

Coursework: Masked Auto-Encoder

In this coursework, you will explore the popular self-supervised masked auto-encoder approach [MAE](#).

The coursework is divided in the following parts:

- **Part A:** Create a dataset and a data module to handle the PneumoniaMNIST dataset.
- **Part B:** Implement MAE utility functions.
- **Part C:** Implement and train a full MAE model.
- **Part D:** Inspect the trained model.

Important: Read the text descriptions carefully and look out for hints and comments indicating a specific 'TODO'. Make sure to add sufficient documentation and comments to your code.

Submission: You are asked to submit two versions of your notebook:

1. You should submit the raw notebook in `.ipynb` format with *all outputs cleared*. Please name your file `coursework.ipynb`.
2. Additionally, you will be asked to submit an exported version of your notebook in `.pdf` format, with *all outputs included*. We will primarily use this version for marking, but we will use the raw notebook to check for correct implementations. Please name this file `coursework_export.pdf`.

Your details

Please add your details below. You can work in groups up to two.

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Setup

```
In [37]: # On Google Colab uncomment the following line to install PyTorch Lightning
#! pip install lightning medmnist timm
```

```
In [1]: import os
import numpy as np
import torch
import torch.nn as nn
import torch.nn.functional as F
import torchvision
import matplotlib.pyplot as plt

from torch.utils.data import DataLoader
```

```

from torchvision import models
from torchvision import transforms
from pytorch_lightning import LightningModule, LightningDataModule, Trainer
from pytorch_lightning.loggers import TensorBoardLogger
from pytorch_lightning.callbacks import ModelCheckpoint, TQDMProgressBar
from torchmetrics.functional import auroc
from PIL import Image
from medmnist.info import INFO
from medmnist.dataset import MedMNIST

```

Part A: Create a dataset and a data module to handle the PneumoniaMNIST dataset.

We will be using the [MedMNIST Pneumonia](#) dataset, which is a medical imaging inspired dataset but with the characteristics of MNIST. This allows efficient experimentation due to the small image size. The dataset contains real chest X-ray images but here downsampled to **28 x 28 pixels**, with binary labels indicating the presence of [Pneumonia](#) (which is an inflammation of the lungs).

Task A-1: Complete the dataset implementation.

You are asked to implement a dataset class `PneumoniaMNISTDataset` suitable for training a classification model. For each sample, your dataset class should return one image and the corresponding label. We won't use the labels during training but for simplicity we will return them for model inspection purposes (part D).

To get you started, we have provided the skeleton of the dataset class in the cell below. Once you have implemented your dataset class, you are asked to run the provided visualisation code to visualise one batch of your training dataloader.

In terms of augmentation, we want to follow what has been done in the original MAE paper, that is **use random cropping (70%-100%) and horizontal flipping only** (see paragraph Data augmentation, page 6 of the [paper](#) for further details). Hint: checkout torchvision transform `RandomResizedCrop`.

```

In [2]: class PneumoniaMNISTDataset(MedMNIST):
        def __init__(self, split = 'train', augmentation: bool = False):
            ''' Dataset class for Pneumonia MNIST.
                The provided init function will automatically download the necessary
                files at the first class initialization.

                :param split: 'train', 'val' or 'test', select subset
            '''
            ...
            self.flag = "pneumoniamnist"
            self.size = 28
            self.size_flag = ""
            self.root = './data/coursework/'
            self.info = INFO[self.flag]
            self.download()

            npz_file = np.load(os.path.join(self.root, "pneumoniamnist.npz"))

```

```

self.split = split

# Load all the images
assert self.split in ['train', 'val', 'test']

self.imgs = npz_file[f'{self.split}_images']
self.labels = npz_file[f'{self.split}_labels']

self.do_augment = augmentation

### TODO: Define here your data augmentation pipeline.
### ADD YOUR CODE HERE

# Transformations
self.transform = transforms.Compose([
    transforms.ToPILImage(), # Convert from numpy to PIL
    transforms.RandomResizedCrop(size=self.size, scale=(0.7, 1.0))
    transforms.RandomHorizontalFlip(p=0.5), # Randomly flip the image
    transforms.ToTensor() # Convert from PIL to tensor
]) if self.do_augment else transforms.ToTensor() # If there is no augmentation

def __len__(self):
    return self.imgs.shape[0]

def __getitem__(self, index):
    ### TODO: Implement the __getitem__ function to return the image
    ### ADD YOUR CODE HERE
    img = self.imgs[index] # Get the image from index
    label = self.labels[index] # Get the label from index
    img = self.transform(img) # Apply the transformation to the image
    return img, label

```

We use a [LightningDataModule](#) for handling your PneumoniaMNIST dataset. No changes needed for this part.

```

In [3]: class PneumoniaMNISTDataModule(LightningDataModule):
    def __init__(self, batch_size: int = 32):
        super().__init__()
        self.batch_size = batch_size
        self.train_set = PneumoniaMNISTDataset(split='train', augmentation=Fa
        self.val_set = PneumoniaMNISTDataset(split='val', augmentation=Fa
        self.test_set = PneumoniaMNISTDataset(split='test', augmentation=Fa

    def train_dataloader(self):
        return DataLoader(dataset=self.train_set, batch_size=self.batch_s

    def val_dataloader(self):
        return DataLoader(dataset=self.val_set, batch_size=self.batch_siz

    def test_dataloader(self):
        return DataLoader(dataset=self.test_set, batch_size=self.batch_si

```

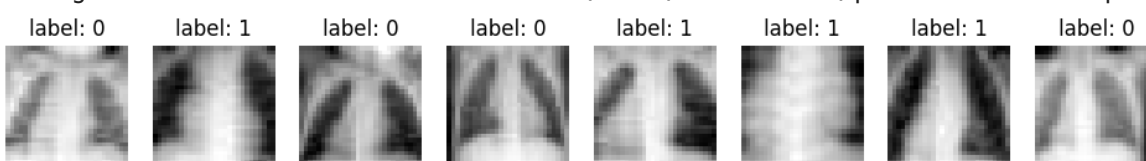
Check dataset implementation.

Run the below cell to visualise a batch of your training dataloader.

In [4]: *# DO NOT MODIFY THIS CELL! IT IS FOR CHECKING THE IMPLEMENTATION ONLY.*

```
# Initialise data module
datamodule = PneumoniaMNISTDataModule()
# Get train dataloader
train_dataloader = datamodule.train_dataloader()
# Get first batch
batch = next(iter(train_dataloader))
# Visualise the images
images, labels = batch
f, ax = plt.subplots(1, 8, figsize=(12,4))
for i in range(8):
    ax[i].imshow(images[i, 0], cmap='gray')
    ax[i].set_title('label: ' + str(labels[i].item()))
    ax[i].axis("off")
```

Using downloaded and verified file: ./data/coursework/pneumoniamnist.npz
 Using downloaded and verified file: ./data/coursework/pneumoniamnist.npz
 Using downloaded and verified file: ./data/coursework/pneumoniamnist.npz



Part B: Implement MAE utility functions.

As we saw in the lecture, Masked Auto-Encoders are based on a [Vision Transformer \(ViT\)](#) architecture. Importantly, the ViT architecture operates on a patch-level, not on the image-level. Hence, to feed the image into the ViT based encoder first we need to divide the images in small patches (typically 16x16 pixels).

In this part, we ask you to write three utility functions:

- **patchify** : takes in a batch of images (N, C, H, W) where N is the batch size, and returns a batch of patches of size (N, L, D) where L is the number of patches fitting in one image and $D = \text{patch_size}^2 * C$.
- **unpatchify** : inverts the above operation, takes in a batch of patches of size (N, L, D) and returns the corresponding batch of images (N, C, H, W).
- **random_masking** : Randomly masks out patches during training to create a self-supervised training task of patch prediction.

Task B-1: Implement **patchify**

```
In [5]: def patchify(imgs, patch_size):
        """
        ### TODO
        ### Write a function that takes the batch of images (N, C, H, W)
        ### and returns a batch of patches (N, L, D) where
        ### L is the number of patches and  $D = \text{patch\_size}^2 * C$ .

        ### This function should throw an error if the H and W of the ori
        image are not divisible by the patch size.
```

```

patch_size: (patch_h, patch_w)
"""
### ADD YOUR CODE HERE

N, C, H, W = imgs.shape
patch_h, patch_w = patch_size

# Check if H and W are divisible by patch size
assert H % patch_h == 0, "H is not divisible by patch_h"
assert W % patch_w == 0, "W is not divisible by patch_w"

# Calculate number of patches
L = (H // patch_h) * (W // patch_w)

# Extract patches
patches = imgs.unfold(2, patch_h, patch_h).unfold(3, patch_w, patch_w)

# Calculate D
D = patch_h * patch_w * C

# Reshape patches to (N, L, D)
patches = patches.reshape(N, L, D)
return patches

```

Let's test our implementation on the first batch of the validation set.

```

In [6]: # Load a batch of validation images
datamodule = PneumoniaMNISTDataModule()
dataloader = datamodule.val_dataloader()
batch = next(iter(dataloader))
images, labels = batch

