# MATHEMATICS METHODS

## MAWA Semester 2 (Units 3 and 4) Examination 2016

## Calculator-free

## Marking Key



The release date for this exam and marking scheme is

- **the end of week 1 of term 4, 2016**

**Section One: Calculator-free** **(54 Marks)**

**Question 1(a)**

Solution

$\ln m = \frac{3}{2} \Rightarrow m = e^{\frac{3}{2}}$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • identifies correct base | 1 |
| • determines correct power | 1 |

**Question 1(b)**

Solution

$\log[(m+3)m] = 1$

$(m+3)m = 10^1$

$m^2 + 3m - 10 = 0$

$(m+5)(m-2) = 0$

$m = -5$ or $2$ but since $m$ has to be greater than zero, $m = 2$ is the only solution.

| Marking key/mathematical behaviours | Marks |
|---|---|
| • applies logarithmic rule for a product correctly | 1 |
| • recognises base 10 | 1 |
| • creates equation with correct trinomial | 1 |
| • solves equation correctly giving the correct value of $m$ | 1 |

**Question 2(a)(i)**

Solution

$\frac{dy}{dx} = \frac{(6x^4 - x^3 + e)(4e^x) - (4e^x)(24x^3 - 3x^2)}{\left(6x^4 - x^3 + e\right)^2}$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • differentiates the 1st term on numerator correctly | 1 |
| • differentiates the 2nd term on numerator correctly | 1 |
| • squares factor on denominator | 1 |




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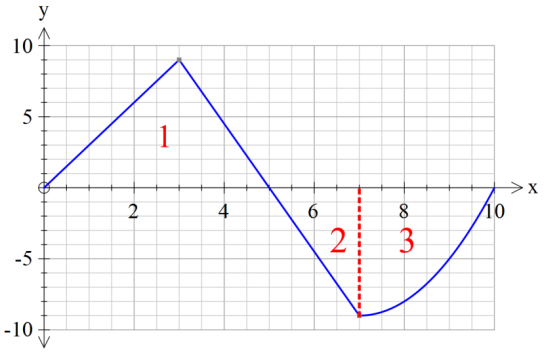
**Question 10(b)**

Solution




$\text{Area } 2 = \frac{1}{2} \times 2 \times 9$

$= 9 \text{ square units}$

$\text{Area } 3 = 50 - 22\frac{1}{2} - 9$

$= 18\frac{1}{2} \text{ square units}$

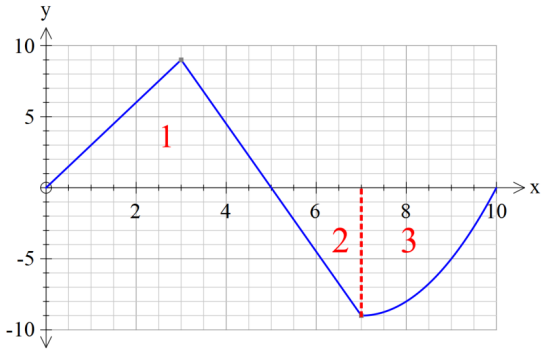
$\int_7^{10} f(x)dx = -\text{ Area } 3$

$= -18\frac{1}{2}$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • calculates area 2 | 1 |
| • calculates area 3 | 1 |
| • determines integral | 1 |

**Question 10(a)**

Solution




$$\int_0^5 f(x)dx = \text{area of triangle (Area 1)}$$

$$= \frac{1}{2} \times 5 \times 9$$

$$= 22\frac{1}{2}$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • identifies integral as area of correct triangle | 1 |
| • determines integral. | 1 |






**Question 2(a)(ii)**

Solution

$$\frac{dy}{dx} = \frac{d}{dx}[\ln(5x^3+3) - \ln(\sin(x))]$$

$$= \frac{d}{dx}[\ln(5x^3+3)] - \frac{d}{dx}[\ln(\sin(x))]$$

$$= \frac{15x^2}{(5x^3+3)} - \frac{\cos(x)}{\sin(x)}$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • applies correctly logarithmic rule for quotients | 1 |
| • differentiates correctly 1st term | 1 |
| • differentiates correctly 2nd term | 1 |

**Question 2(b)**

Solution

Let $u = x^2 - \cos(x) \Rightarrow \frac{du}{dx} = 2x + \sin(x)$ and $\frac{dy}{du} = \frac{e^u}{2}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{e^u}{2} \times (2x + \sin(x)) = \frac{e^{x^2-\cos(x)}}{2}(2x + \sin(x))$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • differentiates correctly to determine1st factor in chain rule | 1 |
| • differentiates correctly to determine 2nd factor in chain rule | 1 |
| • expresses $\frac{dy}{dx}$ in terms of $x$ | 1 |

**Question 3(a)**

| Solution<br>Discrete random variable | |
|---|---|
| Marking key/mathematical behaviours | Marks |
| • determines correct category | 1 |

**Question 3(b)**

| Solution<br>Non-random variable | |
|---|---|
| Marking key/mathematical behaviours | Marks |
| • determines correct category | 1 |

