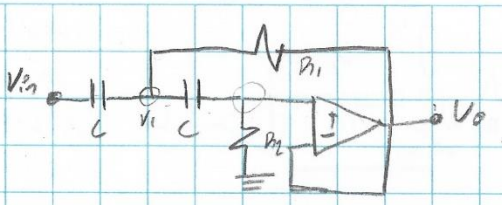
Edgar Lara Arellano Tarea 05N2. Nov 3, 2020

Filtro activo pasa-altos 2do orden



$V^- = V^+ = V_o$

Nodo V_1

$$\frac{V_1 - V_{in}}{1/sC} + \frac{V_1 - V^+}{1/sC} + \frac{V_1 - V_o}{R_1} = 0 \Rightarrow V_1\left(2sC + \frac{1}{R_1}\right) - V_{in}sC - V_o\left(sC + \frac{1}{R_1}\right) = 0 \quad (1)$$

Nodo V^+

$$\frac{V^+ - V_1}{1/sC} + \frac{V^+ - 0}{R_2} = 0 \Rightarrow V^+\left(sC + \frac{1}{R_2}\right) = V_1 sC \Rightarrow V_1 = V_o\left(1 + \frac{1}{sCR_2}\right) \quad (2)$$

(2) en (1)

$$V_o\left(1 + \frac{1}{sCR_2}\right)\left(2sC + \frac{1}{R_1}\right) - V_{in}sC - V_o\left(sC + \frac{1}{R_1}\right) = 0$$

$$V_o\left(2sC + \frac{1}{R_1} + \frac{2}{R_2} + \frac{1}{sCR_1R_2} - sC - \frac{1}{R_1}\right) = V_{in}sC$$

$$V_o\left(sC + \frac{2}{R_2} + \frac{1}{sCR_1R_2}\right) = V_{in}sC \Rightarrow V_o\left(1 + \frac{2}{sCR_2} + \frac{1}{s^2C^2R_1R_2}\right) = V_{in}$$

$$\frac{V_o}{V_{in}} = \frac{1}{\frac{1}{s^2C^2R_1R_2} + \frac{2}{sCR_2} + 1} \cdot \frac{s^2}{s^2} = \frac{s^2}{\frac{1}{C^2R_1R_2} + \frac{2s}{CR_2} + s^2}$$

$$\therefore \boxed{\frac{V_o}{V_{in}} = \frac{s^2}{s^2 + s\left(\frac{2}{CR_2}\right) + \frac{1}{C^2R_1R_2}}} \Longleftrightarrow F(s) = \frac{\omega_o s^2}{s^2 + 2\zeta\omega_o + \omega_o^2}$$

$f_c = 1\,KHz$

$C_1 = 22nF$

$R_1 = ?$

$R_2 = ?$

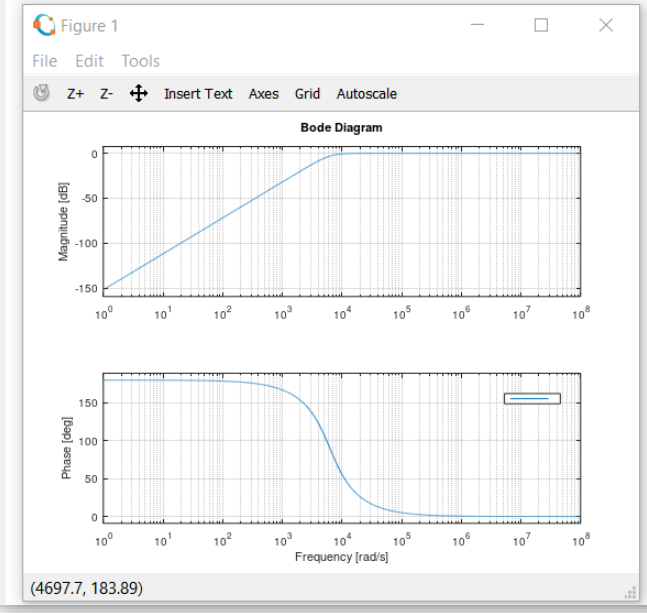
$$\frac{1}{CR_2} = \zeta\omega_o \Rightarrow R_2 = \frac{1}{C\zeta 2\pi f} = \frac{1}{(22nF)(.7072)(2\pi)(1K)} = 10,229.5\,\Omega \cong 10K\Omega$$

