A Statistical Analysis Of Cryptocurrencies

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Abstract

Our goal is to analyze the statistical properties of various cryptocurrencies in different Market Capitalization (Henceforth referred to as Market Cap.) levels. We find suitable distributions to characterize their exchange rates versus the US Dollar and find the best fit parameters. We see that no particular distribution fits all the cryptocurrencies the best, and hence try to understand the results and properties of each currency from their own best fit distributions by conducting log likelihood and information criterion tests on them. We find that for the higher-ranked cryptocurrencies, the laplace distribution fits very well, and the asymmetric student's t and the normal inverse gauss distributions work for the mid-to-lower ranked cryptocurrencies. To further predict the stability and risk factor of each of the currencies, we plot their P-P plots, Value-at-Risk plots and Q-Q plots with the log returns of Euro, to see how they compare to a traditional currency. Finally, we plot their dynamic volatility, and find that the higher ranked currencies like Bitcoin are less volatile. These results are important for investment and risk management purposes.

1 Introduction

Cryptocurrency is a decentralized system of currency which works through distributed ledger technology, typically a blockchain, that serves as a public financial transaction database. Instead of being based on traditional trust, it uses cryptographic methods and a decentralized authority to keep track of its transactions. This leads to many advantages such as high liquidity, lower transaction costs, and anonymity, to name just a few.

A formal definition for a cryptocurrency has been put forward by Jan Lansky, PhD, The University of Finance and Administration according to whom a cryptocurrency should fulfill six requirements:

1. The system does not require a central authority, its state is maintained through distributed consensus.
2. The system keeps an overview of cryptocurrency units and their ownership.
3. The system defines whether new cryptocurrency units can be created. If new cryptocurrency units can be created, the system defines the circumstances of their origin and how to determine the ownership of these new units.
4. Ownership of cryptocurrency units can be proved exclusively cryptographically.
5. The system allows transactions to be performed in which ownership of the cryptographic units is changed. A transaction statement can only be issued by an entity proving the current ownership of these units.

6. If two different instructions for changing the ownership of the same cryptographic units are simultaneously entered, the system performs at most one of them.

The generally accepted oldest cryptocurrency is Bitcoin, with its inception dating back to 2009. By late 2010, the first of what would eventually be dozens of similar cryptocurrencies – including popular alternatives like Litecoin – began appearing. The first public Bitcoin exchanges appeared around this time as well. In late 2012, WordPress became the first major merchant to accept payment in Bitcoin. Others, including Newegg.com (an online electronics retailer), Expedia, and Microsoft, followed. Dozens of merchants started viewing the world's most popular cryptocurrency as a legitimate payment method. Bitcoin and its various contemporaries would keep going up in price until Dec 2017 where the peak would happen (For Bitcoin this was $19,166.98 on Dec 17, 2017). Since then, most cryptocurrencies have fallen in price generally. As of April 2020, Bitcoin sits at around $6000.

As of April 2nd 2020, there are 5,290 different cryptocurrencies (CoinMarketCap 2020). However, Bitcoin, garners over 65.3% of the market, commonly referred to as the BTC Dominance. The top 10 cryptocurrencies are responsible for 90% of the market cap. This shows that most currencies have not garnered nearly as much interest as the top ones have. In our paper, we aim to analyse 8 different cryptocurrencies, at least 4 years old, at various different Market Cap. levels. Cryptocurrencies are generally ranked using their Market Cap. We have used the following cryptocurrencies:

1. Bitcoin - Market Cap. rank 1; Founded January 2009.
2. Ethereum - Market Cap. rank 2; Founded July 2015.
3. Litecoin - Market Cap. rank 7; Founded October 2011.
4. Dogecoin - Market Cap. rank 33; Founded December 2013.
5. Factom - Market Cap. rank 138; Founded November 2014.
6. Nexus - Market Cap. rank 194; Founded September 2014.
7. PhoenixCoin - Market Cap. rank 1126; Founded May 2013
8. Paycoin - Market Cap. rank 1812; Made available to public in January 2015.

All ranks are of 11:27 pm Wednesday, 1 April 2020 Greenwich Mean Time (GMT) according to CoinMarketCap 2020.

Though the days of hype for Cryptocurrencies as being a high risk high reward investment are behind, we aim to look at a more mature side of various cryptocurrencies as a legitimate method of transactions and how its volatility stands today. Using the data, we aim to predict the rise and fall of said cryptocurrencies and their corresponding trends.

When considering Bitcoin, the largest cryptocurrency, much study has already went into the topic. Most of it was during the early days of cryptocurrency and one such study

(Hencic and Gourieroux 2014) indicates that Bitcoin shows local trends which might be because of online trading trends. Another (Sapurki and Kokkinaki 2104) compares the volatility of Bitcoin to several traditional currencies. Though Bitcoin has a higher annualised volatility, it can be considered stable when transaction volume is taken into consideration. However, after the Bitcoin peak of 2017, the peaks have become less abrupt and have started following a more stable trend.

Hence, it is important we look at the volatility of cryptocurrencies. Though different papers define volatility in an array of ways, here, the term volatility will be defined as the spread of daily exchange rates and log returns of the exchange rates of the cryptocurrencies over the time period considered.

The paper is organised as follows — In Section 2, an overview of the data used in the analysis including descriptions and sources is given. Section 3 aims to examine the statistical properties of the cryptocurrencies by fitting a wide range of parametric distributions to the data, and plotting Q-Q, P-P and VaR plots against the best fits. Section 4 provides a discussion of our results. And Finally, Section 5 concludes and summarizes the findings.

Note: Any value which is denoted as 0.000 is not to be taken as zero and instead should be considered as the first three decimal points being zero.

2 Data

The data used in this paper are the historical global price indices of cryptocurrencies and have been gathered from the historical data available in CoinMarketCap 2020. The data has been analysed from October 10th 2015 to August 31st 2019. We deliberately chose October 2015 as the start date as Ethereum started gaining traction around that period. As mentioned before, we have chosen 8 different currencies to get an accurate picture of cryptocurrencies at various market caps. In the following subsection, we aim to give a brief introduction to each of them.

2.1 Bitcoin

The first and most popular of its kind, Bitcoin was founded in 2009 by an unknown person or group of people who went by the same Satoshi Nakamoto. Though the concept of a decentralized cryptography-based currency dates back to the late 80s, Bitcoin was the first realization of the same. The decentralization means that no one user has control over data of the users. All users within the system must adhere to a certain set of rules with each transaction legitimized using proof of work. It uses Blockchain technology to keep track of transactions with longer block chains generally being the accepted ones. As mentioned, though it enables a host of advantages over traditional currency, it has been criticized for its use in illegal transactions, its high electricity consumption, price volatility, and thefts from exchanges. Some economists, including several Nobel laureates, have characterized it as a speculative bubble.

2.2 Ethereum

Though Ethereum shares its foundation with Bitcoin, it aims to be a multipurpose platform with the digital currency Ether being just a component of the application. And while Bitcoin has a cap of 21 million total mined coins, Ether virtually has no limit with the mining time being close to 10 seconds versus Bitcoins' 10 minutes. Its internal code is also Turing complete. Ethereum uses an accounting system where values in Wei are debited from accounts and credited to another, as opposed to Bitcoin's UTXO system, which is more analogous to spending cash and receiving change in return. Their ultimate goal is to completely change the relationship between companies and their audiences.

