Part I: Theory

1.

$$
\begin{gathered}
{x'}^T F x = 0 \\
\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \\
\begin{bmatrix} f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \\
f_{33} = 0
\end{gathered}
$$

As we plug the camera origin point to the Longuet-Higgins equation, we found that the F_{33} element of the fundamental matrix is always zero.

2.

$$
\begin{gathered}
t = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix}, \quad [t]_\times = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \\
E = [t]_\times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \\
l' = Ex = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_x \\ t_x y \end{bmatrix}
\end{gathered}
$$

$$
\begin{bmatrix} \mathbf{x} & \mathbf{y} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -t_x \\ t_x y \end{bmatrix} = -t_x \mathbf{y} + t_x y = 0 \tag{1}
$$

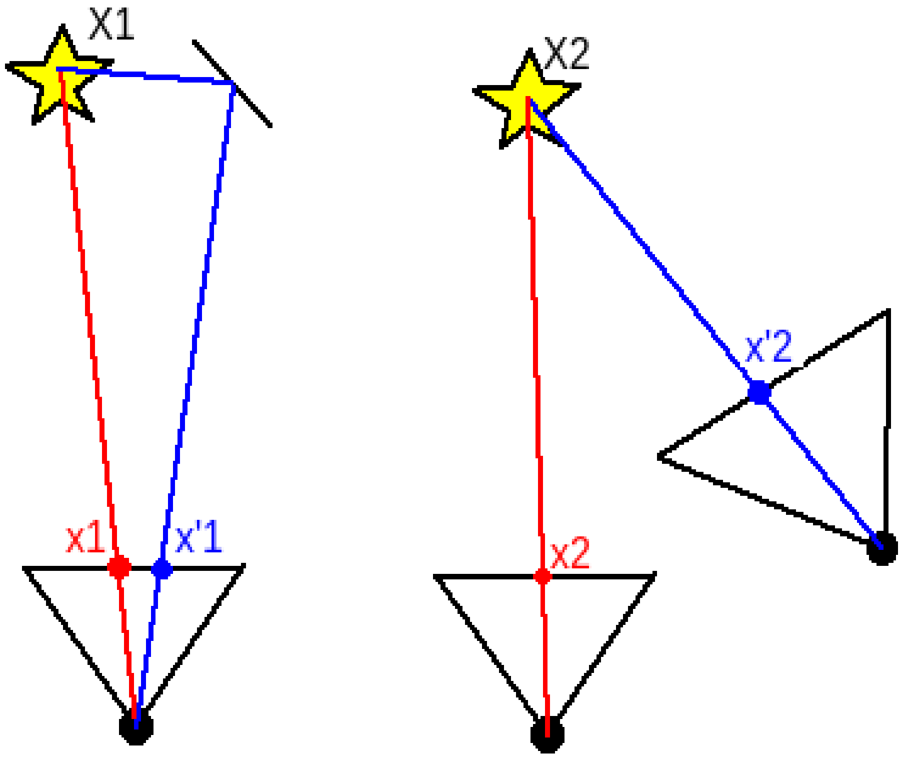
$$
l = E^T x' \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & -t_x & 0 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ t_x \\ -t_x y' \end{bmatrix}
$$

$$
\begin{bmatrix} \mathbf{x} & \mathbf{y} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ t_x \\ -t_x y' \end{bmatrix} = t_x \mathbf{y} - t_x y' = 0 \tag{2}
$$

As the equation (1) and (2) are the equations of epipolar lines, we can see that no x parameter exist which means the lines always lie on the x-axis and remain parallel to it.

