Laboratory 3: Frequency Modulation

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Contents

0 Laboratory Objectives and Student Outcomes

0.1 Laboratory Objectives

This laboratory will assist students in understanding the theoretical foundations of frequency modulation. Students simulate FM communication systems using both passband and baseband system models. Students utilize SDR hardware and the complex envelope function to explore frequency modulation using simple message signals. The dependency between the frequency modulation index and the modulated signal bandwidth is investigated. Students conclude the laboratory by constructing an SDR receiver capable of demodulating an Automatic Picture Transmission [1] waveform.

0.2 Student Outcomes

Upon successful completion of this laboratory, the student will:

- Describe the relationship between the instantaneous angle and the instantaneous frequency of a carrier signal.
- Show how a message signal is used to modulate the frequency of the carrier signal.
- Simulate FM communication systems using both passband and baseband models.
- Explain the relationship between parameters of the modulating signal, such as its amplitude and bandwidth, to characteristics of the resulting frequency modulated signal.
- Verify that Carson's rule is an accurate approximation of the bandwidth of a frequency modulated signal.
- Compute the sideband amplitudes for tone-modulated FM signal as a function of the modulation index.
- Compare the measured spectrum of a tone-modulated FM signal to the theoretical spectrum.
- Implement a frequency demodulator using SDR hardware and verify its operation over a wireless channel.

1 Background

Appendix A provides a summary of key equations and results pertaining to frequency modulation.

2 Investigating a Broadcast FM Radio Signal

In this part of the laboratory, you will demodulate a broadcast FM radio signal and listen to the radio broadcast in real-time.

2.1 Broadcast FM Radio

In the United States, broadcast FM radio stations operate in the range of 88 to 108 MHz. Stations are separated by 200 kHz, and geographically close stations are often separated by 400 kHz. The frequency deviation is 75 kHz.

Radio stations create a composite baseband signal which is the summation of multiple different signals. This composite signal is then used to frequency modulate the carrier. Figure 1 shows the various components of the composite baseband signal and their spectral locations. The mono audio signal, located between 30 Hz and 15 kHz, is the summation of the Left and Right audio channels. The stereo audio signal, located at 23 to 53 kHz, is a double-sideband suppressed carrier (DSB-SC) waveform consisting of the Left minus Right audio channels. This DSB-SC signal is generated by modulation of a 38 kHz carrier which is itself generated by frequency-doubling the 19 kHz pilot tone. The pilot tone is also transmitted by the radio station, as it can be used by the receiver for demodulation. Centered at 57 kHz is the Radio Broadcast Data System (RBDS) signal which consists of digital information such as station identification, song title, and traffic reports.

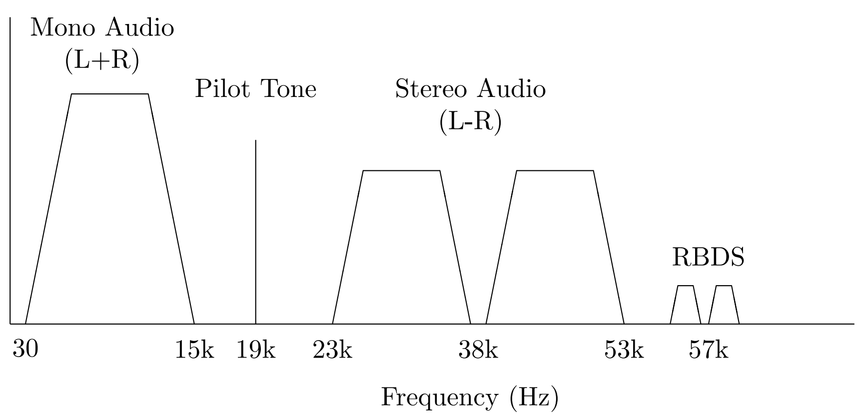


Figure 1: Composite baseband signal for broadcast FM.

2.2 Creating the Simulink model

1. Open a new Simulink model and create the model shown in Figure 2.
   - **RTL-SDR Receiver** configuration:
     - Sampling rate: 240e3
     - Output data type: `single`
     - Samples per frame: 10000

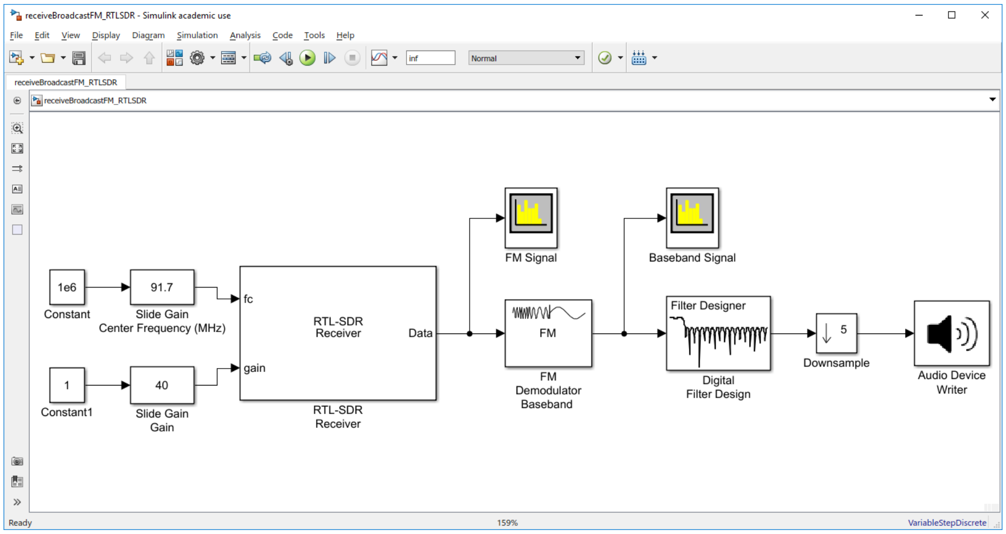


Figure 2: Simulink model for broadcast FM reception.

