Problem 1

Let V be a vector space, and let $\vec{u}, \vec{v}, \vec{w} \in V$. Prove that if $\vec{u} \oplus \vec{w} = \vec{v} \oplus \vec{w}$ then $\vec{u} = \vec{v}$.

Proof. Consider $\vec{v}, \vec{u}, \vec{w} \in V$, and assume $\vec{u} \oplus \vec{w} = \vec{v} \oplus \vec{w}$.

| $\vec{\mathcal{K}} \Theta$ က် = $\vec{\mathcal{V}} \Theta$ က် $\vec{u} \oplus \vec{w} = \vec{v} \oplus \vec{w}$ | Assertion |
|---|-----------|
| by Axiom 5, - WEY $\vec{u} \oplus \vec{u} \oplus \vec{v} = -\vec{u} \oplus \vec{v} \oplus \vec{v}$ | Axiom 3 |
| $(\mathring{\nabla} \oplus \mathring{\vec{\omega}}) \oplus - \mathring{\vec{\omega}} = (\mathring{\vec{v}} \otimes \mathring{\vec{\omega}}) \oplus - \mathring{\vec{\omega}}^{id} \oplus \vec{w} = -\vec{u} \oplus \vec{v} \oplus \vec{w}$ $\vec{w} = (-\vec{u} \oplus \vec{v}) \oplus \vec{w}$ | Axiom 5 |
| Axion 3 $\vec{v} = (-\vec{u} \oplus \vec{v}) \oplus \vec{v}$ | Axiom 4 |
| • 1 → → → | Axiom 4 |
| \mathcal{T} $oldsymbol{\mathfrak{G}}$ $oldsymbol{\mathfrak{G}}$ $oldsymbol{\mathfrak{G}}$ $oldsymbol{\mathfrak{G}}$ $oldsymbol{\mathfrak{G}}$ $oldsymbol{\mathfrak{G}}$ $oldsymbol{\mathfrak{G}}$ | Axiom 5 |
| Letimolian of allitic inverse $\vec{u} = \vec{u}$ | Axiom 5 |
| 2019 = \$0 19 | |
| delighten of additive identity. | |

 $\lambda = \sqrt[7]{}$ Problem 2

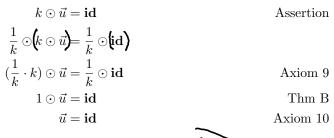
Prove Theorem B.

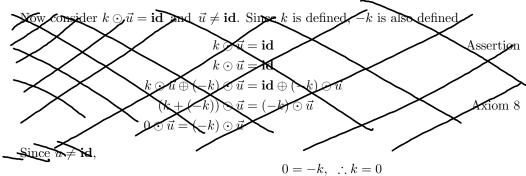
Proof. Let $\vec{u} \in V$ and $k \in \mathbb{R}$. Consider $\mathbf{id} = \vec{u}$:

$$\begin{array}{lll} & & & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ & &$$

Prove Theorem D. If $k \odot \vec{u} = id$, then k = 0 and/or u = id

Proof. Consider $k \odot \vec{u} = \mathbf{id}$ and $k \neq 0$. Since $\frac{1}{k} \neq \frac{1}{0}$, $\frac{1}{k}$ is well-defined.





Problem 4

Prove that there does not exist a real vector space of size 2. Show that there cannot be a vector space of size 2.

Proof. Let $V = \{\vec{u}, \vec{v}\}$ be a vectorspace That is, it satisfies all 10 Axioms.

Axiom 4 states that id exists and is unique. Therefore either $\vec{u} = \text{id}$ or $\vec{v} = \text{id}$. Both cannot be id, so therefore $\vec{u} \neq \vec{v}$ Since $\vec{v} \neq \vec{v}$.

Without the loss of generality, let $\vec{u} = id$.

 $\vec{u} \oplus \vec{v} = \vec{v}$ Axiom 4 $\mathbf{id} \oplus \vec{v} = \vec{v}$ dilive interse exists for all of UV. 50 we know - V + 4 Now consider Axiom 5: Since there is only one other element in $V_{+} = \vec{a}$ must be Since $\vec{v} \neq \mathbf{id}, -\vec{v} \neq \vec{v}$ therefore VEDER Now Jose in 1000010v = ~ (Axiom 10) (1+1)0v = ~ (Axiom 8) Therefore we have from Axiom 4 and 5: 20v= ~=i] by Thm A, ether 2=0 a However, this contradicts with our assertion that $\vec{u} \neq \vec{v}$. V=id, We know 2 +0 and ∴ A vectorspace of size 2 cannot exist. V + id since w=id, on contradiction. ..