

Homework 1

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Task 1 $L = \{ab, c\}$ $A = \{a, b, c\}$

a) $L^0 = \{\lambda\}$ language in power of 0 has only empty word

b) $L^2 = LL = \{ab, c\} \{ab, c\} = \{abab, abbc, cab, c^2\}$

Task 2 $A = \{a, b, c\}$

a) $L = \{b^2\}$ $L^* = L^0 \cup L^1 \cup L^2 \dots = \{\lambda; b^2, b^4, b^6 \dots\}$

b) $L = \{a, b\}$ $L^* = L^0 \cup L^1 \cup L^2 \dots = \{\lambda\} \cup \{a, b\} \cup \{a^2, ab, ba, b^2\} \dots = \{\lambda, a, b, a^2, ab, ba, b^2 \dots\}$

Task 3 $A = \{a, b\}$

a) $r = abb^*a \rightarrow L(r) = L(abb^*a) = L(a)L(b)L(b)^*L(a) = \{a\} \{b\} \{\lambda, b, b^2, b^3 \dots\} \{a\} = \{ab\} \{\lambda, b, b^2, b^3 \dots\} \{a\} = \{ab, ab^2, ab^3 \dots\} \{a\} = \{aba, ab^2a, ab^3a \dots\}$

b) $r = b^*ab^*ab^* = L(b)^*L(a)L(b)^*L(a)L(b)^* = \{\lambda, b, b^2, b^3 \dots\} \{a\} \{\lambda, b, b^2, b^3 \dots\} \{a\} \{\lambda, b, b^2, b^3 \dots\} = \{a, ba, b^2a, b^3a \dots\} \{a\} \{\lambda, b, b^2, b^3 \dots\} = \{a, ba, b^2a, b^3a \dots\} \{a\} \{\lambda, b, b^2, b^3 \dots\}$

c) $r = a^* \cup b^* \rightarrow L(r) = L(a^* \cup b^*) = L(a)^* \cup L(b)^* = \{\lambda, a, a^2 \dots\} \cup \{\lambda, b, b^2 \dots\} = \{\lambda, a, b, a^2, b^2 \dots\}$

d) $r = ab^* \cap a^*$

$L(r)$ is impossible. specifically \cap is not one of the symbols used for regular expressions

Task 4 $A = \{a, b, c\}$ $w = abc$

a) $r = a^* \cup (b \cup c)^*$

$L(r) = L(a^* \cup (b \cup c)^*) = L(a^*) \cup \{x \text{ where } x \text{ is any word consist of } b, c\} = \{\lambda, a, a^2, \dots\} \cup \{x \text{ where } x \text{ is any word in } b, c\}$

w doesn't belongs to $L(r)$ because it is union

b) $r = a^*(b \cup c)^*$

$L(r) = L(a^* \{x \text{ where } x \text{ is any word in } b, c\}) =$

$= \{\lambda, a, a^2, \dots\} \{x \text{ where } x \text{ is any word in } b, c\}$

w belongs to $L(r)$ because there is concatenation

Task 5 $A = \{a, b\}$ $L(r)$ consist of all words w where:

a) w begins with $a^2 = aa$ and ends with $b^2 = bb$

it means that $L(r) = \{a^2\} \{ \text{any word } x \} \{b^2\} =$

$= L(a^2) L(a \cup b)^* L(b^2)$

$r = a^2(a \cup b)^* b^2$

b) w contains an even number of a .

$(aa)^* = \{\lambda, a^2, a^4, a^6, \dots\}$

$L(r) = L(aa)^* L(b)^*$

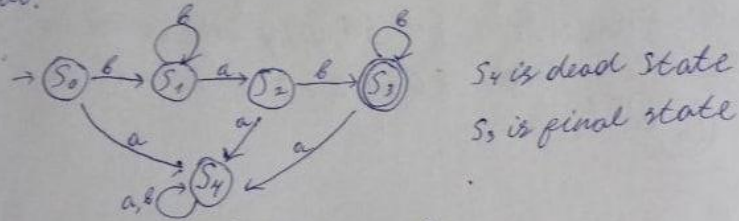
$r = (a^2)^* b^*$

I think it one of possible answers

Task 9 $A = \{a, b\}$

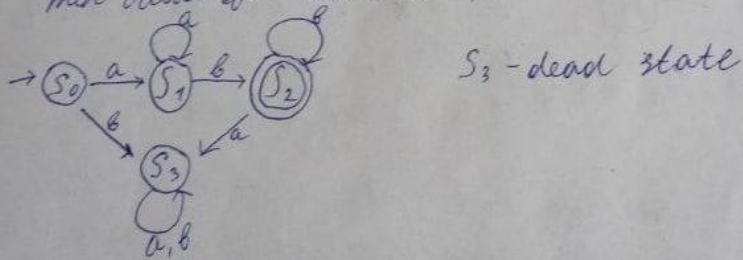
a) $L(M) = \{b^r a b^s \mid r > 0, s > 0\}$

min value of r and s is 1, minimal word is bab .



b) $L(M) = \{a^r b^s \mid r > 0, s > 0\}$

min value of r and s is 1, minimal word is ab .



