Advanced Mathematics

Lab 3: Inhomogeneous differential equations

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anerkannt

72 %

1.1a	1.1b	1.2a	1.2b	1.2c	2	3	exercise points
7	5	10	~~3~~ 2	7	17	24	

Differential equations with constant coefficients

1. Determine the solution of the inhomogeneous differential equations via undetermined parameter, variation of constants or modification rule undetermined parameters: $y_p = e^x(Ax^2+Bx+C)$

$$y'' + y = r(x) \quad \text{for } r(x) \in \left\{ e^x(4x^2 + x), \sin x \right\} \qquad (1.1)$$

$A\cos x + B\sin x$

$$y'' - 2y' + 2y = s(x) \quad \text{for } s(x) \in \left\{ 2x^2, \frac{e^{-2x}}{\sin^4 x}, 8\cosh x \right\} \qquad (1.2)$$

Ax^2+Bx^2+C $\quad A\cosh x + B\sinh x$

(20+33 points)

Variation of constants

G: $y_h = C_1y_1 + C_2y_2$, $y_p = C_1(x)y_1 + C_2(x)y_2$

$\begin{cases} C_1'y_1' + C_2'y_2' = r \\ C_1'y_1 + C_2'y_2 = 0 \end{cases} \Rightarrow \begin{cases} C_1' = \\ C_2' = \end{cases} \Rightarrow y_p$

2. Consider the differential equation

$$-xy'' + (x-2)y' + y = e^x \sin x$$

of lab 1 and solve the inhomogeneous part (17 points)

Solve the inhomogeneous differential equation

$$(x^3 + 2x^2 + x)y'' + (3x^2 + 4x + 1)y' + (x+1)y = \frac{1}{x+1}$$

when the first solution is given by $y_1 = \frac{1}{x+1}$ (30 points)

[V] Variation of constants

iv Assume the homogeneous solutions y_1 and y_2 to be product of two (simple and unknown) functions and solve then the differential equation

$$x^2y'' + 4xy' + (x^2 + 2)y = \frac{1}{1 + \cos x}.$$

v Solve the inhomogeneous Euler differential equation

$$(x-2)^2y'' - 7(x-2)y' + 25y = \sqrt{(x-2)^5}$$

and denote the final answer in real representation!

...ial equations with constant coefficients

1.1

...x)

$$y''+y=e^x(4x^2+x)$$

$\Rightarrow$ consider first the homogeneous equation:

$$y''+y=0.$$ ✓

this is an ODE with constant coefficients, we can get the general solution:

$$y_h = C_1\cos x + C_2 \sin x$$ ✓

variation of constants.

no need to normalization, $y_p = C_1(x)\cos x + C_2(x)\sin x$

consider the equation set,

$$\begin{cases} C_1' y_1 + C_2' y_2 = 0 \\ C_1' y_1' + C_2' y_2' = r \end{cases}$$

in this case

$$\begin{cases} C_1'\cos x + C_2'\sin x = 0 \\ -C_1'\sin x + C_2'\cos x = e^x(4x^2+x) \end{cases}$$

...e Cramer's rule

$$\Rightarrow C_1' = \frac{\det\begin{pmatrix} 0 & \sin x \\ e^x(4x^2+x) & \cos x\end{pmatrix}}{\det\begin{pmatrix}\cos x & \sin x \\ -\sin x & \cos x\end{pmatrix}} = \sin x\, e^x(4x^2+x)$$ ✓

$$C_2' = \frac{\det\begin{pmatrix}\cos x & 0 \\ -\sin x & e^x(4x^2+x)\end{pmatrix}}{\det\begin{pmatrix}\cos x & \sin x \\ -\sin x & \cos x\end{pmatrix}} = \cos x\, e^x(4x^2+x)$$ ✓

...gration by parts, we get give me more step!

