Homework 6

1.2

Determine the values for which the system has no solutions, exactly one solution, or infinitely many solutions

25.
$$\begin{array}{rcrcrcl} x &+& 2y &-& 3z &=& 4 \\ 3x &-& y &+& 5z &=& 2 \\ 4x &+& y &+& (a^2-14)z &=& a+2 \end{array}$$

work.

$$\begin{pmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{pmatrix} \xrightarrow[(-4,-8,12,-16)]{R_3-4R_1} \begin{pmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 0 & -7 & a^2-2 & a-14 \end{pmatrix} \xrightarrow[(-3,-6,9,-12)]{R_2-3R_1}$$

$$\begin{pmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{pmatrix} \xrightarrow[(0,7,-14,10)]{R_3-R_2,-\frac{1}{7}R_2} \begin{pmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & a^2-16 & a-4 \end{pmatrix}$$

R_3 represents the equation $(a^2-16)z = a-4$. If $a = 4$, there are infinitely many solutions, since R_3 leads to a full row of 0's. If $a = -4$, then there is no solution, since R_3 leads to $0 = -8$. If $a \neq \pm 4$, then there is exactly one solution to the system of equations. □

27.
$$\begin{array}{rcrcl} x &+& 2y &=& 1 \\ 2x &+& (a^2-5)y &=& a-1 \end{array}$$

work.

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & a^2-5 & a-1 \end{pmatrix} \xrightarrow[(-2,-4,-2)]{R_2-2R_1} \begin{pmatrix} 1 & 2 & 1 \\ 0 & a^2-9 & a-3 \end{pmatrix}$$

R_2 represents the equation $(a^2-9)y = a-3$. If $a = 3$, there are infinitely many solutions, since R_2 leads to a full row of 0's. If $a = -3$, then there is no solution, since R_2 leads to $0 = -6$. If $a \neq \pm 3$, then there is exactly one solution to the system of equations. □

32. *Reduce* $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix}$ *to rref without introducing fractions at any intermediate stage.*

work.

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix} \xrightarrow[(-2,-1,-3)]{R_3-R_2} \begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 1 & 3 & 2 \end{bmatrix} \xrightarrow[(-2,-6,-4)]{R_1-2R_3} \begin{bmatrix} 0 & -5 & -1 \\ 0 & -2 & -29 \\ 1 & 3 & 2 \end{bmatrix} \xrightarrow[(0,2,29)]{R_1-R_2}$$

$$\begin{bmatrix} 0 & -3 & 28 \\ 0 & -2 & -29 \\ 1 & 3 & 2 \end{bmatrix} \xrightarrow[R_3+R_1]{R_2-R_1} \begin{bmatrix} 0 & -3 & 28 \\ 0 & 1 & -57 \\ 1 & 0 & 30 \end{bmatrix} \xrightarrow[(0,3,-171)]{R_1+3R_2} \begin{bmatrix} 0 & 0 & -143 \\ 0 & 1 & -57 \\ 1 & 0 & 30 \end{bmatrix} \xrightarrow[R_2+57R_1,R_3-30R_1]{-\frac{1}{143}R_1,R_1\leftrightarrow R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

□

1.3

5h. *Calculate* $(C^T B)A^T$, *where* $A = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}$.

work.

$$\begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix} \xrightarrow{C^T} \begin{pmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{pmatrix} \xrightarrow{C^T B} \begin{pmatrix} 1 \cdot 4 + 3 \cdot 0 & 1 \cdot -1 + 3 \cdot 2 \\ 4 \cdot 4 + 1 \cdot 0 & 4 \cdot -1 + 1 \cdot 2 \\ 2 \cdot 4 + 5 \cdot 0 & 2 \cdot -1 + 5 \cdot 2 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 16 & -2 \\ 8 & 8 \end{pmatrix} \xrightarrow{C^T B A^T}$$

$$\begin{pmatrix} 4 & 5 \\ 16 & -2 \\ 8 & 8 \end{pmatrix} \begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix} \xrightarrow{C^T B A^T} \begin{pmatrix} 4 \cdot 3 + 5 \cdot 0 & 4 \cdot -1 + 5 \cdot 2 & 4 \cdot 1 + 5 \cdot 1 \\ 16 \cdot 3 - 2 \cdot 0 & 16 \cdot -1 - 2 \cdot 2 & 16 \cdot 1 - 2 \cdot 1 \\ 8 \cdot 3 + 8 \cdot 0 & 8 \cdot -1 + 8 \cdot 2 & 8 \cdot 1 + 8 \cdot 1 \end{pmatrix} = \begin{pmatrix} 12 & 6 & 9 \\ 48 & -20 & 14 \\ 24 & 8 & 16 \end{pmatrix}$$

□

10. $A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$, $AB = \begin{bmatrix} 67 & 41 & 41 \\ 64 & 21 & 59 \\ 63 & 67 & 57 \end{bmatrix}$, *and* $BA = \begin{bmatrix} 6 & -6 & 70 \\ 6 & 17 & 31 \\ 63 & 41 & 122 \end{bmatrix}$.

a. *express each column vector of AB as a linear combination of the column vectors of A.*

work. Since each column vector of AB is computed using a row of A and a column of B, the linear combination will simply be the corresponding column of B.

1. $6 \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix} + 7 \begin{pmatrix} 7 \\ 4 \\ 9 \end{pmatrix} = \begin{pmatrix} 67 \\ 64 \\ 63 \end{pmatrix}$.

2. $-2 \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix} + 7 \begin{pmatrix} 7 \\ 4 \\ 9 \end{pmatrix} = \begin{pmatrix} 41 \\ 21 \\ 67 \end{pmatrix}$.

3. $4 \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix} + 5 \begin{pmatrix} 7 \\ 4 \\ 9 \end{pmatrix} = \begin{pmatrix} 41 \\ 59 \\ 57 \end{pmatrix}$.

□

b. *express each column vector of BA as a linear combination of the column vectors of B.*

work. Since each column vector of BA is computed using a row of B and a column of A, the linear combination will simply be the corresponding column of A.

1. $3 \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix} + 6 \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} + 0 \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 63 \end{pmatrix}$.

2. $-2 \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix} + 5 \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} + 4 \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ 17 \\ 41 \end{pmatrix}$.

3. $7 \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix} + 4 \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} + 9 \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 70 \\ 31 \\ 122 \end{pmatrix}$.

□

15. *Find all values of k, if any, that satisfy* $\begin{bmatrix} k & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix}$.

work.

$$\begin{bmatrix} k & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} k+1 & k+2 & -1 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = [k(k+1) + k + 2 - 1] = \left[(k+1)^2\right] = [0]$$

$$k + 1 = 0$$
$$k = -1$$

□

22. *description*

24. *description*

27. *description*

1.4

17. *description*

19e. *description*

27. *description*