```

Using downloaded and verified file: ./data/coursework/pneumoniamnist.npz
Using downloaded and verified file: ./data/coursework/pneumoniamnist.npz
Using downloaded and verified file: ./data/coursework/pneumoniamnist.npz

```

In [7]: images.shape

```

```

Out[7]: torch.Size([32, 1, 28, 28])

```

```

In [8]: # Assuming a patch size of (4,4) test your patchify function
# and test that the shape of the outputs corresponds at what is expected
patch_size = (4,4)
patches = patchify(images, (4, 4))

```

```

In [9]: patches.shape

```

```

Out[9]: torch.Size([32, 49, 16])

```

Visualisation of patchify output

Next, we want to check our output visually. In the next cell, plot all the patches of the first image in the batch as a grid of subplots where subplot(i,j) shows patch(i,j) at the right position in the original image. You should be able to recognise the original image.

```

In [10]: # TODO plot all the patches in a subplots grid (with their correct position)
### ADD YOUR CODE HERE

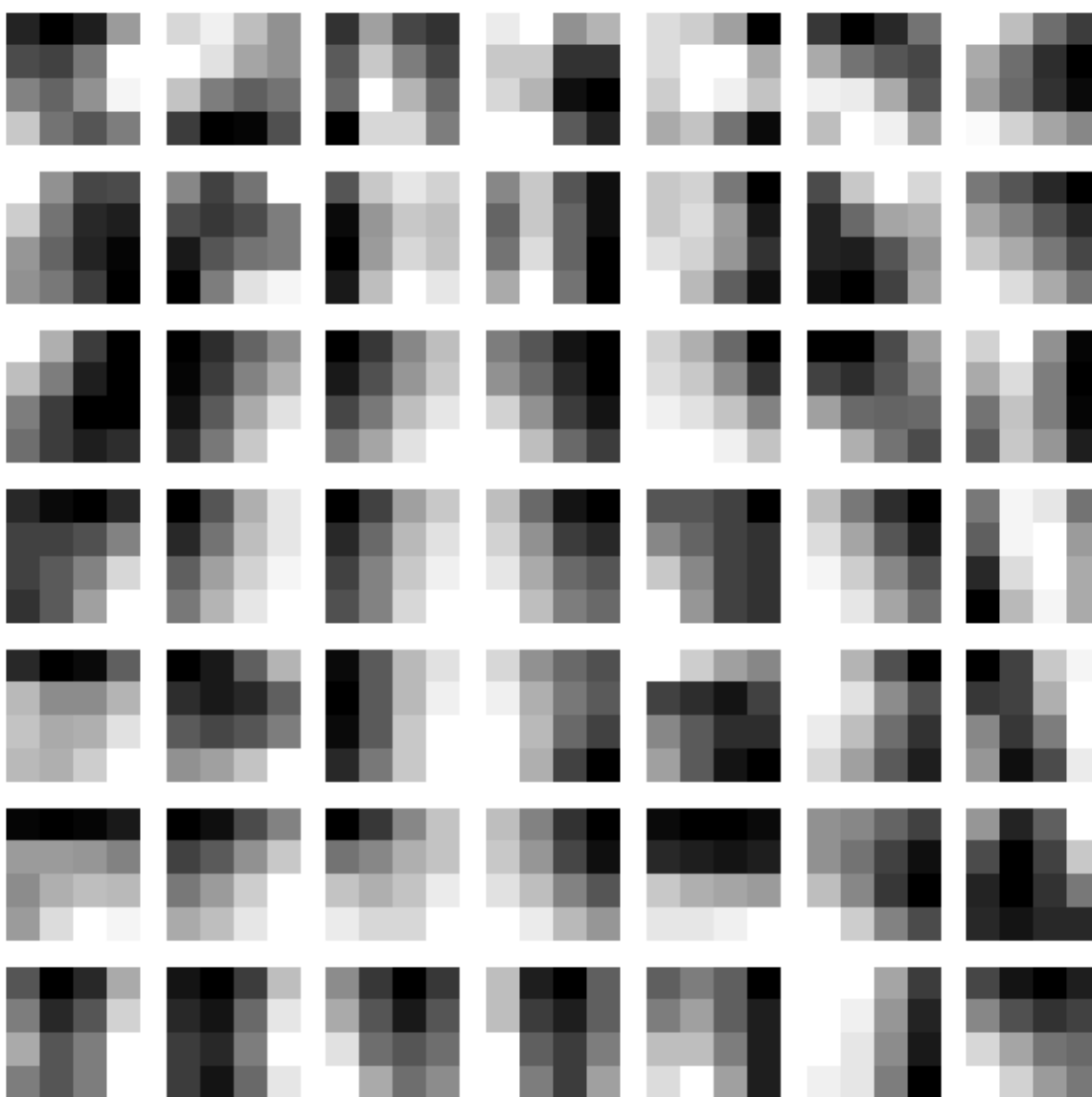
```

```
# Plot the patches of the first image

patches1 = patches[0]
plot_dim = int(np.sqrt(patches1.shape[0]))

_, ax = plt.subplots(plot_dim, plot_dim, figsize=(8,8))

for i in range(plot_dim):
    for j in range(plot_dim):
        ax[i, j].imshow(patches1[i*plot_dim+j].reshape(patch_size[0], patch_size[1]), cmap='gray')
        ax[i, j].axis('off')
```

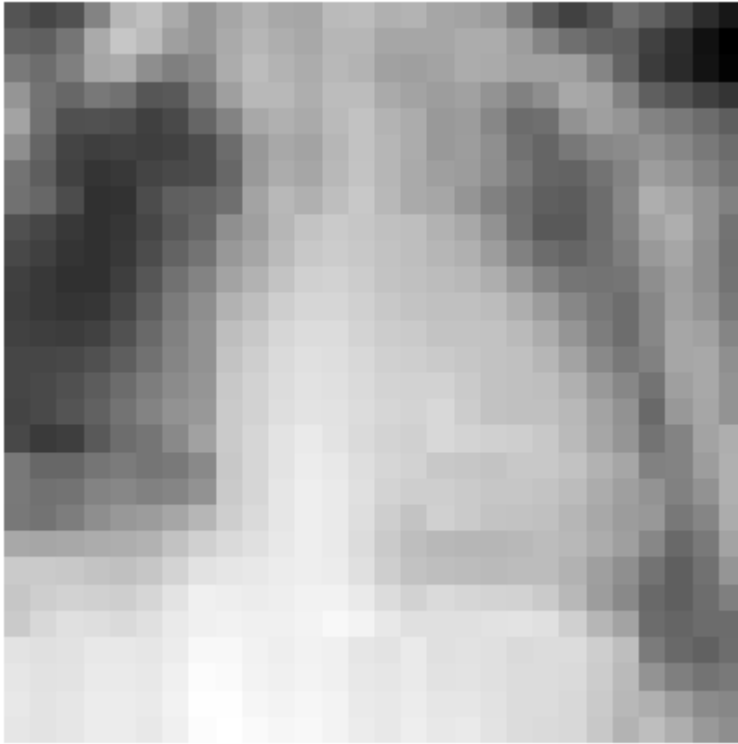


Compare the output with the original image

```
In [11]: # TODO plot the original image for comparison
        ### ADD YOUR CODE HERE

        plt.imshow(images[0][0], cmap='gray')
        plt.axis('off')
```

Out[11]: (-0.5, 27.5, 27.5, -0.5)



Task B-2: Implement `unpatchify`

Next, you are asked to create the reverse function able to take in a batch of patches and return the corresponding batch of images.

```
In [12]: def unpatchify(patches, patch_size, image_size, number_of_channels=1):
    ### TODO
    ### Write a function that takes a batch of patches (N, L, D) where D
    ### and returns the batch of images (N, C, H, W)
    ### ADD YOUR CODE HERE

    N, L, _ = patches.shape
    patch_h, patch_w = patch_size
    img_h, img_w = image_size

    # Check if L matches the expected number of patches
    assert L == (img_h // patch_h) * (img_w // patch_w), "Number of patch

    # Reshape patches to (N, H // patch_h, W // patch_w, patch_h, patch_w)
    patches = patches.view(N, img_h // patch_h, img_w // patch_w, patch_h

    # Permute and reshape to get the final images (N, C, H, W)
    images = patches.permute(0, 5, 1, 3, 2, 4).contiguous().view(N, numbe
    return images
```

