chapter 18

Indexing Structures for Files

In this chapter we assume that a file already exists with some primary organization such as the unordered, ordered, or hashed organizations that were described in Chapter 17. We will describe additional auxiliary **access structures** called **indexes**, which are used to speed up the retrieval of records in response to certain search conditions. The index structures are additional files on disk that provide **secondary access paths**, which provide alternative ways to access the records without affecting the physical placement of records in the primary data file on disk. They enable efficient access to records based on the **indexing fields** that are used to construct the index. Basically, *any field* of the file can be used to create an index, and *multiple indexes* on different fields—as well as indexes on *multiple fields*—can be constructed on the same file. A variety of indexes are possible; each of them uses a particular data structure to speed up the search. To find a record or records in the data file based on a search condition on an indexing field, the index is searched, which leads to pointers to one or more disk blocks in the data file where the required records are located. The most prevalent types of indexes are based on ordered files (single-level indexes) and tree data structures (multilevel indexes, B^+-trees). Indexes can also be constructed based on hashing or other search data structures. We also discuss indexes that are vectors of bits called *bitmap indexes*.

We describe different types of single-level ordered indexes—primary, secondary, and clustering—in Section 18.1. By viewing a single-level index as an ordered file, one can develop additional indexes for it, giving rise to the concept of multilevel indexes. A popular indexing scheme called **ISAM (Indexed Sequential Access Method)** is based on this idea. We discuss multilevel tree-structured indexes in Section 18.2. In Section 18.3 we describe B-trees and B^+-trees, which are data structures that are commonly used in DBMSs to implement dynamically changing multilevel indexes. B^+-trees have become a commonly accepted default structure for

generating indexes on demand in most relational DBMSs. Section 18.4 is devoted to alternative ways to access data based on a combination of multiple keys. In Section 18.5 we discuss hash indexes and introduce the concept of logical indexes, which give an additional level of indirection from physical indexes, allowing for the physical index to be flexible and extensible in its organization. In Section 18.6 we discuss multikey indexing and bitmap indexes used for searching on one or more keys. Section 18.7 summarizes the chapter.

18.1 Types of Single-Level Ordered Indexes

The idea behind an ordered index is similar to that behind the index used in a textbook, which lists important terms at the end of the book in alphabetical order along with a list of page numbers where the term appears in the book. We can search the book index for a certain term in the textbook to find a list of *addresses*—page numbers in this case—and use these addresses to locate the specified pages first and then *search* for the term on each specified page. The alternative, if no other guidance is given, would be to sift slowly through the whole textbook word by word to find the term we are interested in; this corresponds to doing a *linear search*, which scans the whole file. Of course, most books do have additional information, such as chapter and section titles, which help us find a term without having to search through the whole book. However, the index is the only exact indication of the pages where each term occurs in the book.

For a file with a given record structure consisting of several fields (or attributes), an index access structure is usually defined on a single field of a file, called an **indexing field** (or **indexing attribute**).[1] The index typically stores each value of the index field along with a list of pointers to all disk blocks that contain records with that field value. The values in the index are *ordered* so that we can do a *binary search* on the index. If both the data file and the index file are ordered, and since the index file is typically much smaller than the data file, searching the index using a binary search is a better option. Tree-structured multilevel indexes (see Section 18.2) implement an extension of the binary search idea that reduces the search space by 2-way partitioning at each search step, thereby creating a more efficient approach that divides the search space in the file *n*-ways at each stage.

There are several types of ordered indexes. A **primary index** is specified on the *ordering key field* of an **ordered file** of records. Recall from Section 17.7 that an ordering key field is used to *physically order* the file records on disk, and every record has a *unique value* for that field. If the ordering field is not a key field—that is, if numerous records in the file can have the same value for the ordering field—another type of index, called a **clustering index**, can be used. The data file is called a **clustered file** in this latter case. Notice that a file can have at most one physical ordering field, so it can have at most one primary index or one clustering index, *but not both*. A third type of index, called a **secondary index**, can be specified on any

[1]We use the terms *field* and *attribute* interchangeably in this chapter.

nonordering field of a file. A data file can have several secondary indexes in addition to its primary access method. We discuss these types of single-level indexes in the next three subsections.

18.1.1 Primary Indexes

A **primary index** is an ordered file whose records are of fixed length with two fields, and it acts like an access structure to efficiently search for and access the data records in a data file. The first field is of the same data type as the ordering key field—called the **primary key**—of the data file, and the second field is a pointer to a disk block (a block address). There is one **index entry** (or **index record**) in the index file for each *block* in the data file. Each index entry has the value of the primary key field for the *first* record in a block and a pointer to that block as its two field values. We will refer to the two field values of index entry i as $<K(i), P(i)>$.

To create a primary index on the ordered file shown in Figure 17.7, we use the Name field as primary key, because that is the ordering key field of the file (assuming that each value of Name is unique). Each entry in the index has a Name value and a pointer. The first three index entries are as follows:

$<K(1) = (\text{Aaron, Ed}), P(1) = \text{address of block 1}>$

$<K(2) = (\text{Adams, John}), P(2) = \text{address of block 2}>$

$<K(3) = (\text{Alexander, Ed}), P(3) = \text{address of block 3}>$

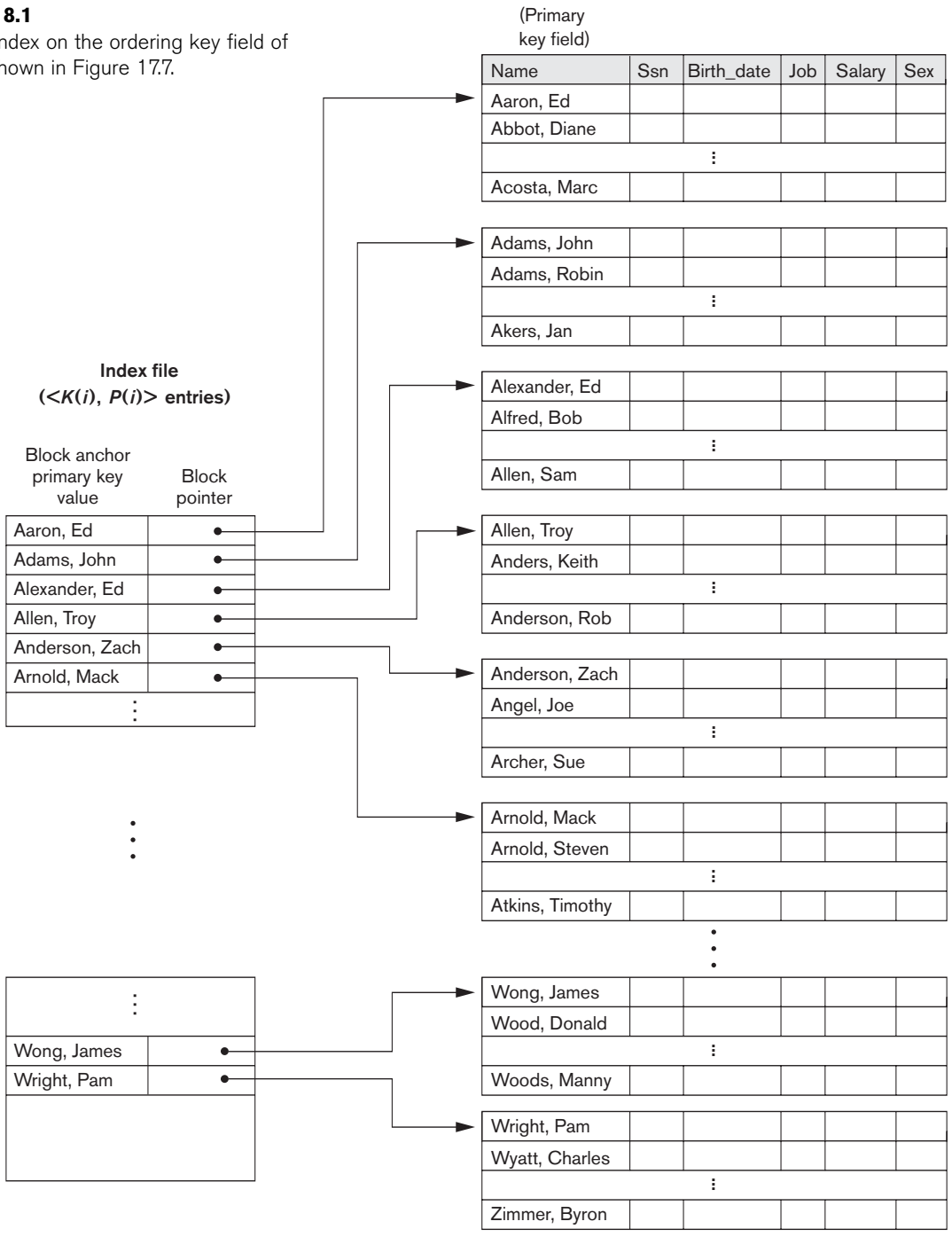
Figure 18.1 illustrates this primary index. The total number of entries in the index is the same as the *number of disk blocks* in the ordered data file. The first record in each block of the data file is called the **anchor record** of the block, or simply the **block anchor**.[2]

Indexes can also be characterized as dense or sparse. A **dense index** has an index entry for *every search key value* (and hence every record) in the data file. A **sparse** (or **nondense**) **index**, on the other hand, has index entries for only some of the search values. A sparse index has fewer entries than the number of records in the file. Thus, a primary index is a nondense (sparse) index, since it includes an entry for each disk block of the data file and the keys of its anchor record rather than for every search value (or every record).

The index file for a primary index occupies a much smaller space than does the data file, for two reasons. First, there are *fewer index entries* than there are records in the data file. Second, each index entry is typically *smaller in size* than a data record because it has only two fields; consequently, more index entries than data records can fit in one block. Therefore, a binary search on the index file requires fewer block accesses than a binary search on the data file. Referring to Table 17.2, note that the binary search for an ordered data file required $\log_2 b$ block accesses. But if the primary index file contains only b_i blocks, then to locate a record with a search key

[2]We can use a scheme similar to the one described here, with the last record in each block (rather than the first) as the block anchor. This slightly improves the efficiency of the search algorithm.

Figure 18.1
Primary index on the ordering key field of the file shown in Figure 17.7.



value requires a binary search of that index and access to the block containing that record: a total of $\log_2 b_i + 1$ accesses.

A record whose primary key value is K lies in the block whose address is $P(i)$, where $K(i) \leq K < K(i+1)$. The ith block in the data file contains all such records because of the physical ordering of the file records on the primary key field. To retrieve a record, given the value K of its primary key field, we do a binary search on the index file to find the appropriate index entry i, and then retrieve the data file block whose address is $P(i)$.[3] Example 1 illustrates the saving in block accesses that is attainable when a primary index is used to search for a record.

