Robust Backstepping Control with Disturbance Rejection for a class of Underactuated Systems

Conference Paper · October 2017

DOI: 10.1109/IECON.2017.8216524

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***Abstract*—This paper is concerned with the problem of stabilization and tracking for a class of underactuated systems subjected to external disturbances. Based on the mathematical model of a 4 degrees of freedom (4DOF) ball and plate system, a robust backstepping controller with disturbance rejection is developed. The proposed controller is capable of handling bounded uncertainties with unknown periodicity affecting the control. A comprehensive comparison between linear quadratic regulator (LQR) and the robust backstepping controller is provided, which affirms the superior performance of the proposed control design.**

***Keywords*—Ball and plate system (BPS), ball and beam system (BBS), underactuated, disturbance rejection, linear quadratic regulator (LQR), backstepping.**

I. INTRODUCTION

Recent advances in nonlinear control algorithms and their applications have spurred greater interest in various applications of control and automation industry. The classical and modern control methods have common characteristics that the controller designs are based on the mathematical model and in nature all the systems have nonlinear time varying dynamics. Ball and Beam System (BBS) is one of the most important and popular test bench for engineers and researchers, which has 2 degrees of freedom (2DOF). In [1], [2] and [3], classical and advanced control methods have been applied to BBS.

The BPS is an extension of the BBS which is a inherently nonlinear, underactuated, unstable and multi-variable system. The BPS is an underactuated electromechanical system with 4 degrees of freedom (4DOF) in which a ball can roll freely along x-y axes. Some of the applications of BPS are in the field of control research, stabilizing platform, modeling and prototyping of toppling control of the rocket, control of aircrafts, etc. The system considered in this work has a pair of two linear hybrid magnetic suspension actuators, moving along z-axis, generates a control torque on a levitating plate which in turn controls the x-y position of the ball lying on the plate. A digital camera determines the position of ball while the plate angles i.e. α inclination angle along x-axis and β inclination angle along y-axis are measured by two potentiometers that are coupled with the plate [4].

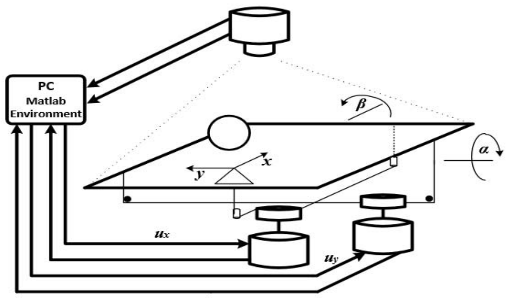

Fig. 1: Experimental setup of ball and plate system

M. Nokhbeh et el in [5] have developed a useful linearized model based on a realistic nonlinear model of BPS by using Euler-Lagrangian equations. The two dimensional electromechanical underactuated BPS are being used as test bench for different classical and modern control techniques [6]. Proportional Integral and Derivative (PID) tuning based radial basis function neural networks are presented and a simplified model of the BPS is derived effectively in [7]. In [8], several control strategies under different system parameters using low cost phototransistor sensors with BPS is investigated. A multi discipline educational purpose BPS is discussed in [9]. The position and trajectory tracking control of a BPS based on fuzzy multi variable control combined with neural network is presented in [10]. Asymptotic stability based on a Lyapunov stability criterion using a nonlinear backstepping control for BPS is provided in [11].

However, it is worth mentioning that the control performance of BPS under matched uncertainty has not been fully investigated up to date, which motivates this study. This paper presents a stabilizing and trajectory tracking control for BPS using a LQR based control and robust backstepping control. The novelty of the robust backstepping control is that it can reject bounded uncertainties with unknown periodicity. It has been demonstrated numerically that the proposed controller has better performance as compared to LQR in the presence of external disturbances.

Rest of the paper is organized as follows. In section 2, the mathematical modeling of BPS is presented. A detailed methodology of the controller design is illustrated in section 3. Section 4 demonstrates the comparison of simulation results of LQR and robust backstepping control in the presence of external disturbances. Finally the conclusion is drawn in section 5.

