Section 4 Groups, p45 #2,3,5,10,11-16 all

In Exercises 1 through 6, determine whether the binary operation $*$ gives a group structure on the given set. If no group results, give the first axiom in order $\mathfrak{G}_1, \mathfrak{G}_2, \mathfrak{G}_3$ from Definition 4.1 that does not hold.

2. Let $*$ be defined on $\mathbb{Z}$ by letting $a * b = ab$.

answer

3. Let $*$ be defined on $2\mathbb{Z} = \{2n : n \in \mathbb{Z}\}$ by letting $a * b = ab$.

answer

5. Let $*$ be defined on the set $\mathbb{R}^*$ of nonzero real numbers by letting $a * b = a/b$.

answer

10. Let n be a positive integer and let $n\mathbb{Z} = \{nm | m \in \mathbb{Z}\}$.

Show the following:

a. $\langle n\mathbb{Z}, +\rangle$ is a group.

answer

b. $\langle n\mathbb{Z}, +\rangle \simeq \langle \mathbb{Z}, +\rangle$.

answer

In exercises 11 through 18, determine whether the given set of matrices under the specified operation, matrix addition or multiplication, is a group.

11. All $n \times n$ diagonal matrices under matrix addition.

answer

12. All $n \times n$ diagonal matrices under matrix multiplication.

answer

13. All $n \times n$ diagonal matricies with no zero diagonal entry under matrix multiplication.

answer

14. All $n \times n$ diagonal matricies with all diagonal entries 1 or -1 under matrix multiplication

answer

15. All $n \times n$ upper-triangular matricies under matrix multiplication.

answer

16. All $n \times n$ upper-triangular matricies under matrix addition.

answer