2.3 Litecoin

Made in 2011 with help from the Bitcoin community, it works on the same principles as Bitcoin with several improvements in various departments. As do many other contemporaries, it significantly decreases confirmation time for transactions and also uses scrypt as a proof-of-work algorithm. In effect, both Ethereum and Litecoin as well as many of the other contenders makes it possible for standard computers to verify transactions vis-à-vis mining Litecoins compared to the industrial-level hardware required for mining bitcoins.

2.4 Dogecoin

Originating from a popular internet meme back in late 2013, it was originally meant as a joke currency by its inventors - an Australian brand and marketing specialist, and a programmer in Portland, Oregon. However, it quickly gained traction and reached a market cap. of US$60 million as early as January 2014. It uses the same architecture as Litecoin enabling similar advantages over Bitcoin such as faster transaction processing times. Compared with other cryptocurrencies, Dogecoin had a fast initial coin production schedule: 100 billion coins were in circulation by mid-2015, with an additional 5.256 billion coins every year thereafter. As of 30 June 2015, the 100 billionth Dogecoin had been mined. While there are few mainstream commercial applications, the currency has gained traction as an Internet tipping system, in which social media users grant Dogecoin tips to other users for providing interesting or noteworthy content. This was popularized after Bitcoin became too expensive for the same.

2.5 Factom

Factom is the first usable blockchain technology to solve real-world business problems by providing an unalterable record-keeping system. The Factom Protocol describes itself as an 'open source, decentralized data integrity protocol built by an international coalition of companies that extends the security of blockchain to any data type'. The protocol is reportedly easy to integrate with any system, thereby providing a low, fixed cost source of indisputable truth and verification for business and government. The fixed cost and dual token design of the protocol reportedly enables subscription models where customers do not have to hold tokens despite their being used automatically in the background. The team views the Factom protocol as a global utility for companies to provide seamless access to applications with no unnecessary interaction with the integrated, underlying technology.

2.6 Nexus (NXS)

Nexus was founded by Colin Cantrell in September 2014 and was initially called CoinShield before changing to Nexus. It started as a basic bitcoin fork, but Nexus has had several activations since it went live. Now it uses two different proof-of-work chains and a proof-of-stake system. And with the new Tritium upgrade, it also has smart-contract functionality with competitive transaction times. Instead of building upon older technologies, Nexus has started from the ground up, whose culmination is known as the Three-Dimensional Chain which transforms the Ledger into a multi-layered processing system, in order to scale the protocol securely with a high degree of decentralization. It chains together cryptographic primitives into a three dimensional immutable object (a 3D block), and has three core dimensions: reputation channels (X), immutability or authenticity (Y), and time (Z). This architecture is being deployed through the TAO framework.

2.7 PhoenixCoin

It is an open source decentralised P2P currency. It was released as a further development of Litecoin. The Phoenixcoin distributed network is proof-of-work based. It generates coins at a decreasing rate and is scheduled to produce 98 million coins. Though it showed promise early on, it ended up in disaster down the road with the 5 forks it went through significantly changing its technical specifications. The forks together, with some of them being unexpected and causing great losses to the miners, led to the eventual fall of the currency.

2.8 Paycoin

Paycoin (XPY) is a decentralised open-source cryptocurrency forked from Bitcoin and launched in late 2014 by the founders of GAWMiner.com, a well-known provider of Bitcoin mining products.Though it made many promises in length in its XPY white paper that was widely distributed on the web four years ago. They made such claims as "Paycoin is the first cryptocurrency to employ a HybridFlex blockchain, expanding on the work of Satoshi Nakamoto, in order to produce a light, efficient, highly-secure blockchain, accessible to nearly all internet connected devices. The HybridFlex model produces a cryptocurrency that offers ease of consumer adoption and use, ease of infrastructure report, while also facilitating large-scale investor entry into the cryptocurrency industry; Augmenting the creation, management and deployment of a competitive global payments and commerce system."

Summary statistics of the exchange rates and log returns of the exchange rates of the seven cryptocurrencies are given in Tables 1 and 2, respectively. Table 1 gives us an estimate of the raw ”worth” of a currency. As seen from the normal and log returns, Dogecoin has the smallest Mean and Median values which agrees with the fact that they are generally used for tipping as opposed to a standardized payment method. It also has the lowest Minimum, Maximum which further enforces this. From the log returns table, we can see that Bitcoin has the smallest value of SD as well as Variance among all of the given cryptocurrencies. Bitcoin, being the most popular, also has the highest Median and Mean values. Similar conclusions can be made about all the other currencies present.

Table 1: Summary statistics of daily exchange rates of Bitcoin, Ethereum, Dogecoin, Litecoin, Nexus, Factom, Paycoin, Phoenixcoin from October 10th 2015 to August 31st 2019

Statistics	Bitcoin	Ethereum	Dogecoin	Litecoin	Nexus	Factom	Paycoin	PhoenixCoin
Minimum	244.195	0.451	0.000	3.015	0.000	0.079	0.001	0.000
Maximum	19531.55	1351.675	0.016	355.48	13.635	75.21	0.179	0.032
Median	3641.782	146.49	0.001	41.54	3.365	5.757	0.013	0.002
Mean	4300.316	215.690	0.002	54.165	0.823	9.997	0.018	0.003
Kurtosis	0.28	2.758	6.094	3.069	24.924	5.700	12.928	6.722
Skewness	0.916	1.647	1.874	1.660	4.099	2.112	2.780	2.202
SD	3939.964	253.243	0.002	61.312	1.365	11.478	0.016	0.004
Variance	15523322	64132.085	0.000	3759.250	1.865	131.760	0.000	0.000
Range	19287.355	1351.223	0.016	352.465	13.634	75.130	0.178	0.032
IQR	6192.666	287.783	0.002	74.782	1.018	12.847	0.013	0.005
Q1	658.227	11.51	0.000	3.9	0.030	2.35	0.008	0.000
Q3	6850.893	299.293	0.003	78.682	1.048	15.1975	0.022	0.005

Table 2: Summary statistics of daily log exchange rates of Bitcoin, Ethereum, Dogecoin, Litecoin, Nexus, Factom, Paycoin, Phoenixcoin from October 10th 2015 to August 31st 2019

Statistics	Bitcoin	Ethereum	Dogecoin	Litecoin	Nexus	Factom	Paycoin	PhoenixCoin
Minimum	-0.195	-0.262	-0.380	-0.391	-0.858	-0.835	-2.150	-1.601
Maximum	0.169	0.325	0.356	0.230	1.01733682	0.303	2.206	0.836
Median	-0.002	-0.001	0.002	0.001	0.001	0.001	0.005	0.008
Mean	0.002	-0.003	-0.002	-0.002	-0.003	-0.002	0.000	-0.002
Kurtosis	3.682	3.684	10.821	9.027	20.444	14.505	36.560	19.897
Skewness	0.206	-0.165	-1.100	-1.016	-0.188	-1.412	-0.446	-1.583
SD	0.032	0.052	0.056	0.047	0.102	0.072	0.191	0.138
Variance	0.001	0.002	0.003	0.002	0.010	0.005	0.036	0.019
Range	0.364	0.587	0.736	0.622	1.875	1.138	4.356	2.438
IQR	0.028	0.044	0.033	0.035	0.074	0.064	0.110	0.101
Q1	-0.017	-0.024	-0.016	-0.017	-0.036	-0.032	-0.050	-0.0475
Q3	0.010	0.019	0.017	0.017	0.038	0.032	0.059	0.053

3 Distributions Fitted

In the previous section, we have briefly examined the summary statistics for both the exchange rates and the log returns of exchange rates of the eight cryptocurrencies. In this section, we aim to provide a visual representation of the distribution of the log returns. Figure 1 shows the daily exchange rate of the eight cryptocurrencies over the years and Figure 2 shows the histograms of the daily log returns of our chosen currencies. From this, we can understand that they show significant deviation from a normal distribution. Therefore, we should try and fit other parametric distributions to the data.

