Nanotechnologies for ICTs

Physics of NanoBiosystems course

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Nanomechanical Resonators Laboratory

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1 Introduction

In the last decades, mechanical sensing platforms have found large applications in many fields. Among all the possible structures, such as QMC, MMR, MC and SMR, in this laboratory, our study is dedicated to two types of suspended micro mechanical resonators: cantilever and bridge, exploited for biomedical analysis and biochemical detection.
The resonance behaviour of both structures can be explained through the so called *Euler-Bernoulli Theory* with the assumption that no damping phenomena occur.

1.1 Aim of the laboratory

The main objectives of this laboratory are to :

- Compute resonance measurements on four resonance beams: two microbridges (both ends clamped) and two microcantilevers (only one clamped end) of different length in order to better grasp the concept of responsivity;
- Compare the accuracy of the analytical method exploited during lectures, comparing its results with the ones obtained through simulations;
- Identify the inserted liquids density through further measurements on the forementioned resonators.

1.2 Instrument equipment

The readout instrument exploited is called *Laser Doppler Vibrometer*. It is made up of an integrated microscope with several lenses, a piezoelectric disc for actuation and a vacuum chamber, necessary for working on vacuum.
As the name suggests, it is based on the Doppler effect, where a laser source is splitted into two beams: one directed to the detector (used as reference beam) and another one directed, through objective lenses, perpendicularly to the sample put under vibration.
Then, the scattered light contains all the information about the vibration of the sample, since its frequency changes according to the velocity of movement of the microresonator. Once the scattered beam interacts with the reference one, we end up with an interference spectrum which gives us the information about the vertical displacement and the velocity of the beam.
The sample needs to be extremely rigid, for this reason diamond or silicon are often used. In our case, glass has been exploited due to ease of processing and cost. The manufacturing of the micro-structure has been done with a femto-second laser able to design the desired shape and break molecular bound inside the bulk. Hydrofluoric acid and potassium hydroxide are then used to generate the channels

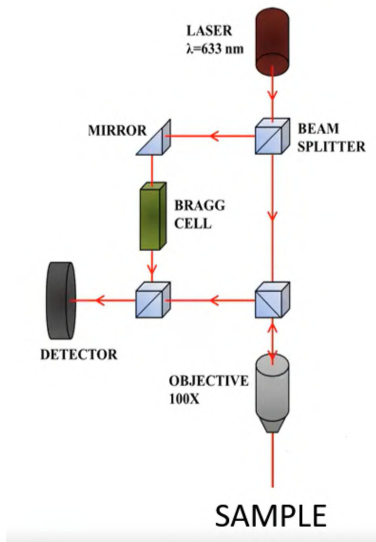


Figure 1: Laser Doppler Vibrometer: internal structure

inside the sample, removing the unnecessary glass. A focus ion beam microscope can be used to check the result of the process.

2 Analytical calculations

In order to best describe the system's response, a small analytical introduction will follow ([2].
Let's consider a transverse load q distributed along a clamped beam, namely a 3-dimensional object whose length is much greater than the other two dimensions.
By defining the internal shear forces inside the beam as V, in static condition, the external force q is: $q = -\frac{dV}{dx}$ in which x is the axis along the length.
V can be related to the bending moments with the relation: $V = -\frac{dM}{dx}$

Due to the elasticity of the beam, the load is responsible for a deflection, that can be described by a curvature $\frac{1}{\rho} = -\frac{M}{EI}$, where E represents the Young's modulus and I the moment of Inertia equal to $I = \frac{1}{12}\ wh^3$ for a rectangular section (w = width = 70μm and h = height = 40μm).
Writing the elastic term equal to $q = EI\frac{d^4w}{dx^4}$, and considering a damping force term and an external force applied on the structure the Euler-Bernoulli equation can be written as:

$$\rho A\frac{\partial^2 w(x,t)}{\partial t^2} + C\frac{\partial w(x,t)}{\partial t} + EI\frac{\partial^4 w(x,t)}{\partial x^4} = F_0 \cos wt$$

In this study, no damping terms or external forces are taken into account.
One or more eigen-frequencies ω_n are expected, and in order to find them, the separation of variables technique is exploited: $\omega_n(x,t) = \chi(x)T(t))$.
The Euler-Bernoulli equation becomes:

$$\rho A\ \chi(x)\frac{\partial^2 T(t)}{\partial t^2} = -EI\ T(t)\frac{\partial^4 \chi(x)}{\partial x^4}$$

It is possible to notice the similarity with the Schrodinger equation with which this theory shares many aspects. Then the two resulting equations can both be considered equal to the same constant:

$$\frac{1}{T}\frac{\partial^2 T}{\partial t^2} = -\frac{EI}{\rho A}\frac{1}{\chi}\frac{\partial^4 \chi}{\partial x^4} = \text{constant} = c$$

The left-hand side has the form of a cosine function, as the external force applied to the system, and it is possible to determine that:

$$c = -w^2$$

The right-hand side leads to the expression below:

$$\varkappa_n^4 = \frac{\rho A}{EI} w_n^2$$

that has a general solution of the form:

$$\chi(x) = a_1 \cos \varkappa x + a_2 \sin \varkappa x + a_3 \cosh \varkappa x + a_4 \sinh \varkappa x$$

where first two terms represent a standing wave and the second two terms represent how clamping influences the response.
Four constants need four boundary conditions for the cantilever to be determined, and others four boundary conditions for the bridge.

2.1 Cantilever solution

Considering a cantilever clamped in a infinite bulk unable to move, the first 2 boundary conditions for this structure at $x = 0$ are:

$$\chi(x = 0) = 0$$

due to no displacement, and

$$\chi'(x = 0) = 0$$

due to small displacement approximation.
The second boundary conditions are in $x = L$:

$$\chi'''(x = L) = 0$$

due to the absence of net force and

$$\chi''''(x = L) = 0$$

due to the absense of net momentum.

These conditions lead to the transcendent equation below:

$$\cos \mathrm{kL} \cdot \cosh \mathrm{kL} + 1 = 0$$

that needs to be solved graphically.
The roots of the equation are fundamental states also called modal numbers that are listed in the table below.

Table 1: Fundamental states

$K_1L =$	$\alpha_1 =$	1.875	
$K_2L =$	$\alpha_2 =$	4.694	
$K_3L =$	$\alpha_3 =$	7.855	
$K_nL =$	$\alpha_n =$	$(2n-1)\,\frac{\pi}{2}$	when $n > 5$

The frequencies of resonance can be determined by using the fundamentals states as:

$$f_n = \frac{(kL)^2}{2\pi\, L^2}\sqrt{\frac{EI}{\rho A}}$$

The total density ρ, in order to compensate the approximation due to a non exact moment of inertia (because of the presence of the channels), during the laboratory was determined considering the mass of the beam without the channels plus the mass of the fluid inside the channels, dividing these two contributes by the total volume of the beam.

$$\rho = \frac{(V_{beam} - V_{channels}) \cdot \rho_{glass} + V_{channels} \cdot \rho_{fluid}}{V_{tot}}$$

where $\rho_{glass} = 2195 kg/m^3$.

ρ_{Air}	$=$	$1.225\ kg/m^3$
ρ_{Vacuum}	$=$	$6.5 \cdot 10^{-27}\ kg/m^3$
ρ_{Water}	$=$	$997\ kg/m^3$
$\rho_{Ethanol}$	$=$	$789\ kg/m^3$
$\rho_{Ethanol-50\%-Water}$	$=$	$893\ kg/m^3$
$\rho_{Ethanol-25\%-Water}$	$=$	$945\ kg/m^3$
$\rho_{Glycerol-10\%-Ethanol}$	$=$	$836.1\ kg/m^3$
$\rho_{Glycerol-25\%-Ethanol}$	$=$	$906.75\ kg/m^3$
$\rho_{Glycerol-10\%-Water}$	$=$	$1.23.3\ kg/m^3$
$\rho_{Acetone}$	$=$	$784\ kg/m^3$
ρ_{Hexane}	$=$	$655\ kg/m^3$

Table 2: Densities of different fluids

2.1.1 Matlab Implementation

In order to perform a comparison between the experimental values and the analytical results of the resonance frequencies, the following Matlab script was implemented.

```
%% Cantilever


E=73e9; % Young's modulus [Pa]
rho_glass=2.195e3; % Glass Density [Kg/m^3]
rho_air=1.225; % Air Density [Kg/m^3]
rho_vacuum=6.5e-27; % Vacuum Density [Kg/m^3]

%measures L>>W>>H
L=500e-6; % Cantilever Length [m]
W=70e-6; % Cantilever Width [m]
H=40e-6; % Cantilever Height [m]
r=5e-6; % Radius of the channel [m]
v_cil=L*pi*r^2; % Volume of the channel [m^3]
v_tot=W*L*H; % Volume of the cantilever [m^3]

rho_eq=((((v_tot)-(2*v_cil))*rho_glass)+((2*v_cil)*rho_air))/(v_tot)
% Equivalent Density in Air [kg/m^3]
rho_eq_vacuum=((((v_tot)-v_chan)*rho_glass)+(v_chan*rho_vacuum))/(v_tot)
% Equivalent Density in Vacuum [kg/m^3]

%Modal Numbers
alfa1=1.875;
alfa2=4.694;
alfa3=7.855;

%frequencies
f1=alfa1^2*H*sqrt(E/(12*rho_eq))/(2*pi*L^2) %Hz
f2=alfa2^2*H*sqrt(E/(12*rho_eq))/(2*pi*L^2)
f3=alfa3^2*H*sqrt(E/(12*rho_eq))/(2*pi*L^2)

f1_vacuum=alfa1^2*H*sqrt(E/(12*rho_eq_vacuum))/(2*pi*L^2) %Hz
f2_vacuum=alfa2^2*H*sqrt(E/(12*rho_eq_vacuum))/(2*pi*L^2)
f3_vacuum=alfa3^2*H*sqrt(E/(12*rho_eq_vacuum))/(2*pi*L^2)
```

The same script has been used for the longer cantilever, by appropriately modifying the constant L.
We found that the first three resonance modes occur at the frequencies listed in Table 3. As can be seen, there are no appreciable differences between the frequencies calculated in air and in vacuum.

Structure	f_1 [MHz]	f_2 [MHz]	f_3 [MHz]
Cantilever 500μm (Air)	0.153	0.961	2.692
Cantilever 500μm (Vacuum)	0.153	0.961	2.692
Cantilever 750μm (Air)	0.068	0.427	1.197
Cantilever 750μm (Vacuum)	0.068	0.427	1.197

Table 3: Analytical resonance frequencies of the cantilevers

It is possible to notice how the length of the structure affects the response of the system. In particular, since longer structures have a higher mass, this results in a shift of the resonance frequencies downward. In fact: $\Delta m = -2\frac{\Delta\omega}{\omega_n} m$ and remember that $\omega = \frac{f}{2\pi}$.

2.2 Bridge solution

Considering a bridge clamped in $x = 0$ and in $x = L$ through an infinite bulk unable to move, the four boundary conditions, to solve the Euler-Bernoulli equation are:

$$\chi(x = 0) = 0$$

$$\chi(x = L) = 0$$

due to no displacement of the edges, and

$$\chi'(x = 0) = 0$$

$$\chi'(x = L) = 0$$

due to the small displacement approximation that implies a slope equal to zero at the edges.
These conditions lead to the transcendent equation below:

$$\cos kL \cdot \cosh kL - 1 = 0$$

Also in this case the roots of the equation are fundamental states that are listed in the table below.

