081100M01002H

图像处理与分析

第六讲: 图像变换 (IV)

图像的正交变换，距离变换

内容提要

- 图像的正交变换
- 图像的距离变换
- 点扩散函数的基本概念
- 概率论基础 (复习)

- 图像的正交变换
- 图像的距离变换
- 点扩散函数的基本概念
- 概率论基础 (复习)

一维离散傅里叶变换(1D-DFT)的定义

$$F(u)=\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \qquad u=0,1,2,\dots,M-1$$

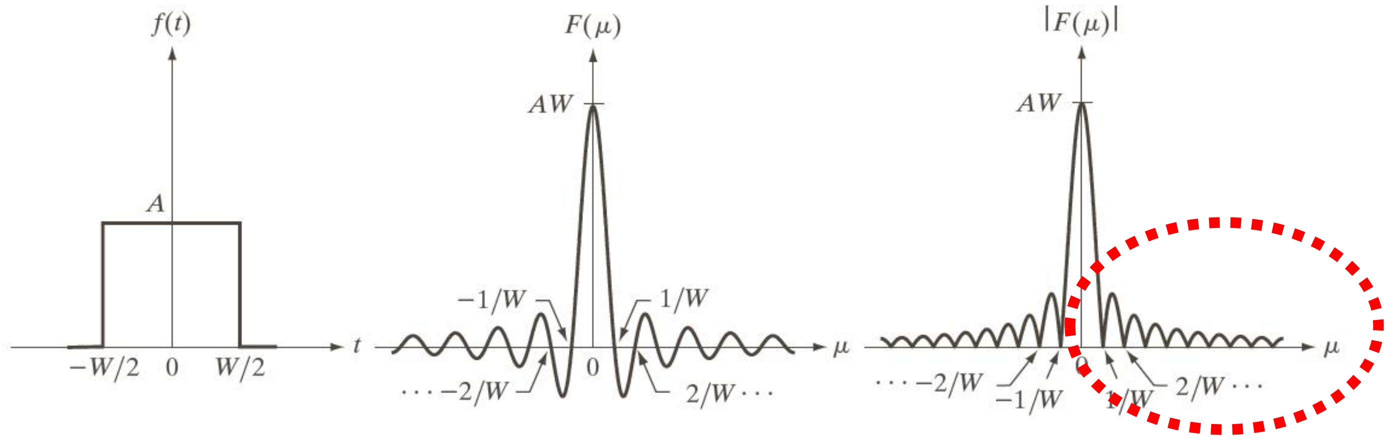
$$f(x)=\frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \qquad x=0,1,2,\dots,M-1$$

二维离散傅里叶变换(2D-DFT)的定义

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

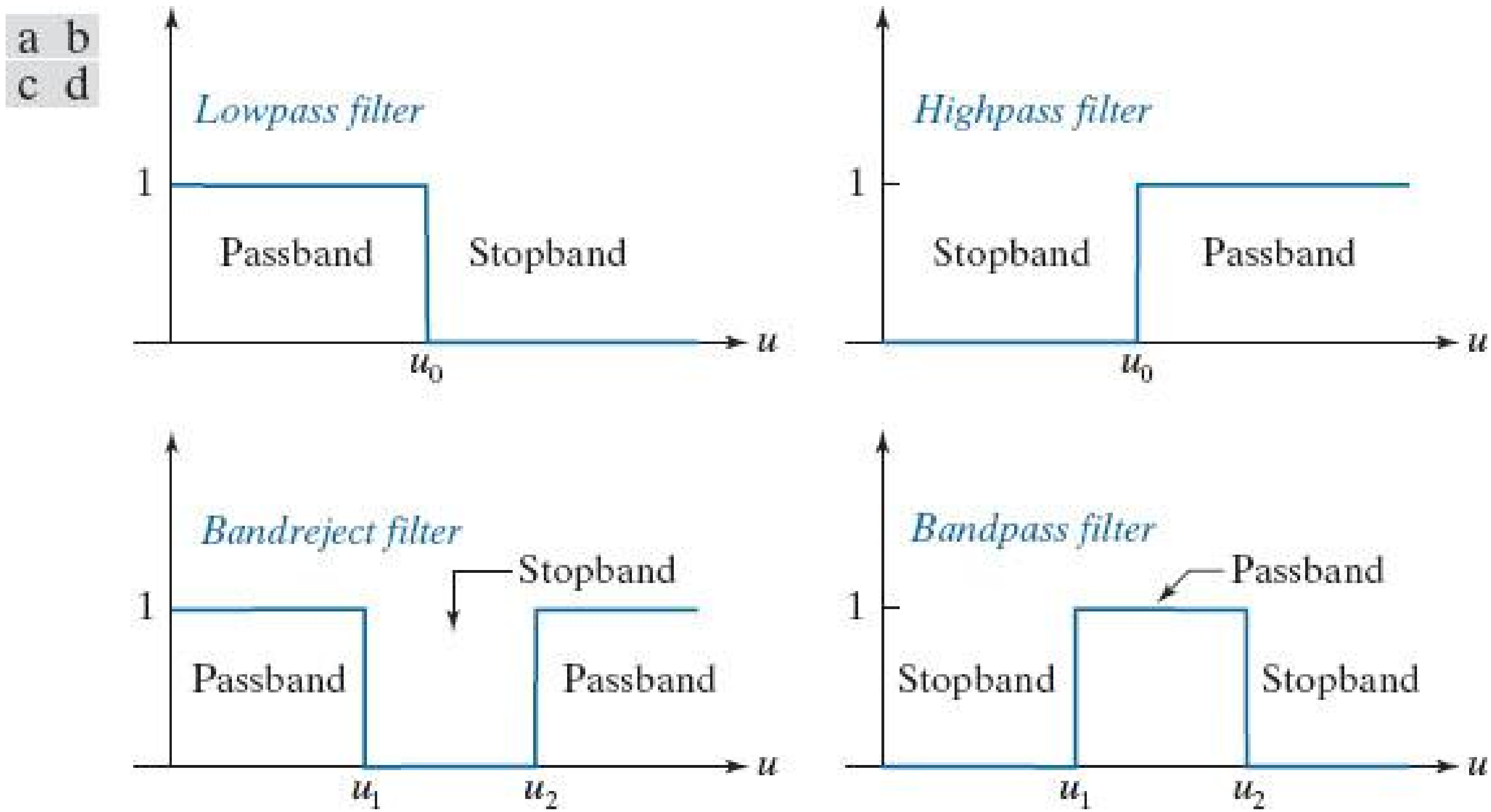
均值滤波器的频率域模型



a b c

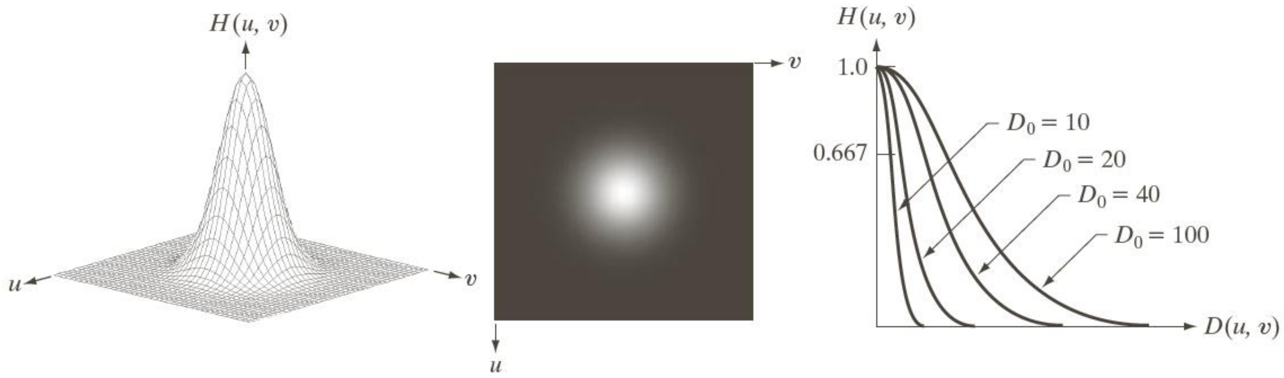
FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

不同类型的频率域滤波器



Transfer functions of ideal 1-D filters in the frequency domain (u denotes frequency). (a) Lowpass filter. (b) Highpass filter. (c) Bandreject filter. (d) Bandpass filter. (As before, we show only positive frequencies for simplicity.)

代表性滤波器：高斯低通滤波器 GLPF



a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

二维离散傅里叶变换(2D-DFT)的定义

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

一维和二维傅里叶变换是正交变换的一个特例，可以直接在时间/空间域获得直观的解释。这些解释具有重要意义。

矢量空间的基本概念 (I)

A ***vector space*** is defined as a nonempty set V of entities called *vectors* and associated scalars that satisfy the conditions outlined in A through C below. A vector space is *real* if the scalars are real numbers; it is *complex* if the scalars are complex numbers.

- Condition A: There is in V an operation called *vector addition*, denoted $\mathbf{x} + \mathbf{y}$, that satisfies:
  1. $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$ for all vectors $\mathbf{x}$ and $\mathbf{y}$ in the space.
  2. $\mathbf{x} + (\mathbf{y} + \mathbf{z}) = (\mathbf{x} + \mathbf{y}) + \mathbf{z}$ for all $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$.
  3. There exists in V a unique vector, called the *zero vector*, and denoted $\mathbf{0}$, such that $\mathbf{x} + \mathbf{0} = \mathbf{x}$ and $\mathbf{0} + \mathbf{x} = \mathbf{x}$ for all vectors $\mathbf{x}$.
  4. For each vector $\mathbf{x}$ in V, there is a unique vector in V, called the *negation* of $\mathbf{x}$, and denoted $-\mathbf{x}$, such that $\mathbf{x} + (-\mathbf{x}) = \mathbf{0}$ and $(-\mathbf{x}) + \mathbf{x} = \mathbf{0}$.

矢量空间的基本概念 (II)

- Condition B: There is in V an operation called *multiplication by a scalar* that associates with each scalar c and each vector $\mathbf{x}$ in V a unique vector called the *product* of c and $\mathbf{x}$, denoted by $c\mathbf{x}$ and $\mathbf{x}c$, and which satisfies:
  1. $c(d\mathbf{x}) = (cd)\mathbf{x}$ for all scalars c and d, and all vectors $\mathbf{x}$.
  2. $(c + d)\mathbf{x} = c\mathbf{x} + d\mathbf{x}$ for all scalars c and d, and all vectors $\mathbf{x}$.
  3. $c(\mathbf{x} + \mathbf{y}) = c\mathbf{x} + c\mathbf{y}$ for all scalars c and all vectors $\mathbf{x}$ and $\mathbf{y}$.

- Condition C: $1\mathbf{x} = \mathbf{x}$ for all vectors $\mathbf{x}$.