3.

$$
\begin{gathered}
x_1 = R_1 x_0 + t_1 \\
x_0 = R_1^{-1}(x_1 - t_1) \\
x_2 = R_2 R_1^{-1}(x_1 - t_1) + t_2 \\
x_2 = (R_2 R_1^{-1}) x_1 + (-R_2 R_1^{-1} t_1 + t_2) \\
R_{rel} = R_2 R_1^{-1} \\
t_{rel} = -R_2 R_1^{-1} t_1 + t_2 \\
E = R_{rel}[t_{rel\times}] \\
F = K^{-T} E K^{-1} \\
F = K^{-T} R_{rel}[t_{rel\times}] K^{-1}
\end{gathered}
$$



4. As the setup shown in figure 1, we number the question's setup as 1 and the normal setup as 2.

$$x_2 = KP_2X_2$$
$$x_2^T = X_2^T P_2^T K^T$$

$$x'_2 = KP'_2X_2$$
$$x'_2 = KTP_2X_2$$
$$x_2'^T = X_2^T P_2^T T^T K^T$$

$$x_2^T F x'_2 = 0$$
$$x_2'^T F^T x_2 = 0$$

$$x_2^T F x'_2 + x_2'^T F^T x_2 = 0$$
$$X_2^T P_2^T K^T FKTP_2X_2 + X_2^T P_2^T T^T K^T F^T KP_2X_2 = 0$$
$$K^T FKT + T^T K^T F^T K = 0$$
$$K^T FK + K^T F^T K = 0$$
$$F + F^T = 0$$
$$F = -F^T$$

Part II: Practice

1 Overview

In this part, we will begin by implementing the two different methods to estimate the fundamental matrix (Section 2). Next, use fundamental matrix and calibrated intrinsics to compute the essential matrix and use this to compute a 3D metric reconstruction from 2D correspondences using triangulation (Section 3). Then, we implement a method to automatically match points taking advantage of epipolar constraints and make a 3D visualization of the results (Section 4). Finally, we implement RANSAC and bundle adjustment to further improve your algorithm (Section5).

2 Fundamental matrix estimation

1. In this part, we implemented the *eightpoint* algorithm to find the fundamental matrix F. Basically, we do the following:
   (a) Normalization
   (b) Stack point to get A
   (c) SVD to find F
   (d) Refine F
   (e) Unnormalization

   Then we recover the fundamental matrix F as :

$$F = \begin{bmatrix} 4.47277513e-09 & 1.21213301e-07 & -1.19108099e-03 \\ 6.86334285e-08 & 3.26769995e-09 & -2.65963407e-05 \\ 1.14422062e-03 & 8.94802848e-06 & 4.12104727e-03 \end{bmatrix}$$

   And able to get output like :

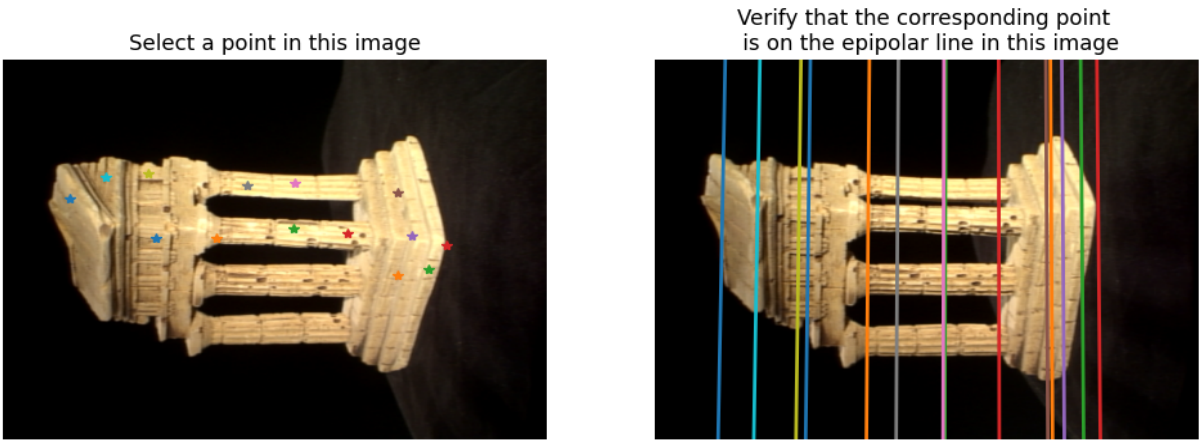


Figure 1: Demo

2.

