MATHEMATICS METHODS Unit 1
2022 Investigation 1
Take Home Section

NAME: SOLUTIONS

Validation Test: Monday 9th May 2022

Weighting: 8%

Transformations of Functions

INSTRUCTIONS:

- It is recommended you work in small groups after school hours and come to tutoring.
- The Take Home part is not worth any marks.
- The Take Home part may be used in the validation test.
- You are encouraged to use your ClassPad to help you to complete the Take Home part.
- **No calculators** will be allowed in the validation test.

Introduction:

A **function** is an equation for which any *x* that can be plugged into the equation will yield exactly one *y* out of the equation. We can draw a graph of a function on the Cartesian plane.

There are certain actions we can perform on a function that change the graph in specific ways. These are called ***TRANSFORMATIONS.***

TRANSFORMATIONS include:

1. Translations: (horizontal or vertical)
2. Dilations: (horizontal or vertical)
3. Reflections: (horizontal or vertical)

Function Notation:

Function notation is the way a function is written. It is meant to be a precise way of giving information about the function without a rather lengthy written explanation.

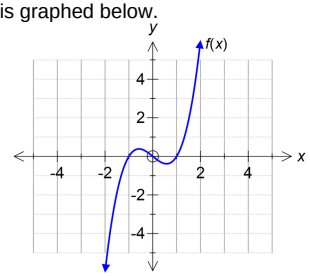
The most popular function notation is $f(x)$ which is read "f of x".
This is NOT the multiplication of f times x.

$f(x) = 3x + 1$ (input value: x; output value: $3x + 1$)

Traditionally, functions are referred to by single letter names, such as f, g, h and so on. Any letter(s), however, may be used to name a function. Examples:

$f(x) = x^2 + 1$ $\quad g(x) = x - 7$ $\quad h(b) = 3b^2 - 2b + 1$ $\quad S(t) = \frac{1}{2}t^2 - 3t + 1$

1. The function $f(x)=x^3-x$ is graphed below.



Use your Classpad to investigate how each of the following changes to $f(x)=x^3-x$ affects the graph. Make sure you consider both **positive** and **negative** numbers. It is recommended that you draw your results on graph paper.

a) Adding a number (constant) to the function. Eg. Adding 2 to the function $f(x)=x^3-x$ results in $f(x)+2=x^3-x+2$.

b) Adding a number to the variable. Eg. Adding the number 2 to the x value in the function $f(x)=x^3-x$
so that every x in the function has been replaced by $(x+2)$. This results in the transformed function:
$f(x+2)=(x+2)^3-(x+2)$.

c) Multiplying the function by a constant Eg. If the function is multiplied by 2, $f(x)$ transforms to $2f(x)$. Thus using $f(x)=x^3-x$ our transformed function becomes $2f(x)=2x^3-2x$.

d) Multiplying the variable by a number. Eg Multiplying the variable by 2 using $f(x)=x^3-x$ our transformed function becomes $f(2x)=¿$ which simplifies to $8x^3-2x$. Consider also multiplying the variable by a fraction <1 Eg. Multiplying the variable by $\frac{1}{2}$ using $f(x)=x^3-x$ our transformed function becomes $f\left(\frac{x}{2}\right)=¿$ which simplifies to $\frac{x^3}{8}-\frac{x}{2}$.