Example 1. Suppose that we have an ordered file with $r = 30{,}000$ records stored on a disk with block size $B = 1024$ bytes. File records are of fixed size and are unspanned, with record length $R = 100$ bytes. The blocking factor for the file would be $bfr = \lfloor (B/R) \rfloor = \lfloor (1024/100) \rfloor = 10$ records per block. The number of blocks needed for the file is $b = \lceil (r/bfr) \rceil = \lceil (30000/10) \rceil = 3000$ blocks. A binary search on the data file would need approximately $\lceil \log_2 b \rceil = \lceil (\log_2 3000) \rceil = 12$ block accesses.

Now suppose that the ordering key field of the file is $V = 9$ bytes long, a block pointer is $P = 6$ bytes long, and we have constructed a primary index for the file. The size of each index entry is $R_i = (9 + 6) = 15$ bytes, so the blocking factor for the index is $bfr_i = \lfloor (B/R_i) \rfloor = \lfloor (1024/15) \rfloor = 68$ entries per block. The total number of index entries r_i is equal to the number of blocks in the data file, which is 3000. The number of index blocks is hence $b_i = \lceil (r_i/bfr_i) \rceil = \lceil (3000/68) \rceil = 45$ blocks. To perform a binary search on the index file would need $\lceil (\log_2 b_i) \rceil = \lceil (\log_2 45) \rceil = 6$ block accesses. To search for a record using the index, we need one additional block access to the data file for a total of $6 + 1 = 7$ block accesses—an improvement over binary search on the data file, which required 12 disk block accesses.

A major problem with a primary index—as with any ordered file—is insertion and deletion of records. With a primary index, the problem is compounded because if we attempt to insert a record in its correct position in the data file, we must not only move records to make space for the new record but also change some index entries, since moving records will change the *anchor records* of some blocks. Using an unordered overflow file, as discussed in Section 17.7, can reduce this problem. Another possibility is to use a linked list of overflow records for each block in the data file. This is similar to the method of dealing with overflow records described with hashing in Section 17.8.2. Records within each block and its overflow linked list can be sorted to improve retrieval time. Record deletion is handled using deletion markers.

18.1.2 Clustering Indexes

If file records are physically ordered on a nonkey field—which *does not* have a distinct value for each record—that field is called the **clustering field** and the data file

[3]Notice that the above formula would not be correct if the data file were ordered on a *nonkey field*; in that case the same index value in the block anchor could be repeated in the last records of the previous block.

is called a **clustered file.** We can create a different type of index, called a **clustering index**, to speed up retrieval of all the records that have the same value for the clustering field. This differs from a primary index, which requires that the ordering field of the data file have a *distinct value* for each record.

A clustering index is also an ordered file with two fields; the first field is of the same type as the clustering field of the data file, and the second field is a disk block pointer. There is one entry in the clustering index for each *distinct value* of the clustering field, and it contains the value and a pointer to the *first block* in the data file that has a record with that value for its clustering field. Figure 18.2 shows an example. Notice that record insertion and deletion still cause problems because the data records are physically ordered. To alleviate the problem of insertion, it is common to reserve a whole block (or a cluster of contiguous blocks) for *each value* of the clustering field; all records with that value are placed in the block (or block cluster). This makes insertion and deletion relatively straightforward. Figure 18.3 shows this scheme.

A clustering index is another example of a *nondense* index because it has an entry for every *distinct value* of the indexing field, which is a nonkey by definition and hence has duplicate values rather than a unique value for every record in the file. There is some similarity between Figures 18.1, 18.2, and 18.3 and Figures 17.11 and 17.12. An index is somewhat similar to dynamic hashing (described in Section 17.8.3) and to the directory structures used for extendible hashing. Both are searched to find a pointer to the data block containing the desired record. A main difference is that an index search uses the values of the search field itself, whereas a hash directory search uses the binary hash value that is calculated by applying the hash function to the search field.

18.1.3 Secondary Indexes

A **secondary index** provides a secondary means of accessing a data file for which some primary access already exists. The data file records could be ordered, unordered, or hashed. The secondary index may be created on a field that is a candidate key and has a unique value in every record, or on a nonkey field with duplicate values. The index is again an ordered file with two fields. The first field is of the same data type as some *nonordering field* of the data file that is an **indexing field**. The second field is either a *block* pointer or a *record* pointer. *Many* secondary indexes (and hence, indexing fields) can be created for the same file—each represents an additional means of accessing that file based on some specific field.

First we consider a secondary index access structure on a key (unique) field that has a *distinct value* for every record. Such a field is sometimes called a **secondary** key; in the relational model, this would correspond to any UNIQUE key attribute or to the primary key attribute of a table. In this case there is one index entry for *each record* in the data file, which contains the value of the field for the record and a pointer either to the block in which the record is stored or to the record itself. Hence, such an index is **dense.**

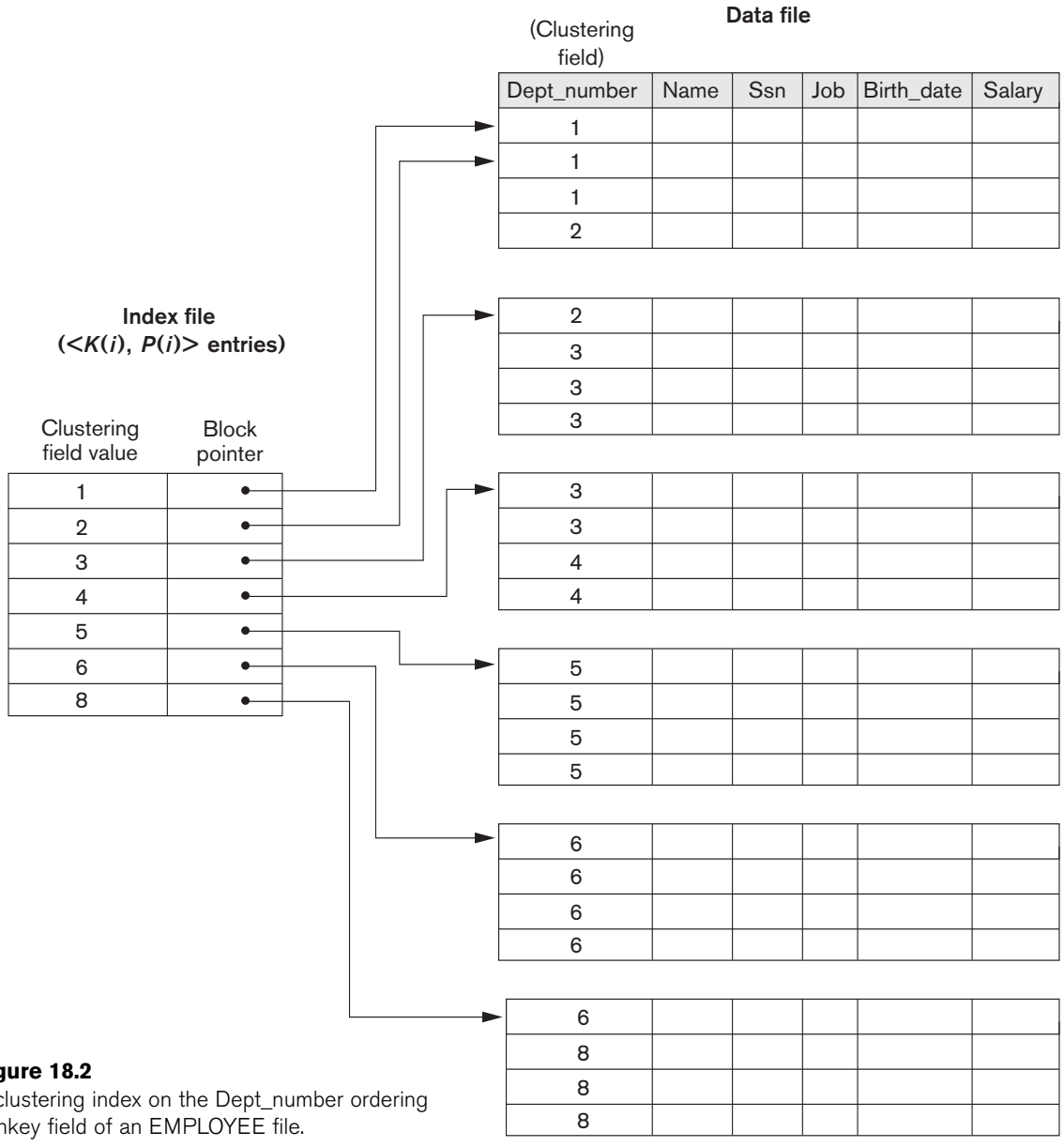


Figure 18.2
A clustering index on the Dept_number ordering nonkey field of an EMPLOYEE file.

Again we refer to the two field values of index entry i as $<K(i), P(i)>$. The entries are **ordered** by value of $K(i)$, so we can perform a binary search. Because the records of the data file are *not* physically ordered by values of the secondary key field, we *cannot* use block anchors. That is why an index entry is created for each record in the data

