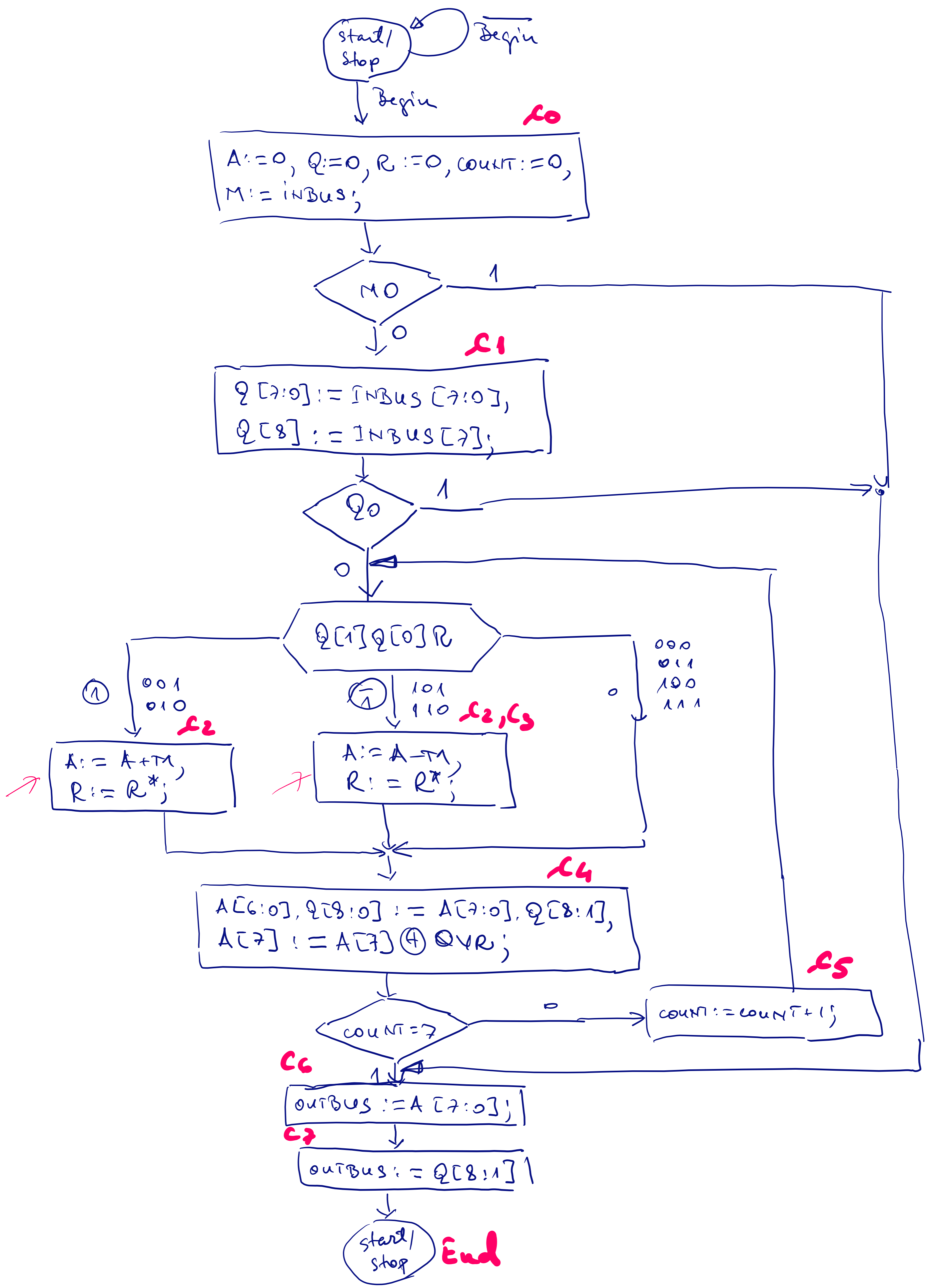
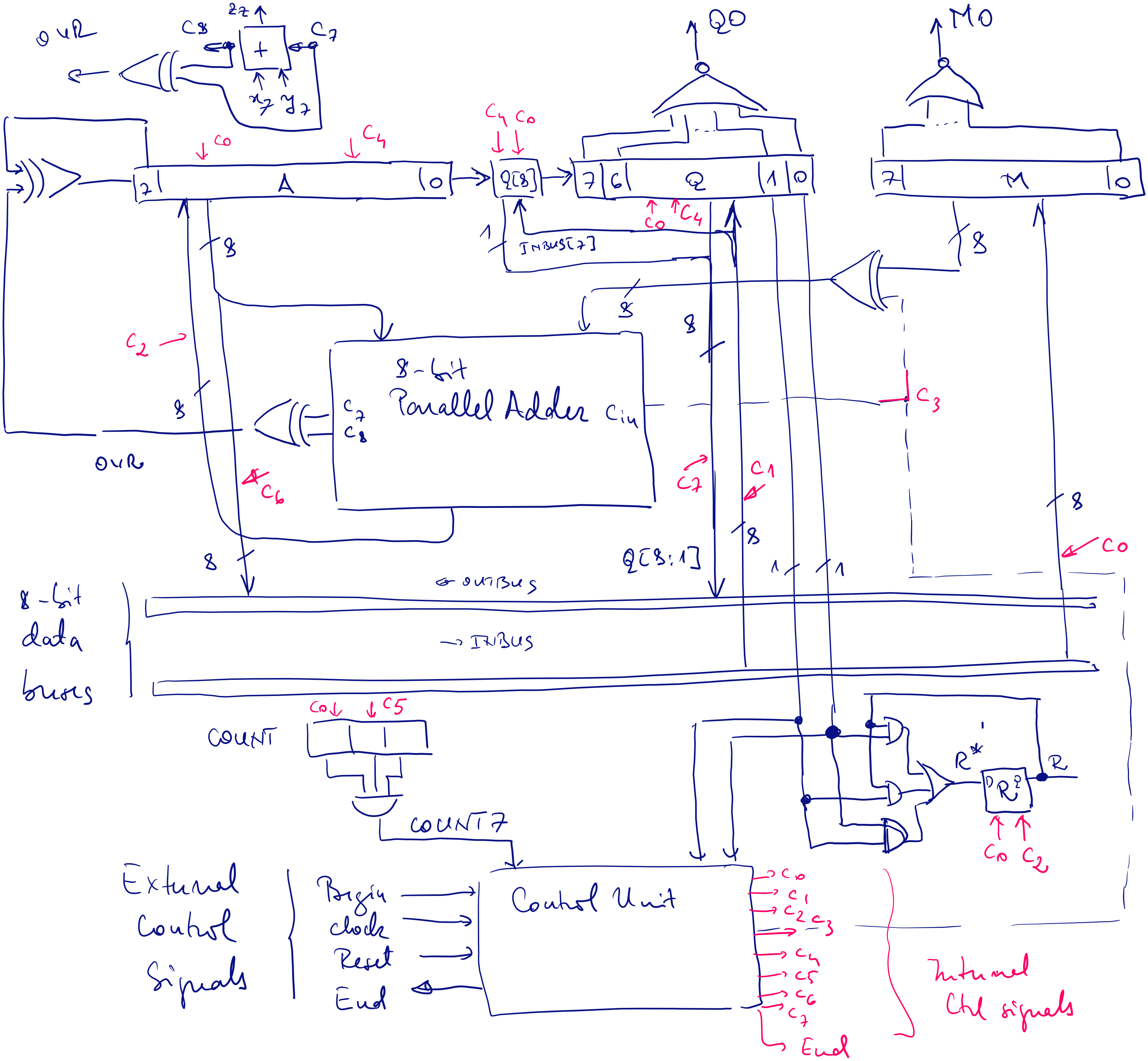
1.2.3 Modified Booth – Hardware implementation





