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Practical Modeling and System Identification of R/C Servo Motors

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***Abstract*— An R/C servo motor is a compact package of DC geared-motor associated with position servo controller. They are widely used in small-sized robotics and mechatronics by virtue of their compactness, easiness-to-use and high power/weight ratio. However, in order to improve control performance of mechatronic systems using R/C servo motors, such as biped robots or under-actuated systems, it is crucial to clarify their mathematical model. In this paper, we propose a simple and realistic internal model of R/C servo motors including the embedded servo controller, and estimate their physical parameters using continuous-time system identification method. We also provide a transfer function model of their reference-to-torque characteristics so that we can estimate the internal torque acting on the load.**

I. Introduction

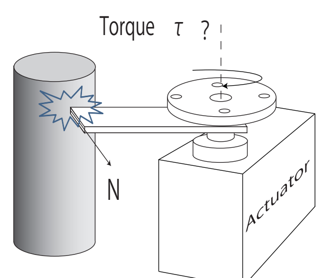
R/C servo motor is a popular name for a sort of compact DC geared-motor packages including motor drivers and position servo controllers, where *R/C* stands for *Radio-Control*(See Fig. 1). R/C servo motors were originally developed for hobby use such as radio-controlled vehicle or aircraft. In the last decade, they have been widely used in the field of robotic systems by virtue of their compactness, high torque-weight ratio, cost performance and easiness-to-use; conversely, high demands raised by recent robotics has been boosting the development of R/C servo motors. Nowadays, R/C servo motor is a reasonable choice to realize compact and less expensive mechatronic systems.

Fig. 1. Overview of an R/C servo motor

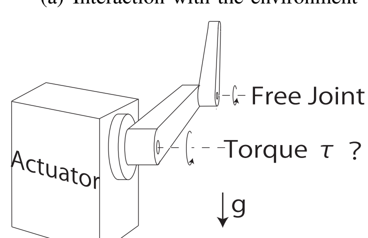
On the other hand, there remains a major problem in achieving satisfactory dynamic performance using the R/C servo motors, mainly due to the existence of *embedded position servo controllers* inside them. One may expect that the embedded servo controller makes it extremely easy to control robotic systems, such as biped robots, since we only have to send pre-designed motion reference patterns to the servo controller. It is true indeed, as long as the robot is moving freely and is full-actuated (i.e., every joint of the robot is properly actuated by an R/C servo motor). However, if the robot is in contact with the floor or other hard objects (Fig. 2(a)), it may not behave as we expected. If worse, it would exhibit a chattering motion caused by the disturbance force from the environment.

Another critical case to be considered is an *under-actuated* robotic system, such as the pendubot[1] or the acrobot[2], where one or more joints are not directly actuated (Fig. 2(b)). Usually the dynamics concerning the under-actuated joints are modeled by Newtonian or Lagrangian equations of motion. For the purpose of precise dynamical analysis and effective controller design, it is thus crucial to know the torques or forces applied by the actuators as well as those by the gravity or the environment. However, it is not available in the case of R/C servo motors, since the torque signal is hidden inside the embedded servo controller. This is a primary reason why R/C servo motors have been rarely used in the studies of control theory in spite of their practical advantages; instead, many control theorists prefer to adopt DC servo motors with current-feedback amplifies capable of torque-command, which are heavy, costly and energy-consuming in general.

Motivated by these observations, we aim at giving a dynamic internal model of R/C servo motors including the embedded servo controller. Our main purpose is to estimate



(a) Interaction with the environment



(b) Under-actuated systems

Fig. 2. Critical situations in R/C servo operation

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the *structure* of the embedded controller, as well as their physical parameters, by means of continuous-time system identification method called SRIVC; it does not require derivative of measurement signals and is quite suitable for systems with limited hardware capability. As a result of the system identification, we also obtain a model of reference-to-torque transfer function so that we can estimate the internal torque acting on the load. The proposed model is simple and realistic enough, so that we can put it in various mechatronic systems.

We stress here that this study is concerned with system identification of 'from position-reference to position' relationship which has been scarcely dealt with, as opposed to some conventional works on 'from velocity-reference to velocity' relationship for DC servo drivers[3].

This paper is organized as follows. In Section 2, we introduce the fundamental electro-mechanical model of DC motor and suggest a hypothetical model for the (unknown) embedded servo controller, to build up a transfer function model of the R/C servo motor. After a brief introduction of continuous-time system identification approach and SRIVC method in Section 3, we perform a series of identification experiments in Section 4 and discuss the results to deduce the internal model of the R/C servo motor. Conclusion of the paper is given in Section 5.