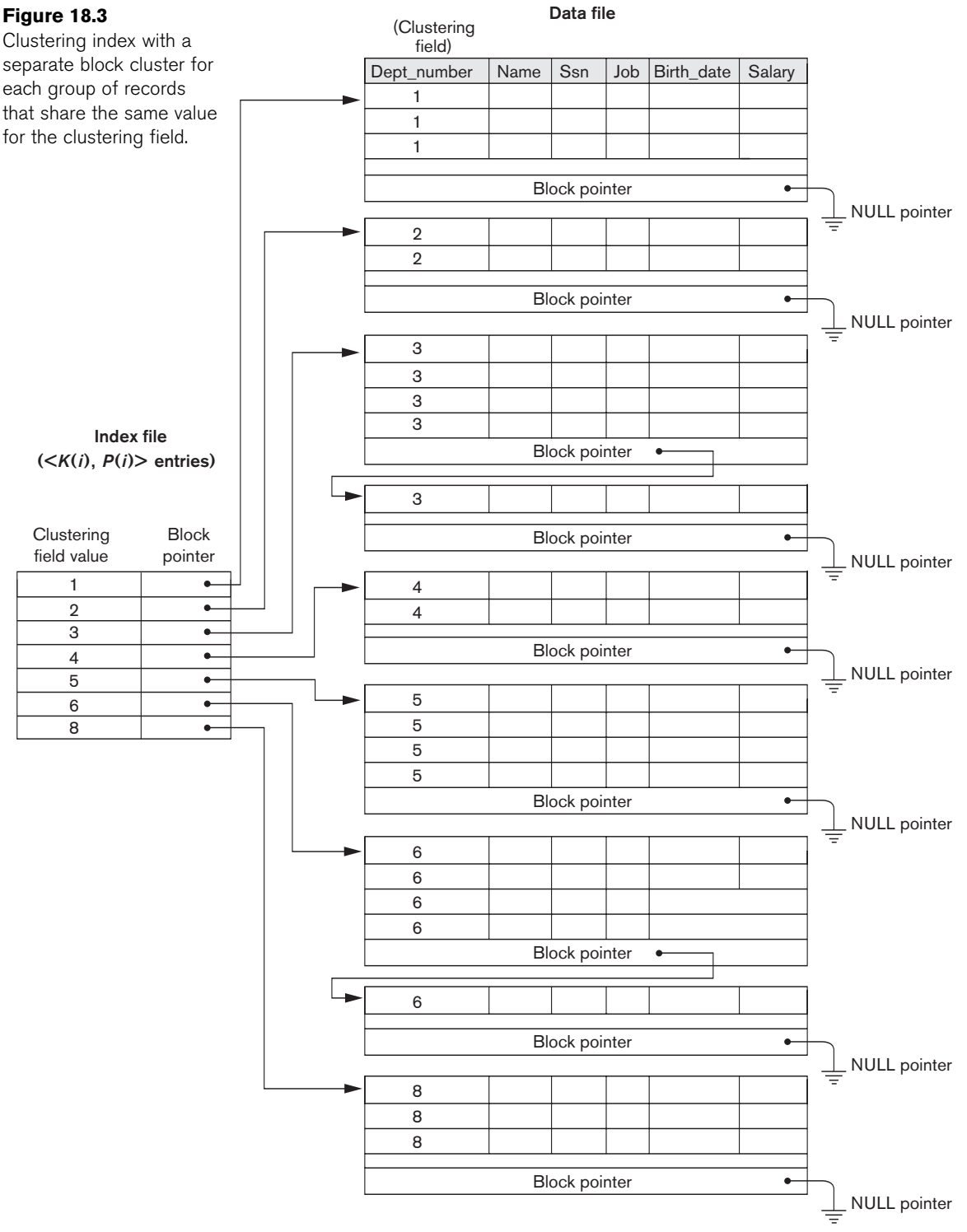
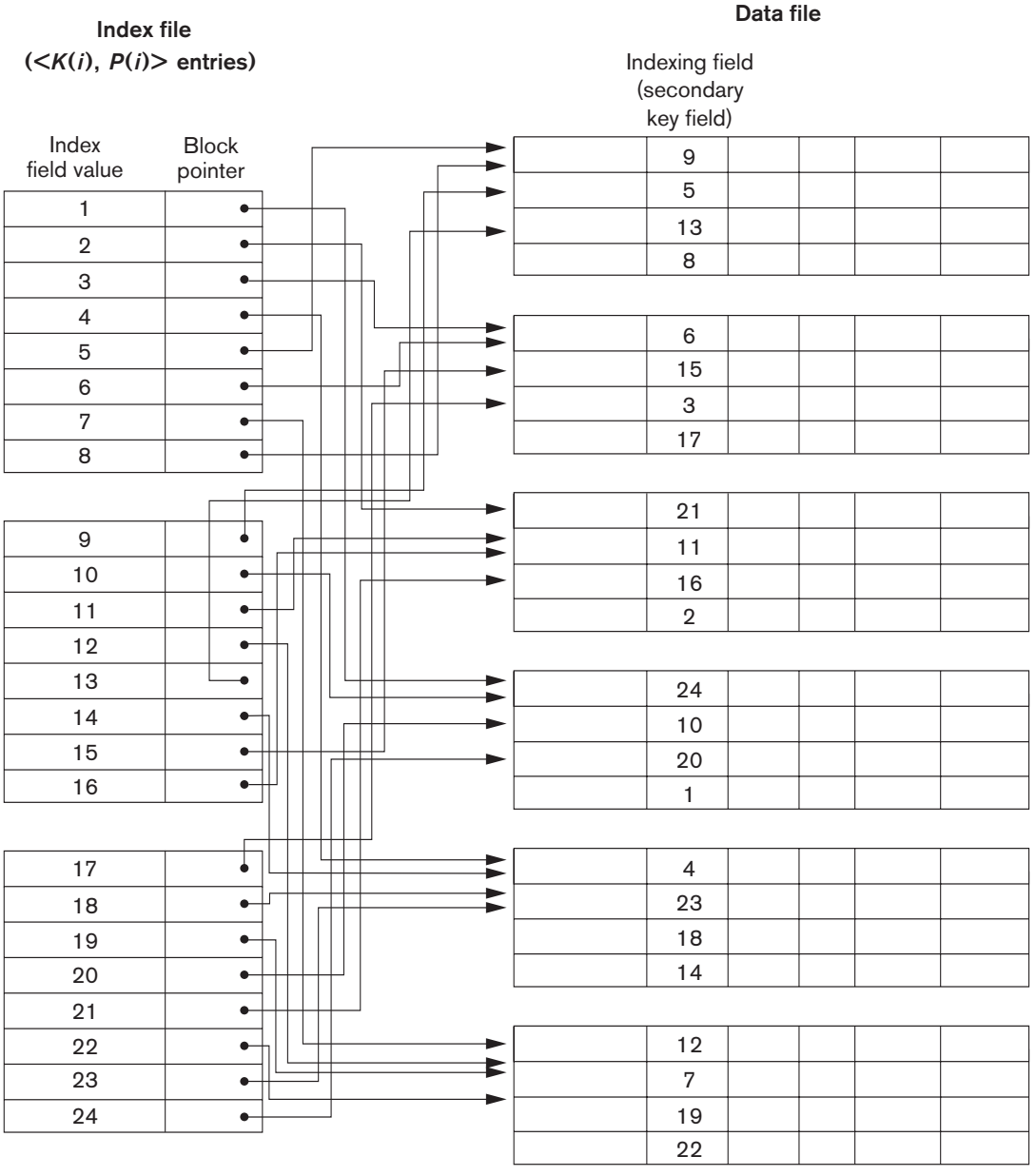


Figure 18.3
Clustering index with a separate block cluster for each group of records that share the same value for the clustering field.

file, rather than for each block, as in the case of a primary index. Figure 18.4 illustrates a secondary index in which the pointers $P(i)$ in the index entries are *block pointers*, not record pointers. Once the appropriate disk block is transferred to a main memory buffer, a search for the desired record within the block can be carried out.

Figure 18.4
A dense secondary index (with block pointers) on a nonordering key field of a file.



A secondary index usually needs more storage space and longer search time than does a primary index, because of its larger number of entries. However, the *improvement* in search time for an arbitrary record is much greater for a secondary index than for a primary index, since we would have to do a *linear search* on the data file if the secondary index did not exist. For a primary index, we could still use a binary search on the main file, even if the index did not exist. Example 2 illustrates the improvement in number of blocks accessed.

Example 2. Consider the file of Example 1 with $r = 30{,}000$ fixed-length records of size $R = 100$ bytes stored on a disk with block size $B = 1024$ bytes. The file has $b = 3000$ blocks, as calculated in Example 1. Suppose we want to search for a record with a specific value for the secondary key—a nonordering key field of the file that is $V = 9$ bytes long. Without the secondary index, to do a linear search on the file would require $b/2 = 3000/2 = 1500$ block accesses on the average. Suppose that we construct a secondary index on that *nonordering key* field of the file. As in Example 1, a block pointer is $P = 6$ bytes long, so each index entry is $R_i = (9 + 6) = 15$ bytes, and the blocking factor for the index is $bfr_i = \lfloor (B/R_i) \rfloor = \lfloor (1024/15) \rfloor = 68$ entries per block. In a dense secondary index such as this, the total number of index entries r_i is equal to the *number of records* in the data file, which is 30,000. The number of blocks needed for the index is hence $b_i = \lceil (r_i/bfr_i) \rceil = \lceil (3000/68) \rceil = 442$ blocks.

A binary search on this secondary index needs $\lceil (\log_2 b_i) \rceil = \lceil (\log_2 442) \rceil = 9$ block accesses. To search for a record using the index, we need an additional block access to the data file for a total of $9 + 1 = 10$ block accesses—a vast improvement over the 1500 block accesses needed on the average for a linear search, but slightly worse than the 7 block accesses required for the primary index. This difference arose because the primary index was nondense and hence shorter, with only 45 blocks in length.

We can also create a secondary index on a *nonkey, nonordering field* of a file. In this case, numerous records in the data file can have the same value for the indexing field. There are several options for implementing such an index:

- Option 1 is to include duplicate index entries with the same $K(i)$ value—one for each record. This would be a dense index.
- Option 2 is to have variable-length records for the index entries, with a repeating field for the pointer. We keep a list of pointers $<P(i, 1), \ldots, P(i, k)>$ in the index entry for $K(i)$—one pointer to each block that contains a record whose indexing field value equals $K(i)$. In either option 1 or option 2, the binary search algorithm on the index must be modified appropriately to account for a variable number of index entries per index key value.
- Option 3, which is more commonly used, is to keep the index entries themselves at a fixed length and have a single entry for each *index field value*, but to create *an extra level of indirection* to handle the multiple pointers. In this nondense scheme, the pointer $P(i)$ in index entry $<K(i), P(i)>$ points to a disk block, which contains a *set of record pointers*; each record pointer in that disk block points to one of the data file records with value $K(i)$ for the indexing field. If some value $K(i)$ occurs in too many records, so that their record pointers cannot fit in a single disk block, a cluster or linked list of blocks is

used. This technique is illustrated in Figure 18.5. Retrieval via the index requires one or more additional block accesses because of the extra level, but the algorithms for searching the index and (more importantly) for inserting of new records in the data file are straightforward. In addition, retrievals on complex selection conditions may be handled by referring to the record pointers, without having to retrieve many unnecessary records from the data file (see Exercise 18.23).