—2

$$\begin{cases} C_1 = C_2 + F_1(x) \\ C_2 = -C_1 + F_2(x)\end{cases}$$

thus we can get

$$\begin{cases} C_1 = \dfrac{(4x^2-7x+3)\cos x - (4x^2+x-4)\sin x}{2}\cdot e^x \\ C_2 = \dfrac{(4x^2-7x+3)\sin x + (4x^2+x-4)\cos x}{2}\cdot e^x \end{cases}$$

$$\Rightarrow y_p = \underbrace{(4x^2-7x+3)e^x}_{2}$$

missing $\frac{1}{2}$

general solution:

$$y = C_1\cos x + C_2\sin x + \underbrace{(4x^2-7x+3)e^x}_{2}$$

—3

$$\underbrace{\int \sin x e^x(4x^2+x)\,dx}_{①} = -\int e^x(4x^2+x)\,d\cos x$$

$\Downarrow$

$$= -\cos x\, e^x(4x^2+x) + \int \underline{\cos x\, e^x(4x^2+x}\ \cancel{\ }+8x+1)\,dx$$

$$= \underbrace{\int \cos x e^x(4x^2+x)dx}_{①} + \underbrace{\int \cos x e^x(8x+1)\,dx}_{②} - \cos x e^x(4x^2+x)$$

$\cancel{\int \cos x e^x(4x^2+x)dx}$

$$\underbrace{\int \cos x\, e^x(8x+1)dx}_{②} = \int e^x(8x+1)\,d\sin x$$

$$= \sin x\, e^x(8x+1) - \int \sin x\, d e^x(8x+1)$$

$$= -\int \sin x e^x(8x+9)\,dx + \sin x e^x(8x+1)$$

$$= -\int \sin x e^x(8x+1)\,dx - 8\underbrace{\int \sin x e^x dx}_{③} + \sin x e^x(8x...$$

$$\begin{cases} \underbrace{\int \sin x e^x dx}_{③} = -\int e^x d\cos x = -\cos x e^x + \underbrace{\int \cos x e^x d...}_{③} \\ \underbrace{\int \cos x e^x dx}_{③} = \int e^x d\sin x = e^x\sin x - \underbrace{\int \sin x e^x dx}_{③} \end{cases}$$

$\Uparrow$

$$\Rightarrow ③ \begin{cases} \int \sin x e^x dx = \ \ldots \\ \int \cos x e^x dx = \ \ldots \end{cases}$$

$$\Rightarrow ② \begin{cases} \int \cos x e^x(8x+1)\,dx = \ldots \\ \int \sin x e^x(8x+1)\,dx = \ldots \end{cases}$$

$$\Rightarrow ① \begin{cases} \int \sin x e^x(4x^2+x)\,dx = \ldots \\ \int \cos x e^x(4x^2+x)\,dx = \ldots \end{cases}$$

b) $y''+y=\sin x$

$\Rightarrow$ consider the homogeneous equation:

$$y''+y=0$$

similar to a) we get the general solution $y_h = C_1\cos x + C_2\sin x$

Then we use variation of constants, $y_p = C_1(x)\cos x + C_2(x)\sin x$

consider the equation set

$$\begin{cases} C_1'y_1 + C_2'y_2 = 0 \\ C_1'y_1' + C_2'y_2' = r \end{cases}$$

in this case $\begin{cases} C_1'\cos x + C_2'\sin x = 0 \\ -C_1'\sin x + C_2'\cos x = \sin x \end{cases}$

$$\int -\sin^2 x\,dx = \cancel{\quad} -\int \frac{1-\cos 2x}{2}dx$$

$$= \int \frac{\cos 2x - 1}{4} d2x = \frac{\sin 2x - 2}{4}$$

$$\int \sin x\cos x\,dx = \frac{1}{2}\sin^2 x + C = -\frac{1}{4}\cos 2x + C$$

Cramer's rule $\Rightarrow$

$$C_1' = \frac{\det\begin{pmatrix} 0 & \sin x \\ \sin x & \cos x \end{pmatrix}}{\det\begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}} = -\sin^2 x$$

$$C_2' = \frac{\det\begin{pmatrix} \cos x & 0 \\ -\sin x & \sin x \end{pmatrix}}{\det\begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}} = \sin x\cos x$$

more steps!