内积空间的基本概念

- 如果矢量空间*V*满足如下条件则是一个内积空间

$$x, y \in V$$

$$\langle x, y \rangle = \langle y, x \rangle^*$$

$$\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$$

$$\langle rx, y \rangle = r \langle x, y \rangle$$

$$\langle x, x \rangle \geq 0 \text{ and } \langle x, x \rangle = 0 \;\; \textit{iff} \;\; x = 0$$

内积空间的示例 I：欧几里得空间

$$\langle x, y\rangle = x^T y = x_0 y_0 + x_0 y_0 + \ldots + x_{N-1} y_{N-1} = \sum_{i=0}^{N-1} x_i y_i$$

$$x_i \in R, y_i \in R, x \in R^{N\times 1}, y \in R^{N\times 1}$$

$$\|x\| = \sqrt{\langle x, x\rangle} = \sqrt{\sum_{i=0}^{N-1} x_i^2}$$

$$\theta = \cos^{-1}\frac{\langle x, y\rangle}{\|x\|\|y\|}$$

内积空间示例II：积分内积空间

- 矢量空间 $C([a,b])$ 其中矢量是定义在 $[a,b]$ 上的连续函数。

$$\langle f(x), g(x) \rangle = \int_a^b f^*(x) g(x) dx$$

内积空间的基和单位正交基

$$W = \left\{ w_0, w_1, w_2, \ldots w_N \right\}$$

$$z = \alpha_0 w_0 + \alpha_1 w_1 + \ldots \alpha_{N-1} w_{N-1}$$

$$\left\langle w_i, z \right\rangle = \alpha_0 \left\langle w_i, w_0 \right\rangle + \alpha_1 \left\langle w_i, w_1 \right\rangle + \ldots \alpha_{N-1} \left\langle w_i, w_{N-1} \right\rangle$$

$$\alpha_i = \frac{\left\langle w_i, z \right\rangle}{\left\langle w_i, w_i \right\rangle}$$

双正交基和单位双正交基

- 一组矢量 $w_0, w_1, w_2, \ldots$ 和一种对偶矢量 $\tilde{w}_0, \tilde{w}_1, \tilde{w}_2, \ldots$ 称为双正交基/单位双正交基如果它们满足如下条件

$$\langle \tilde{w}_k, w_l \rangle = 0 \quad \text{for } k \neq l$$

$$\langle \tilde{w}_k, w_l \rangle = \delta_{kl} = \begin{cases} 0 & \text{for } k \neq l \\ 1 & \text{for } k = l \end{cases}$$

一维傅里叶变换的内积与矩阵表示 (I)

$$T(u)=\sum_{x=0}^{N-1} f(x)e^{-j2\pi ux/N} \qquad u=0,1,2,\ldots,N-1$$

$$f(x)=\frac{1}{N}\sum_{u=0}^{N-1} T(u)e^{j2\pi ux/N} \qquad x=0,1,2,\ldots,N-1$$

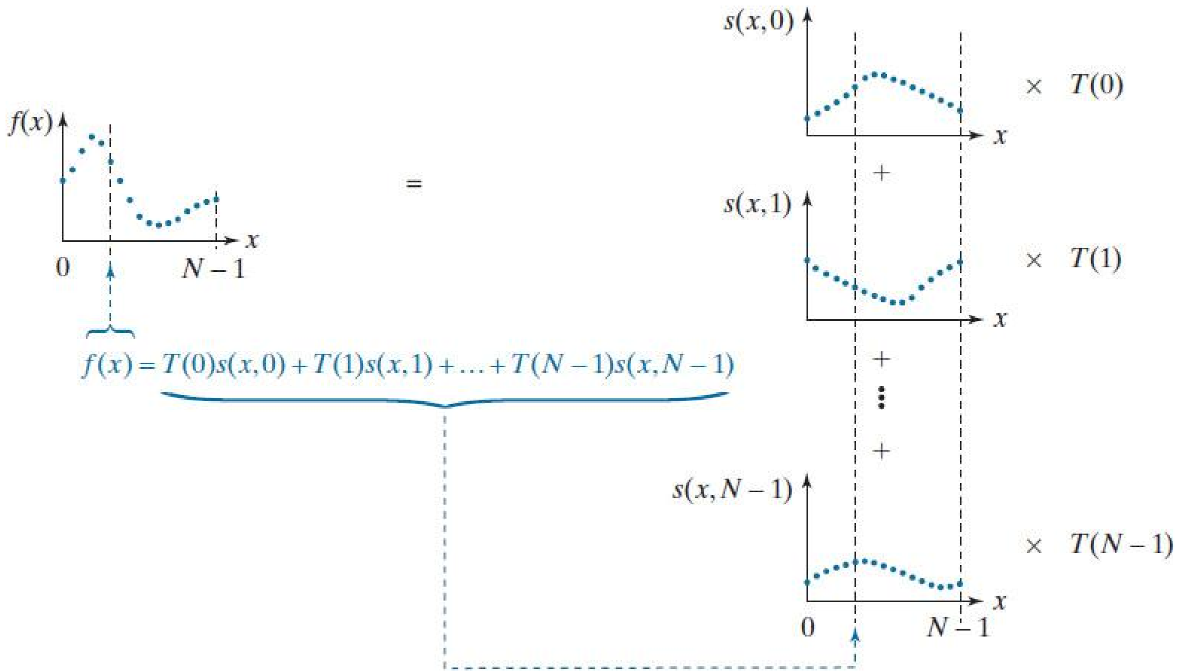
$$T(u)=\sum_{x=0}^{N-1} f(x)\cdot \boxed{r(x,u)}$$ 正向变换核

$$f(x)=\sum_{u=0}^{N-1} T(u)\cdot \boxed{s(x,u)}$$ 逆向变换核

一维傅里叶变换的内积与矩阵表示 (II)

$$f(x)=\sum_{u=0}^{N-1}T(u)\cdot s(x,u)$$

$$=T(0)s(x,0)+T(1)s(x,1)+\ldots+T(N-1)s(x,N-1)$$



一维傅里叶变换的内积与矩阵表示 (III)

$$F = \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-1) \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{N-1} \end{bmatrix} \qquad T = \begin{bmatrix} T(0) \\ T(1) \\ \vdots \\ T(N-1) \end{bmatrix} = \begin{bmatrix} t_0 \\ t_1 \\ \vdots \\ t_{N-1} \end{bmatrix}$$

$$s_u = \begin{bmatrix} s(0,u) \\ s(1,u) \\ \vdots \\ s(N-1,u) \end{bmatrix} = \begin{bmatrix} s_{u,0} \\ s_{u,1} \\ \vdots \\ s_{u,N-1} \end{bmatrix} \quad \text{for } u = 0, 1, \ldots, N-1$$

一维傅里叶变换的内积与矩阵表示 (IV)

$$T = \begin{bmatrix} \langle s_0, f \rangle \\ \langle s_1, f \rangle \\ \vdots \\ \langle s_{N-1}, f \rangle \end{bmatrix} = \begin{bmatrix} s_0^T \\ s_1^T \\ \vdots \\ s_{N-1}^T \end{bmatrix} F = AF$$

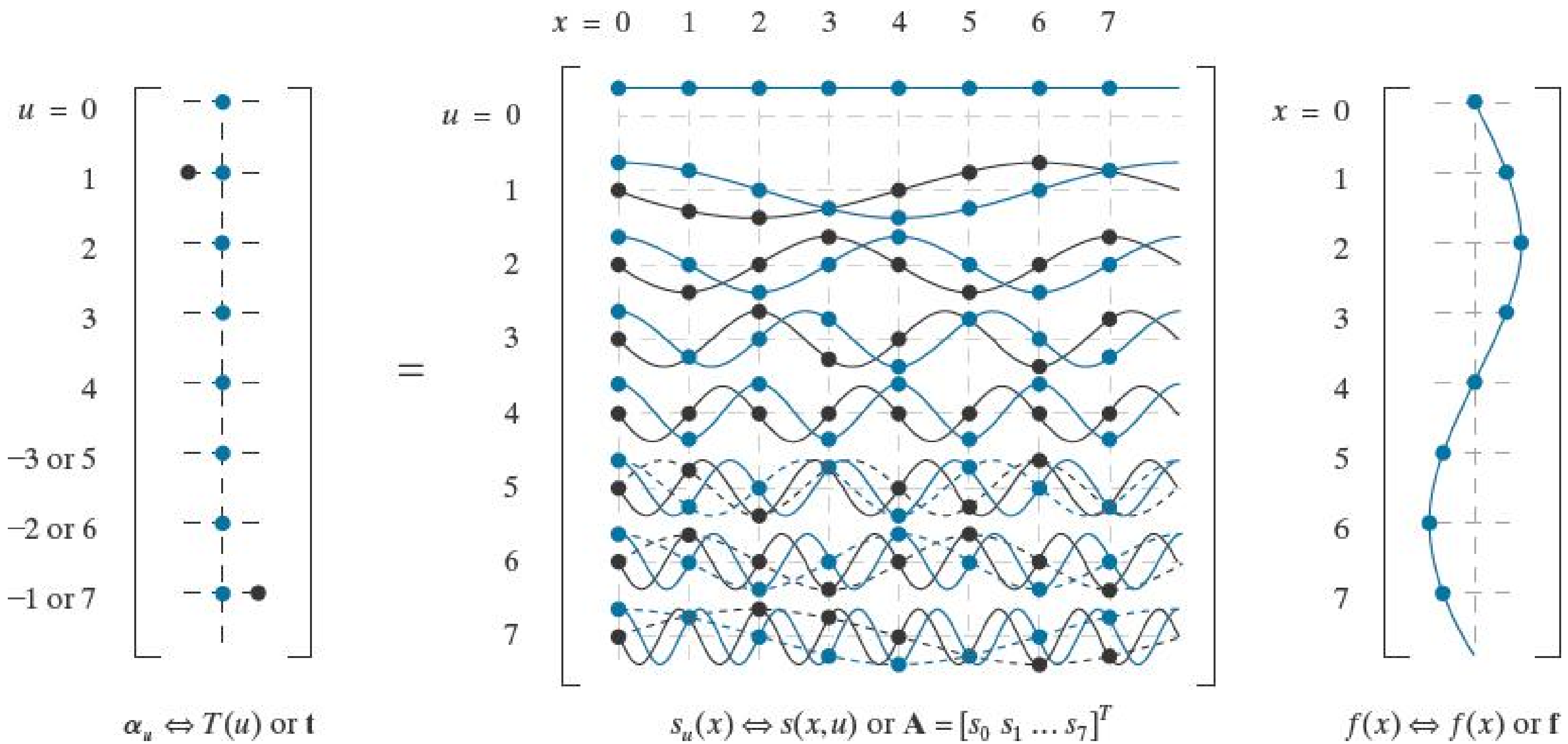
$$A \triangleq \begin{bmatrix} s_0^T \\ s_1^T \\ \vdots \\ s_{N-1}^T \end{bmatrix} = \begin{bmatrix} s_0 & s_1 & \cdots & s_{N-1} \end{bmatrix}$$

一维傅里叶变换的内积与矩阵表示 (V)

$$T = AF$$

$$F = A^T T$$

一维傅里叶变换的内积与矩阵表示 (VI)



Depicting the continuous Fourier series and 8-point DFT of f(x) = sin(2πx) as "matrix multiplications." The real and imaginary parts of all complex quantities are shown in blue and black, respectively. Continuous and discrete functions are represented using lines and dots, respectively. Dashed lines are included to show that $s_5 = s_3^*, s_6 = s_2^*, s_7 = s_1^*$ effectively cutting the maximum frequency of the DFT in half. The negative indices to the left of t are for the Fourier series computation alone.

