X-Nav: Verification of Methods

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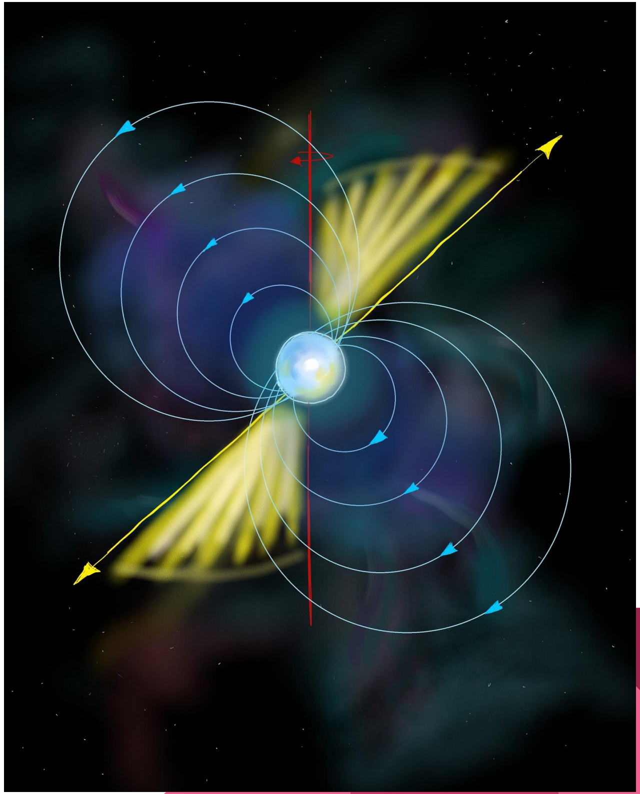
What is X-Nav?

X-Ray Pulsar Navigation, X-Nav, is the idea to use of Pulsars for autonomous navigation through space. Pulsars are small, but extremely dense Neutron stars that act as beacons in deep space. They have very large moments of inertia, causing their periods to be extremely stable. This gives them a possibility of being used as a navigation guide.



Verification of Methods

The task at hand was to recreate and verify the methods presented by different research papers.

Figure of Merit

- Suneel I. Sheikh and Darryll J. Pines. Spacecraft navigation using x-ray pulsars.

Position Estimation

- Suneel I. Sheikh and Darryll J. Pines. Spacecraft navigation using x-ray pulsars, 2006.
- Josep Sala. Feasibility study for a spacecraft navigation system relying on pulsar timing information, 2004.

Why do we need a Figure of Merit?

A Figure of Merit is something that measures the performance of an equation, procedure, etc.

There is a need for a ranking system in order to determine the quality of a Pulsar. By doing so, the pulsars with the strongest and cleanest signals are the ones being used for the position and velocity updates.

The purpose of the FOM is to form the bridge between the Pulsar Catalog and the Position Estimation.

Figure of Merit Equation

The equation used for the calculations is:

$$Q_x = \frac{F_x P_f^2}{W^2 [\, P_f + \left(\frac{W}{P}\right)(1 - P_f)]}$$

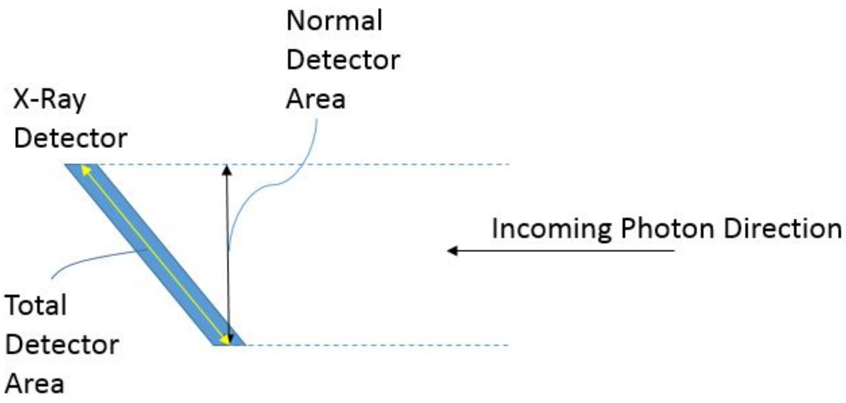
Where:

- **Fx** - the detected photon flux from the pulsar
- **Pf** - The percentage of pulsed photons from the signal
- **P** - The period of the pulsar
- **W** - the measure pulsed width of the pulsar’s signal

FOM: Photon Flux and Period

Photon Flux:

The measured flux is a representation of the amount of photons passing through the x-ray detector per second per unit area.



