# Formelsamling

Formelsamligen utgör bara ett stöd för minnet. Beteckningar förklaras sålunda ej. Ej heller anges förutsättningar för formlernas giltighet.

**Fysikaliska modeller**

Kontinuitetsekvationen

$$\frac{\partial q}{\partial t} + \nabla \cdot \boldsymbol{j} = k.$$

Diffusion

$$\boldsymbol{j} = -D\,\nabla u,$$

$$\frac{\partial u}{\partial t} - D\,\Delta u = k. \qquad \left(\text{Allmännare } \frac{\partial u}{\partial t} - \nabla \cdot (D\,\nabla u) = k.\right)$$

Värmeledning

$$\boldsymbol{j} = -\lambda\,\nabla u, \quad \mathrm{d}q = \rho c\,\mathrm{d}u,$$

$$\frac{\partial u}{\partial t} - a\,\Delta u = \frac{a}{\lambda}k \quad \text{där } a = \frac{\lambda}{\rho c}. \qquad \left(\text{Allmännare } \rho c\,\frac{\partial u}{\partial t} - \nabla \cdot (\lambda\,\nabla u) = k.\right)$$

Elektrostatisk potential

$$\Delta u = -\frac{\rho}{\epsilon\epsilon_0}.$$

Svängande sträng och membran

$$\frac{\partial^2 u}{\partial t^2} - c^2\,\Delta u = \frac{f}{\rho} \quad \text{där } c^2 = \frac{S}{\rho}. \qquad \left(\text{Allmännare } \rho\,\frac{\partial^2 u}{\partial t^2} - \nabla \cdot (S\,\nabla u) = f.\right)$$

Longitudinella svängningar

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = \frac{f}{\rho_l} \quad \text{där } c^2 = \frac{\alpha}{\rho_l}, \quad S = \alpha\,\frac{\partial u}{\partial x}.$$

Svängningar i gaser (ljud)

$$u = \frac{p - p_0}{p_0} \quad \text{(tryckstörning)},$$

$$\frac{\partial^2 u}{\partial t^2} - c^2\,\Delta u = 0 \quad \text{där } c^2 = \frac{\gamma\,p_0}{\rho_0}.$$

För svängningar i gaser (ljud) gäller efter linjärisering att

$$\begin{cases} \dfrac{1}{\gamma}\dfrac{\partial \tilde{p}}{\partial t} + v_0\dfrac{\partial \tilde{v}}{\partial x} = 0, \\[2ex] v_0\dfrac{\partial \tilde{v}}{\partial t} + \dfrac{p_0}{\rho_0}\dfrac{\partial \tilde{p}}{\partial x} = 0, \\[2ex] \tilde{p} = \gamma\tilde{\rho}. \end{cases}$$

där $\tilde{p} = \dfrac{p - p_0}{p_0}$ och $\tilde{v} = \dfrac{v}{v_0}$.

**Vektoranalys**

Gauss formel $\displaystyle\int_\Omega \nabla\cdot\boldsymbol{u}\,\mathrm{d}V = \int_{\partial\Omega} \boldsymbol{u}\cdot\mathrm{d}\boldsymbol{S}.$

Stokes formel $\displaystyle\int_S \nabla\times\boldsymbol{u}\cdot\mathrm{d}\boldsymbol{S} = \int_{\partial S} \boldsymbol{u}\cdot\mathrm{d}\boldsymbol{r}.$

Greens formel I $\displaystyle\int_\Omega \nabla u\cdot\nabla v\,\mathrm{d}V = \int_{\partial\Omega} u\,\frac{\partial v}{\partial\boldsymbol{n}}\,\mathrm{d}S - \int_\Omega u\,\Delta v\,\mathrm{d}V.$

Greens formel II $\displaystyle\int_\Omega (u\,\Delta v - v\,\Delta u)\,\mathrm{d}V = \int_{\partial\Omega}\Big(u\,\frac{\partial v}{\partial\boldsymbol{n}} - v\,\frac{\partial u}{\partial\boldsymbol{n}}\Big)\,\mathrm{d}S.$

Laplaceoperatorn i cylindriska koordinater

$$\begin{aligned}\Delta &= \frac{1}{r}\frac{\partial}{\partial r}\,r\,\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}\\ &= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}.\end{aligned}$$

