Problem 1

Let V be a vector space, and let $\vec{u}, \vec{v}, \vec{w} \in V$. Prove that if $\vec{u} \oplus \vec{w} = \vec{v} \oplus \vec{w}$ then $\vec{u} = \vec{v}$.

Proof. Consider $\vec{v}, \vec{u}, \vec{w} \in V$, and assume $\vec{u} \oplus \vec{w} = \vec{v} \oplus \vec{w}$.

$$\begin{aligned}
\vec{u} \oplus \vec{w} &= \vec{v} \oplus \vec{w} && \text{Assertion} \\
-\vec{u} \oplus \vec{u} \oplus \vec{w} &= -\vec{u} \oplus \vec{v} \oplus \vec{w} && \text{Axiom 3} \\
\mathbf{id} \oplus \vec{w} &= -\vec{u} \oplus \vec{v} \oplus \vec{w} && \text{Axiom 5} \\
\vec{w} &= (-\vec{u} \oplus \vec{v}) \oplus \vec{w} && \text{Axiom 4} \\
\mathbf{id} &= -\vec{u} \oplus \vec{v} && \text{Axiom 4} \\
\vec{v} &= -(-\vec{u}) && \text{Axiom 5} \\
\therefore \vec{v} &= \vec{u} && \text{Axiom 5}
\end{aligned}$$

□

Problem 2

Prove Theorem B.

Proof. □

Problem 3

Prove Theorem D.

Proof. □

Problem 4

Prove that there does not exist a real vector space of size 2.

Proof. □