Section 5 Subgroups, p55 #1-9,11,15,17

In Exercises 1 through 6, determine whether the given subset of the complex numbers is a subgroup of the group $\mathbb{C}$ of complex numbers under addition.

1. $\mathbb{R}$

This is a subgroup. A real number plus a real number will always yield a real number, and the inverse is just the negative of the real number.

2. $\mathbb{Q}^+$

This is not a subgroup. Consider $1 \in \mathbb{Q}^+$. It has no inverse in $\mathbb{Q}^+$.

3. $7\mathbb{Z}$

This is a subgroup. A multiple of 7 plus another multiple of 7 will always result in a multiple of 7, and the inverse is just he negative of the multiple of 7.

4. The set $i\mathbb{R}$ of pure imaginary numbers including 0.

This is a subgroup. An imaginary number plus an imaginary number will always yield an imaginary number, assuming zero is included. The inverse is just the negative of the imaginary number.

5. The set $\pi\mathbb{Q}$ of rational multiples of π.

This is a subgroup. $\pi\frac{a}{b} + \pi\frac{c}{d} = \pi(\frac{a}{b} + \frac{c}{d})$. The inverse will for $\pi\frac{a}{b}$ is just $-\pi\frac{a}{b}$.

6. The set $\{\pi^n : n \in \mathbb{Z}\}$

This is not a subgroup, because it is not closed. Consider $\pi + \pi = 2\pi$. This will not be an integer power of π.