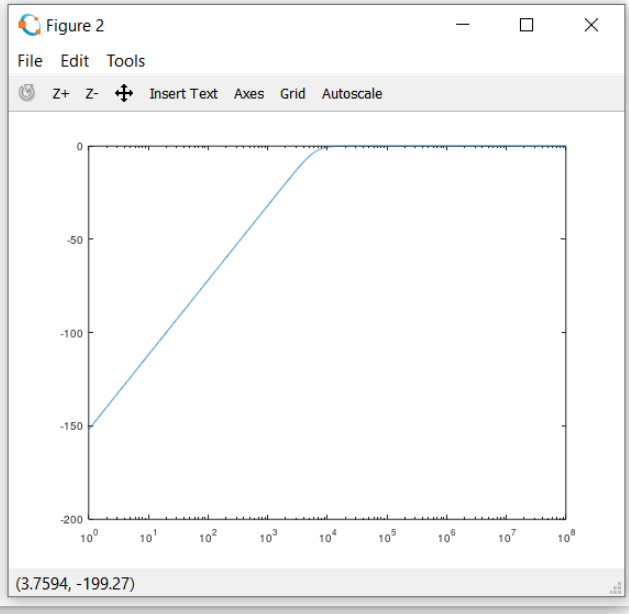
$$\frac{1}{C^2R_1R_2} = (2\pi)^2 f^2 \Rightarrow R_1 = \frac{1}{C^2R_2(2\pi)^2 f^2}$$

$$R_1 = \frac{1}{(22nF)^2(10K)(2\pi)^2(1K)^2} = 5,233.5\,\Omega \cong 5K\Omega$$

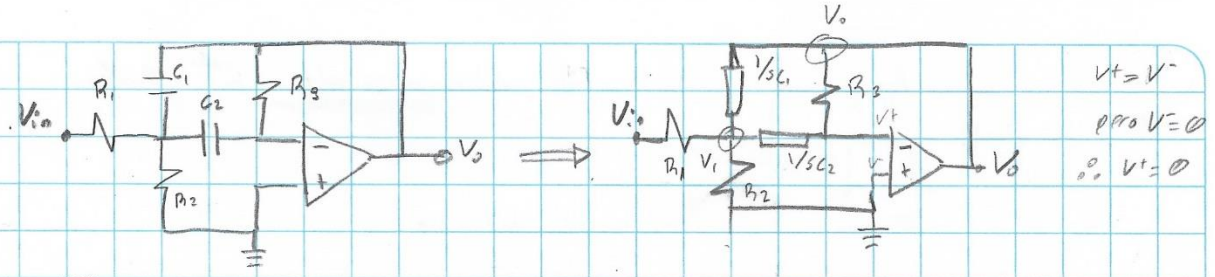
```
clear all
%clc
%clf
%pkg load control

%Definir componentes
fprintf('\nDiseño Filtro pasa-bajas 2do orden\n\n')
fc = input('Introduce frecuencia de corte(fc):  ');
Sigma = input('Introduce el valor de sigma:  ');
C = input('Introdce valor de Capacitor 1 (C1): ');

wc = 2*pi*fc;

R2 = 1/(C*Sigma*2*pi*fc);
R1 = 1/((C^2)*R2*((2*pi)^2)*(fc^2));

fprintf('Capacitor (C): %6.3d\n', C);
fprintf('Resistor 1 (R1): %6.3d\n', R1);
fprintf('Resistor 2 (R2): %6.3d\n', R2);

%Funcion de transferencia
A = 2/(C*R2);
B = 1/(R1*R2*C^2);
fc = (sqrt(B))/(2*pi);
fprintf('Frecuencia de corte %6.2f\n', fc);

%Funcion de transferencia
num = [1 0 0];
den = [1 A B];
w = logspace(0, 8, 1000);
[Mag, Fase, w] = bode(tf(num,den), w);
%bode(tf(num, den), w);
MagdB = 20*log10(Mag);     %Poner la magnitud en decibeles

figure(1)
bode(tf(num, den), w);
```

```
Diseño Filtro pasa-bajas 2do orden

Introduce frecuencia de corte(fc):  1000
Introduce el valor de sigma:  .7072
Introdce valor de Capacitor 1 (C1): 22e-9
Capacitor (C): 2.2e-08
Resistor 1 (R1): 5.12e+03
Resistor 2 (R2): 1.02e+04
Frecuencia de corte 1000.00
>>
```