Period:

The period of the pulsar is the time it takes for the pulsar to make one rotation. This property of a pulsar has been catalogued by the Australian National Telescope Facility (ANTF) and can be directly called into the FOM calculation.

FOM: Pulsed Fraction

The Pulsed Fraction is the measurement of the pulsed component of the signal received from the pulsar. Whereas the non-pulsed component consists of noise and background photons. It uses the bootstrapping algorithm presented by *Swanepoel (1996)* which finds an estimator for the background photon level. Taking the number of photons above this “photon line” over the total photons gives the pulsed fraction.

Pulsed Fraction: Bootstrapping

Step 1: Using the set of folded photon arrival times in ascending order, randomly draw a sample with replacement.

Step 2: Independently repeat step 1 a B number of times.

Step 3: Calculate $\hat{s}$ as defined from variables v_s and w_s.

Step 4: Calculate the non-parametric estimator, $\hat{p}$, the standard error and the confidence interval using the equations defined. The Pulsed Fraction can now also be found.

Pulsed Fraction Examples

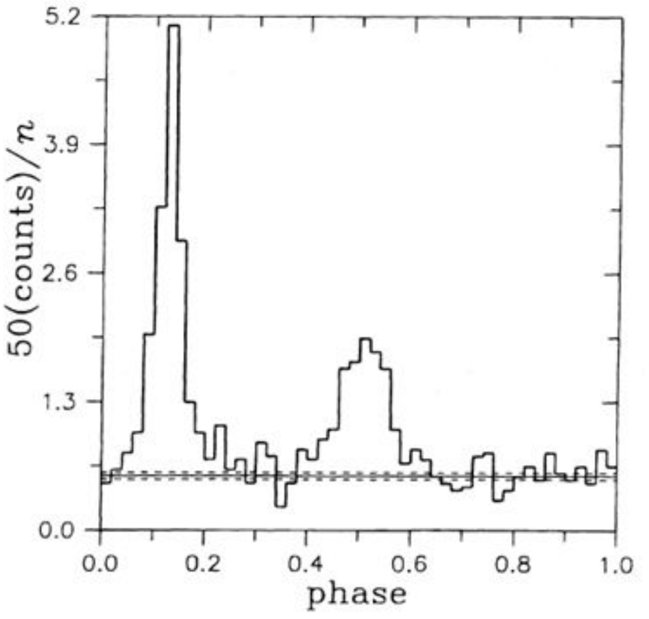
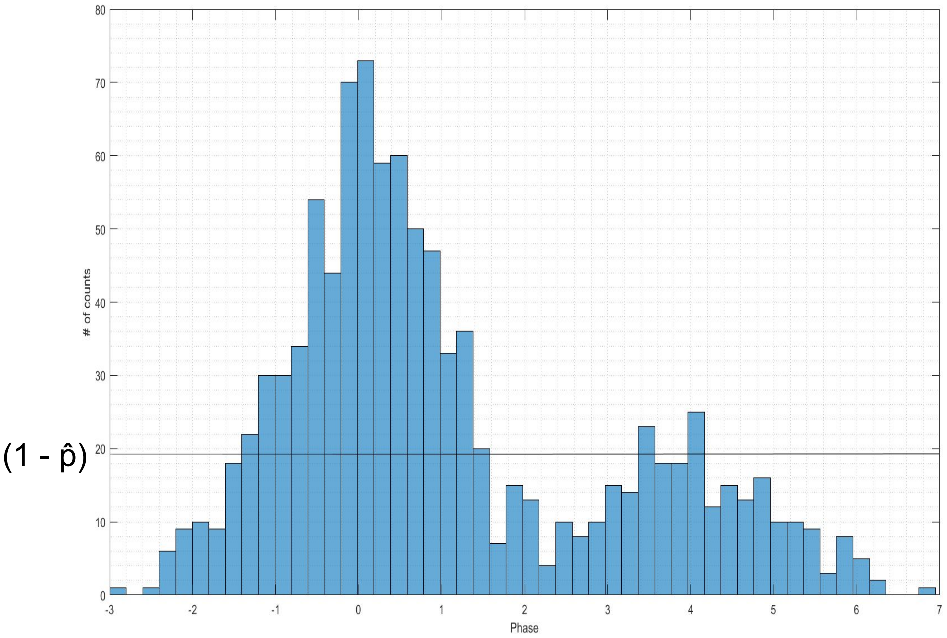