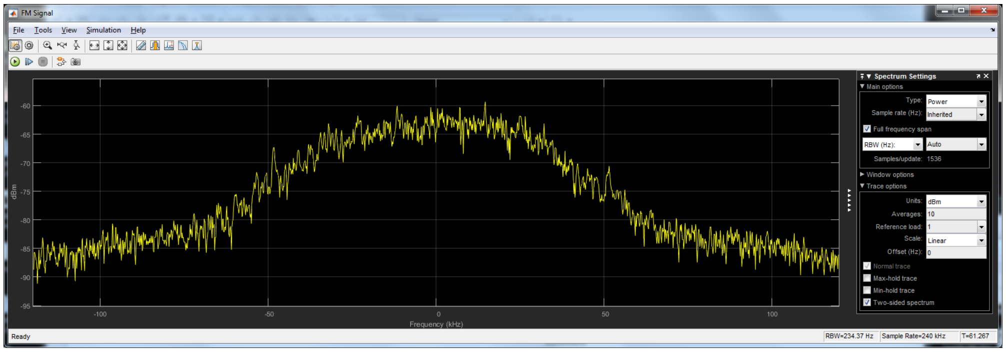
  - Center frequency controlled via `Input port`. Use a **Constant** block (set to 1e6) and a **Slider Gain** (with lower limit 88 and upper limit 108) allowing you to adjust the center frequency while the model is running.
  - Tuner gain controlled via `Input port`. Use a **Constant** block and **Slider Gain**, allowing you to adjust the RTL-SDR receiver gain while the model is running.

- The **FM Demodulator Baseband** block (found in `Communications Toolbox/Modulation/Analog Baseband Modulation`) will perform FM demodulation. Configure for a frequency deviation of 75 kHz, corresponding to the FM broadcast standard in the United States.
- Use the **Digital Filter Design** block to create a lowpass filter that will pass the L+R audio signal. It is suggested to begin with a 40th order Lowpass Equiripple Filter.
- The **Downsample** block (found in `DSP System Toolbox/Signal Operations`) will change the sample rate of the signal. Specify a downsample factor of 5, which results in a sample rate reduction by a factor of 5 at the output of the block. The sample rate for the remainder of the model will be $240e3/5 = 48e3$ which is a standard sample rate for audio signals.
- The **Audio Device Writer** block provides an interface to the audio card in your laptop. The default settings should work reasonably well for this example.

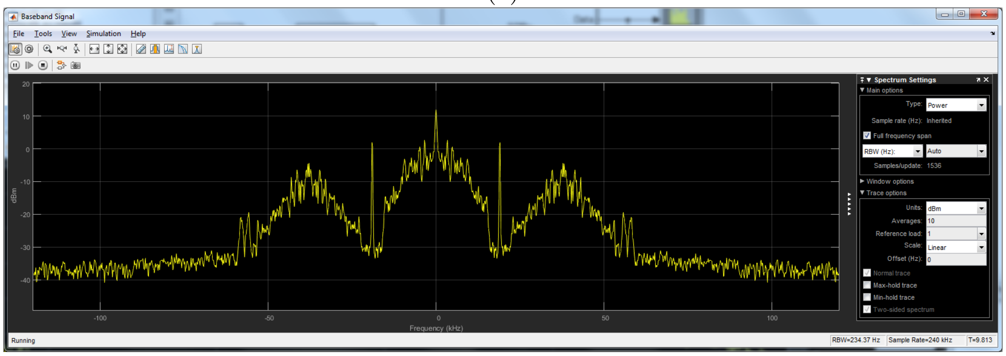
2. Set the simulation time to `inf`.

3. Begin execution by clicking the Play button (`Run`). After a few moments you should see spectrum plots similar to those shown in Figure 3. The spectrum of the FM signal (Figure 3(a)) and the composite baseband waveform should be visible (Figure 3(b)). However, the strength of these components may vary depending on the specific radio station and what is being broadcast at the time (e.g., audio vs. voice). You may need to monitor the spectrum for a few minutes. You may also want to tune to different radio stations.

4. The monaural audio should be playing through your laptop speakers. If the audio is "choppy", you may want to temporarily remove (or comment out) the spectrum analyzer plots from the model to free up computing resources. You may also want to increase the `Samples per frame` setting of the **RTL-SDR Receiver** block.



(a)



(b)

Figure 3: Broadcast FM receiver spectrum plots with center frequency setting of 91.7 MHz. (a) FM signal spectrum. (b) Composite baseband broadcast signal after FM demodulation.

5. You may notice that the audio sounds somewhat bright (or "tinny"). FM broadcasts use a technique called "pre-emphasis" to boost high frequencies in the audio prior to transmission, which combats increased noise levels at high frequencies. FM receivers use an additional filter, called a "de-emphasis" filter to restore the proper high frequency audio levels. For simplicity, this filter has been omitted from our design.

Question 2.1: For an FM radio station of your choosing, submit a screen capture of the spectrum analyzer showing the FM signal. Using Carson's Rule as justification, comment on the signal bandwidth.

Question 2.2: Submit a screen capture of the spectrum analyzer showing the composite baseband signal (after the FM Demodulator). Annotate the image by identifying the various components of the signal.

3 Simulation of a Passband FM Modulator

As discussed during lecture, the instantaneous frequency $f_i(t)$ of a FM communication signal is linearly related to the message $m(t)$

$$f_i(t) = f_c + \frac{k_f}{2\pi} m(t)$$

where f_c is the carrier frequency and k_f is the frequency sensitivity with units rad/(volt $\cdot$ sec), assuming $m(t)$ has units of volts.

