Multivariate Statistics with R

Multi-group CFA and SEM - Lecture 6

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References

- I am not going to reference every slide, but this lecture is heavily influenced by

  Millsap, R.E.(2011). Statistical Approaches to Measurement Invariance. New York: Routledge

  Millsap, R.E. & Kim, H. (In prep). Factorial Invariance Across Multiple Populations in Discrete and Continuous Data. In Paul Irwing, Tom Booth & David J Hughes (Eds). The Wiley Handbook of Psychometrics.

Multi-group CFA: Overview

- Multi-group models, as the names suggest, involve fitting the same model across multiple groups, designated by some categorical identifier.
- Why would we be interested in this?
  - Well if we fit a model across groups, we can test the equivalence of the model across groups in lots of ways.
- Consider multi-group CFA (MG-CFA):
  - We have been talking about ways to replicate solutions.
  - I said last week that CFA was quite a strict test.
  - Arguably multi-group CFA is the strictest test.
- We are asking:

Does a joint model where parameters are constrained to be equal across groups fit our data?

MG-CFA: Typical Examples

- Tests of structure in random groups
- Sex differences
- Age differences
- Differential item functioning & test bias
    - Common in high-stakes settings

MG-CFA: Adding Mean Structure (1)

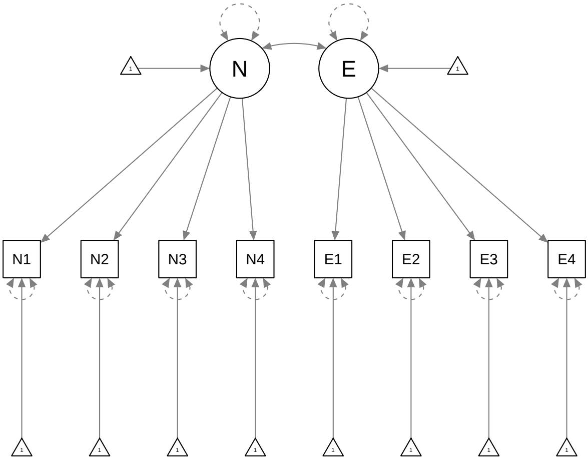
- Lets briefly return to the equation for the factor model and add a small extension.
- Previously we stated the item level equation as:

$$N1 = \lambda_{11} N + \varepsilon_{11}$$

- But we now include a **mean structure** or item itercepts (here τ_{11}). So;

$$N1 = \tau_{11} + \lambda_{11} N + \varepsilon_{11}$$

MG-CFA: Adding Mean Structure (2)



MG-CFA: Adding Mean Structure (3)

- This may all seem complicated, but if we compare the item equation to the standard regression equation:

$$N1 = \tau_{11} + \lambda_{11} N + \varepsilon_{11}$$

$$y_i = \beta_0 + \beta_1 x_1 + \varepsilon 1$$

MG-CFA: Adding Mean Structure (4)

- And if we now extend this to a more general case for multiple measured variables, we get an extension of the factor model in matrix for which includes intercepts:

$$X = \tau + \Lambda W + \Theta$$

- Where
  - X $=$ Matrix of observed scores
  - τ $=$ vector of item intercepts
  - Λ $=$ matrix of factor loadings
  - W $=$ matrix of factor scores
  - Θ $=$ matrix of residual variances

MG-CFA: Bringing in groups (1)

- Where do the the groups come in?
- When we have multiple groups, we are interested in whether the parameters of our model are the same across groups.
- From above, we can see we have a number of different sets of parameters.
- To be precise, we have:
    1. Factor loadings
    2. Item intercepts
    3. Residual variances

MG-CFA: Bringing in groups (2)

- Formally:

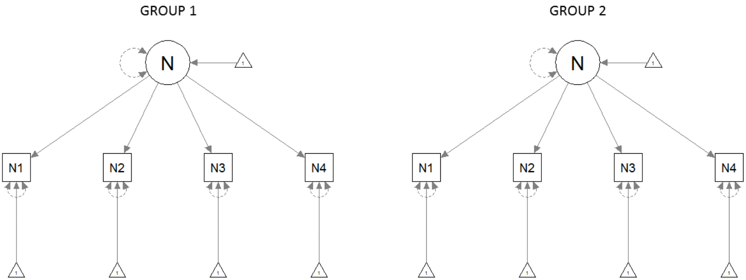
$$P(X|W, V) = P(X|W)$$

- Where;
  - X = data
  - W = latent factors
  - V = a variable defining groups ($\boldsymbol{k}$).
- In words:
  - The probability of the data given the latent variables and group membership, is equal to the probability of the data given the latent variables.
- Tests of this general statement are generally referred to as **measurement invariance**.