变换的直观解释

连续函数的相关函数

$$f \circ g(\Delta x) = \int_{-\infty}^{\infty} f^*(x) g(x) dx$$
$$= \langle f(x), g(x+\Delta x) \rangle$$

$$f \circ g(0) = \int_{-\infty}^{\infty} f^*(x) g(x) dx$$
$$= \langle f(x), g(x) \rangle$$

$$a_u = \langle f, s_u \rangle = f \circ s_u(0)$$

$$f \circ g(m) = \sum_{x=-\infty}^{\infty} f_n^* g_{n+m}$$

$$f \circ g(0) = \langle f, g \rangle$$

$$T(u) = \langle s_u, f \rangle = s_u \circ f(0)$$

二维傅里叶变换的内积与矩阵表示 (I)

$$T(u,v)=\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\,r(x,y,u,v)$$

$$f(x,y)=\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} T(u,v)\,s(x,y,u,v)$$

$$F=\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} T(u,v)\,S_{u,v}$$

$$F=\begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & \cdots & \vdots \\ f(N-1,0) & f(N-1,1) & \cdots & f(N-1,N-1) \end{bmatrix}$$

二维傅里叶变换的内积与矩阵表示 (II)

$$S_{u,v}=\begin{bmatrix} s(0,0,u,v) & s(0,1,u,v) & \cdots & s(0,N-1,u,v) \\ s(1,0,u,v) & s(1,1,u,v) & \cdots & s(1,N-1,u,v) \\ \vdots & \vdots & \cdots & \vdots \\ s(N-1,0,u,v) & s(N-1,1,u,v) & \cdots & s(N-1,N-1,u,v) \end{bmatrix}$$

二维傅里叶变换的内积与矩阵表示 (III)

$$r(x,y,u,v) = r_1(x,u)\,r_2(y,v) \qquad \leftarrow \textit{separable}$$

$$r(x,y,u,v) = r_1(x,u)\,r_1(y,v) \qquad \leftarrow \textit{symmetric}$$

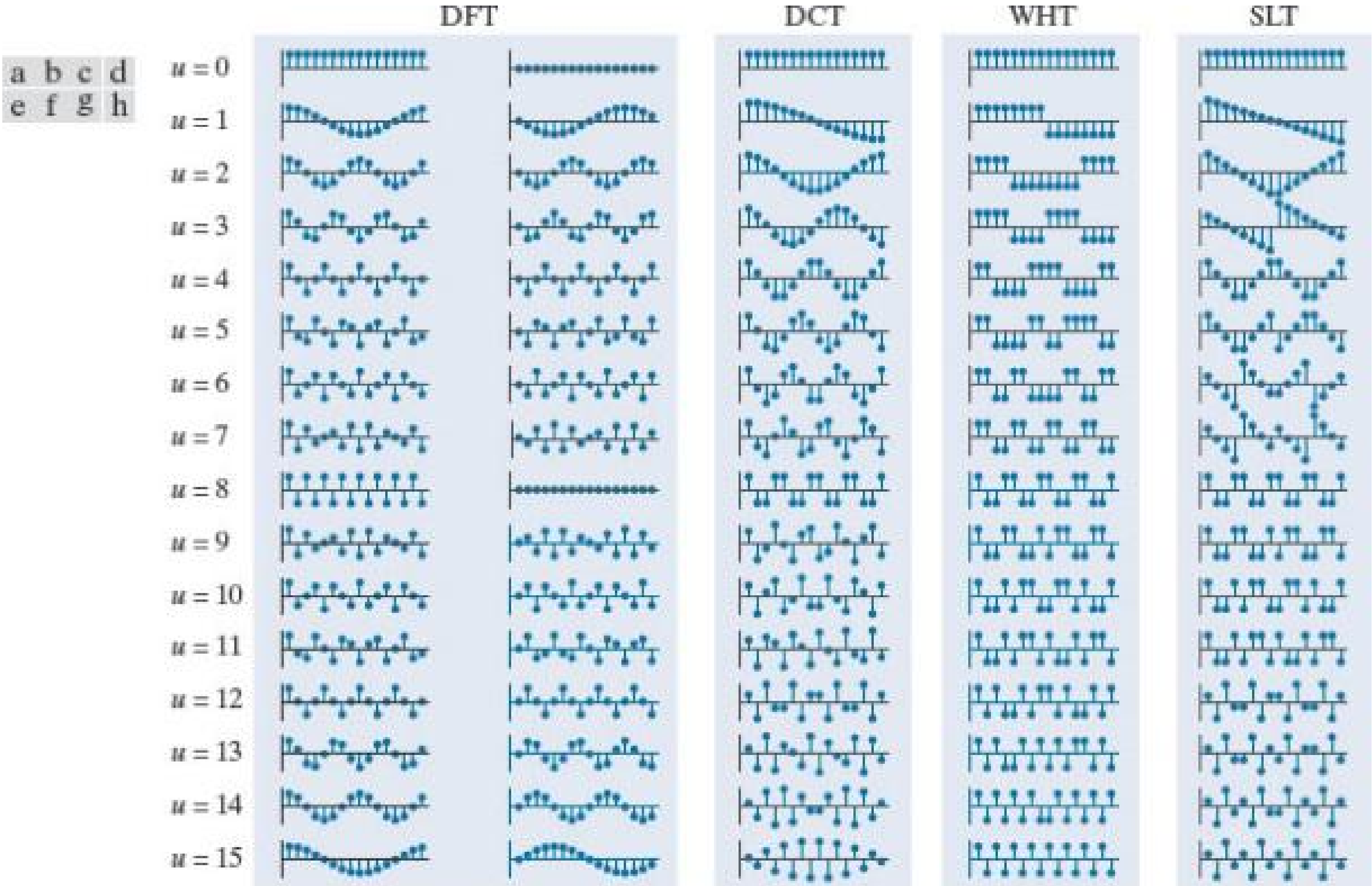
$$T = AFA^T$$

$$F = A^T TA$$

$$T = A_M F A_N^T$$

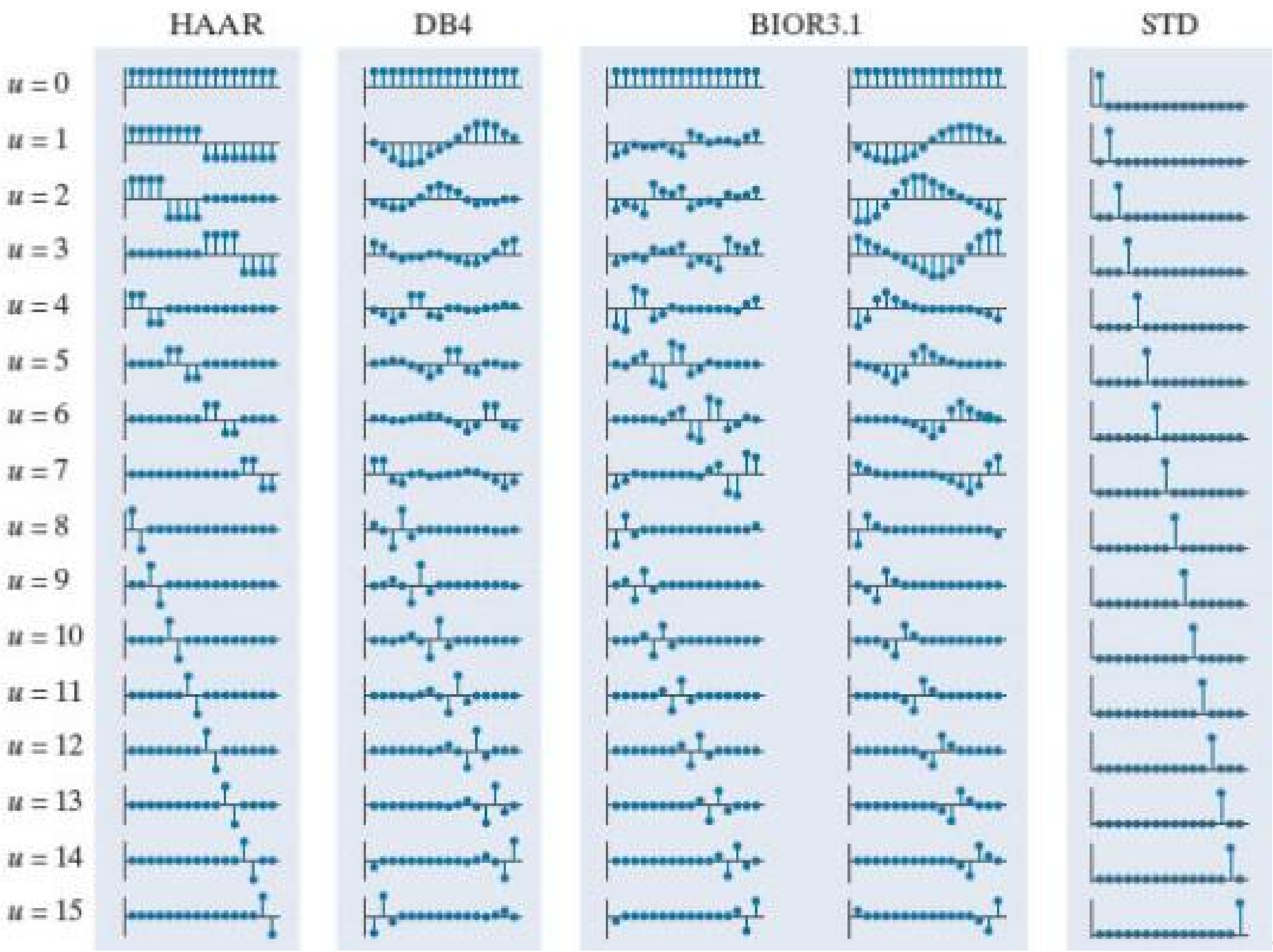
$$F = A_M^T T A_N$$

各种变换基总览



Basis vectors (for N = 16) of some commonly encountered transforms: (a) Fourier basis (real and imaginary parts), (b) discrete Cosine basis, (c) Walsh-Hadamard basis, (d) Slant basis, (e) Haar basis, (f) Daubechies basis, (g) Biorthogonal B-spline basis and its dual, and (h) the standard basis, which is included for reference only (i.e., not used as the basis of a transform).

