Multiple Signal DoA Estimation with Unknown Electromagnetic Coupling using Gaussian Process

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***Abstract*—In this paper, we employ a Gaussian Process Regression method based on the Polynomial Root Finding technique (PRF-GPR) to address the Direction of arrival (DoA) problem. Arrays whose electromagnetic coupling is taken into account in an exact way are employed in our DoA studies here. The Search-free DoA estimation algorithm is modified to accommodate the exact coupling with this learning-based method, PRF-GPR, which is capable of estimating DoA when multiple signals arrive at a receiver array with unknown electromagnetic coupling in its elements. Simulations are provided to verify the effectiveness of the proposed method.**

Keywords—direction of arrival, electromagnetic coupling, Gaussian process regression, polynomial root finding

I. Introduction

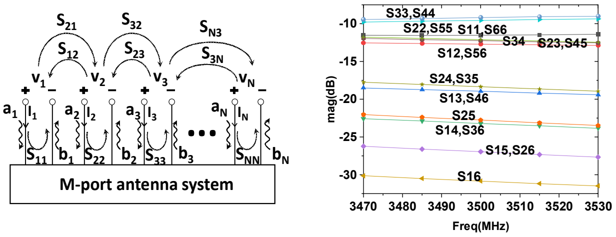
Direction of Arrival (DoA) estimation is a significant technique used in a plethora of applications. A lot of high resolution algorithms have been studied before, including MUSIC method and ESPRIT method. However, some of these methods are based on a simplified, coupling free representation of the array manifold. In the presence of mutual coupling, the performance of simplified array signal processing algorithms is degraded. Taking this issue into consideration, a simple Mutual Coupling Matrix (MCM) for Uniform Linear Arrays (ULA), in the form of a banded symmetric Toeplitz matrix, is proposed in [1]. This model has been used by many investigators to mitigate the perturbations caused by the mutual coupling. Ye *et al.* [2] propose placing auxiliary elements around the original sensors to alleviate the effect of mutual coupling. Unfortunately, this idea requires a large amount of auxiliary sensors. Then, Wang *et al.* [3] construct a selection matrix to truncate the received data, avoiding using extra sensors to construct the auxiliary array. However, the exact electromagnetic coupling is more complicated than the model of a Toeplitz matrix can accommodate and, in several cases, methods based on it fail to estimate DoA very accurately. Impedance matching techniques have been proposed to obtain the correct array manifold. The Conventional Mutual Impedance Method (CMIM) [4] characterizes the effect of mutual coupling. H.T.Hui [5], uses the Receiving-Mutual Impedance Method (RMIM) for the calculation of the open-circuit voltage.

Recently, machine learning methods such as neural networks [6] and Support Vector Regression (SVR) methods [7], [8] have proved powerful for DoA estimation. These methods establish training data with DoA 'labels' (i.e. quantities to be estimated) and then formulate the Multiple Input Single Output (MISO) mapping from array outputs to a signal direction. If we want to classify multiple signals, we need to formulate the MIMO mapping from receive (Rx) array outputs to signal directions. This method is data-driven and the number of training data needs to be large enough to cover different combinations of DOAs. It results in a NP-hard problem (Non-deterministic Polynomial).

Prior work has either ignored electromagnetic coupling among the elements or modeled it in an approximate way. We consider the EM coupling 'exactly' and adjust well known DoA algorithms with GPR in order to handle its effects.

II. Problem Set-up

In our formulation of the problem, we consider the incoming waves as generated by a number of Tx radios each equipped with a single antenna. We assume that the propagation environment is Line of Sight (LoS). Hence, each Tx-Rx pair is connected via a single plane wave.



(a) (b)

Fig. 1 (a) The microwave circuit model of the M-port receiving system. (b) The magnitude of all the EM-exact S-parameters of a 6-port array of dipoles used in the present work.

Ignoring noise and interference, the Rx voltages induced by the incident wave(s) from the Tx radios on the elements of the ULA depicted in Fig. 1 are related by

$$\mathrm{V}_m^{Rx} = \sum_{n}^{N} \mathrm{H}_{mn} \mathrm{V}_n^{Tx} \tag{1}$$

where $m = 1,2,\cdots,M$ is the port index of the Rx ULA, and $n = 1,2,\cdots,N$ is the index of Tx radio. The channel matrix elements, H_{mn}, are calculated via an electromagnetics exact approach [9]. Here we have made the tacit assumption that *N*, the number of Tx radios in (1), is also the number of incoming waves (since we have LoS propagation environment). In this approach, the Rx ULA is represented by its S-par matrix plus the active E-field gains of each port [9]. The latter capture all the mutual coupling and even the termination effects of the antenna elements in the Rx antenna system. The S-par matrix assumes the traditional definition, i.e. $\boldsymbol{b} = \boldsymbol{S} \cdot \boldsymbol{a}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively, represent the incident and reflected power waves into the M-port microwave circuit ports.

