

1) a) add = c (assuming constant c is written in tape k+1)

Label	Condition	TMTM
>	-	$R^0, R^{k+1}, L^0, L^{k+1}, A$
A	$\sigma^0 = \# \wedge \sigma^{k+1} = \#$	h
	$\sigma^0 = \# \wedge \sigma^{k+1} = 0$	R^0, L^0, L^{k+1}, C
	$\sigma^0 = \# \wedge \sigma^{k+1} = 1$	R^0, L^0, L^{k+1}, D
	$\sigma^0 = 0 \wedge \sigma^{k+1} = \#$	L^0, h
	$\sigma^0 = 0 \wedge \sigma^{k+1} = 0$	L^0, L^{k+1}, A
	$\sigma^0 = 0 \wedge \sigma^{k+1} = 1$	L^0, L^{k+1}, A
	$\sigma^0 = 1 \wedge \sigma^{k+1} = \#$	L^0, h
	$\sigma^0 = 1 \wedge \sigma^{k+1} = 0$	L^0, L^{k+1}, A
	$\sigma^0 = 1 \wedge \sigma^{k+1} = 1$	O^0, L^0, L^{k+1}, B
	$\sigma^0 = \# \wedge \sigma^{k+1} = \#$	R^0, L^0, D
	$\sigma^0 = \# \wedge \sigma^{k+1} = 0$	R^0, L^0, L^{k+1}, D
	$\sigma^0 = \# \wedge \sigma^{k+1} = 1$	R^0, L^0, L^{k+1}, E
	$\sigma^0 = 0 \wedge \sigma^{k+1} = \#$	L^0, L^0, h
	$\sigma^0 = 0 \wedge \sigma^{k+1} = 0$	L^0, L^0, L^{k+1}, A
	$\sigma^0 = 0 \wedge \sigma^{k+1} = 1$	L^0, L^{k+1}, B
B	$\sigma^0 = 1 \wedge \sigma^{k+1} = \#$	L^0, B
	$\sigma^0 = 1 \wedge \sigma^{k+1} = 0$	O^0, L^0, L^{k+1}, B
	$\sigma^0 = 1 \wedge \sigma^{k+1} = 1$	L^0, L^{k+1}, B
	$\sigma^0 = x \neq \#$	R^0, x^0, L^0, L^0, C
	$\sigma^0 = \#$	R^0, O^0, L^0, A
C	$\sigma^0 = x \neq \#$	R^0, x^0, L^0, L^0, D
	$\sigma^0 = \#$	R^0, I^0, L^0, A
D	$\sigma^0 = x \neq \#$	R^0, x^0, L^0, L^0, E
	$\sigma^0 = \#$	R^0, O^0, L^0, B

b) jpos s

Label	Condition	TM
>	-	R^0, A
A	$\sigma = \#$	$K := K+1, h$
	$\sigma = 0$	R^0, A
	$\sigma = 1$	$K := S, h$

c) sub j

Label	Condition	TM
>	—	$R_{\#}^0, R_{\#}^1, L^0, L^1, A$
A	$\sigma^0 = \# \wedge \sigma^1 = \#$	h
	$\sigma^0 = \# \wedge \sigma^1 = 0$	$L_{\#}^1, R_{\#}^1, L^0, C$
	$\sigma^0 = \# \wedge \sigma^1 = 1$	$L_{\#}^1, R_{\#}^1, L^0, C$
	$\sigma^0 = 0 \wedge \sigma^1 = \#$	$L_{\#}^0, h$
	$\sigma^0 = 0 \wedge \sigma^1 = 0$	L^0, L^1, A
	$\sigma^0 = 0 \wedge \sigma^1 = 1$	L^0, L^0, B
	$\sigma^0 = 1 \wedge \sigma^1 = \#$	$L_{\#}^1, h$
	$\sigma^0 = 1 \wedge \sigma^1 = 0$	L^0, L^1, A
	$\sigma^0 = 1 \wedge \sigma^1 = 1$	O^0, L^0, L^1, A
	$\sigma^0 = \# \wedge \sigma^1 = \#$	$R_{\#}^0, L^0, C$
B	$\sigma^0 = \# \wedge \sigma^1 = 0$	$L_{\#}^1, R_{\#}^1, L^0, C$
	$\sigma^0 = \# \wedge \sigma^1 = 1$	$L_{\#}^1, R_{\#}^1, L^0, C$
	$\sigma^0 = 0 \wedge \sigma^1 = \#$	L^0, L^0, B
	$\sigma^0 = 0 \wedge \sigma^1 = 0$	L^0, L^0, L^1, B
	$\sigma^0 = 0 \wedge \sigma^1 = 1$	L^0, L^1, B
	$\sigma^0 = 1 \wedge \sigma^1 = \#$	$O^0, L_{\#}^0, h$
	$\sigma^0 = 1 \wedge \sigma^1 = 0$	O^0, L^0, L^1, A
	$\sigma^0 = 1 \wedge \sigma^1 = 1$	L^0, L^1, B
	$\sigma^0 \neq \#$	O^0, L^0, C
	$\sigma^0 = \#$	h
C	$\sigma^0 \neq \#$	O^0, L^0, C
	$\sigma^0 = \#$	h

2) Before constructing the multitape NDTM, I constructed 2 helper TMs in addition to MULT.

 INC^i : increments the binary string on tape i by 1.