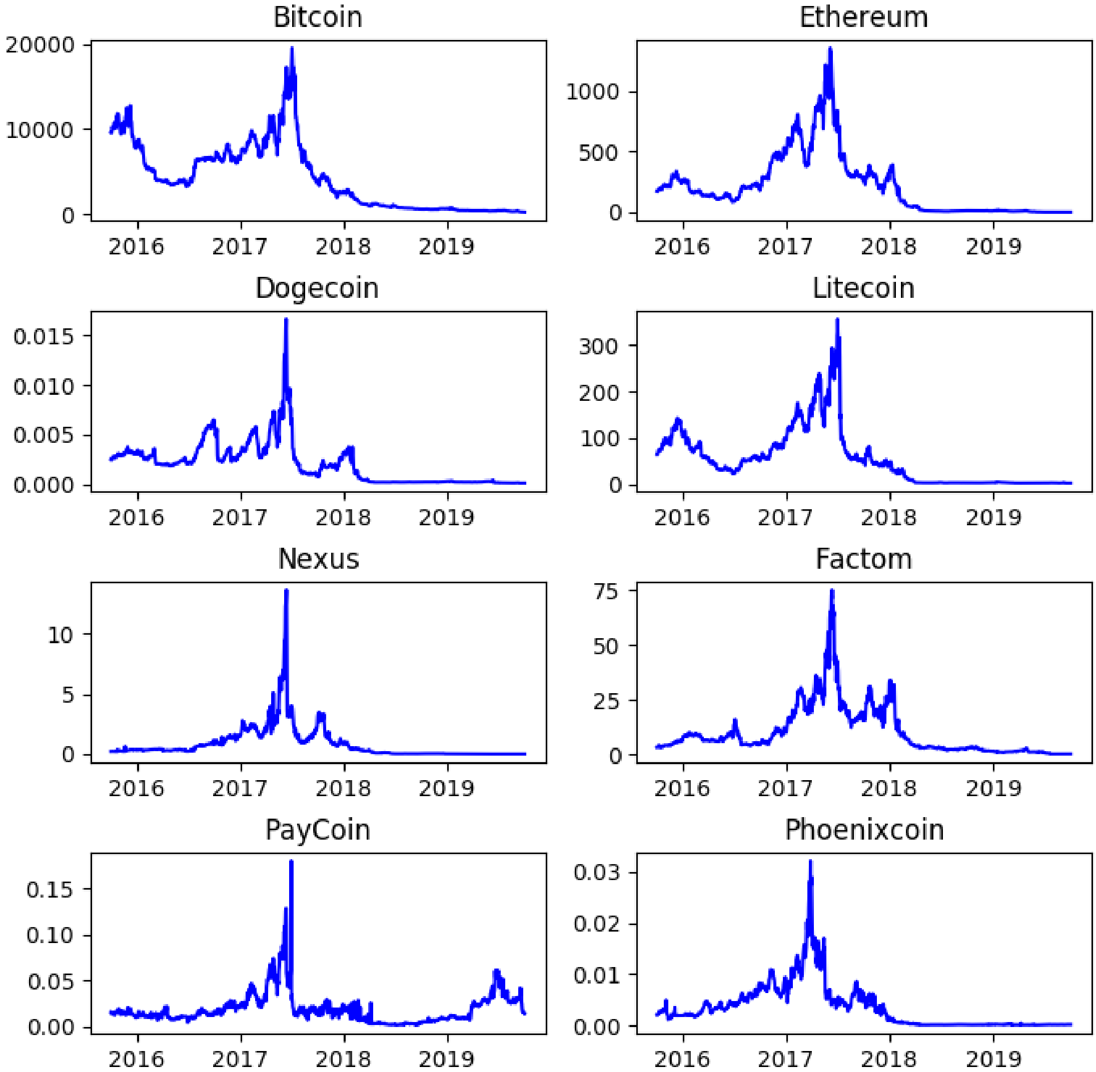


Figure 1: Daily returns of the exchange rates of the eight cryptocurrencies versus the U.S. Dollar from October 10th 2015 to August 31st 2019.

