

No other items may be taken into the examination room. It is **Your responsibility** to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you do have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Important note to candidates

examination

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

To be provided by the candidate

Formula Sheet (extracted from Section One)

This Question/Answer Booklet

Material required/recommended for this section

Working time for this section: 100 minutes

Reading time before commencing work: 10 minutes

Time allowed for this section

In words

Student Number: In figures

Section Two:

Calculator-assumed

3A/3B

MATHEMATICS

Year Group _____

Teachers Name _____

Student Name SOLUTIONS

PERTH MODERN SCHOOL

Question/Answer Booklet

Excellence • Integrity • Dedication



Perth Modern School
Semester One Examination, 2011

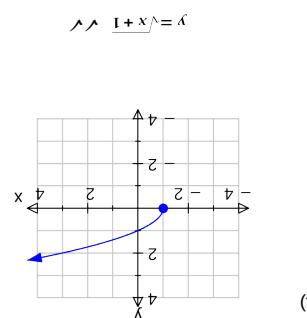
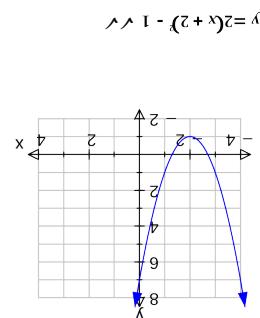
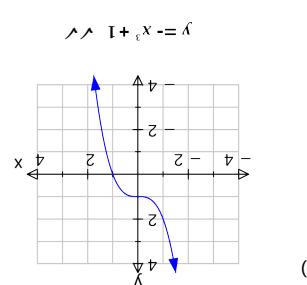
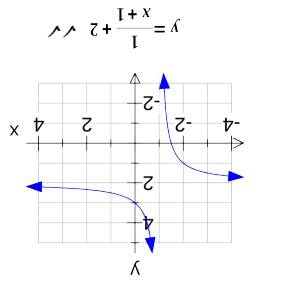
Excellence • Integrity • Dedication

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	11	11	50	40
Section Two Calculator-assumed	15	15	100	80
				120

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.



Determine the equation of the following curves.

Question 1 [2, 2, 2 marks]

Section Two: Calculator-assumed

(80 Marks)

Question 2 [2, 3 marks]

- a) If $X \sim N(2, 1)$ and $\Pr(X < k) = 0.9772$, then find the value of k .

k= 3.9990 (4.d.p) ✓✓

- b) Two food manufacturers sell 650-gram cans of peas. Company A's cans have a mean of 660 grams and a standard deviation of 7 grams, while Company B's cans have a mean of 668 grams and a standard deviation of 14 grams. From which company are you more likely to get a can that weighs 650 grams or more?

For Company A

Pr(X > 650) = 0.9235 ✓

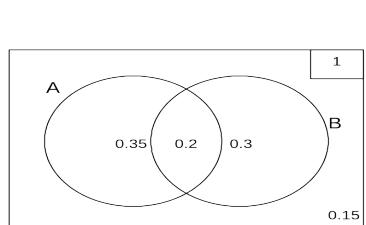
For Company B

Pr(X > 650) = 0.9008 ✓

Company A's cans are preferable according to the given criteria. ✓

Question 3 [½, ½, 1, 2, 1 marks]

The Venn Diagram below shows the probabilities of events A and B occurring.