II. MATHEMATICAL MODELING

To derive the equations of motion for the BPS system, it is assumed that the contact between ball and plate is frictionless and the ball is completely homogenous and symmetric. The experimental setup of BPS is shown in Fig. [1]. The system has 4DOF, 2 for ball x-y position and 2 for plate inclination angles i.e. α and β. To derive the motion equations for the proposed system, the total kinetic energy (T) of the proposed system consists of: (i) plate due to plate rotation, (ii) ball at fixed position due to plate rotation, (iii) ball in vertical direction due to plate rotation, and (iv) ball relative to plate respectively. The equation can be denoted as

$$\begin{aligned}T = \frac{1}{2}m(\dot{x}^2+\dot{y}^2)+\frac{1}{2I_b}(\dot{\alpha}^2+\dot{\beta}^2)+\frac{1}{2I_b}[\frac{(\dot{x}^2+\dot{y}^2)}{r^2}] \\ +\frac{1}{2I_p}(\dot{\alpha}^2+\dot{\beta}^2)+\frac{1}{2}m(x\dot{\alpha}^2+y\dot{\beta}^2)\end{aligned} \tag{1}$$

where T is the total kinetic energy (J), m is the mass of ball (kg), r is the radius of ball (m), I_b is the moment of inertia of ball $(kg.m^2)$ and I_p is the moment of inertia of plate $(kg.m^2)$. The total potential energy (P) of the ball relative to the plate is also a nonlinear function of plate inclination angles and is defined as

$$P = mgxsin\alpha + mgysin\beta. \tag{2}$$

Applying Lagrangian equation (L), kinetic and potential energies of the system can be balanced according to the following relation

$$L = T - P \tag{3}$$

substituting (1) and (2) into (3), the nonlinear differential equations for the BPS are as

$$\ddot{x}(m+\frac{I_b}{r^2}) - m(x\dot{\alpha}+y\dot{\beta})\dot{\alpha} + mgsin\alpha = 0 \tag{4}$$

$$\ddot{y}(m+\frac{I_b}{r^2}) - m(x\dot{\alpha}+y\dot{\beta})\dot{\beta} + mgsin\beta = 0 \tag{5}$$

$$mgxcos\alpha+\ddot{\alpha}(I_p+I_b)+m(x^2\ddot{\alpha}+\dot{x}y\dot{\beta}+x\dot{y}\dot{\beta}+xy\ddot{\beta}) = \tau_x \tag{6}$$

$$mgycos\beta+\ddot{\beta}(I_p+I_b)+m(y^2\ddot{\beta}+\dot{x}y\dot{\alpha}+x\dot{y}\dot{\alpha}+xy\ddot{\alpha}) = \tau_y \tag{7}$$

Equations (4) and (5) show the coupling between states of ball and plate, whereas (6) and (7) show the effect of external torques on the BPS. Torques τ_x and τ_y lavitate the plate while controlling its inclination. The state space representation for the nonlinear dynamical system represented by (4)-(7) with external disturbance in the applied control is given as

$$\dot{x_1} = \dot{x} = x_2 \tag{8}$$

$$\dot{x_2} = \frac{m}{(m+I_b/r^2)}(x_1{x_4}^2 + x_4x_5x_8 - gsinx_3) \tag{9}$$

$$\dot{x_3} = x_4 = \alpha \tag{10}$$

$$\begin{aligned}\dot{x_4} = U_x = \frac{1}{(I_p+I_b+m{x_1}^2)}(\tau_x - mgx_1cosx_3 - \\ m(x_2x_5x_8+x_1x_6x_8+x_1x_5{x_8}^2)) - d1_{ext}(t)\end{aligned} \tag{11}$$

$$\dot{x_5} = \dot{y} = x_6 \tag{12}$$

$$\dot{x_6} = \frac{m}{(m+I_b/r^2)}(x_1{x_8}^2 + x_4x_5x_8 - gsinx_7) \tag{13}$$

$$\dot{x_7} = x_8 = \beta \tag{14}$$

$$\begin{aligned}\dot{x_8} = U_y = \frac{1}{(I_p+I_b+m{x_5}^2)}(\tau_y - mgx_5cosx_7 - \\ m(x_2x_5x_4+x_1x_6x_4+x_1x_5{x_4}^2)) - d2_{ext}(t)\end{aligned} \tag{15}$$

where $[x_1,...,x_8]^T$ is the state vector representing ball position, ball velocity, plate angle and plate angular velocity in x-y axis repectively. Bounded external disturbances $d_{ext}(t)$ in the control input with unknown periodicity are given as

$$d1_{ext}(t) = -\xi_1\sin(x_2) + \xi_2\cos(x_4) - sin(2\pi t) \tag{16}$$

$$d2_{ext}(t) = -\xi_3\sin(x_6) + \xi_4\cos(x_8) - sin(2\pi t) \tag{17}$$

where $\xi = [\xi_1, \xi_2, \xi_3, \xi_4]$ is the vector of maximum bounds on uncertainties.

