HOMEWORK 1 Tasks

Task 1. Consider the language L={ab, c}over A={a, b, c}. Find: (*a*) L^0; (*b*) L^2;

L^0= {λ}

L^2=LL={ab, c}{ab, c}={abab, abc, cab, c_2}

Task 2. Let A={a, b, c}. Find L* where: (*a*) L={b^2}; (*b*) L={a, b};

$L^*=L^0\cup L\cup L^2\cup$....= {λ, b^2, b^4, b^6,   }

Task 3. Let A={a, b}. Describe the language L(r) where:

(a) r=abb*a; (b) r=b*ab*ab*; (c) r=a*∪b*;   (d) r=ab*∩a*.

Solution.

(a) r=abb*a;   L(r)=L(abb*a)=L(a)L(b)[L(b)*] L(a)={a}{b}{λ, b, b^2, b^3,…}{a}=

={ab}{λ, b, b^2, b^3,…}{a}={ab, ab^2, ab^3, ab^4, …}{a}={aba, ab^2a, ab^3a,…}={ab^na | n≥1};

Task 4. Let A={a, b, c} and let w=abc. Whether w belongs to L(r) where:

(a) r=a*∪(b∪c)*;

Solution.

L(r)=L(a*∪(b∪c)*=L(a)* ∪ {x, where x is an arbitrary word in b, c}={λ, a, a^2, a^3,…}∪{x, where x is an arbitrary word in b, c}={λ, a, a^2, a^3,…, x, where x is an arbitrary word in b, c}.

w=abc doesn't belong to L(r).

Task 6. Let A={a, b, c}. Describe L* if: (a) L={a^2};   (b) L={a, b^2};

Solution.

(b) L={a, b^2}; Denote c= b^2 . Then   L*={a, c}*, = {all words in a and b^2}

Task 8. Let A={a, b}. Find a regular expression r such that L(r) consists of all words w where:

(a) w contains exactly three a's.   (b) the number of *a*'s is divisible by 3.

Solution.

(a) w contains exactly three a's.

r=b*ab*ab*ab*

(b) the number of *a*'s is divisible by 3.

r={b^n|n≥0, $(b*ab*ab*ab*)^m$ |m≥1}

Task 9. Let A={a, b}. Construct an automaton M which accepts the language:

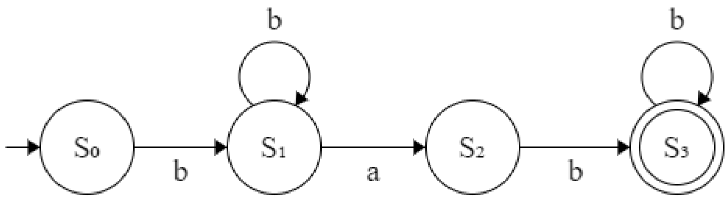
(a) L(M)={b^rab^s |r>0, s>0};   (b) L(M)={a^rb^s |r>0, s>0}.

Solution. https://madebyevan.com/fsm/

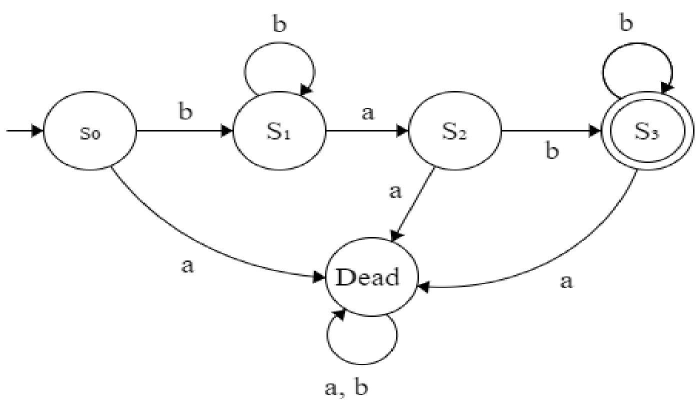
(a) L(M)={$b^r a b^s$ |r>0, s>0}; Min value for r and s is 1. Word with minimal length: bab