Find:

a) $P(A \cup B) = 0.85$ ----- $\frac{1}{2}mk$

b) $P(A \cap B) = 0.2$ ----- $\frac{1}{2}mk$

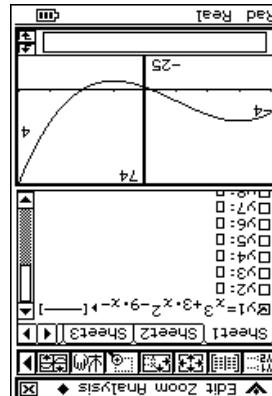
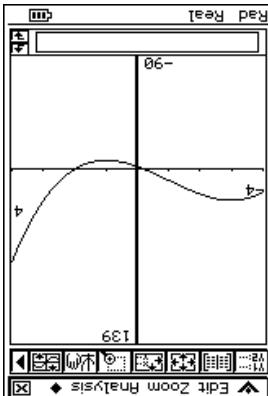
c) $P(\overline{A} \cap B) = 0.8$ ✓-----

d) $P(\overline{A} | B) = 0.6$ ✓-----

e) $P(A | \overline{A} \cup B) = 0$ ✓-----

Below are the graphs of the function $y = x^3 + 3x^2 - 9x - 2$

Question 4 [1, 2, 2, 1 marks]



a) State the domain of the graph as shown in the diagrams above.

$$\text{domain } x: -4 \leq x \leq 4$$

a)

- i) State, to 1 d.p., where necessary, the coordinates of the x and y intercept(s) of the function over the given domain.
 ii) Give the coordinates of any turning points over the given domain and state whether they are a maximum or a minimum.

$$\begin{array}{l} \text{y intercept: } (0, -2) \\ \text{domain} \end{array}$$

x intercepts: (-4.8, 0), (-0.2, 0) and (2, 0) ✓ -4.8 outside

- i) Using an appropriate method:
 ii) State, to 1 d.p., where necessary, the coordinates of the x and y intercept(s) of the function over the given domain.

$$(-1, 9) \checkmark$$

iii)

Give the coordinates of any point(s) of inflection over the given domain.

$$\begin{array}{l} \text{min. t.p.: } (1, -7) \\ \text{max. t.p.: } (-3, 25) \end{array}$$

ii)

Give the coordinates of any turning points over the given domain and state whether they

Question 15 [5 marks]

Amy sits for three tests scoring 56%, 87% and 65%. If the means are 60%, 75% and 51% respectively with corresponding standard deviations of 15.2, 9.3 and 6.5, standardise each test score and determine which test was her best effort.

Is Amy is improving in this subject?

Justifying your answer.

Test 1: $z = \frac{56 - 60}{15.2} = -0.26$ ✓

Test 2: $z = \frac{87 - 75}{9.3} = 1.29$ ✓

Test 3: $z = \frac{65 - 51}{6.5} = 2.15$ ✓

Test 3 was her best effort.

She improved with each test ✓

$$P(X = 10) = 0$$

↗

$$P(8.5 \leq X \leq 9) = 0.0819$$

↗

$$P(X \geq 7 | X \leq 12) = \frac{P(X \leq 12)}{P(X \leq 7)}$$

↗

$$= \frac{0.8413}{0.8413 - 0.0668}$$

↗

$$= 0.8413$$

↗

$$= 0.9206$$

↗

2. Find the following probabilities:
Let X be a random variable which is normally distributed with a mean of 10 and a standard deviation of

Question 6 [1, 1, 3 marks]

- | Is not allergic to grass given they have Eczema? | | Is not allergic to grass but does have Eczema? | |
|--|--|--|-------------------|
| | | 172 ✓ | 172 ✓ |
| iii) | Is not allergic to grass? | 352 ✓ | 560 ✓ |
| ii) | Is not allergic to grass but does have Eczema? | 172 ✓ | 172 ✓ |
| i) | What is the probability that a randomly chosen child | is allergic to grass? | |
| ✓✓✓ -1 / error | | | |
| | TOTAL | 352 | 208 |
| | NO Eczema | 52 | 36 |
| | Eczema positive | 300 | 172 |
| | Allergic to grass | Not Allergic to grass | Allergic to grass |

	Allergic to grass	Not Allergic to grass	TOTAL
Eczema positive	300	172	472
No Eczema	52	36	88
TOTAL	352	208	560

The Department of Environment and Conservation were unable to determine if their toxophilic program at Wellington Dam had been successful. Evidence of this would be an increase in the numbers of previously unrecorded Chuditch, tagged them and then released them. A sample of 30 that was subsequently recaptured, included 4 tagged Chuditch. Assuming that this sample reasonably represents the total Chuditch population, calculate the total Chuditch population.