−3

...ration

$$\Rightarrow C_1 = \frac{\sin 2x - 2x}{4}, \quad C_2 = -\frac{1}{4}\cos 2x$$

$$\Rightarrow y_p = \frac{\sin 2x - 2x}{4}\cdot\cos x + \left(-\frac{1}{4}\cos 2x\right)\cdot\sin x = \frac{\sin x - 2x\cos x}{4}$$

$\Rightarrow$ general solution:

$$y = C_1\cos x + C_2\sin x + \frac{\sin x - 2x\cos x}{4}$$

$$y = C_1\cos x + C_2\sin x - \frac{1}{2}x\cos x$$

a) $y''-2y'+2y=2x^2$

⇒ Consider the homogeneous equation:

$$y''-2y'+2y=0$$

this is an ODE with constant coefficients, thus we can get the general solution.

$$y_h = e^x(C_1\cos x + C_2\sin x)$$

e ~~variation~~ methods of undetermined parameter:

Assume $y_p = k_2x^2+k_1x+k_0$ (with $r(x)=2x^2$).

$y_p' = 2k_2x+k_1$, $y_p''=2k_2$

insert into the ODE

⇒ $2k_2-2(2k_2x+k_1)+2(k_2x^2+k_1x+k_0)=2x^2$

⇒ $k_2=1,\ k_1=2,\ k_0=1$

⇒ $y_p = x^2+2x+1$ is one solution for the ODE

⇒ general solution:

$$y = e^x(C_1\cos x + C_2\sin x) + x^2+2x+1.$$

$$y''-2y'+2y=\frac{e^{-2x}}{\sin^4 x}$$

⇒ Similar to a), we get the general solution of the homogeneous equation

$$y_h = e^x(C_1\cos x + C_2\sin x)$$

+2

$y_1 = e^x\cos x$

$y_2 = e^x\sin x$

e variation of constants: $y_p = e^x[C_1(x)\cos x + C_2(x)\sin x]$

$$\begin{cases} e^x(C_1'\cos x + C_2'\sin x) = 0 \\ e^x(-C_1'\sin x + C_2'\cos x) = \dfrac{e^{-2x}}{\sin^4 x} \end{cases}$$

r's rule ⇒ $C_1' = \dfrac{\det\begin{pmatrix} 0 & \sin x \\ \frac{e^{-3x}}{\sin^4 x} & \cos x \end{pmatrix}}{\det\begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}} = -\dfrac{e^{-3x}}{\sin^3 x}$

$C_2' = \dfrac{\det\begin{pmatrix} \cos x & 0 \\ -\sin x & \frac{e^{-3x}}{\sin^4 x} \end{pmatrix}}{\det\begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}} = \dfrac{e^{-3x}\cos x}{\sin^4 x}$

intergration $\Rightarrow \begin{cases} C_1 = \displaystyle\int -\frac{e^{-2x}}{\sin^3 x}\,dx \\ C_2 = \displaystyle\int \frac{e^{-2x}\cos x}{\sin^4 x}\,dx \end{cases}$

$\Rightarrow$ general solution:

$$y = e^x(C_1\cos x + C_2\sin x) + \int -\frac{e^{-2x}}{\sin^3 x}\,dx \cdot \cos x + \int -\frac{e^{-2x}\cos x}{\sin^4 x}\,dx \cdot \sin x$$

c)

$$y'' - 2y' + 2y = 8\cosh x$$

$\Rightarrow$ Similar to a), b), we get the general solution of the homogeneous ODE.

$$y_h = e^x(C_1\cos x + C_2\sin x)$$

Use "undetermined parameter",

we assume $y_p = A\cosh x + B\sinh x$

then $y_p' = A\sinh x + B\cosh x$, $y_p'' = A\cosh x + B\sinh x$

sert into ODE $\Rightarrow (A\cosh x + B\sinh x) - 2(A\sinh x + B\cosh x) + 2(A\cosh x + B\sinh x) = 8\cosh x$

$\Rightarrow \quad A = \frac{24}{5}, B = \frac{16}{5}$

$\Rightarrow \quad y_p = \frac{24}{5}\cosh x + \frac{16}{5}\sinh x$ is one solution

$\Rightarrow$ general solution:

$$y = e^x(C_1\cos x + C_2\sin x) + \frac{24}{5}\cosh x + \frac{16}{5}\sinh x.$$

Variation of constants

2. $-xy''+(x-2)y'+y=e^x\sin x$

$\Rightarrow$ from lab1 we know the general solution of the homogeneous equation is:

$$y_h = C_1\cdot\frac{1}{x}+C_2\cdot\frac{e^x}{x}$$

(steps: first guess $y_1=\frac{1}{x}$, then get y_2 by "Reduction of order": $y_2=\int\frac{1}{y_1^2}\cdot e^{-\int P dx}dx$;

