

Homework 7

1.4 Inverses; Algebraic Properties of Matrices

28. Show that if a square matrix A satisfies $A^2 - 3A + I = 0$, then $A^{-1} = 3I - A$.

work. Consider $3I - A$.

$$A(3I - A) = 3AI - A^2 = 3A - A^2$$

If $A^2 - 3A + I = 0$, then we can simplify further to determine exactly what $3A - A^2$ equals.

$$\begin{aligned} A^2 - 3A + I &= 0 \\ (3A - A^2) + A^2 - 3A + I &= (3A - A^2) + 0 \\ (3A + (-A^2 + A^2) - 3A) + I &= (3A - A^2) \\ (3A - 3A) + I &= 3A - A^2 \\ I &= 3A - A^2 \\ \therefore I &= A(3I - A) \\ \therefore I &= (3I - A)A \end{aligned}$$

Since $I = A(3I - A)$ and $I = (3I - A)A$, therefore $A^{-1} = 3I - A$ if $A^2 - 3A + I = 0$. □

31. Assuming that all matrices are $n \times n$ and invertible, solve for D :

$$C^T B^{-1} A^2 B A C^{-1} D A^{-2} B^T C^{-2} = C^T.$$

39. Using Matrix Inversion, find the unique solution of the given linear system.

$$\begin{aligned} 3x_1 - 2x_2 &= -1 \\ 4x_1 + 5x_2 &= 3 \end{aligned}$$

53a. Show that if A, B and $A + B$ are invertible matrices with the same size, then

$$A(A^{-1} + B^{-1})B(A + B)^{-1} = I.$$

55. Show that if A is a square matrix such that $A^k = 0$ for some positive integer k , then the matrix $(I - A)$ is invertible and

$$(I - A)^{-1} = I + A + A^2 + \cdots + A^{k-1}.$$

1.5 Elementary Matrices and a Method for Finding A^{-1}

15. Use the inverse algorithm to find the inverse of the given matrix, if the inverse exists.

$$\begin{bmatrix} -1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}.$$

25. Find the inverse of the following 4×4 matrices, where k_1, k_2, k_3, k_4 , and k are all non-zero.

a. $\begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}.$

b. $\begin{bmatrix} k & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & k & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$

27. Find all values of c , if any, for which the given matrix is invertible.

29. Write the given matrix as a product of elementary matrices.

$$\begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix}$$

41. Prove that if A and B are $m \times n$ matrices, then A and B are row equivalent if and only if A and B have the same reduced row echelon form.

1.6 More on Linear Systems and Invertible Matrices

15. Determine conditions on the b_i 's, if any, in order to guarantee that the linear system is consistent.

$$\begin{aligned} x_1 - 2x_2 + 5x_3 &= b_1 \\ 4x_1 - 5x_2 + 8x_3 &= b_2 \\ -3x_1 + 3x_2 - 3x_3 &= b_3 \end{aligned}$$

21. Let $A\vec{x} = \vec{0}$ be a homogenous system of n linear equations in n unknown that has only the trivial solution. Show that if k is any positive integer, then the system $A^k\vec{x} = \vec{0}$ also has only the trivial solution.