

TOWER OF HANOI

Recursive Algorithm

Origins

The french mathematician Édouard Lucas invented the Tower of Hanoi puzzle around 1883.

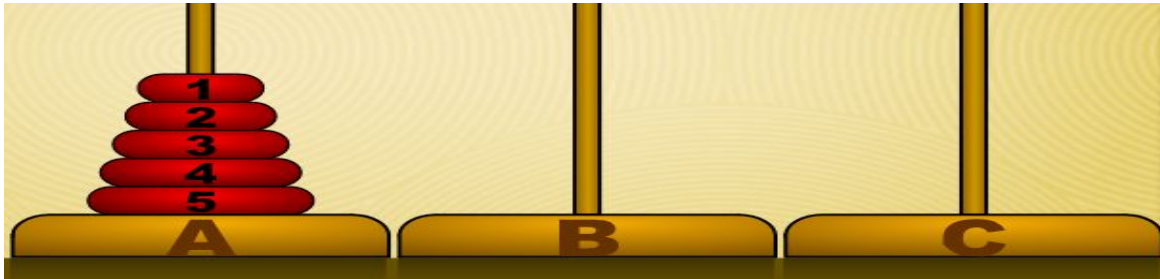
It is associated with a legend of a Hindu temple:

- It was used to increase mental discipline of young priests.
- 64 gold rings stacked on one of three post.
- Recreate the stack on another post: move 1 ring at a time, smaller ring on top of a larger one.
- How long would it take?

At a rate of one movement per second $\rightarrow 2^{64} - 1$ sec. = **585 billion years** > 42 times the age of a universe



Édouard Lucas (1842-1891)



Rules

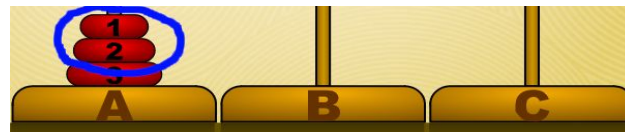
- **Move one ring at a time**
- **No ring may be placed on top of a ring that is smaller than it.**

Activities

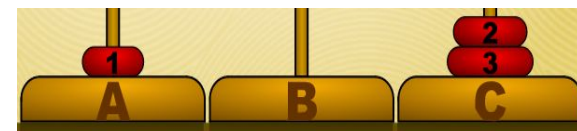
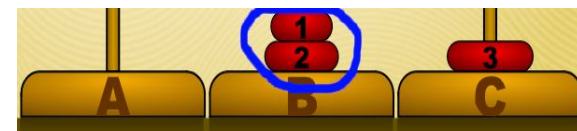
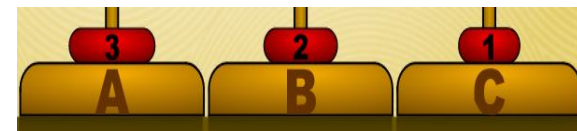
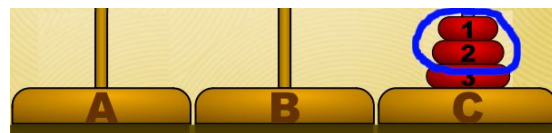
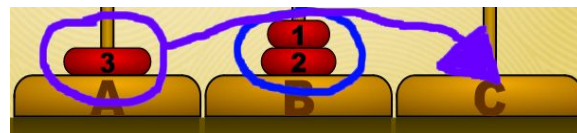
https://www.mathplayground.com/logic_tower_of_hanoi.html (mute the tab so you do not get annoyed by the sound)

1. How many moves for 1 ring from tower A to tower C?
2. How many moves for 2 rings from tower A to tower C?
3. How many moves for 3 rings from tower A to tower C?

