Section 10, p102 #1,3,5,6,9,10,13,15

Computations

1. Find all the cosets of the subgroup $4\mathbb{Z}$ of $\mathbb{Z}$.

$\{4n : n \in \mathbb{Z}\},\ \{4n+1 : n \in \mathbb{Z}\},\ \{4n+2 : n \in \mathbb{Z}\},\ \{4n+3 : n \in \mathbb{Z}\}$

3. Find all the cosets of the subgroup $\langle 2 \rangle$ of $\mathbb{Z}_{12}$.

$\{0, 2, 4, 6, 8, 10\},\ \{1, 3, 5, 7, 9, 11\}$

5. Find all the cosets of the subgroup $\langle 18 \rangle$ of $\mathbb{Z}_{36}$.

$\{n, n+18\}$ for $n \in \{0, 1, 2, \ldots, 17\}$

6. Find all the left cosets of the subgroup $\{\rho_0, \mu_2\}$ of the group D_4 given by Table 8.12.

8.12 Table

	ρ_0	ρ_1	ρ_2	ρ_3	μ_1	μ_2	δ_1	δ_2
ρ_0	ρ_0	ρ_1	ρ_2	ρ_3	μ_1	μ_2	δ_1	δ_2
ρ_1	ρ_1	ρ_2	ρ_3	ρ_0	δ_1	δ_2	μ_2	μ_1
ρ_2	ρ_2	ρ_3	ρ_0	ρ_1	μ_2	μ_1	δ_2	δ_1
ρ_3	ρ_3	ρ_0	ρ_1	ρ_2	δ_2	δ_1	μ_1	μ_2
μ_1	μ_1	δ_2	μ_2	δ_1	ρ_0	ρ_2	ρ_3	ρ_1
μ_2	μ_2	δ_1	μ_1	δ_2	ρ_2	ρ_0	ρ_1	ρ_3
δ_1	δ_1	μ_1	δ_2	μ_2	ρ_1	ρ_3	ρ_0	ρ_2
δ_2	δ_2	μ_2	δ_1	μ_1	ρ_3	ρ_1	ρ_2	ρ_0

Left cosets:

$$\begin{aligned}
\rho_0 H &= \{\rho_0, \mu_2\} = \mu_2 H \\
\rho_1 H &= \{\rho_1, \delta_2\} = \delta_2 H \\
\rho_2 H &= \{\rho_2, \mu_1\} = \mu_1 H \\
\rho_3 H &= \{\rho_3, \delta_1\} = \delta_1 H
\end{aligned}$$

9. Repeat Exercise 6 for the subgroup $\{\rho_0, \rho_2\}$ of D_4

Left cosets:

$$\rho_0 H = \{\rho_0, \rho_2\} = \rho_2 H$$
$$\rho_1 H = \{\rho_1, \rho_3\} = \rho_3 H$$
$$\mu_1 H = \{\mu_1, \mu_2\} = \mu_2 H$$
$$\delta_1 H = \{\delta_1, \delta_2\} = \delta_2 H$$

10. Repeat the preceding exercise, but find the right cosets this time. Are they the same as the left cosets?

Right cosets:

$$H\rho_0 = \{\rho_0, \rho_2\} = H\rho_2$$
$$H\rho_1 = \{\rho_1, \rho_3\} = H\rho_3$$
$$H\mu_1 = \{\mu_1, \mu_2\} = H\mu_2$$
$$H\delta_1 = \{\delta_1, \delta_2\} = H\delta_2$$

Yes, they are the same as the left cosets.

13. Find the index of $struct\mu_1$ in the group S_3, using notation of Example 10.7

$|G : \langle\mu_1\rangle| = 3$, because $|G| = 6 = 3 \cdot 2 = |G : \langle\mu_1\rangle||\langle\mu_1\rangle|$

15. Let $\sigma = (1, 2, 5, 4)(2, 3)$ in S_5. Find the index of $\langle\sigma\rangle$ in S_5.

According to the Theorem of Lagrange, $|G : H| = \frac{|G|}{|H|}$. In this instance, $G = S_5$, so $|G| = 5! = 120$. $\sigma = (1, 2, 5, 4)(2, 3) = (1, 2, 3, 5, 4)$, so $\sigma^5 = \sigma^0$. This means $|\langle\sigma\rangle| = 5$. Thus we have the following.

$$|G : H| = \frac{|G|}{|H|} = \frac{120}{5} = 24$$