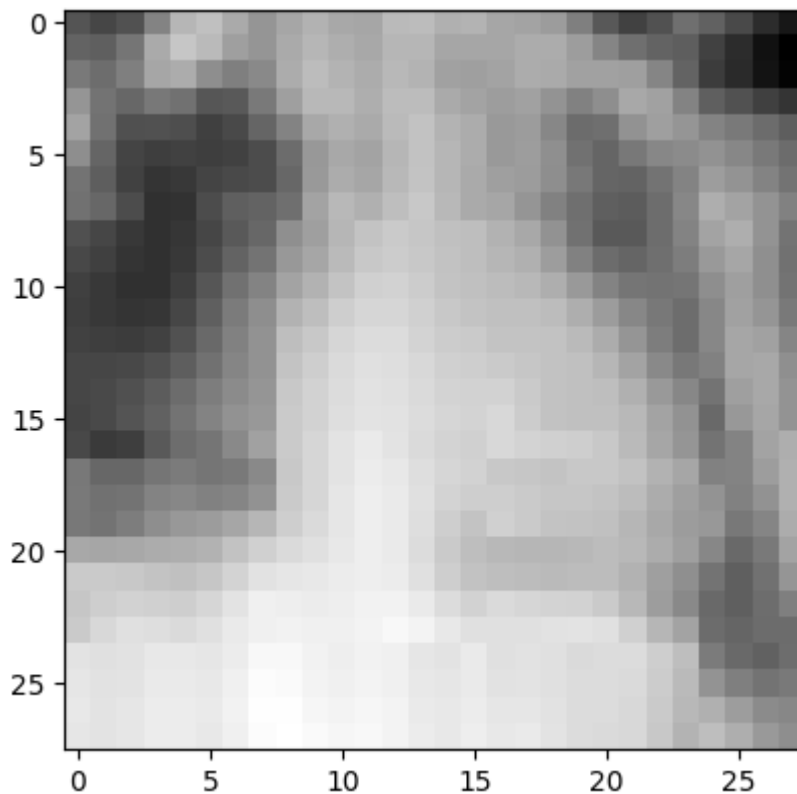
Check that after unpatchifying the patches obtained in the last cells, we get back to the original image batch.

```
In [13]: assert (unpatchify(patches, (4,4), (28,28)) == images).all()

### TODO plot the first image after applying patchify and unpatchify
### ADD YOUR CODE HERE
```

```
# Plot the original image
images = unpatchify(patches, (4, 4), (28, 28))
plt.imshow(images[0][0], cmap='gray')
```

Out[13]: <matplotlib.image.AxesImage at 0x7edabeb58730>



Task B-3: Implement random_masking

Next we need to write the function that will randomly mask out some of the patches for the encoder. We want to follow the approach described in the paper:

Simple implementation. Our MAE pre-training can be implemented efficiently, and importantly, does not require any specialized sparse operations. First we generate a token for every input patch (by linear projection with an added positional embedding). Next we *randomly shuffle* the list of tokens and *remove* the last portion of the list, based on the masking ratio. This process produces a small subset of tokens for the encoder and is equivalent to sampling patches without replacement. After encoding, we append a list of mask tokens to the list of encoded patches, and *unshuffle* this full list (inverting the random shuffle operation) to align all tokens with their targets. The decoder is applied to this full list (with positional embeddings added). As noted, no sparse operations are needed. This simple implementation introduces negligible overhead as the shuffling and unshuffling operations are fast.

Your turn: follow the textual description of the algorithm above as well as the instructions in the following docstring to implement the `random_masking` function.

This function takes the original patched batch of size (N, L, D) as input and returns:

- (a) `patches_kept` : the sequence of non-masked tokens
- (b) `mask` : a binary mask indicating which grid position are masked for every image in the batch
- (c) `ids_restore` : list of indices indicating how to revert the patch shuffling operation used to create the mask.

Hint: the `gather` function in PyTorch could prove handy for this task.

```
In [14]: def random_masking(patches, mask_ratio):
    """
    ### TODO ####
    This function performs the random_masking operation as described

    Args:
        patches: original patched batch of size (N, L, D)
        mask_ratio: float between 0 and 1, the proportion of patches

    Returns:
        patches_kept: tensor (N, L_kept, D) the sequence of non-masked
        mask: tensor (N, L) binary mask indicating which positions are masked
        ids_restore: tensor (N, L) list of indices indicating how to restore
    """

    N, L, D = patches.shape # batch, length, dim
```

```

# Step 1: create noise in [0, 1]
### ADD YOUR CODE HERE

noise = torch.rand(N, L)

# Step 2: sort noise for each sample
### ADD YOUR CODE HERE

noise, indices = noise.sort(dim=1)

# Step 3: store list of indices to revert shuffling operation later
### ADD YOUR CODE HERE

ids_restore = torch.argsort(indices, dim=1)

# Step 4: used shuffled list to keep only a subset of patches
### ADD YOUR CODE HERE

# Calculate the number of patches to keep
L_kept = int(L * (1 - mask_ratio))
# Select the indices of the patches to keep (the first L_kept after shuffle)
indices_keep = indices[:, :L_kept]

# Extract the patches to keep using gather
patches_kept = torch.gather(patches, dim=1, index=indices_keep.unsqueeze(-1))

# Step 5 : generate the binary mask
### ADD YOUR CODE HERE

# First, in the mixed order: the first L_kept are 0 (unmasked) and the rest are 1 (masked)
mask_shuffled = torch.ones(N, L, device=patches.device)
mask_shuffled[:, :L_kept] = 0
# Revert the permutation so that the mask corresponds to the original order
mask = torch.gather(mask_shuffled, dim=1, index=ids_restore)

return patches_kept, mask, ids_restore

```

```
In [15]: patches_kept, mask, ids_restore = random_masking(patches, 0.75)
```

Check the shapes of our outputs. Are there as expected?

```
In [16]: patches_kept.shape, mask.shape, ids_restore.shape
```

```
Out[16]: (torch.Size([32, 12, 16]), torch.Size([32, 49]), torch.Size([32, 49]))
```

The original input is a batch of images represented as patches with a shape of (32, 49, 16), where 32 is the batch size, 49 represents the number of patches per image (which fits a 28×28 image when divided into a 7×7 grid), and 16 is patch_size**2*C. When applying a mask ratio of 0.75, only 25% of the patches are retained. Since 25% of 49 is approximately 12.25, the function truncates this value to 12 patches per image, resulting in a tensor for patches_kept with a shape of (32, 12, 16).

Additionally, the mask tensor is created to indicate which of the 49 patches in each image are masked (represented by 1s) or not (0s), and hence it retains the original patch count, yielding a shape of (32, 49). The ids_restore tensor, which is used to restore the

original order of the patches, also has one entry per original patch, leading to the same shape of (32, 49).

Therefore, all the output shapes are as expected.

Visualisation of random masking

In this cell, we ask you to use the previously implemented functions `patchify`, `unpatchify` and `random_masking` to visualise the first three images in the validation batch at a masking ratio of 75% and 25%. Create a 2 x 3 subplots grids, the first row should be masked at 75%, the second one at 25%

```
In [17]: patch_size = (4,4)
images, _ = next(iter(datamodule.val_data_loader()))
batch_size = images.shape[0]

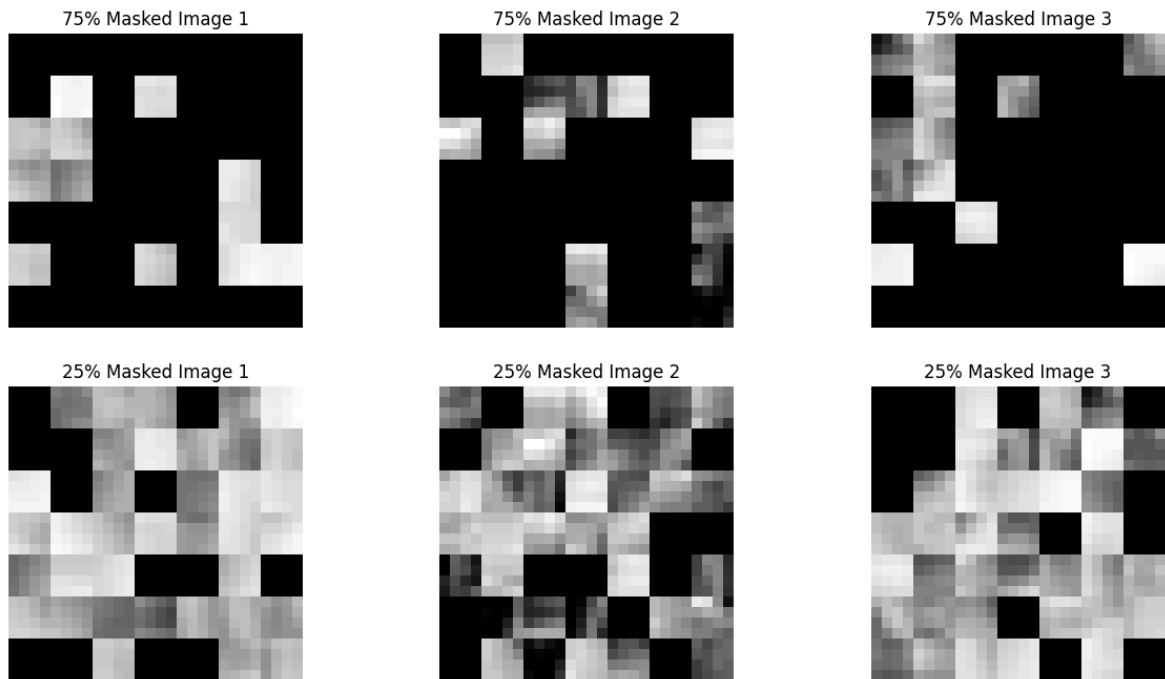
f, ax = plt.subplots(2, 3, figsize=(15, 8))

# Procesamos imagen por imagen para la visualización

# Masking ratio 75%
for i in range(3):
    # Selecciona la i-ésima imagen y agrégle la dimensión batch
    img = images[i].unsqueeze(0) # Shape: (1, C, 28, 28)
    # Divide la imagen en parches
    patches = patchify(img, patch_size) # (1, total_patches, D)
    # Aplica el masking aleatorio (por ejemplo, 75% de parches enmascarados)
    patches_kept, mask, ids_restore = random_masking(patches, 0.75)
    patches_restored = torch.zeros((batch_size, patches.shape[1], patches
    patches_restored.scatter_(1, ids_restore[:, :patches_kept.shape[1]].u
    # Reconstruye la imagen completa usando ids_restore para reinsertar l
    reconstructed = unpatchify(patches_restored, patch_size, (28,28))
    # Para imágenes en escala de grises, extraemos la única canal (recons
    ax[0, i].imshow(reconstructed[0][0].cpu().numpy(), cmap='gray')
    ax[0, i].set_title(f'75% Masked Image {i+1}')
    ax[0, i].axis('off')

# Masking ratio 25%
for i in range(3):
    img = images[i].unsqueeze(0) # (1, C, 28, 28)
    patches = patchify(img, patch_size)
    patches_kept, mask, ids_restore = random_masking(patches, 0.25)
    patches_restored = torch.zeros((batch_size, patches.shape[1], patches
    patches_restored.scatter_(1, ids_restore[:, :patches_kept.shape[1]].u
    reconstructed = unpatchify(patches_restored, patch_size, (28,28))
    ax[1, i].imshow(reconstructed[0][0].cpu().numpy(), cmap='gray')
    ax[1, i].set_title(f'25% Masked Image {i+1}')
    ax[1, i].axis('off')

plt.show()
```