Consider a general sinusoidal signal with angle $\varphi(t)$

$$s(t) = A\cos(\varphi(t))$$

The angle $\varphi(t)$ of the sinusoid is equal to the integral of the instantaneous frequency.

$$\varphi(t) = 2\pi \int_{-\infty}^{t} f_i(\lambda) d\lambda$$

Therefore, we can develop the standard equation for a passband FM waveform as follows

$$\begin{aligned}
s(t) &= \Re\{Ae^{j\varphi(t)}\} \\
&= A\cos\left(\varphi(t)\right) \\
&= A\cos\left(2\pi \int_{-\infty}^{t} f_i(\lambda) d\lambda\right) \\
&= A\cos\left(2\pi \int_{-\infty}^{t} \left[f_c + \frac{k_f}{2\pi} m(\lambda)\right] d\lambda\right) \\
&= A\cos\left(2\pi f_c t + \int_{-\infty}^{t} k_f m(\lambda) d\lambda\right)
\end{aligned}$$

3.1 Constructing the Passband FM Simulink Model

The Simulink model shown in Figure 4 below is a simulation of a passband FM modulator based on the equations above. The model uses several continuous-time blocks. Open a new Simulink model and construct the model as follows:

- The **Signal Generator** (found in `Simulink/Sources`) block creates the message signal $m(t)$. The message amplitude and frequency can be modified while the simulation is running. You must stop the simulation to change the waveform type (e.g., from a sine wave to a square wave). Initially, set the waveform to `square` with an amplitude of 1 and a frequency of 100Hz.
- Use a **Slider Gain** block to control the frequency sensitivity of the modulator. Configure the block for a range of 0 to 500. The parameter can be adjusted while the simulation is running. Note that it has units Hz/(volt $\cdot$ sec).
- Use a **Constant** block to specify the carrier frequency. This will have units Hz.
- Use an **Add** block followed by an **Integrator** (found in `Simulink/Continuous`) block and a **Gain** block set to 2π as shown. The output of the **Gain** block is $\varphi(t)$.
- The carrier amplitude, A, is specified using a **Constant** block. Initially, set this to 1.
- Use a **Magnitude-Angle to Complex** block to form the complex passband signal $Ae^{j\varphi(t)}$. Set the `Output` to `Real`.

- Use a **Complex to Real-Imag** block to form the real-valued passband signal $s(t) = \Re\{Ae^{j\varphi(t)}\}$.
- The **Zero-Order Hold** blocks provide sampling of signals, which in turn allows us to examine signals using **Spectrum Analyzer** and **Time Scope** blocks. Set the `Sample time` parameter to 1/10000. This setting means that the block will sample the continuous signals 10000 times per second. Therefore, the sampled signal can faithfully represent signals having frequency components less than 5000Hz. Any frequencies greater than (or equal to) 5000Hz will alias.

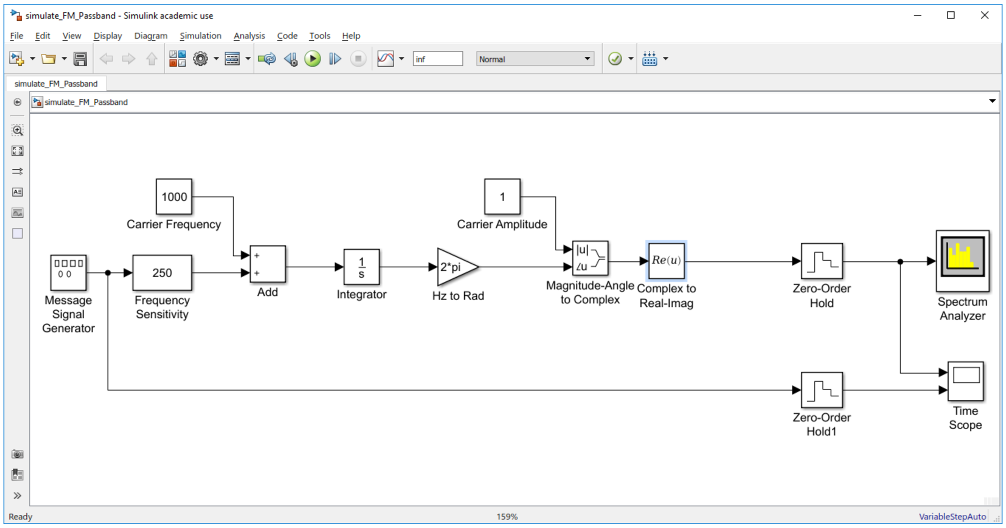


Figure 4: Simulink model for simulating passband FM waveforms.

Experiment with the model by running it and adjusting the parameters. Be sure you understand how the model operates before proceeding.

Question 3.1: Set the frequency sensitivity of the model to zero. What is the resulting passband FM signal? Explain.

Question 3.2: Configure the simulation as follows:

- Frequency Sensitivity = 200
- message Frequency: 0.1 Hz (or some other very small value)
- message Amplitude: 1

Carefully observe the FM signal with the message waveform set to `square`, `sine`, and `sawtooth`. Describe how the message waveform impacts the FM signal.

Question 3.3: Configure the simulation as follows:

- message Wave form: sine
- message Amplitude: 1
- message Frequency: 100 Hz

Carefully observe the FM signal while adjusting the Frequency Sensitivity. Describe how this parameter impacts the FM signal.

Question 3.4: Configure the simulation as follows:

- Frequency Sensitivity = 200
- message Wave form: sine
- message Frequency: 100 Hz

Carefully observe the FM signal while adjusting the message Amplitude. Describe how this parameter impacts the FM signal.

Question 3.5: Configure the simulation as follows:

- Frequency Sensitivity = 100
- message Wave form: sine
- message Frequency: 50 Hz

For these parameters, compute the approximate bandwidth of the passband FM signal using Carson's Rule. Then, measure the bandwidth of the simulated passband FM signal by measuring the range of significant frequencies in its spectrum (consider a frequency component to be significant if its value is within 30dB of the largest peak in the spectrum). Next, repeat the calculation and measurement using a Frequency Sensitivity of 250.

