

Question	Marks	Max	Question	Marks	Max
12		10			
11		10			
10	8	16			8
9	8	15			10
8	15	14			12
7	8	13			11

**Important note to candidates**  
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

**Special items:** drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination.

**Standard items:** pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, ruler, highlighters

**To be provided by the candidate**  
Formula sheet (relinquished from Section One)

**To be provided by the supervisor**  
This Question/Answer booklet

**Materials required/recommended for this section**

**Time allowed for this section**  
Reading time before commencing work: ten minutes  
Working time: one hundred minutes

Your Teacher's Name:

Your Name:

**MATHEMATICS METHODS**  
**UNIT 3 & 4**  
**Section Two:**  
**Calculator-assumed**

**Question/Answer booklet**

**Semester Two Examination, 2022**

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	49	33
Section Two: Calculator-assumed	11	11	100	99	67
<b>Total</b>					<b>100</b>

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

(3 marks)

$$(a) \text{ Show that } k = \frac{\ln 500}{1}$$

The median claim is \$500.

$$f(x) = \begin{cases} 0 & \text{otherwise} \\ \frac{x}{k} & 1 < x < a \end{cases}$$

which has a probability density function defined to be:  
A car insurance company models claims it pays out with a random variable  $\$X$

**Question 8 (8 marks)**

<p>Note: max of 2 marks is no working shown</p> <p>misses for mean and stdv      sets up two equations for mean and stdv      sets up one equation for mean and stdv      determines both z scores      determines one z score</p> <p><b>specific behaviours</b></p> <p><math>\mu = 25.28 \text{ and } \sigma = 3.57</math></p> <p><math>1.8808 = \frac{32 - \mu}{\sigma} \text{ and } -2.8782 = \frac{15 - \mu}{\sigma}</math></p> <p><math>\text{and } P(T &lt; 15) = 0.002 \rightarrow z_1 = -2.8782</math></p> <p><math>P(T &gt; 32) = 0.03 \rightarrow z_2 = 1.8808</math></p>	<p>Determines the mean and standard deviation of <math>T</math>.      the task in less than 15 minutes.      it is known that 3% of students complete the task in at least 32 minutes, while 0.2% complete      assumed to be normally distributed.</p> <p><b>Solution</b></p>
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(5 marks)

**Question 7 (5 marks)**

Working time: 100 minutes.

- Space pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- This section has **eleven** questions. Answer **all** questions. Write your answers in the spaces provided.
- question that you are continuing to answer at the top of the page.
- Continuing an answer: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
  - Planning: if you use the space for planning, indicate this clearly at the top of the page.

(99 Marks)

**Section Two: Calculator-assessed**

<b>Solution</b>
$\int_1^{500} \frac{k}{x} dx = 0.5$ $\therefore \left[ k \ln x  \right]_1^{500} = 0.5$ $k \ln 500 = 0.5 \rightarrow k = \frac{1}{2 \ln 500}$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>► sets up integral equation</li> <li>► integrates and subs limits</li> <li>► shows final result</li> </ul>

- (b) Determine what percentage of claims are less than \$2500. (2 marks)

<b>Solution</b>
$P(X < 2500) = \int_1^{2500} \frac{1}{(2 \ln 500)x} dx$ $= 0.629 = 62.9\%$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>► uses correct limits</li> <li>► obtains correct result</li> </ul>

- (c) The car insurance company determines that 6% of their clients submitted a claim in the past year. A sample of 320 clients is randomly selected. Describe and state the parameters of the sampling distribution of  $\hat{p}$ , the sample proportion of clients who submitted a claim in the past year. (3 marks)

<b>Solution</b>
$sd = \sqrt{\frac{0.06(1 - 0.06)}{320}} = 0.01328$ $\hat{p} \sim N(0.06, 0.01328^2)$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>► states normal</li> </ul>

▪ states mean  
▪ states std dev or variance to at least 3 dp (accept calculation as answer for variance)

End of questions

**Question 9 (9 marks)**

The score when a spinner is spun is given by the discrete random variable  $X$  with the following probability distribution, where  $a$  and  $b$  are probabilities.

