# Supplemental Material for "Can Visual Recognition Benefit from Auxiliary Information in Training?"

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In this supplemental material, we present the proof of Proposition 1 in Section 1 and additional experimental results with the RGBD Object dataset [1] in Section 2.

### 1 Proof of Proposition 1

In this section, we present the proof of the Proposition 1 for Paper 498. First, the existence of the upper bound is proven in Section 1.1, then the proof that the sequence  $f(W(s)), s = 1, 2, \cdots$  is monotonic is presented in Section 1.2. With the Bolzano-Weierstrass theorem and the conclusions of Section 1.1 and Section 1.2, the Proposition 1 is proven.

### 1.1 Proof of the existence of the upper bound $C_u$

From the constraint in Eq. (6) of the paper:

$$\mathbf{W}_{j}^{T} \left[ (1 - \tau_{j}) \frac{1}{n} \mathbf{R}_{j} \mathbf{R}_{j}^{T} + \tau_{j} \mathbf{I}_{n} \right] \mathbf{W}_{j} = \mathbf{I},$$
 (1)

where  $\tau_j$  denotes the pre-specified regularization parameter,  $0 < \tau_j < 1$   $(j = 1, 2, \dots, J)$ , we have

$$(1 - \tau_j) \operatorname{tr} \left( \frac{1}{n} \mathbf{W}_j^T \mathbf{R}_j \mathbf{R}_j^T \mathbf{W}_j \right) + \tau_j \operatorname{tr} \left( \mathbf{W}_j^T \mathbf{W}_j \right) = n,$$
 (2)

and therefore

$$\operatorname{tr}\left(\mathbf{W}_{j}^{T} \mathbf{R}_{j} \mathbf{R}_{j}^{T} \mathbf{W}_{j}\right) \leq \frac{1}{1 - \tau_{i}}, \ 0 < \tau_{j} < 1, \ \forall j = 1, 2, \cdots, J.$$
(3)

In addition, we have

$$\operatorname{tr}\left(\mathbf{W}_{j}^{T}\mathbf{R}_{j}\mathbf{R}_{k}^{T}\mathbf{W}_{k}\right) \leq \frac{1}{2}\left[\operatorname{tr}\left(\mathbf{W}_{j}^{T}\mathbf{R}_{j}\mathbf{R}_{j}^{T}\mathbf{W}_{j}\right) + \operatorname{tr}\left(\mathbf{W}_{k}^{T}\mathbf{R}_{k}\mathbf{R}_{k}^{T}\mathbf{W}_{k}\right)\right]$$
(4)

$$\leq \frac{1}{2} \left[ \frac{1}{1 - \tau_i} + \frac{1}{1 - \tau_k} \right] < \infty, \tag{5}$$

where the inequality (4) follows the property

$$\operatorname{tr}(\mathbf{B}^T \mathbf{A}) \le \frac{1}{2} \left[ \operatorname{tr}(\mathbf{A}^T \mathbf{A}) + \operatorname{tr}(\mathbf{B}^T \mathbf{B}) \right],$$
 (6)

which comes from the fact that  $\operatorname{tr}\left((\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B})\right) \geq 0$ , because the matrix  $(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B}) \succeq 0$ . Since  $\operatorname{tr}\left(\mathbf{W}_j^T \mathbf{R}_j \mathbf{R}_k^T \mathbf{W}_k\right)$  is bounded, , g(x) = x or  $x^2$ , we have

$$g\left(\operatorname{tr}\left(\mathbf{W}_{j}^{T}\mathbf{R}_{j}\mathbf{R}_{k}^{T}\mathbf{W}_{k}\right)\right) \leq \frac{1}{4}\left[\frac{1}{1-\tau_{j}} + \frac{1}{1-\tau_{k}}\right]^{2} < \infty.$$
 (7)

Considering  $c_{ik} = 0$  or 1, we have

$$\sum_{j\neq k}^{J} c_{jk} g\left(\operatorname{tr}\left(\frac{1}{n} \mathbf{W}_{j}^{T} \mathbf{R}_{j} \mathbf{R}_{k}^{T} \mathbf{W}_{k}\right)\right) \leq \frac{1}{4n} \sum_{j\neq k}^{J} \left[\frac{1}{1-\tau_{j}} + \frac{1}{1-\tau_{k}}\right]^{2} < \infty, \quad (8)$$

which shows that the sequence  $f(\mathcal{W}(s)), s = 1, 2, \cdots$  is upper bounded by

$$C_u = \frac{1}{4n} \sum_{j \neq k}^{J} \left[ \frac{1}{1 - \tau_j} + \frac{1}{1 - \tau_k} \right]^2 < \infty.$$
 (9)

## 1.2 Proof that the sequence $f(\mathcal{W}(s)), s = 1, 2, \cdots$ is monotonically increasing

In this section, the monotonic property of the sequence  $f(W(s)), s = 1, 2, \cdots$  is presented. Following [2–4], we first present a Lemma, and then prove that

$$f(\mathcal{W}(s)) \le f(\mathcal{W}(s+1)), \ s = 1, 2, \cdots, \tag{10}$$

where s is the iteration index,  $s = 1, 2, \cdots$ .

