HW1 *(due March 9,2021 Tuesday before recitation)*

1- *Sketch a composed TM* ***M*** *that performs the computation :*

$(s, \Diamond \underline{\#} \omega) \vdash^*_M (h, \Diamond \underline{\#} \omega^c)$

where ω^c *is the compressed version of* ω *where all the characters '\$' within* ω *are removed.*

2- *Problems from the main text book :*

4.1.4, 4.1.7,4.1.12, 4.2.2, 4.3.1, 4.3.2 (a);(b),4.3.6

An optional HW question

Question *– A Turing machine* M_n *operates on an abstract* ***n-dimensional quadrant*** *given by* $X = N \times \ldots \times N = N^n$ *where* $N = \{0,1,2, \ldots ,\}$ *denotes the set of natural numbers. The head can move in positive and negative directions along each of the dimensions; hence it has precisely* ***2n*** *moves at each slot position. Each slot is an* ***n-dimensional cube*** *that carries data represented by a symbol in its alphabet set* Σ. *A standard TM* ***M*** *to simulate* M_n *must assign a unique slot on its tape corresponding to each n-dimensional cubic slot of* M_n.

This is accomplished by defining functions $f_k : N^k \to N$ *defined inductively via functions* $f_1, f_2, \ldots, f_n$ *, each with the range* N^k *for* $k = 1, \ldots, n$ *operating on N to* N^k *as follows :*

$f_1(i_1) = i_1$ *for all* $i_1 \in N$ ***n***

$$f_k(i_1,i_2, \ldots, i_k) := (f_{k-1}(i_1,i_2, \ldots,i_{k-1}) + i_k)(f_{k-1}(i_1,i_2,\ldots,i_{k-1}) + i_k + 1)/2 + i_k, \; k = 2,3,\ldots,n \quad \ldots\ldots (1)$$

Hence each $f_k(i_1,i_2, \ldots,i_k)$ *can be computed sequentially using the inductive recipe above.*

For example $f_2(i_1,i_2) = (i_1+i_2)(i_1+i_2+1)/2 + i_2$ *etc.*

Theorem

The function $\boldsymbol{f_n : N^n \rightarrow N}$ *described above by* ***(1)*** is a ***bijection .***

i.e. :

(i) $\boldsymbol{f_n(i_1,i_2, ..., i_n) = f_n(j_1,j_2, ..., j_n) \Rightarrow i_p = j_p}$ *for* $\boldsymbol{p=1,\dots, n}$ *(hence it is an* ***injection*** *(1-to-1))*

(ii) for every $\boldsymbol{u \in N}$ *there exist* $\boldsymbol{(i_1 ,\dots, i_n) \in N^n}$ *such that* $\boldsymbol{f_n(i_1,i_2, ..., i_n) = u}$ *(hence it is a* ***surjection*** *(onto))*

Using this theorem describe how M can simulate $\boldsymbol{M_n}$*. In particular describe how* ***M*** *moves its head when* $\boldsymbol{M_n}$ *oves its head from slot* $\boldsymbol{(i_1, ..., i_p ... , i_n)}$ *to* $\boldsymbol{(i_1 , \dots , i_p \pm 1, \dots . i_n).}$

(To get a feeling for the problem first solve it for ***n=2 ,*** *namely simulating a 2D-TM !)*

Meanwhile try also to prove the theorem above using induction on n !