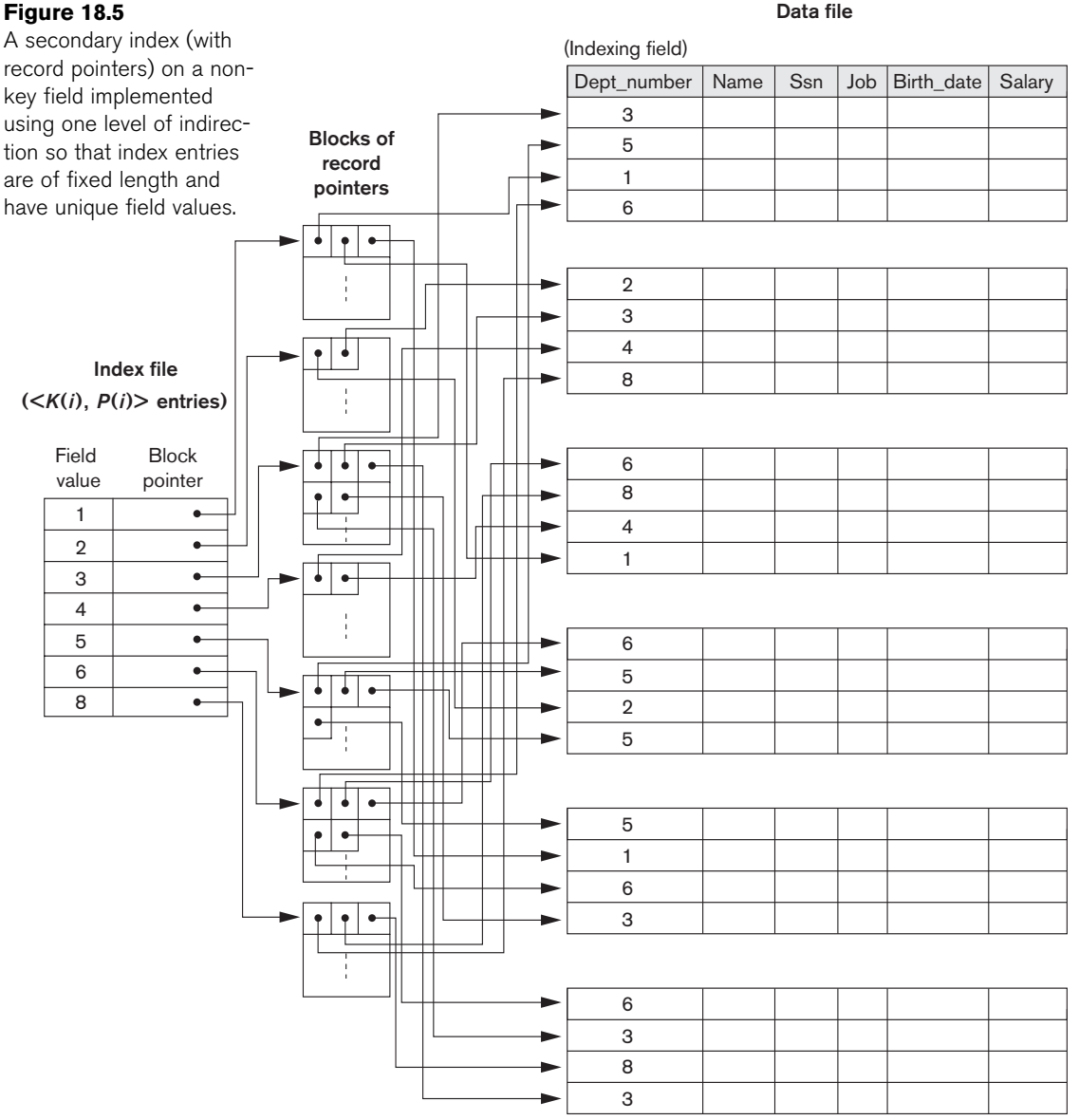


Figure 18.5
A secondary index (with record pointers) on a non-key field implemented using one level of indirection so that index entries are of fixed length and have unique field values.

Notice that a secondary index provides a **logical ordering** on the records by the indexing field. If we access the records in order of the entries in the secondary index, we get them in order of the indexing field. The primary and clustering indexes assume that the field used for **physical ordering** of records in the file is the same as the indexing field.

18.1.4 Summary

To conclude this section, we summarize the discussion of index types in two tables. Table 18.1 shows the index field characteristics of each type of ordered single-level index discussed—primary, clustering, and secondary. Table 18.2 summarizes the properties of each type of index by comparing the number of index entries and specifying which indexes are dense and which use block anchors of the data file.

Table 18.1 Types of Indexes Based on the Properties of the Indexing Field

	Index Field Used for Physical Ordering of the File	Index Field Not Used for Physical Ordering of the File
Indexing field is key	Primary index	Secondary index (Key)
Indexing field is nonkey	Clustering index	Secondary index (NonKey)

Table 18.2 Properties of Index Types

Type of Index	Number of (First-level) Index Entries	Dense or Nondense (Sparse)	Block Anchoring on the Data File
Primary	Number of blocks in data file	Nondense	Yes
Clustering	Number of distinct index field values	Nondense	Yes/no[a]
Secondary (key)	Number of records in data file	Dense	No
Secondary (nonkey)	Number of records[b] or number of distinct index field values[c]	Dense or Nondense	No

[a]Yes if every distinct value of the ordering field starts a new block; no otherwise.
[b]For option 1.
[c]For options 2 and 3.

18.2 Multilevel Indexes

The indexing schemes we have described thus far involve an ordered index file. A binary search is applied to the index to locate pointers to a disk block or to a record (or records) in the file having a specific index field value. A binary search requires approximately ($\log_2 b_i$) block accesses for an index with b_i blocks because each step of the algorithm reduces the part of the index file that we continue to search by a factor of 2. This is why we take the log function to the base 2. The idea behind a **multilevel index** is to reduce the part of the index that we continue to search by bfr_i, the blocking factor for the index, which is larger than 2. Hence, the search space is reduced much faster. The value bfr_i is called the **fan-out** of the multilevel index, and we will refer to it by the symbol ***fo***. Whereas we divide the *record search space* into two halves at each step during a binary search, we divide it n-ways (where n = the fan-out) at each search step using the multilevel index. Searching a multilevel index requires approximately ($\log_{fo} b_i$) block accesses, which is a substantially smaller number than for a binary search if the fan-out is larger than 2. In most cases, the fan-out is much larger than 2.

A multilevel index considers the index file, which we will now refer to as the **first** (or **base**) **level** of a multilevel index, as an *ordered file* with a *distinct value* for each $K(i)$. Therefore, by considering the first-level index file as a sorted data file, we can create a primary index for the first level; this index to the first level is called the **second level** of the multilevel index. Because the second level is a primary index, we can use block anchors so that the second level has one entry for *each block* of the first level. The blocking factor bfr_i for the second level—and for all subsequent levels—is the same as that for the first-level index because all index entries are the same size; each has one field value and one block address. If the first level has r_1 entries, and the blocking factor—which is also the fan-out—for the index is $bfr_i = fo$, then the first level needs $\lceil (r_1/fo) \rceil$ blocks, which is therefore the number of entries r_2 needed at the second level of the index.

We can repeat this process for the second level. The **third level**, which is a primary index for the second level, has an entry for each second-level block, so the number of third-level entries is $r_3 = \lceil (r_2/fo) \rceil$. Notice that we require a second level only if the first level needs more than one block of disk storage, and, similarly, we require a third level only if the second level needs more than one block. We can repeat the preceding process until all the entries of some index level t fit in a single block. This block at the tth level is called the **top** index level.[4] Each level reduces the number of entries at the previous level by a factor of fo—the index fan-out—so we can use the formula $1 \leq (r_1/((fo)^t))$ to calculate t. Hence, a multilevel index with r_1 first-level entries will have approximately t levels, where $t = \lceil (\log_{fo}(r_1)) \rceil$. When searching the

[4]The numbering scheme for index levels used here is the reverse of the way levels are commonly defined for tree data structures. In tree data structures, t is referred to as level 0 (zero), $t - 1$ is level 1, and so on.

index, a single disk block is retrieved at each level. Hence, t disk blocks are accessed for an index search, where t is the *number of index levels*.

The multilevel scheme described here can be used on any type of index—whether it is primary, clustering, or secondary—as long as the first-level index has *distinct values for K(i) and fixed-length entries.* Figure 18.6 shows a multilevel index built over a primary index. Example 3 illustrates the improvement in number of blocks accessed when a multilevel index is used to search for a record.

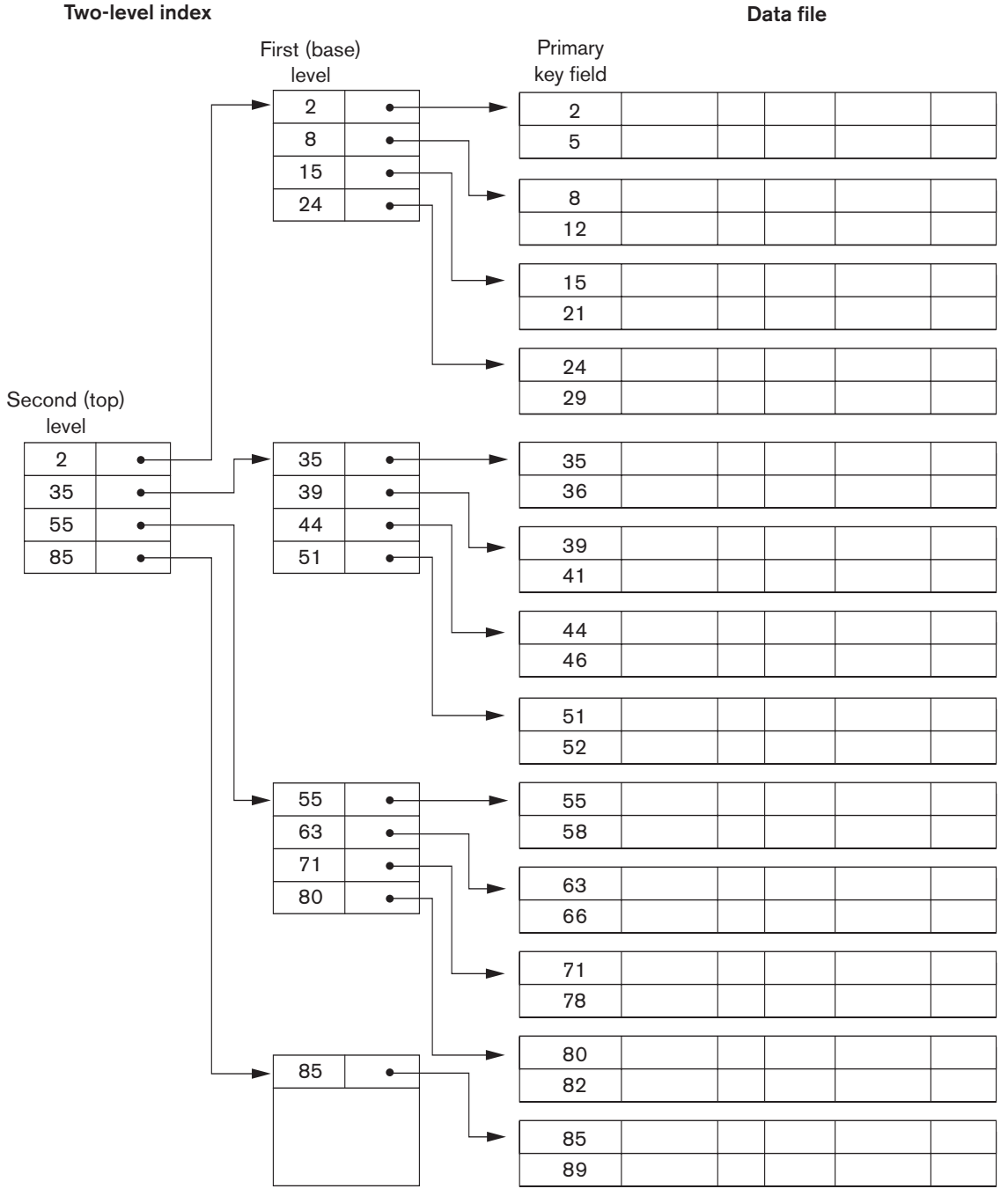
Example 3. Suppose that the dense secondary index of Example 2 is converted into a multilevel index. We calculated the index blocking factor $bfr_i = 68$ index entries per block, which is also the fan-out fo for the multilevel index; the number of first-level blocks $b_1 = 442$ blocks was also calculated. The number of second-level blocks will be $b_2 = \lceil (b_1/fo) \rceil = \lceil (442/68) \rceil = 7$ blocks, and the number of third-level blocks will be $b_3 = \lceil (b_2/fo) \rceil = \lceil (7/68) \rceil = 1$ block. Hence, the third level is the top level of the index, and $t = 3$. To access a record by searching the multilevel index, we must access one block at each level plus one block from the data file, so we need $t + 1 = 3 + 1 = 4$ block accesses. Compare this to Example 2, where 10 block accesses were needed when a single-level index and binary search were used.