Define the function  $r(\mathbf{Y}_j, \mathbf{Y}_k) \stackrel{\text{def}}{=} \operatorname{tr}(\frac{1}{n}\mathbf{Y}_j^T\mathbf{Y}_k) = \operatorname{tr}(\frac{1}{n}\mathbf{W}_j^T\mathbf{R}_j\mathbf{R}_k^T\mathbf{W}_k)$ , therefore,

$$f(\mathbf{W}_1(s), \cdots, \mathbf{W}_J(s)) = \sum_{j,k=1, k \neq j}^J c_{jk} g[r(\mathbf{Y}_j(s), \mathbf{Y}_k(s))]. \tag{11}$$

Using this notation, we have the following Lemma:

Lemma 1. Define

$$f_{j}(\mathbf{W}_{j}) \stackrel{\text{def}}{=} \sum_{k=1}^{j-1} c_{jk} g \left[ r(\mathbf{R}_{j}^{T} \mathbf{W}_{j}, \mathbf{Y}_{k}(s+1)) \right] + \sum_{k=j+1}^{J} c_{jk} g \left[ r(\mathbf{R}_{j}^{T} \mathbf{W}_{j}, \mathbf{Y}_{k}(s)) \right]$$

$$s.t. \ \mathbf{W}_{j}^{T} N_{j} \mathbf{W}_{j} = \mathbf{I},$$

$$(12)$$

then

$$f_j(\mathbf{W}_j(s)) \le f_j(\mathbf{W}_j(s+1)), \ j = 1, \cdots, J. \tag{13}$$

*Proof.* We prove Lemma 1 in two cases, *i.e.*,  $g(x) = x^2$  and g(x) = x. Case 1: when  $g(x) = x^2$ , we have  $f_j(\mathbf{W}_j)$  in the following form,

$$f_{j}(\mathbf{W}_{j}) = \sum_{k=1}^{j-1} c_{jk} \left( r(\mathbf{R}_{j}^{T} \mathbf{W}_{j}, \mathbf{Y}_{k}(s+1)) \right)^{2} + \sum_{k=j+1}^{J} c_{jk} \left( r(\mathbf{R}_{j}^{T} \mathbf{W}_{j}, \mathbf{Y}_{k}(s)) \right)^{2},$$

$$(14)$$

which can be written as

$$f_{j}(\mathbf{W}_{j}(s)) = \frac{1}{n} \sum_{k=1}^{j-1} c_{jk} \theta_{jk}^{(s)} \operatorname{tr}(\mathbf{W}_{j}(s)^{T} \mathbf{R}_{j} \mathbf{Y}_{k}(s+1))$$

$$+ \frac{1}{n} \sum_{k=j+1}^{J} c_{jk} \theta_{jk}^{(s)} \operatorname{tr}(\mathbf{W}_{j}(s)^{T} \mathbf{R}_{j} \mathbf{Y}_{k}(s))$$

$$= \frac{1}{n} \operatorname{tr} \left( \mathbf{W}_{j}(s)^{T} \mathbf{R}_{j} \left( \sum_{k=1}^{j-1} c_{jk} \theta_{jk}^{(s)} \mathbf{Y}_{k}(s+1) + \sum_{k=j+1}^{J} c_{jk} \theta_{jk}^{(s)} \mathbf{Y}_{k}(s) \right) \right),$$

$$(15)$$

where  $\theta_{jk}^{(s)}$  are defined as

$$\theta_{jk}^{(s)} = \begin{cases} r\left(\mathbf{Y}_{j}(s), \mathbf{Y}_{k}(s+1)\right) & \text{if } k=1,\cdots,j-1\\ r\left(\mathbf{Y}_{j}(s), \mathbf{Y}_{k}(s)\right) & \text{if } k=j+1,\cdots,J \end{cases}$$
(17)

Note that in Eq. (16), the term  $\left(\sum_{k=1}^{j-1} c_{jk} \theta_{jk}^{(s)} \mathbf{Y}_k(s+1) + \sum_{k=j+1}^{J} c_{jk} \theta_{jk}^{(s)} \mathbf{Y}_k(s)\right)$  is equivalent to the definition of  $\mathbf{Z}_j(s)$ , hence  $f_j(\mathbf{W}_j)$  can be simplified as  $\frac{1}{n} \operatorname{tr}(\mathbf{W}_j^T \mathbf{R}_j \mathbf{Z}_j(s))$ . Considering the following optimization problem:

$$\max_{\mathbf{W}_j} \frac{1}{n} \operatorname{tr}(\mathbf{W}_j^T \mathbf{R}_j \mathbf{Z}_j(s)), \text{ s.t. } \mathbf{W}_j^T N_j \mathbf{W}_j = \mathbf{I},$$
(18)

whose solution is exactly

$$\mathbf{W}_{j}(s+1) = N_{j}^{-1} \mathbf{R}_{j} \mathbf{Z}_{j}(s) \left( \left[ \mathbf{Z}_{j}(s)^{T} \mathbf{R}_{j}^{T} N_{j}^{-1} \mathbf{R}_{j} \mathbf{Z}_{j}(s) \right]^{1/2} \right)^{\dagger}, \tag{19}$$

we have

$$\operatorname{tr}(\mathbf{W}_{j}(s)^{T} \mathbf{R}_{j} \mathbf{Z}_{j}(s)) \leq \operatorname{tr}(\mathbf{W}_{j}(s+1)^{T} \mathbf{R}_{j} \mathbf{Z}_{j}(s)). \tag{20}$$

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Similarly, the following equations can be obtained:

$$f_{j}(\mathbf{W}_{j}(s)) = \sum_{k=1}^{j-1} c_{jk} \theta_{jk}^{(s)} r(\mathbf{Y}_{j}(s), \mathbf{Y}_{k}(s+1)) + \sum_{k=j+1}^{J} c_{jk} \theta_{jk}^{(s)} r(\mathbf{Y}_{j}(s), \mathbf{Y}_{k}(s))$$

$$\leq \sum_{k=1}^{j-1} c_{jk} \theta_{jk}^{(s)} r(\mathbf{Y}_{j}(s+1), \mathbf{Y}_{k}(s+1))$$

$$+ \sum_{k=j+1}^{J} c_{jk} \theta_{jk}^{(s)} r(\mathbf{Y}_{j}(s+1), \mathbf{Y}_{k}(s)). \tag{21}$$