How well does Carson's Rule approximate the FM bandwidth?

4 Simulation of a Baseband FM Communication System

In this portion of the laboratory, you will investigate FM communication systems through a baseband model. You will also see a clever scheme for non-coherent FM demodulation using the complex envelope.

4.1 Non-coherent FM Demodulation

We know that the message information is contained in the angle of a frequency modulated signal. For the passband FM signal

$$s(t) \quad = \quad A\cos(2\pi f_c t + \theta(t))$$

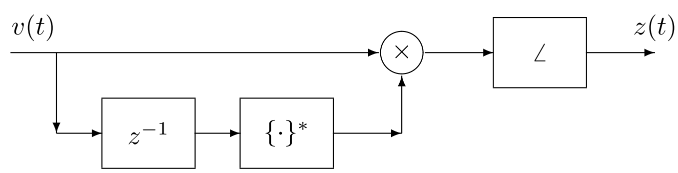
we know that $\theta(t)$ is related to the message $m(t)$

$$\theta(t) \quad = \quad k_f \int_{-\infty}^{t} m(\lambda)d\lambda$$

Thefore, we can recover the message by taking the derivative

$$\frac{d\theta(t)}{dt} \quad = \quad k_f m(t)$$

An approximation to the derivative operation can be realized through the block diagram shown below, where $v(t) = Ae^{j\theta(t)}$ is complex envelope of the received FM signal.



where the block labeled z^{-1} denotes a one sample delay, the block labeled $\{\cdot\}^*$ denotes the complex conjugate operation, and the block labeled $\angle$ outputs the angle of its input. Assuming the time between samples is T_s, the output of the block diagram is

$$\begin{aligned} z(t) &= \angle\{v(t)v^*(t - Ts)\} \\ &= \angle\{Ae^{j\theta(t)}Ae^{-j\theta(t-T_s)}\} \\ &= \angle\{A^2 e^{j[\theta(t)-\theta(t-T_s)]}\} \\ &= \theta(t) - \theta(t - T_s) \end{aligned}$$

Normalized $z(t)$ by the time between samples T_s gives an approximation of the derivative of $\theta(t)$ and therefore the (scaled) message signal.

$$\begin{aligned} \frac{z(t)}{T_s} &\approx \frac{d\theta(t)}{dt} \\ &= k_f m(t) \end{aligned}$$

4.2 Constructing the Baseband FM Simulink Model

Construct the Simulink model shown in Figure 5, which simulates a baseband FM communication systems. The demodulator is based on the approach described in the Section 4.1. The model operates as follows:

- The **Sine Wave** block generates a sinusoidal message signal. Specify a 1000Hz sinusoid and use a `Sample Time` of $1/1e5$. Alternatively, specify this parameter value via a variable defined in `Model Properties -> Callbacks -> InitFcn`.
- The **FM Modulator Baseband** block (found in `Communications Toolbox/Modulation/Analog Baseband Modulation`) performs the FM modulation, producing at its output the baseband FM waveform (complex envelope). Set the `Frequency Deviation (Hz)` parameter to 1000. Alternatively, specify this parameter value via a variable defined in `Model Properties -> Callbacks -> InitFcn`.
- The **AWGN Channel** block (found in `Communications Toolbox -> Channels`) adds white Gaussian noise to the input signal, thus modeling the effects of a noisy communication channel. Set the `Mode` to `Variance from port`, allowing you to adjust the noise power via a **Slider Gain** block and `Constant` block. Configure for a range of 0 to 0.1.
- The **Phase/Frequency Offset** block (found in `Communications System Toolbox/RF Impairments`) applies frequency and/or phase offsets to the signal.
- The FM demodulator is implemented as presented in Section 4.1. Use a **Delay** block (found in `DSP System Toolbox/Signal Operations`), a **Math Function** block configured for `conj`, a **Product** block, and a **Complex to Magnitude-Angle** block (with `Output` set to `Angle`).
- The **Gain** block adjusts the amplitude of the recovered message so that it matches the original message. The proper gain value is $1/(2\pi T_s \tilde{k}_f)$ where T_s is the sample time and $\tilde{k}_f$ is the frequency deviation of the **FM Modulator Baseband** in Hz.
- Add **Spectrum Analyzer** and **Time Scope** blocks to view signals throughout the model.
- Set the Simulation stop time to `inf`.

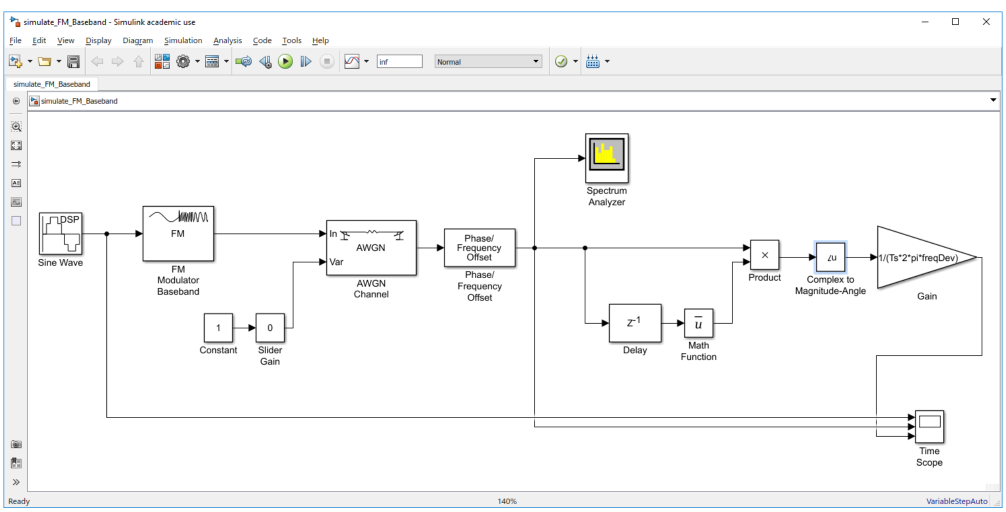


Figure 5: Simulink model for simulating baseband FM.

