

Binomial_MGWR_approaches_tried

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<https://github.com/pysal/mgwr/pull/56>

```
In [3]: import sys
        sys.path.append("C:/Users/msachde1/Downloads/Research/Development/mgwr")

In [4]: import warnings
        warnings.filterwarnings("ignore")
        import pandas as pd
        import numpy as np

        from mgwr.gwr import GWR
        from spglm.family import Gaussian, Binomial, Poisson
        from mgwr.gwr import MGWR
        from mgwr.sel_bw import Sel_BW
        import multiprocessing as mp
        pool = mp.Pool()
        from scipy import linalg
        import numpy.linalg as la
        from scipy import sparse as sp
        from scipy.sparse import linalg as spla
        from spreg.utils import spdots, spmultiply
        from scipy import special
        import libpysal as ps
        import seaborn as sns
        import matplotlib.pyplot as plt
        from copy import deepcopy
        import copy
        from collections import namedtuple
        import spglm
```

Fundamental equation By simple algebraic manipulation, the probability that $Y=1$ is:

$$p = 1 / (1 + \exp(-\beta_k x_{k,i})) \quad (1)$$

(2)

0.0.1 Approaches tried:

1. Changing XB to : $1 / (1 + \text{np.exp}(-1 * \text{np.sum}(\text{np.multiply}(X, \text{params}), \text{axis}=1)))$ - these are the predicted probabilities $\sim(0,1)$
2. Changing XB as above and writing a function to create temp_y as a binary variable using condition 1 if $BX_i > 0$ else 0.

3. Derived manipulations to temp_y as in iwls for Logistic regression as below:

```
v = np.sum(np.multiply(X,params),axis=1)
```

```
mu = 1/(1+(np.exp(-v)))
```

```
z = v + (1/(mu * (1-mu)) * (y-mu)) -- this becomes the temp_y
```

Then a simple linear regression can be run as z as the temp dependent variable

4. Taken from GAM logistic model literature:

$$y = \exp(b_0 + b_1 x_1 + \dots + b_m x_m) / \{1 + \exp(b_0 + b_1 x_1 + \dots + b_m x_m)\}$$

Applying the logistic link function to the probability p (ranging between 0 and 1):

$$p' = \log \{p/(1-p)\}$$

By applying the logistic link function, we can now rewrite the model as:

$$p' = b_0 + b_1 X_1 + \dots + b_m X_m$$

Finally, we substitute the simple single-parameter additive terms to derive the generalized additive logistic model:

$$p' = b_0 + f_1(X_1) + \dots + f_m(X_m)$$

(<http://www.statsoft.com/textbook/generalized-additive-models#gam>)

This is the current approach in the latest commit:

```
XB = 1 / (1 + np.exp (-1*(np.multiply(X,params))))
```

XB is now the probability and is normally distributed

Run MGWR (Gaussian) on this as the dependent variable for the partial models.

0.0.2 Data

Clearwater data - downloaded from link: <https://sgsup.asu.edu/sparc/multiscale-gwr>

```
In [5]: data_p = pd.read_csv("C:/Users/msachde1/Downloads/logistic_mgwr_data/landslides.csv")
```

```
In [6]: data_p.head()
```

```
Out[6]:
```

	UserID	X	Y	Elev	Slope	SinAspct	CosAspct	\
0	1	616168.5625	5201076.5	1450.475	27.44172	0.409126	-0.912478	
1	2	624923.8125	5201008.5	1567.476	21.88343	-0.919245	-0.393685	
2	3	615672.0000	5199187.5	1515.065	38.81030	-0.535024	-0.844837	
3	4	615209.3125	5199112.0	1459.827	26.71631	-0.828548	-0.559918	
4	5	616354.6875	5198945.5	1379.442	27.55271	-0.872281	-0.489005	

```
AbsSouth  Landslid  DistStrm
```

0	24.1499	1	8.506
1	66.8160	1	15.561
2	32.3455	1	41.238
3	55.9499	1	17.539
4	60.7248	1	35.023