Considering that  $c_{jk}$  is either 0 or 1, we hence have  $c_{jk} = c_{jk}^2$ . Applying the Cauchy-Schwartz inequality, we have

$$f_{j}(\mathbf{W}_{j}(s)) \leq \sum_{k=1}^{j-1} c_{jk}^{2} \theta_{jk}^{(s)} r(\mathbf{Y}_{j}(s+1), \mathbf{Y}_{k}(s+1)) + \sum_{k=j+1}^{J} c_{jk}^{2} \theta_{jk}^{(s)} r(\mathbf{Y}_{j}(s+1), \mathbf{Y}_{k}(s))$$

$$\leq \sqrt{\sum_{k=1}^{j-1} c_{jk} \left(\theta_{jk}^{(s)}\right)^{2} + \sum_{k=j+1}^{J} c_{jk} \left(\theta_{jk}^{(s)}\right)^{2}}$$

$$\cdot \sqrt{\sum_{k=1}^{j-1} c_{jk} \left(r(\mathbf{Y}_{j}(s+1), \mathbf{Y}_{k}(s+1))\right)^{2} + \sum_{k=j+1}^{J} c_{jk} \left(r(\mathbf{Y}_{j}(s+1), \mathbf{Y}_{k}(s))\right)^{2}}$$

$$= \sqrt{\sum_{k=1}^{j-1} c_{jk} \left(r(\mathbf{Y}_{j}(s), \mathbf{Y}_{k}(s+1))\right)^{2} + \sum_{k=j+1}^{J} c_{jk} \left(r(\mathbf{Y}_{j}(s), \mathbf{Y}_{k}(s))\right)^{2}}$$

$$\cdot \sqrt{\sum_{k=1}^{j-1} c_{jk} \left(r(\mathbf{Y}_{j}(s+1), \mathbf{Y}_{k}(s+1))\right)^{2} + \sum_{k=j+1}^{J} c_{jk} \left(r(\mathbf{Y}_{j}(s+1), \mathbf{Y}_{k}(s))\right)^{2}}$$

$$= \sqrt{f_{j}(\mathbf{W}_{j}(s))} \cdot \sqrt{f_{j}(\mathbf{W}_{j}(s+1))}. \tag{22}$$

We immediately have  $f_j(\mathbf{W}_j(s)) \leq f_j(\mathbf{W}_j(s+1))$ . This concludes the case 1 scenario.

Case 2: when g(x) = x, we have

$$f_{j}(\mathbf{W}_{j}) = \sum_{k=1}^{j-1} c_{jk} r(\mathbf{R}_{j}^{T} \mathbf{W}_{j}, \mathbf{Y}_{k}(s+1))$$

$$+ \sum_{k=j+1}^{J} c_{jk} r(\mathbf{R}_{j}^{T} \mathbf{W}_{j}, \mathbf{Y}_{k}(s)).$$
(23)

Therefore, we can have exactly the same equation as Eq. (16), except that  $\theta_{jk}^{(s)} \equiv 1$  for all the cases. The same equation as in Eq. (20) can be obtained, which directly implies that  $f_j(\mathbf{W}_j(s)) \leq f_j(\mathbf{W}_j(s+1))$ . This concludes both the case 2 scenario and the entire proof of Lemma.

With the conclusion in Lemma 1, we proceed with the proof that the sequence  $f(W(s)), s = 1, 2, \cdots$  is monotonically increasing. Consider the following subtraction

$$\sum_{j=1}^{J} [f_{j}(\mathbf{W}_{j}(s+1)) - f_{j}(\mathbf{W}_{j}(s))]$$

$$= \sum_{j=1}^{J} \sum_{k=1}^{j-1} c_{jk}g \left[ \operatorname{tr} \left( \frac{1}{n} \mathbf{W}_{j}(s+1)^{T} \mathbf{R}_{j} \mathbf{R}_{k}^{T} \mathbf{W}_{k}(s+1) \right) \right]$$

$$+ \sum_{j=1}^{J} \sum_{k=j+1}^{J} c_{jk}g \left[ \operatorname{tr} \left( \frac{1}{n} \mathbf{W}_{j}(s+1)^{T} \mathbf{R}_{j} \mathbf{R}_{k}^{T} \mathbf{W}_{k}(s) \right) \right]$$

$$- \sum_{j=1}^{J} \sum_{k=1}^{j-1} c_{jk}g \left[ \operatorname{tr} \left( \frac{1}{n} \mathbf{W}_{j}(s)^{T} \mathbf{R}_{j} \mathbf{R}_{k}^{T} \mathbf{W}_{k}(s+1) \right) \right]$$

$$- \sum_{j=1}^{J} \sum_{k=1}^{j-1} c_{jk}g \left[ \operatorname{tr} \left( \frac{1}{n} \mathbf{W}_{j}(s+1)^{T} \mathbf{R}_{j} \mathbf{R}_{k}^{T} \mathbf{W}_{k}(s+1) \right) \right]$$

$$= \frac{1}{2} \left[ \sum_{j,k=1,k\neq j}^{J} c_{jk}g \left[ \operatorname{tr} \left( \frac{1}{n} \mathbf{W}_{j}(s+1)^{T} \mathbf{R}_{j} \mathbf{R}_{k}^{T} \mathbf{W}_{k}(s+1) \right) \right]$$

$$- \sum_{j,k=1,k\neq j}^{J} c_{jk}g \left[ \operatorname{tr} \left( \frac{1}{n} \mathbf{W}_{j}(s)^{T} \mathbf{R}_{j} \mathbf{R}_{k}^{T} \mathbf{W}_{k}(s) \right) \right] \right] \ge 0. \tag{24}$$

