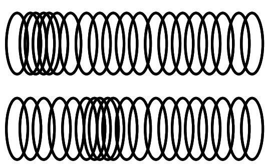
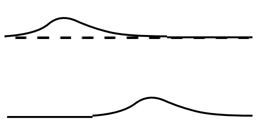
Part 2 WAVE MOTION AND SOUND

The general discussion of wave motion is important because the ideas of wave propagation are ubiquitous. In nearly all areas of science (and therefore real life) energy is transferred via the vibrations that make up waves. Examples of wave motion include waves on strings, water waves, seismic waves,, sound, all electromagnetic radiation including light, heat, x-rays, etc. There are many common elements to all the various types of wave motion that can be described - and these will be pointed out. There are also some differences - especially between the mechanical waves such as waves on strings and sound and all electromagnetic waves - which will be important in some discussions. What is common to all forms of wave motion is the idea that a disturbance is being propagated from one place to another without the necessity for the medium through which the disturbance is being propagated to itself be transported. (We will see what that means shortly.)

It is useful to first classify wave motion into several different categories. Already mentioned are mechanical waves and electromagnetic waves. This discussion will only deal with mechanical waves (although many of the important ideas also apply to electromagnetic waves). Mechanical waves can be either *longitudinal* or *transverse*. The distinction will be whether the disturbance that is being propagated is in the direction of travel of the wave or perpendicular to it.



<u>Longitudinal Waves:</u> When a wave propagates through some medium, if the local displacements of the medium that constitute the disturbance are in the direction of travel of the disturbance, then the wave is l*ongitudinal*. An example of a longitudinal wave is the pulse that can be sent along a stretched slinky by shaking one end of the slinky along its length. The pulse moves along the line of the slinky and ultimately makes the other end move. Notice that in this case, the individual coils of the slinky vibrate back and forth about some equilibrium position, but there is no net movement of the slinky itself. You can think of the slinky as the medium through which the pulse travels - and the wave motion describes the <u>disturbance</u> rather than the slinky. Examples of longitudinal waves are sound waves through the air or compression waves through some solid object.



<u>Transverse Waves</u>: A disturbance that is *perpendicular* to the direction of travel are called *transverse* waves. Examples are waves on strings, surface waves on the water, etc. That is, the wave itself travels along the string or water surface - but displacements of the medium through which the wave travels are perpendicular to the direction of the wave propagation.

In both cases (and in all other forms of wave motion), the disturbance moves through the medium (slinky, string, water, air, whatever....) with only a minimal motion of the medium itself. What is being described in the equations of wave motion is the motion of the disturbance. The *wave speed*, for example, is the speed at which the disturbance moves.

The most general form of the differential equation that describes a mechanical wave is written:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = v^2 \frac{\partial^2 y(x,t)}{\partial x^2}$$

As seen in the derivation for a wave on a string, it is just Newton's second law applied to a small part of the string itself. The most general solution to this equation is *any* function of the argument

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($x \pm vt$) and v is the speed at which the wave propogates. That is, any function of x and t that has the form $y(x,t) = f(x \pm vt)$
would solve the above differential wave equation with v being the speed of the disturbance (*ie,* the wave) through the medium. The specific form of the wave would depend on the source of the disturbance (a hand clap would be different than a tuning fork) - but the wave speed itself would depend on the medium through which the disturbance travelled. The direction of propagation is determined by the sign in the argument. $f(x+vt)$ corresponds to a wave traveling to the *left*, a "-" sign is to the right.

Wave Propagation Speed

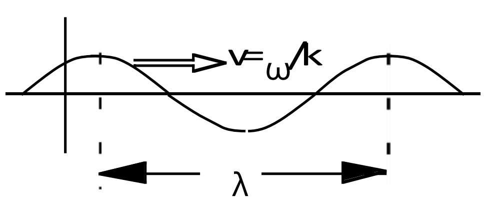
The wave speed is determined by properties of the medium (except for *light* which travels at speed of 3x108 m/s with respect to <u>every</u> observer, not with respect to some medium). In general, the speed of a mechanical wave depends on the resilience (or maybe the "stiffness") of the medium as well as its mass density. The more rigid the medium, *ie* the higher the restoring force as a disturbance moves through the medium, the faster the wave propagation speed. But the higher the mass density - a measure of the medium's *inertia* - the slower the wave speed.

