Instituto Nacional de Telecomunicações - INATEL
2ª Prova de E203-A/B – Circuitos Elétricos III
Prof. Antonio Alves Ferreira Júnior

Aluno: GABARITO

Matrícula:________ **Período:**________ **Curso: EA ( ) EB ( ) EE ( ) ET ( )**

Data: 16/06/2020 **Duração: 90 minutos** **Pontuação: 100 pontos** **Nota:**_______

Formulário:

$i(t)=\frac{dq(t)}{dt}$ $\quad w(t)=\int_{-\infty}^{t} p(t)dt=\int_{-\infty}^{t} v(t)i(t)dt$ $\quad p(t)=v(t)i(t)$ $\quad v_R(t)=Ri_R(t)$ $\quad q=Cv$ $\quad i_c(t)=C\frac{dv_c(t)}{dt}$

$v_c(t)=\frac{1}{C}\int_{t_0}^{t} i_c(t)dt+v_c(0)$ $\quad w_c(t)=\frac{Cv_c^2(t)}{2}$ $\quad \frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\cdots+\frac{1}{C_N}$ $\quad C_{eq}=C_1+C_2+\cdots+C_N$ $\quad \lambda=N\phi$ $\quad \lambda=Li$

$v_L(t)=L\frac{di(t)}{dt}$ $\quad i_L(t)=\frac{1}{L}\int_{t_0}^{t} v_L(t)dt+i(t_0)$ $\quad w_L(t)=\frac{Li_L^2(t)}{2}$ $\quad L_{eq}=L_1+L_2+\cdots+L_N$ $\quad \frac{1}{L_{eq}}=\frac{1}{L_1}+\frac{1}{L_2}+\cdots+\frac{1}{L_N}$

$\frac{dx(t)}{dt}+ax(t)=y(t)$ $\quad x(t)=x_p(t)+x_h(t)$ $\quad x(t)=K_1+K_2e^{-at}$

$b_m\frac{d^m x(t)}{dt^m}+b_{m-1}\frac{d^{m-1}x(t)}{dt^{m-1}}+\cdots+b_1\frac{dx(t)}{dt}+b_0x(t)=a_n\frac{d^n y(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}}+\cdots+a_1\frac{dy(t)}{dt}+a_0y(t)$ $\quad K_1=\frac{A}{a}$ $\quad \tau=\frac{1}{a}$

$s=\sigma+j\omega$ $\quad V_R(s)=RI_R(s)$ $\quad \frac{V_R(s)}{I_R(s)}=R=Z_R(s)$ $\quad I_C(s)=sCV_C(s)-Cv_C(0)$ $\quad V_C(s)=\frac{I_C(s)}{sC}+\frac{v_C(0)}{s}$

$\frac{V_C(s)}{I_C(s)}=\frac{1}{sC}=Z_C(s)$ $\quad V_L(s)=sLI_L(s)-Li_L(0)$ $\quad I_L(s)=\frac{V_L(s)}{sL}+\frac{i_L(0)}{s}$ $\quad Y(s)=Y_f(s)+Y_n(s)$ $\quad \frac{V_L(s)}{I_L(s)}=sL=Z_L(s)$

$Z(s)=\frac{V(s)}{I(s)}=R(s)\pm jX(s)$ $\quad Y(s)=\frac{1}{Z(s)}=\frac{I(s)}{V(s)}=G(s)\pm jB(s)$ $\quad Z(s)=\frac{1}{Y(s)}$ $\quad X_C=-\frac{1}{\omega C}$ $\quad X_L=j\omega L$ $\quad B_L=-\frac{1}{\omega L}$

$B_C=\omega C$ $\quad f(t)\xleftrightarrow{L}F(s)$ $\quad \delta(t)\xleftrightarrow{L}1$ $\quad u(t)\xleftrightarrow{L}\frac{1}{s}$ $\quad e^{-at}\xleftrightarrow{L}\frac{1}{s+a}$ $\quad sen(\omega t)\xleftrightarrow{L}\frac{\omega}{s^2+\omega^2}$

