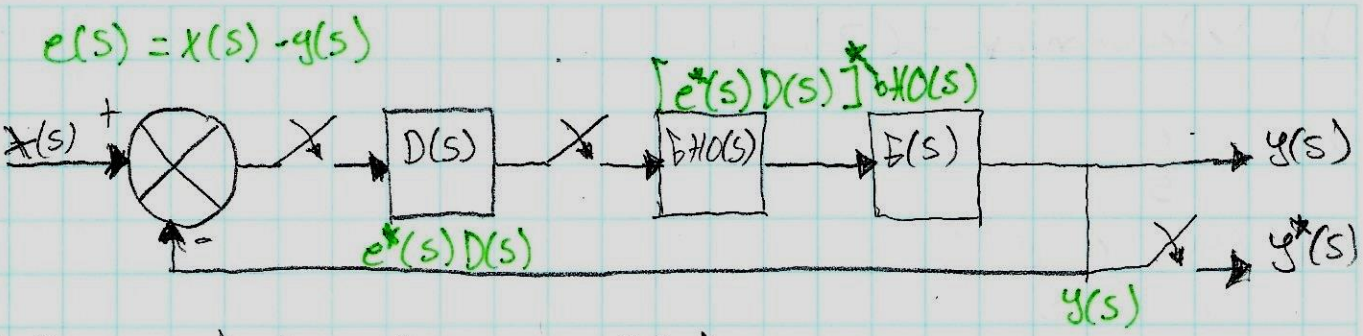
Examen 10/12/2022

SDM



a) Función de transferencia $\frac{y(z)}{X(z)}$

$$y(s) = e^*(s) D^*(s) [G_{HO}(s) G(s)]$$

$$e(s) = X(s) - y(s)$$

$$y^*(s) = e^*(s) D^*(s) [G_{HO}(s) E(s)]^*$$

$$e^*(s) = [X(s) - y(s)]^* = X^*(s) - y^*(s)$$

$$= X^*(s) - [e^*(s) D^*(s) \{G_{HO}(s) G(s)\}^*]$$

$$e^*(s) + e^*(s) D^*(s) [G_{HO}(s) G(s)]^* = X^*(s)$$

$$e^*(s) = \frac{X^*(s)}{1 + D^*(s) [G_{HO}(s) G(s)]^*}$$

$$y^*(s) = \frac{X^*(s) D^*(s) [G_{HO}(s) G(s)]^*}{1 + D^*(s) [G_{HO}(s) G(s)]^*}$$

$$\frac{y^*(s)}{X^*(s)} = \frac{D^*(s) [G_{HO}(s) G(s)]^*}{1 + D^*(s) [G_{HO}(s) G(s)]^*}$$

$$\frac{y(z)}{X(z)} = \frac{D(z) G_{HO}(z)}{1 + D(z) G_{HO}(z)} \quad ; \quad G_{HO}(z) = \mathcal{Z}\left[G_{HO}(s) G(s)\right]$$

b) determinar $Y(z)$ si:

$$X(s) = \frac{1}{s}$$

$$BHO(s) = \frac{1 - e^{-Ts}}{s}$$

$$F(s) = \frac{2[s+2]}{(s+3)(s+2)(s+4)}$$

$$D(s) = 1$$

$$N = 4 + 2(0) + 0 + (0) + 7 + 5(0) + 7 + 1(0)$$
$$N = 4 + 7 + 7 = 18 \therefore$$

$$T = 0.Ns \rightarrow T = 0.18s$$

$$X(z) = \frac{z}{z-1}$$

$$D(z) = 1$$

$$\mathcal{Z}[BHOF(s)] = BHOF(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{2}{(s)(s+4)(s+3)}\right]$$

$$\mathcal{Z}\left[\frac{2}{(s)(s+4)(s+3)}\right] = \mathcal{Z}\left[\frac{1}{s(s+a)(s+b)}\right] = \frac{(Az+B)z}{(z-1)(z-e^{-aT})(z-e^{-bT})}$$

