Finite-temperature critical behavior of long-range quantum Ising models

arXiv:2104.15070

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ICTP

May 25, 2021

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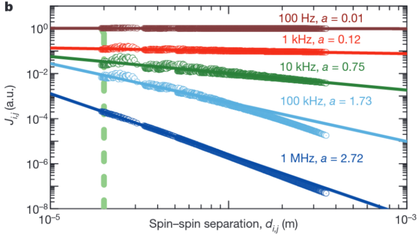
Long Range(LR) interacting quantum systems

- Power-law-decaying interactions($r^{-\alpha}$) can be engineered in experiments
- Analytically and numerically challenging to study
- Raise fundamental questions

Be ions stored in a Penning trap



[Britton et al., 2012]

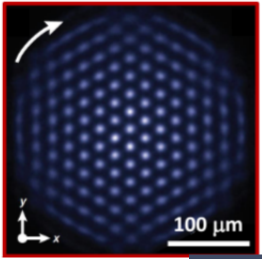
Quantum simulation of Ising or Heisenberg models with power law decaying interactions

Introduction

Long Range(LR) interacting quantum systems

- ☐ Power-law-decaying interactions($r^{-\alpha}$) can be engineered in experiments
- ☐ Analytically and numerically challenging to study
- ☐ Raise fundamental questions

$\sim$ 200 Be ions stored in a Penning trap



[Monroe et al., 2021]

Introduction

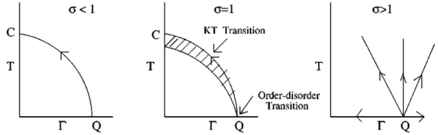
Long Range(LR) interacting quantum systems

- Power-law-decaying interactions($r^{-\alpha}$) can be engineered in experiments
- Analytically and numerically challenging to study
- Raise fundamental questions

1D LR Ising model in a transverse field

$$\alpha = 1 + \sigma$$

Ferromagnetic to paramagnetic



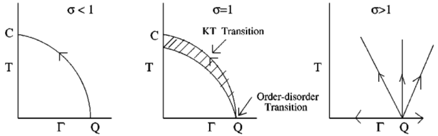
[Dutta et al., 2001]

How interaction range influences

- the critical behavior?
- thermalization?

1D LR Ising model in a transverse field

$$\hat{H} = -\sum_{ij} J_{ij}\sigma_i^z\sigma_j^z - \Gamma\sum_i \sigma_i^x \qquad J_{ij} = \frac{J}{r_{ij}^{1+\sigma}}, \;\; \alpha = 1+\sigma$$



T=0	T>0	
☐ $1 < \alpha \leq 5/3$	☐ $1 < \alpha \leq 3/2$	Mean-Field(MF)
☐ $5/3 < \alpha < 3$	☐ $3/2 < \alpha \leq 2$	Nontrivial
☐ $\alpha \geq 3$	☐ $\alpha > 2$	Short Range (SR)

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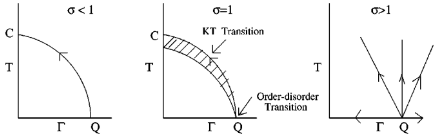

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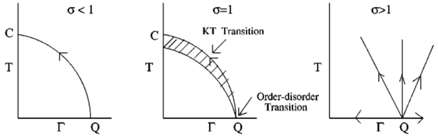
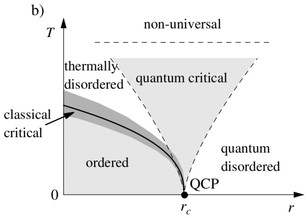


Classical and quantum fluctuations



The critical behavior close to the transition is entirely classical

[Vojta, 2003]

Our work aims to

- Study the 1D LR Ising model in a transverse field
- Answer whether quantum fluctuations change the nature of the thermal phase transition
- Determine universal and non-universal details
  - value of critical exponents
  - position of the critical points
- Use ab initio, non-perturbative solutions of the problem
- Study different α regimes
  - $\alpha = 0.05$, within the extremely long-range region $\alpha < 1$
  - $\alpha = 1.5$, at the boundary between MF and nontrivial behavior

