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FAI Homework 4

Foundations of AI
June 8, 2022

1.

$$\rho(s) = \frac{s}{1 + exp(-s)}$$
$$\rho'(s) = \frac{(1 + exp(-s)) - s(-exp(-s))}{(1 + exp(-s))^2}$$
$$= \frac{exp(-s)(s+1) + 1}{(1 + exp(-s))^2}$$

2.
- Maximum:
  5% of class-4 examples are mislabeled as class-3. And its cost is $20N$
- Minimum:
  5% of class-1 examples are mislabeled as class-2. And its cost is $0.05N$

3.
- Maximum total number of weights:
  $L = 2, d^{(0)} = 10, d^{(1)} = 50, d^{(2)} = 50$
  total number of weights = 3000
  Feed-Forward Neural network
- Minimum total number of weights:
  $L = 2, d^{(0)} = 10, d^{(1)} = 1, d^{(2)} = 99$
  total number of weights = 109
  Feed-Forward Neural network

4. $u(\mathbf{x}) = 1 - 2|\theta_{\mathbf{w}}(\mathbf{x}) - \frac{1}{2}|$, and $0 < \theta_{\mathbf{w}}(\mathbf{x}) < 1$.

   To maximize $u(\mathbf{x})$, we have to make $\theta_{\mathbf{w}}(\mathbf{x})$ as close to $\frac{1}{2}$ as possible.

   Let $f(z) = \frac{1}{1+e^{-z}}$. The problem is to find $\arg\min_z |f(z) - \frac{1}{2}|$.

$$f(z) + f(-z) = \frac{1}{1 + e^{-z}} + \frac{1}{1 + e^{z}}$$
$$= \frac{e^z + 1}{1 + e^z} = 1$$
$$\Rightarrow |f(z) - \frac{1}{2}| = |f(-z) - \frac{1}{2}| = |f(|z|) - \frac{1}{2}|$$

   In addition, $|f(|z|) - \frac{1}{2}|$ increases as $|z|$ becomes larger.

   $\Rightarrow \arg\min_z |f(z) - \frac{1}{2}| = \arg\min_z |z|$

   $\Rightarrow \arg\max_{n=1,2,\ldots,N} u(\mathbf{x}_n) = \arg\min_{n=1,2,\ldots,N} |f(\mathbf{w}^T\mathbf{x}_n) - \frac{1}{2}| = \arg\min_{n=1,2,\ldots,N} |\mathbf{w}^T\mathbf{x}_n|$

5. `PCA fit`:

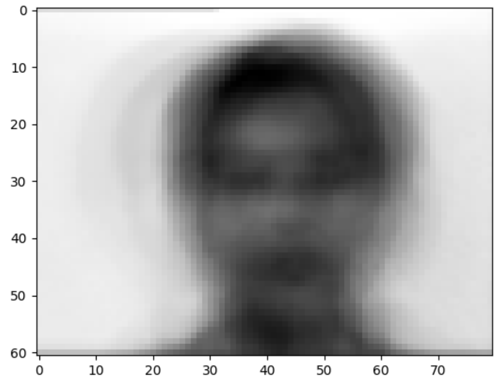


Figure 1: Mean Face

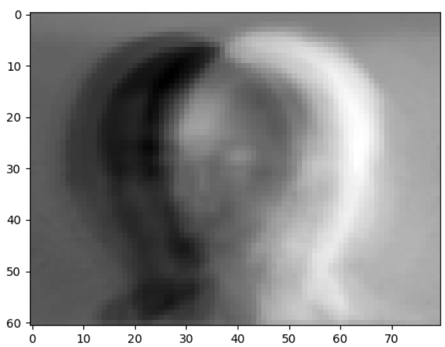


Figure 2: Eigenface 1

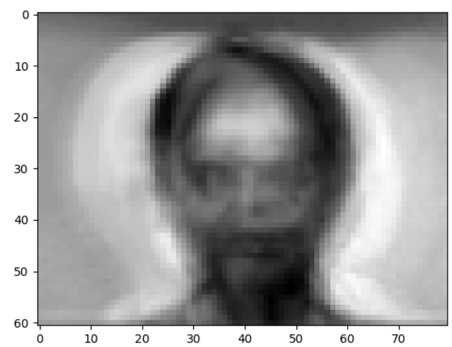


Figure 3: Eigenface 2

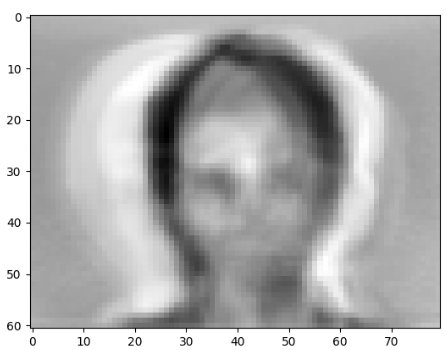


Figure 4: Eigenface 3

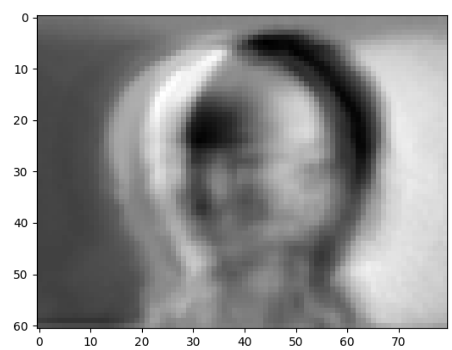
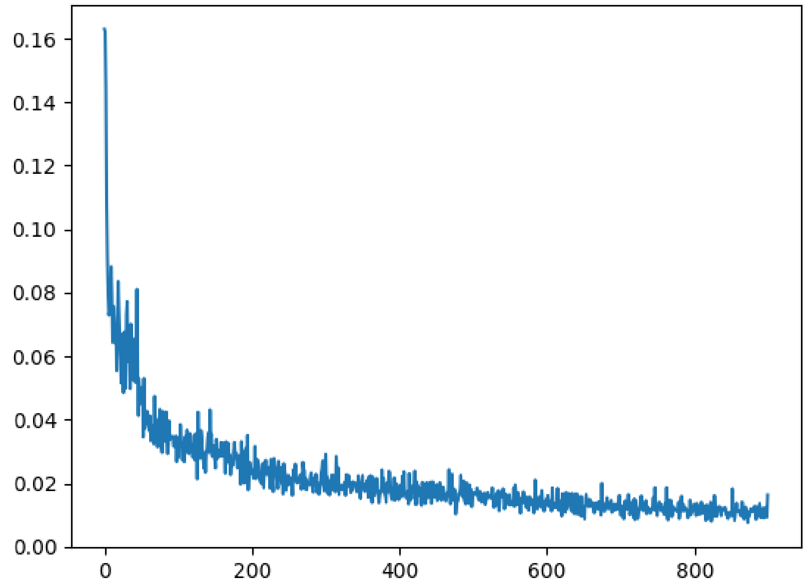
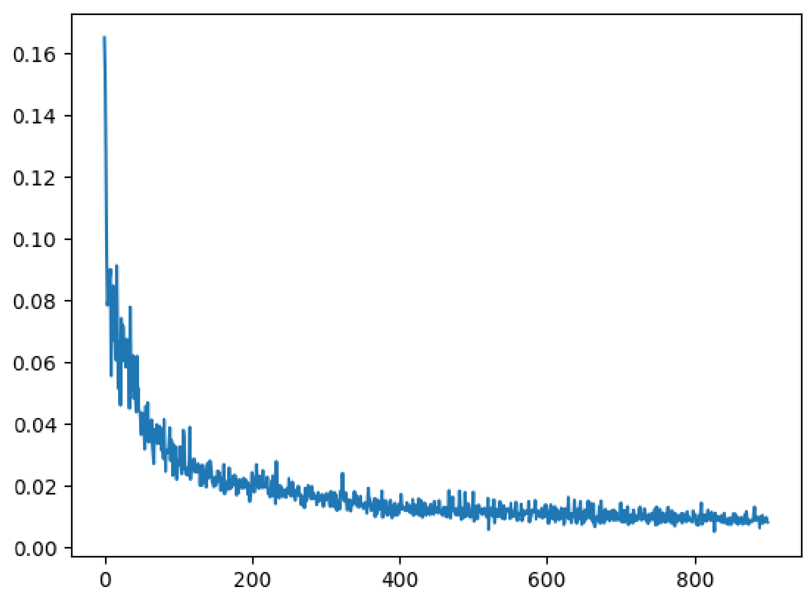