$a = 4$
$b = 3$
$T = 0.18$
$aT = 0.72$
$bT = 0.54$

$$A = \frac{b(1-e^{-aT}) - a(1-e^{-bT})}{ab(b-a)}$$

$$A = \frac{3(1-e^{-0.72}) - 4(1-e^{-0.54})}{(4)(3)(3-4)} = 10.7719m$$

$$B = \frac{ae^{-aT}(1-e^{-bT}) - be^{-bT}(1-e^{-aT})}{ab(b-a)}$$

$$B = \frac{4e^{-0.72}(1-e^{-0.54}) - 3e^{-0.54}(1-e^{-0.72})}{(4)(3)(3-4)} = 7.0741m$$

$$\text{ZHOD}(z) = \frac{2(z-1)}{z}\left[\frac{[10.7719m\,z + 7.0741m]z}{(z-1)(z-0.4867)(z-0.5827)}\right]$$

$$\text{ZHOD}(z) = \frac{2(10.7719m)[z+0.6567]}{(z-0.4867)(z-0.5827)}$$

$$\frac{y(z)}{x(z)} = \frac{21.5438m[z+0.6567]}{(z-0.4867)(z-0.5827)} \Big/ 1 + \left[\frac{21.5438m[z+0.6567]}{(z-0.4867)(z-0.5827)}\right]$$

$$\frac{y(z)}{x(z)} = \frac{21.5438m[z+0.6567]}{(z-0.4867)(z-0.5827) + 21.5438m[z+0.6567]}$$

$$z^2 - 0.4867z - 0.5827z + 0.2836 + 21.5438mz + 14.1478m$$

$$z^2 - 1.0478z + 0.2977$$

$$\frac{y(z)}{x(z)} = \frac{21.5438m[z+0.6567]}{z^2 - 1.0478z + 0.2977}$$

$$y(z) = \frac{21.5438\,m\,(z + 0.6567)\,z}{(z^2 - 1.0478z + 0.2977)(z - 1)}$$

C) Determinar $y(k)$

$$\frac{y(z)}{z} = \frac{21.5438mz + 14.1478m}{(z-1)(z^2 - 1.0478z + 0.3051)}$$

$$\frac{A}{z-1} + \frac{Bz + C}{z^2 - 1.0478z + 0.3051}$$

$$Az^2 - 1.0478Az + 0.3051A + Bz^2 - Bz + Cz - C$$

$A + B = 0 \rightarrow B = -A$

$-1.0478A - B + C = 21.5438m$

$0.3051A - C = 14.1478m \rightarrow C = 0.3051A - 14.1478m$

$-1.0478A + (0.3051A - 14.1478m) + A = 21.5438m$

$0.2573A = 35.6916m \rightarrow A = 0.1387$

$B = -0.1387$

$C = 28.0432m$

$$y(z) = \frac{(0.1387)z}{z-1} + \frac{(-0.1387)z^2 + 28.0432mz}{z^2 - 1.047z + 0.3051}$$

$$y(z) = \frac{(0.1387)z}{z-1} + 0.1382\left[\frac{z^2 - 0.2029z}{z^2 - 1.0478z + 0.3051}\right]$$

$$\mathcal{Z}^{-1}\left[\frac{z^2 - ze^{-aT}\cos bT}{z^2 - 2ze^{aT}\cos bT + e^{-2aT}}\right] = e^{-aTk}\cos bTk$$