Solution for 3 rings



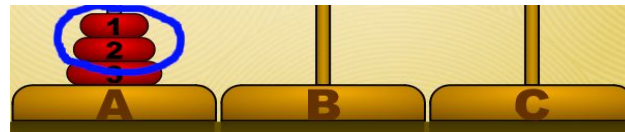
- $T(2, A, B, C)$
 1. Small from A to C
 2. Medium from A to B
 3. Small from C to B



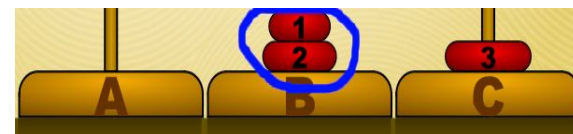
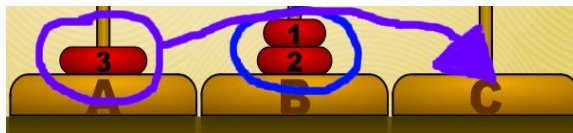
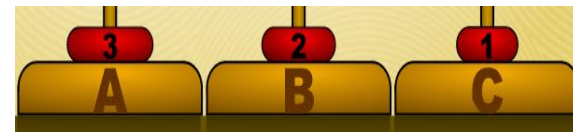
- $T(1, A, C, B)$
 4. Large from A to C

- $T(2, B, C, A)$
 5. Small from B to A
 6. Medium from B to C
 7. Small from A to C

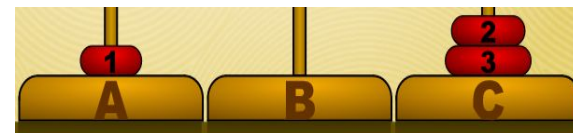
Solution for 3 rings



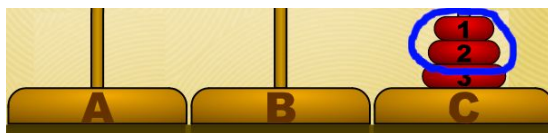
- $T(2, A, B, C)$ {
 1. Small from A to C
 2. Medium from A to B
 3. Small from C to B



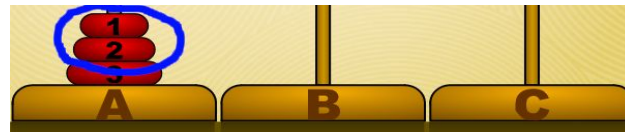
- $T(1, A, C, B)$ 4. Large from A to C



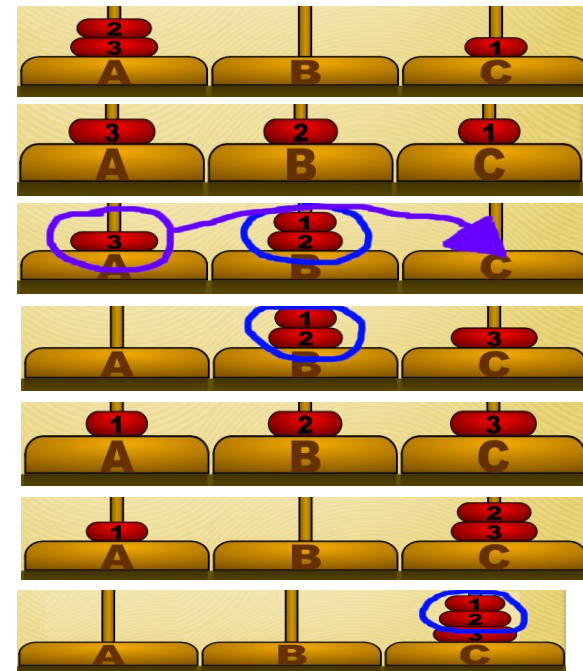
- $T(2, B, C, A)$ {
 5. Small from B to A
 6. Medium from B to C
 7. Small from A to C