The last equation in Eq. (24) follows the Lemma 1. This implies that

$$f(\mathbf{W}_{1}(s), \dots, \mathbf{W}_{J}(s)) \le f(\mathbf{W}_{1}(s+1), \dots, \mathbf{W}_{J}(s+1))$$
 (25)  
i.e.,  $f(\mathcal{W}(s)) \le f(\mathcal{W}(s+1)), \ s = 1, 2, \dots$  (26)

Using Eq. (8), Eq. (26), the bounded sequence  $f(W(s)), s = 1, 2, \cdots$  is monotonically increasing.

According to the Bolzano-Weierstrass theorem, the sequence will converge, i.e., Proposition 1 is proven.

### 2 Additional Experimental Results

With the same RGBD Object dataset and experimental settings from [1] and [5], we have also conducted additional experiments with the state-of-the-art HMP-based features [5]. These additional results are summarized in Table 1–2 (too many entries to fit in a single page).

As is seen in Table 1–2, the overall recognition accuracy improves significantly across all methods, as compared to the EMK-based features [1]. In a large portion of the categories, perfect recognition is achieved even with the naive baseline

**Table 1.** Accuracy Table Part 1 for the Multi-View RGBD Object Instance recognition with HMP features, the highest and second highest values are colored red and blue, respectively. The remaining part is in Table 2.

