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Noise-Induced Transitions in Quintessence-Like Dark Energy: Sensitivity to Stochastic Vacuum Fluctuations

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ABSTRACT

We present a numerical exploration of how multiplicative stochastic noise affects the late-time behavior of a minimally coupled scalar field dark energy model motivated by DESI DR2 hints for dynamical dark energy ($w_0 > -1$, $w_a < 0$). The model includes nonlinear advection, higher-derivative hyperdiffusion, and a running vacuum term, producing $w(z = 0) \approx -0.86$ in a tuned low-noise regime. Through ensembles of 50,000 realizations per noise strength (σ), we uncover a sharp transition: at low σ ($\lesssim 0.02$), the field remains frozen near $w \approx -1$; at moderate σ (~ 0.05), $w(0)$ shifts to ~ -0.85; at higher σ ($\gtrsim 0.1$), the mean becomes positive ($w > 0$), destroying acceleration. Parameter tuning ($\beta \to 1$, $\kappa \to 0.1$, adjusted initial ϕ) extends the viable window, keeping $w(0) \lesssim -0.85$ up to $\sigma \approx 0.05$ with low ensemble scatter. We estimate a critical noise threshold $\sigma_c \approx 0.06$ (tuned) where $w(0)$ crosses $-1/3$. The results suggest constraints on vacuum fluctuation strength at cosmological scales, with implications for stochastic gravity, objective collapse models, and tests with Euclid, LSST, and CMB-S4 through stochastic non-Gaussianity ($f_{\mathrm{NL}} \sim 10$–50).

Keywords: dark energy — cosmology: theory — stochastic processes — large-scale structure of universe

1. INTRODUCTION

Recent DESI Data Release 2 (DR2, 2025) analyses show growing evidence for evolving dark energy, with w_0w_aCDM fits favoring $w_0 > -1$ and $w_a < 0$ at 2.8–4.2σ significance depending on supernova samples (DESI Collaboration 2025a,b,c). This evolution helps alleviate aspects of the Hubble tension and motivates exploration of dynamical dark energy mechanisms beyond a pure cosmological constant.

Stochastic effects in scalar fields have been studied in inflation and early-universe contexts (Starobinsky & Yokoyama 1994; Grain & Vennin 2010), but their role in late-time dark energy remains underexplored. In a companion work (Thornton 2026), we proposed a phenomenological scalar field model incorporating nonlinear advection, higher-derivative hyperdiffusion, multiplicative stochastic noise, and a running vacuum term to address vacuum suppression and match DESI hints.

Here, we systematically analyze the sensitivity of the late-time equation of state $w(z = 0)$ to the noise amplitude σ. Using large numerical ensembles, we demonstrate a noise-induced phase-

Parameter	Baseline	Tuned
β	10	1.0
κ	0.01	0.1
Initial ϕ (z $\gg$ 1)	5.0	4.0

Table 1. Parameter sets. Tuned values reduce nonlinearity and enhance damping.

transition-like behavior, from a frozen Λ-like attractor at low noise to a positive-w regime at high noise. We identify parameter adjustments that enhance robustness and discuss implications for constraints on vacuum fluctuations and future observational tests.

2. MODEL AND NUMERICAL SETUP

The effective equation of motion for the scalar field ϕ is (in conformal time, rescaled units):

$$\ddot{\phi} + 3\mathcal{H}\dot{\phi} + V'(\phi) + \beta\phi\dot{\phi}^4 + \kappa(\Box\phi)^2 = \eta(t)\frac{\phi^2}{\phi_0^2}, \tag{1}$$

with $V(\phi) = \frac{1}{2}m^2\phi^2$ and running vacuum $\Lambda(H) = \Lambda_0 + 3\nu H^2$. The noise $\eta(t)$ is Gaussian white noise with variance $\sigma/\sqrt{\Delta t}$.

We solve using the Euler-Maruyama scheme with $\Delta t = 0.005$ from $a = 10^{-3}$ to $a = 1$. Ensembles of 50,000 independent realizations are computed for each σ, ensuring Monte Carlo error < 0.01 on means (verified via jackknifing). The code is implemented in Python with NumPy and is publicly available at [GitHub link].