Solution is as following:

1. Nondeterministic case:



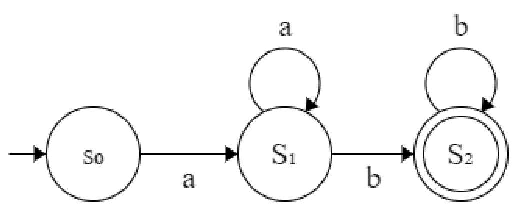
2. Deterministic **case which is not necessary**



Note. Solution is not unique

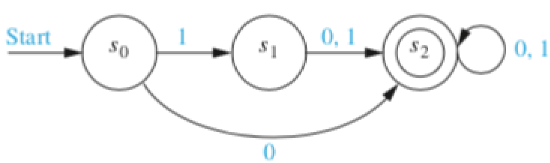
(b) L(M)={$a^r b^s$ |r>0, s>0}.

Nondeterministic solution ⇒



Deterministic solution can also be provided (not required)

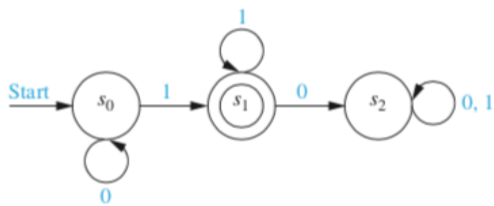
Task 12. Find the language recognized by the given DFSA.



Solution.

L={10x, 11y, 0z | x, y, z are arbitrary words in 0 and 1}

Task 14. Find the language recognized by the given DFSA.



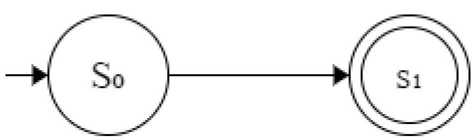
Solution. $L=\{0^n 11^m, \mid n, m \geq 0\}$

Task 16. Show that there is no finite-state automaton **with two states** that recognizes the set of all bit strings that **have one or more 1 bits** and end with a 0.

Proof. Assume the contrary. That is, assume that there exists state-machine with 2 states which recognize set of all bit strings that **have one or more 1 bits** and end with a 0.

Case 1 Start state is final. Then λ is acceptable word – contradiction

Case 2 Start state is not final, second state is a final one.



Subcase:

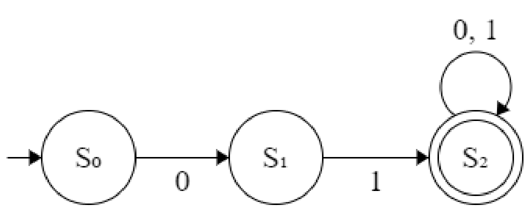
1. We have a loop on 0 at S_1. Then such a loop contains $0^0=\lambda$ therefore transition S_0 to S_1 must on 0. And therefore, we need to have loop at S_0 on b, and therefore word 0 is acceptable – contradiction

(To be continued)

Task 18. Construct a deterministic finite-state automaton (**DFSA**) that recognizes the **set of all bit strings** (=**language**) beginning with 01 (set of Input symbols – I={0,1}).

Solution. Clear that language under consideration is L={01x | x is any word in 0 and 1}. Hence the minimal string recognized by DFSA is 01.

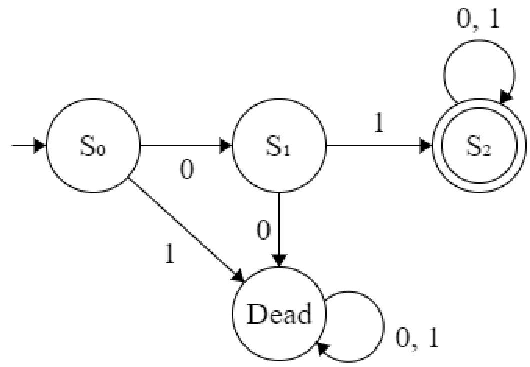
Step 1. NDFSA case is:



Step 2. Transform NDFSA into DFSA

According to the philosophy of relationship between NDFSA and DFSA we need to add "Dead state" and all new transitions we send to the dead state.