$$F_0 = \begin{bmatrix} 6.02167939e-07 & -9.18775130e-08 & 1.10014526e-03 \\ 1.23727632e-06 & 2.10666612e-07 & -1.13857897e-04 \\ -1.68885023e-03 & -2.82271235e-04 & 1.24009267e-01 \end{bmatrix},$$

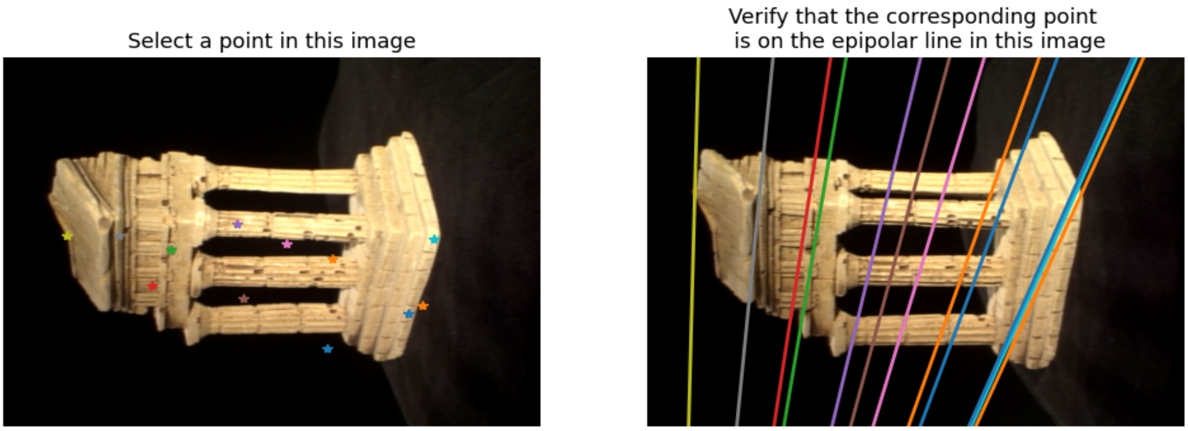
$$F_1 = \begin{bmatrix} -6.83011978e-08 & -2.63561252e-06 & -5.75821499e-04 \\ 2.20895343e-06 & 4.48291578e-08 & -2.74300813e-04 \\ 7.42295288e-04 & 3.26920820e-04 & -2.30023753e-02 \end{bmatrix},$$

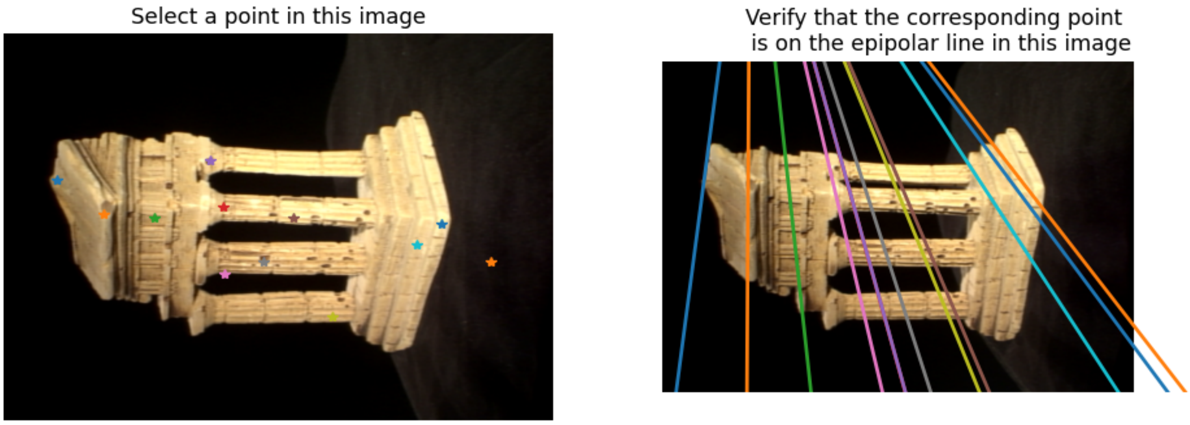
$$F_2 = \begin{bmatrix} -5.80855791e-08 & -2.07925976e-06 & -6.49106234e-05 \\ 1.70276317e-06 & 5.06226292e-08 & -1.70123579e-04 \\ 2.04350896e-04 & 1.95556375e-04 & -1.39673019e-02 \end{bmatrix}$$