Table 4: Fundamental states

$$\begin{array}{llll} K_1L = & \alpha_1 = & 4.730 & \\ K_2L = & \alpha_2 = & 7.853 & \\ K_3L = & \alpha_3 = & 10.996 & \\ K_nL = & \alpha_n = & (2n+1)\,\frac{\pi}{2} & \text{when } n > 3 \end{array}$$

The frequencies of resonance can be determined by using the fundamentals states as:

$$f_n = \frac{(k_n L)^2}{2\pi} \sqrt{\frac{E}{12\rho}} \frac{H}{L^2}$$

2.2.1 Matlab Implementation

By adopting the same implementation steps of the Cantilever case, we proceeded with the mathematical calculation of the first three resonance frequencies.

```
%% Bridge
E=73e9; % Young's Modulus [Pa]
rho_glass=2.195e3; % Glass Density [Kg/m^3]
rho_air=1.225; % Air Density [Kg/m^3]
rho_vacuum=6.5e-27; % Vacuum Density [Kg/m^3]

%measures L>>W>>H
L=500e-6; % Bridge Length [m]
W=70e-6; % Bridge Width [m]
H=40e-6; % Bridge Height [m]
v_tot=L*W*H % Bridge Volume [m^3]

W_chan=30e-6; % Channel Width [m]
h_chan=15e-6; % Channel Height [m]
v_chan=W_chan*h_chan*L; % Channel Volume [m^3]
rho_eq=((((v_tot)-v_chan)*rho_glass)+(v_chan*rho_air))/(v_tot)
% Equivalent Density in Air
rho_eq_vacuum=((((v_tot)-v_chan)*rho_glass)+(v_chan*rho_vacuum))/(v_tot)
% Equivalent Density in Vacuum

%Modal Numbers
alfa1=4.730;
alfa2=7.853;
alfa3=10.996;

f1=alfa1^2*H*sqrt(E/(12*rho_eq))/(2*pi*L^2) %Hz
f2=alfa2^2*H*sqrt(E/(12*rho_eq))/(2*pi*L^2)
f3=alfa3^2*H*sqrt(E/(12*rho_eq))/(2*pi*L^2)

f1_vacuum=alfa1^2*H*sqrt(E/(12*rho_eq_vacuum))/(2*pi*L^2) %Hz
f2_vacuum=alfa2^2*H*sqrt(E/(12*rho_eq_vacuum))/(2*pi*L^2)
f3_vacuum=alfa3^2*H*sqrt(E/(12*rho_eq_vacuum))/(2*pi*L^2)
```

Structure	f_1 [MHz]	f_2 [MHz]	f_3 [MHz]
Bridge 500µm (Air)	1.035	2.854	5.594
Bridge 500µm (Vacuum)	1.035	2.854	5.595
Bridge 750µm (Air)	0.4601	1.268	2.487
Bridge 750µm (Vacuum)	0.4601	1.268	2.487

Table 5: Analytical resonance frequencies of the bridges

Also in this case smaller structures manifest higher resonance frequencies and as seen in the previous case, there are no appreciable differences between the frequencies calculated in air and in vacuum.

3 Experimental

3.1 Bridges

The structure taken into consideration is a bridge, thus a clamped-clamped beam, or double-side clamped beam, with a channel, that has the form of a prism with a rectangular base.

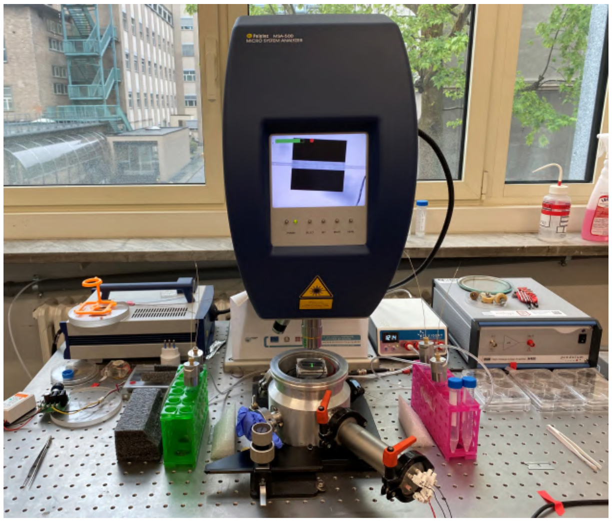


Figure 2: Bridge

Two bridges of different dimensions are used, a longer one with a length of 750µm and a shorter one of 500µm.

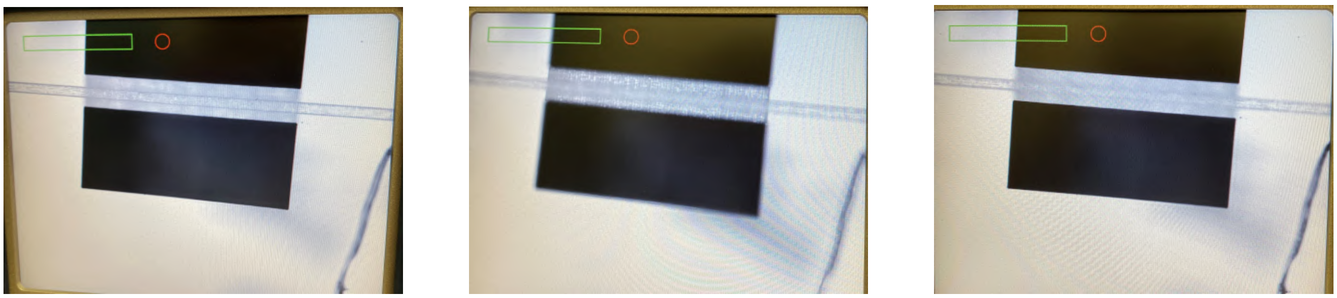
3.1.1 750µm long bridge

The first test was carried out for the longer bridge, with dimensions:

- **Length**: 750µm
- **Width**: 80µm

- **Thickness**: 40µm
- **Channel**: 30µm x 15µm

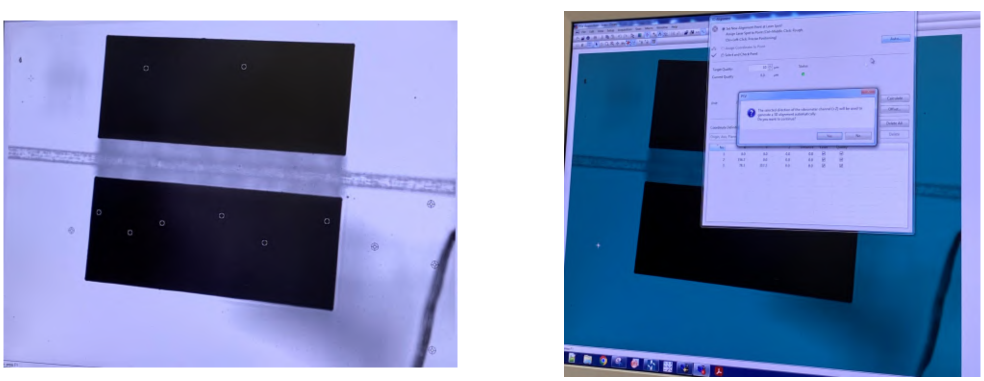
It is placed in the Doppler Laser Vibrometer and the focus is set, as shown in the picture below, where the chosen focus is the last.



(a) Focus on the internal channel (b) Focus on the top (c) Focus on the bottom

Figure 3: Focus on the bridge

The first two used tools are for the alignment in order to "tell the system" where the structure is and to take a reference.
The alignments are both in 2D and 3D, where the first one is manual, and consists in choosing a series of points both in the structure and in the bulk, that the laser will hit, while the second one is automatic.



(a) 2D aligning (b) 3D aligning

Figure 4: 2D and 3D aligning of the bridge

The 2D aligning is needed to create an ideal plane, which is its starting point, while the 3D one measures the position of the structure with respect to the z axis, thus the length of the path of the laser. The third tool, "define scan point", defines the points we want to sample for our test. Successively, the scan grid is created, put on top and fixed, in order for it to have the same size of the channel, and be aligned with it.

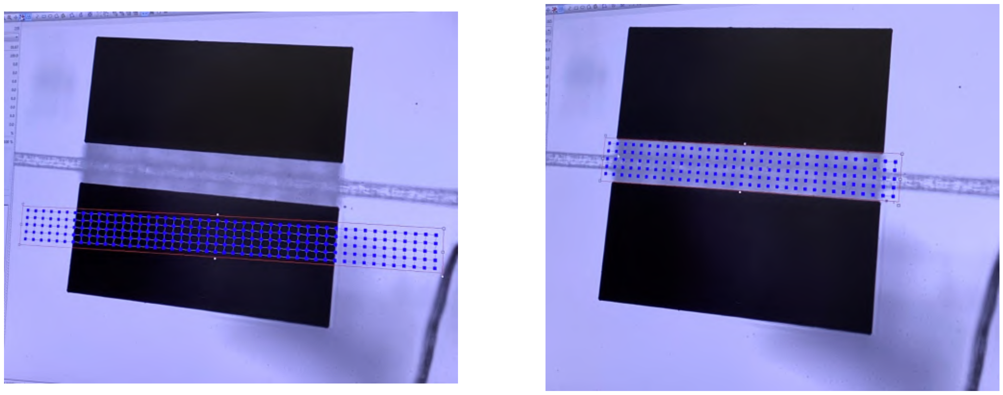


Figure 5: Scan grid positioning and alignment

The points for the rectangle of the grid are both in the bulk and in the structure, and the first are needed to understand if, once measured, a peak is of resonance or noise; in the case of resonance the bulk does not move, or more precisely, its movement is negligible, making the movement of the beam much more noticeable, while in the case of noise, the bulk moves.
Afterwards the acquisition settings were set, such as:

- **averaging count**: measures repeated for each point;
- **decoder** that the system uses to work on the information: for bridges, it is set to "displacement", which allows to get to a sampling of 20MHz, while for bridges, "velocity", which gets up to 2MHz, a sufficient value for the laboratory;
- **range of output**: the measure of the resonator;
- **bandwidth of the frequency**;
- **FFT lines**: the number of samples in the bandwidth (the higher is the bandwidth, the higher is the number of samples);
- **signal**: it is a periodic chirp, thus a pack of functions;
- **actuation tension**: for the cantilever 0.5V, while for the bridge higher.

For example, for the longest gate, the acquisition settings are:

Scan Points		
Total:	175	
Not Measured:	0	0.0 %
Valid:	0	0.0 %
Optimal:	175	100.0 %
Overrange:	0	0.0 %
Invalidated:	0	0.0 %
Disabled:	0	0.0 %
Not Reachable:	0	0.0 %
Hidden:	0	0.0 %
VT Failed:	0	0.0 %
Hardware		
Scanning Head:	MSA-I-500 (OFV-551)	
Firmware version:	1.03	
Junction Box:	MSA-E-500-M2-20	
Firmware Version:	2.000	
Acquisition Board:	Spectrum M2i.3027	
Channels Count:	2	
Highest Sample Frequency:	102.4 MHz	
Firmware Version:		
Function Generator		
Firmware Version:		
Channels Count:	1	
General		
Acquisition Mode:	FFT	
Averaging:	Magnitude	
Averaging count:	5	
Remeasure Automatically:	Not active	
AutoRange:	Not active	
PCA (MIMO):	Not active	
Frequency		
Bandwidth:	2 MHz	
Bandwidth from:	20 kHz	
Bandwidth to:	2 MHz	
Sampling		
FFT Lines:	204800	
Sample frequency:	5.12 MHz	
Sample time:	102.4 ms	
Resolution:	9.765625 Hz	
Trigger		
Source:	Off	
Phase from reference:	On	

(a) Scan grid

Channel Vibrometer (connected to Vibrometer 1)	
Direction:	+Z
Range:	500 mV
Coupling:	DC
Impedance:	1 MOhm
Quantity:	Velocity
Calibration factor:	200e-3 (m/s)/V
Signal Delay:	3.76e-6 s
Filter Type:	No Filter
Int/Diff Quantity:	Velocity
Window:	Rectangle
Signal Enhancement:	Active
Channel Reference 1	
Reference point index:	0
Direction:	+Z
Range:	1 V
Coupling:	DC
Impedance:	1 MOhm
Quantity:	Velocity
Calibration factor:	1 (m/s)/V
Filter Type:	No Filter
Int/Diff Quantity:	Velocity
Window:	Rectangle
Signal Enhancement:	Not active
Signal Enhancement	
Speckle Tracking:	Active
Mode:	Fast
Vibrometer 1	
Controller:	OFV-5000
Firmware version:	3.05
Tracking filter:	Off
Velocity output	
Range:	VD-09 200 mm/s/V
Low pass filter:	Off
High pass filter:	Off
Displacement output	
Range:	DD-300 (Aux) 50 nm/V
Function Generator 1	
Type:	M2i 60xx
Signal:	Periodic Chirp
Amplitude:	500 mV
Offset:	0 V
Amplitude correction:	None
Multiple Channels:	Off

(b) Scan grid positioning and alignment

Figure 6: Acquisition settings of the "long" bridge

After fixing the settings, we switch on the generator, an important move in order not to get only noise, and there are three measure modes: continuous current, single point or whole grid; we opt for the first one.