Adding noise to (1) and concatenating all the M ports and T time instances, the sensor response of the Rx can be modeled as

$$\mathbf{X} = \mathbf{V}^{Rx} + \mathbf{N} = \mathbf{H}\mathbf{V}^{Tx} + \mathbf{N} \tag{2}$$

where $\mathbf{V}^{Rx}$ is the received signal matrix, $\mathbf{V}^{Tx}$ is the source signal matrix, $\mathbf{H}$ is the channel matrix and $\mathbf{N}$ represents the received Gaussian noise matrix with zero mean and variance σ_n^2.

III. Proposed Method

In this section, we propose a learning-based method named PRF-GPR that is more robust to unknown electromagnetic coupling and is capable of estimating the DoA of multiple signals.

A. Polynomial Root Finding Technique

In traditional MUSIC [10], the $M \times M$ array output covariance matrix, $\mathbf{R}$, can be expressed as a superposition of a signal subspace, $\mathbf{U}_s$, and a null subspace, $\mathbf{U}_n$, as

$$\mathbf{R} = \mathbb{E}[\mathbf{X}\mathbf{X}^{\boldsymbol{H}}] = \mathbf{U}_s\boldsymbol{\Lambda}_s\mathbf{U}_s^H + \sigma_n^2\mathbf{U}_n\mathbf{U}_n^H \tag{3}$$

Ignoring mutual coupling, the steering vector $\mathbf{A}(\theta)$ is orthogonal to the null subspace and forms the MUSIC pseudospectrum. That is, for a ULA of inter-element spacing d,

$$\mathbf{A}(\theta) = \left[1, e^{-jkdcos(\theta)}, \cdots, e^{-j(M-1)kdcos(\theta)}\right]^T \tag{4}$$

and

$$\mathbf{A}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}(\theta) = 0 \tag{5}$$

To reduce the computational burden, a search-free PRF method [11] has been proposed, where $\mathbf{A}(\theta)$ is expressed as $\boldsymbol{p}(z) = [1, z, \cdots, z^{M-1}]^T$ and (5) is reduced into the following polynomial equation:

$$\boldsymbol{p}^T\left(\frac{1}{z}\right)\mathbf{U}_n\mathbf{U}_n^H\boldsymbol{p}(z) = 0 \tag{6}$$

Under this imaginary exponential definition of z, the polynomial in (6) is real $\forall$z (since $\mathbf{U}_n\mathbf{U}_n^H$ is Hermitian) and, therefore, its $2M-2$ roots come in complex conjugate pairs. The $2N$ roots closest to the unit circle are the incoming signal solutions and are denoted as $\hat{z}_1, \cdots, \hat{z}_{2N}$. In the absence of mutual coupling, the angle θ_i can be estimated via

$$\theta_i = arccos\left(-\frac{angle(\hat{z}_i)}{kd}\right)\ ;\ i = 1, \cdots, N \tag{7}$$

When unknown electromagnetic coupling is taken into consideration, this method fails to estimate DoA accurately. Gaussian Process Regression is employed to modify this approach and accommodate the exact coupling.

B. Gaussian Process Regression

Gaussian Process (GP) [12] is a non-parametric generalization of Gaussian probability distribution; therefore, we do not have to worry about whether it is possible for the model to fit the data. The traditional GPR has been proven to be an effective tool in the prediction of continuous quantities consistent with the obtained variables [13]. The Gaussian process can be denoted as:

$$f(x) \backsim \mathcal{GP}\big(\mu, k(x, x')\big) \tag{8}$$

where μ is the mean function and $k(x, x')$ is the covariance (kernel) function. The latter can be interpreted as a similarity metric, mapping a pair of inputs (x, x') into a higher dimension. We use the following squared-exponential kernel function to capture the correlation of inputs.

$$k(x, x') = \sigma_f^2 \exp\left(-\frac{\|x - x'\|^2}{2\ell^2}\right) \tag{9}$$

where the hyper-parameters σ_f and ℓ are scale factor and length-scale, respectively. Here both of them have a default value of one.