各种变换基总览



HAAR
DB4
BIOR3.1
STD
u = 0
u = 1
u = 2
u = 3
u = 4
u = 5
u = 6
u = 7
u = 8
u = 9
u = 10
u = 11
u = 12
u = 13
u = 14
u = 15

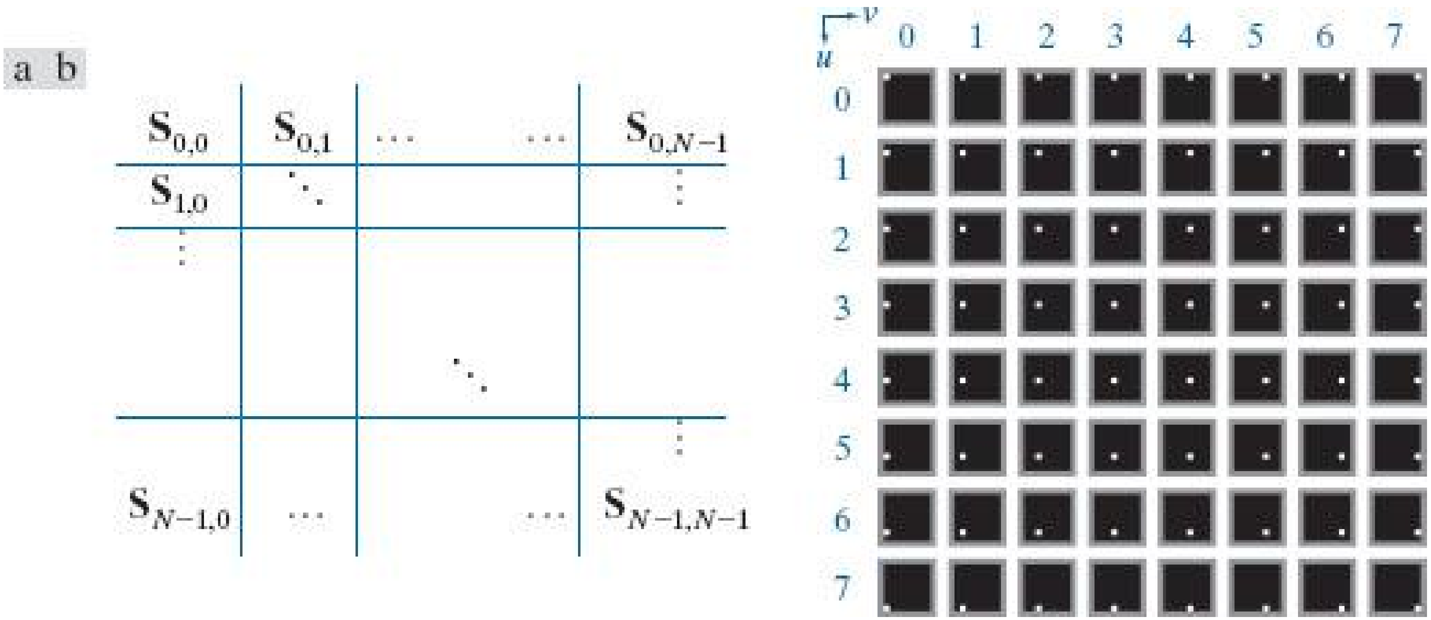
基图像的基本概念

$$F=\sum_{u=0}^{N-1}\sum_{v=0}^{N-1}T(u,v)S_{u,v}$$

$$F=\begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & \cdots & \vdots \\ f(N-1,0) & f(N-1,1) & \cdots & f(N-1,N-1) \end{bmatrix}$$

$$S_{u,v}=\begin{bmatrix} s(0,0,u,v) & s(0,1,u,v) & \cdots & s(0,N-1,u,v) \\ s(1,0,u,v) & s(1,1,u,v) & \cdots & s(1,N-1,u,v) \\ \vdots & \vdots & \cdots & \vdots \\ s(N-1,0,u,v) & s(N-1,1,u,v) & \cdots & s(N-1,N-1,u,v) \end{bmatrix}$$

标准基变换

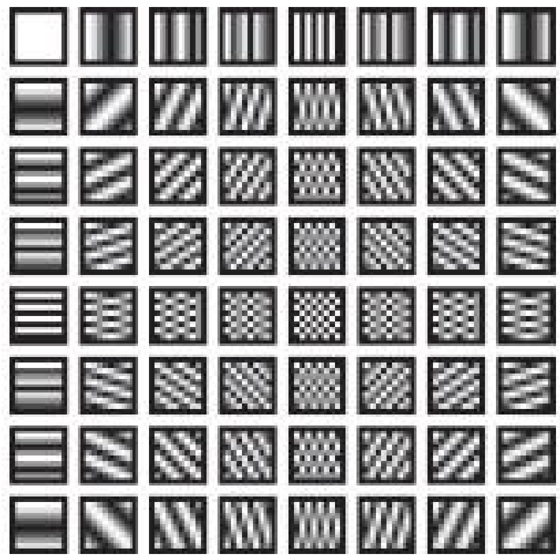


(a) Basis image organization and (b) a standard basis of size 8×8

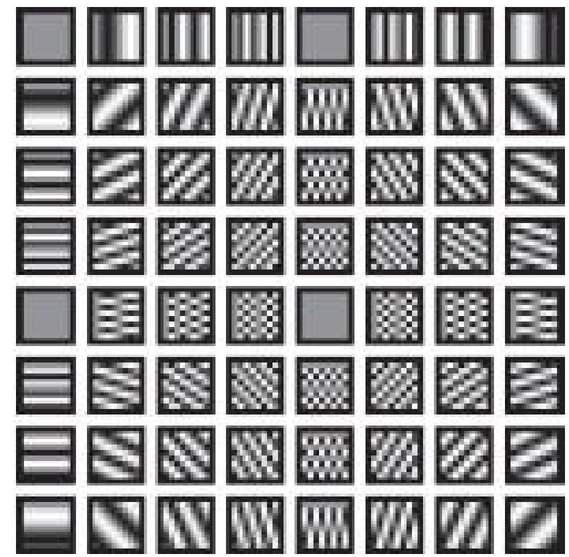
For clarity, a gray border has been added around each basis image. The origin of each basis image (i.e., x = y = 0) is at its top left.

二维傅里叶变换的基图像

$$\frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & -j & -j\omega & -1 & -\omega & j & j\omega \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -j\omega & j & \omega & -1 & j\omega & -j & -\omega \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\omega & -j & j\omega & -1 & \omega & j & -j\omega \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & j\omega & j & -\omega & -1 & -j\omega & -j & \omega \end{bmatrix}$$



实部基图像



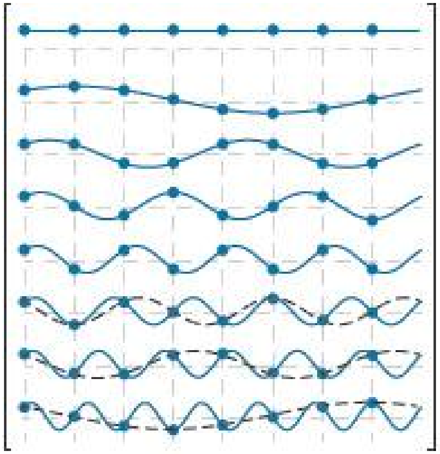
虚部基图像

a b c

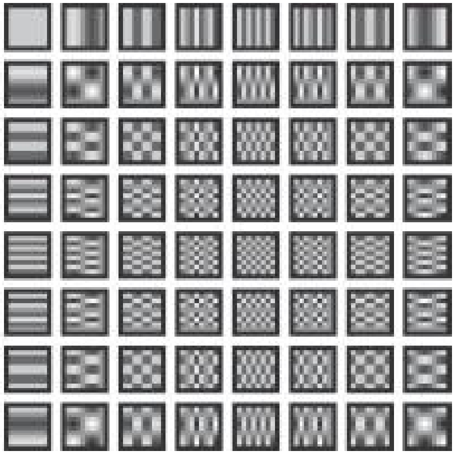
二维Hartley变换的基图像

$$s(x,u)=\frac{1}{\sqrt{N}}\left(\cos\left(\frac{2\pi ux}{N}\right)+\sin\left(\frac{2\pi ux}{N}\right)\right)$$

$$s(x,y,u,v)=\frac{1}{\sqrt{N}}\left(\cos\left(\frac{2\pi ux}{N}\right)+\sin\left(\frac{2\pi ux}{N}\right)\right)\frac{1}{\sqrt{N}}\left(\cos\left(\frac{2\pi vy}{N}\right)+\sin\left(\frac{2\pi vy}{N}\right)\right)$$

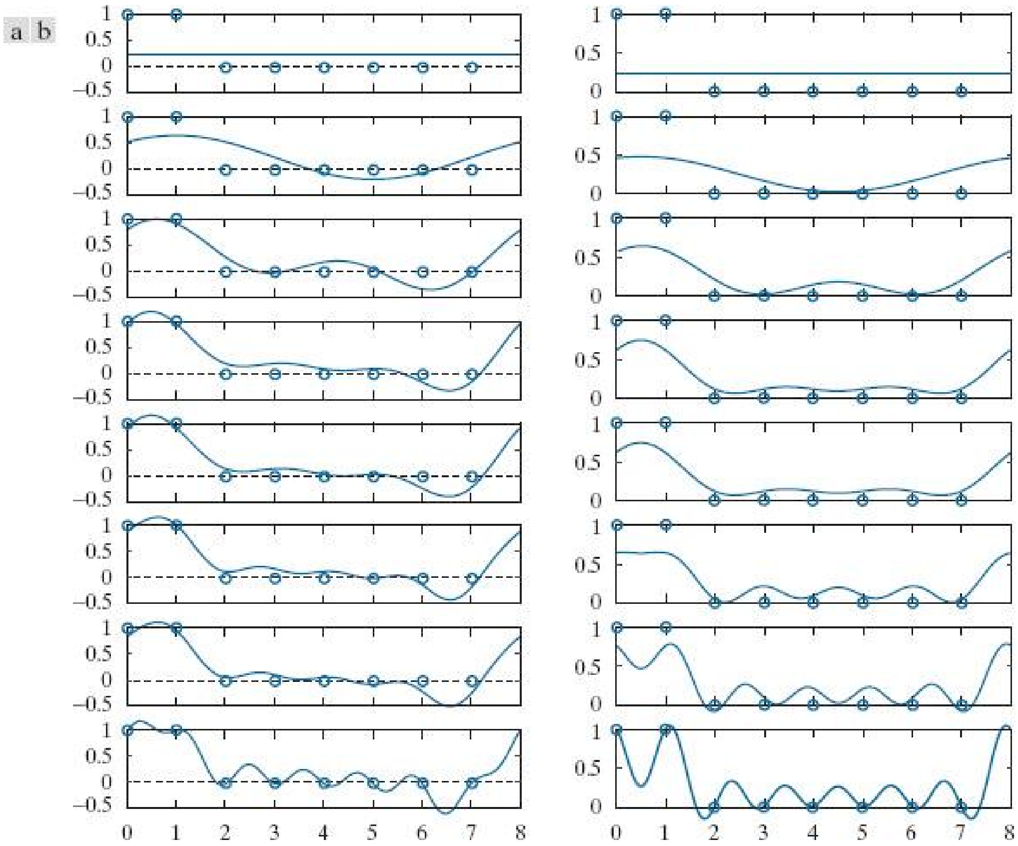


$$\begin{bmatrix} 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\ 0.35 & 0.50 & 0.35 & 0 & -0.35 & -0.50 & -0.35 & 0 \\ 0.35 & 0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 \\ 0.35 & 0 & -0.35 & 0.50 & -0.35 & 0 & 0.35 & -0.50 \\ 0.35 & -0.35 & 0.35 & -0.35 & 0.35 & -0.35 & 0.35 & -0.35 \\ 0.35 & -0.50 & 0.35 & 0 & -0.35 & 0.50 & -0.35 & 0 \\ 0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 & 0.35 \\ 0.35 & 0 & -0.35 & -0.50 & -0.35 & 0 & 0.35 & 0.50 \end{bmatrix}$$



