HW2-Q4

February 17, 2022

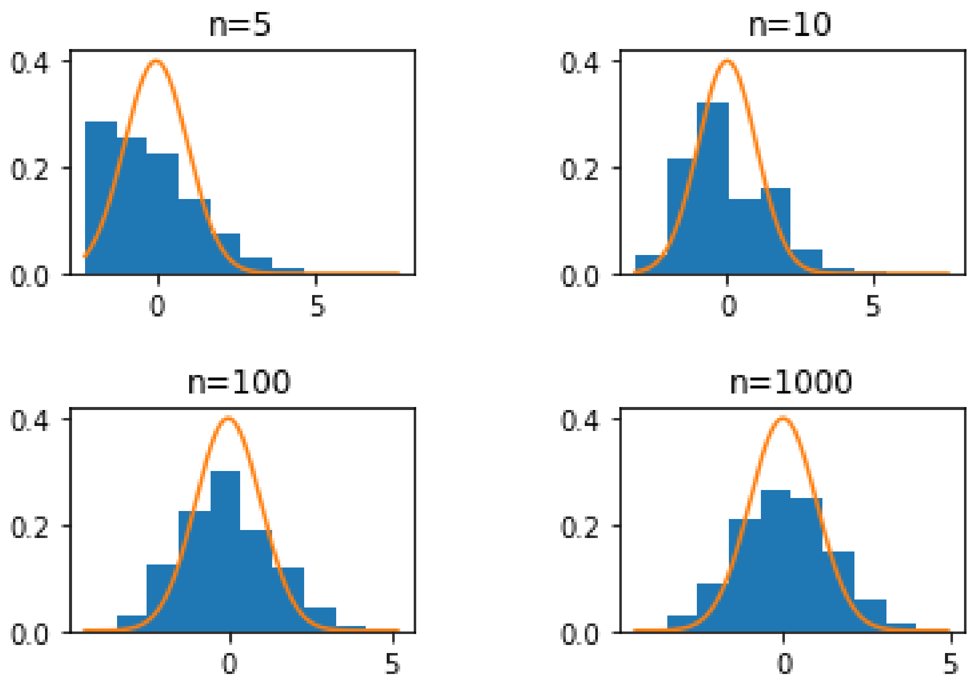
[1]:
```
import numpy as np
import scipy.stats as stats
import matplotlib.pyplot as plt
```

0.1 Possion Distribution

0.1.1 CLT - Possion

[2]:
```
M = 5000
n_choices= [5, 10, 100, 1000]

def rv_pos(n):
    Sn = sum(np.array([np.random.poisson(lam=1/2) for i in range(n)]))
    rv = (Sn - n*(1/2))/((n**(1/2))*0.5)

    return rv


hist_1 = np.zeros((4, M))

for i in range(len(n_choices)):
    hist_1[i] = np.array([rv_pos(n_choices[i]) for j in range(M)])
```

[3]:
```
fig, axs = plt.subplots(2, 2)
plt.subplots_adjust(wspace=0.6, hspace=0.6)

for i in range(len(n_choices)):
    x_space = np.linspace(min(hist_1[i]), max(hist_1[i]), 1000)
    axs[i//2, i%2].hist(hist_1[i], density=True)
    axs[i//2, i%2].plot(x_space, stats.norm(0, 1).pdf(x_space))
    axs[i//2, i%2].set_title("n=" + str(n_choices[i]))
```

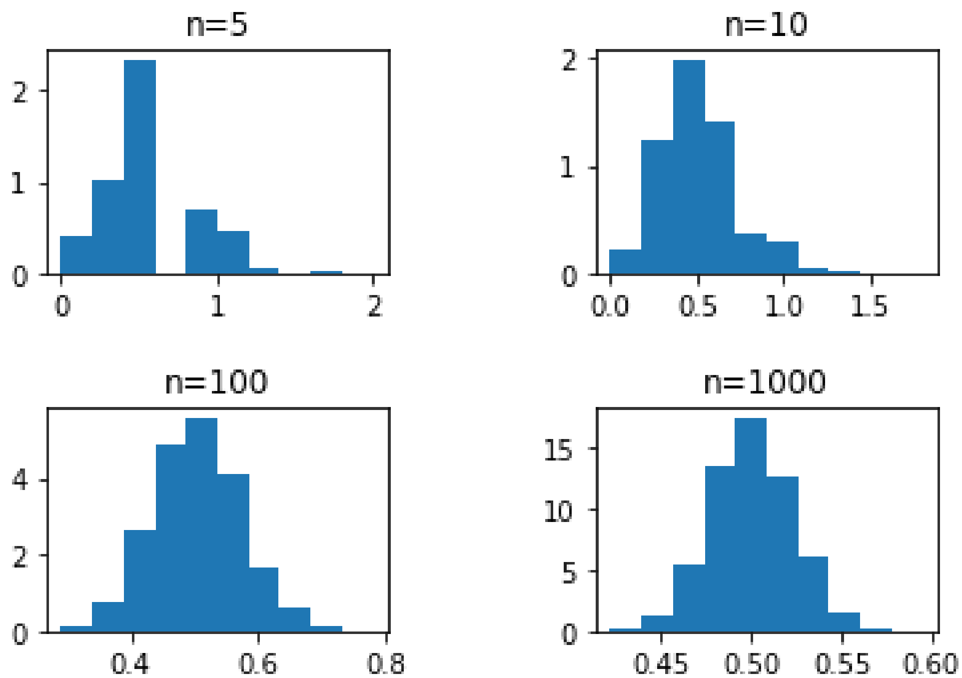


As we can see, when n becomes larger, the distribution of the generated random variable $\frac{S_n - n\cdot\mu}{\sqrt{n}\cdot\sigma}$ is more close to normal distribution, which is the essense of central limit theorem.