$e^{-2aT} = 0.3051 \rightarrow aT = 0.5935 \rightarrow e^{-aT} = 0.5523$

$-2ze^{-aT}\cos bT = -1.047z$

$$\cos bT = \frac{-1.047z}{-2e^{-aT}z} = 0.9477$$

$$bT = \cos^{-1}[0.9477] = 0.3246$$

$$-e^{-aT}\cos bT = -0.5235$$

$$\frac{z^2 - 0.5235z + 0.5235z - 0.2029z}{z^2 - 1.047z + 0.3051}$$

$$\frac{z^2 - 0.5235z}{z^2 - 1.047z + 0.3051} + \frac{0.3206z}{z^2 - 1.047z + 0.3051}$$

$$\mathcal{Z}^{-1}\left[\frac{ze^{-aT}\sin bT}{z^2 - 2ze^{-aT}\cos bT + e^{-2aT}}\right] = e^{-aTk}\sin(bTk)$$

$$e^{-aT}\sin bT = 0.1762$$

$$\frac{z^2 - 0.5235z}{z^2 - 1.047z + 0.3051} + \left(\frac{0.3206}{0.1762}\right)\left[\frac{0.1762z}{z^2 - 1.047z + 0.3051}\right]$$

$$\mathcal{Z}^{-1}\left[\frac{z}{z-1}\right] = 1$$

$$y(k) = 0.1382(1) - e^{aTk}[0.1382\cos(bTk) + 0.2514\sin(bTk)]$$

$$A = \sqrt{a^2 + b^2} = 0.2868$$

$$\theta = \tan^{-1}\left[\frac{b}{a}\right] = 0.5026$$

$$y(k) = 0.1382 - (0.2868)\sin[0.3246k + 0.5026](0.5523)^k$$

e) Lugar Geometrico delas raices.

$$FHOB(z) = \frac{z + 0.6567}{z^2 - 1.0694z + 0.2836}$$

$$\frac{\partial}{\partial z}[FHOB(z)] = \frac{U}{V} = \frac{Vu' - uv'}{V^2}$$

$V = z^2 - 1.0694z + 0.2836$
$V' = 2z - 1.0694$

$U = z + 0.6567$
$U' = 1$

Trabajamos con el numerador

$$(z^2 - 1.0694 + 0.2836)(1) - (z + 0.6567)(2z - 1.0694)$$

$$z^2 - 1.0694 + 0.2836 + (-z - 0.6567)(2z - 1.0694)$$

z^2	z^1	z^0
1	$\cdot$ 1.0694	0.2836
-2	1.0694	0.7022
	-1.3134	
-1	-1.3134	0.9858

$z^2 + 1.3134 - 0.9858 = 0 \therefore z_1 = -1.8471$
$z_2 = 0.5337$

$radio = \frac{z_1 - z_2}{2} \rightarrow r = 1.1904$

$centro = z_1 + r \rightarrow C = -0.6567$

5) Encontrar rango k_p

$D(z) = k_p$

a) Routh

$$1 + \left[\frac{21.5438\,mk\,(z+0.6567)}{(z-0.4867)(z-0.5827)}\right] = 0$$

$$(z-0.4867)(z-0.5827) + 21.5438\,mk\,z + 14.1478\,mk = 0$$

$$P(z) = z^2 - 1.0694z + 21.5438\,mk\,z + 0.2836 + 14.1478\,mk$$

$$z = \frac{1+w}{1-w}$$

$$P(z) = \left[\frac{1+w}{1-w}\right]^2 + \left[-1.0694 + 21.5438\,mk\right]\left(\frac{1+w}{1-w}\right) + \left[0.2836 + 14.1478\,mk\right](1-w)^2$$

multiplicamos $(1-w)^2 \rightarrow (1-w)^2 P(z)$ ∴

$$P(z) = (1+w)^2 + \left[-1.0694 + 21.5438\,mk\right](1-w)^2 + \left[1.2836 + 14.1478\,mk\right](1-w)^2$$