Helper functions - hardcoded here for simplicity in the notebook workflow Please note: A separate `bw_func_b` will not be required when changes will be made in the repository

```
In [7]: kernel='bisquare'
        fixed=False
        spherical=False
        search_method='golden_section'
        criterion='AICc'
        interval=None
        tol=1e-06
        max_iter=500
        X_glob=[]

In [8]: def gwr_func(y, X, bw,family=Gaussian(),offset=None):
        return GWR(coords, y, X, bw, family,offset,kernel=kernel,
                    fixed=fixed, constant=False,
                    spherical=spherical, hat_matrix=False).fit(
                        lite=True, pool=pool)

def gwr_func_g(y, X, bw):
    return GWR(coords, y, X, bw, family=Gaussian(),offset=None,kernel=kernel,
                fixed=fixed, constant=False,
                spherical=spherical, hat_matrix=False).fit(
                    lite=True, pool=pool)

def bw_func_b(coords,y, X):
    selector = Sel_BW(coords,y, X,family=Binomial(),offset=None, X_glob=[],
                        kernel=kernel, fixed=fixed,
                        constant=False, spherical=spherical)
    return selector

def bw_func_p(coords,y, X):
    selector = Sel_BW(coords,y, X,family=Poisson(),offset=off, X_glob=[],
                        kernel=kernel, fixed=fixed,
                        constant=False, spherical=spherical)
    return selector

def bw_func(coords,y,X):
    selector = Sel_BW(coords,y,X,X_glob=[],
                        kernel=kernel, fixed=fixed,
                        constant=False, spherical=spherical)
```

```

    return selector

def sel_func(bw_func, bw_min=None, bw_max=None):
    return bw_func.search(
        search_method=search_method, criterion=criterion,
        bw_min=bw_min, bw_max=bw_max, interval=interval, tol=tol,
        max_iter=max_iter, pool=pool, verbose=False)

```

0.0.3 GWR Binomial model with independent variable, x = slope

```

In [9]: coords = list(zip(data_p['X'],data_p['Y']))
        y = np.array(data_p['Landslid']).reshape((-1,1))
        elev = np.array(data_p['Elev']).reshape((-1,1))
        slope = np.array(data_p['Slope']).reshape((-1,1))
        SinAspct = np.array(data_p['SinAspct']).reshape(-1,1)
        CosAspct = np.array(data_p['CosAspct']).reshape(-1,1)
        X = np.hstack([elev,slope,SinAspct,CosAspct])
        x = SinAspct

        X_std = (X-X.mean(axis=0))/X.std(axis=0)
        x_std = (x-x.mean(axis=0))/x.std(axis=0)
        y_std = (y-y.mean(axis=0))/y.std(axis=0)

In [10]: bw_gwbr=Sel_BW(coords,y,x_std,family=Binomial(),constant=False).search()

In [11]: gwbr_model=GWR(coords,y,x_std,bw=bw_gwbr,family=Binomial(),constant=False).fit()

In [12]: bw_gwbr

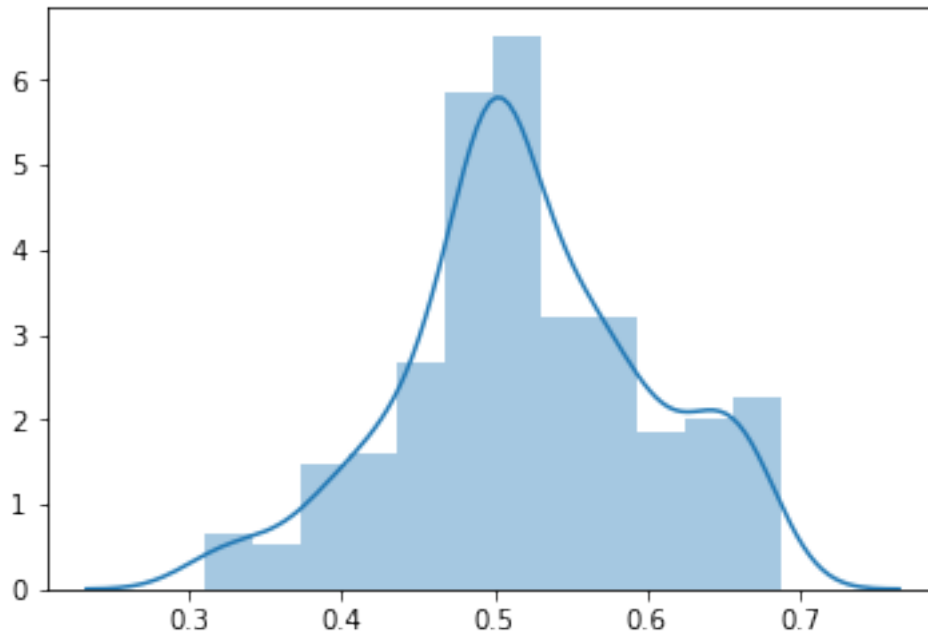
Out[12]: 100.0

In [25]: predy = 1/(1+np.exp(-1*np.sum(gwbr_model.X * gwbr_model.params, axis=1)).reshape(-1, 1))

In [26]: sns.distplot(predy)

Out[26]: <matplotlib.axes._subplots.AxesSubplot at 0x17b8f5dc2b0>