$\cos(\omega t)\xleftrightarrow{L}\frac{s}{s^2+\omega^2}$ $\quad e^{-at}sen(\omega t)\xleftrightarrow{L}\frac{\omega}{(s+a)^2+\omega^2}$ $\quad e^{-at}\cos(\omega t)\xleftrightarrow{L}\frac{s+a}{(s+a)^2+\omega^2}$

$2|K|e^{-\sigma_o t}\cos(\omega_o t+\phi)\xleftrightarrow{L}\frac{K}{s+\sigma_o-j\omega_o}+\frac{K^*}{s+\sigma_o+j\omega_o}$

$f(t)\xleftrightarrow{L}F(s)$ $\quad \delta(t)\xleftrightarrow{L}1$ $\quad u(t)\xleftrightarrow{L}\frac{1}{s}$ $\quad e^{-at}\xleftrightarrow{L}\frac{1}{s+a}$ $\quad sen(\omega t)\xleftrightarrow{L}\frac{\omega}{s^2+\omega^2}$ $\quad \cos(\omega t)\xleftrightarrow{L}\frac{s}{s^2+\omega^2}$

$e^{-at}sen(\omega t)\xleftrightarrow{L}\frac{\omega}{(s+a)^2+\omega^2}$ $\quad e^{-at}\cos(\omega t)\xleftrightarrow{L}\frac{s+a}{(s+a)^2+\omega^2}$

$2|K|e^{-\sigma_o t}\cos(\omega_o t+\phi)\xleftrightarrow{L}\frac{K}{s+\sigma_o-j\omega_o}+\frac{K^*}{s+\sigma_o+j\omega_o}$

$V_1=Z_{11}I_1+Z_{12}I_2$, $V_2=Z_{21}I_1+Z_{22}I_2$ $\quad \det Z=Z_{11}Z_{22}-Z_{12}Z_{21}$ $\quad I_1=Y_{11}V_1+Y_{12}V_2$, $I_2=Y_{21}V_1+Y_{22}V_2$ $\quad \det Y=Y_{11}Y_{22}-Y_{12}Y_{21}$ $\quad V_1=H_{11}I_1+H_{12}V_2$, $I_2=H_{21}I_1+H_{22}V_2$

$\det H=H_{11}H_{22}-H_{12}H_{21}$ $\quad Z_{11}=\frac{Y_{22}}{\det Y}$ $\quad Z_{12}=-\frac{Y_{12}}{\det Y}$ $\quad Z_{21}=-\frac{Y_{21}}{\det Y}$ $\quad Z_{22}=\frac{Y_{11}}{\det Y}$ $\quad Z_{11}=\frac{\det H}{H_{22}}$ $\quad Z_{12}=\frac{H_{12}}{H_{22}}$

$Z_{21}=-\frac{H_{21}}{H_{22}}$ $\quad Z_{22}=\frac{1}{H_{22}}$ $\quad Y_{11}=\frac{Z_{22}}{\det Z}$ $\quad Y_{12}=-\frac{Z_{12}}{\det Z}$ $\quad Y_{21}=-\frac{Z_{21}}{\det Z}$ $\quad Y_{22}=\frac{Z_{11}}{\det Z}$ $\quad Y_{11}=\frac{1}{H_{11}}$ $\quad Y_{12}=-\frac{H_{12}}{H_{11}}$

$Y_{21}=\frac{H_{21}}{H_{11}}$ $\quad Y_{22}=\frac{\det H}{H_{11}}$ $\quad H_{11}=\frac{\det Z}{Z_{22}}$ $\quad H_{12}=\frac{Z_{12}}{Z_{22}}$ $\quad H_{21}=-\frac{Z_{21}}{Z_{22}}$ $\quad H_{22}=\frac{1}{Z_{22}}$ $\quad H_{11}=\frac{1}{Y_{11}}$ $\quad H_{12}=-\frac{Y_{12}}{Y_{11}}$ $\quad H_{21}=\frac{Y_{21}}{Y_{11}}$