II. R/C SERVO MOTOR

In this section, the basic structure of R/C servo motor is described. The R/C servo motor is composed of a DC motor, a potentiometer, an embedded servo controller and an amplifier. The control circuit and the amplifier serve as the controller, and the potentiometer outputs the analog voltage which is proportional to the angle of the DC motor. Usually, R/C servo motor accepts a series of square pulses as its command, and the width of the pulses corresponds to the reference angle of the R/C servo motor, so that we can specify the reference angle by tuning the duty ratio of the pulse. When the reference angle is input into the R/C servo motor, the embedded servo controller computes the control input needed to track the reference angle, and apply the voltage to the DC motor. Fig. 3 shows the block diagram of an R/C servo motor under the assumption that all blocks are linear systems. $K(p)$ and $P(p)$ correspond to the embedded servo controller and the DC motor, respectively. Here, p is the differential operator (i.e. $p \triangleq \frac{d}{dt}$). $u(t)$ is the reference angle, and $\phi(t)$ is the rotation angle of the DC motor. In the next subsection, we develop the internal model of the R/C servo motor, which contains DC motor and the controller explicitly and describes the relationship between $u(t)$ and $\phi(t)$.

A. Model of a DC Motor

First, we begin with a conventional model of the DC motor. The internal structure of the DC motor is depicted in Fig. 4. J is the moment of inertia of the load attached to the drive shaft of the DC motor. The DC motor model $P(p)$ is the transfer function from the input voltage $v(t)$ to the rotation angle $\phi(t)$. Let us derive $P(p)$ from the Kirchhoff's laws and the equation of motion as follows. From the equation of electric circuit,

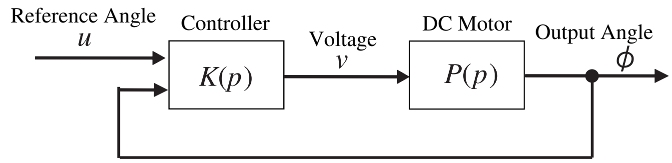

Fig. 3. Block diagram of an R/C servo motor

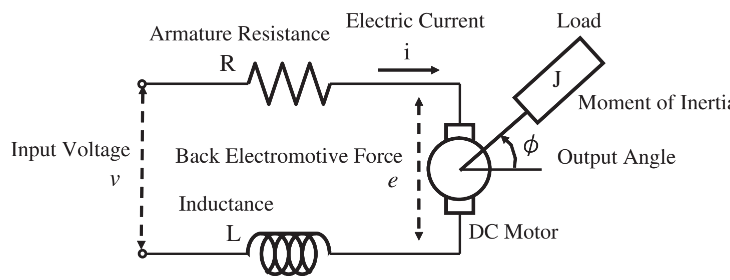

Fig. 4. Electro-Mechanical scheme of a DC motor

$$Ri(t) + L\dot{i}(t) + e(t) = v(t) \tag{1}$$

holds, where $e(t)$ is a back electromotive force generated by the rotation of the motor and satisfies

$$e(t) = k_e\dot{\phi}(t). \tag{2}$$

Also, the torque $\tau(t)$ produced by the motor, which is proportional to the current $i(t)$, satisfies the equation of motion

$$\tau(t) = k_\tau i(t) = J\ddot{\phi}(t) + D_\phi\dot{\phi}(t). \tag{3}$$

Therefore, the transfer function from the input voltage $v(t)$ to the output torque $\tau(t)$ is derived from (1), (2) and (3) as follows.

$$\tau(t) = \frac{k_\tau(Jp + D_\phi)}{(R + Lp)(Jp + D_\phi) + k_e k_\tau} v(t), \tag{4}$$

where $R[\Omega]$ is the armature resistance, L[H] is the inductance, k_e[Vs/rad] is the back electromotive force coefficient, k_τ[Nm/A] is the torque coefficient, D_ϕ[Ns/m] is the viscous friction coefficient.