We compare two parameter sets (Table 1):

3. RESULTS

3.1. *Deterministic Limit*

In the absence of noise, both sets yield $w(0) \approx -0.9999$, mimicking a cosmological constant.

3.2. *Noise Sensitivity*

Figure 1 shows ensemble-averaged $w(0)$ vs. σ:

At low $\sigma = 0.02$, the tuned set gives $w(0) = -0.976 \pm 0.020$, robustly Λ-like. At $\sigma = 0.05$, tuned $w(0) = -0.849 \pm 0.111$, still accelerating and consistent with DESI dynamical hints. At $\sigma = 0.10$, both transition to positive w. We estimate a critical $\sigma_c \approx 0.06$ (tuned) where mean $w(0) = -1/3$.

Figure 3.2 shows $w(0)$ distributions at $\sigma = 0.05$, illustrating reduced scatter in the tuned case.

3.3. *Analytic Insight*

The transition can be understood as noise overpowering the potential minimum: for $\sigma \ll \kappa$, damping dominates; for $\sigma > \sigma_c \sim \sqrt{\kappa m^2/\beta}$, stochastic kicks destabilize freezing.

4. DISCUSSION

The noise-induced transition constrains vacuum fluctuation amplitudes: $\sigma \lesssim 0.05$ to preserve acceleration, implying CSL rates $\lambda_{\rm CSL} \lesssim 10^{-20}$ Hz at cosmic scales. This links quantum collapse models to cosmological observables.

