Homework 7

Prove $A^{-1} = 3I - A$

$A(3I - A) = 3AI - A^2 \quad -1(3A - A^2)$

$I = 3A - A^2$

$-I + I = \emptyset$

then do the direction.

1.4 *Inverses; Algebraic Properties of Matrices*

28. *Show that if a square matrix A satisfies $A^2 - 3A + 1 = 0$, then $A^{-1} = 3I - A$.*

31. *Assuming that all matrices are $n \times n$ and invertible, solve for D:*

$(ABC)^{-1}$

$C^{-1}B^{-1}A^{-1}$

$$\underbrace{C^T B^{-1} A^2 BAC^{-1}}_{(\quad)^{-1}} D \underbrace{A^{-2} B^T}_{(\quad)^{-1}} C^{-2} = C^T.$$

39. *Using Matrix Inversion, find the unique solution of the given linear system.*

$$\begin{pmatrix} 3 & -2 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$3x_1 - 2x_2 = -1$$
$$4x_1 + 5x_2 = 3$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 4 & 5 \end{pmatrix}^{-1}\begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

53a. *Show that if A, B and $A + B$ are invertible matrices with the same size, then*

$$A(A^{-1} + B^{-1})B(A + B)^{-1} = I.$$

55. *Show that if A is a square matrix such that $A^k = 0$ for some positive integer k, then the matrix $(I - A)$ is invertible and*

$$(I - A)^{-1} = I + A + A^2 + \cdots + A^{k-1}.$$

1.5 *Elementary Matrices and a Method for Finding A-1*

rref cannot be form $(I|B)$.

15. *Use the inverse algorithm to find the inverse of the given matrix, if the inverse exists.*

$$\left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} -1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{7}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right]$$

bc singular, then use Big Thm.

25. *Find the inverse of the following 4×4 matrices, where k_1, k_2, k_3, k_4, and k are all non-zero.*

a. $\begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}.$

b. $\begin{bmatrix} k & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & k & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$

27. *Find all values of c, if any, for which the given matrix is invertible.* $\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix}$

29. *Write the given matrix as a product of elementary matrices.*

$$\begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix}$$

41. *Prove that if A and B are $m \times n$ matrices, then A and B are row equivalent if and only if A and B have the same reduced row eschelon form.*

1.6 *More on Linear Systems and Invertible Matrices*

15. *Determine conditions on the b_i's, if any, in order to guarantee that the linear system is consistent.*

$$\begin{aligned} x_1 - 2x_2 + 5x_3 &= b_1 \\ 4x_1 - 5x_2 + 8x_3 &= b_2 \\ -3x_1 + 3x_2 - 3x_3 &= b_3 \end{aligned}$$

21. *Let $A\vec{x} = \vec{0}$ be a homogenous system of n linear equations in n unknown that has only the trivial solution. Show that if k is any positive integer, then the system $A^k\vec{x} = \vec{0}$ also has only the trivial solution.*