Then, the transfer function from the input torque $\tau(t)$ to the output rotation angle $\phi(t)$ can be derived from (3) as the following.

$$\phi(t) = \frac{1}{(Jp + D_\phi)p}\tau(t) \tag{5}$$

Consequently, the transfer function from the input voltage $v(t)$ to the output rotation angle $\phi(t)$ can be derived from (4) and (5) as follows.

$$P(p) = \frac{k_\tau}{((R + Lp)(Jp + D_\phi) + k_e k_\tau)p}$$

TABLE I

DEGREES OF CONTROLLER MODELS

Controller	numerator	denominator
PID	2	4
PI	1	4
PD	1	3
D-P	0	3
P	0	3

B. Model of the embedded Servo Controller

Next, we consider the model of the controller. For most of commercial R/C servo motors, precise structure of the embedded controller is not available. Now, we assume the structure of the controller at first, and verify their validity later on. Candidates of the possible controller structure and corresponding transfer function $K(p) \triangleq \frac{v(t)}{u(t)-\phi(t)}$ are listed as follows.

PID controller

$$K(p) = \frac{K_D p^2 + K_P p + K_I}{p}$$

PI controller

$$K(p) = \frac{K_I p + K_P}{p}$$

PD controller

$$K(p) = K_D p + K_P$$

P controller

$$K(p) = K_P$$

Also, we consider the D-P controller, by which the control input is determined as

$$v(t) = K_P(u(t) - \phi(t)) - K_D p\phi(t). \tag{6}$$

In these controller equations, K_D, K_P and K_I are scalar controller gains.

The selection of these controllers determines the order of the whole RC servo motor model, and their relationship is shown in TABLE I. Therefore, it is possible to identify the controller structure when the order of the R/C servo motor model is estimated.

In the following sections, we intend to estimate the order of the R/C servo motor model to determine the structure of the embedded controller.

III. CONTINUOUS-TIME SYSTEM IDENTIFICATION USING SRIVC

We use SRIVC method [4] [5] [6] as an identification method to estimate the model structure as well as parameters of the system. SRIVC method is an identification method for continuous-time linear time-invariant SISO or MISO systems. In this section, the algorithm for SISO systems is briefly explained.

First, consider the continuous-time system described by the differential equation

$$\begin{aligned} x(t) &= \frac{B(p)}{A(p)} u(t) \\ B(p) &= b_m p^m + b_{m-1} p^{m-1} + \cdots + b_0 \\ A(p) &= p^n + a_{n-1} p^{n-1} + \cdots + a_0 \\ y(t) &= x(t) + w(t) \end{aligned}$$

where $u(t), x(t)$ are the input and the output, respectively. The measurement output $y(t)$ is observed under the influence of the measurement noise $w(t)$. The orders of the numerator and the denominator polynomials are m and n $(n \geq m)$, respectively. Moreover, the coefficient of p^n in the denominator polynomial is set to 1 to determine the all coefficients uniquely.

In the following, we show the algorithm which estimates $\theta = [a_{n-1} \ \dots \ a_0 \ b_m \ \dots \ b_0]^T$ from the input and output data $\{u(t) \ y(t)\}$ observed over $[0, T]$. In practice, only the sampled I/O data observed with sampling interval T_s are available. Let the number of samples be $N+1$, and $t_k \triangleq kT_s$.

STEP 0

Choose the initial estimate of the denominator $\hat{A}_0(p)$.

STEP 1

Calculate $y_f^{(i)}(t)$ and $u_f^{(i)}(t)$ which are obtained by filtering the input signal $u(t)$ and the output signal $y(t)$ as follows.

$$\begin{aligned} y_f^{(i)}(t) &= F_i(p) y(t), \ i = 0, \dots, n \\ u_f^{(i)}(t) &= F_i(p) u(t), \ i = 0, \dots, m \\ F_i(p) &\triangleq \frac{p^i}{\hat{A}_0(p)} \end{aligned}$$

Then, we can derive the parameter estimate $\hat{\theta}$ by minimizing $\{\varepsilon(t_k)\}$ which is defined by

$$\varepsilon(t_k) \triangleq y_f^{(n)}(t_k) - \phi_f^T(t_k)\theta. \tag{7}$$

This is a standard linear regression equation, thus the least-square estimate of $\hat{\theta}$ can be readily calculated.

$$\hat{\theta}_1 = \Big[\sum_{k=0}^{N} \phi_f(t_k)\phi_f^T(t_k)\Big]^{-1} \sum_{k=0}^{N} \phi_f(t_k) y_f^{(n)}(t_k)$$
$$\phi_f(t_k) = [-y_f^{(n-1)}(t_k) \cdots - y_f^{(0)}(t_k) \ u_f^{(m)}(t_k) \cdots u_f^{(0)}(t_k)]^T$$

STEP 2a

Initialize $l = 1$.