We place the laser where we want and the result is the resonance spectrum for the bridge:

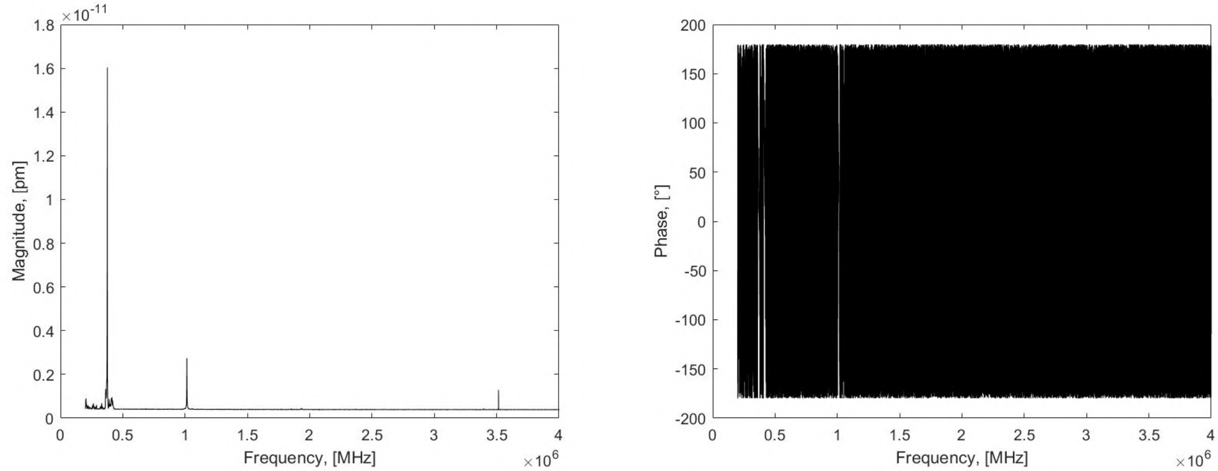


Figure 7: Resonance spectrum: long bridge

It is possible to see two plots: the one on the right is the magnitude as a function of frequency, while the one on the left is the phase as a function of frequency.
On the right graph it is possible to see the three vibrational modes, which correspond to a shift of 180° of the phase, on the left plot.
The first mode corresponds to a resonance frequency of approximately 0.3743MHz:

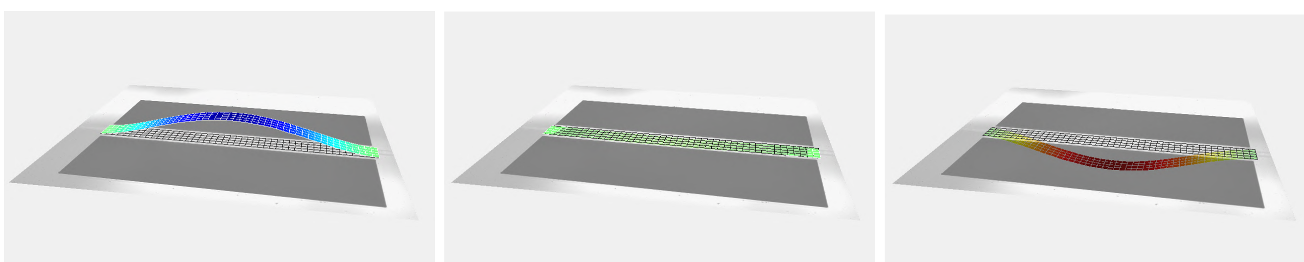


Figure 8: First mode of the long bridge

The second mode corresponds to a resonance frequency of approximately 1.0142 MHz:

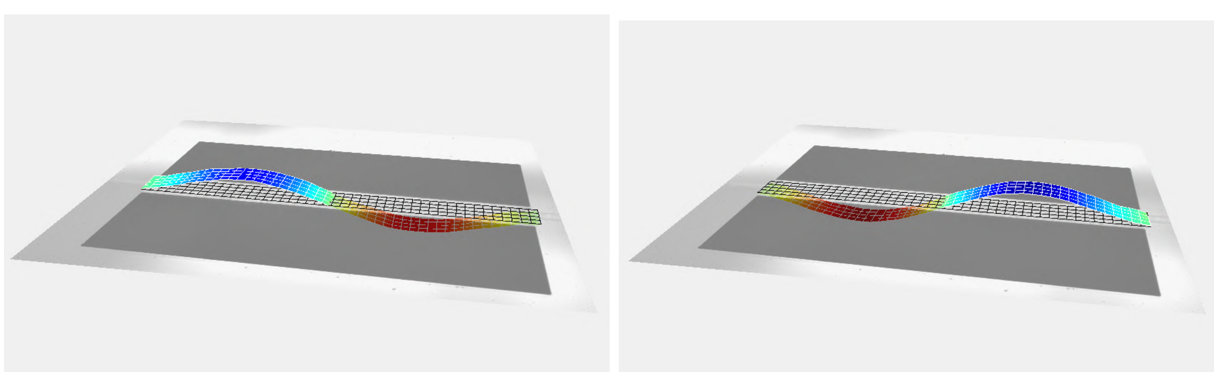


Figure 9: Second mode of the long bridge

The third mode corresponds to a resonance frequency of approximately 1.93377 MHz:

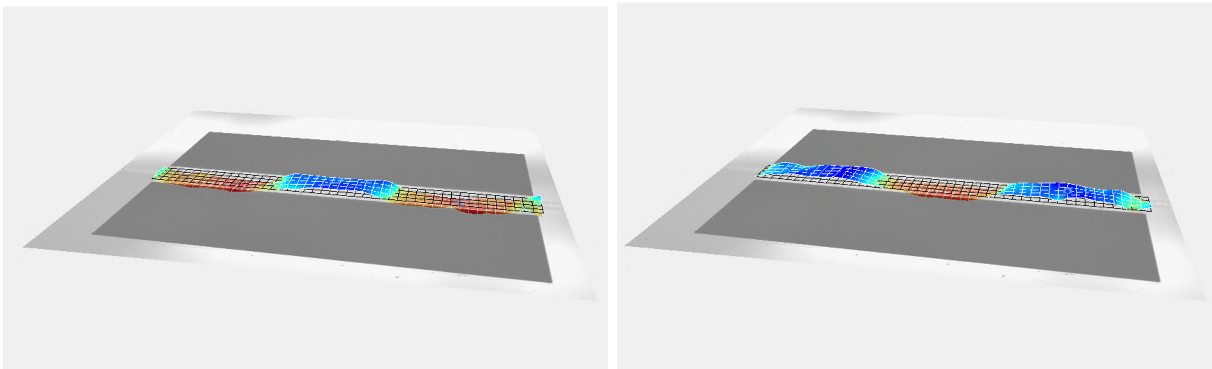


Figure 10: Third mode of the long bridge

For this structure it is also possible to see a torsional mode, which corresponds to a frequency of approximately 1.8521 MHz:

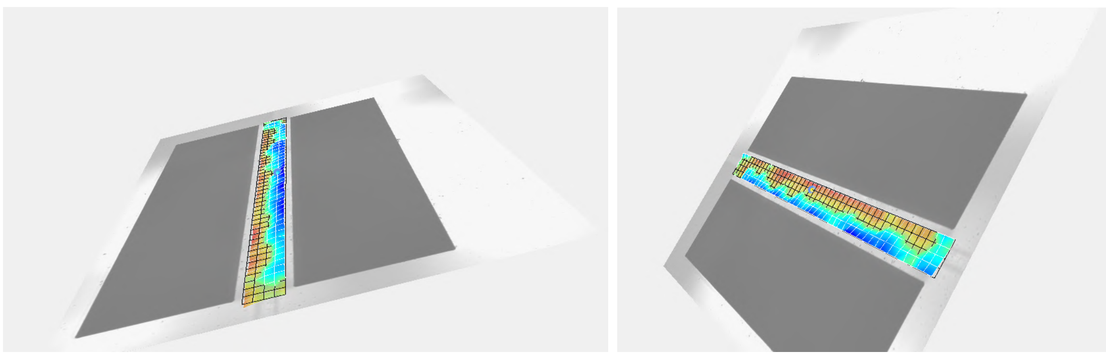


Figure 11: Torsional mode of the long bridge

3.1.2 500µm long bridge

The same steps were followed for the "shorter bridge" with measures:

- **Length**: 500µm
- **Width**: 80µm
- **Thickness**: 40µm
- **Channel**: 30µm x 15µm

The resulting resonance spectra are:

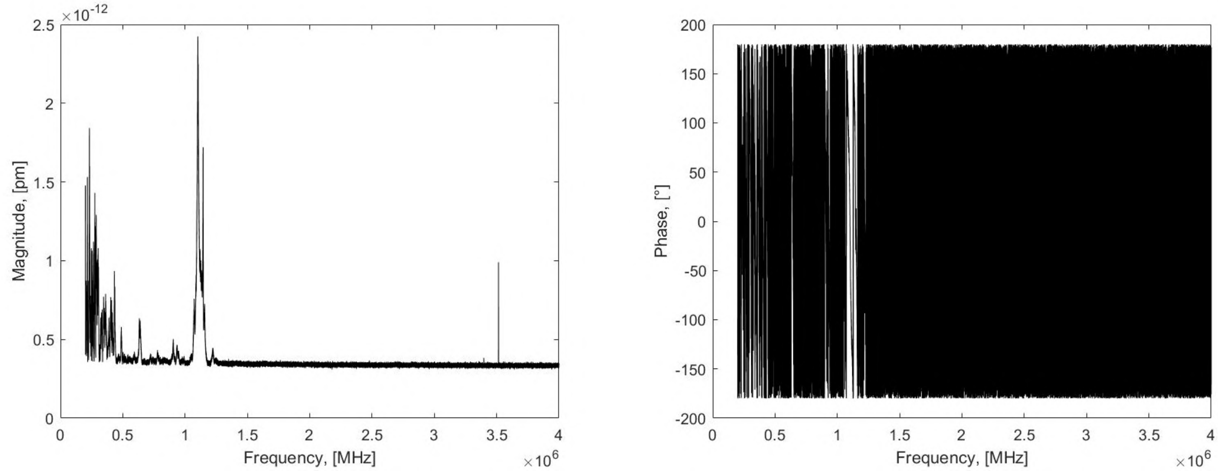


Figure 12: Resonance spectrum: short bridge

The plot of the shorter bridge is far more noisy than the shorter one and that is expected, due to the fact that decreasing the length "distances" us from the ideality with which the beam is studied and analyzed, i.e. length>>width>>thickness.
In this case there is just one vibrational mode, which more or less corresponds to a resonance frequency of 1.1024MHz.

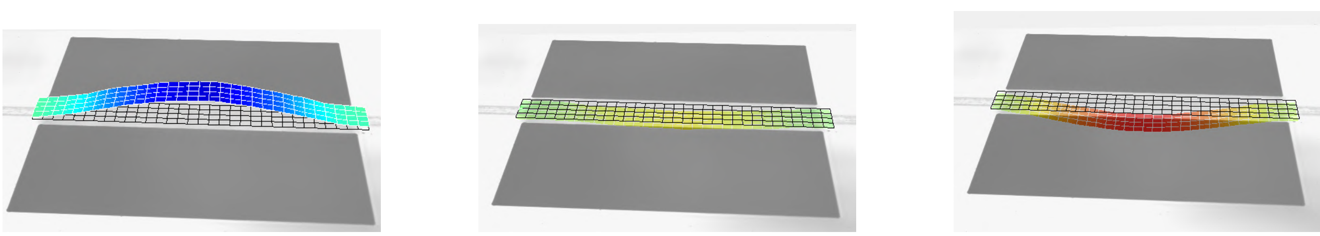


Figure 13: First mode of the short bridge

The measured frequencies for the two bridges, in air, are summed up in the table below:

Structure	**First Mode [Hz]**	**Second Mode [Hz]**	**Third Mode [Hz]**	**First Torsional mode [Hz]**
Bridge 750 µm	$0.3744 \cdot 10^6$	$1.0142 \cdot 10^6$	$1.9334 \cdot 10^6$	$1.8521 \cdot 10^6$
Bridge 500 µm	$1.1024 \cdot 10^6$	N/A	N/A	N/A

Table 6: Experimental resonance frequencies of bridges (in air)

The frequency does not scale with width, but linearly with thickness and inverse to quadratically in length, thus a longer bridge corresponds to lower frequencies.

3.2 Cantilevers

Subsequently, the calculations are performed for two cantilevers, thus one side-clamped beams, with a very different channel from the double-clamped structure previously analyzed, since it presents a cylinder like shape which, close to the free edge of our beam, presents a U shaped turn. This configuration can be easily approximated with two parallel cylindrical channels.

