Set 5 - Lab Report

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Introduction

This experiment explores low pass filters, specifically Butterworth Low-Pass Filter used in analog circuits. With the use of an LMP358 operational amplifier, MATLAB simulations, and experimental measurements with the Analog Discovery 3. Comparing experimental and theoretical values gives insight into real world behaviors of filter behavior. It helps us understand cutoff frequencies and how they are useful.

Working Principle of the Circuit

This lab constructs a Bullenworth Sallen-Key low-pass filter. It is built with an operational amplifier with resistors and capacitors which allows low frequencies and reduces high frequencies. The resistors and calculators set the cutoff frequency and quality of the filter. The operational amplifiers provides a decreasing gain to frequencies above the cutoff, resulting in a smoother wave.

$$\frac{V_o}{V_i} = \frac{1}{R_1R_2C_1C_2s^2+s(R_1C_1+R_2C_1+R_1C_2(1-K))+1}$$

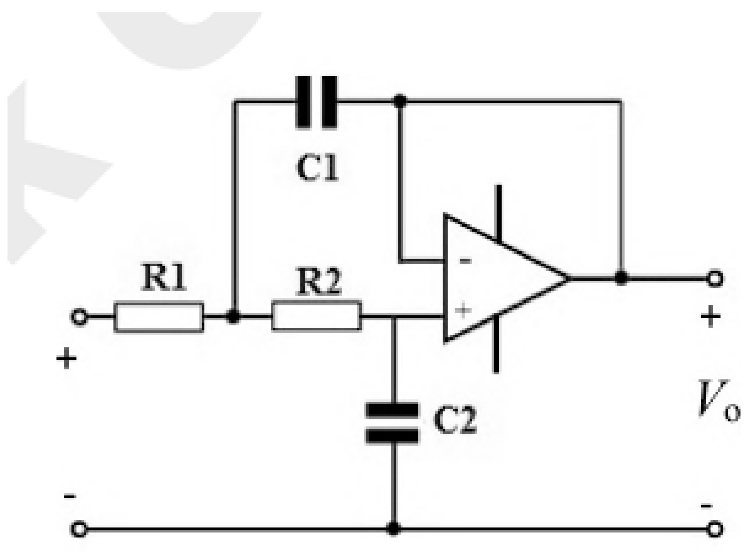

Figure 1: Butterworth Low-Pass Filter

Experiment

1. Derive an expression for the transfer function of the filter.

V_f

$$0 = \frac{V_f + V_i}{R} + \frac{V_f + V_0}{z} + \frac{V_f + V_0}{R}$$

$$0 = \frac{2V_f}{R} + \frac{V_f}{z} - \frac{V_i}{R} - \frac{V_o}{z} - \frac{V_o}{R}$$

V_+

$$0 = \frac{V_o}{z} + \frac{V_o - V_f}{R}$$

$$V_f = \frac{RV_o}{z} + V_o$$

Sub V_+ into V_f

$$0 = \frac{2}{R}\left(\frac{RV_o}{z} + V_o\right) + \frac{1}{z}\left(\frac{RV_o}{z} + V_o\right) - \frac{V_i}{R} - \frac{V_o}{z} - \frac{V_o}{R}$$

$$0 = V_o\left(\frac{2}{z} + \frac{2}{R} + \frac{R}{z^2} + \frac{1}{z} - \frac{1}{z} - \frac{1}{R}\right) - \frac{V_i}{R}$$

$$\frac{V_i}{R} = V_o(2Rsc + 1 + R^2s^2c^2)$$

$$\frac{V_o}{V_i} = \frac{1}{(2Rsc + 1 + R^2s^2c^2)}$$

$$\frac{V_o}{V_i} = \frac{1}{(-10^{-6}w^2 + 2\times10^{-3}jw + 1)}$$

2. Evaluate the filter transfer function abs(V_o/V_i) using the transfer function derived in part (a) for the frequencies shown in the table

Frequency (Hz)	Analytical Gain
50	0.910
100	0.717
200	0.388
500	0.920
1k	0.0247
1.2k	0.0173
1.3k	0.0148
1.4k	0.0128
1.5k	0.0111
1.6k	0.0098
1.7k	0.0087
1.8k	0.0078
1.9k	0.0070
2k	0.0063
5k	0.0010

3. Measure the transfer function using the AD2 board and fill the corresponding components of the table below. Using a sine wave with an amplitude of 2V and offset of 0V ($V_{cc} = \pm5V$).

Frequency (Hz)	Experimental Gain
50	0.912

100	0.725
200	0.410
500	0.109
1k	0.032
1.2k	0.027
1.3k	0.023
1.4k	0.019
1.5k	0.018
1.6k	0.016
1.7k	0.013
1.8k	0.012
1.9k	0.0111
2k	0.0105
5k	0.00738

4. What is the cut-off frequency of this filter?

$$f_c = \frac{1}{2\pi(R_1)(C_1)}$$

$$f_c = \frac{1}{2\pi(10\times10^3)(100\times10^9)} = 159.15$$

5. How do the theoretical and measured results compare? Comment on your results.

Frequency (Hz)	Analytical Gain	Experimental Gain	Percent Difference (%)
50	0.910	0.912	0.22
100	0.717	0.725	1.12
200	0.388	0.410	5.67
500	0.920	0.109	88.15

1k	0.0247	0.032	29.55
1.2k	0.0173	0.027	56.41
1.3k	0.0148	0.023	55.41
1.4k	0.0128	0.019	48.44
1.5k	0.0111	0.018	62.16
1.6k	0.0098	0.016	63.27
1.7k	0.0087	0.013	49.43
1.8k	0.0078	0.012	53.85
1.9k	0.0070	0.0111	58.57
2k	0.0063	0.0105	66.67
5k	0.0010	0.00738	638.00

As shown in the table above, the theoretical and experimental values are very similar for lower frequencies. However, as the frequencies increase, they are increasingly less accurate. This can be seen in the increasing percent difference calculations. This is because the frequencies are moving further away from the cutoff frequency of 159.15 Hz. The greater the frequency from the cutoff frequency, the higher the percent difference.

Discussion

The predictions for the Butterworth low-pass filter are verified from the values in the experiment. The gain behavior is a flat response before the cutoff frequency. On the other hand, when past the cutoff frequency of around 160 Hz, there was a decline in amplitude. This is confirmed by the increasing percent difference values. In other words, the low-pass filter allows lower frequencies under the cutoff frequency through the circuit, but not frequencies above the cutoff frequency.