Measuring Texture and Color in Images

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An RVL Tutorial Presentation

Originally presented in Fall 2016. Code examples updated in January 2018
Corrections in November 2020

CONTENTS

Back to TOC

1: Does the World Really Need Yet Another Tutorial?

- The main reason for this tutorial is for it to serve as a handout for Lecture 15 of my class on Computer Vision at Purdue University. Here is a link to the course website so that you can see for yourself where this lecture belongs in an overall organization of the course:

  `https://engineering.purdue.edu/kak/ComputerVision`

- All of the code examples you see in this tutorial can be downloaded as a gzipped tar archive from

  `https://engineering.purdue.edu/kak/distTextureAndColor/CodeForTextureAndColorTutorial.tar.gz`

- **PLEASE HELP!** Since this still is an early draft of this tutorial (as of December 2019), I am sure it contains typos, inadvertently skipped words (the fingers-on-a-keyboard-not-keeping-up-with-the-brain phenomenon), poor phrasing (the dumb-ass-attack phenomenon), spelling errors (the we-are-losing-our-ability-to-spell-correctly-because-of-auto-spell-checkers phenomenon), and so on. Please let me know (`email: kak@purdue.edu`) if you see any such defects in this document. If you do send email, please

be sure to place the string "Texture and Color" in the Subject line to get past my pretty strong spam filter.

Back to TOC

2: Characterizing Image Textures

- Methods used to characterize image textures fall generally in two categories: statistical and structural. The statistical methods try to figure out how some image property related to its texture may be distributed in the image and then derive numerical texture measures from the computed distributions.

- Structural methods generally investigate the different kinds of periodicities in an image and characterize a texture with the relative spectral energy at different periodicities. [Some readers may argue that "structural" is not the best way to describe what is basically a periodicity analysis of the grayscale (and color) changes. However, as you will see in Section 5, this periodicity analysis is localized to the immediate neighborhoods of the individual pixels. To the extent these within-neighborhood periodicity properties can be used to recognize texture differences between the different regions in an image, I think the word "structural" applies.]

- To elaborate further on the statistical methods, various attempts to characterize image textures over the years are based mostly on extracting the first- and the second-order properties of grayscale and color levels. [By first order, I mean the properties that can be derived directly from the individual pixels — that is, without any cross-comparisons between the pixels. The first-order properties are typically based on the means, the variances, etc., of the pixels. The second-order properties involve comparing two pixels at the same time. The second-order properties, therefore, investigate

how one pixel at some reference location relates statistically to another pixel at a location displaced from the reference location. Some researchers have also looked at characterizing textures with third and higher order properties. These would involve investigating the grayscale and color distributions at three or more pixels whose coordinates must occupy specific positions vis-a-vis one another. Third and higher order texture characterizations have been found to be too complex for a practical characterization of textures.]

- In what follows, GLCM and LBP are examples of texture characterizations based on their second-order statistical properties. On the other hand, the technique based on Gabor filters is an example of the structural approach.

- Any numerical characterization of image textures must possess some essential properties in order to be useful in practical applications:

  – To the maximum extent possible, it must be invariant to changes in image contrast that may be produced by changing or uneven illumination of a scene — assuming that the texture remains more or less the same as perceived by a human. At the least, the numerical characterization must be invariant to monotonic transformation of the grayscale.

  – To the maximum extent possible, it must be invariant to in-plane rotations of the image.

  – It must lend itself to fast computation

- The first invariance is important because one can certainly expect that the illumination conditions under which the training data was collected for a machine learning algorithm may not be the same as the conditions under which the test data was recorded.

- The same goes for the second invariance: It's highly likely that the orientation of the texture you used for training a machine learning algorithm would not be identical to the orientations of the same texture in the test images.

- Some researchers have suggested histogram-equalization as an image normalization tool before subjecting the images to the extraction of texture based properties. Although histogram equalization is a powerful tool for improving the quality of low-contrast images, its usefulness as a normalizer of images prior to texture characterization is open to question. In general, histogram equalization results in a nonlinear transformation of the grayscale and, again in general, such nonlinear transformations can alter the texture in the original images. Additionally, while histogram equalization may balance out the contrast variations in a single image, it does not normalize out image-to-image variations.

- So it is best if the method used for characterizing a texture is mostly independent of the macro-level variations in the contrast

in each image. That is, we want methods that extract texture related information from just the changes in the grayscale at the pixels and their immediate neighborhoods.

Back to TOC

3: Characterizing a Texture with a Gray Level Co-Occurrence Matrix (GLCM)

- The basic idea of GLCM is to estimate the joint probability distribution $P[x1, x2]$ for the grayscale values in an image, where $x1$ is the grayscale value at any randomly selected pixel in the image and $x2$ the grayscale value at another pixel that is at a specific vector distance d from the first pixel. [You are surely familiar with the histogram $P[x]$ of grayscale values in an image: $P[x]$ is the probability that the grayscale value at a randomly chosen pixel in the image will equal x. That is, if you count the number of pixels at the grayscale value x and divide that count by the total number of pixels, you get $P[x]$. Now extend that concept to examining the grayscale values at *two* different pixels that are separated by a displacement vector d. If you count the number of pairs of pixels that are d apart, with one pixel at grayscale value $x1$ and the other at grayscale value $x2$, and you normalize this count by the total number of pairs of pixels that are d apart, you'll get $P[x1, x2]$.] After you have estimated $P[x1, x2]$, the texture can be characterized by the **shape** of this joint distribution.

- Therefore, thinking about the grayscale value at two different pixels that are separated by a fixed displacement vector d is a good place to start for understanding the GLCM approach to texture characterization.

- Let's say we raster scan an image left to right and top to bottom and we examine the grayscale value at each pixel that we encounter and at another pixel that at a displacement d with respect to the first pixel. [Assume that as we are scanning an image, we are currently at the pixel coordinates (i, j). As far as the displacement d is concerned, it could be as simple as pointing to the next pixel, the one at $(i, j+1)$, or, as simple as pointing to the pixel that is one column to the right and one row below. For the first case, $d = (0, 1)$ and for the second case $d = (1, 1)$.]

- As an image is being scanned, the (m, n)-th element of the GLCM matrix records the number of times we have seen the following event: the grayscale value at the current pixel is m while the grayscale value at the d-displaced pixel is n.

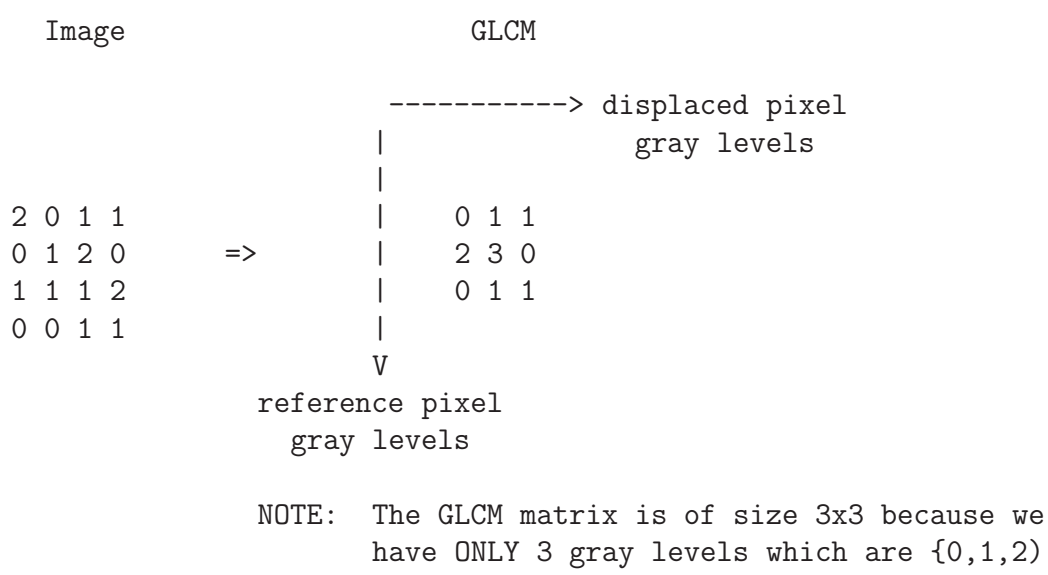
- To illustrate with a toy example, consider the following 4×4 image with pixels whose grayscale values come from the set $\{0, 1, 2\}$:

```
2 0 1 1
0 1 2 0
1 1 1 2
0 0 1 1
```

- And let us assume a displacement vector of

$$d = (1, 1) \tag{1}$$

As we scan the image row by row by visiting each pixel from top left to bottom right, if m is the grayscale value at the current pixel and n the grayscale value at the pixel one position to the right and one row below, we increment $glcm[m][n]$ by 1. Scanning the 4×4 array shown above, we get the following 3×3 GLCM matrix:



- Looking at the first row, what this matrix tells us is that if the reference pixel has grayscale value 0, it is never the case that the displaced pixel also has grayscale value 0. And that there is only one occurrence of the reference pixel having grayscale value 0 while the displaced pixel has grayscale value of 1. And that there is only one occurrence of the reference pixel being of grayscale value 0, while the displaced pixel has grayscale value of 2.

- Looking at the second row of the GLCM matrix shown above, there are two occurrences of the reference pixel being 1 while the displaced pixel has grayscale value 0. And that are three occurrences when both the reference and the displaced pixels have grayscale value of 1. And so on.

- As you would expect, what you get is an asymmetric matrix as shown above. The matrix is asymmetric because, in general, the number of times the grayscale value at the reference pixel is m while the grayscale value at the displaced pixel is n will not be the same for the opposite order of the gray levels at the two pixels.

- Nonetheless, as you will see later, in general one is interested primarily in the fact that two grayscale values, m and n, are associated together because they occur at the two ends of a displacement vector, and one does not want to be concerned with the order of appearance of these two gray levels. When that is the case, it makes sense to create a symmetric GLCM matrix. This can easily be done by the simple expedient of incrementing the element $glcm(n, m)$ when we increment $glcm(m, n)$. For the above example, this yields the result:

```
0 3 1
3 6 1
1 1 2
```

- Here are some interesting observations about GLCM matrices: If you sum the diagonal elements of a normalized GLCM matrix, you get the probability that two pixels in the image that are separate by the displacement vector d will have identical grayscale values — assuming that the GLCM matrix was constructed for a given displacement d. Along the same lines, if you sum any non-diagonal entries in the normalized GLCM matrix that are on a line parallel to the diagonal, you get the probability of finding the grayscale difference at any two pixels separated by d corresponding to the line on which the GLCM elements lie.

- For the example shown above, with a probability of 8/18, two pixels separated by the displacement $d = (1, 1)$ will have identical grayscale values. Similarly, again with a probability of 8/18, two pixels separated by the same d will have their grayscale difference equal to 1. By the same token, with a probability of 2/18, two pixels separated by the same d will have their grayscale difference equal to 2. [**That works out to a level *difference* distribution of** $[8/18, 8/18, 2/18]$**. over the three possible values** $\{0, 1, 2\}$ **for the gray level *differences*.**]

- The Python script in Section 3.3 allows you to experiment with different toy textures in images of arbitrary size and with an arbitrary number of grayscale values. For example, if you set the texture type to `vertical`, the image size to 8 (for an 8×8 array), the number of gray levels to 6, and the displacement

vector to $(1, 1)$, the script yields the output shown below. The first array is the image array created with the `vertical` texture that you specified, and the next array shows the GLCM matrix.

```
Texture type chosen:  vertical
The image:
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]

GLCM:
[0, 0, 0, 0, 0, 49]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[49, 0, 0, 0, 0, 0]

Texture attributes:
    entropy:  1.0
    contrast:  25.0
    homogeneity:  0.166
```

- On the other hand, if in the script of Section 3.3, you set the texture type to `horizontal`, you get the output shown below:

```
Texture type chosen:  horizontal
The image:
[5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0]
```

```
[5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0]

GLCM:
[0, 0, 0, 0, 0, 49]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[49, 0, 0, 0, 0, 0]

Texture attributes:
    entropy:  1.0
    contrast:  25.0
    homogeneity:  0.166
```

which is the same as what you saw for the `vertical` case. This is a consequence of how the GLCM matrix is made symmetric. In any case, we have no problem accepting this result since the two textures are visually the same — even though they are oriented differently.

- It is interesting to see that if you set the texture type to `checkerboard`, you get the following output from the script:

```
Texture type chosen:  checkerboard
The image:
[0, 5, 0, 5, 0, 5, 0, 5]
[5, 0, 5, 0, 5, 0, 5, 0]
[0, 5, 0, 5, 0, 5, 0, 5]
[5, 0, 5, 0, 5, 0, 5, 0]
[0, 5, 0, 5, 0, 5, 0, 5]
[5, 0, 5, 0, 5, 0, 5, 0]
[0, 5, 0, 5, 0, 5, 0, 5]
[5, 0, 5, 0, 5, 0, 5, 0]
```

```
GLCM:
[50, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 48]

Texture attributes:
    entropy:  0.999
    contrast:  0.0
    homogeneity:  0.489
```

- Now our GLCM characterization is different from the previous two cases. This is great since a checkerboard pattern does look visually very different from a ruled surface.

- Here is the output of the script if you choose `random` for the texture type:

```
Texture type chosen:  random
The image:
[1, 5, 5, 0, 0, 1, 1, 3]
[2, 1, 0, 1, 1, 1, 5, 5]
[5, 1, 4, 3, 1, 5, 3, 2]
[1, 1, 5, 1, 5, 2, 2, 1]
[2, 2, 3, 2, 0, 4, 4, 5]
[2, 1, 5, 0, 3, 2, 3, 3]
[4, 5, 1, 5, 0, 5, 5, 5]
[2, 1, 4, 3, 2, 2, 5, 3]

GLCM:
[2, 3, 2, 2, 0, 1]
[3, 6, 4, 4, 3, 6]
[2, 4, 0, 1, 1, 8]
```

```
[2, 4, 1, 0, 2, 4]
[0, 3, 1, 2, 0, 2]
[1, 6, 8, 4, 2, 4]

Texture attributes:
    entropy:  4.70094581328
    contrast:  5.9693877551
    homogeneity:  0.371428571429
```

- Later in Section 3.2 when we talk about how to characterize a GLCM matrix with a small number of attributes, one of the attributes I'll talk about will be "contrast". Here is a texture that is designed specifically to be of low-contrast. You can get it by uncommenting line (A5) in the script. Here is the output for this choice:

```
Texture type chosen:  low_contrast
The image:
[0, 5, 5, 1, 0, 1, 3, 5]
[4, 0, 5, 5, 1, 0, 1, 3]
[0, 4, 0, 5, 5, 1, 0, 1]
[5, 0, 4, 0, 5, 5, 1, 0]
[2, 5, 0, 4, 0, 5, 5, 1]
[3, 2, 5, 0, 4, 0, 5, 5]
[5, 3, 2, 5, 0, 4, 0, 5]
[3, 5, 3, 2, 5, 0, 4, 0]

GLCM:
[30, 0, 0, 0, 0, 0]
[0, 12, 0, 0, 0, 0]
[0, 0, 6, 0, 0, 0]
[0, 0, 0, 6, 0, 0]
[0, 0, 0, 0, 12, 0]
[0, 0, 0, 0, 0, 32]

Texture attributes:
    entropy:  2.28547139622
```

```
contrast:  0.0
homogeneity:  0.69387755102
```

- By the way, if you set the texture type to "None", you get default choices in lines (B19) through (B21) of the script. It is these choices that created a 3×3 GLCM matrix for the 4×4 array that was used at the beginning of this section (see page 8) to explain the basic idea of how one constructs a GLCM matrix.

Back to TOC

3.1: Summary of GLCM Properties

Here is a summary of the GLCM matrix properties:

- GLCM is of size $M \times M$ for an image that has M different gray levels.

- The matrix is symmetric

- Typically, one does NOT construct a GLCM matrix for the full range of grayscale values in an image. For 8-bit grayscale images with its 256 shades of gray (that is, the value of the brightness at each pixel is an integer between 0 and 255, both ends inclusive), you are likely to create a GLCM matrix of size of just 16×16 that corresponds to a re-quantization of the gray levels to just 4 bits (for the purpose of texture characterization).

- Also typically, one may construct multiple GLCM matrices for the same image for different values of the displacement vectors. At the least, one uses the displacement vectors: $(0, 1), (1, 0)$, and $(1, 1)$. [These are the only three possible displacement vectors for a unit Chessboard Distance between the reference pixel and the displaced pixel. (The Chessboard Distance gives us the number of moves required by the King piece in a chess game to get to a square from its current position on the board.) The Chessboard Distance, also known as the Chebyshev Distance or the L_∞ norm, is equal to the max of the coordinate-wise differences between two points. The other distance you frequently run into in digital geometry is the

Cityblock distance, also known as the Manhattan or the Taxicab distance. The Cityblock Distance between any two points is the sum of the absolute differences of the cooordinates of the two points. More formally, the Cityblock distance is referred to as the L_1 norm. The 8 immediate neighbors of a pixel as shown at left below are at a Chessboard Distance of 1 from the pixel. However, as shown at right below, only the two column-wise closest neighbors and the two row-wise closest neighbors of a pixel are at a Cityblock distance of 1:

```
X   X   X                          X

X   P   X                      X   P   X

X   X   X                          X
```

```
   Neighbors at a unit              Neighbors at a unit
Chessboard Distance of P          Cityblock distance from P
(AKA Chebyshev Distance)      (AKA Manhattan or Taxicab Dist.)
 L_inf norm:  ||x||_inf              L_1 norm: ||x||_1
   max of coord diffs                sum of coord diffs
```

In and of itself, the GLCM characterization does not care what distance metric you use for the displacement vector d.]

- To interpret a GLCM matrix as a joint probability distribution, you need to normalize the matrix by dividing its individual elements by the sum of all the elements. That gives you a legitimate joint (or bivariate) probability distribution over two random variables that represent the grayscale values at the two ends of the displacement vector.

Back to TOC

3.2: Deriving Texture Measures from GLCM

- Each element of a GLCM matrix provides too "microscopic" a view of a texture in an image. What we need is a larger "macroscopic" characterization of the texture from the information contained in a GLCM matrix. [Let's say you have constructed a 16×16 GLCM matrix (under the assumption that, regardless of the number of gray levels in an image, you'll place them in just 16 bins for the purpose of texture characterization). Each of the 256 numbers in the GLCM matrix gives you a relative frequency of joint occurrence of the two grayscale values that corresponds to the row-column position of that element in the matrix. Given these 256 GLCM numbers, what we need are a much smaller number of numeric characterizations of the texture that can be derived from the GLCM matrix numbers.]

- Perhaps the most popular characterization of the GLCM matrix is through an entropy value that can be derived when the matrix array is interpreted as a joint probability distribution. This entropy is defined by:

$$Entropy = -\sum_{i=0}^{M-1}\sum_{j=0}^{M-1} P[i,j]\log_2 P[i,j] \tag{2}$$

where $P[i,j]$ is the normalized GLCM matrix. As mentioned at the beginning of Section 3, $P[i,j]$ is the joint probability distribution of the grayscale values at the two ends of the

displacement vector in the image.

- Regarding some properties of entropy that are important from the standpoint of its use as a texture characterizer, it takes on its maximum value when a probability distribution is uniform and its minimum value of 0 when the probability distribution becomes deterministic, meaning that when only a single cell in the GLCM matrix is populated. [The probability distribution would be uniform for a completely random texture. And the probability distribution would become deterministic when all of the grayscale values in the image are identical — that is, when there is no texture in the image.]

- Consider the case when the joint distribution is uniform: Given a 16×16 GLCM matrix, we will have $P[i,j] = 1/256$. In this case, the entropy is

$$\begin{aligned} Entropy &= -\sum_{i=0}^{15}\sum_{j=0}^{15} \frac{1}{256} \log_2 2^{-8} \\ &= -\sum_{i=0}^{15}\sum_{j=0}^{15} \frac{1}{256} (-8) \\ &= 8. \sum_{i=0}^{15}\sum_{j=0}^{15} \frac{1}{256} \\ &= 8 \; bits \end{aligned} \tag{3}$$

- So if you assign all of the grayscale values in an image to 16 bins and it turns out that the entropy calculated from the 16×16

GLCM matrix is 8 bits, you have a completely random texture in the image.

- Now consider the opposite case, that is, when an image has the same grayscale value at all the pixels. In this case, only one cell of the GLCM matrix would be populated — the cell at the $[0, 0]$ element of the matrix. By using the property that $x \cdot \log x$ goes to zero as x approaches 0, it is easy to show that the Entropy becomes zero for this case.

- So, for images whose grayscale values have been placed in 16 bins, we have two bounds on the value of the entropy as derived from a GLCM matrix: a maximum of 8 bits when the texture is completely random and a minimum of 0 when all the pixels have the same grayscale value. For all other textures, the value of entropy will be between these two bounds.

- The main rule to remember here is that the smaller the value of the entropy the more nonuniform a GLCM matrix.

- Here are some other textures attributes that are derived from a GLCM Matrix:

$$
\begin{aligned}
Energy &= \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} P[i,j]^2 \\
Contrast &= \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} (i-j)^2.P[i,j] \\
Homogeneity &= \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} \frac{P[i,j]}{1+|i-j|}
\end{aligned}
\tag{4}
$$

- The first of these, Energy, is also called "Uniformity." Its value is the smallest when all the $P[i,j]$ values are the same — that is, for the case of a completely random texture. Note that since $P[i,j]$ must obey the unit summation constraint, if the values are high for some values of i and j, they must be low elsewhere. In the extreme case, only one cell will have the value $P[i,j]$ equal to 1 and all other cells will be zero. For this extreme case, Energy acquires the largest value of 1. At the other extreme, each cell will have a value of 1/256 for the case a 16x16 GLCM matrix. In this case, Energy will equal $256 \times (1/256)^2 = 1/256$. For all other cases of 16×16 GLCM matrices, Energy will range from the low of 1/256 to the max of 1.