Laplaceoperatorn i sfäriska koordinater

$$\begin{aligned}\Delta &= \frac{1}{r}\frac{\partial^2}{\partial r^2}\,r + \frac{1}{r^2}\,\Lambda = \frac{1}{r^2}\frac{\partial}{\partial r}\,r^2\frac{\partial}{\partial r} + \frac{1}{r^2}\,\Lambda\\ &= \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\,\Lambda,\end{aligned}$$

$$\Lambda = \frac{1}{\sin(\theta)}\frac{\partial}{\partial\theta}\sin(\theta)\frac{\partial}{\partial\theta} + \frac{1}{\sin^2(\theta)}\frac{\partial^2}{\partial\phi^2},$$

$$\Lambda = \frac{\partial}{\partial s}(1-s^2)\frac{\partial}{\partial s} + \frac{1}{1-s^2}\frac{\partial^2}{\partial\phi^2} \quad \text{om } s=\cos(\theta),$$

($\theta$ polardistans, $0<\theta<\pi$, $\phi$ längdgrad, $0\leqslant\phi<2\pi$.

**Ortogonalutvecklingar**

$$(u\,|\,v) = \int_I \overline{u(x)}\,v(x)\,w(x)\,\mathrm{d}x, \qquad \|u\|^2 = (u\,|\,u).$$

Om $(\varphi_j\,|\,\varphi_k) = 0$, $j\neq k$, så $u = \sum c_k(u)\,\varphi_k$ med $c_k(u) = \dfrac{(\varphi_k\,|\,u)}{\rho_k}$, där $\rho_k = (\varphi_k\,|\,\varphi_k)$.

Parseval

$$(u\,|\,v) = \sum\frac{1}{\rho_k}\,\overline{(\varphi_k\,|\,u)}\,(\varphi_k\,|\,v) = \sum\rho_k\,\overline{c_k(u)}\,c_k(v).$$

**Sturm-Liouville**

$$\mathcal{A}u = \frac{1}{w}(-\nabla\cdot(p\,\nabla u) + q\,u).$$

**Speciella funktioner**

*Gammafunktionen och Betafunktionen*

$$\Gamma(z) = \int_0^\infty t^{z-1}\,\mathrm{e}^{-t}\,\mathrm{d}t, \qquad \Gamma(z+1) = z\,\Gamma(z), \qquad \Gamma(n+1) = n!, \qquad \Gamma(1/2) = \sqrt{\pi},$$

$$B(p,q) = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}.$$

*Felfunktion/Error function*

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{e}^{-y^2}\,\mathrm{d}y, \qquad \int_0^\infty \mathrm{e}^{-y^2}\,\mathrm{d}y = \frac{\sqrt{\pi}}{2}.$$

*Besselfunktioner*

$$\mathrm{e}^{\mathrm{i}r\sin(\theta)} = \sum_{-\infty}^{\infty} \mathrm{J}_n(r)\,\mathrm{e}^{\mathrm{i}n\theta},$$

$$\mathrm{J}_n(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{\mathrm{i}(z\sin(\theta)-n\theta)}\,\mathrm{d}\theta, \quad n \text{ heltal},$$

$$\mathrm{J}_\nu(z) = \Big(\frac{z}{2}\Big)^\nu \sum_{k=0}^{\infty} \frac{1}{k!\,\Gamma(k+\nu+1)} \Big(-\frac{z^2}{4}\Big)^k, \quad \nu \neq -1,-2,\dots$$

Bessels differentialekvation

$$u'' + \frac{1}{r}u' + \Big(\lambda - \frac{\nu^2}{r^2}\Big)u = 0$$

har den allmänna lösningen

$$\begin{cases} a\,\mathrm{J}_\nu(\sqrt{\lambda}\,r) + b\,\mathrm{Y}_\nu(\sqrt{\lambda}\,r) & \text{om } \lambda > 0\,, \\ a\,r^\nu + b\,r^{-\nu} & \text{om } \lambda = 0,\ \nu \neq 0, \\ a + b\ln(r) & \text{om } \lambda = \nu = 0. \end{cases}$$