Experiment with the model by running it and adjusting the parameters. Be sure you understand how the model operates before proceeding.

Question 4.1: This question concerns the FM demodulator presented in Section 4.1. Suppose the complex envelope of the received FM signal contains a frequency error f_i. That is,

$$v(t) \quad = \quad Ae^{j(2\pi f_i t+\theta(t))}$$

Determine an equation for the demodulator output $z(t)$.

Question 4.2: Configure the Simulink baseband model to simulate the condition in the previous problem by introducing a frequency error of 1kHz using the **Phase/Frequency Offset** block. Set the noise variance to 0. Examine the spectrum of the baseband FM signal. Then, examine the output of the demodulator in the time-domain and frequency-domain. Include screen captures. Does the simulation match your result in the previous problem? Explain your findings.

Question 4.3: Examine the spectrum of the baseband FM signal while adjusting the noise variance. Also examine the output of the demodulator in both the time-domain and frequency-domain. Summarize your findings.

5 Observing and Demodulating an FM Waveform

In this part of the laboratory you will use your personal SDR device to observe, analyze, and demodulate a frequency modulated (FM) signal. Your instructor will broadcast radio-frequency signals in the 902MHz to 928MHz frequency band.

The broadcast consists of multiple tone-modulated FM signals. That is, the message signals are of the form

$$m(t) = A\cos(2\pi f_m t)$$

giving passband FM signals of the form

$$s(t) = A\cos\left(2\pi f_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right) \tag{1}$$

Each FM waveform uses $A = 1$ volt and $k_f = 2000$ Hz/(volt sec), but f_m varies from waveform to waveform.

Your instructor will assign each student (or group of students) one of the FM waveforms to analyze. Your task is to observe and analyze the FM signal. You will also demodulate it using the FM demodulator presented in Section 4, confirming its operation on a real RF signal.

Construct the Simulink model shown in Figure 6. Several suggestions:

- **RTL-SDR Receiver** configuration:
  - Sampling rate: 240e3
  - Output data type: `single`
  - Samples per frame: 1024
- Adjust the **Center Frequency** to the specific FM waveform you have been assigned.
- The **Digital Filter Design** is used to isolate the FM waveform you have been assigned. Configure this block to implement a lowpass filter. Use a 60th order Equiripple lowpass filter. It is up to you to determine the pass and stop frequencies.

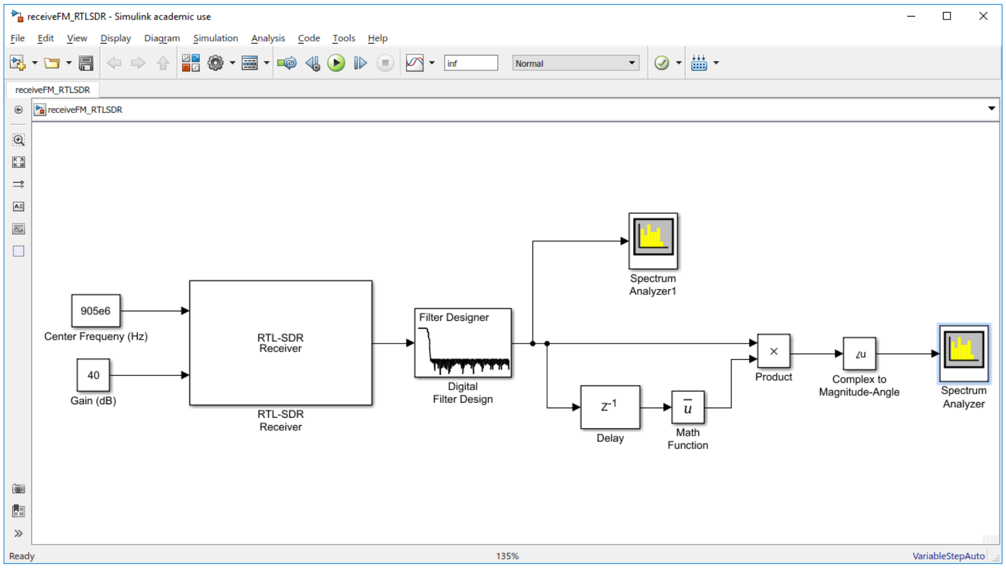


Figure 6: Simulink model for receiving and demodulating FM waveforms.

Question 5.1: Add a **Spectrum Analyzer** block to the model and observe the spectrum of the FM signal (after the lowpass filter). Take a screen capture. Based on this spectrum, what is the message frequency f_m of your waveform? Explain how you arrived at your answer.
Hint: Use the `Zoom In` feature of the spectrum analyzer.

Question 5.2: Observe the spectrum of the output of the FM Demodulator. Take a screen capture. What is the message frequency? Does it agree with your findings in the previous problem?

Question 5.3: Add a **Time Scope** block to the model and observe the received message in the time-domain. Configure the time span to show several cycles of the waveform. Take a screen capture. Comment on the results. Do you observe a DC offset? Explain why or why not.

Question 5.4: Calculate the value of β for your assigned FM signal.

Question 5.5: Based on the values of f_m and β, calculate the power spectrum of the baseband FM signal. Normalize the values so that the value corresponding to $J_0(\beta)$ is 0dB. You are free to use MATLAB for the calculation. Include your MATLAB script.

Question 5.6: Compare your calculated spectrum to the measured spectrum from Question 1. To get accurate measurements, use a **Flat Top** window in the **Spectrum Analzyer** block, which can be set in the `Spectrum Settings -> Window options` panel. Focus on the relative levels between spectral peaks (e.g., the $J_0(\beta)$ and $J_1(\beta)$ peaks).