Notice that we could also have a multilevel primary index, which would be nondense. Exercise 18.18(c) illustrates this case, where we *must* access the data block from the file before we can determine whether the record being searched for is in the file. For a dense index, this can be determined by accessing the first index level (without having to access a data block), since there is an index entry for *every* record in the file.

A common file organization used in business data processing is an ordered file with a multilevel primary index on its ordering key field. Such an organization is called an **indexed sequential file** and was used in a large number of early IBM systems. IBM's **ISAM** organization incorporates a two-level index that is closely related to the organization of the disk in terms of cylinders and tracks (see Section 17.2.1). The first level is a cylinder index, which has the key value of an anchor record for each cylinder of a disk pack occupied by the file and a pointer to the track index for the cylinder. The track index has the key value of an anchor record for each track in the cylinder and a pointer to the track. The track can then be searched sequentially for the desired record or block. Insertion is handled by some form of overflow file that is merged periodically with the data file. The index is recreated during file reorganization.

Algorithm 18.1 outlines the search procedure for a record in a data file that uses a nondense multilevel primary index with t levels. We refer to entry i at level j of the index as $<K_j(i), P_j(i)>$, and we search for a record whose primary key value is K. We assume that any overflow records are ignored. If the record is in the file, there must be some entry at level 1 with $K_1(i) \leq K < K_1(i + 1)$ and the record will be in the block of the data file whose address is $P_1(i)$. Exercise 18.23 discusses modifying the search algorithm for other types of indexes.

Figure 18.6
A two-level primary index resembling ISAM (Indexed Sequential Access Method) organization.



Algorithm 18.1. Searching a Nondense Multilevel Primary Index with t Levels

```
(* We assume the index entry to be a block anchor that is the first key per block. *)
p ← address of top-level block of index;
for j ← t step – 1 to 1 do
    begin
        read the index block (at jth index level) whose address is p;
        search block p for entry i such that K_j (i) ≤ K < K_j(i + 1)
     (* if K_j(i)
        is the last entry in the block, it is sufficient to satisfy K_j(i) ≤ K *);
        p ← P_j(i ) (* picks appropriate pointer at jth index level *)
    end;
    read the data file block whose address is p;
    search block p for record with key = K;
```

As we have seen, a multilevel index reduces the number of blocks accessed when searching for a record, given its indexing field value. We are still faced with the problems of dealing with index insertions and deletions, because all index levels are *physically ordered files*. To retain the benefits of using multilevel indexing while reducing index insertion and deletion problems, designers adopted a multilevel index called a **dynamic multilevel index** that leaves some space in each of its blocks for inserting new entries and uses appropriate insertion/deletion algorithms for creating and deleting new index blocks when the data file grows and shrinks. It is often implemented by using data structures called B-trees and B^+-trees, which we describe in the next section.

18.3 Dynamic Multilevel Indexes Using B-Trees and B^+-Trees

B-trees and B^+-trees are special cases of the well-known search data structure known as a **tree**. We briefly introduce the terminology used in discussing tree data structures. A **tree** is formed of **nodes**. Each node in the tree, except for a special node called the **root**, has one **parent** node and zero or more **child** nodes. The root node has no parent. A node that does not have any child nodes is called a **leaf** node; a nonleaf node is called an **internal** node. The **level** of a node is always one more than the level of its parent, with the level of the root node being *zero*.[5] A **subtree** of a node consists of that node and all its **descendant** nodes—its child nodes, the child nodes of its child nodes, and so on. A precise recursive definition of a subtree is that it consists of a node n and the subtrees of all the child nodes of n. Figure 18.7 illustrates a tree data structure. In this figure the root node is A, and its child nodes are B, C, and D. Nodes E, J, C, G, H, and K are leaf nodes. Since the leaf nodes are at different levels of the tree, this tree is called **unbalanced**.

[5]This standard definition of the level of a tree node, which we use throughout Section 18.3, is different from the one we gave for multilevel indexes in Section 18.2.

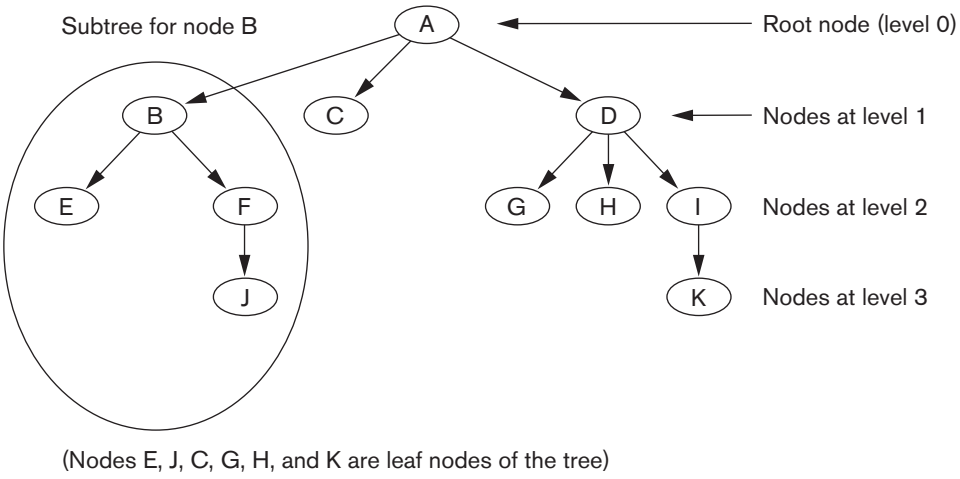


Figure 18.7
A tree data structure that shows an unbalanced tree.

In Section 18.3.1, we introduce search trees and then discuss B-trees, which can be used as dynamic multilevel indexes to guide the search for records in a data file. B-tree nodes are kept between 50 and 100 percent full, and pointers to the data blocks are stored in both internal nodes and leaf nodes of the B-tree structure. In Section 18.3.2 we discuss B+-trees, a variation of B-trees in which pointers to the data blocks of a file are stored only in leaf nodes, which can lead to fewer levels and higher-capacity indexes. In the DBMSs prevalent in the market today, the common structure used for indexing is B+-trees.

18.3.1 Search Trees and B-Trees

A **search tree** is a special type of tree that is used to guide the search for a record, given the value of one of the record's fields. The multilevel indexes discussed in Section 18.2 can be thought of as a variation of a search tree; each node in the multilevel index can have as many as *fo* pointers and *fo* key values, where *fo* is the index fan-out. The index field values in each node guide us to the next node, until we reach the data file block that contains the required records. By following a pointer, we restrict our search at each level to a subtree of the search tree and ignore all nodes not in this subtree.

Search Trees. A search tree is slightly different from a multilevel index. A **search tree of order** p is a tree such that each node contains *at most* $p - 1$ search values and p pointers in the order $<P_1, K_1, P_2, K_2, \ldots, P_{q-1}, K_{q-1}, P_q>$, where $q \leq p$. Each P_i is a pointer to a child node (or a NULL pointer), and each K_i is a search value from some

ordered set of values. All search values are assumed to be unique.[6] Figure 18.8 illustrates a node in a search tree. Two constraints must hold at all times on the search tree:

1. Within each node, $K_1 < K_2 < \ldots < K_{q-1}$.
2. For all values X in the subtree pointed at by P_i, we have $K_{i-1} < X < K_i$ for $1 < i < q$; $X < K_i$ for $i = 1$; and $K_{i-1} < X$ for $i = q$ (see Figure 18.8).

Whenever we search for a value X, we follow the appropriate pointer P_i according to the formulas in condition 2 above. Figure 18.9 illustrates a search tree of order $p = 3$ and integer search values. Notice that some of the pointers P_i in a node may be NULL pointers.

We can use a search tree as a mechanism to search for records stored in a disk file. The values in the tree can be the values of one of the fields of the file, called the **search field** (which is the same as the index field if a multilevel index guides the search). Each key value in the tree is associated with a pointer to the record in the data file having that value. Alternatively, the pointer could be to the disk block containing that record. The search tree itself can be stored on disk by assigning each tree node to a disk block. When a new record is inserted in the file, we must update the search tree by inserting an entry in the tree containing the search field value of the new record and a pointer to the new record.

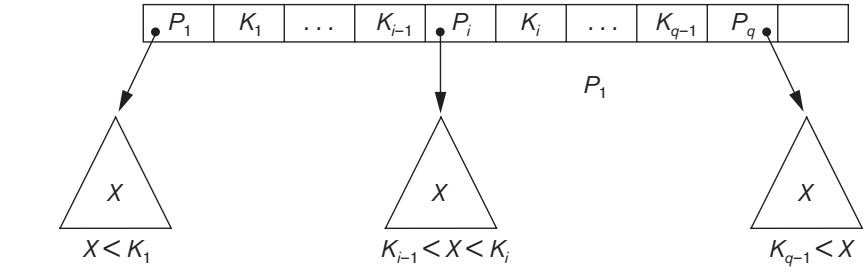


Figure 18.8
A node in a search tree with pointers to subtrees below it.

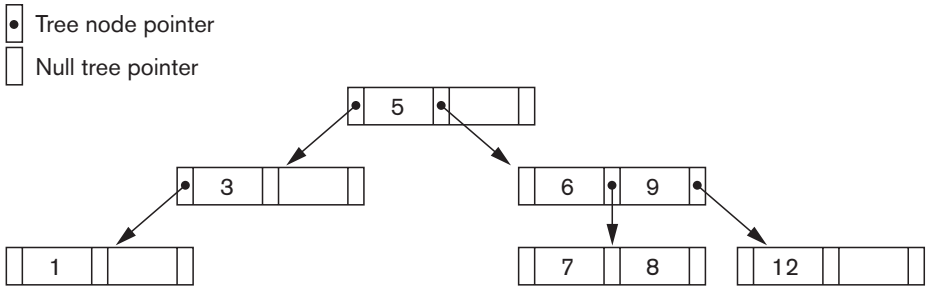


Figure 18.9
A search tree of order $p = 3$.

[6]This restriction can be relaxed. If the index is on a nonkey field, duplicate search values may exist and the node structure and the navigation rules for the tree may be modified.

Algorithms are necessary for inserting and deleting search values into and from the search tree while maintaining the preceding two constraints. In general, these algorithms do not guarantee that a search tree is **balanced**, meaning that all of its leaf nodes are at the same level.[7] The tree in Figure 18.7 is not balanced because it has leaf nodes at levels 1, 2, and 3. The goals for balancing a search tree are as follows:

- To guarantee that nodes are evenly distributed, so that the depth of the tree is minimized for the given set of keys and that the tree does not get skewed with some nodes being at very deep levels
- To make the search speed uniform, so that the average time to find any random key is roughly the same

While minimizing the number of levels in the tree is one goal, another implicit goal is to make sure that the index tree does not need too much restructuring as records are inserted into and deleted from the main file. Thus we want the nodes to be as full as possible and do not want any nodes to be empty if there are too many deletions. Record deletion may leave some nodes in the tree nearly empty, thus wasting storage space and increasing the number of levels. The B-tree addresses both of these problems by specifying additional constraints on the search tree.