Category   SVM SVM2K KCCA   KCCA   FL   FGCCA   FGCCA   FAL   FAL   Bapple   S7.62   93.33   92.86   77.14   92.38   87.62   93.33   93.33   93.31   93.31   93.31   93.31   93.33   94.33   94.06   77.24   94.06		KCCA							
apple         87.62         93.33         92.86         77.14         92.38         87.62         93.33         93.33           ball         100	Category	SVM	SVM2K	KCCA		RGCCA			DCCA
banama         73.74         77.78         81.31         81.31         80.30         75.25         80.30         80.30           bell pepper         82.07         84.06         78.88         77.29         73.31         80.88         73.31         83.27           binder         100         100         100         100         100         100         100           bowl         83.08         86.54         86.54         86.51         87.31         77.69         87.69         887.69           calculator         100         100         100         100         100         100         99.17         99.10	apple	87.62	93.33	92.86		92.38		93.33	93.33
bell pepper         82.07         84.06         78.88         77.29         73.31         80.88         73.31         80.27           binder         100	ball	100	100	100	100	100	100	100	100
binder         100<	banana	73.74	77.78	81.31	81.31	80.30	75.25	80.30	80.30
bowl         83.08         86.54         86.54         86.15         87.31         77.69         87.69         87.69           calculator         100         100         100         99.44         100         100         100         100           camera         99.17         100         100         100         99.17         99.10         00         100         100         100         100         100         100         100         90.68         90.09         90.00         00         100         100         90.88         100         100         100         100 <td>bell pepper</td> <td>82.07</td> <td>84.06</td> <td>78.88</td> <td>77.29</td> <td>73.31</td> <td>80.88</td> <td>73.31</td> <td>83.27</td>	bell pepper	82.07	84.06	78.88	77.29	73.31	80.88	73.31	83.27
calculator         100         100         100         99.44         100         100         100         100         99.17         99.17         99.17         99.17         99.17         cap         100         100         100         100         99.17         99.18         99.18         91.00         100         100         100         100         100         100         100         100         100         100         100         100         100         100         96.67         88.09         98.09         98.00         98.00         98.00         98.00         98.00         98.00         98.00         98.00         98.00         98.00         98.00         98.00         98.00         98.00         99.60         98.00         98.00         99.60         98.00         99.60         99.63         98.00         99.78         99.21         99.60         99.21         99.62	binder	100	100	100	100	100	100	100	100
camera         99.17         100         100         100         100         100         99.17         99.17         99.17         99.17           cap         100         100         100         100         97.08         100         100           cereal box         100	bowl	83.08	86.54	86.54	86.15	87.31	77.69	87.69	87.69
cap         100         100         100         100         97.08         100         100           cereal box         100         98.00         95.33         98.00         99.32         98.00         99.00         99.84         100         100         99.32         99.32         99.32         99.32         99.32         99.32         99.32         99.32 <td>calculator</td> <td>100</td> <td>100</td> <td>100</td> <td>99.44</td> <td>100</td> <td>100</td> <td>100</td> <td>100</td>	calculator	100	100	100	99.44	100	100	100	100
celiphone         98.43         95.29         95.81         94.76         95.81         98.43         95.81         95.81           cereal box         100         100         100         100         100         100         100           comb         92.88         85.45         96.90         94.43         99.69         91.64         99.69         100           comb         96.67         97.33         97.33         96.67         98.00         95.33         98.00         98.00           dry battery         86.34         84.14         85.46         76.65         87.67         85.90         88.55         88.55           flashlight         97.34         97.34         97.37         95.21         100         97.34         100         100           food bag         100         100         100         100         100         100         100         100         100         100         100         100         100         100         100         99.88         100         100         100         99.18         99.99         99.39         99.39         99.39         99.39         99.39         99.39         99.39         99.39         99.39         99.39	camera	99.17	100	100	100	$\boldsymbol{99.17}$	99.17	$\boldsymbol{99.17}$	<b>99.17</b>
cereal box         100         98.00         95.33         98.00         98.00         98.00         95.33         98.00         98.00         98.00         98.00         95.33         98.00         99.33         98.00         90.00         100         100         100         100         100         100         100         100         100         100         100         99.18         99.32         99.32         99.32         99.32         99.32         99.32         99.32         99.32         99.32         99.32         99.32         <	cap	100	100	100	100	100	<b>97.08</b>	100	100
coffee mug         92.88         85.45         96.90         94.43         99.69         91.64         99.69         100           comb         96.67         97.33         97.33         97.33         96.67         98.00         95.33         98.00         98.00           dry battery         86.34         84.14         85.46         76.65         87.67         85.90         88.55         88.55           flashlight         97.34         97.34         97.87         95.21         100         97.34         100         100           food bag         100         100         100         100         100         96.88         100         100           food can         98.08         93.02         94.39         81.67         99.18         98.08         99.32         99.32           food cup         96.32         98.90         97.06         96.32         97.43         97.06         97.43         98.53           food jar         98.73         100         100         91.46         100         97.15         100         100           garlic         90.91         98.40         96.26         95.99         91.98         95.99         95.99	cellphone	98.43	95.29	95.81	94.76	95.81	98.43	95.81	95.81
comb         96.67         97.33         97.33         96.67         98.00         95.33         98.00         98.05           dry battery         86.34         84.14         85.46         76.65         87.67         85.90         88.55         88.55           flashlight         97.34         97.34         97.87         95.21         100         97.34         100         100           food bag         100         99.84         100         100         100         100         100           food can         98.08         93.02         94.39         81.67         99.18         98.08         99.32         99.32           food cup         96.32         98.90         97.06         96.32         97.43         97.06         97.43         98.53           food jar         98.73         100         100         100         100         97.15         100         100           garlic         90.91         98.40         98.40         96.26         95.99         91.98         95.99         95.99           glue stick         100         100         100         100         99.68         100         100           geens         76.76         89.	cereal box	100	100	100	100	100	100	100	100
dry battery         86.34         84.14         85.46         76.65         87.67         85.90         88.55         88.55           flashlight         97.34         97.34         97.87         95.21         100         97.34         100         100           food bag         100         100         100         100         100         100         100           food can         98.08         93.02         94.39         81.67         99.18         98.08         99.32         99.32           food can         96.32         98.90         97.06         96.32         97.43         98.53           food jar         98.73         100         100         91.46         100         97.15         100         100           garlic         90.91         98.40         98.40         96.26         95.99         91.98         95.99         95.99         95.99         91.98         95.99         95.99         91.88         90.27         91.35         96.96         100         100         99.68         100         100         100         100         100         100         100         100         100         100         100         100         100         100	coffee mug	92.88	85.45	96.90	94.43	99.69	91.64	99.69	100
flashlight         97.34         97.34         97.87         95.21         100         97.34         100         100           food bag         100         100         100         100         96.88         100         100           food can         98.08         93.02         94.39         81.67         99.18         98.08         99.32         99.32           food cup         96.32         98.90         97.06         96.32         97.43         97.06         97.43         98.30         99.32         99.32           food cup         96.32         98.90         99.66         96.32         97.43         97.06         97.43         98.50           food jar         98.73         100         100         91.46         100         97.15         100         100           garlic         90.91         98.40         98.40         96.26         95.99         91.98         95.99         95.99         95.99         95.99         91.98         95.99         91.35         95.99         91.35         95.99         91.35         95.99         91.35         95.99         91.35         96.91         90.00         99.63         100         100         100         100	comb	96.67	97.33	97.33	96.67	98.00	95.33	98.00	$\boldsymbol{98.00}$
food bag         100         99.18         98.08         99.32         99.32         99.32         50         60         60.32         97.43         97.06         97.43         98.53         99.32         50         60         99.18         98.08         99.32         99.32         50         60         100         100         100         99.16         100         97.15         100         100         100         90.18         98.73         98.53         98.62         95.99         91.98         95.99         96.83	dry battery	86.34	84.14	85.46	76.65	<b>87.67</b>	85.90	<b>88.55</b>	<b>88.55</b>