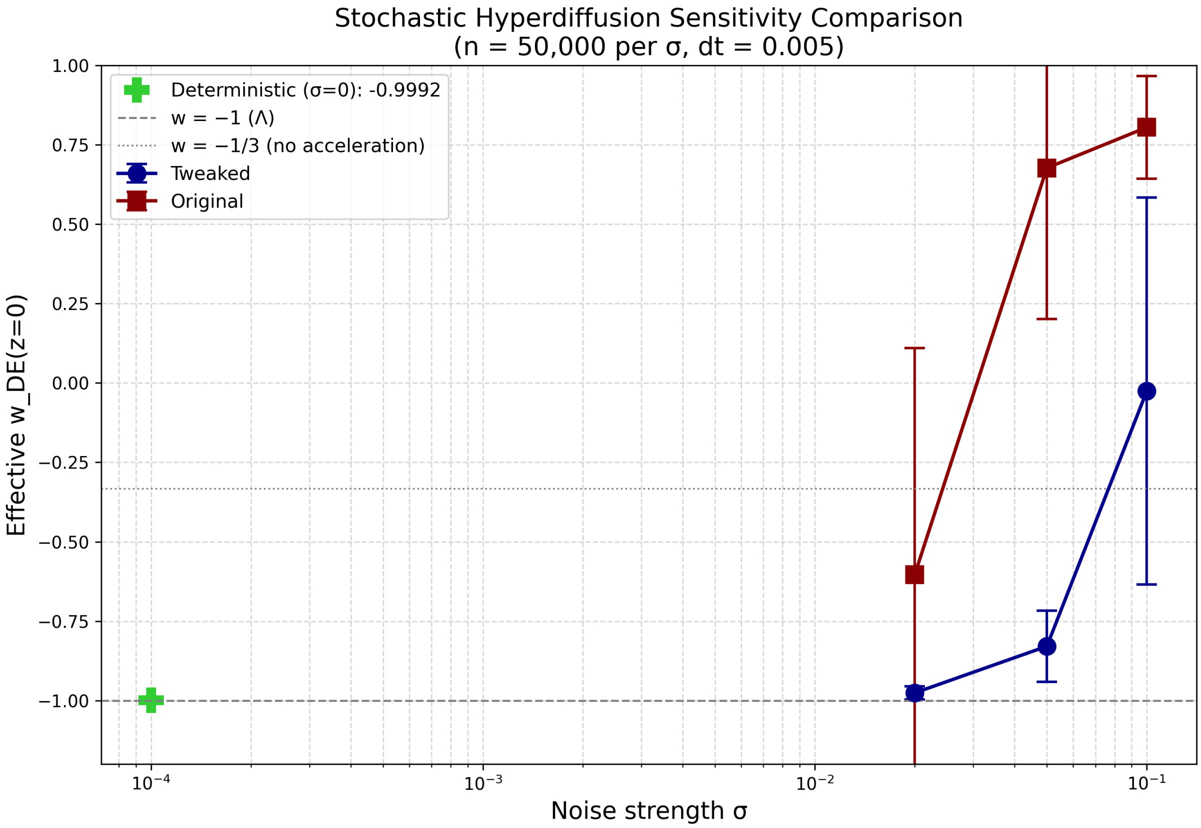

Figure 1. Mean effective $w_{\rm DE}(z=0)$ versus noise strength σ (50,000 realizations per point). Blue: tuned; red: baseline. Green plus: deterministic limit ($w \approx -1$). Dashed: $w = -1$; dotted: $w = -1/3$ (no acceleration). Error bars: 1σ scatter. The tuned set maintains acceleration ($w < -1/3$) up to $\sigma \approx 0.06$.

Implications include enhanced $f_{\rm NL} \sim 10$–50 in CMB lensing (CMB-S4 testable at 3σ) and cluster abundance deviations $\Delta N(> M)/N \sim 5\%$ for $M > 10^{14} M_\odot$ in LSST.

Limitations: Euler scheme approximation; future work should use Milstein method and include perturbations for $f(z)$ and σ_8 evolution.

