Frequency Domain Filtering

Group 3
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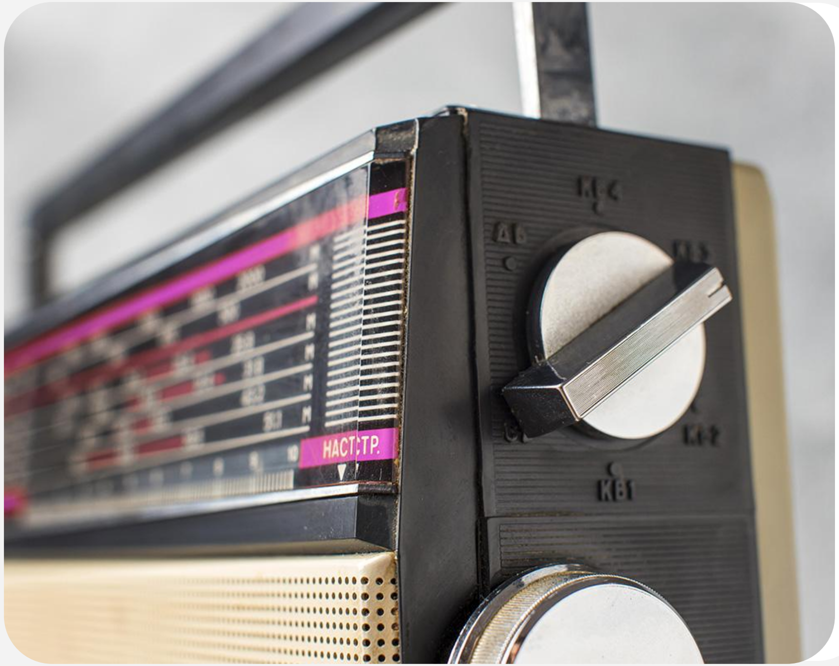
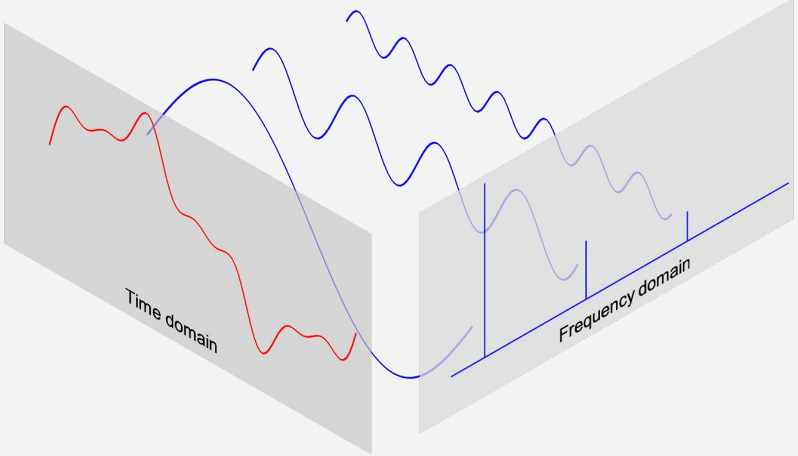
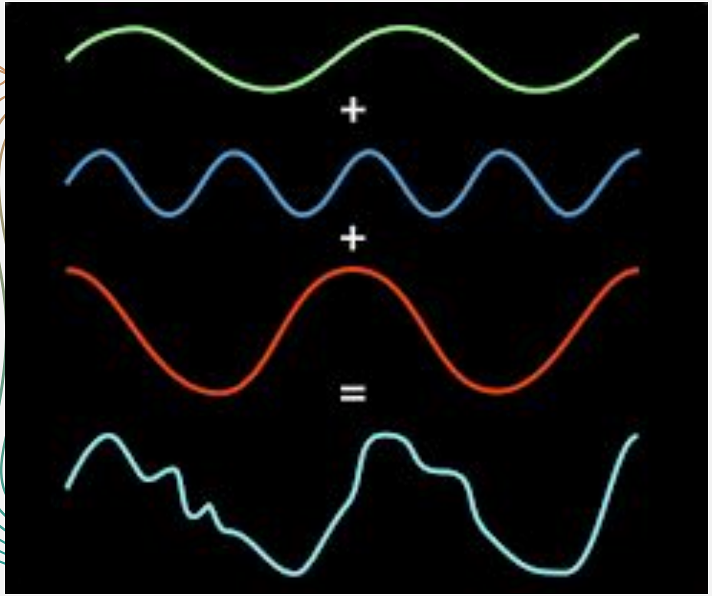


01
Pengantar

Wave Components

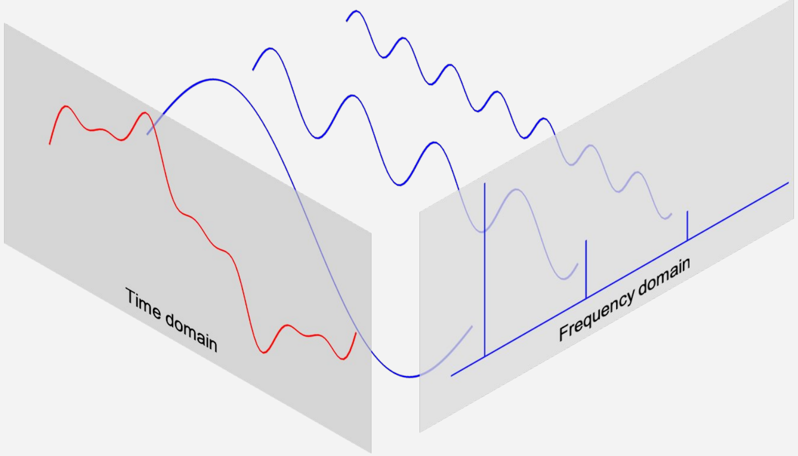
Sinyal **kompleks** terbentuk dari gelombang-gelombang **sederhana**





Fourier Series

Metode **mengdekomposisi sinyal periodik** menjadi banyak gelombang



$$x(t): \quad 0 \le t \le T$$
$$X[k]: \quad k = \dots, -2, -1, 0, 1, 2, \dots$$

$$X[k] \;=\; \frac{1}{T}\int_{t=0}^{T} x(t)\, e^{-jk\omega_0 t} dt$$

$$x(t) \;=\; \sum_{k=-\infty}^{\infty} X[k]\, e^{jk\omega_0 t}$$

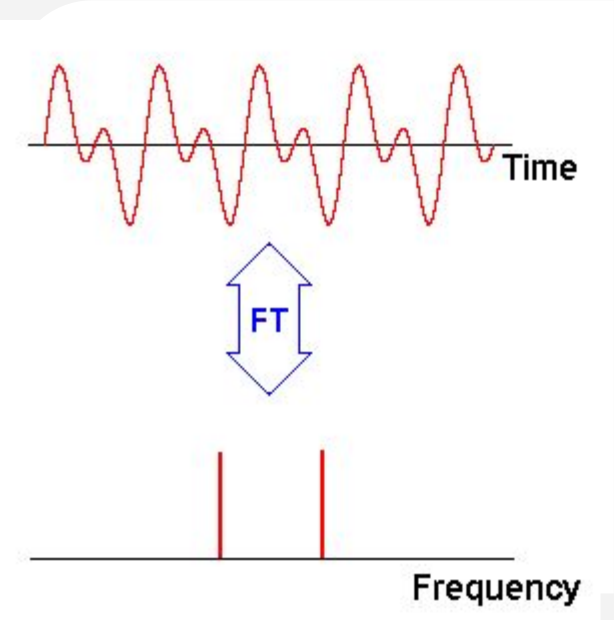
Fourier Transform

Metode melihat sinyal dalam **Domain Frekuensi**

$$x(t): \quad -\infty < t < \infty$$
$$X(\omega): \quad -\infty < \omega < \infty$$

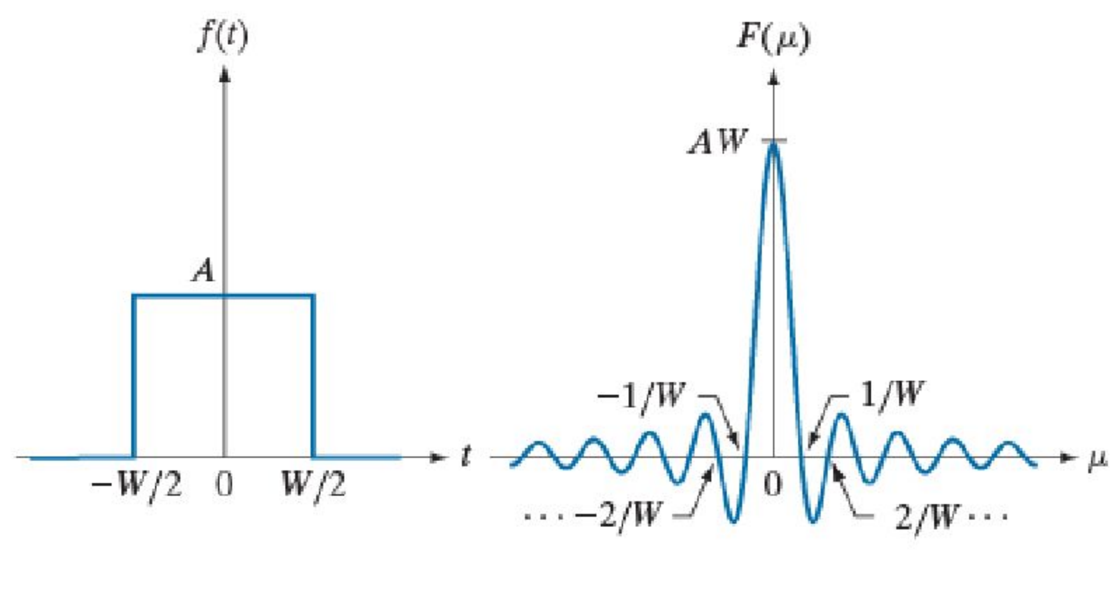
$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(\omega)\, e^{j\omega t} d\omega$$



