Test 2

Problem 1 Let A be an $n \times n$ matrix such that $A^4 + A^2 = O$. Show that

$$(A^2 - A + I)^{-1} = A^2 + A + I$$

Problem 2 In $\mathbb{R}^3$, let $\vec{v}_1 = (-3, 1, 4), \vec{v}_2 = (-4, 2, 5),$ and $\vec{v}_3 = (-1, 0, 2)$. Express $\vec{u} = (5, -4, 2)$ as a linear combination of $\vec{v}_1, \vec{v}_3,$ and $\vec{v}_3$ by finding the MATRIX INVERSE.

Problem 3 Solve for the matrix A if

$$(I - 2A)^{-1} = \begin{pmatrix} 1 & -3 & 3 \\ -2 & 2 & -5 \\ 3 & -8 & 9 \end{pmatrix}.$$

Problem 4 Let

$$A = \begin{pmatrix} 1 & 0 & 3 \\ -2 & 0 & -5 \\ 0 & 2 & 0 \end{pmatrix}.$$

Write A as a product of elementary matrices.

Problem 5 Find the conditions on $b_1, b_2,$ and b_3 such that the system

$$\begin{aligned} 1x_1 - 1x_2 + 1x_3 &= b_1 \\ -4x_1 + 7x_2 + 2x_3 &= b_2 \\ -2x_1 + 3x_2 + 0x_3 &= b_3 \end{aligned}$$

is consistent.

Problem 6 Let A be any $n \times n$ matrix. Prove or disprove: If there exists a positive integer k such that $A^k\vec{x} = \vec{0}$ has only the trivial solution, then A can be written as a product of elementary matrices.

Problem 7 Prove that for all $n \times n$ matrices A, the matrix $A^TA + 2AA^T$ is symmetric.

Problem 8 If A is an $n \times n$ upper-triangular matrix such that $A^5 = O$, find $\mathrm{tr}(A)$.

Problem 9 Does there exist an $n \times n$ matrix A such that for <u>all</u> $n \times 1$ column vector $\vec{b}$, the matrix equation

$$A\vec{x} = \vec{b}$$

has infinitely many solutions? If you think there exists such a matrix A, please write out A; if you think there does not exist such a matrix A, please give a proof.