

(a) $\{A, B, C, D\}$

Find all possible subsets:

$\{A\}^+ = \{A\}$ $\{A B\}^+ = \{A, B, C, D, E, G\}$ $\{A B C\}^+ = \{A, B, C, D, E, G\}$
 $\{B\}^+ = \{B\}$ $\{A C\}^+ = \{A, C, B, D, E, G\}$ $\{A B D\}^+ = \{A, B, C, D, E, G\}$
 $\{C\}^+ = \{C\}$ $\{A D\}^+ = \{A, D, B, C, E, G\}$ $\{B C D\}^+ = \{B, C, D, A, E, G\}$
 $\{D\}^+ = \{D\}$ $\{B C\}^+ = \{B, C\}$ ~~$\{A B C D\}^+$~~
 $\{D D\}^+ = \{D, A, E, G, C\}$
 $\{C D\}^+ = \{C, D\}$

$T = \{ B \rightarrow C, A D \rightarrow C, A B \rightarrow D, A C \rightarrow B,$
 $A C \rightarrow D, B D \rightarrow A, B D \rightarrow C, A B C \rightarrow D,$
 $A B D \rightarrow C, B C D \rightarrow A \}$

(b) $\{A, B, C\}$

$\{A\}^+ = \{A\}$ $\{A B\}^+ = \{A, B, C\}$ ~~$\{A B C\}^+$~~
 $\{B\}^+ = \{B\}$ $\{A C\}^+ = \{A, C, B\}$
 $\{C\}^+ = \{C\}$ $\{B C\}^+ = \{B, C\}$

$T = \{ B \rightarrow C, A B \rightarrow C, A C \rightarrow B, \}$

(c) $\{A, C, E, H\} \rightarrow \{A, C, E, H\} =$

Subsets:

$\{A\}^+$

$\{A, C\}^+$

~~$\{A, C, E\}^+$~~

$\{C\}^+$

~~$\{A, E\}^+$~~

~~$\{A, C, H\}^+$~~

~~$\{E\}^+$~~

~~$\{A, H\}^+$~~

~~$\{C, E, H\}^+$~~

~~$\{H\}^+$~~

~~$\{C, E\}^+$~~

~~$\{A, C, E, H\}^+$~~

~~$\{C, H\}^+$~~

~~$\{E, H\}^+$~~

Let Σ with $\{A, C, E, H\}$

$$\{A\}^+ = \{A\}$$

$$\{C\}^+ = \{C\}$$

$$\{A, C\}^+ = \{A, C, AC, CA, ACC, CCA, \dots\}$$

$$\Sigma^+ = \{A, C, E, H\}$$

(d) $\{A, B, C, E, G\}$

$\{A\}$	$\{A B\}$	$\{A B C\}$
$\{B\}$	$\{A C\}$	$\{A B E\}$
$\{C\}$	$\{A E\}$	$\{A B G\}$
$\{E\}$	$\{A G\}$	$\{B C E\}$
$\{G\}$	$\{B C\}$	$\{B C G\}$
	$\{B E\}$	$\{C E G\}$
	$\{B G\}$	$\{B E G\}$
	$\{C E\}$	$\{A B C E\}$
	$\{C G\}$	$\{A B C G\}$
	$\{E G\}$	$\{B C E G\}$
		$\{A C E G\}$
		$\{A B E G\}$
		$\{A B C E G\}$

E & G not in LHS of any FD of F so left with:

$\{A\}^+ = \{A\}$ $\{A B\}^+ = \{A, B, C, E, G\}$ $\{A B C\}^+ = \{A, B, C, E, G\}$
 $\{B\}^+ = \{B, C\}$ $\{A C\}^+ = \{A, C, B, E, G\}$
 $\{C\}^+ = \{C\}$ $\{B C\}^+ = \{B, C\}$

$T =$ $B \rightarrow C, AB \rightarrow C, AB \rightarrow E, AB \rightarrow G$
 $AC \rightarrow B, AC \rightarrow E, AC \rightarrow G,$
 $ABC \rightarrow E, ABC \rightarrow G$

Question 2

1. $R(A, B, C, D, E)$

FDs: $BDE \rightarrow C, C \rightarrow A, E$

a) minkey: BDE

b) \exists BCNF?