FIG. 4.—Phase histogram for Crab data, with background level $1 - \hat{p}$

Bootstrapping of an artificially created Pulsar



1000 photons, P_f = 52.3%

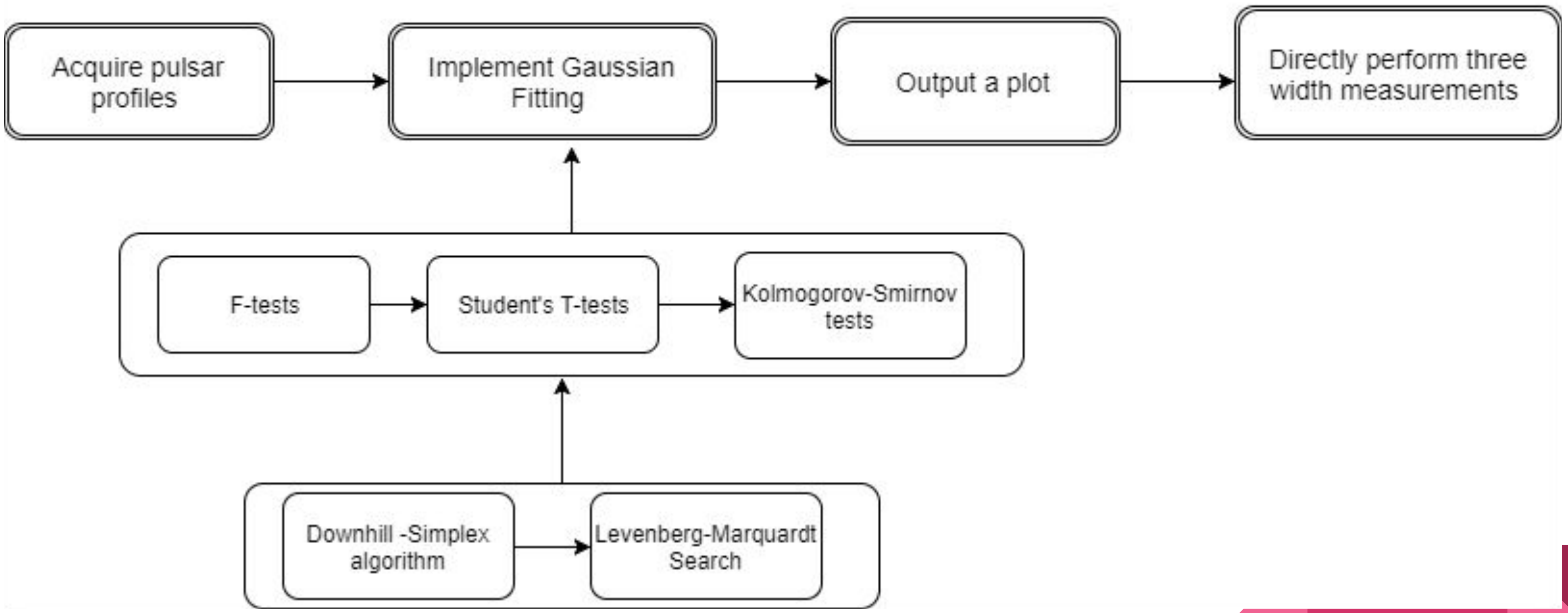
FOM: Pulse Width

The pulse width is one of the identities that helps defines a pulse profile. To have a more accurate estimate of the width of the pulse, we use 3 different width measurements:

- distance between the the maximum peaks
- 50% of the maximum peak
- 10% of the maximum peak

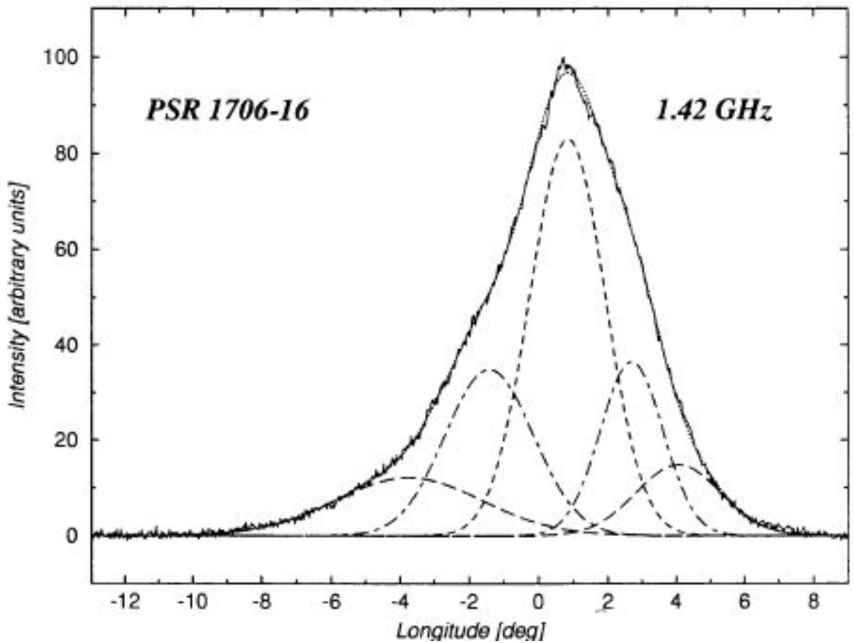
The most common type used in the FOM Calculation is the 50% at maximum peak, otherwise known as Full-Width Half-Max (FWHM).
In order to create the plot, we use a Gaussian fitting procedure that decreases the influence of noise on the measurements.

Pulse Width: Calculation Procedure

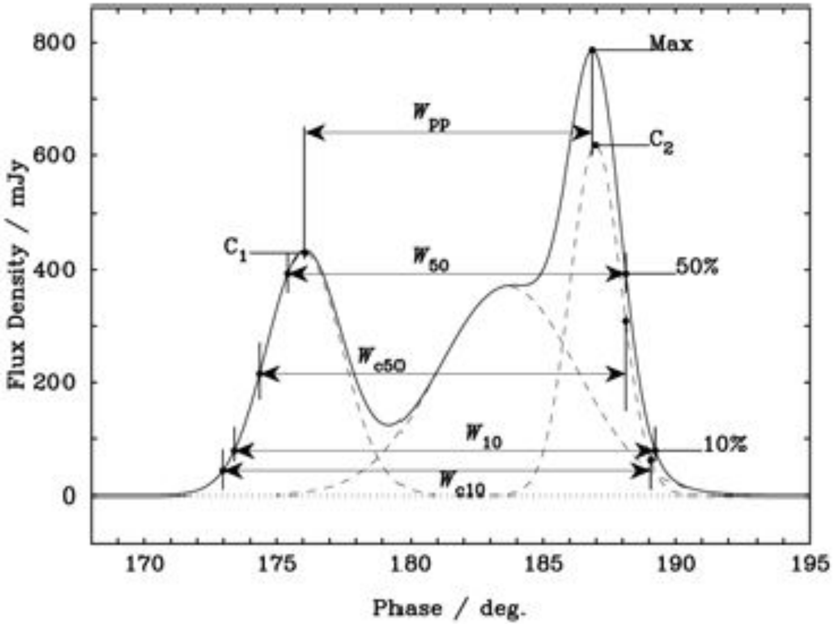


Pulse Width Example

Gaussian Fitting Procedure



The 5 Measured Pulse Widths



FOM: Putting It All Together

Using all of the methods presented for each variable, the Figure of Merit can be calculated for each pulsar and sorted.

