Section 8.12

8.12.1 *Simplify each recurrence reltion as much as possible. The simplified formula for the function should have the same asymptotic growth as the original recurrence relation.*

a. $T(n) = T(n-1) + 5n^3 + 4n$

work.

$$\begin{aligned} T(n) &= T(n-1) + 5n^3 + 4n \\ &= T(n-1) + \Theta(n^3) \\ &= \Theta(n^3) + \Theta(n^3) + \cdots + \Theta(n^3) && [n \text{ times}] \\ &= n \cdot \Theta(n^3) \\ &= \Theta(n^4) \end{aligned}$$

□

b. $T(n) = T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + 7n$

work.

$$\begin{aligned} T(n) &= T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + 7n \\ &= 2T(n/2) + \Theta(n) \\ &= 2(2T(n/4) + \Theta(n)) + \Theta(n) \\ &= \Theta(n) + \Theta(n) + \cdots + \Theta(n) && (\log_2 n \text{ times}) \\ &= \log_2 n \cdot \Theta(n) \\ &= \Theta(n \log n) \end{aligned}$$

□

c. $T(n) = 3 \cdot T(\lfloor n/2 \rfloor) + 14$

work.

$$\begin{aligned} T(n) &= 3 \cdot T(\lfloor n/2 \rfloor) + 14 \\ &= 3T(n/2) + \Theta(1) \\ &= \Theta(n^{\log_2 3}) && (\text{master theorem}) \end{aligned}$$

□

d. $T(n) = 3 \cdot T(\lceil n/3 \rceil) + 4n + 6n \log n$

work.

$$
\begin{aligned}
T(n) &= 3 \cdot T(\lceil n/3 \rceil) + 4n + 6n \log n \\
&= 3 \cdot T(n/3) + \Theta(n \log n) \\
&= 3(3T(n/9) + \Theta(n \log n)) + \Theta(n \log n) \\
&= 9T(n/9) + \Theta(n \log n) + \Theta(n \log n) \\
&= \Theta(n \log n) + \Theta(n \log n) + \cdot + \Theta(n \log n) && (log_3 n \text{ times}) \\
&= \log n \cdot \Theta(n \log n) \\
&= \Theta(n \log^2 n)
\end{aligned}
$$

□

8.12.2

a. *Give the recurrence relation to describe the asymptotic time complexity of your algorithm to compute the sum of the cubes of the first n positive integers.*

```
CubeSum(n)
{
  if(n == 1) return(1);
  return(n**3 + CubeSum(n-1));
}
```

work.

$$
\begin{aligned}
T(n) &= T(n-1) + \Theta(1) \\
T(1) &= \Theta(1)
\end{aligned}
$$

□