Part C: Implement and train a full MAE model.

In this part, you will use the previously defined utility functions along with some helper code that we provide to implement the full training pipeline of Masked Auto-Encoder.

We here provide you with all helper functions for defining positional embeddings and for defining the ViT forward passes. You are asked to link all these pieces together by implementing the MAE forward pass and the loss function computation, along with some visualisation function.

In the following, we provide code for creating the positional embeddings for the ViT. You do not need to implement anything here, just run this cell.

```
In [18]: from functools import partial

import torch
import torch.nn as nn

from timm.models.vision_transformer import PatchEmbed, Block

import numpy as np

def get_2d_sincos_pos_embed(embed_dim, grid_size, cls_token=False):
    """
    grid_size: int of the grid height and width
    return:
    pos_embed: [grid_size*grid_size, embed_dim] or [1+grid_size*grid_size
    """
    if isinstance(grid_size, int):
        grid_size = (grid_size, grid_size)
    grid_h = np.arange(grid_size[0], dtype=np.float32)
    grid_w = np.arange(grid_size[1], dtype=np.float32)
    grid = np.meshgrid(grid_w, grid_h) # here w goes first
    grid = np.stack(grid, axis=0)
```

```

grid = grid.reshape([2, 1, grid_size[0], grid_size[1]])

# use half of dimensions to encode grid_h
emb_h = get_ld_sincos_pos_embed_from_grid(embed_dim // 2, grid[0]) #
emb_w = get_ld_sincos_pos_embed_from_grid(embed_dim // 2, grid[1]) #

pos_embed = np.concatenate([emb_h, emb_w], axis=1) # (H*W, D)

if cls_token:
    pos_embed = np.concatenate([np.zeros([1, embed_dim]), pos_embed],
    return pos_embed

def get_ld_sincos_pos_embed_from_grid(embed_dim, pos):
    """
    embed_dim: output dimension for each position
    pos: a list of positions to be encoded: size (M,)
    out: (M, D)
    """
    assert embed_dim % 2 == 0
    omega = np.arange(embed_dim // 2, dtype=np.float32)
    omega /= embed_dim / 2.0
    omega = 1.0 / 10000**omega # (D/2,)

    pos = pos.reshape(-1) # (M,)
    out = np.einsum("m,d->md", pos, omega) # (M, D/2), outer product

    emb_sin = np.sin(out) # (M, D/2)
    emb_cos = np.cos(out) # (M, D/2)

    emb = np.concatenate([emb_sin, emb_cos], axis=1) # (M, D)
    return emb

```

Task C-1: MAE model implementation

We provide you with the main skeleton for the MAE module. The init function defines the main components for you.

You are asked to fill the blanks in the following functions:

- patchify
- configure_optimizer
- random_masking
- unpatchify
- compute_loss
- forward

For each of these functions we give more detailed instructions in the docstring.

When you have finished implementing these functions, move on to the next cells to start training!

```

In [19]: class MaskedAutoencoderViT(LightningModule):
    """
    Skeleton code for MAE with ViT.
    We provide most of the boiler plate code, including the ViT encoder a

```

decoder forward passes. You are asked to link the pieces together by implementing the pieces of code marked with TODO

```

def __init__(
    self,
    img_size=224,
    patch_size=16,
    in_chans=3,
    embed_dim=1024,
    depth=24,
    num_heads=16,
    decoder_embed_dim=512,
    decoder_depth=8,
    decoder_num_heads=16,
    mlp_ratio=4.0,
):
    super().__init__()

    # MAE encoder definition
    self.embed_dim = embed_dim
    self.in_chans = in_chans
    self.patch_embed = PatchEmbed(img_size, patch_size, in_chans, emb
    num_patches = self.patch_embed.num_patches
    print(self.patch_embed.grid_size)
    self.cls_token = nn.Parameter(torch.zeros(1, 1, embed_dim))
    self.pos_embed = nn.Parameter(
        torch.zeros(1, num_patches + 1, embed_dim), requires_grad=False
    )

    self.blocks = nn.ModuleList(
        [
            Block(
                embed_dim,
                num_heads,
                mlp_ratio,
                qkv_bias=True,
                norm_layer=nn.LayerNorm,
            )
            for i in range(depth)
        ]
    )
    self.norm = nn.LayerNorm(embed_dim)

    # MAE decoder definition
    self.decoder_embed = nn.Linear(embed_dim, decoder_embed_dim, bias
    self.mask_token = nn.Parameter(torch.zeros(1, 1, decoder_embed_dim))
    self.decoder_pos_embed = nn.Parameter(
        torch.zeros(1, num_patches + 1, decoder_embed_dim), requires_
    )

    self.decoder_blocks = nn.ModuleList(
        [
            Block(
                decoder_embed_dim,
                decoder_num_heads,
                mlp_ratio,
                qkv_bias=True,
                norm_layer=nn.LayerNorm,
            )
        ]
    )

```

```

        for i in range(decoder_depth)
    ]
)

self.decoder_norm = nn.LayerNorm(decoder_embed_dim)
self.decoder_pred = nn.Linear(
    decoder_embed_dim, patch_size**2 * in_chans, bias=True
)

# Positional embeddings
pos_embed = get_2d_sincos_pos_embed(
    embed_dim=self.pos_embed.shape[-1],
    grid_size=self.patch_embed.grid_size,
    cls_token=True,
)
self.pos_embed.data.copy_(torch.from_numpy(pos_embed).float().unsqueeze(0))

decoder_pos_embed = get_2d_sincos_pos_embed(
    self.decoder_pos_embed.shape[-1],
    grid_size=self.patch_embed.grid_size,
    cls_token=True,
)
self.decoder_pos_embed.data.copy_(
    torch.from_numpy(decoder_pos_embed).float().unsqueeze(0)
)

def patchify(self, imgs):
    """
    imgs: (N, C, H, W)
    x: (N, L, D)
    """
    ### TODO: Use the previously defined function
    ### ADD YOUR CODE HERE
    N, C, H, W = imgs.shape
    patch_h, patch_w = patch_size

    # Check if H and W are divisible by patch size
    assert H % patch_h == 0, "H is not divisible by patch_h"
    assert W % patch_w == 0, "W is not divisible by patch_w"

    # Calculate number of patches
    L = (H // patch_h) * (W // patch_w)

    # Extract patches
    patches = imgs.unfold(2, patch_h, patch_h).unfold(3, patch_w, patch_w)

    # Calculate D
    D = patch_h * patch_w * C

    # Reshape patches to (N, L, D)
    patches = patches.reshape(N, L, D)

    return patches

def configure_optimizers(self):
    ### TODO: configure the optimiser to be Adam with learning rate 1e-4
    ### ADD YOUR CODE HERE

    return torch.optim.Adam(self.parameters(), lr=1e-4)