food box         100         99.84         100         99.18         98.08         99.32         99.32           food can         98.08         93.02         94.39         81.67         99.18         98.08         99.32         99.32           food cup         96.32         98.90         97.06         96.32         97.43         97.06         97.43         98.53           food jar         98.73         100         100         91.46         100         97.15         100         100           garlic         90.91         98.40         98.40         96.26         95.99         91.98         95.99         95.99           glue stick         100         100         100         100         99.68         100         100           greens         76.76         89.73         89.19         85.41         85.41         81.08         90.27         91.35           hand towel         99.63         100         100         100         99.63         100         100           instant noodles         100         99.51         99.76         99.76         100         100         100         100         100         100         100         100         100 </td <td>flashlight</td> <td>97.34</td> <td>97.34</td> <td>97.87</td> <td>95.21</td> <td>100</td> <td>97.34</td> <td>100</td> <td>100</td>	flashlight	97.34	97.34	97.87	95.21	100	97.34	100	100
food can         98.08         93.02         94.39         81.67         99.18         98.08         99.32         99.32           food cup         96.32         98.90         97.06         96.32         97.43         97.06         97.43         98.53           food jar         98.73         100         100         91.46         100         97.15         100         100           garlic         90.91         98.40         98.40         96.26         95.99         91.98         95.99         95.99           glue stick         100         100         100         100         99.68         100         100           greens         76.76         89.73         89.19         85.41         85.41         81.08         90.27         91.35           hand towel         99.63         100         100         100         99.63         100         100           instant noodles         100         99.51         99.76         99.76         100         100         100         100           kleenex         98.87         97.36         96.58         98.87         98.87         98.87         98.87           lemon         45.02         47.41 <t< td=""><td>food bag</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>96.88</td><td>100</td><td>100</td></t<>	food bag	100	100	100	100	100	96.88	100	100
food cup         96.32         98.90         97.06         96.32         97.43         97.06         97.43         98.53           food jar         98.73         100         100         91.46         100         97.15         100         100           garlic         90.91         98.40         98.40         96.26         95.99         91.98         95.99         95.99           glue stick         100         100         100         100         99.68         100         100           greens         76.76         89.73         89.19         85.41         85.41         81.08         90.27         91.35           hand towel         99.63         100         100         100         100         100         100           instant noodles         100         99.51         99.76         99.76         100         100         100         100           keyboard         94.55         95.05         99.76         99.60         100         100         100         100         100         100         100         100         100         100         100         100         100         100         100         100         90.88         78.87         98.	food box	100	99.84	100	99.84	100	100	100	100
food jar         98.73         100         100         91.46         100         97.15         100         100           garlic         90.91         98.40         98.40         96.26         95.99         91.98         95.99         95.99           glue stick         100         100         100         100         99.68         100         100           greens         76.76         89.73         89.19         85.41         85.41         81.08         90.27         91.35           hand towel         99.63         100         100         100         99.63         100         100           instant noodles         100         99.51         99.76         99.76         100         100         100         100           kleenex         98.87         95.05         95.05         94.55         96.04         94.55         96.53         96.53         96.53         88.77         98.87	food can	98.08	93.02	94.39	81.67	99.18	98.08	<b>99.32</b>	<b>99.32</b>
garlic         90.91         98.40         98.40         100         100         100         100         99.68         100         100           greens         76.76         89.73         89.19         85.41         85.41         81.08         90.27         91.35           hand towel         99.63         100         100         100         100         99.63         100         100           instant noodles         100         99.51         99.76         99.76         100         100         100         100           keyboard         94.55         95.05         95.05         94.55         96.04         94.55         96.53         96.53           kleenex         98.87         97.36         97.36         96.98         98.87         98.87         98.87           lemon         45.02         47.41         47.41         45.82         47.81         44.62         47.81         47.81           light bulb         87.67         97.26         96.58         95.21         91.78         89.04         92.47         92.47           lime         67.78         62.22         61.11         58.33         62.78         68.33         62.78         62.22     <	food cup	96.32	98.90	97.06	96.32	97.43	97.06	97.43	<b>98.53</b>
glue stick         100         100         100         100         99.68         100         100           greens         76.76         89.73         89.19         85.41         85.41         81.08         90.27         91.35           hand towel         99.63         100         100         100         99.63         100         100           instant noodles         100         99.51         99.76         99.76         100         100         100         100           keyboard         94.55         95.05         95.05         94.55         96.04         94.55         96.53         96.53           kleenex         98.87         97.36         96.98         98.87         98.87         98.87           lemon         45.02         47.41         47.41         45.82         47.81         44.62         47.81         47.81           lemon         45.02         47.41         47.41         45.82         47.81         44.62         47.81         47.81           lime         67.78         62.22         61.11         58.33         62.78         68.33         62.78         62.22           marker         100         93.93         98.26         <	food jar	98.73	100	100	91.46	100	97.15	100	100
greens         76.76         89.73         89.19         85.41         85.41         81.08         90.27         91.35           hand towel         99.63         100         100         100         100         99.63         100         100           instant noodles         100         99.51         99.76         99.76         100         100         100         100           keyboard         94.55         95.05         94.55         96.04         94.55         96.53         96.53           kleenex         98.87         97.36         96.98         98.87         98.87         98.87         98.87           lemon         45.02         47.41         47.41         45.82         47.81         44.62         47.81         47.81           lime         67.78         62.22         61.11         58.33         62.78         68.33         62.78         62.22           marker         100         93.93         98.26         96.75         99.57         100         99.78         99.57           mushroom         99.42         100         100         100         100         99.42         100         100           notebook         100         10	garlic	90.91	98.40	98.40	<b>96.26</b>	95.99	91.98	95.99	95.99
hand towel         99.63         100 <t< td=""><td>glue stick</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>99.68</td><td>100</td><td>100</td></t<>	glue stick	100	100	100	100	100	99.68	100	100
instant noodles         100         99.51         99.76         99.76         100         100         100         100           keyboard         94.55         95.05         95.05         94.55         96.04         94.55         96.53         96.53           kleenex         98.87         97.36         97.36         96.98         98.87         98.87         98.87         98.87           lemon         45.02         47.41         47.41         45.82         47.81         44.62         47.81         47.81           lime         67.78         62.22         61.11         58.33         62.78         68.33         62.78         62.22           marker         100         93.93         98.26         96.75         99.57         100         99.78         99.57           mushroom         99.42         100         100         100         100         99.42         100         100           notebook         100         100         100         99.62         100         100         100           onion         98.74         94.64         95.90         95.90         96.53         99.37         98.74         98.74           orange         72.46	greens	76.76	89.73	89.19	85.41	85.41	81.08	$\boldsymbol{90.27}$	91.35
keyboard         94.55         95.05         95.05         94.55         96.04         94.55         96.53         96.53           kleenex         98.87         97.36         97.36         96.98         98.87         98.87         98.87           lemon         45.02         47.41         47.41         45.82         47.81         44.62         47.81         47.81           light bulb         87.67         97.26         96.58         95.21         91.78         89.04         92.47         92.47           lime         67.78         62.22         61.11         58.33         62.78         68.33         62.78         62.22           marker         100         93.93         98.26         96.75         99.57         100         99.78         99.57           mushroom         99.42         100         100         100         100         99.42         100         100           notebook         100         100         100         99.62         100         100         100           orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100         10	hand towel	99.63	100	100	100	100	<b>99.63</b>	100	100
