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1) Assume that the simulated RATM M' has k registers and n instructions. Then our simulator RATM ~~M'~~ will have k registers and $n+1$ external tapes.

→ All k registers and the accumulator will start with 0 as their content.

→ Tape $(n+1)$ will have the exact same contents as the external tape of M'.

→ Tapes $1 \ldots n$ will have the transition functions in binary of n instructions of M'. These n tapes are read-only.

During the simulation:

→ All k registers and the accumulator will have the exact same contents as if M' was running independently at any step of the computation.

→ Depending on the instructions read from tape $(n+1)$, appropriate tape $1 \ldots n$ will be selected and read, then the appropriate actions will be taken to update the registers and accumulator.

This way we can simulate any RATM M' with M.

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5.4.1) a) A Turing Machine $M = (Q, \Sigma, \delta, s, H)$ with using less than k tape squares can be in $\underbrace{|Q|}_{\text{\# of states}} \cdot \underbrace{(k-1)}_{\text{head position}} \cdot \underbrace{|\Sigma|^{k-1}}_{\text{contents of tape}}$ different configurations.

If we simulate M for $|Q| \cdot (k-1) \cdot |\Sigma|^{k-1} + 1$ steps, there will be 3 possible cases:

(i) M entered a halt state
(ii) M repeated a configuration with less than k tape slots.
(iii) M entered a new configuration with k tape slots.

In case (i), we know that the computation has stopped and we know the answer: no!
In case (ii), we know that we have entered an infinite loop and thus will never use k tape squares or more, so the answer is: no!
In case (iii), we have just entered a configuration that uses k tape slots, so the answer is: yes!

In all cases, by simulating the tape a finite number of steps we can answer the question, so the problem is solvable.

b) This is the exact same question as in part a if we set $f(|w|) = k$. Since f is recursive, there is a TM that can compute it. First, let that TM compute $f(|w|)$, and then use $f(|w|)$ as k in the answer in part a.

c) This problem is unsolvable because it is the halting problem. Since we cannot solve whether a TM will halt on an input or not, we cannot tell if it will use finite amount of tape.

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5.4.2) a) This problem is undecidable. Assume it was solvable: Then we could let $q = h$ and solve the halting problem.

b) This problem is undecidable. Assume it was solvable: Let M be a TM. Update the transitions of M to obtain M' by changing:

- $\delta(q', x) = (h, y)$ to $\delta(q', x) = (p, y)$
- $\delta(q', x) = (h, \rightarrow)$ to $\delta(q', x) = (p, \rightarrow)$
- $\delta(q', x) = (h, \leftarrow)$ to $\delta(q', x) = (p, \leftarrow)$

and adding the transition

- $\delta(p, x) = (q, x)$ where $q = h$

Since we assumed that the given prompt is solvable, this would make halting problem solvable for M' and thus for M, which is in contradiction.

c) This problem is solvable. It is enough to check the transition function and see whether there are any rules of the form $\delta(p, x) \rightarrow (q, \{y, \rightarrow, \leftarrow\})$.

d) This problem is undecidable. Assume it was solvable: Also assume M originally does not write a in any of its transitions. Then, update the transitions that lead to the halt state to write a on the tape. This would be the solution to the halting problem for empty initial tape, which is a contradiction.

e) This problem is solvable. Simulate this TM until it moves $|Q|$ squares to the right. It will either:

(i) Write a non-blank symbol.
(ii) Repeat a configuration and enter an infinite loop.
(iii) Move right in an infinite loop.

For case (i), answer is clearly yes.

For case (ii), once a repeated configuration is detected, we know it is in a loop and answer is no.

For case (iii), if we move $|Q|$ squares to the right without writing a symbol, we must have exhausted all the configurations and are moving to the right infinitely. Answer is no.

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f) This problem is solvable. Start the simulation and stop in one of the following cases:

(i) Head moves left: Answer yes!

(ii) A configuration is repeated: We are in an infinite loop, answer no!

(iii) We reach the end of the input and remain in the same configuration twice: We are either stuck in place or moving right infinitely. Answer no!

g) This problem is undecidable. Assume it was solvable: Also assume one of the Turing Machines semidecides the empty set. Then, the other TM would have to semidecide any string. This can be rephrased as does the TM halt for every input provided, which is equivalent to the halting problem. This is a contradiction.

h) This problem is undecidable. Assume it was solvable: Also assume one of the Turing Machines halts on string w. Then, the other TM would have to halt on w, which is equivalent to the halting problem. This is a contradiction.

i) This problem is undecidable. Assume it was solvable: Also assume M semidecides only the empty string which is a finite language. Then, M would have to halt on the empty string, which is equivalent to the halting problem. This is a contradiction.

5.4.3) Assume it was solvable: Then, since M has to enter each of its states when starting on the input w, it also has to enter the halt state. This means that M would have to halt on w, which is equivalent to the halting problem. This is a contradiction.