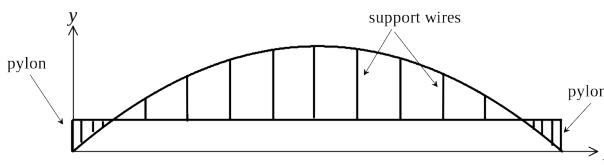
$$\frac{30}{4} = \text{total number of Chuditch}$$

MATHEMATICS 3A/3B Semester One EXAMINATION Calculator Assumed

Question 7 [4, 1, 1, 2, 2 marks]

The Sydney Harbour Bridge has a main steel arch in the shape of a parabola. If the base of the left pylon is taken as the origin then the height y of the arch is given by

$$y = -0.01x^2 + 2.3x, \text{ where } x \text{ is the horizontal distance spanned, from the base of the left pylon.}$$



If the road is situated 40 metres above the base of the pylon, determine (to 2 decimal places):

- a) i) where the arch meets the road

$$x = 18.95\text{m } \checkmark \quad \text{and} \quad x = 211.05\text{m } \checkmark$$

- ii) hence the length of road between the points where this occurs.

$$\therefore \text{Road length} = 211.05 - 18.95\text{m } \checkmark = 192.10\text{m } \checkmark$$

- b) the maximum height of the arch above the road level.

$$\text{Max height of arch} = 132.25\text{m}$$

$$\text{Max ht. of arch} = 132.25 - 40 = 92.25\text{m } \checkmark$$

- c) If the bases of the pylons are at sea level and adjoin each bank, how wide is the expanse of water?

$$230 \text{ m } \checkmark$$

- d) What is the length of the support wires 50 metres from where the arch meets the road?

50 metres from where the arch meets the road is when

$$x = 68.95\text{m and } y = 111.04\text{m } \checkmark$$

$$\therefore \text{length of support wire} = 111.04 - 40 = 71.04\text{m } \checkmark$$

- e) If a worker is painting the arch, where will the worker be (measured from the left end of the arch) if they are 30 metres above the road?

$$\text{i.e. when } y = 70$$

$$x = 36.10\text{m } \checkmark \quad x = 193.90\text{m } \checkmark$$

See next page

Question 13 [3, 2, marks]

- a) Clever Senior High School specialises in mathematics education. The Principal wishes to survey 150 of the student body using a **stratified sample** to determine how well the school is meeting their educational needs.

Year	8	9	10	11	12
Student No.	314	270	205	179	152

- i) Calculate the number of students to be surveyed from each year group. Show full working.

$$\text{Year 8} = \frac{314}{1120} \times 150 = 42 \quad \checkmark$$

$$\text{Year 9} = \frac{270}{1120} \times 150 = 36 \quad \checkmark$$

$$\text{Year 10} = \frac{205}{1120} \times 150 = 27 \text{ accept 28} \quad \checkmark$$

$$\text{Year 11} = \frac{179}{1120} \times 150 = 24 \quad \checkmark$$

$$\text{Year 12} = \frac{152}{1120} \times 150 = 20 \quad \checkmark$$

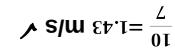
- ii) Justify with clear reasoning why just using thirty students from each year would not give a representative sample.

Different proportion of students in each year group make up the school population.

Therefore, different proportion and hence number of students are required to represent each year group. ✓✓

Question 14 [2 marks]

See next page



(d) What was the average speed on the homeward journey

then total number of arrangements = $24 \times 2 \times 2 \times (3 \times 2 \times 1)$

sugar and milk jug as 1 $2 \times 1 = 2$

2 coffee mugs as one $2 \times 1 = 2$

Consider 4 tea cups as 1

(c) The tea cups must be grouped together, the coffee mugs must be grouped together, the sugar bowl and milk jugs must be grouped together.