Variation of constants: $y_p=C_1(x)\,y_1+C_2(x)\,y_2$.

first normalization $\Rightarrow$ $y''-\frac{x-2}{x}y'-\frac{1}{x}y=-\frac{e^x\sin x}{x}$

consider the equation set:

$$\begin{cases} C_1'y_1+C_2'y_2=0 \\ C_1'y_1'+C_2'y_2'=f \end{cases}$$

in this case, $\begin{cases} C_1'\cdot\frac{1}{x}+C_2'\cdot\frac{e^x}{x}=0 \\ C_1'\cdot(-\frac{1}{x^2})+C_2'\cdot\frac{e^x(x-1)}{x^2}=-\frac{e^x\sin x}{x} \end{cases}$

Cramer's rule $\Rightarrow$ $C_1'=\dfrac{\det\begin{pmatrix}0 & \frac{e^x}{x} \\ -\frac{e^x\sin x}{x} & \frac{e^x(x-1)}{x^2}\end{pmatrix}}{\det\begin{pmatrix}\frac{1}{x} & \frac{e^x}{x} \\ -\frac{1}{x^2} & \frac{e^x(x-1)}{x^2}\end{pmatrix}}=\dfrac{\frac{e^{2x}\sin x}{x^2}}{\frac{e^x}{x^2}}=e^x\sin x$

$C_2'=\dfrac{\det\begin{pmatrix}\frac{1}{x} & 0 \\ -\frac{1}{x^2} & -\frac{e^x\sin x}{x}\end{pmatrix}}{\det\begin{pmatrix}\frac{1}{x} & \frac{e^x}{x} \\ -\frac{1}{x^2} & \frac{e^x(x-1)}{x^2}\end{pmatrix}}=\dfrac{-\frac{e^x\sin x}{x^2}}{\frac{e^x}{x^2}}=-\sin x$

$\Rightarrow C_1=\underline{\int e^x\sin x\,dx}=-\int e^x d\cos x=-e^x\cos x+\int\cos x\cdot e^x dx=-e^x\cos x+\int e^x d\sin x$

$=-e^x\cos x+e^x\sin x-\underline{\int\sin x\, e^x dx}$ $\Rightarrow 2C_1=e^x(\sin x-\cos x)$

$\Rightarrow C_1=\dfrac{e^x(\sin x-\cos x)}{2}$

$C_2=\int-\sin x\,dx=\cos x$

$\Rightarrow y_p=\dfrac{e^x(\sin x+\cos x)}{2x}$

$\Rightarrow$ general solution: $y=\dfrac{C_1}{x}+\dfrac{C_2e^x}{x}+\dfrac{e^x(\sin x+\cos x)}{2x}$

3. $(x^3+2x^2+x)y''+(3x^2+4x+1)y'+(x+1)y=\frac{1}{x+1}$

$\Rightarrow$ since the first solution is given by $y_1=\frac{1}{x+1}$ (the solution of the homogeneous equation)

We assume $y_2=u\cdot y_1=\frac{1}{x+1}\cdot u$. ✓

then $y_2'=-\frac{1}{(x+1)^2}\cdot u+\frac{1}{x+1}\cdot u'$ ✓

$y_2''=\frac{2}{(x+1)^3}u-\frac{2}{(x+1)^2}\cdot u'+\frac{1}{x+1}\cdot u''$ ✓

insert into the ODE:

$(x^3+2x^2+x)\cdot\left(\frac{2}{(x+1)^3}u-\frac{2}{(x+1)^2}u'+\frac{1}{x+1}u''\right)+(3x^2+4x+1)\left(-\frac{1}{(x+1)^2}u+\frac{1}{x+1}u'\right)$

