with(*LinearAlgebra*)

$[$ *&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm,*   **(1)**
BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column,
ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy,
CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal,
DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues,
Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm,
FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic,
GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix,
HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix,
IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary,
JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition,
LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential,
MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower,
MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply,
NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot,
PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank,
RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation,
RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues,
SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis,
SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm,
UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm,
VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip $]$

$$A := \begin{bmatrix} 1 & 15 & 0 \\ 1 & 2 & 15 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A := \begin{bmatrix} 1 & 15 & 0 \\ 1 & 2 & 15 \\ 0 & -1 & 1 \end{bmatrix} \qquad \textbf{(2)}$$

$$Id := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Id := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

1.

$Determinant(A); Trace(A); Transpose(A), A^{-1}; (A+Id).(Transpose(A)-Id)$

$$2$$

$$4$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 15 & 2 & -1 \\ 0 & 15 & 1 \end{bmatrix}, \begin{bmatrix} \dfrac{17}{2} & -\dfrac{15}{2} & \dfrac{225}{2} \\ -\dfrac{1}{2} & \dfrac{1}{2} & -\dfrac{15}{2} \\ -\dfrac{1}{2} & \dfrac{1}{2} & -\dfrac{13}{2} \end{bmatrix}$$

$$\begin{bmatrix} 225 & 17 & -15 \\ 45 & 229 & -3 \\ -15 & 29 & 1 \end{bmatrix} \tag{4}$$

2.

$Eigenvalues(A); Eigenvectors(A)$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -15 & 0 & -15 \\ 0 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix} \tag{5}$$

3.

$J, Q := JordanForm(A, output = ['J', 'Q'])$

$$J, Q := \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 15 & -15 & -14 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix} \tag{6}$$

4.

$Rank(A); Rank(A-Id);$

$$3$$

$$2 \tag{7}$$

5.

$p\_A := CharacteristicPolynomial(A, x); \; fp\_A := Factor(p\_A) \textbf{ mod } 2$

$$p\_A := x^3 - 4x^2 + 5x - 2$$

$$fp\_A := (x+1)^2 x \tag{8}$$

6.

$p\_m := MinimalPolynomial(A, x)$

$$p\_m := x^3 - 4x^2 + 5x - 2 \tag{9}$$

7.

$AIA := (A - Id)^2$

$$AIA := \begin{bmatrix} 15 & 15 & 225 \\ 1 & 1 & 15 \\ -1 & -1 & -15 \end{bmatrix} \tag{10}$$

$Nullspace(AIA) \textbf{ mod } 5$

$$\left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \tag{11}$$

Im(A-I)

$A\_Id := A - Id;$

$$A\_Id := \begin{bmatrix} 0 & 15 & 0 \\ 1 & 1 & 15 \\ 0 & -1 & 0 \end{bmatrix} \tag{12}$$

$Basis([A\_Id[1], A\_Id[2], A\_Id[3]])$

$$\left[\begin{bmatrix} 0 & 15 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 15 \end{bmatrix} \right] \tag{13}$$

8.*a*

GramSchmidt needs eigenvectors as a list, so calculating them.

$e := Eigenvectors(A, output='vectors')$

$$e := \begin{bmatrix} -15 & 0 & -15 \\ 0 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix} \tag{14}$$

$e := Transpose(e)$

$$e := \begin{bmatrix} -15 & 0 & 1 \\ 0 & 0 & 0 \\ -15 & -1 & 1 \end{bmatrix} \tag{15}$$

$V := [e[1], e[2], e[3]]$

$$V := \left[\begin{bmatrix} -15 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -15 & -1 & 1 \end{bmatrix} \right] \tag{16}$$

$g := GramSchmidt(V, normalized)$

$$g := \left[\begin{bmatrix} -\dfrac{15\sqrt{226}}{226} & 0 & \dfrac{\sqrt{226}}{226} \end{bmatrix}, \begin{bmatrix} 0 & -1 & 0 \end{bmatrix} \right] \tag{17}$$

8.b
Doing the same thing but for A-I.

$e := Eigenvectors(A - Id, output = 'vectors')$

$$e := \begin{bmatrix} -15 & 0 & -15 \\ 0 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix} \tag{18}$$

$e := Transpose(e)$

$$e := \begin{bmatrix} -15 & 0 & 1 \\ 0 & 0 & 0 \\ -15 & -1 & 1 \end{bmatrix} \tag{19}$$

$V := [e[1], e[2], e[3]]$

$$V := \left[\begin{bmatrix} -15 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -15 & -1 & 1 \end{bmatrix} \right] \tag{20}$$

$g := GramSchmidt(V, normalized)$

$$g := \left[\begin{bmatrix} -\dfrac{15\sqrt{226}}{226} & 0 & \dfrac{\sqrt{226}}{226} \end{bmatrix}, \begin{bmatrix} 0 & -1 & 0 \end{bmatrix} \right] \tag{21}$$

9

$MatrixExponential(A)$

$$\begin{bmatrix} -29\,e + 15\,e^2 & 15\,e^2 - 15\,e & 225\,e^2 - 450\,e \\ e^2 - e & e^2 & 15\,e^2 - 15\,e \\ -e^2 + 2\,e & -e^2 + e & 31\,e - 15\,e^2 \end{bmatrix} \tag{22}$$

10

$LUDecomposition(A)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & \dfrac{1}{13} & 1 \end{bmatrix}, \begin{bmatrix} 1 & 15 & 0 \\ 0 & -13 & 15 \\ 0 & 0 & -\dfrac{2}{13} \end{bmatrix} \qquad (23)$$

