2.7.1

b. *For every integer n, $n^2 \geq n$*

Proof. Consider cases $n = 0$, $n > 0$, and $n < 0$.
Case 1: $n = 0$

$$\begin{aligned} n^2 &\geq n \\ 0^2 &\geq 0 \\ 0 &\geq 0 \checkmark \end{aligned}$$

Case 2: $n > 0$

$$\begin{aligned} n^2 &\geq n \\ \frac{n^2}{n} &\geq \frac{n}{n} && \text{defined since } n > 0 \\ n &\geq 1 && \text{equivalent to } n \geq 0 \text{ since } n \in \mathbb{Z} \end{aligned}$$

Case 3: $n < 0$

$$\begin{aligned} n^2 &\geq n \\ \frac{n^2}{n} &\leq \frac{n}{n} && \text{sign changes since } n < 0 \\ n &\leq 1 && \text{true since } n < 0 \end{aligned}$$

Since the statement is true for all cases, and the cases completely cover the possibility space, therefore for every integer n, $n^2 \geq n$. □

2.7.2

a. *If x is an integer, then $x^2 + 5x - 1$ is odd*

Proof. Consider cases x is even and x is odd.
Case 1: x is odd, $x = 2k + 1$, for some $k \in \mathbb{Z}$

$$\begin{aligned} (2k+1)^2 + 5(2k+1) - 1 &= 2k^2 + 4k + 1 + 10k + 5 - 1 \\ &= 2k^2 + 14k + 5 \\ \text{odd form} &= 2(k^2 + 7k + 2) + 1 \end{aligned}$$

Case 2: x is even, $x = 2j$, for some $j \in \mathbb{Z}$

$$\begin{aligned} (2j)^2 + 5(2k) - 1 &= 4j^2 + 10k - 1 \\ \text{odd form} &= 2(2j^2 + 5j - 1) + 1 \end{aligned}$$

Since the statement is true for all cases, and the cases completely cover the possibility space, therefore if x is an integer, then $x^2 + 5x - 1$ is odd. □

2.7.3

a. *For any real number x, $|x| \geq 0$*

Proof. Consider cases $x = 0$, $x < 0$, $x > 0$.
Case 1: $x = 0$

$$\begin{aligned} 0 &\geq 0 \\ |0| &\geq 0 \checkmark \end{aligned}$$

Case 2: $x < 0$

$$\begin{aligned} x &\leq 0 & \\ |x| &\geq 0 & \text{since } x = -|x| \end{aligned}$$

Case 3: $x > 0$

$$\begin{aligned} x &\geq 0 & \\ |x| &\geq 0 & \text{since } x = |x| \end{aligned}$$

Since the statement is true for all cases, and the cases completely cover the possibility space, therefore for any real number x, $|x| \geq 0$. $\square$