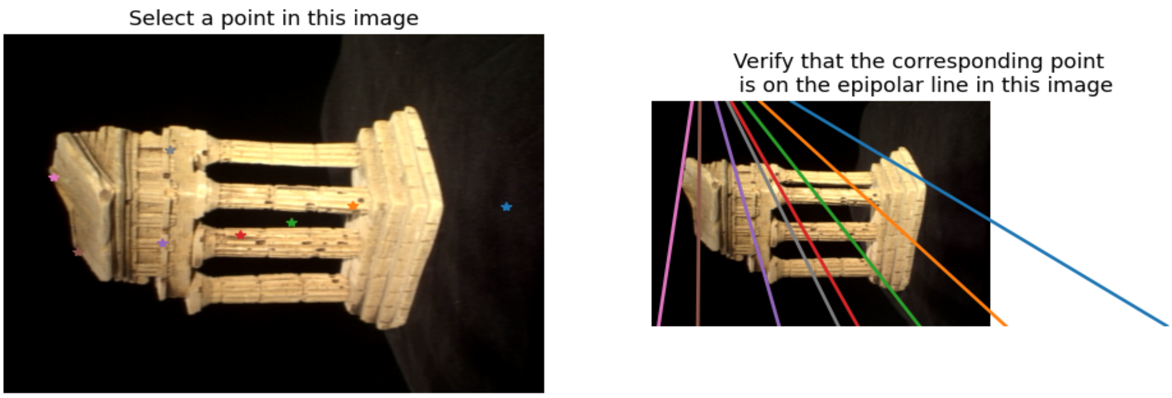
3 Metric Reconstruction

1.

$$E = \begin{bmatrix} 1.03393393e-02 & 2.81212235e-01 & -1.76336730e+00 \\ 1.59228069e-01 & 7.60841719e-03 & -7.69112423e-03 \\ 1.76748987e+00 & 7.08016542e-02 & 3.74299826e-04 \end{bmatrix}$$







The recovered M_2 are :

$$\begin{bmatrix} -0.99903496 & -0.04342569 & -0.00658468 & 0.00438073 \\ -0.04377307 & 0.99673196 & 0.06789195 & -1. \\ 0.0036149 & 0.06811466 & -0.99767095 & 0.09006149 \end{bmatrix},$$
$$\begin{bmatrix} -0.99903496 & -0.04342569 & -0.00658468 & -0.00438073 \\ -0.04377307 & 0.99673196 & 0.06789195 & 1. \\ 0.0036149 & 0.06811466 & -0.99767095 & -0.09006149 \end{bmatrix},$$
$$\begin{bmatrix} 0.99938018 & 0.03481496 & 0.00521351 & 0.00438073 \\ -0.0350305 & 0.96886262 & 0.24510833 & -1. \\ 0.00348226 & -0.24513904 & 0.96948168 & 0.09006149 \end{bmatrix},$$
$$\begin{bmatrix} 0.99938018 & 0.03481496 & 0.00521351 & -0.00438073 \\ -0.0350305 & 0.96886262 & 0.24510833 & 1. \\ 0.00348226 & -0.24513904 & 0.96948168 & -0.09006149 \end{bmatrix}$$

2.