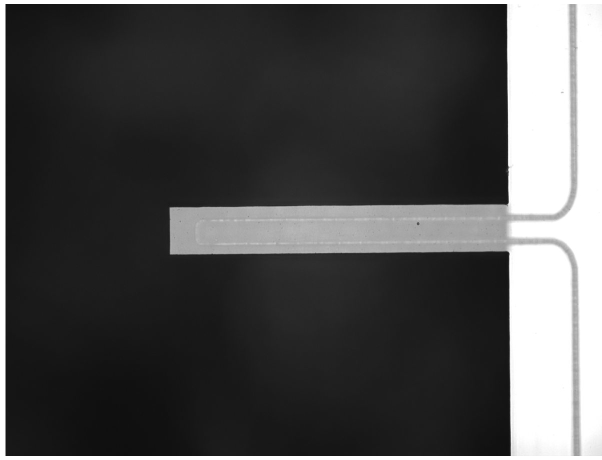


Figure 14: Cantilever

In the software we configure exactly the same parameters, with the only difference that, now, in the "Channels" window we set *Velocity* as the quantity that the Vibrometer will analyse while, in the bridge case, the quantity was set to *Displacement*.
In the following are shown the main setting parameters for the Cantilever microresonator.

```
General

Acquisition Mode:          FFT
Averaging:                 Magnitude
Averaging count:           5
----------------------------------------
Frequency

Bandwidth:                 2 MHz
Bandwidth from:            20 kHz
Bandwidth to:              2 MHz
----------------------------------------
Sampling

FFT Lines:                 204800
Sample frequency:          5.12 MHz
Sample time:               102.4 ms
Resolution:                9.765625 Hz
----------------------------------------
Channel Vibrometer (connected to Vibrometer 1)

Direction:                 +Z
Range:                     200 mV
Coupling:                  DC
Impedance:                 1 MOhm
Quantity:                  Velocity

Channel Reference 1

Reference point index:     0
Direction:                 +Z
Range:                     0.5 V
Coupling:                  DC
Impedance:                 1 MOhm
Quantity:                  Velocity
----------------------------------------
Function Generator 1

Type:                      M2i 60xx
Signal:                    Periodic Chirp
Amplitude:                 500 mV
Offset:                    0 V
Amplitude correction:      None
Multiple Channels:         Off
```

Figure 15: Setting parameters Cantilever

The Cantilevers lengths are, for the longer one: 750µm and for the shorter one : 500µm.

3.2.1 500µm long Cantilever

The first device tested is the shorter Cantilever:

- **Length**: 500µm
- **Width**: 70µm
- **Thickness**: 40µm
- **Channel**: 10µm (Diameter)

The sample is placed in the Laser Doppler Vibrometer in order to begin the procedure for the calculations.
The resulting spectra are the following.

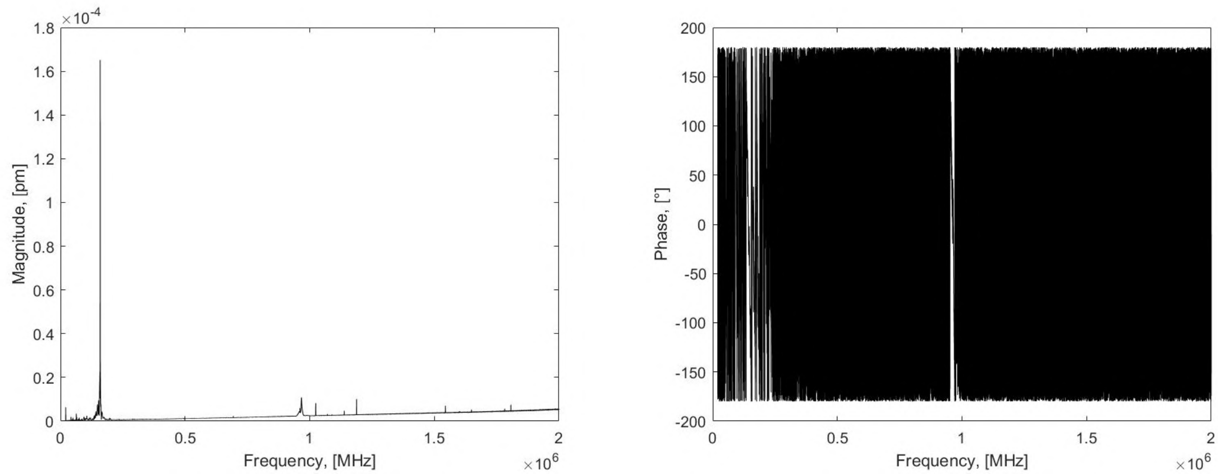


Figure 16: Resonance spectrum: short cantilever

The first mode corresponds to a resonance frequency of around 0.1595 MHz.

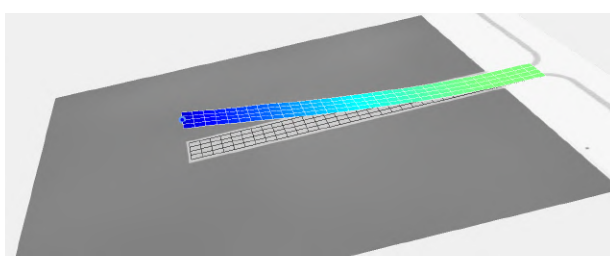


Figure 17: First mode of the short cantilever

The second mode corresponds to a resonance frequency of approximately 0.9667 MHz.

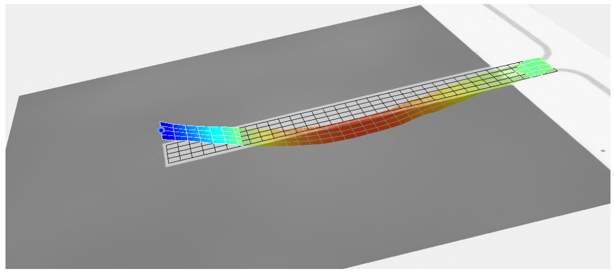


Figure 18: Second mode of the short cantilever

It is also possible to see a torsional mode around 1.5442 MHz, which can be found because a 180° phase shift occurs.

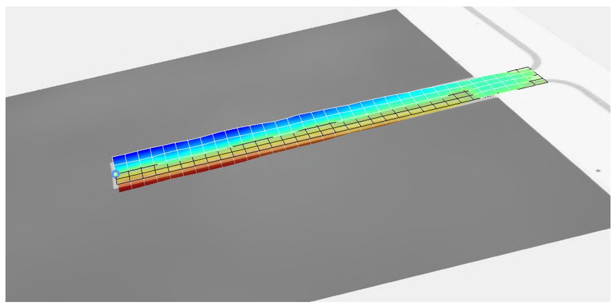


Figure 19: Torsional mode of the short cantilever

3.2.2 750µm long Cantilever

The same exact procedure is carried out for the long Cantilever with measures:

- **Length**: 750µm
- **Width**: 70µm
- **Thickness**: 40µm
- **Channel**: 10µm (Diameter)

The resulting resonance spectra are:

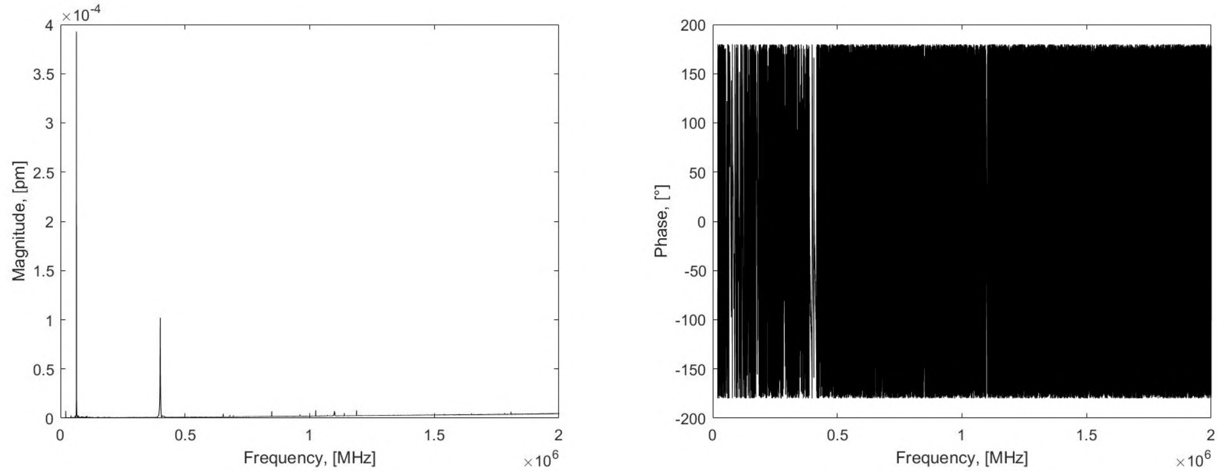


Figure 20: Resonance spectrum: long cantilever

The first mode corresponds to a resonance frequency of 0.0641 MHz.

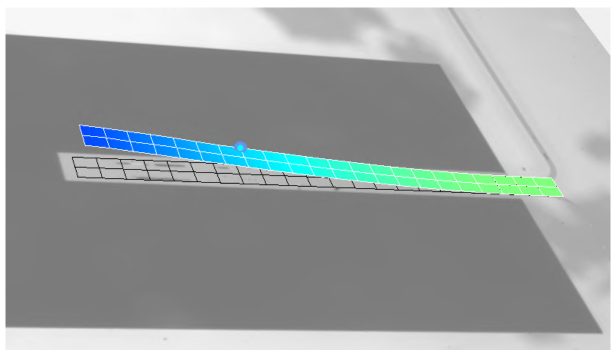


Figure 21: First mode of the long cantilever

The second mode corresponds to a resonance frequency of 0.4004 MHz.

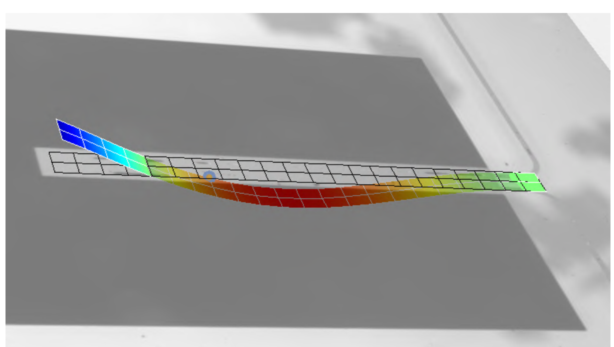


Figure 22: Second mode of the long cantilever

The third mode corresponds to a resonance frequency of 1.0991 MHz.

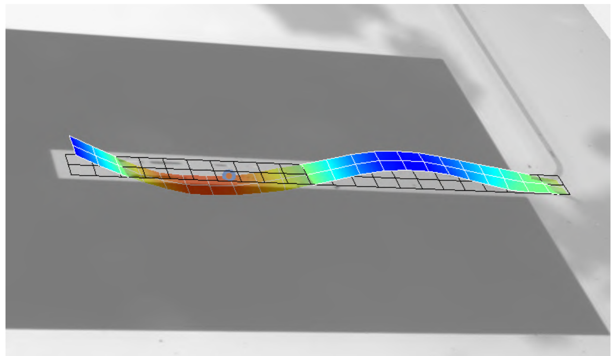


Figure 23: Third mode of the long cantilever

It is also possible to see a torsional mode, at frequency 0.8478 MHz.

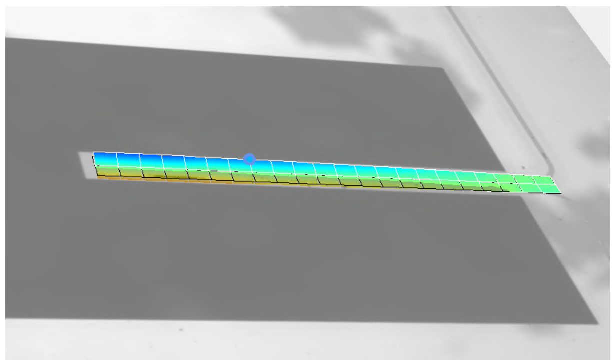


Figure 24: Torsional mode of the long cantilever

Finally, we end up with the following experimental results.

Structure	**First Mode [Hz]**	**Second Mode [Hz]**	**First Torsional mode [Hz]**
Cantilever 500 µm	$0.1595 \cdot 10^6$	$0.9666 \cdot 10^6$	$1.54442 \cdot 10^6$
Cantilever 750 µm	$0.0641 \cdot 10^6$	$0.4004 \cdot 10^6$	$0.8478 \cdot 10^6$

Table 7: Experimental resonance frequencies of cantilevers (in air)

3.3 Comparison with analytical data

In conclusion, in the table below are reported all the eigenfrequency values, evaluated both analytically and experimentally.

Structure	f1 analytic [MHz]	f1 experimental [MHz]	f2 analytic [MHz]	f2 experimental [MHz]
Bridge 500 μm	1.035	1.1024	N/A	N/A
Bridge 750 μm	0.460	0.3744	1.268	1.0142
Cantilever 500 μm	0.153	0.1595	0.961	0.9666
Cantilever 750 μm	0.068	0.0641	0.427	0.4004

Table 8: Comparison between analytical resonance frequencies and experimental data

We can notice that for the microcantilevers there is an almost perfect match between the experimental values and the one computed according to the Euler-Bernoulli equation. Whereas, for what concern the bridges, there is an overestimation of the analytical frequencies, and this is due to the fact that in the Euler-Bernoulli Theory (see chap.[2]) we made the assumption that the damping term is negligible. In reality, every microresonator is subjected to a dumping force, and this is larger for a bridge, since it is clamped on both sides.