a b c

离散余弦变换

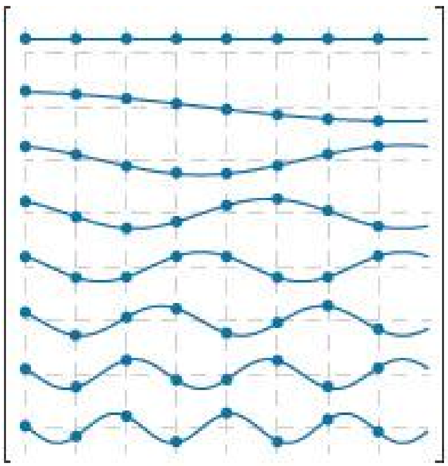


$$s(x,u) = a(u)\cos\left(\frac{(2x+1)u\pi}{2N}\right)$$

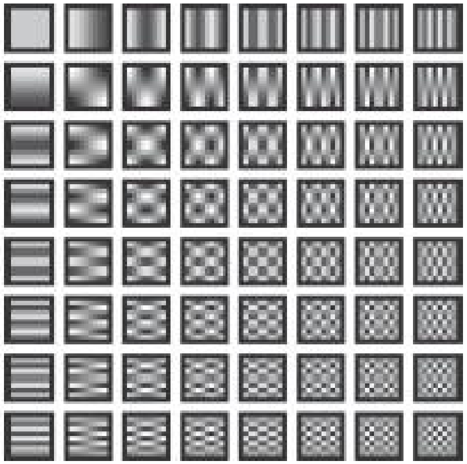
$$a(u) = \begin{cases} \sqrt{\frac{1}{N}} & u = 0 \\ \sqrt{\frac{2}{N}} & u = 1, 2, \ldots, N-1 \end{cases}$$

离散余弦变换

$$s(x,y,u,v)=\alpha(u)\alpha(v)\cos\left(\frac{(2x+1)u\pi}{2N}\right)\cos\left(\frac{(2y+1)v\pi}{2N}\right)$$



$$\begin{bmatrix}
0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\
0.49 & 0.42 & 0.28 & 0.10 & -0.10 & -0.28 & -0.42 & -0.49 \\
0.46 & 0.19 & -0.19 & -0.46 & -0.46 & -0.19 & 0.19 & 0.46 \\
0.42 & -0.10 & -0.49 & -0.28 & 0.28 & 0.49 & 0.10 & -0.42 \\
0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 & 0.35 \\
0.28 & -0.49 & 0.10 & 0.42 & -0.42 & -0.10 & 0.49 & -0.28 \\
0.19 & -0.46 & 0.46 & -0.19 & -0.19 & 0.46 & -0.46 & 0.19 \\
0.10 & -0.28 & 0.42 & -0.49 & 0.49 & -0.42 & 0.28 & -0.10
\end{bmatrix}$$

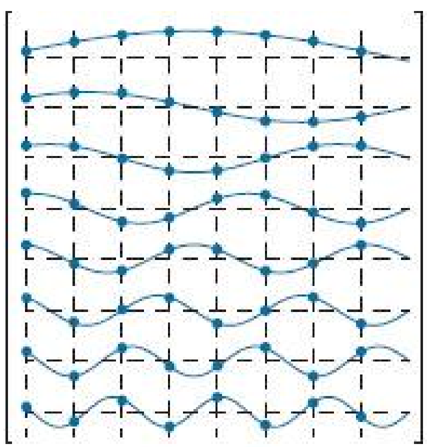


a b c

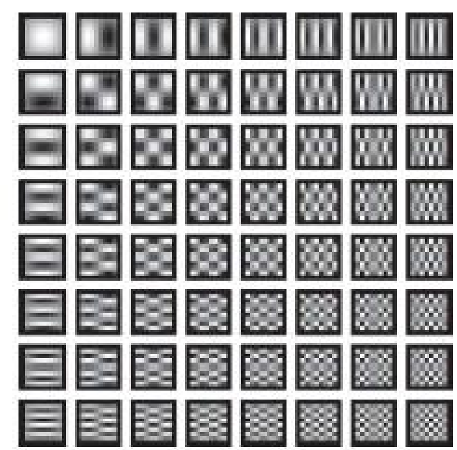
离散正弦变换

$$s(x,u)=\sqrt{\frac{2}{N+1}}\sin\left(\frac{(x+1)(u+1)\pi}{N+1}\right)$$

$$s(x,y,u,v)=\frac{2}{N+1}\sin\left(\frac{(x+1)(u+1)\pi}{N+1}\right)\sin\left(\frac{(y+1)(\mathrm{v}+1)\pi}{N+1}\right)$$



$$\begin{bmatrix}
0.16 & 0.30 & 0.41 & 0.46 & 0.46 & 0.41 & 0.30 & 0.16 \\
0.30 & 0.46 & 0.41 & 0.16 & -0.16 & -0.41 & -0.46 & -0.30 \\
0.41 & 0.41 & 0.00 & -0.41 & -0.41 & 0.00 & 0.41 & 0.41 \\
0.46 & 0.16 & -0.41 & -0.30 & 0.30 & 0.41 & -0.16 & -0.46 \\
0.46 & -0.16 & -0.41 & 0.30 & 0.30 & -0.41 & -0.16 & 0.46 \\
0.41 & -0.41 & 0.00 & 0.41 & -0.41 & -0.00 & 0.41 & -0.41 \\
0.30 & -0.46 & 0.41 & -0.16 & -0.16 & 0.41 & -0.46 & 0.30 \\
0.16 & -0.30 & 0.41 & -0.46 & 0.46 & -0.41 & 0.30 & -0.16
\end{bmatrix}$$



a b c

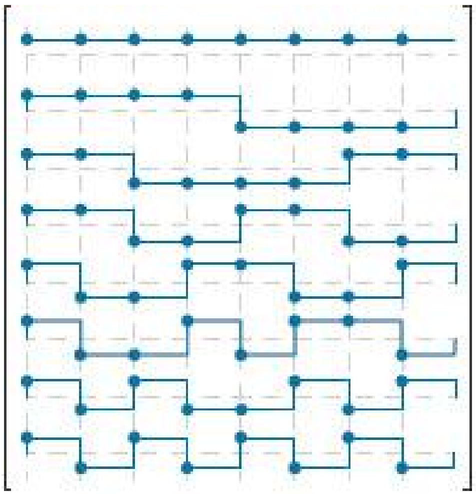
Walsh-Hadamard变换

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

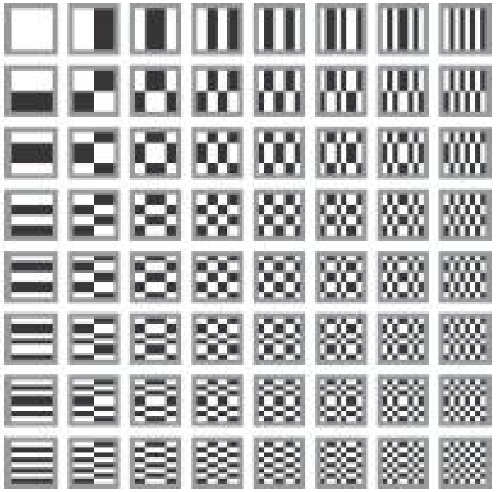
$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

Walsh-Hadamard变换



$$
\begin{bmatrix}
0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\
0.35 & 0.35 & 0.35 & 0.35 & -0.35 & -0.35 & -0.35 & -0.35 \\
0.35 & 0.35 & -0.35 & -0.35 & -0.35 & -0.35 & 0.35 & 0.35 \\
0.35 & 0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 \\
0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 & 0.35 \\
0.35 & -0.35 & -0.35 & 0.35 & -0.35 & 0.35 & 0.35 & -0.35 \\
0.35 & -0.35 & 0.35 & -0.35 & -0.35 & 0.35 & -0.35 & 0.35 \\
0.35 & -0.35 & 0.35 & -0.35 & 0.35 & -0.35 & 0.35 & -0.35
\end{bmatrix}
$$



a b c

Slant斜变换 (I)

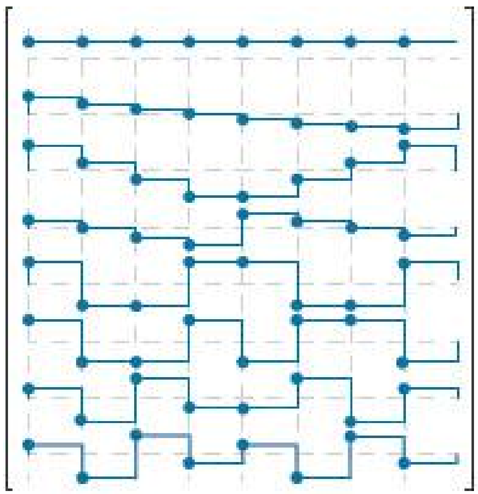
$$S_N = \begin{bmatrix} 1 & 0 & \multirow{2}{*}{0} & 1 & 0 & \multirow{2}{*}{0} \\ a_N & b_N & & -a_N & b_N & \\ \multicolumn{2}{c}{0} & I_{(N/2)-2} & \multicolumn{2}{c}{0} & I_{(N/2)-2} \\ 0 & 1 & \multirow{2}{*}{0} & 0 & -1 & \multirow{2}{*}{0} \\ -b_N & a_N & & b_N & a_N & \\ \multicolumn{2}{c}{0} & I_{(N/2)-2} & \multicolumn{2}{c}{0} & -I_{(N/2)-2} \end{bmatrix} \begin{bmatrix} S_{N/2} & 0 \\ 0 & S_{N/2} \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