- Consider now the Contrast attribute of a texture defined in Eqs. (4). This attribute takes on a low value when the values in the

GLCM matrix are large along and in the vicinity of the diagonal. In the extreme case, if only the diagonal entries in the GLCM matrix are populated, Contrast is 0. As you would expect, when the displacement vector is $(1, 1)$, the value of Contrast for an image that consists of diagonal stripes of constant grayscale values — with each stripe possibly of a different grayscale value — is zero. This should explain the logic used for creating the `low_contrast` texture pattern in lines (B12) through (B17) of the Python script that is shown in the next section.

- Finally, let's talk about the Homogeneity attribute defined in Equation (4). You can think of this attribute as being the opposite of the Contrast attribute. Homogeneity takes a high value when the GLCM matrix is populated mainly along the diagonal. So when Contrast is high, Homogeneity will be low and vice versa. But note that the two are not opposites in the strict sense of the word — since their definitions are not reciprocal. That is, even though they are opposites loosely speaking, we can expect both these attributes to provide non-redundant characterizations of the GLCM matrix.

Back to TOC

3.3: Python Code for Experimenting with GLCM

Shown below is the code that was used to generate the GLCM results you saw previously on pages 11 through 15 of this tutorial.

```
#!/usr/bin/env python

##  GLCM.py
##  Author:   Avi Kak   (kak@purdue.edu)
##  Date:     September 26, 2016


##  Changes on January 21, 2018:
##
##      Code made Python 3 compliant


##  This script was written as a teaching aid for the lecture on "Textures
##  and Color" as a part of my class on Computer Vision at Purdue.  This
##  Python script demonstrates how the Gray Level Co-occurrence Matrix can
##  be used for characterizing image textures.

##  For educational purposes, this script generates five different types of
##  textures -- you make the choice by uncommenting one of the statements in lines
##  (A1) through (A5).  You can also set the size of the image array and number
##  of gray levels to use.

##  The basic definition of GLCM:
##
##              The (m,n)-th element of the matrix is the number of times
##              the reference-pixel gray level is m and the displaced pixel
##              is n.  In order to create a symmetric matrix, when you
##              increment glcm(m,n) because you found the reference pixel to
##              be equal to m and the displaced pixel to be n, you also
##              increment glcm(n,m).
##
##              The main idea in creating a symmetric GLCM matrix is that
##              you only care about the fact that the gray levels m and n
##              occur together at the two ends of the displacement d and
##              that you don't care that one of the two appears at one
##              specific end and the other at the other specific end.

##  HOW TO USE THIS SCRIPT:
##
##     1.   Specify the texture type you want by uncommenting one of the lines (A1)
##          through (A6)
```

```
##
##               Note that if uncomment line (A6), that sets the image size to 4
##               and the number of gray levels of 3 regardless of the choices you
##               make in lines (A7) and (A8)
##
##     2.   Set the image size in line (A7)
##
##     3.   Set the number of gray levels in line (A8)
##
##     4.   Set the displacement vector by uncommenting one of the lines (A9),
#           (A10), or (A11).  However, note that the "low_contrast" choice for
##          the contrast type in line (A5) is low contrast only when the displacement
##          vector is set as in line (A9).

import random
import math
import functools

##  UNCOMMENT THE TEXTURE TYPE YOU WNT:

#texture_type = 'random'                                                    #(A1)
texture_type = 'vertical'                                                 #(A2)
#texture_type = 'horizontal'                                               #(A3)
#texture_type = 'checkerboard'                                             #(A4)
#texture_type = 'low_contrast'                                             #(A5)
#texture_type = None                                                       #(A6)

IMAGE_SIZE = 8                                                            #(A7)
GRAY_LEVELS = 6                                                           #(A8)
displacement = [1,1]                                                      #(A9)
#displacement = [1,0]                                                     #(A10)
#displacement = [0,1]                                                     #(A11)

image = [[0 for _ in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]       #(B1)

if texture_type == 'random':                                              #(B2)
    image = [[random.randint(0,GRAY_LEVELS-1)
                        for _ in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]   #(B3)
elif texture_type == 'diagonal':                                          #(B4)
    image = [[GRAY_LEVELS - 1 if (i+j)%2 == 0 else 0
                        for i in range(IMAGE_SIZE)] for j in range(IMAGE_SIZE)]   #(B5)
elif texture_type == 'vertical':                                          #(B6)
    image = [[GRAY_LEVELS - 1 if i%2 == 0 else 0
                        for i in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]   #(B7)
elif texture_type == 'horizontal':                                        #(B8)
    image = [[GRAY_LEVELS - 1 if j%2 == 0 else 0
                        for i in range(IMAGE_SIZE)] for j in range(IMAGE_SIZE)]   #(B9)
elif texture_type == 'checkerboard':                                      #(B10)
    image = [[GRAY_LEVELS - 1 if (i+j+1)%2 == 0 else 0
                        for i in range(IMAGE_SIZE)] for j in range(IMAGE_SIZE)]   #(B11)
elif texture_type == 'low_contrast':                                      #(B12)
    image[0] = [random.randint(0,GRAY_LEVELS-1) for _ in range(IMAGE_SIZE)]       #(B13)
    for i in range(1,IMAGE_SIZE):                                         #(B14)
        image[i][0] = random.randint(0,GRAY_LEVELS-1)                     #(B15)
        for j in range(1,IMAGE_SIZE):                                     #(B16)
```

```
            image[i][j] = image[i-1][j-1]                                          #(B17)
else:                                                                              #(B18)
    image = [[2, 0, 1, 1],[0, 1, 2, 0],[1, 1, 1, 2],[0, 0, 1, 1]]                 #(B19)
    IMAGE_SIZE = 4                                                                 #(B20)
    GRAY_LEVELS = 3                                                                #(B21)

# CALCULATE THE GLCM MATRIX:

print("Texture type chosen: %s" % texture_type)                                    #(C1)
print("The image: ")                                                              #(C2)
for row in range(IMAGE_SIZE): print(image[row])                                    #(C3)

glcm = [[0 for _ in range(GRAY_LEVELS)] for _ in range(GRAY_LEVELS)]               #(C4)

rowmax = IMAGE_SIZE - displacement[0] if displacement[0] else IMAGE_SIZE -1        #(C5)
colmax = IMAGE_SIZE - displacement[1] if displacement[1] else IMAGE_SIZE -1        #(C6)

for i in range(rowmax):                                                            #(C7)
    for j in range(colmax):                                                        #(C8)
        m, n =  image[i][j], image[i + displacement[0]][j + displacement[1]]       #(C9)
        glcm[m][n] += 1                                                            #(C10)
        glcm[n][m] += 1                                                            #(C11)

print("\nGLCM: ")                                                                  #(C12)
for row in range(GRAY_LEVELS): print(glcm[row])                                    #(C13)

# CALCULATE ATTRIBUTES OF THE GLCM MATRIX:

entropy = energy = contrast = homogeneity = None                                   #(D1)
normalizer = functools.reduce(lambda x,y: x + sum(y), glcm, 0)                     #(D2)
for m in range(len(glcm)):                                                         #(D3)
    for n in range(len(glcm[0])):                                                  #(D4)
        prob = (1.0 * glcm[m][n]) / normalizer                                     #(D5)
        if (prob >= 0.0001) and (prob <= 0.999):                                   #(D6)
            log_prob = math.log(prob,2)                                            #(D7)
        if prob < 0.0001:                                                          #(D8)
            log_prob = 0                                                           #(D9)
        if prob > 0.999:                                                           #(D10)
            log_prob = 0                                                           #(D11)
        if entropy is None:                                                        #(D12)
            entropy = -1.0 * prob * log_prob                                       #(D13)
            continue                                                               #(D14)
        entropy += -1.0 * prob * log_prob                                          #(D15)
        if energy is None:                                                         #(D16)
            energy = prob ** 2                                                     #(D17)
            continue                                                               #(D18)
        energy += prob ** 2                                                        #(D19)
        if contrast is None:                                                       #(D20)
            contrast = ((m - n)**2 ) * prob                                        #(D21)
            continue                                                               #(D22)
        contrast += ((m - n)**2 ) * prob                                           #(D23)
        if homogeneity is None:                                                    #(D24)
            homogeneity = prob / ( ( 1 + abs(m - n) ) * 1.0 )                      #(D25)
            continue                                                               #(D26)
        homogeneity += prob / ( ( 1 + abs(m - n) ) * 1.0 )                         #(D27)
```

```
if abs(entropy) < 0.0000001: entropy = 0.0                                  #(D28)
print("\nTexture attributes: ")                                             #(D29)
print("    entropy: %f" %  entropy)                                         #(D30)
print("    contrast: %f" % contrast)                                       #(D31)
print("    homogeneity: %f" % homogeneity)                                 #(D32)
```

Back to TOC

4: Characterizing Image Textures with Local Binary Pattern (LBP) Histograms

- Another way to create rotationally and grayscale invariant characterizations of image texture is by using Local Binary Pattern (LBP) histograms. Such histograms are invariant to in-plane rotations and to any monotonic transformations of the grayscale.

- The LBP method for characterizing textures was first introduced by T. Ojala, M. Pietikäinen, and T. Mäenpää in their paper "Multiresolution Grayscale and Rotation Invariant Texture Classification with Local Binary Patterns," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 24, no. 7, pp. 971-987, 2002.

- Fundamental to understating the idea of LBP is the notion of a local binary pattern to characterize the grayscale variations around a pixel through runs of 0s and 1s. Experiments with textures have shown that runs that consist of a single run of 0s followed by a single run of 1s carry most of the discriminative information between different kinds of textures.

- The discussion that follows addresses the following different

aspects of LBP:

- How to characterize the local inter-pixel variations in grayscale values by binary patterns in such a way that the patterns are invariant to linear changes in image contrast;

- How to "reduce" a binary patterns resulting from the previous step to their canonical forms that stay invariant to in-plane rotations;

- And, how to focus on "uniform" patterns since they contain most of the inter-texture discriminatory information and, subsequently, how to create a histogram based characterization of a texture.

Back to TOC

4.1: Characterizing a Local Inter-Pixel Grayscale Variation by a Contrast-Change-Invariant Binary Pattern

- Consider the pixel at the location marked by uppercase 'X' in Fig. A below and its 8 neighboring *points* on a unit circle. The exact positions of the neighboring points vis-a-vis the pixel under consideration is given by

$$(\Delta u, \Delta v) = \left(R\cos\left(\frac{2\pi p}{P}\right), R\sin\left(\frac{2\pi p}{P}\right) \right) \qquad p = 0, 1, 2, .., 7 \qquad (5)$$

  with the radius $R = 1$ and with $P = 8$. The point $p = 0$ gives us the neighboring point that is straight down from the pixel under consideration; the point $p = 1$, the neighboring point to the right of the one that is straight down; and so on. Fig. B on the next page shows the neighboring points for the different values of the index p in the equation shown above.

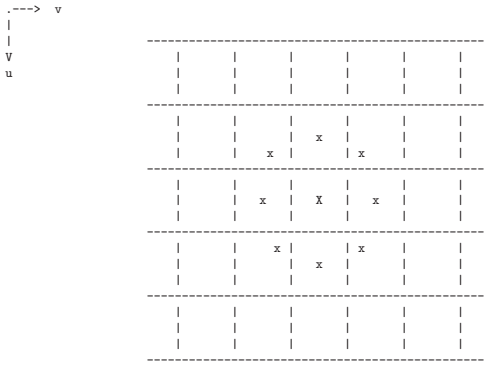


Fig. A

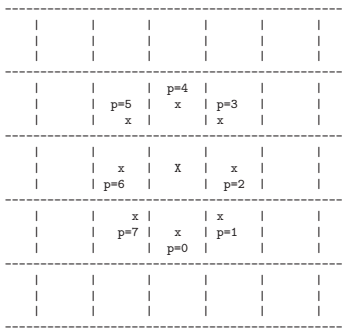
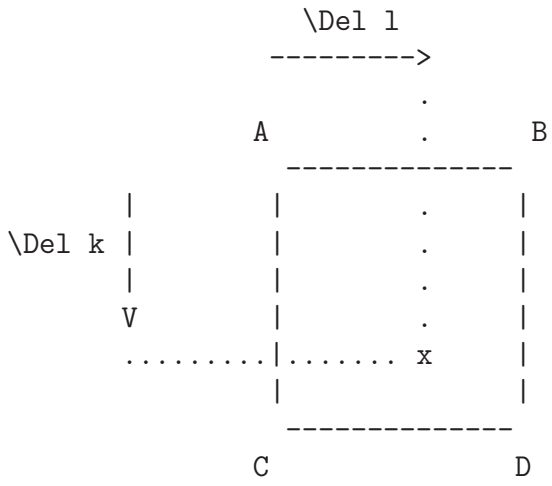


Fig. B

- The grayscale values at the neighborhood points, of which we have 8 when $P = 8$, must be computed with an appropriate interpolation formula. For example, we could use bilinear interpolation to estimate the grayscale value at a neighbor whose coordinates do not coincide with the existing pixel coordinates in the image. [**A neighborhood *point* coincides with a pixel if it is exactly at the center of the pixel.**]

- Let's say that the labels A, B, C, and D at the four corners of the rectangle shown at the top of the next page are the centers of four adjoining pixels and that we want to estimate through interpolation the gray level at the *point* marked 'x' inside the rectangle. Bilinear interpolation says that the gray level at point x can be approximated by

$$\begin{aligned} image(x) \;\approx\; & (1-\Delta k)(1-\Delta l)A \;+\; (1-\Delta k)\Delta l B \\ & +\; \Delta k(1-\Delta l)C \;+\; \Delta k \Delta l D \end{aligned} \tag{6}$$



- The bilinear interpolation formula is based on the assumption that the inter-pixel sampling interval is a unit distance along the horizontal and the vertical axes and that the location of 'x' is a fraction of unity as measured from point A.

- **After estimating the grayscale values at each of the P points on the circle, we threshold these grayscale values with respect to the grayscale value at the pixel at the center of the circle. If the interpolated value at a point is equal or greater than the value at the central pixel, we set that point to 1. Otherwise, we set it to 0.**

- If (i, j) are the coordinates of the pixel at the center of the circle, we now create our binary pattern around the circle through the logic shown below:

```
pattern = []
for p in range(P):
```

```
image_val_at_p =  interpolated value at point p on the circle
if image_val_at_p >= image[i][j]:
    pattern.append(1)
else:
    pattern.append(0)
```

- Consider the following example:

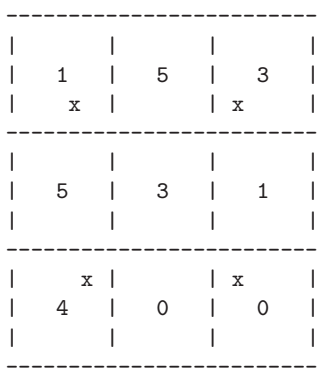


Fig. C

NOTE: 'x' designates those locations on the unit circle where the point does not coincide with the pixel. Cells with no 'x' are those where the points on the circle coincide with those on the circle.

- Going around the central pixel where the grayscale value is 3, we start with p=0 point on the unit-circle neighbors of this pixel. This is the pixel just below the central pixel and the grayscale value there is 0. The next point on circle, for p=1, does not coincide with a pixel (meaning with the center of the rectangle that represents the pixel). This point is in the cell that belongs to the pixel whose grayscale value is also 0. To find by bilinear interpolation the grayscale value at that point, we

see the following situation:

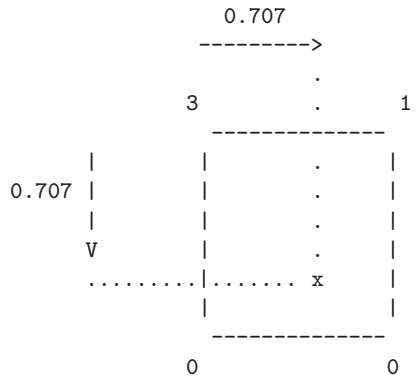


Fig. D

- **In the depiction shown above, note that the gray levels 3, 1, 0, and 0 are at the *centers* of the corresponding pixels.** Therefore, the relationship of the point marked 'x' shown above to the lower right-hand corner of the cell is the same as between the 'x' and the center of the right-most cell in the bottom row of the array on the previous page.

- Using the bilinear interpolation formula shown earlier, we estimate the grayscale value at the point marked 'x' as

$$\begin{aligned} &3 \times (1-0.707) \times (1-0.707) \;+\; 1 \times (1-0.707) \times 0.707 + \\ &0 \times 0.707 \times (1-0.707) \;+\; 0 \times 0.707 \times 0.707 \;\;=\;\; 0.464 \end{aligned} \qquad (7)$$

- If we carry out similar calculations for each of the six remaining points on the circle, we get the following 8 grayscale values on the circle:

```
0.0   0.464   1.0   3.0   5.0   2.828   5.0   3.292
```

Thresholding these values by 3.0, the pixel value at the center of the circle, gives us the binary pattern:

```
0   0   0   1   1   0   1   1
```

That brings us to the issue of how to make such binary patterns invariant to in-plane rotations of the image.

Back to TOC

4.2: Generating Rotation-Invariant Representations from Local Binary Patterns

- Let's say you are imaging a 2D textured surface, such as a brick wall, under the condition that the sensor plane in your camera is parallel to the surface. You have an in-plane rotation of the image if you turn the camera while keeping its sensor plane parallel to the textured surface. As you rotate the camera, say, clockwise, the digital image recorded by the camera would undergo a counter-clockwise rotation through the same angle.

- Under the conditions described above, an in-plane rotation of the image will cause the P points on a circular neighborhood to move along the circle. We now need some way to characterize such a binary pattern for all its possible in-plane rotations. The original authors of LBP proposed circularly rotating the computed binary pattern until the largest number of 0's occupy the most significant positions. This is the same as circularly rotating a binary pattern until it acquires the smallest integer value. We will refer to this representation of a binary pattern as its `minIntVal` representation.

- In a Python implementation, the `minIntVal` form of a binary pattern can easily be found by using my `BitVector` module. In

the code fragment shown below, `pattern` is a binary pattern at a pixel as computed by the algorithm presented in the previous section. In the first statement, we simply initialize a `BitVector` instance with the pattern. In the second statement, we construct a list of the integer values of all circularly shifted versions of the pattern, with each circular shift being to the left by one bit. Finally, in the third statement, we construct a `BitVector` from the smallest integer values calculated in the second statement. This is the bit pattern that characterizes the local texture at the pixel in question in a rotation invariant manner.

```
bv =  BitVector.BitVector( bitlist = pattern )
intvals_for_circular_shifts  =  [int(bv << 1) for _ in range(P)]
minbv = BitVector.BitVector( intVal = min(intvals_for_circular_shifts), \
                                                                  size = P )
```

- I'll next show the `minIntVal` versions of the binary patterns for the image represented by the randomly generated 8x8 array of numbers shown below. The binary patterns will be for the parameter values $R = 1$ and $P = 8$. Here is the image array:

```
The image:

[5, 4, 2, 4, 2, 2, 4, 0]
[4, 2, 1, 2, 1, 0, 0, 2]
[2, 4, 4, 0, 4, 0, 2, 4]
[4, 1, 5, 0, 4, 0, 5, 5]
[0, 4, 4, 5, 0, 0, 3, 2]
[2, 0, 4, 3, 0, 3, 1, 2]
[5, 1, 0, 0, 5, 4, 2, 3]
[1, 0, 0, 4, 5, 5, 0, 1]
```

- The LBP algorithm described in the first section gives us 36 binary patterns for the inner 6x6 subarray of the image array. [The $R = 1$ option requires that we ignore the first and the last rows, with the row index values at 0 and 7, and the first and the last columns, with column index values 0 and 7, for generating the binary patterns.] I have shown the first six of these patterns below; these correspond to the inner six pixels in the second row of the image array:

```
pixel at (1,1):
pattern:  [1, 1, 0, 1, 1, 1, 1, 1]
minbv:  01111111

pixel at (1,2):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111

pixel at (1,3):
pattern:  [0, 1, 0, 1, 1, 1, 0, 1]
minbv:  01010111

pixel at (1,4):
pattern:  [1, 0, 0, 1, 1, 1, 1, 1]
minbv:  00111111

pixel at (1,5):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111

pixel at (1,6):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111

...
...
```

Back to TOC

4.3: Encoding the `minIntVal` Forms of the Local Binary Patterns

- Recall that the binary patterns and the corresponding `minIntVal` versions of those patterns shown on the previous page represent only a pixel-based property, albeit one that is rotationally invariant, of the local grayscale variations in the vicinity of the pixel. **Our ultimate goal is to create an image-based characterization of the texture.**

- Toward that end, we now encode each `minIntVal` binary pattern by a single integer. Since each such integer will represent the local grayscale variations at each pixel, a histogram of all such encodings may subsequently be used to characterize the texture in an image.

- That leads to the question as to what single integer encoding to use for each `minIntVal` binary pattern.

- An obvious answer to this question is to use the integer value of the patterns. For the case of $P = 8$ patterns, that would mean associating an integer value between 0 and 127, both ends inclusive, with each rotationally invariant `minIntVal` pattern. Unfortunately, what speaks against this straightforward

encoding of the patterns is the observation made by the creators of LBP that generally only those `minIntVal` patterns are useful for image-level characterization of a texture that consist of a single run of 0's followed by a single run of 1's — assuming that a pattern has both 0's and 1's. **Such binary patterns were called *uniform* by them.**

- With that observation in mind, the creators of LBP have suggested the following encodings for the patterns:

  – If the `minIntVal` representation of a binary pattern has exactly two runs, that is, a run of 0s followed by a run of 1s, represent the pattern by the number of 1's in the second run. Such encodings would be integers between 1 and $P-1$, both ends inclusive.

