Linear Algebra

Peter Schaefer

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Contents

1 Brief Review

Commonly Used Sets

- $\mathbb{N}$: set of **natural numbers**
  could be *positive* integers
  could be *nonnegative* integers
- $\mathbb{Z}$: set of **integers**
- $\mathbb{Q}$: set of **rational numbers**
- $\mathbb{R}$: set of **real numbers**

Set Building

To denote sets too large to just list, we use **set builder** notation:

$$\{\text{candidate} : \text{condition}\}$$

Examples:

$$\{x \text{ is a fruit} : x \text{ is of yellow color}\}$$
$$\{x \text{ is a human being} : x \text{ is a president of the U.S.}\}$$
$$\{x \text{ is a city} : x \text{ is a capitol of a country}\}$$

Other Notations

- $\forall$: for all
- $\exists$: there exists
- s.t.: such that
- $\rightarrow\leftarrow$: contradiction
- WTS: want to show

2 Real Vector Spaces

A **real vector space** is simply a *nonempty set* that satisfies 10 properties called **10 axioms of a real vector space**.

- $\vec{v} \in$ vector space V can be *anything*
- **Never** assume that an element $\vec{v} \in V$ is an ordered pair

Addition

- denoted by $\oplus$
- simply a map

$$\oplus : V \times V \to V$$

Example of a definition of $\oplus$ for $V = \{\text{apple}, \text{orange}, \text{banana}\}$:

$\oplus$	apple	orange	banana
apple	banana	**banana**	apple
orange	orange	apple	banana
banana	banana	orange	orange

$$\oplus(\text{apple}, \text{orange}) = \text{banana} = \text{apple} \oplus \text{orange}$$

Scalar Multiplication

- denoted by $\odot$
- simply a map
- *must* be $r \times \vec{v}$ for $r \in \mathbb{R}, \vec{v} \in V$

$$\odot : \mathbb{R} \times V \to V$$

Example of a definition of $\odot$ for $V = \{\text{apple}, \text{orange}, \text{banana}\}$:

$$k \odot \text{apple} = \text{orange}, \forall k \in \mathbb{R}$$

$$k \odot \text{orange} = \begin{cases} \text{orange}, & \text{if } k \leq 2, \\ \text{banana}, & \text{if } k > 2, \end{cases}$$

$$k \odot \text{banana} = \begin{cases} \text{banana}, & \text{if } k < -5\sqrt{2}, \\ \text{apple}, & \text{if } -5\sqrt{2} \leq k < 1.2, \\ \text{banana}, & \text{if } k = 1.2, \\ \text{orange}, & \text{if } k > 2, \end{cases}$$

$$\odot(3, \text{orange}) = \text{banana} = 3 \odot \text{orange}$$

10 Good Properties of Addition and Scalar Multiplication

1.