For a transverse wave on a taut string or a longitudinal wave on a stretched "slinky", the wave speed is dependent on the linear mass density of the string (*ie*, the mass per unit length μ) and the tension in the string or the slinky (F). Ie,

$$v = \sqrt{\frac{F}{\mu}}$$

For sound waves through air or water, the wave speed would depend on the compressibility of the medium and the volume mass density (since a disturbance can propagate in all directions in a three dimensional medium). Since water is nearly like a solid with respect to compressions, it should not be surprising that sound travels faster through water than through air (even though the mass density is a thousand times as great).

<u>Harmonic Waves</u>



$y(x,t) = A\sin(kx - \omega t)$ Many important examples of wave motion involve a wave function that is sinusoidal. That is, rather than a single pulse that propagates, the disturbance is in the shape of a

x sine wave. The resulting wave motion description is then

$$y(x,t) = A\sin(kx \pm \omega t)$$

It is not difficult to show that this equation, in fact, is a solution to the differential wave equation by just taking derivatives of $y(x,t)$ and substituting into the differential equation. When that is done, one can see the connection between the wave speed and the constants k and ω. The *wave number*, as k is called, is just 2π divided by the wavelength. The *angular frequency* ω, has the same meaning in wave motion as it does in simple harmonic motion or circular motion. And the wave speed, wavelength, and frequency are always related in the same way.

2π	2π

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$$k = \_\_\_ \quad \text{and} \quad \omega = 2\pi f = \_\_\_ \quad \text{and} \quad v = \lambda f\ \lambda$$
$$T$$

These relationships are always true.

Notice that for any specific time t, the shape of the wave as you move along the string (ie y vs x) is just that of a sine function. And if you consider only that part of the wave at a specific location x, the motion of the medium itself is simple harmonic with an amplitude A and angular frequency ω.

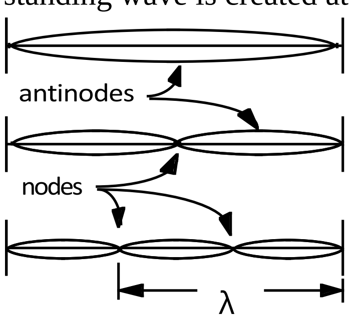
Given the relationships between k, f, T, λ, and ω, the wave function $y(x,t)$ can be written in a variety of ways. They are all equivalent.

$$y(x,t) = A\sin(kx \pm \omega t) = A\sin\left(\frac{2\pi}{\lambda}x \pm 2\pi f t\right) = A\sin k(x \pm vt)$$

The amplitude of the wave is just the maximum displacement of any part of the medium from the equilibrium or undisturbed position. The frequency and period of the oscillation, the wavelength, the wave number, etc., and the wave speed are then related by the equations above. For a wave on a string, the wave speed is still dependent on the mass density and the tension by $v=\sqrt{F/\mu}$

Standing Waves

When a harmonic wave is reflected back on itself, the resulting wave description can be obtained by simply adding two identical waves - one traveling to the right and the other to the left. When that is done, the result is a *standing wave* - a sinusiodal wave that does not travel along the string, but rather just oscillates at the frequency of the source. There are well defined *nodes* , or points of zero amplitude that are separated by half a wavelength. The *antinodes* are the locations of maximum amplitude oscillations where the two waves undergo *constructive interference*. The idea of standing waves is very important in many different fields of study and is closely related to the idea of*resonance* - ie, when a sinusoidal driving force stimulates a system to vibrate in one of its normal modes of oscillation, a standing wave is created at that frequency.



$$y(x,t) = A\sin(kx - \omega t) + A\sin(kx + \omega t) = 2A\sin(kx)\cos(\omega t)$$

The amplitude of the wave as described above is twice the amplitude of each individual component. For a string which is fixed at both ends, node must appear at the ends of the string. Hence a standing wave can only be stimulated when the length is a multiple of half wavelengths. That means only certain frequencies can possibly stimulate a standing wave on such a string. That situation is called a *standing wave resonance*.

The frequencies that are possible for such a standing wave are those for which the length of the string is an integral number of half wavelengths. That is:

$$f_n = \frac{v}{\lambda} = \left(\frac{n}{2L}\right) v = \left(\frac{n}{2L}\right)\sqrt{\frac{F}{\mu}}$$

It is useful to notice how this applies to such things as musical instruments. When a violin string is tightened, for example, all of the normal mode frequencies increase as predicted by the above equation since the tension is increased. Or when a string is shortened - for example, when it is being played and the violinist "fingers" the string - the frequency is also increased since L is decreased.

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Superposition and Interference

In general, the idea of superposition of waves is common to all types of wave motion. The standing wave problem discussed in the waves-on-a-string discussion is simply the result of two identical waves traveling in opposite directions, as shown previously. The superposition of two waves traveling in the *same* direction will lead to the ideas of *constructive and destructive interference* and will be important in both sound and light. But the mathematics is most easily developed using the equations of harmonic waves to describe waves on a string.

