Homework 4

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Let $V = \mathbb{R}^3$ be a vector space with standard addition and scalar multiplication. Use Theorem 3 of Lecture Note 8 to determine whether the following sets are subspaces of V.

b. *The set of vectors of the form* $(a, 1, 1)$

Proof. □

c. *The set of vectors of the form* (a, b, c)*, where* $b = a + c$

Proof. □

d. *The set of vectors of the form* $(a, b, 0)$

Proof. □

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Let $V = P_3$ be the vector space of all polynomials with degree *up to* 3, with standard addition and scalar multiplication. Use Theorem 3 of Lecture Note 8 to determine whether the following sets are subspaces of V.

b. *The set of polynomials* $a_0 + a_1x + a_2x^2 + a_3x^3$ *for which* $a_0 + a_1 + a_2 + a_3 = 0$.

Proof. □

c. *The set of polynomials* $a_0 + a_1x + a_2x^2 + a_3x^3$ *in which* $a_0, a_1, a_2,$ *and* a_3 *are integers.*

Proof. □

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Let $V = F(-\infty, \infty)$ be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$, with standard addition and scalar multiplication. Use Theorem 3 of Lecture Note 8 to determine whether the following sets are subspaces of V.

b. *The set of functions* f *in* $F(-\infty, \infty)$ *for which* $f(0) = 1$.

Proof. □

c. *The set of functions* f *in* $F(-\infty, \infty)$ *for which* $f(-x) = x$

Proof. □

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Let V be a vector space. Let I be a nonempty set (often called the "index set"), and let W_i be a subspace of V for all $i \in I$. Prove that $\bigcap_{i \in I} W_i$, which addition and scalar multiplication inherited from V, is a subspace of V.

Proof. □