STEP 2b

In order to remove the bias of the estimation derived at *STEP 1*, the following instrumental variable $\hat{x}(t)$ is employed.

$$\hat{x}(t) = \frac{B(p, \hat{\theta}_l)}{A(p, \hat{\theta}_l)} u(t)$$

Also, update the filter $F_i(p) = \dfrac{p^i}{A(p,\hat{\theta}_l)}$, and repeat the similar filtering procedure using the new filter $F_i(p)$ as in STEP 1 to obtain $y_f^{(i)}(t)$, $u_f^{(i)}(t)$ and $\hat{x}_f^{(i)}(t)$

Based on these values, compute the estimate as follows.

$$\hat{\theta}_{l+1} = \Big[\sum_{k=0}^{N} \hat{\phi}_f(t_k)\phi_f^T(t_k)\Big]^{-1} \sum_{k=0}^{N} \hat{\phi}_f(t_k) y_f^{(n)}(t_k)$$

$$\phi_f(t_k) = [-y_f^{(n-1)}(t_k) \cdots - y_f^{(0)}(t_k)\ u_f^{(m)}(t_k) \cdots u_f^{(0)}(t_k)]^T$$

$$\hat{\phi}_f(t_k) = [-\hat{x}_f^{(n-1)}(t_k) \cdots - \hat{x}_f^{(0)}(t_k)\ u_f^{(m)}(t_k) \cdots u_f^{(0)}(t_k)]^T$$

STEP 2c

If $\hat{\theta}_l - \hat{\theta}_{l-1}$ gets sufficiently small, let $\hat{\theta}_l$ be the result and proceed to the STEP 3. Otherwise, update $l \leftarrow l+1$ and return to STEP 2b.

STEP 3

Evaluate the validity of the estimated model based on the R_T^2 and YIC criteria, which are defined as follows.

$$R_T^2 = 1 - \frac{\tilde{\sigma_\epsilon}^2}{\tilde{\sigma_y}^2}$$

$$YIC = \ln \frac{\tilde{\sigma_\epsilon}^2}{\tilde{\sigma_y}^2} + \ln \frac{1}{n_p} \sum_{j=1}^{n_p} \frac{\tilde{\sigma_\epsilon}^2 p_{jj}}{\tilde{\theta_j}^2}$$

where $\tilde{\sigma_\epsilon}$: the variance of the model residuals, $\tilde{\sigma_y}$: the variance of the output signal, $\tilde{\theta_j}^2$: the squared value of j-th estimated parameter, p_{jj}: the j-th diagonal element of the estimated parametric error covariance matrix and n_p: the number of parameters to be estimated. The R_T^2 will be recognized as the coefficient of determination based on the simulated model error. The YIC coefficient provides a measure of how well the parameters are defined statistically. These indices roughly imply the accuracy of the estimated model. The model output gets closer to the actual output as the YIC is small and/or R_T^2 is close to 1.

IV. Identification Experiment

In this section, we describe the procedure of our identification experiment and the estimation results.

As shown Fig. 4, an inertial load J is attached to the output axis of the R/C motor HSR-5990GT (Hitec Multiplex Japan). The reference signal is sent to the R/C servo motor from the digital I/O port of the microcomputer SEMB1200A (Shimafuji Electronics Corporation), in the form of PWM code (the reference angle is encoded as the width of a square pulse ;note that it differs from the so-called *PWM control*). The rotation angle $\phi(t)$ is measured by the potentiometer attached to the output axis, and sent to the A/D conversion port of the microcomputer. The sampling interval is set as 4[msec], and the duration of one trial of experiment is 8[sec]. We then estimate the model of the R/C servo motor from the obtained Input/Output data by means of SRIVC identification method.

TABLE II

Compatibility indices for various types of controller models

Controller	YIC	R_T^2
PID	-8.6862	0.9917
PI	-9.0540	0.9912
PD	-10.1788	0.9913
D-P	-12.1709	0.9906

A. Identification of the R/C servo motor; Reference-to-Angle Characteristics

As the reference signal to the R/C servo motor, we selected a series of step functions with the fixed width(0.8[sec]) and two different heights (0.15[rad] and 0.3[rad]), which is shown by the dash curve in Fig. 5. This reference is chosen after several unsuccessfull trials to use the M-series, mixed-frequency sinusoids and other common reference signals used in system identification. One of the reason is that the response of the R/C servo motor is rather quick relative to the capability of the PWM encoding (it takes about 10 millseconds to send one packet of reference signal).