III. CONTROL STRATEGIES

Considering (8)-(11), i.e. state space equations of ball and plate dynamics along x-axis, two control strategies are presented, i) LQR ii) robust backstepping control. LQR is good for linearized model of the systems while to deal with the realistic nonlinear model of BPS, robust backstepping technique along with disturbance rejection is developed. A schematic of the controller architecture is shown in Fig. 2. The following section delineates the design of the proposed controllers.

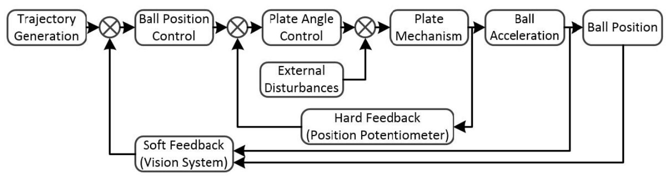

Fig. 2: Architecture of model and control design

A. LQR CONTROL DESIGN

LQR is a linear optimal control technique that maximizes the system performance while minimizing the control effort. The cost function to calculate the state feedback control gains is given as

$$J = \int_0^\infty (X^T(t)Q(t)X(t) + U^T(t)R(t)U(t))dt \tag{18}$$

where $X \in R^{4\times1}$ and $U \in R$ respectively represents matrices corresponding to system state and input for x-axis. $Q(t) \in R^{4\times4}$ and $R(t) \in R$ are positive semi-definite matrix. The optimal gain vector $K \in R^{1\times4}$ is calculated as

$$K = R^{-1}B^TP \tag{19}$$

where $B \in R^{4\times1}$ is the input vector. The symmetric positive definite matrix $P \in R^{4\times4}$ is determined by algebraic Riccati equation as

$$A^T P + PA - PBR^{-1}B^T R + Q = 0 \tag{20}$$

where the system matrix $A \in R^{4\times4}$ can be found by linearizing the state space equations (8)-(11). Using (20), the optimal state feedback control gain vector (19) is calculated. Hence, $\dot{x}(t) = (A - BK)x(t)$ is the closed loop system, in which $U = -Kx(t)$ is the LQR based optimal feedback control law. The control design for y-axis is same like x-axis and is omitted here for brevity.

B. BACKSTEPPING DESIGN

Backstepping technique is developed by Peter v. kokotovic for a class of nonlinear systems. The robust backstepping method is capable of eliminating external disturbances and also globally effective for the stability of nonlinear control systems. Backstepping has a recursive structure, the process terminates when the final control phase is reached. The process which receives its stability through recursitivity is called backstepping. In [12] and [13], a detailed discussion about the design of backstepping controller is provided. In this paper, backstepping technique has been implemented on BPS with improved performance of disturbance rejection. As the mass of the ball is very small compared to the dynamics and acceleration of plate and as there is a coupling effect between two perpendicular directions, so that can be neglected from the state space representation, i.e. $x_4x_5x_8 = 0$.

Definition: Consider the set of nonlinear equations (8)-(15) as follows:

$$\dot{x} = f(x,u), x(0) = x_0, f(0,0) = 0, x \in R^n$$

where $f : u \to R^n$ is a Lipschitz continuous function and u denotes an open set containing the origin. The system is stable in the sense of Lyapunov theory if

1) The transformation from state space to error space i.e. x_1-$x_8 \to e_1$-e_8 and the stabilizing functions α_1-α_8 are Lipschitz continuous over the domain of interest and the 4^{th} derivative of reference signal x_d and y_d are continuous and bounded.