6 Automatic Picture Transmission (APT) Receiver

6.1 Overview

The Automatic Picture Transmission (APT) communication system was developed to broadcast real-time weather satellite imagery to low-cost ground-based receiver stations [1]. The first satellite to broadcast APT was the TIROS-8 (Television Infrared Observational Satellite) which was launched in 1963. Since then, numerous weather satellites have been equipped with APT systems including the National Oceanic and Atmospheric Administration (NOAA) Polar Operational Environmental Satellites (POES) which are still in use today [2].

APT broadcasts can be received whenever a NOAA POES satellite (or other APT equipped satellite) passes overhead, which occurs at least four times in a 24 hour period. Because the modulation scheme and image format is publicly known, anyone with a receiving station can acquire data and extract the satellite imagery. The process is greatly simplified with modern computing tools and low-cost software-defined radios. The image is Figure 7 shows a satellite image received from the NOAA-19 satellite as it passed over the Milwaukee area on April 25th, 2015. The APT broadcast was received using an RTL-SDR and quadrifilar helix antenna.

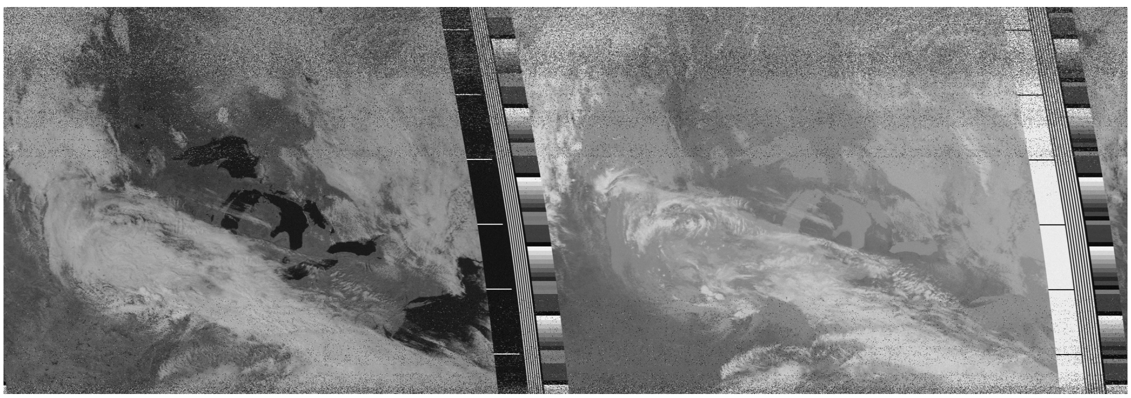
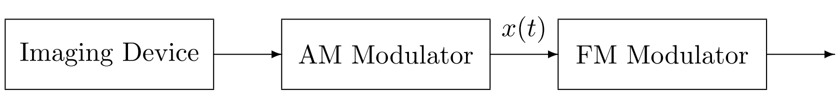


Figure 7: Satellite image received from NOAA 19 weather satellite, south to north mid-afternoon pass, on April 25, 2015 near Milwaukee, WI. Receiving station consisting of an RTL-SDR, quadrifilar helix antenna tuned to 137MHz, and a laptop computer running MATLAB/Simulink. The image shows two multiplexed video channels (visible on LEFT, IR on RIGHT), synchronization bands, and telemetry bands.

In this portion of the lab, you will receive and demodulate an APT communication signal that is being broadcast by your instructor in the laboratory. You will create a Simulink model the demodulate the waveform using your knowledge of frequency and amplitude modulation. The final image formation will occur in MATLAB.

6.2 APT Modulation

The block diagram below shows the modulator of an APT communication system.



- Imaging Device: Outputs image pixels in 8-bit grayscale (values 0 to 255) at the rate of 4160 pixels per second. The image is output as horizontal scan lines (i.e., the image is sent one line after another).
- AM Modulation: The pixel values are used to modulate a 2.4kHz sinusoidal carrier signal. Therefore, we can model $x(t)$ as

  $$x(t) \quad = \quad m(t)\cos(2\pi(2400)t)$$

  where $m(t)$ is the waveform containing the pixel information. Since the pixels are nonnegative, we can consider $m(t) \geq 0$, and therefore $x(t)$ is essentially an AM-LC communication signal. The 2.4kHz sinusoid is referred to as a "sub-carrier".
- FM Modulation: The AM-LC signal $x(t)$ frequency modulates the RF carrier signal. The NOAA POES weather satellites use RF carriers near 137MHz.

6.3 APT Receiver

In this portion of the lab your instructor will broadcast an APT communication signal containing an image. Your instructor will provide you with detailed information concerning the transmission. Your task is to demodulate the signal and recover the image!

Construct the Simulink receiver model shown in Figure 8. This model will demodulate the APT broadcast. You will form the final image using MATLAB.

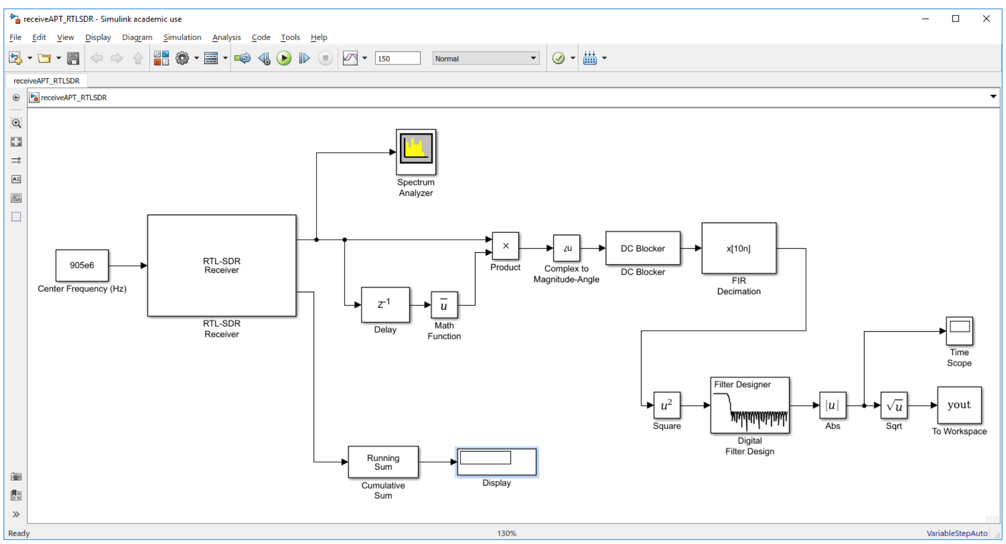
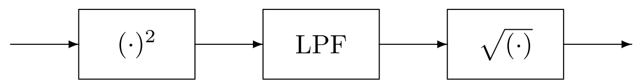


Figure 8: Simulink model for receiving and demodulating the APT broadcast.