Suppose two identical waves are traveling in the same direction, and they differ only in that there is a phase difference between them. That is, the waves $y_1(x,t)$ and $y_2(x,t)$ are the two waves described by

$$y_1(kx-\omega t) \quad \text{and} \quad y_2(kx-\omega t+\delta)$$

The wave descriptions can be rewritten by adding and subtracting $\delta/2$ in each argument. For the sake of the derivation, we can then let $\theta = kx\text{-}wt+\delta/2$. The total combined wave is then written:

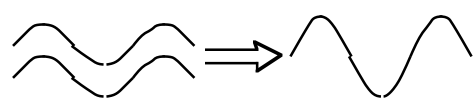
$$y_1(kx-\omega t+\delta/2-\delta/2)+y_2(kx-\omega t+\delta+\delta/2-\delta/2) = y_1(\theta-\delta/2)+y_2(\theta+\delta/2)$$

Using trigonometric identities to break $\sin(\theta\pm\delta/2)$ into products of sines and cosines yields the following result:

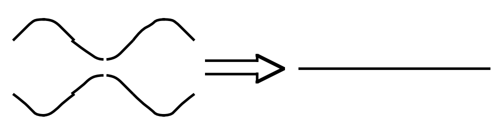
$$y_{total}(x,t) = 2A\cos(\delta/2)\sin(kx-\omega t+\delta/2)$$

This equation represents the combined wave equation. Notice that it simply represents a traveling wave with the same frequency and wavelength as the constituent waves, but with an amplitude $2A\cos(\delta/2)$ that depends on the phase difference δ. When δ.=0, 2π, 4π, etc., the amplitude of the combined wave is $2A$ - that is, the two waves add up *in phase*. When the phase difference is an *odd* multiple of π, then the $\cos(\delta./2)=0$ and there is total cancellation.

Constructive interference: δ.=0, 2π, 4π, etc.



Destructive interference: δ.=π, 3π, etc.



Interference of this type occurs in both sound and light. It is the condition of the problem that identifies the cause (and magnitude) of the phase difference. If two sources of identical waves (on a string, sound, light, it doesn't matter) are out of phase by an amount δ., then the resulting wave pattern could be described by the equation:

$$y_{total}(x,t) = 2A\cos(\delta/2)\sin(kx-\omega t+\delta/2)$$

The phase difference δ. could *also* be a result of a *path difference* between the two sources and the point at which the two waves are detected. If there is a path difference given by Δr, then the phase difference between the two waves is given by $\delta=2\pi\Delta r/l$. So the *total* phase difference is then:

$$\delta_{total} = \delta_{sources} + \delta_{path\ difference} = \delta_{sources} + 2\pi\Delta r/\lambda$$

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This discussion will ultimately be important to understanding the constructive and destructive interference associated with sound waves from two loud speakers as well as, ultimately, the interference (*and* diffraction) of light waves from two slits (an experiment you will do at the very end of the quarter).

Energy transmitted by a wave

Wave propagation transfers energy through the medium in which the wave travels. If a disturbance is transmitted down a string or a spring (or through a solid or across an ocean), the energy associated with the motion of the atoms that make up the system is thus transferred through that medium. The energy per second transmitted by the wave - that is, the average *power* delivered - is proportional to the square of the amplitude of that wave (and also depends on the wave speed and the square of its frequency). The details will depend on the specific type of wave and are not important here - except to recognize that the power delivered is always proportional to the amplitude squared, independent of the type of wave motion under discussion.

$$P_{average} = (constants) v \omega^2 A^2$$

This equation is only meant to indicate the general form of the average power delivered by a wave. The specific type of wave would determine the values of the constants that go into this expression.

<u>SOUND</u>

This study of sound will concentrate on only a few main ideas - sound as an example of a longitunal wave exhibits all the properties that all waves exhibit including a speed that depends on the medium that carries the wave and both interference and diffraction. Sound level measurements (the decibel scale) are related to the energy density in the wave, and the apparent frequency of the sound one hears depends on both the speed of the source and the speed of the listener relative to the speed of sound in air (the Doppler effect). The most general study of sound would include discussions of how sound propagates through air as well as in liquids and solids, our perceptions of sound - which would require understanding the physiology of hearing, and would ultimately lead to the study of musical instruments and the complex study of acoustics.