$x$	-1	0	2	4	5
$P(X = x)$	$b$	$a$	$a$	$a$	$b$

- (a) Explain why  $E(X) = 2$ . (1 mark)

Solution	
The data is symmetrical around the value of $x = 2$	
Specific behaviours	
▪ explains symmetry (accept solving for $a$ & $b$ using two equations)	

- (b) Given that  $\text{Var}(X) = 7.1$ , determine the probabilities  $a$  and  $b$ . (3 marks)

Solution	
$3a + 2b = 1$	
$7.1 = (-1 - 2)^2 b + (0 - 2)^2 a + (2 - 2)^2 a + (4 - 2)^2 a + (5 - 2)^2 b$	
$20a + 26b = 11.1$	
$a = 0.1$ and $b = 0.35$	
Specific behaviours	
▪ uses total prob=1 or equation for $E(x) = 2$	
▪ uses series for variance	
▪ solves for $a$ & $b$	

The discrete random variable  $Y = 10 - 3X$ .

- (c) Find the following:

- (i)  $E(Y)$  (1 mark)

Solution	
$E(Y) = 10 - (3)(2) = 4$	
Specific behaviours	
▪ determines value	

- (ii) Hence, explain why it is not possible to conclude that the lockdown lead to Australians eating more take away food? (2 marks)

Solution
The two confidence intervals overlap, and hence it is possible that the true proportion has not increased. (SCSA approved)
Specific behaviours
✓ States the two confidence intervals overlap. ✓ Explains how this indicates that the true proportion may not have changed. OR ✓ Not every confidence interval contains the true population proportion. ✓ Therefore no inference can be made.

- (f) Assuming that sample proportion does not change, and a 95% confidence interval is used, determine how many people should be included in the survey to ensure that the researchers could conclude that Australians were eating more take away food in 2020 compared to 2019. (3 marks)

Solution
$E = 0.698 - 0.6668 = 0.0312$
$0.0312 > 1.96 \sqrt{\frac{0.698 \times 0.302}{n}}$
$n = 831.89$ (832.07 if unrounded numbers used, 842.66 if 3 decimal places used) <i>i.e. 832 (833 √ 843) people</i>
Specific behaviours
✓ Uses Upper Boundary of previous Confidence Interval to calculate E ✓ Substitutes correct z value, sample proportion, E (followed through) to at least 3 decimal places ✓ Determines n and rounds up to whole number of people