$$C_1 = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{bmatrix} = \begin{bmatrix} - & \mathbf{p}_1^T & - \\ - & \mathbf{p}_2^T & - \\ - & \mathbf{p}_3^T & - \end{bmatrix}$$

$$C_2 = \begin{bmatrix} c'_{11} & c'_{12} & c'_{13} & c'_{14} \\ c'_{21} & c'_{22} & c'_{23} & c'_{24} \\ c'_{31} & c'_{32} & c'_{33} & c'_{34} \end{bmatrix} = \begin{bmatrix} - & \mathbf{p}_1^{\prime T} & - \\ - & \mathbf{p}_2^{\prime T} & - \\ - & \mathbf{p}_3^{\prime T} & - \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}, \quad pts1 = \begin{bmatrix} | & | \\ x & y \\ | & | \end{bmatrix}, \quad pts1 = \begin{bmatrix} | & | \\ x' & y' \\ | & | \end{bmatrix}$$

$$\begin{bmatrix} y\mathbf{p}_3^T - \mathbf{p}_2^T \\ \mathbf{p}_1^T - x\mathbf{p}_3^T \\ y'\mathbf{p}_3^{\prime T} - \mathbf{p}_2^{\prime T} \\ \mathbf{p}_1^{\prime T} - x'\mathbf{p}_3^{\prime T} \end{bmatrix} \mathbf{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\,\mathbf{X} = \mathbf{0}$$

The reprojection error is 4492.29.

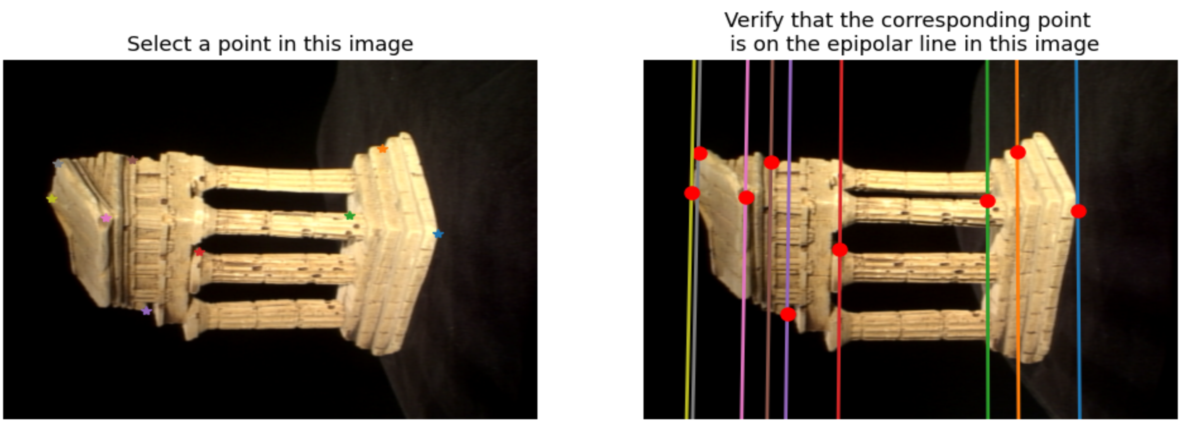
3.

$$M2 = \begin{bmatrix} 0.99938018 & 0.03481496 & 0.00521351 & 0.00438073 \\ -0.0350305 & 0.96886262 & 0.24510833 & -1. \\ 0.00348226 & -0.24513904 & 0.96948168 & 0.09006149 \end{bmatrix}$$