  – Else, if the `minIntVal` representation consists of all 0's, represent it be the encoding 0.

  – Else, if the `minIntVal` representation consists of all 1's, represent it by the encoding P.

  – Else, if the `minIntVal` representation involves more than two runs, encode it by the integer $P+1$.

- The encoding formula presented above requires that we first

extract the individual runs in the `minIntVal` form of a local binary pattern. Fortunately, the `BitVector` module also gives a function `runs()` that returns the runs of 0s and 1's in a bit pattern. When you invoke this function on the `minIntVal` bit pattern, the first run will be formed by the 0s assuming that there are at least two runs in a pattern. In the code fragment shown below, the statements in lines (C36) through (C48) implement the logic presented above for encoding the `minIntVal` representations of the local binary patterns. This code section also shows how we construct a histogram of the $P + 2$ encodings at the same time:

```
lbp_hist = {t:0 for t in range(P+2)}                                                    #(C6)

for i in range(R,rowmax):                                                               #(C7)
    for j in range(R,colmax):                                                           #(C8)
        print("\npixel at (%d,%d):" % (i,j))                                            #(C9)
        pattern = []                                                                    #(C10)
        for p in range(P):                                                              #(C11)
            #  We use the index k to point straight down and l to point to the
            #  right in a circular neighborhood around the point (i,j). And we
            #  use (del_k, del_l) as the offset from (i,j) to the point on the
            #  R-radius circle as p varies.
            del_k,del_l = R*math.cos(2*math.pi*p/P), R*math.sin(2*math.pi*p/P)           #(C12)
            if abs(del_k) < 0.001: del_k = 0.0                                          #(C13)
            if abs(del_l) < 0.001: del_l = 0.0                                          #(C14)
            k, l =  i + del_k, j + del_l                                                #(C15)
            k_base,l_base = int(k),int(l)                                               #(C16)
            delta_k,delta_l = k-k_base,l-l_base                                         #(C17)
            if (delta_k < 0.001) and (delta_l < 0.001):                                 #(C18)
                image_val_at_p = float(image[k_base][l_base])                           #(C19)
            elif (delta_l < 0.001):                                                     #(C20)
                image_val_at_p = (1 - delta_k) * image[k_base][l_base] +  \
                                              delta_k * image[k_base+1][l_base]         #(C21)
            elif (delta_k < 0.001):                                                     #(C22)
                image_val_at_p = (1 - delta_l) * image[k_base][l_base] +  \
                                              delta_l * image[k_base][l_base+1]         #(C23)
            else:                                                                       #(C24)
                image_val_at_p = (1-delta_k)*(1-delta_l)*image[k_base][l_base] + \
                                 (1-delta_k)*delta_l*image[k_base][l_base+1]  + \
                                 delta_k*delta_l*image[k_base+1][l_base+1]  + \
                                 delta_k*(1-delta_l)*image[k_base+1][l_base]            #(C25)
            if image_val_at_p >= image[i][j]:                                           #(C26)
                pattern.append(1)                                                       #(C27)
```

```
            else:                                                                    #(C28)
                pattern.append(0)                                                    #(C29)
        print("pattern: %s" % pattern)                                               #(C30)
        bv =  BitVector.BitVector( bitlist = pattern )                               #(C31)
        intvals_for_circular_shifts  =  [int(bv << 1) for _ in range(P)]             #(C32)
        minbv = BitVector.BitVector( intVal = \
                                min(intvals_for_circular_shifts), size = P )         #(C33)
        print("minbv: %s" % minbv)                                                   #(C34)
        bvruns = minbv.runs()                                                        #(C35)
        encoding = None
        if len(bvruns) > 2:                                                          #(C36)
            lbp_hist[P+1] += 1                                                       #(C37)
            encoding = P+1                                                           #(C38)
        elif len(bvruns) == 1 and bvruns[0][0] == '1':                               #(C39)
            lbp_hist[P] += 1                                                         #(C40)
            encoding = P                                                             #(C41)
        elif len(bvruns) == 1 and bvruns[0][0] == '0':                               #(C42)
            lbp_hist[0] += 1                                                         #(C43)
            encoding = 0                                                             #(C44)
        else:                                                                        #(C45)
            lbp_hist[len(bvruns[1])] += 1                                            #(C46)
            encoding = len(bvruns[1])                                                #(C47)
        print("encoding: %s" % encoding)                                             #(C48)
print("\nLBP Histogram: %s" % lbp_hist)                                               #(C49)
```

- Shown below are the encodings for the local binary patterns for each of the inner 6x6 section of the 8x8 image array presented towards the end of the previous subsection:

```
pixel at (1,1):
pattern:  [1, 1, 0, 1, 1, 1, 1, 1]
minbv:  01111111
encoding:  7

pixel at (1,2):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (1,3):
pattern:  [0, 1, 0, 1, 1, 1, 0, 1]
minbv:  01010111
encoding:  9

pixel at (1,4):
pattern:  [1, 0, 0, 1, 1, 1, 1, 1]
minbv:  00111111
encoding:  6
```

```
pixel at (1,5):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (1,6):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (2,1):
pattern:  [0, 0, 1, 0, 0, 0, 0, 0]
minbv:  00000001
encoding:  1

pixel at (2,2):
pattern:  [1, 0, 0, 0, 0, 0, 1, 0]
minbv:  00000101
encoding:  9

pixel at (2,3):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (2,4):
pattern:  [1, 0, 0, 0, 0, 0, 0, 0]
minbv:  00000001
encoding:  1

pixel at (2,5):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (2,6):
pattern:  [1, 1, 1, 1, 0, 0, 0, 0]
minbv:  00001111
encoding:  4

pixel at (3,1):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (3,2):
pattern:  [0, 0, 0, 0, 0, 0, 0, 0]
minbv:  00000000
encoding:  0

pixel at (3,3):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8
```

```
pixel at (3,4):
pattern:  [0, 0, 0, 0, 1, 0, 0, 0]
minbv:  00000001
encoding:  1

pixel at (3,5):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (3,6):
pattern:  [0, 0, 1, 0, 0, 0, 0, 0]
minbv:  00000001
encoding:  1

pixel at (4,1):
pattern:  [0, 0, 1, 0, 0, 0, 0, 0]
minbv:  00000001
encoding:  1

pixel at (4,2):
pattern:  [1, 0, 1, 0, 1, 0, 1, 0]
minbv:  01010101
encoding:  9

pixel at (4,3):
pattern:  [0, 0, 0, 0, 0, 0, 0, 0]
minbv:  00000000
encoding:  0

pixel at (4,4):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (4,5):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (4,6):
pattern:  [0, 0, 0, 1, 1, 0, 0, 0]
minbv:  00000011
encoding:  2

pixel at (5,1):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (5,2):
pattern:  [0, 0, 0, 1, 1, 0, 0, 0]
minbv:  00000011
encoding:  2
```

```
pixel at (5,3):
pattern:  [0, 0, 0, 0, 1, 1, 1, 0]
minbv:  00000111
encoding:  3

pixel at (5,4):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (5,5):
pattern:  [1, 0, 0, 0, 0, 0, 0, 1]
minbv:  00000011
encoding:  2

pixel at (5,6):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (6,1):
pattern:  [0, 0, 0, 1, 0, 1, 1, 1]
minbv:  00010111
encoding:  9

pixel at (6,2):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (6,3):
pattern:  [1, 1, 1, 1, 1, 1, 1, 1]
minbv:  11111111
encoding:  8

pixel at (6,4):
pattern:  [1, 0, 0, 0, 0, 0, 0, 0]
minbv:  00000001
encoding:  1

pixel at (6,5):
pattern:  [1, 0, 0, 0, 0, 0, 1, 1]
minbv:  00000111
encoding:  3

pixel at (6,6):
pattern:  [0, 0, 1, 1, 0, 1, 1, 1]
minbv:  00110111
encoding:  9

LBP Histogram:  {0: 2, 1: 6, 2: 3, 3: 2, 4: 1, 5: 0, 6: 1, 7: 1, 8: 15, 9: 5}
```

- As displayed in the last line above, when we construct a histogram over the $P + 2$ encodings of the patterns, we get the following result for the random 8x8 image array shown earlier:

```
LBP Histogram:  {0: 2, 1: 6, 2: 3, 3: 2, 4: 1, 5: 0, 6: 1, 7: 1, 8: 15, 9: 5}
```

- Shown below are the LBP histograms for the other cases of textures you can produce with the `LBP.py` script that is presented in the next subsection.

```
Texture type: vertical

[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0]

LBP Histogram:  {0: 0, 1: 0, 2: 0, 3: 0, 4: 0, 5: 0, 6: 0, 7: 0, 8: 18, 9: 18}
```

```
Texture type:: horizontal

[5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0]

LBP Histogram:  {0: 0, 1: 0, 2: 0, 3: 0, 4: 0, 5: 0, 6: 0, 7: 0, 8: 18, 9: 18}
```

```
Texture type chosen:  checkerboard

[0, 5, 0, 5, 0, 5, 0, 5]
[5, 0, 5, 0, 5, 0, 5, 0]
[0, 5, 0, 5, 0, 5, 0, 5]
[5, 0, 5, 0, 5, 0, 5, 0]
[0, 5, 0, 5, 0, 5, 0, 5]
[5, 0, 5, 0, 5, 0, 5, 0]
[0, 5, 0, 5, 0, 5, 0, 5]
[5, 0, 5, 0, 5, 0, 5, 0]

LBP Histogram:  {0: 18, 1: 0, 2: 0, 3: 0, 4: 0, 5: 0, 6: 0, 7: 0, 8: 18, 9: 0}
```

Back to TOC

4.4: Python Code for Experimenting with LBP

All of the LBP based results shown earlier were produced by the Python script shown below.

The script generates elementary textures for you and applies the LBP algorithm to the textures. You can specify the texture you want by uncommenting one of lines in (A1) through (A5).

The script allows you to set any arbitrary size for the image array and the number of gray levels you want in the image.

The script also allows you to give whatever values you wish to the P and R parameters needed by the LBP algorithm.

```
#!/usr/bin/env python

##  LBP.py
##  Author:   Avi Kak   (kak@purdue.edu)
##  Date:     November 1, 2016


##  Changes on January 21, 2018:
##
##      Code made Python 3 compliant


##  This script was written as a teaching aid for the lecture on "Textures and Color"
##  as a part of my class on Computer Vision at Purdue.
##
##  This Python script demonstrates how Local Binary Patterns can be used for
##  characterizing image textures.

##  For educational purposes, this script generates five different types of textures
##  -- you make the choice by uncommenting one of the statements in lines (A1)
##  through (A5).  You can also set the size of the image array and number of gray
##  levels to use.

##  HOW TO USE THIS SCRIPT:
```

```
##
##     1.   Specify the texture type you want by uncommenting one of the lines (A1)
##          through (A5)
##
##     2.   Set the image size in line (A6)
##
##     3.   Set the number of gray levels in line (A7)
##
##     4.   Choose a value for the circle radius R in line (A8)
#3
##     5.   Choose a value for the number of sampling points on the circle in line (A9).

##  Calling syntax:    LBP.py

import random
import math
import BitVector

##  UNCOMMENT THE TEXTURE TYPE YOU WANT:
#texture_type = 'random'                                                    #(A1)
texture_type = 'vertical'                                                 #(A2)
#texture_type = 'horizontal'                                               #(A3)
#texture_type = 'checkerboard'                                             #(A4)
#texture_type = None                                                       #(A5)

IMAGE_SIZE = 8                                                            #(A6)
#IMAGE_SIZE = 4                                                           #(A6)
GRAY_LEVELS = 6                                                           #(A7)
R = 1                  # the parameter R is radius of the circular pattern       #(A8)
P = 8                  # the number of points to sample on the circle            #(A9)


image = [[0 for _ in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]              #(B1)

if texture_type == 'random':                                              #(B2)
    image = [[random.randint(0,GRAY_LEVELS-1)
                  for _ in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]      #(B3)
elif texture_type == 'diagonal':                                          #(B4)
    image = [[GRAY_LEVELS - 1 if (i+j)%2 == 0 else 0
                  for i in range(IMAGE_SIZE)] for j in range(IMAGE_SIZE)]      #(B5)
elif texture_type == 'vertical':                                          #(B6)
    image = [[GRAY_LEVELS - 1 if i%2 == 0 else 0
                  for i in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]      #(B7)
elif texture_type == 'horizontal':                                        #(B8)
    image = [[GRAY_LEVELS - 1 if j%2 == 0 else 0
                  for i in range(IMAGE_SIZE)] for j in range(IMAGE_SIZE)]      #(B9)
elif texture_type == 'checkerboard':                                      #(B10)
    image = [[GRAY_LEVELS - 1 if (i+j+1)%2 == 0 else 0
                  for i in range(IMAGE_SIZE)] for j in range(IMAGE_SIZE)]      #(B11)
else:                                                                     #(B12)
    image = [[1, 5, 3, 1],[5, 3, 1, 4],[4, 0, 0, 0],[2, 3, 4, 5]]         #(B13)
    IMAGE_SIZE = 4                                                        #(B14)
    GRAY_LEVELS = 3                                                       #(B15)

print("Texture type chosen: %s" % texture_type)                           #(C1)
```

```
print("The image: ")                                                          #(C2)
for row in range(IMAGE_SIZE): print(image[row])                               #(C3)

lbp = [[0 for _ in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]             #(C4)
rowmax,colmax = IMAGE_SIZE-R,IMAGE_SIZE-R                                     #(C5)
lbp_hist = {t:0 for t in range(P+2)}                                          #(C6)

for i in range(R,rowmax):                                                     #(C7)
    for j in range(R,colmax):                                                 #(C8)
        print("\npixel at (%d,%d):" % (i,j))                                  #(C9)
        pattern = []                                                          #(C10)
        for p in range(P):                                                    #(C11)
            #  We use the index k to point straight down and l to point to the
            #  right in a circular neighborhood around the point (i,j). And we
            #  use (del_k, del_l) as the offset from (i,j) to the point on the
            #  R-radius circle as p varies.
            del_k,del_l = R*math.cos(2*math.pi*p/P), R*math.sin(2*math.pi*p/P)  #(C12)
            if abs(del_k) < 0.001: del_k = 0.0                                #(C13)
            if abs(del_l) < 0.001: del_l = 0.0                                #(C14)
            k, l =  i + del_k, j + del_l                                      #(C15)
            k_base,l_base = int(k),int(l)                                     #(C16)
            delta_k,delta_l = k-k_base,l-l_base                               #(C17)
            if (delta_k < 0.001) and (delta_l < 0.001):                       #(C18)
                image_val_at_p = float(image[k_base][l_base])                 #(C19)
            elif (delta_l < 0.001):                                           #(C20)
                image_val_at_p = (1 - delta_k) * image[k_base][l_base] +  \
                                          delta_k * image[k_base+1][l_base]   #(C21)
            elif (delta_k < 0.001):                                           #(C22)
                image_val_at_p = (1 - delta_l) * image[k_base][l_base] +  \
                                          delta_l * image[k_base][l_base+1]   #(C23)
            else:                                                             #(C24)
                image_val_at_p = (1-delta_k)*(1-delta_l)*image[k_base][l_base] + \
                                 (1-delta_k)*delta_l*image[k_base][l_base+1]  + \
                                 delta_k*delta_l*image[k_base+1][l_base+1]  + \
                                 delta_k*(1-delta_l)*image[k_base+1][l_base]      #(C25)
            if image_val_at_p >= image[i][j]:                                 #(C26)
                pattern.append(1)                                             #(C27)
            else:                                                             #(C28)
                pattern.append(0)                                             #(C29)
        print("pattern: %s" % pattern)                                        #(C30)
        bv =  BitVector.BitVector( bitlist = pattern )                        #(C31)
        intvals_for_circular_shifts  =  [int(bv << 1) for _ in range(P)]      #(C32)
        minbv = BitVector.BitVector( intVal = \
                                min(intvals_for_circular_shifts), size = P )  #(C33)
        print("minbv: %s" % minbv)                                            #(C34)
        bvruns = minbv.runs()                                                 #(C35)
        encoding = None
        if len(bvruns) > 2:                                                   #(C36)
            lbp_hist[P+1] += 1                                                #(C37)
            encoding = P+1                                                    #(C38)
        elif len(bvruns) == 1 and bvruns[0][0] == '1':                        #(C39)
            lbp_hist[P] += 1                                                  #(C40)
            encoding = P                                                      #(C41)
        elif len(bvruns) == 1 and bvruns[0][0] == '0':                        #(C42)
            lbp_hist[0] += 1                                                  #(C43)
```

```
            encoding = 0                                                    #(C44)
        else:                                                               #(C45)
            lbp_hist[len(bvruns[1])] += 1                                   #(C46)
            encoding = len(bvruns[1])                                       #(C47)
        print("encoding: %s" % encoding)                                    #(C48)
print("\nLBP Histogram: %s" % lbp_hist)                                     #(C49)
```

Back to TOC

5: Characterizing Image Textures with a Gabor Filter Family

- Gabor filters are spatially localized operators for analyzing an image for periodicities at different frequencies and in different directions. To the extent that many image textures are composed of repetitively occurring micro-patterns, that makes them ideal for characterization by Gabor filters.

- You can think of a Gabor filter as a highly localized Fourier transform in which the localization is achieved by applying a Gaussian decay function to the pixels. Whereas the Gaussian weighting gives us the localization needed, the direction of the periodicities in the underlying Fourier kernel allows us to characterize a texture in that direction.

- To underscore the importance of Gabor filters for texture characterization, it is a part of the MPEG-7 multimedia content description standard. The standard specifies description formats for the different components of multi-media objects such as audio, images, and video. It is meant to be used as a complement to the MPEG-4 standard whose primary focus is the standardization of compression routines for multi-media objects. The Gabor filter based texture characterization in

MPEG-7 is meant to facilitate image search and retrieval. [The other two MPEG standards in current use are MPEG-1 and MPEG-2, both concerned primarily with the image resolution to be made available in a video service. MPEG-1, which is the oldest such standard, specifies an image resolution of 352×240 at 30 fps (frames per second). On the other hand, MPEG-2 specifies two image resolutions, 720×480 and 1280×720, both at 60 fps. The acronym MPEG stands for "Moving Picture Experts Group". MPEG is a part of ISO (International Organization for Standardization).]

- The Gabor filter as used in MPEG-7 makes 30 measurements on a texture. These consist of 6 orientations and 5 spatial frequency bands, as shown in the figure on the next page.

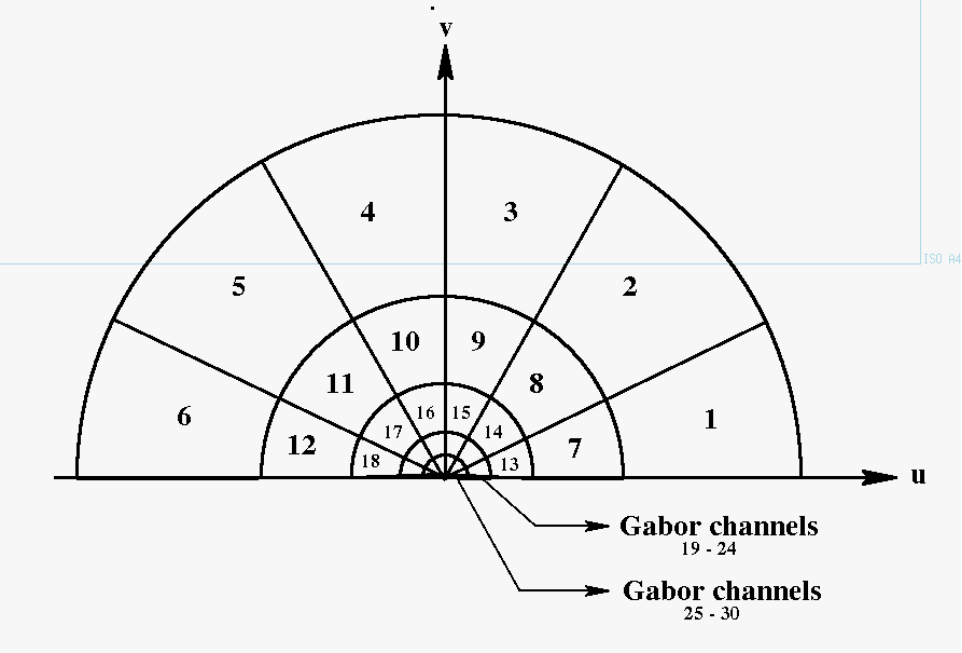


Figure 1: *Defining the Gabor texture channels for the MPEG-7 standard by dividing the spatial frequencies in polar coordinates. The Gabor filter bank in this case consists of Gabor convolutional operators at 6 orientations and 5 spatial frequencies.*

- Don't worry if you do not fully understand what is shown in Figure 1 at this time. The explanation that follows will explain what is meant by the frequencies and the frequency bands in the figure and also what we mean by the orientations of a Gabor filter.

- Suffice it to say at the moment that if f_0 is the frequency associated with the outermost band of frequencies in the figure, the other frequency band boundaries are given by

$$f_s = \frac{f_0}{2^s} \qquad\qquad s = 0, 1, 2, 3, 4 \tag{8}$$

  where $s = 0$ obviously corresponds to the outer boundary of the outermost band. Note that, except for the lowest band, each band represents an octave (since the upper limit of frequencies in each band is twice the lower limit). Here are the frequency bandwidths for the five different bands:

$$
\begin{aligned}
band1 &= \left(\frac{f_0}{2},\ f_0\right) \\
band2 &= \left(\frac{f_0}{4},\ \frac{f_0}{2}\right) \\
band3 &= \left(\frac{f_0}{8},\ \frac{f_0}{4}\right) \\
band4 &= \left(\frac{f_0}{16},\ \frac{f_0}{8}\right) \\
band5 &= \left(0,\ \frac{f_0}{16}\right)
\end{aligned}
$$

- In what follows, I will start with a brief review of the 2D Fourier transform.

