The Mathematics of Lasagne

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06/12/2023

Part I - The Cube Rule of Food[1]

[1] https://cuberule.com/

What is a Sandwich?

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But what about edge-cases!

Are hot dogs sandwiches...?

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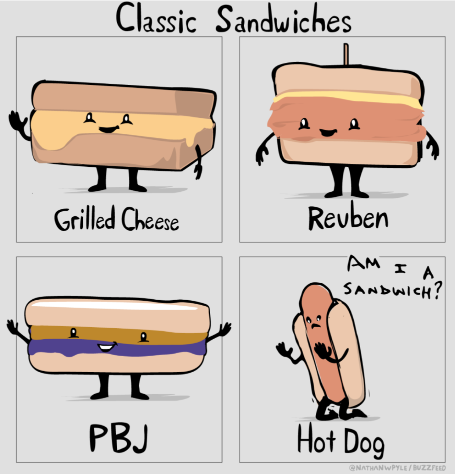


Figure 1: A hot dog experiencing existential dread [1]

The State of New York says "Yes"!

New York State
Department of
Taxation and Finance
Taxpayer Guidance Division

Tax Bulletin
Sales and Use Tax
TB-ST-835
April 13, 2011

Sandwiches

Introduction

Sandwiches are generally subject to sales tax. This bulletin explains what is considered a sandwich for sales tax purposes. Additional resources relating to sales of food and

- gyros;
- hamburgers on buns, rolls, etc.;
- heroes, hoagies, torpedoes, grinders, submarines, and other such sandwiches;
- hot dogs and sausages on buns, rolls, etc.;
- melt sandwiches;
- open-faced sandwiches;
- panini sandwiches;
- Reuben sandwiches; and
- wraps and pita sandwiches.

Figure 2: New York State Tax Code classifying hot dogs as sandwiches

The "Sandwich Alignment Chart" attempted to bring order

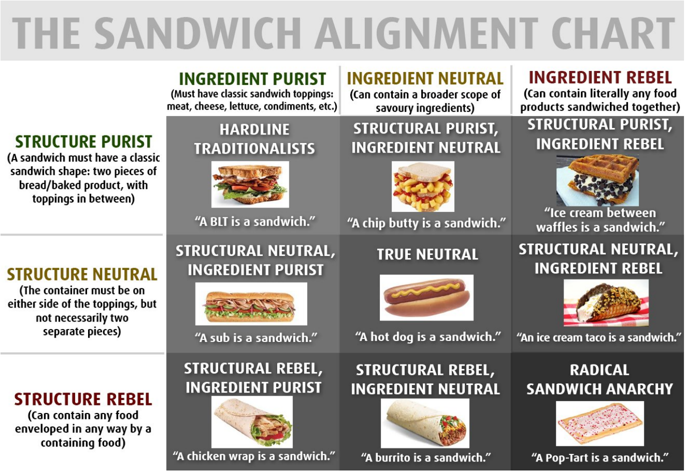


Figure 3: An alignment chart for what is considered a sandwich [2]

But this only led to more chaos...

Until @Phosphatide enlightened the world...

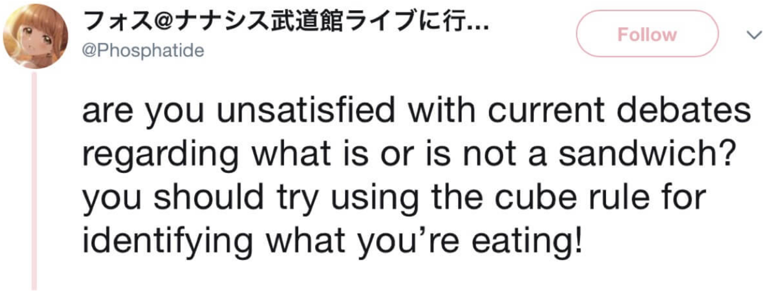
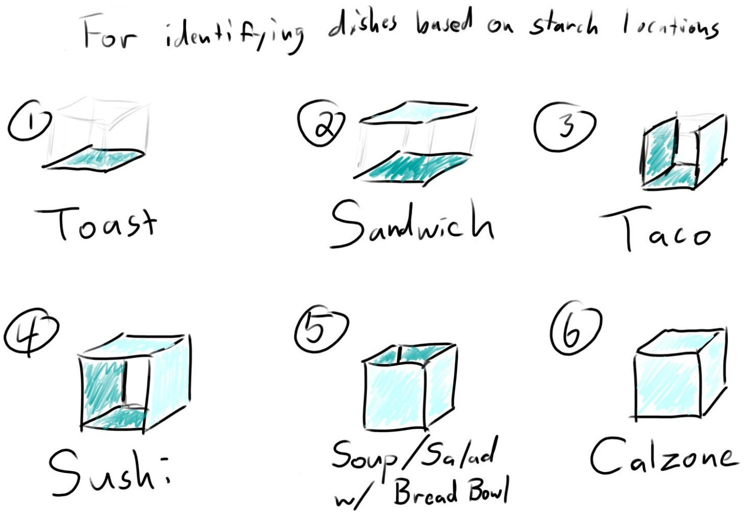