2. Investigate the transformation from Q1 above with other graphs such as:

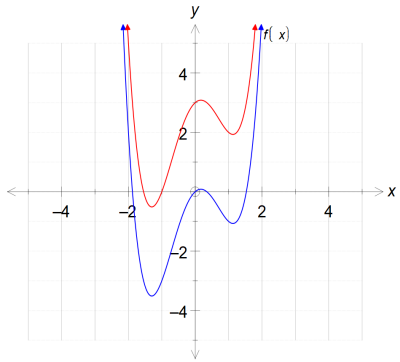
a) $f(x)=\sqrt{x}$

b) $f(x)=\frac{1}{x}$ or $f(x)=x^{-1}$

c) $f(x)=x^2$

PRACTICE PROBLEMS 1

a) Drawn to the right is the graph of $f(x)=x^4-3x^2+x$

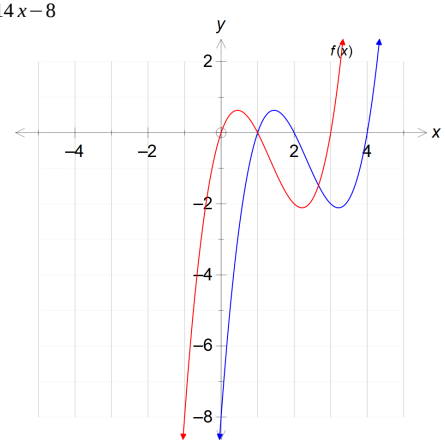


i. Using the terminology for transformations from page 1, describe the transformation on *f(x)* if the equation becomes
$f(x)+3=x^4-3x^2+x+3$.

Vertical translation of 3 units

ii. Add the sketch of $f(x)+3$ to the same set of axes.

b) Drawn to the right is the graph of $f(x)=x^3-7x^2+14x-8$



i. Write down the new equation for $f(x+1)$

$(x+1)^3-7(x+1)^2+14(x+1)-8$

ii. Add the sketch of $f(x+1)$ to the same set of axes.

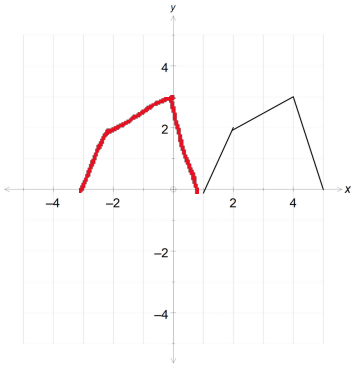
iii. Fully describe the transformation.

Horizontal translation 1 unit to the left

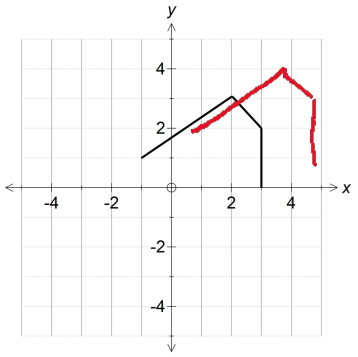
PRACTICE PROBLEMS 2

Draw each transformation on the original axes shown.

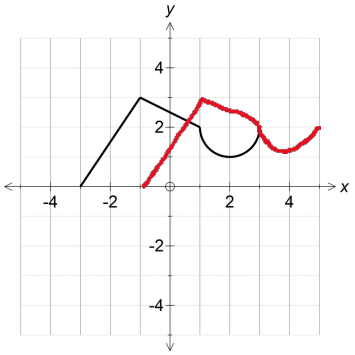
$f(x) \rightarrow f(x+4)$



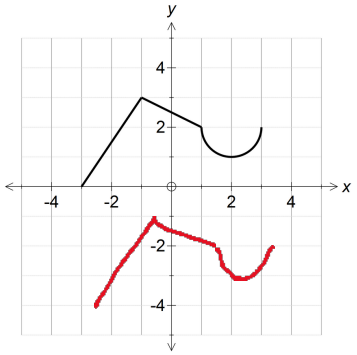
$f(x) \rightarrow f|x|+1$



$f(x) \rightarrow f(x-2)$



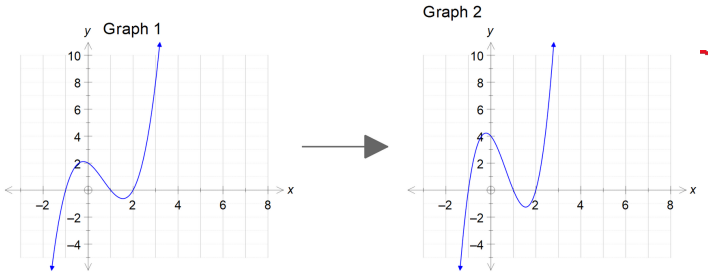
$f(x) \rightarrow f(x-1)-4$



PRACTICE PROBLEMS 7

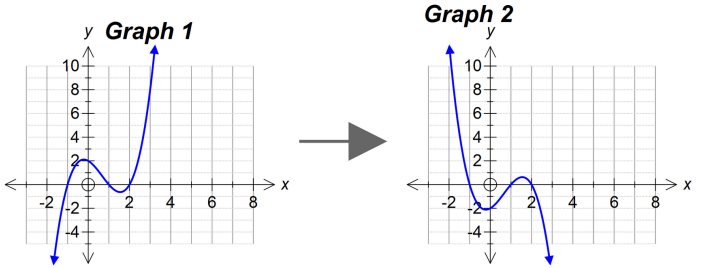
Describe the transformation(s) that take place to create the second graph from the first:

1.



