Section 6, p66 #9,11,17,19,23-25,27,35,37,50

In Exercises 8 through 11, find the number of generators of a cyclic group having the given order

9. 8

8 has prime factorization of 2^3, so generators are $\{1, 3, 5, 7\}$.

11. 60

60 has prime factorization of $2^2 \cdot 3 \cdot 5$, so generators are $\{1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59\}$

In Exercises 17 through 21, find the number of elements in the indicated cyclic group.

17. The cyclic subgroup of $\mathbb{Z}_3 0$, generated by 25

$\langle 25 \rangle = \{25, 20, 15, 10, 5, 0\}$

19. The cyclic subgroup $\langle i \rangle$ of the group $\mathbb{C}^*$ of nonzero complex numbers under multiplication

$\{i, -1, -i, 1\}$

In Exercises 22 through 24, find all the subgroups of the given group, and draw the subgroup diagram for the subgroups.

23. $\mathbb{Z}_{36}$

answer

24. $\mathbb{Z}_8$

answer

In Exercises 25 through 29, find all the orders of the subgroups of the given group.

25. $\mathbb{Z}_6$

answer

27. $\mathbb{Z}_1 2$

answer

In Exercises 33 through 37, either give an example of a group with the property described, or explain why no example exists.

35. A cyclic group having only one generator

answer

37. A finite cyclic group having four generators

answer

Theory

50. Let G be a group and suppose $a \in G$ generates a cyclic subgroup of order 2 and is the *unique* such element. Show that $ax = xa$ for all $x \in G$. Hint: Consider $(xax^{-1})^2$

answer