The solid curve shown in Fig. 5 indicates the response of the R/C servo motor. In overall, it quickly tracks the reference angle with noticeable overshoots and small amount of offsets. Small spikes left in the steady-state seem to be caused by the sensor noise and limited resolution of the A/D converter.

Applying the system identification algorithm to the obtained I/O data, we estimated the transfer function models based on various hypotheses of the model order (shown in TABLE I). The values of the YIC and R_T^2 derived from the estimated models are shown in TABLE II. This result shows the model with D-P controller is the best choice because the index of YIC takes the smallest value with the D-P controller. As for the index of R_T^2, there is no clear difference among the controller candidates.

As a result of the consideration above, we deduce that the embedded controller of the R/C servo motor is *likely to be* the D-P controller. Then, the structure of the R/C servo motor model ($\phi(t)/u(t)$) should be

$$\frac{B_0}{B_3 p^3 + B_2 p^2 + B_1 p + B_0},$$

$$\begin{aligned} B_0 &= K_P k_\tau, & B_1 &= R D_\phi + k_e k_\tau + K_D k_\tau, \\ B_2 &= RJ + D_\phi L, & B_3 &= LJ, \end{aligned}$$

and the transfer function with estimated parameters is

$$\frac{1.409 \times 10^4}{p^3 + 37.46p^2 + 1150p + 1.399 \times 10^4}.$$

Note that the coefficient of p^3 in the denominator of the estimated transfer function is normalized to unity.

The comparison between the output of the estimated model and the measured output is shown in Fig. 6. Also the Bode plot of the estimated model is shown in Fig. 7.

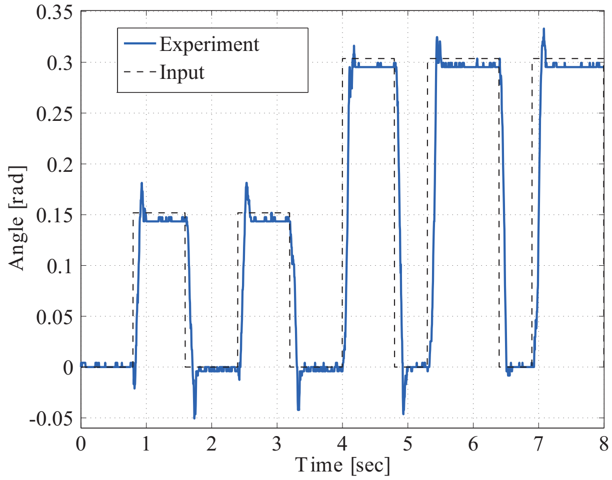

Fig. 5. Reference Signal and the Response of Rotor Angle

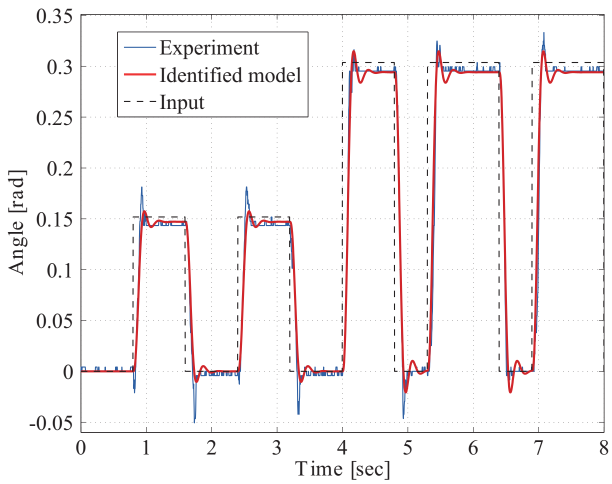

Fig. 6. Response of Experiment and the Estimated Model

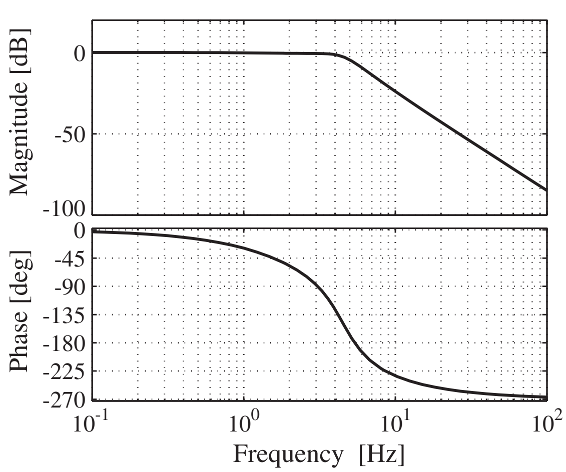

Fig. 7. Bode Plot of the Estimated Model

B. Identification of the Reference-to-Torque Characteristics

Next, let us identify the transfer function of reference-to-torque characteristics of the R/C servo motor, in order to estimate the internal torque acting on the load.