The model

System Hamiltonian

$$\hat{H} = -\frac{V}{K}\sum_{i<j}\frac{\hat{S}_i^z\hat{S}_j^z}{r_{ij}^{\alpha}} - h\sum_i \hat{S}_i^x .$$

$$K \equiv (L-1)^{-1}\sum_{i\neq j} r_{ij}^{-\alpha}$$

$$\hat{S}_i^x = \frac{\hbar}{2}\sigma_i^x$$

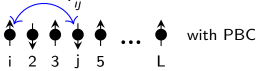
The Kać renormalization factor

- ☐ Ensures the existence of a proper thermodynamic limit
- ☐ Do not affect the universal features of the model

Reduced units Hamiltonian

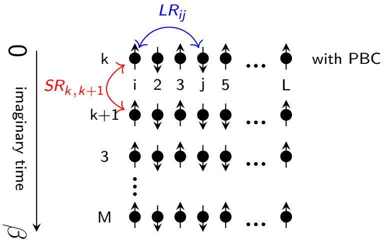
$$\hat{H} = -\sum_{i<j}\frac{V}{K}\frac{\hat{S}_i^z\hat{S}_j^z}{r_{ij}^{\alpha}} - \sum_i \hat{S}_i^x$$

$$J_{ij} = \frac{V}{r_{ij}^{\alpha}}$$



Path Integral Monte Carlo

- The model maps to a classical, anisotropic, 2D Ising model



The mapping is exact up to $O(\beta/M)$ corrections

β up to 1024
L up to 8192
M up to 65536

- Properties of the quantum system are accessed via conventional Monte Carlo simulations

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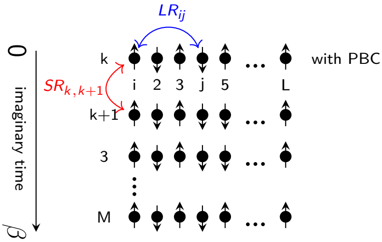

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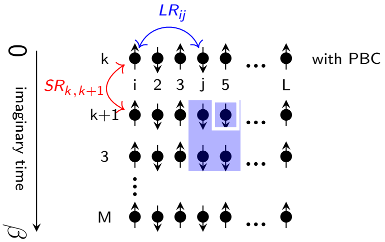


Critical slowing down problem

- Wolff cluster updates [Wolff, 1989] (short-range part)
- Long-range cluster updates [Luijten and Blöte, 1997] (long-range part)

- Properties of the quantum system are accessed via conventional Monte Carlo simulations

PIMC gives direct access to observables like

Binder cumulant

$$U = \frac{1}{2}\left[3 - \frac{\langle m_z^4 \rangle}{\langle m_z^2 \rangle^2}\right]$$

"Classical" Susceptibility

$$\chi = \beta L(\langle m_z^2 \rangle - \langle |m_z| \rangle^2)$$

The finite-size scaling hypothesis (FSS)

$$Q(t, L) = L^{-k\theta_t} g\left(tL^{\theta_t}\right)$$

For $d \geq d_{uc}$

$$\frac{1}{\nu} = \frac{d_{uc}(\alpha)}{d}\theta_t$$

$$T = 0 \quad d_{uc} = \frac{3(\alpha - 1)}{2}$$

$$V \to \infty \quad d_{uc} = 2(\alpha - 1)$$

Ground state phase transition

$$\hat{H} = -\sum_{i<j} \frac{V}{K} \frac{\hat{S}_i^z \hat{S}_j^z}{r_{ij}^{\alpha}} - \sum_i \hat{S}_i^x$$

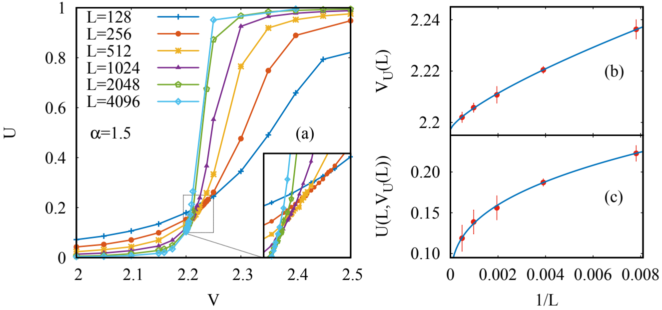
$$U = \frac{1}{2}\left[3 - \frac{\langle m_z^4 \rangle}{\langle m_z^2 \rangle^2}\right] \qquad \alpha = 0.05, 1.5$$

FSS

$$V_U(L) = V_c\left(1 + aL^{-\omega-\theta_t}\right)$$
$$U(L, V_U(L)) = b + cL^{-\omega}$$

[Angelini et al., 2014]

