1 Homework 5

Section 4.2

7

Which of the following are linear combinations of $\vec{u} = (0, -2, 2)$ *and* $\vec{v} = (1, 3, -1)$*?*

a. $(2, 2, 2)$

Proof. Let $k_1, k_2 \in \mathbb{R}$ such that $k_1\vec{u} + k_2\vec{v} = (2, 2, 2)$. That is, $k_1(0, -2, 2) + k_2(1, 3, -1) = (2, 2, 2)$. From this equation, we get a linear system of equations.

$$\begin{aligned} 0k_1 + 1k_2 &= 2 \\ -2k_1 + 3k_2 &= 2 \\ 2k_1 - 1k_2 &= 2 \end{aligned}$$

$$\left(\begin{array}{cc|c} 0 & 1 & 2 \\ -2 & 3 & 2 \\ 2 & -1 & 2 \end{array}\right) \xrightarrow{-\frac{1}{2}R_2} \left(\begin{array}{cc|c} 0 & 1 & 2 \\ 1 & -\frac{3}{2} & -1 \\ 2 & -1 & 2 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c} 1 & -\frac{3}{2} & -1 \\ 0 & 1 & 2 \\ 2 & -1 & 2 \end{array}\right) \xrightarrow[(-2,3,2)]{R_3 - 2R_1} \left(\begin{array}{cc|c} 1 & -\frac{3}{2} & -1 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{array}\right)$$

$$\xrightarrow[(0,-2,-4)]{R_3 - 2R_2} \left(\begin{array}{cc|c} 1 & -\frac{3}{2} & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right) \xrightarrow[(0,\frac{3}{2},3)]{R_1 + \frac{3}{2}R_2} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right)$$

This augmented matrix represents the following equations:

$$\begin{aligned} k_1 + 0k_2 &= 2 \\ 0k_1 + k_2 &= 2 \\ 0 + 0 &= 0 \end{aligned} \qquad \begin{aligned} k_1 &= 2 \\ k_2 &= 2 \end{aligned}$$

This means that $(2, 2, 2)$ is a linear combination of $\{\vec{u}, \vec{v}\}$, when $k_1 = 2$ and $k_2 = 2$. □

c. $(0, 4, 5)$

8

a. *description*

c. *description*

9

a. *description*

c. *description*

10

a. *description*

d. *description*

Section 1.2

4