Section 8.8

8.8.1

a. *Give a recursive definition for strings of properly nested parentheses and curly braces. For example, $\{\}\{\}$ is properly nested but $\{\}$ is not properly nested. The empty string should not be included in your definition.*

- Basis: λ has properly nested parentheses and curly braces.
- Recursive rules: If u and v are properly nested sequences of parentheses and curly braces then:
  1. $(u),\ \{u\},\ (\{u\}),$ and $\{(u)\}$ is properly nested.
  2. uv is properly nested.
- Exclusion statement: a string is properly nested only if it given in the basis or can be constructed by applying the recursive rules to the string in the basis.

8.8.2

Let $A = \{a, b\}$.

a. *Give a recursive definition for A^*.*

- Basis: $\lambda \in A^*$.
- Recursive rules: If $u \in A^*$ then:
  1. ub and $ua \in A^*$.
  2. bu and $au \in A^*$.
- Exclusion statement: a string is in A^* if it given in the basis or can be constructed by applying the recursive rules to the string in the basis.

b. *The set A^+ is the set of strings over the alphabet $\{a, b\}$ of length at least 1. That is $A^+ = A^* - \{\lambda\}$. Give a recursive definition for A^+.*

- Basis: a and $b \in A^+$.
- Recursive rules: If $u \in A^+$ then:
  1. ub and $ua \in A^+$.
  2. bu and $au \in A^+$.
- Exclusion statement: a string is in A^+ if it given in the basis or can be constructed by applying the recursive rules to the strings in the basis.

c. *Let S be the set of all strings from A^* in which there is no b before an a. For example, the strings λ, aa, bbb, $aabbbb$ all belong to S, but $aabab \notin S$. Give a recursive definition for the set S.*

- Basis: $\lambda \in S$.
- Recursive rules: If $u \in S$ then:
  1. $au \in S$.
  2. $ub \in S$.
- Exclusion statement: a string is in S if it given in the basis or can be constructed by applying the recursive rules to the string in the basis.

8.8.4

Give a recursive definition for each subset of the binary strings. A string x should be in the recursively defined set if and only if x has the property described.

a. *The set S consists of all strings with an even number of 1's.*

- Basis: $\lambda \in S$.
- Recursive rules: If $u \in S$ then:
  1. $1u1,\ 11u,$ and $u11 \in S$.
  2. $0u$ and $u0 \in S$.
- Exclusion statement: a string is in S if it given in the basis or can be constructed by applying the recursive rules to the string in the basis.

b. *The set S is the set of all binary strings that are palindromes. A string is a palindrome if it is equal to its reverse. For example,* 0110 *and* 11011 *are both palindromes.*

- Basis: $\lambda,\ 0,$ and $1 \in S$.
- Recursive rules: If $u \in S$ then:
  1. $1u1 \in S$.
  2. $0u0 \in S$.
- Exclusion statement: a string is in S if it given in the basis or can be constructed by applying the recursive rules to the string in the basis.

c. *The set S consists of all strings that have the same number of 0's and 1's.*

- Basis: $\lambda \in S$.
- Recursive rules: If u and $v \in S$ then:
  1. $1u0 \in S$.
  2. $0u1 \in S$.
  3. $uv \in S$.
- Exclusion statement: a string is in S if it given in the basis or can be constructed by applying the recursive rules to the string in the basis.