$+(x+1)\cdot\frac{1}{x+1}u=0$

✓

$\Rightarrow\ x(x+1)u''+(x+1)u'=0$

$\Rightarrow\ u=\ln|x|$

$\Rightarrow\ y_2=-\frac{1}{x+1}\ln|x|$ ✓

$\Rightarrow$ the general solution of the homogeneous ODE is:

$$y_h=\frac{C_1+C_2\ln|x|}{x+1}=\frac{c_1+c_2\ln|x|}{x+1}$$ ✓

too many $\left\{\begin{matrix}c_1\\c_2\end{matrix}\right.$ use another letter

variation of constants": $y_p=C_1(x)\cdot\frac{1}{x+1}+C_2(x)\cdot\frac{\ln|x|}{x+1}$ ✓

consider the equation set:

$\begin{cases}C_1'y_1+C_2'y_2=0\\C_1'y_1'+C_2'y_2'=r\end{cases}$, in this case $\begin{cases}C_1'\frac{1}{x+1}+C_2'\frac{\ln|x|}{x+1}=0\\C_1'\frac{-1}{(x+1)^2}+C_2'\cdot\frac{\frac{x+1}{x}-\ln|x|}{(x+1)^2}=\frac{1}{x(x+1)^3}\end{cases}$

$\Rightarrow\begin{cases}C_1'=\dfrac{\det\begin{pmatrix}0&\frac{\ln|x|}{x+1}\\\frac{1}{x(x+1)^3}&\frac{\frac{x+1}{x}-\ln|x|}{(x+1)^2}\end{pmatrix}}{\det\begin{pmatrix}\frac{1}{x+1}&\frac{\ln|x|}{x+1}\\-\frac{1}{(x+1)^2}&\frac{\frac{x+1}{x}-\ln|x|}{(x+1)^2}\end{pmatrix}}=-\dfrac{\frac{\ln|x|}{x(x+1)^4}}{\frac{1}{x(x+1)^3}}=-\dfrac{\ln|x|}{(x+1)^2}\\[2ex]C_2'=\dfrac{\det\begin{pmatrix}\frac{1}{x+1}&0\\-\frac{1}{(x+1)^2}&\frac{1}{x(x+1)^3}\end{pmatrix}}{W}=\dfrac{\frac{1}{x(x+1)^4}}{\frac{1}{x(x+1)^3}}=\dfrac{1}{(x+1)^2}\end{cases}$ ✓

$\Rightarrow\begin{cases}C_1=-\ln\frac{|x|}{x+1}+\frac{\ln x}{x+1}\text{ (int by parts)}\\C_2=-\frac{1}{x+1}\end{cases}$ ✓

−

general solution $y=\frac{C_1+C_2\ln|x|}{x+1}+\frac{\ln\frac{x}{x+1}}{x+1}-\frac{2\ln|x|}{(x+1)^2}$

−2

Markus An

1) Variation of Constants

$$x^2y''+4xy'+(x^2+2)y=\frac{1}{1+\cos x}$$

$\Rightarrow$ Consider the homogeneous equation: $x^2y''+4xy'+(x^2+2)y=0$

With the hint from the question, we assume $y_1=x^k\sin x$

insert into the ODE:

$$x^2\cdot\left(k(k-1)x^{k-2}\sin x+kx^{k-1}\cos x+kx^{k-1}\cos x-x^k\sin x\right)$$
$$+4x\cdot\left(kx^{k-1}\sin x+x^k\cos x\right)+(x^2+2)\cdot x^k\sin x=0$$

$\Rightarrow$ $k=-2$

thus $y_1=x^{-2}\sin x$ is one solution

Similarly we assume $y_2=x^k\cos x$, and fortunately we get $y_2=x^{-2}\cos x$

Hence the general solution of the homogeneous ODE is:

$$y_h=\frac{C_1\sin x+C_2\cos x}{x^2}$$

en we use "variation of constants", and get:

$$\begin{cases} C_1'\frac{\sin x}{x^2}+C_2'\cdot\frac{\cos x}{x^2}=0 \\ C_1'\cdot\frac{x\cos x-2\sin x}{x^3}+C_2'\cdot\frac{-x\sin x-2\cos x}{x^3}=\frac{1}{x^2(1+\cos x)}\end{cases}$$