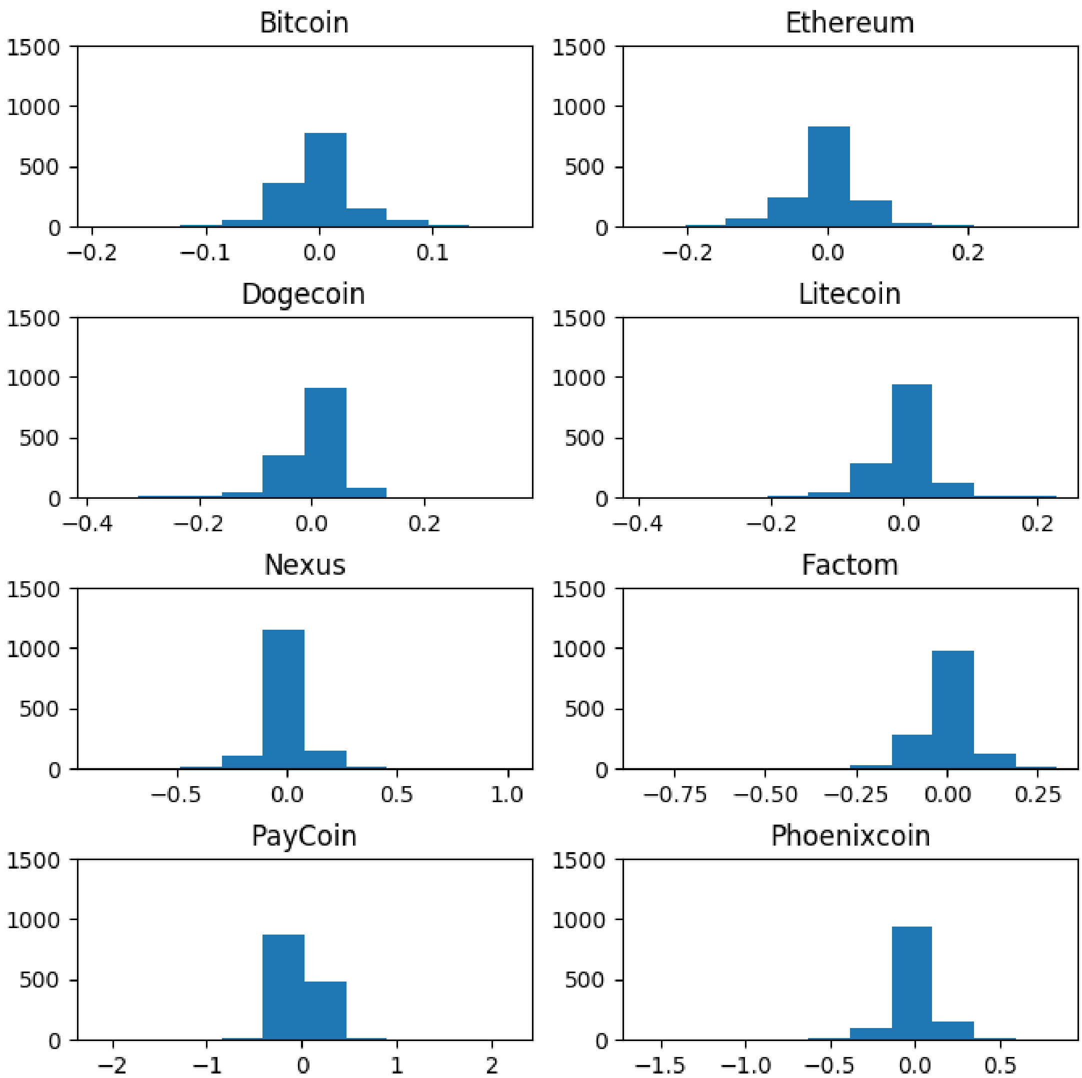


Figure 2: Histograms of daily log returns of the exchange rates of the eight cryptocurrencies versus the U.S. Dollar from October 10th 2015 to August 31st 2019.

Let X be used to denote a continuous random variable which represents the log returns of the exchange rate of the cryptocurrency of interest and f(x) be the probability density function (pdf) of X. Let X follow one of the following given distributions:

1. **Student's t Distribution**

$$f(x) = \frac{\kappa(\nu)}{\sigma}\left[1 + \frac{(x-\mu)^2}{\sigma^2\nu}\right]^{-(1+\upsilon)/2}$$

$$for - \infty < x < -\infty, -\infty < \mu < -\infty, \sigma > 0, \nu > 0$$

where,

$$\kappa(\nu) = \sqrt{\nu}B(\nu/2, 1/2)$$

and B is the beta function.

2. **Laplace Distribution**

$$f(x) = \frac{1}{2\sigma} exp\left(\frac{-|x-\mu|}{\sigma}\right)$$

$$for -\infty < x < -\infty, -\infty < \mu < -\infty, \sigma > 0$$

3. **Asymmetric Student's t Distribution**

$$f(x) = \frac{1}{\sigma}\begin{cases} \frac{\alpha}{\alpha^*}\kappa(\nu_1)\left\{1+\frac{1}{\nu_1}\left[\frac{x-\mu}{2\sigma\alpha^*}\right]^2\right\}^{-\frac{\nu_1+1}{2}} & x \le \mu \\ \frac{1-\alpha}{1-\alpha^*}\kappa(\nu_2)\left\{1+\frac{1}{\nu_2}\left[\frac{x-\mu}{2\sigma(1-\alpha^*)}\right]^2\right\}^{-\frac{\nu_2+1}{2}} & x > \mu \end{cases}$$

$$for -\infty < x < -\infty, -\infty < \mu < -\infty, 0 < \alpha < 1, \nu_1 > 0, \nu_2 > 0$$

where,

$$\alpha^* = \frac{\alpha\kappa(\nu_1)}{\alpha\kappa(\nu_1) + (1-\alpha)\kappa(\nu_2)}$$

4. **Normal Inverse Gaussian Distribution**

$$f(x) = \frac{(\gamma/\delta)^\lambda \alpha}{\sqrt{2\pi}\kappa_{-1/2}(\delta\gamma)} exp\left[\beta(x-\mu)\right]\left[\delta^2 + (x-\mu)^2)\right]^{-1}\kappa_{-1}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)$$

$$for -\infty < x < -\infty, -\infty < \mu < -\infty, \delta > 0, \alpha > 0, \beta > 0$$

where,

$$\gamma = \sqrt{\alpha^2 - \beta 2}$$

and

$$K_\nu(.)$$

denotes the modified Bessel function of the second kind of order ν defined by,

$$f(x) = \begin{cases} \frac{\pi csc(\pi\nu)}{2}\left[I\_\nu(x) - I_\nu(x)\right] & if \nu \notin \mathbb{Z} \\ \lim_{\mu\to\nu}\kappa_\mu(x) & if \nu \in \mathbb{Z} \end{cases}$$

where $I_\nu(.)$ denotes the modified Bessel function of the kind of order ν defined by

$$I_\nu(x) = \sum_{k=0}^{\infty}\frac{1}{\tau(k+\nu+1)k!}\left(\frac{x}{2}\right)^{2k+\nu}$$

where $\tau(.)$ denotes the gamma function defined by

$$\tau(a) = \int_0^\infty t^{a-1} exp(-t)dt$$

5. **Beta Distribution**

$$f(x) = \frac{1}{B} x^{\alpha-1}(1-x)^{\beta-1}$$

$$\alpha > 0, \beta > 0$$

where, B is the beta function defined by,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt$$

All distributions except Laplace are heavy-tailed here. This is is expected as they are common when dealing with financial data. The asymmetric student's t distribution is a generalization of the student's t distribution with the scale as well as the tail parameter allowed to be on two different sides of μ. This distribution becomes useful if the positive log returns have a different heavy tail behaviour compared to the negative log return behaviour.

Maximum likelihood Estimation was used to fit each distribution.

Discrimination among the non-nested distributions were made using various criterion as mentioned below:

- **Akaike Information Criterion defined by**

$$AIC = 2k - 2\ln L(\hat{\omega})$$

- **Bayesian Information criterion defined by**

$$BIC = k\ln L - 2\ln L(\hat{\omega})$$

- **Corrected Akaike Information Criterion defined by**

$$AICc = AIC + \frac{2k(k+1)}{n-k-1}$$

- **Hannan-Quinn Criterion defined by**

$$HQC = -2\ln L(\hat{\omega}) + 2k\ln\ln n$$

All these criteria utilize the maximum likelihood estimate with a smaller value of said criteria resulting in a better fit. For AIC, the larger the log likelihood, the better the fit is. However, it imposes a penalty as the number of parameters increase and it discourages over-fitting. BIC is similar with the only difference being the definition of the penalty term. In general, the Hannan-Quinn criterion penalises models with a greater number of parameters when compared to both the Akaike and Bayesian information criteria.

We can also use the K-S test as a Goodness of Fit measure:

- **The K-S Test defined by**

$$KS = \sup_x \left| \frac{1}{n} \sum_{i=1}^{n} I\{x_i < x\} - \hat{F}(x) \right|$$

where $I(.)$ denotes the indicator function and $\hat{F}(.)$ the maximum likelihood estimate of F(x).

Once again, the smaller the value, the better the fit.

4 Results

In this section, we shall see the best fit distributions of the cryptocurrencies and the results for the log returns of the cryptocurrencies we have seen in Section 2, and the results of the tests on the distributions discussed in Section 3.

4.1 Best-fit distributions and results

To fit the distributions to the data, we used functions that employed Maximum Likelihood Estimation to find the best fit parameters. All five distributions were fitted against the data of each cryptocurrency. The values of AIC, BIC, AICc, HQC and the log likelihoods were used to compare models. The best fit distributions of the data along with their parameter estimates are given in Tables 3 and 4.