Normuttryck

$$\int_0^R \Big|\mathrm{J}_\nu\Big(\frac{r}{R}\alpha_{\nu k}\Big)\Big|^2 r\,\mathrm{d}r = \frac{R^2}{2}\,\mathrm{J}_{\nu+1}(\alpha_{\nu k})^2 = \frac{R^2}{2}\,\mathrm{J}'_\nu(\alpha_{\nu k})^2.$$

*Nollställen till Besselfunktioner* $\mathrm{J}_n(x)$, $\mathrm{J}_n(\alpha_{nk}) = 0$.

| $k\backslash n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2,405 | 3,832 | 5,136 | 6,380 | 7,588 | 8,771 | 9,936 | 11,086 | 12,225 | 13,354 | 14,475 |
| 2 | 5,520 | 7,016 | 8,417 | 9,761 | 11,065 | 12,339 | 13,589 | 14,821 | 16,038 | 17,241 | 18,433 |
| 3 | 8,654 | 10,173 | 11,620 | 13,015 | 14,372 | 15,700 | 17,004 | 18,288 | 19,554 | 20,807 | 22,047 |
| 4 | 11,791 | 13,324 | 14,796 | 16,223 | 17,616 | 18,980 | 20,321 | 21,641 | 22,945 | 24,234 | 25,509 |
| 5 | 14,931 | 16,471 | 17,960 | 19,409 | 20,827 | 22,218 | 23,586 | 24,935 | 26,267 | 27,584 | 28,887 |
| 6 | 18,071 | 19,616 | 21,117 | 22,583 | 24,019 | 25,430 | 26,820 | 28,191 | 29,546 | 30,885 | 32,212 |
| 7 | 21,212 | 22,760 | 24,270 | 25,748 | 27,199 | 28,627 | 30,034 | 31,423 | 32,796 | 34,154 | 35,500 |
| 8 | 24,352 | 25,904 | 27,421 | 28,908 | 30,371 | 31,812 | 33,233 | 34,637 | 36,026 | 37,400 | 38,762 |
| 9 | 27,493 | 29,047 | 30,569 | 32,065 | 33,537 | 34,989 | 36,422 | 37,839 | 39,240 | 40,628 | 42,004 |
| 10 | 30,635 | 32,190 | 33,716 | 35,219 | 36,699 | 38,160 | 39,603 | 41,031 | 42,444 | 43,844 | 45,232 |

*Nollställen till* $\mathrm{J}'_n(x)$, $\mathrm{J}'_n(\alpha'_{nk}) = 0$.

| $k\backslash n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0,000 | 1,841 | 3,054 | 4,201 | 5,317 | 6,416 | 7,501 | 8,578 | 9,647 | 10,711 | 11,771 |
| 2 | 3,832 | 5,331 | 6,706 | 8,015 | 9,282 | 10,520 | 11,735 | 12,932 | 14,115 | 15,287 | 16,448 |
| 3 | 7,016 | 8,536 | 9,969 | 11,346 | 12,682 | 13,987 | 15,268 | 16,529 | 17,774 | 19,005 | 20,223 |
| 4 | 10,173 | 11,706 | 13,170 | 14,586 | 15,964 | 17,313 | 18,637 | 19,942 | 21,229 | 22,501 | 23,761 |
| 5 | 13,324 | 14,864 | 16,347 | 17,789 | 19,196 | 20,575 | 21,932 | 23,268 | 24,587 | 25,891 | 27,182 |
| 6 | 16,471 | 18,015 | 19,513 | 20,972 | 22,401 | 23,804 | 25,184 | 26,545 | 27,889 | 29,219 | 30,534 |
| 7 | 19,616 | 21,164 | 22,672 | 24,145 | 25,590 | 27,010 | 28,410 | 29,791 | 31,155 | 32,505 | 33,842 |
| 8 | 22,760 | 24,311 | 25,826 | 27,310 | 28,768 | 30,203 | 31,618 | 33,015 | 34,397 | 35,764 | 37,118 |
| 9 | 25,904 | 27,457 | 28,978 | 30,470 | 31,938 | 33,385 | 34,813 | 36,224 | 37,620 | 39,002 | 40,371 |