2) The Lyapunov function candidate must satisfy the criteria, $V(e) = 0, \forall e = 0$; $V(e) > 0, \forall e > 0$ and $\dot{V}(e) < 0, \forall e \neq 0$. Based on the above definition, the design of robust backstepping control proceeds as follows

Step 1: Define the error variable for state x_1 as

$$e_1 = x_1 - x_d \tag{21}$$

where x_d is desired trajectory of ball along x-axis. Define the error variable for state x_2 as

$$e_2 = x_2 - \alpha_1 \tag{22}$$

where α_1 is the 1^{st} stabilizing function which is to be determined. Substituting (8) and (22) into the time derivative of (21), we get

$$\dot{e_1} = e_2 + \alpha_1 - \dot{x_d} \tag{23}$$

Choose the Lyapunov function candidate for e_1 as

$$V_1 = \frac{1}{2}{e_1}^2 \tag{24}$$

Differentiating (24) and substituting (23), we get

$$\dot{V_1} = e_1(e_2 + \alpha_1 - \dot{x_d}) \tag{25}$$

Let

$$-c_1e_1 = \alpha_1 - \dot{x_d} \tag{26}$$

where $c_1 > 0$, substituting α_1 from (26) in (23), we have

$$\dot{e_1} = -c_1e_1 + e_2 \tag{27}$$

Differentiating (26) and using (27), $\dot{\alpha_1}$ becomes

$$\dot{\alpha_1} = {c_1}^2e_1 - c_1e_1 + \ddot{x_d} \tag{28}$$

Taking the time derivative of (24) and substituting (21) and (27), $\dot{V_1}$ becomes

$$\dot{V_1} = -c_1{e_1}^2 + e_1e_2 \tag{29}$$

Clearly, $\dot{V_1}$ is not negative definite in step 1.

Step 2: Modify the Lyaponov function candidate as

$$V_2 = \frac{1}{2}({e_1}^2 + {e_2}^2) \tag{30}$$

by substituting (9) and (28), the time derivative of (22) becomes

$$\dot{e_2} = Ax_1{x_4}^2 - Agsinx_3 - {c_1}^2e_1 + c_1e_2 - \ddot{x_d} \tag{31}$$

where $A = m/(m + I_b/r^2)$. Assume $\sin x_3$ be a virtual control and choose 2^{nd} stabilizing function as α_2, we can write $\sin x_3 = \alpha_2$. Let the error variable for state x_3 be the difference between $sinx_3$ and α_2. Thus, we can write

$$e_3 = sinx_3 - \alpha_2 \tag{32}$$

Subtituting (32) into (31), we get

$$\dot{e_2} = Ax_1{x_4}^2 - Ag(e_3 + \alpha_2) - {c_1}^2e_1 + c_1e_2 - \ddot{x_d} \tag{33}$$

Substituting (27) and (33), the time derivative of (30) yields

$$\begin{aligned}\dot{V_2} = & -c_1{e_1}^2 - Age_2e_3 + Ax_1{x_4}^2e_2 + e_2(e_1 - Ag\alpha_2 \\ & - {c_1}^2e_1 + c_1e_2 - \ddot{x_d})\end{aligned} \tag{34}$$

Let

$$-c_2e_2 = e_1 - Ag\alpha_2 - {c_1}^2e_1 + c_1e_2 - \ddot{x_d} \tag{35}$$

where $c_2 > 0$. From (35), α_2 can be written as

$$\alpha_2 = \frac{1}{Ag}(c_2e_2 + e_1 - {c_1}^2e_1 + c_1e_2 - \ddot{x_d}) \tag{36}$$

Substituting (36) into (33), we get

$$\dot{e_2} = Ax_1{x_4}^2 - Age_3 - e_1 + c_2e_2 \tag{37}$$

Now substituting (27), (36) and (37) into the time derivative of (30) yields

$$\dot{V_2} = -c_1{e_1}^2 - c_2{e_2}^2 - Age_2e_3 + Ac_1{x_4}^2e_2 \tag{38}$$

Clearly, $\dot{V_2}$ is again not negative definite in step 2.

Step 3: Modify the Lyaponov candidate function as

$$V_3 = \frac{1}{2}({e_1}^2 + {e_2}^2 + {e_3}^2) \tag{39}$$

Substituting (10) into the time derivative of (32), we get

$$\dot{e_3} = x_4 cosx_3 - \dot{\alpha_2} \tag{40}$$

Assume $x_4 \cos x_3$ be a virtual control and choose 3^{rd} stabilizing function as α_3, we can write $x_4 \cos x_3 = \alpha_3$. Let the error variable for state x_4 be the difference between $x_4 \cos x_3$ and α_3. Thus, we can write

$$e_4 = x_4 cosx_3 - \alpha_3 \tag{41}$$

Taking the time derivative of (36) and substituting (41), (40) can be written as

$$\begin{aligned}\dot{e_3} = e_4 + \alpha_3 - \frac{1}{A_g}[(c_1 + k_2)(-Age_3 - e_1 - c_2e_2) + \\ (1 - {c_1}^2)(-c_1e_1 + e_2) - \ddot{x_d}] - \frac{1}{g}(c_1 + c_2)x_1{x_4}^2\end{aligned} \tag{42}$$