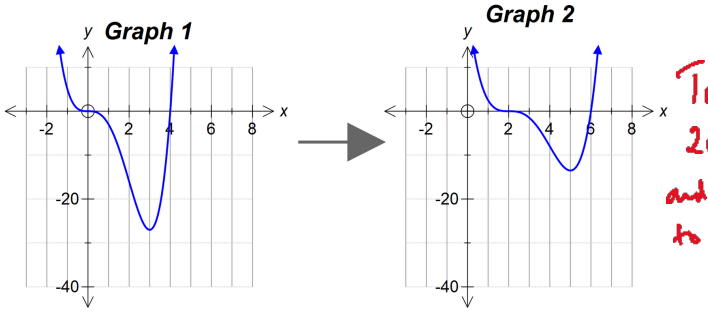
Dilation parallel to y-axis SF 2

2.



Reflection in x-axis

3.



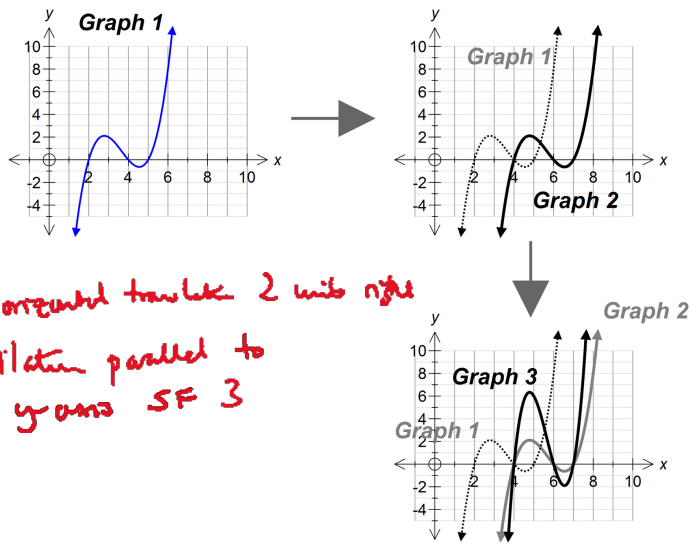
Translation 2 units right and dilation parallel to y-axis SF ½

END OF INVESTIGATION

PRACTICE PROBLEMS 6

Describe ***in order*** the transformation that take place from graph 1 to graph 3 in each case:

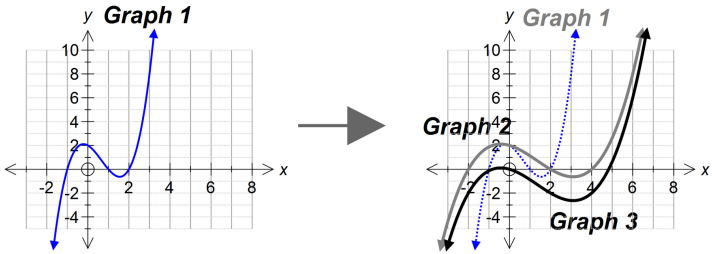
1.



Horizontal translation 2 units right

Dilation parallel to y-axis SF 3

2.