Table 3: Best-fit distributions for the top tier cryptocurrencies along with their parameter estimates

Cryptocurrency	**Best-Fit Distribution**	**Parameter Estimates**
Bitcoin	Laplace	$\mu = -0.00236$
		$\sigma = 0.02192$
Ethereum	Laplace	$\mu = -0.00135$
		$\sigma = 0.03571$
Dogecoin	Normal inverse Gaussian	$\mu = 0.11945$
		$\sigma = -0.02653$
		$\alpha = 0.00241$
		$\beta = 0.01998$
Litecoin	Asymmetric Student's T	$\mu = 1.86933$
		$\sigma = -0.08195$
		$\nu 1 = 0.00174$
		$\nu 2 = 0.02162$

Table 4: Best-fit distributions for the bottom tier cryptocurrencies along with their parameter estimates

Cryptocurrency	Best-Fit Distribution	Parameter Estimates
Nexus	Asymmetric Student's T	$\mu = 0.28729$
		$\sigma = -0.07008$
		$\nu 1 = 0.00903$
		$\nu 2 = 0.05001$
Factom	Normal Inverse Gauss	$\mu = 0.43917$
		$\sigma = -0.05533$
		$\alpha = 0.00368$
		$\beta = 0.04732$
Paycoin	Normal Inverse Gauss	$\mu = 0.11886$
		$\sigma = -0.00657$
		$\alpha = 0.00342$
		$\beta = 0.06298$
Phoenixcoin	Normal Inverse Gauss	$\mu = 0.20735$
		$\sigma = -0.03720$
		$\alpha = 0.00919$
		$\beta = 0.06181$

As evident from the results, we can see that there is not one distribution that fits all the datasets best. Bitcoin and Ethereum, the highest and the second highest ranked cryptocurrencies currently, follow a Laplace distribution, while Litecoin and Nexus follow an Asymmetric Student's T distribution. The lowest ranked currencies Paycoin and Phoenixcoin, along with moderately ranked Factom and Dogecoin follow a Normal Inverse Gauss distribution. We further bring these distributions under analysis by plotting QQ and PP plots, and applying the KS test.

Table 5: Fitted distributions and results of the daily log returns on exchange rates of **Bitcoin**

Distributions	-lnL	AIC	BIC	AICc	HQC
Student's T	-2999.00112	-5992.00224	-5983.48401	-5991.98530	-5986.10869
Laplace	-3022.16292	-6040.32584	-6029.80761	-6040.31738	-6036.39681
Asymmetric Students T	-2999.14269	-5990.28539	-5983.76716	-5990.25714	-5982.42733
Normal inverse Gaussian	-3013.88896	-6013.25970	-6019.77793	-6019.74968	-6011.91986
Beta	-2857.45948	-5700.40073	-5706.918966	-5706.89071	-5699.06090

Table 6: Fitted distributions and results of the daily log returns on exchange rates of **Ethereum**

Distributions	-lnL	AIC	BIC	AICc	HQC
Student's T	-2313.32487	-4620.64974	-4612.13151	-4620.63280	-4614.75619
Laplace	-2329.38316	-4654.76633	-4644.24810	-4654.75787	-4650.83730
Asymmetric Students T	-2316.84652	-4625.69305	-4619.17482	-4625.66480	-4617.83498
Normal inverse Gaussian	2303.68324	-4592.84826	-4599.36649	-4599.33824	-4591.50843
Beta	-2171.63473	-4328.75123	-4335.26946	-4335.24121	-4327.41139

Table 7: Fitted distributions and results of the daily log returns on exchange rates of **Dogecoin**

Distributions	-lnL	AIC	BIC	AICc	HQC
Student's T	-2491.675836	-4977.351673	-4968.833441	-4977.334736	-4971.458124
Laplace	-2438.888332	-4873.776664	-4863.258432	-4873.768201	-4869.847632
Asymmetric Students T	-2493.313799	-4978.627599	-4972.109366	-4978.59935	-4970.769534
Normal inverse Gaussian	2510.370209	-5006.222186	-5012.740418	-5012.71217	-5004.882354
Beta	-2080.882109	-4147.245986	-4153.764218	-4153.73597	-4145.906154

Table 8: Fitted distributions and results of the daily log returns on exchange rates of **Litecoin**

Distributions	-lnL	AIC	BIC	AICc	HQC
Student's T	-2604.693373	-5203.386746	-5194.868514	-5203.369809	-5197.493197
Laplace	-2595.476949	-5186.953898	-5176.435665	-5186.945435	-5183.024865
Asymmetric Students T	-2605.3693	-5202.7386	-5196.220368	-5202.710352	-5194.880536
Normal inverse Gaussian	-2620.427048	-5226.335865	-5232.854097	-5232.825848	-5224.996032
Beta	-2331.416892	-4648.315552	-4654.833784	-4654.805536	-4646.97572

Table 9: Fitted distributions and results of the daily log returns on exchange rates of **Nexus**

Distributions	-lnL	AIC	BIC	AICc	HQC
Student's T	-1600.967963	-3195.935925	-3187.417693	-3195.918988	-3190.042377
Laplace	-1561.49173	-3118.98346	-3108.465228	-3118.974997	-3115.054428
Asymmetric Students T	-1604.724441	-3201.448882	-3194.930649	-3201.420633	-3193.590817
Normal inverse Gaussian	-1600.711326	-3186.904419	-3193.422651	-3193.394403	-3185.564587
Beta	-1220.978346	-2427.438459	-2433.956692	-2433.928443	-2426.098627

Table 10: Fitted distributions and results of the daily log returns on exchange rates of **Factom**

Distributions	-lnL	AIC	BIC	AICc	HQC
Student's T	-1892.279505	-3778.55901	-3770.040778	-3778.542073	-3772.665462
Laplace	-1890.325194	-3776.650389	-3766.132156	-3776.641926	-3772.721356
Asymmetric Students T	-1893.826481	-3779.652962	-3773.13473	-3779.624714	-3771.794898
Normal inverse Gaussian	-1899.138995	-3783.759757	-3790.27799	-3790.249741	-3782.419925
Beta	-1706.318279	3420.636558	3427.154791	3420.664807	3428.494623

Table 11: Fitted distributions and results of the daily log returns on exchange rates of **Paycoin**

Distributions	-lnL	AIC	BIC	AICc	HQC
Student's T	-891.4851846	-1776.970369	-1768.504903	-1776.952978	-1771.098667
Laplace	-823.9927622	-1643.985524	-1633.520058	-1643.976835	-1640.071056
Asymmetric Students T	-891.6771694	-1775.354339	-1768.888873	-1775.325332	-1767.525403
Normal inverse Gaussian	-896.6917347	-1778.918003	-1785.383469	-1785.354463	-1777.554533
Beta	-325.6786755	-636.8918848	-643.3573511	-643.3283445	-635.5284151

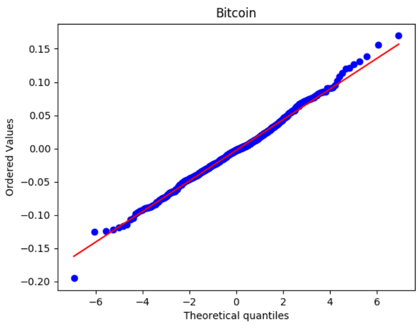
Table 12: Fitted distributions and results of the daily log returns on exchange rates of **PhoenixCoin**

Distributions	-lnL	AIC	BIC	AICc	HQC
Student's T	-960.1513289	-1914.302658	-1906.110871	-1914.282708	-1908.545561
Laplace	-945.673124	-1887.346248	-1877.154462	-1887.336281	-1883.508183
Asymmetric Students T	-962.4343068	-1916.868614	-1910.676827	-1916.835336	-1909.192484
Normal inverse Gaussian	-970.9475237	-1927.703261	-1933.895047	-1933.86177	-1926.218918
Beta	-694.3356185	-1374.479451	-1380.671237	-1380.637959	-1372.995107

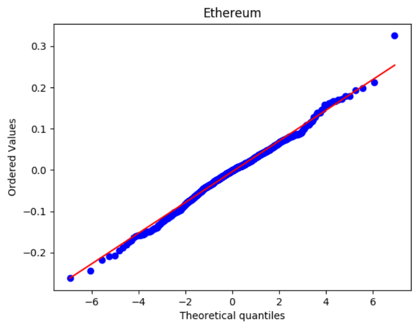
From the best fit distributions, we can see that Ethereum and Bitcoin have lighter tails when compared to the rest, who show semi-heavy tails. It had been found that the Laplace distribution can usually provide good fits to financial data, and is used to model even stock growth rates and prices fluctuations. Traditional currencies are usually known to exhibit heavy tails, so the cryptocurrencies having comparatively lighter tails is an important result.

