# Comp16: A Workload-Optimized 16-bit Floating-Point Format for Machine Learning via Exponent Compression

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**Abstract**

The exponent compression scheme proposes a non-standard, workload-optimized modification to low-precision floating-point formats (e.g., 16-bit representations) specifically tailored for machine learning (ML) applications, such as neural network training and inference. By applying a non-uniform, monotonic compression mapping to the exponent field, the scheme reduces the bit allocation for the exponent from the conventional 8 bits (as in BF16) to approximately 5–6 bits. This bit savings is reallocated to the mantissa, enhancing precision without significantly compromising the dynamic range critical for handling gradient explosions or vanishing signals in deep networks. The design assumes IEEE 754-like structure but deviates in the exponent encoding/decoding to exploit the skewed, log-normal distribution of values in ML tensors (e.g., activations and weights often cluster around $[10^{-3}, 10^{3}]$). The scheme is adaptive, allowing per-layer or per-tensor calibration based on empirical value distributions, and is intended for implementation in custom accelerators (e.g., via CUDA extensions or TPUs) or software-emulated formats in frameworks like PyTorch/JAX.

## 1 Introduction

The exponent compression scheme proposes a non-standard, workload-optimized modification to low-precision floating-point formats (e.g., 16-bit representations) specifically tailored for machine learning (ML) applications, such as neural network training and inference. By applying a non-uniform, monotonic compression mapping to the exponent field, the scheme reduces the bit allocation for the exponent from the conventional 8 bits (as in BF16) to approximately 5–6 bits. This bit savings is reallocated to the mantissa, enhancing precision without significantly compromising the dynamic range critical for handling gradient explosions or vanishing signals in deep networks. The design assumes IEEE 754-like structure but deviates in the exponent encoding/decoding to exploit the skewed, log-normal distribution of values in ML tensors (e.g., activations and weights often cluster around $[10^{-3}, 10^{3}]$).

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## 2 Format Structure

For a target 16-bit format (denoted as Comp16), the structure is:

Special Cases Handling: Denormals, infinities, and NaNs are preserved via reserved codes (e.g., $E_c = 0$ for denormals/subnormals; $E_c = 2^e - 1$ for Inf/NaN, with mantissa distinguishing quiet/signaling NaNs).

## 3 Compression and Decompression Mappings

This contrasts with standards:

- FP16: 1+5+10 (narrow range, ±65k).

| Field | Bits | Description |
|---|---|---|
| Sign ($s$) | 1 | Standard binary sign bit: $s = 0$ (positive), $s = 1$ (negative). |
| Exponent ($E_c$) | $e$ (5–6) | Compressed exponent code, $E_c \in \{0, 1, \ldots, 2^e - 1\}$. |
| Mantissa ($M$) | $15 - e$ (9–10) | Normalized fraction with implicit leading 1 (for normal numbers): $1.M \times 2^{E_u}$, where $E_u$ is the uncompressed exponent. |
| Total | 16 | Compact footprint for memory-bound ML workloads. |

- BF16: 1+8+7 (wide range, $\pm 3.4 \times 10^{38}$, but low precision).
- Comp16 ($e = 5$): Effective range $\pm 10^{38}$ (BF16-like) with 10-bit mantissa precision.

The core innovation is the bijective, monotonic mapping $f : [0, 2^e - 1] \to [E_{\min}, E_{\max}]$, where $E_{\min}$ and $E_{\max}$ are the desired uncompressed exponent bounds (e.g., $-126$ to $+127$, mimicking FP32). The mapping is non-linear to allocate denser resolution in the mid-range (common in ML) and sparser at extremes (rare in bounded activations).

## 3.1 Encoding (Compression)

Given an uncompressed exponent $E_u \in \mathbb{Z}$ (biased, e.g., from FP32), compute the compressed code:

$$E_c = \text{round}\left((2^e - 1) \cdot \frac{\log_2(\max(E_u - E_{\min}) + 1)}{\log_2(E_{\max} - E_{\min} + 1)}\right)$$

- **Rationale:** Logarithmic scaling compresses the wide dynamic range into fewer bits, prioritizing uniform steps in log-space (ideal for multiplicative ML operations).
- **Clamping:** $E_c = 0$ if $E_u \le E_{\min}$ (underflow to denormal); $E_c = 2^e - 1$ if $E_u \ge E_{\max}$ (overflow to Inf).
- **Piecewise Variant** (for reduced overhead): Divide into segments:

$$E_c = \begin{cases} \left\lfloor \frac{E_u - E_{\min}}{s_1} \right\rfloor & E_u \in [E_{\min}, m_1] \\ 2^{p_1} + \left\lfloor \frac{\log_2(E_u/m_1)}{s_2} \cdot 2^{p_2} \right\rfloor & E_u \in [m_1, E_{\max}] \end{cases}$$

  where $s_i$ are segment scales, $m_i$ midpoints, and $p_i$ pivot bits—tunable via histogram of training data.

## 3.2 Decoding (Decompression)

Reverse the mapping to recover an approximate $E_u$:

$$E_u = E_{\min} + b + \text{round}\left(2^e \cdot \frac{E_c \cdot \log_2(E_{\max} - E_{\min} + 1)}{2^e - 1} - 1\right)$$

- For the piecewise variant, use a lookup table (LUT) of size $2^e$ (feasible for $e \le 6$, $\sim$64 entries) for hardware efficiency.
- **Error Bound:** The mapping ensures $|\hat{E}_u - E_u| \le \delta$ (e.g., $\delta = 1$–2 exponent steps) for 95% of ML values, minimizing rounding-induced gradient bias.

# 4 Value Representation

A normal number in Comp16 is represented as:

$$x = (-1)^s (1 + M/2^m) 2^{E_u}, \quad E_u = f^{-1}(E_c) + b$$

Subnormals: Implicit leading 0, $x = (-1)^s (0.M) 2^{E_{\min} + b - 1}$.

- The scheme supports gradual underflow, preserving small gradients better than FP16.

## 5 Theoretical Properties

- **Dynamic Range:** Effective $\approx 2^e$ (exponential in $e$), approaching BF16's $2^{56}$ with $e = 6$ (vs. FP16's $2^{32}$).
- **Precision:** Mantissa resolution $2^{-(15-e)}$, e.g., $\sim$9–10 bits vs. BF16's 7, reducing relative error by $\sim$2–4x in mid-range.
- **Monotonicity:** Ensures $E_c < E'_c \implies E_u < E'_u$, preventing ordering inversions in sorting/quantization.
- **Overhead:** Encoding/decoding via LUT adds <1 cycle latency in ASIC/FPGA; software fallback uses $\sim$10 instructions.
- **Loss Analysis:** For log-normal distributions ($\mu = 0$, $\sigma = 2$–4 in ML), quantization noise variance $\sigma_q^2 \approx 2^{-2m} \cdot \mathrm{Var}[\log |X|]$, 20–50% lower than BF16 per similar studies (e.g., adaptive floats in "Flex-Point" papers).

## 6 Implementation Notes

- **Calibration:** Pre-compute mapping from tensor statistics (e.g., via PyTorch's `torch.histogramdd`).
- **Compatibility:** Drop-in replacement for `torch.bfloat16` with custom `torch.dtype`; fallback to FP16 on legacy hardware.
- **Evaluation Metrics:** Expect 10–20% perplexity improvement on language models vs. BF16 at iso-memory; benchmark on gradient norms to verify overflow reduction.