Transfer functions from the voltage $v(t)$ and the rotation angle $\phi(t)$ to the torque $\tau(t)$ is easily derived from (1), (2) and (3) as

$$\tau(t) = \frac{k_\tau}{Lp + R}v(t) - \frac{k_e k_\tau p}{Lp + R}\phi(t). \tag{8}$$

Based on our conclusion of the last subsection that the embedded controller of the R/C servo motor is a D-P controller, we substitute the D-P control law (6) for (8), then the transfer function from the rotation angle $\phi(t)$ and the reference angle $u(t)$ to the torque $\tau(t)$ is obtained as

$$\tau(t) = \frac{-(k_e k_\tau + K_D k_\tau)p - K_P k_\tau}{Lp + R}\phi(t) + \frac{K_P k_e}{Lp + R}u(t). \tag{9}$$

Here we have to know all the cofficients appeared in the transfer function above, however, the parameters that we estimated in the previous experiment (B_0, B_1, B_2 and B_3) are not sufficient for this purpose. Now, we intend to perform another identification experiment for slightly modified system, with another ineartial load $J + \delta J$ is different from the former one, where the amount of difference δJ is known. In this case, the transfer function of the R/C servo motor model (from $u(t)$ to $\phi(t)$) turns to be

$$\frac{B_0}{B_3' p^3 + B_2' p^2 + B_1 p + B_0},$$

$$B_0 = K_P k_\tau, \qquad B_1 = RD_\phi + k_e k_\tau + K_D k_\tau,$$
$$B_2 = R(J + \delta J) + D_\phi L, \quad B_3 = L(J + \delta J).$$

The transfer function with estimated parameters, where the experiment procedure is the same as previous and δJ = 0.0022[kgm], results in

$$\frac{1.813 \times 10^4}{p^3 + 45.29p^2 + 1413p + 1.797 \times 10^4}.$$

Combining this result with the previous one, we can partially derive the physical parameters by algebraic computation, as follows:

$$J = \frac{B_3}{B_3' - B_3}\delta J, \qquad D_\phi = (\frac{B_2}{B_3} - \frac{B_2'}{B_3'})\frac{J(J + \delta J)}{\delta J},$$
$$\frac{R}{L} = \frac{B_2}{B_3} - \frac{D_\phi}{J}, \qquad \frac{K_P k_\tau}{L} = \frac{B_0}{B_3}J,$$
$$\frac{k_e k_\tau + K_D k_\tau}{L} = (\frac{B_1}{B_3} - \frac{R}{L}\frac{D_\phi}{J})J.$$

The obtained parameters are

$$\begin{aligned} J &= 0.0099[\text{kg}\cdot\text{m}] \\ D_\phi &= 0.2009[\text{N/m}] \\ R/L &= 17.46 \\ K_P k_\tau / L &= 140,9 \\ (k_e k_\tau + K_D k_\tau)/L &= 8.008 \end{aligned}$$

Note that we are not able to know the individual parameters $k_e, k_\tau, K_P, K_D, R, L$.

Based on the derived values and (9), the reference-to-torque transfer function is

$$\tau(t) = -\frac{8.008p - 140.9}{p + 17.46}\phi(t) + \frac{140.9}{p + 17.46}u(t). \qquad (10)$$

The transfer function enables us to compute the torque generated by the R/C servo motor.

V. CONCLUSION

In this paper, we proposed a simple and realistic model of the R/C servo motor where the input is the reference angle and the output is the angle. Based on the result of continuous-time system identification experiment under various relative-degree assumptions, we deduced that the internal model of the embedded servo controller is of type D-P (Differentiation-first PD control). We also gave a model of reference-to-torque transfer function so that we can estimate the internal torque acting on the load. All sorts of R/C servo motors can be identified, basically, using the approach we showed in this paper.

Finally, let us re-emphasize the benefit we could obtain from this study. We have a mathematical *model* of the R/C servo motor; now the control/robotics researchers can deal with compact mechatronic systems using R/C servo motors (such as hobby humanoids), with the full aid of advanced control/robotic theories. The authors are currently engaged in modeling and system identification of under-actuated mechatronic systems including the R/C servo motors.

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