Curves $U(V, L)$ for system sizes L and $2L$ cross at size-dependent points $V = V_U(L)$



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$\alpha = 0.05, 1.5$

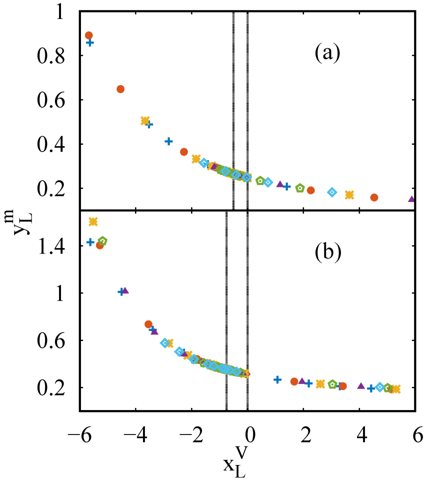
Data collapse

$$m_z^2 \sim L^{-2\beta_m \theta_t} \cdot f\left[L^{+\theta_t}\left(V_c - V\right)\right]$$

$$y_L^m \equiv m_z^2(L) L^{+2\beta_m \theta_t}$$

$$x_L^V \equiv \left(V_c - V\right) L^{\theta_t}$$

[Sandvik, 2010]



Ground state phase transition

α	V_c (BC)	V_c (DC)	θ_t (BC)	θ_t (DC)	$2\beta_m\theta_t$ (DC)
0.05	1.9997(4)	1.9999	0.50(7)	0.688	0.68
1.50	2.1972(7)	2.1981	0.39(6)	0.64	0.715

Comparison of the results

- SSE[1] predictions in [Koziol et al., 2021]

α	θ_t	$2\beta\theta_t$
1.5	~ 0.667	~ 0.667

$\rightarrow$ MF

- MF predictions in [Botet and Jullien, 1983]

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0	2/3	2/3

[1]Stochastic Series Expansion (Monte Carlo)

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Finite temperature phase transition

$$\hat{H} = -\sum_{i<j} \frac{V}{K} \frac{\hat{S}_i^z \hat{S}_j^z}{r_{ij}^{\alpha}} - \sum_i \hat{S}_i^x$$

$$U = \frac{1}{2}\left[3 - \frac{\langle m_z^4 \rangle}{\langle m_z^2 \rangle^2}\right]$$

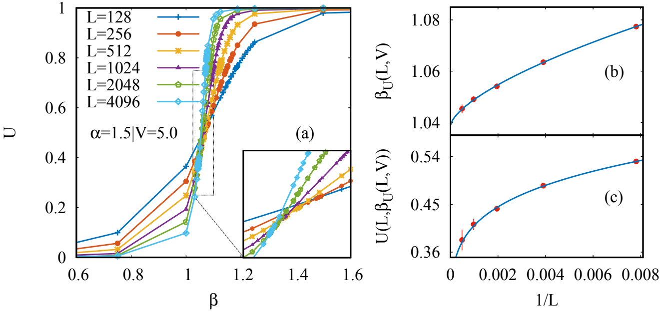
$\alpha = 0.05, 1.5$ | $V = 2.5, 3, 3.5, 5$

FSS

$$\beta_U(L, V) = \beta_c \left(1 + aL^{-\omega-\theta_t}\right)$$

$$U(L, \beta_U(L, V)) = b + cL^{-\omega}$$

[Angelini et al., 2014]



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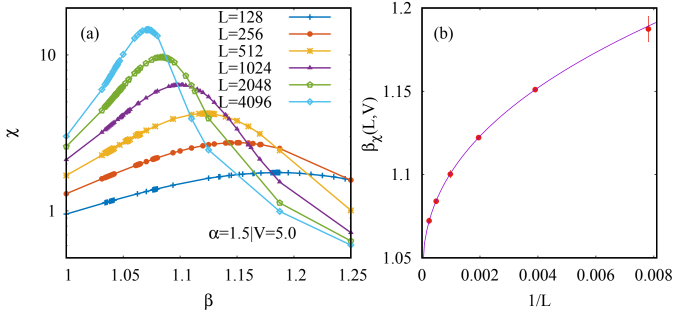
$$\chi = \beta L(\langle m_z^2 \rangle - \langle |m_z| \rangle^2)$$

FSS

$$\beta_\chi(L, V) = \beta_c + fL^{-\theta_t}$$

[Sandvik, 2010]