Differentiating (39), we get

$$\begin{aligned}\dot{V_3} = & -c_1{e_1}^2 - c_2{e_2}^2 + e_3e_4 + e_3(-Age_2 + \alpha_3 - \frac{1}{Ag} \\ & [(c_1 + c_2)(-Age_3 - e_1 - k_2e_2) + (1 - {c_1}^2)(-c_1e_1 \\ & + e_2) - \ddot{x_d}]) + Ax_1{x_4}^2e_2 - \frac{1}{g}(c_1 + c_2)x_1{x_4}^2e_3\end{aligned} \tag{43}$$

Let

$$\begin{aligned}& -Age_2 + \alpha_3 - \frac{1}{Ag}[(c_1 + c_2)(-Age_3 - e_1 - c_2e_2) + \\ & (1 - {c_1}^2)(-c_1e_1 + e_2) - \ddot{x_d}] = -c_3e_3\end{aligned} \tag{44}$$

where $c_3 > 0$. From (44), α_3 can be written as

$$\begin{aligned}\alpha_3 = -c_3e_3 + Age_2 + \frac{1}{Ag}[(c_1 + c_2)(-Age_3 - e_1 - c_2e_2) \\ + (1 - {c_1}^2)(-c_1e_1 + e_2) - \ddot{x_d}]\end{aligned} \tag{45}$$

Substituting (45) into (42), we get

$$\dot{e_3} = -c_3e_3 + e_4 + Age_2 - \frac{1}{g}(c_1 + c_2)x_1{x_4}^2 \tag{46}$$

Substituting (27), (37) and (46) into the time derivative of (39), we get

$$\begin{aligned}\dot{V_3} = & -c_1{e_1}^2 - c_2{e_2}^2 - c_3{e_3}^2 + e_3e_4 + Ax_1{x_4}^2e_2 - \\ & \frac{1}{g}(c_1 + c_2)x_1{x_4}^2e_3\end{aligned} \tag{47}$$

Clearly, $\dot{V_3}$ is again not negative definite in step 3.

Step 4: Modify the Lyaponov candidate function as

$$V_4 = \frac{1}{2}({e_1}^2 + {e_2}^2 + {e_3}^2 + {e_4}^2) \tag{48}$$

Taking the time derivative of (45) and substituting (10) and (11) into the time derivative of (41) yields

$$\begin{aligned}\dot{e_4} = & U_x cosx_3 - {x_4}^2 sinx_3 + (c_1 + c_2 + c_3)[-c_3e_3 + e_4 \\ & + Age_2 - \frac{1}{g}(c_1 + c_2)x_1{x_4}^2] - [Ag + \frac{1}{Ag}(1 - {c_1}^2 \\ & - c_1c_2 - {c_2}^2)](Ax_1{x_4}^2 - Age_3 - e_1 - c_2e_2) + \\ & \frac{1}{Ag}(2c_1 + c_2 - {c_1}^3)(-c_1e_1 + e_2) + \frac{1}{Ag}\dddot{x_d}\end{aligned} \tag{49}$$

Substituting (27), (37), (46) and (49) into the time derivative of (48), we get

$$\begin{aligned}\dot{V_4} = & -c_1{e_1}^2 - c_2{e_2}^2 - c_3{e_3}^2 + e_4(U_x cosx_3 - {x_4}^2 sinx_3 \\ & + (c_1 + c_2 + k_3)e_4 + [A^2g^2 + 2 - ({c_1}^2 - {c_2}^2 - {c_3}^2 \\ & + c_1c_2 + c_2c_3 + c_3c_1)]e_3 + [Ag(c_1 + 2c_2 + c_3) + \\ & \frac{1}{Ag}(2c_2 - {c_1}^2c_2 - c_1{c_2}^2 - {c_2}^3 + 2c_1 - {c_1}^3)]e_2 + [Ag \\ & + \frac{1}{Ag}(1 - 3{c_1}^2 - 2c_1c_2 - {c_2}^2 + {c_1}^4)]e_1 + \frac{1}{Ag}x_d^4) \\ & + Ax_1{x_4}^2e_2 - \frac{1}{g}(c_1 + c_2)x_1{x_4}^2e_3 - [A^2g + \frac{1}{g} \\ & (1 + c_1c_2 + c_1c_3 + c_2c_3)]x_1{x_4}^2e_4\end{aligned} \tag{50}$$