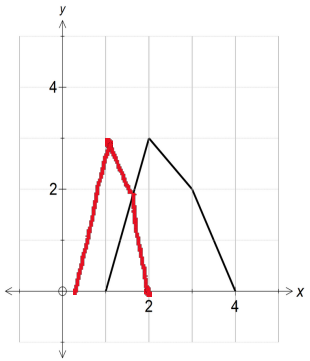
Dilation parallel to x-axis SF 2

Translation 2 units vertically down

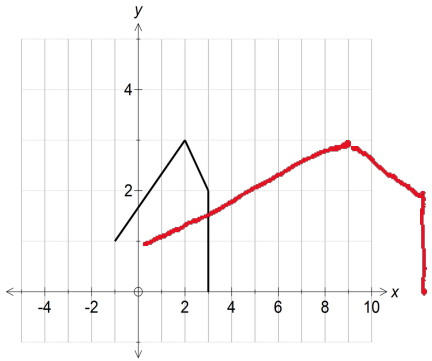
PRACTICE PROBLEMS 3

Draw each transformation on the original axes shown.

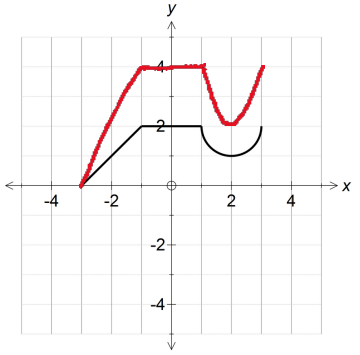
$f(x) \to f(2x)$



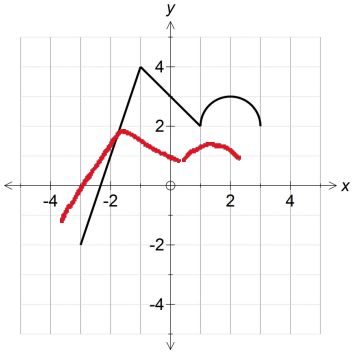
$f(x) \to f\left(\frac{1}{3}x\right)$



$f(x) \to 2f(x)$



$f(x) \to \frac{1}{2}f(x)$



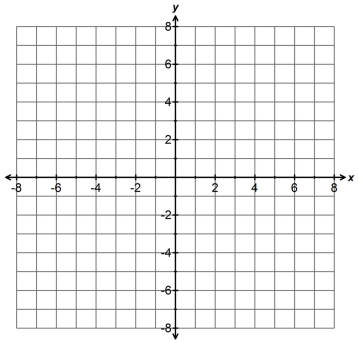
PRACTICE PROBLEMS 4 USE YOUR CLASSPAD

For each of the following:

a Use your classpad or otherwise to draw each of these original functions.
b Write down the new equation according to the transformation shown.
c Sketch the graph of both the original function and the transformation on the axes provided.

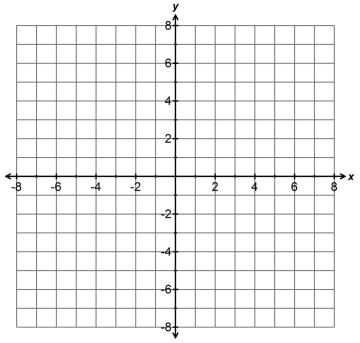
$f(x)=(x-1)(x-2)(x-4)$ with transformation $f(x)\to 2f(x)$.

$2f(x)=2(x-1)(x-2)(x-4)$



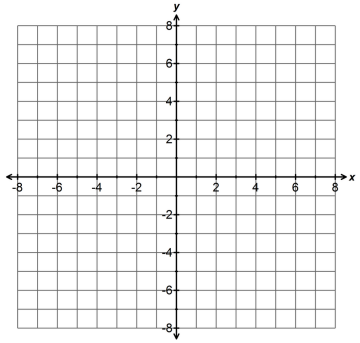
$f(x)=x^3-3x$ with transformation $f(x)\to -f(x)$.

$-f(x)=$ ¿ $-x^3+3x$



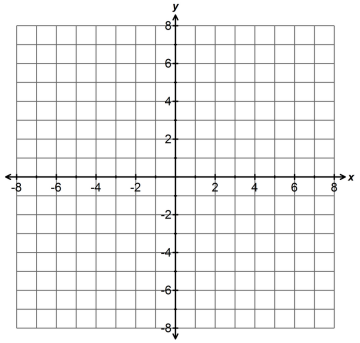
$f(x)=\frac{1}{x-3}$ with transformation $f(x)\to f(x)+2$