MG-CFA: Measurement Invariance

- We discuss measurement invariance with respect to different levels which correspond to our parameters.
  1. Pattern factor loadings: **configural invariance**
  2. Magnitude of factor loadings: **metric invariance**
  3. Item intercepts: **scalar invariance**
  4. Residual variances: **strict invariance**
- The labels differ a little across papers/discussions, but the levels are the same.
- Different questions may require different levels of invariance.

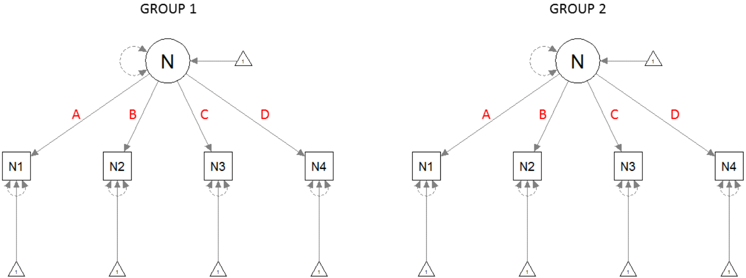
MG-CFA: Configural Invariance (1)



MG-CFA: Configural Invariance (2)

- What we are testing:
  1. That the number of factors is identical in both groups.
  2. That the non-zero elements in Λ_k are identical.
     - This is only a concern with more than1 factor.
- Identification:
  - Fix a factor loading in both groups to 1.
    - Issue here, is this item invariant?
  - Constain the factor means to 0 in one group
- Note big difference here is we need to think about identification of our factor model, and our means structure.

MG-CFA: Metric Invariance (1)

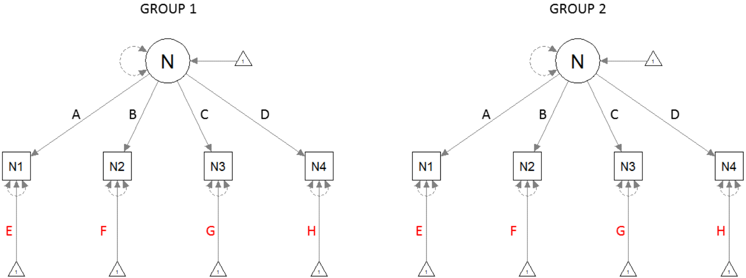


$$\Lambda_k = \Lambda$$

MG-CFA: Metric Invariance (2)

- What we are testing:
  1. That the magnitude of the estimates of the factor loadings are equal across groups.
     - Metric invariance helps us to ensure the interpretation of the factors is the same across groups.
     - Metric invariance is necessary but not sufficient here.
- Identification:
  - We retain our identification constraints from the previous model.
- **Note**: There are many different options for identification when conducitng invariance analyses, here is just one set.

MG-CFA: Scalar Invariance (1)

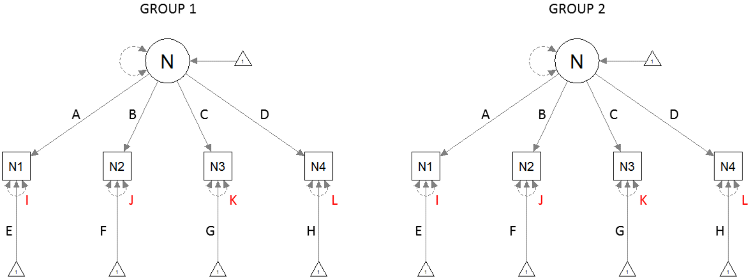


$$\tau_k = \tau$$

MG-CFA: Scalar Invariance (2)

- What we are testing:
  1. That the intercepts of the items are equal across groups.
     - By this point all elements of our regression model involving the latent variable and item are fixed; the loading (or weight) and the intercept.
- Identification:
  - We retain our identification constraints from the previous model.

MG-CFA: Strict Invariance (1)



$$\Theta_k = \Theta$$

MG-CFA: Strict Invariance (2)

- What we are testing:
  1. That the residual variances are equal across groups.
- The residual captures true unique variance and error variance.
  - A difference across groups in true unique variance may suggest differences in, say, understanding of idiosyncractic features like item wording.
  - If the difference is error variance, then lack of invariance may suggest different item reliability across groups.
- Formally, loadings and residual invariance is needed for true interpretation of factor variances and covariances.
  - Remains some discussion concerning other models it may be required for.