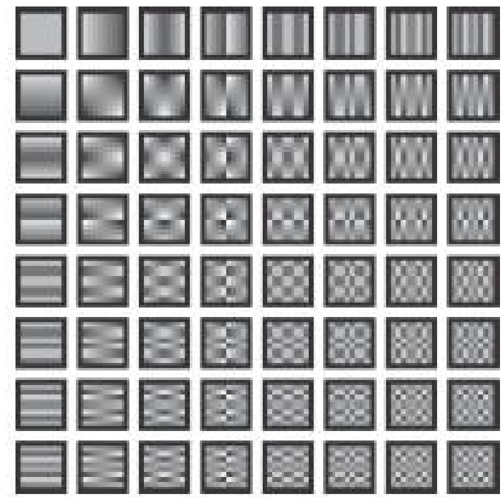
$$a_N = \left[\frac{3N^2}{4\left(N^2-1\right)}\right]^{1/2}, \quad b_N = \left[\frac{N^2-4}{4\left(N^2-1\right)}\right]^{1/2}$$

$$A_{SL} = \frac{1}{\sqrt{4}} S_4 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ \frac{3}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{-1}{\sqrt{5}} & \frac{-3}{\sqrt{5}} \\ 1 & -1 & -1 & 1 \\ \frac{1}{\sqrt{5}} & \frac{-3}{\sqrt{5}} & \frac{3}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}$$

Slant变换 (II)

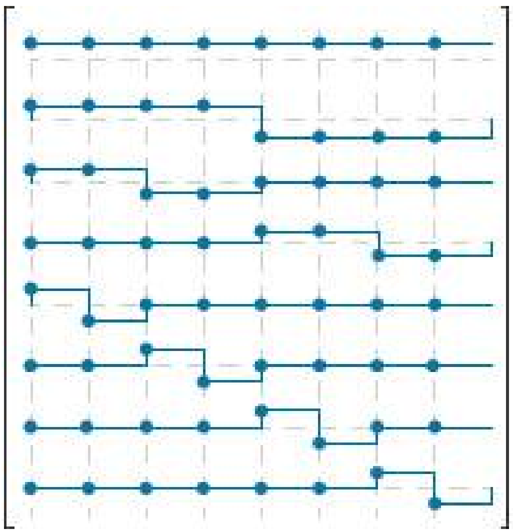


$$
\begin{bmatrix}
0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\
0.54 & 0.39 & 0.23 & 0.08 & -0.08 & -0.23 & -0.39 & -0.54 \\
0.47 & 0.16 & -0.16 & -0.47 & -0.47 & -0.16 & 0.16 & 0.47 \\
0.24 & -0.04 & -0.31 & -0.59 & 0.59 & 0.31 & 0.04 & -0.24 \\
0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 & 0.35 \\
0.35 & -0.35 & -0.35 & 0.35 & -0.35 & 0.35 & 0.35 & -0.35 \\
0.16 & -0.47 & 0.47 & -0.16 & -0.16 & 0.47 & -0.47 & 0.16 \\
0.16 & -0.47 & 0.47 & -0.16 & 0.16 & -0.47 & 0.47 & -0.16
\end{bmatrix}
$$



a b c

Haar变换的定义



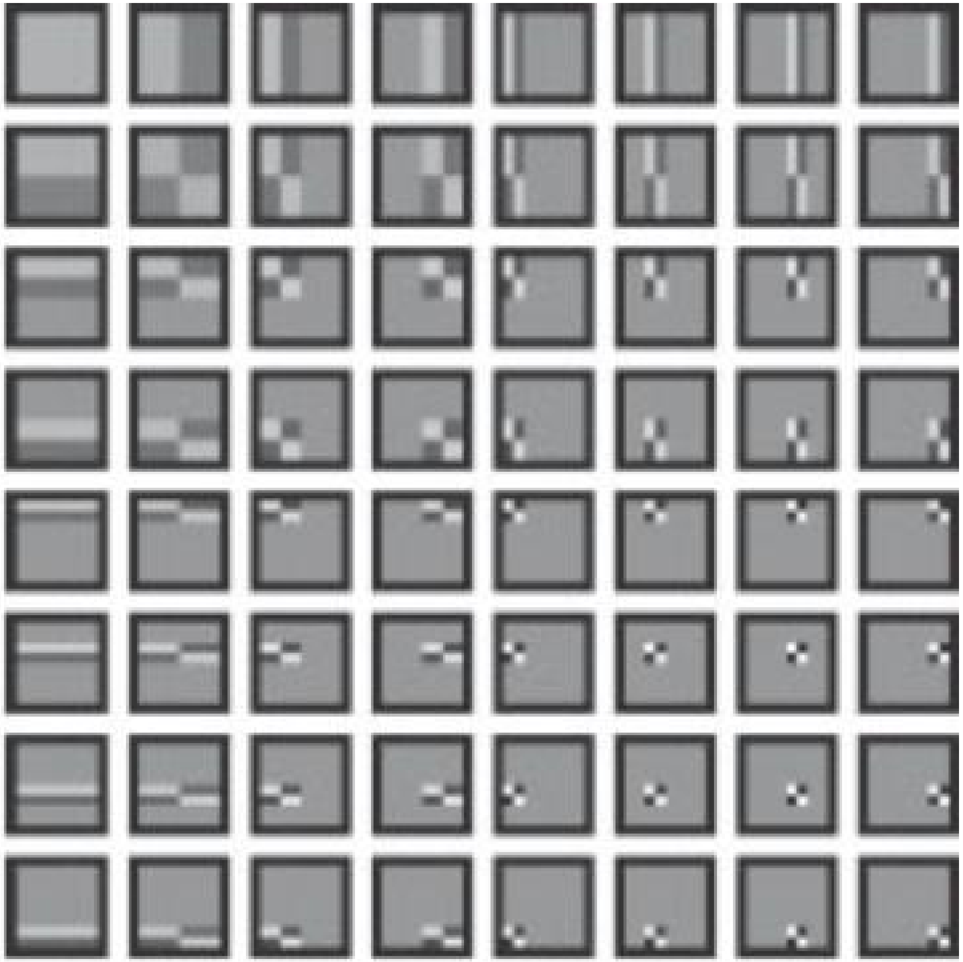
$$\begin{bmatrix}
0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\
0.35 & 0.35 & 0.35 & 0.35 & -0.35 & -0.35 & -0.35 & -0.35 \\
0.50 & 0.50 & -0.50 & -0.50 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.50 & 0.50 & -0.50 & -0.50 \\
0.71 & -0.71 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.71 & -0.71 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.71 & -0.71 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.71 & -0.71
\end{bmatrix}$$

a b c

$$h_u(x)=\begin{cases} 1 & u=0 \text{ and } 0\le x<1 \\ 2^{p/2} & u>0 \text{ and } q/2^p \le x < (q+0.5)/2^p \\ -2^{p/2} & u>0 \text{ and } (q+0.5)/2^p \le x < (q+1)/2^p \\ 0 & \text{otherwise} \end{cases}$$

已知的最早最简单的单位正交基

Haar变换



- 图像的正交变换
- 图像的距离变换
- 点扩散函数的基本概念
- 概率论基础 (复习)

为什么要使用图像距离变换？

- 在比较二进制图像时，即使很局部的差别也会造成大的区别。
- 距离变换可以减缓图像特征的变化。
- 在计算机视觉中有广泛应用。

如何计算图像距离变换？

- 定义 $$DT_{(P)}[x] = \min_{y \in P} dist(x, y)$$
- 在计算机图形学，机器人学与人工智能的应用中经常使用相距边界的最短距离。
- 在计算机视觉中，有时距离定义为相距某个特征的最短距离。

不同的距离定义 (I)

0	0	0
0	1	0
0	0	0

Image

1.41	1.0	1.41
1.0	0.0	1.0
1.41	1.0	1.41

Distance Transform

Euclidean $D_{Euclidean} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

0	0	0
0	1	0
0	0	0

Image

2	1	2
1	0	1
2	1	2

Distance Transform

Citi block $D_{City} = |x_2 - x_1| + |y_2 - y_1|$

0	0	0
0	1	0
0	0	0

Image

1	1	1
1	0	1
1	1	1

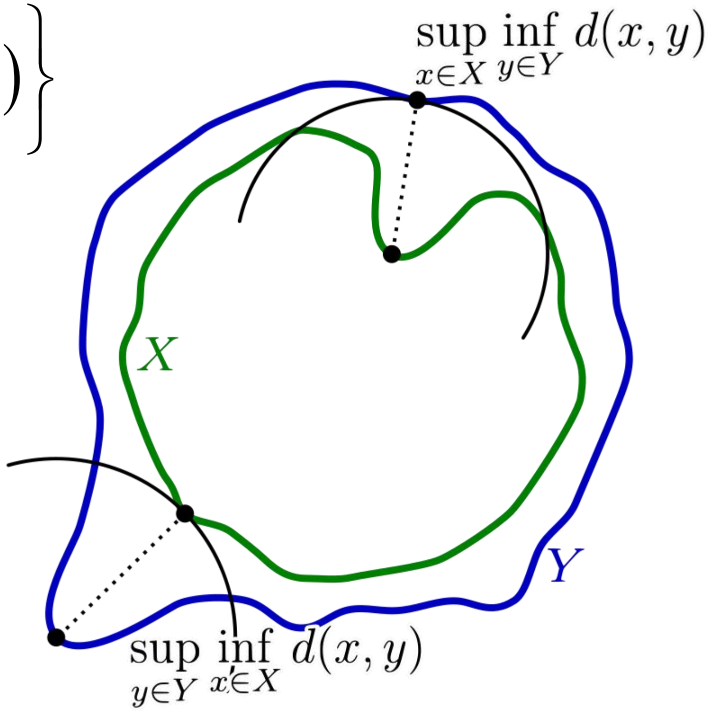
Distance Transform

Chessboard $D_{Chess} = \max\left(|x_2 - x_1|, |y_2 - y_1|\right)$

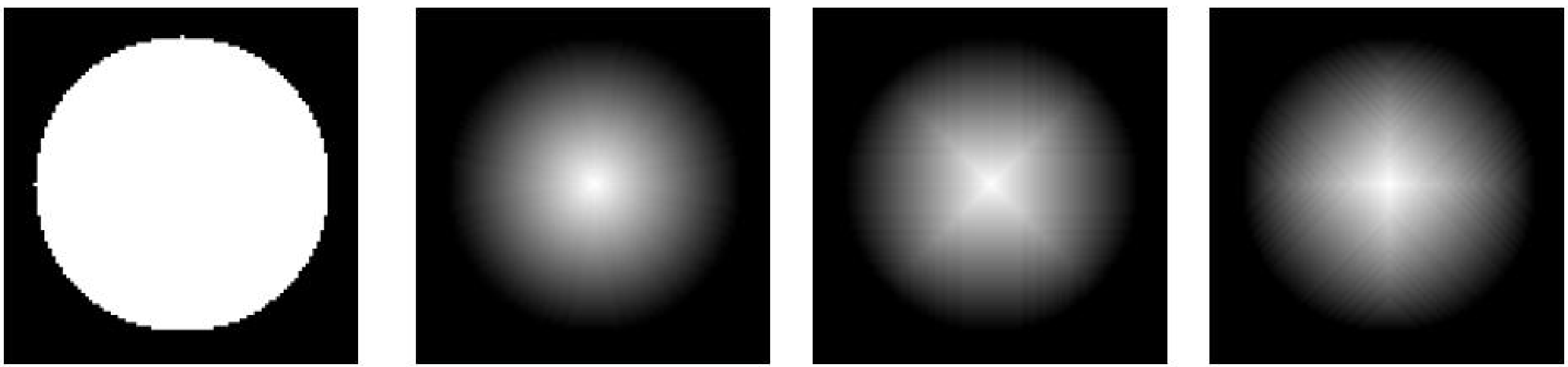
不同的距离定义 (II)

$$d_H(X,Y) = \max\left\{\sup_{x\in X}\inf_{y\in Y} d(x,y), \sup_{y\in Y}\inf_{x\in X} d(x,y)\right\}$$

