Graded Assignment #2

1 [2 points each]

a. List the elements of $\langle f \rangle$ in S_6 where $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 3 & 2 & 5 & 4 \end{pmatrix}$

$\langle f \rangle$ consists of $\{f, f^2, f^3, \mathbf{id}\}$ where

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 3 & 2 & 5 & 4 \end{pmatrix} \qquad f^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 3 & 1 & 5 & 2 \end{pmatrix}$$

$$f^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 6 & 5 & 1 \end{pmatrix} \qquad \mathbf{id} = f^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$$

b. If $f(x) = x + 1$, describe the cyclic subgroup $\langle f \rangle$ of $\langle S_{\mathbb{R}}, \circ \rangle$.

$\langle f \rangle$ consists of $\{\mathbf{id} = f^0, f^1, f^2, \ldots\}$, where for $x \in \mathbb{R}$

$$f^n(x) = x + n$$

c. If $f(x) = x + 1$, describe the cyclic subgroup $\langle f \rangle$ of $\langle F(\mathbb{R}), + \rangle$.

$\langle f \rangle$ consists of $\{\mathbf{id} = f^0, f^1, f^2, \ldots\}$, where for $x \in \mathbb{R}$

$$f^n(x) = nx + n$$

2 [4 points] The subgroup of S_5 generated by $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 5 & 3 \end{pmatrix}$ has six elements. List them, using f and/or g as appropriate, and write the operation table of this subgroup.

Since each of these permutations are disjoint, the resulting subgroup generated by them will be Abelian. Let us first look at the permutations of f and g individually.

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix} \qquad g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 5 & 3 \end{pmatrix}$$

$$\mathbf{id} = f^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \qquad g^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 3 & 4 \end{pmatrix}$$

$$\mathbf{id} = g^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

Knowing these identities, let us determine the final two elements of this subgroup.

$$fg = gf = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 5 & 3 \end{pmatrix} \qquad fg^2 = g^2 f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 3 & 4 \end{pmatrix}$$

The subgroup generated is the set $\{\mathbf{id}, f, g, g^2, fg, fg^2\}$, with operation table

$\circ$	$\mathbf{id}$	f	g	g^2	fg	fg^2
$\mathbf{id}$	$\mathbf{id}$	f	g	g^2	fg	fg^2

3 [2 points each] You must justify your answers for parts (b) and (c).

a. Compute the following product in S_9 and write your answers as a permutation tabular form: $(1,4,7)(1,6,7,8)(7,4,1,3,2)$.

answer

b. Determine whether the following is even or odd: $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 1 & 5 & 6 & 2 & 3 & 8 \end{pmatrix}$.

answer

c. Determine whether the following is even or odd: $(1,2,7,6)(3,2,4,1)(7,8,1,2)$.

answer

4 [4 points] Let H and K be subgroups of a group G. Define $\sim$ on G by $a \sim b$ if and only if $a = hbk$ for some $h \in H$ and some $k \in K$. Prove that $\sim$ is an equivalence relation on G. The equivalence classes of this equivalence relation are called *double cosets*.

answer