ME2 Computing- Session 3: Numerical Integration

Learning outcomes:

- Being able to compute numerically the integration of a proper integral
- Being able to compute numerical integration for a set of points not positioned equidistantly

Before you start

In your H drive create a folder *H:\ME2MCP\Session3*and work within it.

Task A: Trapezium rule for functions with equidistant nodes

1. Write a Python function, *trapzeqd*, receiving a set of points *x* and *y*, and outputting the numerical integral of *y* within the interval specified by *x*. Assume that the nodes *x* are equidistant.
   Test the Python function by integrating:

$$I = \int_0^b \frac{1}{\sqrt{x^{18.10} + 2021}} dx$$

   in the interval *x = [0 : b=2]* with 5 nodes and then with 11 nodes.

2. Increase the interval of integration with *b = 10, 100, 1000, 10000*, and recompute the integral with same number of nodes (5).
   Plot the values of *I* vs *b*.

3. Repeat the numerical integration for the intervals in Part 2, but retaining the same interval *h = 0.5*, i.e. by increasing progressively the number of nodes.
   Replot the values of *I* vs *b*.

Task B: Numerical integration of diverging improper integrals

4. Recompute the numerical integrations as in Task A2 and A3, but with the integrand function:

$$I = \int_0^b \frac{1}{\sqrt{x^{18.10} - 2021}}$$

Task C: Trapezium rule for functions with non-equidistant nodes

1. Write another Python function, *trapz*, receiving a set of points *x* and *y*, and outputting the numerical integral of *y* within the interval specified by *x*. The values in x might not be distanced at same intervals.

Task D: The river Thames basin in London

The files *(xn.txt, yn.txt)* and *(xs.txt, ys.txt)* contain the *(x,y)* spatial coordinates of the north and south bank, respectively, of the river Thames (units in meters), within the Central London region (between Chiswick and Woolwich).

1. Read in the data from the files and plot the two banks of the river together, to visualise the shape of the basin. (To plot with aspect ratio 1:1 for the two axes use *pl.axis('equal')*, after plotting.

2. Compute the surface occupied by the basin in Km^2.

Task E: Multiple integrals*: *the domes of Samarkand in Uzbekistan

A two-dimensional integral has the form of:

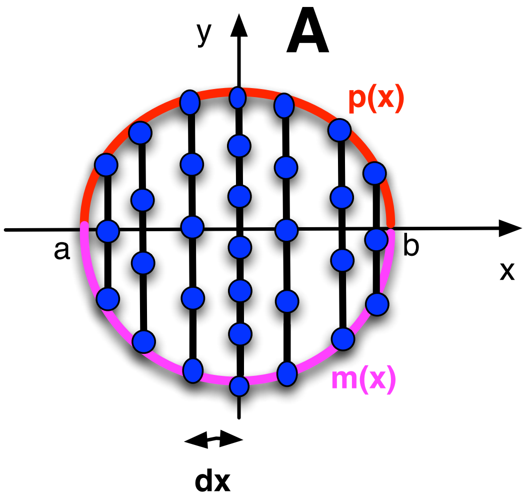
$$I = \iint\limits_A z(x,y)dA = \int_a^b dx \int_{m(x)}^{p(x)} z(x,y)dy = \int_a^b G(x)dx$$

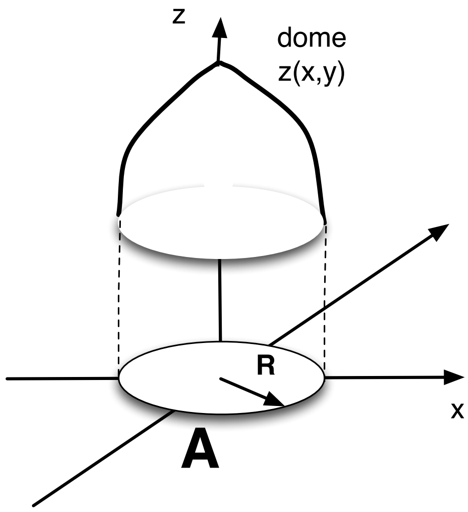
where A is the domain of integration.

The two-dimensional integral can be computed numerically, by applying the trapezium method twice. Firstly, the integral

$$G(x) = \int_{m(x)}^{p(x)} z(x,y)dy$$

is computed for all values of x. Then, the total integral is obtained as: $I = \int_a^b G(x)dx$.





1. Compute the volume of the dome described by $z(x,y) = \sqrt{R - \sqrt{x^2 + y^2}}$ in the domain A: $x^2 + y^2 < R^2$, with R = 5, using equidistant intervals both along the *x* and *y* axes, i.e. *dx = dy = 0.05*.

2. When $z(x,y) = \sqrt{R^2 - x^2 - y^2}$ the dome is a perfect hemisphere. For this case, you can check your computed volume against the exact value.

3. Plot the function $z(x,y)$.