The results in the Sheikh Paper have the FOM normalized to pulsar B0531+21. When normalizing the calculated data, the same results are found.

Pulsar Name	Figure of Merit
B0531+21	0.002354
B1937+21	0.0007978
B1821-24	0.00040481
B1957+20	4.8818e-05
B0540-69	3.4846e-06

Signal to Noise Ratio

The Signal-to-Noise Ratio (SNR) is the measurement of the ratio of the incoming signal with respect to the background noise. Having a high SNR is necessary for X-Nav in order to minimize error in the detection of the pulse.

$$= \frac{F_X A p_f t_{\text{obs}}}{\sqrt{[B_X + F_X(1 - p_f)](A t_{\text{obs}} d) + F_X A p_f t_{\text{obs}}}}$$

- A, detector area
- Bx, x-ray background radiation
- t_{obs} , observation time
- d, duty cycle

What can you do with the SNR?

Using the SNR, we can make the calculation of the estimation of the one sigma value of the time of arrival of the pulse:

$$\sigma_{\mathrm{TOA}} = \tfrac{1}{2}W \,/\, \mathrm{SNR}$$

Uses:

- Creating the pulse profile of a pulsar
- Generating a test signal of a pulsar
- Estimating position error

Position Estimation: Delta-Correction Method

Using the Delta-Correction Method, the matlab code created will take an estimated position and correct its position using pulsars. The equation is as follows:

$$\delta t_b = \delta t_{\mathrm{obs}} + \frac{\hat{\boldsymbol{n}} \cdot \delta \boldsymbol{r}}{c}$$

- δt_b - error in arrival at time of arrival at the Sun (reference point)
- δt_{obs} - error of observation (timing error/clock drift)
- δr - error in position

Position Estimation: Methods (how)

In order to solve for both time and position, at least 4 pulsars must be used.

$$\begin{bmatrix} \delta x \\ \delta y \\ \delta z \\ \delta t \end{bmatrix} = c \begin{bmatrix} x_1 & y_1 & z_1 & -c \\ \vdots & \vdots & \vdots & \vdots \\ x_4 & y_4 & z_4 & -c \end{bmatrix}^{-1} \begin{bmatrix} \delta t_{b_1} & -\delta t_{obs_1} \\ \vdots & \vdots \\ \delta t_{b_4} & -\delta t_{obs_4} \end{bmatrix}$$

$$\boldsymbol{\delta r} = cN^{-1}(\boldsymbol{\delta t_b} - \boldsymbol{\delta t_{obs}})$$

In order to solve using any number of pulsars, at least square was used.

$$\boldsymbol{\delta r} = c(N^T N)^{-1} N^T (\boldsymbol{\delta t_b} - \boldsymbol{\delta t_{obs}})$$