Non Periodic Fourier Transform

Transformasi Fourier untuk sinyal non-periodik: Konten **frekuensi kontinu**

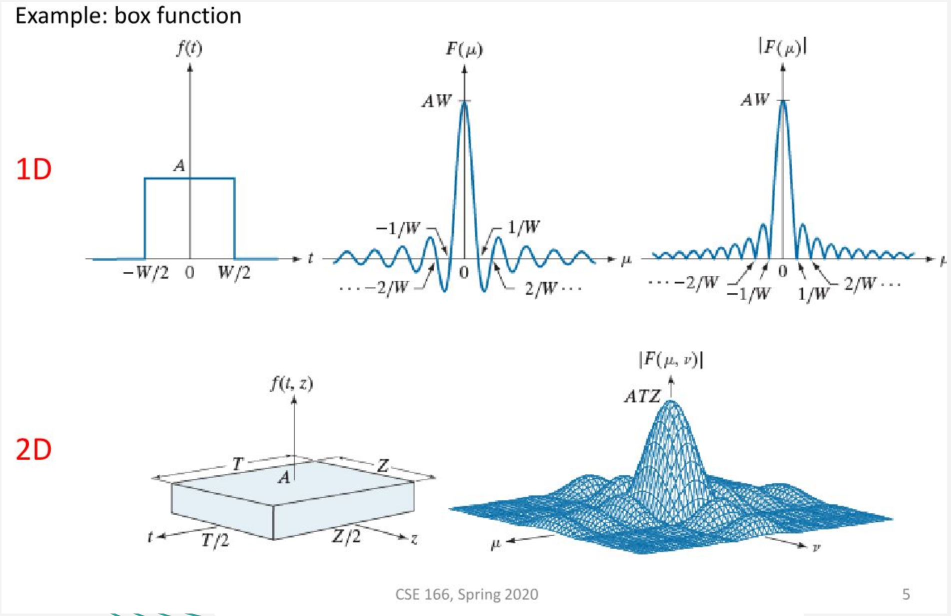


2D Fourier Transform

Transformasi Fourier 2D: Box function **dua dimensi**

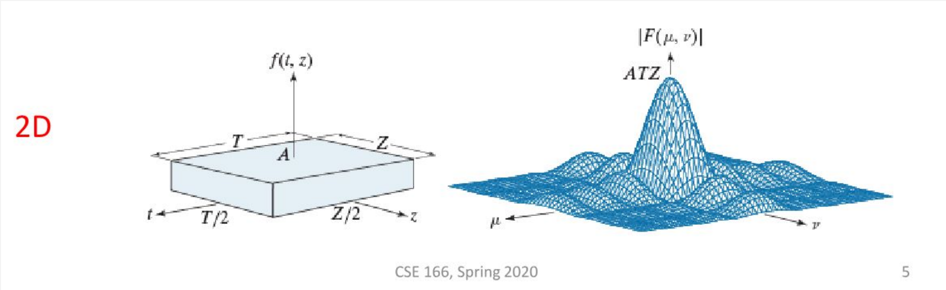
$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$
$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

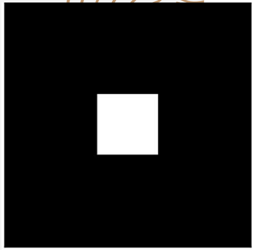
$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dxdy$$
$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} dudv$$

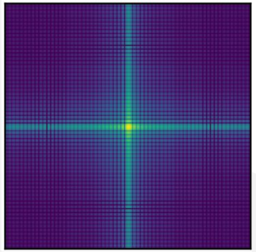


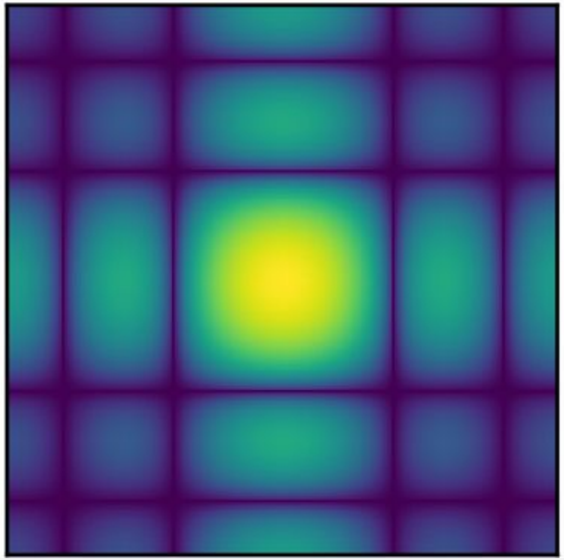
2d Image

Contoh aplikasi Transformasi Fourier 2D pada gambar









DFT

2D discrete Fourier transform (DFT)

- (Forward) Fourier transform

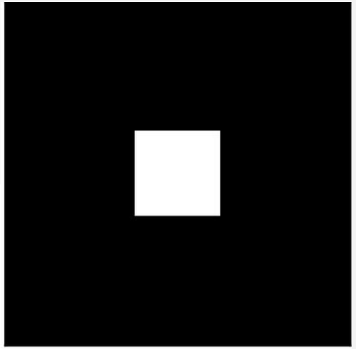
$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

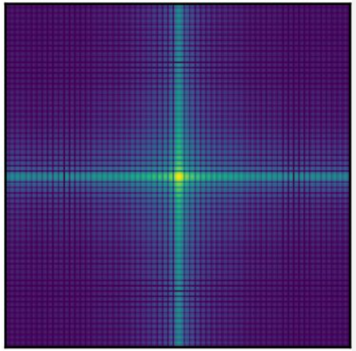
$$u = 0, 1, 2, \ldots, M-1 \text{ and } v = 0, 1, 2, \ldots, N-1$$

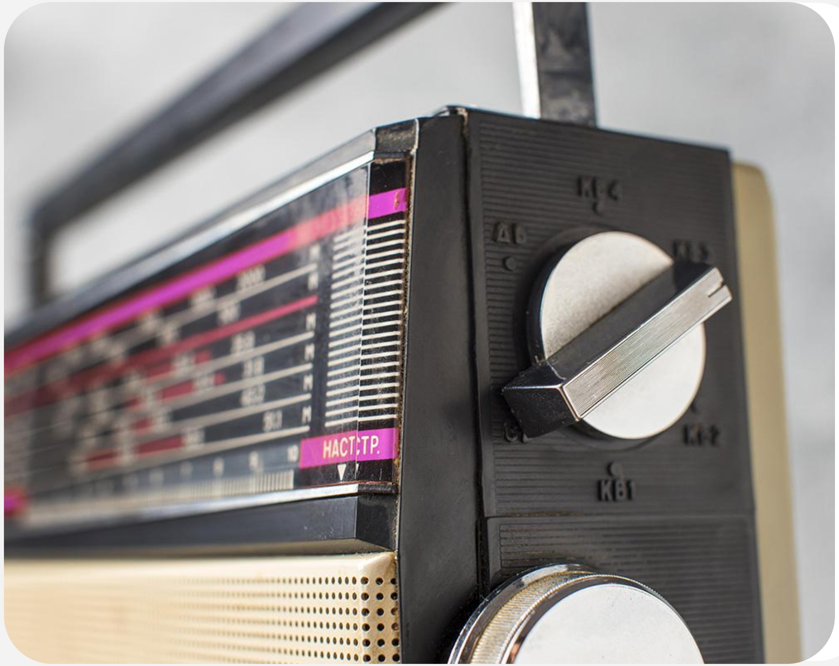
- Inverse Fourier transform

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M + vy/N)}$$

$$x = 0, 1, 2, \ldots, M-1 \text{ and } y = 0, 1, 2, \ldots, N-1$$







02

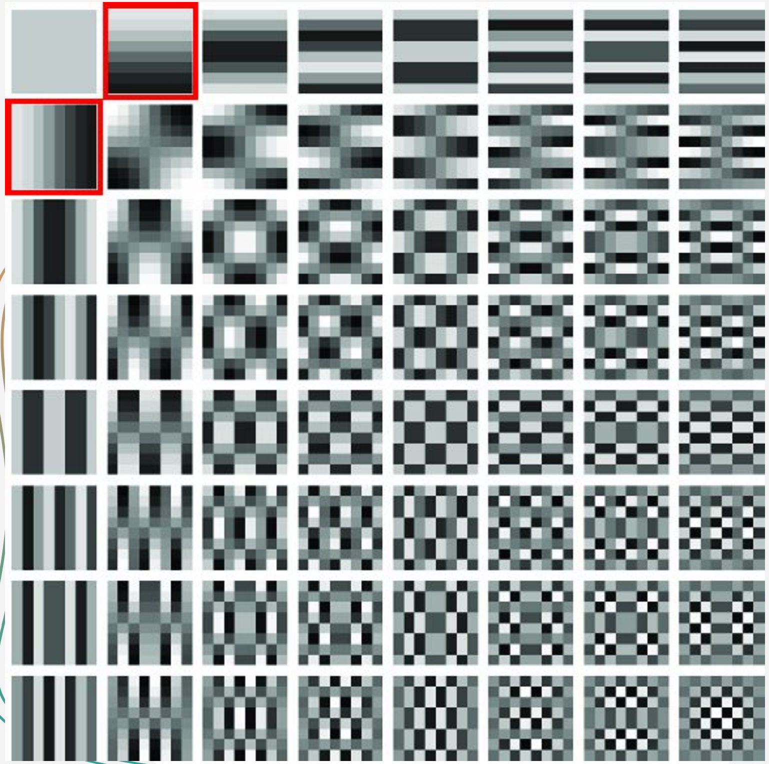
Balik Ke Materi Sebelumnya

DCT

Discrete Fourier Transform

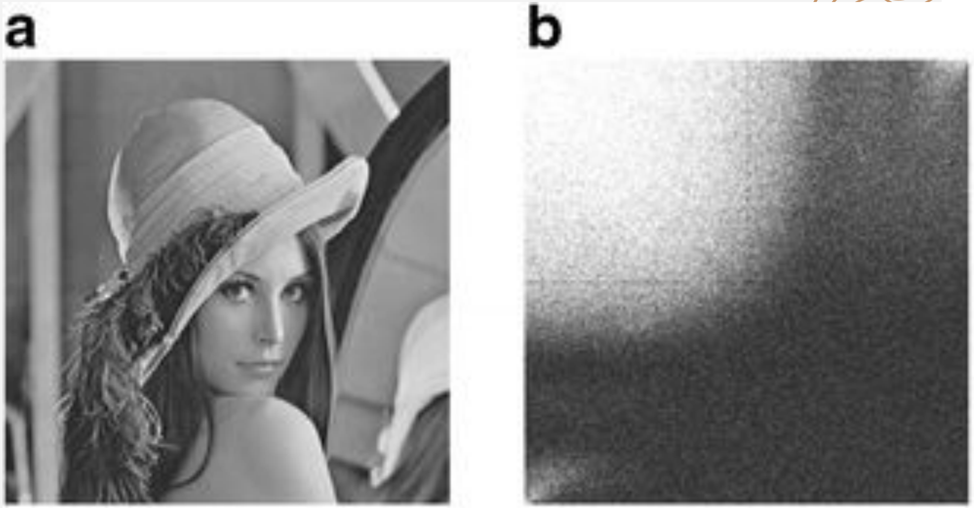
$$\mathrm{DCT}(i, j) = \frac{1}{\sqrt{2N}} C(i)\, C(j) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \mathrm{pixel}(x, y)\, \mathrm{COS}\left[\frac{(2x+1)i\pi}{2N}\right] \mathrm{COS}\left[\frac{(2y+1)j\pi}{2N}\right]$$