<p><b>Solution</b></p> <p>C: Undercoverage, will not sample students who don't visit pool after school.</p> <p>B: Students may not be at home / bias towards those not at the pool.</p> <p>A: Non-response, students might not want to divulge information when asked.</p> <p>A: Self-selection, only sample students who don't see link in newspaper.</p> <p><b>C</b></p>	<p><b>Specific behaviours</b></p> <p><math>0.6578 \leq p \leq 0.7382</math></p> <p><math>\sqrt{D} \text{ determines confidence interval to at least } 3 \text{ decimal places}</math></p> <p><b>Solution</b></p>
<p>(a) Briefly discuss a source of bias in each sampling method and suggest a better sampling procedure.</p> <p>C Ask students who turn up to the pool after school.</p> <p>B Visit local homes chosen at random and ask students who live there.</p> <p>A Create an online survey and publish a link to it in the local newspaper.</p> <p>The manager of a swimming pool wanted to confirm their estimate that 25% of local school students visited the pool at least once a month. The manager considered the following three ways of selecting a sample:</p> <p><b>Quesiton 10 (7 marks)</b></p>	<p><b>Specific behaviours</b></p> <p><math>p = 0.698</math></p> <p>As the new sample proportion is above the upper boundary of the confidence interval in part (b)(ii).</p> <p><math>\checkmark</math> States new sample proportion is ABOVE the upper bound of the confidence interval (must state it is above, rather than outside).</p> <p><math>\checkmark</math> Determines new sample proportion is <math>\sqrt{D}</math> above the upper bound of the confidence interval (must state it is above, rather than outside).</p> <p><b>Solution</b></p>
<p>(b) Explain why the researchers might conclude that the lockdown lead to Australians eating more take away food?</p> <p>The 2020 survey of 500 Australians aged 14 and above found that 349 people ate take away food at least once a week.</p> <p>The researchers conducted another survey in 2020 to see if nationwide lockdowns had changed the eating habits of Australians.</p> <p>For the rest of this question, assume the above sample is random and unbiased, and was conducted by a group of scientific researchers.</p> <p><b>Quesiton 10 (7 marks)</b></p>	<p><b>Specific behaviours</b></p> <p><math>\checkmark</math> Clearly EXPLAIN WHY this is biased (logic / prediction / specific to scenario)</p> <p><math>\checkmark</math> Clearly IDENTIFY the SOURCE of the bias (location / venue / food court).</p> <p><b>Solution</b></p>
<p>(c) The spinner is spun once. Find <math>P(Y &gt; X)</math>.</p> <p><math>Y &gt; X \therefore 10 - 3X &gt; X</math></p> <p><math>X &lt; 2.5 \therefore X = -1, 0, 2</math></p> <p><math>P(Y &gt; X) = 2a + b = 0.55 \quad (or \frac{20}{11})</math></p> <p><b>Quesiton 10 (7 marks)</b></p>	<p><b>Specific behaviours</b></p> <p><math>\checkmark</math> Clearly EXPLAIN WHY this is biased (logic / prediction / specific to scenario)</p> <p><math>\checkmark</math> Clearly IDENTIFY the SOURCE of the bias (location / venue / food court).</p> <p><b>Solution</b></p>

C: Convenience, only sample students who visit pool after school.  
 C: Non-response, students might not want to divulge information when asked.

#### Specific behaviours

- discusses a source of bias in A
- discusses a source of bias in B
- discusses a source of bias in C
- describes procedure involving random sampling from whole population

- (b) It was found that 42 out of a random sample of 120 students visited the centre at least once a week. Determine the 95% confidence interval for the proportion based on this data and use it to comment on the manager's estimate. (3 marks)

<b>Solution</b>
$p = \frac{42}{120} = 0.35, 0.35 \pm 1.96 \sqrt{\frac{0.35(1-0.35)}{120}} \approx (0.2647, 0.4353)$

The 95% confidence interval does not contain the manager's estimate of 0.25, and it suggests that the true value of the proportion is likely to be **higher** than 25%.

#### Specific behaviours

- indicates correct method to construct confidence interval
- correct confidence interval (to at least 2 dp)
- uses interval to dispute manager's estimate

#### Question 17

**(16 marks)**

A 2019 survey of 1000 Australians aged 14 and above found that 637 people in the sample indicated that they ate take away food at least once a week.

- (a) Determine the sample proportion for this sample. (1 mark)

<b>Solution</b>
$\hat{p} = 0.637$
<b>Specific behaviours</b>
✓ Determines sample proportion.

The survey report included a 95% confidence interval for the population proportion of Australians who ate take away food at least once a week.

- (b) (i) Complete the boxes below to form a 95% confidence interval for the population proportion of Australians who ate take away food at least once a week. (2 marks)

$$0.637 - \boxed{1.96} \times \sqrt{\frac{0.637 \times 0.363}{1000}} \leq p \leq 0.637 + \boxed{1.96} \times \sqrt{\frac{0.637 \times 0.363}{1000}}$$

<b>Solution</b>
<b>Specific behaviours</b>
✓ Uses z value for confidence interval. ✓ Writes in standard deviation (accept $\sqrt{\frac{0.637 \times 0.363}{1000}} = \sqrt{0.000231} = 0.01520623$ ).

- (ii) Hence, determine a 95% confidence interval for the population proportion of Australians who ate take away food at least once a week. (1 mark)

<b>Solution</b>
$0.6072 \leq p \leq 0.6668$
<b>Specific behaviours</b>
✓ Determines confidence interval to at least 3 decimal places

- (c) Identify and explain a possible source of bias with the following two sample schemes.