$$R^* = R \cdot x_i + R \cdot x_{i+1} + x_i \cdot x_{i+1}$$

Q[0]   Q[1]

1.3 Speeding up multiplication with higher radix

1.3.1 Redundant sets of digits

Booth

Q[0]	Q[-1]	OP
0	0	0
0	1	1
1	0	$\bar{1}$
1	1	0

Radix 2

Conventional: $\{0, 1\}$

Redundant: $\{\bar{1}, 0, 1\}$

Example

$$\begin{array}{ccccc} 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & \bar{1} & 0 & 0 & 1 \end{array} = 1\times 2^0 + 0\times 2^1 + 0\times 2^2 - 1\times 2^3 + 1\times 2^4 =$$

$$= 16 - 8 + 1 = 9$$

Encoding

$$\begin{array}{ccccc} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 1 \end{array} - \quad = 9_{ten} \checkmark$$

$0 \to \begin{matrix}0\\0\end{matrix}$; $1 \to \begin{matrix}1\\0\end{matrix}$; $\bar{1} \to \begin{matrix}0\\1\end{matrix}$

Radix 4

Conventional: $\{0, 1, 2, 3\}$

Redundant: $\{\bar{2}, \bar{1}, 0, 1, 2\}$

Example

$$\begin{array}{ccccc} 4^4 & 4^3 & 4^2 & 4^1 & 4^0 \\ 1 & 2 & 0 & \bar{2} & \bar{1} \end{array} = 1\times 4^4 + 2\times 4^3 - \underline{2\times 4^1} - 1\times 4^0 = 379_{ten}$$

$$\begin{array}{r} 256 + \\ 128 \\ \hline 384 - \\ 9 \\ \hline 375 \end{array} \checkmark$$

$0 \to \begin{matrix}00\\00\end{matrix}$; $1 \to \begin{matrix}01\\00\end{matrix}$

$2 \to \begin{matrix}10\\00\end{matrix}$; $\bar{1} \to \begin{matrix}00\\01\end{matrix}$

$\bar{2} \to \begin{matrix}00\\10\end{matrix}$

Encoding

$$\begin{array}{ccccc} 01 & 10 & 00 & 00 & 00 \\ 00 & 00 & 00 & 10 & 01 \\ \hline 01 & 01 & 11 & 01 & 11 \end{array} - \quad \checkmark$$

$$\begin{array}{r} 23 \\ 32 \\ 64 \\ 256 \\ \hline 375_{ten} \end{array} \checkmark$$

1.3.2 Radix-4 Booth algorithm

x_i	x_{i-1}	OP
0	0	0
0	1	1
1	0	$\bar{1}$
1	1	1

$0 \to$ no.op
$1 \to +Y$
$\bar{1} \to -Y$
$2 \to +2Y$
$\bar{2} \to -2Y$

x_{i+1}	x_i	x_{i-1}	OP	
0	0	0	0	$0\times 2^i + 0\times 2^{i+1}$
0	0	1	1	$1\times 2^i + 0\times 2^{i+1} = 1\times 2^i$
0	1	0	1	$-1\times 2^i + 1\times 2^{i+1} = 1\times 2^i$
0	1	1	2	$0\times 2^i + 1\times 2^{i+1} = 2\times 2^i$
1	0	0	$\bar{2}$	$0\times 2^i - 1\times 2^{i+1} = -2\times 2^i$
1	0	1	$\bar{1}$	$1\times 2^i - 1\times 2^{i+1} = -1\times 2^i$
1	1	0	$\bar{1}$	$-1\times 2^i + 0\times 2^{i+1} = -1\times 2^i$
1	1	1		$0\times 2^i + 0\times 2^{i+1} = 0$

$105 - 64 = 41$, $41 - 32 = 9$

$79 - 64 = 15$

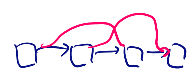
$X = -105$

$Y = -79$

$X = 1110\ 1001_{SM}$
$= 1001\ 0111_{C_2}$

$Y = 1100\ 1111_{SM}$
$= 1011\ 0001_{C_2}$

$+M = 1\ 1011\ 0001$
$-M = 0\ 0100\ 1111$
$2M = 1\ 0110\ 0010$
$-2M = 0\ 1001\ 1110$



COUNT	A	Q	Q[-1]	M
00	00000 0000 0	1001 0111	0	1011 0001
+	00100 1111		$-M$	
	00100 1111			
	00001 0011	1110 0101	1	
01 +	10110 0010		$+2M$	
	10111 0101			
	11101 1101	0111 1001	0	
10 +	11011 0001		$+M$	
	11000 1110			
	11110 0011	1001 1110	0	
11 +	01001 1110		$-2M$	
	01000 0001	1001 1110	0	
	0 0010 0000	0110 0111	1	

Result A[7:0].Q[7:0]

-105×-79
945
735
$+8295$

$39 +$
64
8192
$+8295$ ✓

1.3.3 HW Implementation of the Radix-4 Booth algorithm