$\{BDE\}^+ = \{B, D, E, C, A\}$ ✓

$\{C\}^+ = \{C, A, E\}$ ✗

Not in BCNF

$R1(C, A, E)$

$R2(D, B, E)$

Compute FDs: $R1: C \rightarrow A, E$

$\{A\}^+ = \{A\}$

$\{C\}^+ = \{C, A, E\}$

$\{E\}^+ = \{E\}$

$T = \{C \rightarrow A, C \rightarrow E\}$

C is the key.

$\{C\}^+ = \{C, A, E\}$

So $R1$ is in BCNF

Compute FDs for R2

5 attributes

(A, B, C, D, E)

$$\{B\}^+ = \{B\} \leftarrow \{B, C\} \leftarrow \{B, C, A, E\} \leftarrow \{B, C, A, E, D\}$$

$$\{C\}^+ = \{C, A, E\} \quad \{B, D\}^+ = \{B, D\}$$

$$\{D\}^+ = \{D\} \quad \{C, D\}^+ = \{C, D, A, E\}$$

$$T = \{C \rightarrow A, C \rightarrow E, C, D \rightarrow A, C, D \rightarrow E, B, D\}$$

$$\models A \mid C \rightarrow A \mid E, \{B\} = \{C, D, B\}$$

$$\models \{B, A, C, D\} = \{B, D\}$$

$$\{C\}^+ =$$

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Compute FDs for R2

$$\{B\}^+ = \{B\} \leftarrow \{B, C\} = \{B, C, A, E, D\}$$

$$\{C\}^+ = \{C, A, E\} \quad \{B, D\}^+ = \{B, D\}$$

$$\{D\}^+ = \{D\} \quad \{C, D\}^+ = \{C, D, A, E\}$$

No FDs so BCNF

$$\{B\}^+ = \{B, C, A, E, D\}$$

$$\{C, A, E, D\} = \{B, D\}$$

c) FDs were not produced by decompositions so no

not ok

2. $S(A, B, C, D, E, F)$ (unique)

FDs: $B \rightarrow CE, CD \rightarrow AD$

min {key} BF

Following this logic as B gives you C, E,

C can be used to get A, D. No FD returns F so it must be a key.

b. BCNF?

$\{B\}^+ = \{B, C, E, A, D\}$

$\{C\}^+ = \{C, A, D\}$

So S is not in BCNF, must decompose relation.

$R_1(C, A, D)$

$R_2(C, B, E, F)$

Compute FDs for R_1 .

$\{C\}^+ = \{C, A, D\}$

$\{CA\}^+ = \{C, A, D\}$

~~$\{CA\}^+$~~

~~$\{A\}^+$~~

$\{CD\}^+ = \{C, A, D\}$

~~$\{D\}^+$~~

~~$\{AD\}^+$~~

C is key. $\{C\}^+ = \{C, A, D\}$, $\tau = \{C \rightarrow AD\}$

R_1 is in BCNF

Compute FDs for RZ: (A, B, C, D, E)

~~Let~~ $\{A, B\} \rightarrow \{C, D, E\}$ $\{C, B, E\} \rightarrow \{A, D\}$

$\{B\} \rightarrow \{C, D, E\}$ $\{C, B, E\} \rightarrow \{A, D\}$

~~Let~~ $\{C, E\} \rightarrow \{A, D\}$ $\{B, E, F\} \rightarrow \{C, D, E, A\}$

~~Let~~ $\{B, E\} \rightarrow \{A, D\}$ $\{C, B, E, F\}$

Now we have $\{B, E\} \rightarrow \{A, D\}$ and $\{C, B, E, F\}$

$T = \{B \rightarrow C, E\}$

Key: B, E $\{C, A, D, F\} \rightarrow \{B\}$

$\{B, E\} \rightarrow \{C, D, E, A\}$ $\{C, B, E, F\}$

And RZ is BCNF

Yes, $T = \{B \rightarrow C, E\}$ is a key for RZ

(A, B, C, D, E)

(A, B, C, D, E)