Label	Condition	TM
>	—	$R_{\#}^1, L^1, A$
A	$\sigma = \#$	$R_{\#}^1, L^1, B$
	$\sigma = 0$	$L^1, L_{\#}^1, h$
	$\sigma = 1$	O^1, L^1, A
B	$\sigma = x \neq \#$	R^1, x^1, L^1, L^1, B
	$\sigma = \#$	R^1, L^1, L^1, h

 $COPY^{ij}$: copies the string from tape i to tape j .

Label	Condition	TM
>	—	$R_{\#}^1, L^1, A$
A	$\sigma^j \neq \#$	$\#^j, L$
	$\sigma^j = \#$	R^1, R^1, B
B	$\sigma^i = x \neq \#$	x^j, R^1, R^1, B
	$\sigma^i = \#$	$L_{\#}^1, L_{\#}^1, h$

I constructed a 5-tape NDTM with initial configuration: $(\#, \#, \#10, \#10, \#w)$.

My TM will decide if w is composite or not. When this TM rejects, it means w is prime.

Label	Condition	TM
> A	$C^3 < w$	INC^3, A
	$C^4 < w$	INC^4, A
	—	$COPY^{31}, COPY^{42}, MULT, R^1, R^5, B$
	$\sigma^1 = \sigma^5 \neq \#$	R^1, R^5, B
	$\sigma^1 = \sigma^5 = \#$	$L_{\#}^1, L_{\#}^5, h_{yes}$
	else	h_{no}

 $C^3 < w$ means binary integer on tape 3 smaller than w
 $C^4 < w$ means binary integer on tape 4 smaller than w

4.5.1) a) I constructed a single-tape NDTM M :

Label	Condition	TM
>	—	R.A
A	$\sigma = a$	R.A
	$\sigma = b$	R.B
B	$\sigma = b$	R.C
C	$\sigma = b$	R.C
	$\sigma = b$	R.D
D	$\sigma = a$	R.D
	$\sigma = \#$	h

*It is assumed that the conditions not indicated get stuck for the given label.

b) I constructed a 4-tape NDTM M as well as a helper 2-tape NDTM CR^j ,

Start convention for M : $(\#ww^2uu^2, \#, \#, \#)$

Start convention for CR^j : $(\#ww^2, \#)$ {decides ww^2 : $w \in \{a, b\}^*$ }

CR^j :

Label	Condition	TM
>	—	$R^1.A$
A	$\sigma^1 = \#$	h
	$\sigma^1 \neq \#$	$R^1.B$
B	$\sigma^1 \neq \#$	$R^1.B$
	$\sigma^1 \neq \#$	$R^1.C$
C	$\sigma^1 = x \neq \#$	$\#^1.x^3.R^1.R^3.C$
	$\sigma^1 = \#$	$L^1.R^1.L^3.D$
D	$\sigma^1 = \sigma^3 \neq \#$	$R^1.L^3.D$
	$\sigma^1 = \sigma^3 = \#$	h

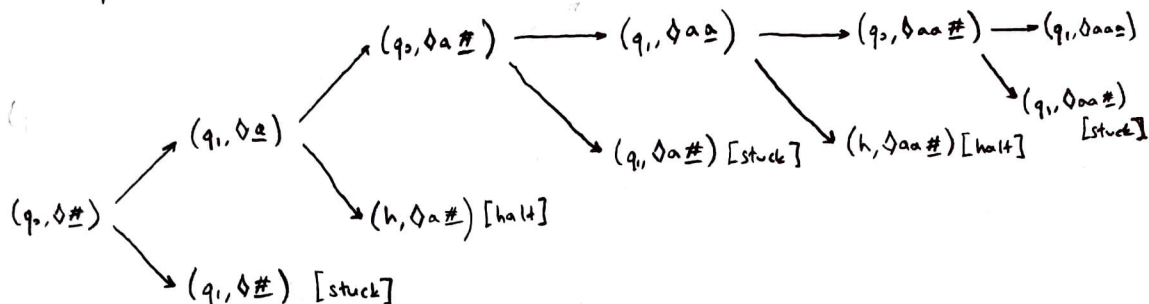
*It is assumed that the conditions not indicated get stuck for the given label

M :

Label	Condition	TM
>	—	$R^1.A$
A	$\sigma^1 = \#$	h
	$\sigma^1 \neq \#$	$R^1.B$
	$\sigma^1 \neq \#$	$R^1.C$
B	$\sigma^1 \neq \#$	$R^1.B$
	$\sigma^1 \neq \#$	$R^1.R^3.C$
C	$\sigma^1 = x \neq \#$	$\#^1.x^3.R^1.R^3.C$
	$\sigma^1 = \#$	$L^1.D$
D	$\sigma^1 = \#$	$L^1.D$
	$\sigma^1 \neq \#$	$L^1.E$
E	—	$CR^{12}.CR^{34}.h$

*It is assumed that the conditions not indicated get stuck for the given label

4.5.2) Computation tree:



When M starts at this configuration, it generates all strings of form aa^* and halts for each such string. In all other computation paths, it gets stuck.

$$r \leq |K| \cdot (|S| + 2)$$

$$r \leq 3 \cdot (2 + 2)$$

$$r \leq 12 \text{ "upper bound"}$$

However, not all state-condition pairs are given in M . r in this case is 2 at most.