浙江大学爱丁堡大学联合学院
ZJU-UoE Institute

Sampling distributions and The Central Limit Theorem

ADS 2, Lecture 3

Melanie Stefan - melanie.stefan@ed.ac.uk

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Pre-lecture version

This lecture contains a lot of questions that I will ask you to think about in class. Providing the answers beforehand would defeat that purpose. Therefore, the version of the slides available to you before the lecture will not contain all of the information that is presented in the lecture.

A complete version will be uploaded to Learn after the lecture. In the meantime, here is a picture of a rather beautiful dragon.

By Caseman - own work, Pentax *ist D, reduced, Public Domain, `https://commons.wikimedia.org/w/index.php?curid=1658439`

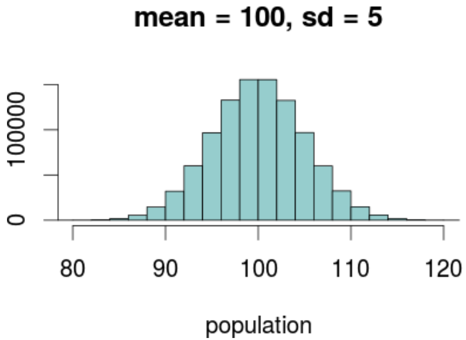
Before we start ...

Please congratulate our class reps:

Before we start . . .

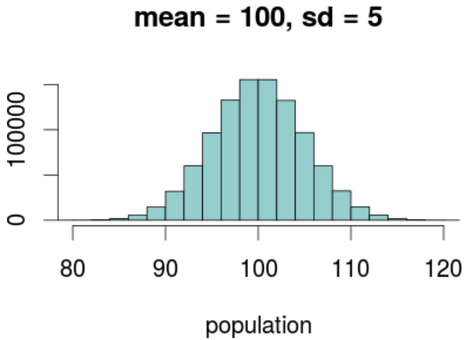
Please congratulate our class reps: Adele, Alana, and Jeff

What's up with normal distributions?



We talk about normal distributions a lot.

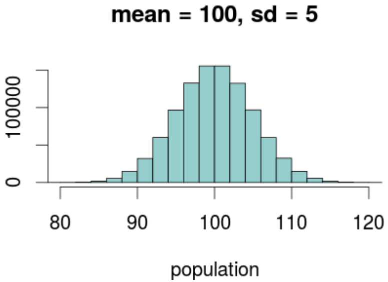
What's up with normal distributions?

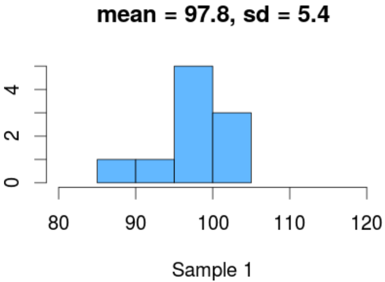


We talk about normal distributions a lot. But why, actually?

This lecture is about . . .

Properties of sampling distributions;
the normal distribution, its properties and why it is special.





Learning Objectives

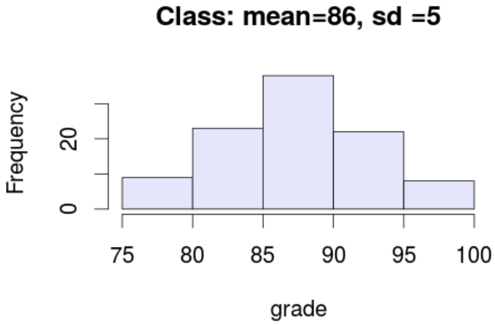
After this lecture, you should be able to . . .

- Define the standard error of the mean
- Compare sampling distributions and underlying population distributions
- Describe a normal distribution and explain its importance
- Explain the Central Limit Theorem

Outline

From Problem Set 1

Create a “virtual class” of 100 exam grades with a mean of 86 and standard deviation of 5

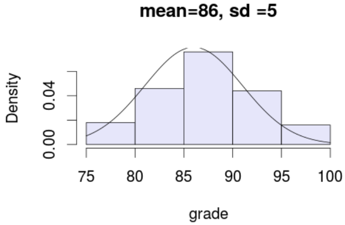


Notation

How would you read this?

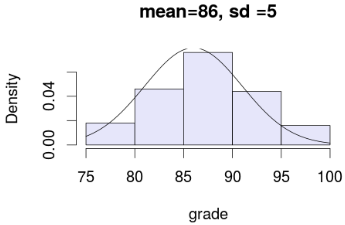
$$X \sim \mathcal{N}(\mu, \sigma^2)$$

Features of a normal distribution



Properties of a normal distribution

Features of a normal distribution



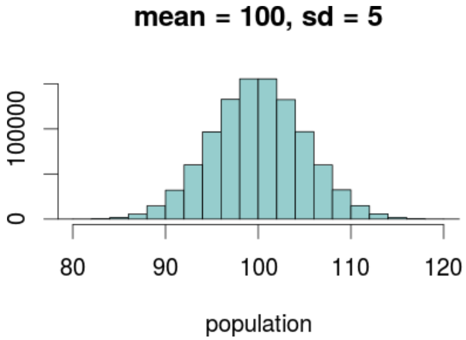
In problem set 1, we asked:

- How many students are more than one standard deviation away from the mean (less than 81 or more than 91)?
- How many students are more than 2 standard deviations away?