's rule

$$\Rightarrow C_1'=\frac{\det\begin{pmatrix}0 & \frac{\cos x}{x^2}\\ \frac{1}{x^2(\cos x+1)} & \frac{-x\sin x-2\cos x}{x^3}\end{pmatrix}}{W}=\frac{-\frac{\cos x}{x^4(\cos x+1)}}{-\frac{1}{x^4}}=\frac{\cos x}{\cos x+1}$$

$$C_2'=\frac{\det\begin{pmatrix}\frac{\sin x}{x^2} & 0\\ \frac{x\cos x-2\sin x}{x^3} & \frac{1}{x^2(1+\cos x)}\end{pmatrix}}{W}=\frac{\frac{\sin x}{x^4(1+\cos x)}}{-\frac{1}{x^4}}=-\frac{\sin x}{1+\cos x}$$

$$\Rightarrow C_1=\int\frac{\cos x}{\cos x+1}dx=\int\left(1-\frac{1}{\cos x+1}\right)dx=\int\left(1-\frac{1}{2\cos^2\frac{x}{2}}\right)dx=x-\tan\frac{x}{2}$$

$$C_2=\int-\frac{\sin x}{1+\cos x}dx=\int\frac{1}{1+\cos x}d\cos x=\ln|\cos x+1|$$

absolute value because $\cos x+1\geq 1$

general solution:

$$y=\frac{C_1\sin x+C_2\cos x}{x^2}+\frac{x-\tan\frac{x}{2}}{x^2}\sin x+\frac{\ln|\cos x+1|}{x^2}\cos x$$

$$(x-2)^2 y'' - 7(x-2)y' + 25y = \sqrt{(x-2)^5}$$

consider the homogeneous equation: $(x-2)^2y'' - 7(x-2)y' + 25y = 0$

since this is an Euler-Cauchy ODE,

consider the characteristic equation

more step in your exam.

$$\lambda^2 - 8\lambda + 25 = 0$$

$$\Rightarrow \quad \lambda = 4 \pm 3i$$

$\Rightarrow$ the general solution of the ODE:

$$\cancel{y_h = (x-2)^4\left(c_1\cos[3(x-2)] + c_2\sin[3(x-2)]\right)}$$

$$y_h = c_1(x-2)^{4+3i} + c_2(x-2)^{4-3i}$$

tion of constants:

$$\begin{cases} c_1'\cdot(x-2)^{4+3i} + c_2'(x-2)^{4-3i} = 0 \\ c_1'\cdot(4+3i)(x-2)^{3+3i} + c_2'\cdot(4-3i)(x-2)^{3-3i} = (x-2)^{\frac{1}{2}} \end{cases}$$

rule

$\Rightarrow$

$$\begin{cases} c_1' = \dfrac{\det\begin{pmatrix} 0 & (x-2)^{4-3i} \\ (x-2)^{\frac{1}{2}} & (4-3i)(x-2)^{3-3i}\end{pmatrix}}{W} = \dfrac{-(x-2)^{\frac{9}{2}-3i}}{-6i(x-2)^7} = -\dfrac{i}{6}\cdot(x-2)^{-\frac{5}{2}-3i} \\ c_2' = \dfrac{\det\begin{pmatrix} (x-2)^{4+3i} & 0 \\ (4+3i)(x-2)^{3+3i} & (x-2)^{\frac{1}{2}}\end{pmatrix}}{W} = \dfrac{(x-2)^{\frac{9}{2}+3i}}{-6i(x-2)^7} = \dfrac{i}{6}\cdot(x-2)^{-\frac{5}{2}+3i} \end{cases}$$

$$c_1 = \frac{2+i}{45}(x-2)^{-\frac{3}{2}-3i}, \quad c_2 = \frac{2-i}{45}(x-2)^{-\frac{3}{2}+3i}$$

$$y = c_1(x-2)^{4+3i} + c_2(x-2)^{4-3i} + \frac{2+i}{45}\cdot(x-2)^{\frac{5}{2}} + \frac{2-i}{45}\cdot(x-2)^{\frac{5}{2}}$$

e in real representation:

$$y = (x-2)^4\left(c_1\cos(3x-6) + c_2\sin(3x-6)\right) + \frac{4}{45}(x-2)^{\frac{5}{2}}$$

$$y = (x-2)^4\left(c_1\cos(3\ln|x-2|) + c_2\sin(3\ln|x-2|)\right) + \frac{4}{45}(x-2$$