$$H_{22}=\frac{\det Y}{Y_{11}} \quad Z_1=\frac{Y_{22}+Y_{12}}{\det Y} \quad Z_2=-\frac{Y_{12}}{\det Y} \quad Z_3=\frac{Y_{11}+Y_{12}}{\det Y} \quad Y_1=\frac{Z_{22}-Z_{12}}{\det Z} \quad Y_2=\frac{Z_{12}}{\det Z} \quad Y_3=\frac{Z_{11}-Z_{12}}{\det Z} \quad A_v=\frac{V_2}{V_g}$$

$$A_{vZ}=\frac{Z_L Z_{21}}{(Z_L+Z_{22})(Z_{11}+Z_g)-Z_{12}Z_{21}} \quad A_{vY}=-\frac{Y_{21}}{(Y_L+Y_{22})(1+Z_gY_{11})-Z_gY_{12}Y_{21}} \quad A_{vH}=-\frac{H_{21}}{(Y_L+H_{22})(Z_g+H_{11})-H_{12}H_{21}}$$

$$A_i=\frac{I_2}{I_1} \quad A_{iZ}=-\frac{Z_{21}}{Z_L+Z_{22}} \quad A_{iY}=\frac{Y_LY_{21}}{Y_{11}(Y_L+Y_{22})-Y_{12}Y_{21}} \quad A_{iH}=\frac{Y_LH_{21}}{Y_L+H_{22}} \quad A_p=|A_v||A_i| \quad Z_{in}=\frac{V_g}{I_1}$$

$$Z_{inZ}=Z_g+Z_{11}-\frac{Z_{12}Z_{21}}{Z_L+Z_{22}} \quad Z_{inY}=\frac{(1+Z_gY_{11})(Y_L+Y_{22})-Z_gY_{12}Y_{21}}{Y_{11}(Y_{22}+Y_L)-Y_{12}Y_{21}} \quad Z_{inH}=Z_g+H_{11}-\frac{Z_LH_{12}H_{21}}{1+Z_LH_{22}} \quad Z_{out}=\frac{V_2}{I_2}$$

$$Z_{outZ}=Z_{22}-\frac{Z_{12}Z_{21}}{Z_g+Z_{11}} \quad Z_{outY}=\frac{1+Z_gY_{11}}{Y_{22}(1+Z_gY_{11})-Z_gY_{12}Y_{21}} \quad Z_{outH}=\frac{Z_g+H_{11}}{H_{22}(Z_g+H_{11})-H_{12}H_{21}}$$

$$y(t)=h(t)*x(t) \quad Y(s)=H(s)X(s) \quad H(s)=\frac{Y(s)}{X(s)} \quad A_v(s)=\frac{V_o(s)}{V_i(s)} \quad A_i(s)=\frac{I_o(s)}{I_i(s)} \quad A_p(s)=\frac{P_o(s)}{P_i(s)} \quad Z(s)=\frac{V_o(s)}{I_i(s)}$$

$$Y(s)=\frac{I_o(s)}{V_i(s)} \qquad a_n\frac{d^n y(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}}+\cdots+a_1\frac{dy(t)}{dt}+a_0y(t)=b_m\frac{d^m x(t)}{dt^m}+b_{m-1}\frac{d^{m-1}x(t)}{dt^{m-1}}+\cdots+b_1\frac{dx(t)}{dt}+b_0x(t)$$

$$H(s)=\frac{Y(s)}{X(s)}=\frac{b_ms^m+b_{m-1}s^{m-1}+\ldots+b_1s+b_0}{a_ns^n+b_{n-1}s^{n-1}+\ldots+a_1s+a_0} \quad H(s)=\frac{Y(s)}{X(s)}=k\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} \quad k=\frac{b}{a} \quad s=\sigma+j\omega$$

$$H(s)=H_1(s)H_2(s)H_3(s)\cdots H_n(s) \quad H(s)=H_1(s)+H_2(s)+H_3(s)+\cdots+H_n(s) \quad \omega_{zi}=|z_i| \quad \omega_{pj}=|p_j| \quad \tau=\frac{1}{\omega_{pj}}$$