2) Filtro activo pasa-banda 2do Orden NOV 3, 2020



Nodo V_1

$$\frac{V_1 - V_{in}}{R_1} + \frac{V_1 - V^+}{R_2} + \frac{V_1 - V_o}{1/sC_1} + \frac{V_1 - V^-}{1/sC_2} = 0 \Longrightarrow \frac{V_1 - V_{in}}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_o}{1/sC_1} + \frac{V_1}{1/sC_2} = 0$$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_1 + sC_2\right) - \frac{V_{in}}{R_1} - V_o sC_1 = 0 \quad (1)$$

Nodo V^-

$$\frac{V^- - V_o}{R_3} + \frac{V^- - V_1}{1/sC_2} = 0 \Longrightarrow V^-\left(\frac{1}{R_3} + sC_2\right) - \frac{V_o}{R_3} - V_1 sC_2 = 0 \Longrightarrow V_1 = V^-\left(\frac{1}{sC_2R_3} + 1\right) - \frac{V_o}{sC_2R_3} \quad (2)$$

(2) en (1)

$$-V_o\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_1 + sC_2\right)\left(\frac{1}{sC_2R_3}\right) - \frac{V_{in}}{R_1} - V_o sC_1 = 0 \Longrightarrow -V_o\left(\frac{1}{sC_2R_1R_3} + \frac{1}{sC_2R_2R_3} + \frac{C_1}{C_2R_3} + \frac{1}{R_3} + sC_1\right) = \frac{V_{in}}{R_1}$$

$$-V_o\left(\frac{1}{sC_2R_3} + \frac{R_1}{sC_2R_2R_3} + \frac{C_1R_1}{C_2R_3} + \frac{R_1}{R_3} + sC_1R_1\right) = V_{in} \Longrightarrow \frac{V_o}{V_{in}} = \frac{1}{\frac{1}{sC_2R_3} + \frac{R_1}{sC_2R_2R_3} + \frac{C_1R_1}{C_2R_3} + \frac{R_1}{R_3} + sC_1R_1} \cdot \frac{s/C_1R_1}{s/C_1R_1}$$

$$\frac{V_o}{V_{in}} = \frac{-s/C_1R_1}{\frac{1}{C_1C_2R_1R_3} + \frac{1}{C_1C_2R_2R_3} + \frac{s}{C_2R_3} + \frac{s}{C_1R_3} + s^2}$$

$$\frac{V_o}{V_{in}} = \frac{-s/C_1R_1}{s^2 + s\left(\frac{1}{C_2R_3} + \frac{1}{C_1R_3}\right) + \left(\frac{1}{C_1C_2R_1R_3} + \frac{1}{C_1C_2R_2R_3}\right)}$$

$$\therefore \boxed{\frac{V_o}{V_{in}} = \frac{s\left(\frac{-1}{C_1R_1}\right)}{s^2 + s\left(\frac{C_1 + C_2}{C_1C_2R_3}\right) + \left(\frac{R_2 + R_1}{C_1C_2R_1R_2R_3}\right)}} \Longleftrightarrow \boxed{F(s) = \frac{-As}{s^2 + sB + C}}$$