*Sfäriska Besselfunktioner*

Differentialekvationen

$$u'' + \frac{2}{z}u' + \Big(\lambda - \frac{\ell(\ell+1)}{z^2}\Big)u = 0$$

har den allmänna lösningen

$$\begin{cases} a\,\mathrm{j}_\ell(\sqrt{\lambda}\,z) + b\,\mathrm{y}_\ell(\sqrt{\lambda}\,z) & \text{om } \lambda > 0, \\ a\,z^\ell + b\,z^{-\ell-1} & \text{om } \lambda = 0,\ \ell \neq -1/2, \\ \dfrac{a + b\ln(z)}{\sqrt{z}} & \text{om } \lambda = 0,\ \ell = -1/2, \end{cases}$$

där

$$\mathrm{j}_\ell(z) = \sqrt{\frac{\pi}{2z}}\ \mathrm{J}_{\ell+1/2}(z), \quad \mathrm{y}_\ell(z) = \sqrt{\frac{\pi}{2z}}\ \mathrm{Y}_{\ell+1/2}(z)\,.$$

Speciellt är

$$\mathrm{j}_0(z) = \frac{\sin(z)}{z}, \qquad \mathrm{j}_1(z) = \frac{\sin(z) - z\cos(z)}{z^2},$$
$$\mathrm{y}_0(z) = -\frac{\cos(z)}{z}, \quad \mathrm{y}_1(z) = -\frac{\cos(z) + z\sin(z)}{z^2}.$$

*Legendrefunktioner*

Legendrepolynomen $(\mathrm{P}_\ell)_0^\infty$ är ortogonala i $\mathrm{L}_2(I),\ I = (-1,1)$.
Legendres differentialekvation

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big((1-x^2)\frac{\mathrm{d}u}{\mathrm{d}x}\Big) + \ell(\ell+1)\,u = 0, \quad \ell = 0,1,2,\ldots$$

har allmänna lösningen

$$a\,\mathrm{P}_\ell(x) + b\,\mathrm{Q}_\ell(x)$$

där $\mathrm{Q}_\ell$ ej är begränsad i $(-1,1)$ och

$$\mathrm{P}_\ell(x) = \frac{1}{2^\ell \ell!}\,\mathrm{D}^\ell (x^2-1)^\ell.$$

Rekursionsformel för Legendrepolynom:

$$\mathrm{P}_0(x) = 1\,, \quad \mathrm{P}_1(x) = x\,, \quad \mathrm{P}_{\ell+1}(x) = \frac{2\ell+1}{\ell+1}\,x\,\mathrm{P}_\ell(x) - \frac{\ell}{\ell+1}\,\mathrm{P}_{\ell-1}(x).$$

Associerade Legrendreekvationen

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big((1-x^2)\frac{\mathrm{d}u}{\mathrm{d}x}\Big) - \frac{m^2}{1-x^2}\,u + \ell(\ell+1)\,u = 0$$

har allmänna lösningen

$$a\,\mathrm{P}_\ell^m(x) + b\,\mathrm{Q}_\ell^m(x)$$

där $\mathrm{Q}_\ell^m$ ej är begränsad och

$$\mathrm{P}_\ell^m = (1-x^2)^{m/2}\,\mathrm{D}^m\,\mathrm{P}_\ell(x).$$

**Greenfunktioner**

Fundamentallösningar till Laplaceoperatorn ($-\Delta \mathrm{K} = \delta$)

$$\mathrm{K}(\boldsymbol{x}) = -\frac{1}{2\pi}\ln|\boldsymbol{x}| \quad \text{i } \mathbb{R}^2,$$

$$\mathrm{K}(\boldsymbol{x}) = \frac{1}{4\pi|\boldsymbol{x}|} \quad \text{i } \mathbb{R}^3.$$

Poissonkärnor

$$\mathrm{P}(r,\theta) = \frac{1}{2\pi}\frac{1-r^2}{1+r^2-2r\cos(\theta)} \quad \text{(enhetscirkeln)},$$

$$\mathrm{P}(x,y) = \frac{1}{\pi}\frac{y}{x^2+y^2} \quad \text{(halvplanet } y>0\text{)}.$$

Greenfunktion för Dirichlets problem

$$\begin{cases} -\Delta_x \mathrm{G}(\boldsymbol{x},\boldsymbol{\alpha}) = \delta_\alpha(\boldsymbol{x}), & \boldsymbol{x}\in\Omega, \\ \mathrm{G}(\boldsymbol{x},\boldsymbol{\alpha}) = 0, & \boldsymbol{x}\in\partial\Omega. \end{cases}$$