```

```

def unpatchify(self, x):
    """
    x: (N, L, D)
    imgs: (N, C, H, W)
    """
    ### TODO: Use the previously defined function
    ### ADD YOUR CODE HERE

    N, L, _ = x.shape
    number_of_channels = self.in_chans
    patch_h, patch_w = self.patch_embed.patch_size
    img_h, img_w = self.patch_embed.img_size

    # Check if L matches the expected number of patches
    assert L == (img_h // patch_h) * (img_w // patch_w), "Number of p

    # Reshape patches to (N, H // patch_h, W // patch_w, patch_h, pat
    patches = x.view(N, img_h // patch_h, img_w // patch_w, patch_h,

    # Permute and reshape to get the final images (N, C, H, W)
    imgs = patches.permute(0, 5, 1, 3, 2, 4).contiguous().view(N, num
    return imgs

def random_masking(self, x, mask_ratio):
    #def random_masking(patches, mask_ratio):

    """
    Perform per-sample random masking by per-sample shuffling.
    Per-sample shuffling is done by argsort random noise.
    x: [N, L, D], sequence
    """
    ### TODO: Use the previously defined function
    ### ADD YOUR CODE HERE

    x = x.to(self.pos_embed.device)
    N, L, D = x.shape # batch, length, dim

    # Step 1: create noise in [0, 1]

    noise = torch.rand(N, L).to(x.device)

    # Step 2: sort noise for each sample

    noise, indices = noise.sort(dim=1)

    # Step 3: store list of indices to revert shuffling operation lat

    ids_restore = torch.argsort(indices, dim=1)

    # Step 4: used shuffled list to keep only a subset of patches

    # Calculate the number of patches to keep
    L_kept = int(L * (1 - mask_ratio))
    # Select the indices of the patches to keep (the first L_kept aft
    indices_keep = indices[:, :L_kept]

    # Extract the patches to keep using gather
    patches_kept = torch.gather(x, dim=1, index=indices_keep.unsqueeze

```



```

# Step 5 : generate the binary mask

# First, in the mixed order: the first L_kept are 0 (unmasked) and
mask_shuffled = torch.ones(N, L, device=x.device)
mask_shuffled[:, :L_kept] = 0
# Revert the permutation so that the mask corresponds to the orig
mask = torch.gather(mask_shuffled, dim=1, index=ids_restore)

return patches_kept, mask, ids_restore

def forward_encoder(self, x, mask_ratio):
    """
    Forward function for the encoding part.
    """
    # embed patches (use self.patch_embed)
    x = self.patch_embed(x)

    # add pos embed w/o cls token
    x = x + self.pos_embed[:, 1:, :]

    # masking: length -> length * mask_ratio
    x, mask, ids_restore = self.random_masking(x, mask_ratio)

    # append cls token
    cls_token = self.cls_token + self.pos_embed[:, :1, :]
    cls_tokens = cls_token.expand(x.shape[0], -1, -1)
    x = torch.cat((cls_tokens, x), dim=1)

    # apply Transformer blocks
    for blk in self.blocks:
        x = blk(x)
    x = self.norm(x)

    return x, mask, ids_restore

def forward_decoder(self, x, ids_restore):
    """
    Forward function for the decoding part.
    """
    # embed tokens
    x = self.decoder_embed(x)

    # append mask tokens to sequence
    mask_tokens = self.mask_token.repeat(
        x.shape[0], ids_restore.shape[1] + 1 - x.shape[1], 1
    )
    x_ = torch.cat([x[:, 1:, :], mask_tokens], dim=1) # no cls token
    x_ = torch.gather(
        x_, dim=1, index=ids_restore.unsqueeze(-1).repeat(1, 1, x.sha
    ) # unshuffle
    x = torch.cat([x[:, :1, :], x_], dim=1) # append cls token

    # add pos embed
    x = x + self.decoder_pos_embed

    # apply Transformer blocks
    for blk in self.decoder_blocks:
        x = blk(x)
    x = self.decoder_norm(x)

```

```

    # predictor projection
    x = self.decoder_pred(x)

    # remove cls token
    x = x[:, 1:, :]

    return x

def compute_loss(self, target_patches, pred_patches, mask):
    """
    This function returns the MAE loss value for a given batch.
    Should be MSE loss over masked patches
    Args:
        target_patches: [N, L, D] ground truth patches
        pred_patches: [N, L, D] predicted patches
        mask: [N, L] binary mask indicating which patches are masked
    """
    ### TODO
    ### ADD YOUR CODE HERE

    mask = mask.unsqueeze(2) # (N, L, 1) Add third dimension to mask
    masked_pred_patches = pred_patches * mask # Masked predicted patches
    masked_target_patches = target_patches * mask # Masked target patches
    loss = F.mse_loss(masked_pred_patches, masked_target_patches) # L
    return loss

def forward(self, imgs, mask_ratio=0.75):
    """
    Forward function
    Args:
        imgs: batch of [N, C, H, W] images
        mask_ratio: masking ratio to use for the encoder

    Returns:
        predicted_patches [N, L, D], where D = patch_size[0]*patch_size
        mask [N, L]
    """
    ### TODO
    ### ADD YOUR CODE HERE

    # Forward pass through the encoder
    x, mask, ids_restore = self.forward_encoder(imgs, mask_ratio)

    # Forward pass through the decoder
    predicted_patches = self.forward_decoder(x, ids_restore)

    return predicted_patches, mask

def training_step(self, batch, batch_idx):
    images = batch[0]
    predicted_patches, mask = self(images)
    target_patches = self.patchify(images)
    loss = self.compute_loss(target_patches, predicted_patches, mask)
    self.log('loss_train', loss, prog_bar=True)

    if batch_idx == 0:
        images_output = self.unpatchify(predicted_patches * mask.unsqueeze(2))
        grid = torchvision.utils.make_grid(images[0:4], nrow=4, normalize=True)
        self.logger.experiment.add_image('train_images_input', grid, batch_idx)
        grid = torchvision.utils.make_grid(images_output[0:4], nrow=4, normalize=True)
        self.logger.experiment.add_image('train_images_output', grid, batch_idx)

```

```

        self.logger.experiment.add_image('train_images_output', grid,
        grid = torchvision.utils.make_grid(self.unpatchify(target_patches),
        self.logger.experiment.add_image('train_patches_target', grid,
        grid_predicted = torchvision.utils.make_grid(self.unpatchify(predicted_patches),
        self.logger.experiment.add_image('train_patches_predicted', grid,
    return loss

def validation_step(self, batch, batch_idx):
    images = batch[0]
    predicted_patches, mask = self(batch[0])
    target_patches = self.patchify(images)
    loss = self.compute_loss(target_patches, predicted_patches, mask)

    self.log('loss_val', loss, prog_bar=True)

def get_class_embeddings(self, images):
    """
    Return the class embeddings extracted from the encoder
    for each image in the batch.
    This function is meant to be used at inference, we do not mask
    any patches.
    """
    embeddings, _, _ = self.forward_encoder(images, mask_ratio=0)
    return embeddings[:, 0, :]

def predict_step(self, batch, batch_idx):
    images, labels = batch[0], batch[1]
    return {'embeddings': self.get_class_embeddings(images), 'labels': labels}

```

Next, we define a tiny toy ViT architecture for you to use in this coursework. This is much smaller than standard ViT architectures but will allow you to train your MAE rapidly on a single GPU. Note that we use again a patch size of 4 given the small resolution of the input images.

```

In [20]: def mae_vit_toy_patch4_dec256d4b():
    """
    Creates a toy ViT with patch size 4.
    """
    model = MaskedAutoencoderViT(
        in_chans=1,
        img_size=28,
        patch_size=4,
        embed_dim=384,
        depth=6,
        num_heads=6,
        decoder_embed_dim=256,
        decoder_depth=4,
        decoder_num_heads=8,
        mlp_ratio=4,
    )
    return model