Diseño del Filtro activo paso-banda 2do orden

En este caso $C_1 = C_2 = C$ entonces

$$\frac{C_1 + C_2}{C_1 C_2 R_3} = 2\zeta\omega_0 \Rightarrow \frac{2C}{C^2 R_3} = 2\zeta\omega_0 \Rightarrow \frac{1}{CR_3} = \zeta\omega_0$$

Por tanto $$\boxed{R_3 = \frac{1}{C\zeta\omega_0}}$$

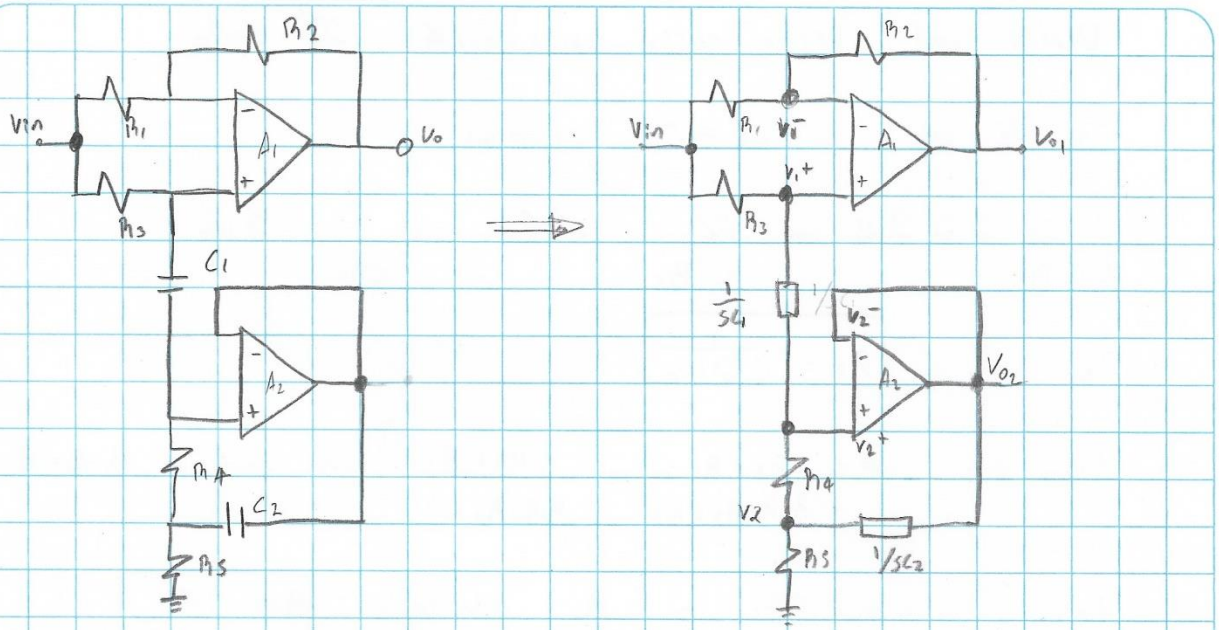
Ahora bien $$\omega^2 = \frac{R_2 + R_1}{R_1 R_2 R_3 C_1 C_2} = \frac{R_1 + R_2}{R_1 R_2 R_3 C^2}$$ si suponemos $R_1 = R_2 = R$

Entonces $$\omega^2 = \frac{2R}{R^2 R_3 C_2} = \frac{2}{R R_3 C_2}$$ Por tanto $$\boxed{R = \frac{2}{\omega^2 R_3 C_2}}$$

