MAT 369 Introduction to Graph Theory

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1 Introduction

1.1 Graphs and Graph Models

Graph Definition

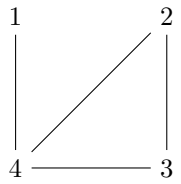
A (simple) **graph** is an ordered pair (V, E) where

- V is a nonempty set of objects called "vertices"
- E is a set containing some two-subsets of V called "edges". E may be empty.

Graphs are often represented pictorially. For example consider

$$G = (V, E) \text{ where } V = \{1, 2, 3, 4, 5\} \text{ and } E = \{\{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$$

G :



- Vertices 1 and 4 are **adjacent** because they are joined by an edge.
- Vertex 2 and edge $2-3$ are **indicent**.
- Edges $2-3$ and $3-4$ are **adjacent**.

Order Definition

The **order** of a graph G is $|V(G)|$, or the number of vertices.

Size Definition

The **size** of a graph G is $|E(G)|$, or the number of edges.

2 Degrees

3 Isomorphic Graphs

4 Trees