Received between the common         98.87         97.36         97.36         96.98         98.87         98.87         98.87         98.87           lemon         45.02         47.41         47.41         45.82         47.81         44.62         47.81         47.81           light bulb         87.67         97.26         96.58         95.21         91.78         89.04         92.47         92.47           lime         67.78         62.22         61.11         58.33         62.78         68.33         62.78         62.22           marker         100         93.93         98.26         96.75         99.57         100         99.78         99.57           mushroom         99.42         100         100         100         100         99.42         100         100           notebook         100         100         100         99.62         100         100         100           orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100         100         100         100         100         100         100         100         100         100 <td< td=""><td>instant noodles</td><td>100</td><td>99.51</td><td>99.76</td><td><b>99.76</b></td><td>100</td><td>100</td><td>100</td><td>100</td></td<>	instant noodles	100	99.51	99.76	<b>99.76</b>	100	100	100	100
lemon         45.02         47.41         47.41         45.82         47.81         44.62         47.81         47.81           light bulb         87.67         97.26         96.58         95.21         91.78         89.04         92.47         92.47           lime         67.78         62.22         61.11         58.33         62.78         68.33         62.78         62.22           marker         100         93.93         98.26         96.75         99.57         100         99.78         99.57           mushroom         99.42         100         100         100         100         99.42         100         100           notebook         100         100         100         99.62         100         100         100           onion         98.74         94.64         95.90         95.90         96.53         99.37         98.74         98.74           orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100         100         100         100         100         100         100         100         100         100         100	keyboard	94.55	95.05	95.05	94.55	96.04	94.55	96.53	96.53
light bulb         87.67         97.26         96.58         95.21         91.78         89.04         92.47         92.47           lime         67.78         62.22         61.11         58.33         62.78         68.33         62.78         62.22           marker         100         93.93         98.26         96.75         99.57         100         99.78         99.57           mushroom         99.42         100         100         100         99.62         100         100         100           notebook         100         100         100         99.62         100         100         100           onion         98.74         94.64         95.90         95.90         96.53         99.37         98.74         98.74           orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100	kleenex	98.87	<b>97.36</b>	97.36	96.98	$\boldsymbol{98.87}$	$\boldsymbol{98.87}$	$\boldsymbol{98.87}$	$\boldsymbol{98.87}$
lime         67.78         62.22         61.11         58.33         62.78         68.33         62.78         62.22           marker         100         93.93         98.26         96.75         99.57         100         99.78         99.57           mushroom         99.42         100         100         100         99.42         100         100           notebook         100         100         100         99.62         100         100         100           onion         98.74         94.64         95.90         95.90         96.53         99.37         98.74         98.74           orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100	lemon	45.02	<b>47.41</b>	47.41	45.82	47.81	44.62	47.81	47.81
marker         100         93.93         98.26         96.75         99.57         100         99.78         99.57           mushroom         99.42         100         100         100         100         99.42         100         100           notebook         100         100         100         99.62         100         100         100           onion         98.74         94.64         95.90         95.90         96.53         99.37         98.74         98.74           orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100	light bulb	87.67	97.26	96.58	95.21	91.78	89.04	92.47	92.47
mushroom         99.42         100         100         100         100         99.42         100         100           notebook         100         100         100         99.62         100         100         100           onion         98.74         94.64         95.90         95.90         96.53         99.37         98.74         98.74           orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100         10	lime	<b>67.78</b>	62.22	61.11	58.33	62.78	<b>68.33</b>	62.78	62.22
notebook         100         100         100         100         99.62         100         100         100           onion         98.74         94.64         95.90         95.90         96.53         99.37         98.74         98.74           orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100         100         100         100         100         100         100         100           pear         90.04         93.24         90.75         90.04         91.81         88.61         91.81         93.95           pitcher         100	marker	100	93.93	98.26	96.75	99.57	100	<b>99.78</b>	99.57
onion         98.74         94.64         95.90         95.90         96.53         99.37         98.74         98.74           orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100         100         100         100         100         100         100         100           pear         90.04         93.24         90.75         90.04         91.81         88.61         91.81         93.95           pitcher         100         <	mushroom	99.42	100	100	100	100	<b>99.42</b>	100	100
orange         72.46         97.58         99.03         98.07         96.62         69.57         99.03         99.03           peach         100         100         100         100         100         100         100         100           pear         90.04         93.24         90.75         90.04         91.81         88.61         91.81         93.95           pitcher         100         100         100         100         100         100         100         100         100         100         100         100         100         100         99.32         99.34         90.32         99.34         90.32         99.34         92.58 </td <td>notebook</td> <td>100</td> <td>100</td> <td>100</td> <td>100</td> <td><b>99.62</b></td> <td>100</td> <td>100</td> <td>100</td>	notebook	100	100	100	100	<b>99.62</b>	100	100	100
peach         100 </td <td>onion</td> <td>98.74</td> <td>94.64</td> <td>95.90</td> <td>95.90</td> <td>96.53</td> <td><b>99.37</b></td> <td><b>98.74</b></td> <td>98.74</td>	onion	98.74	94.64	95.90	95.90	96.53	<b>99.37</b>	<b>98.74</b>	98.74
pear         90.04         93.24         90.75         90.04         91.81         88.61         91.81         93.95           pitcher         100	orange	72.46	97.58	99.03	98.07	96.62	69.57	99.03	99.03
pitcher         100         100         100         100         100         100         100         100         100         100         100         99.32         95.59         100         99.32         99.32         99.32         99.32         99.35         92.58         99.36         90.39         90.39         93.45         92.58         93.45 <t< td=""><td>peach</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td></t<>	peach	100	100	100	100	100	100	100	100
plate         99.66         100         100         99.32         95.59         100         99.32         99.32           pliers         92.58         89.96         90.39         90.39         93.45         92.58         93.45         93.45           potato         63.71         71.81         74.13         65.64         65.25         67.57         71.04         71.43           rubber eraser         69.61         75.49         71.57         69.61         70.59         74.02         70.59         71.08           scissors         100         100         100         100         100         100         100           shampoo         99.35         99.68         99.68         98.39         99.68         94.19         99.68         99.68	pear	90.04	93.24	90.75	90.04	91.81	88.61	91.81	<b>93.95</b>
pliers         92.58         89.96         90.39         90.39         93.45         92.58         93.45         93.45           potato         63.71         71.81         74.13         65.64         65.25         67.57         71.04         71.43           rubber eraser         69.61         75.49         71.57         69.61         70.59         74.02         70.59         71.08           scissors         100         100         100         100         100         100         100           shampoo         99.35         99.68         99.68         98.39         99.68         94.19         99.68         99.68	pitcher	100	100	100	100	100	100	100	100
potato         63.71         71.81         74.13         65.64         65.25         67.57         71.04         71.43           rubber eraser         69.61         75.49         71.57         69.61         70.59         74.02         70.59         71.08           scissors         100         100         100         100         100         100         100           shampoo         99.35         99.68         99.68         98.39         99.68         94.19         99.68         99.68	plate	99.66	100	100	99.32	95.59	100	99.32	99.32
rubber eraser     69.61     75.49     71.57     69.61     70.59     74.02     70.59     71.08       scissors     100     100     100     100     100     100     100     100       shampoo     99.35     99.68     99.68     98.39     99.68     94.19     99.68     99.68	pliers	<b>92.58</b>	89.96	90.39	90.39	93.45	<b>92.58</b>		93.45
scissors         100         100         100         100         100         100         100         100         100         100         100         100         99.68         <	potato	63.71	71.81	74.13	65.64	65.25	67.57	71.04	71.43
shampoo <b>99.35 99.68 99.68</b> 98.39 <b>99.68</b> 94.19 <b>99.68 99.68</b>	rubber eraser	69.61	75.49	71.57	69.61	70.59	<b>74.02</b>	70.59	71.08
	scissors	100	100	100	100	100	100	100	100
soda can <b>100 100 99.55</b> 99.10 <b>100 100 100 100</b>	shampoo	99.35	99.68	99.68	98.39	99.68	94.19	99.68	99.68
	soda can	100	100	99.55	99.10	100	100	100	100

