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CS407 HW6

0) $M = (Q, \Sigma, \delta, s, H)$

where $Q = \{s, q, h\}$
$\Sigma = \Sigma'$ (Σ of M')
$H = H'$ (H of M')

δ:

State	Input	Action
$>s$	—	$R.q$
q	$\sigma = \#$	h
	$\sigma \neq \#$	$M'.R.q$

1) Assume M_{12} is a TM that reduces L_1 to L_2 in polynomial time.
Assume M_{23} is a TM that reduces L_2 to L_3 in polynomial time.
Assume $w \in L_1$.

Then, if we apply M_{12} on w, the resulting $w' \in L_2$
Then, if we apply M_{23} on w', the resulting $w'' \in L_3$

Similarly, if we apply the composition $M_{12}.M_{23}$ on w, the resulting $w'' \in L_3$.
Since, the composition of M_{12} and M_{23} is also a polynomial time machine, we can call it M_{13} and say that M_{13} is a TM that reduces L_1 to L_3 in polynomial time. ($M_{13} = M_{12}.M_{23}$)

2) $\Rightarrow$: SAT has a solution.

If SAT has a solution, then each of the clauses are evaluated to true, meaning that each clause contains at least one literal with value true. Pick one of the true literals from each clause into a set C. C is our consistent set because all the literals in C have value true, meaning that if $x \in C$, $x^c \notin C$.

$\Leftarrow$: There exists a consistent set C.
Assign value true to every literal in C. This will evaluate every clause to true, resulting in a solution to SAT problem.

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3) IS $\propto$ CLIQUE

Say that IS is given on a graph $G=(V,E)$.

Construct a graph $G'=(V',E')$ in polynomial time where $V'=V$ and $E'=\{$all the edges missing in $E\}$

Then, there is an IS of size k in G iff there is a CLIQUE of size k in G'.

IS $\propto$ NC

Say that IS is given on a graph $G=(V,E)$.

Take the same graph for NC. If there exists a NC containing vertices $V-S$, then S is an independent set because if there was an edge uv where $u,v\in S$, then $V-S$ would not be a NC.

Therefore, there is an IS of size $|S|$ in G iff there is a NC of size $|V-S|$ in G.

SAT $\propto$ MAXSAT

An instance of SAT has a set P of boolean literals and a set F of clauses.

Formulate MAXSAT such that it has a set $P'=P$ of boolean literals and a set $F'=F$ of clauses and at least $k=|F|$ of these clauses (all of them) are satisfied.

Then, SAT will have a solution iff MAXSAT formulated in this way will have a solution.

HC $\propto$ UHC

Say that HC is given on a directed graph $G=(V,E)$

Construct an undirected graph $G'=(V',E')$ where $V'=\{v,v' \mid v\in V\}$ and $E'=\{uv' \mid \vec{uv}\in E\}\cup\{vv' \mid v\in V\}$. Basically, duplicate each vertex in V. One of each such vertices will be the entry vertex and the other one will be the exit vertex. For each edge from u to v in G, connect u's exit vertex with v's entry vertex. Additionally, connect each entry and exit vertex pair.

Then, HC will have a solution in G iff UHC will have a solution in G'.

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4) a) Create a graph $G=(V,E)$ where there is a vertex $v \in V$ for each Boolean literal and its complement (eg. $x, x' \in V$). Then for each clause $(x,y) \equiv \neg x \Rightarrow y \equiv \neg y \Rightarrow x$ add an edge from x' to y and an edge from y' to x. After this graph is constructed if there is a path from x to x' and from x' to x, then there is no solution to this problem.

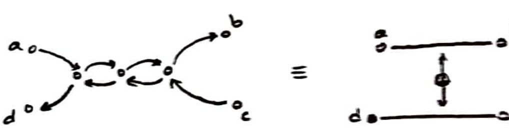
b) $2SAT \in P$ because construction of G is polynomial time and computing the reachability matrix is also polynomial time. From the reachability matrix we find the solution.

5) <u>$EC \propto KS$</u>

We transform every subset in F into a $|U|$ digit number in base $|U|$ such that if u_i is in that subset then ith digit is 1, otherwise ith digit is zero. Then we set k to be a $|U|$ digit number in base $|U|$ with all digits equal to 1.

Then, if we can select a set of those numbers whose sum equals to k that is also the solution for the EC problem.

<u>$EC \propto HC$</u>

First define an XOR widget: 

What the XOR widget does is, exactly one of the edges must be used in HC.

Now, generate a graph $G=(V,E)$.

Suppose the EC problem has elements $u_1, \dots, u_n$. Add vertices $u_1, \dots, u_n$ and an additional vertex u_{n+1} to V. Suppose the EC problem has subsets $S_1, \dots, S_m$. Add vertices $S_1, \dots, S_m$ and an additional vertex S_{m+1} to V. Vertex set V is completed.

For the edge set E, first add an edge from u_n to S_1 and from S_m to u_1. Add 2 edges from S_i to S_{i+1} $\forall i < m$, where one of the edges will be called short edge and the other long edge. Add an edge from a_i to a_{i+1} for each subset a_{i+1} appears in $\forall i < n$. Each such edge corresponds to a subset. Connect these edges to the long edge of the corresponding subset with an XOR widget.

Then, if the created graph G has a HC, there is a solution for the EC problem.

Solutions can be seen on the next page.

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EC Solution:

$B = \{S_0, S_4\} \subseteq F$ where $S_0 = \{u_0, u_3, u_4\}$ and $S_4 = \{s_1, s_2\}$

KS Solution:

$R = \{S_0, S_4\} \subseteq P$ where $S_0 = 10011$ and $S_4 = 01100$ and $S_0 + S_4 = 11111 = K$

HC Solution:

The graph below is generated. The HC is indicated with bold edges.