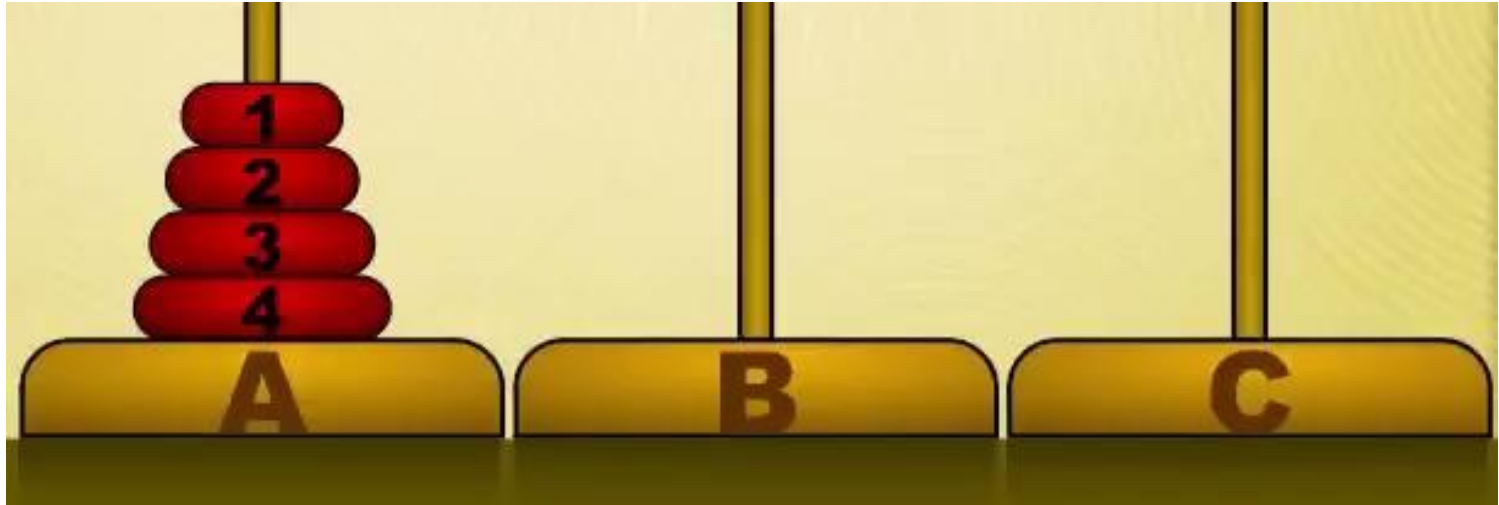
Solution for 3 rings



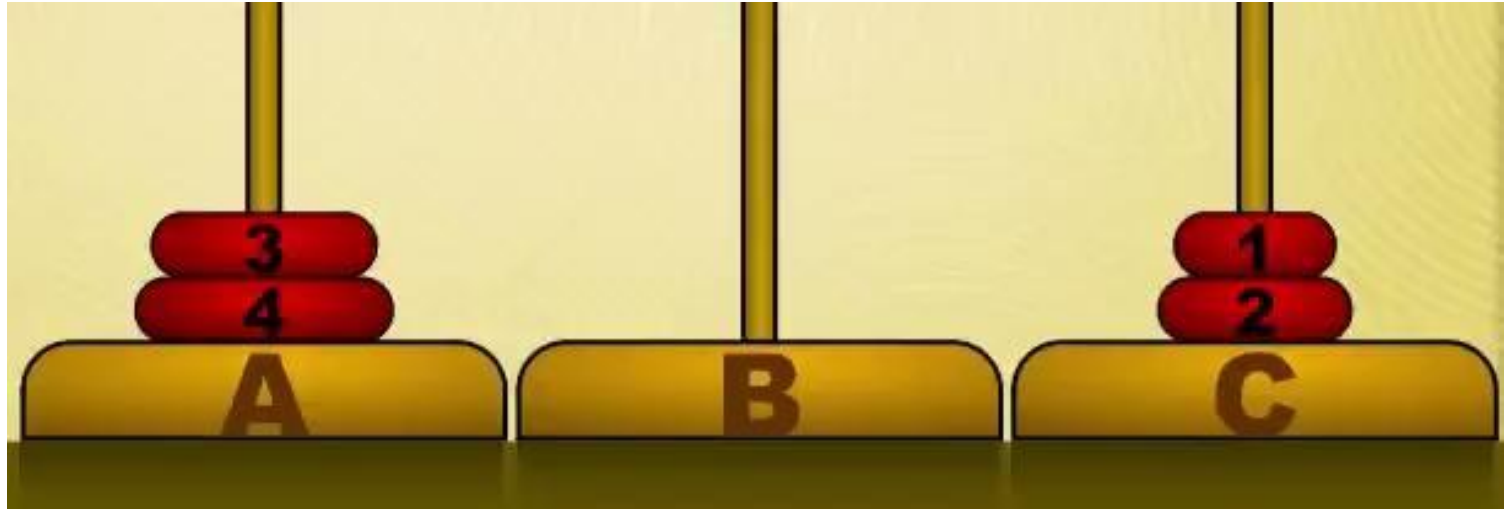
- | | | | | |
|---------------|---------------|-----------------------|---|---------------|
| T(3, A, C, B) | T(2, A, B, C) | 1. Small from A to C | ⇒ | T(1, A, C, B) |
| | | 2. Medium from A to B | ⇒ | T(1, A, B, C) |
| | | 3. Small from C to B | ⇒ | T(1, C, B, A) |
| | T(1, A, C, B) | 4. Large from A to C | | |
| | | 5. Small from B to A | ⇒ | T(1, B, A, C) |
| | | 6. Medium from B to C | ⇒ | T(1, B, C, A) |
| | | 7. Small from A to C | ⇒ | T(1, A, C, B) |