Figure 5: A tweet introducing the "Cube Rule of Food"[3]

The Cube Rule of Food



The Cube Rule of Food

- Two foodstuffs are isomorphic under the "Cube Rule of Food" iff the location of their starch content as mapped onto a cube are the same
- This partitions the set of all foodstuffs into equivalence classes based on the location of their starch content[2]
  - Foodstuffs can be referred to interchangeably within their equivalence class, for example: *A slice of toast* ***is*** *Pizza*

[2] Will need some additional special cases to cover the entire set

The Cube Rule of Food – Toast

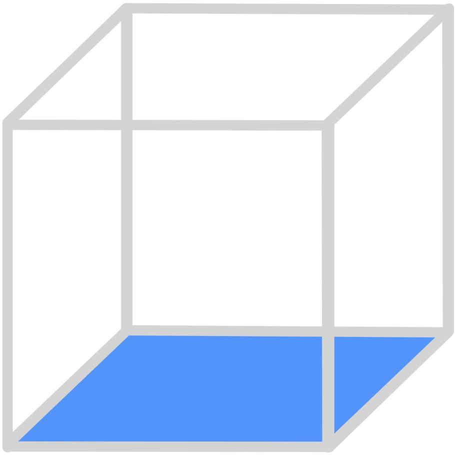


Figure 6: The starch locations of the "Toast" equivalence class

(a) Pizza

(b) Nigiri Sushi

(c) A slice of Pumpkin Pie
(i.e. bent toast)

The Cube Rule of Food – Sandwich

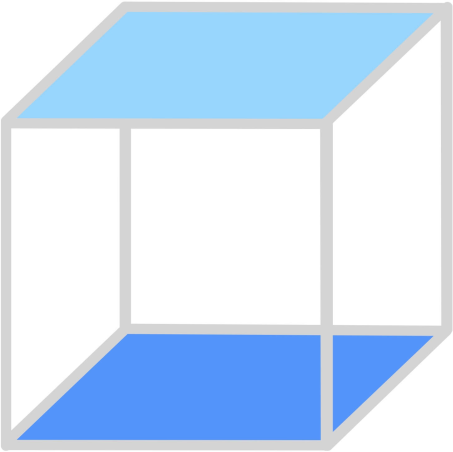


Figure 8: The starch locations of the “Sandwich” equivalence class

Examples of Sandwiches

(a) Quesadilla

(b) Victoria Sponge Cake

The Cube Rule of Food – Taco

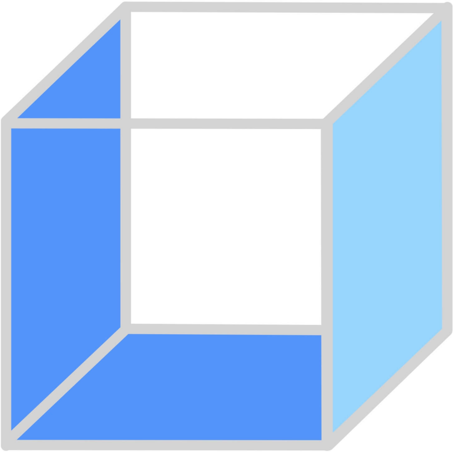


Figure 10: The starch locations of the "Taco" equivalence class

Examples of Tacos

(a) Hot dog

(b) Sub sandwich (uncut)

(c) A slice of pie (a taco on its side)