4 Vacuum analyses

The measurements of both long bridge and cantilever, thus L=750μm, were repeated under vacuum conditions.

In the *Introduction* Chapter we have said that, in a microstructure, the resonance modes are linked to their mass m0, defined as the mass in vacuum. Since in this case no other masses are applied, we should expect higher frequency values and higher sensitivity, according to the formula $S_m = \frac{\Delta f_n}{\Delta m} = -\frac{f_0}{2m_0}$

4.1 750μm long bridge

The resonance spectrum of the bridge are shown in the following.

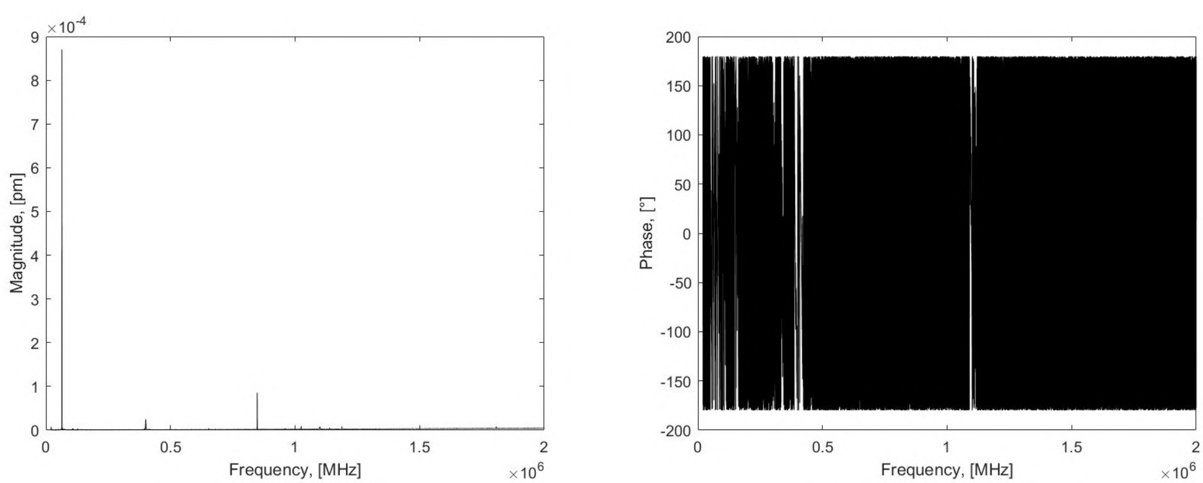


Figure 25: Resonance measurement: long bridge in vacuum

For a bandwidth of 2MHz, there are two picks, representative of the two resonance modes respectively at: 0.3767 MHz and 1.0151 MHz.

4.2 750 µm long cantilever

The resonance spectrum of the cantilever are shown in the following.

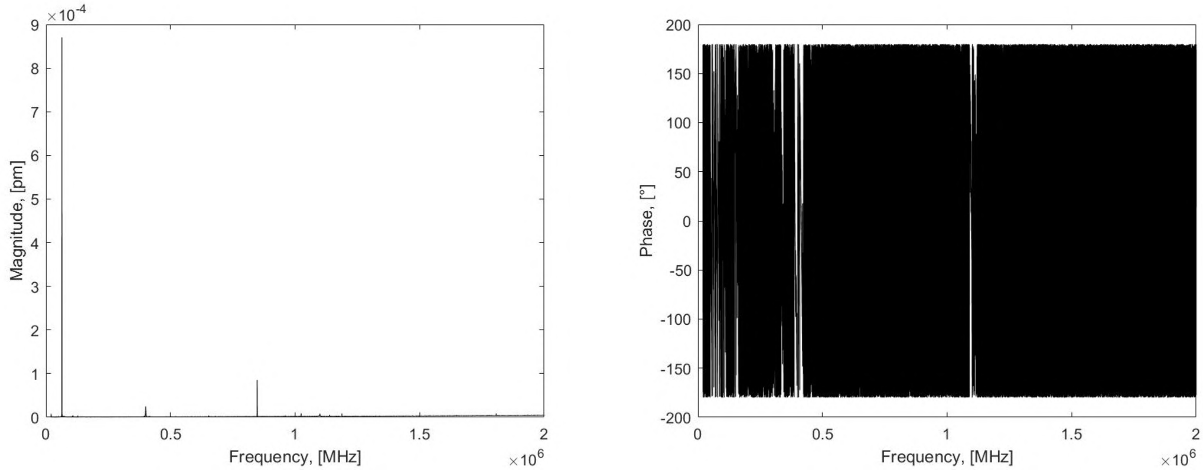


Figure 26: Resonance measurement: long cantilever in vacuum

Also in this case, for a bandwidth of 2MHz, there are two peaks, representative of the two resonance modes respectively at: 0.0642 MHz and 0.4007 MHz.

Structure	**f1 [MHz]**	**f2 [MHz]**
Bridge, air	0.3744	1.0142
Bridge, vacuum	0.3767	1.0151
Cantilever, air	0.0641	0.4004
Cantilever, vacuum	0.0642	0.4007

Table 9: Resonance mode comparison in air and vacuum, long sample

In conclusion, we can say that there is a downward shift in the frequency spectrum when we move from the vacuum case to the air one. However, this difference is minimal because we are dealing with microstructure, so the presence of air is not so detrimental for the resonators' performances.

Part II

Lorentzian fits and Q factors

5 Lorentzian Fits

At the base of every frequency calculation for experimental data a Lorentzian fit (or some other kind of fit) is needed. In our case this task was carried out through Matlab and in particular with an open-source function called "lorentzfit" whose documentation is found at [1]. Beyond just the resonance frequency calculation, the quality factor Q was also estimated for each structure and for each (measureable) mode both in vaccum and in air.
Operatively, all the exported data containing the frequency response was saved in a variable. Successively, depending on the mode, the range of frequencies that was fed to the script was changed so that it only included the relevant peaks. This is performed via the script below (where maximum and minimum are chosen manually by looking at the whole frequency spectrum):

```
indici_X_ridotto=find(data(:,1)>minimum & data(:,1)<maximum);%frequency
%interval of mode under exam

    for i=1:size(indici_X_ridotto,1)%reconstruction of X and Y

        ind=indici_X_ridotto(i,1);
        Y_ridotto(i,1)=data(ind,2);
        X_ridotto(i,1)=data(ind,1);
    end
```

Now, the actual fit can be computed using the aforementioned function:

```
 %parameters for lorentzian
    p3 = ((max(X_ridotto)-min(X_ridotto))/den_p3)^2;
    p1 = max(Y_ridotto)*p3;
    p2 = X_ridotto(I);
    c=min(Y_ridotto);
    p0 = [p1,p2,p3,c];

    %actual lorentzian fit
    lor_bridge=lorentzfit(X_ridotto,Y_ridotto,p0);
```

The **resonance frequency** is simply obtained by taking the maximum of the Lorentzian function:

```
    [M,I]= max(lor_bridge);
    f_r=X_ridotto(I);
```

The measured values of the resonance frequencies are tabulated below (table 10):

Structure	**First Mode [Hz]**	**Second Mode [Hz]**	**Third Mode [Hz]**	**First Torsional mode [Hz]**
Bridge 750 μm AIR	$3.7646 \cdot 10^5$	$1.0142 \cdot 10^6$	$1.9332 \cdot 10^6$	N/A
Bridge 750 μm VACUUM	$3.7673 \cdot 10^5$	$1.0151 \cdot 10^6$	$1.9332 \cdot 10^6$	$1.8517 \cdot 10^6$
Bridge 500 μm AIR	$1.1024 \cdot 10^6$	N/A	N/A	N/A
Cantilever 750μm AIR	$6.4111 \cdot 10^4$	$4.0045 \cdot 10^5$	$1.0992 \cdot 10^6$	$8.4775 \cdot 10^5$
Cantilever 750μm VACUUM	$6.4160 \cdot 10^4$	$4.0070 \cdot 10^5$	$1.0997 \cdot 10^6$	$8.4797 \cdot 10^5$
Cantilever 500μm AIR	$1.5951 \cdot 10^5$	$9.6668 \cdot 10^5$	N/A	N/A

Table 10: Resonance frequencies

The quality of each of these fits can be estimated via the linear correlation coefficient, in particular the closer this value to 1 the better the fit. Practically this was done through the ”corrcoeff” function:

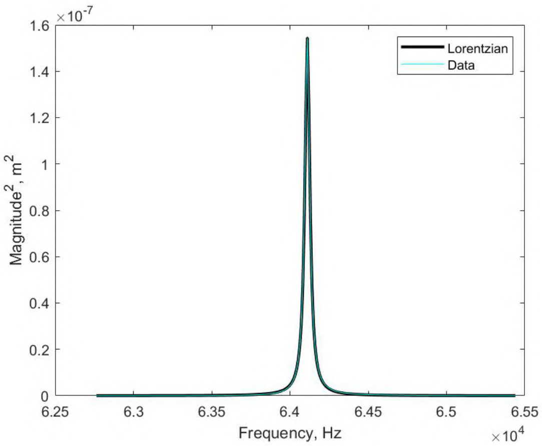
```
data=[lor_bridge Y_ridotto];
R=corrcoef(data);
```

Here is a table with the correlation coefficient for each structure (table 11):

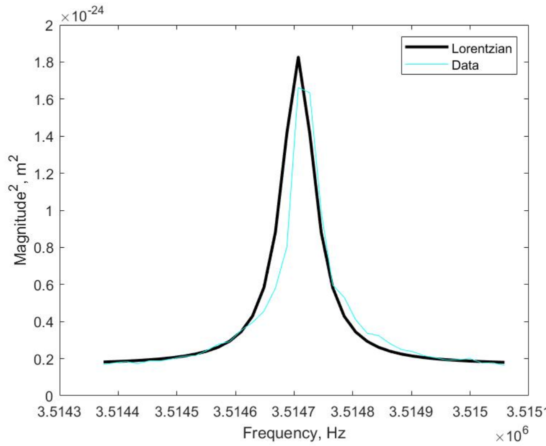
Structure	**First Mode**	**Second Mode**	**Third Mode**	**First Torsional mode**
Bridge 750 μm AIR	0.980378	0.996609	0.946081	N/A
Bridge 750 μm VACUUM	0.998563	0.982526	0.989769	0.988935
Bridge 500 μm AIR	0.939175	N/A	N/A	N/A
Cantilever 750μm AIR	0.999423	0.996123	0.998071	0.998935
Cantilever 750μm VACUUM	0.998566	0.994080	0.963623	0.998713
Cantilever 500μm AIR	0.985783	0.982112	N/A	N/A

Table 11: Correlation coefficients

This value has a very clear graphical meaning which can be appreciated by comparing the best fits (highest R) with the worst ones (lower R). In our case, we can take as the best one the first mode of the 750µm long Cantilever in air (R=0.999423) and as the worst one the third mode of the 750µm long Bridge in air (R=0.946081). The graphs are reported below (figure 27).



(a) Best fit



(b) Worst fit

Figure 27: Comparison of fits

The reasons behind such a difference are multiple, probably the main one is the number of sampling points. In fact the plot on the right deals with values in the order of 10^{-24}Hz and the width of the curve is extremely small which means that the number of sampled points is very small as well. Another important point is the lack of simmetry of the curve, as we can see the data is very asymmetric while the Lorentzian curve is, by definition, symmetric. In summary, one expects to find the most reliable measurements when the correlation coefficient is the highest.