4.2 Q-Q Plots

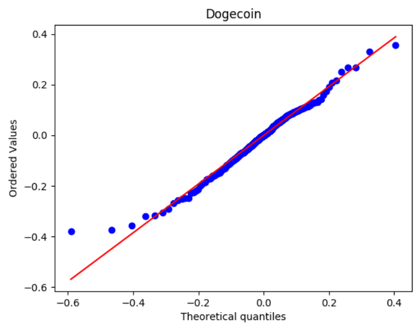
A Q-Q plot is a scatterplot created by plotting two sets of quantiles against one another. It is used to graphically check if the data follows a particular theoretical distribution. If both the quantiles follow the same distribution, we see that the points follow a straight line.



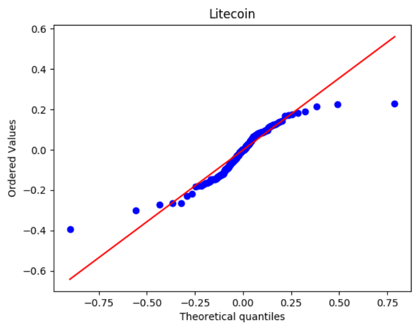
(a) Bitcoin



(b) Ethereum

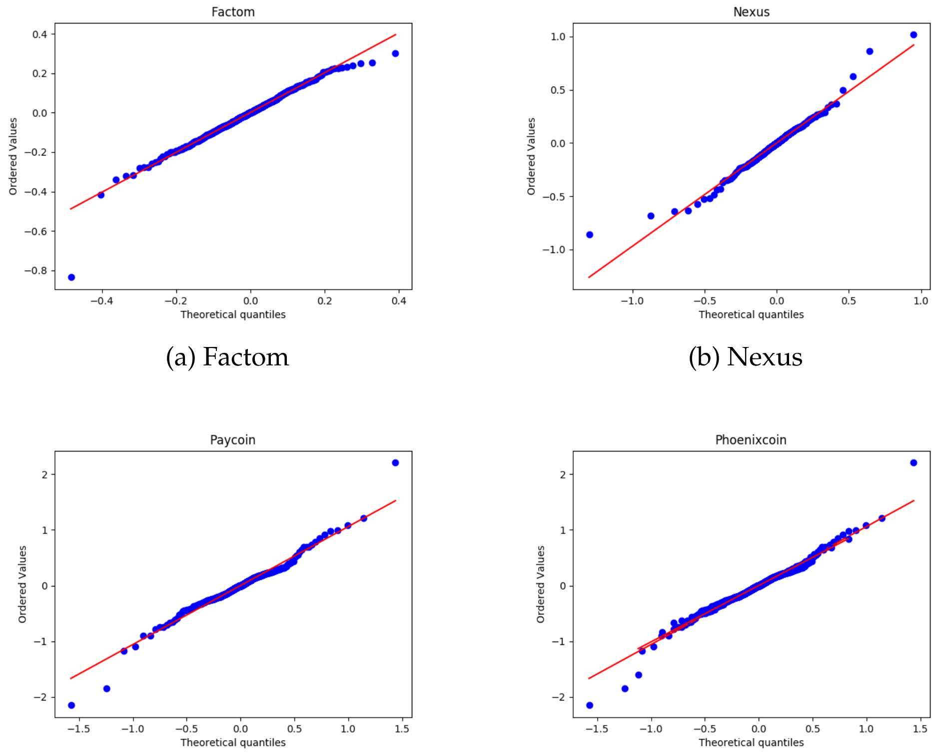


(c) Dogecoin



(d) Litecoin

Figure 3: Q-Q plots of the daily log returns of each cryptocurrency according to their best fit distributions



(a) Factom (b) Nexus

(c) Paycoin (d) Phoenixcoin

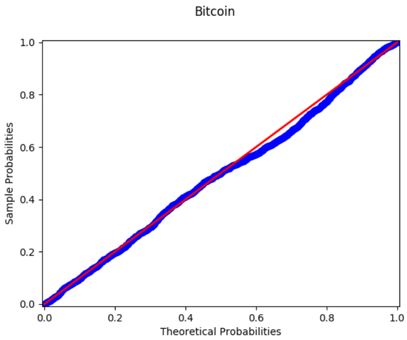
Figure 4: Q-Q plots of the daily log returns cont.

The Q-Q plots for the data is shown in Figure 3. From the Q-Q plot, we can see that the best fit distribution captures the middle part of the data well for Bitcoin, but does not do as well on the ends. For Ethereum, The distribution fits the data decently everywhere except the extreme top. For Dogecoin, the distribution does not captures a part of the lower end very well, but gives a good fit from the middle to almost the upper end. From Litecoin, again we observe that the distribution deviates from the data on the ends, fitting well in the middle. Only the upper part deviates from the straight line for Factom. Like, Litecoin, the distribution does not fit as well at the ends. Except for a little of the bottom and top parts, the distribution fit well almost throught for both Paycoin and Phoenixcoin.

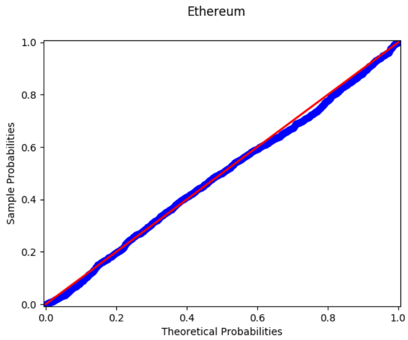
4.3 P-P plots

A P-P plots two cumulative distribution functions against each other. It is used to check how closely two datasets or distributions agree with each other. They are usually used to evaluate the skewness of distributions.

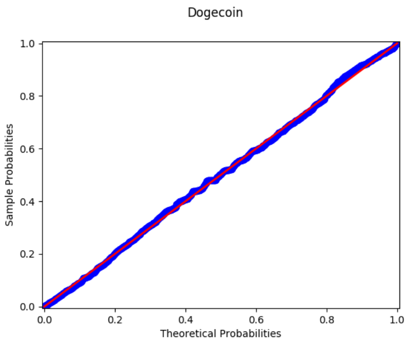
From Figure 4, we see that the best fit distributions capture all parts well, except for Bitcoin, in which there is a very slight deviation in the middle.