Y además la frecuencia de corte se calcula como

$$\boxed{f_c = \frac{\sqrt{\omega^2}}{2\pi} = \frac{\sqrt{\frac{R_1 + R_2}{C_1 C_2 R_1 R_2 R_3}}}{2\pi}}$$

3) Filtro activo 2do orden Rechazo de Banda



Nodo V_1^-

$$\frac{V_1^- - V_{in}}{R_1} + \frac{V_1^- - V_o}{R_2} = 0 \Rightarrow V_1^-\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - \frac{V_{in}}{R_1} - \frac{V_o}{R_2} = 0 \Rightarrow V_1^-(R_2+R_1) - V_{in}R_2 - V_o R_1 = 0 \quad (1)$$

Nodo V_1^+

$$\frac{V_1^+ - V_{in}}{R_3} + \frac{V_1^+ - V_2^+}{1/SC_1} = 0 \Rightarrow V_1^+\left(\frac{1}{R_3}+SC_1\right) - \frac{V_{in}}{R_3} - V_2^+ SC_1 = 0 \quad (2)$$

Nodo V_2^+

$$\frac{V_2^+ - V_1^+}{1/SC_1} + \frac{V_2^+ - V_2}{R_4} = 0 \Rightarrow V_2^+\left(SC_1 + \frac{1}{R_4}\right) - V_1^+ SC_1 - \frac{V_2}{R_4} = 0 \quad (3)$$

Nodo V_2

$$\frac{V_2 - V_2^+}{R_4} + \frac{V_2 - V_{o2}}{1/SC_2} + \frac{V_2 - 0}{R_5} = 0 \Rightarrow V_2\left(\frac{1}{R_4}+SC_2+\frac{1}{R_5}\right) - \frac{V_2^+}{R_4} - V_{o2}SC_2 = 0 \quad (4)$$

Nodo V_2^- (5)

$$V_2^- = V_{o2}$$

Debido a que $V_2^- = V_2^+$ y que $V_2^- = V_{o2}$ entonces $V_2^+ = V_{o2}$ (6)

Por tanto la ecn ④ pasa a

$$V_2\left(\frac{1}{R_4}+\frac{1}{R_5}+SC_2\right) = V_2^+\left(\frac{1}{R_4}+SC_2\right) \Rightarrow V_2 \quad \text{todo por } R_4R_5$$

$$V_2\left(R_4+R_5+SC_2R_4R_5\right) = V_2^+\left(R_5+SC_2R_4R_5\right) \quad \text{en fracc}$$

$$V_2 = V_2^+\left(\frac{R_5+SC_2R_4R_5}{R_4+R_5+SC_2R_4R_5}\right) = V_2^+\left(1-\frac{R_4}{R_4+R_5+SC_2R_4R_5}\right) \quad (4')$$

⑥' en ③

$$V_2^+\left(\frac{1}{R_4}+SC_1\right) - \frac{V_2^+}{R_4} - V_2^+\left(\frac{1}{SC_2R_4R_5+R_4+R_5}\right) - V_1^+SC_1 = 0$$

$$V_2^+\left(SC_1+\frac{1}{SC_2R_4R_5+R_4+R_5}\right) = V_1^+SC_1$$

$$V_2^+ = V_1^+\left(\frac{1}{1+SC_1(SC_2R_4R_5+R_4+R_5)}\right) \quad (3')$$

Ahora la ecn ② pasa a

$$V_1^+(1+SC_1R_3) = V_{in} + V_1^+ SC_1R_3 / \left(1+1/SC_1(SC_2R_4R_5+R_4+R_5)\right)$$

$$V_1^+\left[1+SC_1R_3\left(1-\frac{1}{1+SC_1(SC_2R_4R_5+R_4+R_5)}\right)\right] = V_{in} \quad \text{entre es}$$

$$V_1^+ = V_{in}\left[\frac{1}{1+SC_1R_3\left(1-\frac{1}{1+SC_1(SC_2R_4R_5+R_5+R_4)}\right)}\right] \quad (2')$$