0.1.2 LNN - Possion

[4]:
```
def rv_pos_lln(n):
    Xn_bar = np.array([np.random.poisson(lam=1/2) for i in range(n)]).mean()
    return Xn_bar

hist_2 = np.zeros((4, M))

for i in range(len(n_choices)):
    hist_2[i] = np.array([rv_pos_lln(n_choices[i]) for j in range(M)])
```

[5]:
```
fig, axs = plt.subplots(2, 2)
plt.subplots_adjust(wspace=0.6, hspace=0.6)

for i in range(len(n_choices)):
    x_space = np.linspace(min(hist_2[i]), max(hist_2[i]), 1000)
    axs[i//2, i%2].hist(hist_2[i], density=True)
    axs[i//2, i%2].set_title("n=" + str(n_choices[i]))
```



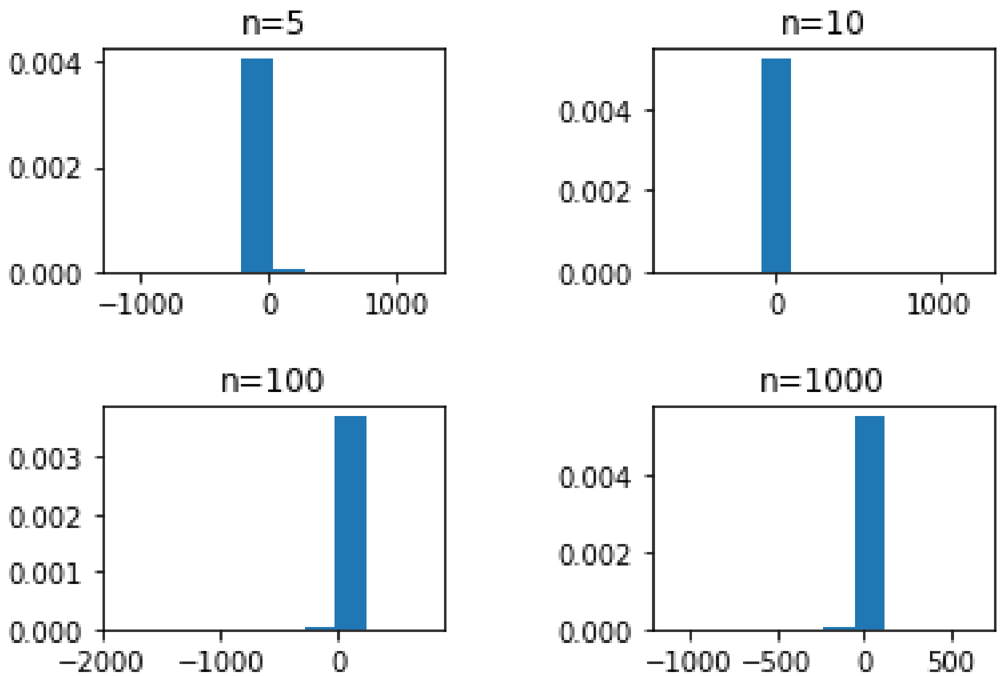
As we can see, as n grows, $\bar{X_n}$ is more concentrated to the mean of X_1, which is 1. ~ The reuslt is consistent with **Law of Large Numbers**

0.2 Cauchy

[6]:
```
def rv_cauchy(n):
    Sn = sum(np.array([np.random.standard_cauchy() for i in range(n)]))
    # rv = (Sn - n*(1/2))/((n**(1/2))*0.5)
    # Since mean and variance is essentially missing here
    # We do not normalize it
    return Sn/n

hist_3 = np.zeros((4, M))

for i in range(len(n_choices)):
    hist_3[i] = np.array([rv_cauchy(n_choices[i]) for j in range(M)])
```

[7]:
```
fig, axs = plt.subplots(2, 2)
plt.subplots_adjust(wspace=0.6, hspace=0.6)

for i in range(len(n_choices)):
    x_space = np.linspace(min(hist_3[i]), max(hist_3[i]), 1000)
    axs[i//2, i%2].hist(hist_3[i], density=True)
    # axs[i//2, i%2].plot(x_space, stats.norm(0, 1).pdf(x_space))
```

```
    axs[i//2, i%2].set_title("n=" + str(n_choices[i]))
```



As we can see, the mean of cauchy distributions do not behave like other distributions, which conerges to the mean as n grows large. This is perhaps because the mean and variance of cauchy distribution is essentially missing.