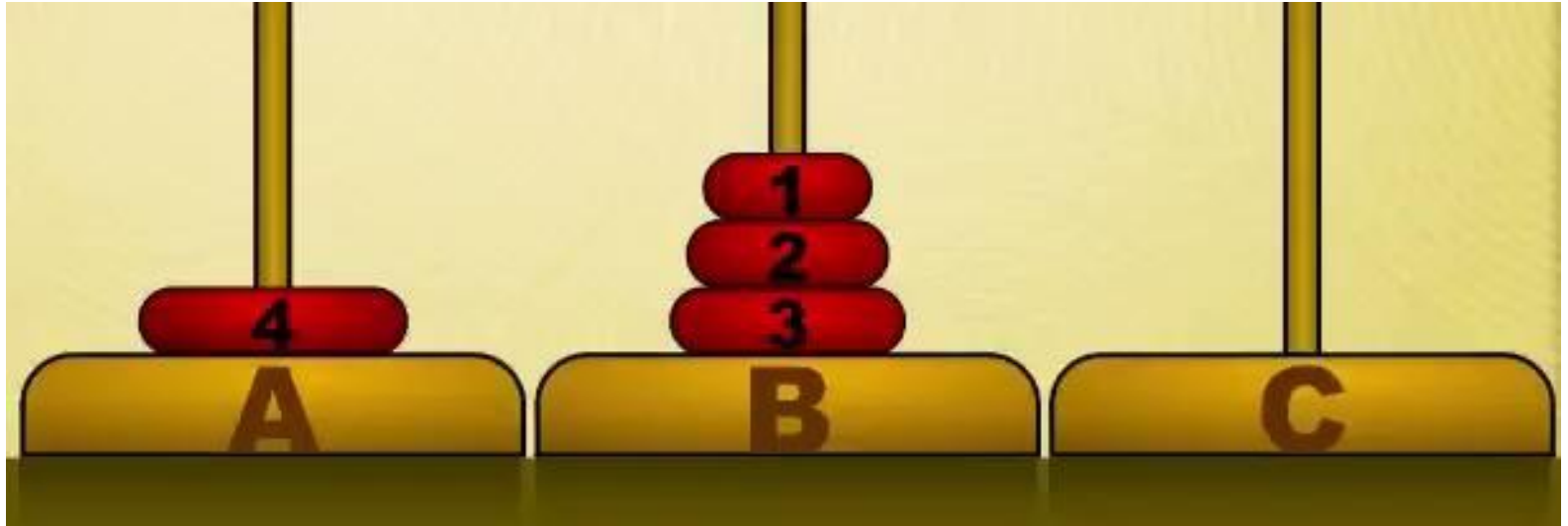
Solution for 4 ring (first step)