Because of the centrifugal acceleration terms $Ax_1{x_4}^2 = Ar\dot{\theta}^2$ the system fails to have a well defined relative degree even after step 4. We proceed with the control design by neglecting the centrifugal acceleration term $Ax_1{x_4}^2$ as in [14]. Let

$$\begin{aligned}-k_4e_4 = & U_x cosx_3 - {x_4}^2 sinx_3 + (c_1 + c_2 + c_3)e_4 + \\ & [A^2g^2 + 2 - ({c_1}^2 - {c_2}^2 - {c_3}^2 + c_1c_2 + c_2c_3 \\ & + c_3c_1)]e_3 + [Ag(c_1 + 2c_2 + c_3) + \frac{1}{Ag}(2c_2 - \\ & {c_1}^2c_2 - c_1{c_2}^2 - {c_2}^3 + 2c_1 - {c_1}^3)]e_2 + [Ag + \\ & \frac{1}{Ag}(1 - 3{c_1}^2 - 2c_1c_2 - {c_2}^2 + {c_1}^4)]e_1 + \frac{1}{Ag}x_d^4\end{aligned} \tag{51}$$

Now to satisfy the negative definiteness of (50), the control input U_x is designed as

$$\begin{aligned}U_x = & \frac{1}{cosx_3}[-c_4e_4 + {x_4}^2 sinx_3 - (c_1 + c_2 + c_3)e_4 - \\ & [A^2g^2 + 2 - ({c_1}^2 - {c_2}^2 - {c_3}^2 + c_1c_2 + c_2c_3 \\ & + c_3c_1)]e_3 - [Ag(c_1 + 2c_2 + c_3) + \frac{1}{Ag}(2c_2 - \\ & {c_1}^2c_2 - c_1{c_2}^2 - {c_2}^3 + 2c_1 - {c_1}^3)]e_2 - [Ag + \\ & \frac{1}{Ag}(1 - 3{c_1}^2 - 2c_1c_2 - {c_2}^2 + {c_1}^4)]e_1 - \frac{1}{Ag}x_d^4] \\ & -\zeta|e_4|^\beta sign(e_4)\end{aligned} \tag{52}$$

where ζ and β are the positive constants. The robust term $-\zeta|e_4|^\beta sign(e_4)$ is taken for the rejection of external disturbance.

Substituting (52) into (50), $\dot{V_4}$ can be rendered negative definite as

$$\dot{V_4} = -c_1{e_1}^2 - c_2{e_2}^2 - c_3{c_3}^2 - c_4{e_4}^2 - \frac{\zeta}{\|e_4\|}{e_4}^2 \tag{53}$$

Hence, the control U_x fulfills the stability criteria and the errors from e_1 - e_4 converge to zero in finite time. The controller design approach for the y-axis has the same procedure as that of the controller design approach for x-axis and is omitted here for brevity. The control law U_y for y-axis is given as

$$\begin{aligned} U_y = & \frac{1}{cosx_7}[-c_8e_8 + {x_8}^2 sinx_7 - (c_5 + c_6 + c_7)e_8 - \\ & [A^2g^2 + 2 - ({c_5}^2 - {c_6}^2 - {c_7}^2 + c_5c_6 + c_6c_7 \\ & + c_7c_5)]e_7 - [Ag(c_5 + 2c_6 + c_7) + \frac{1}{Ag}(2c_6 - \\ & {c_5}^2c_6 - c_5{c_6}^2 - {c_6}^3 + 2c_5 - {c_5}^3)]e_6 - [Ag + \\ & \frac{1}{Ag}(1 - 3{c_5}^2 - 2c_5c_6 - {c_6}^2 + {c_5}^4)]e_5 - \frac{1}{Ag}y_d^4] \\ & -\bar{\zeta}|e_8|^{\bar{\beta}} sign(e_8) \end{aligned} \tag{54}$$

where $\bar{\zeta}$ and $\bar{\beta}$ are positive constants and y_d is the desired trajectory along y-axis. Note that x_3 and x_7 i.e. the angles of plate along x and y axes respectively should not be equal to $90°$ in order to avoid singularities in the controller (52) and (54). The Lyapunov candidate function $\dot{V_8}$ for y-axis becomes

$$\dot{V_8} = -c_5{e_5}^2 - c_6{e_6}^2 - c_7{e_7}^2 - c_8{e_8}^2 - \frac{\bar{\zeta}}{\|e_8\|}{e_8}^2 \tag{55}$$