$$H(s)=\frac{N(s)}{s+\omega_{p1}} \quad H(s)=\frac{As+B}{s+\omega_{p1}}=A+\frac{B-A\omega_{p1}}{s+\omega_{p1}} \quad y(t)=y_f(t)+(B-A\omega_{p1})e^{-\omega_{p1}t} \quad H(s)=\frac{N(s)}{(s+\omega_{p1})(s+\omega_{p2})}$$

$$H(s)=\frac{N(s)}{s^2+2\xi\omega_ns+\omega_n^2} \quad \omega_n=\sqrt{\omega_{p1}\omega_{p2}} \quad s=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1} \quad y(t)=y_f(t)+Ae^{-\omega_{p1}t}+Be^{-\omega_{p2}t}$$

$$s=-\xi\omega_n\pm j\omega_n\sqrt{1-\xi^2}=-\sigma_0\pm j\omega_0 \quad y(t)=y_f(t)+2|K_1|e^{-\sigma_0t}\cos(\omega_0t+\phi) \quad s=-\xi\omega_n$$

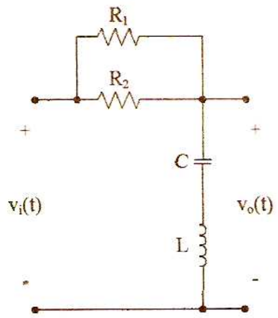
$$y(t)=y_f(t)+(K_1+K_2t)e^{-\omega_{p1}t} \quad Y(j\omega)=H(j\omega)X(j\omega) \quad H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}$$

$$H(j\omega)=k\frac{(j\omega-z_1)(j\omega-z_2)\cdots(j\omega-z_m)}{(j\omega-p_1)(j\omega-p_2)\cdots(j\omega-p_n)} \quad H(j\omega)=\mathrm{Re}\{H(j\omega)\}+j\,\mathrm{Im}\{H(j\omega)\}=|H(j\omega)|\angle\phi(\omega)=|H(j\omega)|e^{j\phi(\omega)}$$

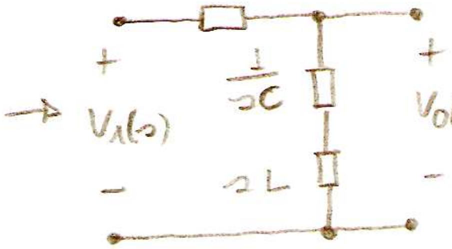
$$|H(j\omega)|=\sqrt{\mathrm{Re}^2\{H(j\omega)\}+\mathrm{Im}^2\{H(j\omega)\}} \quad \phi(\omega)=\arctan\left[\frac{\mathrm{Im}\{H(j\omega)\}}{\mathrm{Re}\{H(j\omega)\}}\right] \quad BW=\frac{\omega_n}{Q}=2\xi\omega_n \quad Q=\frac{\omega_n}{BW}=\frac{1}{2\xi}$$

Questões

1) (50 pontos) Para o circuito a seguir, tem-se que a largura de faixa é igual a 50rad/s, C = 1mF, L = 0,1H e R_1 = 30Ω. Determinar: a) o valor de R_2 (15 pontos); b) os valores dos módulos dos ganhos de tensão nas frequências angulares iguais a 0rads/s, 10rad/s, 100rad/s, 1000rad/s e tendendo a infinito (25 pontos); c) o tipo de filtro (10 pontos). Não serão aceitas respostas sem as soluções e as devidas justificativas.



$R_{eq} = R_1 // R_2$



$BW = 50 \, rad/s$

$C = 1mF$

$L = 0{,}1H$

$R_1 = 30\Omega$

$$Av(s) = \frac{sL + \frac{1}{sC}}{sL + \frac{1}{sC} + R_{eq}} = \frac{\frac{s^2LC+1}{sC}}{\frac{s^2LC + sR_{eq}C + 1}{sC}} = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R_{eq}}{L}s + \frac{1}{LC}} = \frac{s^2 + 10^4}{s^2 + 10R_{eq}s + 10^4}$$