- (i) The interviewer said they were from a company that offers a healthy meal delivery service. (2 marks)

<b>Solution</b>
This is biased because of the interviewer mentioned healthy meals. People may be reluctant to be honest about how much fast food they eat as they may be judged so less people would admit to eating take away food as it is regarded as not healthy.

**Solution**

The sampling is random (each observation is independent).

The sample size is sufficiently large (typically 30 or more).

Population proportion is not too near zero nor one.

- (b) Explain why the retailer can expect the distribution of  $p$  to closely approximate normally in this case. (3 marks)
- The retailer takes a large number of random samples of 150 parts from its sales data and records the proportion  $p$  of returned parts in each sample. Under certain circumstances, the distribution of  $p$  will approximately normality.

**Solution**

$\bullet$  indicates correct binomial probability to calculate correct probability

$p(X \leq 13) = 0.2264$

$0.15 \times 88 = 13.2$

**Specific behaviours**

- (ii) Determine the probability that less than 15% of the parts sold in this batch will be returned. (2 marks)

**Solution**

$X$  is binomially distributed with parameters  $n=88$  and  $p=0.185$ .

or

$X \sim B(88, 0.185)$

$\bullet$  states binomial

**Specific behaviours**

$\bullet$  states correct parameters

- (a) Let the random variable  $X$  be the number of parts returned when a batch of 88 parts are sold. (2 marks)

- An online retailer of auto parts knows that on average, 18.5% of parts sold will be returned. (10 marks)

#### Question 11

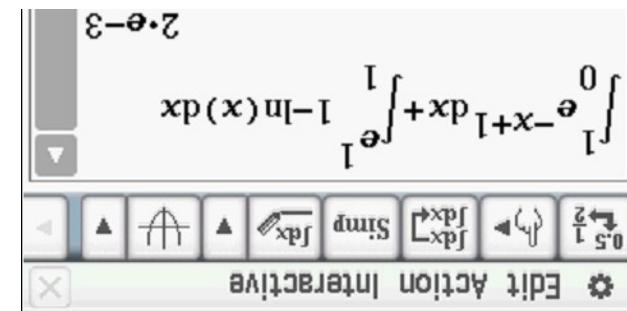
**Solution**

The new area is a reflection of the old area in the line  $y=x$ .  
 Hence  $A = 2 \left( e^{-\frac{1}{2}} - \frac{1}{2} \right) = 2e^{-\frac{1}{2}} \text{ units}^2$

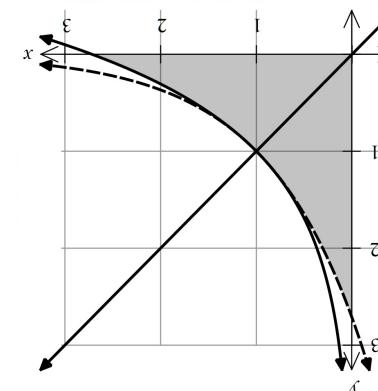
**Specific behaviours**

$y=e^{-x+1}$  or states two integrals as above with exact limits.  
 $y=e^{-x+1}$  or states two integrals which is consistent with part (c).  
 (max 1 mark if answer only as must justify for full marks)

Explains how to find the area by recognising that the graph is the inverse of  $y=e^{-x+1}$



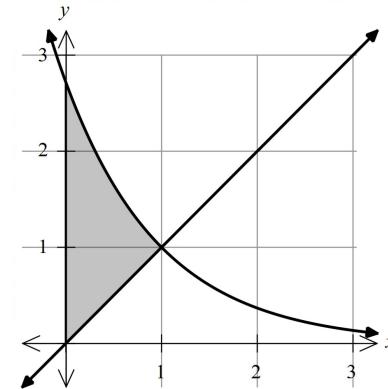
- (d) Determine the exact shaded area shown on the graph, justifying your answer. (2 marks)



Specific behaviours	
states samples are randomly selected	
states sample size sufficiently large	
Mentions central limit Theorem OR $p$ not too close to zero nor one.	