χ is expected to display peaks at size-dependent temperatures $\beta_\chi(L, V)$



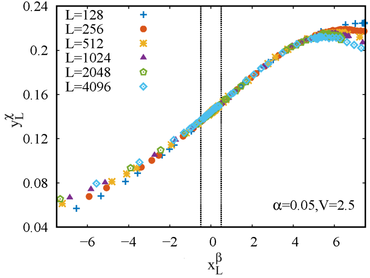
Finite temperature phase transition

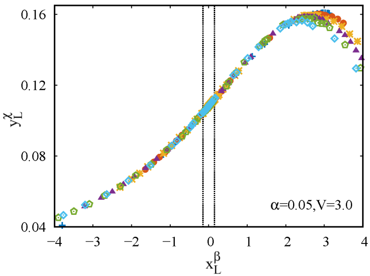
Data collapse

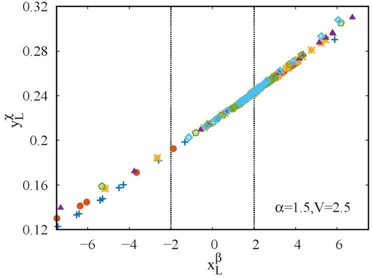
$$\chi \sim L^{\gamma\theta_t} \cdot f(L^{\theta_t}(\beta_c - \beta))$$

$$y_L^{\chi} \equiv \chi(L)L^{-\gamma\theta_t}$$
$$x_L^{\beta} \equiv (\beta_c - \beta)L^{\theta_t}$$

[Sandvik, 2010]







Finite temperature phase transition

		β_c			θ_t			$\gamma\theta_t$
	V	U	χ	χ_{dc}	U	χ	χ_{dc}	χ_{dc}
$\alpha = 0.05$	$V = 2.5$	2.2007(4)	2.23(1)	2.20	/	0.72(4)*	0.51	0.505
	$V = 3.0$	1.6120(7)	1.61(1)	1.612	/	0.54(3)	0.485	0.515
	$V = 3.5$	1.299(1)	1.303(3)	1.303	/	0.54(2)	0.49	0.523
	$V = 5.0$	0.8474(2)*	0.844(2)	0.8491	0.5(1)	0.47(2)	0.50	0.524
$\alpha = 1.50$	$V = 2.5$	3.21(1)	3.351(9)	3.229	0.49(7)	0.75(1)*	0.50	0.516
	$V = 3.0$	2.109(1)*	2.12(1)	2.115	0.50(2)	0.48(3)	0.52	0.538
	$V = 3.5$	1.647(6)	1.646(5)	1.650	0.5(2)	0.46(2)	0.52	0.545
	$V = 5.0$	1.039(1)	1.035(1)	1.041	0.44(7)	0.41(1)	0.530	0.550

Comparison of the results

- MC[2] predictions in [Luijten and Blöte, 1997]

α	V	θ_t	$\gamma\theta_t$
1.5	∞	~ 0.5	~ 0.5

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[2] Classical Monte Carlo

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Criticality and phase diagram

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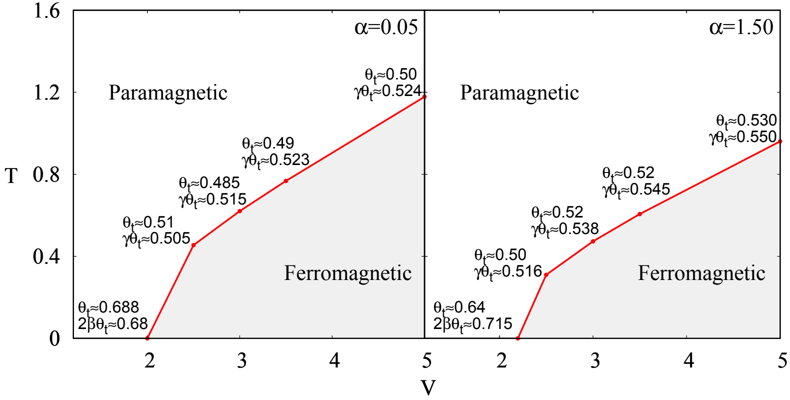

Conclusions

- We study the long-range quantum Ising chain
  - Characterising the phase diagram and critical behavior
- We find essential agreement with existing predictions when possible
  - We find compatibility of our $\alpha = 0.05$ results with the fully-connected universal properties
  - The non-zero T transition follows the classical prediction

Acknowledgements

Adriano Angelone

Marcello Dalmonte

Markus Heyl

Thank you for your attention

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