Task 11

a) $\{0\}^*$ is recognized because S_0 is final state

b) $\{0\} \{0\}^*$ is recognized
 $\{0\} \{0\}^*$ will stand S_0

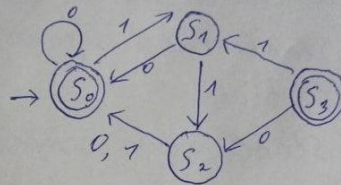
c) $\{1\} \{0\}^* = \{1, 10, 100, \dots\}$

recognized because it moves to S_1 then returns to S_0 which is final state

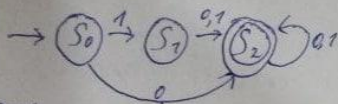
d) $\{01\}^* = \{\lambda, 01, 0101, \dots\}$ is not recognized S_1 is not final

e) $\{0\}^* \{1\}^*$ is not recognized, not all words recognized.

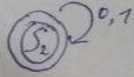
f) $\{1\} \{0, 1\}^*$ is not recognized, not all words recognized.



Task 12



When we are on state S_2 no matter what will be next because we stand on S_2 .



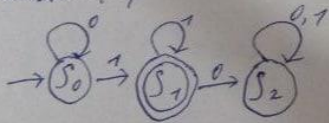
we see that there is 3 possible ways to move to S_2

$\{10\} \{x \mid x \text{ any word in } 0,1\}$

$\{11\} \{y \mid y \text{ any word in } 0,1\}$

$\{0\} \{z \mid z \text{ any word in } 0,1\}$

Task 14



$L(M) = \{0^n 1^m \mid n, m \geq 0\}$

the quickest way to accept S_1

is 1^* , but we also can 01

or 001 , and on S_1 if we choose

1 we will return to S_1

Task 16

Assume the contrary. That is, exist state machine with 2 states which recognize set of all bits strings that have one or more 1 bits and end with 0

Case 1: Start state is final. Then λ is acceptable -

- contradiction

Case 2: Start state is not final, second state is final



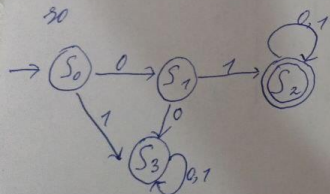
We have a loop on 0 at S_1 then such a loop contains $0^0 = \lambda$ there transition S_0 to S_1 must on 0. Therefore we need a loop at S_0 and word 0 is acceptable - contradiction

Task 18

Construct DFSA that recognize set of all bit strings beginning 01

01 is beginning so $L(M) = \{01x \mid x \text{ is any word in } 0 \text{ and } 1\}$

so



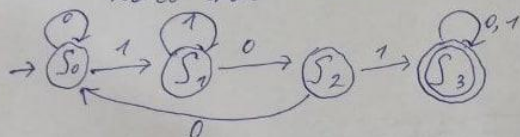
S_3 is dead state

Task 19

Task 19

Construct DFSA that recognize the set of all bit strings contains 101

We know that minimal word is 101



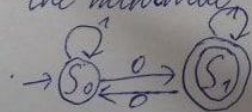
at S_2 if we read 0 we go to S_0 because the sequence 101 is broken

S	1	0
S_0	S_1	S_0
S_1	S_1	S_2
S_2	S_3	S_0
S_3	S_3	S_3

Task 22

DFSA that recognize the set of all bit strings that contains odd number of 0.

the minimal word is 0 so

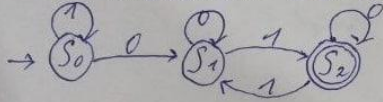


loops when 1 because no matter number of 1.

Task 23

DFSA that recognize set of bit strings consisting of a 0 followed by string with odd number of 1.

the minimal word is 01



$$L(M) = \{0^n 1^m \mid n > 0, m > 0 \text{ and } m \text{ is odd}\}$$

S	0	1
S ₀	S ₁	S ₀
S ₁	S ₁	S ₂
S ₂	S ₂	S ₁

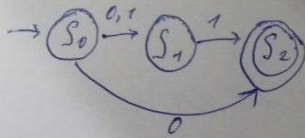
Task 24

DFSA that recognize set of bit strings that begin and end with 11.

$$L(M) = \{11\} \{0, 1\}^* \{11\}$$

so the diagram

Task 25



the ways to move to S₂ are

01, 11, 0

and there are not other words

so

$$L = \{01, 11, 0\}$$