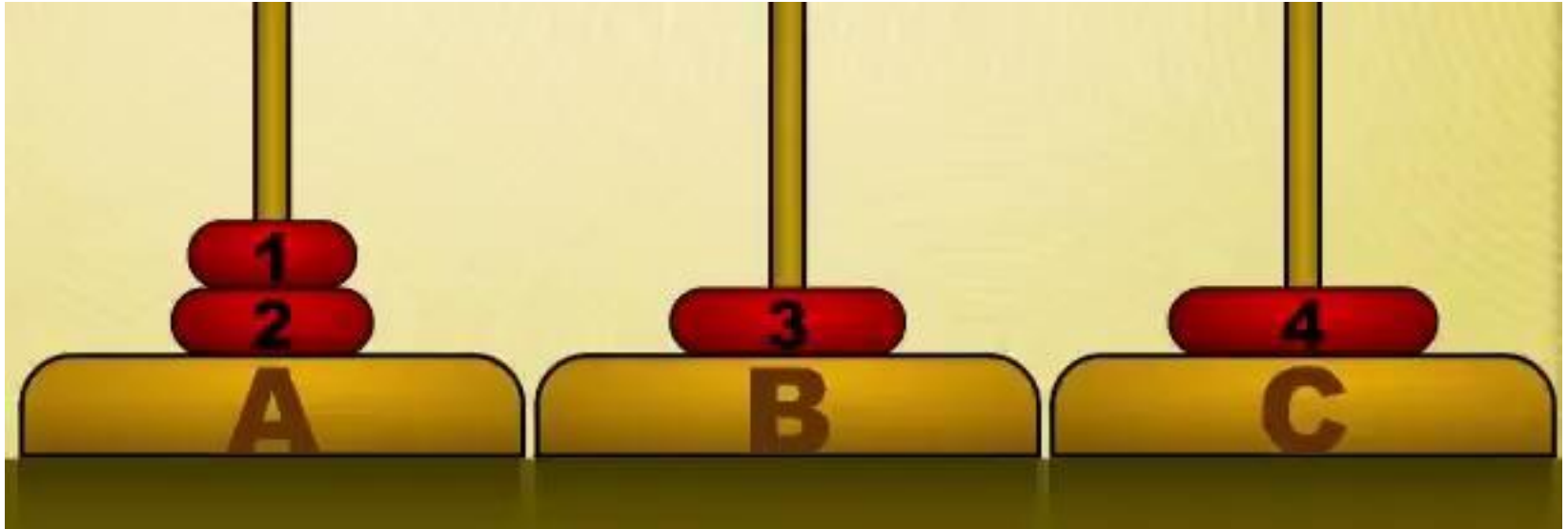
Solution for 4 ring (second step)