```

Task C-2: MAE training

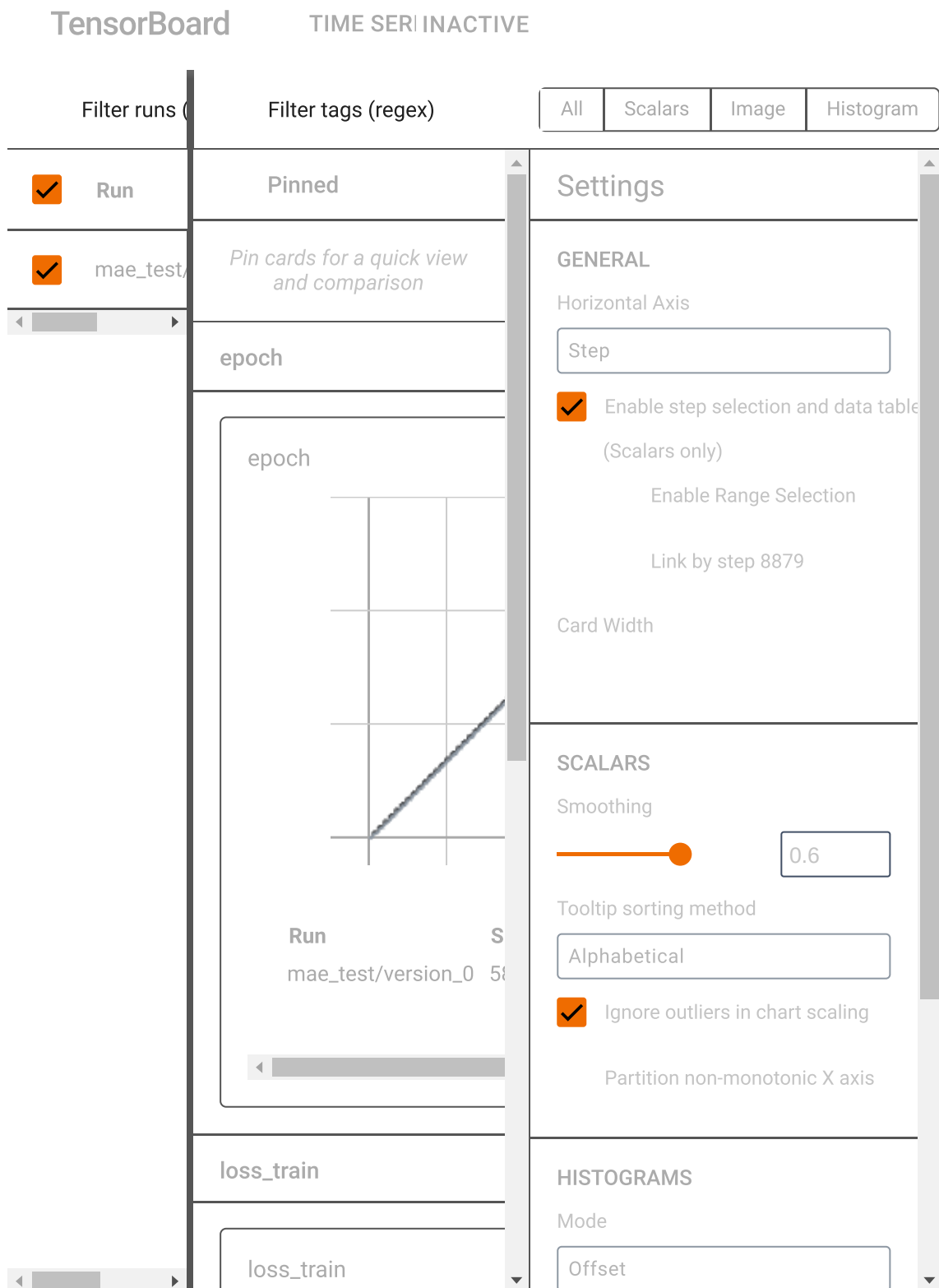
Tensorboard logging

Load tensorboard, you should be able to monitor training and validation loss as well as your reconstructed training images.

IMPORTANT keep the output of the cell, your submitted notebook should show tensorbard as well!

```
In [47]: %reload_ext tensorboard
%tensorboard --logdir './lightning_logs/coursework/'
```

Reusing TensorBoard on port 6006 (pid 7543), started 0:06:46 ago. (Use '!kill 7543' to kill it.)



We provide the training code, just run this cell and wait...

```
In [ ]: seed_everything(33, workers=True)

data = PneumoniaMNISTDataModule(batch_size=32)

device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
print(device)

model = mae_vit_toy_patch4_dec256d4b()

trainer = Trainer(
    max_epochs=60, # Increased a bit because of the jump at the end in th
    accelerator='auto',
    devices=1,
    logger=TensorBoardLogger(save_dir='./lightning_logs/coursework/', nam
)
trainer.fit(model=model, datamodule=data)
```

Seed set to 33

Using downloaded and verified file: ./data/coursework/pneumoniarnist.npz

Using downloaded and verified file: ./data/coursework/pneumoniarnist.npz

Using downloaded and verified file: ./data/coursework/pneumoniarnist.npz

GPU available: True (cuda), used: True

TPU available: False, using: 0 TPU cores

HPU available: False, using: 0 HPUs

You are using a CUDA device ('NVIDIA GeForce RTX 4060 Laptop GPU') that has Tensor Cores. To properly utilize them, you should set `torch.set_float32_matmul_precision('medium' | 'high')` which will trade-off precision for performance. For more details, read https://pytorch.org/docs/stable/generated/torch.set_float32_matmul_precision.html#torch.set_float32_matmul_precision

LOCAL_RANK: 0 - CUDA_VISIBLE_DEVICES: [0]

cuda

(7, 7)

	Name	Type	Params	Mode
0	patch_embed	PatchEmbed	6.5 K	train
1	blocks	ModuleList	10.6 M	train
2	norm	LayerNorm	768	train
3	decoder_embed	Linear	98.6 K	train
4	decoder_blocks	ModuleList	3.2 M	train
5	decoder_norm	LayerNorm	512	train
6	decoder_pred	Linear	4.1 K	train
	other params	n/a	32.6 K	n/a
13.9 M	Trainable params			
32.0 K	Non-trainable params			
13.9 M	Total params			
55.796	Total estimated model params size (MB)			
219	Modules in train mode			
0	Modules in eval mode			

Sanity Checking: | 0/? [00:00<?, ?it/s]

```
/home/eder/miniconda3/envs/mlimaging/lib/python3.10/site-packages/pytorch_lightning/trainer/connectors/data_connector.py:425: The 'train_dataloader' does not have many workers which may be a bottleneck. Consider increasing the value of the `num_workers` argument` to `num_workers=11` in the `DataLoader` to improve performance.
```

[illegible]

```

Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]
Validation: |           | 0/? [00:00<?, ?it/s]

```

```
`Trainer.fit` stopped: `max_epochs=60` reached.
```

Part D: Inspect the trained model.

In this last part, we ask you to analyse the feature embeddings (or representations) obtained from your trained model with t-SNE, similar to the tutorial on model inspection. Let's see if your model learned anything useful!

Task D-1: Inspect and compare the learned feature representations of your trained model.

Compare the feature embeddings of your trained model to embeddings obtained with a randomly initialised (untrained) model. Create some scatter plot visualisations and describe your findings with a few sentences.

```
In [23]: from sklearn.manifold import TSNE
import seaborn as sns
from sklearn import decomposition
import pandas as pd
```

Let's get the representations from our trained model:

```
In [24]: class MaskedAutoencoderViTEmbeddings(MaskedAutoencoderViT):
    def __init__(
        self,
        img_size=224,
        patch_size=16,
        in_chans=3,
        embed_dim=1024,
        depth=24,
        num_heads=16,
        decoder_embed_dim=512,
        decoder_depth=8,
        decoder_num_heads=16,
        mlp_ratio=4.0,
    ):
        super().__init__(img_size, patch_size, in_chans, embed_dim, depth)
        self.embeddings = [] # list where we still store the embeddings

    def get_embedding(self, x, mask_ratio=0.75):
        x, _, _ = self.forward_encoder(x, mask_ratio)
        return x.view(x.size(0), -1)

    def on_test_start(self):
```

```

        self.embeddings = [] # clear the list of embeddings at the start

    def test_step(self, batch, batch_idx):
        imgs, _ = batch
        emb = self.get_embedding(imgs)
        self.embeddings.append(emb)

```

```

In [38]: model_dir = './lightning_logs/coursework/mae_test/version_0/checkpoints/e

model_modified = MaskedAutoencoderViTEmbeddings.load_from_checkpoint(mode
                                                deco

trainer.test(model=model_modified, datamodule=data)
embeddings = torch.cat(model_modified.embeddings, dim=0).cpu().numpy()
print(embeddings.shape)

```

LOCAL_RANK: 0 - CUDA_VISIBLE_DEVICES: [0]

(7, 7)

/home/eder/miniconda3/envs/mlimaging/lib/python3.10/site-packages/pytorch_lightning/trainer/connectors/data_connector.py:425: PossibleUserWarning:

The 'test_dataloader' does not have many workers which may be a bottleneck. Consider increasing the value of the 'num_workers' argument to 'num_workers=11' in the 'DataLoader' to improve performance.

Testing: | | 0/? [00:00<?, ?it/s]
(624, 4992)

```

In [39]: # Create a dataframe with the class labels
labels = np.array([data.test_set[i][1] for i in range(0, len(data.test_set)
df = pd.DataFrame(labels, columns=['class_label'])

# Perform PCA on the embeddings
pca = decomposition.PCA(n_components=0.95, whiten=False)
embeddings_pca = pca.fit_transform(embeddings)

print("Embedding shape after PCA: ", embeddings_pca.shape)

# Add the PCA components to the dataframe
df['features - PCA 1'] = embeddings_pca[:,0]
df['features - PCA 2'] = embeddings_pca[:,1]
df['features - PCA 3'] = embeddings_pca[:,2]
df['features - PCA 4'] = embeddings_pca[:,3]

# Perform t-SNE on the embeddings
embeddings_tsne = TSNE(n_components=2, init='random', learning_rate='auto')

print("Embedding shape after PCA and TSNE: ", embeddings_tsne.shape)

# Add the t-SNE components to the dataframe
df['features - t-SNE 1'] = embeddings_tsne[:,0]
df['features - t-SNE 2'] = embeddings_tsne[:,1]

df.head() # showing the first five entries in the dataframe

```

Embedding shape after PCA: (624, 293)

Embedding shape after PCA and TSNE: (624, 2)