Here N is (nx4), δt_b and δt_{obs} are (nx1), where n is the number of pulsars

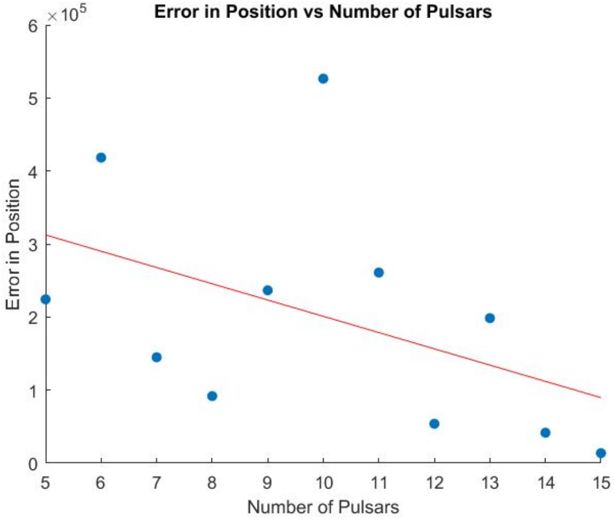
Position Estimation: Errors

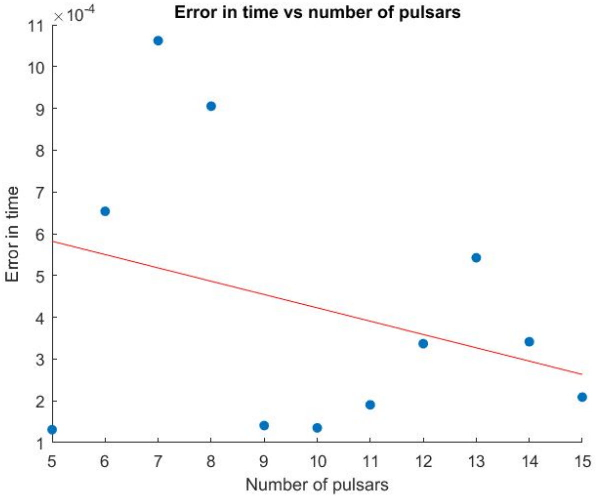
While considering many errors, we chose the following to account for:

- Error in position of the pulsar
- Error in clock drift
- Error in phase

Position Estimation: Simulation results

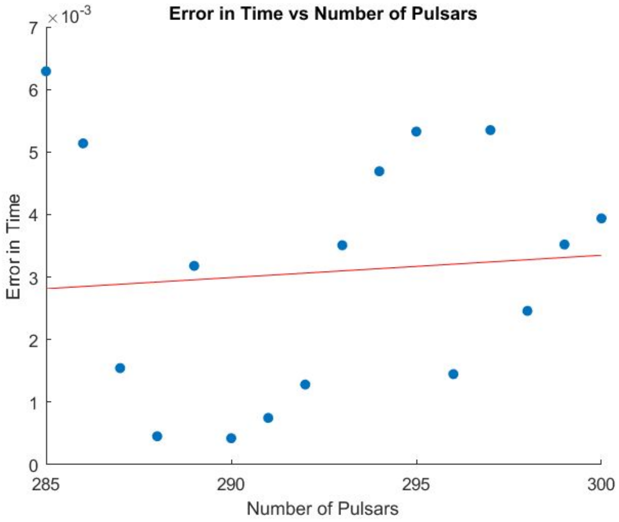
After repeating the calculation many times in a Monte Carlo simulation, the estimated using 15 pulsars was on the order of 10^5 m:

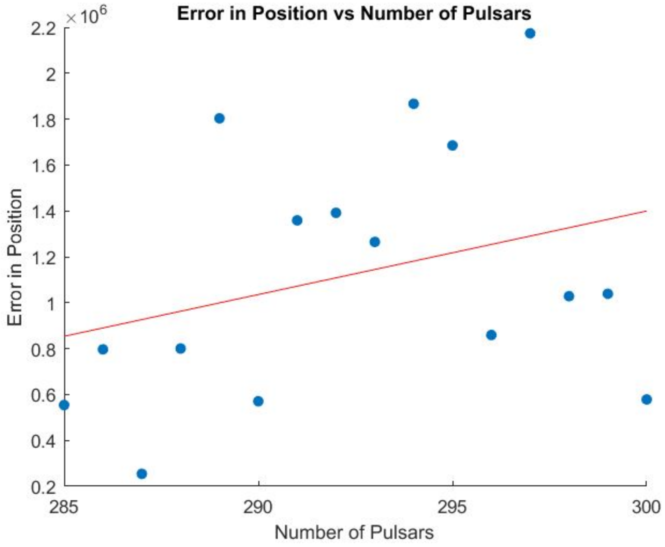




Position Estimation: Simulation results(cont.)

When using all 300 the error increased to the order of 10^6 m. Time increased the error to the order of 10^{-3} s:





References

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[4] Josep Sala. Feasibility study for a spacecraft navigation system relying on pulsar timing information, 2004.

[5] Suneel I. Sheikh and Darryll J. Pines. Spacecraft navigation using x-ray pulsars. Journal of Guidance, Control, and Dynamics, 29(1):49–63, 2006.

[6] J. W. H. Swanepoel. Estimation of the strength of a periodic signal from photon arrival times. Pages 261–264, 1996.