Homework 7

1.4 Inverses; Algebraic Properties of Matrices

Prove
$$A^{-1} = 3I - A$$

$$A(3I - A) = 3AI - A^{2} - 1(3A - A^{2})$$

$$I = 3A - A^{2}$$

28. Show that if a square matrix A satisfies $A^2 - 3A + 1 = 0$, then $A^{-1} = 3I - A$.

 $-I+I=\emptyset$

31. Assuming that all matrices are $n \times n$ and invertible, solve for D:

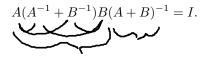
$$(ABC)^{-1}$$
 $C^{T}B^{-1}A^{2}BAC^{-1}DA^{-2}B^{T}C^{-2} = C^{T}$

39. Using Matrix Inversion, find the unique solution of the given linear system.

$$\begin{pmatrix} 3 - 2 \\ 4 - 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \begin{pmatrix} 3x_1 - 2x_2 = -1 \\ 4x_1 + 5x_2 = 3 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 - 2 \\ 4 - 5 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

53a. Show that if A, B and A + B are invertible matrices with the same size, then

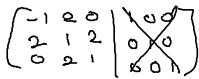


55. Show that if A is a square matrix such that $A^k = 0$ for some positive integer k, then the matrix (I - A) is invertible and

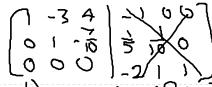
$$(I-A)^{-1} = I + A + A^2 + \dots + A^{k-1}.$$

1.5 Elementary Matrices and a Method for Finding A-1

15. Use the inverse algorithm to find the inverse of the given matrix, if the inverse exists.



$$\begin{bmatrix} -1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

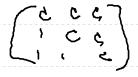


25. Find the inverse of the following 4×4 matrices, where k_1, k_2, k_3, k_4 , and k are all non-zero.

$$\boldsymbol{a.} \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}.$$

$$b. \begin{bmatrix} k & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & k & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

27. Find all values of c, if any, for which the given matrix is invertible.



29. Write the given matrix as a product of elementary matrices.

$$\begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix}$$

- **41.** Prove that if A and B are $m \times n$ matrices, then A and B are row equivalent if and only if A and B have the same reduced row eschelon form.
- 1.6 More on Linear Systems and Invertible Matrices
- 15. Determine conditions on the b_i's, if any, in order to guarantee that the linear system is consistent.

$$x_1 - 2x_2 + 5x_3 = b_1$$
$$4x_1 - 5x_2 + 8x_3 = b_2$$
$$-3x_1 + 3x_2 - 3x_3 = b_3$$

21. Let $A\vec{x} = \vec{0}$ be a homogenous system of n linear equations in n unknown that has only the trivial solution. Show that if k is any positive integer, then the system $A^k\vec{x} = \vec{0}$ also has only the trivial solution.