B-Trees. The B-tree has additional constraints that ensure that the tree is always balanced and that the space wasted by deletion, if any, never becomes excessive. The algorithms for insertion and deletion, though, become more complex in order to maintain these constraints. Nonetheless, most insertions and deletions are simple processes; they become complicated only under special circumstances—namely, whenever we attempt an insertion into a node that is already full or a deletion from a node that makes it less than half full. More formally, a **B-tree of order *p***, when used as an access structure on a *key field* to search for records in a data file, can be defined as follows:

1. Each internal node in the B-tree (Figure 18.10(a)) is of the form
   $$<P_1, <K_1, Pr_1>, P_2, <K_2, Pr_2>, ..., <K_{q-1}, Pr_{q-1}>, P_q>$$
   where $q \leq p$. Each P_i is a **tree pointer**—a pointer to another node in the B-tree. Each Pr_i is a **data pointer**[8]—a pointer to the record whose search key field value is equal to K_i (or to the data file block containing that record).
2. Within each node, $K_1 < K_2 < ... < K_{q-1}$.
3. For all search key field values X in the subtree pointed at by P_i (the ith subtree, see Figure 18.10(a)), we have:
   $$K_{i-1} < X < K_i \text{ for } 1 < i < q; X < K_i \text{ for } i = 1; \text{ and } K_{i-1} < X \text{ for } i = q.$$
4. Each node has at most p tree pointers.

[7]The definition of *balanced* is different for binary trees. Balanced binary trees are known as *AVL trees*.

[8]A data pointer is either a block address or a record address; the latter is essentially a block address and a record offset within the block.

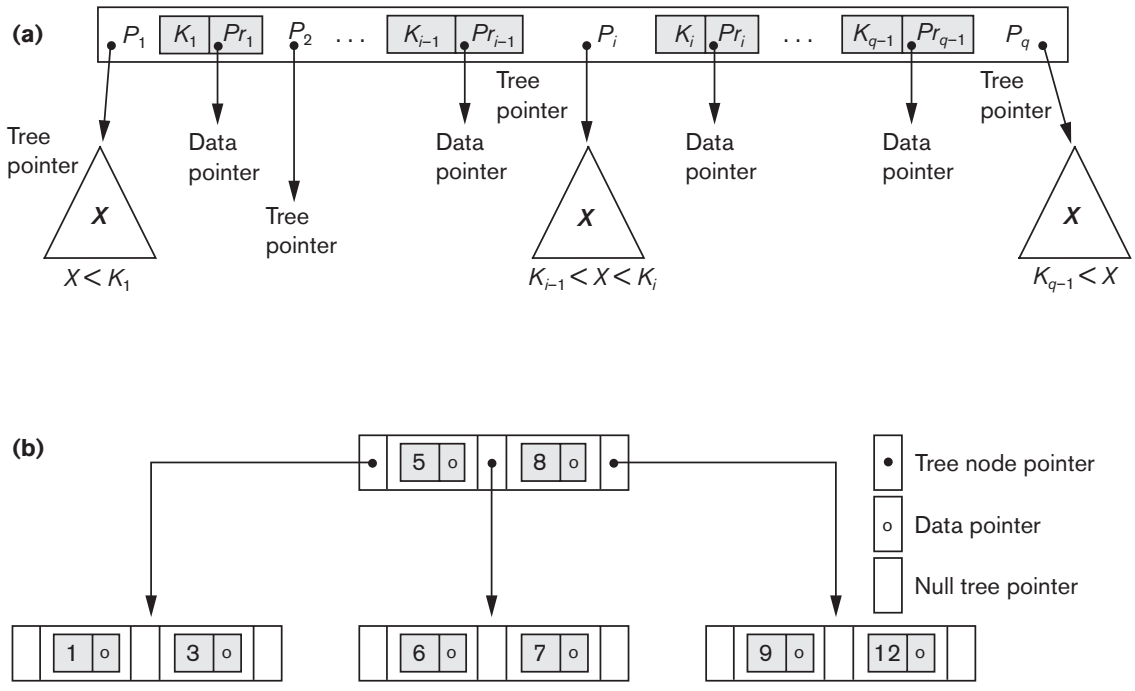


Figure 18.10
B-tree structures. (a) A node in a B-tree with $q - 1$ search values. (b) A B-tree of order $p = 3$.The values were inserted in the order 8, 5, 1, 7, 3, 12, 9, 6.

5. Each node, except the root and leaf nodes, has at least $\lceil (p/2) \rceil$ tree pointers. The root node has at least two tree pointers unless it is the only node in the tree.
6. A node with q tree pointers, $q \leq p$, has $q - 1$ search key field values (and hence has $q - 1$ data pointers).
7. All leaf nodes are at the same level. Leaf nodes have the same structure as internal nodes except that all of their *tree pointers* P_i are NULL.

Figure 18.10(b) illustrates a B-tree of order $p = 3$. Notice that all search values K in the B-tree are unique because we assumed that the tree is used as an access structure on a key field. If we use a B-tree *on a nonkey field*, we must change the definition of the file pointers Pr_i to point to a block—or a cluster of blocks—that contain the pointers to the file records. This extra level of indirection is similar to option 3, discussed in Section 18.1.3, for secondary indexes.

A B-tree starts with a single root node (which is also a leaf node) at level 0 (zero). Once the root node is full with $p - 1$ search key values and we attempt to insert another entry in the tree, the root node splits into two nodes at level 1. Only the middle value is kept in the root node, and the rest of the values are split evenly

between the other two nodes. When a nonroot node is full and a new entry is inserted into it, that node is split into two nodes at the same level, and the middle entry is moved to the parent node along with two pointers to the new split nodes. If the parent node is full, it is also split. Splitting can propagate all the way to the root node, creating a new level if the root is split. We do not discuss algorithms for B-trees in detail in this book,[9] but we outline search and insertion procedures for B^+-trees in the next section.

If deletion of a value causes a node to be less than half full, it is combined with its neighboring nodes, and this can also propagate all the way to the root. Hence, deletion can reduce the number of tree levels. It has been shown by analysis and simulation that, after numerous random insertions and deletions on a B-tree, the nodes are approximately 69 percent full when the number of values in the tree stabilizes. This is also true of B^+-trees. If this happens, node splitting and combining will occur only rarely, so insertion and deletion become quite efficient. If the number of values grows, the tree will expand without a problem—although splitting of nodes may occur, so some insertions will take more time. Each B-tree node can have *at most p* tree pointers, $p - 1$ data pointers, and $p - 1$ search key field values (see Figure 18.10(a)).

In general, a B-tree node may contain additional information needed by the algorithms that manipulate the tree, such as the number of entries q in the node and a pointer to the parent node. Next, we illustrate how to calculate the number of blocks and levels for a B-tree.

Example 4. Suppose that the search field is a nonordering key field, and we construct a B-tree on this field with $p = 23$. Assume that each node of the B-tree is 69 percent full. Each node, on the average, will have $p * 0.69 = 23 * 0.69$ or approximately 16 pointers and, hence, 15 search key field values. The **average fan-out** $fo = 16$. We can start at the root and see how many values and pointers can exist, on the average, at each subsequent level:

Root:	1 node	15 key entries	16 pointers
Level 1:	16 nodes	240 key entries	256 pointers
Level 2:	256 nodes	3840 key entries	4096 pointers
Level 3:	4096 nodes	61,440 key entries	

At each level, we calculated the number of key entries by multiplying the total number of pointers at the previous level by 15, the average number of entries in each node. Hence, for the given block size, pointer size, and search key field size, a two-level B-tree holds 3840 + 240 + 15 = 4095 entries on the average; a three-level B-tree holds 65,535 entries on the average.

B-trees are sometimes used as **primary file organizations**. In this case, *whole records* are stored within the B-tree nodes rather than just the <search key, record pointer> entries. This works well for files with a relatively *small number of records* and a *small*

[9]For details on insertion and deletion algorithms for B-trees, consult Ramakrishnan and Gehrke [2003].

record size. Otherwise, the fan-out and the number of levels become too great to permit efficient access.

In summary, B-trees provide a multilevel access structure that is a balanced tree structure in which each node is at least half full. Each node in a B-tree of order p can have at most $p - 1$ search values.

18.3.2 B+-Trees

Most implementations of a dynamic multilevel index use a variation of the B-tree data structure called a **B+-tree**. In a B-tree, every value of the search field appears once at some level in the tree, along with a data pointer. In a B+-tree, data pointers are stored *only at the leaf nodes* of the tree; hence, the structure of leaf nodes differs from the structure of internal nodes. The leaf nodes have an entry for *every* value of the search field, along with a data pointer to the record (or to the block that contains this record) if the search field is a key field. For a nonkey search field, the pointer points to a block containing pointers to the data file records, creating an extra level of indirection.

The leaf nodes of the B+-tree are usually linked to provide ordered access on the search field to the records. These leaf nodes are similar to the first (base) level of an index. Internal nodes of the B+-tree correspond to the other levels of a multilevel index. Some search field values from the leaf nodes are *repeated* in the internal nodes of the B+-tree to guide the search. The structure of the *internal nodes* of a B+-tree of order p (Figure 18.11(a)) is as follows:

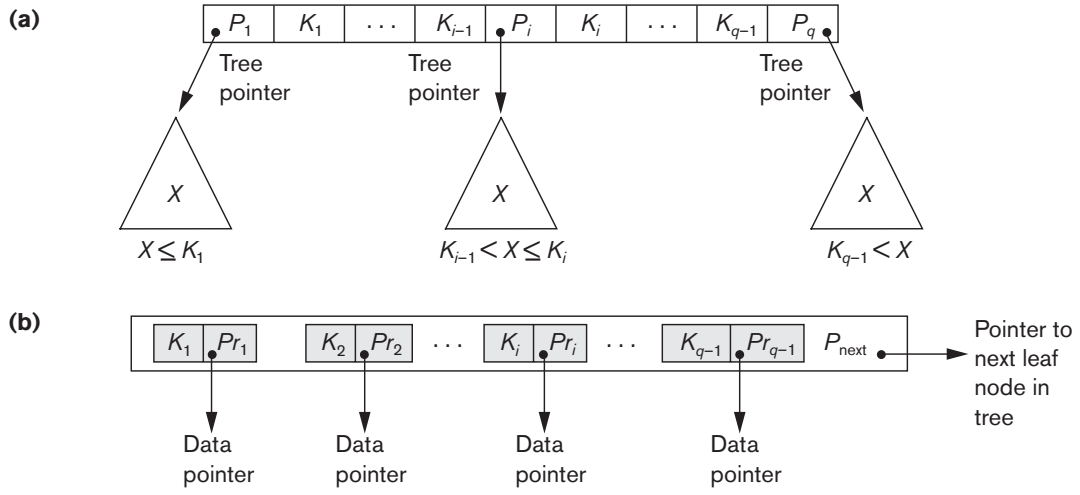
1. Each internal node is of the form
   $$<P_1, K_1, P_2, K_2, ..., P_{q-1}, K_{q-1}, P_q>$$
   where $q \leq p$ and each P_i is a **tree pointer**.
2. Within each internal node, $K_1 < K_2 < ... < K_{q-1}$.
3. For all search field values X in the subtree pointed at by P_i, we have $K_{i-1} < X \leq K_i$ for $1 < i < q$; $X \leq K_i$ for $i = 1$; and $K_{i-1} < X$ for $i = q$ (see Figure 18.11(a)).[10]
4. Each internal node has at most p tree pointers.
5. Each internal node, except the root, has at least $\lceil (p/2) \rceil$ tree pointers. The root node has at least two tree pointers if it is an internal node.
6. An internal node with q pointers, $q \leq p$, has $q - 1$ search field values.