- (c) State the parameters of the normal distribution that  $\hat{p}$  approximates and use this distribution to determine the probability that the proportion of returns in a random sample of 150 parts is less than 15%. (3 marks)

Solution	
$\hat{p} \sim N(\mu_{\hat{p}}, \sigma_{\hat{p}}^2)$	
$\mu_{\hat{p}} = p = 0.185$	
$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.185(1-0.185)}{150}} \approx 0.0317, \sigma_{\hat{p}}^2 \approx 0.0010052$	
Hence normally distributed with mean 0.185 and standard deviation 0.0317.	
$P(\hat{p} < 0.15) = 0.1348$	
Specific behaviours	
states mean of distribution	
states standard deviation or variance of distribution	
correct probability	



- (b) Show that the two graphs intersect when  $x=1$ . (1 mark)

Solution	
When $x=1, y = e^{-1+1} = 1$	
Specific behaviours	
✓ Substitutes in $x=1$ and shows coordinates are the same.	

- (c) Determine the exact shaded area shown on the graph. (3 marks)

Solution	
$A = \int_0^1 e^{-x+1} dx - \int_0^1 x dx$	
$\therefore [-e^{-x+1}]_0^1 - \frac{1}{2}$	
$\therefore [-1 - (-e)] - \frac{1}{2}$	
$\therefore \left(e - \frac{3}{2}\right) \text{ units}^2$	
Specific behaviours	
✓ Writes down an appropriate integral to find the required area.	
✓ Integrates and substitutes in boundaries.	
✓ Determines area.	

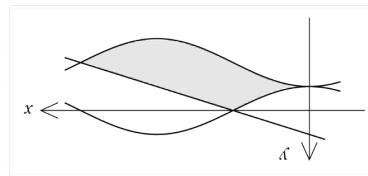
The axes below show the graphs of  $y=x$  and  $y=1-\ln x$ . The graph of  $y=e^{-x+1}$  is included as a dotted line.

<p><b>Specific behaviours</b></p> <p>writes correct integral for area between <math>x=0</math> and <math>x=2.5</math></p> <p>writes first integral</p> <p>evaluates second integral and states area of shaded region</p>
<p><b>Solution</b></p> <p>Using CAS, <math>f=h</math> when <math>x=2.5</math> and <math>g=h</math> when <math>x=7.5</math>.</p>

- (b) Determine the area of the shaded region enclosed by the three functions. (4 marks)

<p><b>Specific behaviours</b></p> <p>writes integral (may preface with negative sign - see last mark)</p> <p>evaluates integral</p> <p>clearly details with negative value of integral to obtain area</p>
<p><b>Solution</b></p> <p>Hence area is <math>\frac{25\sqrt{2}}{\pi} + 25 \approx 36.3</math> sq units.</p>

- (a) Determine the area between  $y=f(x)$ , the  $x$ -axis,  $x=3.75$  and  $x=5$ . (3 marks)



**Question 12**  
Functions  $f$ ,  $g$  and  $h$  are defined by

$$f(x) = 10 \cos\left(\frac{5}{\pi}x\right) - 20g(x) = -10 \cos\left(\frac{5}{\pi}x\right)$$

$$h(x) = 10 - 4x.$$

The graphs of these functions are shown to the right.