Expandimos y Factorizamos

$$P(z) = 1 + 2w + w^2$$
$$+$$
$$\left[-1.0694 + 21.5438\,mk\right](1-w^2)$$
$$\left[0.2836 + 14.1478\,mk\right](1-2w+w^2)$$

w^2	w^1	w^0
1	2	1

1.0694 $\quad$ -1.0694

$-21.5438k$ $\quad$ $21.5438mk$

0.2836 $\quad$ -0.5672 $\quad$ 0.2836

$14.1478mk$ $\quad$ $-28.2956mk$ $\quad$ $14.1478mk$

$2.353 - 7.396mk$ | $1.4328 - 0.02829k$ | $0.2142 + 0.0356916k$

$$k_1 = \frac{-2.353}{-7.396} = 318.1449$$

$$k_2 = \frac{-1.4328}{-0.02829456} = 50.6368$$

$$k_3 = \frac{-0.2142}{0.0356916} = -6.0014$$

$$0 \leq k_p \leq 50.6368$$

b) Jury

$$P(z) = z^2 + [-1.0694 + 21.5438mk]z + [0.2836 + 14.1478mk]$$

i) $|a_n| < a_0$

$$0.2836 + 14.1478mk < 1$$

$$k < \frac{1 - 0.2836}{14.1478m} \rightarrow k < 50.6368$$

ii) $P(1) > 0$

$(1)^2 + [-1.0694 + 21.5438mk](1) + [0.2836 + 14.1478mk]$

$1 - 1.0694 + 21.5438mk + 0.2836 + 14.1478mk$

$0.2142 + 35.6916mk > 0$

$k > \frac{-0.2142}{35.6916m} \rightarrow k > -6.0014$

iii) $P(-1) > 0$

$(-1)^2 + [-1.0694 + 21.5438mk](-1) + [0.2836 + 14.1478mk] > 0$

$1 + 1.0694 - 21.5438mk + 0.2836 + 14.1478mk < 0$

$2.353 - 7.396mk < 0$

$k < \frac{-2.353}{-7.396m} \rightarrow k < 318.1449$

$0 \leq k_p \leq 50.6368$

g) Encontrar k_p y k_i con menos sobre paso (5 valores)

$$BHOB(z) = \frac{z + 0.6567}{(z - 0.4867)(z - 0.5827)}$$

$P_1 = 0.4867$
$P_2 = 0.5827$

$$z - \frac{2k_p - k_i T}{2k_p + k_i T} = z - P_x \therefore P_1 = \frac{2k_p - k_i T}{2k_p + k_i T}$$

Despejamos k_i

$$k_{i_1} = \frac{2k_p [1 - P_1]}{T [P_1 + 1]} =$$

$$k_{i_2} = \frac{2k_p [1 - P_2]}{T [P_2 + 1} =$$

$0 \leq k_p \leq 41.3$
$T = 0.18\ s$

con $k_p = 10$

$k_1 = 38.3623$
$k_2 = 29.2959$

con $k_p = 5$

$k_1 = 19.1811$
$k_2 = 14.6479$

con $k_p = 21$

$k_1 = 80.5609$
$k_2 = 61.5214$

con $k_p = 29$

$k_1 = 111.2508$
$k_2 = 84.9581$

con $k_p = 33$

$k_1 = 126.5958$
$k_2 = 96.6765$

Comprobando con CC la mejor opción es el valor de $k_p = 5$ y $k_i = 14.6479$

#) valores de k_p, k_i, k_d

$A = 1$
$B = -(P_1 + P_2)$
$C = P_1 P_2$

$$[2k_pT + k_iT^2 + 2k_d]z^2 + [k_iT^2 - 2k_pT - 4k_d]z + 2k_d$$
$$Az^2 \qquad + Bz \qquad + C$$

$P_1 = 0.4867$; $P_2 = 0.5827$; $T = 0.18s$

$$2k_pT + k_iT^2 + 2k_d = 1$$

$$k_iT^2 - 2k_pT - 4k_d = -(P_1 + P_2)$$

$$2k_d = P_1 P_2$$

$$0.36k_p + 0.0324k_i + 2k_d = 1$$
$$-0.36k_p + 0.0324k_i - 4k_d = -1.0694$$
$$2k_d = 0.2836$$

$k_p = 1.6925$
$k_i = 3.3055$
$k_d = 0.1418$