The structure of the *leaf nodes* of a B+-tree of order p (Figure 18.11(b)) is as follows:

1. Each leaf node is of the form
   $$<<K_1, Pr_1>, <K_2, Pr_2>, ..., <K_{q-1}, Pr_{q-1}>, P_{next}>$$
   where $q \leq p$, each Pr_i is a data pointer, and P_{next} points to the next *leaf node* of the B+-tree.

[10]Our definition follows Knuth (1998). One can define a B+-tree differently by exchanging the $<$ and $\leq$ symbols ($K_{i-1} \leq X < K_i$; $K_{q-1} \leq X$), but the principles remain the same.

Figure 18.11
The nodes of a B+-tree. (a) Internal node of a B+-tree with $q - 1$ search values. (b) Leaf node of a B+-tree with $q - 1$ search values and $q - 1$ data pointers.



2. Within each leaf node, $K_1 \leq K_2 \ldots, K_{q-1}, q \leq p$.
3. Each Pr_i is a **data pointer** that points to the record whose search field value is K_i or to a file block containing the record (or to a block of record pointers that point to records whose search field value is K_i if the search field is not a key).
4. Each leaf node has at least $\lceil (p/2) \rceil$ values.
5. All leaf nodes are at the same level.

The pointers in internal nodes are *tree pointers* to blocks that are tree nodes, whereas the pointers in leaf nodes are *data pointers* to the data file records or blocks—except for the P_{next} pointer, which is a tree pointer to the next leaf node. By starting at the leftmost leaf node, it is possible to traverse leaf nodes as a linked list, using the P_{next} pointers. This provides ordered access to the data records on the indexing field. A P_{previous} pointer can also be included. For a B+-tree on a nonkey field, an extra level of indirection is needed similar to the one shown in Figure 18.5, so the *Pr* pointers are block pointers to blocks that contain a set of record pointers to the actual records in the data file, as discussed in option 3 of Section 18.1.3.

Because entries in the *internal nodes* of a B+-tree include search values and tree pointers without any data pointers, more entries can be packed into an internal node of a B+-tree than for a similar B-tree. Thus, for the same block (node) size, the order p will be larger for the B+-tree than for the B-tree, as we illustrate in Example 5. This can lead to fewer B+-tree levels, improving search time. Because the structures for internal and for leaf nodes of a B+-tree are different, the order p can be different. We

will use p to denote the order for *internal nodes* and p_{leaf} to denote the order for *leaf nodes*, which we define as being the maximum number of data pointers in a leaf node.

Example 5. To calculate the order p of a B^+-tree, suppose that the search key field is $V = 9$ bytes long, the block size is $B = 512$ bytes, a record pointer is $Pr = 7$ bytes, and a block pointer is $P = 6$ bytes. An internal node of the B^+-tree can have up to p tree pointers and $p - 1$ search field values; these must fit into a single block. Hence, we have:

$$(p * P) + ((p - 1) * V) \leq B$$
$$(P * 6) + ((P - 1) * 9) \leq 512$$
$$(15 * p) \leq 521$$

We can choose p to be the largest value satisfying the above inequality, which gives $p = 34$. This is larger than the value of 23 for the B-tree (it is left to the reader to compute the order of the B-tree assuming same size pointers), resulting in a larger fan-out and more entries in each internal node of a B^+-tree than in the corresponding B-tree. The leaf nodes of the B^+-tree will have the same number of values and pointers, except that the pointers are data pointers and a next pointer. Hence, the order p_{leaf} for the leaf nodes can be calculated as follows:

$$(p_{leaf} * (Pr + V)) + P \leq B$$
$$(p_{leaf} * (7 + 9)) + 6 \leq 512$$
$$(16 * p_{leaf}) \leq 506$$

It follows that each leaf node can hold up to $p_{leaf} = 31$ key value/data pointer combinations, assuming that the data pointers are record pointers.

As with the B-tree, we may need additional information—to implement the insertion and deletion algorithms—in each node. This information can include the type of node (internal or leaf), the number of current entries q in the node, and pointers to the parent and sibling nodes. Hence, before we do the above calculations for p and p_{leaf}, we should reduce the block size by the amount of space needed for all such information. The next example illustrates how we can calculate the number of entries in a B^+-tree.

Example 6. Suppose that we construct a B^+-tree on the field in Example 5. To calculate the approximate number of entries in the B^+-tree, we assume that each node is 69 percent full. On the average, each internal node will have $34 * 0.69$ or approximately 23 pointers, and hence 22 values. Each leaf node, on the average, will hold $0.69 * p_{leaf} = 0.69 * 31$ or approximately 21 data record pointers. A B^+-tree will have the following average number of entries at each level:

Root:	1 node	22 key entries	23 pointers
Level 1:	23 nodes	506 key entries	529 pointers
Level 2:	529 nodes	11,638 key entries	12,167 pointers
Leaf level:	12,167 nodes	255,507 data record pointers	

For the block size, pointer size, and search field size given above, a three-level B+-tree holds up to 255,507 record pointers, with the average 69 percent occupancy of nodes. Compare this to the 65,535 entries for the corresponding B-tree in Example 4. This is the main reason that B+-trees are preferred to B-trees as indexes to database files.

Search, Insertion, and Deletion with B+-Trees. Algorithm 18.2 outlines the procedure using the B+-tree as the access structure to search for a record. Algorithm 18.3 illustrates the procedure for inserting a record in a file with a B+-tree access structure. These algorithms assume the existence of a key search field, and they must be modified appropriately for the case of a B+-tree on a nonkey field. We illustrate insertion and deletion with an example.

Algorithm 18.2. Searching for a Record with Search Key Field Value K, Using a B+-tree

```
n ← block containing root node of B+-tree;
read block n;
while (n is not a leaf node of the B+-tree) do
   begin
   q ← number of tree pointers in node n;
   if K ≤ n.K1 (*n.Ki refers to the ith search field value in node n*)
      then n ← n.P1 (*n.Pi refers to the ith tree pointer in node n*)
      else if K > n.Kq–1
         then n ← n.Pq
         else begin
            search node n for an entry i such that n.Ki–1 < K ≤ n.Ki;
            n ← n.Pi
            end;
   read block n
   end;
search block n for entry (Ki, Pri) with K = Ki; (* search leaf node *)
if found
   then read data file block with address Pri and retrieve record
   else the record with search field value K is not in the data file;
```

Algorithm 18.3. Inserting a Record with Search Key Field Value K in a B+-tree of Order p

```
n ← block containing root node of B+-tree;
read block n; set stack S to empty;
while (n is not a leaf node of the B+-tree) do
   begin
   push address of n on stack S;
      (*stack S holds parent nodes that are needed in case of split*)
   q ← number of tree pointers in node n;
   if K ≤ n.K1 (*n.Ki refers to the ith search field value in node n*)
```

```
        then n ← n.P1 (*n.Pi refers to the ith tree pointer in node n*)
        else if K > n.Kq–1
            then n ← n.Pq
            else begin
                search node n for an entry i such that n.Ki–1 < K ≤ n.Ki;
                n ← n.Pi
                end;
        read block n
    end;
search block n for entry (Ki,Pri) with K = Ki; (*search leaf node n*)
if found
    then record already in file; cannot insert
    else (*insert entry in B+-tree to point to record*)
        begin
        create entry (K, Pr) where Pr points to the new record;
        if leaf node n is not full
            then insert entry (K, Pr) in correct position in leaf node n
            else begin (*leaf node n is full with pleaf record pointers; is split*)
                copy n to temp (*temp is an oversize leaf node to hold extra
                                    entries*);
                insert entry (K, Pr) in temp in correct position;
                (*temp now holds pleaf + 1 entries of the form (Ki, Pri)*)
                new ← a new empty leaf node for the tree; new.Pnext ← n.Pnext ;
                j ← ⌈(pleaf + 1)/2⌉;
                n ← first j entries in temp (up to entry (Kj, Prj)); n.Pnext ← new;
                new ← remaining entries in temp; K ← Kj ;
                (*now we must move (K, new) and insert in parent internal node;
                  however, if parent is full, split may propagate*)
                finished ← false;
                repeat
                if stack S is empty
                    then (*no parent node; new root node is created for the tree*)
                        begin
                        root ← a new empty internal node for the tree;
                        root ← <n, K, new>; finished ← true;
                        end
                    else begin
                        n ← pop stack S;
                        if internal node n is not full
                            then
                                begin (*parent node not full; no split*)
                                insert (K, new) in correct position in internal node n;
                                finished ← true
                                end
                            else begin (*internal node n is full with p tree pointers;
                                        overflow condition; node is split*)
```

```
                copy n to temp (*temp is an oversize internal node*);
                insert (K, new) in temp in correct position;
                (*temp now has p + 1 tree pointers*)
                new ← a new empty internal node for the tree;
                j ← ⌊((p + 1)/2)⌋;
                n ← entries up to tree pointer P_j in temp;
                (*n contains <P_1, K_1, P_2, K_2, ..., P_{j-1}, K_{j-1}, P_j >*)
                new ← entries from tree pointer P_{j+1} in temp;
                (*new contains < P_{j+1}, K_{j+1}, ..., K_{p-1}, P_p, K_p, P_{p+1} >*)
                K ← K_j
                (*now we must move (K, new) and insert in parent
                  internal node*)
                end
            end
        until finished
        end;
end;
```

Figure 18.12 illustrates insertion of records in a B+-tree of order $p = 3$ and $p_{leaf} = 2$. First, we observe that the root is the only node in the tree, so it is also a leaf node. As soon as more than one level is created, the tree is divided into internal nodes and leaf nodes. Notice that *every key value must exist at the leaf level*, because all data pointers are at the leaf level. However, only some values exist in internal nodes to guide the search. Notice also that every value appearing in an internal node also appears as *the rightmost value* in the leaf level of the subtree pointed at by the tree pointer to the left of the value.

When a *leaf node* is full and a new entry is inserted there, the node *overflows* and must be split. The first $j = \lceil ((p_{leaf} + 1)/2) \rceil$ entries in the original node are kept there, and the remaining entries are moved to a new leaf node. The jth search value is replicated in the parent internal node, and an extra pointer to the new node is created in the parent. These must be inserted in the parent node in their correct sequence. If the parent internal node is full, the new value will cause it to overflow also, so it must be split. The entries in the internal node up to P_j—the jth tree pointer after inserting the new value and pointer, where $j = \lfloor ((p + 1)/2) \rfloor$—are kept, while the jth search value is moved to the parent, not replicated. A new internal node will hold the entries from P_{j+1} to the end of the entries in the node (see Algorithm 18.3). This splitting can propagate all the way up to create a new root node and hence a new level for the B+-tree.