Sound as a wave

The general principles studied in the discussion of wave motion apply equally well to sound. That includes, of course, the most general relationships between wave speed, wavelength, and frequency: That is, $v = \lambda f$

where λ represents the wavelength and f is the frequency. The wave speed v is determined by properties of the medium - which we will consider is air. The wave itself is longitudinal rather than transverse as are waves on strings and the surface waves on water. But the waves propagate in all directions as a speed determined by the compressibility and mass density of the air. The propagation of a disturbance in air is described by a differential equation of the same form as for transverse waves on a taut string. So the solutions to the equation are necessarily of the same form as well. That is, a disturbance in air is governed by a wave equation of the form

$$\frac{\partial^2 y(x,t)}{\partial t^2} = v^2 \frac{\partial^2 y(x,t)}{\partial x^2}$$

which has solutions representing waves traveling a speed v that depends on the *bulk modulus* B (which in turn depends on pressure and temperature) and on the mass density ρ of the air. That is, the disturbance that propagates through the air that we call "sound" can be described with the same type of wave equation that was used to describe waves on strings. And the speed of those waves depends on properties of the medium through which they travel.

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$$v = \sqrt{B/\rho}$$

where B plays the same role as the tension in the string and ρ is the mass per unit volume of the air rather than the linear mass density of the string which supported a transverse wave.

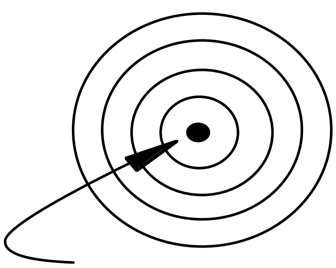
In air, the wave speed can be related to the air temperature by

$$v = \sqrt{\frac{\gamma RT}{M}} = (331 \text{ m/s}) \sqrt{\frac{T}{273}}$$

where γ is a constant for air, R is the Universal Gas Constant, M is the average molecular mass of air and the temperature is the Absolute temperature on the Kelvin scale. The equation can be simplified in terms of the speed of sound in air at 273 K (ie, ice point).

Sound Intensity

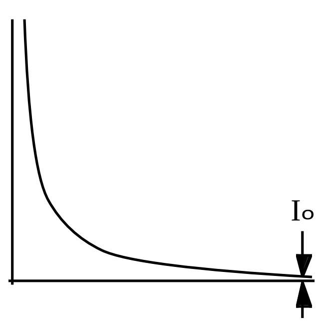
The *intensity* of a wave is simply the energy per unit time that is transferred per unit area of a surface that the wave impinges on. But energy per time is just the power that is delivered by the source. And that energy is distributed over an ever increasing area as the wave propagates away from the source. Assuming a point source of sound, with waves spreading outward in spherical wave fronts - and assuming no energy dissipation as the wave propagates through the air - the intensity decreases as the inverse square of the distance from the source as the energy is spread over an ever increasing spherical surface. So the intensity, or power per unit area, is simply given by



Intensity in watts/m2 $$I = \frac{P_{avg}}{4\pi r^2}$$

where P_{avg} is the power emitted by the source and r is the distance from the source. The intensity I would be measured in watts per square meter.

In practice, the intensity of a sound is much more complicated, since the above expression assumes a point source of sound that spreads out uniformly in all directions. Ignored in this expression is the absorption of sound by the air itself and reflections from surfaces that the sound encounters.



I(r) Graphing the intensity as a function of distance from the source shows how intensity diminishes as the energy of the sound waves is spread over an ever increasing area. As the distance from the sound source increases, the intensity decreases until it would be undetectable. The weakest sound intensity that most humans can hear is about 10^{-12} w/m^2. That level is called the *threshold of hearing* and is assigned the symbol I_o. All other sound intensities can be related to I_o. That is, a sound intensity one hundred times the threshold of hearing would be written $I=10^2 I_o$., etc. The loudest sounds that humans can endure over any prolonged time - although that level is physically uncomfortable - has an intensity of about one watt/m^2 and is called the *threshold of pain*. And we can detect much larger intensities even than that - although they can cause permanent hearing loss.

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The Relationship Between Intensity and Loudness - the Sound Level in Decibels

Because human hearing covers such an enormously large range in intensities (perhaps 15 orders of magnitude!) the measure of *sound levels* makes use a logarithmic scale called the *decibel* scale which more accurately reflects our *perception* of the loudness of sound.

The *sound level* (sometimes called the *intensity level* - a term which I think is too easily confused with "intensity" which is measured in w/m^2) associated with any sound is determined by the intensity ratio relative to the threshold of hearing and is measured in decibels. That is, the sound level in decibels is defined by

$$\text{Sound level in decibels } \beta = 10 \log\left(\frac{I}{I_o}\right)$$

It is useful to notice that the sound level of the threshold of hearing would be given as 0 db, since log(1)=0. It is also useful to notice that the sound intensity that humans find physically uncomfortable (called the *threshold of pain*) is about 1 w/m^2 - which is 10^{12} times the threshold of hearing. The corresponding sound level is thus 10 log (10^{12}), or 120 db.

