MAT 260 LINEAR ALGEBRA
LECTURE 1

WING HONG TONY WONG

Brief review — set notations

Most of modern mathematics are built upon set theory. Hence, to learn modern subjects such as linear algebra, we must first understand the basic set language.

Here are some commonly used sets in mathematics and in this course.

- $\mathbb{N}$: set of **natural numbers** (I use this interchangeably with **positive integers**, but some mathematicians use this interchangeably with nonnegative integers).
- $\mathbb{Z}$: set of **integers**.
- $\mathbb{Q}$: set of **rational numbers**.
- $\mathbb{R}$: set of **real numbers**.

To denote other sets, we often use the **set builder notation**:

$$\{\text{candidate} : \text{condition}\}.$$

For example,

- $\{x \text{ is a fruit} : x \text{ is of yellow color}\}$.
- $\{x \text{ is a human being} : x \text{ is a president of the U.S.}\}$.
- $\{x \text{ is a city} : x \text{ is a capital of a country}\}$.

Exercise 1.
(a) Write the set notation for all rational numbers less than 4 and greater than 1.
(b) Write a set of your own using the set builder notation. Be creative!

Question 2.
After specifying a set using set builder notation, how do we verify whether an element is in the set or not?

Question 3.
Let A and B be sets. If A is a **subset** of B, we write $A \subseteq B$ or $B \supseteq A$ (I never use $A \subset B$ to avoid confusion).
(a) What is the formal definition of $A \subseteq B$?
(b) What is the formal definition of $A = B$?

Date: Monday, January 23, 2023.

Question 4. Let X be a set and $A, B \subseteq X$.
(a) What is the formal definition of the **union** of A and B, i.e. $A \cup B$?
(b) What is the formal definition of the **intersection** of A and B, i.e. $A \cap B$?
Let I be a nonempty set, and let $A_i \subseteq X$ for all $i \in I$.
(c) What is the formal definition of $\bigcup_{i \in I} A_i$?
(d) What is the formal definition of $\bigcap_{i \in I} A_i$?

Another commonly used type of sets in this course is the **product set**. Let X be a set and $A, B \subseteq X$.

$$A \times B = \{(x, y) : x \in A \text{ and } y \in B\}.$$

Furthermore, we use A^2 to denote $A \times A$, and A^3 to denote $A \times A \times A$, etc.

Question 5. Let X be a set and $A, B, C, D \subseteq X$.
(a) Is it true that $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$? If yes, prove it; if not, try to find out what we can say about them.
(a) Is it true that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$? If yes, prove it; if not, try to find out what we can say about them.

Notations in this course

- $\forall$: for all.
- $\exists$: there exists.
- s.t.: such that.
- $\rightarrow\leftarrow$: contradiction.
- WTS: want to show.