Hausdorff



图像的距离变换 (II)



Euclidean City Chessboard

https://www.cs.cornell.edu/courses/cs664/2008sp/handouts/cs664-7-dtrans.pdf

图像的距离变换 (II)

0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0

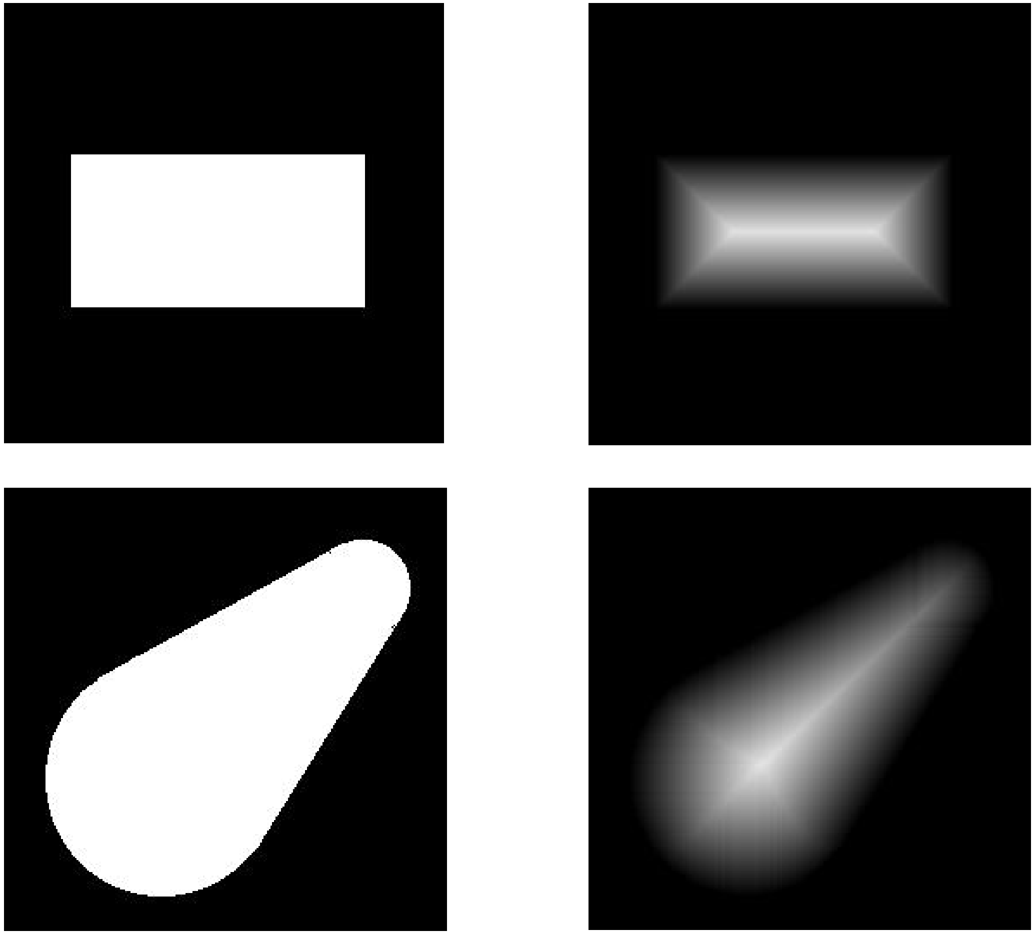
$\Rightarrow$

0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0
0	1	2	2	2	2	1	0
0	1	2	3	3	2	1	0
0	1	2	2	2	2	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0

定义 距离为与最近边界的距离

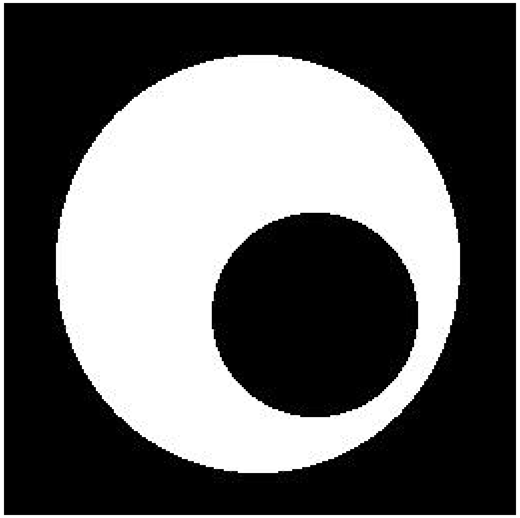
https://homepages.inf.ed.ac.uk/rbf/HIPR2/distance.htm

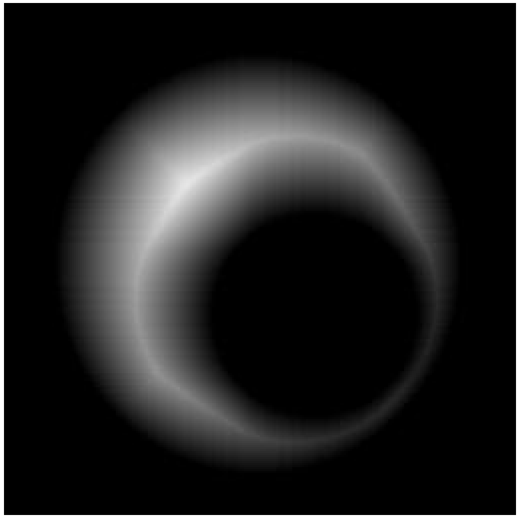
距离变换结果示例 (I)



https://homepages.inf.ed.ac.uk/rbf/HIPR2/distance.htm

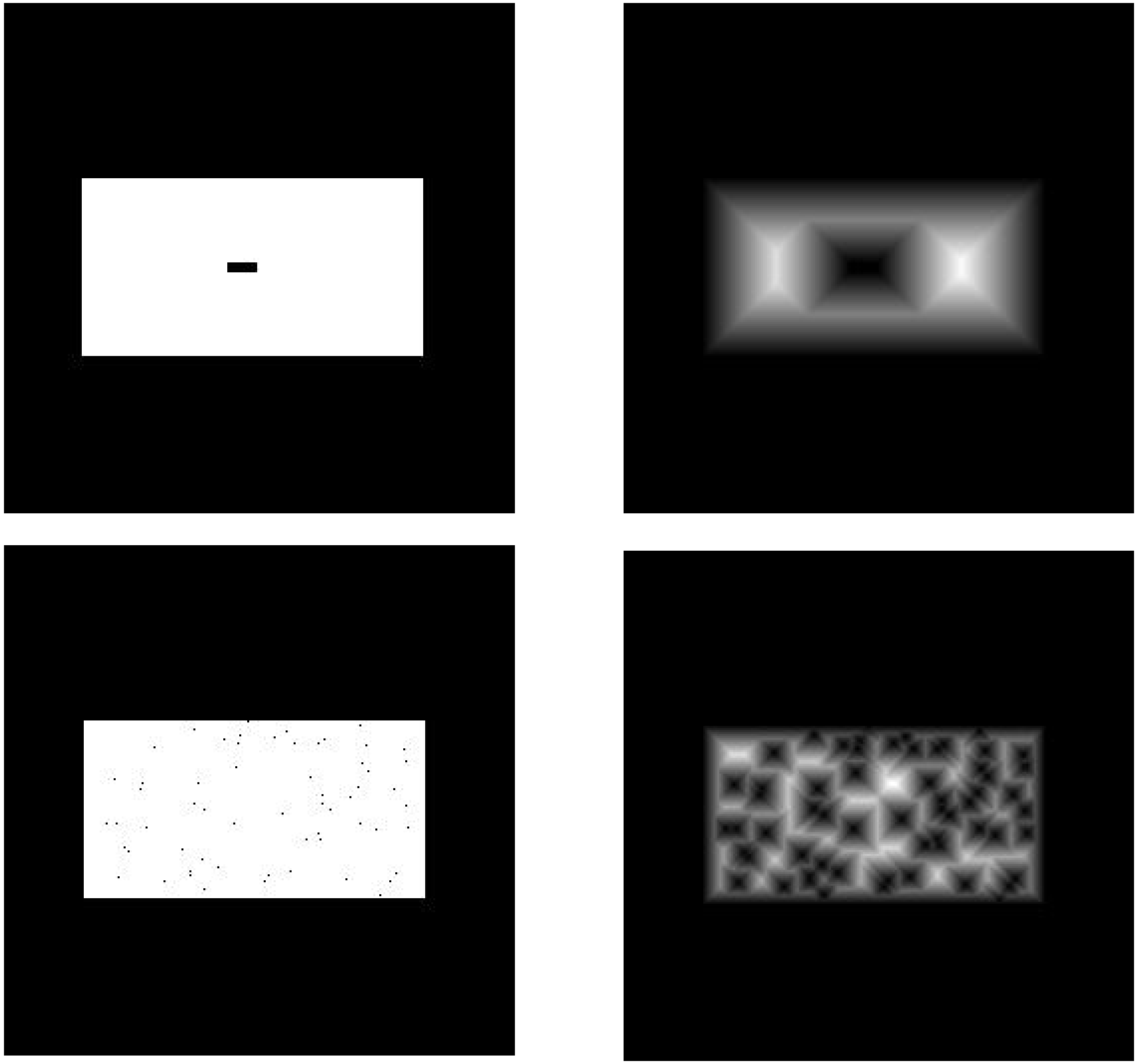
距离变换结果示例 (II)





https://homepages.inf.ed.ac.uk/rbf/HIPR2/distance.htm

距离变换的局限性



- 图像的正交变换
- 图像的距离变换
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A Microscope as a Linear System

- A light microscope can be considered as a linear system.

- A linear system satisfies the following two conditions
  - Homogeneity
  - Additivity

- Homogeneity

$$\boxed{\begin{array}{c} x(t) \rightarrow y(t) \\ \Downarrow \\ k \cdot x(t) \rightarrow k \cdot y(t) \end{array}}$$

- Additivity

$$\boxed{\begin{array}{c} x_1(t) \rightarrow y_1(t) \\ x_2(t) \rightarrow y_2(t) \\ \Downarrow \\ x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t) \end{array}}$$

How to Characterize a Linear System

- A linear system can be characterized by
  - Impulse response
  - Frequency response

- Impulse response of a microscope: point spread function

$$I(x,y) = O(x,y) \otimes psf(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} O(u,v)\cdot psf(x-u,y-v)\,du\,dv$$

- Frequency response of a microscope: optical transfer function

$$F\{I(x,y)\} = F\{O(x,y)\}\cdot F\{psf(x,y)\} = F\{O(x,y)\}\cdot OTF(\cdot)$$

Airy Disk

- Airy (after George Biddell Airy) disk is the diffraction pattern of a point feature under a circular aperture.