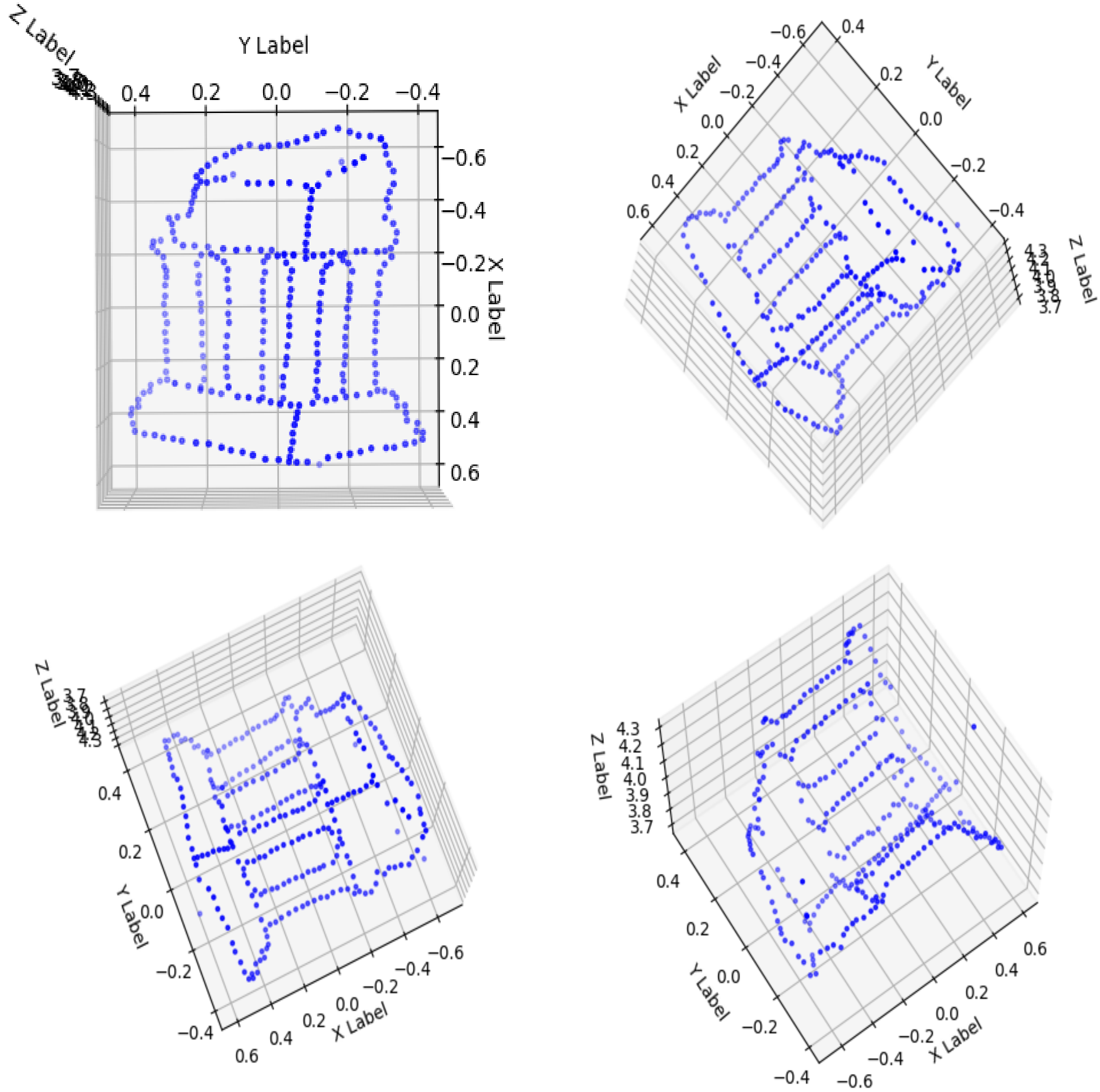
$$C2 = \begin{bmatrix} 1.52051038e+03 & -2.11777737e+01 & 3.01020318e+02 & 3.38878506e+01 \\ -5.25933703e+01 & 1.41786999e+03 & 6.13346750e+02 & -1.50366652e+03 \\ 3.48226351e-03 & -2.45139043e-01 & 9.69481678e-01 & 9.00614919e-02 \end{bmatrix}$$