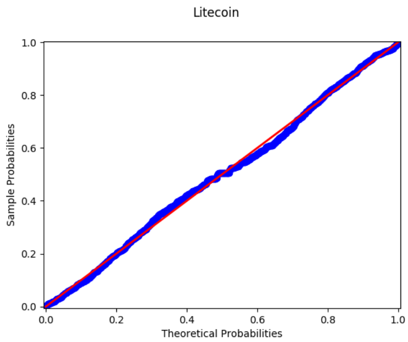
(a) Bitcoin



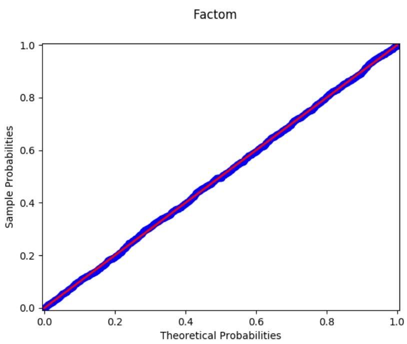
(b) Ethereum



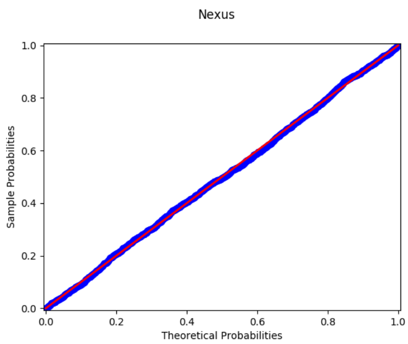
(c) Dogecoin



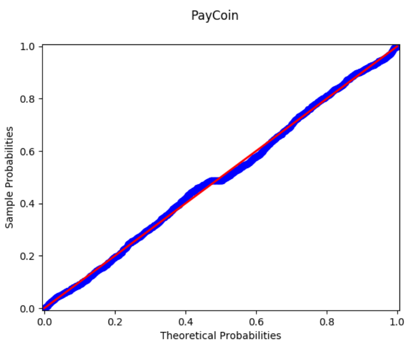
(d) Litecoin



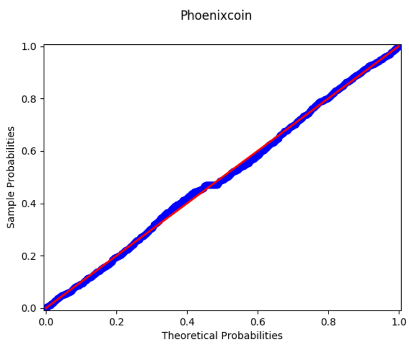
(e) Factom



(f) Nexus



(g) Paycoin



(h) Phoenixcoin

Figure 5: P-P plots of the daily logreturns of each cryptocurrency according to their best fit distributions

4.4 GoF

The table provides the results of KS test on the best fit distributions on each of the currencies. The fits are mostly adequate.

Table 13: Fitted distributions and results of the daily log returns on exchange rates of **Bitcoin**

Cryptocurrency	K-S Test Values
Bitcoin	0.043574189200156654
Ethereum	0.0340460899416426
Dogecoin	0.023778513373374444
Litecoin	0.033853734074813846
Nexus	0.01590736589033137
Factom	0.012104758626290013
Paycoin	0.026904685881658896
PhoenixCoin	0.02517224056285783

4.5 VaR Plots

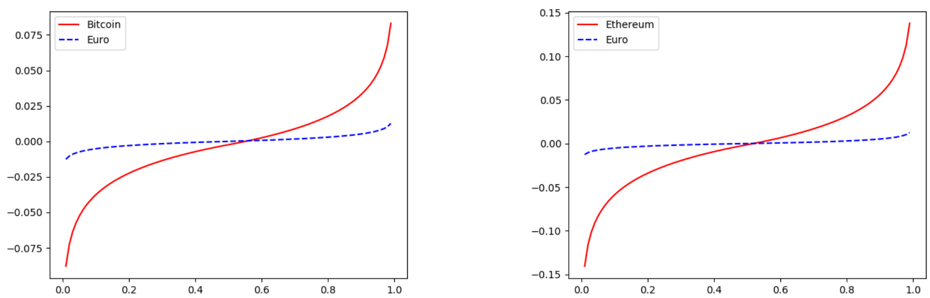
Value-at-risk is a popular financial risk measure and a statistical method that quantifies the risk level associated with a portfolio. If $F()$ denotes the cdf of a best fitting distributions, then VaR is correspondingly defines as:

$$VaR(q) = F^{-1}(q)$$

for

$$0 < q < 1$$

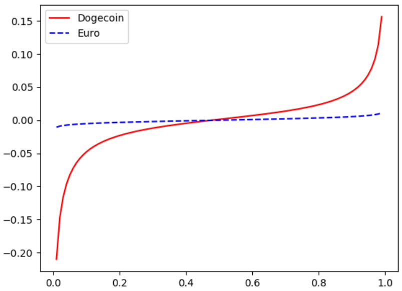
In the VaR plots, the daily log returns of the Euro is also plotted against the same best fit distributions to offer some comparison to traditonal currencies. Looking at the plots in Figure 5, it is clear that all of the cryptocurrencies have a higher risk factor than Euro.



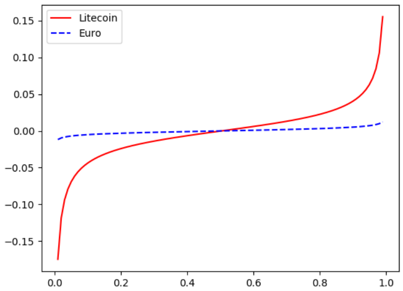
(a) Bitcoin

(b) Ethereum

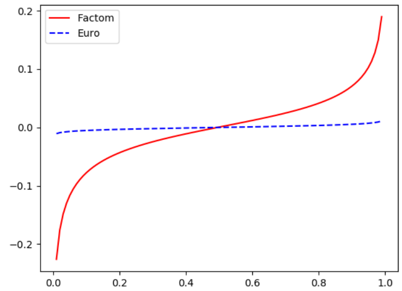
Figure 6: VaR plots of the daily log returns of each cryptocurrency according to their best fit distributions compared to the daily log returns of Euro fitted to the same distribution



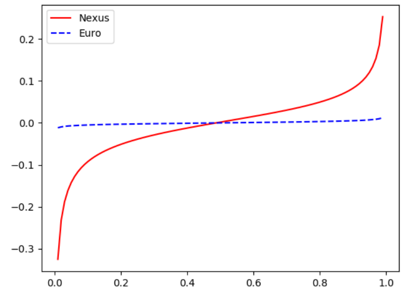
(a) Dogecoin



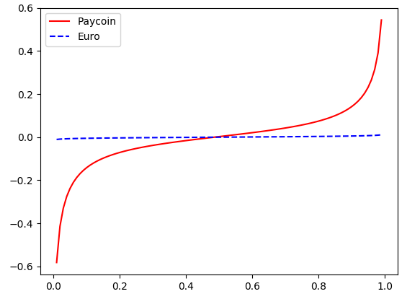
(b) Litecoin



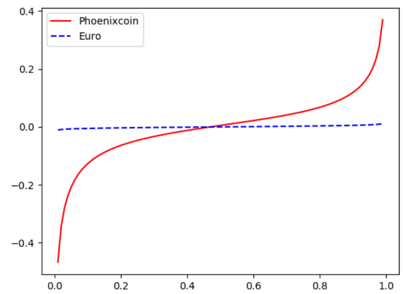
(c) Factom



(d) Nexus



(e) Paycoin

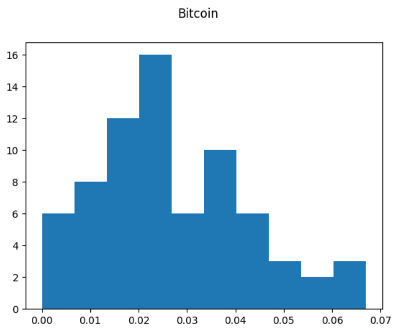


(f) Phoenixcoin

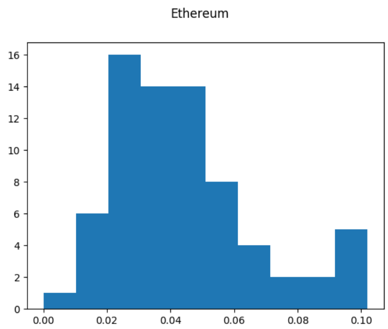
Figure 7: VaR plots of the daily log returns of each cryptocurrency cont.