```

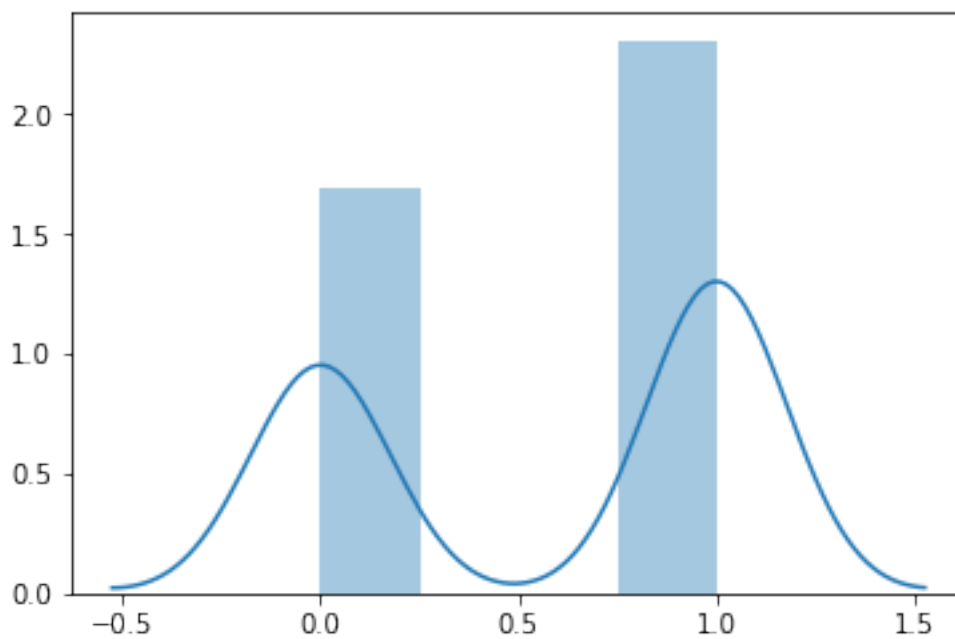


```
In [28]: (predy==gwbr_model.predy).all()
```

```
Out[28]: True
```

```
In [29]: sns.distplot(gwbr_model.y)
```

```
Out[29]: <matplotlib.axes._subplots.AxesSubplot at 0x17b8ecf4080>
```