6 Q factors

Having found the Lorentzian fits one can estimate the Q factor, a value indicating the sharpness of the resonance peak. The method utilized to compute this value is the *-3dB bandwidth method* that is based on the definition of Q in electrical resonant circuits where the quality factor is given by:

$$Q = \frac{\omega_r}{\Delta\omega_{-3dB}} \tag{1}$$

What this implies, from a practical standpoint, is:

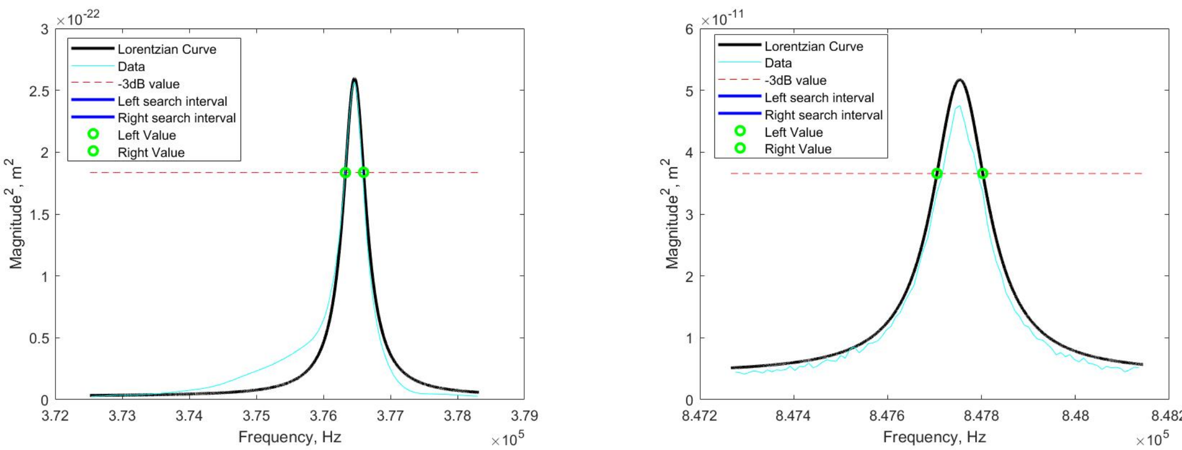
- Find the peak of the Lorentzian.
- Divide the peak by $\sqrt{2}$ to get the –3dB value.
- Look for points (y coordinate) in the Lorentzian curve that are as close as possible to the value found in the previous point.
- Obtain the frequency (x coordinate) corresponding to the 2 points and calculate the difference.

- Compute Q.

The first two steps of the list are fairly straightforward and are carried out through:

```
valore_3dB = max(lor_bridge)/sqrt(2);% -3dB theoretical value
```

For what concerns the third step the process is slightly more complex since we're dealing with a sampled curve. This means that there won't be the exact value at -3dB but one that is very close. For completeness the error on this estimation was computed and all the details of the Matlab script are explained in Appendix A. To get a visual sense of the values that were extracted, a graph was developed in which all the main quantities of interest are shown, as an example two are shown below.



(a) First Mode, Bridge 750 in Air (b) Torsional Mode,Cantilever 750 in Air

Figure 28: Examples of -3dB calculation

The green points are the ones whose frequency (x coordinate) is then used to compute the denominator of the quality factor. As mentioned earlier the relative error on the -3dB value was computed (basically the difference in y coordinates between the red dotted line and both of the green points). The results are reported in the table below (12).

Structure	First Mode	Second Mode	Third Mode	First Torsional mode
Bridge AIR(750µm)	Left=0.062% Right=0.052%	Left=0.14% Right=0.05%	Left=0.04% Right=0.0002%	Left=N/A Right=N/A
Bridge VACUUM(750µm)	Left=0.04% Right=0.28%	Left=0.02% Right=0.003%	Left=0.07% Right=0.01%	Left=0.07% Right=0.02%
Bridge AIR(500µm)	Left=0.01% Right=0.15%	Left=N/A Right=N/A	Left=N/A Right=N/A	Left=N/A Right=N/A
Cantilever AIR(750µm)	Left=0.57% Right=0.16%	Left=0.19% Right=0.11%	Left=0.01% Right=0.003%	Left=0.02% Right=0.04%
Cantilever VACUUM(750µm)	Left=0.04% Right=0.1%	Left=0.04% Right=0.01%	Left=0.02% Right=0.03%	Left=0.24% Right=0.06%
Cantilever AIR(500µm)	Left=0.07% Right=0.01%	Left=0.04% Right=0.04%	Left=N/A Right=N/A=%	Left=N/A Right=N/A

Table 12: Relative errors on -3dB value

At this point the Q factor can be calculated through (Once again the Matlab script is fully explained in appendix A):

```
Q=p2/(asse(indici(2))-asse(indici(1)));
```

The table (13) containing all the Q factors is:

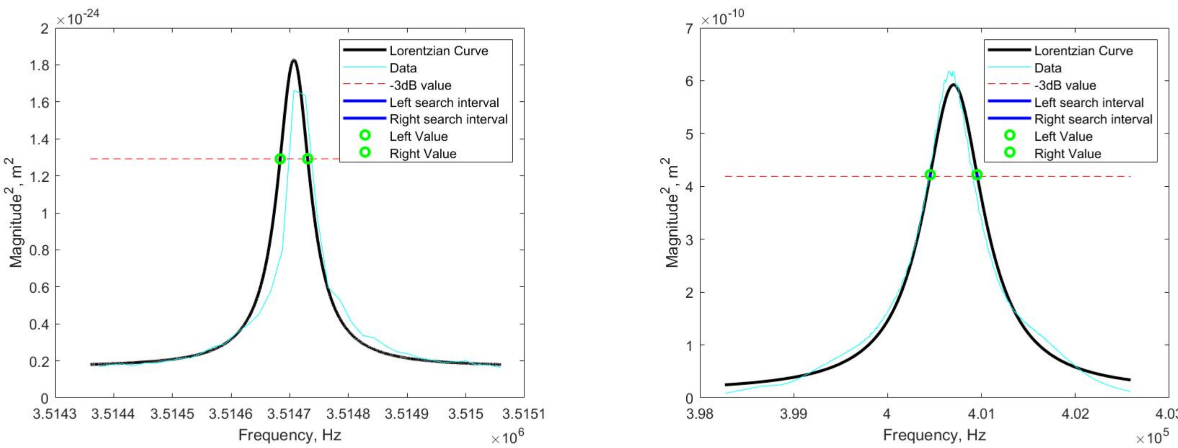
Structure	First Mode	Second Mode	Third Mode	First Torsional mode
Bridge 750 µm AIR	1389.748199	1133.070201	75044.961121	N/A
Bridge 750 µm VACUUM	2566.326936	2469.085391	926.923419	5285.537622
Bridge 500 µm AIR	159.023372	N/A	N/A	N/A
Cantilever 750µm AIR	2566.270226	1009.507848	760.480671	8697.788787
Cantilever 750µm VACUUM	26289.822860	804.523780	1279.670400	94171.154929
Cantilever 500µm AIR	2691.898059	548.158380	N/A	N/A

Table 13: Q factors

In general we expect the Q factor to be larger in vacuum due to the lesser damping and overall sharper peaks. Looking at the table the only two cases that don't respect this are:

- Bridge 750µm Third mode
- Cantilever 750µm Second mode

Looking at the fits of these modes the reason behind this irregularity becomes immediately apparent.



(a) Third Mode, Bridge 750 in Air (b) Second Mode,Cantilever 750 in Vacuum

Figure 29: Problematic fits

For what concerns the third mode of the bridge in air, it is crucial to remember that this is the worst fit between all of them, which is clearly highlighted by the poor correlation coefficient reported in a previous table. This necessarily implies a poor estimation of the Q factor.
A similar reasoning can be used for the second mode of the Cantilever in vaccum. In fact, the data is very noisy and even though it has a more "lorentzian behaviour" then the previous one it still results in a poor computation of Q. In particular, looking at the graph one can see that the $\Delta\omega_{-3dB}$ is overestimated which leads to a lower Q than the real one.

7 Theoretical mass responsivity

The theoretical mass responsivity is calculated as:

$$S_m = -\frac{f}{2m} \tag{2}$$

where f is the frequency of the mode and m is the real mass of the device.
We calculated both an analytical responsivity, exploiting the analytical frequency calculated with Matlab, and a "hybrid" one, exploiting the measured frequency. The calculation was carried out just for the first mode.
As expected, and as shown in Table 10, longer bridges correspond to lower frequencies, thus being the mass difference between the two structures minimal with respect to the difference of frequency, in absolute value, a longer bridge corresponds to a lower theoretical mass responsivity.

Structure	**Analytical** [$kg^{-1}s^{-1}$]	**Hybrid** [$kg^{-1}s^{-1}$]
Bridge 750 µm AIR	$-5.9459 \cdot 10^{13}$	$-4.8650 \cdot 10^{13}$
Bridge 750 µm VACUUM	$-5.9468 \cdot 10^{13}$	$-4.8690 \cdot 10^{13}$
Bridge 500 µm AIR	$-2.0067 \cdot 10^{14}$	$-2.1369 \cdot 10^{14}$
Cantilever 750 µm AIR	$-7.8346 \cdot 10^{12}$	$-7.3673 \cdot 10^{12}$
Cantilever 750 µm VACUUM	$-7.8350 \cdot 10^{12}$	$-7.3732 \cdot 10^{12}$
Cantilever 500µm AIR	$-2.6442 \cdot 10^{13}$	$-2.7495 \cdot 10^{13}$

Table 14: Theoretical mass responsivity

Part III

Fluids

8 Suspended channel micro-resonator liquid analysis

The analysis is performed with a 500µm long bridge with a bigger channel than the bridges previously used, 45µmx15µm, applying different liquids inside the channel.
Such liquids are solutions, composed of a total of 10mL of ethanol (C_2H_5OH) and water.
The six analyses are:

- air (empty nanochannel)
- water 100%
- ethanol 25%, water 75%
- ethanol 50%, water 50%
- ethanol, 100%
- unknown liquid

Changing the liquid will change the mass of the bridge, being the nanochannel fixed, leading to a change in the resonance.
The workstation is composed of the same device used previously, the Doppler laser vibrometer, using the same chip of the first analyses, but with gold chromium electrodes that arrive up to the bridge; such electrodes were fabricated for electrophoresis inside suspended channels, for trapping cells or particles inside the channel.

Figure 30: Work station for the liquid analysis

The measure is taken for a single point, in vacuum, with the laser in the centre.
The steps for each liquid are: disconnect the tiny tube of the chosen pipette from the electrode, open the pipette, empty it and fill it with 1 mL of the liquid of the measurement, and release that mL inside a becker just to clean; then fill the pipette again with 3mL of the chosen liquid, close it and attack it to the chip, normal to it. Subsequently set the reference frequency to 500mBar, start the continous current measure, and set the pressure on the third channel that will get to the microfluidics to get the liquid into the chip. There are four inlets/outlets, but for the chosen analysis, it is sufficient to send just from one towards the other three, (while for example when working with a solid dispersed in a liquid, all of the inlets are necessary). When it is stabilized, the pressure is decreased to 100mBar (an elevated pressure would influence the peak in the resonance spectrum); once it has reached the regime, the measurement can be stopped. The pressure is then set to 0 mBar, in order to change the liquid.

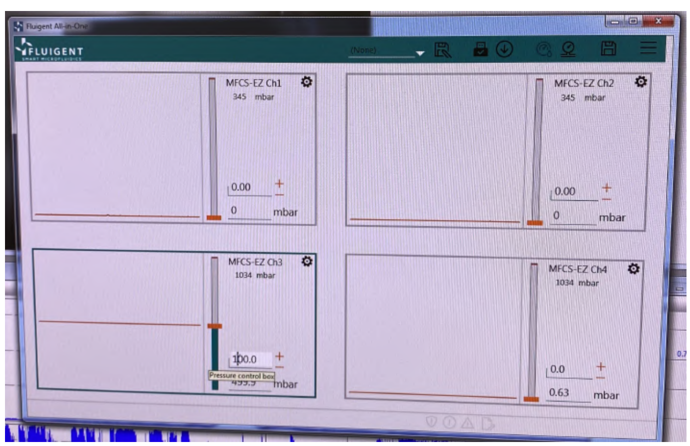
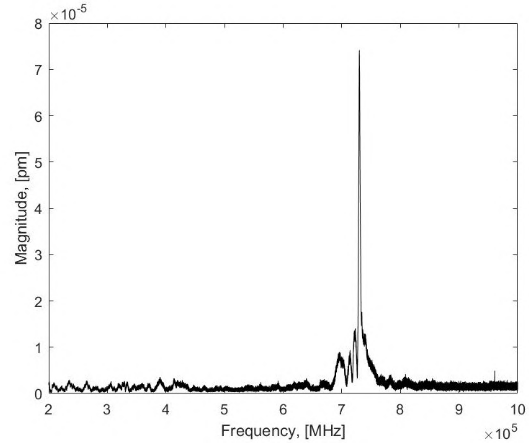


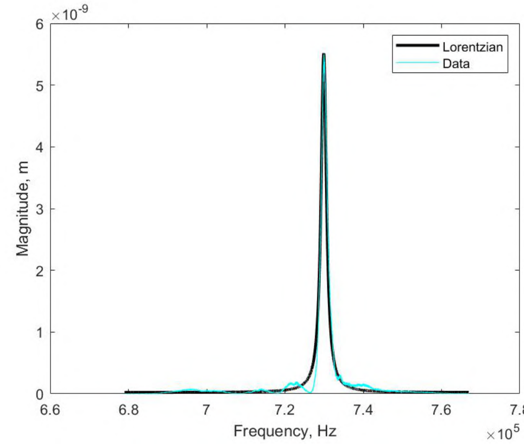
Figure 31: Four channels and pressure

The resulting frequency spectra and their relative Lorentzian fits of the first mode are shown below.