Om $-\Delta u = f$ i $\Omega$, $u = g$ på $\partial\Omega$ så

$$u(\boldsymbol{x}) = \int_\Omega \mathrm{G}(\boldsymbol{x},\boldsymbol{\alpha})\, f(\boldsymbol{\alpha})\,\mathrm{d}V_\alpha - \int_{\partial\Omega}\frac{\partial \mathrm{G}}{\partial \boldsymbol{n}_\alpha}(\boldsymbol{x},\boldsymbol{\alpha})\, g(\boldsymbol{\alpha})\,\mathrm{d}S_\alpha.$$

Konjugerade punkter med avseende på cirkeln (sfären) $|\boldsymbol{x}| = \rho$

$$|\boldsymbol{\alpha}||\tilde{\boldsymbol{\alpha}}| = \rho^2,$$

$$|\boldsymbol{x}-\boldsymbol{\alpha}| = \frac{|\boldsymbol{\alpha}|}{\rho}|\boldsymbol{x}-\tilde{\boldsymbol{\alpha}}| \quad \text{då } |\boldsymbol{x}| = \rho.$$

Värmeledning

$$\begin{cases} \mathrm{G}(x,t) = \dfrac{1}{\sqrt{4\pi a t}}\,\mathrm{e}^{-x^2/4at}, & x\in\mathbb{R},\ t>0, \\ \dfrac{\partial \mathrm{G}}{\partial t} - a\dfrac{\partial^2 \mathrm{G}}{\partial x^2} = 0, & x\in\mathbb{R},\ t>0, \\ \mathrm{G}(x,0) = \delta(x), & x\in\mathbb{R}. \end{cases}$$

**Vågutbredning**

d'Alembert

$$\begin{cases} u(x,t) = \dfrac{1}{2}(g(x-ct)+g(x+ct)) + \dfrac{1}{2c}\displaystyle\int_{x-ct}^{x+ct} h(y)\,\mathrm{d}y, \\ g(x) = u(x,0),\ h(x) = u_t(x,0). \end{cases}$$

Karakteristikor

$$\begin{cases} a_{11}u''_{xx} + 2a_{12}u''_{xy} + a_{22}u''_{yy} + F(x,y,u,u_x,u_y) = 0, \\ a_{11}\,\mathrm{d}y^2 - 2a_{12}\,\mathrm{d}x\,\mathrm{d}y + a_{22}\,\mathrm{d}x^2 = 0. \end{cases}$$

Kvasilinjära

$$\begin{cases} \alpha\dfrac{\partial u}{\partial x} + \beta\dfrac{\partial u}{\partial y} = f, \\ u(x_0,y_0) = u_0(x_0,y_0), \quad \text{för } g(x_0,y_0) = 0, \end{cases} \qquad \begin{cases} \dot{x} = \alpha, & x(0) = x_0, \\ \dot{y} = \beta, & y(0) = y_0, \\ \dot{z} = f, & z(0) = u_0(x_0,y_0). \end{cases}$$

**Fouriertransformer**

$$\mathcal{F}f(\xi) = \widehat{f}(\xi) = \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\xi x} f(x)\,\mathrm{d}x,$$

$$(\mathcal{F}^{-1}\widehat{f})(x) = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\xi x}\,\widehat{f}(\xi)\,\mathrm{d}\xi.$$

Parsevals formel

$$\int_{-\infty}^{\infty} \overline{f(x)}\,g(x)\,\mathrm{d}x = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{\widehat{f}(\xi)}\,\widehat{g}(\xi)\,\mathrm{d}\xi.$$

