Calculus I, II, III

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Contents

1 General Algebraic Concepts

Order of Operations

1. **P**arenthesis
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3. **F**unctions and **R**oots
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5. **A**ddition and **S**ubtraction

Properties of Exponents

Property	Symbolic Form
Product of Powers	$b^r \cdot b^s = b^{r+s}$
Quotient of Powers	$\frac{b^r}{b^q} = b^{r-s}$
Power of a Power	$(b^r)^s = b^{rs}$
Power of a Product	$(ab)^r = a^r b^r$
Power of a Quotient	$(\frac{a}{b})^r = \frac{a^r}{b^r}$
Negative Exponents	$a^{-r} = \frac{1}{a^r}$ or $\frac{1}{a^{-r}} = a^r$
	$\left(\frac{a}{b}\right)^{-r} = \left(\frac{b}{a}\right)^r = \frac{b^r}{a^r}$
Fractional Exponents	$\sqrt[d]{a} = a^{\frac{1}{d}}$
	$(\sqrt[d]{a})^n = \sqrt[d]{a^n} = a^{\frac{n}{d}}$

Ration Root Theorem

A rational root of a polynomial function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

is of the form:

$$\pm \frac{p}{q} = \pm \frac{\text{a factor of last term, } a_0}{\text{a factor of first term, } a_n}$$

Where a_{n-0} are integers.

Trigonometry

Property	Formula
Reciprocal	$\sin\theta = \frac{1}{\csc\theta} \quad \cos\theta = \frac{1}{\sec\theta} \quad \tan\theta = \frac{1}{\cot\theta}$
Pythagorean	$\sin^2 a + \cos^2 a = 1 \quad 1+\tan^2 a = \sec^2 a \quad 1+\cot^2 a = \csc^2 a$
Ratio	$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$
Opposite Angle	$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta \quad \tan(-\theta) = -\tan\theta$
	$\csc(-\theta) = -\csc\theta \quad \sec(-\theta) = \sec\theta \quad \cot(-\theta) = -\cot\theta$
Sum/Difference	$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta \quad \sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$
of Angles	$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta \quad \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$
	$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta} \quad \tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}$
Double Angle	$\sin(2\theta) = 2\sin\theta\cos\theta \quad \tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$
	$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
Half Angle	$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \quad \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

2 Limits and their Properties

3 Differentiation

4 Applications of Differentiation

5 Integration

6 Differential Equations

7 Applications of Integration