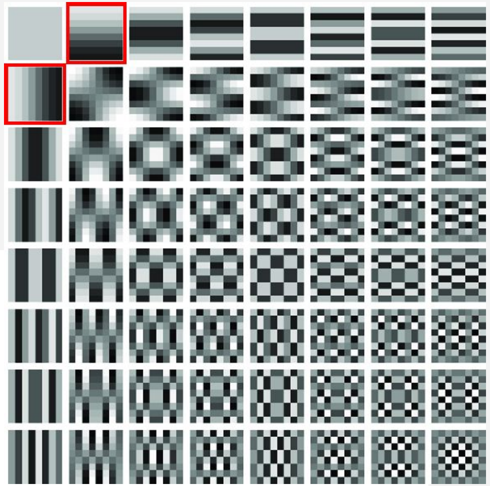
$$C(x) = \frac{1}{\sqrt{2}} \text{ if } x \text{ is } 0, \text{ else } 1 \text{ if } x > 0$$

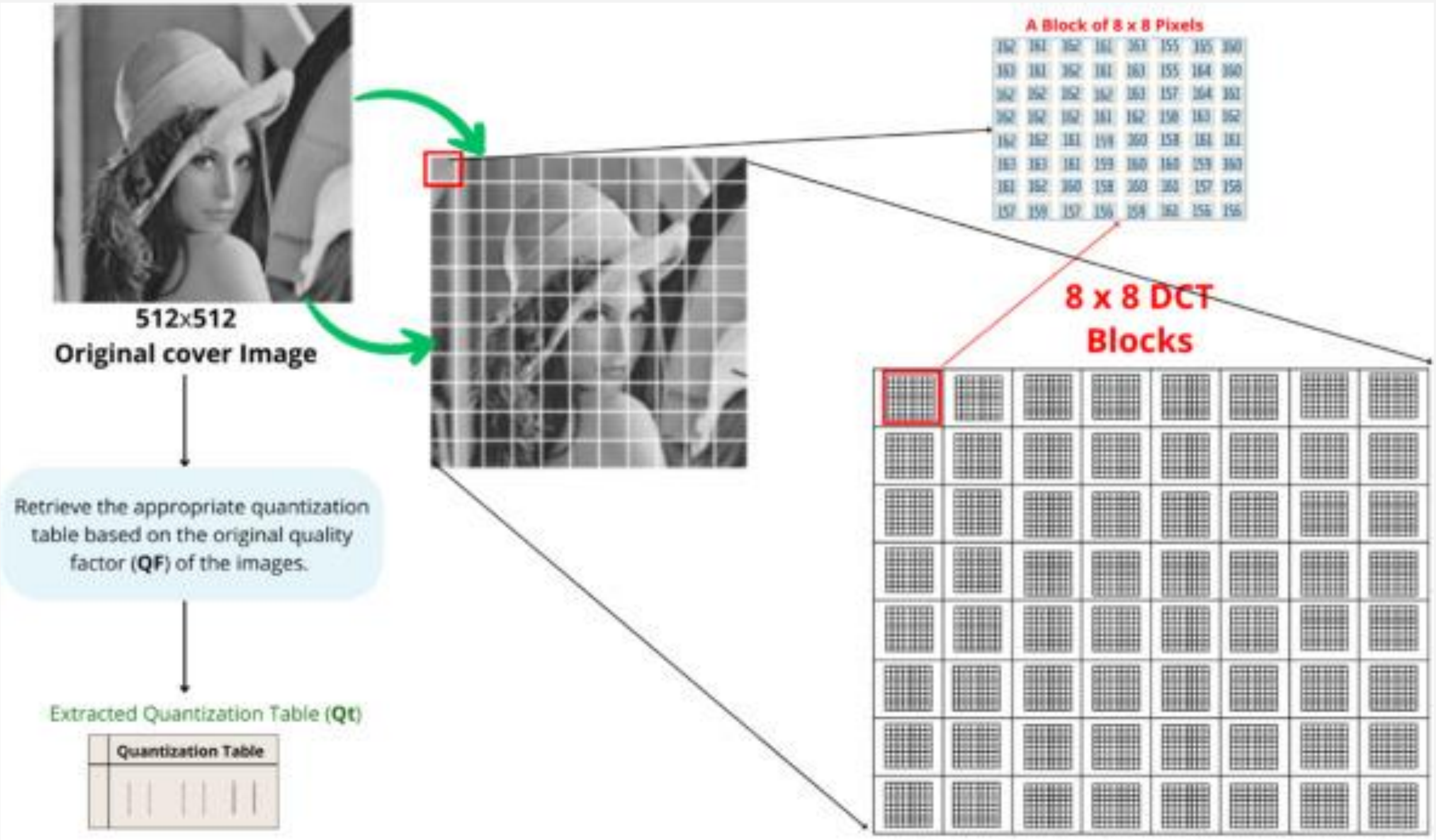


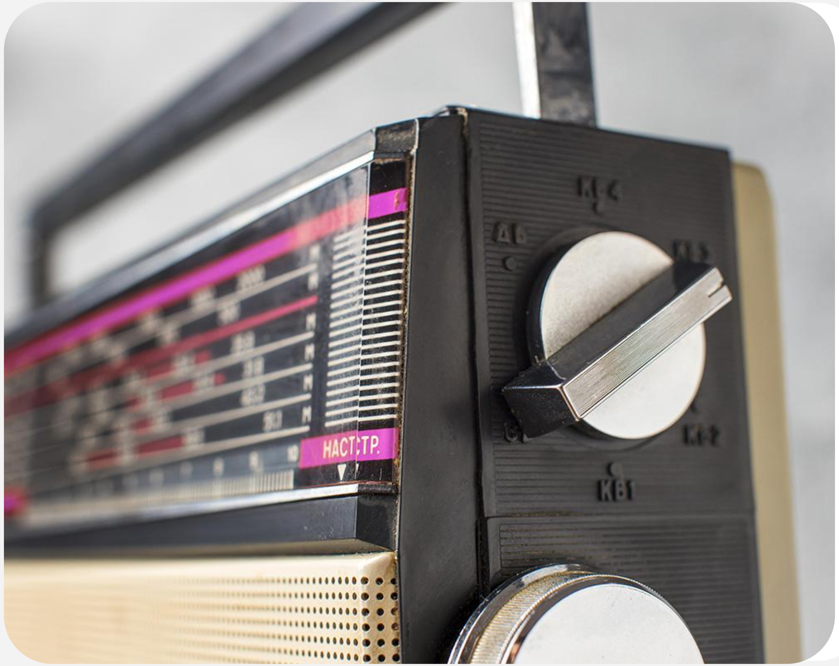
DCT

Contoh gambar dan representasi DCT-nya






A Block of 8 x 8 Pixels
8 x 8 DCT Blocks
512×512
Original cover Image
Retrieve the appropriate quantization table based on the original quality factor (QF) of the images.
Extracted Quantization Table (Qt)
Quantization Table