- **RTL-SDR Receiver** configuration:
  - Sampling rate: 240e3
  - Output data type: `single`
  - Samples per frame: 1000
  - "Lost samples output port" enabled. This signal will be non-zero when samples are lost. Connect this output as shown to **Cumulative Sum** and **Display** blocks, which will count any samples that are lost during the recording. It is important that no samples are lost. Consult your instructor if samples are being lost.

- Note that the model first does FM demodulation using the approach described in Section 4. The **DC Blocker** block is included to remove any DC offset. The default settings for this block should work well for this example.
- The **FIR Decimation** block is used to reduce the sample rate by a factor of 10.
- The AM Demodulation is performed using a "Square-Law Demodulator", which is illustrated in the block diagram below [1] It is similar to the AM demodulation techniques you studied in the previous lab.



Note that if the input signal to this demodulator is the AM-LC signal

$$s(t) \quad = \quad [A + \tilde{m}(t)]\cos(2\pi f_c t)$$

then

$$\begin{aligned} s^2(t) &= [A + \tilde{m}(t)]^2 \cos^2(2\pi f_c t) \\ &= [A + \tilde{m}(t)]^2 \left(\frac{1}{2} + \frac{1}{2}\cos(2\pi(2f_c)t)\right) \\ &= \frac{1}{2}[A + \tilde{m}(t)]^2 + \frac{1}{2}[A + \tilde{m}(t)]^2 \cos(2\pi(2f_c)t) \end{aligned}$$

Then, the LPF removes the component centered at $2f_c$. It is suggested to use a 60th order Equiripple lowpass filter. It is up to you to determine the pass and stop frequencies. *Hint:* If $[A + \tilde{m}(t)]$ has bandwidth B, then $[A + \tilde{m}(t)]^2$ has bandwidth $2B$.

Assuming $[A + \tilde{m}(t)]$ is nonnegative, the square-root operation recovers the message. The Simulink model uses **Math Function** blocks to implement the squaring and square root operations. The model includes an absolute value operation (via another **Math Function** block) to correct for any small negative signal values resulting from the lowpass filter implemented by the **Digital Filter Design** block.

- The **To Workspace** block writes data to the MATLAB workspace. Set the `Limit data points to last` parameter to `inf` and set the `Save format` to `Array`.
  - After running the model, you will see an array named `yout` (or whatever `Variable name` is specified in the block parameters) in the MATLAB workspace.
  - You can save the MATLAB workspace, and therefore your recorded data, using the `save` function in MATLAB. You can then restore the workspace at a later time using the `load` function.
- Set the stop time of the simulation so that your recording will contain at least one complete image. For example, if the image is 800 by 600 pixels transmitted at 4160 pixels per second, then the transmission will take almost 2 minutes.

[1]Note that the square-law demodulator requires the carrier frequency (in our case, the sub-carrier at 2.4kHz) to be at least twice the bandwidth of the message signal, otherwise distortion can occur. Your instructor's broadcast has been generated with this condition in mind.

Done correctly, you will now have a vector of samples in MATLAB that contain the image information. Now, you must write a MATLAB script that forms the image based on these samples. The following sequence of steps is suggested:

1. Resample the data to 4160 samples per second. Doing so gives exactly one sample per pixel. Use MATLAB's `resample` function to perform the resampling.
2. Reorganize the vector of pixels into a matrix using MATLAB's `reshape` function. This matrix should have the same number of columns as the number of pixels per line of the image. The number of rows in the matrix will depend on the length of your recording. Because your data most likely contains partial lines of the image, you may want to truncate the data to only those lines that are complete. You probably also want to align the synchronization and telemetry bands to one side of the image.
3. View the image using MATLAB's `imshow` function, which will display a matrix as a grayscale image. An example image is shown below in Figure 9

Figure 9: Example APT image recovered from instructor broadcast.

Question 6.1: Carefully observe the spectrum of the APT broadcast signal. Take a screen capture. Explain and interpret what is seen in the spectrum.

Question 6.2: Include a **Spectrum Analyzer** capture of the signal at the output of the FM demodulator. Explain and interpret what is seen in the spectrum.

Question 6.3: What pass and stop frequencies did you choose for the lowpass filter in the AM demodulator? Explain your choices.

Question 6.4: Include a **Spectrum Analyzer** capture of the signal at the output of the AM demodulator. Explain and interpret what is seen in the spectrum.

Question 6.5: Submit the image you recovered from the APT broadcast. Provide the MATLAB code listing you used to create the image from the vector of samples produced by the Simulink receiver.

