Homogeneous Dirichlet : $U_t = DU_{xx}$ $u(0,x) = \phi(x) = \sin\left(\frac{\pi x}{L}\right)$

$u(t,x) = e^{-\frac{D\pi^2}{L^2}t} \cdot \sin\left(\frac{\pi}{L}x\right)$ $u(t,0) = 0$

$u(t,L) = 0$

Inhomogeneous Dirichlet : $U_t = U_{xx} + x$ $u(0,x) = \phi(x) = 3\sin(2\pi/L\, x) + 2(1-x)$.

$u(t,x) = 3\sin(2\pi x) \cdot e^{-4\pi^2 t} + 2 + (t-2)x$ $u(t,0) = 2$

$u(t,L) = t$.

Neumann : in x (homogeneous) $U_t = DU_{xx}$ $u(x,0) = \phi(x) = \cos\left(\frac{\pi}{a}x\right)$

$U_x(0,t) = U_x(a,t) = 0$

$u(t,x) = e^{-\frac{\pi^2}{a^2}Dt} \cdot \cos\left(\frac{\pi x}{a}\right)$

Find analytical Solution to Neumann in t

" " " inhomogeneous in x.

" " " t.

Mixed: $U_t = DU_{xx}$ $U(0,x) = \phi(x) = 1 - \cos\left(\frac{3}{2L}\pi x\right)$

Neumann in x

homogeneous in Neumann. $U_x(0,t) = 0$ $u(L,t) = 1$.

$u(t,x) = 1 - \cos\left(\frac{3}{2L}\pi x\right) e^{-D\left(\frac{9\pi^2}{4L^2}\right)t}$

r.h.S funct. dep on x (homo Dirich) $U_t = DU_{xx} + f(x)$ $\phi(x) = \sin\left(\frac{2\pi x}{a}\right)$

$u(t,0) = 0$ $u(t,a) = 0$ $f(x) = \sin\left(\frac{\pi}{a}x\right)$

$u(t,x) = \frac{a^2}{D\pi^2}\left(1 - e^{-\frac{D\pi^2 t}{a^2}}\right)\sin\left(\frac{\pi}{a}x\right) + e^{-\frac{4D\pi^2}{a^2}t} \sin\left(\frac{2\pi}{a}x\right)$

r.h.S funct. dep on x & t. (Homo Dirich.) $U_t = DU_{xx} + f(x,t)$ $\phi(x) = \sin\left(\frac{2\pi x}{a}\right)$

$U(t,0) = 0$ $u(t,a) = 0$ $f(x) = e^{-\frac{D\pi^2}{a^2}t} \sin\left(\frac{\pi}{a}x\right)$

$u(x,t) = \sin\left(\frac{\pi x}{a}\right) \cdot t \cdot e^{-\frac{D\pi^2}{a^2}t} + e^{-D\cdot 4 \cdot \frac{\pi^2 t}{a^2}} \cdot \sin\left(\frac{2\pi}{a}x\right)$

Periodic boundary condition : (inf. domain).

$U_t = DU_{xx}$ $u(x,0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$.

$U(x,t) = \frac{1}{\sqrt{2\pi + Dt/\pi}} \cdot e^{-\frac{x^2}{2\left(1 + \frac{Dt}{2\pi^2}\right)}}$

Variable kappa in t: $U_t = t\, U_{xx}$ $u(0,x) = \phi(x) = \sin\left(\frac{\pi}{L}x\right)$

$u(t,0) = u(t,L) = 0$

$u(x,t) = \sin\left(\frac{\pi}{L}x\right) \cdot e^{-\frac{\pi^2 t^2}{2L^2}}$