When comparing two different sound intensities, say I_1 and I_2 the *difference* in sound levels would be given by $\Delta\beta=\beta_2-\beta_1=10 \log (I_2/I_o)-10 \log (I_1/I_o)$, which would reduce to

$$\Delta\beta = 10 \log\left(\frac{I_2}{I_1}\right)$$

When the source of sound is a "point source" - ie, the sound spreads out uniformly in all directions - the intensity is given by $I = P/4\pi r2$. So the sound level difference between the two points at different distances from the same source is given by

$$\Delta\beta = 10 \log\left(\frac{r_1^2}{r_2^2}\right) = 20 \log\left(\frac{r_1}{r_2}\right)$$

Examples:

- A factor of two change in *intensity* is a 3 db change in *sound level*. That is, if $I_2 = 2I_1$, then the sound level difference is $\Delta\beta$=10 log 2=3 db, since log 2 = 0.3.
- A change in *sound level* of N db corresponds to a change in *intensity* of a factor of $10^{N/10}$. That is, if $\Delta\beta=N$, then $N/10 = \log (I_2/I_1)$, so $I_2 = 10^{N/10} I_1$. Notice that this means that each change in sound level of 10 db is a factor of ten change in intensity. A 20 db increase in sound level is 100 times "louder" - or more intense - than the original!
- Moving twice as far from an omnidirectional sound source reduces the intensity of the sound by a factor of 4 (since r is squared in the denominator of the intensity expression) and hence the sound level decreases by 6 db. That is, $\Delta\beta=10 \log (r_1^2/r_2^2)=10 \log (1/4) = -6$ db

INTERFERENCE

The discussion of interference of sound waves will appear in three different types of problems, the applications of which will appear quite different even though there is a great similarity in the development of the ideas. We will consider interference between two identical sound waves that are constrained to travel in a straight line, but in opposite directions. This is identical in principle to the

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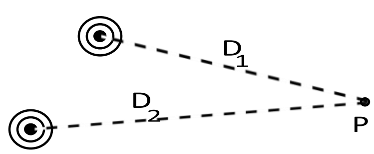
standing waves on a string problem. Secondly, we will consider two identical sound waves that arrive at the same point from two different sources which are separated from each other (as the sound from two loudspeakers driven by the same signal). In general, those sources can either be exactly in phase - or there could be a phase difference between the signals produced at the speakers. This problem is identical to the superposition and interference of two waves on a string traveling in the same direction. The resulting wave depended on the phase difference between the two signals. Finally, when two sources have slightly different frequencies, there is no fixed phase difference between them and the super-position of the two signals results in a varying intensity wave with a "beat frequency" that depends on the difference in the individual frequencies that are being added.

The general principle, of course, is the *superposition principle* - that is, waves can be added to form a new wave which depends on the properties and characteristics of the original waves. This idea treated earlier applies equally well to longitudinal waves. Adding two waves $y_1(k_1x\pm\omega_1t)$ and $y_2(k_2x\pm\omega_2t+\delta)$ results in a wave equation that represents one of three possibilities: A standing wave if y_1 and y_2 have the same frequency (and hence wavelengths) and are traveling in opposite directions; either constructive or destructive interference at a particular location, depending on the phase angle δ at that location; and a wave whose amplitude varies if the two frequencies are different. We will consider each of those cases.

<u>Interference from Two Coherent Sources</u>



S1 the same The sound frequency heard depends from two on identical the relative sound phase sources of the two producingwaves at the location of the listener - call it point P. The problem has already been generally discussed in the section on wave motion. If

S $y_1(\omega t)$ and $y_2(\omega t+\delta)$ represent the two arriving waves at P, with y_2
2 out of phase with respect to y_1 by the phase angle δ, the signal that is heard can be represented by

$$y_{total}(t) = 2A\cos(\delta/2)\sin(\omega t+\delta/2)$$

This equation represents the combined signal at point P. Notice that has the same frequency (and hence wavelength) as the constituent waves, but with an amplitude $2A\cos(\delta/2)$ that depends on the phase difference δ. When δ=0, 2π, 4π, etc., the amplitude of the combined wave is $2A$ - that is, the two waves add up *in phase*. When the phase difference is an *odd* multiple of π, then the $\cos(\delta/2)=0$ and there is total cancellation. That is, total *constructive interference* occurs when the waves are in phase and *destructive interference* occurs when they are 180° out of phase.