= 1440✓

(b) the sugar bowl must go on one end and the milk jug must go on the other end.

(aa) there are no restrictions. $= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$ ✓

In how many ways can 8 new items she has purchased be arranged, in a row, if she has 8 new items?

There are 6 people in Jord's family, four who drink tea and two who drink coffee. Jord has purchased three new cups, two new mugs and a matching sugar bowl and milk jug. Each cup and mug has the name of the person in the family who drinks it printed on the front.

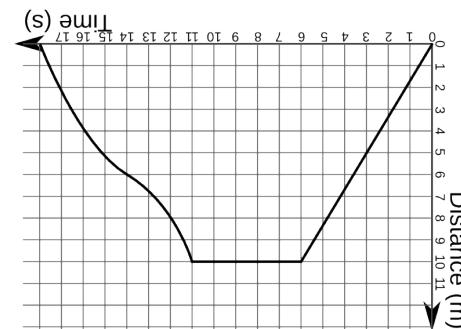
Question 9

$$\frac{\overline{C_2 D_1} \cdot \overline{C_2 E_1}}{\overline{q_1 q_2} \cdot \overline{q_1 q_3}} = \frac{\overline{C_2 D_1} \cdot \overline{C_2 F_1}}{\overline{q_1 q_2} \cdot \overline{q_1 q_4}}$$

Simplify and express with positive indices [*Show All Working*]

Question 8 [4 marks]

At one stage the cymene were some distance from their mother and she travelled out to bring them back to the nest. How far were the cymene from their mother?



A Western Australian black swan (Cygnus atratus) may never travel far from its mother. Below is a graph of the movement of a nestling swan (body swan). The distance she travels is in metres (m) from the nest and the time taken is in seconds (s).

Question 12 [1, 1, 1, 1 marks]

Question 10 [1, 1, 2 marks]

A tin of a new brand of coffee claims to hold 150 grams. In fact, the weight of the coffee in the tin is normally distributed with a mean of 151 grams and a standard deviation of 2 grams.

Determine the probability that a tin of this brand of coffee holds

a) exactly 152 grams of coffee. = 0 ✓

b) at least 149 grams of coffee. = $P(X > 149) = 0.84$ ✓

c) at least 149 grams of coffee given that it holds no more than 152 grams.

$$= P(X > 149 | X < 152) = \frac{P(X > 149) - P(X < 152)}{P(X < 152)}$$

$$= \frac{0.5328}{0.6914} \quad \checkmark$$

$$= 0.7705 \quad \checkmark$$

Question 11 [5 marks]

The high school 'Battle of the Bands' is judged on three criteria: musical ability, T.V. Appeal and an audience vote. Two specialist judges give a rating on their area of speciality only (One judge per criterion) and there is an overall audience vote. Each category is rated out of 100 and a weighting is applied to each category.

The categories and weightings are as follows.

	Categories		
	Musical Ability	T.V. Appeal	Audience Vote
Weights→	5	3	3
Dunridge SHS	95	76	87
Belcraig SHS	75	92	96
Leemton SHS	98	57	46
Willing SHS	65	98	92
Church Mod SHS	84	94	92

Rank the schools in order with the school with the highest weighted mean ranked first. Show full working.

DSHS $95 \times 5 + 76 \times 3 + 87 \times 3 = 964$ ✓ ②

BSHS $75 \times 5 + 92 \times 3 + 96 \times 3 = 939$ ✓ ③

LHS $98 \times 5 + 57 \times 3 + 46 \times 3 = 799$ ✓ ⑤

WSHS $65 \times 5 + 98 \times 3 + 92 \times 3 = 895$ ✓ ④

CMSHS $84 \times 5 + 94 \times 3 + 92 \times 3 = 978$ ✓ ①