Figure 5: Eigenface 4

6. `Autoencoder fit loss`:

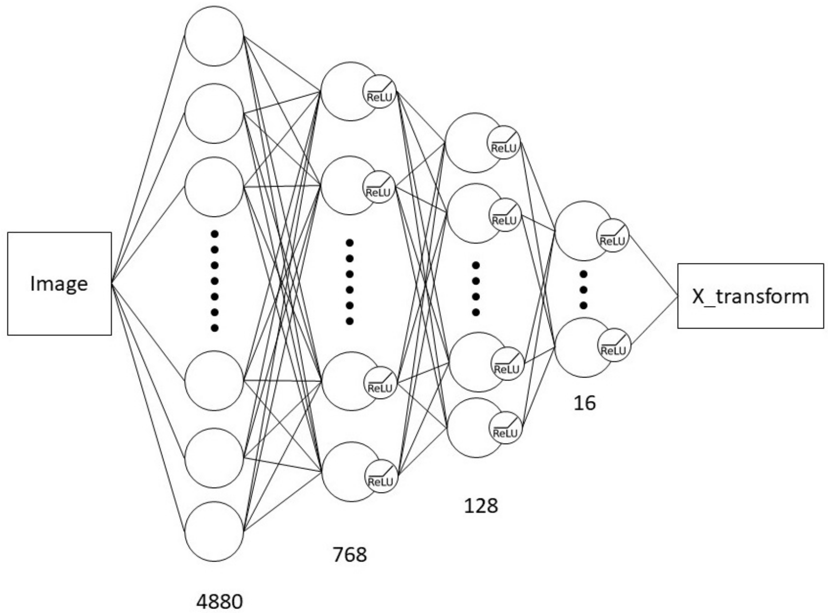


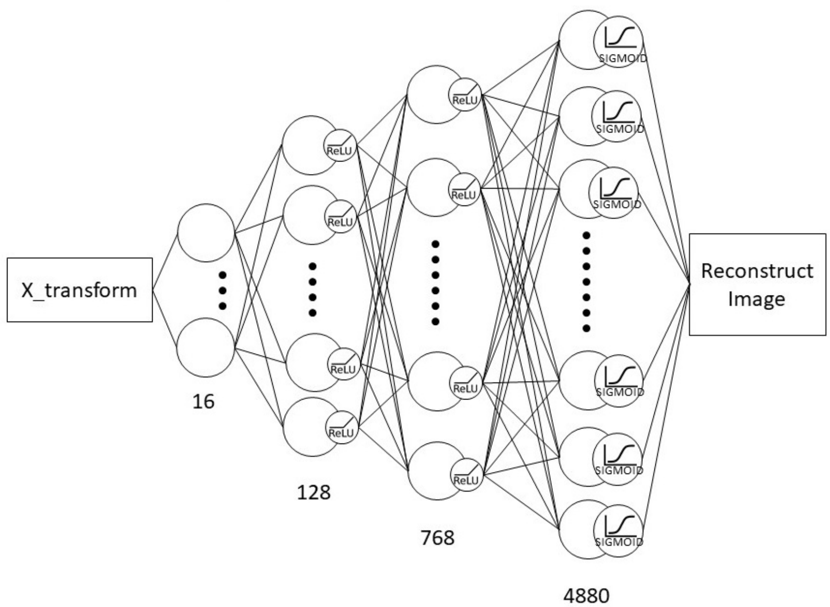
7. `DenoisingAutoencoder fit loss`:



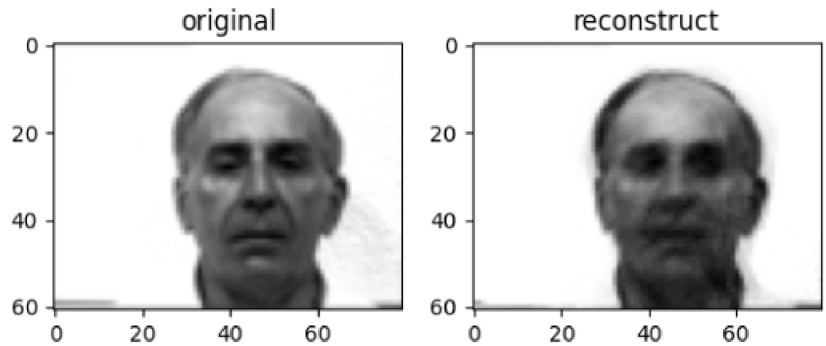
8. (a) sample_architecture

- Architecture:





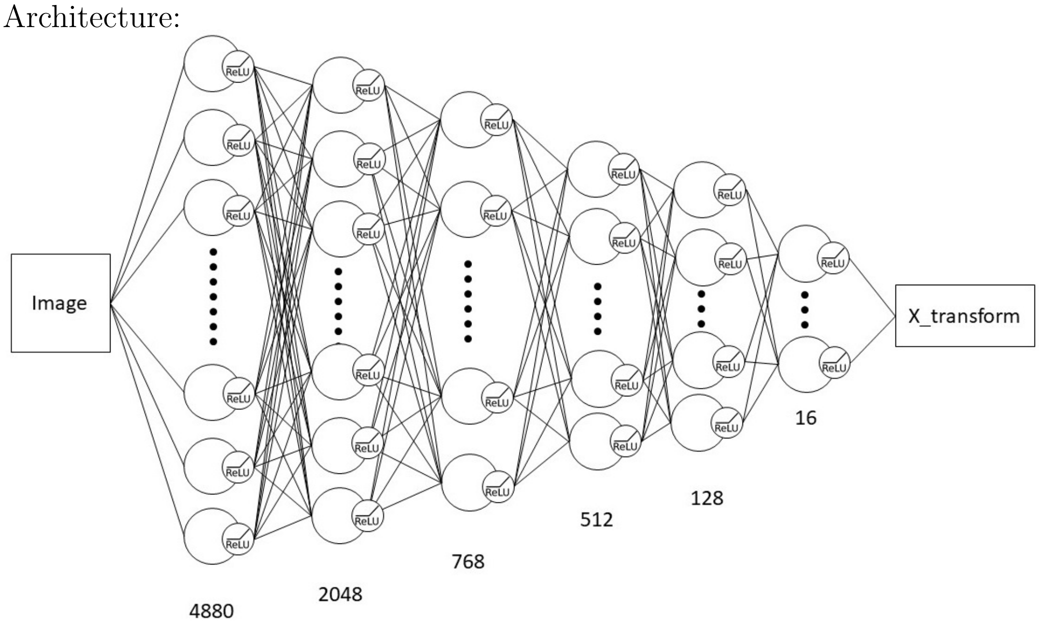
- Original image v.s. Reconstructed image