A Frequency Modulation

1. The complex envelope for an angle modulated waveform is

$$g(t) = Ae^{j\theta(t)} \tag{2}$$

which gives the passband signal

$$\begin{aligned} s(t) &= \Re\{g(t)e^{j2\pi f_c t}\} & (3)\\ &= \Re\{Ae^{j\theta(t)}e^{j2\pi f_c t}\} & (4)\\ &= A\cos(2\pi f_c t + \theta(t)) & (5)\end{aligned}$$

Note that the instantaneous angle is $\varphi(t) = 2\pi f_c t + \theta(t)$ and therefore the instantaneous frequency $f_i(t)$ is

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi}\frac{d\varphi(t)}{dt} & (6)\\ &= f_c + \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right] & (7)\end{aligned}$$

The peak frequency deviation from the carrier, Δf, is

$$\Delta f = \max\left\{\frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]\right\} \tag{8}$$

2. For the case of phase modulation (PM), the phase $\theta(t)$ is proportional to the message signal $m(t)$

$$\theta(t) = k_p m(t) \tag{9}$$

where k_p is the phase sensitivity and, assuming $m(t)$ has units of volts, has units $\frac{\text{rad}}{\text{volt}}$.

3. For the case of frequency modulation (FM), the phase $\theta(t)$ is proportional to the integral of the message signal $m(t)$

$$\theta(t) = k_f \int_{-\infty}^{t} m(\lambda)d\lambda \tag{10}$$

where k_f is the frequency sensitivity and, assuming $m(t)$ has units of volts, has units $\frac{\text{rad}}{\text{volt}\cdot\text{sec}}$.
The instantaneous frequency of the passband signal is

$$\begin{aligned} f_i(t) &= f_c + \frac{1}{2\pi}k_f m(t) & (11)\\ & & (12)\end{aligned}$$

and therefore the peak frequency deviation is

$$\Delta f = \frac{1}{2\pi}k_f m_p \tag{13}$$

where $m_p = \max\{m(t)\}$.
Define the frequency modulation index as

$$\beta = \frac{\Delta f}{B} \tag{14}$$

where B is the bandwidth of the message signal $m(t)$.

4. Carson's Rule states that the bandwidth of a passband FM signal is approximately $2(B + \Delta f)$ where B is the bandwidth of the message signal $m(t)$.

5. A frequency discriminator, shown in the block diagram below, can be used to demodulate passband FM signals.

Given the passband FM signal

$$s(t) = A\cos\left(2\pi f_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right) \tag{15}$$

the output of the derivative block is

$$\frac{ds(t)}{dt} = -A\left[2\pi f_c + k_f m(t)\right]\sin\left(2\pi f_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right). \tag{16}$$

Note that this signal can be interpreted as containing being both amplitude modulated and frequency modulated. The output of the envelope detector is just the AM envelope

$$A\left[2\pi f_c + k_f m(t)\right] \tag{17}$$

and the DC Block removes the DC term, leaving the recovered signal $k_f m(t)$ which is proportional to the message.

6. Consider the sinusoidal message signal

$$m(t) = A\cos(2\pi f_m t) \tag{18}$$

For the case of FM, we have

$$\begin{aligned}
\theta(t) &= k_f \int_{-\infty}^{t} m(\lambda)d\lambda & (19)\\
&= k_f A \int_{-\infty}^{t} \cos(2\pi f_m \lambda)d\lambda & (20)\\
&= \frac{k_f A}{2\pi f_m}\sin(2\pi f_m t) & (21)\\
&= \beta\sin(2\pi f_m t) & (22)
\end{aligned}$$

where the last equality results because $A = m_p$, $B = f_m$, and therefore $\frac{k_f A}{2\pi f_m} = \frac{\Delta f}{B} = \beta$. The complex envelope for the FM signal is

$$\begin{aligned}
g(t) &= Ae^{j\theta(t)} & (23)\\
&= Ae^{j\beta\sin(2\pi f_m t)} & (24)
\end{aligned}$$

Since $g(t)$ is a periodic function with period $T_m = 1/f_m$, $g(t)$ has Fourier series

$$g(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi\frac{n}{T_m}t} \tag{25}$$

$$\tag{26}$$

The Fourier coefficients c_n are

$$\begin{aligned}
c_n &= \frac{1}{T_m}\int_{-T_m/2}^{T_m/2} g(t)e^{-j2\pi\frac{n}{T_m}t} &(27)\\
&= \frac{A}{T_m}\int_{-T_m/2}^{T_m/2} e^{j\beta\sin(2\pi f_m t)}e^{-j2\pi\frac{n}{T_m}t} &(28)\\
&= &(29)\\
&\vdots \\
&= A\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta sin(\phi)-n\phi)}d\phi\right] &(30)\\
&= AJ_n(\beta) &(31)
\end{aligned}$$

where the integral $J_n(\beta)$ is the Bessel function of the first kind of order n. The Fourier transform of $g(t)$ is then

$$\begin{aligned}
G(f) &= \sum_{n=-\infty}^{\infty} c_n\delta(f-nf_m) &(32)\\
&= A\sum_{n=-\infty}^{\infty} J_n(\beta)\delta(f-nf_m) &(33)
\end{aligned}$$

and the Fourier transform of the passband FM signal is

$$S(f) = \frac{1}{2}\left[G(f-f_c)+G^*(-f-f_c)\right] \quad (34)$$

The MATLAB function `besselj` can be used to calculate values of $J_n(\beta)$.

References

[1] National Oceanic and Atmospheric Administration. *User's Guide for Building and Operating Environmental Satellite Receiving Stations*, February 2009.

[2] NASA. Polar Operational Environmental Satellites. https://poes.gsfc.nasa.gov/.

[3] Mathworks. Communications Toolbox. Online Resource. https://www.mathworks.com/products/communications.html.

[4] Mathworks. USRP Support Package from Communications Toolbox. Online Resource. http://www.mathworks.com/discovery/sdr/usrp.html.

[5] Mathworks. RTL-SDR Support Package from Communications Toolbox. Online Resource. https://www.mathworks.com/hardware-support/rtl-sdr.html.

[6] Leon W. Couch III. *Modern Communication Systems*. 1995.

[7] Simon Haykin. *Communication Systems*. 4th edition, 2001.

[8] B.P. Lathi and Zhi Ding. *Modern Digital and Analog Communication Systems*. 5th edition, 2019.