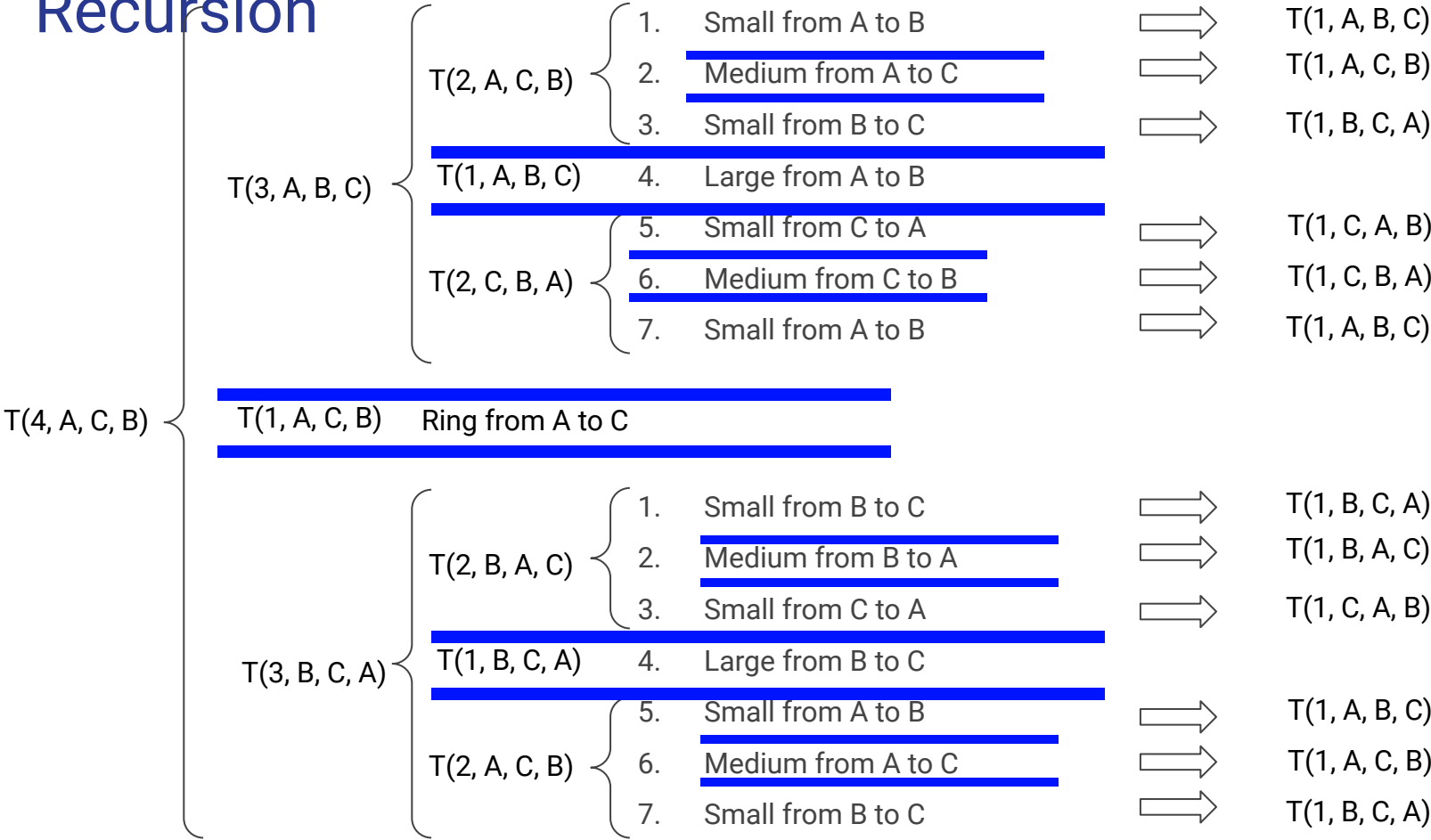
Solution for 4 ring (third step)



Solution for 4 ring (last step)