4 3D Visualization

1. Screenshot of *epipolarMatchGUIwith* some detected correspondences.



2. Screenshots of the 3D visualization.

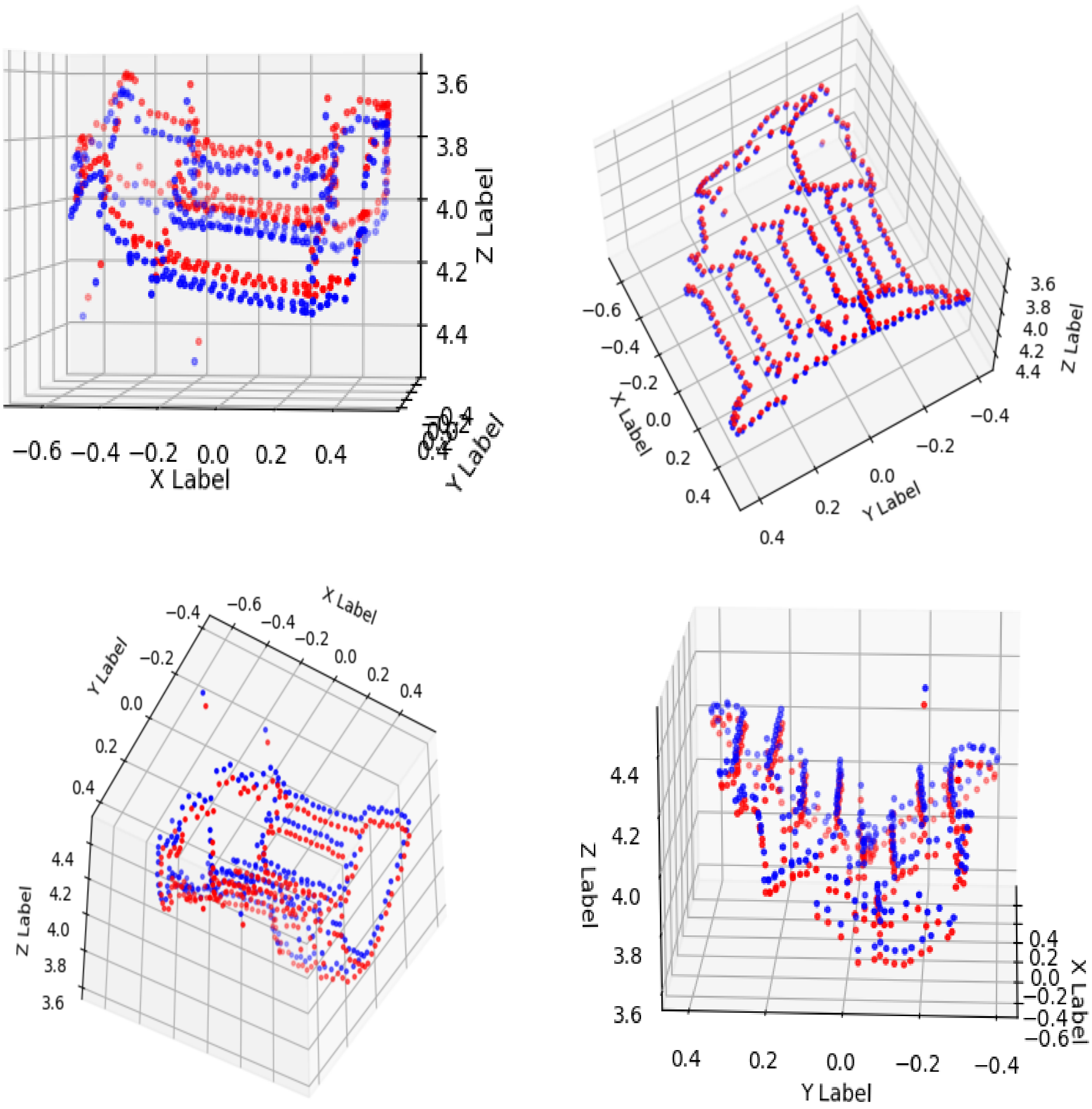


5 Bundle Adjustment

1. The RANSAC method will run totally 100 iterations and each iteration will random choose seven pairs of points to calculate the fundamental matrix F. Then the fundamental matrix is used to calculate the epipolar line on image2. Calculating the distance from each point to the corresponding epipolar line. When the distance is below the threshold, it is regarded as an inlier.

2. The Rodrigues formula is implemented.

3. The reprojection error with initial M2 and P is 4492.290014500242; The reprojection error with the optimized matrices is 13.639709436778343.


X Label
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