- It has the following form

$$y = \left[\frac{2J_1(x)}{x} \right]^2$$

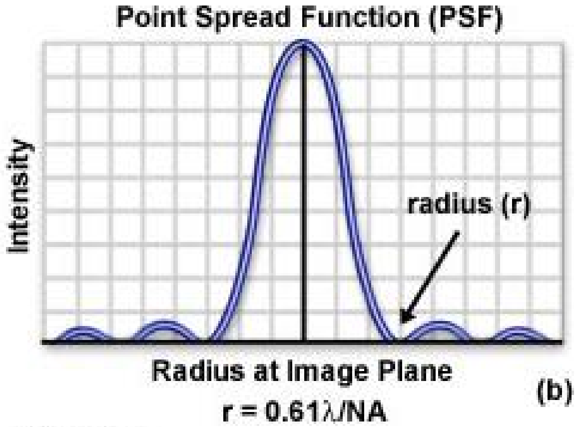


Figure 1

$J_1(x)$ is a Bessel function of the first kind.

- Detailed derivation is given in

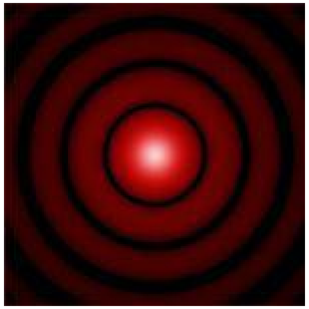
Born & Wolf, Principles of Optics, 7th ed., pp. 439-441.

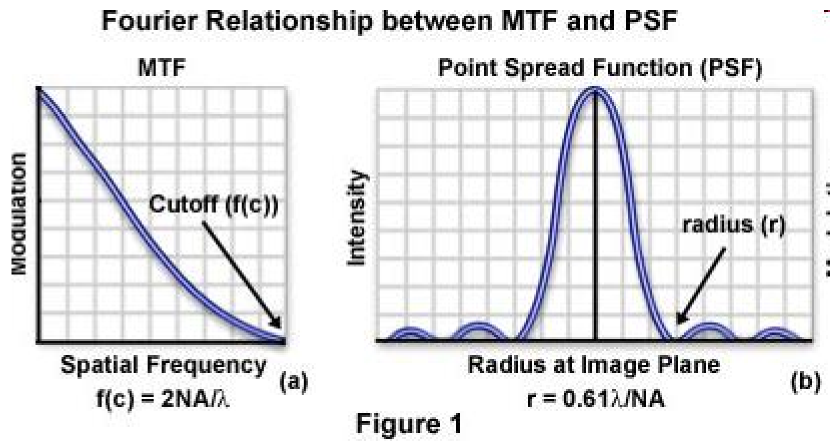
Microscope Image Formation (I)

- Microscope image formation can be modeled as a convolution with the PSF.

$$I(x,y) = O(x,y) \otimes psf(x,y)$$

$$F\{I(x,y)\} = F\{O(x,y)\} \cdot F\{psf(x,y)\}$$

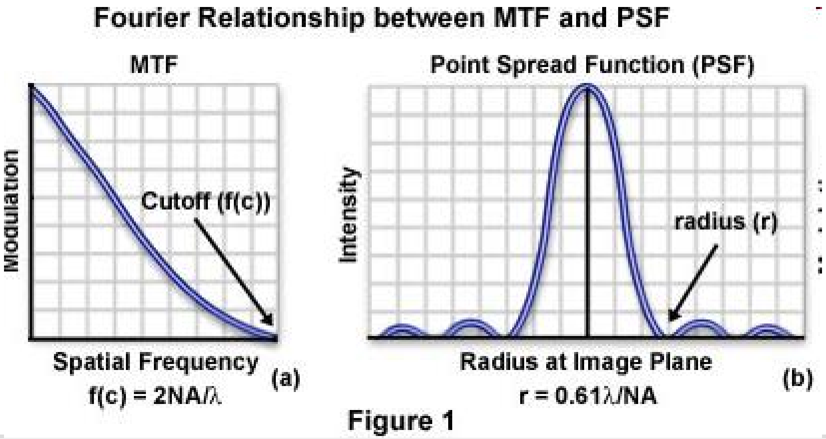




http://micro.magnet.fsu.edu/primer/java/mtf/airydisksize/index.html

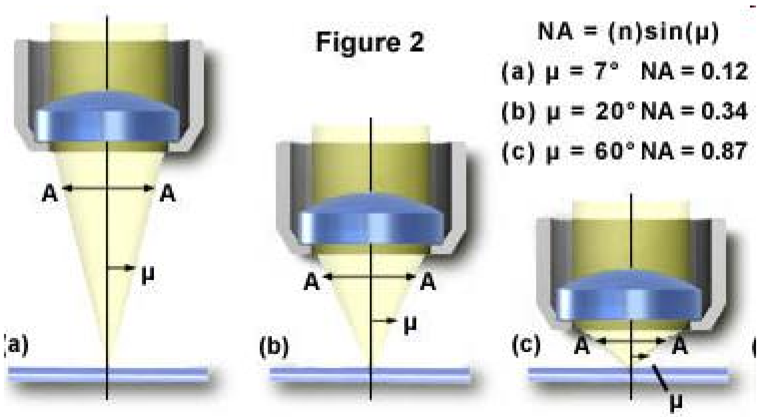
Microscope Image Formation (II)

- The impulse response of the microscope is called its point spread function (PSF).
- The transfer function of a microscope is called its optical transfer function (OTF).
- The PSF has the shape of an Airy Disk.



Numerical Aperture

- Numerical aperture (NA) determines microscope resolution and light collection power.



$$NA = n \cdot sin\ \mu$$

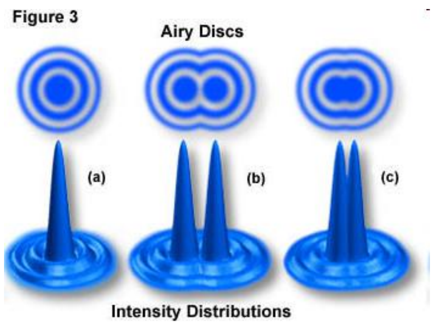
n: refractive index of the medium between the lens and the specimen

μ: half of the angular aperture

Different Definition of Light Microscopy Resolution Limit (Demo)

- Rayleigh limit

$$D = \frac{0.61\lambda}{NA}$$



- Sparrow limit

$$D = \frac{0.47\lambda}{NA}$$

http://www.microscopy.fsu.edu/primer/java/imageformation/rayleighdisks/index.html

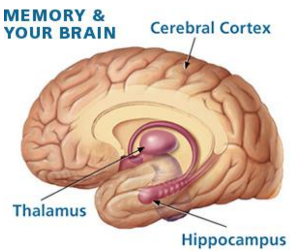
Summary: High Resolution Microscopy

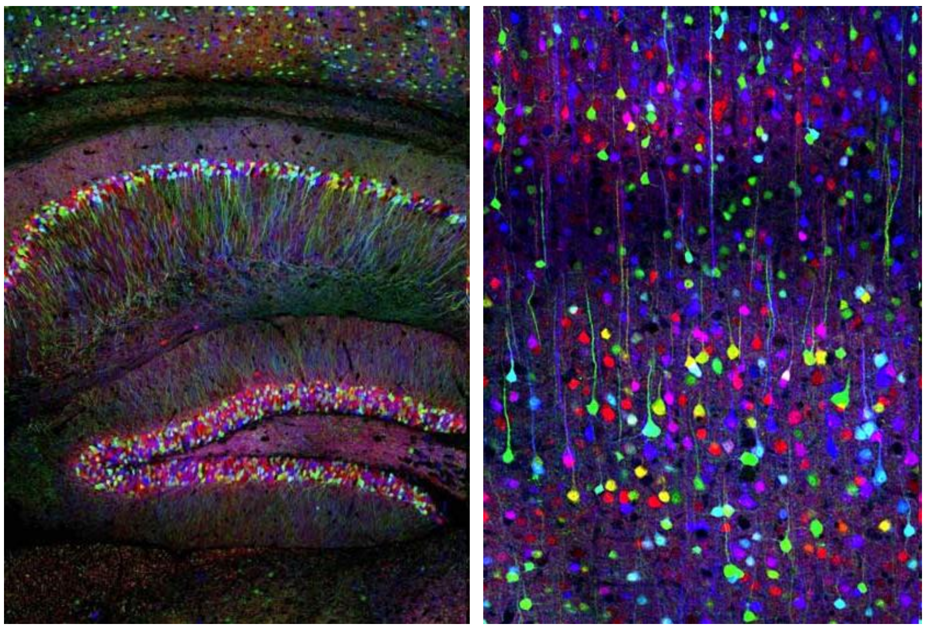
- Size of cellular features are typically on the scale of a micron or smaller.

- To resolve such features require
  - Shorter wavelength (electron microscopy)
  - High numerical aperture (resolution)
  - High magnification (spatial sampling)

$$D = \frac{0.61\lambda}{NA}$$

Introducing Fluorescence Microscopy Images





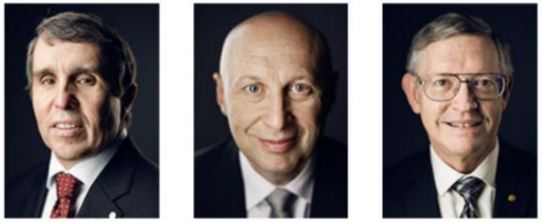
Livet J, Weissman TA, Kang H, *et al. Nature* **450**: 56–62, 2007

The Nobel Prize in Chemistry 2008

"for the discovery and development of the green fluorescent protein, GFP"

Osamu Shimomura	Martin Chalfie	Roger Y. Tsien
Photo: U. Montan	Photo: U. Montan	Photo: U. Montan
1/3 of the prize	1/3 of the prize	1/3 of the prize
USA	USA	USA
Marine Biological Laboratory (MBL) Woods Hole, MA, USA; Boston University Medical School Massachusetts, MA, USA	Columbia University New York, NY, USA	University of California San Diego, CA, USA; Howard Hughes Medical Institute
b. 1928 (in Kyoto, Japan)	b. 1947	b. 1952

The Nobel Prize in Chemistry 2014



Eric Betzig	Stefan W. Hell	William E. Moerner
Photo: A. Mahmoud	Photo: A. Mahmoud	Photo: A. Mahmoud
Prize share: 1/3	Prize share: 1/3	Prize share: 1/3

The Nobel Prize in Chemistry 2014 was awarded jointly to Eric Betzig, Stefan W. Hell and William E. Moerner *"for the development of super-resolved fluorescence microscopy".*

- 图像的正交变换
- 图像的距离变换
- 点扩散函数的基本概念
- 概率论基础 (复习)

概率论复习