MATH-UA.0234 Mathematical Statistics Spring 2022
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Problem Set 7

Submission:
Thursday, 04/14/2022, until 1 PM, to be uploaded on the NYU Brightspace course homepage.

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1. **Confidence intervals for normally distributed data** [4 Points]

   Suppose that in an experiment 5 data points are observed:

   $$x_1 = 2.3, \qquad x_2 = 1.9, \qquad x_3 = 2.0, \qquad x_4 = 1.8, \qquad x_5 = 2,1.$$

   Suppose that these data points are realizations of i.i.d. random variables $X_1, ..., X_5$, where $X_1 \sim \mathcal{N}(\mu, \sigma^2)$ with *unknown* μ and σ^2.

   (a) Find a 95%-confidence interval for μ.

   (b) Explain why for general $X_1, ..., X_n \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mu, \sigma^2)$ random variables, $\alpha \in (0,1)$, the interval

   $$S(\mathbf{X}) = \left[\frac{n-1}{\chi^2_{n-1,1-\frac{\alpha}{2}}} S_n^2, \frac{n-1}{\chi^2_{n-1,\frac{\alpha}{2}}} S_n^2 \right], \qquad S_n^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \overline{X}_n)^2$$

   is a $(1-\alpha)$-confidence interval for σ^2, when $\chi^2_{k,\beta}$ describes the β-quantile of the χ^2 distribution with k degrees of freedom. Calculate a 95%-confindence interval for σ^2 based on the observed data in the concrete example.
   Hint: Use Theorem 2.7, (i).

2. **Asymptotic confidence interval for the Poisson distribution** [4 points]

   Let $X_1, ..., X_n \overset{\text{i.i.d.}}{\sim} Pois(\lambda)$, with unknown $\lambda > 0$.

   (a) Calculate the Fisher information $I(\lambda)$.

   (b) For $\alpha \in (0,1)$, find an asymptotic $(1-\alpha)$-confidence interval for λ.

3. **Asymptotic confidence interval for the Exponential distribution** [4 points]

   Let $X_1, ..., X_n \overset{\text{i.i.d.}}{\sim} \mathcal{E}(\theta)$, with unknown $\theta > 0$.

   (a) Calculate the Fisher information $I(\theta)$.

   (b) For $\alpha \in (0,1)$, find an asymptotic $(1-\alpha)$-confidence interval for θ.

4. **Exact confidence interval for the Exponential distribution** [4 points]

   Let $X_1, ..., X_n \overset{\text{i.i.d.}}{\sim} \mathcal{E}(\theta)$, with unknown $\theta > 0$.

   (a) Show that $2\theta \sum_{i=1}^n X_i$ follows a χ^2_{2n} distribution.

   (b) Use (a) to find an *exact* $(1-\alpha)$-confidence interval for θ, for $\alpha \in (0,1)$.