8.1 Air



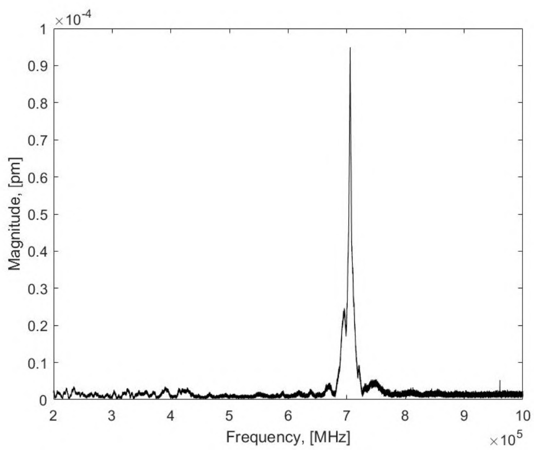
(a) Full spectrum



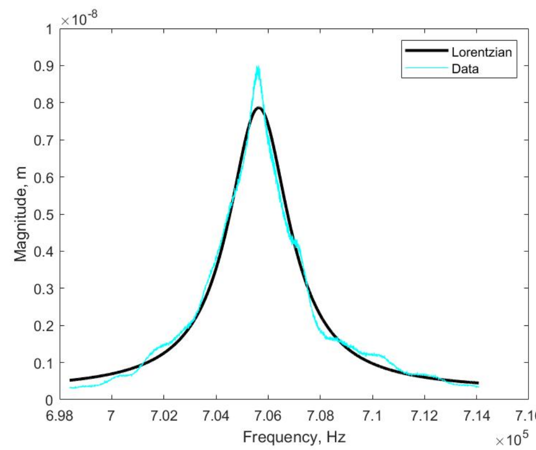
(b) First mode

Figure 32: Resonance spectrum and first mode fit: air

8.2 Water



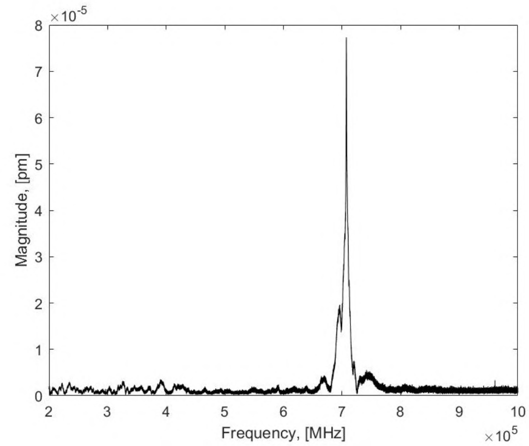
(a) Full spectrum



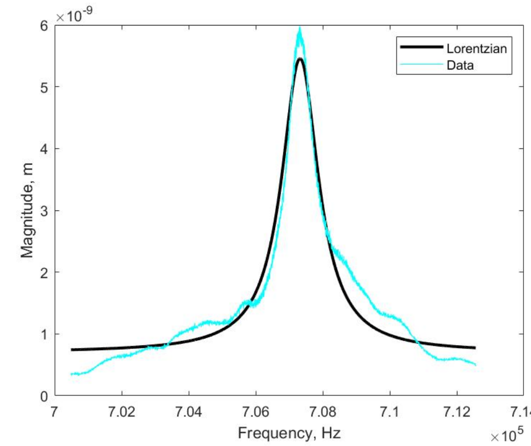
(b) First mode

Figure 33: Resonance spectrum and first mode fit: water

8.3 Ethanol 25%



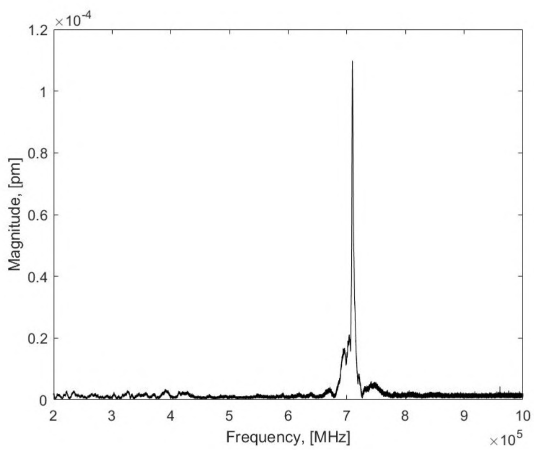
(a) Full spectrum



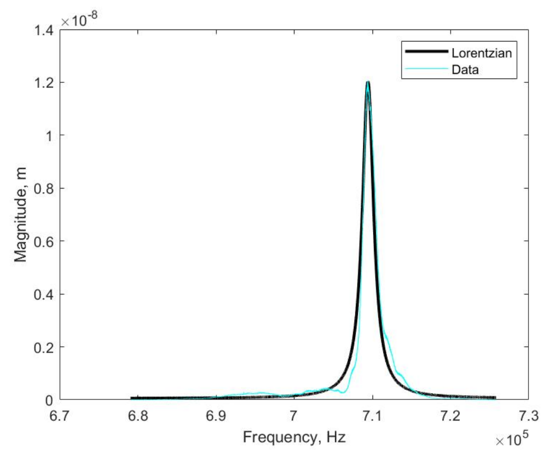
(b) First mode

Figure 34: Resonance spectrum and first mode fit: ethanol 25%

8.4 Ethanol 50%



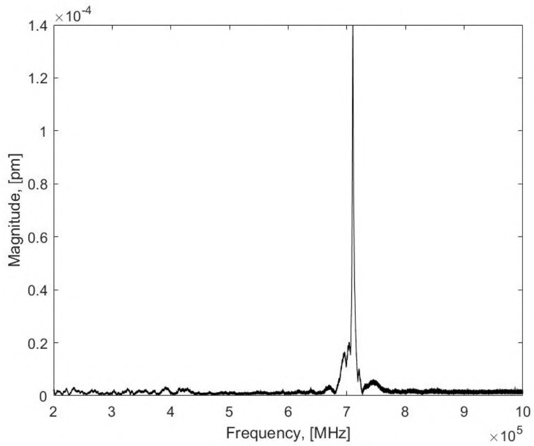
(a) Full spectrum



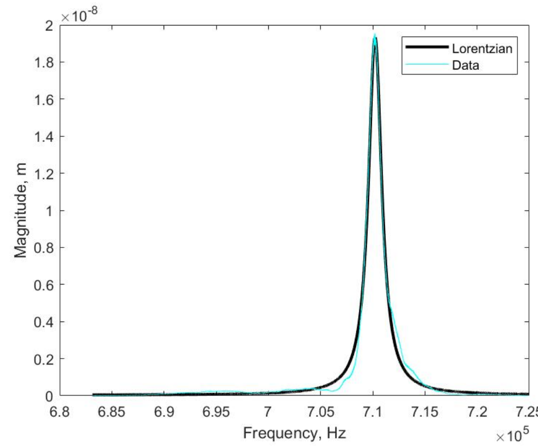
(b) First mode

Figure 35: Resonance spectrum and first mode fit: ethanol 50%

8.5 Ethanol 100%



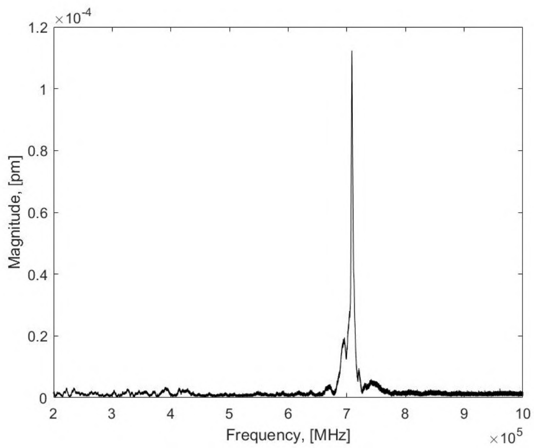
(a) Full spectrum



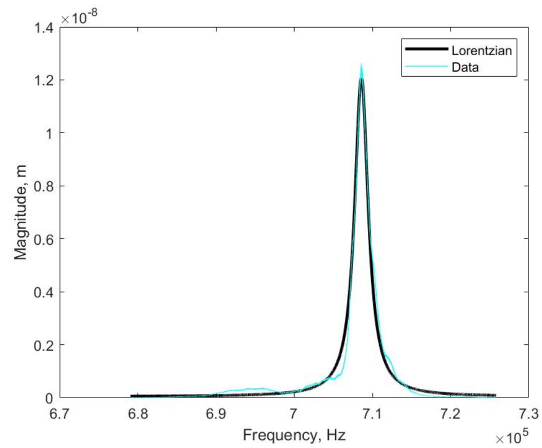
(b) First mode

Figure 36: Resonance spectrum and first mode fit: ethanol 100%

8.6 Unknown liquid



(a) Full spectrum



(b) First mode

Figure 37: Resonance spectrum and first mode fit: unknown liquid

All the measured first mode frequencies and the calculated quality factor are listed in the table below:

Liquid	**Frequency** [MHz]	**Quality factor**
Air	0.7299	674.9305
Water	0.7056	76.0010
Ethanol 25%	0.7073	742.0466
Ethanol 50%	0.7094	621.7065
Ehtanol 100%	0.7102	713.5468
Unknown liquid	0.7086	517.1222

Table 15: Calculated first mode frequencies and quality factors

9 Calibration Plot and Sensitivity

In the interest of finding the nature of the analyzed unknown fluid, it is imperative the Calibration Plot evaluation which is simply given by the interpolation of the obtained data.

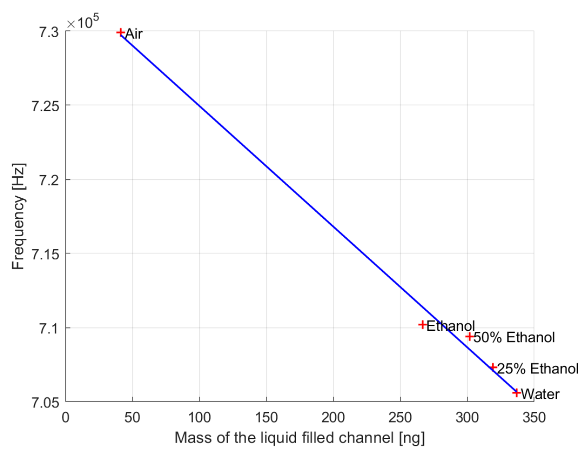


Figure 38: Calibration Plot

The following table states the computed masses for the analyzed fluids :

Liquid	**Frequency** [MHz]	**Mass** [ng]
Air	0.7299	$4.13438 \cdot 10^{-1}$
Water	0.7056	336.90
Ethanol 25%	0.7073	319.33
Ethanol 50%	0.7094	301.76
Ehtanol 100%	0.7102	266.61

Table 16: First mode frequencies and calculated masses of the fluid filled channel

In order to obtain the mass values it was implemented the following matlab code sections:

- Compound fluid density evaluation

```
air_dens=1.22e3;%g/m^3
ethanol_dens=789.95e3;
water_dens=998.23e3;
eth25_dens=((0.25*ethanol_dens)+(0.75*water_dens));
eth50_dens=((0.5*ethanol_dens)+(0.5*water_dens));
```

- Mass evaluation

```
mass_air=v_chan*air_dens; %4.134375e-10
mass_water=v_chan*water_dens; %3.369026251e-7
mass_eth25=v_chan*eth25_dens;%3.193291e-7
mass_eth50=v_chan*eth50_dens;%3.01755375e-7
mass_ethanol=v_chan*ethanol_dens;%2.666081251e-7
```

where vchan is the channel volume

- Interpolation

```
x=polyfit(mass,freq,1);
f=polyval(x,mass);
```

The interpolation polynomial (line in this case) presents a Slope=-81.4474.
Once obtained the Calibration plot, stating the unknown liquid chemical nature is very straightforward: Since the experimental first mode resonance frequency is : 0.7086 [MHz]; making use of the Calibration plot, the mass for the unknown liquid is found to be : 299.64 [ng], and its density : 887822.2 [g/m^3]. Thereafter, evaluating the densities of the given unknown liquid candidates, the closest one in Density is found to be Glycerole 25% in Ethanol which presents an analytical density of : 907462.5 [g/m^3] and a mass: 212.89 [ng].
The complete comparison between the unknown liquid and all the contenders is in the Table below:

Liquid	**Density** [g/m^3]	**Mass** [ng]
Unknown	887822.2	299.64
Glycerol 10% Ethanol	836955.0	282.47
Glycerol 25% Ethanol	907462.5	306.27
Glycerol 10% Water	1024400	345.74
Acetone	784000	264.60
Hexane	655000	221.06

Table 17: First mode frequencies and calculated masses of the fluid filled channel

A Appendix: Matlab Code

The whole system that performs the Lorentzian fit and evaluates the Q factor revolves around three scripts. In particular, there is a core function 'Q_eval' that does the fitting and calculates the parameters of interest. Then there is another function called 'total' that iteratively calls the Q_eval function to find the best fitting parameters. For each structure the 'total' function is called once and gives all the required values.
We'll start from the core function 'Q_eval' then explain the function 'total' and then the final script.