Back to TOC

5.1: A Brief Review of 2D Fourier Transform

- Let's start with the expression for the 2D Fourier transform of an image:

$$G(u,v) \;=\; \int_{x=-\infty}^{\infty}\int_{y=-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}dxdy \tag{9}$$

  and its inverse Fourier transform:

$$g(x,y) \;=\; \int_{-\infty}^{\infty} G(u,v)e^{j2\pi(ux+vy)}dudv \tag{10}$$

- If we assume that our image consists of a single sinusoidal wave that exhibits a periodicity of u_0 cycles per unit length along the x-axis and v_0 cycles per unit length along the y-axis, then

$$G(u,v) \;=\; \delta(u-u_0, v-v_0) \tag{11}$$

  where $\delta(u,v)$ is a dirac delta function that is non-zero only at the point (u_0, v_0) in the (u,v)-plane. As is to be expected, the image $g(x,y)$ for such a $G(u,v)$ would be given by

$$\begin{aligned} g(x,y) &= e^{j2\pi(u_0x+v_0y)} \\ &= \cos\Big(2\pi(u_0x+v_0y)\Big) + j\sin\Big(2\pi(u_0x+v_0y)\Big) \end{aligned} \tag{12}$$

The real part (and also the imaginary part) of the expression on the right above is a two-dimensional sinusoidal wave (think of an ocean wave) in the (x, y) plane that cuts the x-axis at u-cycles per unit length and the y-axis at v cycles per unit length.

- As mentioned above, in the Fourier transform $G(u, v)$, u is the frequency of the 2D sinusoid as seen along the x-axis and v the frequency of the same along the y-axis. The frequency along the direction in the (x, y) plane along which the sinusoid changes more rapidly (in terms of cycles per unit length) is given by

$$f = \sqrt{u^2 + v^2} \tag{13}$$

and the direction θ of the maximum rate of change is given by the angle

$$\theta = arctan(v/u) \tag{14}$$

that is subtended with the x-axis. If we wish, we can express the frequency components u and v in polar coordinates:

$$\begin{aligned} u &= f * \cos(\theta) \\ v &= f * \sin(\theta) \end{aligned} \tag{15}$$

- In the polar representation of the frequencies, the formula for the Fourier transform shown at the beginning of this section can be expressed as

$$G(f, \theta) = \int_{r=0}^{\infty} \int_{\theta=0}^{2\pi} g(x, y) e^{-j2\pi f \Big(x\cos(\theta) + y\sin(\theta) \Big)} dxdy \tag{16}$$

Back to TOC

5.2: The Gabor Filter Operator

- The convolutional operator for the continuous form a Gabor filter is given by:

$$h(x, y; u, v) = \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}} e^{-j2\pi(ux+vy)} \tag{17}$$

  where u is the spatial frequency along x and v is the spatial frequency along y. That is, if you looked at only those points of $h(x, y; u, v)$ that are on the x-axis, you will see a sinusoid with a frequency of u cycles per unit length. And, if you did the same along the y-axis, you will see a sinusoid with a frequency of v cycles per unit length.

- Being a convolutional operator, we slide it to each pixel in the image f(x,y), flip it with respect to the x and the y axes, multiply the grayscale values at the pixels with the corresponding values in the above operator and integrate the products:

$$g(x, y; u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x - \alpha, y - \beta; u, v) d\alpha d\beta \tag{18}$$

- If you substitute the expression for $h(x, y; u, v)$ in the above equation — but without the exponential decay term — what

you'll get will be the same as the 2D Fourier transform (except for a phase shift at each of the frequencies).

- What that implies is that the best way to interpret the Gabor operator is to think of it as a localized Fourier transform of an image where, for the calculation at each locale in the image, you weight the pixels away from the locale according to the exponential decay factor. Obviously, the decay rate with respect to the pixel coordinates is controlled by the "standard deviation" σ

- What is interesting is that this Gaussian weighting in the space domain results in Gaussian weighting in the frequency domain. To see this, the Fourier transform of

$$h(x, y; u_0, v_0) = \frac{1}{\sqrt{(\pi)}\sigma} e^{-\frac{1}{2}\frac{x^2+y^2}{2\sigma^2}} e^{-j2\pi(u_0 x + v_0 y)} \tag{19}$$

is given by

$$H(u, v; x, y) = \frac{\sigma^2}{2} e^{-\pi\sigma^2\Big((u-u_0)^2+(v-v_0)^2\Big)} e^{j2\pi(u_0 x + v_0 y)} \tag{20}$$

- What that shows is that each Gabor operator involves a band of frequencies around the central frequency (u_0, v_0) of the filter operator. The width of this frequency band is inversely proportional to the width of the space domain operator. This should clarify by what it means to create a Gabor filter for each of the 30 "channels" shown earlier in Figure 1 for how the

Gabor filter is used in MPEG-7.

- With regard to its implementation, it is more common to express the Gabor filter operator in terms of the polar coordinates in the frequency domain.

$$h(x,y;f,\theta) \;=\; \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{1}{2}\frac{x^2+y^2}{2\sigma^2}} e^{-j2\pi f(x\cos(\theta)+y\sin(\theta))} \tag{21}$$

- To remind the reader, when we convolve an image $g(x,y)$ with this operator, at each pixel we will find the component of the localized Gaussian weighted Fourier transform at the frequency f along the direction given by θ with respect to the x axis. The form shown above can be broken into the real part and the imaginary part:

$$\begin{aligned} h_{re}(x,y;f,\theta) &= \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{1}{2}\frac{x^2+y^2}{2\sigma^2}} \cos\Big(2\pi f(x\cos(\theta)+y\sin(\theta))\Big) \\ h_{im}(x,y;f,\theta) &= \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{1}{2}\frac{x^2+y^2}{2\sigma^2}} \sin\Big(2\pi f(x\cos(\theta)+y\sin(\theta))\Big) \end{aligned} \tag{22}$$

For discrete implementation, we can represent the real and the imaginary parts as

$$\begin{aligned} h_{re}(i,j;f,\theta) &= \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{1}{2}\frac{i^2+j^2}{2\sigma^2}} \cos\Big(2\pi f(i\cos(\theta)+j\sin(\theta))\Big) \\ h_{im}(i,j;f,\theta) &= \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{1}{2}\frac{i^2+j^2}{2\sigma^2}} \sin\Big(2\pi f(i\cos(\theta)+j\sin(\theta))\Big) \end{aligned} \tag{23}$$

where i represents the row index and the j the column index. Note that we commonly think of the x-axis as being along the horizontal, going from left to right, and the y-axis as being along the vertical, going from bottom to top. However, in the discrete version, we think of the i index as representing the rows (that is, i is along the vertical, going from top to bottom, and j is along the horizontal, going from left to right. The origin in the continuous case is usually at the center of the (x, y) plane. The origin in the discrete case is at the upper left corner.

- An important issue for the discrete case is how far one should go from the origin for a given value of σ. Obviously, the smaller the value of σ, the faster the drop-off in terms of the weighting given to the pixels in the image, and, therefore, the smaller the effective footprint of the operator on the (x, y) plane. A commonly used rule of thumb for the discrete case is to let i and j cover a bounding box that goes from $-int(3\sigma)$ to $int(3\sigma)$ along the x and the y axes.

- As to purpose played by the coefficient

$$\frac{1}{\sqrt{\pi}\sigma} \tag{24}$$

its purpose is to ensure that the "energy" of the operator is unity. For the continuous case, the energy constraint is defined as

$$\int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} |h(x,y)|^2 = 1 \tag{25}$$

For the discrete case, you'd hopefully get a good approximation to the constraint with the summation shown below:

$$\sum_{i=-int(3\sigma)}^{int(3\sigma)} \sum_{j=-int(3\sigma)}^{int(3\sigma)} |h(i,j)|^2 \approx 1 \tag{26}$$

- The next subsection presents a Python script, `Gabor.py`, that first generates a Gabor filter bank and then applies it to some internally created image arrays. In that script, the basic Gabor operator is defined by the function `gabor()` that is reproduced below. As you can see, the statements in lines (B12) and (B13) correspond to the two definitions shown previously in Eq. (23).

```
def gabor(sigma, theta, f, size):                                          #(B1)
    assert  size >= 6 * sigma, \
      "\n\nThe size of the Gabor operator must be at least 6 times sigma"    #(B2)
    W =  size                                  # Gabor operator Window width #(B3)
```

```
    coef = 1.0 / (math.sqrt(math.pi) * sigma)                                   #(B4)
    ivals = range(-(W//2), W//2+1)                                              #(B5)
    jvals = range(-(W//2), W//2+1)                                              #(B6)
    greal = [[0.0 for _ in jvals] for _ in ivals]                               #(B7)
    gimag = [[0.0 for _ in jvals] for _ in ivals]                               #(B8)
    energy = 0.0                                                                #(B9)
    for i in ivals:                                                             #(B10)
        for j in jvals:                                                         #(B11)
            greal[i][j] = coef * math.exp(-((i**2 + j**2)/(2.0*sigma**2))) *\
                  math.cos(2*math.pi*(f/(1.0*W))*(i*math.cos(theta) +
                                                  j * math.sin(theta)))         #(B12)
            gimag[i][j] = coef * math.exp(-((i**2 + j**2)/(2.0*sigma**2))) *\
                  math.sin(2*math.pi*(f/(1.0*W))*(i*math.cos(theta) +
                                                  j * math.sin(theta)))         #(B13)
            energy += greal[i][j] ** 2  +  gimag[i][j] ** 2                     #(B14)
    normalizer_r = functools.reduce(lambda x,y: x + sum(y), greal, 0)           #(B15)
    normalizer_i = functools.reduce(lambda x,y: x + sum(y), gimag, 0)           #(B16)
    print("\nnormalizer for the real part: %f" % normalizer_r)                  #(B17)
    print("normalizer for the imaginary part: %.10f" % normalizer_i)           #(B18)
    print("energy: %f" % energy)                                                #(B19)
    return (greal, gimag)                                                       #(B20)
```

- In the `Gabor.py` script in the next subsection, the function shown above is used to create a bank of Gabor filters using the function shown below:

```
def generate_filter_bank(sigma, size, how_many_frequencies,
                                                    how_many_directions): #(C1)
    filter_bank = {f : {d : None for d in range(how_many_directions)}
                                 for f in range(how_many_frequencies)}      #(C2)
    for freq in range(1,how_many_frequencies):                              #(C3)
        for direc in range(how_many_directions):                            #(C4)
            filter_bank[freq][direc] = \
                gabor(sigma, direc*math.pi/how_many_directions, 2*freq, size) #(C5)
    return filter_bank                                                      #(C6)
```

- In the function shown above, note the multiplier 2 for the variable `freq` and the multiplier π/`how_many_directions`. So if we choose 3 for `how_many_frequencies`, we will be testing the periodicities at 2, 4, 6 cycles per window width. Similarly, if we choose 4 for `how_many_directions`, we will be choosing the

angles 0, $\pi/4$, $\pi/2$, and $3\pi/4$ for the directions.

- The two function defined above, `gabor()` and `generate_filter_bank()`, generate the convolutional operators presented in Figure 2 **As you can see, the rows in the figure are paired, with the fist row in each pair for the real part of the Gabor convolutional operator and the second row for the imaginary part.** The first two rows are for the spatial frequency of 2 cycles per operator width; the next two rows for 4 cycles per operator width; and the bottom two rows for 6 cycles per operator width. The four columns stand for the four spatial directions of the operator as listed in the previous bullet.

- The Gabor filter bank shown in the figure was used to measure the texture in the following 32×32 image array:

```
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
[5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0, 5, 0]
```

we get the following characterization of the texture with a Gabor filter family:

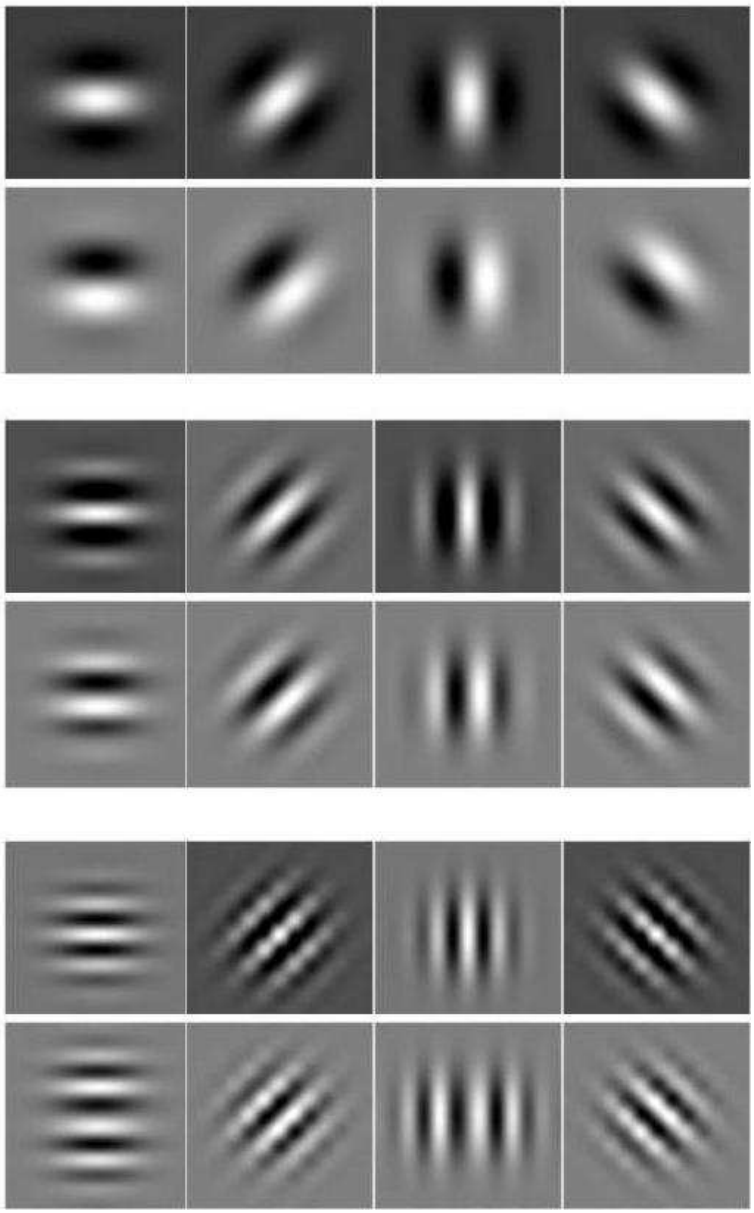


Figure 2: *This Gabor filter bank was generated by my Python script* `Gabor.py` *presented in the next subsection. See the narrative for the parameters associated with each convolutional operator shown above.*

```
Gabor filter output:

[2.7185178980, 2.802698256, 4.382719891, 2.802698252]
[0.0035060595, 0.119809025, 7.525635518, 0.119809025]
[0.0011640249, 0.004196268, 7.099408827, 0.004196268]
```

- **About the output shown, your first question is likely to be:** Since the result obtained by convolving an image with an operator is again an image (which may be of smaller size depending on how you deal with the boundary pixels), how come we are not seeing 12 different output images, with one output image for each of the 12 convolutional operators?
  Answer: Each number shown above is a summation of all the output values obtained for each of the 12 operators. To be even more precise, each convolutional operator has a real part and an imaginary part. For each operator, we convolve the input image with the real and the imaginary parts separately, sum the output values obtained, and then take the positive square-root of the sum of the squares of those two numbers.

- In the output shown above, each of the three rows corresponds to a different frequency, in terms of cycles per unit Gabor window size, and each column corresponds to a different direction of the sinusoids. The first column corresponds to the 0 radians direction — this is when the sinusoidal wave has crests and valleys parallel the horizontal axis, which would be perpendicular to how the texture is oriented in the image array. The second column corresponds to an orientation of $\pi/4$, which

again does not correspond to the orientation of the image texture under test. However, the third column does correspond to the sinusoids that have the same orientation as the image texture. Finally, the last column is for the sinusoids that are oriented at $3\pi/4$ with respect to the i axis, which again does not correspond to the image texture orientation.

- The fact that the third column has the highest values is consistent with the orientation of the texture in the image.

- As for the frequencies, the frequencies we are looking for are at 2 cycle per window width in the first row, 4 cycles per window width in the second row, and 6 cycles per window width in the third row. Since Gabor window is 13 pixels on the side and the image is 32 pixels on the side, and since we have alternating high and low pixels in the image, we can expect the frequency components to be relatively strong around 6 cycles per window width. This is borne out by the results shown above.

- Let's now see what happens to the output characterization of the texture when we rotate the image by 90 degrees, as in the array shown below:

```
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
```

```
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
```

We now get the following output:

```
[4.3827198915, 2.80269825, 2.71851789, 2.802698256]
[7.5256355186, 0.11980902, 0.00350605, 0.119809025]
[7.0994088273, 0.00419626, 0.00116402, 0.004196268]
```

- In the output shown above, recall again that the first column for the orientation of 0 radians with respect to the vertical, the second for 45 degrees, the third for 90 degrees, and the last for 135 degrees, all with respect to the vertical and going counter-clockwise. We see that the highest spectral energy is now at 0 degrees which is consistent with the new orientation of the texture.

- With regard to the rows shown above, the first row is for 2 cycles per window width, the second for 4 cycles per window width, and the third for 6 cycles per window width. Since the pattern of the alternations in the texture is the same as before (albeit its new rotation), we again expect the maximum spectral energy to be roughly around 6 cycles per window width. This is borne out by the output shown above.

- Finally, let's examine the texture characterization for the following 32×32 random array of pixels:

```
[2, 3, 3, 0, 3, 0, 5, 5, 2, 0, 1, 4, 2, 2, 4, 2, 1, 4, 1, 1, 0, 4, 3, 3, 4, 1, 1, 2, 3, 2, 1, 1]
[2, 5, 5, 5, 5, 2, 2, 2, 2, 2, 1, 0, 5, 3, 0, 4, 4, 2, 0, 3, 1, 5, 0, 4, 5, 5, 4, 1, 1, 5, 0, 1]
[5, 0, 0, 3, 3, 5, 1, 0, 3, 5, 1, 1, 5, 3, 2, 4, 1, 1, 5, 3, 4, 1, 3, 1, 0, 4, 4, 5, 2, 3, 5, 3]
[0, 4, 1, 5, 5, 4, 5, 2, 0, 1, 0, 2, 0, 3, 2, 5, 1, 4, 1, 0, 3, 5, 1, 3, 2, 0, 4, 3, 5, 5, 3, 5]
[3, 1, 1, 0, 5, 5, 5, 2, 5, 1, 0, 0, 5, 1, 2, 1, 0, 3, 2, 4, 0, 1, 0, 3, 0, 1, 5, 1, 0, 2, 4, 0]
[0, 0, 4, 5, 1, 5, 4, 3, 4, 1, 3, 1, 0, 3, 4, 5, 0, 5, 4, 0, 1, 5, 2, 2, 5, 4, 3, 1, 5, 4, 5, 1]
[1, 3, 3, 2, 1, 2, 2, 2, 1, 3, 0, 5, 5, 2, 1, 3, 3, 2, 2, 2, 3, 5, 3, 3, 2, 3, 2, 2, 5, 1, 1, 0]
[3, 5, 3, 2, 4, 0, 2, 4, 1, 4, 0, 1, 4, 3, 0, 0, 1, 2, 5, 5, 4, 4, 5, 2, 5, 3, 5, 2, 0, 1, 2, 3]
[5, 3, 1, 3, 0, 4, 5, 4, 4, 2, 4, 0, 5, 5, 3, 2, 0, 4, 2, 4, 2, 3, 1, 2, 4, 2, 1, 3, 2, 2, 0, 2]
[4, 3, 3, 2, 5, 2, 4, 0, 1, 4, 1, 2, 2, 5, 3, 5, 4, 1, 5, 2, 1, 3, 5, 4, 5, 5, 3, 0, 5, 2, 4, 5]
[0, 2, 0, 4, 0, 5, 0, 4, 0, 2, 5, 2, 2, 4, 0, 5, 5, 1, 0, 4, 5, 4, 5, 1, 4, 3, 5, 4, 2, 4, 1, 5]
[2, 0, 0, 5, 1, 0, 1, 3, 4, 3, 2, 4, 3, 4, 2, 5, 5, 0, 3, 0, 5, 3, 1, 5, 1, 0, 1, 0, 3, 4, 4, 3]
[3, 0, 0, 4, 1, 5, 3, 0, 2, 2, 5, 3, 3, 3, 4, 2, 5, 0, 1, 1, 0, 0, 2, 5, 0, 5, 4, 3, 5, 4, 2, 4]
[3, 2, 1, 3, 3, 2, 3, 2, 1, 2, 4, 2, 1, 2, 1, 3, 5, 0, 3, 2, 3, 2, 0, 3, 1, 3, 2, 4, 3, 4, 1, 1]
[4, 5, 3, 2, 3, 2, 1, 2, 0, 1, 0, 0, 1, 1, 4, 0, 1, 0, 3, 3, 4, 1, 4, 0, 2, 0, 4, 0, 2, 3, 0, 5]
[2, 1, 5, 4, 0, 1, 3, 1, 3, 2, 4, 3, 5, 4, 1, 3, 3, 4, 3, 4, 0, 5, 1, 4, 2, 5, 3, 4, 3, 5, 5, 0]
[5, 2, 3, 4, 0, 0, 2, 0, 4, 0, 1, 2, 2, 2, 3, 1, 1, 0, 1, 4, 1, 1, 4, 3, 0, 2, 3, 3, 5, 0, 4, 2]
[4, 4, 5, 3, 3, 4, 5, 1, 2, 3, 5, 5, 3, 0, 5, 0, 1, 5, 0, 5, 1, 4, 2, 3, 4, 4, 0, 0, 3, 2, 3, 3]
[3, 3, 0, 1, 1, 4, 4, 5, 5, 5, 1, 1, 3, 3, 2, 3, 2, 0, 1, 3, 0, 1, 0, 5, 2, 3, 4, 0, 3, 0, 0, 4]
[4, 5, 1, 5, 3, 2, 0, 5, 0, 4, 4, 2, 5, 5, 5, 3, 5, 4, 2, 0, 0, 1, 1, 0, 1, 2, 0, 0, 1, 4, 0, 0]
[1, 1, 0, 1, 1, 0, 3, 1, 3, 3, 1, 5, 5, 4, 0, 1, 1, 5, 4, 1, 2, 0, 2, 3, 2, 1, 4, 0, 5, 3, 2, 2]
[4, 4, 3, 0, 0, 0, 1, 1, 0, 3, 0, 4, 2, 0, 0, 0, 3, 1, 1, 2, 0, 5, 5, 3, 4, 3, 1, 1, 4, 1, 4, 2]
[5, 5, 5, 4, 5, 5, 5, 1, 5, 5, 4, 3, 2, 1, 3, 3, 2, 3, 0, 3, 5, 4, 3, 0, 0, 1, 4, 4, 2, 2, 5, 2]
[0, 4, 3, 4, 3, 0, 0, 1, 3, 5, 5, 1, 4, 5, 2, 0, 0, 3, 3, 5, 1, 3, 0, 3, 3, 5, 5, 1, 2, 5, 5, 2]
[2, 5, 4, 4, 0, 3, 4, 1, 0, 3, 1, 4, 2, 3, 4, 3, 1, 1, 0, 1, 2, 3, 5, 0, 0, 0, 5, 3, 0, 5, 0, 1]
[5, 0, 5, 5, 1, 4, 4, 1, 0, 1, 0, 1, 3, 1, 1, 1, 1, 2, 5, 4, 0, 0, 4, 4, 5, 4, 2, 0, 4, 0, 1, 1]
[2, 5, 0, 0, 1, 4, 3, 2, 5, 5, 3, 0, 3, 0, 1, 3, 3, 1, 0, 2, 5, 1, 3, 1, 5, 0, 3, 5, 3, 5, 0, 5]
[3, 1, 3, 2, 0, 3, 1, 4, 5, 1, 2, 4, 5, 3, 4, 0, 4, 2, 2, 3, 3, 0, 1, 5, 3, 1, 2, 0, 5, 0, 3, 5]
[4, 3, 1, 5, 2, 1, 4, 5, 4, 3, 0, 5, 3, 2, 0, 0, 4, 0, 5, 2, 0, 1, 0, 1, 0, 0, 0, 3, 0, 2, 4, 1]
[3, 4, 3, 0, 3, 1, 3, 2, 5, 4, 1, 1, 2, 1, 3, 4, 3, 1, 1, 1, 5, 3, 0, 2, 4, 0, 1, 0, 4, 3, 2, 5]
[4, 3, 1, 1, 0, 5, 0, 1, 0, 4, 0, 0, 4, 5, 5, 5, 2, 1, 0, 0, 3, 0, 4, 4, 0, 4, 2, 0, 1, 3, 0, 1]
[2, 1, 1, 4, 3, 4, 0, 0, 5, 1, 3, 4, 2, 3, 2, 1, 1, 5, 5, 0, 1, 4, 5, 4, 0, 0, 0, 1, 1, 0, 5, 4]
```

