Homework 8

4.3

4 Which of the following sets of vector in P_2 are linearly dependent?

a. $2 - x + 4x^2, \quad 3 + 6x + 2x^2, \quad 2 + 10x - 4x^2$

answer

c. $3 + x + x^2, \quad 2 - x + 5x^2, \quad 4 - 3x^2$

answer

9 For which real values of λ do the following vectors form a linearly dependent set in $\mathbb{R}^3$?

$$v_1 = \left(\lambda, -\frac{1}{2}, -\frac{1}{2}\right), \quad v_2 = \left(-\frac{1}{2}, \lambda, -\frac{1}{2}\right), \quad v_3 = \left(-\frac{1}{2}, -\frac{1}{2}, \lambda\right)$$

13 Show that if $S = \{v_1, v_2, \ldots, v_r\}$ is a linearly dependent set of vectors in a vector space V, and if $v_{r+1}, \ldots, v_n$ are any vectors in V that are not in S, then $\{v_1, v_2, \ldots, v_r, v_{r+1}, \ldots, v_n\}$ is also linearly dependent.

21 The functions $f_1(x) = x$ and $f_2(x) = \cos x$ are linearly independent in $F(-\infty, \infty)$ because neither function is a scalar multiple of the other. Confirm the linear independence using Wroński's test.

4.4

4 Which of the following form bases for P_2?

a. $1 - 3x + 2x^2, \quad 1 + x + 4x^2, \quad 1 - 7x$

answer

7 Find the coordinate vector of $\vec{w}$ relative to the basis $S = \{\vec{u}_1, \vec{u}_2\}$ for $\mathbb{R}^2$.

b. $\vec{u}_1 = (2, -4), \quad \vec{u}_2 = (3, 8); \quad \vec{w} = (1, 1)$

answer

c. $\vec{u}_1 = (1, 1), \quad \vec{u}_2 = (0, 2); \quad \vec{w} = (a, b)$

answer

12 Show that $\{A_1, A_2, A_3, A_4\}$ is a basis for $\mathcal{M}_{22}$, and express A as a linear combination of the basis vectors.

$$A_1 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}; \quad A = \begin{bmatrix} 6 & 2 \\ 5 & 3 \end{bmatrix}$$

4.5

3 Find a basis for the solution space of the homogeneous linear system, and find the dimension of that space.

$$\begin{aligned} 1x_1 - 4x_2 + 3x_3 - 1x_4 &= 0 \\ 2x_1 - 8x_2 + 6x_3 - 2x_4 &= 0 \end{aligned}$$

7 Find bases for the following subspaces of $\mathbb{R}^3$.

a. The plane $3x - 2y + 5z = 0$.

answer

b. The plane $x - y = 0$.

answer

c. The lines $x = 2t, \quad y = -t, \quad z = 4t$.

answer

c. All the vectors of the form (a, b, c), where $b = a + c$.

answer

11

a. Show that the set W of all polynomials in P_2 such that $p(1) = 0$ is a subspace of P_2.

answer

b. Make a conjecture about the dimesion of W.

answer

c. Confirm you conjecture by finding a basis for W.

answer

18 Let S be a basis for an n-dimesional vector space V. show that if $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_r$ form a linearly independent set of vectors in V, then the coordinate vectors $(\vec{v}_1)_S, (\vec{v}_2)_S, \ldots, (\vec{v}_r)_S$ form a linearly independent set in $\mathbb{R}^n$, and conversely.