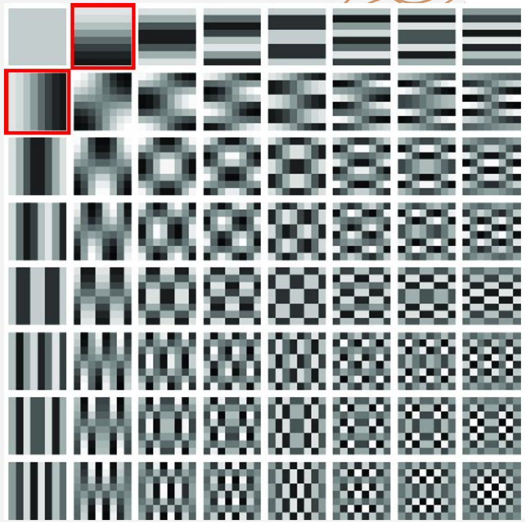


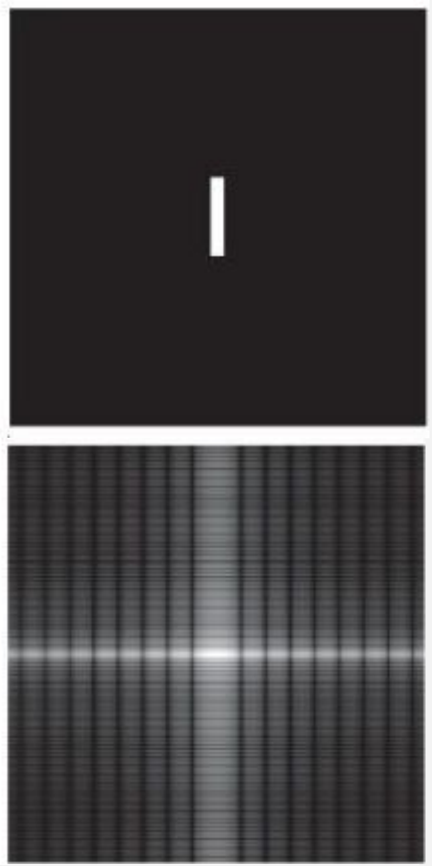
03

Phase and Aliasing

Phase

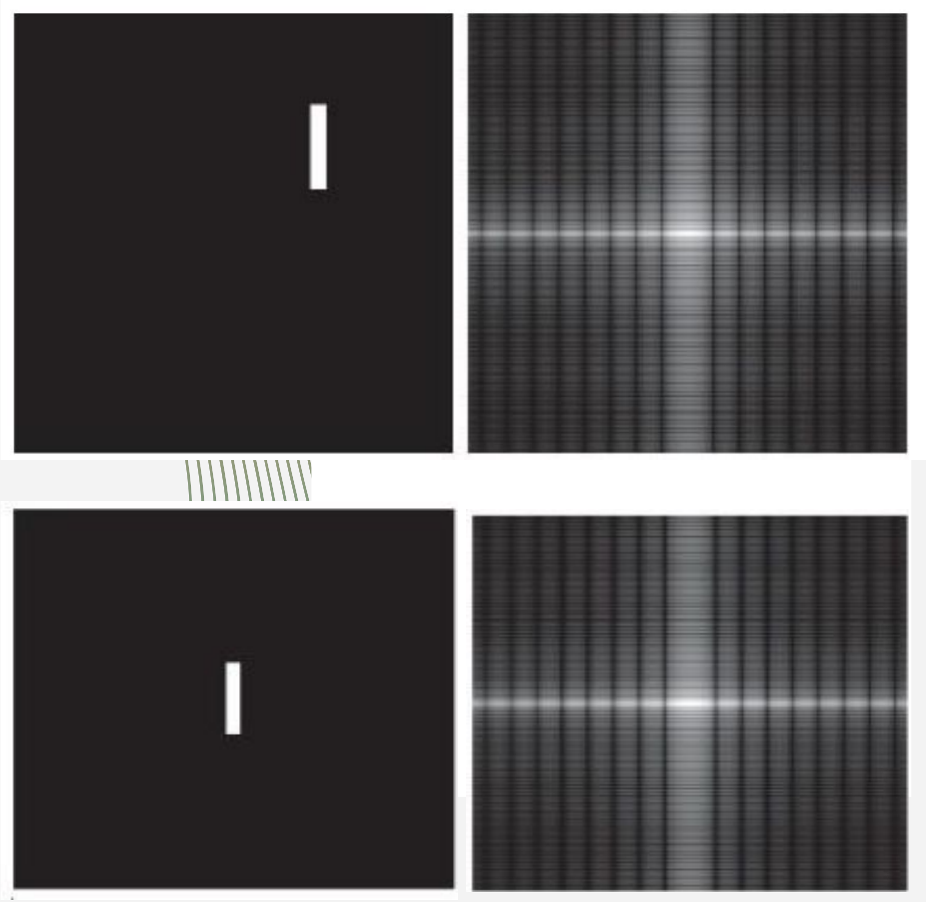
Keperluan fase dalam DFT





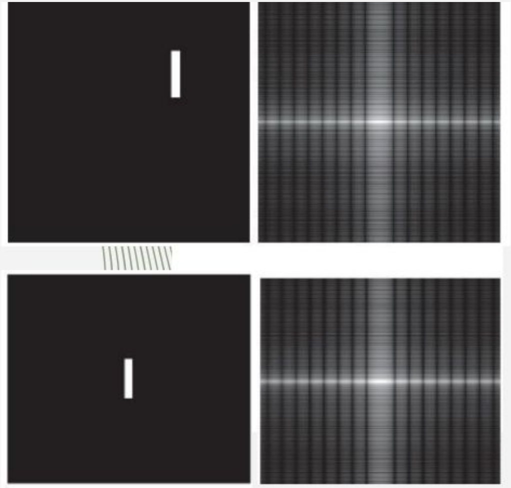
Phase

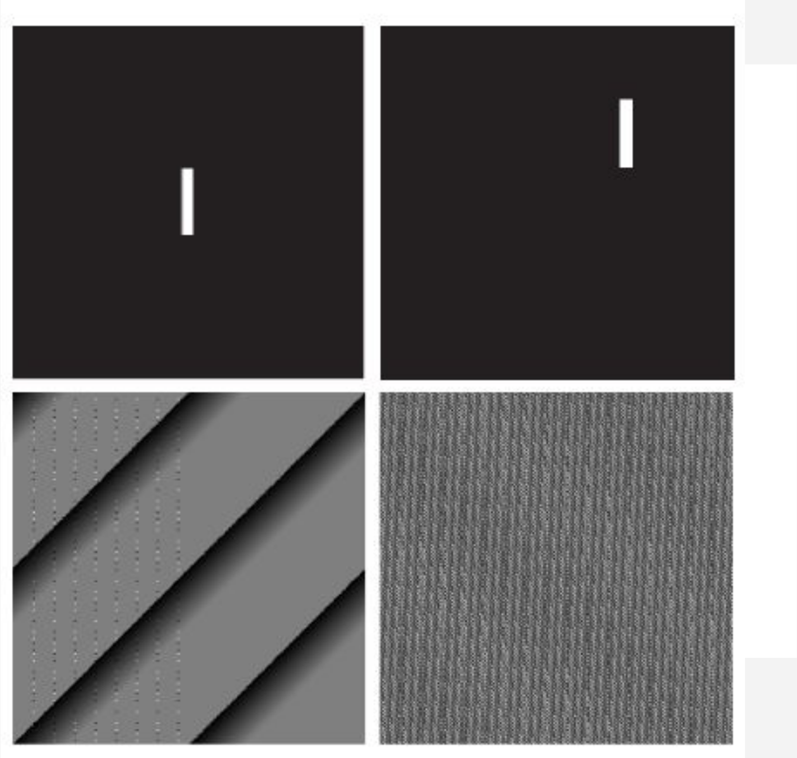
Gambar yang berbeda dapat menghasilkan hasil yang sama dalam domain frekuensi



Phase

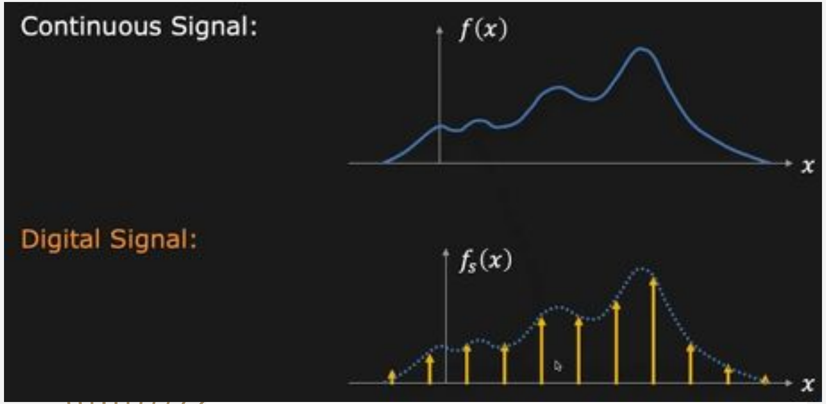
Gambar yang berbeda menghasilkan fase yang berbeda

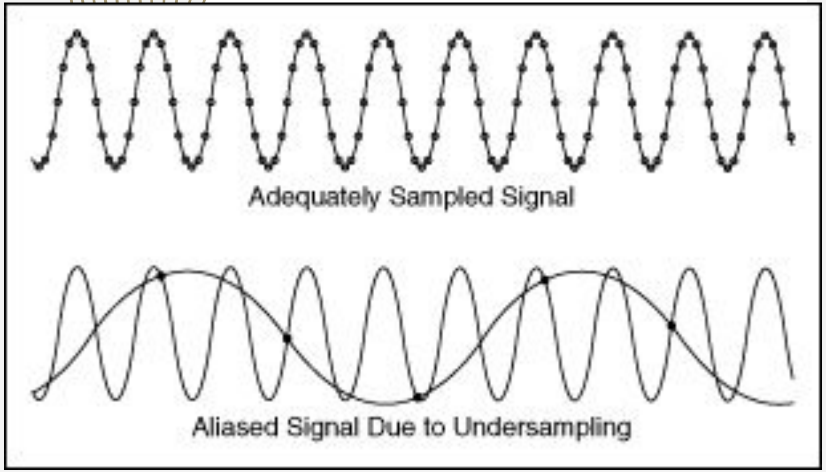




Sampling 1D

Jika sampling rate terlalu rendah, signal tidak dapat direpresentasi dengan baik (Aliasing)