Recursion



Recursion

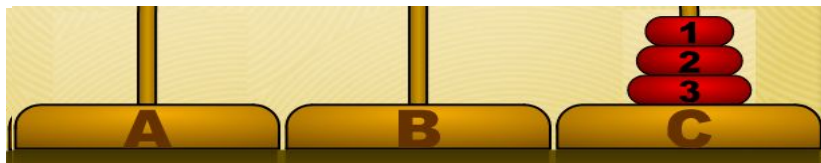
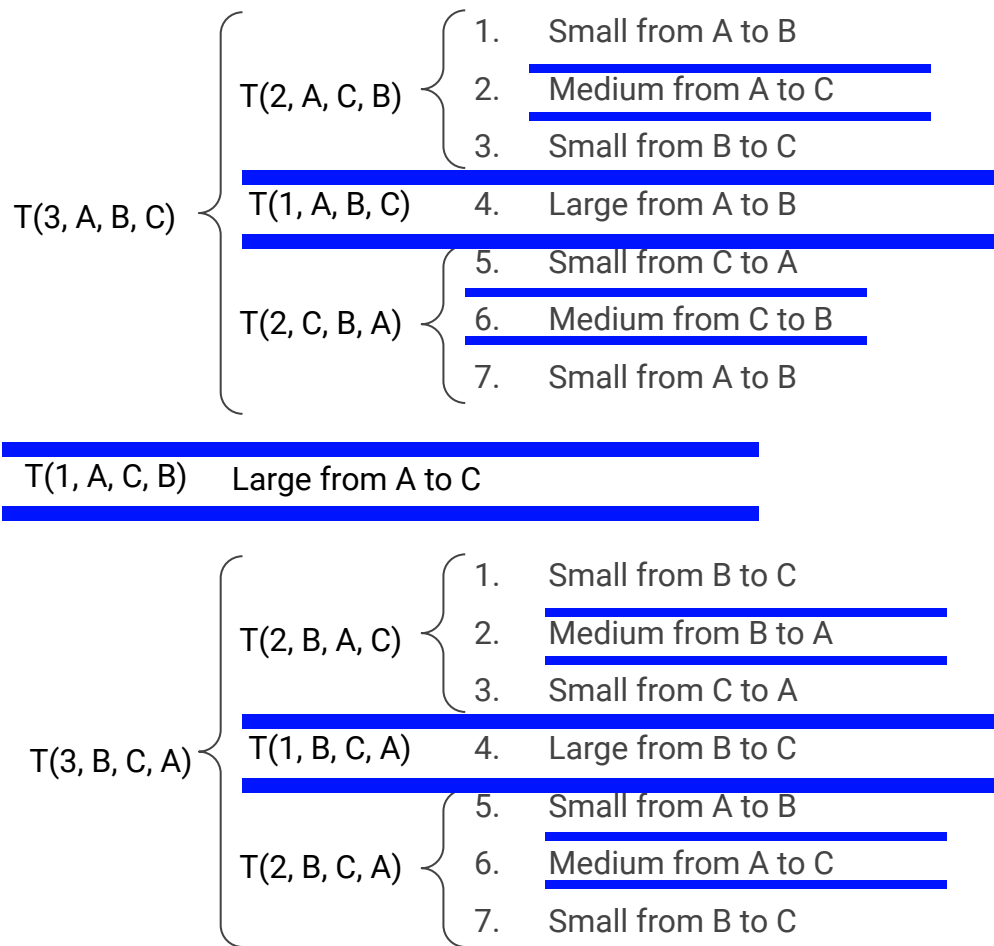
$T(n, A, C, B)$:

$T(n-1, A, B, C)$

Move ring from A to C

$T(n-1, B, C, A)$

$T(4, A, C, B)$



Numbers of moves

Minimal number of moves required
 $= 2^n - 1$

Where, n = number of rings.

For 3 rings: $2^3 - 1 = 7$ movements

Number of rings (N)	Number of Moves ((2**N)-1)	2**N
1	1	1
2	3	4
3	7	8
4	15	16
5	31	32
6	63	64
7	127	128
8	255	256

Coding Time !!!

Write a function to move n rings from source rod to destination rod, print the moves of each ring.

```
public static void hanoi(int n, char source_rod, char destination_rod, char aux_rod)
{
    YOUR CODE HERE
}
```

Output with 3 rings:

Move ring 1 from source A to destination C
Move ring 2 from source A to destination B
Move ring 1 from source C to destination B
Move ring 3 from source A to destination C
Move ring 1 from source B to destination A
Move ring 2 from source B to destination C
Move ring 1 from source A to destination C