The Cube Rule of Food – Sushi

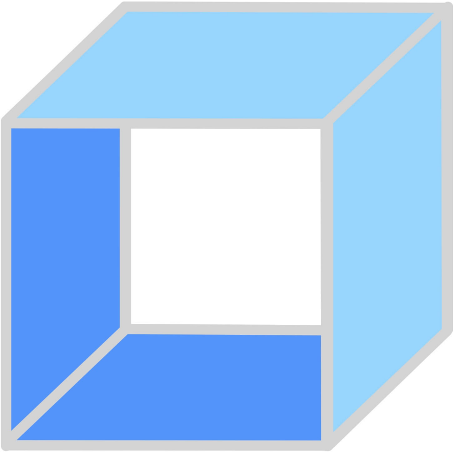


Figure 12: The starch locations of the "Sushi" equivalence class

Examples of Sushi

(a) Enchilada

(b) Falafel

(c) Pigs in blankets

The Cube Rule of Food – Quiche

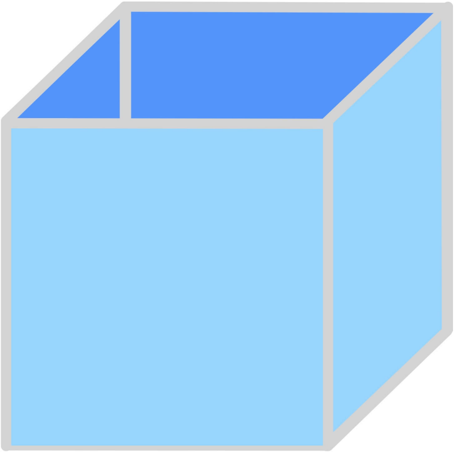


Figure 14: The starch locations of the "Quiche" equivalence class

Examples of Quiche

(a) Cheesecake

(b) Deep dish pizza

(c) Soup bread bowl

The Cube Rule of Food – Calzone

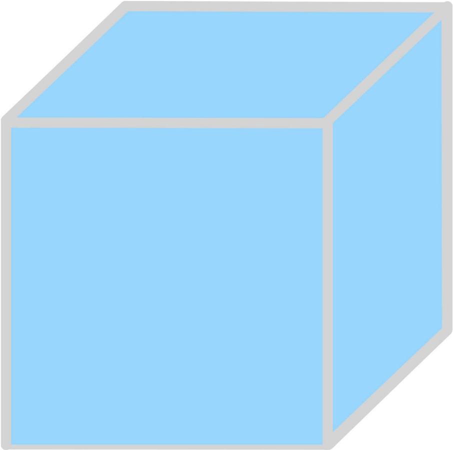


Figure 16: The starch locations of the “Calzone” equivalence class

Examples of Calzone

(a) Pie (whole)

(b) Burrito

(c) Dumplings

Is this enough?

- For these groupings to be equivalence classes, their union must cover the entire set
- In the initial set of rulings, this is not true!
  - For example foodstuffs with no starch, such as salads, are not in any of the groupings
- To address this, we need to introduce a couple more classes to capture the foods which don't conform to our (beautiful) system

The Cube Rule of Food – Salad

Figure 18: The starch locations of the “Salad” equivalence class

Examples of Salads

(a) Steak

(b) Chocolate

(c) Soup (wet salad)

The Cube Rule of Food – Nachos

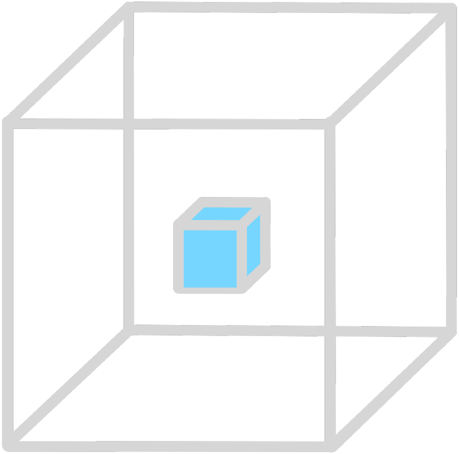
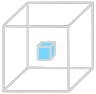


Figure 20: The starch locations of the “Nachos” equivalence class

Examples of Nachos



(a) Salad (with croutons)

(b) Poutine

The Cube Rule of Food – Cake

Figure 22: The starch locations of the "Cake" grouping of foodstuffs

Examples of Cakes

(a) Big Mac™

(b) A stack of pancakes

Is ***this*** enough?