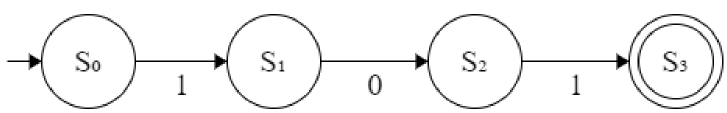
Below is a Deterministic machine (**solution**):



Task 19. Construct a deterministic finite-state automaton that recognizes the **set of all bit strings** that contains the string **101**.

Examples of acceptable words: 0**101**1100, 0001**101**000**101**111 **101**0001111 λ**101**λ

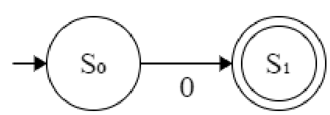
Solution. Acceptable String with minimal length is **101.** Thus a required NDFSA contains the following part as a skeleton



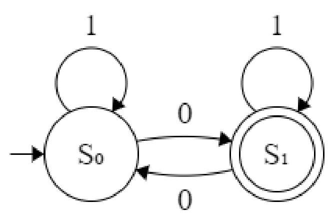
For the remaining part See lecture Notes Exercise Set 3.

Task 22. Construct a deterministic finite-state automaton that recognizes **the set of all bit strings** that contains an odd number of 0s.

Solution. Acceptable String with minimal length is **0.** Thus, a required NDFSA contains the following part as a skeleton



Next, DFSA is



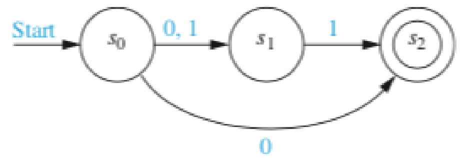
Task 23. Construct a finite-state automaton that recognizes the set of bit strings consisting of a 0 followed by a string with an odd number of 1s.

See lecture Notes Exercise Set 3

Task 24. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that begin and end with 11.

See lecture Notes Exercise Set 3

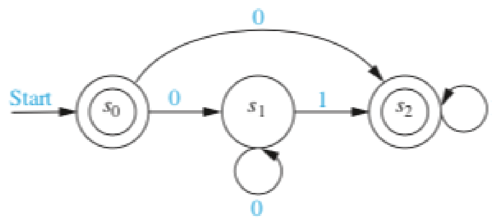
Task 25. Find the language recognized by the given NDFSA M.



Solution.

L={01, 11, 0}

Task 27. Find the language recognized by the given NDFSA M.

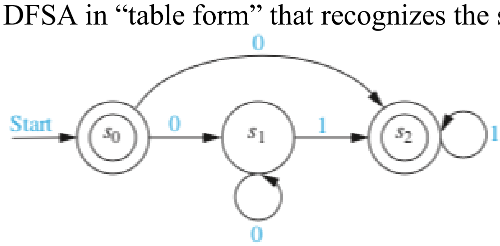


Solution.

$L=\{\lambda, 01^k, 00^m11^n \mid k\geq 0, m\geq 0, n \geq 0 \}$

Task 28.

1. Find a DFSA in "table form" that recognizes the same language as the NDFSA in task 25.



2. Then build up state diagram for the DFSA created.

To be cont

Table 1 (NDFSA)

State	f: Input 0	f: Input 1
s_0	s_1, s_2	
s_1	s_1	s_2
s_2		s_2
Final states: s_0 , s_2		

Table 2 (DFSA)

State	f: Input 0	f: Input 1	New states produced (theorem 2)