$f(x)+2=$ ¿ $\frac{1}{x-3}+2$



$f(x)=\sqrt{x-4}$ with transformation $f(x)\to f(x+2)$

$f(x+2)=$ ¿ $\sqrt{x-2}$

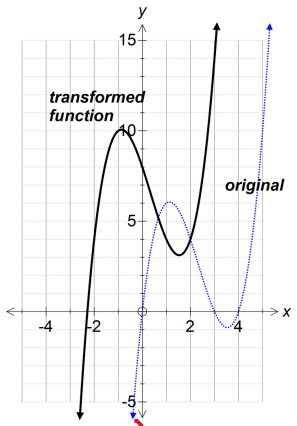


PRACTICE PROBLEMS 5

Describe ***in order*** the transformations shown by the following functions:

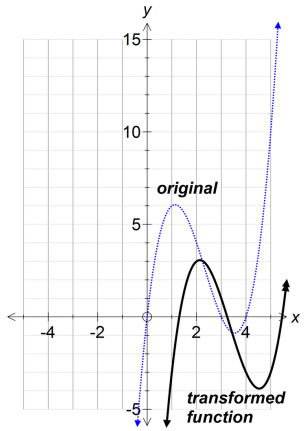
1. $f(x)\to -f(x)+3$
2. $f(x)\to 2f(x)-1$
3. $f(x)\to f(x-1)+4$
4. $f(x)\to 2f(x-3)$
5. $x^2(x-5)\to(x-2)^2(x-2-5)-1$
6. $\frac{1}{x+3}\to\frac{1}{2x+6}+5$
7. $\sqrt{x}\to 2\sqrt{x-4}$
8. $x^3(x-1)\to 4(x+1)^3x$

1. Reflect in x-axis and translate 3 units vertically up
2. Dilation parallel to y-axis SF 2 then translation 1 unit vertically down
3. Horizontal translation 1 unit right then translation 4 units vertically up or vice-versa
4. Horizontal translation 3 units right then vertical dilation SF 2 parallel to y-axis
5. Horizontal translation 2 units right then vertical translation 1 unit down
6. Dilation parallel to y-axis SF $\frac{1}{2}$ then vertical translation 5 up
7. Horizontal translation 4 units right then dilation parallel to y-axis SF 2
8. Horizontal translation 1 unit left then dilation parallel to y-axis SF 4



$f(x) \rightarrow$ $f(x+2)+4$

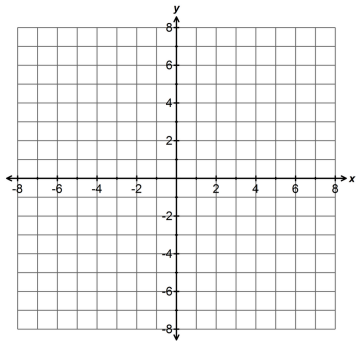
Translation through $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$



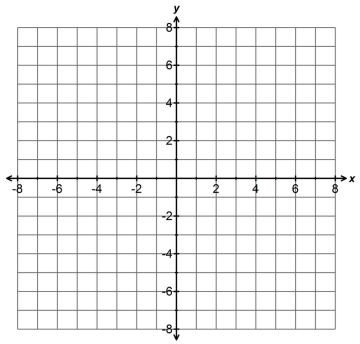
$f(x) \rightarrow$ $f(x-1)-3$

Translation through $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$f(x)=(x+1)(x+2)(x-1)$ with transformation
$f(x) \rightarrow f(x-1)$
$f(x-1)=x(x+1)(x-2)$



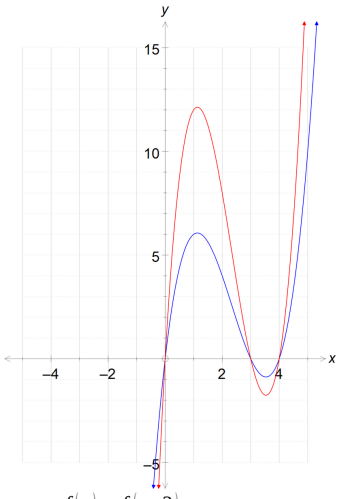
$f(x)=(x+1)^2(x+2)(x-1)$ with transformation
$f(x) \rightarrow f(-x)$
$f(-x)=(-x+1)(-x+2)(-x-1)$