  The characterization for the array shown above consists of the following matrix:

```
[3.037470073, 3.01976483, 2.92454278, 2.98770835]
[1.632409589, 1.40793174, 1.51748497, 1.49434399]
[1.509071087, 1.43654545, 1.24886371, 1.69980807]
```

- About the output shown above, note first that the characterization now appears to be orientation independent. Recall again that the columns represent the orientation of the filter, with the first column representing 0 degrees and the last 135 degrees with respect to the vertical.

- With regard to the rows shown above, which represent the frequencies, we have more or less uniform energy in the frequencies at 4 and 6 cycles per window length in the second and the third rows, and elevated energies at 2 cycles per

sequence length in the first row. This would be general pattern you would see in a non-zero-mean image array — most of the spectral energy would be focused towards the lowest frequencies.

Back to TOC

5.3: Python Code for Experimenting with Gabor Filter Banks

All of the Gabor based results shown earlier were produced by the Python script shown below.

The script generates elementary textures for you and applies Gabor filter bank to the image array. You can specify the texture you want by uncommenting one of lines in (A4) through (A8).

The script allows you to set any arbitrary size for the image array and the number of grayscale values you want in the image.

Note that you must also set the size of the Gabor σ and the size of the convolutional operator in lines (A29) and (A30).

```
#!/usr/bin/env python

##  Gabor.py
##  Author:   Avi Kak   (kak@purdue.edu)
##  Date:     October 15, 2016


##  bugfix and changes on January 21, 2018:
##
##      See the fix in Line (E7) that was needed to make the code work with the more recent
##      Pillow library for PIL.
##
##      Additionally, the code should now be Python 3 compliant.


##  This script was written as a teaching aid for the lecture on "Textures and Color" as
##  a part of my class on Computer Vision at Purdue.
##
##  This Python script demonstrates how Gabor Filtering can be used for characterizing
##  image textures.

##  Just for the sake of playing with the code, this script generates five different types
```

```
##  of "textures".   You make the choice by uncommenting one of the statements in lines
##  (A4) through (A8).  You can also set the size of the image array and number of grayscale
##  values to use in lines in lines (A9) and (A10)

##  HOW TO USE THIS SCRIPT:
##
##     1.   Specify the texture type you want by uncommenting one of the lines (A4) through (A8)
##
##     2.   Set the image size in line (A9)
##
##     3.   Set the number of gray levels in line (A10)
##
##     4.   Set the size of the Gabor sigma in line (A29)
##
##     5.   Set the size of the Gabor convolutional operator in line (A30).

##  Call syntax:   Gabor.py


import random
import math
import sys, glob, os
if sys.version_info[0] == 3:
    import tkinter as Tkinter
    from tkinter.constants import *
else:
    import Tkinter
    from Tkconstants import *
from PIL import Image
from PIL import ImageDraw
from PIL import ImageTk
import functools

debug = True                                                                    #(A1)

def main():                                                                     #(A2)
    if debug:                                                                   #(A3)
        ##  UNCOMMENT THE TEXTURE TYPE YOU WANT:
#        texture_type = 'random'                                                 #(A4)
        texture_type = 'vertical'                                              #(A5)
#        texture_type = 'horizontal'                                            #(A6)
#        texture_type = 'checkerboard'                                          #(A7)
#        texture_type = None                                                    #(A8)
        IMAGE_SIZE = 32                                                         #(A9)
        GRAY_LEVELS = 6                                                         #(A10)
        image = [[0 for _ in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]     #(A11)
        if texture_type == 'random':                                            #(A12)
            image = [[random.randint(0,GRAY_LEVELS-1)
                    for _ in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]     #(A13)
        elif texture_type == 'diagonal':                                        #(A14)
            image = [[GRAY_LEVELS - 1 if (i+j)%2 == 0 else 0
                    for i in range(IMAGE_SIZE)] for j in range(IMAGE_SIZE)]     #(A15)
        elif texture_type == 'vertical':                                        #(A16)
            image = [[GRAY_LEVELS - 1 if i%2 == 0 else 0
                    for i in range(IMAGE_SIZE)] for _ in range(IMAGE_SIZE)]     #(A17)
```

```
    elif texture_type == 'horizontal':                                          #(A18)
        image = [[GRAY_LEVELS - 1 if j%2 == 0 else 0
                for i in range(IMAGE_SIZE)] for j in range(IMAGE_SIZE)]         #(A19)
    elif texture_type == 'checkerboard':                                        #(A20)
        image = [[GRAY_LEVELS - 1 if (i+j+1)%2 == 0 else 0
                for i in range(IMAGE_SIZE)] for j in range(IMAGE_SIZE)]         #(A21)
    else:                                                                       #(A22)
        sys.exit("You must satisfy a texture type by uncommenting one" +
                 "of lines (A1) through (A5).")                                 #(A23)
    print("Texture type chosen: %s" % texture_type)                             #(A24)
    print("The image: ")                                                        #(A25)
    for row in range(IMAGE_SIZE): print(image[row])                             #(A26)
else:                                                                           #(A27)
    sys.exit("Code for actual images goes here. Yet to be coded.")              #(A28)

gabor_sigma = 2.0                                                               #(A29)
gabor_size = 13           # must be odd                                         #(A30)
assert gabor_size % 2 == 1, "\n\nGabor filter size needs to be odd"             #(A31)
rowmin = colmin = gabor_size//2                                                 #(A32)
rowmax = colmax = IMAGE_SIZE - gabor_size//2                                    #(A33)
how_many_frequencies = 4                                                        #(A34)
how_many_directions = 4                                                         #(A35)
gabor_filter_bank = generate_filter_bank(gabor_sigma, gabor_size,
                           how_many_frequencies, how_many_directions)           #(A36)
directory_name = "filters" + str(gabor_size)                                    #(A37)
if os.path.isdir(directory_name):                                               #(A38)
    list(map(os.remove, glob.glob(directory_name + '/*.jpg')))                  #(A39)
else:                                                                           #(A40)
    os.mkdir(directory_name)                                                    #(A41)
for item in glob.glob(directory_name + "/*"): os.remove(item)                   #(A42)
filter_outputs = [[0.0 for _ in range(how_many_directions)]
                               for _ in range(how_many_frequencies)]            #(A43)
for freq in range(1,how_many_frequencies):                                      #(A44)
    for direc in range(how_many_directions):                                    #(A45)
        print("\n\nfilter for freq=%d and direction=%d:" % (freq,direc))        #(A46)
        print("\nop_real:")                                                     #(A47)
        op_real,op_imag = gabor_filter_bank[freq][direc]                        #(A48)
        display_gabor(op_real)                                                  #(A49)
        display_and_save_gabor_as_image(op_real,directory_name,
                                "real_freq=%d_direc=%d"%(freq,direc))           #(A50)
        print("\nop_imag:")                                                     #(A51)
        display_gabor(op_imag)                                                  #(A52)
        display_and_save_gabor_as_image(op_imag,directory_name,
                                 "imag_freq=%d_direc=%d"%(freq,direc))          #(A53)
        for i in range(rowmin,rowmax):                                          #(A54)
            for j in range(colmin,colmax):                                      #(A55)
                if debug: print("\n\nFor new pixel at (%d,%d):" % (i,j))       #(A56)
                real_part,imag_part = 0.0,0.0                                   #(A57)
                for k in range(-(gabor_size//2), gabor_size//2+1):              #(A58)
                    for l in range(-(gabor_size//2), gabor_size//2+1):          #(A59)
                        real_part += \
                          image[i-(gabor_size//2)+k][j-(gabor_size//2)+l] * \
                         op_real[-(gabor_size//2)+k][-(gabor_size//2)+l]        #(A60)
                        imag_part += \
                          image[i-(gabor_size//2)+k][j-(gabor_size//2)+l] * \
```

```
                            op_imag[-(gabor_size//2)+k][-(gabor_size//2)+l]  #(A61)
                    filter_outputs[freq][direc] += \
                         math.sqrt( real_part**2 + imag_part**2 )            #(A62)
    filter_outputs = list(map(lambda x: x / (1.0*(rowmax-rowmin)*(colmax-colmin)),
          filter_outputs[freq]) for freq in range(1,how_many_frequencies))    #(A63)
    print("\nGabor filter output:\n")                                         #(A64)
    for freq in range(len(filter_outputs)): print(list(filter_outputs[freq])) #(A65)

def gabor(sigma, theta, f, size):                                           #(B1)
    assert  size >= 6 * sigma, \
      "\n\nThe size of the Gabor operator must be at least 6 times sigma"   #(B2)
    W =  size                                  # Gabor operator Window width #(B3)
    coef = 1.0 / (math.sqrt(math.pi) * sigma)                               #(B4)
    ivals = range(-(W//2), W//2+1)                                          #(B5)
    jvals = range(-(W//2), W//2+1)                                          #(B6)
    greal = [[0.0 for _ in jvals] for _ in ivals]                           #(B7)
    gimag = [[0.0 for _ in jvals] for _ in ivals]                           #(B8)
    energy = 0.0                                                            #(B9)
    for i in ivals:                                                         #(B10)
        for j in jvals:                                                     #(B11)
            greal[i][j] = coef * math.exp(-((i**2 + j**2)/(2.0*sigma**2))) *\
                  math.cos(2*math.pi*(f/(1.0*W))*(i*math.cos(theta) +
                                                  j * math.sin(theta)))     #(B12)
            gimag[i][j] = coef * math.exp(-((i**2 + j**2)/(2.0*sigma**2))) *\
                  math.sin(2*math.pi*(f/(1.0*W))*(i*math.cos(theta) +
                                                  j * math.sin(theta)))     #(B13)
            energy += greal[i][j] ** 2  +  gimag[i][j] ** 2                 #(B14)
    normalizer_r = functools.reduce(lambda x,y: x + sum(y), greal, 0)       #(B15)
    normalizer_i = functools.reduce(lambda x,y: x + sum(y), gimag, 0)       #(B16)
    print("\nnormalizer for the real part: %f" % normalizer_r)               #(B17)
    print("normalizer for the imaginary part: %.10f" % normalizer_i)        #(B18)
    print("energy: %f" % energy)                                            #(B19)
    return (greal, gimag)                                                   #(B20)

def generate_filter_bank(sigma, size, how_many_frequencies,
                                                    how_many_directions): #(C1)
    filter_bank = {f : {d : None for d in range(how_many_directions)}
                                 for f in range(how_many_frequencies)}      #(C2)
    for freq in range(1,how_many_frequencies):                              #(C3)
        for direc in range(how_many_directions):                            #(C4)
            filter_bank[freq][direc] = \
                gabor(sigma, direc*math.pi/how_many_directions, 2*freq, size) #(C5)
    return filter_bank                                                      #(C6)

def display_gabor(oper):                                                    #(D1)
    height,width = len(oper), len(oper[0])                                  #(D2)
    for row in range(-(height//2), height//2+1):                            #(D3)
        sys.stdout.write("\n")
        for col in range(-(width//2), width//2+1):                          #(D4)
            sys.stdout.write("%5.2f" % oper[row][col])
    sys.stdout.write("\n")

def display_and_save_gabor_as_image(oper, directory_name, what_type):       #(E1)
    height,width = len(oper), len(oper[0])                                  #(E2)
    maxVal = max(list(map(max, oper)))                                      #(E3)
```

```
    minVal = min(list(map(min, oper)))                                          #(E4)
    print("maxVal: %f" % maxVal)                                                #(E5)
    print("minVal: %f" % minVal)                                                #(E6)
#    newimage = Image.new("L", (width,height), 0.0)                              #(E7)
    newimage = Image.new("L", (width,height), 0)                               #(E7)
    for i in range(-(height//2), height//2+1):                                  #(E8)
        for j in range(-(width//2),width//2+1):                                 #(E9)
            if abs(maxVal-minVal) > 0:                                          #(E10)
                displayVal = int((oper[i][j] - minVal) *
                                                 (255/(maxVal-minVal)))         #(E11)
            else:                                                               #(E12)
                displayVal = 0                                                  #(E13)
            newimage.putpixel((j+width//2,i+height//2), displayVal)             #(E14)
    displayImage3(newimage,directory_name, what_type, what_type +
                                    "  (close window when done viewing)")      #(E15)

def displayImage3(argimage, directory_name, what_type, title=""):               #(F1)
    '''
    Displays the argument image in its actual size.  The display stays on until the
    user closes the window.  If you want a display that automatically shuts off after
    a certain number of seconds, use the method displayImage().
    '''
    width,height = argimage.size                                                #(F2)
    tk = Tkinter.Tk()                                                           #(F3)
    winsize_x,winsize_y = None,None                                             #(F4)
    screen_width,screen_height = \
                            tk.winfo_screenwidth(),tk.winfo_screenheight()      #(F5)
    if screen_width <= screen_height:                                           #(F6)
        winsize_x = int(0.5 * screen_width)                                     #(F7)
        winsize_y = int(winsize_x * (height * 1.0 / width))                     #(F8)
    else:                                                                       #(F9)
        winsize_y = int(0.5 * screen_height)                                    #(F10)
        winsize_x = int(winsize_y * (width * 1.0 / height))                     #(F11)
    display_image = argimage.resize((winsize_x,winsize_y), Image.ANTIALIAS)     #(F12)
    image_name = directory_name + "/" + what_type                               #(F13)
    display_image.save(image_name + ".jpg")                                     #(F14)
    tk.title(title)                                                             #(F15)
    frame = Tkinter.Frame(tk, relief=RIDGE, borderwidth=2)                      #(F16)
    frame.pack(fill=BOTH,expand=1)                                              #(F17)
    photo_image = ImageTk.PhotoImage( display_image )                           #(F18)
    label = Tkinter.Label(frame, image=photo_image)                             #(F19)
    label.pack(fill=X, expand=1)                                                #(F20)
    tk.mainloop()                                                               #(F21)

main()                                                                          #(G1)
```

Back to TOC

6: Dealing with Color in Images

- On the face of it, learning about color and how it should be dealt with in images couldn't be simpler. You could say that all you need to know is that, typically, a color image is represented by three 8-bit integers at each pixel, one for each of the red, green, and blue channels. So one could think of a color image as three separate gray-scale images, one for each color channel.

- Unfortunately, it is not as simple as that.

- Let's say you want to apply a segmentation algorithm to a color image. Considering that you have three color values at each pixel, you have to first decide what to apply the segmentation algorithm to. Should you be applying it to each color channel separately? Or should you base your segmentation on something else entirely — perhaps an attribute that could be inferred from the color channels? [Along the same lines, suppose you want to characterize the texture in a color image. Should you be applying the texture characterization algorithm to each color channel separately? If you do, how would you combine the three characterizations? Yes, one could possibly "concatenate" the three characterizations. But might there be other ways to represent color images so that such a concatenation would become unnecessary?]

- If you are an individual who knows a bit about art, your

reaction to the comments made above is likely to be: It has been known for centuries that the human experience of color is based primarily on its hue, saturation, and brightness, and, not at all on the RGB color components per se. So you will say why not represent a color directly by its hue, saturation, and brightness attributes? [By the way, one of the acronyms, HSI, HSV, or HSL, is likely to be used for the representation of color when you are working directly with hue (H), saturation (S), and brightness (I/V/L), where I stands for intensity, V for value, and L for lightness.]

- As it turns out, in order to become competent with color in images, you really have no choice but to learn of the different ways in which color can be represented. You obviously need to know about RGB since that is important to how hardware records and displays images. In addition, you must also learn the HSI/V/L representations, since these are used extensively in software systems when it is important create effects that are important to the human experience of color. And if you are trying to come to terms with the nonlinear aspects of color vision, you must also understand the L*a*b* representation of color. [In the L*a*b* representation, L* represents lightness (conceptually the same thing as brightness), a* the red-green value, and b* the yellow-blue value. See the explanation under "Opponent Color Spaces" in Section 6.3 of this tutorial for what is meant by "red-green" and "yellow-blue" values. **The L*a*b* space has become very important for the purpose of data visualization.** It is now well known that the changes in data are best visualized if they correspond to proportional changes in the L* value in the color graphic used for visualization.]

- Just to give you a simple example of how certain computer

vision operations become simpler when you operate in the HSV color space, as opposed to the RGB color space, our goal vis-a-vis the RGB image on the left in Figure 3 is to segment out the fruitlets. The segmentation on the right was carried out in the HSV color space by accepting all pixels whose hue values are between 0° and 30°. You will learn later why hue is measured in degrees.

Figure 3: *The goal here is to segment out the fruitlets from the image on the left. The segmentation shown at right is based on just hue filtering. We accept all pixels whose hue values are between* 0° *and* 30°. *The image on the left was supplied by Dr. Peter Hirst, Professor of Horticulture, Purdue University. And the segmentation of the fruitlets on the right was carried out by invoking the function* `apply_color_filter_hsv()` *of my Python based Watershed module, Version 2.0.4.*

Back to TOC

6.1: What Makes Learning About Color So Frustrating

- Human perception of color is so complex and so nonlinear that it cannot be described fully by any single computational model — certainly not by any single linear model.

- Considering the nonlinearities involved, it is indeed amazing how much mileage we have gotten out of the RGB and the closely related HSI, HSV, HSL, etc., models. All these models can be thought of as linear approximations to a nonlinear phenomenon. [For an example of the nonlinearity of our color vision: When you darken the color orange, you get the color brown. However, when you darken the color blue, it remains blue.] Attempts at coping with the nonlinear aspects of color vision have resulted in color spaces such as L*a*b*. These color spaces are based on the **Opponent Color Modeling** of our color vision. On the other hand, RGB, HSI/V/L, etc., are based on the **Trichromatic Model** of our color vision.

- In addition to having to deal with the above mentioned modeling complexities, what makes learning about color so frustrating for an engineer are the nature of approximations that go into the computational models. Here is a case in point: You will see frequent references to the RGB *cube* for the

representation of color. This representation obviously implies that the R, G, and B components of color are orthogonal in some sense. But are they really orthogonal in any sense at all?

- As it turns out, we assume the RGB cube to be orthogonal because that makes it easier to derive formulas for transformation between different types of color spaces. This assumption also makes it easier to visualize the three primary color components (R, G, and B) when talking about colors. However, as you will see later on in this tutorial, in a more "absolute" representation of color — known as the X,Y,Z space — the RGB representation is definitely non-orthogonal.