$QRDecomposition(A)$

$$\begin{bmatrix} \dfrac{\sqrt{2}}{2} & \dfrac{13\sqrt{38}}{114} & -\dfrac{\sqrt{19}}{57} \\[2mm] \dfrac{\sqrt{2}}{2} & -\dfrac{13\sqrt{38}}{114} & \dfrac{\sqrt{19}}{57} \\[2mm] 0 & -\dfrac{\sqrt{38}}{57} & -\dfrac{13\sqrt{19}}{57} \end{bmatrix}, \begin{bmatrix} \sqrt{2} & \dfrac{17\sqrt{2}}{2} & \dfrac{15\sqrt{2}}{2} \\[2mm] 0 & \dfrac{3\sqrt{38}}{2} & -\dfrac{197\sqrt{38}}{114} \\[2mm] 0 & 0 & \dfrac{2\sqrt{19}}{57} \end{bmatrix} \qquad (24)$$

11

$evalf(Norm(A, 2))$

$$16.09555372 \qquad (25)$$

$ConditionNumber(A)$

$$2313 \qquad (26)$$

12

$$C := \begin{bmatrix} aa & bb & cc \\ dd & ee & ff \\ gg & hh & ii \end{bmatrix}; C := C^2$$

$$C := \begin{bmatrix} aa & bb & cc \\ dd & ee & ff \\ gg & hh & ii \end{bmatrix}$$

$$C := \begin{bmatrix} aa^2 + bb\,dd + cc\,gg & aa\,bb + bb\,ee + cc\,hh & aa\,cc + bb\,ff + cc\,ii \\ dd\,aa + ee\,dd + ff\,gg & bb\,dd + ee^2 + ff\,hh & dd\,cc + ee\,ff + ff\,ii \\ gg\,aa + hh\,dd + ii\,gg & gg\,bb + hh\,ee + ii\,hh & cc\,gg + ff\,hh + ii^2 \end{bmatrix} \qquad (27)$$

>

$LinearSolve(A, C)$

$$\left[\left[\frac{17}{2}aa^2 + \frac{17}{2}bb\,dd + \frac{17}{2}cc\,gg - \frac{15}{2}dd\,aa - \frac{15}{2}ee\,dd - \frac{15}{2}ff\,gg + \frac{225}{2}gg\,aa\right.\right. \qquad (28)$$

$$+ \frac{225}{2} \, hh \, dd + \frac{225}{2} \, ii \, gg, \frac{17}{2} \, aa \, bb + \frac{17}{2} \, bb \, ee + \frac{17}{2} \, cc \, hh - \frac{15}{2} \, bb \, dd - \frac{15}{2} \, ee^2$$

$$- \frac{15}{2} \, ff \, hh + \frac{225}{2} \, gg \, bb + \frac{225}{2} \, hh \, ee + \frac{225}{2} \, ii \, hh, \frac{17}{2} \, aa \, cc + \frac{17}{2} \, bb \, ff + \frac{17}{2} \, cc \, ii$$

$$- \frac{15}{2} \, dd \, cc - \frac{15}{2} \, ee \, ff - \frac{15}{2} \, ff \, ii + \frac{225}{2} \, cc \, gg + \frac{225}{2} \, ff \, hh + \frac{225}{2} \, ii^2 \Bigg],$$

$$\Bigg[-\frac{1}{2} \, aa^2 + \frac{1}{2} \, dd \, aa - \frac{1}{2} \, bb \, dd - \frac{1}{2} \, cc \, gg + \frac{1}{2} \, ee \, dd + \frac{1}{2} \, ff \, gg - \frac{15}{2} \, gg \, aa$$

$$- \frac{15}{2} \, hh \, dd - \frac{15}{2} \, ii \, gg, -\frac{1}{2} \, aa \, bb + \frac{1}{2} \, bb \, dd - \frac{1}{2} \, bb \, ee - \frac{1}{2} \, cc \, hh + \frac{1}{2} \, ee^2$$

$$+ \frac{1}{2} \, ff \, hh - \frac{15}{2} \, gg \, bb - \frac{15}{2} \, hh \, ee - \frac{15}{2} \, ii \, hh, -\frac{1}{2} \, aa \, cc - \frac{1}{2} \, bb \, ff + \frac{1}{2} \, dd \, cc$$

$$- \frac{1}{2} \, cc \, ii + \frac{1}{2} \, ee \, ff + \frac{1}{2} \, ff \, ii - \frac{15}{2} \, cc \, gg - \frac{15}{2} \, ff \, hh - \frac{15}{2} \, ii^2 \Bigg],$$

$$\Bigg[-\frac{13}{2} \, gg \, aa - \frac{13}{2} \, hh \, dd - \frac{13}{2} \, ii \, gg - \frac{1}{2} \, aa^2 + \frac{1}{2} \, dd \, aa - \frac{1}{2} \, bb \, dd - \frac{1}{2} \, cc \, gg$$

$$+ \frac{1}{2} \, ee \, dd + \frac{1}{2} \, ff \, gg, -\frac{13}{2} \, gg \, bb - \frac{13}{2} \, hh \, ee - \frac{13}{2} \, ii \, hh - \frac{1}{2} \, aa \, bb + \frac{1}{2} \, bb \, dd$$

$$- \frac{1}{2} \, bb \, ee - \frac{1}{2} \, cc \, hh + \frac{1}{2} \, ee^2 + \frac{1}{2} \, ff \, hh, -\frac{13}{2} \, cc \, gg - \frac{13}{2} \, ff \, hh - \frac{13}{2} \, ii^2 - \frac{1}{2} \, aa \, cc$$

$$- \frac{1}{2} \, bb \, ff + \frac{1}{2} \, dd \, cc - \frac{1}{2} \, cc \, ii + \frac{1}{2} \, ee \, ff + \frac{1}{2} \, ff \, ii \Bigg] \Bigg]$$