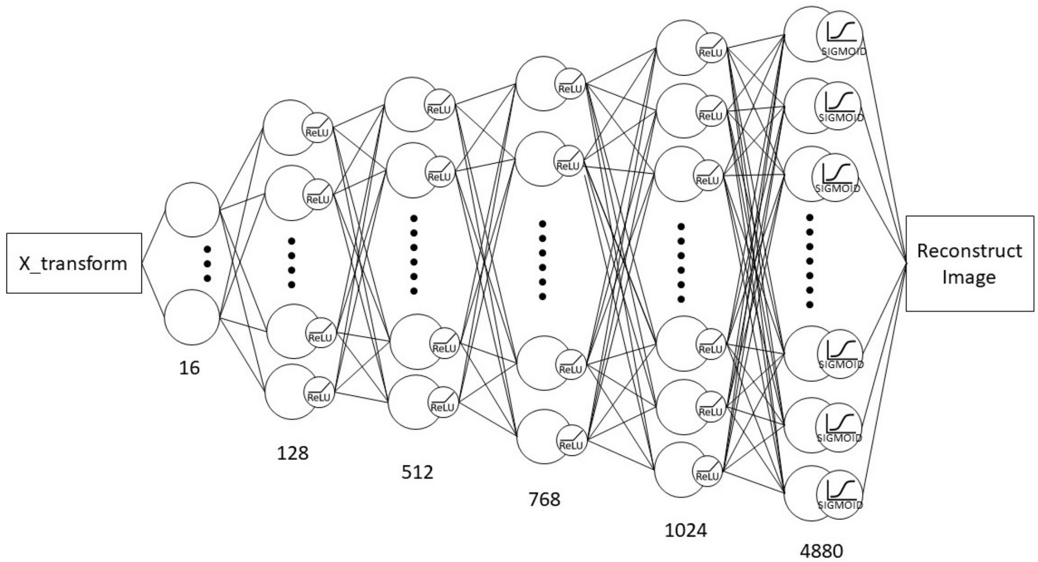


- Mean Square Error (MSE) and Validation Accuracy (VA)
  MSE: 0.0017002755, VA: 0.7
- Findings and discussion
  This is the original model provided in the sample code. The model performs well, with 0.0017 mean square error.

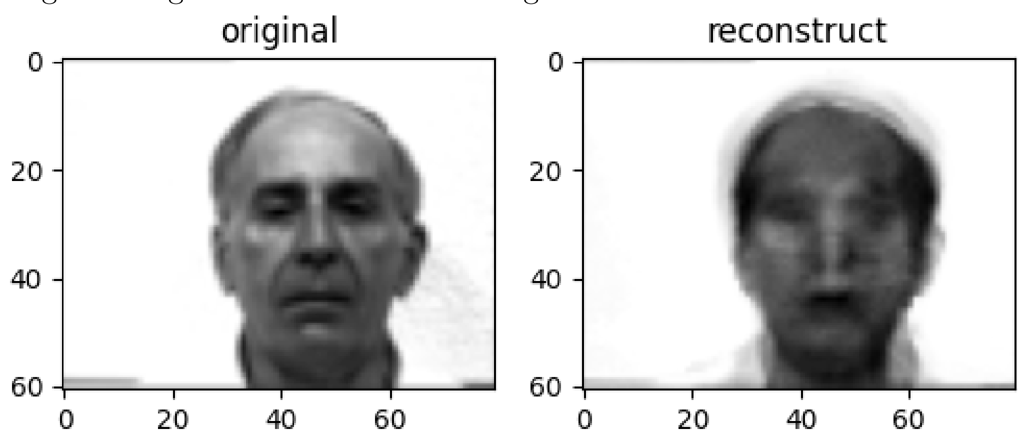
(b) architecture2

- Architecture:





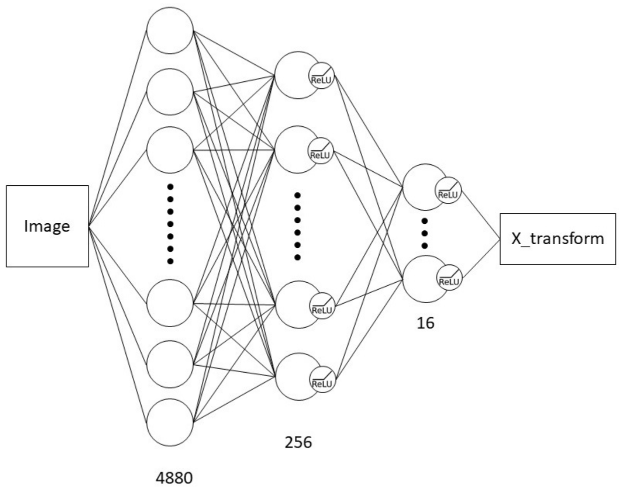
- Original image v.s. Reconstructed image