The phase difference δ can be a result of the sound sources themselves being out of phase (called δ_{souce}). That is, they could be producing exactly the same frequency, but one source could be out of phase with the other. (This can occur, for example, when the polarity of the speaker wires in a stereo system are reversed between the right and left speakers. In that case, when the speakers are driven with the same signal, one speaker will be 180° out of phase with respect to the other.) The phase difference δ could *also* be a result of a *path difference* between the two sources and the point at which the two waves are detected. If there is a path difference given by Δr, then the phase difference between the two waves is given by $\delta=2\pi\Delta r/\lambda$ In general, the *total* phase difference is then given by:

$$\delta_{total} = \delta_{sources} + \delta_{path\ difference} = \delta_{sources} + 2\pi\Delta r/\lambda$$

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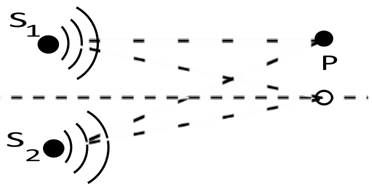


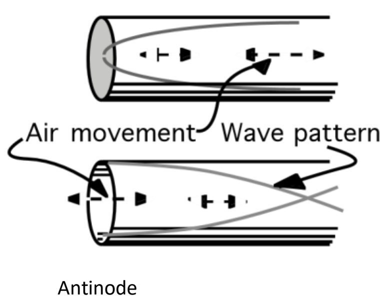
It should be noted that if the sources are in phase, then only the difference in distances from the speakers to the listener will create a phase difference. But the phase difference due to a path difference is not a fixed quantity, but rather depends on the wavelength (hence the frequency) of the sound being produced. So the locations of interference maxima and minima depend on the frequencies being produced. Along the center line, the two signals are necessarily in phase

if δ_{souce}=0.

It should also be noted that when two waves add constructively, it is the *amplitudes* that add. That is, when total constructive interference occurs, the combined amplitude is twice the amplitude of each of the individual waves. Since the *intensity* of the wave is proportional to the square of the amplitude, the combined intensity where constructive interference occurs is four times the intensity (I_o) of either of the individual waves. Although that seems like it violates some energy conservation principle, it is in fact necessary to preserve that principle. That is, the total transmitted power from both sources must be accounted for. But where cancellation due to destructive interference occurs, the intensity (hence power) is zero. So the intensity must be twice the total of the individual intensities ($2I_o$) where constructive interference occurs - or $4I_o$.

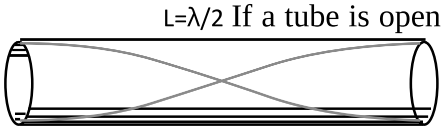
Standing Wave Resonances in Air Columns

The standing sound wave problem requires a one-dimensional sound wave - as in a sound wave created inside a hollow tube or pipe. Consider a sound wave traveling in a tube which encounters an identical wave in the opposite direction. This is identical to the situation when a wave on a string "sees" its reflection from the end of the string - which leads to standing waves. The difference here is that when a string is fixed at both ends, there must be a node at each end of the string. But a tube can have either a *node* or an *antinode* at the ends depending on whether it is closed or open at that end.

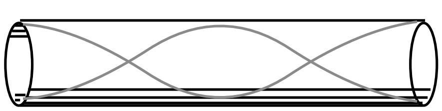


Since sound is a longitudinal wave, air must be able to move along the axis of the tube. This results in the special condition that an *open* end of a tube must be at an *antinode* of any standing wave whereas a *closed* end must be a *node* (since air cannot move in-and-out of a closed ended tube). So the relationship between the length of a tube and the wavelength of the sound when a standing wave occurs depends on whether the tube has open or closed ends. Standing waves can be supported in a tube (or air column) only if the tube length are the wavelength of the sound in the tube are related in certain specific ways.

Open-Open (or Closed-Closed) Tube



If a tube is open at both ends, antinodes must appear at the ends of the tube for a standing wave to occur - and that means the tube length must be a multiple of half-wavelengths (ie, the distance between antinodes will always be λ/2 or λ or 3λ/2, etc). Of course, if the tube is closed at both ends, nodes appear at the ends and the condition is the same (and identical to the standing wave resonances on a taut string). But with a tube closed at both ends, it is not obvious how the wave would be generated nor whether the resonance could be heard outside the tube! The condition for resonance and the frequencies at which standing waves can be supported in a tube of length *L* are given by



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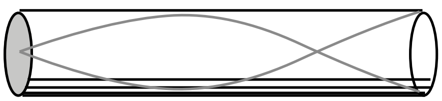
$$L = n\frac{\lambda}{2} \quad \text{and} \quad f_n = \frac{v}{\lambda} = n\left(\frac{v}{2L}\right)$$

where v is the speed of sound in air (ie, in the air column contained within the tube). Again, this is the same condition as for standing waves on a string - and the frequencies are just multiples of the fundamental frequency given by ($v/2L$) for the tube open at both ends.