MG-CFA: Evaluating MG-CFA

- The set of measurement invariance levels provide a group of *nested models*.
- A model is said to be nested when:
  1. It uses the same variables.
  2. It uses the same sample.
  3. The models differ in the estimated parameters.
     - If one model can be specified as a more constrained version of the other.
- When we fix a parameter in a model, it is considered to be *nested* within the model with that parameter freely estimated.
- Given this, we can test the difference in fit between our models in a number of ways.

MG-CFA: Difference in Fit Criteria

- χ^2 *difference* test.
  - A significant value for the test indicates the model fit is significantly worse.
- Based on Chen (2007) invariance is considered to hold if:
  - $\Delta CFI \leq -.01$
  - $\Delta RMSEA \leq .015$

MG-CFA: Invariance in R (1)

```
# Configural Invariance
config <- cfa(model, data, group = "Sex")

# Metric
metric <- cfa(model, data, group = "Sex",
              group.equal=c("loadings"))

# Scalar
scalar <- cfa(model, data, group = "Sex",
              group.equal=c("loadings", "intercepts"))

# Strict
metric <- cfa(model, data, group = "Sex",
              group.equal=c("loadings", "intercepts",
                            "residuals"))
```

MG-CFA: Invariance in R (2)

- The above code let's us build our invariance models sequentially.
- We could then use the `semTools` function `compareFit()` to consider the fit across models.
- We could also use the function `measurementInvariance()` from `semTools`.
  - This automates the analysis and runs sequentially more constrained models.
  - It also provides fit comparisons.

MG-CFA: Partial Invariance (1)

- What if model fit suggests that invariance does not hold?
- As with all model building, we could apply different strategies:
  - Backwards exploration: Start with all values fixed, and gradually free parameters.
  - Forwards exploration: Start with all free and gradually constrain.
- In the context of invariance, backwards is most often used.

MG-CFA: Partial Invariance (2)

- We can use modification indices to identify the constrained parameters we need to free.
- Once we have done this, our model is referred to as partially invariant.
- If we free a loading, we would also allow it's intercept and residual to be free.
- Interpretation of partially invariant models is much debated.

MG-CFA: Partial Invariance (3)

```
# Configural Invariance
config <- cfa(model, data, group = "Sex")

# Metric
metric <- cfa(model, data, group = "Sex",
              group.equal=c("loadings"))

# Partial Metric
# Free the loading of N2
metric <- cfa(model, data, group = "Sex",
              group.equal=c("loadings"),
              group.partial=c("N=~N2"))
```

MG-CFA: Levels of invariance and subsequent analyses

- We can use invariance to test different things.
- These may require different levels of invariance to hold.
- For example:
  - Tests of latent mean differences require scalar invariance to hold.
  - Tests of regression paths in broader SEM (see below) require metric invariance to hold.

MG-CFA: Categorical Data

- There is not time in this overview lecture to discuss all the details of invariance for categorical data.
- A few simple points can be made:
  1. Instead of item intercepts, we deal with item thresholds
     - Point at which we move from one category response to the next.
  2. This means we have different sets of invariance constraints.
  3. It also means we have different sets of identification constraints.

SEM: Overview

- By this point, you should have a decent grasp of CFA and regression.
- When we discuss SEM models, we are really talkig about the integration of these two things.
- A primary advantage of SEM is the simultaneous estimation of measurement (CFA) and structural (regression) models.
- But more simply, SEM allows models with multiple dependent variables, simple tests of parameter constraints, model fit information etc etc.

SEM: Terminology

- Though not completely uniform, you may see the following labels for models:
  - **Path Model**: This is a SEM which does not include latent variables (measurement model/CFA).
    - This is simply a regression model with multiple dependent variables.
  - **Measurement Model**: This is simply the CFA models we have been discussing.
  - **Structural Model**: This is our full model which includes latent variables and the regression paths between them.

SEM: A confirmatory method (1)

- SEM when used to test specific models is confirmatory (like we discussed with CFA)
- But model modification is common especially when a model does not meet conventions for model fit.
  - Modification is often based on quantitative model derived measures such as modification indices.
  - Modification indices provide information on model improvement if a given parameter is added to the model.
- After modification the model is no longer confirmatory, it is exploratory.

