

# Discrete Math for Computer Science

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## Contents

<b>1</b>	<b>Logic</b>	<b>2</b>
1.1	Propositions and Logical Operations . . . . .	2
1.2	Evaluating Compound Propositions . . . . .	2
1.3	Conditional Statements . . . . .	2
1.4	Logical Equivalence . . . . .	3
1.5	Laws of Propositional Logic . . . . .	3
1.6	Predicates and Quantifiers . . . . .	3
1.7	Quantified Statements . . . . .	3
1.8	DeMorgan's law for Quantified Statements . . . . .	3
1.9	Nested Quantifiers . . . . .	3
1.10	More Nested Quantifiers . . . . .	3
1.11	Logical Reasoning . . . . .	3
1.12	Rules of Inference with Propositions . . . . .	3
1.13	Rules of Interence with Quantifiers . . . . .	3

# 1 Logic

## 1.1 Propositions and Logical Operations

**Proposition:** a statement that is either true or false.

Some examples include "It is raining today" and " $3 \cdot 8 = 20$ ".

However, not all statements are propositions, such as "open the door"

Name	Symbol	alternate name	$p$	$q$	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$
NOT	$\neg$	negation	T	T	F	T	T	F
AND	$\wedge$	conjunction	T	F	F	F	T	T
OR	$\vee$	dijunction	F	T	T	F	T	T
XOR	$\oplus$	exclusive or	F	F	T	F	F	F

XOR is very useful for encryption and binary arithmetic.

## 1.2 Evaluating Compound Propositions

$p$  : The weather is bad.

$p \wedge q$  : The weather is bad *and* the trip is cancelled

$q$  : The trip is cancelled.  $\triangleright$

$p \vee q$  : The weather is bad *or* the trip is cancelled

$r$  : The trip is delayed.

$p \wedge (q \oplus r)$  : The weather is bad *and* either the trip is cancelled *or* delayed

**Order of Evaluation**  $\neg$ , then  $\wedge$ , then  $\vee$ , but parenthesis always help for clarity.

Example Truth Table:	$p$	$q$	$p \wedge q$	$\neg q$	$(p \wedge q) \oplus \neg q$
	T	T	T	F	T
	T	F	F	T	T
	F	T	F	F	F
	F	F	F	T	T

## 1.3 Conditional Statements

$p \rightarrow q$  where  $p$  is the hypothesis and  $q$  is the conclusion

Format	Terminology	
$p \rightarrow q$	given	
$\neg q \rightarrow \neg p$	contrapositive	given $p \rightarrow q \equiv \neg q \rightarrow \neg p$ contrapositive
$q \rightarrow p$	converse	inverse $\neg p \rightarrow \neg q \equiv q \rightarrow p$ converse
$\neg p \rightarrow \neg q$	inverse	

$p$	$q$	$p \rightarrow q$		Phrase	Logic
T	T	T	$p$ is a <u>sufficient</u> condition for $q$ $q$ is a <u>necessary</u> condition for $p$	$q$ if $p$	$p \rightarrow q$
T	F	F		$q$ only if $p$	$q \rightarrow p$
F	T	T		$q$ if and only if $p$	$p \leftrightarrow q$
F	F	T			

**Order of Operations:**  $p \wedge q \rightarrow r \equiv (p \wedge q) \rightarrow r$

- 1.4 Logical Equivalence
- 1.5 Laws of Propositional Logic
- 1.6 Predicates and Quantifiers
- 1.7 Quantified Statements
- 1.8 DeMorgan's law for Quantified Statements
- 1.9 Nested Quantifiers
- 1.10 More Nested Quantifiers
- 1.11 Logical Reasoning
- 1.12 Rules of Inference with Propositions
- 1.13 Rules of Interence with Quantifiers