- Once you realize that the orthogonality of the RGB cube is really just for convenience, you are left to wonder how much trust you should place in the color-space transformation formulas that are based on tilting the cube in a vector space and then deriving equations for hue, saturation, and intensity from the tilted cube. [You even wonder as to why you should believe that the colors on the surface of the tilted cube are the purest (meaning the most saturated) hues.]

- If you are an engineer trying to understand color, another challenge you face is wrapping your head around what are known as *spectral colors*. These are colors that would be produced by, say, a tunable laser. You quickly find out that spectral colors are just a very small subset of all the colors that humans are capable of experiencing. In general, the hue

attribute of the human experience of color is based as much on the shape of the spectrum of wavelengths in the light incident on the eye as it does on the individual frequencies, but you never get a good sense of precise nature of this dependence. [If we optically combine the light emitted from a pure red laser with the light emitted from a pure blue laser, a human observer would perceive this combined light as magenta. On the other hand, if we could produce a spectrally pure light at a wavelength halfway between blue and red, that light would not be perceived as magenta. In other words, both the average wavelength and the shape of the spectral distribution are important for characterizing hue.]

- Another source of frustration in learning about color is the importance of ratios in almost every aspect of color. It is generally believed that the sensation of color that we experience is determined primarily by the proportion of each of the three primary colors in the light incident on the eye. [These would generally be the officially formulated X, Y, and Z components that are deemed to be abstract. These could also be the R, G, and B components] As you will see later in this tutorial, this implies that the experience of color is described mostly by the points *on a single plane* inside an XYZ or the RGB cube. Presumably, the colors on this plane are the different possible hues. But it is not clear as to what extent folks such as artists would agree with that. [On account of its importance, this plane is called the ***Chromaticity Plane.*** Since all points on a plane possess only two degrees of freedom, everything conveyed by the chromaticity plane can also be conveyed by its projection on either the XY, or the YZ, or the XZ plane. Such a projection is referred to as the ***Chromatic Diagram.***]

- Last but not the least is the great difficulty of making reliable

measurements of color if the goal is for these measurements to reflect the "true" colors of the object surfaces in a scene.

- The color that is captured by a camera depends significantly on three factors: (1) the spectral composition of the illumination; and (2) the light reflection properties of the object surfaces (some surfaces are more specularly reflecting and others more diffuse); and (3) the "true" color of the object surface (which depends on what portion of the light spectrum is absorbed by the surface). **If you are looking at a scene that you are very familiar with, your brain will do a great job of compensating for the confounding effects of these three effects. However, a computer vision algorithm must deal with all of the complexities created by these effects.** The guidance that the literature offers on how to address these effects is entirely much too ad hoc.

- Finally, here are some additional sources of frustration when wrapping your head around the literature on color: The words "color" and "hue" are often used interchangeably, especially in explanations of what is depicted by Chromaticity Planes and Chromaticity Diagrams. Whereas the word "chroma" in the context of HSV and HSL spaces stands for basically the same thing as saturation, the closely related word "chromaticity" is more about depicting hues than saturation.

- As to why some of the issues I have mentioned above are

frustrating will become clearer in the sections that follow. Note that the color related discussion in the rest of this tutorial draws significantly from the following sources:

1. Margaretha Schwarz, Lynne Grewe, and Avi Kak, "Representation of Color and Segmentation of Color Images," *Purdue University Technical Report,* 1-1-1994. Paper 171, January 1994.
`http://docs.lib.purdue.edu/ecetr/171`

2. Lynne Grewe and A. C. Kak, " Interactive Learning of a Multi-Attribute Hash Table Classifier for Fast Object Recognition," *Computer Vision and Image Understanding,* pp. 387-416, Vol. 61, No. 3, 1995.

3. Jae Byung Park and Avinash C. Kak, "A New Color Representation for Non-White Illumination Conditions," *Purdue University Technical Report,* 6-1-2005, Paper 8, June 2005.
`http://docs.lib.purdue.edu/ecetr/8`

Back to TOC

6.2: Our Trichromatic Vision and the RGB Model of Color

- Although hue, saturation, and brightness describe naturally the different aspects of color sensation experienced by humans, they are not always useful for designing engineering systems.

- From an engineering perspective, a more useful description of color is inspired by the fact that the retina of the human eye contains three different types of photo receptors, with each type possessing a different spectral sensitivity pattern. [These photo receptors are commonly called cone cells.] By spectral sensitivity pattern I mean the response of a photo receptor to incoming monochromatic light at different wavelengths. The fact that the eye contains three different types of cone cells is referred to as the **trichromaticity property** of the human visual system.

- The three spectral patterns — designated S, M, and L, which stand for "Short", "Medium", and "Long" wavelengths — for the three types of photo receptors are shown in Figure 4. [As you can tell from the figure, these three spectral responses do not correspond to any particular colors as we experience them. The L photo receptors are just as sensitive to monochromatic green as to monochromatic yellow. The M photo receptors are primarily sensitive to green, but their sensitivity to red is not insignificant. The S

photo receptors are sensitive mostly to monochromatic blue. In the past, L, M, and S photo receptors have been loosely associated with R (for red), G (for green), and B (for blue).] The three spectral responses shown in the figure have commonly been denoted $h_R(\lambda)$, $h_G(\lambda)$, and $h_B(\lambda)$ in the literature.

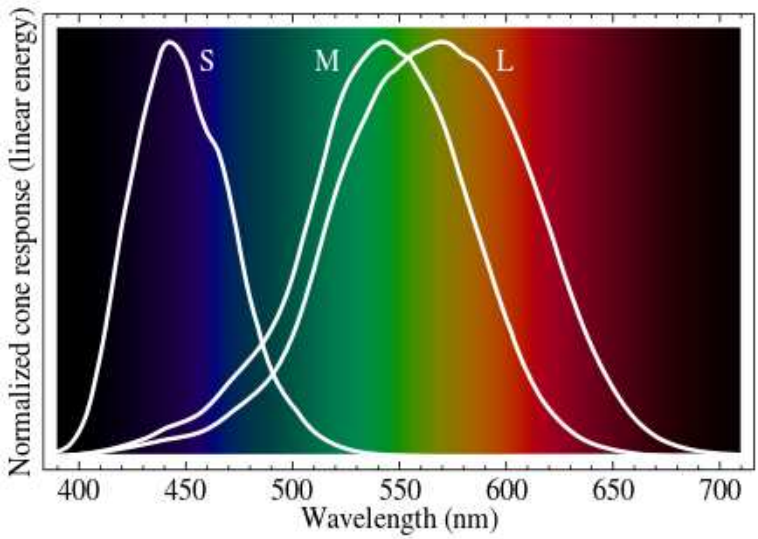


Figure 4: *The retina of the human eye contains three different types of photo receptors for color vision. The spectral responses of the three types are shown here. This figure is from the Wikipedia page on "Color Vision".*

- Now assume for a moment that our goal is to design a vision sensor that would help a backend processor perceive colors the way the humans do. Ignoring for a moment what happens at the higher levels in the brain, the trichromaticity of our visual system would imply that if we could embed in the sensor three

optical filters whose spectral properties are the same as $h_R(\lambda)$, $h_G(\lambda)$, and $h_B(\lambda)$, all that the sensor would need to do would be to generate the following three numbers at each pixel (since, presumably, that is what the cone cells in the retina do):

$$\begin{aligned} R &= \int f_{ill}(\lambda) f_{ref}(\lambda) h_R(\lambda) d\lambda \\ G &= \int f_{ill}(\lambda) f_{ref}(\lambda) h_G(\lambda) d\lambda \\ B &= \int f_{ill}(\lambda) f_{ref}(\lambda) h_B(\lambda) d\lambda \end{aligned} \tag{27}$$

where $f_{ill}(\lambda)$ is the spectral composition of the illumination and $f_{ref}(\lambda)$ the spectral definition of the surface reflectivity in the direction of the camera.

- Unfortunately, this logic for mimicking the human visual system is not practical since it is not possible to create optical filters with exactly the same spectral responses as for the three types of cone cells in the retina. [And, even if we could, there would remain the issue of mimicking what the human brain does with the RGB values thus generated — since the higher level processing of color in the visual cortex of the brain remains largely a mystery.]

- Since it is not practical to design optical filters with the kinds of responses shown in Figure 4 , approximations are necessary. The common approximation consists of using filters with "single line" responses for the three filters. [That typically means the R filter passes

through light at or in close vicinity to the 700 nm wavelength, the wavelength of the pure red hue. Similarly, the G filter passes through light at or in close vicinity to the 545 nm wavelength, the wavelength of the pure green hue. And, the B filter passes through light at or in close vicinity to the 480 nm wavelength, the wavelength of the pure blue hue.] Mathematically, this commonly used engineering approximation is tantamount to saying that the filter functions $h_R(\lambda)$, $h_G(\lambda)$ and $h_B(\lambda)$ in Eq. (27) are replaced by the Dirac Delta functions $\delta(\lambda - \lambda_R)$, $\delta(\lambda - \lambda_G)$, and $\delta(\lambda - \lambda_B)$, respectively.

- The primary justification for the above mentioned approximation — the approximation of using "single-line" filters as opposed to the filters whose spectral responses would be akin to those of the cone cells in our retina — goes like this: Say we use a vision sensor based on the above approximation to represent a given color by three numbers R, G, and B. Now suppose we mix three spectrally pure light beams, one red, one green, and one blue, in the same *proportion* as the numbers R, G, and B. If a human observer sees this composite beam, the color experienced by the human would be *roughly* the same as the color in the original light beam on the vision sensor. The words most open to questioning in this justification are *proportion* and *roughly*; the former implies using only two degree of freedom in the synthesis of the composite beam and the latter is open to any interpretation.

- As it turns out, a number of spectrally pure colors corresponding to different wavelengths cannot be synthesized by

mixing the pure R, G, and B components. This fact is made evident by Figure 5 where it is shown how much red, how much green, and how much blue must be mixed in order to generate a spectrally pure color at a given wavelength. The significance to be associated with the wavelength band where the $\bar{r}(\lambda)$ function becomes negative is most interesting: No spectrally pure color in this band can be produced by any combination of red, green, and blue. The negative values for $\bar{r}(\lambda)$ mean that if we were to combine the red hue in proportion equal to the negative value of $\bar{r}(\lambda)$ to the green and the blue hues in proportion to the positive values of $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$, respectively, we would obtain a color that, as seen by the eye, would be the same as the spectrally pure color at that wavelength. This would obviously be impossible since you cannot have a negative proportion of a color component.

- Spectrally pure colors, although they represent only a fraction of all possible colors, are obviously of engineering significance since they can be generated with relative ease in a laboratory for measurement and standardization. Since, as explained in the previous bullet, it is not possible to generate even all the spectrally pure colors by mixing red, green, and blue, these three colors can really not be thought of as being *true primary colors*. [**We refer to a set of three colors as the *true* primary colors if all other colors that are normally seen by humans can be synthesized by mixing them.**]

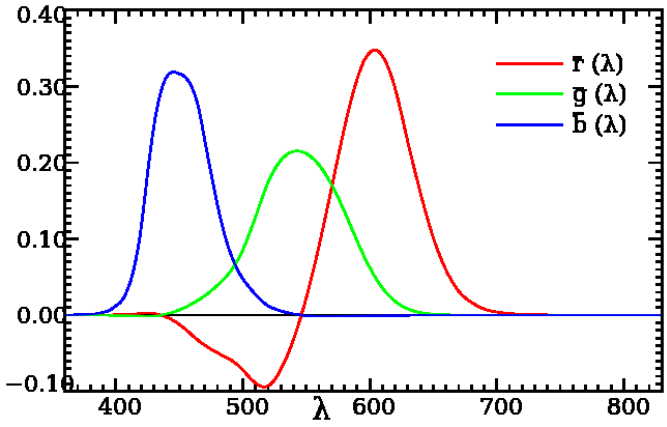


Figure 5: *This figure shows how much red, how much green, and how much blue is needed at each wavelength to match the spectral color at that wavelength. Obviously, the wavelengths where the curve goes negative means that it is not possible to match those spectral colors with a combination of R, G, and B. This figure is from the Wikipedia page on "CIE 1931 color space".*

- It is the above stated inability to generate even the spectral colors from R, G, and B that has led to the designation of three "abstract" colors, denoted X, Y, and Z, as the primary colors. All spectral colors in the visible band can be generated by a mixture of the XYZ primaries. To repeat for emphasis, only the spectrally pure colors can be generated by combining the X, Y, Z primary colors. It may **not** be possible to generate a non-spectral color, such as magenta, by combining the X, Y, Z primaries.

Back to TOC

6.3: Color Spaces

- So far I have mentioned three different types of color spaces: RGB, HSI/V/L, and XYZ. Each of these color spaces amounts to representing color by a point in three dimensions.

- A color space may also be two dimensional, which may come as a surprise since color is inherently three dimensional. A prime example of a 2D representation of color is the Chromaticity Space. 2D representations of color, as in a Chromaticity Space, are intended to show only the range of hues that can be generated by a given trio of primary colors.

- In the rest of this section, I'll first explain the idea of the Chromaticity Space. Subsequently, I'll discuss the relationship between different types of 3D color spaces.

The Chromaticity Space:

- As I mentioned previously, the XYZ space does a good job of representing spectrally pure colors. In the XYZ space, an arbitrary spectral color C may be represented as

$$C = X\vec{X} + Y\vec{Y} + Z\vec{Z} \tag{28}$$

where X, Y, Z are the components of the color along the three primaries that are symbolically represented by $\vec{X}$, $\vec{Y}$ and $\vec{Z}$.

- If the spectral distribution $P(\lambda)$ of a color C is known, the components X, Y, and Z may be found from

$$\begin{aligned} X &= k\int P(\lambda)\bar{x}(\lambda)d\lambda \\ Y &= k\int P(\lambda)\bar{y}(\lambda)d\lambda \\ Z &= k\int P(\lambda)\bar{z}(\lambda)d\lambda \end{aligned} \tag{29}$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are functions of the wavelength as shown in Figure 6, and k is a value that depends on the device. For example, a self-luminous object like a CRT has a $k = 680$ lumen/watt. Using vector notion, we may therefore say

$$C = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{30}$$

- I'll now introduce the notion of the Chromaticity Space. Representation of color in chromaticity space is based on an

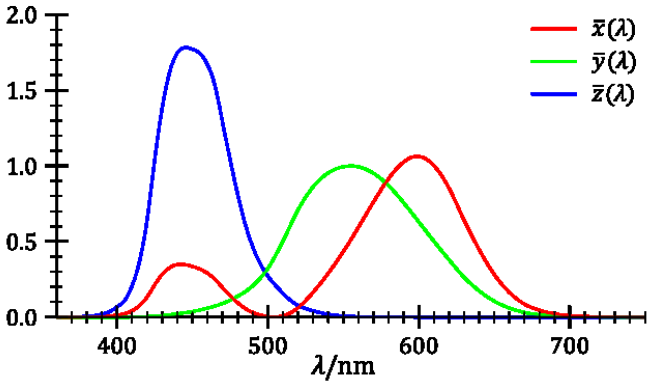


Figure 6: *This figure shows how much of each of the primaries X, Y, and Z it takes to produce a spectral color. This figure is from the Wikipedia page on "CIE 1931 color space".*

empirically verifiable premise that a large number of colors can be produced by mixing the X, Y, and Z *in just the right proportions, without regard to their absolute values.* To make this notion more precise, consider the following normalized versions of X, Y , and Z values in the XYZ space:

$$\begin{aligned} x &= \frac{X}{X+Y+Z} \\ y &= \frac{Y}{X+Y+Z} \\ z &= \frac{Z}{X+Y+Z} \end{aligned} \tag{31}$$

- With the normalization shown above, the lowercase x, y, and z

will always obey the following constraint:

$$x + y + z = 1 \tag{32}$$

- What is of engineering significance here is that as long as X, Y and Z are in the same ratio vis-a-vis one another, the values of x, y and z will remain unchanged and the constraint shown above will be satisfied.

- To give the reader a greater insight into the transformation from XYZ to xyz, consider the following two dimensional case:

$$\begin{aligned} u' &= \frac{u}{u+v} \\ v' &= \frac{v}{u+v} \end{aligned} \tag{33}$$

  Just by substitution, the reader can easily verify that for all the points along the line OA in the (u, v) space in Figure 7, the corresponding u' and v' values will be at A'. Similarly, all the points on the line OB will be mapped to the point B', and so on. [In other words, the entire positive quadrant in the (u, v) plane will be mapped to the line PQ. Mathematically, this fact is inferred readily from the observation that $u' + v'$ must equal 1 for all mapped points.]

- Extending our two dimensional example to the three dimensional case of XYZ (Figure 8), we see that for all the

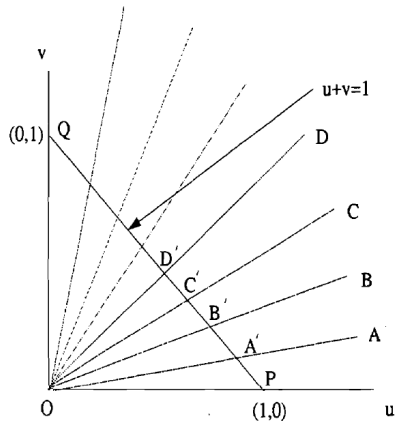


Figure 7: *This figure shows how a normalization of the coordinates in a 2D plane can map all the first quadrant points to the line PQ.*

values of X, Y, and Z, the value of x, y, and z will be confined to the planar surface denoted by P, Q and S in Figure 8. *This plane is referred to as the Chromaticity Plane.* The projection of this planar surface on one of the orthogonal planes constitutes the *Chromaticity Diagram*. Usually, the projection on the XY plane is taken.

- Remember the main justification for using x, y and z is that a large majority of at least the pure spectral colors can be produced simply by maintaining the right proportionalities between the three constituents X, Y, and Z. Translating into our diagram of Figure 8, that means that each point on the planar surface bounded by P, Q, and S corresponds to some color. The colors at the different points of this plane are shown in Figure 9 in the form of a Chromaticity Plane.

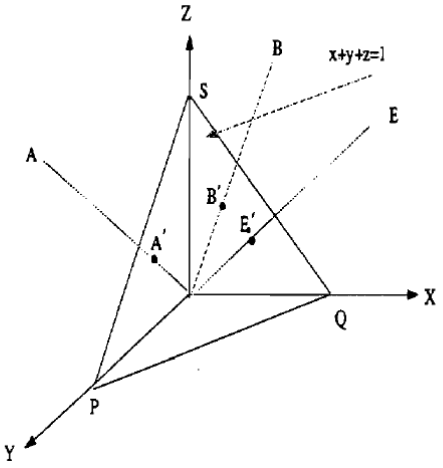


Figure 8: *The Chromaticity Plane is formed by mapping all of the first quadrant values in the XYZ space to the plane defined by* $X + Y + Z = 1$.

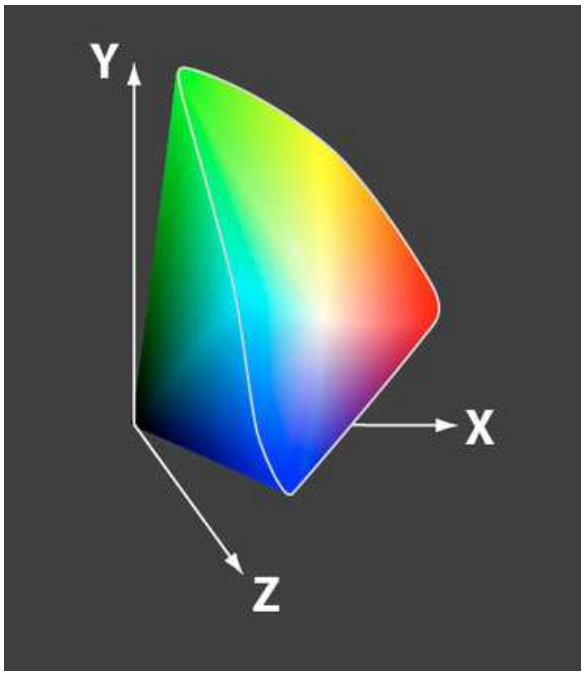


Figure 9: *The Chromaticity Plane representation of color. This figure is from http://www.iotalartiste.com/demarche.html*

- When these color on a Chromaticity Plane are projected on to the XY plane, we get the corresponding Chromaticity Diagram, as shown in Figure 10. This figure points to the main use of the color depiction through Chromaticity Planes or Diagrams. The triangle that is shown inside the overall rounded cone corresponds to the sRGB color space. [sRGB is the RGB color space as standardized by CEI. The prefix letter "s" stands for "standardized".] The colors that are outside the sRGB gamut, while discernible to humans, cannot be reproduced by typical electronic displays. [By gamut, we mean the colors that can be created through a linear mixture of R, G, and B.] The colors at the periphery of the large rounded cone are the pure spectral colors. So if you could tune the wavelength of a laser, you would move around the rounded perimeter shown in the diagram.

- The boundary colors in the chromaticity depictions of the sort in Figures 9 and 10 correspond to those colors that can be produced by spectrally pure light. For example, if we were to take a tunable laser and change its wavelength continuously from one end of the visible spectrum to the other, as shown in Figure 10 we would obtain colors corresponding to a clockwise traversal of the boundary of the large rounded cone. So, the interior colors of this cone correspond to those that would require combining laser beams of different colors.