A.1 Q_eval.m

The function starts with:

```
function [Q,errore_sx,errore_dx,f_r,R,flag_indici]=
Q_eval2(minimum,maximum,data,den_p3,den_val,flag)

flag_indici=0;

  indici_X_ridotto=find(data(:,1)>minimum & data(:,1)<maximum);%frequency interval of the mode

  for i=1:size(indici_X_ridotto,1)%reconstruction of X and Y

    ind=indici_X_ridotto(i,1);
    Y_ridotto(i,1)=data(ind,2);
    X_ridotto(i,1)=data(ind,1);
  end
```

The **inputs** are:

- minimum, the minimum of the frequency interval that contains the peak;
- maximum, the maximum of the frequency interval that contains the peak;
- data, a Nx2 matrix that contains the frequency spectrum. In particular the first column contains the frequency sampled points while the second has the amplitude values;
- den_p3 is the denominator of a parameter successively used in the lorentzian fit;
- den_val is a parameter used to shorten the search interval for values similar to the -3dB one in the computation of Q;
- flag is a control value used by the function 'total' that calls the 'Q_eval' function.This control value can either be 1 or 0 and is used to plot the graphs and print the values of interest only in the final iteration, when all the fitting parameters have been optimized.

The **outputs** are:

- Q, the quality factor.
- errore_sx, the error on the -3dB value in the left side of the lorentzian;
- errore_dx,the error on the -3dB value in the right side of the lorentzian;
- f_r, resonance frequency;
- R, linear correlation coefficient to estimate the quality of the fit;
- flag_indici, a control value that tells the calling function 'total' when the best parameters have been found.

'flag_indici' is immediately set to 0 because the best parameters obviously haven't been found yet. Then, 'indici_X_ridotto' finds the frequency interval that contains the peak. Note that minimum and maximum are set manually by looking at the spectrum. Then a for loop creates two vectors 'Y ridotto' and 'X ridotto' that are basically the peak since they contain the amplitude and frequency of the peak respectively.

```
if flag
        figure(2)
        plot(X_ridotto,Y_ridotto) %plot of first mode
        xlabel('Frequency, Hz')
        ylabel('Magnitude^2, m^2')
    end
    %Looking for maximum of data
    [M,I]=max(Y_ridotto);%get the index

    X_ridotto(I);%get the x value
```

Here the mode in question is plotted in figure 2, only if the flag parameter was set to 1 by the caller function meaning that it is the last iteration and results have to be shown. Successively the script looks for the maximum of the peak and finds the frequency value associated to it.

```
%parameters for lorentzian
    p3 = ((max(X_ridotto)-min(X_ridotto))/den_p3)^2;
    p1 = max(Y_ridotto)*p3;
    p2 = X_ridotto(I);
    c=min(Y_ridotto);
    p0 = [p1,p2,p3,c];

    %actual lorentzian fit
    [lor_bridge params resnorm residual jacobian]=lorentzfit(X_ridotto,Y_ridotto,p0);
```

Now the actual Lorentzian fit is performed with the function 'lorentzfit' [1]. Four 'starting' parameters are defined, basically these serve as initial guesses for the fitting function to obtain the best result. Note that p2 is the frequency of the peak.

```
 if flag

        %overlay of lorentzian and data
        figure(3)
        plot(X_ridotto,lor_bridge,'k','LineWidth',2)
        hold on
        plot(X_ridotto,Y_ridotto,'c','LineWidth',0.1)
        legend('Lorentzian','Data')
        xlabel('Frequency, Hz')
        ylabel('Magnitude, m')
        hold off
    end
    asse=linspace(minimum,maximum,10000);
    lorentz= params(1)./((asse-params(2)).^2 +params(3)) +params(4);
```

If the iteration in question is the final one the Lorentzian fit is plotted over the actual data. Then, since the mesh fed to the fitting function might be coarse, a much more finely sampled vector of frequency is created. With this vector a much smoother Lorentzian is obtained exploiting the parameters found by the fitting function.

```
valore_3dB = max(lorentz)/sqrt(2);%valore teorico a -3dB
    %estremi intervallo di ricerca per valore uguale a valore_3dB
    val_min=(max(lorentz)/sqrt(2))-(max(lorentz)/sqrt(2))/den_val;
    val_max=(max(lorentz)/sqrt(2))+(max(lorentz)/sqrt(2))/den_val;
    indici=find(lorentz>val_min & lorentz<val_max);%trovo indici corrispondenti
```

Starting from the newly found smooth Lorentzian, the -3dB value is defined. Then, one must consider that we're dealing with a sampled function so it's almost impossible that our data has the exact value we're looking for which, in this case, is the -3dB one. Therefore a range of amplitudes centered at the -3dB value is defined by the two limits 'val_min' and 'val_max'. Note that each half of the interval is modulated by 'den_val' that was introduced previously. Finally by using the Matlab function 'find' we can look for values within that range.

```
 %correctly identify indici even if there are values close together
    if(size(indici,2)==2)
        flag_indici=1;
    elseif(size(indici,1)==3 && (abs(indici(1)-indici(2))==1) )
        indici=[indici(2) indici(3)];
```

```
        flag_indici=1;
    elseif(size(indici,1)==3 && (abs(indici(2)-indici(3))==1) )
        indici=[indici(1) indici(2)];
        flag_indici=1;
    elseif(size(indici,1)==4 && (abs(indici(1)-indici(2))==1) && (abs(indici(3)-indici(4))==1)  )
        indici=[indici(2) indici(3)];
        flag_indici=1;
    end
```

Sometimes it might happen that there are multiple points very close together in the search interval so a manual elimination is needed.
Now the Q factor can be calculated:

```
valore_lor_dx=lorentz(indici(2));%valore trovato sulla lorentziana su cui poi si calcola il
%delta che va a denominatore di Q
    valore_lor_sx=lorentz(indici(1));

    errore_dx= abs(valore_lor_dx-valore_3dB)*100/valore_lor_dx;
    errore_sx= abs(valore_lor_sx-valore_3dB)*100/valore_lor_sx;%errore percentuale sulla misura
    %del valore a -3dB
    Q=p2/(asse(indici(2))-asse(indici(1)));
```

Before the actual computation of Q the two amplitude values that were identified on the Lorentzian are defined. A relative error is then calculated and then finally Q is found.

```
if flag
    figure(4)
    plot(asse,lorentz,'k','LineWidth',2)
    hold on
    plot(X_ridotto,Y_ridotto,'c','LineWidth',0.1)
    plot(asse,retta,'r--')%retta valore ideale a -3dB
    plot(x_dx,intervallo,'b','LineWidth',2)%intervallo dove cerco valori simili a quello a -3dB
    %a destra
    plot(x_sx,intervallo,'b','LineWidth',2)%intervallo dove cerco valori simili a quello a -3dB
    %a sinistra
    plot(asse(indici(1)),valore_lor_sx,'g o','LineWidth',2)
    plot(asse(indici(2)),valore_lor_dx,'g o','LineWidth',2)
    legend('Lorentzian Curve','Data', '-3dB value','Left search interval','Right search interval'
    'Left Value','Right Value')
    xlabel('Frequency, Hz')
    ylabel('Magnitude, m')
```

```
    hold off
end
```

Here a figure with all the previously defined elements is plotted, just like the one below.

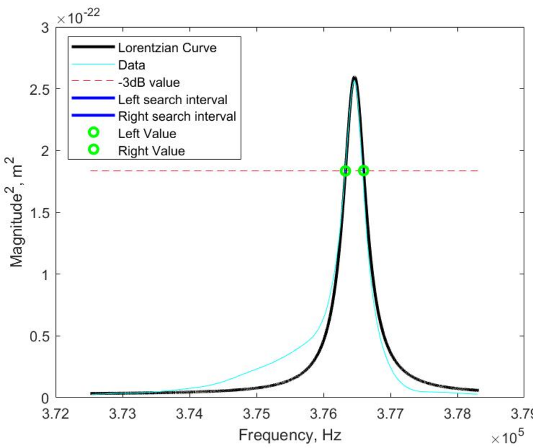


Figure 39: Example of figure 4. First Mode, Bridge 750 in Air

The resonance frequency is finally calculated along with the correlation coefficient.Then the values relevant for the analysis are printed:

```
[M,I]= max(lorentz);
    f_r=asse(I);
    data=[lor_bridge Y_ridotto];
    R=corrcoef(data);
    if flag
        fprintf('Q: %f \n',Q)
        fprintf('errore_sx: %f \n',errore_sx)
        fprintf('errore_dx: %f \n',errore_dx)
        fprintf('f_r: %f [Hz]\n',f_r)
        fprintf('R: %f \n',R(2,1))

    end
```

A.2 total.m

Due to the repetitiveness of the work for each structure a Matlab function called 'total' was created that has the purpose of iteratively running 'Q_eval' until the best fit is found. Two parameters are optimized: **den_p3** and **den_val**.

```
function [Q,errore_sx,errore_dx,f_r,R,flag_indici]=total(minimum,maximum,data)


den_p3=linspace(1,1000,1000);
```

```
den_val=linspace(1,3000,1000);
flag=0;%flag per fare i grafici o meno
```

As we can see the input and output parameters are the same as before. The parameters over which the optimization is performed are defined in a wide range of values so that the most suitable one can be found. It is worth mentioning that 'den_p3' in some sense modulates how narrow the Lorentzian is while 'den_val' simply makes smaller the interval over which the -3dB values are searched.

```
%look for best fit, starting with p3 parameter
massimo=0;
for i=1:500
[Q,errore_sx,errore_dx,f_r,R,flag_indici]=Q_eval2(minimum,maximum,data,den_p3(i),
den_val(1),flag);
    if(R(2,1)>massimo)
        massimo=R(2,1);
        den_p3_final=den_p3(i);
    end
end
```

First off, the best fit is found by choosing the best p3 starting parameter of the Lorentzian fit. Practically this is done by looking for the maximum R value between all the various calls to 'Q_eval'. Note that den_val is set to the first value which is also the broadest so that a match within the -3dB search interval is certainly found and no error occurs. This can be done because the -3dB value only becomes relevant once the Q factor has to be calculated but, up till now, what we want is the best fit possible.
After this the focus shifts to finding Q:

```
%now I look for the minimum value of the denominator in the search of -3dB
%value
i=1;
while flag_indici==0
    [Q,errore_sx,errore_dx,f_r,R,flag_indici]=
    Q_eval2(minimum,maximum,data,den_p3_final,den_val(i),flag);
    den_val_final=den_val(i);
    i=i+1;
end
flag=1;
```

Basically the range over which we look for the -3dB value is reduced until there are only 2 points left, no better resolution can be achieved.
Finally, having saved the optimized parameters, we launch the last iteration and plot all the figures (note that flag now is set to 1).

```
[Q,errore_sx,errore_dx,f_r,R,flag_indici]=
Q_eval2(minimum,maximum,data,den_p3_final,den_val_final,flag);
```

A.3 q_factor.m

All these elements are put together in the 'mother' script that calls the total.m function for each structure after minimum and maximum are properly set:

```
%% Bridge 500
clc
close all
clear all
data = readmatrix('ScanBridge500.txt');
data(:,2)=data(:,2).^2;

figure(1)
plot(data(:,1),data(:,2));%full plot of gathered data
xlabel('Frequency, Hz')
ylabel('Magnitude, m')
title('Full Spectrum')

minimum=1014120;
maximum=1195570;

[Q,errore_sx,errore_dx,f_r,R,flag_indici]=total(minimum,maximum,data);
```

References

[1] *Lorentzian fit function.* URL:
`https://it.mathworks.com/matlabcentral/fileexchange/33775-lorentzfit-x-y-varargin`.

[2] Michael Lee Roukes Silvan Schmid Luis Guillermo Villanueva. *Fundamentals of Nanomechanical Resonators*. Springer Cham, 2016.