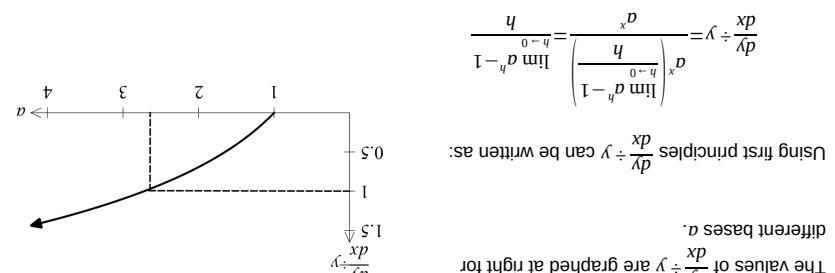
(7 marks)

<p><b>Specific behaviours</b></p> <p>writes correct integral for area between <math>x=0</math> and <math>x=2.5</math></p> <p>writes first integral</p> <p>evaluates first integral</p> <p>writes correct integral for area between <math>x=2.5</math> and <math>x=7.5</math></p> <p>writes correct integral for area between <math>x=0</math> and <math>x=7.5</math></p>
<p><b>Solution</b></p> <p><math>A = \int_{2.5}^0 g(x) - f(x) dx + \int_{2.5}^{7.5} h(x) - f(x) dx = \left(50 - \frac{100}{\pi}\right) + \left(50 + \frac{100}{\pi}\right) = 100</math> sq units</p>

The axes below show the graphs of  $y=x$  and  $y=e^{-x+1}$ .

<p><b>Specific behaviours</b></p> <p><math>a=e</math></p>
<p><b>Solution</b></p>

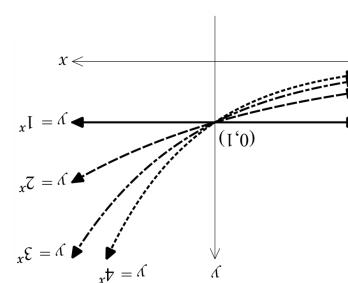
- (a) State the value of  $a$ , for which  $\lim_{h \rightarrow 0} \frac{h}{a^h - 1} = 1$ . (1 mark)



Using first principles  $\frac{dy}{dx}$  can be written as:

The values of  $\frac{dy}{dx}$  are graphed at right for different bases  $a$ .

$$\lim_{h \rightarrow 0} \frac{h}{a^h - 1} = \frac{h}{\lim_{h \rightarrow 0} a^h - 1} = \frac{h}{\lim_{h \rightarrow 0} \frac{d}{dh} a^h - 1} = \frac{h}{\lim_{h \rightarrow 0} h \frac{d}{dh} a^h - 1} = \frac{h}{\lim_{h \rightarrow 0} h a^{h-1} \ln a - 1} = \frac{h}{\ln a - 1}$$



For each of these curves the value of  $\frac{dy}{dx}$  is constant for all values of  $x$ .

The family of curves with equation  $y=a^x$  where  $a>0$ , are exponential curves, and are shown at right.

**Question 16**

**Question 13****(9 marks)**

In a random sample of 225 adult female Australians, 72 were born overseas. This data is to be used to construct a 90% confidence interval for the proportion of adult female Australians born overseas.

- (a) Determine the margin of error for the 90% confidence interval. (3 marks)

<b>Solution</b>
$p = 72 \div 225 = 0.32, \sigma = \sqrt{\frac{0.32(1-0.32)}{225}} = 0.0311$ $z_{0.9} = 1.645, E = 1.645 \times 0.0311 = 0.0512$
<b>Specific behaviours</b>
<input checked="" type="checkbox"/> correct proportion <input checked="" type="checkbox"/> correct standard deviation of sample proportion <input checked="" type="checkbox"/> correct margin of error

- (b) State the 90% confidence interval. (1 mark)

<b>Solution</b>
$p \pm E \rightarrow (0.2688, 0.3712)$
<b>Specific behaviours</b>
<input checked="" type="checkbox"/> correct interval

