Mini project 2 in Matrix Theory, ht 2023

The aim of the project is to study the possibilities of Maple in linear algebra, which can be used later (at least) for the home exam.

To get started   Even for the beginners Maple is easy to use with a lot of examples and good help. If, for example, you want to know how to construct a matrix in Maple it is sufficient to write
?matrix
at the input and press Enter. You get a long description, probably more detailed than necassary for beginners, but most importantly you get at the end of the text a lot of good examples, which are easy to follow and copy. Besides that you will find useful references to other related topics.
Now close the help-window and look to the left in your screen. You should see a button with the word matrix that allows you to create new matrices. If you press this button a fold down menu appears where you can specify size and insert the matrix into the worksheet. In order to do this first write $A :=$ at the prompt and then press insert matrix. Note the assignment sign $:=$. The sign $=$ without semicolon is used for other constructions.
You can change the elements a_{ij} in the matrix to arbitrary numbers or expressions (use the mouse or the TAB button on the keyboard to jump to the next element). Press Enter when you are ready and you get your matrix in a very nice output. Write $B := A^2$ in the next input line and get the expected result. Note that you can write several commands in one line, but they should be separated by semicolons and it is a good habit to write a semicolon after every command, e.g. $B := A^2; C := A + B;$
Maple has a very convenient package called LinearAlgebra (in one word with Capitals!) and the aim of this project is to learn part of it. Write the command
with(LinearAlgebra);
to include the package into your work and press Enter. Maple lists the functions of the package. You can ask Maple using sign ? about every command in this package (try e.g. ?JordanForm) to get the complete description with many examples. But it is easier to start with general: write ?LinearAlgebra and be introduced into the package!

The project   Use the package to do the following.
Consider the matrix

$$A := \begin{pmatrix} 1 & x & 0 \\ 1 & y & x \\ 0 & -1 & 1 \end{pmatrix},$$

where x is the day of your birthday and y is the month.
Find (using the apropiate commands from the library):

1. $\det(A), \operatorname{tr} A, A^T, A^{-1}, (A + I) * (A^T - I);$

2. The eigenvalues and eigenvectors (try to understand how to interpret the answer from Maple).

3. Jordan form and the corresponding transition matrix.

4. The rank of A and $A - I$;

5. The characteristic polynomial $p_A(x)$. Factorize it using the command factor;

6. The minimal polynomial $\pi_A(x)$;

7. The basis of $\mathrm{Ker}(A - I)^2$ and $\mathrm{Im}(A - I)$;

8. The **orthonormal** basis in $Im(A - I)$ (using Gram-Schmidt algorithm, choose correct options!);

9. The exponent e^A;

10. The LU-decomposition and QR-decomposition of A;

11. The norm $\|A\|_2$ and the condition number of A; (use evalf if you want to see the value as a number)

12. A matrix C such that $C^2 = A$. Here (depending of your approach) you maybe need more then one command.

Take away all inputs and outputs that are not connected with the project (clean the input line and press the Enter for this). Save your session as pdf file (use Export in File menu) and submit it through the canvas page. The last day for submission is 30/11.