**Question 3(c)**

| Solution<br>Continuous random variable | |
|---|---|
| Marking key/mathematical behaviours | Marks |
| • determines correct category | 1 |






**Question 9(c)(i)**

Solution

$n_2$ is larger than $n_1$

To increase confidence a larger interval is required for a stable sample size. Increasing n reduces the standard error and thus the interval can remain the same.

| Marking key/mathematical behaviours | Marks |
|---|---|
| • states $n_2$ is larger with reason | 1 |
| • states correct reason | 1 |

**Question 9(c)(ii)**

Solution

$$E_1 = 1 \times \sqrt{\frac{m(1-m)}{n_1}}$$

$$E_2 = 1.5 \times \sqrt{\frac{m(1-m)}{n_2}}$$

Same interval so $E_1 = E_2$

$$\sqrt{\frac{m(1-m)}{n_1}} = 1.5 \times \sqrt{\frac{m(1-m)}{n_2}}$$

$$\frac{m(1-m)}{n_1} = (1.5)^2 \frac{m(1-m)}{n_2}$$

$$\frac{n_2}{n_1} = 2.25$$

$$n_2 = 2.25 n_1$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • equates $E_1$ and $E_2$ | 1 |
| • squares both sides | 1 |
| • states relationship | 1 |

**Question 9(a)(ii)**

Solution

$$E = 2 \times \sqrt{\frac{\frac{1}{5}(1-\frac{1}{5})}{100}}$$
$$= 2 \times \sqrt{\frac{4}{2500}}$$
$$= 2 \times \frac{2}{50}$$
$$= 0.08$$

95% CI is $(0.12, 0.28)$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • substitutes values for z, n and p | 1 |
| • simplifies square root | 1 |
| • simplifies E | 1 |
| • states interval | 1 |

**Question 9(b)**

Solution

$$E = 1 \times \sqrt{\frac{m(1-m)}{n_1}}$$

68% CI is $(m - \sqrt{\frac{m(1-m)}{n_1}}, m + \sqrt{\frac{m(1-m)}{n_1}})$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • determines E | 1 |
| • states confidence interval. | 1 |






**Question 4**

Solution

$$k\int_0^1 x - \frac{x^3}{3}\, dx = 1$$
$$k\left[\frac{x^2}{2} - \frac{x^4}{12}\right]_0^1 = 1$$
$$k\left[\frac{1}{2} - \frac{1}{12}\right] = 1 \quad \Rightarrow \quad k = \frac{12}{5}$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • sets up integral and equates to one | 1 |
| • integrates correctly | 1 |
| • evaluates integral correctly | 1 |
| • calculates the value of $k$ | 1 |
| | |

**Question 5**

Solution

$$p(1-p) = \left(\frac{\sqrt{3}}{4}\right)^2 = \frac{3}{16}$$
$$16p^2 - 16p + 3 = 0$$
$$(4p-1)(4p-3) = 0 \quad \Rightarrow \quad p = \frac{1}{4} \text{ or } p = \frac{3}{4}$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • sets up equation using variance of a Bernoulli distribution | 1 |
| • derives quadratic equation | 1 |
| • factorises trinomial | 1 |
| • solves correctly for $p$ | 1 |

**Question 6(a)**

Solution

Function is valid for $x > -3$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • correctly states the values of $x$ for which the function is valid | 1 |

**Question 6(b)**

Solution

$$\frac{dy}{dx} = \frac{2}{2x+6} = 4 \Rightarrow \frac{2x+6}{2} = \frac{1}{4} \Rightarrow x+3 = \frac{1}{4} \Rightarrow x = -2.75$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • differentiates correctly | 1 |
| • solves equation correctly | 1 |

**Question 7(a)**

Solution

| $y$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(Y=y)$ | **0** | $\boldsymbol{k}$ | $4k$ | $\boldsymbol{9k}$ | $\boldsymbol{16k}$ |

| Marking key/mathematical behaviours | Marks |
|---|---|
| • correctly completes two values | 1 |
| • correctly completes 4 values | 1 |

**Question 7(b)**

Solution

$$k + 4k + 9k + 16k = 1$$

$$30k = 1 \Rightarrow k = \frac{1}{30}$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • sums probabilities equal to one | 1 |
| • correctly solves equation for $k$ | 1 |

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**Question 8**

Solution

$$f(x) = \int f'(x)\,dx$$

$$= \int 2xe^{3x^2-1}\,dx$$

$$= \frac{1}{3}e^{3x^2-1} + c$$

since $f(0) = 0$:

$$0 = \frac{1}{3}e^{-1} + c$$

$$c = -\frac{1}{3e}$$

$$f(x) = \frac{1}{3}e^{3x^2-1} - \frac{1}{3e}$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • determines indefinite integral | 1 |
| • substitutes initial conditions to calculate the constant $c$ | 1 |
| • states f($x$) | 1 |

**Question 9 (a)(i)**

Solution

$$\hat{p} = \frac{20}{100} = \frac{1}{5}$$

| Marking key/mathematical behaviours | Marks |
|---|---|
| • determines the proportion | 1 |