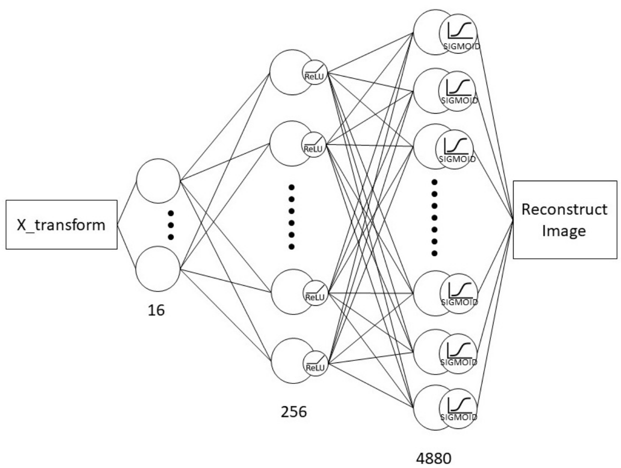


- Mean Square Error (MSE) and Validation Accuracy (VA)
  MSE: 0.011790511, VA: 0.6
- Findings and discussion
  Initially, I expected that it would result in a better image. But it turned out that the deeper model did not have a better outcome. The MSE is about 0.0118, and the image is quite blurry. It was probably because the model overfit the noise.

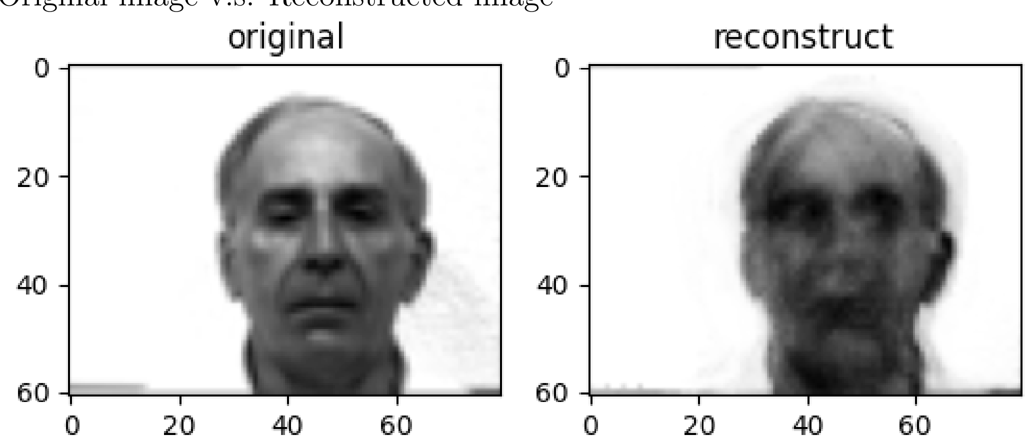
(c) architecture3

- Architecture:





- Original image v.s. Reconstructed image

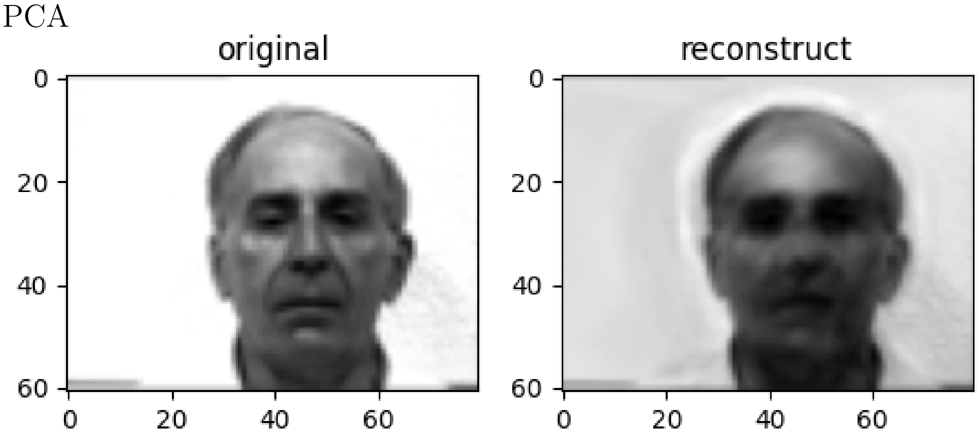


- Mean Square E rror (MSE) and Validation Accuracy (VA)
  MSE: 0.008102858, VA: 0.5333333333333333

- Findings and discussion
  I expected that the shallower model would have a worse reconstruction image than the original model. The result showed that my expectation was right. But the MSE of the shallower model (0.0081) is better than the MSE of the deeper model (0.0118). That was out of my expectation.

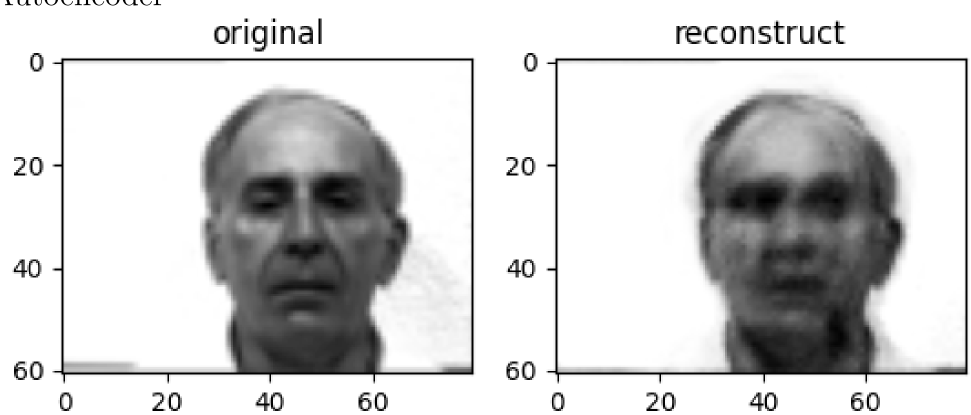
9. Original image v.s. Reconstructed image

- PCA



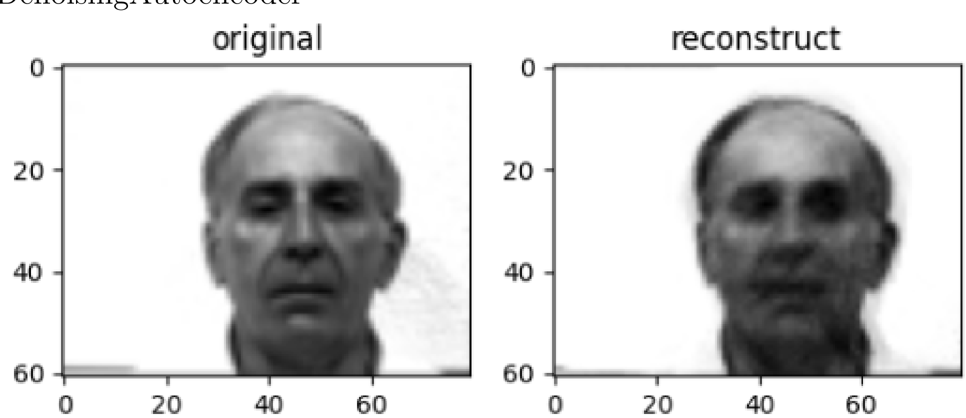
Mean Squared Error (MSE)= 0.0023547083

- Autoencoder



Mean Squared Error (MSE)= 0.0029266474

- DenoisingAutoencoder



Mean Squared Error (MSE)= 0.0017002755

10. Validation accuracy:
    - PCA: 0.9
    - Autoencoder: 0.7333333333333333
    - DenoisingAutoencoder: 0.7