Finally, the Lyapunov function candidate $\dot{V}$ for the complete system becomes

$$\begin{aligned} \dot{V} = & -c_1{e_1}^2 - c_2{e_2}^2 - c_3{e_3}^2 - c_4{e_4}^2 - \frac{\zeta}{\|e_4\|}{e_4}^2 - c_5{e_5}^2 \\ & - c_6{e_6}^2 - c_7{e_7}^2 - c_8{e_8}^2 - \frac{\bar{\zeta}}{\|e_8\|}{e_8}^2 \end{aligned} \tag{56}$$

Hence, by applying controls U_x and U_y, it is proved that $\dot{V}$ is negative definite.

IV. Simulation Results

In this section, the simulation results of the proposed controllers are shown. The simulation environment used is Matlab/Simulink. Comprehensive results of LQR and robust backstepping technique with disturbance rejection are provided. To test the performance of proposed controller, bounded disturbances of unknown periodicity is applied to the system. It is shown in the results that the proposed robust backstepping controller has superior performance over LQR in the presence of disturbances. The system performance is verified by two conditions: (a) Stability Test (b) Trajectory Tracking Test.

A. Stability Test

For stability test, regardless of the initial conditions, the ball position should be stabilized at the origin of the plate i.e. $x_d = 0$ and $y_d = 0$. To verify the robustness of the system, bounded disturbances with unknown periodicity (16) and (17) are applied to the system. By applying the robust backstepping control, the plate changes its deflection angles around x-y axes and finally the ball is stabilized at the plate origin. Fig. (3) shows the x-y position of ball and Fig. (4) shows the deflection angle of plate where the initial position of ball is $x_0, y_0 = 0.1m$ along x and y axes respectively. From the curves, it can be seen that the proposed controller has a much better transient and steady state performance.

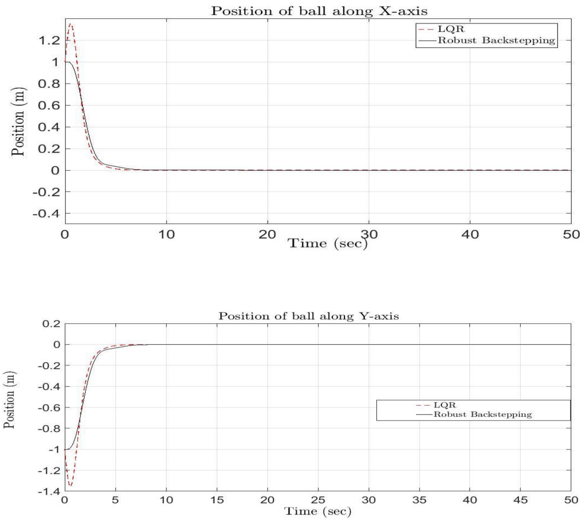

Fig. 3: x-y position of ball for stability test

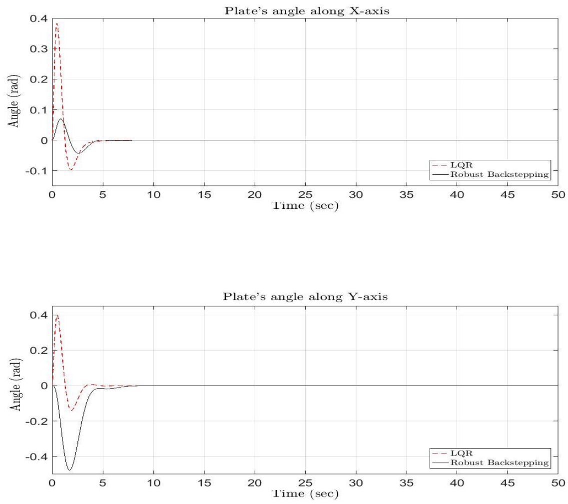

Fig. 4: Plate deflection angle

B. Trajectory Tracking Test

The desired trajectory for tracking along x-axis is $x_d = A_m sin\omega t$ and along y-axis is $y_d = A_m cos\omega t$ where A_m is 0.1m and $\omega = 0.1rad/s$. Fig. (5) shows the tracking of the ball position trajectory and Fig. (6) shows the plate deflection angle. The disturbances (16) and (17) are also applied in this case. It is obvious from the curves that LQR has a lag and overshoot in tracking the position trajectory while the proposed controller tracks the position trajectory efficiently.