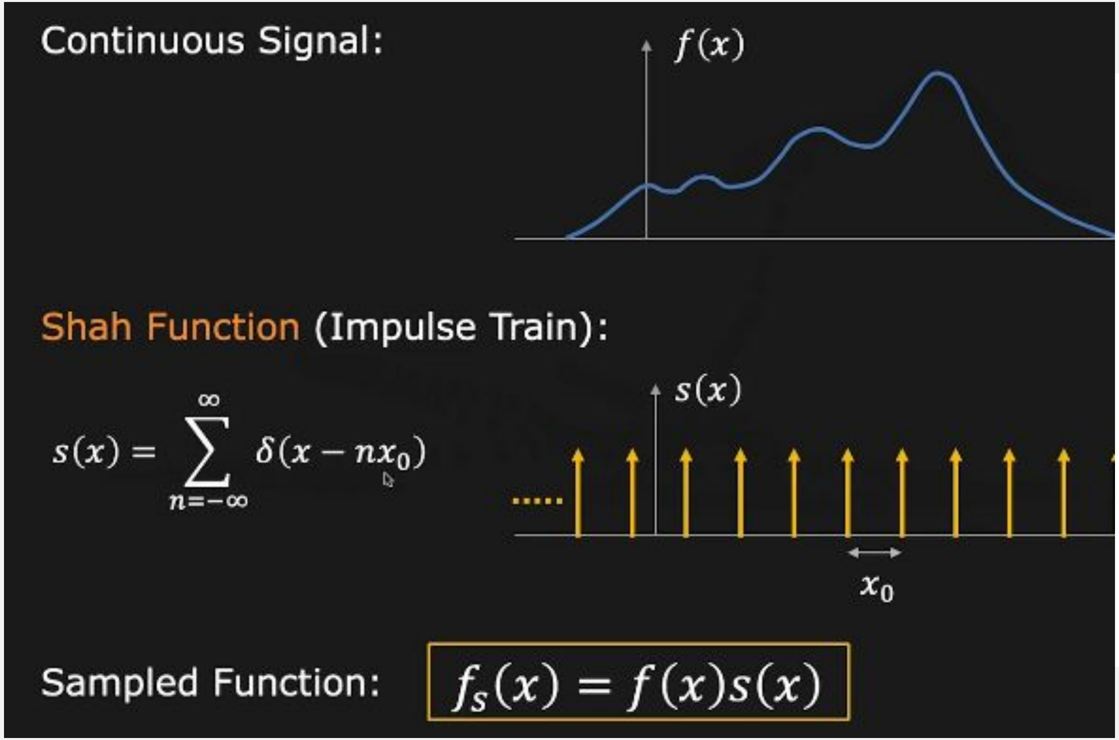




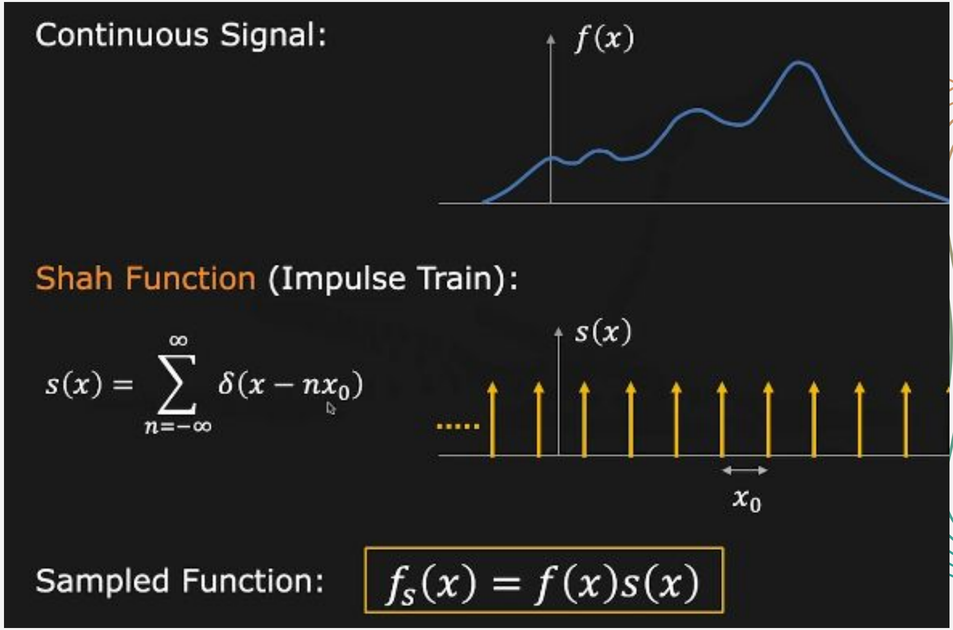
Sampling 2D

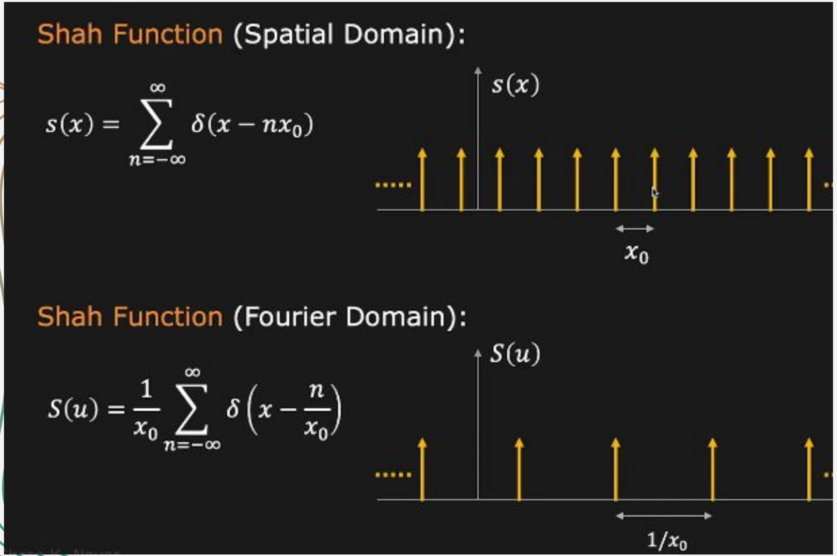
Aliasing pada 2 dimensi menyebabkan artifak bergelombang

Fungsi untuk Sampling



Fungsi untuk Sampling





Fungsi untuk Sampling

Sampled Signal:

$$f_s(x) = f(x)s(x) = f(x)\sum \delta(x - nx_0)$$

$$F_s(u) = F(u) * S(u) = F(u) * \frac{1}{x_0}\sum \delta(u - n/x_0)$$

For example:

If $u_{max} \le \frac{1}{2x_0}$

Fungsi untuk Sampling

Sampled Signal:

$$f_s(x) = f(x)s(x) = f(x)\sum \delta(x - nx_0)$$

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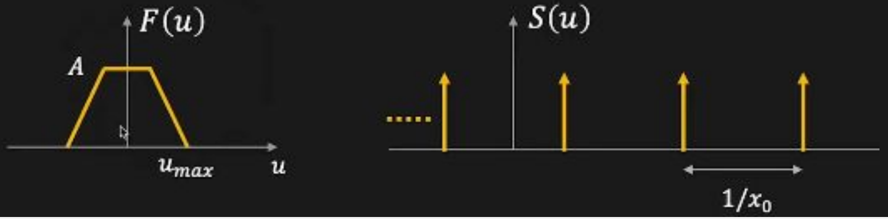
For example:



If $u_{max} \leq \frac{1}{2x_0}$

If $u_{max} > \frac{1}{2x_0}$

Aliasing

Nyquist Theorem

Can we recover $f(x)$ from $f_s(x)$? In other words, can we recover $F(u)$ from $F_s(u)$?

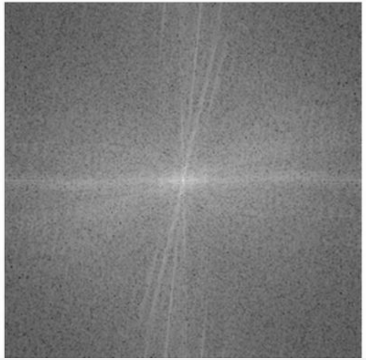
Only if $u_{max} \leq \frac{1}{2x_0}$ (Nyquist Frequency)

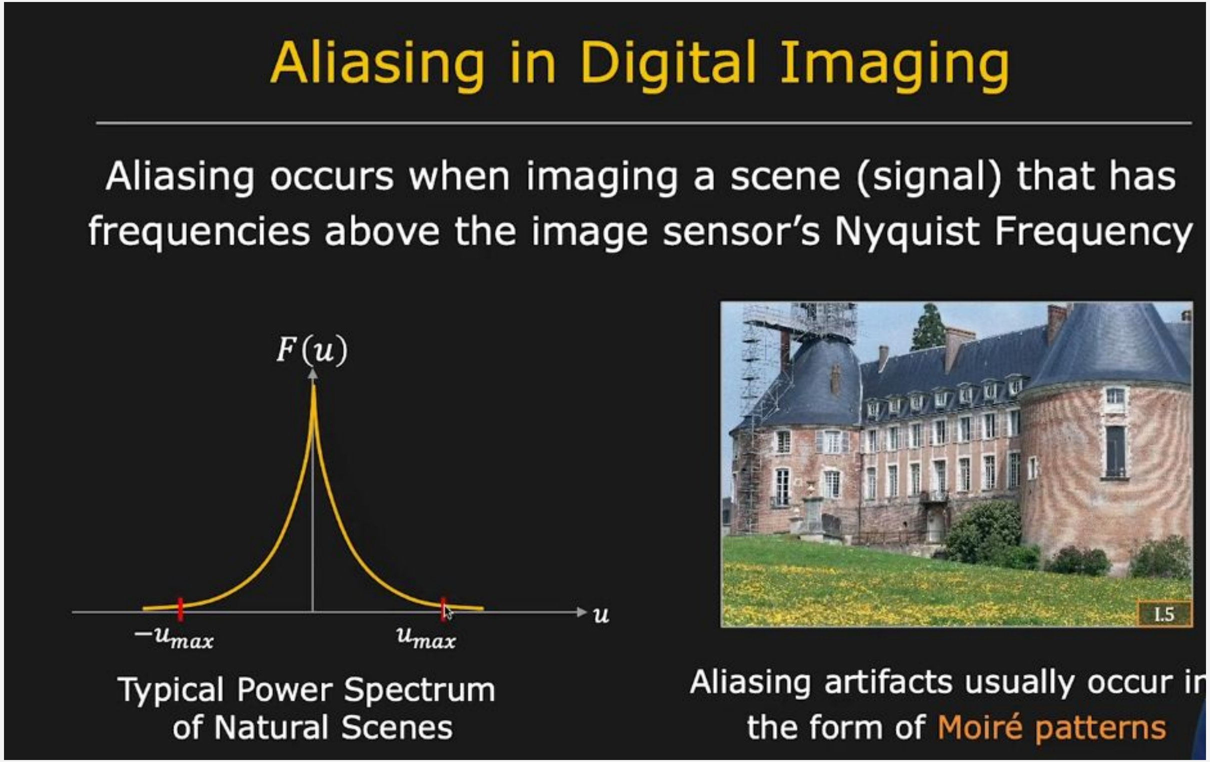
$$F(u) = F_s(u)C(u)$$

$$C(u) = \begin{cases} x_0, & |u| < 1/2 \\ 0, & Otherwis \end{cases}$$

$$\boxed{f(x) = IFT\big(F(u)\big)}$$

Aliasing Image

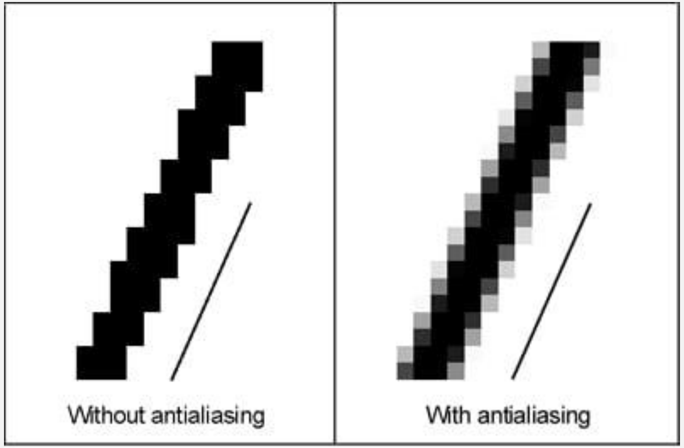


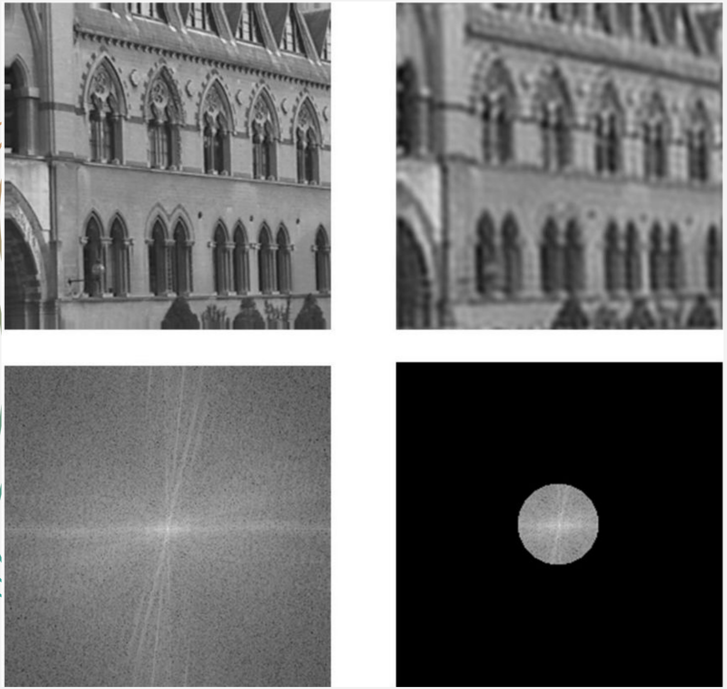


Minimizing Aliasing

Minimizing the Effects of Aliasing

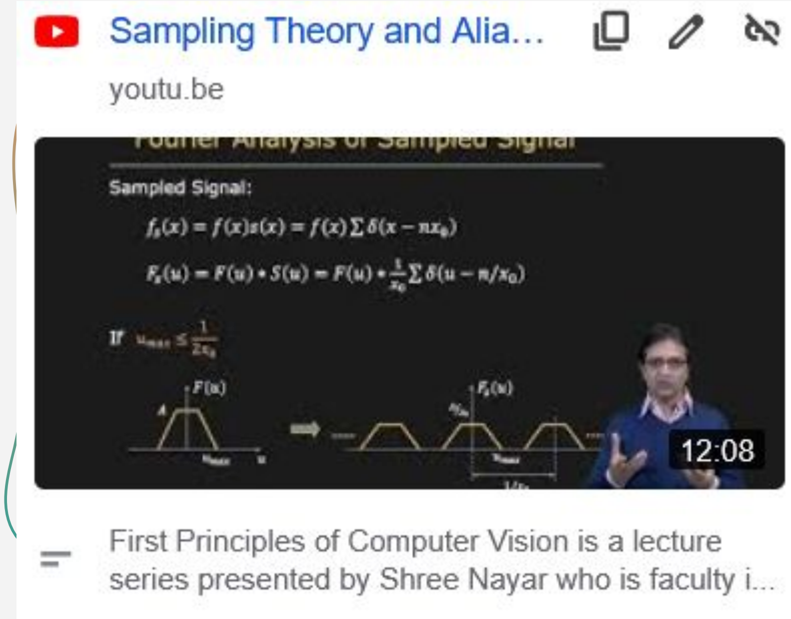
Band Limit: Clip the signal above the Nyquist frequency. Effectively, "blur" the scene before sampling.