SEM: A confirmatory method (2)

- Poses problems:
  - Are the additional parameters capitalizing on chance (sample specific)?
  - Are the additional parameters theoretically justified?
- Modified models require replication - rarely seen in published research.

SEM: Possible SEM Specifications

- If you can think of a model, you can estimate it in SEM:
  1. Simple regression or path models
  2. CFA
  3. Multigroup, invariance models
  4. Mixture models
  5. Growth curve models (like MLM)
  6. Behaviour genetic models
  7. Latent class models
  8. and on and on and on
- SEM also has many options for estimation and missing data readily incorporated into software.

SEM: Identification

- In general, when latent variables are present, identification is focussed on these.
- We have discussed this in the CFA lectures.
- Assuming the measurement model is identified, and you are not trying to estimate every regression parameter, you model will likely be OK.
  - This is of course an imprecise statement but a reasonable rule of thumb.
- Things to watch out for:
  - Specifying recursive paths

SEM: Estimation and Evaluation

- Very much the same as our discussion with CFA.
- Many estimators available with ML a sensible choice unless you have specific variable types or non-normality.
- Models assessed initially based on fit.
- And subsquently assessed on the magnitude and significance of parameters of interest.

SEM: A small example

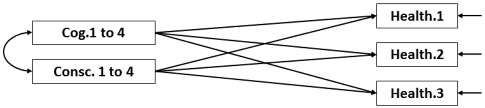
- Research Question: Are the effects of cognitive ability and conscientiousness on health outcomes mediated by body-mass index (BMI)?
- We have:
  - 4 measures of cognitive ability.
  - 4 conscientiousness items
  - 3 health outcomes
  - And BMI

SEM: Path Models (1)

```
Path.model = '
health1 ~ Cog + Con
health2 ~ Cog + Con
health3 ~ Cog + Con
'

sem(Path.model, data)
```

SEM: Path Models (2)



SEM: Path Models (3)

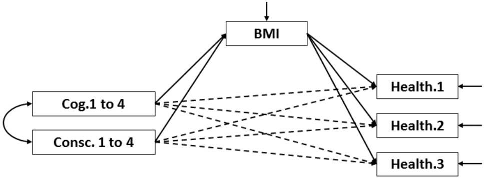
```
Mediation = '
health1 ~ BMI + Cog + Con
health2 ~ BMI + Cog + Con
health3 ~ BMI + Cog + Con

BMI ~ Cog
BMI ~ Con

Cog ~~ Con
'


sem(Mediation, data)
```

SEM: Path Models (4)



SEM: With Measurement Models (1)

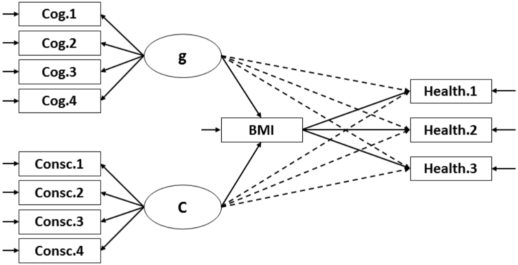
```
Full = '
g =~ Cog.1 + Cog.2 + Cog.3 + Cog.4
C =~ Consc.1 + Consc.2 + Consc.3 + Consc.4

BMI ~ g + C

health1 ~ BMI + g + C
health2 ~ BMI + g + C
health3 ~ BMI + g + C
'

sem(Full, data)
```

SEM: With Measurement Models (2)



SEM: MG-SEM

- In the same way as we tested parameters in a MG-CFA, so we can do the same in MG-SEM.
- If we have a path model, this is a simple contraint.
- If we have a measurement model, we need first to establish measurement invariance of our latent variables.
  - We need to check we have “the same” latent variable in both groups, before we see if the relations between them are the same.

And so we come to the end!

- And that is all I have to say.
- Thank you for your time and your attention.
- I hope the course has been. . . .
  - Interesting?? If not. . .
  - Useful?? If not. . . ..
  - Tolerable!
- Good luck to you all from here on in. I am sure I will be seeing you around the place.

And so we come to the end!

- If anyone wants to talk stats come and see me.
  - e.g. Dissertations
- Keep an eye out for the invited guest talks we have coming up
  - Mirjam Moerbeek: Power Analysis - 19th May
  - Joop Hox: Integratig multi-level models & SEM - TBC
  - Sascha Eskcamp (we hope): Experience sampling data analysis - TBC