4.6 Dynamic Volatility

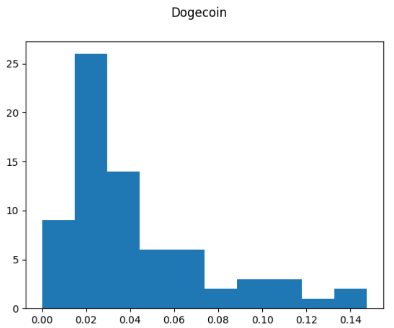
Volatility is not always a fixed parameter and often changes with time. So, it becomes necessary to compute the dynamic volatility of the same. Therefore, we shall treat volatility as a random variable. For the eight cryptocurrencies, we computed standard deviations of daily log returns of the exchange rates over windows of width 20 days. Histograms of these standard deviations are shown in Figure 8. We can see that the range of volatility is lowest for Bitcoin and generally increases with a decrease in popularity. Most coins shown here were popular in the first year of their release which is reflected in these histograms.



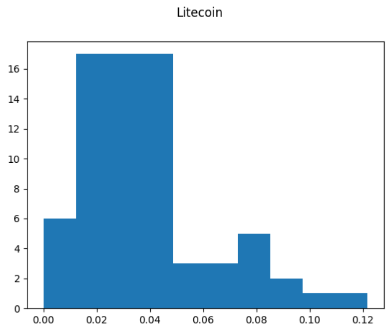
(a) Bitcoin



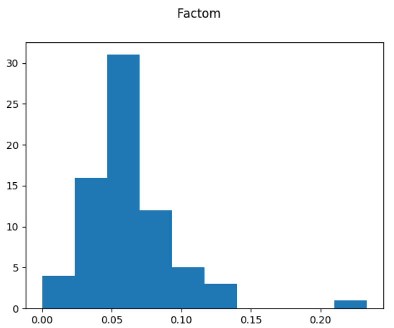
(b) Ethereum



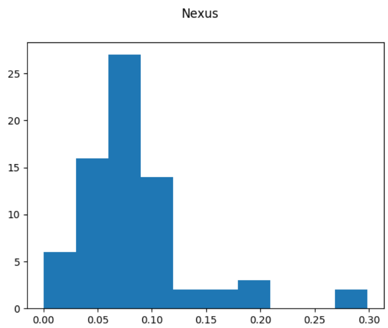
(c) Dogecoin



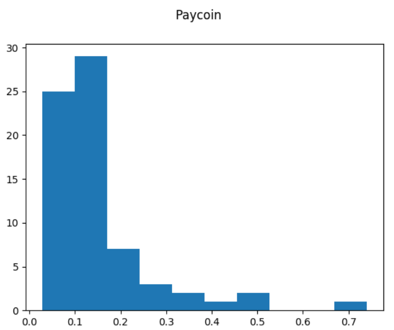
(d) Litecoin



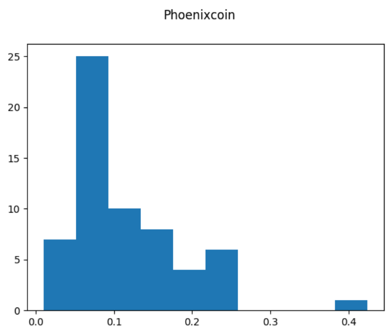
(e) Factom



(f) Nexus



(g) Paycoin



(h) Phoenixcoin

Figure 8: Histogram of daily log returns of the exchange rates of the cryptocurrencies over every 20 days

5 Conclusions

The exchange rate of the top eight cryptocurrencies versus the U.S. Dollar have been analyzed using popular parametric distributions in finance. Most Cryptocurrencies seem to exhibit a heavy tail (except the two most popular ones) w.r.t the data from 2015 to 2019. Using the discrimination criteria of the log likelihood, AIC, AICc, BIC and HQC, the results obtained show that none of the distributions used give the best fit jointly across the data for all of the cryptocurrencies. Instead, we find that the Laplace distribution gives the best fit for the Bitcoin and Ethereum; the Normal Inverse Gaussian distribution gives the best fit for Dogecoin, Factom, Paycoin, and PheonixCoin; and the asymmetric student's t gives the best fit for Litecoin and Nexus. Using these results, we may compute the VaR for risk management and as a consequence, for investment purposes. We have further extrapolated on earlier studies regarding the same with interesting results. The mature, stable cryptocurrencies have started to develop a light tail when compared to earlier studies where they generally exhibited a heavier tail. The consequences of this are not yet certain but are important. Scope for future work regarding the same is heavy.

Notes:

1. The code for the all the plots and figures are available in this GitHub repo: GitHub Repository

2. The code has been done completely in python. For convenience, it has been split into Code1.py and Code2.py. Each of the .py files have comments on the top stating the results produced by the code. The text files of the code are also available.

3. The fits of the distributions that haven't been included here will also be there in the repo.

4. All the results of data analysis in this project report has been generated by us, and not taken from any external sources. Values1.txt and Values2.txt are files used to store values during the execution of the code. They may not contain all the results given in the paper as we have constantly been updating the files while executing the program to reduce the clutter of data.

References

https://www.sec.gov/oiea/investor-alerts-bulletins/investoralertsiabitcoin.html

http://si-journal.org/index.php/JSI/article/viewFile/335/325

https://cftc.gov/Bitcoin/index.htm

https://topbrokers.trade/guide/basics-cryptocurrency/history-of-cryptocurrency/

https://coinmarketcap.com/

Founding dates via official sites and archived versions of the same.

https://cointelegraph.com/ethereum-for-beginners/what-is-ethereum

https://litecoin.com/en/about

https://www.abc.net.au/pm/content/2013/s3931812.htm

https://www.factom.com/factom-blockchain/

https://www.researchgate.net/publication/317274073AStatisticalAnalysisofCryptocurrenciespf17

https://www.factom.com/assets/docs/FactomWhitepaperv1.2.pdf

https://nexus.io/3dc

http://phoenixcoin.org/wiki.html

http://phoenixcoin.org/archive/UNOCSpressrelease07-11-13.pdf

https://www.coindesk.com/phenixcoin-ashes-developer-goes-awol

http://www.dognose.com/ paycoin/paycoin-whitepaper.pdf

https://data.library.virginia.edu/understanding-q-q-plots/

https://en.wikipedia.org/wiki/P%E2%80%93Pplot

https://in.mathworks.com/help/risk/value-at-risk-estimation-and-backtesting-1.html