**Table 2.** Continued from Table 1: Accuracy Table Part 2 for the Multi-View RGBD Object Instance recognition with HMP features, the highest and second highest values are colored red and blue, respectively.

Category	SVM	SVM2K	KCCA	KCCA+L	RGCCA	RGCCA+L	RGCCA+AI	DCCA
sponge	99.49	100	100	99.83	99.83	99.83	99.83	99.83
stapler	80.12	73.59	76.56	76.56	81.90	81.90	$\bf 82.20$	82.49
tomato	69.69	81.59	83.85	<b>83.85</b>	80.45	68.56	80.45	80.45
tooth brush	99.48	100	99.48	96.91	98.97	99.48	98.97	98.97
tooth paste	100	100	100	100	100	100	100	100
water bottle	97.32	95.98	95.71	94.64	96.25	96.78	96.25	96.25
average	92.62	93.35	93.87	91.85	93.93	92.40	94.35	94.62

SVM algorithm, However, the advantage of the proposed method is revealed in some of the challenging categories.

Overall, with the HMP-based features, "KCCA+L" and the "RGCCA+L" algorithms cannot match the SVM baseline, and the "SVM2K" and "KCCA" algorithms are only marginally better than the baseline. The "RGCCA" and "RGCCA+AL" algorithms offer some improvements, while the proposed DCCA algorithm achieves the highest overall recognition accuracy. Considering there are more than 13,000 testing samples, the two percent performance improvement means correctly classifying an additional amount of more than 200 samples.

#### References

- 1. Lai, K., Bo, L., Ren, X., Fox, D.: A large-scale hierarchical multi-view rgb-d object dataset. In: Robotics and Automation (ICRA), 2011 IEEE International Conference on, IEEE (2011) 1817–1824
- Tenenhaus, A., Tenenhaus, M.: Regularized generalized canonical correlation analysis. Psychometrika 76 (2011) 257–284
- 3. Tenenhaus, A.: Kernel generalized canonical correlation analysis. Actes des 42è journées de Statistique (2010)
- 4. Hanafi, M.: Pls path modelling: computation of latent variables with the estimation mode b. Computational Statistics **22** (2007) 275–292
- 5. Bo, L., Ren, X., Fox, D.: Unsupervised feature learning for rgb-d based object recognition. ISER, June (2012)