Figure 18.13 illustrates deletion from a B+-tree. When an entry is deleted, it is always removed from the leaf level. If it happens to occur in an internal node, it must also be removed from there. In the latter case, the value to its left in the leaf node must replace it in the internal node because that value is now the rightmost entry in the subtree. Deletion may cause **underflow** by reducing the number of entries in the leaf node to below the minimum required. In this case, we try to find a sibling leaf node—a leaf node directly to the left or to the right of the node with underflow—

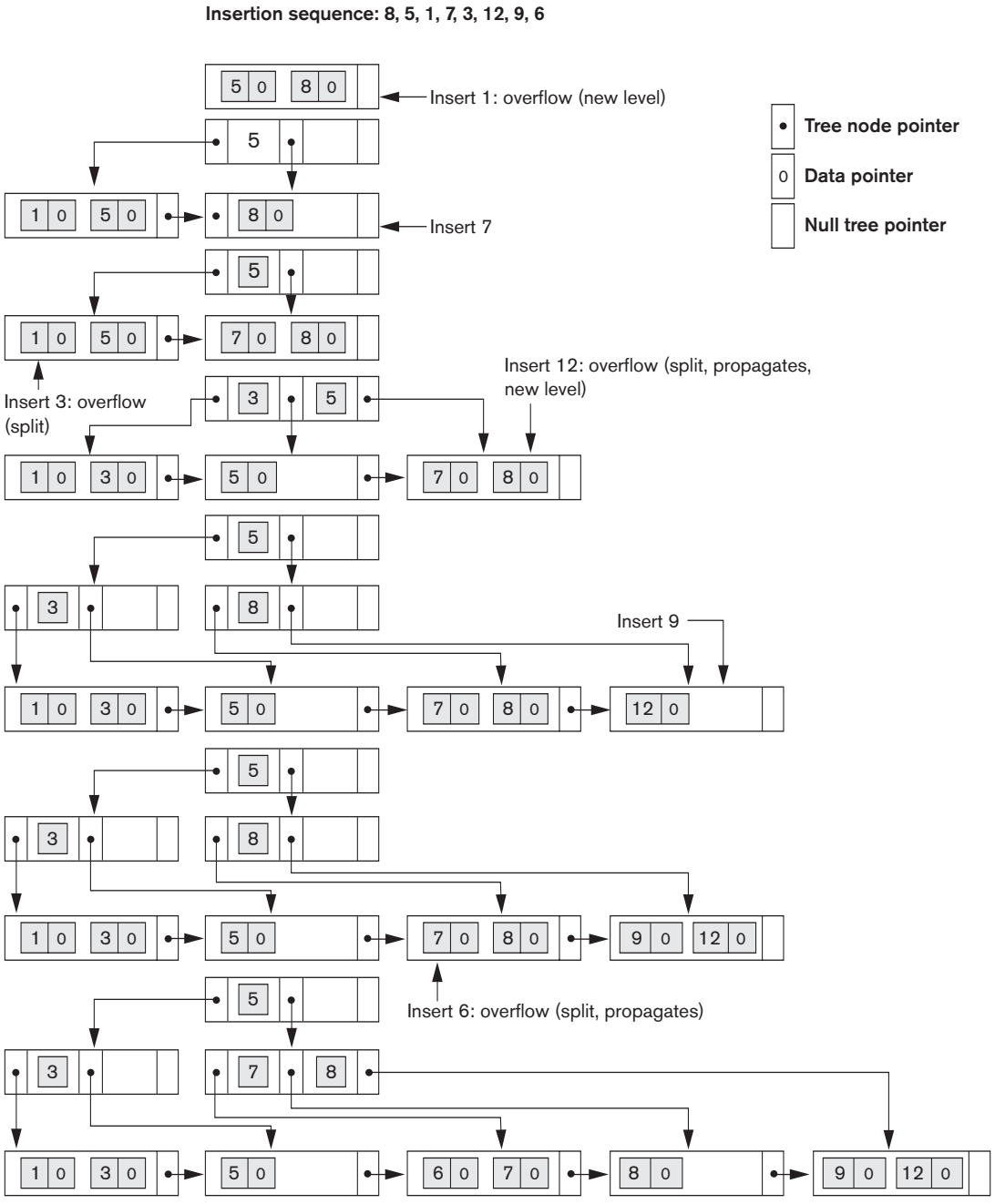


Figure 18.12
An example of insertion in a B$^+$-tree with $p = 3$ and $p_{leaf} = 2$.

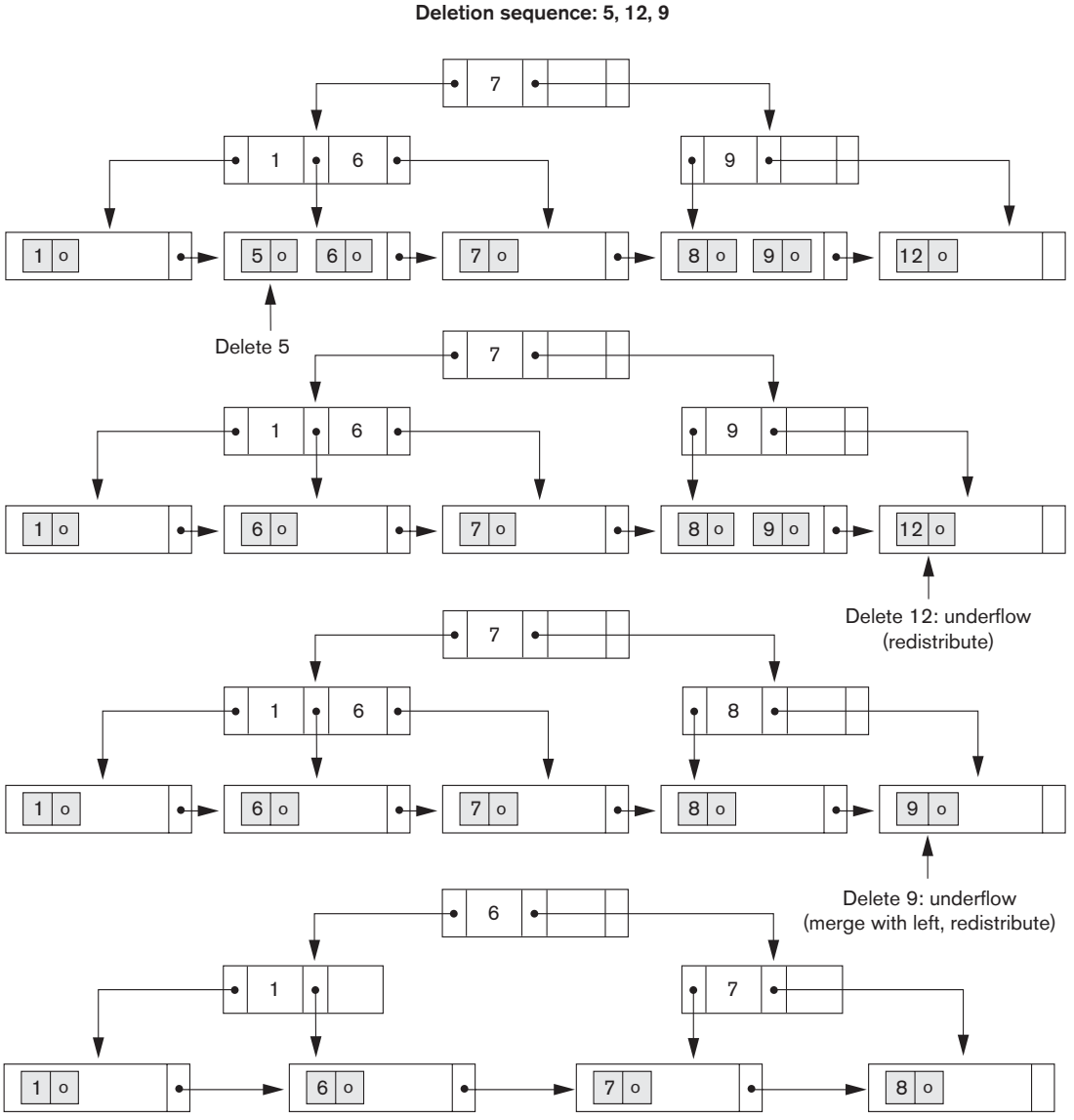


Figure 18.13
An example of deletion from a B+-tree.

and redistribute the entries among the node and its **sibling** so that both are at least half full; otherwise, the node is merged with its siblings and the number of leaf nodes is reduced. A common method is to try to **redistribute** entries with the left sibling; if this is not possible, an attempt to redistribute with the right sibling is

made. If this is also not possible, the three nodes are merged into two leaf nodes. In such a case, underflow may propagate to **internal** nodes because one fewer tree pointer and search value are needed. This can propagate and reduce the tree levels.

Notice that implementing the insertion and deletion algorithms may require parent and sibling pointers for each node, or the use of a stack as in Algorithm 18.3. Each node should also include the number of entries in it and its type (leaf or internal). Another alternative is to implement insertion and deletion as recursive procedures.[11]

Variations of B-Trees and B$^+$-Trees. To conclude this section, we briefly mention some variations of B-trees and B$^+$-trees. In some cases, constraint 5 on the B-tree (or for the internal nodes of the B$^+$–tree, except the root node), which requires each node to be at least half full, can be changed to require each node to be at least two-thirds full. In this case the B-tree has been called a **B*-tree**. In general, some systems allow the user to choose a **fill factor** between 0.5 and 1.0, where the latter means that the B-tree (index) nodes are to be completely full. It is also possible to specify two fill factors for a B$^+$-tree: one for the leaf level and one for the internal nodes of the tree. When the index is first constructed, each node is filled up to approximately the fill factors specified. Some investigators have suggested relaxing the requirement that a node be half full, and instead allow a node to become completely empty before merging, to simplify the deletion algorithm. Simulation studies show that this does not waste too much additional space under randomly distributed insertions and deletions.

18.4 Indexes on Multiple Keys

In our discussion so far, we have assumed that the primary or secondary keys on which files were accessed were single attributes (fields). In many retrieval and update requests, multiple attributes are involved. If a certain combination of attributes is used frequently, it is advantageous to set up an access structure to provide efficient access by a key value that is a combination of those attributes.

For example, consider an EMPLOYEE file containing attributes Dno (department number), Age, Street, City, Zip_code, Salary and Skill_code, with the key of Ssn (Social Security number). Consider the query: *List the employees in department number 4 whose age is 59*. Note that both Dno and Age are nonkey attributes, which means that a search value for either of these will point to multiple records. The following alternative search strategies may be considered:

1. Assuming Dno has an index, but Age does not, access the records having Dno = 4 using the index, and then select from among them those records that satisfy Age = 59.

[11]For more details on insertion and deletion algorithms for B$^+$ trees, consult Ramakrishnan and Gehrke [2003].