Closed-Open Tube

$L=\lambda/4$

$L=3\lambda/4$ $f_2=3$ f_1



If the tube has one open and one closed end, there must be a node at the closed end and an antinode at the open end for resonance to occur. Since the distance between a node and an antinode is either $\lambda/4$,

is different than for the open-open tube. In this case, the length of the tube is necessarily an odd multiple of quarter wavelengths, or

$3\lambda/4$, $5\lambda/4$, etc. the resonance condition

given by

$$L = m\frac{\lambda}{4} = (2n+1)\frac{\lambda}{4},$$

where m is an odd integer represented by ($2n$+1) for any integer n. So the frequencies at which resonance can occur are

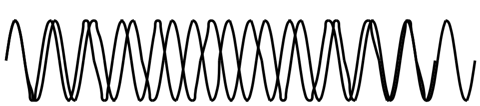
$$f_n = \frac{v}{\lambda} = m\left(\frac{v}{4L}\right) = (2n+1)f_1$$

where f_1 is the fundamental frequency given by ($v/4L$) for the tube closed at one end.

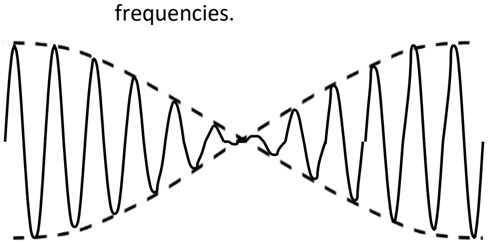
Two Sources of Different Frequencies - Beats

Finally, the effect of two different frequencies interfering creates a very interesting effect. That is, adding two sine waves which are slightly different in frequency ($f1$ and $f2$) creates another sine wave with a frequency equal to the average of the original sounds (ie, $favg = (f1 + f2)/2$). But the amplitude of the resulting wave varies with a frequency equal to the difference of the two frequencies - called the *beat frequency*.

$$f_{beat} = f_2 - f_1 \quad \text{or} \quad \omega_{beat} = 2\pi f_{beat} = \omega_2 - \omega_1$$



Two similar waves of different frequencies.



The two waves add constructively and destructively - resulting in beats

Mathematically, this is just a result of adding $A\sin\omega_1 t$ and $A\sin\omega_2 t$. The result (using trigonometric identities) is just

$$A\sin\omega_1 t + A\sin\omega_2 t = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)\sin\left(\frac{\omega_1 + \omega_2}{2}t\right)$$

which is of the form $B(t)\sin(\omega_{avg}t)$ with $B(t)=A\cos(\omega_{beat}t/2)$. The sound that one would hear is at the average frequency. However, the intensity varies *at* the beat frequency (and not $\omega_{beat}/2$) since the sound depends on the magnitude but not the sign of the amplitude and the cosine function thus has two "maxima" per cycle.

The beat frequency effect is not strictly an interference phenomenon in the same sense as the interference of coherent

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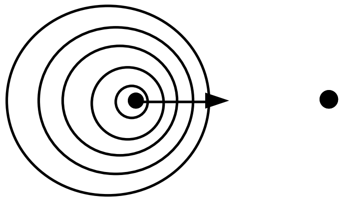
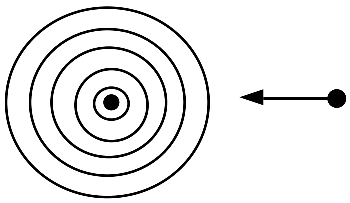
waves. There are no maxima or minima (constructive or destructive interference). The effect is most easily recognized when the two sound sources are quite close together in frequency - so that the average is very similar in tone to either of the sources (tuning forks, for example) themselves. But when both sources sounding, the amplitude varies noticably - and is easily recognizable when the sources differ in frequency by only a few hertz. When the frequency difference is more than about ten hertz, the beat frequency is too high for the ear to detect as a variation in amplitude - and the two distinct frequencies are heard.