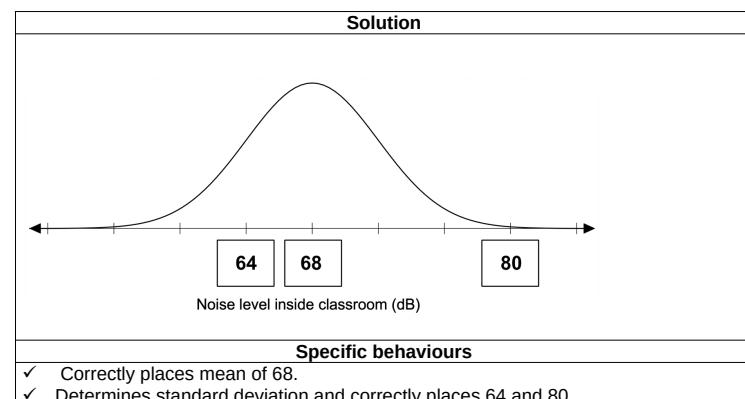
- (c) The 90% confidence interval for the proportion of adult male Australians born overseas constructed from another random sample was (0.288, 0.412). Determine the number of adult males who were born overseas in this sample. (5 marks)

<b>Solution</b>
$E =  0.412 - 0.288  \div 2 = 0.062, p = 0.288 + 0.062 = 0.35$ $\sqrt{\frac{0.35(1-0.35)}{n}} = \frac{0.062}{1.645} \rightarrow n = 160$ $X = 160 \times 0.35 = 56 \text{ males.}$
<b>Specific behaviours</b>
<input checked="" type="checkbox"/> calculates $p$ and $E$ <input checked="" type="checkbox"/> uses sample proportion stdev expression <input checked="" type="checkbox"/> calculates sample size $n$

- Substitutes into standardised score formula.
- Determines new mean.

High school students spend 45-75% of their time in the classroom listening to their teacher or classmates. Hence, classrooms can be prone to high noise levels. The noise levels inside a large number of busy classrooms were found to be normally distributed with a mean of 68 dB, and variance of 16 dB.

- (f) Write a number in each box to provide to indicate the scale of the distribution. (2 marks)