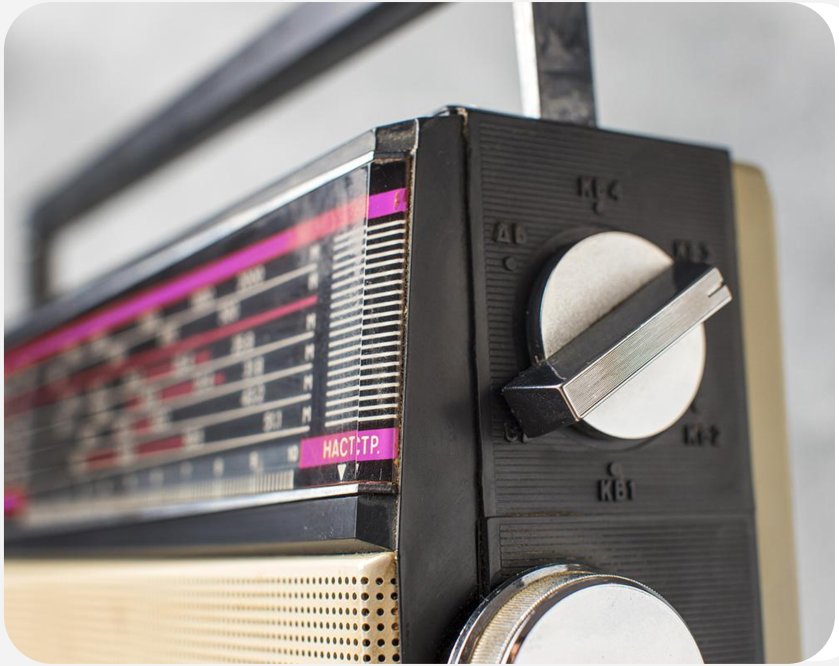




Referensi Aliasing

https://youtu.be/YFZsxY_2_l4?si=L_DZBk7EtBCEbVk_

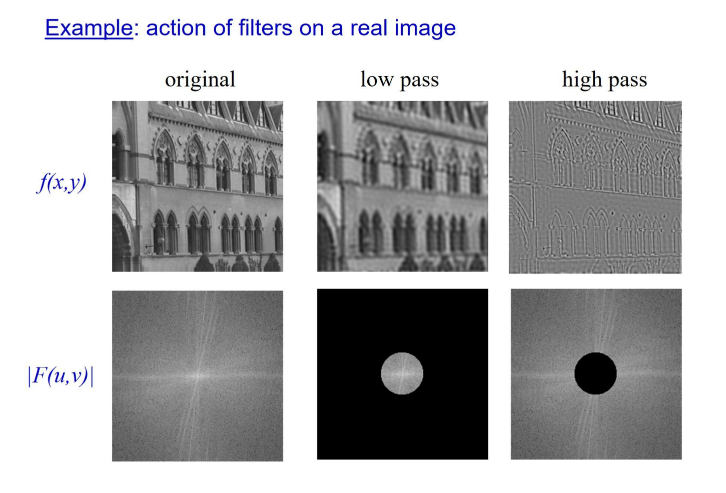




04 Intro to Frequency Domain Filtering

What is Frequency Domain Filtering

- Analyzes the frequency components of an image
- Opposite of Spatial Domain
- Values are represented in a range of frequencies
- Useful for noise reduction, compression and feature detection



Why Frequency Domain Filtering?

- Access to frequency components of an image
- Noise Reduction
- Compression
- Feature Detection
- Efficient Manipulation

How To Filter in the Frequency Domain

Convert

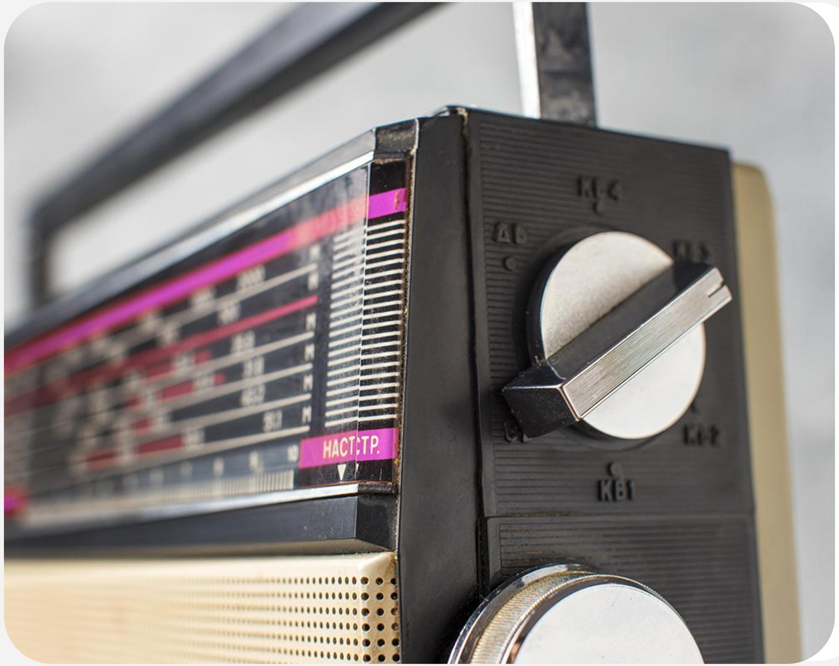
Convert the image into its frequency components using FT

Filter

Apply a filter to modify the frequency components as desired

Convert Back

Convert the filtered result back to the spatial domain using IFT



05

Image Smoothing in Frequency Domain

Image Smoothing?

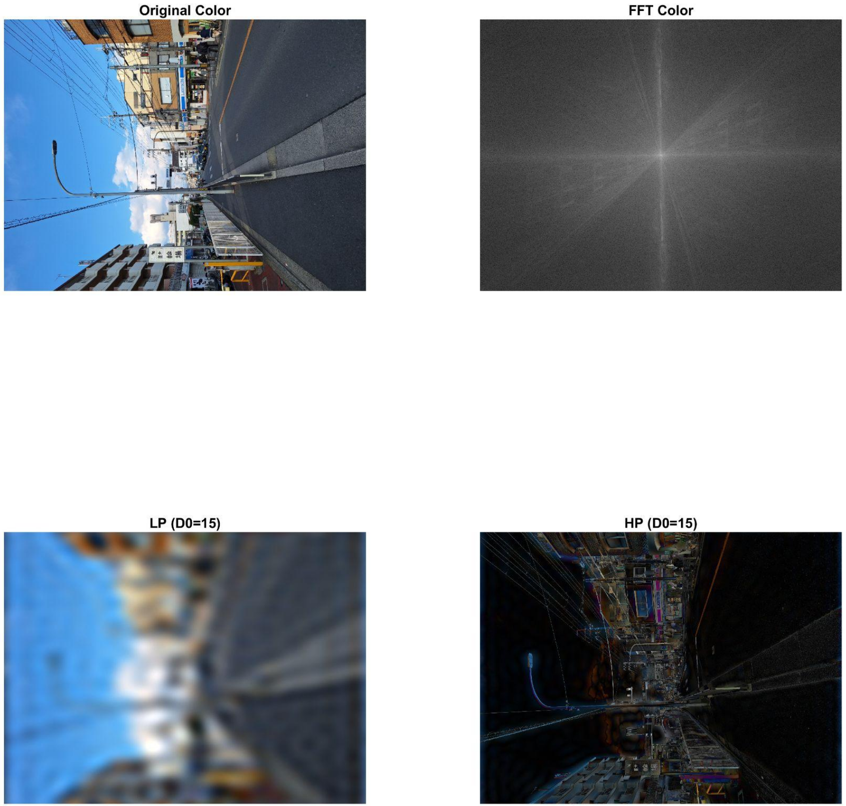
- Image smoothing is a technique to blur an image
- Reduces noise and detail
- Creates a smoother appearance
- Opposite of sharpening
- Low pass filters are commonly used for image smoothing

How To: Smoothing in Frequency Domain

- Take the Fourier Transform of the image.
- Multiply the result by a low-pass filter.
- Take the Inverse Fourier Transform of the result.

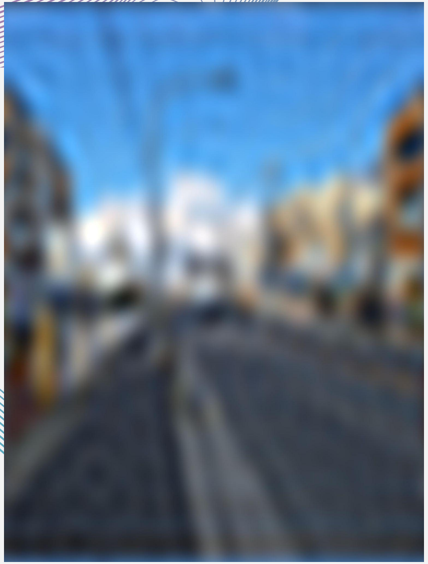
Ideal Low-Pass Filters

- Ideal low-pass filters (ILPF) pass frequencies below a cutoff frequency without attenuation
- ILPF completely attenuates frequencies above the cutoff
- Creates a sharp transition between passband and stopband
- Not physically realizable
- Serves as a theoretical reference point