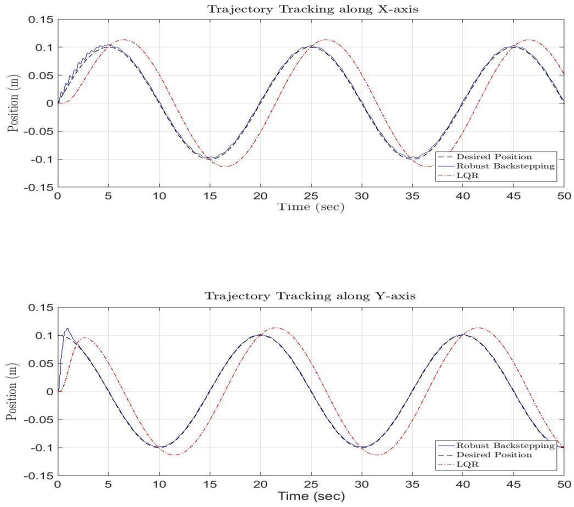

Fig. 5: x-y position of ball for trajectory tracking test

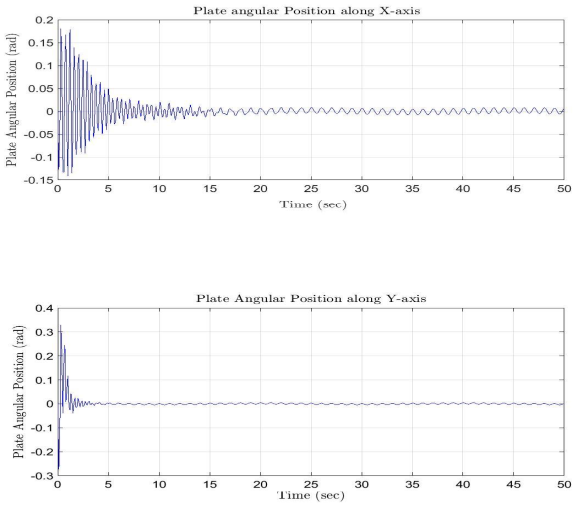

Fig. 6: Plate deflection angle

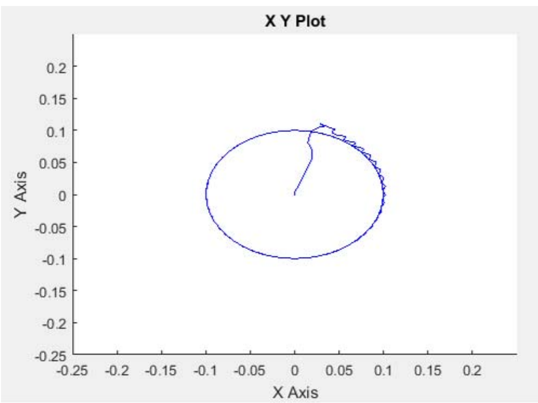

Fig. 7: Circular trajectory of ball

The tracking time for LQR and robust backstepping control are approximately $2.4sec$ and $1.4sec$ respectively. The output of the trajectory tracking along x-axis and y-axis gives a circular trajectory with a radius of $0.1m$ shown in Fig. (7). The controller parameters for LQR are $diag(Q) = [60, 50, 80, 50]$; $K = [13.1171, 16.9622, 59.0154, 14.7659]$ and $R = 0.5$ and for robust backstepping controller the parameters are chosen as $c_1 = 4.8, c_2 = 48, c_3 = 40, c_4 = 3.5$.

V. Conclusion

In this paper, robust backstepping control technique with disturbance rejection and its comparison with LQR for the position and trajectory tracking of BPS is presented. Mathematical modeling of the system along with detailed design of LQR and robust backstepping controllers are provided. Based on Lyapunov stability criteria, the proposed robust backstepping controller with disturbance rejection is designed in four steps. Comprehensive comparison of transient and steady state performance is also provided in the presence of bounded external disturbances with unknown periodicity. The results verify that the robust backstepping controller delivers a better performance in terms of stability and trajectory tracking.

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