MAT 340 Differential Equations

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1 Introduction

2 First-order Ordinary Differential Equations

A first-order ordinary differential equation takes the form

$$\frac{dy}{dx} = f(x, y), \qquad x \in I$$

2.1 Separable Equations

Assume $f(x, y)$ be separable, meaning that

$$f(x, y) = g(x) \cdot h(y)$$

for some g and h. Then,

$$\begin{aligned} \frac{dy}{dx} &= g(x)h(x) \\ \frac{dy}{h(y)} &= g(x)dx \\ \int \frac{dy}{h(y)} &= \int g(x)dx \end{aligned}$$

Letting $H(y) = \int \frac{dy}{h(y)}$ and $G(x) = \int g(x)dx$, the solution to the first-order ODE is then

$$H(y) = G(x) + C$$

Example

from pg. 46, #8

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{xy^3} = \frac{1}{x} \cdot \frac{1}{y^3} \\ \int y^3 dy &= \int \frac{dx}{x} \\ \frac{y^4}{4} + C_1 &= \ln|x| + C_2 \\ y^4 &= 4\ln|x| + K \end{aligned}$$

2.2 Linear Equations

A **linear** first-order equation is an equation that can be expressed in the form

$$a_1(x)\frac{dy}{dx} + a_0(x)y = b(x),$$

where $a_1(x), a_0(x)$, and $b(x)$ depend only on the independent variable x, not on y. If $a_0(x)$ is identically zero, then the above equation reduces to

$$\begin{aligned} a_1(x)\frac{dy}{dx} &= b(x) && \text{(assume } a_0(x) = 0\text{)} \\ y(x) &= \int \frac{b(x)}{a_0(x)} dx + C && \text{(as long as } a_1(x) \neq 0\text{)} \end{aligned}$$