Generalized Fixed Effects Model for Panel Data

Minsheng Liu

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Consider the following model:

$$y_{t,i} = \vec{x}_{t,i} \cdot \vec{\beta} + \vec{\phi}_t \cdot \vec{\alpha}_i + \vec{\psi}_i \cdot \vec{\gamma}_t + \epsilon_{i,t} \,,$$

where i ranges from 1 to n, t ranges from 1 to t, and $\vec{\beta} \in \mathbb{R}^m$.

Equivalently, the model can be expressed as

$$\mathbf{Y} = \sum_{i=1}^{m} \beta i \cdot \mathbf{X}_i + \mathbf{\Phi A} + (\mathbf{\Psi \Gamma})^\top + \mathbf{E} \,,$$

where

1. $\mathbf{Y}, \mathbf{X}_i, \mathbf{E} \in \mathcal{M}_{i,t}(\mathbb{R})$;
2. $\mathbf{\Phi} \in \mathcal{M}_{t,p}(\mathbb{R})$ is given, $\mathbf{A} \in \mathcal{M}_{p,i}(\mathbb{R})$ is unknown. They are called as time-observable individual-specific effects (tois);
3. $\mathbf{\Psi} \in \mathcal{M}_{i,q}(\mathbb{R})$ is given, $\mathbf{\Gamma} \in \mathcal{M}_{q,t}(\mathbb{R})$ is unknown. They are called as individual-observable time-specific effects (iots).

Note that time andindividual fixed effects can be expressed by adding a column of 1s in Φ and Ψ, respectively.

We will apply the method of alternate projections. Let

$$\mathbf{P}_{\mathsf{tois}} = \Phi(\Phi^\top \Phi)^{-1} \Phi^\top \,,$$
$$\mathbf{P}_{\mathsf{iots}} = \Psi(\Psi^\top \Psi)^{-1} \Psi^\top \,,$$

be the projection matrices for Φ and Ψ respectively. Define tois : $\{1, \ldots, n\} \to (\mathcal{M}_{t,i}(\mathbb{R}) \to \mathcal{M}_{t,i}(\mathbb{R}))$ and $\mathsf{iots} : \{1, \ldots, t\} \to (\mathcal{M}_{t,i}(\mathbb{R}) \to \mathcal{M}_{t,i}(\mathbb{R}))$ as follows[1]:

$$\mathsf{tois}(i)(\mathbf{M}) = \mathbf{M}[\mathbf{M}_{,i} \mapsto \mathbf{M}_{,i} - \mathbf{P}_{\mathsf{tois}} \mathbf{M}_{,i}] \,,$$
$$\mathsf{iots}(t)(\mathbf{M}) = \mathbf{M}[\mathbf{M}_{t,} \mapsto \mathbf{M}_{t,} - \mathbf{M}_{t,} \mathbf{P}_{\mathsf{iots}}^\top] \,.$$

Essentially, $\mathsf{tois}(i, \mathbf{M})$ "demeans" the i-th column of $\mathbf{M}$—namely the individual i; $\mathsf{iots}(t, \mathbf{M})$ "demeans" the t-th row of $\mathbf{M}$—namely the time t.

For each iteration, let $\mathbf{M}$ range from $\mathbf{Y}, \mathbf{X}_1, \ldots, \mathbf{Y}$. we have

$$\hat{\mathbf{M}} = ((\mathsf{iots}(t) \circ \cdots \circ \mathsf{iots}(1)) \circ (\mathsf{tois}(n) \circ \cdots \circ \mathsf{tois}(1)))(\mathbf{M}) \,.$$

The algorithm stops after all $\mathbf{M}$ stablizes.

[1] As in R, we use $\mathbf{M}_{,i}$ to denote the i-th column and $\mathbf{M}_{t,}$ to denote the t-th row. The notation $\mathbf{M}[X \mapsto Y]$ means the matrix M, with its row or column as specified by X replaced by Y.

Both tois and iots are so-called *higher order functions*. Namely, it produces a function $\mathcal{M}_{t,i}(\mathbb{R}) \to \mathcal{M}_{t,i}(\mathbb{R})$ with a given i or t.