Section 1.1 1-12, Section 1.2 1-16

1.1

In Problems 1-12, a differential equation is given along with the field or problem area in which it arises. Classify each as an ordinary differential equation (ODE) or a partial differential equation (PDE), give the order, and indicate the independent and dependent variables. If the equation is an ordinary differential equation, indicate whether the equation is linear or nonlinear.

1. $5\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 9x = 2\cos 3t$

ODE linear, order 2, independent vars are t, dependent vars are x

2. $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

ODE nonlinear, order 2, independent vars are x, dependent vars are y

3. $\frac{dy}{dx} = \frac{y(2-3x)}{x(1-3y)}$

ODE nonlinear, order 1, independent vars are x, dependent vars are y

4. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

PDE, order 2, independent vars are x, dependent vars are y

5. $y\left[1 + \left(\frac{dy}{dx}\right)^2\right] = C$

ODE nonlinear, order 1, independent vars are x, y, dependent vars are u

6. $\frac{dx}{dt} = k(4-x)(1-x)$, where k is a constant

ODE nonlinear, order 1, independent vars are t, dependent vars are x

7. $\frac{dp}{dt} = kp(P-p)$, where k and P are constants

ODE nonlinear, order 1, independent vars are t, dependent vars are p

8. $\sqrt{1-y}\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$

ODE nonlinear, order 2, independent vars are x, dependent vars are y

9. $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$

ODE nonlinear, order 2, independent vars are x, dependent vars are y

10. $8\frac{d^4y}{dx^4} = x(1-x)$

ODE nonlinear, order 4, independent vars are x, dependent vars are y

11. $\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r} + kN$, where k is a constant

PDE, order 2, independent vars are t, r, dependent vars are N

12. $\frac{d^2y}{dx^2} - 0.1(1-y^2)\frac{dy}{dx} + 9y = 0$

ODE nonlinear, order 2, independent vars are x, dependent vars are y

1.2

1

(a). Show that $\phi(x) = x^2$ is an explicit solution to $x\frac{dy}{dx} = 2y$

answer

(b). Show that $\phi(x) = e^x - x$ is an explicit solution to $\frac{dy}{dx} + y^2 = e^{2x} + (1-2x)e^x + x^2 - 1$

answer

(c). Show that $\phi(x) = x^2 - x^{-1}$ is an explicit solution to $x^2\frac{d^2y}{dx^2} = 2y$ on the interval $(0, \infty)$

answer

2

(a). Show that $y^2 + x - 3 = 0$ is an implicit solution to $\frac{dy}{dx} = -\frac{1}{2y}$ on the interval $(-\infty, 3)$.

answer

(b). Show that $xy^3 - xy^3\sin x = 1$ is an implicit solution to $\frac{dy}{dx} = \frac{(x\cos x + \sin x - 1)y}{3(x - x\sin x)}$ on the interval $(0, \pi/2)$.

answer

In Problems 3-8, determine whether the given function is a solution to the given differential equation.

3. $y = \sin x + x^2, \qquad \frac{d^2y}{dx^2} + y = x^2 + 2$

answer

4. $x = 2\cos t - 3\sin t, \qquad x'' + x = 0$

answer

5. $\theta = 2e^{3t} - e^{2t}, \qquad \frac{d^2\theta}{dt^2} - \theta\frac{d\theta}{dt} + 3\theta = -2e^{2t}$

answer

6. $x = \cos 2t, \qquad \frac{dx}{dt} + tx = \sin 2t$

answer

7. $y = e^{2x} - 3e^{-x}, \qquad \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

answer

8. $y = 3\sin 2x + e^{-x}, \qquad y'' + 4y = 5e^{-x}$

answer

In Problems 9-13, determine whether the given relation is an implicit solution to the given differential equation. Assume that the relationship does define y implicitly as a function of x and use implicit differentiation.

9. $x^2 + y^2 = 4, \qquad \frac{dy}{dx} = \frac{x}{y}$

answer

10. $y - \ln y = x^2 + 1, \qquad \frac{dy}{dx} = \frac{2xy}{y-1}$

answer

11. $e^{xy} + y = x - 1, \qquad \frac{dy}{dx} = \frac{e^{-xy}-y}{e^{-xy}+x}$

answer

12. $x^2 - \sin(x+y) = 1, \qquad \frac{dy}{dx} = 2x\sec(x+y) - 1$

answer

13. $\sin y + xy - x^3 = 2, \qquad y'' = \frac{6xy' + (y')^3 \sin y - 2(y')^2}{3x^2 - y}$

answer

14. Show that $\phi(x) = c_1 \sin x + c_2 \cos x$ is a solution to $\frac{d^2y}{dx^2} + y = 0$ for any choice of the constants c_1 and c_2. Thus, $c_1 \sin x + c_2 \cos x$ is a two-parameter family of solutions to the differential equation.

answer

15. Verify that $\phi(x) = \frac{2}{1-ce^x}$, where c is an arbitrary constant, is a one-parameter family of solutions to $\frac{dy}{dx} = \frac{y(y-2)}{2}$. Graph the solution curves corresponding to $c = 0, \pm 1, \pm 2$, using the same coordinate axes.

answer

16. Verify that $x^2+cy^2=1$, where c is an arbitrary nonzero constant, is a one-parameter family of implicit solutions to $\frac{dy}{dx}=\frac{xy}{x^2-1}$ and graph several of the solution curves using the same coordinate axes.

answer