**PERTH MODERN SCHOOL**

**UNIT 3C/3D MAS – 2011**

**TEST 1 – POLAR COORDINATES & COMPLEX NUMBERS**

**NAME: ________________________________________ DATE: _______________**

**[To achieve full marks and to allow assessment of particular outcomes, working and reasoning should be shown.]**
**[A maximum of 2 marks will be deducted for incorrect rounding, units, etc.]**

**This is *Resource Rich – 50 minutes for 53 marks*:**

1. [1, 1, 1, 1 = 4 marks]
   Convert:
   a) (4,-6) into polar coordinates with $-180^\circ < \theta \leq 180^\circ$.

   b) $(3,-\sqrt{3})$ into ***exact*** polar coordinates with $-\pi < \theta \leq \pi$.

   c) $[4,35^\circ]$ into Cartesian coordinates.

   d) $[8,-\frac{3}{4}\pi]$ into ***exact*** Cartesian coordinates.

2. [3, 3 = 6 marks]
   Clearly show how you obtain your answers, find:
   a) the distance between [20,-210°] and [ $\sqrt{5}$ ,-50°].

   b) the ***exact*** distance between $\left[\frac{\sqrt{5}}{3}, -\frac{2\pi}{3}\right]$ and $\left[10, -\frac{7\pi}{6}\right]$ .

3. [8 marks]

   Find the ***exact*** distance between ( $\sqrt{3k}$ , $\sqrt{\mathbf{k}}$ ) and $\left[\sqrt{\mathbf{k}}, \frac{5\pi}{6}\right]$ . Draw a diagram.

4. [3, 2 = 5 marks]

a) Find, in ***exact*** form, the modulus and principal argument of $-\sqrt{3} + i$, and hence rewrite $-\sqrt{3} + i$ in ***exact*** polar (***cis***) form.

b) Convert $2\ cis \left( \frac{\pi}{4} \right)$ into ***exact*** algebraic Cartesian/rectangular form.

5. [3, 3 = 6 marks]

Evaluate, giving answers in ***exact*** form:

a) $4\ cis\ \frac{\pi}{3} \times 2\ cis\ \frac{3\pi}{4}$

b) $\dfrac{4\ cis \left( -\frac{5\pi}{6} \right)}{2\ cis \left( \frac{5\pi}{6} \right)}$

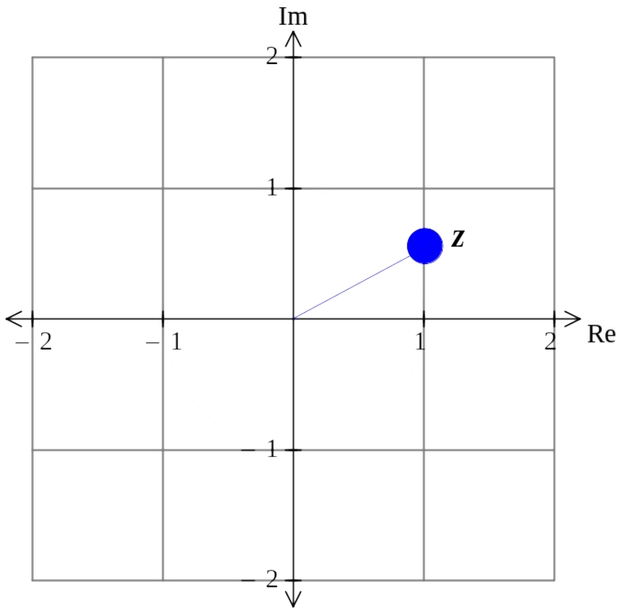
6. [4 marks]

Given $z = 2\ cis\ \frac{\pi}{4}$, express $z^{-1}$ and $\bar{z}$ in ***exact*** polar and rectangular form.

7. [2, 2, 2, 2 = 8 marks]

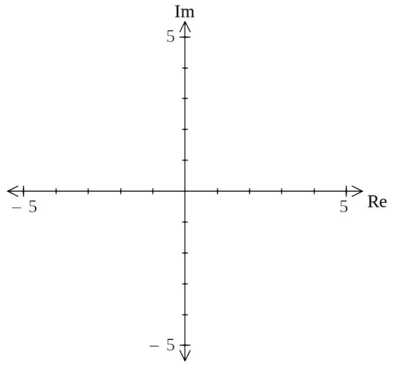
The Argand diagram below shows the point representing the complex number $z$ where $|\ z\ | > 1$. Plot on the same diagram, the points representing the complex numbers:

a) $\bar{z}$ b) $iz$ c) $z^2$ d) $\frac{1}{z}$

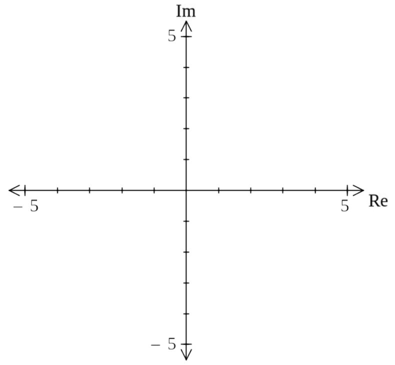


8. [3, 3, 3, 3 = 12 marks]
Sketch on an Argand diagram, the locus of the point $z = x + iy$, satisfying each of the following conditions. In each case, give the Cartesian equation or inequality of the locus.

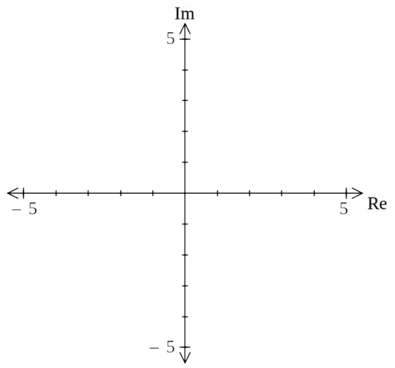
a) $\text{Re}(z) = -2$




b) $\text{Im}(z) = \text{Re}(z)$




c) $\text{Re}(z) + 2\text{Im}(z) > 3$




d) $\text{Re}(z).\text{Im}(z) = 1$