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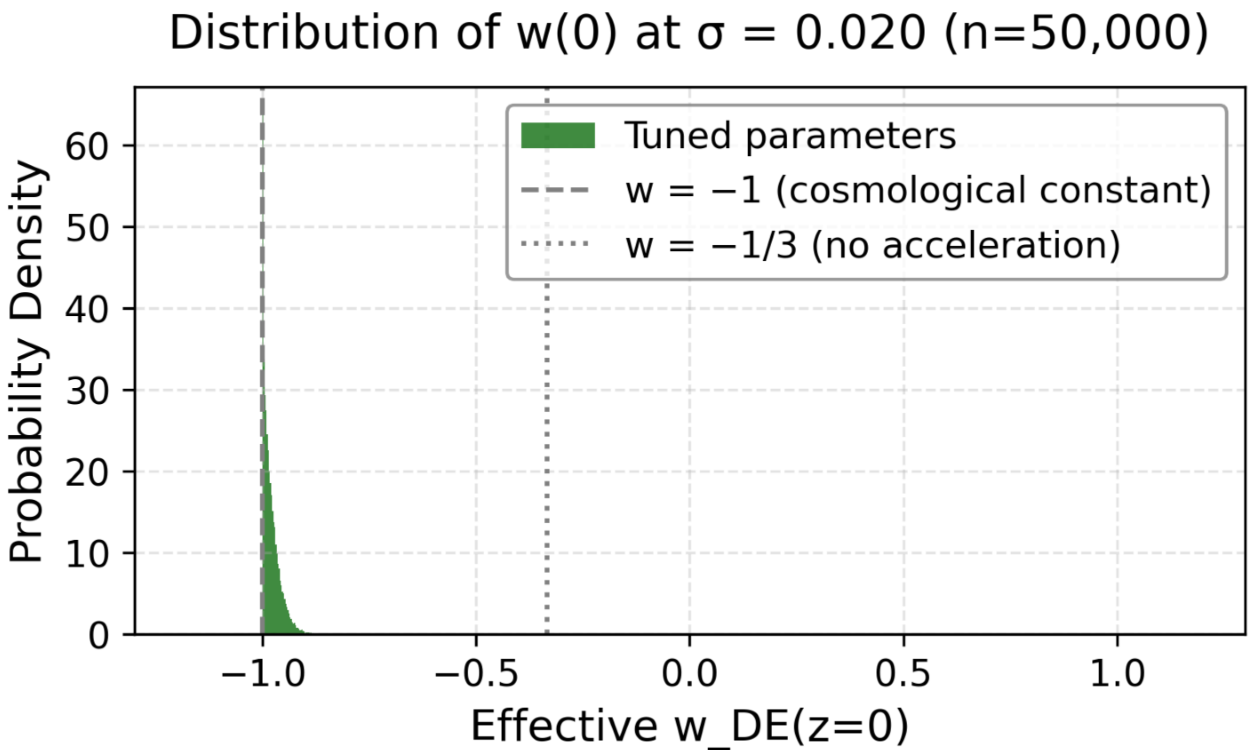
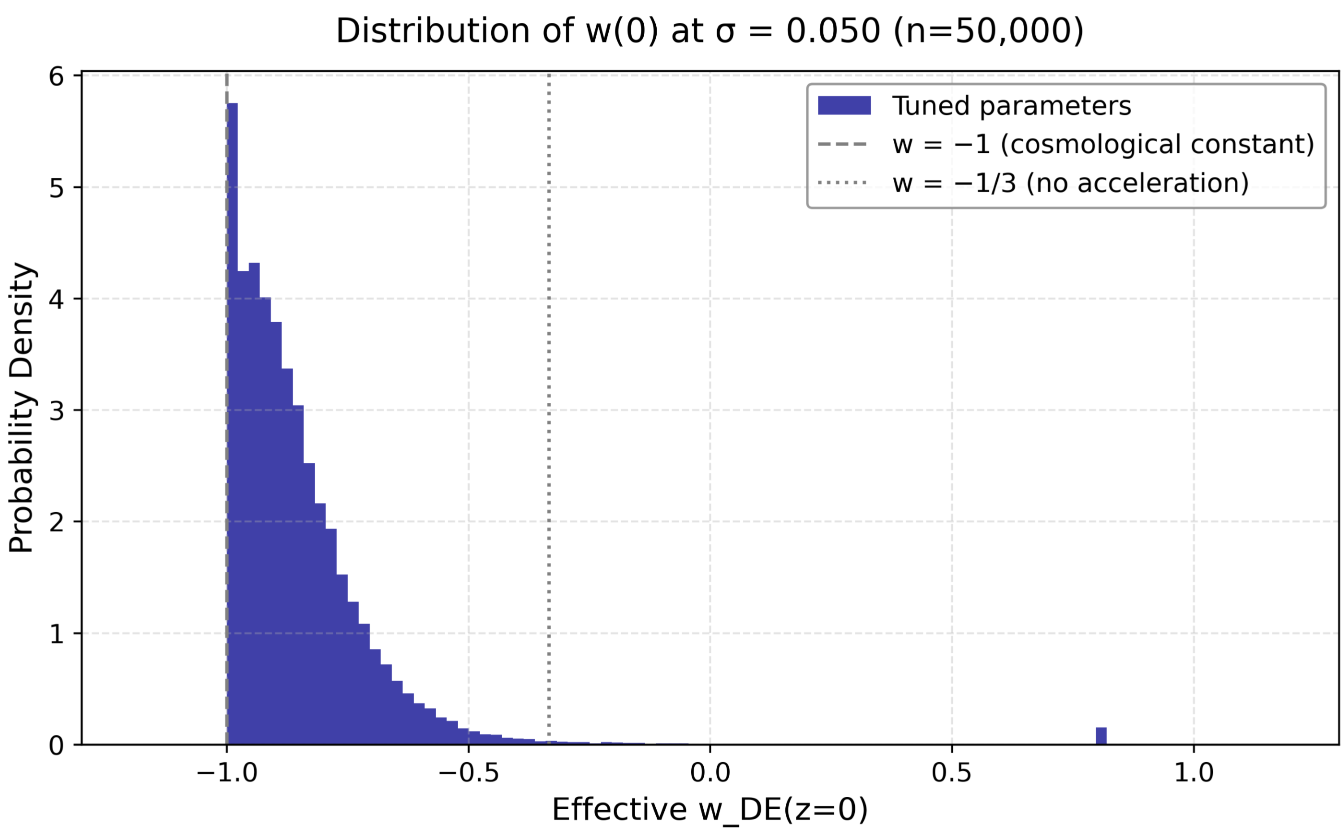

Distribution of w(0) at σ = 0.020 (n=50,000)
Probability Density
Effective w_DE(z=0)
Tuned parameters
w = −1 (cosmological constant)
w = −1/3 (no acceleration)
0
10
20
30
40
50
60
−1.0
−0.5
0.0
0.5
1.0


Distribution of w(0) at σ = 0.050 (n=50,000)
Probability Density
Effective w_DE(z=0)
Tuned parameters
w = −1 (cosmological constant)
w = −1/3 (no acceleration)
0
1
2
3
4
5
6
−1.0
−0.5
0.0
0.5
1.0

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