For a real GP f, training data is denoted as:

$$\mathcal{D} = \{(x_i, y_i)\}_{i=1}^N \tag{10}$$

where $x_i \in \mathbb{R}^d$ is an input vector of dimension d. $y_i \in \mathbb{R}$ denotes a scalar output or target generated by a nonlinear function $f(x_i)$ with an additive Gaussian error in the value of the angle $\epsilon_i \sim \mathcal{N}(0, \delta^2)$

$$y_i = f(x_i) + \epsilon_i \tag{11}$$

In the paper, we assume polynomial roots as the input vector x_i and true DoA as target y_i.

We assume that the pairs $\mathcal{D}$ satisfy the GP, in other words, they have a joint Gaussian distribution, i.e.

$$[f(x_1), f(x_2), \cdots, f(x_N)]^T \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{K}) \tag{12}$$

where $\boldsymbol{\mu} = [\mu(x_1), \mu(x_2), \cdots, \mu(x_N)]^T$ is a vector of mean function, and $\boldsymbol{K}$ is a matrix of $N \times N$ with $\boldsymbol{K}_{ij} = k(x_i, x_j)$. The problem is now to infer target values $\boldsymbol{f}_*$ for unknown inputs $\boldsymbol{Z}$ with $\boldsymbol{Z} = [z_1, \cdots, z_P]^T$. According to GP, the joint distribution of the training outputs, $\boldsymbol{y}$, and the test GP $\boldsymbol{f}_*$ is

$$\begin{bmatrix}\boldsymbol{y}\\ \boldsymbol{f}_*\end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix}\boldsymbol{\mu}(\boldsymbol{X})\\ \boldsymbol{\mu}(\boldsymbol{Z})\end{bmatrix}, \begin{bmatrix}\boldsymbol{K}(\boldsymbol{X},\boldsymbol{X}) + \delta^2\mathbf{I} & \boldsymbol{K}(\boldsymbol{X},\boldsymbol{Z})\\ \boldsymbol{K}(\boldsymbol{Z},\boldsymbol{X}) & \boldsymbol{K}(\boldsymbol{Z},\boldsymbol{Z})\end{bmatrix}\right) \tag{13}$$

Without loss of generality and for computational simplicity, the $\boldsymbol{\mu}(\boldsymbol{X})$ and $\boldsymbol{\mu}(\boldsymbol{Z})$ are assumed to be zero. Based on the conditional property of Gaussian distribution, the predictive distribution of $\boldsymbol{f}_*$ corresponding to $\boldsymbol{Z}$ can be inferred as

$$\boldsymbol{f}_*|\boldsymbol{X}, \boldsymbol{y}, \boldsymbol{Z} \sim \mathcal{N}\big(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}\big) \tag{14}$$

The predicted mean $\hat{\boldsymbol{\mu}}$ and derivation $\hat{\boldsymbol{\Sigma}}$ in (14) should be:

$$\hat{\boldsymbol{\mu}} = \boldsymbol{K}(\boldsymbol{Z},\boldsymbol{X})(\boldsymbol{K}(\boldsymbol{X},\boldsymbol{X}) + \delta^2\mathbf{I})^{-1}\boldsymbol{y} \tag{15}$$

$$\hat{\boldsymbol{\Sigma}} = \boldsymbol{K}(\boldsymbol{Z},\boldsymbol{Z}) - \boldsymbol{K}(\boldsymbol{Z},\boldsymbol{X})\left(\boldsymbol{K}(\boldsymbol{X},\boldsymbol{X}) + \delta^2\mathbf{I}\right)^{-1}\boldsymbol{K}(\boldsymbol{X},\boldsymbol{Z}) \tag{16}$$

In order to get more accurate predictive distribution, the hyper-parameters of $\beta = \{\delta, \sigma_f, \ell\}$ need to be optimized from the training data $\mathcal{D}$. For hyper-parameter optimization, we use an optimizer introduced in [12] to minimize the negative log of the marginal likelihood.