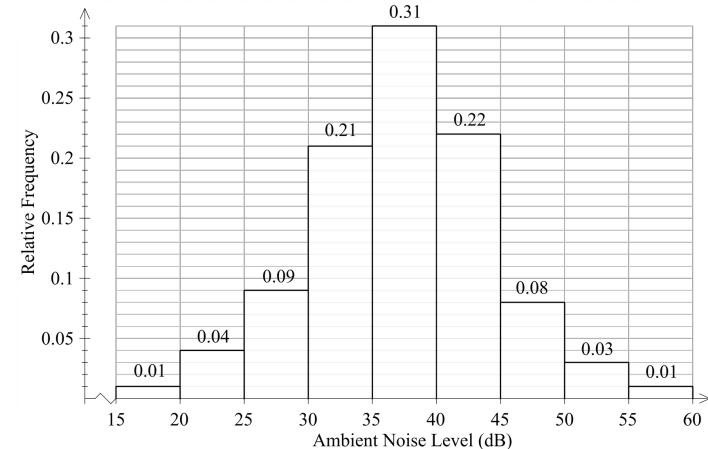
<p><b>Question 14 (8 marks)</b></p> <p>Correctly determines probability.</p> <p>States Z quantile</p> <p>Corrects one feature of the histogram that supports using a normal distribution to model the ambient noise levels.</p>	<p><b>Solution</b></p> <p>The graph has a bell shaped curve</p> <p>Refers to the shape of the distribution (Bell shape or almost symmetrical)</p> <p>Describes one feature of the histogram that supports using a normal distribution to model the ambient noise levels.</p>
<p>(a) Clearly show that <math>V(t) = -3t + c</math>.</p> <p>When humans sneeze the speed of the droplets expelleed decrease at a constant rate of <math>3 \text{ ms}^{-2}</math>.</p> <p>To stop the spread of Covid it is advised to cover your nose when sneezing.</p> <p>Correctly determines probability.</p> <p>States Z quantile</p> <p>Describes one feature of the histogram that supports using a normal distribution to model the ambient noise levels.</p>	<p><b>Solution</b></p> <p>For the data, the mean ambient noise level is <math>37.3 \text{ dB}</math> and the standard deviation is <math>7.1 \text{ dB}</math>.</p> <p>(d) (i) Using a normal distribution determine the probability a classroom has an ambient noise level of between <math>30</math> and <math>45 \text{ dB}</math>.</p> <p><math>P(30 \leq Y \leq 45) = 0.790</math></p> <p><math>Y \sim N(37.3, 7.1^2)</math></p> <p>Using the histogram above, determine the probability that a classroom as an ambient noise level of between <math>30</math> and <math>45 \text{ dB}</math>, and explain whether this supports using a normal distribution to model this data.</p> <p>(ii) Since <math>0.74</math> is close to <math>0.790</math>, it is appropriate to model this data with a normal distribution.</p> <p><math>P(30 &lt; X &lt; 45) = 0.74</math></p> <p><b>Solution</b></p> <p>Assuming the data can be modelled using a normal distribution with a mean of <math>37.3 \text{ dB}</math> and the standard deviation of <math>7.1 \text{ dB}</math>, determine</p> <p>(e) the value to which the mean ambient noise level have to be reduced in order to ensure that at most <math>2\%</math> of the classrooms would fail to meet the standard.</p> <p>Assume that the standard deviation remains unchanged.</p> <p>(3 marks)</p>
<p>(b) Determine, in terms of <math>c</math>, how long it takes the droplets to come to rest.</p> <p>Integrates <math>a = -3</math>. (or <math>V'(t) = -3</math>)</p> <p>Recognises <math>a = -3</math>.</p> <p>Sets velocity to zero and solves for <math>t</math>.</p> <p><b>Solution</b></p>	<p><b>Solution</b></p> <p>Comparing probability to part (d)(i) and concludes it is appropriate.</p> <p>Determines probability from histogram.</p> <p>Specifies behaviours</p> <p>Assume that the standard deviation remains unchanged.</p> <p>(3 marks)</p>
<p>(c) (i) Write down an integral expression for <math>d</math>, the distance travelled by the droplets from <math>t=0</math> until they come to rest.</p> <p><math>d = \int_0^3   -3t   dt</math></p> <p>Recognises distance is the integral of the absolute value of velocity function.</p> <p>Integrals includes boundaries from part (d).</p> <p>The New Scientist magazine reported in 2020 that some droplets in a sneeze can travel upwards of <math>8 \text{ m}</math>.</p> <p>Use your answer to part (c)(i) to find the initial speed of a sneeze,</p> <p>correct to two decimal places.</p> <p>(3 marks)</p>	<p><b>Solution</b></p> <p>Integrates standardised score.</p> <p><math>P(Z &gt; z) = 0.02</math></p> <p><math>z = 2.0537</math></p> <p><math>2.0537 = \frac{45 - \mu}{7.1}</math></p> <p><math>\mu = 30.4 \text{ dB}</math></p> <p>Determines standardised score.</p> <p>(3 marks)</p>

Specific behaviours
✓ Correctly integrates expression in part (c)(i).
✓ Substitutes in boundaries and solves for $c$ .
✓ States initial speed with unit

### Question 15 (13 marks)

Current Australian standards currently recommend the maximum ambient noise level for an empty classroom is 45 dB.

Acoustic consultants tested the ambient noise level in the classrooms of a newly built school. Their results are shown in the relative frequency histogram below.



- (a) Determine the proportion of classrooms that fail to meet the standard. (1 mark)

Solution
0.12
Specific behaviours
✓ Determines correct proportion.

- (b) Determine the probability that a classroom had an ambient noise level greater than 42.5 dB, given that it has an ambient noise level within 10 dB of the standard. (3 marks)

Solution
$P(X > 42.5   35 < X < 55)$
$\frac{P(42.5 < X < 55)}{P(35 < X < 55)}$
$\frac{\frac{1}{2} \times 0.22 + 0.08 + 0.03}{0.31 + 0.22 + 0.08 + 0.03}$
$\frac{0.22}{0.64} = \frac{11}{32} = 0.34375$
Specific behaviours
✓ Correctly interprets conditional probability.
✓ Correctly determines $P(42.5 < X < 55)$ .