Outline

From Problem Set 2

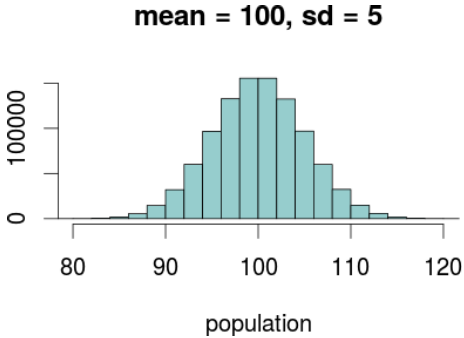
Take samples of size 5 from a normal distribution, record mean and standard deviation.



Is our sample likely to have a higher or lower standard deviation than the population? Why? How does this relate to sample size?

From Problem Set 2

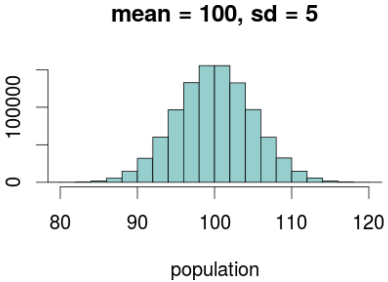
Take samples of size 5 from a normal distribution, record mean



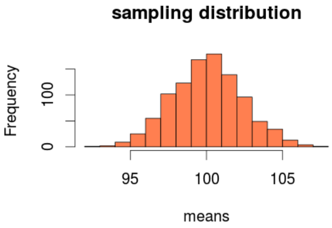
If you do this repeatedly, the distribution of sample means is called the sampling distribution

Sampling distribution

Population:



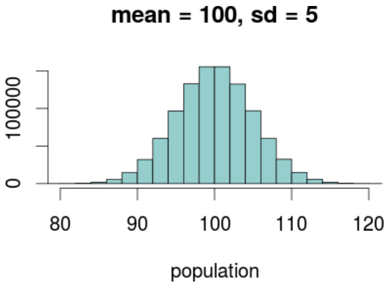
Sampling distribution (n=5)



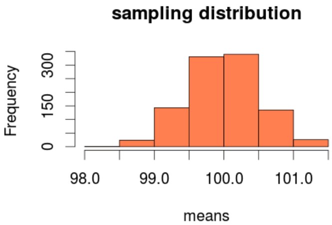
Where is the sampling distribution centred? How much spread is there?

Sampling distribution

Population:



Sampling distribution (n=100)



Fine, but all we (usually) have is one sample . . .

How do we know how good a guess our sample mean is for the true population mean?

Fine, but all we (usually) have is one sample . . .

How do we know how good a guess our sample mean is for the true population mean?
The **Standard Error of the Mean** (SEM) is a measure of how well your sample mean estimates the true population mean.

$$SEM = \frac{sd}{\sqrt{n}}$$

sd . . . standard deviation
n . . . sample size

What happens if n increases? What happens if sd increases?

What's the difference between Standard Error of the Mean and Standard Deviation?

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Outline

Why do we like normal distributions so much?

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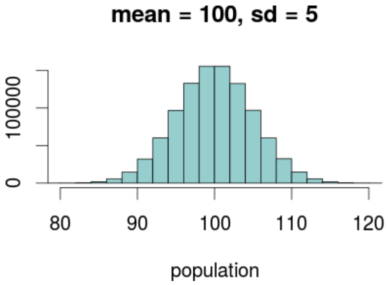
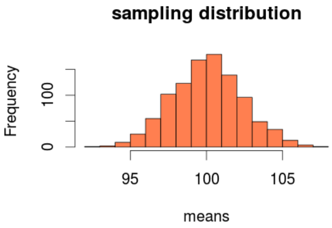
Let's recap:

Population:



Sampling distribution (n=5)



Why do we like normal distributions so much?

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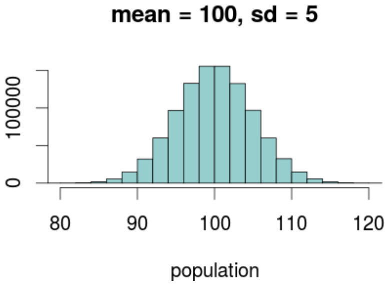
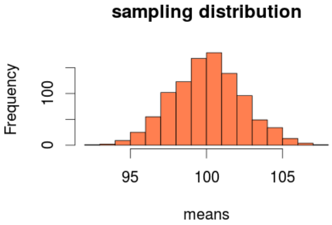


Sampling distribution (n=5)



But what happens if we are not sampling from a normal distribution?

Central Limit Theorem

The Central Limit Theorem

For sample means

Even if a population is not normally distributed, the sampling distribution (for large enough samples) will tend to be normal

More general

If we take n independent random variables from *any distribution*, and take their (normalised) sum, then that sum will tend towards a normal distribution with increasing n.

Maybe you have seen this in real life before?

The Central Limit Theorem

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Why do we like normal distributions so much?

What questions do you have?

Now, you should be able to . . .

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