Discrete Math for Computer Science

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Contents

1 Logic

1.1 Propositions and Logical Operations

Proposition: a statement that is either true or false.

Some examples include "It is raining today" and "$3 \cdot 8 = 20$".

However, not all statements are propositions, such as "open the door"

Name	**Symbol**	**alternate name**
NOT	$\neg$	negation
AND	$\wedge$	conjunction
OR	$\vee$	dijunction
XOR	$\oplus$	exclusive or

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$
T	T	F	T	T	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	F	F	F

XOR is very useful for encryption and binary arithmetic.

1.2 Evaluating Compound Propositions

p : The weather is bad.

q : The trip is cancelled.

r : The trip is delayed.

$\triangleright$

$p \wedge q$: The weather is bad *and* the trip is cancelled

$p \vee q$: The weather is bad *or* the trip is cancelled

$p \wedge (q \oplus r)$: The weather is bad *and* either the trip is cancelled *or* delayed

Order of Evaluation $\neg$, then $\wedge$, then $\vee$, but parenthesis always help for clarity.

Example Truth Table:

p	q	$p \wedge q$	$\neg q$	$(p \wedge q) \oplus \neg q$
T	T	T	F	T
T	F	F	T	T
F	T	F	F	F
F	F	F	T	T

1.3 Conditional Statements

$p \to q$ where p is the hypothesis and q is the conclusion

Format	Terminology
$p \to q$	given
$\neg q \to \neg p$	contrapostive
$q \to p$	converse
$\neg p \to \neg q$	inverse

given	$p \to q$	$\equiv$	$\neg q \to \neg p$	contrapostive
inverse	$\neg p \to \neg q$	$\equiv$	$q \to p$	converse

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is a sufficient condition for q

q is a necessary condition for p

Phrase	Logic
q if p	$p \to q$
q only if p	$q \to p$
q if and only if p	$p \leftrightarrow q$

Order of Operations: $p \wedge q \to r \equiv (p \wedge q) \to r$