$$\xrightarrow{\mathcal{F}}$$

| | | |
|---|---|---|
| (1) | $\lambda f(x) + \mu g(x)$ | $\lambda \widehat{f}(\xi) + \mu \hat{g}(\xi)$ |
| (2) | $f(ax)$ | $\frac{1}{\lvert a\rvert}\,\widehat{f}\left(\frac{\xi}{a}\right)$ |
| (3) | $f(x - x_0)$ | $\mathrm{e}^{-\mathrm{i}x_0\xi}\,\widehat{f}(\xi)$ |
| (4) | $\mathrm{e}^{\mathrm{i}\xi_0 x} f(x)$ | $\widehat{f}(\xi - \xi_0)$ |
| (5) | $f'(x)$ | $\mathrm{i}\xi\,\widehat{f}(\xi)$ |
| (6) | $x f(x)$ | $\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\xi}\,\widehat{f}(\xi)$ |
| (7) | $(f * g)(x)$ | $\widehat{f}(\xi)\,\widehat{g}(\xi)$ |
| (8) | $\delta$ | $1$ |
| (9) | $1$ | $2\pi\,\delta$ |
| (10) | $\mathrm{e}^{-x}\,\theta(x)$ | $\frac{1}{1+\mathrm{i}\xi}$ |
| (11) | $\mathrm{e}^{-\lvert x\rvert}$ | $\frac{2}{1+\xi^2}$ |
| (12) | $\frac{1}{1+x^2}$ | $\pi\,\mathrm{e}^{-\lvert\xi\rvert}$ |
| (13) | $\mathrm{e}^{-x^2}$ | $\sqrt{\pi}\,\mathrm{e}^{-\xi^2/4}$ |
| (14) | $\theta(x+1) - \theta(x-1)$ | $2\,\frac{\sin(\xi)}{\xi}$ |
| (15) | $\theta(x)$ | $\frac{1}{\mathrm{i}}\,\mathrm{pv}\left(\frac{1}{\xi}\right) + \pi\,\delta$ |

$$\theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0, \end{cases} \qquad \theta' = \delta, \qquad \mathrm{sgn}(x) = \begin{cases} 1, & x > 0, \\ -1, & x < 0, \end{cases}$$

$$f(x)\,\delta = f(0)\,\delta, \qquad f(x)\,\delta' = f(0)\,\delta' - f'(0)\,\delta\,.$$

**Laplacetransformer**

$$\mathcal{L} f(s) = \mathcal{L}_{\mathrm{II}} f(s) = \int_{-\infty}^{\infty} \mathrm{e}^{-st} f(t)\,\mathrm{d}t, \qquad \alpha < \operatorname{Re} s < \beta, \quad s = \sigma + \mathrm{i}\omega,$$

$$f(t) = \frac{1}{2\pi\mathrm{i}} \int_{\sigma-\mathrm{i}\infty}^{\sigma+\mathrm{i}\infty} \mathrm{e}^{st} F(s)\,\mathrm{d}s, \qquad \alpha < \sigma < \beta,$$

$$\mathcal{F} f(\omega) = \mathcal{L}_{\mathrm{II}} f(\mathrm{i}\omega),$$

$$\mathcal{L}_{\mathrm{I}} f = \mathcal{L}_{\mathrm{II}}(\theta f).$$

$$\xrightarrow{\mathcal{L}_{\mathrm{II}}}$$

| | | |
|---|---|---|
| (16) | $\lambda f(t) + \mu g(t)$ | $\lambda F(s) + \mu G(s)$ |
| (17) | $f(at)$ | $\frac{1}{\lvert a\rvert} F\left(\frac{s}{a}\right)$ |
| (18) | $f(t - t_0)$ | $\mathrm{e}^{-t_0 s} F(s)$ |
| (19) | $\mathrm{e}^{at} f(t)$ | $F(s - a)$ |
| (20) | $f'(t)$ | $s F(s)$ |
| (21) | $t f(t)$ | $-\frac{\mathrm{d}}{\mathrm{d}s} F(s)$ |
| (22) | $(f * g)(t)$ | $F(s)\, G(s)$ |
| (23) | $\theta(t) f'(t)$ | $s\, \mathcal{L}_{\mathrm{II}}(\theta f)(s) - f(0)$ |
| (24) | $\delta$ | $1$ |
| (25) | $\theta(t)$ | $\frac{1}{s}, \quad \sigma > 0$ |
| (26) | $\theta(t) - 1$ | $\frac{1}{s}, \quad \sigma < 0$ |
| (27) | $t^k\, \mathrm{e}^{at}\, \theta(t)$ | $\frac{k!}{(s-a)^{k+1}}, \quad \sigma > \operatorname{Re}(a)$ |
| (28) | $\sin(bt)\, \theta(t)$ | $\frac{b}{s^2 + b^2}, \quad \sigma > 0$ |
| (29) | $\cos(bt)\, \theta(t)$ | $\frac{s}{s^2 + b^2}, \quad \sigma > 0$ |
| (30) | $\mathrm{e}^{-t^2}$ | $\sqrt{\pi}\, \mathrm{e}^{s^2/4}$ |
| (31) | $t^{\alpha-1}\, \theta(t)$ | $\frac{\Gamma(\alpha)}{s^\alpha}, \quad \operatorname{Re}(\alpha) > 0,\ \operatorname{Re}(s) > 0$ |
| (32) | $\frac{\lvert a\rvert}{\sqrt{4\pi}} \frac{\mathrm{e}^{-a^2/4t}}{t^{3/2}}\, \theta(t)$ | $\mathrm{e}^{-\lvert a\rvert\sqrt{s}}$ |
| (33) | $\frac{1}{\sqrt{\pi t}}\, \mathrm{e}^{-a^2/4t}\, \theta(t)$ | $\frac{\mathrm{e}^{-\lvert a\rvert\sqrt{s}}}{\sqrt{s}}$ |