- Figure 11 again shows the Chromaticity Plane conceptually in the positive quadrant of the XYZ space. Note that this time we

Figure 10: *Projection of the Chromaticity Plane on the XY-plane gives us the Chromaticity Diagram. The triangle in the middle is the sRGB color gamut that most electronic displays are capable of displaying. The larger rounded cone is what the human eye can see. The colors on the periphery of the large rounded cone are the spectral colors. As you change the wavelength of a monochromatic light beam over the entire spectral range visible to humans, you will move along the rounded perimeter shown in the diagram. The arc in the middle shows what is known as the Planckian Locus. The numbers shown along the arc are the temperatures to which you must raise an incandescent black body for it to emit the light at that point in the diagram.*

have roughly marked the points on the plane that would correspond to the **pure red, green, and the blue** hues.

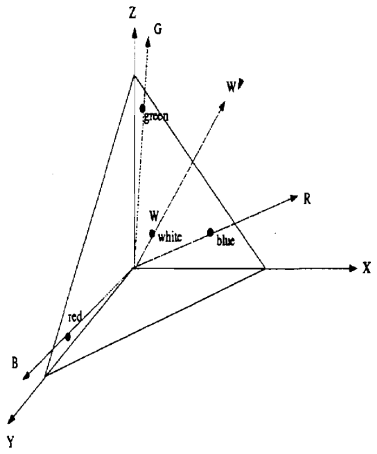


Figure 11: *Chromaticity Plane that shows the locations of red, green, and blue on the plane.*

- With reference to the Chromaticity Planes in Figures 9 , 10, and 11, it stands to reason that a mixture of any two colors that can be represented by points on the chromaticity plane will lie on a line joining those two points. Therefore, it follows that any mixture of red, green, and blue will result in a color that will correspond to a point in the triangle formed by the red, green, and blue points. [As mentioned earlier, such a triangle is called a color gamut.] It should therefore be clear from Figures 9 , 10, and 11 that a vision sensor or display device that represents colors by mixtures of red, green, and blue will certainly not cover all the colors on the chromaticity plane.

RGB Space:

- Commonly used hardware devices for capturing and displaying color images use the RGB representation, meaning that each color is expressed by a mixture of red, green, and blue.

- Strictly speaking, as made clear by Figures 10 and 11, the positive "quadrant" of the RGB space is a subset of the positive quadrant the XYZ space. This fact is made clear by Figure 11 where we have drawn the X, Y, and Z vectors in an orthogonal frame and the vectors corresponding to R, G, and B are then shown by drawing lines through the corresponding points on the chromaticity plane. [Theoretically speaking, the three RGB vectors span the same space as the XYZ vectors. However, since colors can only be made from positive mixtures, the positive "quadrant" of the RGB space is a subset of the positive quadrant of the XYZ space.]

- The R, G, and B components of color are typically expressed as 8-bit integers. That is, the value of each color component is an integer in the range $[0, 255]$.

- Figure 12 shows the colors on the surface of the RGB cube as the cube is viewed inwards from the positive quadrant. [It may seem like a contradiction that while the colors white, red, green, and blue are coplanar in Figure 11, they are non-coplanar in the RGB cube of Figure 12. This contradiction is not real for the following reason: In the chromaticity depiction, every color is forced to lie on the chromaticity plane of Figure 11 whether or not it really does. In other

words, *the chromaticity coordinates only give us the relative proportions of the primaries in a mixture but not their true values.* So, it may well be that to produce the color white we may have to be at the point W', but, as far the chromaticity depiction is concerned, the color white will be represented by the point W.]

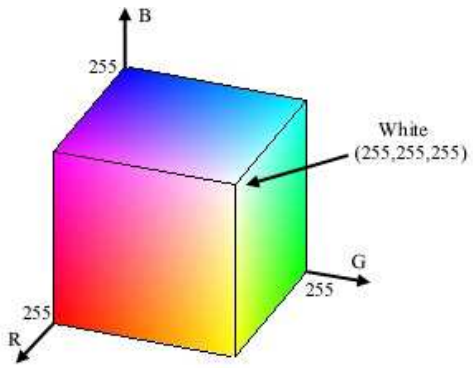


Figure 12: *The RGB space with an orthonormal representation of the three primary colors. This figure is from http://radio.feld.cvut.cz/matlab/toolbox/images/color4.html*

- Although not directly useful in everyday color processing, one should note that linear transformations are available that readily convert from XYZ values into RGB values and vice versa. The relationship between the XYZ space and the RGB space can be expressed as :

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.618 & 0.177 & 0.205 \\ 0.299 & 0.587 & 0.114 \\ 0.000 & 0.056 & 0.944 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{34}$$

- The reader might wonder that if a linear transformation can convert any XYZ value into a corresponding RGB value, and if it is possible to generate all pure spectral colors by mixing XYZ primaries, why it is not possible to generate all such colors with mixtures of R, G, and B. **The answer is that there is no guarantee that such transformations would yield positive weights for the RGB colors.**

HSI Space:

- We have already mentioned the HSI (Hue, Saturation, Intensity) scheme for representing colors. The HSI coordinates are cylindrical, with H being represented by the azimuthal angle measured around the vertical axis, S by the outward radial distance, and I the height along the vertical (Figure 13).

- As you would expect, $[0.0, 360.0]$ in degrees is the value range for H. And the value range is $[0.0, 1.0]$ for both S and I.

- The red hue is placed at $H = 0°$, the green at $H = 120°$, and the blue at $H = 240°$. The point $I = 1$ on the vertical axis corresponds to pure white, and the origin to the black color since the intensity will be zero there.

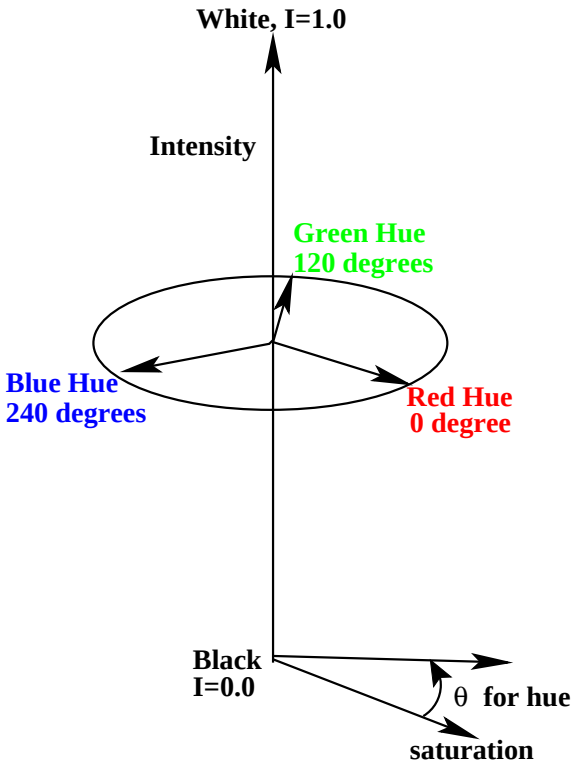


Figure 13: *The HSI space*

- Since the HSI coordinates are more intuitive to humans and since RGB is what is used by most digital devices for image capture and display, it is important to establish a transformation between the two spaces.

- To set up a transformation between RGB and HSI, we line up the diagonal of the the RGB cube in Figure 12 with the vertical axis of Figure 13. We obviously want the $(255, 255, 255)$ point of the RGB cube to become coincident with the $I = 1$ point on the vertical axis of the HSI cylindrical representation since these two points in their respective spaces represent the color white (Figure 14). [IMPORTANT: When aligning the RGB cube with the HSI coodinates, the R, G, and B values are mapped to the $[0, 1]$ interval from their original $[0, 255$ value range.]

- In Figure 14, the rotational orientation of the RGB cube is such that the red axis of the cube is in same azimuthal plane (the $H = O^\circ$ plane) that contains the red hue in HSI. This will automatically cause the green axis of the RGB cube to fall on the azimuthal plane corresponding to $H = 120^\circ$. Recall that the point at $H = 120^\circ$, $S = 1$, and $I = 1$ designates green in the HSI system. Similarly, for the blue axis of the RGB frame.

- One may now write down the transformation equations that take a color point from RGB to HSI and vice versa. To facilitate the derivation of these equations, let's momentarily assume that the HSI system is embedded in a Cartesian frame whose axes

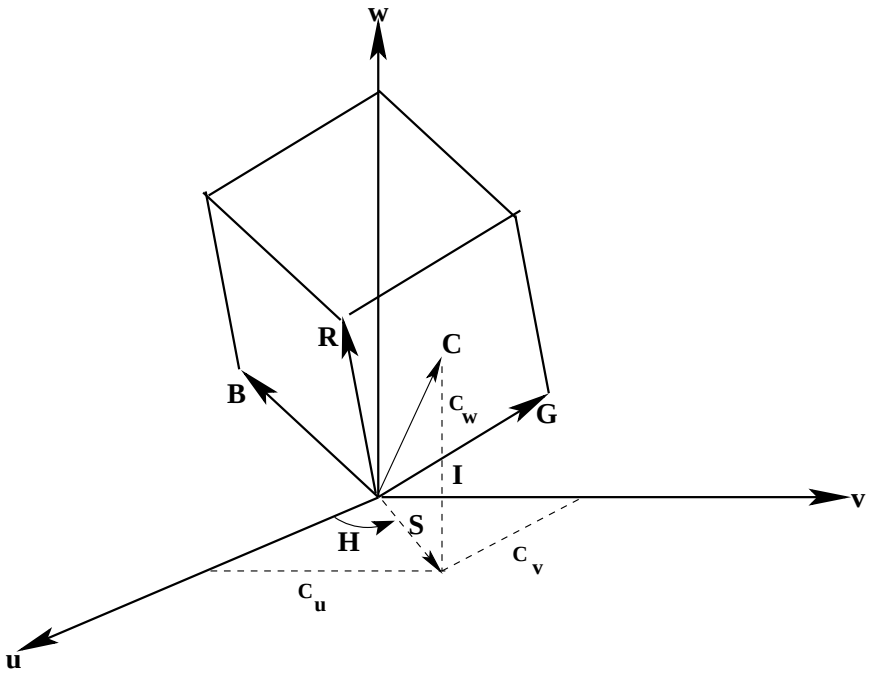


Figure 14: *The RGB cube is tilted and rotated in the manner shown here for deriving a relationship between RGB and HSI coordinates. The longest diagonal of the cube becomes aligned with the vertical Intensity (I) axis in HSI. The Hue (H) coordinates of HSI is measured counterclockwise from the projection of the RGB cube's red corner on the horizontal plane. Saturation (S) is the radial distance from the vertical axis.*

are labeled U, V and W, as shown in Figure 14. Consider now an arbitrary color point C, whose projections on the U, V, W axes are given by C_u, C_v, and C_w. Clearly, then the HSI values associated with this color point are given by

$$\begin{aligned} H &= \tan^{-1}\frac{C_v}{C_u} \\ S &= \sqrt{C_u^2 + C_v^2} \\ I &= C_w \end{aligned} \tag{35}$$

- To express the transformation from RGB to HSI, all we now need do is express an arbitrary point C in the RGB points in the UVW frame. Let the coordinates of this point in the RGB frame be given by R, G, and B and the corresponding coordinates in the UVW frame by C_u, C_v, and C_w. [IMPORTANT: In the formulas here, we assume that the value of R, G, and B are normalized to the $[0, 1]$ range.] Then,

$$\begin{aligned} C_u &= \sqrt{\frac{2R - G - B}{6}} \\ C_v &= \sqrt{\frac{G - B}{2}} \\ C_w &= \frac{R + G + B}{3} \end{aligned} \tag{36}$$

- Since the arctan in Eq. (35) for H is a multivalued function, the value of H needs to be specified more precisely. Ambiguities are resolved by using the following set of formulas for H:

$$\begin{aligned} H &= \cos^{-1}\left(\frac{C_u}{\sqrt{C_u^2 + C_v^2}}\right) \qquad if\ G >= B \\ &= 360\ -\ \cos^{-1}\left(\frac{C_u}{\sqrt{C_u^2 + C_v^2}}\right) \qquad if\ G < B \end{aligned} \tag{37}$$

- Since there is no guarantee that the computed value for S will not exceed 1, it is normalized as follows:

$$S\ =\ \frac{S}{S_{max}} \tag{38}$$

which results in the following equation for S :

$$S\ =\ 1\ -\ \frac{3 \cdot min(R,G,B)}{R+G+B} \tag{39}$$

Although useful in an approximate sense, there are serious conceptual shortcomings in the above derivation of the transformation. First and foremost, we assumed the RGB space to be orthogonal. As mentioned earlier, while the R, G, and B vectors may be linearly independent in the XYZ space, they are certainly not orthogonal. Their orthogonality in Figure 12 was meant simply to make easier the visualization of color distributions.

- So, strictly speaking, to derive a transformation between the RGB space and HSI space, one must derive transformation equations that relate the RGB vectors, as shown in Figure 11, with the HSI space as shown in Figure 13, the transformation being controlled on the one hand by the alignment of the white points in the two spaces and, on the other, by the requirements that the R, G, and B vectors in Figure 11 pass through the the designated points for the three colors in the HSI cylinder in Figure 13. Such a transformation would be very complex for obvious reasons. However, it is possible to make reasonable approximations.

- The goal of what I have described so far was to give the reader a sense how one can set up a connection between the RGB and the HSI color spaces. Using this reasoning, along with some refinements related to explicitly factoring in the piecewise continuous nature of the boundary of the RGB cube, I'll show below the transformation equations that have actually been implemented in several software libraries: [These and other transformations like the one shown below are taken from `http://dystopiancode.blogspot.com` with some modifications by me to reflect how they are actually implemented in code.]

- In the RGB to HSI transformation shown below, the input consists of a triplet of values (R, G, B), with each color component expressed as a floating point number in the $[0, 1]$ range. And the output of the transformation is a triplet of values (H, S, I), with H expressed in degrees.

$$
\begin{aligned}
M &= \max(R,G,B) \\
m &= \min(R,G,B) \\
c &= M - m \\
I &= \frac{R+G+B}{3} \\
H &= \begin{cases} 60\Big(\frac{G-B}{c} \; mod \; 6\Big) & if \; M = R, \; c \neq 0 \\ 60\Big(\frac{B-R}{c} + 2\Big) & if \; M = G, \; c \neq 0 \\ 60\Big(\frac{R-G}{c} + 4\Big) & if \; M = B, \; c \neq 0 \\ 0.0 & if \; C = 0 \end{cases} \\
S &= \begin{cases} 0 & if \; c = 0 \\ 1 - \frac{m}{l} & if \; c \neq 0 \end{cases}
\end{aligned}
\tag{40}
$$

- Note that when the three color components, R, G, and B are equal, you are supposed to get a pure gray point on the vertical axis of the HSI coordinate system. That's what the formulas shown above do. In such cases, c is zero. That causes H to be set to 0, S to also be set to 0, and I to be set to one of the R, G, B values (since they will all be the same). Yes, the point $H = 0^\circ$ means pure red. However, that is not an issue since it is at a radial distance of 0 from the vertical axis. Also note that using the condition $c \neq 0$ in the formulas for H protects us against division by 0 when all three color components are the same.

- Shown below are the formulas that take you from HSI to RGB. In these formulas, we represent a point in the HSI space by the triplet (H, S, I). The variable h_r stands for the value of H in

radians (as opposed to degrees) and the variable h_k is the corresponding angle offset in each of the 120° segments of the hue circle. As you know from the previous discussion, the 0° point for H corresponds to pure red, the 120° point to pure green, and the 240° point to pure blue. In terms of radians these points are at 0, $2\pi/3$ and $4\pi/3$ radians. [Strictly speaking, I should not have shown the expression for calculating the three variables (x, y, z) in a single line — since the calculation for z requires that you first have calculated the values for x and y.] The input to the transformation shown below consists of a triplet of values (H, S, I), with H expressed in degrees. And the output of the transformation consists of a triplet of values (R, G, B) with each color component expressed as a floating point value in the $[0, 1]$ range.

$$
\begin{aligned}
h_r &= 0.0174532925 \times H \\
h_k &= \begin{bmatrix} h_r & if\ 0 \leq h_r \leq \frac{2}{3}\pi \\ h_r - \frac{2}{3}\pi & if\ \frac{2}{3}\pi < h_r \leq \frac{4}{3}\pi \\ h_r - \frac{4}{3}\pi & if\ \frac{4}{3}\pi < h_r < 2\pi \end{bmatrix} \\
(x, y, z) &= \left(\frac{1-S}{3},\ \frac{1 + S\cos(h_k)}{3\cos(\frac{\pi}{3} - h_k)},\ 1 - (x+y) \right) \\
(x', y', z') &= \Big(3 \times I \times x,\ \ 3 \times I \times y,\ \ 3 \times I \times z \Big) \\
(R, G, B) &= \begin{bmatrix} (I, I, I) & if\ S = 0 \\ (y', z', x') & if\ 0 \leq h_r \leq \frac{2}{3}\pi \\ (x', y', z') & if\ \frac{2}{3}\pi < h_r \leq \frac{4}{3}\pi \\ (z', y', x') & if\ \frac{4}{3}\pi < h_r < 2\pi \end{bmatrix}
\end{aligned}
\tag{41}
$$

HSV And HSL Spaces:

- The HSI representation of color tends to underestimate the intensity values at the pixels. In the HSI model, the intensity at a pixel is supposed to capture the "energy" of the light your eye wants to associate with that pixel. However, when you set $I = (R + G + B)/3$, you are basically setting I to the average of the "energies" that you may associate separately with the three color channels. If you really think about it, that does not seem like the correct thing to do. [Let's say that an image display looks very bright — meaning that its pixels pack much light "energy" —because of the bright reds in the image. And let's also assume that there is no blue or green in the image. By setting $I = R/3$ in this case, the HSI formula would discount the "energy" coming off the pixels by two-thirds — which really does not make a whole lot of sense. The fact that HSI mutes the I values unless it sees strength in all three color channels can be highly problematic for computer vision algorithms. For extracting the shape information, many computer vision algorithms depend primarily on the intensity attributes at the pixels.]

- This shortcoming of HSI is remedied in the HSV and HSL models of color vision. So as not to cause any confusion between different ways of calculating the intensities, what is I in HSI is represented by V for "Value" in HSV, and by L for "Lightness" in HSL. Whereas the calculation of H remains the same in HSV and HSL as as in HSI, the formulas for how V and L are calculated are very different, as you would expect. The different ways of calculating the intensity result in a small modification to the formulas for calculating the saturation. [The value ranges for the three variables in both HSV and HSL remain the same as for HSI. That is, H values span $[0.0, 360.0]$, S values span $[0.0, 1.0]$; and both V and L values span $[0.0, 1.0]$.]

- The V in HSV is calculated as $max(R, G, B)$. And the L in HSL is calculated as $\frac{max(R,G,B)+min(R,G,B)}{2}$.

- The S in HSV is calculated as $\frac{max(R,G,B)-min(R,G,B)}{V}$. And the same in HSL is calculated as $\frac{max(R,G,B)-min(R,G,B)}{1-abs(2L-1)}$.

- Figure 15 on the next page shows a rather dramatic comparison of the intensity values calculated by the different color models.

- Since HSV tends to be employed more commonly compared to HSL, I'll show below how the transformation formulas as they have actually been implemented in several software libraries for going from RGB to HSV and back.

- Here are the formulas that take you from RGB to HSV. The input to the transformation shown below consists of a triplet of values (R, G, B) with the individual color components normalized to the $[0, 1]$ range.