DOPPLER EFFECT



Moving source



When a car or train passes you, the frequency of the sound that you hear varies from a higher pitch as it approaches to a lower pitch as it recedes from you. (Do not confuse this effect with the change in intensity or sound level as it approaches and then recedes.) This Doppler effect is a common occurance in everyday life when a source of sound is moving with respect to an observer. There are really two different causes for the effect. When the source of the sound is moving, the sound waves are compressed in front of the source and expanded behind. But the sound still travels with respect to the air at the speed of sound. So an observer in front of the source hears a higher frequency than the source is emitting, since the wave fronts arrive closer together than if the source were not moving, whereas an observer behind the moving source hears a lower Moving observer frequency.

If the source is stationary but the observer is moving, a similar effect occurs for a different reason. The wavefronts spread out from the source uniformly. But if the observer is moving toward the source, he or she encounters wavefronts more often - hence hears a higher frequency. If the observer is moving away from the source, the wavefronts arrive less often - hence a lower frequency.

All of this can be summarized mathematically. The frequency one hears (f) is <u>always</u> the speed at which the wave is traveling with respect to the observer (v_{rel}) divided by the wavelength (λ) that is encountered by the observer. That is, $f=v_{rel}/\lambda$.

The *relative speed* v_{rel} is just the difference betweeen the speed of sound relative to the air and the speed of the observer (u_{obs}) relative to the air, or

$$v_{rel}=v_o \pm u_{obs}$$

where v_o is the speed of sound and u_{obs} is the speed of the observer. Whether the + or - sign is used depends on the direction of the observer's motion relative to the source - to be discussed shortly.

The wavelength that the observer encounters is the wavelength in front or behind the source, depending on whether the source is moving toward or away from the observer. Hence

$$\lambda =(v_o \pm u_{source})/f_o \ ,$$

where f_o is the frequency the source produces and u_{source} is the speed of the source. Combining these results yields

$$\boxed{f = f_0 \left(\frac{v_o \pm u_{observer}}{v_o \pm u_{source}} \right)}$$

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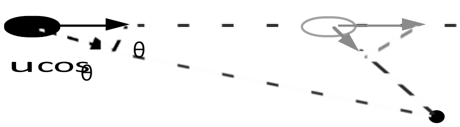
Notice that in this form, all cases are included. That is, if the source is moving, the denominator is modified by the speed of the source. If the observer is moving, the numerator is modified. And, of course, if both are moving, then both are affected. The choice of whether to use the + or - sign in each case can be made by deciding whether the speed of the source or of the observer has the effect of increasing or decreasing the frequency that would be heard depending on whether the motion of either tends to decrease or increase the separation between the source and observer. If the gap between them is closing, the sound is Doppler shifted to a higher frequency, whereas if the gap is increasing, the sound is Doppler shifted to a lower frequency - regardless of which is moving. The choice of sign is then made to yield that result.

For example, if you are stationary, and a car or train passes you, the frequency as it approaches would be given by

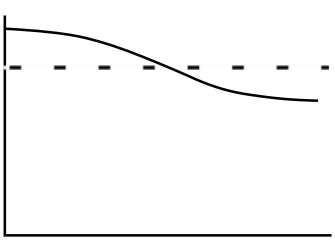
$$f = f_0\left(\frac{v_o}{v_o - u_{source}}\right) = f_0\left(\frac{1}{1 - u_{source}/v_o}\right)$$

since, as an observer, your speed is zero and the source is moving toward you so the frequency you hear is higher than that produced. After it passes, the sign in the denominator would change resulting in a lower frequency than f_o. Had you been moving as well, your speed would have modified the numerator - with the choice of sign still determined by whether you were moving toward or away from the source.



Seldom is the problem really one-dimensional, of course. That is, when a train passes, you are not directly in front of the train (!). So the speed that affects the sound you hear is the component of the speed along the line that joins you with the source - and that component changes continuously. The component of the speed that matters is obtained by multiplying the source speed by the cosine of the angle between the direction of travel of the source and location of the observer.

$$f = f_0\left(\frac{v_o}{v_o - u_{source}\cos\theta}\right)$$



Notice that in that form, the sign change is automatic - ie, the cosine changes sign as θ goes from less than π/2 to greater than π/2. When θ=π/2, the source is moving neither toward nor away from you, so you would hear the true frequency of the source. Notice that this explains what you actually hear as a moving source - car, train, it doesn't matter - goes past you and the frequency of the sound varies continuously from a higher pitch to a lower one. You would hear the frequency of the source at the moment it is moving perpendicular to the line of sight to the vehicle - ie, just as it passes you.

TAKE YOUR TIME AND UNDERSTAND THIS PHYSICS…REMEMBER EVEN ALBERT READ ALL THIS BEFORE HE BECAME THE FATHER OF PHYSICS

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