Para la ecn ① pasa a:

$$V_{in}\left(\frac{R_2+R_1}{1+SC_1R_3\left(1-\frac{1}{1+SC_1(SC_2R_4R_5+R_4+R_5)}\right)}-R_2\right) = V_0R_1$$

$$\frac{V_0}{V_{in}} = \frac{R_2+R_1}{R_1+SC_1R_1R_3\left(1-\frac{1}{1+SC_1(SC_2R_4R_5+R_4+R_5)}\right)} - \frac{R_2}{R_1} \quad (1')$$

Para lo siguiente usaremos una técnica del álgebra:

$$1-\frac{1}{1+\frac{1}{A}} = 1-\frac{1}{\frac{A}{A}+\frac{1}{A}} = 1-\frac{1}{\frac{1+A}{A}} = 1-\frac{A}{1+A} = \frac{1+A-A}{1+A} = \boxed{\frac{1}{1+A}}$$

entonces aplicando en (1)

$$\frac{V_o}{V_{in}} = \frac{R_1+R_2}{R_1+sC_1R_1R_3\left(\frac{1}{1+sC_1(sC_2R_4R_5+R_4+R_5)}\right)} - \frac{R_2}{R_1}$$

donde por simplicidad

$$B = 1+sC_1(sC_2R_4R_5+R_4+R_5) = 1+sC_1(R_4+R_5)+s^2C_1C_2R_4R_5$$

entonces

$$\frac{V_o}{V_{in}} = \frac{R_1+R_2}{R_1\left(1+\frac{sC_1R_3}{B}\right)} - \frac{R_2}{R_1} = \frac{R_1+R_2}{R_1\left(\frac{B+sC_1R_3}{B}\right)} - \frac{R_2}{R_1}$$

$$= \frac{B(R_1+R_2)}{R_1(B+sC_1R_3)} - \frac{R_2}{R_1}\cdot\frac{(B+sC_1R_3)}{(B+sC_1R_3)} = \frac{BR_1 - sC_1R_2R_3}{R_1(B+sC_1R_3)}$$

Ahora sustituyendo de nuevo el valor de B.

$$\frac{V_o}{V_{in}} = \frac{1+sC_1(R_4+R_5)+s^2C_1C_2R_4R_5 - \frac{R_2R_3}{R_1}sC_1}{1+sC_1(R_4+R_5)+s^2C_1C_2R_4R_5 - R_3sC_1}\cdot\frac{1/C_1C_2R_4R_5}{1/C_1C_2R_4R_5}$$

$$\frac{V_o}{V_{in}} = \frac{\frac{1}{C_1C_2R_4R_5}+s\frac{R_4+R_5}{C_2R_4R_5}+s^2-s\frac{R_2R_3}{C_2R_1R_4R_5}}{\frac{1}{C_1C_2R_4R_5}+s\frac{R_4+R_5}{C_2R_4R_5}+s^2-s\frac{R_3}{C_2R_4R_5}}$$

y esto es finalmente

$$\boxed{\frac{V_o}{V_{in}} = \frac{s^2+s\left(\frac{R_4+R_5-R_3\left(\frac{R_2}{R_1}\right)}{C_2R_4R_5}\right)+\frac{1}{C_1C_2R_4R_5}}{s^2+s\left(\frac{R_4+R_5-R_3}{C_2R_4R_5}\right)+\frac{1}{C_1C_2R_4R_5}}}$$

Para diseñar basta con seleccionar $R_1=R_2=R_3$ y $R_4=R_5=R$ entonces,

$$C_2 = \frac{2Q}{\pi f R} \quad ; \quad C_1 = \frac{1}{(2\pi f)^2R^2C_2}$$

donde además $f = \frac{1}{2\pi R\sqrt{C_1C_2}}$