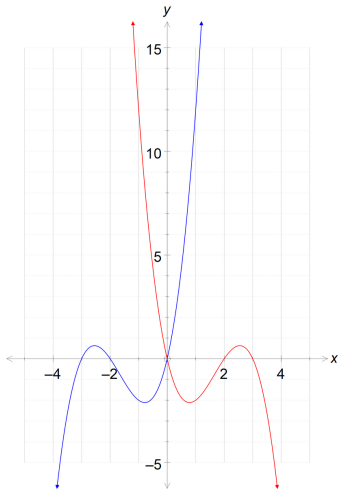
Sketch the graphs of the following transformations:

$f(x) \rightarrow 2f(x)$

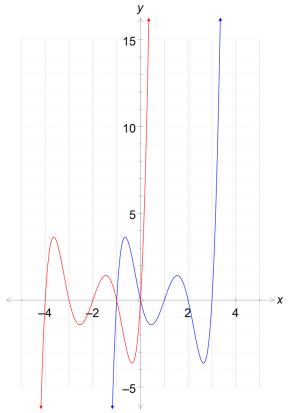
$f(x) \rightarrow f(-x)$

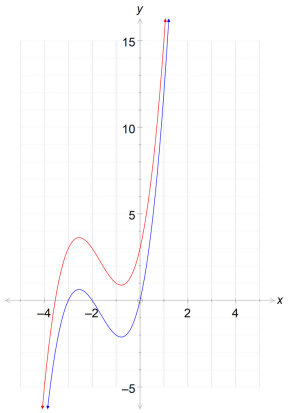


$f(x) \rightarrow f(x+3)$

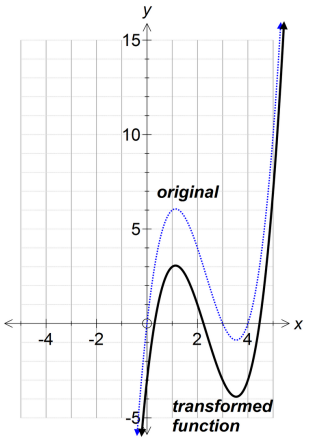


$f(x) \rightarrow f(x)+3$



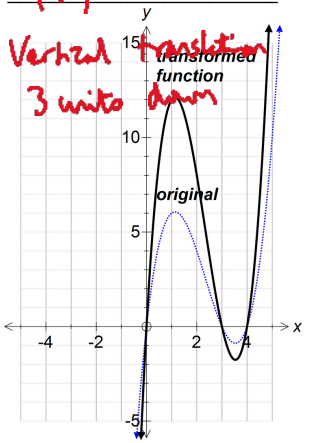


For each of the following, describe the transformation in words AND using function notation (e.g. $f(x) \rightarrow 2f(x)$ or $f(x) \rightarrow f(x-3)$)



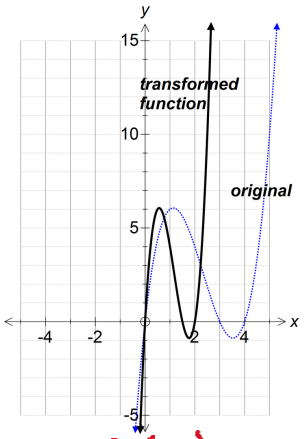
$f(x) \rightarrow$ f(x) - 3

Vertical translation 3 units down



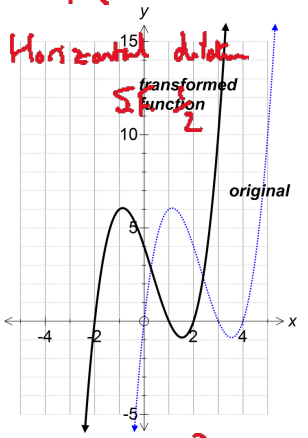
$f(x) \rightarrow$ 2 f(x)

Vertical dilation SF 2



$f(x) \rightarrow$ f(2x)

Horizontal dilation SF $\frac{1}{2}$



$f(x) \rightarrow$ f(x + 2)

Horizontal translation 2 units left