IV. Simulation

In this section, numerical simulation results for the DoA are presented to illustrate the performance of our proposed method. A standard ULA of $M = 6$ sensors with element spacing $d = 0.4\lambda$ is considered (i.e. kd=0.8π) with λ being the wavelength of the incoming waves. The antenna is designed at the frequency of 3.5GHz. The electromagnetic coupling is considered exactly via a Method of Moments (MoM) full wave simulation of the dipole ULA as a 6-port

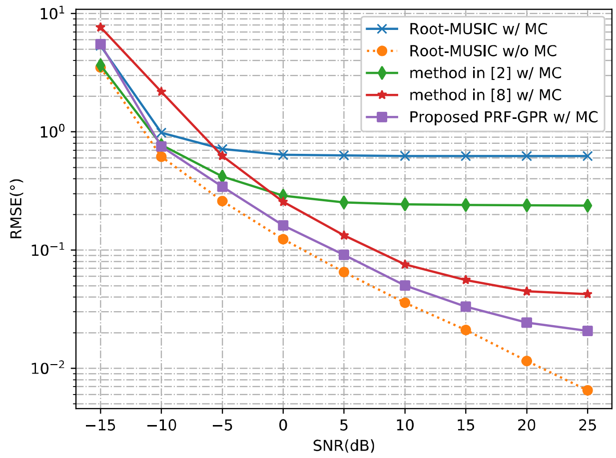
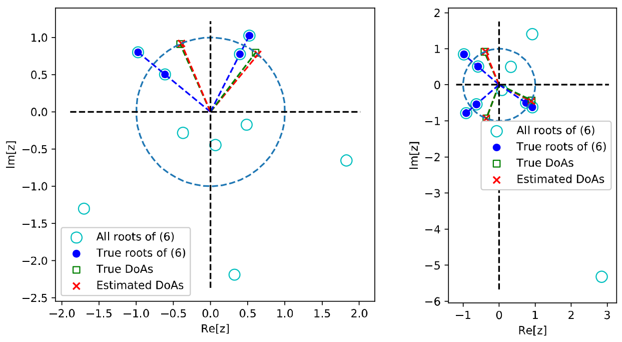


Fig. 2 The RMSE of DoA estimation with different methods versus SNR. The number of snapshots is T=1000. The number of training data is L=200.



(a) Two DoAs (b) Three DoAs

Fig. 3 The effect of mutual coupling (T=1000, L=200, SNR=5).

antenna system. The Root Mean Square Error (RMSE) of the DoA angle is used as the performance metric.

The set of 1000 different DoA angles that are uniformly distributed in the range of $[-180°, 0°]$ is generated by geometric means. In the training phase, a small amount of DoAs are chosen uniformly as the training dataset. We then chose the remaining DoAs to form the test dataset which is compared to the estimated DoA in the testing stage. The generation of the training data set would form the so called “calibration” of the array under real life deployment of our proposed method.

In Fig. 2 we compare the performance of different methods versus SNR. Previous works, e.g. [1], [8], take only the nearest neighbor mutual coupling into account and approximate it by a single number (i.e. ignoring the presence in the array of those nearest neighbors). In effect, they multiply the steering vector with a Toeplitz matrix. We consider the EM-exact mutual coupling as described by the full wave S-par matrix of the Rx 6-port antenna system. In all four cases labeled “w/MC”, our steering vector is the EM-exact channel matrix, $\mathbf{H}$. The curve indicated as “method in [2] w/MC” uses an auxiliary array to decouple the mutual coupling effect. The curve indicated as “ method in [8] w/MC” uses the same idea as in [8] augmented by our full wave EM-exact mutual coupling. It considers the upper diagonal elements of the covariance matrix as features. However, in cases of multiple incoming waves, this is a NP-hard problem. The Root-MUSIC algorithm suffers severe degradation in the presence of unknown electromagnetic coupling. The “Root-MUSIC w/o MC” is unrealizable as it assumes that there is no coupling in the antennas. Therefore, its performance shown here cannot be obtained in practice. Our proposed method has overall performance superior to others in the presence of mutual coupling.

In Fig. 3, two and three equal magnitude sources are considered at 5 dB SNR. The 2M-2 roots, see (6), are shown. The 2N roots that are closest to the unit circle are the solutions with the effect of the EM coupling. The estimated DoAs of the proposed method are close to the true DoAs. The proposed method achieves good accuracy (Table I).

TABLE I MULTIPLE SIGNAL DOA ESTIMATION WITH MUTUAL COUPLING

	N=2		N=3		
True DoAs (°)	-39.00	-68.99	-39.00	-100.01	-138.99
MUSIC w/MC (°)	-16.59	-64.61	-18.83	-103.35	-161.49
Proposed method (°)	-39.53	-69.59	-40.20	-100.78	-137.95

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