Multi_bw changes

```
In [18]: def multi_bw(init,coords,y, X, n, k, family=Gaussian(),offset=None, tol=1e-06, max_it=100,
                verbose=True):

    if multi_bw_min==None:
        multi_bw_min = multi_bw_min*X.shape[1]

    if multi_bw_max==None:
        multi_bw_max = multi_bw_max*X.shape[1]

    if isinstance(family,spglm.family.Poisson):
        bw = sel_func(bw_func_p(coords,y,X))
        optim_model=gwr_func(y,X,bw,family=Poisson(),offset=offset)
        err = optim_model.resid_response.reshape((-1, 1))
        param = optim_model.params
        #This change for the Poisson model follows from equation (1) above
        XB = offset*np.exp(np.multiply(param, X))

    elif isinstance(family,spglm.family.Binomial):
        bw = sel_func(bw_func_b(coords,y,X))
        optim_model=gwr_func(y,X,bw,family=Binomial())
        err = optim_model.resid_response.reshape((-1, 1))
        param = optim_model.params
        XB = 1/(1+np.exp(-1*np.multiply(optim_model.params,X)))
        print("first family: "+str(optim_model.family))

    else:
        bw=sel_func(bw_func(coords,y,X))
        optim_model=gwr_func(y,X,bw)
        err = optim_model.resid_response.reshape((-1, 1))
        param = optim_model.params
        XB = np.multiply(param, X)

    bw_gwr = bw
    XB=XB

    if rss_score:
        rss = np.sum((err)**2)
    iters = 0
    scores = []
    delta = 1e6
    BWs = []
```

```

bw_stable_counter = np.ones(k)
bws = np.empty(k)

try:
    from tqdm.auto import tqdm #if they have it, let users have a progress bar
except ImportError:

    def tqdm(x, desc=''): #otherwise, just passthrough the range
        return x

for iters in tqdm(range(1, max_iter + 1), desc='Backfitting'):
    new_XB = np.zeros_like(X)
    neww_XB = np.zeros_like(X)

    params = np.zeros_like(X)

    for j in range(k):
        temp_y = XB[:, j].reshape((-1, 1))
        temp_y = temp_y + err

        temp_X = X[:, j].reshape((-1, 1))

        #The step below will not be necessary once the bw_func is changed in the
        if isinstance(family, spglm.family.Poisson):

            bw_class = bw_func_p(coords, temp_y, temp_X)

        else:
            bw_class = bw_func(coords, temp_y, temp_X)
            print(bw_class.family)

        if np.all(bw_stable_counter == bws_same_times):
            #If in backfitting, all bws not changing in bws_same_times (default 3)
            bw = bws[j]
        else:
            bw = sel_func(bw_class, multi_bw_min[j], multi_bw_max[j])
            if bw == bws[j]:
                bw_stable_counter[j] += 1
            else:
                bw_stable_counter = np.ones(k)

        optim_model = gwr_func_g(temp_y, temp_X, bw)
        print(optim_model.family)

        err = optim_model.resid_response.reshape((-1, 1))
        param = optim_model.params.reshape((-1, ))

        new_XB[:, j] = optim_model.predy.reshape(-1)

```

```

        params[:, j] = param
        bws[j] = bw

    num = np.sum((new_XB - XB)**2) / n
    print("num = "+str(num))
    den = np.sum(np.sum(new_XB, axis=1)**2)
    score = (num / den)**0.5
    print(score)
    XB = new_XB

    if rss_score:
        print("here")
        predy = 1/(1+np.exp(-1*np.sum(X * params, axis=1).reshape(-1, 1)))
        new_rss = np.sum((y - predy)**2)
        score = np.abs((new_rss - rss) / new_rss)
        rss = new_rss
        scores.append(deepcopy(score))
        delta = score
        print(delta)
        Bws.append(deepcopy(bws))

    if verbose:
        print("Current iteration:", iters, ",SOC:", np.round(score, 7))
        print("Bandwidths:", ', '.join([str(bw) for bw in bws]))

    if delta < tol:
        break

    print("iters = "+str(iters))
    opt_bws = Bws[-1]
    print("opt_bws = "+str(opt_bws))
    print(bw_gwr)
    return (opt_bws, np.array(Bws), np.array(scores), params, err, bw_gwr)

```

In [21]: mgwbr = multi_bw(init=None,coords=coords,y=y, X=x_std, n=239, k=x.shape[1], family=Binomial)

```

first family: <spglm.family.Binomial object at 0x0000017B8F302588>
<spglm.family.Gaussian object at 0x0000017B8DA45940>
<spglm.family.Gaussian object at 0x0000017B8F341828>
num = 0.2261860620412264
0.1493438159109546
here
0.019261197296944955
Current iteration: 1 ,SOC: 0.0192612
Bandwidths: 82.0
<spglm.family.Gaussian object at 0x0000017B8DA45940>
<spglm.family.Gaussian object at 0x0000017B8F2BD4A8>
num = 0.0

```



```
0.0
here
0.0
Current iteration: 2 ,SOC: 0.0
Bandwidths: 82.0
iters = 2
opt_bws = [82.]
100.0
```

```
In [22]: param = mgwbr[3]
```

```
In [23]: predy = 1/(1+np.exp(-1*np.sum(x_std * param, axis=1).reshape(-1, 1)))
```

```
In [24]: sns.distplot(predy)
```

```
Out[24]: <matplotlib.axes._subplots.AxesSubplot at 0x17b8f341860>
```

