

Section 8.17

8.17.2 *Applying the Master Theorem: Give the asymptotic growth of $T(n)$ using Θ notation.*

a. $T(n) = 4T(n/3) + \Theta(n)$ $\frac{4}{3^1} > 1$, $\therefore T(n) = \Theta(n^{\log_3 4})$

b. $T(n) = 4T(n/4) + \Theta(\sqrt{n})$ $\frac{4}{4^{0.5}} > 1$, $\therefore T(n) = \Theta(n^{\log_4 4}) = \Theta(n)$

c. $T(n) = 4T(n/2) + \Theta(n^2)$ $\frac{4}{2^2} = 1$, $\therefore T(n) = \Theta(n^2 \log n)$

d. $T(n) = 4T(n/2) + \Theta(n^3)$ $\frac{4}{2^3} < 1$, $\therefore T(n) = \Theta(n^3)$

e. $T(n) = 2T(n/3) + \Theta(n)$ $\frac{2}{3^1} < 1$, $\therefore T(n) = \Theta(n)$

f. $T(n) = 2T(n/3) + \Theta(1)$ $\frac{2}{3^0} > 1$, $\therefore T(n) = \Theta(n^{\log_3 2})$

g. $T(n) = 7T(n/4) + \Theta(n^2)$ $\frac{7}{4^2} < 1$, $\therefore T(n) = \Theta(n^2)$

h. $T(n) = 7T(n/4) + \Theta(n)$ $\frac{7}{4^1} > 1$, $\therefore T(n) = \Theta(n^{\log_4 7}) = \Theta(1)$

i. $T(n) = 2T(n/4) + \Theta(\sqrt{n})$ $\frac{2}{4^{0.5}} = 1$, $\therefore T(n) = \Theta(\sqrt{n} \cdot \log n)$

j. $T(n) = 3T(n/3) + \Theta(1)$ $\frac{3}{3^0} > 1$, $\therefore T(n) = \Theta(n^{\log_3 3}) = \Theta(n)$
