Section 3, p34 1-7odd, 17, 21, 25

Computations

1. What three things must be check to determine whether a function $\phi : S \mapsto S'$ is an isomorphism of a binary structure $\langle S, * \rangle$ with $\langle S', *' \rangle$?

ϕ must be *one-to-one*, *onto*, and *operation preserving.*

In Exercises 2 through 10, determine whether the given map ϕ is an isomorphism of the first binary structure with the second. If it is not an isomorphism, why not?

3. $\langle \mathbb{Z}, + \rangle$ with $\langle \mathbb{Z}, + \rangle$ where $\phi(n) = 2n$ for $n \in \mathbb{Z}$

An isomorphism must be one-to-one, onto, and operation preserving.

Proof. (a) One-to-one: Assume $\phi(n_1) = \phi(n_2)$ for some $n_1, n_2 \in \mathbb{Z}$.

$$\begin{aligned} \phi(n_1) &= \phi(n_2) \\ 2n_1 &= 2n_2 \\ n_1 &= n_2 \end{aligned}$$

Thus ϕ is one-to-one.

(b) Onto: Let $y \in \mathbb{Z}$. Let us find $n \in \mathbb{Z}$ such that $y = \phi(x)$

$$\begin{aligned} y &= \phi(n) \\ y &= 2n \\ y/2 &= n \end{aligned}$$

$y/2$ is not always an integer, so we cannot say that ϕ is onto. Therefore, ϕ is not an isomorphism in this case.

□

5. $\langle \mathbb{Q}, + \rangle$ with $\langle \mathbb{Q}, + \rangle$ where $\phi(x) = x/2$ for $x \in \mathbb{Q}$

answer

7. $\langle \mathbb{R}, \cdot \rangle$ with $\langle \mathbb{R}, \cdot \rangle$ where $\phi(x) = x^3$ for $x \in \mathbb{R}$

answer

17. The map $\phi : \mathbb{Z} \mapsto \mathbb{Z}$ defined by $\phi(n) = n + 1$ for $n \in \mathbb{Z}$ is one to one and onto $\mathbb{Z}$.

Give the definition of a binary operation $*$ on $\mathbb{Z}$ such that ϕ is an isomorphism mapping

a. $\langle \mathbb{Z}, \cdot \rangle$ onto $\langle \mathbb{Z}, * \rangle$

answer

b. $\langle \mathbb{Z}, * \rangle$ onto $\langle \mathbb{Z}, \cdot \rangle$

answer

In Exercises 21 and 22, correct the definition of the italicized term without reference to the text, if correction is needed, so that it is in a form acceptable for publication.

21. A function $\phi : S \mapsto S'$ is an *isomorphism* if and only if $\phi(a * b) = \phi(a) *' \phi(b)$.

answer

25. Continuing the ideas of Exercise 24 can a binary structure have the left identity element e_L and a right identity element e_R where $e_L \neq e_R$? If so, given an example, using an operation on a finite set S. If not, prove that it is impossible.

answer