Calculus I, II, III

Peter Schaefer

Highschool - Kutztown

Contents

1 General Algebraic Concepts

Order of Operations

1. **P**arenthesis
2. **E**xponents and **I**nverse
3. **F**unctions and **R**oots
4. **M**ultiplication and **D**ivision
5. **A**ddition and **S**ubtraction

Properties of Exponents

Property	Symbolic Form
Product of Powers	$b^r \cdot b^s = b^{r+s}$
Quotient of Powers	$\frac{b^r}{b^q} = b^{r-s}$
Power of a Power	$(b^r)^s = b^{rs}$
Power of a Product	$(ab)^r = a^r b^r$
Power of a Quotient	$(\frac{a}{b})^r = \frac{a^r}{b^r}$
Negative Exponents	$a^{-r} = \frac{1}{a^r}$ or $\frac{1}{a^{-r}} = a^r$
	$\left(\frac{a}{b}\right)^{-r} = \left(\frac{b}{a}\right)^r = \frac{b^r}{a^r}$
Fractional Exponents	$\sqrt[d]{a} = a^{\frac{1}{d}}$
	$(\sqrt[d]{a})^n = \sqrt[d]{a^n} = a^{\frac{n}{d}}$

Ration Root Theorem

A rational root of a polynomial function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

is of the form:

$$\pm\frac{p}{q} = \pm\frac{\text{a factor of last term, } a_0}{\text{a factor of first term, } a_n}$$

Where a_{n-0} are integers.

Trigonometry

Property	Formula
Reciprocal	$\sin\theta = \frac{1}{\csc\theta} \quad \cos\theta = \frac{1}{\sec\theta} \quad \tan\theta = \frac{1}{\cot\theta}$
Pythagorean	$\sin^2 a + \cos^2 a = 1 \quad 1 + \tan^2 a = \sec^2 a \quad 1 + \cot^2 a = \csc^2 a$
Ratio	$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$
Opposite Angle	$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta \quad \tan(-\theta) = -\tan\theta$
	$\csc(-\theta) = -\csc\theta \quad \sec(-\theta) = \sec\theta \quad \cot(-\theta) = -\cot\theta$
Sum/Difference	$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta \quad \sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$
of Angles	$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta \quad \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$
	$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta} \quad \tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}$
Double Angle	$\sin(2\theta) = 2\sin\theta\cos\theta \quad \tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$
	$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
Half Angle	$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \quad \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

2 Limits and their Properties

3 Differentiation

Common Differentiation Identities

General Differentiation

Variables	$a,\ n \in \mathbb{R},\ f,\ g$ is a function
Power Rule	$\frac{d}{dx}x^n = nx^{n-1}$
Constant	$\frac{d}{dx}a = 0$
Scalar	$\frac{d}{dx}af = af'$
Sum Rule	$\frac{d}{dx}f \pm g = f' \pm g'$
Product Rule	$\frac{d}{dx}fg = fg' + f'g$
Quotient Rule	$\frac{d}{dx}\frac{f}{g} = \frac{gf' - g'f}{g^2}$

Trigonometric

$$\frac{d}{dx}\sin x = \cos x$$
$$\frac{d}{dx}\cos x = -\sin x$$
$$\frac{d}{dx}\tan x = \sec^2 x$$
$$\frac{d}{dx}\cot x = -\csc^2 x$$
$$\frac{d}{dx}\sec x = \sec x \tan x$$
$$\frac{d}{dx}\csc x = -\csc x \cot x$$

Inverse Trigonometric

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$$
$$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$$

Other Differentiation

$$\frac{d}{dx}a^x = a^x \cdot \ln a$$
$$\frac{d}{dx}e^x = e^x$$
$$\frac{d}{dx}\log_a x = \frac{1}{\ln_a x}$$
$$\frac{d}{dx}\ln x = \frac{1}{x}$$
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$$

4 Applications of Differentiation

5 Integration

Common Integration Identities

Common Integrals

$$\int a\ dx = ax + C$$
$$\int x^n\ dx = \frac{x^{n+1}}{n+1} + C,\ n \neq 1$$
$$\int \frac{1}{x}\ dx = \ln|x| + C$$
$$\int e^x\ dx = e^x + C$$
$$\int a^x\ dx = \frac{a^x}{\ln a} + C,\ a > 0,\ a \neq 1$$

Trigonometric Integrals

$$\int \sin x\ dx = -\cos x + C$$
$$\int \cos x\ dx = \sin x + C$$
$$\int \sec^2 x\ dx = \tan x + C$$
$$\int \csc^2 x\ dx = -\cot x + C$$
$$\int \sec x \tan x\ dx = \sec x + C$$
$$\int \csc x \cot x\ dx = -\csc x + C$$

Inverse Trigonometric Integrals

$$\int \frac{1}{\sqrt{a^x - x^2}}\ dx = \sin^{-1}\frac{x}{a} + C \qquad \int \frac{1}{a^2 + x^2}\ dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \qquad \int \frac{1}{|x|}\sqrt{x^2 - 1}\ dx = \sec^{-1} x + C$$

6 Differential Equations

7 Applications of Integration