Out[39]:

	class_label	features - PCA 1	features - PCA 2	features - PCA 3	features - PCA 4	features - t-SNE 1	features - t-SNE 2
0	1	18.458714	6.261166	0.538575	10.214960	8.963068	10.153724
1	0	-10.724750	-8.332768	-14.331533	-13.483197	-8.658085	4.787352
2	1	9.716640	-15.274928	-9.090676	-4.383893	0.027103	15.896848
3	0	16.009182	-15.863500	-11.250609	10.440196	-9.291687	14.415173
4	1	-15.072647	7.944145	19.252541	-8.597525	1.458526	-8.276349

```
In [40]: # Convert the images into a numpy array and reshape them
images = np.array([data.test_set[i][0] for i in range(0, len(data.test_set))])
images = images.reshape(images.shape[0], -1) # linearize the 28x28 Pneumo

print(images.shape)

# Perform PCA on the images
pca = decomposition.PCA(n_components=0.95, whiten=False)
images_pca = pca.fit_transform(images)

# Add the PCA components to the dataframe
df['images - PCA 1'] = images_pca[:,0]
df['images - PCA 2'] = images_pca[:,1]
df['images - PCA 3'] = images_pca[:,2]
df['images - PCA 4'] = images_pca[:,3]

print("Image shape after PCA: ", images_pca.shape)

# Perform t-SNE on the images
images_tsne = TSNE(n_components=2, init='random', learning_rate='auto').fit_transform(images_pca)

# Add the t-SNE components to the dataframe
df['images - t-SNE 1'] = images_tsne[:,0]
df['images - t-SNE 2'] = images_tsne[:,1]

print("Image shape after PCA and TSNE: ", images_tsne.shape)

df.head() # showing the first five entries in the dataframe
```

(624, 784)

Image shape after PCA: (624, 64)

Image shape after PCA and TSNE: (624, 2)

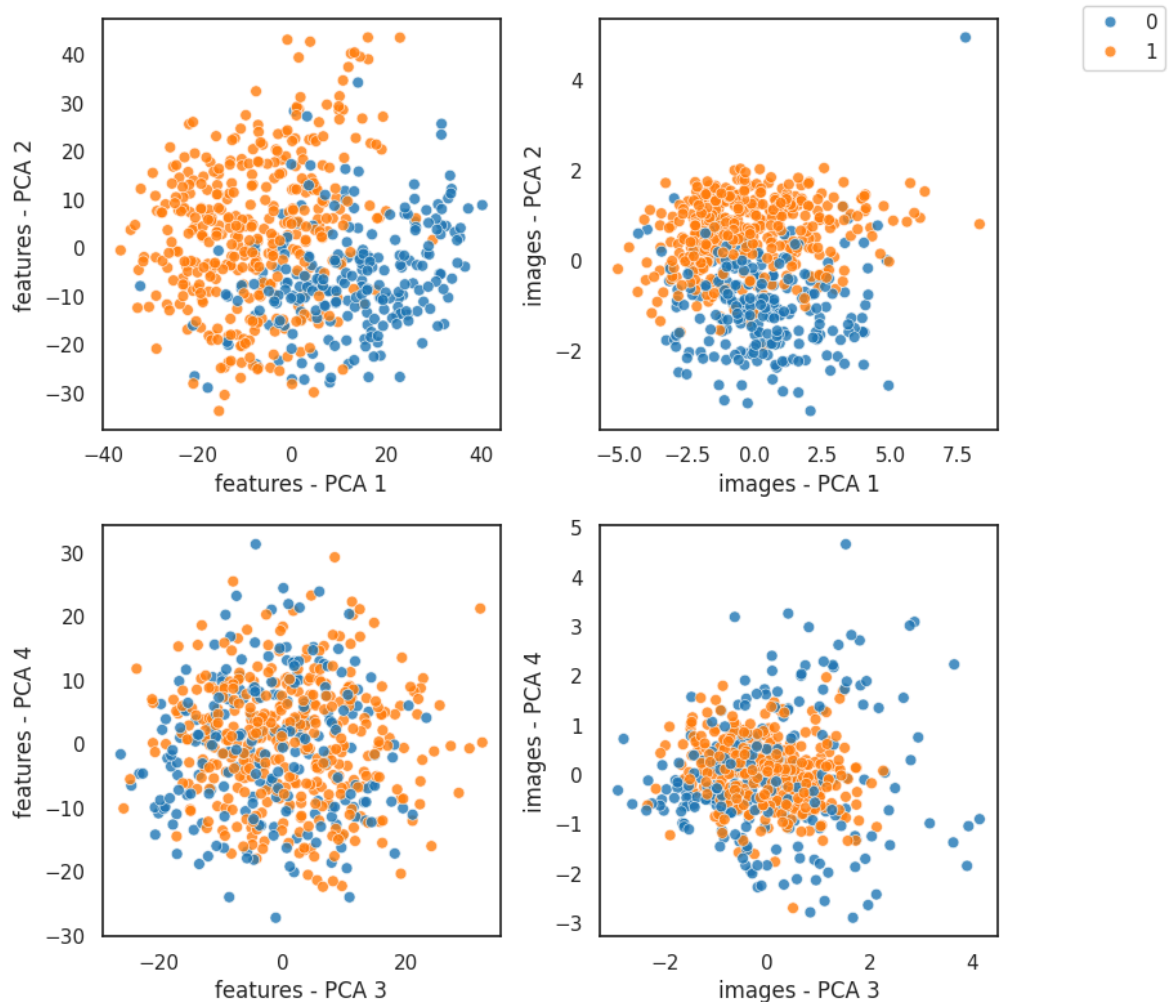
Out[40]:

	class_label	features - PCA 1	features - PCA 2	features - PCA 3	features - PCA 4	features - t-SNE 1	features - t-SNE 2	
0	1	18.458714	6.261166	0.538575	10.214960	8.963068	10.153724	1
1	0	-10.724750	-8.332768	-14.331533	-13.483197	-8.658085	4.787352	-1
2	1	9.716640	-15.274928	-9.090676	-4.383893	0.027103	15.896848	-1
3	0	16.009182	-15.863500	-11.250609	10.440196	-9.291687	14.415173	C
4	1	-15.072647	7.944145	19.252541	-8.597525	1.458526	-8.276349	-C

```
In [41]: # Define the plotting parameters
alpha = 0.8
```

```
style = 'o'  
markersize = 40  
color_palette = 'tab10'  
kind = 'scatter'
```

```
In [42]: fig, axs = plt.subplots(2, 2, figsize=(8, 8))  
sns.set_theme(style="white")  
  
# First plot: we leave the legend active to extract it later  
ax0 = axs[0, 0]  
sns.scatterplot(ax=axs[0, 0], data=df, x='features - PCA 1', y='features  
                hue='class_label', alpha=alpha, marker=style, s=markersiz  
                palette=color_palette)  
# Extract the handles and labels  
handles, labels = ax0.get_legend_handles_labels()  
# Remove the legend from the first subplot  
if ax0.get_legend() is not None:  
    ax0.get_legend().remove()  
  
# Second plot  
sns.scatterplot(ax=axs[0, 1], data=df, x='images - PCA 1', y='images - PC  
                hue='class_label', alpha=alpha, marker=style, s=markersiz  
                palette=color_palette, legend=False)  
  
# Third plot  
sns.scatterplot(ax=axs[1, 0], data=df, x='features - PCA 3', y='features  
                hue='class_label', alpha=alpha, marker=style, s=markersiz  
                palette=color_palette, legend=False)  
  
# Fourth plot  
sns.scatterplot(ax=axs[1, 1], data=df, x='images - PCA 3', y='images - PC  
                hue='class_label', alpha=alpha, marker=style, s=markersiz  
                palette=color_palette, legend=False)  
  
# Create the global legend  
fig.legend(handles, labels, loc='upper right', bbox_to_anchor=(1.15, 1))  
plt.tight_layout()  
plt.show()
```