Original

D0 = 15

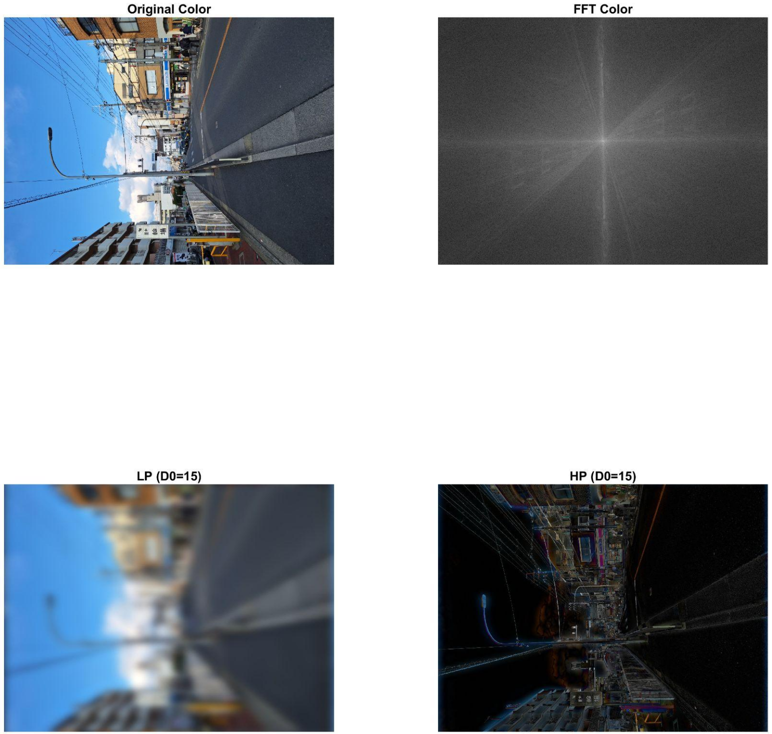


D0 = 30

D0 = 80

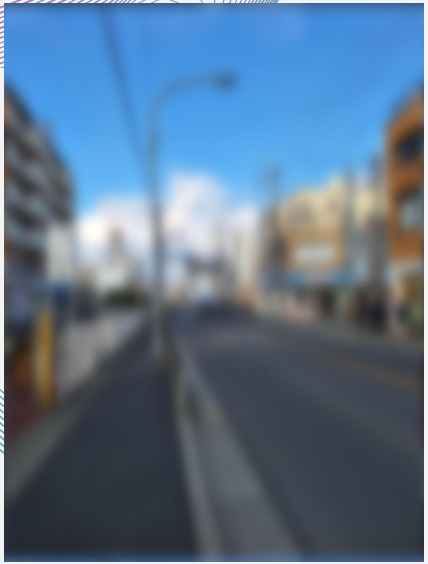
Butterworth Low-Pass Filters

- Smooths image by reducing high frequency components
- Blurs edges and fine details
- Shape of the Butterworth filter is controlled by its order
- Higher order -> sharper cutoff
- Simple and efficient to implement



Original

D0 = 15



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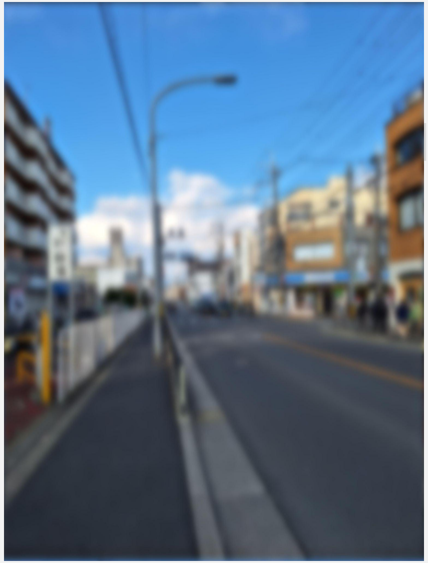
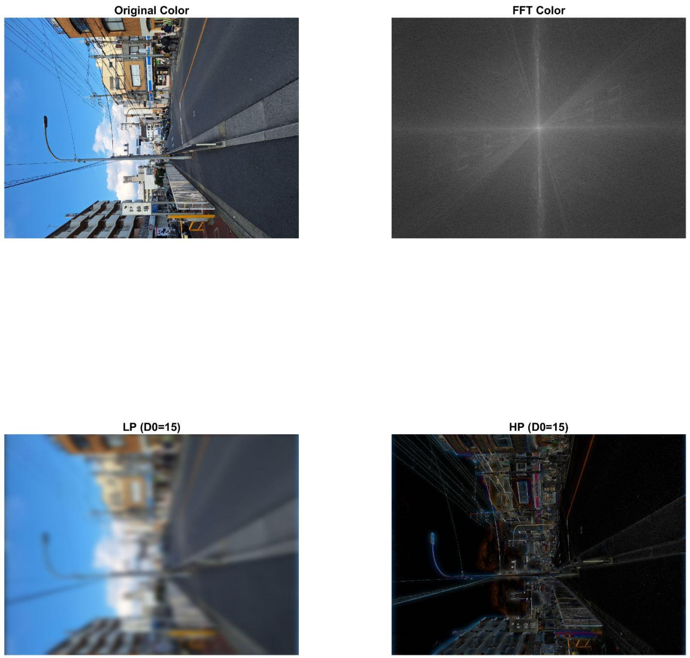


D0 = 80

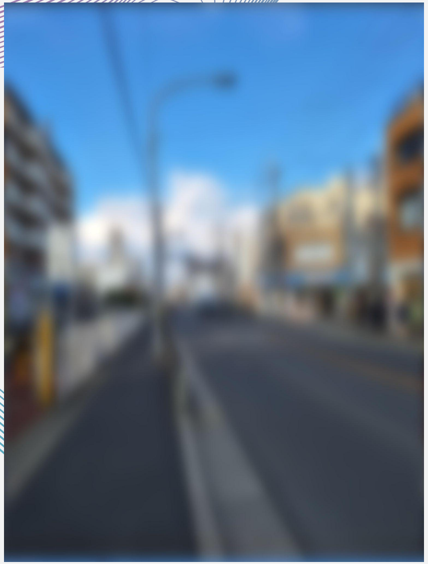
Gaussian Low-Pass Filters

- Gaussian Low Pass Filter (GLPF) is used to blur images and reduce noise.
- GLPF has a smoother cut off compared to Ideal Low Pass Filter (ILPF).
- Smoothing is controlled by the standard deviation.
- Higher standard deviation -> More smoothing.
- Shape of the filter is a bell curve.



Original

D0 = 15



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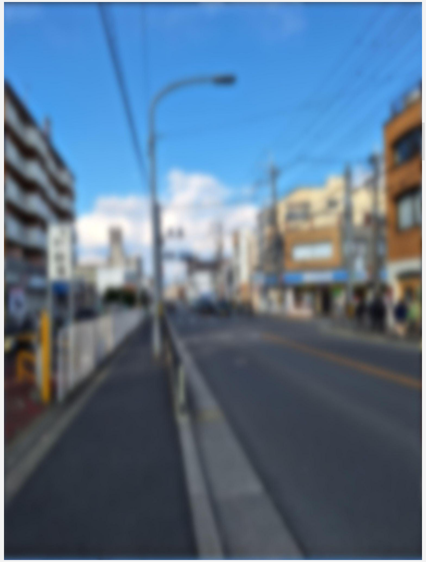

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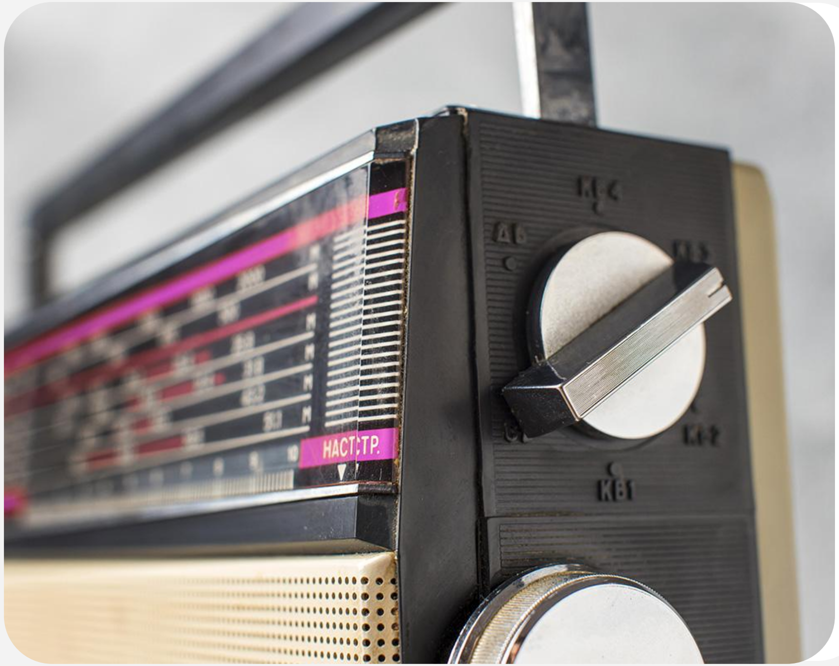


06

Image Sharpening in Frequency Domain

What is Image Sharpening?

- Image sharpening is a technique used to enhance the details and edges of an image.
- Opposite of image smoothing.
- Makes the image appear clearer and more focused.
- High-pass filters are commonly used for image sharpening.

How To Sharpen In The Frequency Domain

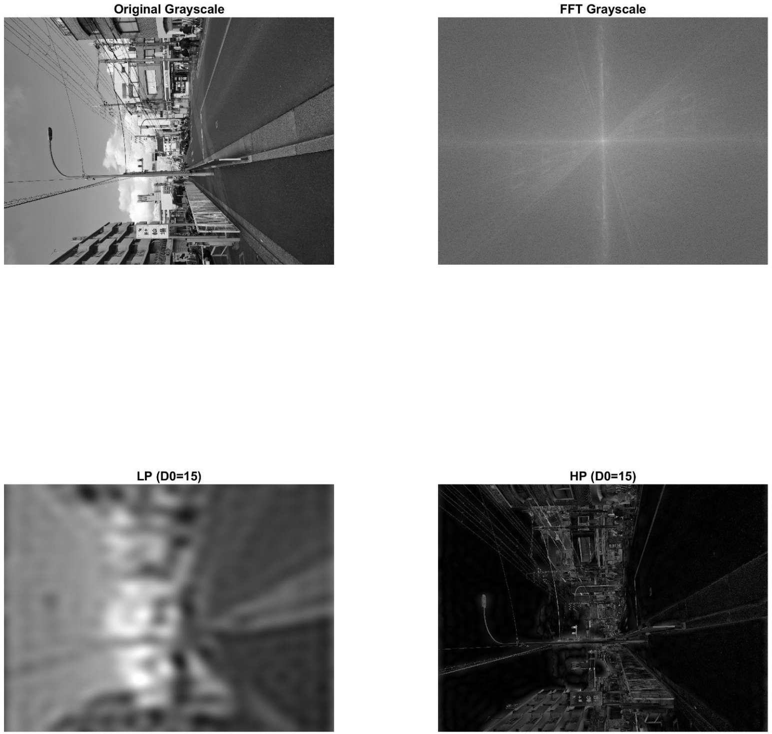
01 — Take the Fourier Transform of the image.

02 — Multiply the result by a high-pass filter.

03 — Take the Inverse Fourier Transform of the result.