- I'm not sure how you would actually prove this!
- However, if you take a generous view of equivalence class membership, this is probably enough to uniquely cover all foods

Part II - A (brief) introduction to Group Theory

What is a group?

- Intuitively, a group is an algebraic structure consisting of both:

1. A set of items
2. An operation which combines two of its elements to form a third element

What is a group?

More formally, a group is defined as:

- A set of elements, G
- A binary operation $\bullet$ which maps two elements $a, b \in G$ to another element $c = a \bullet b \in G$ in the set
- Where the following properties hold:
  1. **Closure** – $\forall a, b \in G, \quad a \bullet b \in G$
  2. **Associativity** – $\forall a, b, c \in G, \quad (a \bullet b) \bullet c = a \bullet (b \bullet c)$
  3. **Identity element** – $\exists e \in G \forall a \in G, \quad e \bullet a = a \bullet e = a$
  4. **Inverse element** – $\forall a \in G \exists b \in G, \quad a \bullet b = b \bullet a = e$

What if some of the properties don't hold?

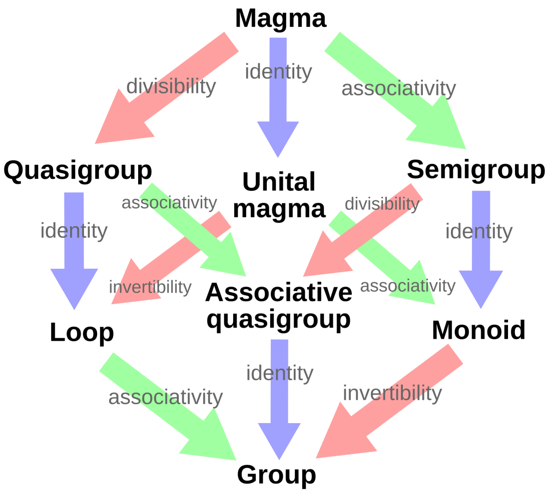


Figure 24: Algebraic structures between magmas and groups [4]

Why are groups interesting?

Groups have many applications for understanding the Real World™

- Modelling physical phenomena
  - Crystals
  - Hydrogen atoms
  - Three of the four known fundamental forces in the universe
- Public key cryptography
- And many more...[3]

[3] https://tvtropes.org/.../ChekhovsGun

Part III - Defining Lasagne

Figure 25: Garfield loves Lasagne [5]

What is a Lasagne?

Figure 26: A portion of Lasagne

Is this still Lasagne?

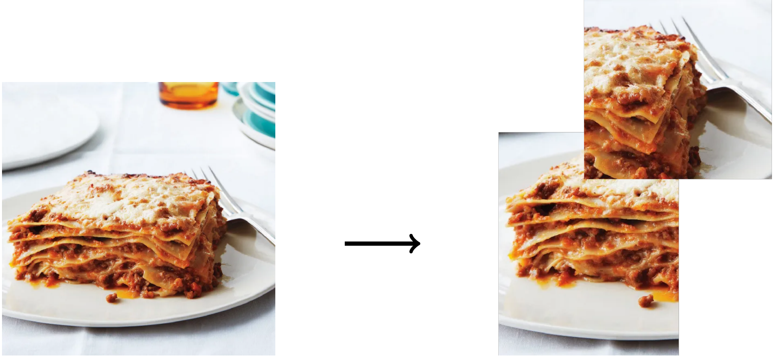


Figure 27: If you cut a portion of Lasagne in half, and stack one half on top of the other – it is still Lasagne!

Does Lasagne form a group?

- G is the set containing all Lasagnes with a non-negative number of layers, i.e. $G = \{$ n-layer Lasagne $\mid n \in \mathbb{N}_0\}$
- The binary operation $\bullet$ is stacking two Lasagnes, one atop the other
- Do the four properties hold?
  1. **Closure**
  2. **Associativity**
  3. **Identity element**
  4. **Inverse element**

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- Does Lasagne have an identity element under the stacking operation?
  - Yes! The identity element is the empty Lasagne, a Lasagne with no layers
- Does Lasagne have inverse elements under the stacking operation?
  - No! By counter-example, there is no Lasagne with a non-negative integer number of layers which you could stack on a single-layer Lasagne to get the empty Lasagne

So what does it form?