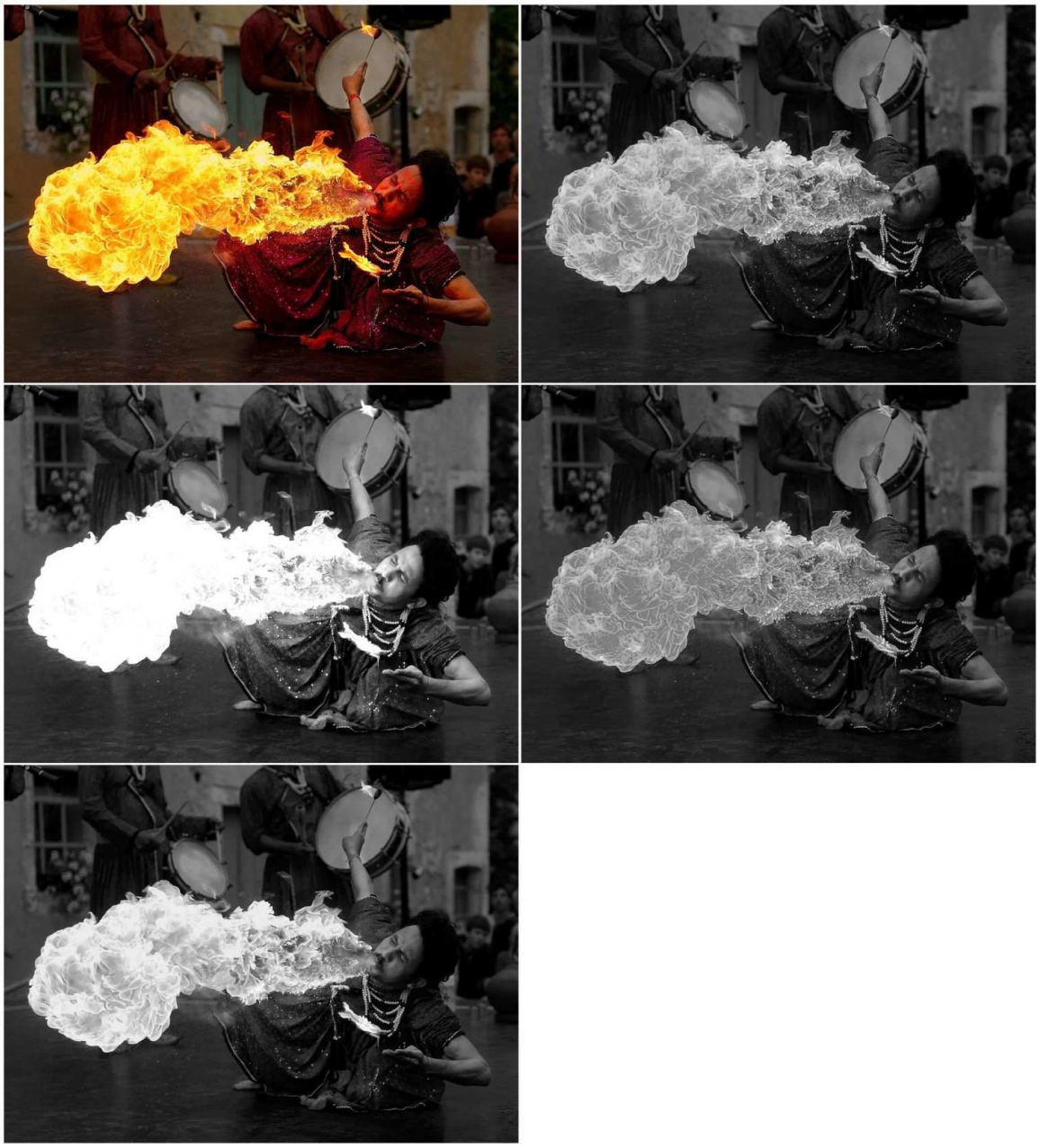


Figure 15: *The left image in the topmost row is the original image. The right image in the same row is the depiction of I values in the HSI model. In the second row, the left image shows the V in the HSV representation of the colors and the right image the L in the HSL representation. The image shown in the last row is the $L*$ component in the CIE L*a*b* representation of color. All these images are from the Wikipedia page on "HSL and HSV".*

$$
\begin{aligned}
M &= \max(R,G,B) \\
m &= \min(R,G,B) \\
c &= M - m \\
V &= M \\
H &= \begin{cases} 60\Big(\frac{G-B}{c} \; mod \; 6\Big) & if \; M = R, \; c \neq 0 \\ 60\Big(\frac{B-R}{c} + 2\Big) & if \; M = G, \; c \neq 0 \\ 60\Big(\frac{R-G}{c} + 4\Big) & if \; M = B, \; c \neq 0 \\ 0 & if \; c = 0 \end{cases} \\
S &= \frac{c}{V}
\end{aligned}
\tag{42}
$$

- In the formulas shown above, note how we protect the calculation of H against division by 0 when R, G, and B have exactly the same values. We know that when you mix these primary colors in equal amounts, you get pure gray, which is a point on the vertical axis of the HSV coordinate frame. When $c = 0$, these formulas return $(M, 0, 0)$, which is a point on the vertical axis of the HSV space.

- And, finally, shown are the corresponding formulas for going from HSV to RGB. The input to the transformation shown below consists of triplet of values (H, S, V). Note that the returned values for the individual colors in the triplet (R, G, B) are in the range $[0, 1]$.

$$
\begin{aligned}
c &= V \times S \\
m &= V - c \\
x &= c\Big(1 - \Big|\frac{H}{60}\ mod\ 2\ -\ 1\Big|\Big) \\
(R,G,B) &= \begin{cases}
(c+m, x+m, m) & if\ 0 \leq H \leq 60 \\
(x+m, c+m, m) & if\ 60 < H \leq 120 \\
(m, c+m, x+m) & if\ 120 < H \leq 180 \\
(m, x+m, c+m) & if\ 180 < H \leq 240 \\
(x+m, m, c+m) & if\ 240 < H \leq 300 \\
(c+m, m, x+m) & if\ 300 < H \leq 360
\end{cases}
\end{aligned}
\tag{43}
$$

Opponent Color Spaces

- It has been suggested by many researchers who work on the psychophysics of color vision that the human experience of color is marked by what are referred to as the opponent colors. Opponent colors are based on the notion that the cells in the retina collaborate to form receptor complexes and, through the receptor complexes, the three independent components of color that the eye can see are along the red-green, blue-yellow, and black-white dimensions. [The "red-green", "blue-yellow", and "black-white" are referred to as *opponent pairs*. You can think of red as opposed to green, blue as opposed to yellow, and black as opposed to white. The human brain wants to receive a single signal for each of the three opposites.]

- That is, as shown in Figure 16, the visual cortex in the brain gets three independent signals from the retina: one regarding how much of the red-green dimension exists at a point in the scene; a second regarding how much of the blue-yellow

dimension exists; and a third regarding how much of the black-white dimension exists at a scene point. The receptor complexes that perceive these three different dimensions of the color are referred to as the opponent cells. [The fact that we can see orange, whose three independent components would be red, yellow, and white, and the fact that we cannot see reddish-green or yellowish-blue supports the opponent color model. Since the two color components in a reddish-green mixture are along the same dimension, the red-green receptor complex is unable to discern them separately.] Note the very important fact that if there was not considerable overlap between the spectral sensitivities shown in Figure 4 for L, M, and S, it would not be possible for a receptor complex to see the colors in the opponent manner.

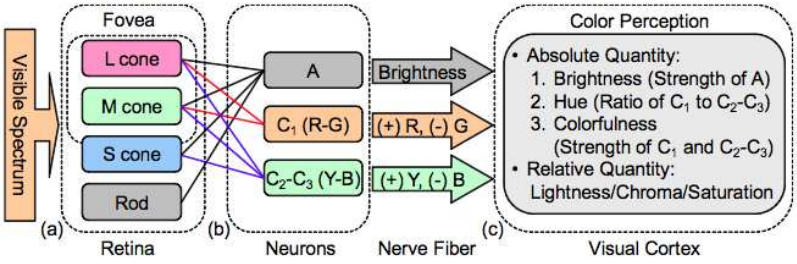


Figure 16: *This figure depicts how the output produced by the L, M, and S cone cells (whose spectral responses were shown previously in Figure 4) are combined to produce the three opponent color signals for the visual cortex in the brain. This figure is from the Wikipedia page on "Opponent Processes".*

- The L*a*b* is currently the most widely used opponent color model, L stands for luminance (what $L*$ is to the L*a*b* model, I is to HSI, V is to HSV, L to HSL, etc.) and $a*$ and $b*$

stand for the two color opponent dimensions.

- Since L*a*b* is a nonlinear color space, the transformation equations that go between it and the other color spaces we have covered so far are computationally complex. For example, to go from RGB to L*a*b*, you must locate the color value in the "absolute" color space XYZ, and then apply nonlinear functions on the result values to get the $L*$, $a*$, and $b*$ values. The Wikipedia page on "Lab color space" for a good summary of what is required by such transformations.

CMY Space:

- Understanding color in RGB space is important because digital cameras produce R, G, and B signals and because display monitors take R, G, and B signals to output color images. Understanding color in HSI/HSV/HSL space is important because it is in that space we as humans understand color the best.

- However, When it comes to making hardcopies of color images, one must understand color in what is referred to as Cyan, Magenta, Yellow space or the CMY space.

- Just as red, green, and blue are additive "primaries" (meaning that we may attempt to express any emitted color as an additive mixture of these three colors), **cyan, magenta, and yellow are *subtractive primaries***. To explain what that means, shown in Figure 17 on the next page is the ***color wheel for pigment hues.*** [On a color wheel, the pigments corresponding to any two hues that are one hue apart may be mixed to yield a pigment whose hue is in the middle. For example, if we mix together the cyan and magenta pigments, we obtain the blue pigment.]

- When we say a pigment hue on the color wheel of Figure 17 is subtractive, what we mean is that if we shine white light on a surface coated with that pigment, it will absorb from the white

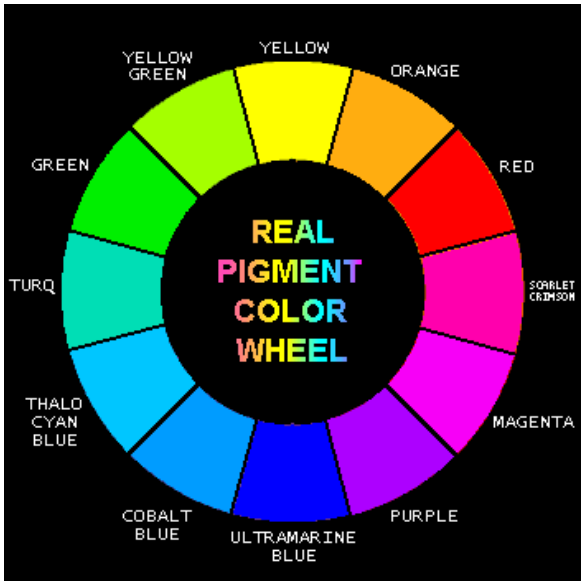


Figure 17: *The color wheel for pigment hues. The colors that are diametrically opposites are subtractive vis-a-vis each other. The opposite color pairs are considered to be complementary. The two colors that are immediately adjacent to each color are considered to be analogous. Finally, the three colors at the vertices of an equilateral inside the wheel are considered to form a triad. From* `http://www.realcolorwheel.com/newcolorwheel.htm`

light the hue at the opposite point on the color circle. Assume that a surface is coated with the cyan pigment. Now assume that a beam of white light rich in all hues is incident on the surface. The pigment will absorb the red hue and the distribution of hues in the reflected light will carry all hues except red. Since the reflected light must again be thought of as emitted light, we must add the hues on the color wheel to determine the color of the reflected beam. **The hues that are opposite on the color wheel add up to white.** Therefore, the reflected light will bear the color of cyan (since its opposite, red, would be absorbed by the pigment), in addition to containing a significant extent of white light. In other words, the reflected light will be an unsaturated version of cyan. [**This explanation should also point to the fact that the colors seen in the white light that is *reflected* off colored object surfaces can never appear to be intense since much of the white light remains in what is reflected. In other words, you have to use emitted light for constructing brilliant displays — as you might have noticed from the more modern billboards along highways these days.**]

- Just as a video monitor has separate light emitters for red, green, and blue, a hard copy printer has separate "pens" for cyan, magenta, and yellow. Suppose a pixel produced by a color camera has the following components of R, G, and B: $R = 0.8$, $G = 0.3$, and $B = 0.4$, where for the sake of the explanation here, we have assumed the range $[0.0, 1.0]$ for the three color components. When this pixel is displayed on a color calibrated video monitor, these RGB values will cause the monitor pixel to

possess the right color. However, when the same pixel is sent for display to a hardcopy printer, the values sent to the cyan, magenta, and yellow pens must be

$$\begin{aligned} C &= 0.2 = 1 - R \\ M &= 0.7 = 1 - G \\ Y &= 0.6 = 1 - B \end{aligned} \tag{44}$$

- With the CMY values set as shown above, C being 0.2 will cause the deposited cyan pigment to absorb 0.2 fraction of the red hue in the supposedly white illumination of the hardcopy surface, releasing 0.8 of the red hue in the direction of the observer. Similarly, with M and Y for the generation of the correct values of G and B for an observer.

- This transformation between RGB and the CMY values required to produce the correct RGB in light reflected from a hardcopy is expressed succinctly by

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{45}$$

Back to TOC

6.4: The Great Difficulty of Measuring the True Color of an Object Surface

- From the standpoint of computer vision, our goal is obviously to associate with each pixel in an image a color value that represents the true color of the object surface that gave rise to the pixel.

- Unfortunately, what makes the measurement of that color complicated are the following factors:

  - The direction of illumination. When illumination is not purely diffuse, the color of the light coming off an object surface depends on the direction of the incident illumination. In general, now the light coming off the surface will have a specular component and a diffuse component. The "true" color of a surface is normally associated with just the diffuse light coming off the surface.

  - The reflection properties of the object surface with regard to how much of the incident light is reflected specularly and how through diffuse processes. This would depend on the roughness (texture) of the surface.

  - The color of the illumination. The color of the light coming off a surface depends significantly on the color of the illumination.

  - The intensity of the illumination. The dependence on the intensity means that, if you are recording an image of an outdoor scene

during daytime, the color measurements for same surface will be different depending on whether it is directly in the sunlight or whether it is in a shadow. [This is partly the same phenomenon as captured by the nonlinearities of the human color vision. See the earlier discussion on the Opponent Color Spaces such as L*a*b* in the previous section.]

– Color filtering by the hardware. The spectral properties of the three color filters used by the camera that are used to break incoming light into ostensibly the R, G, and B components.

- That the color of the light coming off an object surface depends significantly on the color composition of the illumination is illustrated by Figure 18. As you can see in the figure, a yellow ball looks green under mostly blue illumination. This figure also shows how one can algorithmically correct for illumination induced distortion of the color of a surface. The result shown in (c) of the figure was obtained through the use of an *illumination-adaptive* color space that was introduced in 2005 by Park and Kak in the report "*A New Color Representation for Non-White Illumination Conditions*," that was cited at the end of Section 6.1.

- For another example of illumination dependency of the image recorded by a digital camera, shown in the top row of Figure 19 are four different images for the same plastic toy. These images were recorded under the following illuminations: (1) florescent, (2) blue incandescent, (3) green incandescent, and (4) tungston.

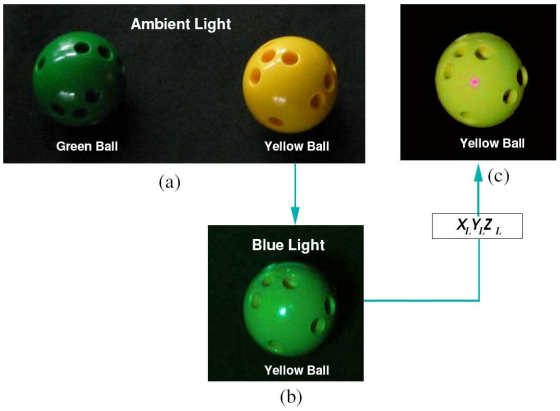


Figure 18: *(a) A green ball and a yellow ball imaged under ambient white-light illumination. (b) The yellow ball imaged under blue illumination. It now looks quite a bit like the green ball. (c) The image of (b) after its processing by the algorithm in the 2005 Park and Kak report titled "A New Color Representation for Non-White Illumination Conditions" that was cited at the end of Section 6.1.*

The second row demonstrates the extent to which the true color of the toy was restored in each case by the illumination-adaptive color representation of Park and Kak mentioned in the previous bullet.

- Regarding the dependence of the measured color on the measuring device itself, note that the photoelectric cells in the camera imaging sensor (which may either be a CCD sensor or a CMOS sensor) use color filters centered around the R, G, and B wavelengths. So the 3-dimensional measurement of the color at each pixel also depends on the spectral properties of these filters. [The three measurements at each pixel may be recorded by a single sensor chip that contains a mosaic of cells in which groupings of three adjacent cells are used for measuring the R, G, B components

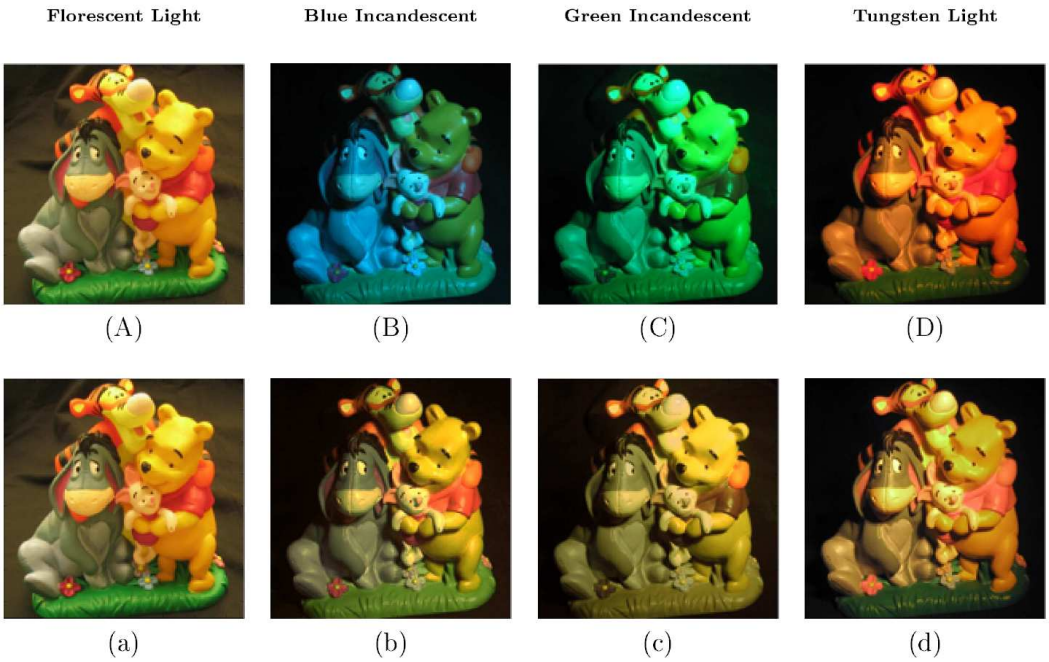


Figure 19: *The photos in the top row are of the same plastic toy and were recorded by the same digital camera using four different illumination sources: (1) florescent, (2) blue incandescent, (3) green incandescent, and (4) tungston. The second row demonstrates the extent to which the true color of the toy was restored in each case by the illumination-adaptive color representation method of Park and Kak.*

separately. An incoming light ray is deflected to each of three cells in a grouping and light for each cell passed through a color-sensitive filter specific to that cell. An alternative consists of using three separate sensor chips, one for each color component, and a set of prisms that are shaped to deflect a particular color component to each chip.]

- Figure 20 points to the issues involved in the dependence of the measured color on the direction of the illumination. The main issue here is that the diffuse light that is reflected off a surface (which is the light we want a camera to capture for characterizing the color of the surface) has a cosine dependence on the angle of illumination to the perpendicular to the surface. [For a given incident light beam, we have shown reflected light as consisting of specular and diffuse components, the components being additive. The specular and the diffuse components of the reflected light are also known as the **surface component** and the **body component**, respectively. Light reflected by homogeneous materials such as metals is dominated by the surface components, whereas the light reflected by inhomogeneous material is dominated by the body component.]

- The dependence of the reflected diffuse color light on the angle of illumination is illustrated with some actual measurements in Figure 22. The two results shown in that figure are for the pink and the yellow objects in the pile shown in Figure 21. For the two experimental results shown in Figure 22, the position/angle of the source of illumination and the camera were kept fixed while the surface was tilted in increments of 15°.

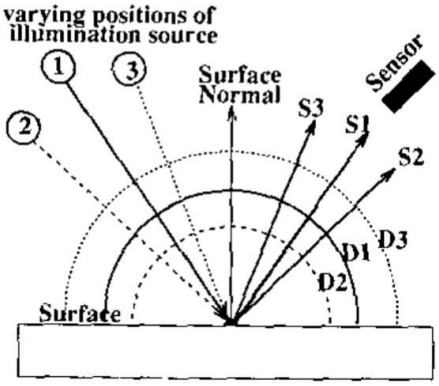


Figure 20: *S_i represents the specular reflection resulting from the illumination source i. The diffuse reflection from any illumination source radiates out in all directions. The radius of the arc labeled D_i represents the magnitude of the diffuse reflection from the illumination source i. Note that this magnitude depends on the cosine of the angle the illumination source makes with the surface.*

Figure 21: *The yellow and the white objects shown in this pile were used for demonstrating the dependence of the measured color on the orientation of a surface.*

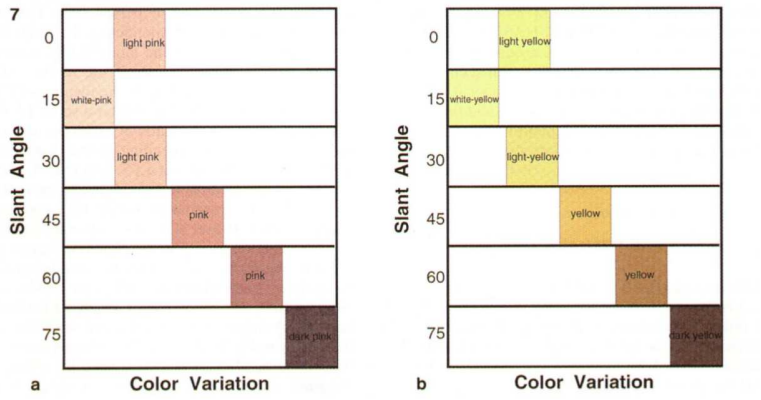


Figure 22: *The color recorded by a camera as an object is rotated in increments of* 15°. *The position and the direction of both the illumination source and the camera were kept fixed for the results shown. This figure is taken from the Grewe and Kak publication cited at the end of Section 6.1.*

- To fully appreciate the results shown in Figure 22, note that at the relatively small slant angle of 15°, which is also the angle of incidence, we would expect the specular component of the reflected light to be the strongest, as can be seen in both the results in the figure.

- When the camera is situated in a direction where the angle of reflection is nearly the same as the angle of incidence, the surface component will usually dominate and the spectral composition of this component will be approximately the same as that of the illumination source. Since our illumination source for the measurement of color is white light, the color shown in Figure 22 for the slant angle of 15° has the most white in it.

  [The angle between the illumination source and the optical axis of the camera is approximately 30° and,

when the surface is at slant of 15°, the camera registers the largest specular component. This is explained approximately by the law of reflection, which says that the angle of reflection subtended by the specular component of light with respect to the surface normal must equal the angle of incidence of the illumination light with respect to the normal. More accurately, this phenomenon is explained by the Torrance-Sparrow model of reflection.]

- In the results shown in Figure 22, when the slant angle of the surface becomes large, the reflected light consists mostly of the body component. But the intensity of this light diminishes rapidly as the surface slant angle approaches 90°. This explains the "darkness" of the result shown for the surface tilt angle of 75°. This diminishing effect is a result of the fact that if we assume a perfectly Lambertian surface, the magnitude of the body component varies as a cosine of the angle of incidence.