Section 2.5

2.5.1

b. For every integer n, if n^3 is even, then n is even.

Proof. Assume that n is odd. That is $n = 2k + 1$, for some integer k.

$$\begin{aligned} n^3 = (2k+1)^3 &= (4k^2 + 4k + 1)(2k + 1) \\ &= 8k^3 + 8k^2 + 2k + 4k^2 + 4k + 1 \\ &= 8k^3 + 12k^2 + 4k + 1 \\ &= 2(4k^3 + 6k^2 + 2k) + 1 \\ &= 2j + 1 \text{ for } j = 4k^3 + 6k^2 + 2k \end{aligned}$$

Therefore n^3 takes the form of an odd number. Therefore, through contrapositive, for every integer n, if n^3 is even, then n is even. □

c. For every integer n, if $5n + 3$ is even, then n is odd.

Proof. Assume that n is even. That is $n = 2k$, for some integer k.

$$\begin{aligned} 5n + 3 = 5(2k) + 3 &= 10k + 3 \\ &= 2(5k + 1) + 1 \\ &= 2j + 1 \text{ for } j = 5k + 1 \end{aligned}$$

Therefore $5n+3$ takes the form of an odd number. Therefore, through contrapositive, for every integer n, if $5n + 3$ is even, then n is odd. □

2.5.2

a. If x and y are integers such that $3 \nmid xy$, then $3 \nmid x$.

Proof. Assume that $3 \nmid x$. That is, $x = 3k$, for some integer k. Let y be an integer.

$$\begin{aligned} x &= 3k \\ xy &= 3ky \end{aligned}$$

$$3 \mid xy \equiv 3 \mid 3ky$$

Since 3 divides 3. Therefore, through contrapositive, if x and y are integer such that $3 \nmid xy$, then $3 \nmid x$ □

b. For any integers $x, y,$ and z if $x \mid y$ and $x \nmid z$, then $x \nmid (y + z)$.

Proof. Let $x \mid (y + z)$. That is, $y + z = kx$, for some integer k. Additionally, let $x \mid y$. That is, $y = jx$, for some integer j.

$$y + z = jx + z$$

$$\begin{aligned} jx + z &= kx \\ z &= kx - jx = x(k - j) \end{aligned}$$

$$\therefore x \mid z \equiv x \mid x(k - j)$$

Since $x \mid x(k-j)$, therefore $x \mid z$. Therefore, through contrapositive, for any integers $x, y,$ and z if $x \mid y$ and $x \nmid z$, then $x \nmid (y+z)$. $\square$