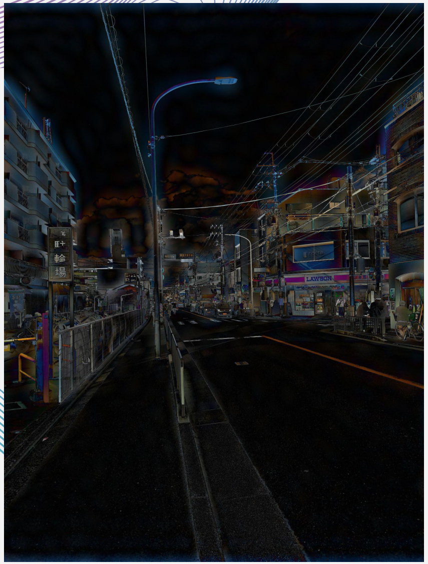
Ideal High-Pass Filters

- Ideal high-pass filters (IHPF) pass frequencies above a cutoff frequency without attenuation.
- IHPF completely attenuates frequencies below the cutoff.
- Creates a sharp transition between passband and stopband.
- Not physically realizable.
- Serves as a theoretical reference point.

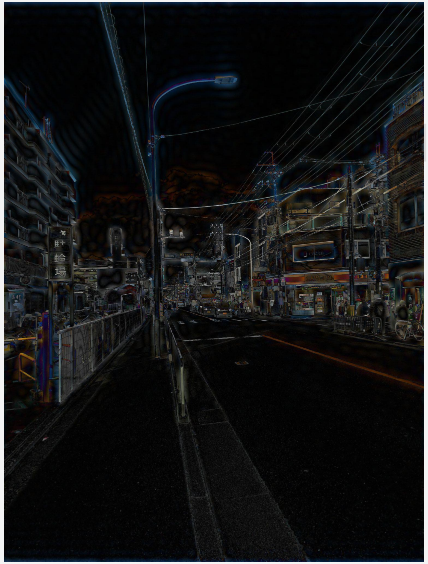


Original

D0 = 15



D0 = 30



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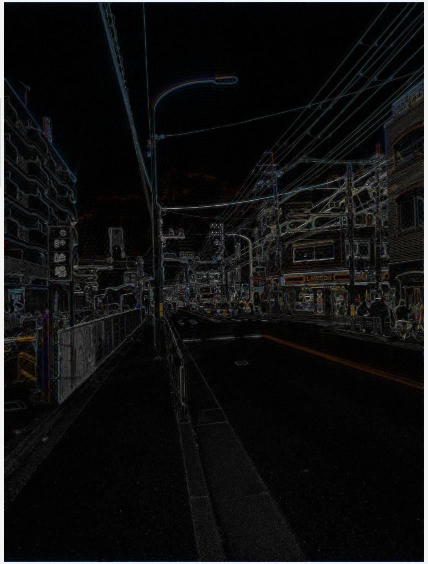
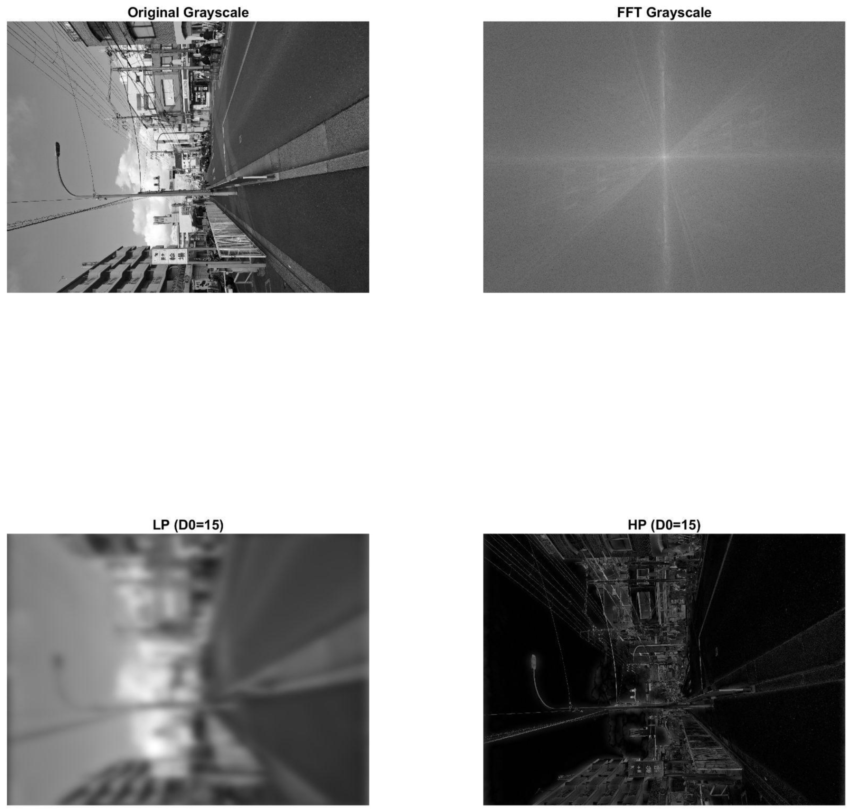


Butterworth High-Pass Filters

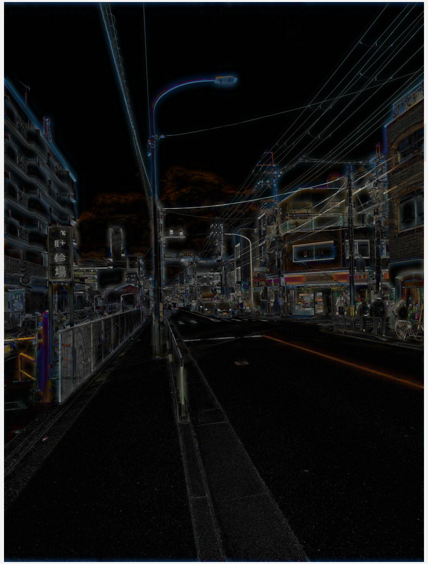
- Used for sharpening images by attenuating low-frequency components.
- Allows high-frequency components to pass through, emphasizing edges and details.
- Shape of the filter is controlled by its order.
- Higher order --> sharper cutoff, but may introduce ringing artifacts.
- Less abrupt transition compared to ideal high-pass filters, reducing ringing artifacts.



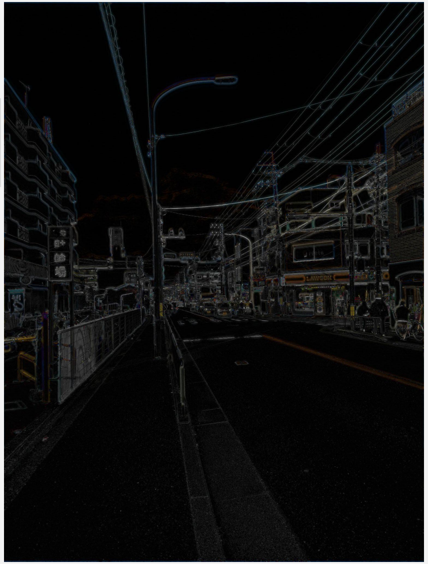
Original

D0 = 15

D0 = 30

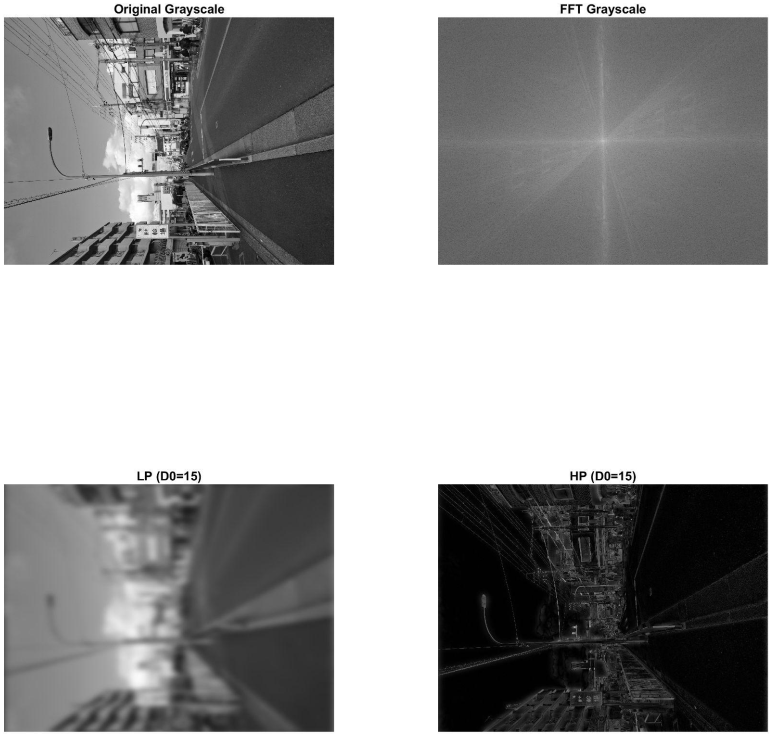


D0 = 80



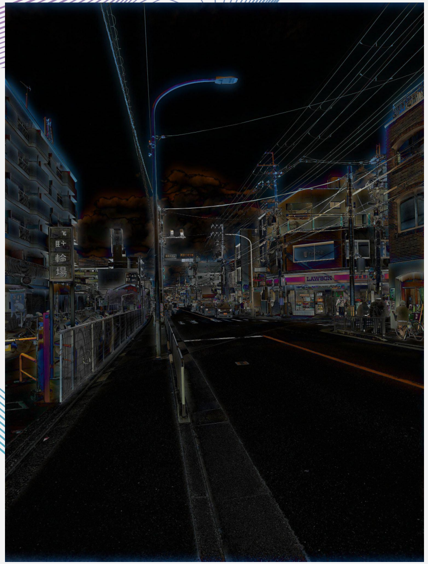
Gaussian High-Pass Filters

- Gaussian High Pass Filter (GHPF) is used to sharpen images and enhance details.
- GHPF attenuates low-frequency components and allows high-frequency components to pass through.
- Smoothing is controlled by the standard deviation.
- Lower standard deviation -> More sharpening.
- Shape of the filter is a bell curve.

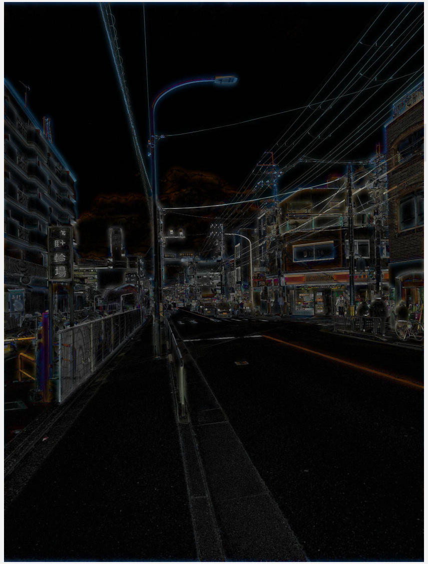


Original

D0 = 15



D0 = 30



D0 = 80