## Fourierserier

$$f(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{ik\omega t} = c_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega t) + b_k \sin(k\omega t), \qquad \omega T = 2\pi,$$

$$c_k = \frac{1}{T}\int_{\text{period}} e^{-ik\omega t} f(t)\,dt, \qquad \begin{cases} a_k = \dfrac{2}{T}\displaystyle\int_{\text{period}} \cos(k\omega t)\, f(t)\,dt, \\ b_k = \dfrac{2}{T}\displaystyle\int_{\text{period}} \sin(k\omega t)\, f(t)\,dt, \end{cases}$$

$$\begin{cases} a_k = c_k + c_{-k}, \\ b_k = i(c_k - c_{-k}), \end{cases} \qquad \begin{cases} c_k = \dfrac{1}{2}(a_k - ib_k), \\ c_{-k} = \dfrac{1}{2}(a_k + ib_k). \end{cases}$$

*Parsevals formel*

$$\frac{1}{T}\int_{\text{period}} \overline{f(t)} g(t)\,dt = \sum_{k=-\infty}^{\infty} \overline{c_k(f)} c_k(g),$$

$$\frac{1}{T}\int_{\text{period}} |f(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |c_k|^2, \qquad \frac{1}{T}\int_{\text{period}} |f(t)|^2\,dt = |c_0|^2 + \frac{1}{2}\sum_{k=1}^{\infty}(|a_k|^2 + |b_k|^2).$$

## Halvperiodutvecklingar

*Cosinusserie*

$$f(x) = c_0 + \sum_{k=1}^{\infty} \alpha_k \cos\Big(\frac{k\pi}{L}x\Big),$$

$$\alpha_k = \frac{2}{L}\int_0^L f(x)\cos\Big(\frac{k\pi}{L}x\Big)\,dx,$$

$$c_0 = \frac{1}{L}\int_0^L f(x)\,dx.$$

*Sinusserie*

$$f(x) = \sum_{k=1}^{\infty} \beta_k \sin\Big(\frac{k\pi}{L}x\Big),$$

$$\beta_k = \frac{2}{L}\int_0^L f(x)\sin\Big(\frac{k\pi}{L}x\Big)\,dx,$$

## Några trigonometriska formler

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta),$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\Big(\frac{\alpha+\beta}{2}\Big)\cos\Big(\frac{\alpha-\beta}{2}\Big),$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\Big(\frac{\alpha+\beta}{2}\Big)\sin\Big(\frac{\alpha-\beta}{2}\Big),$$

$$\sin(\alpha) + \sin(\beta) = 2\sin\Big(\frac{\alpha+\beta}{2}\Big)\cos\Big(\frac{\alpha-\beta}{2}\Big),$$

$$\sin(\alpha) - \sin(\beta) = 2\cos\Big(\frac{\alpha+\beta}{2}\Big)\sin\Big(\frac{\alpha-\beta}{2}\Big),$$

$$a\cos(\alpha) + b\sin(\alpha) = c\cos(\alpha-\gamma), \qquad c = \sqrt{a^2+b^2}, \quad \tan(\gamma) = b/a.$$