Section 8, p83 # 11-17, 23-26

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix} \qquad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix} \qquad \mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$$

Let A be a set and let $\sigma \in S_A$. For a fixed $a \in A$, the set

$$\mathcal{O}_{a.\sigma} = \{\sigma^n(a) : n \in \mathbb{Z}\}$$

is the **orbit** of a **under** σ. In Exercises 11 through 13, find the orbit of 1 under the permutation defined prior to Exercise 1.

11. σ

$1 \mapsto 3 \mapsto 4 \mapsto 5 \mapsto 6 \mapsto 2 \mapsto 1$

12. τ

$1 \mapsto 2 \mapsto 4 \mapsto 3 \mapsto 1$

13. μ

$1 \mapsto 5 \mapsto 1$

14. In Table 8.8, we used $\rho_0, \rho_1, \rho_2, \mu_1, \mu_2, \mu_3$ as the names of the 6 elements of S_3. Some authors use the notations $\epsilon, \rho, \rho^2, \phi, \rho\phi, \rho^2\phi$ for these elements. Verify *geometrically* that their six expression do give all of S_3.

8.8 Table

	ρ_0	ρ_1	ρ_2	μ_1	μ_2	μ_3
ρ_0	ρ_0	ρ_1	ρ_2	μ_1	μ_2	μ_3
ρ_1	ρ_1	ρ_2	ρ_0	μ_3	μ_1	μ_2
ρ_2	ρ_2	ρ_0	ρ_1	μ_2	μ_3	μ_1
μ_1	μ_1	μ_2	μ_3	ρ_0	ρ_1	ρ_2
μ_2	μ_2	μ_3	μ_1	ρ_2	ρ_0	ρ_1
μ_3	μ_3	μ_1	μ_2	ρ_1	ρ_2	ρ_0

They do, with isomorphism X, where

$$X(\rho_0) = \epsilon \qquad X(\mu_1) = \phi$$
$$X(\rho_1) = \rho \qquad X(\mu_2) = \rho\phi$$
$$X(\rho_2) = \rho^2 \qquad X(\mu_3) = \rho^2\phi$$

15. With reference to Exercise 14, give a similar alternative labeling for the 8 elements of D_4 in Table 8.12

8.12 Table

	ρ_0	ρ_1	ρ_2	ρ_3	μ_1	μ_2	δ_1	δ_2
ρ_0	ρ_0	ρ_1	ρ_2	ρ_3	μ_1	μ_2	δ_1	δ_2
ρ_1	ρ_1	ρ_2	ρ_3	ρ_0	δ_1	δ_2	μ_2	μ_1
ρ_2	ρ_2	ρ_3	ρ_0	ρ_1	μ_2	μ_1	δ_2	δ_1
ρ_3	ρ_3	ρ_0	ρ_1	ρ_2	δ_2	δ_1	μ_1	μ_2
μ_1	μ_1	δ_2	μ_2	δ_1	ρ_0	ρ_2	ρ_3	ρ_1
μ_2	μ_2	δ_1	μ_1	δ_2	ρ_2	ρ_0	ρ_1	ρ_3
δ_1	δ_1	μ_1	δ_2	μ_2	ρ_1	ρ_3	ρ_0	ρ_2
δ_2	δ_2	μ_2	δ_1	μ_1	ρ_3	ρ_1	ρ_2	ρ_0

Alternative labeling:

$$\rho_0 \mapsto \epsilon \qquad \mu_1 \mapsto \phi$$
$$\rho_1 \mapsto \rho \qquad \mu_2 \mapsto \rho^2\phi$$
$$\rho_2 \mapsto \rho^2 \qquad \delta_1 \mapsto \rho^3\phi$$
$$\rho_3 \mapsto \rho^3 \qquad \delta_2 \mapsto \rho\phi$$

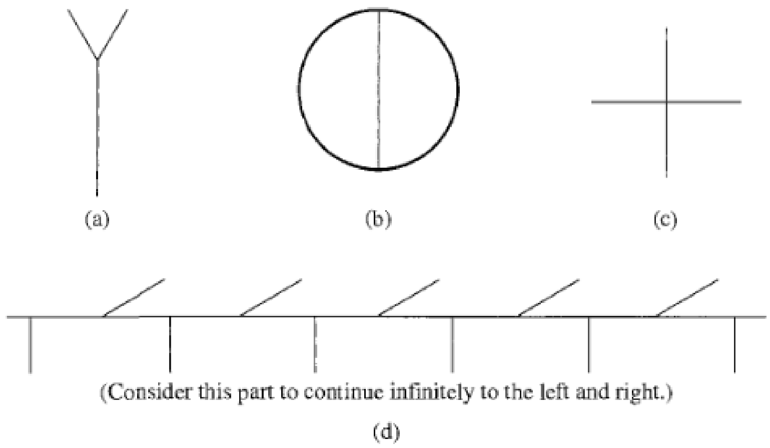
16. Find the number of elements in the set $\{\sigma \in S_4 : \sigma(3) = 3\}$.

Rationale: Since 3 is fixed such that $\sigma(3) = 3$, there are 3! ways to arrange the remaining three elements. So $|\{\sigma \in S_4 : \sigma(3) = 3\}| = 3! = 6$.

17. Find the number of elements in the set $\{\sigma \in S_5 : \sigma(2) = 5\}$.

Rationale: Since 2 is fixed such that $\sigma(2) = 5$, there are 4! ways to arrange the remaining four elements. So $|\{\sigma \in S_5 : \sigma(2) = 5\}| = 4! = 24$.

In this section we discussed the group of symmetries of an equilateral triangle and of a square. In Exercises 23 through 26, give a group that we have discussed in the text that is isomorphic to the group of symmetries of the indicated figure. You may want to label some special points on the figure, write some permutations corresponding to symmetries, and compute some products of permutations.



8.21 Figure

23. The figure in Fig. 8.21 (a)

This figure's symmetries are equivalent to S_2. Consider labeling the top left branch 1, and the top right branch 2.

24. The figure in Fig. 8.21 (b)

This figure's symmetries are equivalent to D_4, excluding diagonal reflection.

Consider labeling, according to the hours of a clock, the following: 12 o'clock as 1, 3 o'clock as 2, 6 o'clock as 3, and 9 o'clock as 4.

25. The figure in Fig. 8.21 (c)

This figure's symmetries are equivalent to D_4. Consider labeling the ends with 1, 2, 3, and 4.

26. The figure in Fig. 8.21 (d)

This figure's symmetries are equivalent to $\langle \mathbb{Z}, + \rangle$. Consider picking a starting point as 0, and shifting to the left until the figure is symmetric increases the labeling by 1, and shifting to the right decreases the labeling by 1.