Graded Assignment #1

1 [2 points each] Which of the following are binary operations on the given sets? If it is not an operation, explain why.

(a). $S = \mathbb{R}^+$ with $a * b = a \ln b$

answer

(b). $S = \mathbb{R}$ where $a * b$ is the root of the equation $x^2 - a^2b^2 = 0$

answer

2 [2 points each] Consider the binary operation $*$ defined on $\mathbb{R}^+$ by $a * b = \frac{ab}{a+b+1}$

(a). Is $*$ commutative? Explain.

answer

(b). Is $*$ associative? Explain.

answer

3 [3 points] Let E denote the set of all even integers. Prove that $\langle \mathbb{Z}, + \rangle \simeq \langle E, + \rangle$.

4 [3 points each] Prove that isomorphism is an equivalence relation among binary structures. To do this, you need to prove the following three properties:

(a). Reflexive: Every binary structure is isomorphic to itself. Hint: let $\langle S, * \rangle$ be a binary structure and define $\phi : S \to S$ by $\phi(x) = x$. Prove that ϕ is an isomorphism.

answer

(b). Symmetric: For binary structures $\langle S_1, * \rangle$ and $\langle S_2, * \rangle$, if $S_1 \simeq S_2$ then $S_2 \simeq S_1$. Hint: assume $\phi : S_1 \to S_2$ is an isomorphism and prove that $\phi^{-1} : S_2 \to S_1$ is also an isomorphism.

answer

(c). Transtiive: For binary structures $\langle S_1, * \rangle$, $\langle S_2, *' \rangle$, and $\langle S_3, *'' \rangle$, if $S_1 \simeq S_2$ and $S_2 \simeq S_3$ then $S_1 \simeq S_3$. Hind: assume $\phi_1 : S_1 \to S_2$ and $\phi_2 : S_2 \to S_3$ are isomorphisms and prove that $\phi_2 \circ \phi_1 : S_1 \to S_3$ is also an isomorphism.

answer