e) = 0 \quad \checkmark$$

$$\therefore 2x = 0 \text{ or } e^{x^4} - e = 0$$

$$e^{x^4} = e$$

$$x^4 = 1$$

$$x = \pm 1$$

$$\therefore \text{Soln } x = 0, \pm 1 \quad \checkmark$$

3

End of questions

Question 4

(6 marks)

$X \sim B[n, p]$, with $\text{Var}(X) = 1.5$ and $E(X) = 2$.

(a) Determine the value of n and p .

(4 marks)

(b) Determine an expression for $P(X \geq 1)$. Do not simplify.

(2 marks)

See next page

Question 5

(5 marks)

A population grows continuously such that $\frac{dP}{dt} = 0.09P$, where P is the size of the population t years after observation commenced. When observation commenced, the size of the population was 350 000.

(a) Determine an expression for P in terms of t .

(1 mark)

(b) How long will it take for the population to reach 1 000 000?

(2 marks)

(c) A second population is increasing at the same rate as that from part (a), however the initial population is unknown. How long will it take for this population to double in size?

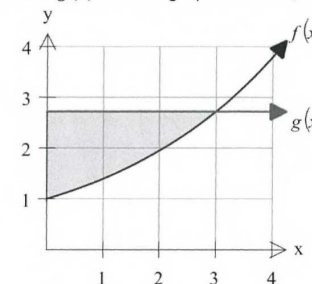
(2 marks)

See next page

Question 2

(3 marks)

The functions $f(x) = e^{\frac{x}{3}}$ and $g(x) = e$ are graphed below, intersecting at $(3, e)$.



Determine the area bound by the two curves and the y-axis.

$$= \int_0^3 e - e^{\frac{x}{3}} dx \quad \checkmark$$

$$= \left[ex - 3e^{\frac{x}{3}} \right]_0^3 \quad \checkmark$$

$$= 3e - 3e^1 - (e \times 0 - 3e^0)$$

$$= 3e - 3e + 3e^0$$

$$= \underline{3} \text{ units}^2 \quad \checkmark$$

3

See next page

(d) $\int_{\frac{\pi}{2}}^0 2 + \cos \frac{x}{2} dx$, giving your answer as an exact value.

(3 marks)

$$= \left[2x + 2 \sin \frac{x}{2} \right]_{\frac{\pi}{2}}^0$$

$$= 2 \frac{\pi}{2} + 2 \sin \frac{\pi}{2} - \left(2(0) + 2 \sin \frac{0}{2} \right)$$

$$= \pi + 2 \sin \frac{\pi}{2} - 0$$

$$= \pi + 2 \times \sqrt{2}$$

$$= \pi + \sqrt{2}$$

See next page

3

Question 6

(5 marks)

A particle Q, with an initial displacement of 3m moves along a straight line. Its velocity $v \text{ ms}^{-1}$ after t seconds is given by $v(t) = 2 - 4e^{-0.5t}$.

(a) Determine:

(i) The displacement of the particle in terms of t .

(2 marks)

(ii) The acceleration of the particle when it is at rest.

(2 marks)

(b) Describe the speed of Q for large values of t .

(1 mark)

See next page

Question 7

(8 marks)

Consider the discrete probability distribution shown below

x	-1	0	1	2	3
$P(X=x)$	0.25	0.3	0.1	p	q

(a) Determine:

i) $P(X > -1)$

(1 mark)

ii) $P(X = -1 | X \leq 1)$

(1 mark)

(b) Given that $E(X) = 0.85$ determine:

i) the values of p and q .

(3 marks)

ii) $E(2X - 1)$

(1 mark)

See next page

Question 1

(9 marks)

Evaluate the following.

(a) $\int_{-1}^2 \frac{d}{dx} x^2 e^{x^3} dx$

(2 marks)

$$= \left[x^2 \cdot e^{x^3} \right]_{-1}^2 \quad \checkmark$$

$$= (2)^2 \cdot e^8 - (-1)^2 \cdot e^{-1}$$

$$= 4e^8 - \frac{1}{e} \quad \checkmark$$

(b) $\frac{d}{dx} (\sin(2x) + e^{x^3})$

(2 marks)

$$2 \cos 2x + 3x^2 e^{x^3}$$

(c) $\int -3xe^{2x^2} dx$

(2 marks)

$$= -\frac{3}{4} \int 4x \cdot e^{2x^2} dx$$

$$= -\frac{3}{4} e^{2x^2} + C$$

See next page

6

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See next page

End of questions

- (c) Given that $SD(X) = 1.6$, determine:

i) $SD(3X)$ (1 mark)

ii) $Var(3X)$ (1 mark)
- Question 8 (6 marks)**

It is known that 3% of cars manufactured in a particular assembly line will have some kind of defect.

(a) If a random sample of 7 cars is selected for testing, find that probability that:

i) no cars have a defect. (1 mark)

ii) exactly 3 cars have a defect. (1 mark)

iii) no cars have a defect, given that less than 5 cars have a defect. (2 marks)

(b) What is the largest number of cars that can be selected in a random sample such that the probability of there being at least 1 defective car is less than 20%. (2 marks)

Additional working space

Question number: _____

Christ Church
Grammar School2017
UNIT TEST 3**MATHEMATICS METHODS Year 12****Section One:****Calculator-free**Student name _____ *- SOLUTIONS -*

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes

Working time for this section: 15 minutes

Marks available: 15 marks

Materials required/recommended for this section***To be provided by the supervisor***

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