$$Av(j\omega) = \frac{10^4 - \omega^2}{10^4 - \omega^2 + j10\omega R_{eq}} \quad , \quad |Av(j\omega)| = \frac{\sqrt{(10^4-\omega^2)^2}}{\sqrt{(10^4-\omega^2)^2 + (10\omega R_{eq})^2}}$$

a) $BW = 10R_{eq} \rightarrow 50 = 10R_{eq} \rightarrow R_{eq} = 5\Omega \rightarrow R_2 = \frac{R_{eq}R_1}{R_1 - R_{eq}} \rightarrow \boxed{R_2 = 6\Omega}$

b) $|Av(j0)| = 1$

$|Av(j10)| = 0{,}998$

$|Av(j100)| = 0$

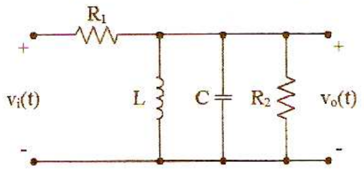
$|Av(j1000)| = 0{,}998$

$|Av(j\infty)| = 1$

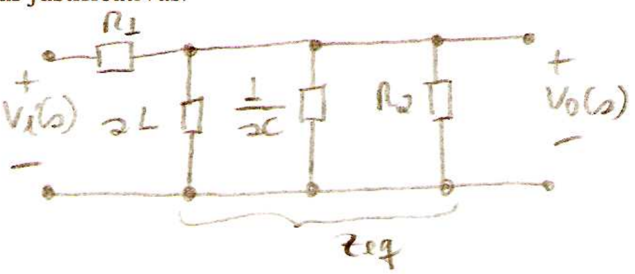
c) FRF de 2ª ordem

Respostas a caneta	a)	$R_2 = 6\Omega$
	b)	1 / 0,998 / 0 / 0,998 / 1
	c)	FRF de 2ª ordem

2) (50 pontos) Para o circuito a seguir, tem-se que $C = 1\mu F$, $L = 1H$, $R_1 = 1k\Omega$ e $R_2 = 500\Omega$. Determinar: a) o valor da frequência angular de ressonância (10 pontos); b) o valor da largura de faixa em rad/s (10 pontos); c) o valor do fator de amortecimento (10 pontos); d) classificar o circuito quanto ao amortecimento (10 pontos); e) o tipo de filtro (10 pontos). Não serão aceitas respostas sem as soluções e as devidas justificativas.



$\rightarrow$



$$Y_{eq} = \frac{1}{sL} + sC + \frac{1}{R_2} = \frac{R_2 + s^2 R_2 LC + sL}{s R_2 L}, \quad Z_{eq} = \frac{1}{Y_{eq}} = \frac{s R_2 L}{s^2 R_2 LC + sL + R_2}$$

$$A_v(s) = \frac{\dfrac{sR_2L}{s^2R_2LC+sL+R_2}}{\dfrac{sR_2L}{s^2R_2LC+sL+R_2} + R_1} = \frac{\dfrac{sR_2L}{s^2R_2LC+sL+R_2}}{\dfrac{sR_2L+s^2R_1R_2LC+sR_1L+R_1R_2}{s^2R_2LC+sL+R_2}} =$$

$$= \frac{sR_2L}{s^2R_1R_2LC+(R_1+R_2)Ls+R_1R_2} = \frac{\frac{1}{R_1C}s}{s^2+\left(\frac{R_1+R_2}{R_1R_2C}\right)s+\frac{1}{LC}} = \frac{10^3 s}{s^2+3000s+10^6}$$

a) $\omega_n^2 = 10^6 \rightarrow \boxed{\omega_n = 10^3 \text{ rad/s}}$

b) $\boxed{BW = 3000 \text{ rad/s}}$

c) $BW = 2\xi\omega_n \rightarrow \boxed{\xi = 1,5}$

d) superamortecido

e) FPF de 2ª ordem

Respostas a caneta	a)	$\omega_n = 1000$ rad/s
	b)	$BW = 3000$ rad/s
	c)	$\xi = 1,5$
	d)	superamortecido
	e)	FPF de 2ª ordem