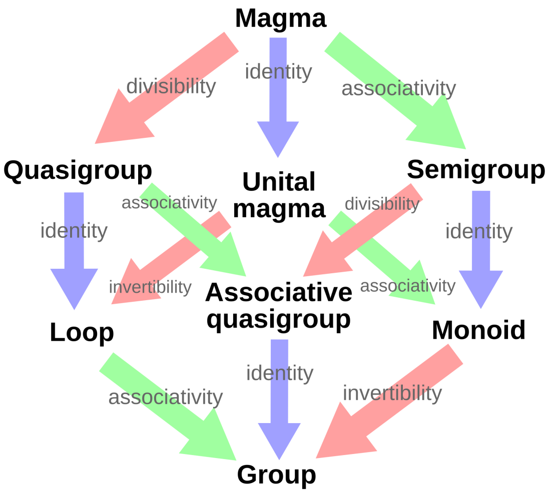

Figure 28: Algebraic structures between magmas and groups [4]

Lasagne is a monoid under the stacking operation!

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However, it is not a monad, since it is not in the category of endofunctors...

The Lasagne monoid in Haskell

```haskell
newtype Lasagne = Lasagne Int
    deriving (Show, Num)

-- The stacking operating can be considered integer
-- addition of the number of layers
instance Semigroup Lasagne where
    (<>) = (+)

-- The identity element is the empty (zero-layer) Lasagne
instance Monoid Lasagne where
    mempty = Lasagne 0

-- Stacking 5 and 6 layers gives 11 layers:
--
-- ghci> Lasagne 5 <> Lasagne 6
-- Lasagne 11
```

Extending the Cube Rule of Food

- Now that we know Lasagne is a monoid, we can use it to extend the "Cube Rule of Food"!
  - Salad is isomorphic to the identity element of the Lasagne monoid
  - Pizza is isomorphic to the single-layer element of the Lasagne monoid
  - Sandwiches are isomorphic to the double-layer element in the monoid

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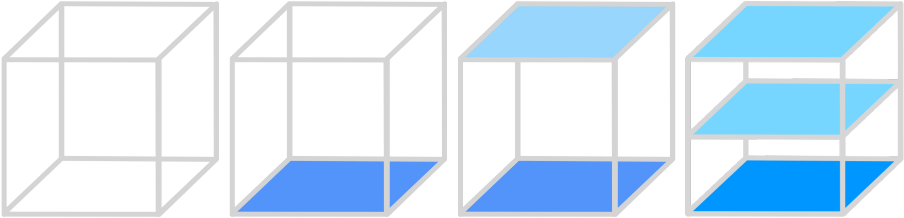
  - Sandwiches are isomorphic to the double-layer element in the monoid

  - **In fact, Lasagne forms a rigorous definition of Cake!**



(a) Salad as the identity Lasagne

(b) Toast as a single-layer Lasagne

(c) Sandwiches as a double-layer Lasagne

(d) Triple-layer cakes as a Lasagne

Figure 29: Other equivalence classes as elements of the Lasagne Monoid

Why is this useful?

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It isn't...

It isn't...

But I think it is funny, and maybe you did too...

Thanks for listening!

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I refuse to answer any questions...

Bibliography I

[1] Nathan W. Pyle. *Classic Sandwiches*. en. URL: https://imgur.com/a/R2Hru (visited on 10/17/2023).

[2] @matttomic. *The Sandwich Alignment Chart*. en-GB. URL: https://twitter.com/matttomic/status/859117370455060481 (visited on 10/17/2023).

[3] @Phosphatide. *The Cube Rule*. en-GB. URL: https://twitter.com/Phosphatide/status/974067376894328833 (visited on 10/17/2023).

[4] Wikimedia Commons. *Algebraic structures between magmas and groups*. Dec. 2020. URL: https://commons.wikimedia.org/wiki/File:Algebraic_structures_-_magma_to_group.svg (visited on 10/18/2023).

Bibliography II

[5] Jim Davis. *Garfield Loves Lasagna (food)*. en. URL: https://garfield.fandom.com/wiki/Lasagna_(food) (visited on 10/18/2023).