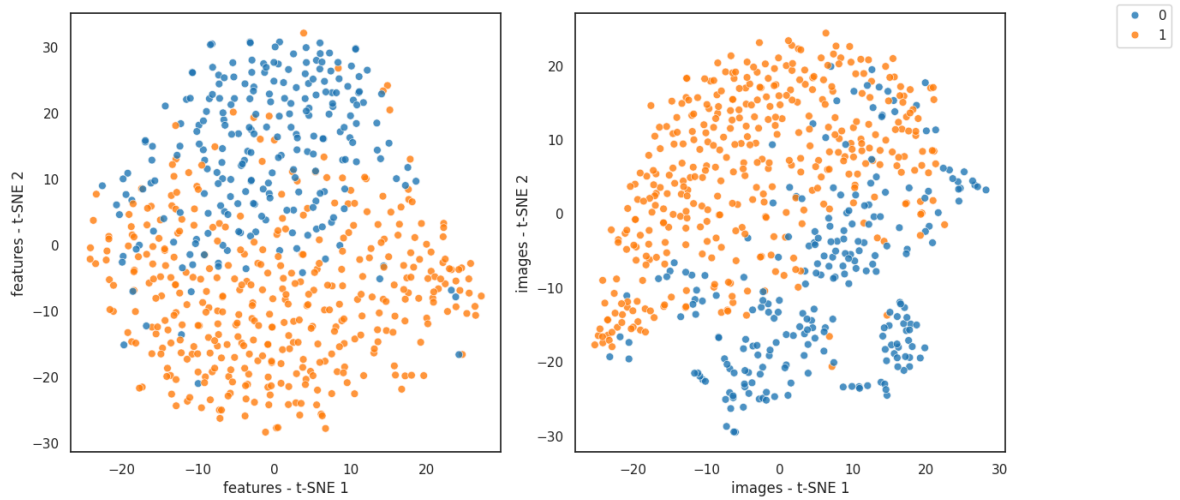


```
In [43]: # Know we plot the results for the t-SNE embeddings
fig, axs = plt.subplots(1, 2, figsize=(12, 6))
sns.set_theme(style="white")

ax0 = axs[0]
sns.scatterplot(ax=ax0, data=df, x='features - t-SNE 1', y='features - t-SNE 2',
                hue='class_label', alpha=alpha, marker=style, s=markersize,
                palette=color_palette)
handles, labels = ax0.get_legend_handles_labels()
if ax0.get_legend() is not None:
    ax0.get_legend().remove()

sns.scatterplot(ax=axs[1], data=df, x='images - t-SNE 1', y='images - t-SNE 2',
                hue='class_label', alpha=alpha, marker=style, s=markersize,
                palette=color_palette, legend=False)

# Create the global legend
fig.legend(handles, labels, loc='upper right', bbox_to_anchor=(1.15, 1))
plt.tight_layout()
plt.show()
```



```
In [44]: import matplotlib as mpl
import plotly.graph_objs as go
import plotly.express as px
from matplotlib import cm
from ipywidgets import Output, HBox
```

```
In [45]: def rgb_to_hex(rgb):
    return '#{02x}{02x}{02x}'.format(rgb[0], rgb[1], rgb[2])

color = cm.tab10(np.linspace(0, 1, 10))
colorlist = [(np.array(mpl.colors.to_rgb(c))*255).astype(int).tolist() for c in color]
colors = [rgb_to_hex(colorlist[label]) for label in df.class_label.values]
```

```
In [46]: x = 'features - t-SNE 1'
y = 'features - t-SNE 2'

out = Output()
@out.capture(clear_output=True)
def handle_click(trace, points, state):
    idx = df.index.values[points.point_inds[0]]
    img = images[idx, :]

    s = [8] * len(df)
    for i in points.point_inds:
        s[i] = 16
    with fig.batch_update():
        scatter.marker.size = s

    f, ax = plt.subplots(1,1, figsize=(4,4))
    ax.imshow(img.reshape((28,28)), cmap='gray')
    ax.axis('off')
    plt.show(f)

fig = go.FigureWidget(px.scatter(df, x=x, y=y, template='plotly_white', height=600))
fig.update_layout(width=600, height=600)
scatter = fig.data[0]
scatter.on_click(handle_click)
scatter.marker.size = [8] * len(df)
scatter.marker.color = colors

HBox([fig, out])
```

```
Out[46]: HBox(children=(FigureWidget({
    'data': [{'customdata': array([[1],
                                     [0],
                                     ...
```

Let's compare with the representation of an untrained model.

```
In [34]: # Create the untrained model
untrained_model = MaskedAutoencoderViTEmbeddings(in_chans=1, img_size=28,
                                                    decoder=decoder)

trainer.test(model=untrained_model, datamodule=data)
embeddings = torch.cat(untrained_model.embeddings, dim=0).cpu().numpy()
print(embeddings.shape)
```

```
LOCAL_RANK: 0 - CUDA_VISIBLE_DEVICES: [0]
(7, 7)
```

```
/home/eder/miniconda3/envs/mlimaging/lib/python3.10/site-packages/pytorch_
lightning/trainer/connectors/data_connector.py:425: PossibleUserWarning:
```

The 'test_dataloader' does not have many workers which may be a bottleneck. Consider increasing the value of the 'num_workers' argument to 'num_workers=11' in the 'DataLoader' to improve performance.

```
Testing: |          | 0/? [00:00<?, ?it/s]
(624, 4992)
```

```
In [35]: # Create a dataframe with the class labels for the untrained model
labels = np.array([data.test_set[i][1] for i in range(0, len(data.test_set))])
df = pd.DataFrame(labels, columns=['class_label'])

# Perform PCA on the embeddings
pca = decomposition.PCA(n_components=0.95, whiten=False)
embeddings_pca = pca.fit_transform(embeddings)

print(embeddings_pca.shape)

# Add the PCA components to the dataframe
df['features - PCA 1'] = embeddings_pca[:,0]
df['features - PCA 2'] = embeddings_pca[:,1]
df['features - PCA 3'] = embeddings_pca[:,2]
df['features - PCA 4'] = embeddings_pca[:,3]

# Perform t-SNE on the embeddings
embeddings_tsne = TSNE(n_components=2, init='random', learning_rate='auto')

print(embeddings_tsne.shape)

# Add the t-SNE components to the dataframe
df['features - t-SNE 1'] = embeddings_tsne[:,0]
df['features - t-SNE 2'] = embeddings_tsne[:,1]

df.head() # showing the first five entries in the dataframe
```

(624, 62)

 $(624, 2)$

Out[35]:

	class_label	features - PCA 1	features - PCA 2	features - PCA 3	features - PCA 4	features - t-SNE 1	features - t-SNE 2
0	1	0.085608	-2.314194	3.707670	-2.359124	1.803094	-1.547044
1	0	0.716355	5.428017	3.517769	4.926601	-5.493159	13.958353
2	1	-1.473587	4.459114	3.500272	-0.861529	-18.851530	4.424058
3	0	0.858930	-8.287944	-3.508290	4.701985	10.046527	-13.687622
4	1	-1.767120	-5.695096	-8.738058	3.342774	6.433622	-9.634429

```

In [36]: fig, axs = plt.subplots(1, 3, figsize=(18, 6))
sns.set_theme(style="white")

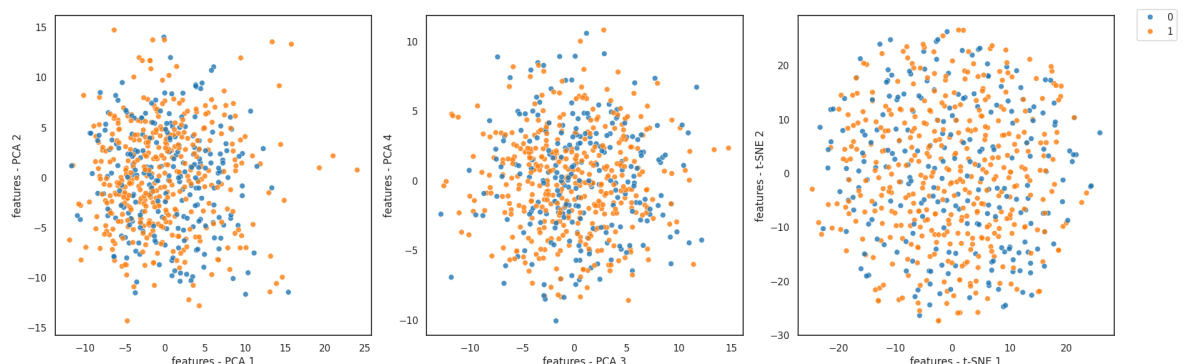
# Firts plot
ax0 = axs[0]
sns.scatterplot(ax=ax0, data=df, x='features - PCA 1', y='features - PCA 2',
                hue='class_label', alpha=alpha, marker=style, s=markersize,
                palette=color_palette)
handles, labels = ax0.get_legend_handles_labels()
# Remove the legend from the first subplot
if ax0.get_legend() is not None:
    ax0.get_legend().remove()

# Sedond plot
sns.scatterplot(ax=axs[1], data=df, x='features - PCA 3', y='features - PCA 4',
                hue='class_label', alpha=alpha, marker=style, s=markersize,
                palette=color_palette, legend=False)

# Third plot
sns.scatterplot(ax=axs[2], data=df, x='features - t-SNE 1', y='features - t-SNE 2',
                hue='class_label', alpha=alpha, marker=style, s=markersize,
                palette=color_palette, legend=False)

# General legend
fig.legend(handles, labels, loc='upper right', bbox_to_anchor=(1.05, 1))
plt.tight_layout()
plt.show()

```



Summarise your observations...

The scatter plots of the first two PCA components reveal a significantly better separation of class labels compared to the third and fourth components. A similar trend appears when PCA is applied directly to the raw images—the first two dimensions yield

much clearer class separation, suggesting that these dimensions are the most informative.

Furthermore, when t-SNE is applied to the PCA outputs, the visual discrimination of the labels is further enhanced. In the case of the embeddings, t-SNE produces a more distinct separation between the two groups, although some misclassified instances still persist. A comparable improvement is evident when t-SNE is applied to the PCA-reduced images, with the classes becoming more distinguishable even though a few samples remain ambiguous. This outcome is intuitive since PCA alone may discard certain dimensions that contain relevant information, whereas t-SNE, by capturing the nonlinear structure in the data, offers a more holistic visualization.

In stark contrast, the visualizations derived from the embeddings of an untrained model—both via PCA and t-SNE—show the classes to be completely intermixed. This stark difference underscores the critical role of training in enabling the model to learn discriminative features.

The interactive visualizations also reveal that while some misclassifications are understandable due to the inherent ambiguity of certain samples, others are less intuitive. Although the trained model exhibits a marked improvement over its untrained counterpart, the presence of misclassified instances in the t-SNE plots indicates that there is still room for refinement.

Rather than simply increasing the number of epochs—which would probably further reduce the training loss as we see in the validation plot—a better improvement could involve incorporating a contrastive learning objective into the training process. By integrating a contrastive loss (or even a triplet loss), the model would be encouraged to pull together similar samples while pushing apart dissimilar ones in the latent space, potentially resulting in a more robust and discriminative feature representation. This approach could lead to a more finely tuned model that better separates the classes in its embeddings.